

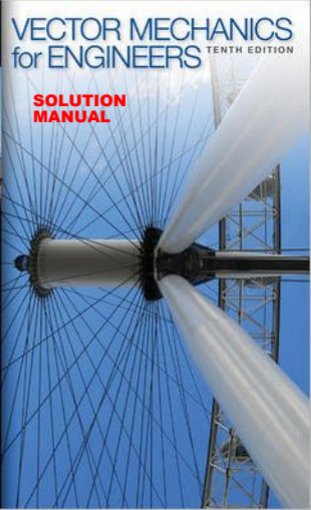
VECTOR MECHANICS for ENGINEERS

TENTH EDITION

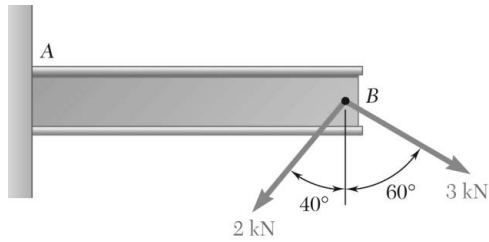
**SOLUTION
MANUAL**

STATICS

Beer | Johnston | Mazurek



CHAPTER 2

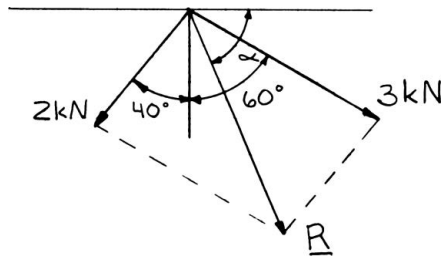


PROBLEM 2.1

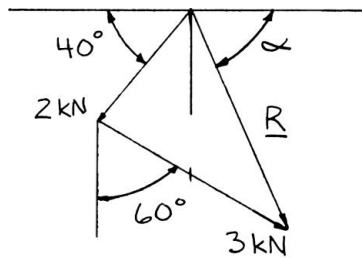
Two forces are applied at point B of beam AB . Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:



(b) Triangle rule:



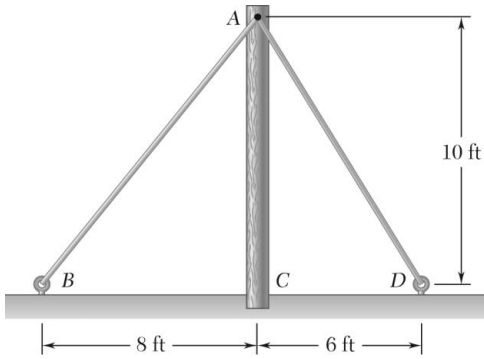
We measure:

$$R = 3.30 \text{ kN}, \quad \alpha = 66.6^\circ$$

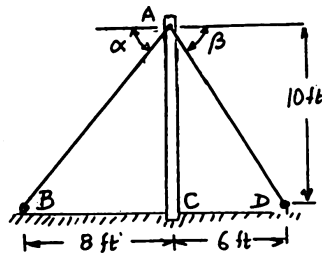
$$R = 3.30 \text{ kN} \quad \swarrow 66.6^\circ \quad \blacktriangleleft$$

PROBLEM 2.2

The cable stays AB and AD help support pole AC . Knowing that the tension is 120 lb in AB and 40 lb in AD , determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at A using (a) the parallelogram law, (b) the triangle rule.



SOLUTION

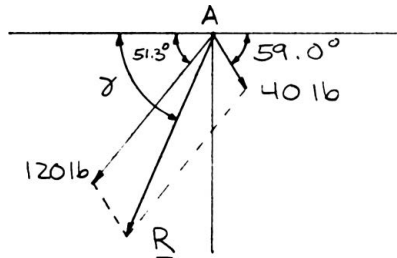


We measure:

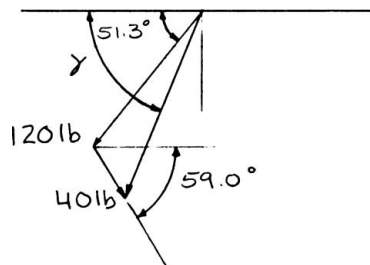
$$\alpha = 51.3^\circ$$

$$\beta = 59.0^\circ$$

(a) Parallelogram law:



(b) Triangle rule:

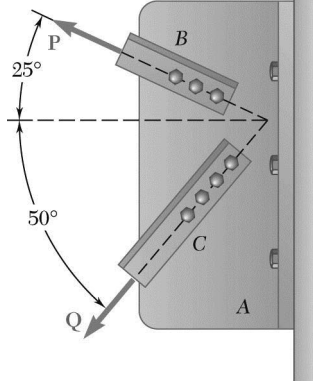


We measure:

$$R = 139.1 \text{ lb}, \quad \gamma = 67.0^\circ$$

$$R = 139.1 \text{ lb} \nearrow 67.0^\circ \blacktriangleleft$$

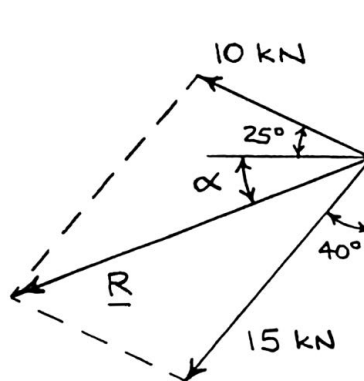
PROBLEM 2.3



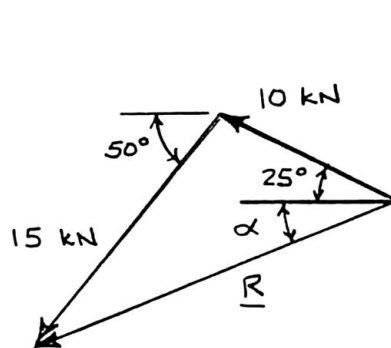
Two structural members B and C are bolted to bracket A . Knowing that both members are in tension and that $P = 10 \text{ kN}$ and $Q = 15 \text{ kN}$, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:



(b) Triangle rule:

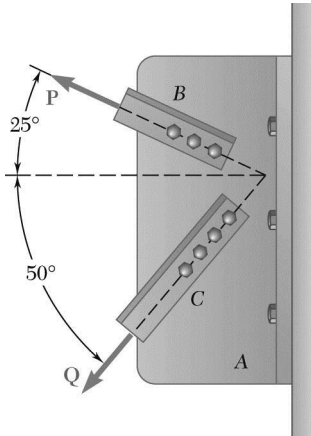


We measure:

$$R = 20.1 \text{ kN}, \quad \alpha = 21.2^\circ$$

$$\mathbf{R} = 20.1 \text{ kN} \nearrow 21.2^\circ \blacktriangleleft$$

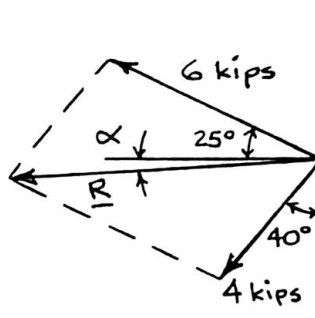
PROBLEM 2.4



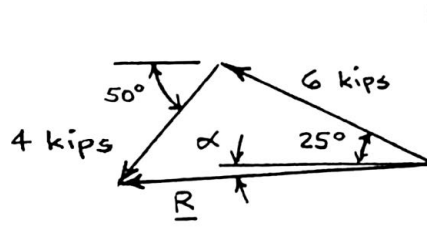
Two structural members B and C are bolted to bracket A . Knowing that both members are in tension and that $P = 6$ kips and $Q = 4$ kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:



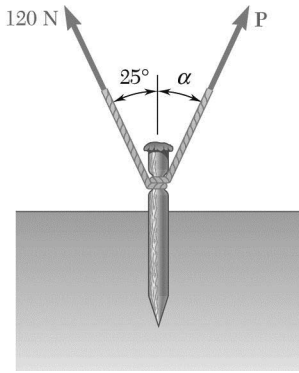
(b) Triangle rule:



We measure:

$$R = 8.03 \text{ kips}, \quad \alpha = 3.8^\circ$$

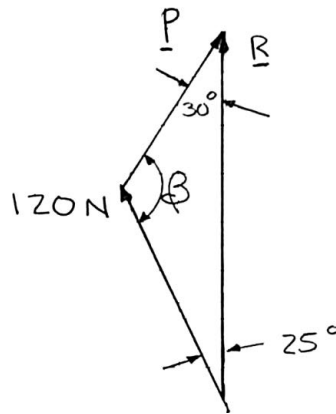
$$R = 8.03 \text{ kips} \nearrow 3.8^\circ \blacktriangleleft$$



PROBLEM 2.5

A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30^\circ$, determine by trigonometry (a) the magnitude of the force \mathbf{P} so that the resultant force exerted on the stake is vertical, (b) the corresponding magnitude of the resultant.

SOLUTION



Using the triangle rule and the law of sines:

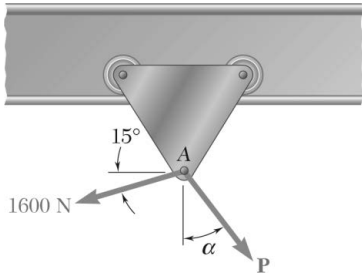
$$(a) \quad \frac{120 \text{ N}}{\sin 30^\circ} = \frac{P}{\sin 25^\circ} \quad P = 101.4 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad 30^\circ + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 30^\circ$$

$$= 125^\circ$$

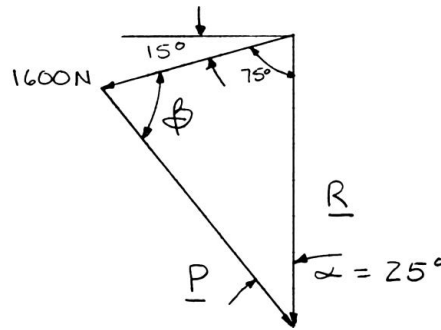
$$\frac{120 \text{ N}}{\sin 30^\circ} = \frac{R}{\sin 125^\circ} \quad R = 196.6 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.6

A trolley that moves along a horizontal beam is acted upon by two forces as shown. (a) Knowing that $\alpha = 25^\circ$, determine by trigonometry the magnitude of the force \mathbf{P} so that the resultant force exerted on the trolley is vertical. (b) What is the corresponding magnitude of the resultant?

SOLUTION



Using the triangle rule and the law of sines:

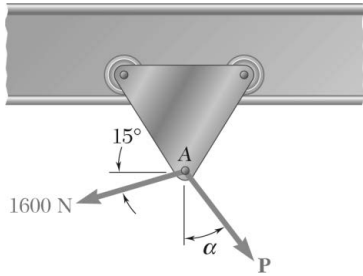
$$(a) \quad \frac{1600 \text{ N}}{\sin 25^\circ} = \frac{P}{\sin 75^\circ} \quad P = 3660 \text{ N} \blacktriangleleft$$

$$(b) \quad 25^\circ + \beta + 75^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 75^\circ$$

$$= 80^\circ$$

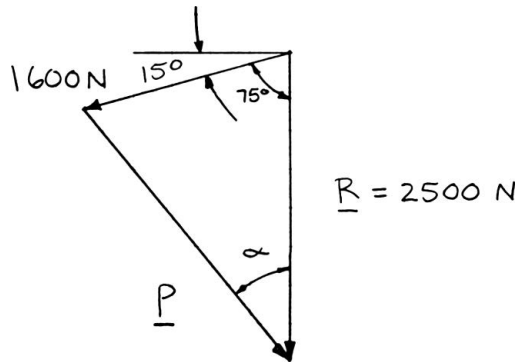
$$\frac{1600 \text{ N}}{\sin 25^\circ} = \frac{R}{\sin 80^\circ} \quad R = 3730 \text{ N} \blacktriangleleft$$



PROBLEM 2.7

A trolley that moves along a horizontal beam is acted upon by two forces as shown. Determine by trigonometry the magnitude and direction of the force \mathbf{P} so that the resultant is a vertical force of 2500 N.

SOLUTION



Using the law of cosines:

$$P^2 = (1600 \text{ N})^2 + (2500 \text{ N})^2 - 2(1600 \text{ N})(2500 \text{ N})\cos 75^\circ$$

$$P = 2596 \text{ N}$$

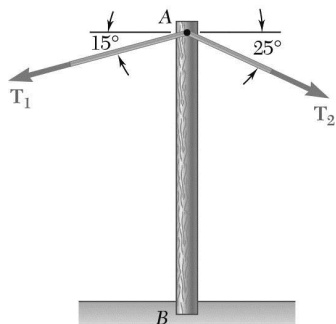
Using the law of sines:

$$\frac{\sin \alpha}{1600 \text{ N}} = \frac{\sin 75^\circ}{2596 \text{ N}}$$

$$\alpha = 36.5^\circ$$

\mathbf{P} is directed $90^\circ - 36.5^\circ$ or 53.5° below the horizontal.

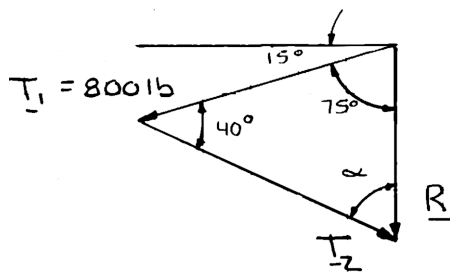
$$\mathbf{P} = 2600 \text{ N} \swarrow 53.5^\circ \blacktriangleleft$$



PROBLEM 2.8

A telephone cable is clamped at A to the pole AB . Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry (a) the required tension T_2 in the right-hand portion if the resultant \mathbf{R} of the forces exerted by the cable at A is to be vertical, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION



Using the triangle rule and the law of sines:

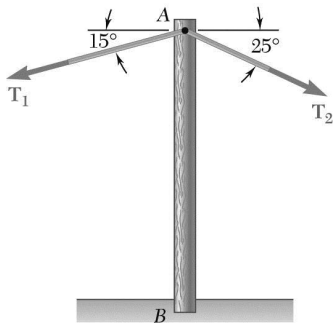
$$\begin{aligned} (a) \quad 75^\circ + 40^\circ + \alpha &= 180^\circ \\ \alpha &= 180^\circ - 75^\circ - 40^\circ \\ &= 65^\circ \end{aligned}$$

$$\frac{800 \text{ lb}}{\sin 65^\circ} = \frac{T_2}{\sin 75^\circ}$$

$$T_2 = 853 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad \frac{800 \text{ lb}}{\sin 65^\circ} = \frac{R}{\sin 40^\circ}$$

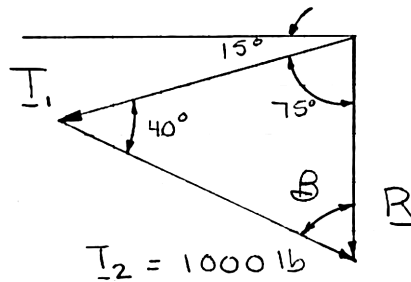
$$R = 567 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.9

A telephone cable is clamped at A to the pole AB . Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000$ lb, determine by trigonometry (a) the required tension T_1 in the left-hand portion if the resultant \mathbf{R} of the forces exerted by the cable at A is to be vertical, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION

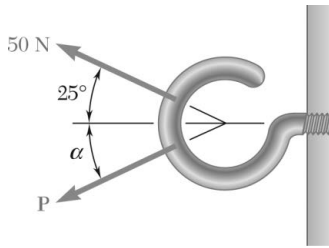


Using the triangle rule and the law of sines:

$$\begin{aligned} (a) \quad 75^\circ + 40^\circ + \beta &= 180^\circ \\ \beta &= 180^\circ - 75^\circ - 40^\circ \\ &= 65^\circ \end{aligned}$$

$$\frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{T_1}{\sin 65^\circ} \quad T_1 = 938 \text{ lb} \blacktriangleleft$$

$$(b) \quad \frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{R}{\sin 40^\circ} \quad R = 665 \text{ lb} \blacktriangleleft$$



PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of \mathbf{P} is 35 N, determine by trigonometry (a) the required angle α if the resultant \mathbf{R} of the two forces applied to the support is to be horizontal, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION

Using the triangle rule and law of sines:

$$(a) \quad \frac{\sin \alpha}{50 \text{ N}} = \frac{\sin 25^\circ}{35 \text{ N}}$$

$$\sin \alpha = 0.60374$$

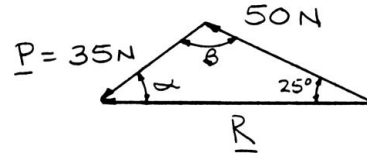
$$\alpha = 37.138^\circ$$

$$(b) \quad \alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

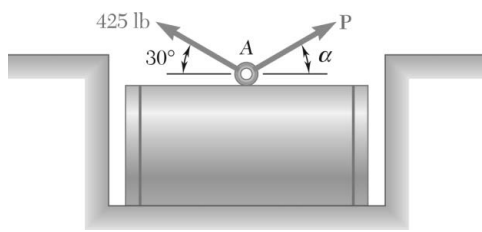
$$= 117.862^\circ$$

$$\frac{R}{\sin 117.862^\circ} = \frac{35 \text{ N}}{\sin 25^\circ}$$



$$\alpha = 37.1^\circ \quad \blacktriangleleft$$

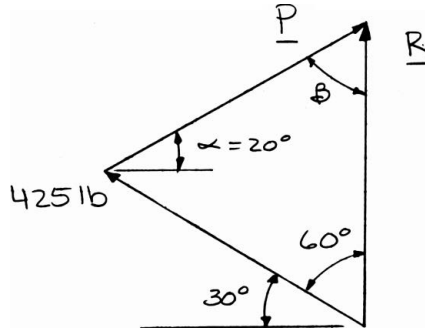
$$R = 73.2 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20^\circ$, determine by trigonometry (a) the required magnitude of the force \mathbf{P} if the resultant \mathbf{R} of the two forces applied at A is to be vertical, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION

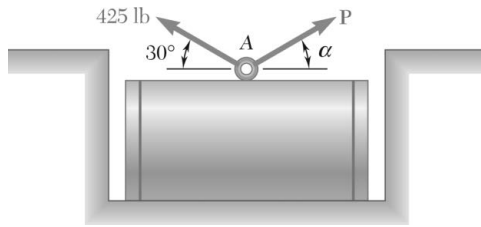


Using the triangle rule and the law of sines:

$$\begin{aligned} (a) \quad \beta + 50^\circ + 60^\circ &= 180^\circ \\ \beta &= 180^\circ - 50^\circ - 60^\circ \\ &= 70^\circ \end{aligned}$$

$$\frac{425 \text{ lb}}{\sin 70^\circ} = \frac{P}{\sin 60^\circ} \qquad P = 392 \text{ lb} \quad \blacktriangleleft$$

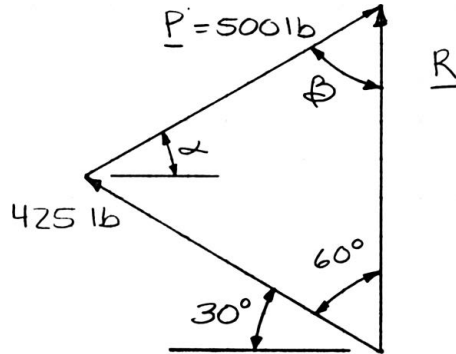
$$(b) \quad \frac{425 \text{ lb}}{\sin 70^\circ} = \frac{R}{\sin 50^\circ} \qquad R = 346 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of \mathbf{P} is 500 lb, determine by trigonometry (a) the required angle α if the resultant \mathbf{R} of the two forces applied at A is to be vertical, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION



Using the triangle rule and the law of sines:

$$(a) \quad (\alpha + 30^\circ) + 60^\circ + \beta = 180^\circ$$

$$\beta = 180^\circ - (\alpha + 30^\circ) - 60^\circ$$

$$\beta = 90^\circ - \alpha$$

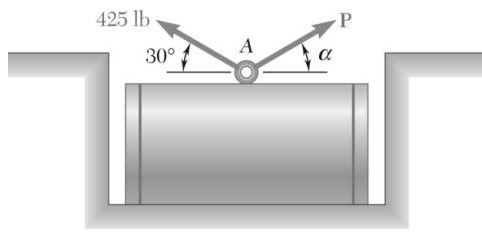
$$\frac{\sin(90^\circ - \alpha)}{425 \text{ lb}} = \frac{\sin 60^\circ}{500 \text{ lb}}$$

$$90^\circ - \alpha = 47.402^\circ$$

$$\alpha = 42.6^\circ \quad \blacktriangleleft$$

$$(b) \quad \frac{R}{\sin(42.598^\circ + 30^\circ)} = \frac{500 \text{ lb}}{\sin 60^\circ}$$

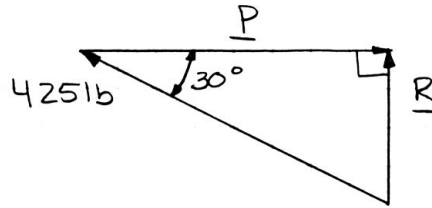
$$R = 551 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.13

A steel tank is to be positioned in an excavation. Determine by trigonometry (a) the magnitude and direction of the smallest force \mathbf{P} for which the resultant \mathbf{R} of the two forces applied at A is vertical, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION



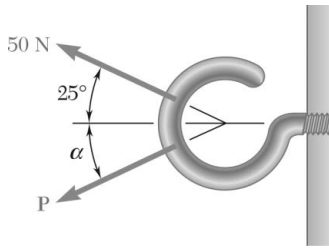
The smallest force P will be perpendicular to R .

(a) $P = (425 \text{ lb}) \cos 30^\circ$

$\mathbf{P} = 368 \text{ lb} \rightarrow \blacktriangleleft$

(b) $R = (425 \text{ lb}) \sin 30^\circ$

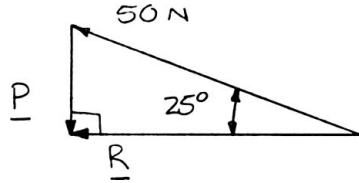
$R = 213 \text{ lb} \blacktriangleleft$



PROBLEM 2.14

For the hook support of Prob. 2.10, determine by trigonometry (a) the magnitude and direction of the smallest force \mathbf{P} for which the resultant \mathbf{R} of the two forces applied to the support is horizontal, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION



The smallest force P will be perpendicular to R .

(a) $P = (50 \text{ N}) \sin 25^\circ$

$\mathbf{P} = 21.1 \text{ N} \downarrow \blacktriangleleft$

(b) $R = (50 \text{ N}) \cos 25^\circ$

$R = 45.3 \text{ N} \blacktriangleleft$

PROBLEM 2.15

Solve Problem 2.2 by trigonometry.

PROBLEM 2.2 The cable stays AB and AD help support pole AC . Knowing that the tension is 120 lb in AB and 40 lb in AD , determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at A using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

Using the triangle rule:

$$\alpha + \beta + \psi = 180^\circ$$

$$38.66^\circ + 30.96^\circ + \psi = 180^\circ$$

$$\psi = 110.38^\circ$$

Using the law of cosines:

$$R^2 = (120 \text{ lb})^2 + (40 \text{ lb})^2 - 2(120 \text{ lb})(40 \text{ lb}) \cos 110.38^\circ$$

$$R = 139.08 \text{ lb}$$

Using the law of sines:

$$\frac{\sin \gamma}{40 \text{ lb}} = \frac{\sin 110.38^\circ}{139.08 \text{ lb}}$$

$$\gamma = 15.64^\circ$$

$$\phi = (90^\circ - \alpha) + \gamma$$

$$\phi = (90^\circ - 38.66^\circ) + 15.64^\circ$$

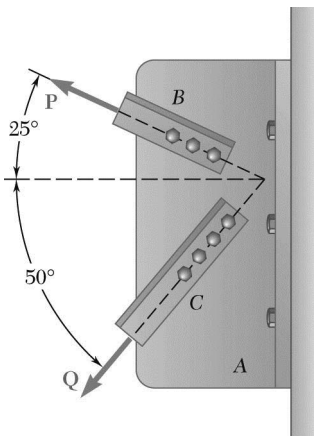
$$\phi = 66.98^\circ$$

$R = 139.1 \text{ lb} \nearrow 67.0^\circ \blacktriangleleft$

PROBLEM 2.16

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two structural members B and C are bolted to bracket A . Knowing that both members are in tension and that $P = 6$ kips and $Q = 4$ kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.



SOLUTION

Using the force triangle and the laws of cosines and sines:

We have:

$$\begin{aligned}\gamma &= 180^\circ - (50^\circ + 25^\circ) \\ &= 105^\circ\end{aligned}$$

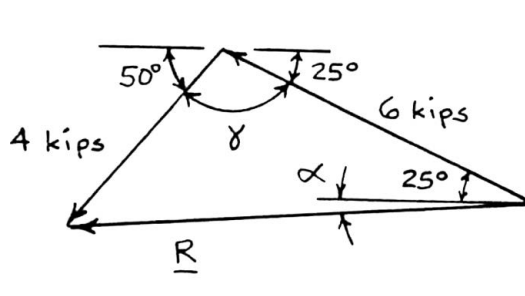
Then

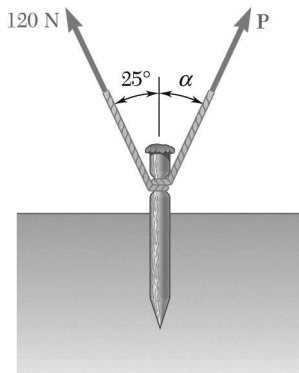
$$\begin{aligned}R^2 &= (4 \text{ kips})^2 + (6 \text{ kips})^2 - 2(4 \text{ kips})(6 \text{ kips})\cos 105^\circ \\ &= 64.423 \text{ kips}^2 \\ R &= 8.0264 \text{ kips}\end{aligned}$$

And

$$\begin{aligned}\frac{4 \text{ kips}}{\sin(25^\circ + \alpha)} &= \frac{8.0264 \text{ kips}}{\sin 105^\circ} \\ \sin(25^\circ + \alpha) &= 0.48137 \\ 25^\circ + \alpha &= 28.775^\circ \\ \alpha &= 3.775^\circ\end{aligned}$$

$$R = 8.03 \text{ kips} \nearrow 3.8^\circ \blacktriangleleft$$



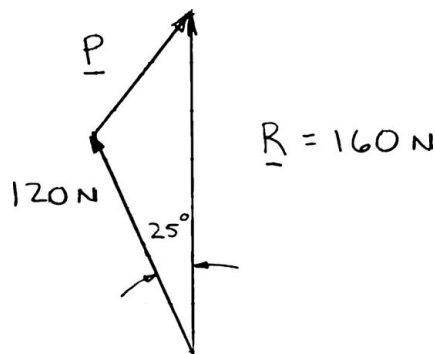


PROBLEM 2.17

For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 160 N.

PROBLEM 2.5 A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30^\circ$, determine by trigonometry (a) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (b) the corresponding magnitude of the resultant.

SOLUTION



Using the laws of cosines and sines:

$$P^2 = (120 \text{ N})^2 + (160 \text{ N})^2 - 2(120 \text{ N})(160 \text{ N}) \cos 25^\circ$$

$$P = 72.096 \text{ N}$$

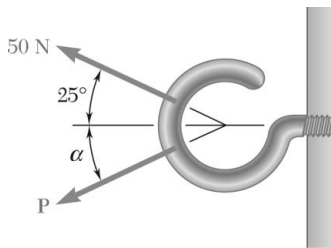
And

$$\frac{\sin \alpha}{120 \text{ N}} = \frac{\sin 25^\circ}{72.096 \text{ N}}$$

$$\sin \alpha = 0.70343$$

$$\alpha = 44.703^\circ$$

$$\mathbf{P} = 72.1 \text{ N} \nearrow 44.7^\circ \blacktriangleleft$$



PROBLEM 2.18

For the hook support of Prob. 2.10, knowing that $P = 75 \text{ N}$ and $\alpha = 50^\circ$, determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

PROBLEM 2.10 Two forces are applied as shown to a hook support. Knowing that the magnitude of \mathbf{P} is 35 N , determine by trigonometry (a) the required angle α if the resultant \mathbf{R} of the two forces applied to the support is to be horizontal, (b) the corresponding magnitude of \mathbf{R} .

SOLUTION

Using the force triangle and the laws of cosines and sines:

We have
$$\beta = 180^\circ - (50^\circ + 25^\circ)$$

$$= 105^\circ$$

Then
$$R^2 = (75 \text{ N})^2 + (50 \text{ N})^2$$

$$- 2(75 \text{ N})(50 \text{ N}) \cos 105^\circ$$

$$R^2 = 10,066.1 \text{ N}^2$$

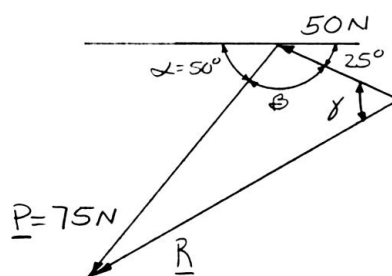
$$R = 100.330 \text{ N}$$

and
$$\frac{\sin \gamma}{75 \text{ N}} = \frac{\sin 105^\circ}{100.330 \text{ N}}$$

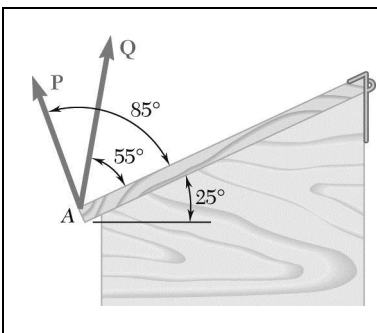
$$\sin \gamma = 0.72206$$

$$\gamma = 46.225^\circ$$

Hence:
$$\gamma - 25^\circ = 46.225^\circ - 25^\circ = 21.225^\circ$$



$$\mathbf{R} = 100.3 \text{ N} \nearrow 21.2^\circ \blacktriangleleft$$



PROBLEM 2.19

Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that $P = 48\text{ N}$ and $Q = 60\text{ N}$, determine by trigonometry the magnitude and direction of the resultant of the two forces.

SOLUTION

Using the force triangle and the laws of cosines and sines:

We have
$$\gamma = 180^\circ - (20^\circ + 10^\circ) = 150^\circ$$

Then
$$R^2 = (48\text{ N})^2 + (60\text{ N})^2 - 2(48\text{ N})(60\text{ N})\cos 150^\circ$$

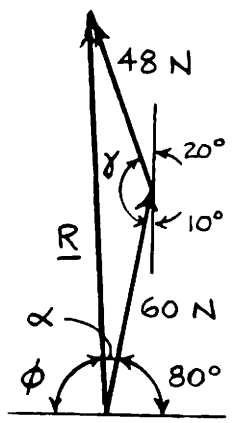
$$R = 104.366\text{ N}$$

and
$$\frac{48\text{ N}}{\sin \alpha} = \frac{104.366\text{ N}}{\sin 150^\circ}$$

$$\sin \alpha = 0.22996$$

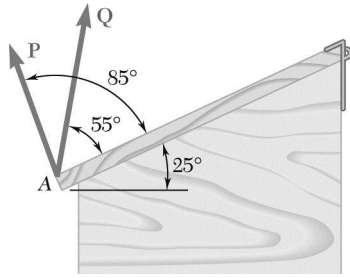
$$\alpha = 13.2947^\circ$$

Hence:
$$\phi = 180^\circ - \alpha - 80^\circ = 180^\circ - 13.2947^\circ - 80^\circ = 86.705^\circ$$



$R = 104.4\text{ N} \searrow 86.7^\circ \blacktriangleleft$

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PROBLEM 2.20

Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that $P = 60 \text{ N}$ and $Q = 48 \text{ N}$, determine by trigonometry the magnitude and direction of the resultant of the two forces.

SOLUTION

Using the force triangle and the laws of cosines and sines:

We have
$$\gamma = 180^\circ - (20^\circ + 10^\circ) = 150^\circ$$

Then
$$R^2 = (60 \text{ N})^2 + (48 \text{ N})^2 - 2(60 \text{ N})(48 \text{ N})\cos 150^\circ$$

$$R = 104.366 \text{ N}$$

and
$$\frac{60 \text{ N}}{\sin \alpha} = \frac{104.366 \text{ N}}{\sin 150^\circ}$$

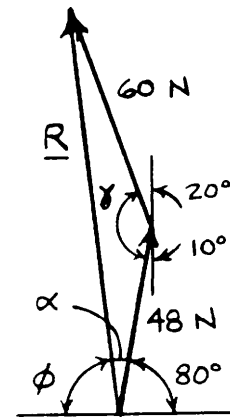
$$\sin \alpha = 0.28745$$

$$\alpha = 16.7054^\circ$$

Hence:
$$\phi = 180^\circ - \alpha - 80^\circ$$

$$= 180^\circ - 16.7054^\circ - 80^\circ$$

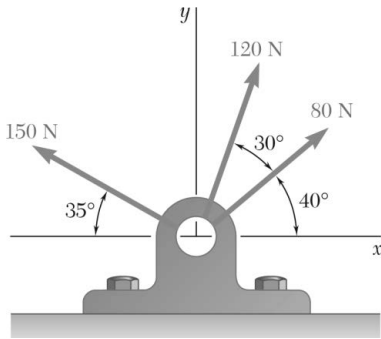
$$= 83.295^\circ$$



$R = 104.4 \text{ N} \nearrow 83.3^\circ \blacktriangleleft$

PROBLEM 2.21

Determine the x and y components of each of the forces shown.



SOLUTION

80-N Force:

$$F_x = +(80 \text{ N}) \cos 40^\circ$$

$$F_x = 61.3 \text{ N} \blacktriangleleft$$

$$F_y = +(80 \text{ N}) \sin 40^\circ$$

$$F_y = 51.4 \text{ N} \blacktriangleleft$$

120-N Force:

$$F_x = +(120 \text{ N}) \cos 70^\circ$$

$$F_x = 41.0 \text{ N} \blacktriangleleft$$

$$F_y = +(120 \text{ N}) \sin 70^\circ$$

$$F_y = 112.8 \text{ N} \blacktriangleleft$$

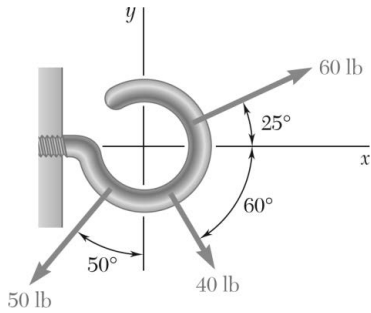
150-N Force:

$$F_x = -(150 \text{ N}) \cos 35^\circ$$

$$F_x = -122.9 \text{ N} \blacktriangleleft$$

$$F_y = +(150 \text{ N}) \sin 35^\circ$$

$$F_y = 86.0 \text{ N} \blacktriangleleft$$



PROBLEM 2.22

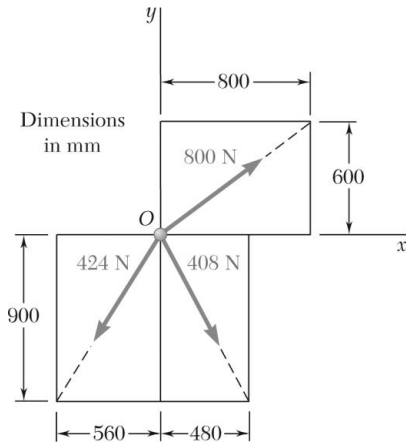
Determine the x and y components of each of the forces shown.

SOLUTION

40-lb Force:	$F_x = +(40 \text{ lb}) \cos 60^\circ$	$F_x = 20.0 \text{ lb} \blacktriangleleft$
	$F_y = -(40 \text{ lb}) \sin 60^\circ$	$F_y = -34.6 \text{ lb} \blacktriangleleft$
50-lb Force:	$F_x = -(50 \text{ lb}) \sin 50^\circ$	$F_x = -38.3 \text{ lb} \blacktriangleleft$
	$F_y = -(50 \text{ lb}) \cos 50^\circ$	$F_y = -32.1 \text{ lb} \blacktriangleleft$
60-lb Force:	$F_x = +(60 \text{ lb}) \cos 25^\circ$	$F_x = 54.4 \text{ lb} \blacktriangleleft$
	$F_y = +(60 \text{ lb}) \sin 25^\circ$	$F_y = 25.4 \text{ lb} \blacktriangleleft$

PROBLEM 2.23

Determine the x and y components of each of the forces shown.



SOLUTION

Compute the following distances:

$$OA = \sqrt{(600)^2 + (800)^2} \\ = 1000 \text{ mm}$$

$$OB = \sqrt{(560)^2 + (900)^2} \\ = 1060 \text{ mm}$$

$$OC = \sqrt{(480)^2 + (900)^2} \\ = 1020 \text{ mm}$$

800-N Force:

$$F_x = +(800 \text{ N}) \frac{800}{1000}$$

$$F_x = +640 \text{ N} \quad \blacktriangleleft$$

$$F_y = +(800 \text{ N}) \frac{600}{1000}$$

$$F_y = +480 \text{ N} \quad \blacktriangleleft$$

424-N Force:

$$F_x = -(424 \text{ N}) \frac{560}{1060}$$

$$F_x = -224 \text{ N} \quad \blacktriangleleft$$

$$F_y = -(424 \text{ N}) \frac{900}{1060}$$

$$F_y = -360 \text{ N} \quad \blacktriangleleft$$

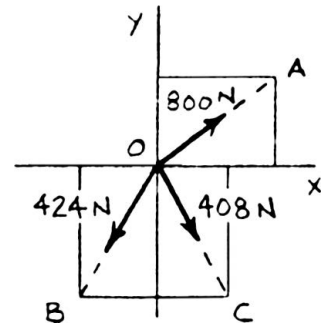
408-N Force:

$$F_x = +(408 \text{ N}) \frac{480}{1020}$$

$$F_x = +192.0 \text{ N} \quad \blacktriangleleft$$

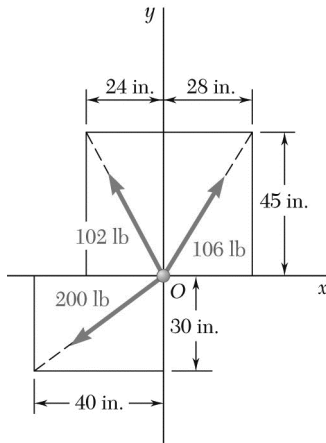
$$F_y = -(408 \text{ N}) \frac{900}{1020}$$

$$F_y = -360 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.24

Determine the x and y components of each of the forces shown.



SOLUTION

Compute the following distances:

$$OA = \sqrt{(24 \text{ in.})^2 + (45 \text{ in.})^2} \\ = 51.0 \text{ in.}$$

$$OB = \sqrt{(28 \text{ in.})^2 + (45 \text{ in.})^2} \\ = 53.0 \text{ in.}$$

$$OC = \sqrt{(40 \text{ in.})^2 + (30 \text{ in.})^2} \\ = 50.0 \text{ in.}$$

102-lb Force:	$F_x = -102 \text{ lb} \frac{24 \text{ in.}}{51.0 \text{ in.}}$	$F_x = -48.0 \text{ lb} \blacktriangleleft$
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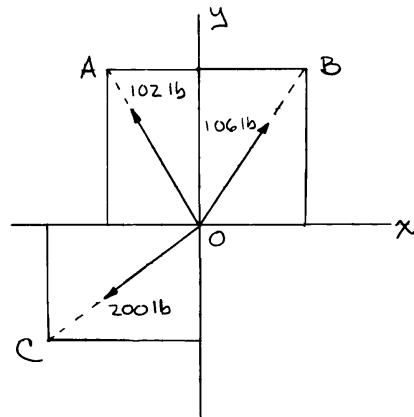
106-lb Force:	$F_y = +102 \text{ lb} \frac{45 \text{ in.}}{51.0 \text{ in.}}$	$F_y = +90.0 \text{ lb} \blacktriangleleft$
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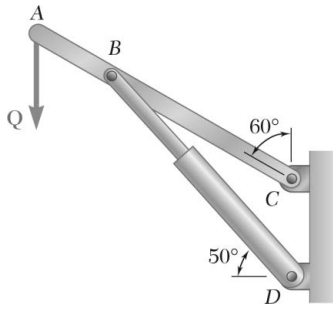
106-lb Force:	$F_x = +106 \text{ lb} \frac{28 \text{ in.}}{53.0 \text{ in.}}$	$F_x = +56.0 \text{ lb} \blacktriangleleft$
---------------	---	---

200-lb Force:	$F_y = +106 \text{ lb} \frac{45 \text{ in.}}{53.0 \text{ in.}}$	$F_y = +90.0 \text{ lb} \blacktriangleleft$
---------------	---	---

200-lb Force:	$F_x = -200 \text{ lb} \frac{40 \text{ in.}}{50.0 \text{ in.}}$	$F_x = -160.0 \text{ lb} \blacktriangleleft$
---------------	---	--

200-lb Force:	$F_y = -200 \text{ lb} \frac{30 \text{ in.}}{50.0 \text{ in.}}$	$F_y = -120.0 \text{ lb} \blacktriangleleft$
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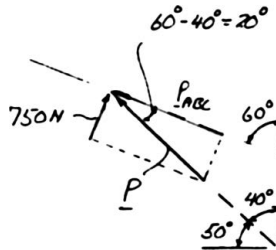




PROBLEM 2.25

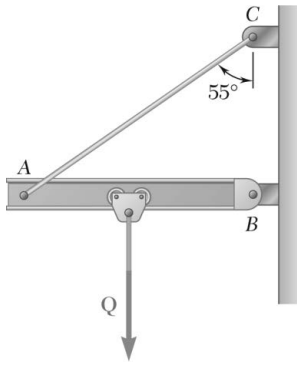
The hydraulic cylinder BD exerts on member ABC a force \mathbf{P} directed along line BD . Knowing that \mathbf{P} must have a 750-N component perpendicular to member ABC , determine (a) the magnitude of the force \mathbf{P} , (b) its component parallel to ABC .

SOLUTION



(a) $750 \text{ N} = P \sin 20^\circ$
 $P = 2192.9 \text{ N}$ $P = 2190 \text{ N} \blacktriangleleft$

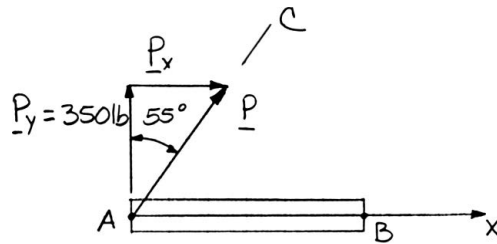
(b) $P_{ABC} = P \cos 20^\circ$
 $= (2192.9 \text{ N}) \cos 20^\circ$ $P_{ABC} = 2060 \text{ N} \blacktriangleleft$



PROBLEM 2.26

Cable AC exerts on beam AB a force \mathbf{P} directed along line AC . Knowing that \mathbf{P} must have a 350-lb vertical component, determine (a) the magnitude of the force \mathbf{P} , (b) its horizontal component.

SOLUTION



(a)

$$P = \frac{P_y}{\cos 55^\circ}$$

$$= \frac{350 \text{ lb}}{\cos 55^\circ}$$

$$= 610.21 \text{ lb}$$

$$P = 610 \text{ lb} \quad \blacktriangleleft$$

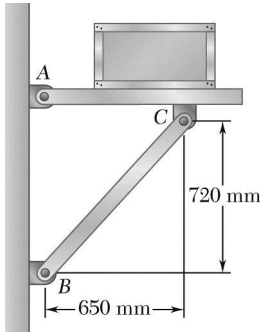
(b)

$$P_x = P \sin 55^\circ$$

$$= (610.21 \text{ lb}) \sin 55^\circ$$

$$= 499.85 \text{ lb}$$

$$P_x = 500 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.27

Member BC exerts on member AC a force \mathbf{P} directed along line BC . Knowing that \mathbf{P} must have a 325-N horizontal component, determine (a) the magnitude of the force \mathbf{P} , (b) its vertical component.

SOLUTION

$$BC = \sqrt{(650 \text{ mm})^2 + (720 \text{ mm})^2}$$

$$= 970 \text{ mm}$$

(a)

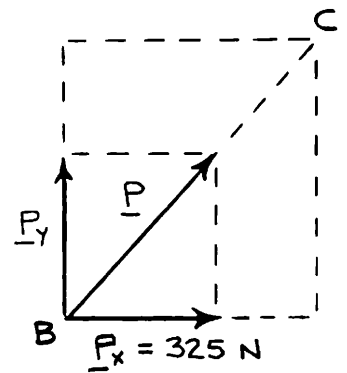
$$P_x = P \left(\frac{650}{970} \right)$$

or

$$P = P_x \left(\frac{970}{650} \right)$$

$$= 325 \text{ N} \left(\frac{970}{650} \right)$$

$$= 485 \text{ N}$$



$$P = 485 \text{ N} \blacktriangleleft$$

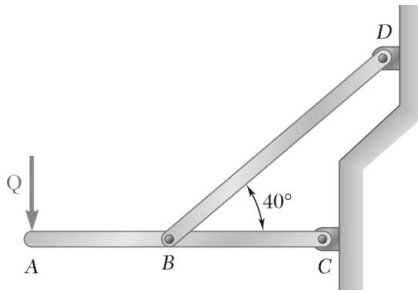
(b)

$$P_y = P \left(\frac{720}{970} \right)$$

$$= 485 \text{ N} \left(\frac{720}{970} \right)$$

$$= 360 \text{ N}$$

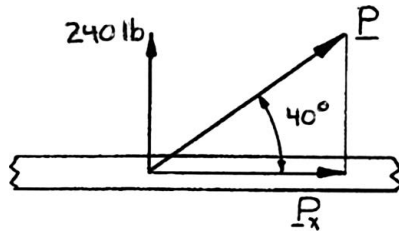
$$P_y = 360 \text{ N} \blacktriangleleft$$



PROBLEM 2.28

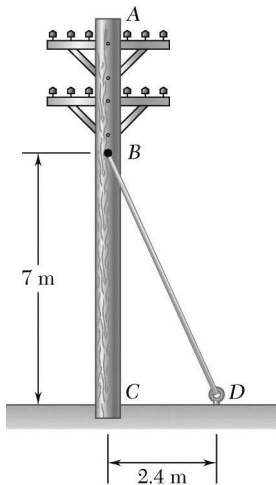
Member BD exerts on member ABC a force \mathbf{P} directed along line BD . Knowing that \mathbf{P} must have a 240-lb vertical component, determine (a) the magnitude of the force \mathbf{P} , (b) its horizontal component.

SOLUTION



(a)
$$P = \frac{P_y}{\sin 40^\circ} = \frac{240 \text{ lb}}{\sin 40^\circ} \quad \text{or } P = 373 \text{ lb} \blacktriangleleft$$

(b)
$$P_x = \frac{P_y}{\tan 40^\circ} = \frac{240 \text{ lb}}{\tan 40^\circ} \quad \text{or } P_x = 286 \text{ lb} \blacktriangleleft$$



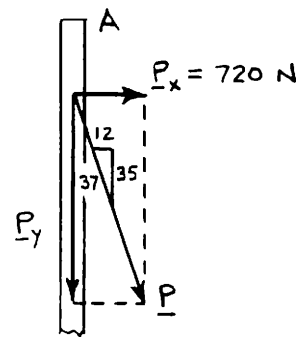
PROBLEM 2.29

The guy wire BD exerts on the telephone pole AC a force \mathbf{P} directed along BD . Knowing that \mathbf{P} must have a 720-N component perpendicular to the pole AC , determine (a) the magnitude of the force \mathbf{P} , (b) its component along line AC .

SOLUTION

(a)

$$\begin{aligned}
 P &= \frac{37}{12} P_x \\
 &= \frac{37}{12} (720 \text{ N}) \\
 &= 2220 \text{ N}
 \end{aligned}$$



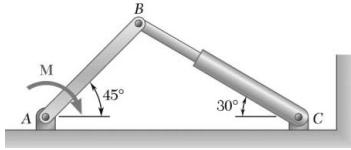
$$P = 2.22 \text{ kN} \quad \blacktriangleleft$$

(b)

$$\begin{aligned}
 P_y &= \frac{35}{12} P_x \\
 &= \frac{35}{12} (720 \text{ N}) \\
 &= 2100 \text{ N}
 \end{aligned}$$

$$P_y = 2.10 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 2.30



The hydraulic cylinder BC exerts on member AB a force \mathbf{P} directed along line BC . Knowing that \mathbf{P} must have a 600-N component perpendicular to member AB , determine (a) the magnitude of the force \mathbf{P} , (b) its component along line AB .

SOLUTION

$$180^\circ = 45^\circ + \alpha + 90^\circ + 30^\circ$$

$$\alpha = 180^\circ - 45^\circ - 90^\circ - 30^\circ$$

$$= 15^\circ$$

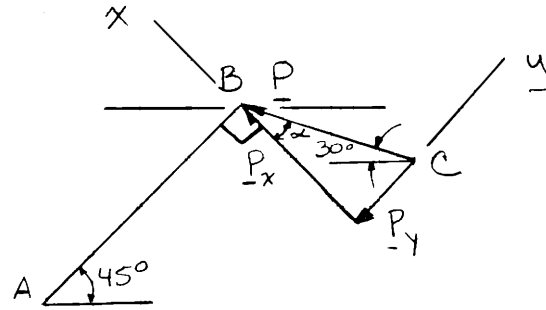
(a)

$$\cos \alpha = \frac{P_x}{P}$$

$$P = \frac{P_x}{\cos \alpha}$$

$$= \frac{600 \text{ N}}{\cos 15^\circ}$$

$$= 621.17 \text{ N}$$



$$P = 621 \text{ N} \quad \blacktriangleleft$$

(b)

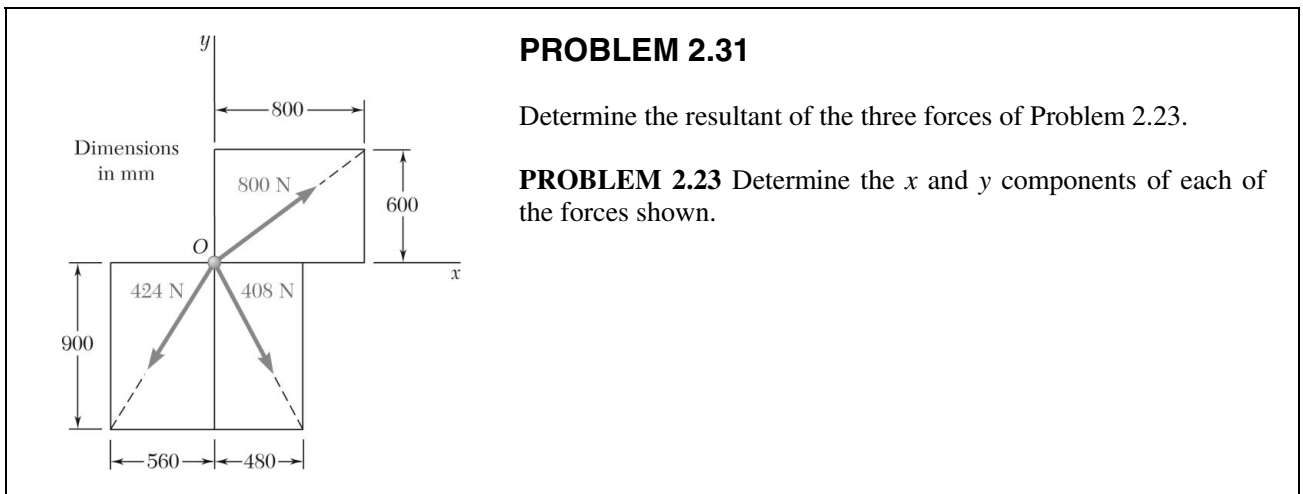
$$\tan \alpha = \frac{P_y}{P_x}$$

$$P_y = P_x \tan \alpha$$

$$= (600 \text{ N}) \tan 15^\circ$$

$$= 160.770 \text{ N}$$

$$P_y = 160.8 \text{ N} \quad \blacktriangleleft$$



SOLUTION

Components of the forces were determined in Problem 2.23:

Force	x Comp. (N)	y Comp. (N)
800 lb	+640	+480
424 lb	-224	-360
408 lb	+192	-360
	$R_x = +608$	$R_y = -240$

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$

$$= (608 \text{ lb})\mathbf{i} + (-240 \text{ lb})\mathbf{j}$$

$$\tan \alpha = \frac{R_y}{R_x}$$

$$= \frac{240}{608}$$

$$\alpha = 21.541^\circ$$

$$R = \frac{240 \text{ N}}{\sin(21.541^\circ)}$$

$$= 653.65 \text{ N}$$

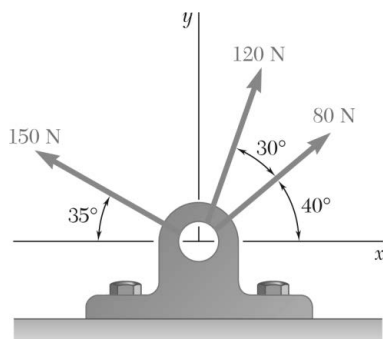
$$\mathbf{R} = 654 \text{ N} \searrow 21.5^\circ \blacktriangleleft$$

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PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.21.

PROBLEM 2.21 Determine the x and y components of each of the forces shown.



SOLUTION

Components of the forces were determined in Problem 2.21:

Force	x Comp. (N)	y Comp. (N)
80 N	+61.3	+51.4
120 N	+41.0	+112.8
150 N	-122.9	+86.0
	$R_x = -20.6$	$R_y = +250.2$

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$
$$= (-20.6 \text{ N})\mathbf{i} + (250.2 \text{ N})\mathbf{j}$$

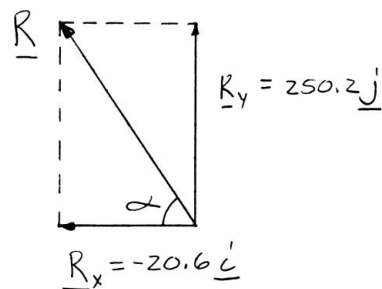
$$\tan \alpha = \frac{R_y}{R_x}$$

$$\tan \alpha = \frac{250.2 \text{ N}}{20.6 \text{ N}}$$

$$\tan \alpha = 12.1456$$

$$\alpha = 85.293^\circ$$

$$R = \frac{250.2 \text{ N}}{\sin 85.293^\circ}$$



$$\mathbf{R} = 251 \text{ N} \nearrow 85.3^\circ \blacktriangleleft$$

The diagram shows a ring with a central hole. Three forces are applied to the ring: a 50 lb force pointing down and to the left at an angle of 50 degrees from the vertical y-axis; a 40 lb force pointing down and to the right at an angle of 60 degrees from the horizontal x-axis; and a 60 lb force pointing up and to the right at an angle of 25 degrees from the horizontal x-axis. A coordinate system with x and y axes is centered on the ring.

PROBLEM 2.33

Determine the resultant of the three forces of Problem 2.22.

PROBLEM 2.22 Determine the x and y components of each of the forces shown.

SOLUTION

Force	x Comp. (lb)	y Comp. (lb)
40 lb	+20.00	-34.64
50 lb	-38.30	-32.14
60 lb	+54.38	+25.36
	$R_x = +36.08$	$R_y = -41.42$

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$

$$= (+36.08 \text{ lb})\mathbf{i} + (-41.42 \text{ lb})\mathbf{j}$$

$$\tan \alpha = \frac{R_y}{R_x}$$

$$\tan \alpha = \frac{41.42 \text{ lb}}{36.08 \text{ lb}}$$

$$\tan \alpha = 1.14800$$

$$\alpha = 48.942^\circ$$

$$R = \frac{41.42 \text{ lb}}{\sin 48.942^\circ}$$

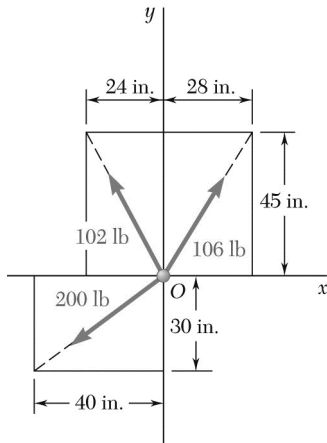
The diagram shows a right-angled triangle representing the resultant force. The horizontal leg is labeled $R_x = 36.08 \underline{i}$ and the vertical leg is labeled $R_y = -41.42 \underline{j}$. The hypotenuse is labeled \underline{R} . The angle α is shown between the horizontal leg and the hypotenuse.

$\mathbf{R} = 54.9 \text{ lb} \searrow 48.9^\circ \blacktriangleleft$

PROBLEM 2.34

Determine the resultant of the three forces of Problem 2.24.

PROBLEM 2.24 Determine the x and y components of each of the forces shown.



SOLUTION

Components of the forces were determined in Problem 2.24:

Force	x Comp. (lb)	y Comp. (lb)
102 lb	-48.0	+90.0
106 lb	+56.0	+90.0
200 lb	-160.0	-120.0
	$R_x = -152.0$	$R_y = 60.0$

$$\begin{aligned} \mathbf{R} &= R_x \mathbf{i} + R_y \mathbf{j} \\ &= (-152 \text{ lb}) \mathbf{i} + (60.0 \text{ lb}) \mathbf{j} \end{aligned}$$

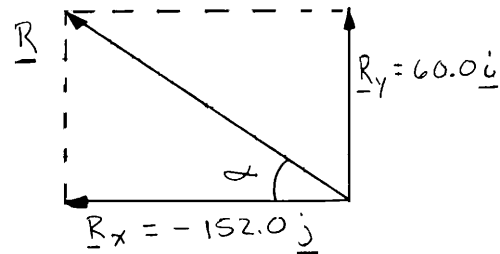
$$\tan \alpha = \frac{R_y}{R_x}$$

$$\tan \alpha = \frac{60.0 \text{ lb}}{152.0 \text{ lb}}$$

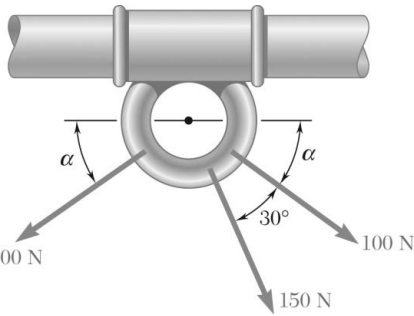
$$\tan \alpha = 0.39474$$

$$\alpha = 21.541^\circ$$

$$R = \frac{60.0 \text{ lb}}{\sin 21.541^\circ}$$



$$\mathbf{R} = 163.4 \text{ lb} \nearrow 21.5^\circ \nwarrow$$



PROBLEM 2.35

Knowing that $\alpha = 35^\circ$, determine the resultant of the three forces shown.

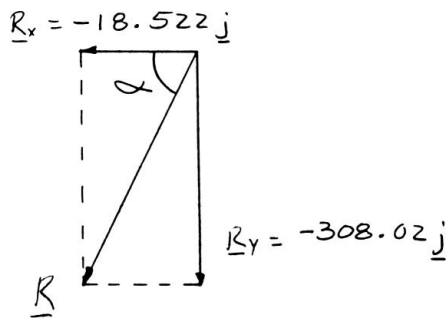
SOLUTION

100-N Force: $F_x = +(100 \text{ N}) \cos 35^\circ = +81.915 \text{ N}$
 $F_y = -(100 \text{ N}) \sin 35^\circ = -57.358 \text{ N}$

150-N Force: $F_x = +(150 \text{ N}) \cos 65^\circ = +63.393 \text{ N}$
 $F_y = -(150 \text{ N}) \sin 65^\circ = -135.946 \text{ N}$

200-N Force: $F_x = -(200 \text{ N}) \cos 35^\circ = -163.830 \text{ N}$
 $F_y = -(200 \text{ N}) \sin 35^\circ = -114.715 \text{ N}$

Force	x Comp. (N)	y Comp. (N)
100 N	+81.915	-57.358
150 N	+63.393	-135.946
200 N	-163.830	-114.715
	$R_x = -18.522$	$R_y = -308.02$



$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$

$$= (-18.522 \text{ N})\mathbf{i} + (-308.02 \text{ N})\mathbf{j}$$

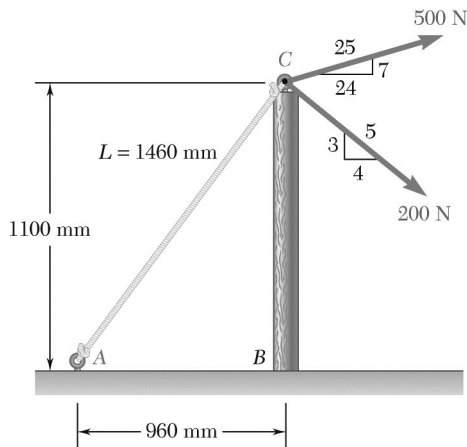
$$\tan \alpha = \frac{R_y}{R_x}$$

$$= \frac{308.02}{18.522}$$

$$\alpha = 86.559^\circ$$

$$R = \frac{308.02 \text{ N}}{\sin 86.559}$$

$$\mathbf{R} = 309 \text{ N} \nearrow 86.6^\circ \blacktriangleleft$$



PROBLEM 2.36

Knowing that the tension in rope AC is 365 N, determine the resultant of the three forces exerted at point C of post BC.

SOLUTION

Determine force components:

Cable force AC:

$$F_x = -(365 \text{ N}) \frac{960}{1460} = -240 \text{ N}$$

$$F_y = -(365 \text{ N}) \frac{1100}{1460} = -275 \text{ N}$$

500-N Force:

$$F_x = (500 \text{ N}) \frac{24}{25} = 480 \text{ N}$$

$$F_y = (500 \text{ N}) \frac{7}{25} = 140 \text{ N}$$

200-N Force:

$$F_x = (200 \text{ N}) \frac{4}{5} = 160 \text{ N}$$

$$F_y = -(200 \text{ N}) \frac{3}{5} = -120 \text{ N}$$

and

$$R_x = \Sigma F_x = -240 \text{ N} + 480 \text{ N} + 160 \text{ N} = 400 \text{ N}$$

$$R_y = \Sigma F_y = -275 \text{ N} + 140 \text{ N} - 120 \text{ N} = -255 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

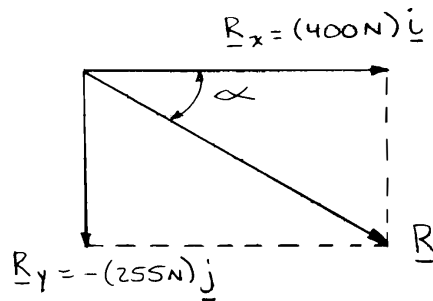
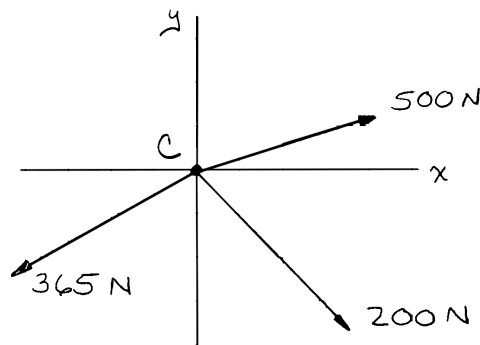
$$= \sqrt{(400 \text{ N})^2 + (-255 \text{ N})^2}$$

$$= 474.37 \text{ N}$$

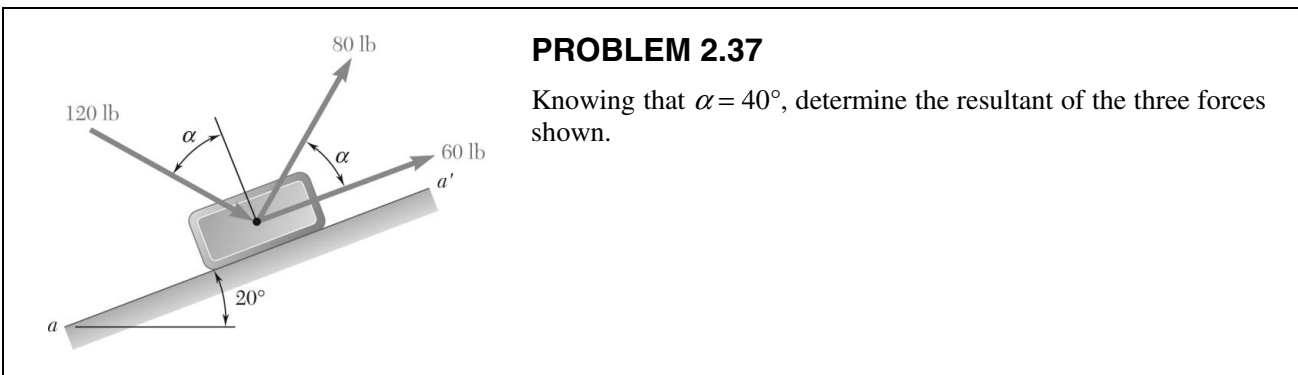
Further:

$$\tan \alpha = \frac{255}{400}$$

$$\alpha = 32.5^\circ$$



$$\mathbf{R} = 474 \text{ N} \searrow 32.5^\circ \blacktriangleleft$$



PROBLEM 2.37

Knowing that $\alpha = 40^\circ$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force: $F_x = (60 \text{ lb}) \cos 20^\circ = 56.382 \text{ lb}$
 $F_y = (60 \text{ lb}) \sin 20^\circ = 20.521 \text{ lb}$

80-lb Force: $F_x = (80 \text{ lb}) \cos 60^\circ = 40.000 \text{ lb}$
 $F_y = (80 \text{ lb}) \sin 60^\circ = 69.282 \text{ lb}$

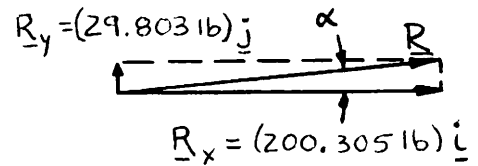
120-lb Force: $F_x = (120 \text{ lb}) \cos 30^\circ = 103.923 \text{ lb}$
 $F_y = -(120 \text{ lb}) \sin 30^\circ = -60.000 \text{ lb}$

and $R_x = \Sigma F_x = 200.305 \text{ lb}$
 $R_y = \Sigma F_y = 29.803 \text{ lb}$

$$R = \sqrt{(200.305 \text{ lb})^2 + (29.803 \text{ lb})^2}$$

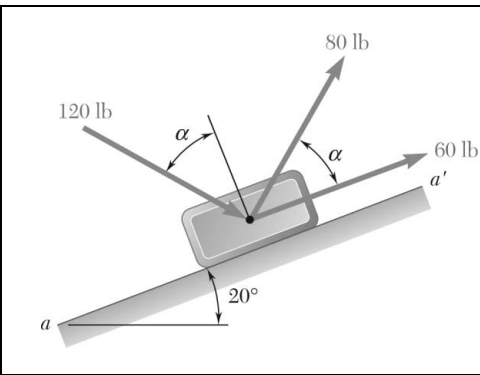
$$= 202.510 \text{ lb}$$

Further: $\tan \alpha = \frac{29.803}{200.305}$
 $\alpha = \tan^{-1} \frac{29.803}{200.305}$
 $= 8.46^\circ$



$R = 203 \text{ lb} \angle 8.46^\circ \blacktriangleleft$

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PROBLEM 2.38

Knowing that $\alpha = 75^\circ$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force: $F_x = (60 \text{ lb}) \cos 20^\circ = 56.382 \text{ lb}$
 $F_y = (60 \text{ lb}) \sin 20^\circ = 20.521 \text{ lb}$

80-lb Force: $F_x = (80 \text{ lb}) \cos 95^\circ = -6.9725 \text{ lb}$
 $F_y = (80 \text{ lb}) \sin 95^\circ = 79.696 \text{ lb}$

120-lb Force: $F_x = (120 \text{ lb}) \cos 5^\circ = 119.543 \text{ lb}$
 $F_y = (120 \text{ lb}) \sin 5^\circ = 10.459 \text{ lb}$

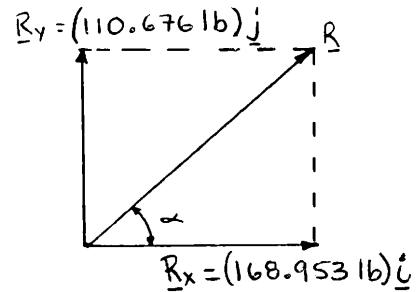
Then $R_x = \Sigma F_x = 168.953 \text{ lb}$
 $R_y = \Sigma F_y = 110.676 \text{ lb}$

and $R = \sqrt{(168.953 \text{ lb})^2 + (110.676 \text{ lb})^2}$
 $= 201.976 \text{ lb}$

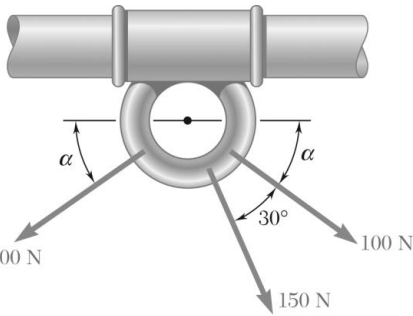
$$\tan \alpha = \frac{110.676}{168.953}$$

$$\tan \alpha = 0.65507$$

$$\alpha = 33.228^\circ$$



$$\mathbf{R} = 202 \text{ lb} \angle 33.2^\circ \blacktriangleleft$$



PROBLEM 2.39

For the collar of Problem 2.35, determine (a) the required value of α if the resultant of the three forces shown is to be vertical, (b) the corresponding magnitude of the resultant.

SOLUTION

$$\begin{aligned}
 R_x &= \Sigma F_x \\
 &= (100 \text{ N}) \cos \alpha + (150 \text{ N}) \cos(\alpha + 30^\circ) - (200 \text{ N}) \cos \alpha \\
 R_x &= -(100 \text{ N}) \cos \alpha + (150 \text{ N}) \cos(\alpha + 30^\circ)
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 R_y &= \Sigma F_y \\
 &= -(100 \text{ N}) \sin \alpha - (150 \text{ N}) \sin(\alpha + 30^\circ) - (200 \text{ N}) \sin \alpha \\
 R_y &= -(300 \text{ N}) \sin \alpha - (150 \text{ N}) \sin(\alpha + 30^\circ)
 \end{aligned} \tag{2}$$

(a) For \mathbf{R} to be vertical, we must have $R_x = 0$. We make $R_x = 0$ in Eq. (1):

$$\begin{aligned}
 -100 \cos \alpha + 150 \cos(\alpha + 30^\circ) &= 0 \\
 -100 \cos \alpha + 150(\cos \alpha \cos 30^\circ - \sin \alpha \sin 30^\circ) &= 0 \\
 29.904 \cos \alpha &= 75 \sin \alpha
 \end{aligned}$$

$$\begin{aligned}
 \tan \alpha &= \frac{29.904}{75} \\
 &= 0.39872 \\
 \alpha &= 21.738^\circ
 \end{aligned}$$

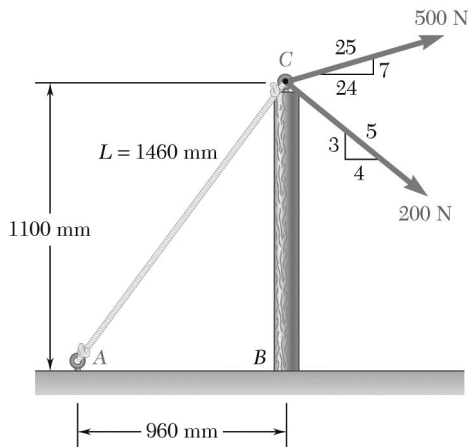
$$\alpha = 21.7^\circ \blacktriangleleft$$

(b) Substituting for α in Eq. (2):

$$\begin{aligned}
 R_y &= -300 \sin 21.738^\circ - 150 \sin 51.738^\circ \\
 &= -228.89 \text{ N}
 \end{aligned}$$

$$R = |R_y| = 228.89 \text{ N}$$

$$R = 229 \text{ N} \blacktriangleleft$$



PROBLEM 2.40

For the post of Prob. 2.36, determine (a) the required tension in rope AC if the resultant of the three forces exerted at point C is to be horizontal, (b) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x = -\frac{960}{1460}T_{AC} + \frac{24}{25}(500 \text{ N}) + \frac{4}{5}(200 \text{ N})$$

$$R_x = -\frac{48}{73}T_{AC} + 640 \text{ N} \quad (1)$$

$$R_y = \Sigma F_y = -\frac{1100}{1460}T_{AC} + \frac{7}{25}(500 \text{ N}) - \frac{3}{5}(200 \text{ N})$$

$$R_y = -\frac{55}{73}T_{AC} + 20 \text{ N} \quad (2)$$

(a) For \mathbf{R} to be horizontal, we must have $R_y = 0$.

Set $R_y = 0$ in Eq. (2):

$$-\frac{55}{73}T_{AC} + 20 \text{ N} = 0$$

$$T_{AC} = 26.545 \text{ N}$$

$$T_{AC} = 26.5 \text{ N} \quad \blacktriangleleft$$

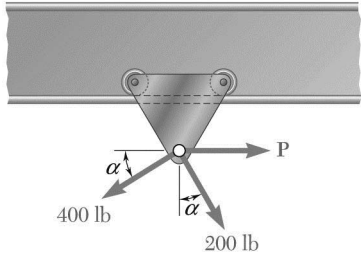
(b) Substituting for T_{AC} into Eq. (1) gives

$$R_x = -\frac{48}{73}(26.545 \text{ N}) + 640 \text{ N}$$

$$R_x = 622.55 \text{ N}$$

$$R = R_x = 623 \text{ N}$$

$$R = 623 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.41

A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40^\circ$, determine (a) the required magnitude of the force \mathbf{P} if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \pm \sum F_x = P + (200 \text{ lb}) \sin 40^\circ - (400 \text{ lb}) \cos 40^\circ$$

$$R_x = P - 177.860 \text{ lb} \quad (1)$$

$$R_y = \pm \sum F_y = (200 \text{ lb}) \cos 40^\circ + (400 \text{ lb}) \sin 40^\circ$$

$$R_y = 410.32 \text{ lb} \quad (2)$$

(a) For \mathbf{R} to be vertical, we must have $R_x = 0$.

Set

$$R_x = 0 \text{ in Eq. (1)}$$

$$0 = P - 177.860 \text{ lb}$$

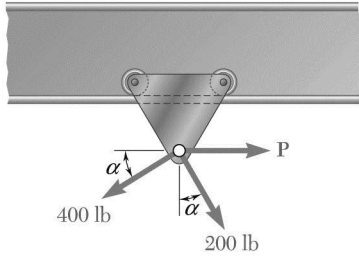
$$P = 177.860 \text{ lb}$$

$$P = 177.9 \text{ lb} \quad \blacktriangleleft$$

(b) Since \mathbf{R} is to be vertical:

$$R = R_y = 410 \text{ lb}$$

$$R = 410 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.42

A hoist trolley is subjected to the three forces shown. Knowing that $P = 250$ lb, determine (a) the required value of α if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \overset{+}{\rightarrow} \Sigma F_x = 250 \text{ lb} + (200 \text{ lb}) \sin \alpha - (400 \text{ lb}) \cos \alpha$$

$$R_x = 250 \text{ lb} + (200 \text{ lb}) \sin \alpha - (400 \text{ lb}) \cos \alpha \quad (1)$$

$$R_y = \overset{+}{\downarrow} \Sigma F_y = (200 \text{ lb}) \cos \alpha + (400 \text{ lb}) \sin \alpha$$

(a) For \mathbf{R} to be vertical, we must have $R_x = 0$.

Set $R_x = 0$ in Eq. (1)

$$0 = 250 \text{ lb} + (200 \text{ lb}) \sin \alpha - (400 \text{ lb}) \cos \alpha$$

$$(400 \text{ lb}) \cos \alpha = (200 \text{ lb}) \sin \alpha + 250 \text{ lb}$$

$$2 \cos \alpha = \sin \alpha + 1.25$$

$$4 \cos^2 \alpha = \sin^2 \alpha + 2.5 \sin \alpha + 1.5625$$

$$4(1 - \sin^2 \alpha) = \sin^2 \alpha + 2.5 \sin \alpha + 1.5625$$

$$0 = 5 \sin^2 \alpha + 2.5 \sin \alpha - 2.4375$$

Using the quadratic formula to solve for the roots gives

$$\sin \alpha = 0.49162$$

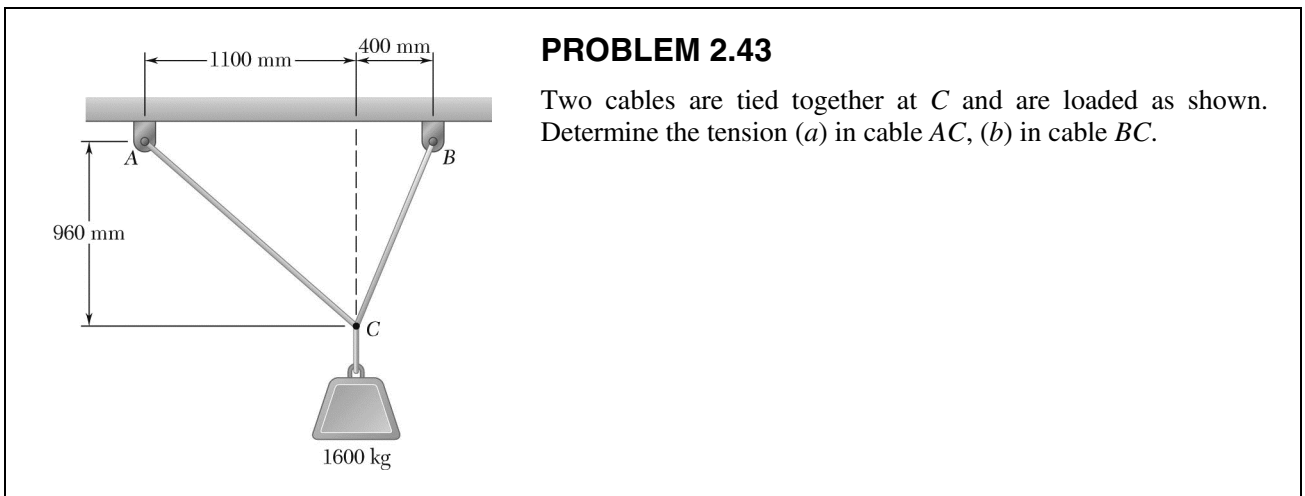
or $\alpha = 29.447^\circ$

$$\alpha = 29.4^\circ \blacktriangleleft$$

(b) Since \mathbf{R} is to be vertical:

$$R = R_y = (200 \text{ lb}) \cos 29.447^\circ + (400 \text{ lb}) \sin 29.447^\circ$$

$$\mathbf{R} = 371 \text{ lb} \blacktriangleleft$$



PROBLEM 2.43

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

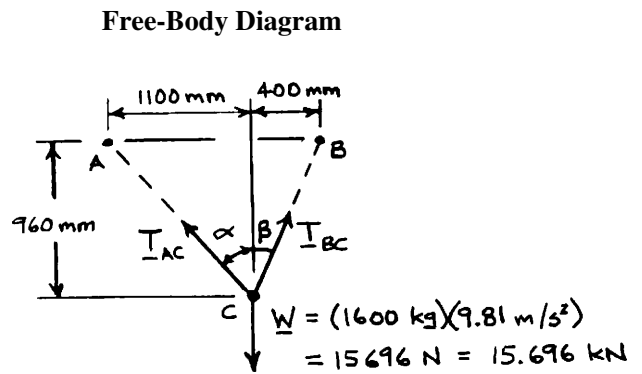
SOLUTION

$$\tan \alpha = \frac{1100}{960}$$

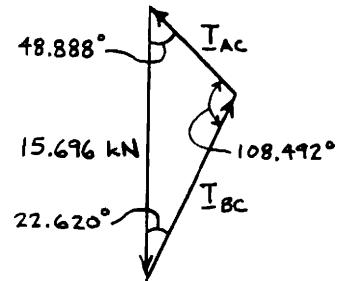
$$\alpha = 48.888^\circ$$

$$\tan \beta = \frac{400}{960}$$

$$\beta = 22.620^\circ$$



Force Triangle



Law of sines:

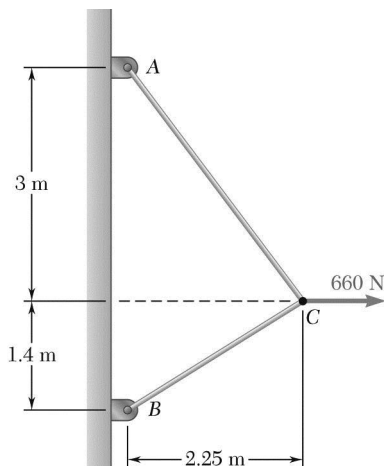
$$\frac{T_{AC}}{\sin 22.620^\circ} = \frac{T_{BC}}{\sin 48.888^\circ} = \frac{15.696 \text{ kN}}{\sin 108.492^\circ}$$

(a) $T_{AC} = \frac{15.696 \text{ kN}}{\sin 108.492^\circ} (\sin 22.620^\circ)$ $T_{AC} = 6.37 \text{ kN} \blacktriangleleft$

(b) $T_{BC} = \frac{15.696 \text{ kN}}{\sin 108.492^\circ} (\sin 48.888^\circ)$ $T_{BC} = 12.47 \text{ kN} \blacktriangleleft$

PROBLEM 2.44

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC , (b) in cable BC .



SOLUTION

$$\tan \alpha = \frac{3}{2.25}$$

$$\alpha = 53.130^\circ$$

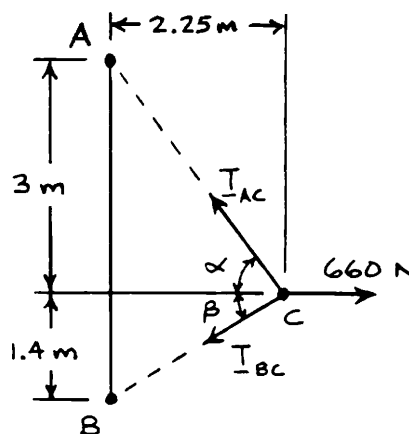
$$\tan \beta = \frac{1.4}{2.25}$$

$$\beta = 31.891^\circ$$

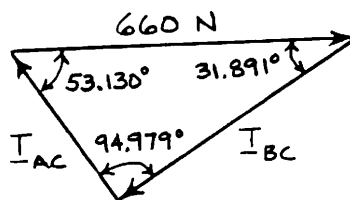
Law of sines:

$$\frac{T_{AC}}{\sin 31.891^\circ} = \frac{T_{BC}}{\sin 53.130^\circ} = \frac{660 \text{ N}}{\sin 94.979^\circ}$$

Free-Body Diagram



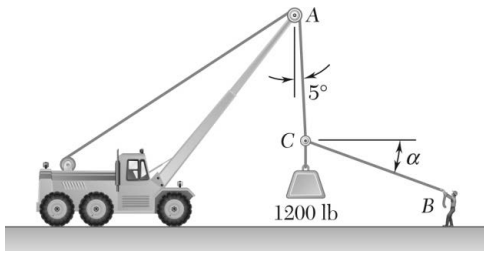
Force-Triangle



(a) $T_{AC} = \frac{660 \text{ N}}{\sin 94.979^\circ} (\sin 31.891^\circ)$ $T_{AC} = 350 \text{ N} \blacktriangleleft$

(b) $T_{BC} = \frac{660 \text{ N}}{\sin 94.979^\circ} (\sin 53.130^\circ)$ $T_{BC} = 530 \text{ N} \blacktriangleleft$

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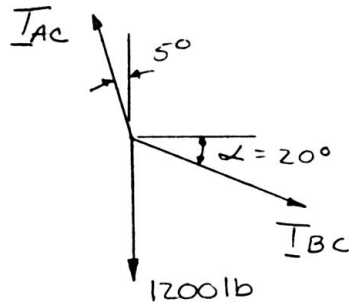


PROBLEM 2.45

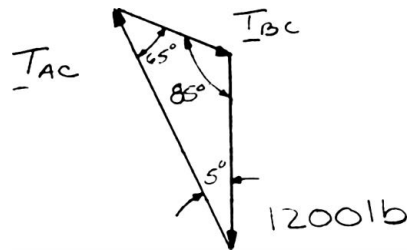
Knowing that $\alpha = 20^\circ$, determine the tension (a) in cable AC, (b) in rope BC.

SOLUTION

Free-Body Diagram



Force Triangle



Law of sines:

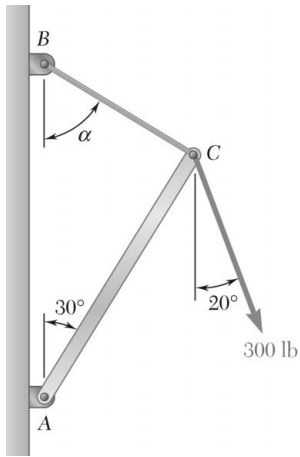
$$\frac{T_{AC}}{\sin 110^\circ} = \frac{T_{BC}}{\sin 5^\circ} = \frac{1200 \text{ lb}}{\sin 65^\circ}$$

(a) $T_{AC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 110^\circ$ $T_{AC} = 1244 \text{ lb} \blacktriangleleft$

(b) $T_{BC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 5^\circ$ $T_{BC} = 115.4 \text{ lb} \blacktriangleleft$

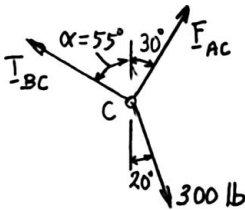
PROBLEM 2.46

Knowing that $\alpha = 55^\circ$ and that boom AC exerts on pin C a force directed along line AC , determine (a) the magnitude of that force, (b) the tension in cable BC .

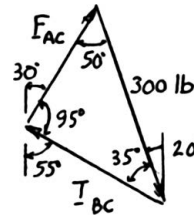


SOLUTION

Free-Body Diagram



Force Triangle

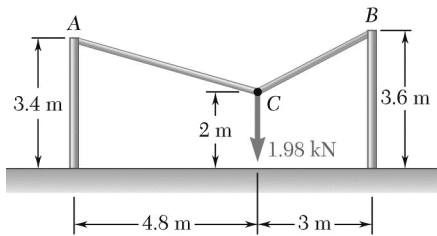


Law of sines:

$$\frac{F_{AC}}{\sin 35^\circ} = \frac{T_{BC}}{\sin 50^\circ} = \frac{300 \text{ lb}}{\sin 95^\circ}$$

(a) $F_{AC} = \frac{300 \text{ lb}}{\sin 95^\circ} \sin 35^\circ$ $F_{AC} = 172.7 \text{ lb} \blacktriangleleft$

(b) $T_{BC} = \frac{300 \text{ lb}}{\sin 95^\circ} \sin 50^\circ$ $T_{BC} = 231 \text{ lb} \blacktriangleleft$



PROBLEM 2.47

Two cables are tied together at C and loaded as shown. Determine the tension (a) in cable AC , (b) in cable BC .

SOLUTION

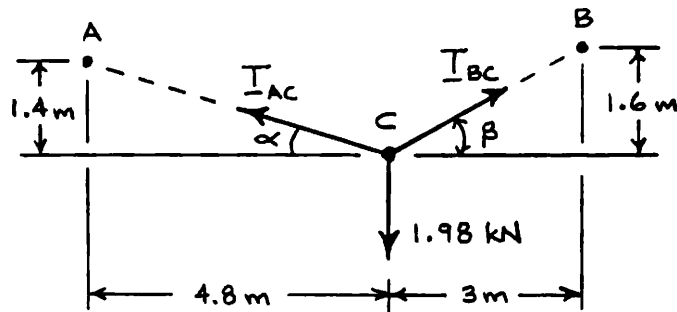
Free-Body Diagram

$$\tan \alpha = \frac{1.4}{4.8}$$

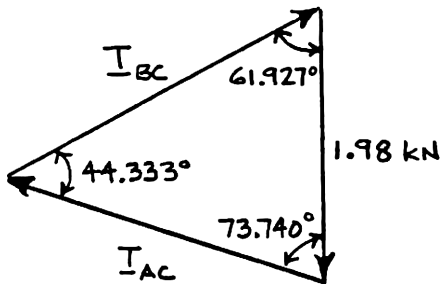
$$\alpha = 16.2602^\circ$$

$$\tan \beta = \frac{1.6}{3}$$

$$\beta = 28.073^\circ$$



Force Triangle

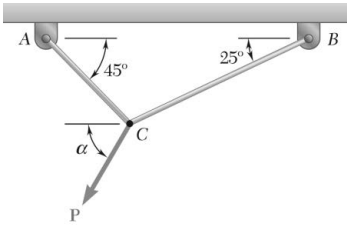


Law of sines:

$$\frac{T_{AC}}{\sin 61.927^\circ} = \frac{T_{BC}}{\sin 73.740^\circ} = \frac{1.98 \text{ kN}}{\sin 44.333^\circ}$$

$$(a) \quad T_{AC} = \frac{1.98 \text{ kN}}{\sin 44.333^\circ} \sin 61.927^\circ \quad T_{AC} = 2.50 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad T_{BC} = \frac{1.98 \text{ kN}}{\sin 44.333^\circ} \sin 73.740^\circ \quad T_{BC} = 2.72 \text{ kN} \quad \blacktriangleleft$$

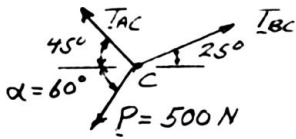


PROBLEM 2.48

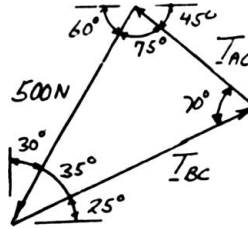
Two cables are tied together at C and are loaded as shown. Knowing that $\mathbf{P} = 500 \text{ N}$ and $\alpha = 60^\circ$, determine the tension in (a) in cable AC , (b) in cable BC .

SOLUTION

Free-Body Diagram



Force Triangle

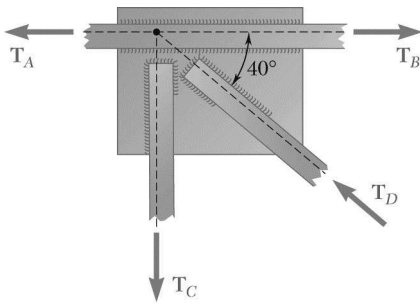


Law of sines:

$$\frac{T_{AC}}{\sin 35^\circ} = \frac{T_{BC}}{\sin 75^\circ} = \frac{500 \text{ N}}{\sin 70^\circ}$$

$$(a) \quad T_{AC} = \frac{500 \text{ N}}{\sin 70^\circ} \sin 35^\circ \quad T_{AC} = 305 \text{ N} \quad \blacktriangleleft$$

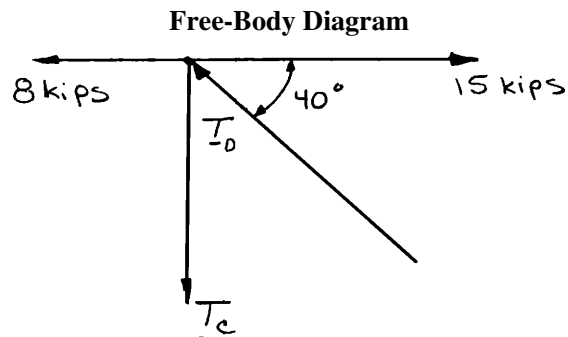
$$(b) \quad T_{BC} = \frac{500 \text{ N}}{\sin 70^\circ} \sin 75^\circ \quad T_{BC} = 514 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.49

Two forces of magnitude $T_A = 8$ kips and $T_B = 15$ kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_C and T_D .

SOLUTION



$$\rightarrow \Sigma F_x = 0 \quad 15 \text{ kips} - 8 \text{ kips} - T_D \cos 40^\circ = 0$$

$$T_D = 9.1379 \text{ kips}$$

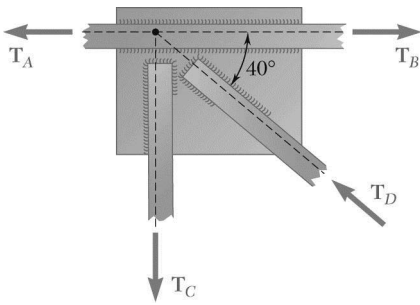
$$+\uparrow \Sigma F_y = 0 \quad T_D \sin 40^\circ - T_C = 0$$

$$(9.1379 \text{ kips}) \sin 40^\circ - T_C = 0$$

$$T_C = 5.8737 \text{ kips}$$

$$T_C = 5.87 \text{ kips} \quad \blacktriangleleft$$

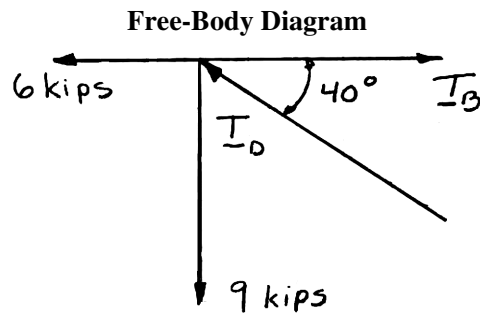
$$T_D = 9.14 \text{ kips} \quad \blacktriangleleft$$



PROBLEM 2.50

Two forces of magnitude $T_A = 6$ kips and $T_C = 9$ kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_B and T_D .

SOLUTION



$$\pm \rightarrow \Sigma F_x = 0$$

$$T_B - 6 \text{ kips} - T_D \cos 40^\circ = 0 \quad (1)$$

$$\uparrow \Sigma F_y = 0$$

$$T_D \sin 40^\circ - 9 \text{ kips} = 0$$

$$T_D = \frac{9 \text{ kips}}{\sin 40^\circ}$$

$$T_D = 14.0015 \text{ kips}$$

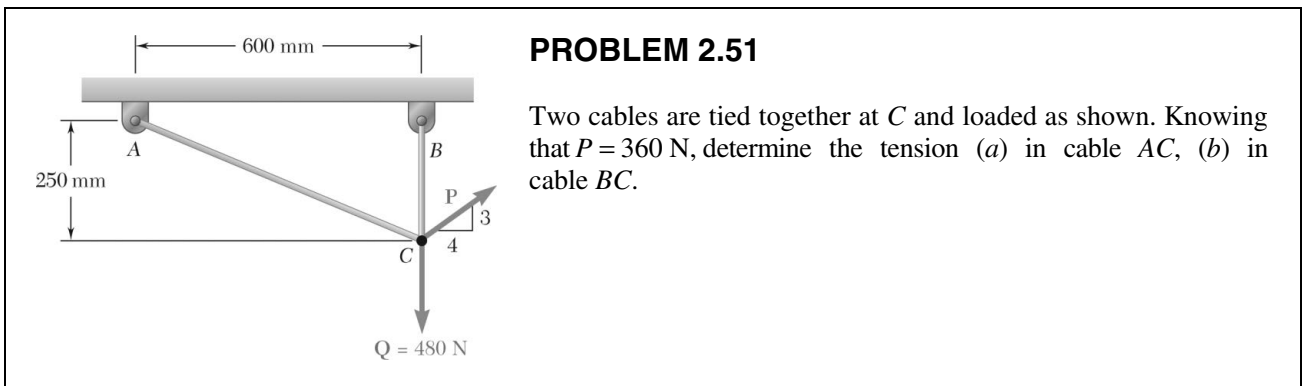
Substituting for T_D into Eq. (1) gives:

$$T_B - 6 \text{ kips} - (14.0015 \text{ kips}) \cos 40^\circ = 0$$

$$T_B = 16.7258 \text{ kips}$$

$$T_B = 16.73 \text{ kips} \quad \blacktriangleleft$$

$$T_D = 14.00 \text{ kips} \quad \blacktriangleleft$$



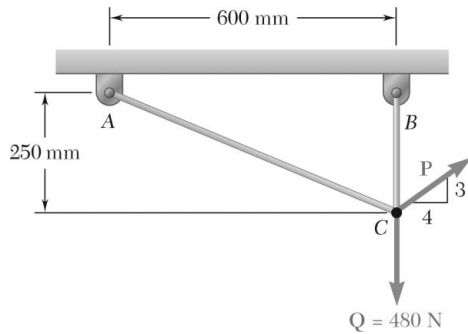
SOLUTION

Free Body: C

(a) $\Sigma F_x = 0: -\frac{12}{13}T_{AC} + \frac{4}{5}(360\text{ N}) = 0$ $T_{AC} = 312\text{ N} \blacktriangleleft$

(b) $\Sigma F_y = 0: \frac{5}{13}(312\text{ N}) + T_{BC} + \frac{3}{5}(360\text{ N}) - 480\text{ N} = 0$

$T_{BC} = 480\text{ N} - 120\text{ N} - 216\text{ N}$ $T_{BC} = 144\text{ N} \blacktriangleleft$

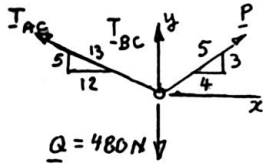


PROBLEM 2.52

Two cables are tied together at C and loaded as shown. Determine the range of values of P for which both cables remain taut.

SOLUTION

Free Body: C



$$\Sigma F_x = 0: -\frac{12}{13}T_{AC} + \frac{4}{5}P = 0$$

$$T_{AC} = \frac{13}{15}P \quad (1)$$

$$\Sigma F_y = 0: \frac{5}{13}T_{AC} + T_{BC} + \frac{3}{5}P - 480 \text{ N} = 0$$

Substitute for T_{AC} from (1):

$$\left(\frac{5}{13}\right)\left(\frac{13}{15}\right)P + T_{BC} + \frac{3}{5}P - 480 \text{ N} = 0$$

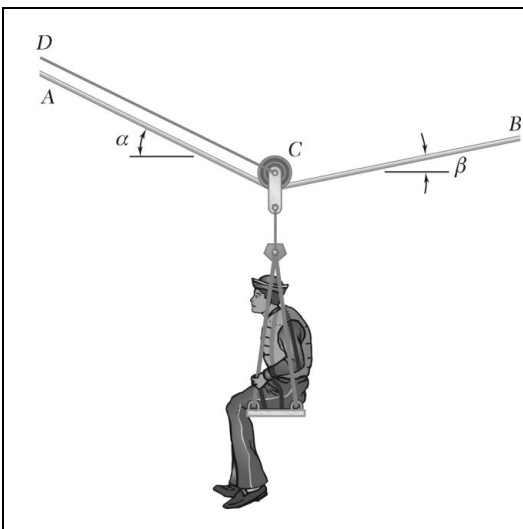
$$T_{BC} = 480 \text{ N} - \frac{14}{15}P \quad (2)$$

From (1), $T_{AC} > 0$ requires $P > 0$.

From (2), $T_{BC} > 0$ requires $\frac{14}{15}P < 480 \text{ N}$, $P < 514.29 \text{ N}$

Allowable range:

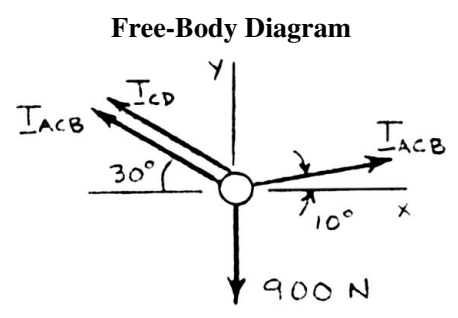
$$0 < P < 514 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.53

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable ACB and is pulled at a constant speed by cable CD . Knowing that $\alpha = 30^\circ$ and $\beta = 10^\circ$ and that the combined weight of the boatswain's chair and the sailor is 900 N , determine the tension (a) in the support cable ACB , (b) in the traction cable CD .

SOLUTION

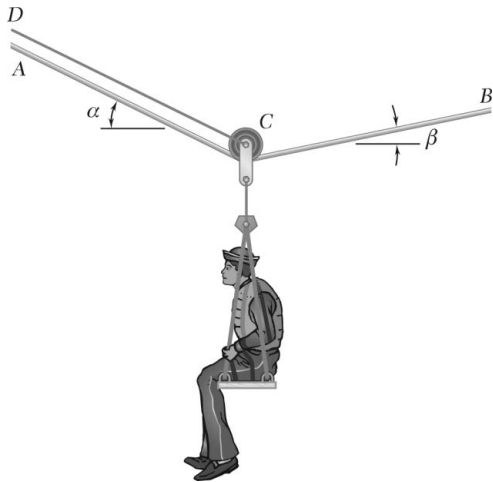


$$\begin{aligned} \rightarrow \Sigma F_x = 0: & T_{ACB} \cos 10^\circ - T_{ACB} \cos 30^\circ - T_{CD} \cos 30^\circ = 0 \\ & T_{CD} = 0.137158 T_{ACB} \end{aligned} \tag{1}$$

$$\begin{aligned} \uparrow \Sigma F_y = 0: & T_{ACB} \sin 10^\circ + T_{ACB} \sin 30^\circ + T_{CD} \sin 30^\circ - 900 = 0 \\ & 0.67365 T_{ACB} + 0.5 T_{CD} = 900 \end{aligned} \tag{2}$$

- (a) Substitute (1) into (2): $0.67365 T_{ACB} + 0.5(0.137158 T_{ACB}) = 900$
 $T_{ACB} = 1212.56\text{ N}$ $T_{ACB} = 1213\text{ N} \blacktriangleleft$
- (b) From (1): $T_{CD} = 0.137158(1212.56\text{ N})$ $T_{CD} = 166.3\text{ N} \blacktriangleleft$

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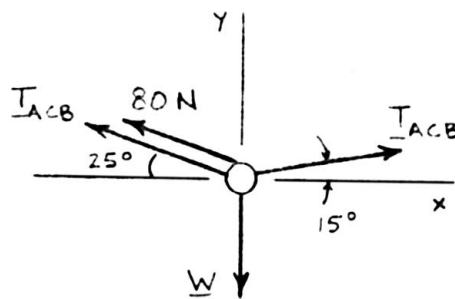


PROBLEM 2.54

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable ACB and is pulled at a constant speed by cable CD . Knowing that $\alpha = 25^\circ$ and $\beta = 15^\circ$ and that the tension in cable CD is 80 N , determine (a) the combined weight of the boatswain's chair and the sailor, (b) in tension in the support cable ACB .

SOLUTION

Free-Body Diagram



$$\pm \rightarrow \Sigma F_x = 0: T_{ACB} \cos 15^\circ - T_{ACB} \cos 25^\circ - (80\text{ N}) \cos 25^\circ = 0$$

$$T_{ACB} = 1216.15\text{ N}$$

$$+\uparrow \Sigma F_y = 0: (1216.15\text{ N}) \sin 15^\circ + (1216.15\text{ N}) \sin 25^\circ$$

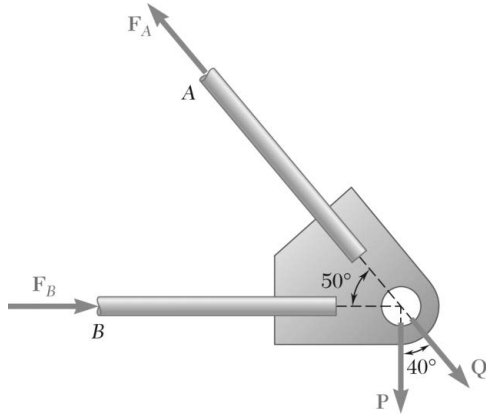
$$+ (80\text{ N}) \sin 25^\circ - W = 0$$

$$W = 862.54\text{ N}$$

$$(a) \quad W = 863\text{ N} \quad \blacktriangleleft$$

$$(b) \quad T_{ACB} = 1216\text{ N} \quad \blacktriangleleft$$

PROBLEM 2.55



Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 500$ lb and $Q = 650$ lb, determine the magnitudes of the forces exerted on the rods **A** and **B**.

SOLUTION

Resolving the forces into x - and y -directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\begin{aligned} \mathbf{R} = & -(500 \text{ lb})\mathbf{j} + [(650 \text{ lb})\cos 50^\circ]\mathbf{i} \\ & - [(650 \text{ lb})\sin 50^\circ]\mathbf{j} \\ & + F_B\mathbf{i} - (F_A \cos 50^\circ)\mathbf{i} + (F_A \sin 50^\circ)\mathbf{j} = 0 \end{aligned}$$

In the y -direction (one unknown force):

$$-500 \text{ lb} - (650 \text{ lb})\sin 50^\circ + F_A \sin 50^\circ = 0$$

Thus,

$$\begin{aligned} F_A &= \frac{500 \text{ lb} + (650 \text{ lb})\sin 50^\circ}{\sin 50^\circ} \\ &= 1302.70 \text{ lb} \end{aligned}$$

$$F_A = 1303 \text{ lb} \quad \blacktriangleleft$$

In the x -direction:

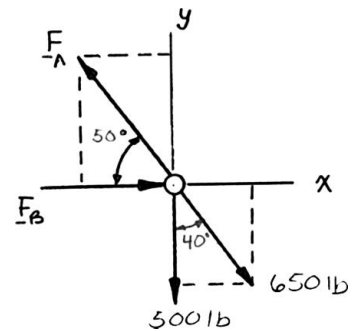
$$(650 \text{ lb})\cos 50^\circ + F_B - F_A \cos 50^\circ = 0$$

Thus,

$$\begin{aligned} F_B &= F_A \cos 50^\circ - (650 \text{ lb})\cos 50^\circ \\ &= (1302.70 \text{ lb})\cos 50^\circ - (650 \text{ lb})\cos 50^\circ \\ &= 419.55 \text{ lb} \end{aligned}$$

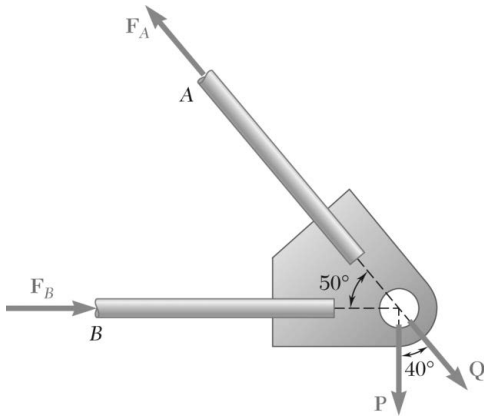
$$F_B = 420 \text{ lb} \quad \blacktriangleleft$$

Free-Body Diagram



PROBLEM 2.56

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods **A** and **B** are $F_A = 750$ lb and $F_B = 400$ lb, determine the magnitudes of **P** and **Q**.



SOLUTION

Resolving the forces into x - and y -directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components: $\mathbf{R} = -P\mathbf{j} + Q \cos 50^\circ\mathbf{i} - Q \sin 50^\circ\mathbf{j}$
 $-[(750 \text{ lb}) \cos 50^\circ]\mathbf{i}$
 $+ [(750 \text{ lb}) \sin 50^\circ]\mathbf{j} + (400 \text{ lb})\mathbf{i}$

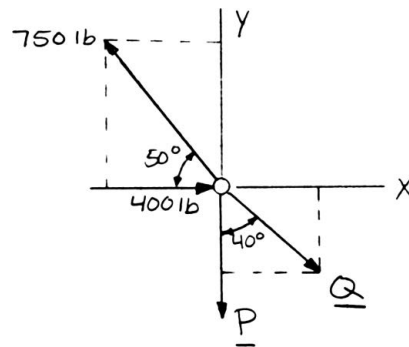
In the x -direction (one unknown force):

$$Q \cos 50^\circ - [(750 \text{ lb}) \cos 50^\circ] + 400 \text{ lb} = 0$$
$$Q = \frac{(750 \text{ lb}) \cos 50^\circ - 400 \text{ lb}}{\cos 50^\circ}$$
$$= 127.710 \text{ lb}$$

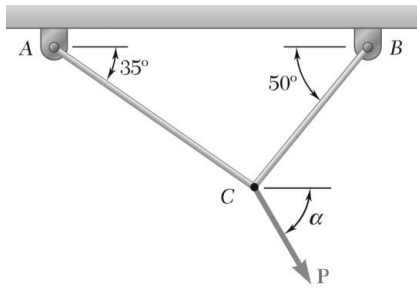
In the y -direction:

$$-P - Q \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ = 0$$
$$P = -Q \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ$$
$$= -(127.710 \text{ lb}) \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ$$
$$= 476.70 \text{ lb}$$

Free-Body Diagram



$$P = 477 \text{ lb}; \quad Q = 127.7 \text{ lb} \quad \blacktriangleleft$$

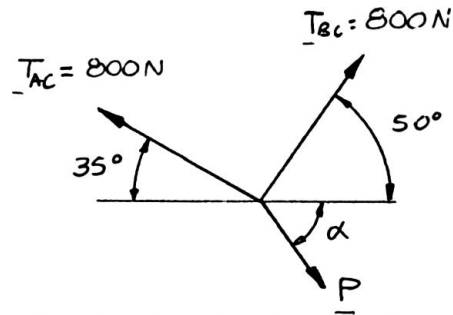


PROBLEM 2.57

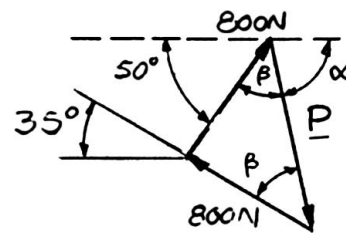
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N , determine
 (a) the magnitude of the largest force P that can be applied at C ,
 (b) the corresponding value of α .

SOLUTION

Free-Body Diagram: C



Force Triangle



Force triangle is isosceles with

$$2\beta = 180^\circ - 85^\circ$$

$$\beta = 47.5^\circ$$

(a)

$$P = 2(800\text{ N})\cos 47.5^\circ = 1081\text{ N}$$

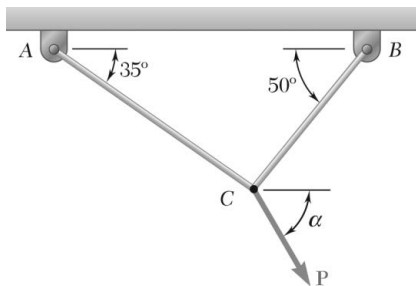
Since $P > 0$, the solution is correct.

$$P = 1081\text{ N} \quad \blacktriangleleft$$

(b)

$$\alpha = 180^\circ - 50^\circ - 47.5^\circ = 82.5^\circ$$

$$\alpha = 82.5^\circ \quad \blacktriangleleft$$

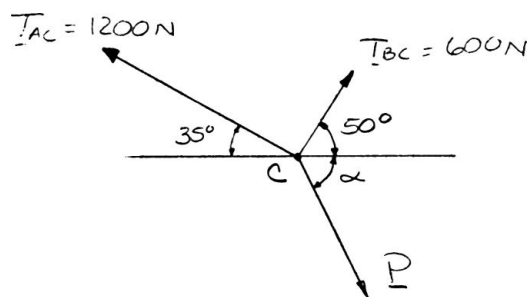


PROBLEM 2.58

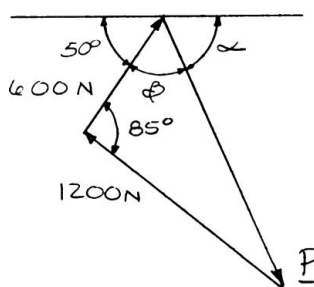
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable AC and 600 N in cable BC , determine (a) the magnitude of the largest force \mathbf{P} that can be applied at C , (b) the corresponding value of α .

SOLUTION

Free-Body Diagram



Force Triangle



(a) Law of cosines:
$$P^2 = (1200 \text{ N})^2 + (600 \text{ N})^2 - 2(1200 \text{ N})(600 \text{ N})\cos 85^\circ$$

$$P = 1294.02 \text{ N}$$

Since $P > 1200 \text{ N}$, the solution is correct.

$$P = 1294 \text{ N} \quad \blacktriangleleft$$

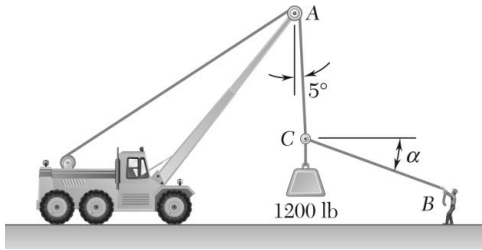
(b) Law of sines:

$$\frac{\sin \beta}{1200 \text{ N}} = \frac{\sin 85^\circ}{1294.02 \text{ N}}$$

$$\beta = 67.5^\circ$$

$$\alpha = 180^\circ - 50^\circ - 67.5^\circ$$

$$\alpha = 62.5^\circ \quad \blacktriangleleft$$



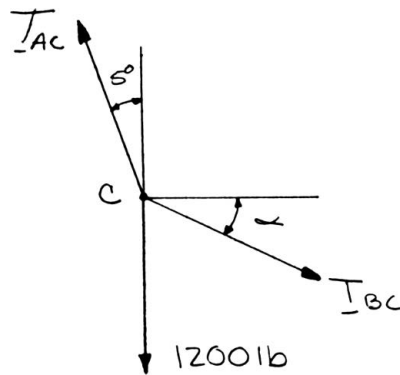
PROBLEM 2.59

For the situation described in Figure P2.45, determine (a) the value of α for which the tension in rope BC is as small as possible, (b) the corresponding value of the tension.

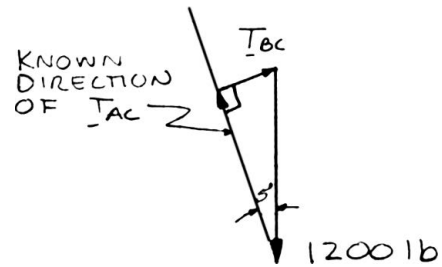
PROBLEM 2.45 Knowing that $\alpha = 20^\circ$, determine the tension (a) in cable AC , (b) in rope BC .

SOLUTION

Free-Body Diagram



Force Triangle

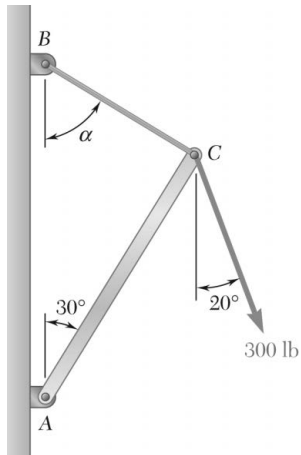


To be smallest, T_{BC} must be perpendicular to the direction of T_{AC} .

- | | | | |
|-----|-------|---|-------------------------------|
| (a) | Thus, | $\alpha = 5^\circ$ | $\alpha = 5.00^\circ$ ◀ |
| (b) | | $T_{BC} = (1200 \text{ lb}) \sin 5^\circ$ | $T_{BC} = 104.6 \text{ lb}$ ◀ |

PROBLEM 2.60

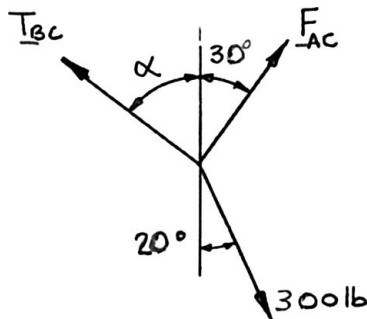
For the structure and loading of Problem 2.46, determine (a) the value of α for which the tension in cable BC is as small as possible, (b) the corresponding value of the tension.



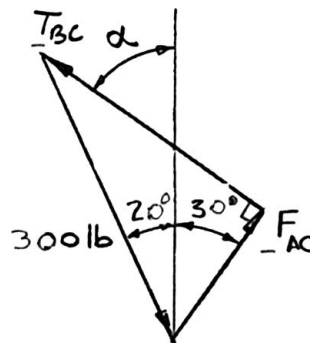
SOLUTION

T_{BC} must be perpendicular to F_{AC} to be as small as possible.

Free-Body Diagram: C



Force Triangle is a right triangle

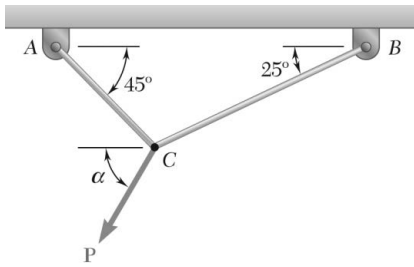


To be a minimum, T_{BC} must be perpendicular to F_{AC} .

(a) We observe: $\alpha = 90^\circ - 30^\circ$ $\alpha = 60.0^\circ$ ◀

(b) $T_{BC} = (300 \text{ lb}) \sin 50^\circ$

or $T_{BC} = 229.81 \text{ lb}$ $T_{BC} = 230 \text{ lb}$ ◀

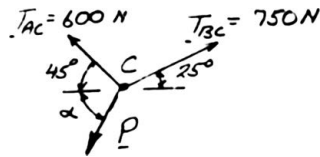


PROBLEM 2.61

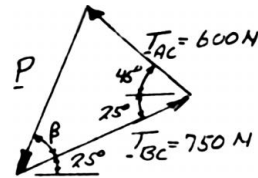
For the cables of Problem 2.48, it is known that the maximum allowable tension is 600 N in cable AC and 750 N in cable BC. Determine (a) the maximum force \mathbf{P} that can be applied at C, (b) the corresponding value of α .

SOLUTION

Free-Body Diagram



Force Triangle



(a) Law of cosines

$$P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25^\circ + 45^\circ)$$

$$P = 784.02 \text{ N}$$

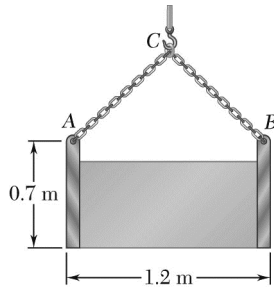
$$P = 784 \text{ N} \quad \blacktriangleleft$$

(b) Law of sines

$$\frac{\sin \beta}{600 \text{ N}} = \frac{\sin(25^\circ + 45^\circ)}{784.02 \text{ N}}$$

$$\beta = 46.0^\circ \quad \therefore \alpha = 46.0^\circ + 25^\circ$$

$$\alpha = 71.0^\circ \quad \blacktriangleleft$$

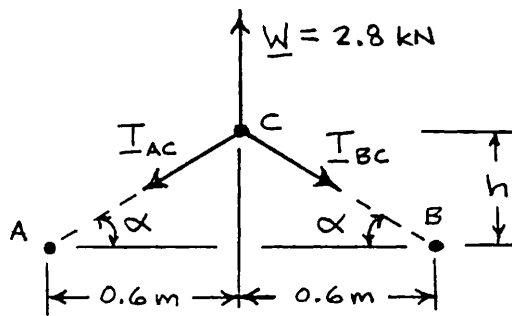


PROBLEM 2.62

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling ACB that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

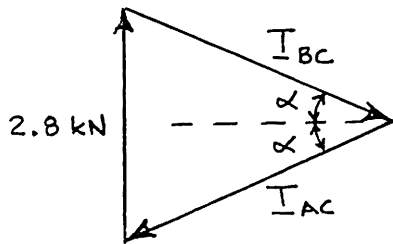
SOLUTION

Free-Body Diagram



$$\tan \alpha = \frac{h}{0.6 \text{ m}} \quad (1)$$

Isosceles Force Triangle

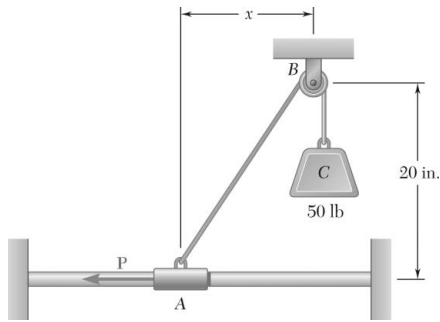


$$\begin{aligned} \text{Law of sines: } \sin \alpha &= \frac{\frac{1}{2}(2.8 \text{ kN})}{T_{AC}} \\ T_{AC} &= 5 \text{ kN} \\ \sin \alpha &= \frac{\frac{1}{2}(2.8 \text{ kN})}{5 \text{ kN}} \\ \alpha &= 16.2602^\circ \end{aligned}$$

$$\text{From Eq. (1): } \tan 16.2602^\circ = \frac{h}{0.6 \text{ m}} \quad \therefore h = 0.175000 \text{ m}$$

$$\begin{aligned} \text{Half length of chain} = AC &= \sqrt{(0.6 \text{ m})^2 + (0.175 \text{ m})^2} \\ &= 0.625 \text{ m} \end{aligned}$$

$$\text{Total length: } = 2 \times 0.625 \text{ m} \quad 1.250 \text{ m} \quad \blacktriangleleft$$

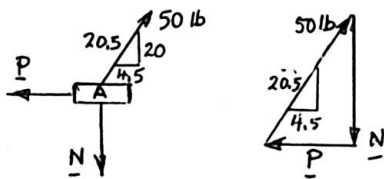


PROBLEM 2.63

Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force \mathbf{P} required to maintain the equilibrium of the collar when (a) $x = 4.5$ in., (b) $x = 15$ in.

SOLUTION

(a) **Free Body: Collar A**

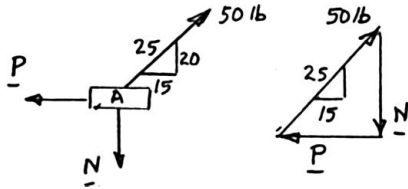


Force Triangle

$$\frac{P}{4.5} = \frac{50 \text{ lb}}{20.5}$$

$$P = 10.98 \text{ lb} \quad \blacktriangleleft$$

(b) **Free Body: Collar A**



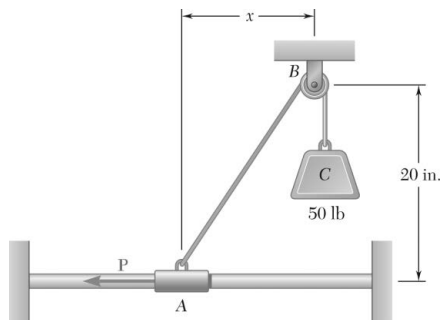
Force Triangle

$$\frac{P}{15} = \frac{50 \text{ lb}}{25}$$

$$P = 30.0 \text{ lb} \quad \blacktriangleleft$$

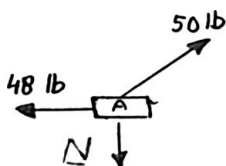
PROBLEM 2.64

Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 48$ lb.



SOLUTION

Free Body: Collar A



Force Triangle

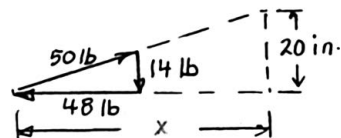


$$N^2 = (50)^2 - (48)^2 = 196$$

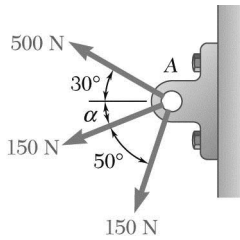
$$N = 14.00 \text{ lb}$$

Similar Triangles

$$\frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}}$$



$$x = 68.6 \text{ in.} \blacktriangleleft$$

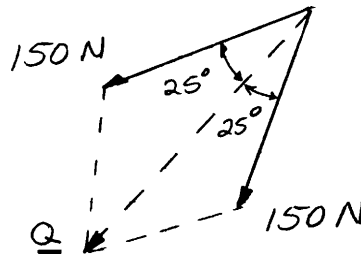


PROBLEM 2.65

Three forces are applied to a bracket as shown. The directions of the two 150-N forces may vary, but the angle between these forces is always 50° . Determine the range of values of α for which the magnitude of the resultant of the forces acting at A is less than 600 N.

SOLUTION

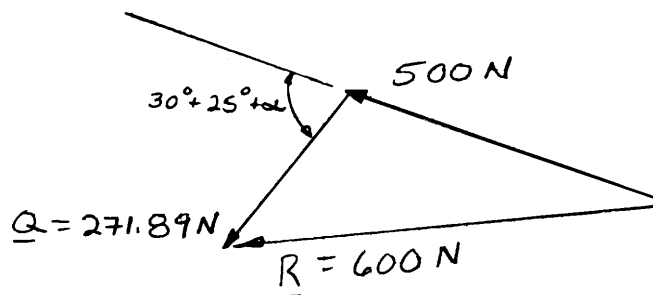
Combine the two 150-N forces into a resultant force Q :



$$Q = 2(150 \text{ N}) \cos 25^\circ$$

$$= 271.89 \text{ N}$$

Equivalent loading at A:



Using the law of cosines:

$$(600 \text{ N})^2 = (500 \text{ N})^2 + (271.89 \text{ N})^2 + 2(500 \text{ N})(271.89 \text{ N}) \cos(55^\circ + \alpha)$$

$$\cos(55^\circ + \alpha) = 0.132685$$

Two values for α :

$$55^\circ + \alpha = 82.375$$

$$\alpha = 27.4^\circ$$

or

$$55^\circ + \alpha = -82.375^\circ$$

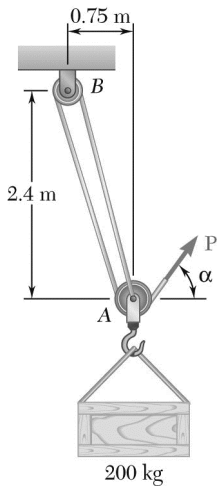
$$55^\circ + \alpha = 360^\circ - 82.375^\circ$$

$$\alpha = 222.6^\circ$$

For $R < 600 \text{ lb}$:

$$27.4^\circ < \alpha < 222.6 \quad \blacktriangleleft$$

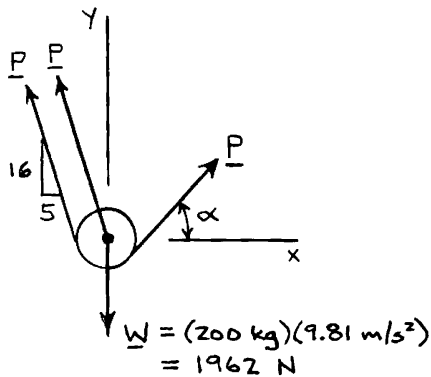
PROBLEM 2.66



A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force \mathbf{P} that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

SOLUTION

Free-Body Diagram: Pulley A



$$\pm \rightarrow \Sigma F_x = 0: \quad -2P \left(\frac{5}{\sqrt{281}} \right) + P \cos \alpha = 0$$

$$\cos \alpha = 0.59655$$

$$\alpha = \pm 53.377^\circ$$

For $\alpha = +53.377^\circ$:

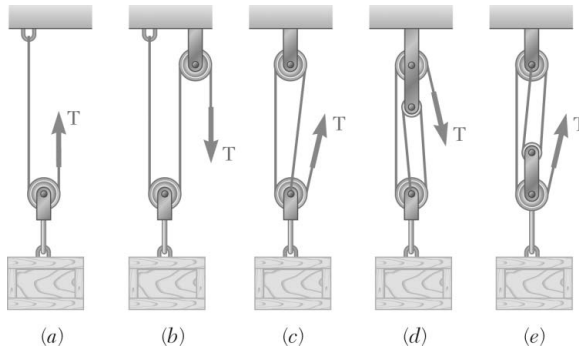
$$+\uparrow \Sigma F_y = 0: \quad 2P \left(\frac{16}{\sqrt{281}} \right) + P \sin 53.377^\circ - 1962 \text{ N} = 0$$

$$\mathbf{P} = 724 \text{ N} \nearrow 53.4^\circ \blacktriangleleft$$

For $\alpha = -53.377^\circ$:

$$+\uparrow \Sigma F_y = 0: \quad 2P \left(\frac{16}{\sqrt{281}} \right) + P \sin(-53.377^\circ) - 1962 \text{ N} = 0$$

$$\mathbf{P} = 1773 \text{ N} \searrow 53.4^\circ \blacktriangleleft$$

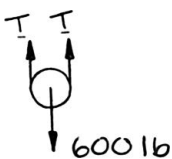


PROBLEM 2.67

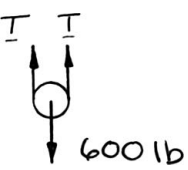
A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

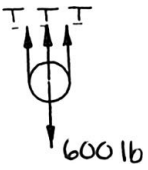
Free-Body Diagram of Pulley

- (a) 
$$+\uparrow \Sigma F_y = 0: 2T - (600 \text{ lb}) = 0$$

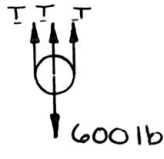
$$T = \frac{1}{2}(600 \text{ lb})$$

$$T = 300 \text{ lb} \quad \blacktriangleleft$$
- (b) 
$$+\uparrow \Sigma F_y = 0: 2T - (600 \text{ lb}) = 0$$

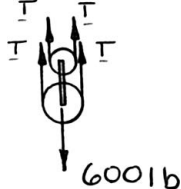
$$T = \frac{1}{2}(600 \text{ lb})$$

$$T = 300 \text{ lb} \quad \blacktriangleleft$$
- (c) 
$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

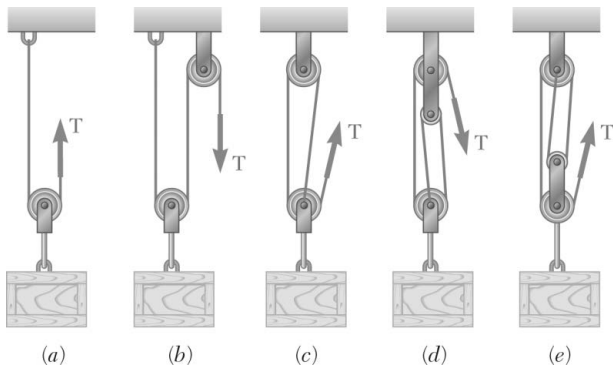
$$T = 200 \text{ lb} \quad \blacktriangleleft$$
- (d) 
$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

$$T = 200 \text{ lb} \quad \blacktriangleleft$$
- (e) 
$$+\uparrow \Sigma F_y = 0: 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$$T = 150.0 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.68

Solve Parts *b* and *d* of Problem 2.67, assuming that the free end of the rope is attached to the crate.

PROBLEM 2.67 A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley and Crate

(b)

$$+\uparrow \Sigma F_y = 0: \quad 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

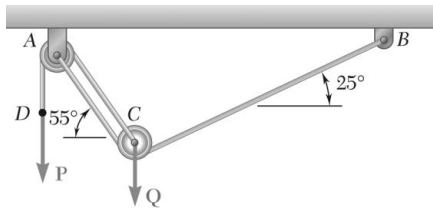
$$T = 200 \text{ lb} \quad \blacktriangleleft$$

(d)

$$+\uparrow \Sigma F_y = 0: \quad 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$$T = 150.0 \text{ lb} \quad \blacktriangleleft$$

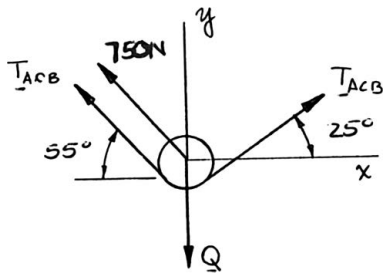


PROBLEM 2.69

A load **Q** is applied to the pulley **C**, which can roll on the cable **ACB**. The pulley is held in the position shown by a second cable **CAD**, which passes over the pulley **A** and supports a load **P**. Knowing that $P = 750 \text{ N}$, determine (a) the tension in cable **ACB**, (b) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley **C**



$$(a) \quad \pm \rightarrow \Sigma F_x = 0: \quad T_{ACB}(\cos 25^\circ - \cos 55^\circ) - (750 \text{ N})\cos 55^\circ = 0$$

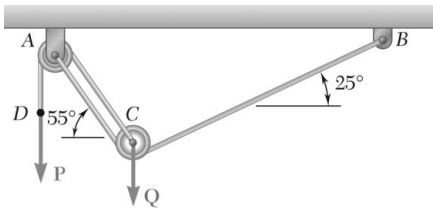
$$\text{Hence:} \quad T_{ACB} = 1292.88 \text{ N}$$

$$T_{ACB} = 1293 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad + \uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25^\circ + \sin 55^\circ) + (750 \text{ N})\sin 55^\circ - Q = 0$$

$$(1292.88 \text{ N})(\sin 25^\circ + \sin 55^\circ) + (750 \text{ N})\sin 55^\circ - Q = 0$$

$$\text{or} \quad Q = 2219.8 \text{ N} \quad Q = 2220 \text{ N} \quad \blacktriangleleft$$

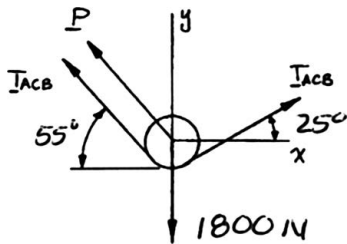


PROBLEM 2.70

An 1800-N load **Q** is applied to the pulley **C**, which can roll on the cable **ACB**. The pulley is held in the position shown by a second cable **CAD**, which passes over the pulley **A** and supports a load **P**. Determine (a) the tension in cable **ACB**, (b) the magnitude of load **P**.

SOLUTION

Free-Body Diagram: Pulley C



$$\pm \rightarrow \Sigma F_x = 0: T_{ACB}(\cos 25^\circ - \cos 55^\circ) - P \cos 55^\circ = 0$$

$$\text{or} \quad P = 0.58010T_{ACB} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: T_{ACB}(\sin 25^\circ + \sin 55^\circ) + P \sin 55^\circ - 1800 \text{ N} = 0$$

$$\text{or} \quad 1.24177T_{ACB} + 0.81915P = 1800 \text{ N} \quad (2)$$

(a) Substitute Equation (1) into Equation (2):

$$1.24177T_{ACB} + 0.81915(0.58010T_{ACB}) = 1800 \text{ N}$$

$$\text{Hence:} \quad T_{ACB} = 1048.37 \text{ N}$$

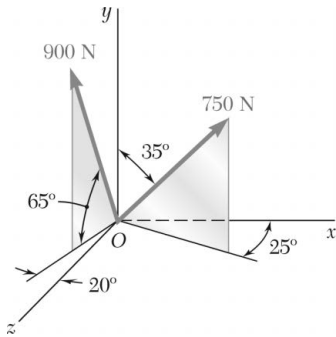
$$T_{ACB} = 1048 \text{ N} \quad \blacktriangleleft$$

(b) Using (1), $P = 0.58010(1048.37 \text{ N}) = 608.16 \text{ N}$

$$P = 608 \text{ N} \quad \blacktriangleleft$$

PROBLEM 2.71

Determine (a) the x , y , and z components of the 900-N force, (b) the angles θ_x , θ_y , and θ_z that the force forms with the coordinate axes.



SOLUTION

$$F_h = F \cos 65^\circ$$

$$= (900 \text{ N}) \cos 65^\circ$$

$$F_h = 380.36 \text{ N}$$

(a)

$$F_x = F_h \sin 20^\circ$$

$$= (380.36 \text{ N}) \sin 20^\circ$$

$$F_x = -130.091 \text{ N},$$

$$F_x = -130.1 \text{ N} \quad \blacktriangleleft$$

$$F_y = F \sin 65^\circ$$

$$= (900 \text{ N}) \sin 65^\circ$$

$$F_y = +815.68 \text{ N},$$

$$F_y = +816 \text{ N} \quad \blacktriangleleft$$

$$F_z = F_h \cos 20^\circ$$

$$= (380.36 \text{ N}) \cos 20^\circ$$

$$F_z = +357.42 \text{ N}$$

$$F_z = +357 \text{ N} \quad \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{-130.091 \text{ N}}{900 \text{ N}}$$

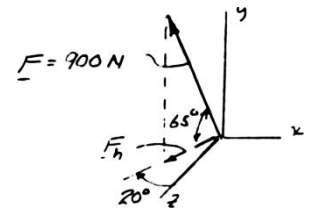
$$\theta_x = 98.3^\circ \quad \blacktriangleleft$$

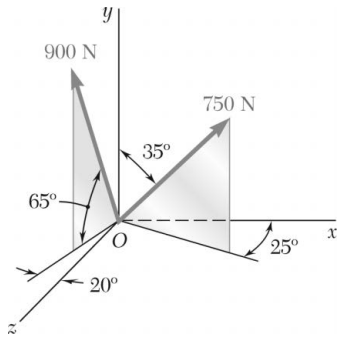
$$\cos \theta_y = \frac{F_y}{F} = \frac{+815.68 \text{ N}}{900 \text{ N}}$$

$$\theta_y = 25.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{+357.42 \text{ N}}{900 \text{ N}}$$

$$\theta_z = 66.6^\circ \quad \blacktriangleleft$$

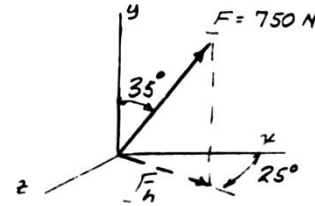




PROBLEM 2.72

Determine (a) the x , y , and z components of the 750-N force, (b) the angles θ_x , θ_y , and θ_z that the force forms with the coordinate axes.

SOLUTION



$$\begin{aligned} F_h &= F \sin 35^\circ \\ &= (750 \text{ N}) \sin 35^\circ \\ F_h &= 430.18 \text{ N} \end{aligned}$$

$$\begin{aligned} (a) \quad F_x &= F_h \cos 25^\circ \\ &= (430.18 \text{ N}) \cos 25^\circ \end{aligned}$$

$$F_x = +389.88 \text{ N},$$

$$F_x = +390 \text{ N} \blacktriangleleft$$

$$\begin{aligned} F_y &= F \cos 35^\circ \\ &= (750 \text{ N}) \cos 35^\circ \end{aligned}$$

$$F_y = +614.36 \text{ N},$$

$$F_y = +614 \text{ N} \blacktriangleleft$$

$$\begin{aligned} F_z &= F_h \sin 25^\circ \\ &= (430.18 \text{ N}) \sin 25^\circ \end{aligned}$$

$$F_z = +181.802 \text{ N}$$

$$F_z = +181.8 \text{ N} \blacktriangleleft$$

$$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{+389.88 \text{ N}}{750 \text{ N}}$$

$$\theta_x = 58.7^\circ \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{+614.36 \text{ N}}{750 \text{ N}}$$

$$\theta_y = 35.0^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{+181.802 \text{ N}}{750 \text{ N}}$$

$$\theta_z = 76.0^\circ \blacktriangleleft$$

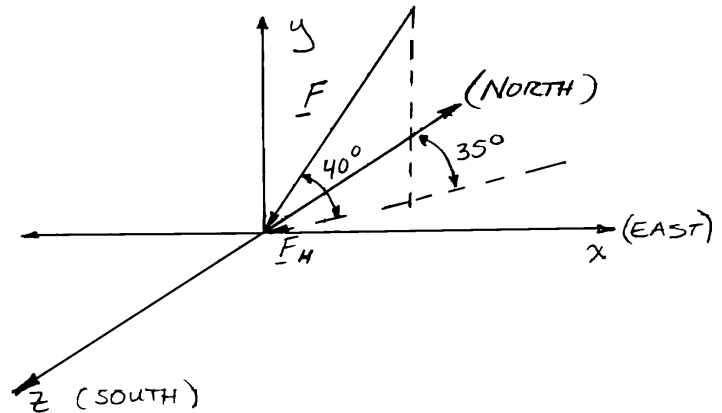
PROBLEM 2.73

A gun is aimed at a point A located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N , determine (a) the x , y , and z components of that force, (b) the values of the angles θ_x , θ_y , and θ_z defining the direction of the recoil force. (Assume that the x , y , and z axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force

$$\begin{aligned} F &= 400\text{ N} \\ \therefore F_H &= (400\text{ N})\cos 40^\circ \\ &= 306.42\text{ N} \end{aligned}$$



(a)

$$\begin{aligned} F_x &= -F_H \sin 35^\circ \\ &= -(306.42\text{ N})\sin 35^\circ \\ &= -175.755\text{ N} \end{aligned}$$

$$F_x = -175.8\text{ N} \blacktriangleleft$$

$$\begin{aligned} F_y &= -F \sin 40^\circ \\ &= -(400\text{ N})\sin 40^\circ \\ &= -257.12\text{ N} \end{aligned}$$

$$F_y = -257\text{ N} \blacktriangleleft$$

$$\begin{aligned} F_z &= +F_H \cos 35^\circ \\ &= +(306.42\text{ N})\cos 35^\circ \\ &= +251.00\text{ N} \end{aligned}$$

$$F_z = +251\text{ N} \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{-175.755\text{ N}}{400\text{ N}}$$

$$\theta_x = 116.1^\circ \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-257.12\text{ N}}{400\text{ N}}$$

$$\theta_y = 130.0^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{251.00\text{ N}}{400\text{ N}}$$

$$\theta_z = 51.1^\circ \blacktriangleleft$$

PROBLEM 2.74

Solve Problem 2.73, assuming that point A is located 15° north of west and that the barrel of the gun forms an angle of 25° with the horizontal.

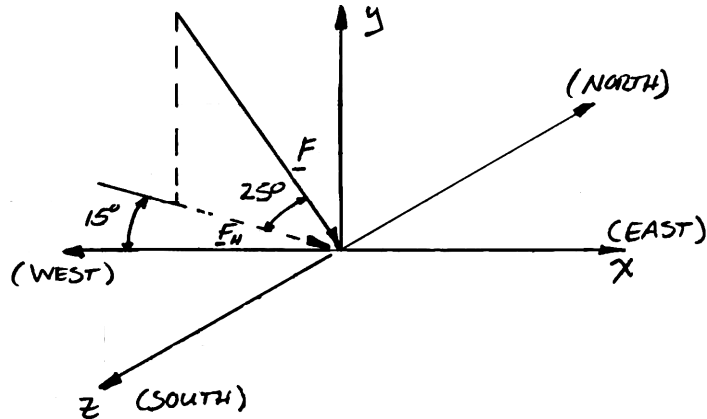
PROBLEM 2.73 A gun is aimed at a point A located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine (a) the x, y, and z components of that force, (b) the values of the angles θ_x , θ_y , and θ_z defining the direction of the recoil force. (Assume that the x, y, and z axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force

$$F = 400 \text{ N}$$

$$\begin{aligned} \therefore F_H &= (400 \text{ N}) \cos 25^\circ \\ &= 362.52 \text{ N} \end{aligned}$$



(a)

$$\begin{aligned} F_x &= +F_H \cos 15^\circ \\ &= +(362.52 \text{ N}) \cos 15^\circ \\ &= +350.17 \text{ N} \end{aligned}$$

$$F_x = +350 \text{ N} \blacktriangleleft$$

$$\begin{aligned} F_y &= -F \sin 25^\circ \\ &= -(400 \text{ N}) \sin 25^\circ \\ &= -169.047 \text{ N} \end{aligned}$$

$$F_y = -169.0 \text{ N} \blacktriangleleft$$

$$\begin{aligned} F_z &= +F_H \sin 15^\circ \\ &= +(362.52 \text{ N}) \sin 15^\circ \\ &= +93.827 \text{ N} \end{aligned}$$

$$F_z = +93.8 \text{ N} \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{+350.17 \text{ N}}{400 \text{ N}}$$

$$\theta_x = 28.9^\circ \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-169.047 \text{ N}}{400 \text{ N}}$$

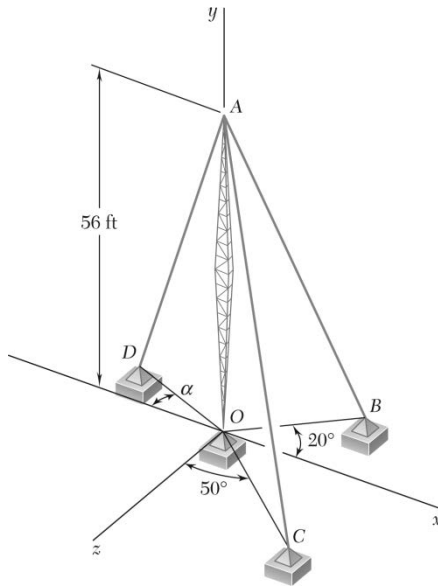
$$\theta_y = 115.0^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{+93.827 \text{ N}}{400 \text{ N}}$$

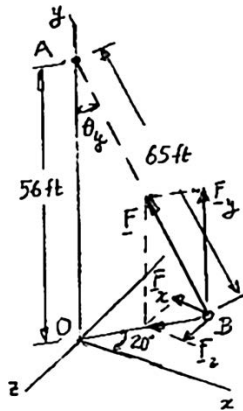
$$\theta_z = 76.4^\circ \blacktriangleleft$$

PROBLEM 2.75

Cable AB is 65 ft long, and the tension in that cable is 3900 lb. Determine (a) the x , y , and z components of the force exerted by the cable on the anchor B , (b) the angles θ_x , θ_y , and θ_z defining the direction of that force.



SOLUTION



From triangle AOB :

$$\begin{aligned}\cos \theta_y &= \frac{56 \text{ ft}}{65 \text{ ft}} \\ &= 0.86154 \\ \theta_y &= 30.51^\circ\end{aligned}$$

(a)

$$\begin{aligned}F_x &= -F \sin \theta_y \cos 20^\circ \\ &= -(3900 \text{ lb}) \sin 30.51^\circ \cos 20^\circ\end{aligned}$$

$$F_x = -1861 \text{ lb} \quad \blacktriangleleft$$

$$F_y = +F \cos \theta_y = (3900 \text{ lb})(0.86154)$$

$$F_y = +3360 \text{ lb} \quad \blacktriangleleft$$

$$F_z = +(3900 \text{ lb}) \sin 30.51^\circ \sin 20^\circ$$

$$F_z = +677 \text{ lb} \quad \blacktriangleleft$$

(b)

$$\cos \theta_x = \frac{F_x}{F} = -\frac{1861 \text{ lb}}{3900 \text{ lb}} = -0.4771$$

$$\theta_x = 118.5^\circ \quad \blacktriangleleft$$

From above:

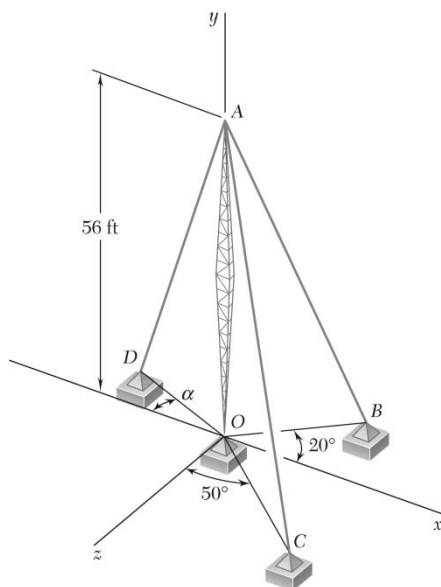
$$\theta_y = 30.51^\circ$$

$$\theta_y = 30.5^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = +\frac{677 \text{ lb}}{3900 \text{ lb}} = +0.1736$$

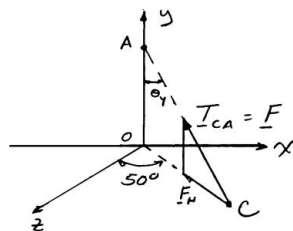
$$\theta_z = 80.0^\circ \quad \blacktriangleleft$$

PROBLEM 2.76



Cable AC is 70 ft long, and the tension in that cable is 5250 lb. Determine (a) the x , y , and z components of the force exerted by the cable on the anchor C , (b) the angles θ_x , θ_y , and θ_z defining the direction of that force.

SOLUTION



In triangle AOB :

$$\begin{aligned} AC &= 70 \text{ ft} \\ OA &= 56 \text{ ft} \\ F &= 5250 \text{ lb} \end{aligned}$$

$$\cos \theta_y = \frac{56 \text{ ft}}{70 \text{ ft}}$$

$$\theta_y = 36.870^\circ$$

$$\begin{aligned} F_H &= F \sin \theta_y \\ &= (5250 \text{ lb}) \sin 36.870^\circ \\ &= 3150.0 \text{ lb} \end{aligned}$$

$$(a) \quad F_x = -F_H \sin 50^\circ = -(3150.0 \text{ lb}) \sin 50^\circ = -2413.04 \text{ lb} \quad F_x = -2413 \text{ lb} \quad \blacktriangleleft$$

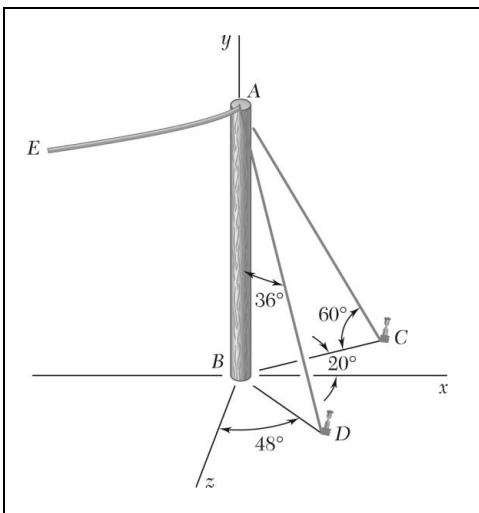
$$F_y = +F \cos \theta_y = +(5250 \text{ lb}) \cos 36.870^\circ = +4200.0 \text{ lb} \quad F_y = +4200 \text{ lb} \quad \blacktriangleleft$$

$$F_z = -F_H \cos 50^\circ = -3150 \cos 50^\circ = -2024.8 \text{ lb} \quad F_z = -2025 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{-2413.04 \text{ lb}}{5250 \text{ lb}} \quad \theta_x = 117.4^\circ \quad \blacktriangleleft$$

$$\text{From above:} \quad \theta_y = 36.870^\circ \quad \theta_y = 36.9^\circ \quad \blacktriangleleft$$

$$\theta_z = \frac{F_z}{F} = \frac{-2024.8 \text{ lb}}{5250 \text{ lb}} \quad \theta_z = 112.7^\circ \quad \blacktriangleleft$$



PROBLEM 2.77

The end of the coaxial cable AE is attached to the pole AB , which is strengthened by the guy wires AC and AD . Knowing that the tension in wire AC is 120 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles θ_x , θ_y , and θ_z that the force forms with the coordinate axes.

SOLUTION

(a)

$$F_x = (120 \text{ lb}) \cos 60^\circ \cos 20^\circ$$

$$F_x = 56.382 \text{ lb} \qquad F_x = +56.4 \text{ lb} \blacktriangleleft$$

$$F_y = -(120 \text{ lb}) \sin 60^\circ$$

$$F_y = -103.923 \text{ lb} \qquad F_y = -103.9 \text{ lb} \blacktriangleleft$$

$$F_z = -(120 \text{ lb}) \cos 60^\circ \sin 20^\circ$$

$$F_z = -20.521 \text{ lb} \qquad F_z = -20.5 \text{ lb} \blacktriangleleft$$

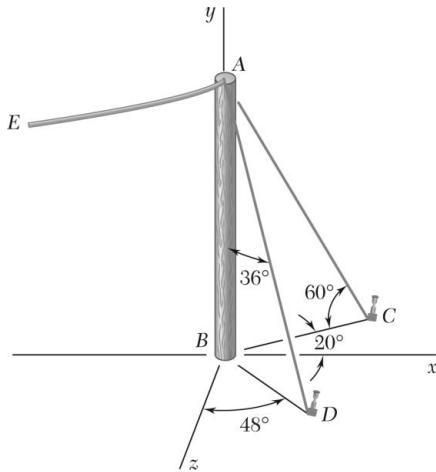
(b)

$$\cos \theta_x = \frac{F_x}{F} = \frac{56.382 \text{ lb}}{120 \text{ lb}} \qquad \theta_x = 62.0^\circ \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-103.923 \text{ lb}}{120 \text{ lb}} \qquad \theta_y = 150.0^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-20.52 \text{ lb}}{120 \text{ lb}} \qquad \theta_z = 99.8^\circ \blacktriangleleft$$

PROBLEM 2.78



The end of the coaxial cable AE is attached to the pole AB , which is strengthened by the guy wires AC and AD . Knowing that the tension in wire AD is 85 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles θ_x , θ_y , and θ_z that the force forms with the coordinate axes.

SOLUTION

$$(a) \quad F_x = (85 \text{ lb}) \sin 36^\circ \sin 48^\circ = 37.129 \text{ lb} \quad F_x = 37.1 \text{ lb} \quad \blacktriangleleft$$

$$F_y = -(85 \text{ lb}) \cos 36^\circ = -68.766 \text{ lb} \quad F_y = -68.8 \text{ lb} \quad \blacktriangleleft$$

$$F_z = (85 \text{ lb}) \sin 36^\circ \cos 48^\circ = 33.431 \text{ lb} \quad F_z = 33.4 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{37.129 \text{ lb}}{85 \text{ lb}} \quad \theta_x = 64.1^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-68.766 \text{ lb}}{85 \text{ lb}} \quad \theta_y = 144.0^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{33.431 \text{ lb}}{85 \text{ lb}} \quad \theta_z = 66.8^\circ \quad \blacktriangleleft$$

PROBLEM 2.79

Determine the magnitude and direction of the force $\mathbf{F} = (690 \text{ lb})\mathbf{i} + (300 \text{ lb})\mathbf{j} - (580 \text{ lb})\mathbf{k}$.

SOLUTION

$$\mathbf{F} = (690 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} - (580 \text{ N})\mathbf{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(690 \text{ N})^2 + (300 \text{ N})^2 + (-580 \text{ N})^2}$$

$$= 950 \text{ N}$$

$$F = 950 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{690 \text{ N}}{950 \text{ N}}$$

$$\theta_x = 43.4^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{300 \text{ N}}{950 \text{ N}}$$

$$\theta_y = 71.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-580 \text{ N}}{950 \text{ N}}$$

$$\theta_z = 127.6^\circ \quad \blacktriangleleft$$

PROBLEM 2.80

Determine the magnitude and direction of the force $\mathbf{F} = (650 \text{ N})\mathbf{i} - (320 \text{ N})\mathbf{j} + (760 \text{ N})\mathbf{k}$.

SOLUTION

$$\mathbf{F} = (650 \text{ N})\mathbf{i} - (320 \text{ N})\mathbf{j} + (760 \text{ N})\mathbf{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(650 \text{ N})^2 + (-320 \text{ N})^2 + (760 \text{ N})^2} \qquad F = 1050 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{650 \text{ N}}{1050 \text{ N}} \qquad \theta_x = 51.8^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-320 \text{ N}}{1050 \text{ N}} \qquad \theta_y = 107.7^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{760 \text{ N}}{1050 \text{ N}} \qquad \theta_z = 43.6^\circ \quad \blacktriangleleft$$

PROBLEM 2.81

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 75^\circ$ and $\theta_z = 130^\circ$. Knowing that the y component of the force is +300 lb, determine (a) the angle θ_y , (b) the other components and the magnitude of the force.

SOLUTION

$$\begin{aligned}\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z &= 1 \\ \cos^2(75^\circ) + \cos^2 \theta_y + \cos^2(130^\circ) &= 1 \\ \cos \theta_y &= \pm 0.72100\end{aligned}$$

(a) Since $F_y > 0$, we choose $\cos \theta_y = +0.72100$ $\therefore \theta_y = 43.9^\circ \blacktriangleleft$

(b) $F_y = F \cos \theta_y$
 $300 \text{ lb} = F(0.72100)$
 $F = 416.09 \text{ lb}$ $F = 416 \text{ lb} \blacktriangleleft$
 $F_x = F \cos \theta_x = 416.09 \text{ lb} \cos 75^\circ$ $F_x = +107.7 \text{ lb} \blacktriangleleft$
 $F_z = F \cos \theta_z = 416.09 \text{ lb} \cos 130^\circ$ $F_z = -267 \text{ lb} \blacktriangleleft$

PROBLEM 2.82

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 55^\circ$ and $\theta_z = 45^\circ$. Knowing that the x component of the force is -500 N, determine (a) the angle θ_x , (b) the other components and the magnitude of the force.

SOLUTION

$$\begin{aligned}\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z &= 1 \\ \cos^2 \theta_x + \cos^2 55^\circ + \cos^2 45^\circ &= 1 \\ \cos \theta_x &= \pm 0.41353\end{aligned}$$

(a) Since $F_x < 0$, we choose $\cos \theta_x = -0.41353$ $\therefore \theta_x = 114.4^\circ \blacktriangleleft$

(b)

$$\begin{aligned}F_x &= F \cos \theta_x \\ -500 \text{ N} &= F(-0.41353) \\ F &= 1209.10 \text{ N} && F = 1209.1 \text{ N} \blacktriangleleft \\ F_y &= F \cos \theta_y = 1209.10 \text{ N} \cos 55^\circ && F_y = +694 \text{ N} \blacktriangleleft \\ F_z &= F \cos \theta_z = 1209.10 \text{ N} \cos 45^\circ && F_z = +855 \text{ N} \blacktriangleleft\end{aligned}$$

PROBLEM 2.83

A force \mathbf{F} of magnitude 230 N acts at the origin of a coordinate system. Knowing that $\theta_x = 32.5^\circ$, $F_y = -60$ N, and $F_z > 0$, determine (a) the components F_x and F_z , (b) the angles θ_y and θ_z .

SOLUTION

(a) We have

$$F_x = F \cos \theta_x = (230 \text{ N}) \cos 32.5^\circ \qquad F_x = -194.0 \text{ N} \blacktriangleleft$$

Then:

$$F_x = 193.980 \text{ N}$$

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(230 \text{ N})^2 = (193.980 \text{ N})^2 + (-60 \text{ N})^2 + F_z^2$$

Hence:

$$F_z = +\sqrt{(230 \text{ N})^2 - (193.980 \text{ N})^2 - (-60 \text{ N})^2} \qquad F_z = 108.0 \text{ N} \blacktriangleleft$$

(b)

$$F_z = 108.036 \text{ N}$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{-60 \text{ N}}{230 \text{ N}} = -0.26087 \qquad \theta_y = 105.1^\circ \blacktriangleleft$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{108.036 \text{ N}}{230 \text{ N}} = 0.46972 \qquad \theta_z = 62.0^\circ \blacktriangleleft$$

PROBLEM 2.84

A force \mathbf{F} of magnitude 210 N acts at the origin of a coordinate system. Knowing that $F_x = 80$ N, $\theta_z = 151.2^\circ$, and $F_y < 0$, determine (a) the components F_y and F_z , (b) the angles θ_x and θ_y .

SOLUTION

$$\begin{aligned} (a) \quad F_z &= F \cos \theta_z = (210 \text{ N}) \cos 151.2^\circ \\ &= -184.024 \text{ N} \qquad F_z = -184.0 \text{ N} \blacktriangleleft \end{aligned}$$

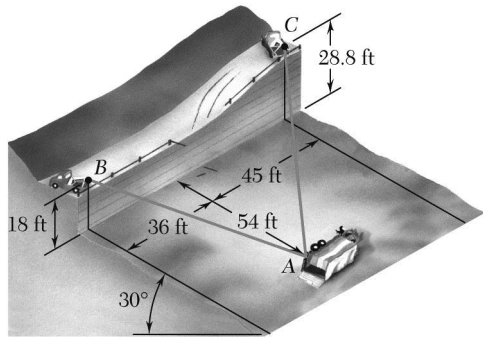
$$\text{Then:} \quad F^2 = F_x^2 + F_y^2 + F_z^2$$

$$\text{So:} \quad (210 \text{ N})^2 = (80 \text{ N})^2 + (F_y)^2 + (184.024 \text{ N})^2$$

$$\begin{aligned} \text{Hence:} \quad F_y &= -\sqrt{(210 \text{ N})^2 - (80 \text{ N})^2 - (184.024 \text{ N})^2} \\ &= -61.929 \text{ N} \qquad F_y = -62.0 \text{ lb} \blacktriangleleft \end{aligned}$$

$$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{80 \text{ N}}{210 \text{ N}} = 0.38095 \qquad \theta_x = 67.6^\circ \blacktriangleleft$$

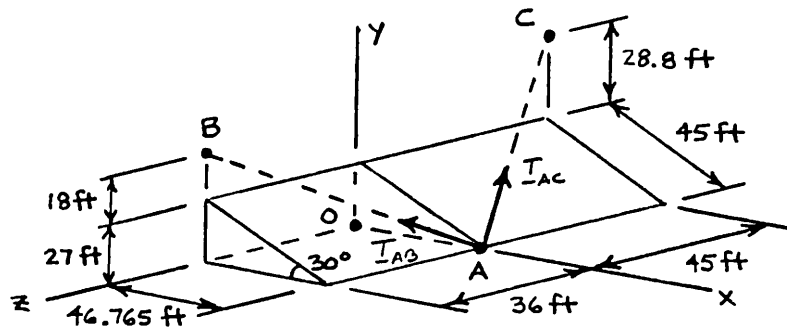
$$\cos \theta_y = \frac{F_y}{F} = \frac{61.929 \text{ N}}{210 \text{ N}} = -0.29490 \qquad \theta_y = 107.2^\circ \blacktriangleleft$$



PROBLEM 2.85

In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in cable AB is 2 kips, determine the components of the force exerted at A by the cable.

SOLUTION



$$AB = 74.216 \text{ ft}$$

$$AC = 85.590 \text{ ft}$$

Cable AB :

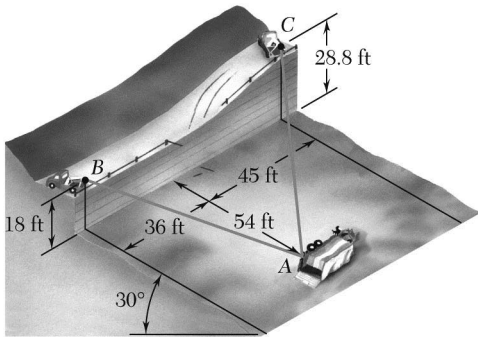
$$\lambda_{AB} = \frac{\overline{AB}}{AB} = \frac{(-46.765 \text{ ft})\mathbf{i} + (45 \text{ ft})\mathbf{j} + (36 \text{ ft})\mathbf{k}}{74.216 \text{ ft}}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = \frac{-46.765\mathbf{i} + 45\mathbf{j} + 36\mathbf{k}}{74.216}$$

$$(T_{AB})_x = -1.260 \text{ kips} \quad \blacktriangleleft$$

$$(T_{AB})_y = +1.213 \text{ kips} \quad \blacktriangleleft$$

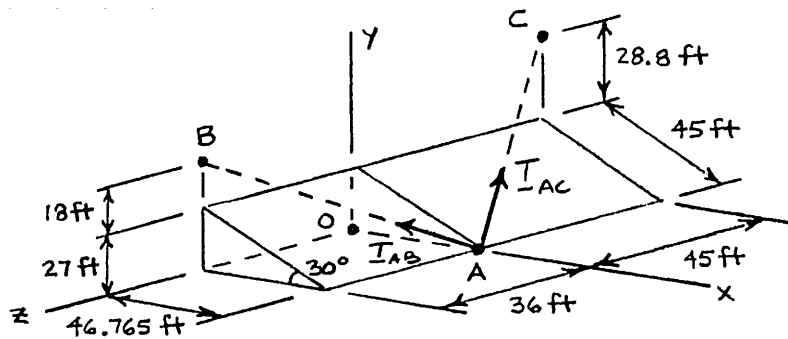
$$(T_{AB})_z = +0.970 \text{ kips} \quad \blacktriangleleft$$



PROBLEM 2.86

In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in cable AC is 1.5 kips, determine the components of the force exerted at A by the cable.

SOLUTION



$$\underline{AB} = 74.216 \text{ ft}$$

$$\underline{AC} = 85.590 \text{ ft}$$

Cable AC :

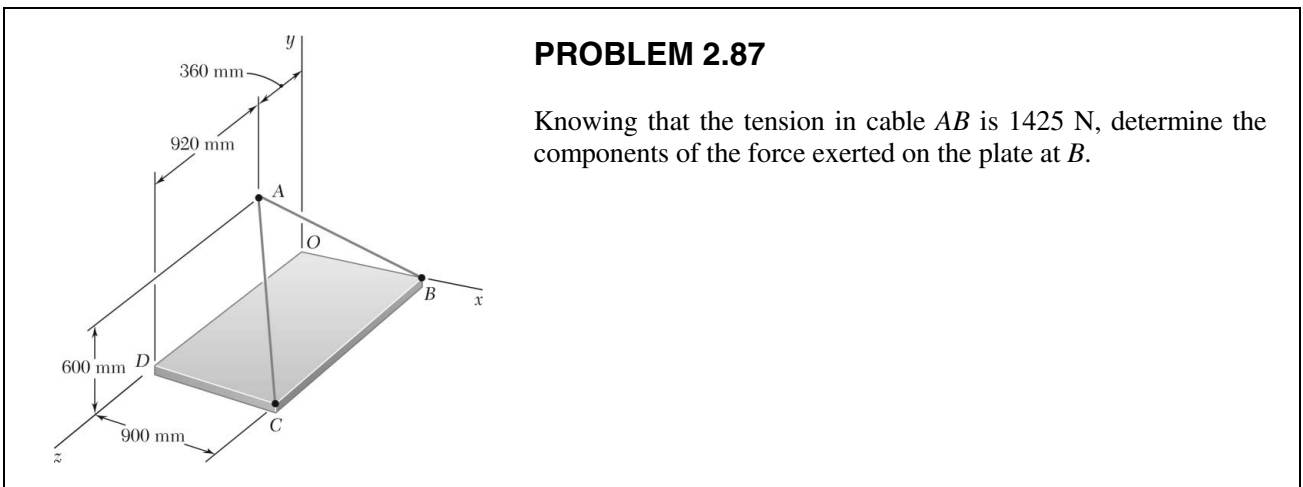
$$\lambda_{AC} = \frac{\overline{AC}}{AC} = \frac{(-46.765 \text{ ft})\mathbf{i} + (55.8 \text{ ft})\mathbf{j} + (-45 \text{ ft})\mathbf{k}}{85.590 \text{ ft}}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = (1.5 \text{ kips}) \frac{-46.765\mathbf{i} + 55.8\mathbf{j} - 45\mathbf{k}}{85.590}$$

$$(T_{AC})_x = -0.820 \text{ kips} \quad \blacktriangleleft$$

$$(T_{AC})_y = +0.978 \text{ kips} \quad \blacktriangleleft$$

$$(T_{AC})_z = -0.789 \text{ kips} \quad \blacktriangleleft$$



PROBLEM 2.87

Knowing that the tension in cable AB is 1425 N, determine the components of the force exerted on the plate at B .

SOLUTION

$$\overline{BA} = -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

$$BA = \sqrt{(900 \text{ mm})^2 + (600 \text{ mm})^2 + (360 \text{ mm})^2}$$

$$= 1140 \text{ mm}$$

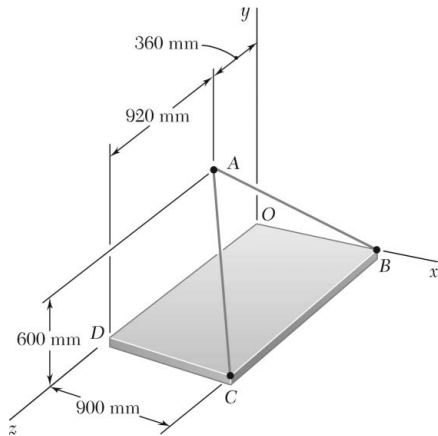
$$\mathbf{T}_{BA} = T_{BA} \lambda_{BA}$$

$$= T_{BA} \frac{\overline{BA}}{BA}$$

$$\mathbf{T}_{BA} = \frac{1425 \text{ N}}{1140 \text{ mm}} [-(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}]$$

$$= -(1125 \text{ N})\mathbf{i} + (750 \text{ N})\mathbf{j} + (450 \text{ N})\mathbf{k}$$

$$(T_{BA})_x = -1125 \text{ N}, \quad (T_{BA})_y = 750 \text{ N}, \quad (T_{BA})_z = 450 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.88

Knowing that the tension in cable AC is 2130 N, determine the components of the force exerted on the plate at C.

SOLUTION

$$\overline{CA} = -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}$$

$$CA = \sqrt{(900 \text{ mm})^2 + (600 \text{ mm})^2 + (920 \text{ mm})^2}$$

$$= 1420 \text{ mm}$$

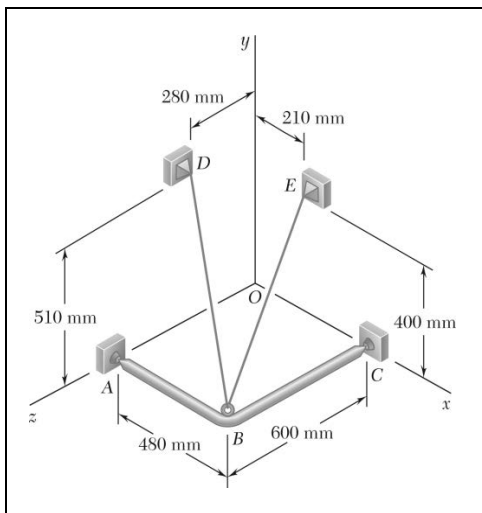
$$\mathbf{T}_{CA} = T_{CA} \lambda_{CA}$$

$$= T_{CA} \frac{\overline{CA}}{CA}$$

$$\mathbf{T}_{CA} = \frac{2130 \text{ N}}{1420 \text{ mm}} [-(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}]$$

$$= -(1350 \text{ N})\mathbf{i} + (900 \text{ N})\mathbf{j} - (1380 \text{ N})\mathbf{k}$$

$$(T_{CA})_x = -1350 \text{ N}, \quad (T_{CA})_y = 900 \text{ N}, \quad (T_{CA})_z = -1380 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.89

A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B . Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D .

SOLUTION

$$\overline{DB} = (480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}$$

$$DB = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm})^2 + (320 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

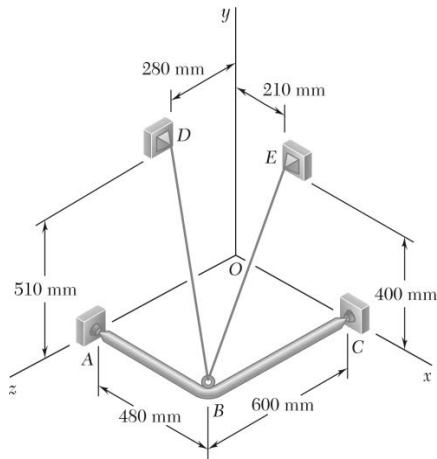
$$\mathbf{F} = F \lambda_{DB}$$

$$= F \frac{\overline{DB}}{DB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}} [(480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}]$$

$$= (240 \text{ N})\mathbf{i} - (255 \text{ N})\mathbf{j} + (160 \text{ N})\mathbf{k}$$

$$F_x = +240 \text{ N}, \quad F_y = -255 \text{ N}, \quad F_z = +160.0 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.90

For the frame and cable of Problem 2.89, determine the components of the force exerted by the cable on the support at E.

PROBLEM 2.89 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B . Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D .

SOLUTION

$$\overline{EB} = (270 \text{ mm})\mathbf{i} - (400 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$EB = \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

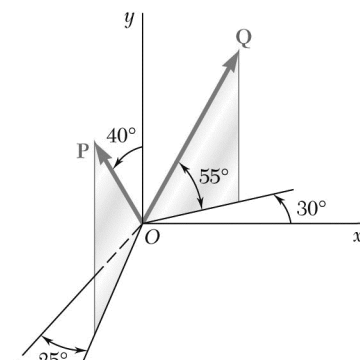
$$\mathbf{F} = F\lambda_{EB}$$

$$= F \frac{\overline{EB}}{EB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}} [(270 \text{ mm})\mathbf{i} - (400 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (135 \text{ N})\mathbf{i} - (200 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k}$$

$$F_x = +135.0 \text{ N}, \quad F_y = -200 \text{ N}, \quad F_z = +300 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.91

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 600 \text{ N}$ and $Q = 450 \text{ N}$.

SOLUTION

$$\begin{aligned} \mathbf{P} &= (600 \text{ N})[\sin 40^\circ \sin 25^\circ \mathbf{i} + \cos 40^\circ \mathbf{j} + \sin 40^\circ \cos 25^\circ \mathbf{k}] \\ &= (162.992 \text{ N})\mathbf{i} + (459.63 \text{ N})\mathbf{j} + (349.54 \text{ N})\mathbf{k} \\ \mathbf{Q} &= (450 \text{ N})[\cos 55^\circ \cos 30^\circ \mathbf{i} + \sin 55^\circ \mathbf{j} - \cos 55^\circ \sin 30^\circ \mathbf{k}] \\ &= (223.53 \text{ N})\mathbf{i} + (368.62 \text{ N})\mathbf{j} - (129.055 \text{ N})\mathbf{k} \\ \mathbf{R} &= \mathbf{P} + \mathbf{Q} \\ &= (386.52 \text{ N})\mathbf{i} + (828.25 \text{ N})\mathbf{j} + (220.49 \text{ N})\mathbf{k} \\ R &= \sqrt{(386.52 \text{ N})^2 + (828.25 \text{ N})^2 + (220.49 \text{ N})^2} \\ &= 940.22 \text{ N} \qquad R = 940 \text{ N} \quad \blacktriangleleft \end{aligned}$$

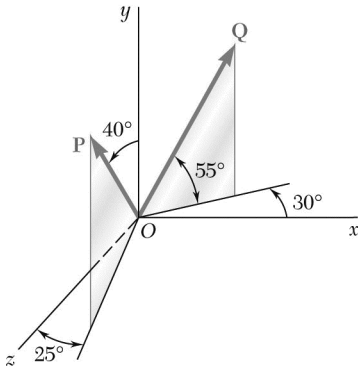
$$\cos \theta_x = \frac{R_x}{R} = \frac{386.52 \text{ N}}{940.22 \text{ N}} \qquad \theta_x = 65.7^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{828.25 \text{ N}}{940.22 \text{ N}} \qquad \theta_y = 28.2^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{220.49 \text{ N}}{940.22 \text{ N}} \qquad \theta_z = 76.4^\circ \quad \blacktriangleleft$$

PROBLEM 2.92

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 450 \text{ N}$ and $Q = 600 \text{ N}$.



SOLUTION

$$\begin{aligned}\mathbf{P} &= (450 \text{ N})[\sin 40^\circ \sin 25^\circ \mathbf{i} + \cos 40^\circ \mathbf{j} + \sin 40^\circ \cos 25^\circ \mathbf{k}] \\ &= (122.244 \text{ N})\mathbf{i} + (344.72 \text{ N})\mathbf{j} + (262.154 \text{ N})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{Q} &= (600 \text{ N})[\cos 55^\circ \cos 30^\circ \mathbf{i} + \sin 55^\circ \mathbf{j} - \cos 55^\circ \sin 30^\circ \mathbf{k}] \\ &= (298.04 \text{ N})\mathbf{i} + (491.49 \text{ N})\mathbf{j} - (172.073 \text{ N})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{R} &= \mathbf{P} + \mathbf{Q} \\ &= (420.28 \text{ N})\mathbf{i} + (836.21 \text{ N})\mathbf{j} + (90.081 \text{ N})\mathbf{k}\end{aligned}$$

$$\begin{aligned}R &= \sqrt{(420.28 \text{ N})^2 + (836.21 \text{ N})^2 + (90.081 \text{ N})^2} \\ &= 940.21 \text{ N}\end{aligned}$$

$$R = 940 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{R_x}{R} = \frac{420.28}{940.21}$$

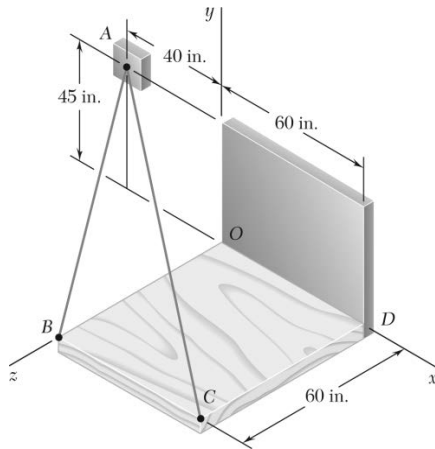
$$\theta_x = 63.4^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{836.21}{940.21}$$

$$\theta_y = 27.2^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{90.081}{940.21}$$

$$\theta_z = 84.5^\circ \quad \blacktriangleleft$$



PROBLEM 2.93

Knowing that the tension is 425 lb in cable AB and 510 lb in cable AC , determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overline{AB} = (40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.}$$

$$\overline{AC} = (100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (425 \text{ lb}) \left[\frac{(40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{85 \text{ in.}} \right]$$

$$\mathbf{T}_{AB} = (200 \text{ lb})\mathbf{i} - (225 \text{ lb})\mathbf{j} + (300 \text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (510 \text{ lb}) \left[\frac{(100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{125 \text{ in.}} \right]$$

$$\mathbf{T}_{AC} = (408 \text{ lb})\mathbf{i} - (183.6 \text{ lb})\mathbf{j} + (244.8 \text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (608 \text{ lb})\mathbf{i} - (408.6 \text{ lb})\mathbf{j} + (544.8 \text{ lb})\mathbf{k}$$

Then:

$$R = 912.92 \text{ lb}$$

$$R = 913 \text{ lb} \quad \blacktriangleleft$$

and

$$\cos \theta_x = \frac{608 \text{ lb}}{912.92 \text{ lb}} = 0.66599$$

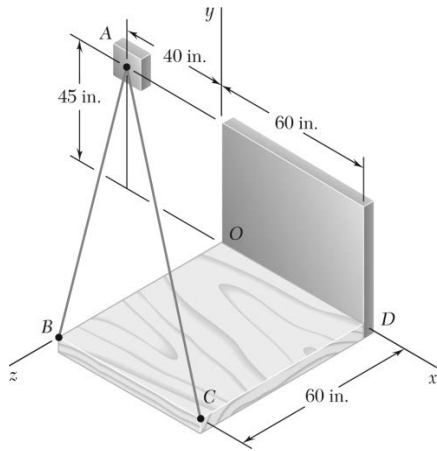
$$\theta_x = 48.2^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{408.6 \text{ lb}}{912.92 \text{ lb}} = -0.44757$$

$$\theta_y = 116.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{544.8 \text{ lb}}{912.92 \text{ lb}} = 0.59677$$

$$\theta_z = 53.4^\circ \quad \blacktriangleleft$$



PROBLEM 2.94

Knowing that the tension is 510 lb in cable AB and 425 lb in cable AC , determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

$$\overline{AB} = (40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.}$$

$$\overline{AC} = (100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (510 \text{ lb}) \left[\frac{(40 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{85 \text{ in.}} \right]$$

$$\mathbf{T}_{AB} = (240 \text{ lb})\mathbf{i} - (270 \text{ lb})\mathbf{j} + (360 \text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (425 \text{ lb}) \left[\frac{(100 \text{ in.})\mathbf{i} - (45 \text{ in.})\mathbf{j} + (60 \text{ in.})\mathbf{k}}{125 \text{ in.}} \right]$$

$$\mathbf{T}_{AC} = (340 \text{ lb})\mathbf{i} - (153 \text{ lb})\mathbf{j} + (204 \text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (580 \text{ lb})\mathbf{i} - (423 \text{ lb})\mathbf{j} + (564 \text{ lb})\mathbf{k}$$

Then:

$$R = 912.92 \text{ lb}$$

$$R = 913 \text{ lb} \quad \blacktriangleleft$$

and

$$\cos \theta_x = \frac{580 \text{ lb}}{912.92 \text{ lb}} = 0.63532$$

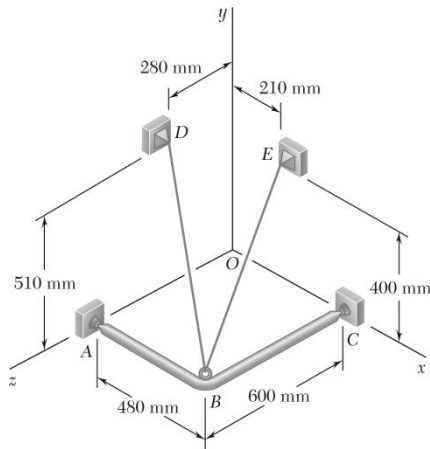
$$\theta_x = 50.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{-423 \text{ lb}}{912.92 \text{ lb}} = -0.46335$$

$$\theta_y = 117.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{564 \text{ lb}}{912.92 \text{ lb}} = 0.61780$$

$$\theta_z = 51.8^\circ \quad \blacktriangleleft$$



PROBLEM 2.95

For the frame of Problem 2.89, determine the magnitude and direction of the resultant of the forces exerted by the cable at B knowing that the tension in the cable is 385 N.

PROBLEM 2.89 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B . Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D .

SOLUTION

$$\overline{BD} = -(480 \text{ mm})\mathbf{i} + (510 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}$$

$$BD = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm})^2 + (320 \text{ mm})^2} = 770 \text{ mm}$$

$$\begin{aligned} \mathbf{F}_{BD} &= T_{BD} \lambda_{BD} = T_{BD} \frac{\overline{BD}}{BD} \\ &= \frac{(385 \text{ N})}{(770 \text{ mm})} [-(480 \text{ mm})\mathbf{i} + (510 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}] \\ &= -(240 \text{ N})\mathbf{i} + (255 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k} \end{aligned}$$

$$\overline{BE} = -(270 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} - (600 \text{ mm})\mathbf{k}$$

$$BE = \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2} = 770 \text{ mm}$$

$$\begin{aligned} \mathbf{F}_{BE} &= T_{BE} \lambda_{BE} = T_{BE} \frac{\overline{BE}}{BE} \\ &= \frac{(385 \text{ N})}{(770 \text{ mm})} [-(270 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} - (600 \text{ mm})\mathbf{k}] \\ &= -(135 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k} \end{aligned}$$

$$\mathbf{R} = \mathbf{F}_{BD} + \mathbf{F}_{BE} = -(375 \text{ N})\mathbf{i} + (455 \text{ N})\mathbf{j} - (460 \text{ N})\mathbf{k}$$

$$R = \sqrt{(375 \text{ N})^2 + (455 \text{ N})^2 + (460 \text{ N})^2} = 747.83 \text{ N}$$

$$R = 748 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{-375 \text{ N}}{747.83 \text{ N}}$$

$$\theta_x = 120.1^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{455 \text{ N}}{747.83 \text{ N}}$$

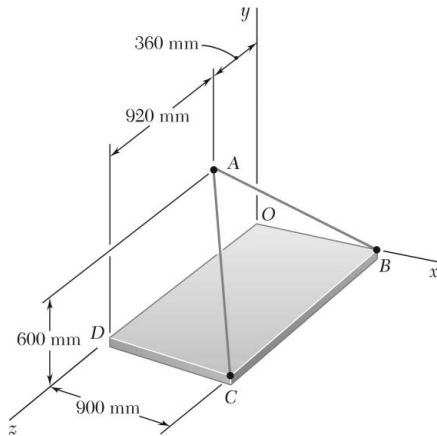
$$\theta_y = 52.5^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{-460 \text{ N}}{747.83 \text{ N}}$$

$$\theta_z = 128.0^\circ \quad \blacktriangleleft$$

PROBLEM 2.96

For the cables of Problem 2.87, knowing that the tension is 1425 N in cable AB and 2130 N in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.



SOLUTION

$$T_{AB} = -T_{BA} \quad (\text{use results of Problem 2.87})$$

$$(T_{AB})_x = +1125 \text{ N} \quad (T_{AB})_y = -750 \text{ N} \quad (T_{AB})_z = -450 \text{ N}$$

$$T_{AC} = -T_{CA} \quad (\text{use results of Problem 2.88})$$

$$(T_{AC})_x = +1350 \text{ N} \quad (T_{AC})_y = -900 \text{ N} \quad (T_{AC})_z = +1380 \text{ N}$$

Resultant:

$$R_x = \Sigma F_x = +1125 + 1350 = +2475 \text{ N}$$

$$R_y = \Sigma F_y = -750 - 900 = -1650 \text{ N}$$

$$R_z = \Sigma F_z = -450 + 1380 = +930 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

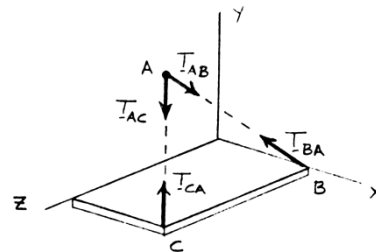
$$= \sqrt{(+2475)^2 + (-1650)^2 + (+930)^2}$$

$$= 3116.6 \text{ N}$$

$$\cos \theta_x = \frac{R_x}{R} = \frac{+2475}{3116.6}$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{-1650}{3116.6}$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{+930}{3116.6}$$

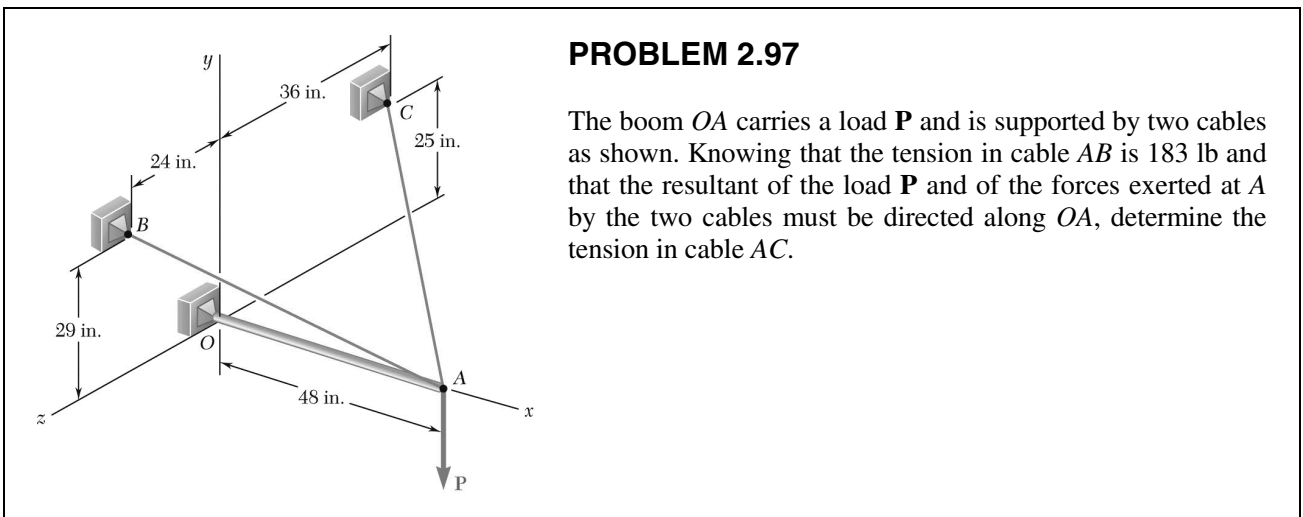


$$R = 3120 \text{ N} \quad \blacktriangleleft$$

$$\theta_x = 37.4^\circ \quad \blacktriangleleft$$

$$\theta_y = 122.0^\circ \quad \blacktriangleleft$$

$$\theta_z = 72.6^\circ \quad \blacktriangleleft$$



PROBLEM 2.97

The boom OA carries a load P and is supported by two cables as shown. Knowing that the tension in cable AB is 183 lb and that the resultant of the load P and of the forces exerted at A by the two cables must be directed along OA , determine the tension in cable AC .

SOLUTION

Cable AB : $T_{AB} = 183 \text{ lb}$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (183 \text{ lb}) \frac{(-48 \text{ in.})\mathbf{i} + (29 \text{ in.})\mathbf{j} + (24 \text{ in.})\mathbf{k}}{61 \text{ in.}}$$

$$\mathbf{T}_{AB} = -(144 \text{ lb})\mathbf{i} + (87 \text{ lb})\mathbf{j} + (72 \text{ lb})\mathbf{k}$$

Cable AC : $\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(-48 \text{ in.})\mathbf{i} + (25 \text{ in.})\mathbf{j} + (-36 \text{ in.})\mathbf{k}}{65 \text{ in.}}$

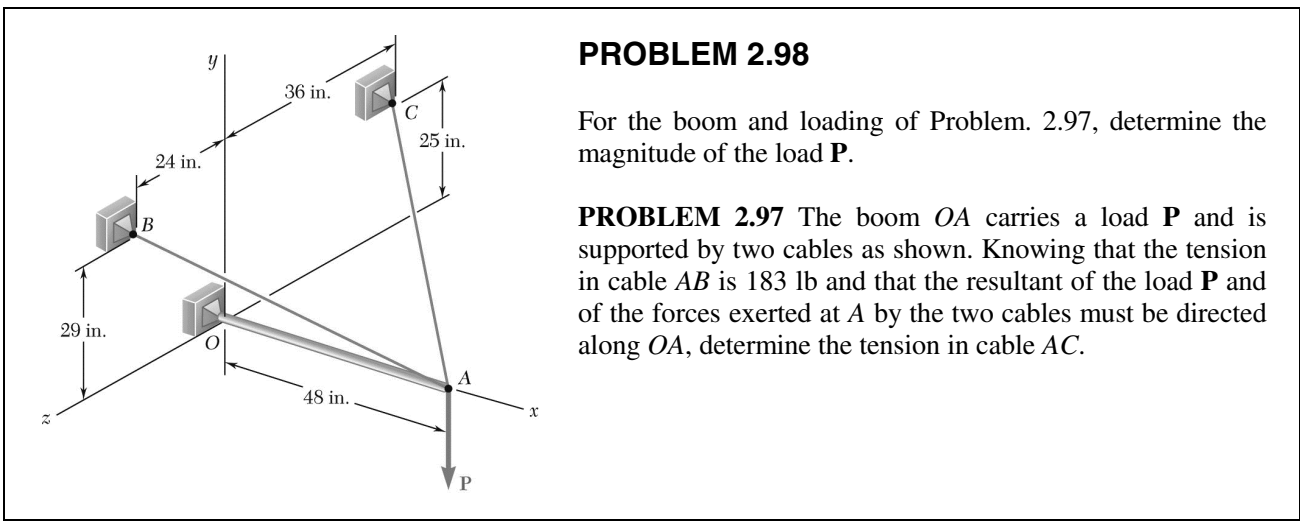
$$\mathbf{T}_{AC} = -\frac{48}{65} T_{AC} \mathbf{i} + \frac{25}{65} T_{AC} \mathbf{j} - \frac{36}{65} T_{AC} \mathbf{k}$$

Load P : $\mathbf{P} = P \mathbf{j}$

For resultant to be directed along OA , i.e., x -axis

$$R_z = 0: \quad \Sigma F_z = (72 \text{ lb}) - \frac{36}{65} T'_{AC} = 0 \qquad T_{AC} = 130.0 \text{ lb} \quad \blacktriangleleft$$

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PROBLEM 2.98

For the boom and loading of Problem. 2.97, determine the magnitude of the load **P**.

PROBLEM 2.97 The boom *OA* carries a load **P** and is supported by two cables as shown. Knowing that the tension in cable *AB* is 183 lb and that the resultant of the load **P** and of the forces exerted at *A* by the two cables must be directed along *OA*, determine the tension in cable *AC*.

SOLUTION

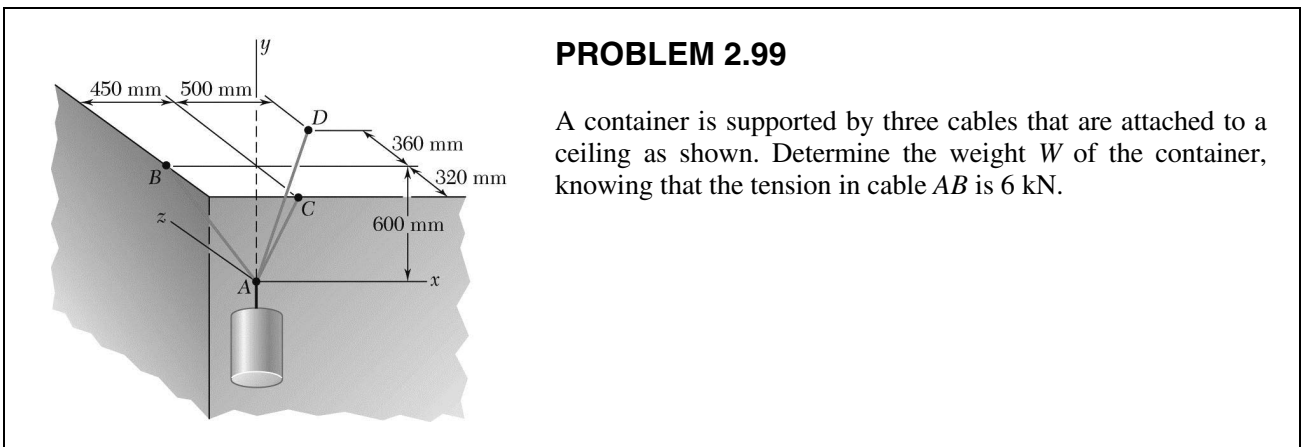
See Problem 2.97. Since resultant must be directed along *OA*, i.e., the *x*-axis, we write

$$R_y = 0: \quad \Sigma F_y = (87 \text{ lb}) + \frac{25}{65} T_{AC} - P = 0$$

$T_{AC} = 130.0 \text{ lb}$ from Problem 2.97.

Then $(87 \text{ lb}) + \frac{25}{65} (130.0 \text{ lb}) - P = 0$ $P = 137.0 \text{ lb} \blacktriangleleft$

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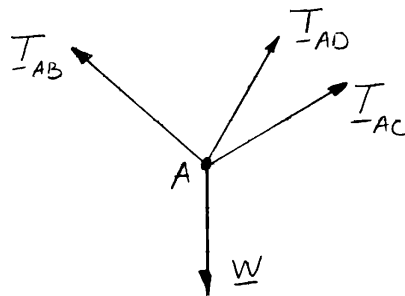


PROBLEM 2.99

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container, knowing that the tension in cable AB is 6 kN.

SOLUTION

Free-Body Diagram at A:



The forces applied at A are:

\mathbf{T}_{AB} , \mathbf{T}_{AC} , \mathbf{T}_{AD} , and W

where $\mathbf{W} = W\mathbf{j}$. To express the other forces in terms of the unit vectors \mathbf{i} , \mathbf{j} , \mathbf{k} , we write

$$\overline{AB} = -(450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} \quad AB = 750 \text{ mm}$$

$$\overline{AC} = +(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 680 \text{ mm}$$

$$\overline{AD} = +(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AD = 860 \text{ mm}$$

and

$$\begin{aligned} \mathbf{T}_{AB} &= \lambda_{AB}T_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \frac{-(450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}}{750 \text{ mm}} \\ &= \left(-\frac{45}{75}\mathbf{i} + \frac{60}{75}\mathbf{j} \right) T_{AB} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AC} &= \lambda_{AC}T_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}}{680 \text{ mm}} \\ &= \left(\frac{60}{68}\mathbf{j} - \frac{32}{68}\mathbf{k} \right) T_{AC} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AD} &= \lambda_{AD}T_{AD} = T_{AD} \frac{\overline{AD}}{AD} = T_{AD} \frac{(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}}{860 \text{ mm}} \\ &= \left(\frac{50}{86}\mathbf{i} + \frac{60}{86}\mathbf{j} + \frac{36}{86}\mathbf{k} \right) T_{AD} \end{aligned}$$

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PROBLEM 2.99 (Continued)

Equilibrium condition: $\Sigma F = 0: \therefore \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

Substituting the expressions obtained for \mathbf{T}_{AB} , \mathbf{T}_{AC} , and \mathbf{T}_{AD} ; factoring \mathbf{i} , \mathbf{j} , and \mathbf{k} ; and equating each of the coefficients to zero gives the following equations:

From \mathbf{i} :
$$-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0 \quad (1)$$

From \mathbf{j} :
$$\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0 \quad (2)$$

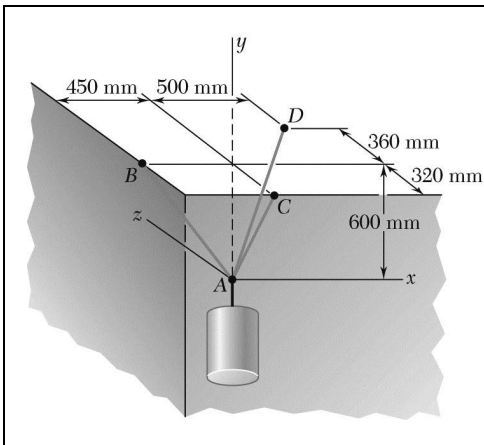
From \mathbf{k} :
$$-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0 \quad (3)$$

Setting $T_{AB} = 6 \text{ kN}$ in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 6.1920 \text{ kN}$$

$$T_{AD} = 5.5080 \text{ kN}$$

$$W = 13.98 \text{ kN} \quad \blacktriangleleft$$



PROBLEM 2.100

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container, knowing that the tension in cable AD is 4.3 kN.

SOLUTION

See Problem 2.99 for the figure and analysis leading to the following set of linear algebraic equations:

$$-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0 \quad (1)$$

$$\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0 \quad (2)$$

$$-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0 \quad (3)$$

Setting $T_{AD} = 4.3$ kN into the above equations gives

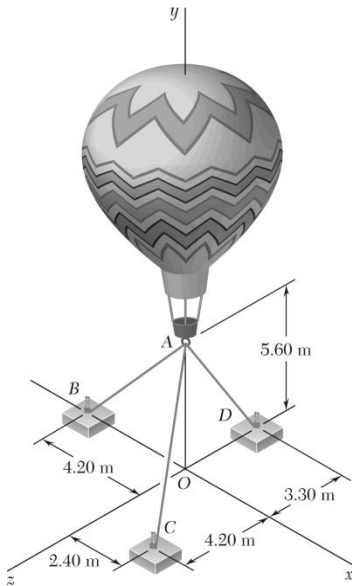
$$T_{AB} = 4.1667 \text{ kN}$$

$$T_{AC} = 3.8250 \text{ kN}$$

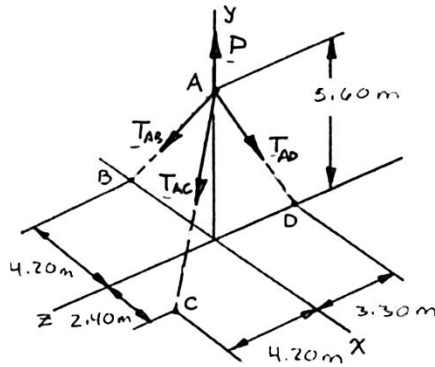
$$W = 9.71 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 2.101

Three cables are used to tether a balloon as shown. Determine the vertical force \mathbf{P} exerted by the balloon at A knowing that the tension in cable AD is 481 N.



SOLUTION



The forces applied at A are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}, \text{ and } \mathbf{P}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, we write

$$\begin{aligned} \overline{AB} &= -(4.20 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} & AB &= 7.00 \text{ m} \\ \overline{AC} &= (2.40 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} + (4.20 \text{ m})\mathbf{k} & AC &= 7.40 \text{ m} \\ \overline{AD} &= -(5.60 \text{ m})\mathbf{j} - (3.30 \text{ m})\mathbf{k} & AD &= 6.50 \text{ m} \end{aligned}$$

and

$$\begin{aligned} \mathbf{T}_{AB} &= T_{AB}\lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB} \\ \mathbf{T}_{AC} &= T_{AC}\lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (0.32432\mathbf{i} - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC} \\ \mathbf{T}_{AD} &= T_{AD}\lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD} \end{aligned}$$

PROBLEM 2.101 (Continued)

Equilibrium condition: $\Sigma F = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for \mathbf{T}_{AB} , \mathbf{T}_{AC} , and \mathbf{T}_{AD} and factoring \mathbf{i} , \mathbf{j} , and \mathbf{k} :

$$\begin{aligned} &(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j} \\ &+ (0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0 \end{aligned}$$

Equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

Setting $T_{AD} = 481 \text{ N}$ in (2) and (3), and solving the resulting set of equations gives

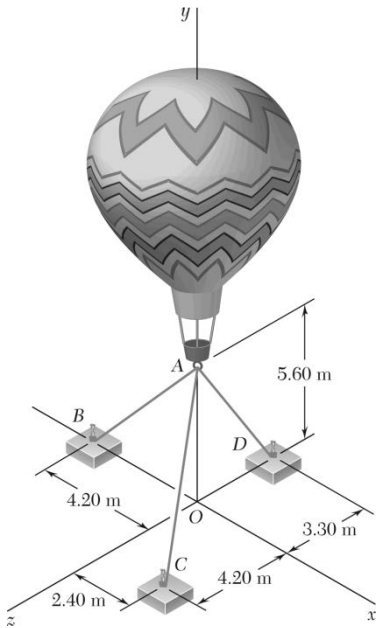
$$T_{AC} = 430.26 \text{ N}$$

$$T_{AD} = 232.57 \text{ N}$$

$$\mathbf{P} = 926 \text{ N} \uparrow \leftarrow$$

PROBLEM 2.102

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.



SOLUTION

See Problem 2.101 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

From Eq. (1): $T_{AB} = 0.54053T_{AC}$

From Eq. (3): $T_{AD} = 1.11795T_{AC}$

Substituting for T_{AB} and T_{AD} in terms of T_{AC} into Eq. (2) gives

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

$$2.1523T_{AC} = P; \quad P = 800 \text{ N}$$

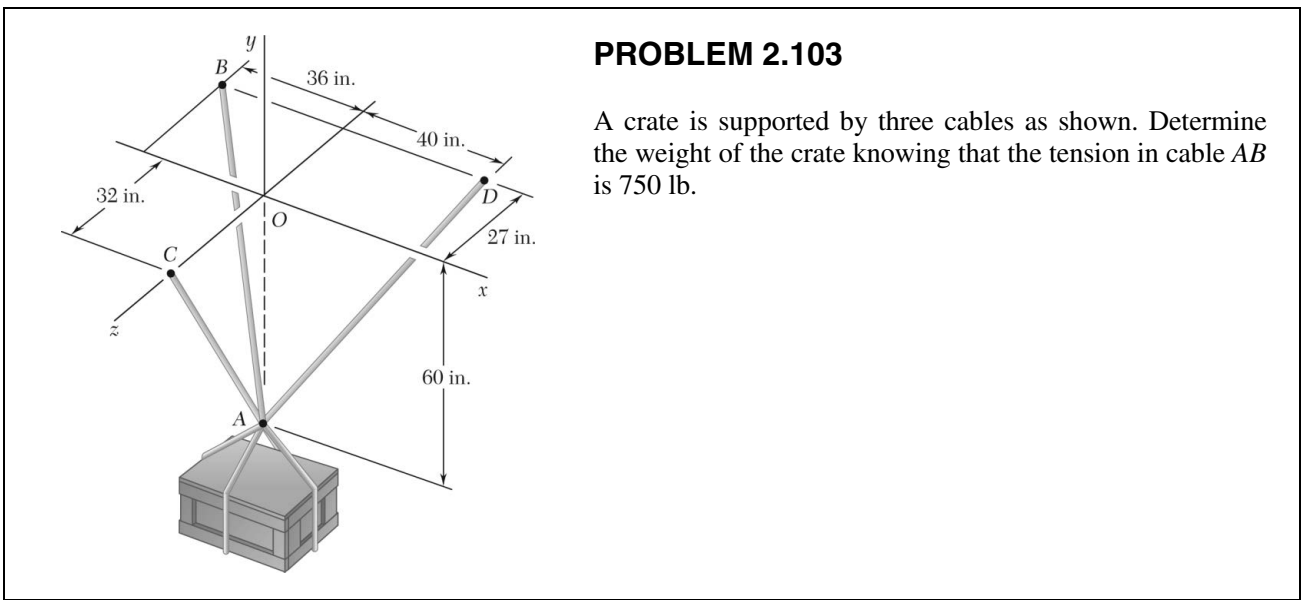
$$\begin{aligned} T_{AC} &= \frac{800 \text{ N}}{2.1523} \\ &= 371.69 \text{ N} \end{aligned}$$

Substituting into expressions for T_{AB} and T_{AD} gives

$$T_{AB} = 0.54053(371.69 \text{ N})$$

$$T_{AD} = 1.11795(371.69 \text{ N})$$

$$T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.103

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AB is 750 lb.

SOLUTION

The forces applied at A are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD} \text{ and } \mathbf{W}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, we write

$$\overline{AB} = -(36 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$

$$AB = 75 \text{ in.}$$

$$\overline{AC} = (60 \text{ in.})\mathbf{j} + (32 \text{ in.})\mathbf{k}$$

$$AC = 68 \text{ in.}$$

$$\overline{AD} = (40 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$

$$AD = 77 \text{ in.}$$

and

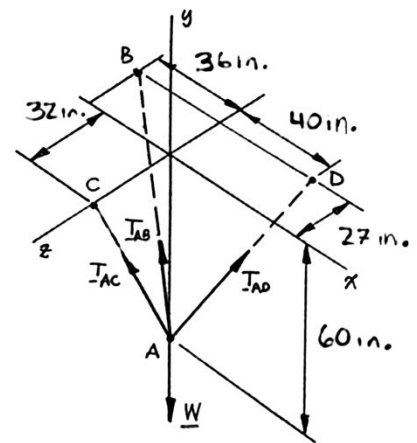
$$\begin{aligned} \mathbf{T}_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} \\ &= (-0.48\mathbf{i} + 0.8\mathbf{j} - 0.36\mathbf{k})T_{AB} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} \\ &= (0.88235\mathbf{j} + 0.47059\mathbf{k})T_{AC} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} \\ &= (0.51948\mathbf{i} + 0.77922\mathbf{j} - 0.35065\mathbf{k})T_{AD} \end{aligned}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$



PROBLEM 2.103 (Continued)

Substituting the expressions obtained for T_{AB} , T_{AC} , and T_{AD} and factoring \mathbf{i} , \mathbf{j} , and \mathbf{k} :

$$\begin{aligned} &(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j} \\ &+ (-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0 \end{aligned}$$

Equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} :

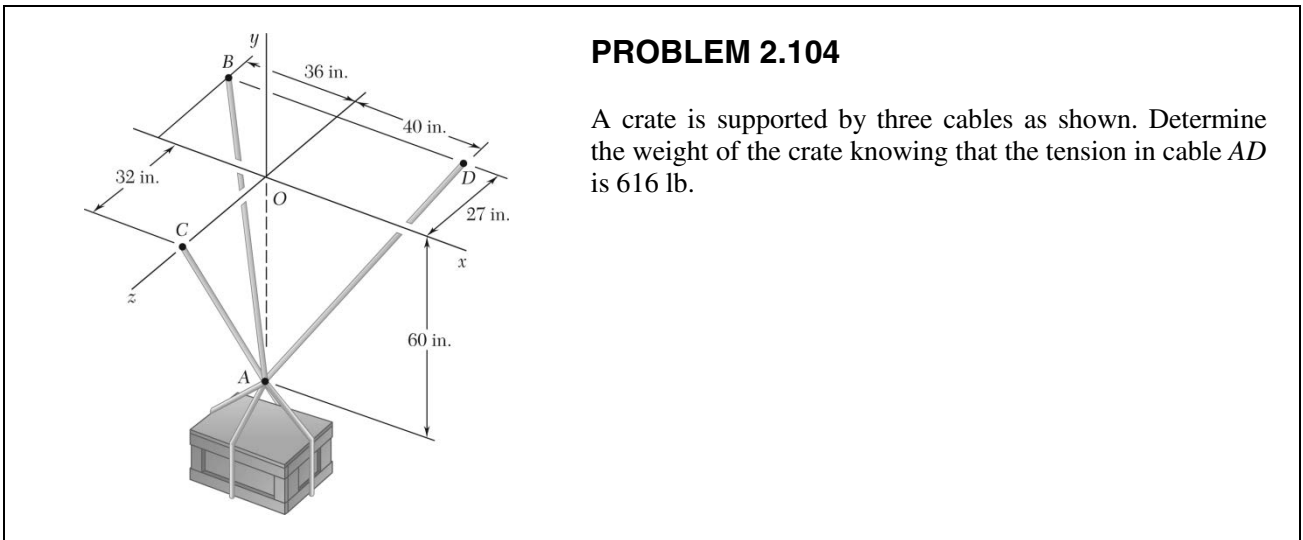
$$\begin{aligned} -0.48T_{AB} + 0.51948T_{AD} &= 0 \\ 0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W &= 0 \\ -0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} &= 0 \end{aligned}$$

Substituting $T_{AB} = 750$ lb in Equations (1), (2), and (3) and solving the resulting set of equations, using conventional algorithms for solving linear algebraic equations, gives:

$$T_{AC} = 1090.1 \text{ lb}$$

$$T_{AD} = 693 \text{ lb}$$

$$W = 2100 \text{ lb} \blacktriangleleft$$



PROBLEM 2.104

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AD is 616 lb.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0 \tag{1}$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \tag{2}$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \tag{3}$$

Substituting $T_{AD} = 616$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 667.67 \text{ lb}$$

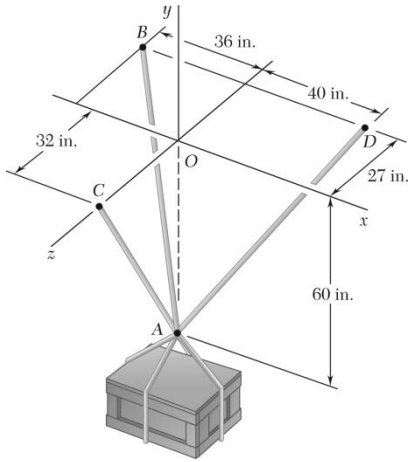
$$T_{AC} = 969.00 \text{ lb}$$

$W = 1868 \text{ lb} \blacktriangleleft$

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PROBLEM 2.105

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AC is 544 lb.



SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0 \quad (1)$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \quad (2)$$

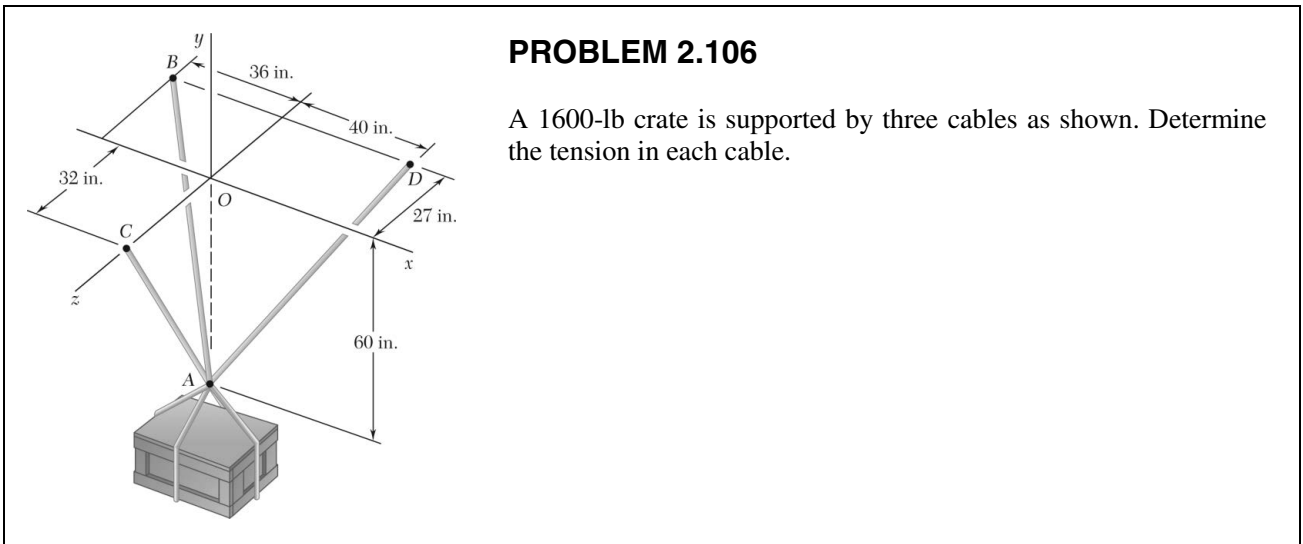
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \quad (3)$$

Substituting $T_{AC} = 544$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 374.27 \text{ lb}$$

$$T_{AD} = 345.82 \text{ lb}$$

$$W = 1049 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.106

A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0 \tag{1}$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \tag{2}$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \tag{3}$$

Substituting $W = 1600$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$$T_{AB} = 571 \text{ lb} \blacktriangleleft$$

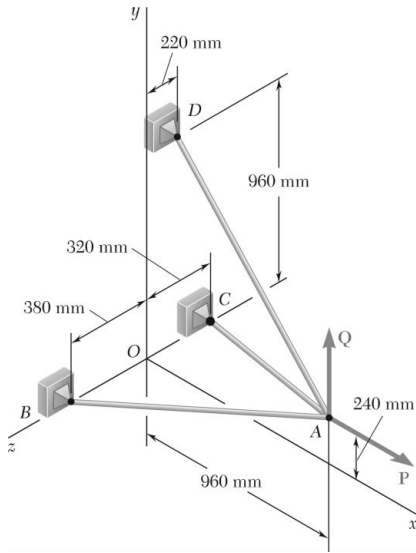
$$T_{AC} = 830 \text{ lb} \blacktriangleleft$$

$$T_{AD} = 528 \text{ lb} \blacktriangleleft$$

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PROBLEM 2.107

Three cables are connected at A, where the forces \mathbf{P} and \mathbf{Q} are applied as shown. Knowing that $Q=0$, find the value of P for which the tension in cable AD is 305 N.



SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P\mathbf{i}$$

$$\overline{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overline{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overline{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \left(-\frac{48}{53} \mathbf{i} - \frac{12}{53} \mathbf{j} + \frac{19}{53} \mathbf{k} \right)$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \left(-\frac{12}{13} \mathbf{i} - \frac{3}{13} \mathbf{j} - \frac{4}{13} \mathbf{k} \right)$$

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD} \lambda_{AD} = \frac{305 \text{ N}}{1220 \text{ mm}} [(-960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k}] \\ &= -(240 \text{ N})\mathbf{i} + (180 \text{ N})\mathbf{j} - (55 \text{ N})\mathbf{k} \end{aligned}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring \mathbf{i} , \mathbf{j} , \mathbf{k} , and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad P = \frac{48}{53} T_{AB} + \frac{12}{13} T_{AC} + 240 \text{ N} \quad (1)$$

$$\mathbf{j}: \quad \frac{12}{53} T_{AB} + \frac{3}{13} T_{AC} = 180 \text{ N} \quad (2)$$

$$\mathbf{k}: \quad \frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} = 55 \text{ N} \quad (3)$$

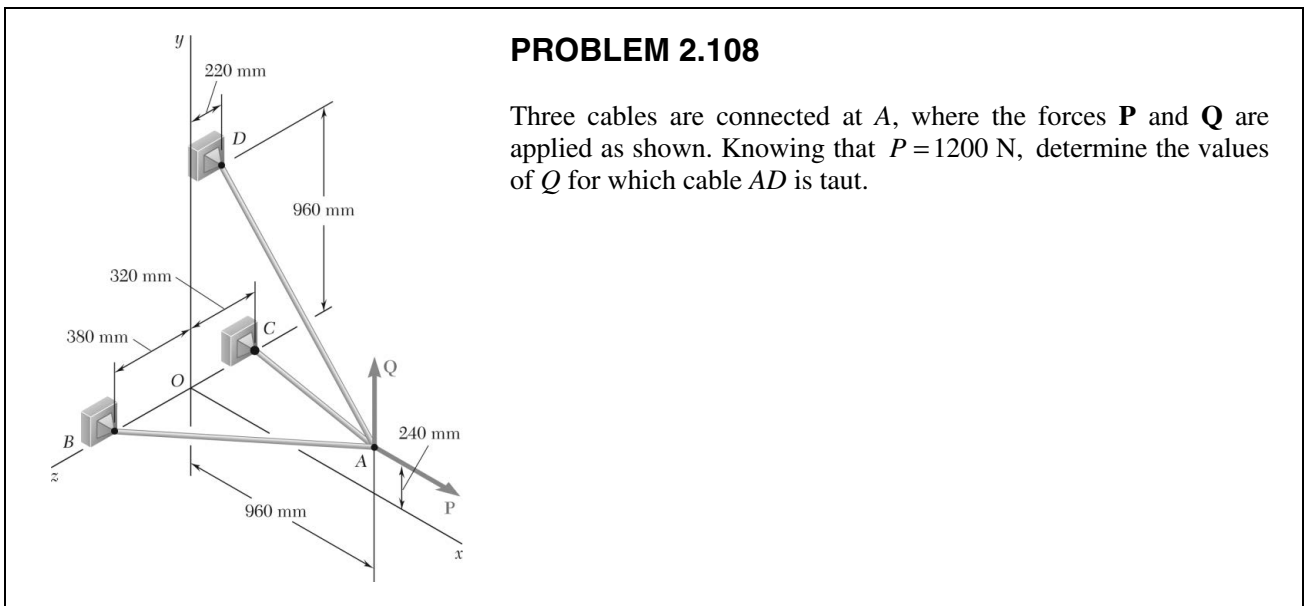
Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71 \text{ N}$$

$$T_{AC} = 341.71 \text{ N}$$

$$P = 960 \text{ N} \quad \blacktriangleleft$$

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PROBLEM 2.108

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $P = 1200 \text{ N}$, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\Sigma \mathbf{F}_A = 0$: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200 \text{ N})\mathbf{i} = 0$

$$\overline{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overline{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k} \right) T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k} \right) T_{AC}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring **i**, **j**, **k**, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200 \text{ N} = 0 \quad (1)$$

$$\mathbf{j}: -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \quad (2)$$

$$\mathbf{k}: \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \quad (3)$$

Solving the resulting system of linear equations using conventional algorithms gives:

$$T_{AB} = 605.71 \text{ N}$$

$$T_{AC} = 705.71 \text{ N}$$

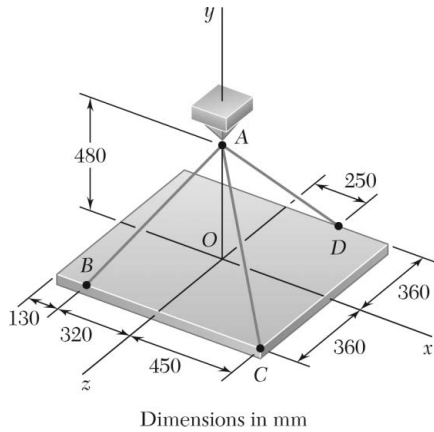
$$Q = 300.00 \text{ N}$$

$$0 \leq Q < 300 \text{ N} \blacktriangleleft$$

Note: This solution assumes that Q is directed upward as shown ($Q \geq 0$), if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460 \text{ N}$.

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PROBLEM 2.109



A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.

SOLUTION

We note that the weight of the plate is equal in magnitude to the force \mathbf{P} exerted by the support on Point A.

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overline{AB} = -(320 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AB = 680 \text{ mm}$$

$$\overline{AC} = (450 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AC = 750 \text{ mm}$$

$$\overline{AD} = (250 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad AD = 650 \text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = \left(-\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k} \right) T_{AB}$$

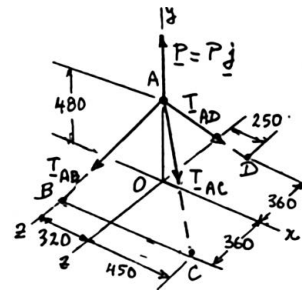
$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = \left(\frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k} \right) T_{AD}$$

Substituting into the Eq. $\Sigma \mathbf{F} = 0$ and factoring \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$\begin{aligned} & \left(-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} \right) \mathbf{i} \\ & + \left(-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P \right) \mathbf{j} \\ & + \left(\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} \right) \mathbf{k} = 0 \end{aligned}$$

Free Body A:



PROBLEM 2.109 (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Making $T_{AC} = 60$ N in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36 \text{ N} + \frac{5}{13}T_{AD} = 0 \quad (1')$$

$$\frac{9}{17}T_{AB} + 28.8 \text{ N} - \frac{7.2}{13}T_{AD} = 0 \quad (3')$$

Multiply (1') by 9, (3') by 8, and add:

$$554.4 \text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0 \text{ N}$$

Substitute into (1') and solve for T_{AB} :

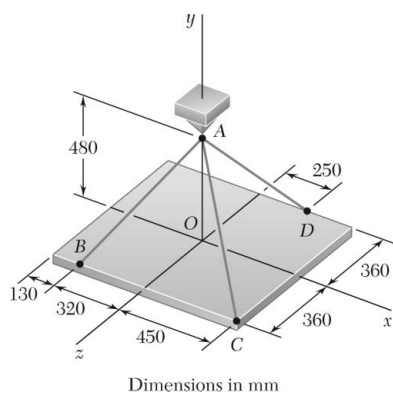
$$T_{AB} = \frac{17}{8} \left(36 + \frac{5}{13} \times 572 \right) \quad T_{AB} = 544.0 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P :

$$\begin{aligned} P &= \frac{12}{17}(544 \text{ N}) + 0.64(60 \text{ N}) + \frac{9.6}{13}(572 \text{ N}) \\ &= 844.8 \text{ N} \end{aligned}$$

Weight of plate = $P = 845$ N ◀

PROBLEM 2.110



A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 520 N, determine the weight of the plate.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$-\frac{12}{17}T_{AB} + 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \quad (2)$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Making $T_{AD} = 520$ N in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + 200 \text{ N} = 0 \quad (1')$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - 288 \text{ N} = 0 \quad (3')$$

Multiply (1') by 9, (3') by 8, and add:

$$9.24T_{AC} - 504 \text{ N} = 0 \quad T_{AC} = 54.5455 \text{ N}$$

Substitute into (1') and solve for T_{AB} :

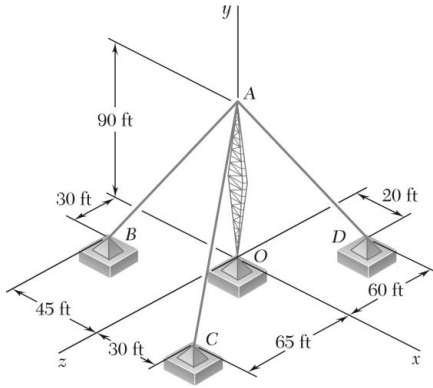
$$T_{AB} = \frac{17}{8}(0.6 \times 54.5455 + 200) \quad T_{AB} = 494.545 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P :

$$\begin{aligned} P &= \frac{12}{17}(494.545 \text{ N}) + 0.64(54.5455 \text{ N}) + \frac{9.6}{13}(520 \text{ N}) \\ &= 768.00 \text{ N} \end{aligned}$$

Weight of plate = $P = 768$ N ◀

PROBLEM 2.111



A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AB is 630 lb, determine the vertical force \mathbf{P} exerted by the tower on the pin at A.

SOLUTION

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

$$\overline{AB} = -45\mathbf{i} - 90\mathbf{j} + 30\mathbf{k} \quad AB = 105 \text{ ft}$$

$$\overline{AC} = 30\mathbf{i} - 90\mathbf{j} + 65\mathbf{k} \quad AC = 115 \text{ ft}$$

$$\overline{AD} = 20\mathbf{i} - 90\mathbf{j} - 60\mathbf{k} \quad AD = 110 \text{ ft}$$

We write

$$\begin{aligned} \mathbf{T}_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} \\ &= \left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k} \right) T_{AB} \end{aligned}$$

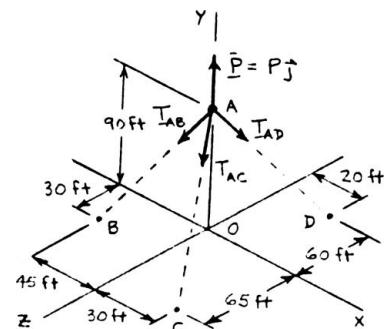
$$\begin{aligned} \mathbf{T}_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} \\ &= \left(\frac{6}{23}\mathbf{i} - \frac{18}{23}\mathbf{j} + \frac{13}{23}\mathbf{k} \right) T_{AC} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AD} &= T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} \\ &= \left(\frac{2}{11}\mathbf{i} - \frac{9}{11}\mathbf{j} - \frac{6}{11}\mathbf{k} \right) T_{AD} \end{aligned}$$

Substituting into the Eq. $\Sigma \mathbf{F} = 0$ and factoring \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$\begin{aligned} &\left(-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} \right) \mathbf{i} \\ &+ \left(-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P \right) \mathbf{j} \\ &+ \left(\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} \right) \mathbf{k} = 0 \end{aligned}$$

Free Body A:



PROBLEM 2.111 (Continued)

Setting the coefficients of **i**, **j**, **k**, equal to zero:

$$\mathbf{i}: \quad -\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3)$$

Set $T_{AB} = 630$ lb in Eqs. (1) – (3):

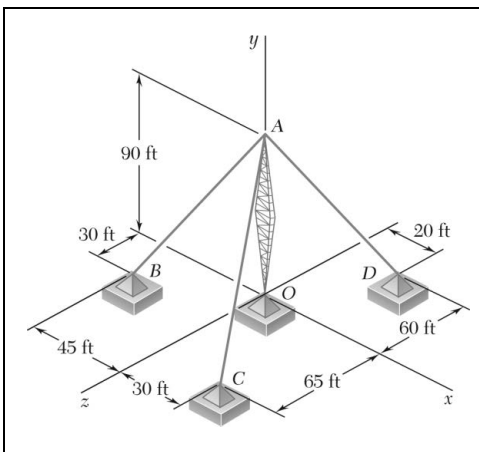
$$-270 \text{ lb} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1')$$

$$-540 \text{ lb} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \quad (2')$$

$$180 \text{ lb} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3')$$

Solving, $T_{AC} = 467.42$ lb $T_{AD} = 814.35$ lb $P = 1572.10$ lb

$P = 1572$ lb ◀



PROBLEM 2.112

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B , C , and D . If the tension in wire AC is 920 lb, determine the vertical force \mathbf{P} exerted by the tower on the pin at A .

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1)$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \quad (2)$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3)$$

Substituting for $T_{AC} = 920$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB} + 240 \text{ lb} + \frac{2}{11}T_{AD} = 0 \quad (1')$$

$$-\frac{6}{7}T_{AB} - 720 \text{ lb} - \frac{9}{11}T_{AD} + P = 0 \quad (2')$$

$$\frac{2}{7}T_{AB} + 520 \text{ lb} - \frac{6}{11}T_{AD} = 0 \quad (3')$$

Solving,

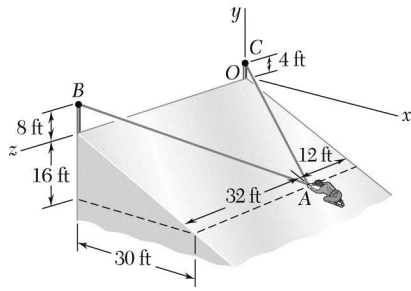
$$T_{AB} = 1240.00 \text{ lb}$$

$$T_{AD} = 1602.86 \text{ lb}$$

$$P = 3094.3 \text{ lb}$$

$$P = 3090 \text{ lb} \blacktriangleleft$$

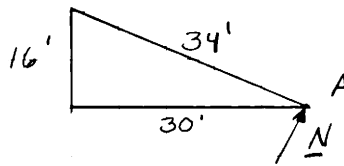
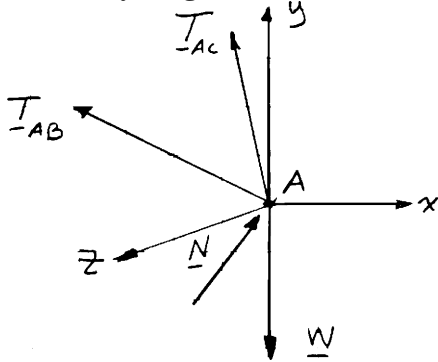
PROBLEM 2.113



In trying to move across a slippery icy surface, a 180-lb man uses two ropes AB and AC . Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Free-Body Diagram at A



$$\mathbf{N} = N \left(\frac{16}{34} \mathbf{i} + \frac{30}{34} \mathbf{j} \right)$$

$$\text{and } \mathbf{W} = W \mathbf{j} = -(180 \text{ lb}) \mathbf{j}$$

$$\begin{aligned} \mathbf{T}_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(-30 \text{ ft}) \mathbf{i} + (20 \text{ ft}) \mathbf{j} - (12 \text{ ft}) \mathbf{k}}{38 \text{ ft}} \\ &= T_{AC} \left(-\frac{15}{19} \mathbf{i} + \frac{10}{19} \mathbf{j} - \frac{6}{19} \mathbf{k} \right) \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \frac{(-30 \text{ ft}) \mathbf{i} + (24 \text{ ft}) \mathbf{j} + (32 \text{ ft}) \mathbf{k}}{50 \text{ ft}} \\ &= T_{AB} \left(-\frac{15}{25} \mathbf{i} + \frac{12}{25} \mathbf{j} + \frac{16}{25} \mathbf{k} \right) \end{aligned}$$

Equilibrium condition: $\Sigma \mathbf{F} = 0$

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0$$

PROBLEM 2.113 (Continued)

Substituting the expressions obtained for T_{AB} , T_{AC} , N , and W ; factoring \mathbf{i} , \mathbf{j} , and \mathbf{k} ; and equating each of the coefficients to zero gives the following equations:

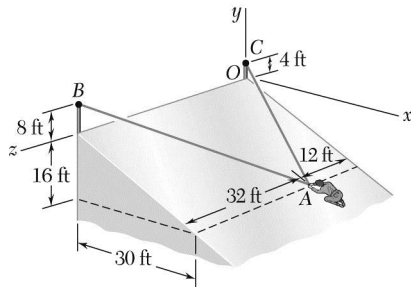
$$\text{From } \mathbf{i}: \quad -\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \quad (1)$$

$$\text{From } \mathbf{j}: \quad \frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (180 \text{ lb}) = 0 \quad (2)$$

$$\text{From } \mathbf{k}: \quad \frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} = 0 \quad (3)$$

Solving the resulting set of equations gives:

$$T_{AB} = 31.7 \text{ lb}; \quad T_{AC} = 64.3 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.114

Solve Problem 2.113, assuming that a friend is helping the man at A by pulling on him with a force $\mathbf{P} = -(60 \text{ lb})\mathbf{k}$.

PROBLEM 2.113 In trying to move across a slippery icy surface, a 180-lb man uses two ropes AB and AC. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Refer to Problem 2.113 for the figure and analysis leading to the following set of equations, Equation (3) being modified to include the additional force $\mathbf{P} = -(60 \text{ lb})\mathbf{k}$.

$$-\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \quad (1)$$

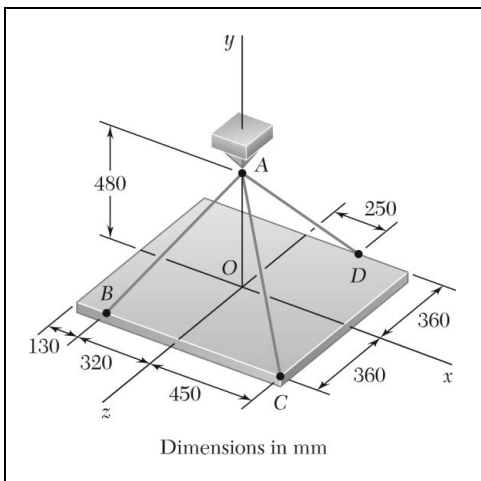
$$\frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (180 \text{ lb}) = 0 \quad (2)$$

$$\frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} - (60 \text{ lb}) = 0 \quad (3)$$

Solving the resulting set of equations simultaneously gives:

$$T_{AB} = 99.0 \text{ lb} \quad \blacktriangleleft$$

$$T_{AC} = 10.55 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.115

For the rectangular plate of Problems 2.109 and 2.110, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below. Setting $P = 792 \text{ N}$ gives:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + 792 \text{ N} = 0 \quad (2)$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Solving Equations (1), (2), and (3) by conventional algorithms gives

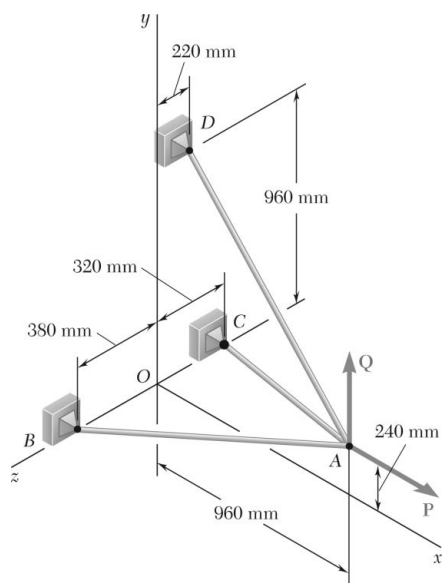
$$T_{AB} = 510.00 \text{ N} \qquad T_{AB} = 510 \text{ N} \blacktriangleleft$$

$$T_{AC} = 56.250 \text{ N} \qquad T_{AC} = 56.2 \text{ N} \blacktriangleleft$$

$$T_{AD} = 536.25 \text{ N} \qquad T_{AD} = 536 \text{ N} \blacktriangleleft$$

PROBLEM 2.116

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880 \text{ N}$ and $Q = 0$.



SOLUTION

$$\Sigma \mathbf{F}_A = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} + \mathbf{Q} = 0$$

Where

$$\mathbf{P} = P\mathbf{i} \text{ and } \mathbf{Q} = Q\mathbf{j}$$

$$\overline{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overline{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overline{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k} \right)$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k} \right)$$

$$\mathbf{T}_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = T_{AD} \left(-\frac{48}{61}\mathbf{i} + \frac{36}{61}\mathbf{j} - \frac{11}{61}\mathbf{k} \right)$$

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $P = (2880 \text{ N})\mathbf{i}$ and $Q = 0$, and setting the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} equal to 0, we obtain the following three equilibrium equations:

$$\mathbf{i}: -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \quad (1)$$

$$\mathbf{j}: -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} = 0 \quad (2)$$

$$\mathbf{k}: \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \quad (3)$$

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PROBLEM 2.116 (Continued)

Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 1340.14 \text{ N}$$

$$T_{AC} = 1025.12 \text{ N}$$

$$T_{AD} = 915.03 \text{ N}$$

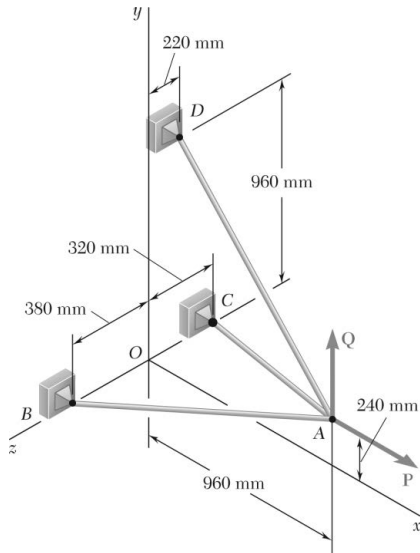
$$T_{AB} = 1340 \text{ N} \quad \blacktriangleleft$$

$$T_{AC} = 1025 \text{ N} \quad \blacktriangleleft$$

$$T_{AD} = 915 \text{ N} \quad \blacktriangleleft$$

PROBLEM 2.117

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880 \text{ N}$ and $Q = 576 \text{ N}$.



SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \quad (1)$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \quad (2)$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \quad (3)$$

Setting $P = 2880 \text{ N}$ and $Q = 576 \text{ N}$ gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \quad (1')$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + 576 \text{ N} = 0 \quad (2')$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \quad (3')$$

Solving the resulting set of equations using conventional algorithms gives:

$$T_{AB} = 1431.00 \text{ N}$$

$$T_{AC} = 1560.00 \text{ N}$$

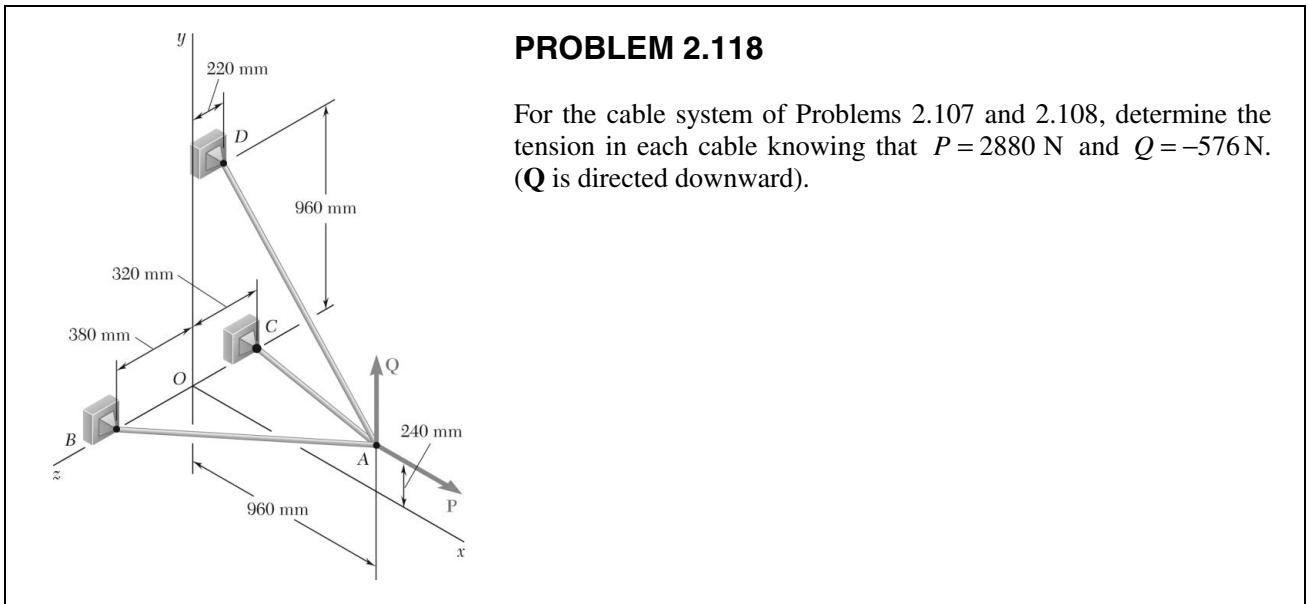
$$T_{AD} = 183.010 \text{ N}$$

$$T_{AB} = 1431 \text{ N} \quad \blacktriangleleft$$

$$T_{AC} = 1560 \text{ N} \quad \blacktriangleleft$$

$$T_{AD} = 183.0 \text{ N} \quad \blacktriangleleft$$

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SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \tag{1}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \tag{2}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3}$$

Setting $P = 2880 \text{ N}$ and $Q = -576 \text{ N}$ gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \tag{1'}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} - 576 \text{ N} = 0 \tag{2'}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3'}$$

Solving the resulting set of equations using conventional algorithms gives:

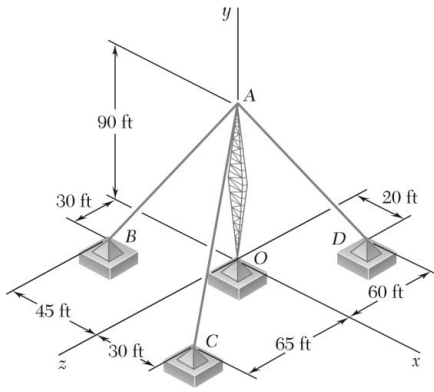
$T_{AB} = 1249.29 \text{ N}$
 $T_{AC} = 490.31 \text{ N}$
 $T_{AD} = 1646.97 \text{ N}$

$T_{AB} = 1249 \text{ N} \blacktriangleleft$
 $T_{AC} = 490 \text{ N} \blacktriangleleft$
 $T_{AD} = 1647 \text{ N} \blacktriangleleft$

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PROBLEM 2.119

For the transmission tower of Problems 2.111 and 2.112, determine the tension in each guy wire knowing that the tower exerts on the pin at A an upward vertical force of 2100 lb.



SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1)$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \quad (2)$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3)$$

Substituting for $P = 2100$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \quad (1')$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + 2100 \text{ lb} = 0 \quad (2')$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \quad (3')$$

$$T_{AB} = 841.55 \text{ lb}$$

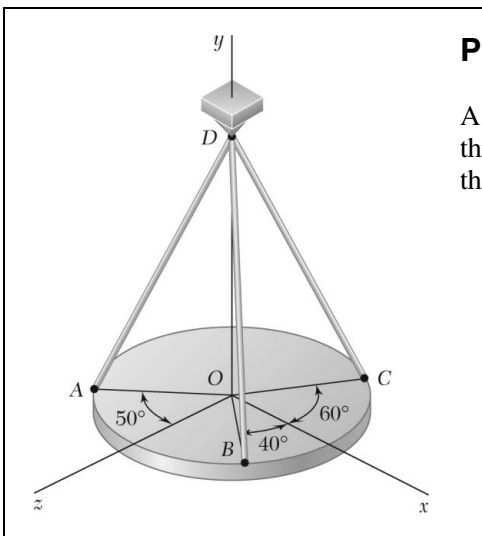
$$T_{AC} = 624.38 \text{ lb}$$

$$T_{AD} = 1087.81 \text{ lb}$$

$$T_{AB} = 842 \text{ lb} \quad \blacktriangleleft$$

$$T_{AC} = 624 \text{ lb} \quad \blacktriangleleft$$

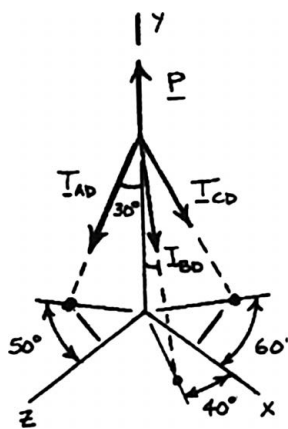
$$T_{AD} = 1088 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.120

A horizontal circular plate weighing 60 lb is suspended as shown from three wires that are attached to a support at D and form 30° angles with the vertical. Determine the tension in each wire.

SOLUTION



$$\Sigma F_x = 0:$$

$$-T_{AD}(\sin 30^\circ)(\sin 50^\circ) + T_{BD}(\sin 30^\circ)(\cos 40^\circ) + T_{CD}(\sin 30^\circ)(\cos 60^\circ) = 0$$

Dividing through by $\sin 30^\circ$ and evaluating:

$$-0.76604T_{AD} + 0.76604T_{BD} + 0.5T_{CD} = 0 \quad (1)$$

$$\Sigma F_y = 0: -T_{AD}(\cos 30^\circ) - T_{BD}(\cos 30^\circ) - T_{CD}(\cos 30^\circ) + 60 \text{ lb} = 0$$

or

$$T_{AD} + T_{BD} + T_{CD} = 69.282 \text{ lb} \quad (2)$$

$$\Sigma F_z = 0: T_{AD} \sin 30^\circ \cos 50^\circ + T_{BD} \sin 30^\circ \sin 40^\circ - T_{CD} \sin 30^\circ \sin 60^\circ = 0$$

or

$$0.64279T_{AD} + 0.64279T_{BD} - 0.86603T_{CD} = 0 \quad (3)$$

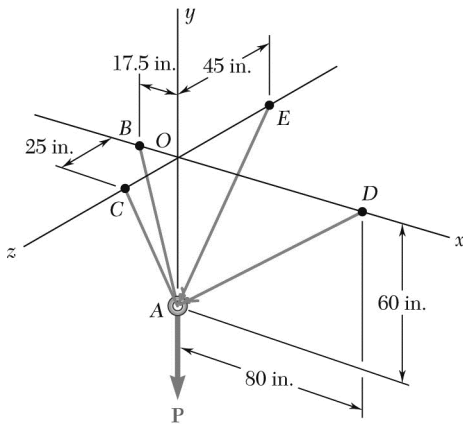
Solving Equations (1), (2), and (3) simultaneously:

$$T_{AD} = 29.5 \text{ lb} \quad \blacktriangleleft$$

$$T_{BD} = 10.25 \text{ lb} \quad \blacktriangleleft$$

$$T_{CD} = 29.5 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 2.121



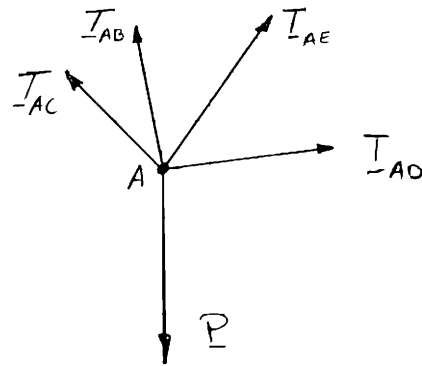
Cable BAC passes through a frictionless ring A and is attached to fixed supports at B and C , while cables AD and AE are both tied to the ring and are attached, respectively, to supports at D and E . Knowing that a 200-lb vertical load \mathbf{P} is applied to ring A , determine the tension in each of the three cables.

SOLUTION

Since T_{BAC} = tension in cable BAC , it follows that

$$T_{AB} = T_{AC} = T_{BAC}$$

Free Body Diagram at A:



$$\mathbf{T}_{AB} = T_{BAC} \lambda_{AB} = T_{BAC} \frac{(-17.5 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j}}{62.5 \text{ in.}} = T_{BAC} \left(\frac{-17.5}{62.5} \mathbf{i} + \frac{60}{62.5} \mathbf{j} \right)$$

$$\mathbf{T}_{AC} = T_{BAC} \lambda_{AC} = T_{BAC} \frac{(60 \text{ in.})\mathbf{i} + (25 \text{ in.})\mathbf{k}}{65 \text{ in.}} = T_{BAC} \left(\frac{60}{65} \mathbf{j} + \frac{25}{65} \mathbf{k} \right)$$

$$\mathbf{T}_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{(80 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j}}{100 \text{ in.}} = T_{AD} \left(\frac{4}{5} \mathbf{i} + \frac{3}{5} \mathbf{j} \right)$$

$$\mathbf{T}_{AE} = T_{AE} \lambda_{AE} = T_{AE} \frac{(60 \text{ in.})\mathbf{j} - (45 \text{ in.})\mathbf{k}}{75 \text{ in.}} = T_{AE} \left(\frac{4}{5} \mathbf{j} - \frac{3}{5} \mathbf{k} \right)$$

PROBLEM 2.121 (Continued)

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $\mathbf{P} = (-200 \text{ lb})\mathbf{j}$, and setting the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} equal to ϕ , we obtain the following three equilibrium equations:

$$\text{From } \mathbf{i}: -\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} = 0 \quad (1)$$

$$\text{From } \mathbf{j}: \left(\frac{60}{62.5} + \frac{60}{65} \right) T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - 200 \text{ lb} = 0 \quad (2)$$

$$\text{From } \mathbf{k}: \frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} = 0 \quad (3)$$

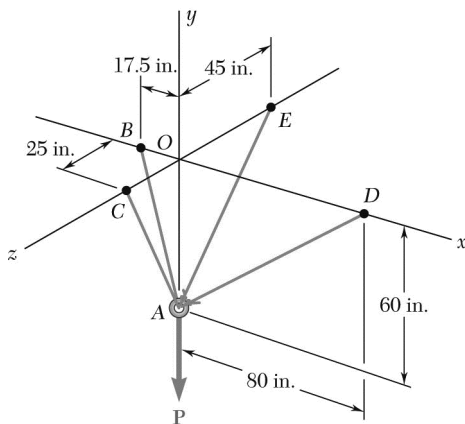
Solving the system of linear equations using conventional algorithms gives:

$$T_{BAC} = 76.7 \text{ lb}; T_{AD} = 26.9 \text{ lb}; T_{AE} = 49.2 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 2.122

Knowing that the tension in cable AE of Prob. 2.121 is 75 lb, determine (a) the magnitude of the load \mathbf{P} , (b) the tension in cables BAC and AD .

PROBLEM 2.121 Cable BAC passes through a frictionless ring A and is attached to fixed supports at B and C , while cables AD and AE are both tied to the ring and are attached, respectively, to supports at D and E . Knowing that a 200-lb vertical load \mathbf{P} is applied to ring A , determine the tension in each of the three cables.



SOLUTION

Refer to the solution to Problem 2.121 for the figure and analysis leading to the following set of equilibrium equations, Equation (2) being modified to include $P\mathbf{j}$ as an unknown quantity:

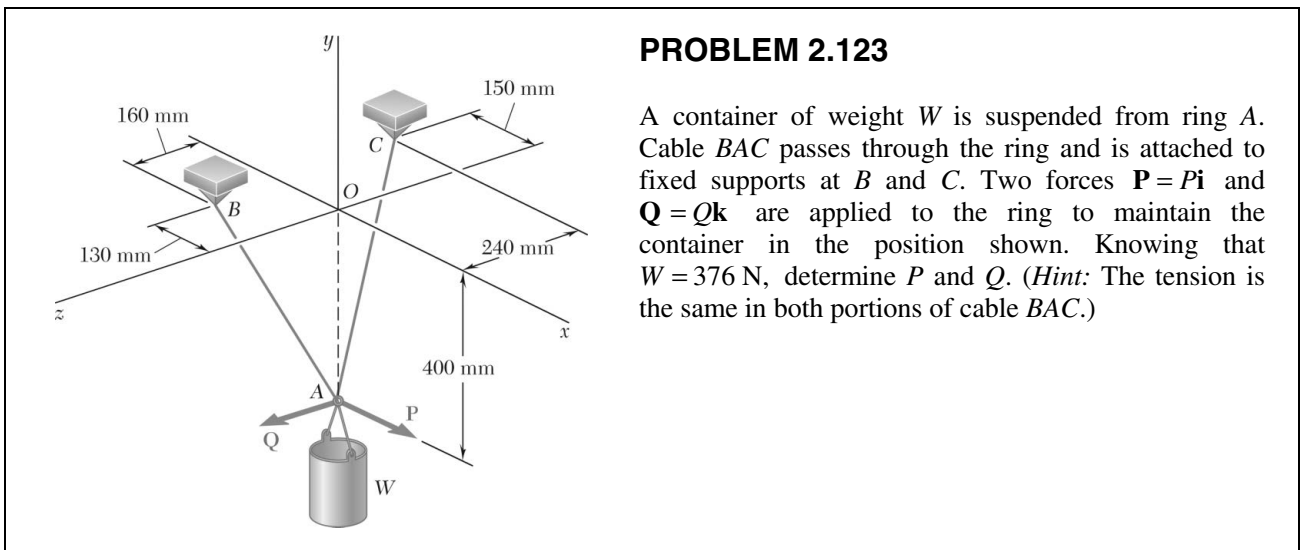
$$-\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} = 0 \quad (1)$$

$$\left(\frac{60}{62.5} + \frac{60}{65}\right)T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - P = 0 \quad (2)$$

$$\frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} = 0 \quad (3)$$

Substituting for $T_{AE} = 75$ lb and solving simultaneously gives:

$$P = 305 \text{ lb}; \quad T_{BAC} = 117.0 \text{ lb}; \quad T_{AD} = 40.9 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 2.123

A container of weight W is suspended from ring A . Cable BAC passes through the ring and is attached to fixed supports at B and C . Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376 \text{ N}$, determine P and Q . (*Hint:* The tension is the same in both portions of cable BAC .)

SOLUTION

$$\begin{aligned} \mathbf{T}_{AB} &= T\lambda_{AB} \\ &= T \frac{\overline{AB}}{AB} \\ &= T \frac{(-130 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} + (160 \text{ mm})\mathbf{k}}{450 \text{ mm}} \\ &= T \left(-\frac{13}{45}\mathbf{i} + \frac{40}{45}\mathbf{j} + \frac{16}{45}\mathbf{k} \right) \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{AC} &= T\lambda_{AC} \\ &= T \frac{\overline{AC}}{AC} \\ &= T \frac{(-150 \text{ mm})\mathbf{i} + (400 \text{ mm})\mathbf{j} + (-240 \text{ mm})\mathbf{k}}{490 \text{ mm}} \\ &= T \left(-\frac{15}{49}\mathbf{i} + \frac{40}{49}\mathbf{j} - \frac{24}{49}\mathbf{k} \right) \end{aligned}$$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

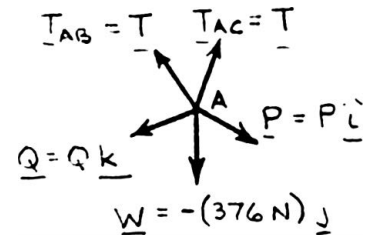
Setting coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} equal to zero:

$$\mathbf{i}: \quad -\frac{13}{45}T - \frac{15}{49}T + P = 0 \qquad 0.59501T = P \qquad (1)$$

$$\mathbf{j}: \quad +\frac{40}{45}T + \frac{40}{49}T - W = 0 \qquad 1.70521T = W \qquad (2)$$

$$\mathbf{k}: \quad +\frac{16}{45}T - \frac{24}{49}T + Q = 0 \qquad 0.134240T = Q \qquad (3)$$

Free-Body A:



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PROBLEM 2.123 (Continued)

Data:

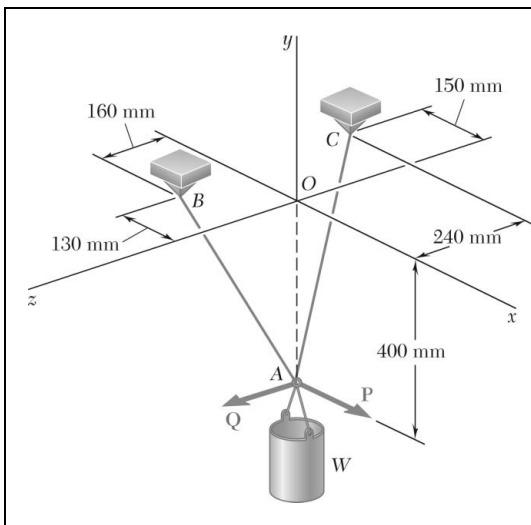
$$W = 376 \text{ N} \quad 1.70521T = 376 \text{ N} \quad T = 220.50 \text{ N}$$

$$0.59501(220.50 \text{ N}) = P$$

$$P = 131.2 \text{ N} \quad \blacktriangleleft$$

$$0.134240(220.50 \text{ N}) = Q$$

$$Q = 29.6 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.124

For the system of Problem 2.123, determine W and Q knowing that $P = 164 \text{ N}$.

PROBLEM 2.123 A container of weight W is suspended from ring A . Cable BAC passes through the ring and is attached to fixed supports at B and C . Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376 \text{ N}$, determine P and Q . (*Hint*: The tension is the same in both portions of cable BAC .)

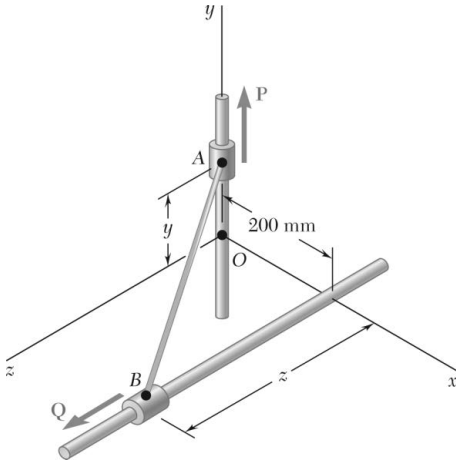
SOLUTION

Refer to Problem 2.123 for the figure and analysis resulting in Equations (1), (2), and (3) for P , W , and Q in terms of T below. Setting $P = 164 \text{ N}$ we have:

Eq. (1):	$0.59501T = 164 \text{ N}$	$T = 275.63 \text{ N}$
Eq. (2):	$1.70521(275.63 \text{ N}) = W$	$W = 470 \text{ N} \blacktriangleleft$
Eq. (3):	$0.134240(275.63 \text{ N}) = Q$	$Q = 37.0 \text{ N} \blacktriangleleft$

PROBLEM 2.125

Collars *A* and *B* are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341 \text{ N})\mathbf{j}$ is applied to collar *A*, determine (a) the tension in the wire when $y = 155 \text{ mm}$, (b) the magnitude of the force \mathbf{Q} required to maintain the equilibrium of the system.



SOLUTION

For both Problems 2.125 and 2.126:

$$(AB)^2 = x^2 + y^2 + z^2$$

Here $(0.525 \text{ m})^2 = (0.20 \text{ m})^2 + y^2 + z^2$

or $y^2 + z^2 = 0.23563 \text{ m}^2$

Thus, when y given, z is determined,

Now
$$\begin{aligned} \lambda_{AB} &= \frac{\overline{AB}}{AB} \\ &= \frac{1}{0.525 \text{ m}} (0.20\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m} \\ &= 0.38095\mathbf{i} - 1.90476y\mathbf{j} + 1.90476z\mathbf{k} \end{aligned}$$

Where y and z are in units of meters, m.

From the F.B. Diagram of collar *A*: $\Sigma \mathbf{F} = 0: N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$

Setting the \mathbf{j} coefficient to zero gives $P - (1.90476y)T_{AB} = 0$

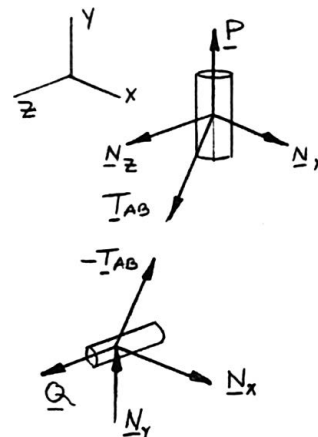
With
$$\begin{aligned} P &= 341 \text{ N} \\ T_{AB} &= \frac{341 \text{ N}}{1.90476y} \end{aligned}$$

Now, from the free body diagram of collar *B*: $\Sigma \mathbf{F} = 0: N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\lambda_{AB} = 0$

Setting the \mathbf{k} coefficient to zero gives $Q - T_{AB}(1.90476z) = 0$

And using the above result for T_{AB} , we have
$$Q = T_{AB}z = \frac{341 \text{ N}}{(1.90476)y} (1.90476z) = \frac{(341 \text{ N})(z)}{y}$$

Free-Body Diagrams of Collars:



PROBLEM 2.125 (Continued)

Then from the specifications of the problem, $y = 155 \text{ mm} = 0.155 \text{ m}$

$$z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2$$

$$z = 0.46 \text{ m}$$

and

$$\begin{aligned} (a) \quad T_{AB} &= \frac{341 \text{ N}}{0.155(1.90476)} \\ &= 1155.00 \text{ N} \end{aligned}$$

or

$$T_{AB} = 1155 \text{ N} \blacktriangleleft$$

and

$$\begin{aligned} (b) \quad Q &= \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})} \\ &= (1012.00 \text{ N}) \end{aligned}$$

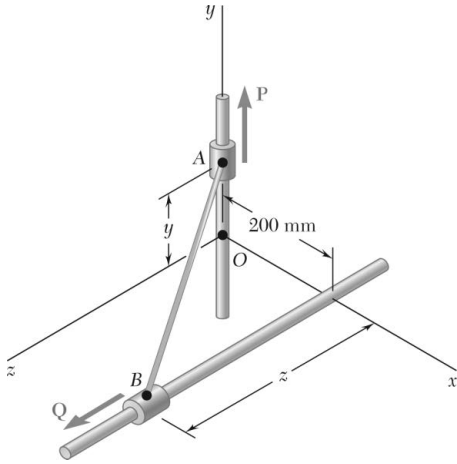
or

$$Q = 1012 \text{ N} \blacktriangleleft$$

PROBLEM 2.126

Solve Problem 2.125 assuming that $y = 275$ mm.

PROBLEM 2.125 Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341 \text{ N})\mathbf{j}$ is applied to collar A , determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force \mathbf{Q} required to maintain the equilibrium of the system.



SOLUTION

From the analysis of Problem 2.125, particularly the results:

$$y^2 + z^2 = 0.23563 \text{ m}^2$$

$$T_{AB} = \frac{341 \text{ N}}{1.90476y}$$

$$Q = \frac{341 \text{ N}}{y} z$$

With $y = 275 \text{ mm} = 0.275 \text{ m}$, we obtain:

$$z^2 = 0.23563 \text{ m}^2 - (0.275 \text{ m})^2$$

$$z = 0.40 \text{ m}$$

and

$$(a) \quad T_{AB} = \frac{341 \text{ N}}{(1.90476)(0.275 \text{ m})} = 651.00$$

or

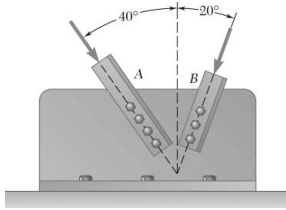
$$T_{AB} = 651 \text{ N} \blacktriangleleft$$

and

$$(b) \quad Q = \frac{341 \text{ N}(0.40 \text{ m})}{(0.275 \text{ m})}$$

or

$$Q = 496 \text{ N} \blacktriangleleft$$



PROBLEM 2.127

Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member A and 10 kN in member B , determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members A and B .

SOLUTION

Using the force triangle and the laws of cosines and sines, we have

$$\begin{aligned}\gamma &= 180^\circ - (40^\circ + 20^\circ) \\ &= 120^\circ\end{aligned}$$

Then

$$\begin{aligned}R^2 &= (15 \text{ kN})^2 + (10 \text{ kN})^2 \\ &\quad - 2(15 \text{ kN})(10 \text{ kN})\cos 120^\circ \\ &= 475 \text{ kN}^2 \\ R &= 21.794 \text{ kN}\end{aligned}$$

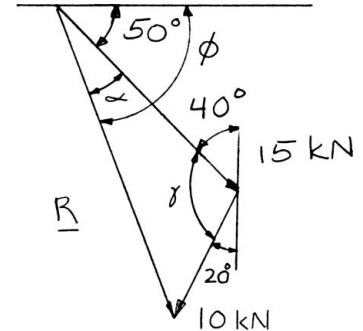
and

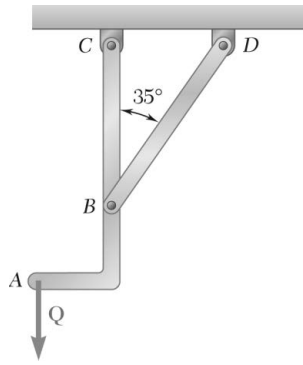
$$\begin{aligned}\frac{10 \text{ kN}}{\sin \alpha} &= \frac{21.794 \text{ kN}}{\sin 120^\circ} \\ \sin \alpha &= \left(\frac{10 \text{ kN}}{21.794 \text{ kN}} \right) \sin 120^\circ \\ &= 0.39737 \\ \alpha &= 23.414\end{aligned}$$

Hence:

$$\phi = \alpha + 50^\circ = 73.414$$

$$\mathbf{R} = 21.8 \text{ kN} \swarrow 73.4^\circ \blacktriangleleft$$

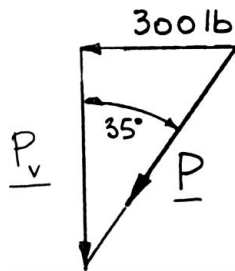




PROBLEM 2.128

Member BD exerts on member ABC a force \mathbf{P} directed along line BD . Knowing that \mathbf{P} must have a 300-lb horizontal component, determine (a) the magnitude of the force \mathbf{P} , (b) its vertical component.

SOLUTION



(a)

$$P \sin 35^\circ = 300 \text{ lb}$$

$$P = \frac{300 \text{ lb}}{\sin 35^\circ}$$

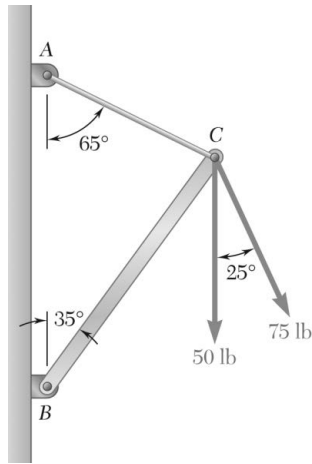
$$P = 523 \text{ lb} \blacktriangleleft$$

(b) Vertical component

$$P_v = P \cos 35^\circ$$

$$= (523 \text{ lb}) \cos 35^\circ$$

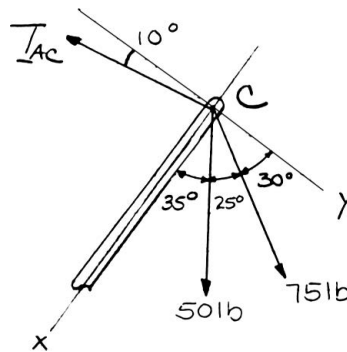
$$P_v = 428 \text{ lb} \blacktriangleleft$$



PROBLEM 2.129

Determine (a) the required tension in cable AC, knowing that the resultant of the three forces exerted at Point C of boom BC must be directed along BC, (b) the corresponding magnitude of the resultant.

SOLUTION



Using the x and y axes shown:

$$R_x = \Sigma F_x = T_{AC} \sin 10^\circ + (50 \text{ lb}) \cos 35^\circ + (75 \text{ lb}) \cos 60^\circ$$

$$= T_{AC} \sin 10^\circ + 78.458 \text{ lb} \quad (1)$$

$$R_y = \Sigma F_y = (50 \text{ lb}) \sin 35^\circ + (75 \text{ lb}) \sin 60^\circ - T_{AC} \cos 10^\circ$$

$$R_y = 93.631 \text{ lb} - T_{AC} \cos 10^\circ \quad (2)$$

(a) Set $R_y = 0$ in Eq. (2):

$$93.631 \text{ lb} - T_{AC} \cos 10^\circ = 0$$

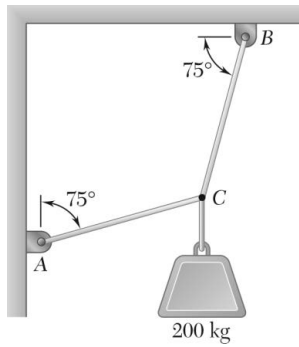
$$T_{AC} = 95.075 \text{ lb} \quad T_{AC} = 95.1 \text{ lb} \blacktriangleleft$$

(b) Substituting for T_{AC} in Eq. (1):

$$R_x = (95.075 \text{ lb}) \sin 10^\circ + 78.458 \text{ lb}$$

$$= 94.968 \text{ lb}$$

$$R = R_x \quad R = 95.0 \text{ lb} \blacktriangleleft$$

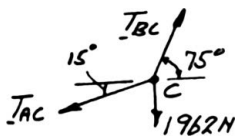


PROBLEM 2.130

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC , (b) in cable BC .

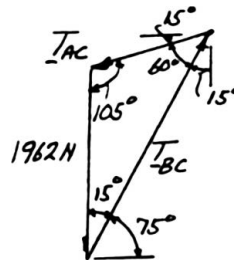
SOLUTION

Free-Body Diagram



$$\begin{aligned}
 W &= mg \\
 &= (200 \text{ kg})(9.81 \text{ m/s}^2) \\
 &= 1962 \text{ N}
 \end{aligned}$$

Force Triangle



Law of sines:

$$\frac{T_{AC}}{\sin 15^\circ} = \frac{T_{BC}}{\sin 105^\circ} = \frac{1962 \text{ N}}{\sin 60^\circ}$$

$$(a) \quad T_{AC} = \frac{(1962 \text{ N}) \sin 15^\circ}{\sin 60^\circ} \quad T_{AC} = 586 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad T_{BC} = \frac{(1962 \text{ N}) \sin 105^\circ}{\sin 60^\circ} \quad T_{BC} = 2190 \text{ N} \quad \blacktriangleleft$$

PROBLEM 2.131

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8 \text{ kN}$ and $F_B = 16 \text{ kN}$, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection

$$\Sigma F_x = 0: \frac{3}{5}F_B - F_C - \frac{4}{5}F_A = 0$$

With

$$F_A = 8 \text{ kN}$$

$$F_B = 16 \text{ kN}$$

$$F_C = \frac{4}{5}(16 \text{ kN}) - \frac{4}{5}(8 \text{ kN}) \qquad F_C = 6.40 \text{ kN} \blacktriangleleft$$

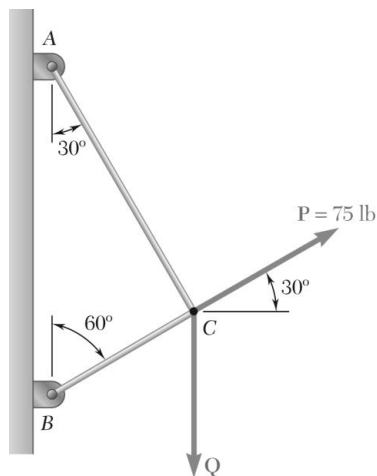
$$\Sigma F_y = 0: -F_D + \frac{3}{5}F_B - \frac{3}{5}F_A = 0$$

With F_A and F_B as above:

$$F_D = \frac{3}{5}(16 \text{ kN}) - \frac{3}{5}(8 \text{ kN}) \qquad F_D = 4.80 \text{ kN} \blacktriangleleft$$

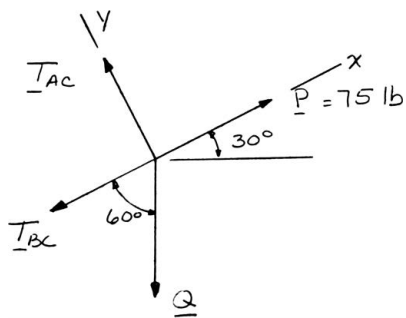
PROBLEM 2.132

Two cables tied together at C are loaded as shown. Determine the range of values of Q for which the tension will not exceed 60 lb in either cable.



SOLUTION

Free-Body Diagram



$$\Sigma F_x = 0: -T_{BC} - Q \cos 60^\circ + 75 \text{ lb} = 0$$

$$T_{BC} = 75 \text{ lb} - Q \cos 60^\circ \quad (1)$$

$$\Sigma F_y = 0: T_{AC} - Q \sin 60^\circ = 0$$

$$T_{AC} = Q \sin 60^\circ \quad (2)$$

Requirement: $T_{AC} \leq 60 \text{ lb}$:

From Eq. (2): $Q \sin 60^\circ \leq 60 \text{ lb}$

$$Q \leq 69.3 \text{ lb}$$

Requirement: $T_{BC} \leq 60 \text{ lb}$:

From Eq. (1): $75 \text{ lb} - Q \sin 60^\circ \leq 60 \text{ lb}$

$$Q \geq 30.0 \text{ lb} \quad 30.0 \text{ lb} \leq Q \leq 69.3 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 2.133

A horizontal circular plate is suspended as shown from three wires that are attached to a support at D and form 30° angles with the vertical. Knowing that the x component of the force exerted by wire AD on the plate is 110.3 N, determine (a) the tension in wire AD , (b) the angles θ_x , θ_y , and θ_z that the force exerted at A forms with the coordinate axes.

SOLUTION

(a) $F_x = F \sin 30^\circ \sin 50^\circ = 110.3$ N (Given)

$$F = \frac{110.3 \text{ N}}{\sin 30^\circ \sin 50^\circ} = 287.97 \text{ N} \quad F = 288 \text{ N} \quad \blacktriangleleft$$

(b) $\cos \theta_x = \frac{F_x}{F} = \frac{110.3 \text{ N}}{287.97 \text{ N}} = 0.38303 \quad \theta_x = 67.5^\circ \quad \blacktriangleleft$

$$F_y = F \cos 30^\circ = 249.39$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{249.39 \text{ N}}{287.97 \text{ N}} = 0.86603 \quad \theta_y = 30.0^\circ \quad \blacktriangleleft$$

$$F_z = -F \sin 30^\circ \cos 50^\circ$$

$$= -(287.97 \text{ N}) \sin 30^\circ \cos 50^\circ$$

$$= -92.552 \text{ N}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-92.552 \text{ N}}{287.97 \text{ N}} = -0.32139 \quad \theta_z = 108.7^\circ \quad \blacktriangleleft$$

PROBLEM 2.134

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 55^\circ$ and $\theta_z = 45^\circ$. Knowing that the x component of the force is -500 lb, determine (a) the angle θ_x , (b) the other components and the magnitude of the force.

SOLUTION

(a) We have

$$(\cos \theta_x)^2 + (\cos \theta_y)^2 + (\cos \theta_z)^2 = 1 \Rightarrow (\cos \theta_x)^2 = 1 - (\cos \theta_y)^2 - (\cos \theta_z)^2$$

Since $F_x < 0$, we must have $\cos \theta_x < 0$.

Thus, taking the negative square root, from above, we have

$$\cos \theta_x = -\sqrt{1 - (\cos 55^\circ)^2 - (\cos 45^\circ)^2} = 0.41353 \qquad \theta_x = 114.4^\circ \blacktriangleleft$$

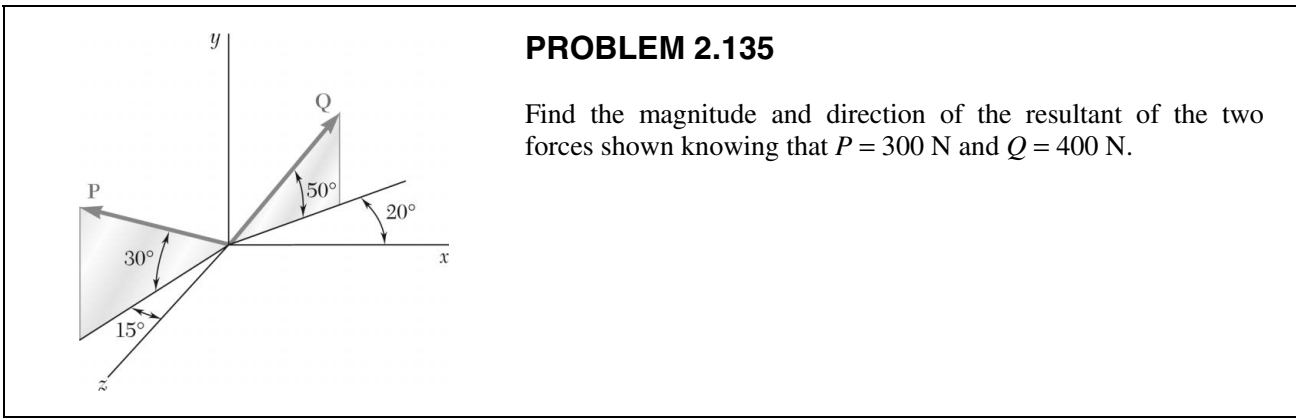
(b) Then

$$F = \frac{F_x}{\cos \theta_x} = \frac{500 \text{ lb}}{0.41353} = 1209.10 \text{ lb} \qquad F = 1209 \text{ lb} \blacktriangleleft$$

and

$$F_y = F \cos \theta_y = (1209.10 \text{ lb}) \cos 55^\circ \qquad F_y = 694 \text{ lb} \blacktriangleleft$$

$$F_z = F \cos \theta_z = (1209.10 \text{ lb}) \cos 45^\circ \qquad F_z = 855 \text{ lb} \blacktriangleleft$$



PROBLEM 2.135

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 300 \text{ N}$ and $Q = 400 \text{ N}$.

SOLUTION

$$\begin{aligned} \mathbf{P} &= (300 \text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}] \\ &= -(67.243 \text{ N})\mathbf{i} + (150 \text{ N})\mathbf{j} + (250.95 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{Q} &= (400 \text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}] \\ &= (400 \text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985\mathbf{k}] \\ &= (241.61 \text{ N})\mathbf{i} + (306.42 \text{ N})\mathbf{j} - (87.939 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{R} &= \mathbf{P} + \mathbf{Q} \\ &= (174.367 \text{ N})\mathbf{i} + (456.42 \text{ N})\mathbf{j} + (163.011 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} R &= \sqrt{(174.367 \text{ N})^2 + (456.42 \text{ N})^2 + (163.011 \text{ N})^2} \\ &= 515.07 \text{ N} \end{aligned}$$

$$R = 515 \text{ N} \quad \blacktriangleleft$$

$$\cos \theta_x = \frac{R_x}{R} = \frac{174.367 \text{ N}}{515.07 \text{ N}} = 0.33853$$

$$\theta_x = 70.2^\circ \quad \blacktriangleleft$$

$$\cos \theta_y = \frac{R_y}{R} = \frac{456.42 \text{ N}}{515.07 \text{ N}} = 0.88613$$

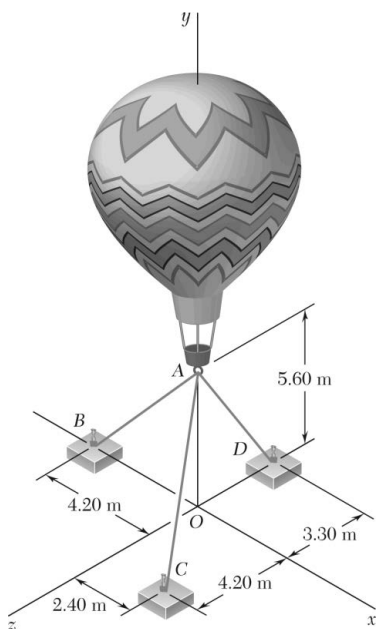
$$\theta_y = 27.6^\circ \quad \blacktriangleleft$$

$$\cos \theta_z = \frac{R_z}{R} = \frac{163.011 \text{ N}}{515.07 \text{ N}} = 0.31648$$

$$\theta_z = 71.5^\circ \quad \blacktriangleleft$$

PROBLEM 2.136

Three cables are used to tether a balloon as shown. Determine the vertical force \mathbf{P} exerted by the balloon at A knowing that the tension in cable AC is 444 N.



SOLUTION

See Problem 2.101 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

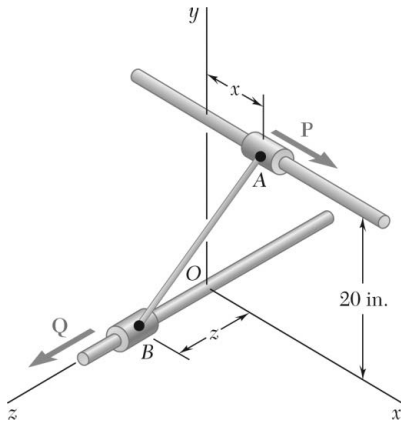
Substituting $T_{AC} = 444 \text{ N}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240 \text{ N}$$

$$T_{AD} = 496.36 \text{ N}$$

$$\mathbf{P} = 956 \text{ N} \uparrow \blacktriangleleft$$

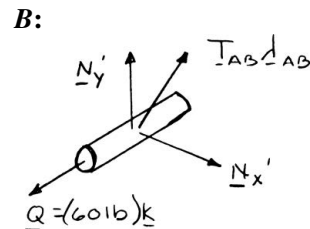
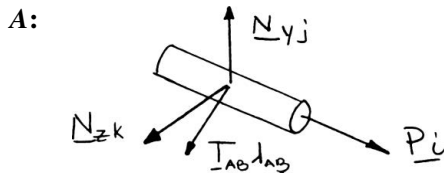
PROBLEM 2.137



Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force \mathbf{Q} is applied to collar B as shown, determine (a) the tension in the wire when $x = 9$ in., (b) the corresponding magnitude of the force \mathbf{P} required to maintain the equilibrium of the system.

SOLUTION

Free-Body Diagrams of Collars:



$$\lambda_{AB} = \frac{\overline{AB}}{AB} = \frac{-x\mathbf{i} - (20 \text{ in.})\mathbf{j} + z\mathbf{k}}{25 \text{ in.}}$$

Collar A : $\Sigma \mathbf{F} = 0: P\mathbf{i} + N_y\mathbf{j} + N_z\mathbf{k} + T_{AB}\lambda_{AB} = 0$

Substitute for λ_{AB} and set coefficient of \mathbf{i} equal to zero:

$$P - \frac{T_{AB}x}{25 \text{ in.}} = 0 \quad (1)$$

Collar B : $\Sigma \mathbf{F} = 0: (60 \text{ lb})\mathbf{k} + N'_x\mathbf{i} + N'_y\mathbf{j} - T_{AB}\lambda_{AB} = 0$

Substitute for λ_{AB} and set coefficient of \mathbf{k} equal to zero:

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \quad (2)$$

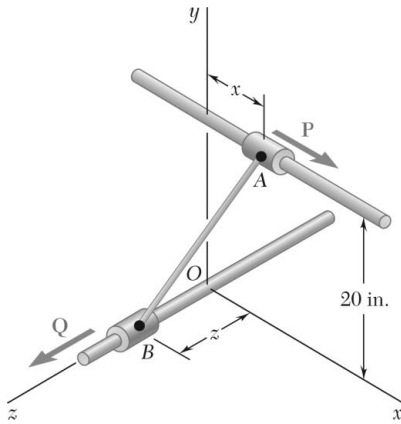
(a) $x = 9 \text{ in.}$ $(9 \text{ in.})^2 + (20 \text{ in.})^2 + z^2 = (25 \text{ in.})^2$
 $z = 12 \text{ in.}$

From Eq. (2): $\frac{60 \text{ lb} - T_{AB}(12 \text{ in.})}{25 \text{ in.}} = 0$ $T_{AB} = 125.0 \text{ lb} \blacktriangleleft$

(b) From Eq. (1): $P = \frac{(125.0 \text{ lb})(9 \text{ in.})}{25 \text{ in.}}$ $P = 45.0 \text{ lb} \blacktriangleleft$

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PROBLEM 2.138



Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances x and z for which the equilibrium of the system is maintained when $P = 120$ lb and $Q = 60$ lb.

SOLUTION

See Problem 2.137 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{25 \text{ in.}} = 0 \quad (1)$$

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \quad (2)$$

For $P = 120$ lb, Eq. (1) yields

$$T_{AB}x = (25 \text{ in.})(20 \text{ lb}) \quad (1')$$

From Eq. (2):

$$T_{AB}z = (25 \text{ in.})(60 \text{ lb}) \quad (2')$$

Dividing Eq. (1') by (2'),

$$\frac{x}{z} = 2 \quad (3)$$

Now write

$$x^2 + z^2 + (20 \text{ in.})^2 = (25 \text{ in.})^2 \quad (4)$$

Solving (3) and (4) simultaneously,

$$4z^2 + z^2 + 400 = 625$$

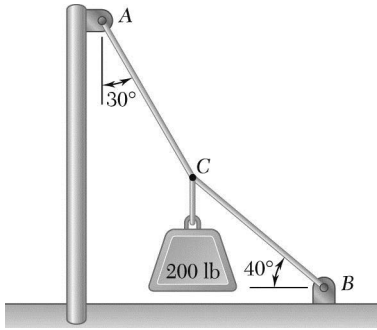
$$z^2 = 45$$

$$z = 6.7082 \text{ in.}$$

From Eq. (3):

$$\begin{aligned} x &= 2z = 2(6.7082 \text{ in.}) \\ &= 13.4164 \text{ in.} \end{aligned}$$

$$x = 13.42 \text{ in.}, \quad z = 6.71 \text{ in.} \quad \blacktriangleleft$$

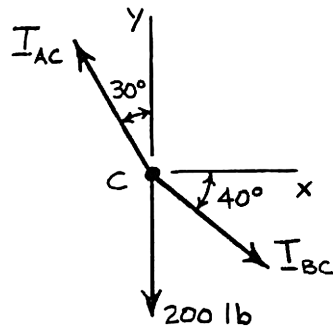


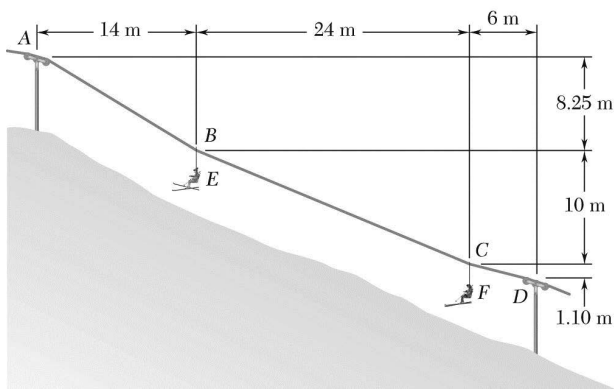
PROBLEM 2F1

Two cables are tied together at C and loaded as shown. Draw the free-body diagram needed to determine the tension in AC and BC .

SOLUTION

Free-Body Diagram of Point C :



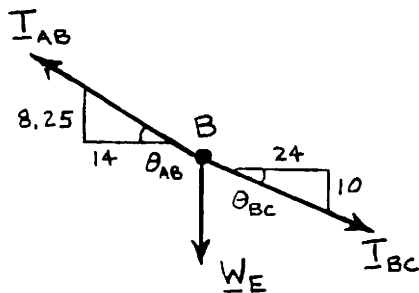


PROBLEM 2.F2

A chairlift has been stopped in the position shown. Knowing that each chair weighs 250 N and that the skier in chair *E* weighs 765 N, draw the free-body diagrams needed to determine the weight of the skier in chair *F*.

SOLUTION

Free-Body Diagram of Point *B*:



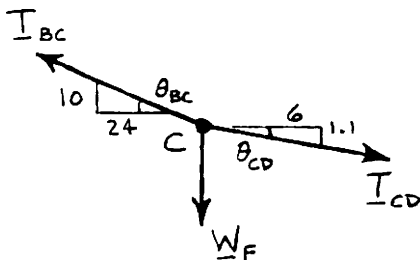
$$W_E = 250 \text{ N} + 765 \text{ N} = 1015 \text{ N}$$

$$\theta_{AB} = \tan^{-1} \frac{8.25}{14} = 30.510^\circ$$

$$\theta_{BC} = \tan^{-1} \frac{10}{24} = 22.620^\circ$$

Use this free body to determine T_{AB} and T_{BC} .

Free-Body Diagram of Point *C*:



$$\theta_{CD} = \tan^{-1} \frac{1.1}{6} = 10.3889^\circ$$

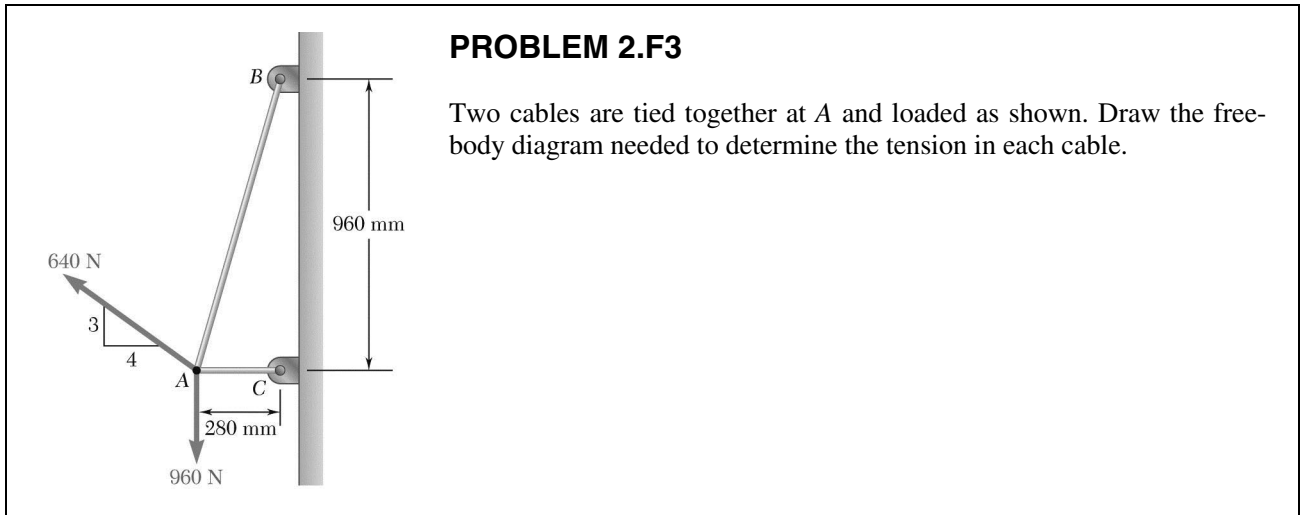
Use this free body to determine T_{CD} and W_F .

Then weight of skier W_S is found by

$$W_S = W_F - 250 \text{ N} \quad \blacktriangleleft$$

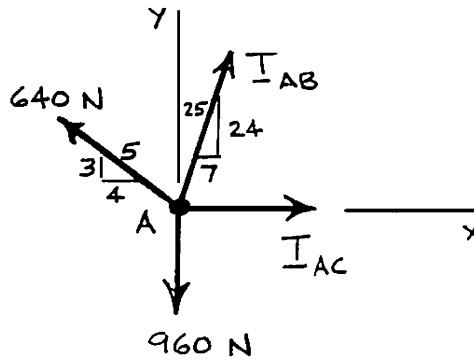
PROBLEM 2.F3

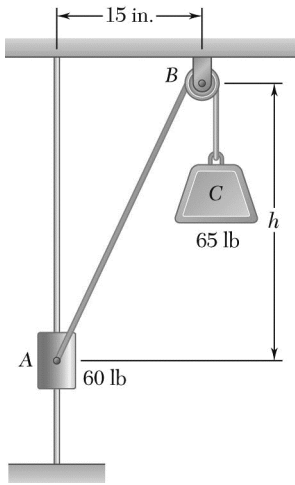
Two cables are tied together at A and loaded as shown. Draw the free-body diagram needed to determine the tension in each cable.



SOLUTION

Free-Body Diagram of Point A:



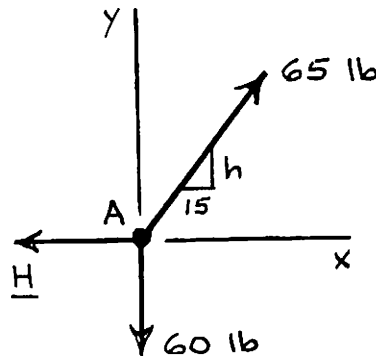


PROBLEM 2.F4

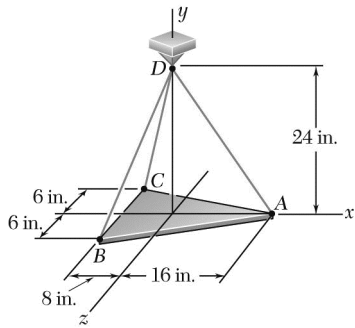
The 60-lb collar *A* can slide on a frictionless vertical rod and is connected as shown to a 65-lb counterweight *C*. Draw the free-body diagram needed to determine the value of *h* for which the system is in equilibrium.

SOLUTION

Free-Body Diagram of Point *A*:



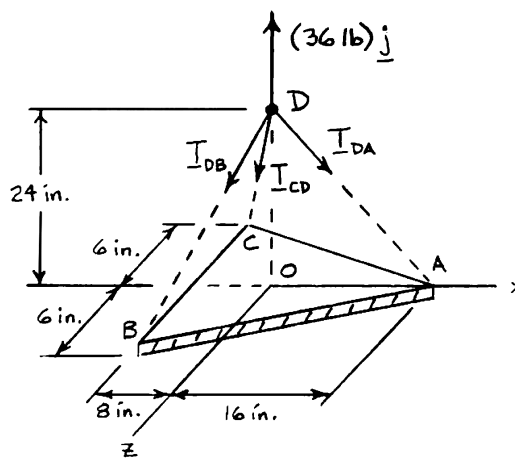
PROBLEM 2.F5



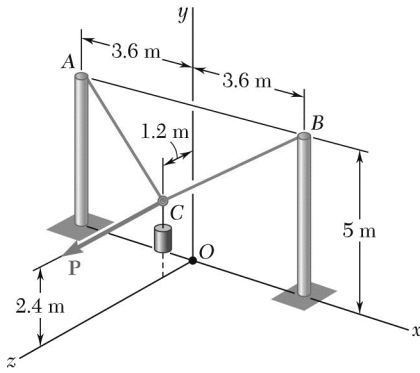
A 36-lb triangular plate is supported by three cables as shown. Draw the free-body diagram needed to determine the tension in each wire.

SOLUTION

Free-Body Diagram of Point *D*:



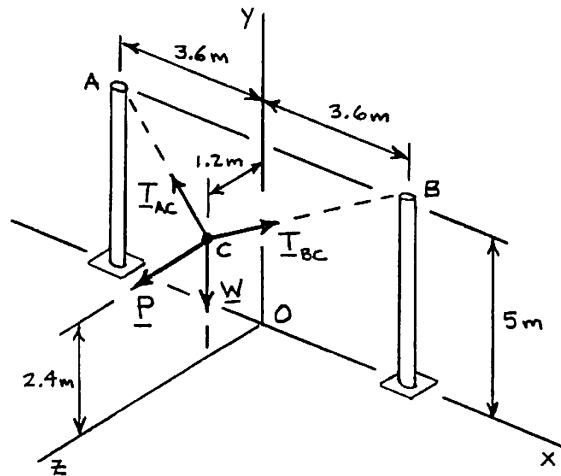
PROBLEM 2.F6



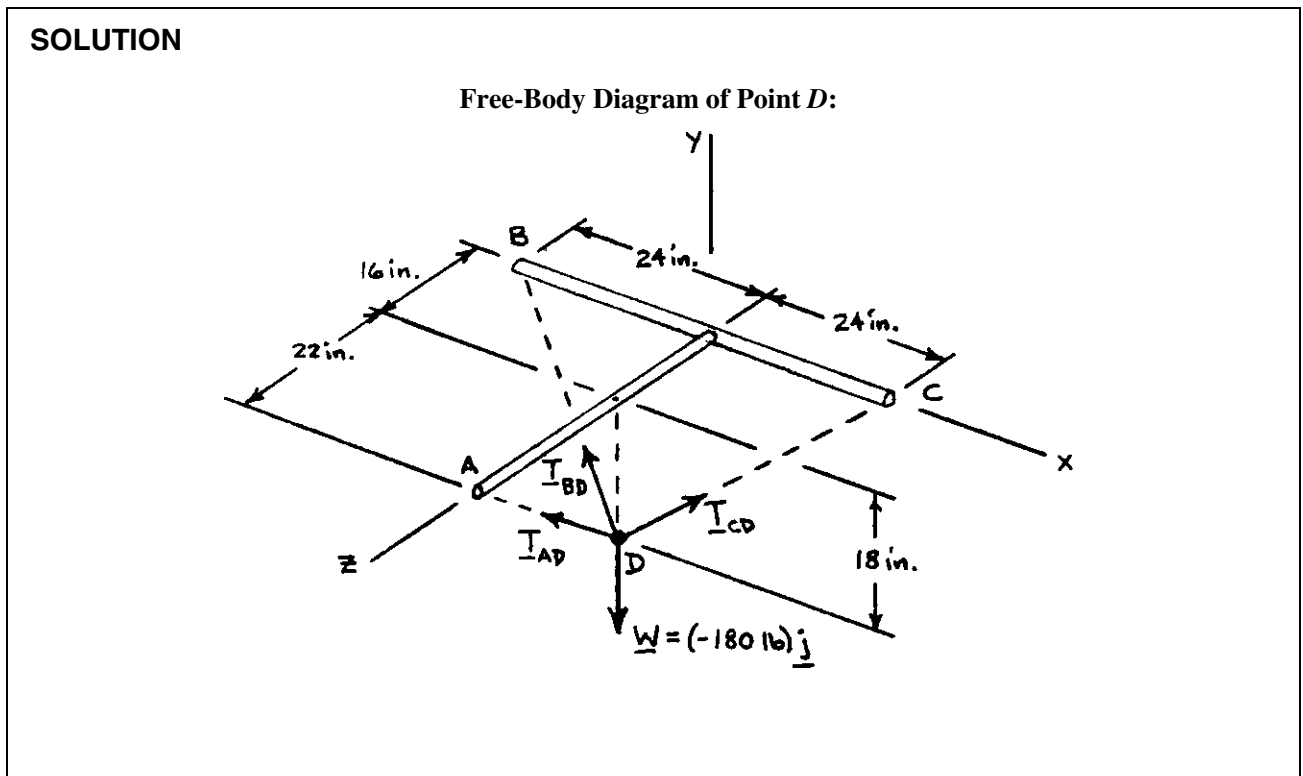
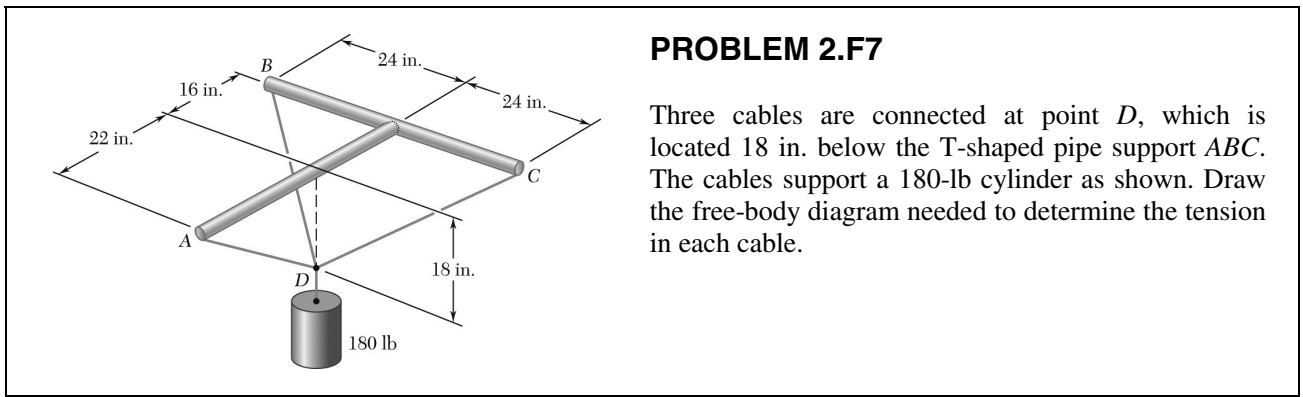
A 70-kg cylinder is supported by two cables AC and BC , which are attached to the top of vertical posts. A horizontal force \mathbf{P} , perpendicular to the plane containing the posts, holds the cylinder in the position shown. Draw the free-body diagram needed to determine the magnitude of \mathbf{P} and the force in each cable.

SOLUTION

Free-Body Diagram of Point C:

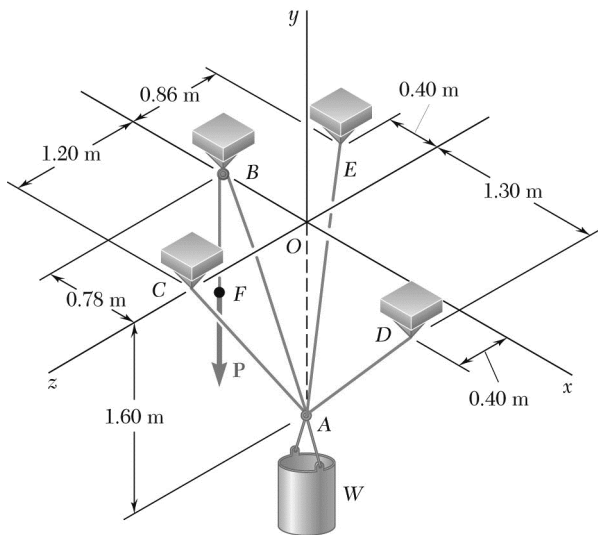


$$\begin{aligned} W &= (70 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 686.7 \text{ N} \\ \mathbf{W} &= -(686.7 \text{ N})\mathbf{j} \end{aligned}$$



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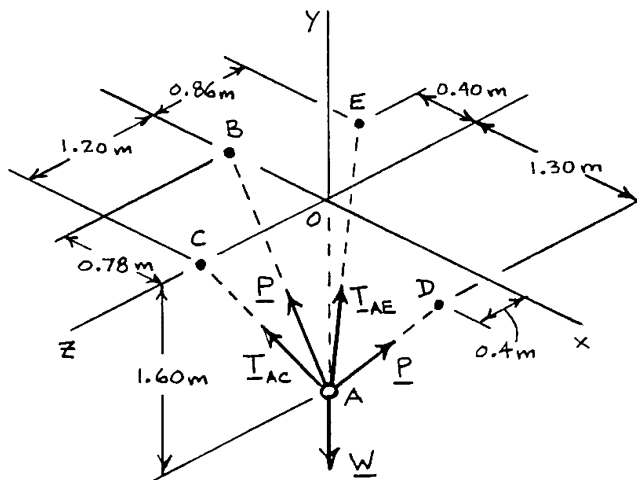
PROBLEM 2.F8



A 100-kg container is suspended from ring A, to which cables AC and AE are attached. A force P is applied to end F of a third cable that passes over a pulley at B and through ring A and then is attached to a support at D . Draw the free-body diagram needed to determine the magnitude of P . (Hint: The tension is the same in all portions of cable $FBAD$.)

SOLUTION

Free-Body Diagram of Ring A:

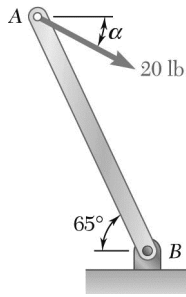


$$W = (100 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 981 \text{ N}$$

$$\mathbf{W} = -(981 \text{ N})\mathbf{j}$$

CHAPTER 3

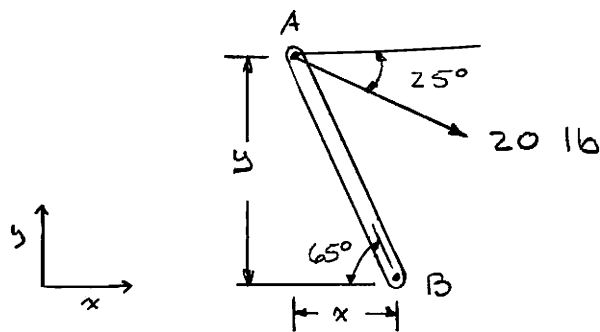


PROBLEM 3.1

A 20-lb force is applied to the control rod AB as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25^\circ$, determine the moment of the force about Point B by resolving the force into horizontal and vertical components.

SOLUTION

Free-Body Diagram of Rod AB :



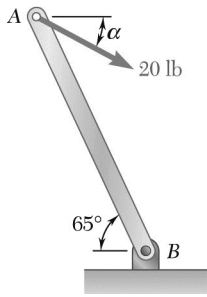
$$\begin{aligned} x &= (9 \text{ in.}) \cos 65^\circ \\ &= 3.8036 \text{ in.} \\ y &= (9 \text{ in.}) \sin 65^\circ \\ &= 8.1568 \text{ in.} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= F_x \mathbf{i} + F_y \mathbf{j} \\ &= (20 \text{ lb} \cos 25^\circ) \mathbf{i} + (-20 \text{ lb} \sin 25^\circ) \mathbf{j} \\ &= (18.1262 \text{ lb}) \mathbf{i} - (8.4524 \text{ lb}) \mathbf{j} \end{aligned}$$

$$\mathbf{r}_{A/B} = \overline{BA} = (-3.8036 \text{ in.}) \mathbf{i} + (8.1568 \text{ in.}) \mathbf{j}$$

$$\begin{aligned} \mathbf{M}_B &= \mathbf{r}_{A/B} \times \mathbf{F} \\ &= (-3.8036 \mathbf{i} + 8.1568 \mathbf{j}) \times (18.1262 \mathbf{i} - 8.4524 \mathbf{j}) \\ &= 32.150 \mathbf{k} - 147.852 \mathbf{k} \\ &= -115.702 \text{ lb-in.} \end{aligned}$$

$$\mathbf{M}_B = 115.7 \text{ lb-in.} \quad \blacktriangleleft$$

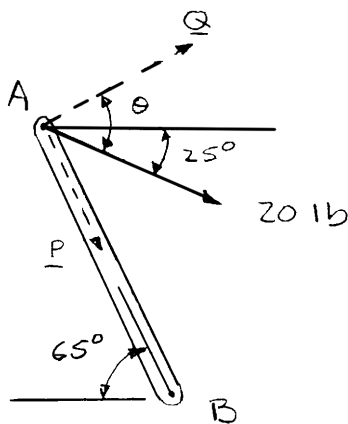


PROBLEM 3.2

A 20-lb force is applied to the control rod AB as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25^\circ$, determine the moment of the force about Point B by resolving the force into components along AB and in a direction perpendicular to AB .

SOLUTION

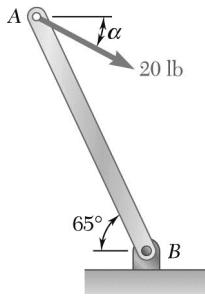
Free-Body Diagram of Rod AB :



$$\begin{aligned}\theta &= 90^\circ - (65^\circ - 25^\circ) \\ &= 50^\circ\end{aligned}$$

$$\begin{aligned}Q &= (20 \text{ lb}) \cos 50^\circ \\ &= 12.8558 \text{ lb} \\ M_B &= Q(9 \text{ in.}) \\ &= (12.8558 \text{ lb})(9 \text{ in.}) \\ &= 115.702 \text{ lb-in.}\end{aligned}$$

$$M_B = 115.7 \text{ lb-in.} \quad \curvearrowleft$$

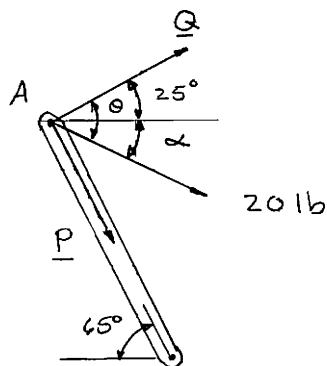


PROBLEM 3.3

A 20-lb force is applied to the control rod AB as shown. Knowing that the length of the rod is 9 in. and that the moment of the force about B is 120 lb·in. clockwise, determine the value of α .

SOLUTION

Free-Body Diagram of Rod AB :



$$\alpha = \theta - 25^\circ$$

$$Q = (20 \text{ lb}) \cos \theta$$

and $M_B = (Q)(9 \text{ in.})$

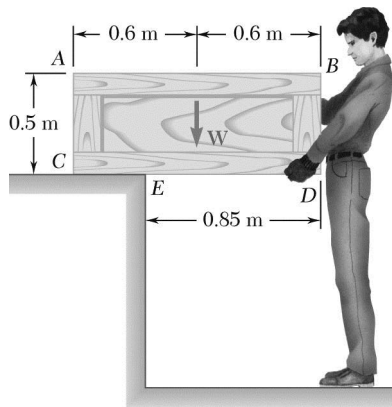
Therefore, $120 \text{ lb-in.} = (20 \text{ lb})(\cos \theta)(9 \text{ in.})$

$$\cos \theta = \frac{120 \text{ lb-in.}}{180 \text{ lb-in.}}$$

or $\theta = 48.190^\circ$

Therefore, $\alpha = 48.190^\circ - 25^\circ$

$$\alpha = 23.2^\circ \blacktriangleleft$$



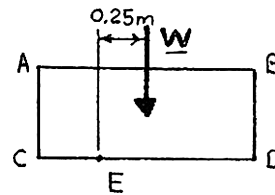
PROBLEM 3.4

A crate of mass 80 kg is held in the position shown. Determine
 (a) the moment produced by the weight \mathbf{W} of the crate about E ,
 (b) the smallest force applied at B that creates a moment of equal magnitude and opposite sense about E .

SOLUTION

(a) By definition, $W = mg = 80 \text{ kg}(9.81 \text{ m/s}^2) = 784.8 \text{ N}$

We have $\Sigma M_E: M_E = (784.8 \text{ N})(0.25 \text{ m})$



$M_E = 196.2 \text{ N} \cdot \text{m} \curvearrowleft$

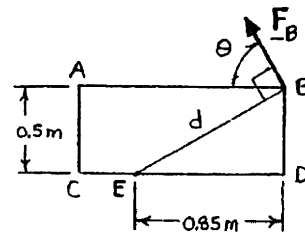
(b) For the force at B to be the smallest, resulting in a moment (M_E) about E , the line of action of force \mathbf{F}_B must be perpendicular to the line connecting E to B . The sense of \mathbf{F}_B must be such that the force produces a counterclockwise moment about E .

Note: $d = \sqrt{(0.85 \text{ m})^2 + (0.5 \text{ m})^2} = 0.98615 \text{ m}$

We have $\Sigma M_E: 196.2 \text{ N} \cdot \text{m} = F_B(0.98615 \text{ m})$

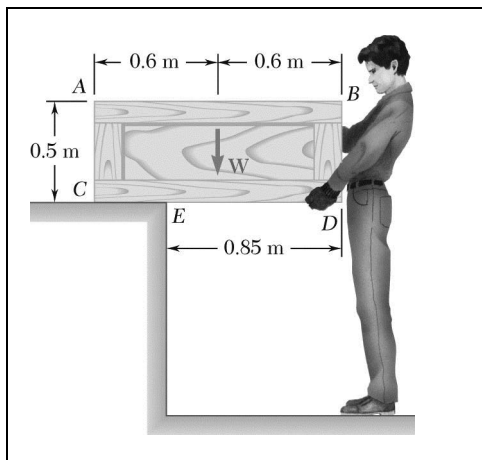
$F_B = 198.954 \text{ N}$

and $\theta = \tan^{-1}\left(\frac{0.85 \text{ m}}{0.5 \text{ m}}\right) = 59.534^\circ$



$F_B = 199.0 \text{ N} \curvearrowleft 59.5^\circ$

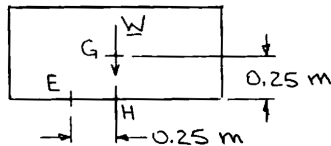
or



PROBLEM 3.5

A crate of mass 80 kg is held in the position shown. Determine (a) the moment produced by the weight W of the crate about E , (b) the smallest force applied at A that creates a moment of equal magnitude and opposite sense about E , (c) the magnitude, sense, and point of application on the bottom of the crate of the smallest vertical force that creates a moment of equal magnitude and opposite sense about E .

SOLUTION

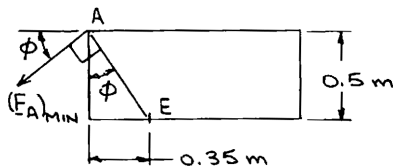


First note. . .

$$W = mg = (80 \text{ kg})(9.81 \text{ m/s}^2) = 784.8 \text{ N}$$

(a) We have $M_E = r_{H/E} W = (0.25 \text{ m})(784.8 \text{ N}) = 196.2 \text{ N} \cdot \text{m}$ or $\mathbf{M}_E = 196.2 \text{ N} \cdot \text{m}$ ◀

(b)



For F_A to be minimum, it must be perpendicular to the line joining Points A and E . Then with F_A directed as shown, we have $(-M_E) = r_{A/E} (F_A)_{\min}$.

Where $r_{A/E} = \sqrt{(0.35 \text{ m})^2 + (0.5 \text{ m})^2} = 0.61033 \text{ m}$

then $196.2 \text{ N} \cdot \text{m} = (0.61033 \text{ m})(F_A)_{\min}$

or $(F_A)_{\min} = 321 \text{ N}$

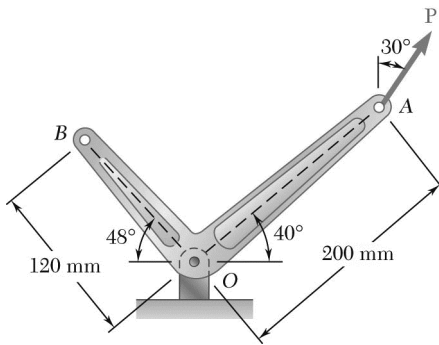
Also $\tan \phi = \frac{0.35 \text{ m}}{0.5 \text{ m}}$ or $\phi = 35.0^\circ$ $(F_A)_{\min} = 321 \text{ N} \nearrow 35.0^\circ$ ◀

(c) For F_{vertical} to be minimum, the perpendicular distance from its line of action to Point E must be maximum. Thus, apply $(F_{\text{vertical}})_{\min}$ at Point D , and then

$$(-M_E) = r_{D/E} (F_{\text{vertical}})_{\min}$$

$$196.2 \text{ N} \cdot \text{m} = (0.85 \text{ m})(F_{\text{vertical}})_{\min}$$

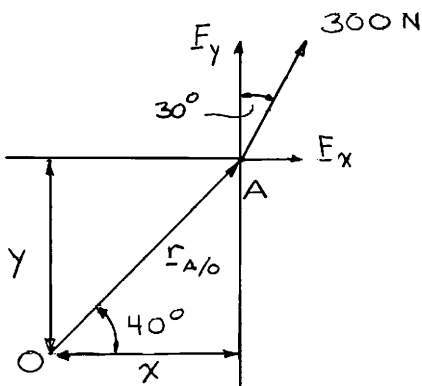
or $(F_{\text{vertical}})_{\min} = 231 \text{ N} \uparrow$ at Point D ◀



PROBLEM 3.6

A 300-N force \mathbf{P} is applied at Point A of the bell crank shown. (a) Compute the moment of the force \mathbf{P} about O by resolving it into horizontal and vertical components. (b) Using the result of part (a), determine the perpendicular distance from O to the line of action of \mathbf{P} .

SOLUTION



$$x = (0.2 \text{ m}) \cos 40^\circ$$

$$= 0.153209 \text{ m}$$

$$y = (0.2 \text{ m}) \sin 40^\circ$$

$$= 0.128558 \text{ m}$$

$$\therefore \mathbf{r}_{A/O} = (0.153209 \text{ m})\mathbf{i} + (0.128558 \text{ m})\mathbf{j}$$

$$(a) \quad F_x = (300 \text{ N}) \sin 30^\circ$$

$$= 150 \text{ N}$$

$$F_y = (300 \text{ N}) \cos 30^\circ$$

$$= 259.81 \text{ N}$$

$$\mathbf{F} = (150 \text{ N})\mathbf{i} + (259.81 \text{ N})\mathbf{j}$$

$$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{F}$$

$$= (0.153209\mathbf{i} + 0.128558\mathbf{j}) \text{ m} \times (150\mathbf{i} + 259.81\mathbf{j}) \text{ N}$$

$$= (39.805\mathbf{k} - 19.2837\mathbf{k}) \text{ N} \cdot \text{m}$$

$$= (20.521 \text{ N} \cdot \text{m})\mathbf{k}$$

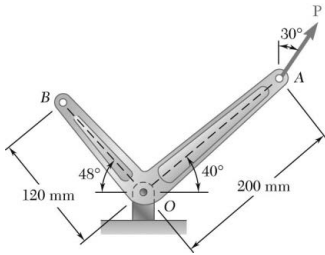
$$\mathbf{M}_O = 20.5 \text{ N} \cdot \text{m} \curvearrowleft$$

$$(b) \quad M_O = Fd$$

$$20.521 \text{ N} \cdot \text{m} = (300 \text{ N})(d)$$

$$d = 0.068403 \text{ m}$$

$$d = 68.4 \text{ mm} \curvearrowleft$$



PROBLEM 3.7

A 400-N force \mathbf{P} is applied at Point A of the bell crank shown. (a) Compute the moment of the force \mathbf{P} about O by resolving it into components along line OA and in a direction perpendicular to that line. (b) Determine the magnitude and direction of the smallest force \mathbf{Q} applied at B that has the same moment as \mathbf{P} about O .

SOLUTION

(a) Portion OA of crank:

$$\theta = 90^\circ - 30^\circ - 40^\circ$$

$$\theta = 20^\circ$$

$$S = P \sin \theta$$

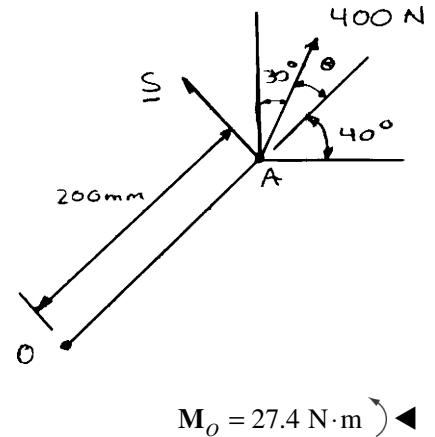
$$= (400 \text{ N}) \sin 20^\circ$$

$$= 136.81 \text{ N}$$

$$M_O = r_{O/A} S$$

$$= (0.2 \text{ m})(136.81 \text{ N})$$

$$= 27.362 \text{ N}\cdot\text{m}$$



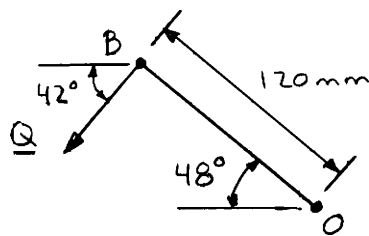
(b) Smallest force \mathbf{Q} must be perpendicular to OB .

Portion OB of crank:

$$M_O = r_{O/B} Q$$

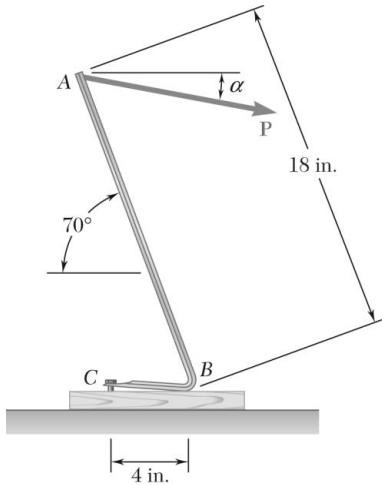
$$M_O = (0.120 \text{ m}) Q$$

$$27.362 \text{ N}\cdot\text{m} = (0.120 \text{ m}) Q$$



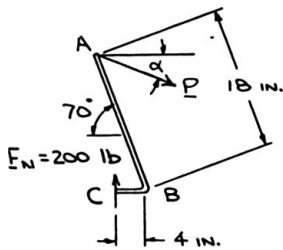
$$Q = 228 \text{ N} \nearrow 42.0^\circ$$

PROBLEM 3.8



It is known that a vertical force of 200 lb is required to remove the nail at C from the board. As the nail first starts moving, determine (a) the moment about B of the force exerted on the nail, (b) the magnitude of the force \mathbf{P} that creates the same moment about B if $\alpha = 10^\circ$, (c) the smallest force \mathbf{P} that creates the same moment about B.

SOLUTION



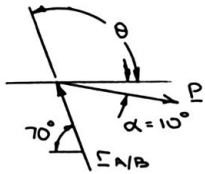
(a) We have $M_B = r_{CB} F_N$
 $= (4 \text{ in.})(200 \text{ lb})$
 $= 800 \text{ lb} \cdot \text{in.}$

or $M_B = 800 \text{ lb} \cdot \text{in.}$ ◀

(b) By definition, $M_B = r_{AB} P \sin \theta$
 $\theta = 10^\circ + (180^\circ - 70^\circ)$
 $= 120^\circ$

Then $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) \times P \sin 120^\circ$

or $P = 51.3 \text{ lb}$ ◀



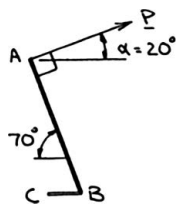
(c) For \mathbf{P} to be minimum, it must be perpendicular to the line joining Points A and B. Thus, \mathbf{P} must be directed as shown.

Thus $M_B = d P_{\min}$
 $d = r_{A/B}$

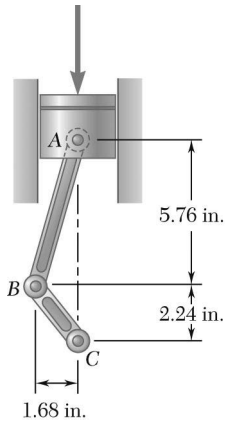
or $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) P_{\min}$

or $P_{\min} = 44.4 \text{ lb}$

$\mathbf{P}_{\min} = 44.4 \text{ lb}$ ◀ 20°



PROBLEM 3.9

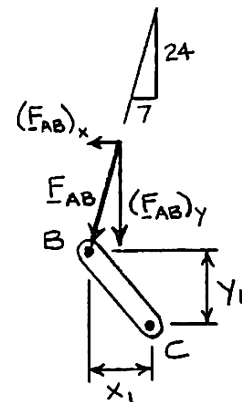


It is known that the connecting rod AB exerts on the crank BC a 500-lb force directed down and to the left along the centerline of AB . Determine the moment of the force about C .

SOLUTION

Using (a):

$$\begin{aligned} M_C &= y_1(F_{AB})_x + x_1(F_{AB})_y \\ &= (2.24 \text{ in.})\left(\frac{7}{25} \times 500 \text{ lb}\right) + (1.68 \text{ in.})\left(\frac{24}{25} \times 500 \text{ lb}\right) \\ &= 1120 \text{ lb} \cdot \text{in.} \end{aligned}$$

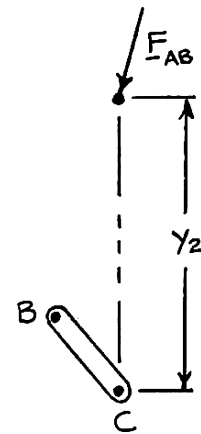


(a)

$$M_C = 1.120 \text{ kip} \cdot \text{in.} \quad \curvearrowright \blacktriangleleft$$

Using (b):

$$\begin{aligned} M_C &= y_2(F_{AB})_x \\ &= (8 \text{ in.})\left(\frac{7}{25} \times 500 \text{ lb}\right) \\ &= 1120 \text{ lb} \cdot \text{in.} \end{aligned}$$

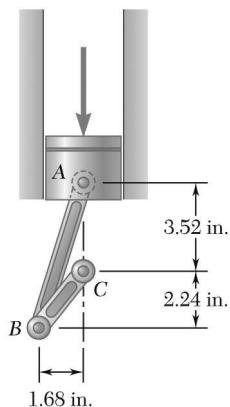


(b)

$$M_C = 1.120 \text{ kip} \cdot \text{in.} \quad \curvearrowright \blacktriangleleft$$

PROBLEM 3.10

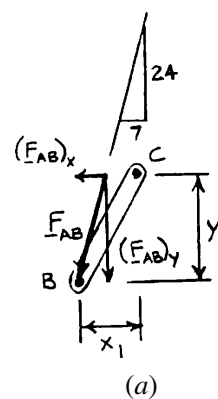
It is known that the connecting rod AB exerts on the crank BC a 500-lb force directed down and to the left along the centerline of AB . Determine the moment of the force about C .



SOLUTION

Using (a):

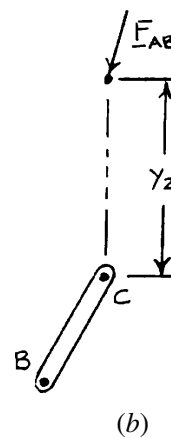
$$\begin{aligned} M_C &= -y_1(F_{AB})_x + x_1(F_{AB})_y \\ &= -(2.24 \text{ in.})\left(\frac{7}{25} \times 500 \text{ lb}\right) + (1.68 \text{ in.})\left(\frac{24}{25} \times 500 \text{ lb}\right) \\ &= +492.8 \text{ lb} \cdot \text{in.} \end{aligned}$$



$$M_C = 493 \text{ lb} \cdot \text{in.} \curvearrowleft$$

Using (b):

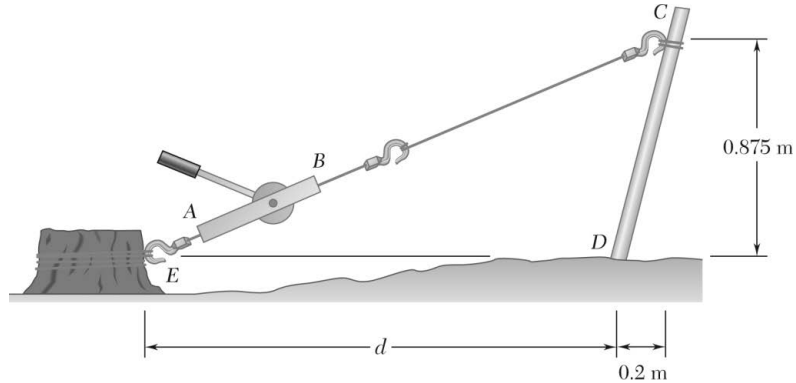
$$\begin{aligned} M_C &= y_2(F_{AB})_x \\ &= (3.52 \text{ in.})\left(\frac{7}{25} \times 500 \text{ lb}\right) \\ &= +492.8 \text{ lb} \cdot \text{in.} \end{aligned}$$



$$M_C = 493 \text{ lb} \cdot \text{in.} \curvearrowleft$$

PROBLEM 3.11

A winch puller AB is used to straighten a fence post. Knowing that the tension in cable BC is 1040 N and length d is 1.90 m, determine the moment about D of the force exerted by the cable at C by resolving that force into horizontal and vertical components applied (a) at Point C , (b) at Point E .



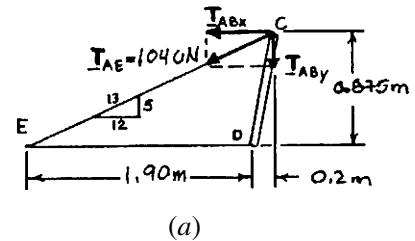
SOLUTION

(a) Slope of line: $EC = \frac{0.875 \text{ m}}{1.90 \text{ m} + 0.2 \text{ m}} = \frac{5}{12}$

Then $T_{ABx} = \frac{12}{13}(T_{AB})$
 $= \frac{12}{13}(1040 \text{ N})$
 $= 960 \text{ N}$

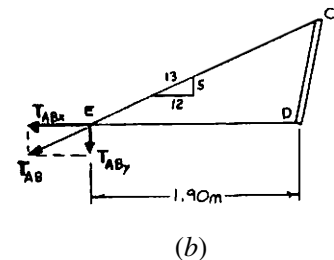
and $T_{ABy} = \frac{5}{13}(1040 \text{ N})$
 $= 400 \text{ N}$

Then $M_D = T_{ABx}(0.875 \text{ m}) - T_{ABy}(0.2 \text{ m})$
 $= (960 \text{ N})(0.875 \text{ m}) - (400 \text{ N})(0.2 \text{ m})$
 $= 760 \text{ N} \cdot \text{m}$



or $M_D = 760 \text{ N} \cdot \text{m}$ ◀

(b) We have $M_D = T_{ABx}(y) + T_{ABy}(x)$
 $= (960 \text{ N})(0) + (400 \text{ N})(1.90 \text{ m})$
 $= 760 \text{ N} \cdot \text{m}$

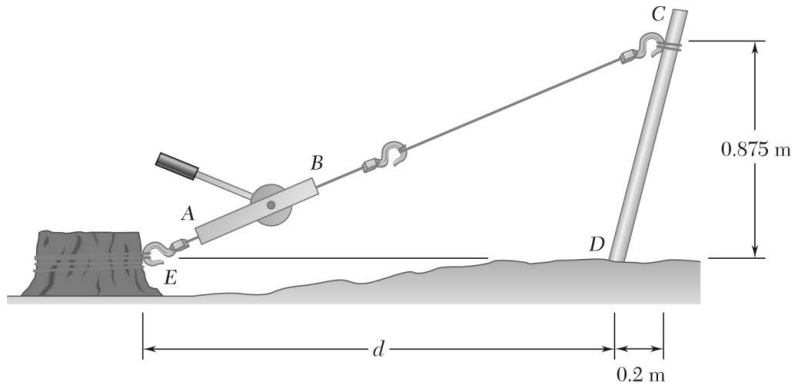


or $M_D = 760 \text{ N} \cdot \text{m}$ ◀

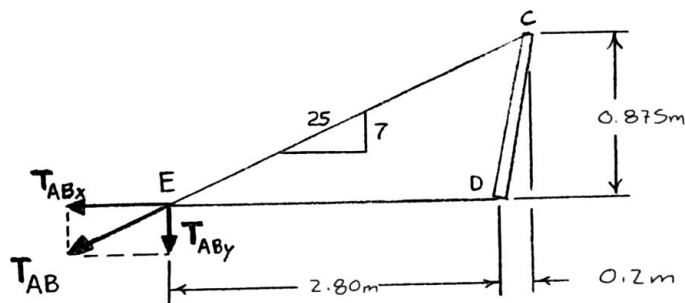
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PROBLEM 3.12

It is known that a force with a moment of $960 \text{ N} \cdot \text{m}$ about D is required to straighten the fence post CD . If $d = 2.80 \text{ m}$, determine the tension that must be developed in the cable of winch puller AB to create the required moment about Point D .



SOLUTION



Slope of line:

$$EC = \frac{0.875 \text{ m}}{2.80 \text{ m} + 0.2 \text{ m}} = \frac{7}{24}$$

Then

$$T_{ABx} = \frac{24}{25} T_{AB}$$

and

$$T_{ABy} = \frac{7}{25} T_{AB}$$

We have

$$M_D = T_{ABx}(y) + T_{ABy}(x)$$

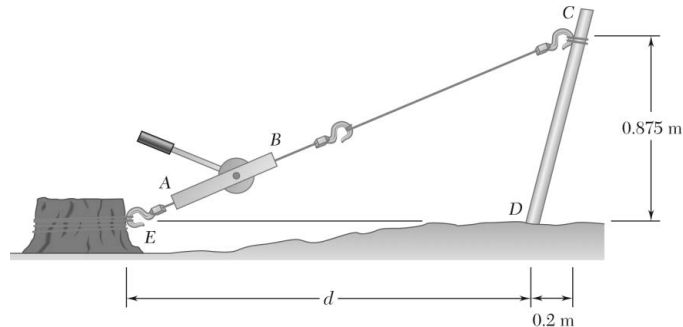
$$960 \text{ N} \cdot \text{m} = \frac{24}{25} T_{AB}(0) + \frac{7}{25} T_{AB}(2.80 \text{ m})$$

$$T_{AB} = 1224 \text{ N}$$

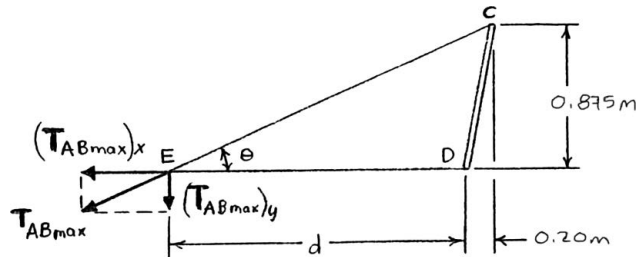
$$\text{or } T_{AB} = 1224 \text{ N} \blacktriangleleft$$

PROBLEM 3.13

It is known that a force with a moment of $960 \text{ N} \cdot \text{m}$ about D is required to straighten the fence post CD . If the capacity of winch puller AB is 2400 N , determine the minimum value of distance d to create the specified moment about Point D .



SOLUTION



The minimum value of d can be found based on the equation relating the moment of the force \mathbf{T}_{AB} about D :

$$M_D = (T_{AB\max})_y (d)$$

where

$$M_D = 960 \text{ N} \cdot \text{m}$$

$$(T_{AB\max})_y = T_{AB\max} \sin \theta = (2400 \text{ N}) \sin \theta$$

Now

$$\sin \theta = \frac{0.875 \text{ m}}{\sqrt{(d + 0.20)^2 + (0.875)^2} \text{ m}}$$

$$960 \text{ N} \cdot \text{m} = 2400 \text{ N} \left[\frac{0.875}{\sqrt{(d + 0.20)^2 + (0.875)^2}} \right] (d)$$

or

$$\sqrt{(d + 0.20)^2 + (0.875)^2} = 2.1875d$$

or

$$(d + 0.20)^2 + (0.875)^2 = 4.7852d^2$$

or

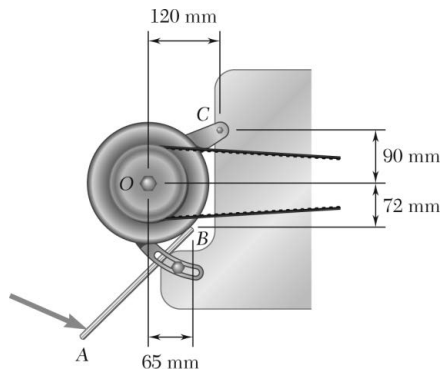
$$3.7852d^2 - 0.40d - 0.8056 = 0$$

Using the quadratic equation, the minimum values of d are 0.51719 m and -0.41151 m .

Since only the positive value applies here, $d = 0.51719 \text{ m}$

or $d = 517 \text{ mm}$ ◀

PROBLEM 3.14



A mechanic uses a piece of pipe AB as a lever when tightening an alternator belt. When he pushes down at A , a force of 485 N is exerted on the alternator at B . Determine the moment of that force about bolt C if its line of action passes through O .

SOLUTION

We have

$$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$$

Noting the direction of the moment of each force component about C is clockwise,

$$M_C = xF_{By} + yF_{Bx}$$

where

$$x = 120 \text{ mm} - 65 \text{ mm} = 55 \text{ mm}$$

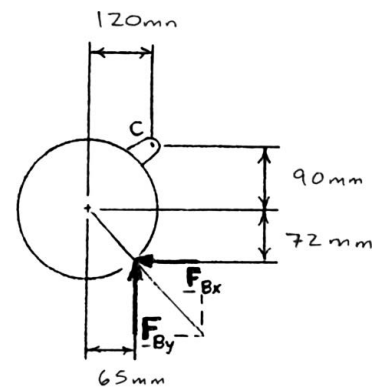
$$y = 72 \text{ mm} + 90 \text{ mm} = 162 \text{ mm}$$

and

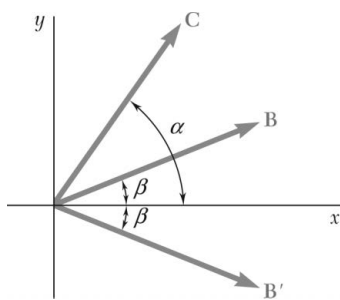
$$F_{Bx} = \frac{65}{\sqrt{(65)^2 + (72)^2}} (485 \text{ N}) = 325 \text{ N}$$

$$F_{By} = \frac{72}{\sqrt{(65)^2 + (72)^2}} (485 \text{ N}) = 360 \text{ N}$$

$$\begin{aligned} M_C &= (55 \text{ mm})(360 \text{ N}) + (162)(325 \text{ N}) \\ &= 72450 \text{ N} \cdot \text{m} \\ &= 72.450 \text{ N} \cdot \text{m} \end{aligned}$$



$$\text{or } \mathbf{M}_C = 72.5 \text{ N} \cdot \text{m} \curvearrowright$$



PROBLEM 3.15

Form the vector products $\mathbf{B} \times \mathbf{C}$ and $\mathbf{B}' \times \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta).$$

SOLUTION

Note:

$$\mathbf{B} = B(\cos \beta \mathbf{i} + \sin \beta \mathbf{j})$$

$$\mathbf{B}' = B(\cos \beta \mathbf{i} - \sin \beta \mathbf{j})$$

$$\mathbf{C} = C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$$

By definition, $|\mathbf{B} \times \mathbf{C}| = BC \sin(\alpha - \beta)$ (1)

$$|\mathbf{B}' \times \mathbf{C}| = BC \sin(\alpha + \beta)$$
 (2)

Now $\mathbf{B} \times \mathbf{C} = B(\cos \beta \mathbf{i} + \sin \beta \mathbf{j}) \times C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$

$$= BC(\cos \beta \sin \alpha - \sin \beta \cos \alpha) \mathbf{k}$$
 (3)

and $\mathbf{B}' \times \mathbf{C} = B(\cos \beta \mathbf{i} - \sin \beta \mathbf{j}) \times C(\cos \alpha \mathbf{i} + \sin \alpha \mathbf{j})$

$$= BC(\cos \beta \sin \alpha + \sin \beta \cos \alpha) \mathbf{k}$$
 (4)

Equating the magnitudes of $\mathbf{B} \times \mathbf{C}$ from Equations (1) and (3) yields:

$$BC \sin(\alpha - \beta) = BC(\cos \beta \sin \alpha - \sin \beta \cos \alpha)$$
 (5)

Similarly, equating the magnitudes of $\mathbf{B}' \times \mathbf{C}$ from Equations (2) and (4) yields:

$$BC \sin(\alpha + \beta) = BC(\cos \beta \sin \alpha + \sin \beta \cos \alpha)$$
 (6)

Adding Equations (5) and (6) gives:

$$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2 \cos \beta \sin \alpha$$

$$\text{or } \sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha + \beta) + \frac{1}{2} \sin(\alpha - \beta) \blacktriangleleft$$

PROBLEM 3.16

The vectors \mathbf{P} and \mathbf{Q} are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (a) $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, (b) $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

SOLUTION

(a) We have

$$A = |\mathbf{P} \times \mathbf{Q}|$$

where

$$\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$$

$$\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$$

Then

$$\begin{aligned}\mathbf{P} \times \mathbf{Q} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 3 & -3 \\ 2 & 2 & 5 \end{vmatrix} \\ &= [(15 + 6)\mathbf{i} + (-6 + 35)\mathbf{j} + (-14 - 6)\mathbf{k}] \\ &= (21)\mathbf{i} + (29)\mathbf{j} - (20)\mathbf{k}\end{aligned}$$

$$A = \sqrt{(20)^2 + (29)^2 + (-20)^2} \quad \text{or } A = 41.0 \blacktriangleleft$$

(b) We have

$$A = |\mathbf{P} \times \mathbf{Q}|$$

where

$$\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$$

$$\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$$

Then

$$\begin{aligned}\mathbf{P} \times \mathbf{Q} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -5 & -2 \\ -2 & 5 & -1 \end{vmatrix} \\ &= [(5 + 10)\mathbf{i} + (4 + 6)\mathbf{j} + (30 - 10)\mathbf{k}] \\ &= (15)\mathbf{i} + (10)\mathbf{j} + (20)\mathbf{k}\end{aligned}$$

$$A = \sqrt{(15)^2 + (10)^2 + (20)^2} \quad \text{or } A = 26.9 \blacktriangleleft$$

PROBLEM 3.17

A plane contains the vectors \mathbf{A} and \mathbf{B} . Determine the unit vector normal to the plane when \mathbf{A} and \mathbf{B} are equal to, respectively, (a) $\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$, (b) $3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $-2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$.

SOLUTION

(a) We have

$$\lambda = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

where

$$\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$$

$$\mathbf{B} = 4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$$

Then

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -5 \\ 4 & -7 & -5 \end{vmatrix} \\ &= (-10 - 35)\mathbf{i} + (20 + 5)\mathbf{j} + (-7 - 8)\mathbf{k} \\ &= 15(3\mathbf{i} - \mathbf{j} - \mathbf{k})\end{aligned}$$

and

$$|\mathbf{A} \times \mathbf{B}| = 15\sqrt{(-3)^2 + (-1)^2 + (-1)^2} = 15\sqrt{11}$$

$$\lambda = \frac{15(-3\mathbf{i} - \mathbf{j} - \mathbf{k})}{15\sqrt{11}}$$

$$\text{or } \lambda = \frac{1}{\sqrt{11}}(-3\mathbf{i} - \mathbf{j} - \mathbf{k}) \blacktriangleleft$$

(b) We have

$$\lambda = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

where

$$\mathbf{A} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$$

Then

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 2 \\ -2 & 6 & -4 \end{vmatrix} \\ &= (12 - 12)\mathbf{i} + (-4 + 12)\mathbf{j} + (18 - 6)\mathbf{k} \\ &= (8\mathbf{j} + 12\mathbf{k})\end{aligned}$$

and

$$|\mathbf{A} \times \mathbf{B}| = 4\sqrt{(2)^2 + (3)^2} = 4\sqrt{13}$$

$$\lambda = \frac{4(2\mathbf{j} + 3\mathbf{k})}{4\sqrt{13}}$$

$$\text{or } \lambda = \frac{1}{\sqrt{13}}(2\mathbf{j} + 3\mathbf{k}) \blacktriangleleft$$

PROBLEM 3.18

A line passes through the Points (20 m, 16 m) and (-1 m, -4 m). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

SOLUTION

$$d_{AB} = \sqrt{[20 \text{ m} - (-1 \text{ m})]^2 + [16 \text{ m} - (-4 \text{ m})]^2}$$
$$= 29 \text{ m}$$

Assume that a force \mathbf{F} , or magnitude F (N), acts at Point A and is directed from A to B.

Then

$$\mathbf{F} = F\lambda_{AB}$$

where

$$\lambda_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{d_{AB}}$$
$$= \frac{1}{29}(21\mathbf{i} + 20\mathbf{j})$$

By definition,

$$\mathbf{M}_O = |\mathbf{r}_A \times \mathbf{F}| = dF$$

where

$$\mathbf{r}_A = -(1 \text{ m})\mathbf{i} - (4 \text{ m})\mathbf{j}$$

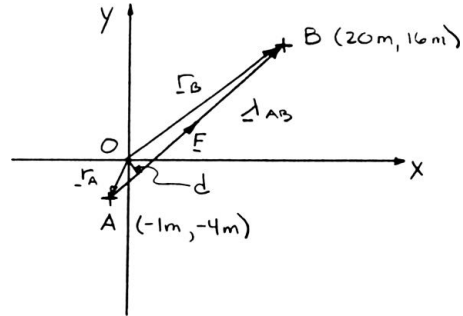
Then

$$\mathbf{M}_O = [-(-1 \text{ m})\mathbf{i} - (4 \text{ m})\mathbf{j}] \times \frac{F}{29 \text{ m}} [(21 \text{ m})\mathbf{i} + (20 \text{ m})\mathbf{j}]$$
$$= \frac{F}{29} [-(20)\mathbf{k} + (84)\mathbf{k}]$$
$$= \left(\frac{64}{29} F \right) \mathbf{k} \text{ N} \cdot \text{m}$$

Finally,

$$\left(\frac{64}{29} F \right) = d(F)$$
$$d = \frac{64}{29} \text{ m}$$

$$d = 2.21 \text{ m} \quad \blacktriangleleft$$



PROBLEM 3.19

Determine the moment about the origin O of the force $\mathbf{F} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ that acts at a Point A . Assume that the position vector of A is (a) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, (b) $\mathbf{r} = -8\mathbf{i} + 6\mathbf{j} - 10\mathbf{k}$, (c) $\mathbf{r} = 8\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$.

SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$\begin{aligned} \text{(a)} \quad \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -4 \\ 4 & -3 & 5 \end{vmatrix} \\ &= (15 - 12)\mathbf{i} + (-16 - 10)\mathbf{j} + (-6 - 12)\mathbf{k} & \mathbf{M}_O = 3\mathbf{i} - 26\mathbf{j} - 18\mathbf{k} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 6 & -10 \\ 4 & -3 & 5 \end{vmatrix} \\ &= (30 - 30)\mathbf{i} + (-40 + 40)\mathbf{j} + (24 - 24)\mathbf{k} & \mathbf{M}_O = 0 \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & -6 & 5 \\ 4 & -3 & 5 \end{vmatrix} \\ &= (-30 + 15)\mathbf{i} + (20 - 40)\mathbf{j} + (-24 + 24)\mathbf{k} & \mathbf{M}_O = -15\mathbf{i} - 20\mathbf{j} \quad \blacktriangleleft \end{aligned}$$

Note: The answer to Part (b) could have been anticipated since the elements of the last two rows of the determinant are proportional.

PROBLEM 3.20

Determine the moment about the origin O of the force $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ that acts at a Point A . Assume that the position vector of A is (a) $\mathbf{r} = 3\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$, (b) $\mathbf{r} = \mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$, (c) $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$.

SOLUTION

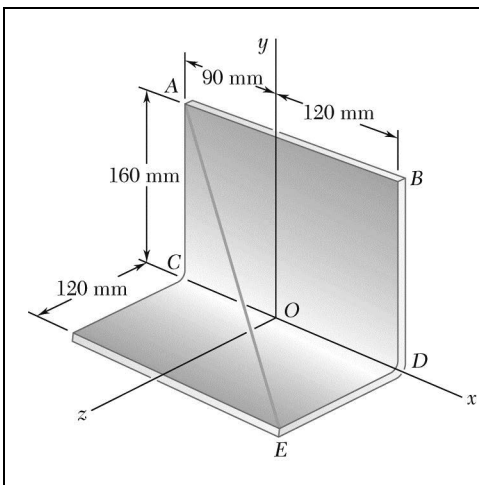
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$\begin{aligned} \text{(a)} \quad \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 5 \\ 2 & 3 & -4 \end{vmatrix} \\ &= (24 - 15)\mathbf{i} + (10 + 12)\mathbf{j} + (9 + 12)\mathbf{k} & \mathbf{M}_O = 9\mathbf{i} + 22\mathbf{j} + 21\mathbf{k} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & -2 \\ 2 & 3 & -4 \end{vmatrix} \\ &= (16 + 6)\mathbf{i} + (-4 + 4)\mathbf{j} + (3 + 8)\mathbf{k} & \mathbf{M}_O = 22\mathbf{i} + 11\mathbf{k} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -8 \\ 2 & 3 & -4 \end{vmatrix} \\ &= (-24 + 24)\mathbf{i} + (-16 + 16)\mathbf{j} + (12 - 12)\mathbf{k} & \mathbf{M}_O = 0 \quad \blacktriangleleft \end{aligned}$$

Note: The answer to Part (c) could have been anticipated since the elements of the last two rows of the determinant are proportional.



PROBLEM 3.21

The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (a) on corner A , (b) on corner E .

SOLUTION

$$\overline{AE} = (0.21 \text{ m})\mathbf{i} - (0.16 \text{ m})\mathbf{j} + (0.12 \text{ m})\mathbf{k}$$

$$AE = \sqrt{(0.21 \text{ m})^2 + (-0.16 \text{ m})^2 + (0.12 \text{ m})^2} = 0.29 \text{ m}$$

$$\begin{aligned} (a) \quad \mathbf{F}_A &= F_A \lambda_{AE} = F \frac{\overline{AE}}{AE} \\ &= (435 \text{ N}) \frac{0.21\mathbf{i} - 0.16\mathbf{j} + 0.12\mathbf{k}}{0.29} \\ &= (315 \text{ N})\mathbf{i} - (240 \text{ N})\mathbf{j} + (180 \text{ N})\mathbf{k} \\ \mathbf{r}_{A/O} &= -(0.09 \text{ m})\mathbf{i} + (0.16 \text{ m})\mathbf{j} \end{aligned}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.09 & 0.16 & 0 \\ 315 & -240 & 180 \end{vmatrix}$$

$$= 28.8\mathbf{i} + 16.20\mathbf{j} + (21.6 - 50.4)\mathbf{k}$$

$$\mathbf{M}_O = (28.8 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{j} - (28.8 \text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$

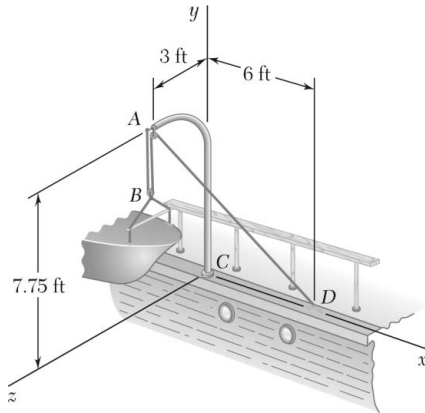
$$\begin{aligned} (b) \quad \mathbf{F}_E &= -\mathbf{F}_A = -(315 \text{ N})\mathbf{i} + (240 \text{ N})\mathbf{j} - (180 \text{ N})\mathbf{k} \\ \mathbf{r}_{E/O} &= (0.12 \text{ m})\mathbf{i} + (0.12 \text{ m})\mathbf{k} \end{aligned}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0.12 \\ -315 & 240 & -180 \end{vmatrix}$$

$$= -28.8\mathbf{i} + (-37.8 + 21.6)\mathbf{j} + 28.8\mathbf{k}$$

$$\mathbf{M}_O = -(28.8 \text{ N}\cdot\text{m})\mathbf{i} - (16.20 \text{ N}\cdot\text{m})\mathbf{j} + (28.8 \text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.22



A small boat hangs from two davits, one of which is shown in the figure. The tension in line $ABAD$ is 82 lb. Determine the moment about C of the resultant force \mathbf{R}_A exerted on the davit at A .

SOLUTION

We have

$$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD}$$

where

$$\mathbf{F}_{AB} = -(82 \text{ lb})\mathbf{j}$$

and

$$\mathbf{F}_{AD} = F_{AD} \frac{\overline{AD}}{AD} = (82 \text{ lb}) \frac{6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k}}{10.25}$$

$$\mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (62 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

Thus

$$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (226 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

Also

$$\mathbf{r}_{AC} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$$

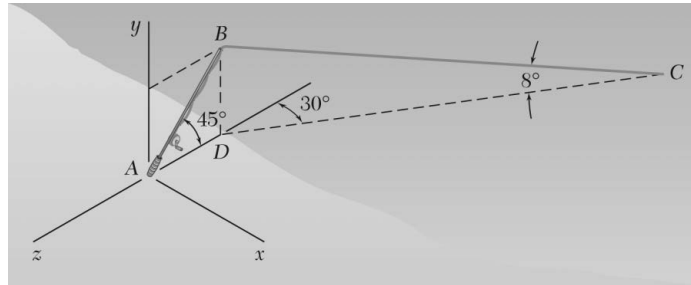
Using Eq. (3.21):

$$\begin{aligned} \mathbf{M}_C &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7.75 & 3 \\ 48 & -226 & -24 \end{vmatrix} \\ &= (492 \text{ lb} \cdot \text{ft})\mathbf{i} + (144.0 \text{ lb} \cdot \text{ft})\mathbf{j} - (372 \text{ lb} \cdot \text{ft})\mathbf{k} \end{aligned}$$

$$\mathbf{M}_C = (492 \text{ lb} \cdot \text{ft})\mathbf{i} + (144.0 \text{ lb} \cdot \text{ft})\mathbf{j} - (372 \text{ lb} \cdot \text{ft})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.23

A 6-ft-long fishing rod AB is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about A of the force exerted by the line at B .



SOLUTION

We have

$$T_{xz} = (6 \text{ lb}) \cos 8^\circ = 5.9416 \text{ lb}$$

Then

$$T_x = T_{xz} \sin 30^\circ = 2.9708 \text{ lb}$$

$$T_y = T_{xz} \sin 8^\circ = -0.83504 \text{ lb}$$

$$T_z = T_{xz} \cos 30^\circ = -5.1456 \text{ lb}$$

Now

$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$$

where

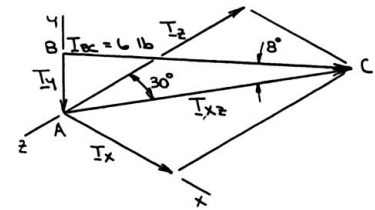
$$\begin{aligned} \mathbf{r}_{B/A} &= (6 \sin 45^\circ)\mathbf{j} - (6 \cos 45^\circ)\mathbf{k} \\ &= \frac{6 \text{ ft}}{\sqrt{2}}(\mathbf{j} - \mathbf{k}) \end{aligned}$$

Then

$$\begin{aligned} \mathbf{M}_A &= \frac{6}{\sqrt{2}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix} \\ &= \frac{6}{\sqrt{2}}(-5.1456 - 0.83504)\mathbf{i} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{j} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{k} \end{aligned}$$

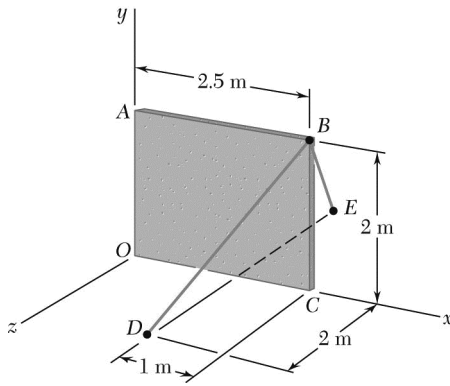
or

$$\mathbf{M}_A = -(25.4 \text{ lb} \cdot \text{ft})\mathbf{i} - (12.60 \text{ lb} \cdot \text{ft})\mathbf{j} - (12.60 \text{ lb} \cdot \text{ft})\mathbf{k}$$



PROBLEM 3.24

A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about Point O of the force exerted by the cable at B .



SOLUTION

$$\mathbf{F} = F \frac{\overline{BD}}{BD} \quad \text{where } F = 900 \text{ N}$$

$$\overline{BD} = -(1 \text{ m})\mathbf{i} - (2 \text{ m})\mathbf{j} + (2 \text{ m})\mathbf{k}$$

$$BD = \sqrt{(-1 \text{ m})^2 + (-2 \text{ m})^2 + (2 \text{ m})^2}$$
$$= 3 \text{ m}$$

$$\mathbf{F} = (900 \text{ N}) \frac{-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}}{3}$$
$$= -(300 \text{ N})\mathbf{i} - (600 \text{ N})\mathbf{j} + (600 \text{ N})\mathbf{k}$$

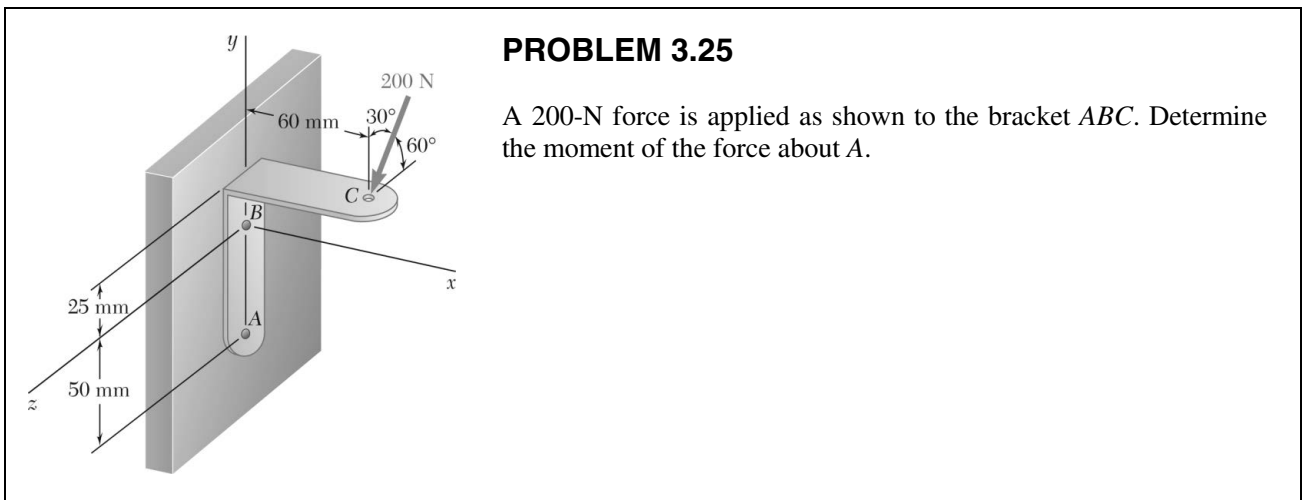
$$\mathbf{r}_{B/O} = (2.5 \text{ m})\mathbf{i} + (2 \text{ m})\mathbf{j}$$

$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 2 & 0 \\ -300 & -600 & 600 \end{vmatrix}$$

$$= 1200\mathbf{i} - 1500\mathbf{j} + (-1500 + 600)\mathbf{k}$$

$$\mathbf{M}_O = (1200 \text{ N}\cdot\text{m})\mathbf{i} - (1500 \text{ N}\cdot\text{m})\mathbf{j} - (900 \text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.25

A 200-N force is applied as shown to the bracket ABC . Determine the moment of the force about A .

SOLUTION

We have

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$$

where

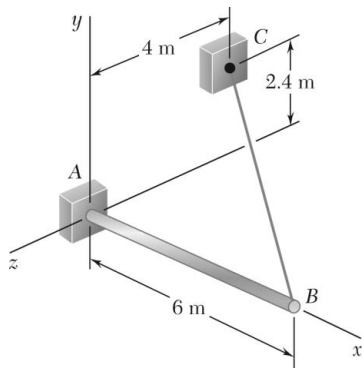
$$\mathbf{r}_{C/A} = (0.06 \text{ m})\mathbf{i} + (0.075 \text{ m})\mathbf{j}$$

$$\mathbf{F}_C = -(200 \text{ N})\cos 30^\circ\mathbf{j} + (200 \text{ N})\sin 30^\circ\mathbf{k}$$

Then

$$\begin{aligned} \mathbf{M}_A &= 200 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30^\circ & \sin 30^\circ \end{vmatrix} \\ &= 200[(0.075 \sin 30^\circ)\mathbf{i} - (0.06 \sin 30^\circ)\mathbf{j} - (0.06 \cos 30^\circ)\mathbf{k}] \end{aligned}$$

$$\text{or } \mathbf{M}_A = (7.50 \text{ N}\cdot\text{m})\mathbf{i} - (6.00 \text{ N}\cdot\text{m})\mathbf{j} - (10.39 \text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.26

The 6-m boom AB has a fixed end A . A steel cable is stretched from the free end B of the boom to a Point C located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about A of the force exerted by the cable at B .

SOLUTION

First note
$$d_{BC} = \sqrt{(-6)^2 + (2.4)^2 + (-4)^2}$$

$$= 7.6 \text{ m}$$

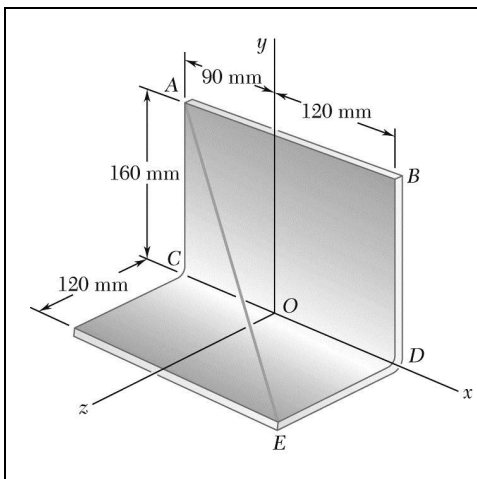
Then
$$\mathbf{T}_{BC} = \frac{2.5 \text{ kN}}{7.6} (-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

We have
$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$$

where
$$\mathbf{r}_{B/A} = (6 \text{ m})\mathbf{i}$$

Then
$$\mathbf{M}_A = (6 \text{ m})\mathbf{i} \times \frac{2.5 \text{ kN}}{7.6} (-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

or
$$\mathbf{M}_A = (7.89 \text{ kN}\cdot\text{m})\mathbf{j} + (4.74 \text{ kN}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.27

In Prob. 3.21, determine the perpendicular distance from point O to wire AE .

PROBLEM 3.21 The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (a) on corner A , (b) on corner E .

SOLUTION

From the solution to Prob. 3.21

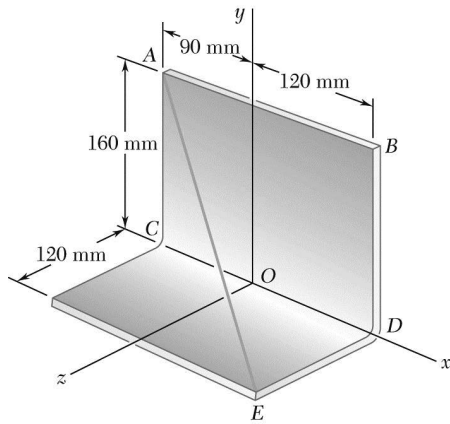
$$\mathbf{M}_O = (28.8 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{j} - (28.8 \text{ N}\cdot\text{m})\mathbf{k}$$

$$\begin{aligned} M_O &= \sqrt{(28.8)^2 + (16.20)^2 + (28.8)^2} \\ &= 43.8329 \text{ N}\cdot\text{m} \end{aligned}$$

But $M_O = F_A d$ or $d = \frac{M_O}{F_A}$

$$\begin{aligned} d &= \frac{43.8329 \text{ N}\cdot\text{m}}{435 \text{ N}} \\ &= 0.100765 \text{ m} \end{aligned}$$

$$d = 100.8 \text{ mm} \blacktriangleleft$$



PROBLEM 3.28

In Prob. 3.21, determine the perpendicular distance from point B to wire AE .

PROBLEM 3.21 The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (a) on corner A , (b) on corner E .

SOLUTION

From the solution to Prob. 3.21

$$\mathbf{F}_A = (315 \text{ N})\mathbf{i} - (240 \text{ N})\mathbf{j} + (180 \text{ N})\mathbf{k}$$

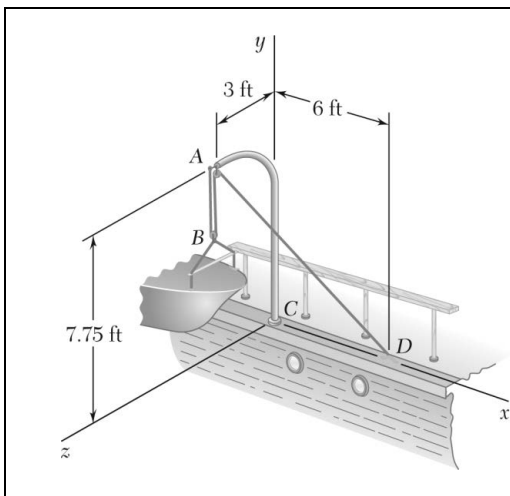
$$\mathbf{r}_{A/B} = -(0.210 \text{ m})\mathbf{i}$$

$$\begin{aligned} \mathbf{M}_B &= \mathbf{r}_{A/B} \times \mathbf{F}_A = -0.21\mathbf{i} \times (315\mathbf{i} - 240\mathbf{j} + 180\mathbf{k}) \\ &= 50.4\mathbf{k} + 37.8\mathbf{j} \end{aligned}$$

$$\begin{aligned} M_B &= \sqrt{(50.4)^2 + (37.8)^2} \\ &= 63.0 \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} M_B = F_A d \quad \text{or} \quad d &= \frac{M_B}{F_A} \\ d &= \frac{63.0 \text{ N}\cdot\text{m}}{435 \text{ N}} \\ &= 0.144829 \text{ m} \end{aligned}$$

$$d = 144.8 \text{ mm} \blacktriangleleft$$



PROBLEM 3.29

In Problem 3.22, determine the perpendicular distance from point C to portion AD of the line $ABAD$.

PROBLEM 3.22 A small boat hangs from two davits, one of which is shown in the figure. The tension in line $ABAD$ is 82 lb. Determine the moment about C of the resultant force \mathbf{R}_A exerted on the davit at A .

SOLUTION

First compute the moment about C of the force \mathbf{F}_{DA} exerted by the line on D :

From Problem 3.22:

$$\begin{aligned}\mathbf{F}_{DA} &= -\mathbf{F}_{AD} \\ &= -(48 \text{ lb})\mathbf{i} + (62 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_C &= \mathbf{r}_{D/C} \times \mathbf{F}_{DA} \\ &= (6 \text{ ft})\mathbf{i} \times [-(48 \text{ lb})\mathbf{i} + (62 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}] \\ &= -(144 \text{ lb} \cdot \text{ft})\mathbf{j} + (372 \text{ lb} \cdot \text{ft})\mathbf{k}\end{aligned}$$

$$\begin{aligned}M_C &= \sqrt{(144)^2 + (372)^2} \\ &= 398.90 \text{ lb} \cdot \text{ft}\end{aligned}$$

Then

$$\mathbf{M}_C = F_{DA}d$$

Since

$$F_{DA} = 82 \text{ lb}$$

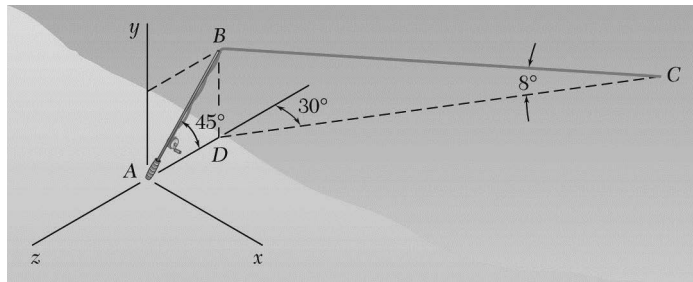
$$\begin{aligned}d &= \frac{M_C}{F_{DA}} \\ &= \frac{398.90 \text{ lb} \cdot \text{ft}}{82 \text{ lb}}\end{aligned}$$

$$d = 4.86 \text{ ft} \quad \blacktriangleleft$$

PROBLEM 3.30

In Prob. 3.23, determine the perpendicular distance from point A to a line drawn through points B and C .

PROBLEM 3.23 A 6-ft-long fishing rod AB is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about A of the force exerted by the line at B .



SOLUTION

From the solution to Prob. 3.23:

$$\mathbf{M}_A = -(25.4 \text{ lb} \cdot \text{ft})\mathbf{i} - (12.60 \text{ lb} \cdot \text{ft})\mathbf{j} - (12.60 \text{ lb} \cdot \text{ft})\mathbf{k}$$

$$\begin{aligned} M_A &= \sqrt{(-25.4)^2 + (-12.60)^2 + (-12.60)^2} \\ &= 31.027 \text{ lb} \cdot \text{ft} \end{aligned}$$

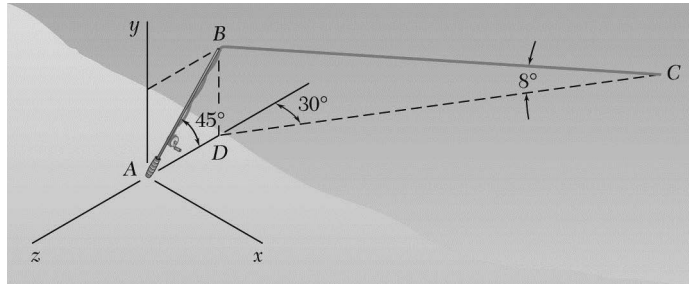
$$\begin{aligned} M_A = T_{BC}d \quad \text{or} \quad d &= \frac{M_A}{T_{BC}} \\ &= \frac{31.027 \text{ lb} \cdot \text{ft}}{6 \text{ lb}} \\ &= 5.1712 \text{ ft} \end{aligned}$$

$$d = 5.17 \text{ ft} \quad \blacktriangleleft$$

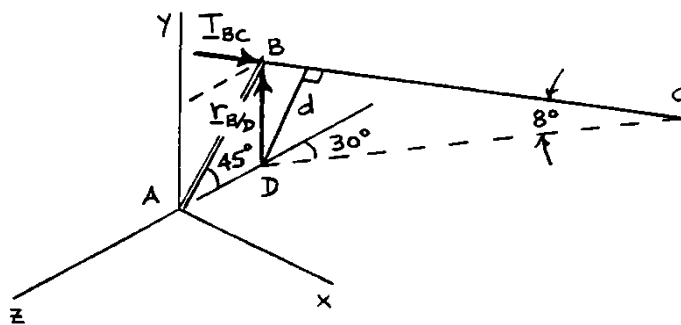
PROBLEM 3.31

In Prob. 3.23, determine the perpendicular distance from point D to a line drawn through points B and C .

PROBLEM 3.23 A 6-ft-long fishing rod AB is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about A of the force exerted by the line at B .



SOLUTION



$$\overline{AB} = 6 \text{ ft}$$

$$T_{BC} = 6 \text{ lb}$$

We have $|\mathbf{M}_D| = T_{BC}d$

where d = perpendicular distance from D to line BC .

$$\mathbf{M}_D = \mathbf{r}_{B/D} \times \mathbf{T}_{BC} \quad \mathbf{r}_{B/D} = (6 \sin 45^\circ \text{ ft})\mathbf{j} = (4.2426 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BC}: (T_{BC})_x = (6 \text{ lb}) \cos 8^\circ \sin 30^\circ = 2.9708 \text{ lb}$$

$$(T_{BC})_y = -(6 \text{ lb}) \sin 8^\circ = -0.83504 \text{ lb}$$

$$(T_{BC})_z = -(6 \text{ lb}) \cos 8^\circ \cos 30^\circ = -5.1456 \text{ lb}$$

$$\mathbf{T}_{BC} = (2.9708 \text{ lb})\mathbf{i} - (0.83504 \text{ lb})\mathbf{j} - (5.1456 \text{ lb})\mathbf{k}$$

$$\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.2426 & 0 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$$

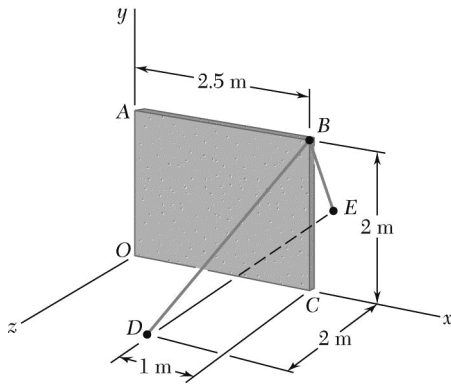
$$= -(21.831 \text{ lb} \cdot \text{ft})\mathbf{i} - (12.6039 \text{ lb} \cdot \text{ft})\mathbf{j}$$

$$|\mathbf{M}_D| = \sqrt{(-21.831)^2 + (-12.6039)^2} = 25.208 \text{ lb} \cdot \text{ft}$$

$$25.208 \text{ lb} \cdot \text{ft} = (6 \text{ lb})d$$

$$d = 4.20 \text{ ft} \quad \blacktriangleleft$$

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PROBLEM 3.32

In Prob. 3.24, determine the perpendicular distance from point O to cable BD .

PROBLEM 3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about Point O of the force exerted by the cable at B .

SOLUTION

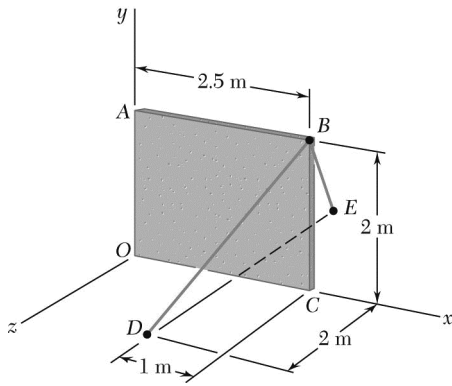
From the solution to Prob. 3.24 we have

$$\mathbf{M}_O = (1200 \text{ N}\cdot\text{m})\mathbf{i} - (1500 \text{ N}\cdot\text{m})\mathbf{j} - (900 \text{ N}\cdot\text{m})\mathbf{k}$$

$$M_O = \sqrt{(1200)^2 + (-1500)^2 + (-900)^2} = 2121.3 \text{ N}\cdot\text{m}$$

$$\begin{aligned} M_O &= Fd & d &= \frac{M_O}{F} \\ & & &= \frac{2121.3 \text{ N}\cdot\text{m}}{900 \text{ N}} \end{aligned}$$

$$d = 2.36 \text{ m} \quad \blacktriangleleft$$



PROBLEM 3.33

In Prob. 3.24, determine the perpendicular distance from point C to cable BD .

PROBLEM 3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about Point O of the force exerted by the cable at B .

SOLUTION

From the solution to Prob. 3.24 we have

$$\mathbf{F} = -(300 \text{ N})\mathbf{i} - (600 \text{ N})\mathbf{j} + (600 \text{ N})\mathbf{k}$$

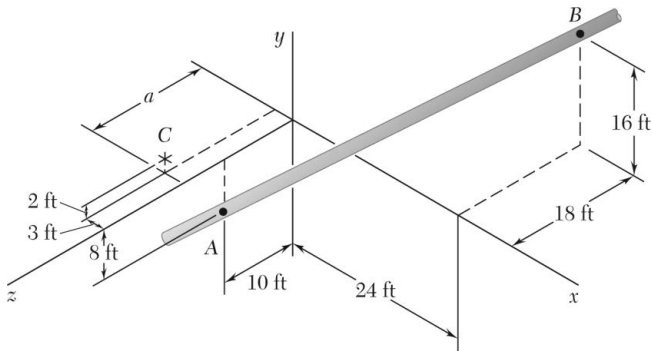
$$\mathbf{r}_{B/C} = (2 \text{ m})\mathbf{j}$$

$$\begin{aligned} \mathbf{M}_C &= \mathbf{r}_{B/C} \times \mathbf{F} = (2 \text{ m})\mathbf{j} \times (-300 \text{ N}\mathbf{i} - 600 \text{ N}\mathbf{j} + 600 \text{ N}\mathbf{k}) \\ &= (600 \text{ N} \cdot \text{m})\mathbf{k} + (1200 \text{ N} \cdot \text{m})\mathbf{i} \end{aligned}$$

$$M_C = \sqrt{(600)^2 + (1200)^2} = 1341.64 \text{ N} \cdot \text{m}$$

$$\begin{aligned} M_C &= Fd & d &= \frac{M_C}{F} \\ & & &= \frac{1341.64 \text{ N} \cdot \text{m}}{900 \text{ N}} \end{aligned}$$

$$d = 1.491 \text{ m} \quad \blacktriangleleft$$



PROBLEM 3.34

Determine the value of a that minimizes the perpendicular distance from Point C to a section of pipeline that passes through Points A and B .

SOLUTION

Assuming a force \mathbf{F} acts along AB ,

$$|\mathbf{M}_C| = |\mathbf{r}_{A/C} \times \mathbf{F}| = F(d)$$

where

$d =$ perpendicular distance from C to line AB

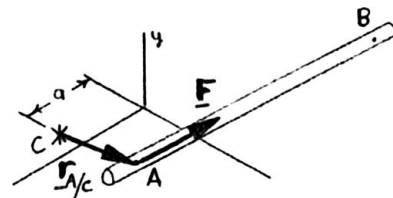
$$\begin{aligned} \mathbf{F} &= \lambda_{AB} \mathbf{F} \\ &= \frac{(24 \text{ ft})\mathbf{i} + (24 \text{ ft})\mathbf{j} - (28 \text{ ft})\mathbf{k}}{\sqrt{(24)^2 + (24)^2 + (28)^2}} F \end{aligned}$$

$$= \frac{F}{11} (6)\mathbf{i} + (6)\mathbf{j} - (7)\mathbf{k}$$

$$\mathbf{r}_{A/C} = (3 \text{ ft})\mathbf{i} - (10 \text{ ft})\mathbf{j} - (a - 10 \text{ ft})\mathbf{k}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -10 & 10a \\ 6 & 6 & -7 \end{vmatrix} \frac{F}{11}$$

$$= [(10 + 6a)\mathbf{i} + (81 - 6a)\mathbf{j} + 78\mathbf{k}] \frac{F}{11}$$



Since

$$|\mathbf{M}_C| = \sqrt{|\mathbf{r}_{A/C} \times \mathbf{F}|^2} \quad \text{or} \quad |\mathbf{r}_{A/C} \times \mathbf{F}|^2 = (dF)^2$$

$$\frac{1}{121} (10 + 6a)^2 + (81 - 6a)^2 + (78)^2 = d^2$$

Setting $\frac{d}{da}(d^2) = 0$ to find a to minimize d :

$$\frac{1}{121} [2(6)(10 + 6a) + 2(-6)(81 - 6a)] = 0$$

Solving

$$a = 5.92 \text{ ft}$$

or $a = 5.92 \text{ ft} \blacktriangleleft$

PROBLEM 3.35

Given the vectors $\mathbf{P} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{Q} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, and $\mathbf{S} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, compute the scalar products $\mathbf{P} \cdot \mathbf{Q}$, $\mathbf{P} \cdot \mathbf{S}$, and $\mathbf{Q} \cdot \mathbf{S}$.

SOLUTION

$$\begin{aligned}\mathbf{P} \cdot \mathbf{Q} &= (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) \\ &= (3)(4) + (-1)(5) + (2)(-3) \\ &= 12 - 5 - 6\end{aligned}$$

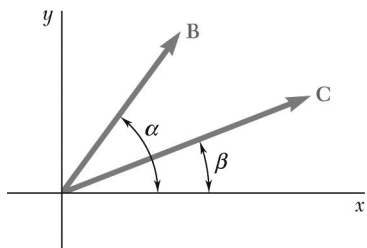
$$\mathbf{P} \cdot \mathbf{Q} = +1 \blacktriangleleft$$

$$\begin{aligned}\mathbf{P} \cdot \mathbf{S} &= (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (-2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \\ &= (3)(-2) + (-1)(3) + (2)(-1) \\ &= -6 - 3 - 2\end{aligned}$$

$$\mathbf{P} \cdot \mathbf{S} = -11 \blacktriangleleft$$

$$\begin{aligned}\mathbf{Q} \cdot \mathbf{S} &= (4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) \cdot (-2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \\ &= (4)(-2) + (5)(3) + (-3)(-1) \\ &= -8 + 15 + 3\end{aligned}$$

$$\mathbf{Q} \cdot \mathbf{S} = +10 \blacktriangleleft$$



PROBLEM 3.36

Form the scalar product $\mathbf{B} \cdot \mathbf{C}$ and use the result obtained to prove the identity

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$$

SOLUTION

$$\mathbf{B} = B \cos \alpha \mathbf{i} + B \sin \alpha \mathbf{j} \quad (1)$$

$$\mathbf{C} = C \cos \beta \mathbf{i} + C \sin \beta \mathbf{j} \quad (2)$$

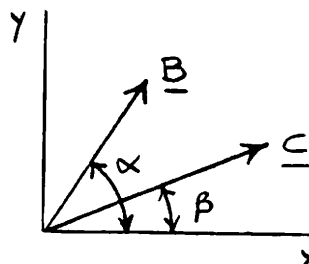
By definition:

$$\mathbf{B} \cdot \mathbf{C} = BC \cos(\alpha - \beta) \quad (3)$$

From (1) and (2):

$$\begin{aligned} \mathbf{B} \cdot \mathbf{C} &= (B \cos \alpha \mathbf{i} + B \sin \alpha \mathbf{j}) \cdot (C \cos \beta \mathbf{i} + C \sin \beta \mathbf{j}) \\ &= BC(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \end{aligned} \quad (4)$$

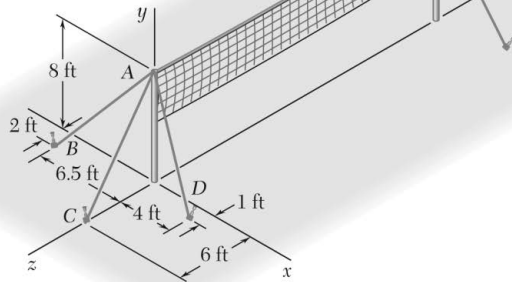
Equating the right-hand members of (3) and (4),



$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad \blacktriangleleft$$

PROBLEM 3.37

Consider the volleyball net shown. Determine the angle formed by guy wires AB and AC .



SOLUTION

First note:

$$AB = \sqrt{(-6.5)^2 + (-8)^2 + (2)^2} = 10.5 \text{ ft}$$

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2} = 10 \text{ ft}$$

and

$$\overline{AB} = -(6.5 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$$

$$\overline{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$

By definition,

$$\overline{AB} \cdot \overline{AC} = (AB)(AC)\cos\theta$$

or

$$(-6.5\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) \cdot (-8\mathbf{j} + 6\mathbf{k}) = (10.5)(10)\cos\theta$$

$$(-6.5)(0) + (-8)(-8) + (2)(6) = 105\cos\theta$$

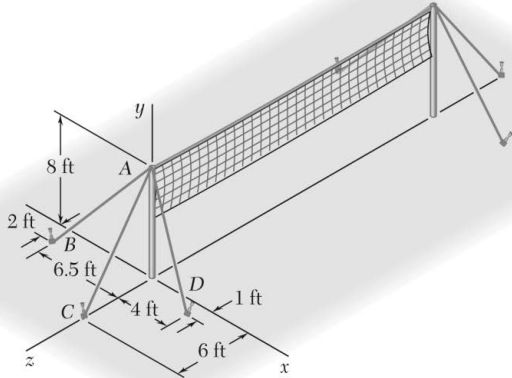
or

$$\cos\theta = 0.72381$$

$$\text{or } \theta = 43.6^\circ \blacktriangleleft$$

PROBLEM 3.38

Consider the volleyball net shown. Determine the angle formed by guy wires AC and AD .



SOLUTION

First note:

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2} \\ = 10 \text{ ft}$$

$$AD = \sqrt{(4)^2 + (-8)^2 + (1)^2} \\ = 9 \text{ ft}$$

and

$$\overline{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k} \\ \overline{AD} = (4 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (1 \text{ ft})\mathbf{k}$$

By definition,

$$\overline{AC} \cdot \overline{AD} = (AC)(AD) \cos \theta$$

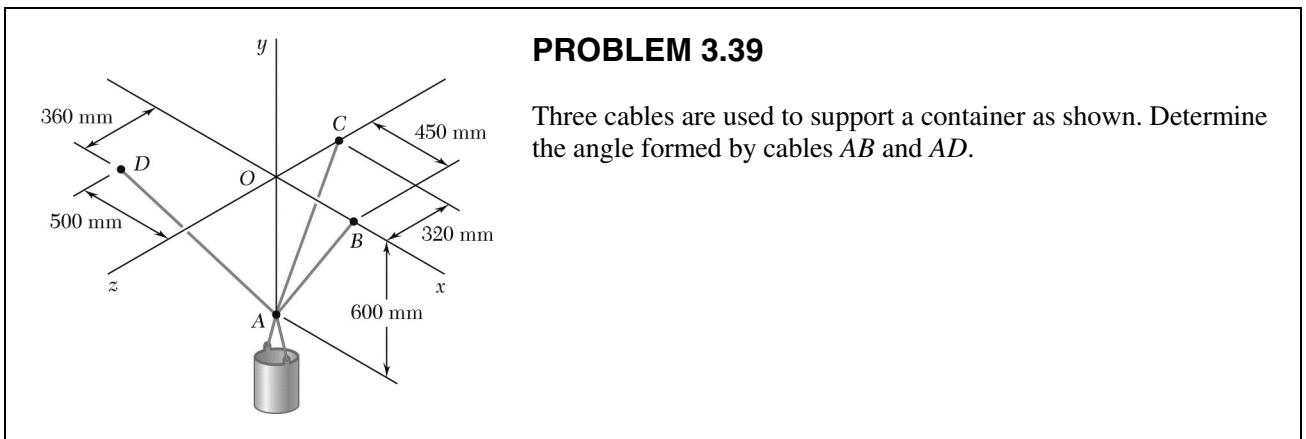
or

$$(-8\mathbf{j} + 6\mathbf{k}) \cdot (4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) = (10)(9) \cos \theta \\ (0)(4) + (-8)(-8) + (6)(1) = 90 \cos \theta$$

or

$$\cos \theta = 0.77778$$

$$\text{or } \theta = 38.9^\circ \blacktriangleleft$$



PROBLEM 3.39

Three cables are used to support a container as shown. Determine the angle formed by cables AB and AD .

SOLUTION

First note:

$$AB = \sqrt{(450 \text{ mm})^2 + (600 \text{ mm})^2}$$

$$= 750 \text{ mm}$$

$$AD = \sqrt{(-500 \text{ mm})^2 + (600 \text{ mm})^2 + (360 \text{ mm})^2}$$

$$= 860 \text{ mm}$$

and

$$\overline{AB} = (450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}$$

$$\overline{AD} = (-500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

By definition,

$$\overline{AB} \cdot \overline{AD} = (AB)(AD)\cos \theta$$

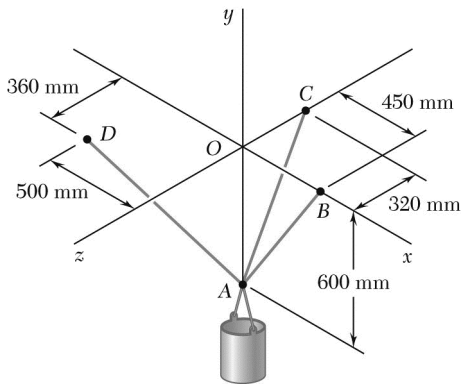
$$(450\mathbf{i} + 600\mathbf{j}) \cdot (-500\mathbf{i} - 600\mathbf{j} + 360\mathbf{k}) = (750)(860)\cos \theta$$

$$(450)(-500) + (600)(600) + (0)(360) = (750)(860)\cos \theta$$

or $\cos \theta = 0.20930$ $\theta = 77.9^\circ \blacktriangleleft$

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PROBLEM 3.40



Three cables are used to support a container as shown. Determine the angle formed by cables AC and AD .

SOLUTION

First note:

$$AC = \sqrt{(600 \text{ mm})^2 + (-320 \text{ mm})^2}$$
$$= 680 \text{ mm}$$

$$AD = \sqrt{(-500 \text{ mm})^2 + (600 \text{ mm})^2 + (360 \text{ mm})^2}$$
$$= 860 \text{ mm}$$

and

$$\overline{AC} = (600 \text{ mm})\mathbf{j} + (-320 \text{ mm})\mathbf{k}$$

$$\overline{AD} = (-500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

By definition,

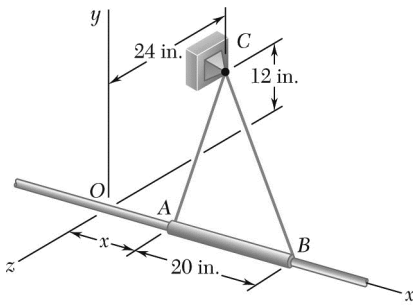
$$\overline{AC} \cdot \overline{AD} = (AC)(AD) \cos \theta$$

$$(600\mathbf{j} - 320\mathbf{k}) \cdot (-500\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}) = (680)(860) \cos \theta$$

$$0(-500) + (600)(600) + (-320)(360) = (680)(860) \cos \theta$$

$$\cos \theta = 0.41860$$

$$\theta = 65.3^\circ \blacktriangleleft$$



PROBLEM 3.41

The 20-in. tube AB can slide along a horizontal rod. The ends A and B of the tube are connected by elastic cords to the fixed point C . For the position corresponding to $x = 11$ in., determine the angle formed by the two cords (a) using Eq. (3.32), (b) applying the law of cosines to triangle ABC .

SOLUTION

(a) Using Eq. (3.32):

$$\overline{CA} = 11\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CA = \sqrt{(11)^2 + (-12)^2 + (24)^2} = 29 \text{ in.}$$

$$\overline{CB} = 31\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CB = \sqrt{(31)^2 + (-12)^2 + (24)^2} = 41 \text{ in.}$$

$$\begin{aligned} \cos \theta &= \frac{\overline{CA} \cdot \overline{CB}}{(CA)(CB)} \\ &= \frac{(11\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}) \cdot (31\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})}{(29)(41)} \\ &= \frac{(11)(31) + (-12)(-12) + (24)(24)}{(29)(41)} \\ &= 0.89235 \end{aligned}$$

$$\theta = 26.8^\circ \blacktriangleleft$$

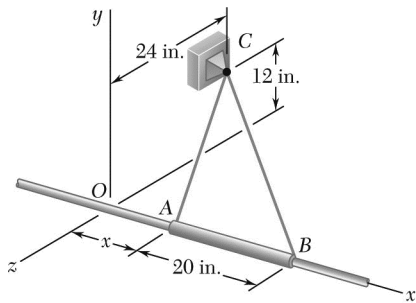
(b) Law of cosines:

$$(AB)^2 = (CA)^2 + (CB)^2 - 2(CA)(CB)\cos \theta$$

$$(20)^2 = (29)^2 + (41)^2 - 2(29)(41)\cos \theta$$

$$\cos \theta = 0.89235$$

$$\theta = 26.8^\circ \blacktriangleleft$$



PROBLEM 3.42

Solve Prob. 3.41 for the position corresponding to $x = 4$ in.

PROBLEM 3.41 The 20-in. tube AB can slide along a horizontal rod. The ends A and B of the tube are connected by elastic cords to the fixed point C . For the position corresponding to $x = 11$ in., determine the angle formed by the two cords (a) using Eq. (3.32), (b) applying the law of cosines to triangle ABC .

SOLUTION

(a) Using Eq. (3.32):

$$\overline{CA} = 4\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CA = \sqrt{(4)^2 + (-12)^2 + (24)^2} = 27.129 \text{ in.}$$

$$\overline{CB} = 24\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CB = \sqrt{(24)^2 + (-12)^2 + (24)^2} = 36 \text{ in.}$$

$$\begin{aligned} \cos \theta &= \frac{\overline{CA} \cdot \overline{CB}}{(CA)(CB)} \\ &= \frac{(4\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}) \cdot (24\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})}{(27.129)(36)} \\ &= 0.83551 \end{aligned}$$

$$\theta = 33.3^\circ \blacktriangleleft$$

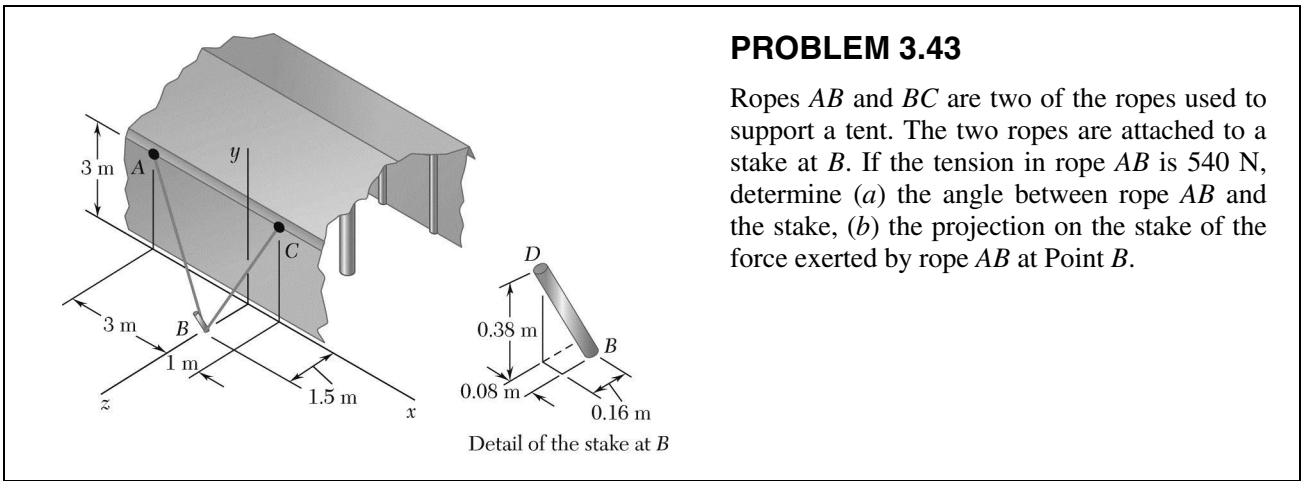
(b) Law of cosines:

$$(AB)^2 = (CA)^2 + (CB)^2 - 2(CA)(CB)\cos \theta$$

$$(20)^2 = (27.129)^2 + (36)^2 - 2(27.129)(36)\cos \theta$$

$$\cos \theta = 0.83551$$

$$\theta = 33.3^\circ \blacktriangleleft$$



SOLUTION

First note:

$$BA = \sqrt{(-3)^2 + (3)^2 + (-1.5)^2} = 4.5 \text{ m}$$

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

Then

$$\mathbf{T}_{BA} = \frac{T_{BA}}{4.5} (-3\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$$

$$= \frac{T_{BA}}{3} (-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

$$\boldsymbol{\lambda}_{BD} = \frac{\overline{BD}}{BD} = \frac{1}{0.42} (-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$$

$$= \frac{1}{21} (-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$$

(a) We have $\mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD} = T_{BA} \cos \theta$

or $\frac{T_{BA}}{3} (-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \frac{1}{21} (-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BA} \cos \theta$

or $\cos \theta = \frac{1}{63} [(-2)(-4) + (2)(19) + (-1)(8)]$

$$= 0.60317$$

or $\theta = 52.9^\circ \blacktriangleleft$

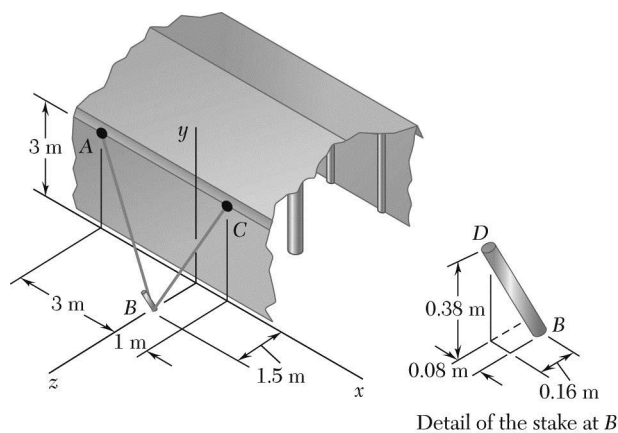
(b) We have $(T_{BA})_{BD} = \mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD}$

$$= T_{BA} \cos \theta$$

$$= (540 \text{ N})(0.60317)$$

or $(T_{BA})_{BD} = 326 \text{ N} \blacktriangleleft$

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PROBLEM 3.44

Ropes AB and BC are two of the ropes used to support a tent. The two ropes are attached to a stake at B . If the tension in rope BC is 490 N, determine (a) the angle between rope BC and the stake, (b) the projection on the stake of the force exerted by rope BC at Point B .

SOLUTION

First note:

$$BC = \sqrt{(1)^2 + (3)^2 + (-1.5)^2} = 3.5 \text{ m}$$

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

$$\begin{aligned} \mathbf{T}_{BC} &= \frac{T_{BC}}{3.5} (\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k}) \\ &= \frac{T_{BC}}{7} (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) \end{aligned}$$

$$\begin{aligned} \lambda_{BD} &= \frac{\overline{BD}}{BD} = \frac{1}{0.42} (-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k}) \\ &= \frac{1}{21} (-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) \end{aligned}$$

$$(a) \quad \mathbf{T}_{BC} \cdot \lambda_{BD} = T_{BC} \cos \theta$$

$$\frac{T_{BC}}{7} (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) \cdot \frac{1}{21} (-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BC} \cos \theta$$

$$\begin{aligned} \cos \theta &= \frac{1}{147} [(2)(-4) + (6)(19) + (-3)(8)] \\ &= 0.55782 \end{aligned}$$

$$\theta = 56.1^\circ \quad \blacktriangleleft$$

$$\begin{aligned} (b) \quad (T_{BC})_{BD} &= \mathbf{T}_{BC} \cdot \lambda_{BD} \\ &= T_{BC} \cos \theta \\ &= (490 \text{ N})(0.55782) \end{aligned}$$

$$(T_{BC})_{BD} = 273 \text{ N} \quad \blacktriangleleft$$

PROBLEM 3.45

Given the vectors $\mathbf{P} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{Q} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, and $\mathbf{S} = S_x\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, determine the value of S_x for which the three vectors are coplanar.

SOLUTION

If \mathbf{P} , \mathbf{Q} , and \mathbf{S} are coplanar, then \mathbf{P} must be perpendicular to $(\mathbf{Q} \times \mathbf{S})$.

$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) = 0$$

(or, the volume of a parallelepiped defined by \mathbf{P} , \mathbf{Q} , and \mathbf{S} is zero).

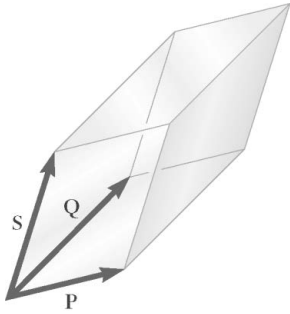
Then

$$\begin{vmatrix} 4 & -2 & 3 \\ 2 & 4 & -5 \\ S_x & -1 & 2 \end{vmatrix} = 0$$

or

$$32 + 10S_x - 6 - 20 + 8 - 12S_x = 0$$

$$S_x = 7 \quad \blacktriangleleft$$



PROBLEM 3.46

Determine the volume of the parallelepiped of Fig. 3.25 when

(a) $\mathbf{P} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{Q} = -2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, and $\mathbf{S} = 7\mathbf{i} + \mathbf{j} - \mathbf{k}$,

(b) $\mathbf{P} = 5\mathbf{i} - \mathbf{j} + 6\mathbf{k}$, $\mathbf{Q} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and $\mathbf{S} = -3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

SOLUTION

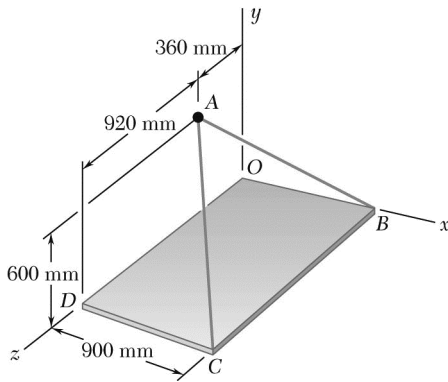
Volume of a parallelepiped is found using the mixed triple product.

$$\begin{aligned}
 (a) \quad \text{Vol.} &= \mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) \\
 &= \begin{vmatrix} 4 & -3 & 2 \\ -2 & -5 & 1 \\ 7 & 1 & -1 \end{vmatrix} \text{ in.}^3 \\
 &= (20 - 21 - 4 + 70 + 6 - 4) \\
 &= 67
 \end{aligned}$$

or Volume = 67.0 ◀

$$\begin{aligned}
 (b) \quad \text{Vol.} &= \mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) \\
 &= \begin{vmatrix} 5 & -1 & 6 \\ 2 & 3 & 1 \\ -3 & -2 & 4 \end{vmatrix} \text{ in.}^3 \\
 &= (60 + 3 - 24 + 54 + 8 + 10) \\
 &= 111
 \end{aligned}$$

or Volume = 111.0 ◀



PROBLEM 3.47

Knowing that the tension in cable AB is 570 N, determine the moment about each of the coordinate axes of the force exerted on the plate at B .

SOLUTION

$$\overline{BA} = (-900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

$$BA = \sqrt{(-900)^2 + (600)^2 + (360)^2} = 1140 \text{ mm}$$

$$\mathbf{F}_B = F_B \frac{\overline{BA}}{BA}$$

$$= (570 \text{ N}) \frac{-900\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}}{1140}$$

$$= -(450 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (180 \text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.9 \text{ m})\mathbf{i}$$

$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F}_B = 0.9\mathbf{i} \times (-450\mathbf{i} + 300\mathbf{j} + 180\mathbf{k})$$

$$= 270\mathbf{k} - 162\mathbf{j}$$

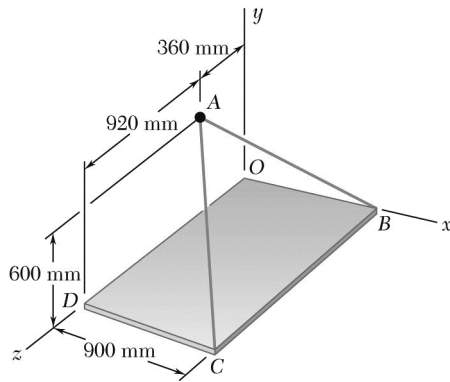
$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= -(162 \text{ N}\cdot\text{m})\mathbf{j} + (270 \text{ N}\cdot\text{m})\mathbf{k}$$

Therefore,

$$M_x = 0, \quad M_y = -162.0 \text{ N}\cdot\text{m}, \quad M_z = +270 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 3.48



Knowing that the tension in cable AC is 1065 N, determine the moment about each of the coordinate axes of the force exerted on the plate at C.

SOLUTION

$$\overline{CA} = (-900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (-920 \text{ mm})\mathbf{k}$$

$$CA = \sqrt{(-900)^2 + (600)^2 + (-920)^2} = 1420 \text{ mm}$$

$$\mathbf{F}_C = F_C \frac{\overline{CA}}{CA}$$

$$= (1065 \text{ N}) \frac{-900\mathbf{i} + 600\mathbf{j} - 920\mathbf{k}}{1420}$$

$$= -(675 \text{ N})\mathbf{i} + (450 \text{ N})\mathbf{j} - (690 \text{ N})\mathbf{k}$$

$$\mathbf{r}_C = (0.9 \text{ m})\mathbf{i} + (1.28 \text{ m})\mathbf{k}$$

Using Eq. (3.19):

$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 1.28 \\ -675 & 450 & -690 \end{vmatrix}$$

$$\mathbf{M}_O = -(576 \text{ N}\cdot\text{m})\mathbf{i} - (243 \text{ N}\cdot\text{m})\mathbf{j} + (405 \text{ N}\cdot\text{m})\mathbf{k}$$

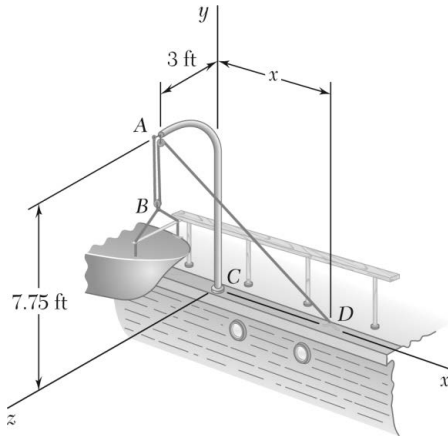
But

$$\mathbf{M}_O = M_x \mathbf{i} + M_y \mathbf{j} + M_z \mathbf{k}$$

Therefore,

$$M_x = -576 \text{ N}\cdot\text{m}, \quad M_y = -243 \text{ N}\cdot\text{m}, \quad M_z = +405 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 3.49



A small boat hangs from two davits, one of which is shown in the figure. It is known that the moment about the z -axis of the resultant force \mathbf{R}_A exerted on the davit at A must not exceed 279 lb·ft in absolute value. Determine the largest allowable tension in line $ABAD$ when $x = 6$ ft.

SOLUTION

First note: $\mathbf{R}_A = 2\mathbf{T}_{AB} + \mathbf{T}_{AD}$

Also note that only \mathbf{T}_{AD} will contribute to the moment about the z -axis.

Now $AD = \sqrt{(6)^2 + (-7.75)^2 + (-3)^2}$
 $= 10.25$ ft

Then $\mathbf{T}_{AD} = T \frac{\overline{AD}}{AD}$
 $= \frac{T}{10.25} (6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$

Now $M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$

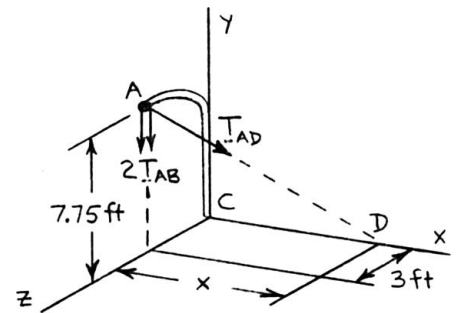
where $\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$

Then for T_{\max} ,

$$279 = \begin{vmatrix} T_{\max} & 0 & 0 & 1 \\ 10.25 & 0 & 7.75 & 3 \\ 6 & -7.75 & -3 & \end{vmatrix}$$

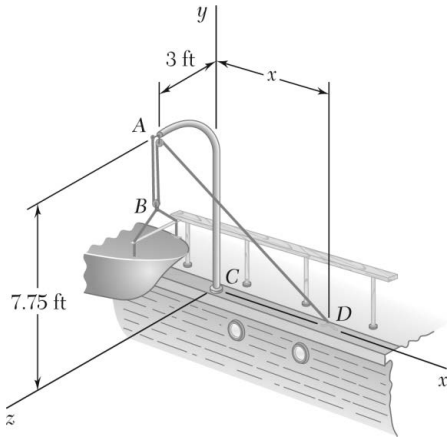
$$= \frac{T_{\max}}{10.25} |(1)(7.75)(6)|$$

or $T_{\max} = 61.5$ lb ◀



PROBLEM 3.50

For the davit of Problem 3.49, determine the largest allowable distance x when the tension in line $ABAD$ is 60 lb.



SOLUTION

From the solution of Problem 3.49, \mathbf{T}_{AD} is now

$$\begin{aligned}\mathbf{T}_{AD} &= T \frac{\overline{AD}}{AD} \\ &= \frac{60 \text{ lb}}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}} (x\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})\end{aligned}$$

Then $M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$ becomes

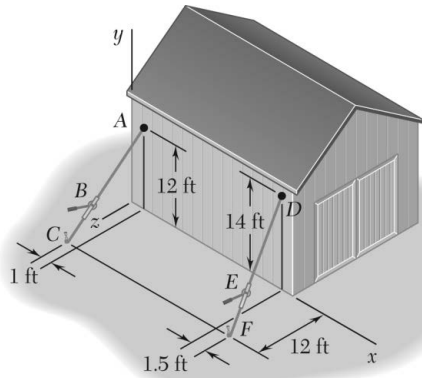
$$\begin{aligned}279 &= \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ x & -7.75 & -3 \end{vmatrix} \\ &= \frac{60}{\sqrt{x^2 + 69.0625}} |-(1)(7.75)(x)| \\ 279\sqrt{x^2 + 69.0625} &= 465x \\ 0.6\sqrt{x^2 + 69.0625} &= x\end{aligned}$$

Squaring both sides:

$$0.36x^2 + 24.8625 = x^2$$

$$x^2 = 38.848$$

$$x = 6.23 \text{ ft} \blacktriangleleft$$



PROBLEM 3.51

A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x -axis of the forces exerted by the cables on the barn at Points A and D is equal to $4728 \text{ lb} \cdot \text{ft}$, determine the magnitude of \mathbf{T}_{DE} when $T_{AB} = 255 \text{ lb}$.

SOLUTION

The moment about the x -axis due to the two cable forces can be found using the z components of each force acting at their intersection with the xy plane (A and D). The x components of the forces are parallel to the x -axis, and the y components of the forces intersect the x -axis. Therefore, neither the x or y components produce a moment about the x -axis.

We have

$$\Sigma M_x: (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$$

where

$$\begin{aligned} (T_{AB})_z &= \mathbf{k} \cdot \mathbf{T}_{AB} \\ &= \mathbf{k} \cdot (T_{AB} \lambda_{AB}) \\ &= \mathbf{k} \cdot \left[255 \text{ lb} \left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17} \right) \right] \\ &= 180 \text{ lb} \end{aligned}$$

$$\begin{aligned} (T_{DE})_z &= \mathbf{k} \cdot \mathbf{T}_{DE} \\ &= \mathbf{k} \cdot (T_{DE} \lambda_{DE}) \\ &= \mathbf{k} \cdot \left[T_{DE} \left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5} \right) \right] \\ &= 0.64865T_{DE} \end{aligned}$$

$$y_A = 12 \text{ ft}$$

$$y_D = 14 \text{ ft}$$

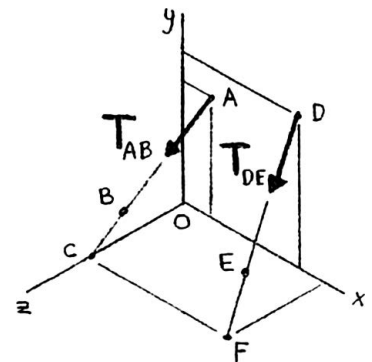
$$M_x = 4728 \text{ lb} \cdot \text{ft}$$

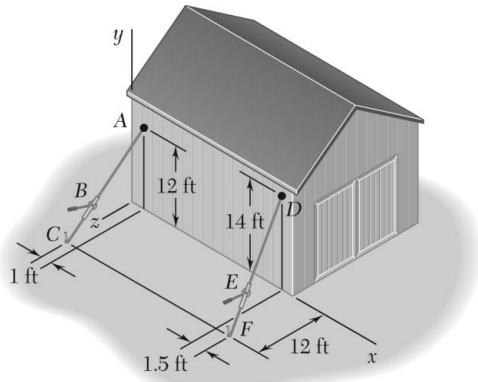
$$(180 \text{ lb})(12 \text{ ft}) + (0.64865T_{DE})(14 \text{ ft}) = 4728 \text{ lb} \cdot \text{ft}$$

and

$$T_{DE} = 282.79 \text{ lb}$$

$$\text{or } T_{DE} = 283 \text{ lb} \blacktriangleleft$$





PROBLEM 3.52

Solve Problem 3.51 when the tension in cable AB is 306 lb.

PROBLEM 3.51 A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x -axis of the forces exerted by the cables on the barn at Points A and D is equal to $4728 \text{ lb} \cdot \text{ft}$, determine the magnitude of \mathbf{T}_{DE} when $T_{AB} = 255 \text{ lb}$.

SOLUTION

The moment about the x -axis due to the two cable forces can be found using the z components of each force acting at the intersection with the xy plane (A and D). The x components of the forces are parallel to the x -axis, and the y components of the forces intersect the x -axis. Therefore, neither the x or y components produce a moment about the x -axis.

We have

$$\Sigma M_x: (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$$

Where

$$\begin{aligned} (T_{AB})_z &= \mathbf{k} \cdot \mathbf{T}_{AB} \\ &= \mathbf{k} \cdot (T_{AB} \lambda_{AB}) \\ &= \mathbf{k} \cdot \left[306 \text{ lb} \left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17} \right) \right] \\ &= 216 \text{ lb} \end{aligned}$$

$$\begin{aligned} (T_{DE})_z &= \mathbf{k} \cdot \mathbf{T}_{DE} \\ &= \mathbf{k} \cdot (T_{DE} \lambda_{DE}) \\ &= \mathbf{k} \cdot \left[T_{DE} \left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5} \right) \right] \\ &= 0.64865T_{DE} \end{aligned}$$

$$y_A = 12 \text{ ft}$$

$$y_D = 14 \text{ ft}$$

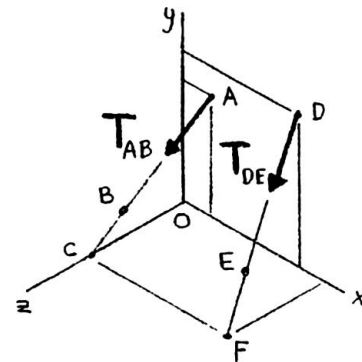
$$M_x = 4728 \text{ lb} \cdot \text{ft}$$

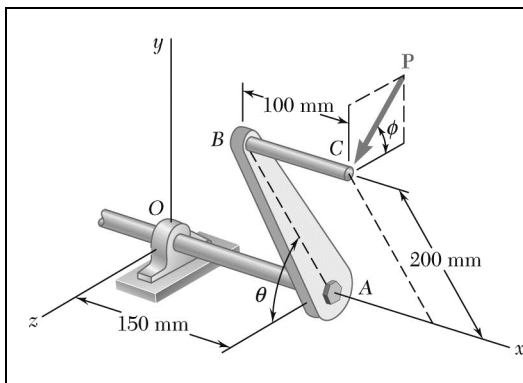
$$(216 \text{ lb})(12 \text{ ft}) + (0.64865T_{DE})(14 \text{ ft}) = 4728 \text{ lb} \cdot \text{ft}$$

and

$$T_{DE} = 235.21 \text{ lb}$$

$$\text{or } T_{DE} = 235 \text{ lb} \blacktriangleleft$$





PROBLEM 3.53

A single force \mathbf{P} acts at C in a direction perpendicular to the handle BC of the crank shown. Knowing that $M_x = +20 \text{ N} \cdot \text{m}$ and $M_y = -8.75 \text{ N} \cdot \text{m}$, and $M_z = -30 \text{ N} \cdot \text{m}$, determine the magnitude of \mathbf{P} and the values of ϕ and θ .

SOLUTION

$$\mathbf{r}_C = (0.25 \text{ m})\mathbf{i} + (0.2 \text{ m})\sin\theta\mathbf{j} + (0.2 \text{ m})\cos\theta\mathbf{k}$$

$$\mathbf{P} = -P\sin\phi\mathbf{j} + P\cos\phi\mathbf{k}$$

$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2\sin\theta & 0.2\cos\theta \\ 0 & -P\sin\phi & P\cos\phi \end{vmatrix}$$

Expanding the determinant, we find

$$M_x = (0.2)P(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$M_x = (0.2)P\sin(\theta + \phi) \quad (1)$$

$$M_y = -(0.25)P\cos\phi \quad (2)$$

$$M_z = -(0.25)P\sin\phi \quad (3)$$

Dividing Eq. (3) by Eq. (2) gives: $\tan\phi = \frac{M_z}{M_y}$ (4)

$$\tan\phi = \frac{-30 \text{ N} \cdot \text{m}}{-8.75 \text{ N} \cdot \text{m}}$$

$$\phi = 73.740$$

$$\phi = 73.7^\circ \blacktriangleleft$$

Squaring Eqs. (2) and (3) and adding gives:

$$M_y^2 + M_z^2 = (0.25)^2 P^2 \quad \text{or} \quad P = 4\sqrt{M_y^2 + M_z^2} \quad (5)$$

$$P = 4\sqrt{(8.75)^2 + (30)^2}$$

$$= 125.0 \text{ N}$$

$$P = 125.0 \text{ N} \blacktriangleleft$$

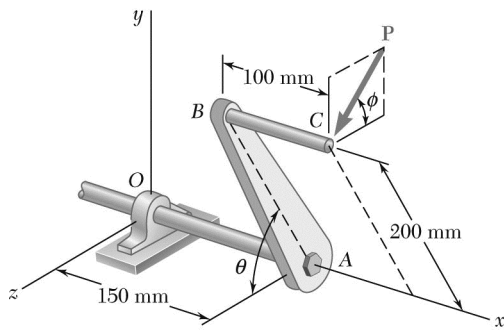
Substituting data into Eq. (1):

$$(+20 \text{ N} \cdot \text{m}) = 0.2 \text{ m}(125.0 \text{ N})\sin(\theta + \phi)$$

$$(\theta + \phi) = 53.130^\circ \quad \text{and} \quad (\theta + \phi) = 126.87^\circ$$

$$\theta = -20.6^\circ \quad \text{and} \quad \theta = 53.1^\circ$$

$$Q = 53.1^\circ \blacktriangleleft$$



PROBLEM 3.54

A single force \mathbf{P} acts at C in a direction perpendicular to the handle BC of the crank shown. Determine the moment M_x of \mathbf{P} about the x -axis when $\theta = 65^\circ$, knowing that $M_y = -15 \text{ N} \cdot \text{m}$ and $M_z = -36 \text{ N} \cdot \text{m}$.

SOLUTION

See the solution to Prob. 3.53 for the derivation of the following equations:

$$M_x = (0.2)P \sin(\theta + \phi) \quad (1)$$

$$\tan \phi = \frac{M_z}{M_y} \quad (4)$$

$$P = 4\sqrt{M_y^2 + M_z^2} \quad (5)$$

Substituting for known data gives:

$$\tan \phi = \frac{-36 \text{ N} \cdot \text{m}}{-15 \text{ N} \cdot \text{m}}$$

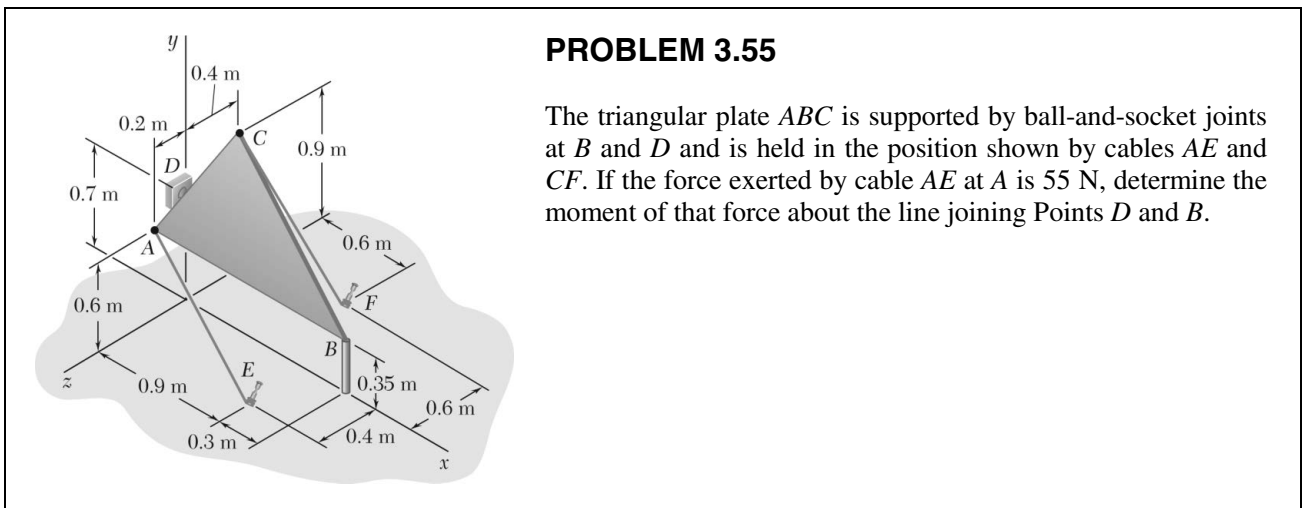
$$\phi = 67.380^\circ$$

$$P = 4\sqrt{(-15)^2 + (-36)^2}$$

$$P = 156.0 \text{ N}$$

$$\begin{aligned} M_x &= 0.2 \text{ m}(156.0 \text{ N})\sin(65^\circ + 67.380^\circ) \\ &= 23.047 \text{ N} \cdot \text{m} \end{aligned}$$

$$M_x = 23.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 3.55

The triangular plate ABC is supported by ball-and-socket joints at B and D and is held in the position shown by cables AE and CF . If the force exerted by cable AE at A is 55 N , determine the moment of that force about the line joining Points D and B .

SOLUTION

First note:

$$\mathbf{T}_{AE} = T_{AE} \frac{\overline{AE}}{AE}$$

$$AE = \sqrt{(0.9)^2 + (-0.6)^2 + (0.2)^2} = 1.1\text{ m}$$

Then

$$\begin{aligned} \mathbf{T}_{AE} &= \frac{55\text{ N}}{1.1} (0.9\mathbf{i} - 0.6\mathbf{j} + 0.2\mathbf{k}) \\ &= 5[(9\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (2\text{ N})\mathbf{k}] \end{aligned}$$

Also,

$$\begin{aligned} DB &= \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2} \\ &= 1.25\text{ m} \end{aligned}$$

Then

$$\begin{aligned} \lambda_{DB} &= \frac{\overline{DB}}{DB} \\ &= \frac{1}{1.25} (1.2\mathbf{i} - 0.35\mathbf{j}) \\ &= \frac{1}{25} (24\mathbf{i} - 7\mathbf{j}) \end{aligned}$$

Now

$$M_{DB} = \lambda_{DB} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{AE})$$

where

$$\mathbf{r}_{A/D} = -(0.1\text{ m})\mathbf{j} + (0.2\text{ m})\mathbf{k}$$

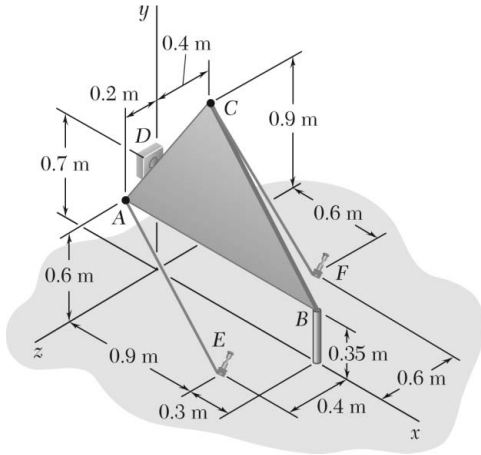
Then

$$\begin{aligned} M_{DB} &= \frac{1}{25} (5) \begin{vmatrix} 24 & -7 & 0 \\ 0 & -0.1 & 0.2 \\ 9 & -6 & 2 \end{vmatrix} \\ &= \frac{1}{5} (-4.8 - 12.6 + 28.8) \end{aligned}$$

$$\text{or } M_{DB} = 2.28\text{ N}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 3.56

The triangular plate ABC is supported by ball-and-socket joints at B and D and is held in the position shown by cables AE and CF . If the force exerted by cable CF at C is 33 N, determine the moment of that force about the line joining Points D and B .



SOLUTION

First note:

$$\mathbf{T}_{CF} = T_{CF} \frac{\overline{CF}}{CF}$$

$$CF = \sqrt{(0.6)^2 + (-0.9)^2 + (-0.2)^2} = 1.1 \text{ m}$$

Then

$$\begin{aligned} \mathbf{T}_{CF} &= \frac{33 \text{ N}}{1.1} (0.6\mathbf{i} - 0.9\mathbf{j} + 0.2\mathbf{k}) \\ &= 3[(6 \text{ N})\mathbf{i} - (9 \text{ N})\mathbf{j} - (2 \text{ N})\mathbf{k}] \end{aligned}$$

Also,

$$\begin{aligned} DB &= \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2} \\ &= 1.25 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} \lambda_{DB} &= \frac{\overline{DB}}{DB} \\ &= \frac{1}{1.25} (1.2\mathbf{i} - 0.35\mathbf{j}) \\ &= \frac{1}{25} (24\mathbf{i} - 7\mathbf{j}) \end{aligned}$$

Now

$$M_{DB} = \lambda_{DB} \cdot (\mathbf{r}_{C/D} \times \mathbf{T}_{CF})$$

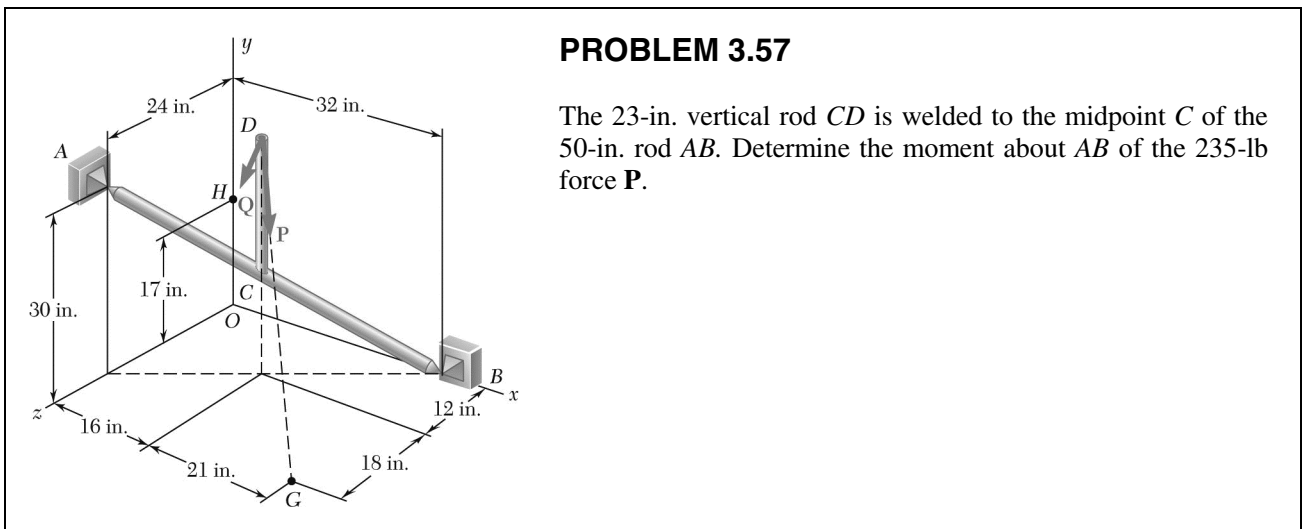
where

$$\mathbf{r}_{C/D} = (0.2 \text{ m})\mathbf{j} - (0.4 \text{ m})\mathbf{k}$$

Then

$$\begin{aligned} M_{DB} &= \frac{1}{25} (3) \begin{vmatrix} 24 & -7 & 0 \\ 0 & 0.2 & -0.4 \\ 6 & -9 & -2 \end{vmatrix} \\ &= \frac{3}{25} (-9.6 + 16.8 - 86.4) \end{aligned}$$

$$\text{or } M_{DB} = -9.50 \text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.57

The 23-in. vertical rod CD is welded to the midpoint C of the 50-in. rod AB . Determine the moment about AB of the 235-lb force \mathbf{P} .

SOLUTION

$$\overline{AB} = (32 \text{ in.})\mathbf{i} - (30 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50 \text{ in.}$$

$$\lambda_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

We shall apply the force \mathbf{P} at Point G :

$$\mathbf{r}_{G/B} = (5 \text{ in.})\mathbf{i} + (30 \text{ in.})\mathbf{k}$$

$$\overline{DG} = (21 \text{ in.})\mathbf{i} - (38 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$DG = \sqrt{(21)^2 + (-38)^2 + (18)^2} = 47 \text{ in.}$$

$$\mathbf{P} = P \frac{\overline{DG}}{DG} = (235 \text{ lb}) \frac{21\mathbf{i} - 38\mathbf{j} + 18\mathbf{k}}{47}$$

$$\mathbf{P} = (105 \text{ lb})\mathbf{i} - (190 \text{ lb})\mathbf{j} + (90 \text{ lb})\mathbf{k}$$

The moment of \mathbf{P} about AB is given by Eq. (3.46):

$$\mathbf{M}_{AB} = \lambda_{AB} \cdot (\mathbf{r}_{G/B} \times \mathbf{P}) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ 5 \text{ in.} & 0 & 30 \text{ in.} \\ 105 \text{ lb} & -190 \text{ lb} & 90 \text{ lb} \end{vmatrix}$$

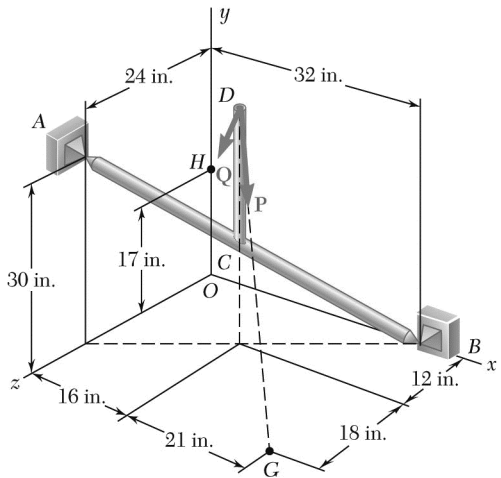
$$\begin{aligned} \mathbf{M}_{AB} &= 0.64[0 - (30 \text{ in.})(-190 \text{ lb})] \\ &\quad - 0.60[(30 \text{ in.})(105 \text{ lb}) - (5 \text{ in.})(90 \text{ lb})] \\ &\quad - 0.48[(5 \text{ in.})(-190 \text{ lb}) - 0] \\ &= +2484 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$\mathbf{M}_{AB} = +207 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

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PROBLEM 3.58

The 23-in. vertical rod CD is welded to the midpoint C of the 50-in. rod AB . Determine the moment about AB of the 174-lb force \mathbf{Q} .



SOLUTION

$$\overline{AB} = (32 \text{ in.})\mathbf{i} - (30 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50 \text{ in.}$$

$$\lambda_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

We shall apply the force \mathbf{Q} at Point H :

$$\mathbf{r}_{H/B} = -(32 \text{ in.})\mathbf{i} + (17 \text{ in.})\mathbf{j}$$

$$\overline{DH} = -(16 \text{ in.})\mathbf{i} - (21 \text{ in.})\mathbf{j} - (12 \text{ in.})\mathbf{k}$$

$$DH = \sqrt{(16)^2 + (-21)^2 + (-12)^2} = 29 \text{ in.}$$

$$\mathbf{Q} = \frac{\overline{DH}}{DH} = (174 \text{ lb}) \frac{-16\mathbf{i} - 21\mathbf{j} - 12\mathbf{k}}{29}$$

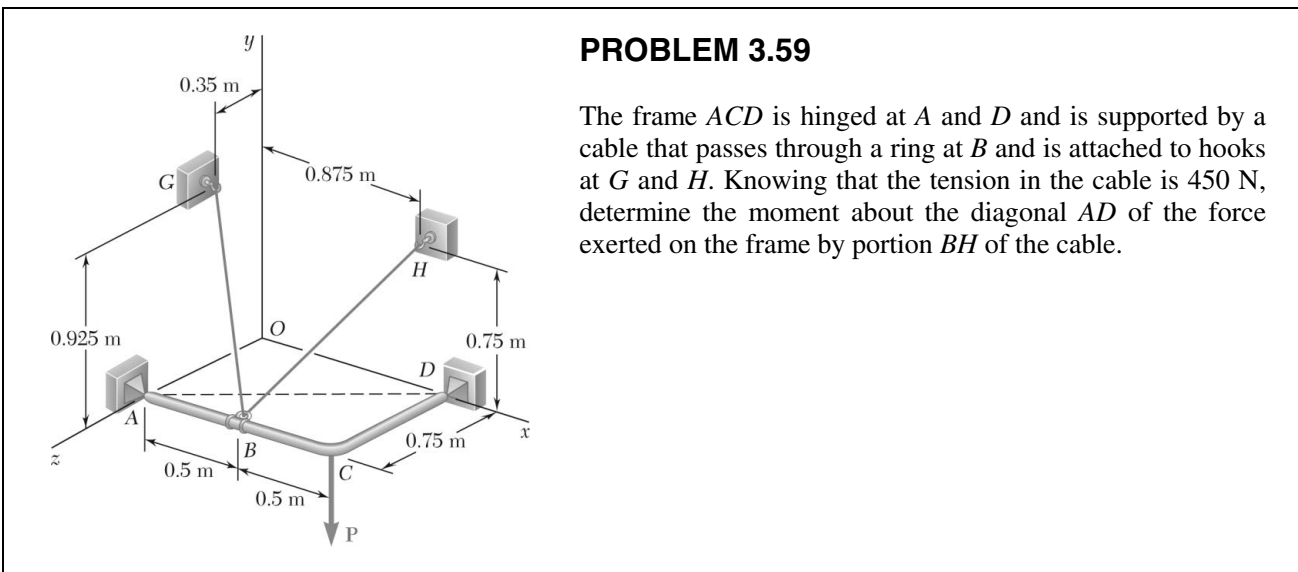
$$\mathbf{Q} = -(96 \text{ lb})\mathbf{i} - (126 \text{ lb})\mathbf{j} - (72 \text{ lb})\mathbf{k}$$

The moment of \mathbf{Q} about AB is given by Eq. (3.46):

$$\mathbf{M}_{AB} = \lambda_{AB} \cdot (\mathbf{r}_{H/B} \times \mathbf{Q}) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ -32 \text{ in.} & 17 \text{ in.} & 0 \\ -96 \text{ lb} & -126 \text{ lb} & -72 \text{ lb} \end{vmatrix}$$

$$\begin{aligned} \mathbf{M}_{AB} &= 0.64[(17 \text{ in.})(-72 \text{ lb}) - 0] \\ &\quad - 0.60[(0 - (-32 \text{ in.})(-72 \text{ lb})] \\ &\quad - 0.48[(-32 \text{ in.})(-126 \text{ lb}) - (17 \text{ in.})(-96 \text{ lb})] \\ &= -2119.7 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$\mathbf{M}_{AB} = 176.6 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$



PROBLEM 3.59

The frame ACD is hinged at A and D and is supported by a cable that passes through a ring at B and is attached to hooks at G and H . Knowing that the tension in the cable is 450 N , determine the moment about the diagonal AD of the force exerted on the frame by portion BH of the cable.

SOLUTION

$$M_{AD} = \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH})$$

Where

$$\lambda_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}$$

and

$$d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2}$$

$$= 1.125\text{ m}$$

Then

$$\mathbf{T}_{BH} = \frac{450\text{ N}}{1.125}(0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$$

$$= (150\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$$

Finally,

$$M_{AD} = \frac{1}{5} \begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(300)]$$

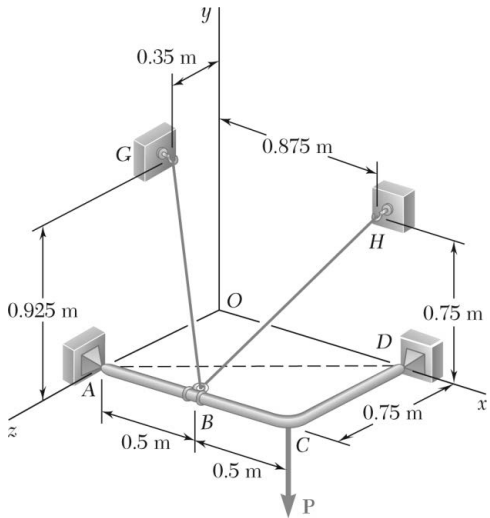
or $M_{AD} = -90.0\text{ N}\cdot\text{m} \blacktriangleleft$

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PROBLEM 3.60

In Problem 3.59, determine the moment about the diagonal AD of the force exerted on the frame by portion BG of the cable.

PROBLEM 3.59 The frame ACD is hinged at A and D and is supported by a cable that passes through a ring at B and is attached to hooks at G and H . Knowing that the tension in the cable is 450 N, determine the moment about the diagonal AD of the force exerted on the frame by portion BH of the cable.



SOLUTION

$$M_{AD} = \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG})$$

Where

$$\lambda_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{j}$$

and

$$BG = \sqrt{(-0.5)^2 + (0.925)^2 + (-0.4)^2}$$

$$= 1.125 \text{ m}$$

Then

$$\mathbf{T}_{BG} = \frac{450 \text{ N}}{1.125}(-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k})$$

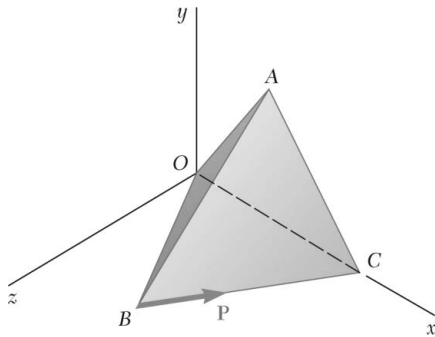
$$= -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$$

Finally,

$$M_{AD} = \frac{1}{5} \begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ -200 & 370 & -160 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(370)]$$

$$M_{AD} = -111.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 3.61

A regular tetrahedron has six edges of length a . A force \mathbf{P} is directed as shown along edge BC . Determine the moment of \mathbf{P} about edge OA .

SOLUTION

We have

$$M_{OA} = \lambda_{OA} \cdot (\mathbf{r}_{C/O} \times \mathbf{P})$$

From triangle OBC :

$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30^\circ = \frac{a}{2} \left(\frac{1}{\sqrt{3}} \right) = \frac{a}{2\sqrt{3}}$$

Since

$$(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA)_z^2$$

or

$$a^2 = \left(\frac{a}{2} \right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}} \right)^2$$

$$(OA)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$

Then

$$\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$$

and

$$\lambda_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$$

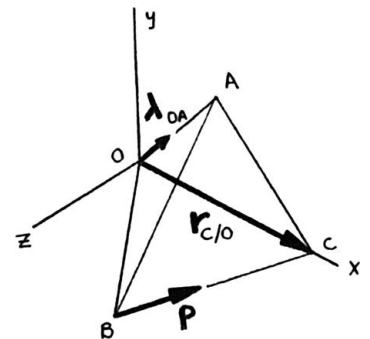
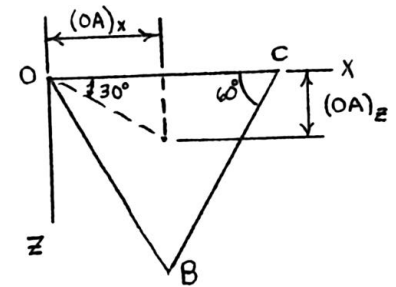
$$\mathbf{P} = \lambda_{BC}P = \frac{(a \sin 30^\circ)\mathbf{i} - (a \cos 30^\circ)\mathbf{k}}{a} (P) = \frac{P}{2}(\mathbf{i} - \sqrt{3}\mathbf{k})$$

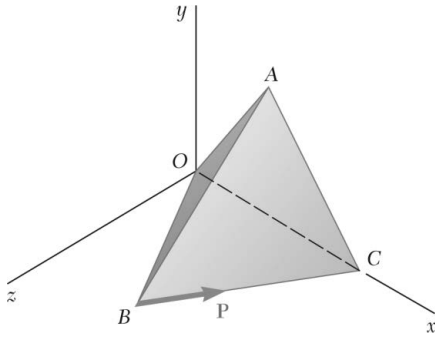
$$\mathbf{r}_{C/O} = a\mathbf{i}$$

$$M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix} (a) \left(\frac{P}{2} \right)$$

$$= \frac{aP}{2} \left(-\sqrt{\frac{2}{3}} \right) (1)(-\sqrt{3}) = \frac{aP}{\sqrt{2}}$$

$$M_{OA} = \frac{aP}{\sqrt{2}} \blacktriangleleft$$





PROBLEM 3.62

A regular tetrahedron has six edges of length a . (a) Show that two opposite edges, such as OA and BC , are perpendicular to each other. (b) Use this property and the result obtained in Problem 3.61 to determine the perpendicular distance between edges OA and BC .

SOLUTION

(a) For edge OA to be perpendicular to edge BC ,

$$\overline{OA} \cdot \overline{BC} = 0$$

From triangle OBC : $(OA)_x = \frac{a}{2}$

$$(OA)_z = (OA)_x \tan 30^\circ = \frac{a}{2} \left(\frac{1}{\sqrt{3}} \right) = \frac{a}{2\sqrt{3}}$$

$$\overline{OA} = \left(\frac{a}{2} \right) \mathbf{i} + (OA)_y \mathbf{j} + \left(\frac{a}{2\sqrt{3}} \right) \mathbf{k}$$

and

$$\begin{aligned} \overline{BC} &= (a \sin 30^\circ) \mathbf{i} - (a \cos 30^\circ) \mathbf{k} \\ &= \frac{a}{2} \mathbf{i} - \frac{a\sqrt{3}}{2} \mathbf{k} = \frac{a}{2} (\mathbf{i} - \sqrt{3} \mathbf{k}) \end{aligned}$$

Then

$$\left[\frac{a}{2} \mathbf{i} + (OA)_y \mathbf{j} + \left(\frac{a}{2\sqrt{3}} \right) \mathbf{k} \right] \cdot (\mathbf{i} - \sqrt{3} \mathbf{k}) \frac{a}{2} = 0$$

or

$$\begin{aligned} \frac{a^2}{4} + (OA)_y(0) - \frac{a^2}{4} &= 0 \\ \overline{OA} \cdot \overline{BC} &= 0 \end{aligned}$$

so that

\overline{OA} is perpendicular to \overline{BC} . ◀

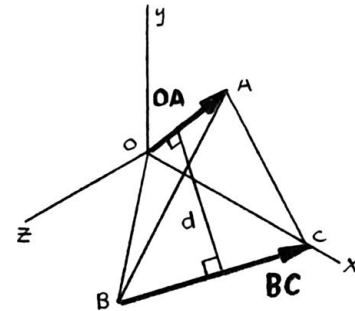
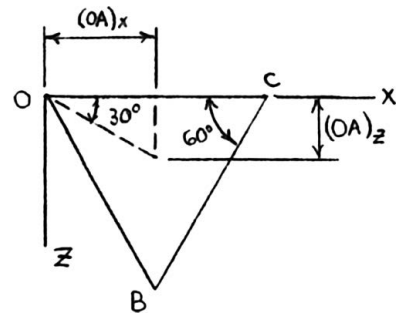
(b) We have $M_{OA} = Pd$, with P acting along BC and d the perpendicular distance from \overline{OA} to \overline{BC} .

From the results of Problem 3.57,

$$M_{OA} = \frac{Pa}{\sqrt{2}}$$

$$\frac{Pa}{\sqrt{2}} = Pd$$

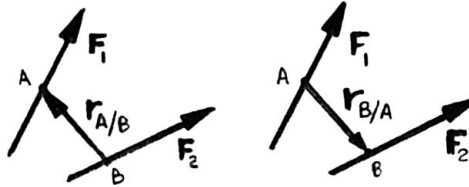
or $d = \frac{a}{\sqrt{2}}$ ◀



PROBLEM 3.63

Two forces \mathbf{F}_1 and \mathbf{F}_2 in space have the same magnitude F . Prove that the moment of \mathbf{F}_1 about the line of action of \mathbf{F}_2 is equal to the moment of \mathbf{F}_2 about the line of action of \mathbf{F}_1 .

SOLUTION



First note that

$$\mathbf{F}_1 = F_1 \boldsymbol{\lambda}_1 \quad \text{and} \quad \mathbf{F}_2 = F_2 \boldsymbol{\lambda}_2$$

Let $M_1 =$ moment of \mathbf{F}_2 about the line of action of \mathbf{F}_1 and $M_2 =$ moment of \mathbf{F}_1 about the line of action of \mathbf{F}_2 .

Now, by definition,

$$\begin{aligned} M_1 &= \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \mathbf{F}_2) \\ &= \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2) F_2 \\ M_2 &= \boldsymbol{\lambda}_2 \cdot (\mathbf{r}_{A/B} \times \mathbf{F}_1) \\ &= \boldsymbol{\lambda}_2 \cdot (\mathbf{r}_{A/B} \times \boldsymbol{\lambda}_1) F_1 \end{aligned}$$

Since

$$\begin{aligned} F_1 &= F_2 = F \quad \text{and} \quad \mathbf{r}_{A/B} = -\mathbf{r}_{B/A} \\ M_1 &= \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2) F \\ M_2 &= \boldsymbol{\lambda}_2 \cdot (-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1) F \end{aligned}$$

Using Equation (3.39):

$$\boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2) = \boldsymbol{\lambda}_2 \cdot (-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1)$$

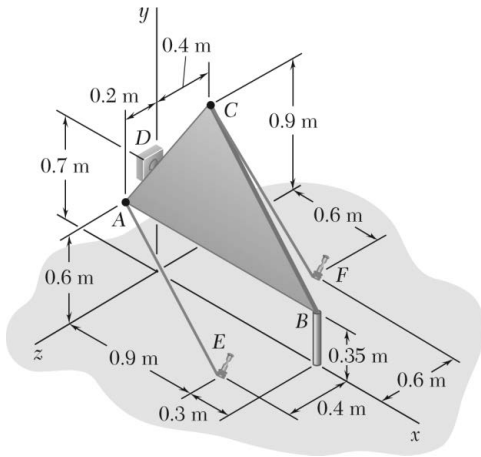
so that

$$M_2 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2) F \qquad M_{12} = M_{21} \blacktriangleleft$$

PROBLEM 3.64

In Problem 3.55, determine the perpendicular distance between cable AE and the line joining Points D and B .

PROBLEM 3.55 The triangular plate ABC is supported by ball-and-socket joints at B and D and is held in the position shown by cables AE and CF . If the force exerted by cable AE at A is 55 N, determine the moment of that force about the line joining Points D and B .



SOLUTION

From the solution to Problem 3.55:

$$\mathbf{T}_{AE} = 55 \text{ N}$$

$$\mathbf{T}_{AE} = 5[(9 \text{ N})\mathbf{i} - (6 \text{ N})\mathbf{j} + (2 \text{ N})\mathbf{k}]$$

$$|M_{DB}| = 2.28 \text{ N} \cdot \text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of \mathbf{T}_{AE} will contribute to the moment of \mathbf{T}_{AE} about line \overline{DB} .

Now

$$(\mathbf{T}_{AE})_{\text{parallel}} = \mathbf{T}_{AE} \cdot \boldsymbol{\lambda}_{DB}$$

$$= 5(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

$$= \frac{1}{5}[(9)(24) + (-6)(-7)]$$

$$= 51.6 \text{ N}$$

Also,

$$\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{parallel}} + (\mathbf{T}_{AE})_{\text{perpendicular}}$$

so that

$$(\mathbf{T}_{AE})_{\text{perpendicular}} = \sqrt{(55)^2 + (51.6)^2} = 19.0379 \text{ N}$$

Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{AE})_{\text{perpendicular}}$ are perpendicular, it follows that

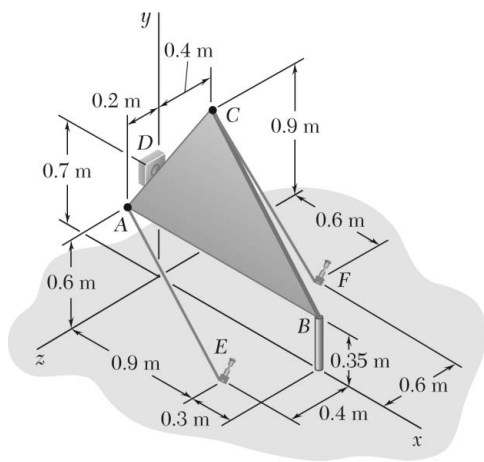
$$M_{DB} = d(\mathbf{T}_{AE})_{\text{perpendicular}}$$

or

$$2.28 \text{ N} \cdot \text{m} = d(19.0379 \text{ N})$$

$$d = 0.119761$$

$$d = 0.1198 \text{ m} \blacktriangleleft$$



PROBLEM 3.65

In Problem 3.56, determine the perpendicular distance between cable CF and the line joining Points D and B .

PROBLEM 3.56 The triangular plate ABC is supported by ball-and-socket joints at B and D and is held in the position shown by cables AE and CF . If the force exerted by cable CF at C is 33 N , determine the moment of that force about the line joining Points D and B .

SOLUTION

From the solution to Problem 3.56:

$$\mathbf{T}_{CF} = 33\text{ N}$$

$$\mathbf{T}_{CF} = 3[(6\text{ N})\mathbf{i} - (9\text{ N})\mathbf{j} - (2\text{ N})\mathbf{k}]$$

$$|M_{DB}| = 9.50\text{ N} \cdot \text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of \mathbf{T}_{CF} will contribute to the moment of \mathbf{T}_{CF} about line \overline{DB} .

Now

$$(\mathbf{T}_{CF})_{\text{parallel}} = \mathbf{T}_{CF} \cdot \boldsymbol{\lambda}_{DB}$$

$$= 3(6\mathbf{i} - 9\mathbf{j} - 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

$$= \frac{3}{25}[(6)(24) + (-9)(-7)]$$

$$= 24.84\text{ N}$$

Also,

$$\mathbf{T}_{CF} = (\mathbf{T}_{CF})_{\text{parallel}} + (\mathbf{T}_{CF})_{\text{perpendicular}}$$

so that

$$(\mathbf{T}_{CF})_{\text{perpendicular}} = \sqrt{(33)^2 - (24.84)^2}$$

$$= 21.725\text{ N}$$

Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{CF})_{\text{perpendicular}}$ are perpendicular, it follows that

$$|M_{DB}| = d(T_{CF})_{\text{perpendicular}}$$

or

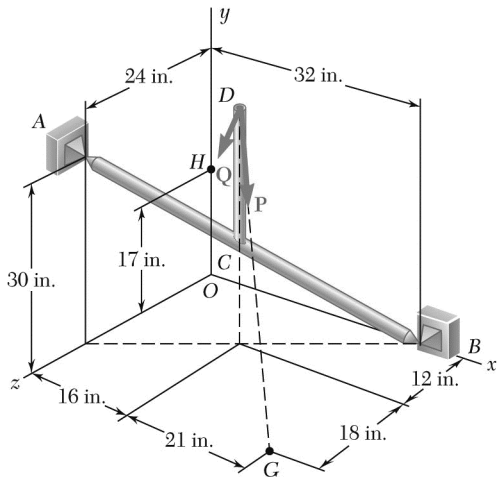
$$9.50\text{ N} \cdot \text{m} = d \times 21.725\text{ N}$$

or $d = 0.437\text{ m}$ ◀

PROBLEM 3.66

In Prob. 3.57, determine the perpendicular distance between rod AB and the line of action of \mathbf{P} .

PROBLEM 3.57 The 23-in. vertical rod CD is welded to the midpoint C of the 50-in. rod AB . Determine the moment about AB of the 235-lb force \mathbf{P} .



SOLUTION

$$\overline{AB} = (32 \text{ in.})\mathbf{i} - (30 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50 \text{ in.}$$

$$\lambda_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

$$\lambda_P = \frac{\mathbf{P}}{P} = \frac{105\mathbf{i} - 190\mathbf{j} + 90\mathbf{k}}{235}$$

Angle θ between AB and \mathbf{P} :

$$\begin{aligned} \cos \theta &= \lambda_{AB} \cdot \lambda_P \\ &= (0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}) \cdot \frac{105\mathbf{i} - 190\mathbf{j} + 90\mathbf{k}}{235} \\ &= 0.58723 \\ \therefore \theta &= 54.039^\circ \end{aligned}$$

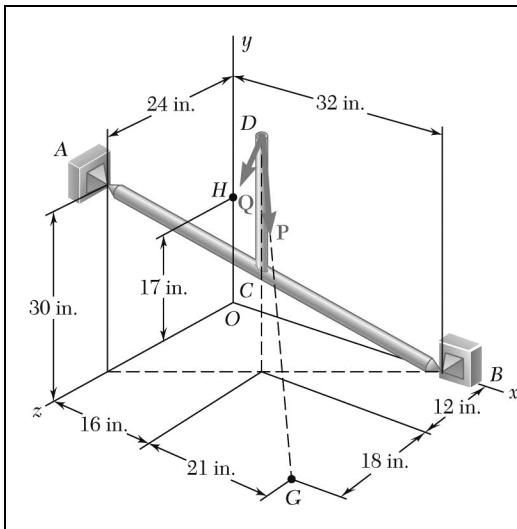
The moment of \mathbf{P} about AB may be obtained by multiplying the projection of \mathbf{P} on a plane perpendicular to AB by the perpendicular distance d from AB to \mathbf{P} :

$$\mathbf{M}_{AB} = (P \sin \theta)d$$

From the solution to Prob. 3.57: $\mathbf{M}_{AB} = 207 \text{ lb} \cdot \text{ft} = 2484 \text{ lb} \cdot \text{in.}$

We have $2484 \text{ lb} \cdot \text{in.} = (235 \text{ lb})(\sin 54.039^\circ)d$

$$d = 13.06 \text{ in.} \blacktriangleleft$$



PROBLEM 3.67

In Prob. 3.58, determine the perpendicular distance between rod AB and the line of action of \mathbf{Q} .

PROBLEM 3.58 The 23-in. vertical rod CD is welded to the midpoint C of the 50-in. rod AB . Determine the moment about AB of the 174-lb force \mathbf{Q} .

SOLUTION

$$\overline{AB} = (32 \text{ in.})\mathbf{i} - (30 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50 \text{ in.}$$

$$\lambda_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

$$\lambda_Q = \frac{\mathbf{Q}}{Q} = \frac{-96\mathbf{i} - 126\mathbf{j} - 72\mathbf{k}}{174}$$

Angle θ between AB and \mathbf{Q} :

$$\begin{aligned} \cos \theta &= \lambda_{AB} \cdot \lambda_Q \\ &= (0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}) \cdot \frac{(-96\mathbf{i} - 126\mathbf{j} - 72\mathbf{k})}{174} \\ &= 0.28000 \\ \therefore \theta &= 73.740^\circ \end{aligned}$$

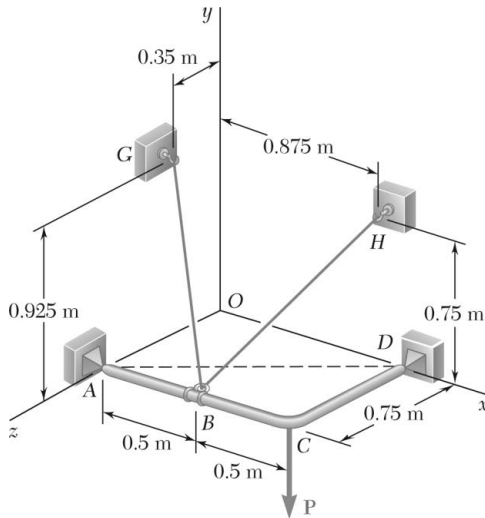
The moment of \mathbf{Q} about AB may be obtained by multiplying the projection of \mathbf{Q} on a plane perpendicular to AB by the perpendicular distance d from AB to \mathbf{Q} :

$$\mathbf{M}_{AB} = (Q \sin \theta)d$$

From the solution to Prob. 3.58: $\mathbf{M}_{AB} = 176.6 \text{ lb} \cdot \text{ft} = 2119.2 \text{ lb} \cdot \text{in.}$

$$2119.2 \text{ lb} \cdot \text{in.} = (174 \text{ lb})(\sin 73.740^\circ)d$$

$$d = 12.69 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 3.68

In Problem 3.59, determine the perpendicular distance between portion BH of the cable and the diagonal AD .

PROBLEM 3.59 The frame ACD is hinged at A and D and is supported by a cable that passes through a ring at B and is attached to hooks at G and H . Knowing that the tension in the cable is 450 N , determine the moment about the diagonal AD of the force exerted on the frame by portion BH of the cable.

SOLUTION

From the solution to Problem 3.59:

$$T_{BH} = 450\text{ N}$$

$$\mathbf{T}_{BH} = (150\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$$

$$|M_{AD}| = 90.0\text{ N}\cdot\text{m}$$

$$\lambda_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of \mathbf{T}_{BH} will contribute to the moment of \mathbf{T}_{BH} about line \overline{AD} .

Now

$$\begin{aligned} (T_{BH})_{\text{parallel}} &= \mathbf{T}_{BH} \cdot \lambda_{AD} \\ &= (150\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}) \cdot \frac{1}{5}(4\mathbf{i} - 3\mathbf{k}) \\ &= \frac{1}{5}[(150)(4) + (-300)(-3)] \\ &= 300\text{ N} \end{aligned}$$

Also,

$$\mathbf{T}_{BH} = (\mathbf{T}_{BH})_{\text{parallel}} + (\mathbf{T}_{BH})_{\text{perpendicular}}$$

so that

$$(T_{BH})_{\text{perpendicular}} = \sqrt{(450)^2 - (300)^2} = 335.41\text{ N}$$

Since λ_{AD} and $(\mathbf{T}_{BH})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{BH})_{\text{perpendicular}}$$

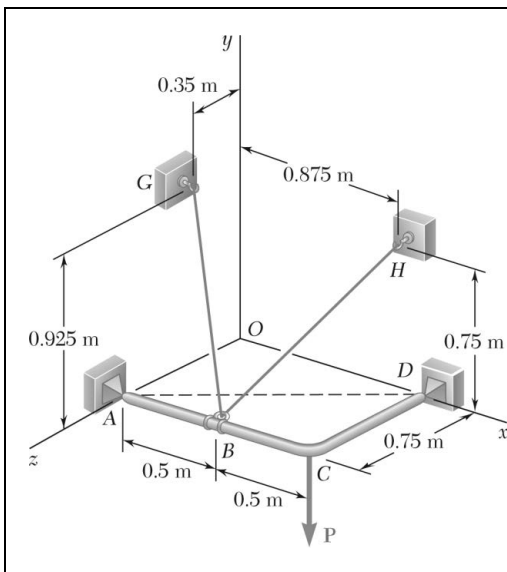
or

$$90.0\text{ N}\cdot\text{m} = d(335.41\text{ N})$$

$$d = 0.26833\text{ m}$$

$$d = 0.268\text{ m} \blacktriangleleft$$

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PROBLEM 3.69

In Problem 3.60, determine the perpendicular distance between portion BG of the cable and the diagonal AD .

PROBLEM 3.60 In Problem 3.59, determine the moment about the diagonal AD of the force exerted on the frame by portion BG of the cable.

SOLUTION

From the solution to Problem 3.60: $\mathbf{T}_{BG} = 450 \text{ N}$
 $\mathbf{T}_{BG} = -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$

$$|M_{AD}| = 111 \text{ N} \cdot \text{m}$$

$$\lambda_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of \mathbf{T}_{BG} will contribute to the moment of \mathbf{T}_{BG} about line \overline{AD} .

Now

$$\begin{aligned} (T_{BG})_{\text{parallel}} &= \mathbf{T}_{BG} \cdot \lambda_{AD} \\ &= (-200\mathbf{i} + 370\mathbf{j} - 160\mathbf{k}) \cdot \frac{1}{5}(4\mathbf{i} - 3\mathbf{k}) \\ &= \frac{1}{5}[(-200)(4) + (-160)(-3)] \\ &= -64 \text{ N} \end{aligned}$$

Also,
$$\bar{\mathbf{T}}_{BG} = (\mathbf{T}_{BG})_{\text{parallel}} + (\mathbf{T}_{BG})_{\text{perpendicular}}$$

so that
$$(\mathbf{T}_{BG})_{\text{perpendicular}} = \sqrt{(450)^2 - (-64)^2} = 445.43 \text{ N}$$

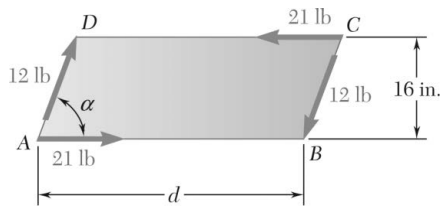
Since λ_{AD} and $(\mathbf{T}_{BG})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(\mathbf{T}_{BG})_{\text{perpendicular}}$$

or
$$111 \text{ N} \cdot \text{m} = d(445.43 \text{ N})$$

$$d = 0.24920 \text{ m} \qquad d = 0.249 \text{ m} \blacktriangleleft$$

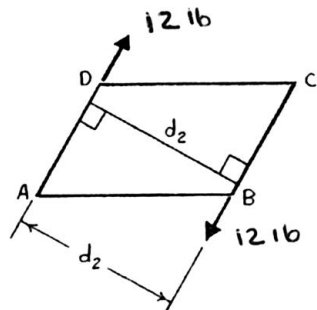
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PROBLEM 3.70

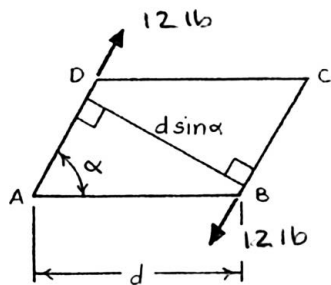
A plate in the shape of a parallelogram is acted upon by two couples. Determine (a) the moment of the couple formed by the two 21-lb forces, (b) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero, (c) the value of α if the resultant couple is 72 lb·in. clockwise and d is 42 in.

SOLUTION



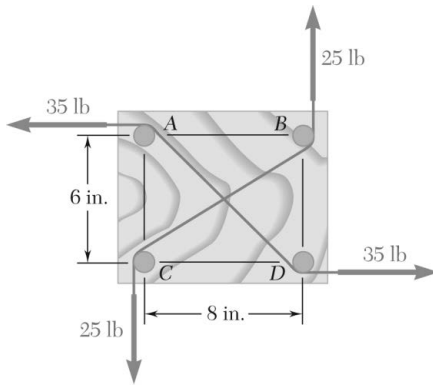
(a) We have $M_1 = d_1 F_1$
 where $d_1 = 16 \text{ in.}$
 $F_1 = 21 \text{ lb}$
 $M_1 = (16 \text{ in.})(21 \text{ lb})$
 $= 336 \text{ lb}\cdot\text{in.}$ or $M_1 = 336 \text{ lb}\cdot\text{in.}$ ◀

(b) We have $M_1 + M_2 = 0$
 or $336 \text{ lb}\cdot\text{in.} - d_2(12 \text{ lb}) = 0$ $d_2 = 28.0 \text{ in.}$ ◀



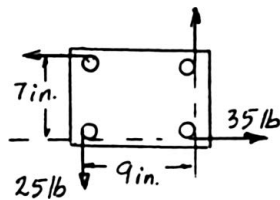
(c) We have $M_{\text{total}} = M_1 + M_2$
 or $-72 \text{ lb}\cdot\text{in.} = 336 \text{ lb}\cdot\text{in.} - (42 \text{ in.})(\sin \alpha)(12 \text{ lb})$
 $\sin \alpha = 0.80952$
 and $\alpha = 54.049^\circ$ or $\alpha = 54.0^\circ$ ◀

PROBLEM 3.71



Four 1-in.-diameter pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. (a) Determine the resultant couple acting on the board. (b) If only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with the minimum tension in the string? (c) What is the value of that minimum tension?

SOLUTION



$$\begin{aligned} (a) \quad +\curvearrowright M &= (35 \text{ lb})(7 \text{ in.}) + (25 \text{ lb})(9 \text{ in.}) \\ &= 245 \text{ lb} \cdot \text{in.} + 225 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$M = 470 \text{ lb} \cdot \text{in.} \quad \curvearrowright \blacktriangleleft$$

(b) With only one string, pegs A and D , or B and C should be used. We have

$$\tan \theta = \frac{6}{8} \quad \theta = 36.9^\circ \quad 90^\circ - \theta = 53.1^\circ$$

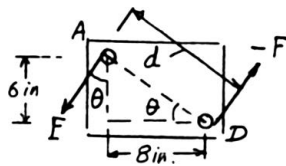
Direction of forces:

With pegs A and D :

$$\theta = 53.1^\circ \quad \blacktriangleleft$$

With pegs B and C :

$$\theta = 53.1^\circ \quad \blacktriangleleft$$



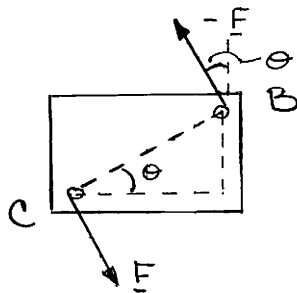
(c) The distance between the centers of the two pegs is

$$\sqrt{8^2 + 6^2} = 10 \text{ in.}$$

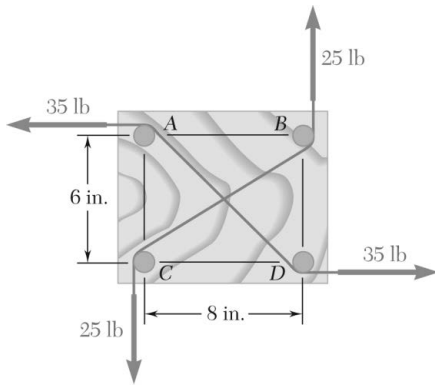
Therefore, the perpendicular distance d between the forces is

$$\begin{aligned} d &= 10 \text{ in.} + 2\left(\frac{1}{2} \text{ in.}\right) \\ &= 11 \text{ in.} \end{aligned}$$

$$\text{We must have} \quad M = Fd \quad 470 \text{ lb} \cdot \text{in.} = F(11 \text{ in.}) \quad F = 42.7 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 3.72

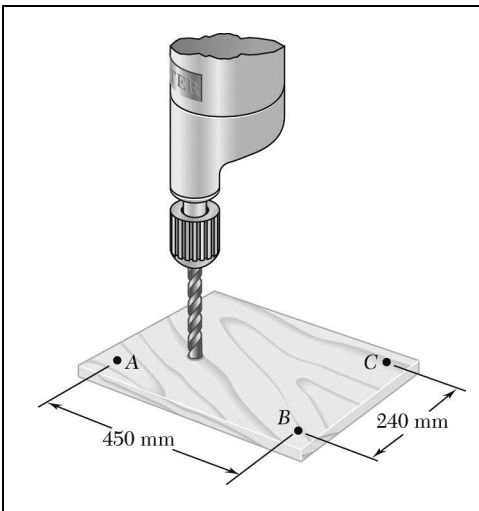


Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is 485 lb·in. counterclockwise.

SOLUTION

$$M = d_{AD}F_{AD} + d_{BC}F_{BC}$$
$$485 \text{ lb} \cdot \text{in.} = [(6 + d) \text{ in.}](35 \text{ lb}) + [(8 + d) \text{ in.}](25 \text{ lb})$$

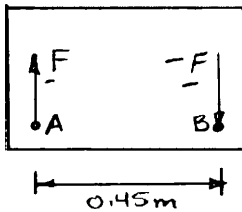
$$d = 1.250 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 3.73

A piece of plywood in which several holes are being drilled successively has been secured to a workbench by means of two nails. Knowing that the drill exerts a 12-N·m couple on the piece of plywood, determine the magnitude of the resulting forces applied to the nails if they are located (a) at A and B, (b) at B and C, (c) at A and C.

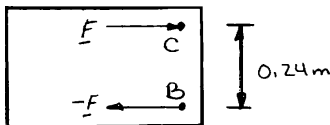
SOLUTION



$$(a) \quad M = Fd$$

$$12 \text{ N} \cdot \text{m} = F(0.45 \text{ m})$$

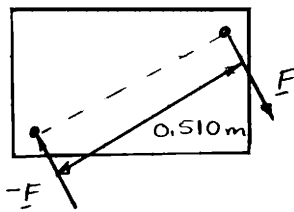
$$F = 26.7 \text{ N} \quad \blacktriangleleft$$



$$(b) \quad M = Fd$$

$$12 \text{ N} \cdot \text{m} = F(0.24 \text{ m})$$

$$F = 50.0 \text{ N} \quad \blacktriangleleft$$



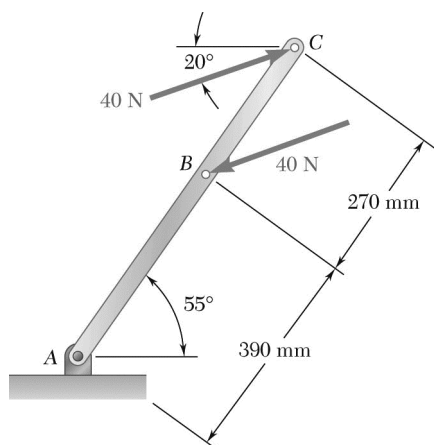
$$(c) \quad M = Fd \quad d = \sqrt{(0.45 \text{ m})^2 + (0.24 \text{ m})^2}$$

$$= 0.510 \text{ m}$$

$$12 \text{ N} \cdot \text{m} = F(0.510 \text{ m})$$

$$F = 23.5 \text{ N} \quad \blacktriangleleft$$

PROBLEM 3.74



Two parallel 40-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (a) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (b) by using the perpendicular distance between the two forces, (c) by summing the moments of the two forces about Point A.

SOLUTION

(a) We have $\Sigma \mathbf{M}_B: -d_1 C_x + d_2 C_y = M$

where

$$d_1 = (0.270 \text{ m}) \sin 55^\circ = 0.22117 \text{ m}$$

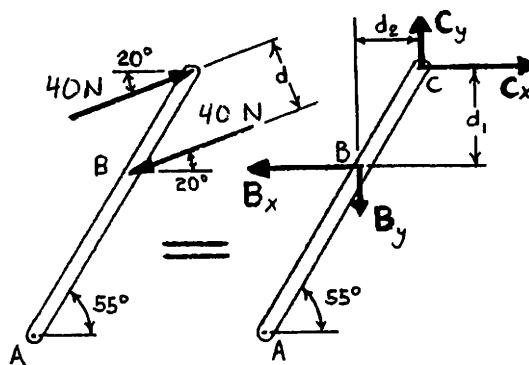
$$d_2 = (0.270 \text{ m}) \cos 55^\circ = 0.154866 \text{ m}$$

$$C_x = (40 \text{ N}) \cos 20^\circ = 37.588 \text{ N}$$

$$C_y = (40 \text{ N}) \sin 20^\circ = 13.6808 \text{ N}$$

$$\mathbf{M} = -(0.22117 \text{ m})(37.588 \text{ N})\mathbf{k} + (0.154866 \text{ m})(13.6808 \text{ N})\mathbf{k} = -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$$

or $M = 6.19 \text{ N}\cdot\text{m}$ ◀



(b) We have $\mathbf{M} = Fd(-\mathbf{k})$
 $= 40 \text{ N}[(0.270 \text{ m}) \sin(55^\circ - 20^\circ)](-\mathbf{k})$
 $= -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$

or $M = 6.19 \text{ N}\cdot\text{m}$ ◀

(c) We have $\Sigma \mathbf{M}_A: \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

$$M = (0.390 \text{ m})(40 \text{ N}) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ -\cos 20^\circ & -\sin 20^\circ & 0 \end{vmatrix}$$

$$+ (0.660 \text{ m})(40 \text{ N}) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ \cos 20^\circ & \sin 20^\circ & 0 \end{vmatrix}$$

$$= (8.9478 \text{ N}\cdot\text{m} - 15.1424 \text{ N}\cdot\text{m})\mathbf{k}$$

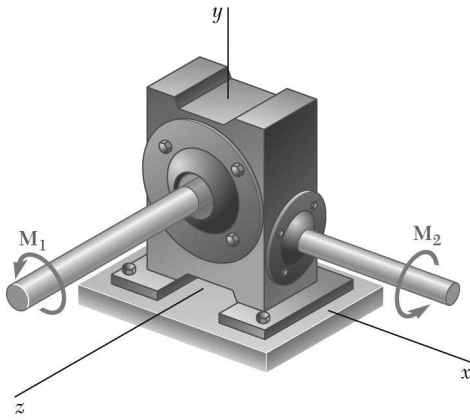
$$= -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$$

or $M = 6.19 \text{ N}\cdot\text{m}$ ◀

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PROBLEM 3.75

The two shafts of a speed-reducer unit are subjected to couples of magnitude $M_1 = 15 \text{ lb}\cdot\text{ft}$ and $M_2 = 3 \text{ lb}\cdot\text{ft}$, respectively. Replace the two couples with a single equivalent couple, specifying its magnitude and the direction of its axis.



SOLUTION

$$M_1 = (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$M_2 = (3 \text{ lb}\cdot\text{ft})\mathbf{i}$$

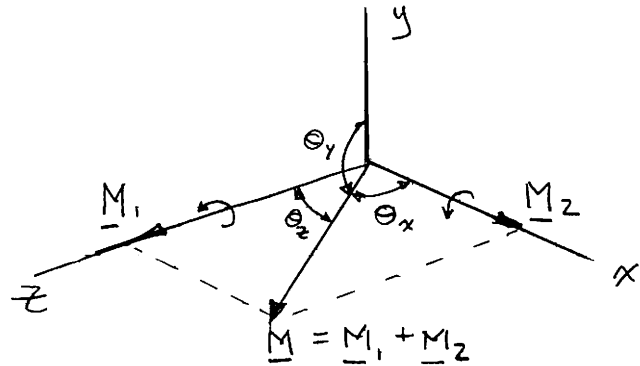
$$\begin{aligned} M &= \sqrt{M_1^2 + M_2^2} \\ &= \sqrt{(15)^2 + (3)^2} \\ &= 15.30 \text{ lb}\cdot\text{ft} \end{aligned}$$

$$\tan \theta_x = \frac{15}{3} = 5$$

$$\theta_x = 78.7^\circ$$

$$\theta_y = 90^\circ$$

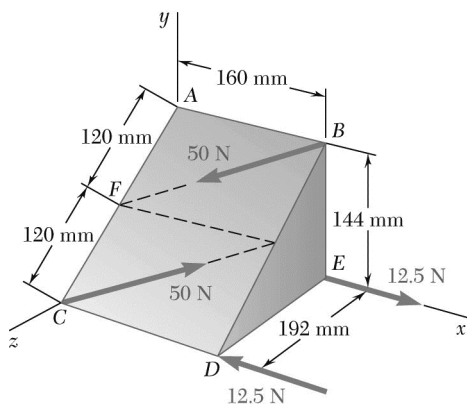
$$\begin{aligned} \theta_z &= 90^\circ - 78.7^\circ \\ &= 11.30^\circ \end{aligned}$$



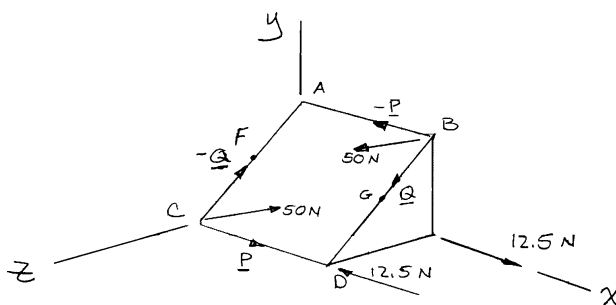
$$M = 15.30 \text{ lb}\cdot\text{ft}; \theta_x = 78.7^\circ, \theta_y = 90.0^\circ, \theta_z = 11.30^\circ \blacktriangleleft$$

PROBLEM 3.76

Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.



SOLUTION



Replace the couple in the $ABCD$ plane with two couples P and Q shown:

$$P = (50 \text{ N}) \frac{CD}{CG} = (50 \text{ N}) \left(\frac{160 \text{ mm}}{200 \text{ mm}} \right) = 40 \text{ N}$$

$$Q = (50 \text{ N}) \frac{CF}{CG} = (50 \text{ N}) \left(\frac{120 \text{ mm}}{200 \text{ mm}} \right) = 30 \text{ N}$$

Couple vector \mathbf{M}_1 perpendicular to plane $ABCD$:

$$+\curvearrowright M_1 = (40 \text{ N})(0.24 \text{ m}) - (30 \text{ N})(0.16 \text{ m}) = 4.80 \text{ N} \cdot \text{m}$$

Couple vector \mathbf{M}_2 in the xy plane:

$$+\curvearrowright M_2 = -(12.5 \text{ N})(0.192 \text{ m}) = -2.40 \text{ N} \cdot \text{m}$$

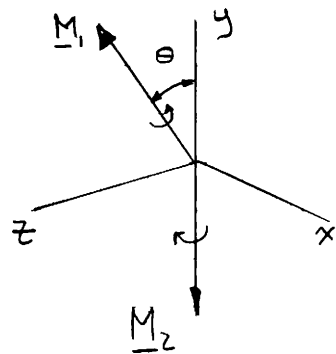
$$\tan \theta = \frac{144 \text{ mm}}{192 \text{ mm}} \quad \theta = 36.870^\circ$$

$$\mathbf{M}_1 = (4.80 \cos 36.870^\circ) \mathbf{j} + (4.80 \sin 36.870^\circ) \mathbf{k} \\ = 3.84 \mathbf{j} + 2.88 \mathbf{k}$$

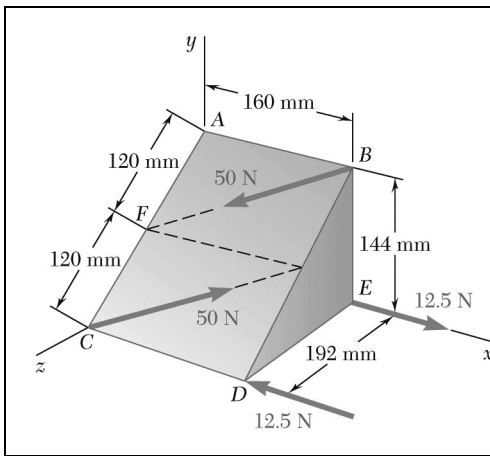
$$\mathbf{M}_2 = -2.40 \mathbf{j}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = 1.44 \mathbf{j} + 2.88 \mathbf{k}$$

$$\mathbf{M} = 3.22 \text{ N} \cdot \text{m}; \quad \theta_x = 90.0^\circ, \quad \theta_y = 53.1^\circ, \quad \theta_z = 36.9^\circ \quad \blacktriangleleft$$



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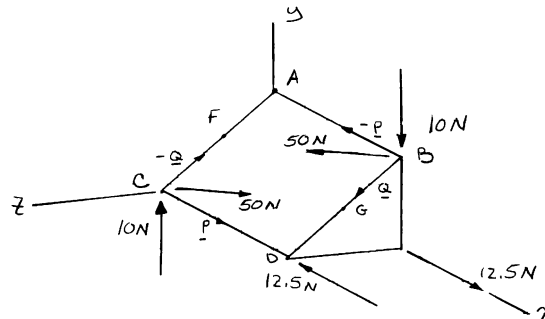


PROBLEM 3.77

Solve Prob. 3.76, assuming that two 10-N vertical forces have been added, one acting upward at C and the other downward at B .

PROBLEM 3.76 Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Replace the couple in the $ABCD$ plane with two couples P and Q shown.

$$P = (50 \text{ N}) \frac{CD}{CG} = (50 \text{ N}) \left(\frac{160 \text{ mm}}{200 \text{ mm}} \right) = 40 \text{ N}$$

$$Q = (50 \text{ N}) \frac{CF}{CG} = (50 \text{ N}) \left(\frac{120 \text{ mm}}{200 \text{ mm}} \right) = 30 \text{ N}$$

Couple vector \mathbf{M}_1 perpendicular to plane $ABCD$.

$$+\curvearrowright M_1 = (40 \text{ N})(0.24 \text{ m}) - (30 \text{ N})(0.16 \text{ m}) = 4.80 \text{ N} \cdot \text{m}$$

$$\tan \theta = \frac{144 \text{ mm}}{192 \text{ mm}} \quad \theta = 36.870^\circ$$

$$\begin{aligned} \mathbf{M}_1 &= (4.80 \cos 36.870^\circ) \mathbf{j} + (4.80 \sin 36.870^\circ) \mathbf{k} \\ &= 3.84 \mathbf{j} + 2.88 \mathbf{k} \end{aligned}$$

$$\begin{aligned} +\curvearrowright M_2 &= -(12.5 \text{ N})(0.192 \text{ m}) = -2.40 \text{ N} \cdot \text{m} \\ &= -2.40 \mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{M}_3 &= \mathbf{r}_{B/C} \times \mathbf{M}_3; \mathbf{r}_{B/C} = (0.16 \text{ m}) \mathbf{i} + (0.144 \text{ m}) \mathbf{j} - (0.192 \text{ m}) \mathbf{k} \\ &= (0.16 \text{ m}) \mathbf{i} + (0.144 \text{ m}) \mathbf{j} - (0.192 \text{ m}) \mathbf{k} \times (-10 \text{ N}) \mathbf{j} \\ &= -1.92 \mathbf{i} - 1.6 \mathbf{k} \end{aligned}$$

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PROBLEM 3.77 (Continued)

$$\begin{aligned} M &= M_1 + M_2 + M_3 = (3.84\mathbf{j} + 2.88\mathbf{k}) - 2.40\mathbf{j} + (-1.92\mathbf{i} - 1.6\mathbf{k}) \\ &= -(1.92 \text{ N}\cdot\text{m})\mathbf{i} + (1.44 \text{ N}\cdot\text{m})\mathbf{j} + (1.28 \text{ N}\cdot\text{m})\mathbf{k} \end{aligned}$$

$$M = \sqrt{(-1.92)^2 + (1.44)^2 + (1.28)^2} = 2.72 \text{ N}\cdot\text{m} \quad M = 2.72 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$\cos \theta_x = -1.92/2.72$$

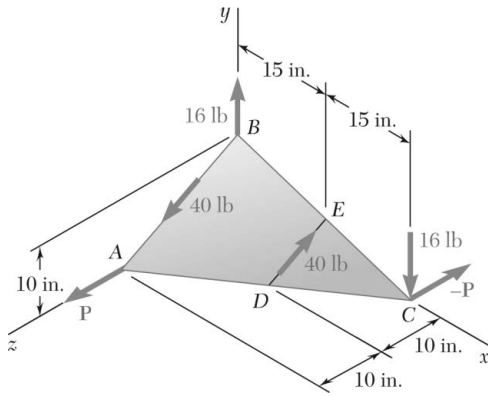
$$\cos \theta_y = 1.44/2.72$$

$$\cos \theta_z = 1.28/2.72$$

$$\theta_x = 134.9^\circ \quad \theta_y = 58.0^\circ \quad \theta_z = 61.9^\circ \quad \blacktriangleleft$$

PROBLEM 3.78

If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.



SOLUTION

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2; \quad F_1 = 16 \text{ lb}, \quad F_2 = 40 \text{ lb}$$

$$\mathbf{M}_1 = \mathbf{r}_C \times \mathbf{F}_1 = (30 \text{ in.})\mathbf{i} \times [-(16 \text{ lb})\mathbf{j}] = -(480 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$\mathbf{M}_2 = \mathbf{r}_{E/B} \times \mathbf{F}_2; \quad \mathbf{r}_{E/B} = (15 \text{ in.})\mathbf{i} - (5 \text{ in.})\mathbf{j}$$

$$d_{DE} = \sqrt{(0)^2 + (5)^2 + (10)^2} = 5\sqrt{5} \text{ in.}$$

$$\mathbf{F}_2 = \frac{40 \text{ lb}}{5\sqrt{5}} (5\mathbf{j} - 10\mathbf{k})$$

$$= 8\sqrt{5} [(1 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}]$$

$$\mathbf{M}_2 = 8\sqrt{5} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 8\sqrt{5} [(10 \text{ lb} \cdot \text{in.})\mathbf{i} + (30 \text{ lb} \cdot \text{in.})\mathbf{j} + (15 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

$$\mathbf{M} = -(480 \text{ lb} \cdot \text{in.})\mathbf{k} + 8\sqrt{5} [(10 \text{ lb} \cdot \text{in.})\mathbf{i} + (30 \text{ lb} \cdot \text{in.})\mathbf{j} + (15 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

$$= (178.885 \text{ lb} \cdot \text{in.})\mathbf{i} + (536.66 \text{ lb} \cdot \text{in.})\mathbf{j} - (211.67 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$$

$$= 603.99 \text{ lb} \cdot \text{in.}$$

$$M = 604 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

$$\lambda_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.29617\mathbf{i} + 0.88852\mathbf{j} - 0.35045\mathbf{k}$$

$$\cos \theta_x = 0.29617$$

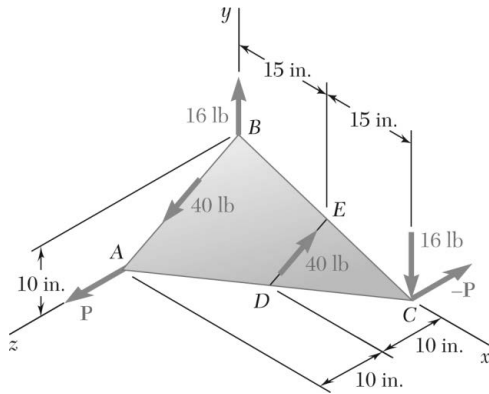
$$\cos \theta_y = 0.88852$$

$$\cos \theta_z = -0.35045$$

$$\theta_x = 72.8^\circ \quad \theta_y = 27.3^\circ \quad \theta_z = 110.5^\circ \quad \blacktriangleleft$$

PROBLEM 3.79

If $P = 20$ lb, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.



SOLUTION

From the solution to Problem. 3.78:

$$16\text{-lb force: } M_1 = -(480 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$40\text{-lb force: } M_2 = 8\sqrt{5}[(10 \text{ lb} \cdot \text{in.})\mathbf{i} + (30 \text{ lb} \cdot \text{in.})\mathbf{j} + (15 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

$$\begin{aligned} P = 20 \text{ lb } M_3 &= \mathbf{r}_C \times P \\ &= (30 \text{ in.})\mathbf{i} \times (20 \text{ lb})\mathbf{k} \\ &= (600 \text{ lb} \cdot \text{in.})\mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 \\ &= -(480)\mathbf{k} + 8\sqrt{5}(10\mathbf{i} + 30\mathbf{j} + 15\mathbf{k}) + 600\mathbf{j} \\ &= (178.885 \text{ lb} \cdot \text{in.})\mathbf{i} + (1136.66 \text{ lb} \cdot \text{in.})\mathbf{j} - (211.67 \text{ lb} \cdot \text{in.})\mathbf{k} \end{aligned}$$

$$\begin{aligned} M &= \sqrt{(178.885)^2 + (1136.66)^2 + (211.67)^2} \\ &= 1169.96 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$M = 1170 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

$$\lambda_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.152898\mathbf{i} + 0.97154\mathbf{j} - 0.180921\mathbf{k}$$

$$\cos \theta_x = 0.152898$$

$$\cos \theta_y = 0.97154$$

$$\cos \theta_z = -0.180921$$

$$\theta_x = 81.2^\circ \quad \theta_y = 13.70^\circ \quad \theta_z = 100.4^\circ \quad \blacktriangleleft$$

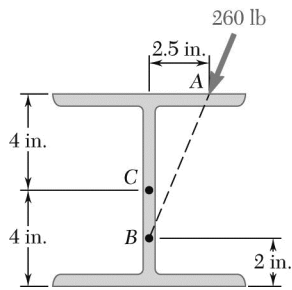
PROBLEM 3.80

In a manufacturing operation, three holes are drilled simultaneously in a workpiece. If the holes are perpendicular to the surfaces of the workpiece, replace the couples applied to the drills with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$\begin{aligned}
 \mathbf{M} &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 \\
 &= (1.5 \text{ N}\cdot\text{m})(-\cos 20^\circ \mathbf{j} + \sin 20^\circ \mathbf{k}) - (1.5 \text{ N}\cdot\text{m})\mathbf{j} \\
 &\quad + (1.75 \text{ N}\cdot\text{m})(-\cos 25^\circ \mathbf{j} + \sin 25^\circ \mathbf{k}) \\
 &= -(4.4956 \text{ N}\cdot\text{m})\mathbf{j} + (0.22655 \text{ N}\cdot\text{m})\mathbf{k} \\
 M &= \sqrt{(0)^2 + (-4.4956)^2 + (0.22655)^2} \\
 &= 4.5013 \text{ N}\cdot\text{m} \qquad \qquad \qquad M = 4.50 \text{ N}\cdot\text{m} \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 \lambda_{\text{axis}} &= \frac{\mathbf{M}}{M} = -(0.99873\mathbf{j} + 0.050330\mathbf{k}) \\
 \cos \theta_x &= 0 \\
 \cos \theta_y &= -0.99873 \\
 \cos \theta_z &= 0.050330 \qquad \qquad \qquad \theta_x = 90.0^\circ, \theta_y = 177.1^\circ, \theta_z = 87.1^\circ \blacktriangleleft
 \end{aligned}$$



PROBLEM 3.81

A 260-lb force is applied at A to the rolled-steel section shown. Replace that force with an equivalent force-couple system at the center C of the section.

SOLUTION

$$AB = \sqrt{(2.5 \text{ in.})^2 + (6.0 \text{ in.})^2} = 6.50 \text{ in.}$$

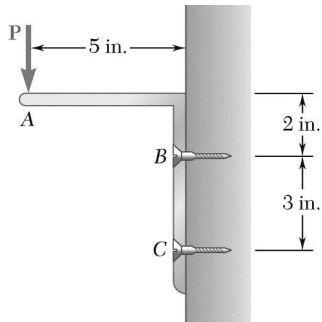
$$\sin \alpha = \frac{2.5 \text{ in.}}{6.5 \text{ in.}} = \frac{5}{13}$$

$$\cos \alpha = \frac{6.0 \text{ in.}}{6.5 \text{ in.}} = \frac{12}{13} \quad \alpha = 22.6^\circ$$

$$\begin{aligned} \mathbf{F} &= -F \sin \alpha \mathbf{i} - F \cos \alpha \mathbf{j} \\ &= -(260 \text{ lb}) \frac{5}{13} \mathbf{i} - (260 \text{ lb}) \frac{12}{13} \mathbf{j} \\ &= -(100.0 \text{ lb}) \mathbf{i} - (240 \text{ lb}) \mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{M}_C &= \mathbf{r}_{A/C} \times \mathbf{F} \\ &= (2.5\mathbf{i} + 4.0\mathbf{j}) \times (-100.0\mathbf{i} - 240\mathbf{j}) \\ &= 400\mathbf{k} - 600\mathbf{k} \\ &= -(200 \text{ lb} \cdot \text{in.}) \mathbf{k} \end{aligned}$$

$$\mathbf{F} = 260 \text{ lb} \nearrow 67.4^\circ; \quad \mathbf{M}_C = 200 \text{ lb} \cdot \text{in.} \curvearrowleft$$



PROBLEM 3.82

A 30-lb vertical force \mathbf{P} is applied at A to the bracket shown, which is held by screws at B and C . (a) Replace \mathbf{P} with an equivalent force-couple system at B . (b) Find the two horizontal forces at B and C that are equivalent to the couple obtained in part a.

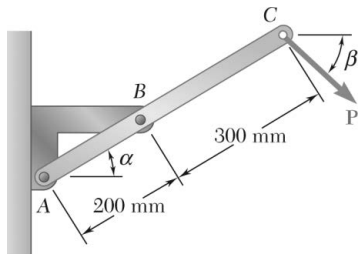
SOLUTION

$$(a) \quad M_B = (30 \text{ lb})(5 \text{ in.}) \\ = 150.0 \text{ lb} \cdot \text{in.}$$

$$\mathbf{F} = 30.0 \text{ lb} \downarrow, \quad \mathbf{M}_B = 150.0 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$

$$(b) \quad B = C = \frac{150 \text{ lb} \cdot \text{in.}}{3.0 \text{ in.}} = 50.0 \text{ lb}$$

$$\mathbf{B} = 50.0 \text{ lb} \leftarrow; \quad \mathbf{C} = 50.0 \text{ lb} \rightarrow \blacktriangleleft$$



PROBLEM 3.83

The force \mathbf{P} has a magnitude of 250 N and is applied at the end C of a 500-mm rod AC attached to a bracket at A and B . Assuming $\alpha = 30^\circ$ and $\beta = 60^\circ$, replace \mathbf{P} with (a) an equivalent force-couple system at B , (b) an equivalent system formed by two parallel forces applied at A and B .

SOLUTION

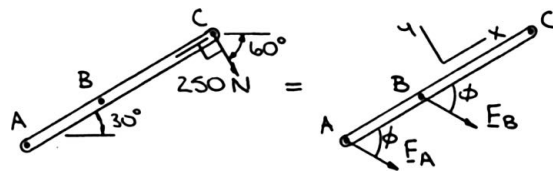
(a) Equivalence requires $\Sigma \mathbf{F}: \mathbf{F} = \mathbf{P}$ or $\mathbf{F} = 250 \text{ N} \searrow 60^\circ$

$$\Sigma M_B: M = -(0.3 \text{ m})(250 \text{ N}) = -75 \text{ N} \cdot \text{m}$$

The equivalent force-couple system at B is

$$\mathbf{F}_B = 250 \text{ N} \searrow 60^\circ \qquad \mathbf{M}_B = 75.0 \text{ N} \cdot \text{m} \curvearrowleft$$

(b) We require



Equivalence then requires

$$\Sigma F_x: 0 = F_A \cos \phi + F_B \cos \phi$$

$$F_A = -F_B \text{ or } \cos \phi = 0$$

$$\Sigma F_y: -250 = -F_A \sin \phi - F_B \sin \phi$$

Now if $F_A = -F_B \Rightarrow -250 = 0$, reject.

$$\cos \phi = 0$$

or $\phi = 90^\circ$

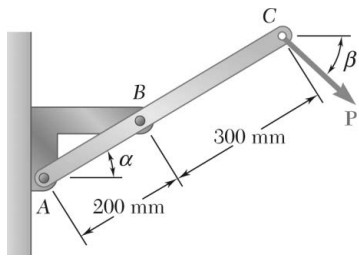
and $F_A + F_B = 250$

Also, $\Sigma M_B: -(0.3 \text{ m})(250 \text{ N}) = (0.2 \text{ m})F_A$

or $F_A = -375 \text{ N}$

and $F_B = 625 \text{ N}$

$$\mathbf{F}_A = 375 \text{ N} \nearrow 60^\circ \qquad \mathbf{F}_B = 625 \text{ N} \searrow 60.0^\circ \curvearrowleft$$

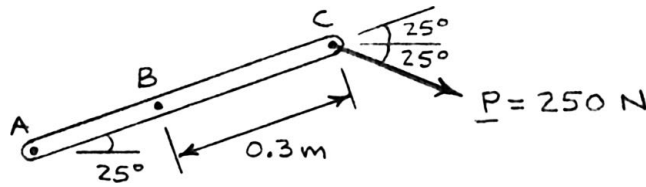


PROBLEM 3.84

Solve Problem 3.83, assuming $\alpha = \beta = 25^\circ$.

PROBLEM 3.83 The force \mathbf{P} has a magnitude of 250 N and is applied at the end C of a 500-mm rod AC attached to a bracket at A and B . Assuming $\alpha = 30^\circ$ and $\beta = 60^\circ$, replace \mathbf{P} with (a) an equivalent force-couple system at B , (b) an equivalent system formed by two parallel forces applied at A and B .

SOLUTION



(a) Equivalence requires

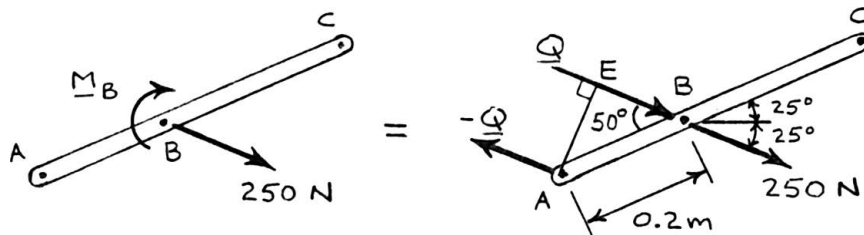
$$\Sigma \mathbf{F}: \mathbf{F}_B = \mathbf{P} \quad \text{or} \quad \mathbf{F}_B = 250 \text{ N} \searrow 25.0^\circ$$

$$\Sigma \mathbf{M}_B: M_B = -(0.3 \text{ m})[(250 \text{ N}) \sin 50^\circ] = -57.453 \text{ N} \cdot \text{m}$$

The equivalent force-couple system at B is

$$\mathbf{F}_B = 250 \text{ N} \searrow 25.0^\circ \qquad \mathbf{M}_B = 57.5 \text{ N} \cdot \text{m} \curvearrowleft$$

(b) We require

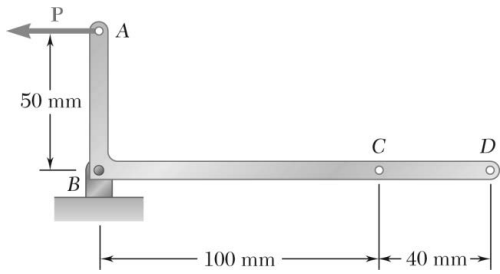


Equivalence requires

$$\begin{aligned} M_B &= d_{AE} Q \quad (0.3 \text{ m})[(250 \text{ N}) \sin 50^\circ] \\ &= [(0.2 \text{ m}) \sin 50^\circ] Q \\ Q &= 375 \text{ N} \end{aligned}$$

Adding the forces at B :

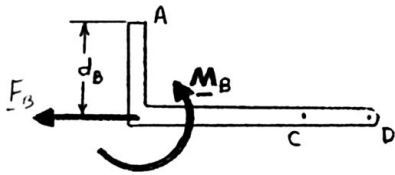
$$\mathbf{F}_A = 375 \text{ N} \searrow 25.0^\circ \qquad \mathbf{F}_B = 625 \text{ N} \searrow 25.0^\circ \curvearrowleft$$



PROBLEM 3.85

The 80-N horizontal force \mathbf{P} acts on a bell crank as shown. (a) Replace \mathbf{P} with an equivalent force-couple system at B . (b) Find the two vertical forces at C and D that are equivalent to the couple found in part a .

SOLUTION



(a) Based on $\Sigma F: F_B = F = 80 \text{ N}$ or $\mathbf{F}_B = 80.0 \text{ N} \leftarrow$

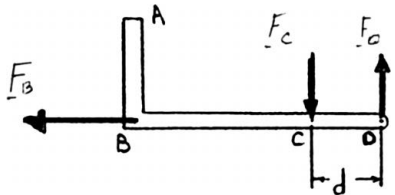
$\Sigma M: M_B = Fd_B$

$= 80 \text{ N} (0.05 \text{ m})$

$= 4.0000 \text{ N} \cdot \text{m}$

or $\mathbf{M}_B = 4.00 \text{ N} \cdot \text{m} \curvearrowleft$

(b) If the two vertical forces are to be equivalent to \mathbf{M}_B , they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then with F_C and F_D acting as shown,

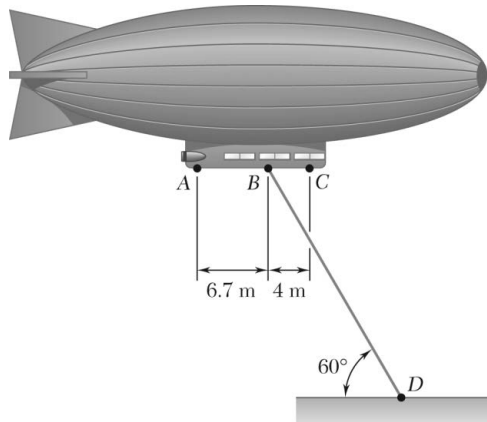
$\Sigma M: M_D = F_C d$

$4.0000 \text{ N} \cdot \text{m} = F_C (0.04 \text{ m})$

$F_C = 100.000 \text{ N}$ or $\mathbf{F}_C = 100.0 \text{ N} \downarrow$

$\Sigma F_y: 0 = F_D - F_C$

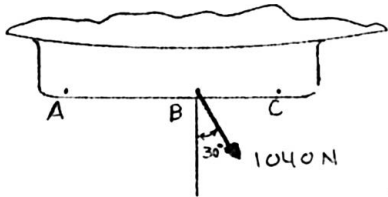
$F_D = 100.000 \text{ N}$ or $\mathbf{F}_D = 100.0 \text{ N} \uparrow$



PROBLEM 3.86

A dirigible is tethered by a cable attached to its cabin at B . If the tension in the cable is 1040 N , replace the force exerted by the cable at B with an equivalent system formed by two parallel forces applied at A and C .

SOLUTION



Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: (1040\text{ N})\sin 30^\circ = F_A \sin \alpha + F_B \sin \alpha \quad (1)$$

$$\Sigma F_y: -(1040\text{ N})\cos 30^\circ = -F_A \cos \alpha - F_B \cos \alpha \quad (2)$$

Dividing Equation (1) by Equation (2),

$$\frac{(1040\text{ N})\sin 30^\circ}{-(1040\text{ N})\cos 30^\circ} = \frac{(F_A + F_B)\sin \alpha}{-(F_A + F_B)\cos \alpha}$$

Simplifying yields $\alpha = 30^\circ$.

Based on

$$\Sigma M_C: [(1040\text{ N})\cos 30^\circ](4\text{ m}) = (F_A \cos 30^\circ)(10.7\text{ m})$$

$$F_A = 388.79\text{ N}$$

or

$$\mathbf{F}_A = 389\text{ N} \swarrow 60.0^\circ \blacktriangleleft$$

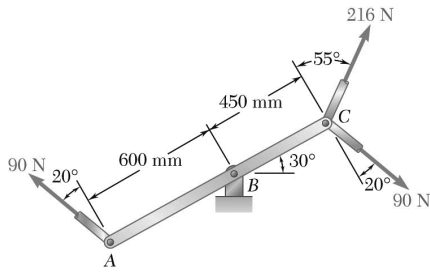
Based on

$$\Sigma M_A: -[(1040\text{ N})\cos 30^\circ](6.7\text{ m}) = (F_C \cos 30^\circ)(10.7\text{ m})$$

$$F_C = 651.21\text{ N}$$

or

$$\mathbf{F}_C = 651\text{ N} \swarrow 60.0^\circ \blacktriangleleft$$



PROBLEM 3.87

Three control rods attached to a lever ABC exert on it the forces shown. (a) Replace the three forces with an equivalent force-couple system at B . (b) Determine the single force that is equivalent to the force-couple system obtained in part a , and specify its point of application on the lever.

SOLUTION

- (a) First note that the two 90-N forces form a couple. Then

$$\mathbf{F} = 216 \text{ N} \nearrow \theta$$

where

$$\theta = 180^\circ - (60^\circ + 55^\circ) = 65^\circ$$

and

$$\begin{aligned} M &= \Sigma M_B \\ &= (0.450 \text{ m})(216 \text{ N}) \cos 55^\circ - (1.050 \text{ m})(90 \text{ N}) \cos 20^\circ \\ &= -33.049 \text{ N}\cdot\text{m} \end{aligned}$$

The equivalent force-couple system at B is

$$\mathbf{F} = 216 \text{ N} \nearrow 65.0^\circ; \quad \mathbf{M} = 33.0 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$

- (b) The single equivalent force \mathbf{F}' is equal to \mathbf{F} . Further, since the sense of \mathbf{M} is clockwise, \mathbf{F}' must be applied between A and B . For equivalence,

$$\Sigma M_B: \quad M = aF' \cos 55^\circ$$

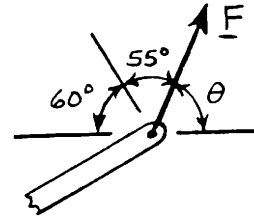
where a is the distance from B to the point of application of \mathbf{F}' . Then

$$-33.049 \text{ N}\cdot\text{m} = -a(216 \text{ N}) \cos 55^\circ$$

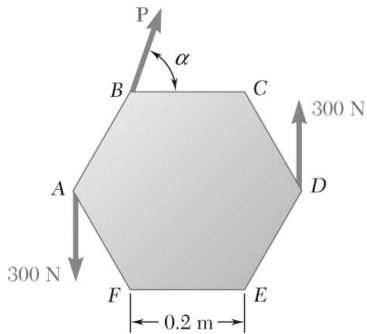
$$a = 0.26676 \text{ m}$$

or

$$\mathbf{F}' = 216 \text{ N} \nearrow 65.0^\circ \text{ applied to the lever } 267 \text{ mm to the left of } B \blacktriangleleft$$



PROBLEM 3.88



A hexagonal plate is acted upon by the force \mathbf{P} and the couple shown. Determine the magnitude and the direction of the smallest force \mathbf{P} for which this system can be replaced with a single force at E .

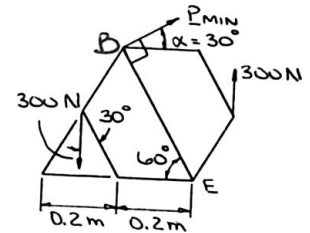
SOLUTION

From the statement of the problem, it follows that $\Sigma M_E = 0$ for the given force-couple system. Further, for \mathbf{P}_{\min} , we must require that \mathbf{P} be perpendicular to \mathbf{r}_{BE} . Then

$$\begin{aligned} \Sigma M_E: & (0.2 \sin 30^\circ + 0.2)\text{m} \times 300 \text{ N} \\ & + (0.2 \text{ m}) \sin 30^\circ \times 300 \text{ N} \\ & - (0.4 \text{ m}) P_{\min} = 0 \end{aligned}$$

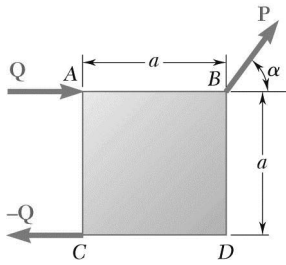
or

$$P_{\min} = 300 \text{ N}$$



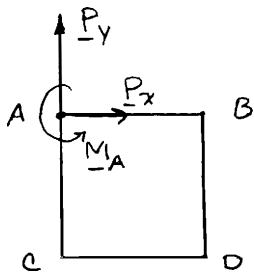
$$\mathbf{P}_{\min} = 300 \text{ N} \nearrow 30.0^\circ \blacktriangleleft$$

PROBLEM 3.89



A force and couple act as shown on a square plate of side $a = 25$ in. Knowing that $P = 60$ lb, $Q = 40$ lb, and $\alpha = 50^\circ$, replace the given force and couple by a single force applied at a point located (a) on line AB , (b) on line AC . In each case determine the distance from A to the point of application of the force.

SOLUTION

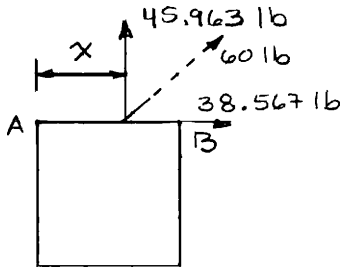


Replace the given force-couple system with an equivalent force-couple system at A .

$$P_x = (60 \text{ lb})(\cos 50^\circ) = 38.567 \text{ lb}$$

$$P_y = (60 \text{ lb})(\sin 50^\circ) = 45.963 \text{ lb}$$

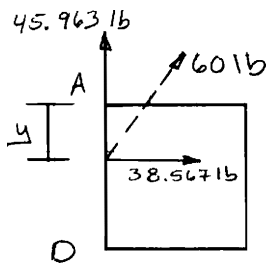
$$\begin{aligned} +\curvearrowright M_A &= P_y a - Qa \\ &= (45.963 \text{ lb})(25 \text{ in.}) - (40 \text{ lb})(25 \text{ in.}) \\ &= 149.075 \text{ lb} \cdot \text{in.} \end{aligned}$$



(a) Equating moments about A gives:

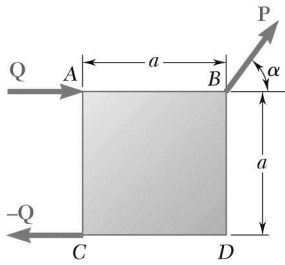
$$\begin{aligned} 149.075 \text{ lb} \cdot \text{in.} &= (45.963 \text{ lb})x \\ x &= 3.24 \text{ in.} \end{aligned}$$

$\mathbf{P} = 60.0 \text{ lb} \angle 50.0^\circ; 3.24 \text{ in. from } A \blacktriangleleft$



(b) $149.075 \text{ lb} \cdot \text{in.} = (38.567 \text{ lb})y$
 $y = 3.87 \text{ in.}$

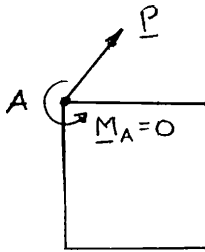
$\mathbf{P} = 60.0 \text{ lb} \angle 50.0^\circ; 3.87 \text{ in. below } A \blacktriangleleft$



PROBLEM 3.90

The force and couple shown are to be replaced by an equivalent single force. Knowing that $P = 2Q$, determine the required value of α if the line of action of the single equivalent force is to pass through (a) Point A, (b) Point C.

SOLUTION

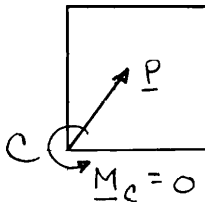


(a) We must have $M_A = 0$

$$(P \sin \alpha)a - Q(a) = 0$$

$$\sin \alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\alpha = 30.0^\circ \blacktriangleleft$$



(b) We must have $M_C = 0$

$$(P \sin \alpha)a - (P \cos \alpha)a - Q(a) = 0$$

$$\sin \alpha - \cos \alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\sin \alpha = \cos \alpha + \frac{1}{2} \quad (1)$$

$$\sin^2 \alpha = \cos^2 \alpha + \cos \alpha + \frac{1}{4}$$

$$1 - \cos^2 \alpha = \cos^2 \alpha + \cos \alpha + \frac{1}{4}$$

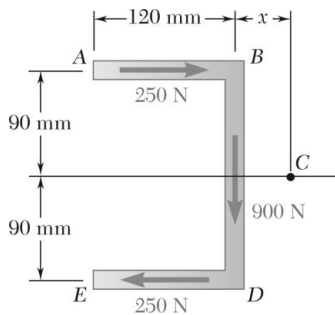
$$2 \cos^2 \alpha + \cos \alpha - 0.75 = 0 \quad (2)$$

Solving the quadratic in $\cos \alpha$:

$$\cos \alpha = \frac{-1 \pm \sqrt{7}}{4} \quad \alpha = 65.7^\circ \text{ or } 155.7^\circ$$

Only the first value of α satisfies Eq. (1),

$$\text{therefore } \alpha = 65.7^\circ \blacktriangleleft$$

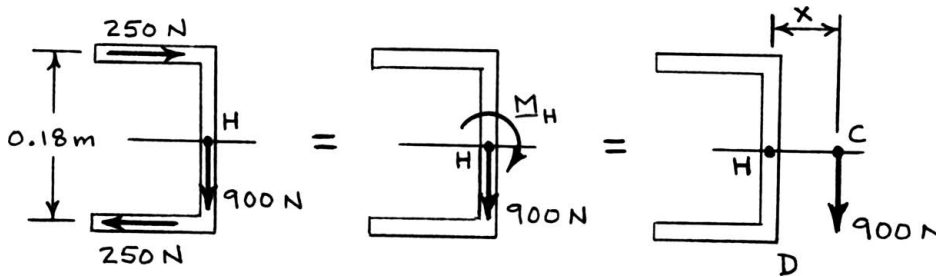


PROBLEM 3.91

The shearing forces exerted on the cross section of a steel channel can be represented by a 900-N vertical force and two 250-N horizontal forces as shown. Replace this force and couple with a single force \mathbf{F} applied at Point C , and determine the distance x from C to line BD . (Point C is defined as the *shear center* of the section.)

SOLUTION

Replace the 250-N forces with a couple and move the 900-N force to Point C such that its moment about H is equal to the moment of the couple



$$M_H = (0.18)(250 \text{ N})$$

$$= 45 \text{ N} \cdot \text{m}$$

Then

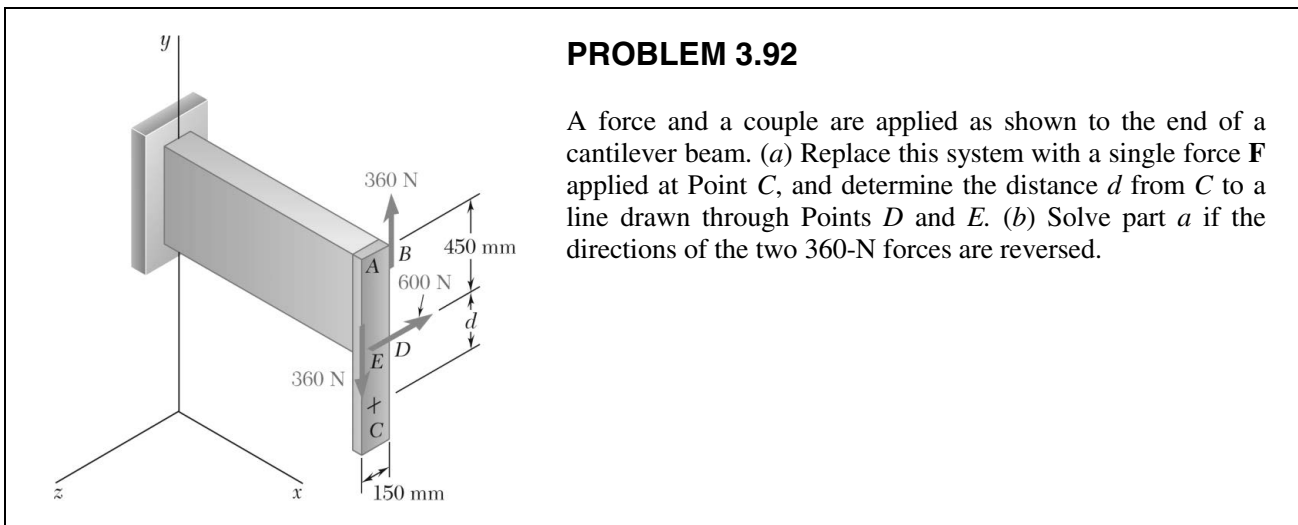
$$M_H = x(900 \text{ N})$$

or

$$45 \text{ N} \cdot \text{m} = x(900 \text{ N})$$

$$x = 0.05 \text{ m}$$

$$\mathbf{F} = 900 \text{ N} \downarrow \quad x = 50.0 \text{ mm} \leftarrow$$



PROBLEM 3.92

A force and a couple are applied as shown to the end of a cantilever beam. (a) Replace this system with a single force \mathbf{F} applied at Point C, and determine the distance d from C to a line drawn through Points D and E. (b) Solve part a if the directions of the two 360-N forces are reversed.

SOLUTION

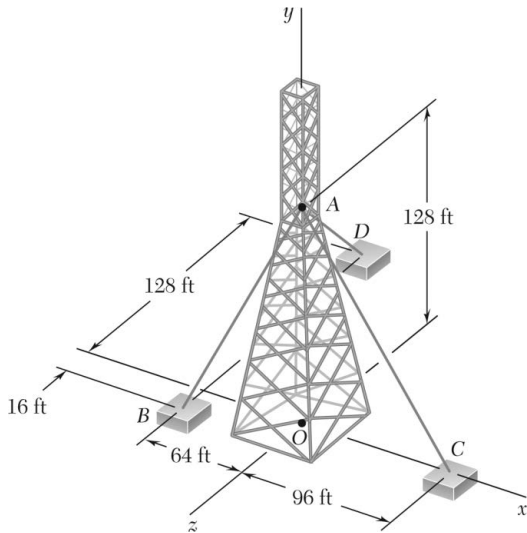
(a) We have $\Sigma \mathbf{F}: \mathbf{F} = (360 \text{ N})\mathbf{j} - (360 \text{ N})\mathbf{j} - (600 \text{ N})\mathbf{k}$
or $\mathbf{F} = -(600 \text{ N})\mathbf{k} \blacktriangleleft$

and $\Sigma M_D: (360 \text{ N})(0.15 \text{ m}) = (600 \text{ N})(d)$
 $d = 0.09 \text{ m}$
or $d = 90.0 \text{ mm below } ED \blacktriangleleft$

(b) We have from part a: $\mathbf{F} = -(600 \text{ N})\mathbf{k} \blacktriangleleft$

and $\Sigma M_D: -(360 \text{ N})(0.15 \text{ m}) = -(600 \text{ N})(d)$
 $d = 0.09 \text{ m}$
or $d = 90.0 \text{ mm above } ED \blacktriangleleft$

PROBLEM 3.93



An antenna is guyed by three cables as shown. Knowing that the tension in cable AB is 288 lb, replace the force exerted at A by cable AB with an equivalent force-couple system at the center O of the base of the antenna.

SOLUTION

We have

$$d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144 \text{ ft}$$

Then

$$\begin{aligned} \mathbf{T}_{AB} &= \frac{288 \text{ lb}}{144} (-64\mathbf{i} - 128\mathbf{j} + 16\mathbf{k}) \\ &= (32 \text{ lb})(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) \end{aligned}$$

Now

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB} \\ &= 128\mathbf{j} \times 32(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) \\ &= (4096 \text{ lb} \cdot \text{ft})\mathbf{i} + (16,384 \text{ lb} \cdot \text{ft})\mathbf{k} \end{aligned}$$

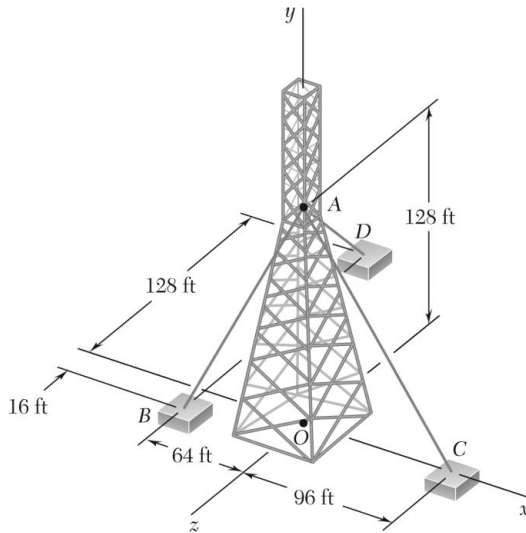
The equivalent force-couple system at O is

$$\mathbf{F} = -(128.0 \text{ lb})\mathbf{i} - (256 \text{ lb})\mathbf{j} + (32.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{M} = (4.10 \text{ kip} \cdot \text{ft})\mathbf{i} + (16.38 \text{ kip} \cdot \text{ft})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.94

An antenna is guyed by three cables as shown. Knowing that the tension in cable AD is 270 lb, replace the force exerted at A by cable AD with an equivalent force-couple system at the center O of the base of the antenna.



SOLUTION

We have

$$d_{AD} = \sqrt{(-64)^2 + (-128)^2 + (-128)^2} \\ = 192 \text{ ft}$$

Then

$$\mathbf{T}_{AD} = \frac{270 \text{ lb}}{192} (-64\mathbf{i} - 128\mathbf{j} + 128\mathbf{k}) \\ = (90 \text{ lb})(-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

Now

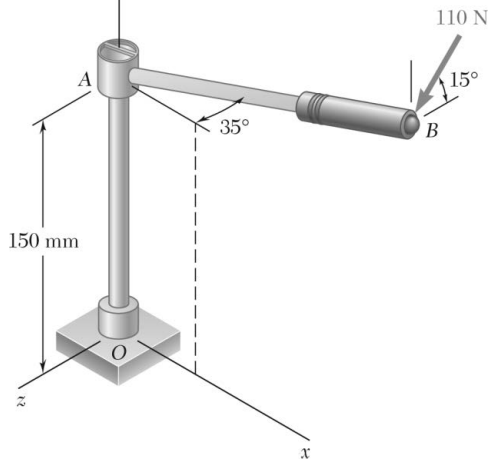
$$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AD} \\ = 128\mathbf{j} \times 90(-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \\ = -(23,040 \text{ lb} \cdot \text{ft})\mathbf{i} + (11,520 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at O is

$$\mathbf{F} = -(90.0 \text{ lb})\mathbf{i} - (180.0 \text{ lb})\mathbf{j} - (180.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

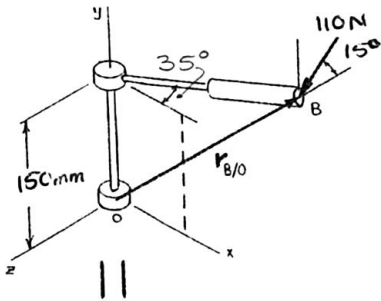
$$\mathbf{M} = -(23.0 \text{ kip} \cdot \text{ft})\mathbf{i} + (11.52 \text{ kip} \cdot \text{ft})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.95



A 110-N force acting in a vertical plane parallel to the yz -plane is applied to the 220-mm-long horizontal handle AB of a socket wrench. Replace the force with an equivalent force-couple system at the origin O of the coordinate system.

SOLUTION



We have $\Sigma \mathbf{F}: \mathbf{P}_B = \mathbf{F}$

where
$$\mathbf{P}_B = 110 \text{ N}[-(\sin 15^\circ)\mathbf{j} + (\cos 15^\circ)\mathbf{k}]$$

$$= -(28.470 \text{ N})\mathbf{j} + (106.252 \text{ N})\mathbf{k}$$

or $\mathbf{F} = -(28.5 \text{ N})\mathbf{j} + (106.3 \text{ N})\mathbf{k} \blacktriangleleft$

We have $\Sigma M_O: \mathbf{r}_{B/O} \times \mathbf{P}_B = \mathbf{M}_O$

where
$$\mathbf{r}_{B/O} = [(0.22 \cos 35^\circ)\mathbf{i} + (0.15)\mathbf{j} - (0.22 \sin 35^\circ)\mathbf{k}] \text{ m}$$

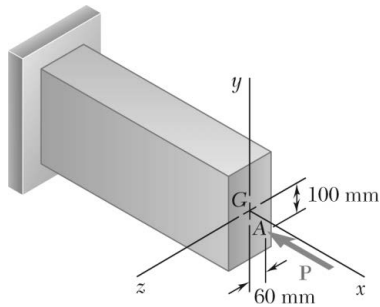
$$= (0.180213 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j} - (0.126187 \text{ m})\mathbf{k}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.180213 & 0.15 & 0.126187 \\ 0 & -28.5 & 106.3 \end{vmatrix} \text{ N} \cdot \text{m} = \mathbf{M}_O$$

$$\mathbf{M}_O = [(12.3487)\mathbf{i} - (19.1566)\mathbf{j} - (5.1361)\mathbf{k}] \text{ N} \cdot \text{m}$$

or $\mathbf{M}_O = (12.35 \text{ N} \cdot \text{m})\mathbf{i} - (19.16 \text{ N} \cdot \text{m})\mathbf{j} - (5.13 \text{ N} \cdot \text{m})\mathbf{k} \blacktriangleleft$

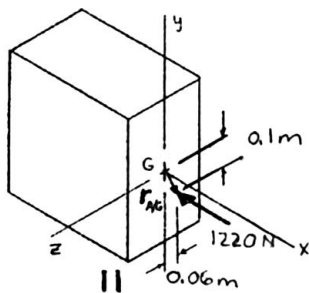




PROBLEM 3.96

An eccentric, compressive 1220-N force \mathbf{P} is applied to the end of a cantilever beam. Replace \mathbf{P} with an equivalent force-couple system at G .

SOLUTION



We have

$$\Sigma \mathbf{F}: -(1220 \text{ N})\mathbf{i} = \mathbf{F}$$

$$\mathbf{F} = -(1220 \text{ N})\mathbf{i} \quad \blacktriangleleft$$

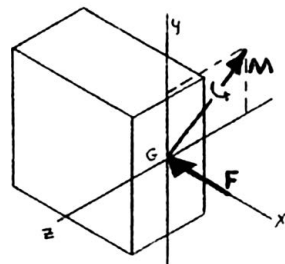
Also, we have

$$\Sigma \mathbf{M}_G: \mathbf{r}_{AG} \times \mathbf{P} = \mathbf{M}$$

$$1220 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.1 & -0.06 \\ -1 & 0 & 0 \end{vmatrix} \text{ N} \cdot \text{m} = \mathbf{M}$$

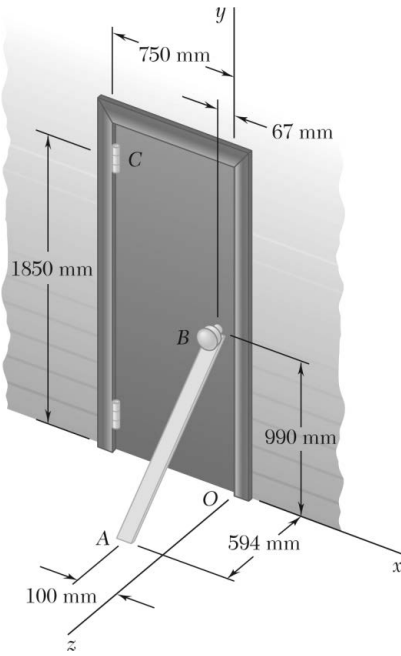
$$\mathbf{M} = (1220 \text{ N} \cdot \text{m})[(-0.06)(-1)\mathbf{j} - (-0.1)(-1)\mathbf{k}]$$

$$\text{or } \mathbf{M} = (73.2 \text{ N} \cdot \text{m})\mathbf{j} - (122 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.97

To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at B a 175-N force directed along line AB . Replace that force with an equivalent force-couple system at C .



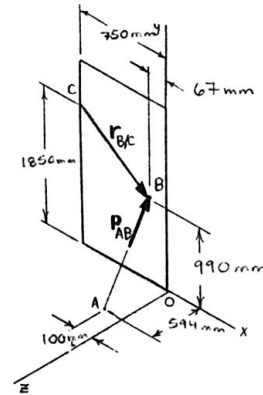
SOLUTION

We have

$$\Sigma \mathbf{F}: \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\begin{aligned} \mathbf{P}_{AB} &= \lambda_{AB} P_{AB} \\ &= \frac{(33 \text{ mm})\mathbf{i} + (990 \text{ mm})\mathbf{j} - (594 \text{ mm})\mathbf{k}}{1155.00 \text{ mm}} (175 \text{ N}) \end{aligned}$$



$$\text{or } \mathbf{F}_C = (5.00 \text{ N})\mathbf{i} + (150.0 \text{ N})\mathbf{j} - (90.0 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

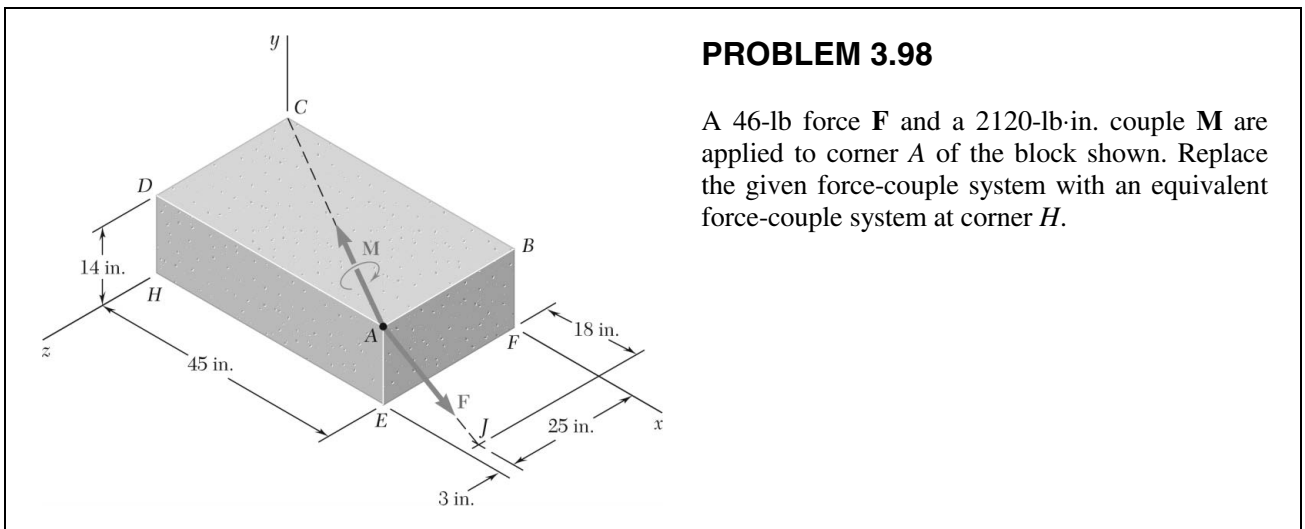
We have

$$\Sigma \mathbf{M}_C: \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$$

$$\begin{aligned} \mathbf{M}_C &= 5 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.683 & -0.860 & 0 \\ 1 & 30 & -18 \end{vmatrix} \text{ N} \cdot \text{m} \\ &= (5) \{ (-0.860)(-18)\mathbf{i} - (0.683)(-18)\mathbf{j} \\ &\quad + [(0.683)(30) - (0.860)(1)]\mathbf{k} \} \end{aligned}$$

$$\text{or } \mathbf{M}_C = (77.4 \text{ N} \cdot \text{m})\mathbf{i} + (61.5 \text{ N} \cdot \text{m})\mathbf{j} + (106.8 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$

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SOLUTION

We have $d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}$

Then
$$\mathbf{F} = \frac{46 \text{ lb}}{23} (18\mathbf{i} - 14\mathbf{j} - 3\mathbf{k})$$

$$= (36 \text{ lb})\mathbf{i} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$$

Also $d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}$

Then
$$\mathbf{M} = \frac{2120 \text{ lb} \cdot \text{in.}}{53} (-45\mathbf{i} - 28\mathbf{k})$$

$$= -(1800 \text{ lb} \cdot \text{in.})\mathbf{i} - (1120 \text{ lb} \cdot \text{in.})\mathbf{k}$$

Now
$$\mathbf{M}' = \mathbf{M} + \mathbf{r}_{A/H} \times \mathbf{F}$$

where
$$\mathbf{r}_{A/H} = (45 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$$

Then
$$\mathbf{M}' = (-1800\mathbf{i} - 1120\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}$$

$$= (-1800\mathbf{i} - 1120\mathbf{k}) + \{ [(14)(-6)]\mathbf{i} + [-(45)(-6)]\mathbf{j} + [(45)(-28) - (14)(36)]\mathbf{k} \}$$

$$= (-1800 - 84)\mathbf{i} + (270)\mathbf{j} + (-1120 - 1764)\mathbf{k}$$

$$= -(1884 \text{ lb} \cdot \text{in.})\mathbf{i} + (270 \text{ lb} \cdot \text{in.})\mathbf{j} - (2884 \text{ lb} \cdot \text{in.})\mathbf{k}$$

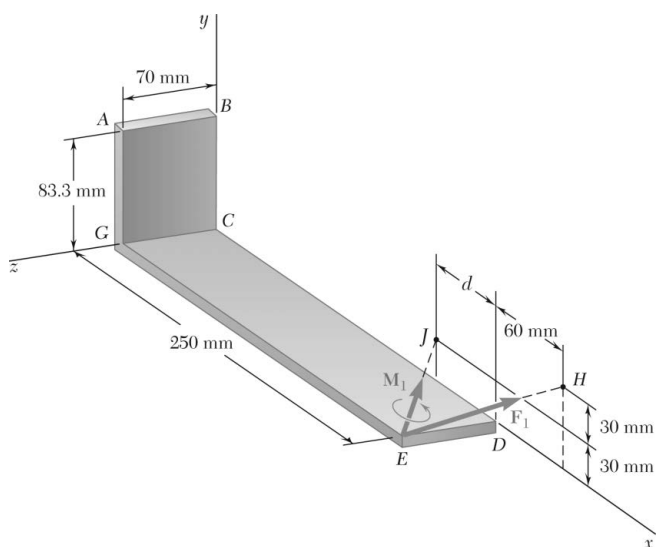
$$= -(157 \text{ lb} \cdot \text{ft})\mathbf{i} + (22.5 \text{ lb} \cdot \text{ft})\mathbf{j} - (240 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at H is
$$\mathbf{F}' = (36.0 \text{ lb})\mathbf{i} - (28.0 \text{ lb})\mathbf{j} - (6.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}' = -(157.0 \text{ lb} \cdot \text{ft})\mathbf{i} + (22.5 \text{ lb} \cdot \text{ft})\mathbf{j} - (240 \text{ lb} \cdot \text{ft})\mathbf{k} \blacktriangleleft$$

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PROBLEM 3.99



A 77-N force \mathbf{F}_1 and a 31-N · m couple \mathbf{M}_1 are applied to corner E of the bent plate shown. If \mathbf{F}_1 and \mathbf{M}_1 are to be replaced with an equivalent force-couple system $(\mathbf{F}_2, \mathbf{M}_2)$ at corner B and if $(M_2)_z = 0$, determine (a) the distance d , (b) \mathbf{F}_2 and \mathbf{M}_2 .

SOLUTION

(a) We have

$$\begin{aligned} \Sigma M_{Bz}: M_{2z} &= 0 \\ \mathbf{k} \cdot (\mathbf{r}_{H/B} \times \mathbf{F}_1) + M_{1z} &= 0 \end{aligned} \quad (1)$$

where

$$\mathbf{r}_{H/B} = (0.31 \text{ m})\mathbf{i} - (0.0233)\mathbf{j}$$

$$\begin{aligned} \mathbf{F}_1 &= \lambda_{EH} F_1 \\ &= \frac{(0.06 \text{ m})\mathbf{i} + (0.06 \text{ m})\mathbf{j} - (0.07 \text{ m})\mathbf{k}}{0.11 \text{ m}} (77 \text{ N}) \\ &= (42 \text{ N})\mathbf{i} + (42 \text{ N})\mathbf{j} - (49 \text{ N})\mathbf{k} \end{aligned}$$

$$M_{1z} = \mathbf{k} \cdot \mathbf{M}_1$$

$$\begin{aligned} \mathbf{M}_1 &= \lambda_{EJ} M_1 \\ &= \frac{-d\mathbf{i} + (0.03 \text{ m})\mathbf{j} - (0.07 \text{ m})\mathbf{k}}{\sqrt{d^2 + 0.0058}} (31 \text{ N} \cdot \text{m}) \end{aligned}$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(-0.07 \text{ m})(31 \text{ N} \cdot \text{m})}{\sqrt{d^2 + 0.0058}} = 0$$

Solving for d , Equation (1) reduces to

$$(13.0200 + 0.9786) - \frac{2.17 \text{ N} \cdot \text{m}}{\sqrt{d^2 + 0.0058}} = 0$$

from which

$$d = 0.1350 \text{ m}$$

$$\text{or } d = 135.0 \text{ mm} \quad \blacktriangleleft$$

PROBLEM 3.99 (Continued)

(b) $\mathbf{F}_2 = \mathbf{F}_1 = (42\mathbf{i} + 42\mathbf{j} - 49\mathbf{k}) \text{ N}$ or $\mathbf{F}_2 = (42.0 \text{ N})\mathbf{i} + (42.0 \text{ N})\mathbf{j} - (49.0 \text{ N})\mathbf{k} \blacktriangleleft$

$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(0.1350)\mathbf{i} + 0.03\mathbf{j} - 0.07\mathbf{k}}{0.155000} (31 \text{ N} \cdot \text{m})$$

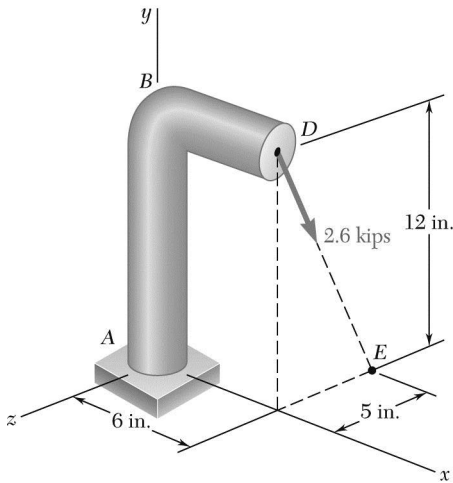
$$= (1.14170\mathbf{i} + 15.1900\mathbf{j} + 13.9986\mathbf{k}) \text{ N} \cdot \text{m}$$

$$+ (-27.000\mathbf{i} + 6.000\mathbf{j} - 14.000\mathbf{k}) \text{ N} \cdot \text{m}$$

$$\mathbf{M}_2 = -(25.858 \text{ N} \cdot \text{m})\mathbf{i} + (21.190 \text{ N} \cdot \text{m})\mathbf{j}$$

$$\text{or } \mathbf{M}_2 = -(25.9 \text{ N} \cdot \text{m})\mathbf{i} + (21.2 \text{ N} \cdot \text{m})\mathbf{j} \blacktriangleleft$$

PROBLEM 3.100



A 2.6-kip force is applied at Point D of the cast iron post shown. Replace that force with an equivalent force-couple system at the center A of the base section.

SOLUTION

$$\overline{DE} = -(12 \text{ in.})\mathbf{j} - (5 \text{ in.})\mathbf{k}; \quad DE = 13.00 \text{ in.}$$

$$\mathbf{F} = (2.6 \text{ kips}) \frac{\overline{DE}}{DE}$$

$$\mathbf{F} = (2.6 \text{ kips}) \frac{-12\mathbf{j} - 5\mathbf{k}}{13}$$

$$\mathbf{F} = -(2.40 \text{ kips})\mathbf{j} - (1.000 \text{ kip})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{M}_A = \mathbf{r}_{D/A} \times \mathbf{F}$$

where

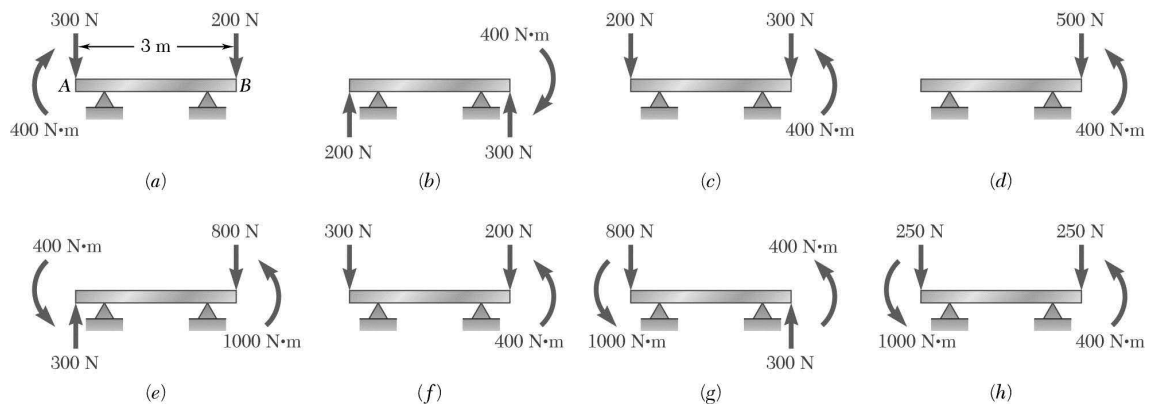
$$\mathbf{r}_{D/A} = (6 \text{ in.})\mathbf{i} + (12 \text{ in.})\mathbf{j}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 \text{ in.} & 12 \text{ in.} & 0 \\ 0 & -2.4 \text{ kips} & -1.0 \text{ kips} \end{vmatrix}$$

$$\mathbf{M}_A = -(12.00 \text{ kip} \cdot \text{in.})\mathbf{i} + (6.00 \text{ kip} \cdot \text{in.})\mathbf{j} - (14.40 \text{ kip} \cdot \text{in.})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.101

A 3-m-long beam is subjected to a variety of loadings. (a) Replace each loading with an equivalent force-couple system at end A of the beam. (b) Which of the loadings are equivalent?



SOLUTION

(a) (a) We have

$$\Sigma F_y: -300 \text{ N} - 200 \text{ N} = R_a$$

$$\text{or } R_a = 500 \text{ N} \downarrow \blacktriangleleft$$

and

$$\Sigma M_A: -400 \text{ N}\cdot\text{m} - (200 \text{ N})(3 \text{ m}) = M_a$$

$$\text{or } M_a = 1000 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$

(b) We have

$$\Sigma F_y: 200 \text{ N} + 300 \text{ N} = R_b$$

$$\text{or } R_b = 500 \text{ N} \uparrow \blacktriangleleft$$

and

$$\Sigma M_A: -400 \text{ N}\cdot\text{m} + (300 \text{ N})(3 \text{ m}) = M_b$$

$$\text{or } M_b = 500 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$

(c) We have

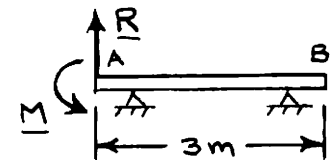
$$\Sigma F_y: -200 \text{ N} - 300 \text{ N} = R_c$$

$$\text{or } R_c = 500 \text{ N} \downarrow \blacktriangleleft$$

and

$$\Sigma M_A: 400 \text{ N}\cdot\text{m} - (300 \text{ N})(3 \text{ m}) = M_c$$

$$\text{or } M_c = 500 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$



PROBLEM 3.101 (Continued)

(d) We have $\Sigma F_Y: -500 \text{ N} = R_d$ or $\mathbf{R}_d = 500 \text{ N} \downarrow \blacktriangleleft$

and $\Sigma M_A: 400 \text{ N} \cdot \text{m} - (500 \text{ N})(3 \text{ m}) = M_d$ or $\mathbf{M}_d = 1100 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$

(e) We have $\Sigma F_Y: 300 \text{ N} - 800 \text{ N} = R_e$ or $\mathbf{R}_e = 500 \text{ N} \downarrow \blacktriangleleft$

and $\Sigma M_A: 400 \text{ N} \cdot \text{m} + 1000 \text{ N} \cdot \text{m} - (800 \text{ N})(3 \text{ m}) = M_e$ or $\mathbf{M}_e = 1000 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$

(f) We have $\Sigma F_Y: -300 \text{ N} - 200 \text{ N} = R_f$ or $\mathbf{R}_f = 500 \text{ N} \downarrow \blacktriangleleft$

and $\Sigma M_A: 400 \text{ N} \cdot \text{m} - (200 \text{ N})(3 \text{ m}) = M_f$ or $\mathbf{M}_f = 200 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$

(g) We have $\Sigma F_Y: -800 \text{ N} + 300 \text{ N} = R_g$ or $\mathbf{R}_g = 500 \text{ N} \downarrow \blacktriangleleft$

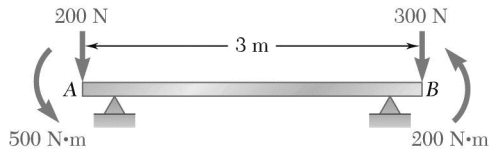
and $\Sigma M_A: 1000 \text{ N} \cdot \text{m} + 400 \text{ N} \cdot \text{m} + (300 \text{ N})(3 \text{ m}) = M_g$ or $\mathbf{M}_g = 2300 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$

(h) We have $\Sigma F_Y: -250 \text{ N} - 250 \text{ N} = R_h$ or $\mathbf{R}_h = 500 \text{ N} \downarrow \blacktriangleleft$

and $\Sigma M_A: 1000 \text{ N} \cdot \text{m} + 400 \text{ N} \cdot \text{m} - (250 \text{ N})(3 \text{ m}) = M_h$ or $\mathbf{M}_h = 650 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$

(b) Therefore, loadings (a) and (e) are equivalent.

PROBLEM 3.102



A 3-m-long beam is loaded as shown. Determine the loading of Prob. 3.101 that is equivalent to this loading.

SOLUTION

We have

$$\Sigma F_y: -200 \text{ N} - 300 \text{ N} = R$$

or

$$R = 500 \text{ N} \downarrow$$

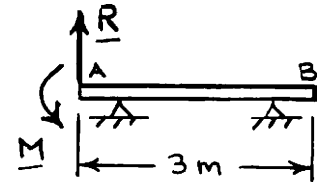
and

$$\Sigma M_A: 500 \text{ N}\cdot\text{m} + 200 \text{ N}\cdot\text{m} - (300 \text{ N})(3 \text{ m}) = M$$

or

$$M = 200 \text{ N}\cdot\text{m} \curvearrowright$$

Problem 3.101 equivalent force-couples at A:



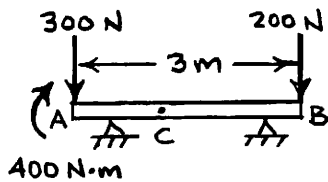
Case	\bar{R}	\bar{M}
(a)	500 N \downarrow	1000 N·m \curvearrowright
(b)	500 N \uparrow	500 N·m \curvearrowright
(c)	500 N \downarrow	500 N·m \curvearrowright
(d)	500 N \downarrow	1100 N·m \curvearrowright
(e)	500 N \downarrow	1000 N·m \curvearrowright
(f)	500 N \downarrow	200 N·m \curvearrowright
(g)	500 N \downarrow	2300 N·m \curvearrowright
(h)	500 N \downarrow	650 N·m \curvearrowright

Equivalent to case (f) of Problem 3.101 ◀

PROBLEM 3.103

Determine the single equivalent force and the distance from Point A to its line of action for the beam and loading of (a) Prob. 3.101a, (b) Prob. 3.101b, (c) Prob. 3.102.

SOLUTION



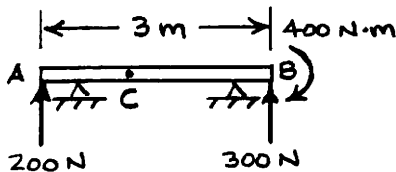
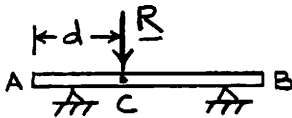
For equivalent single force at distance d from A:

(a) We have $\Sigma F_y: -300 \text{ N} - 200 \text{ N} = R$

or $R = 500 \text{ N} \downarrow \blacktriangleleft$

and $\Sigma M_C: -400 \text{ N}\cdot\text{m} + (300 \text{ N})(d) - (200 \text{ N})(3 - d) = 0$

or $d = 2.00 \text{ m} \blacktriangleleft$

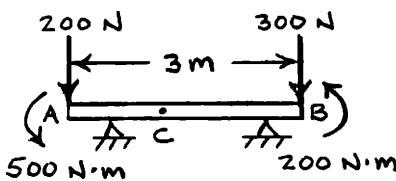
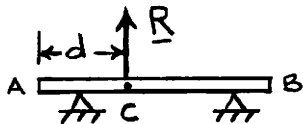


(b) We have $\Sigma F_y: 200 \text{ N} + 300 \text{ N} = R$

or $R = 500 \text{ N} \uparrow \blacktriangleleft$

and $\Sigma M_C: -400 \text{ N}\cdot\text{m} - (200 \text{ N})(d) + (300 \text{ N})(3 - d) = 0$

or $d = 1.000 \text{ m} \blacktriangleleft$

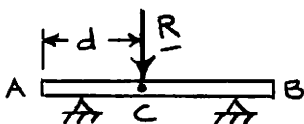


(c) We have $\Sigma F_y: -200 \text{ N} - 300 \text{ N} = R$

or $R = 500 \text{ N} \downarrow \blacktriangleleft$

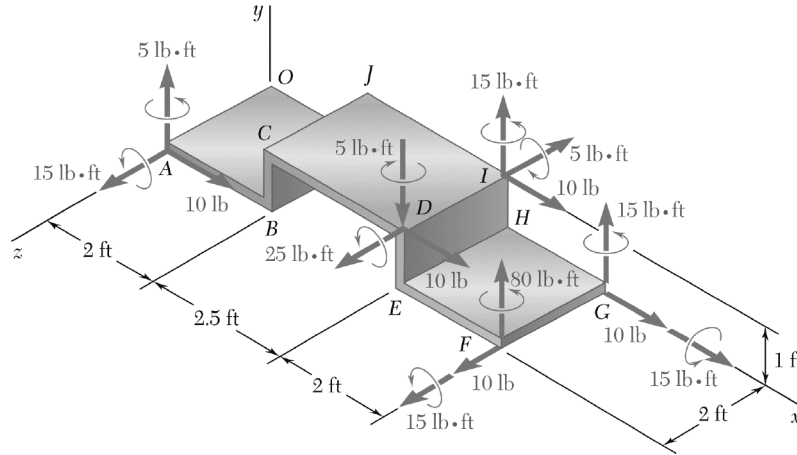
and $\Sigma M_C: 500 \text{ N}\cdot\text{m} + 200 \text{ N}\cdot\text{m} + (200 \text{ N})(d) - (300 \text{ N})(3 - d) = 0$

or $d = 0.400 \text{ m} \blacktriangleleft$



PROBLEM 3.104

Five separate force-couple systems act at the corners of a piece of sheet metal, which has been bent into the shape shown. Determine which of these systems is equivalent to a force $\mathbf{F} = (10 \text{ lb})\mathbf{i}$ and a couple of moment $\mathbf{M} = (15 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}$ located at the origin.



SOLUTION

First note that the force-couple system at F cannot be equivalent because of the direction of the force [The force of the other four systems is $(10 \text{ lb})\mathbf{i}$]. Next, move each of the systems to the origin O ; the forces remain unchanged.

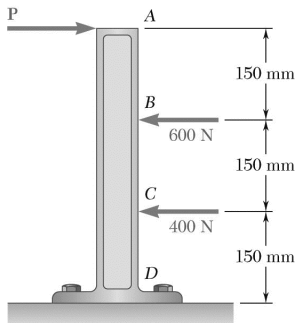
$$\begin{aligned} A: \quad \mathbf{M}_A &= \Sigma \mathbf{M}_O = (5 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k} + (2 \text{ ft})\mathbf{k} \times (10 \text{ lb})\mathbf{i} \\ &= (25 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k} \end{aligned}$$

$$\begin{aligned} D: \quad \mathbf{M}_D &= \Sigma \mathbf{M}_O = -(5 \text{ lb} \cdot \text{ft})\mathbf{j} + (25 \text{ lb} \cdot \text{ft})\mathbf{k} \\ &\quad + [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}] \times (10 \text{ lb})\mathbf{i} \\ &= (15 \text{ lb} \cdot \text{ft})\mathbf{i} + (15 \text{ lb} \cdot \text{ft})\mathbf{k} \end{aligned}$$

$$G: \quad \mathbf{M}_G = \Sigma \mathbf{M}_O = (15 \text{ lb} \cdot \text{ft})\mathbf{i} + (15 \text{ lb} \cdot \text{ft})\mathbf{j}$$

$$\begin{aligned} I: \quad \mathbf{M}_I &= \Sigma \mathbf{M}_I = (15 \text{ lb} \cdot \text{ft})\mathbf{j} - (5 \text{ lb} \cdot \text{ft})\mathbf{k} \\ &\quad + [(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j}] \times (10 \text{ lb})\mathbf{j} \\ &= (15 \text{ lb} \cdot \text{ft})\mathbf{j} - (15 \text{ lb} \cdot \text{ft})\mathbf{k} \end{aligned}$$

The equivalent force-couple system is the system at corner D . ◀

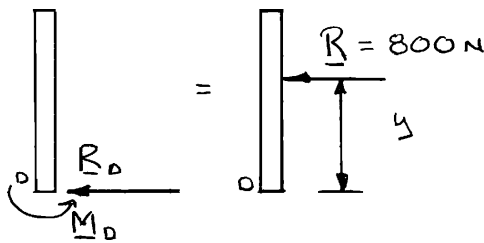


PROBLEM 3.105

Three horizontal forces are applied as shown to a vertical cast iron arm. Determine the resultant of the forces and the distance from the ground to its line of action when (a) $P = 200$ N, (b) $P = 2400$ N, (c) $P = 1000$ N.

SOLUTION

(a)



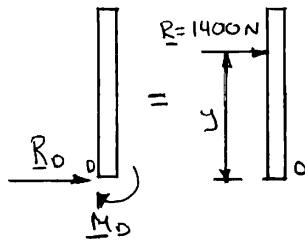
$$+\rightarrow R_D = +200 \text{ N} - 600 \text{ N} - 400 \text{ N} = -800 \text{ N}$$

$$+\curvearrowright M_D = -(200 \text{ N})(0.450 \text{ m}) + (600 \text{ N})(0.300 \text{ m}) + (400 \text{ N})(0.1500 \text{ m}) \\ = +150.0 \text{ N}\cdot\text{m}$$

$$y = \frac{M_D}{R} = \frac{150 \text{ N}\cdot\text{m}}{800 \text{ N}} = 0.1875 \text{ m}$$

$$\mathbf{R} = 800 \text{ N} \leftarrow; y = 187.5 \text{ mm} \blacktriangleleft$$

(b)



$$+\rightarrow R_D = +2400 \text{ N} - 600 \text{ N} - 400 \text{ N} = +1400 \text{ N}$$

$$+\curvearrowright M_D = -(2400 \text{ N})(0.450 \text{ m}) + (600 \text{ N})(0.300 \text{ m}) + (400 \text{ N})(0.1500 \text{ m}) \\ = -840 \text{ N}\cdot\text{m}$$

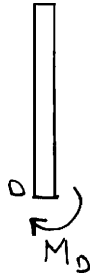
$$y = \frac{M_D}{R} = \frac{840 \text{ N}\cdot\text{m}}{1400 \text{ N}} = 0.600 \text{ m}$$

$$\mathbf{R} = 1400 \text{ N} \rightarrow; y = 600 \text{ mm} \blacktriangleleft$$

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PROBLEM 3.105 (Continued)

(c)

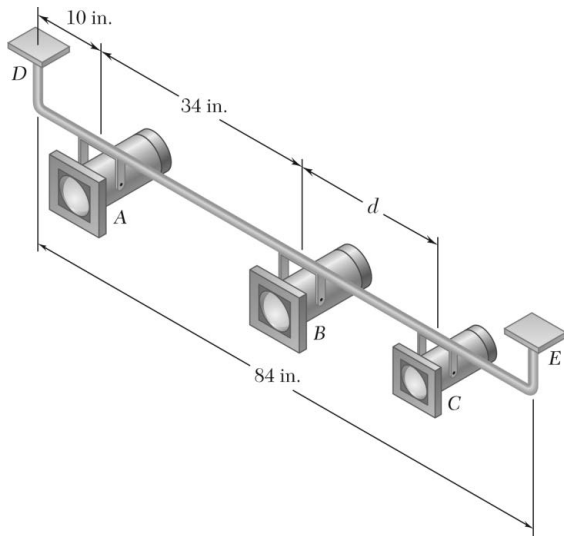


$$+\rightarrow R_D = +1000 - 600 - 400 = 0$$

$$\begin{aligned} +\curvearrowright M_D &= -(1000 \text{ N})(0.450 \text{ m}) + (600 \text{ N})(0.300 \text{ m}) + (400 \text{ N})(0.1500 \text{ m}) \\ &= -210 \text{ N}\cdot\text{m} \end{aligned}$$

$\therefore y = \infty$ System reduces to a couple.

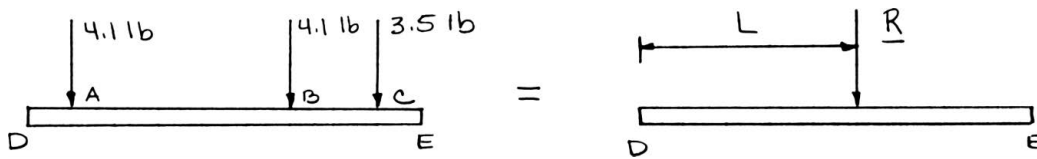
$$\mathbf{M}_D = 210 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$



PROBLEM 3.106

Three stage lights are mounted on a pipe as shown. The lights at *A* and *B* each weigh 4.1 lb, while the one at *C* weighs 3.5 lb. (a) If $d = 25$ in., determine the distance from *D* to the line of action of the resultant of the weights of the three lights. (b) Determine the value of d so that the resultant of the weights passes through the midpoint of the pipe.

SOLUTION



For equivalence,

$$\Sigma F_y: -4.1 - 4.1 - 3.5 = -R \quad \text{or} \quad \mathbf{R = 11.7 \text{ lb} \downarrow}$$

$$\Sigma F_D: -(10 \text{ in.})(4.1 \text{ lb}) - (44 \text{ in.})(4.1 \text{ lb}) - [(4.4 + d) \text{ in.}](3.5 \text{ lb}) = -(L \text{ in.})(11.7 \text{ lb})$$

or $375.4 + 3.5d = 11.7L \quad (d, L \text{ in. in.})$

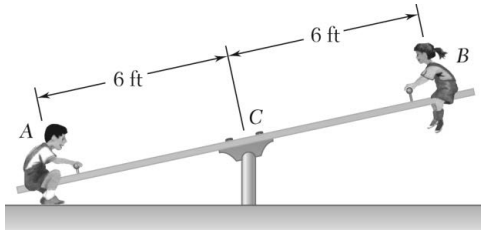
(a) $d = 25 \text{ in.}$

We have $375.4 + 3.5(25) = 11.7L \quad \text{or} \quad L = 39.6 \text{ in.}$

The resultant passes through a point 39.6 in. to the right of *D*. ◀

(b) $L = 42 \text{ in.}$

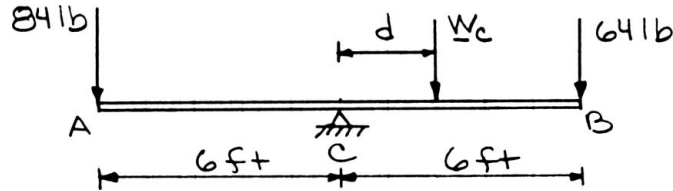
We have $375.4 + 3.5d = 11.7(42) \quad \text{or} \quad d = 33.1 \text{ in.} \quad \blacktriangleleft$



PROBLEM 3.107

The weights of two children sitting at ends A and B of a seesaw are 84 lb and 64 lb, respectively. Where should a third child sit so that the resultant of the weights of the three children will pass through C if she weighs (a) 60 lb, (b) 52 lb.

SOLUTION



(a) For the resultant weight to act at C , $\Sigma M_C = 0$ $W_C = 60$ lb

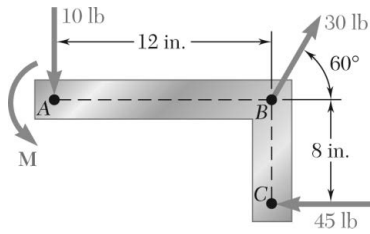
Then $(84 \text{ lb})(6 \text{ ft}) - 60 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.00$ ft to the right of C ◀

(b) For the resultant weight to act at C , $\Sigma M_C = 0$ $W_C = 52$ lb

Then $(84 \text{ lb})(6 \text{ ft}) - 52 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.31$ ft to the right of C ◀



PROBLEM 3.108

A couple of magnitude $M = 54 \text{ lb} \cdot \text{in.}$ and the three forces shown are applied to an angle bracket. (a) Find the resultant of this system of forces. (b) Locate the points where the line of action of the resultant intersects line AB and line BC .

SOLUTION

(a) We have $\Sigma \mathbf{F}: \mathbf{R} = (-10\mathbf{j}) + (30 \cos 60^\circ)\mathbf{i}$
 $+ 30 \sin 60^\circ\mathbf{j} + (-45\mathbf{i})$
 $= -(30 \text{ lb})\mathbf{i} + (15.9808 \text{ lb})\mathbf{j}$

or $\mathbf{R} = 34.0 \text{ lb} \nearrow 28.0^\circ \blacktriangleleft$

(b) First reduce the given forces and couple to an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_B)$ at B .

We have $\Sigma M_B: M_B = (54 \text{ lb} \cdot \text{in.}) + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb})$
 $= -186 \text{ lb} \cdot \text{in.}$

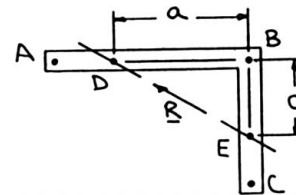
Then with \mathbf{R} at D , $\Sigma M_B: -186 \text{ lb} \cdot \text{in.} = a(15.9808 \text{ lb})$

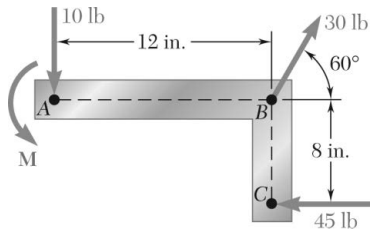
or $a = 11.64 \text{ in.}$

and with \mathbf{R} at E , $\Sigma M_B: -186 \text{ lb} \cdot \text{in.} = C(30 \text{ lb})$

or $C = 6.2 \text{ in.}$

The line of action of \mathbf{R} intersects line AB 11.64 in. to the left of B and intersects line BC 6.20 in. below B .





PROBLEM 3.109

A couple \mathbf{M} and the three forces shown are applied to an angle bracket. Find the moment of the couple if the line of action of the resultant of the force system is to pass through (a) Point A, (b) Point B, (c) Point C.

SOLUTION

In each case, we must have $\mathbf{M}_I^R = 0$

$$(a) \quad +\curvearrowright M_A^B = \Sigma M_A = M + (12 \text{ in.})[(30 \text{ lb}) \sin 60^\circ] - (8 \text{ in.})(45 \text{ lb}) = 0$$

$$M = +48.231 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 48.2 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$

$$(b) \quad +\curvearrowright M_B^R = \Sigma M_B = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb}) = 0$$

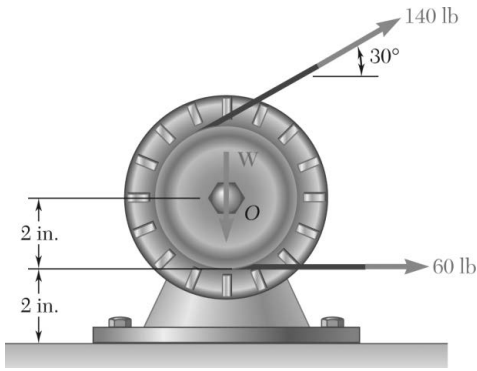
$$M = +240 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 240 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$

$$(c) \quad +\curvearrowright M_C^R = \Sigma M_C = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})[(30 \text{ lb}) \cos 60^\circ] = 0$$

$$M = 0$$

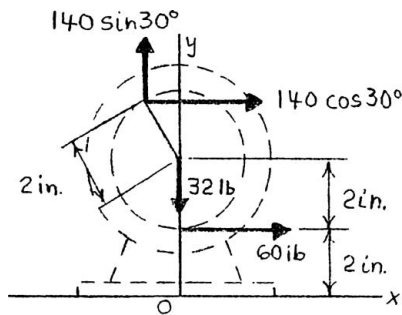
$$\mathbf{M} = 0 \blacktriangleleft$$



PROBLEM 3.110

A 32-lb motor is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.

SOLUTION



We have

$$\Sigma \mathbf{F}: (60 \text{ lb})\mathbf{i} - (32 \text{ lb})\mathbf{j} + (140 \text{ lb})(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) = \mathbf{R}$$

$$\mathbf{R} = (181.244 \text{ lb})\mathbf{i} + (38.0 \text{ lb})\mathbf{j}$$

$$\text{or } \mathbf{R} = 185.2 \text{ lb } \angle 11.84^\circ \blacktriangleleft$$

We have

$$\Sigma M_O: \Sigma M_O = xR_y$$

$$-[(140 \text{ lb}) \cos 30^\circ][(4 + 2 \cos 30^\circ) \text{ in.}] - [(140 \text{ lb}) \sin 30^\circ][(2 \text{ in.}) \sin 30^\circ]$$

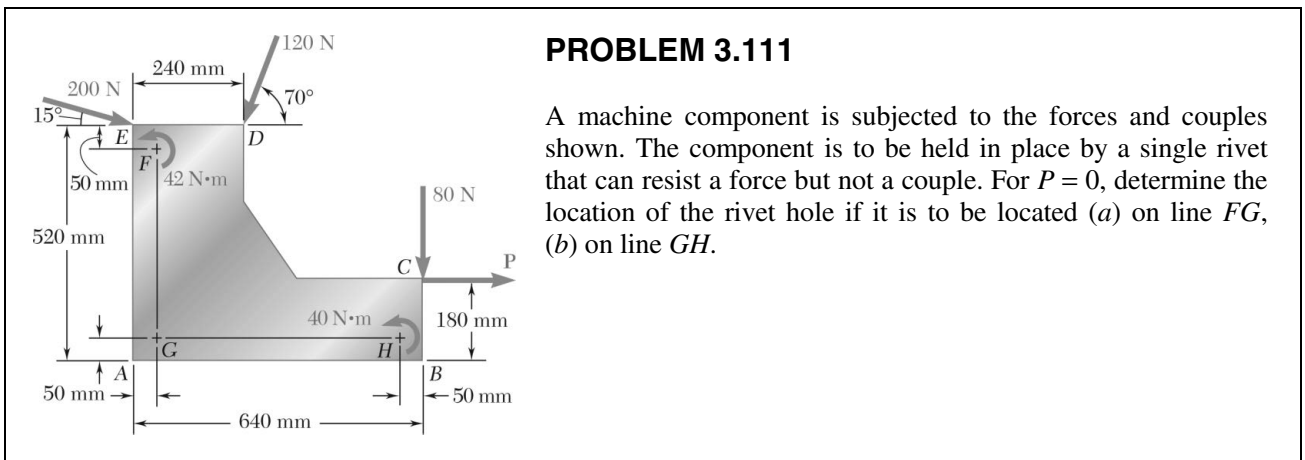
$$- (60 \text{ lb})(2 \text{ in.}) = x(38.0 \text{ lb})$$

$$x = \frac{1}{38.0}(-694.97 - 70.0 - 120) \text{ in.}$$

and

$$x = -23.289 \text{ in.}$$

Or resultant intersects the base (x -axis) 23.3 in. to the left of the vertical centerline (y -axis) of the motor. \blacktriangleleft

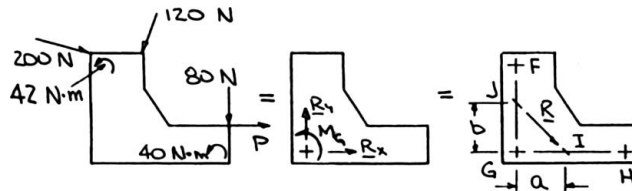


PROBLEM 3.111

A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (a) on line FG , (b) on line GH .

SOLUTION

We have



First replace the applied forces and couples with an equivalent force-couple system at G .

Thus, $\Sigma F_x: 200 \cos 15^\circ - 120 \cos 70^\circ + P = R_x$

or $R_x = (152.142 + P) \text{ N}$

$\Sigma F_y: -200 \sin 15^\circ - 120 \sin 70^\circ - 80 = R_y$

or $R_y = -244.53 \text{ N}$

$\Sigma M_G: -(0.47 \text{ m})(200 \text{ N}) \cos 15^\circ + (0.05 \text{ m})(200 \text{ N}) \sin 15^\circ$
 $+ (0.47 \text{ m})(120 \text{ N}) \cos 70^\circ - (0.19 \text{ m})(120 \text{ N}) \sin 70^\circ$
 $- (0.13 \text{ m})(P \text{ N}) - (0.59 \text{ m})(80 \text{ N}) + 42 \text{ N} \cdot \text{m}$
 $+ 40 \text{ N} \cdot \text{m} = M_G$

or $M_G = -(55.544 + 0.13P) \text{ N} \cdot \text{m} \tag{1}$

Setting $P = 0$ in Eq. (1):

Now with \mathbf{R} at I , $\Sigma M_G: -55.544 \text{ N} \cdot \text{m} = -a(244.53 \text{ N})$

or $a = 0.227 \text{ m}$

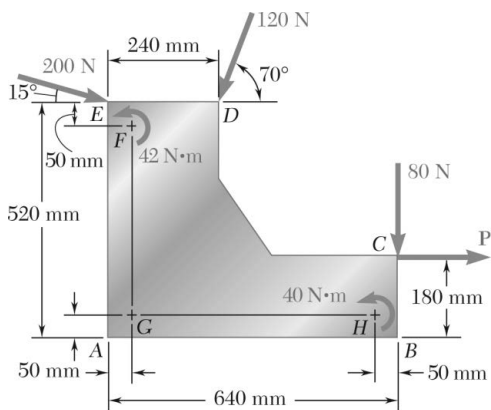
and with \mathbf{R} at J , $\Sigma M_G: -55.544 \text{ N} \cdot \text{m} = -b(152.142 \text{ N})$

or $b = 0.365 \text{ m}$

(a) The rivet hole is 0.365 m above G . ◀

(b) The rivet hole is 0.227 m to the right of G . ◀

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PROBLEM 3.112

Solve Problem 3.111, assuming that $P = 60$ N.

PROBLEM 3.111 A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (a) on line FG , (b) on line GH .

SOLUTION

See the solution to Problem 3.111 leading to the development of Equation (1):

$$M_G = -(55.544 + 0.13P) \text{ N} \cdot \text{m}$$

and

$$R_x = (152.142 + P) \text{ N}$$

For

$$P = 60 \text{ N}$$

we have

$$\begin{aligned} R_x &= (152.142 + 60) \\ &= 212.14 \text{ N} \end{aligned}$$

$$\begin{aligned} M_G &= -[55.544 + 0.13(60)] \\ &= -63.344 \text{ N} \cdot \text{m} \end{aligned}$$

Then with \mathbf{R} at I ,

$$\Sigma M_G: -63.344 \text{ N} \cdot \text{m} = -a(244.53 \text{ N})$$

or

$$a = 0.259 \text{ m}$$

and with \mathbf{R} at J ,

$$\Sigma M_G: -63.344 \text{ N} \cdot \text{m} = -b(212.14 \text{ N})$$

or

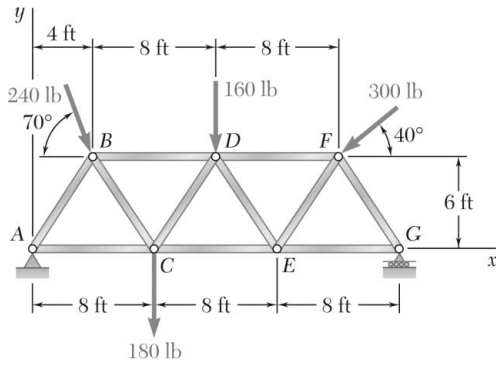
$$b = 0.299 \text{ m}$$

(a) The rivet hole is 0.299 m above G .



(b) The rivet hole is 0.259 m to the right of G .





PROBLEM 3.113

A truss supports the loading shown. Determine the equivalent force acting on the truss and the point of intersection of its line of action with a line drawn through Points A and G.

SOLUTION

We have

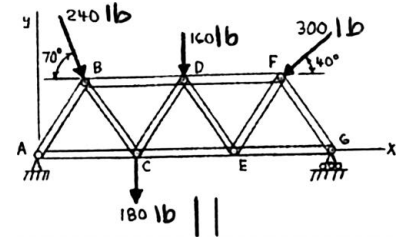
$$\mathbf{R} = \Sigma \mathbf{F}$$

$$\mathbf{R} = (240 \text{ lb})(\cos 70^\circ \mathbf{i} - \sin 70^\circ \mathbf{j}) - (160 \text{ lb})\mathbf{j} \\ + (300 \text{ lb})(-\cos 40^\circ \mathbf{i} - \sin 40^\circ \mathbf{j}) - (180 \text{ lb})\mathbf{j}$$

$$\mathbf{R} = -(147.728 \text{ lb})\mathbf{i} - (758.36 \text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} \\ = \sqrt{(147.728)^2 + (758.36)^2} \\ = 772.62 \text{ lb}$$

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right) \\ = \tan^{-1} \left(\frac{-758.36}{-147.728} \right) \\ = 78.977^\circ$$



or $\mathbf{R} = 773 \text{ lb} \nearrow 79.0^\circ$ ◀

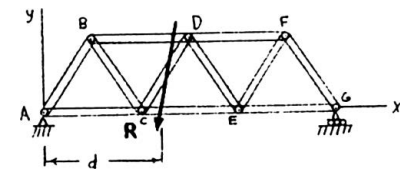
We have

$$\Sigma M_A = dR_y$$

where

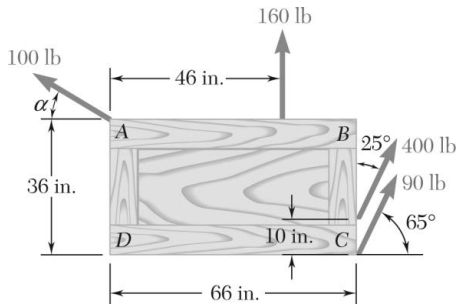
$$\Sigma M_A = -[240 \text{ lb} \cos 70^\circ](6 \text{ ft}) - [240 \text{ lb} \sin 70^\circ](4 \text{ ft}) \\ - (160 \text{ lb})(12 \text{ ft}) + [300 \text{ lb} \cos 40^\circ](6 \text{ ft}) \\ - [300 \text{ lb} \sin 40^\circ](20 \text{ ft}) - (180 \text{ lb})(8 \text{ ft}) \\ = -7232.5 \text{ lb} \cdot \text{ft}$$

$$d = \frac{-7232.5 \text{ lb} \cdot \text{ft}}{-758.36 \text{ lb}} \\ = 9.5370 \text{ ft}$$



or $d = 9.54 \text{ ft}$ to the right of A ◀

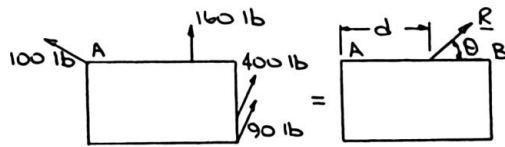
PROBLEM 3.114



Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line AB , determine (a) the equivalent force and the distance from A to the point of application of the force when $\alpha = 30^\circ$, (b) the value of α so that the single equivalent force is applied at Point B .

SOLUTION

We have



(a) For equivalence, $\Sigma F_x: -100 \cos 30^\circ + 400 \cos 65^\circ + 90 \cos 65^\circ = R_x$

or $R_x = 120.480 \text{ lb}$

$$\Sigma F_y: 100 \sin \alpha + 160 + 400 \sin 65^\circ + 90 \sin 65^\circ = R_y$$

or $R_y = (604.09 + 100 \sin \alpha) \text{ lb}$ (1)

With $\alpha = 30^\circ$, $R_y = 654.09 \text{ lb}$

Then $R = \sqrt{(120.480)^2 + (654.09)^2}$ $\tan \theta = \frac{654.09}{120.480}$
 $= 665 \text{ lb}$ or $\theta = 79.6^\circ$

$$\Sigma M_A: (46 \text{ in.})(160 \text{ lb}) + (66 \text{ in.})(400 \text{ lb}) \sin 65^\circ + (26 \text{ in.})(400 \text{ lb}) \cos 65^\circ + (66 \text{ in.})(90 \text{ lb}) \sin 65^\circ + (36 \text{ in.})(90 \text{ lb}) \cos 65^\circ = d(654.09 \text{ lb})$$

or $\Sigma M_A = 42,435 \text{ lb} \cdot \text{in.}$ and $d = 64.9 \text{ in.}$ $R = 665 \text{ lb} \angle 79.6^\circ \blacktriangleleft$

and \mathbf{R} is applied 64.9 in. to the right of A . \blacktriangleleft

(b) We have $d = 66 \text{ in.}$

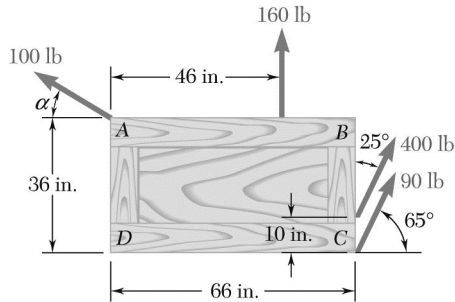
Then $\Sigma M_A: 42,435 \text{ lb} \cdot \text{in} = (66 \text{ in.})R_y$

or $R_y = 642.95 \text{ lb}$

Using Eq. (1): $642.95 = 604.09 + 100 \sin \alpha$ or $\alpha = 22.9^\circ \blacktriangleleft$

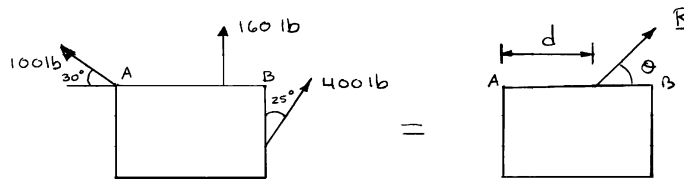
PROBLEM 3.115

Solve Prob. 3.114, assuming that the 90-lb force is removed.



PROBLEM 3.114 Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line AB , determine (a) the equivalent force and the distance from A to the point of application of the force when $\alpha = 30^\circ$, (b) the value of α so that the single equivalent force is applied at Point B .

SOLUTION

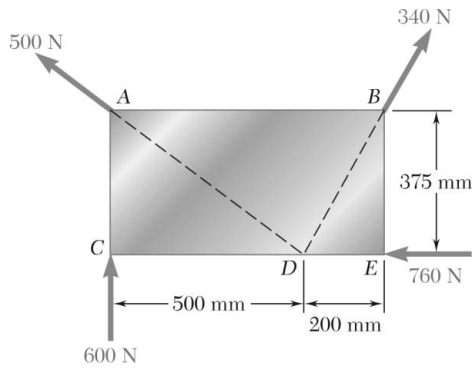


(a) For equivalence, $\Sigma F_x: -(100 \text{ lb}) \cos 30^\circ + (400 \text{ lb}) \sin 25^\circ = R_x$
 or $R_x = 82.445 \text{ lb}$
 $\Sigma F_y: 160 \text{ lb} + (100 \text{ lb}) \sin 30^\circ + (400 \text{ lb}) \cos 25^\circ = R_y$
 or $R_y = 572.52 \text{ lb}$
 $R = \sqrt{(82.445)^2 + (572.52)^2} = 578.43 \text{ lb}$
 $\tan \theta = \frac{572.52}{82.445} \quad \text{or} \quad \theta = 81.806^\circ$
 $\Sigma M_A: (46 \text{ in.})(160 \text{ lb}) + (66 \text{ in.})(400 \text{ lb}) \cos 25^\circ + (26 \text{ in.})(400 \text{ lb}) \sin 25^\circ$
 $= d(527.52 \text{ lb})$
 $d = 62.3 \text{ in.}$

$\mathbf{R} = 578 \text{ lb} \nearrow 81.8^\circ$ and is applied 62.3 in. to the right of A . ◀

(b) We have $d = 66.0 \text{ in.}$ For R applied at B ,
 $\Sigma M_A: R_y(66 \text{ in.}) = (160 \text{ lb})(46 \text{ in.}) + (66 \text{ in.})(400 \text{ lb}) \cos 25^\circ + (26 \text{ in.})(400 \text{ lb}) \sin 25^\circ$
 $R_y = 540.64 \text{ lb}$
 $\Sigma F_y: 160 \text{ lb} + (100 \text{ lb}) \sin \alpha + (400 \text{ lb}) \cos 25^\circ = 540.64 \text{ lb}$

$\alpha = 10.44^\circ$ ◀



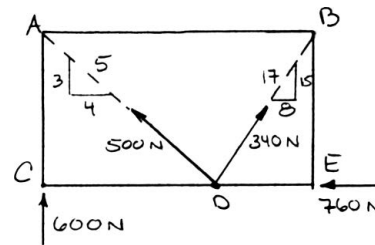
PROBLEM 3.116

Four forces act on a 700×375 -mm plate as shown. (a) Find the resultant of these forces. (b) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION

(a)

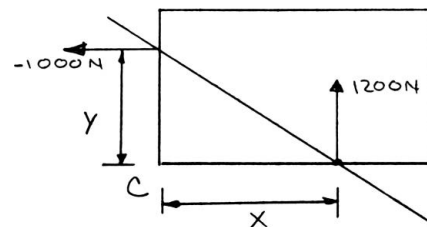
$$\begin{aligned} \mathbf{R} &= \Sigma \mathbf{F} \\ &= (-400 \text{ N} + 160 \text{ N} - 760 \text{ N})\mathbf{i} \\ &\quad + (600 \text{ N} + 300 \text{ N} + 300 \text{ N})\mathbf{j} \\ &= -(1000 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j} \\ R &= \sqrt{(1000 \text{ N})^2 + (1200 \text{ N})^2} \\ &= 1562.09 \text{ N} \\ \tan \theta &= \left(-\frac{1200 \text{ N}}{1000 \text{ N}} \right) \\ &= -1.20000 \\ \theta &= -50.194^\circ \end{aligned}$$



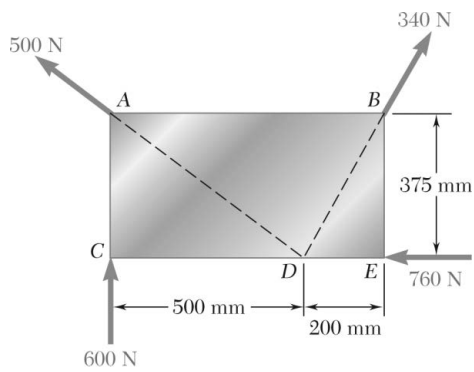
$$\mathbf{R} = 1562 \text{ N} \nearrow 50.2^\circ \blacktriangleleft$$

(b)

$$\begin{aligned} \mathbf{M}_C^R &= \Sigma \mathbf{r} \times \mathbf{F} \\ &= (0.5 \text{ m})\mathbf{i} \times (300 \text{ N} + 300 \text{ N})\mathbf{j} \\ &= (300 \text{ N} \cdot \text{m})\mathbf{k} \\ (300 \text{ N} \cdot \text{m})\mathbf{k} &= x\mathbf{i} \times (1200 \text{ N})\mathbf{j} \\ x &= 0.25000 \text{ m} \\ x &= 250 \text{ mm} \\ (300 \text{ N} \cdot \text{m}) &= y\mathbf{j} \times (-1000 \text{ N})\mathbf{i} \\ y &= 0.30000 \text{ m} \\ y &= 300 \text{ mm} \end{aligned}$$



Intersection 250 mm to right of C and 300 mm above C \blacktriangleleft



PROBLEM 3.117

Solve Problem 3.116, assuming that the 760-N force is directed to the right.

PROBLEM 3.116 Four forces act on a 700 × 375-mm plate as shown. (a) Find the resultant of these forces. (b) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION

(a)

$$\mathbf{R} = \Sigma \mathbf{F}$$

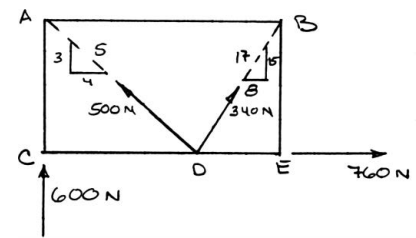
$$= (-400\text{ N} + 160\text{ N} + 760\text{ N})\mathbf{i} \\ + (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j} \\ = (520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}$$

$$R = \sqrt{(520\text{ N})^2 + (1200\text{ N})^2} = 1307.82\text{ N}$$

$$\tan \theta = \left(\frac{1200\text{ N}}{520\text{ N}} \right) = 2.3077$$

$$\theta = 66.5714^\circ$$

$$\mathbf{R} = 1308\text{ N} \angle 66.6^\circ \blacktriangleleft$$



(b)

$$\mathbf{M}_C^R = \Sigma \mathbf{r} \times \mathbf{F}$$

$$= (0.5\text{ m})\mathbf{i} \times (300\text{ N} + 300\text{ N})\mathbf{j} \\ = (300\text{ N} \cdot \text{m})\mathbf{k}$$

$$(300\text{ N} \cdot \text{m})\mathbf{k} = x\mathbf{i} \times (1200\text{ N})\mathbf{j}$$

$$x = 0.25000\text{ m}$$

or

$$x = 0.250\text{ mm}$$

$$(300\text{ N} \cdot \text{m})\mathbf{k} = [x'\mathbf{i} + (0.375\text{ m})\mathbf{j}] \times [(520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}]$$

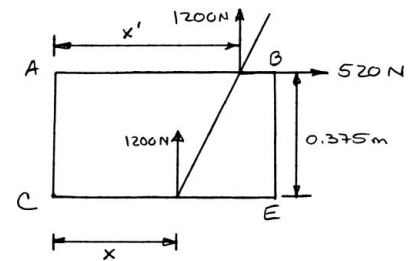
$$= (1200x' - 195)\mathbf{k}$$

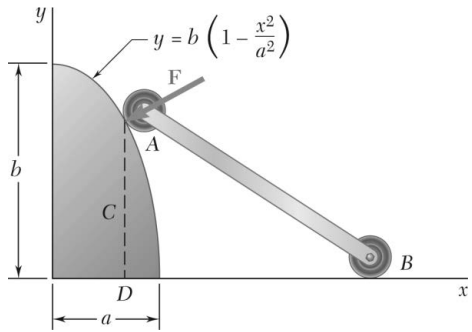
$$x' = 0.41250\text{ m}$$

or

$$x' = 412.5\text{ mm}$$

Intersection 412 mm to the right of A and 250 mm to the right of C \blacktriangleleft





PROBLEM 3.118

As follower AB rolls along the surface of member C , it exerts a constant force \mathbf{F} perpendicular to the surface. (a) Replace \mathbf{F} with an equivalent force-couple system at Point D obtained by drawing the perpendicular from the point of contact to the x -axis. (b) For $a = 1$ m and $b = 2$ m, determine the value of x for which the moment of the equivalent force-couple system at D is maximum.

SOLUTION

(a) The slope of any tangent to the surface of member C is

$$\frac{dy}{dx} = \frac{d}{dx} \left[b \left(1 - \frac{x^2}{a^2} \right) \right] = \frac{-2b}{a^2} x$$

Since the force \mathbf{F} is perpendicular to the surface,

$$\tan \alpha = - \left(\frac{dy}{dx} \right)^{-1} = \frac{a^2}{2b} \left(\frac{1}{x} \right)$$

For equivalence,

$$\Sigma F: \mathbf{F} = \mathbf{R}$$

$$\Sigma M_D: (F \cos \alpha)(y_A) = M_D$$

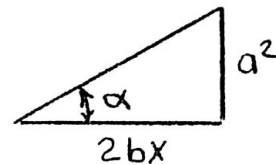
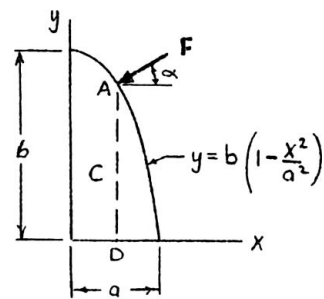
where

$$\cos \alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$

$$y_A = b \left(1 - \frac{x^2}{a^2} \right)$$

$$M_D = \frac{2Fb^2 \left(x - \frac{x^3}{a^2} \right)}{\sqrt{a^4 + 4b^2 x^2}}$$

Therefore, the equivalent force-couple system at D is



$$\mathbf{R} = F \nearrow \tan^{-1} \left(\frac{a^2}{2bx} \right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2 \left(x - \frac{x^3}{a^2} \right)}{\sqrt{a^4 + 4b^2 x^2}} \blacktriangleleft$$

PROBLEM 3.118 (Continued)

(b) To maximize M , the value of x must satisfy $\frac{dM}{dx} = 0$

where for $a = 1 \text{ m}$, $b = 2 \text{ m}$

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

$$\frac{dM}{dx} = 8F \frac{\sqrt{1 + 16x^2}(1 - 3x^2) - (x - x^3) \left[\frac{1}{2}(32x)(1 + 16x^2)^{-1/2} \right]}{(1 + 16x^2)^2} = 0$$

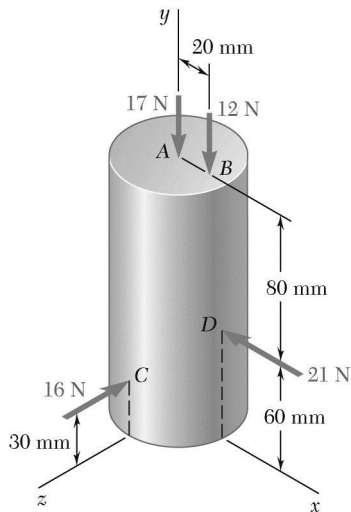
$$(1 + 16x^2)(1 - 3x^2) - 16x(x - x^3) = 0$$

or $32x^4 + 3x^2 - 1 = 0$

$$x^2 = \frac{-3 \pm \sqrt{9 - 4(32)(-1)}}{2(32)} = 0.136011 \text{ m}^2 \quad \text{and} \quad -0.22976 \text{ m}^2$$

Using the positive value of x^2 : $x = 0.36880 \text{ m}$ or $x = 369 \text{ mm} \blacktriangleleft$

PROBLEM 3.119



As plastic bushings are inserted into a 60-mm-diameter cylindrical sheet metal enclosure, the insertion tools exert the forces shown on the enclosure. Each of the forces is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at C .

SOLUTION

For equivalence,

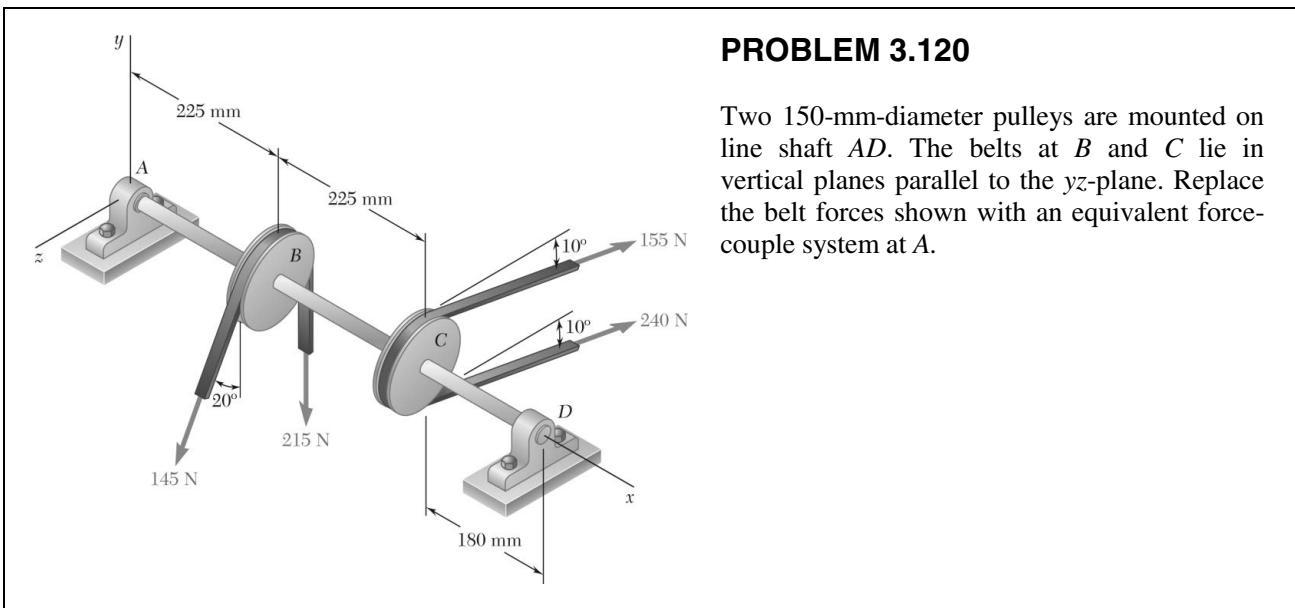
$$\begin{aligned}\Sigma \mathbf{F}: \quad \mathbf{R} &= \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D \\ &= -(17 \text{ N})\mathbf{j} - (12 \text{ N})\mathbf{j} - (16 \text{ N})\mathbf{k} - (21 \text{ N})\mathbf{i} \\ &= -(21 \text{ N})\mathbf{i} - (29 \text{ N})\mathbf{j} - (16 \text{ N})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\Sigma M_C: \quad \mathbf{M} &= \mathbf{r}_{A/C} \times \mathbf{F}_A + \mathbf{r}_{B/C} \times \mathbf{F}_B + \mathbf{r}_{D/C} \times \mathbf{F}_D \\ M &= [(0.11 \text{ m})\mathbf{j} - (0.03 \text{ m})\mathbf{k}] \times [-(17 \text{ N})\mathbf{j}] \\ &\quad + [(0.02 \text{ m})\mathbf{i} + (0.11 \text{ m})\mathbf{j} - (0.03 \text{ m})\mathbf{k}] \times [-(12 \text{ N})\mathbf{j}] \\ &\quad + [(0.03 \text{ m})\mathbf{i} + (0.03 \text{ m})\mathbf{j} - (0.03 \text{ m})\mathbf{k}] \times [-(21 \text{ N})\mathbf{i}] \\ &= -(0.51 \text{ N}\cdot\text{m})\mathbf{i} + [-(0.24 \text{ N}\cdot\text{m})\mathbf{k} - (0.36 \text{ N}\cdot\text{m})\mathbf{i}] \\ &\quad + [(0.63 \text{ N}\cdot\text{m})\mathbf{k} + (0.63 \text{ N}\cdot\text{m})\mathbf{j}]\end{aligned}$$

\therefore The equivalent force-couple system at C is

$$\mathbf{R} = -(21.0 \text{ N})\mathbf{i} - (29.0 \text{ N})\mathbf{j} - (16.00 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{M} = -(0.870 \text{ N}\cdot\text{m})\mathbf{i} + (0.630 \text{ N}\cdot\text{m})\mathbf{j} + (0.390 \text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$



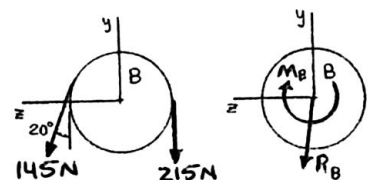
PROBLEM 3.120

Two 150-mm-diameter pulleys are mounted on line shaft *AD*. The belts at *B* and *C* lie in vertical planes parallel to the *yz*-plane. Replace the belt forces shown with an equivalent force-couple system at *A*.

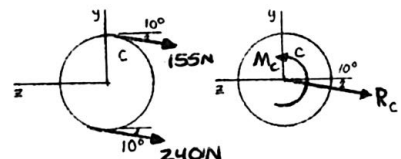
SOLUTION

Equivalent force-couple at each pulley:

Pulley *B*: $\mathbf{R}_B = (145 \text{ N})(-\cos 20^\circ \mathbf{j} + \sin 20^\circ \mathbf{k}) - 215 \text{ N} \mathbf{j}$
 $= -(351.26 \text{ N})\mathbf{j} + (49.593 \text{ N})\mathbf{k}$
 $\mathbf{M}_B = -(215 \text{ N} - 145 \text{ N})(0.075 \text{ m})\mathbf{i}$
 $= -(5.25 \text{ N} \cdot \text{m})\mathbf{i}$



Pulley *C*: $\mathbf{R}_C = (155 \text{ N} + 240 \text{ N})(-\sin 10^\circ \mathbf{j} - \cos 10^\circ \mathbf{k})$
 $= -(68.591 \text{ N})\mathbf{j} - (389.00 \text{ N})\mathbf{k}$
 $\mathbf{M}_C = (240 \text{ N} - 155 \text{ N})(0.075 \text{ m})\mathbf{i}$
 $= (6.3750 \text{ N} \cdot \text{m})\mathbf{i}$



Then $\mathbf{R} = \mathbf{R}_B + \mathbf{R}_C = -(419.85 \text{ N})\mathbf{j} - (339.41 \text{ N})\mathbf{k}$ or $\mathbf{R} = (420 \text{ N})\mathbf{j} - (339 \text{ N})\mathbf{k} \blacktriangleleft$

$$\mathbf{M}_A = \mathbf{M}_B + \mathbf{M}_C + \mathbf{r}_{B/A} \times \mathbf{R}_B + \mathbf{r}_{C/A} \times \mathbf{R}_C$$

$$= -(5.25 \text{ N} \cdot \text{m})\mathbf{i} + (6.3750 \text{ N} \cdot \text{m})\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.225 & 0 & 0 \\ 0 & -351.26 & 49.593 \end{vmatrix} \text{ N} \cdot \text{m}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0 \\ 0 & -68.591 & -389.00 \end{vmatrix} \text{ N} \cdot \text{m}$$

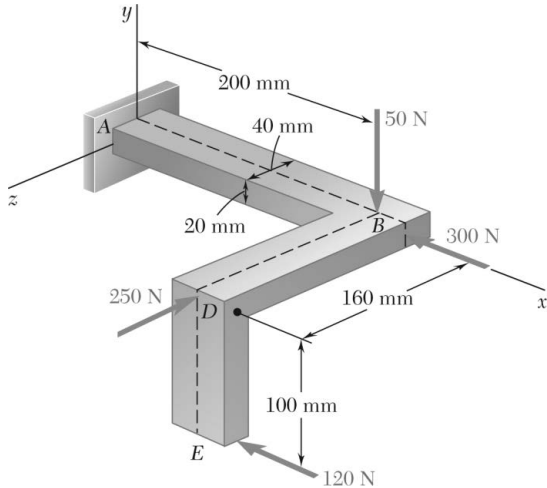
$$= (1.12500 \text{ N} \cdot \text{m})\mathbf{i} + (163.892 \text{ N} \cdot \text{m})\mathbf{j} - (109.899 \text{ N} \cdot \text{m})\mathbf{k}$$

or $\mathbf{M}_A = (1.125 \text{ N} \cdot \text{m})\mathbf{i} + (163.9 \text{ N} \cdot \text{m})\mathbf{j} - (109.9 \text{ N} \cdot \text{m})\mathbf{k} \blacktriangleleft$

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PROBLEM 3.121

Four forces are applied to the machine component $ABDE$ as shown. Replace these forces with an equivalent force-couple system at A .



SOLUTION

$$\mathbf{R} = -(50 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{i} - (120 \text{ N})\mathbf{i} - (250 \text{ N})\mathbf{k}$$

$$\mathbf{R} = -(420 \text{ N})\mathbf{i} - (50 \text{ N})\mathbf{j} - (250 \text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.2 \text{ m})\mathbf{j}$$

$$\mathbf{r}_D = (0.2 \text{ m})\mathbf{j} + (0.16 \text{ m})\mathbf{k}$$

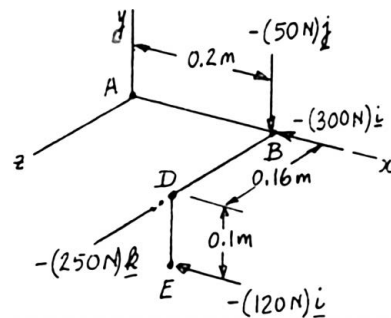
$$\mathbf{r}_E = (0.2 \text{ m})\mathbf{j} - (0.1 \text{ m})\mathbf{j} + (0.16 \text{ m})\mathbf{k}$$

$$\mathbf{M}_A^R = \mathbf{r}_B \times [-(300 \text{ N})\mathbf{i} - (50 \text{ N})\mathbf{j}] + \mathbf{r}_D \times (-250 \text{ N})\mathbf{k} + \mathbf{r}_E \times (-120 \text{ N})\mathbf{i}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 \text{ m} & 0 & 0 \\ -300 \text{ N} & -50 \text{ N} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 \text{ m} & 0 & 0.16 \text{ m} \\ 0 & 0 & -250 \text{ N} \end{vmatrix}$$

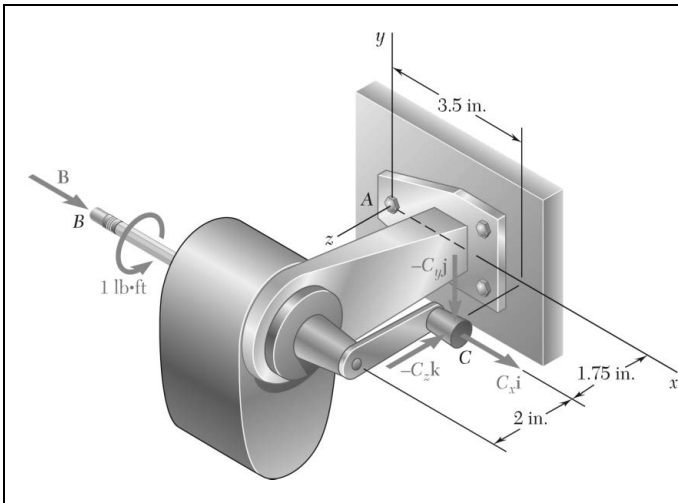
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 \text{ m} & -0.1 \text{ m} & 0.16 \text{ m} \\ -120 \text{ N} & 0 & 0 \end{vmatrix}$$

$$= -(10 \text{ N} \cdot \text{m})\mathbf{k} + (50 \text{ N} \cdot \text{m})\mathbf{j} - (19.2 \text{ N} \cdot \text{m})\mathbf{j} - (12 \text{ N} \cdot \text{m})\mathbf{k}$$



Force-couple system at A is

$$\mathbf{R} = -(420 \text{ N})\mathbf{i} - (50 \text{ N})\mathbf{j} - (250 \text{ N})\mathbf{k} \quad \mathbf{M}_A^R = (30.8 \text{ N} \cdot \text{m})\mathbf{j} - (220 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.122

While using a pencil sharpener, a student applies the forces and couple shown. (a) Determine the forces exerted at B and C knowing that these forces and the couple are equivalent to a force-couple system at A consisting of the force $\mathbf{R} = (2.6 \text{ lb})\mathbf{i} + R_y\mathbf{j} - (0.7 \text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_A^R = M_x\mathbf{i} + (1.0 \text{ lb} \cdot \text{ft})\mathbf{j} - (0.72 \text{ lb} \cdot \text{ft})\mathbf{k}$. (b) Find the corresponding values of R_y and M_x .

SOLUTION

(a) From the statement of the problem, equivalence requires

$$\Sigma \mathbf{F}: \mathbf{B} + \mathbf{C} = \mathbf{R}$$

or

$$\Sigma F_x: B_x + C_x = 2.6 \text{ lb} \quad (1)$$

$$\Sigma F_y: -C_y = R_y \quad (2)$$

$$\Sigma F_z: -C_z = -0.7 \text{ lb} \quad \text{or} \quad C_z = 0.7 \text{ lb}$$

and

$$\Sigma \mathbf{M}_A: (\mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{M}_B) + \mathbf{r}_{C/A} \times \mathbf{C} = \mathbf{M}_A^R$$

or

$$\Sigma M_x: (1 \text{ lb} \cdot \text{ft}) + \left(\frac{1.75}{12} \text{ ft}\right)(C_y) = M_x \quad (3)$$

$$\Sigma M_y: \left(\frac{3.75}{12} \text{ ft}\right)(B_x) + \left(\frac{1.75}{12} \text{ ft}\right)(C_x) + \left(\frac{3.5}{12} \text{ ft}\right)(0.7 \text{ lb}) = 1 \text{ lb} \cdot \text{ft}$$

or

$$3.75B_x + 1.75C_x = 9.55$$

Using Eq. (1):

$$3.75B_x + 1.75(2.6B_x) = 9.55$$

or

$$B_x = 2.5 \text{ lb}$$

and

$$C_x = 0.1 \text{ lb}$$

$$\Sigma M_z: -\left(\frac{3.5}{12} \text{ ft}\right)(C_y) = -0.72 \text{ lb} \cdot \text{ft}$$

or

$$C_y = 2.4686 \text{ lb}$$

$$\mathbf{B} = (2.50 \text{ lb})\mathbf{i} \quad \mathbf{C} = (0.1000 \text{ lb})\mathbf{i} - (2.47 \text{ lb})\mathbf{j} - (0.700 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

(b) Eq. (2) \Rightarrow

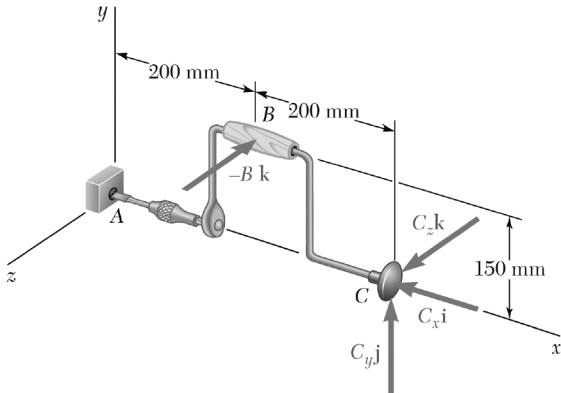
$$R_y = -2.47 \text{ lb} \quad \blacktriangleleft$$

Using Eq. (3):

$$1 + \left(\frac{1.75}{12}\right)(2.4686) = M_x \quad \text{or} \quad M_x = 1.360 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

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PROBLEM 3.123



A blade held in a brace is used to tighten a screw at A . (a) Determine the forces exerted at B and C , knowing that these forces are equivalent to a force-couple system at A consisting of $\mathbf{R} = -(30 \text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$ and $\mathbf{M}_A^R = -(12 \text{ N} \cdot \text{m})\mathbf{i}$. (b) Find the corresponding values of R_y and R_z . (c) What is the orientation of the slot in the head of the screw for which the blade is least likely to slip when the brace is in the position shown?

SOLUTION

(a) Equivalence requires

$$\Sigma \mathbf{F}: \mathbf{R} = \mathbf{B} + \mathbf{C}$$

or

$$-(30 \text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -B\mathbf{k} + (-C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$

Equating the \mathbf{i} coefficients:

$$\mathbf{i}: -30 \text{ N} = -C_x \quad \text{or} \quad C_x = 30 \text{ N}$$

Also,

$$\Sigma \mathbf{M}_A^R: \mathbf{M}_A^R = \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{C/A} \times \mathbf{C}$$

or

$$-(12 \text{ N} \cdot \text{m})\mathbf{i} = [(0.2 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j}] \times (-B)\mathbf{k} \\ + (0.4 \text{ m})\mathbf{i} \times [-(30 \text{ N})\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}]$$

Equating coefficients:

$$\mathbf{i}: -12 \text{ N} \cdot \text{m} = -(0.15 \text{ m})B \quad \text{or} \quad B = 80 \text{ N}$$

$$\mathbf{k}: 0 = (0.4 \text{ m})C_y \quad \text{or} \quad C_y = 0$$

$$\mathbf{j}: 0 = (0.2 \text{ m})(80 \text{ N}) - (0.4 \text{ m})C_z \quad \text{or} \quad C_z = 40 \text{ N}$$

$$\mathbf{B} = -(80.0 \text{ N})\mathbf{k} \quad \mathbf{C} = -(30.0 \text{ N})\mathbf{i} + (40.0 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

(b) Now we have for the equivalence of forces

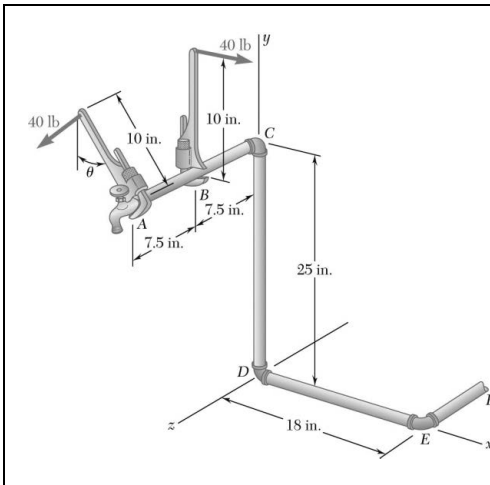
$$-(30 \text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -(80 \text{ N})\mathbf{k} + [(-30 \text{ N})\mathbf{i} + (40 \text{ N})\mathbf{k}]$$

Equating coefficients:

$$\mathbf{j}: R_y = 0 \quad \quad \quad R_y = 0 \quad \blacktriangleleft$$

$$\mathbf{k}: R_z = -80 + 40 \quad \quad \quad \text{or} \quad R_z = -40.0 \text{ N} \quad \blacktriangleleft$$

(c) First note that $\mathbf{R} = -(30 \text{ N})\mathbf{i} - (40 \text{ N})\mathbf{k}$. Thus, the screw is best able to resist the lateral force R_z when the slot in the head of the screw is vertical. ◀



PROBLEM 3.124

In order to unscrew the tapped faucet A, a plumber uses two pipe wrenches as shown. By exerting a 40-lb force on each wrench, at a distance of 10 in. from the axis of the pipe and in a direction perpendicular to the pipe and to the wrench, he prevents the pipe from rotating, and thus avoids loosening or further tightening the joint between the pipe and the tapped elbow C. Determine (a) the angle θ that the wrench at A should form with the vertical if elbow C is not to rotate about the vertical, (b) the force-couple system at C equivalent to the two 40-lb forces when this condition is satisfied.

SOLUTION

We first reduce the given forces to force-couple systems at A and B, noting that

$$\begin{aligned} |\mathbf{M}_A| &= |\mathbf{M}_B| = (40 \text{ lb})(10 \text{ in.}) \\ &= 400 \text{ lb} \cdot \text{in.} \end{aligned}$$

We now determine the equivalent force-couple system at C.

$$\mathbf{R} = (40 \text{ lb})(1 - \cos \theta)\mathbf{i} - (40 \text{ lb})\sin \theta\mathbf{j} \quad (1)$$

$$\begin{aligned} \mathbf{M}_C^R &= \mathbf{M}_A + \mathbf{M}_B + (15 \text{ in.})\mathbf{k} \times [-(40 \text{ lb})\cos \theta\mathbf{i} - (40 \text{ lb})\sin \theta\mathbf{j}] \\ &\quad + (7.5 \text{ in.})\mathbf{k} \times (40 \text{ lb})\mathbf{i} \\ &= +400 - 400 - 600\cos \theta\mathbf{j} + 600\sin \theta\mathbf{i} + 300\mathbf{j} \\ &= (600 \text{ lb} \cdot \text{in.})\sin \theta\mathbf{i} + (300 \text{ lb} \cdot \text{in.})(1 - 2\cos \theta)\mathbf{j} \end{aligned} \quad (2)$$

(a) For no rotation about vertical, y component of \mathbf{M}_C^R must be zero.

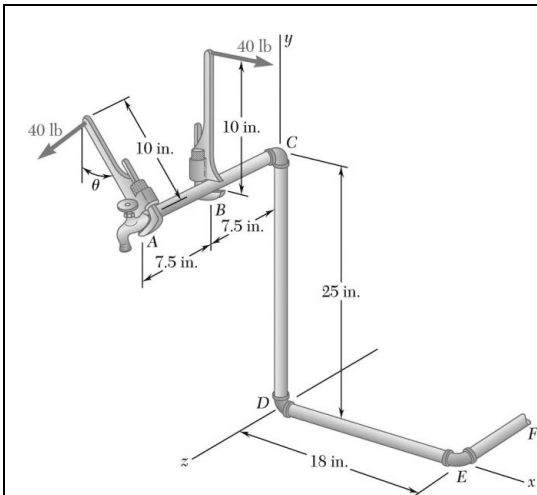
$$\begin{aligned} 1 - 2\cos \theta &= 0 \\ \cos \theta &= 1/2 \end{aligned}$$

$$\theta = 60.0^\circ \quad \blacktriangleleft$$

(b) For $\theta = 60.0^\circ$ in Eqs. (1) and (2),

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}; \quad \mathbf{M}_C^R = (519.62 \text{ lb} \cdot \text{in.})\mathbf{i}$$

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.6 \text{ lb})\mathbf{j}; \quad \mathbf{M}_C^R = (520 \text{ lb} \cdot \text{in.})\mathbf{i} \quad \blacktriangleleft$$



PROBLEM 3.125

Assuming $\theta = 60^\circ$ in Prob. 3.124, replace the two 40-lb forces with an equivalent force-couple system at D and determine whether the plumber's action tends to tighten or loosen the joint between (a) pipe CD and elbow D , (b) elbow D and pipe DE . Assume all threads to be right-handed.

PROBLEM 3.124 In order to unscrew the tapped faucet A , a plumber uses two pipe wrenches as shown. By exerting a 40-lb force on each wrench, at a distance of 10 in. from the axis of the pipe and in a direction perpendicular to the pipe and to the wrench, he prevents the pipe from rotating, and thus avoids loosening or further tightening the joint between the pipe and the tapped elbow C . Determine (a) the angle θ that the wrench at A should form with the vertical if elbow C is not to rotate about the vertical, (b) the force-couple system at C equivalent to the two 40-lb forces when this condition is satisfied.

SOLUTION

The equivalent force-couple system at C for $\theta = 60^\circ$ was obtained in the solution to Prob. 3.124:

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}$$

$$\mathbf{M}_C^R = (519.62 \text{ lb} \cdot \text{in.})\mathbf{i}$$

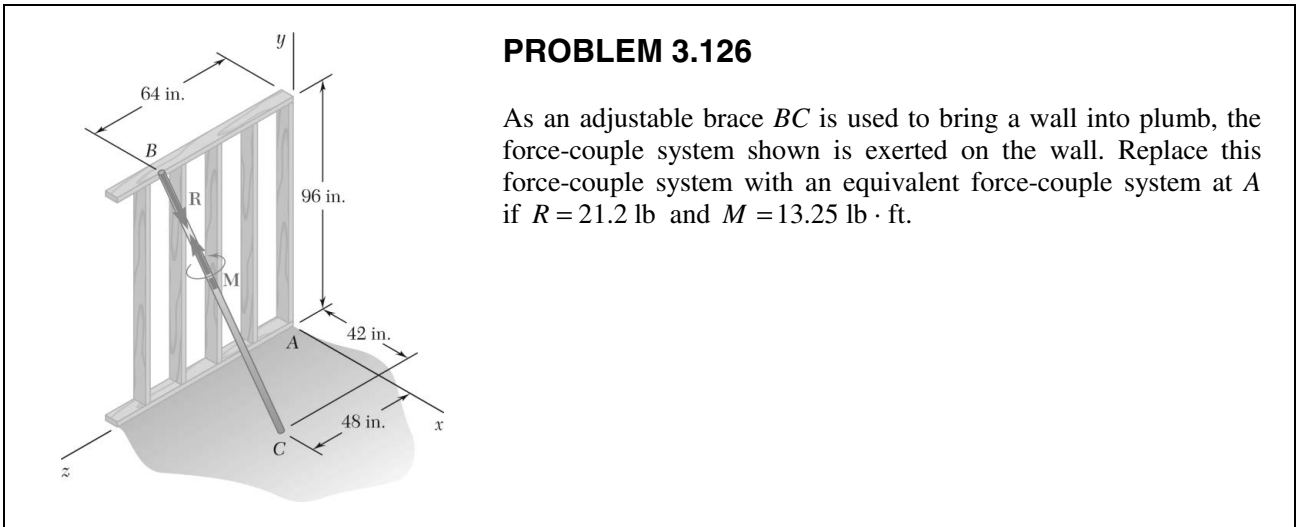
The equivalent force-couple system at D is made of \mathbf{R} and \mathbf{M}_D^R where

$$\begin{aligned} \mathbf{M}_D^R &= \mathbf{M}_C^R + \mathbf{r}_{C/D} \times \mathbf{R} \\ &= (519.62 \text{ lb} \cdot \text{in.})\mathbf{i} + (25.0 \text{ in.})\mathbf{j} \times [(20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}] \\ &= (519.62 \text{ lb} \cdot \text{in.})\mathbf{i} - (500 \text{ lb} \cdot \text{in.})\mathbf{k} \end{aligned}$$

Equivalent force-couple at D :

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.6 \text{ lb})\mathbf{j}; \quad \mathbf{M}_D^R = (520 \text{ lb} \cdot \text{in.})\mathbf{i} - (500 \text{ lb} \cdot \text{in.})\mathbf{k} \quad \blacktriangleleft$$

- (a) Since \mathbf{M}_D^R has no component along the y -axis, the plumber's action will neither loosen nor tighten the joint between pipe CD and elbow. \blacktriangleleft
- (b) Since the x component of \mathbf{M}_D^R is \curvearrowright , the plumber's action will tend to tighten the joint between elbow and pipe DE . \blacktriangleleft



PROBLEM 3.126

As an adjustable brace BC is used to bring a wall into plumb, the force-couple system shown is exerted on the wall. Replace this force-couple system with an equivalent force-couple system at A if $R = 21.2 \text{ lb}$ and $M = 13.25 \text{ lb} \cdot \text{ft}$.

SOLUTION

We have $\Sigma \mathbf{F}: \mathbf{R} = \mathbf{R}_A = R\lambda_{BC}$

where $\lambda_{BC} = \frac{(42 \text{ in.})\mathbf{i} - (96 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{106 \text{ in.}}$

$$\mathbf{R}_A = \frac{21.2 \text{ lb}}{106} (42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

or $\mathbf{R}_A = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$

We have $\Sigma \mathbf{M}_A: \mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$

where $\mathbf{r}_{C/A} = (42 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{k} = \frac{1}{12} (42\mathbf{i} + 48\mathbf{k}) \text{ ft}$

$$= (3.5 \text{ ft})\mathbf{i} + (4.0 \text{ ft})\mathbf{k}$$

$$\mathbf{R} = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$$

$$\mathbf{M} = -\lambda_{BC}M$$

$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106} (13.25 \text{ lb} \cdot \text{ft})$$

$$= (-5.25 \text{ lb} \cdot \text{ft})\mathbf{i} + (12 \text{ lb} \cdot \text{ft})\mathbf{j} + (2 \text{ lb} \cdot \text{ft})\mathbf{k}$$

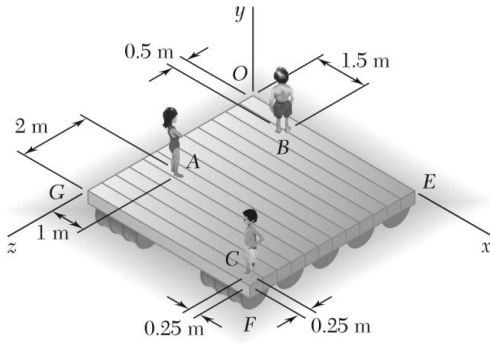
Then $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{ lb} \cdot \text{ft} + (-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k}) \text{ lb} \cdot \text{ft} = \mathbf{M}_A$

$$\mathbf{M}_A = (71.55 \text{ lb} \cdot \text{ft})\mathbf{i} + (56.80 \text{ lb} \cdot \text{ft})\mathbf{j} - (65.20 \text{ lb} \cdot \text{ft})\mathbf{k}$$

or $\mathbf{M}_A = (71.6 \text{ lb} \cdot \text{ft})\mathbf{i} + (56.8 \text{ lb} \cdot \text{ft})\mathbf{j} - (65.2 \text{ lb} \cdot \text{ft})\mathbf{k}$ ◀

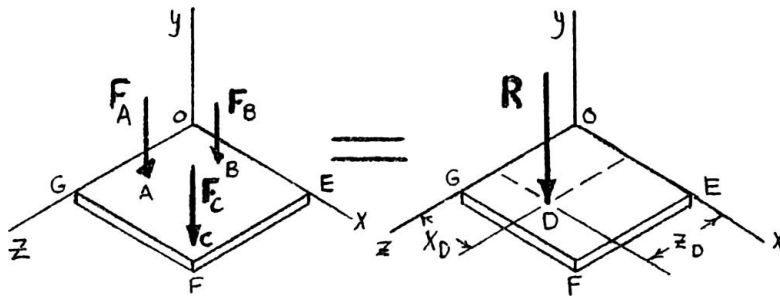
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PROBLEM 3.127



Three children are standing on a 5×5-m raft. If the weights of the children at Points A , B , and C are 375 N, 260 N, and 400 N, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



We have

$$\begin{aligned}\Sigma \mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C &= \mathbf{R} \\ -(375 \text{ N})\mathbf{j} - (260 \text{ N})\mathbf{j} - (400 \text{ N})\mathbf{j} &= \mathbf{R} \\ -(1035 \text{ N})\mathbf{j} &= \mathbf{R}\end{aligned}$$

$$\text{or } R = 1035 \text{ N} \quad \blacktriangleleft$$

We have

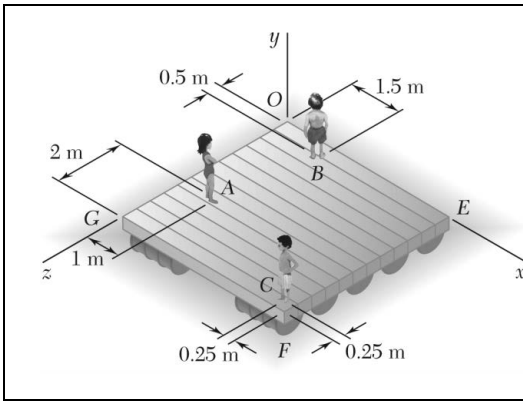
$$\begin{aligned}\Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) &= R(z_D) \\ (375 \text{ N})(3 \text{ m}) + (260 \text{ N})(0.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m}) &= (1035 \text{ N})(z_D) \\ z_D &= 3.0483 \text{ m}\end{aligned}$$

$$\text{or } z_D = 3.05 \text{ m} \quad \blacktriangleleft$$

We have

$$\begin{aligned}\Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) &= R(x_D) \\ 375 \text{ N}(1 \text{ m}) + (260 \text{ N})(1.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m}) &= (1035 \text{ N})(x_D) \\ x_D &= 2.5749 \text{ m}\end{aligned}$$

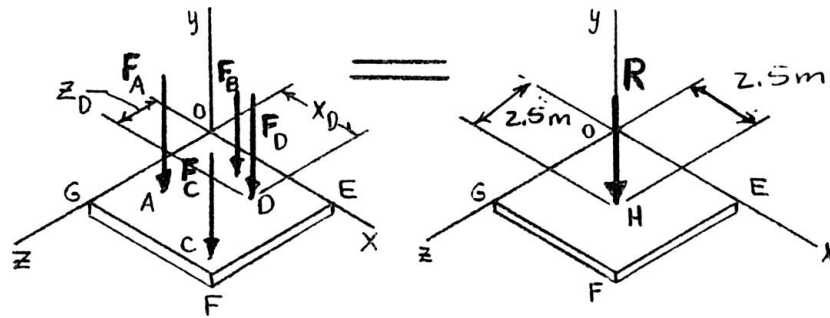
$$\text{or } x_D = 2.57 \text{ m} \quad \blacktriangleleft$$



PROBLEM 3.128

Three children are standing on a 5×5-m raft. The weights of the children at Points A, B, and C are 375 N, 260 N, and 400 N, respectively. If a fourth child of weight 425 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



We have

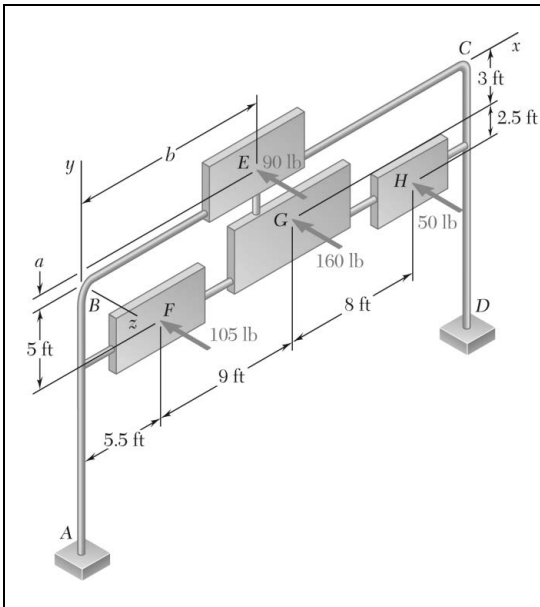
$$\begin{aligned} \Sigma \mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C &= \mathbf{R} \\ -(375 \text{ N})\mathbf{j} - (260 \text{ N})\mathbf{j} - (400 \text{ N})\mathbf{j} - (425 \text{ N})\mathbf{j} &= \mathbf{R} \\ \mathbf{R} &= -(1460 \text{ N})\mathbf{j} \end{aligned}$$

We have

$$\begin{aligned} \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) &= R(z_H) \\ (375 \text{ N})(3 \text{ m}) + (260 \text{ N})(0.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m}) \\ + (425 \text{ N})(z_D) &= (1460 \text{ N})(2.5 \text{ m}) \\ z_D &= 1.16471 \text{ m} \qquad \text{or } z_D = 1.165 \text{ m} \blacktriangleleft \end{aligned}$$

We have

$$\begin{aligned} \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) &= R(x_H) \\ (375 \text{ N})(1 \text{ m}) + (260 \text{ N})(1.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m}) \\ + (425 \text{ N})(x_D) &= (1460 \text{ N})(2.5 \text{ m}) \\ x_D &= 2.3235 \text{ m} \qquad \text{or } x_D = 2.32 \text{ m} \blacktriangleleft \end{aligned}$$

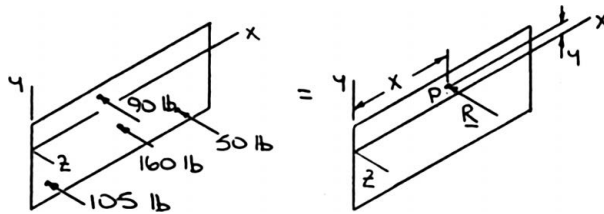


PROBLEM 3.129

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine the magnitude and the point of application of the resultant of the four wind forces when $a = 1$ ft and $b = 12$ ft.

SOLUTION

We have



Assume that the resultant \mathbf{R} is applied at Point P whose coordinates are $(x, y, 0)$.

Equivalence then requires

$$\Sigma F_z: -105 - 90 - 160 - 50 = -R$$

$$\text{or } R = 405 \text{ lb} \blacktriangleleft$$

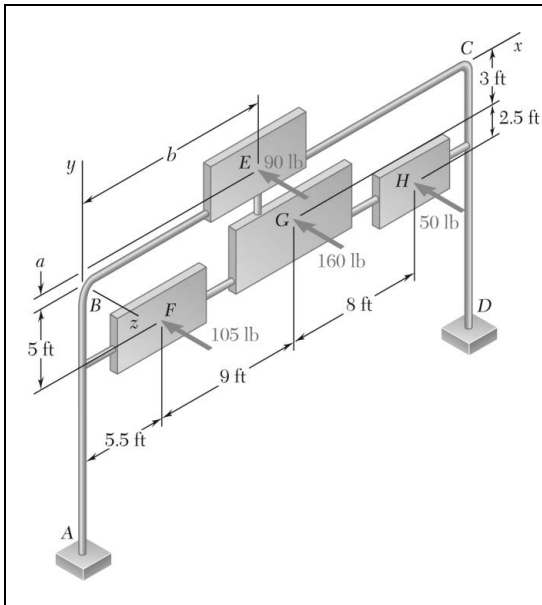
$$\Sigma M_x: (5 \text{ ft})(105 \text{ lb}) - (1 \text{ ft})(90 \text{ lb}) + (3 \text{ ft})(160 \text{ lb}) + (5.5 \text{ ft})(50 \text{ lb}) = -y(405 \text{ lb})$$

or $y = -2.94 \text{ ft}$

$$\Sigma M_y: (5.5 \text{ ft})(105 \text{ lb}) + (12 \text{ ft})(90 \text{ lb}) + (14.5 \text{ ft})(160 \text{ lb}) + (22.5 \text{ ft})(50 \text{ lb}) = -x(405 \text{ lb})$$

or $x = 12.60 \text{ ft}$

\mathbf{R} acts 12.60 ft to the right of member AB and 2.94 ft below member BC . \blacktriangleleft



PROBLEM 3.130

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine a and b so that the point of application of the resultant of the four forces is at G .

SOLUTION

Since \mathbf{R} acts at G , equivalence then requires that $\Sigma \mathbf{M}_G$ of the applied system of forces also be zero. Then at

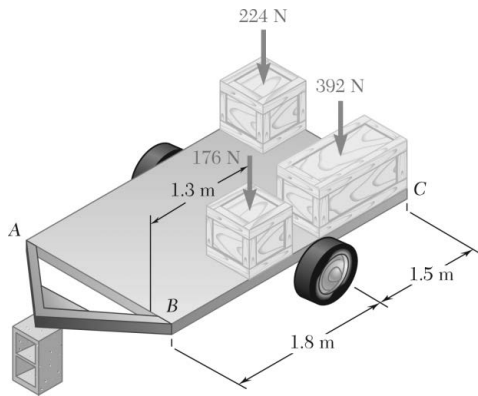
$$G: \Sigma M_x: -(a + 3) \text{ ft} \times (90 \text{ lb}) + (2 \text{ ft})(105 \text{ lb}) \\ + (2.5 \text{ ft})(50 \text{ lb}) = 0$$

$$\text{or } a = 0.722 \text{ ft} \blacktriangleleft$$

$$\Sigma M_y: -(9 \text{ ft})(105 \text{ lb}) - (14.5 - b) \text{ ft} \times (90 \text{ lb}) \\ + (8 \text{ ft})(50 \text{ lb}) = 0$$

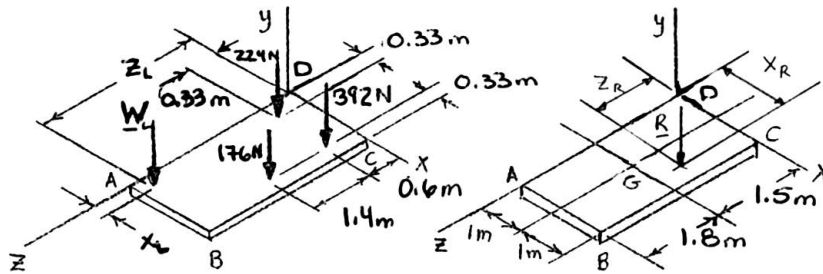
$$\text{or } b = 20.6 \text{ ft} \blacktriangleleft$$

PROBLEM 3.131*



A group of students loads a 2×3.3 -m flatbed trailer with two $0.66 \times 0.66 \times 0.66$ -m boxes and one $0.66 \times 0.66 \times 1.2$ -m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66 \times 0.66 \times 1.2$ -m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (Hint: Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.66 \times 0.66 \times 1.2$ -m box should be placed adjacent to one of the edges of the trailer with the 0.66×0.66 -m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

We have $\Sigma \mathbf{F}: -(224 \text{ N})\mathbf{j} - (392 \text{ N})\mathbf{j} - (176 \text{ N})\mathbf{j} = \mathbf{R}$
 $\mathbf{R} = -(792 \text{ N})\mathbf{j}$

We have $\Sigma M_z: -(224 \text{ N})(0.33 \text{ m}) - (392 \text{ N})(1.67 \text{ m}) - (176 \text{ N})(1.67 \text{ m}) = (-792 \text{ N})(x)$
 $x_R = 1.29101 \text{ m}$

We have $\Sigma M_x: (224 \text{ N})(0.33 \text{ m}) + (392 \text{ N})(0.6 \text{ m}) + (176 \text{ N})(2.0 \text{ m}) = (792 \text{ N})(z)$
 $z_R = 0.83475 \text{ m}$

From the statement of the problem, it is known that the resultant of \mathbf{R} from the original loading and the lightest load \mathbf{W} passes through G , the point of intersection of the two center lines. Thus, $\Sigma \mathbf{M}_G = 0$.

Further, since the lightest load \mathbf{W} is to be as small as possible, the fourth box should be placed as far from G as possible without the box overhanging the trailer. These two requirements imply

$$(0.33 \text{ m} \leq x \leq 1 \text{ m}) (1.5 \text{ m} \leq z \leq 2.97 \text{ m})$$

PROBLEM 3.131* (Continued)

With $x_L = 0.33$ m

at $G: \Sigma M_z: (1 - 0.33) \text{ m} \times W_L - (1.29101 - 1) \text{ m} \times (792 \text{ N}) = 0$

or $W_L = 344.00 \text{ N}$

Now we must check if this is physically possible,

at $G: \Sigma M_x: (z_L - 1.5) \text{ m} \times 344 \text{ N} - (1.5 - 0.83475) \text{ m} \times (792 \text{ N}) = 0$

or $z_L = 3.032 \text{ m}$

which is **not** acceptable.

With $z_L = 2.97$ m:

at $G: \Sigma M_x: (2.97 - 1.5) \text{ m} \times W_L - (1.5 - 0.83475) \text{ m} \times (792 \text{ N}) = 0$

or $W_L = 358.42 \text{ N}$

Now check if this is physically possible,

at $G: \Sigma M_z: (1 - x_L) \text{ m} \times (358.42 \text{ N}) - (1.29101 - 1) \text{ m} \times (792 \text{ N}) = 0$

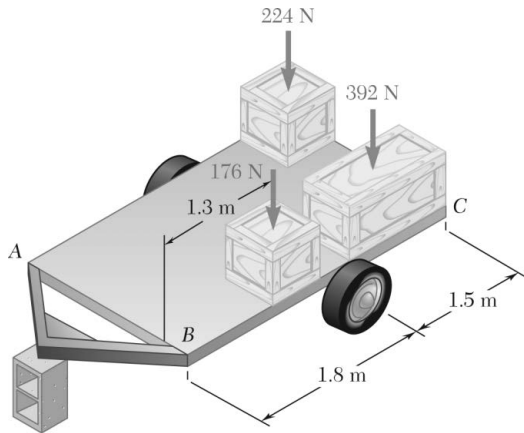
or $x_L = 0.357 \text{ m}$ ok!

The minimum weight of the fourth box is $W_L = 358 \text{ N}$ ◀

And it is placed on end A (0.66×0.66 -m side down) along side AB with the center of the box 0.357 m from side AD . ◀

PROBLEM 3.132*

Solve Problem 3.131 if the students want to place as much weight as possible in the fourth box and at least one side of the box must coincide with a side of the trailer.



PROBLEM 3.131* A group of students loads a 2×3.3 -m flatbed trailer with two $0.66 \times 0.66 \times 0.66$ -m boxes and one $0.66 \times 0.66 \times 1.2$ -m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66 \times 0.66 \times 1.2$ -m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION

First replace the three known loads with a single equivalent force \mathbf{R} applied at coordinate $(x_R, 0, z_R)$.

Equivalence requires

$$\Sigma F_y: -224 - 392 - 176 = -R$$

or
$$\mathbf{R} = 792 \text{ N} \downarrow$$

$$\Sigma M_x: (0.33 \text{ m})(224 \text{ N}) + (0.6 \text{ m})(392 \text{ N}) + (2 \text{ m})(176 \text{ N}) = z_R(792 \text{ N})$$

or
$$z_R = 0.83475 \text{ m}$$

$$\Sigma M_z: -(0.33 \text{ m})(224 \text{ N}) - (1.67 \text{ m})(392 \text{ N}) - (1.67 \text{ m})(176 \text{ N}) = x_R(792 \text{ N})$$

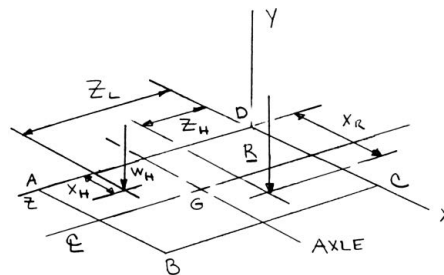
or
$$x_R = 1.29101 \text{ m}$$

From the statement of the problem, it is known that the resultant of \mathbf{R} and the heaviest loads \mathbf{W}_H passes through G , the point of intersection of the two center lines. Thus,

$$\Sigma \mathbf{M}_G = 0$$

Further, since \mathbf{W}_H is to be as large as possible, the fourth box should be placed as close to G as possible while keeping one of the sides of the box coincident with a side of the trailer. Thus, the two limiting cases are

$$x_H = 0.6 \text{ m} \quad \text{or} \quad z_H = 2.7 \text{ m}$$



PROBLEM 3.132* (Continued)

Now consider these two possibilities.

With $x_H = 0.6$ m

at $G: \Sigma M_z: (1 - 0.6) \text{ m} \times W_H - (1.29101 - 1) \text{ m} \times (792 \text{ N}) = 0$

or $W_H = 576.20 \text{ N}$

Checking if this is physically possible

at $G: \Sigma M_x: (z_H - 1.5) \text{ m} \times (576.20 \text{ N}) - (1.5 - 0.83475) \text{ m} \times (792 \text{ N}) = 0$

or $z_H = 2.414 \text{ m}$

which is acceptable.

With $z_H = 2.7$ m

at $G: \Sigma M_x: (2.7 - 1.5) \text{ m} \times W_H - (1.5 - 0.83475) \text{ m} \times (792 \text{ N}) = 0$

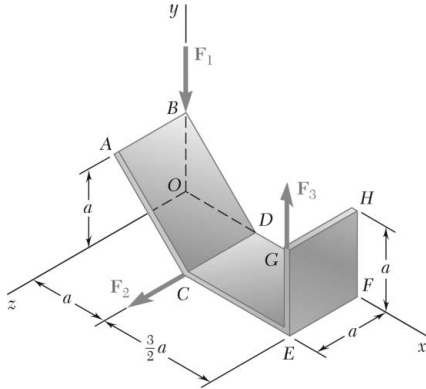
or $W_H = 439 \text{ N}$

Since this is less than the first case, the maximum weight of the fourth box is

$$W_H = 576 \text{ N} \quad \blacktriangleleft$$

and it is placed with a 0.66×1.2 -m side down, a 0.66 -m edge along side AD , and the center 2.41 m from side DC . \blacktriangleleft

PROBLEM 3.133*



A piece of sheet metal is bent into the shape shown and is acted upon by three forces. If the forces have the same magnitude P , replace them with an equivalent wrench and determine (a) the magnitude and the direction of the resultant force \mathbf{R} , (b) the pitch of the wrench, (c) the axis of the wrench.

SOLUTION

First reduce the given forces to an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_O^R)$ at the origin.

We have

$$\Sigma \mathbf{F}: -P\mathbf{j} + P\mathbf{j} + P\mathbf{k} = \mathbf{R}$$

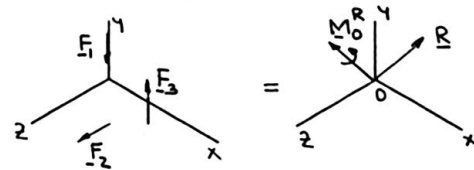
or

$$\mathbf{R} = P\mathbf{k}$$

$$\Sigma \mathbf{M}_O: -(aP)\mathbf{j} + \left[-(aP)\mathbf{i} + \left(\frac{5}{2}aP \right)\mathbf{k} \right] = \mathbf{M}_O^R$$

or

$$\mathbf{M}_O^R = aP \left(-\mathbf{i} - \mathbf{j} + \frac{5}{2}\mathbf{k} \right)$$



(a) Then for the wrench,

$$R = P \blacktriangleleft$$

and

$$\lambda_{\text{axis}} = \frac{\mathbf{R}}{R} = \mathbf{k}$$

$$\cos \theta_x = 0 \quad \cos \theta_y = 0 \quad \cos \theta_z = 1$$

or

$$\theta_x = 90^\circ \quad \theta_y = 90^\circ \quad \theta_z = 0^\circ \quad \blacktriangleleft$$

(b) Now

$$\begin{aligned} M_1 &= \lambda_{\text{axis}} \cdot \mathbf{M}_O^R \\ &= \mathbf{k} \cdot aP \left(-\mathbf{i} - \mathbf{j} + \frac{5}{2}\mathbf{k} \right) \\ &= \frac{5}{2}aP \end{aligned}$$

Then

$$P = \frac{M_1}{R} = \frac{\frac{5}{2}aP}{P}$$

$$\text{or } P = \frac{5}{2}a \quad \blacktriangleleft$$

PROBLEM 3.133* (Continued)

- (c) The components of the wrench are $(\mathbf{R}, \mathbf{M}_1)$, where $\mathbf{M}_1 = M_1 \lambda_{\text{axis}}$, and the axis of the wrench is assumed to intersect the xy -plane at Point Q , whose coordinates are $(x, y, 0)$. Thus, we require

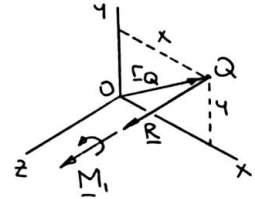
$$\mathbf{M}_z = \mathbf{r}_Q \times \mathbf{R}_R$$

where

$$\mathbf{M}_z = \mathbf{M}_O \times \mathbf{M}_1$$

Then

$$aP \left(-\mathbf{i} - \mathbf{j} + \frac{5}{2} \mathbf{k} \right) - \frac{5}{2} aP \mathbf{k} = (x\mathbf{i} + y\mathbf{j}) + P\mathbf{k}$$



Equating coefficients:

$$\mathbf{i}: -aP = yP \quad \text{or} \quad y = -a$$

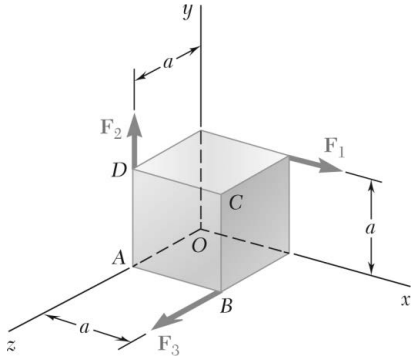
$$\mathbf{j}: -aP = -xP \quad \text{or} \quad x = a$$

The axis of the wrench is parallel to the z -axis and intersects the xy -plane at

$$x = a, y = -a. \quad \blacktriangleleft$$

PROBLEM 3.134*

Three forces of the same magnitude P act on a cube of side a as shown. Replace the three forces by an equivalent wrench and determine (a) the magnitude and direction of the resultant force \mathbf{R} , (b) the pitch of the wrench, (c) the axis of the wrench.



SOLUTION

Force-couple system at O :

$$\mathbf{R} = P\mathbf{i} + P\mathbf{j} + P\mathbf{k} = P(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$\begin{aligned} \mathbf{M}_O^R &= a\mathbf{j} \times P\mathbf{i} + a\mathbf{k} \times P\mathbf{j} + a\mathbf{i} \times P\mathbf{k} \\ &= -Pa\mathbf{k} - Pa\mathbf{i} - Pa\mathbf{j} \end{aligned}$$

$$\mathbf{M}_O^R = -Pa(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

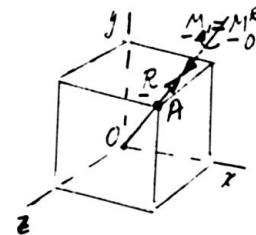
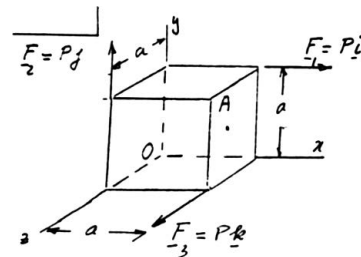
Since \mathbf{R} and \mathbf{M}_O^R have the same direction, they form a wrench with $\mathbf{M}_1 = \mathbf{M}_O^R$. Thus, the axis of the wrench is the diagonal OA . We note that

$$\cos \theta_x = \cos \theta_y = \cos \theta_z = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7^\circ$$

$$M_1 = M_O^R = -Pa\sqrt{3}$$

$$\text{Pitch} = p = \frac{M_1}{R} = \frac{-Pa\sqrt{3}}{P\sqrt{3}} = -a$$



(a)

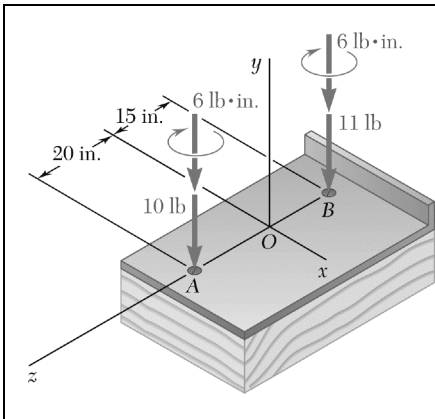
$$R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7^\circ \quad \blacktriangleleft$$

(b)

$$-a \quad \blacktriangleleft$$

(c)

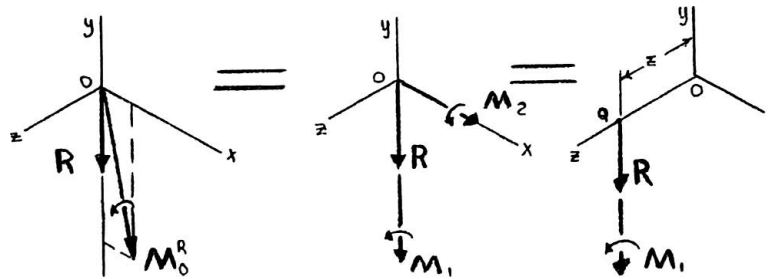
Axis of the wrench is diagonal OA . \blacktriangleleft



PROBLEM 3.135*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (a) the resultant force \mathbf{R} , (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the xz -plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma \mathbf{F}: -(10 \text{ lb})\mathbf{j} - (11 \text{ lb})\mathbf{j} = \mathbf{R}$

$$\mathbf{R} = -(21 \text{ lb})\mathbf{j}$$

We have $\Sigma \mathbf{M}_O: \Sigma (\mathbf{r}_O \times \mathbf{F}) + \Sigma \mathbf{M}_C = \mathbf{M}_O^R$

$$\begin{aligned} \mathbf{M}_O^R &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 20 \\ 0 & -10 & 0 \end{vmatrix} \text{ lb}\cdot\text{in.} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -15 \\ 0 & -11 & 0 \end{vmatrix} \text{ lb}\cdot\text{in.} - (12 \text{ lb}\cdot\text{in.})\mathbf{j} \\ &= (35 \text{ lb}\cdot\text{in.})\mathbf{i} - (12 \text{ lb}\cdot\text{in.})\mathbf{j} \end{aligned}$$

(a) $\mathbf{R} = -(21 \text{ lb})\mathbf{j}$ or $\mathbf{R} = -(21.0 \text{ lb})\mathbf{j} \blacktriangleleft$

(b) We have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \quad \lambda_R = \frac{\mathbf{R}}{R}$

$$= (-\mathbf{j}) \cdot [(35 \text{ lb}\cdot\text{in.})\mathbf{i} - (12 \text{ lb}\cdot\text{in.})\mathbf{j}]$$

$$= 12 \text{ lb}\cdot\text{in.} \quad \text{and} \quad \mathbf{M}_1 = -(12 \text{ lb}\cdot\text{in.})\mathbf{j}$$

and pitch $p = \frac{M_1}{R} = \frac{12 \text{ lb}\cdot\text{in.}}{21 \text{ lb}} = 0.57143 \text{ in.}$ or $p = 0.571 \text{ in.} \blacktriangleleft$

PROBLEM 3.135* (Continued)

(c) We have

$$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (35 \text{ lb} \cdot \text{in.})\mathbf{i}$$

We require

$$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(35 \text{ lb} \cdot \text{in.})\mathbf{i} = (x\mathbf{i} + z\mathbf{k}) \times [-(21 \text{ lb})\mathbf{j}]$$

$$35\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

From \mathbf{i} :

$$35 = 21z$$

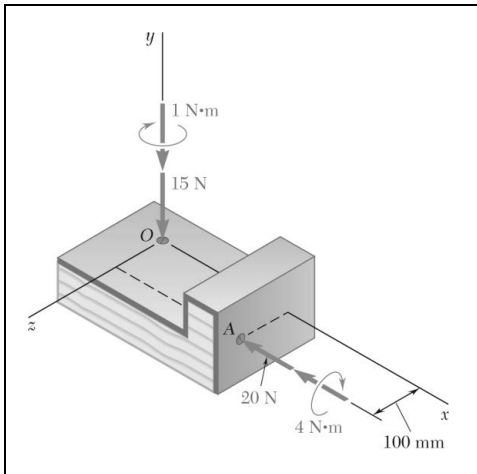
$$z = 1.66667 \text{ in.}$$

From \mathbf{k} :

$$0 = -21x$$

$$x = 0$$

The axis of the wrench is parallel to the y -axis and intersects the xz -plane at $x = 0$, $z = 1.667$ in. ◀



PROBLEM 3.136*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (a) the resultant force \mathbf{R} , (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the xz -plane.

SOLUTION

First, reduce the given force system to a force-couple system.

We have $\Sigma \mathbf{F}: -(20 \text{ N})\mathbf{i} - (15 \text{ N})\mathbf{j} = \mathbf{R} \quad R = 25 \text{ N}$

We have $\Sigma \mathbf{M}_O: \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma \mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = -20 \text{ N}(0.1 \text{ m})\mathbf{j} - (4 \text{ N} \cdot \text{m})\mathbf{i} - (1 \text{ N} \cdot \text{m})\mathbf{j}$$

$$= -(4 \text{ N} \cdot \text{m})\mathbf{i} - (3 \text{ N} \cdot \text{m})\mathbf{j}$$

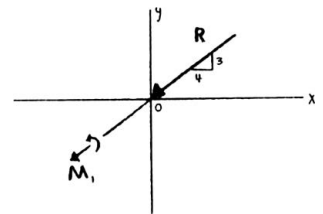
(a) $\mathbf{R} = -(20.0 \text{ N})\mathbf{i} - (15.00 \text{ N})\mathbf{j} \quad \blacktriangleleft$

(b) We have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \quad \lambda = \frac{\mathbf{R}}{R}$

$$= (-0.8\mathbf{i} - 0.6\mathbf{j}) \cdot [-(4 \text{ N} \cdot \text{m})\mathbf{i} - (3 \text{ N} \cdot \text{m})\mathbf{j}]$$

$$= 5 \text{ N} \cdot \text{m}$$

Pitch: $p = \frac{M_1}{R} = \frac{5 \text{ N} \cdot \text{m}}{25 \text{ N}} = 0.200 \text{ m}$



or $p = 0.200 \text{ m} \quad \blacktriangleleft$

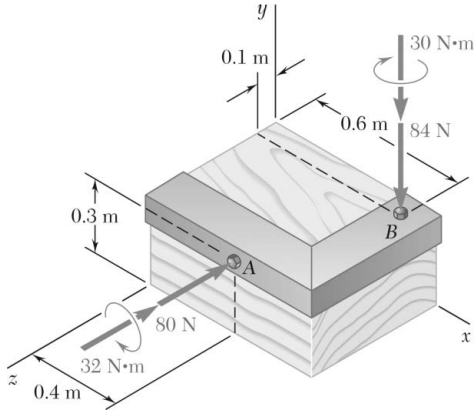
(c) From above, note that

$$\mathbf{M}_1 = \mathbf{M}_O^R$$

Therefore, the axis of the wrench goes through the origin. The line of action of the wrench lies in the xy -plane with a slope of

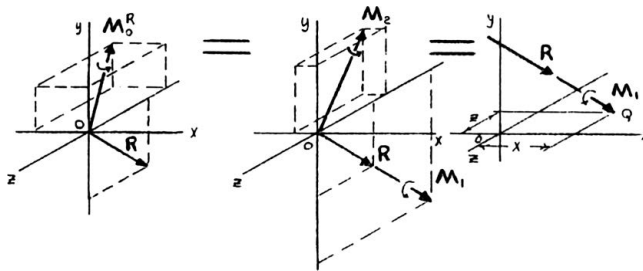
$$y = \frac{3}{4}x \quad \blacktriangleleft$$

PROBLEM 3.137*



Two bolts at A and B are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (a) the resultant \mathbf{R} , (b) the pitch of the single equivalent wrench, (c) the point where the axis of the wrench intersects the xz -plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma \mathbf{F}: -(84 \text{ N})\mathbf{j} - (80 \text{ N})\mathbf{k} = \mathbf{R} \quad R = 116 \text{ N}$

and $\Sigma \mathbf{M}_O: \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma \mathbf{M}_C = \mathbf{M}_O^R$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0 & 0.1 \\ 0 & 84 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & 0.3 & 0 \\ 0 & 0 & 80 \end{vmatrix} + (-30\mathbf{j} - 32\mathbf{k}) \text{ N} \cdot \text{m} = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = -(15.6 \text{ N} \cdot \text{m})\mathbf{i} + (2 \text{ N} \cdot \text{m})\mathbf{j} - (82.4 \text{ N} \cdot \text{m})\mathbf{k}$$

(a) $\mathbf{R} = -(84.0 \text{ N})\mathbf{j} - (80.0 \text{ N})\mathbf{k} \quad \blacktriangleleft$

(b) We have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \quad \lambda_R = \frac{\mathbf{R}}{R}$

$$= \frac{-84\mathbf{j} - 80\mathbf{k}}{116} \cdot [-(15.6 \text{ N} \cdot \text{m})\mathbf{i} + (2 \text{ N} \cdot \text{m})\mathbf{j} - (82.4 \text{ N} \cdot \text{m})\mathbf{k}]$$

$$= 55.379 \text{ N} \cdot \text{m}$$

and $\mathbf{M}_1 = M_1 \lambda_R = -(40.102 \text{ N} \cdot \text{m})\mathbf{j} - (38.192 \text{ N} \cdot \text{m})\mathbf{k}$

Then pitch $p = \frac{M_1}{R} = \frac{55.379 \text{ N} \cdot \text{m}}{116 \text{ N}} = 0.47741 \text{ m} \quad \text{or } p = 0.477 \text{ m} \quad \blacktriangleleft$

PROBLEM 3.137* (Continued)

(c) We have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$
 $\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = [(-15.6\mathbf{i} + 2\mathbf{j} - 82.4\mathbf{k}) - (40.102\mathbf{j} - 38.192\mathbf{k})] \text{ N} \cdot \text{m}$
 $= -(15.6 \text{ N} \cdot \text{m})\mathbf{i} + (42.102 \text{ N} \cdot \text{m})\mathbf{j} - (44.208 \text{ N} \cdot \text{m})\mathbf{k}$

We require $\mathbf{M}_2 = \mathbf{r}_{O/O} \times \mathbf{R}$
 $(-15.6\mathbf{i} + 42.102\mathbf{j} - 44.208\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times (84\mathbf{j} - 80\mathbf{k})$
 $= (84z)\mathbf{i} + (80x)\mathbf{j} - (84x)\mathbf{k}$

From **i**: $-15.6 = 84z$
 $z = -0.185714 \text{ m}$

or $z = -0.1857 \text{ m}$

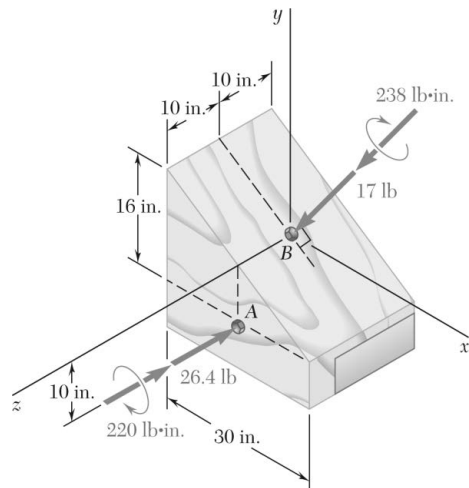
From **k**: $-44.208 = -84x$
 $x = 0.52629 \text{ m}$

or $x = 0.526 \text{ m}$

The axis of the wrench intersects the xz -plane at

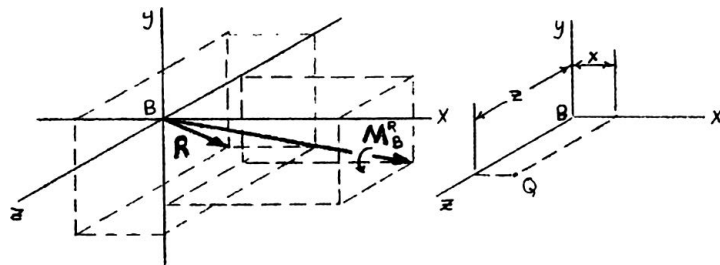
$$x = 0.526 \text{ m} \quad y = 0 \quad z = -0.1857 \text{ m} \quad \blacktriangleleft$$

PROBLEM 3.138*



Two bolts at A and B are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (a) the resultant \mathbf{R} , (b) the pitch of the single equivalent wrench, (c) the point where the axis of the wrench intersects the xz -plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin at B .

$$(a) \quad \text{We have} \quad \Sigma \mathbf{F}: \quad -(26.4 \text{ lb})\mathbf{k} - (17 \text{ lb})\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = \mathbf{R}$$

$$\mathbf{R} = -(8.00 \text{ lb})\mathbf{i} - (15.00 \text{ lb})\mathbf{j} - (26.4 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\text{and} \quad R = 31.4 \text{ lb}$$

$$\text{We have} \quad \Sigma \mathbf{M}_B: \quad \mathbf{r}_{AB} \times \mathbf{F}_A + \mathbf{M}_A + \mathbf{M}_B = \mathbf{M}_B^R$$

$$\mathbf{M}_B^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 0 \\ 0 & 0 & -26.4 \end{vmatrix} - 220\mathbf{k} - 238\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = 264\mathbf{i} - 220\mathbf{k} - 14(8\mathbf{i} + 15\mathbf{j})$$

$$\mathbf{M}_B^R = (152 \text{ lb} \cdot \text{in.})\mathbf{i} - (210 \text{ lb} \cdot \text{in.})\mathbf{j} - (220 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$(b) \quad \text{We have}$$

$$\begin{aligned} M_1 &= \lambda_R \cdot \mathbf{M}_O^R \quad \lambda_R = \frac{\mathbf{R}}{R} \\ &= \frac{-8.00\mathbf{i} - 15.00\mathbf{j} - 26.4\mathbf{k}}{31.4} \cdot [(152 \text{ lb} \cdot \text{in.})\mathbf{i} - (210 \text{ lb} \cdot \text{in.})\mathbf{j} - (220 \text{ lb} \cdot \text{in.})\mathbf{k}] \\ &= 246.56 \text{ lb} \cdot \text{in.} \end{aligned}$$

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PROBLEM 3.138* (Continued)

and $\mathbf{M}_1 = M_1 \lambda_R = -(62.818 \text{ lb} \cdot \text{in.})\mathbf{i} - (117.783 \text{ lb} \cdot \text{in.})\mathbf{j} - (207.30 \text{ lb} \cdot \text{in.})\mathbf{k}$

Then pitch $p = \frac{M_1}{R} = \frac{246.56 \text{ lb} \cdot \text{in.}}{31.4 \text{ lb}} = 7.8522 \text{ in.}$ or $p = 7.85 \text{ in.}$ ◀

(c) We have $\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$
 $\mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (152\mathbf{i} - 210\mathbf{j} - 220\mathbf{k}) - (-62.818\mathbf{i} - 117.783\mathbf{j} - 207.30\mathbf{k})$
 $= (214.82 \text{ lb} \cdot \text{in.})\mathbf{i} - (92.217 \text{ lb} \cdot \text{in.})\mathbf{j} - (12.7000 \text{ lb} \cdot \text{in.})\mathbf{k}$

We require $\mathbf{M}_2 = \mathbf{r}_{QB} \times \mathbf{R}$

$$214.82\mathbf{i} - 92.217\mathbf{j} - 12.7000\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -8 & -15 & -26.4 \end{vmatrix}$$

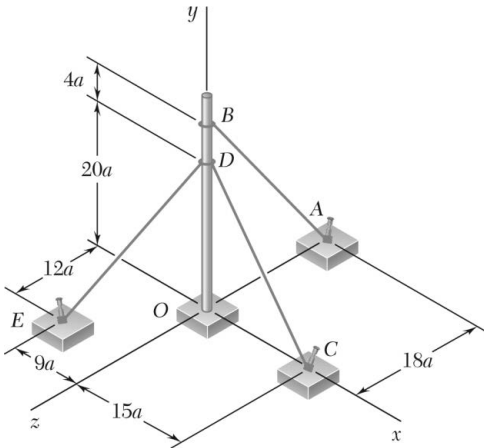
$$= (15z)\mathbf{i} - (8z)\mathbf{j} + (26.4x)\mathbf{j} - (15x)\mathbf{k}$$

From \mathbf{i} : $214.82 = 15z$ $z = 14.3213 \text{ in.}$

From \mathbf{k} : $-12.7000 = -15x$ $x = 0.84667 \text{ in.}$

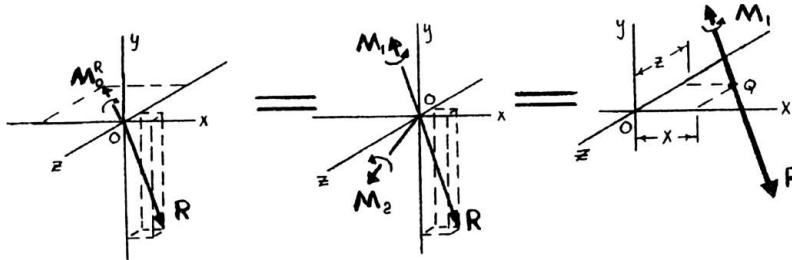
The axis of the wrench intersects the xz -plane at $x = 0.847 \text{ in.}$ $y = 0$ $z = 14.32 \text{ in.}$ ◀

PROBLEM 3.139*



A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P , replace the forces exerted on the pole with an equivalent wrench and determine (a) the resultant force \mathbf{R} , (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the xz -plane.

SOLUTION



(a) First reduce the given force system to a force-couple at the origin.

We have $\Sigma \mathbf{F}: P\lambda_{BA} + P\lambda_{DC} + P\lambda_{DE} = \mathbf{R}$

$$\mathbf{R} = P \left[\left(\frac{4}{5} \mathbf{j} - \frac{3}{5} \mathbf{k} \right) + \left(\frac{3}{5} \mathbf{i} - \frac{4}{5} \mathbf{j} \right) + \left(\frac{-9}{25} \mathbf{i} - \frac{4}{5} \mathbf{j} + \frac{12}{25} \mathbf{k} \right) \right]$$

$$\mathbf{R} = \frac{3P}{25} (2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \quad \blacktriangleleft$$

$$R = \frac{3P}{25} \sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25} P$$

We have $\Sigma \mathbf{M}: \Sigma (\mathbf{r}_O \times P) = \mathbf{M}_O^R$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5} \mathbf{j} - \frac{3P}{5} \mathbf{k} \right) + (20a)\mathbf{j} \times \left(\frac{3P}{5} \mathbf{i} - \frac{4P}{5} \mathbf{j} \right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25} \mathbf{i} - \frac{4P}{5} \mathbf{j} + \frac{12P}{25} \mathbf{k} \right) = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \frac{24Pa}{5} (-\mathbf{i} - \mathbf{k})$$

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PROBLEM 3.139* (Continued)

(b) We have

$$M_1 = \lambda_R \cdot \mathbf{M}_O^R$$

where

$$\lambda_R = \frac{\mathbf{R}}{R} = \frac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \frac{25}{27\sqrt{5}P} = \frac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

Then

$$M_1 = \frac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \frac{-8Pa}{15\sqrt{5}}$$

and pitch

$$p = \frac{M_1}{R} = \frac{-8Pa}{15\sqrt{5}} \left(\frac{25}{27\sqrt{5}P} \right) = \frac{-8a}{81} \quad \text{or } p = -0.0988a \quad \blacktriangleleft$$

(c)

$$\mathbf{M}_1 = M_1 \lambda_R = \frac{-8Pa}{15\sqrt{5}} \left(\frac{1}{9\sqrt{5}} \right) (2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \frac{8Pa}{675} (-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$$

$$\text{Then } \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \frac{8Pa}{675}(-430\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$$

We require

$$\mathbf{M}_2 = \mathbf{r}_{O'O} \times \mathbf{R}$$

$$\begin{aligned} \left(\frac{8Pa}{675} \right) (-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) &= (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25} \right) (2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \\ &= \left(\frac{3P}{25} \right) [20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}] \end{aligned}$$

From \mathbf{i} :

$$8(-403) \frac{Pa}{675} = 20z \left(\frac{3P}{25} \right) \quad z = -1.99012a$$

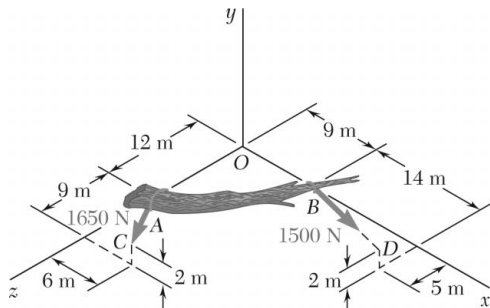
From \mathbf{k} :

$$8(-406) \frac{Pa}{675} = -20x \left(\frac{3P}{25} \right) \quad x = 2.0049a$$

The axis of the wrench intersects the xz -plane at

$$x = 2.00a, z = -1.990a \quad \blacktriangleleft$$

PROBLEM 3.140*



Two ropes attached at A and B are used to move the trunk of a fallen tree. Replace the forces exerted by the ropes with an equivalent wrench and determine (a) the resultant force \mathbf{R} , (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the yz -plane.

SOLUTION

(a) First replace the given forces with an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_O^R)$ at the origin.

We have

$$d_{AC} = \sqrt{(6)^2 + (2)^2 + (9)^2} = 11 \text{ m}$$

$$d_{BD} = \sqrt{(14)^2 + (2)^2 + (5)^2} = 15 \text{ m}$$

Then

$$\begin{aligned} \mathbf{T}_{AC} &= \frac{1650 \text{ N}}{11} = (6\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}) \\ &= (900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k} \end{aligned}$$

and

$$\begin{aligned} \mathbf{T}_{BD} &= \frac{1500 \text{ N}}{15} = (14\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) \\ &= (1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k} \end{aligned}$$

Equivalence then requires

$$\begin{aligned} \Sigma \mathbf{F}: \quad \mathbf{R} &= \mathbf{T}_{AC} + \mathbf{T}_{BD} \\ &= (900\mathbf{i} + 300\mathbf{j} + 1350\mathbf{k}) \\ &\quad + (1400\mathbf{i} + 200\mathbf{j} + 500\mathbf{k}) \\ &= (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} \Sigma \mathbf{M}_O: \quad \mathbf{M}_O^R &= \mathbf{r}_A \times \mathbf{T}_{AC} + \mathbf{r}_B \times \mathbf{T}_{BD} \\ &= (12 \text{ m})\mathbf{k} \times [(900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k}] \\ &\quad + (9 \text{ m})\mathbf{j} \times [(1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k}] \\ &= -(3600)\mathbf{i} + (10,800 - 4500)\mathbf{j} + (1800)\mathbf{k} \\ &= -(3600 \text{ N} \cdot \text{m})\mathbf{i} + (6300 \text{ N} \cdot \text{m})\mathbf{j} + (1800 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

The components of the wrench are $(\mathbf{R}, \mathbf{M}_1)$, where

$$\mathbf{R} = (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.140* (Continued)

(b) We have

$$R = 100\sqrt{(23)^2 + (5)^2 + (18.5)^2} = 2993.7 \text{ N}$$

Let

$$\lambda_{\text{axis}} = \frac{\mathbf{R}}{R} = \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k})$$

Then

$$\begin{aligned} M_1 &= \lambda_{\text{axis}} \cdot \mathbf{M}_O^R \\ &= \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \cdot (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k}) \\ &= \frac{1}{0.29937}[(23)(-36) + (5)(63) + (18.5)(18)] \\ &= -601.26 \text{ N} \cdot \text{m} \end{aligned}$$

Finally,

$$P = \frac{M_1}{R} = \frac{-601.26 \text{ N} \cdot \text{m}}{2993.7 \text{ N}}$$

or $P = -0.201 \text{ m}$ ◀

(c) We have

$$\begin{aligned} M_1 &= M_1 \lambda_{\text{axis}} \\ &= (-601.26 \text{ N} \cdot \text{m}) \times \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \end{aligned}$$

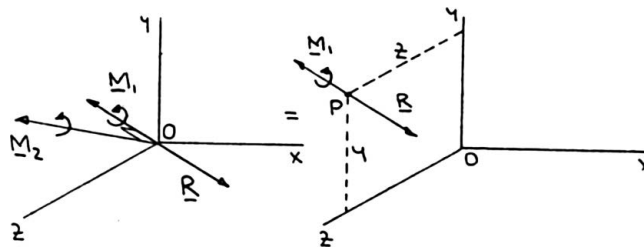
or

$$\mathbf{M}_1 = -(461.93 \text{ N} \cdot \text{m})\mathbf{i} - (100.421 \text{ N} \cdot \text{m})\mathbf{j} - (371.56 \text{ N} \cdot \text{m})\mathbf{k}$$

Now

$$\begin{aligned} \mathbf{M}_2 &= \mathbf{M}_O^R - \mathbf{M}_1 \\ &= (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k}) \\ &\quad - (-461.93\mathbf{i} - 100.421\mathbf{j} - 371.56\mathbf{k}) \\ &= -(3138.1 \text{ N} \cdot \text{m})\mathbf{i} + (6400.4 \text{ N} \cdot \text{m})\mathbf{j} + (2171.6 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

For equivalence:



PROBLEM 3.140* (Continued)

Thus, we require $\mathbf{M}_2 = \mathbf{r}_p \times \mathbf{R}$ $\mathbf{r} = (y\mathbf{j} + z\mathbf{k})$

Substituting:

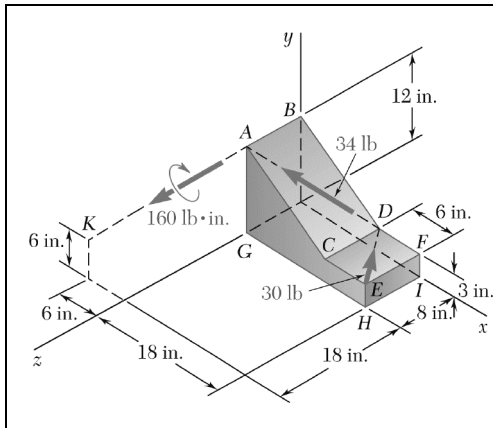
$$-3138.1\mathbf{i} + 6400.4\mathbf{j} + 2171.6\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 2300 & 500 & 1850 \end{vmatrix}$$

Equating coefficients:

$$\mathbf{j}: 6400.4 = 2300z \quad \text{or} \quad z = 2.78 \text{ m}$$

$$\mathbf{k}: 2171.6 = -2300y \quad \text{or} \quad y = -0.944 \text{ m}$$

The axis of the wrench intersects the yz -plane at $y = -0.944 \text{ m}$ $z = 2.78 \text{ m}$ ◀



PROBLEM 3.141*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force \mathbf{R} . If it can, determine \mathbf{R} and the point where the line of action of \mathbf{R} intersects the yz -plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the yz -plane.

SOLUTION

First determine the resultant of the forces at D . We have

$$d_{DA} = \sqrt{(-12)^2 + (9)^2 + (8)^2} = 17 \text{ in.}$$

$$d_{ED} = \sqrt{(-6)^2 + (0)^2 + (-8)^2} = 10 \text{ in.}$$

Then

$$\begin{aligned} \mathbf{F}_{DA} &= \frac{34 \text{ lb}}{17} = (-12\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}) \\ &= -(24 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} + (16 \text{ lb})\mathbf{k} \end{aligned}$$

and

$$\begin{aligned} \mathbf{F}_{ED} &= \frac{30 \text{ lb}}{10} = (-6\mathbf{i} - 8\mathbf{k}) \\ &= -(18 \text{ lb})\mathbf{i} - (24 \text{ lb})\mathbf{k} \end{aligned}$$

Then

$$\begin{aligned} \Sigma \mathbf{F}: \mathbf{R} &= \mathbf{F}_{DA} + \mathbf{F}_{ED} \\ &= (-24\mathbf{i} + 18\mathbf{j} + 16\mathbf{k} + (-18\mathbf{i} - 24\mathbf{k})) \\ &= -(42 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} - (8 \text{ lb})\mathbf{k} \end{aligned}$$

For the applied couple

$$d_{AK} = \sqrt{(-6)^2 + (-6)^2 + (18)^2} = 6\sqrt{11} \text{ in.}$$

Then

$$\begin{aligned} \mathbf{M} &= \frac{160 \text{ lb} \cdot \text{in.}}{6\sqrt{11}} (-6\mathbf{i} - 6\mathbf{j} + 18\mathbf{k}) \\ &= \frac{160}{\sqrt{11}} [-(1 \text{ lb} \cdot \text{in.})\mathbf{i} - (1 \text{ lb} \cdot \text{in.})\mathbf{j} + (3 \text{ lb} \cdot \text{in.})\mathbf{k}] \end{aligned}$$

To be able to reduce the original forces and couple to a single equivalent force, \mathbf{R} and \mathbf{M} must be perpendicular. Thus

$$\mathbf{R} \cdot \mathbf{M} \stackrel{?}{=} 0$$

PROBLEM 3.141* (Continued)

Substituting

$$(-42\mathbf{i} + 18\mathbf{j} - 8\mathbf{k}) \cdot \frac{160}{\sqrt{11}}(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \stackrel{?}{=} 0$$

or

$$\frac{160}{\sqrt{11}}[(-42)(-1) + (18)(-1) + (-8)(3)] \stackrel{?}{=} 0$$

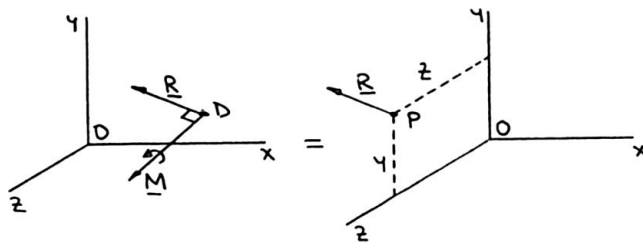
or

$$0 \neq 0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force.

$$\mathbf{R} = -(42.0 \text{ lb})\mathbf{i} + (18.00 \text{ lb})\mathbf{j} - (8.00 \text{ lb})\mathbf{k}$$

Then for equivalence,



Thus, we require

$$\mathbf{M} = \mathbf{r}_{PD} \times \mathbf{R}$$

where

$$\mathbf{r}_{PD} = -(12 \text{ in.})\mathbf{i} + [(y - 3) \text{ in.}]\mathbf{j} + (z \text{ in.})\mathbf{k}$$

Substituting:

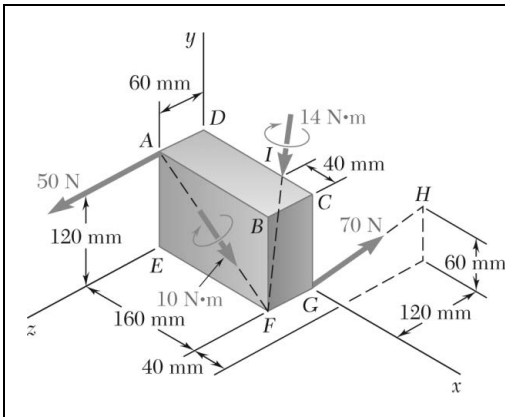
$$\begin{aligned} \frac{160}{\sqrt{11}}(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -12 & (y - 3) & z \\ -42 & 18 & -8 \end{vmatrix} \\ &= [(y - 3)(-8) - (z)(18)]\mathbf{i} \\ &\quad + [(z)(-42) - (-12)(-8)]\mathbf{j} \\ &\quad + [(-12)(18) - (y - 3)(-42)]\mathbf{k} \end{aligned}$$

Equating coefficients:

$$\mathbf{j}: -\frac{160}{\sqrt{11}} = -42z - 96 \quad \text{or} \quad z = -1.137 \text{ in.}$$

$$\mathbf{k}: \frac{480}{\sqrt{11}} = -216 + 42(y - 3) \quad \text{or} \quad y = 11.59 \text{ in.}$$

The line of action of **R** intersects the *yz*-plane at $x = 0$ $y = 11.59 \text{ in.}$ $z = -1.137 \text{ in.}$



PROBLEM 3.142*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force \mathbf{R} . If it can, determine \mathbf{R} and the point where the line of action of \mathbf{R} intersects the yz -plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the yz -plane.

SOLUTION

First, reduce the given force system to a force-couple at the origin.

We have $\Sigma \mathbf{F}: \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$

$$\begin{aligned} \mathbf{R} &= (50 \text{ N})\mathbf{k} + 70 \text{ N} \left[\frac{(40 \text{ mm})\mathbf{i} + (60 \text{ mm})\mathbf{j} - (120 \text{ mm})\mathbf{k}}{140 \text{ mm}} \right] \\ &= (20 \text{ N})\mathbf{i} + (30 \text{ N})\mathbf{j} - (10 \text{ N})\mathbf{k} \end{aligned}$$

and

$$R = 37.417 \text{ N}$$

We have $\Sigma \mathbf{M}_O: \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma \mathbf{M}_C = \mathbf{M}_O^R$

$$\begin{aligned} \mathbf{M}_O^R &= [(0.12 \text{ m})\mathbf{j} \times (50 \text{ N})\mathbf{k}] + \left\{ (0.16 \text{ m})\mathbf{i} \times [(20 \text{ N})\mathbf{i} + (30 \text{ N})\mathbf{j} - (60 \text{ N})\mathbf{k}] \right\} \\ &\quad + (10 \text{ N} \cdot \text{m}) \left[\frac{(160 \text{ mm})\mathbf{i} - (120 \text{ mm})\mathbf{j}}{200 \text{ mm}} \right] \\ &\quad + (14 \text{ N} \cdot \text{m}) \left[\frac{(40 \text{ mm})\mathbf{i} - (120 \text{ mm})\mathbf{j} + (60 \text{ mm})\mathbf{k}}{140 \text{ mm}} \right] \\ \mathbf{M}_O^R &= (18 \text{ N} \cdot \text{m})\mathbf{i} - (8.4 \text{ N} \cdot \text{m})\mathbf{j} + (10.8 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

To be able to reduce the original forces and couples to a single equivalent force, \mathbf{R} and \mathbf{M} must be perpendicular. Thus, $\mathbf{R} \cdot \mathbf{M} = 0$.

Substituting

$$(20\mathbf{i} + 30\mathbf{j} - 10\mathbf{k}) \cdot (18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k}) \stackrel{?}{=} 0$$

or

$$(20)(18) + (30)(-8.4) + (-10)(10.8) \stackrel{?}{=} 0$$

or

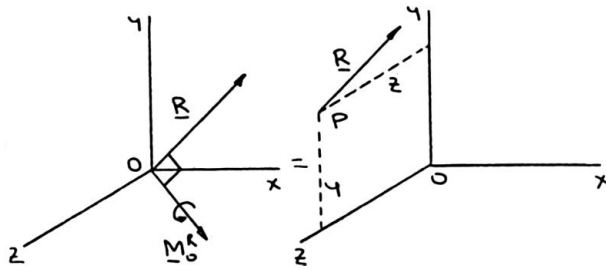
$$0 \checkmark = 0$$

\mathbf{R} and \mathbf{M} are perpendicular so that the given system can be reduced to the single equivalent force.

$$\mathbf{R} = (20.0 \text{ N})\mathbf{i} + (30.0 \text{ N})\mathbf{j} - (10.00 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.142* (Continued)

Then for equivalence,



Thus, we require

$$\mathbf{M}_O^R = \mathbf{r}_p \times \mathbf{R} \quad \mathbf{r}_p = y\mathbf{j} + z\mathbf{k}$$

Substituting:

$$18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 20 & 30 & -10 \end{vmatrix}$$

Equating coefficients:

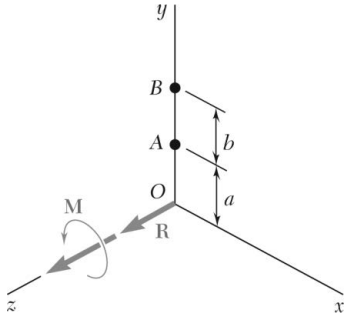
$$\mathbf{j}: -8.4 = 20z \quad \text{or} \quad z = -0.42 \text{ m}$$

$$\mathbf{k}: 10.8 = -20y \quad \text{or} \quad y = -0.54 \text{ m}$$

The line of action of \mathbf{R} intersects the yz -plane at $x = 0 \quad y = -0.540 \text{ m} \quad z = -0.420 \text{ m}$ ◀

PROBLEM 3.143*

Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the y -axis and applied respectively at A and B .



SOLUTION

Express the forces at A and B as

$$\mathbf{A} = A_x \mathbf{i} + A_z \mathbf{k}$$

$$\mathbf{B} = B_x \mathbf{i} + B_z \mathbf{k}$$

Then, for equivalence to the given force system,

$$\Sigma F_x: A_x + B_x = 0 \quad (1)$$

$$\Sigma F_z: A_z + B_z = R \quad (2)$$

$$\Sigma M_x: A_z(a) + B_z(a+b) = 0 \quad (3)$$

$$\Sigma M_z: -A_x(a) - B_x(a+b) = M \quad (4)$$

From Equation (1),

$$B_x = -A_x$$

Substitute into Equation (4):

$$-A_x(a) + A_x(a+b) = M$$

$$A_x = \frac{M}{b} \quad \text{and} \quad B_x = -\frac{M}{b}$$

From Equation (2),

$$B_z = R - A_z$$

and Equation (3),

$$A_z a + (R - A_z)(a+b) = 0$$

$$A_z = R \left(1 + \frac{a}{b} \right)$$

and

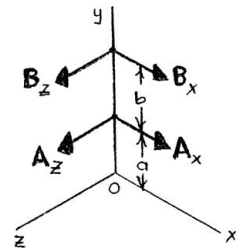
$$B_z = R - R \left(1 + \frac{a}{b} \right)$$

$$B_z = -\frac{a}{b} R$$

Then

$$\mathbf{A} = \left(\frac{M}{b} \right) \mathbf{i} + R \left(1 + \frac{a}{b} \right) \mathbf{k} \quad \blacktriangleleft$$

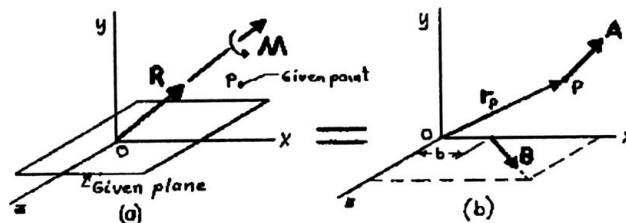
$$\mathbf{B} = -\left(\frac{M}{b} \right) \mathbf{i} - \left(\frac{a}{b} R \right) \mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.144*

Show that, in general, a wrench can be replaced with two forces chosen in such a way that one force passes through a given point while the other force lies in a given plane.

SOLUTION



First, choose a coordinate system so that the xy -plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (\mathbf{R} , \mathbf{M}) of the wrench are known, it follows that the scalar components of \mathbf{R} and \mathbf{M} are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let \mathbf{A} be the force passing through the given Point P and \mathbf{B} be the force that lies in the given plane. Let b be the x -axis intercept of \mathbf{B} .

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} + R_z \mathbf{k} \quad \text{and} \quad \mathbf{M} = M_x \mathbf{i} + M_y \mathbf{j} + M_z \mathbf{k}$$

while the unknown forces \mathbf{A} and \mathbf{B} can be expressed as

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x \mathbf{i} + B_z \mathbf{k}$$

Since the position vector of Point P is given, it follows that the scalar components (x , y , z) of the position vector \mathbf{r}_P are also known.

Then, for equivalence of the two systems,

$$\Sigma F_x: R_x = A_x + B_x \tag{1}$$

$$\Sigma F_y: R_y = A_y \tag{2}$$

$$\Sigma F_z: R_z = A_z + B_z \tag{3}$$

$$\Sigma M_x: M_x = yA_z - zA_y \tag{4}$$

$$\Sigma M_y: M_y = zA_x - xA_z - bB_z \tag{5}$$

$$\Sigma M_z: M_z = xA_y - yA_x \tag{6}$$

Based on the above six independent equations for the six unknowns (A_x , A_y , A_z , B_x , B_z , b), there exists a unique solution for \mathbf{A} and \mathbf{B} .

From Equation (2),

$$A_y = R_y \quad \blacktriangleleft$$

PROBLEM 3.144* (Continued)

Equation (6):
$$A_x = \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (1):
$$B_x = R_x - \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (4):
$$A_z = \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

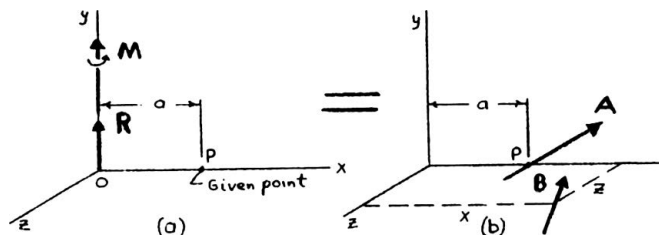
Equation (3):
$$B_z = R_z - \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

Equation (5):
$$b = \frac{(xM_x + yM_y + zM_z)}{(M_x - yR_z + zR_y)} \blacktriangleleft$$

PROBLEM 3.145*

Show that a wrench can be replaced with two perpendicular forces, one of which is applied at a given point.

SOLUTION



First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures *a* and *b*.

We have $\mathbf{R} = R\mathbf{j}$ and $\mathbf{M} = M\mathbf{j}$ and are known.

The unknown forces \mathbf{A} and \mathbf{B} can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The distance a is known. It is assumed that force \mathbf{B} intersects the xz -plane at $(x, 0, z)$. Then for equivalence,

$$\Sigma F_x: \quad 0 = A_x + B_x \quad (1)$$

$$\Sigma F_y: \quad R = A_y + B_y \quad (2)$$

$$\Sigma F_z: \quad 0 = A_z + B_z \quad (3)$$

$$\Sigma M_x: \quad 0 = -zB_y \quad (4)$$

$$\Sigma M_y: \quad M = -aA_z - xB_z + zB_x \quad (5)$$

$$\Sigma M_z: \quad 0 = aA_y + xB_y \quad (6)$$

Since \mathbf{A} and \mathbf{B} are made perpendicular,

$$\mathbf{A} \cdot \mathbf{B} = 0 \quad \text{or} \quad A_xB_x + A_yB_y + A_zB_z = 0 \quad (7)$$

There are eight unknowns: $A_x, A_y, A_z, B_x, B_y, B_z, x, z$

But only seven independent equations. Therefore, *there exists an infinite number of solutions.*

Next, consider Equation (4): $0 = -zB_y$

If $B_y = 0$, Equation (7) becomes $A_xB_x + A_zB_z = 0$

Using Equations (1) and (3), this equation becomes $A_x^2 + A_z^2 = 0$

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PROBLEM 3.145* (Continued)

Since the components of \mathbf{A} must be real, a nontrivial solution is not possible. Thus, it is required that $B_y \neq 0$, so that from Equation (4), $z = 0$.

To obtain one possible solution, arbitrarily let $A_x = 0$.

(Note: Setting A_y , A_z , or B_z equal to zero results in unacceptable solutions.)

The defining equations then become

$$0 = B_x \quad (1)'$$

$$R = A_y + B_y \quad (2)$$

$$0 = A_z + B_z \quad (3)$$

$$M = -aA_z - xB_z \quad (5)'$$

$$0 = aA_y + xB_y \quad (6)$$

$$A_y B_y + A_z B_z = 0 \quad (7)'$$

Then Equation (2) can be written

$$A_y = R - B_y$$

Equation (3) can be written

$$B_z = -A_z$$

Equation (6) can be written

$$x = -\frac{aA_y}{B_y}$$

Substituting into Equation (5)',

$$M = -aA_z - \left(-a\frac{R - B_y}{B_y}\right)(-A_z)$$

or

$$A_z = -\frac{M}{aR} B_y \quad (8)$$

Substituting into Equation (7)',

$$(R - B_y)B_y + \left(-\frac{M}{aR} B_y\right)\left(\frac{M}{aR} B_y\right) = 0$$

or

$$B_y = \frac{a^2 R^3}{a^2 R^2 + M^2}$$

Then from Equations (2), (8), and (3),

$$A_y = R - \frac{a^2 R^2}{a^2 R^2 + M^2} = \frac{RM^2}{a^2 R^2 + M^2}$$

$$A_z = -\frac{M}{aR} \left(\frac{a^2 R^3}{a^2 R^2 + M^2}\right) = -\frac{aR^2 M}{a^2 R^2 + M^2}$$

$$B_z = \frac{aR^2 M}{a^2 R^2 + M^2}$$

PROBLEM 3.145* (Continued)

In summary,

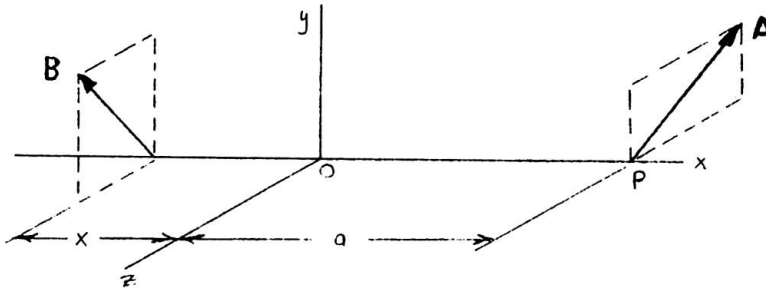
$$\mathbf{A} = \frac{RM}{a^2R^2 + M^2}(M\mathbf{j} - aR\mathbf{k}) \quad \blacktriangleleft$$

$$\mathbf{B} = \frac{aR^2}{a^2R^2 + M^2}(aR\mathbf{j} + M\mathbf{k}) \quad \blacktriangleleft$$

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

Lastly, if $R > 0$ and $M > 0$, it follows from the equations found for \mathbf{A} and \mathbf{B} that $A_y > 0$ and $B_y > 0$.

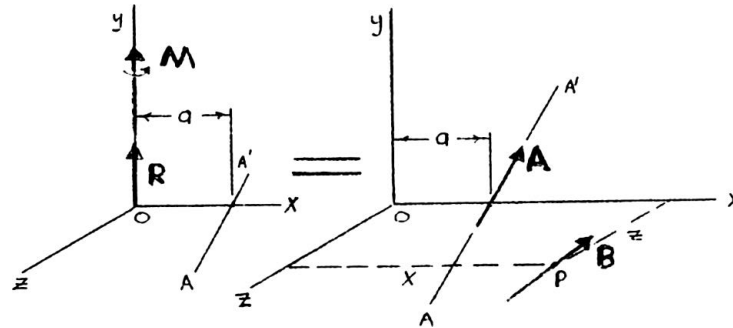
From Equation (6), $x < 0$ (assuming $a > 0$). Then, as a consequence of letting $A_x = 0$, force \mathbf{A} lies in a plane parallel to the yz -plane and to the right of the origin, while force \mathbf{B} lies in a plane parallel to the yz -plane but to the left to the origin, as shown in the figure below.



PROBLEM 3.146*

Show that a wrench can be replaced with two forces, one of which has a prescribed line of action.

SOLUTION



First, choose a rectangular coordinate system where one axis coincides with the axis of the wrench and another axis intersects the prescribed line of action (AA'). Note that it has been assumed that the line of action of force \mathbf{B} intersects the xz -plane at Point $P(x, 0, z)$. Denoting the known direction of line AA' by

$$\lambda_A = \lambda_x \mathbf{i} + \lambda_y \mathbf{j} + \lambda_z \mathbf{k}$$

it follows that force \mathbf{A} can be expressed as

$$\mathbf{A} = A\lambda_A = A(\lambda_x \mathbf{i} + \lambda_y \mathbf{j} + \lambda_z \mathbf{k})$$

Force \mathbf{B} can be expressed as

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

Next, observe that since the axis of the wrench and the prescribed line of action AA' are known, it follows that the distance a can be determined. In the following solution, it is assumed that a is known.

Then for equivalence,

$$\Sigma F_x: 0 = A\lambda_x + B_x \quad (1)$$

$$\Sigma F_y: R = A\lambda_y + B_y \quad (2)$$

$$\Sigma F_z: 0 = A\lambda_z + B_z \quad (3)$$

$$\Sigma M_x: 0 = -zB_y \quad (4)$$

$$\Sigma M_y: M = -aA\lambda_z + zB_x - xB_z \quad (5)$$

$$\Sigma M_x: 0 = -aA\lambda_y + xB_y \quad (6)$$

Since there are six unknowns (A, B_x, B_y, B_z, x, z) and six independent equations, it will be possible to obtain a solution.

PROBLEM 3.146* (Continued)

Case 1: Let $z = 0$ to satisfy Equation (4).

Now Equation (2): $A\lambda_y = R - B_y$

Equation (3): $B_z = -A\lambda_z$

Equation (6): $x = -\frac{aA\lambda_y}{B_y} = -\left(\frac{a}{B_y}\right)(R - B_y)$

Substitution into Equation (5):

$$M = -aA\lambda_z - \left[-\left(\frac{a}{B_y}\right)(R - B_y)(-A\lambda_z) \right]$$

$$A = -\frac{1}{\lambda_z} \left(\frac{M}{aR} \right) B_y$$

Substitution into Equation (2):

$$R = -\frac{1}{\lambda_z} \left(\frac{M}{aR} \right) B_y \lambda_y + B_y$$

$$B_y = \frac{\lambda_z a R^2}{\lambda_z a R - \lambda_y M}$$

Then

$$A = -\frac{MR}{\lambda_z a R - \lambda_y M} = \frac{R}{\lambda_y - \frac{aR}{M} \lambda_z}$$

$$B_x = -A\lambda_x = \frac{\lambda_x MR}{\lambda_z a R - \lambda_y M}$$

$$B_z = -A\lambda_z = \frac{\lambda_z MR}{\lambda_z a R - \lambda_y M}$$

In summary,

$$\mathbf{A} = \frac{P}{\lambda_y - \frac{aR}{M} \lambda_z} \lambda_A \mathbf{i} \quad \blacktriangleleft$$

$$\mathbf{B} = \frac{R}{\lambda_z a R - \lambda_y M} (\lambda_x M \mathbf{i} + \lambda_z a R \mathbf{j} + \lambda_z M \mathbf{k}) \quad \blacktriangleleft$$

and

$$x = a \left(1 - \frac{R}{B_y} \right)$$

$$= a \left[1 - R \left(\frac{\lambda_z a R - \lambda_y M}{\lambda_z a R^2} \right) \right]$$

$$\text{or } x = \frac{\lambda_y M}{\lambda_z R} \quad \blacktriangleleft$$

Note that for this case, the lines of action of both **A** and **B** intersect the x -axis.

PROBLEM 3.146* (Continued)

Case 2: Let $B_y = 0$ to satisfy Equation (4).

Now Equation (2):
$$A = \frac{R}{\lambda_y}$$

Equation (1):
$$B_x = -R \left(\frac{\lambda_x}{\lambda_y} \right)$$

Equation (3):
$$B_z = -R \left(\frac{\lambda_z}{\lambda_y} \right)$$

Equation (6): $aA\lambda_y = 0$ which requires $a = 0$

Substitution into Equation (5):

$$M = z \left[-R \left(\frac{\lambda_x}{\lambda_y} \right) \right] - x \left[-R \left(\frac{\lambda_z}{\lambda_y} \right) \right] \quad \text{or} \quad \lambda_z x - \lambda_x z = \left(\frac{M}{R} \right) \lambda_y$$

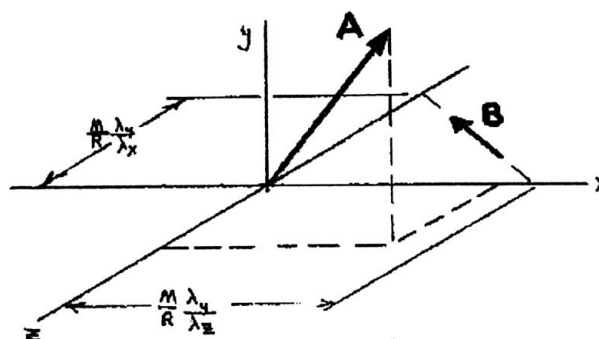
This last expression is the equation for the line of action of force **B**.

In summary,

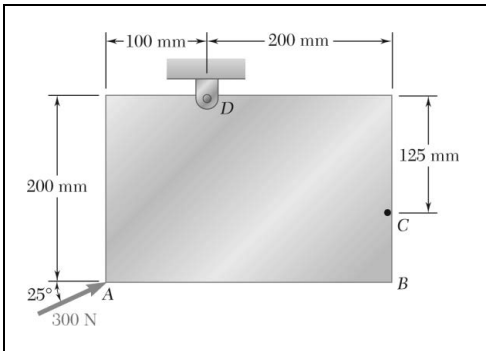
$$\mathbf{A} = \left(\frac{R}{\lambda_y} \right) \lambda_A \quad \blacktriangleleft$$

$$\mathbf{B} = \left(\frac{R}{\lambda_y} \right) (-\lambda_x \mathbf{i} - \lambda_z \mathbf{k}) \quad \blacktriangleleft$$

Assuming that $\lambda_x, \lambda_y, \lambda_z > 0$, the equivalent force system is as shown below.



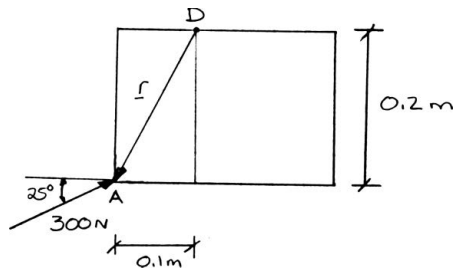
Note that the component of **A** in the xz -plane is parallel to **B**.



PROBLEM 3.147

A 300-N force is applied at A as shown. Determine (a) the moment of the 300-N force about D, (b) the smallest force applied at B that creates the same moment about D.

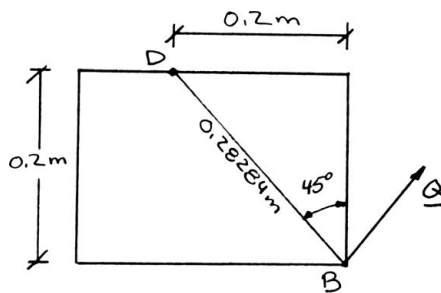
SOLUTION



$$\begin{aligned}
 (a) \quad F_x &= (300 \text{ N}) \cos 25^\circ \\
 &= 271.89 \text{ N} \\
 F_y &= (300 \text{ N}) \sin 25^\circ \\
 &= 126.785 \text{ N} \\
 \mathbf{F} &= (271.89 \text{ N})\mathbf{i} + (126.785 \text{ N})\mathbf{j} \\
 \mathbf{r} &= \overline{DA} = -(0.1 \text{ m})\mathbf{i} - (0.2 \text{ m})\mathbf{j}
 \end{aligned}$$

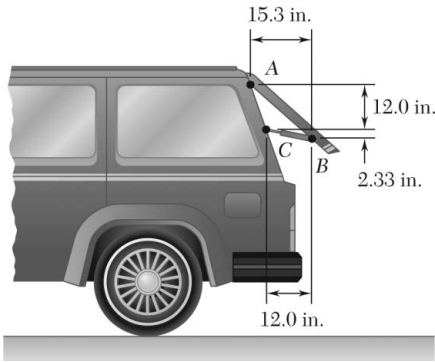
$$\begin{aligned}
 \mathbf{M}_D &= \mathbf{r} \times \mathbf{F} \\
 \mathbf{M}_D &= [-(0.1 \text{ m})\mathbf{i} - (0.2 \text{ m})\mathbf{j}] \times [(271.89 \text{ N})\mathbf{i} + (126.785 \text{ N})\mathbf{j}] \\
 &= -(12.6785 \text{ N} \cdot \text{m})\mathbf{k} + (54.378 \text{ N} \cdot \text{m})\mathbf{k} \\
 &= (41.700 \text{ N} \cdot \text{m})\mathbf{k}
 \end{aligned}$$

$$\mathbf{M}_D = 41.7 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$



(b) The smallest force Q at B must be perpendicular to \overline{DB} at 45°

$$\begin{aligned}
 \mathbf{M}_D &= Q(\overline{DB}) \\
 41.700 \text{ N} \cdot \text{m} &= Q(0.28284 \text{ m}) \quad Q = 147.4 \text{ N} \curvearrowright 45.0^\circ \blacktriangleleft
 \end{aligned}$$



PROBLEM 3.148

The tailgate of a car is supported by the hydraulic lift BC . If the lift exerts a 125-lb force directed along its centerline on the ball and socket at B , determine the moment of the force about A .

SOLUTION

First note

$$d_{CB} = \sqrt{(12.0 \text{ in.})^2 + (2.33 \text{ in.})^2}$$

$$= 12.2241 \text{ in.}$$

Then

$$\cos \theta = \frac{12.0 \text{ in.}}{12.2241 \text{ in.}}$$

$$\sin \theta = \frac{2.33 \text{ in.}}{12.2241 \text{ in.}}$$

and

$$\mathbf{F}_{CB} = F_{CB} \cos \theta \mathbf{i} - F_{CB} \sin \theta \mathbf{j}$$

$$= \frac{125 \text{ lb}}{12.2241 \text{ in.}} [(12.0 \text{ in.}) \mathbf{i} - (2.33 \text{ in.}) \mathbf{j}]$$

Now

$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where

$$\mathbf{r}_{B/A} = (15.3 \text{ in.}) \mathbf{i} - (12.0 \text{ in.} + 2.33 \text{ in.}) \mathbf{j}$$

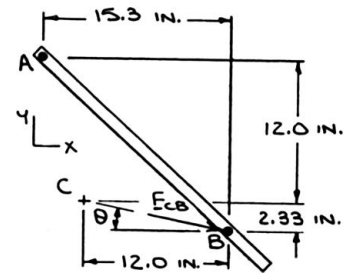
$$= (15.3 \text{ in.}) \mathbf{i} - (14.33 \text{ in.}) \mathbf{j}$$

Then

$$\mathbf{M}_A = [(15.3 \text{ in.}) \mathbf{i} - (14.33 \text{ in.}) \mathbf{j}] \times \frac{125 \text{ lb}}{12.2241 \text{ in.}} (12.0 \mathbf{i} - 2.33 \mathbf{j})$$

$$= (1393.87 \text{ lb} \cdot \text{in.}) \mathbf{k}$$

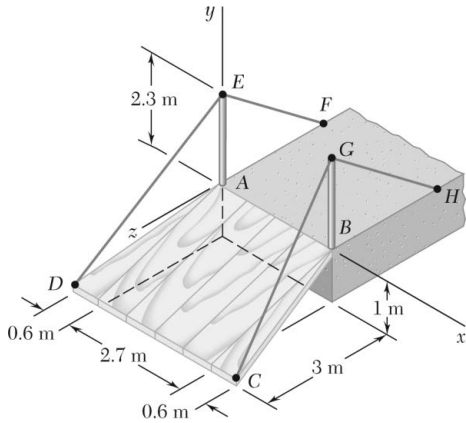
$$= (116.156 \text{ lb} \cdot \text{ft}) \mathbf{k}$$



$$\text{or } \mathbf{M}_A = 116.2 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

PROBLEM 3.149

The ramp $ABCD$ is supported by cables at corners C and D . The tension in each of the cables is 810 N. Determine the moment about A of the force exerted by (a) the cable at D , (b) the cable at C .



SOLUTION

(a) We have

$$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$$

where

$$\mathbf{r}_{E/A} = (2.3 \text{ m})\mathbf{j}$$

$$\begin{aligned} \mathbf{T}_{DE} &= \lambda_{DE} T_{DE} \\ &= \frac{(0.6 \text{ m})\mathbf{i} + (3.3 \text{ m})\mathbf{j} - (3 \text{ m})\mathbf{k}}{\sqrt{(0.6)^2 + (3.3)^2 + (3)^2} \text{ m}} (810 \text{ N}) \\ &= (108 \text{ N})\mathbf{i} + (594 \text{ N})\mathbf{j} - (540 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{M}_A &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.3 & 0 \\ 108 & 594 & -540 \end{vmatrix} \text{ N} \cdot \text{m} \\ &= -(1242 \text{ N} \cdot \text{m})\mathbf{i} - (248.4 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

$$\text{or } \mathbf{M}_A = -(1242 \text{ N} \cdot \text{m})\mathbf{i} - (248 \text{ N} \cdot \text{m})\mathbf{k} \blacktriangleleft$$

(b) We have

$$\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$$

where

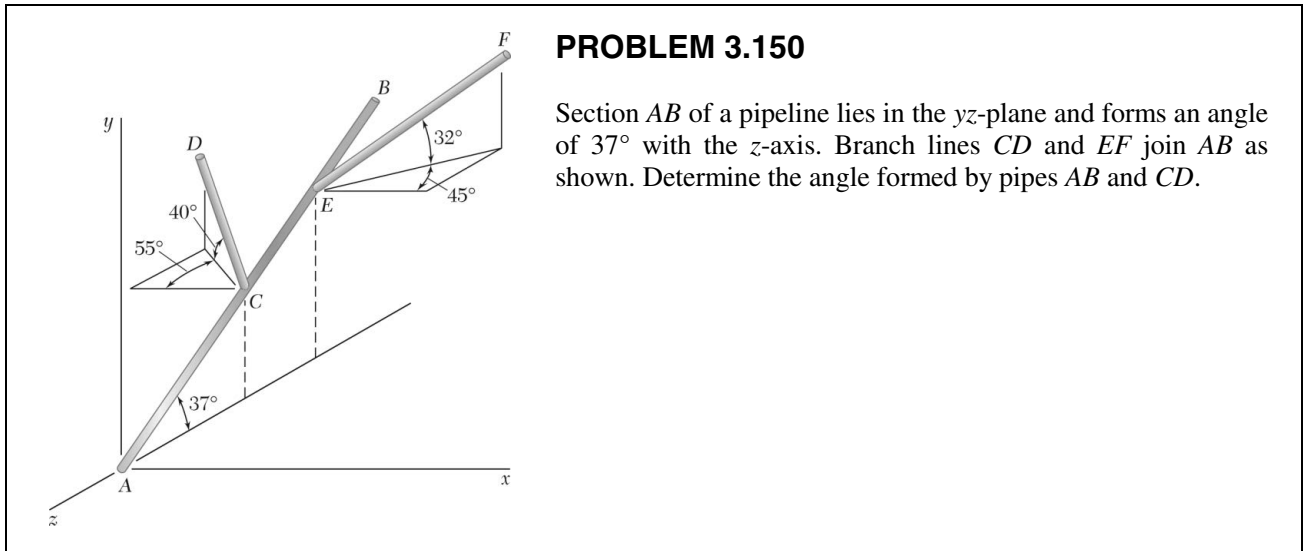
$$\mathbf{r}_{G/A} = (2.7 \text{ m})\mathbf{i} + (2.3 \text{ m})\mathbf{j}$$

$$\begin{aligned} \mathbf{T}_{CG} &= \lambda_{CG} T_{CG} \\ &= \frac{-(.6 \text{ m})\mathbf{i} + (3.3 \text{ m})\mathbf{j} - (3 \text{ m})\mathbf{k}}{\sqrt{(.6)^2 + (3.3)^2 + (3)^2} \text{ m}} (810 \text{ N}) \\ &= -(108 \text{ N})\mathbf{i} + (594 \text{ N})\mathbf{j} - (540 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{M}_A &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 2.3 & 0 \\ -108 & 594 & -540 \end{vmatrix} \text{ N} \cdot \text{m} \\ &= -(1242 \text{ N} \cdot \text{m})\mathbf{i} + (1458 \text{ N} \cdot \text{m})\mathbf{j} + (1852 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

$$\text{or } \mathbf{M}_A = -(1242 \text{ N} \cdot \text{m})\mathbf{i} + (1458 \text{ N} \cdot \text{m})\mathbf{j} + (1852 \text{ N} \cdot \text{m})\mathbf{k} \blacktriangleleft$$

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PROBLEM 3.150

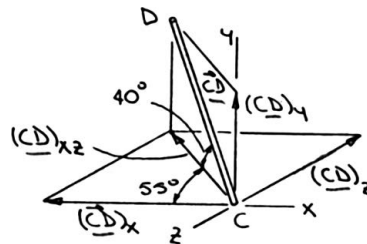
Section AB of a pipeline lies in the yz -plane and forms an angle of 37° with the z -axis. Branch lines CD and EF join AB as shown. Determine the angle formed by pipes AB and CD .

SOLUTION

First note

$$\overline{AB} = AB(\sin 37^\circ \mathbf{j} - \cos 37^\circ \mathbf{k})$$

$$\overline{CD} = CD(-\cos 40^\circ \cos 55^\circ \mathbf{j} + \sin 40^\circ \mathbf{j} - \cos 40^\circ \sin 55^\circ \mathbf{k})$$



Now

$$\overline{AB} \cdot \overline{CD} = (AB)(CD) \cos \theta$$

or

$$AB(\sin 37^\circ \mathbf{j} - \cos 37^\circ \mathbf{k}) \cdot CD(-\cos 40^\circ \cos 55^\circ \mathbf{j} + \sin 40^\circ \mathbf{j} - \cos 40^\circ \sin 55^\circ \mathbf{k})$$

$$= (AB)(CD) \cos \theta$$

or

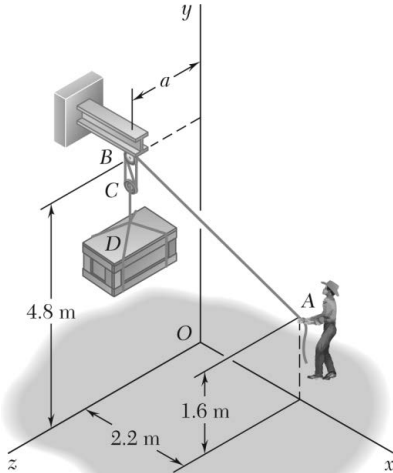
$$\cos \theta = (\sin 37^\circ)(\sin 40^\circ) + (-\cos 37^\circ)(-\cos 40^\circ \sin 55^\circ)$$

$$= 0.88799$$

or $\theta = 27.4^\circ \blacktriangleleft$

PROBLEM 3.151

To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B . Knowing that the moments about the y and the z axes of the force exerted at B by portion AB of the rope are, respectively, $120 \text{ N} \cdot \text{m}$ and $-460 \text{ N} \cdot \text{m}$, determine the distance a .



SOLUTION

First note $\overline{BA} = (2.2 \text{ m})\mathbf{i} - (3.2 \text{ m})\mathbf{j} - (a \text{ m})\mathbf{k}$

Now $\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{T}_{BA}$

where $\mathbf{r}_{A/D} = (2.2 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j}$

$$\mathbf{T}_{BA} = \frac{T_{BA}}{d_{BA}} (2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k}) \text{ (N)}$$

Then
$$\mathbf{M}_D = \frac{T_{BA}}{d_{BA}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$$

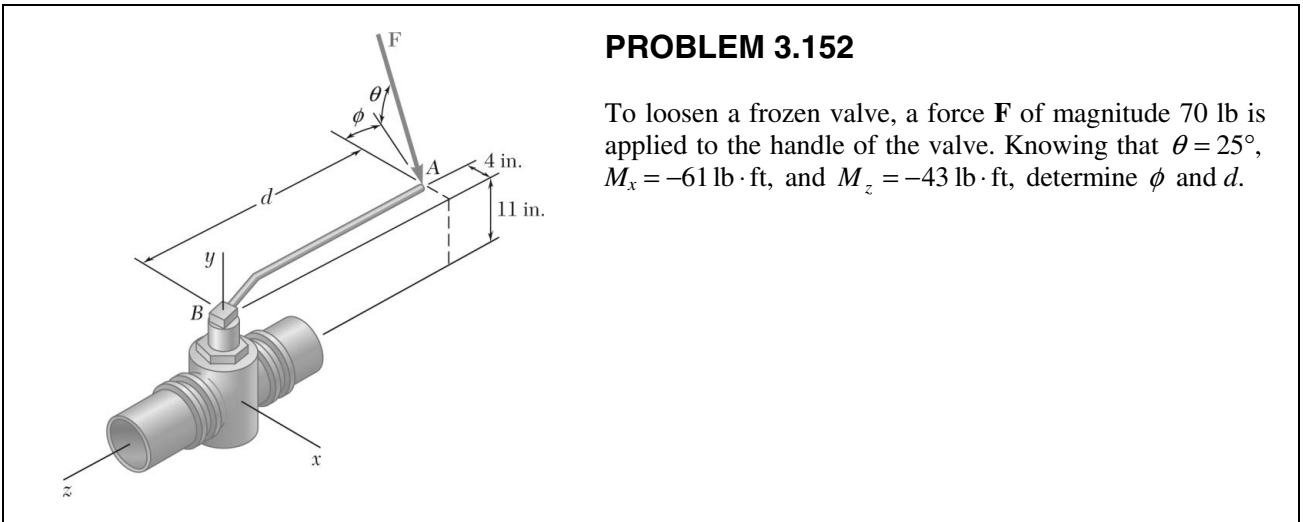
$$= \frac{T_{BA}}{d_{BA}} \{-1.6a\mathbf{i} + 2.2a\mathbf{j} + [(2.2)(-3.2) - (1.6)(2.2)]\mathbf{k}\}$$

Thus $M_y = 2.2 \frac{T_{BA}}{d_{BA}} a \text{ (N} \cdot \text{m)}$

$$M_z = -10.56 \frac{T_{BA}}{d_{BA}} \text{ (N} \cdot \text{m)}$$

Then forming the ratio $\frac{M_y}{M_z}$

$$\frac{120 \text{ N} \cdot \text{m}}{-460 \text{ N} \cdot \text{m}} = \frac{2.2 \frac{T_{BA}}{d_{BA}} \text{ (N} \cdot \text{m)}}{-10.56 \frac{T_{BA}}{d_{BA}} \text{ (N} \cdot \text{m)}} \quad \text{or } a = 1.252 \text{ m} \blacktriangleleft$$



PROBLEM 3.152

To loosen a frozen valve, a force \mathbf{F} of magnitude 70 lb is applied to the handle of the valve. Knowing that $\theta = 25^\circ$, $M_x = -61 \text{ lb}\cdot\text{ft}$, and $M_z = -43 \text{ lb}\cdot\text{ft}$, determine ϕ and d .

SOLUTION

We have $\Sigma \mathbf{M}_O: \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$

where $\mathbf{r}_{A/O} = -(4 \text{ in.})\mathbf{i} + (11 \text{ in.})\mathbf{j} - (d)\mathbf{k}$
 $\mathbf{F} = F(\cos \theta \cos \phi \mathbf{i} - \sin \theta \mathbf{j} + \cos \theta \sin \phi \mathbf{k})$

For $F = 70 \text{ lb}, \theta = 25^\circ$
 $\mathbf{F} = (70 \text{ lb})[(0.90631 \cos \phi)\mathbf{i} - 0.42262\mathbf{j} + (0.90631 \sin \phi)\mathbf{k}]$

$$\mathbf{M}_O = (70 \text{ lb}) \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -d \\ -0.90631 \cos \phi & -0.42262 & 0.90631 \sin \phi \end{vmatrix} \text{ in.}$$

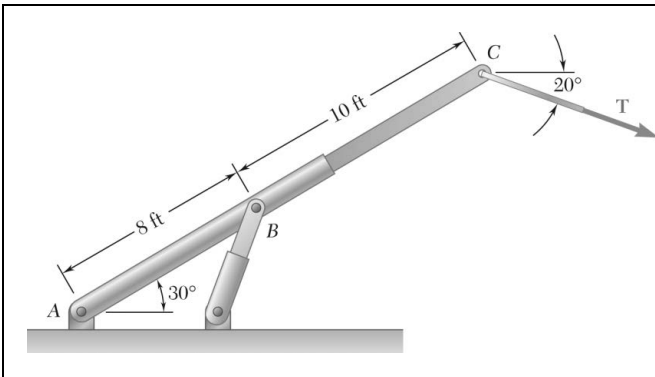
$$= (70 \text{ lb})[(9.9694 \sin \phi - 0.42262d)\mathbf{i} + (-0.90631d \cos \phi + 3.6252 \sin \phi)\mathbf{j} + (1.69048 - 9.9694 \cos \phi)\mathbf{k}] \text{ in.}$$

and $M_x = (70 \text{ lb})(9.9694 \sin \phi - 0.42262d) \text{ in.} = -(61 \text{ lb}\cdot\text{ft})(12 \text{ in./ft}) \tag{1}$
 $M_y = (70 \text{ lb})(-0.90631d \cos \phi + 3.6252 \sin \phi) \text{ in.} \tag{2}$
 $M_z = (70 \text{ lb})(1.69048 - 9.9694 \cos \phi) \text{ in.} = -43 \text{ lb}\cdot\text{ft}(12 \text{ in./ft}) \tag{3}$

From Equation (3): $\phi = \cos^{-1}\left(\frac{634.33}{697.86}\right) = 24.636^\circ$ or $\phi = 24.6^\circ \blacktriangleleft$

From Equation (1): $d = \left(\frac{1022.90}{29.583}\right) = 34.577 \text{ in.}$ or $d = 34.6 \text{ in.} \blacktriangleleft$

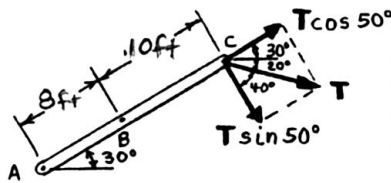
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PROBLEM 3.153

The tension in the cable attached to the end C of an adjustable boom ABC is 560 lb. Replace the force exerted by the cable at C with an equivalent force-couple system (a) at A , (b) at B .

SOLUTION

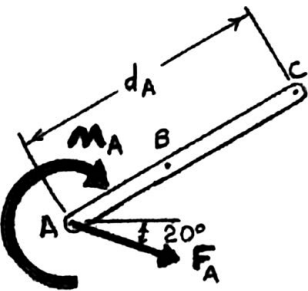


(a) Based on $\Sigma F: F_A = T = 560 \text{ lb}$

or $F_A = 560 \text{ lb} \searrow 20.0^\circ \blacktriangleleft$

$$\begin{aligned} \Sigma M_A: M_A &= (T \sin 50^\circ)(d_A) \\ &= (560 \text{ lb}) \sin 50^\circ (18 \text{ ft}) \\ &= 7721.7 \text{ lb} \cdot \text{ft} \end{aligned}$$

or $M_A = 7720 \text{ lb} \cdot \text{ft} \curvearrowright \blacktriangleleft$

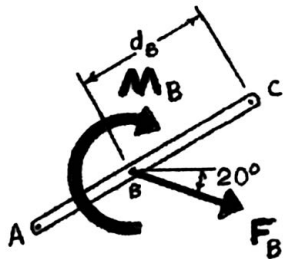


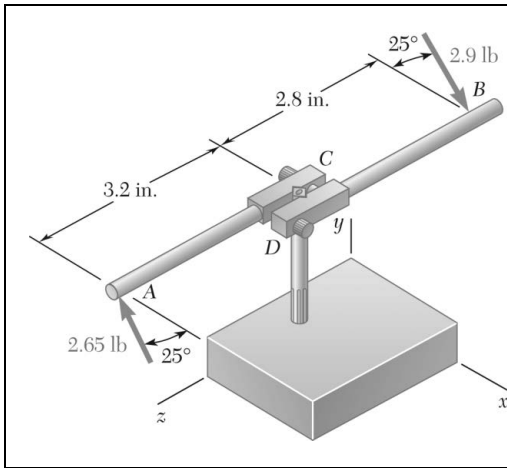
(b) Based on $\Sigma F: F_B = T = 560 \text{ lb}$

or $F_B = 560 \text{ lb} \searrow 20.0^\circ \blacktriangleleft$

$$\begin{aligned} \Sigma M_B: M_B &= (T \sin 50^\circ)(d_B) \\ &= (560 \text{ lb}) \sin 50^\circ (10 \text{ ft}) \\ &= 4289.8 \text{ lb} \cdot \text{ft} \end{aligned}$$

or $M_B = 4290 \text{ lb} \cdot \text{ft} \curvearrowright \blacktriangleleft$





PROBLEM 3.154

While tapping a hole, a machinist applies the horizontal forces shown to the handle of the tap wrench. Show that these forces are equivalent to a single force, and specify, if possible, the point of application of the single force on the handle.

SOLUTION

Since the forces at A and B are parallel, the force at B can be replaced with the sum of two forces with one of the forces equal in magnitude to the force at A except with an opposite sense, resulting in a force-couple.

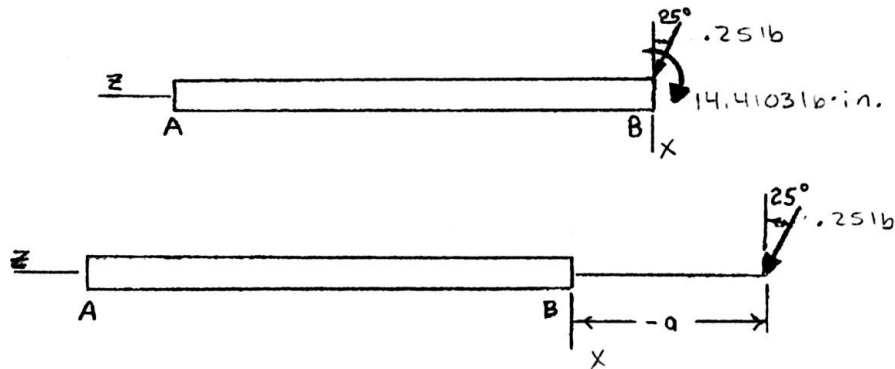
We have $F_B = 2.9 \text{ lb} - 2.65 \text{ lb} = 0.25 \text{ lb}$, where the 2.65-lb force is part of the couple. Combining the two parallel forces,

$$M_{\text{couple}} = (2.65 \text{ lb})[(3.2 \text{ in.} + 2.8 \text{ in.}) \cos 25^\circ]$$

$$= 14.4103 \text{ lb} \cdot \text{in.}$$

and

$$M_{\text{couple}} = 14.4103 \text{ lb} \cdot \text{in.}$$



A single equivalent force will be located in the negative z direction.

Based on $\Sigma M_B: -14.4103 \text{ lb} \cdot \text{in.} = [(0.25 \text{ lb}) \cos 25^\circ](a)$

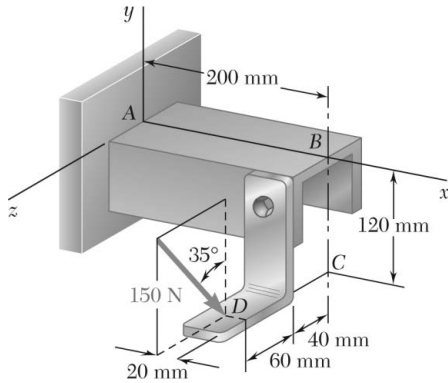
$$a = 63.600 \text{ in.}$$

$$\mathbf{F}' = (0.25 \text{ lb})(\cos 25^\circ \mathbf{i} + \sin 25^\circ \mathbf{k})$$

$\mathbf{F}' = (0.227 \text{ lb})\mathbf{i} + (0.1057 \text{ lb})\mathbf{k}$ and is applied on an extension of handle BD at a distance of 63.6 in. to the right of B .

PROBLEM 3.155

Replace the 150-N force with an equivalent force-couple system at A.



SOLUTION

Equivalence requires $\Sigma \mathbf{F}: \mathbf{F} = (150 \text{ N})(-\cos 35^\circ \mathbf{j} - \sin 35^\circ \mathbf{k})$
 $= -(122.873 \text{ N})\mathbf{j} - (86.036 \text{ N})\mathbf{k}$

$$\Sigma \mathbf{M}_A: \mathbf{M} = \mathbf{r}_{D/A} \times \mathbf{F}$$

where $\mathbf{r}_{D/A} = (0.18 \text{ m})\mathbf{i} - (0.12 \text{ m})\mathbf{j} + (0.1 \text{ m})\mathbf{k}$

Then
$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.18 & -0.12 & 0.1 \\ 0 & -122.873 & -86.036 \end{vmatrix} \text{ N} \cdot \text{m}$$

$$= [(-0.12)(-86.036) - (0.1)(-122.873)]\mathbf{i}$$

$$+ [-(0.18)(-86.036)]\mathbf{j}$$

$$+ [(0.18)(-122.873)]\mathbf{k}$$

$$= (22.6 \text{ N} \cdot \text{m})\mathbf{i} + (15.49 \text{ N} \cdot \text{m})\mathbf{j} - (22.1 \text{ N} \cdot \text{m})\mathbf{k}$$

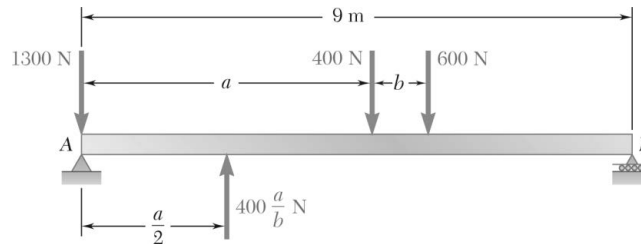
The equivalent force-couple system at A is

$$\mathbf{F} = -(122.9 \text{ N})\mathbf{j} - (86.0 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

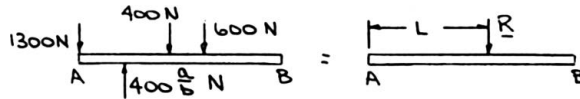
$$\mathbf{M} = (22.6 \text{ N} \cdot \text{m})\mathbf{i} + (15.49 \text{ N} \cdot \text{m})\mathbf{j} - (22.1 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.156

A beam supports three loads of given magnitude and a fourth load whose magnitude is a function of position. If $b = 1.5$ m and the loads are to be replaced with a single equivalent force, determine (a) the value of a so that the distance from support A to the line of action of the equivalent force is maximum, (b) the magnitude of the equivalent force and its point of application on the beam.



SOLUTION



For equivalence,

$$\Sigma F_y: -1300 + 400 \frac{a}{b} - 400 - 600 = -R$$

or

$$R = \left(2300 - 400 \frac{a}{b} \right) \text{ N} \quad (1)$$

$$\Sigma M_A: \frac{a}{2} \left(400 \frac{a}{b} \right) - a(400) - (a+b)(600) = -LR$$

or

$$L = \frac{1000a + 600b - 200 \frac{a^2}{b}}{2300 - 400 \frac{a}{b}}$$

Then with

$$b = 1.5 \text{ m} \quad L = \frac{10a + 9 - \frac{4}{3}a^2}{23 - \frac{8}{3}a} \quad (2)$$

where a, L are in m.

(a) Find value of a to maximize L .

$$\frac{dL}{da} = \frac{\left(10 - \frac{8}{3}a \right) \left(23 - \frac{8}{3}a \right) - \left(10a + 9 - \frac{4}{3}a^2 \right) \left(-\frac{8}{3} \right)}{\left(23 - \frac{8}{3}a \right)^2}$$

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PROBLEM 3.156 (Continued)

or
$$230 - \frac{184}{3}a - \frac{80}{3}a + \frac{64}{9}a^2 + \frac{80}{3}a + 24 - \frac{32}{9}a^2 = 0$$

or
$$16a^2 - 276a + 1143 = 0$$

Then
$$a = \frac{276 \pm \sqrt{(-276)^2 - 4(16)(1143)}}{2(16)}$$

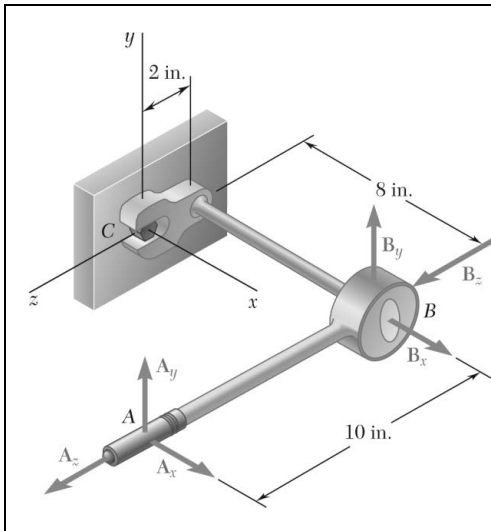
or
$$a = 10.3435 \text{ m} \quad \text{and} \quad a = 6.9065 \text{ m}$$

Since $AB = 9 \text{ m}$, a must be less than 9 m $a = 6.91 \text{ m} \blacktriangleleft$

(b) Using Eq. (1),
$$R = 2300 - 400 \frac{6.9065}{1.5} \quad \text{or} \quad R = 458 \text{ N} \blacktriangleleft$$

and using Eq. (2),
$$L = \frac{10(6.9065) + 9 - \frac{4}{3}(6.9065)^2}{23 - \frac{8}{3}(6.9065)} = 3.16 \text{ m}$$

R is applied 3.16 m to the right of A. \blacktriangleleft



PROBLEM 3.157

A mechanic uses a crowfoot wrench to loosen a bolt at C . The mechanic holds the socket wrench handle at Points A and B and applies forces at these points. Knowing that these forces are equivalent to a force-couple system at C consisting of the force $\mathbf{C} = (8 \text{ lb})\mathbf{i} + (4 \text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_C = (360 \text{ lb} \cdot \text{in.})\mathbf{i}$, determine the forces applied at A and at B when $A_z = 2 \text{ lb}$.

SOLUTION

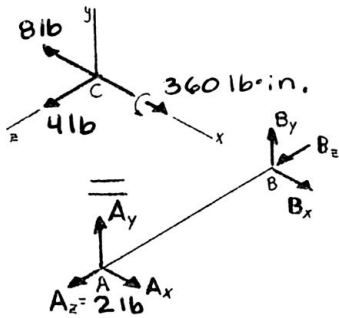
We have

$$\Sigma \mathbf{F}: \quad \mathbf{A} + \mathbf{B} = \mathbf{C}$$

or

$$F_x: \quad A_x + B_x = 8 \text{ lb}$$

$$B_x = -(A_x + 8 \text{ lb}) \quad (1)$$



$$\Sigma F_y: \quad A_y + B_y = 0$$

$$\text{or} \quad A_y = -B_y \quad (2)$$

$$\Sigma F_z: \quad 2 \text{ lb} + B_z = 4 \text{ lb}$$

$$B_z = 2 \text{ lb} \quad (3)$$

or

We have

$$\Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{B} + \mathbf{r}_{A/C} \times \mathbf{A} = \mathbf{M}_C$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 2 \\ B_x & B_y & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 8 \\ A_x & A_y & 2 \end{vmatrix} \text{ lb} \cdot \text{in.} = (360 \text{ lb} \cdot \text{in.})\mathbf{i}$$

or

$$(2B_y - 8A_y)\mathbf{i} + (2B_x - 16 + 8A_x - 16)\mathbf{j}$$

$$+ (8B_y + 8A_y)\mathbf{k} = (360 \text{ lb} \cdot \text{in.})\mathbf{i}$$

From

$$\mathbf{i}\text{-coefficient:} \quad 2B_y - 8A_y = 360 \text{ lb} \cdot \text{in.} \quad (4)$$

$$\mathbf{j}\text{-coefficient:} \quad -2B_x + 8A_x = 32 \text{ lb} \cdot \text{in.} \quad (5)$$

$$\mathbf{k}\text{-coefficient:} \quad 8B_y + 8A_y = 0 \quad (6)$$

PROBLEM 3.157 (Continued)

From Equations (2) and (4): $2B_y - 8(-B_y) = 360$

$$B_y = 36 \text{ lb} \quad A_y = 36 \text{ lb}$$

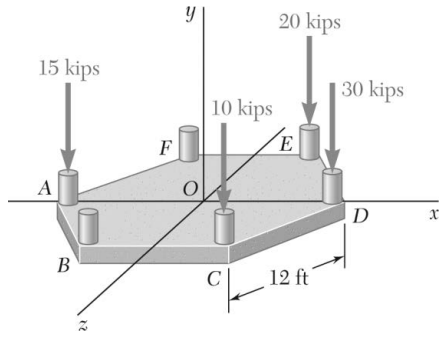
From Equations (1) and (5): $2(-A_x - 8) + 8A_x = 32$

$$A_x = 1.6 \text{ lb}$$

From Equation (1): $B_x = -(1.6 + 8) = -9.6 \text{ lb}$

$$\mathbf{A} = (1.600 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{B} = -(9.60 \text{ lb})\mathbf{i} + (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.158

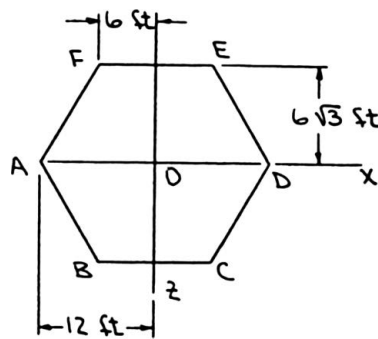
A concrete foundation mat in the shape of a regular hexagon of side 12 ft supports four column loads as shown. Determine the magnitudes of the additional loads that must be applied at B and F if the resultant of all six loads is to pass through the center of the mat.

SOLUTION

From the statement of the problem, it can be concluded that the six applied loads are equivalent to the resultant \mathbf{R} at O . It then follows that

$$\Sigma \mathbf{M}_O = 0 \quad \text{or} \quad \Sigma M_x = 0 \quad \Sigma M_z = 0$$

For the applied loads:



Then $\Sigma M_x = 0$: $(6\sqrt{3} \text{ ft})F_B + (6\sqrt{3} \text{ ft})(10 \text{ kips}) - (6\sqrt{3} \text{ ft})(20 \text{ kips}) - (6\sqrt{3} \text{ ft})F_F = 0$

or $F_B - F_F = 10$ (1)

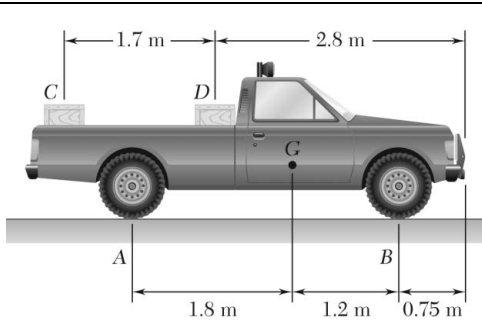
$\Sigma M_z = 0$: $(12 \text{ ft})(15 \text{ kips}) + (6 \text{ ft})F_B - (6 \text{ ft})(10 \text{ kips}) - (12 \text{ ft})(30 \text{ kips}) - (6 \text{ ft})(20 \text{ kips}) + (6 \text{ ft})F_F = 0$

or $F_B + F_F = 60$ (2)

Then Eqs. (1) + (2) \Rightarrow $F_B = 35.0 \text{ kips} \downarrow \blacktriangleleft$

and $F_F = 25.0 \text{ kips} \downarrow \blacktriangleleft$

CHAPTER 4

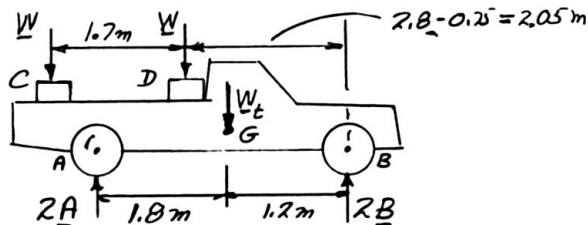


PROBLEM 4.1

Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (a) rear wheels A, (b) front wheels B.

SOLUTION

Free-Body Diagram:



$$W = (350 \text{ kg})(9.81 \text{ m/s}^2) = 3.4335 \text{ kN}$$

$$W_t = (1400 \text{ kg})(9.81 \text{ m/s}^2) = 13.7340 \text{ kN}$$

(a) Rear wheels: $\rightarrow \Sigma M_B = 0: W(1.7 \text{ m} + 2.05 \text{ m}) + W(2.05 \text{ m}) + W_t(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$

$$(3.4335 \text{ kN})(3.75 \text{ m}) + (3.4335 \text{ kN})(2.05 \text{ m}) + (13.7340 \text{ kN})(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$A = +6.0659 \text{ kN}$$

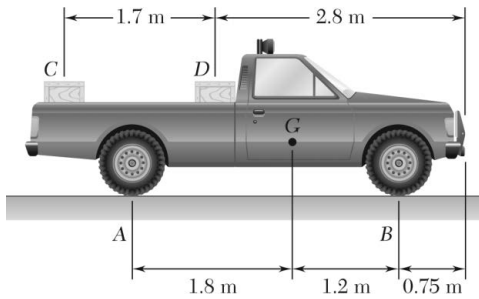
$$A = 6.07 \text{ kN} \uparrow \blacktriangleleft$$

(b) Front wheels: $\uparrow \Sigma F_y = 0: -W - W - W_t + 2A + 2B = 0$

$$-3.4335 \text{ kN} - 3.4335 \text{ kN} - 13.7340 \text{ kN} + 2(6.0659 \text{ kN}) + 2B = 0$$

$$B = +4.2346 \text{ kN}$$

$$B = 4.23 \text{ kN} \uparrow \blacktriangleleft$$



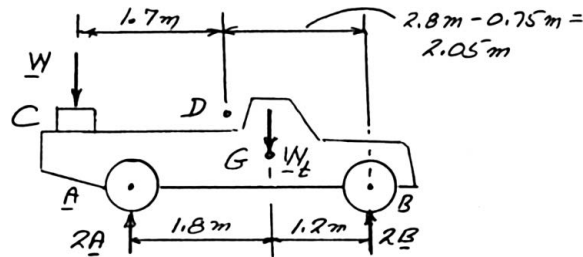
PROBLEM 4.2

Solve Problem 4.1, assuming that crate D is removed and that the position of crate C is unchanged.

PROBLEM 4.1 Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (a) rear wheels A , (b) front wheels B .

SOLUTION

Free-Body Diagram:



$$W = (350 \text{ kg})(9.81 \text{ m/s}^2) = 3.4335 \text{ kN}$$

$$W_i = (1400 \text{ kg})(9.81 \text{ m/s}^2) = 13.7340 \text{ kN}$$

(a) Rear wheels: $+\circlearrowleft \Sigma M_B = 0: W(1.7 \text{ m} + 2.05 \text{ m}) + W_i(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$

$$(3.4335 \text{ kN})(3.75 \text{ m}) + (13.7340 \text{ kN})(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$$

$$A = +4.8927 \text{ kN}$$

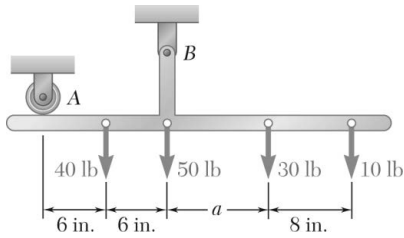
$$\mathbf{A} = 4.89 \text{ kN} \uparrow \blacktriangleleft$$

(b) Front wheels: $+\uparrow \Sigma M_y = 0: -W - W_i + 2A + 2B = 0$

$$-3.4335 \text{ kN} - 13.7340 \text{ kN} + 2(4.8927 \text{ kN}) + 2B = 0$$

$$B = +3.6911 \text{ kN}$$

$$\mathbf{B} = 3.69 \text{ kN} \uparrow \blacktriangleleft$$

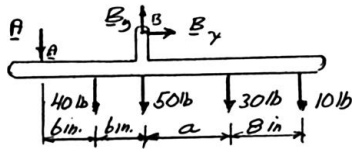


PROBLEM 4.3

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



$$\pm \rightarrow \Sigma F_x = 0: B_x = 0$$

$$+\curvearrowright \Sigma M_B = 0: (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12} \quad (1)$$

$$+\curvearrowright \Sigma M_A = 0: -(40 \text{ lb})(6 \text{ in.}) - (50 \text{ lb})(12 \text{ in.}) - (30 \text{ lb})(a + 12 \text{ in.}) - (10 \text{ lb})(a + 20 \text{ in.}) + (12 \text{ in.})B_y = 0$$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since

$$B_x = 0, \quad B = \frac{(1400 + 40a)}{12} \quad (2)$$

(a) For $a = 10$ in.,

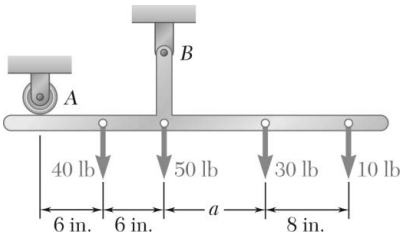
$$\text{Eq. (1):} \quad A = \frac{(40 \times 10 - 160)}{12} = +20.0 \text{ lb} \quad \mathbf{A} = 20.0 \text{ lb} \downarrow \blacktriangleleft$$

$$\text{Eq. (2):} \quad B = \frac{(1400 + 40 \times 10)}{12} = +150.0 \text{ lb} \quad \mathbf{B} = 150.0 \text{ lb} \uparrow \blacktriangleleft$$

(b) For $a = 7$ in.,

$$\text{Eq. (1):} \quad A = \frac{(40 \times 7 - 160)}{12} = +10.00 \text{ lb} \quad \mathbf{A} = 10.00 \text{ lb} \downarrow \blacktriangleleft$$

$$\text{Eq. (2):} \quad B = \frac{(1400 + 40 \times 7)}{12} = +140.0 \text{ lb} \quad \mathbf{B} = 140.0 \text{ lb} \uparrow \blacktriangleleft$$



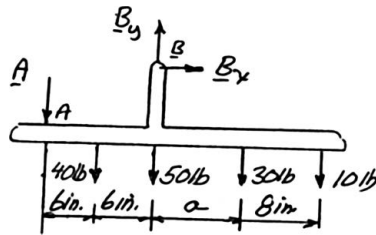
PROBLEM 4.4

For the bracket and loading of Problem 4.3, determine the smallest distance a if the bracket is not to move.

PROBLEM 4.3 A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



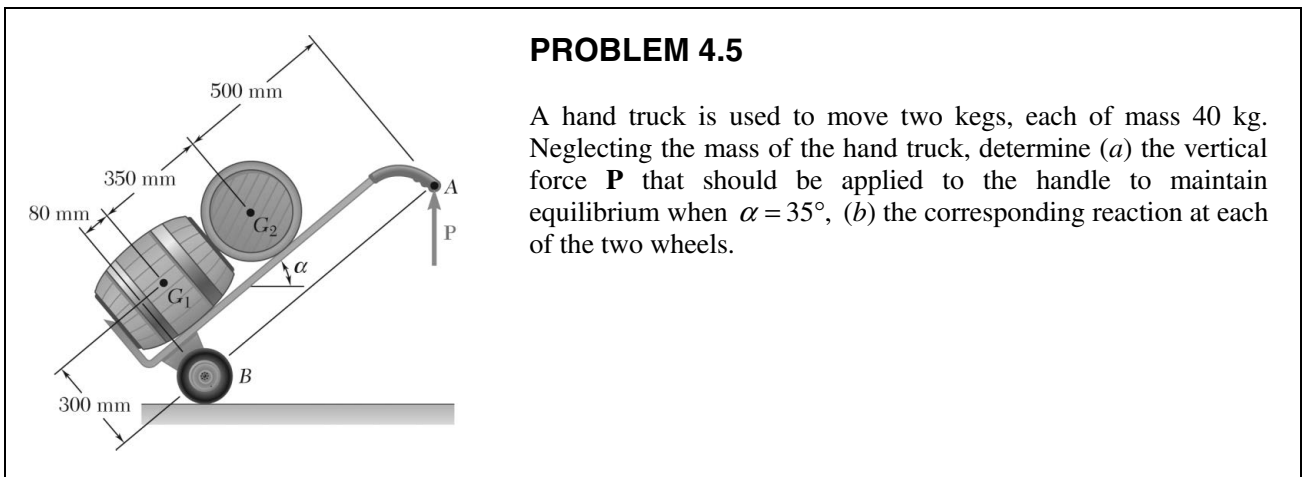
For no motion, reaction at A must be downward or zero; smallest distance a for no motion corresponds to $A = 0$.

$$+\circlearrowleft \Sigma M_B = 0: (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12}$$

$$A = 0: (40a - 160) = 0$$

$$a = 4.00 \text{ in.} \blacktriangleleft$$



PROBLEM 4.5

A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (a) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35^\circ$, (b) the corresponding reaction at each of the two wheels.

SOLUTION

$$W = mg = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.40 \text{ N}$$

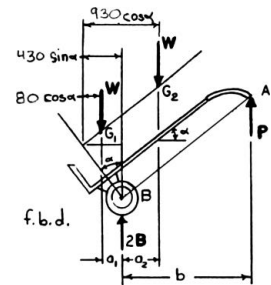
$$a_1 = (300 \text{ mm})\sin\alpha - (80 \text{ mm})\cos\alpha$$

$$a_2 = (430 \text{ mm})\cos\alpha - (300 \text{ mm})\sin\alpha$$

$$b = (930 \text{ mm})\cos\alpha$$

From free-body diagram of hand truck,

Free-Body Diagram:



Dimensions in mm

$$+\curvearrowright \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2W + 2B = 0 \quad (2)$$

For

$$\alpha = 35^\circ$$

$$a_1 = 300 \sin 35^\circ - 80 \cos 35^\circ = 106.541 \text{ mm}$$

$$a_2 = 430 \cos 35^\circ - 300 \sin 35^\circ = 180.162 \text{ mm}$$

$$b = 930 \cos 35^\circ = 761.81 \text{ mm}$$

(a) From Equation (1):

$$P(761.81 \text{ mm}) - 392.40 \text{ N}(180.162 \text{ mm}) + 392.40 \text{ N}(106.54 \text{ mm}) = 0$$

$$P = 37.921 \text{ N}$$

$$\text{or } \mathbf{P} = 37.9 \text{ N } \uparrow \blacktriangleleft$$

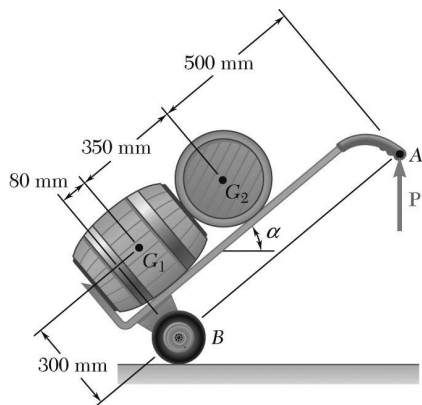
(b) From Equation (2):

$$37.921 \text{ N} - 2(392.40 \text{ N}) + 2B = 0$$

$$\text{or } \mathbf{B} = 373 \text{ N } \uparrow \blacktriangleleft$$

PROBLEM 4.6

Solve Problem 4.5 when $\alpha = 40^\circ$.



PROBLEM 4.5 A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (a) the vertical force \mathbf{P} that should be applied to the handle to maintain equilibrium when $\alpha = 35^\circ$, (b) the corresponding reaction at each of the two wheels.

SOLUTION

$$W = mg = (40 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 392.40 \text{ N}$$

$$a_1 = (300 \text{ mm})\sin\alpha - (80 \text{ mm})\cos\alpha$$

$$a_2 = (430 \text{ mm})\cos\alpha - (300 \text{ mm})\sin\alpha$$

$$b = (930 \text{ mm})\cos\alpha$$

From F.B.D.:

$$+\circlearrowleft \Sigma M_B = 0: P(b) - W(a_2) + W(a_1) = 0$$

$$P = W(a_2 - a_1)/b$$

$$+\uparrow \Sigma F_y = 0: -W - W + P + 2B = 0$$

$$B = W - \frac{1}{2}P$$

For $\alpha = 40^\circ$:

$$a_1 = 300 \sin 40^\circ - 80 \cos 40^\circ = 131.553 \text{ mm}$$

$$a_2 = 430 \cos 40^\circ - 300 \sin 40^\circ = 136.563 \text{ mm}$$

$$b = 930 \cos 40^\circ = 712.42 \text{ mm}$$

(a) From Equation (1):
$$P = \frac{392.40 \text{ N} (0.136563 \text{ m} - 0.131553 \text{ m})}{0.71242 \text{ m}}$$

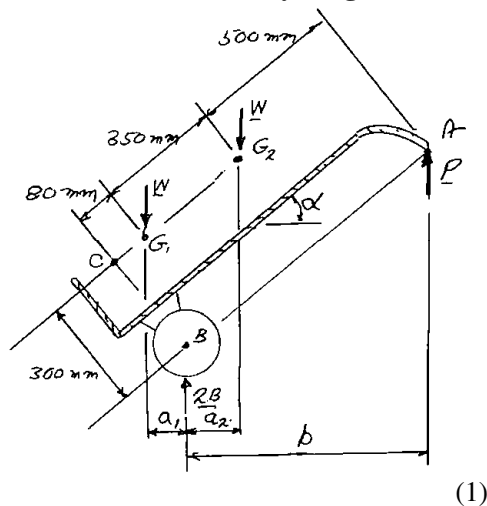
$$P = 2.7595 \text{ N}$$

$$\mathbf{P} = 2.76 \text{ N} \uparrow \leftarrow$$

(b) From Equation (2):
$$B = 392.40 \text{ N} - \frac{1}{2} (2.7595 \text{ N})$$

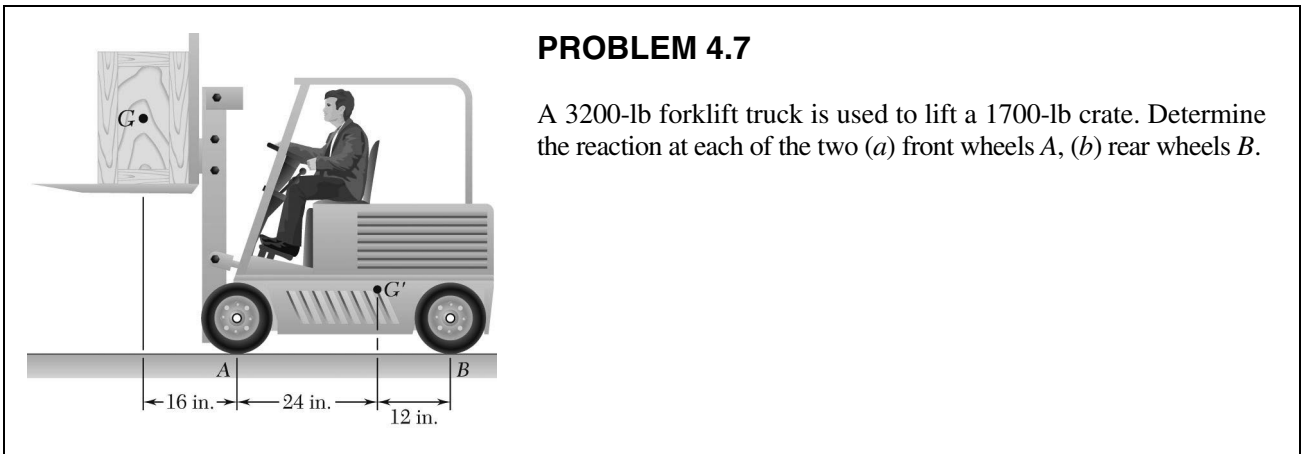
$$\mathbf{B} = 391 \text{ N} \uparrow \leftarrow$$

Free-Body Diagram:



(1)

(2)

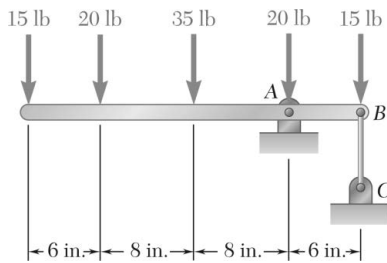


SOLUTION

Free-Body Diagram:

(a) Front wheels:
$$+\curvearrowright \Sigma M_B = 0: (1700 \text{ lb})(52 \text{ in.}) + (3200 \text{ lb})(12 \text{ in.}) - 2A(36 \text{ in.}) = 0$$
$$A = +1761.11 \text{ lb} \qquad \mathbf{A = 1761 \text{ lb} \uparrow \blacktriangleleft}$$

(b) Rear wheels:
$$+\uparrow \Sigma F_y = 0: -1700 \text{ lb} - 3200 \text{ lb} + 2(1761.11 \text{ lb}) + 2B = 0$$
$$B = +688.89 \text{ lb} \qquad \mathbf{B = 689 \text{ lb} \uparrow \blacktriangleleft}$$

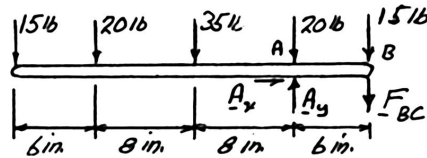


PROBLEM 4.8

For the beam and loading shown, determine (a) the reaction at A, (b) the tension in cable BC.

SOLUTION

Free-Body Diagram:



(a) Reaction at A: $\Sigma F_x = 0: A_x = 0$

$\curvearrowright \Sigma M_B = 0: (15 \text{ lb})(28 \text{ in.}) + (20 \text{ lb})(22 \text{ in.}) + (35 \text{ lb})(14 \text{ in.})$
 $+ (20 \text{ lb})(6 \text{ in.}) - A_y(6 \text{ in.}) = 0$

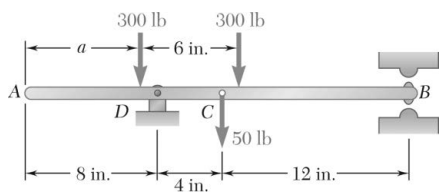
$$A_y = +245 \text{ lb} \quad \mathbf{A = 245 \text{ lb} \uparrow \blacktriangleleft}$$

(b) Tension in BC: $\curvearrowright \Sigma M_A = 0: (15 \text{ lb})(22 \text{ in.}) + (20 \text{ lb})(16 \text{ in.}) + (35 \text{ lb})(8 \text{ in.})$
 $- (15 \text{ lb})(6 \text{ in.}) - F_{BC}(6 \text{ in.}) = 0$

$$F_{BC} = +140.0 \text{ lb} \quad \mathbf{F_{BC} = 140.0 \text{ lb} \blacktriangleleft}$$

Check: $+\uparrow \Sigma F_y = 0: -15 \text{ lb} - 20 \text{ lb} + 35 \text{ lb} - 20 \text{ lb} + A - F_{BC} = 0$
 $-105 \text{ lb} + 245 \text{ lb} - 140.0 = 0$

$$0 = 0 \quad (\text{Checks})$$

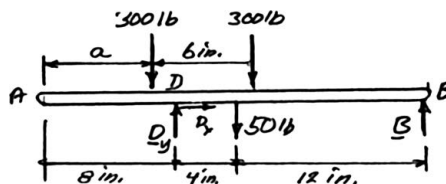


PROBLEM 4.9

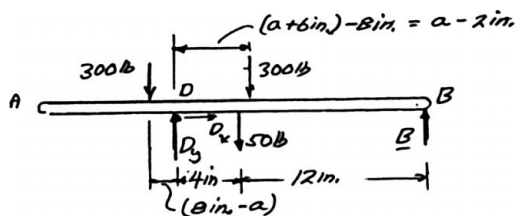
For the beam and loading shown, determine the range of the distance a for which the reaction at B does not exceed 100 lb downward or 200 lb upward.

SOLUTION

Assume B is positive when directed \uparrow .



Sketch showing distance from D to forces.



$$\sum M_D = 0: (300 \text{ lb})(8 \text{ in.} - a) - (300 \text{ lb})(a - 2 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) + 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = \frac{(2800 + 16B)}{600} \quad (1)$$

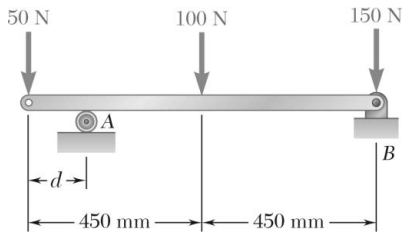
For $B = 100 \text{ lb} \downarrow = -100 \text{ lb}$, Eq. (1) yields:

$$a \geq \frac{[2800 + 16(-100)]}{600} = \frac{1200}{600} = 2 \text{ in.} \quad a \geq 2.00 \text{ in.} \quad \triangleleft$$

For $B = 200 \uparrow = +200 \text{ lb}$, Eq. (1) yields:

$$a \leq \frac{[2800 + 16(200)]}{600} = \frac{6000}{600} = 10 \text{ in.} \quad a \leq 10.00 \text{ in.} \quad \triangleleft$$

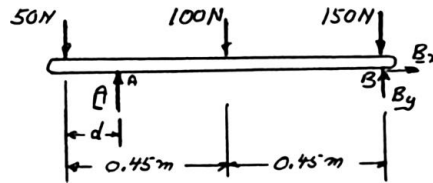
Required range: $2.00 \text{ in.} \leq a \leq 10.00 \text{ in.}$ \blacktriangleleft



PROBLEM 4.10

The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: (50 \text{ N})d - (100 \text{ N})(0.45 \text{ m} - d) - (150 \text{ N})(0.9 \text{ m} - d) + B(0.9 \text{ m} - d) = 0$$

$$50d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180 \text{ N} \cdot \text{m} - (0.9 \text{ m})B}{300A - B} \quad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: (50 \text{ N})(0.9 \text{ m}) - A(0.9 \text{ m} - d) + (100 \text{ N})(0.45 \text{ m}) = 0$$

$$45 - 0.9A + Ad + 45 = 0$$

$$d = \frac{(0.9 \text{ m})A - 90 \text{ N} \cdot \text{m}}{A} \quad (2)$$

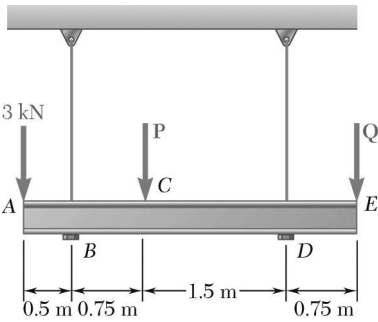
Since $B \leq 180 \text{ N}$, Eq. (1) yields

$$d \geq \frac{180 - (0.9)180}{300 - 180} = \frac{18}{120} = 0.15 \text{ m} \quad d \geq 150.0 \text{ mm} \triangleleft$$

Since $A \leq 180 \text{ N}$, Eq. (2) yields

$$d \leq \frac{(0.9)180 - 90}{180} = \frac{72}{180} = 0.40 \text{ m} \quad d \leq 400 \text{ mm} \triangleleft$$

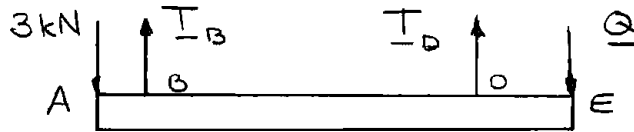
Range: $150.0 \text{ mm} \leq d \leq 400 \text{ mm} \blacktriangleleft$



PROBLEM 4.11

Three loads are applied as shown to a light beam supported by cables attached at B and D . Neglecting the weight of the beam, determine the range of values of Q for which neither cable becomes slack when $P = 0$.

SOLUTION



$$+\curvearrowright \Sigma M_B = 0: (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750) T_D \quad (1)$$

$$+\curvearrowright \Sigma M_D = 0: (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00) T_B \quad (2)$$

For cable B not to be slack, $T_B \geq 0$, and from Eq. (2),

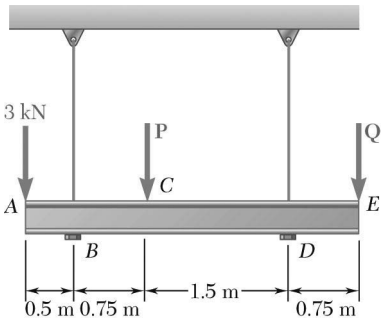
$$Q \leq 11.00 \text{ kN}$$

For cable D not to be slack, $T_D \geq 0$, and from Eq. (1),

$$Q \geq 0.500 \text{ kN}$$

For neither cable to be slack,

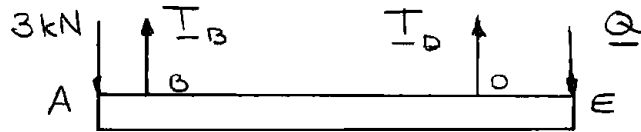
$$0.500 \text{ kN} \leq Q \leq 11.00 \text{ kN} \quad \blacktriangleleft$$



PROBLEM 4.12

Three loads are applied as shown to a light beam supported by cables attached at B and D . Knowing that the maximum allowable tension in each cable is 4 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750) T_D \quad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00) T_B \quad (2)$$

For $T_B \leq 4.00 \text{ kN}$, Eq. (2) yields

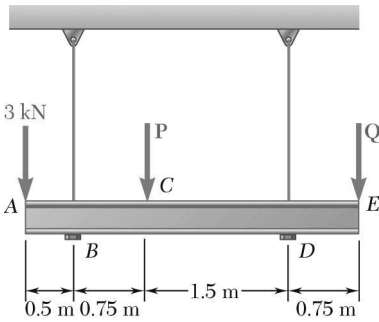
$$Q \geq 11.00 \text{ kN} - 3.00(4.00 \text{ kN}) \quad Q \geq -1.000 \text{ kN}$$

For $T_D \leq 4.00 \text{ kN}$, Eq. (1) yields

$$Q \leq 0.500 \text{ kN} + 0.750(4.00 \text{ kN}) \quad Q \leq 3.50 \text{ kN}$$

For loading to be safe, cables must also not be slack. Combining with the conditions obtained in Problem 4.11,

$$0.500 \text{ kN} \leq Q \leq 3.50 \text{ kN} \quad \blacktriangleleft$$

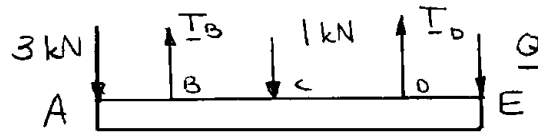


PROBLEM 4.13

For the beam of Problem 4.12, determine the range of values of Q for which the loading is safe when $P = 1$ kN.

PROBLEM 4.12 Three loads are applied as shown to a light beam supported by cables attached at B and D . Knowing that the maximum allowable tension in each cable is 4 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when $P = 0$.

SOLUTION



$$+\curvearrowright \Sigma M_B = 0: (3.00 \text{ kN})(0.500 \text{ m}) - (1.000 \text{ kN})(0.750 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.250 \text{ kN} + 0.75 T_D \quad (1)$$

$$+\curvearrowright \Sigma M_D = 0: (3.00 \text{ kN})(2.75 \text{ m}) + (1.000 \text{ kN})(1.50 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 13.00 \text{ kN} - 3.00 T_B \quad (2)$$

For the loading to be safe, cables must not be slack and tension must not exceed 4.00 kN.

Making $0 \leq T_B \leq 4.00 \text{ kN}$ in Eq. (2), we have

$$13.00 \text{ kN} - 3.00(4.00 \text{ kN}) \leq Q \leq 13.00 \text{ kN} - 3.00(0)$$

$$1.000 \text{ kN} \leq Q \leq 13.00 \text{ kN} \quad (3)$$

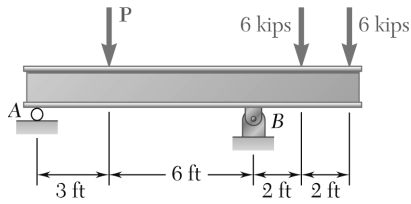
Making $0 \leq T_D \leq 4.00 \text{ kN}$ in Eq. (1), we have

$$0.250 \text{ kN} + 0.750(0) \leq Q \leq 0.250 \text{ kN} + 0.750(4.00 \text{ kN})$$

$$0.250 \text{ kN} \leq Q \leq 3.25 \text{ kN} \quad (4)$$

Combining Eqs. (3) and (4),

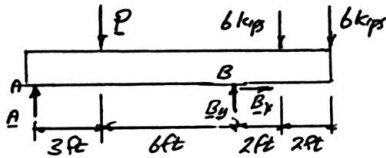
$$1.000 \text{ kN} \leq Q \leq 3.25 \text{ kN} \quad \blacktriangleleft$$



PROBLEM 4.14

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe, knowing that the maximum allowable value of each of the reactions is 30 kips and that the reaction at A must be directed upward.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\mathbf{B} = B_y \uparrow$$

$$+\curvearrowright \Sigma M_A = 0: \quad -P(3 \text{ ft}) + B(9 \text{ ft}) - (6 \text{ kips})(11 \text{ ft}) - (6 \text{ kips})(13 \text{ ft}) = 0$$

$$P = 3B - 48 \text{ kips} \quad (1)$$

$$+\curvearrowright \Sigma M_B = 0: \quad -A(9 \text{ ft}) + P(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (6 \text{ kips})(4 \text{ ft}) = 0$$

$$P = 1.5A + 6 \text{ kips} \quad (2)$$

Since $B \leq 30$ kips, Eq. (1) yields

$$P \leq (3)(30 \text{ kips}) - 48 \text{ kips} \quad P \leq 42.0 \text{ kips} \quad \triangleleft$$

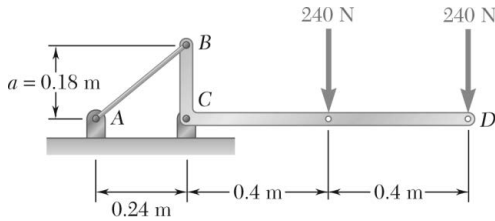
Since $0 \leq A \leq 30$ kips, Eq. (2) yields

$$0 + 6 \text{ kips} \leq P \leq (1.5)(30 \text{ kips}) + 6 \text{ kips}$$

$$6.00 \text{ kips} \leq P \leq 51.0 \text{ kips} \quad \triangleleft$$

Range of values of P for which beam will be safe:

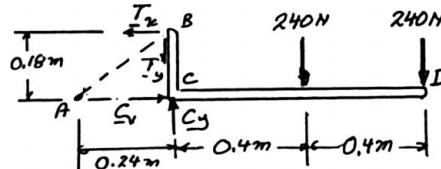
$$6.00 \text{ kips} \leq P \leq 42.0 \text{ kips} \quad \blacktriangleleft$$



PROBLEM 4.15

The bracket BCD is hinged at C and attached to a control cable at B . For the loading shown, determine (a) the tension in the cable, (b) the reaction at C .

SOLUTION



At B :

$$\frac{T_y}{T_x} = \frac{0.18 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{3}{4} T_x \quad (1)$$

(a) $\sum M_C = 0: T_x(0.18 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$

$$T_x = +1600 \text{ N}$$

From Eq. (1):

$$T_y = \frac{3}{4}(1600 \text{ N}) = 1200 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000 \text{ N} \quad T = 2.00 \text{ kN} \quad \blacktriangleleft$$

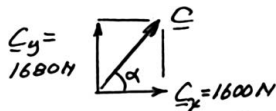
(b) $\sum F_x = 0: C_x - T_x = 0$

$$C_x - 1600 \text{ N} = 0 \quad C_x = +1600 \text{ N}$$

$$C_x = 1600 \text{ N} \rightarrow$$

$$\sum F_y = 0: C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$



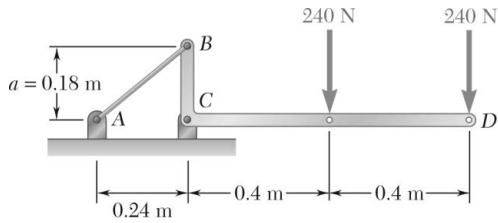
$$C_y = +1680 \text{ N}$$

$$C_y = 1680 \text{ N} \uparrow$$

$$\alpha = 46.4^\circ$$

$$C = 2320 \text{ N}$$

$$C = 2.32 \text{ kN} \nearrow 46.4^\circ \quad \blacktriangleleft$$

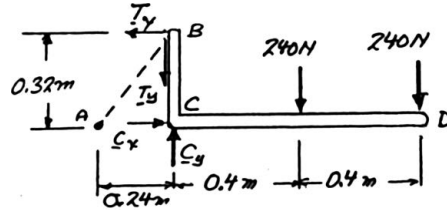


PROBLEM 4.16

Solve Problem 4.15, assuming that $a = 0.32$ m.

PROBLEM 4.15 The bracket BCD is hinged at C and attached to a control cable at B . For the loading shown, determine (a) the tension in the cable, (b) the reaction at C .

SOLUTION



At B :

$$\frac{T_y}{T_x} = \frac{0.32 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{4}{3} T_x$$

$$+\curvearrowright \Sigma M_C = 0: T_x(0.32 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = 900 \text{ N}$$

From Eq. (1):

$$T_y = \frac{4}{3}(900 \text{ N}) = 1200 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500 \text{ N} \quad T = 1.500 \text{ kN} \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: C_x - T_x = 0$$

$$C_x - 900 \text{ N} = 0 \quad C_x = +900 \text{ N} \quad C_x = 900 \text{ N} \rightarrow$$

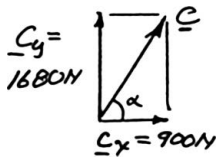
$$+\uparrow \Sigma F_y = 0: C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

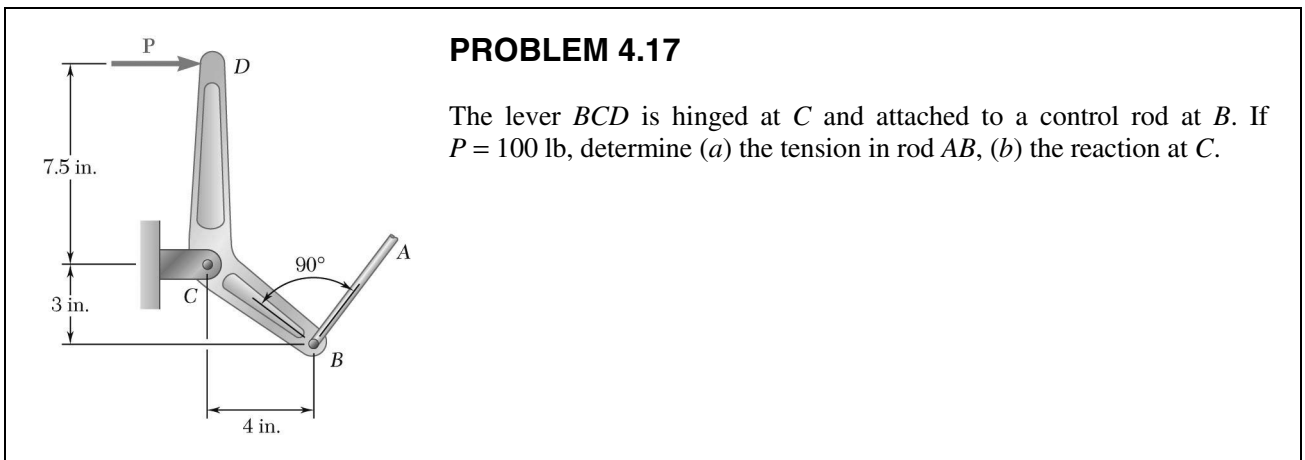
$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$

$$C_y = +1680 \text{ N} \quad C_y = 1680 \text{ N} \uparrow$$

$$\alpha = 61.8^\circ$$

$$C = 1906 \text{ N} \quad C = 1.906 \text{ kN} \blacktriangleleft 61.8^\circ$$





PROBLEM 4.17

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. If $P = 100$ lb, determine (a) the tension in rod *AB*, (b) the reaction at *C*.

SOLUTION

Free-Body Diagram:

$BC = 5 \text{ in.}$

(a) $\sum M_C = 0: T(5 \text{ in.}) - (100 \text{ lb})(7.5 \text{ in.}) = 0$

$T = 150.0 \text{ lb} \blacktriangleleft$

(b) $\sum F_x = 0: C_x + 100 \text{ lb} + \frac{3}{5}(150.0 \text{ lb}) = 0$

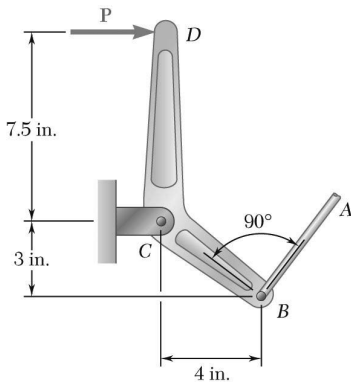
$C_x = -190 \text{ lb} \quad C_x = 190 \text{ lb} \blacktriangleleft$

$\sum F_y = 0: C_y + \frac{4}{5}(150.0 \text{ lb}) = 0$

$C_y = -120 \text{ lb} \quad C_y = 120 \text{ lb} \downarrow$

$\alpha = 32.3^\circ \quad C = 225 \text{ lb} \blacktriangleright 32.3^\circ \blacktriangleleft$

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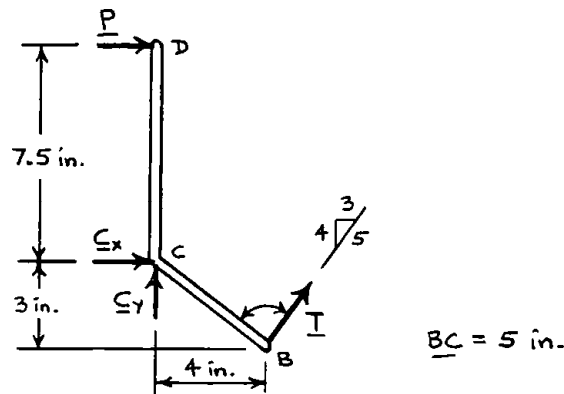


PROBLEM 4.18

The lever BCD is hinged at C and attached to a control rod at B . Determine the maximum force \mathbf{P} that can be safely applied at D if the maximum allowable value of the reaction at C is 250 lb.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: T(5 \text{ in.}) - P(7.5 \text{ in.}) = 0$$

$$T = 1.5P$$

$$\pm \rightarrow \Sigma F_x = 0: P + C_x + \frac{3}{5}(1.5P) = 0$$

$$C_x = -1.9P \quad C_x = 1.9P \leftarrow$$

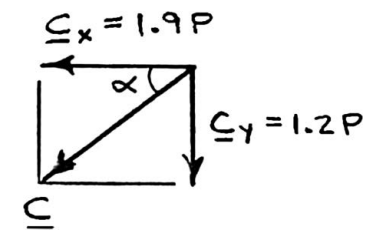
$$+\uparrow \Sigma F_y = 0: C_y + \frac{4}{5}(1.5P) = 0$$

$$C_y = -1.2P \quad C_y = 1.2P \downarrow$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{(1.9P)^2 + (1.2P)^2}$$

$$C = 2.2472P$$



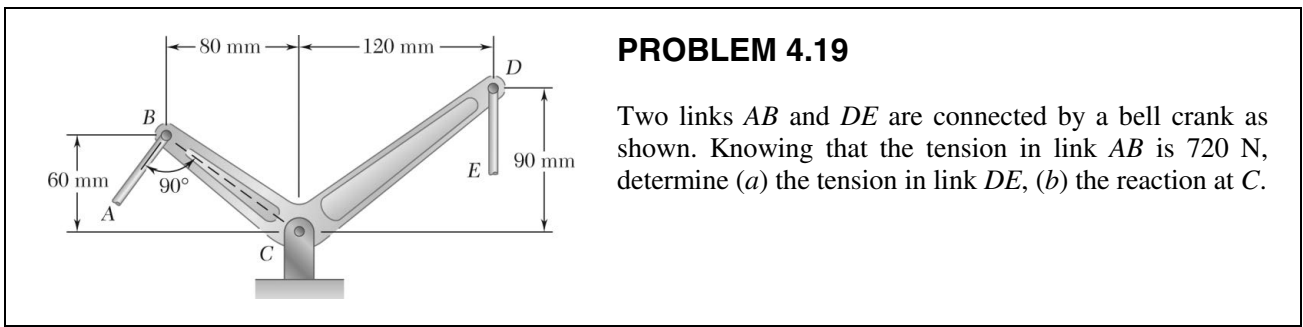
For $C = 250 \text{ lb}$,

$$250 \text{ lb} = 2.2472P$$

$$P = 111.2 \text{ lb}$$

$$\mathbf{P} = 111.2 \text{ lb} \rightarrow \blacktriangleleft$$

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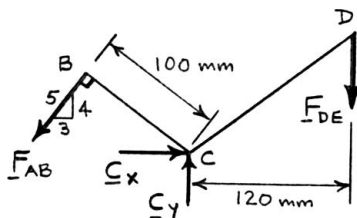


PROBLEM 4.19

Two links *AB* and *DE* are connected by a bell crank as shown. Knowing that the tension in link *AB* is 720 N, determine (a) the tension in link *DE*, (b) the reaction at *C*.

SOLUTION

Free-Body Diagram:



$$+\curvearrowright \Sigma M_C = 0: F_{AB}(100 \text{ mm}) - F_{DE}(120 \text{ mm}) = 0$$

$$F_{DE} = \frac{5}{6} F_{AB} \tag{1}$$

(a) For

$$F_{AB} = 720 \text{ N}$$

$$F_{DE} = \frac{5}{6}(720 \text{ N}) \qquad F_{DE} = 600 \text{ N} \blacktriangleleft$$

(b)

$$+\rightarrow \Sigma F_x = 0: -\frac{3}{5}(720 \text{ N}) + C_x = 0$$

$$C_x = +432 \text{ N}$$

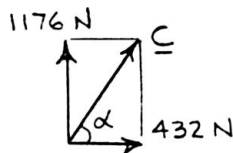
$$+\uparrow \Sigma F_y = 0: -\frac{4}{5}(720 \text{ N}) + C_y - 600 \text{ N} = 0$$

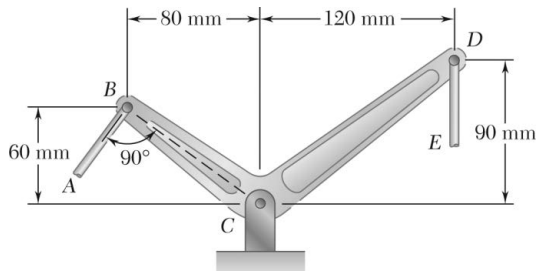
$$C_y = +1176 \text{ N}$$

$$C = 1252.84 \text{ N}$$

$$\alpha = 69.829^\circ$$

$$C = 1253 \text{ N} \nearrow 69.8^\circ \blacktriangleleft$$





PROBLEM 4.20

Two links AB and DE are connected by a bell crank as shown. Determine the maximum force that may be safely exerted by link AB on the bell crank if the maximum allowable value for the reaction at C is 1600 N.

SOLUTION

See solution to Problem 4.15 for F.B.D. and derivation of Eq. (1).

$$F_{DE} = \frac{5}{6} F_{AB} \quad (1)$$

$$\pm \rightarrow \Sigma F_x = 0: \quad -\frac{3}{5} F_{AB} + C_x = 0 \quad C_x = \frac{3}{5} F_{AB}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5} F_{AB} + C_y - F_{DE} = 0$$

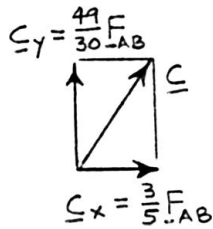
$$-\frac{4}{5} F_{AB} + C_y - \frac{5}{6} F_{AB} = 0$$

$$C_y = \frac{49}{30} F_{AB}$$

$$C = \sqrt{C_x^2 + C_y^2}$$

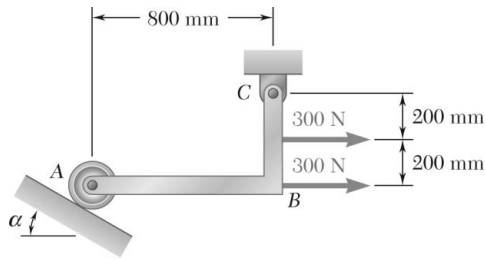
$$= \frac{1}{30} \sqrt{(49)^2 + (18)^2} F_{AB}$$

$$C = 1.74005 F_{AB}$$



For $C = 1600 \text{ N}$, $1600 \text{ N} = 1.74005 F_{AB}$

$F_{AB} = 920 \text{ N} \blacktriangleleft$



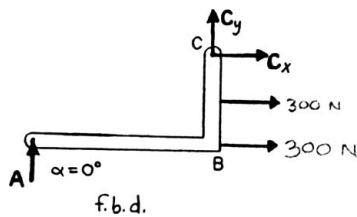
PROBLEM 4.21

Determine the reactions at A and C when (a) $\alpha = 0$, (b) $\alpha = 30^\circ$.

SOLUTION

(a) $\alpha = 0$

From F.B.D. of member ABC:



$$+\curvearrowright \Sigma M_C = 0: (300 \text{ N})(0.2 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) - A(0.8 \text{ m}) = 0$$

$$A = 225 \text{ N} \quad \text{or} \quad \mathbf{A = 225 \text{ N} \uparrow \blacktriangleleft}$$

$$+\uparrow \Sigma F_y = 0: C_y + 225 \text{ N} = 0$$

$$C_y = -225 \text{ N} \quad \text{or} \quad \mathbf{C_y = 225 \text{ N} \downarrow}$$

$$+\rightarrow \Sigma F_x = 0: 300 \text{ N} + 300 \text{ N} + C_x = 0$$

$$C_x = -600 \text{ N} \quad \text{or} \quad \mathbf{C_x = 600 \text{ N} \leftarrow}$$

Then

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(600)^2 + (225)^2} = 640.80 \text{ N}$$

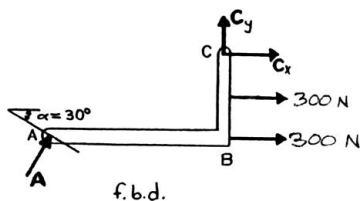
and

$$\theta = \tan^{-1} \left(\frac{C_y}{C_x} \right) = \tan^{-1} \left(\frac{-225}{-600} \right) = 20.556^\circ$$

$$\text{or} \quad \mathbf{C = 641 \text{ N} \nearrow 20.6^\circ \blacktriangleleft}$$

(b) $\alpha = 30^\circ$

From F.B.D. of member ABC:



$$+\curvearrowright \Sigma M_C = 0: (300 \text{ N})(0.2 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) - (A \cos 30^\circ)(0.8 \text{ m}) + (A \sin 30^\circ)(20 \text{ in.}) = 0$$

$$A = 365.24 \text{ N} \quad \text{or} \quad \mathbf{A = 365 \text{ N} \nearrow 60.0^\circ \blacktriangleleft}$$

$$+\rightarrow \Sigma F_x = 0: 300 \text{ N} + 300 \text{ N} + (365.24 \text{ N}) \sin 30^\circ + C_x = 0$$

$$C_x = -782.62$$

PROBLEM 4.21 (Continued)

$$+\uparrow \Sigma F_y = 0: C_y + (365.24 \text{ N}) \cos 30^\circ = 0$$

$$C_y = -316.31 \text{ N} \quad \text{or} \quad C_y = 316 \text{ N} \downarrow$$

Then

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(782.62)^2 + (316.31)^2} = 884.12 \text{ N}$$

and

$$\theta = \tan^{-1} \left(\frac{C_y}{C_x} \right) = \tan^{-1} \left(\frac{-316.31}{-782.62} \right) = 22.007^\circ$$

or

$$C = 884 \text{ N} \nearrow 22.0^\circ \blacktriangleleft$$

PROBLEM 4.22

Determine the reactions at A and B when (a) $\alpha = 0$, (b) $\alpha = 90^\circ$, (c) $\alpha = 30^\circ$.

SOLUTION

(a) $\alpha = 0$

$$+\curvearrowright \Sigma M_A = 0: B(20 \text{ in.}) - 75 \text{ lb}(10 \text{ in.}) = 0$$

$$B = 37.5 \text{ lb}$$

$$\pm \rightarrow \Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y - 75 \text{ lb} + 37.5 \text{ lb} = 0$$

$$A_y = 37.5 \text{ lb}$$

$A = B = 37.5 \text{ lb} \uparrow \blacktriangleleft$

(b) $\alpha = 90^\circ$

$$+\curvearrowright \Sigma M_A = 0: B(12 \text{ in.}) - 75 \text{ lb}(10 \text{ in.}) = 0$$

$$B = 62.5 \text{ lb}$$

$$\pm \rightarrow \Sigma F_x = 0: A_x - B = 0$$

$$A_x = 62.5 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: A_y - 75 \text{ lb} = 0$$

$$A_y = 75 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(62.5 \text{ lb})^2 + (75 \text{ lb})^2}$$

$$= 97.6 \text{ lb}$$

$$\tan \theta = \frac{75}{62.5}$$

$$\theta = 50.2^\circ$$

$A = 97.6 \text{ lb} \nearrow 50.2^\circ; B = 62.51 \text{ lb} \leftarrow \blacktriangleleft$

PROBLEM 4.22 (Continued)

(c) $\alpha = 30^\circ$

$$+\curvearrowright \Sigma M_A = 0: (B \cos 30^\circ)(20 \text{ in.}) + (B \sin 30^\circ)(12 \text{ in.}) - (75 \text{ lb})(10 \text{ in.}) = 0$$

$$B = 32.161 \text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: A_x - (32.161) \sin 30^\circ = 0$$

$$A_x = 16.0805 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: A_y + (32.161) \cos 30^\circ - 75 = 0$$

$$A_y = 47.148 \text{ lb}$$

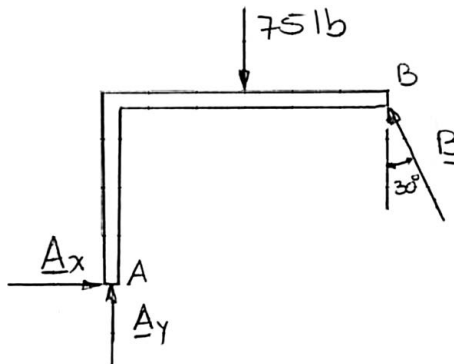
$$A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(16.0805)^2 + (47.148)^2}$$

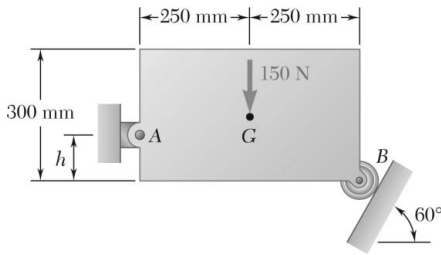
$$= 49.8 \text{ lb}$$

$$\tan \theta = \frac{47.148}{16.0805}$$

$$\theta = 71.2^\circ$$



$$\mathbf{A} = 49.8 \text{ lb} \nearrow 71.2^\circ; \quad \mathbf{B} = 32.2 \text{ lb} \searrow 60.0^\circ \blacktriangleleft$$

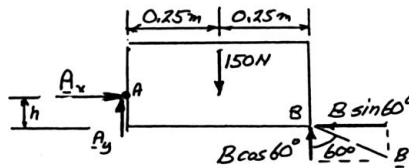


PROBLEM 4.23

Determine the reactions at A and B when (a) $h = 0$,
(b) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: (B \cos 60^\circ)(0.5 \text{ m}) - (B \sin 60^\circ)h - (150 \text{ N})(0.25 \text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866h} \quad (1)$$

(a) When $h = 0$,

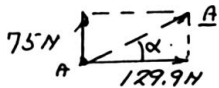
From Eq. (1): $B = \frac{37.5}{0.25} = 150 \text{ N} \quad \mathbf{B} = 150.0 \text{ N} \searrow 30.0^\circ \blacktriangleleft$

$$+\rightarrow \Sigma F_x = 0: A_x - B \sin 60^\circ = 0$$

$$A_x = (150) \sin 60^\circ = 129.9 \text{ N} \quad \mathbf{A}_x = 129.9 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 150 + B \cos 60^\circ = 0$$

$$A_y = 150 - (150) \cos 60^\circ = 75 \text{ N} \quad \mathbf{A}_y = 75 \text{ N} \uparrow$$



$$\alpha = 30^\circ$$

$$\mathbf{A} = 150.0 \text{ N} \nearrow 30.0^\circ \blacktriangleleft$$

(b) When $h = 200$ mm = 0.2 m,

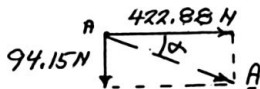
From Eq. (1): $B = \frac{37.5}{0.25 - 0.866(0.2)} = 488.3 \text{ N} \quad \mathbf{B} = 488 \text{ N} \searrow 30.0^\circ \blacktriangleleft$

$$+\rightarrow \Sigma F_x = 0: A_x - B \sin 60^\circ = 0$$

$$A_x = (488.3) \sin 60^\circ = 422.88 \text{ N} \quad \mathbf{A}_x = 422.88 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 150 + B \cos 60^\circ = 0$$

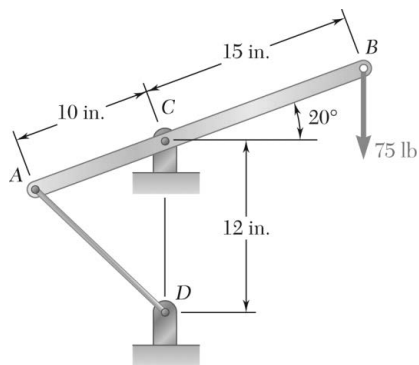
$$A_y = 150 - (488.3) \cos 60^\circ = -94.15 \text{ N} \quad \mathbf{A}_y = 94.15 \text{ N} \downarrow$$



$$\alpha = 12.55^\circ$$

$$\mathbf{A} = 433 \text{ N} \searrow 12.55^\circ \blacktriangleleft$$

PROBLEM 4.24



A lever AB is hinged at C and attached to a control cable at A . If the lever is subjected to a 75-lb vertical force at B , determine (a) the tension in the cable, (b) the reaction at C .

SOLUTION

Geometry:

$$x_{AC} = (10 \text{ in.}) \cos 20^\circ = 9.3969 \text{ in.}$$

$$y_{AC} = (10 \text{ in.}) \sin 20^\circ = 3.4202 \text{ in.}$$

$$\Rightarrow y_{DA} = 12 \text{ in.} - 3.4202 \text{ in.} = 8.5798 \text{ in.}$$

$$\alpha = \tan^{-1} \left(\frac{y_{DA}}{x_{AC}} \right) = \tan^{-1} \left(\frac{8.5798}{9.3969} \right) = 42.397^\circ$$

$$\beta = 90^\circ - 20^\circ - 42.397^\circ = 27.603^\circ$$

Equilibrium for lever:

$$(a) \quad +\curvearrowright \Sigma M_C = 0: \quad T_{AD} \cos 27.603^\circ (10 \text{ in.}) - (75 \text{ lb}) [(15 \text{ in.}) \cos 20^\circ] = 0$$

$$T_{AD} = 119.293 \text{ lb}$$

$$T_{AD} = 119.3 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad \pm \rightarrow \Sigma F_x = 0: \quad C_x + (119.293 \text{ lb}) \cos 42.397^\circ = 0$$

$$C_x = -88.097 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75 \text{ lb} - (119.293 \text{ lb}) \sin 42.397^\circ = 0$$

$$C_y = 155.435$$

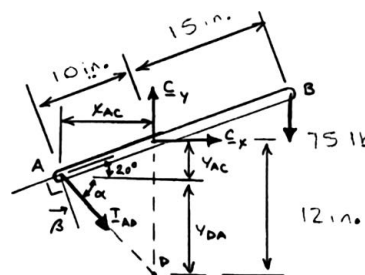
Thus,

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-88.097)^2 + (155.435)^2} = 178.665 \text{ lb}$$

and

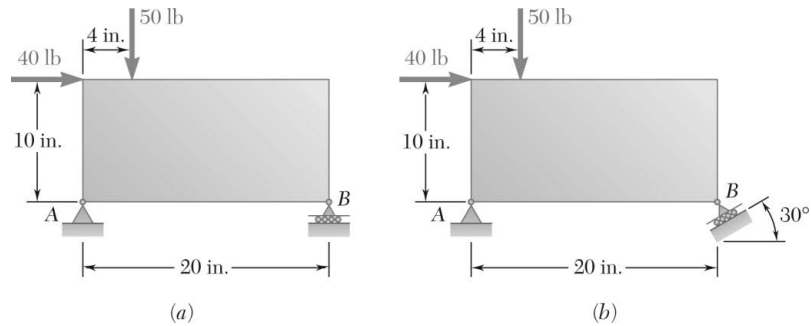
$$\theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1} \frac{155.435}{-88.097} = 60.456^\circ \quad \mathbf{C} = 178.7 \text{ lb} \nearrow 60.5^\circ \quad \blacktriangleleft$$

Free-Body Diagram:



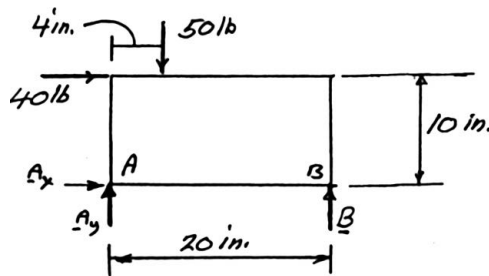
PROBLEM 4.25

For each of the plates and loadings shown, determine the reactions at A and B .



SOLUTION

(a) Free-Body Diagram:



$$+\curvearrowright \Sigma M_A = 0: B(20 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$B = +30 \text{ lb}$$

$$\mathbf{B} = 30.0 \text{ lb} \uparrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: A_x + 40 \text{ lb} = 0$$

$$A_x = -40 \text{ lb}$$

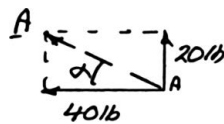
$$\mathbf{A}_x = 40.0 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + B - 50 \text{ lb} = 0$$

$$A_y + 30 \text{ lb} - 50 \text{ lb} = 0$$

$$A_y = +20 \text{ lb}$$

$$\mathbf{A}_y = 20.0 \text{ lb} \uparrow$$



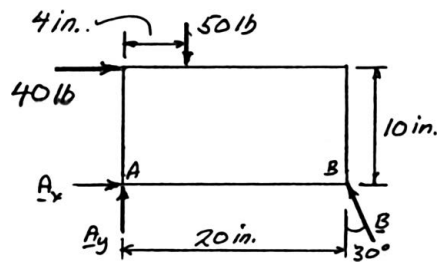
$$\alpha = 26.56^\circ$$

$$A = 44.72 \text{ lb}$$

$$\mathbf{A} = 44.7 \text{ lb} \searrow 26.6^\circ \blacktriangleleft$$

PROBLEM 4.25 (Continued)

(b) **Free-Body Diagram:**



$$+\curvearrowright \Sigma M_A = 0: (B \cos 30^\circ)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) = 0$$

$$B = 34.64 \text{ lb}$$

$$\mathbf{B} = 34.6 \text{ lb} \searrow 60.0^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: A_x - B \sin 30^\circ + 40 \text{ lb}$$

$$A_x - (34.64 \text{ lb}) \sin 30^\circ + 40 \text{ lb} = 0$$

$$A_x = -22.68 \text{ lb}$$

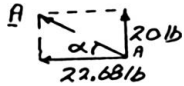
$$\mathbf{A}_x = 22.68 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + B \cos 30^\circ - 50 \text{ lb} = 0$$

$$A_y + (34.64 \text{ lb}) \cos 30^\circ - 50 \text{ lb} = 0$$

$$A_y = +20 \text{ lb}$$

$$\mathbf{A}_y = 20.0 \text{ lb} \uparrow$$



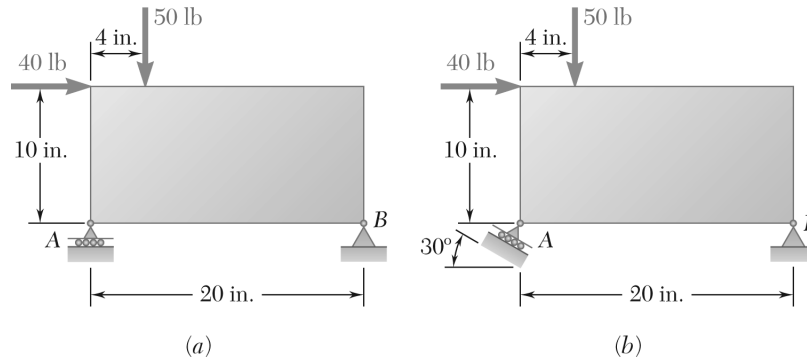
$$\alpha = 41.4^\circ$$

$$\mathbf{A} = 30.24 \text{ lb}$$

$$\mathbf{A} = 30.2 \text{ lb} \searrow 41.4^\circ \blacktriangleleft$$

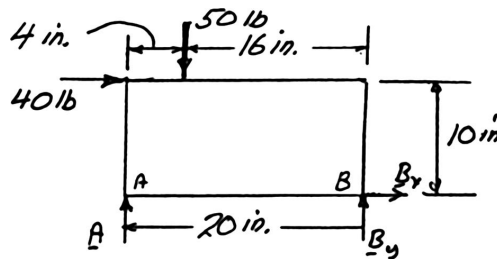
PROBLEM 4.26

For each of the plates and loadings shown, determine the reactions at A and B.



SOLUTION

(a) Free-Body Diagram:



$$+\curvearrowright \Sigma M_B = 0: A(20 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$A = +20 \text{ lb}$$

$$\mathbf{A} = 20.0 \text{ lb } \uparrow \blacktriangleleft$$

$$\pm \rightarrow \Sigma F_x = 0: 40 \text{ lb} + B_x = 0$$

$$B_x = -40 \text{ lb}$$

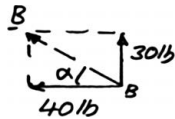
$$\mathbf{B}_x = 40 \text{ lb } \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A + B_y - 50 \text{ lb} = 0$$

$$20 \text{ lb} + B_y - 50 \text{ lb} = 0$$

$$B_y = +30 \text{ lb}$$

$$\mathbf{B}_y = 30 \text{ lb } \uparrow$$



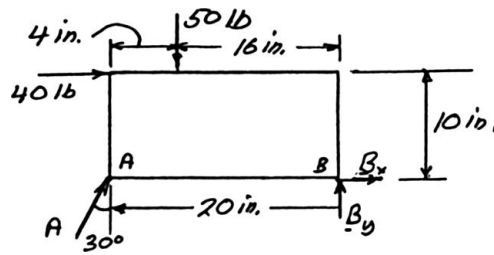
$$\alpha = 36.87^\circ$$

$$B = 50 \text{ lb}$$

$$\mathbf{B} = 50.0 \text{ lb } \searrow 36.9^\circ \blacktriangleleft$$

PROBLEM 4.26 (Continued)

(b)



$$+\curvearrowright \Sigma M_A = 0: \quad -(A \cos 30^\circ)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) = 0$$

$$A = 23.09 \text{ lb}$$

$$\mathbf{A} = 23.1 \text{ lb} \nearrow 60.0^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad A \sin 30^\circ + 40 \text{ lb} + B_x = 0$$

$$(23.09 \text{ lb}) \sin 30^\circ + 40 \text{ lb} + B_x = 0$$

$$B_x = -51.55 \text{ lb}$$

$$\mathbf{B}_x = 51.55 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A \cos 30^\circ + B_y - 50 \text{ lb} = 0$$

$$(23.09 \text{ lb}) \cos 30^\circ + B_y - 50 \text{ lb} = 0$$

$$B_y = +30 \text{ lb}$$

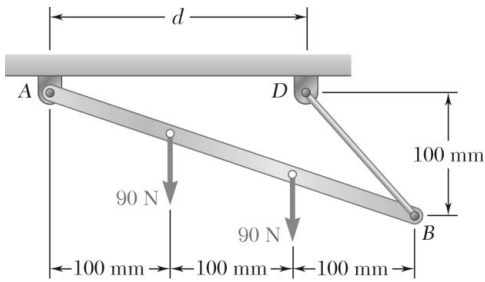
$$\mathbf{B}_y = 30 \text{ lb} \uparrow$$

$$\alpha = 30.2^\circ$$

$$B = 59.64 \text{ lb}$$

$$\mathbf{B} = 59.6 \text{ lb} \searrow 30.2^\circ \blacktriangleleft$$



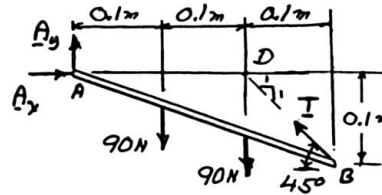


PROBLEM 4.27

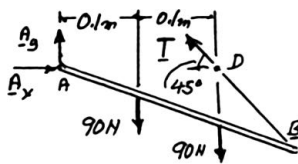
A rod AB hinged at A and attached at B to cable BD supports the loads shown. Knowing that $d = 200$ mm, determine (a) the tension in cable BD , (b) the reaction at A .

SOLUTION

Free-Body Diagram:



(a) Move T along BD until it acts at Point D .



$$+\curvearrowright \Sigma M_A = 0: (T \sin 45^\circ)(0.2 \text{ m}) + (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 190.919 \text{ N}$$

$$T = 190.9 \text{ N} \quad \blacktriangleleft$$

(b)

$$+\rightarrow \Sigma F_x = 0: A_x - (190.919 \text{ N}) \cos 45^\circ = 0$$

$$A_x = +135.0 \text{ N}$$

$$A_x = 135.0 \text{ N} \quad \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 90 \text{ N} - 90 \text{ N} + (190.919 \text{ N}) \sin 45^\circ = 0$$

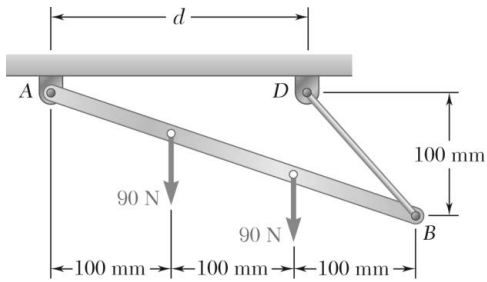
$$A_y = +45.0 \text{ N}$$

$$A_y = 45.0 \text{ N} \quad \uparrow$$



$$A = 142.3 \text{ N} \quad \nearrow 18.43^\circ \quad \blacktriangleleft$$

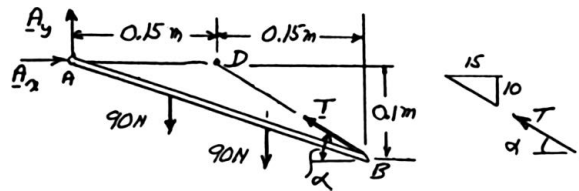
PROBLEM 4.28



A rod AB , hinged at A and attached at B to cable BD , supports the loads shown. Knowing that $d = 150$ mm, determine (a) the tension in cable BD , (b) the reaction at A .

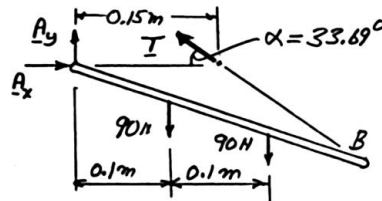
SOLUTION

Free-Body Diagram:



$$\tan \alpha = \frac{10}{15}; \quad \alpha = 33.690^\circ$$

(a) Move T along BD until it acts at Point D .



$$+\curvearrowright \Sigma M_A = 0: (T \sin 33.690^\circ)(0.15 \text{ m}) - (90 \text{ N})(0.1 \text{ m}) - (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 324.50 \text{ N}$$

$$T = 324 \text{ N} \quad \blacktriangleleft$$

(b) $\rightarrow \Sigma F_x = 0: A_x - (324.50 \text{ N}) \cos 33.690^\circ = 0$

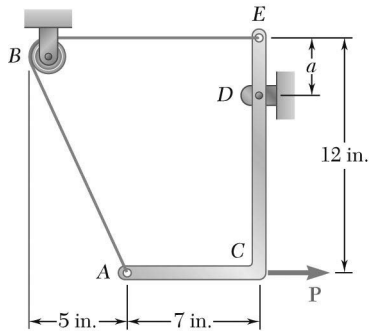
$$A_x = +270 \text{ N}$$

$$A_x = 270 \text{ N} \quad \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 90 \text{ N} - 90 \text{ N} + (324.50 \text{ N}) \sin 33.690^\circ = 0$$

$$A_y = 0$$

$$A = 270 \text{ N} \quad \rightarrow \quad \blacktriangleleft$$

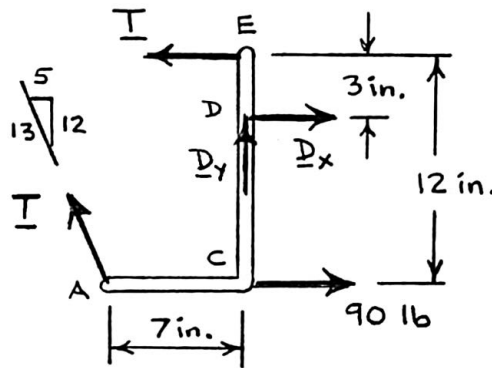


PROBLEM 4.29

A force P of magnitude 90 lb is applied to member $ACDE$, which is supported by a frictionless pin at D and by the cable ABE . Since the cable passes over a small pulley at B , the tension may be assumed to be the same in portions AB and BE of the cable. For the case when $a = 3$ in., determine (a) the tension in the cable, (b) the reaction at D .

SOLUTION

Free-Body Diagram:



$$(a) \quad +\circlearrowleft \Sigma M_D = 0: \quad (90 \text{ lb})(9 \text{ in.}) - \frac{5}{13}T(9 \text{ in.}) - \frac{12}{13}T(7 \text{ in.}) + T(3 \text{ in.}) = 0$$

$$T = 117 \text{ lb}$$

$$T = 117.0 \text{ lb} \quad \blacktriangleleft$$

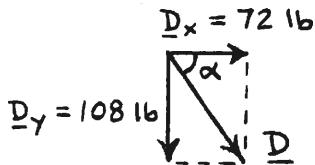
$$(b) \quad \pm \Sigma F_x = 0: \quad D_x - 117 \text{ lb} - \frac{5}{13}(117 \text{ lb}) + 90 = 0$$

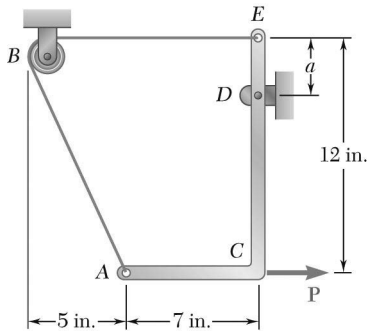
$$D_x = +72 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + \frac{12}{13}(117 \text{ lb}) = 0$$

$$D_y = -108 \text{ lb}$$

$$D = 129.8 \text{ lb} \quad \blacktriangleleft 56.3^\circ$$





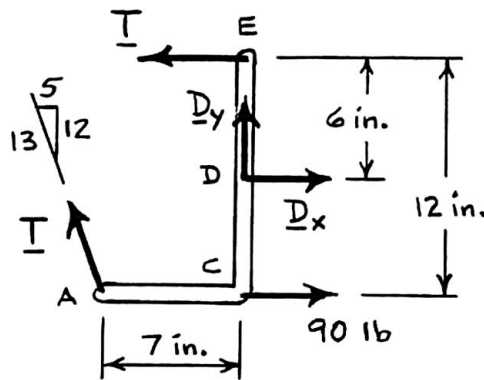
PROBLEM 4.30

Solve Problem 4.29 for $a = 6$ in.

PROBLEM 4.29 A force P of magnitude 90 lb is applied to member $ACDE$, which is supported by a frictionless pin at D and by the cable ABE . Since the cable passes over a small pulley at B , the tension may be assumed to be the same in portions AB and BE of the cable. For the case when $a = 3$ in., determine (a) the tension in the cable, (b) the reaction at D .

SOLUTION

Free-Body Diagram:



$$(a) \quad +\curvearrowright \Sigma M_D = 0: \quad (90 \text{ lb})(6 \text{ in.}) - \frac{5}{13}T(6 \text{ in.}) - \frac{12}{13}T(7 \text{ in.}) + T(6 \text{ in.}) = 0$$

$$T = 195 \text{ lb}$$

$$T = 195.0 \text{ lb} \quad \blacktriangleleft$$

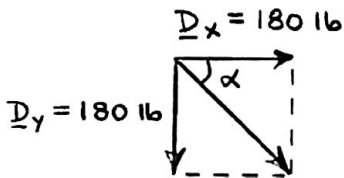
$$(b) \quad \pm \Sigma F_x = 0: \quad D_x - 195 \text{ lb} - \frac{5}{13}(195 \text{ lb}) + 90 = 0$$

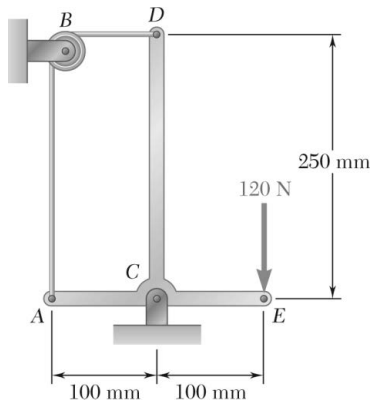
$$D_x = +180 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + \frac{12}{13}(195 \text{ lb}) = 0$$

$$D_y = -180 \text{ lb}$$

$$D = 255 \text{ lb} \quad \swarrow 45.0^\circ \quad \blacktriangleleft$$



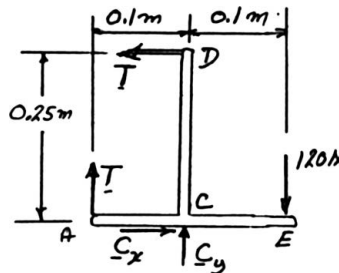


PROBLEM 4.31

Neglecting friction, determine the tension in cable ABD and the reaction at support C .

SOLUTION

Free-Body Diagram:



$$+\curvearrowright \Sigma M_C = 0: T(0.25 \text{ m}) - T(0.1 \text{ m}) - (120 \text{ N})(0.1 \text{ m}) = 0$$

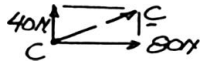
$$T = 80.0 \text{ N} \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: C_x - 80 \text{ N} = 0 \quad C_x = +80 \text{ N}$$

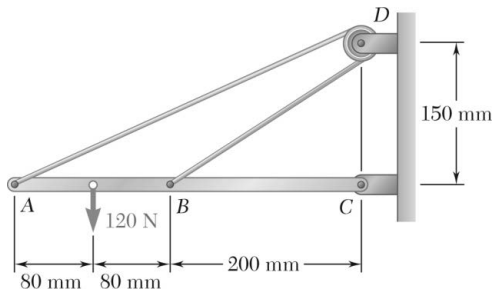
$$C_x = 80.0 \text{ N} \quad \rightarrow$$

$$+\uparrow \Sigma F_y = 0: C_y - 120 \text{ N} + 80 \text{ N} = 0 \quad C_y = +40 \text{ N}$$

$$C_y = 40.0 \text{ N} \quad \uparrow$$



$$C = 89.4 \text{ N} \quad \nearrow 26.6^\circ \quad \blacktriangleleft$$



PROBLEM 4.32

Neglecting friction and the radius of the pulley, determine (a) the tension in cable ADB, (b) the reaction at C.

SOLUTION

Geometry:

Distance: $AD = \sqrt{(0.36)^2 + (0.150)^2} = 0.39 \text{ m}$

Distance: $BD = \sqrt{(0.2)^2 + (0.15)^2} = 0.25 \text{ m}$

Equilibrium for beam:

$$(a) \quad +\curvearrowright \Sigma M_C = 0: \quad (120 \text{ N})(0.28 \text{ m}) - \left(\frac{0.15}{0.39}T\right)(0.36 \text{ m}) - \left(\frac{0.15}{0.25}T\right)(0.2 \text{ m}) = 0$$

$$T = 130.000 \text{ N}$$

$$\text{or } T = 130.0 \text{ N} \blacktriangleleft$$

$$(b) \quad \pm \Sigma F_x = 0: \quad C_x + \left(\frac{0.36}{0.39}\right)(130.000 \text{ N}) + \left(\frac{0.2}{0.25}\right)(130.000 \text{ N}) = 0$$

$$C_x = -224.00 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + \left(\frac{0.15}{0.39}\right)(130.00 \text{ N}) + \left(\frac{0.15}{0.25}\right)(130.00 \text{ N}) - 120 \text{ N} = 0$$

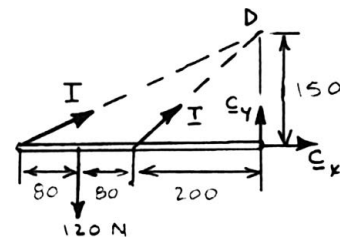
$$C_y = -8.0000 \text{ N}$$

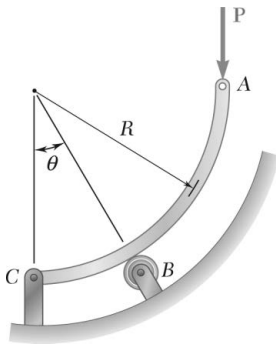
Thus, $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-224)^2 + (-8)^2} = 224.14 \text{ N}$

and $\theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1} \frac{8}{224} = 2.0454^\circ$ $C = 224 \text{ N} \nearrow 2.05^\circ \blacktriangleleft$

Free-Body Diagram:

Dimensions in mm





PROBLEM 4.33

Rod ABC is bent in the shape of an arc of circle of radius R . Knowing the $\theta = 30^\circ$, determine the reaction (a) at B , (b) at C .

SOLUTION

Free-Body Diagram: $\curvearrowright \Sigma M_D = 0: C_x(R) - P(R) = 0$

$$C_x = +P$$

$$\rightarrow \Sigma F_x = 0: C_x - B \sin \theta = 0$$

$$P - B \sin \theta = 0$$

$$B = P / \sin \theta$$

$$\mathbf{B} = \frac{P}{\sin \theta} \searrow \theta$$

$$\uparrow \Sigma F_y = 0: C_y + B \cos \theta - P = 0$$

$$C_y + (P / \sin \theta) \cos \theta - P = 0$$

$$C_y = P \left(1 - \frac{1}{\tan \theta} \right)$$

For $\theta = 30^\circ$,

(a) $B = P / \sin 30^\circ = 2P$ $\mathbf{B} = 2P \searrow 60.0^\circ \blacktriangleleft$

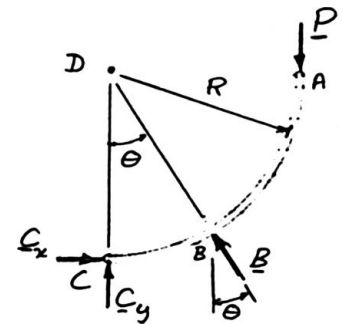
(b) $C_x = +P$ $C_x = P \rightarrow$

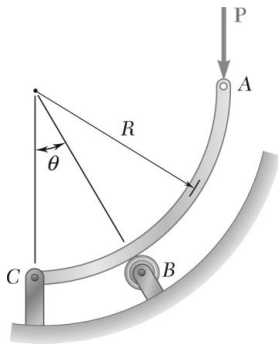


$$C_y = P(1 - 1/\tan 30^\circ) = -0.732/P$$

$$C_y = 0.7321P \downarrow$$

$$\mathbf{C} = 1.239P \searrow 36.2^\circ \blacktriangleleft$$





PROBLEM 4.34

Rod ABC is bent in the shape of an arc of circle of radius R . Knowing that $\theta = 60^\circ$, determine the reaction (a) at B , (b) at C .

SOLUTION

See the solution to Problem 4.33 for the free-body diagram and analysis leading to the following expressions:

$$C_x = +P$$

$$C_y = P \left(1 - \frac{1}{\tan \theta} \right)$$

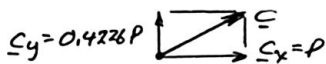
$$B = \frac{P}{\sin \theta}$$

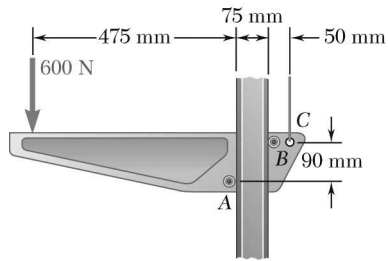
For $\theta = 60^\circ$,

(a) $B = P/\sin 60^\circ = 1.1547P$ $\mathbf{B} = 1.155P \nearrow 30.0^\circ \blacktriangleleft$

(b) $C_x = +P$ $C_x = P \rightarrow$ $C_y = 0.4226P \downarrow$

$C_y = P(1 - 1/\tan 60^\circ) = +0.4226P$ $\mathbf{C} = 1.086P \nearrow 22.9^\circ \blacktriangleleft$



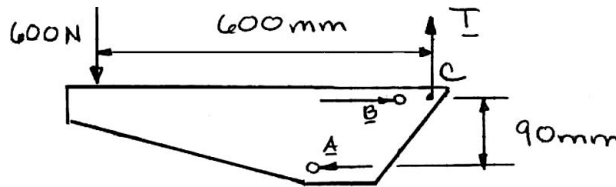


PROBLEM 4.35

A movable bracket is held at rest by a cable attached at C and by frictionless rollers at A and B . For the loading shown, determine (a) the tension in the cable, (b) the reactions at A and B .

SOLUTION

Free-Body Diagram:



$$(a) \quad +\uparrow \Sigma F_y = 0: \quad T - 600 \text{ N} = 0$$

$$T = 600 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad \rightarrow \Sigma F_x = 0: \quad B - A = 0 \quad \therefore B = A$$

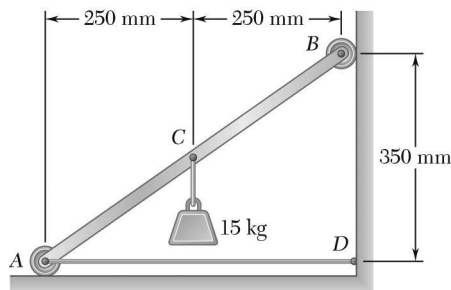
Note that the forces shown form two couples.

$$+\curvearrowright \Sigma M = 0: \quad (600 \text{ N})(600 \text{ mm}) - A(90 \text{ mm}) = 0$$

$$A = 4000 \text{ N}$$

$$\therefore B = 4000 \text{ N}$$

$$\mathbf{A} = 4.00 \text{ kN} \quad \blacktriangleleft; \quad \mathbf{B} = 4.00 \text{ kN} \quad \blacktriangleright$$

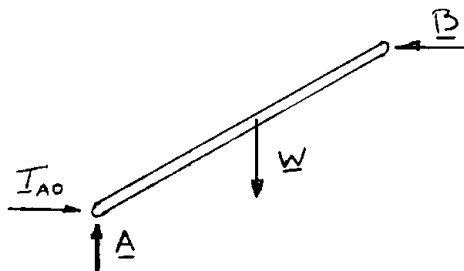


PROBLEM 4.36

A light bar AB supports a 15-kg block at its midpoint C . Rollers at A and B rest against frictionless surfaces, and a horizontal cable AD is attached at A . Determine (a) the tension in cable AD , (b) the reactions at A and B .

SOLUTION

Free-Body Diagram:



$$W = (15 \text{ kg})(9.81 \text{ m/s}^2) = 147.150 \text{ N}$$

$$(a) \quad \pm \rightarrow \Sigma F_x = 0: \quad T_{AD} - 105.107 \text{ N} = 0$$

$$T_{AD} = 105.1 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad + \uparrow \Sigma F_y = 0: \quad A - W = 0$$

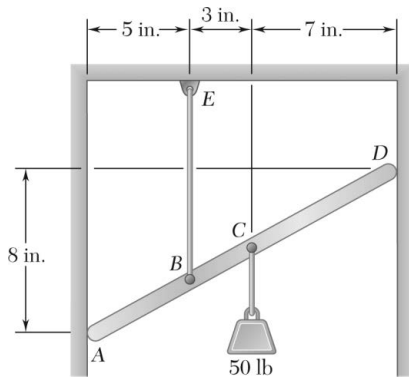
$$A - 147.150 \text{ N} = 0$$

$$A = 147.2 \text{ N} \quad \uparrow \blacktriangleleft$$

$$+ \curvearrowright \Sigma M_A = 0: \quad B(350 \text{ mm}) - (147.150 \text{ N})(250 \text{ mm}) = 0$$

$$B = 105.107 \text{ N}$$

$$B = 105.1 \text{ N} \quad \leftarrow \blacktriangleleft$$

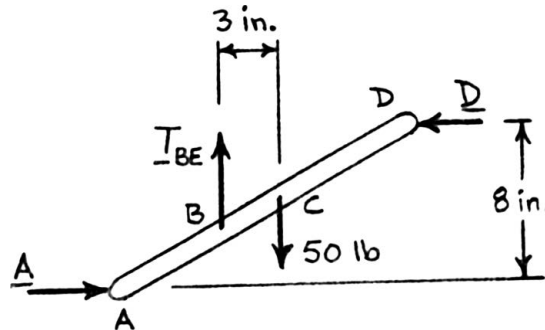


PROBLEM 4.37

A light bar AD is suspended from a cable BE and supports a 50-lb block at C . The ends A and D of the bar are in contact with frictionless vertical walls. Determine the tension in cable BE and the reactions at A and D .

SOLUTION

Free-Body Diagram:



$$\Sigma F_x = 0: \quad A = D$$

$$\Sigma F_y = 0:$$

$$T_{BE} = 50.0 \text{ lb} \quad \blacktriangleleft$$

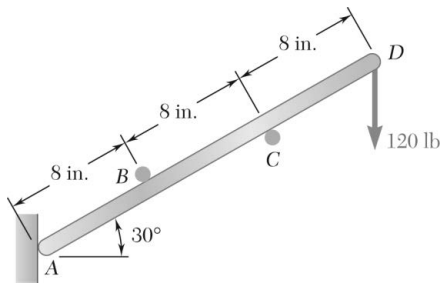
We note that the forces shown form two couples.

$$+\curvearrowright \Sigma M = 0: \quad A(8 \text{ in.}) - (50 \text{ lb})(3 \text{ in.}) = 0$$

$$A = 18.75 \text{ lb}$$

$$A = 18.75 \text{ lb} \quad \rightarrow$$

$$D = 18.75 \text{ lb} \quad \leftarrow \blacktriangleleft$$

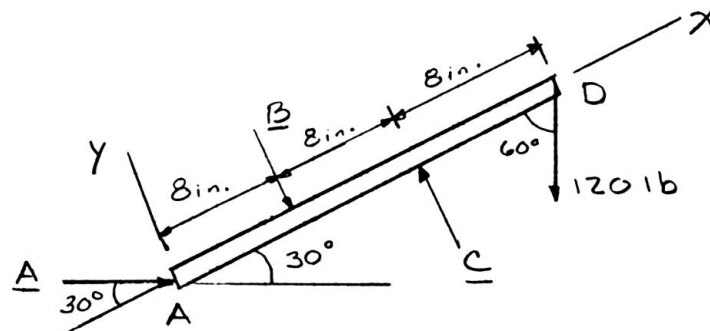


PROBLEM 4.38

A light rod AD is supported by frictionless pegs at B and C and rests against a frictionless wall at A . A vertical 120-lb force is applied at D . Determine the reactions at A , B , and C .

SOLUTION

Free-Body Diagram:



$$\nearrow \Sigma F_x = 0: A \cos 30^\circ - (120 \text{ lb}) \cos 60^\circ = 0$$

$$A = 69.28 \text{ lb}$$

$$A = 69.3 \text{ lb} \rightarrow \blacktriangleleft$$

$$\begin{aligned} + \curvearrowright \Sigma M_B = 0: & C(8 \text{ in.}) - (120 \text{ lb})(16 \text{ in.}) \cos 30^\circ \\ & + (69.28 \text{ lb})(8 \text{ in.}) \sin 30^\circ = 0 \end{aligned}$$

$$C = 173.2 \text{ lb}$$

$$C = 173.2 \text{ lb} \nearrow 60.0^\circ \blacktriangleleft$$

$$\begin{aligned} + \curvearrowright \Sigma M_C = 0: & B(8 \text{ in.}) - (120 \text{ lb})(8 \text{ in.}) \cos 30^\circ \\ & + (69.28 \text{ lb})(16 \text{ in.}) \sin 30^\circ = 0 \end{aligned}$$

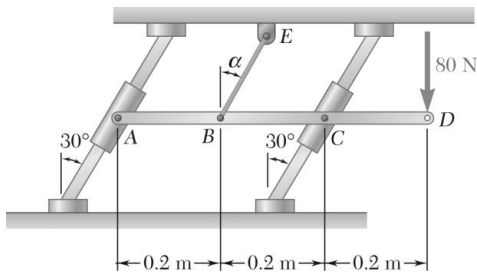
$$B = 34.6 \text{ lb}$$

$$B = 34.6 \text{ lb} \searrow 60.0^\circ \blacktriangleleft$$

Check:

$$\nearrow \Sigma F_y = 0: 173.2 - 34.6 - (69.28) \sin 30^\circ - (120) \sin 60^\circ = 0$$

$$0 = 0 \quad (\text{check})$$

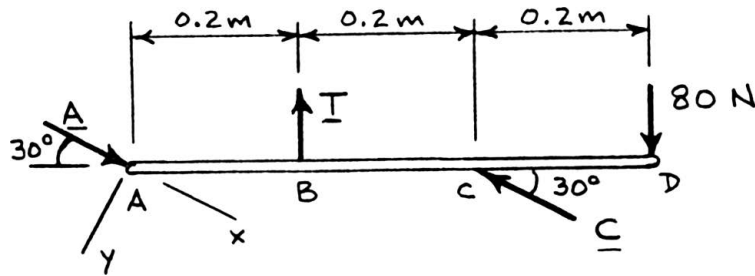


PROBLEM 4.39

Bar AD is attached at A and C to collars that can move freely on the rods shown. If the cord BE is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at A and C .

SOLUTION

Free-Body Diagram:



$$\uparrow \Sigma F_y = 0: -T \cos 30^\circ + (80 \text{ N}) \cos 30^\circ = 0$$

$$T = 80 \text{ N}$$

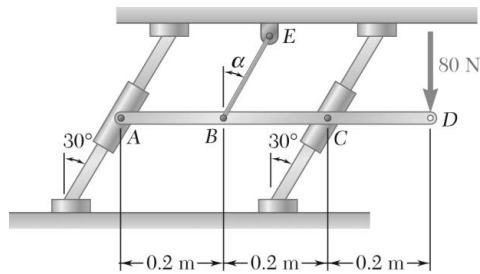
$$T = 80.0 \text{ N} \quad \blacktriangleleft$$

$$\curvearrowright \Sigma M_C = 0: (A \sin 30^\circ)(0.4 \text{ m}) - (80 \text{ N})(0.2 \text{ m}) - (80 \text{ N})(0.2 \text{ m}) = 0$$

$$A = +160 \text{ N} \quad \mathbf{A} = 160.0 \text{ N} \quad \swarrow 30.0^\circ \quad \blacktriangleleft$$

$$\curvearrowright \Sigma M_A = 0: (80 \text{ N})(0.2 \text{ m}) - (80 \text{ N})(0.6 \text{ m}) + (C \sin 30^\circ)(0.4 \text{ m}) = 0$$

$$C = +160 \text{ N} \quad \mathbf{C} = 160.0 \text{ N} \quad \searrow 30.0^\circ \quad \blacktriangleleft$$



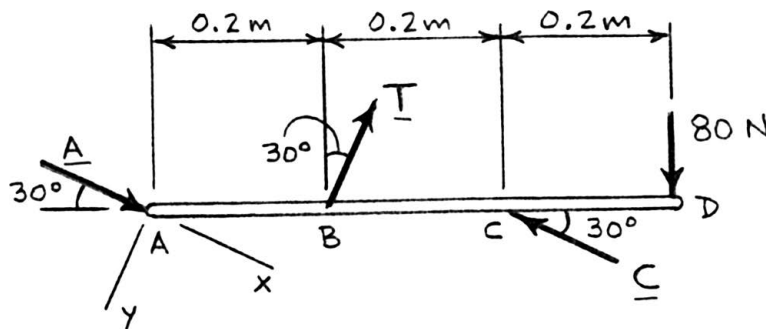
PROBLEM 4.40

Solve Problem 4.39 if the cord BE is parallel to the rods ($\alpha = 30^\circ$).

PROBLEM 4.39 Bar AD is attached at A and C to collars that can move freely on the rods shown. If the cord BE is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at A and C.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: -T + (80 \text{ N}) \cos 30^\circ = 0$$

$$T = 69.282 \text{ N}$$

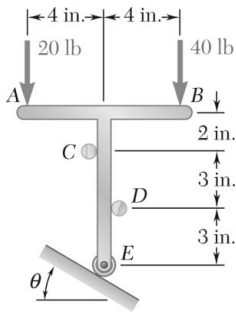
$$T = 69.3 \text{ N} \quad \blacktriangleleft$$

$$+\curvearrowright \Sigma M_C = 0: -(69.282 \text{ N}) \cos 30^\circ (0.2 \text{ m}) - (80 \text{ N})(0.2 \text{ m}) + (A \sin 30^\circ)(0.4 \text{ m}) = 0$$

$$A = +140.000 \text{ N} \quad A = 140.0 \text{ N} \quad \swarrow 30.0^\circ \quad \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: +(69.282 \text{ N}) \cos 30^\circ (0.2 \text{ m}) - (80 \text{ N})(0.6 \text{ m}) + (C \sin 30^\circ)(0.4 \text{ m}) = 0$$

$$C = +180.000 \text{ N} \quad C = 180.0 \text{ N} \quad \searrow 30.0^\circ \quad \blacktriangleleft$$

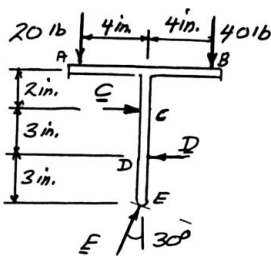


PROBLEM 4.41

The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D . Neglecting the effect of friction, determine the reactions at C , D , and E when $\theta = 30^\circ$.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: E \cos 30^\circ - 20 - 40 = 0$$

$$E = \frac{60 \text{ lb}}{\cos 30^\circ} = 69.282 \text{ lb}$$

$$\mathbf{E} = 69.3 \text{ lb } \swarrow 60.0^\circ \blacktriangleleft$$

$$+\curvearrowright \Sigma M_D = 0: (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.}) - C(3 \text{ in.}) + E \sin 30^\circ(3 \text{ in.}) = 0$$

$$-80 - 3C + 69.282(0.5)(3) = 0$$

$$C = 7.9743 \text{ lb}$$

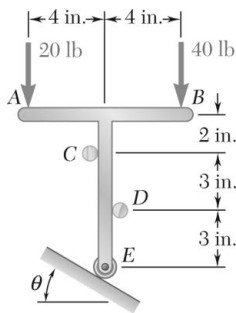
$$\mathbf{C} = 7.97 \text{ lb } \rightarrow \blacktriangleleft$$

$$\Sigma F_x = 0: E \sin 30^\circ + C - D = 0$$

$$(69.282 \text{ lb})(0.5) + 7.9743 \text{ lb} - D = 0$$

$$D = 42.615 \text{ lb}$$

$$\mathbf{D} = 42.6 \text{ lb } \leftarrow \blacktriangleleft$$

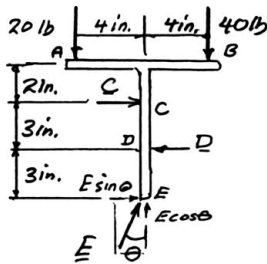


PROBLEM 4.42

The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D . Neglecting the effect of friction, determine (a) the smallest value of θ for which the equilibrium of the bracket is maintained, (b) the corresponding reactions at C , D , and E .

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E \cos \theta - 20 - 40 = 0$$

$$E = \frac{60}{\cos \theta} \quad (1)$$

$$+\curvearrowright \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.}) - C(3 \text{ in.})$$

$$+ \left(\frac{60}{\cos \theta} \sin \theta \right) 3 \text{ in.} = 0$$

$$C = \frac{1}{3}(180 \tan \theta - 80)$$

(a) For $C = 0$,

$$180 \tan \theta = 80$$

$$\tan \theta = \frac{4}{9} \quad \theta = 23.962^\circ$$

$$\theta = 24.0^\circ \quad \blacktriangleleft$$

From Eq. (1):

$$E = \frac{60}{\cos 23.962^\circ} = 65.659$$

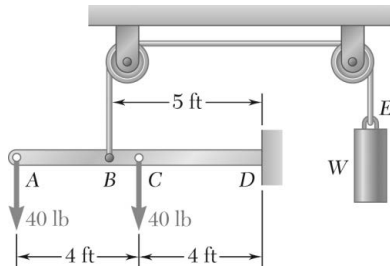
$$+\rightarrow \Sigma F_x = 0: \quad -D + C + E \sin \theta = 0$$

$$D = (65.659) \sin 23.962 = 26.666 \text{ lb}$$

(b)

$$C = 0 \quad D = 26.7 \text{ lb} \quad \leftarrow$$

$$E = 65.7 \text{ lb} \quad \nearrow 66.0^\circ \quad \blacktriangleleft$$



PROBLEM 4.43

Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE that is attached to the counterweight W . Determine the reaction at D when (a) $W = 100$ lb, (b) $W = 90$ lb.

SOLUTION

(a)

$$W = 100 \text{ lb}$$

From F.B.D. of beam AD :

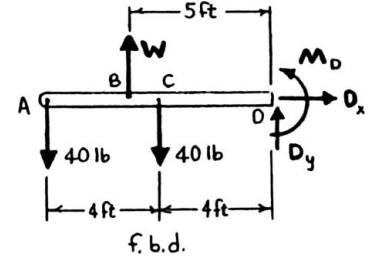
$$\rightarrow \Sigma F_x = 0: D_x = 0$$

$$+\uparrow \Sigma F_y = 0: D_y - 40 \text{ lb} - 40 \text{ lb} + 100 \text{ lb} = 0$$

$$D_y = -20.0 \text{ lb}$$

$$+\curvearrowright \Sigma M_D = 0: M_D - (100 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$M_D = 20.0 \text{ lb} \cdot \text{ft}$$



$$\text{or } \mathbf{D} = 20.0 \text{ lb} \downarrow \leftarrow$$

$$\text{or } \mathbf{M}_D = 20.0 \text{ lb} \cdot \text{ft} \curvearrowright \leftarrow$$

(b)

$$W = 90 \text{ lb}$$

From F.B.D. of beam AD :

$$\rightarrow \Sigma F_x = 0: D_x = 0$$

$$+\uparrow \Sigma F_y = 0: D_y + 90 \text{ lb} - 40 \text{ lb} - 40 \text{ lb} = 0$$

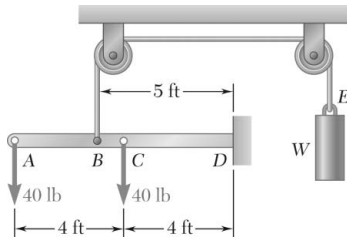
$$D_y = -10.00 \text{ lb}$$

$$+\curvearrowright \Sigma M_D = 0: M_D - (90 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$M_D = -30.0 \text{ lb} \cdot \text{ft}$$

$$\text{or } \mathbf{D} = 10.00 \text{ lb} \downarrow \leftarrow$$

$$\text{or } \mathbf{M}_D = 30.0 \text{ lb} \cdot \text{ft} \curvearrowright \leftarrow$$



PROBLEM 4.44

For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed $40 \text{ lb} \cdot \text{ft}$.

SOLUTION

For W_{\min} ,

$$M_D = -40 \text{ lb} \cdot \text{ft}$$

From F.B.D. of beam AD : $\rightarrow \Sigma M_D = 0$: $(40 \text{ lb})(8 \text{ ft}) - W_{\min}(5 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) - 40 \text{ lb} \cdot \text{ft} = 0$

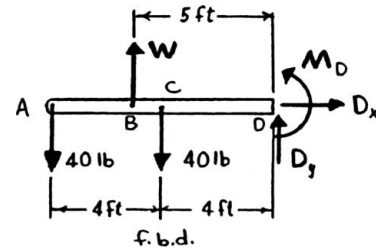
$$W_{\min} = 88.0 \text{ lb}$$

For W_{\max} ,

$$M_D = 40 \text{ lb} \cdot \text{ft}$$

From F.B.D. of beam AD : $\rightarrow \Sigma M_D = 0$: $(40 \text{ lb})(8 \text{ ft}) - W_{\max}(5 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) + 40 \text{ lb} \cdot \text{ft} = 0$

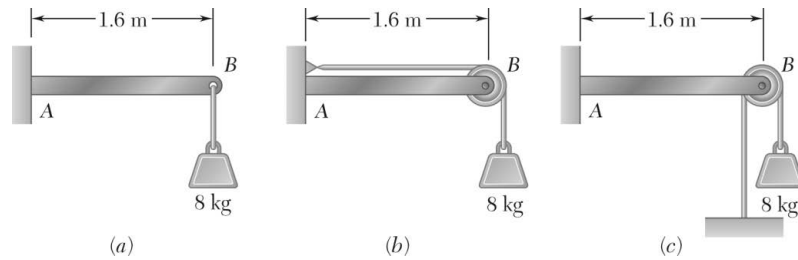
$$W_{\max} = 104.0 \text{ lb}$$



or $88.0 \text{ lb} \leq W \leq 104.0 \text{ lb}$ ◀

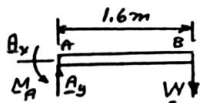
PROBLEM 4.45

An 8-kg mass can be supported in the three different ways shown. Knowing that the pulleys have a 100-mm radius, determine the reaction at A in each case.



SOLUTION

$$W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.480 \text{ N}$$



$$(a) \quad \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W = 0$$

$$A_y = 78.480 \text{ N} \uparrow$$

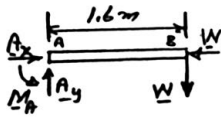
$$+\curvearrowright \Sigma M_A = 0: \quad M_A - W(1.6 \text{ m}) = 0$$

$$M_A = +(78.480 \text{ N})(1.6 \text{ m})$$

$$M_A = 125.568 \text{ N} \cdot \text{m} \curvearrowright$$

$$A = 78.5 \text{ N} \uparrow$$

$$M_A = 125.6 \text{ N} \cdot \text{m} \curvearrowleft$$



$$(b) \quad \pm \Sigma F_x = 0: \quad A_x - W = 0$$

$$A_x = 78.480 \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W = 0$$

$$A_y = 78.480 \leftarrow$$

$$A = (78.480 \text{ N})\sqrt{2} = 110.987 \text{ N} \nearrow 45^\circ$$

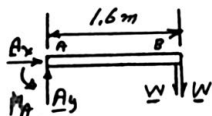
$$+\curvearrowright \Sigma M_A = 0: \quad M_A - W(1.6 \text{ m}) = 0$$

$$M_A = +(78.480 \text{ N})(1.6 \text{ m})$$

$$M_A = 125.568 \text{ N} \cdot \text{m} \curvearrowright$$

$$A = 111.0 \text{ N} \nearrow 45^\circ$$

$$M_A = 125.6 \text{ N} \cdot \text{m} \curvearrowleft$$



$$(c) \quad \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 2W = 0$$

$$A_y = 2W = 2(78.480 \text{ N}) = 156.960 \text{ N} \uparrow$$

$$+\curvearrowright \Sigma M_A = 0: \quad M_A - 2W(1.6 \text{ m}) = 0$$

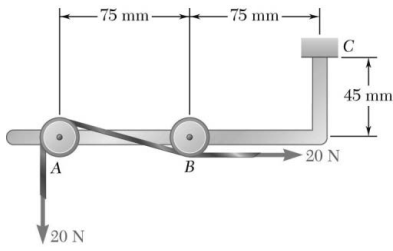
$$M_A = +2(78.480 \text{ N})(1.6 \text{ m})$$

$$M_A = 251.14 \text{ N} \cdot \text{m} \curvearrowright$$

$$A = 157.0 \text{ N} \uparrow$$

$$M_A = 251 \text{ N} \cdot \text{m} \curvearrowleft$$

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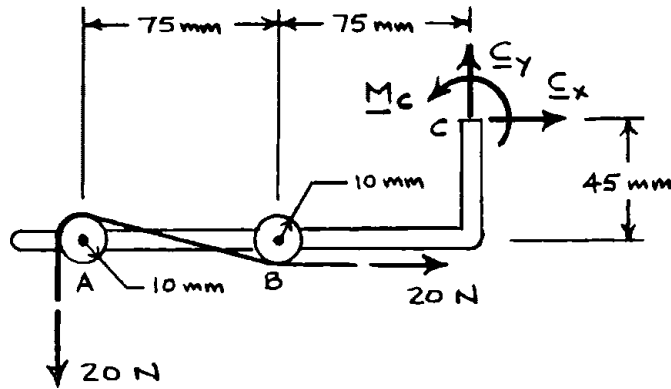


PROBLEM 4.46

A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at C.

SOLUTION

Free-Body Diagram:



$$\pm \rightarrow \Sigma F_x = 0: C_x + (20 \text{ N}) = 0 \quad C_x = -20 \text{ N}$$

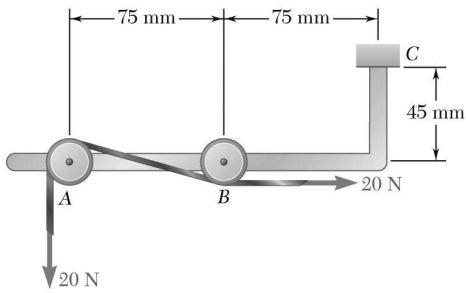
$$+ \uparrow \Sigma F_y = 0: C_y - (20 \text{ N}) = 0 \quad C_y = +20 \text{ N}$$

$$C = 28.3 \text{ N} \nearrow 45.0^\circ \blacktriangleleft$$

$$+ \curvearrowright \Sigma M_C = 0: M_C + (20 \text{ N})(0.160 \text{ m}) + (20 \text{ N})(0.055 \text{ m}) = 0$$

$$M_C = -4.30 \text{ N} \cdot \text{m}$$

$$M_C = 4.30 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$



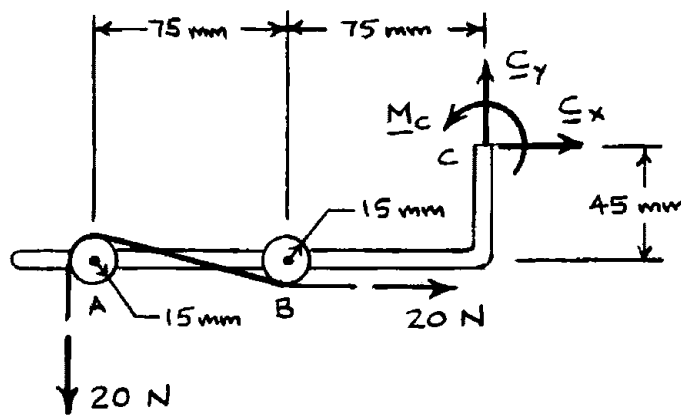
PROBLEM 4.47

Solve Problem 4.46, assuming that 15-mm-radius pulleys are used.

PROBLEM 4.46 A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at C.

SOLUTION

Free-Body Diagram:



$$\pm \rightarrow \Sigma F_x = 0: C_x + (20 \text{ N}) = 0 \quad C_x = -20 \text{ N}$$

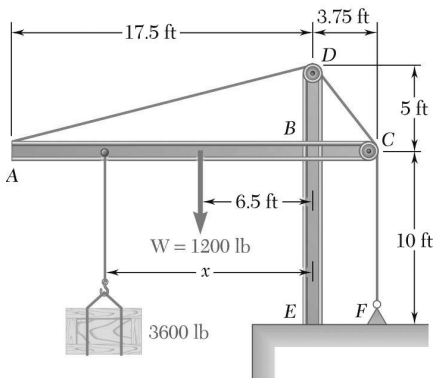
$$+ \uparrow \Sigma F_y = 0: C_y - (20 \text{ N}) = 0 \quad C_y = +20 \text{ N}$$

$$C = 28.3 \text{ N} \searrow 45.0^\circ \blacktriangleleft$$

$$+ \curvearrowright \Sigma M_C = 0: M_C + (20 \text{ N})(0.165 \text{ m}) + (20 \text{ N})(0.060 \text{ m}) = 0$$

$$M_C = -4.50 \text{ N} \cdot \text{m}$$

$$M_C = 4.50 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$

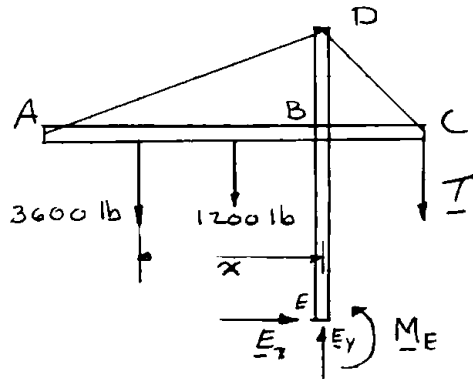


PROBLEM 4.48

The rig shown consists of a 1200-lb horizontal member ABC and a vertical member DBE welded together at B . The rig is being used to raise a 3600-lb crate at a distance $x = 12$ ft from the vertical member DBE . If the tension in the cable is 4 kips, determine the reaction at E , assuming that the cable is (a) anchored at F as shown in the figure, (b) attached to the vertical member at a point located 1 ft above E .

SOLUTION

Free-Body Diagram:



$$+\curvearrowright M_E = 0: M_E + (3600 \text{ lb})x + (1200 \text{ lb})(6.5 \text{ ft}) - T(3.75 \text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \quad (1)$$

(a) For $x = 12$ ft and $T = 4000$ lbs,

$$\begin{aligned} M_E &= 3.75(4000) - 3600(12) - 7800 \\ &= 36,000 \text{ lb} \cdot \text{ft} \end{aligned}$$

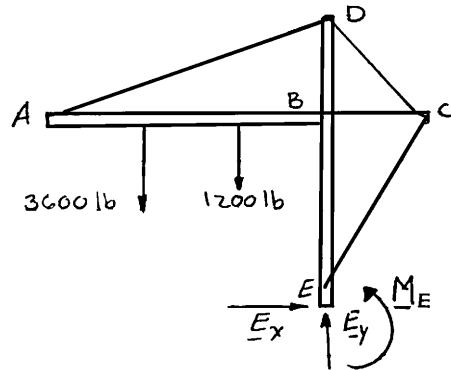
$$+\rightarrow \Sigma F_x = 0 \quad \therefore E_x = 0$$

$$+\uparrow \Sigma F_y = 0: E_y - 3600 \text{ lb} - 1200 \text{ lb} - 4000 = 0$$

$$E_y = 8800 \text{ lb}$$

$$\mathbf{E} = 8.80 \text{ kips } \uparrow; \mathbf{M}_E = 36.0 \text{ kip} \cdot \text{ft } \curvearrowleft$$

PROBLEM 4.48 (Continued)



(b)

$$+\curvearrowright \Sigma M_E = 0: M_E + (3600 \text{ lb})(12 \text{ ft}) + (1200 \text{ lb})(6.5 \text{ ft}) = 0$$

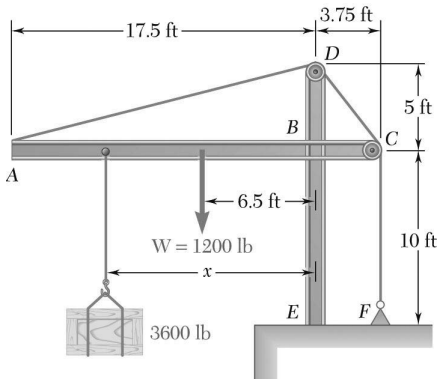
$$M_E = -51,000 \text{ lb} \cdot \text{ft}$$

$$+\rightarrow \Sigma F_x = 0 \quad \therefore E_x = 0$$

$$+\uparrow \Sigma F_y = 0: E_y - 3600 \text{ lb} - 1200 \text{ lb} = 0$$

$$E_y = 4800 \text{ lb}$$

$$\mathbf{E} = 4.80 \text{ kips} \uparrow; \mathbf{M}_E = 51.0 \text{ kip} \cdot \text{ft} \curvearrowleft$$

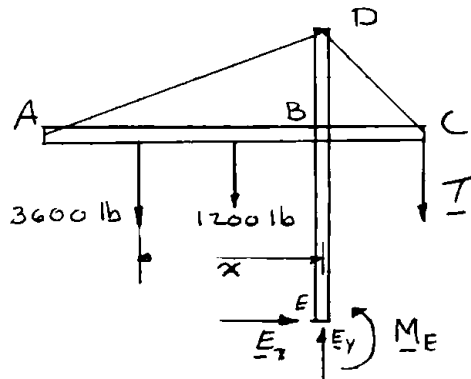


PROBLEM 4.49

For the rig and crate of Prob. 4.48, and assuming that cable is anchored at F as shown, determine (a) the required tension in cable $ADCF$ if the maximum value of the couple at E as x varies from 1.5 to 17.5 ft is to be as small as possible, (b) the corresponding maximum value of the couple.

SOLUTION

Free-Body Diagram:



$$+\curvearrowright M_E = 0: M_E + (3600 \text{ lb})x + (1200 \text{ lb})(6.5 \text{ ft}) - T(3.75 \text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \quad (1)$$

For $x = 1.5$ ft, Eq. (1) becomes

$$(M_E)_1 = 3.75T - 3600(1.5) - 7800 \quad (2)$$

For $x = 17.5$ ft, Eq. (1) becomes

$$(M_E)_2 = 3.75T - 3600(17.5) - 7800$$

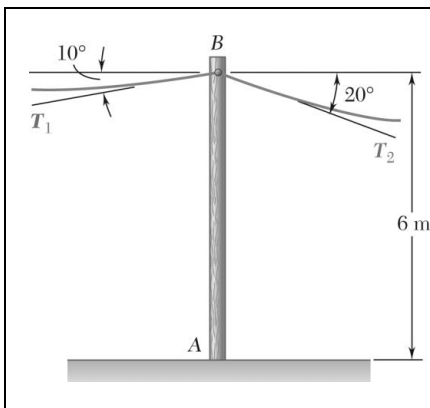
(a) For smallest max value of $|M_E|$, we set

$$(M_E)_1 = -(M_E)_2$$

$$3.75T - 13,200 = -3.75T + 70,800 \quad T = 11.20 \text{ kips} \quad \blacktriangleleft$$

(b) From Equation (2), then

$$M_E = 3.75(11.20) - 13,200 \quad |M_E| = 28.8 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

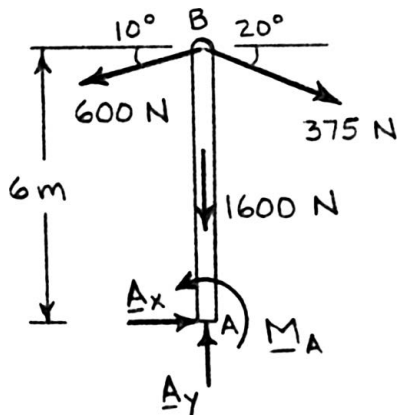


PROBLEM 4.50

A 6-m telephone pole weighing 1600 N is used to support the ends of two wires. The wires form the angles shown with the horizontal axis and the tensions in the wires are, respectively, $T_1 = 600$ N and $T_2 = 375$ N. Determine the reaction at the fixed end A.

SOLUTION

Free-Body Diagram:



$$\rightarrow \Sigma F_x = 0: A_x + (375 \text{ N}) \cos 20^\circ - (600 \text{ N}) \cos 10^\circ = 0$$

$$A_x = +238.50 \text{ N}$$

$$\uparrow \Sigma F_y = 0: A_y - 1600 \text{ N} - (600 \text{ N}) \sin 10^\circ - (375 \text{ N}) \sin 20^\circ = 0$$

$$A_y = +1832.45 \text{ N}$$

$$A = \sqrt{238.50^2 + 1832.45^2}$$

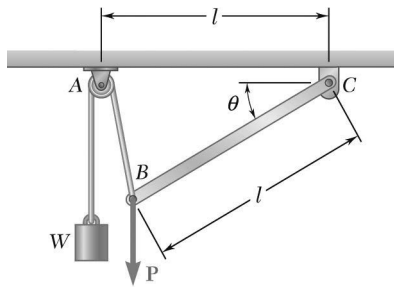
$$\theta = \tan^{-1} \frac{1832.45}{238.50}$$

$$A = 1848 \text{ N} \nearrow 82.6^\circ \blacktriangleleft$$

$$\curvearrowright \Sigma M_A = 0: M_A + (600 \text{ N}) \cos 10^\circ (6 \text{ m}) - (375 \text{ N}) \cos 20^\circ (6 \text{ m}) = 0$$

$$M_A = -1431.00 \text{ N} \cdot \text{m}$$

$$M_A = 1431 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$

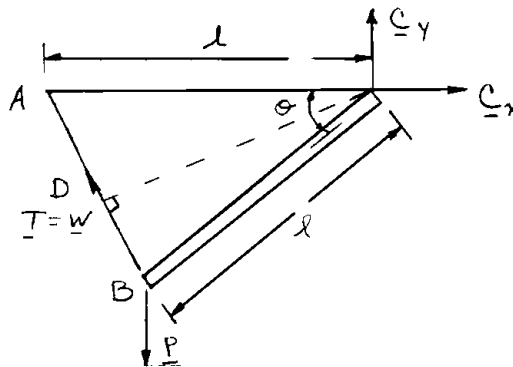


PROBLEM 4.51

A vertical load P is applied at end B of rod BC . (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P , l , and the counterweight W . (b) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION

Free-Body Diagram:



(a) Triangle ABC is isosceles. We have

$$CD = (BC) \cos\left(\frac{\theta}{2}\right) = l \cos\left(\frac{\theta}{2}\right)$$

$$+\circlearrowleft \Sigma M_C = 0: P(l \cos \theta) - W\left(l \cos \frac{\theta}{2}\right) = 0$$

Setting $\cos \theta = 2 \cos^2 \frac{\theta}{2} - 1$:

$$Pl \left(2 \cos^2 \frac{\theta}{2} - 1 \right) - Wl \cos \frac{\theta}{2} = 0$$

$$\cos^2 \frac{\theta}{2} - \left(\frac{W}{2P} \right) \cos \frac{\theta}{2} - \frac{1}{2} = 0$$

$$\cos \frac{\theta}{2} = \frac{1}{4} \left(\frac{W}{P} \pm \sqrt{\frac{W^2}{P^2} + 8} \right)$$

$$\theta = 2 \cos^{-1} \left[\frac{1}{4} \left(\frac{W}{P} \pm \sqrt{\frac{W^2}{P^2} + 8} \right) \right] \quad \blacktriangleleft$$

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PROBLEM 4.51 (Continued)

(b) For $P = 2W$,

$$\cos \frac{\theta}{2} = \frac{1}{4} \left(\frac{1}{2} \pm \sqrt{\frac{1}{4} + 8} \right) = \frac{1}{8} (1 \pm \sqrt{33})$$

$$\cos \frac{\theta}{2} = 0.84307 \quad \text{and} \quad \cos \frac{\theta}{2} = -0.59307$$

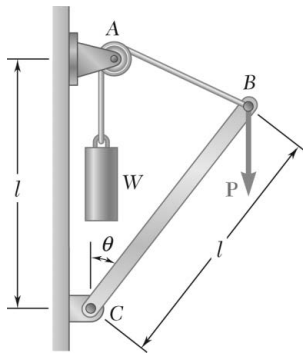
$$\frac{\theta}{2} = 32.534^\circ$$

$$\frac{\theta}{2} = 126.375^\circ$$

$$\theta = 65.1^\circ$$

$$\theta = 252.75^\circ \quad (\text{discard})$$

$$\theta = 65.1^\circ \quad \blacktriangleleft$$



PROBLEM 4.52

A vertical load P is applied at end B of rod BC . (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P , l , and the counterweight W . (b) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION

(a) Triangle ABC is isosceles. We have

$$CD = (BC) \cos \frac{\theta}{2} = l \cos \frac{\theta}{2}$$

$$+\circlearrowleft \Sigma M_C = 0: W \left(l \cos \frac{\theta}{2} \right) - P(l \sin \theta) = 0$$

$$\text{Setting } \sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}: Wl \cos \frac{\theta}{2} - 2Pl \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 0$$

$$W - 2P \sin \frac{\theta}{2} = 0$$

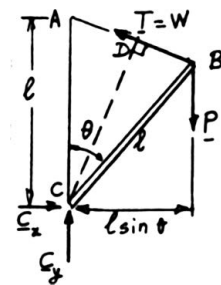
(b) For $P = 2W$,

$$\sin \frac{\theta}{2} = \frac{W}{2P} = \frac{W}{4W} = 0.25$$

$$\frac{\theta}{2} = 14.5^\circ$$

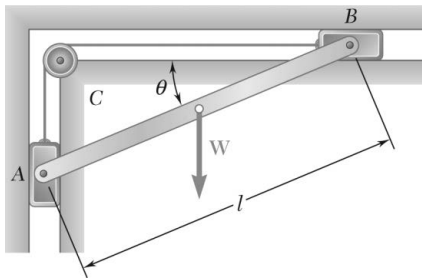
or

$$\frac{\theta}{2} = 165.5^\circ \quad \theta = 331^\circ \quad (\text{discard})$$



$$\theta = 2 \sin^{-1} \left(\frac{W}{2P} \right) \blacktriangleleft$$

$$\theta = 29.0^\circ \blacktriangleleft$$



PROBLEM 4.53

A slender rod AB , of weight W , is attached to blocks A and B , which move freely in the guides shown. The blocks are connected by an elastic cord that passes over a pulley at C . (a) Express the tension in the cord in terms of W and θ . (b) Determine the value of θ for which the tension in the cord is equal to $3W$.

SOLUTION

(a) From F.B.D. of rod AB :

$$+\curvearrowright \Sigma M_C = 0: T(l \sin \theta) + W \left[\left(\frac{1}{2} \right) \cos \theta \right] - T(l \cos \theta) = 0$$

$$T = \frac{W \cos \theta}{2(\cos \theta - \sin \theta)}$$

Dividing both numerator and denominator by $\cos \theta$,

$$T = \frac{W}{2} \left(\frac{1}{1 - \tan \theta} \right)$$

$$\text{or } T = \frac{\left(\frac{W}{2} \right)}{(1 - \tan \theta)} \blacktriangleleft$$

(b) For $T = 3W$,

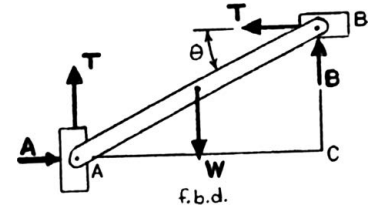
$$3W = \frac{\left(\frac{W}{2} \right)}{(1 - \tan \theta)}$$

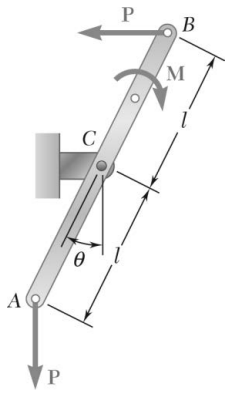
$$1 - \tan \theta = \frac{1}{6}$$

or

$$\theta = \tan^{-1} \left(\frac{5}{6} \right) = 39.806^\circ$$

$$\text{or } \theta = 39.8^\circ \blacktriangleleft$$



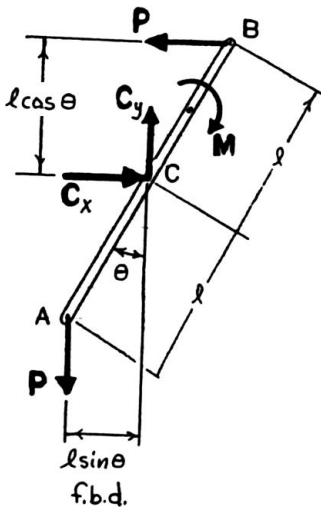


PROBLEM 4.54

Rod AB is acted upon by a couple M and two forces, each of magnitude P . (a) Derive an equation in θ , P , M , and l that must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150 \text{ N} \cdot \text{m}$, $P = 200 \text{ N}$, and $l = 600 \text{ mm}$.

SOLUTION

Free-Body Diagram:



(a) From free-body diagram of rod AB :

$$+\circlearrowleft \sum M_C = 0: P(l \cos \theta) + P(l \sin \theta) - M = 0$$

$$\text{or } \sin \theta + \cos \theta = \frac{M}{Pl} \quad \blacktriangleleft$$

(b) For $M = 150 \text{ lb} \cdot \text{in.}$, $P = 20 \text{ lb}$, and $l = 6 \text{ in.}$,

$$\sin \theta + \cos \theta = \frac{150 \text{ lb} \cdot \text{in.}}{(20 \text{ lb})(6 \text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\sin^2 \theta + \cos^2 \theta = 1$

$$\sin \theta + (1 - \sin^2 \theta)^{1/2} = 1.25$$

$$(1 - \sin^2 \theta)^{1/2} = 1.25 - \sin \theta$$

$$1 - \sin^2 \theta = 1.5625 - 2.5 \sin \theta + \sin^2 \theta$$

$$2 \sin^2 \theta - 2.5 \sin \theta + 0.5625 = 0$$

Using quadratic formula

$$\sin \theta = \frac{-(-2.5) \pm \sqrt{(625) - 4(2)(0.5625)}}{2(2)}$$

$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\sin \theta = 0.95572$ and $\sin \theta = 0.29428$

$$\theta = 72.886^\circ \quad \text{and} \quad \theta = 17.1144^\circ$$

$$\text{or } \theta = 17.11^\circ \quad \text{and} \quad \theta = 72.9^\circ \quad \blacktriangleleft$$

PROBLEM 4.55

Solve Sample Problem 4.5, assuming that the spring is unstretched when $\theta = 90^\circ$.

SOLUTION

First note: $T = \text{tension in spring} = ks$

where $s = \text{deformation of spring}$
 $= r\beta$
 $F = kr\beta$

From F.B.D. of assembly: $\sum M_O = 0: W(l \cos \beta) - F(r) = 0$

or $Wl \cos \beta - kr^2 \beta = 0$

$$\cos \beta = \frac{kr^2}{Wl} \beta$$

For $k = 250 \text{ lb/in.}$
 $r = 3 \text{ in.}$
 $l = 8 \text{ in.}$
 $W = 400 \text{ lb}$

$$\cos \beta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})} \beta$$

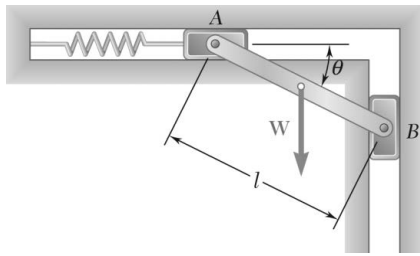
or $\cos \beta = 0.703125 \beta$

Solving numerically, $\beta = 0.89245 \text{ rad}$

or $\beta = 51.134^\circ$

Then $\theta = 90^\circ + 51.134^\circ = 141.134^\circ$ or $\theta = 141.1^\circ \blacktriangleleft$

f. b. d.

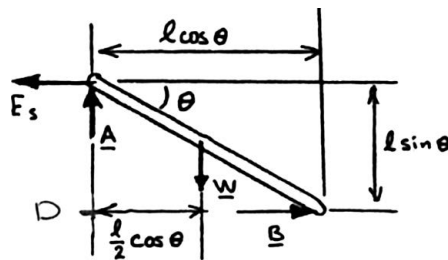


PROBLEM 4.56

A slender rod AB , of weight W , is attached to blocks A and B that move freely in the guides shown. The constant of the spring is k , and the spring is unstretched when $\theta = 0$. (a) Neglecting the weight of the blocks, derive an equation in W , k , l , and θ that must be satisfied when the rod is in equilibrium. (b) Determine the value of θ when $W = 75$ lb, $l = 30$ in., and $k = 3$ lb/in.

SOLUTION

Free-Body Diagram:



Spring force:

$$F_s = ks = k(l - l \cos \theta) = kl(1 - \cos \theta)$$

$$(a) \quad +\curvearrowright \Sigma M_D = 0: \quad F_s(l \sin \theta) - W\left(\frac{l}{2} \cos \theta\right) = 0$$

$$kl(1 - \cos \theta)l \sin \theta - \frac{W}{2}l \cos \theta = 0$$

$$kl(1 - \cos \theta) \tan \theta - \frac{W}{2} = 0 \quad \text{or} \quad (1 - \cos \theta) \tan \theta = \frac{W}{2kl} \quad \blacktriangleleft$$

(b) For given values of

$$W = 75 \text{ lb}$$

$$l = 30 \text{ in.}$$

$$k = 3 \text{ lb/in.}$$

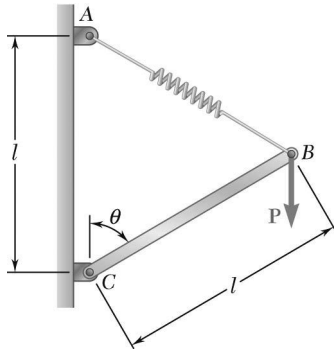
$$\begin{aligned} (1 - \cos \theta) \tan \theta &= \tan \theta - \sin \theta \\ &= \frac{75 \text{ lb}}{2(3 \text{ lb/in.})(30 \text{ in.})} \\ &= 0.41667 \end{aligned}$$

Solving numerically,

$$\theta = 49.710^\circ$$

or

$$\theta = 49.7^\circ \quad \blacktriangleleft$$

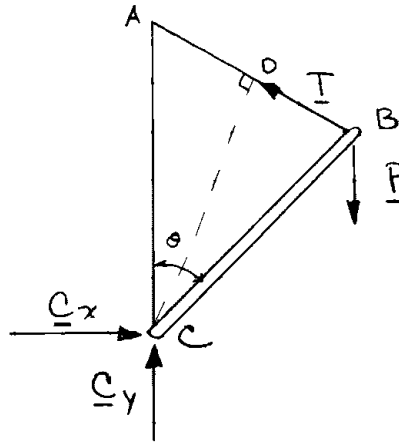


PROBLEM 4.57

A vertical load \mathbf{P} is applied at end B of rod BC . The constant of the spring is k , and the spring is unstretched when $\theta = 60^\circ$. (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position terms of P , k , and l . (b) Determine the value of θ corresponding to equilibrium if $P = \frac{1}{4} kl$.

SOLUTION

Free-Body Diagram:



(a) Triangle ABC is isosceles. We have

$$AB = 2(AD) = 2l \sin\left(\frac{\theta}{2}\right); \quad CD = l \cos\left(\frac{\theta}{2}\right)$$

Elongation of spring: $x = (AB)_\theta - (AB)_{\theta = 60^\circ}$

$$= 2l \sin\left(\frac{\theta}{2}\right) - 2l \sin 30^\circ$$

$$T = kx = 2kl \left(\sin \frac{\theta}{2} - \frac{1}{2} \right)$$

$$\sum M_C = 0: \quad T \left(l \cos \frac{\theta}{2} \right) - P(l \sin \theta) = 0$$

PROBLEM 4.57 (Continued)

$$2kl\left(\sin\frac{\theta}{2} - \frac{1}{2}\right)l\cos\frac{\theta}{2} - Pl\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = 0$$

$$\cos\frac{\theta}{2} = 0 \quad \text{or} \quad 2(kl - P)\sin\frac{\theta}{2} - kl = 0$$

$$\theta = 180^\circ \text{ (trivial)} \quad \sin\frac{\theta}{2} = \frac{\frac{1}{2}kl}{kl - P}$$

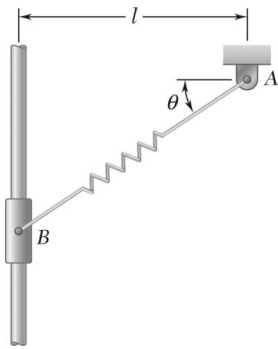
$$\theta = 2\sin^{-1}\left[\frac{1}{2}kl/(kl - P)\right] \blacktriangleleft$$

(b) For $P = \frac{1}{4}kl$,

$$\sin\frac{\theta}{2} = \frac{\frac{1}{2}kl}{\frac{3}{4}kl} = \frac{2}{3}$$

$$\frac{\theta}{2} = 41.8^\circ$$

$$\theta = 83.6^\circ \blacktriangleleft$$



PROBLEM 4.58

A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k , and the spring is unstretched when $\theta = 0$. (a) Derive an equation in θ , W , k , and l that must be satisfied when the collar is in equilibrium. (b) Knowing that $W = 300$ N, $l = 500$ mm, and $k = 800$ N/m, determine the value of θ corresponding to equilibrium.

SOLUTION

First note:

where

$$T = ks$$

k = spring constant

s = elongation of spring

$$= \frac{l}{\cos \theta} - l$$

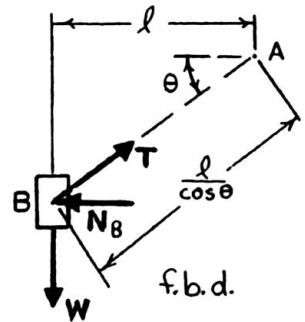
$$= \frac{l}{\cos \theta} (1 - \cos \theta)$$

$$T = \frac{kl}{\cos \theta} (1 - \cos \theta)$$

(a) From F.B.D. of collar B : $+\uparrow \Sigma F_y = 0: T \sin \theta - W = 0$

or

$$\frac{kl}{\cos \theta} (1 - \cos \theta) \sin \theta - W = 0$$



$$\text{or } \tan \theta - \sin \theta = \frac{W}{kl} \quad \blacktriangleleft$$

(b) For

$$W = 3 \text{ lb}$$

$$l = 6 \text{ in.}$$

$$k = 8 \text{ lb/ft}$$

$$l = \frac{6 \text{ in.}}{12 \text{ in./ft}} = 0.5 \text{ ft}$$

$$\tan \theta - \sin \theta = \frac{3 \text{ lb}}{(8 \text{ lb/ft})(0.5 \text{ ft})} = 0.75$$

Solving numerically,

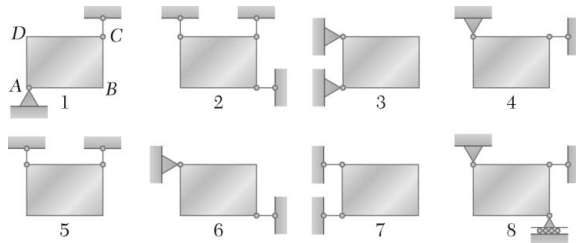
$$\theta = 57.957^\circ$$

or

$$\theta = 58.0^\circ \quad \blacktriangleleft$$

PROBLEM 4.59

Eight identical 500×750 -mm rectangular plates, each of mass $m = 40$ kg, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. In each case, determine whether (a) the plate is completely, partially, or improperly constrained, (b) the reactions are statically determinate or indeterminate, (c) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions.

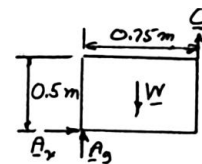


SOLUTION

1. Three non-concurrent, non-parallel reactions:

- (a) Plate: completely constrained
 (b) Reactions: determinate
 (c) Equilibrium maintained

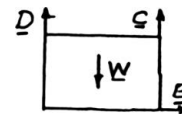
$$A = C = 196.2 \text{ N} \uparrow$$



2. Three non-concurrent, non-parallel reactions:

- (a) Plate: completely constrained
 (b) Reactions: determinate
 (c) Equilibrium maintained

$$B = 0, \quad C = D = 196.2 \text{ N} \uparrow$$

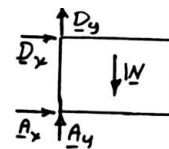


3. Four non-concurrent, non-parallel reactions:

- (a) Plate: completely constrained
 (b) Reactions: indeterminate
 (c) Equilibrium maintained

$$A_x = 294 \text{ N} \rightarrow, \quad D_x = 294 \text{ N} \leftarrow$$

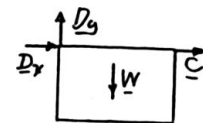
$$(A_y + D_y = 392 \text{ N} \uparrow)$$



4. Three concurrent reactions (through D):

- (a) Plate: improperly constrained
 (b) Reactions: indeterminate
 (c) No equilibrium

$$(\Sigma M_D \neq 0)$$

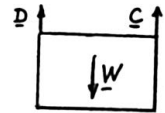


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PROBLEM 4.59 (Continued)

5. Two reactions:

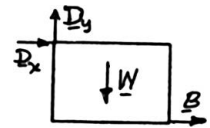
- (a) Plate: partial constraint
- (b) Reactions: determinate
- (c) Equilibrium maintained



$$C = D = 196.2 \text{ N } \uparrow$$

6. Three non-concurrent, non-parallel reactions:

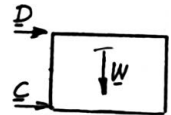
- (a) Plate: completely constrained
- (b) Reactions: determinate
- (c) Equilibrium maintained



$$B = 294 \text{ N } \rightarrow, \quad D = 491 \text{ N } \nearrow 53.1^\circ$$

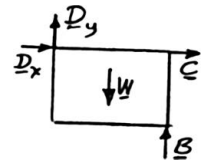
7. Two reactions:

- (a) Plate: improperly constrained
- (b) Reactions determined by dynamics
- (c) No equilibrium ($\Sigma F_y \neq 0$)



8. Four non-concurrent, non-parallel reactions:

- (a) Plate: completely constrained
- (b) Reactions: indeterminate
- (c) Equilibrium maintained

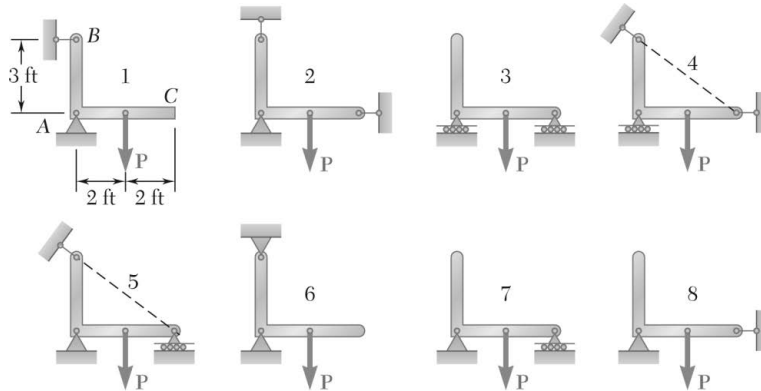


$$B = D_y = 196.2 \text{ N } \uparrow$$

$$(C + D_x = 0)$$

PROBLEM 4.60

The bracket ABC can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. For each case, answer the questions listed in Problem 4.59, and, wherever possible, compute the reactions, assuming that the magnitude of the force P is 100 lb.

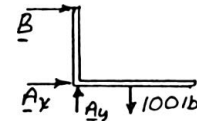


SOLUTION

1. Three non-concurrent, non-parallel reactions:

- (a) Bracket: complete constraint
 (b) Reactions: determinate
 (c) Equilibrium maintained

$$\mathbf{A} = 120.2 \text{ lb } \nearrow 56.3^\circ, \quad \mathbf{B} = 66.7 \text{ lb } \leftarrow$$



2. Four concurrent, reactions (through A):

- (a) Bracket: improper constraint
 (b) Reactions: indeterminate
 (c) No equilibrium

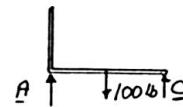
$$(\Sigma M_A \neq 0)$$



3. Two reactions:

- (a) Bracket: partial constraint
 (b) Reactions: indeterminate
 (c) Equilibrium maintained

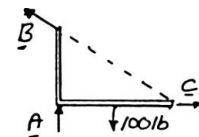
$$\mathbf{A} = 50 \text{ lb } \uparrow, \quad \mathbf{C} = 50 \text{ lb } \uparrow$$



4. Three non-concurrent, non-parallel reactions:

- (a) Bracket: complete constraint
 (b) Reactions: determinate
 (c) Equilibrium maintained

$$\mathbf{A} = 50 \text{ lb } \uparrow, \quad \mathbf{B} = 83.3 \text{ lb } \searrow 36.9^\circ, \quad \mathbf{C} = 66.7 \text{ lb } \rightarrow$$



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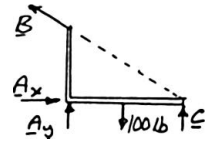
PROBLEM 4.60 (Continued)

5. Four non-concurrent, non-parallel reactions:

(a) Bracket: complete constraint

(b) Reactions: indeterminate

(c) Equilibrium maintained $(\sum M_C = 0) \mathbf{A}_y = 50 \text{ lb} \uparrow$



6. Four non-concurrent, non-parallel reactions:

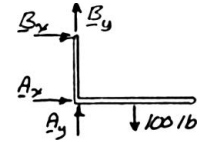
(a) Bracket: complete constraint

(b) Reactions: indeterminate

(c) Equilibrium maintained

$$\mathbf{A}_x = 66.7 \text{ lb} \rightarrow \quad \mathbf{B}_x = 66.7 \text{ lb} \leftarrow$$

$$(\mathbf{A}_y + \mathbf{B}_y = 100 \text{ lb} \uparrow)$$



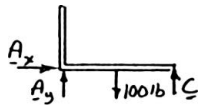
7. Three non-concurrent, non-parallel reactions:

(a) Bracket: complete constraint

(b) Reactions: determinate

(c) Equilibrium maintained

$$\mathbf{A} = \mathbf{C} = 50 \text{ lb} \uparrow$$

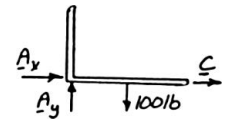


8. Three concurrent, reactions (through A)

(a) Bracket: improper constraint

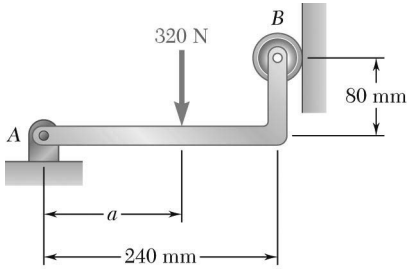
(b) Reactions: indeterminate

(c) No equilibrium $(\sum M_A \neq 0)$



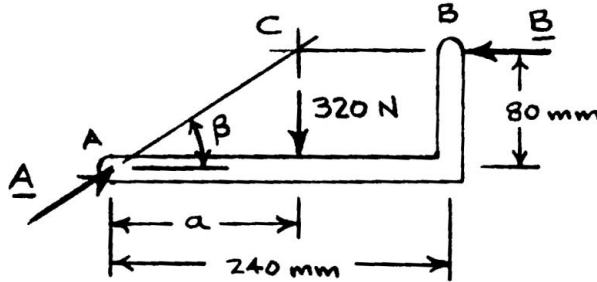
PROBLEM 4.61

Determine the reactions at A and B when $a = 150$ mm.

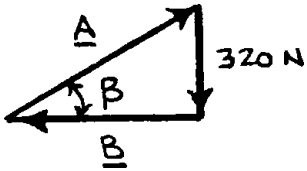


SOLUTION

Free-Body Diagram:



Force triangle



$$\tan \beta = \frac{80 \text{ mm}}{a} = \frac{80 \text{ mm}}{150 \text{ mm}}$$
$$\beta = 28.072^\circ$$

$$A = \frac{320 \text{ N}}{\sin 28.072^\circ}$$

$$A = 680 \text{ N} \swarrow 28.1^\circ \blacktriangleleft$$

$$B = \frac{320 \text{ N}}{\tan 28.072^\circ}$$

$$B = 600 \text{ N} \leftarrow \blacktriangleleft$$

PROBLEM 4.62

Determine the value of a for which the magnitude of the reaction at B is equal to 800 N.

SOLUTION

Free-Body Diagram:

Force triangle

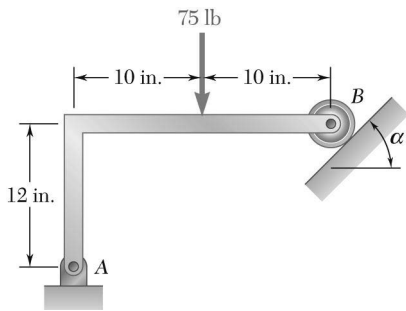
$$\tan \beta = \frac{80 \text{ mm}}{a} \quad a = \frac{80 \text{ mm}}{\tan \beta} \quad (1)$$

From force triangle:

$$\tan \beta = \frac{320 \text{ N}}{800 \text{ N}} = 0.4$$

From Eq. (1):

$$a = \frac{80 \text{ mm}}{0.4} \quad a = 200 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 4.63

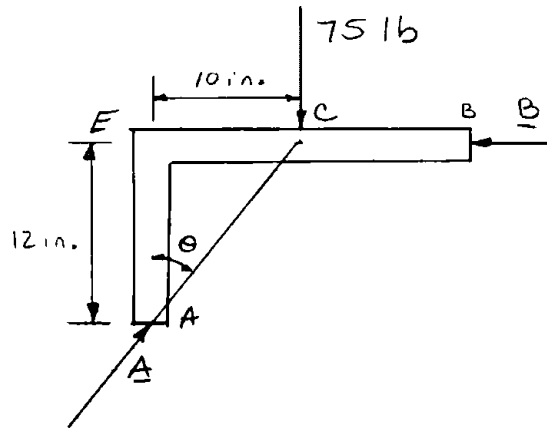
Using the method of Sec. 4.7, solve Problem 4.22b.

PROBLEM 4.22 Determine the reactions at A and B when (a) $\alpha = 0$, (b) $\alpha = 90^\circ$, (c) $\alpha = 30^\circ$.

SOLUTION

Free-Body Diagram:

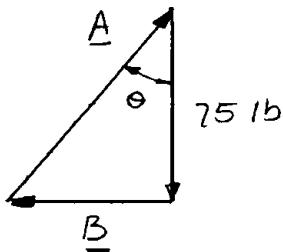
(Three-force body)



The line of action at A must pass through C, where **B** and the 75-lb load intersect.

In triangle ACE: $\tan \theta = \frac{10 \text{ in.}}{12 \text{ in.}}$ $\theta = 39.806^\circ$

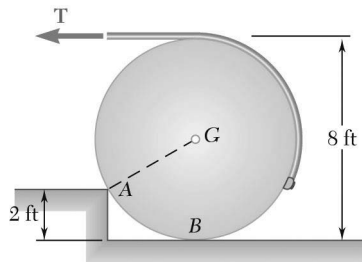
Force triangle



$$B = (75 \text{ lb}) \tan 39.806^\circ = 62.5 \text{ lb}$$

$$A = \frac{75 \text{ lb}}{\cos 39.806^\circ} = 97.6^\circ$$

$A = 97.6 \text{ lb} \nearrow 50.2^\circ$; $B = 62.5 \text{ lb} \leftarrow$

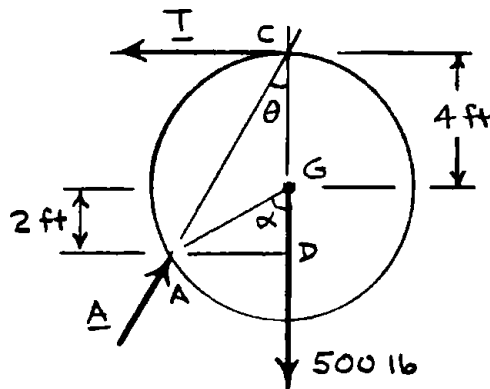


PROBLEM 4.64

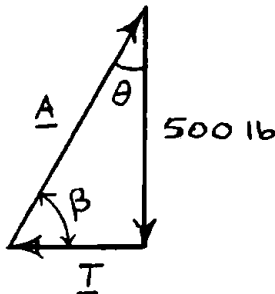
A 500-lb cylindrical tank, 8 ft in diameter, is to be raised over a 2-ft obstruction. A cable is wrapped around the tank and pulled horizontally as shown. Knowing that the corner of the obstruction at A is rough, find the required tension in the cable and the reaction at A.

SOLUTION

Free-Body Diagram:



Force triangle



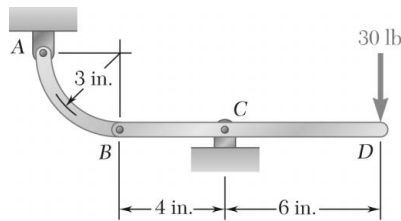
$$\cos \alpha = \frac{GD}{AG} = \frac{2 \text{ ft}}{4 \text{ ft}} = 0.5 \quad \alpha = 60^\circ$$

$$\theta = \frac{1}{2} \alpha = 30^\circ \quad (\beta = 60^\circ)$$

$$T = (500 \text{ lb}) \tan 30^\circ \quad T = 289 \text{ lb}$$

$$A = \frac{500 \text{ lb}}{\cos 30^\circ}$$

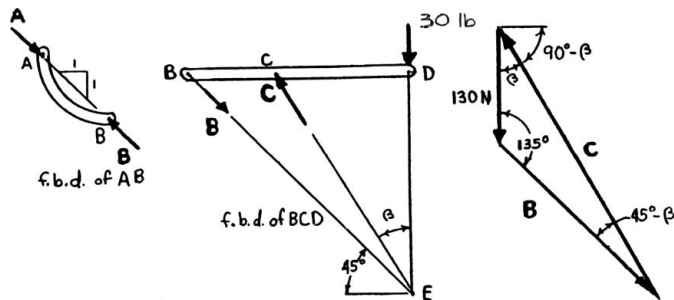
$$A = 577 \text{ lb} \angle 60.0^\circ$$



PROBLEM 4.65

For the frame and loading shown, determine the reactions at A and C.

SOLUTION



Since member AB is acted upon by two forces, A and B , they must be colinear, have the same magnitude, and be opposite in direction for AB to be in equilibrium. The force B acting at B of member BCD will be equal in magnitude but opposite in direction to force B acting on member AB . Member BCD is a three-force body with member forces intersecting at E . The F.B.D.'s of members AB and BCD illustrate the above conditions. The force triangle for member BCD is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1} \left(\frac{6 \text{ in.}}{10 \text{ in.}} \right) = 30.964^\circ$$

Applying the law of sines to the force triangle for member BCD ,

$$\frac{30 \text{ lb}}{\sin(45^\circ - \beta)} = \frac{B}{\sin \beta} = \frac{C}{\sin 135^\circ}$$

or

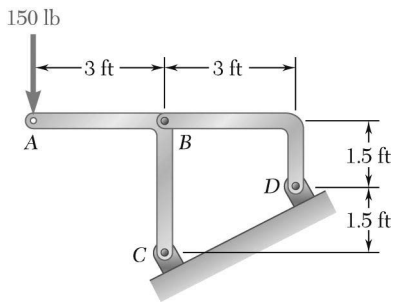
$$\frac{30 \text{ lb}}{\sin 14.036^\circ} = \frac{B}{\sin 30.964^\circ} = \frac{C}{\sin 135^\circ}$$

$$A = B = \frac{(30 \text{ lb}) \sin 30.964^\circ}{\sin 14.036^\circ} = 63.641 \text{ lb}$$

or $A = 63.6 \text{ lb} \searrow 45.0^\circ \blacktriangleleft$

and
$$C = \frac{(30 \text{ lb}) \sin 135^\circ}{\sin 14.036^\circ} = 87.466 \text{ lb}$$

or $C = 87.5 \text{ lb} \nearrow 59.0^\circ \blacktriangleleft$



PROBLEM 4.66

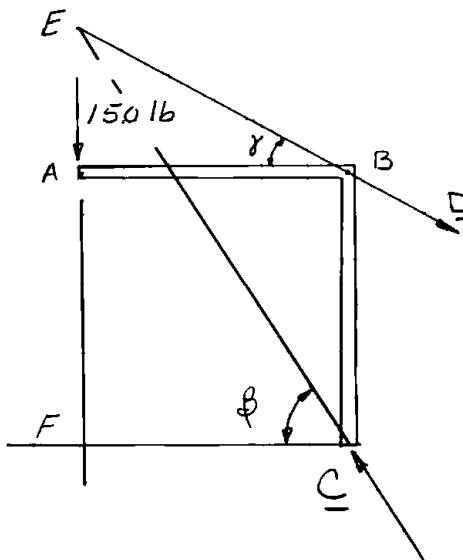
For the frame and loading shown, determine the reactions at C and D .

SOLUTION

Since BD is a two-force member, the reaction at D must pass through Points B and D .

Free-Body Diagram:

(Three-force body)



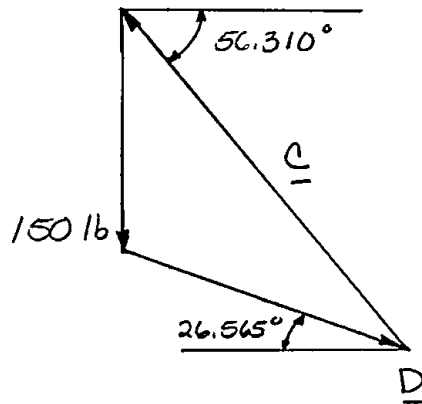
Reaction at C must pass through E , where the reaction at D and the 150-lb load intersect.

Triangle CEF: $\tan \beta = \frac{4.5 \text{ ft}}{3 \text{ ft}} \quad \beta = 56.310^\circ$

Triangle ABE: $\tan \gamma = \frac{1}{2} \quad \gamma = 26.565^\circ$

PROBLEM 4.66 (Continued)

Force Triangle



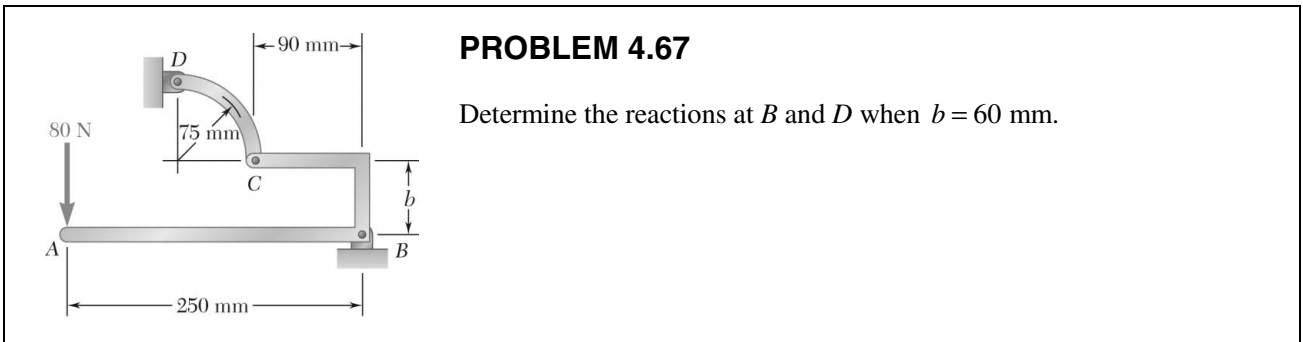
Law of sines:

$$\frac{150 \text{ lb}}{\sin 29.745^\circ} = \frac{C}{\sin 116.565^\circ} = \frac{D}{\sin 33.690^\circ}$$

$$C = 270.42 \text{ lb,}$$

$$D = 167.704 \text{ lb}$$

$$C = 270 \text{ lb} \searrow 56.3^\circ; \quad D = 167.7 \text{ lb} \swarrow 26.6^\circ \blacktriangleleft$$



SOLUTION

Since CD is a two-force member, the line of action of reaction at D must pass through Points C and D . 45°

Free-Body Diagram:
(Three-force body)

Reaction at B must pass through E , where the reaction at D and the 80-N force intersect.

$$\tan \beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348^\circ$$

Force triangle

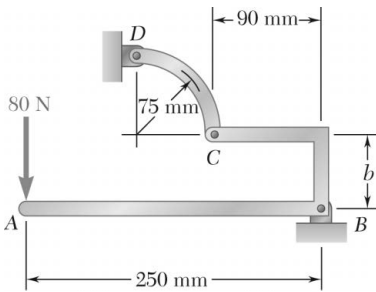
Law of sines:

$$\frac{80 \text{ N}}{\sin 3.652^\circ} = \frac{B}{\sin 45^\circ} = \frac{D}{\sin 131.348^\circ}$$

$$B = 888.0 \text{ N}$$

$$D = 942.8 \text{ N}$$

$B = 888 \text{ N} \searrow 41.3^\circ \quad D = 943 \text{ N} \nearrow 45.0^\circ \blacktriangleleft$



PROBLEM 4.68

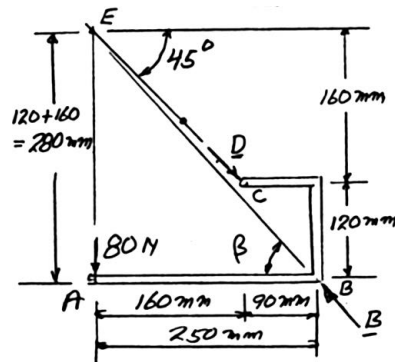
Determine the reactions at B and D when $b = 120$ mm.

SOLUTION

Since CD is a two-force member, line of action of reaction at D must pass through C and D .

Free-Body Diagram:

(Three-force body)

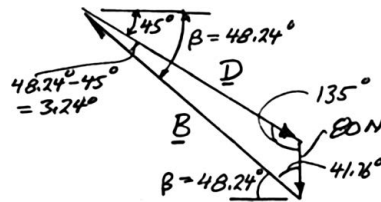


Reaction at B must pass through E , where the reaction at D and the 80-N force intersect.

$$\tan \beta = \frac{280 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 48.24^\circ$$

Force triangle



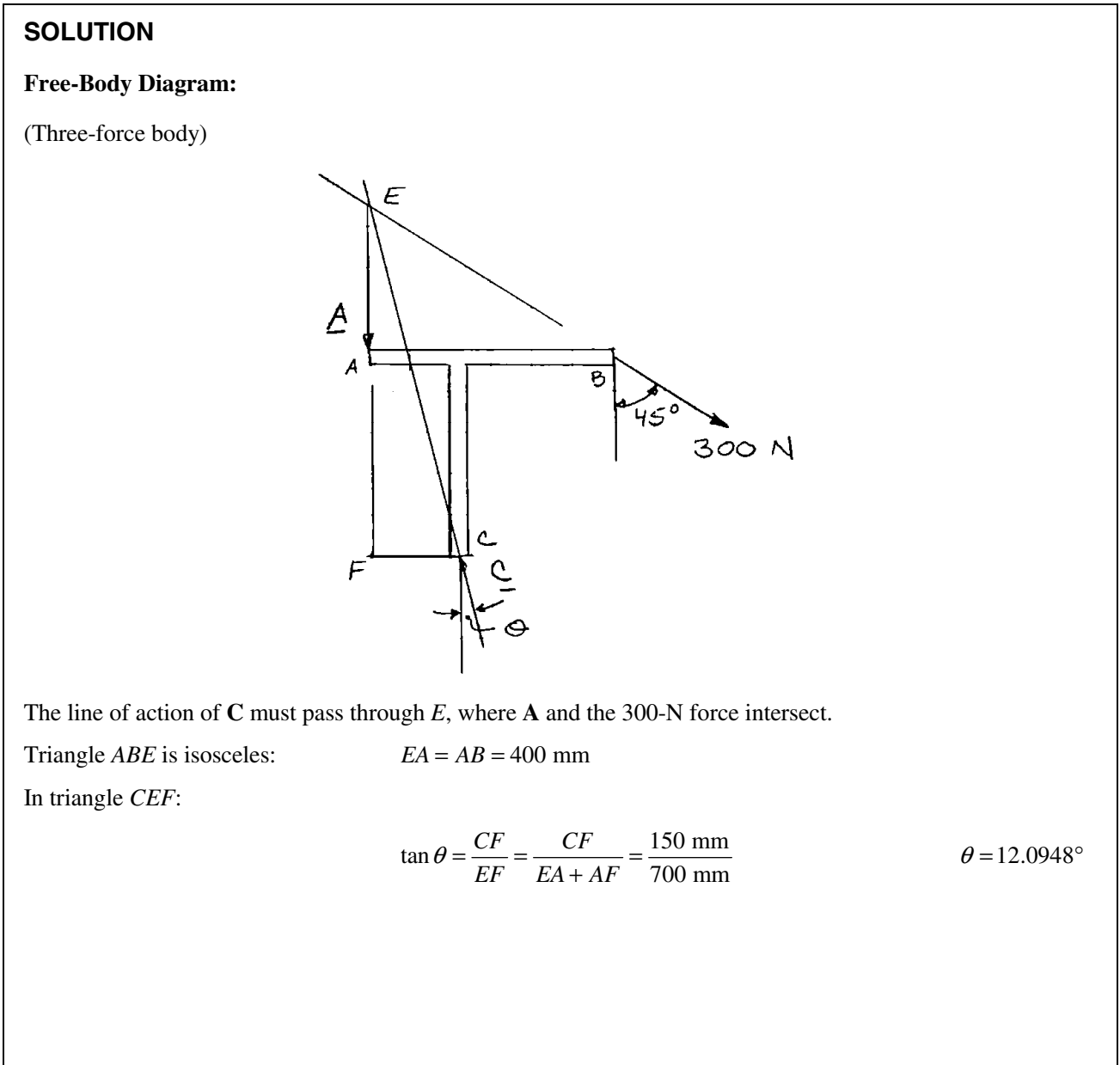
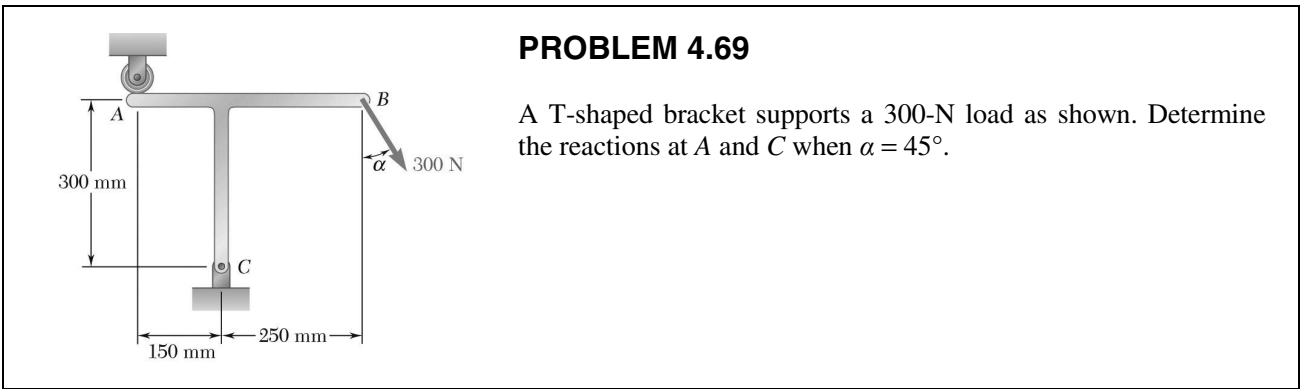
Law of sines:

$$\frac{80 \text{ N}}{\sin 3.24^\circ} = \frac{B}{\sin 135^\circ} = \frac{D}{\sin 41.76^\circ}$$

$$B = 1000.9 \text{ N}$$

$$D = 942.8 \text{ N}$$

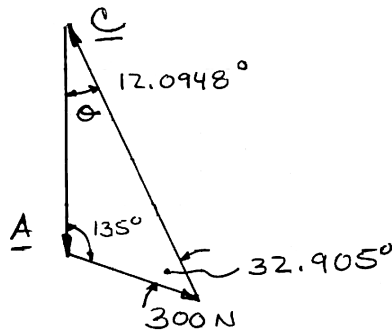
$$\mathbf{B} = 1001 \text{ N} \searrow 48.2^\circ \quad \mathbf{D} = 943 \text{ N} \swarrow 45.0^\circ \blacktriangleleft$$



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PROBLEM 4.69 (Continued)

Force Triangle

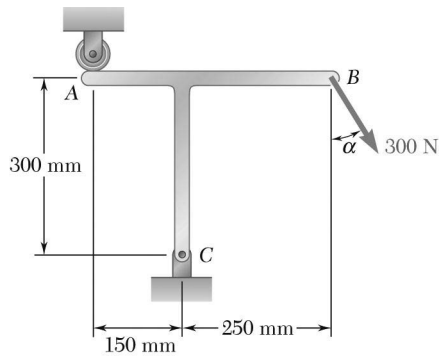


Law of sines:

$$\frac{A}{\sin 32.905^\circ} = \frac{C}{\sin 135^\circ} = \frac{300 \text{ N}}{\sin 12.0948^\circ}$$

$$A = 778 \text{ N} \downarrow; \quad C = 1012 \text{ N} \nearrow 77.9^\circ \blacktriangleleft$$

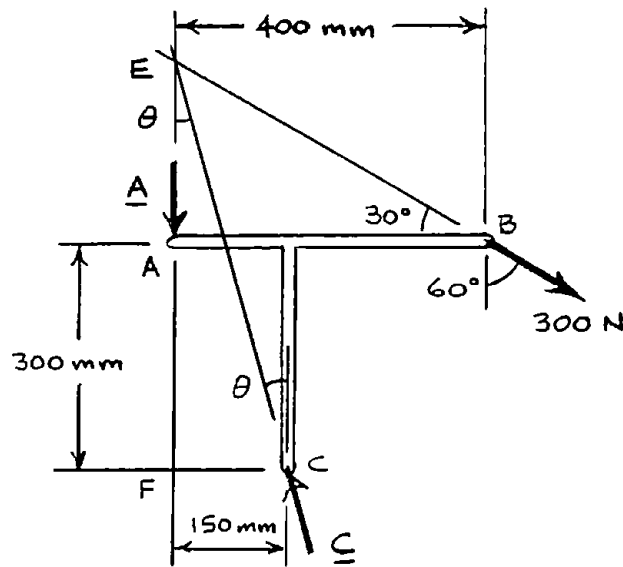
PROBLEM 4.70



A T-shaped bracket supports a 300-N load as shown. Determine the reactions at A and C when $\alpha = 60^\circ$.

SOLUTION

Free-Body Diagram:



$$\begin{aligned}EA &= (400 \text{ mm}) \tan 30^\circ \\ &= 230.94 \text{ mm}\end{aligned}$$

In triangle CEF :

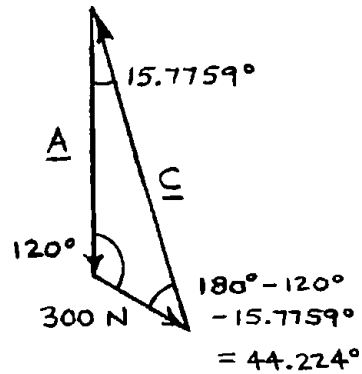
$$\tan \theta = \frac{CF}{EF} = \frac{CF}{EA + AF}$$

$$\begin{aligned}\tan \theta &= \frac{150}{230.94 + 300} \\ \theta &= 15.7759^\circ\end{aligned}$$

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PROBLEM 4.70 (Continued)

Force Triangle



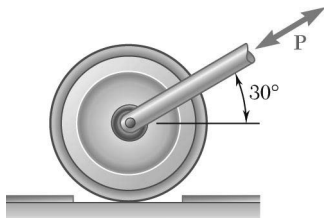
Law of sines:

$$\frac{A}{\sin 44.224^\circ} = \frac{C}{\sin 120^\circ} = \frac{300\text{ N}}{\sin 15.7759^\circ}$$

$$A = 770\text{ N}$$

$$C = 956\text{ N}$$

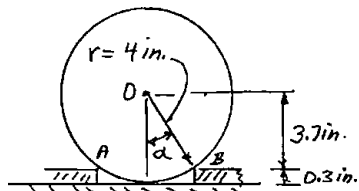
$$A = 770\text{ N} \downarrow; \quad C = 956\text{ N} \nearrow 74.2^\circ \blacktriangleleft$$



PROBLEM 4.71

A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force \mathbf{P} required to move the roller onto the tiles if the roller is (a) pushed to the left, (b) pulled to the right.

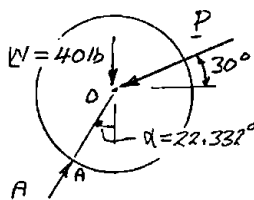
SOLUTION



Geometry: For each case as roller comes into contact with tile,

$$\alpha = \cos^{-1} \frac{3.7 \text{ in.}}{4 \text{ in.}}$$

$$\alpha = 22.332^\circ$$

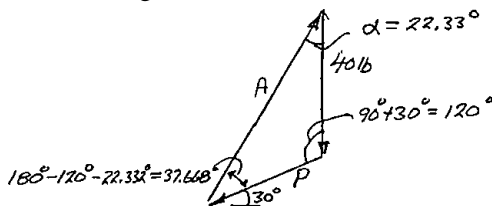


- (a) Roller pushed to left (three-force body):
Forces must pass through O .

Law of sines: $\frac{40 \text{ lb}}{\sin 37.668^\circ} = \frac{P}{\sin 22.332^\circ}; P = 24.87 \text{ lb}$

$$\mathbf{P} = 24.9 \text{ lb } \nearrow 30.0^\circ \blacktriangleleft$$

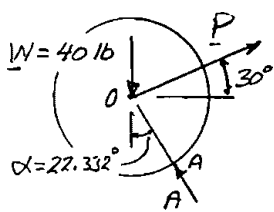
Force Triangle



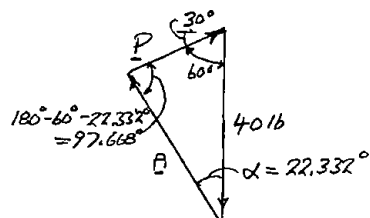
- (b) Roller pulled to right (three-force body):
Forces must pass through O .

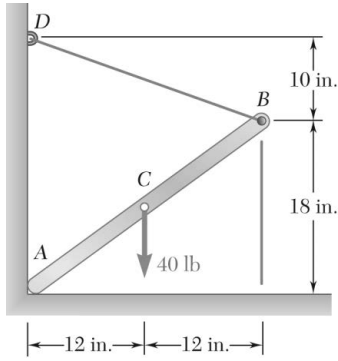
Law of sines: $\frac{40 \text{ lb}}{\sin 97.668^\circ} = \frac{P}{\sin 22.332^\circ}; P = 15.3361 \text{ lb}$

$$\mathbf{P} = 15.34 \text{ lb } \nearrow 30.0^\circ \blacktriangleleft$$



Force Triangle



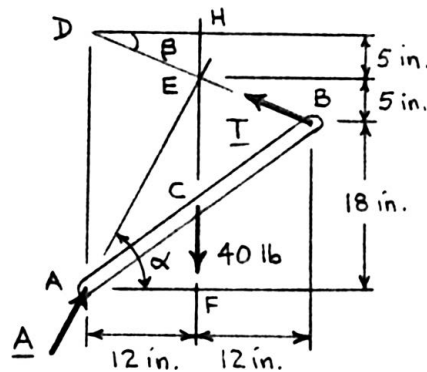


PROBLEM 4.72

One end of rod AB rests in the corner A and the other end is attached to cord BD . If the rod supports a 40-lb load at its midpoint C , find the reaction at A and the tension in the cord.

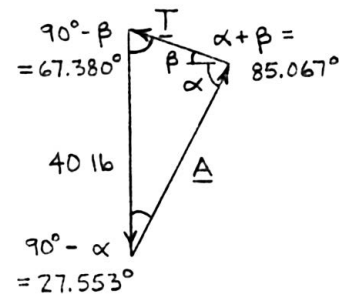
SOLUTION

Free-Body Diagram: (Three-force body)



The line of action of reaction at A must pass through E , where T and the 40-lb load intersect.

Force triangle



$$\tan \alpha = \frac{EF}{AF} = \frac{23}{12}$$

$$\alpha = 62.447^\circ$$

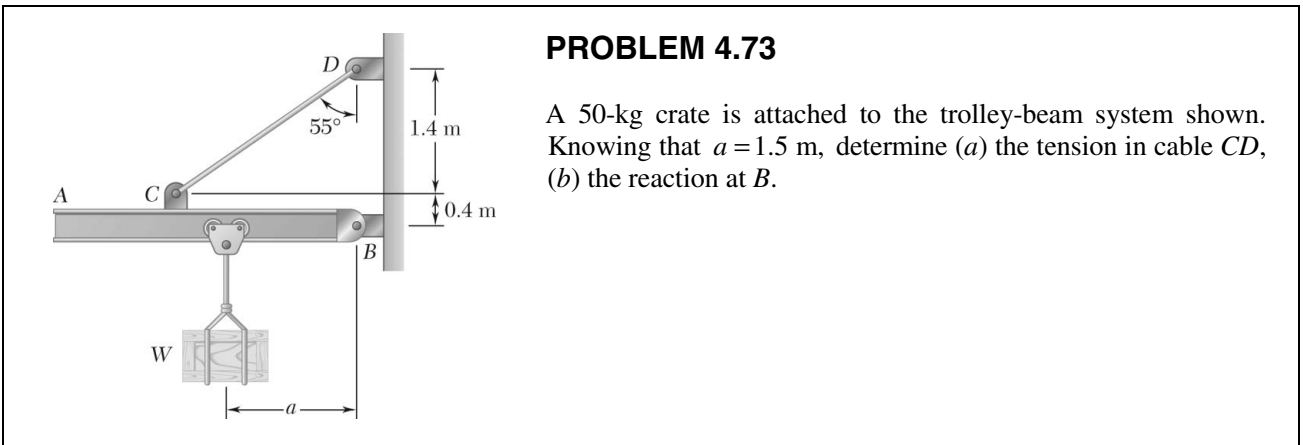
$$\tan \beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620^\circ$$

$$\frac{A}{\sin 67.380^\circ} = \frac{T}{\sin 27.553^\circ} = \frac{40 \text{ lb}}{\sin 85.067^\circ}$$

$$A = 37.1 \text{ lb} \nearrow 62.4^\circ \blacktriangleleft$$

$$T = 18.57 \text{ lb} \blacktriangleleft$$



PROBLEM 4.73

A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (a) the tension in cable CD, (b) the reaction at B.

SOLUTION

Three-force body: \mathbf{W} and \mathbf{T}_{CD} intersect at E .

$$\tan \beta = \frac{0.7497 \text{ m}}{1.5 \text{ m}}$$

$$\beta = 26.556^\circ$$

Three forces intersect at E .

$$W = (50 \text{ kg}) 9.81 \text{ m/s}^2 = 490.50 \text{ N}$$

Law of sines:

$$\frac{490.50 \text{ N}}{\sin 61.556^\circ} = \frac{T_{CD}}{\sin 63.444^\circ} = \frac{B}{\sin 55^\circ}$$

$$T_{CD} = 498.99 \text{ N}$$

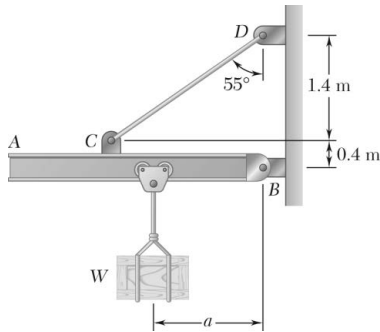
$$B = 456.96 \text{ N}$$

Force triangle

(a) $T_{CD} = 499 \text{ N} \blacktriangleleft$

(b) $B = 457 \text{ N} \blacktriangleright 26.6^\circ \blacktriangleleft$

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PROBLEM 4.74

Solve Problem 4.73, assuming that $a = 3$ m.

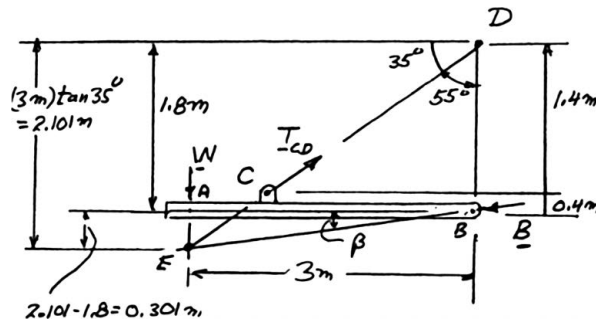
PROBLEM 4.73 A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (a) the tension in cable CD, (b) the reaction at B.

SOLUTION

W and T_{CD} intersect at E .

Free-Body Diagram:

Three-Force Body



$$\tan \beta = \frac{AE}{AB} = \frac{0.301 \text{ m}}{3 \text{ m}}$$

$$\beta = 5.7295^\circ$$

Three forces intersect at E .

$$W = (50 \text{ kg}) 9.81 \text{ m/s}^2$$

$$= 490.50 \text{ N}$$

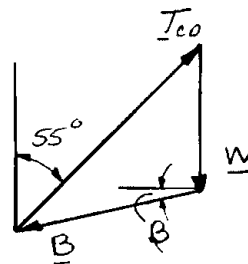
Law of sines:

$$\frac{490.50 \text{ N}}{\sin 29.271^\circ} = \frac{T_{CD}}{\sin 95.730^\circ} = \frac{B}{\sin 55^\circ}$$

$$T_{CD} = 998.18 \text{ N}$$

$$B = 821.76 \text{ N}$$

Force Triangle

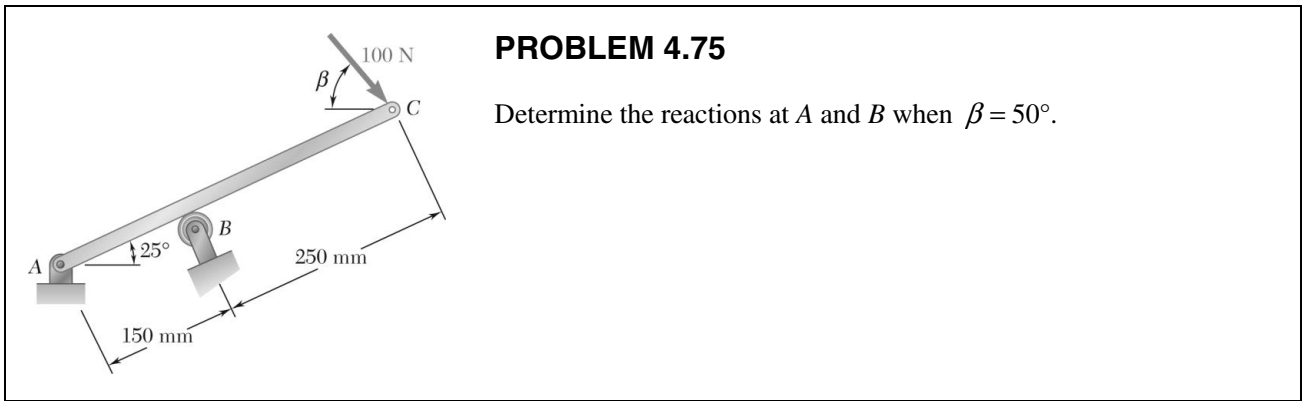


(a)

$$T_{CD} = 998 \text{ N} \blacktriangleleft$$

(b)

$$B = 822 \text{ N} \nearrow 5.73^\circ \blacktriangleleft$$



PROBLEM 4.75

Determine the reactions at A and B when $\beta = 50^\circ$.

SOLUTION

Reaction **A** must pass through Point **D** where the 100-N force and **B** intersect.

In right $\triangle BCD$:

$$\alpha = 90^\circ - 75^\circ = 15^\circ$$

$$BD = 250 \tan 75^\circ = 933.01 \text{ mm}$$

In right $\triangle ABD$:

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.1333^\circ$$

Law of sines:

$$\frac{100 \text{ N}}{\sin 9.1333^\circ} = \frac{A}{\sin 15^\circ} = \frac{B}{\sin 155.867^\circ}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

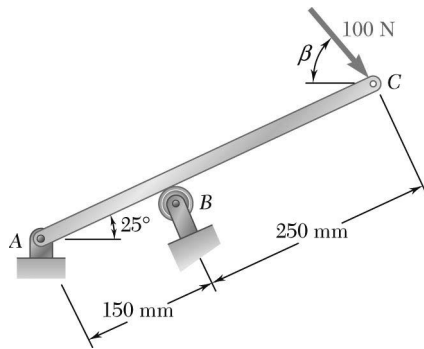
Free-Body Diagram: (Three-force body)

Dimensions in mm

Force Triangle

$A = 163.1 \text{ N} \searrow 74.1^\circ \quad B = 258 \text{ N} \nearrow 65.0^\circ \blacktriangleleft$

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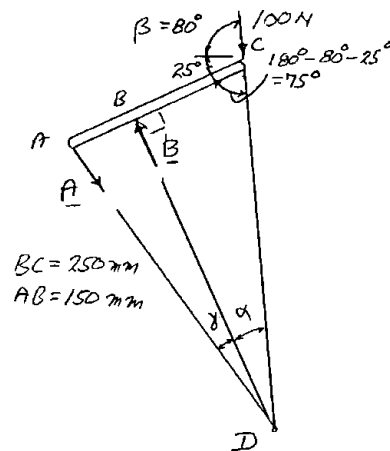
PROBLEM 4.76

Determine the reactions at A and B when $\beta = 80^\circ$.

SOLUTION

Free-Body Diagram:

(Three-force body)



Reaction A must pass through D where the 100-N force and B intersect.

In right triangle BCD:

$$\alpha = 90^\circ - 75^\circ = 15^\circ$$

$$BD = BC \tan 75^\circ = 250 \tan 75^\circ$$

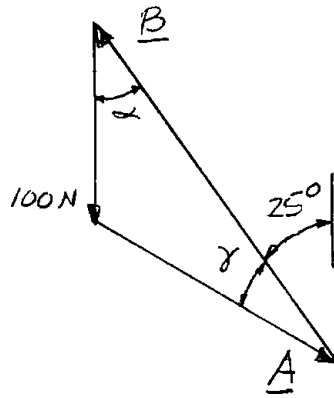
$$BD = 933.01 \text{ mm}$$

In right triangle ABD:

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}} \quad \gamma = 9.1333^\circ$$

PROBLEM 4.76 (Continued)

Force Triangle

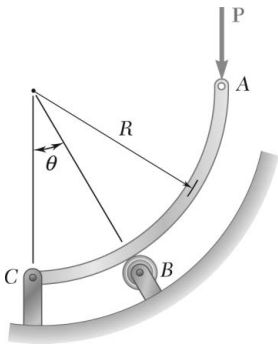


Law of sines:

$$\frac{100 \text{ N}}{\sin 9.1333^\circ} = \frac{A}{\sin 15^\circ} = \frac{B}{\sin 155.867^\circ}$$

$$A = 163.1 \text{ N} \searrow 55.9^\circ \blacktriangleleft$$

$$B = 258 \text{ N} \searrow 65.0^\circ \blacktriangleleft$$



PROBLEM 4.77

Knowing that $\theta = 30^\circ$, determine the reaction (a) at B, (b) at C.

SOLUTION

Reaction at C must pass through D where force P and reaction at B intersect.

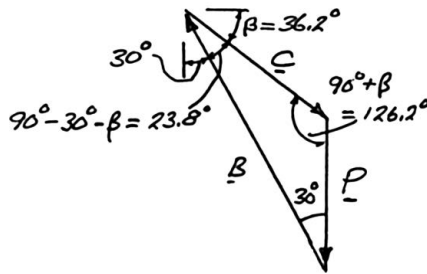
In $\triangle CDE$:

$$\tan \beta = \frac{(\sqrt{3}-1)R}{R}$$

$$= \sqrt{3}-1$$

$$\beta = 36.2^\circ$$

Force Triangle



Law of sines:

$$\frac{P}{\sin 23.8^\circ} = \frac{B}{\sin 126.2^\circ} = \frac{C}{\sin 30^\circ}$$

$$B = 2.00P$$

$$C = 1.239P$$

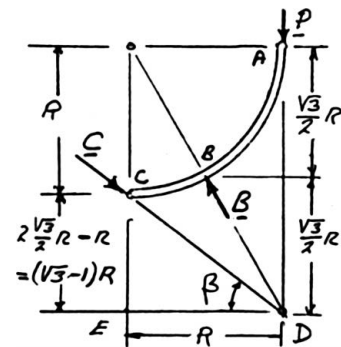
(a)

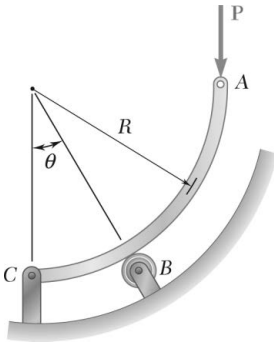
$$B = 2P \nearrow 60.0^\circ \blacktriangleleft$$

(b)

$$C = 1.239P \searrow 36.2^\circ \blacktriangleleft$$

Free-Body Diagram: (Three-force body)





PROBLEM 4.78

Knowing that $\theta = 60^\circ$, determine the reaction (a) at B, (b) at C.

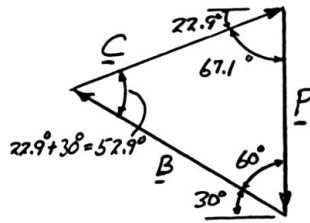
SOLUTION

Reaction at C must pass through D where force P and reaction at B intersect.

In $\triangle CDE$:

$$\begin{aligned} \tan \beta &= \frac{R - \frac{R}{\sqrt{3}}}{R} \\ &= 1 - \frac{1}{\sqrt{3}} \\ \beta &= 22.9^\circ \end{aligned}$$

Force Triangle



Law of sines:

$$\begin{aligned} \frac{P}{\sin 52.9^\circ} &= \frac{B}{\sin 67.1^\circ} = \frac{C}{\sin 60^\circ} \\ B &= 1.155P \\ C &= 1.086P \end{aligned}$$

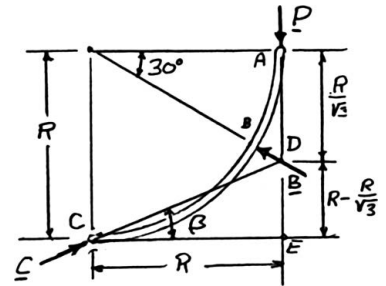
(a)

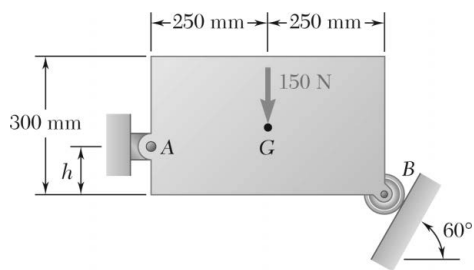
$$B = 1.155P \nearrow 30.0^\circ \blacktriangleleft$$

(b)

$$C = 1.086P \nearrow 22.9^\circ \blacktriangleleft$$

Free-Body Diagram:
(Three-force body)





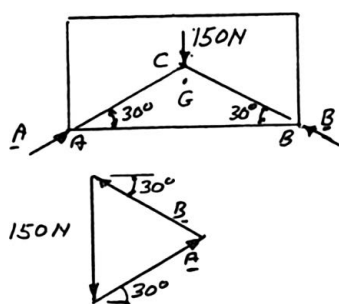
PROBLEM 4.79

Using the method of Section 4.7, solve Problem 4.23.

PROBLEM 4.23 Determine the reactions at A and B when
 (a) $h = 0$, (b) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



(a) $h = 0$

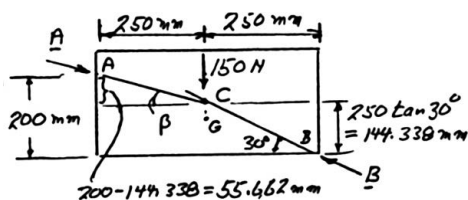
Reaction A must pass through C where the 150-N weight and B intersect.

Force triangle is equilateral.

$$A = 150.0 \text{ N} \swarrow 30.0^\circ \blacktriangleleft$$

$$B = 150.0 \text{ N} \searrow 30.0^\circ \blacktriangleleft$$

(b) $h = 200$ mm



$$\tan \beta = \frac{55.662}{250}$$

$$\beta = 12.5521^\circ$$

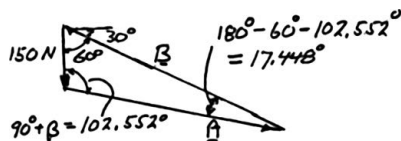
Law of sines:

$$\frac{150 \text{ N}}{\sin 17.448^\circ} = \frac{A}{\sin 60^\circ} = \frac{B}{\sin 102.552^\circ}$$

$$A = 433.24 \text{ N}$$

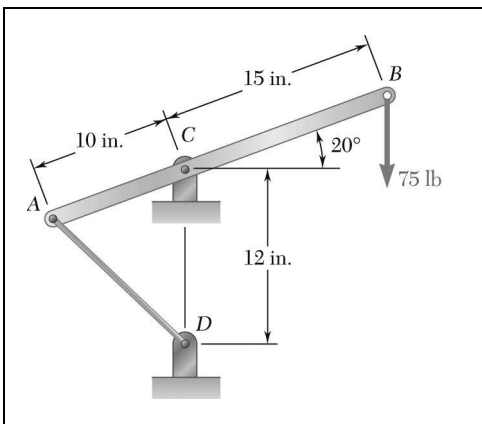
$$B = 488.31 \text{ N}$$

Force Triangle



$$A = 433 \text{ N} \swarrow 12.55^\circ \blacktriangleleft$$

$$B = 488 \text{ N} \searrow 30.0^\circ \blacktriangleleft$$

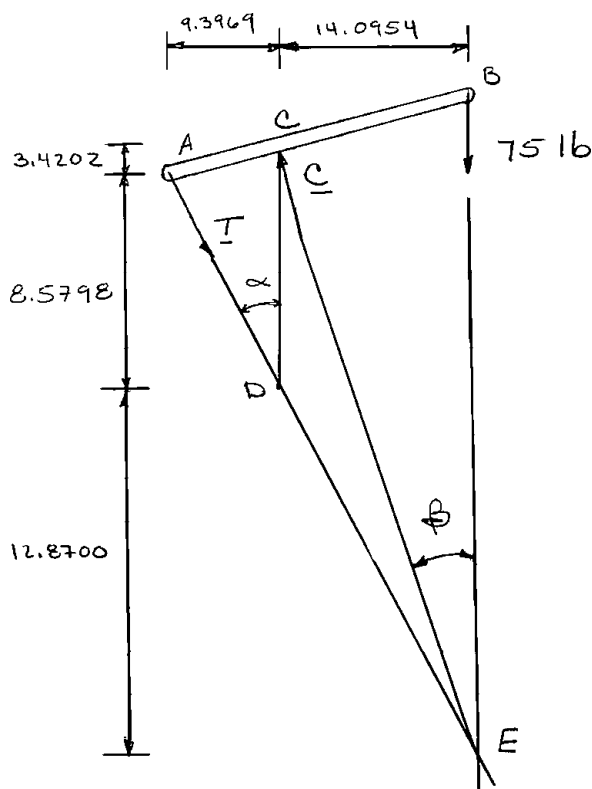


PROBLEM 4.80

Using the method of Section 4.7, solve Problem 4.24.

PROBLEM 4.24 A lever AB is hinged at C and attached to a control cable at A . If the lever is subjected to a 75-lb vertical force at B , determine (a) the tension in the cable, (b) the reaction at C .

SOLUTION



Free-Body Diagram:

Dimensions in in.

Reaction at C must pass through E , where the 75-lb force and T intersect.

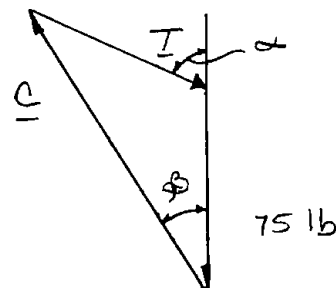
$$\tan \alpha = \frac{9.3969 \text{ in.}}{8.5798 \text{ in.}}$$

$$\alpha = 47.602^\circ$$

$$\tan \beta = \frac{14.0954 \text{ in.}}{24.870 \text{ in.}}$$

$$\beta = 29.543^\circ$$

Force Triangle

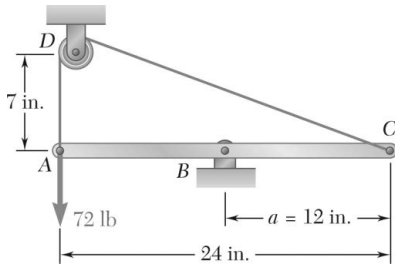


Law of sines:

$$\frac{75 \text{ lb}}{\sin 18.0590^\circ} = \frac{T}{\sin 29.543^\circ} = \frac{C}{\sin 132.398^\circ}$$

- (a) $T = 119.3 \text{ lb} \blacktriangleleft$
- (b) $C = 178.7 \text{ lb} \blacktriangleright 60.5^\circ \blacktriangleleft$

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PROBLEM 4.81

Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D . The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B .

SOLUTION

Reaction at B must pass through D .

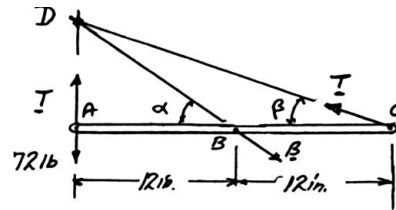
$$\tan \alpha = \frac{7 \text{ in.}}{12 \text{ in.}}$$

$$\alpha = 30.256^\circ$$

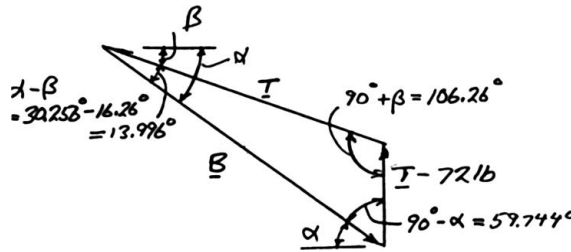
$$\tan \beta = \frac{7 \text{ in.}}{24 \text{ in.}}$$

$$\beta = 16.26^\circ$$

Free-Body Diagram:



Force Triangle



Law of sines:

$$\frac{T}{\sin 59.744^\circ} = \frac{T - 72 \text{ lb}}{\sin 13.996^\circ} = \frac{B}{\sin 106.26^\circ}$$

$$T(\sin 13.996^\circ) = (T - 72 \text{ lb})(\sin 59.744^\circ)$$

$$T(0.24185) = (T - 72)(0.86378)$$

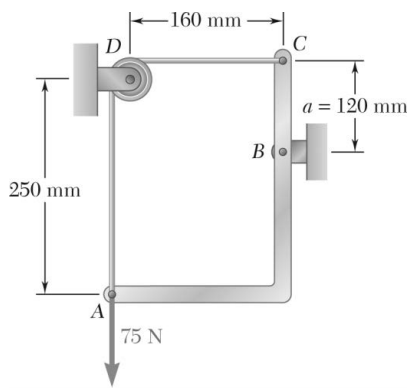
$$T = 100.00 \text{ lb}$$

$$T = 100.0 \text{ lb} \quad \blacktriangleleft$$

$$B = (100 \text{ lb}) \frac{\sin 106.26^\circ}{\sin 59.744^\circ}$$

$$= 111.14 \text{ lb}$$

$$B = 111.1 \text{ lb} \quad \blacktriangleleft 30.3^\circ \quad \blacktriangleleft$$

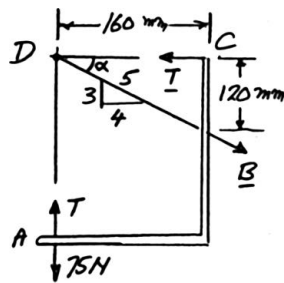


PROBLEM 4.82

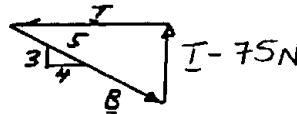
Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D . The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B .

SOLUTION

Free-Body Diagram:



Force Triangle



Reaction at B must pass through D .

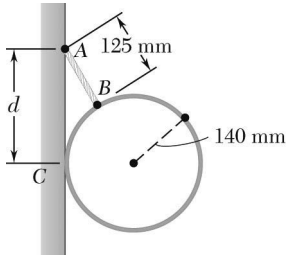
$$\tan \alpha = \frac{120}{160}; \quad \alpha = 36.9^\circ$$

$$\frac{T}{4} = \frac{T - 75 \text{ N}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; \quad T = 300 \text{ N}$$

$$B = \frac{5}{4}T = \frac{5}{4}(300 \text{ N}) = 375 \text{ N}$$

$$\mathbf{B} = 375 \text{ N} \searrow 36.9^\circ$$



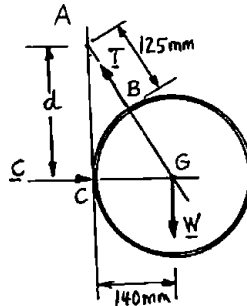
PROBLEM 4.83

A thin ring of mass 2 kg and radius $r = 140$ mm is held against a frictionless wall by a 125-mm string AB . Determine (a) the distance d , (b) the tension in the string, (c) the reaction at C .

SOLUTION

Free-Body Diagram:

(Three-force body)



The force T exerted at B must pass through the center G of the ring, since C and W intersect at that point. Thus, points A , B , and G are in a straight line.

(a) From triangle ACG :

$$d = \sqrt{(AG)^2 - (CG)^2}$$

$$= \sqrt{(265 \text{ mm})^2 - (140 \text{ mm})^2}$$

$$= 225.00 \text{ mm}$$

$$W = (2 \text{ kg})(9.81 \text{ m/s}^2) = 19.6200 \text{ N}$$

Law of sines:

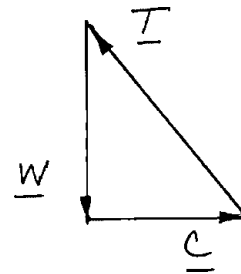
$$\frac{T}{265 \text{ mm}} = \frac{C}{140 \text{ mm}} = \frac{19.6200 \text{ N}}{225.00 \text{ mm}}$$

(b)

(c)

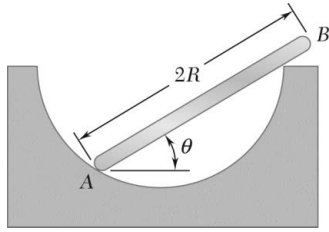
$$d = 225 \text{ mm} \quad \blacktriangleleft$$

Force Triangle



$$T = 23.1 \text{ N} \quad \blacktriangleleft$$

$$C = 12.21 \text{ N} \quad \rightarrow \blacktriangleleft$$



PROBLEM 4.84

A uniform rod AB of length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION

Based on the F.B.D., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O , the center of the circle, and since force C is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA .

$$\alpha = 2\theta$$

The horizontal projections of AE , (x_{AE}), and AG , (x_{AG}), are equal.

$$x_{AE} = x_{AG} = x_A$$

or

$$(AE) \cos 2\theta = (AG) \cos \theta$$

and

$$(2R) \cos 2\theta = R \cos \theta$$

Now

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

then

$$4 \cos^2 \theta - 2 = \cos \theta$$

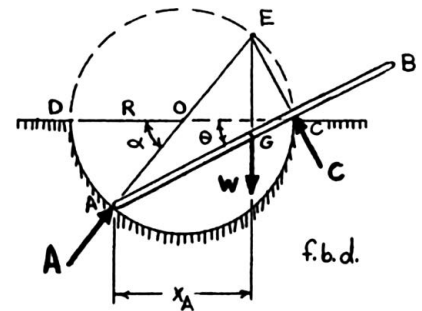
or

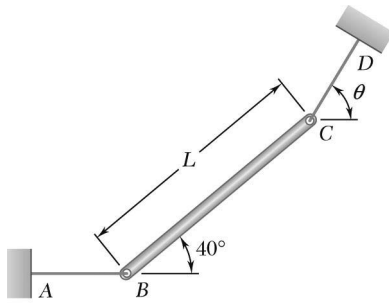
$$4 \cos^2 \theta - \cos \theta - 2 = 0$$

Applying the quadratic equation,

$$\cos \theta = 0.84307 \quad \text{and} \quad \cos \theta = -0.59307$$

$$\theta = 32.534^\circ \quad \text{and} \quad \theta = 126.375^\circ \quad (\text{Discard}) \quad \text{or} \quad \theta = 32.5^\circ \quad \blacktriangleleft$$





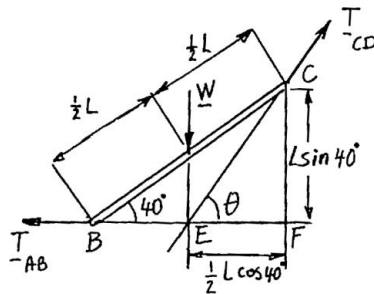
PROBLEM 4.85

A slender rod BC of length L and weight W is held by two cables as shown. Knowing that cable AB is horizontal and that the rod forms an angle of 40° with the horizontal, determine (a) the angle θ that cable CD forms with the horizontal, (b) the tension in each cable.

SOLUTION

Free-Body Diagram:

(Three-force body)



(a) The line of action of T_{CD} must pass through E , where T_{AB} and W intersect.

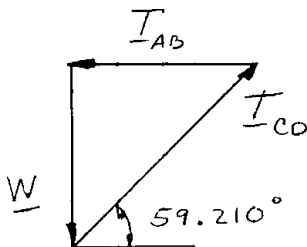
$$\begin{aligned}\tan \theta &= \frac{CF}{EF} \\ &= \frac{L \sin 40^\circ}{\frac{1}{2} L \cos 40^\circ} \\ &= 2 \tan 40^\circ \\ &= 59.210^\circ\end{aligned}$$

$$\theta = 59.2^\circ \blacktriangleleft$$

(b) Force Triangle

$$\begin{aligned}T_{AB} &= W \tan 30.790^\circ \\ &= 0.59588W\end{aligned}$$

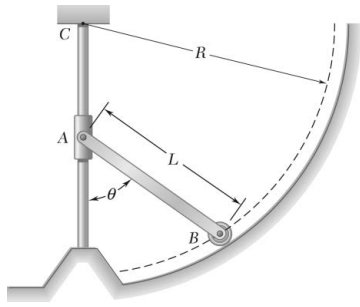
$$T_{AB} = 0.596W \blacktriangleleft$$



$$\begin{aligned}T_{CD} &= \frac{W}{\cos 30.790^\circ} \\ &= 1.16408W\end{aligned}$$

$$T_{CD} = 1.164W \blacktriangleleft$$

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PROBLEM 4.86

A slender rod of length L and weight W is attached to a collar at A and is fitted with a small wheel at B . Knowing that the wheel rolls freely along a cylindrical surface of radius R , and neglecting friction, derive an equation in θ , L , and R that must be satisfied when the rod is in equilibrium.

SOLUTION

Reaction \mathbf{B} must pass through D where \mathbf{B} and \mathbf{W} intersect.

Note that $\triangle ABC$ and $\triangle BGD$ are similar.

$$AC = AE = L \cos \theta$$

In $\triangle ABC$:

$$(CE)^2 + (BE)^2 = (BC)^2$$

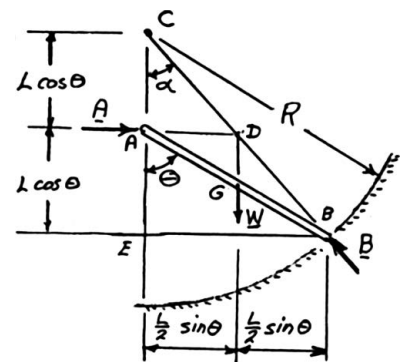
$$(2L \cos \theta)^2 + (L \sin \theta)^2 = R^2$$

$$\left(\frac{R}{L}\right)^2 = 4 \cos^2 \theta + \sin^2 \theta$$

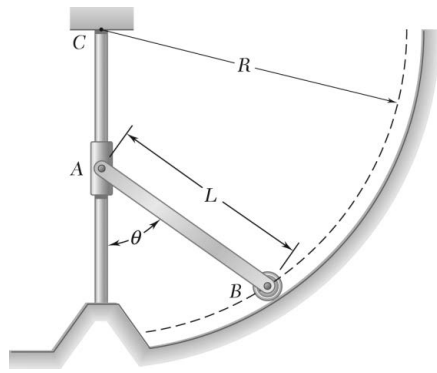
$$\left(\frac{R}{L}\right)^2 = 4 \cos^2 \theta + 1 - \cos^2 \theta$$

$$\left(\frac{R}{L}\right)^2 = 3 \cos^2 \theta + 1$$

Free-Body Diagram (Three-force body)



$$\cos^2 \theta = \frac{1}{3} \left[\left(\frac{R}{L}\right)^2 - 1 \right] \blacktriangleleft$$



PROBLEM 4.87

Knowing that for the rod of Problem 4.86, $L = 15$ in., $R = 20$ in., and $W = 10$ lb, determine (a) the angle θ corresponding to equilibrium, (b) the reactions at A and B.

SOLUTION

See the solution to Problem 4.86 for the free-body diagram and analysis leading to the following equation:

$$\cos^2 \theta = \frac{1}{3} \left[\left(\frac{R}{L} \right)^2 - 1 \right]$$

For $L = 15$ in., $R = 20$ in., and $W = 10$ lb,

$$(a) \quad \cos^2 \theta = \frac{1}{3} \left[\left(\frac{20 \text{ in.}}{15 \text{ in.}} \right)^2 - 1 \right]; \quad \theta = 59.39^\circ \quad \theta = 59.4^\circ \blacktriangleleft$$

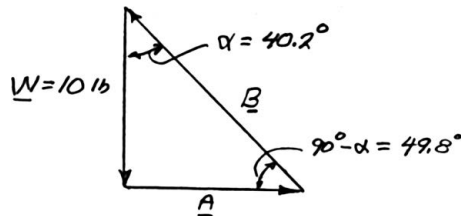
In $\triangle ABC$:

$$\tan \alpha = \frac{BE}{CE} = \frac{L \sin \theta}{2L \cos \theta} = \frac{1}{2} \tan \theta$$

$$\tan \alpha = \frac{1}{2} \tan 59.39^\circ = 0.8452$$

$$\alpha = 40.2^\circ$$

Force Triangle

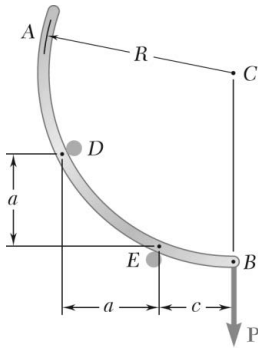


$$A = W \tan \alpha = (10 \text{ lb}) \tan 40.2^\circ = 8.45 \text{ lb}$$

$$B = \frac{W}{\cos \alpha} = \frac{(10 \text{ lb})}{\cos 40.2^\circ} = 13.09 \text{ lb}$$

$$(b) \quad \mathbf{A} = 8.45 \text{ lb} \rightarrow \blacktriangleleft$$

$$\mathbf{B} = 13.09 \text{ lb} \nearrow 49.8^\circ \blacktriangleleft$$



PROBLEM 4.88

Rod AB is bent into the shape of an arc of circle and is lodged between two pegs D and E . It supports a load P at end B . Neglecting friction and the weight of the rod, determine the distance c corresponding to equilibrium when $a = 20$ mm and $R = 100$ mm.

SOLUTION

Since $y_{ED} = x_{ED} = a$,

slope of ED is $\sphericalangle 45^\circ$;

slope of HC is $\sphericalangle 45^\circ$.

Also

$$DE = \sqrt{2}a$$

and

$$DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$$

For triangles DHC and EHC ,

$$\sin \beta = \frac{\frac{a}{\sqrt{2}}}{R} = \frac{a}{\sqrt{2}R}$$

Now

$$c = R \sin(45^\circ - \beta)$$

For

$$a = 20 \text{ mm} \quad \text{and} \quad R = 100 \text{ mm}$$

$$\sin \beta = \frac{20 \text{ mm}}{\sqrt{2}(100 \text{ mm})}$$

$$= 0.141421$$

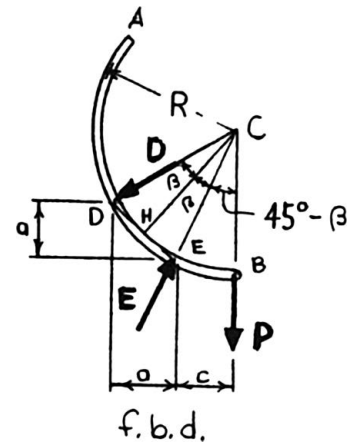
$$\beta = 8.1301^\circ$$

and

$$c = (100 \text{ mm}) \sin(45^\circ - 8.1301^\circ)$$

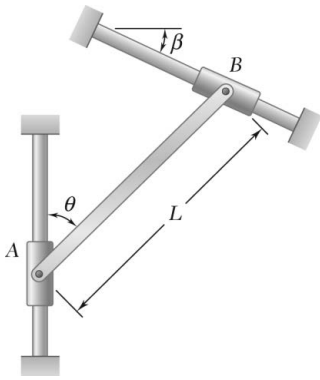
$$= 60.00 \text{ mm}$$

Free-Body Diagram:



$$\text{or } c = 60.0 \text{ mm} \quad \blacktriangleleft$$

PROBLEM 4.89



A slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β .

SOLUTION

As shown in the free-body diagram of the slender rod AB , the three forces intersect at C . From the force geometry:

$$\tan \beta = \frac{x_{GB}}{y_{AB}}$$

where

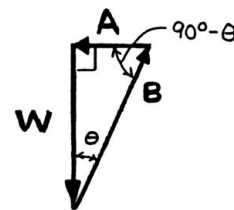
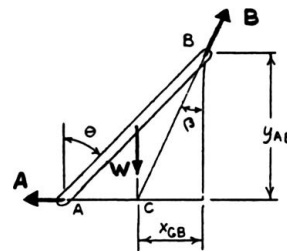
$$y_{AB} = L \cos \theta$$

and

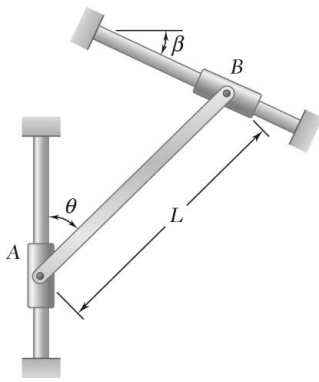
$$x_{GB} = \frac{1}{2} L \sin \theta$$

$$\begin{aligned} \tan \beta &= \frac{\frac{1}{2} L \sin \theta}{L \cos \theta} \\ &= \frac{1}{2} \tan \theta \end{aligned}$$

Free-Body Diagram:



$$\text{or } \tan \theta = 2 \tan \beta \quad \blacktriangleleft$$



PROBLEM 4.90

An 8-kg slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 30^\circ$, determine (a) the angle θ that the rod forms with the vertical, (b) the reactions at A and B.

SOLUTION

- (a) As shown in the free-body diagram of the slender rod AB , the three forces intersect at C . From the geometry of the forces:

$$\tan \beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2} L \sin \theta$$

and

$$y_{BC} = L \cos \theta$$

$$\tan \beta = \frac{1}{2} \tan \theta$$

or

$$\tan \theta = 2 \tan \beta$$

For

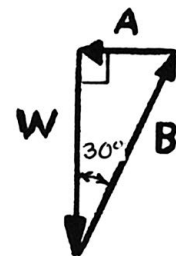
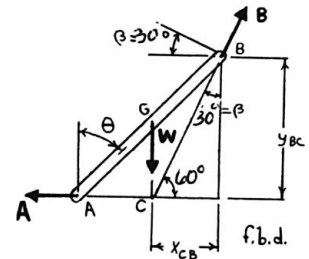
$$\beta = 30^\circ$$

$$\tan \theta = 2 \tan 30^\circ$$

$$= 1.15470$$

$$\theta = 49.107^\circ$$

Free-Body Diagram:



or $\theta = 49.1^\circ \blacktriangleleft$

- (b) $W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.480 \text{ N}$

From force triangle:

$$\begin{aligned} A &= W \tan \beta \\ &= (78.480 \text{ N}) \tan 30^\circ \\ &= 45.310 \text{ N} \end{aligned}$$

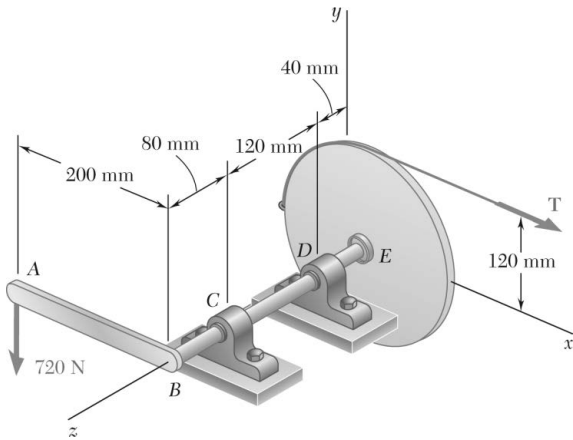
or $A = 45.3 \text{ N} \leftarrow \blacktriangleleft$

and

$$B = \frac{W}{\cos \beta} = \frac{78.480 \text{ N}}{\cos 30^\circ} = 90.621 \text{ N}$$

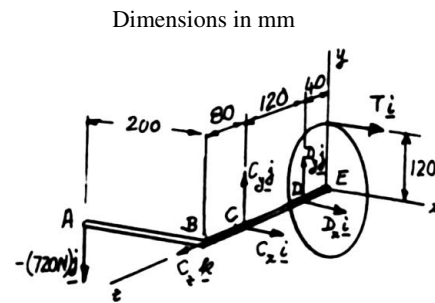
or $B = 90.6 \text{ N} \nearrow 60.0^\circ \blacktriangleleft$

PROBLEM 4.91



A 200-mm lever and a 240-mm-diameter pulley are welded to the axle BE that is supported by bearings at C and D . If a 720-N vertical load is applied at A when the lever is horizontal, determine (a) the tension in the cord, (b) the reactions at C and D . Assume that the bearing at D does not exert any axial thrust.

SOLUTION



We have six unknowns and six equations of equilibrium. —OK

$$\begin{aligned}\Sigma \mathbf{M}_C = 0: & \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 200\mathbf{i}) \times (-720\mathbf{j}) = 0 \\ & \quad -120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\mathbf{k} - 160T\mathbf{j} + 57.6 \times 10^3\mathbf{i} + 144 \times 10^3\mathbf{k} = 0\end{aligned}$$

Equating to zero the coefficients of the unit vectors:

$$\mathbf{k}: \quad -120T + 144 \times 10^3 = 0 \quad (a) \quad T = 1200 \text{ N} \quad \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y + 57.6 \times 10^3 = 0 \quad D_y = -480 \text{ N}$$

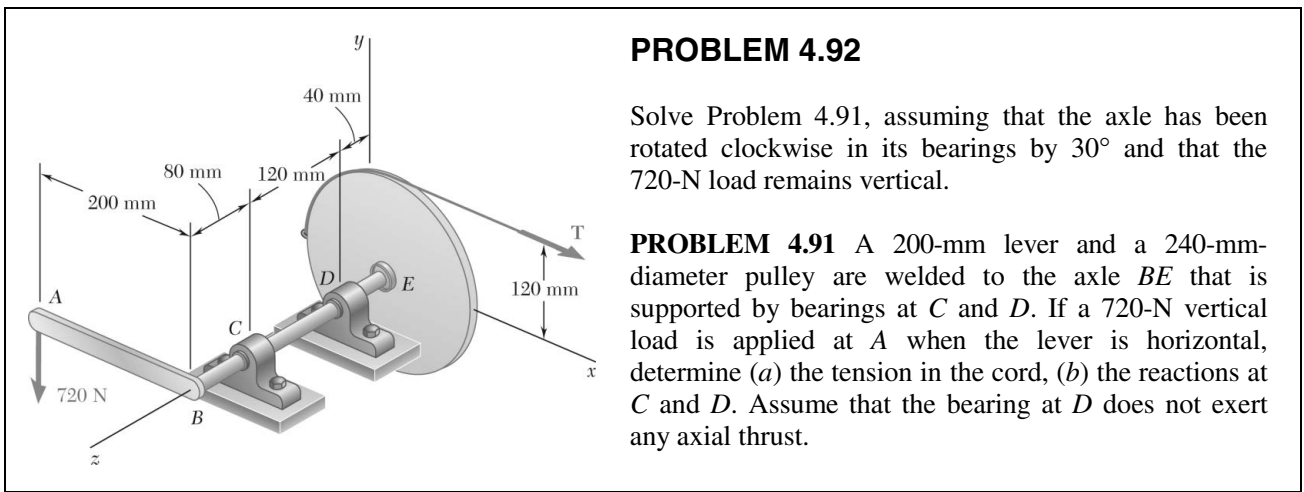
$$\mathbf{j}: \quad -120D_x - 160(1200 \text{ N}) = 0 \quad D_x = -1600 \text{ N}$$

$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \quad C_x = 1600 - 1200 = 400 \text{ N}$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \quad C_y = 480 + 720 = 1200 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

$$(b) \quad \mathbf{C} = (400 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j}; \quad \mathbf{D} = -(1600 \text{ N})\mathbf{i} - (480 \text{ N})\mathbf{j} \quad \blacktriangleleft$$



PROBLEM 4.92

Solve Problem 4.91, assuming that the axle has been rotated clockwise in its bearings by 30° and that the 720-N load remains vertical.

PROBLEM 4.91 A 200-mm lever and a 240-mm-diameter pulley are welded to the axle BE that is supported by bearings at C and D. If a 720-N vertical load is applied at A when the lever is horizontal, determine (a) the tension in the cord, (b) the reactions at C and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

Dimensions in mm

We have six unknowns and six equations of equilibrium.

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 173.21\mathbf{i}) \times (-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\mathbf{k} - 160T\mathbf{j} + 57.6 \times 10^3\mathbf{i} + 124.71 \times 10^3\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{k}: \quad -120T + 124.71 \times 10^3 = 0 \quad T = 1039.2 \text{ N} \quad T = 1039 \text{ N} \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y + 57.6 \times 10^3 = 0 \quad D_y = -480 \text{ N}$$

$$\mathbf{j}: \quad -120D_x - 160(1039.2) \quad D_x = -1385.6 \text{ N}$$

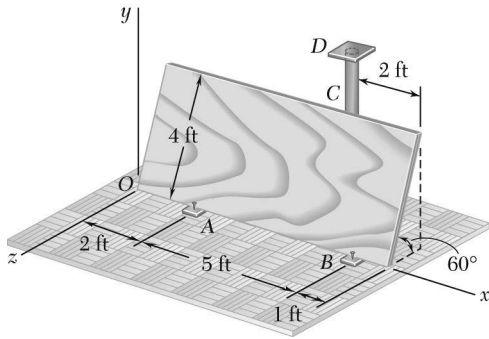
$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \quad C_x = 1385.6 - 1039.2 = 346.4$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \quad C_y = 480 + 720 = 1200 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

(b) $\mathbf{C} = (346 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j} \quad \mathbf{D} = -(1386 \text{ N})\mathbf{i} - (480 \text{ N})\mathbf{j} \blacktriangleleft$

PROBLEM 4.93

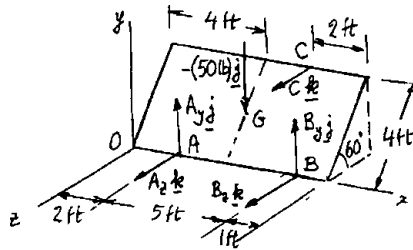


A 4×8 -ft sheet of plywood weighing 40 lb has been temporarily propped against column CD . It rests at A and B on small wooden blocks and against protruding nails. Neglecting friction at all surfaces of contact, determine the reactions at A , B , and C .

SOLUTION

Free-Body Diagram:

We have five unknowns and six equations of equilibrium. Plywood sheet is free to move in x direction, but equilibrium is maintained ($\Sigma F_x = 0$).



$$\Sigma M_A = 0: \mathbf{r}_{B/A} \times (B_y \mathbf{j} + B_z \mathbf{k}) + \mathbf{r}_{C/A} \times C \mathbf{k} + \mathbf{r}_{G/A} \times (-40 \text{ lb}) \mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4 \sin 60^\circ & -4 \cos 60^\circ \\ 0 & 0 & C \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 \sin 60^\circ & -2 \cos 60^\circ \\ 0 & -40 & 0 \end{vmatrix} = 0$$

$$(4C \sin 60^\circ - 80 \cos 60^\circ) \mathbf{i} + (-5B_z - 4C) \mathbf{j} + (5B_y - 80) \mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 4C \sin 60^\circ - 80 \cos 60^\circ = 0 \quad C = 11.5470 \text{ lb}$$

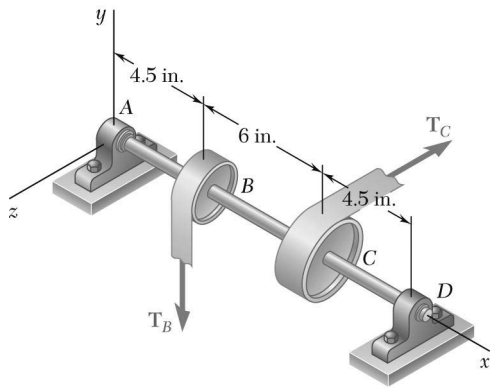
$$\mathbf{j}: \quad -5B_z - 4C = 0 \quad B_z = 9.2376 \text{ lb}$$

$$\mathbf{k}: \quad 5B_y - 80 = 0 \quad B_y = 16.0000 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 40 = 0 \quad A_y = 40 - 16.0000 = 24.000 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + B_z + C = 0 \quad A_z = 9.2376 - 11.5470 = -2.3094 \text{ lb}$$

$$\mathbf{A} = (24.0 \text{ lb}) \mathbf{j} - (2.31 \text{ lb}) \mathbf{k}; \quad \mathbf{B} = (16.00 \text{ lb}) \mathbf{j} - (9.24 \text{ lb}) \mathbf{k}; \quad \mathbf{C} = (11.55 \text{ lb}) \mathbf{k} \quad \blacktriangleleft$$

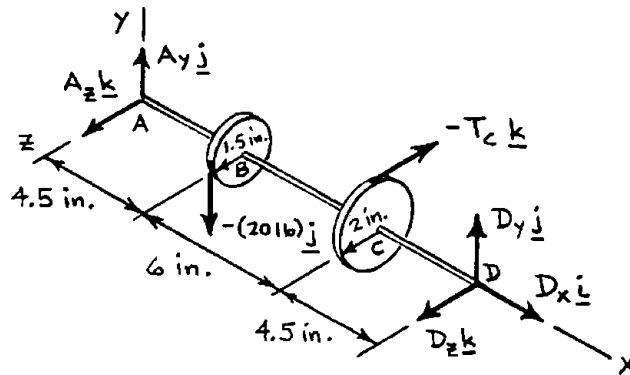


PROBLEM 4.94

Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 1.5 in. and the radius of spool C is 2 in. Knowing that $T_B = 20$ lb and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Free-Body Diagram:



We have six unknowns and six equations of equilibrium.

$$\begin{aligned} \Sigma M_A = 0: & (4.5\mathbf{i} + 1.5\mathbf{k}) \times (-20\mathbf{j}) + (10.5\mathbf{i} + 2\mathbf{j}) \times (-T_C\mathbf{k}) + (15\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0 \\ & -90\mathbf{k} + 30\mathbf{i} + 10.5T_C\mathbf{j} - 2T_C\mathbf{i} + 15D_y\mathbf{k} - 15D_z\mathbf{j} = 0 \end{aligned}$$

Equate coefficients of unit vectors to zero:

$$\bar{\mathbf{i}}: \quad 30 - 2T_C = 0 \quad T_C = 15 \text{ lb}$$

$$\bar{\mathbf{j}}: \quad 10.5T_C - 15D_z = 0 \quad 10.5(15) - 15D_z = 0 \quad D_z = 10.5 \text{ lb}$$

$$\bar{\mathbf{k}}: \quad -90 + 15D_y = 0 \quad D_y = 6 \text{ lb}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

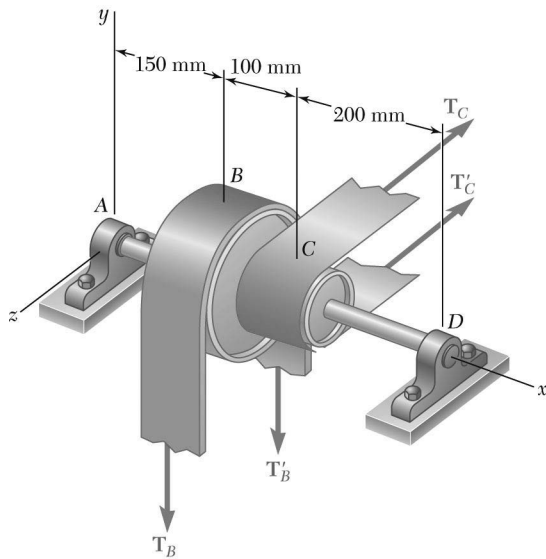
$$\Sigma F_y = 0: \quad A_y + D_y - 20 \text{ lb} = 0 \quad A_y = 20 - 6 = 14 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 15 \text{ lb} = 0 \quad A_z = 15 - 10.5 = 4.5 \text{ lb}$$

$$\mathbf{A} = (14.00 \text{ lb})\mathbf{j} + (4.50 \text{ lb})\mathbf{k}; \quad \mathbf{D} = (6.00 \text{ lb})\mathbf{j} + (10.50 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

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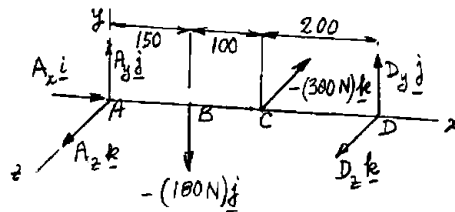
PROBLEM 4.95



Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at A and D . The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt B and 150 N in both portions of belt C , determine the reactions at A and D . Assume that the bearing at D does not exert any axial thrust.

SOLUTION

We replace T_B and T'_B by their resultant $(-180\text{ N})\mathbf{j}$ and T_C and T'_C by their resultant $(-300\text{ N})\mathbf{k}$.



Dimensions in mm

We have five unknowns and six equations of equilibrium. Axle AD is free to rotate about the x -axis, but equilibrium is maintained ($\Sigma M_x = 0$).

$$\Sigma \mathbf{M}_A = 0: (150\mathbf{i}) \times (-180\mathbf{j}) + (250\mathbf{i}) \times (-300\mathbf{k}) + (450\mathbf{i}) \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-27 \times 10^3 \mathbf{k} + 75 \times 10^3 \mathbf{j} + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating coefficients of \mathbf{j} and \mathbf{k} to zero,

$$\mathbf{j}: 75 \times 10^3 - 450D_z = 0 \qquad D_z = 166.7 \text{ N}$$

$$\mathbf{k}: -27 \times 10^3 + 450D_y = 0 \qquad D_y = 60.0 \text{ N}$$

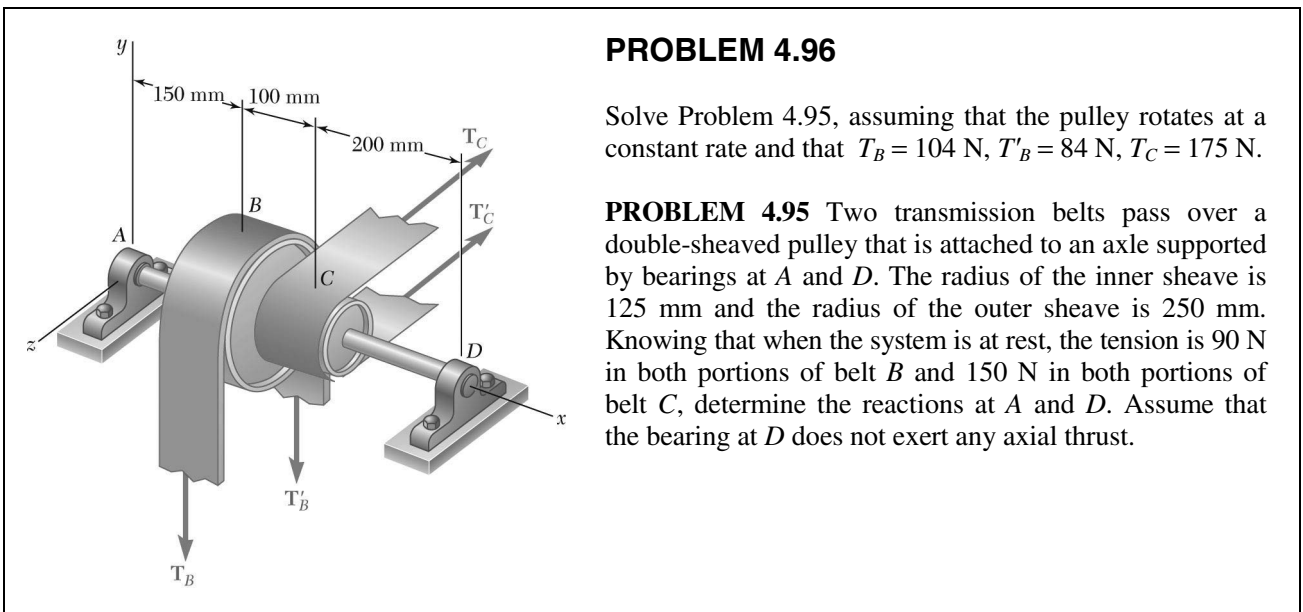
$$\Sigma F_x = 0: A_x = 0$$

$$\Sigma F_y = 0: A_y + D_y - 180 \text{ N} = 0 \qquad A_y = 180 - 60 = 120.0 \text{ N}$$

$$\Sigma F_z = 0: A_z + D_z - 300 \text{ N} = 0 \qquad A_z = 300 - 166.7 = 133.3 \text{ N}$$

$$\mathbf{A} = (120.0 \text{ N})\mathbf{j} + (133.3 \text{ N})\mathbf{k}; \quad \mathbf{D} = (60.0 \text{ N})\mathbf{j} + (166.7 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

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PROBLEM 4.96

Solve Problem 4.95, assuming that the pulley rotates at a constant rate and that $T_B = 104 \text{ N}$, $T'_B = 84 \text{ N}$, $T_C = 175 \text{ N}$.

PROBLEM 4.95 Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at A and D. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt B and 150 N in both portions of belt C, determine the reactions at A and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

We have six unknowns and six equations of equilibrium. —OK

$$\begin{aligned} \Sigma \mathbf{M}_A = 0: & (150\mathbf{i} + 250\mathbf{k}) \times (-104\mathbf{j}) + (150\mathbf{i} - 250\mathbf{k}) \times (-84\mathbf{j}) \\ & + (250\mathbf{i} + 125\mathbf{j}) \times (-175\mathbf{k}) + (250\mathbf{i} - 125\mathbf{j}) \times (-T'_C) \\ & + 450\mathbf{i} \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0 \\ & -150(104 + 84)\mathbf{k} + 250(104 - 84)\mathbf{i} + 250(175 + T'_C)\mathbf{j} - 125(175 - T'_C) \\ & + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0 \end{aligned}$$

Equating the coefficients of the unit vectors to zero,

$$\begin{aligned} \mathbf{i}: & 250(104 - 84) - 125(175 - T'_C) = 0 & 175 = T'_C = 40 & T'_C = 135; \\ \mathbf{j}: & 250(175 + 135) - 450D_z = 0 & D_z = 172.2 \text{ N} \\ \mathbf{k}: & -150(104 + 84) + 450D_y = 0 & D_y = 62.7 \text{ N} \end{aligned}$$

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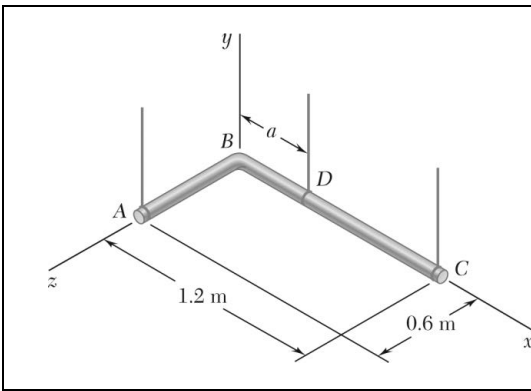
PROBLEM 4.96 (Continued)

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y - 104 - 84 + 62.7 = 0 \quad A_y = 125.3 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z - 175 - 135 + 172.2 = 0 \quad A_z = 137.8 \text{ N}$$

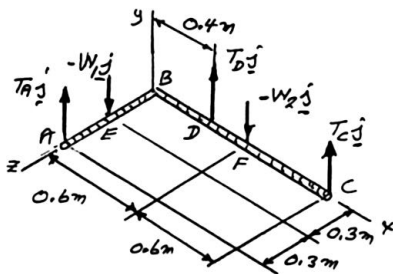
$$\mathbf{A} = (125.3 \text{ N})\mathbf{j} + (137.8 \text{ N})\mathbf{k}; \quad \mathbf{D} = (62.7 \text{ N})\mathbf{j} + (172.2 \text{ N})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 4.97

Two steel pipes AB and BC , each having a mass per unit length of 8 kg/m , are welded together at B and supported by three wires. Knowing that $a = 0.4 \text{ m}$, determine the tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$

$$W_2 = 1.2m'g$$

$$\begin{aligned} \Sigma M_D = 0: \quad & \mathbf{r}_{A/D} \times T_A \mathbf{j} + \mathbf{r}_{E/D} \times (-W_1 \mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2 \mathbf{j}) + \mathbf{r}_{C/D} \times T_C \mathbf{j} = 0 \\ & (-0.4\mathbf{i} + 0.6\mathbf{k}) \times T_A \mathbf{j} + (-0.4\mathbf{i} + 0.3\mathbf{k}) \times (-W_1 \mathbf{j}) + 0.2\mathbf{i} \times (-W_2 \mathbf{j}) + 0.8\mathbf{i} \times T_C \mathbf{j} = 0 \\ & -0.4T_A \mathbf{k} - 0.6T_A \mathbf{i} + 0.4W_1 \mathbf{k} + 0.3W_1 \mathbf{i} - 0.2W_2 \mathbf{k} + 0.8T_C \mathbf{k} = 0 \end{aligned}$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}(0.6m'g) = 0.3m'g$$

$$\begin{aligned} \mathbf{k}: \quad & -0.4T_A + 0.4W_1 - 0.2W_2 + 0.8T_C = 0 \\ & -0.4(0.3m'g) + 0.4(0.6m'g) - 0.2(1.2m'g) + 0.8T_C = 0 \end{aligned}$$

$$T_C = \frac{(0.12 - 0.24 - 0.24)m'g}{0.8} = 0.15m'g$$

$$\begin{aligned} \Sigma F_y = 0: \quad & T_A + T_C + T_D - W_1 - W_2 = 0 \\ & 0.3m'g + 0.15m'g + T_D - 0.6m'g - 1.2m'g = 0 \\ & T_D = 1.35m'g \end{aligned}$$

$$m'g = (8 \text{ kg/m})(9.81 \text{ m/s}^2) = 78.48 \text{ N/m}$$

$$T_A = 0.3m'g = 0.3 \times 78.45$$

$$T_A = 23.5 \text{ N} \quad \blacktriangleleft$$

$$T_B = 0.15m'g = 0.15 \times 78.45$$

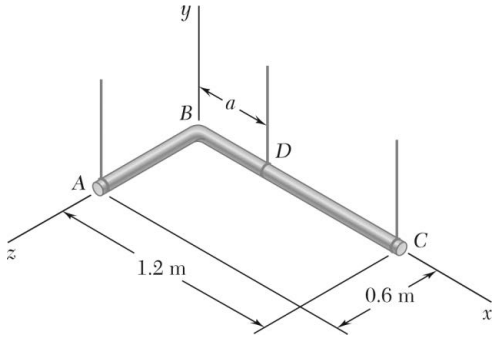
$$T_B = 11.77 \text{ N} \quad \blacktriangleleft$$

$$T_C = 1.35m'g = 1.35 \times 78.45$$

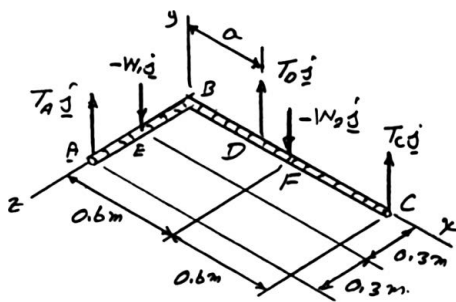
$$T_C = 105.9 \text{ N} \quad \blacktriangleleft$$

PROBLEM 4.98

For the pipe assembly of Problem 4.97, determine (a) the largest permissible value of a if the assembly is not to tip, (b) the corresponding tension in each wire.



SOLUTION



$$W_1 = 0.6m'g$$

$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \mathbf{r}_{A/D} \times T_A \mathbf{j} + \mathbf{r}_{E/D} \times (-W_1 \mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2 \mathbf{j}) + \mathbf{r}_{C/D} \times T_C \mathbf{j} = 0$$

$$(-a\mathbf{i} + 0.6\mathbf{k}) \times T_A \mathbf{j} + (-a\mathbf{i} + 0.3\mathbf{k}) \times (-W_1 \mathbf{j}) + (0.6 - a)\mathbf{i} \times (-W_2 \mathbf{j}) + (1.2 - a)\mathbf{i} \times T_C \mathbf{j} = 0$$

$$-T_A a \mathbf{k} - 0.6T_A \mathbf{i} + W_1 a \mathbf{k} + 0.3W_1 \mathbf{i} - W_2(0.6 - a)\mathbf{k} + T_C(1.2 - a)\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}(0.6m'g) = 0.3m'g$$

$$\mathbf{k}: -T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3m'ga + 0.6m'ga - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a} \quad \text{For maximum } a \text{ and no tipping, } T_C = 0.$$

$$(a) \quad -0.3a + 1.2(0.6 - a) = 0$$

$$-0.3a + 0.72 - 1.2a = 0$$

$$1.5a = 0.72$$

$$a = 0.480 \text{ m} \quad \blacktriangleleft$$

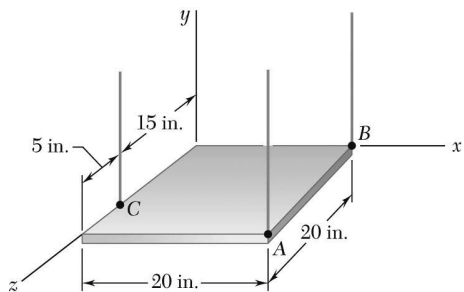
PROBLEM 4.98 (Continued)

(b) Reactions:

$$m'g = (8 \text{ kg/m}) 9.81 \text{ m/s}^2 = 78.48 \text{ N/m}$$
$$T_A = 0.3m'g = 0.3 \times 78.48 = 23.544 \text{ N} \qquad T_A = 23.5 \text{ N} \blacktriangleleft$$
$$\Sigma F_y = 0: T_A + T_C + T_D - W_1 - W_2 = 0$$
$$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$$
$$T_D = 1.8m'g - T_A = 1.8 \times 78.48 - 23.544 = 117.72 \text{ N} \qquad T_D = 117.7 \text{ N} \blacktriangleleft$$

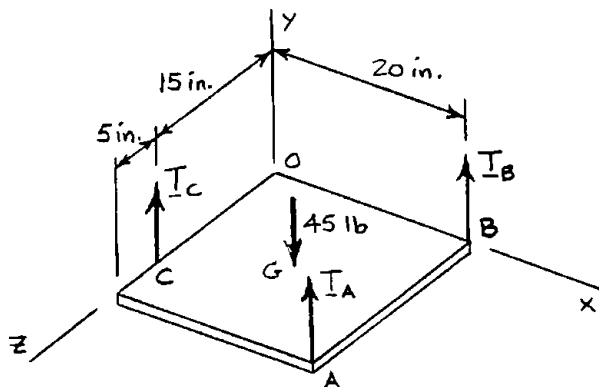
PROBLEM 4.99

The 45-lb square plate shown is supported by three vertical wires. Determine the tension in each wire.



SOLUTION

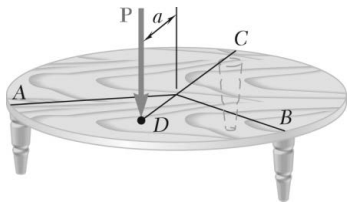
Free-Body Diagram:



$$\begin{aligned} \Sigma M_B = 0: \quad & \mathbf{r}_{C/B} \times T_C \mathbf{j} + \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{G/B} \times (-45 \text{ lb}) \mathbf{j} = 0 \\ & [-(20 \text{ in.})\mathbf{i} + (15 \text{ in.})\mathbf{k}] \times T_C \mathbf{j} + (20 \text{ in.})\mathbf{k} \times T_A \mathbf{j} \\ & + [-(10 \text{ in.})\mathbf{i} + (10 \text{ in.})\mathbf{k}] \times [-(45 \text{ lb})\mathbf{j}] = 0 \\ & -20T_C \mathbf{k} - 15T_C \mathbf{i} - 20T_A \mathbf{i} + 450\mathbf{k} + 450\mathbf{i} = 0 \end{aligned}$$

Equating to zero the coefficients of the unit vectors,

$$\begin{aligned} \mathbf{k}: \quad & -20T_C + 450 = 0 & T_C = 22.5 \text{ lb} \quad \blacktriangleleft \\ \mathbf{i}: \quad & -15(22.5) - 20T_A + 450 = 0 & T_A = 5.625 \text{ lb} \quad \blacktriangleleft \\ \Sigma F_y = 0: \quad & T_A + T_B + T_C - 45 \text{ lb} = 0 \\ & 5.625 \text{ lb} + T_B + 22.5 \text{ lb} - 45 \text{ lb} = 0 & T_B = 16.875 \text{ lb} \quad \blacktriangleleft \\ & & T_A = 5.63 \text{ lb}; T_B = 16.88 \text{ lb}; T_C = 22.5 \text{ lb} \quad \blacktriangleleft \end{aligned}$$



PROBLEM 4.100

The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load \mathbf{P} of magnitude 100 lb is applied to the top of the table at D . Determine the maximum value of a if the table is not to tip over. Show, on a sketch, the area of the table over which \mathbf{P} can act without tipping the table.

SOLUTION

$$r = 2 \text{ ft} \quad b = r \sin 30^\circ = 1 \text{ ft}$$

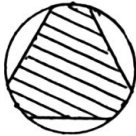
We shall sum moments about AB .

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a-1)100]$$

If table is not to tip, $C \geq 0$.

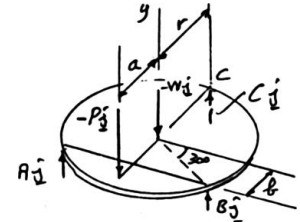


$$[30 - (a-1)100] \geq 0$$

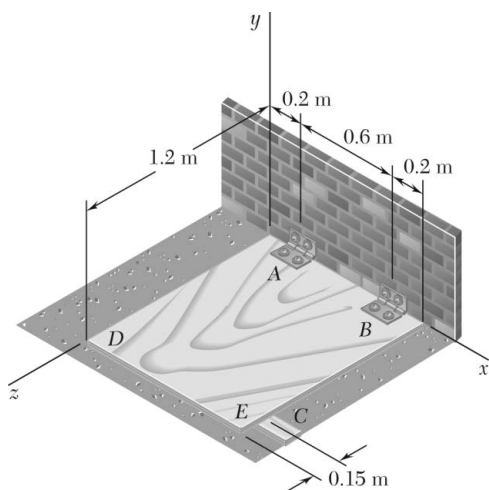
$$30 \geq (a-1)100$$

$$a-1 \leq 0.3 \quad a \leq 1.3 \text{ ft} \quad a = 1.300 \text{ ft}$$

Only \perp distance from P to AB matters. Same condition must be satisfied for each leg. P must be located in shaded area for no tipping.

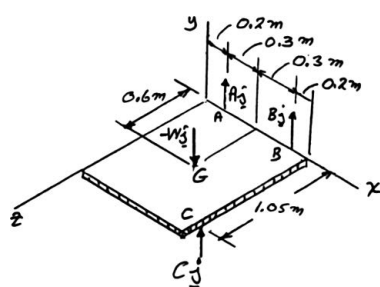


PROBLEM 4.101



An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (a) at *A*, (b) at *B*, (c) at *C*.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$

$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$

$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg})9.81$$

$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right)176.58 \text{ N} = 100.90 \text{ N}$$

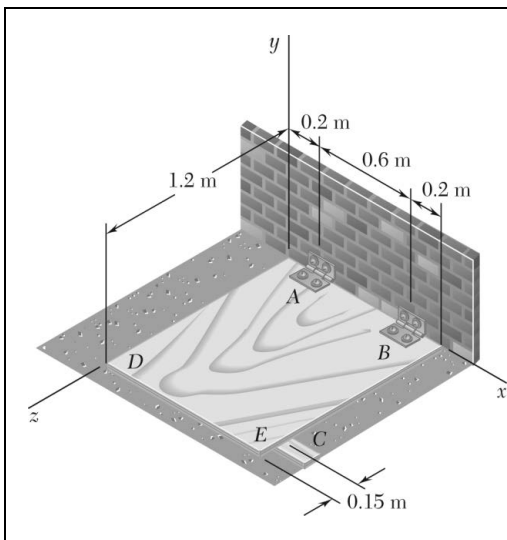
$$\mathbf{k}: 0.6B + 0.8C - 0.3W = 0$$

$$0.6B + 0.8(100.90 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \quad B = -46.24 \text{ N}$$

$$\Sigma F_y = 0: A + B + C - W = 0$$

$$A - 46.24 \text{ N} + 100.90 \text{ N} + 176.58 \text{ N} = 0 \quad A = 121.92 \text{ N}$$

$$(a) A = 121.9 \text{ N} \quad (b) B = -46.2 \text{ N} \quad (c) C = 100.9 \text{ N} \quad \blacktriangleleft$$

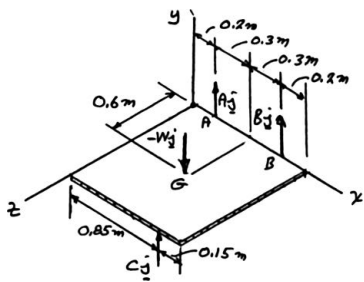


PROBLEM 4.102

Solve Problem 4.101, assuming that the small block C is moved and placed under edge DE at a point 0.15 m from corner E .

PROBLEM 4.101 An opening in a floor is covered by a 1×1.2 -m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C . Determine the vertical component of the reaction (a) at A , (b) at B , (c) at C .

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$

$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$

$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg}) 9.81 \text{ m/s}^2$$

$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -1.2C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.2}\right) 176.58 \text{ N} = 88.29 \text{ N}$$

$$\mathbf{k}: \quad 0.6B + 0.65C - 0.3W = 0$$

$$0.6B + 0.65(88.29 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \quad B = -7.36 \text{ N}$$

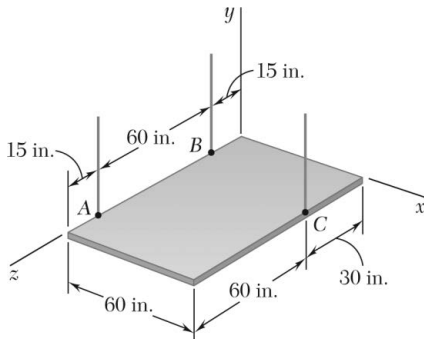
$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 7.36 \text{ N} + 88.29 \text{ N} - 176.58 \text{ N} = 0 \quad A = 95.648 \text{ N}$$

$$(a) \quad A = 95.6 \text{ N} \quad (b) \quad B = -7.36 \text{ N} \quad (c) \quad C = 88.3 \text{ N} \quad \blacktriangleleft$$

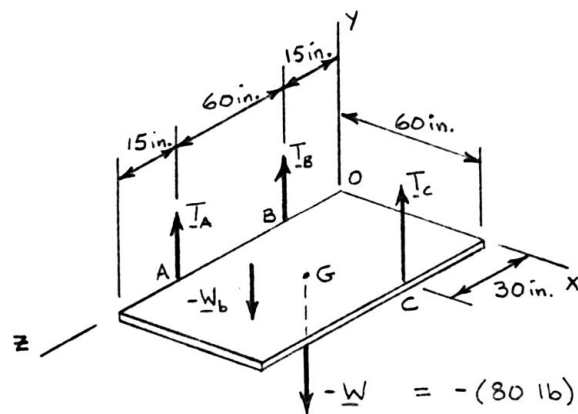
PROBLEM 4.103

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the tension in each wire.



SOLUTION

Free-Body Diagram:



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{C/B} \times T_C \mathbf{j} + \mathbf{r}_{G/B} \times (-80 \text{ lb}) \mathbf{j} = 0$$

$$(60 \text{ in.}) \mathbf{k} \times T_A \mathbf{j} + [(60 \text{ in.}) \mathbf{i} + (15 \text{ in.}) \mathbf{k}] \times T_C \mathbf{j} + [(30 \text{ in.}) \mathbf{i} + (30 \text{ in.}) \mathbf{k}] \times (-80 \text{ lb}) \mathbf{j} = 0$$

$$-60T_A \mathbf{i} + 60T_C \mathbf{k} - 15T_C \mathbf{i} - 2400 \mathbf{k} + 2400 \mathbf{i} = 0$$

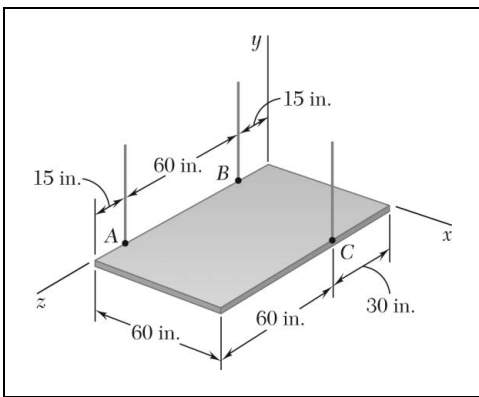
Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad 60T_A - 15(40) + 2400 = 0 \qquad T_A = 30.0 \text{ lb} \quad \blacktriangleleft$$

$$\mathbf{k}: \quad 60T_C - 2400 = 0 \qquad T_C = 40.0 \text{ lb} \quad \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 80 \text{ lb} = 0$$

$$30 \text{ lb} + T_B + 40 \text{ lb} - 80 \text{ lb} = 0 \qquad T_B = 10.00 \text{ lb} \quad \blacktriangleleft$$

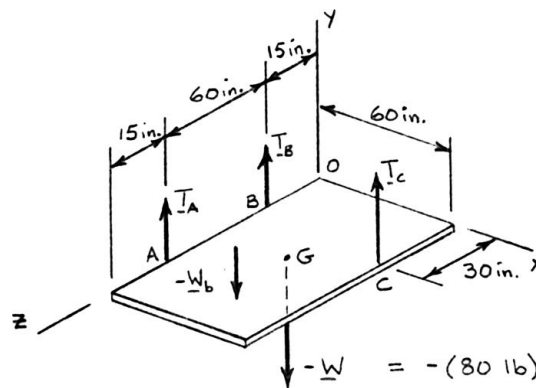


PROBLEM 4.104

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:



Let $-W_b \mathbf{j}$ be the weight of the block and x and z the block's coordinates.

Since tensions in wires are equal, let

$$T_A = T_B = T_C = T$$

$$\Sigma M_O = 0: (\mathbf{r}_A \times T\mathbf{j}) + (\mathbf{r}_B \times T\mathbf{j}) + (\mathbf{r}_C \times T\mathbf{j}) + \mathbf{r}_G \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

$$\text{or } (75\mathbf{k}) \times T\mathbf{j} + (15\mathbf{k}) \times T\mathbf{j} + (60\mathbf{i} + 30\mathbf{k}) \times T\mathbf{j} + (30\mathbf{i} + 45\mathbf{k}) \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

$$\text{or } -75T\mathbf{i} - 15T\mathbf{i} + 60T\mathbf{k} - 30T\mathbf{i} - 30W\mathbf{k} + 45W\mathbf{i} - W_b \times \mathbf{k} + W_b z\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: -120T + 45W + W_b z = 0 \quad (1)$$

$$\mathbf{k}: 60T - 30W - W_b x = 0 \quad (2)$$

$$\text{Also, } \Sigma F_y = 0: 3T - W - W_b = 0 \quad (3)$$

$$\text{Eq. (1) + 40 Eq. (3): } 5W + (z - 40)W_b = 0 \quad (4)$$

$$\text{Eq. (2) - 20 Eq. (3): } -10W - (x - 20)W_b = 0 \quad (5)$$

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PROBLEM 4.104 (Continued)

Solving Eqs. (4) and (5) for W_b/W and recalling that $0 \leq x \leq 60$ in., $0 \leq z \leq 90$ in.,

$$\text{Eq. (4):} \quad \frac{W_b}{W} = \frac{5}{40-z} \geq \frac{5}{40-0} = 0.125$$

$$\text{Eq. (5):} \quad \frac{W_b}{W} = \frac{10}{20-x} \geq \frac{10}{20-0} = 0.5$$

Thus, $(W_b)_{\min} = 0.5W = 0.5(80) = 40$ lb

$$(W_b)_{\min} = 40.0 \text{ lb} \quad \blacktriangleleft$$

Making $W_b = 0.5W$ in Eqs. (4) and (5):

$$5W + (z-40)(0.5W) = 0$$

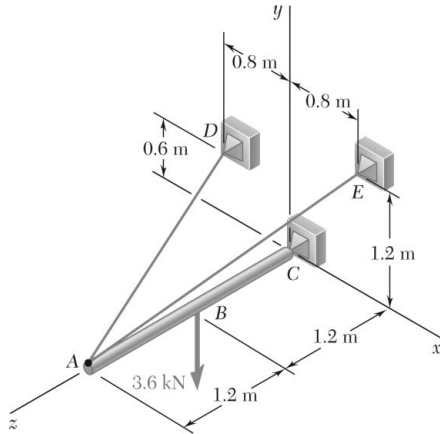
$$z = 30.0 \text{ in.} \quad \blacktriangleleft$$

$$-10W - (x-20)(0.5W) = 0$$

$$x = 0 \text{ in.} \quad \blacktriangleleft$$

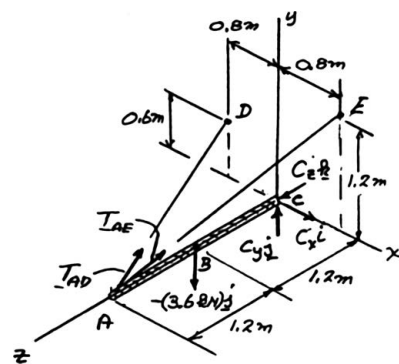
PROBLEM 4.105

A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.



SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\mathbf{r}_B = 1.2\mathbf{k}$$

$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overline{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \quad AD = 2.6 \text{ m}$$

$$\overline{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \quad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overline{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overline{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3.6\text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6\text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \quad (1)$$

$$\mathbf{j}: -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \quad (2)$$

From Eq. (1): $-0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$

$$1.54286T_{AE} = 4.32$$

$$T_{AE} = 2.800 \text{ kN}$$

$$T_{AE} = 2.800 \text{ kN} \quad \blacktriangleleft$$

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PROBLEM 4.105 (Continued)

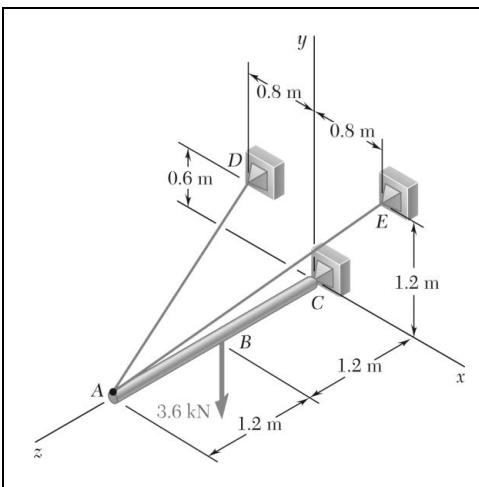
From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600 \text{ kN}$ $T_{AD} = 2.60 \text{ kN} \blacktriangleleft$

$$\Sigma F_x = 0: C_x - \frac{0.8}{2.6}(2.6 \text{ kN}) + \frac{0.8}{2.8}(2.8 \text{ kN}) = 0 \quad C_x = 0$$

$$\Sigma F_y = 0: C_y + \frac{0.6}{2.6}(2.6 \text{ kN}) + \frac{1.2}{2.8}(2.8 \text{ kN}) - (3.6 \text{ kN}) = 0 \quad C_y = 1.800 \text{ kN}$$

$$\Sigma F_z = 0: C_z - \frac{2.4}{2.6}(2.6 \text{ kN}) - \frac{2.4}{2.8}(2.8 \text{ kN}) = 0 \quad C_z = 4.80 \text{ kN}$$

$$\mathbf{C} = (1.800 \text{ kN})\mathbf{j} + (4.80 \text{ kN})\mathbf{k} \blacktriangleleft$$



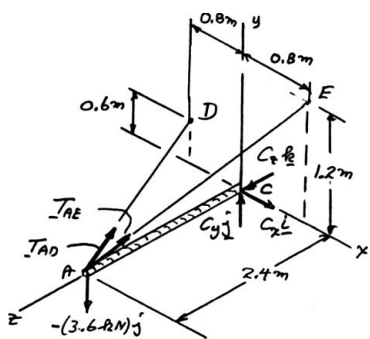
PROBLEM 4.106

Solve Problem 4.105, assuming that the 3.6-kN load is applied at Point A.

PROBLEM 4.105 A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\overline{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \quad AD = 2.6 \text{ m}$$

$$\overline{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \quad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overline{AD}}{AD} = \frac{T_{AD}}{2.6} (-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overline{AE}}{AE} = \frac{T_{AE}}{2.8} (0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_A \times (-3.6 \text{ kN})\mathbf{j}$$

Factor r_A : $\mathbf{r}_A \times (\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3.6 \text{ kN})\mathbf{j})$

or $\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3 \text{ kN})\mathbf{j} = 0$ (Forces concurrent at A)

Coefficient of i: $-\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$

$$T_{AD} = \frac{2.6}{2.8} T_{AE} \quad (1)$$

Coefficient of j: $\frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6 \text{ kN} = 0$

$$\frac{2.6}{2.8} T_{AE} \left(\frac{0.6}{2.6} \right) + \frac{1.2}{2.8} T_{AE} - 3.6 \text{ kN} = 0$$

$$T_{AE} \left(\frac{0.6 + 1.2}{2.8} \right) = 3.6 \text{ kN}$$

$$T_{AE} = 5.600 \text{ kN}$$

$$T_{AE} = 5.60 \text{ kN} \quad \blacktriangleleft$$

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PROBLEM 4.106 (Continued)

From Eq. (1): $T_{AD} = \frac{2.6}{2.8}(5.6) = 5.200 \text{ kN}$ $T_{AD} = 5.20 \text{ kN} \blacktriangleleft$

$$\Sigma F_x = 0: C_x - \frac{0.8}{2.6}(5.2 \text{ kN}) + \frac{0.8}{2.8}(5.6 \text{ kN}) = 0 \quad C_x = 0$$

$$\Sigma F_y = 0: C_y + \frac{0.6}{2.6}(5.2 \text{ kN}) + \frac{1.2}{2.8}(5.6 \text{ kN}) - 3.6 \text{ kN} = 0 \quad C_y = 0$$

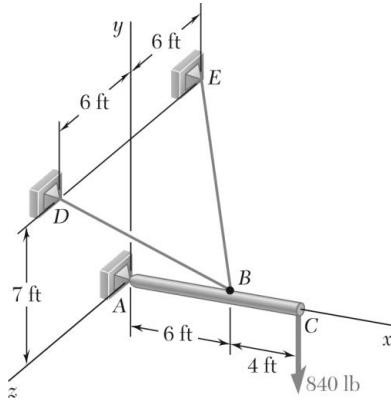
$$\Sigma F_z = 0: C_z - \frac{2.4}{2.6}(5.2 \text{ kN}) - \frac{2.4}{2.8}(5.6 \text{ kN}) = 0 \quad C_z = 9.60 \text{ kN}$$

$$\mathbf{C} = (9.60 \text{ kN})\mathbf{k} \blacktriangleleft$$

Note: Since the forces and reaction are concurrent at A, we could have used the methods of Chapter 2.

PROBLEM 4.107

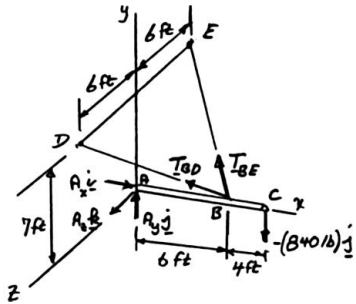
A 10-ft boom is acted upon by the 840-lb force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at A.



SOLUTION

We have five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_x = 0$).

Free-Body Diagram:



$$\overline{BD} = (-6 \text{ ft})\mathbf{i} + (7 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k} \quad BD = 11 \text{ ft}$$

$$\overline{BE} = (-6 \text{ ft})\mathbf{i} + (7 \text{ ft})\mathbf{j} - (6 \text{ ft})\mathbf{k} \quad BE = 11 \text{ ft}$$

$$T_{BD} = T_{BD} \frac{\overline{BD}}{BD} = \frac{T_{BD}}{11} (-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$T_{BE} = T_{BE} \frac{\overline{BE}}{BE} = \frac{T_{BE}}{11} (-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times T_{BD} + \mathbf{r}_B \times T_{BE} + \mathbf{r}_C \times (-840\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11} (-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11} (-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-840\mathbf{j}) = 0$$

$$\frac{42}{11} T_{BD} \mathbf{k} - \frac{36}{11} T_{BD} \mathbf{j} + \frac{42}{11} T_{BE} \mathbf{k} + \frac{36}{11} T_{BE} \mathbf{j} - 8400 \mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -\frac{36}{11} T_{BD} + \frac{36}{11} T_{BE} = 0 \quad T_{BE} = T_{BD}$$

$$\mathbf{k}: \quad \frac{42}{11} T_{BD} + \frac{42}{11} T_{BE} - 8400 = 0$$

$$2 \left(\frac{42}{11} T_{BD} \right) = 8400$$

$$T_{BD} = 1100 \text{ lb} \quad \blacktriangleleft$$

$$T_{BE} = 1100 \text{ lb} \quad \blacktriangleleft$$

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PROBLEM 4.107 (Continued)

$$\Sigma F_x = 0: A_x - \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_x = 1200 \text{ lb}$$

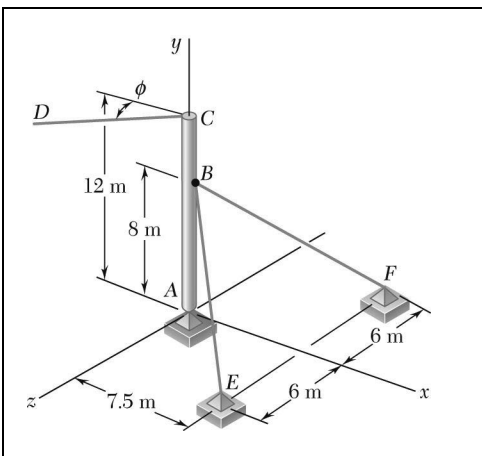
$$\Sigma F_y = 0: A_y + \frac{7}{11}(1100 \text{ lb}) + \frac{7}{11}(1100 \text{ lb}) - 840 \text{ lb} = 0$$

$$A_y = -560 \text{ lb}$$

$$\Sigma F_z = 0: A_z + \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_z = 0$$

$$\mathbf{A} = (1200 \text{ lb})\mathbf{i} - (560 \text{ lb})\mathbf{j} \quad \blacktriangleleft$$

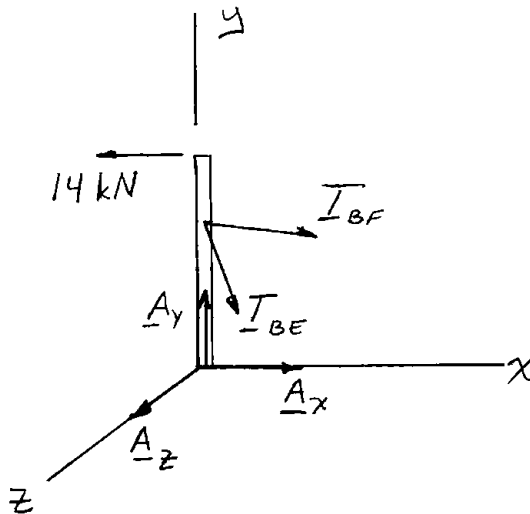


PROBLEM 4.108

A 12-m pole supports a horizontal cable CD and is held by a ball and socket at A and two cables BE and BF . Knowing that the tension in cable CD is 14 kN and assuming that CD is parallel to the x -axis ($\phi = 0$), determine the tension in cables BE and BF and the reaction at A .

SOLUTION

Free-Body Diagram:



There are five unknowns and six equations of equilibrium. The pole is free to rotate about the y -axis, but equilibrium is maintained under the given loading ($\sum M_y = 0$).

Resolve \overline{BE} and \overline{BF} into components:

$$\overline{BE} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} + (6 \text{ m})\mathbf{k} \quad BE = 12.5 \text{ m}$$

$$\overline{BF} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} - (6 \text{ m})\mathbf{k} \quad BF = 12.5 \text{ m}$$

Express \mathbf{T}_{BE} and \mathbf{T}_{BF} in terms of components:

$$\mathbf{T}_{BE} = T_{BE} \frac{\overline{BE}}{BE} = T_{BE} (0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) \quad (1)$$

$$\mathbf{T}_{BF} = T_{BF} \frac{\overline{BF}}{BF} = T_{BF} (0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) \quad (2)$$

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PROBLEM 4.108 (Continued)

$$\Sigma M_A = 0: \mathbf{r}_{B/A} \times T_{BE} + \mathbf{r}_{B/A} \times T_{BF} + \mathbf{r}_{C/A} \times (-14 \text{ kN})\mathbf{i} = 0$$

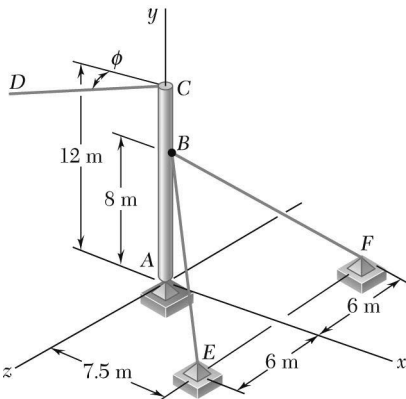
$$8\mathbf{j} \times T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) + 8\mathbf{j} \times T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) + 12\mathbf{j} \times (-14\mathbf{i}) = 0$$

$$-4.8 T_{BE}\mathbf{k} + 3.84 T_{BE}\mathbf{i} - 4.8 T_{BF}\mathbf{k} - 3.84 T_{BF}\mathbf{i} + 168\mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

\mathbf{i} :	$3.84T_{BE} - 3.84T_{BF} = 0$	$T_{BE} = T_{BF}$
\mathbf{k} :	$-4.8T_{BE} - 4.8T_{BF} + 168 = 0$	$T_{BE} = T_{BF} = 17.50 \text{ kN} \blacktriangleleft$
$\Sigma F_x = 0$:	$A_x + 2(0.60)(17.50 \text{ kN}) - 14 \text{ kN} = 0$	$A_x = 7.00 \text{ kN}$
$\Sigma F_y = 0$:	$A_y - 2(0.64)(17.50 \text{ kN}) = 0$	$A_y = 22.4 \text{ kN}$
$\Sigma F_z = 0$:	$A_z + 0 = 0$	$A_z = 0$
		$\mathbf{A} = -(7.00 \text{ kN})\mathbf{i} + (22.4 \text{ kN})\mathbf{j} \blacktriangleleft$

Because of the symmetry, we could have noted at the outset that $T_{BF} = T_{BE}$ and eliminated one unknown.



PROBLEM 4.109

Solve Problem 4.108, assuming that cable CD forms an angle $\phi = 25^\circ$ with the vertical xy plane.

PROBLEM 4.108 A 12-m pole supports a horizontal cable CD and is held by a ball and socket at A and two cables BE and BF . Knowing that the tension in cable CD is 14 kN and assuming that CD is parallel to the x -axis ($\phi = 0$), determine the tension in cables BE and BF and the reaction at A .

SOLUTION

Free-Body Diagram:

$$\overline{BE} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} + (6 \text{ m})\mathbf{k}$$

$$BE = 12.5 \text{ m}$$

$$\overline{BF} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} - (6 \text{ m})\mathbf{k}$$

$$BF = 12.5 \text{ m}$$

$$\mathbf{T}_{BE} = T_{BE} \frac{\overline{BE}}{BE} = T_{BE} (0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k})$$

$$\mathbf{T}_{BF} = T_{BF} \frac{\overline{BF}}{BF} = T_{BF} (0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k})$$

$$\Sigma M_A = 0: \mathbf{r}_{B/A} \times \mathbf{T}_{BE} + \mathbf{r}_{B/A} \times \mathbf{T}_{BF} + \mathbf{r}_{C/A} \times \mathbf{T}_{CD} = 0$$

$$8\mathbf{j} \times T_{BE} (0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) + 8\mathbf{j} \times T_{BF} (0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) + 12\mathbf{j} \times (19 \text{ kN})(-\cos 25^\circ \mathbf{i} + \sin 25^\circ \mathbf{k}) = 0$$

$$-4.8T_{BE} \mathbf{k} + 3.84T_{BE} \mathbf{i} - 4.8T_{BF} \mathbf{k} - 3.84T_{BF} \mathbf{i} + 152.6 \mathbf{k} - 71.00 \mathbf{i} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: 3.84T_{BE} - 3.84T_{BF} + 71.00 = 0; \quad T_{BF} - T_{BE} = 18.4896$$

$$\mathbf{k}: -4.8T_{BE} - 4.8T_{BF} + 152.26 = 0; \quad T_{BF} + T_{BE} = 31.721$$

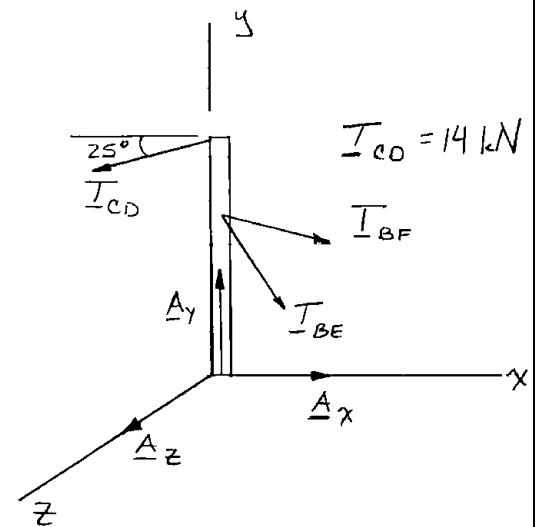
Solving simultaneously, $T_{BE} = 6.6157 \text{ kN}; \quad T_{BF} = 25.105 \text{ kN}$

$$T_{BE} = 6.62 \text{ kN}; \quad T_{BF} = 25.1 \text{ kN} \quad \blacktriangleleft$$

$$\Sigma F_x = 0: A_x + (0.60)(T_{BF} + T_{BE}) - 14 \cos 25^\circ = 0$$

$$A_x = 12.6883 - 0.60(31.7207)$$

$$A_x = -6.34 \text{ kN}$$



PROBLEM 4.109 (Continued)

$$\Sigma F_y = 0: A_y - (0.64)(T_{BF} + T_{BE}) = 0$$

$$A_y = 0.64(31.721)$$

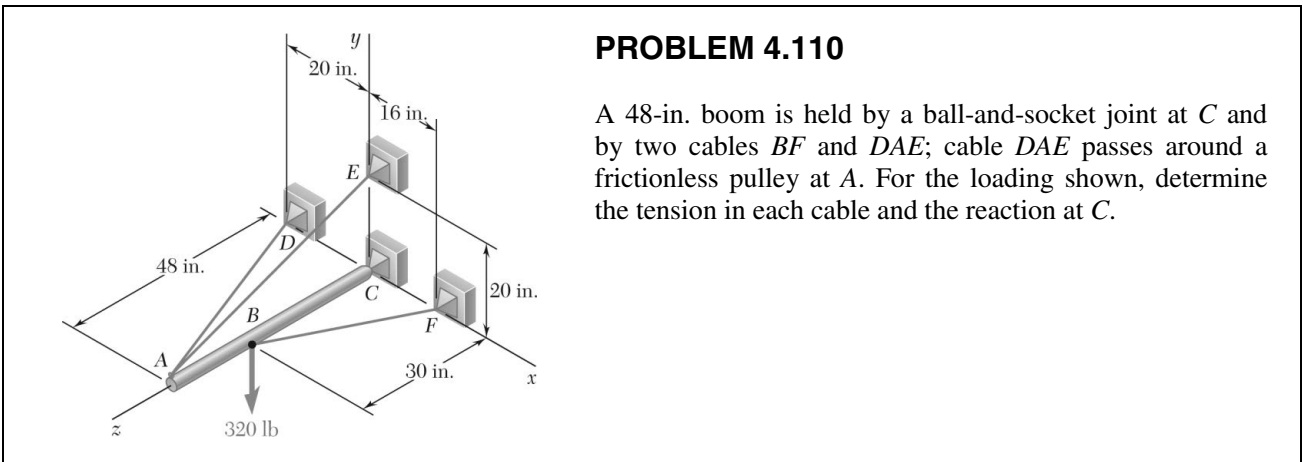
$$A_y = 20.3 \text{ kN}$$

$$\Sigma F_z = 0: A_z - 0.48(T_{BF} - T_{BE}) + 14 \sin 25^\circ = 0$$

$$A_z = 0.48(18.4893) - 5.9167$$

$$A_z = 2.96 \text{ kN}$$

$$\mathbf{A} = -(6.34 \text{ kN})\mathbf{i} + (20.3 \text{ kN})\mathbf{j} + (2.96 \text{ kN})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 4.110

A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:

Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).

$T =$ Tension in both parts of cable DAE.

$r_B = 30\mathbf{k}$
 $r_A = 48\mathbf{k}$

$\overline{AD} = -20\mathbf{i} - 48\mathbf{k} \quad AD = 52 \text{ in.}$
 $\overline{AE} = 20\mathbf{j} - 48\mathbf{k} \quad AE = 52 \text{ in.}$
 $\overline{BF} = 16\mathbf{i} - 30\mathbf{k} \quad BF = 34 \text{ in.}$

$\mathbf{T}_{AD} = T \frac{\overline{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$
 $\mathbf{T}_{AE} = T \frac{\overline{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$
 $\mathbf{T}_{BF} = T_{BF} \frac{\overline{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$

$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_B \times (-320 \text{ lb})\mathbf{j} = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix} \frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix} \frac{T_{BF}}{17} + (30\mathbf{k}) \times (-320\mathbf{j}) = 0$$

Coefficient of \mathbf{i} : $-\frac{240}{13}T + 9600 = 0 \quad T = 520 \text{ lb}$

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PROBLEM 4.110 (Continued)

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(520) \quad T_{BD} = 680 \text{ lb}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $-\frac{20}{52}(520) + \frac{8}{17}(680) + C_x = 0$

$$-200 + 320 + C_x = 0 \quad C_x = -120 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(520) - 320 + C_y = 0$

$$200 - 320 + C_y = 0 \quad C_y = 120 \text{ lb}$$

Coefficient of **k**: $-\frac{48}{52}(520) - \frac{48}{52}(520) - \frac{30}{34}(680) + C_z = 0$

$$-480 - 480 - 600 + C_z = 0$$

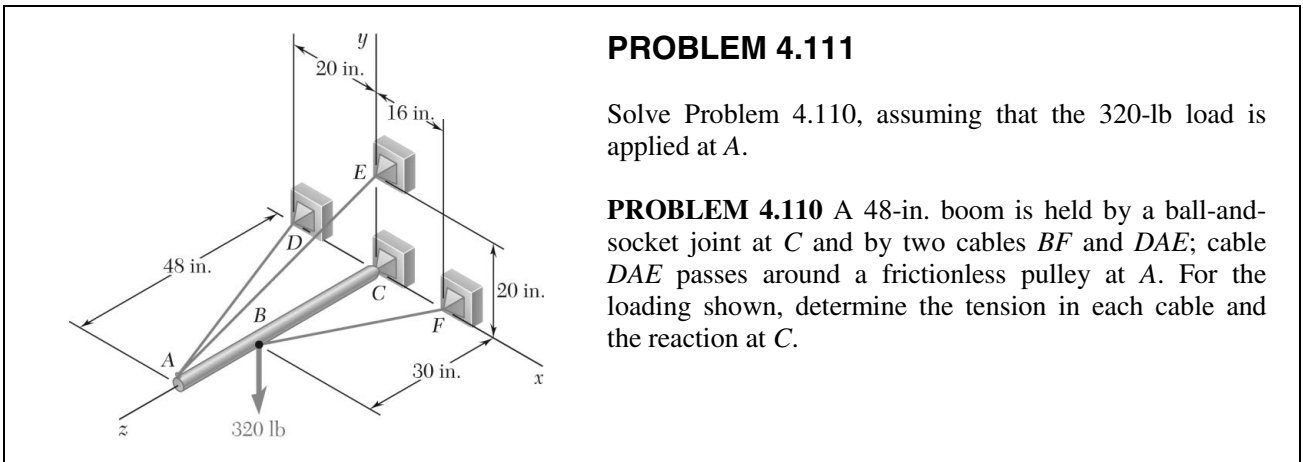
$$C_z = 1560 \text{ lb}$$

Answers: $T_{DAE} = T$

$$T_{DAE} = 520 \text{ lb} \quad \blacktriangleleft$$

$$T_{BD} = 680 \text{ lb} \quad \blacktriangleleft$$

$$\mathbf{C} = -(120.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j} + (1560 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 4.111

Solve Problem 4.110, assuming that the 320-lb load is applied at A.

PROBLEM 4.110 A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:

Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).

T = tension in both parts of cable DAE.

$\mathbf{r}_B = 30\mathbf{k}$
 $\mathbf{r}_A = 48\mathbf{k}$

$\overline{AD} = -20\mathbf{i} - 48\mathbf{k} \quad AD = 52 \text{ in.}$
 $\overline{AE} = 20\mathbf{j} - 48\mathbf{k} \quad AE = 52 \text{ in.}$
 $\overline{BF} = 16\mathbf{i} - 30\mathbf{k} \quad BF = 34 \text{ in.}$

$\mathbf{T}_{AD} = T \frac{\overline{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$
 $\mathbf{T}_{AE} = T \frac{\overline{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$
 $\mathbf{T}_{BF} = T_{BF} \frac{\overline{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$

$\Sigma M_C = 0: \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_A \times (-320 \text{ lb})\mathbf{j} = 0$

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix} \frac{T}{13}$	$+$	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix} \frac{T}{13}$	$+$	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix} \frac{T_{BF}}{17}$	$+$	$48\mathbf{k} \times (-320\mathbf{j}) = 0$
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Coefficient of \mathbf{i} : $-\frac{240}{13}T + 15,360 = 0 \quad T = 832 \text{ lb}$

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PROBLEM 4.111 (Continued)

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \quad T_{BD} = 1088 \text{ lb}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$

$$-320 + 512 + C_x = 0 \quad C_x = -192 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(832) - 320 + C_y = 0$

$$320 - 320 + C_y = 0 \quad C_y = 0$$

Coefficient of **k**: $-\frac{48}{52}(832) - \frac{48}{52}(832) - \frac{30}{34}(1088) + C_z = 0$

$$-768 - 768 - 960 + C_z = 0 \quad C_z = 2496 \text{ lb}$$

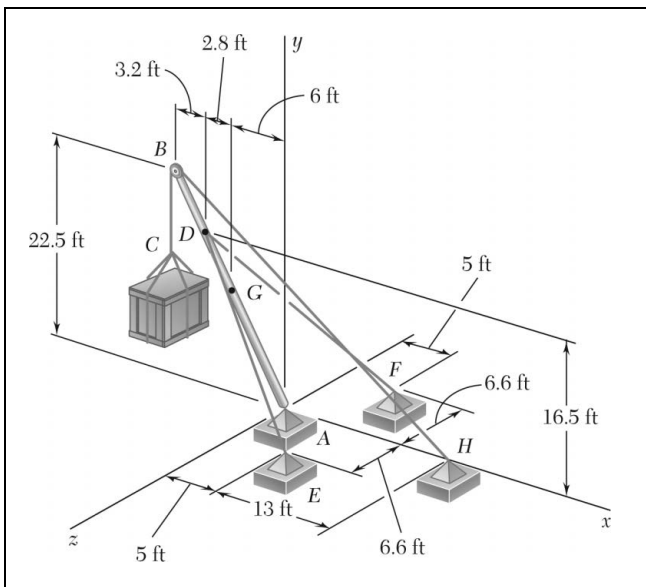
Answers:

$$T_{DAE} = T$$

$$T_{DAE} = 832 \text{ lb} \quad \blacktriangleleft$$

$$T_{BD} = 1088 \text{ lb} \quad \blacktriangleleft$$

$$\mathbf{C} = -(192.0 \text{ lb})\mathbf{i} + (2496 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

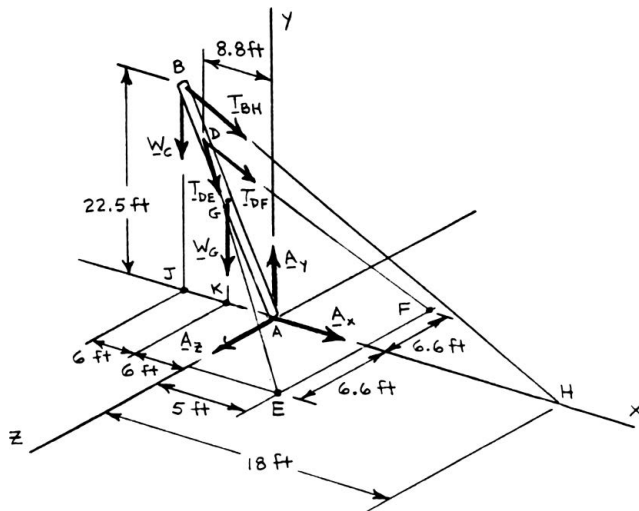


PROBLEM 4.112

A 600-lb crate hangs from a cable that passes over a pulley B and is attached to a support at H . The 200-lb boom AB is supported by a ball-and-socket joint at A and by two cables DE and DF . The center of gravity of the boom is located at G . Determine (a) the tension in cables DE and DF , (b) the reaction at A .

SOLUTION

Free-Body Diagram:



$$W_C = 600 \text{ lb}$$

$$W_G = 200 \text{ lb}$$

We have five unknowns ($T_{DE}, T_{DF}, A_x, A_y, A_z$) and five equilibrium equations. The boom is free to spin about the AB axis, but equilibrium is maintained, since $\Sigma M_{AB} = 0$.

We have

$$\overline{BH} = (30 \text{ ft})\mathbf{i} - (22.5 \text{ ft})\mathbf{j} \quad BH = 37.5 \text{ ft}$$

$$\begin{aligned} \overline{DE} &= (13.8 \text{ ft})\mathbf{i} - \frac{8.8}{12}(22.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k} \\ &= (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k} \quad DE = 22.5 \text{ ft} \end{aligned}$$

$$\overline{DF} = (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} - (6.6 \text{ ft})\mathbf{k} \quad DF = 22.5 \text{ ft}$$

PROBLEM 4.112 (Continued)

Thus:
$$\mathbf{T}_{BH} = T_{BH} \frac{\overline{BH}}{BH} = (600 \text{ lb}) \frac{30\mathbf{i} - 22.5\mathbf{j}}{37.5} = (480 \text{ lb})\mathbf{i} - (360 \text{ lb})\mathbf{j}$$

$$\mathbf{T}_{DE} = T_{DE} \frac{\overline{DE}}{DE} = \frac{T_{DE}}{22.5} (13.8\mathbf{i} - 16.5\mathbf{j} + 6.6\mathbf{k})$$

$$\mathbf{T}_{DF} = T_{DF} \frac{\overline{DF}}{DF} = \frac{T_{DF}}{22.5} (13.8\mathbf{i} - 16.5\mathbf{j} - 6.6\mathbf{k})$$

(a) $\Sigma \mathbf{M}_A = 0: (\mathbf{r}_J \times \mathbf{W}_C) + (\mathbf{r}_K \times \mathbf{W}_G) + (\mathbf{r}_H \times \mathbf{T}_{BH}) + (\mathbf{r}_E \times \mathbf{T}_{DE}) + (\mathbf{r}_F \times \mathbf{T}_{DF}) = 0$

$$- (12\mathbf{i}) \times (-600\mathbf{j}) - (6\mathbf{i}) \times (-200\mathbf{j}) + (18\mathbf{i}) \times (480\mathbf{i} - 360\mathbf{j})$$

$$+ \frac{T_{DE}}{22.5} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 6.6 \\ 13.8 & -16.5 & 6.6 \end{vmatrix} + \frac{T_{DF}}{22.5} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & -6.6 \\ 13.8 & -16.5 & -6.6 \end{vmatrix} = 0$$

or
$$7200\mathbf{k} + 1200\mathbf{k} - 6480\mathbf{k} + 4.84(T_{DE} - T_{DF})\mathbf{i}$$

$$+ \frac{58.08}{22.5}(T_{DE} - T_{DF})\mathbf{j} - \frac{82.5}{22.5}(T_{DE} + T_{DF})\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

i or **j**: $T_{DE} - T_{DF} = 0 \quad T_{DE} = T_{DF}^*$

k: $7200 + 1200 - 6480 - \frac{82.5}{22.5}(2T_{DE}) = 0 \quad T_{DE} = 261.82 \text{ lb}$

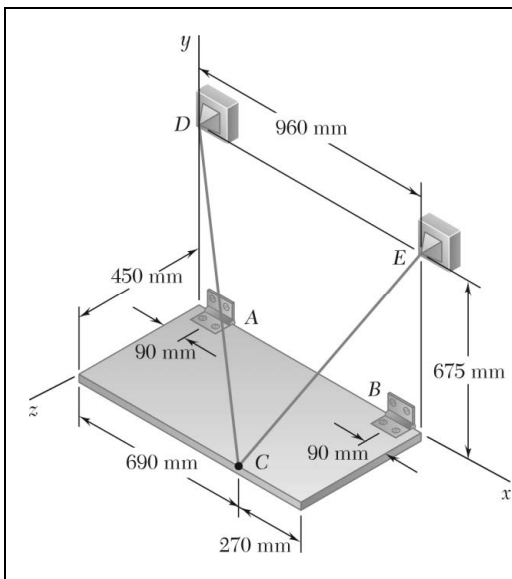
$T_{DE} = T_{DF} = 262 \text{ lb} \quad \blacktriangleleft$

(b) $\Sigma F_x = 0: A_x + 480 + 2\left(\frac{13.8}{22.5}\right)(261.82) = 0 \quad A_x = -801.17 \text{ lb}$

$\Sigma F_y = 0: A_y - 600 - 200 - 360 - 2\left(\frac{16.5}{22.5}\right)(261.82) = 0 \quad A_y = 1544.00 \text{ lb}$

$\Sigma F_z = 0: A_z = 0 \quad \mathbf{A} = -(801 \text{ lb})\mathbf{i} + (1544 \text{ lb})\mathbf{j} \quad \blacktriangleleft$

*Remark: The fact is that $T_{DE} = T_{DF}$ could have been noted at the outset from the symmetry of structure with respect to xy plane.



PROBLEM 4.113

A 100-kg uniform rectangular plate is supported in the position shown by hinges A and B and by cable DCE that passes over a frictionless hook at C . Assuming that the tension is the same in both parts of the cable, determine (a) the tension in the cable, (b) the reactions at A and B . Assume that the hinge at B does not exert any axial thrust.

SOLUTION

$$\mathbf{r}_{B/A} (960 - 180)\mathbf{i} = 780\mathbf{i}$$

$$\begin{aligned} \mathbf{r}_{G/A} &= \left(\frac{960}{2} - 90 \right) \mathbf{i} + \frac{450}{2} \mathbf{k} \\ &= 390\mathbf{i} + 225\mathbf{k} \end{aligned}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

T = Tension in cable DCE

$$\overline{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \quad CD = 1065 \text{ mm}$$

$$\overline{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \quad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065} (-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

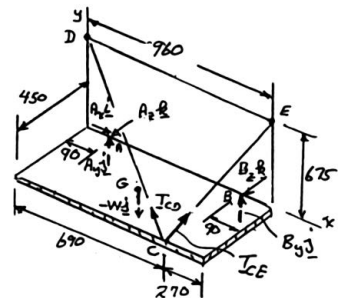
$$\mathbf{T}_{CE} = \frac{T}{855} (270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} \\ + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Dimensions in mm



PROBLEM 4.113 (Continued)

Coefficient of **i**: $-(450)(675)\frac{T}{1065} - (450)(675)\frac{T}{855} + 220.73 \times 10^3 = 0$

$T = 344.64 \text{ N} \qquad T = 345 \text{ N} \blacktriangleleft$

Coefficient of **j**: $(-690 \times 450 + 600 \times 450)\frac{344.64}{1065} + (270 \times 450 + 600 \times 450)\frac{344.64}{855} - 780B_z = 0$

$B_z = 185.516 \text{ N}$

Coefficient of **k**: $(600)(675)\frac{344.64}{1065} + (600)(675)\frac{344.64}{855} - 382.59 \times 10^3 + 780B_y = 0 \quad B_y = 113.178 \text{ N}$

$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (185.5 \text{ N})\mathbf{k} \blacktriangleleft$

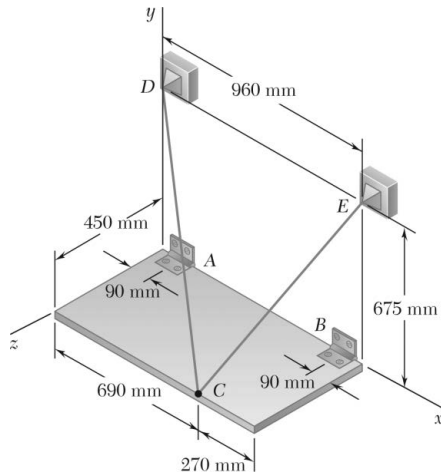
$\Sigma \mathbf{F} = 0: \mathbf{A} + \mathbf{B} + \mathbf{T}_{CD} + \mathbf{T}_{CE} + \mathbf{W} = 0$

Coefficient of **i**: $A_x - \frac{690}{1065}(344.64) + \frac{270}{855}(344.64) = 0 \qquad A_x = 114.5 \text{ N}$

Coefficient of **j**: $A_y + 113.178 + \frac{675}{1065}(344.64) + \frac{675}{855}(344.64) - 981 = 0 \quad A_y = 377 \text{ N}$

Coefficient of **k**: $A_z + 185.516 - \frac{450}{1065}(344.64) - \frac{450}{855}(344.64) = 0 \qquad A_z = 141.5 \text{ N}$

$\mathbf{A} = (114.5 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} + (144.5 \text{ N})\mathbf{k} \blacktriangleleft$



PROBLEM 4.114

Solve Problem 4.113, assuming that cable DCE is replaced by a cable attached to Point E and hook C .

PROBLEM 4.113 A 100-kg uniform rectangular plate is supported in the position shown by hinges A and B and by cable DCE that passes over a frictionless hook at C . Assuming that the tension is the same in both parts of the cable, determine (a) the tension in the cable, (b) the reactions at A and B . Assume that the hinge at B does not exert any axial thrust.

SOLUTION

See solution to Problem 4.113 for free-body diagram and analysis leading to the following:

$$CD = 1065 \text{ mm}$$

$$CE = 855 \text{ mm}$$

Now,

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma \mathbf{M}_A = 0: \mathbf{r}_{CIA} \times \mathbf{T}_{CE} + \mathbf{r}_{GIA} \times (-W\mathbf{j}) + \mathbf{r}_{BIA} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of \mathbf{i} : $-(450)(675)\frac{T}{855} + 220.73 \times 10^3 = 0$

$$T = 621.31 \text{ N}$$

$$T = 621 \text{ N} \quad \blacktriangleleft$$

Coefficient of \mathbf{j} : $(270 \times 450 + 600 \times 450)\frac{621.31}{855} - 780B_z = 0 \quad B_z = 364.74 \text{ N}$

Coefficient of \mathbf{k} : $(600)(675)\frac{621.31}{855} - 382.59 \times 10^3 + 780B_y = 0 \quad B_y = 113.186 \text{ N}$

$$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (365 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

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PROBLEM 4.114 (Continued)

$$\Sigma \mathbf{F} = 0: \mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{W} = 0$$

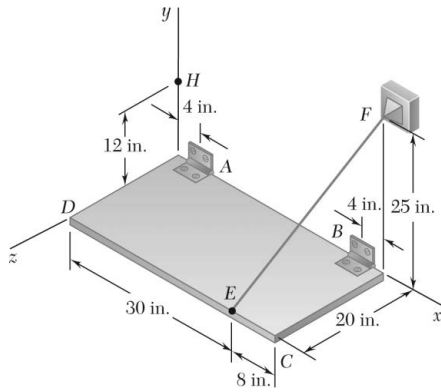
Coefficient of **i**: $A_x + \frac{270}{855}(621.31) = 0$ $A_x = -196.2 \text{ N}$

Coefficient of **j**: $A_y + 113.186 + \frac{675}{855}(621.31) - 981 = 0$ $A_y = 377.3 \text{ N}$

Coefficient of **k**: $A_z + 364.74 - \frac{450}{855}(621.31) = 0$ $A_z = -37.7 \text{ N}$

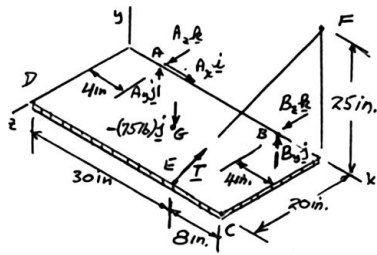
$$\mathbf{A} = -(196.2 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} - (37.7 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.115



The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF . Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B .

SOLUTION



$$\mathbf{r}_{B/A} = (38 - 8)\mathbf{i} = 30\mathbf{i}$$

$$\begin{aligned}\mathbf{r}_{E/A} &= (30 - 4)\mathbf{i} + 20\mathbf{k} \\ &= 26\mathbf{i} + 20\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{r}_{G/A} &= \frac{38}{2}\mathbf{i} + 10\mathbf{k} \\ &= 19\mathbf{i} + 10\mathbf{k}\end{aligned}$$

$$\overline{EF} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$EF = 33 \text{ in.}$$

$$\mathbf{T} = T \frac{\overline{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$\text{Coefficient of } \mathbf{i}: \quad -(25)(20) \frac{T}{33} + 750 = 0: \quad T = 49.5 \text{ lb} \quad \blacktriangleleft$$

$$\text{Coefficient of } \mathbf{j}: \quad (160 + 520) \frac{49.5}{33} - 30B_z = 0: \quad B_z = 34 \text{ lb}$$

$$\text{Coefficient of } \mathbf{k}: \quad (26)(25) \frac{49.5}{33} - 1425 + 30B_y = 0: \quad B_y = 15 \text{ lb} \quad \mathbf{B} = (15.00 \text{ lb})\mathbf{j} + (34.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

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PROBLEM 4.115 (Continued)

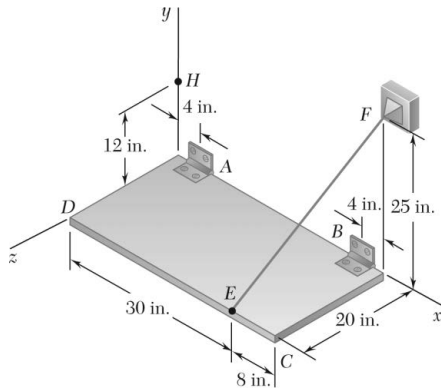
$$\Sigma \mathbf{F} = 0: \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x + \frac{8}{33}(49.5) = 0 \quad A_x = -12.00 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{25}{33}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z + 34 - \frac{20}{33}(49.5) = 0 \quad A_z = -4.00 \text{ lb}$

$$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} - (4.00 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

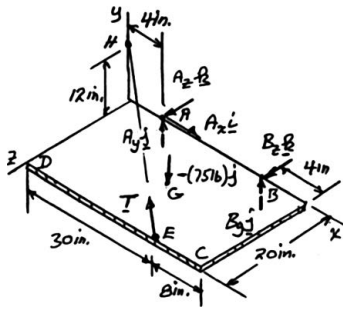


PROBLEM 4.116

Solve Problem 4.115, assuming that cable EF is replaced by a cable attached at points E and H .

PROBLEM 4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF . Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B .

SOLUTION



$$\mathbf{r}_{B/A} = (38 - 8)\mathbf{i} = 30\mathbf{i}$$

$$\begin{aligned} \mathbf{r}_{E/A} &= (30 - 4)\mathbf{i} + 20\mathbf{k} \\ &= 26\mathbf{i} + 20\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{G/A} &= \frac{38}{2}\mathbf{i} + 10\mathbf{k} \\ &= 19\mathbf{i} + 10\mathbf{k} \end{aligned}$$

$$\overline{EH} = -30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k}$$

$$EH = 38 \text{ in.}$$

$$\mathbf{T} = T \frac{\overline{EH}}{EH} = \frac{T}{38}(-30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ -30 & 12 & -20 \end{vmatrix} \frac{T}{38} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

$$\text{Coefficient of } \mathbf{i}: \quad -(12)(20)\frac{T}{38} + 750 = 0 \quad T = 118.75 \quad T = 118.8 \text{ lb} \quad \blacktriangleleft$$

$$\text{Coefficient of } \mathbf{j}: \quad (-600 + 520)\frac{118.75}{38} - 30B_z = 0 \quad B_z = -8.33 \text{ lb}$$

$$\text{Coefficient of } \mathbf{k}: \quad (26)(12)\frac{118.75}{38} - 1425 + 30B_y = 0 \quad B_y = 15.00 \text{ lb} \quad \mathbf{B} = (15.00 \text{ lb})\mathbf{j} - (8.33 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

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PROBLEM 4.116 (Continued)

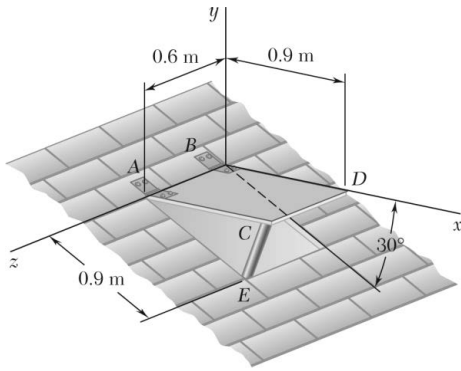
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x - \frac{30}{38}(118.75) = 0 \quad A_x = 93.75 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{12}{38}(118.75) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z - 8.33 - \frac{20}{38}(118.75) = 0 \quad A_z = 70.83 \text{ lb}$

$$\mathbf{A} = (93.8 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} + (70.8 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 4.117

A 20-kg cover for a roof opening is hinged at corners A and B . The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace CE . Determine (a) the magnitude of the force exerted by the brace, (b) the reactions at the hinges. Assume that the hinge at A does not exert any axial thrust.

SOLUTION

Force exerted by CE :

$$\mathbf{F} = F(\cos 75^\circ)\mathbf{i} + F(\sin 75^\circ)\mathbf{j}$$

$$\mathbf{F} = F(0.25882\mathbf{i} + 0.96593\mathbf{j})$$

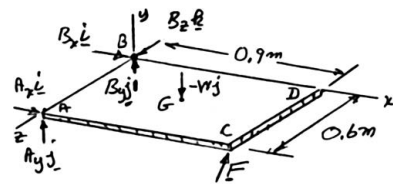
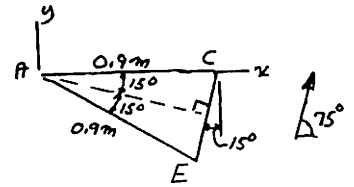
$$W = mg = 20 \text{ kg}(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$\mathbf{r}_{A/B} = 0.6\mathbf{k}$$

$$\mathbf{r}_{C/B} = 0.9\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{r}_{G/B} = 0.45\mathbf{i} + 0.3\mathbf{k}$$

$$\mathbf{F} = F(0.25882\mathbf{i} + 0.96593\mathbf{j})$$



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times (-196.2\mathbf{j}) + \mathbf{r}_{C/B} \times \mathbf{F} + \mathbf{r}_{A/B} \times \mathbf{A} = 0$$

$$(a) \quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0.6 \\ 0.25882 & +0.96593 & 0 \end{vmatrix} F + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

$$\text{Coefficient of } \mathbf{i}: \quad +58.86 - 0.57956F - 0.6A_y = 0 \quad (1)$$

$$\text{Coefficient of } \mathbf{j}: \quad +0.155292F + 0.6A_x = 0 \quad (2)$$

$$\text{Coefficient of } \mathbf{k}: \quad -88.29 + 0.86934F = 0: \quad F = 101.56 \text{ N}$$

$$\text{From Eq. (2):} \quad +58.86 - 0.57956(101.56) - 0.6A_y = 0 \quad A_y = 0$$

$$\text{From Eq. (3):} \quad +0.155292(101.56) + 0.6A_x = 0 \quad A_x = -26.286 \text{ N}$$

$$F = (101.6 \text{ N}) \quad \blacktriangleleft$$

$$(b) \quad \Sigma \mathbf{F}: \quad \mathbf{A} + \mathbf{B} + \mathbf{F} - W\mathbf{j} = 0$$

$$\text{Coefficient of } \mathbf{i}: \quad 26.286 + B_x + 0.25882(101.56) = 0 \quad B_x = 0$$

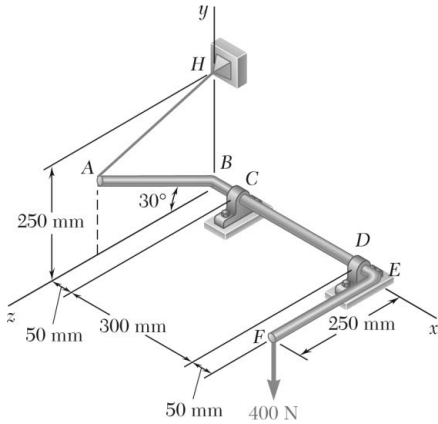
$$\text{Coefficient of } \mathbf{j}: \quad B_y + 0.96593(101.56) - 196.2 = 0 \quad B_y = 98.1 \text{ N}$$

$$\text{Coefficient of } \mathbf{k}: \quad B_z = 0 \quad \mathbf{A} = -(26.3 \text{ N})\mathbf{i}; \quad \mathbf{B} = (98.1 \text{ N})\mathbf{j} \quad \blacktriangleleft$$

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PROBLEM 4.118

The bent rod $ABEF$ is supported by bearings at C and D and by wire AH . Knowing that portion AB of the rod is 250 mm long, determine (a) the tension in wire AH , (b) the reactions at C and D . Assume that the bearing at D does not exert any axial thrust.

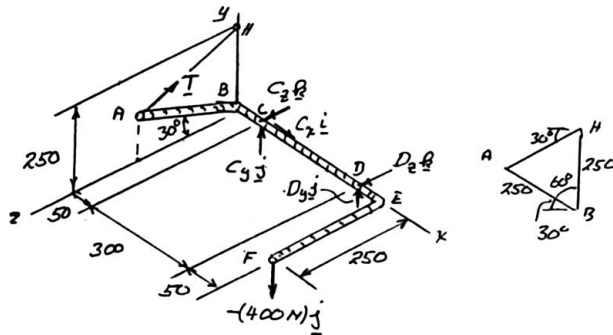


SOLUTION

Free-Body Diagram:

$\triangle ABH$ is equilateral.

Dimensions in mm



$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$

$$\mathbf{r}_{D/C} = 300\mathbf{i}$$

$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$

$$\mathbf{T} = T(\sin 30^\circ)\mathbf{j} - T(\cos 30^\circ)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \mathbf{r}_{H/C} \times \mathbf{T} + \mathbf{r}_D \times \mathbf{D} + \mathbf{r}_{F/C} \times (-400\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

Coefficient \mathbf{i} : $-216.5T + 100 \times 10^3 = 0$

$$T = 461.9 \text{ N}$$

$$T = 462 \text{ N} \quad \blacktriangleleft$$

Coefficient of \mathbf{j} : $-43.3T - 300D_z = 0$

$$-43.3(461.9) - 300D_z = 0 \quad D_z = -66.67 \text{ N}$$

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PROBLEM 4.118 (Continued)

Coefficient of **k**: $-25T + 300D_y - 140 \times 10^3 = 0$

$$-25(461.9) + 300D_y - 140 \times 10^3 = 0 \quad D_y = 505.1 \text{ N}$$

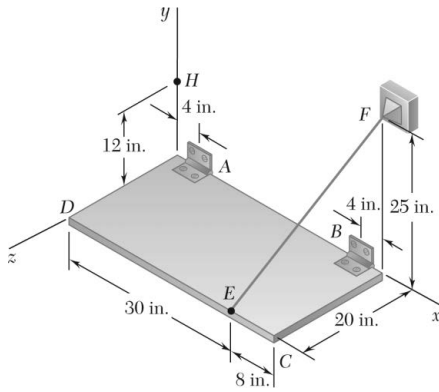
$$\mathbf{D} = (505 \text{ N})\mathbf{j} - (66.7 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{T} - 400\mathbf{j} = 0$$

Coefficient **i**: $C_x = 0$ $C_x = 0$

Coefficient **j**: $C_y + (461.9)0.5 + 505.1 - 400 = 0$ $C_y = -336 \text{ N}$

Coefficient **k**: $C_z - (461.9)0.866 - 66.67 = 0$ $C_z = 467 \text{ N}$ $\mathbf{C} = -(336 \text{ N})\mathbf{j} + (467 \text{ N})\mathbf{k} \quad \blacktriangleleft$

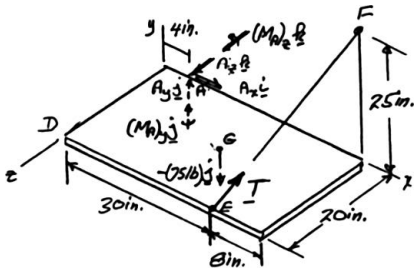


PROBLEM 4.119

Solve Problem 4.115, assuming that the hinge at B is removed and that the hinge at A can exert couples about axes parallel to the y and z axes.

PROBLEM 4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF . Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B .

SOLUTION



$$\mathbf{r}_{E/A} = (30 - 4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = (0.5 \times 38)\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$$

$$\overline{AE} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$AE = 33 \text{ in.}$$

$$\mathbf{T} = T \frac{\overline{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + (M_A)_y \mathbf{j} + (M_A)_z \mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y \mathbf{j} + (M_A)_z \mathbf{k} = 0$$

Coefficient of \mathbf{i} : $-(20)(25) \frac{T}{33} + 750 = 0 \quad T = 49.5 \text{ lb} \blacktriangleleft$

Coefficient of \mathbf{j} : $(160 + 520) \frac{49.5}{33} + (M_A)_y = 0 \quad (M_A)_y = -1020 \text{ lb} \cdot \text{in.}$

Coefficient of \mathbf{k} : $(26)(25) \frac{49.5}{33} - 1425 + (M_A)_z = 0 \quad (M_A)_z = 450 \text{ lb} \cdot \text{in.}$

$$\Sigma \mathbf{F} = 0: A + T - 75\mathbf{j} = 0 \quad \mathbf{M}_A = -(1020 \text{ lb} \cdot \text{in.})\mathbf{j} + (450 \text{ lb} \cdot \text{in.})\mathbf{k} \blacktriangleleft$$

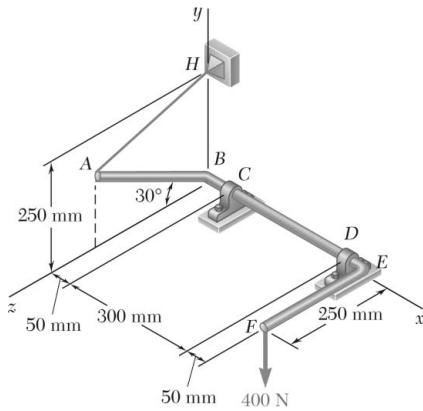
Coefficient of \mathbf{i} : $A_x + \frac{8}{33}(49.5) = 0 \quad A_x = 12.00 \text{ lb}$

Coefficient of \mathbf{j} : $A_y + \frac{25}{33}(49.5) - 75 = 0 \quad A_y = 37.5 \text{ lb}$

Coefficient of \mathbf{k} : $A_z - \frac{20}{33}(49.5) = 0 \quad A_z = 30.0 \text{ lb}$

$$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (37.5 \text{ lb})\mathbf{j} + (30.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

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PROBLEM 4.120

Solve Problem 4.118, assuming that the bearing at D is removed and that the bearing at C can exert couples about axes parallel to the y and z axes.

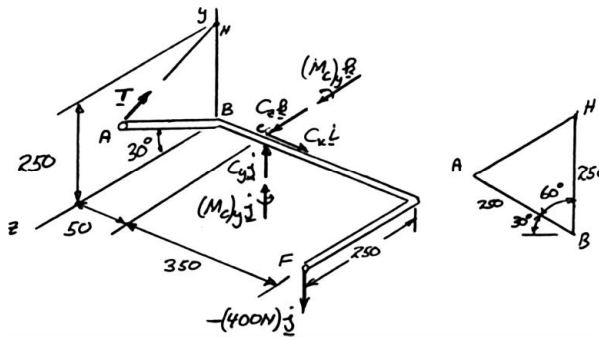
PROBLEM 4.118 The bent rod $ABEF$ is supported by bearings at C and D and by wire AH . Knowing that portion AB of the rod is 250 mm long, determine (a) the tension in wire AH , (b) the reactions at C and D . Assume that the bearing at D does not exert any axial thrust.

SOLUTION

Free-Body Diagram:

$\triangle ABH$ is equilateral.

Dimensions in mm



$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$

$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$

$$\mathbf{T} = T(\sin 30^\circ)\mathbf{j} - T(\cos 30^\circ)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times T + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

Coefficient of \mathbf{i} : $+100 \times 10^3 - 216.5T = 0 \quad T = 461.9 \text{ N}$

$T = 462 \text{ N} \quad \blacktriangleleft$

Coefficient of \mathbf{j} : $-43.3(461.9) + (M_C)_y = 0$

$$(M_C)_y = 20 \times 10^3 \text{ N} \cdot \text{mm}$$

$$(M_C)_y = 20.0 \text{ N} \cdot \text{m}$$

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PROBLEM 4.120 (Continued)

Coefficient of **k**: $-140 \times 10^3 - 25(461.9) + (M_C)_z = 0$

$$(M_C)_z = 151.54 \times 10^3 \text{ N} \cdot \text{mm}$$

$$(M_C)_z = 151.5 \text{ N} \cdot \text{m}$$

$$\Sigma F = 0: C + T - 400\mathbf{j} = 0$$

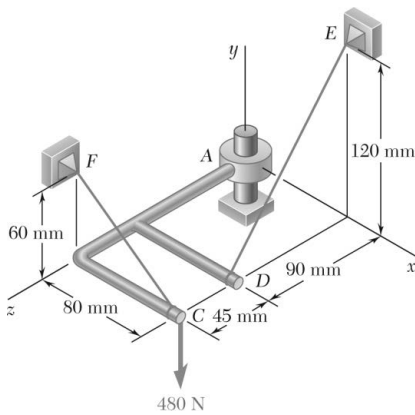
$$\mathbf{M}_C = (20.0 \text{ N} \cdot \text{m})\mathbf{j} + (151.5 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$

Coefficient of **i**: $C_x = 0$

Coefficient of **j**: $C_y + 0.5(461.9) - 400 = 0 \quad C_y = 169.1 \text{ N}$

Coefficient of **k**: $C_z - 0.866(461.9) = 0 \quad C_z = 400 \text{ N}$

$$\mathbf{C} = (169.1 \text{ N})\mathbf{j} + (400 \text{ N})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 4.121

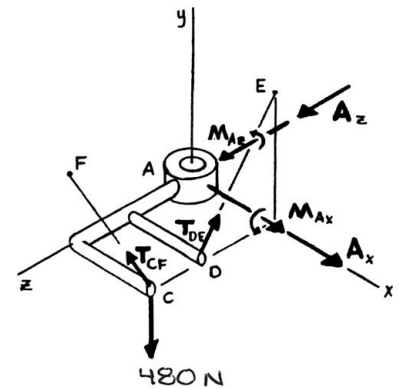
The assembly shown is welded to collar A that fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y -axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:

First note:

$$\begin{aligned} \mathbf{T}_{CF} &= \lambda_{CF} T_{CF} = \frac{-(0.08 \text{ m})\mathbf{i} + (0.06 \text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2} \text{ m}} T_{CF} \\ &= T_{CF} (-0.8\mathbf{i} + 0.6\mathbf{j}) \\ \mathbf{T}_{DE} &= \lambda_{DE} T_{DE} = \frac{(0.12 \text{ m})\mathbf{j} - (0.09 \text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2} \text{ m}} T_{DE} \\ &= T_{DE} (0.8\mathbf{j} - 0.6\mathbf{k}) \end{aligned}$$



(a) From F.B.D. of assembly:

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480 \text{ N} = 0$$

$$\text{or} \quad 0.6T_{CF} + 0.8T_{DE} = 480 \text{ N} \quad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135 \text{ m}) + (0.6T_{DE})(0.08 \text{ m}) = 0$$

$$\text{or} \quad T_{DE} = 2.25T_{CF} \quad (2)$$

Substituting Equation (2) into Equation (1),

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480 \text{ N}$$

$$T_{CF} = 200.00 \text{ N}$$

$$\text{or} \quad T_{CF} = 200 \text{ N} \quad \blacktriangleleft$$

$$\text{and from Equation (2):} \quad T_{DE} = 2.25(200.00 \text{ N}) = 450.00$$

$$\text{or} \quad T_{DE} = 450 \text{ N} \quad \blacktriangleleft$$

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PROBLEM 4.121 (Continued)

(b) From F.B.D. of assembly:

$$\Sigma F_z = 0: A_z - (0.6)(450.00 \text{ N}) = 0 \quad A_z = 270.00 \text{ N}$$

$$\Sigma F_x = 0: A_x - (0.8)(200.00 \text{ N}) = 0 \quad A_x = 160.000 \text{ N}$$

$$\text{or } \mathbf{A} = (160.0 \text{ N})\mathbf{i} + (270 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

$$\Sigma M_x = 0: M_{A_x} + (480 \text{ N})(0.135 \text{ m}) - [(200.00 \text{ N})(0.6)](0.135 \text{ m}) \\ - [(450 \text{ N})(0.8)](0.09 \text{ m}) = 0$$

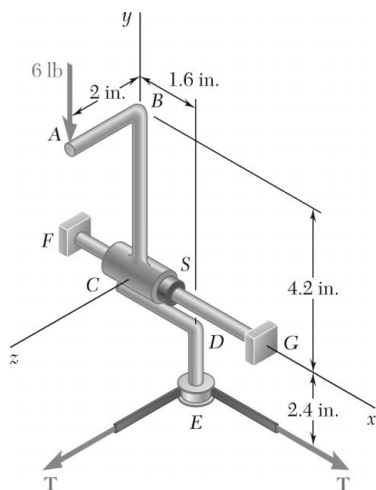
$$M_{A_x} = -16.2000 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: M_{A_z} - (480 \text{ N})(0.08 \text{ m}) + [(200.00 \text{ N})(0.6)](0.08 \text{ m}) \\ + [(450 \text{ N})(0.8)](0.08 \text{ m}) = 0$$

$$M_{A_z} = 0$$

$$\text{or } \mathbf{M}_A = -(16.20 \text{ N}\cdot\text{m})\mathbf{i} \quad \blacktriangleleft$$

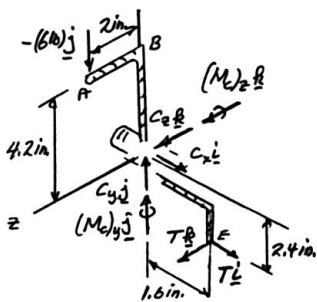
PROBLEM 4.122



The assembly shown is used to control the tension T in a tape that passes around a frictionless spool at E . Collar C is welded to rods ABC and CDE . It can rotate about shaft FG but its motion along the shaft is prevented by a washer S . For the loading shown, determine (a) the tension T in the tape, (b) the reaction at C .

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\Sigma M_C = 0: \mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

$$(4.2\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\mathbf{j}) \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

Coefficient of \mathbf{i} : $12 - 2.4T = 0$

$T = 5.00 \text{ lb} \quad \blacktriangleleft$

Coefficient of \mathbf{j} : $-1.6(5 \text{ lb}) + (M_C)_y = 0 \quad (M_C)_y = 8 \text{ lb} \cdot \text{in.}$

Coefficient of \mathbf{k} : $2.4(5 \text{ lb}) + (M_C)_z = 0 \quad (M_C)_z = -12 \text{ lb} \cdot \text{in.}$

$\mathbf{M}_C = (8.00 \text{ lb} \cdot \text{in.})\mathbf{j} - (12.00 \text{ lb} \cdot \text{in.})\mathbf{k} \quad \blacktriangleleft$

$$\Sigma F = 0: C_x \mathbf{i} + C_y \mathbf{j} + C_z \mathbf{k} - (6 \text{ lb})\mathbf{j} + (5 \text{ lb})\mathbf{i} + (5 \text{ lb})\mathbf{k} = 0$$

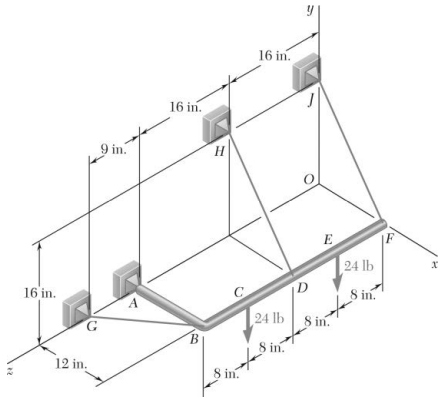
Equate coefficients of unit vectors to zero.

$C_x = -5 \text{ lb} \quad C_y = 6 \text{ lb} \quad C_z = -5 \text{ lb} \quad \blacktriangleleft$

$\mathbf{C} = -(5.00 \text{ lb})\mathbf{i} + (6.00 \text{ lb})\mathbf{j} - (5.00 \text{ lb})\mathbf{k} \quad \blacktriangleleft$

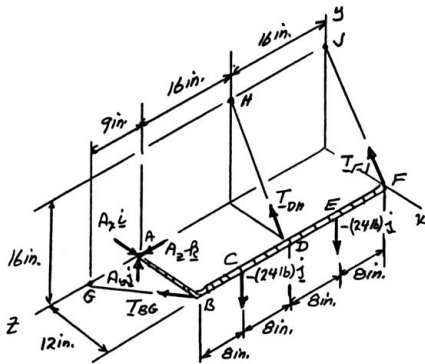
PROBLEM 4.123

The rigid L-shaped member ABF is supported by a ball-and-socket joint at A and by three cables. For the loading shown, determine the tension in each cable and the reaction at A .



SOLUTION

Free-Body Diagram:



$$\begin{aligned} \mathbf{r}_{B/A} &= 12\mathbf{i} \\ \mathbf{r}_{F/A} &= 12\mathbf{j} - 8\mathbf{k} \\ \mathbf{r}_{D/A} &= 12\mathbf{i} - 16\mathbf{k} \\ \mathbf{r}_{E/A} &= 12\mathbf{i} - 24\mathbf{k} \\ \mathbf{r}_{F/A} &= 12\mathbf{i} - 32\mathbf{k} \\ \overline{BG} &= -12\mathbf{i} + 9\mathbf{k} \\ BG &= 15 \text{ in.} \\ \lambda_{BG} &= -0.8\mathbf{i} + 0.6\mathbf{k} \end{aligned}$$

$$\overline{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \lambda_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\overline{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \lambda_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BG} \lambda_{BG} + \mathbf{r}_{D/A} \times \mathbf{T}_{DH} \lambda_{DH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FJ} \lambda_{FJ} \\ + \mathbf{r}_{F/A} \times (-24\mathbf{j}) + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ} \\ + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

$$\text{Coefficient of } \mathbf{i}: \quad +12.8T_{DH} + 25.6T_{FJ} - 192 - 576 = 0 \quad (1)$$

$$\text{Coefficient of } \mathbf{k}: \quad +9.6T_{DH} + 9.6T_{FJ} - 288 - 288 = 0 \quad (2)$$

$$\frac{3}{4} \text{ Eq. (1) - Eq. (2):} \quad 9.6T_{FJ} = 0 \quad T_{FJ} = 0 \quad \blacktriangleleft$$

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PROBLEM 4.123 (Continued)

From Eq. (1): $12.8T_{DH} - 268 = 0$ $T_{DH} = 60 \text{ lb} \blacktriangleleft$

Coefficient of **j**: $-7.2T_{BG} + (16 \times 0.6)(60.0 \text{ lb}) = 0$ $T_{BG} = 80.0 \text{ lb} \blacktriangleleft$

$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + T_{BG}\lambda_{BG} + T_{DH}\lambda_{DH} + T_{FJ} - 24\mathbf{j} - 24\mathbf{j} = 0$

Coefficient of **i**: $A_x + (80)(-0.8) + (60.0)(-0.6) = 0$ $A_x = 100.0 \text{ lb}$

Coefficient of **j**: $A_y + (60.0)(0.8) - 24 - 24 = 0$ $A_y = 0$

Coefficient of **k**: $A_z + (80.0)(+0.6) = 0$ $A_z = -48.0 \text{ lb}$

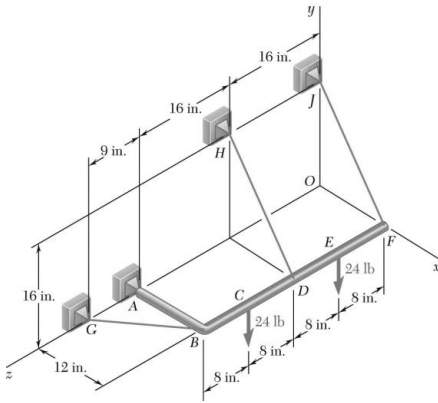
$\mathbf{A} = (100.0 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j} \blacktriangleleft$

Note: The value $A_y = 0$ can be confirmed by considering $\Sigma M_{BF} = 0$.

PROBLEM 4.124

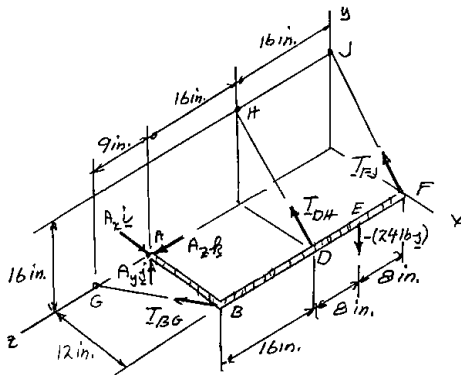
Solve Problem 4.123, assuming that the load at C has been removed.

PROBLEM 4.123 The rigid L-shaped member ABF is supported by a ball-and-socket joint at A and by three cables. For the loading shown, determine the tension in each cable and the reaction at A.



SOLUTION

Free-Body Diagram:



$$\begin{aligned} \mathbf{r}_{B/A} &= 12\mathbf{i} \\ \mathbf{r}_{D/A} &= 12\mathbf{i} - 16\mathbf{k} \\ \mathbf{r}_{E/A} &= 12\mathbf{i} - 24\mathbf{k} \\ \mathbf{r}_{F/A} &= 12\mathbf{i} - 32\mathbf{k} \end{aligned}$$

$$\begin{aligned} \overline{BG} &= -12\mathbf{i} + 9\mathbf{k}; \quad BG = 15 \text{ in.}; \quad \boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k} \\ \overline{DH} &= -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \boldsymbol{\lambda}_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j} \\ \overline{FJ} &= -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \boldsymbol{\lambda}_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j} \end{aligned}$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times T_{BG} \boldsymbol{\lambda}_{BG} + \mathbf{r}_{D/A} \times T_{DH} \boldsymbol{\lambda}_{DH} + \mathbf{r}_{F/A} \times T_{FJ} \boldsymbol{\lambda}_{FJ} + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

$$\mathbf{i}: \quad +12.8T_{DH} + 25.6T_{FJ} - 576 = 0 \quad (1)$$

$$\mathbf{k}: \quad +9.6T_{DH} + 9.6T_{FJ} - 288 = 0 \quad (2)$$

PROBLEM 4.124 (Continued)

Multiply Eq. (1) by $\frac{3}{4}$ and subtract Eq. (2):

$$9.6T_{FJ} - 144 = 0 \qquad T_{FJ} = 15.00 \text{ lb} \blacktriangleleft$$

From Eq. (1):

$$12.8T_{DH} + 25.6(15.00) - 576 = 0 \qquad T_{DH} = 15.00 \text{ lb} \blacktriangleleft$$

$$\mathbf{j}: \quad -7.2T_{BG} + (16)(0.6)(15) + (32)(0.6)(15) = 0$$

$$-7.2T_{BG} + 432 = 0 \qquad T_{BG} = 60.0 \text{ lb} \blacktriangleleft$$

$$\Sigma F = 0: \quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DA}\boldsymbol{\lambda}_{DH} + T_{FJ}\boldsymbol{\lambda}_{FJ} - 24\mathbf{j} = 0$$

$$\mathbf{i}: \quad A_x + (60)(-0.8) + (15)(-0.6) + (15)(-0.6) = 0 \qquad A_x = 66.0 \text{ lb}$$

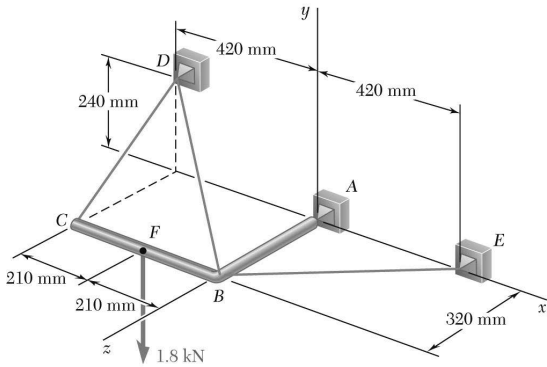
$$\mathbf{j}: \quad A_y + (15)(0.8) + (15)(0.8) - 24 = 0 \qquad A_y = 0$$

$$\mathbf{k}: \quad A_z + (60)(0.6) = 0 \qquad A_z = -36.0 \text{ lb}$$

$$\mathbf{A} = (66.0 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

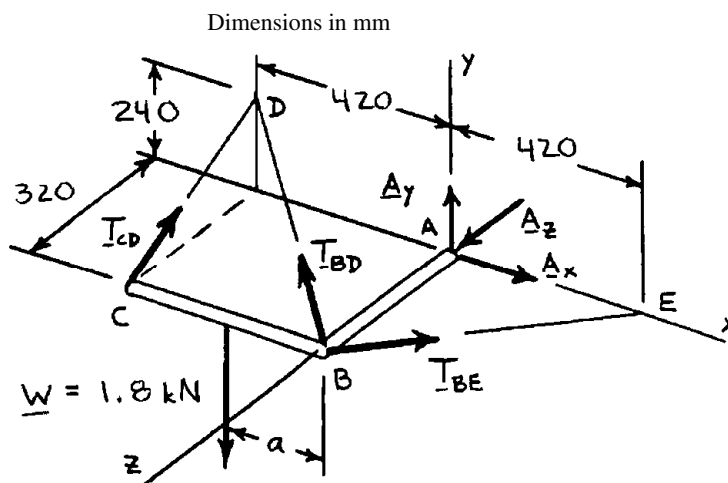
PROBLEM 4.125

The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. If a 1.8-kN load is applied at F , determine the tension in each cable.



SOLUTION

Free-Body Diagram:



In this problem:

$$a = 210 \text{ mm}$$

We have

$$\overline{CD} = (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad CD = 400 \text{ mm}$$

$$\overline{BD} = -(420 \text{ mm})\mathbf{i} + (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad BD = 580 \text{ mm}$$

$$\overline{BE} = (420 \text{ mm})\mathbf{i} - (320 \text{ mm})\mathbf{k} \quad BE = 528.02 \text{ mm}$$

Thus,

$$T_{CD} = T_{CD} \frac{\overline{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$

$$T_{BD} = T_{BD} \frac{\overline{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$

$$T_{BE} = T_{BE} \frac{\overline{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma \mathbf{M}_A = 0: (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

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PROBLEM 4.125 (Continued)

Noting that

$$\mathbf{r}_C = -(420 \text{ mm})\mathbf{i} + (320 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = (320 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_W = -a\mathbf{i} + (320 \text{ mm})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -420 & 0 & 320 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD} \\ + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 320 \\ 0 & -1.8 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad -192T_{CD} - 132.413T_{BD} + 576 = 0 \quad (1)$$

$$\mathbf{j}: \quad -336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \quad (2)$$

$$\mathbf{k}: \quad -252T_{CD} + 1.8a = 0 \quad (3)$$

Recalling that $a = 210 \text{ mm}$, Eq. (3) yields

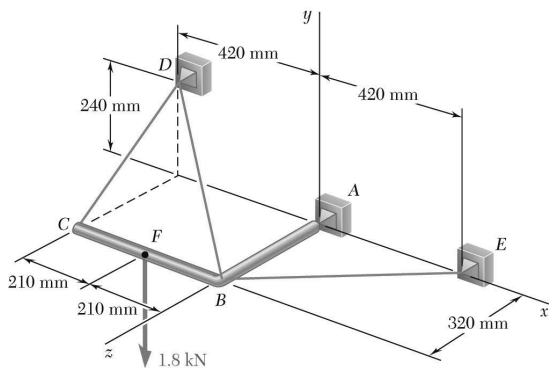
$$T_{CD} = \frac{1.8(210)}{252} = 1.500 \text{ kN} \quad T_{CD} = 1.500 \text{ kN} \quad \blacktriangleleft$$

$$\text{From Eq. (1):} \quad -192(1.5) - 132.413T_{BD} + 576 = 0$$

$$T_{BD} = 2.1751 \text{ kN} \quad T_{BD} = 2.18 \text{ kN} \quad \blacktriangleleft$$

$$\text{From Eq. (2):} \quad -336(1.5) - 231.72(2.1751) + 254.53T_{BE} = 0$$

$$T_{BE} = 3.9603 \text{ kN} \quad T_{BE} = 3.96 \text{ kN} \quad \blacktriangleleft$$



PROBLEM 4.126

Solve Problem 4.125, assuming that the 1.8-kN load is applied at C.

PROBLEM 4.125 The rigid L-shaped member ABC is supported by a ball-and-socket joint at A and by three cables. If a 1.8-kN load is applied at F , determine the tension in each cable.

SOLUTION

See solution of Problem 4.125 for free-body diagram and derivation of Eqs. (1), (2), and (3):

$$-192T_{CD} - 132.413T_{BD} + 576 = 0 \quad (1)$$

$$-336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \quad (2)$$

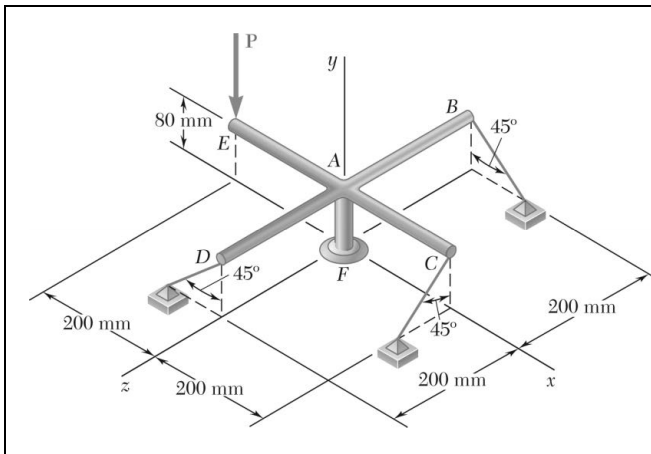
$$-252T_{CD} + 1.8a = 0 \quad (3)$$

In this problem, the 1.8-kN load is applied at C and we have $a = 420$ mm. Carrying into Eq. (3) and solving for T_{CD} ,

$$T_{CD} = 3.00 \quad T_{CD} = 3.00 \text{ kN} \blacktriangleleft$$

From Eq. (1): $-(192)(3) - 132.413T_{BD} + 576 = 0 \quad T_{BD} = 0 \blacktriangleleft$

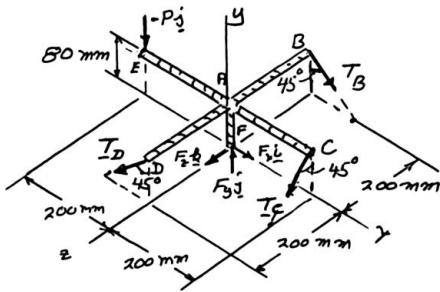
From Eq. (2): $-336(3) - 0 + 254.53T_{BE} = 0 \quad T_{BE} = 3.96 \text{ kN} \blacktriangleleft$



PROBLEM 4.127

The assembly shown consists of an 80-mm rod AF that is welded to a cross consisting of four 200-mm arms. The assembly is supported by a ball-and-socket joint at F and by three short links, each of which forms an angle of 45° with the vertical. For the loading shown, determine (a) the tension in each link, (b) the reaction at F .

SOLUTION



$$\mathbf{r}_{E/F} = -200\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_B = T_B(\mathbf{i} - \mathbf{j})/\sqrt{2} \quad \mathbf{r}_{B/F} = 80\mathbf{j} - 200\mathbf{k}$$

$$\mathbf{T}_C = T_C(-\mathbf{j} + \mathbf{k})/\sqrt{2} \quad \mathbf{r}_{C/F} = 200\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_D = T_D(-\mathbf{i} + \mathbf{j})/\sqrt{2} \quad \mathbf{r}_{D/F} = 80\mathbf{j} + 200\mathbf{k}$$

$$\Sigma M_F = 0: \mathbf{r}_{B/F} \times \mathbf{T}_B + \mathbf{r}_{C/F} \times \mathbf{T}_C + \mathbf{r}_{D/F} \times \mathbf{T}_D + \mathbf{r}_{E/F} \times (-P\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & -200 \\ 1 & -1 & 0 \end{vmatrix} \frac{T_B}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 80 & 0 \\ 0 & -1 & 1 \end{vmatrix} \frac{T_C}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix} \frac{T_D}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero and multiply each equation by $\sqrt{2}$.

$$\mathbf{i}: \quad -200T_B + 80T_C + 200T_D = 0 \quad (1)$$

$$\mathbf{j}: \quad -200T_B - 200T_C - 200T_D = 0 \quad (2)$$

$$\mathbf{k}: \quad -80T_B - 200T_C + 80T_D + 200\sqrt{2}P = 0 \quad (3)$$

$$\frac{80}{200}(2): \quad -80T_B - 80T_C - 80T_D = 0 \quad (4)$$

$$\text{Eqs. (3) + (4):} \quad -160T_B - 280T_C + 200\sqrt{2}P = 0 \quad (5)$$

$$\text{Eqs. (1) + (2):} \quad -400T_B - 120T_C = 0$$

$$T_B = -\frac{120}{400}T_C - 0.3T_C \quad (6)$$

PROBLEM 4.127 (Continued)

$$\text{Eqs. (6)} \rightarrow \text{(5):} \quad -160(-0.3T_C) - 280T_C + 200\sqrt{2}P = 0$$

$$-232T_C + 200\sqrt{2}P = 0$$

$$T_C = 1.2191P$$

$$T_C = 1.219P \quad \blacktriangleleft$$

$$\text{From Eq. (6):} \quad T_B = -0.3(1.2191P) = -0.36574 = P$$

$$T_B = -0.366P \quad \blacktriangleleft$$

$$\text{From Eq. (2):} \quad -200(-0.3657P) - 200(1.2191P) - 200T_{\theta D} = 0$$

$$T_D = -0.8534P$$

$$T_D = -0.853P \quad \blacktriangleleft$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F} + \mathbf{T}_B + \mathbf{T}_C + \mathbf{T}_D - P\mathbf{j} = 0$$

$$\mathbf{i}: \quad F_x + \frac{(-0.36574P)}{\sqrt{2}} - \frac{(-0.8534P)}{\sqrt{2}} = 0$$

$$F_x = -0.3448P \quad F_x = -0.345P$$

$$\mathbf{j}: \quad F_y - \frac{(-0.36574P)}{\sqrt{2}} - \frac{(1.2191P)}{\sqrt{2}} - \frac{(-0.8534P)}{\sqrt{2}} - 200 = 0$$

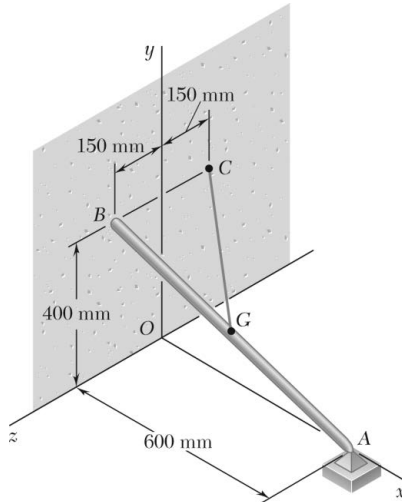
$$F_y = P \quad F_y = P$$

$$\mathbf{k}: \quad F_z + \frac{(1.2191P)}{\sqrt{2}} = 0$$

$$F_z = -0.8620P \quad F_z = -0.862P$$

$$\mathbf{F} = -0.345P\mathbf{i} + P\mathbf{j} - 0.862P\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 4.128



The uniform 10-kg rod AB is supported by a ball-and-socket joint at A and by the cord CG that is attached to the midpoint G of the rod. Knowing that the rod leans against a frictionless vertical wall at B , determine (a) the tension in the cord, (b) the reactions at A and B .

SOLUTION

Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AB} = 0$).

$$\begin{aligned} W &= mg \\ &= (10 \text{ kg})9.81 \text{ m/s}^2 \\ W &= 98.1 \text{ N} \end{aligned}$$

$$\overline{GC} = -300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k} \quad GC = 425 \text{ mm}$$

$$\mathbf{T} = T \frac{\overline{GC}}{GC} = \frac{T}{425}(-300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k})$$

$$\mathbf{r}_{B/A} = -600\mathbf{i} + 400\mathbf{j} + 150\mathbf{k}$$

$$\mathbf{r}_{G/A} = -300\mathbf{i} + 200\mathbf{j} + 75\mathbf{k}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{G/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

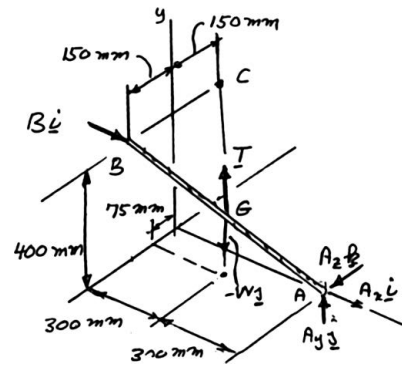
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -600 & 400 & 150 \\ B & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ -300 & 200 & -225 \end{vmatrix} \frac{T}{425} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ 0 & -98.1 & 0 \end{vmatrix}$$

Coefficient of \mathbf{i} : $(-105.88 - 35.29)T + 7357.5 = 0$

$$T = 52.12 \text{ N}$$

$$T = 52.1 \text{ N} \quad \blacktriangleleft$$

Free-Body Diagram:



PROBLEM 4.128 (Continued)

$$\text{Coefficient of } \mathbf{j}: 150B - (300 \times 75 + 300 \times 225) \frac{52.12}{425} = 0$$

$$B = 73.58 \text{ N}$$

$$\mathbf{B} = (73.6 \text{ N})\mathbf{i} \quad \blacktriangleleft$$

$$\Sigma \mathbf{F} = 0: \mathbf{A} + \mathbf{B} + \mathbf{T} - W\mathbf{j} = 0$$

$$\text{Coefficient of } \mathbf{i}: A_x + 73.58 - 52.15 \frac{300}{425} = 0$$

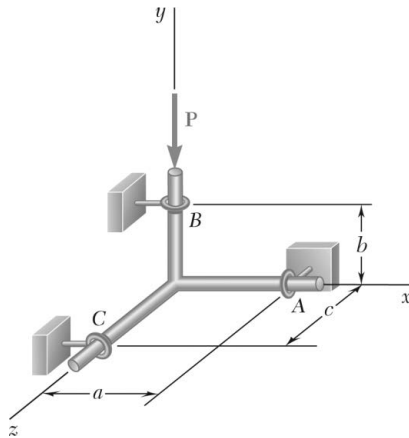
$$A_x = -36.8 \text{ N} \quad \blacktriangleleft$$

$$\text{Coefficient of } \mathbf{j}: A_y + 52.15 \frac{200}{425} - 98.1 = 0$$

$$A_y = 73.6 \text{ N} \quad \blacktriangleleft$$

$$\text{Coefficient of } \mathbf{k}: A_z - 52.15 \frac{225}{425} = 0$$

$$A_z = 27.6 \text{ N} \quad \blacktriangleleft$$



PROBLEM 4.129

Three rods are welded together to form a “corner” that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240 \text{ lb}$, $a = 12 \text{ in.}$, $b = 8 \text{ in.}$, and $c = 10 \text{ in.}$

SOLUTION

From F.B.D. of weldment:

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{AO} \times \mathbf{A} + \mathbf{r}_{BO} \times \mathbf{B} + \mathbf{r}_{CO} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{i} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) = 0$$

From **i**-coefficient: $8B_z - 10C_y = 0$

or $B_z = 1.25C_y$ (1)

j-coefficient: $-12A_z + 10C_x = 0$

or $C_x = 1.2A_z$ (2)

k-coefficient: $12A_y - 8B_x = 0$

or $B_x = 1.5A_y$ (3)

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

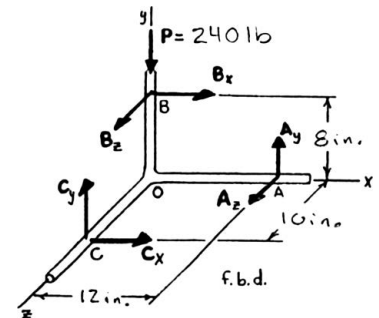
or $(B_x + C_x)\mathbf{i} + (A_y + C_y - 240 \text{ lb})\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$

From **i**-coefficient: $B_x + C_x = 0$

or $C_x = -B_x$ (4)

j-coefficient: $A_y + C_y - 240 \text{ lb} = 0$

or $A_y + C_y = 240 \text{ lb}$ (5)



PROBLEM 4.129 (Continued)

$$\mathbf{k}\text{-coefficient: } A_z + B_z = 0$$

$$\text{or } A_z = -B_z \quad (6)$$

Substituting C_x from Equation (4) into Equation (2),

$$-B_z = 1.2A_z \quad (7)$$

Using Equations (1), (6), and (7),

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25} \left(\frac{B_x}{1.2} \right) = \frac{B_x}{1.5} \quad (8)$$

From Equations (3) and (8):

$$C_y = \frac{1.5A_y}{1.5} \quad \text{or} \quad C_y = A_y$$

and substituting into Equation (5),

$$\begin{aligned} 2A_y &= 240 \text{ lb} \\ A_y &= C_y = 120 \text{ lb} \end{aligned} \quad (9)$$

Using Equation (1) and Equation (9),

$$B_z = 1.25(120 \text{ lb}) = 150.0 \text{ lb}$$

Using Equation (3) and Equation (9),

$$B_x = 1.5(120 \text{ lb}) = 180.0 \text{ lb}$$

$$\text{From Equation (4): } C_x = -180.0 \text{ lb}$$

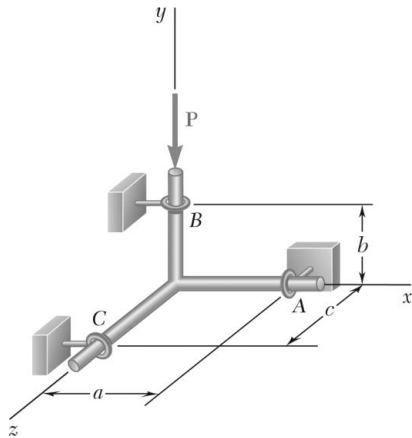
$$\text{From Equation (6): } A_z = -150.0 \text{ lb}$$

Therefore,

$$\mathbf{A} = (120.0 \text{ lb})\mathbf{j} - (150.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{B} = (180.0 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{C} = -(180.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j} \quad \blacktriangleleft$$



PROBLEM 4.130

Solve Problem 4.129, assuming that the force \mathbf{P} is removed and is replaced by a couple $\mathbf{M} = +(600 \text{ lb} \cdot \text{in.})\mathbf{j}$ acting at B .

PROBLEM 4.129 Three rods are welded together to form a “corner” that is supported by three eyebolts. Neglecting friction, determine the reactions at A , B , and C when $P = 240 \text{ lb}$, $a = 12 \text{ in.}$, $b = 8 \text{ in.}$, and $c = 10 \text{ in.}$

SOLUTION

From F.B.D. of weldment:

$$\Sigma \mathbf{M}_O = 0: \mathbf{r}_{AO} \times \mathbf{A} + \mathbf{r}_{BO} \times \mathbf{B} + \mathbf{r}_{CO} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} + (600 \text{ lb} \cdot \text{in.})\mathbf{j} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{j} - 8B_x\mathbf{k}) + (-10C_x\mathbf{i} + 10C_y\mathbf{j}) + (600 \text{ lb} \cdot \text{in.})\mathbf{j} = 0$$

From \mathbf{i} -coefficient: $8B_z - 10C_x = 0$

or $C_y = 0.8B_z$ (1)

\mathbf{j} -coefficient: $-12A_z + 10C_x + 600 = 0$

or $C_x = 1.2A_z - 60$ (2)

\mathbf{k} -coefficient: $12A_y - 8B_x = 0$

or $B_x = 1.5A_y$ (3)

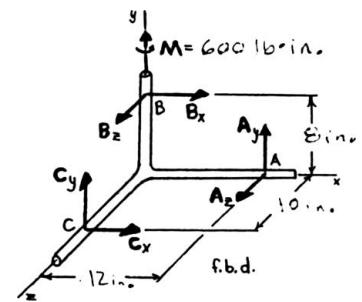
$$\Sigma \mathbf{F} = 0: \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$(B_x + C_x)\mathbf{i} + (A_y + C_y)\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$$

From \mathbf{i} -coefficient: $C_x = -B_x$ (4)

\mathbf{j} -coefficient: $C_y = -A_y$ (5)

\mathbf{k} -coefficient: $A_z = -B_z$ (6)



PROBLEM 4.130 (Continued)

Substituting C_x from Equation (4) into Equation (2),

$$A_z = 50 - \left(\frac{B_x}{1.2} \right) \quad (7)$$

Using Equations (1), (6), and (7),

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3} \right) B_x - 40 \quad (8)$$

From Equations (3) and (8):

$$C_y = A_y - 40$$

Substituting into Equation (5),

$$2A_y = 40$$

$$A_y = 20.0 \text{ lb}$$

From Equation (5):

$$C_y = -20.0 \text{ lb}$$

Equation (1):

$$B_z = -25.0 \text{ lb}$$

Equation (3):

$$B_x = 30.0 \text{ lb}$$

Equation (4):

$$C_x = -30.0 \text{ lb}$$

Equation (6):

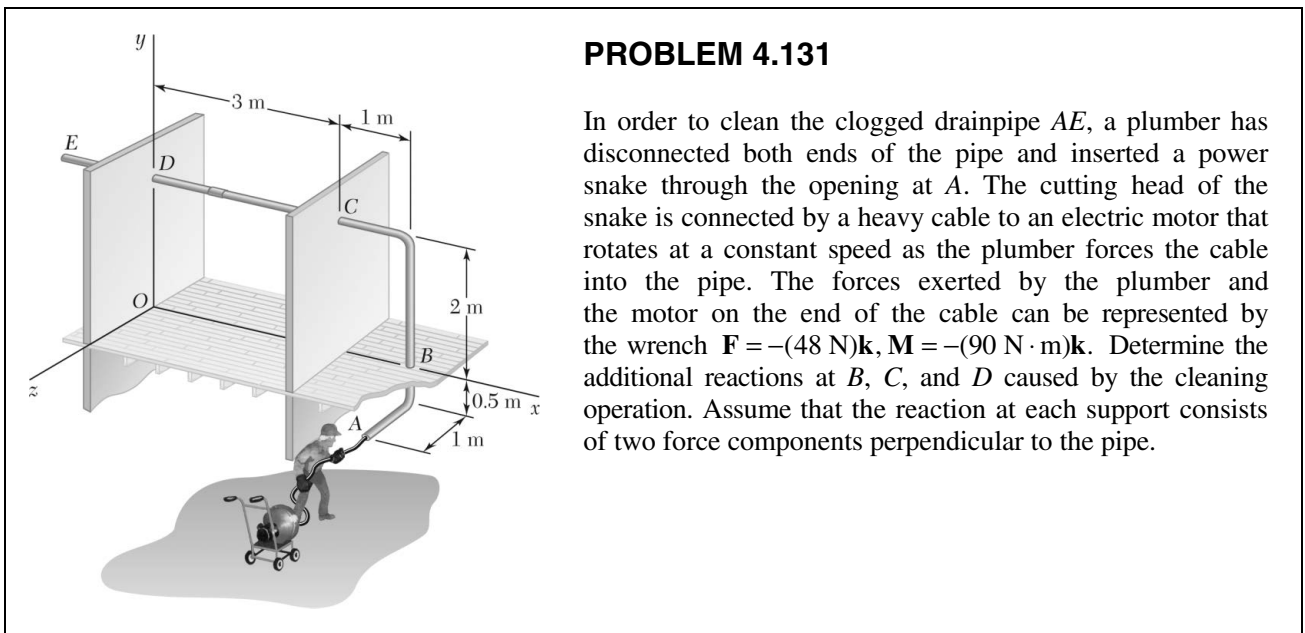
$$A_z = 25.0 \text{ lb}$$

Therefore,

$$\mathbf{A} = (20.0 \text{ lb})\mathbf{j} + (25.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{B} = (30.0 \text{ lb})\mathbf{i} - (25.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{C} = -(30.0 \text{ lb})\mathbf{i} - (20.0 \text{ lb})\mathbf{j} \quad \blacktriangleleft$$

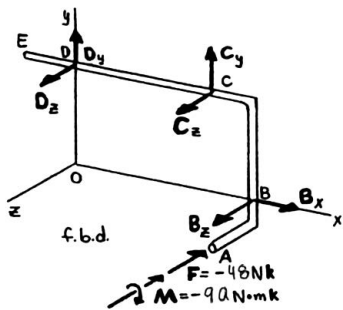


PROBLEM 4.131

In order to clean the clogged drainpipe AE , a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at A . The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48 \text{ N})\mathbf{k}$, $\mathbf{M} = -(90 \text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at B , C , and D caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From F.B.D. of pipe assembly $ABCD$:



$$\Sigma F_x = 0: B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: (48 \text{ N})(2.5 \text{ m}) - B_z(2 \text{ m}) = 0$$

$$B_z = 60.0 \text{ N}$$

$$\text{and } \mathbf{B} = (60.0 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: C_y(3 \text{ m}) - 90 \text{ N}\cdot\text{m} = 0$$

$$C_y = 30.0 \text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: -C_z(3 \text{ m}) - (60.0 \text{ N})(4 \text{ m}) + (48 \text{ N})(4 \text{ m}) = 0$$

$$C_z = -16.00 \text{ N}$$

$$\text{and } \mathbf{C} = (30.0 \text{ N})\mathbf{j} - (16.00 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

$$\Sigma F_y = 0: D_y + 30.0 = 0$$

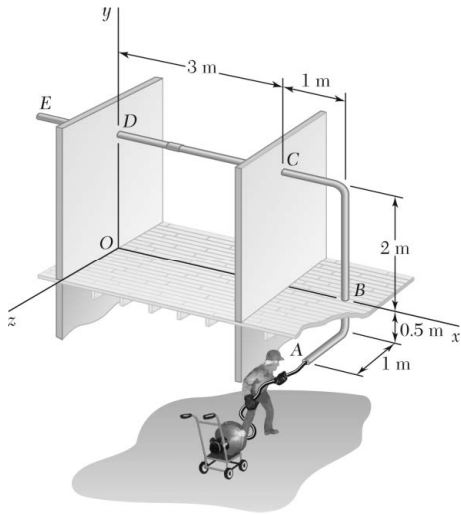
$$D_y = -30.0 \text{ N}$$

$$\Sigma F_z = 0: D_z - 16.00 \text{ N} + 60.0 \text{ N} - 48 \text{ N} = 0$$

$$D_z = 4.00 \text{ N}$$

$$\text{and } \mathbf{D} = -(30.0 \text{ N})\mathbf{j} + (4.00 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

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PROBLEM 4.132

Solve Problem 4.131, assuming that the plumber exerts a force $\mathbf{F} = -(48 \text{ N})\mathbf{k}$ and that the motor is turned off ($\mathbf{M} = 0$).

PROBLEM 4.131 In order to clean the clogged drainpipe AE , a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at A . The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48 \text{ N})\mathbf{k}$, $\mathbf{M} = -(90 \text{ N} \cdot \text{m})\mathbf{k}$. Determine the additional reactions at B , C , and D caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From F.B.D. of pipe assembly $ABCD$:

$$\Sigma F_x = 0: B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: (48 \text{ N})(2.5 \text{ m}) - B_z(2 \text{ m}) = 0$$

$$B_z = 60.0 \text{ N}$$

$$\text{and } \mathbf{B} = (60.0 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: C_y(3 \text{ m}) - B_x(2 \text{ m}) = 0$$

$$C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: C_z(3 \text{ m}) - (60.0 \text{ N})(4 \text{ m}) + (48 \text{ N})(4 \text{ m}) = 0$$

$$C_z = -16.00 \text{ N}$$

$$\text{and } \mathbf{C} = -(16.00 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_y = 0: D_y + C_y = 0$$

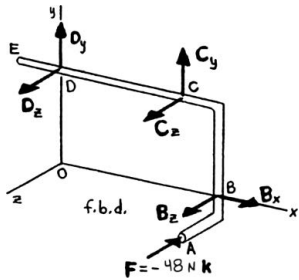
$$D_y = 0$$

$$\Sigma F_z = 0: D_z + B_z + C_z - F = 0$$

$$D_z + 60.0 \text{ N} - 16.00 \text{ N} - 48 \text{ N} = 0$$

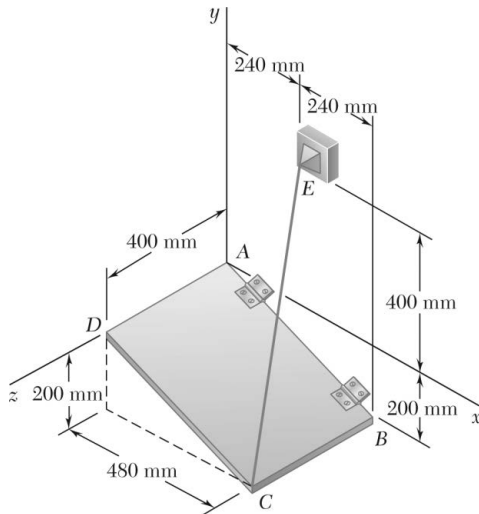
$$D_z = 4.00 \text{ N}$$

$$\text{and } \mathbf{D} = (4.00 \text{ N})\mathbf{k} \blacktriangleleft$$



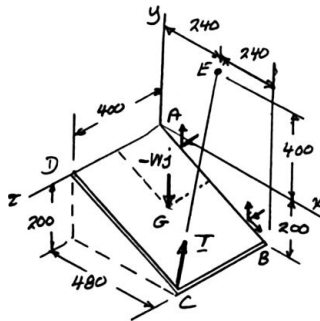
PROBLEM 4.133

The 50-kg plate $ABCD$ is supported by hinges along edge AB and by wire CE . Knowing that the plate is uniform, determine the tension in the wire.



SOLUTION

Free-Body Diagram:



$$W = mg = (50 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 490.5 \text{ N}$$

$$\overline{CE} = -240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k}$$

$$CE = 760 \text{ mm}$$

$$\mathbf{T} = T \frac{\overline{CE}}{CE} = \frac{T}{760}(-240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k})$$

$$\lambda_{AB} = \frac{\overline{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

$$\Sigma \mathbf{M}_{AB} = 0: \lambda_{AB} \cdot (\mathbf{r}_{E/A} \times \mathbf{T}) + \lambda_{AB} \cdot (\mathbf{r}_{G/A} \times -W\mathbf{j}) = 0$$

$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ -240 & 600 & -400 \end{vmatrix} \frac{T}{13 \times 20} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400) \frac{T}{760} + 12 \times 200W = 0$$

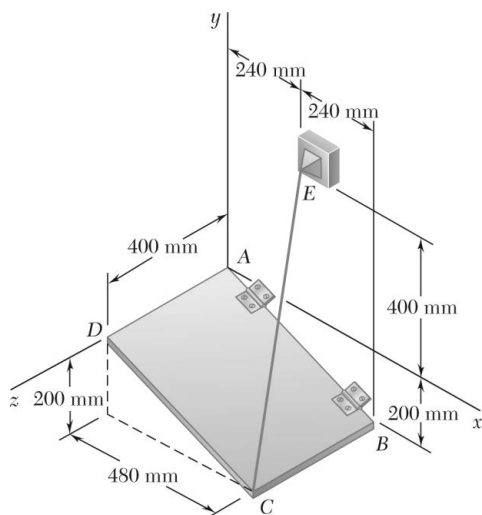
$$T = 0.76W = 0.76(490.5 \text{ N}) \quad T = 373 \text{ N} \quad \blacktriangleleft$$

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PROBLEM 4.134

Solve Problem 4.133, assuming that wire CE is replaced by a wire connecting E and D .

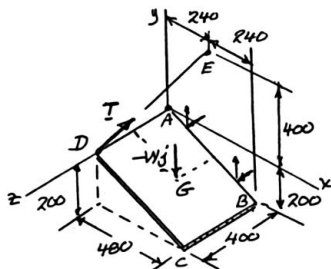
PROBLEM 4.133 The 50-kg plate $ABCD$ is supported by hinges along edge AB and by wire CE . Knowing that the plate is uniform, determine the tension in the wire.



SOLUTION

Free-Body Diagram:

Dimensions in mm



$$W = mg = (50 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 490.50 \text{ N}$$

$$\overline{DE} = -240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k}$$

$$DE = 614.5 \text{ mm}$$

$$\mathbf{T} = T \frac{\overline{DE}}{DE} = \frac{T}{614.5} (240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k})$$

$$\lambda_{AB} = \frac{\overline{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13} (12\mathbf{i} - 5\mathbf{j})$$

$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

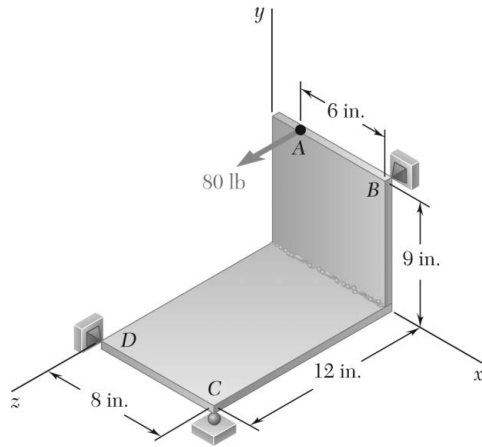
$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 614.5} + \begin{vmatrix} 12 & 5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400) \frac{T}{614.5} + 12 \times 200 \times W = 0$$

$$T = 0.6145W = 0.6145(490.50 \text{ N})$$

$$T = 301 \text{ N} \quad \blacktriangleleft$$

PROBLEM 4.135



Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at B and D and by a ball on a horizontal surface at C . For the loading shown, determine the reaction at C .

SOLUTION

First note:

$$\begin{aligned}\lambda_{BD} &= \frac{-(6 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j} + (12 \text{ in.})\mathbf{k}}{\sqrt{(6)^2 + (9)^2 + (12)^2} \text{ in.}} \\ &= \frac{1}{16.1555} (-6\mathbf{i} - 9\mathbf{j} + 12\mathbf{k}) \\ \mathbf{r}_{AB} &= -(6 \text{ in.})\mathbf{i} \\ \mathbf{P} &= (80 \text{ lb})\mathbf{k} \\ \mathbf{r}_{CD} &= (8 \text{ in.})\mathbf{i} \\ \mathbf{C} &= (C)\mathbf{j}\end{aligned}$$

From the F.B.D. of the plates:

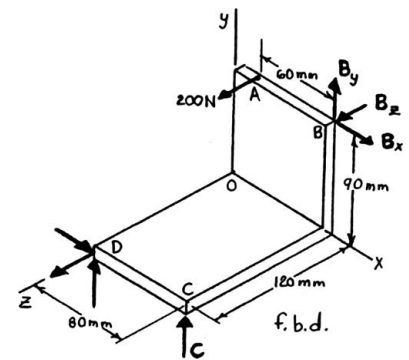
$$\Sigma M_{BD} = 0: \lambda_{BD} \cdot (\mathbf{r}_{AB} \times \mathbf{P}) + \lambda_{BD} \cdot (\mathbf{r}_{CD} \times \mathbf{C}) = 0$$

$$\begin{vmatrix} -6 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} 6(80) \\ 16.1555 \end{bmatrix} + \begin{vmatrix} -6 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \begin{bmatrix} C(8) \\ 16.1555 \end{bmatrix} = 0$$

$$(-9)(6)(80) + (12)(8)C = 0$$

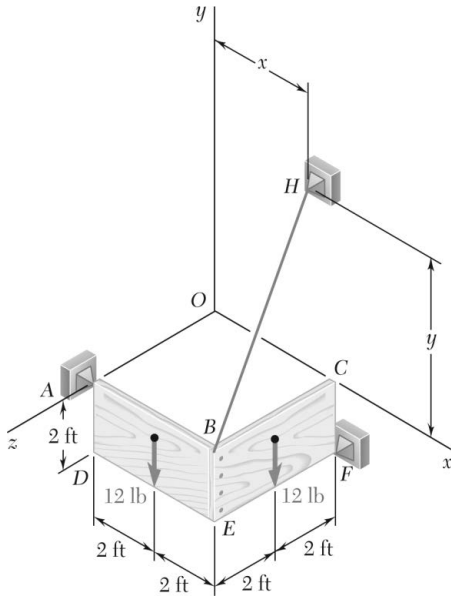
$$C = 45.0 \text{ lb}$$

$$\text{or } \mathbf{C} = (45.0 \text{ lb})\mathbf{j} \blacktriangleleft$$



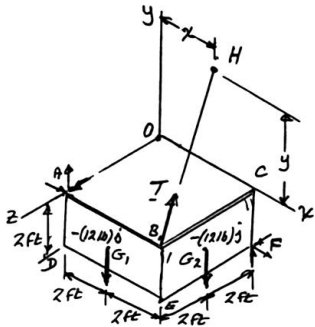
PROBLEM 4.136

Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.



SOLUTION

Free-Body Diagram:



$$\overline{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} \quad AF = 6 \text{ ft}$$

$$\lambda_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \lambda_{AF} \cdot (\mathbf{r}_{G_1/A} \times (-12\mathbf{j})) + \lambda_{AF} \cdot (\mathbf{r}_{G_2/A} \times (-12\mathbf{j})) + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2 \times 2 \times 12) \frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12) \frac{1}{3} + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \quad \text{or} \quad \mathbf{T} \cdot (\lambda_{AF} \times \mathbf{r}_{B/A}) = -32 \quad (1)$$

PROBLEM 4.136 (Continued)

Projection of \mathbf{T} on $(\lambda_{AF} \times \mathbf{r}_{B/A})$ is constant. Thus, T_{\min} is parallel to

$$\lambda_{AF} \times \mathbf{r}_{B/A} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i} = \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k})$$

Corresponding unit vector is $\frac{1}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k})$.

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k}) \frac{1}{\sqrt{5}} \quad (2)$$

From Eq. (1): $\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \left[\frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i} \right] = -32$

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k}) = -32$$

$$\frac{T}{3\sqrt{5}}(16 + 4) = -32 \quad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$$

$$T = 10.7331 \text{ lb}$$

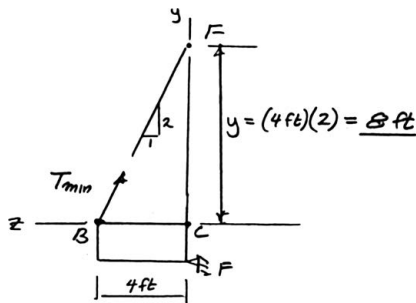
From Eq. (2):

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k}) \frac{1}{\sqrt{5}}$$

$$= 4.8\sqrt{5}(-2\mathbf{j} + \mathbf{k}) \frac{1}{\sqrt{5}}$$

$$\mathbf{T}_{\min} = -(9.6 \text{ lb})\mathbf{j} + (4.8 \text{ lb})\mathbf{k}$$

Since T_{\min} has no \mathbf{i} component, wire BH is parallel to the yz plane, and $x = 4$ ft.



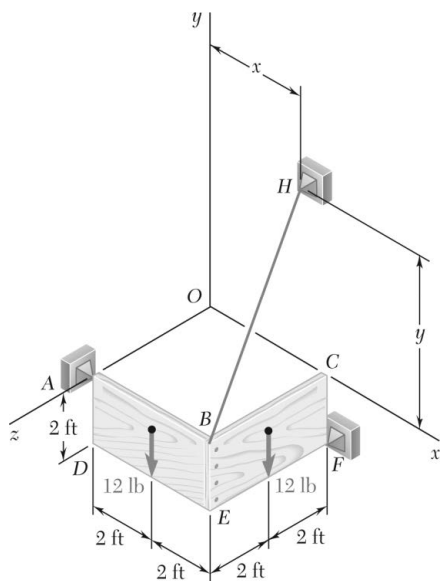
(a) $x = 4.00 \text{ ft}; \quad y = 8.00 \text{ ft} \quad \blacktriangleleft$

(b) $T_{\min} = 10.73 \text{ lb} \quad \blacktriangleleft$

PROBLEM 4.137

Solve Problem 4.136, subject to the restriction that H must lie on the y -axis.

PROBLEM 4.136 Two 2×4 -ft plywood panels, each of weight 12 lb , are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH . Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.



SOLUTION

$$\overline{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$\lambda_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \lambda_{AF} \cdot (\mathbf{r}_{G_1/A} \times (-12\mathbf{j})) + \lambda_{AF} \cdot (\mathbf{r}_{G_2/A} \times (-12\mathbf{j})) + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

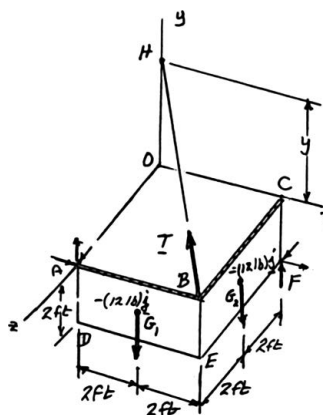
$$(2 \times 2 \times 12) \frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12) \frac{1}{3} + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \quad (1)$$

$$\overline{BH} = -4\mathbf{i} + y\mathbf{j} - 4\mathbf{k} \quad BH = (32 + y^2)^{1/2}$$

$$\mathbf{T} = T \frac{\overline{BH}}{BH} = T \frac{-4\mathbf{i} + y\mathbf{j} - 4\mathbf{k}}{(32 + y^2)^{1/2}}$$

Free-Body Diagram:



PROBLEM 4.137 (Continued)

From Eq. (1):

$$\lambda_{AF} \cdot (\mathbf{r}_{B/A} \times T) = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & 0 \\ -4 & y & -4 \end{vmatrix} \frac{T}{3(32 + y^2)^{1/2}} = -32$$

$$(-16 - 8y)T = -3 \times 32(32 + y^2)^{1/2} \quad T = 96 \frac{(32 + y^2)^{1/2}}{8y + 16} \quad (2)$$

$$\frac{dT}{dy} = 0: \quad 96 \frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y) + (32+y^2)^{1/2}(8)}{(8y+16)^2}$$

Numerator = 0:

$$(8y + 16)y = (32 + y^2)8$$

$$8y^2 + 16y = 32 \times 8 + 8y^2$$

$$x = 0 \text{ ft}; y = 16.00 \text{ ft} \quad \blacktriangleleft$$

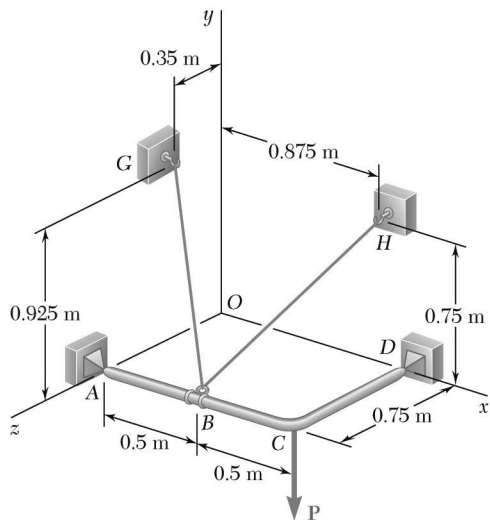
From Eq. (2):

$$T = 96 \frac{(32 + 16^2)^{1/2}}{8 \times 16 + 16} = 11.3137 \text{ lb}$$

$$T_{\min} = 11.31 \text{ lb} \quad \blacktriangleleft$$

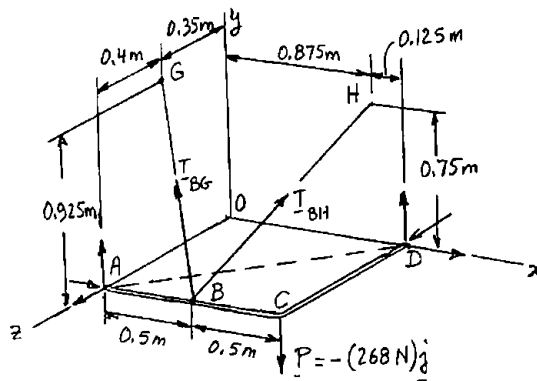
PROBLEM 4.138

The frame ACD is supported by ball-and-socket joints at A and D and by a cable that passes through a ring at B and is attached to hooks at G and H . Knowing that the frame supports at Point C a load of magnitude $P = 268$ N, determine the tension in the cable.



SOLUTION

Free-Body Diagram:



$$\lambda_{AD} = \frac{\overline{AD}}{AD} = \frac{(1 \text{ m})\mathbf{i} - (0.75 \text{ m})\mathbf{k}}{1.25 \text{ m}}$$

$$\lambda_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\begin{aligned} \mathbf{T}_{BG} &= T_{BG} \frac{\overline{BG}}{BG} \\ &= T_{BG} \frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{BH} &= T_{BH} \frac{\overline{BH}}{BH} \\ &= T_{BH} \frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125} \end{aligned}$$

PROBLEM 4.138 (Continued)

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1 \text{ m})\mathbf{i}; \quad \mathbf{P} = -(268 \text{ N})\mathbf{j}$$

To eliminate the reactions at A and D , we shall write

$$\Sigma \mathbf{M}_{AD} = 0: \quad \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG}) + \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH}) + \lambda_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{P}) = 0 \quad (1)$$

Substituting for terms in Eq. (1) and using determinants,

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix} \frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix} \frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

Multiplying all terms by (-1.125) ,

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \quad (2)$$

For this problem,

$$T_{BG} = T_{BH} = T$$

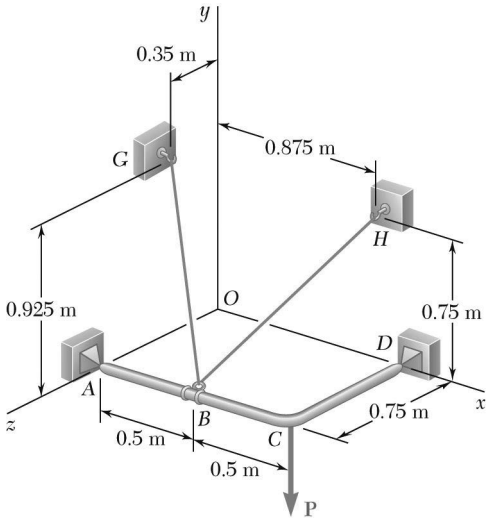
$$(0.27750 + 0.22500)T = 180.900$$

$$T = 360 \text{ N} \blacktriangleleft$$

PROBLEM 4.139

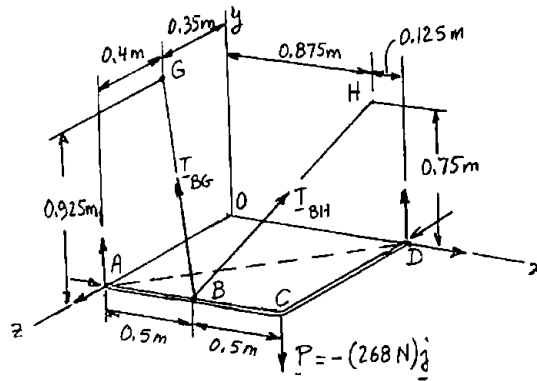
Solve Prob. 4.138, assuming that cable GBH is replaced by a cable GB attached at G and B .

PROBLEM 4.138 The frame ACD is supported by ball-and-socket joints at A and D and by a cable that passes through a ring at B and is attached to hooks at G and H . Knowing that the frame supports at Point C a load of magnitude $P = 268$ N, determine the tension in the cable.



SOLUTION

Free-Body Diagram:



$$\lambda_{AD} = \frac{\overline{AD}}{AD} = \frac{(1 \text{ m})\mathbf{i} - (0.75 \text{ m})\mathbf{k}}{1.25 \text{ m}}$$

$$\lambda_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\begin{aligned} T_{BG} &= T_{BG} \frac{\overline{BG}}{BG} \\ &= T_{BG} \frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125} \end{aligned}$$

$$\begin{aligned} T_{BH} &= T_{BH} \frac{\overline{BH}}{BH} \\ &= T_{BH} \frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125} \end{aligned}$$

PROBLEM 4.139 (Continued)

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1 \text{ m})\mathbf{i}; \quad \mathbf{P} = -(268 \text{ N})\mathbf{j}$$

To eliminate the reactions at A and D , we shall write

$$\Sigma \mathbf{M}_{AD} = 0: \quad \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG}) + \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH}) + \lambda_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{P}) = 0 \quad (1)$$

Substituting for terms in Eq. (1) and using determinants,

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix} \frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix} \frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

Multiplying all terms by (-1.125) ,

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \quad (2)$$

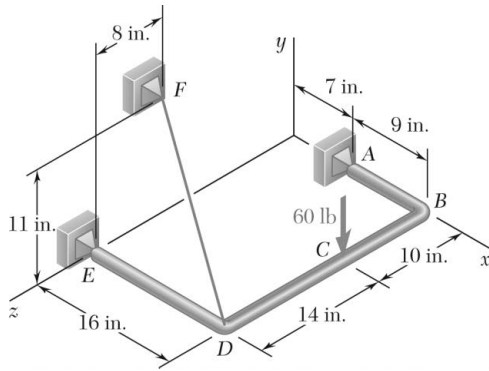
For this problem, $T_{BH} = 0$.

Thus, Eq. (2) reduces to

$$0.27750T_{BG} = 180.900 \qquad T_{BG} = 652 \text{ N} \blacktriangleleft$$

PROBLEM 4.140

The bent rod $ABDE$ is supported by ball-and-socket joints at A and E and by the cable DF . If a 60-lb load is applied at C as shown, determine the tension in the cable.



SOLUTION

$$\overline{DF} = -16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k} \quad DF = 21 \text{ in.}$$

$$\mathbf{T} = T \frac{\overline{DE}}{DF} = \frac{T}{21}(-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

$$\mathbf{r}_{D/E} = 16\mathbf{i}$$

$$\mathbf{r}_{C/E} = 16\mathbf{i} - 14\mathbf{k}$$

$$\lambda_{EA} = \frac{\overline{EA}}{EA} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$$

$$\Sigma M_{EA} = 0: \lambda_{EA} \cdot (\mathbf{r}_{D/E} \times \mathbf{T}) + \lambda_{EA} \cdot (\mathbf{r}_{C/E} \cdot (-60\mathbf{j})) = 0$$

$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T}{21 \times 25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

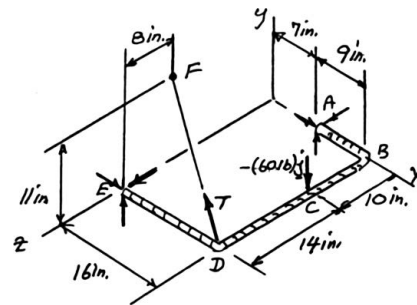
$$-\frac{24 \times 16 \times 11}{21 \times 25} T + \frac{-7 \times 14 \times 60 + 24 \times 16 \times 60}{25} = 0$$

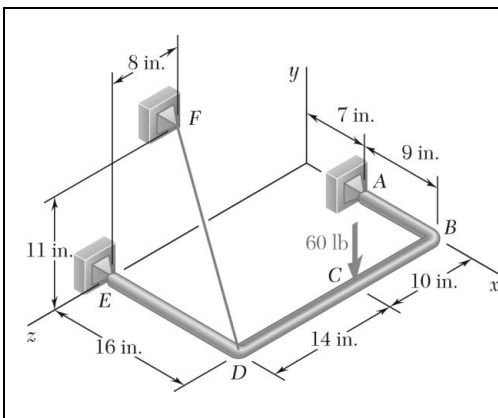
$$201.14T + 17,160 = 0$$

$$T = 85.314 \text{ lb}$$

$$T = 85.3 \text{ lb} \quad \blacktriangleleft$$

Free-Body Diagram:





PROBLEM 4.141

Solve Problem 4.140, assuming that cable DF is replaced by a cable connecting B and F .

SOLUTION

$$\mathbf{r}_{B/A} = 9\mathbf{i}$$

$$\mathbf{r}_{C/A} = 9\mathbf{i} + 10\mathbf{k}$$

$$\overline{BF} = -16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k} \quad BF = 25.16 \text{ in.}$$

$$\mathbf{T} = T \frac{\overline{BF}}{BF} = \frac{T}{25.16} (-16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k})$$

$$\lambda_{AE} = \frac{\overline{AE}}{AE} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$$

$$\Sigma M_{AE} = 0: \quad \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \lambda_{AE} \cdot (\mathbf{r}_{C/A} \cdot (-60\mathbf{j})) = 0$$

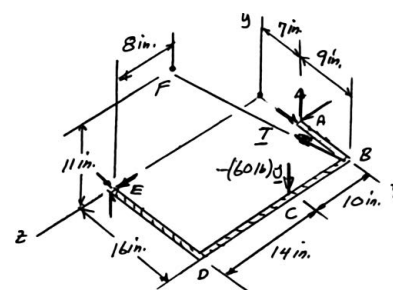
$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix} \frac{T}{25 \times 25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 9 \times 11}{25 \times 25.16} T + \frac{24 \times 9 \times 60 + 7 \times 10 \times 60}{25} = 0$$

$$94.436T - 17,160 = 0$$

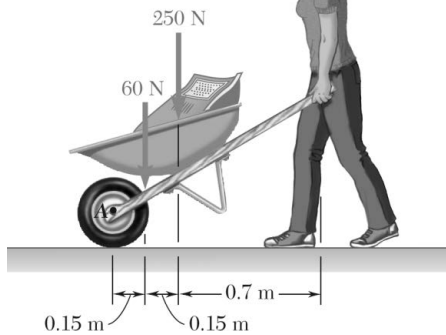
$$T = 181.7 \text{ lb} \quad \blacktriangleleft$$

Free-Body Diagram:



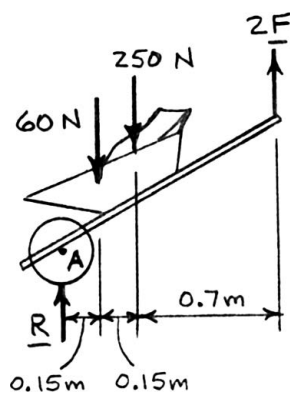
PROBLEM 4.142

A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?



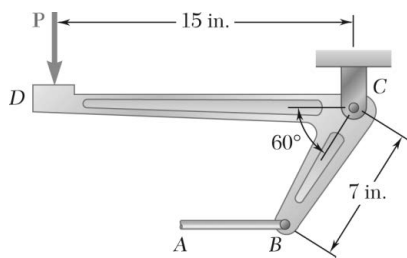
SOLUTION

Free-Body Diagram:



$$+\curvearrowright \Sigma M_A = 0: (2F)(1 \text{ m}) - (60 \text{ N})(0.15 \text{ m}) - (250 \text{ N})(0.3 \text{ m}) = 0$$

$$F = 42.0 \text{ N} \uparrow \blacktriangleleft$$

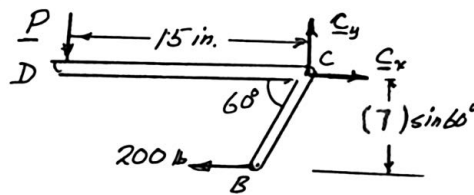


PROBLEM 4.143

The required tension in cable AB is 200 lb. Determine (a) the vertical force P that must be applied to the pedal, (b) the corresponding reaction at C .

SOLUTION

Free-Body Diagram:



$$BC = 7 \text{ in.}$$

$$(a) \quad +\curvearrowright \Sigma M_C = 0: \quad P(15 \text{ in.}) - (200 \text{ lb})(6.062 \text{ in.}) = 0$$

$$P = 80.83 \text{ lb}$$

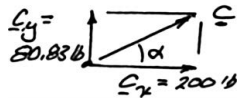
$$\mathbf{P} = 80.8 \text{ lb} \downarrow \blacktriangleleft$$

$$(b) \quad \pm \rightarrow \Sigma F_x = 0: \quad C_x - 200 \text{ lb} = 0$$

$$\mathbf{C}_x = 200 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - P = 0 \quad C_y - 80.83 \text{ lb} = 0$$

$$\mathbf{C}_y = 80.83 \text{ lb} \uparrow$$

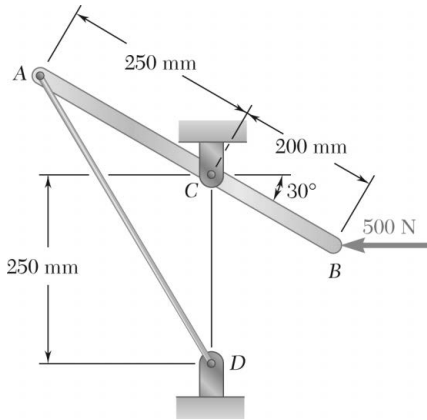


$$\alpha = 22.0^\circ$$

$$C = 215.7 \text{ lb}$$

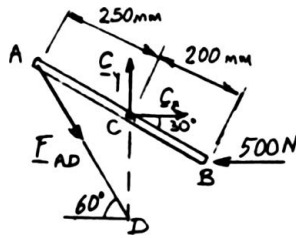
$$\mathbf{C} = 216 \text{ lb} \nearrow 22.0^\circ \blacktriangleleft$$

PROBLEM 4.144



A lever AB is hinged at C and attached to a control cable at A . If the lever is subjected to a 500-N horizontal force at B , determine (a) the tension in the cable, (b) the reaction at C .

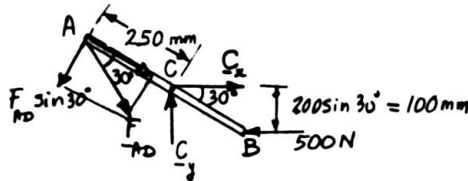
SOLUTION



Triangle ACD is isosceles with $\sphericalangle C = 90^\circ + 30^\circ = 120^\circ$ $\sphericalangle A = \sphericalangle D = \frac{1}{2}(180^\circ - 120^\circ) = 30^\circ$.

Thus, DA forms angle of 60° with the horizontal axis.

(a) We resolve F_{AD} into components along AB and perpendicular to AB .

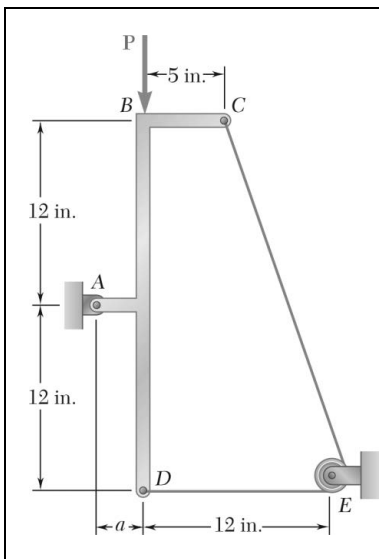


$$+\curvearrowright \Sigma M_C = 0: (F_{AD} \sin 30^\circ)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0 \quad F_{AD} = 400 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad +\rightarrow \Sigma F_x = 0: -(400 \text{ N}) \cos 60^\circ + C_x - 500 \text{ N} = 0 \quad C_x = +300 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: -(400 \text{ N}) \sin 60^\circ + C_y = 0 \quad C_y = +346.4 \text{ N}$$

$$C = 458 \text{ N} \quad \blacktriangleleft 49.1^\circ$$

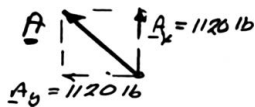
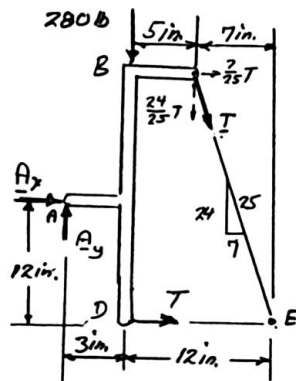


PROBLEM 4.145

A force P of magnitude 280 lb is applied to member $ABCD$, which is supported by a frictionless pin at A and by the cable CED . Since the cable passes over a small pulley at E , the tension may be assumed to be the same in portions CE and ED of the cable. For the case when $a = 3$ in., determine (a) the tension in the cable, (b) the reaction at A .

SOLUTION

Free-Body Diagram:



$$(a) \quad +\curvearrowright \Sigma M_A = 0: \quad -(280 \text{ lb})(8 \text{ in.})$$

$$T(12 \text{ in.}) - \frac{7}{25}T(12 \text{ in.})$$

$$- \frac{24}{25}T(8 \text{ in.}) = 0$$

$$(12 - 11.04)T = 840$$

$$T = 875 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad +\rightarrow \Sigma F_x = 0: \quad \frac{7}{25}(875 \text{ lb}) + 875 \text{ lb} + A_x = 0$$

$$A_x = -1120$$

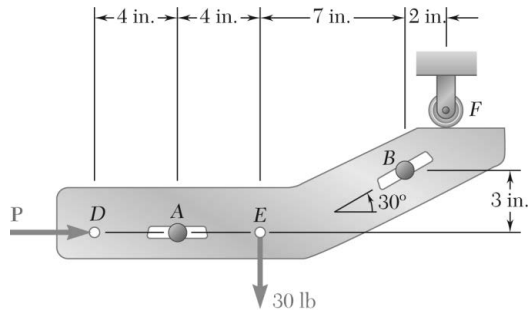
$$A_x = 1120 \text{ lb} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 280 \text{ lb} - \frac{24}{25}(875 \text{ lb}) = 0$$

$$A_y = +1120$$

$$A_y = 1120 \text{ lb} \quad \uparrow$$

$$A = 1584 \text{ lb} \quad \searrow 45.0^\circ \quad \blacktriangleleft$$

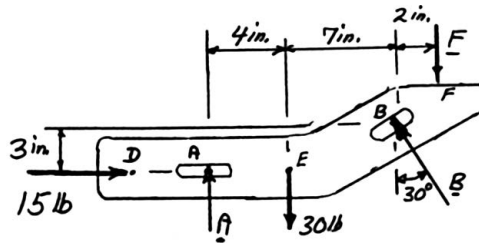


PROBLEM 4.146

Two slots have been cut in plate DEF , and the plate has been placed so that the slots fit two fixed, frictionless pins A and B . Knowing that $P = 15$ lb, determine (a) the force each pin exerts on the plate, (b) the reaction at F .

SOLUTION

Free-Body Diagram:



$$(a) \quad \pm \rightarrow \Sigma F_x = 0: \quad 15 \text{ lb} - B \sin 30^\circ = 0 \qquad \mathbf{B = 30.0 \text{ lb} \nearrow 60.0^\circ \blacktriangleleft}$$

$$(b) \quad + \curvearrowright \Sigma M_A = 0: \quad -(30 \text{ lb})(4 \text{ in.}) + B \sin 30^\circ(3 \text{ in.}) + B \cos 30^\circ(11 \text{ in.}) - F(13 \text{ in.}) = 0$$

$$-120 \text{ lb} \cdot \text{in.} + (30 \text{ lb}) \sin 30^\circ(3 \text{ in.}) + (30 \text{ lb}) \cos 30^\circ(11 \text{ in.}) - F(13 \text{ in.}) = 0$$

$$F = +16.2145 \text{ lb}$$

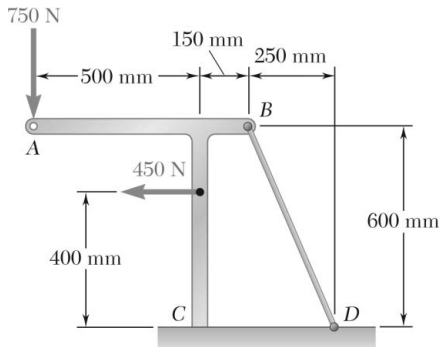
$$\mathbf{F = 16.21 \text{ lb} \downarrow \blacktriangleleft}$$

$$(a) \quad + \uparrow \Sigma F_y = 0: \quad A - 30 \text{ lb} + B \cos 30^\circ - F = 0$$

$$A - 30 \text{ lb} + (30 \text{ lb}) \cos 30^\circ - 16.2145 \text{ lb} = 0$$

$$A = +20.23 \text{ lb}$$

$$\mathbf{A = 20.2 \text{ lb} \uparrow \blacktriangleleft}$$



PROBLEM 4.147

Knowing that the tension in wire BD is 1300 N, determine the reaction at the fixed support C of the frame shown.

SOLUTION

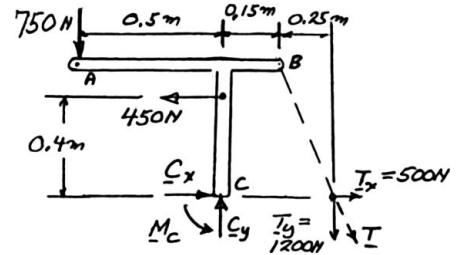
$$T = 1300 \text{ N}$$

$$T_x = \frac{5}{13}T$$

$$= 500 \text{ N}$$

$$T_y = \frac{12}{13}T$$

$$= 1200 \text{ N}$$



$$\rightarrow \Sigma M_x = 0: C_x - 450 \text{ N} + 500 \text{ N} = 0 \quad C_x = -50 \text{ N}$$

$$C_x = 50 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: C_y - 750 \text{ N} - 1200 \text{ N} = 0 \quad C_y = +1950 \text{ N}$$

$$C_y = 1950 \text{ N} \uparrow$$

$$C_x = 50 \text{ N} \leftarrow$$

$$C_y = 1950 \text{ N} \uparrow$$

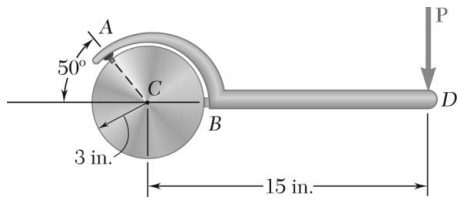
$$C = 1951 \text{ N} \nearrow 88.5^\circ \blacktriangleleft$$

$$+\curvearrowright \Sigma M_C = 0: M_C + (750 \text{ N})(0.5 \text{ m}) + (4.50 \text{ N})(0.4 \text{ m})$$

$$- (1200 \text{ N})(0.4 \text{ m}) = 0$$

$$M_C = -75.0 \text{ N} \cdot \text{m}$$

$$M_C = 75.0 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$



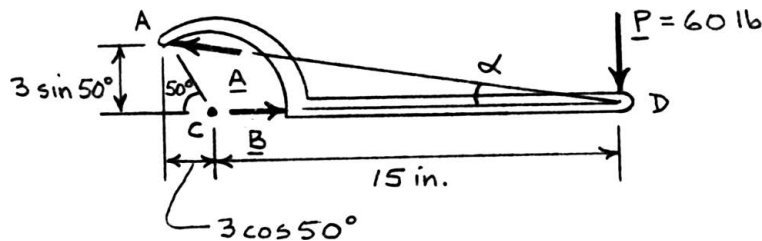
PROBLEM 4.148

The spanner shown is used to rotate a shaft. A pin fits in a hole at *A*, while a flat, frictionless surface rests against the shaft at *B*. If a 60-lb force *P* is exerted on the spanner at *D*, find the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:

(Three-force body)



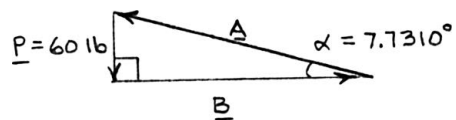
The line of action of **A** must pass through *D*, where **B** and **P** intersect.

$$\begin{aligned}\tan \alpha &= \frac{3 \sin 50^\circ}{3 \cos 50^\circ + 15} \\ &= 0.135756 \\ \alpha &= 7.7310^\circ\end{aligned}$$

$$\begin{aligned}A &= \frac{60 \text{ lb}}{\sin 7.7310^\circ} \\ &= 446.02 \text{ lb}\end{aligned}$$

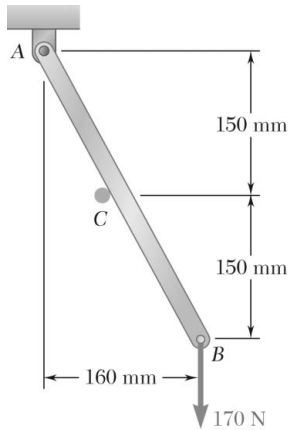
$$\begin{aligned}B &= \frac{60 \text{ lb}}{\tan 7.7310^\circ} \\ &= 441.97 \text{ lb}\end{aligned}$$

Force triangle



$$A = 446 \text{ lb} \nearrow 7.73^\circ \blacktriangleleft$$

$$B = 442 \text{ lb} \rightarrow \blacktriangleleft$$



PROBLEM 4.149

Rod AB is supported by a pin and bracket at A and rests against a frictionless peg at C . Determine the reactions at A and C when a 170-N vertical force is applied at B .

SOLUTION

The reaction at A must pass through D where C and the 170-N force intersect.

$$\tan \alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

$$\alpha = 28.07^\circ$$

We note that triangle ABD is isosceles (since $AC = BC$) and, therefore,

$$\sphericalangle CAD = \alpha = 28.07^\circ$$

Also, since $CD \perp CB$, reaction C forms angle $\alpha = 28.07^\circ$ with the horizontal axis.

We note that A forms angle 2α with the vertical axis. Thus, A and C form angle

$$180^\circ - (90^\circ - \alpha) - 2\alpha = 90^\circ - \alpha$$

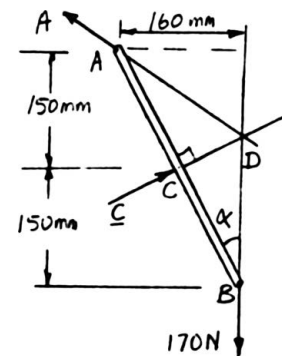
Force triangle is isosceles, and we have

$$A = 170 \text{ N}$$

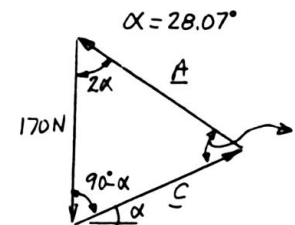
$$C = 2(170 \text{ N}) \sin \alpha$$

$$= 160.0 \text{ N}$$

Free-Body Diagram: (Three-force body)



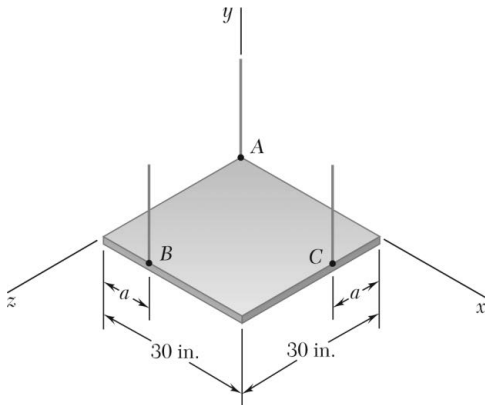
Force triangle



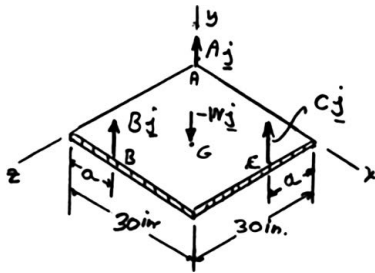
$$A = 170.0 \text{ N} \nearrow 33.9^\circ; \quad C = 160.0 \text{ N} \swarrow 28.1^\circ \blacktriangleleft$$

PROBLEM 4.150

The 24-lb square plate shown is supported by three vertical wires. Determine (a) the tension in each wire when $a = 10$ in., (b) the value of a for which the tension in each wire is 8 lb.



SOLUTION



$$\begin{aligned} \mathbf{r}_{B/A} &= a\mathbf{i} + 30\mathbf{k} \\ \mathbf{r}_{C/A} &= 30\mathbf{i} + a\mathbf{k} \\ \mathbf{r}_{G/A} &= 15\mathbf{i} + 15\mathbf{k} \end{aligned}$$

By symmetry, $B = C$.

$$\begin{aligned} \Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) &= 0 \\ (a\mathbf{i} + 30\mathbf{k}) \times B\mathbf{j} + (30\mathbf{i} + a\mathbf{k}) \times C\mathbf{j} + (15\mathbf{i} + 15\mathbf{k}) \times (-W\mathbf{j}) &= 0 \\ Bak - 30B\mathbf{i} + 30B\mathbf{k} - Bai - 15W\mathbf{k} + 15W\mathbf{i} &= 0 \end{aligned}$$

Equate coefficient of unit vector \mathbf{i} to zero:

$$\mathbf{i}: \quad -30B - Ba + 15W = 0$$

$$B = \frac{15W}{30+a} \quad C = B = \frac{15W}{30+a} \quad (1)$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A + 2 \left[\frac{15W}{30+a} \right] - W = 0; \quad A = \frac{aW}{30+a} \quad (2)$$

(a) For $a = 10$ in.

From Eq. (1): $C = B = \frac{15(24 \text{ lb})}{30+10} = 9.00 \text{ lb}$

From Eq. (2): $A = \frac{10(24 \text{ lb})}{30+10} = 6.00 \text{ lb}$

$$A = 6.00 \text{ lb}; \quad B = C = 9.00 \text{ lb} \quad \blacktriangleleft$$

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PROBLEM 4.150 (Continued)

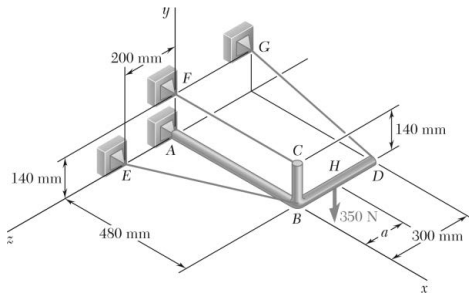
(b) For tension in each wire = 8 lb,

From Eq. (1): $8 \text{ lb} = \frac{15(24 \text{ lb})}{30 + a}$

$$30 \text{ in.} + a = 45$$

$$a = 15.00 \text{ in.} \blacktriangleleft$$

PROBLEM 4.151



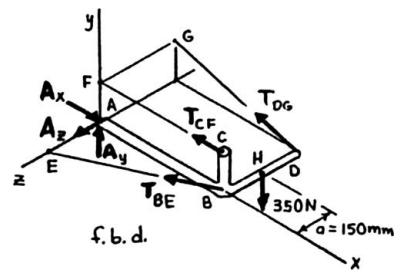
Frame $ABCD$ is supported by a ball-and-socket joint at A and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at A .

SOLUTION

First note:

$$\begin{aligned} \mathbf{T}_{DG} &= \lambda_{DG} T_{DG} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.14 \text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2} \text{ m}} T_{DG} \\ &= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG} \\ &= \frac{T_{DG}}{25} (24\mathbf{i} + 7\mathbf{j}) \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{BE} &= \lambda_{BE} T_{BE} = \frac{-(0.48 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2} \text{ m}} T_{BE} \\ &= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE} \\ &= \frac{T_{BE}}{13} (-12\mathbf{j} + 5\mathbf{k}) \end{aligned}$$



From F.B.D. of frame $ABCD$:

$$\Sigma M_x = 0: \left(\frac{7}{25} T_{DG} \right) (0.3 \text{ m}) - (350 \text{ N})(0.15 \text{ m}) = 0$$

or $T_{DG} = 625 \text{ N} \blacktriangleleft$

$$\Sigma M_y = 0: \left(\frac{24}{25} \times 625 \text{ N} \right) (0.3 \text{ m}) - \left(\frac{5}{13} T_{BE} \right) (0.48 \text{ m}) = 0$$

or $T_{BE} = 975 \text{ N} \blacktriangleleft$

$$\Sigma M_z = 0: T_{CF} (0.14 \text{ m}) + \left(\frac{7}{25} \times 625 \text{ N} \right) (0.48 \text{ m}) - (350 \text{ N})(0.48 \text{ m}) = 0$$

or $T_{CF} = 600 \text{ N} \blacktriangleleft$

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PROBLEM 4.151 (Continued)

$$\Sigma F_x = 0: A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600 \text{ N} - \left(\frac{12}{13} \times 975 \text{ N}\right) - \left(\frac{24}{25} \times 625 \text{ N}\right) = 0$$

$$A_x = 2100 \text{ N}$$

$$\Sigma F_y = 0: A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 625 \text{ N}\right) - 350 \text{ N} = 0$$

$$A_y = 175.0 \text{ N}$$

$$\Sigma F_z = 0: A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 975 \text{ N}\right) = 0$$

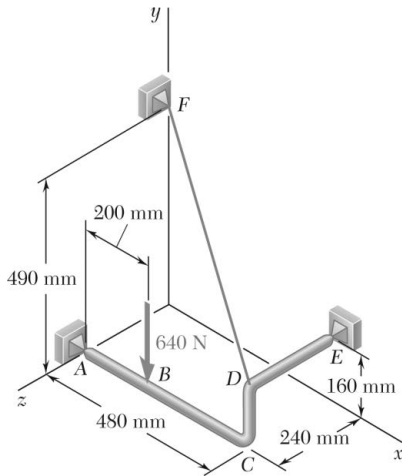
$$A_z = -375 \text{ N}$$

Therefore,

$$\mathbf{A} = (2100 \text{ N})\mathbf{i} + (175.0 \text{ N})\mathbf{j} - (375 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.152

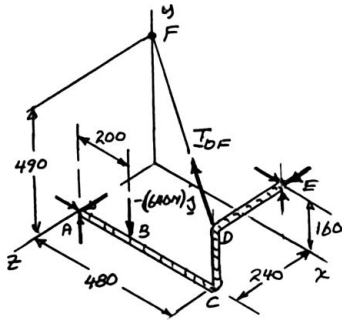
The pipe $ACDE$ is supported by ball-and-socket joints at A and E and by the wire DF . Determine the tension in the wire when a 640-N load is applied at B as shown.



SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overline{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\lambda_{AE} = \frac{\overline{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\lambda_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{D/A} = 480\mathbf{i} + 160\mathbf{j}$$

$$\overline{DF} = -480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}; \quad DF = 630 \text{ mm}$$

$$\mathbf{T}_{DF} = T_{DF} \frac{\overline{DF}}{DF} = T_{DF} \frac{-480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}}{630} = T_{DF} \frac{-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}}{21}$$

$$\Sigma M_{AE} = \lambda_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DF}) + \lambda_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DF}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16}{21 \times 7} T_{DF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-1120T_{DF} + 384 \times 10^3 = 0$$

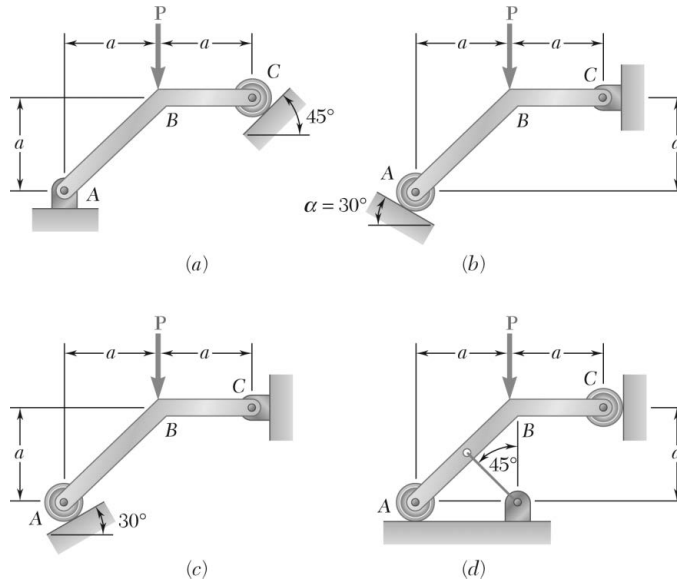
$$T_{DF} = 342.86 \text{ N}$$

$$T_{DF} = 343 \text{ N} \quad \blacktriangleleft$$

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PROBLEM 4.153

A force P is applied to a bent rod ABC , which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(a)

$$+\curvearrowright \Sigma M_A = 0: -Pa + (C \sin 45^\circ)2a + (\cos 45^\circ)a = 0$$

$$3 \frac{C}{\sqrt{2}} = P \quad C = \frac{\sqrt{2}}{3} P \quad C = 0.471P \nearrow 45^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: A_x - \left(\frac{\sqrt{2}}{3} P \right) \frac{1}{\sqrt{2}} \quad A_x = \frac{P}{3} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - P + \left(\frac{\sqrt{2}}{3} P \right) \frac{1}{\sqrt{2}} \quad A_y = \frac{2P}{3} \uparrow$$

$$A = 0.745P \nearrow 63.4^\circ \blacktriangleleft$$

(b)

$$+\curvearrowright \Sigma M_C = 0: +Pa - (A \cos 30^\circ)2a + (A \sin 30^\circ)a = 0$$

$$A(1.732 - 0.5) = P \quad A = 0.812P$$

$$+\rightarrow \Sigma F_x = 0: (0.812P) \sin 30^\circ + C_x = 0 \quad C_x = -0.406P$$

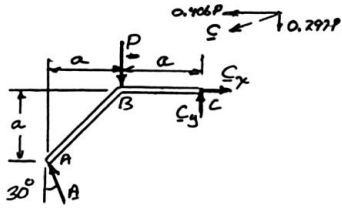
$$+\uparrow \Sigma F_y = 0: (0.812P) \cos 30^\circ - P + C_y = 0 \quad C_y = -0.297P$$

$$C = 0.503P \searrow 36.2^\circ \blacktriangleleft$$

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PROBLEM 4.153 (Continued)

(c)



$$+\curvearrowright \Sigma M_C = 0: +Pa - (A \cos 30^\circ)2a + (A \sin 30^\circ)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

$$A = 0.448P \nearrow 60.0^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: -(0.448P) \sin 30^\circ + C_x = 0 \quad C_x = 0.224P \rightarrow$$

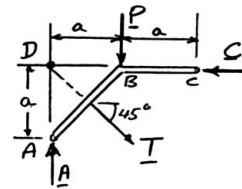
$$+\uparrow \Sigma F_y = 0: (0.448P) \cos 30^\circ - P + C_y = 0 \quad C_y = 0.612P \uparrow$$

$$C = 0.652P \nearrow 69.9^\circ \blacktriangleleft$$

(d) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point **D**.

$$+\curvearrowright \Sigma M_D = 0: P_a = 0$$

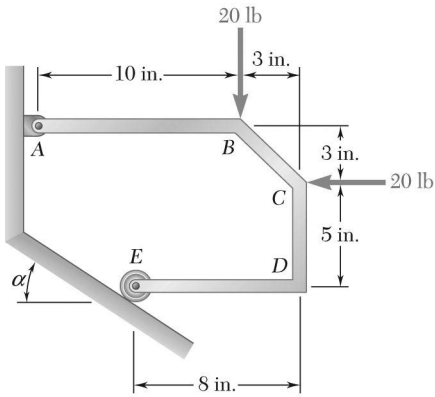
Equilibrium is not maintained.



Rod is improperly constrained. \blacktriangleleft

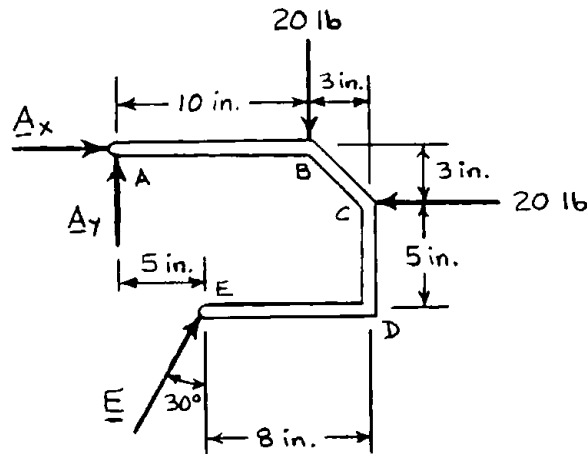
PROBLEM 4.F1

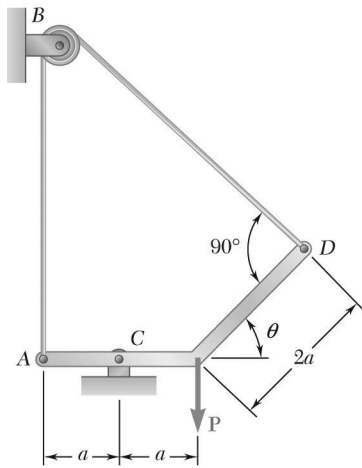
For the frame and loading shown, draw the free-body diagram needed to determine the reactions at A and E when $\alpha = 30^\circ$.



SOLUTION

Free-Body Diagram of Frame:



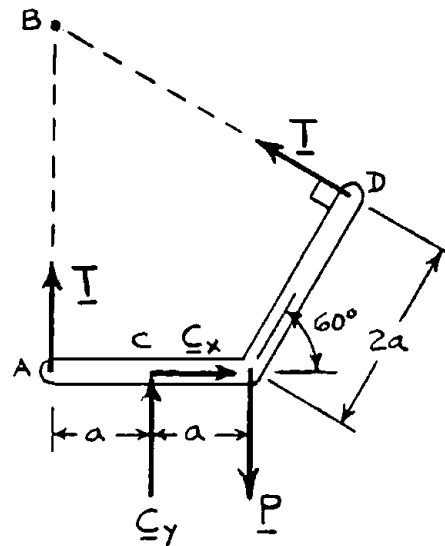


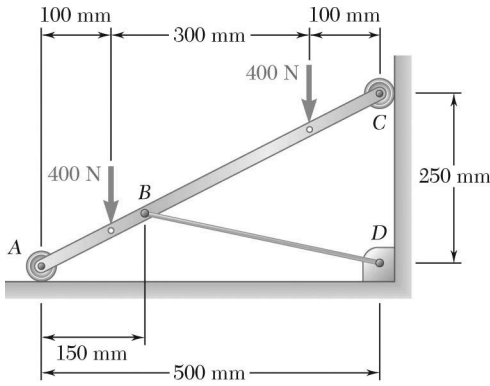
PROBLEM 4.F2

Neglecting friction, draw the free-body diagram needed to determine the tension in cable ABD and the reaction at C when $\theta = 60^\circ$.

SOLUTION

Free-Body Diagram of Member ACD :



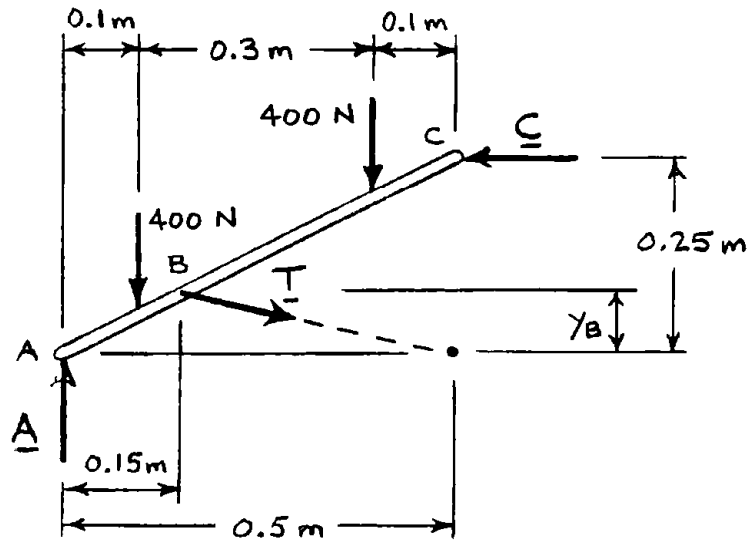


PROBLEM 4.F3

Bar AC supports two 400-N loads as shown. Rollers at A and C rest against frictionless surfaces and a cable BD is attached at B. Draw the free-body diagram needed to determine the tension in cable BD and the reactions at A and C.

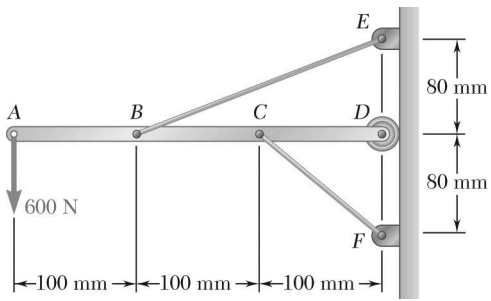
SOLUTION

Free-Body Diagram of Bar AC:



Note: By similar triangles

$$\frac{y_B}{0.25 \text{ m}} = \frac{0.15 \text{ m}}{0.5 \text{ m}} \quad y_B = 0.075 \text{ m}$$

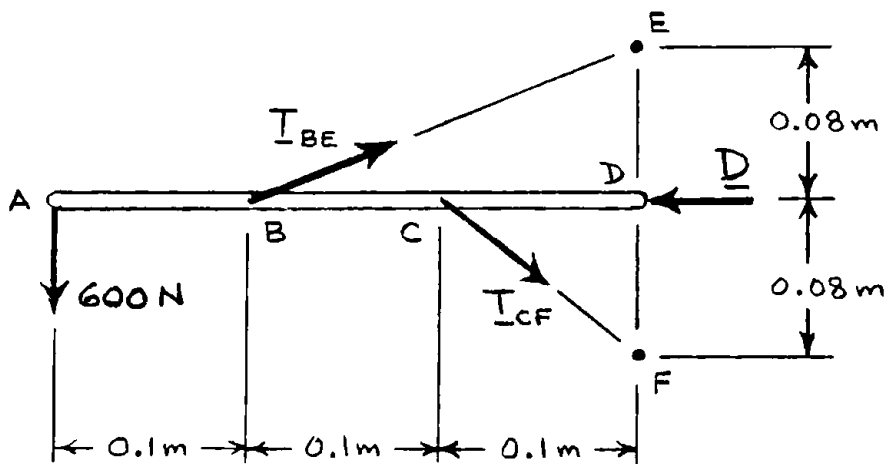


PROBLEM 4.F4

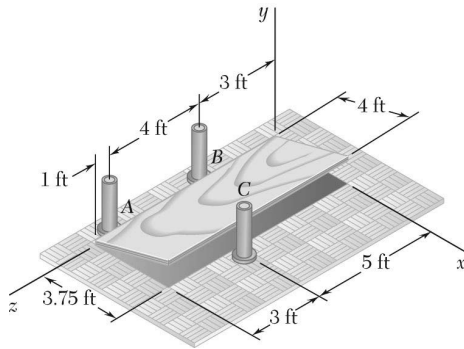
Draw the free-body diagram needed to determine the tension in each cable and the reaction at D .

SOLUTION

Free-Body Diagram of Member $ABCD$:



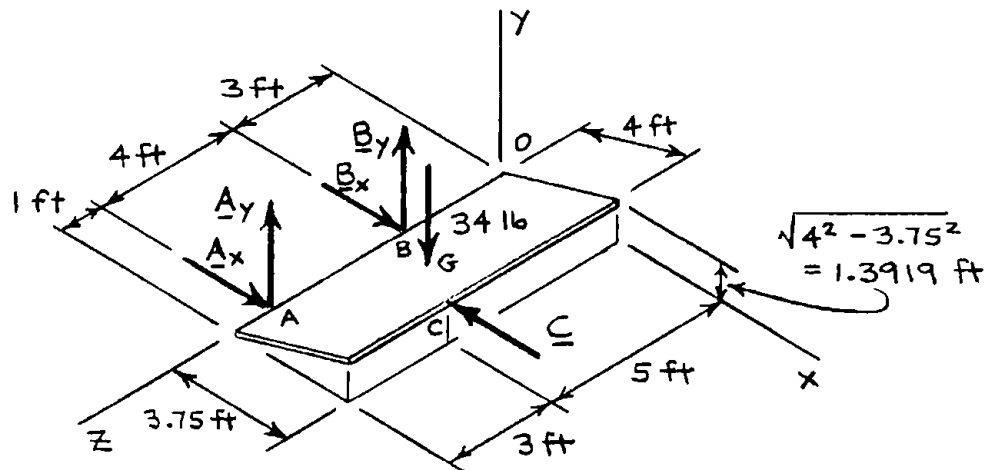
PROBLEM 4.F5



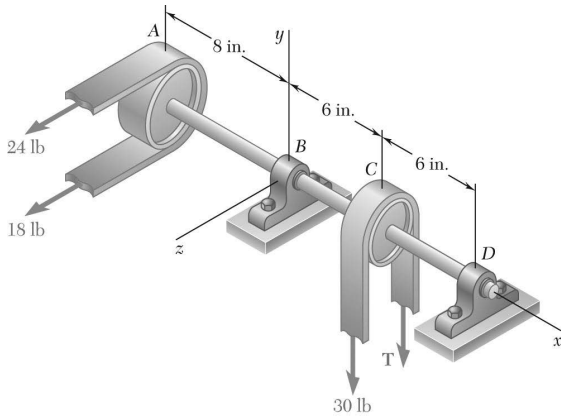
A 4×8 -ft sheet of plywood weighing 34 lb has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars at A and B and its upper edge leans against pipe C . Neglecting friction on all surfaces, draw the free-body diagram needed to determine the reactions at A , B , and C .

SOLUTION

Free-Body Diagram of Plywood sheet:



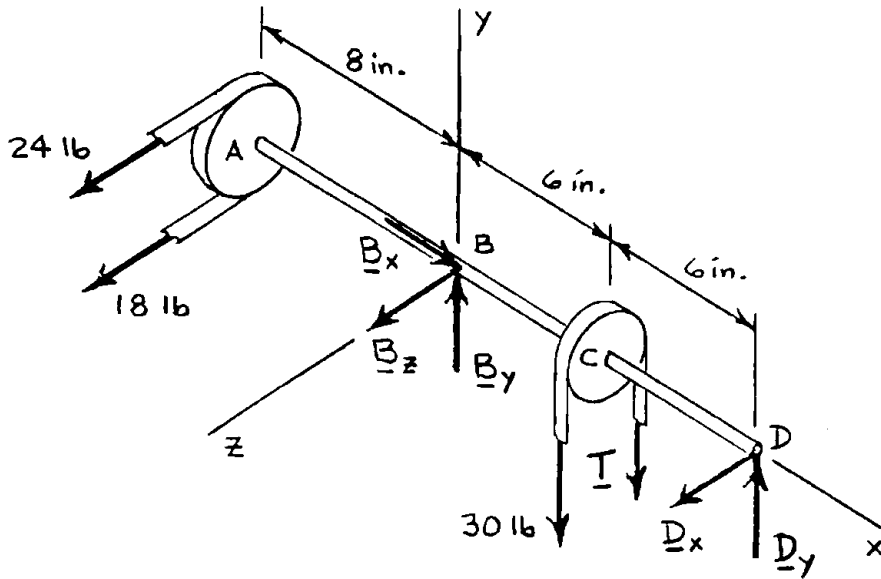
PROBLEM 4.F6



Two transmission belts pass over sheaves welded to an axle supported by bearings at B and D. The sheave at A has a radius of 2.5 in. and the sheave at C has a radius of 2 in. Knowing that the system rotates at a constant rate, draw the free-body diagram needed to determine the tension T and the reactions at B and D. Assume that the bearing at D does not exert any axial thrust and neglect the weights of the sheaves and axle.

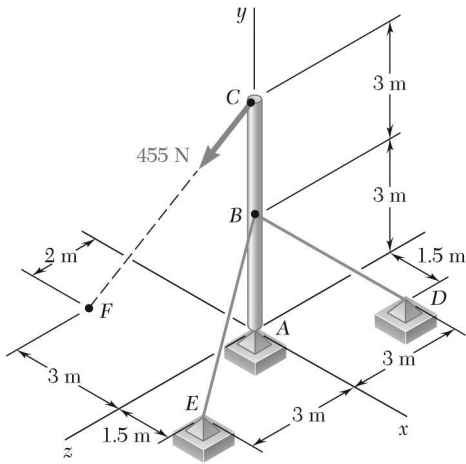
SOLUTION

Free-Body Diagram of axle-sheave system:



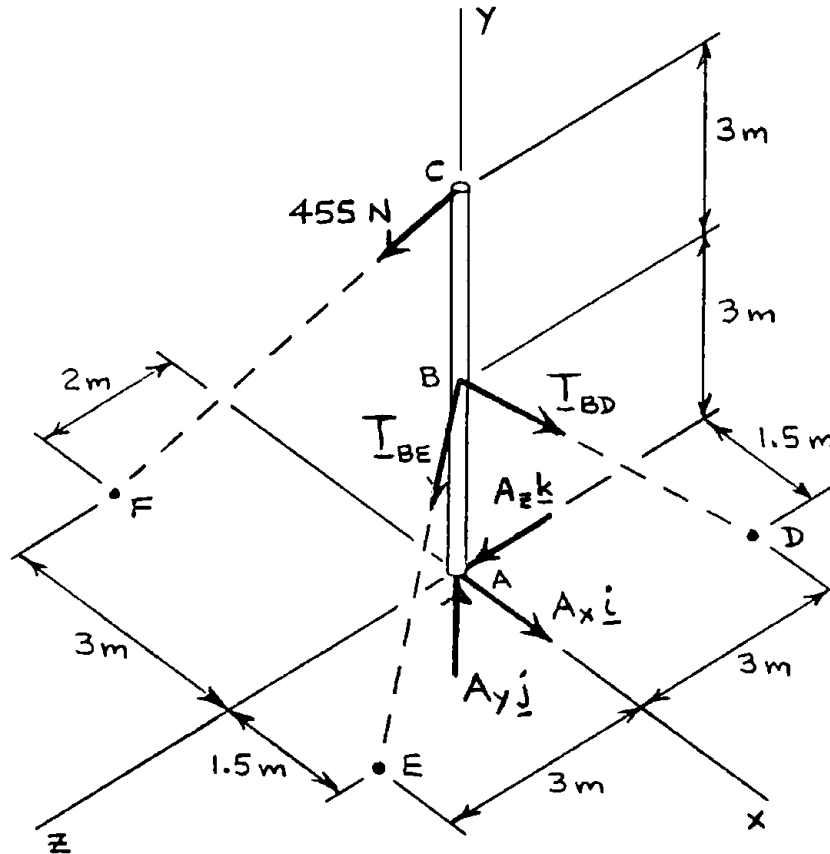
PROBLEM 4.F7

The 6-m pole ABC is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at A and by two cables BD and BE . Draw the free-body diagram needed to determine the tension in each cable and the reaction at A .



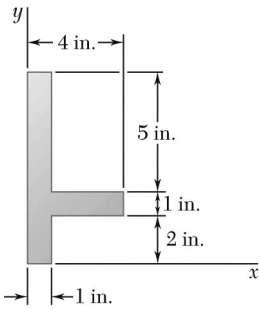
SOLUTION

Free-Body Diagram of Pole:



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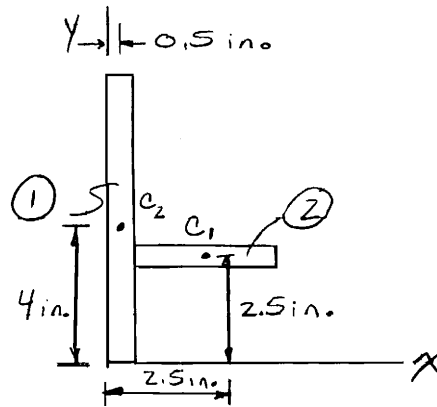
CHAPTER 5



PROBLEM 5.1

Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	\bar{x}, in	\bar{y}, in	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	8	0.5	4	4	32
2	3	2.5	2.5	7.5	7.5
Σ	11			11.5	39.5

$$\bar{X} \Sigma A = \Sigma \bar{x} A$$

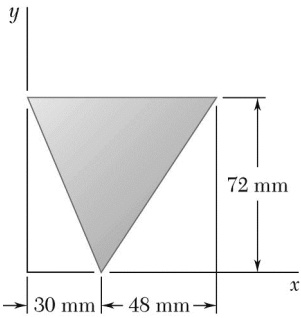
$$\bar{X}(11 \text{ in}^2) = 11.5 \text{ in}^3$$

$$\bar{X} = 1.045 \text{ in.} \quad \blacktriangleleft$$

$$\bar{Y} \Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}(11) = 39.5$$

$$\bar{Y} = 3.59 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 5.2

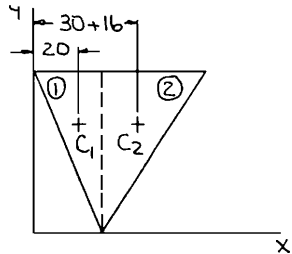
Locate the centroid of the plane area shown.

SOLUTION

For the area as a whole, it can be concluded by observation that

$$\bar{Y} = \frac{2}{3}(72 \text{ mm})$$

or $\bar{Y} = 48.0 \text{ mm} \blacktriangleleft$



Dimensions in mm

	A, mm^2	\bar{x}, mm	$\bar{x}A, \text{mm}^3$
1	$\frac{1}{2} \times 30 \times 72 = 1080$	20	21,600
2	$\frac{1}{2} \times 48 \times 72 = 1728$	46	79,488
Σ	2808		101,088

Then $\bar{X}A = \Sigma \bar{x}A$

$$\bar{X}(2808) = 101,088$$

or $\bar{X} = 36.0 \text{ mm} \blacktriangleleft$

PROBLEM 5.3

Locate the centroid of the plane area shown.

SOLUTION

	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$126 \times 54 = 6804$	9	27	61,236	183,708
2	$\frac{1}{2} \times 126 \times 30 = 1890$	30	64	56,700	120,960
3	$\frac{1}{2} \times 72 \times 48 = 1728$	48	-16	82,944	-27,648
Σ	10,422			200,880	277,020

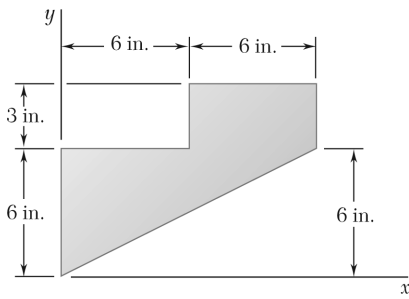
Then $\bar{X} \Sigma A = \Sigma \bar{x} A$

$\bar{X} (10,422 \text{ m}^2) = 200,880 \text{ mm}^2$ or $\bar{X} = 19.27 \text{ mm} \blacktriangleleft$

and $\bar{Y} \Sigma A = \Sigma \bar{y} A$

$\bar{Y} (10,422 \text{ m}^2) = 270,020 \text{ mm}^3$ or $\bar{Y} = 26.6 \text{ mm} \blacktriangleleft$

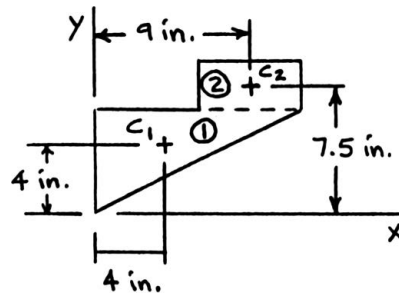
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PROBLEM 5.4

Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	\bar{x}, in	\bar{y}, in	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\frac{1}{2}(12)(6) = 36$	4	4	144	144
2	$(6)(3) = 18$	9	7.5	162	135
Σ	54			306	279

Then

$$\bar{X}A = \Sigma \bar{x}A$$

$$\bar{X}(54) = 306$$

$$\bar{X} = 5.67 \text{ in.} \blacktriangleleft$$

$$\bar{Y}A = \Sigma \bar{y}A$$

$$\bar{Y}(54) = 279$$

$$\bar{Y} = 5.17 \text{ in.} \blacktriangleleft$$

PROBLEM 5.5

Locate the centroid of the plane area shown.

SOLUTION

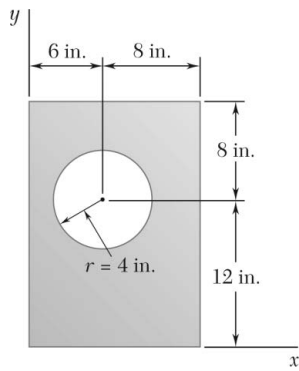
By symmetry, $\bar{X} = \bar{Y}$

	Component	A, in ²	x̄, in.	x̄A, in ³
I	Quarter circle	$\frac{\pi}{4}(10)^2 = 78.54$	4.2441	333.33
II	Square	$-(5)^2 = -25$	2.5	-62.5
Σ		53.54		270.83

$\bar{X}\Sigma A = \Sigma \bar{x}A: \bar{X}(53.54 \text{ in}^2) = 270.83 \text{ in}^3$
 $\bar{X} = 5.0585 \text{ in.}$

$\bar{X} = \bar{Y} = 5.06 \text{ in.} \blacktriangleleft$

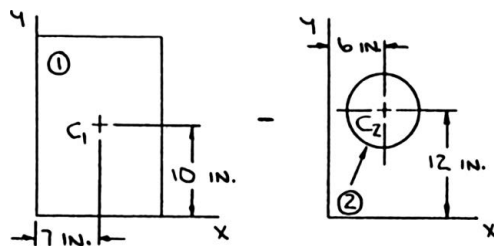
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PROBLEM 5.6

Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	-301.59	-603.19
Σ	229.73			1658.41	2196.8

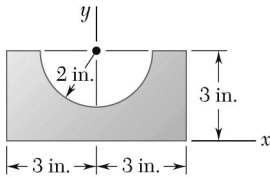
Then

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1658.41}{229.73}$$

$$\bar{X} = 7.22 \text{ in.} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{2196.8}{229.73}$$

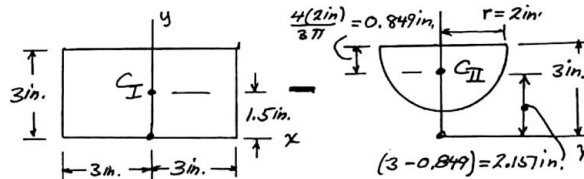
$$\bar{Y} = 9.56 \text{ in.} \blacktriangleleft$$



PROBLEM 5.7

Locate the centroid of the plane area shown.

SOLUTION



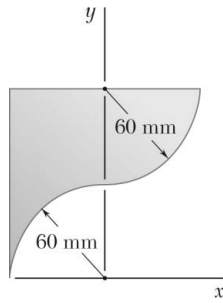
By symmetry, $\bar{X} = 0$

	Component	A, in^2	$\bar{y}, \text{in.}$	$\bar{y}A, \text{in}^3$
I	Rectangle	$(3)(6) = 18$	1.5	27.0
II	Semicircle	$-\frac{\pi}{2}(2)^2 = -6.28$	2.151	-13.51
Σ		11.72		13.49

$$\begin{aligned}\bar{Y} \Sigma A &= \Sigma \bar{y}A \\ \bar{Y}(11.72 \text{ in.}^2) &= 13.49 \text{ in}^3 \\ \bar{Y} &= 1.151 \text{ in.}\end{aligned}$$

$$\bar{X} = 0$$

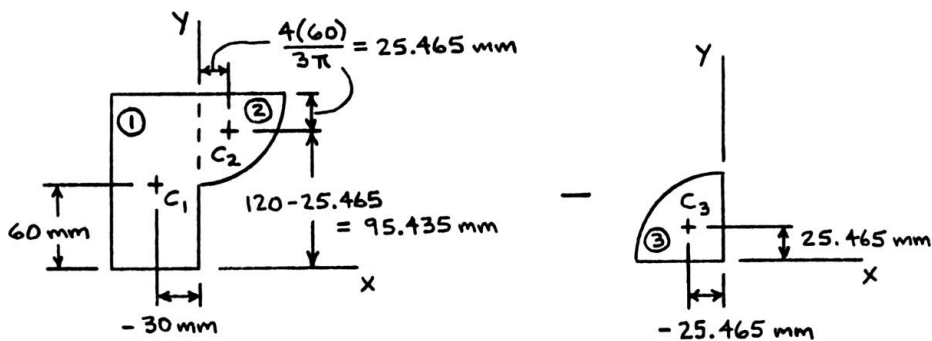
$$\bar{Y} = 1.151 \text{ in.} \blacktriangleleft$$



PROBLEM 5.8

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$(60)(120) = 7200$	-30	60	-216×10^3	432×10^3
2	$\frac{\pi}{4}(60)^2 = 2827.4$	25.465	95.435	72.000×10^3	269.83×10^3
3	$-\frac{\pi}{4}(60)^2 = -2827.4$	-25.465	25.465	72.000×10^3	-72.000×10^3
Σ	7200			-72.000×10^3	629.83×10^3

Then

$$\bar{X}A = \Sigma \bar{x}A \quad \bar{X}(7200) = -72.000 \times 10^3 \quad \bar{X} = -10.00 \text{ mm} \blacktriangleleft$$

$$\bar{Y}A = \Sigma \bar{y}A \quad \bar{Y}(7200) = 629.83 \times 10^3 \quad \bar{Y} = 87.5 \text{ mm} \blacktriangleleft$$

PROBLEM 5.9

Locate the centroid of the plane area shown.

SOLUTION

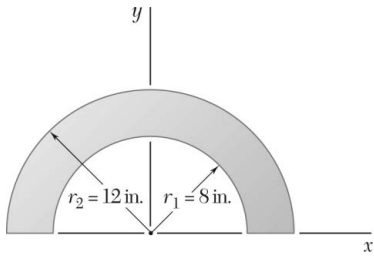
	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{1}{2}(120)(75) = 4500$	80	25	360×10^3	112.5×10^3
2	$(75)(75) = 5625$	157.5	37.5	885.94×10^3	210.94×10^3
3	$-\frac{\pi}{4}(75)^2 = -4417.9$	163.169	43.169	-720.86×10^3	-190.716×10^3
Σ	5707.1			525.08×10^3	132.724×10^3

Then

$$\bar{X}A = \Sigma \bar{x}A \quad \bar{X}(5707.1) = 525.08 \times 10^3 \quad \bar{X} = 92.0 \text{ mm} \blacktriangleleft$$

$$\bar{Y}A = \Sigma \bar{y}A \quad \bar{Y}(5707.1) = 132.724 \times 10^3 \quad \bar{Y} = 23.3 \text{ mm} \blacktriangleleft$$

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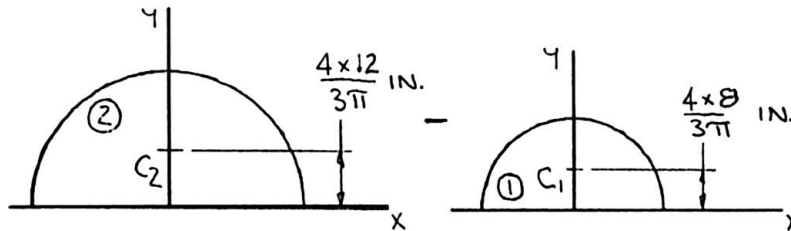
PROBLEM 5.10

Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies

$$\bar{X} = 0 \quad \blacktriangleleft$$

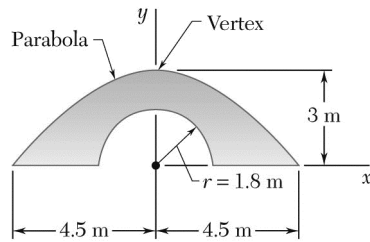


	A, in^2	$\bar{y}, \text{in.}$	$\bar{y}A, \text{in}^3$
1	$-\frac{\pi(8)^2}{2} = -100.531$	3.3953	-341.33
2	$\frac{\pi(12)^2}{2} = 226.19$	5.0930	1151.99
Σ	125.659		810.66

Then

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{810.66 \text{ in}^3}{125.66 \text{ in}^2}$$

$$\text{or } \bar{Y} = 6.45 \text{ in.} \quad \blacktriangleleft$$



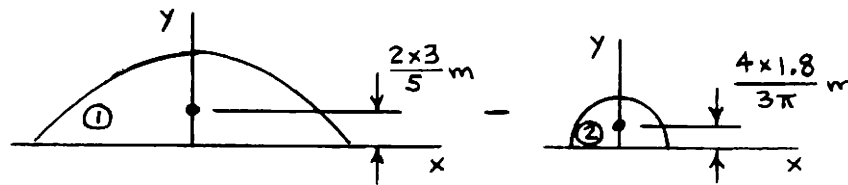
PROBLEM 5.11

Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies

$$\bar{X} = 0 \quad \blacktriangleleft$$

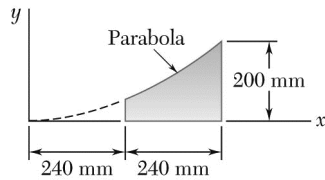


	A, m^2	\bar{y}, m	$\bar{y}A, \text{m}^3$
1	$\frac{4}{3} \times 4.5 \times 3 = 18$	1.2	21.6
2	$-\frac{\pi}{2} (1.8)^2 = -5.0894$	0.76394	-3.8880
Σ	12.9106		17.7120

Then

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{17.7120 \text{ m}^3}{12.9106 \text{ m}^2}$$

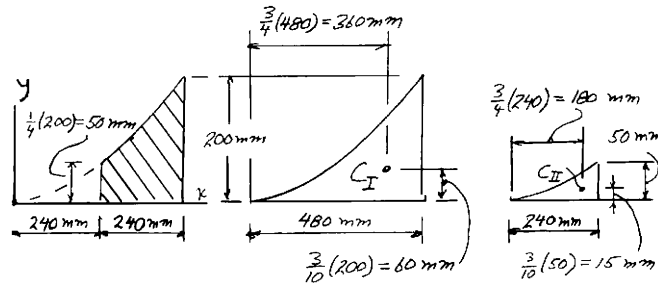
$$\text{or } \bar{Y} = 1.372 \text{ m} \quad \blacktriangleleft$$



PROBLEM 5.12

Locate the centroid of the plane area shown.

SOLUTION



	Area mm ²	\bar{x} , mm	\bar{y} , mm	$\bar{x}A$, mm ³	$\bar{y}A$, mm ³
1	$\frac{1}{3}(200)(480) = 32 \times 10^3$	360	60	11.52×10^6	1.92×10^6
2	$-\frac{1}{3}(50)(240) = 4 \times 10^3$	180	15	-0.72×10^6	-0.06×10^6
Σ	28×10^3			10.80×10^6	1.86×10^6

$$\bar{X} \Sigma A = \Sigma \bar{x} A: \quad \bar{X} (28 \times 10^3 \text{ mm}^2) = 10.80 \times 10^6 \text{ mm}^3$$

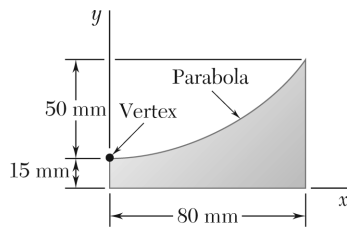
$$\bar{X} = 385.7 \text{ mm}$$

$$\bar{X} = 386 \text{ mm} \quad \blacktriangleleft$$

$$\bar{Y} \Sigma A = \Sigma \bar{y} A: \quad \bar{Y} (28 \times 10^3 \text{ mm}^2) = 1.86 \times 10^6 \text{ mm}^3$$

$$\bar{Y} = 66.43 \text{ mm}$$

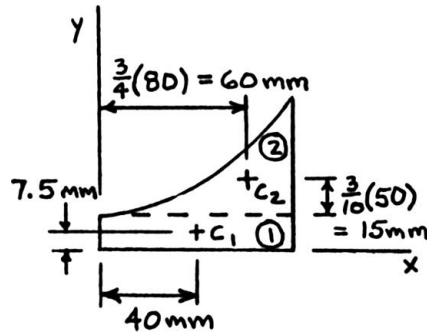
$$\bar{Y} = 66.4 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 5.13

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$(15)(80) = 1200$	40	7.5	48×10^3	9×10^3
2	$\frac{1}{3}(50)(80) = 1333.33$	60	30	80×10^3	40×10^3
Σ	2533.3			128×10^3	49×10^3

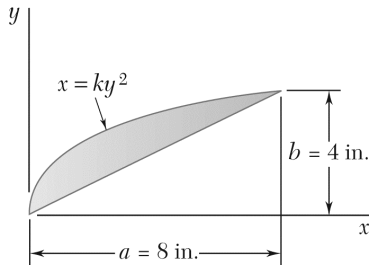
Then

$$\bar{X}A = \Sigma \bar{x}A$$

$$\bar{X}(2533.3) = 128 \times 10^3 \qquad \bar{X} = 50.5 \text{ mm} \blacktriangleleft$$

$$\bar{Y}A = \Sigma \bar{y}A$$

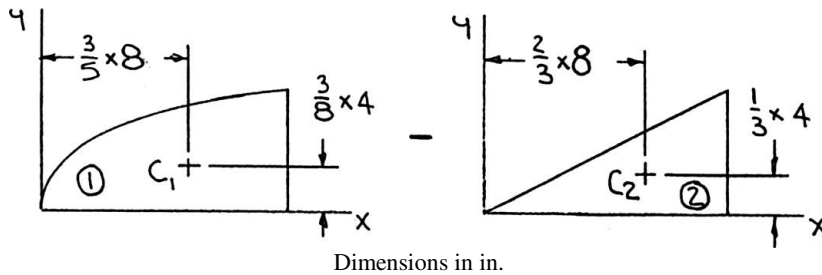
$$\bar{Y}(2533.3) = 49 \times 10^3 \qquad \bar{Y} = 19.34 \text{ mm} \blacktriangleleft$$



PROBLEM 5.14

Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\frac{2}{3}(4)(8) = 21.333$	4.8	1.5	102.398	32.000
2	$-\frac{1}{2}(4)(8) = -16.0000$	5.3333	1.33333	85.333	-21.333
Σ	5.3333			17.0650	10.6670

Then

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(5.3333 \text{ in}^2) = 17.0650 \text{ in}^3$$

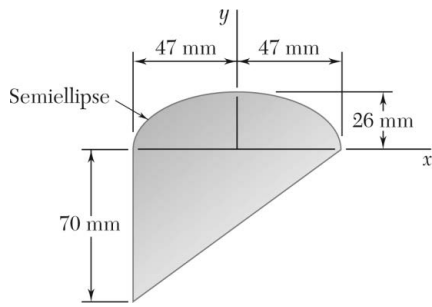
$$\text{or } \bar{X} = 3.20 \text{ in. } \blacktriangleleft$$

and

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(5.3333 \text{ in}^2) = 10.6670 \text{ in}^3$$

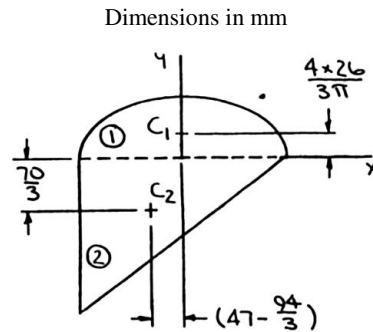
$$\text{or } \bar{Y} = 2.00 \text{ in. } \blacktriangleleft$$



PROBLEM 5.15

Locate the centroid of the plane area shown.

SOLUTION



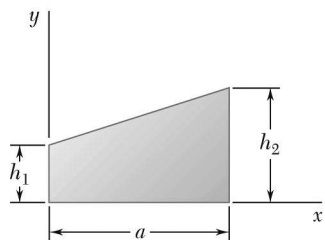
	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{\pi}{2} \times 47 \times 26 = 1919.51$	0	11.0347	0	21,181
2	$\frac{1}{2} \times 94 \times 70 = 3290$	-15.6667	-23.333	-51,543	-76,766
Σ	5209.5			-51,543	-55,584

Then

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{-51,543}{5209.5} \qquad \bar{X} = -9.89 \text{ mm} \blacktriangleleft$$

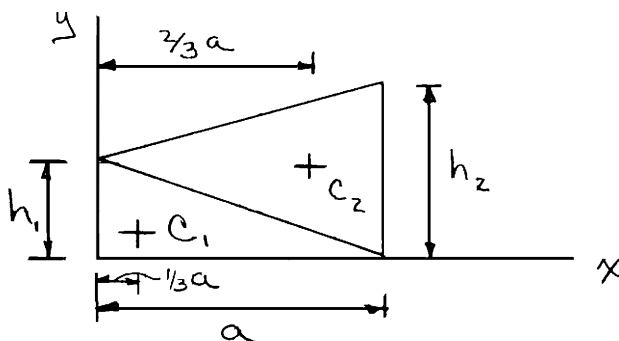
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{-55,584}{5209.5} \qquad \bar{Y} = -10.67 \text{ mm} \blacktriangleleft$$

PROBLEM 5.16



Determine the x coordinate of the centroid of the trapezoid shown in terms of h_1 , h_2 , and a .

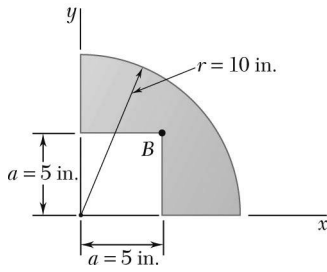
SOLUTION



	A	\bar{x}	$\bar{x}A$
1	$\frac{1}{2}h_1a$	$\frac{1}{3}a$	$\frac{1}{6}h_1a^2$
2	$\frac{1}{2}h_2a$	$\frac{2}{3}a$	$\frac{2}{6}h_2a^2$
Σ	$\frac{1}{2}a(h_1 + h_2)$		$\frac{1}{6}a^2(h_1 + 2h_2)$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{\frac{1}{6}a^2(h_1 + 2h_2)}{\frac{1}{2}a(h_1 + h_2)}$$

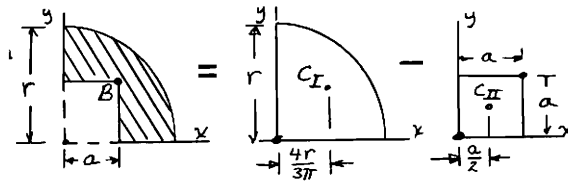
$$\bar{X} = \frac{1}{3}a \frac{h_1 + 2h_2}{h_1 + h_2} \blacktriangleleft$$



PROBLEM 5.17

For the plane area of Problem 5.5, determine the ratio a/r so that the centroid of the area is located at point B .

SOLUTION



By symmetry, $\bar{X} = \bar{Y}$. For centroid to be at B , $\bar{X} = a$.

		Area	\bar{x}	$\bar{x}A$
I	Quarter circle	$\frac{1}{4}\pi r^2$	$\frac{4r}{3\pi}$	$\frac{1}{3}r^3$
II	Square	$-a^2$	$\frac{1}{2}a$	$-\frac{1}{2}a^3$
Σ		$\frac{\pi}{4}r^2 - a^2$		$\frac{1}{3}r^3 - \frac{1}{2}a^3$

$$\bar{X} \Sigma A = \Sigma \bar{x} A: \quad \bar{X} \left(\frac{\pi}{4} r^2 - a^2 \right) = \frac{1}{3} r^3 - \frac{1}{2} a^3$$

$$\text{Set } \bar{X} = a: \quad a \left(\frac{\pi}{4} r^2 - a^2 \right) = \frac{1}{3} r^3 - \frac{1}{2} a^3$$

$$\frac{1}{2} a^3 - \frac{\pi}{4} r^2 a + \frac{1}{3} r^3 = 0$$

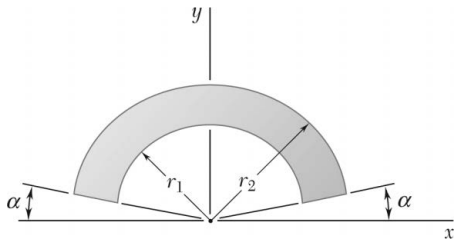
Divide by $\frac{1}{2} r^3$:

$$\left(\frac{a}{r} \right)^3 - \frac{\pi}{2} \frac{a}{r} + \frac{2}{3} = 0$$

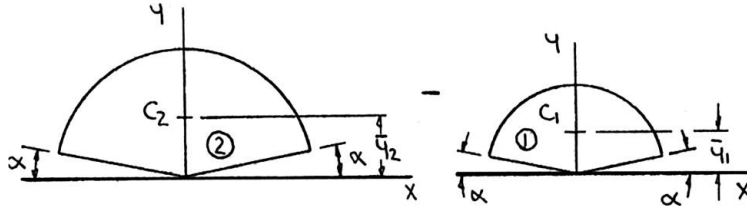
$$\frac{a}{r} = 0.508 \quad \blacktriangleleft$$

PROBLEM 5.18

Determine the y coordinate of the centroid of the shaded area in terms of r_1 , r_2 , and α .



SOLUTION



First, determine the location of the centroid.

From Figure 5.8A:

$$\bar{y}_2 = \frac{2}{3} r_2 \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)} \quad A_2 = \left(\frac{\pi}{2} - \alpha\right) r_2^2$$

$$= \frac{2}{3} r_2 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)}$$

Similarly,

$$\bar{y}_1 = \frac{2}{3} r_1 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \quad A_1 = \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

Then

$$\Sigma \bar{y}A = \frac{2}{3} r_2 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_2^2 \right] - \frac{2}{3} r_1 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_1^2 \right]$$

$$= \frac{2}{3} (r_2^3 - r_1^3) \cos \alpha$$

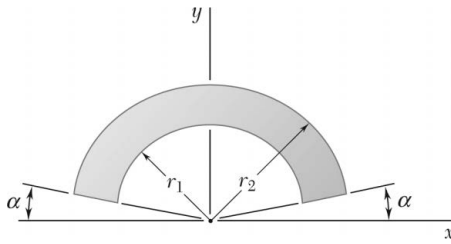
and

$$\Sigma A = \left(\frac{\pi}{2} - \alpha\right) r_2^2 - \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

$$= \left(\frac{\pi}{2} - \alpha\right) (r_2^2 - r_1^2)$$

Now $\bar{Y} \Sigma A = \Sigma \bar{y}A$

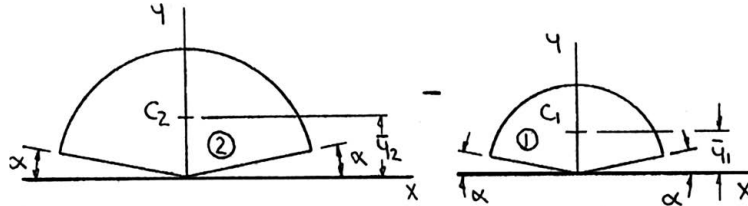
$$\bar{Y} \left[\left(\frac{\pi}{2} - \alpha\right) (r_2^2 - r_1^2) \right] = \frac{2}{3} (r_2^3 - r_1^3) \cos \alpha \quad \bar{Y} = \frac{2}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) \left(\frac{2 \cos \alpha}{\pi - 2\alpha} \right) \blacktriangleleft$$



PROBLEM 5.19

Show that as r_1 approaches r_2 , the location of the centroid approaches that for an arc of circle of radius $(r_1 + r_2)/2$.

SOLUTION



First, determine the location of the centroid.

From Figure 5.8A:

$$\bar{y}_2 = \frac{2}{3} r_2 \frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)} \quad A_2 = \left(\frac{\pi}{2} - \alpha\right) r_2^2$$

$$= \frac{2}{3} r_2 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)}$$

Similarly,

$$\bar{y}_1 = \frac{2}{3} r_1 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \quad A_1 = \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

Then

$$\Sigma \bar{y}A = \frac{2}{3} r_2 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_2^2 \right] - \frac{2}{3} r_1 \frac{\cos \alpha}{\left(\frac{\pi}{2} - \alpha\right)} \left[\left(\frac{\pi}{2} - \alpha\right) r_1^2 \right]$$

$$= \frac{2}{3} (r_2^3 - r_1^3) \cos \alpha$$

and

$$\Sigma A = \left(\frac{\pi}{2} - \alpha\right) r_2^2 - \left(\frac{\pi}{2} - \alpha\right) r_1^2$$

$$= \left(\frac{\pi}{2} - \alpha\right) (r_2^2 - r_1^2)$$

Now $\bar{Y} \Sigma A = \Sigma \bar{y}A$

$$\bar{Y} \left[\left(\frac{\pi}{2} - \alpha\right) (r_2^2 - r_1^2) \right] = \frac{2}{3} (r_2^3 - r_1^3) \cos \alpha$$

$$\bar{Y} = \frac{2}{3} \left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \right) \left(\frac{2 \cos \alpha}{\pi - 2\alpha} \right)$$

PROBLEM 5.19 (Continued)

Using Figure 5.8B, \bar{Y} of an arc of radius $\frac{1}{2}(r_1 + r_2)$ is

$$\begin{aligned}\bar{Y} &= \frac{1}{2}(r_1 + r_2) \frac{\sin(\frac{\pi}{2} - \alpha)}{(\frac{\pi}{2} - \alpha)} \\ &= \frac{1}{2}(r_1 + r_2) \frac{\cos \alpha}{(\frac{\pi}{2} - \alpha)}\end{aligned}\quad (1)$$

Now

$$\begin{aligned}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} &= \frac{(r_2 - r_1)(r_2^2 + r_1 r_2 + r_1^2)}{(r_2 - r_1)(r_2 + r_1)} \\ &= \frac{r_2^2 + r_1 r_2 + r_1^2}{r_2 + r_1}\end{aligned}$$

Let

$$\begin{aligned}r_2 &= r + \Delta \\ r_1 &= r - \Delta\end{aligned}$$

Then

$$r = \frac{1}{2}(r_1 + r_2)$$

and

$$\begin{aligned}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} &= \frac{(r + \Delta)^2 + (r + \Delta)(r - \Delta)(r - \Delta)^2}{(r + \Delta) + (r - \Delta)} \\ &= \frac{3r^2 + \Delta^2}{2r}\end{aligned}$$

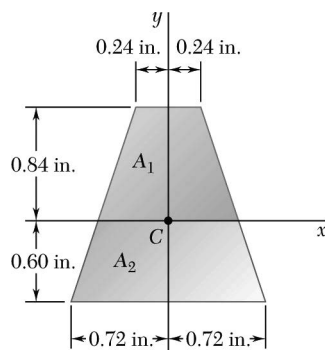
In the limit as $\Delta \rightarrow 0$ (i.e., $r_1 = r_2$), then

$$\begin{aligned}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} &= \frac{3}{2}r \\ &= \frac{3}{2} \times \frac{1}{2}(r_1 + r_2)\end{aligned}$$

So that

$$\bar{Y} = \frac{2}{3} \times \frac{3}{4}(r_1 + r_2) \frac{\cos \alpha}{\frac{\pi}{2} - \alpha} \quad \text{or} \quad \bar{Y} = (r_1 + r_2) \frac{\cos \alpha}{\pi - 2\alpha} \blacktriangleleft$$

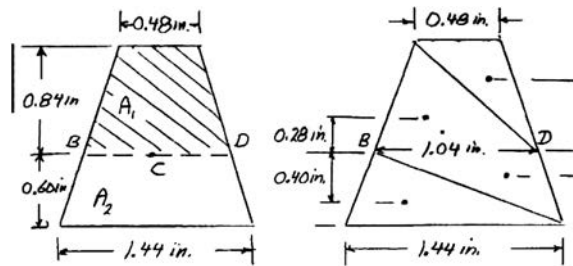
which agrees with Equation (1).



PROBLEM 5.20

The horizontal x -axis is drawn through the centroid C of the area shown, and it divides the area into two component areas A_1 and A_2 . Determine the first moment of each component area with respect to the x -axis, and explain the results obtained.

SOLUTION



Length of BD :

$$BD = 0.48 \text{ in.} + (1.44 \text{ in.} - 0.48 \text{ in.}) \frac{0.84 \text{ in.}}{0.84 \text{ in.} \times 0.60 \text{ in.}} = 0.48 + 0.56 = 1.04 \text{ in.}$$

Area above x -axis (consider two triangular areas):

$$\begin{aligned} Q_1 &= \Sigma \bar{y}A = (0.28 \text{ in.}) \left[\frac{1}{2} (0.84 \text{ in.})(1.04 \text{ in.}) \right] + (0.56 \text{ in.}) \left[\frac{1}{2} (0.84 \text{ in.})(0.48 \text{ in.}) \right] \\ &= 0.122304 \text{ in}^3 + 0.112896 \text{ in}^3 \end{aligned}$$

$$Q_1 = 0.2352 \text{ in}^3 \quad \blacktriangleleft$$

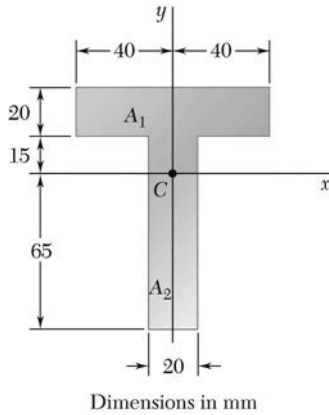
Area below x -axis:

$$\begin{aligned} Q_2 &= \Sigma \bar{y}A = -(0.40 \text{ in.}) \left[\frac{1}{2} (0.60 \text{ in.})(1.44 \text{ in.}) \right] - (0.20 \text{ in.}) \left[\frac{1}{2} (0.60 \text{ in.}) \right] \\ &= -0.1728 \text{ in}^3 - 0.0624 \text{ in}^3 \end{aligned}$$

$$Q_2 = -0.2352 \text{ in}^3 \quad \blacktriangleleft$$

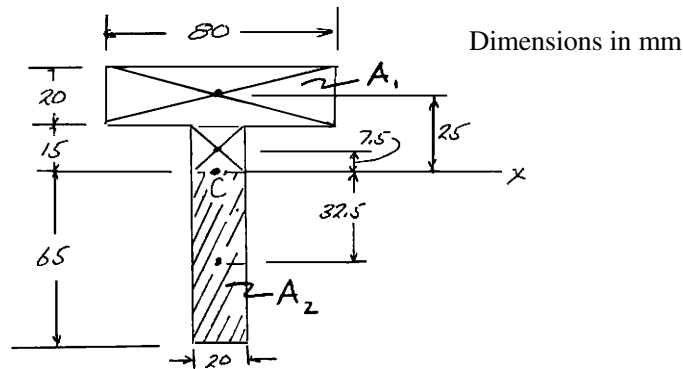
$$|Q_1| = |Q_2|, \text{ since } C \text{ is centroid and thus, } Q = \Sigma \bar{y}A = 0$$

PROBLEM 5.21



The horizontal x -axis is drawn through the centroid C of the area shown, and it divides the area into two component areas A_1 and A_2 . Determine the first moment of each component area with respect to the x -axis, and explain the results obtained.

SOLUTION



Area above x -axis (Area A_1):

$$Q_1 = \Sigma \bar{y} A = (25)(20 \times 80) + (7.5)(15 \times 20)$$

$$= 40 \times 10^3 + 2.25 \times 10^3$$

$$Q_1 = 42.3 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

Area below x -axis (Area A_2):

$$Q_2 = \Sigma \bar{y} A = (-32.5)(65 \times 20)$$

$$Q_2 = -42.3 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

$|Q_1| = |Q_2|$, since C is centroid and thus, $Q = \Sigma \bar{y} A = 0$

PROBLEM 5.22

A composite beam is constructed by bolting four plates to four $60 \times 60 \times 12$ -mm angles as shown. The bolts are equally spaced along the beam, and the beam supports a vertical load. As proved in mechanics of materials, the shearing forces exerted on the bolts at A and B are proportional to the first moments with respect to the centroidal x -axis of the red-shaded areas shown, respectively, in parts *a* and *b* of the figure. Knowing that the force exerted on the bolt at A is 280 N, determine the force exerted on the bolt at B.

SOLUTION

From the problem statement, F is proportional to Q_x .

Therefore,

$$\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}, \text{ or } F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

For the first moments,

$$(Q_x)_A = \left(225 + \frac{12}{2} \right) (300 \times 12)$$

$$= 831,600 \text{ mm}^3$$

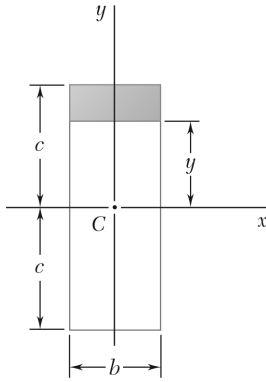
$$(Q_x)_B = (Q_x)_A + 2 \left(225 - \frac{12}{2} \right) (48 \times 12) + 2(225 - 30)(12 \times 60)$$

$$= 1,364,688 \text{ mm}^3$$

Then

$$F_B = \frac{1,364,688}{831,600} (280 \text{ N}) \quad \text{or } F_B = 459 \text{ N} \blacktriangleleft$$

PROBLEM 5.23



The first moment of the shaded area with respect to the x -axis is denoted by Q_x .
 (a) Express Q_x in terms of b , c , and the distance y from the base of the shaded area to the x -axis. (b) For what value of y is Q_x maximum, and what is that maximum value?

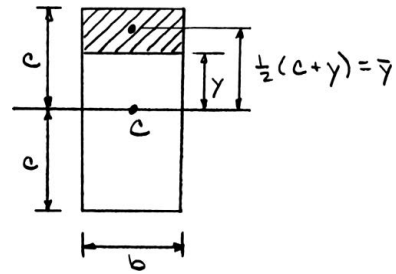
SOLUTION

Shaded area:

$$A = b(c - y)$$

$$Q_x = \bar{y}A$$

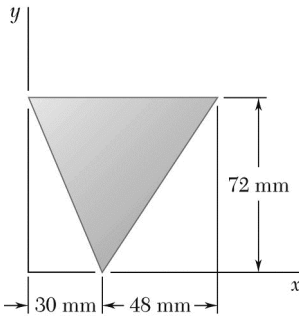
$$= \frac{1}{2}(c + y)[b(c - y)]$$



(a) $Q_x = \frac{1}{2}b(c^2 - y^2)$ ◀

(b) For Q_{\max} , $\frac{dQ}{dy} = 0$ or $\frac{1}{2}b(-2y) = 0$ $y = 0$ ◀

For $y = 0$, $(Q_x) = \frac{1}{2}bc^2$ $(Q_x) = \frac{1}{2}bc^2$ ◀

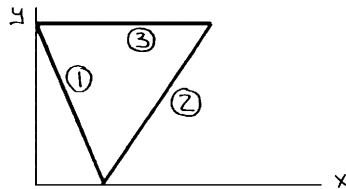


PROBLEM 5.24

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



	$L, \text{ mm}$	$\bar{x}, \text{ mm}$	$\bar{y}, \text{ mm}$	$\bar{y}L, \text{ mm}^2$	$\bar{y}L, \text{ mm}^2$
1	$\sqrt{30^2 + 72^2} = 78$	15	36	1170.0	2808.0
2	$\sqrt{48^2 + 72^2} = 86.533$	54	36	4672.8	3115.2
3	78	39	72	3042.0	5616.0
Σ	242.53			8884.8	11,539.2

Then

$$\bar{X} \Sigma L = \Sigma \bar{x} L$$

$$\bar{X} (242.53) = 8884.8$$

$$\text{or } \bar{X} = 36.6 \text{ mm} \blacktriangleleft$$

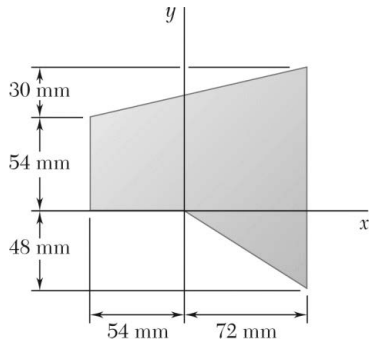
and

$$\bar{Y} \Sigma L = \Sigma \bar{y} L$$

$$\bar{Y} (242.53) = 11,539.2$$

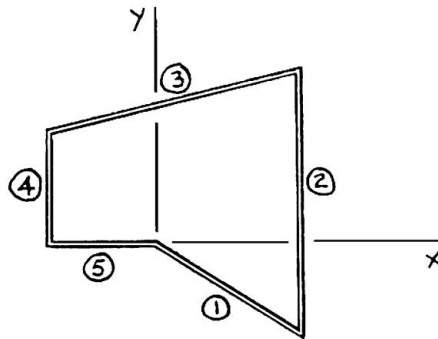
$$\text{or } \bar{Y} = 47.6 \text{ mm} \blacktriangleleft$$

PROBLEM 5.25



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



	$L, \text{ mm}$	$\bar{x}, \text{ mm}$	$\bar{y}, \text{ mm}$	$\bar{x}L, \text{ mm}^2$	$\bar{y}L, \text{ mm}^2$
1	$\sqrt{72^2 + 48^2} = 86.533$	36	-24	3115.2	-2076.8
2	132	72	18	9504.0	2376.0
3	$\sqrt{126^2 + 30^2} = 129.522$	9	69	1165.70	8937.0
4	54	-54	27	-2916.0	1458.0
5	54	-27	0	-1458.0	0
Σ	456.06			9410.9	10,694.2

Then

$$\bar{X} \Sigma L = \Sigma \bar{x} L$$

$$\bar{X} (456.06) = 9410.9$$

$$\text{or } \bar{X} = 20.6 \text{ mm} \blacktriangleleft$$

$$\bar{Y} \Sigma L = \Sigma \bar{y} L$$

$$\bar{Y} (456.06) = 10,694.2$$

$$\text{or } \bar{Y} = 23.4 \text{ mm} \blacktriangleleft$$

The diagram shows a composite shape in the first quadrant of a Cartesian coordinate system. The shape is bounded by the x-axis, a vertical line at x=12, a horizontal line at y=9, a horizontal line at y=3 from x=6 to x=12, a vertical line at x=6 from y=3 to y=9, and a slanted line from (0,0) to (12,9). The total width is 12 inches and the total height is 9 inches.

PROBLEM 5.26

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire thus formed.

SOLUTION

The perimeter of the shape is divided into six segments, labeled 1 through 6, starting from the origin and moving counter-clockwise:

- Segment 1: Slanted line from (0,0) to (12,9)
- Segment 2: Vertical line from (12,9) to (12,7.5)
- Segment 3: Horizontal line from (12,7.5) to (6,7.5)
- Segment 4: Vertical line from (6,7.5) to (6,6)
- Segment 5: Horizontal line from (6,6) to (0,6)
- Segment 6: Vertical line from (0,6) to (0,0)

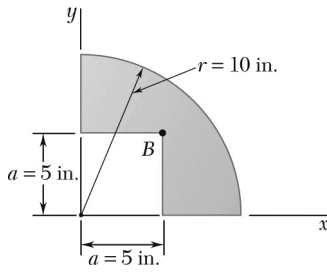
	$L, \text{ in.}$	$\bar{x}, \text{ in.}$	$\bar{y}, \text{ in.}$	$\bar{x}L, \text{ in}^2$	$\bar{y}L, \text{ in}^2$
1	$\sqrt{12^2 + 6^2} = 13.4164$	6	3	80.498	40.249
2	3	12	7.5	36	22.5
3	6	9	9	54	54.0
4	3	6	7.5	18	22.5
5	6	3	6	18	36.0
6	6	0	3	0	18.0
Σ	37.416			206.50	193.249

Then

$$\bar{X} \Sigma L = \Sigma \bar{x} L \quad \bar{X}(37.416) = 206.50 \quad \bar{X} = 5.52 \text{ in.} \blacktriangleleft$$

$$\bar{Y} \Sigma L = \Sigma \bar{y} L \quad \bar{Y}(37.416) = 193.249 \quad \bar{Y} = 5.16 \text{ in.} \blacktriangleleft$$

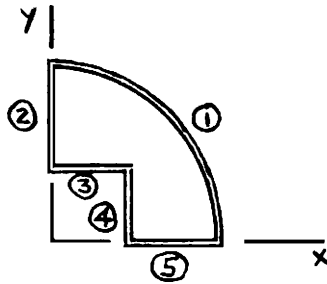
PROBLEM 5.27



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

By symmetry, $\bar{X} = \bar{Y}$.

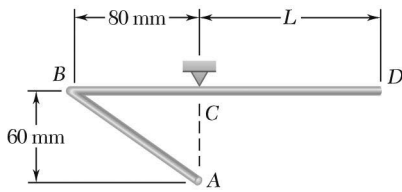


	L , in.	\bar{x} , in.	$\bar{y}L$, in ²
1	$\frac{1}{2}\pi(10) = 15.7080$	$\frac{2(10)}{\pi} = 6.3662$	100
2	5	0	0
3	5	2.5	12.5
4	5	5	25
5	5	7.5	37.5
Σ	35.708		175

Then

$$\bar{X} \Sigma L = \Sigma \bar{x} L \quad \bar{X}(35.708) = 175$$

$$\bar{X} = \bar{Y} = 4.90 \text{ in.} \quad \blacktriangleleft$$



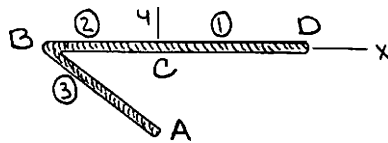
PROBLEM 5.28

The homogeneous wire $ABCD$ is bent as shown and is attached to a hinge at C . Determine the length L for which portion BCD of the wire is horizontal.

SOLUTION

First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through C . Further, because the wire is homogeneous, the center of gravity of the wire will coincide with the centroid of the corresponding line. Thus,

$$\bar{X} = 0 \text{ so that } \Sigma \bar{X} L = 0$$

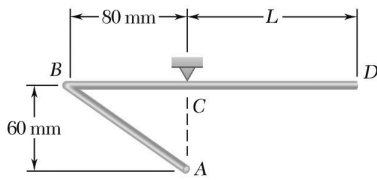


Then

$$\frac{L}{2} + (-40 \text{ mm})(80 \text{ mm}) + (-40 \text{ mm})(100 \text{ mm}) = 0$$

$$L^2 = 14,400 \text{ mm}^2$$

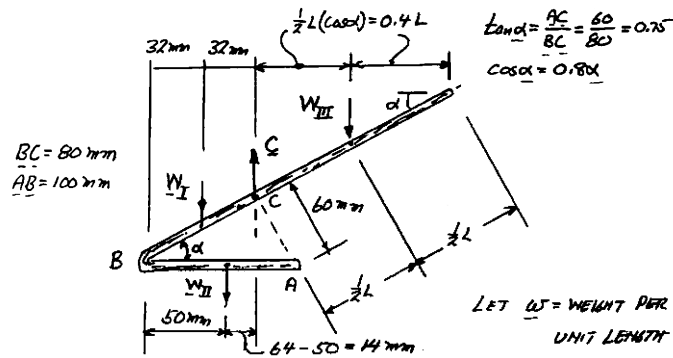
$$L = 120.0 \text{ mm} \blacktriangleleft$$



PROBLEM 5.29

The homogeneous wire $ABCD$ is bent as shown and is attached to a hinge at C . Determine the length L for which portion AB of the wire is horizontal.

SOLUTION

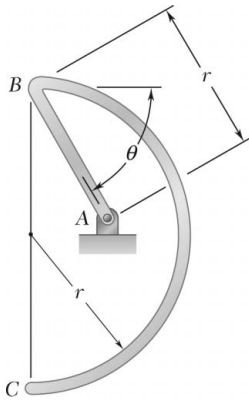


$$W_I = 80w \quad W_{II} = 100w \quad W_{III} = Lw$$

$$+\circlearrowleft \Sigma M_C = 0: (80w)(32) + (100w)(14) - (Lw)(0.4L) = 0$$

$$L^2 = 9900$$

$$L = 99.5 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 5.30

The homogeneous wire ABC is bent into a semicircular arc and a straight section as shown and is attached to a hinge at A . Determine the value of θ for which the wire is in equilibrium for the indicated position.

SOLUTION

First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through A . Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line. Thus,

$$\bar{X} = 0$$

so that

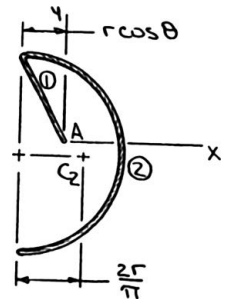
$$\Sigma \bar{x}L = 0$$

Then

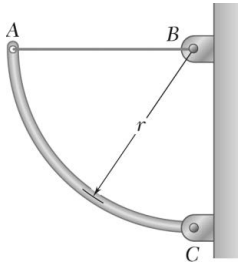
$$\left(-\frac{1}{2}r \cos \theta\right)(r) + \left(\frac{2r}{\pi} - r \cos \theta\right)(\pi r) = 0$$

or

$$\begin{aligned} \cos \theta &= \frac{4}{1 + 2\pi} \\ &= 0.54921 \end{aligned}$$



$$\text{or } \theta = 56.7^\circ \blacktriangleleft$$



PROBLEM 5.31

A uniform circular rod of weight 8 lb and radius 10 in. is attached to a pin at C and to the cable AB. Determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION

For quarter circle,

$$\bar{r} = \frac{2r}{\pi}$$

$$(a) \quad +\curvearrowright \Sigma M_C = 0: \quad W \left(\frac{2r}{\pi} \right) - Tr = 0$$

$$T = W \left(\frac{2}{\pi} \right) = (8 \text{ lb}) \left(\frac{2}{\pi} \right)$$

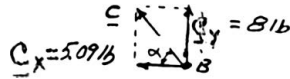
$$T = 5.09 \text{ lb} \quad \blacktriangleleft$$

$$(b) \quad +\rightarrow \Sigma F_x = 0: \quad T - C_x = 0 \quad 5.09 \text{ lb} - C_x = 0$$

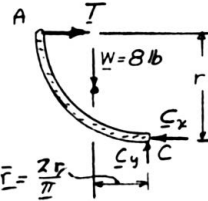
$$C_x = 5.09 \text{ lb} \quad \leftarrow$$

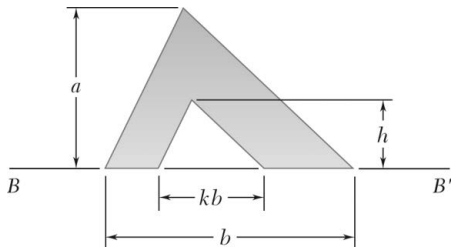
$$+\uparrow \Sigma F_y = 0: \quad C_y - W = 0 \quad C_y - 8 \text{ lb} = 0$$

$$C_y = 8 \text{ lb} \quad \uparrow$$



$$C = 9.48 \text{ lb} \quad \nearrow 57.5^\circ \quad \blacktriangleleft$$

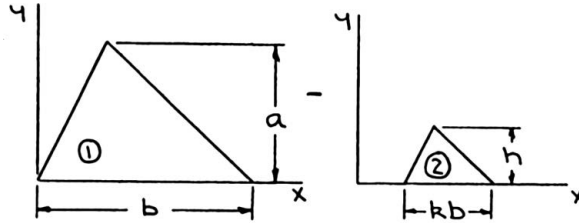




PROBLEM 5.32

Determine the distance h for which the centroid of the shaded area is as far above line BB' as possible when (a) $k = 0.10$, (b) $k = 0.80$.

SOLUTION



	A	\bar{y}	$\bar{y}A$
1	$\frac{1}{2}ba$	$\frac{1}{3}a$	$\frac{1}{6}a^2b$
2	$-\frac{1}{2}(kb)h$	$\frac{1}{3}h$	$-\frac{1}{6}kbh^2$
Σ	$\frac{b}{2}(a - kh)$		$\frac{b}{6}(a^2 - kh^2)$

Then

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y} \left[\frac{b}{2}(a - kh) \right] = \frac{b}{6}(a^2 - kh^2)$$

or

$$\bar{Y} = \frac{a^2 - kh^2}{3(a - kh)} \quad (1)$$

and

$$\frac{d\bar{Y}}{dh} = \frac{1}{3} \frac{-2kh(a - kh) - (a^2 - kh^2)(-k)}{(a - kh)^2} = 0$$

or

$$2h(a - kh) - a^2 + kh^2 = 0 \quad (2)$$

Simplifying Eq. (2) yields

$$kh^2 - 2ah + a^2 = 0$$

PROBLEM 5.32 (Continued)

Then

$$h = \frac{2a \pm \sqrt{(-2a)^2 - 4(k)(a^2)}}{2k}$$
$$= \frac{a}{k} [1 \pm \sqrt{1-k}]$$

Note that only the negative root is acceptable since $h < a$. Then

(a)

$$k = 0.10$$

$$h = \frac{a}{0.10} [1 - \sqrt{1-0.10}]$$

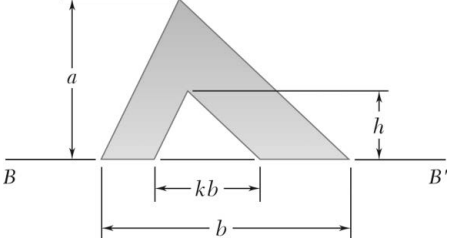
$$\text{or } h = 0.513a \blacktriangleleft$$

(b)

$$k = 0.80$$

$$h = \frac{a}{0.80} [1 - \sqrt{1-0.80}]$$

$$\text{or } h = 0.691a \blacktriangleleft$$



PROBLEM 5.33

Knowing that the distance h has been selected to maximize the distance \bar{y} from line BB' to the centroid of the shaded area, show that $\bar{y} = 2h/3$.

SOLUTION

See solution to Problem 5.32 for analysis leading to the following equations:

$$\bar{Y} = \frac{a^2 - kh^2}{3(a - kh)} \tag{1}$$

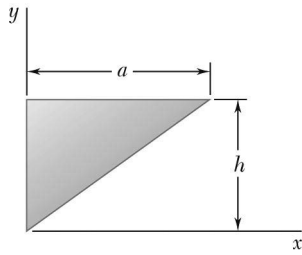
$$2h(a - kh) - a^2 + kh^2 = 0 \tag{2}$$

Rearranging Eq. (2) (which defines the value of h which maximizes \bar{Y}) yields

$$a^2 - kh^2 = 2h(a - kh)$$

Then substituting into Eq. (1) (which defines \bar{Y}),

$$\bar{Y} = \frac{1}{3(a - kh)} \times 2h(a - kh) \qquad \text{or } \bar{Y} = \frac{2}{3}h \blacktriangleleft$$



PROBLEM 5.34

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h .

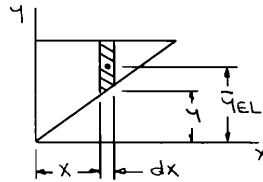
SOLUTION

We have

$$y = \frac{h}{a}x$$

and

$$\begin{aligned} dA &= (h - y)dx \\ &= h\left(1 - \frac{x}{a}\right)dx \end{aligned}$$



$$\bar{x}_{EL} = x$$

$$\begin{aligned} \bar{y}_{EL} &= \frac{1}{2}(h + y) \\ &= \frac{h}{2}\left(1 + \frac{x}{a}\right) \end{aligned}$$

Then

$$A = \int dA = \int_0^a h\left(1 - \frac{x}{a}\right)dx = h\left[x - \frac{x^2}{2a}\right]_0^a = \frac{1}{2}ah$$

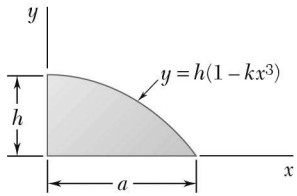
and

$$\int \bar{x}_{EL} dA = \int_0^a x\left[h\left(1 - \frac{x}{a}\right)\right]dx = h\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a = \frac{1}{6}a^2h$$

$$\begin{aligned} \int \bar{y}_{EL} dA &= \int_0^a \frac{h}{2}\left(1 + \frac{x}{a}\right)\left[h\left(1 - \frac{x}{a}\right)\right]dx \\ &= \frac{h^2}{2} \int_0^a \left(1 - \frac{x^2}{a^2}\right)dx = \frac{h^2}{2}\left[x - \frac{x^3}{3a^2}\right]_0^a = \frac{1}{3}ah^2 \end{aligned}$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x}\left(\frac{1}{2}ah\right) = \frac{1}{6}a^2h \quad \bar{x} = \frac{2}{3}a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y}\left(\frac{1}{2}ah\right) = \frac{1}{3}ah^2 \quad \bar{y} = \frac{2}{3}h \quad \blacktriangleleft$$



PROBLEM 5.35

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h .

SOLUTION

$$y = h(1 - kx^3)$$

For $x = a$, $y = 0$.

$$0 = h(1 - ka^3)$$

$$\therefore k = \frac{1}{a^3}$$

$$y = h\left(1 - \frac{x^3}{a^3}\right)$$

$$\bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2}y \quad dA = ydx$$

$$A = \int dA = \int_0^a ydx = \int_0^a h\left(1 - \frac{x^3}{a^3}\right)dx = h\left[x - \frac{x^4}{4a^3}\right]_0^a = \frac{3}{4}ah$$

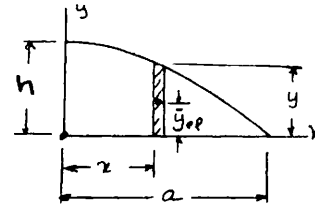
$$\int \bar{x}_{EL}dA = \int_0^a xydx = \int_0^a h\left(x - \frac{x^4}{a^3}\right)dx = h\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]_0^a = \frac{3}{10}a^2h$$

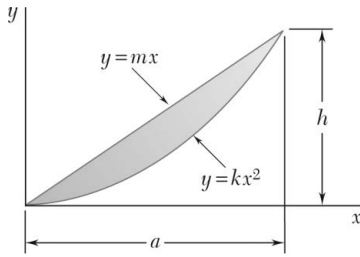
$$\int \bar{y}_{EL}dA = \int_0^a \left(\frac{1}{2}y\right)ydx = \frac{1}{2}\int_0^a h^2\left(1 - \frac{x^3}{a^3}\right)dx = \frac{h^2}{2}\int_0^a \left(1 - \frac{2x^3}{a^3} + \frac{x^6}{a^6}\right)dx$$

$$= \frac{h^2}{2}\left[x - \frac{x^4}{2a^3} + \frac{x^7}{7a^6}\right]_0^a = \frac{9}{28}ah^2$$

$$\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{3}{4}ah\right) = \frac{3}{10}a^2h \quad \bar{x} = \frac{2}{5}a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\frac{3}{4}ah\right) = \frac{9}{28}ah^2 \quad \bar{y} = \frac{3}{7}h \quad \blacktriangleleft$$





PROBLEM 5.36

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h .

SOLUTION

At (a, h) ,

$$y_1: h = ka^2$$

or

$$k = \frac{h}{a^2}$$

$$y_2: h = ma$$

or

$$m = \frac{h}{a}$$

Now

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)$$

and

$$\begin{aligned} dA &= (y_2 - y_1)dx = \left[\frac{h}{a}x - \frac{h}{a^2}x^2 \right] dx \\ &= \frac{h}{a^2}(ax - x^2)dx \end{aligned}$$

Then

$$A = \int dA = \int_0^a \frac{h}{a^2}(ax - x^2)dx = \frac{h}{a^2} \left[\frac{a}{2}x^2 - \frac{1}{3}x^3 \right]_0^a = \frac{1}{6}ah$$

and

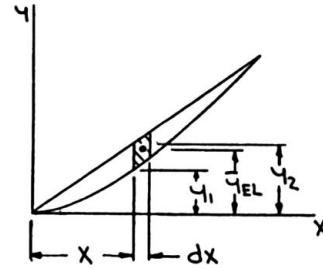
$$\int \bar{x}_{EL} dA = \int_0^a x \left[\frac{h}{a^2}(ax - x^2) \right] dx = \frac{h}{a^2} \left[\frac{a}{3}x^3 - \frac{1}{4}x^4 \right]_0^a = \frac{1}{12}a^2h$$

$$\int \bar{y}_{EL} dA = \int \frac{1}{2}(y_1 + y_2)[(y_2 - y_1)dx] = \int \frac{1}{2}(y_2^2 - y_1^2)dx$$

$$= \frac{1}{2} \int_0^a \left(\frac{h^2}{a^2}x^2 - \frac{h^2}{a^4}x^4 \right) dx$$

$$= \frac{1}{2} \frac{h^2}{a^4} \left[\frac{a^2}{3}x^3 - \frac{1}{5}x^5 \right]_0^a$$

$$= \frac{1}{15}ah^2$$



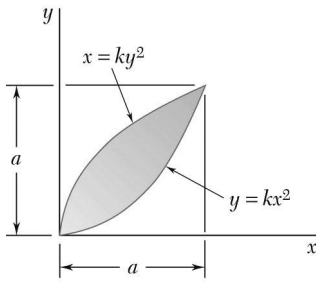
PROBLEM 5.36 (Continued)

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{1}{6} ah \right) = \frac{1}{12} a^2 h$$

$$\bar{x} = \frac{1}{2} a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{1}{6} ah \right) = \frac{1}{15} ah^2$$

$$\bar{y} = \frac{2}{5} h \quad \blacktriangleleft$$



PROBLEM 5.37

Determine by direct integration the centroid of the area shown.

SOLUTION

$$y_2 = \sqrt{\frac{x}{k}}, \quad y_1 = kx^2$$

But

$$a = ka^2, \text{ thus, } k = \frac{1}{a^2}$$

$$y_2 = \sqrt{ax}, \quad y_1 = \frac{x^2}{a}$$

$$\bar{x}_{EL} = x$$

$$dA = (y_2 - y_1)dx = \left(\sqrt{ax} - \frac{x^2}{a} \right) dx$$

$$A = \int dA = \int_0^a \left(\sqrt{ax} - \frac{x^2}{a} \right) dx$$

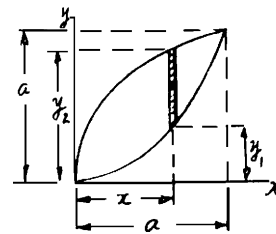
$$= \left[\frac{2}{3} \sqrt{ax}^{3/2} - \frac{x^3}{3a} \right]_0^a = \frac{1}{3} a^2$$

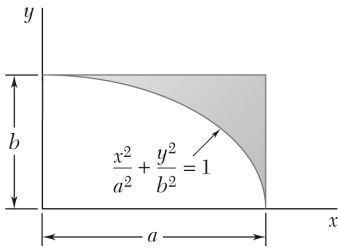
$$\int \bar{x}_{EL} dA = \int_0^a x \left(\sqrt{ax} - \frac{x^2}{a} \right) dx = \int_0^a \left(\sqrt{ax}^{3/2} - \frac{x^3}{a} \right) dx = \left[\frac{2}{5} \sqrt{ax}^{5/2} - \frac{x^4}{4a} \right]_0^a = \frac{3}{20} a^3$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{1}{3} a^2 \right) = \frac{3}{20} a^3 \quad \bar{x} = \frac{9a}{20}$$

By symmetry,

$$\bar{y} = \bar{x} = \frac{9a}{20} \blacktriangleleft$$





PROBLEM 5.38

Determine by direct integration the centroid of the area shown.

SOLUTION

For the element (EL) shown,

$$y = \frac{b}{a} \sqrt{a^2 - x^2}$$

and

$$\begin{aligned} dA &= (b - y)dx \\ &= \frac{b}{a} (a - \sqrt{a^2 - x^2}) dx \end{aligned}$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}(y + b)$$

$$= \frac{b}{2a} (a + \sqrt{a^2 - x^2})$$

Then

$$A = \int dA = \int_0^a \frac{b}{a} (a - \sqrt{a^2 - x^2}) dx$$

To integrate, let

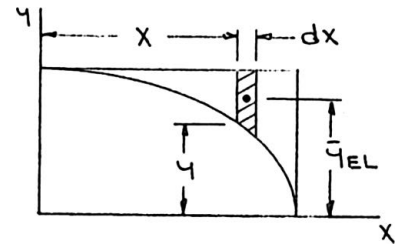
$$x = a \sin \theta: \quad \sqrt{a^2 - x^2} = a \cos \theta, \quad dx = a \cos \theta d\theta$$

Then

$$\begin{aligned} A &= \int_0^{\pi/2} \frac{b}{a} (a - a \cos \theta) (a \cos \theta d\theta) \\ &= \frac{b}{a} \left[a^2 \sin \theta - a^2 \left(\frac{\theta}{2} + \sin \frac{2\theta}{4} \right) \right]_0^{\pi/2} \\ &= ab \left(1 - \frac{\pi}{4} \right) \end{aligned}$$

and

$$\begin{aligned} \int \bar{x}_{EL} dA &= \int_0^a x \left[\frac{b}{a} (a - \sqrt{a^2 - x^2}) dx \right] \\ &= \frac{b}{a} \left[\left(\frac{a}{2} x^2 + \frac{1}{3} (a^2 - x^2)^{3/2} \right) \right]_0^{\pi/2} \\ &= \frac{1}{6} a^3 b \end{aligned}$$

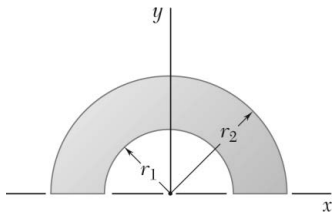


PROBLEM 5.38 (Continued)

$$\begin{aligned}\int \bar{y}_{EL} dA &= \int_0^a \frac{b}{2a} (a + \sqrt{a^2 - x^2}) \left[\frac{b}{a} (a - \sqrt{a^2 - x^2}) dx \right] \\ &= \frac{b^2}{2a^2} \int_0^a (x^2) dx = \frac{b^2}{2a^2} \left(\frac{x^3}{3} \right) \Big|_0^a \\ &= \frac{1}{6} ab^2\end{aligned}$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left[ab \left(1 - \frac{\pi}{4} \right) \right] = \frac{1}{6} a^2 b \quad \text{or } \bar{x} = \frac{2a}{3(4 - \pi)} \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left[ab \left(1 - \frac{\pi}{4} \right) \right] = \frac{1}{6} ab^2 \quad \text{or } \bar{y} = \frac{2b}{3(4 - \pi)} \blacktriangleleft$$



PROBLEM 5.39

Determine by direct integration the centroid of the area shown.

SOLUTION

First note that symmetry implies

$$\bar{x} = 0 \quad \blacktriangleleft$$

For the element (EL) shown,

$$\bar{y}_{EL} = \frac{2r}{\pi} \quad (\text{Figure 5.8B})$$

$$dA = \pi r dr$$

Then

$$A = \int dA = \int_{r_1}^{r_2} \pi r dr = \pi \left(\frac{r^2}{2} \right) \Big|_{r_1}^{r_2} = \frac{\pi}{2} (r_2^2 - r_1^2)$$

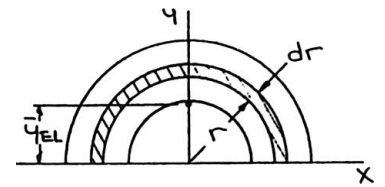
and

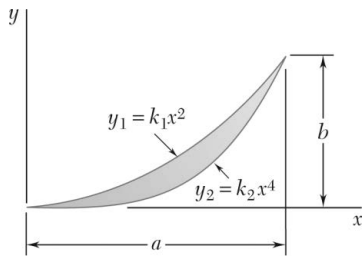
$$\int \bar{y}_{EL} dA = \int_{r_1}^{r_2} \frac{2r}{\pi} (\pi r dr) = 2 \left(\frac{1}{3} r^3 \right) \Big|_{r_1}^{r_2} = \frac{2}{3} (r_2^3 - r_1^3)$$

So

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left[\frac{\pi}{2} (r_2^2 - r_1^2) \right] = \frac{2}{3} (r_2^3 - r_1^3)$$

$$\text{or } \bar{y} = \frac{4}{3\pi} \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} \quad \blacktriangleleft$$





PROBLEM 5.40

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b .

SOLUTION

$$y_1 = k_1 x^2 \quad \text{but} \quad b = k_1 a^2 \quad y_1 = \frac{b}{a^2} x^2$$

$$y_2 = k_2 x^4 \quad \text{but} \quad b = k_2 a^4 \quad y_2 = \frac{b}{a^4} x^4$$

$$dA = (y_2 - y_1) dx = \frac{b}{a^2} \left(x^2 - \frac{x^4}{a^2} \right) dx$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)$$

$$= \frac{b}{2a^2} \left(x^2 + \frac{x^4}{a^2} \right)$$

$$A = \int dA = \frac{b}{a^2} \int_0^a \left(x^2 - \frac{x^4}{a^2} \right) dx$$

$$= \frac{b}{a^2} \left[\frac{x^3}{3} - \frac{x^5}{5a^2} \right]_0^a$$

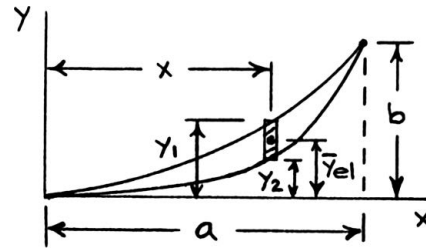
$$= \frac{2}{15} ba$$

$$\int \bar{x}_{EL} dA = \int_0^a x \frac{b}{a^2} \left(x^2 - \frac{x^4}{a^2} \right) dx$$

$$= \frac{b}{a^2} \int_0^a \left(x^3 - \frac{x^5}{a^2} \right) dx$$

$$= \frac{b}{a^2} \left[\frac{x^4}{4} - \frac{x^6}{6a^2} \right]_0^a$$

$$= \frac{1}{12} a^2 b$$

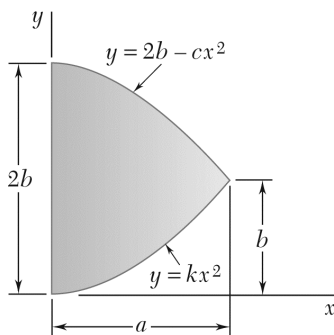


PROBLEM 5.40 (Continued)

$$\begin{aligned}\int \bar{y}_{EL} dA &= \int_0^a \frac{b}{2a^2} \left(x^2 + \frac{x^4}{a^2} \right) \frac{b}{a^2} \left(x^2 - \frac{x^4}{a^2} \right) dx \\ &= \frac{b^2}{2a^4} \int_0^a \left(x^4 - \frac{x^8}{a^4} \right) dx \\ &= \frac{b^2}{2a^4} \left[\frac{x^5}{5} - \frac{x^9}{9a^4} \right]_0^a = \frac{2}{45} ab^2\end{aligned}$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{2}{15} ba \right) = \frac{1}{12} a^2 b \quad \bar{x} = \frac{5}{8} a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{2}{15} ba \right) = \frac{2}{45} ab^2 \quad \bar{y} = \frac{1}{3} b \quad \blacktriangleleft$$



PROBLEM 5.41

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b .

SOLUTION

First note that symmetry implies

$$\bar{y} = b \quad \blacktriangleleft$$

At $x = a$, $y = b$

$$y_1: b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

Then

$$y_1 = \frac{b}{a^2} x^2$$

$$y_2: b = 2b - ca^2$$

or

$$c = \frac{b}{a^2}$$

Then

$$y_2 = b \left(2 - \frac{x^2}{a^2} \right)$$

Now

$$\begin{aligned} dA &= (y_2 - y_1) dx = \left[b \left(2 - \frac{x^2}{a^2} \right) - \frac{b}{a^2} x^2 \right] dx \\ &= 2b \left(1 - \frac{x^2}{a^2} \right) dx \end{aligned}$$

and

$$\bar{x}_{EL} = x$$

Then

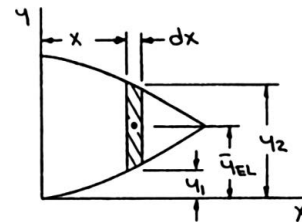
$$A = \int dA = \int_0^a 2b \left(1 - \frac{x^2}{a^2} \right) dx = 2b \left[x - \frac{x^3}{3a^2} \right]_0^a = \frac{4}{3} ab$$

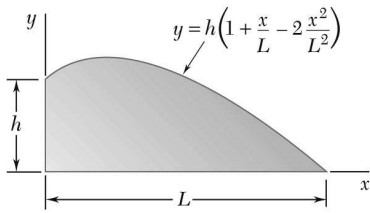
and

$$\int \bar{x}_{EL} dA = \int_0^a x \left[2b \left(1 - \frac{x^2}{a^2} \right) dx \right] = 2b \left[\frac{x^2}{2} - \frac{x^4}{4a^2} \right]_0^a = \frac{1}{2} a^2 b$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{4}{3} ab \right) = \frac{1}{2} a^2 b$$

$$\bar{x} = \frac{3}{8} a \quad \blacktriangleleft$$

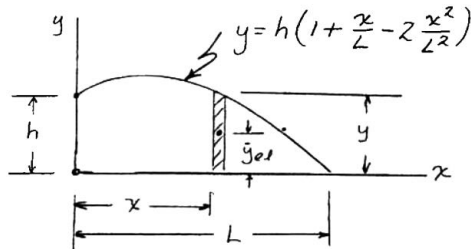




PROBLEM 5.42

Determine by direct integration the centroid of the area shown.

SOLUTION



$$\bar{x}_{EL} = x \quad \bar{y}_{EL} = \frac{1}{2}y \quad dA = y dx$$

$$A = \int dA = \int_0^L h \left(1 + \frac{x}{L} - 2 \frac{x^2}{L^2} \right) dx = h \left[x + \frac{x^2}{2L} - \frac{2x^3}{3L^2} \right]_0^L = \frac{5}{6}hL$$

$$\begin{aligned} \int x_{EL} dA &= \int_0^L xh \left(1 + \frac{x}{L} - 2 \frac{x^2}{L^2} \right) dx = h \int_0^L \left(x + \frac{x^2}{L} - 2 \frac{x^3}{L^2} \right) dx \\ &= h \left[\frac{x^2}{2} + \frac{1}{3} \frac{x^3}{L} - \frac{2}{4} \frac{x^4}{L^2} \right]_0^L = \frac{1}{3}hL^2 \end{aligned}$$

$$\bar{x}A = \int x_{EL} dA: \quad \bar{x} \left(\frac{5}{6}hL \right) = \frac{1}{3}hL^2$$

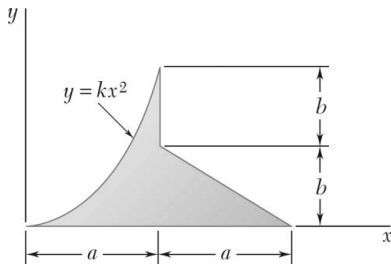
$$\bar{x} = \frac{2}{5}L \quad \blacktriangleleft$$

$$A = \frac{5}{6}hL \quad \bar{y}_{EL} = \frac{1}{2}y \quad y = h \left(1 + \frac{x}{L} - 2 \frac{x^2}{L^2} \right)$$

$$\begin{aligned} \int \bar{y}_{EL} dA &= \frac{1}{2} \int y^2 dx = \frac{h^2}{2} \int_0^L \left(1 + \frac{x}{L} - 2 \frac{x^2}{L^2} \right)^2 dx \\ &= \frac{h^2}{2} \int_0^L \left(1 + \frac{x^2}{L^2} + 4 \frac{x^4}{L^4} + 2 \frac{x}{L} - 4 \frac{x^2}{L^2} - 4 \frac{x^3}{L^3} \right) dx \\ &= \frac{h^2}{2} \left[x + \frac{x^3}{3L^2} + \frac{4x^5}{5L^4} + \frac{x^2}{L} - \frac{4x^3}{3L^2} - \frac{x^4}{L^3} \right]_0^L = \frac{4}{10}h^2L \end{aligned}$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{5}{6}hL \right) = \frac{4}{10}h^2L$$

$$\bar{y} = \frac{12}{25}h \quad \blacktriangleleft$$



PROBLEM 5.43

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b .

SOLUTION

For y_1 at $x = a$, $y = 2b$, $2b = ka^2$, or $k = \frac{2b}{a^2}$

Then $y_1 = \frac{2b}{a^2}x^2$

By observation, $y_2 = -\frac{b}{a}(x + 2b) = b\left(2 - \frac{x}{a}\right)$

Now $\bar{x}_{EL} = x$

and for $0 \leq x \leq a$, $\bar{y}_{EL} = \frac{1}{2}y_1 = \frac{b}{a^2}x^2$ and $dA = y_1 dx = \frac{2b}{a^2}x^2 dx$

For $a \leq x \leq 2a$, $\bar{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}\left(2 - \frac{x}{a}\right)$ and $dA = y_2 dx = b\left(2 - \frac{x}{a}\right) dx$

Then
$$A = \int dA = \int_0^a \frac{2b}{a^2}x^2 dx + \int_a^{2a} b\left(2 - \frac{x}{a}\right) dx$$

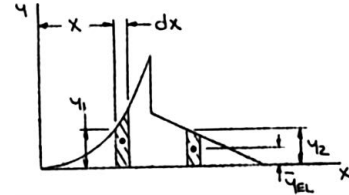
$$= \frac{2b}{a^2} \left[\frac{x^3}{3} \right]_0^a + b \left[-\frac{a}{2} \left(2 - \frac{x}{a}\right)^2 \right]_a^{2a} = \frac{7}{6}ab$$

and
$$\int \bar{x}_{EL} dA = \int_0^a x \left(\frac{2b}{a^2}x^2 dx \right) + \int_a^{2a} x \left[b\left(2 - \frac{x}{a}\right) dx \right]$$

$$= \frac{2b}{a^2} \left[\frac{x^4}{4} \right]_0^a + b \left[x^2 - \frac{x^3}{3a} \right]_a^{2a}$$

$$= \frac{1}{2}a^2b + b \left\{ \left[(2a)^2 - (a)^2 \right] + \frac{1}{3a} \left[(2a^2) - (a)^3 \right] \right\}$$

$$= \frac{7}{6}a^2b$$



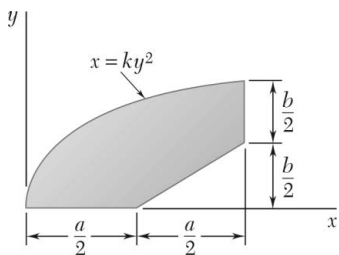
PROBLEM 5.43 (Continued)

$$\begin{aligned}\int \bar{y}_{EL} dA &= \int_0^a \frac{b}{a^2} x^2 \left[\frac{2b}{a^2} x^2 dx \right] + \int_0^{2a} \frac{b}{2} \left(2 - \frac{x}{a} \right) \left[b \left(2 - \frac{x}{a} \right) dx \right] \\ &= \frac{2b^2}{a^4} \left[\frac{x^5}{5} \right]_0^a + \frac{b^2}{2} \left[-\frac{a}{3} \left(2 - \frac{x}{a} \right)^3 \right]_a^{2a} \\ &= \frac{17}{30} ab^2\end{aligned}$$

Hence,

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{7}{6} ab \right) = \frac{7}{6} a^2 b \quad \bar{x} = a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{7}{6} ab \right) = \frac{17}{30} ab^2 \quad \bar{y} = \frac{17}{35} b \quad \blacktriangleleft$$



PROBLEM 5.44

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b .

SOLUTION

For y_2 at $x = a$, $y = b$, $a = kb^2$, or $k = \frac{a}{b^2}$

Then $y_2 = \frac{b}{\sqrt{a}} x^{1/2}$

Now $\bar{x}_{EL} = x$

and for $0 \leq x \leq \frac{a}{2}$, $\bar{y}_{EL} = \frac{y_2}{2} = \frac{b}{2} \frac{x^{1/2}}{\sqrt{a}}$

$$dA = y_2 dx = b \frac{x^{1/2}}{\sqrt{a}} dx$$

For $\frac{a}{2} \leq x \leq a$, $\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2) = \frac{b}{2} \left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}} \right)$

$$dA = (y_2 - y_1) dx = b \left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2} \right) dx$$

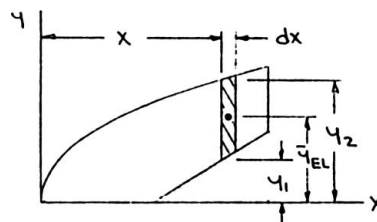
Then $A = \int dA = \int_0^{a/2} b \frac{x^{1/2}}{\sqrt{a}} dx + \int_{a/2}^a b \left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2} \right) dx$

$$= \frac{b}{\sqrt{a}} \left[\frac{2}{3} x^{3/2} \right]_0^{a/2} + b \left[\frac{2}{3} \frac{x^{3/2}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2} x \right]_{a/2}^a$$

$$= \frac{2}{3} \frac{b}{\sqrt{a}} \left[\left(\frac{a}{2} \right)^{3/2} + (a)^{3/2} - \left(\frac{a}{2} \right)^{3/2} \right]$$

$$+ b \left\{ -\frac{1}{2a} \left[(a^2) - \left(\frac{a}{2} \right)^2 \right] + \frac{1}{2} \left[(a) - \left(\frac{a}{2} \right) \right] \right\}$$

$$= \frac{13}{24} ab$$



PROBLEM 5.44 (Continued)

and

$$\begin{aligned} \int \bar{x}_{EL} dA &= \int_0^{a/2} x \left(b \frac{x^{1/2}}{\sqrt{a}} dx \right) + \int_{a/2}^a x \left[b \left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2} \right) \right] dx \\ &= \frac{b}{\sqrt{a}} \left[\frac{2}{5} x^{5/2} \right]_0^{a/2} + b \left[\frac{2}{5} \frac{x^{5/2}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^4}{4} \right]_{a/2}^a \\ &= \frac{2}{5} \frac{b}{\sqrt{a}} \left[\left(\frac{a}{2} \right)^{5/2} + (a)^{5/2} - \left(\frac{a}{2} \right)^{5/2} \right] \\ &\quad + b \left\{ -\frac{1}{3a} \left[(a)^3 - \left(\frac{a}{2} \right)^3 \right] + \frac{1}{4} \left[(a)^2 - \left(\frac{a}{2} \right)^2 \right] \right\} \\ &= \frac{71}{240} a^2 b \end{aligned}$$

$$\begin{aligned} \int \bar{y}_{EL} dA &= \int_0^{a/2} \frac{a/2}{2} \frac{b x^{1/2}}{\sqrt{a}} \left[b \frac{x^{1/2}}{\sqrt{a}} dx \right] \\ &\quad + \int_{a/2}^a \frac{b}{2} \left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}} \right) \left[b \left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2} \right) dx \right] \\ &= \frac{b^2}{2a} \left[\frac{1}{2} x^2 \right]_0^{a/2} + \frac{b^2}{2} \left[\left(\frac{x^2}{2a} - \frac{1}{3a} \left(\frac{x}{a} - \frac{1}{2} \right)^3 \right) \right]_{a/2}^a \\ &= \frac{b}{4a} \left[\left(\frac{a}{2} \right)^2 + (a)^2 - \left(\frac{a}{2} \right)^2 \right] - \frac{b^2}{6a} \left(\frac{a}{2} - \frac{1}{2} \right)^3 \\ &= \frac{11}{48} ab^2 \end{aligned}$$

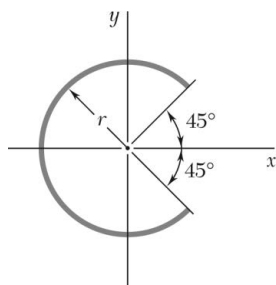
Hence,

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{13}{24} ab \right) = \frac{71}{240} a^2 b$$

$$\bar{x} = \frac{17}{130} a = 0.546a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{13}{24} ab \right) = \frac{11}{48} ab^2$$

$$\bar{y} = \frac{11}{26} b = 0.423b \quad \blacktriangleleft$$



PROBLEM 5.45

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid.

SOLUTION

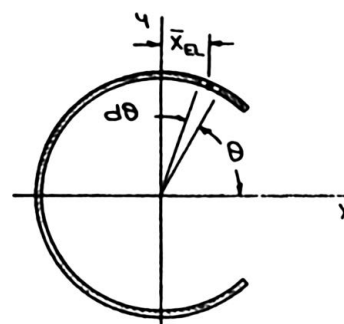
First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line.

Now $\bar{x}_{EL} = r \cos \theta$ and $dL = r d\theta$

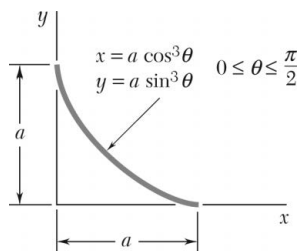
Then
$$L = \int dL = \int_{\pi/4}^{7\pi/4} r d\theta = r[\theta]_{\pi/4}^{7\pi/4} = \frac{3}{2}\pi r$$

and
$$\begin{aligned} \int \bar{x}_{EL} dL &= \int_{\pi/4}^{7\pi/4} r \cos \theta (r d\theta) \\ &= r^2 [\sin \theta]_{\pi/4}^{7\pi/4} \\ &= r^2 \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) \\ &= -r^2 \sqrt{2} \end{aligned}$$

Thus
$$\bar{x}L = \int \bar{x} dL: \quad \bar{x} \left(\frac{3}{2}\pi r \right) = -r^2 \sqrt{2}$$



$$\bar{x} = -\frac{2\sqrt{2}}{3\pi} r \quad \blacktriangleleft$$



PROBLEM 5.46

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid.

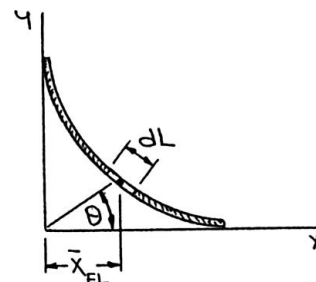
SOLUTION

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line.

Now $\bar{x}_{EL} = a \cos^3 \theta$ and $dL = \sqrt{dx^2 + dy^2}$

where $x = a \cos^3 \theta: dx = -3a \cos^2 \theta \sin \theta d\theta$
 $y = a \sin^3 \theta: dy = 3a \sin^2 \theta \cos \theta d\theta$

Then $dL = [(-3a \cos^2 \theta \sin \theta d\theta)^2 + (3a \sin^2 \theta \cos \theta d\theta)^2]^{1/2}$
 $= 3a \cos \theta \sin \theta (\cos^2 \theta + \sin^2 \theta)^{1/2} d\theta$
 $= 3a \cos \theta \sin \theta d\theta$



$$L = \int dL = \int_0^{\pi/2} 3a \cos \theta \sin \theta d\theta = 3a \left[\frac{1}{2} \sin^2 \theta \right]_0^{\pi/2}$$

$$= \frac{3}{2} a$$

and $\int \bar{x}_{EL} dL = \int_0^{\pi/2} a \cos^3 \theta (3a \cos \theta \sin \theta d\theta)$

$$= 3a^2 \left[-\frac{1}{5} \cos^5 \theta \right]_0^{\pi/2} = \frac{3}{5} a^2$$

Hence, $\bar{x}L = \int \bar{x}_{EL} dL: \bar{x} \left(\frac{3}{2} a \right) = \frac{3}{5} a^2$ $\bar{x} = \frac{2}{5} a \blacktriangleleft$

Alternative Solution:

$$x = a \cos^3 \theta \Rightarrow \cos^2 \theta = \left(\frac{x}{a} \right)^{2/3}$$

$$y = a \sin^3 \theta \Rightarrow \sin^2 \theta = \left(\frac{y}{a} \right)^{2/3}$$

$$\left(\frac{x}{a} \right)^{2/3} + \left(\frac{y}{a} \right)^{2/3} = 1 \quad \text{or} \quad y = (a^{2/3} - x^{2/3})^{3/2}$$

PROBLEM 5.46 (Continued)

Then
$$\frac{dy}{dx} = (a^{2/3} - x^{2/3})^{1/2} (-x^{-1/3})$$

Now
$$\bar{x}_{EL} = x$$

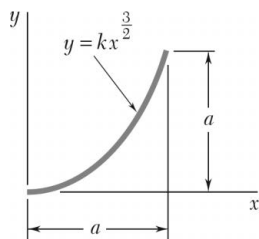
and
$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$dx = \left\{ 1 + \left[(a^{2/3} - x^{2/3})^{1/2} (-x^{-1/3}) \right]^2 \right\}^{1/2} dx$$

Then
$$L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}} dx = a^{1/3} \left[\frac{3}{2} x^{2/3} \right]_0^a = \frac{3}{2} a$$

and
$$\int \bar{x}_{EL} dL = \int_0^a x \left(\frac{a^{1/3}}{x^{1/3}} dx \right) = a^{1/3} \left[\frac{3}{5} x^{5/3} \right]_0^a = \frac{3}{5} a^2$$

Hence
$$\bar{x}L = \int \bar{x}_{EL} dL: \quad \bar{x} \left(\frac{3}{2} a \right) = \frac{3}{5} a^2 \qquad \bar{x} = \frac{2}{5} a \quad \blacktriangleleft$$



PROBLEM 5.47*

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid. Express your answer in terms of a .

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

We have at $x = a$, $y = a$, $a = ka^{3/2}$, or $k = \frac{1}{\sqrt{a}}$

Then $y = \frac{1}{\sqrt{a}}x^{3/2}$

and $\frac{dy}{dx} = \frac{3}{2\sqrt{a}}x^{1/2}$

Now $\bar{x}_{EL} = x$

and
$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \left[1 + \left(\frac{3}{2\sqrt{a}}x^{1/2}\right)^2\right]^{1/2} dx$$

$$= \frac{1}{2\sqrt{a}}\sqrt{4a + 9x} dx$$

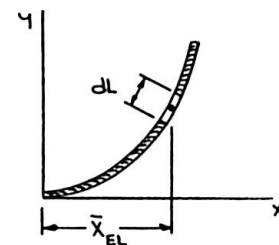
Then
$$L = \int dL = \int_0^a \frac{1}{2\sqrt{a}}\sqrt{4a + 9x} dx$$

$$= \frac{1}{2\sqrt{a}} \left[\frac{2}{3} \times \frac{1}{9} (4a + 9x)^{3/2} \right]_0^a$$

$$= \frac{a}{27} [(13)^{3/2} - 8]$$

$$= 1.43971a$$

and
$$\int \bar{x}_{EL} dL = \int_0^a x \left[\frac{1}{2\sqrt{a}}\sqrt{4a + 9x} dx \right]$$



PROBLEM 5.47* (Continued)

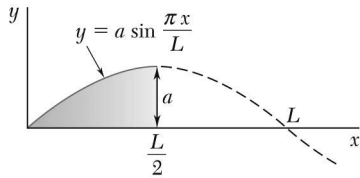
Use integration by parts with

$$\begin{aligned}u &= x & dv &= \sqrt{4a + 9x} \, dx \\ du &= dx & v &= \frac{2}{27}(4a + 9x)^{3/2}\end{aligned}$$

Then

$$\begin{aligned}\int \bar{x}_{EL} dL &= \frac{1}{2\sqrt{a}} \left\{ \left[x \times \frac{2}{27}(4a + 9x)^{3/2} \right]_0^a - \int_0^a \frac{2}{27}(4a + 9x)^{3/2} dx \right\} \\ &= \frac{(13)^{3/2}}{27} a^2 - \frac{1}{27\sqrt{a}} \left[\frac{2}{45}(4a + 9x)^{5/2} \right]_0^a \\ &= \frac{a^2}{27} \left\{ (13)^{3/2} - \frac{2}{45} [(13)^{5/2} - 32] \right\} \\ &= 0.78566a^2\end{aligned}$$

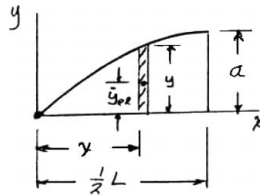
$$\bar{x}L = \int x_{EL} dL: \quad \bar{x}(1.43971a) = 0.78566a^2 \quad \text{or } \bar{x} = 0.546a \quad \blacktriangleleft$$



PROBLEM 5.48

Determine by direct integration the centroid of the area shown.

SOLUTION



$$y = a \sin \frac{\pi x}{L}$$

$$\bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2}y, \quad dA = y dx$$

$$A = \int_0^{L/2} y dx = a \int_0^{L/2} \sin \frac{\pi x}{L} dx$$

$$A = a \left[\frac{L}{\pi} \left(-\cos \frac{\pi x}{L} \right) \right]_0^{L/2} = \frac{aL}{\pi}$$

$$\int \bar{x}_{EL} dA = \int_0^{L/2} xy dx = \int_0^{L/2} xa \sin \frac{\pi x}{L} dx$$

Setting $u = \frac{\pi x}{L}$, we have $x = \frac{L}{\pi}u$, $dx = \frac{L}{\pi}du$,

$$\int \bar{x}_{EL} dA = \int_0^{\pi/2} \left(\frac{L}{\pi}u \right) a \sin u \left(\frac{L}{\pi} du \right) = a \left(\frac{L}{\pi} \right)^2 \int_0^{\pi/2} u \sin u du$$

Integrating by parts,

$$\int \bar{x}_{EL} dA = a \left(\frac{L}{\pi} \right)^2 \left\{ [-u \cos u]_0^{\pi/2} + \int_0^{\pi/2} \cos u du \right\} = \frac{aL^2}{\pi^2}$$

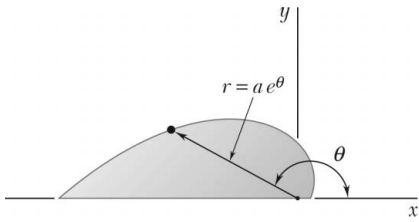
$$\begin{aligned} \int \bar{y}_{EL} dA &= \int_0^{L/2} \frac{1}{2} y^2 dx = \frac{1}{2} a^2 \int_0^{L/2} \sin^2 \frac{\pi x}{L} dx = \frac{a^2 L}{2\pi} \int_0^{\pi/2} \sin^2 u du \\ &= \frac{a^2 L}{2\pi^2} \int_0^{\pi/2} \frac{1}{2} (1 - \cos 2u) du = \frac{a^2 L}{4\pi} \left[u - \frac{1}{2} \sin 2u \right]_0^{\pi/2} = \frac{1}{8} a^2 L \end{aligned}$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{aL}{\pi} \right) = \frac{aL^2}{\pi^2} \qquad \bar{x} = \frac{L}{\pi} \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{aL}{\pi} \right) = \frac{1}{8} a^2 L \qquad \bar{y} = \frac{\pi}{8} a \blacktriangleleft$$

PROBLEM 5.49*

Determine by direct integration the centroid of the area shown.



SOLUTION

We have

$$\bar{x}_{EL} = \frac{2}{3}r \cos \theta = \frac{2}{3}ae^{\theta} \cos \theta$$

$$\bar{y}_{EL} = \frac{2}{3}r \sin \theta = \frac{2}{3}ae^{\theta} \sin \theta$$

and

$$dA = \frac{1}{2}(r)(rd\theta) = \frac{1}{2}a^2e^{2\theta}d\theta$$

Then

$$\begin{aligned} A &= \int dA = \int_0^{\pi} \frac{1}{2}a^2e^{2\theta}d\theta = \frac{1}{2}a^2 \left[\frac{1}{2}e^{2\theta} \right]_0^{\pi} \\ &= \frac{1}{4}a^2(e^{2\pi} - 1) \\ &= 133.623a^2 \end{aligned}$$

and

$$\begin{aligned} \int \bar{x}_{EL}dA &= \int_0^{\pi} \frac{2}{3}ae^{\theta} \cos \theta \left(\frac{1}{2}a^2e^{2\theta}d\theta \right) \\ &= \frac{1}{3}a^3 \int_0^{\pi} e^{3\theta} \cos \theta d\theta \end{aligned}$$

To proceed, use integration by parts, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta}d\theta$$

$$dv = \cos \theta d\theta \quad \text{and} \quad v = \sin \theta$$

Then

$$\int e^{3\theta} \cos \theta d\theta = e^{3\theta} \sin \theta - \int \sin \theta (3e^{3\theta}d\theta)$$

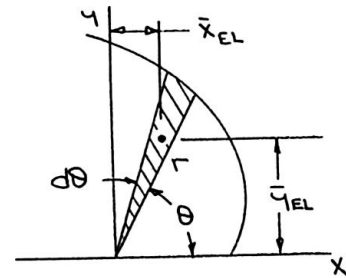
Now let

$$u = e^{3\theta} \quad \text{then} \quad du = 3e^{3\theta}d\theta$$

$$dv = \sin \theta d\theta, \quad \text{then} \quad v = -\cos \theta$$

Then

$$\int e^{3\theta} \cos \theta d\theta = e^{3\theta} \sin \theta - 3 \left[-e^{3\theta} \cos \theta - \int (-\cos \theta)(3e^{3\theta}d\theta) \right]$$



PROBLEM 5.49* (Continued)

so that

$$\int e^{3\theta} \cos \theta d\theta = \frac{e^{3\theta}}{10} (\sin \theta + 3 \cos \theta)$$

$$\int x_{EL} dA = \frac{1}{3} a^3 \left[\frac{e^{3\theta}}{10} (\sin \theta + 3 \cos \theta) \right]_0^\pi$$

$$= \frac{a^3}{30} (-3e^{3\pi} - 3) = -1239.26a^3$$

Also,

$$\int \bar{y}_{EL} dA = \int_0^\pi \frac{2}{3} a e^\theta \sin \theta \left(\frac{1}{2} a^2 e^{2\theta} d\theta \right)$$

$$= \frac{1}{3} a^3 \int_0^\pi e^{3\theta} \sin \theta d\theta$$

Use integration by parts, as above, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta} d\theta$$

$$dv = \int \sin \theta d\theta \quad \text{and} \quad v = -\cos \theta$$

Then

$$\int e^{3\theta} \sin \theta d\theta = -e^{3\theta} \cos \theta - \int (-\cos \theta)(3e^{3\theta} d\theta)$$

so that

$$\int e^{3\theta} \sin \theta d\theta = \frac{e^{3\theta}}{10} (-\cos \theta + 3 \sin \theta)$$

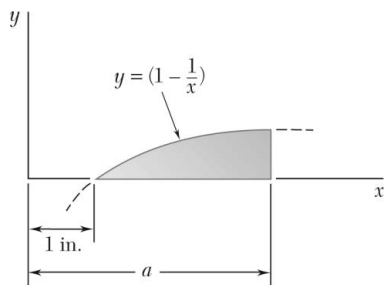
$$\int \bar{y}_{EL} dA = \frac{1}{3} a^3 \left[\frac{e^{3\theta}}{10} (-\cos \theta + 3 \sin \theta) \right]_0^\pi$$

$$= \frac{a^3}{30} (e^{3\pi} + 1) = 413.09a^3$$

Hence,

$$\bar{x}A = \int x_{EL} dA: \quad \bar{x}(133.623a^2) = -1239.26a^3 \quad \text{or} \quad \bar{x} = -9.27a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y}(133.623a^2) = 413.09a^3 \quad \text{or} \quad \bar{y} = 3.09a \quad \blacktriangleleft$$



PROBLEM 5.50

Determine the centroid of the area shown when $a = 2$ in.

SOLUTION

We have

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$$

and

$$dA = ydx = \left(1 - \frac{1}{x}\right)dx$$

Then

$$A = \int dA = \int_1^a \left(1 - \frac{1}{x}\right) dx = [x - \ln x]_1^a = (a - \ln a - 1) \text{ in}^2$$

and

$$\int \bar{x}_{EL} dA = \int_1^a x \left[\left(1 - \frac{1}{x}\right) dx \right] = \left[\frac{x^2}{2} - x \right]_1^a = \left(\frac{a^2}{2} - a + \frac{1}{2} \right) \text{ in}^3$$

$$\begin{aligned} \int \bar{y}_{EL} dA &= \int_1^a \frac{1}{2} \left(1 - \frac{1}{x}\right) \left[\left(1 - \frac{1}{x}\right) dx \right] = \frac{1}{2} \int_1^a \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx \\ &= \frac{1}{2} \left[x - 2 \ln x - \frac{1}{x} \right]_1^a = \frac{1}{2} \left(a - 2 \ln a - \frac{1}{a} \right) \text{ in}^3 \end{aligned}$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} = \frac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1} \text{ in.}$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} = \frac{a - 2 \ln a - \frac{1}{a}}{2(a - \ln a - 1)} \text{ in.}$$

Find \bar{x} and \bar{y} when $a = 2$ in.

We have

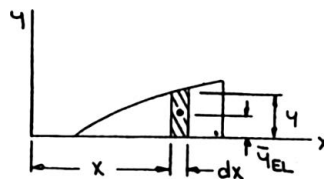
$$\bar{x} = \frac{\frac{1}{2}(2)^2 - 2 + \frac{1}{2}}{2 - \ln 2 - 1}$$

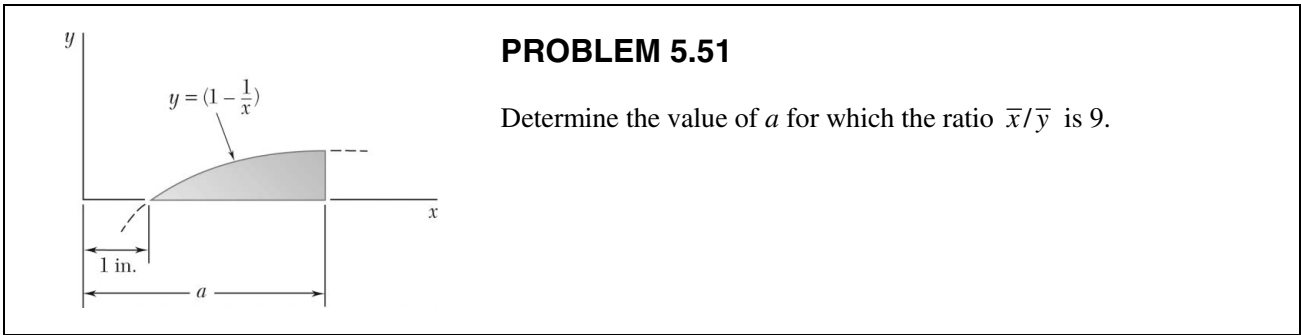
$$\text{or } \bar{x} = 1.629 \text{ in. } \blacktriangleleft$$

and

$$\bar{y} = \frac{2 - 2 \ln 2 - \frac{1}{2}}{2(2 - \ln 2 - 1)}$$

$$\text{or } \bar{y} = 0.1853 \text{ in. } \blacktriangleleft$$





PROBLEM 5.51

Determine the value of a for which the ratio \bar{x}/\bar{y} is 9.

SOLUTION

We have $\bar{x}_{EL} = x$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$$

and $dA = y dx = \left(1 - \frac{1}{x}\right) dx$

Then $A = \int dA = \int_1^a \left(1 - \frac{1}{x}\right) \frac{dx}{2} = [x - \ln x]_1^a$

$$= (a - \ln a - 1) \text{ in}^2$$

and $\int \bar{x}_{EL} dA = \int_1^a x \left[\left(1 - \frac{1}{x}\right) dx \right] = \left[\frac{x^2}{2} - x \right]_1^a$

$$= \left(\frac{a^2}{2} - a + \frac{1}{2} \right) \text{ in}^3$$

$$\int \bar{y}_{EL} dA = \int_1^a \frac{1}{2} \left(1 - \frac{1}{x}\right) \left[\left(1 - \frac{1}{x}\right) dx \right] = \frac{1}{2} \int_1^a \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx$$

$$= \frac{1}{2} \left[x - 2 \ln x - \frac{1}{x} \right]_1^a$$

$$= \frac{1}{2} \left(a - 2 \ln a - \frac{1}{a} \right) \text{ in}^3$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} = \frac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1} \text{ in.}$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} = \frac{a - 2 \ln a - \frac{1}{a}}{2(a - \ln a - 1)} \text{ in.}$$

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PROBLEM 5.51 (Continued)

Find a so that $\frac{\bar{x}}{\bar{y}} = 9$.

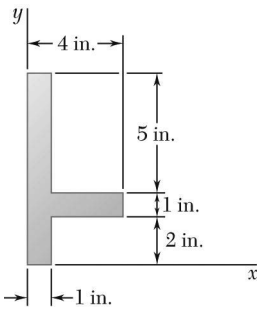
We have
$$\frac{\bar{x}}{\bar{y}} = \frac{\bar{x}A}{\bar{y}A} = \frac{\int \bar{x}_{EL} dA}{\int \bar{y}_{EL} dA}$$

Then
$$\frac{\frac{1}{2}a^2 - a + \frac{1}{2}}{\frac{1}{2}\left(a - 2 \ln a - \frac{1}{a}\right)} = 9$$

or
$$a^3 - 11a^2 + a + 18a \ln a + 9 = 0$$

Using trial and error or numerical methods, and ignoring the trivial solution $a = 1$ in., we find

$$a = 1.901 \text{ in.} \quad \text{and} \quad a = 3.74 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 5.52

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.1 about (a) the x -axis, (b) the y -axis.

SOLUTION

From the solution of Problem 5.1, we have

$$A = 11 \text{ in}^2$$

$$\Sigma \bar{x}A = 11.5 \text{ in}^3$$

$$\Sigma \bar{y}A = 39.5 \text{ in}^3$$

Applying the theorems of Pappus-Guldinus, we have

(a) Rotation about the x -axis:

$$\begin{aligned} \text{Volume} &= 2\pi \bar{y}_{\text{area}} A = 2\pi \Sigma \bar{y}A \\ &= 2\pi(39.5 \text{ in}^3) \end{aligned}$$

$$\text{or Volume} = 248 \text{ in}^3 \blacktriangleleft$$

$$\begin{aligned} \text{Area} &= 2\pi \bar{y}_{\text{line}} L = 2\pi \Sigma (\bar{y}_{\text{line}}) L \\ &= 2\pi(\bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4 + \bar{y}_5 L_5 + \bar{y}_6 L_6 + \bar{y}_7 L_7 + \bar{y}_8 L_8) \\ &= 2\pi[(1)(2) + (2)(3) + (2.5)(1) + (3)(3) + (5.5)(5) + (8)(1) + (4)(8)] \end{aligned}$$

$$\text{or Area} = 547 \text{ in}^2 \blacktriangleleft$$

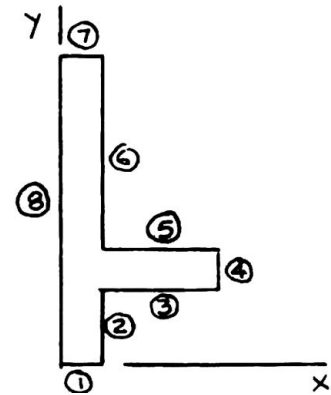
(b) Rotation about the y -axis:

$$\begin{aligned} \text{Volume} &= 2\pi \bar{x}_{\text{area}} A = 2\pi \Sigma \bar{x}A \\ &= 2\pi(11.5 \text{ in}^3) \end{aligned}$$

$$\text{or Volume} = 72.3 \text{ in}^3 \blacktriangleleft$$

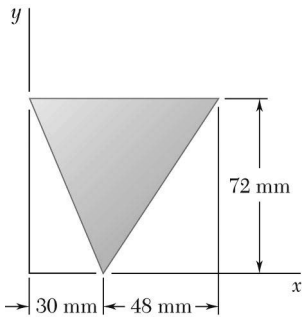
$$\begin{aligned} \text{Area} &= 2\pi \bar{x}_{\text{line}} L = 2\pi \Sigma (\bar{x}_{\text{line}}) L \\ &= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5 + \bar{x}_6 L_6 + \bar{x}_7 L_7) \\ &= 2\pi[(0.5)(1) + (1)(2) + (2.5)(3) + (4)(1) + (2.5)(3) + (1)(5) + (0.5)(1)] \end{aligned}$$

$$\text{or Area} = 169.6 \text{ in}^2 \blacktriangleleft$$



PROBLEM 5.53

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.2 about (a) the line $y = 72$ mm, (b) the x -axis.



SOLUTION

From the solution of Problem 5.2, we have

$$A = 2808 \text{ mm}^2$$

$$\bar{x} = 36 \text{ mm}$$

$$\bar{y} = 48 \text{ mm}$$

Applying the theorems of Pappus-Guldinus, we have

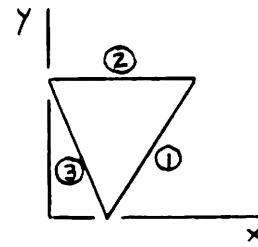
(a) Rotation about the line $y = 72$ mm:

$$\begin{aligned} \text{Volume} &= 2\pi(72 - \bar{y})A \\ &= 2\pi(72 - 48)(2808) \end{aligned}$$

$$\begin{aligned} \text{Area} &= 2\pi\bar{y}_{\text{line}}L \\ &= 2\pi\Sigma(\bar{y}_{\text{line}})L \\ &= 2\pi(\bar{y}_1L_1 + \bar{y}_3L_3) \end{aligned}$$

where \bar{y}_1 and \bar{y}_3 are measured with respect to line $y = 72$ mm.

$$\text{Area} = 2\pi \left[(36) \left(\sqrt{48^2 + 72^2} \right) + (36) \left(\sqrt{30^2 + 72^2} \right) \right]$$



$$\text{Volume} = 423 \times 10^3 \text{ mm}^3 \quad \blacktriangleleft$$

$$\text{Area} = 37.2 \times 10^3 \text{ mm}^2 \quad \blacktriangleleft$$

(b) Rotation about the x -axis:

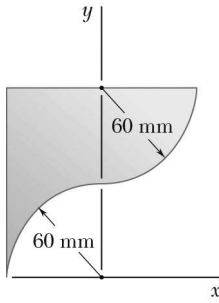
$$\begin{aligned} \text{Volume} &= 2\pi\bar{y}_{\text{area}}A \\ &= 2\pi(48)(2808) \end{aligned}$$

$$\begin{aligned} \text{Area} &= 2\pi\bar{y}_{\text{line}}L = 2\pi\Sigma(\bar{y}_{\text{line}})L \\ &= 2\pi(\bar{y}_1L_1 + \bar{y}_2L_2 + \bar{y}_3L_3) \\ &= 2\pi \left[(36) \left(\sqrt{48^2 + 72^2} \right) + (72)(78) + (36) \left(\sqrt{30^2 + 72^2} \right) \right] \end{aligned}$$

$$\text{Volume} = 847 \times 10^3 \text{ mm}^3 \quad \blacktriangleleft$$

$$\text{Area} = 72.5 \times 10^3 \text{ mm}^2 \quad \blacktriangleleft$$

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PROBLEM 5.54

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.8 about (a) the line $x = -60$ mm, (b) the line $y = 120$ mm.

SOLUTION

From the solution of Problem 5.8, we have

$$\begin{aligned} A &= 7200 \text{ mm}^2 \\ \Sigma \bar{x}A &= -72 \times 10^3 \text{ mm}^3 \\ \Sigma \bar{y}A &= 629.83 \times 10^3 \text{ mm}^3 \end{aligned}$$

Applying the theorems of Pappus-Guldinus, we have

(a) Rotation about line $x = -60$ mm:

$$\begin{aligned} \text{Volume} &= 2\pi(\bar{x} + 60)A = 2\pi(\Sigma \bar{x}A + 60A) \\ &= 2\pi[-72 \times 10^3 + 60(7200)] \end{aligned}$$

$$\text{Volume} = 2.26 \times 10^6 \text{ mm}^3 \quad \blacktriangleleft$$

$$\begin{aligned} \text{Area} &= 2\pi \bar{x}_{\text{line}} L = 2\pi \Sigma(\bar{x}_{\text{line}})L \\ &= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3) \end{aligned}$$

$$= 2\pi \left[\left(60 - \frac{2(60)}{\pi} \right) \left(\frac{\pi(60)}{2} \right) + \left(60 + \frac{2(60)}{\pi} \right) \left(\frac{\pi(60)}{2} \right) + (60)(120) \right]$$

where $\bar{x}_1, \bar{x}_2, \bar{x}_3$ are measured with respect to line $x = -60$ mm.

$$\text{Area} = 116.3 \times 10^3 \text{ mm}^2 \quad \blacktriangleleft$$

(b) Rotation about line $y = 120$ mm:

$$\begin{aligned} \text{Volume} &= 2\pi(120 - \bar{y})A = 2\pi(120A - \Sigma \bar{y}A) \\ &= 2\pi[120(7200) - 629.83 \times 10^3] \end{aligned}$$

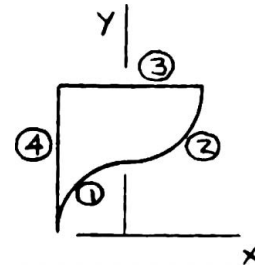
$$\text{Volume} = 1.471 \times 10^6 \text{ mm}^3 \quad \blacktriangleleft$$

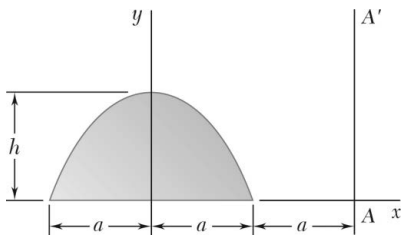
$$\begin{aligned} \text{Area} &= 2\pi \bar{y}_{\text{line}} L = 2\pi \Sigma(\bar{y}_{\text{line}})L \\ &= 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_4 L_4) \end{aligned}$$

where $\bar{y}_1, \bar{y}_2, \bar{y}_4$ are measured with respect to line $y = 120$ mm.

$$\text{Area} = 2\pi \left[\left(120 - \frac{2(60)}{\pi} \right) \left(\frac{\pi(60)}{2} \right) + \left(\frac{2(60)}{\pi} \right) \left(\frac{\pi(60)}{2} \right) + (60)(120) \right]$$

$$\text{Area} = 116.3 \times 10^3 \text{ mm}^2 \quad \blacktriangleleft$$





PROBLEM 5.55

Determine the volume of the solid generated by rotating the parabolic area shown about (a) the x -axis, (b) the axis AA' .

SOLUTION

First, from Figure 5.8a, we have

$$A = \frac{4}{3}ah$$

$$\bar{y} = \frac{2}{5}h$$

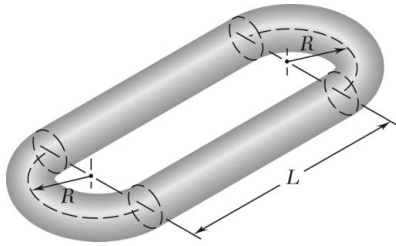
Applying the second theorem of Pappus-Guldinus, we have

(a) Rotation about the x -axis:

$$\begin{aligned} \text{Volume} &= 2\pi\bar{y}A \\ &= 2\pi\left(\frac{2}{5}h\right)\left(\frac{4}{3}ah\right) \end{aligned} \quad \text{or Volume} = \frac{16}{15}\pi ah^2 \blacktriangleleft$$

(b) Rotation about the line AA' :

$$\begin{aligned} \text{Volume} &= 2\pi(2a)A \\ &= 2\pi(2a)\left(\frac{4}{3}ah\right) \end{aligned} \quad \text{or Volume} = \frac{16}{3}\pi a^2h \blacktriangleleft$$



PROBLEM 5.56

Determine the volume and the surface area of the chain link shown, which is made from a 6-mm-diameter bar, if $R = 10$ mm and $L = 30$ mm.

SOLUTION

The area A and circumference C of the cross section of the bar are

$$A = \frac{\pi}{4}d^2 \quad \text{and} \quad C = \pi d.$$

Also, the semicircular ends of the link can be obtained by rotating the cross section through a horizontal semicircular arc of radius R . Now, applying the theorems of Pappus-Guldinus, we have for the volume V ,

$$\begin{aligned} V &= 2(V_{\text{side}}) + 2(V_{\text{end}}) \\ &= 2(AL) + 2(\pi RA) \\ &= 2(L + \pi R)A \end{aligned}$$

or

$$\begin{aligned} V &= 2[30 \text{ mm} + \pi(10 \text{ mm})] \left[\frac{\pi}{4} (6 \text{ mm})^2 \right] \\ &= 3470 \text{ mm}^3 \end{aligned}$$

$$\text{or } V = 3470 \text{ mm}^3 \quad \blacktriangleleft$$

For the area A ,

$$\begin{aligned} A &= 2(A_{\text{side}}) + 2(A_{\text{end}}) \\ &= 2(CL) + 2(\pi RC) \\ &= 2(L + \pi R)C \end{aligned}$$

or

$$\begin{aligned} A &= 2[30 \text{ mm} + \pi(10 \text{ mm})][\pi(6 \text{ mm})] \\ &= 2320 \text{ mm}^2 \end{aligned}$$

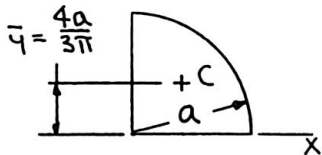
$$\text{or } A = 2320 \text{ mm}^2 \quad \blacktriangleleft$$

PROBLEM 5.57

Verify that the expressions for the volumes of the first four shapes in Figure 5.21 on Page 264 are correct.

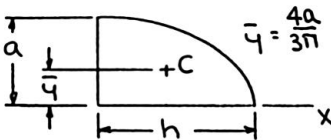
SOLUTION

Following the second theorem of Pappus-Guldinus, in each case, a specific generating area A will be rotated about the x -axis to produce the given shape. Values of \bar{y} are from Figure 5.8a.



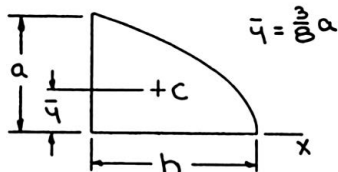
- (1) Hemisphere: the generating area is a quarter circle.

$$\text{We have } V = 2\pi \bar{y} A = 2\pi \left(\frac{4a}{3\pi} \right) \left(\frac{\pi}{4} a^2 \right) \quad \text{or } V = \frac{2}{3} \pi a^3 \blacktriangleleft$$



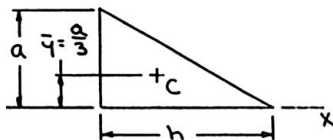
- (2) Semiellipsoid of revolution: the generating area is a quarter ellipse.

$$\text{We have } V = 2\pi \bar{y} A = 2\pi \left(\frac{4a}{3\pi} \right) \left(\frac{\pi}{4} ha \right) \quad \text{or } V = \frac{2}{3} \pi a^2 h \blacktriangleleft$$



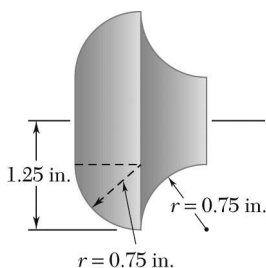
- (3) Paraboloid of revolution: the generating area is a quarter parabola.

$$\text{We have } V = 2\pi \bar{y} A = 2\pi \left(\frac{3}{8} a \right) \left(\frac{2}{3} ah \right) \quad \text{or } V = \frac{1}{2} \pi a^2 h \blacktriangleleft$$



- (4) Cone: the generating area is a triangle.

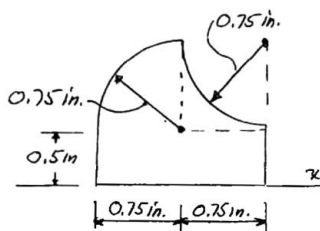
$$\text{We have } V = 2\pi \bar{y} A = 2\pi \left(\frac{a}{3} \right) \left(\frac{1}{2} ha \right) \quad \text{or } V = \frac{1}{3} \pi a^2 h \blacktriangleleft$$



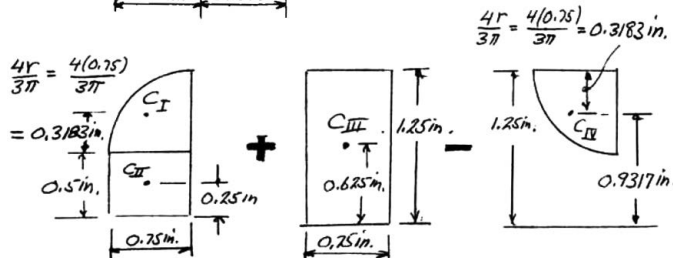
PROBLEM 5.58

Determine the volume and weight of the solid brass knob shown, knowing that the specific weight of brass is 0.306 lb/in^3 .

SOLUTION



Volume of knob is obtained by rotating area at left about the x -axis. Consider area as made of components shown below.



	Area, in^2	\bar{y} , in.	$\bar{y}A$, in^3
1	$\frac{\pi}{4}(0.75)^2 = 0.4418$	0.8183	0.3615
2	$(0.5)(0.75) = 0.375$	0.25	0.0938
3	$(1.25)(0.75) = 0.9375$	0.625	0.5859
4	$-\frac{\pi}{4}(0.75)^2 = -0.4418$	0.9317	-0.4116
Σ			0.6296

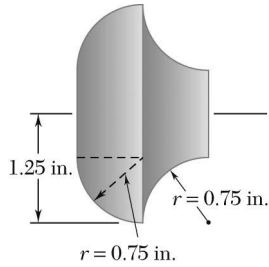
$$V = 2\pi\Sigma\bar{y}A = 2\pi(0.6296 \text{ in}^3) = 3.9559 \text{ in}^3$$

$$V = 3.96 \text{ in}^3 \quad \blacktriangleleft$$

$$W = \gamma V = (0.306 \text{ lb/in}^3)(3.9559 \text{ in}^3)$$

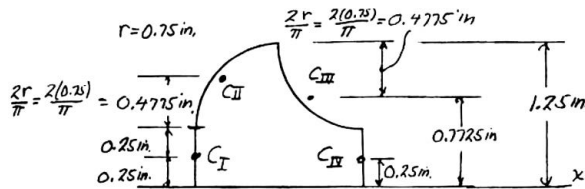
$$W = 1.211 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 5.59



Determine the total surface area of the solid brass knob shown.

SOLUTION



Area is obtained by rotating lines shown about the x -axis.

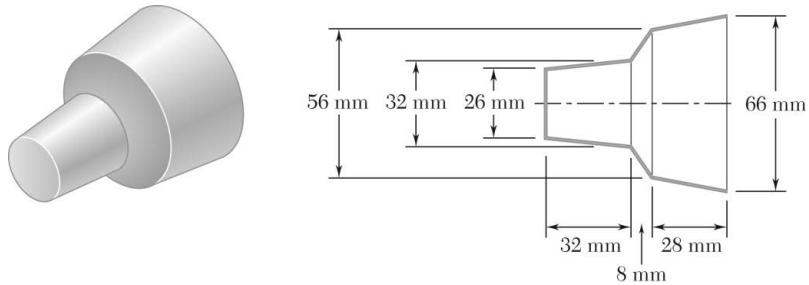
	L , in.	\bar{y} , in.	$\bar{y}L$, in ²
1	0.5	0.25	0.1250
2	$\frac{\pi}{2}(0.75) = 1.1781$	0.9775	1.1516
3	$\frac{\pi}{2}(0.75) = 1.1781$	0.7725	0.9101
4	0.5	0.25	0.1250
Σ			2.3117

$$A = 2\pi\Sigma\bar{y}L = 2\pi(2.3117 \text{ in}^2)$$

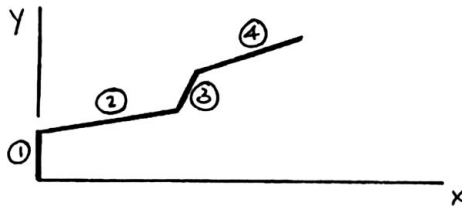
$$A = 14.52 \text{ in}^2 \blacktriangleleft$$

PROBLEM 5.60

The aluminum shade for the small high-intensity lamp shown has a uniform thickness of 1 mm. Knowing that the density of aluminum is 2800 kg/m^3 , determine the mass of the shade.



SOLUTION



The mass of the lamp shade is given by

$$m = \rho V = \rho A t$$

where A is the surface area and t is the thickness of the shade. The area can be generated by rotating the line shown about the x -axis. Applying the first theorem of Pappus Guldinus, we have

$$\begin{aligned} A &= 2\pi \bar{y}L = 2\pi \sum \bar{y}L \\ &= 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4) \end{aligned}$$

or

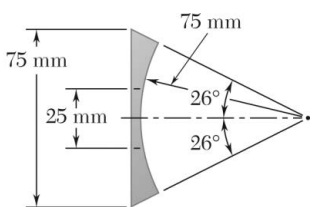
$$\begin{aligned} A &= 2\pi \left[\frac{13 \text{ mm}}{2} (13 \text{ mm}) + \left(\frac{13+16}{2} \right) \text{ mm} \times \sqrt{(32 \text{ mm})^2 + (3 \text{ mm})^2} \right. \\ &\quad \left. + \left(\frac{16+28}{2} \right) \text{ mm} \times \sqrt{(8 \text{ mm})^2 + (12 \text{ mm})^2} \right. \\ &\quad \left. + \left(\frac{28+33}{2} \right) \text{ mm} \times \sqrt{(28 \text{ mm})^2 + (5 \text{ mm})^2} \right] \\ &= 2\pi(84.5 + 466.03 + 317.29 + 867.51) \\ &= 10,903.4 \text{ mm}^2 \end{aligned}$$

Then

$$\begin{aligned} m &= \rho A t \\ &= (2800 \text{ kg/m}^3)(10.9034 \times 10^{-3} \text{ m}^2)(0.001 \text{ m}) \end{aligned}$$

or

$$m = 0.0305 \text{ kg} \quad \blacktriangleleft$$



PROBLEM 5.61

The escutcheon (a decorative plate placed on a pipe where the pipe exits from a wall) shown is cast from brass. Knowing that the density of brass is 8470 kg/m^3 , determine the mass of the escutcheon.

SOLUTION

The mass of the escutcheon is given by $m = (\text{density})V$, where V is the volume. V can be generated by rotating the area A about the x -axis.

From the figure:

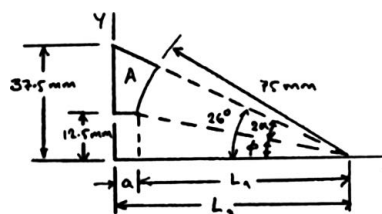
$$L_1 = \sqrt{75^2 - 12.5^2} = 73.9510 \text{ m}$$

$$L_2 = \frac{37.5}{\tan 26^\circ} = 76.8864 \text{ mm}$$

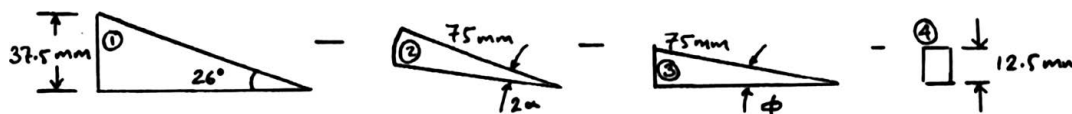
$$a = L_2 - L_1 = 2.9324 \text{ mm}$$

$$\phi = \sin^{-1} \frac{12.5}{75} = 9.5941^\circ$$

$$\alpha = \frac{26^\circ - 9.5941^\circ}{2} = 8.2030^\circ = 0.143168 \text{ rad}$$



Area A can be obtained by combining the following four areas:



Applying the second theorem of Pappus-Guldinus and using Figure 5.8a, we have

$$V = 2\pi \bar{y}A = 2\pi \Sigma \bar{y}A$$

Seg.	$A, \text{ mm}^2$	$\bar{y}, \text{ mm}$	$\bar{y}A, \text{ mm}^3$
1	$\frac{1}{2}(76.886)(37.5) = 1441.61$	$\frac{1}{3}(37.5) = 12.5$	18,020.1
2	$-\alpha(75)^2 = -805.32$	$\frac{2(75)\sin \alpha}{3\alpha} \sin(\alpha + \phi) = 15.2303$	-12,265.3
3	$-\frac{1}{2}(73.951)(12.5) = -462.19$	$\frac{1}{3}(12.5) = 4.1667$	-1925.81
4	$-(2.9354)(12.5) = -36.693$	$\frac{1}{2}(12.5) = 6.25$	-229.33
Σ			3599.7

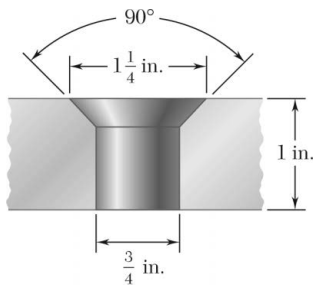
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PROBLEM 5.61 (Continued)

Then

$$\begin{aligned}V &= 2\pi\Sigma\bar{y}A \\ &= 2\pi(3599.7 \text{ mm}^3) \\ &= 22,618 \text{ mm}^3 \\ m &= (\text{density})V \\ &= (8470 \text{ kg/m}^3)(22.618 \times 10^{-6} \text{ m}^3) \\ &= 0.191574 \text{ kg}\end{aligned}$$

or $m = 0.1916 \text{ kg}$ ◀

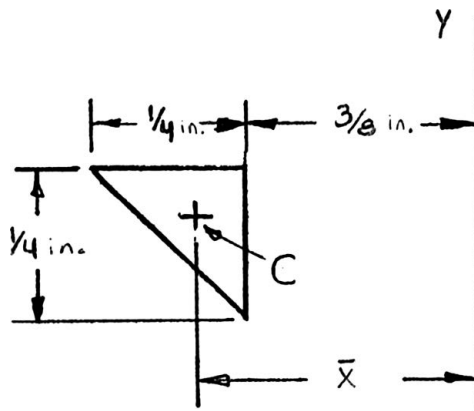


PROBLEM 5.62

A $\frac{3}{4}$ -in.-diameter hole is drilled in a piece of 1-in.-thick steel; the hole is then countersunk as shown. Determine the volume of steel removed during the countersinking process.

SOLUTION

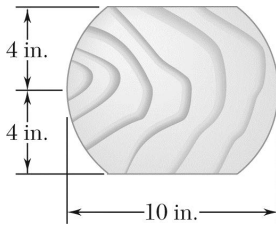
The required volume can be generated by rotating the area shown about the y -axis. Applying the second theorem of Pappus-Guldinus, we have



$$V = 2\pi \bar{x} A$$

$$= 2\pi \left[\frac{3}{8} + \frac{1}{3} \left(\frac{1}{4} \right) \text{ in.} \right] \times \left[\frac{1}{2} \times \frac{1}{4} \text{ in.} \times \frac{1}{4} \text{ in.} \right]$$

$$V = 0.0900 \text{ in}^3 \blacktriangleleft$$



PROBLEM 5.63

Knowing that two equal caps have been removed from a 10-in.-diameter wooden sphere, determine the total surface area of the remaining portion.

SOLUTION

The surface area can be generated by rotating the line shown about the y -axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \bar{X}L = 2\pi \Sigma \bar{x}L$$

$$= 2\pi(2\bar{x}_1L_1 + \bar{x}_2L_2)$$

Now

$$\tan \alpha = \frac{4}{3}$$

or

$$\alpha = 53.130^\circ$$

Then

$$\bar{x}_2 = \frac{5 \text{ in.} \times \sin 53.130^\circ}{53.130^\circ \times \frac{\pi}{180^\circ}}$$

$$= 4.3136 \text{ in.}$$

and

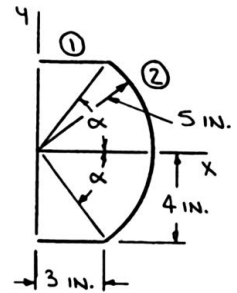
$$L_2 = 2 \left(53.130^\circ \times \frac{\pi}{180^\circ} \right) (5 \text{ in.})$$

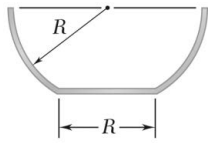
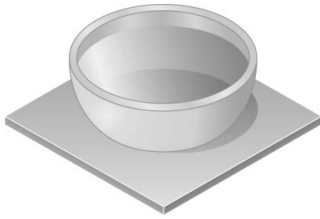
$$= 9.2729 \text{ in.}$$

$$A = 2\pi \left[2 \left(\frac{3}{2} \text{ in.} \right) (3 \text{ in.}) + (4.3136 \text{ in.})(9.2729 \text{ in.}) \right]$$

or

$$A = 308 \text{ in}^2 \blacktriangleleft$$



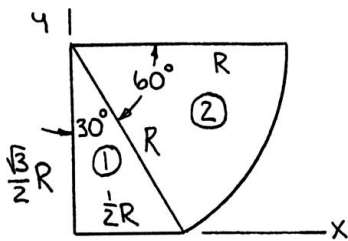


PROBLEM 5.64

Determine the capacity, in liters, of the punch bowl shown if $R = 250$ mm.

SOLUTION

The volume can be generated by rotating the triangle and circular sector shown about the y -axis. Applying the second theorem of Pappus-Guldinus and using Figure 5.8a, we have



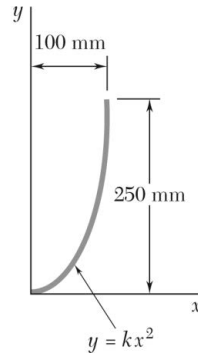
$$\begin{aligned}
 V &= 2\pi \bar{x}A = 2\pi \Sigma \bar{x}A \\
 &= 2\pi(\bar{x}_1 A_1 + \bar{x}_2 A_2) \\
 &= 2\pi \left[\left(\frac{1}{3} \times \frac{1}{2} R \right) \left(\frac{1}{2} \times \frac{1}{2} R \times \frac{\sqrt{3}}{2} R \right) + \left(\frac{2R \sin 30^\circ}{3 \times \frac{\pi}{6}} \right) \left(\frac{\pi}{6} R^2 \right) \right] \\
 &= 2\pi \left(\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}} \right) \\
 &= \frac{3\sqrt{3}}{8} \pi R^3 \\
 &= \frac{3\sqrt{3}}{8} \pi (0.25 \text{ m})^3 \\
 &= 0.031883 \text{ m}^3
 \end{aligned}$$

Since

$$10^3 \text{ l} = 1 \text{ m}^3$$

$$V = 0.031883 \text{ m}^3 \times \frac{10^3 \text{ l}}{1 \text{ m}^3}$$

$$V = 31.91 \text{ l} \blacktriangleleft$$



PROBLEM 5.65*

The shade for a wall-mounted light is formed from a thin sheet of translucent plastic. Determine the surface area of the outside of the shade, knowing that it has the parabolic cross section shown.

SOLUTION

First note that the required surface area A can be generated by rotating the parabolic cross section through π radians about the y -axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = \pi \bar{x} L$$

Now at

$$x = 100 \text{ mm}, \quad y = 250 \text{ mm}$$

$$250 = k(100)^2 \quad \text{or} \quad k = 0.025 \text{ mm}^{-1}$$

and

$$x_{EL} = x$$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

where

$$\frac{dy}{dx} = 2kx$$

Then

$$dL = \sqrt{1 + 4k^2 x^2} dx$$

We have

$$xL = \int x_{EL} dL = \int_0^{100} x \left(\sqrt{1 + 4k^2 x^2} dx \right)$$

$$xL = \left[\frac{1}{3} \frac{1}{4k^2} (1 + 4k^2 x^2)^{3/2} \right]_0^{100}$$

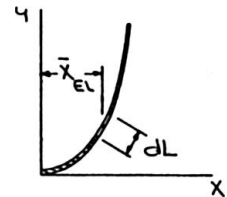
$$= \frac{1}{12} \frac{1}{(0.025)^2} \left\{ [1 + 4(0.025)^2 (100)^2]^{3/2} - (1)^{3/2} \right\}$$

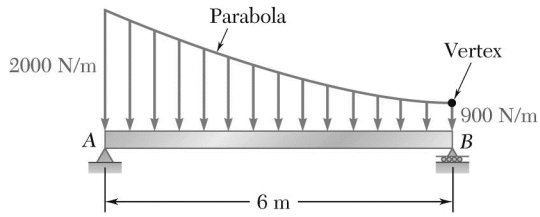
$$= 17,543.3 \text{ mm}^2$$

Finally,

$$A = \pi(17,543.3 \text{ mm}^2)$$

$$\text{or } A = 55.1 \times 10^3 \text{ mm}^2 \blacktriangleleft$$

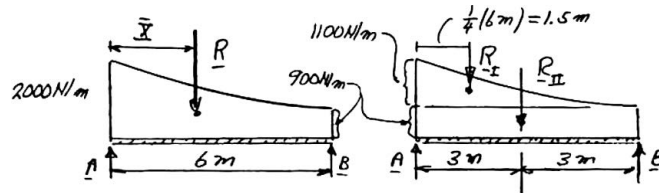




PROBLEM 5.66

For the beam and loading shown, determine (a) the magnitude and location of the resultant of the distributed load, (b) the reactions at the beam supports.

SOLUTION



$$(a) \quad R_I = \frac{1}{3}(1100 \text{ N/m})(6 \text{ m}) = 2200 \text{ N}$$

$$R_{II} = (900 \text{ N/m})(6 \text{ m}) = 5400 \text{ N}$$

$$R = R_I + R_{II} = 2200 + 5400 = 7600 \text{ N}$$

$$XR = \Sigma xR: \quad X(7600) = (2200)(1.5) + (5400)(3)$$

$$X = 2.5658 \text{ m}$$

$$\mathbf{R} = 7.60 \text{ kN} \downarrow, \quad X = 2.57 \text{ m} \blacktriangleleft$$

$$(b) \quad +\curvearrowright \Sigma M_A = 0: \quad B(6 \text{ m}) - (7600 \text{ N})(2.5658 \text{ m}) = 0$$

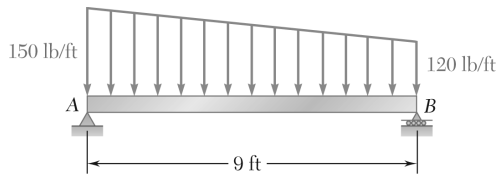
$$B = 3250.0 \text{ N}$$

$$\mathbf{B} = 3.25 \text{ kN} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + 3250.0 \text{ N} - 7600 \text{ N} = 0$$

$$A = 4350.0 \text{ N}$$

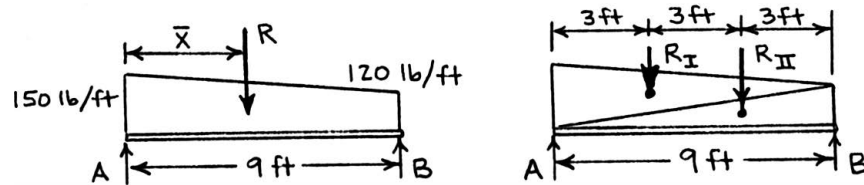
$$\mathbf{A} = 4.35 \text{ kN} \uparrow \blacktriangleleft$$



PROBLEM 5.67

For the beam and loading shown, determine (a) the magnitude and location of the resultant of the distributed load, (b) the reactions at the beam supports.

SOLUTION



$$R_I = \frac{1}{2}(150 \text{ lb/ft})(9 \text{ ft}) = 675 \text{ lb}$$

$$R_{II} = \frac{1}{2}(120 \text{ lb/ft})(9 \text{ ft}) = 540 \text{ lb}$$

$$R = R_I + R_{II} = 675 + 540 = 1215 \text{ lb}$$

$$\bar{X}R = \Sigma \bar{x}R: \quad \bar{X}(1215) = (3)(675) + (6)(540) \quad \bar{X} = 4.3333 \text{ ft}$$

(a) $R = 1215 \text{ lb} \downarrow$ $\bar{X} = 4.33 \text{ ft} \blacktriangleleft$

(b) Reactions: $+\curvearrowright \Sigma M_A = 0: B(9 \text{ ft}) - (1215 \text{ lb})(4.3333 \text{ ft}) = 0$

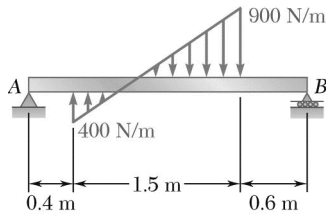
$$B = 585.00 \text{ lb}$$

$$B = 585 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A + 585.00 \text{ lb} - 1215 \text{ lb} = 0$$

$$A = 630.00 \text{ lb}$$

$$A = 630 \text{ lb} \uparrow \blacktriangleleft$$

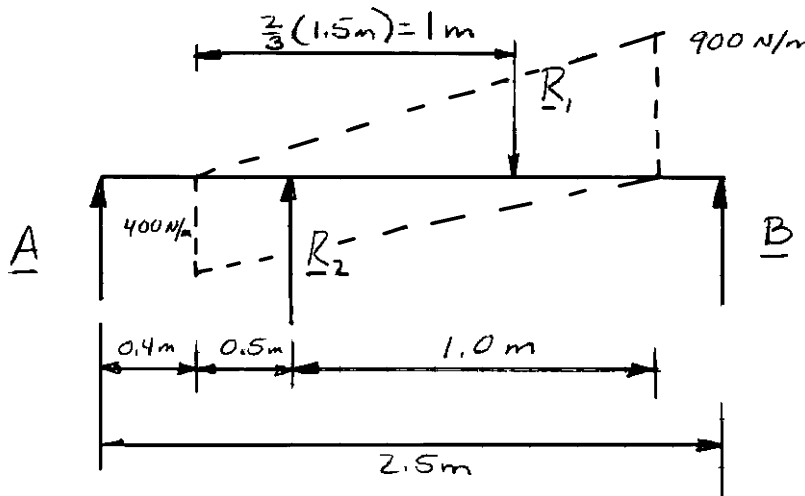


PROBLEM 5.68

Determine the reactions at the beam supports for the given loading.

SOLUTION

First replace the given loading by the loadings shown below. Both loadings are equivalent since they are both defined by a linear relation between load and distance and have the same values at the end points.



$$R_1 = \frac{1}{2}(900 \text{ N/m})(1.5 \text{ m}) = 675 \text{ N}$$

$$R_2 = \frac{1}{2}(400 \text{ N/m})(1.5 \text{ m}) = 300 \text{ N}$$

$$+\curvearrowright \Sigma M_A = 0: \quad -(675 \text{ N})(1.4 \text{ m}) + (300 \text{ N})(0.9 \text{ m}) + B(2.5 \text{ m}) = 0$$

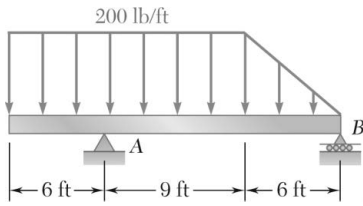
$$B = 270 \text{ N}$$

$$B = 270 \text{ N} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A - 675 \text{ N} + 300 \text{ N} + 270 \text{ N} = 0$$

$$A = 105.0 \text{ N}$$

$$A = 105.0 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 5.69

Determine the reactions at the beam supports for the given loading.

SOLUTION

$$R_I = (200 \text{ lb/ft})(15 \text{ ft})$$

$$R_I = 3000 \text{ lb}$$

$$R_{II} = \frac{1}{2}(200 \text{ lb/ft})(6 \text{ ft})$$

$$R_{II} = 600 \text{ lb}$$

$$+\circlearrowleft \Sigma M_A = 0: -(3000 \text{ lb})(1.5 \text{ ft}) - (600 \text{ lb})(9 \text{ ft} + 2 \text{ ft}) + B(15 \text{ ft}) = 0$$

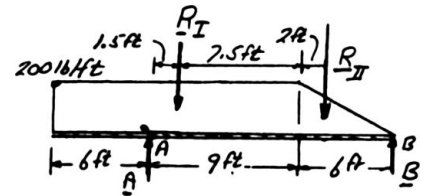
$$B = 740 \text{ lb}$$

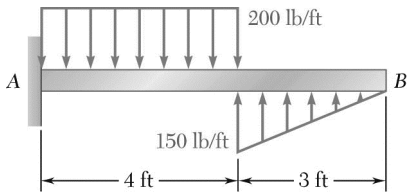
$$B = 740 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A + 740 \text{ lb} - 3000 \text{ lb} - 600 \text{ lb} = 0$$

$$A = 2860 \text{ lb}$$

$$A = 2860 \text{ lb} \uparrow \blacktriangleleft$$





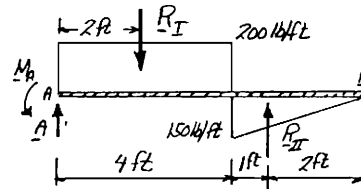
PROBLEM 5.70

Determine the reactions at the beam supports for the given loading.

SOLUTION

$$R_I = (200 \text{ lb/ft})(4 \text{ ft}) = 800 \text{ lb}$$

$$R_{II} = \frac{1}{2}(150 \text{ lb/ft})(3 \text{ ft}) = 225 \text{ lb}$$

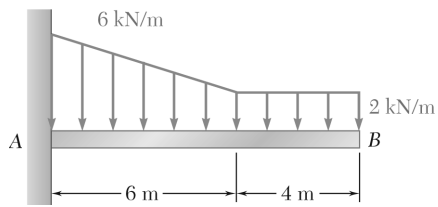


$$+\uparrow \Sigma F_y = 0: \quad A - 800 \text{ lb} + 225 \text{ lb} = 0$$

$$A = 575 \text{ lb} \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: \quad M_A - (800 \text{ lb})(2 \text{ ft}) + (225 \text{ lb})(5 \text{ ft}) = 0$$

$$M_A = 475 \text{ lb} \cdot \text{ft} + \curvearrowright \blacktriangleleft$$



PROBLEM 5.71

Determine the reactions at the beam supports for the given loading.

SOLUTION

$$R_I = \frac{1}{2}(4 \text{ kN/m})(6 \text{ m})$$

$$= 12 \text{ kN}$$

$$R_{II} = (2 \text{ kN/m})(10 \text{ m})$$

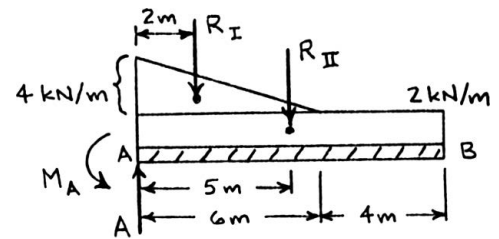
$$= 20 \text{ kN}$$

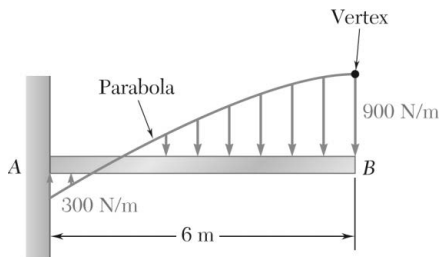
$$+\uparrow \Sigma F_y = 0: \quad A - 12 \text{ kN} - 20 \text{ kN} = 0$$

$$A = 32.0 \text{ kN} \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: \quad M_A - (12 \text{ kN})(2 \text{ m}) - (20 \text{ kN})(5 \text{ m}) = 0$$

$$M_A = 124.0 \text{ kN} \cdot \text{m} \curvearrowright \blacktriangleleft$$



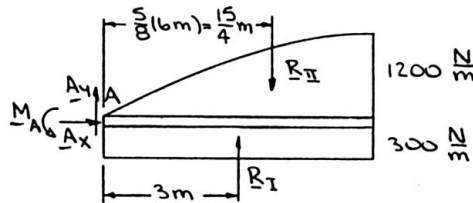


PROBLEM 5.72

Determine the reactions at the beam supports for the given loading.

SOLUTION

First replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a parabolic relation between load and distance and the values at the end points are the same.



We have

$$R_I = (6 \text{ m})(300 \text{ N/m}) = 1800 \text{ N}$$

$$R_{II} = \frac{2}{3}(6 \text{ m})(1200 \text{ N/m}) = 4800 \text{ N}$$

Then

$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y + 1800 \text{ N} - 4800 \text{ N} = 0$$

or

$$A_y = 3000 \text{ N}$$

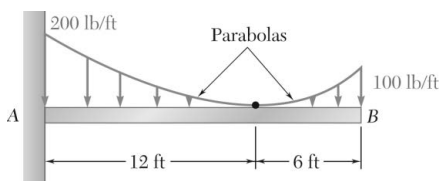
$$\mathbf{A} = 3.00 \text{ kN} \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: M_A + (3 \text{ m})(1800 \text{ N}) - \left(\frac{15}{4} \text{ m}\right)(4800 \text{ N}) = 0$$

or

$$M_A = 12.6 \text{ kN} \cdot \text{m}$$

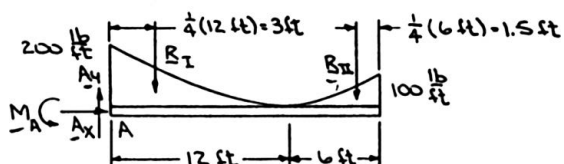
$$\mathbf{M}_A = 12.60 \text{ kN} \cdot \text{m} \curvearrowright \blacktriangleleft$$



PROBLEM 5.73

Determine the reactions at the beam supports for the given loading.

SOLUTION



We have

$$R_I = \frac{1}{3}(12 \text{ ft})(200 \text{ lb/ft}) = 800 \text{ lb}$$

$$R_{II} = \frac{1}{3}(6 \text{ ft})(100 \text{ lb/ft}) = 200 \text{ lb}$$

Then

$$+\rightarrow \Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y - 800 \text{ lb} - 200 \text{ lb} = 0$$

or

$$A_y = 1000 \text{ lb}$$

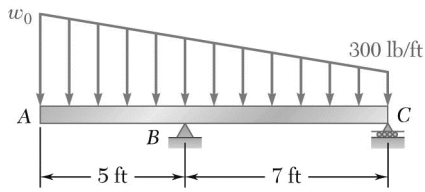
$$A = 1000 \text{ lb} \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: M_A - (3 \text{ ft})(800 \text{ lb}) - (16.5 \text{ ft})(200 \text{ lb}) = 0$$

or

$$M_A = 5700 \text{ lb} \cdot \text{ft}$$

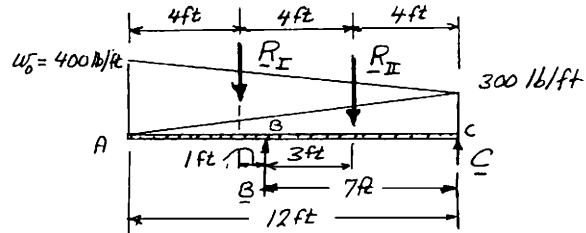
$$M_A = 5700 \text{ lb} \cdot \text{ft} \curvearrowright \blacktriangleleft$$



PROBLEM 5.74

Determine the reactions at the beam supports for the given loading when $w_0 = 400$ lb/ft.

SOLUTION



$$R_I = \frac{1}{2} w_0 (12 \text{ ft}) = \frac{1}{2} (400 \text{ lb/ft})(12 \text{ ft}) = 2400 \text{ lb}$$

$$R_{II} = \frac{1}{2} (300 \text{ lb/ft})(12 \text{ ft}) = 1800 \text{ lb}$$

$$+\curvearrowright \Sigma M_B = 0: (2400 \text{ lb})(1 \text{ ft}) - (1800 \text{ lb})(3 \text{ ft}) + C(7 \text{ ft}) = 0$$

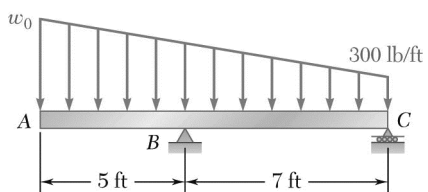
$$C = 428.57 \text{ lb}$$

$$C = 429 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B + 428.57 \text{ lb} - 2400 \text{ lb} - 1800 \text{ lb} = 0$$

$$B = 3771 \text{ lb}$$

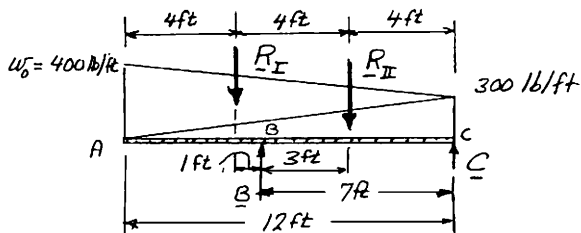
$$B = 3770 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 5.75

Determine (a) the distributed load w_0 at the end A of the beam ABC for which the reaction at C is zero, (b) the corresponding reaction at B.

SOLUTION



For w_0 ,

$$R_I = \frac{1}{2} w_0 (12 \text{ ft}) = 6 w_0$$

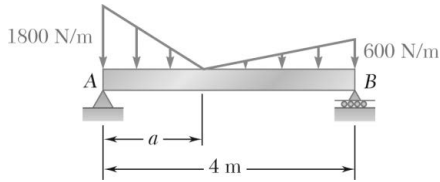
$$R_{II} = \frac{1}{2} (300 \text{ lb/ft})(12 \text{ ft}) = 1800 \text{ lb}$$

(a) For $C = 0$, $\rightarrow \Sigma M_B = 0$: $(6 w_0)(1 \text{ ft}) - (1800 \text{ lb})(3 \text{ ft}) = 0$ $w_0 = 900 \text{ lb/ft}$ ◀

(b) Corresponding value of R_I :

$$R_I = 6(900) = 5400 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: B - 5400 \text{ lb} - 1800 \text{ lb} = 0$$
 $B = 7200 \text{ lb}$ ↑ ◀

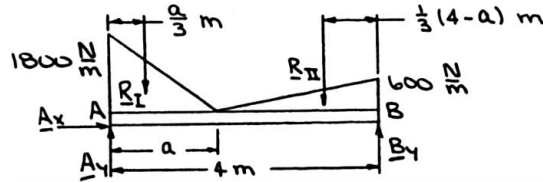


PROBLEM 5.76

Determine (a) the distance a so that the vertical reactions at supports A and B are equal, (b) the corresponding reactions at the supports.

SOLUTION

(a)



We have

$$R_I = \frac{1}{2}(a \text{ m})(1800 \text{ N/m}) = 900a \text{ N}$$

$$R_{II} = \frac{1}{2}[(4-a) \text{ m}](600 \text{ N/m}) = 300(4-a) \text{ N}$$

Then

$$+\uparrow \Sigma F_y = 0: A_y - 900a - 300(4-a) + B_y = 0$$

or

$$A_y + B_y = 1200 + 600a$$

Now

$$A_y = B_y \Rightarrow A_y = B_y = 600 + 300a \text{ (N)} \quad (1)$$

Also,

$$+\curvearrowright \Sigma M_B = 0: -(4 \text{ m})A_y + \left[\left(4 - \frac{a}{3} \right) \text{ m} \right] [(900a) \text{ N}]$$

$$+ \left[\frac{1}{3}(4-a) \text{ m} \right] [300(4-a) \text{ N}] = 0$$

or

$$A_y = 400 + 700a - 50a^2 \quad (2)$$

Equating Eqs. (1) and (2),

$$600 + 300a = 400 + 700a - 50a^2$$

or

$$a^2 - 8a + 4 = 0$$

Then

$$a = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2}$$

or

$$a = 0.53590 \text{ m}$$

$$a = 7.4641 \text{ m}$$

Now

$$a \leq 4 \text{ m} \Rightarrow$$

$$a = 0.536 \text{ m} \blacktriangleleft$$

PROBLEM 5.76 (Continued)

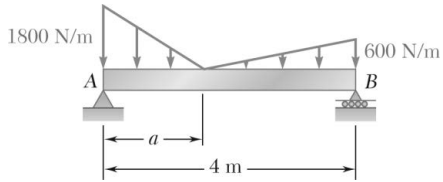
(b) We have

$$\overset{+}{\rightarrow} \Sigma F_x = 0: A_x = 0$$

From Eq. (1):

$$\begin{aligned} A_y &= B_y \\ &= 600 + 300(0.53590) \\ &= 761 \text{ N} \end{aligned}$$

$$\mathbf{A} = \mathbf{B} = 761 \text{ N} \uparrow \blacktriangleleft$$

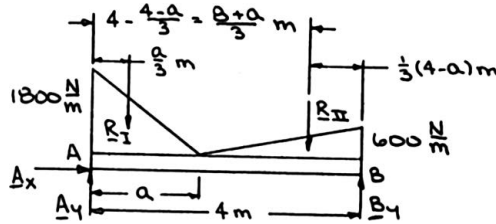


PROBLEM 5.77

Determine (a) the distance a so that the reaction at support B is minimum, (b) the corresponding reactions at the supports.

SOLUTION

(a)



We have
$$R_I = \frac{1}{2}(a \text{ m})(1800 \text{ N/m}) = 900a \text{ N}$$

$$R_{II} = \frac{1}{2}[(4 - a)\text{m}](600 \text{ N/m}) = 300(4 - a) \text{ N}$$

Then
$$+\circlearrowleft \Sigma M_A = 0: -\left(\frac{a}{3}\text{m}\right)(900a \text{ N}) - \left(\frac{8+a}{3}\text{m}\right)[300(4 - a)\text{N}] + (4 \text{ m})B_y = 0$$

or
$$B_y = 50a^2 - 100a + 800 \quad (1)$$

Then
$$\frac{dB_y}{da} = 100a - 100 = 0 \quad \text{or } a = 1.000 \text{ m} \quad \blacktriangleleft$$

(b) From Eq. (1):
$$B_y = 50(1)^2 - 100(1) + 800 = 750 \text{ N} \quad \mathbf{B} = 750 \text{ N} \uparrow \quad \blacktriangleleft$$

and
$$+\rightarrow \Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y - 900(1) \text{ N} - 300(4 - 1) \text{ N} + 750 \text{ N} = 0$$

or
$$A_y = 1050 \text{ N} \quad \mathbf{A} = 1050 \text{ N} \uparrow \quad \blacktriangleleft$$

PROBLEM 5.78

A beam is subjected to a linearly distributed downward load and rests on two wide supports BC and DE , which exert uniformly distributed upward loads as shown. Determine the values of w_{BC} and w_{DE} corresponding to equilibrium when $w_A = 600 \text{ N/m}$.

SOLUTION

We have

$$R_I = \frac{1}{2}(6 \text{ m})(600 \text{ N/m}) = 1800 \text{ N}$$

$$R_{II} = \frac{1}{2}(6 \text{ m})(1200 \text{ N/m}) = 3600 \text{ N}$$

$$R_{BC} = (0.8 \text{ m})(w_{BC} \text{ N/m}) = (0.8w_{BC}) \text{ N}$$

$$R_{DE} = (1.0 \text{ m})(w_{DE} \text{ N/m}) = (w_{DE}) \text{ N}$$

Then

$$+\curvearrowright \Sigma M_G = 0: \quad -(1 \text{ m})(1800 \text{ N}) - (3 \text{ m})(3600 \text{ N}) + (4 \text{ m})(w_{DE} \text{ N}) = 0$$

or

$$w_{DE} = 3150 \text{ N/m} \quad \blacktriangleleft$$

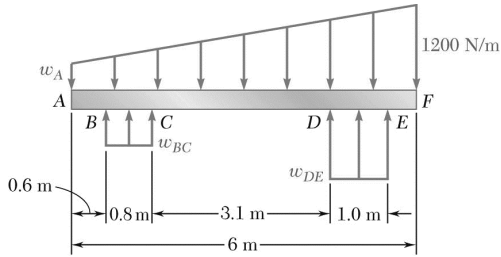
and

$$+\uparrow \Sigma F_y = 0: \quad (0.8w_{BC}) \text{ N} - 1800 \text{ N} - 3600 \text{ N} + 3150 \text{ N} = 0$$

or

$$w_{BC} = 2812.5 \text{ N/m} \qquad \qquad \qquad w_{BC} = 2810 \text{ N/m} \quad \blacktriangleleft$$

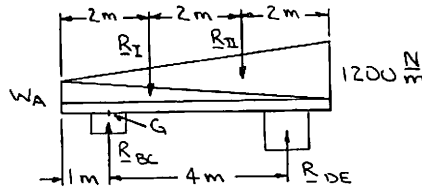
PROBLEM 5.79



A beam is subjected to a linearly distributed downward load and rests on two wide supports BC and DE , which exert uniformly distributed upward loads as shown. Determine (a) the value of w_A so that $w_{BC} = w_{DE}$, (b) the corresponding values of w_{BC} and w_{DE} .

SOLUTION

(a)



We have
$$R_I = \frac{1}{2}(6 \text{ m})(w_A \text{ N/m}) \cdot (3 w_A) \text{ N}$$

$$R_{II} = \frac{1}{2}(6 \text{ m})(1200 \text{ N/m}) = 3600 \text{ N}$$

$$R_{BC} = (0.8 \text{ m})(w_{BC} \text{ N/m}) = (0.8 w_{BC}) \text{ N}$$

$$R_{DE} = (1 \text{ m})(w_{DE} \text{ N/m}) = (w_{DE}) \text{ N}$$

Then
$$+\uparrow \Sigma F_y = 0: (0.8 w_{BC}) \text{ N} - (3 w_A) \text{ N} - 3600 \text{ N} + (w_{DE}) \text{ N} = 0$$

or
$$0.8 w_{BC} + w_{DE} = 3600 + 3 w_A$$

Now
$$w_{BC} = w_{DE} \Rightarrow w_{BC} = w_{DE} = 2000 + \frac{5}{3} w_A \quad (1)$$

Also,
$$+\circlearrowleft \Sigma M_G = 0: -(1 \text{ m})(3 w_A \text{ N}) - (3 \text{ m})(3600 \text{ N}) + (4 \text{ m})(w_{DE} \text{ N}) = 0$$

or
$$w_{DE} = 2700 + \frac{3}{4} w_A \quad (2)$$

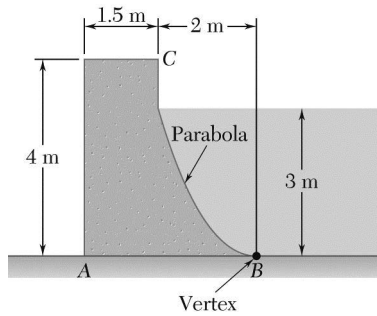
Equating Eqs. (1) and (2),

$$2000 + \frac{5}{3} w_A = 2700 + \frac{3}{4} w_A$$

or
$$w_A = \frac{8400}{11} \text{ N/m} \quad w_A = 764 \text{ N/m} \quad \blacktriangleleft$$

(b) Eq. (1) \Rightarrow
$$w_{BC} = w_{DE} = 2000 + \frac{5}{3} \left(\frac{8400}{11} \right)$$

or
$$w_{BC} = w_{DE} = 3270 \text{ N/m} \quad \blacktriangleleft$$

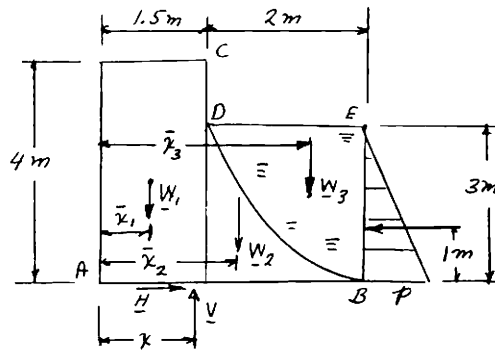


PROBLEM 5.80

The cross section of a concrete dam is as shown. For a 1-m-wide dam section, determine (a) the resultant of the reaction forces exerted by the ground on the base AB of the dam, (b) the point of application of the resultant of part a, (c) the resultant of the pressure forces exerted by the water on the face BC of the dam.

SOLUTION

- (a) Consider free body made of dam and section BDE of water. (Thickness = 1 m)



$$p = (3 \text{ m})(10 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$W_1 = (1.5 \text{ m})(4 \text{ m})(1 \text{ m})(2.4 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 144.26 \text{ kN}$$

$$W_2 = \frac{1}{3}(2 \text{ m})(3 \text{ m})(1 \text{ m})(2.4 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 47.09 \text{ kN}$$

$$W_3 = \frac{2}{3}(2 \text{ m})(3 \text{ m})(1 \text{ m})(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 39.24 \text{ kN}$$

$$P = \frac{1}{2}Ap = \frac{1}{2}(3 \text{ m})(1 \text{ m})(3 \text{ m})(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 44.145 \text{ kN}$$

$$+\rightarrow \Sigma F_x = 0: H - 44.145 \text{ kN} = 0$$

$$H = 44.145 \text{ kN}$$

$$\mathbf{H} = 44.1 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: V - 141.26 - 47.09 - 39.24 = 0$$

$$V = 227.6 \text{ kN}$$

$$\mathbf{V} = 228 \text{ kN} \uparrow \blacktriangleleft$$

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PROBLEM 5.80 (Continued)

$$\bar{x}_1 = \frac{1}{2}(1.5 \text{ m}) = 0.75 \text{ m}$$

$$\bar{x}_2 = 1.5 \text{ m} + \frac{1}{4}(2 \text{ m}) = 2 \text{ m}$$

$$\bar{x}_3 = 1.5 \text{ m} + \frac{5}{8}(2 \text{ m}) = 2.75 \text{ m}$$

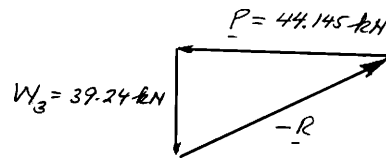
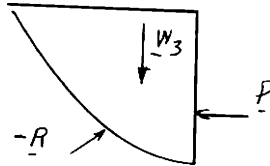
$$+\circlearrowleft \Sigma M_A = 0: \quad xV - \Sigma \bar{x}W + P(1 \text{ m}) = 0$$

$$\begin{aligned} x(227.6 \text{ kN}) - (141.26 \text{ kN})(0.75 \text{ m}) - (47.09 \text{ kN})(2 \text{ m}) \\ - (39.24 \text{ kN})(2.75 \text{ m}) + (44.145 \text{ kN})(1 \text{ m}) = 0 \\ x(227.6 \text{ kN}) - 105.9 - 94.2 - 107.9 + 44.145 = 0 \\ x(227.6) - 263.9 = 0 \end{aligned}$$

$$x = 1.159 \text{ m (to right of A)} \blacktriangleleft$$

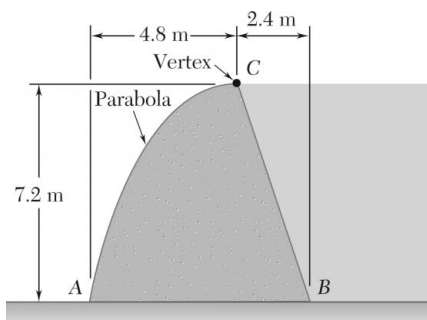
(b) Resultant of face BC:

Consider free body of section *BDE* of water.



$$-\mathbf{R} = 59.1 \text{ kN} \nearrow 41.6^\circ$$

$$\mathbf{R} = 59.1 \text{ kN} \swarrow 41.6^\circ \blacktriangleleft$$

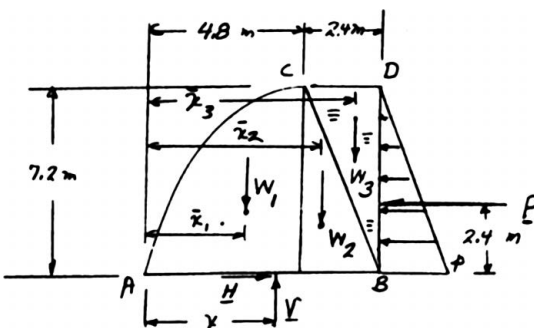


PROBLEM 5.81

The cross section of a concrete dam is as shown. For a 1-m-wide dam section, determine (a) the resultant of the reaction forces exerted by the ground on the base AB of the dam, (b) the point of application of the resultant of part a, (c) the resultant of the pressure forces exerted by the water on the face BC of the dam.

SOLUTION

- (a) Consider free body made of dam and triangular section of water shown. (Thickness = 1 m.)



$$p = (7.2 \text{ m})(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$W_1 = \frac{2}{3}(4.8 \text{ m})(7.2 \text{ m})(1 \text{ m})(2.4 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$= 542.5 \text{ kN}$$

$$W_2 = \frac{1}{2}(2.4 \text{ m})(7.2 \text{ m})(1 \text{ m})(2.4 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$= 203.4 \text{ kN}$$

$$W_3 = \frac{1}{2}(2.4 \text{ m})(7.2 \text{ m})(1 \text{ m})(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$= 84.8 \text{ kN}$$

$$P = \frac{1}{2}Ap = \frac{1}{2}(7.2 \text{ m})(1 \text{ m})(7.2 \text{ m})(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)$$

$$= 254.3 \text{ kN}$$

$$+\rightarrow \Sigma F_x = 0: \quad H - 254.3 \text{ kN} = 0 \qquad \qquad \qquad \mathbf{H = 254 \text{ kN} \rightarrow \blacktriangleleft}$$

$$+\uparrow \Sigma F_y = 0: \quad V - 542.5 - 203.4 - 84.8 = 0$$

$$\qquad \qquad \qquad \mathbf{V = 830.7 \text{ kN}} \qquad \qquad \qquad \mathbf{V = 831 \text{ kN} \uparrow \blacktriangleleft}$$

PROBLEM 5.81 (Continued)

(b) $\bar{x}_1 = \frac{5}{8}(4.8 \text{ m}) = 3 \text{ m}$
 $\bar{x}_2 = 4.8 + \frac{1}{3}(2.4) = 5.6 \text{ m}$
 $\bar{x}_3 = 4.8 + \frac{2}{3}(2.4) = 6.4 \text{ m}$

$\rightarrow \Sigma M_A = 0: xV - \Sigma \bar{x}W + P(2.4 \text{ m}) = 0$
 $x(830.7 \text{ kN}) - (3 \text{ m})(542.5 \text{ kN}) - (5.6 \text{ m})(203.4 \text{ kN})$
 $\quad - (6.4 \text{ m})(84.8 \text{ kN}) + (2.4 \text{ m})(254.3 \text{ kN}) = 0$
 $x(830.7) - 1627.5 - 1139.0 - 542.7 + 610.3 = 0$
 $x(830.7) - 2698.9 = 0$

$x = 3.25 \text{ m}$ (to right of A) ◀

(c) Resultant on face BC:
 Direct computation:

$P = \rho gh = (10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(7.2 \text{ m})$
 $P = 70.63 \text{ kN/m}^2$

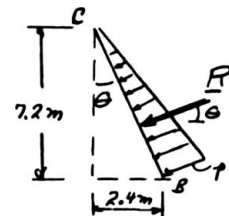
$BC = \sqrt{(2.4)^2 + (7.2)^2}$
 $= 7.589 \text{ m}$

$\theta = 18.43^\circ$

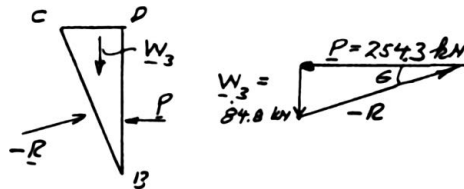
$R = \frac{1}{2} PA$

$= \frac{1}{2} (70.63 \text{ kN/m}^2)(7.589 \text{ m})(1 \text{ m})$

$R = 268 \text{ kN}$ ↗ 18.43° ◀

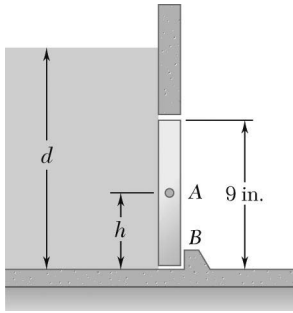


Alternate computation: Use free body of water section BCD.



$-R = 268 \text{ kN}$ ↗ 18.43°

$R = 268 \text{ kN}$ ↗ 18.43° ◀



PROBLEM 5.82

An automatic valve consists of a 9×9 -in. square plate that is pivoted about a horizontal axis through A located at a distance $h = 3.6$ in. above the lower edge. Determine the depth of water d for which the valve will open.

SOLUTION

Since valve is 9 in. wide, $w = 9p = 9\gamma h$, where all dimensions are in inches.

$$w_1 = 9\gamma(d - 9), \quad w_2 = 9\gamma d$$

$$P_I = \frac{1}{2}(9 \text{ in.})w_1 = \frac{1}{2}(9)(9\gamma)(d - 9)$$

$$P_{II} = \frac{1}{2}(9 \text{ in.})w_2 = \frac{1}{2}(9)(9\gamma d)$$

Valve opens when $B = 0$.

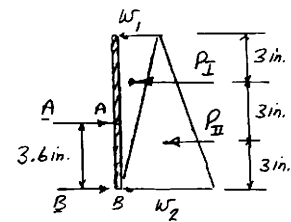
$$+\circlearrowleft \Sigma M_A = 0: \quad P_I(6 \text{ in.} - 3.6 \text{ in.}) - P_{II}(3.6 \text{ in.} - 3 \text{ in.}) = 0$$

$$\left[\frac{1}{2}(9)(9\gamma)(d - 9) \right] (2.4) - \left[\frac{1}{2}(9)(9\gamma d) \right] (0.6) = 0$$

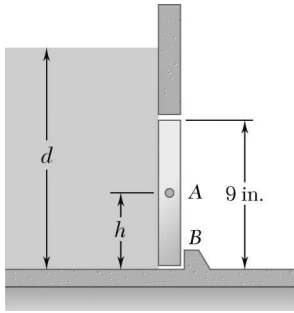
$$(d - 9)(2.4) - d(0.6) = 0$$

$$1.8d - 21.6 = 0$$

$$d = 12.00 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 5.83



An automatic valve consists of a 9×9 -in. square plate that is pivoted about a horizontal axis through A . If the valve is to open when the depth of water is $d = 18$ in., determine the distance h from the bottom of the valve to the pivot A .

SOLUTION

Since valve is 9 in. wide, $w = 9p = 9\gamma h$, where all dimensions are in inches.

$$w_1 = 9\gamma(d - 9)$$

$$w_2 = 9\gamma d$$

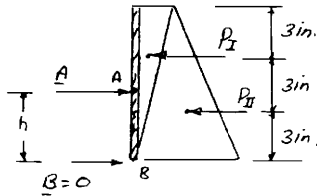
For $d = 18$ in.,

$$w_1 = 9\gamma(18 - 9) = 81\gamma$$

$$w_2 = 9\gamma(18) = 162\gamma$$

$$P_I = \frac{1}{2}(9)(9\gamma)(18 - 9) = \frac{1}{2}(729\gamma)$$

$$P_{II} = \frac{1}{2}(9)(9\gamma)(18) = 729\gamma$$



Valve opens when $B = 0$.

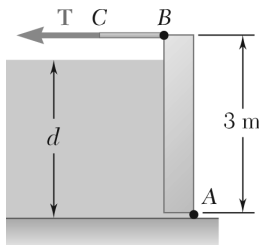
$$+\circlearrowleft \Sigma M_A = 0: P_I(6 - h) - P_{II}(h - 3) = 0$$

$$\frac{1}{2}729\gamma(6 - h) - 729(h - 3) = 0$$

$$3 - \frac{1}{2}h - h + 3 = 0$$

$$6 - 1.5h = 0$$

$$h = 4.00 \text{ in.} \blacktriangleleft$$



PROBLEM 5.84

The $3 \times 4\text{-m}$ side AB of a tank is hinged at its bottom A and is held in place by a thin rod BC . The maximum tensile force the rod can withstand without breaking is 200 kN , and the design specifications require the force in the rod not to exceed 20 percent of this value. If the tank is slowly filled with water, determine the maximum allowable depth of water d in the tank.

SOLUTION

Consider the free-body diagram of the side.

We have
$$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$$

Now
$$+\circlearrowleft \Sigma M_A = 0: \quad hT - \frac{d}{3}P = 0$$

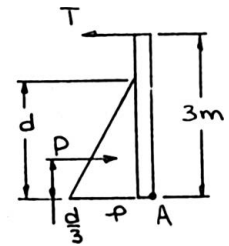
where
$$h = 3\text{ m}$$

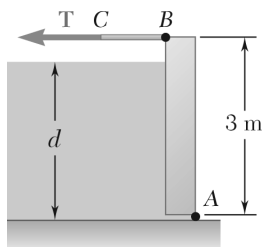
Then for d_{max} ,

$$(3\text{ m})(0.2 \times 200 \times 10^3\text{ N}) - \frac{d_{max}}{3} \left[\frac{1}{2}(4\text{ m} \times d_{max}) \times (10^3\text{ kg/m}^3 \times 9.81\text{ m/s}^2 \times d_{max}) \right] = 0$$

or
$$120\text{ N}\cdot\text{m} - 6.54d_{max}^3\text{ N/m}^2 = 0$$

or
$$d_{max} = 2.64\text{ m} \quad \blacktriangleleft$$





PROBLEM 5.85

The 3×4 -m side of an open tank is hinged at its bottom A and is held in place by a thin rod BC . The tank is to be filled with glycerine, whose density is 1263 kg/m^3 . Determine the force T in the rod and the reactions at the hinge after the tank is filled to a depth of 2.9 m .

SOLUTION

Consider the free-body diagram of the side.

We have

$$\begin{aligned}
 P &= \frac{1}{2}Ap = \frac{1}{2}A(\rho gd) \\
 &= \frac{1}{2}[(2.9 \text{ m})(4 \text{ m})] [(1263 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.9 \text{ m})] \\
 &= 208.40 \text{ kN}
 \end{aligned}$$

Then

$$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

$$+\curvearrowright \Sigma M_A = 0: \quad (3 \text{ m})T - \left(\frac{2.9}{3} \text{ m}\right)(208.4 \text{ kN}) = 0$$

or

$$T = 67.151 \text{ kN}$$

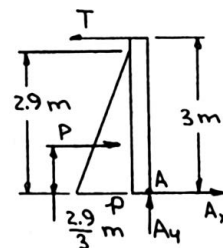
$$T = 67.2 \text{ kN} \leftarrow \blacktriangleleft$$

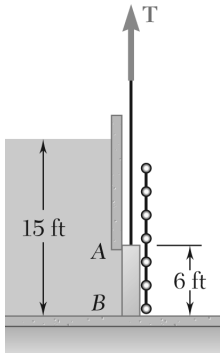
$$+\rightarrow \Sigma F_x = 0: \quad A_x + 208.40 \text{ kN} - 67.151 \text{ kN} = 0$$

or

$$A_x = -141.249 \text{ kN}$$

$$A = 141.2 \text{ kN} \leftarrow \blacktriangleleft$$





PROBLEM 5.86

The friction force between a 6×6 -ft square sluice gate AB and its guides is equal to 10 percent of the resultant of the pressure forces exerted by the water on the face of the gate. Determine the initial force needed to lift the gate if it weighs 1000 lb.

SOLUTION

Consider the free-body diagram of the gate.

Now

$$P_I = \frac{1}{2} A p_I = \frac{1}{2} [(6 \times 6) \text{ ft}^2] [(62.4 \text{ lb/ft}^3)(9 \text{ ft})]$$

$$= 10,108.8 \text{ lb}$$

$$P_{II} = \frac{1}{2} A p_{II} = \frac{1}{2} [(6 \times 6) \text{ ft}^2] [(62.4 \text{ lb/ft}^3)(15 \text{ ft})]$$

$$= 16,848 \text{ lb}$$

Then

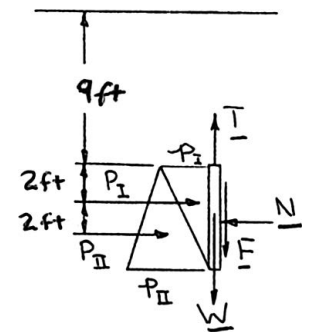
$$F = 0.1P = 0.1(P_I + P_{II})$$

$$= 0.1(10,108.8 + 16,848) \text{ lb}$$

$$= 2695.7 \text{ lb}$$

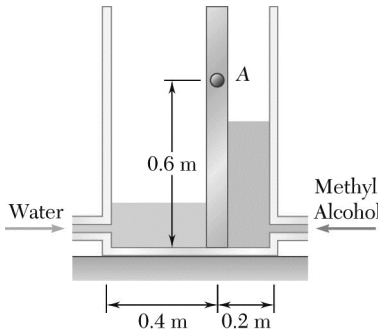
Finally

$$+\uparrow \Sigma F_y = 0: \quad T - 2695.7 \text{ lb} - 1000 \text{ lb} = 0$$



$$\text{or } T = 3.70 \text{ kips} \uparrow \blacktriangleleft$$

PROBLEM 5.87



A tank is divided into two sections by a 1×1 -m square gate that is hinged at A. A couple of magnitude $490 \text{ N} \cdot \text{m}$ is required for the gate to rotate. If one side of the tank is filled with water at the rate of $0.1 \text{ m}^3/\text{min}$ and the other side is filled simultaneously with methyl alcohol (density $\rho_{ma} = 789 \text{ kg/m}^3$) at the rate of $0.2 \text{ m}^3/\text{min}$, determine at what time and in which direction the gate will rotate.

SOLUTION

Consider the free-body diagram of the gate.

First note $V = A_{\text{base}}d$ and $V = rt$.

Then
$$d_W = \frac{0.1 \text{ m}^3/\text{min} \times t(\text{min})}{(0.4 \text{ m})(1 \text{ m})} = 0.25t(\text{m})$$

$$d_{MA} = \frac{0.2 \text{ m}^3/\text{min} \times t(\text{min})}{(0.2 \text{ m})(1 \text{ m})} = t(\text{m})$$

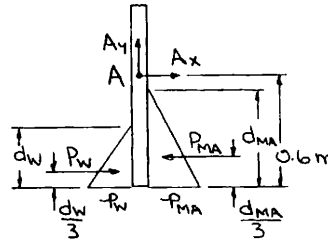
Now
$$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho gh)$$
 so that

$$P_W = \frac{1}{2}[(0.25t) \text{ m} \times (1 \text{ m})][(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25t) \text{ m}]$$

$$= 306.56t^2 \text{ N}$$

$$P_{MA} = \frac{1}{2}[(t) \text{ m} \times (1 \text{ m})][(789 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(t) \text{ m}]$$

$$= 3870t^2 \text{ N}$$



Now assume that the gate will rotate clockwise and when $d_{MA} \leq 0.6 \text{ m}$. When rotation of the gate is impending, we require

$$\Sigma M_A: M_R = \left(0.6 \text{ m} - \frac{1}{3}d_{MA}\right)P_{MA} - \left(0.6 \text{ m} - \frac{1}{3}d_W\right)P_W$$

Substituting

$$490 \text{ N} \cdot \text{m} = \left(0.6 - \frac{1}{3}t\right) \text{ m} \times (3870t^2) \text{ N} - \left(0.6 - \frac{1}{3} \times 0.25t\right) \text{ m} \times (306.56t^2) \text{ N}$$

PROBLEM 5.87 (Continued)

Simplifying

$$1264.45t^3 - 2138.1t^2 + 490 = 0$$

Solving (positive roots only)

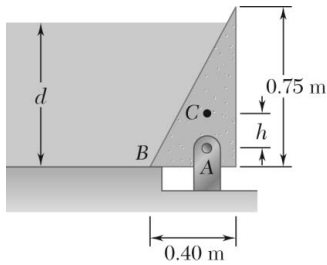
$$t = 0.59451 \text{ min} \text{ and } t = 1.52411 \text{ min}$$

Now check assumption using the smaller root. We have

$$d_{MA} = (t) \text{ m} = 0.59451 \text{ m} < 0.6 \text{ m}$$

$$\therefore t = 0.59451 \text{ min} = 35.7 \text{ s} \blacktriangleleft$$

and the gate rotates clockwise. \blacktriangleleft



PROBLEM 5.88

A prismaticly shaped gate placed at the end of a freshwater channel is supported by a pin and bracket at A and rests on a frictionless support at B. The pin is located at a distance $h = 0.10$ m below the center of gravity C of the gate. Determine the depth of water d for which the gate will open.

SOLUTION

First note that when the gate is about to open (clockwise rotation is impending), $B_y \rightarrow 0$ and the line of action of the resultant \mathbf{P} of the pressure forces passes through the pin at A. In addition, if it is assumed that the gate is homogeneous, then its center of gravity C coincides with the centroid of the triangular area. Then

$$a = \frac{d}{3} - (0.25 - h)$$

and

$$b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$$

Now

$$\frac{a}{b} = \frac{8}{15}$$

so that

$$\frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

Simplifying yields

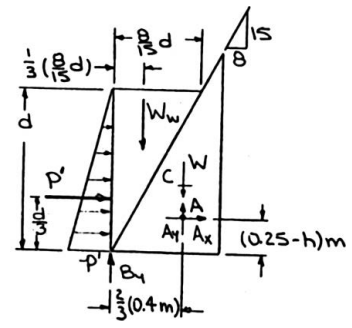
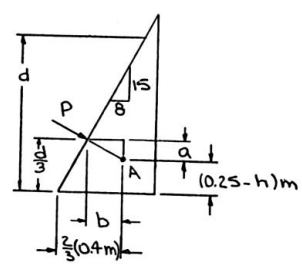
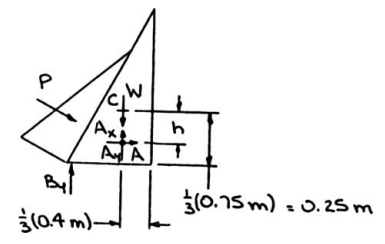
$$\frac{289}{45}d + 15h = \frac{70.6}{12} \tag{1}$$

Alternative solution:

Consider a free body consisting of a 1-m thick section of the gate and the triangular section BDE of water above the gate.

Now

$$\begin{aligned} P' &= \frac{1}{2} A p' = \frac{1}{2} (d \times 1 \text{ m}) (\rho g d) \\ &= \frac{1}{2} \rho g d^2 \text{ (N)} \\ W' &= \rho g V = \rho g \left(\frac{1}{2} \times \frac{8}{15} d \times d \times 1 \text{ m} \right) \\ &= \frac{4}{15} \rho g d^2 \text{ (N)} \end{aligned}$$



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PROBLEM 5.88 (Continued)

Then with $B_y = 0$ (as explained above), we have

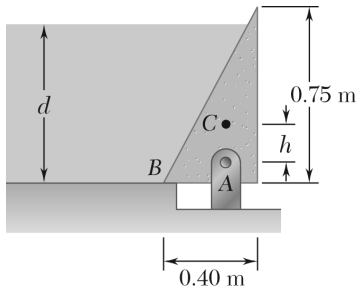
$$+\curvearrowright \Sigma M_A = 0: \left[\frac{2}{3}(0.4) - \frac{1}{3} \left(\frac{8}{15} d \right) \right] \left(\frac{4}{15} \rho g d^2 \right) - \left[\frac{d}{3} - (0.25 - h) \right] \left(\frac{1}{2} \rho g d^2 \right) = 0$$

Simplifying yields
$$\frac{289}{45} d + 15h = \frac{70.6}{12}$$

as above.

Find d :
$$h = 0.10 \text{ m}$$

Substituting into Eq. (1),
$$\frac{289}{45} d + 15(0.10) = \frac{70.6}{12} \quad \text{or } d = 0.683 \text{ m} \quad \blacktriangleleft$$



PROBLEM 5.89

A prismaticly shaped gate placed at the end of a freshwater channel is supported by a pin and bracket at A and rests on a frictionless support at B . Determine the distance h if the gate is to open when $d = 0.75$ m.

SOLUTION

First note that when the gate is about to open (clockwise rotation is impending), $B_y \rightarrow 0$ and the line of action of the resultant \mathbf{P} of the pressure forces passes through the pin at A . In addition, if it is assumed that the gate is homogeneous, then its center of gravity C coincides with the centroid of the triangular area. Then

$$a = \frac{d}{3} - (0.25 - h)$$

and

$$b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$$

Now

$$\frac{a}{b} = \frac{8}{15}$$

so that

$$\frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

Simplifying yields

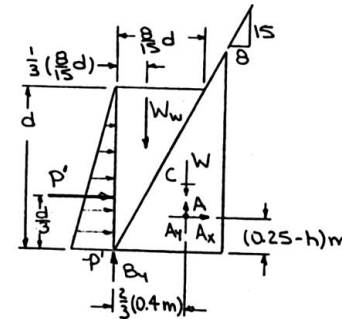
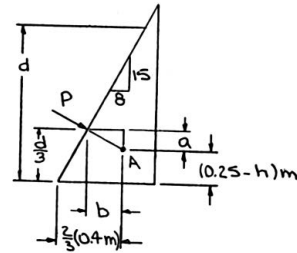
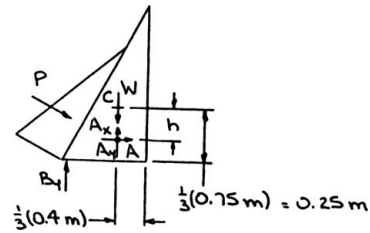
$$\frac{289}{45}d + 15h = \frac{70.6}{12} \quad (1)$$

Alternative solution:

Consider a free body consisting of a 1-m thick section of the gate and the triangular section BDE of water above the gate.

Now

$$\begin{aligned} P' &= \frac{1}{2} A p' = \frac{1}{2} (d \times 1 \text{ m}) (\rho g d) \\ &= \frac{1}{2} \rho g d^2 \text{ (N)} \\ W' &= \rho g V = \rho g \left(\frac{1}{2} \times \frac{8}{15} d \times d \times 1 \text{ m} \right) \\ &= \frac{4}{15} \rho g d^2 \text{ (N)} \end{aligned}$$



PROBLEM 5.89 (Continued)

Then with $B_y = 0$ (as explained above), we have

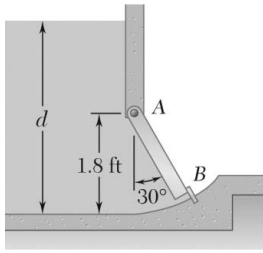
$$+\circlearrowleft \Sigma M_A = 0: \left[\frac{2}{3}(0.4) - \frac{1}{3}\left(\frac{8}{15}d\right) \right] \left(\frac{4}{15} \rho g d^2 \right) - \left[\frac{d}{3} - (0.25 - h) \right] \left(\frac{1}{2} \rho g d^2 \right) = 0$$

Simplifying yields $\frac{289}{45}d + 15h = \frac{70.6}{12}$

as above.

Find h : $d = 0.75 \text{ m}$

Substituting into Eq. (1), $\frac{289}{45}(0.75) + 15h = \frac{70.6}{12}$ or $h = 0.0711 \text{ m} \blacktriangleleft$



PROBLEM 5.90

The square gate AB is held in the position shown by hinges along its top edge A and by a shear pin at B . For a depth of water $d = 3.5$ ft, determine the force exerted on the gate by the shear pin.

SOLUTION

First consider the force of the water on the gate. We have

$$P = \frac{1}{2} Ap$$

$$= \frac{1}{2} A(\gamma h)$$

Then

$$P_I = \frac{1}{2} (1.8 \text{ ft})^2 (62.4 \text{ lb/ft}^3) (1.7 \text{ ft})$$

$$= 171.850 \text{ lb}$$

$$P_{II} = \frac{1}{2} (1.8 \text{ ft})^2 (62.4 \text{ lb/ft}^3) \times (1.7 + 1.8 \cos 30^\circ) \text{ ft}$$

$$= 329.43 \text{ lb}$$

Now

$$\Sigma M_A = 0: \left(\frac{1}{3} L_{AB} \right) P_I + \left(\frac{2}{3} L_{AB} \right) P_{II} - L_{AB} F_B = 0$$

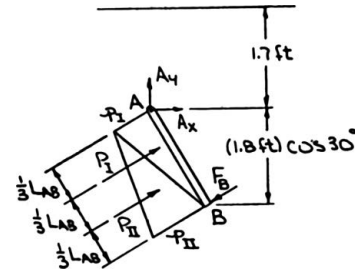
or

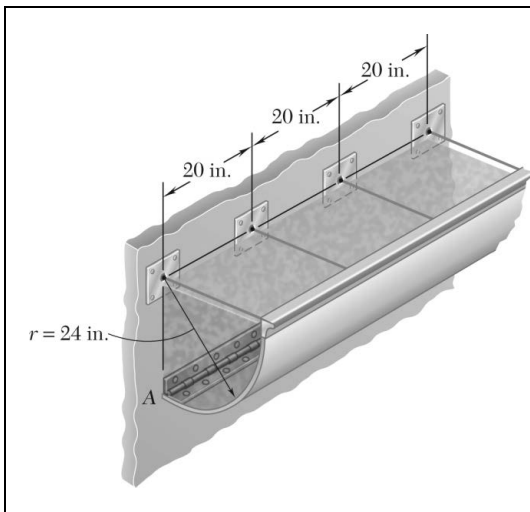
$$\frac{1}{3} (171.850 \text{ lb}) + \frac{2}{3} (329.43 \text{ lb}) - F_B = 0$$

or

$$F_B = 276.90 \text{ lb}$$

$$\mathbf{F}_B = 277 \text{ lb} \angle 30.0^\circ \blacktriangleleft$$





PROBLEM 5.91

A long trough is supported by a continuous hinge along its lower edge and by a series of horizontal cables attached to its upper edge. Determine the tension in each of the cables, at a time when the trough is completely full of water.

SOLUTION

Consider free body consisting of 20-in. length of the trough and water.

$l = 20\text{-in.}$ length of free body

$$W = \gamma v = \gamma \left[\frac{\pi}{4} r^2 l \right]$$

$$P_A = \gamma r$$

$$P = \frac{1}{2} P_A r l = \frac{1}{2} (\gamma r) r l = \frac{1}{2} \gamma r^2 l$$

$$+\circlearrowleft \Sigma M_A = 0: Tr - Wr - P \left(\frac{1}{3} r \right) = 0$$

$$Tr - \left(\gamma \frac{\pi}{4} r^2 l \right) \left(\frac{4r}{3\pi} \right) - \left(\frac{1}{2} \gamma r^2 l \right) \left(\frac{1}{3} r \right) = 0$$

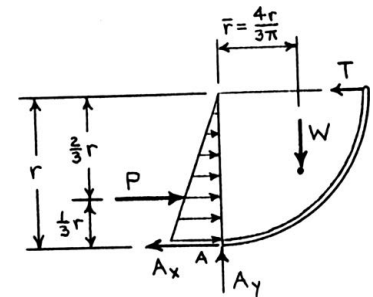
$$T = \frac{1}{3} \gamma r^2 l + \frac{1}{6} \gamma r^2 l = \frac{1}{2} \gamma r^2 l$$

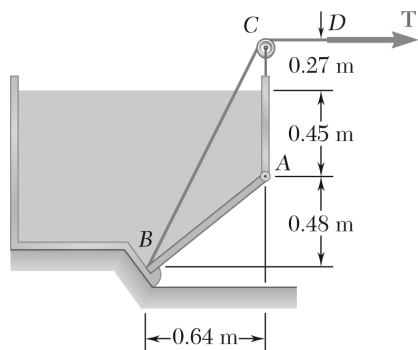
Data: $\gamma = 62.4 \text{ lb/ft}^3$ $r = \frac{24}{12} \text{ ft} = 2 \text{ ft}$ $l = \frac{20}{12} \text{ ft}$

Then $T = \frac{1}{2} (62.4 \text{ lb/ft}^3) (2 \text{ ft})^2 \left(\frac{20}{12} \text{ ft} \right)$

$$= 208.00 \text{ lb}$$

$$T = 208 \text{ lb} \quad \blacktriangleleft$$





PROBLEM 5.92

A $0.5 \times 0.8\text{-m}$ gate AB is located at the bottom of a tank filled with water. The gate is hinged along its top edge A and rests on a frictionless stop at B . Determine the reactions at A and B when cable BCD is slack.

SOLUTION

First consider the force of the water on the gate.

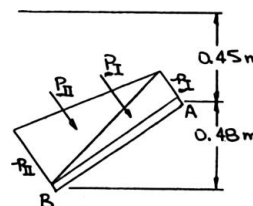
We have
$$P = \frac{1}{2} Ap = \frac{1}{2} A(\rho gh)$$

so that
$$P_I = \frac{1}{2} [(0.5 \text{ m})(0.8 \text{ m})] \times [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.45 \text{ m})]$$

$$= 882.9 \text{ N}$$

$$P_{II} = \frac{1}{2} [(0.5 \text{ m})(0.8 \text{ m})] \times [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.93 \text{ m})]$$

$$= 1824.66 \text{ N}$$



Reactions at A and B when $T = 0$:

We have

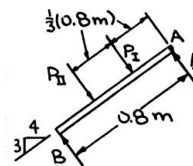
$$+\curvearrowright \Sigma M_A = 0: \frac{1}{3}(0.8 \text{ m})(882.9 \text{ N}) + \frac{2}{3}(0.8 \text{ m})(1824.66 \text{ N}) - (0.8 \text{ m})B = 0$$

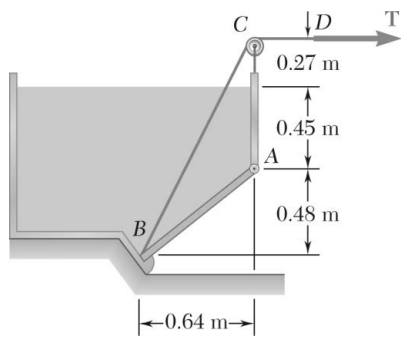
or
$$B = 1510.74 \text{ N}$$

or
$$B = 1511 \text{ N} \searrow 53.1^\circ \blacktriangleleft$$

$$+\searrow \Sigma F = 0: A + 1510.74 \text{ N} - 882.9 \text{ N} - 1824.66 \text{ N} = 0$$

or
$$A = 1197 \text{ N} \searrow 53.1^\circ \blacktriangleleft$$





PROBLEM 5.93

A 0.5×0.8 -m gate AB is located at the bottom of a tank filled with water. The gate is hinged along its top edge A and rests on a frictionless stop at B . Determine the minimum tension required in cable BCD to open the gate.

SOLUTION

First consider the force of the water on the gate.

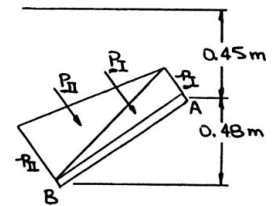
We have
$$P = \frac{1}{2} Ap = \frac{1}{2} A(\rho gh)$$

so that
$$P_I = \frac{1}{2} [(0.5 \text{ m})(0.8 \text{ m})] \times [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.45 \text{ m})]$$

$$= 882.9 \text{ N}$$

$$P_{II} = \frac{1}{2} [(0.5 \text{ m})(0.8 \text{ m})] \times [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.93 \text{ m})]$$

$$= 1824.66 \text{ N}$$



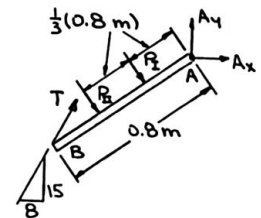
T to open gate:

First note that when the gate begins to open, the reaction at $B \rightarrow 0$.

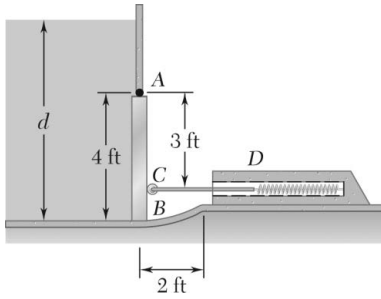
Then
$$+\circlearrowleft \Sigma M_A = 0: \frac{1}{3}(0.8 \text{ m})(882.9 \text{ N}) + \frac{2}{3}(0.8 \text{ m})(1824.66 \text{ N})$$

$$-(0.45 + 0.27) \text{ m} \times \left(\frac{8}{17} T \right) = 0$$

or
$$235.44 + 973.152 - 0.33882T = 0$$



or $T = 3570 \text{ N} \blacktriangleleft$



PROBLEM 5.94

A 4×2-ft gate is hinged at A and is held in position by rod CD. End D rests against a spring whose constant is 828 lb/ft. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod CD on the gate remains horizontal, determine the minimum depth of water d for which the bottom B of the gate will move to the end of the cylindrical portion of the floor.

SOLUTION

First determine the forces exerted on the gate by the spring and the water when B is at the end of the cylindrical portion of the floor.

We have $\sin \theta = \frac{2}{4} \quad \theta = 30^\circ$

Then $x_{SP} = (3 \text{ ft}) \tan 30^\circ$

and $F_{SP} = kx_{SP}$
 $= 828 \text{ lb/ft} \times 3 \text{ ft} \times \tan 30^\circ$
 $= 1434.14 \text{ lb}$

Assume $d \geq 4 \text{ ft}$

We have $P = \frac{1}{2} A p = \frac{1}{2} A (\gamma h)$

Then $P_I = \frac{1}{2} [(4 \text{ ft})(2 \text{ ft})] \times [(62.4 \text{ lb/ft}^3)(d - 4) \text{ ft}]$
 $= 249.6(d - 4) \text{ lb}$

$$P_{II} = \frac{1}{2} [(4 \text{ ft})(2 \text{ ft})] \times [(62.4 \text{ lb/ft}^3)(d - 4 + 4 \cos 30^\circ)]$$

$$= 249.6(d - 0.53590^\circ) \text{ lb}$$

For d_{\min} so that the gate opens, $W = 0$

Using the above free-body diagrams of the gate, we have

$$+\curvearrowright \Sigma M_A = 0: \left(\frac{4}{3} \text{ ft}\right) [249.6(d - 4) \text{ lb}] + \left(\frac{8}{3} \text{ ft}\right) [249.6(d - 0.53590) \text{ lb}]$$

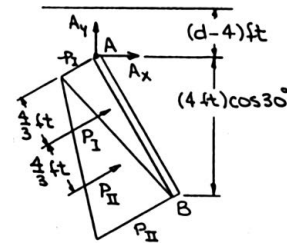
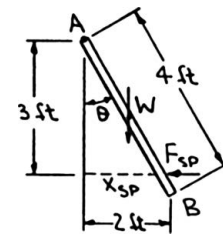
$$- (3 \text{ ft})(1434.14 \text{ lb}) = 0$$

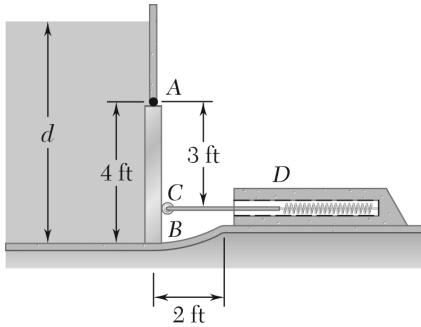
or $(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 = 0$

or $d = 6.00 \text{ ft}$

$d \geq 4 \text{ ft} \Rightarrow$ assumption correct

$d = 6.00 \text{ ft} \blacktriangleleft$





PROBLEM 5.95

Solve Problem 5.94 if the gate weighs 1000 lb.

PROBLEM 5.94 A 4×2-ft gate is hinged at A and is held in position by rod CD. End D rests against a spring whose constant is 828 lb/ft. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod CD on the gate remains horizontal, determine the minimum depth of water d for which the bottom B of the gate will move to the end of the cylindrical portion of the floor.

SOLUTION

First determine the forces exerted on the gate by the spring and the water when B is at the end of the cylindrical portion of the floor.

We have $\sin \theta = \frac{2}{4} \quad \theta = 30^\circ$

Then $x_{SP} = (3 \text{ ft}) \tan 30^\circ$

and $F_{SP} = kx_{SP} = 828 \text{ lb/ft} \times 3 \text{ ft} \times \tan 30^\circ$
 $= 1434.14 \text{ lb}$

Assume $d \geq 4 \text{ ft}$

We have $P = \frac{1}{2} A p = \frac{1}{2} A (\gamma h)$

Then $P_I = \frac{1}{2} [(4 \text{ ft})(2 \text{ ft})] \times [(62.4 \text{ lb/ft}^3)(d - 4) \text{ ft}]$
 $= 249.6(d - 4) \text{ lb}$

$$P_{II} = \frac{1}{2} [(4 \text{ ft})(2 \text{ ft})] \times [(62.4 \text{ lb/ft}^3)(d - 4 + 4 \cos 30^\circ)]$$

$$= 249.6(d - 0.53590^\circ) \text{ lb}$$

For d_{\min} so that the gate opens, $W = 1000 \text{ lb}$

Using the above free-body diagrams of the gate, we have

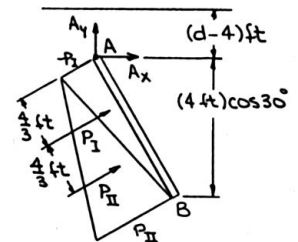
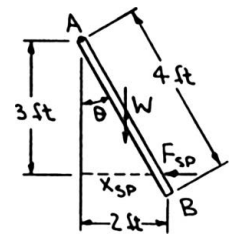
$$\begin{aligned} \sum M_A = 0: & \left(\frac{4}{3} \text{ ft}\right) [249.6(d - 4) \text{ lb}] + \left(\frac{8}{3} \text{ ft}\right) [249.6(d - 0.53590) \text{ lb}] \\ & - (3 \text{ ft})(1434.14 \text{ lb}) - (1 \text{ ft})(1000 \text{ lb}) = 0 \end{aligned}$$

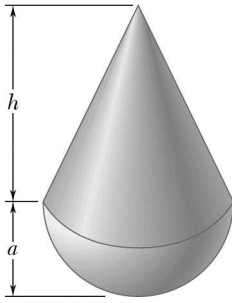
or $(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 - 1000 = 0$

or $d = 7.00 \text{ ft}$

$d \geq 4 \text{ ft} \Rightarrow$ assumption correct

$d = 7.00 \text{ ft} \blacktriangleleft$

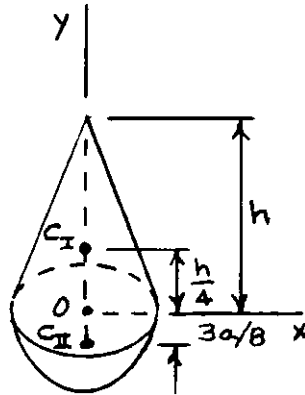




PROBLEM 5.96

A hemisphere and a cone are attached as shown. Determine the location of the centroid of the composite body when (a) $h = 1.5a$, (b) $h = 2a$.

SOLUTION



	V	\bar{y}	$\bar{y}V$
Cone I	$\frac{1}{3}\pi a^2 h$	$\frac{h}{4}$	$\frac{1}{12}\pi a^2 h^2$
Hemisphere II	$\frac{2}{3}\pi a^3$	$-\frac{3a}{8}$	$-\frac{1}{4}\pi a^4$

$$V = \frac{1}{3}\pi a^2(h + 2a)$$

$$\Sigma \bar{y}V = \frac{1}{12}\pi a^2(h^2 - 3a^2)$$

(a) For $h = 1.5a$,

$$V = \frac{1}{3}\pi a^2(1.5a + 2a) = \frac{7}{6}\pi a^3$$

$$\Sigma \bar{y}V = \frac{1}{12}\pi a^2[(1.5a)^2 - 3a^2] = -\frac{1}{16}\pi a^4$$

$$\bar{Y}V = \Sigma \bar{y}V: \bar{Y}\left(\frac{7}{6}\pi a^3\right) = -\frac{1}{16}\pi a^4 \quad \bar{Y} = -\frac{3}{56}a$$

Centroid is $0.0536a$ below base of cone. ◀

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PROBLEM 5.96 (Continued)

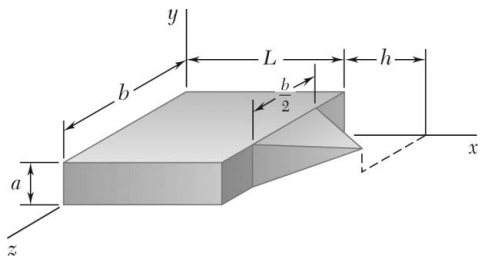
(b) For $h = 2a$,

$$V = \frac{1}{3}\pi a^2(2a + 2a) = \frac{4}{3}\pi a^3$$

$$\Sigma \bar{y}V = \frac{1}{12}\pi a^2[(2a)^2 - 3a^2] = \frac{1}{12}\pi a^4$$

$$\bar{Y}V = \Sigma \bar{y}V: \bar{Y}\left(\frac{4}{3}\pi a^3\right) = \frac{1}{12}\pi a^4 \quad \bar{Y} = \frac{1}{16}a$$

Centroid is $0.0625a$ above base of cone. ◀



PROBLEM 5.97

Consider the composite body shown. Determine (a) the value of \bar{x} when $h = L/2$, (b) the ratio h/L for which $\bar{x} = L$.

SOLUTION

	V	\bar{x}	$\bar{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L + \frac{1}{4}h$	$\frac{1}{6}abh\left(L + \frac{1}{4}h\right)$

Then

$$\Sigma V = ab\left(L + \frac{1}{6}h\right)$$

$$\Sigma \bar{x}V = \frac{1}{6}ab\left[3L^2 + h\left(L + \frac{1}{4}h\right)\right]$$

Now

$$\bar{X}\Sigma V = \Sigma \bar{x}V$$

so that

$$\bar{X}\left[ab\left(L + \frac{1}{6}h\right)\right] = \frac{1}{6}ab\left(3L^2 + hL + \frac{1}{4}h^2\right)$$

or

$$\bar{X}\left(1 + \frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3 + \frac{h}{L} + \frac{1}{4}\frac{h^2}{L^2}\right) \quad (1)$$

(a) $\bar{X} = ?$ when $h = \frac{1}{2}L$.

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1),

$$\bar{X}\left[1 + \frac{1}{6}\left(\frac{1}{2}\right)\right] = \frac{1}{6}L\left[3 + \left(\frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

or

$$\bar{X} = \frac{57}{104}L \quad \bar{X} = 0.548L \quad \blacktriangleleft$$

PROBLEM 5.97 (Continued)

(b) $\frac{h}{L} = ?$ when $\bar{X} = L$.

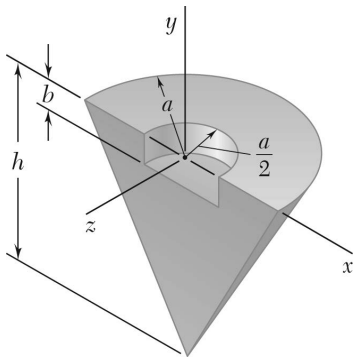
Substituting into Eq. (1),
$$L\left(1 + \frac{1}{6} \frac{h}{L}\right) = \frac{1}{6} L\left(3 + \frac{h}{L} + \frac{1}{4} \frac{h^2}{L^2}\right)$$

or
$$1 + \frac{1}{6} \frac{h}{L} = \frac{1}{2} + \frac{1}{6} \frac{h}{L} + \frac{1}{24} \frac{h^2}{L^2}$$

or
$$\frac{h^2}{L^2} = 12 \qquad \frac{h}{L} = 2\sqrt{3} \blacktriangleleft$$

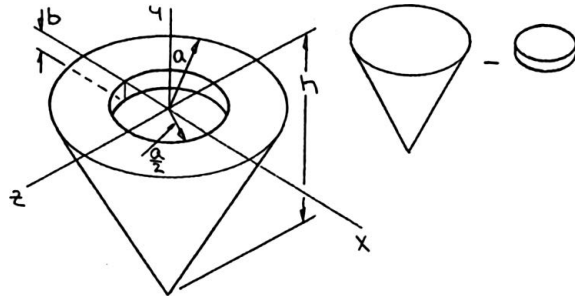
PROBLEM 5.98

Determine the y coordinate of the centroid of the body shown.



SOLUTION

First note that the values of \bar{Y} will be the same for the given body and the body shown below. Then



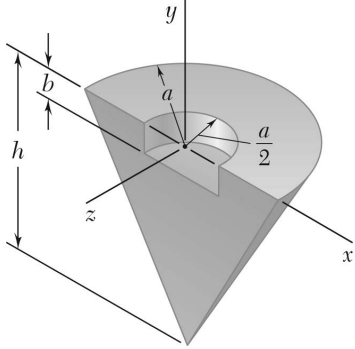
	V	\bar{y}	$\bar{y}V$
Cone	$\frac{1}{3}\pi a^2 h$	$-\frac{1}{4}h$	$-\frac{1}{12}\pi a^2 h^2$
Cylinder	$-\pi\left(\frac{a}{2}\right)^2 b = -\frac{1}{4}\pi a^2 b$	$-\frac{1}{2}b$	$\frac{1}{8}\pi a^2 b^2$
Σ	$\frac{\pi}{12}a^2(4h - 3b)$		$-\frac{\pi}{24}a^2(2h^2 - 3b^2)$

We have

$$\bar{Y}\Sigma V = \Sigma \bar{y}V$$

Then

$$\bar{Y}\left[\frac{\pi}{12}a^2(4h - 3b)\right] = -\frac{\pi}{24}a^2(2h^2 - 3b^2) \quad \text{or} \quad \bar{Y} = -\frac{2h^2 - 3b^2}{2(4h - 3b)} \blacktriangleleft$$

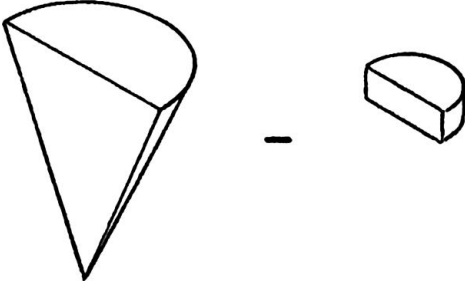


PROBLEM 5.99

Determine the z coordinate of the centroid of the body shown. (*Hint:* Use the result of Sample Problem 5.13.)

SOLUTION

First note that the body can be formed by removing a half cylinder from a half cone, as shown.



	V	\bar{z}	$\bar{z}V$
Half cone	$\frac{1}{6}\pi a^2 h$	$-\frac{a}{\pi}$	$-\frac{1}{6}a^3 h$
Half cylinder	$-\frac{\pi}{2}\left(\frac{a}{2}\right)^2 b = -\frac{\pi}{8}a^2 b$	$-\frac{4}{3\pi}\left(\frac{a}{2}\right) = -\frac{2a}{3\pi}$	$\frac{1}{12}a^3 b$
Σ	$\frac{\pi}{24}a^2(4h - 3b)$		$-\frac{1}{12}a^3(2h - b)$

From Sample Problem 5.13:

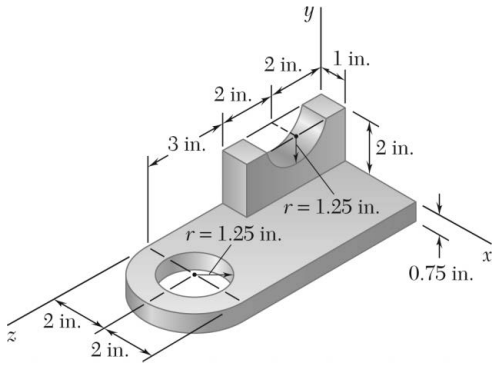
We have
$$\bar{Z}\Sigma V = \Sigma \bar{z}V$$

Then
$$\bar{Z}\left[\frac{\pi}{24}a^2(4h - 3b)\right] = -\frac{1}{12}a^3(2h - b) \quad \text{or} \quad \bar{Z} = -\frac{a}{\pi}\left(\frac{4h - 2b}{4h - 3b}\right) \blacktriangleleft$$

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PROBLEM 5.100

For the machine element shown, locate the y coordinate of the center of gravity.



SOLUTION

For half-cylindrical hole,

$$r = 1.25 \text{ in.}$$

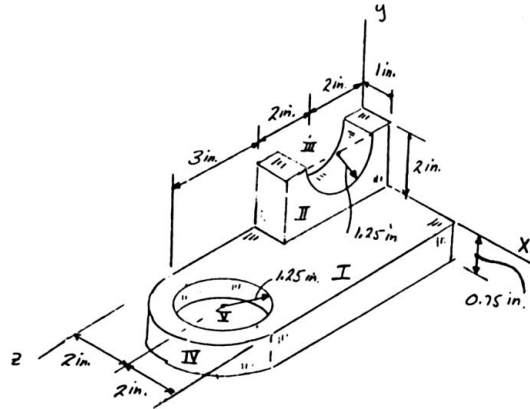
$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi}$$

$$= 1.470 \text{ in.}$$

For half-cylindrical plate,

$$r = 2 \text{ in.}$$

$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85 \text{ in.}$$



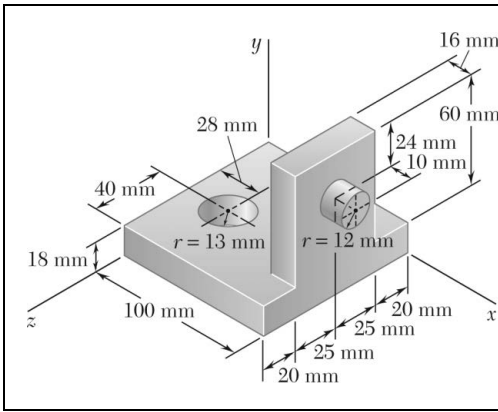
		V, in^3	$\bar{y}, \text{in.}$	$\bar{z}, \text{in.}$	$\bar{y}V, \text{in}^4$	$\bar{z}V, \text{in}^4$
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	-0.375	3.5	-7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	-(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	-3.607	-4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	-0.375	-7.85	-1.767	36.99
V	-(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	-0.375	7	1.381	-25.77
	Σ	27.58			-3.868	95.81

$$\bar{Y}\Sigma V = \Sigma \bar{y}V$$

$$\bar{Y}(27.58 \text{ in}^3) = -3.868 \text{ in}^4$$

$$\bar{Y} = -0.1403 \text{ in.} \blacktriangleleft$$

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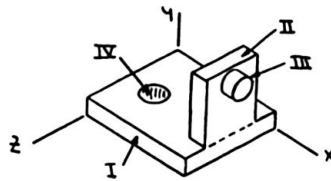


PROBLEM 5.101

For the machine element shown, locate the y coordinate of the center of gravity.

SOLUTION

First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.



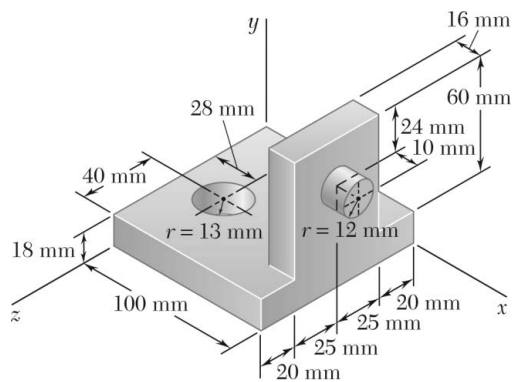
	V, mm^3	\bar{x}, mm	\bar{y}, mm	$\bar{x}V, \text{mm}^4$	$\bar{y}V, \text{mm}^4$
I	$(100)(18)(90) = 162,000$	50	9	8,100,000	1,458,000
II	$(16)(60)(50) = 48,000$	92	48	4,416,000	2,304,000
III	$\pi(12)^2(10) = 4523.9$	105	54	475,010	244,290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	-267,590	-86,010
Σ	204,967.2			12,723,420	3,920,280

We have

$$\bar{Y}\Sigma V = \Sigma \bar{y}V$$

$$\bar{Y}(204,967.2 \text{ mm}^3) = 3,920,280 \text{ mm}^4$$

$$\text{or } \bar{Y} = 19.13 \text{ mm} \blacktriangleleft$$

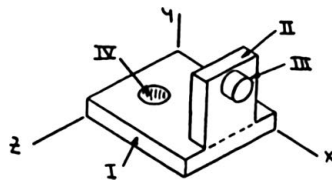


PROBLEM 5.102

For the machine element shown, locate the x coordinate of the center of gravity.

SOLUTION

First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.



	V, mm^3	\bar{x}, mm	\bar{y}, mm	$\bar{x}V, \text{mm}^4$	$\bar{y}V, \text{mm}^4$
I	$(100)(18)(90) = 162,000$	50	9	8,100,000	1,458,000
II	$(16)(60)(50) = 48,000$	92	48	4,416,000	2,304,000
III	$\pi(12)^2(10) = 4523.9$	105	54	475,010	244,290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	-267,590	-86,010
Σ	204,967.2			12,723,420	3,920,280

We have

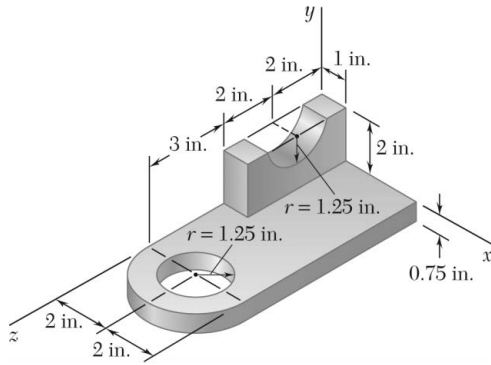
$$\bar{X}\Sigma V = \Sigma \bar{x}V$$

$$\bar{X}(204,967.2 \text{ mm}^3) = 12,723,420 \text{ mm}^4$$

$$\bar{X} = 62.1 \text{ mm} \blacktriangleleft$$

PROBLEM 5.103

For the machine element shown, locate the z coordinate of the center of gravity.



SOLUTION

For half-cylindrical hole,

$$r = 1.25 \text{ in.}$$

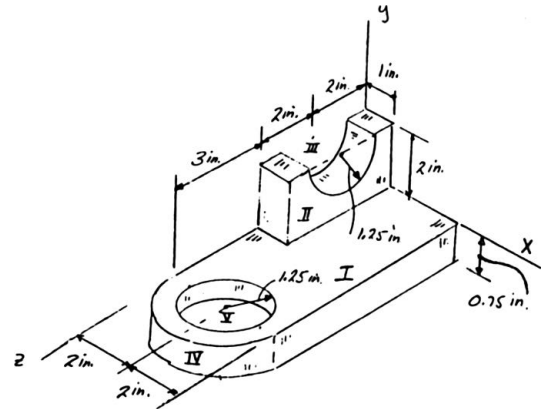
$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi}$$

$$= 1.470 \text{ in.}$$

For half-cylindrical plate,

$$r = 2 \text{ in.}$$

$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85 \text{ in.}$$



		V, in^3	$\bar{y}, \text{in.}$	$\bar{z}, \text{in.}$	$\bar{y}V, \text{in}^4$	$\bar{z}V, \text{in}^4$
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	-0.375	3.5	-7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	-(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	-3.607	-4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	-0.375	-7.85	-1.767	36.99
V	-(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	-0.375	7	1.381	-25.77
	Σ	27.58			-3.868	95.81

Now

$$\bar{Z}\Sigma V = \bar{z}V$$

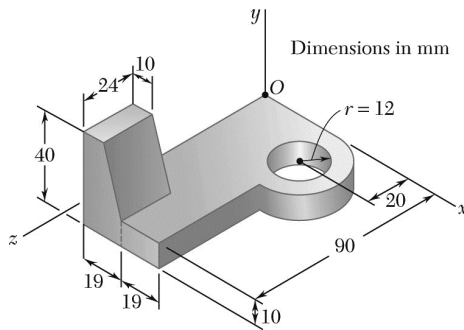
$$\bar{Z}(27.58 \text{ in}^3) = 95.81 \text{ in}^4$$

$$\bar{Z} = 3.47 \text{ in.} \blacktriangleleft$$

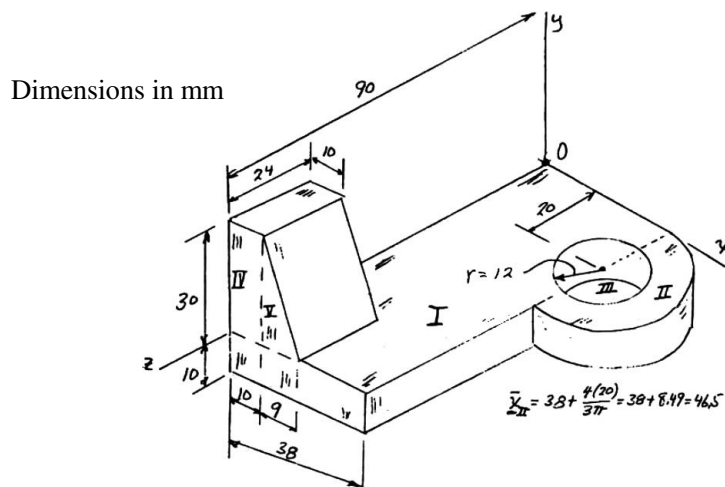
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PROBLEM 5.104

For the machine element shown, locate the x coordinate of the center of gravity.



SOLUTION



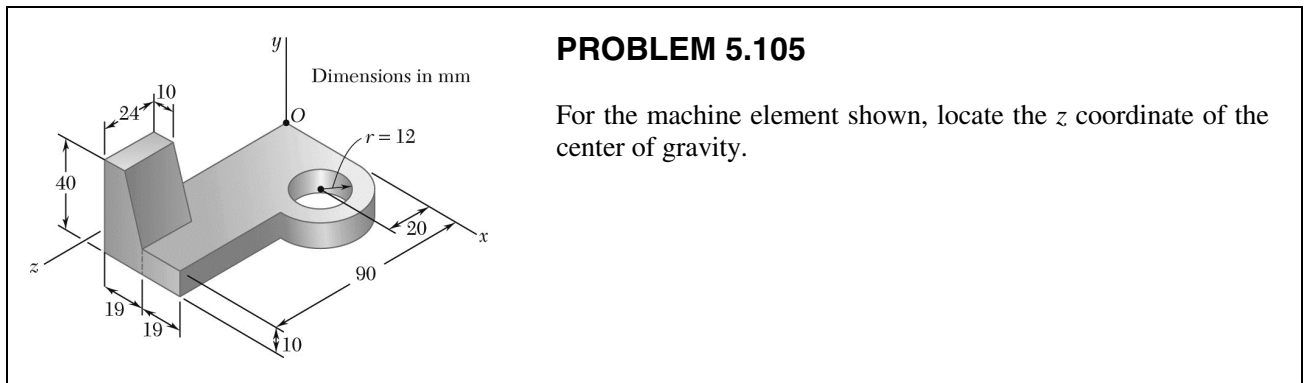
		V, mm^3	\bar{x}, mm	\bar{z}, mm	$\bar{x}V, \text{mm}^4$	$\bar{z}V, \text{mm}^4$
I	Rectangular plate	$(10)(90)(38) = 34.2 \times 10^3$	19	45	649.8×10^3	1539×10^3
II	Half cylinder	$\frac{\pi}{2}(20)^2(10) = 6.2832 \times 10^3$	46.5	20	292.17×10^3	125.664×10^3
III	-(Cylinder)	$-\pi(12)^2(10) = -4.5239 \times 10^3$	38	20	-171.908×10^3	-90.478×10^3
IV	Rectangular prism	$(30)(10)(24) = 7.2 \times 10^3$	5	78	36×10^3	561.6×10^3
V	Triangular prism	$\frac{1}{2}(30)(9)(24) = 3.24 \times 10^3$	13	78	42.12×10^3	252.72×10^3
	Σ	46.399×10^3			848.18×10^3	2388.5×10^3

$$\bar{X} \Sigma V = \Sigma \bar{x} V$$

$$\bar{X} = \frac{\Sigma \bar{x} V}{\Sigma V} = \frac{848.18 \times 10^3 \text{ mm}^4}{46.399 \times 10^3 \text{ mm}^3}$$

$$\bar{X} = 18.28 \text{ mm} \blacktriangleleft$$

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SOLUTION

		V, mm^3	\bar{x}, mm	\bar{z}, mm	$\bar{x}V, \text{mm}^4$	$\bar{z}V, \text{mm}^4$
I	Rectangular plate	$(10)(90)(38) = 34.2 \times 10^3$	19	45	649.8×10^3	1539×10^3
II	Half cylinder	$\frac{\pi}{2}(20)^2(10) = 6.2832 \times 10^3$	46.5	20	292.17×10^3	125.664×10^3
III	-(Cylinder)	$-\pi(12)^2(10) = -4.5239 \times 10^3$	38	20	-171.908×10^3	-90.478×10^3
IV	Rectangular prism	$(30)(10)(24) = 7.2 \times 10^3$	5	78	36×10^3	561.6×10^3
V	Triangular prism	$\frac{1}{2}(30)(9)(24) = 3.24 \times 10^3$	13	78	42.12×10^3	252.72×10^3
	Σ	46.399×10^3			848.18×10^3	2388.5×10^3

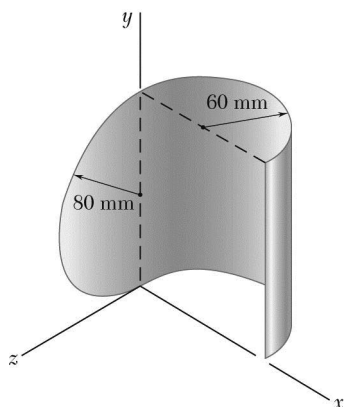
$\bar{Z}\Sigma V = \Sigma \bar{z}V$

$\bar{Z} = \frac{\Sigma \bar{z}V}{\Sigma V} = \frac{2388.5 \times 10^3 \text{ mm}^4}{46.399 \times 10^3 \text{ mm}^3}$ $\bar{Z} = 51.5 \text{ mm} \blacktriangleleft$

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PROBLEM 5.106

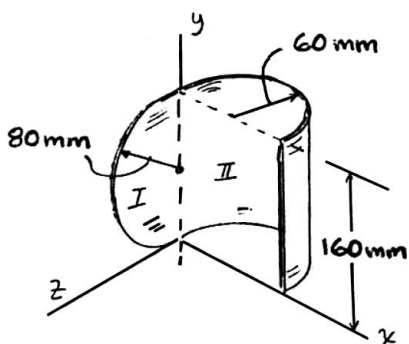
Locate the center of gravity of the sheet-metal form shown.



SOLUTION

By symmetry,

$$\bar{Y} = 80.0 \text{ mm} \quad \blacktriangleleft$$



$$\bar{z}_I = \frac{4(80)}{3\pi} = 33.953 \text{ mm}$$

$$\bar{z}_{II} = -\frac{2(60)}{\pi} = -38.197 \text{ mm}$$

	A, mm^2	\bar{x}, mm	\bar{z}, mm	$\bar{x}A, \text{mm}^3$	$\bar{z}A, \text{mm}^3$
I	$\frac{\pi}{2}(80)^2 = 10,053$	0	33.953	0	341.33×10^3
II	$\pi(60)(160) = 30,159$	60	-38.197	1809.54×10^3	-1151.98×10^3
Σ	40,212			1809.54×10^3	-810.65×10^3

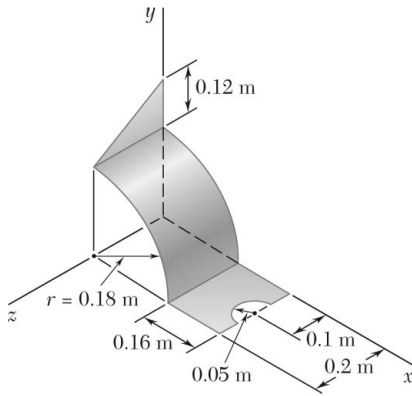
$$\bar{X} \Sigma A = \Sigma \bar{x}A: \quad \bar{X}(40,212) = 1809.54 \times 10^3 \quad \bar{X} = 45.0 \text{ mm} \quad \blacktriangleleft$$

$$\bar{Z} \Sigma A = \Sigma \bar{z}A: \quad \bar{Z}(40,212) = -810.65 \times 10^3 \quad \bar{Z} = -20.2 \text{ mm} \quad \blacktriangleleft$$

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PROBLEM 5.107

Locate the center of gravity of the sheet-metal form shown.



SOLUTION

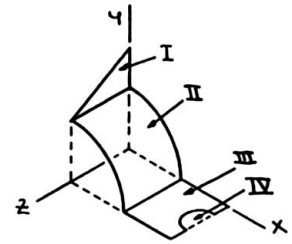
First assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.

$$\bar{y}_I = 0.18 + \frac{1}{3}(0.12) = 0.22 \text{ m}$$

$$\bar{z}_I = \frac{1}{3}(0.2 \text{ m})$$

$$\bar{x}_{II} = \bar{y}_{II} = \frac{2 \times 0.18}{\pi} = \frac{0.36}{\pi} \text{ m}$$

$$\begin{aligned} \bar{x}_{IV} &= 0.34 - \frac{4 \times 0.05}{3\pi} \\ &= 0.31878 \text{ m} \end{aligned}$$



	A, m^2	\bar{x}, m	\bar{y}, m	\bar{z}, m	$\bar{x}A, \text{m}^3$	$\bar{y}A, \text{m}^3$	$\bar{z}A, \text{m}^3$
I	$\frac{1}{2}(0.2)(0.12) = 0.012$	0	0.22	$\frac{0.2}{3}$	0	0.00264	0.0008
II	$\frac{\pi}{2}(0.18)(0.2) = 0.018\pi$	$\frac{0.36}{\pi}$	$\frac{0.36}{\pi}$	0.1	0.00648	0.00648	0.005655
III	$(0.16)(0.2) = 0.032$	0.26	0	0.1	0.00832	0	0.0032
IV	$-\frac{\pi}{2}(0.05)^2 = -0.00125\pi$	0.31878	0	0.1	-0.001258	0	-0.000393
Σ	0.096622				0.013542	0.00912	0.009262

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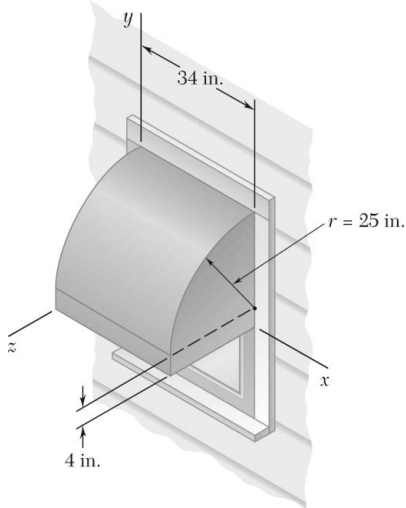
PROBLEM 5.107 (Continued)

We have

$$\bar{X}\Sigma V = \Sigma \bar{x}V: \bar{X}(0.096622 \text{ m}^2) = 0.013542 \text{ m}^3 \quad \text{or } \bar{X} = 0.1402 \text{ m} \blacktriangleleft$$
$$\bar{Y}\Sigma V = \Sigma \bar{y}V: \bar{Y}(0.096622 \text{ m}^2) = 0.00912 \text{ m}^3 \quad \text{or } \bar{Y} = 0.0944 \text{ m} \blacktriangleleft$$
$$\bar{Z}\Sigma V = \Sigma \bar{z}V: \bar{Z}(0.096622 \text{ m}^2) = 0.009262 \text{ m}^3 \quad \text{or } \bar{Z} = 0.0959 \text{ m} \blacktriangleleft$$

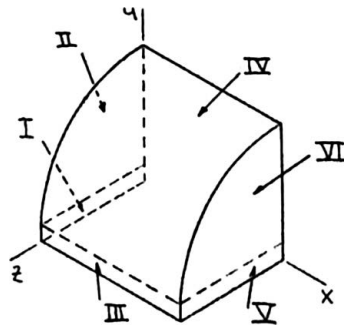
PROBLEM 5.108

A window awning is fabricated from sheet metal of uniform thickness. Locate the center of gravity of the awning.



SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the awning coincides with the centroid of the corresponding area.



$$\bar{y}_{II} = \bar{y}_{VI} = 4 + \frac{(4)(25)}{3\pi} = 14.6103 \text{ in.}$$

$$\bar{z}_{II} = \bar{z}_{VI} = \frac{(4)(25)}{3\pi} = \frac{100}{3\pi} \text{ in.}$$

$$\bar{y}_{IV} = 4 + \frac{(2)(25)}{\pi} = 19.9155 \text{ in.}$$

$$\bar{z}_{IV} = \frac{(2)(25)}{\pi} = \frac{50}{\pi} \text{ in.}$$

$$A_{II} = A_{VI} = \frac{\pi}{4} (25)^2 = 490.87 \text{ in}^2$$

$$A_{IV} = \frac{\pi}{2} (25)(34) = 1335.18 \text{ in}^2$$

	A, in^2	$\bar{y}, \text{in.}$	$\bar{z}, \text{in.}$	$\bar{y}A, \text{in}^3$	$\bar{z}A, \text{in}^3$
I	$(4)(25) = 100$	2	12.5	200	1250
II	490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
III	$(4)(34) = 136$	2	25	272	3400
IV	1335.18	19.9155	$\frac{50}{\pi}$	26,591	21,250
V	$(4)(25) = 100$	2	12.5	200	1250
VI	490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
Σ	2652.9			41,607	37,567

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PROBLEM 5.108 (Continued)

Now, symmetry implies

$$\bar{X} = 17.00 \text{ in.} \blacktriangleleft$$

and

$$\bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}(2652.9 \text{ in}^2) = 41,607 \text{ in}^3$$

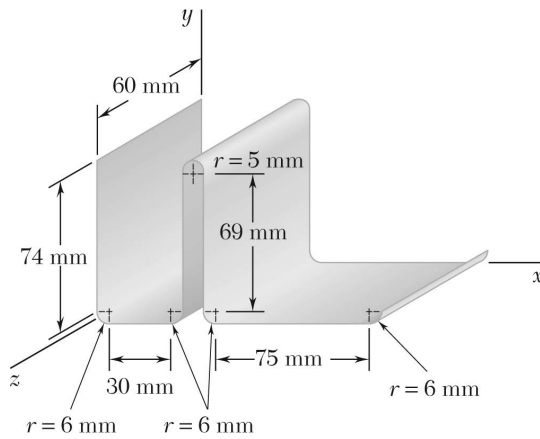
$$\text{or } \bar{Y} = 15.68 \text{ in.} \blacktriangleleft$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A: \quad \bar{Z}(2652.9 \text{ in}^2) = 37,567 \text{ in}^3$$

$$\text{or } \bar{Z} = 14.16 \text{ in.} \blacktriangleleft$$

PROBLEM 5.109

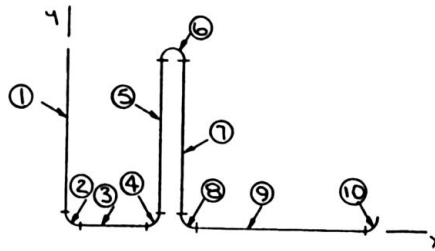
A thin sheet of plastic of uniform thickness is bent to form a desk organizer. Locate the center of gravity of the organizer.



SOLUTION

First assume that the plastic is homogeneous so that the center of gravity of the organizer will coincide with the centroid of the corresponding area. Now note that symmetry implies

$$\bar{Z} = 30.0 \text{ mm} \quad \blacktriangleleft$$



$$\bar{x}_2 = 6 - \frac{2 \times 6}{\pi} = 2.1803 \text{ mm}$$

$$\bar{x}_4 = 36 + \frac{2 \times 6}{\pi} = 39.820 \text{ mm}$$

$$\bar{x}_8 = 58 - \frac{2 \times 6}{\pi} = 54.180 \text{ mm}$$

$$\bar{x}_{10} = 133 + \frac{2 \times 6}{\pi} = 136.820 \text{ mm}$$

$$\bar{y}_2 = \bar{y}_4 = \bar{y}_8 = \bar{y}_{10} = 6 - \frac{2 \times 6}{\pi} = 2.1803 \text{ mm}$$

$$\bar{y}_6 = 75 + \frac{2 \times 5}{\pi} = 78.183 \text{ mm}$$

$$A_2 = A_4 = A_8 = A_{10} = \frac{\pi}{2} \times 6 \times 60 = 565.49 \text{ mm}^2$$

$$A_6 = \pi \times 5 \times 60 = 942.48 \text{ mm}^2$$

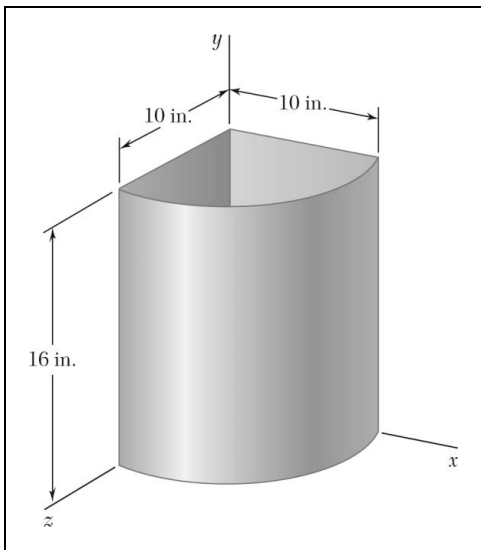
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PROBLEM 5.109 (Continued)

	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$(74)(60) = 4440$	0	43	0	190,920
2	565.49	2.1803	2.1803	1233	1233
3	$(30)(60) = 1800$	21	0	37,800	0
4	565.49	39.820	2.1803	22,518	1233
5	$(69)(60) = 4140$	42	40.5	173,880	167,670
6	942.48	47	78.183	44,297	73,686
7	$(69)(60) = 4140$	52	40.5	215,280	167,670
8	565.49	54.180	2.1803	30,638	1233
9	$(75)(60) = 4500$	95.5	0	429,750	0
10	565.49	136.820	2.1803	77,370	1233
Σ	22,224.44			1,032,766	604,878

We have $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(22,224.44 \text{ mm}^2) = 1,032,766 \text{ mm}^3$ or $\bar{X} = 46.5 \text{ mm} \blacktriangleleft$

$\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(22,224.44 \text{ mm}^2) = 604,878 \text{ mm}^3$ or $\bar{Y} = 27.2 \text{ mm} \blacktriangleleft$



PROBLEM 5.110

A wastebasket, designed to fit in the corner of a room, is 16 in. high and has a base in the shape of a quarter circle of radius 10 in. Locate the center of gravity of the wastebasket, knowing that it is made of sheet metal of uniform thickness.

SOLUTION

By symmetry,

$$\bar{X} = \bar{Z}$$

For III (Cylindrical surface),

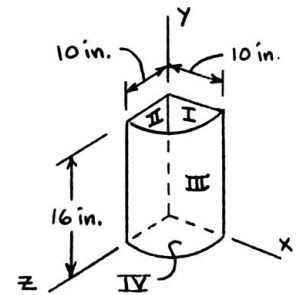
$$\bar{x} = \frac{2r}{\pi} = \frac{2(10)}{\pi} = 6.3662 \text{ in.}$$

$$A = \frac{\pi}{2} rh = \frac{\pi}{2} (10)(16) = 251.33 \text{ in}^2$$

For IV (Quarter-circle bottom),

$$\bar{x} = \frac{4r}{3\pi} = \frac{4(10)}{3\pi} = 4.2441 \text{ in.}$$

$$A = \frac{\pi}{4} r^2 = \frac{\pi}{4} (10)^2 = 78.540 \text{ in}^2$$



	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
I	$(10)(16) = 160$	5	8	800	1280
II	$(10)(16) = 160$	0	8	0	1280
III	251.33	6.3662	8	1600.0	2010.6
IV	78.540	4.2441	0	333.33	0
Σ	649.87			2733.3	4570.6

$$\bar{X} \Sigma A = \Sigma \bar{x}A: \quad \bar{X} (649.87 \text{ in}^2) = 2733.3 \text{ in}^3$$

$$\bar{X} = 4.2059 \text{ in.}$$

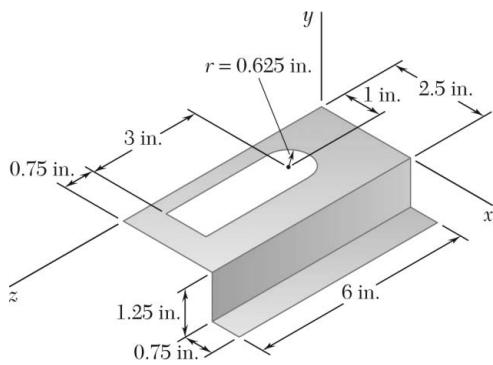
$$\bar{X} = \bar{Z} = 4.21 \text{ in.} \quad \blacktriangleleft$$

$$\bar{Y} \Sigma A = \Sigma \bar{y}A: \quad \bar{Y} (649.87 \text{ in}^2) = 4570.6 \text{ in}^3$$

$$\bar{Y} = 7.0331 \text{ in.}$$

$$\bar{Y} = 7.03 \text{ in.} \quad \blacktriangleleft$$

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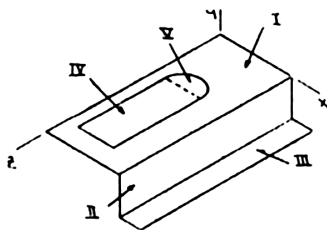


PROBLEM 5.111

A mounting bracket for electronic components is formed from sheet metal of uniform thickness. Locate the center of gravity of the bracket.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the bracket coincides with the centroid of the corresponding area. Then (see diagram)



$$\begin{aligned}\bar{z}_V &= 2.25 - \frac{4(0.625)}{3\pi} \\ &= 1.98474 \text{ in.} \\ A_V &= -\frac{\pi}{2}(0.625)^2 \\ &= -0.61359 \text{ in}^2\end{aligned}$$

	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{z}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$	$\bar{z}A, \text{in}^3$
I	$(2.5)(6) = 15$	1.25	0	3	18.75	0	45
II	$(1.25)(6) = 7.5$	2.5	-0.625	3	18.75	-4.6875	22.5
III	$(0.75)(6) = 4.5$	2.875	-1.25	3	12.9375	-5.625	13.5
IV	$-\left(\frac{5}{4}\right)(3) = -3.75$	1.0	0	3.75	3.75	0	-14.0625
V	-0.61359	1.0	0	1.98474	0.61359	0	-1.21782
Σ	22.6364				46.0739	10.3125	65.7197

We have

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(22.6364 \text{ in}^2) = 46.0739 \text{ in}^3 \quad \text{or} \quad \bar{X} = 2.04 \text{ in.} \quad \blacktriangleleft$$

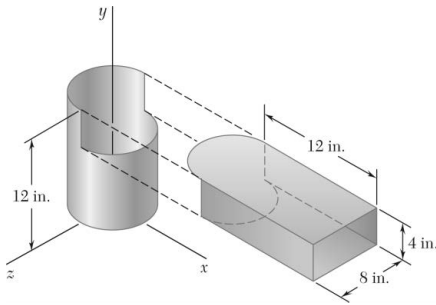
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(22.6364 \text{ in}^2) = -10.3125 \text{ in}^3 \quad \text{or} \quad \bar{Y} = -0.456 \text{ in.} \quad \blacktriangleleft$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A$$

$$\bar{Z}(22.6364 \text{ in}^2) = 65.7197 \text{ in}^3 \quad \text{or} \quad \bar{Z} = 2.90 \text{ in.} \quad \blacktriangleleft$$

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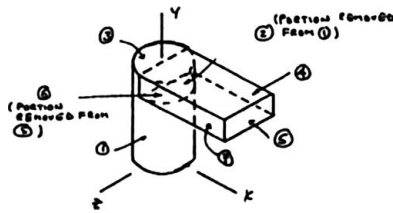
PROBLEM 5.112

An 8-in.-diameter cylindrical duct and a 4 × 8-in. rectangular duct are to be joined as indicated. Knowing that the ducts were fabricated from the same sheet metal, which is of uniform thickness, locate the center of gravity of the assembly.

SOLUTION

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

By symmetry, $\bar{z} = 0$.



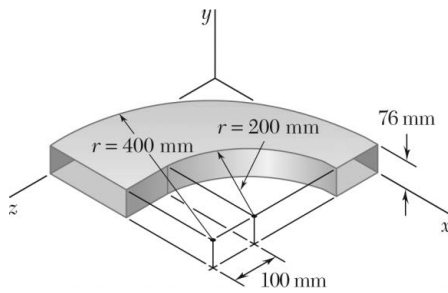
	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\pi(8)(12) = 96\pi$	0	6	0	576π
2	$-\frac{\pi}{2}(8)(4) = -16\pi$	$\frac{2(4)}{\pi} = \frac{8}{\pi}$	10	-128	-160π
3	$\frac{\pi}{2}(4)^2 = 8\pi$	$-\frac{4(4)}{3\pi} = -\frac{16}{3\pi}$	12	-42.667	96π
4	$(8)(12) = 96$	6	12	576	1152
5	$(8)(12) = 96$	6	8	576	768
6	$-\frac{\pi}{2}(4)^2 = -8\pi$	$\frac{4(4)}{3\pi} = \frac{16}{3\pi}$	8	-42.667	-64π
7	$(4)(12) = 48$	6	10	288	480
8	$(4)(12) = 48$	6	10	288	480
Σ	539.33			1514.6	4287.4

Then
$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1514.67}{539.33} \text{ in.} \quad \text{or } \bar{X} = 2.81 \text{ in.} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{4287.4}{539.33} \text{ in.} \quad \text{or } \bar{Y} = 7.95 \text{ in.} \blacktriangleleft$$

PROBLEM 5.113

An elbow for the duct of a ventilating system is made of sheet metal of uniform thickness. Locate the center of gravity of the elbow.



SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also, note that the shape of the duct implies

$$\bar{Y} = 38.0 \text{ mm} \blacktriangleleft$$

Note that $\bar{x}_I = \bar{z}_I = 400 - \frac{2}{\pi}(400) = 145.352 \text{ mm}$

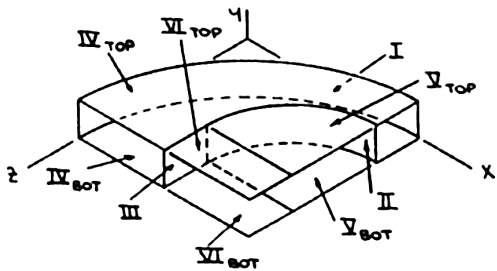
$$\bar{x}_{II} = 400 - \frac{2}{\pi}(200) = 272.68 \text{ mm}$$

$$\bar{z}_{II} = 300 - \frac{2}{\pi}(200) = 172.676 \text{ mm}$$

$$\bar{x}_{IV} = \bar{z}_{IV} = 400 - \frac{4}{3\pi}(400) = 230.23 \text{ mm}$$

$$\bar{x}_V = 400 - \frac{4}{3\pi}(200) = 315.12 \text{ mm}$$

$$\bar{z}_V = 300 - \frac{4}{3\pi}(200) = 215.12 \text{ mm}$$



Also note that the corresponding top and bottom areas will contribute equally when determining \bar{x} and \bar{z} .

Thus,

	A, mm^2	\bar{x}, mm	\bar{z}, mm	$\bar{x}A, \text{mm}^3$	$\bar{z}A, \text{mm}^3$
I	$\frac{\pi}{2}(400)(76) = 47,752$	145.352	145.352	6,940,850	6,940,850
II	$\frac{\pi}{2}(200)(76) = 23,876$	272.68	172.676	6,510,510	4,122,810
III	$100(76) = 7600$	200	350	1,520,000	2,660,000
IV	$2\left(\frac{\pi}{4}\right)(400)^2 = 251,327$	230.23	230.23	57,863,020	57,863,020
V	$-2\left(\frac{\pi}{4}\right)(200)^2 = -62,832$	315.12	215.12	-19,799,620	-13,516,420
VI	$-2(100)(200) = -40,000$	300	350	-12,000,000	-14,000,000
Σ	227,723			41,034,760	44,070,260

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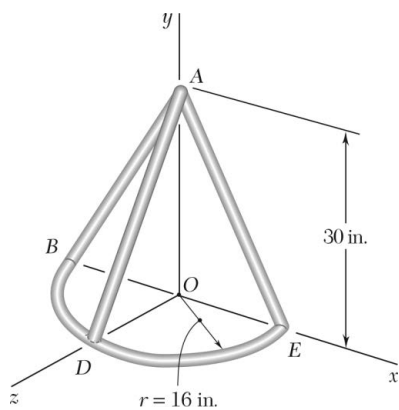
PROBLEM 5.113 (Continued)

We have

$$\bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(227,723 \text{ mm}^2) = 41,034,760 \text{ mm}^3 \quad \text{or } \bar{X} = 180.2 \text{ mm} \blacktriangleleft$$
$$\bar{Z}\Sigma A = \Sigma \bar{z}A: \quad \bar{Z}(227,723 \text{ mm}^2) = 44,070,260 \text{ mm}^3 \quad \text{or } \bar{Z} = 193.5 \text{ mm} \blacktriangleleft$$

PROBLEM 5.114

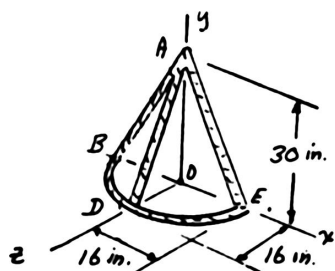
Locate the center of gravity of the figure shown, knowing that it is made of thin brass rods of uniform diameter.



SOLUTION

By symmetry,

$$\bar{X} = 0 \quad \blacktriangleleft$$



	L , in.	\bar{y} , in.	\bar{z} , in.	$\bar{y}L$, in ²	$\bar{z}L$, in ²
AB	$\sqrt{30^2 + 16^2} = 34$	15	0	510	0
AD	$\sqrt{30^2 + 16^2} = 34$	15	8	510	272
AE	$\sqrt{30^2 + 16^2} = 34$	15	0	510	0
BDE	$\pi(16) = 50.265$	0	$\frac{2(16)}{\pi} = 10.186$	0	512
Σ	152.265			1530	784

$$\bar{Y}\Sigma L = \Sigma \bar{y}L: \quad \bar{Y}(152.265 \text{ in.}) = 1530 \text{ in}^2$$

$$\bar{Y} = 10.048 \text{ in.}$$

$$\bar{Y} = 10.05 \text{ in.} \quad \blacktriangleleft$$

$$\bar{Z}\Sigma L = \Sigma \bar{z}L: \quad \bar{Z}(152.265 \text{ in.}) = 784 \text{ in}^2$$

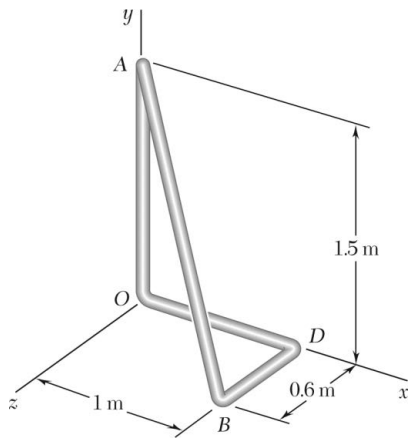
$$\bar{Z} = 5.149 \text{ in.}$$

$$\bar{Z} = 5.15 \text{ in.} \quad \blacktriangleleft$$

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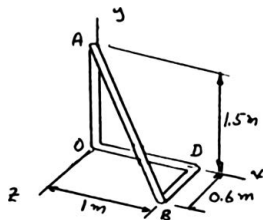
PROBLEM 5.115

Locate the center of gravity of the figure shown, knowing that it is made of thin brass rods of uniform diameter.



SOLUTION

Uniform rod:



$$AB^2 = (1 \text{ m})^2 + (0.6 \text{ m})^2 + (1.5 \text{ m})^2$$

$$AB = 1.9 \text{ m}$$

	$L, \text{ m}$	$\bar{x}, \text{ m}$	$\bar{y}, \text{ m}$	$\bar{z}, \text{ m}$	$\bar{x}L, \text{ m}^2$	$\bar{y}L, \text{ m}^2$	$\Sigma L, \text{ m}$
AB	1.9	0.5	0.75	0.3	0.95	1.425	0.57
BD	0.6	1.0	0	0.3	0.60	0	0.18
DO	1.0	0.5	0	0	0.50	0	0
OA	1.5	0	0.75	0	0	1.125	0
Σ	5.0				2.05	2.550	0.75

$$\bar{X} \Sigma L = \Sigma \bar{x} L: \quad \bar{X}(5.0 \text{ m}) = 2.05 \text{ m}^2 \quad \bar{X} = 0.410 \text{ m} \quad \blacktriangleleft$$

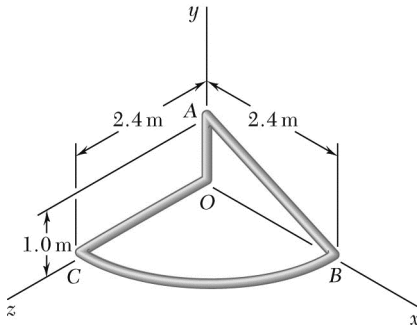
$$\bar{Y} \Sigma L = \Sigma \bar{y} L: \quad \bar{Y}(5.0 \text{ m}) = 2.55 \text{ m}^2 \quad \bar{Y} = 0.510 \text{ m} \quad \blacktriangleleft$$

$$\bar{Z} \Sigma L = \Sigma \bar{z} L: \quad \bar{Z}(5.0 \text{ m}) = 0.75 \text{ m}^2 \quad \bar{Z} = 0.1500 \text{ m} \quad \blacktriangleleft$$

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PROBLEM 5.116

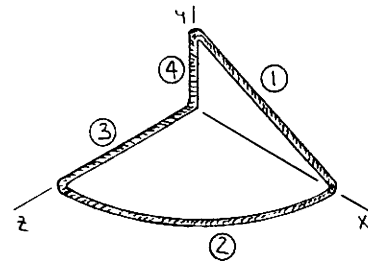
A thin steel wire of uniform cross section is bent into the shape shown. Locate its center of gravity.



SOLUTION

First assume that the wire is homogeneous so that its center of gravity will coincide with the centroid of the corresponding line.

$$\bar{x}_2 = \bar{z}_2 = \frac{2 \times 2.4}{\pi} = \frac{4.8}{\pi} \text{ m}$$



	$L, \text{ m}$	$\bar{x}, \text{ m}$	$\bar{y}, \text{ m}$	$\bar{z}, \text{ m}$	$\bar{x}L, \text{ m}^2$	$\bar{y}L, \text{ m}^2$	$\bar{z}L, \text{ m}^2$
1	2.6	1.2	0.5	0	3.12	1.3	0
2	$\frac{\pi}{2} \times 2.4 = 1.2\pi$	$\frac{4.8}{\pi}$	0	$\frac{4.8}{\pi}$	5.76	0	5.76
3	2.4	0	0	1.2	0	0	2.88
4	1.0	0	0.5	0	0	0.5	0
Σ	9.7699				8.88	1.8	8.64

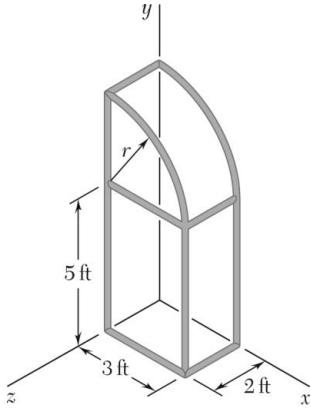
We have $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(9.7699 \text{ m}) = 8.88 \text{ m}^2$ or $\bar{X} = 0.909 \text{ m} \blacktriangleleft$

$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(9.7699 \text{ m}) = 1.8 \text{ m}^2$ or $\bar{Y} = 0.1842 \text{ m} \blacktriangleleft$

$\bar{Z}\Sigma L = \Sigma \bar{z}L$: $\bar{Z}(9.7699 \text{ m}) = 8.64 \text{ m}^2$ or $\bar{Z} = 0.884 \text{ m} \blacktriangleleft$

PROBLEM 5.117

The frame of a greenhouse is constructed from uniform aluminum channels. Locate the center of gravity of the portion of the frame shown.

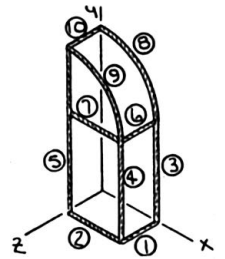


SOLUTION

First assume that the channels are homogeneous so that the center of gravity of the frame will coincide with the centroid of the corresponding line.

$$\bar{x}_8 = \bar{x}_9 = \frac{2 \times 3}{\pi} = \frac{6}{\pi} \text{ ft}$$

$$\bar{y}_8 = \bar{y}_9 = 5 + \frac{2 \times 3}{\pi} = 6.9099 \text{ ft}$$



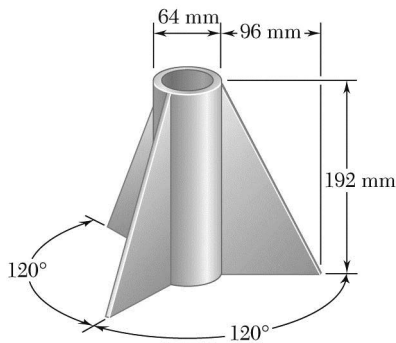
	$L, \text{ ft}$	$\bar{x}, \text{ ft}$	$\bar{y}, \text{ ft}$	$\bar{z}, \text{ ft}$	$\bar{x}L, \text{ ft}^2$	$\bar{y}L, \text{ ft}^2$	$\bar{z}L, \text{ ft}^2$
1	2	3	0	1	6	0	2
2	3	1.5	0	2	4.5	0	6
3	5	3	2.5	0	15	12.5	0
4	5	3	2.5	2	15	12.5	10
5	8	0	4	2	0	32	16
6	2	3	5	1	6	10	2
7	3	1.5	5	2	4.5	15	6
8	$\frac{\pi}{2} \times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	0	9	32.562	0
9	$\frac{\pi}{2} \times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	2	9	32.562	9.4248
10	2	0	8	1	0	16	2
Σ	39.4248				69	163.124	53.4248

We have $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(39.4248 \text{ ft}) = 69 \text{ ft}^2$ or $\bar{X} = 1.750 \text{ ft}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(39.4248 \text{ ft}) = 163.124 \text{ ft}^2$ or $\bar{Y} = 4.14 \text{ ft}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z}L$: $\bar{Z}(39.4248 \text{ ft}) = 53.4248 \text{ ft}^2$ or $\bar{Z} = 1.355 \text{ ft}$ ◀

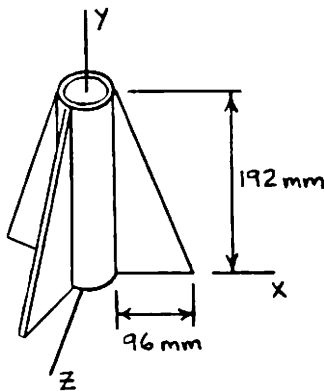
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PROBLEM 5.118

Three brass plates are brazed to a steel pipe to form the flagpole base shown. Knowing that the pipe has a wall thickness of 8 mm and that each plate is 6 mm thick, determine the location of the center of gravity of the base. (Densities: brass = 8470 kg/m^3 ; steel = 7860 kg/m^3 .)

SOLUTION



Since brass plates are equally spaced, we note that the center of gravity lies on the y-axis.

Thus,

$$\bar{x} = \bar{z} = 0 \quad \blacktriangleleft$$

Steel pipe:

$$\begin{aligned} V &= \frac{\pi}{4} [(0.064 \text{ m})^2 - (0.048 \text{ m})^2] (0.192 \text{ m}) \\ &= 270.22 \times 10^{-6} \text{ m}^3 \\ m &= \rho V = (7860 \text{ kg/m}^3) (270.22 \times 10^{-6} \text{ m}^3) \\ &= 2.1239 \text{ kg} \end{aligned}$$

Each brass plate:

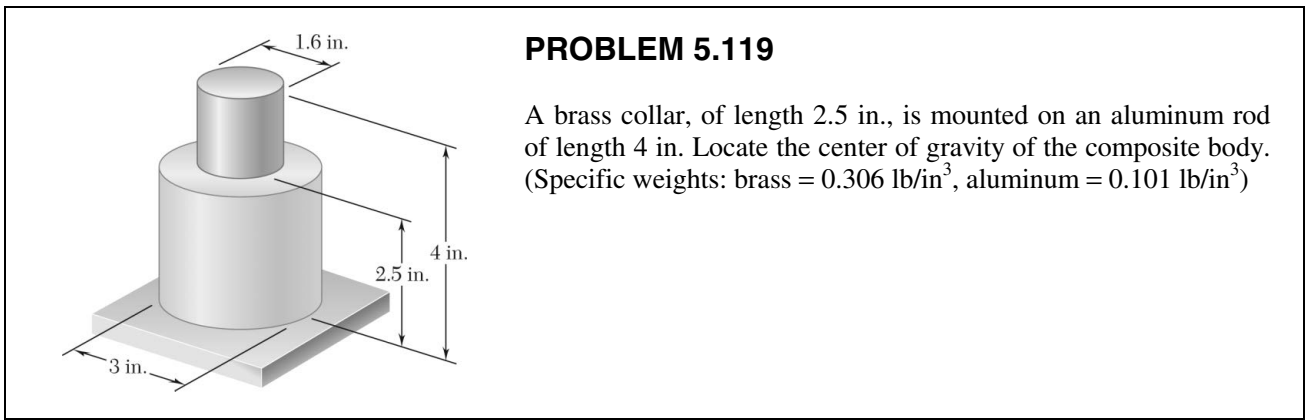
$$\begin{aligned} V &= \frac{1}{2} (0.096 \text{ m})(0.192 \text{ m})(0.006 \text{ m}) = 55.296 \times 10^{-6} \text{ m}^3 \\ m &= \rho V = (8470 \text{ kg/m}^3) (55.296 \times 10^{-6} \text{ m}^3) = 0.46836 \text{ kg} \end{aligned}$$

Flagpole base:

$$\begin{aligned} \Sigma m &= 2.1239 \text{ kg} + 3(0.46836 \text{ kg}) = 3.5290 \text{ kg} \\ \Sigma \bar{y} m &= (0.096 \text{ m})(2.1239 \text{ kg}) + 3[(0.064 \text{ m})(0.46836 \text{ kg})] = 0.29382 \text{ kg} \cdot \text{m} \\ \bar{Y} \Sigma m &= \Sigma \bar{y} m: \quad \bar{Y} (3.5290 \text{ kg}) = 0.29382 \text{ kg} \cdot \text{m} \end{aligned}$$

$$\bar{Y} = 0.083259 \text{ m}$$

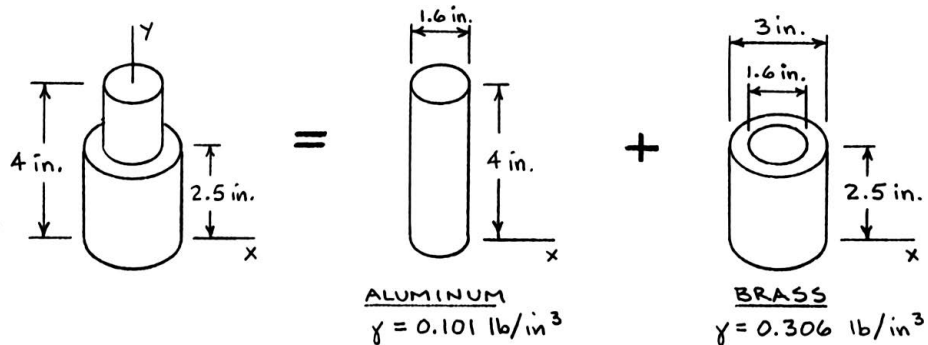
$$\bar{Y} = 83.3 \text{ mm above the base} \quad \blacktriangleleft$$



PROBLEM 5.119

A brass collar, of length 2.5 in., is mounted on an aluminum rod of length 4 in. Locate the center of gravity of the composite body. (Specific weights: brass = 0.306 lb/in³, aluminum = 0.101 lb/in³)

SOLUTION



Aluminum rod:

$$\begin{aligned}
 W &= \gamma V \\
 &= (0.101 \text{ lb/in}^3) \left[\frac{\pi}{4} (1.6 \text{ in.})^2 (4 \text{ in.}) \right] \\
 &= 0.81229 \text{ lb}
 \end{aligned}$$

Brass collar:

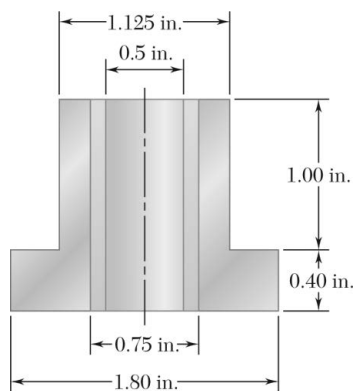
$$\begin{aligned}
 W &= \gamma V \\
 &= (0.306 \text{ lb/in}^3) \frac{\pi}{4} [(3 \text{ in.})^2 - (1.6 \text{ in.})^2] (2.5 \text{ in.}) \\
 &= 3.8693 \text{ lb}
 \end{aligned}$$

Component	W (lb)	\bar{y} (in.)	$\bar{y}W$ (lb · in.)
Rod	0.81229	2	1.62458
Collar	3.8693	1.25	4.8366
Σ	4.6816		6.4612

$$\bar{Y} \Sigma W = \Sigma \bar{y} W: \bar{Y} (4.6816 \text{ lb}) = 6.4612 \text{ lb} \cdot \text{in.}$$

$$\bar{Y} = 1.38013 \text{ in.}$$

$$\bar{Y} = 1.380 \text{ in.} \blacktriangleleft$$



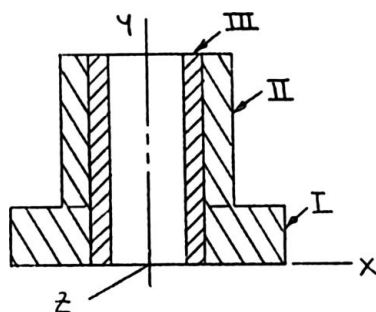
PROBLEM 5.120

A bronze bushing is mounted inside a steel sleeve. Knowing that the specific weight of bronze is 0.318 lb/in^3 and of steel is 0.284 lb/in^3 , determine the location of the center of gravity of the assembly.

SOLUTION

First, note that symmetry implies

$$\bar{X} = \bar{Z} = 0 \quad \blacktriangleleft$$



Now

$$W = (\rho g)V$$

$$\bar{y}_I = 0.20 \text{ in.} \quad W_I = (0.284 \text{ lb/in}^3) \left\{ \left(\frac{\pi}{4} \right) [(1.8^2 - 0.75^2) \text{ in}^2] (0.4 \text{ in.}) \right\} = 0.23889 \text{ lb}$$

$$\bar{y}_{II} = 0.90 \text{ in.} \quad W_{II} = (0.284 \text{ lb/in}^3) \left\{ \left(\frac{\pi}{4} \right) [(1.125^2 - 0.75^2) \text{ in}^2] (1 \text{ in.}) \right\} = 0.156834 \text{ lb}$$

$$\bar{y}_{III} = 0.70 \text{ in.} \quad W_{III} = (0.318 \text{ lb/in}^3) \left\{ \left(\frac{\pi}{4} \right) [(0.75^2 - 0.5^2) \text{ in}^2] (1.4 \text{ in.}) \right\} = 0.109269 \text{ lb}$$

We have

$$\bar{Y} \Sigma W = \Sigma \bar{y} W$$

$$\bar{Y} = \frac{(0.20 \text{ in.})(0.23889 \text{ lb}) + (0.90 \text{ in.})(0.156834 \text{ lb}) + (0.70 \text{ in.})(0.109269 \text{ lb})}{0.23889 \text{ lb} + 0.156834 \text{ lb} + 0.109269 \text{ lb}}$$

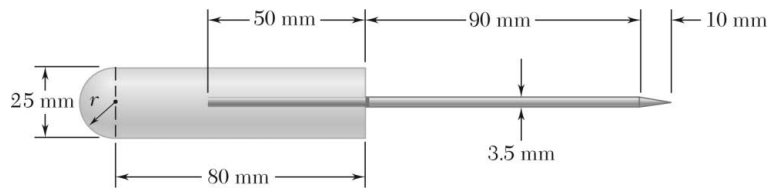
or

$$\bar{Y} = 0.526 \text{ in.} \quad \blacktriangleleft$$

(above base)

PROBLEM 5.121

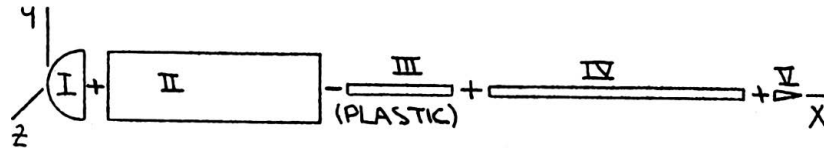
A scratch awl has a plastic handle and a steel blade and shank. Knowing that the density of plastic is 1030 kg/m^3 and of steel is 7860 kg/m^3 , locate the center of gravity of the awl.



SOLUTION

First, note that symmetry implies

$$\bar{Y} = \bar{Z} = 0 \quad \blacktriangleleft$$



$$\bar{x}_I = \frac{5}{8}(12.5 \text{ mm}) = 7.8125 \text{ mm}$$

$$W_I = (1030 \text{ kg/m}^3) \left(\frac{2\pi}{3} \right) (0.0125 \text{ m})^3$$

$$= 4.2133 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{II} = 52.5 \text{ mm}$$

$$W_{II} = (1030 \text{ kg/m}^3) \left(\frac{\pi}{4} \right) (0.025 \text{ m})^2 (0.08 \text{ m})$$

$$= 40.448 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{III} = 92.5 \text{ mm} - 25 \text{ mm} = 67.5 \text{ mm}$$

$$W_{III} = -(1030 \text{ kg/m}^3) \left(\frac{\pi}{4} \right) (0.0035 \text{ m})^2 (0.05 \text{ m})$$

$$= -0.49549 \times 10^{-3} \text{ kg}$$

$$\bar{x}_{IV} = 182.5 \text{ mm} - 70 \text{ mm} = 112.5 \text{ mm}$$

$$W_{IV} = (7860 \text{ kg/m}^3) \left(\frac{\pi}{4} \right) (0.0035 \text{ m})^2 (0.14 \text{ m})^2 = 10.5871 \times 10^{-3} \text{ kg}$$

$$\bar{x}_V = 182.5 \text{ mm} + \frac{1}{4}(10 \text{ mm}) = 185 \text{ mm}$$

$$W_V = (7860 \text{ kg/m}^3) \left(\frac{\pi}{3} \right) (0.00175 \text{ m})^2 (0.01 \text{ m}) = 0.25207 \times 10^{-3} \text{ kg}$$

PROBLEM 5.121 (Continued)

	W, kg	x̄, mm	x̄W, kg · mm
I	4.123×10^{-3}	7.8125	32.916×10^{-3}
II	40.948×10^{-3}	52.5	2123.5×10^{-3}
III	-0.49549×10^{-3}	67.5	-33.447×10^{-3}
IV	10.5871×10^{-3}	112.5	1191.05×10^{-3}
V	0.25207×10^{-3}	185	46.633×10^{-3}
Σ	55.005×10^{-3}		3360.7×10^{-3}

We have $\bar{X} \Sigma W = \Sigma \bar{x} W$: $\bar{X} (55.005 \times 10^{-3} \text{ kg}) = 3360.7 \times 10^{-3} \text{ kg} \cdot \text{mm}$

or

$$\bar{X} = 61.1 \text{ mm} \quad \blacktriangleleft$$

(from the end of the handle)

PROBLEM 5.122

Determine by direct integration the values of \bar{x} for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A hemisphere

SOLUTION

Choose as the element of volume a disk of radius r and thickness dx . Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x^2 + y^2 = a^2$ so that $r^2 = a^2 - x^2$ and then

$$dV = \pi(a^2 - x^2)dx$$

Component 1:

$$\begin{aligned} V_1 &= \int_0^{a/2} \pi(a^2 - x^2)dx = \pi \left[a^2 x - \frac{x^3}{3} \right]_0^{a/2} \\ &= \frac{11}{24} \pi a^3 \end{aligned}$$

and

$$\begin{aligned} \int_1 \bar{x}_{EL} dV &= \int_0^{a/2} x [\pi(a^2 - x^2)dx] \\ &= \pi \left[a^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_0^{a/2} \\ &= \frac{7}{64} \pi a^4 \end{aligned}$$

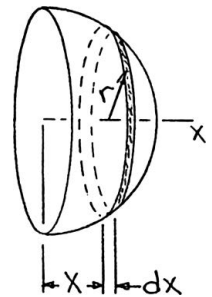
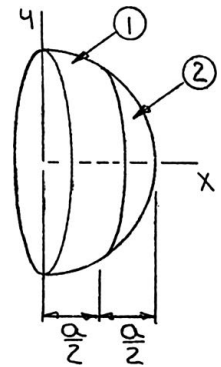
Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1 \left(\frac{11}{24} \pi a^3 \right) = \frac{7}{64} \pi a^4$$

$$\text{or } \bar{x}_1 = \frac{21}{88} a \quad \blacktriangleleft$$

Component 2:

$$\begin{aligned} V_2 &= \int_{a/2}^a \pi(a^2 - x^2)dx = \pi \left[a^2 x - \frac{x^3}{3} \right]_{a/2}^a \\ &= \pi \left\{ \left[a^2(a) - \frac{a^3}{3} \right] - \left[a^2 \left(\frac{a}{2} \right) - \frac{\left(\frac{a}{2} \right)^3}{3} \right] \right\} \\ &= \frac{5}{24} \pi a^3 \end{aligned}$$



PROBLEM 5.122 (Continued)

and

$$\begin{aligned}\int_2 \bar{x}_{EL} dV &= \int_{a/2}^a x [\pi(a^2 - x^2) dx] = \pi \left[a^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_{a/2}^a \\ &= \pi \left\{ \left[a^2 \frac{(a)^2}{2} - \frac{(a)^4}{4} \right] - \left[a^2 \frac{(\frac{a}{2})^2}{2} - \frac{(\frac{a}{2})^4}{4} \right] \right\} \\ &= \frac{9}{64} \pi a^4\end{aligned}$$

Now

$$\bar{x}_2 V_2 = \int_2 \bar{x}_{EL} dV: \quad \bar{x}_2 \left(\frac{5}{24} \pi a^3 \right) = \frac{9}{64} \pi a^4 \quad \text{or} \quad \bar{x}_2 = \frac{27}{40} a \quad \blacktriangleleft$$

PROBLEM 5.123

Determine by direct integration the values of \bar{x} for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A semiellipsoid of revolution

SOLUTION

Choose as the element of volume a disk of radius r and thickness dx . Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $\frac{x^2}{h^2} + \frac{y^2}{a^2} = 1$ so that

$$r^2 = \frac{a^2}{h^2}(h^2 - x^2)$$

and then

$$dV = \pi \frac{a^2}{h^2}(h^2 - x^2) dx$$

Component 1:

$$\begin{aligned} V_1 &= \int_0^{h/2} \pi \frac{a^2}{h^2}(h^2 - x^2) dx = \pi \frac{a^2}{h^2} \left[h^2 x - \frac{x^3}{3} \right]_0^{h/2} \\ &= \frac{11}{24} \pi a^2 h \end{aligned}$$

and

$$\begin{aligned} \int_1 \bar{x}_{EL} dV &= \int_0^{h/2} x \left[\pi \frac{a^2}{h^2}(h^2 - x^2) dx \right] \\ &= \pi \frac{a^2}{h^2} \left[h^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_0^{h/2} \\ &= \frac{7}{64} \pi a^2 h^2 \end{aligned}$$

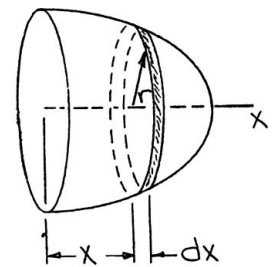
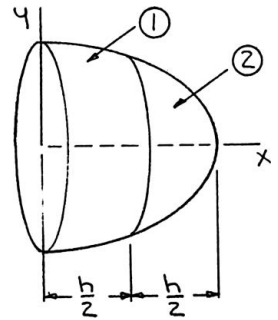
Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1 \left(\frac{11}{24} \pi a^2 h \right) = \frac{7}{64} \pi a^2 h^2$$

$$\text{or } \bar{x}_1 = \frac{21}{88} h \quad \blacktriangleleft$$

Component 2:

$$\begin{aligned} V_2 &= \int_{h/2}^h \pi \frac{a^2}{h^2}(h^2 - x^2) dx = \pi \frac{a^2}{h^2} \left[h^2 x - \frac{x^3}{3} \right]_{h/2}^h \\ &= \pi \frac{a^2}{h^2} \left\{ \left[h^2(h) - \frac{(h)^3}{3} \right] - \left[h^2 \left(\frac{h}{2} \right) - \frac{\left(\frac{h}{2} \right)^3}{3} \right] \right\} \\ &= \frac{5}{24} \pi a^2 h \end{aligned}$$



PROBLEM 5.123 (Continued)

and

$$\begin{aligned}\int_2 \bar{x}_{EL} dV &= \int_{h/2}^h x \left[\pi \frac{a^2}{h^2} (h^2 - x^2) dx \right] \\ &= \pi \frac{a^2}{h^2} \left[h^2 \frac{x^2}{2} - \frac{x^4}{4} \right]_{h/2}^h \\ &= \pi \frac{a^2}{h^2} \left\{ \left[h^2 \frac{(h)^2}{2} - \frac{(h)^4}{4} \right] - \left[h^2 \frac{(\frac{h}{2})^2}{2} - \frac{(\frac{h}{2})^4}{4} \right] \right\} \\ &= \frac{9}{64} \pi a^2 h^2\end{aligned}$$

Now

$$\bar{x}_2 V_2 = \int_2 \bar{x}_{EL} dV: \quad \bar{x}_2 \left(\frac{5}{24} \pi a^2 h \right) = \frac{9}{64} \pi a^2 h^2 \quad \text{or} \quad \bar{x}_2 = \frac{27}{40} h \quad \blacktriangleleft$$

PROBLEM 5.124

Determine by direct integration the values of \bar{x} for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A paraboloid of revolution

SOLUTION

Choose as the element of volume a disk of radius r and thickness dx . Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x = h - \frac{h}{a^2} y^2$ so that $r^2 = \frac{a^2}{h}(h-x)$.

and then

$$dV = \pi \frac{a^2}{h}(h-x)dx$$

Component 1:

$$\begin{aligned} V_1 &= \int_0^{h/2} \pi \frac{a^2}{h}(h-x)dx \\ &= \pi \frac{a^2}{h} \left[hx - \frac{x^2}{2} \right]_0^{h/2} \\ &= \frac{3}{8} \pi a^2 h \end{aligned}$$

and

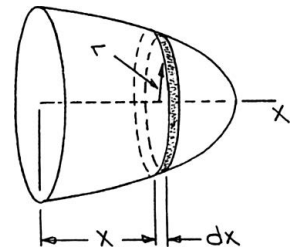
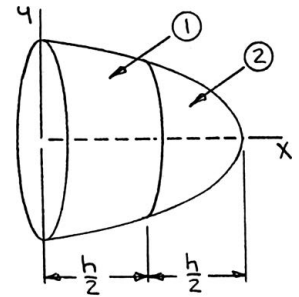
$$\begin{aligned} \int_1 \bar{x}_{EL} dV &= \int_0^{h/2} x \left[\pi \frac{a^2}{h}(h-x)dx \right] \\ &= \pi \frac{a^2}{h} \left[h \frac{x^2}{2} - \frac{x^3}{3} \right]_0^{h/2} = \frac{1}{12} \pi a^2 h^2 \end{aligned}$$

Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1 \left(\frac{3}{8} \pi a^2 h \right) = \frac{1}{12} \pi a^2 h^2 \quad \text{or} \quad \bar{x}_1 = \frac{2}{9} h \quad \blacktriangleleft$$

Component 2:

$$\begin{aligned} V_2 &= \int_{h/2}^h \pi \frac{a^2}{h}(h-x)dx = \pi \frac{a^2}{h} \left[hx - \frac{x^2}{2} \right]_{h/2}^h \\ &= \pi \frac{a^2}{h} \left\{ \left[h(h) - \frac{(h)^2}{2} \right] - \left[h \left(\frac{h}{2} \right) - \frac{\left(\frac{h}{2} \right)^2}{2} \right] \right\} \\ &= \frac{1}{8} \pi a^2 h \end{aligned}$$



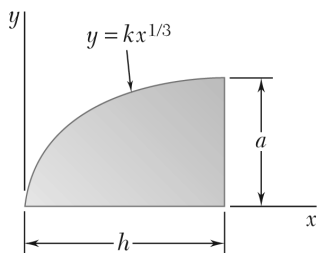
PROBLEM 5.124 (Continued)

and

$$\begin{aligned}\int_2 \bar{x}_{EL} dV &= \int_{h/2}^h x \left[\pi \frac{a^2}{h} (h-x) dx \right] = \pi \frac{a^2}{h} \left[h \frac{x^2}{2} - \frac{x^3}{3} \right]_{h/2}^h \\ &= \pi \frac{a^2}{h} \left\{ \left[h \frac{(h)^2}{2} - \frac{(h)^3}{3} \right] - \left[h \frac{(\frac{h}{2})^2}{2} - \frac{(\frac{h}{2})^3}{3} \right] \right\} \\ &= \frac{1}{12} \pi a^2 h^2\end{aligned}$$

Now

$$\bar{x}_2 V_2 = \int_2 \bar{x}_{EL} dV: \quad \bar{x}_2 \left(\frac{1}{8} \pi a^2 h \right) = \frac{1}{12} \pi a^2 h^2 \quad \text{or} \quad \bar{x}_2 = \frac{2}{3} h \quad \blacktriangleleft$$



PROBLEM 5.125

Locate the centroid of the volume obtained by rotating the shaded area about the x -axis.

SOLUTION

First note that symmetry implies

$$\bar{y} = 0 \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

Choose as the element of volume a disk of radius r and thickness dx . Then

$$dV = \pi r^2 dx, \quad x_{EL} = x$$

Now

$$r = kx^{1/3}$$

so that

$$dV = \pi k^2 x^{2/3} dx$$

at $x = h$, $y = a$,

$$a = kh^{1/3}$$

or

$$k = \frac{a^3}{h}$$

Then

$$dV = \pi \frac{a^2}{h^{2/3}} x^{2/3} dx$$

and

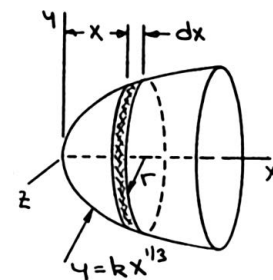
$$\begin{aligned} V &= \int_0^h \pi \frac{a^2}{h^{2/3}} x^{2/3} dx \\ &= \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{5} x^{5/3} \right]_0^h \\ &= \frac{3}{5} \pi a^2 h \end{aligned}$$

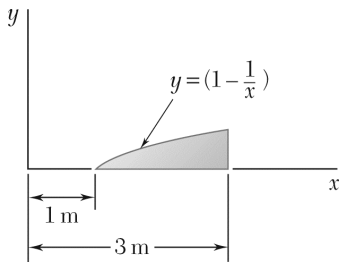
Also

$$\begin{aligned} \int \bar{x}_{EL} dV &= \int_0^h x \left(\pi \frac{a^2}{h^{2/3}} x^{2/3} dx \right) = \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{8} x^{8/3} \right]_0^h \\ &= \frac{3}{8} \pi a^2 h^2 \end{aligned}$$

Now

$$\bar{x}V = \int \bar{x} dV: \quad \bar{x} \left(\frac{3}{5} \pi a^2 h \right) = \frac{3}{8} \pi a^2 h^2 \quad \text{or} \quad \bar{x} = \frac{5}{8} h \quad \blacktriangleleft$$





PROBLEM 5.126

Locate the centroid of the volume obtained by rotating the shaded area about the x -axis.

SOLUTION

First, note that symmetry implies

$$\bar{y} = 0 \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

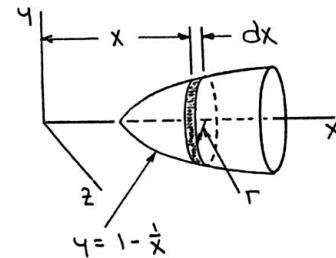
Choose as the element of volume a disk of radius r and thickness dx .

Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

Now $r = 1 - \frac{1}{x}$ so that

$$\begin{aligned} dV &= \pi \left(1 - \frac{1}{x}\right)^2 dx \\ &= \pi \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx \end{aligned}$$



Then

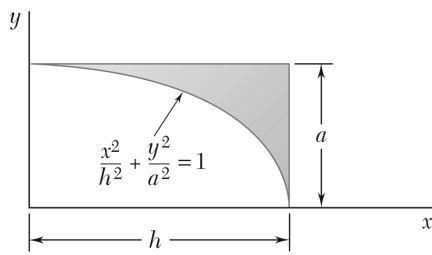
$$\begin{aligned} V &= \int_1^3 \pi \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx = \pi \left[x - 2 \ln x - \frac{1}{x} \right]_1^3 \\ &= \pi \left[\left(3 - 2 \ln 3 - \frac{1}{3}\right) - \left(1 - 2 \ln 1 - \frac{1}{1}\right) \right] \\ &= (0.46944\pi) \text{ m}^3 \end{aligned}$$

and

$$\begin{aligned} \int \bar{x}_{EL} dV &= \int_1^3 x \left[\pi \left(1 - \frac{2}{x} + \frac{1}{x^2}\right) dx \right] = \pi \left[\frac{x^2}{2} - 2x + \ln x \right]_1^3 \\ &= \pi \left\{ \left[\frac{3^2}{2} - 2(3) + \ln 3 \right] - \left[\frac{1^3}{2} - 2(1) + \ln 1 \right] \right\} \\ &= (1.09861\pi) \text{ m} \end{aligned}$$

Now

$$\bar{x}V = \int \bar{x}_{EL} dV: \quad \bar{x}(0.46944\pi \text{ m}^3) = 1.09861\pi \text{ m}^4 \quad \text{or } \bar{x} = 2.34 \text{ m} \quad \blacktriangleleft$$



PROBLEM 5.127

Locate the centroid of the volume obtained by rotating the shaded area about the line $x = h$.

SOLUTION

First, note that symmetry implies

$$\bar{x} = h \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

Choose as the element of volume a disk of radius r and thickness dx . Then

$$dV = \pi r^2 dy, \quad \bar{y}_{EL} = y$$

Now $x^2 = \frac{h^2}{a^2}(a^2 - y^2)$ so that $r = h - \frac{h}{a}\sqrt{a^2 - y^2}$.

Then

$$dV = \pi \frac{h^2}{a^2} \left(a - \sqrt{a^2 - y^2} \right)^2 dy$$

and

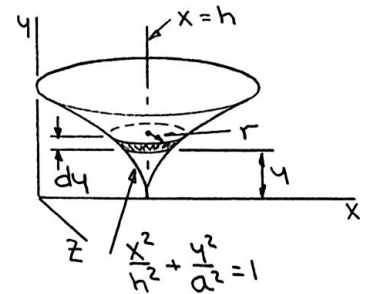
$$V = \int_0^a \pi \frac{h^2}{a^2} \left(a - \sqrt{a^2 - y^2} \right)^2 dy$$

Let

$$y = a \sin \theta \Rightarrow dy = a \cos \theta d\theta$$

Then

$$\begin{aligned} V &= \pi \frac{h^2}{a^2} \int_0^{\pi/2} \left(a - \sqrt{a^2 - a^2 \sin^2 \theta} \right)^2 a \cos \theta d\theta \\ &= \pi \frac{h^2}{a^2} \int_0^{\pi/2} \left[a^2 - 2a(a \cos \theta) + (a^2 - a^2 \sin^2 \theta) \right] a \cos \theta d\theta \\ &= \pi a h^2 \int_0^{\pi/2} (2 \cos \theta - 2 \cos^2 \theta - \sin^2 \theta \cos \theta) d\theta \\ &= \pi a h^2 \left[2 \sin \theta - 2 \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) - \frac{1}{3} \sin^3 \theta \right]_0^{\pi/2} \\ &= \pi a h^2 \left[2 - 2 \left(\frac{\pi}{2} \right) - \frac{1}{3} \right] \\ &= 0.095870 \pi a h^2 \end{aligned}$$



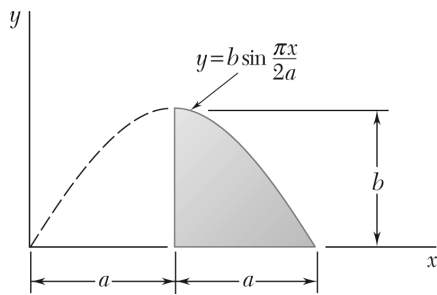
PROBLEM 5.127 (Continued)

and

$$\begin{aligned}\int \bar{y}_{EL} dV &= \int_0^a y \left[\pi \frac{h^2}{a^2} \left(a - \sqrt{a^2 - y^2} \right)^2 dy \right] \\ &= \pi \frac{h^2}{a^2} \int_0^a \left(2a^2 y - 2ay\sqrt{a^2 - y^2} - y^3 \right) dy \\ &= \pi \frac{h^2}{a^2} \left[a^2 y^2 + \frac{2}{3} a (a^2 - y^2)^{3/2} - \frac{1}{4} y^4 \right]_0^a \\ &= \pi \frac{h^2}{a^2} \left\{ \left[a^2 (a)^2 - \frac{1}{4} a^4 \right] - \left[\frac{2}{3} a (a^2)^{3/2} \right] \right\} \\ &= \frac{1}{12} \pi a^2 h^2\end{aligned}$$

Now

$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y}(0.095870\pi ah^2) = \frac{1}{12}\pi a^2 h^2 \quad \text{or } \bar{y} = 0.869a \quad \blacktriangleleft$$



PROBLEM 5.128*

Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the x -axis.

SOLUTION

First, note that symmetry implies

$$\bar{y} = 0 \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

Choose as the element of volume a disk of radius r and thickness dx .

Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

Now

$$r = b \sin \frac{\pi x}{2a}$$

so that

$$dV = \pi b^2 \sin^2 \frac{\pi x}{2a} dx$$

Then

$$\begin{aligned} V &= \int_a^{2a} \pi b^2 \sin^2 \frac{\pi x}{2a} dx \\ &= \pi b^2 \left[\frac{x}{2} - \frac{\sin \frac{\pi x}{2a}}{2 \frac{\pi}{a}} \right]_a^{2a} \\ &= \pi b^2 \left[\left(\frac{2a}{2} \right) - \left(\frac{a}{2} \right) \right] \\ &= \frac{1}{2} \pi a b^2 \end{aligned}$$

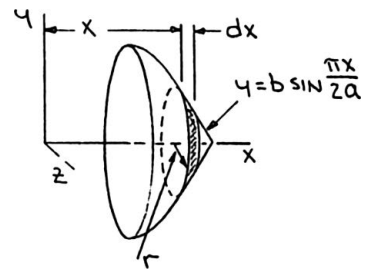
and

$$\int \bar{x}_{EL} dV = \int_a^{2a} x \left(\pi b^2 \sin^2 \frac{\pi x}{2a} dx \right)$$

Use integration by parts with

$$u = x \quad dV = \sin^2 \frac{\pi x}{2a}$$

$$du = dx \quad V = \frac{x}{2} - \frac{\sin \frac{\pi x}{a}}{\frac{2\pi}{a}}$$



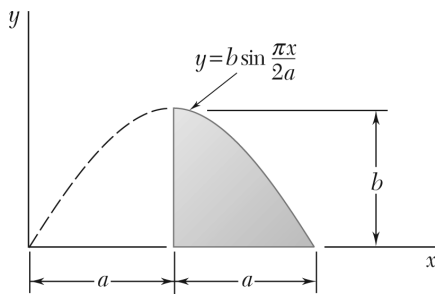
PROBLEM 5.128* (Continued)

Then

$$\begin{aligned}
 \int \bar{x}_{EL} dV &= \pi b^2 \left\{ \left[x \left(\frac{x}{2} - \frac{\sin \frac{\pi x}{a}}{2\pi/a} \right) \right]_a^{2a} - \int_a^{2a} \left(\frac{x}{2} - \frac{\sin \frac{\pi x}{a}}{2\pi/a} \right) dx \right\} \\
 &= \pi b^2 \left\{ \left[2a \left(\frac{2a}{2} \right) - a \left(\frac{a}{2} \right) \right] - \left[\frac{1}{4} x^2 + \frac{a^2}{2\pi^2} \cos \frac{\pi x}{a} \right]_a^{2a} \right\} \\
 &= \pi b^2 \left\{ \left(\frac{3}{2} a^2 \right) - \left[\frac{1}{4} (2a)^2 + \frac{a^2}{2\pi^2} - \frac{1}{4} (a)^2 + \frac{a^2}{2\pi^2} \right] \right\} \\
 &= \pi a^2 b^2 \left(\frac{3}{4} - \frac{1}{\pi^2} \right) \\
 &= 0.64868\pi a^2 b^2
 \end{aligned}$$

Now

$$\bar{x}V = \int \bar{x}_{EL} dV: \quad \bar{x} \left(\frac{1}{2} \pi a b^2 \right) = 0.64868\pi a^2 b^2 \quad \text{or } \bar{x} = 1.297a \quad \blacktriangleleft$$



PROBLEM 5.129*

Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the y -axis. (*Hint:* Use a thin cylindrical shell of radius r and thickness dr as the element of volume.)

SOLUTION

First note that symmetry implies

$$\bar{x} = 0 \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

Choose as the element of volume a cylindrical shell of radius r and thickness dr .

Then

$$dV = (2\pi r)(y)(dr), \quad \bar{y}_{EL} = \frac{1}{2}y$$

Now

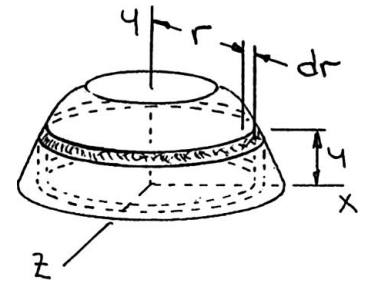
$$y = b \sin \frac{\pi r}{2a}$$

so that

$$dV = 2\pi br \sin \frac{\pi r}{2a} dr$$

Then

$$V = \int_a^{2a} 2\pi br \sin \frac{\pi r}{2a} dr$$



Use integration by parts with

$$u = rd \quad dv = \sin \frac{\pi r}{2a} dr$$

$$du = dr \quad v = -\frac{2a}{\pi} \cos \frac{\pi r}{2a}$$

Then

$$V = 2\pi b \left\{ \left[(r) \left(-\frac{2a}{\pi} \cos \frac{\pi r}{2a} \right) \right]_a^{2a} - \int_a^{2a} \left(\frac{2a}{\pi} \cos \frac{\pi r}{2a} \right) dr \right\}$$

$$= 2\pi b \left\{ -\frac{2a}{\pi} [(2a)(-1)] + \left[\frac{4a^2}{\pi^2} \sin \frac{\pi r}{2a} \right]_a^{2a} \right\}$$

$$V = 2\pi b \left(\frac{4a^2}{\pi} - \frac{4a^2}{\pi^2} \right)$$

$$= 8a^2 b \left(1 - \frac{1}{\pi} \right)$$

$$= 5.4535a^2 b$$

PROBLEM 5.129* (Continued)

Also

$$\begin{aligned}\int \bar{y}_{EL} dV &= \int_a^{2a} \left(\frac{1}{2} b \sin \frac{\pi r}{2a} \right) \left(2\pi b r \sin \frac{\pi r}{2a} dr \right) \\ &= \pi b^2 \int_a^{2a} r \sin^2 \frac{\pi r}{2a} dr\end{aligned}$$

Use integration by parts with

$$\begin{aligned}u &= r & dv &= \sin^2 \frac{\pi r}{2a} dr \\ du &= dr & v &= \frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{\frac{2\pi}{a}}\end{aligned}$$

Then

$$\begin{aligned}\int \bar{y}_{EL} dV &= \pi b^2 \left\{ \left[\left(r \right) \left(\frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{\frac{2\pi}{a}} \right) \right]_a^{2a} - \int_a^{2a} \left(\frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{\frac{2\pi}{a}} \right) dr \right\} \\ &= \pi b^2 \left\{ \left[\left(2a \right) \left(\frac{2a}{2} \right) - \left(a \right) \left(\frac{a}{2} \right) \right] - \left[\frac{r^2}{4} + \frac{a^2}{2\pi^2} \cos \frac{\pi r}{a} \right]_a^{2a} \right\} \\ &= \pi b^2 \left\{ \frac{3}{2} a^2 - \left[\frac{(2a)^2}{4} + \frac{a^2}{2\pi^2} - \frac{(a)^2}{4} + \frac{a^2}{2\pi^2} \right] \right\} \\ &= \pi a^2 b^2 \left(\frac{3}{4} - \frac{1}{\pi^2} \right) \\ &= 2.0379 a^2 b^2\end{aligned}$$

Now

$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y}(5.4535a^2b) = 2.0379a^2b^2 \quad \text{or } \bar{y} = 0.374b \quad \blacktriangleleft$$

PROBLEM 5.130*

Show that for a regular pyramid of height h and n sides ($n = 3, 4, \dots$) the centroid of the volume of the pyramid is located at a distance $h/4$ above the base.

SOLUTION

Choose as the element of a horizontal slice of thickness dy . For any number N of sides, the area of the base of the pyramid is given by

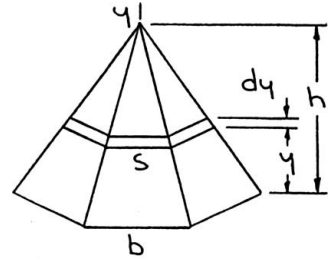
$$A_{\text{base}} = kb^2$$

where $k = k(N)$; see note below. Using similar triangles, we have

$$\frac{s}{b} = \frac{h-y}{h}$$

or

$$s = \frac{b}{h}(h-y)$$



Then

$$dV = A_{\text{slice}} dy = ks^2 dy = k \frac{b^2}{h^2} (h-y)^2 dy$$

and

$$\begin{aligned} V &= \int_0^h k \frac{b^2}{h^2} (h-y)^2 dy = k \frac{b^2}{h^2} \left[-\frac{1}{3} (h-y)^3 \right]_0^h \\ &= \frac{1}{3} kb^2 h \end{aligned}$$

Also,

$$\bar{y}_{EL} = y$$

so that

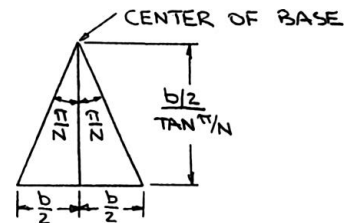
$$\begin{aligned} \int \bar{y}_{EL} dV &= \int_0^h y \left[k \frac{b^2}{h^2} (h-y)^2 dy \right] = k \frac{b^2}{h^2} \int_0^h (h^2 y - 2hy^2 + y^3) dy \\ &= k \frac{b^2}{h^2} \left[\frac{1}{2} h^2 y^2 - \frac{2}{3} hy^3 + \frac{1}{4} y^4 \right]_0^h = \frac{1}{12} kb^2 h^2 \end{aligned}$$

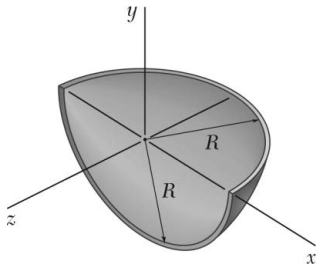
Now

$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y} \left(\frac{1}{3} kb^2 h \right) = \frac{1}{12} kb^2 h^2 \quad \text{or } \bar{y} = \frac{1}{4} h \quad \text{Q.E.D.} \quad \blacktriangleleft$$

Note:

$$\begin{aligned} A_{\text{base}} &= N \left(\frac{1}{2} \times b \times \frac{b}{2 \tan \frac{\pi}{N}} \right) \\ &= \frac{N}{4 \tan \frac{\pi}{N}} b^2 \\ &= k(N) b^2 \end{aligned}$$





PROBLEM 5.131

Determine by direct integration the location of the centroid of one-half of a thin, uniform hemispherical shell of radius R .

SOLUTION

First note that symmetry implies

$$\bar{x} = 0 \quad \blacktriangleleft$$

The element of area dA of the shell shown is obtained by cutting the shell with two planes parallel to the xy plane. Now

$$dA = (\pi r)(R d\theta)$$

$$\bar{y}_{EL} = -\frac{2r}{\pi}$$

where

$$r = R \sin \theta$$

so that

$$dA = \pi R^2 \sin \theta d\theta$$

$$\bar{y}_{EL} = -\frac{2R}{\pi} \sin \theta$$

Then

$$\begin{aligned} A &= \int_0^{\pi/2} \pi R^2 \sin \theta d\theta = \pi R^2 [-\cos \theta]_0^{\pi/2} \\ &= \pi R^2 \end{aligned}$$

and

$$\begin{aligned} \int \bar{y}_{EL} dA &= \int_0^{\pi/2} \left(-\frac{2R}{\pi} \sin \theta \right) (\pi R^2 \sin \theta d\theta) \\ &= -2R^3 \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} \\ &= -\frac{\pi}{2} R^3 \end{aligned}$$

Now

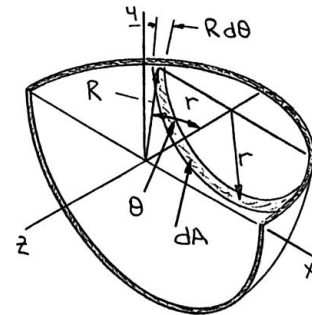
$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y}(\pi R^2) = -\frac{\pi}{2} R^3$$

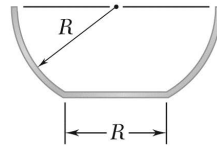
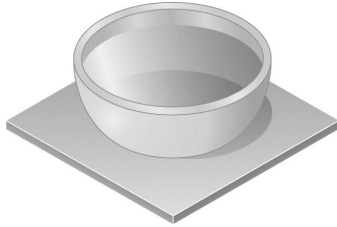
$$\text{or } \bar{y} = -\frac{1}{2} R \quad \blacktriangleleft$$

Symmetry implies

$$\bar{z} = \bar{y}$$

$$\bar{z} = -\frac{1}{2} R \quad \blacktriangleleft$$





PROBLEM 5.132

The sides and the base of a punch bowl are of uniform thickness t . If $t \ll R$ and $R = 250$ mm, determine the location of the center of gravity of (a) the bowl, (b) the punch.

SOLUTION

(a) Bowl:

First note that symmetry implies

$$\bar{x} = 0 \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

for the coordinate axes shown below. Now assume that the bowl may be treated as a shell; the center of gravity of the bowl will coincide with the centroid of the shell. For the walls of the bowl, an element of area is obtained by rotating the arc ds about the y -axis. Then

$$dA_{\text{wall}} = (2\pi R \sin \theta)(R d\theta)$$

and

$$(\bar{y}_{EL})_{\text{wall}} = -R \cos \theta$$

Then

$$\begin{aligned} A_{\text{wall}} &= \int_{\pi/6}^{\pi/2} 2\pi R^2 \sin \theta d\theta \\ &= 2\pi R^2 [-\cos \theta]_{\pi/6}^{\pi/2} \\ &= \pi\sqrt{3}R^2 \end{aligned}$$

and

$$\begin{aligned} \bar{y}_{\text{wall}} A_{\text{wall}} &= \int (\bar{y}_{EL})_{\text{wall}} dA \\ &= \int_{\pi/6}^{\pi/2} (-R \cos \theta)(2\pi R^2 \sin \theta d\theta) \\ &= \pi R^3 [\cos^2 \theta]_{\pi/6}^{\pi/2} \\ &= -\frac{3}{4}\pi R^3 \end{aligned}$$

By observation,

$$A_{\text{base}} = \frac{\pi}{4}R^2, \quad \bar{y}_{\text{base}} = -\frac{\sqrt{3}}{2}R$$

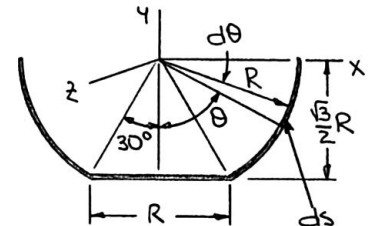
Now

$$\bar{y}\Sigma A = \Sigma \bar{y}A$$

$$\text{or} \quad \bar{y} \left(\pi\sqrt{3}R^2 + \frac{\pi}{4}R^2 \right) = -\frac{3}{4}\pi R^3 + \frac{\pi}{4}R^2 \left(-\frac{\sqrt{3}}{2}R \right)$$

$$\text{or} \quad \bar{y} = -0.48763R \quad R = 250 \text{ mm}$$

$$\bar{y} = -121.9 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 5.132 (Continued)

(b) Punch:

First note that symmetry implies

$$\bar{x} = 0 \quad \blacktriangleleft$$

$$\bar{z} = 0 \quad \blacktriangleleft$$

and that because the punch is homogeneous, its center of gravity will coincide with the centroid of the corresponding volume. Choose as the element of volume a disk of radius x and thickness dy . Then

$$dV = \pi x^2 dy, \quad \bar{y}_{EL} = y$$

Now

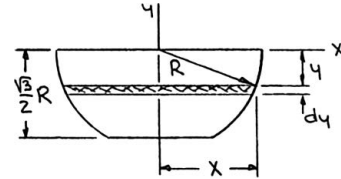
$$x^2 + y^2 = R^2$$

so that

$$dV = \pi(R^2 - y^2)dy$$

Then

$$\begin{aligned} V &= \int_{-\sqrt{3}/2 R}^0 \pi(R^2 - y^2) dy \\ &= \pi \left[R^2 y - \frac{1}{3} y^3 \right]_{-\sqrt{3}/2 R}^0 \\ &= -\pi \left[R^2 \left(-\frac{\sqrt{3}}{2} R \right) - \frac{1}{3} \left(-\frac{\sqrt{3}}{2} R \right)^3 \right] = \frac{3}{8} \pi \sqrt{3} R^3 \end{aligned}$$



and

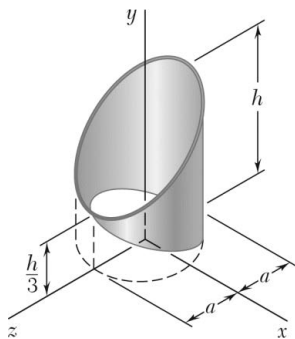
$$\begin{aligned} \int \bar{y}_{EL} dV &= \int_{-\sqrt{3}/2 R}^0 (y) \left[\pi(R^2 - y^2) dy \right] \\ &= \pi \left[\frac{1}{2} R^2 y^2 - \frac{1}{4} y^4 \right]_{-\sqrt{3}/2 R}^0 \\ &= -\pi \left[\frac{1}{2} R^2 \left(-\frac{\sqrt{3}}{2} R \right)^2 - \frac{1}{4} \left(-\frac{\sqrt{3}}{2} R \right)^4 \right] = -\frac{15}{64} \pi R^4 \end{aligned}$$

Now

$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y} \left(\frac{3}{8} \pi \sqrt{3} R^3 \right) = -\frac{15}{64} \pi R^4$$

or

$$\bar{y} = -\frac{5}{8\sqrt{3}} R \quad R = 250 \text{ mm} \quad \bar{y} = -90.2 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 5.133

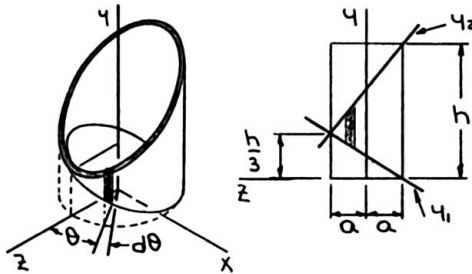
Locate the centroid of the section shown, which was cut from a thin circular pipe by two oblique planes.

SOLUTION

First note that symmetry implies

$$\bar{x} = 0 \quad \blacktriangleleft$$

Assume that the pipe has a uniform wall thickness t and choose as the element of volume a vertical strip of width $ad\theta$ and height $(y_2 - y_1)$. Then



$$dV = (y_2 - y_1)ta d\theta, \quad \bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)\bar{z}_{EL} = z$$

Now

$$y_1 = \frac{h}{3}z + \frac{h}{6} \qquad y_2 = -\frac{2h}{3}z + \frac{2}{3}h$$

$$= \frac{h}{6a}(z + a) \qquad = \frac{h}{3a}(-z + 2a)$$

and

$$z = a \cos \theta$$

Then

$$(y_2 - y_1) = \frac{h}{3a}(-a \cos \theta + 2a) - \frac{h}{6a}(a \cos \theta + a)$$

$$= \frac{h}{2}(1 - \cos \theta)$$

and

$$(y_1 + y_2) = \frac{h}{6a}(a \cos \theta + a) + \frac{h}{3a}(-a \cos \theta + 2a)$$

$$= \frac{h}{6}(5 - \cos \theta)$$

$$dV = \frac{ah}{2}(1 - \cos \theta)d\theta \quad \bar{y}_{EL} = \frac{h}{12}(5 - \cos \theta), \quad \bar{z}_{EL} = a \cos \theta$$

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PROBLEM 5.133 (Continued)

Then
$$V = 2 \int_0^\pi \frac{ah t}{2} (1 - \cos \theta) d\theta = ah t [\theta - \sin \theta]_0^\pi$$

$$= \pi ah t$$

and
$$\int \bar{y}_{EL} dV = 2 \int_0^\pi \frac{h}{12} (5 - \cos \theta) \left[\frac{ah t}{2} (1 - \cos \theta) d\theta \right]$$

$$= \frac{ah^2 t}{12} \int_0^\pi (5 - 6 \cos \theta + \cos^2 \theta) d\theta$$

$$= \frac{ah^2 t}{12} \left[5\theta - 6 \sin \theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^\pi$$

$$= \frac{11}{24} \pi ah^2 t$$

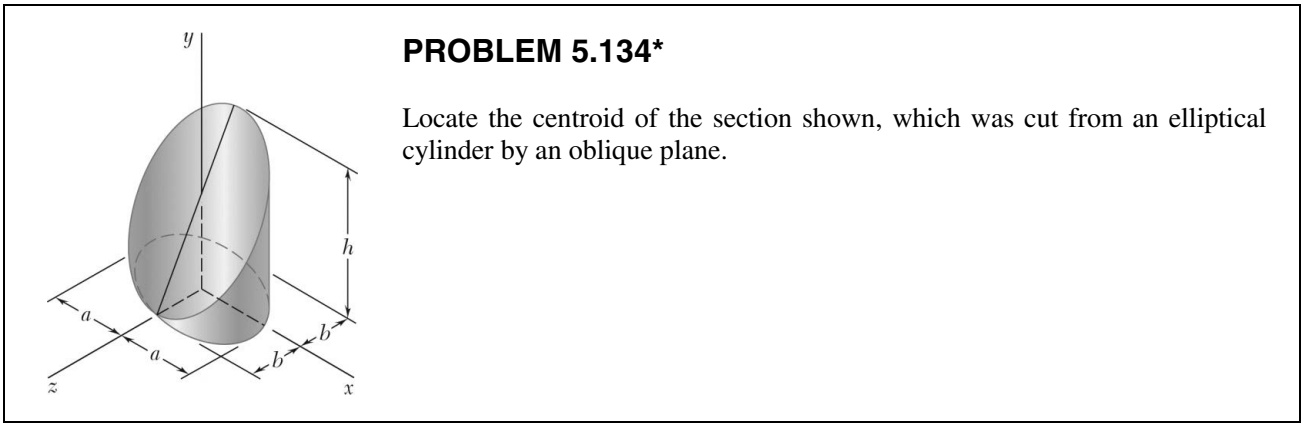
$$\int \bar{z}_{EL} dV = 2 \int_0^\pi a \cos \theta \left[\frac{ah t}{2} (1 - \cos \theta) d\theta \right]$$

$$= a^2 ht \left[\sin \theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^\pi$$

$$= -\frac{1}{2} \pi a^2 ht$$

Now
$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y}(\pi ah t) = \frac{11}{24} \pi ah^2 t$$
 or $\bar{y} = \frac{11}{24} h \blacktriangleleft$

and
$$\bar{z}V = \int \bar{z}_{EL} dV: \quad \bar{z}(\pi ah t) = -\frac{1}{2} \pi a^2 ht$$
 or $\bar{z} = -\frac{1}{2} a \blacktriangleleft$



PROBLEM 5.134*

Locate the centroid of the section shown, which was cut from an elliptical cylinder by an oblique plane.

SOLUTION

First note that symmetry implies $x = 0$ ◀

Choose as the element of volume a vertical slice of width $2x$, thickness dz , and height y . Then

$$dV = 2xy \, dz, \quad \bar{y}_{EL} = \frac{1}{24}, \quad \bar{z}_{EL} = z$$

Now $x = \frac{a}{b} \sqrt{b^2 - z^2}$

and $y = -\frac{h/2}{b} z + \frac{h}{2} = \frac{h}{2b}(b - z)$

Then $V = \int_{-b}^b \left(2 \frac{a}{b} \sqrt{b^2 - z^2} \right) \left[\frac{h}{2b}(b - z) \right] dz$

Let $z = b \sin \theta \quad dz = b \cos \theta \, d\theta$

Then
$$V = \frac{ah}{b^2} \int_{-\pi/2}^{\pi/2} (b \cos \theta) [b(1 - \sin \theta)] b \cos \theta \, d\theta$$

$$= abh \int_{-\pi/2}^{\pi/2} (\cos^2 \theta - \sin \theta \cos^2 \theta) \, d\theta$$

$$= abh \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{1}{3} \cos^3 \theta \right]_{-\pi/2}^{\pi/2}$$

$$V = \frac{1}{2} \pi abh$$

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PROBLEM 5.134* (Continued)

and
$$\int \bar{y}_{EL} dV = \int_{-b}^b \left[\frac{1}{2} \times \frac{h}{2b} (b-z) \right] \left\{ \left(2 \frac{a}{b} \sqrt{b^2 - z^2} \right) \left[\frac{h}{2b} (b-z) \right] dz \right\}$$

$$= \frac{1}{4} \frac{ah^2}{b^3} \int_{-b}^b (b-z)^2 \sqrt{b^2 - z^2} dz$$

Let
$$z = b \sin \theta \quad dz = b \cos \theta d\theta$$

Then
$$\int \bar{y}_{EL} dV = \frac{1}{4} \frac{ah^2}{b^3} \int_{-\pi/2}^{\pi/2} [b(1 - \sin \theta)]^2 (b \cos \theta) \times (b \cos \theta d\theta)$$

$$= \frac{1}{4} abh^2 \int_{-\pi/2}^{\pi/2} (\cos^2 \theta - 2 \sin \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta) d\theta$$

Now
$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta) \quad \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

so that
$$\sin^2 \theta \cos^2 \theta = \frac{1}{4}(1 - \cos^2 2\theta)$$

Then
$$\int \bar{y}_{EL} dV = \frac{1}{4} abh^2 \int_{-\pi/2}^{\pi/2} \left[\cos^2 \theta - 2 \sin \theta \cos^2 \theta + \frac{1}{4}(1 - \cos^2 2\theta) \right] d\theta$$

$$= \frac{1}{4} abh^2 \left[\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) + \frac{1}{3} \cos^3 \theta + \frac{1}{4} \theta - \frac{1}{4} \left(\frac{\theta}{2} + \frac{\sin 4\theta}{8} \right) \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{5}{32} \pi abh^2$$

Also,
$$\int \bar{z}_{EL} dV = \int_{-b}^b z \left\{ 2 \frac{a}{b} \sqrt{a^2 - z^2} \left[\frac{h}{2b} (b-z) \right] dz \right\}$$

$$= \frac{ah}{b^2} \int_{-b}^b z(b-z) \sqrt{b^2 - z^2} dz$$

Let
$$z = b \sin \theta \quad dz = b \cos \theta d\theta$$

Then
$$\int \bar{z}_{EL} dV = \frac{ah}{b^2} \int_{-\pi/2}^{\pi/2} (b \sin \theta) [b(1 - \sin \theta)] (b \cos \theta) \times (b \cos \theta d\theta)$$

$$= ab^2 h \int_{-\pi/2}^{\pi/2} (\sin \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta) d\theta$$

Using
$$\sin^2 \theta \cos^2 \theta = \frac{1}{4}(1 - \cos^2 2\theta)$$
 from above,

$$\int \bar{z}_{EL} dV = ab^2 h \int_{-\pi/2}^{\pi/2} \left[\sin \theta \cos^2 \theta - \frac{1}{4}(1 - \cos^2 2\theta) \right] d\theta$$

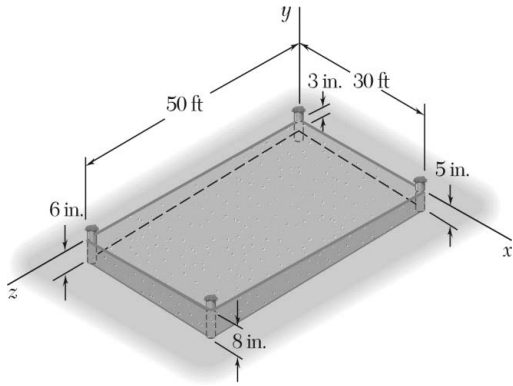
$$= ab^2 h \left[-\frac{1}{3} \cos^3 \theta - \frac{1}{4} \theta + \frac{1}{4} \left(\frac{\theta}{2} + \frac{\sin 4\theta}{8} \right) \right]_{-\pi/2}^{\pi/2} = -\frac{1}{8} \pi ab^2 h$$

PROBLEM 5.134* (Continued)

Now $\bar{y}V = \int \bar{y}_{EL} dV: \bar{y} \left(\frac{1}{2} \pi abh \right) = \frac{5}{32} \pi abh^2$ or $\bar{y} = \frac{5}{16} h \blacktriangleleft$

and $\bar{z}V = \int \bar{z}_{EL} dV: \bar{z} \left(\frac{1}{2} \pi abh \right) = -\frac{1}{8} \pi ab^2 h$ or $\bar{z} = -\frac{1}{4} b \blacktriangleleft$

PROBLEM 5.135



After grading a lot, a builder places four stakes to designate the corners of the slab for a house. To provide a firm, level base for the slab, the builder places a minimum of 3 in. of gravel beneath the slab. Determine the volume of gravel needed and the x coordinate of the centroid of the volume of the gravel. (*Hint:* The bottom surface of the gravel is an oblique plane, which can be represented by the equation $y = a + bx + cz$.)

SOLUTION

The centroid can be found by integration. The equation for the bottom of the gravel is $y = a + bx + cz$, where the constants a , b , and c can be determined as follows:

For $x = 0$ and $z = 0$, $y = -3$ in., and therefore,

$$-\frac{3}{12} \text{ ft} = a, \text{ or } a = -\frac{1}{4} \text{ ft}$$

For $x = 30$ ft and $z = 0$, $y = -5$ in., and therefore,

$$-\frac{5}{12} \text{ ft} = -\frac{1}{4} \text{ ft} + b(30 \text{ ft}), \text{ or } b = -\frac{1}{180}$$

For $x = 0$ and $z = 50$ ft, $y = -6$ in., and therefore,

$$-\frac{6}{12} \text{ ft} = -\frac{1}{4} \text{ ft} + c(50 \text{ ft}), \text{ or } c = -\frac{1}{200}$$

Therefore,

$$y = -\frac{1}{4} \text{ ft} - \frac{1}{180}x - \frac{1}{200}z$$

Now

$$\bar{x} = \frac{\int x_{EL} dV}{V}$$

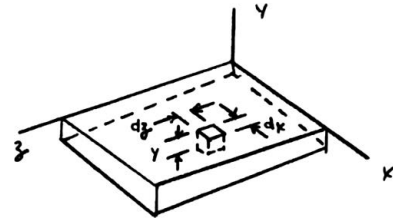
A volume element can be chosen as

$$dV = |y| dx dz$$

or

$$dV = \frac{1}{4} \left(1 + \frac{1}{45}x + \frac{1}{50}z \right) dx dz$$

and

$$\bar{x}_{EL} = x$$


PROBLEM 5.135 (Continued)

Then

$$\begin{aligned}\int x_{EL} dV &= \int_0^{50} \int_0^{30} \frac{x}{4} \left(1 + \frac{1}{45}x + \frac{1}{50}z \right) dx dz \\ &= \frac{1}{4} \int_0^{50} \left[\frac{x^2}{2} + \frac{1}{135}x^3 + \frac{z}{100}x^2 \right]_0^{30} dz \\ &= \frac{1}{4} \int_0^{50} (650 + 9z) dz \\ &= \frac{1}{4} \left[650z + \frac{9}{2}z^2 \right]_0^{50} \\ &= 10937.5 \text{ ft}^4\end{aligned}$$

The volume is

$$\begin{aligned}V \int dV &= \int_0^{50} \int_0^{30} \frac{1}{4} \left(1 + \frac{1}{45}x + \frac{1}{50}z \right) dx dz \\ &= \frac{1}{4} \int_0^{50} \left[x + \frac{1}{90}x^2 + \frac{z}{50}x \right]_0^{30} dz \\ &= \frac{1}{4} \int_0^{50} \left(40 + \frac{3}{5}z \right) dz \\ &= \frac{1}{4} \left[40z + \frac{3}{10}z^2 \right]_0^{50} \\ &= 687.50 \text{ ft}^3\end{aligned}$$

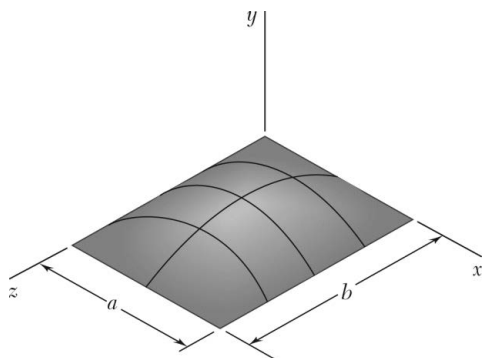
Then

$$\bar{x} = \frac{\int \bar{x}_{EL} dV}{V} = \frac{10937.5 \text{ ft}^4}{687.5 \text{ ft}^3} = 15.9091 \text{ ft}$$

Therefore,

$$V = 688 \text{ ft}^3 \blacktriangleleft$$

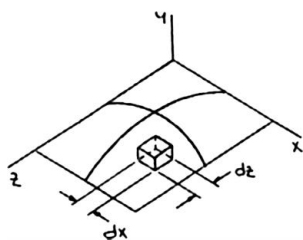
$$\bar{x} = 15.91 \text{ ft} \blacktriangleleft$$



PROBLEM 5.136

Determine by direct integration the location of the centroid of the volume between the xz plane and the portion shown of the surface $y = 16h(ax - x^2)(bz - z^2)/a^2b^2$.

SOLUTION



First note that symmetry implies

$$\bar{x} = \frac{a}{2} \blacktriangleleft$$

$$\bar{z} = \frac{b}{2} \blacktriangleleft$$

Choose as the element of volume a filament of base $dx \times dz$ and height y . Then

$$dV = y \, dx \, dz, \quad \bar{y}_{EL} = \frac{1}{2} y$$

or

$$dV = \frac{16h}{a^2b^2} (ax - x^2)(bz - z^2) \, dx \, dz$$

Then

$$V = \int_0^b \int_0^a \frac{16h}{a^2b^2} (ax - x^2)(bz - z^2) \, dx \, dz$$

$$\begin{aligned} V &= \frac{16h}{a^2b^2} \int_0^b (bz - z^2) \left[\frac{a}{z} x^2 - \frac{1}{3} x^3 \right]_0^a dz \\ &= \frac{16h}{a^2b^2} \left[\frac{a}{2} (a^2) - \frac{1}{3} (a)^3 \right] \left[\frac{b}{2} z^2 - \frac{1}{3} z^3 \right]_0^b \\ &= \frac{8ah}{3b^2} \left[\frac{b}{2} (b)^2 - \frac{1}{3} (b)^3 \right] \\ &= \frac{4}{9} abh \end{aligned}$$

PROBLEM 5.136 (Continued)

and

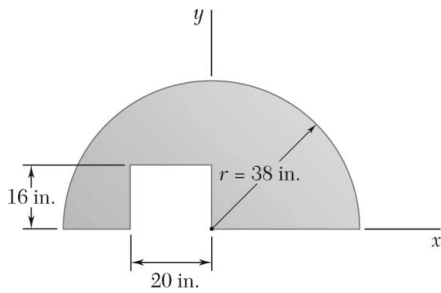
$$\begin{aligned}
 \int \bar{y}_{EL} dV &= \int_0^b \int_0^a \frac{1}{2} \left[\frac{16h}{a^2 b^2} (ax - x^2)(bz - z^2) \right] \left[\frac{16h}{a^2 b^2} (ax - x^2)(bz - z^2) dx dz \right] \\
 &= \frac{128h^2}{a^4 b^4} \int_0^b \int_0^a (a^2 x^2 - 2ax^3 + x^4)(b^2 z^2 - 2bz^3 + z^4) dx dz \\
 &= \frac{128h^2}{a^2 b^4} \int_0^b (b^2 z^2 - 2bz^3 + z^4) \left[\frac{a^2}{3} x^3 - \frac{a}{2} x^4 + \frac{1}{5} x^5 \right]_0^a dz \\
 &= \frac{128h^2}{a^4 b^4} \left[\frac{a^2}{3} (a)^3 - \frac{a}{2} (a)^4 + \frac{1}{5} (a)^5 \right] \left[\frac{b^2}{3} z^3 - \frac{b}{z} z^4 + \frac{1}{5} z^5 \right]_0^b \\
 &= \frac{64ah^2}{15b^4} \left[\frac{b^3}{3} (b)^3 - \frac{b}{2} (b)^4 + \frac{1}{5} (b)^5 \right] = \frac{32}{225} abh^2
 \end{aligned}$$

Now

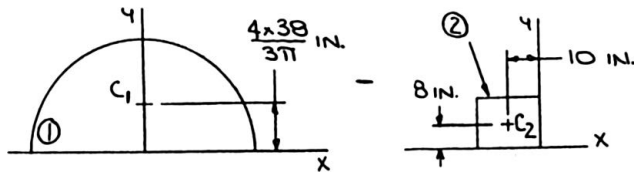
$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y} \left(\frac{4}{9} abh \right) = \frac{32}{225} abh^2 \quad \text{or} \quad \bar{y} = \frac{8}{25} h \quad \blacktriangleleft$$

PROBLEM 5.137

Locate the centroid of the plane area shown.



SOLUTION



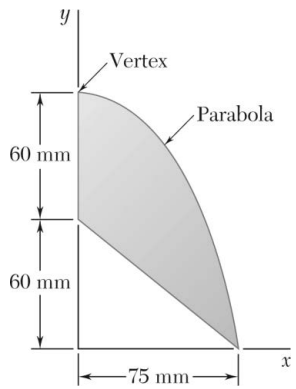
	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\frac{\pi}{2}(38)^2 = 2268.2$	0	16.1277	0	36,581
2	$-20 \times 16 = 320$	-10	8	3200	-2560
Σ	1948.23			3200	34,021

Then
$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{3200}{1948.23} \quad \bar{X} = 1.643 \text{ in.} \blacktriangleleft$$

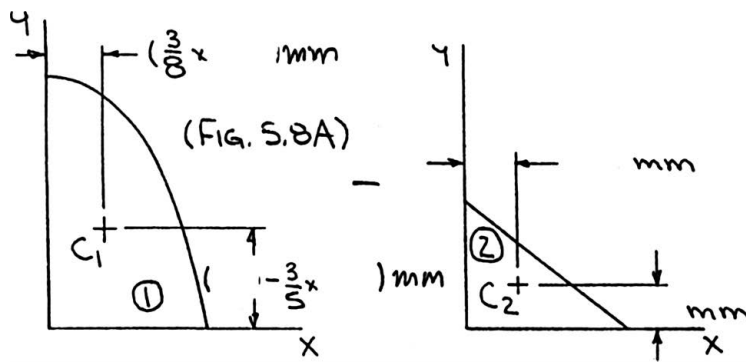
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{34,021}{1948.23} \quad \bar{Y} = 17.46 \text{ in.} \blacktriangleleft$$

PROBLEM 5.138

Locate the centroid of the plane area shown.



SOLUTION



	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$\frac{2}{3}(75)(120) = 6000$	28.125	48	168,750	288,000
2	$-\frac{1}{2}(75)(60) = -2250$	25	20	-56,250	-45,000
Σ	3750			112,500	243,000

Then

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(3750 \text{ mm}^2) = 112,500 \text{ mm}^3$$

$$\text{or } \bar{X} = 30.0 \text{ mm} \blacktriangleleft$$

and

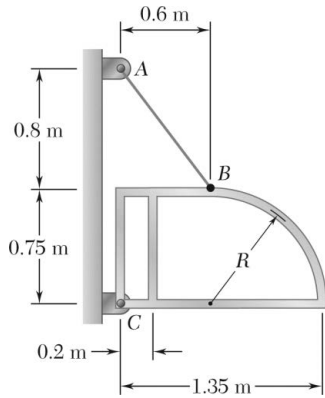
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(3750 \text{ mm}^2) = 243,000 \text{ mm}^3$$

$$\text{or } \bar{Y} = 64.8 \text{ mm} \blacktriangleleft$$

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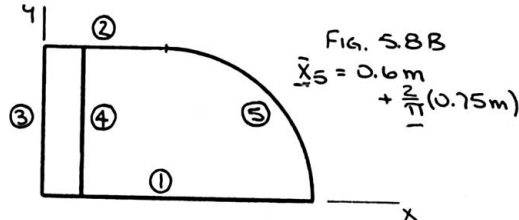
PROBLEM 5.139



The frame for a sign is fabricated from thin, flat steel bar stock of mass per unit length 4.73 kg/m. The frame is supported by a pin at C and by a cable AB. Determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION

First note that because the frame is fabricated from uniform bar stock, its center of gravity will coincide with the centroid of the corresponding line.



	L, m	\bar{x}, m	$\bar{x}L, m^2$
1	1.35	0.675	0.91125
2	0.6	0.3	0.18
3	0.75	0	0
4	0.75	0.2	0.15
5	$\frac{\pi}{2}(0.75) = 1.17810$	1.07746	1.26936
Σ	4.62810		2.5106

Then

$$\bar{X}\Sigma L = \Sigma \bar{x}L$$

$$\bar{X}(4.62810) = 2.5106$$

or

$$\bar{X} = 0.54247 m$$

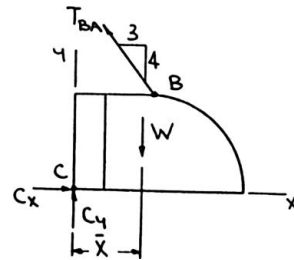
The free-body diagram of the frame is then

where

$$W = (m'\Sigma L)g$$

$$= 4.73 \text{ kg/m} \times 4.62810 \text{ m} \times 9.81 \text{ m/s}^2$$

$$= 214.75 \text{ N}$$



PROBLEM 5.139 (Continued)

Equilibrium then requires

$$(a) \quad \Sigma M_C = 0: (1.55 \text{ m})\left(\frac{3}{5}T_{BA}\right) - (0.54247 \text{ m})(214.75 \text{ N}) = 0$$

$$\text{or} \quad T_{BA} = 125.264 \text{ N} \quad \text{or} \quad T_{BA} = 125.3 \text{ N} \quad \blacktriangleleft$$

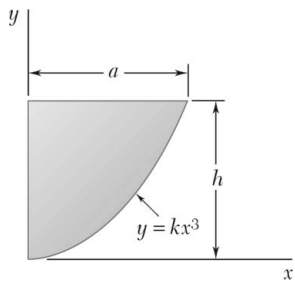
$$(b) \quad \Sigma F_x = 0: C_x - \frac{3}{5}(125.264 \text{ N}) = 0$$

$$\text{or} \quad C_x = 75.158 \text{ N} \quad \rightarrow$$

$$\Sigma F_y = 0: C_y + \frac{4}{5}(125.264 \text{ N}) - (214.75 \text{ N}) = 0$$

$$\text{or} \quad C_y = 114.539 \text{ N} \quad \uparrow$$

$$\text{Then} \quad C = 137.0 \text{ N} \quad \nearrow 56.7^\circ \quad \blacktriangleleft$$



PROBLEM 5.140

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h .

SOLUTION

For the element (EL) shown,

$$\text{at } x = a, y = h, \quad h = ka^3 \quad \text{or} \quad k = \frac{h}{a^3}$$

$$\text{Then} \quad x = \frac{a}{h^{1/3}} y^{1/3}$$

$$\text{Now} \quad dA = x dy = \frac{a}{h^{1/3}} y^{1/3} dy$$

$$\bar{x}_{EL} = \frac{1}{2} x = \frac{1}{2} \frac{a}{h^{1/3}} y^{1/3}$$

$$\bar{y}_{EL} = y$$

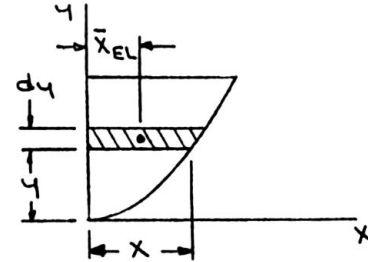
$$\text{Then} \quad A = \int dA = \int_0^h \frac{a}{h^{1/3}} y^{1/3} dy = \frac{3}{4} \frac{a}{h^{1/3}} \left(y^{4/3} \right) \Big|_0^h = \frac{3}{4} ah$$

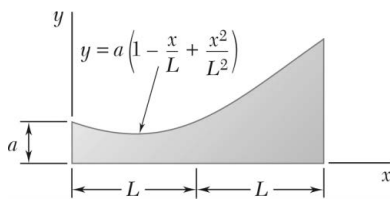
$$\text{and} \quad \int \bar{x}_{EL} dA = \int_0^h \frac{1}{2} \frac{a}{h^{1/3}} y^{1/3} \left(\frac{a}{h^{1/3}} y^{1/3} dy \right) = \frac{1}{2} \frac{a}{h^{2/3}} \left(\frac{3}{5} y^{5/3} \right) \Big|_0^h = \frac{3}{10} a^2 h$$

$$\int \bar{y}_{EL} dA = \int_0^h y \left(\frac{a}{h^{1/3}} y^{1/3} dy \right) = \frac{a}{h^{1/3}} \left(\frac{3}{7} y^{7/3} \right) \Big|_0^h = \frac{3}{7} ah^2$$

$$\text{Hence} \quad \bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{3}{4} ah \right) = \frac{3}{10} a^2 h \quad \bar{x} = \frac{2}{5} a \quad \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{3}{4} ah \right) = \frac{3}{7} ah^2 \quad \bar{y} = \frac{4}{7} h \quad \blacktriangleleft$$





PROBLEM 5.141

Determine by direct integration the centroid of the area shown.

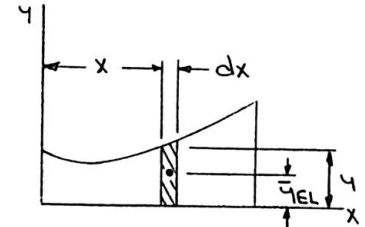
SOLUTION

We have

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2} \left(1 - \frac{x}{L} + \frac{x^2}{L^2} \right)$$

$$dA = y dx = a \left(1 - \frac{x}{L} + \frac{x^2}{L^2} \right) dx$$



Then

$$A = \int dA = \int_0^{2L} a \left(1 - \frac{x}{L} + \frac{x^2}{L^2} \right) dx = a \left[x - \frac{x^2}{2L} + \frac{x^3}{3L^2} \right]_0^{2L}$$

$$= \frac{8}{3} aL$$

and

$$\int \bar{x}_{EL} dA = \int_0^{2L} x \left[a \left(1 - \frac{x}{L} + \frac{x^2}{L^2} \right) dx \right] = a \left[\frac{x^2}{2} - \frac{x^3}{3L} + \frac{x^4}{4L^2} \right]_0^{2L}$$

$$= \frac{10}{3} aL^2$$

$$\int \bar{y}_{EL} dA = \int_0^{2L} \frac{a}{2} \left(1 - \frac{x}{L} + \frac{x^2}{L^2} \right) \left[a \left(1 - \frac{x}{L} + \frac{x^2}{L^2} \right) dx \right]$$

$$= \frac{a^2}{2} \int_0^{2L} \left(1 - 2\frac{x}{L} + 3\frac{x^2}{L^2} - 2\frac{x^3}{L^3} + \frac{x^4}{L^4} \right) dx$$

$$= \frac{a^2}{2} \left[x - \frac{x^2}{L} + \frac{x^3}{L^2} - \frac{x^4}{2L^3} + \frac{x^5}{5L^4} \right]_0^{2L}$$

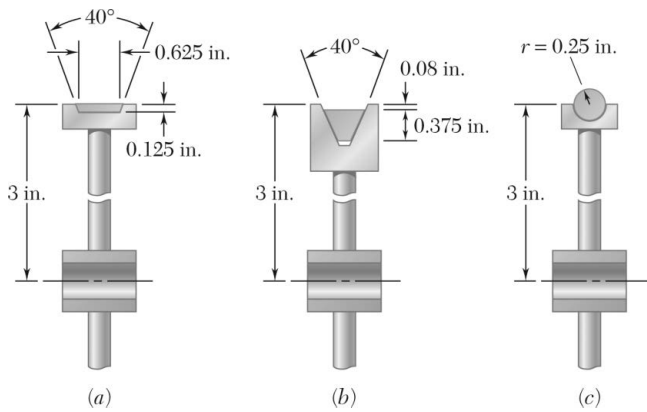
$$= \frac{11}{5} a^2 L$$

Hence,

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x} \left(\frac{8}{3} aL \right) = \frac{10}{3} aL^2 \quad \bar{x} = \frac{5}{4} L \quad \blacktriangleleft$$

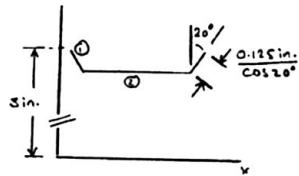
$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y} \left(\frac{1}{8} a \right) = \frac{11}{5} a^2 \quad \bar{y} = \frac{33}{40} a \quad \blacktriangleleft$$

PROBLEM 5.142



Three different drive belt profiles are to be studied. If at any given time each belt makes contact with one-half of the circumference of its pulley, determine the *contact area* between the belt and the pulley for each design.

SOLUTION



SOLUTION

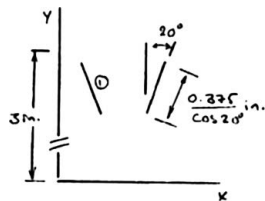
Applying the first theorem of Pappus-Guldinus, the contact area A_C of a belt is given by

$$A_C = \pi \bar{y} L = \pi \sum \bar{y} L$$

where the individual lengths are the lengths of the belt cross section that are in contact with the pulley.

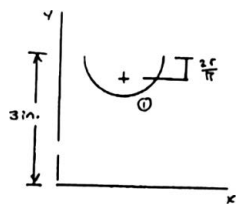
$$(a) \quad A_C = \pi [2(\bar{y}_1 L_1) + \bar{y}_2 L_2] \\ = \pi \left\{ 2 \left[\left(3 - \frac{0.125}{2} \right) \text{in.} \right] \left[\frac{0.125 \text{ in.}}{\cos 20^\circ} \right] + [(3 - 0.125) \text{ in.}] (0.625 \text{ in.}) \right\}$$

$$\text{or} \quad A_C = 8.10 \text{ in}^2 \quad \blacktriangleleft$$



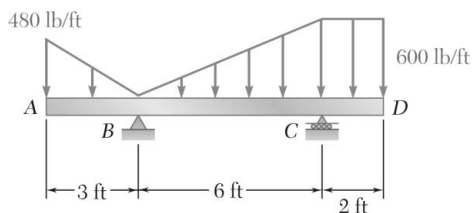
$$(b) \quad A_C = \pi [2(\bar{y}_1 L_1)] \\ = 2\pi \left[\left(3 - 0.08 - \frac{0.375}{2} \right) \text{in.} \right] \left(\frac{0.375 \text{ in.}}{\cos 20^\circ} \right)$$

$$\text{or} \quad A_C = 6.85 \text{ in}^2 \quad \blacktriangleleft$$



$$(c) \quad A_C = \pi [2(\bar{y}_1 L_1)] \\ = \pi \left[\left(3 - \frac{2(0.25)}{\pi} \right) \text{in.} \right] [\pi (0.25 \text{ in.})]$$

$$\text{or} \quad A_C = 7.01 \text{ in}^2 \quad \blacktriangleleft$$



PROBLEM 5.143

Determine the reactions at the beam supports for the given loading.

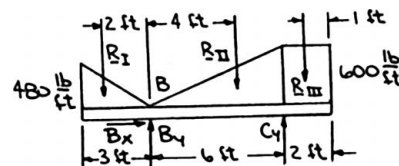
SOLUTION

We have

$$R_I = \frac{1}{2}(3 \text{ ft})(480 \text{ lb/ft}) = 720 \text{ lb}$$

$$R_{II} = \frac{1}{2}(6 \text{ ft})(600 \text{ lb/ft}) = 1800 \text{ lb}$$

$$R_{III} = (2 \text{ ft})(600 \text{ lb/ft}) = 1200 \text{ lb}$$



Then

$$\rightarrow \Sigma F_x = 0: \quad B_x = 0$$

$$\curvearrowright \Sigma M_B = 0: \quad (2 \text{ ft})(720 \text{ lb}) - (4 \text{ ft})(1800 \text{ lb}) + (6 \text{ ft})C_y - (7 \text{ ft})(1200 \text{ lb}) = 0$$

or

$$C_y = 2360 \text{ lb}$$

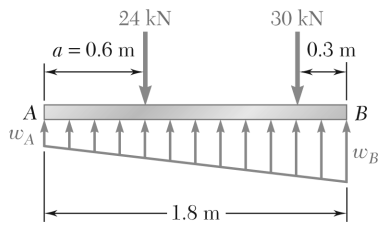
$$C = 2360 \text{ lb} \uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad -720 \text{ lb} + B_y - 1800 \text{ lb} + 2360 \text{ lb} - 1200 \text{ lb} = 0$$

or

$$B_y = 1360 \text{ lb}$$

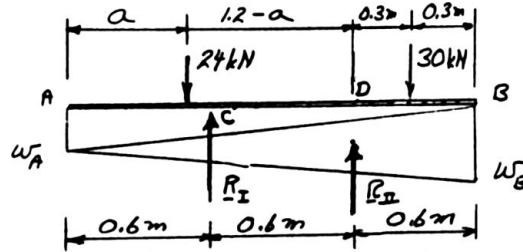
$$B = 1360 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 5.144

The beam AB supports two concentrated loads and rests on soil that exerts a linearly distributed upward load as shown. Determine the values of w_A and w_B corresponding to equilibrium.

SOLUTION



$$R_I = \frac{1}{2} w_A (1.8 \text{ m}) = 0.9 w_A$$

$$R_{II} = \frac{1}{2} w_B (1.8 \text{ m}) = 0.9 w_B$$

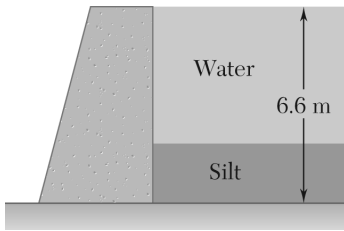
$$+\circlearrowleft \Sigma M_D = 0: (24 \text{ kN})(1.2 - a) - (30 \text{ kN})(0.3 \text{ m}) - (0.9 w_A)(0.6 \text{ m}) = 0 \quad (1)$$

For $a = 0.6 \text{ m}$,

$$24(1.2 - 0.6) - (30)(0.3) - 0.54 w_A = 0$$

$$14.4 - 9 - 0.54 w_A = 0 \quad w_A = 10.00 \text{ kN/m} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -24 \text{ kN} - 30 \text{ kN} + 0.9(10 \text{ kN/m}) + 0.9 w_B = 0 \quad w_B = 50.0 \text{ kN/m} \quad \blacktriangleleft$$



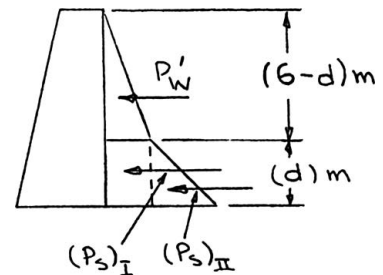
PROBLEM 5.145

The base of a dam for a lake is designed to resist up to 120 percent of the horizontal force of the water. After construction, it is found that silt (that is equivalent to a liquid of density $\rho_s = 1.76 \times 10^3 \text{ kg/m}^3$) is settling on the lake bottom at the rate of 12 mm/year. Considering a 1-m-wide section of dam, determine the number of years until the dam becomes unsafe.

SOLUTION

First determine force on dam without the silt,

$$\begin{aligned}
 P_w &= \frac{1}{2} A_{p_w} = \frac{1}{2} A(\rho gh) \\
 &= \frac{1}{2} [(6.6 \text{ m})(1 \text{ m})] [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6.6 \text{ m})] \\
 &= 213.66 \text{ kN} \\
 P_{\text{allow}} &= 1.2P_w = (1.5)(213.66 \text{ kN}) = 256.39 \text{ kN}
 \end{aligned}$$



Next determine the force P' on the dam face after a depth d of silt has settled.

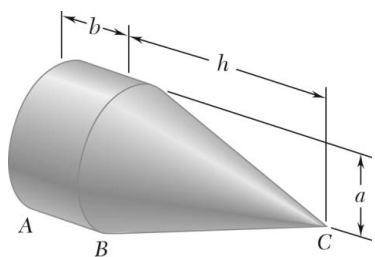
$$\begin{aligned}
 \text{We have } P'_w &= \frac{1}{2} [(6.6 - d) \text{ m} \times (1 \text{ m})] [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6.6 - d) \text{ m}] \\
 &= 4.905(6.6 - d)^2 \text{ kN} \\
 (P_s)_I &= [d(1 \text{ m})] [(10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6.6 - d) \text{ m}] \\
 &= 9.81(6.6d - d^2) \text{ kN} \\
 (P_s)_{II} &= \frac{1}{2} [d(1 \text{ m})] [(1.76 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(d) \text{ m}] \\
 &= 8.6328d^2 \text{ kN} \\
 P' &= P'_w + (P_s)_I + (P_s)_{II} = [4.905(43.560 - 13.2000d + d^2) \\
 &\quad + 9.81(6.6d - d^2) + 8.6328d^2] \text{ kN} \\
 &= [3.7278d^2 + 213.66] \text{ kN}
 \end{aligned}$$

Now it's required that $P' = P_{\text{allow}}$ to determine the maximum value of d .

$$(3.7278d^2 + 213.66) \text{ kN} = 256.39 \text{ kN}$$

$$\text{or } d = 3.3856 \text{ m}$$

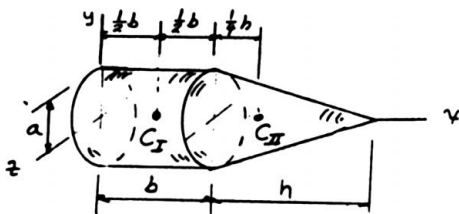
$$\text{Finally, } 3.3856 \text{ m} = 12 \times 10^{-3} \frac{\text{m}}{\text{year}} \times N \quad \text{or } N = 282 \text{ years} \blacktriangleleft$$



PROBLEM 5.146

Determine the location of the centroid of the composite body shown when (a) $h = 2b$, (b) $h = 2.5b$.

SOLUTION



	V	\bar{x}	$\bar{x}V$
Cylinder I	$\pi a^2 b$	$\frac{1}{2}b$	$\frac{1}{2}\pi a^2 b^2$
Cone II	$\frac{1}{3}\pi a^2 h$	$b + \frac{1}{4}h$	$\frac{1}{3}\pi a^2 h \left(b + \frac{1}{4}h \right)$

$$V = \pi a^2 \left(b + \frac{1}{3}h \right)$$

$$\Sigma \bar{x}V = \pi a^2 \left(\frac{1}{2}b^2 + \frac{1}{3}hb + \frac{1}{12}h^2 \right)$$

(a) For $h = 2b$,

$$V = \pi a^2 \left[b + \frac{1}{3}(2b) \right] = \frac{5}{3}\pi a^2 b$$

$$\Sigma \bar{x}V = \pi a^2 \left[\frac{1}{2}b^2 + \frac{1}{3}(2b)b + \frac{1}{12}(2b)^2 \right]$$

$$= \pi a^2 b^2 \left[\frac{1}{2} + \frac{2}{3} + \frac{1}{3} \right] = \frac{3}{2}\pi a^2 b^2$$

$$\bar{X}V = \Sigma \bar{x}V: \quad \bar{X} \left(\frac{5}{3}\pi a^2 b \right) = \frac{3}{2}\pi a^2 b^2 \quad \bar{X} = \frac{9}{10}b$$

Centroid is $\frac{1}{10}b$ to left of base of cone. ◀

PROBLEM 5.146 (Continued)

(b) For $h = 2.5b$,
$$V = \pi a^2 \left[b + \frac{1}{3}(2.5b) \right] = 1.8333\pi a^2 b$$

$$\begin{aligned}\Sigma \bar{x}V &= \pi a^2 \left[\frac{1}{2}b^2 + \frac{1}{3}(2.5b)b + \frac{1}{12}(2.5b)^2 \right] \\ &= \pi a^2 b^2 [0.5 + 0.8333 + 0.52083] \\ &= 1.85416\pi a^2 b^2\end{aligned}$$

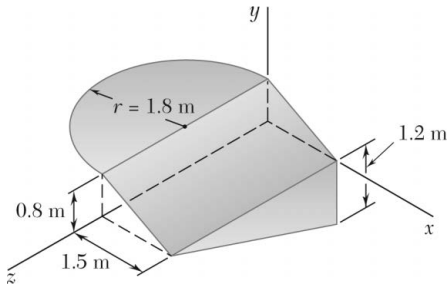
$$\bar{X}V = \Sigma \bar{x}V: \quad \bar{X}(1.8333\pi a^2 b) = 1.85416\pi a^2 b^2 \quad \bar{X} = 1.01136b$$

Centroid is $0.01136b$ to right of base of cone. ◀

Note: Centroid is at base of cone for $h = \sqrt{6}b = 2.449b$.

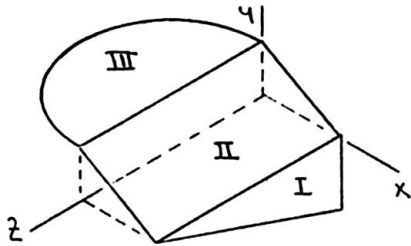
PROBLEM 5.147

Locate the center of gravity of the sheet-metal form shown.



SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.



$$\bar{y}_I = -\frac{1}{3}(1.2) = -0.4 \text{ m}$$

$$\bar{z}_I = \frac{1}{3}(3.6) = 1.2 \text{ m}$$

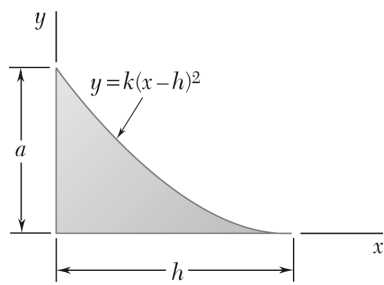
$$\bar{x}_{III} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi} \text{ m}$$

	A, m^2	\bar{x}, m	\bar{y}, m	\bar{z}, m	$\bar{x}A, \text{m}^3$	$\bar{y}A, \text{m}^3$	$\bar{z}A, \text{m}^3$
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	-0.4	1.2	3.24	-0.864	2.592
II	$(3.6)(1.7) = 6.12$	0.75	0.4	1.8	4.59	2.448	11.016
III	$\frac{\pi}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	-3.888	4.0715	9.1609
Σ	13.3694				3.942	5.6555	22.769

We have $\bar{X}\Sigma V = \Sigma\bar{x}V$: $\bar{X}(13.3694 \text{ m}^2) = 3.942 \text{ m}^3$ or $\bar{X} = 0.295 \text{ m} \blacktriangleleft$

$\bar{Y}\Sigma V = \Sigma\bar{y}V$: $\bar{Y}(13.3694 \text{ m}^2) = 5.6555 \text{ m}^3$ or $\bar{Y} = 0.423 \text{ m} \blacktriangleleft$

$\bar{Z}\Sigma V = \Sigma\bar{z}V$: $\bar{Z}(13.3694 \text{ m}^2) = 22.769 \text{ m}^3$ or $\bar{Z} = 1.703 \text{ m} \blacktriangleleft$



PROBLEM 5.148

Locate the centroid of the volume obtained by rotating the shaded area about the x -axis.

SOLUTION

First note that symmetry implies

$$\bar{y} = 0 \quad \blacktriangleleft$$

and

$$\bar{z} = 0 \quad \blacktriangleleft$$

We have

$$y = k(X - h)^2$$

At $x = 0, y = a,$

$$a = k(-h)^2$$

or

$$k = \frac{a}{h^2}$$

Choose as the element of volume a disk of radius r and thickness dx . Then

$$dV = \pi r^2 dx, \quad \bar{X}_{EL} = x$$

Now

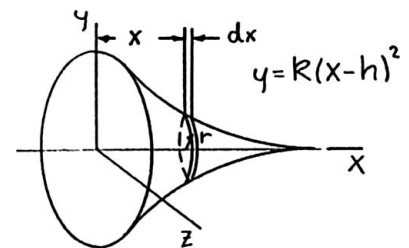
$$r = \frac{a}{h^2}(x - h)^2$$

so that

$$dV = \pi \frac{a^2}{h^4}(x - h)^4 dx$$

Then

$$\begin{aligned} V &= \int_0^h \pi \frac{a^2}{h^4}(x - h)^4 dx = \frac{\pi a^2}{5 h^4} [(x - h)^5]_0^h \\ &= \frac{1}{5} \pi a^2 h \end{aligned}$$



and

$$\begin{aligned} \int \bar{x}_{EL} dV &= \int_0^h x \left[\pi \frac{a^2}{h^4}(x - h)^4 dx \right] \\ &= \pi \frac{a^2}{h^4} \int_0^h (x^5 - 4hx^4 + 6h^2x^3 - 4h^3x^2 + h^4x) dx \\ &= \pi \frac{a^2}{h^4} \left[\frac{1}{6}x^6 - \frac{4}{5}hx^5 + \frac{3}{2}h^2x^4 - \frac{4}{3}h^3x^3 + \frac{1}{2}h^4x^2 \right]_0^h \\ &= \frac{1}{30} \pi a^2 h^2 \end{aligned}$$

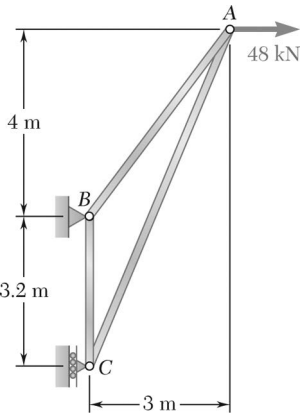
Now

$$\bar{x}V = \int \bar{x}_{EL} dV: \quad \bar{x} \left(\frac{\pi a^2 h}{5} \right) = \frac{\pi}{30} a^2 h^2$$

$$\text{or } \bar{x} = \frac{1}{6} h \quad \blacktriangleleft$$

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CHAPTER 6



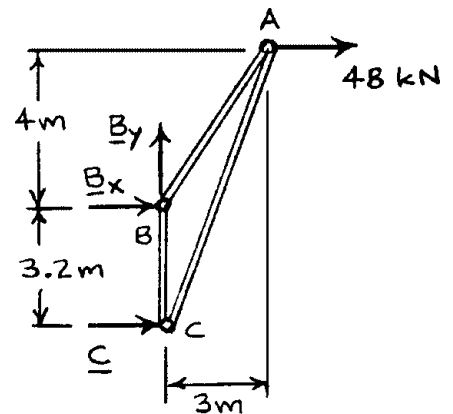
PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

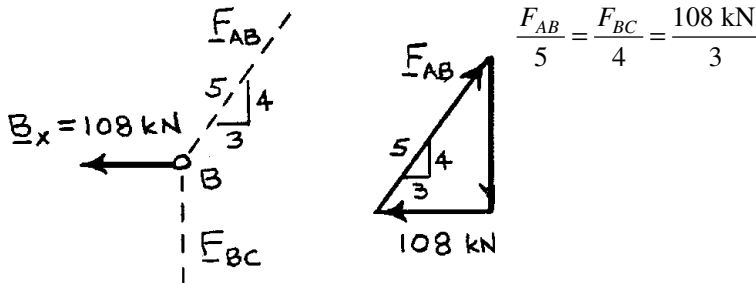
SOLUTION

Free body: Entire truss:

$$\begin{aligned}
 +\uparrow \Sigma F_y = 0: & \quad B_y = 0 \quad B_y = 0 \\
 +\curvearrowright \Sigma M_C = 0: & \quad -B_x(3.2 \text{ m}) - (48 \text{ kN})(7.2 \text{ m}) = 0 \\
 & \quad B_x = -108 \text{ kN} \quad B_x = 108 \text{ kN} \leftarrow \\
 +\rightarrow \Sigma F_x = 0: & \quad C - 108 \text{ kN} + 48 \text{ kN} = 0 \\
 & \quad C = 60 \text{ kN} \quad C = 60 \text{ kN} \rightarrow
 \end{aligned}$$



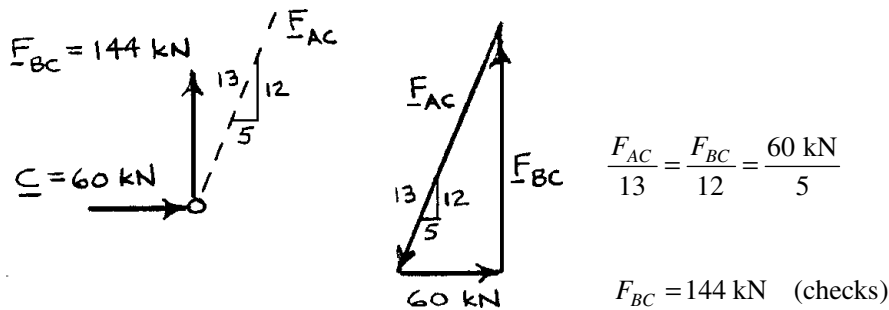
Free body: Joint B:



$$F_{AB} = 180.0 \text{ kN} \quad T \leftarrow$$

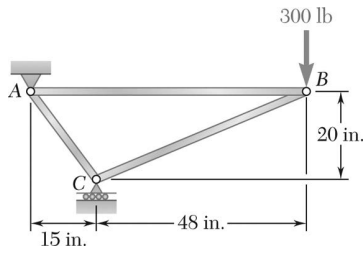
$$F_{BC} = 144.0 \text{ kN} \quad T \leftarrow$$

Free body: Joint C:



$$F_{AC} = 156.0 \text{ kN} \quad C \leftarrow$$

$$F_{BC} = 144 \text{ kN} \quad (\text{checks})$$



PROBLEM 6.2

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

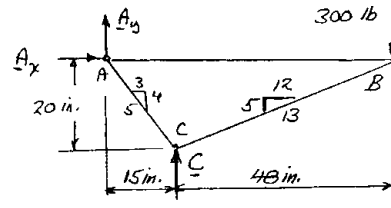
SOLUTION

Reactions:

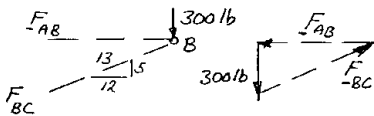
$$\Sigma M_A = 0: C = 1260 \text{ lb } \uparrow$$

$$\Sigma F_x = 0: A_x = 0$$

$$\Sigma F_y = 0: A_y = 960 \text{ lb } \downarrow$$



Joint B:

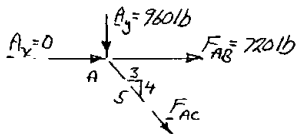


$$\frac{F_{AB}}{12} = \frac{F_{BC}}{13} = \frac{300 \text{ lb}}{5}$$

$$F_{AB} = 720 \text{ lb } T \quad \blacktriangleleft$$

$$F_{BC} = 780 \text{ lb } C \quad \blacktriangleleft$$

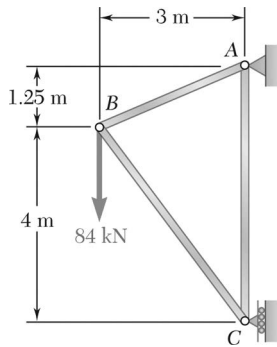
Joint A:



$$+\uparrow \Sigma F_y = 0: -960 \text{ lb} - \frac{4}{5} F_{AC} = 0$$

$$F_{AC} = 1200 \text{ lb} \quad F_{AC} = 1200 \text{ lb } C \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: 720 \text{ lb} - (1200 \text{ lb}) \frac{3}{5} = 0 \quad (\text{checks})$$



PROBLEM 6.3

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

$$AB = \sqrt{3^2 + 1.25^2} = 3.25 \text{ m}$$

$$BC = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

Reactions:

$$+\circlearrowleft \Sigma M_A = 0: (84 \text{ kN})(3 \text{ m}) - C(5.25 \text{ m}) = 0$$

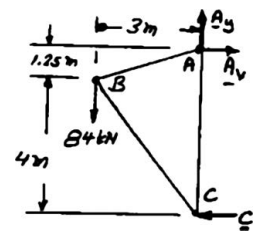
$$C = 48 \text{ kN} \leftarrow$$

$$\pm \rightarrow \Sigma F_x = 0: A_x - C = 0$$

$$A_x = 48 \text{ kN} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y - 84 \text{ kN} = 0$$

$$A_y = 84 \text{ kN} \uparrow$$



Joint A:

$$\pm \rightarrow \Sigma F_x = 0: 48 \text{ kN} - \frac{12}{13} F_{AB} = 0$$

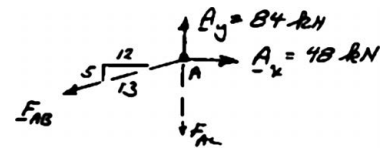
$$F_{AB} = +52 \text{ kN}$$

$$F_{AB} = 52.0 \text{ kN} \quad T \blacktriangleleft$$

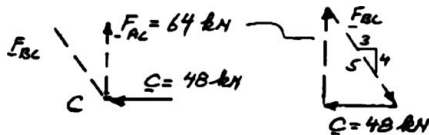
$$+\uparrow \Sigma F_y = 0: 84 \text{ kN} - \frac{5}{13}(52 \text{ kN}) - F_{AC} = 0$$

$$F_{AC} = +64.0 \text{ kN}$$

$$F_{AC} = 64.0 \text{ kN} \quad T \blacktriangleleft$$

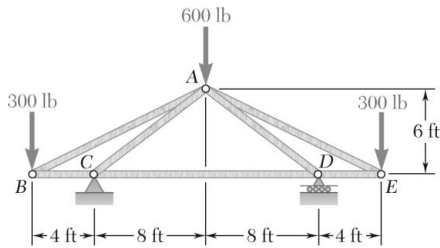


Joint C:



$$\frac{F_{BC}}{5} = \frac{48 \text{ kN}}{3}$$

$$F_{BC} = 80.0 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.4

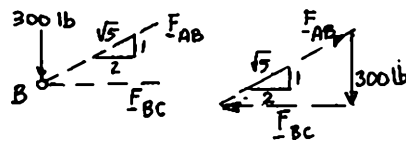
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

From the symmetry of the truss and loading, we find

$$C = D = 600 \text{ lb } \uparrow$$

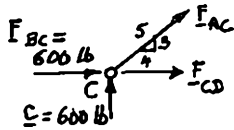


Free body: Joint B:

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{BC}}{2} = \frac{300 \text{ lb}}{1}$$

$$F_{AB} = 671 \text{ lb } T \quad F_{BC} = 600 \text{ lb } C \leftarrow$$

Free body: Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5} F_{AC} + 600 \text{ lb} = 0$$

$$F_{AC} = -1000 \text{ lb}$$

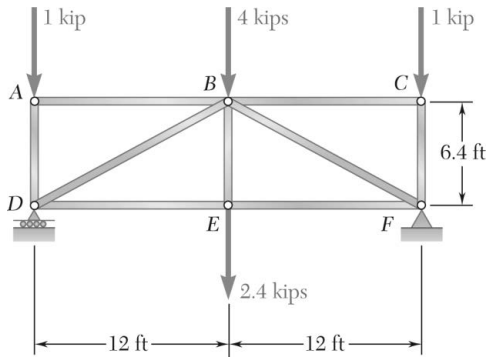
$$F_{AC} = 1000 \text{ lb } C \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: \quad \frac{4}{5} (-1000 \text{ lb}) + 600 \text{ lb} + F_{CD} = 0$$

$$F_{CD} = 200 \text{ lb } T \leftarrow$$

From symmetry:

$$F_{AD} = F_{AC} = 1000 \text{ lb } C, \quad F_{AE} = F_{AB} = 671 \text{ lb } T, \quad F_{DE} = F_{BC} = 600 \text{ lb } C \leftarrow$$

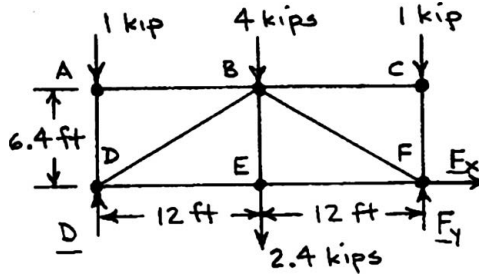


PROBLEM 6.5

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Reactions:



$$+\curvearrowright \Sigma M_D = 0: F_y(24) - (4 + 2.4)(12) - (1)(24) = 0$$

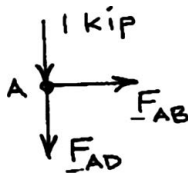
$$F_y = 4.2 \text{ kips} \uparrow$$

$$\Sigma F_x = 0: F_x = 0$$

$$+\uparrow \Sigma F_y = 0: D - (1 + 4 + 1 + 2.4) + 4.2 = 0$$

$$D = 4.2 \text{ kips} \uparrow$$

Joint A:



$$\Sigma F_x = 0: F_{AB} = 0$$

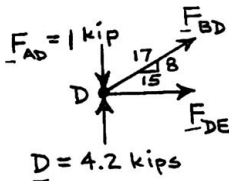
$$F_{AB} = 0 \quad \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -1 - F_{AD} = 0$$

$$F_{AD} = -1 \text{ kip}$$

$$F_{AD} = 1.000 \text{ kip} \quad C \quad \leftarrow$$

Joint D:



$$+\uparrow \Sigma F_y = 0: -1 + 4.2 + \frac{8}{17} F_{BD} = 0$$

$$F_{BD} = -6.8 \text{ kips}$$

$$F_{BD} = 6.80 \text{ kips} \quad C \quad \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: \frac{15}{17}(-6.8) + F_{DE} = 0$$

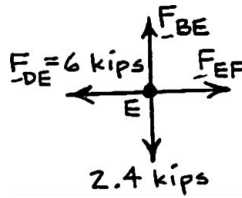
$$F_{DE} = +6 \text{ kips}$$

$$F_{DE} = 6.00 \text{ kips} \quad T \quad \leftarrow$$

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PROBLEM 6.5 (Continued)

Joint E:

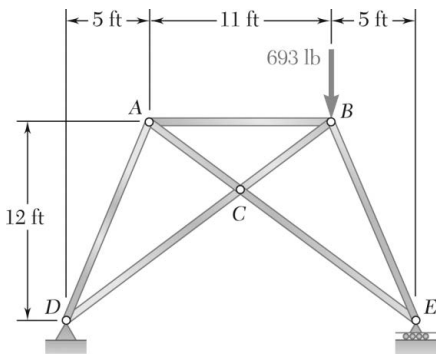


$$+\uparrow \Sigma F_y = 0: F_{BE} - 2.4 = 0$$

$$F_{BE} = +2.4 \text{ kips}$$

$$F_{BE} = 2.40 \text{ kips } T \quad \blacktriangleleft$$

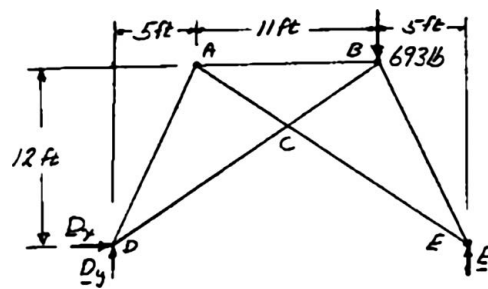
Truss and loading symmetrical about \mathcal{C} .



PROBLEM 6.6

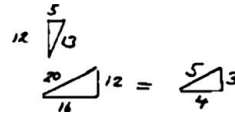
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



$$AD = \sqrt{5^2 + 12^2} = 13 \text{ ft}$$

$$BCD = \sqrt{12^2 + 16^2} = 20 \text{ ft}$$



Reactions:

$$\Sigma F_x = 0: D_x = 0$$

$$+\circlearrowleft \Sigma M_E = 0: D_y(21 \text{ ft}) - (693 \text{ lb})(5 \text{ ft}) = 0$$

$$D_y = 165 \text{ lb } \uparrow$$

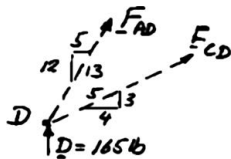
$$+\uparrow \Sigma F_y = 0: 165 \text{ lb} - 693 \text{ lb} + E = 0$$

$$E = 528 \text{ lb } \uparrow$$

Joint D:

$$\leftarrow + \Sigma F_x = 0: \frac{5}{13} F_{AD} + \frac{4}{5} F_{DC} = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \frac{12}{13} F_{AD} + \frac{3}{5} F_{DC} + 165 \text{ lb} = 0 \quad (2)$$



Solving Eqs. (1) and (2) simultaneously,

$$F_{AD} = -260 \text{ lb}$$

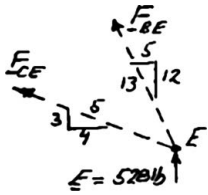
$$F_{AD} = 260 \text{ lb } \quad C \quad \blacktriangleleft$$

$$F_{DC} = +125 \text{ lb}$$

$$F_{DC} = 125 \text{ lb } \quad T \quad \blacktriangleleft$$

PROBLEM 6.6 (Continued)

Joint E:



$$\leftarrow + \Sigma F_x = 0: \quad \frac{5}{13} F_{BE} + \frac{4}{5} F_{CE} = 0 \quad (3)$$

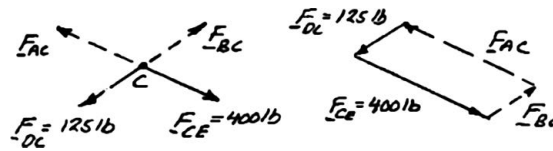
$$+ \uparrow \Sigma F_y = 0: \quad \frac{12}{13} F_{BE} + \frac{3}{5} F_{CE} + 528 \text{ lb} = 0 \quad (4)$$

Solving Eqs. (3) and (4) simultaneously,

$$F_{BE} = -832 \text{ lb} \qquad F_{BE} = 832 \text{ lb} \quad C \blacktriangleleft$$

$$F_{CE} = +400 \text{ lb} \qquad F_{CE} = 400 \text{ lb} \quad T \blacktriangleleft$$

Joint C:

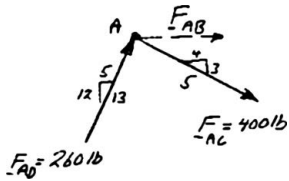


Force polygon is a parallelogram (see Fig. 6.11, p. 209).

$$F_{AC} = 400 \text{ lb} \quad T \blacktriangleleft$$

$$F_{BC} = 125.0 \text{ lb} \quad T \blacktriangleleft$$

Joint A:

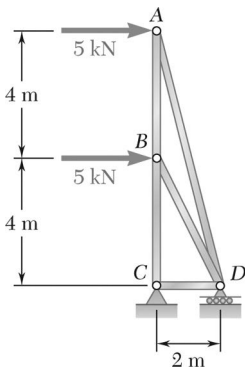


$$\leftarrow + \Sigma F_x = 0: \quad \frac{5}{13} (260 \text{ lb}) + \frac{4}{5} (400 \text{ lb}) + F_{AB} = 0$$

$$F_{AB} = -420 \text{ lb} \qquad F_{AB} = 420 \text{ lb} \quad C \blacktriangleleft$$

$$+ \uparrow \Sigma F_y = 0: \quad \frac{12}{13} (260 \text{ lb}) - \frac{3}{5} (400 \text{ lb}) = 0$$

$$0 = 0 \quad (\text{Checks})$$



PROBLEM 6.7

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:

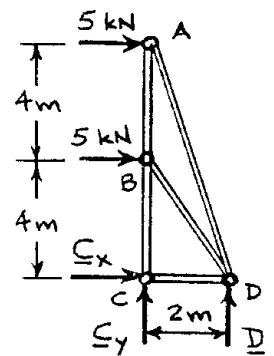
$$\pm \rightarrow \Sigma F_x = 0: C_x + 2(5 \text{ kN}) = 0$$

$$C_x = -10 \text{ kN} \quad \bar{C}_x = 10 \text{ kN} \leftarrow$$

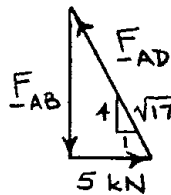
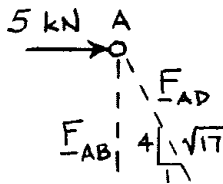
$$+\curvearrowright \Sigma M_C = 0: D(2 \text{ m}) - (5 \text{ kN})(8 \text{ m}) - (5 \text{ kN})(4 \text{ m}) = 0$$

$$D = +30 \text{ kN} \quad \bar{D} = 30 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: C_y + 30 \text{ kN} = 0 \quad C_y = -30 \text{ kN} \quad \bar{C}_y = 30 \text{ kN} \downarrow$$



Free body: Joint A:

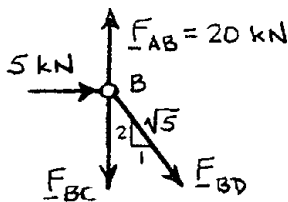


$$\frac{F_{AB}}{4} = \frac{F_{AD}}{\sqrt{17}} = \frac{5 \text{ kN}}{1}$$

$$F_{AB} = 20.0 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{AD} = 20.6 \text{ kN} \quad C \quad \blacktriangleleft$$

Free body: Joint B:



$$\pm \rightarrow \Sigma F_x = 0: 5 \text{ kN} + \frac{1}{\sqrt{5}} F_{BD} = 0$$

$$F_{BD} = -5\sqrt{5} \text{ kN}$$

$$F_{BD} = 11.18 \text{ kN} \quad C \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 20 \text{ kN} - F_{BC} - \frac{2}{\sqrt{5}} (-5\sqrt{5} \text{ kN}) = 0$$

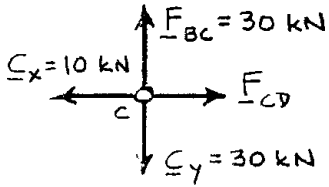
$$F_{BC} = +30 \text{ kN}$$

$$F_{BC} = 30.0 \text{ kN} \quad T \quad \blacktriangleleft$$

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PROBLEM 6.7 (Continued)

Free body: Joint C:

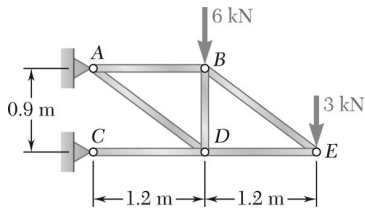


$$\rightarrow \Sigma F_x = 0: F_{CD} - 10 \text{ kN} = 0$$

$$F_{CD} = +10 \text{ kN}$$

$$F_{CD} = 10.00 \text{ kN } T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 30 \text{ kN} - 30 \text{ kN} = 0 \text{ (checks)}$$



PROBLEM 6.8

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

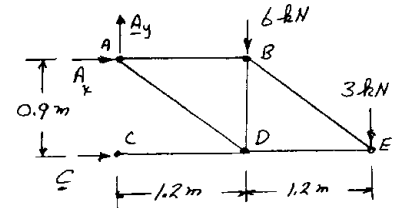
SOLUTION

Reactions:

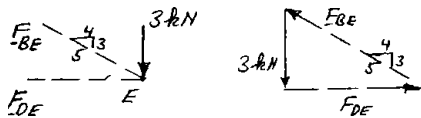
$$\Sigma M_C = 0: A_x = 16 \text{ kN} \leftarrow$$

$$\Sigma F_y = 0: A_y = 9 \text{ kN} \uparrow$$

$$\Sigma F_x = 0: C = 16 \text{ kN} \rightarrow$$



Joint E:

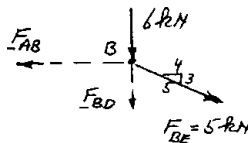


$$\frac{F_{BE}}{5} = \frac{F_{DE}}{4} = \frac{3 \text{ kN}}{3}$$

$$F_{BE} = 5.00 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{DE} = 4.00 \text{ kN} \quad C \quad \blacktriangleleft$$

Joint B:



$$+\rightarrow \Sigma F_x = 0: \frac{4}{5}(5 \text{ kN}) - F_{AB} = 0$$

$$F_{AB} = +4 \text{ kN}$$

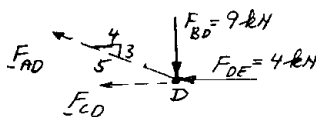
$$F_{AB} = 4.00 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -6 \text{ kN} - \frac{3}{5}(5 \text{ kN}) - F_{BD} = 0$$

$$F_{BD} = -9 \text{ kN}$$

$$F_{BD} = 9.00 \text{ kN} \quad C \quad \blacktriangleleft$$

Joint D:



$$+\uparrow \Sigma F_y = 0: -9 \text{ kN} + \frac{3}{5}F_{AD} = 0$$

$$F_{AD} = +15 \text{ kN}$$

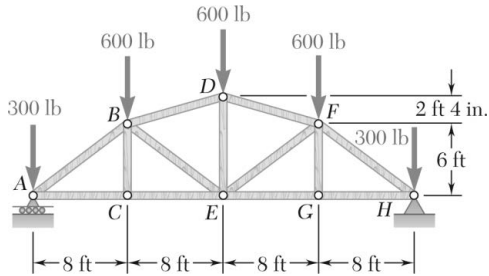
$$F_{AD} = 15.00 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: -4 \text{ kN} - \frac{4}{5}(15 \text{ kN}) - F_{CD} = 0$$

$$F_{CD} = -16 \text{ kN}$$

$$F_{CD} = 16.00 \text{ kN} \quad C \quad \blacktriangleleft$$

PROBLEM 6.9



Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION

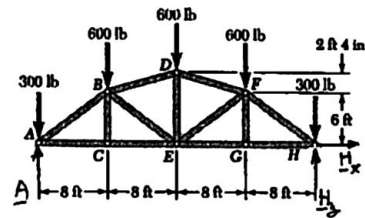
Free body: Truss:

$$\Sigma F_x = 0: \quad H_x = 0$$

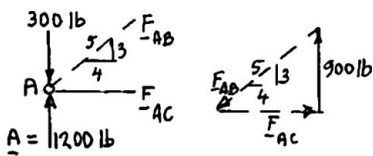
Because of the symmetry of the truss and loading,

$$A = H_y = \frac{1}{2} \text{ total load}$$

$$A = H_y = 1200 \text{ lb} \uparrow$$



Free body: Joint A:



$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

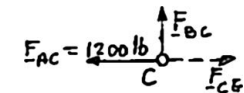
$$F_{AB} = 1500 \text{ lb} \quad C \leftarrow$$

$$F_{AC} = 1200 \text{ lb} \quad T \leftarrow$$

Free body: Joint C:

BC is a zero-force member.

$$F_{BC} = 0$$



$$F_{CE} = 1200 \text{ lb} \quad T \leftarrow$$

Free body: Joint B:

$$\pm \rightarrow \Sigma F_x = 0: \quad \frac{24}{25} F_{BD} + \frac{4}{5} F_{BE} + \frac{4}{5} (1500 \text{ lb}) = 0$$

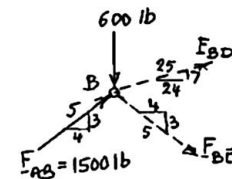
or

$$24F_{BD} + 20F_{BE} = -30,000 \text{ lb} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{7}{25} F_{BD} - \frac{3}{5} F_{BE} + \frac{3}{5} (1500) - 600 = 0$$

or

$$7F_{BD} - 15F_{BE} = -7,500 \text{ lb} \quad (2)$$



PROBLEM 6.9 (Continued)

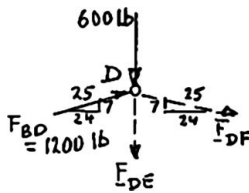
Multiply Eq. (1) by 3, Eq. (2) by 4, and add:

$$100F_{BD} = -120,000 \text{ lb} \qquad F_{BD} = 1200 \text{ lb} \quad C \quad \blacktriangleleft$$

Multiply Eq. (1) by 7, Eq. (2) by -24 , and add:

$$500F_{BE} = -30,000 \text{ lb} \qquad F_{BE} = 60.0 \text{ lb} \quad C \quad \blacktriangleleft$$

Free body: Joint D:



$$+\rightarrow \Sigma F_x = 0: \quad \frac{24}{25}(1200 \text{ lb}) + \frac{24}{25}F_{DF} = 0$$

$$F_{DF} = -1200 \text{ lb} \qquad F_{DF} = 1200 \text{ lb} \quad C \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{7}{25}(1200 \text{ lb}) - \frac{7}{25}(-1200 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$$F_{DE} = 72.0 \text{ lb} \qquad F_{DE} = 72.0 \text{ lb} \quad T \quad \blacktriangleleft$$

Because of the symmetry of the truss and loading, we deduce that

$$F_{EF} = F_{BE} \qquad F_{EF} = 60.0 \text{ lb} \quad C \quad \blacktriangleleft$$

$$F_{EG} = F_{CE} \qquad F_{EG} = 1200 \text{ lb} \quad T \quad \blacktriangleleft$$

$$F_{FG} = F_{BC} \qquad F_{FG} = 0 \quad \blacktriangleleft$$

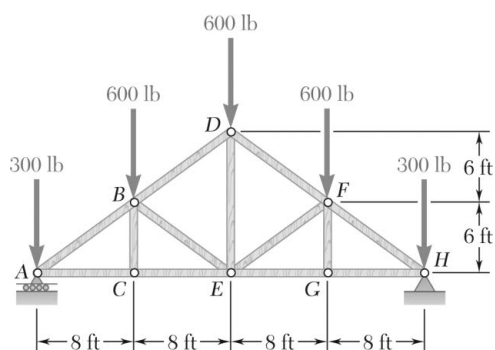
$$F_{FH} = F_{AB} \qquad F_{FH} = 1500 \text{ lb} \quad C \quad \blacktriangleleft$$

$$F_{GH} = F_{AC} \qquad F_{GH} = 1200 \text{ lb} \quad T \quad \blacktriangleleft$$

Note: Compare results with those of Problem 6.11.

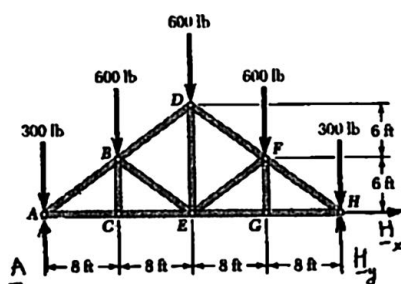
PROBLEM 6.10

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.



SOLUTION

Free body: Truss:



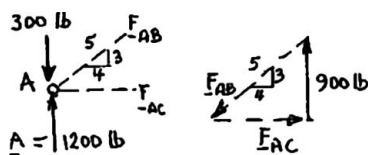
$$\Sigma F_x = 0: H_x = 0$$

Because of the symmetry of the truss and loading,

$$A = H_y = \frac{1}{2} \text{ total load}$$

$$A = H_y = 1200 \text{ lb} \uparrow$$

Free body: Joint A:



$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$$F_{AB} = 1500 \text{ lb} \quad C \leftarrow$$

$$F_{AC} = 1200 \text{ lb} \quad T \leftarrow$$

Free body: Joint C:

BC is a zero-force member.



$$F_{BC} = 0$$

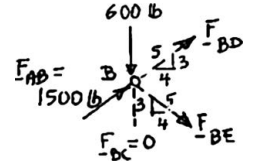
$$F_{CE} = 1200 \text{ lb} \quad T \leftarrow$$

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PROBLEM 6.10 (Continued)

Free body: Joint B:

$$\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(1500 \text{ lb}) = 0$$



or

$$F_{BD} + F_{BE} = -1500 \text{ lb} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500 \text{ lb}) - 600 \text{ lb} = 0$$

or

$$F_{BD} - F_{BE} = -500 \text{ lb} \quad (2)$$

Add Eqs. (1) and (2):

$$2F_{BD} = -2000 \text{ lb} \quad F_{BD} = 1000 \text{ lb} \quad C \leftarrow$$

Subtract Eq. (2) from Eq. (1):

$$2F_{BE} = -1000 \text{ lb} \quad F_{BE} = 500 \text{ lb} \quad C \leftarrow$$

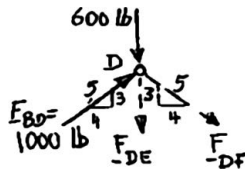
Free Body: Joint D:

$$\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}(1000 \text{ lb}) + \frac{4}{5}F_{DF} = 0$$

$$F_{DF} = -1000 \text{ lb} \quad F_{DF} = 1000 \text{ lb} \quad C \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(1000 \text{ lb}) - \frac{3}{5}(-1000 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$$F_{DE} = +600 \text{ lb} \quad F_{DE} = 600 \text{ lb} \quad T \leftarrow$$



Because of the symmetry of the truss and loading, we deduce that

$$F_{EF} = F_{BE} \quad F_{EF} = 500 \text{ lb} \quad C \leftarrow$$

$$F_{EG} = F_{CE} \quad F_{EG} = 1200 \text{ lb} \quad T \leftarrow$$

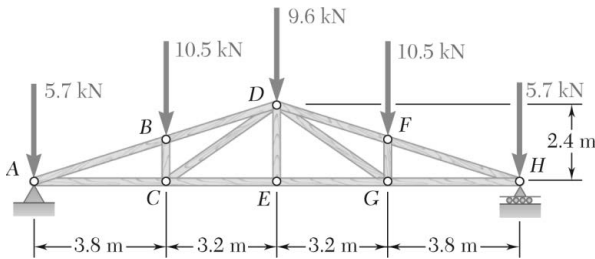
$$F_{FG} = F_{BC} \quad F_{FG} = 0 \quad \leftarrow$$

$$F_{FH} = F_{AB} \quad F_{FH} = 1500 \text{ lb} \quad C \leftarrow$$

$$F_{GH} = F_{AC} \quad F_{GH} = 1200 \text{ lb} \quad T \leftarrow$$

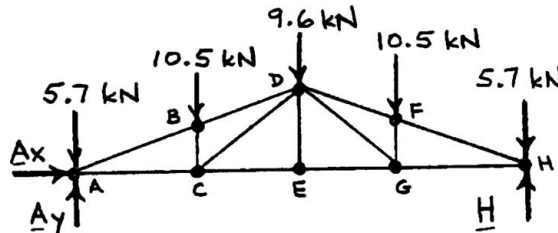
PROBLEM 6.11

Determine the force in each member of the Pratt roof truss shown. State whether each member is in tension or compression.



SOLUTION

Free body: Truss:

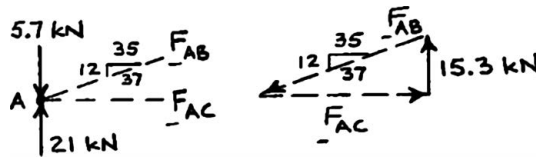


$$\Sigma F_x = 0: A_x = 0$$

Due to symmetry of truss and load,

$$A_y = H_y = \frac{1}{2} \text{ total load} = 21 \text{ kN} \uparrow$$

Free body: Joint A:



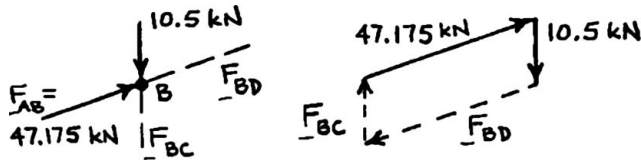
$$\frac{F_{AB}}{37} = \frac{F_{AC}}{35} = \frac{15.3 \text{ kN}}{12}$$

$$F_{AB} = 47.175 \text{ kN} \quad F_{AC} = 44.625 \text{ kN}$$

$$F_{AB} = 47.2 \text{ kN} \quad C \leftarrow$$

$$F_{AC} = 44.6 \text{ kN} \quad T \leftarrow$$

Free body: Joint B:



From force polygon:

$$F_{BD} = 47.175 \text{ kN}, \quad F_{BC} = 10.5 \text{ kN}$$

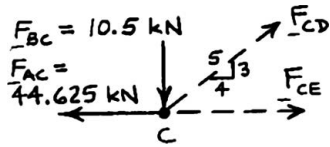
$$F_{BC} = 10.50 \text{ kN} \quad C \leftarrow$$

$$F_{BD} = 47.2 \text{ kN} \quad C \leftarrow$$

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PROBLEM 6.11 (Continued)

Free body: Joint C:



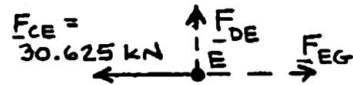
$$+\uparrow \Sigma F_y = 0: \frac{3}{5} F_{CD} - 10.5 = 0$$

$$F_{CD} = 17.50 \text{ kN } T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: F_{CE} + \frac{4}{5}(17.50) - 44.625 = 0$$

$$F_{CE} = 30.625 \text{ kN} \quad F_{CE} = 30.6 \text{ kN } T \quad \blacktriangleleft$$

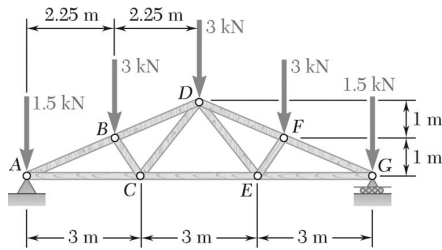
Free body: Joint E:



DE is a zero-force member.

$$F_{DE} = 0 \quad \blacktriangleleft$$

Truss and loading symmetrical about Φ .



PROBLEM 6.12

Determine the force in each member of the Fink roof truss shown. State whether each member is in tension or compression.

SOLUTION

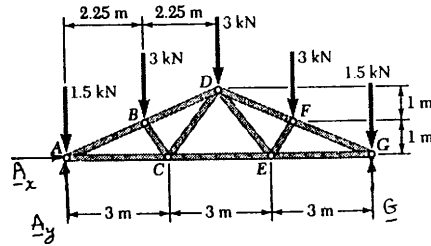
Free body: Truss:

$$\Sigma F_x = 0: A_x = 0$$

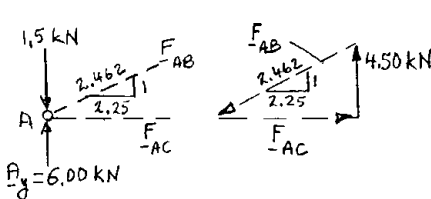
Because of the symmetry of the truss and loading,

$$A_y = G = \frac{1}{2} \text{ total load}$$

$$A_y = G = 6.00 \text{ kN } \uparrow$$



Free body: Joint A:



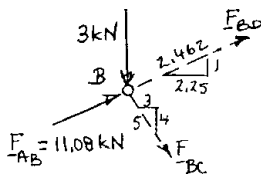
$$\frac{F_{AB}}{2.462} = \frac{F_{AC}}{2.25} = \frac{4.50 \text{ kN}}{1}$$

$$F_{AB} = 11.08 \text{ kN } \quad C \quad \blacktriangleleft$$

$$F_{AC} = 10.125 \text{ kN}$$

$$F_{AC} = 10.13 \text{ kN } \quad T \quad \blacktriangleleft$$

Free body: Joint B:



$$+\rightarrow \Sigma F_x = 0: \frac{3}{5} F_{BC} + \frac{2.25}{2.462} F_{BD} + \frac{2.25}{2.462} (11.08 \text{ kN}) = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0: -\frac{4}{5} F_{BC} + \frac{F_{BD}}{2.462} + \frac{11.08 \text{ kN}}{2.462} - 3 \text{ kN} = 0 \quad (2)$$

Multiply Eq. (2) by -2.25 and add to Eq. (1):

$$\frac{12}{5} F_{BC} + 6.75 \text{ kN} = 0 \quad F_{BC} = -2.8125 \quad F_{BC} = 2.81 \text{ kN } \quad C \quad \blacktriangleleft$$

Multiply Eq. (1) by 4, Eq. (2) by 3, and add:

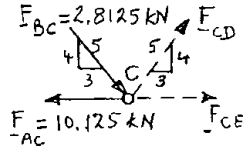
$$\frac{12}{2.462} F_{BD} + \frac{12}{2.462} (11.08 \text{ kN}) - 9 \text{ kN} = 0$$

$$F_{BD} = -9.2335 \text{ kN}$$

$$F_{BD} = 9.23 \text{ kN } \quad C \quad \blacktriangleleft$$

PROBLEM 6.12 (Continued)

Free body: Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5} F_{CD} - \frac{4}{5} (2.8125 \text{ kN}) = 0$$

$$F_{CD} = 2.8125 \text{ kN}, \quad F_{CD} = 2.81 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad F_{CE} - 10.125 \text{ kN} + \frac{3}{5} (2.8125 \text{ kN}) + \frac{3}{5} (2.8125 \text{ kN}) = 0$$

$$F_{CE} = +6.7500 \text{ kN} \quad F_{CE} = 6.75 \text{ kN} \quad T \quad \blacktriangleleft$$

Because of the symmetry of the truss and loading, we deduce that

$$F_{DE} = F_{CD} \quad F_{CD} = 2.81 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{DF} = F_{BD} \quad F_{DF} = 9.23 \text{ kN} \quad C \quad \blacktriangleleft$$

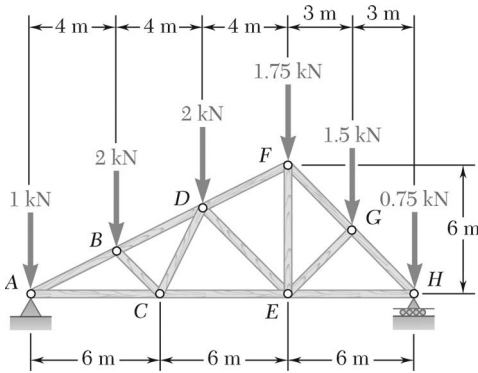
$$F_{EF} = F_{BC} \quad F_{EF} = 2.81 \text{ kN} \quad C \quad \blacktriangleleft$$

$$F_{EG} = F_{AC} \quad F_{EG} = 10.13 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{FG} = F_{AB} \quad F_{FG} = 11.08 \text{ kN} \quad C \quad \blacktriangleleft$$

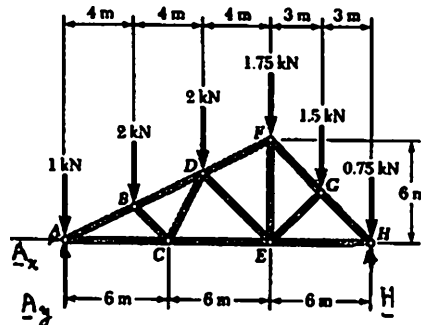
PROBLEM 6.13

Using the method of joints, determine the force in each member of the double-pitch roof truss shown. State whether each member is in tension or compression.



SOLUTION

Free body: Truss:



$$+\circlearrowleft \Sigma M_A = 0: H(18 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (2 \text{ kN})(8 \text{ m}) - (1.75 \text{ kN})(12 \text{ m}) - (1.5 \text{ kN})(15 \text{ m}) - (0.75 \text{ kN})(18 \text{ m}) = 0$$

$$H = 4.50 \text{ kN} \uparrow$$

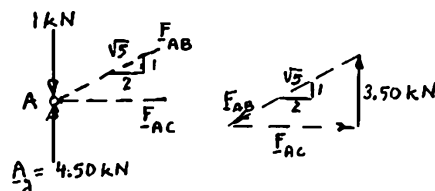
$$\Sigma F_x = 0: A_x = 0$$

$$\Sigma F_y = 0: A_y + H - 9 = 0$$

$$A_y = 9 - 4.50$$

$$A_y = 4.50 \text{ kN} \uparrow$$

Free body: Joint A:



$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{3.50 \text{ kN}}{1}$$

$$F_{AB} = 7.8262 \text{ kN} \quad C$$

$$F_{AB} = 7.83 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 7.00 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.13 (Continued)

Free body: Joint B:

$$\rightarrow \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} (7.8262 \text{ kN}) + \frac{1}{\sqrt{2}} F_{BC} = 0$$

or $F_{BD} + 0.79057 F_{BC} = -7.8262 \text{ kN}$ (1)

$$\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}} (7.8262 \text{ kN}) - \frac{1}{\sqrt{2}} F_{BC} - 2 \text{ kN} = 0$$

or $F_{BD} - 1.58114 F_{BC} = -3.3541$ (2)

Multiply Eq. (1) by 2 and add Eq. (2):

$$3F_{BD} = -19.0065$$

$$F_{BD} = -6.3355 \text{ kN}$$

$$F_{BD} = 6.34 \text{ kN} \quad C \blacktriangleleft$$

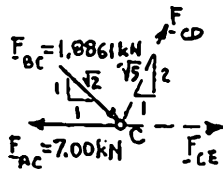
Subtract Eq. (2) from Eq. (1):

$$2.37111 F_{BC} = -4.4721$$

$$F_{BC} = -1.8861 \text{ kN}$$

$$F_{BC} = 1.886 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C:



$$\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}} (1.8861 \text{ kN}) = 0$$

$$F_{CD} = +1.4911 \text{ kN}$$

$$F_{CD} = 1.491 \text{ kN} \quad T \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: F_{CE} - 7.00 \text{ kN} + \frac{1}{\sqrt{2}} (1.8861 \text{ kN}) + \frac{1}{\sqrt{5}} (1.4911 \text{ kN}) = 0$$

$$F_{CE} = +5.000 \text{ kN}$$

$$F_{CE} = 5.00 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint D:

$$\rightarrow \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}} (6.3355 \text{ kN}) - \frac{1}{\sqrt{5}} (1.4911 \text{ kN}) = 0$$

or $F_{DF} + 0.79057 F_{DE} = -5.5900 \text{ kN}$ (1)

$$\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}} (6.3355 \text{ kN}) - \frac{2}{\sqrt{5}} (1.4911 \text{ kN}) - 2 \text{ kN} = 0$$

or $F_{DF} - 0.79057 F_{DE} = -1.1188 \text{ kN}$ (2)

Add Eqs. (1) and (2):

$$2F_{DF} = -6.7088 \text{ kN}$$

$$F_{DF} = -3.3544 \text{ kN}$$

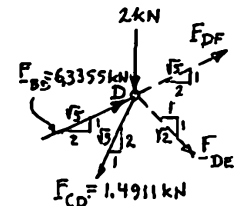
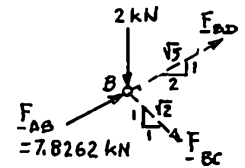
$$F_{DF} = 3.35 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (2) from Eq. (1):

$$1.58114 F_{DE} = -4.4712 \text{ kN}$$

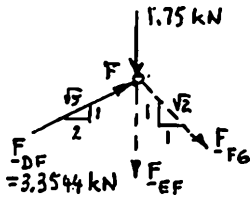
$$F_{DE} = -2.8278 \text{ kN}$$

$$F_{DE} = 2.83 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.13 (Continued)

Free body: Joint F:



$$\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}} (3.3544 \text{ kN}) = 0$$

$$F_{FG} = -4.243 \text{ kN}$$

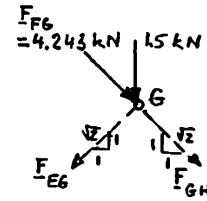
$$F_{FG} = 4.24 \text{ kN} \quad C \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -F_{EF} - 1.75 \text{ kN} + \frac{1}{\sqrt{5}} (3.3544 \text{ kN}) - \frac{1}{\sqrt{2}} (-4.243 \text{ kN}) = 0$$

$$F_{EF} = 2.750 \text{ kN}$$

$$F_{EF} = 2.75 \text{ kN} \quad T \quad \blacktriangleleft$$

Free body: Joint G:



$$\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} + \frac{1}{\sqrt{2}} (4.243 \text{ kN}) = 0$$

or

$$F_{GH} - F_{EG} = -4.243 \text{ kN} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: -\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} - \frac{1}{\sqrt{2}} (4.243 \text{ kN}) - 1.5 \text{ kN} = 0$$

or

$$F_{GH} + F_{EG} = -6.364 \text{ kN} \quad (2)$$

Add Eqs. (1) and (2):

$$2F_{GH} = -10.607$$

$$F_{GH} = -5.303$$

$$F_{GH} = 5.30 \text{ kN} \quad C \quad \blacktriangleleft$$

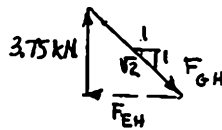
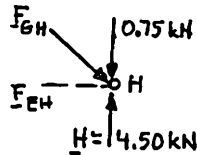
Subtract Eq. (1) from Eq. (2):

$$2F_{EG} = -2.121 \text{ kN}$$

$$F_{EG} = -1.0605 \text{ kN}$$

$$F_{EG} = 1.061 \text{ kN} \quad C \quad \blacktriangleleft$$

Free body: Joint H:



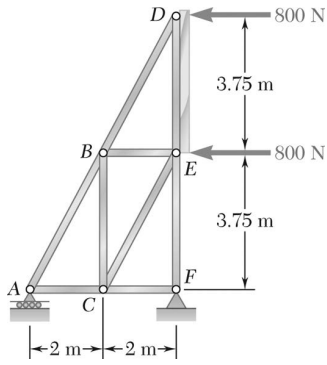
$$\frac{F_{EH}}{1} = \frac{3.75 \text{ kN}}{1}$$

$$F_{EH} = 3.75 \text{ kN} \quad T \quad \blacktriangleleft$$

We can also write

$$\frac{F_{GH}}{\sqrt{2}} = \frac{3.75 \text{ kN}}{1}$$

$$F_{GH} = 5.30 \text{ kN} \quad C \quad (\text{Checks})$$

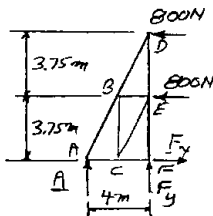


PROBLEM 6.14

The truss shown is one of several supporting an advertising panel. Determine the force in each member of the truss for a wind load equivalent to the two forces shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:



$$+\circlearrowleft \Sigma M_F = 0: (800 \text{ N})(7.5 \text{ m}) + (800 \text{ N})(3.75 \text{ m}) - A(2 \text{ m}) = 0$$

$$A = +2250 \text{ N} \quad A = 2250 \text{ N} \uparrow$$

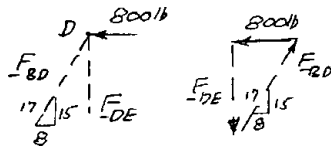
$$+\uparrow \Sigma F_y = 0: 2250 \text{ N} + F_y = 0$$

$$F_y = -2250 \text{ N} \quad F_y = 2250 \text{ N} \downarrow$$

$$\pm \rightarrow \Sigma F_x = 0: -800 \text{ N} - 800 \text{ N} + F_x = 0$$

$$F_x = +1600 \text{ N} \quad F_x = 1600 \text{ N} \rightarrow$$

Joint D:

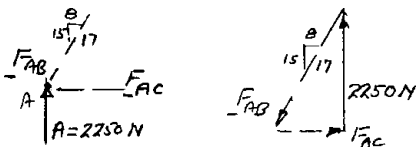


$$\frac{800 \text{ N}}{8} = \frac{F_{DE}}{15} = \frac{F_{BD}}{17}$$

$$F_{BD} = 1700 \text{ N} \quad C \blacktriangleleft$$

$$F_{DE} = 1500 \text{ N} \quad T \blacktriangleleft$$

Joint A:

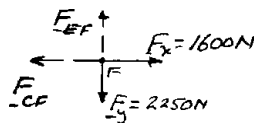


$$\frac{2250 \text{ N}}{15} = \frac{F_{AB}}{17} = \frac{F_{AC}}{8}$$

$$F_{AB} = 2250 \text{ N} \quad C \blacktriangleleft$$

$$F_{AC} = 1200 \text{ N} \quad T \blacktriangleleft$$

Joint F:



$$\pm \rightarrow \Sigma F_x = 0: 1600 \text{ N} - F_{CF} = 0$$

$$F_{CF} = +1600 \text{ N}$$

$$F_{CF} = 1600 \text{ N} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: F_{EF} - 2250 \text{ N} = 0$$

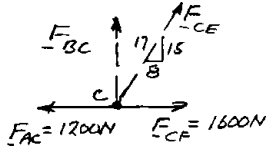
$$F_{EF} = +2250 \text{ N}$$

$$F_{EF} = 2250 \text{ N} \quad T \blacktriangleleft$$

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PROBLEM 6.14 (Continued)

Joint C:



$$\rightarrow \Sigma F_x = 0: \quad \frac{8}{17} F_{CE} - 1200 \text{ N} + 1600 \text{ N} = 0$$

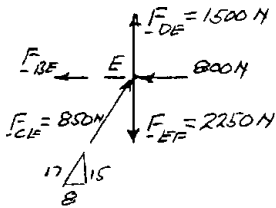
$$F_{CE} = -850 \text{ N} \qquad F_{CE} = 850 \text{ N} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BC} + \frac{15}{17} F_{CE} = 0$$

$$F_{BC} = -\frac{15}{17} F_{CE} = -\frac{15}{17} (-850 \text{ N})$$

$$F_{BC} = +750 \text{ N} \qquad F_{BC} = 750 \text{ N} \quad T \blacktriangleleft$$

Joint E:

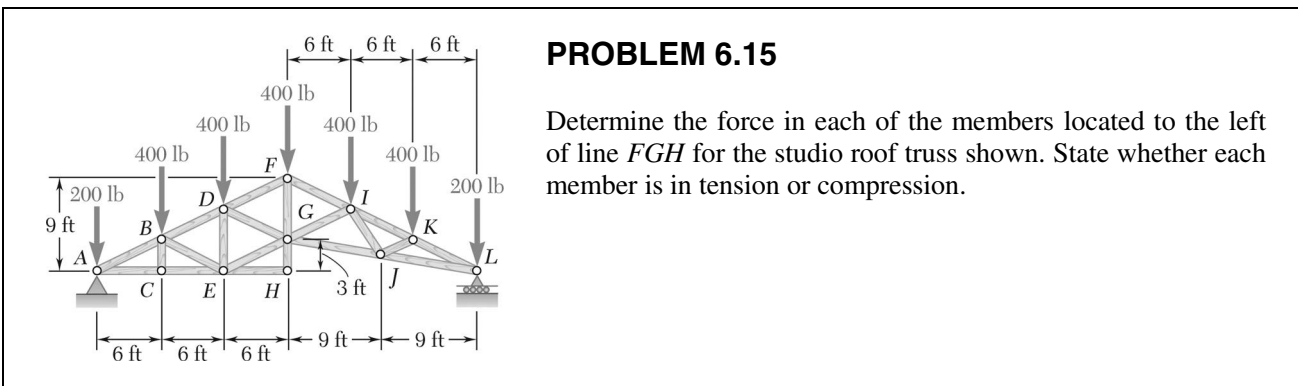


$$\rightarrow \Sigma F_x = 0: \quad -F_{BE} - 800 \text{ N} + \frac{8}{17} (850 \text{ N}) = 0$$

$$F_{BE} = -400 \text{ N} \qquad F_{BE} = 400 \text{ N} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 1500 \text{ N} - 2250 \text{ N} + \frac{15}{17} (850 \text{ N}) = 0$$

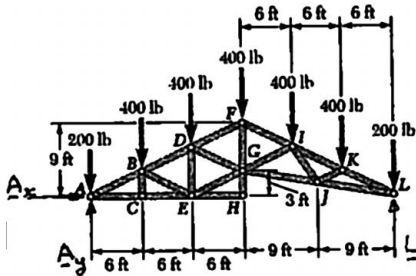
$$0 = 0 \quad (\text{checks})$$



PROBLEM 6.15

Determine the force in each of the members located to the left of line FGH for the studio roof truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss: $\Sigma F_x = 0: A_x = 0$

Because of symmetry of loading,

$$A_y = L_y = \frac{1}{2} \text{ total load}$$

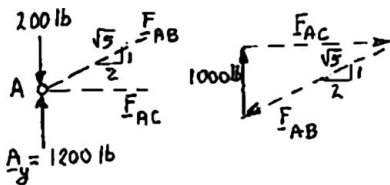
$$A_y = L_y = 1200 \text{ lb } \uparrow$$

Zero-Force Members: Examining joints C and H , we conclude that BC , EH , and GH are zero-force members. Thus,

$$F_{BC} = F_{EH} = 0 \quad \blacktriangleleft$$

Also, $F_{CE} = F_{AC} \quad (1)$

Free body: Joint A : $\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{1000 \text{ lb}}{1}$
 $F_{AB} = 2236 \text{ lb } \quad C$



$$F_{AB} = 2240 \text{ lb } \quad C \quad \blacktriangleleft$$

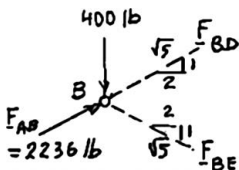
$$F_{AC} = 2000 \text{ lb } \quad T \quad \blacktriangleleft$$

From Eq. (1): $F_{CE} = 2000 \text{ lb } \quad T \quad \blacktriangleleft$

Free body: Joint B :

$$\rightarrow \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}} (2236 \text{ lb}) = 0$$

or $F_{BD} + F_{BE} = -2236 \text{ lb} \quad (2)$



$$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}} (2236 \text{ lb}) - 400 \text{ lb} = 0$$

or $F_{BD} - F_{BE} = -1342 \text{ lb} \quad (3)$

PROBLEM 6.15 (Continued)

Add Eqs. (2) and (3): $2F_{BD} = -3578 \text{ lb} \quad F_{BD} = 1789 \text{ lb} \quad C \blacktriangleleft$

Subtract Eq. (3) from Eq. (1): $2F_{BE} = -894 \text{ lb} \quad F_{BE} = 447 \text{ lb} \quad C \blacktriangleleft$

Free body: Joint E:

$$\rightarrow \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}} (447 \text{ lb}) - 2000 \text{ lb} = 0$$

$$F_{EG} = 1789 \text{ lb} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} + \frac{1}{\sqrt{5}} (1789 \text{ lb}) - \frac{1}{\sqrt{5}} (447 \text{ lb}) = 0$$

$$F_{DE} = -600 \text{ lb} \quad F_{DE} = 600 \text{ lb} \quad C \blacktriangleleft$$

Free body: Joint D:

$$\rightarrow \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}} (1789 \text{ lb}) = 0$$

or $F_{DF} + F_{DG} = -1789 \text{ lb} \quad (4)$

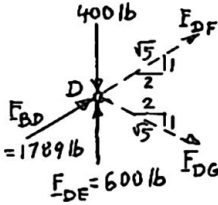
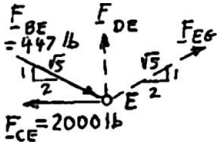
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}} (1789 \text{ lb})$$

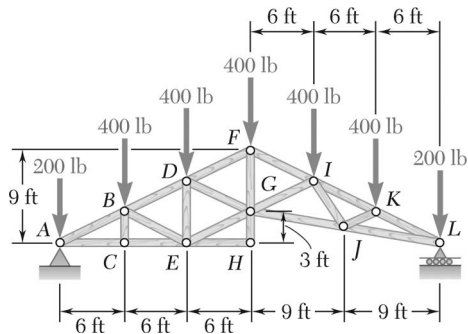
$$+ 600 \text{ lb} - 400 \text{ lb} = 0$$

or $F_{DF} - F_{DG} = -2236 \text{ lb} \quad (5)$

Add Eqs. (4) and (5): $2F_{DF} = -4025 \text{ lb} \quad F_{DF} = 2010 \text{ lb} \quad C \blacktriangleleft$

Subtract Eq. (5) from Eq. (4): $2F_{DG} = 447 \text{ lb} \quad F_{DG} = 224 \text{ lb} \quad T \blacktriangleleft$





PROBLEM 6.16

Determine the force in member FG and in each of the members located to the right of FG for the studio roof truss shown. State whether each member is in tension or compression.

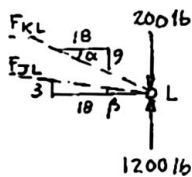
SOLUTION

Reaction at L : Because of the symmetry of the loading,

$$L = \frac{1}{2} \text{ total load, } \mathbf{L} = 1200 \text{ lb } \uparrow$$

(See $F.B.$ diagram to the left for more details.)

Free body: Joint L :

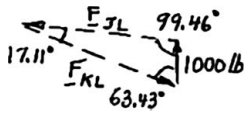


$$\alpha = \tan^{-1} \frac{9}{18} = 26.57^\circ$$

$$\beta = \tan^{-1} \frac{3}{18} = 9.46^\circ$$

$$\frac{F_{JL}}{\sin 63.43^\circ} = \frac{F_{KL}}{\sin 99.46^\circ} = \frac{1000 \text{ lb}}{\sin 17.11^\circ} \quad F_{JL} = 3040 \text{ lb } T \quad \blacktriangleleft$$

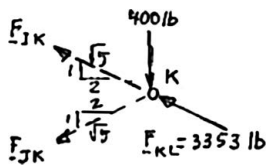
$$F_{KL} = 3352.7 \text{ lb } C \quad F_{KL} = 3350 \text{ lb } C \quad \blacktriangleleft$$



Free body: Joint K :

$$\pm \rightarrow \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}} F_{IK} - \frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}} (3352.7 \text{ lb}) = 0$$

$$\text{or} \quad F_{IK} + F_{JK} = -3352.7 \text{ lb} \quad (1)$$



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{IK} - \frac{1}{\sqrt{5}} F_{JK} + \frac{1}{\sqrt{5}} (3352.7) - 400 = 0$$

$$\text{or} \quad F_{IK} - F_{JK} = -2458.3 \text{ lb} \quad (2)$$

$$\text{Add Eqs. (1) and (2):} \quad 2F_{IK} = -5811.0$$

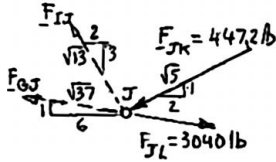
$$F_{IK} = -2905.5 \text{ lb} \quad F_{IK} = 2910 \text{ lb } C \quad \blacktriangleleft$$

$$\text{Subtract Eq. (2) from Eq. (1):} \quad 2F_{JK} = -894.4$$

$$F_{JK} = -447.2 \text{ lb} \quad F_{JK} = 447 \text{ lb } C \quad \blacktriangleleft$$

PROBLEM 6.16 (Continued)

Free body: Joint J:



$$+\rightarrow \Sigma F_x = 0: -\frac{2}{\sqrt{13}}F_{IJ} - \frac{6}{\sqrt{37}}F_{GJ} + \frac{6}{\sqrt{37}}(3040 \text{ lb}) - \frac{2}{\sqrt{5}}(447.2) = 0 \quad (3)$$

$$+\uparrow \Sigma F_y = 0: \frac{3}{\sqrt{13}}F_{IJ} + \frac{1}{\sqrt{37}}F_{GJ} - \frac{1}{\sqrt{37}}(3040 \text{ lb}) - \frac{1}{\sqrt{5}}(447.2) = 0 \quad (4)$$

Multiply Eq. (4) by 6 and add to Eq. (3):

$$\frac{16}{\sqrt{13}}F_{IJ} - \frac{8}{\sqrt{5}}(447.2) = 0$$

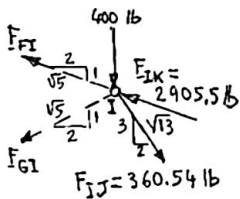
$$F_{IJ} = 360.54 \text{ lb} \quad F_{IJ} = 361 \text{ lb} \quad T \quad \blacktriangleleft$$

Multiply Eq. (3) by 3, Eq. (4) by 2, and add:

$$-\frac{16}{\sqrt{37}}(F_{GJ} - 3040) - \frac{8}{\sqrt{5}}(447.2) = 0$$

$$F_{GJ} = 2431.7 \text{ lb} \quad F_{GJ} = 2430 \text{ lb} \quad T \quad \blacktriangleleft$$

Free body: Joint I:



$$+\rightarrow \Sigma F_x = 0: -\frac{2}{\sqrt{5}}F_{FI} - \frac{2}{\sqrt{5}}F_{GI} - \frac{2}{\sqrt{5}}(2905.5) + \frac{2}{\sqrt{13}}(360.54) = 0$$

or
$$F_{FI} + F_{GI} = -2681.9 \text{ lb} \quad (5)$$

$$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}}F_{FI} - \frac{1}{\sqrt{5}}F_{GI} + \frac{1}{\sqrt{5}}(2905.5) - \frac{3}{\sqrt{13}}(360.54) - 400 = 0$$

or
$$F_{FI} - F_{GI} = -1340.3 \text{ lb} \quad (6)$$

Add Eqs. (5) and (6):
$$2F_{FI} = -4022.2$$

$$F_{FI} = -2011.1 \text{ lb} \quad F_{FI} = 2010 \text{ lb} \quad C \quad \blacktriangleleft$$

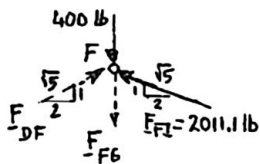
Subtract Eq. (6) from Eq. (5):
$$2F_{GI} = -1341.6 \text{ lb} \quad F_{GI} = 671 \text{ lb} \quad C \quad \blacktriangleleft$$

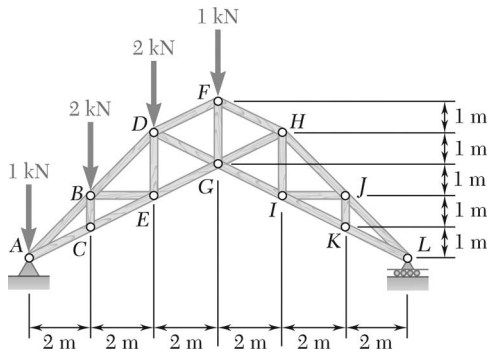
Free body: Joint F:

From
$$\Sigma F_x = 0: F_{DF} = F_{FI} = 2011.1 \text{ lb} \quad C$$

$$+\uparrow \Sigma F_y = 0: F_{FG} - 400 \text{ lb} + 2\left(\frac{1}{\sqrt{5}}2011.1 \text{ lb}\right) = 0$$

$$F_{FG} = +1400 \text{ lb} \quad F_{FG} = 1400 \text{ lb} \quad T \quad \blacktriangleleft$$



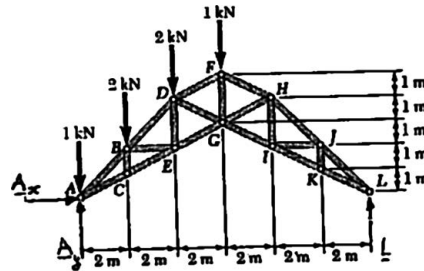


PROBLEM 6.17

Determine the force in each of the members located to the left of FG for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free Body: Truss:



$$\Sigma F_x = 0: A_x = 0$$

$$+\curvearrowright \Sigma M_L = 0: (1 \text{ kN})(12 \text{ m}) + (2 \text{ kN})(10 \text{ m}) + (2 \text{ kN})(8 \text{ m}) + (1 \text{ kN})(6 \text{ m}) - A_y(12 \text{ m}) = 0$$

$$A_y = 4.50 \text{ kN} \uparrow$$

We note that BC is a zero-force member:

$$F_{BC} = 0 \quad \blacktriangleleft$$

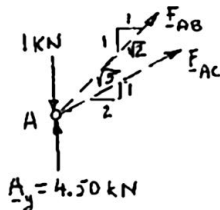
Also,

$$F_{CE} = F_{AC} \quad (1)$$

Free body: Joint A:

$$+\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{2}} F_{AB} + \frac{2}{\sqrt{5}} F_{AC} = 0 \quad (2)$$

$$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{5}} F_{AC} + 3.50 \text{ kN} = 0 \quad (3)$$



Multiply Eq. (3) by -2 and add Eq. (2):

$$-\frac{1}{\sqrt{2}} F_{AB} - 7 \text{ kN} = 0 \quad F_{AB} = 9.90 \text{ kN} \quad C \quad \blacktriangleleft$$

PROBLEM 6.17 (Continued)

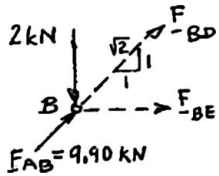
Subtract Eq. (3) from Eq. (2):

$$\frac{1}{\sqrt{5}} F_{AC} - 3.50 \text{ kN} = 0 \quad F_{AC} = 7.826 \text{ kN} \quad F_{AC} = 7.83 \text{ kN} \quad T \quad \blacktriangleleft$$

From Eq. (1):

$$F_{CE} = F_{AC} = 7.826 \text{ kN} \quad F_{CE} = 7.83 \text{ kN} \quad T \quad \blacktriangleleft$$

Free body: Joint B:



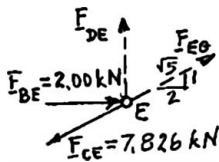
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}} F_{BD} + \frac{1}{\sqrt{2}} (9.90 \text{ kN}) - 2 \text{ kN} = 0$$

$$F_{BD} = -7.071 \text{ kN} \quad F_{BD} = 7.07 \text{ kN} \quad C \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad F_{BE} + \frac{1}{\sqrt{2}} (9.90 - 7.071) \text{ kN} = 0$$

$$F_{BE} = -2.000 \text{ kN} \quad F_{BE} = 2.00 \text{ kN} \quad C \quad \blacktriangleleft$$

Free body: Joint E:



$$+\rightarrow \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} (F_{EG} - 7.826 \text{ kN}) + 2.00 \text{ kN} = 0$$

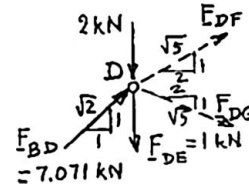
$$F_{EG} = 5.590 \text{ kN} \quad F_{EG} = 5.59 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} - \frac{1}{\sqrt{5}} (7.826 - 5.590) \text{ kN} = 0$$

$$F_{DE} = 1.000 \text{ kN} \quad F_{DE} = 1.000 \text{ kN} \quad T \quad \blacktriangleleft$$

Free body: Joint D:

$$+\rightarrow \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} (F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}} (7.071 \text{ kN})$$



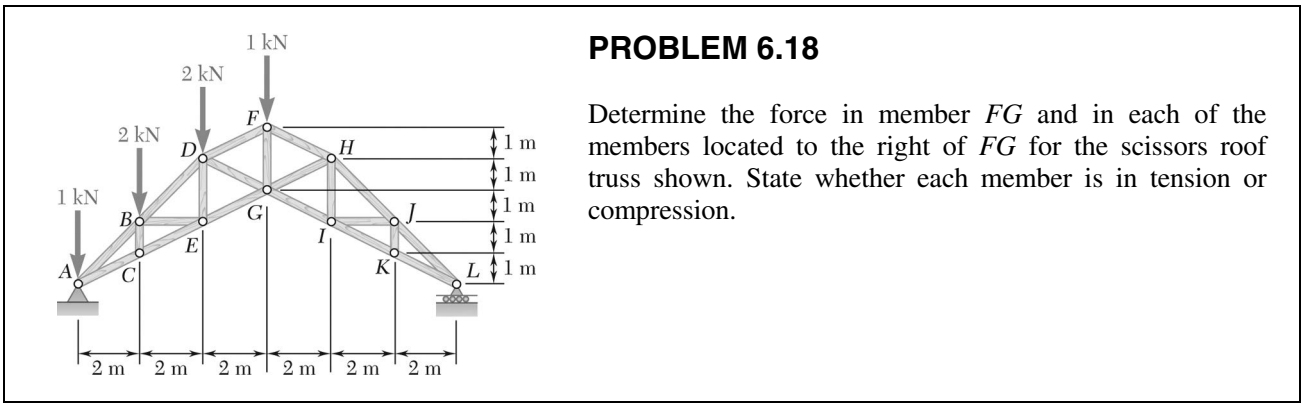
$$\text{or} \quad F_{DF} + F_{DG} = -5.590 \text{ kN} \quad (4)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} (F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}} (7.071 \text{ kN}) = 2 \text{ kN} - 1 \text{ kN} = 0$$

$$\text{or} \quad F_{DE} - F_{DG} = -4.472 \quad (5)$$

$$\text{Add Eqs. (4) and (5):} \quad 2F_{DF} = -10.062 \text{ kN} \quad F_{DF} = 5.03 \text{ kN} \quad C \quad \blacktriangleleft$$

$$\text{Subtract Eq. (5) from Eq. (4):} \quad 2F_{DG} = -1.1180 \text{ kN} \quad F_{DG} = 0.559 \text{ kN} \quad C \quad \blacktriangleleft$$



PROBLEM 6.18

Determine the force in member *FG* and in each of the members located to the right of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

$$+\circlearrowleft \sum M_A = 0: \quad L(12 \text{ m}) - (2 \text{ kN})(2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (1 \text{ kN})(6 \text{ m}) = 0$$

$$L = 1.500 \text{ kN} \uparrow$$

Angles:

$$\tan \alpha = 1 \quad \alpha = 45^\circ$$

$$\tan \beta = \frac{1}{2} \quad \beta = 26.57^\circ$$

Zero-force members:

Examining successively joints *K*, *J*, and *I*, we note that the following members to the right of *FG* are zero-force members: *JK*, *IJ*, and *HI*.

Thus, $F_{HI} = F_{IJ} = F_{JK} = 0 \quad \blacktriangleleft$

We also note that

$$F_{GI} = F_{IK} = F_{KL} \quad (1)$$

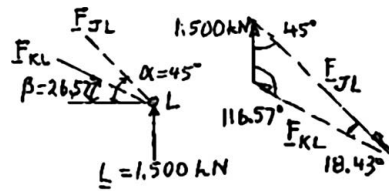
and

$$F_{HJ} = F_{JL} \quad (2)$$

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PROBLEM 6.18 (Continued)

Free body: Joint L :



$$\frac{F_{JL}}{\sin 116.57^\circ} = \frac{F_{KL}}{\sin 45^\circ} = \frac{1.500 \text{ kN}}{\sin 18.43^\circ}$$

$$F_{JL} = 4.2436 \text{ kN}$$

$$F_{JL} = 4.24 \text{ kN} \quad C \quad \blacktriangleleft$$

$$F_{KL} = 3.35 \text{ kN} \quad T \quad \blacktriangleleft$$

From Eq. (1):

$$F_{GI} = F_{IK} = F_{KL}$$

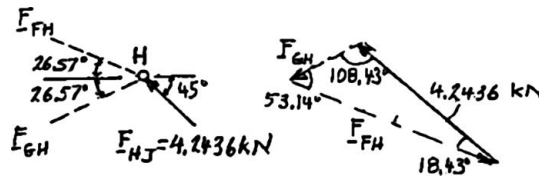
$$F_{GI} = F_{IK} = 3.35 \text{ kN} \quad T \quad \blacktriangleleft$$

From Eq. (2):

$$F_{HJ} = F_{JL} = 4.2436 \text{ kN}$$

$$F_{HJ} = 4.24 \text{ kN} \quad C \quad \blacktriangleleft$$

Free body: Joint H :

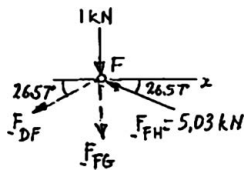


$$\frac{F_{FH}}{\sin 108.43^\circ} = \frac{F_{GH}}{\sin 18.43^\circ} = \frac{4.2436}{\sin 53.14^\circ}$$

$$F_{FH} = 5.03 \text{ kN} \quad C \quad \blacktriangleleft$$

$$F_{GH} = 1.677 \text{ kN} \quad T \quad \blacktriangleleft$$

Free body: Joint F :



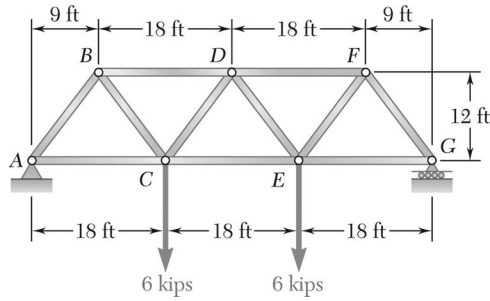
$$\rightarrow \Sigma F_x = 0: \quad -F_{DF} \cos 26.57^\circ - (5.03 \text{ kN}) \cos 26.57^\circ = 0$$

$$F_{DF} = -5.03 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{FG} - 1 \text{ kN} + (5.03 \text{ kN}) \sin 26.57^\circ - (-5.03 \text{ kN}) \sin 26.57^\circ = 0$$

$$F_{FG} = 3.500 \text{ kN}$$

$$F_{FG} = 3.50 \text{ kN} \quad T \quad \blacktriangleleft$$



PROBLEM 6.19

Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

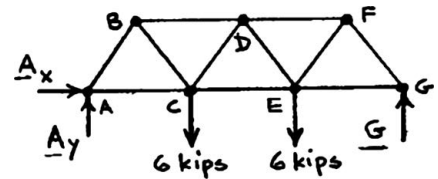
SOLUTION

Free body: Truss:

$$\sum F_x = 0: A_x = 0$$

Due to symmetry of truss and loading,

$$A_y = G = \frac{1}{2} \text{ total load} = 6 \text{ kips} \uparrow$$

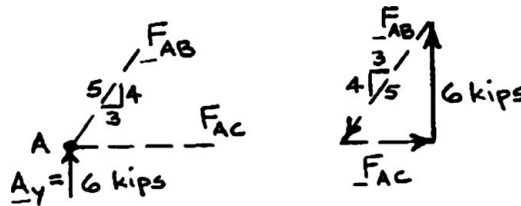


Free body: Joint A:

$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{6 \text{ kips}}{4}$$

$$F_{AB} = 7.50 \text{ kips} \quad C \blacktriangleleft$$

$$F_{AC} = 4.50 \text{ kips} \quad T \blacktriangleleft$$

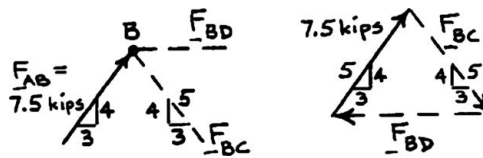


Free body: Joint B:

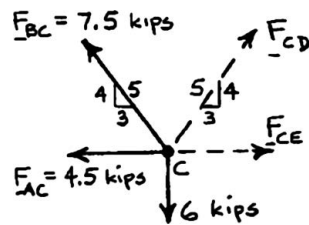
$$\frac{F_{BC}}{5} = \frac{F_{BD}}{6} = \frac{7.5 \text{ kips}}{5}$$

$$F_{BC} = 7.50 \text{ kips} \quad T \blacktriangleleft$$

$$F_{BD} = 9.00 \text{ kips} \quad C \blacktriangleleft$$



Free body: Joint C:



$$+\uparrow \sum F_y = 0: \frac{4}{5}(7.5) + \frac{4}{5}F_{CD} - 6 = 0$$

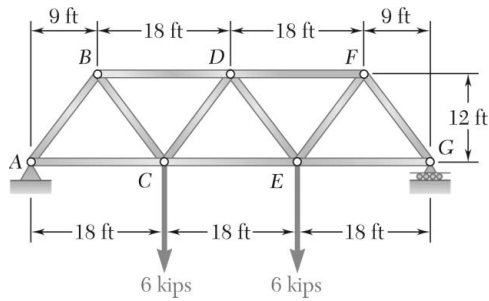
$$F_{CD} = 0 \quad C \blacktriangleleft$$

$$+\rightarrow \sum F_x = 0: F_{CE} - 4.5 - \frac{3}{5}(7.5) = 0$$

$$+\uparrow F_{CE} = +9 \text{ kips}$$

$$F_{CE} = 9.00 \text{ kips} \quad T \blacktriangleleft$$

Truss and loading is symmetrical about Φ .



PROBLEM 6.20

Solve Problem 6.19 assuming that the load applied at E has been removed.

PROBLEM 6.19 Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

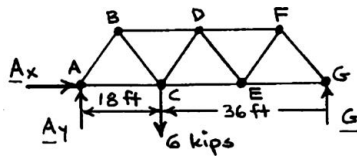
SOLUTION

Free body: Truss:

$$\Sigma F_x = 0: A_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: 6(36) - A_y(54) = 0 \quad A_y = 4 \text{ kips } \uparrow$$

$$+\uparrow \Sigma F_y = 0: 4 - 6 + G = 0 \quad G = 2 \text{ kips } \uparrow$$

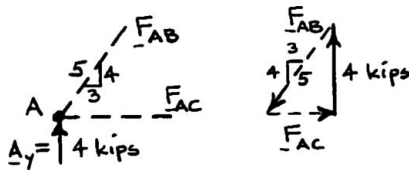


Free body: Joint A:

$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{4 \text{ kips}}{4}$$

$$F_{AB} = 5.00 \text{ kips } \quad C \leftarrow$$

$$F_{AC} = 3.00 \text{ kips } \quad T \leftarrow$$

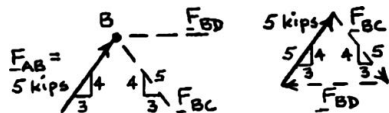


Free body Joint B:

$$\frac{F_{BC}}{5} = \frac{F_{BD}}{6} = \frac{5 \text{ kips}}{5}$$

$$F_{BC} = 5.00 \text{ kips } \quad T \leftarrow$$

$$F_{BD} = 6.00 \text{ kips } \quad C \leftarrow$$



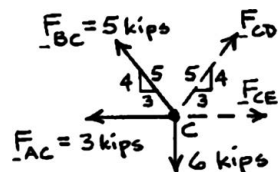
Free body Joint C:

$$+\circlearrowleft \Sigma M_y = 0: \frac{4}{5}(5) + \frac{4}{5}F_{CD} - 6 = 0 \quad F_{CD} = 2.50 \text{ kips } \quad T \leftarrow$$

$$F_{CD} = 2.50 \text{ kips } \quad T \leftarrow$$

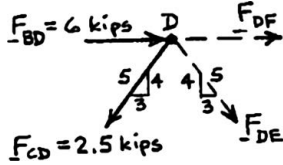
$$+\uparrow \Sigma F_x = 0: F_{CE} + \frac{3}{5}(2.5) - \frac{3}{5}(5) - 3 = 0 \quad F_{CE} = 4.50 \text{ kips } \quad T \leftarrow$$

$$F_{CE} = 4.50 \text{ kips } \quad T \leftarrow$$



PROBLEM 6.20 (Continued)

Free body: Joint D:



$$+\uparrow \Sigma F_y = 0: -\frac{4}{5}(2.5) - \frac{4}{5}F_{DE} = 0$$

$$F_{DE} = -2.5 \text{ kips}$$

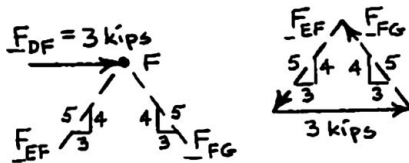
$$F_{DE} = 2.50 \text{ kips } C \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: F_{DF} + 6 - \frac{3}{5}(2.5) - \frac{3}{5}(2.5) = 0$$

$$F_{DF} = -3 \text{ kips}$$

$$F_{DF} = 3.00 \text{ kips } C \quad \blacktriangleleft$$

Free body: Joint F:



$$\frac{F_{EF}}{5} = \frac{F_{FG}}{5} = \frac{3 \text{ kips}}{6}$$

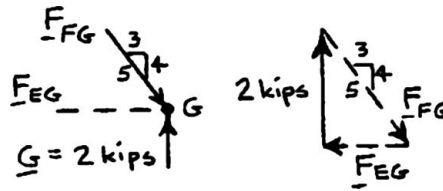
$$F_{EF} = 2.50 \text{ kips } T \quad \blacktriangleleft$$

$$F_{FG} = 2.50 \text{ kips } C \quad \blacktriangleleft$$

Free body: Joint G:

$$\frac{F_{EG}}{3} = \frac{2 \text{ kips}}{4}$$

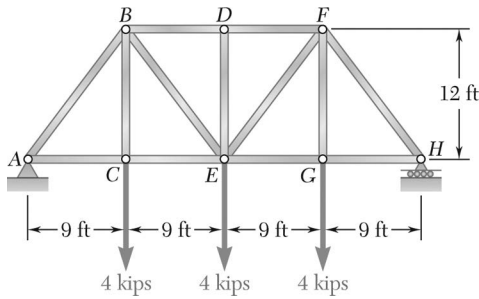
$$F_{EG} = 1.500 \text{ kips } T \quad \blacktriangleleft$$



Also,

$$\frac{F_{FG}}{5} = \frac{2 \text{ kips}}{4}$$

$$F_{FG} = 2.50 \text{ kips } C \quad (\text{Checks})$$



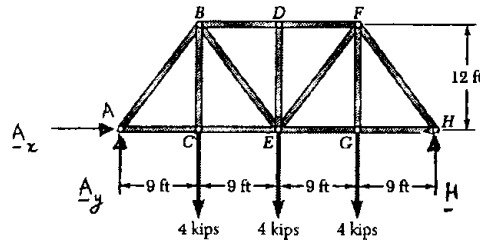
PROBLEM 6.21

Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

$$\begin{aligned} \pm \Sigma F_x = 0: & \quad A_x = 0 \\ + \Sigma M_A = 0: & \quad H(36 \text{ ft}) - (4 \text{ kips})(9 \text{ ft}) \\ & \quad - (4 \text{ kips})(18 \text{ ft}) - (4 \text{ kips})(27 \text{ ft}) = 0 \\ & \quad H = 6 \text{ kips} \uparrow \\ + \Sigma F_y = 0: & \quad A_y + 6 \text{ kips} - 12 \text{ kips} = 0 \quad A_y = 6 \text{ kips} \uparrow \end{aligned}$$



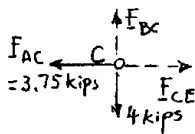
Free body: Joint A:

$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{6 \text{ kips}}{4}$$

$$F_{AB} = 7.50 \text{ kips} \quad C \blacktriangleleft$$

$$F_{AC} = 4.50 \text{ kips} \quad T \blacktriangleleft$$

Free body: Joint C:



$$\Sigma F_x = 0: \quad F_{CE} = 4.50 \text{ kips} \quad T \blacktriangleleft$$

$$\Sigma F_y = 0: \quad F_{BC} = 4.00 \text{ kips} \quad T \blacktriangleleft$$

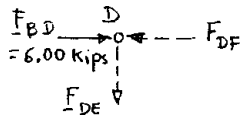
Free body: Joint B:

$$\begin{aligned} + \Sigma F_y = 0: & \quad -\frac{4}{5} F_{BE} + \frac{4}{5} (7.50 \text{ kips}) - 4.00 \text{ kips} = 0 \\ & \quad F_{BE} = 2.50 \text{ kips} \quad T \blacktriangleleft \\ + \Sigma F_x = 0: & \quad \frac{8}{5} (7.50 \text{ kips}) + \frac{3}{5} (2.50 \text{ kips}) + F_{BD} = 0 \\ & \quad F_{BD} = -6.00 \text{ kips} \quad F_{BD} = 6.00 \text{ kips} \quad C \blacktriangleleft \end{aligned}$$

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PROBLEM 6.21 (Continued)

Free body: Joint D:



We note that DE is a zero-force member:

$$F_{DE} = 0 \quad \blacktriangleleft$$

Also,

$$F_{DF} = 6.00 \text{ kips} \quad C \quad \blacktriangleleft$$

From symmetry:

$$F_{FE} = F_{BE}$$

$$F_{EF} = 2.50 \text{ kips} \quad T \quad \blacktriangleleft$$

$$F_{EG} = F_{CE}$$

$$F_{EG} = 4.50 \text{ kips} \quad T \quad \blacktriangleleft$$

$$F_{FG} = F_{BC}$$

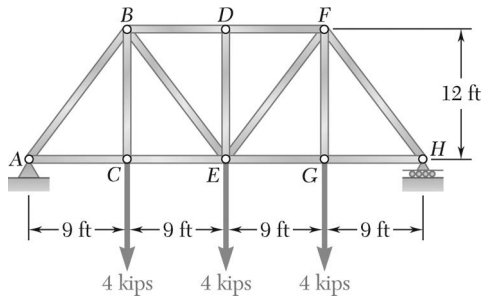
$$F_{FG} = 4.00 \text{ kips} \quad T \quad \blacktriangleleft$$

$$F_{FH} = F_{AB}$$

$$F_{FH} = 7.50 \text{ kips} \quad C \quad \blacktriangleleft$$

$$F_{GH} = F_{AC}$$

$$F_{GH} = 4.50 \text{ kips} \quad T \quad \blacktriangleleft$$



PROBLEM 6.22

Solve Problem 6.21 assuming that the load applied at G has been removed.

PROBLEM 6.21 Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

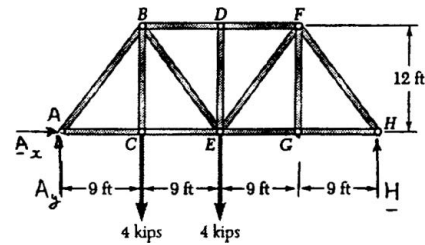
$$\begin{aligned} \Sigma F_x = 0: \quad A_x &= 0 \\ +\curvearrowright \Sigma M_A = 0: \quad H(36 \text{ ft}) - (4 \text{ kips})(9 \text{ ft}) - (4 \text{ kips})(18 \text{ ft}) &= 0 \\ \mathbf{H} &= 3.00 \text{ kips} \uparrow \\ +\uparrow \Sigma F_y = 0: \quad A_y + 5.00 \text{ kips} &\uparrow \end{aligned}$$

We note that DE and FG are zero-force members.

Therefore,

$$\text{Also,} \quad F_{BD} = F_{DF}$$

$$\text{and} \quad F_{EG} = F_{GH}$$

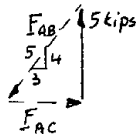
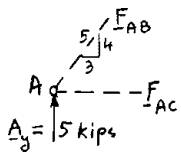


$$F_{DE} = 0, \quad F_{FG} = 0 \quad \blacktriangleleft$$

$$(1)$$

$$(2)$$

Free body: Joint A:

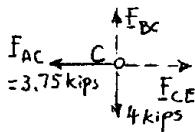


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{5 \text{ kips}}{4}$$

$$F_{AB} = 6.25 \text{ kips} \quad C \quad \blacktriangleleft$$

$$F_{AC} = 3.75 \text{ kips} \quad T \quad \blacktriangleleft$$

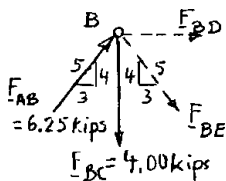
Free body: Joint C:



$$\Sigma F_x = 0: \quad F_{CE} = 3.75 \text{ kips} \quad T \quad \blacktriangleleft$$

$$\Sigma F_y = 0: \quad F_{BC} = 4.00 \text{ kips} \quad T \quad \blacktriangleleft$$

Free body: Joint B:



$$+\uparrow \Sigma F_x = 0: \quad \frac{4}{5}(6.25 \text{ kips}) - 4.00 \text{ kips} - \frac{4}{5}F_{BE} = 0$$

$$F_{BE} = 1.250 \text{ kips} \quad T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad F_{BD} + \frac{3}{5}(6.25 \text{ kips}) + \frac{3}{5}(1.250 \text{ kips}) = 0$$

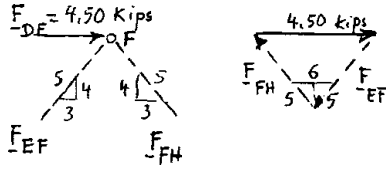
$$F_{BD} = -4.50 \text{ kips}$$

$$F_{BD} = 4.50 \text{ kips} \quad C \quad \blacktriangleleft$$

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PROBLEM 6.22 (Continued)

Free body: Joint F:



We recall that $F_{FG} = 0$, and from Eq. (1) that

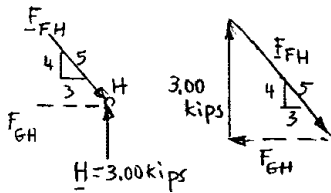
$$F_{DF} = F_{BD} \qquad F_{DF} = 4.50 \text{ kips } C \blacktriangleleft$$

$$\frac{F_{EF}}{5} = \frac{F_{FH}}{5} = \frac{4.50 \text{ kips}}{6}$$

$$F_{EF} = 3.75 \text{ kips } T \blacktriangleleft$$

$$F_{FH} = 3.75 \text{ kips } C \blacktriangleleft$$

Free body: Joint H:



$$\frac{F_{GH}}{3} = \frac{3.00 \text{ kips}}{4}$$

$$F_{GH} = 2.25 \text{ kips } T \blacktriangleleft$$

Also,

$$\frac{F_{FH}}{5} = \frac{3.00 \text{ kips}}{4}$$

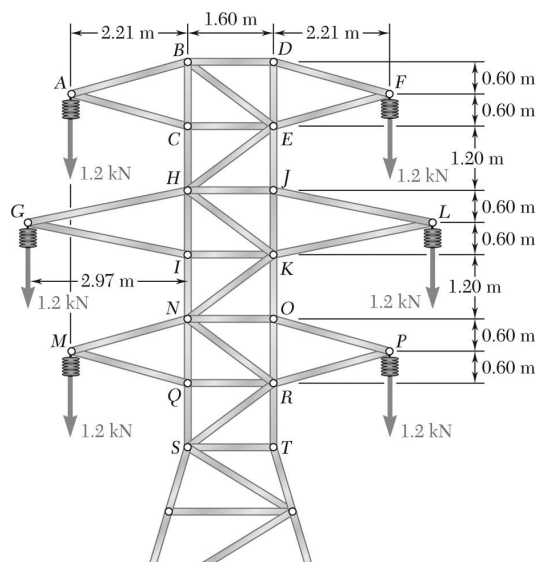
$$F_{FH} = 3.75 \text{ kips } C \text{ (checks)}$$

From Eq. (2):

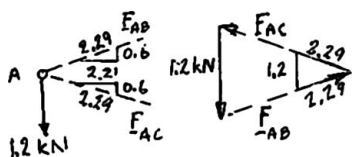
$$F_{EG} = F_{GH} \qquad F_{EG} = 2.25 \text{ kips } T \blacktriangleleft$$

PROBLEM 6.23

The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above HJ . State whether each member is in tension or compression.



SOLUTION

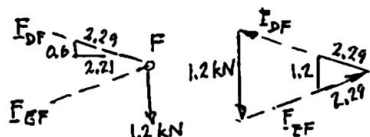


Free body: Joint A:

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2 \text{ kN}}{1.2}$$

$$F_{AB} = 2.29 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{AC} = 2.29 \text{ kN} \quad C \quad \blacktriangleleft$$

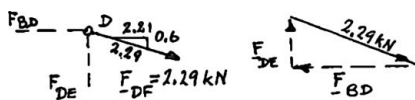


Free body: Joint F:

$$\frac{F_{DF}}{2.29} = \frac{F_{EF}}{2.29} = \frac{1.2 \text{ kN}}{2.1}$$

$$F_{DF} = 2.29 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{EF} = 2.29 \text{ kN} \quad C \quad \blacktriangleleft$$



Free body: Joint D:

$$\frac{F_{BD}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.29 \text{ kN}}{2.29}$$

$$F_{BD} = 2.21 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{DE} = 0.600 \text{ kN} \quad C \quad \blacktriangleleft$$

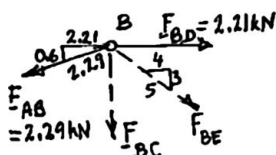
Free body: Joint B:

$$\rightarrow \Sigma F_x = 0: \quad \frac{4}{5} F_{BE} + 2.21 \text{ kN} - \frac{2.21}{2.29} (2.29 \text{ kN}) = 0$$

$$F_{BE} = 0 \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{3}{5} (0) - \frac{0.6}{2.29} (2.29 \text{ kN}) = 0$$

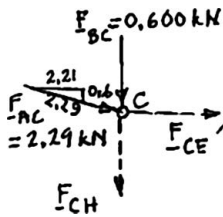
$$F_{BC} = -0.600 \text{ kN} \quad F_{BC} = 0.600 \text{ kN} \quad C \quad \blacktriangleleft$$



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PROBLEM 6.23 (Continued)

Free body: Joint C:



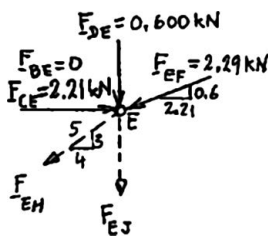
$$\rightarrow \Sigma F_x = 0: F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = -2.21 \text{ kN} \quad F_{CE} = 2.21 \text{ kN} \quad C \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -F_{CH} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -1.200 \text{ kN} \quad F_{CH} = 1.200 \text{ kN} \quad C \leftarrow$$

Free body: Joint E:

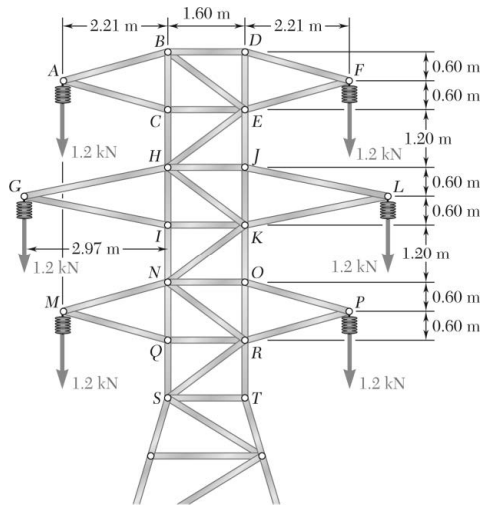


$$\rightarrow \Sigma F_x = 0: 2.21 \text{ kN} - \frac{2.21}{2.29}(2.29 \text{ kN}) - \frac{4}{5}F_{EH} = 0$$

$$F_{EH} = 0 \quad \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -F_{EJ} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) - 0 = 0$$

$$F_{EJ} = -1.200 \text{ kN} \quad F_{EJ} = 1.200 \text{ kN} \quad C \leftarrow$$

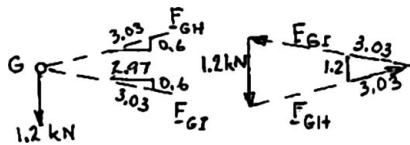


PROBLEM 6.24

For the tower and loading of Problem 6.23 and knowing that $F_{CH} = F_{EJ} = 1.2$ kN *C* and $F_{EH} = 0$, determine the force in member *HJ* and in each of the members located between *HJ* and *NO*. State whether each member is in tension or compression.

PROBLEM 6.23 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

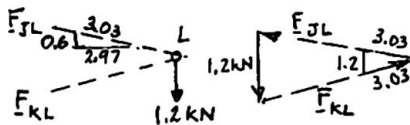


Free body: Joint *G*:

$$\frac{F_{GH}}{3.03} = \frac{F_{GI}}{3.03} = \frac{1.2}{1.2}$$

$$F_{GH} = 3.03 \text{ kN } T \leftarrow$$

$$F_{GI} = 3.03 \text{ kN } C \leftarrow$$

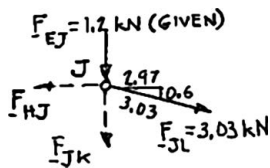


Free body: Joint *L*:

$$\frac{F_{JL}}{3.03} = \frac{F_{KL}}{3.03} = \frac{1.2}{1.2}$$

$$F_{JL} = 3.03 \text{ kN } T \leftarrow$$

$$F_{KL} = 3.03 \text{ kN } C \leftarrow$$



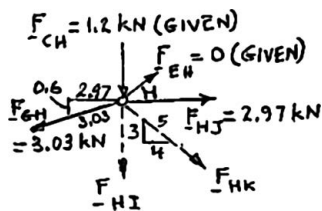
Free body: Joint *J*:

$$\rightarrow \Sigma F_x = 0: -F_{HJ} + \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{HJ} = 2.97 \text{ kN } T \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -F_{JK} - 1.2 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{JK} = -1.800 \text{ kN} \quad F_{JK} = 1.800 \text{ kN } C \leftarrow$$



Free body: Joint *H*:

$$\rightarrow \Sigma F_x = 0: \frac{4}{5}F_{HK} + 2.97 \text{ kN} - \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

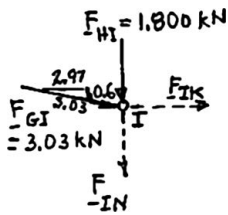
$$F_{HK} = 0 \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -F_{HI} - 1.2 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) - \frac{3}{5}(0) = 0$$

$$F_{HI} = -1.800 \text{ kN} \quad F_{HI} = 1.800 \text{ kN } C \leftarrow$$

PROBLEM 6.24 (Continued)

Free body: Joint I:



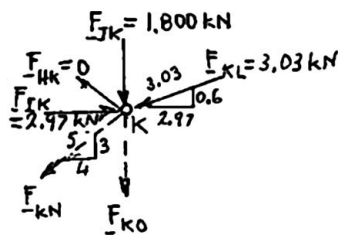
$$\rightarrow \Sigma F_x = 0: F_{IK} + \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{IK} = -2.97 \text{ kN} \quad F_{IK} = 2.97 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -F_{IN} - 1.800 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{IN} = -2.40 \text{ kN} \quad F_{IN} = 2.40 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint K:

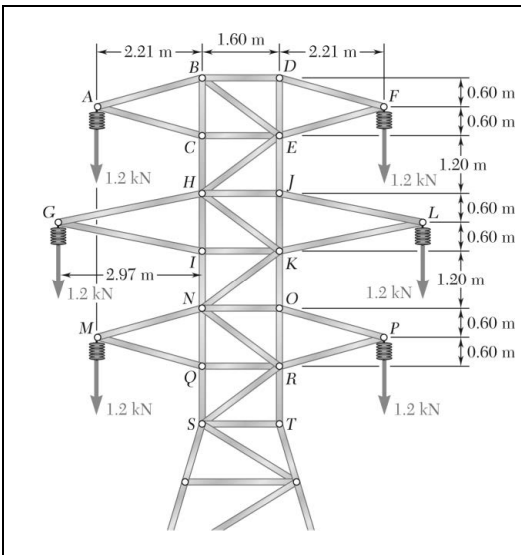


$$\rightarrow \Sigma F_x = 0: -\frac{4}{5}F_{KN} + 2.97 \text{ kN} - \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{KN} = 0 \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -F_{KO} - \frac{0.6}{3.03}(3.03 \text{ kN}) - 1.800 \text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{KO} = -2.40 \text{ kN} \quad F_{KO} = 2.40 \text{ kN} \quad C \blacktriangleleft$$

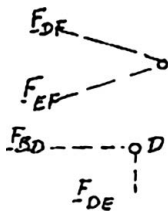


PROBLEM 6.25

Solve Problem 6.23 assuming that the cables hanging from the right side of the tower have fallen to the ground.

PROBLEM 6.23 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION



Zero-Force Members:

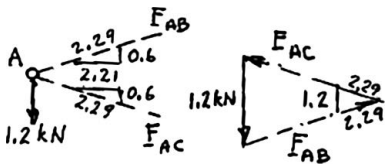
Considering joint *F*, we note that *DF* and *EF* are zero-force members:

$$F_{DF} = F_{EF} = 0 \quad \blacktriangleleft$$

Considering next joint *D*, we note that *BD* and *DE* are zero-force members:

$$F_{BD} = F_{DE} = 0 \quad \blacktriangleleft$$

Free body: Joint A:

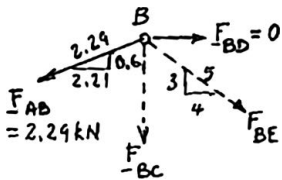


$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2 \text{ kN}}{1.2}$$

$$F_{AB} = 2.29 \text{ kN} \quad T \quad \blacktriangleleft$$

$$F_{AC} = 2.29 \text{ kN} \quad C \quad \blacktriangleleft$$

Free body: Joint B:



$$+\rightarrow \Sigma F_x = 0: \quad \frac{4}{5} F_{BE} - \frac{2.21}{2.29} (2.29 \text{ kN}) = 0$$

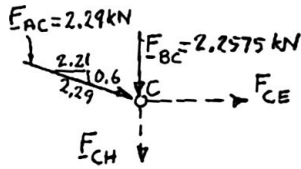
$$F_{BE} = 2.7625 \text{ kN} \quad F_{BE} = 2.76 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{0.6}{2.29} (2.29 \text{ kN}) - \frac{3}{5} (2.7625 \text{ kN}) = 0$$

$$F_{BC} = -2.2575 \text{ kN} \quad F_{BC} = 2.26 \text{ kN} \quad C \quad \blacktriangleleft$$

PROBLEM 6.25 (Continued)

Free body: Joint C:



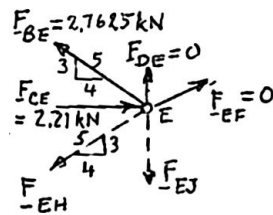
$$\rightarrow \Sigma F_x = 0: F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = 2.21 \text{ kN} \quad C \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -F_{CH} - 2.2575 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -2.8575 \text{ kN} \quad F_{CH} = 2.86 \text{ kN} \quad C \leftarrow$$

Free body: Joint E:

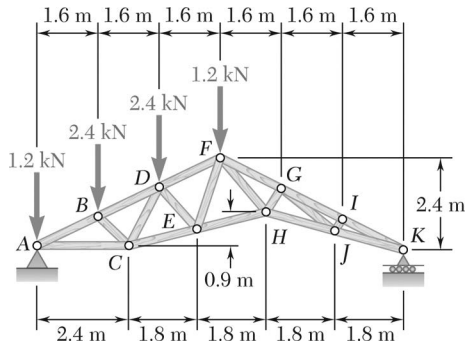


$$\rightarrow \Sigma F_x = 0: -\frac{4}{5}F_{EH} - \frac{4}{5}(2.7625 \text{ kN}) + 2.21 \text{ kN} = 0$$

$$F_{EH} = 0 \quad C \leftarrow$$

$$+\uparrow \Sigma F_y = 0: -F_{EJ} + \frac{3}{5}(2.7625 \text{ kN}) - \frac{3}{5}(0) = 0$$

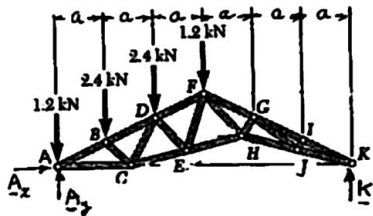
$$F_{EJ} = +1.6575 \text{ kN} \quad F_{EJ} = 1.658 \text{ kN} \quad T \leftarrow$$



PROBLEM 6.26

Determine the force in each of the members connecting joints A through F of the vaulted roof truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss:

$$\sum F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \sum M_K = 0: \quad (1.2 \text{ kN})6a + (2.4 \text{ kN})5a + (2.4 \text{ kN})4a + (1.2 \text{ kN})3a$$

$$-A_y(6a) = 0 \quad A_y = 5.40 \text{ kN} \uparrow$$

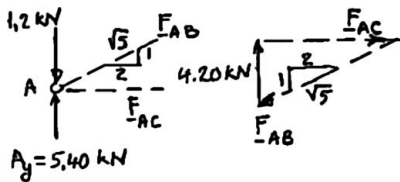
Free body: Joint A:

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{4.20 \text{ kN}}{1}$$

$$F_{AB} = 9.3915 \text{ kN}$$

$$F_{AB} = 9.39 \text{ kN} \quad C \leftarrow$$

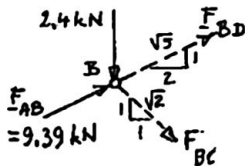
$$F_{AC} = 8.40 \text{ kN} \quad T \leftarrow$$



Free body: Joint B:

$$+\rightarrow \sum F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{2}} F_{BC} + \frac{2}{\sqrt{5}} (9.3915) = 0 \quad (1)$$

$$+\uparrow \sum F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{2}} F_{BC} + \frac{1}{\sqrt{5}} (9.3915) - 2.4 = 0 \quad (2)$$



Add Eqs. (1) and (2):

$$\frac{3}{\sqrt{5}} F_{BD} + \frac{3}{\sqrt{5}} (9.3915 \text{ kN}) - 2.4 \text{ kN} = 0$$

$$F_{BD} = -7.6026 \text{ kN}$$

$$F_{BD} = 7.60 \text{ kN} \quad C \leftarrow$$

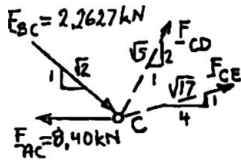
Multiply Eq. (2) by -2 and add Eq. (1):

$$\frac{3}{\sqrt{2}} F_B + 4.8 \text{ kN} = 0$$

$$F_{BC} = -2.2627 \text{ kN}$$

$$F_{BC} = 2.26 \text{ kN} \quad C \leftarrow$$

PROBLEM 6.26 (Continued)



Free body: Joint C:

$$\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{5}} F_{CD} + \frac{4}{\sqrt{17}} F_{CE} + \frac{1}{\sqrt{2}} (2.2627) - 8.40 = 0 \quad (3)$$

$$+\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{5}} F_{CD} + \frac{1}{\sqrt{17}} F_{CE} - \frac{1}{\sqrt{2}} (2.2627) = 0 \quad (4)$$

Multiply Eq. (4) by -4 and add Eq. (1):

$$-\frac{7}{\sqrt{5}} F_{CD} + \frac{5}{\sqrt{2}} (2.2627) - 8.40 = 0$$

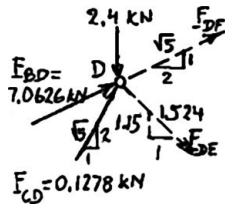
$$F_{CD} = -0.1278 \text{ kN} \quad F_{CD} = 0.128 \text{ kN} \quad C \blacktriangleleft$$

Multiply Eq. (1) by 2 and subtract Eq. (2):

$$\frac{7}{\sqrt{17}} F_{CE} + \frac{3}{\sqrt{2}} (2.2627) - 2(8.40) = 0$$

$$F_{CE} = 7.068 \text{ kN} \quad F_{CE} = 7.07 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint D:



$$\begin{aligned} \rightarrow \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{1.524} F_{DE} + \frac{2}{\sqrt{5}} (7.6026) \\ + \frac{1}{\sqrt{5}} (0.1278) = 0 \end{aligned} \quad (5)$$

$$\begin{aligned} +\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{DF} - \frac{1.15}{1.524} F_{DE} + \frac{1}{\sqrt{5}} (7.6026) \\ + \frac{2}{\sqrt{5}} (0.1278) - 2.4 = 0 \end{aligned} \quad (6)$$

Multiply Eq. (5) by 1.15 and add Eq. (6):

$$\frac{3.30}{\sqrt{5}} F_{DF} + \frac{3.30}{\sqrt{5}} (7.6026) + \frac{3.15}{\sqrt{5}} (0.1278) - 2.4 = 0$$

$$F_{DF} = -6.098 \text{ kN} \quad F_{DF} = 6.10 \text{ kN} \quad C \blacktriangleleft$$

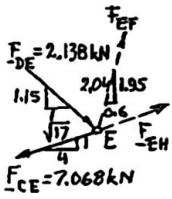
Multiply Eq. (6) by -2 and add Eq. (5):

$$\frac{3.30}{1.524} F_{DE} - \frac{3}{\sqrt{5}} (0.1278) + 4.8 = 0$$

$$F_{DE} = -2.138 \text{ kN} \quad F_{DE} = 2.14 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.26 (Continued)

Free body: Joint E :

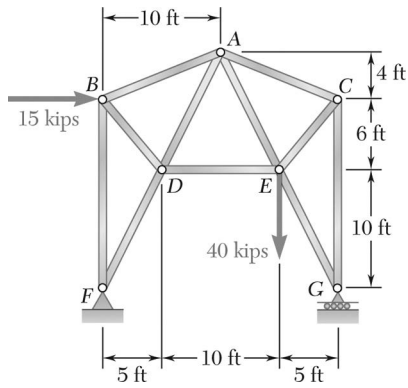


$$\rightarrow \Sigma F_x = 0: \frac{0.6}{2.04} F_{EF} + \frac{4}{\sqrt{17}} (F_{EH} - F_{CE}) + \frac{1}{1.524} (2.138) = 0 \quad (7)$$

$$+\uparrow \Sigma F_y = 0: \frac{1.95}{2.04} F_{EF} + \frac{1}{\sqrt{17}} (F_{EH} - F_{CE}) - \frac{1.15}{1.524} (2.138) = 0 \quad (8)$$

Multiply Eq. (8) by 4 and subtract Eq. (7):

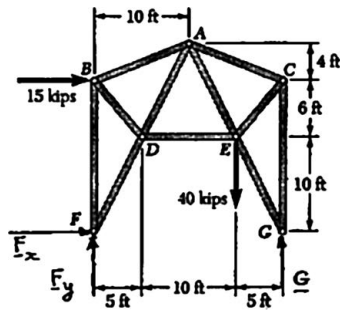
$$\frac{7.2}{2.04} F_{EF} - 7.856 \text{ kN} = 0 \quad F_{EF} = 2.23 \text{ kN } T \quad \blacktriangleleft$$



PROBLEM 6.27

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss:

$$+\circlearrowleft \Sigma M_F = 0: G(20 \text{ ft}) - (15 \text{ kips})(16 \text{ ft}) - (40 \text{ kips})(15 \text{ ft}) = 0$$

$$G = 42 \text{ kips} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: F_x + 15 \text{ kips} = 0$$

$$F_x = 15 \text{ kips} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: F_y - 40 \text{ kips} + 42 \text{ kips} = 0$$

$$F_y = 2 \text{ kips} \downarrow$$

Free body: Joint F:

$$+\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{5}} F_{DF} - 15 \text{ kips} = 0$$

$$F_{DF} = 33.54 \text{ kips} \quad F_{DF} = 33.5 \text{ kips} \quad T \quad \blacktriangleleft$$

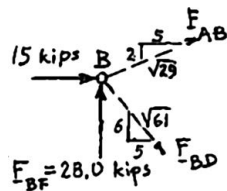
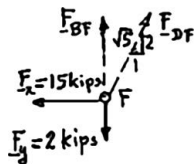
$$+\uparrow \Sigma F_y = 0: F_{BF} - 2 \text{ kips} + \frac{2}{\sqrt{5}} (33.54 \text{ kips}) = 0$$

$$F_{BF} = -28.00 \text{ kips} \quad F_{BF} = 28.0 \text{ kips} \quad C \quad \blacktriangleleft$$

Free body: Joint B:

$$+\rightarrow \Sigma F_x = 0: \frac{5}{\sqrt{29}} F_{AB} + \frac{5}{\sqrt{61}} F_{BD} + 15 \text{ kips} = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{29}} F_{AB} - \frac{6}{\sqrt{61}} F_{BD} + 28 \text{ kips} = 0 \quad (2)$$



PROBLEM 6.27 (Continued)

Multiply Eq. (1) by 6, Eq. (2) by 5, and add:

$$\frac{40}{\sqrt{29}} F_{AB} + 230 \text{ kips} = 0$$

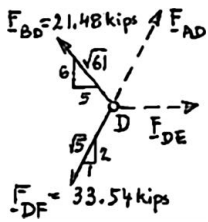
$$F_{AB} = -30.96 \text{ kips} \quad F_{AB} = 31.0 \text{ kips} \quad C \quad \blacktriangleleft$$

Multiply Eq. (1) by 2, Eq. (2) by -5 , and add:

$$\frac{40}{\sqrt{61}} F_{BD} - 110 \text{ kips} = 0$$

$$F_{BD} = 21.48 \text{ kips} \quad F_{BD} = 21.5 \text{ kips} \quad T \quad \blacktriangleleft$$

Free body: Joint D:



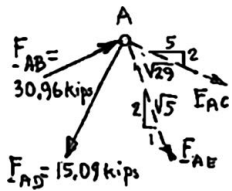
$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{AD} - \frac{2}{\sqrt{5}} (33.54) + \frac{6}{\sqrt{61}} (21.48) = 0$$

$$F_{AD} = 15.09 \text{ kips} \quad T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad F_{DE} + \frac{1}{\sqrt{5}} (15.09 - 33.54) - \frac{5}{\sqrt{61}} (21.48) = 0$$

$$F_{DE} = 22.0 \text{ kips} \quad T \quad \blacktriangleleft$$

Free body: Joint A:



$$+\rightarrow \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}} F_{AC} + \frac{1}{\sqrt{5}} F_{AE} + \frac{5}{\sqrt{29}} (30.36) - \frac{1}{\sqrt{5}} (15.09) = 0 \quad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2}{\sqrt{29}} F_{AC} - \frac{2}{\sqrt{5}} F_{AE} + \frac{2}{\sqrt{29}} (30.96) - \frac{2}{\sqrt{5}} (15.09) = 0 \quad (4)$$

Multiply Eq. (3) by 2 and add Eq. (4):

$$\frac{8}{\sqrt{29}} F_{AC} + \frac{12}{\sqrt{29}} (30.96) - \frac{4}{\sqrt{5}} (15.09) = 0$$

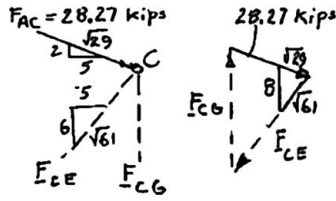
$$F_{AC} = -28.27 \text{ kips}, \quad F_{AC} = 28.3 \text{ kips} \quad C \quad \blacktriangleleft$$

Multiply Eq. (3) by 2, Eq. (4) by 5, and add:

$$-\frac{8}{\sqrt{5}} F_{AE} + \frac{20}{\sqrt{29}} (30.96) - \frac{12}{\sqrt{5}} (15.09) = 0$$

$$F_{AE} = 9.50 \text{ kips} \quad T \quad \blacktriangleleft$$

PROBLEM 6.27 (Continued)



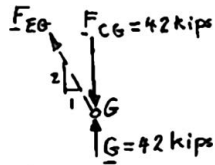
Free body: Joint C:

From force triangle:

$$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{28.27 \text{ kips}}{\sqrt{29}}$$

$$F_{CE} = 41.0 \text{ kips } T \quad \blacktriangleleft$$

$$F_{CG} = 42.0 \text{ kips } C \quad \blacktriangleleft$$

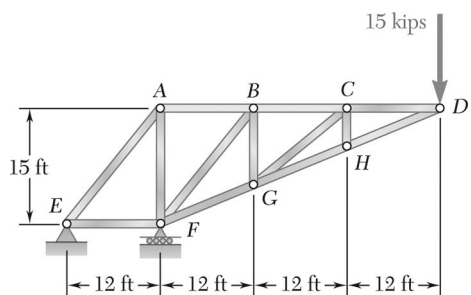


Free body: Joint G:

$$\pm \rightarrow \Sigma F_x = 0:$$

$$F_{EG} = 0 \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 42 \text{ kips} - 42 \text{ kips} = 0 \quad (\text{Checks})$$



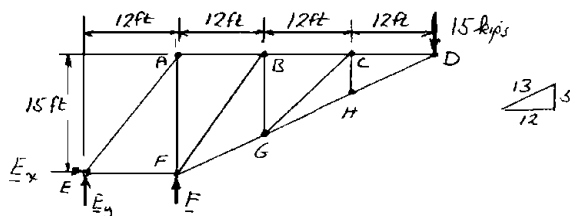
PROBLEM 6.28

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

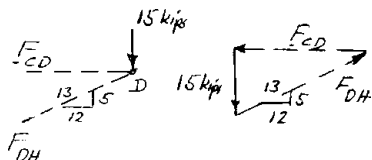
SOLUTION

Reactions:

$$\begin{aligned} \Sigma F_x = 0: & \quad E_x = 0 \\ \Sigma M_F = 0: & \quad E_y = 45 \text{ kips} \downarrow \\ \Sigma F_y = 0: & \quad F = 60 \text{ kips} \uparrow \end{aligned}$$



Joint D:

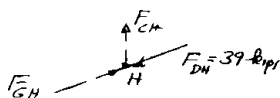


$$\frac{F_{CD}}{12} = \frac{F_{DH}}{13} = \frac{15 \text{ kips}}{5}$$

$$F_{CD} = 36.0 \text{ kips} \quad T \quad \blacktriangleleft$$

$$F_{DH} = 39.0 \text{ kips} \quad C \quad \blacktriangleleft$$

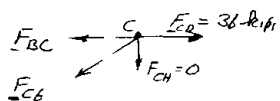
Joint H:



$$\nearrow \Sigma F = 0: \quad F_{CH} = 0 \quad \blacktriangleleft$$

$$\nearrow \Sigma F = 0: \quad F_{GH} = 39.0 \text{ kips} \quad C \quad \blacktriangleleft$$

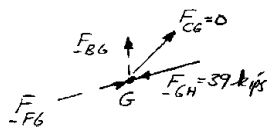
Joint C:



$$\uparrow \Sigma F = 0: \quad F_{CG} = 0 \quad \blacktriangleleft$$

$$\rightarrow \Sigma F = 0: \quad F_{BC} = 36.0 \text{ kips} \quad T \quad \blacktriangleleft$$

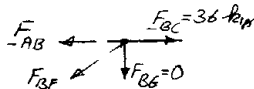
Joint G:



$$\nearrow \Sigma F = 0: \quad F_{BG} = 0 \quad \blacktriangleleft$$

$$\nearrow \Sigma F = 0: \quad F_{FG} = 39.0 \text{ kips} \quad C \quad \blacktriangleleft$$

Joint B:



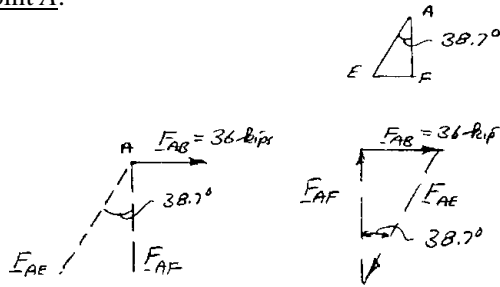
$$\uparrow \Sigma F = 0: \quad F_{BF} = 0 \quad \blacktriangleleft$$

$$\rightarrow \Sigma F = 0: \quad F_{AB} = 36.0 \text{ kips} \quad T \quad \blacktriangleleft$$

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PROBLEM 6.28 (Continued)

Joint A:



$$AE = \sqrt{12^2 + 15^2} = 19.21 \text{ ft}$$

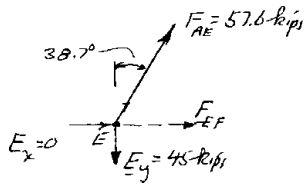
$$\tan 38.7^\circ = \frac{36 \text{ kips}}{F_{AF}}$$

$$F_{AF} = 45.0 \text{ kips} \quad C \blacktriangleleft$$

$$\sin 38.7^\circ = \frac{36 \text{ kips}}{F_{AE}}$$

$$F_{AE} = 57.6 \text{ kips} \quad T \blacktriangleleft$$

Joint E:



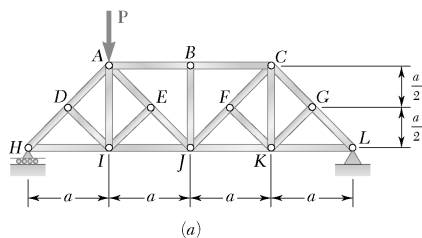
$$\rightarrow \Sigma F_x = 0: \quad + (57.6 \text{ kips}) \sin 38.7^\circ + F_{EF} = 0$$

$$F_{EF} = -36.0 \text{ kips} \quad F_{EF} = 36.0 \text{ kips} \quad C \blacktriangleleft$$

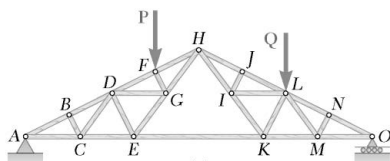
$$\uparrow \Sigma F_y = 0: \quad (57.6 \text{ kips}) \cos 38.7^\circ - 45 \text{ kips} = 0 \quad (\text{Checks})$$

PROBLEM 6.29

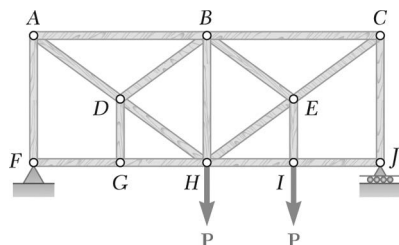
Determine whether the trusses of Problems 6.31a, 6.32a, and 6.33a are simple trusses.



(a)



(a)



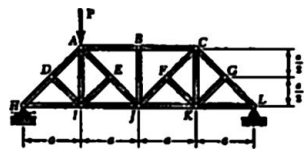
(a)

SOLUTION

Truss of Problem 6.31a:

Starting with triangle *HDI* and adding two members at a time, we obtain successively joints *A*, *E*, *J*, and *B*, but cannot go further. Thus, this truss

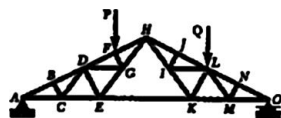
is not a simple truss. ◀



Truss of Problem 6.32a:

Starting with triangle *ABC* and adding two members at a time, we obtain joints *D*, *E*, *G*, *F*, and *H*, but cannot go further. Thus, this truss

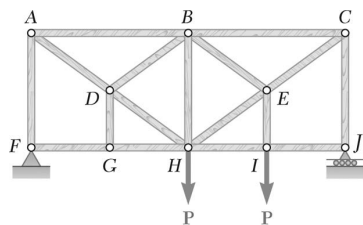
is not a simple truss. ◀



Truss of Problem 6.33a:

Starting with triangle *ABD* and adding two members at a time, we obtain successively joints *H*, *G*, *F*, *E*, *I*, *C*, and *J*, thus completing the truss.

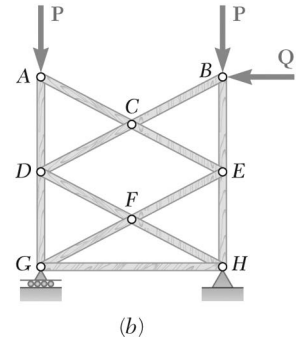
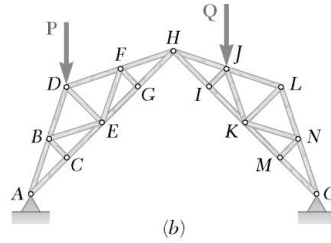
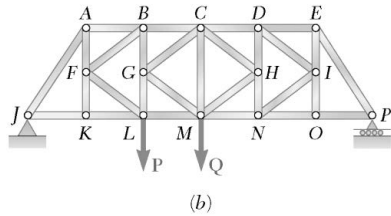
Therefore, this is a simple truss. ◀



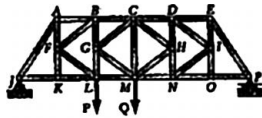
(a)

PROBLEM 6.30

Determine whether the trusses of Problems 6.31*b*, 6.32*b*, and 6.33*b* are simple trusses.



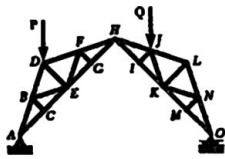
SOLUTION



Truss of Problem 6.31*b*:

Starting with triangle CGM and adding two members at a time, we obtain successively joints B, L, F, A, K, J , then H, D, N, I, E, O , and P , thus completing the truss.

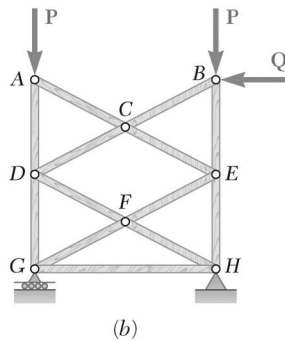
Therefore, this truss is a simple truss. ◀



Truss of Problem 6.32*b*:

Starting with triangle ABC and adding two members at a time, we obtain successively joints E, D, F, G , and H , but cannot go further. Thus, this truss

is not a simple truss. ◀



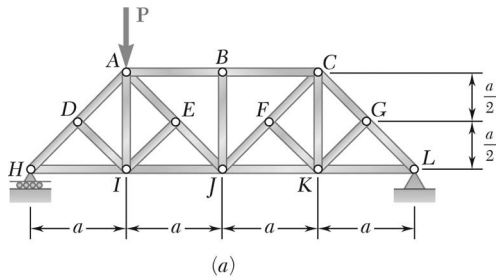
Truss of Problem 6.33*b*:

Starting with triangle GFH and adding two members at a time, we obtain successively joints D, E, C, A , and B , thus completing the truss.

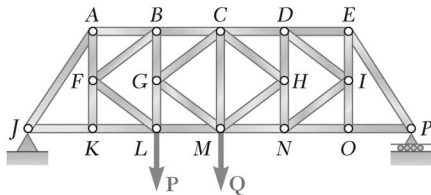
Therefore, this is a simple truss. ◀

PROBLEM 6.31

For the given loading, determine the zero-force members in each of the two trusses shown.



(a)



(b)

SOLUTION

Truss (a):

FB: Joint B: $F_{BJ} = 0$

FB: Joint D: $F_{DI} = 0$

FB: Joint E: $F_{EI} = 0$

FB: Joint I: $F_{AI} = 0$

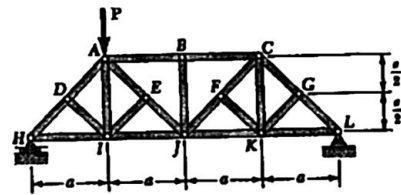
FB: Joint F: $F_{FK} = 0$

FB: Joint G: $F_{GK} = 0$

FB: Joint K: $F_{CK} = 0$

The zero-force members, therefore, are

$AI, BJ, CK, DI, EI, FK, GK$ ◀



(a)

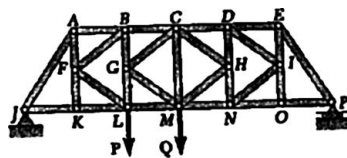
Truss (b):

FB: Joint K: $F_{FK} = 0$

FB: Joint O: $F_{IO} = 0$

The zero-force members, therefore, are

FK and IO ◀

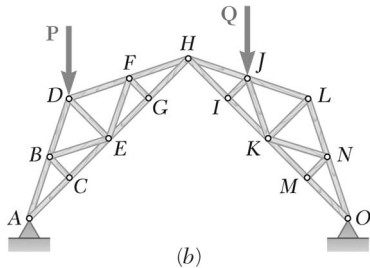
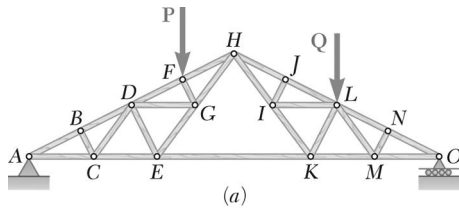


(b)

All other members are either in tension or compression.

PROBLEM 6.32

For the given loading, determine the zero-force members in each of the two trusses shown.



SOLUTION

Truss (a):

$$FB: \text{Joint } B: F_{BC} = 0$$

$$FB: \text{Joint } C: F_{CD} = 0$$

$$FB: \text{Joint } J: F_{IJ} = 0$$

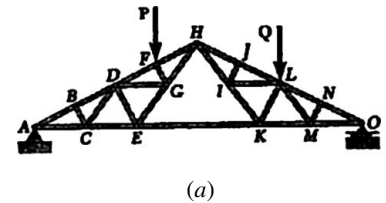
$$FB: \text{Joint } I: F_{IL} = 0$$

$$FB: \text{Joint } N: F_{MN} = 0$$

$$FB: \text{Joint } M: F_{LM} = 0$$

The zero-force members, therefore, are

BC, CD, IJ, IL, LM, MN ◀



Truss (b):

$$FB: \text{Joint } C: F_{BC} = 0$$

$$FB: \text{Joint } B: F_{BE} = 0$$

$$FB: \text{Joint } G: F_{FG} = 0$$

$$FB: \text{Joint } F: F_{EF} = 0$$

$$FB: \text{Joint } E: F_{DE} = 0$$

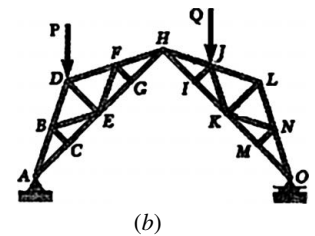
$$FB: \text{Joint } I: F_{IJ} = 0$$

$$FB: \text{Joint } M: F_{MN} = 0$$

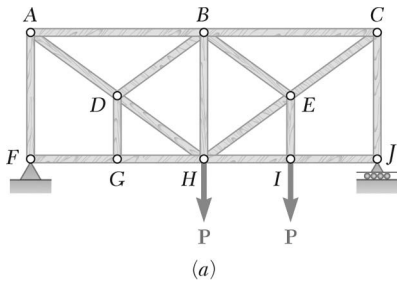
$$FB: \text{Joint } N: F_{KN} = 0$$

The zero-force members, therefore, are

$BC, BE, DE, EF, FG, IJ, KN, MN$ ◀

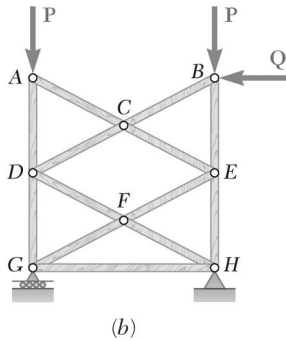


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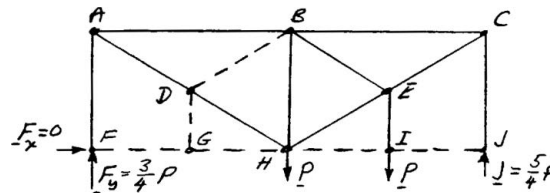
PROBLEM 6.33

For the given loading, determine the zero-force members in each of the two trusses shown.



SOLUTION

Truss (a):

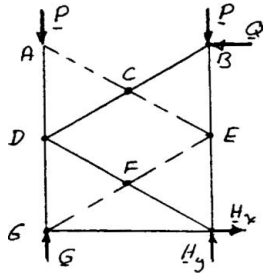


Note: Reaction at F is vertical ($F_x = 0$).

Joint G : $\downarrow \Sigma F = 0$,	$F_{DG} = 0 \blacktriangleleft$
Joint D : $\nearrow \Sigma F = 0$,	$F_{DB} = 0 \blacktriangleleft$
Joint F : $\rightarrow \Sigma F = 0$,	$F_{FG} = 0 \blacktriangleleft$
Joint G : $\rightarrow \Sigma F = 0$,	$F_{GH} = 0 \blacktriangleleft$
Joint J : $\rightarrow \Sigma F = 0$,	$F_{IJ} = 0 \blacktriangleleft$
Joint I : $\rightarrow \Sigma F = 0$,	$F_{HI} = 0 \blacktriangleleft$

PROBLEM 6.33 (Continued)

Truss (b):



Joint A: $\rightarrow \Sigma F = 0,$

$F_{AC} = 0 \blacktriangleleft$

Joint C: $\searrow \Sigma F = 0,$

$F_{CE} = 0 \blacktriangleleft$

Joint E: $\rightarrow \Sigma F = 0,$

$F_{EF} = 0 \blacktriangleleft$

Joint F: $\swarrow \Sigma F = 0,$

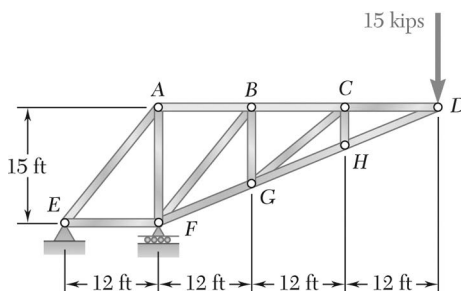
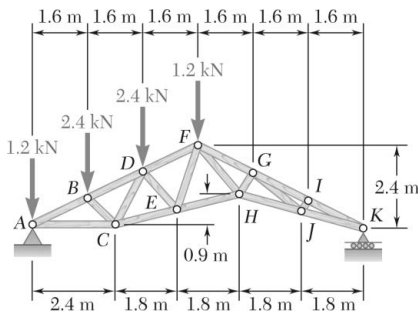
$F_{FG} = 0 \blacktriangleleft$

Joint G: $\rightarrow \Sigma F = 0,$

$F_{GH} = 0 \blacktriangleleft$

PROBLEM 6.34

Determine the zero-force members in the truss of (a) Problem 6.26, (b) Problem 6.28.



SOLUTION

(a) Truss of Problem 6.26:

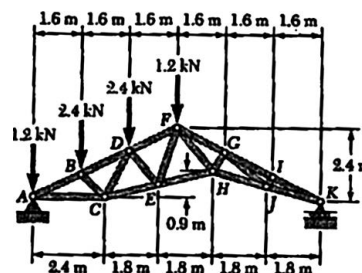
$$FB: \text{Joint } I: F_{IJ} = 0$$

$$FB: \text{Joint } J: F_{GJ} = 0$$

$$FB: \text{Joint } G: F_{GH} = 0$$

The zero-force members, therefore, are

GH, GJ, IJ ◀



(b) Truss of Problem 6.28:

$$FB: \text{Joint } B: F_{BF} = 0$$

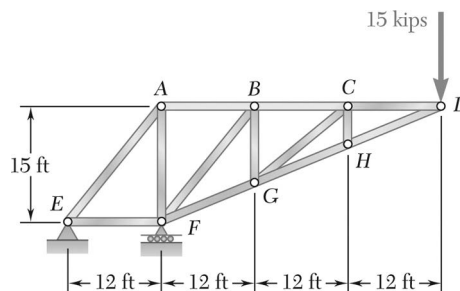
$$FB: \text{Joint } B: F_{BG} = 0$$

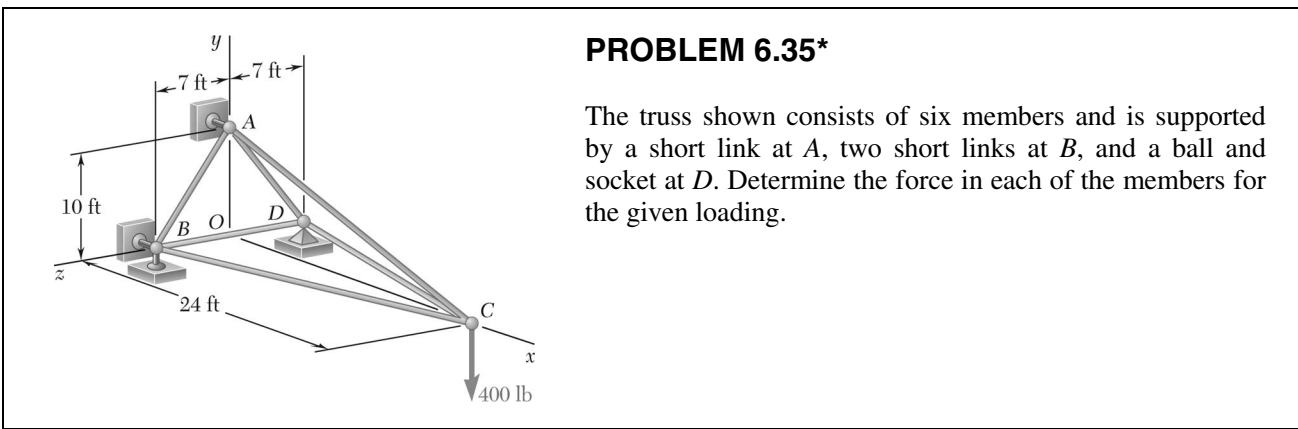
$$FB: \text{Joint } C: F_{CG} = 0$$

$$FB: \text{Joint } C: F_{CH} = 0$$

The zero-force members, therefore, are

BF, BG, CG, CH ◀





PROBLEM 6.35*

The truss shown consists of six members and is supported by a short link at A, two short links at B, and a ball and socket at D. Determine the force in each of the members for the given loading.

SOLUTION

Free body: Truss:

From symmetry:

$$D_x = B_x \quad \text{and} \quad D_y = B_y$$

$$\Sigma M_z = 0: \quad -A(10 \text{ ft}) - (400 \text{ lb})(24 \text{ ft}) = 0$$

$$A = -960 \text{ lb}$$

$$\Sigma F_x = 0: \quad B_x + D_x + A = 0$$

$$2B_x - 960 \text{ lb} = 0, \quad B_x = 480 \text{ lb}$$

$$\Sigma F_y = 0: \quad B_y + D_y - 400 \text{ lb} = 0$$

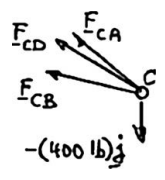
$$2B_y = 400 \text{ lb}$$

$$B_y = +200 \text{ lb}$$

Thus,

$$\mathbf{B} = (480 \text{ lb})\mathbf{i} + (200 \text{ lb})\mathbf{j} \quad \triangleleft$$

Free body: C:

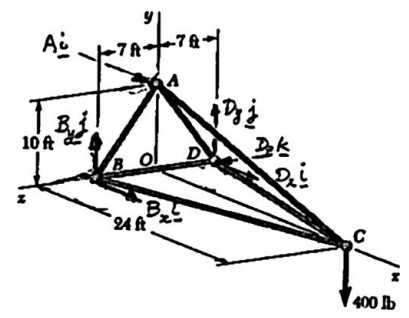


$$F_{CA} = F_{AC} \frac{\overline{CA}}{CA} = \frac{F_{AC}}{26} (-24\mathbf{i} + 10\mathbf{j})$$

$$F_{CB} = F_{BC} \frac{\overline{CB}}{CB} = \frac{F_{BC}}{25} (-24\mathbf{i} + 7\mathbf{k})$$

$$F_{CD} = F_{CD} \frac{\overline{CD}}{CD} = \frac{F_{CD}}{25} (-24\mathbf{i} - 7\mathbf{k})$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} - (400 \text{ lb})\mathbf{j} = 0$$



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PROBLEM 6.35* (Continued)

Substituting for \mathbf{F}_{CA} , \mathbf{F}_{CB} , \mathbf{F}_{CD} , and equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$\mathbf{i}: \quad -\frac{24}{26}F_{AC} - \frac{24}{25}(F_{BC} + F_{CD}) = 0 \quad (1)$$

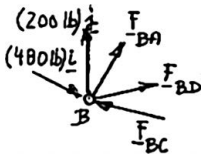
$$\mathbf{j}: \quad \frac{10}{26}F_{AC} - 400 \text{ lb} = 0 \quad F_{AC} = 1040 \text{ lb} \quad T \quad \blacktriangleleft$$

$$\mathbf{k}: \quad \frac{7}{25}(F_{BC} - F_{CD}) = 0 \quad F_{CD} = F_{BC}$$

Substitute for F_{AC} and F_{CD} in Eq. (1):

$$-\frac{24}{26}(1040 \text{ lb}) - \frac{24}{25}(2F_{BC}) = 0 \quad F_{BC} = -500 \text{ lb} \quad F_{BC} = F_{CD} = 500 \text{ lb} \quad C \quad \blacktriangleleft$$

Free body: B:



$$F_{BC} = (500 \text{ lb}) \frac{\overline{CB}}{CB} = -(480 \text{ lb})\mathbf{i} + (140 \text{ lb})\mathbf{k}$$

$$F_{BA} = F_{AB} \frac{\overline{BA}}{BA} = \frac{F_{AB}}{12.21}(10\mathbf{j} - 7\mathbf{k})$$

$$F_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BD} + \mathbf{F}_{BC} + (480 \text{ lb})\mathbf{i} + (200 \text{ lb})\mathbf{j} = 0$$

Substituting for \mathbf{F}_{BA} , \mathbf{F}_{BD} , \mathbf{F}_{BC} and equating to zero the coefficients of \mathbf{j} and \mathbf{k} :

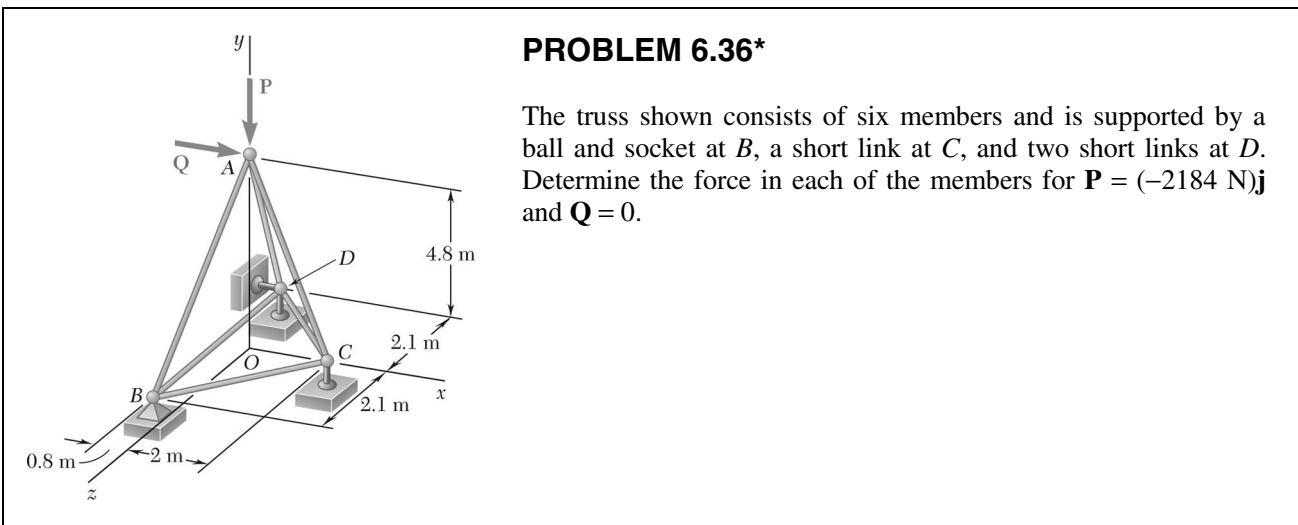
$$\mathbf{j}: \quad \frac{10}{12.21}F_{AB} + 200 \text{ lb} = 0 \quad F_{AB} = -244.2 \text{ lb} \quad F_{AB} = 244 \text{ lb} \quad C \quad \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{7}{12.21}F_{AB} - F_{BD} + 140 \text{ lb} = 0$$

$$F_{BD} = -\frac{7}{12.21}(-244.2 \text{ lb}) + 140 \text{ lb} = +280 \text{ lb} \quad F_{BD} = 280 \text{ lb} \quad T \quad \blacktriangleleft$$

From symmetry:

$$F_{AD} = F_{AB} \quad F_{AD} = 244 \text{ lb} \quad C \quad \blacktriangleleft$$



PROBLEM 6.36*

The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = (-2184 \text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION

Free body: Truss:

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0: 2B_x = 0$$

$$B_x = D_x = 0$$

$$\Sigma F_z = 0: B_z = 0$$

$$\Sigma M_{cz} = 0: -2B_y(2.8 \text{ m}) + (2184 \text{ N})(2 \text{ m}) = 0$$

$$B_y = 780 \text{ N}$$

Thus,

$$\mathbf{B} = (780 \text{ N})\mathbf{j} \quad \triangleleft$$

Free body: A:

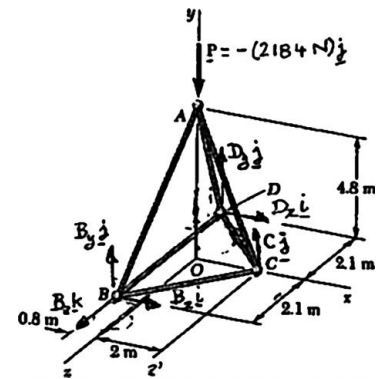


$$F_{AB} = F_{AB} \frac{\overline{AB}}{AB} = \frac{F_{AB}}{5.30} (-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC} \frac{\overline{AC}}{AC} = \frac{F_{AC}}{5.20} (2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD} \frac{\overline{AD}}{AD} = \frac{F_{AD}}{5.30} (+0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\Sigma \mathbf{F} = 0: \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2184 \text{ N})\mathbf{j} = 0$$



PROBLEM 6.36* (Continued)

Substituting for \mathbf{F}_{AB} , \mathbf{F}_{AC} , \mathbf{F}_{AD} , and equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB} + F_{AD}) + \frac{2}{5.20}F_{AC} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB} + F_{AD}) - \frac{4.8}{5.20}F_{AC} - 2184 \text{ N} = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB} - F_{AD}) = 0 \quad F_{AD} = F_{AB}$$

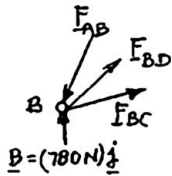
Multiply Eq. (1) by -6 and add Eq. (2):

$$-\left(\frac{16.8}{5.20}\right)F_{AC} - 2184 \text{ N} = 0, \quad F_{AC} = -676 \text{ N} \quad F_{AC} = 676 \text{ N} \quad C \leftarrow$$

Substitute for F_{AC} and F_{AD} in Eq. (1):

$$-\left(\frac{0.8}{5.30}\right)2F_{AB} + \left(\frac{2}{5.20}\right)(-676 \text{ N}) = 0, \quad F_{AB} = -861.25 \text{ N} \quad F_{AB} = F_{AD} = 861 \text{ N} \quad C \leftarrow$$

Free body: B:



$$\mathbf{F}_{AB} = (861.25 \text{ N}) \frac{\overline{AB}}{AB} = -(130 \text{ N})\mathbf{i} - (780 \text{ N})\mathbf{j} + (341.25 \text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC} = F_{BC} \left(\frac{2.8\mathbf{i} - 2.1\mathbf{k}}{3.5} \right) = F_{BC} (0.8\mathbf{i} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{BC} + \mathbf{F}_{BD} + (780 \text{ N})\mathbf{j} = 0$$

Substituting for \mathbf{F}_{AB} , \mathbf{F}_{BC} , \mathbf{F}_{BD} and equating to zero the coefficients of \mathbf{i} and \mathbf{k} ,

$$\mathbf{i}: \quad -130 \text{ N} + 0.8F_{BC} = 0 \quad F_{BC} = +162.5 \text{ N} \quad F_{BC} = 162.5 \text{ N} \quad T \leftarrow$$

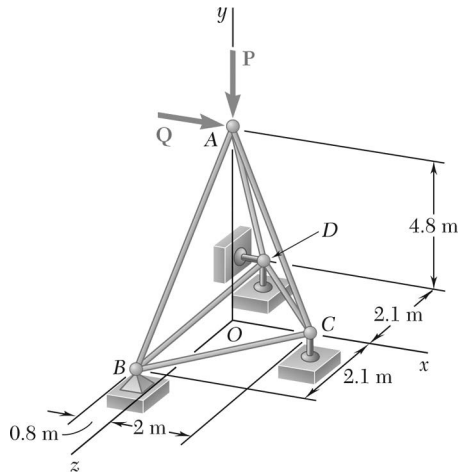
$$\mathbf{k}: \quad 341.25 \text{ N} - 0.6F_{BC} - F_{BD} = 0$$

$$F_{BD} = 341.25 - 0.6(162.5) = +243.75 \text{ N} \quad F_{BD} = 244 \text{ N} \quad T \leftarrow$$

From symmetry: $F_{CD} = F_{BC} \quad F_{CD} = 162.5 \text{ N} \quad T \leftarrow$

PROBLEM 6.37*

The truss shown consists of six members and is supported by a ball and socket at B , a short link at C , and two short links at D . Determine the force in each of the members for $P = 0$ and $Q = (2968 \text{ N})\mathbf{i}$.



SOLUTION

Free body: Truss:

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0: 2B_x + 2968 \text{ N} = 0$$

$$B_x = D_x = -1484 \text{ N}$$

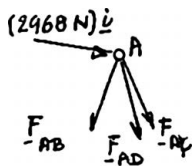
$$\Sigma M_{cz'} = 0: -2B_y(2.8 \text{ m}) - (2968 \text{ N})(4.8 \text{ m}) = 0$$

$$B_y = -2544 \text{ N}$$

Thus,

$$\mathbf{B} = -(1484 \text{ N})\mathbf{i} - (2544 \text{ N})\mathbf{j} \triangleleft$$

Free body: A:

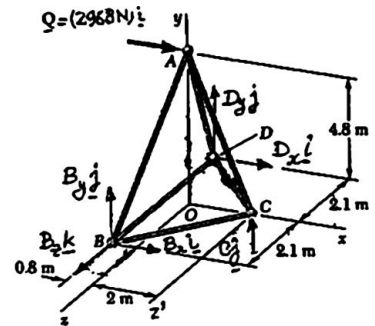


$$\begin{aligned} F_{AB} &= F_{AB} \frac{\overline{AB}}{AB} \\ &= \frac{F_{AB}}{5.30} (-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k}) \end{aligned}$$

$$F_{AC} = F_{AC} \frac{\overline{AC}}{AC} = \frac{F_{AC}}{5.20} (2\mathbf{i} - 4.8\mathbf{j})$$

$$\begin{aligned} F_{AD} &= F_{AD} \frac{\overline{AD}}{AD} \\ &= \frac{F_{AD}}{5.30} (-0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k}) \end{aligned}$$

$$\Sigma \mathbf{F} = 0: \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + (2968 \text{ N})\mathbf{i} = 0$$



PROBLEM 6.37* (Continued)

Substituting for \mathbf{F}_{AB} , \mathbf{F}_{AC} , \mathbf{F}_{AD} , and equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} ,

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB} + F_{AD}) + \frac{2}{5.20}F_{AC} + 2968 \text{ N} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB} + F_{AD}) - \frac{4.8}{5.20}F_{AC} = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB} - F_{AD}) = 0 \quad F_{AD} = F_{AB}$$

Multiply Eq. (1) by -6 and add Eq. (2):

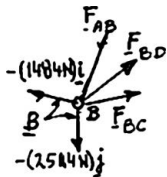
$$-\left(\frac{16.8}{5.20}\right)F_{AC} - 6(2968 \text{ N}) = 0, \quad F_{AC} = -5512 \text{ N} \quad F_{AC} = 5510 \text{ N} \quad C \blacktriangleleft$$

Substitute for F_{AC} and F_{AD} in Eq. (2):

$$-\left(\frac{4.8}{5.30}\right)2F_{AB} - \left(\frac{4.8}{5.20}\right)(-5512 \text{ N}) = 0, \quad F_{AB} = +2809 \text{ N}$$

$$F_{AB} = F_{AD} = 2810 \text{ N} \quad T \blacktriangleleft$$

Free body: B:



$$\mathbf{F}_{AB} = (2809 \text{ N}) \frac{\overline{BA}}{BA} = (424 \text{ N})\mathbf{i} + (2544 \text{ N})\mathbf{j} - (1113 \text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC} = F_{BC} \left(\frac{2.8\mathbf{i} - 2.1\mathbf{k}}{3.5} \right) = F_{BC}(0.8\mathbf{i} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{BC} + \mathbf{F}_{BD} - (1484 \text{ N})\mathbf{i} - (2544 \text{ N})\mathbf{j} = 0$$

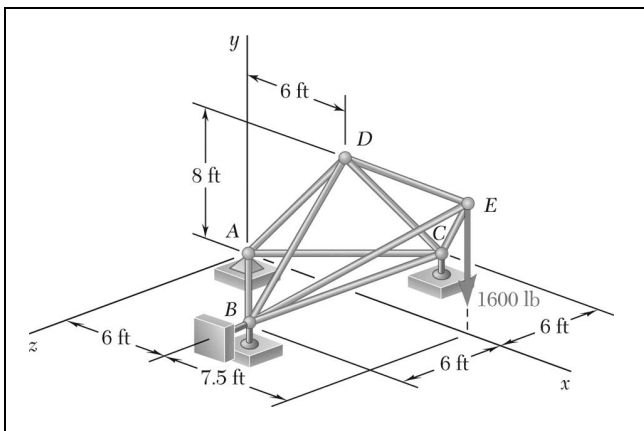
Substituting for \mathbf{F}_{AB} , \mathbf{F}_{BC} , \mathbf{F}_{BD} and equating to zero the coefficients of \mathbf{i} and \mathbf{k} ,

$$\mathbf{i}: \quad +24 \text{ N} + 0.8F_{BC} - 1484 \text{ N} = 0, \quad F_{BC} = +1325 \text{ N} \quad F_{BC} = 1325 \text{ N} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad -1113 \text{ N} - 0.6F_{BC} - F_{BD} = 0$$

$$F_{BD} = -1113 \text{ N} - 0.6(1325 \text{ N}) = -1908 \text{ N}, \quad F_{BD} = 1908 \text{ N} \quad C \blacktriangleleft$$

From symmetry: $F_{CD} = F_{BC} \quad F_{CD} = 1325 \text{ N} \quad T \blacktriangleleft$



PROBLEM 6.38*

The truss shown consists of nine members and is supported by a ball and socket at A, two short links at B, and a short link at C. Determine the force in each of the members for the given loading.

SOLUTION

Free body: Truss:

From symmetry:

$$A_z = B_z = 0$$

$$\Sigma F_x = 0: A_x = 0$$

$$\Sigma M_{BC} = 0: A_y(6 \text{ ft}) + (1600 \text{ lb})(7.5 \text{ ft}) = 0$$

$$A_y = -2000 \text{ lb}$$

$$\mathbf{A} = -(2000 \text{ lb})\mathbf{j} \quad \triangleleft$$

From symmetry:

$$B_y = C$$

$$\Sigma F_y = 0: 2B_y - 2000 \text{ lb} - 1600 \text{ lb} = 0$$

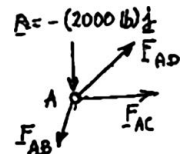
$$B_y = 1800 \text{ lb}$$

$$\mathbf{B} = (1800 \text{ lb})\mathbf{j} \quad \triangleleft$$

Free body: A:

$$\Sigma \mathbf{F} = 0: \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2000 \text{ lb})\mathbf{j} = 0$$

$$F_{AB} \frac{\mathbf{i} + \mathbf{k}}{\sqrt{2}} + F_{AC} \frac{\mathbf{i} - \mathbf{k}}{\sqrt{2}} + F_{AD}(0.6\mathbf{i} + 0.8\mathbf{j}) - (2000 \text{ lb})\mathbf{j} = 0$$



Factoring \mathbf{i} , \mathbf{j} , \mathbf{k} and equating their coefficient to zero,

$$\frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{2}} F_{AC} + 0.6 F_{AD} = 0 \quad (1)$$

$$0.8 F_{AD} - 2000 \text{ lb} = 0$$

$$F_{AD} = 2500 \text{ lb} \quad T \quad \blacktriangleleft$$

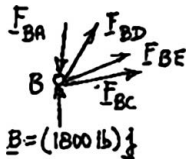
$$\frac{1}{\sqrt{2}} F_{AB} - \frac{1}{\sqrt{2}} F_{AC} = 0 \quad F_{AC} = F_{AB}$$

PROBLEM 6.38* (Continued)

Substitute for F_{AD} and F_{AC} into Eq. (1):

$$\frac{2}{\sqrt{2}} F_{AB} + 0.6(2500 \text{ lb}) = 0, \quad \mathbf{F}_{AB} = -1060.7 \text{ lb}, \quad F_{AB} = F_{AC} = 1061 \text{ lb} \quad C \blacktriangleleft$$

Free body: B :



$$\mathbf{F}_{BA} = F_{AB} \frac{\overline{BA}}{BA} = +(1060.7 \text{ lb}) \frac{\mathbf{i} + \mathbf{k}}{\sqrt{2}} = (750 \text{ lb})(\mathbf{i} + \mathbf{k})$$

$$\mathbf{F}_{BC} = -F_{BC} \mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}(0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BE} = F_{BE} \frac{\overline{BE}}{BE} = \frac{F_{BE}}{12.5} (7.5\mathbf{i} + 8\mathbf{j} - 6\mathbf{k})$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_{BE} + (1800 \text{ lb})\mathbf{j} = 0$$

Substituting for \mathbf{F}_{BA} , \mathbf{F}_{BC} , \mathbf{F}_{BD} , and \mathbf{F}_{BE} and equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} ,

$$\mathbf{i}: \quad 750 \text{ lb} + \left(\frac{7.5}{12.5}\right) F_{BE} = 0, \quad F_{BE} = -1250 \text{ lb} \quad F_{BE} = 1250 \text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{j}: \quad 0.8 F_{BD} + \left(\frac{8}{12.5}\right)(-1250 \text{ lb}) + 1800 \text{ lb} = 0 \quad F_{BD} = 1250 \text{ lb} \quad C \blacktriangleleft$$

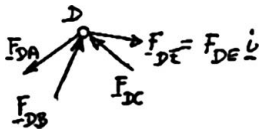
$$\mathbf{k}: \quad 750 \text{ lb} - F_{BC} - 0.6(-1250 \text{ lb}) - \frac{6}{12.5}(-1250 \text{ lb}) = 0$$

$$F_{BC} = 2100 \text{ lb} \quad T \blacktriangleleft$$

From symmetry:

$$F_{BD} = F_{CD} = 1250 \text{ lb} \quad C \blacktriangleleft$$

Free body: D :



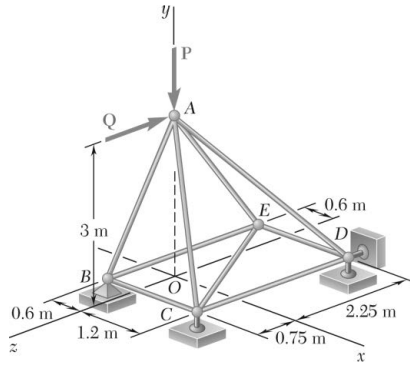
$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} + F_{DE} \mathbf{i} = 0$$

We now substitute for \mathbf{F}_{DA} , \mathbf{F}_{DB} , \mathbf{F}_{DC} and equate to zero the coefficient of \mathbf{i} . Only \mathbf{F}_{DA} contains \mathbf{i} and its coefficient is

$$-0.6 F_{AD} = -0.6(2500 \text{ lb}) = -1500 \text{ lb}$$

$$\mathbf{i}: \quad -1500 \text{ lb} + F_{DE} = 0 \quad F_{DE} = 1500 \text{ lb} \quad T \blacktriangleleft$$

PROBLEM 6.39*



The truss shown consists of nine members and is supported by a ball and socket at B , a short link at C , and two short links at D . (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) Determine the force in each member for $\mathbf{P} = (-1200 \text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION

Free body: Truss:

$$\begin{aligned} \Sigma \mathbf{M}_B = 0: & 1.8\mathbf{i} \times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k}) \times (D_y\mathbf{j} + D_z\mathbf{k}) \\ & + (0.6\mathbf{i} - 0.75\mathbf{k}) \times (-1200\mathbf{j}) = 0 \\ & -1.8C\mathbf{k} + 1.8D_y\mathbf{k} - 1.8D_z\mathbf{j} \\ & + 3D_y\mathbf{i} - 720\mathbf{k} - 900\mathbf{i} = 0 \end{aligned}$$

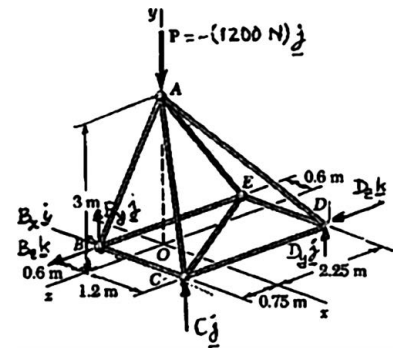
Equate to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad 3D_y - 900 = 0, \quad D_y = 300 \text{ N}$$

$$\mathbf{j}: \quad D_z = 0,$$

$$\mathbf{k}: \quad 1.8C + 1.8(300) - 720 = 0$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{B} + 300\mathbf{j} + 100\mathbf{j} - 1200\mathbf{j} = 0$$

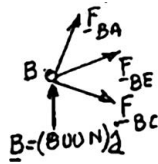


$$\mathbf{D} = (300 \text{ N})\mathbf{j} \quad \blacktriangleleft$$

$$\mathbf{C} = (100 \text{ N})\mathbf{j} \quad \blacktriangleleft$$

$$\mathbf{B} = (800 \text{ N})\mathbf{j} \quad \blacktriangleleft$$

Free body: B :



$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BE} + (800 \text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{BA} = F_{AB} \frac{\overline{BA}}{BA} = \frac{F_{AB}}{3.15} (0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{i}$$

$$\mathbf{F}_{BE} = -F_{BE}\mathbf{k}$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$$\mathbf{j}: \quad \left(\frac{3}{3.15} \right) F_{AB} + 800 \text{ N} = 0, \quad F_{AB} = -840 \text{ N}, \quad F_{AB} = 840 \text{ N} \quad \blacktriangleleft$$

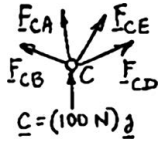
$$\mathbf{i}: \quad \left(\frac{0.6}{3.15} \right) (-840 \text{ N}) + F_{BC} = 0 \quad F_{BC} = 160.0 \text{ N} \quad \blacktriangleleft$$

$$\mathbf{k}: \quad \left(-\frac{0.75}{3.15} \right) (-840 \text{ N}) - F_{BE} = 0 \quad F_{BE} = 200 \text{ N} \quad \blacktriangleleft$$

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PROBLEM 6.39* (Continued)

Free body: C:



$$\Sigma \mathbf{F} = 0: \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} + \mathbf{F}_{CE} + (100 \text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{CA} = F_{AC} \frac{\overline{CA}}{CA} = \frac{F_{AC}}{3.317} (-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{CB} = -(160 \text{ N})\mathbf{i}$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = F_{CE} \frac{\overline{CE}}{CE} = \frac{F_{CE}}{3.499} (-1.8\mathbf{i} - 3\mathbf{k})$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$$\mathbf{j}: \quad \left(\frac{3}{3.317} \right) F_{AC} + 100 \text{ N} = 0, \quad F_{AC} = -110.57 \text{ N} \quad F_{AC} = 110.6 \text{ N} \quad C \blacktriangleleft$$

$$\mathbf{i}: \quad -\frac{1.2}{3.317} (-110.57) - 160 - \frac{1.8}{3.499} F_{CE} = 0, \quad F_{CE} = -233.3 \quad F_{CE} = 233 \text{ N} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{0.75}{3.317} (-110.57) - F_{CD} - \frac{3}{3.499} (-233.3) = 0 \quad F_{CD} = 225 \text{ N} \quad T \blacktriangleleft$$

Free body: D:



$$\Sigma \mathbf{F} = 0: \mathbf{F}_{DA} + \mathbf{F}_{DC} + \mathbf{F}_{DE} + (300 \text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{DA} = F_{AD} \frac{\overline{DA}}{DA} = \frac{F_{AD}}{3.937} (-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

$$\mathbf{F}_{DC} = F_{CD}\mathbf{k} = (225 \text{ N})\mathbf{k} \quad F_{DE} = -F_{DE}\mathbf{i}$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

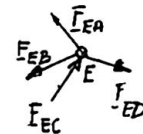
$$\mathbf{j}: \quad \left(\frac{3}{3.937} \right) F_{AD} + 300 \text{ N} = 0, \quad F_{AD} = -393.7 \text{ N}, \quad F_{AD} = 394 \text{ N} \quad C \blacktriangleleft$$

$$\mathbf{i}: \quad \left(-\frac{1.2}{3.937} \right) (-393.7 \text{ N}) - F_{DE} = 0 \quad F_{DE} = 120.0 \text{ N} \quad T \blacktriangleleft$$

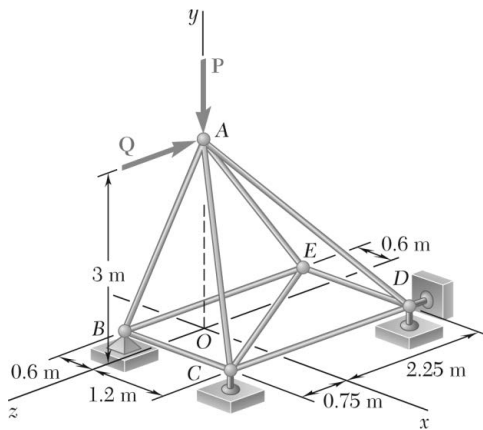
$$\mathbf{k}: \quad \left(\frac{2.25}{3.937} \right) (-393.7 \text{ N}) + 225 \text{ N} = 0 \quad (\text{Checks})$$

Free body: E:

Member AE is the only member at E which does not lie in the xz plane. Therefore, it is a zero-force member.



$$F_{AE} = 0 \quad \blacktriangleleft$$



PROBLEM 6.40*

Solve Problem 6.39 for $\mathbf{P} = 0$ and $\mathbf{Q} = (-900 \text{ N})\mathbf{k}$.

PROBLEM 6.39* The truss shown consists of nine members and is supported by a ball and socket at B, a short link at C, and two short links at D. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) Determine the force in each member for $\mathbf{P} = (-1200 \text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION

Free body: Truss:

$$\begin{aligned} \Sigma \mathbf{M}_B = 0: & 1.8\mathbf{i} \times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k}) \times (D_y\mathbf{j} + D_z\mathbf{k}) \\ & + (0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k}) \times (-900\text{N})\mathbf{k} = 0 \\ & 1.8C\mathbf{k} + 1.8D_y\mathbf{k} - 1.8D_z\mathbf{j} \\ & + 3D_y\mathbf{i} + 540\mathbf{j} - 2700\mathbf{i} = 0 \end{aligned}$$

Equate to zero the coefficient of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\begin{aligned} 3D_y - 2700 &= 0 & D_y &= 900 \text{ N} \\ -1.8D_z + 540 &= 0 & D_z &= 300 \text{ N} \\ 1.8C + 1.8D_y &= 0 & C &= -D_y = -900 \text{ N} \end{aligned}$$

Thus,

$$\mathbf{C} = -(900 \text{ N})\mathbf{j} \quad \mathbf{D} = (900 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k}$$

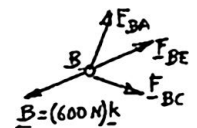
$$\Sigma \mathbf{F} = 0: \quad \mathbf{B} - 900\mathbf{j} + 900\mathbf{j} + 300\mathbf{k} - 900\mathbf{k} = 0$$

$$\mathbf{B} = (600 \text{ N})\mathbf{k}$$

Free body: B:

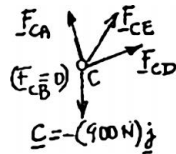
Since \mathbf{B} is aligned with member BE,

$$F_{AB} = F_{BC} = 0,$$



$$F_{BE} = 600 \text{ N} \quad T$$

Free body: C:



$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{F}_{CE} - (900 \text{ N})\mathbf{j} = 0, \quad \text{with}$$

PROBLEM 6.40* (Continued)

$$\mathbf{F}_{CA} = F_{AC} \frac{\overline{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = F_{CE} \frac{\overline{CE}}{CE} = \frac{F_{CE}}{3.499}(-1.8\mathbf{i} - 3\mathbf{k})$$

Substitute and equate to zero the coefficient of **j, i, k**:

$$\mathbf{j}: \quad \left(\frac{3}{3.317}\right)F_{AC} - 900 \text{ N} = 0, \quad F_{AC} = 995.1 \text{ N} \quad F_{AC} = 995 \text{ N} \quad T \quad \blacktriangleleft$$

$$\mathbf{i}: \quad -\frac{1.2}{3.317}(995.1) - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -699.8 \text{ N} \quad F_{CE} = 700 \text{ N} \quad C \quad \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{0.75}{3.317}(995.1) - F_{CD} - \frac{3}{3.499}(-699.8) = 0 \quad F_{CD} = 375 \text{ N} \quad T \quad \blacktriangleleft$$

Free body: D:

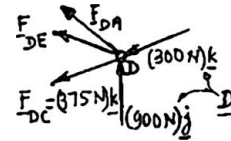
$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DE} + (375 \text{ N})\mathbf{k} + (900 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k} = 0$$

with

$$\mathbf{F}_{DA} = F_{AD} \frac{\overline{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

and

$$F_{DE} = -F_{DE} \mathbf{i}$$



Substitute and equate to zero the coefficient **j, i, k**:

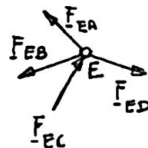
$$\mathbf{j}: \quad \left(\frac{3}{3.937}\right)F_{AD} + 900 \text{ N} = 0, \quad F_{AD} = -1181.1 \text{ N} \quad F_{AD} = 1181 \text{ N} \quad C \quad \blacktriangleleft$$

$$\mathbf{i}: \quad -\left(\frac{1.2}{3.937}\right)(-1181.1 \text{ N}) - F_{DE} = 0 \quad F_{DE} = 360 \text{ N} \quad T \quad \blacktriangleleft$$

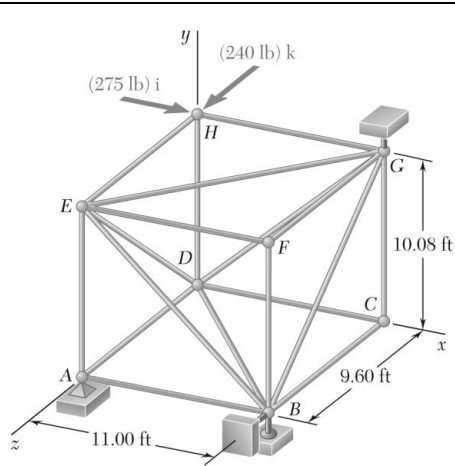
$$\mathbf{k}: \quad \left(\frac{2.25}{3.937}\right)(-1181.1 \text{ N} + 375 \text{ N} + 300 \text{ N}) = 0 \quad (\text{Checks})$$

Free body: E:

Member *AE* is the only member at *E* which does not lie in the *xz* plane. Therefore, it is a zero-force member.



$$F_{AE} = 0 \quad \blacktriangleleft$$



PROBLEM 6.41*

The truss shown consists of 18 members and is supported by a ball and socket at A , two short links at B , and one short link at G . (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) For the given loading, determine the force in each of the six members joined at E .

SOLUTION

(a) Check simple truss.

- (1) Start with tetrahedron $BEFG$.
- (2) Add members BD , ED , GD joining at D .
- (3) Add members BA , DA , EA joining at A .
- (4) Add members DH , EH , GH joining at H .
- (5) Add members BC , DC , GC joining at C .

Truss has been completed: It is a simple truss.

Free body: Truss:

Check constraints and reactions.

Six unknown reactions—ok; however, supports at A and B constrain truss to rotate about AB and support at G prevents such a rotation. Thus,

Truss is completely constrained and reactions are statically determinate.

Determination of reactions:

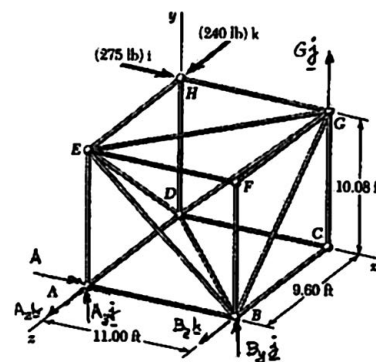
$$\begin{aligned} \Sigma \mathbf{M}_A = 0: & 11\mathbf{i} \times (B_y\mathbf{j} + B_z\mathbf{k}) + (11\mathbf{i} - 9.6\mathbf{k}) \times G\mathbf{j} \\ & + (10.08\mathbf{j} - 9.6\mathbf{k}) \times (275\mathbf{i} + 240\mathbf{k}) = 0 \\ 11B_y\mathbf{k} - 11B_z\mathbf{j} + 11G\mathbf{k} + 9.6G\mathbf{i} - (10.08)(275)\mathbf{k} \\ & + (10.08)(240)\mathbf{i} - (9.6)(275)\mathbf{j} = 0 \end{aligned}$$

Equate to zero the coefficient of \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$\mathbf{i}: 9.6G + (10.08)(240) = 0 \quad G = -252 \text{ lb} \quad \mathbf{G} = (-252 \text{ lb})\mathbf{j} \triangleleft$$

$$\mathbf{j}: -11B_z - (9.6)(275) = 0 \quad B_z = -240 \text{ lb}$$

$$\mathbf{k}: 11B_y + 11(-252) - (10.08)(275) = 0, \quad B_y = 504 \text{ lb} \quad \mathbf{B} = (504 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k} \triangleleft$$



PROBLEM 6.41* (Continued)

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + (504 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k} - (252 \text{ lb})\mathbf{j} \\ + (275 \text{ lb})\mathbf{i} + (240 \text{ lb})\mathbf{k} = 0$$

$$\mathbf{A} = -(275 \text{ lb})\mathbf{i} - (252 \text{ lb})\mathbf{j} \quad \triangleleft$$

Zero-force members.

The determination of these members will facilitate our solution.

FB: C: Writing $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$ yields $F_{BC} = F_{CD} = F_{CG} = 0 \quad \triangleleft$

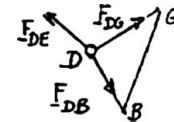
FB: F: Writing $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$ yields $F_{BF} = F_{EF} = F_{FG} = 0 \quad \triangleleft$

FB: A: Since $A_z = 0$, writing $\Sigma F_z = 0$ yields $F_{AD} = 0 \quad \triangleleft$

FB: H: Writing $\Sigma F_y = 0$ yields $F_{DH} = 0 \quad \triangleleft$

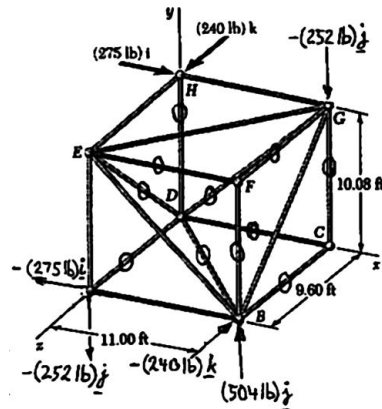
FB: D: Since $F_{AD} = F_{CD} = F_{DH} = 0$, we need consider only members DB, DE , and DG .

Since F_{DE} is the only force not contained in plane BDG , it must be zero. Simple reasonings show that the other two forces are also zero.



$$F_{BD} = F_{DE} = F_{DG} = 0 \quad \triangleleft$$

The results obtained for the reactions at the supports and for the zero-force members are shown on the figure below. Zero-force members are indicated by a zero ("0").



(b) Force in each of the members joined at E.

We already found that

$$F_{DE} = F_{EF} = 0 \quad \triangleleft$$

Free body: A: $\Sigma F_y = 0$ yields $F_{AE} = 252 \text{ lb} \quad T \quad \triangleleft$

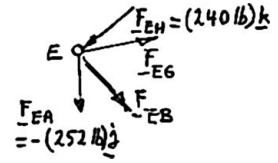
Free body: H: $\Sigma F_z = 0$ yields $F_{EH} = 240 \text{ lb} \quad C \quad \triangleleft$

PROBLEM 6.41* (Continued)

Free body: E:

$$\Sigma \mathbf{F} = 0: \mathbf{F}_{EB} + \mathbf{F}_{EG} + (240 \text{ lb})\mathbf{k} - (252 \text{ lb})\mathbf{j} = 0$$

$$\frac{F_{BE}}{14.92}(11\mathbf{i} - 10.08\mathbf{j}) + \frac{F_{EG}}{14.6}(11\mathbf{i} - 9.6\mathbf{k}) + 240\mathbf{k} - 252\mathbf{j} = 0$$



Equate to zero the coefficient of \mathbf{y} and \mathbf{k} :

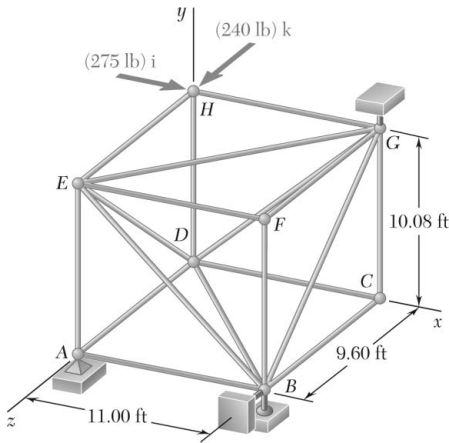
$$\mathbf{j}: -\left(\frac{10.08}{14.92}\right)F_{BE} - 252 = 0$$

$$F_{BE} = 373 \text{ lb } C \quad \blacktriangleleft$$

$$\mathbf{k}: -\left(\frac{9.6}{14.6}\right)F_{EG} + 240 = 0$$

$$F_{EG} = 365 \text{ lb } T \quad \blacktriangleleft$$

PROBLEM 6.42*



The truss shown consists of 18 members and is supported by a ball and socket at A , two short links at B , and one short link at G . (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) For the given loading, determine the force in each of the six members joined at G .

SOLUTION

See solution to Problem 6.41 for part (a) and for reactions and zero-force members.

(b) Force in each of the members joined at G .

We already know that

$$F_{CG} = F_{DG} = F_{FG} = 0 \quad \blacktriangleleft$$

$$\text{yields } F_{GH} = 275 \text{ lb} \quad C \quad \blacktriangleleft$$

Free body: H : $\Sigma F_x = 0$

Free body: G : $\Sigma \mathbf{F} = 0: \mathbf{F}_{GB} + \mathbf{F}_{GE} + (275 \text{ lb})\mathbf{i} - (252 \text{ lb})\mathbf{j} = 0$

$$\frac{F_{BG}}{13.92}(-10.08\mathbf{j} + 9.6\mathbf{k}) + \frac{F_{EG}}{14.6}(-11\mathbf{i} + 9.6\mathbf{k}) + 275\mathbf{i} - 252\mathbf{j} = 0$$

Equate to zero the coefficient of \mathbf{i} , \mathbf{j} , \mathbf{k} :

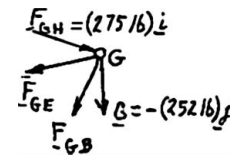
$$\mathbf{i}: -\left(\frac{11}{14.6}\right)F_{EG} + 275 = 0$$

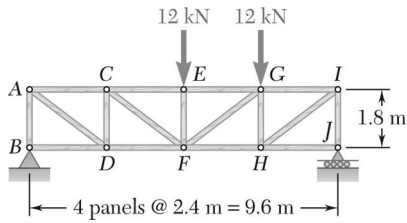
$$F_{EG} = 365 \text{ lb} \quad T \quad \blacktriangleleft$$

$$\mathbf{j}: -\left(\frac{10.08}{13.92}\right)F_{BG} - 252 = 0$$

$$F_{BG} = 348 \text{ lb} \quad C \quad \blacktriangleleft$$

$$\mathbf{k}: \left(\frac{9.6}{13.92}\right)(-348) + \left(\frac{9.6}{14.6}\right)(365) = 0 \quad (\text{Checks})$$



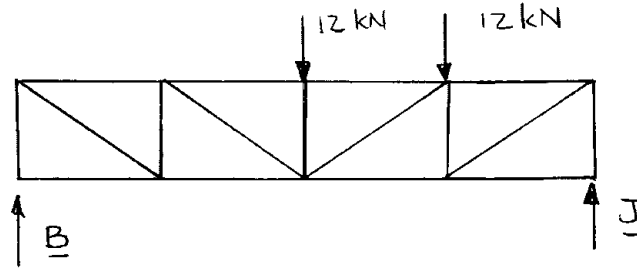


PROBLEM 6.43

Determine the force in members CD and DF of the truss shown.

SOLUTION

Reactions:



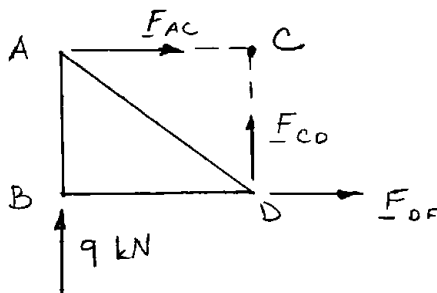
$$+\circlearrowleft \sum M_J = 0: (12 \text{ kN})(4.8 \text{ m}) + (12 \text{ kN})(2.4 \text{ m}) - B(9.6 \text{ m}) = 0$$

$$B = 9.00 \text{ kN}$$

$$+\uparrow \sum F_y = 0: 9.00 \text{ kN} - 12.00 \text{ kN} - 12.00 \text{ kN} + J = 0$$

$$J = 15.00 \text{ kN}$$

Member CD :



$$+\uparrow \sum F_y = 0: 9.00 \text{ kN} + F_{CD} = 0$$

$$F_{CD} = 9.00 \text{ kN} \quad C \blacktriangleleft$$

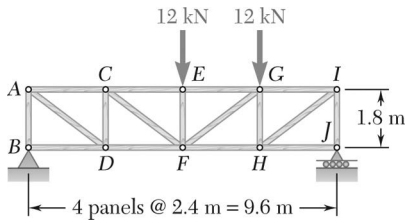
Member DF :

$$+\circlearrowleft \sum M_C = 0: F_{DF}(1.8 \text{ m}) - (9.00 \text{ kN})(2.4 \text{ m}) = 0$$

$$F_{DF} = 12.00 \text{ kN} \quad T \blacktriangleleft$$

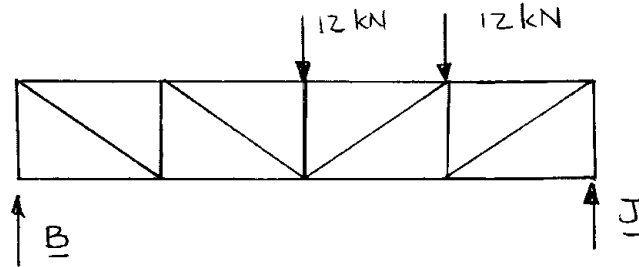
PROBLEM 6.44

Determine the force in members FG and FH of the truss shown.



SOLUTION

Reactions:



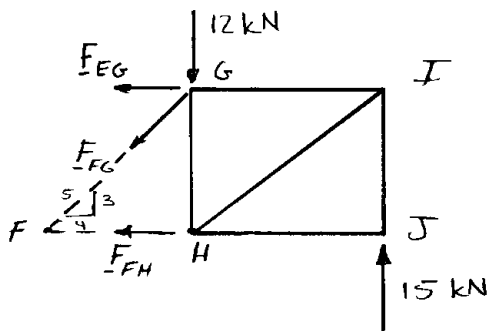
$$+\circlearrowleft \Sigma M_J = 0: (12 \text{ kN})(4.8 \text{ m}) + (12 \text{ kN})(2.4 \text{ m}) - B(9.6 \text{ m}) = 0$$

$$B = 9.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: 9.00 \text{ kN} - 12.00 \text{ kN} - 12.00 \text{ kN} + J = 0$$

$$J = 15.00 \text{ kN}$$

Member FG :



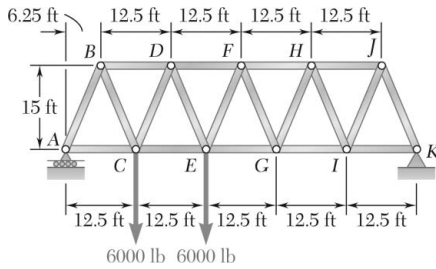
$$+\uparrow \Sigma F_y = 0: -\frac{3}{5}F_{FG} - 12.00 \text{ kN} + 15.00 \text{ kN} = 0$$

$$F_{FG} = 5.00 \text{ kN } T \leftarrow$$

Member FH :

$$+\circlearrowleft \Sigma M_G = 0: (15.00 \text{ kN})(2.4 \text{ m}) - F_{FH}(1.8 \text{ m}) = 0$$

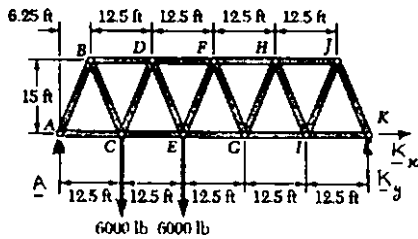
$$F_{FH} = 20.0 \text{ kN } T \leftarrow$$



PROBLEM 6.45

A Warren bridge truss is loaded as shown. Determine the force in members CE , DE , and DF .

SOLUTION



Free body: Truss:

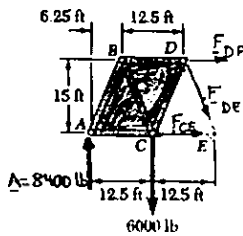
$$\rightarrow \Sigma F_x = 0: \quad k_x = 0$$

$$\begin{aligned} +\curvearrowright \Sigma M_A = 0: & \quad k_y(6.25 \text{ ft}) - (6000 \text{ lb})(12.5 \text{ ft}) \\ & \quad - (6000 \text{ lb})(25 \text{ ft}) = 0 \end{aligned}$$

$$k = k_y = 3600 \text{ lb} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + 3600 \text{ lb} - 6000 \text{ lb} - 6000 \text{ lb} = 0$$

$$A = 8400 \text{ lb} \uparrow \triangleleft$$



We pass a section through members CE , DE , and DF and use the free body shown.

$$\begin{aligned} +\curvearrowright \Sigma M_D = 0: & \quad F_{CE}(15 \text{ ft}) - (8400 \text{ lb})(18.75 \text{ ft}) \\ & \quad + (6000 \text{ lb})(6.25 \text{ ft}) = 0 \end{aligned}$$

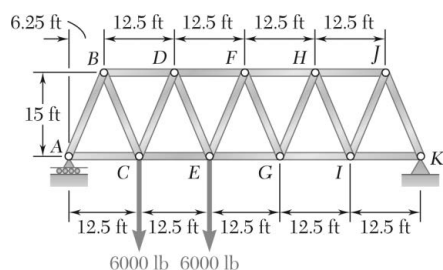
$$F_{CE} = +8000 \text{ lb} \quad F_{CE} = 8000 \text{ lb} \quad T \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 8400 \text{ lb} - 6000 \text{ lb} - \frac{15}{16.25} F_{DE} = 0$$

$$F_{DE} = +2600 \text{ lb} \quad F_{DE} = 2600 \text{ lb} \quad T \triangleleft$$

$$\begin{aligned} +\curvearrowright \Sigma M_E = 0: & \quad 6000 \text{ lb}(12.5 \text{ ft}) - (8400 \text{ lb})(25 \text{ ft}) \\ & \quad - F_{DF}(15 \text{ ft}) = 0 \end{aligned}$$

$$F_{DF} = -9000 \text{ lb} \quad F_{DF} = 9000 \text{ lb} \quad C \triangleleft$$



PROBLEM 6.46

A Warren bridge truss is loaded as shown. Determine the force in members EG , FG , and FH .

SOLUTION

See solution of Problem 6.45 for free-body diagram of truss and determination of reactions:

$$A = 8400 \text{ lb}$$

$$K = 3600 \text{ lb} \quad \triangleleft$$

We pass a section through members EG , FG , and FH , and use the free body shown.

$$+\curvearrowright \Sigma M_F = 0: (3600 \text{ lb})(31.25 \text{ ft}) - F_{EG}(15 \text{ ft}) = 0$$

$$F_{EG} = +7500 \text{ lb}$$

$$F_{EG} = 7500 \text{ lb} \quad T \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \frac{15}{16.25} F_{FG} + 3600 \text{ lb} = 0$$

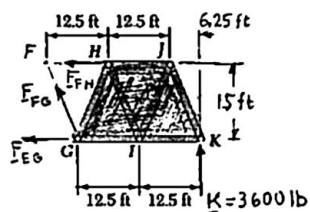
$$F_{FG} = -3900 \text{ lb}$$

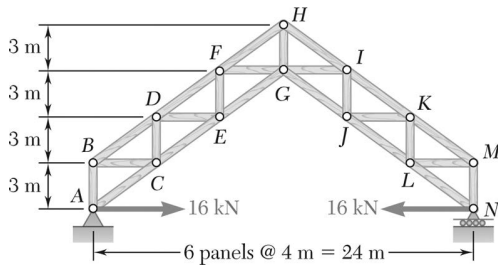
$$F_{FG} = 3900 \text{ lb} \quad C \quad \triangleleft$$

$$+\curvearrowright \Sigma M_G = 0: F_{FH}(15 \text{ ft}) + (3600 \text{ lb})(25 \text{ ft}) = 0$$

$$F_{FH} = -6000 \text{ lb}$$

$$F_{FH} = 6000 \text{ lb} \quad C \quad \triangleleft$$

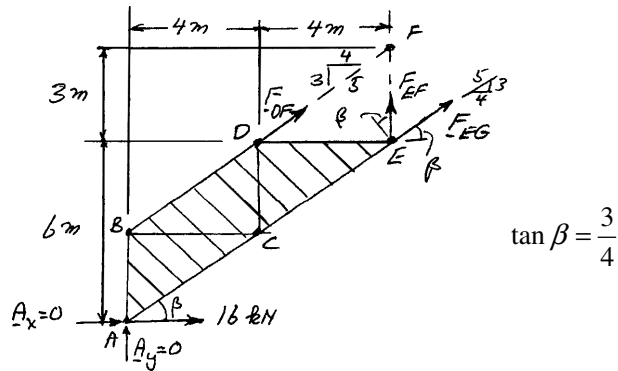




PROBLEM 6.47

Determine the force in members DF , EF , and EG of the truss shown.

SOLUTION



Reactions:

$$A = N = 0$$

Member DF : $\rightarrow \sum M_E = 0: + (16 \text{ kN})(6 \text{ m}) - \frac{3}{5} F_{DF}(4 \text{ m}) = 0$

$$F_{DF} = +40 \text{ kN}$$

$$F_{DF} = 40.0 \text{ kN } T \blacktriangleleft$$

Member EF : $+\nearrow \sum F = 0: (16 \text{ kN}) \sin \beta - F_{EF} \cos \beta = 0$

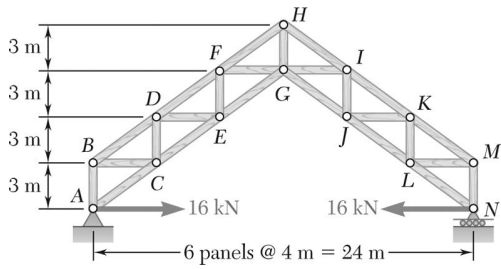
$$F_{EF} = 16 \tan \beta = 16(0.75) = 12 \text{ kN}$$

$$F_{EF} = 12.00 \text{ kN } T \blacktriangleleft$$

Member EG : $\rightarrow \sum M_F = 0: (16 \text{ kN})(9 \text{ m}) + \frac{4}{5} F_{EG}(3 \text{ m}) = 0$

$$F_{EG} = -60 \text{ kN}$$

$$F_{EG} = 60.0 \text{ kN } C \blacktriangleleft$$



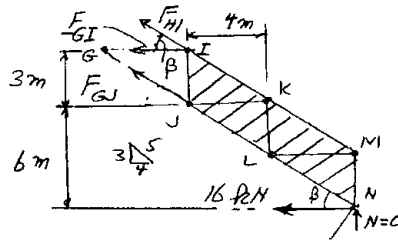
PROBLEM 6.48

Determine the force in members GI , GJ , and HI of the truss shown.

SOLUTION

Reactions:

$$A = N = 0$$



Member GI : $+\nearrow \sum F = 0: (16 \text{ kN}) \sin \beta + F_{GI} \sin \beta = 0$

$$F_{GI} = -16 \text{ kN}$$

$$F_{GI} = 16.00 \text{ kN} \quad C \blacktriangleleft$$

Member GJ : $+\curvearrowright \sum M_I = 0: -(16 \text{ kN})(9 \text{ m}) - \frac{4}{5} F_{GJ}(3 \text{ m}) = 0$

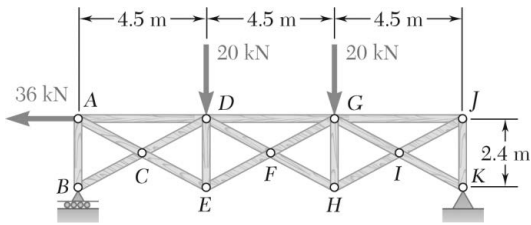
$$F_{GJ} = -60 \text{ kN}$$

$$F_{GJ} = 60.0 \text{ kN} \quad C \blacktriangleleft$$

Member HI : $+\curvearrowright \sum M_G = 0: -(16 \text{ kN})(9 \text{ m}) + \frac{3}{5} F_{HI}(4 \text{ m}) = 0$

$$F_{HI} = +60 \text{ kN}$$

$$F_{HI} = 60.0 \text{ kN} \quad T \blacktriangleleft$$

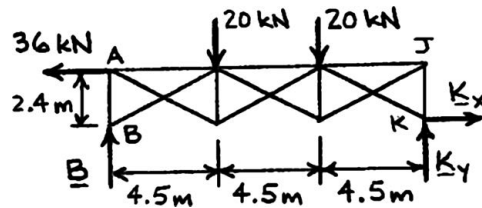


PROBLEM 6.49

Determine the force in members AD , CD , and CE of the truss shown.

SOLUTION

Reactions:

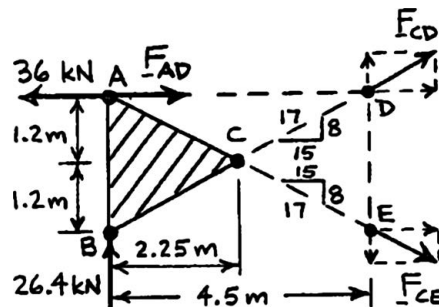


$$+\circlearrowleft \Sigma M_K = 0: 36(2.4) - B(13.5) + 20(9) + 20(4.5) = 0$$

$$B = 26.4 \text{ kN } \uparrow$$

$$+\rightarrow \Sigma F_x = 0: -36 + K_x = 0 \quad K_x = 36 \text{ kN } \rightarrow$$

$$+\uparrow \Sigma F_y = 0: 26.4 - 20 - 20 + K_y = 0 \quad K_y = 13.6 \text{ kN } \uparrow$$



$$+\circlearrowleft \Sigma M_C = 0: 36(1.2) - 26.4(2.25) - F_{AD}(1.2) = 0$$

$$F_{AD} = -13.5 \text{ kN}$$

$$F_{AD} = 13.5 \text{ kN } \quad C \blacktriangleleft$$

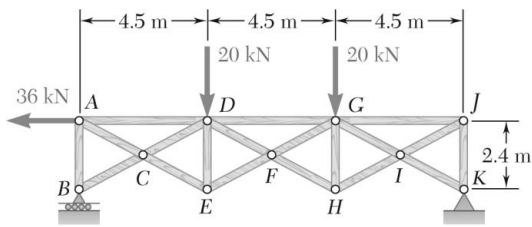
$$+\circlearrowleft \Sigma M_A = 0: \left(\frac{8}{17} F_{CD} \right) (4.5) = 0$$

$$F_{CD} = 0 \quad \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \left(\frac{15}{17} F_{CE} \right) (2.4) - 26.4(4.5) = 0$$

$$F_{CE} = +56.1 \text{ kN}$$

$$F_{CE} = 56.1 \text{ kN } \quad T \blacktriangleleft$$



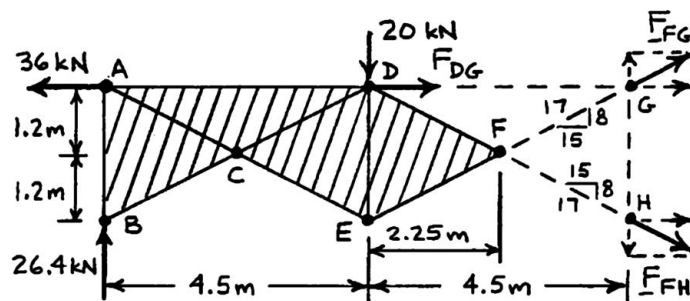
PROBLEM 6.50

Determine the force in members DG , FG , and FH of the truss shown.

SOLUTION

See the solution to Problem 6.49 for free-body diagram and analysis to determine the reactions at the supports B and K .

$$B = 26.4 \text{ kN } \uparrow; \quad K_x = 36.0 \text{ kN } \rightarrow; \quad K_y = 13.60 \text{ kN } \uparrow$$



$$+\circlearrowleft \Sigma M_F = 0: \quad 36(1.2) - 26.4(6.75) + 20(2.25) - F_{DG}(1.2) = 0$$

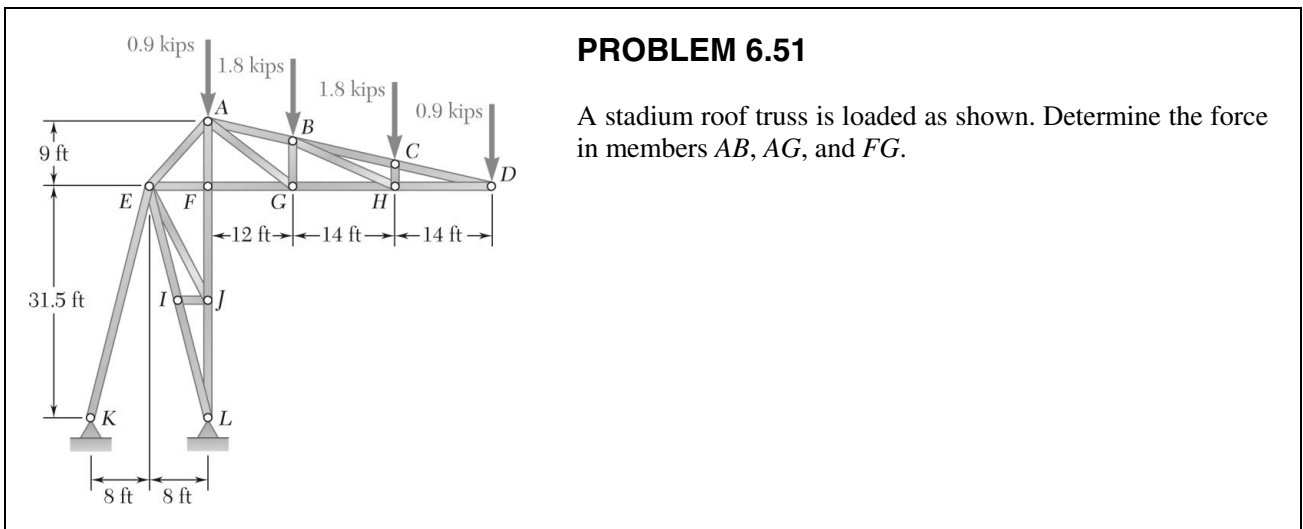
$$F_{DG} = -75 \text{ kN} \quad F_{DG} = 75.0 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -26.4(4.5) + \left(\frac{8}{17} F_{FG}\right)(4.5) = 0$$

$$F_{FG} = +56.1 \text{ kN} \quad F_{FG} = 56.1 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad 20(4.5) - 26.4(9) + \left(\frac{15}{17} F_{FH}\right)(2.4) = 0$$

$$F_{FH} = +69.7 \text{ kN} \quad F_{FH} = 69.7 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.51

A stadium roof truss is loaded as shown. Determine the force in members AB, AG, and FG.

SOLUTION

We pass a section through members AB, AG, and FG, and use the free body shown.

$$+\curvearrowright \Sigma M_G = 0: \left(\frac{40}{41} F_{AB} \right) (6.3 \text{ ft}) - (1.8 \text{ kips})(14 \text{ ft}) - (0.9 \text{ kips})(28 \text{ ft}) = 0$$

$$F_{AB} = +8.20 \text{ kips} \qquad F_{AB} = 8.20 \text{ kips} \quad T \quad \blacktriangleleft$$

$$+\curvearrowright \Sigma M_D = 0: -\left(\frac{3}{5} F_{AG} \right) (28 \text{ ft}) + (1.8 \text{ kips})(28 \text{ ft}) + (1.8 \text{ kips})(14 \text{ ft}) = 0$$

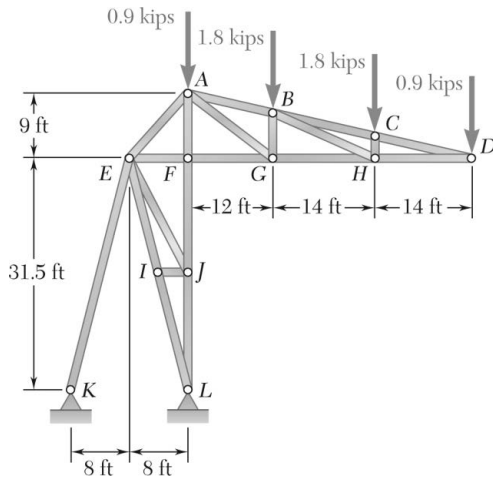
$$F_{AG} = +4.50 \text{ kips} \qquad F_{AG} = 4.50 \text{ kips} \quad T \quad \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: -F_{FG} (9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) - (0.9 \text{ kips})(40 \text{ ft}) = 0$$

$$F_{FG} = -11.60 \text{ kips} \qquad F_{FG} = 11.60 \text{ kips} \quad C \quad \blacktriangleleft$$

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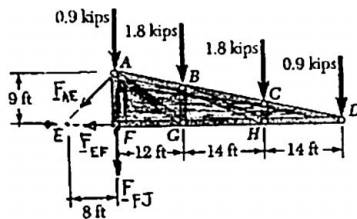
PROBLEM 6.52



A stadium roof truss is loaded as shown. Determine the force in members AE , EF , and FJ .

SOLUTION

We pass a section through members AE , EF , and FJ , and use the free body shown.



$$+\circlearrowleft \Sigma M_F = 0: \left(\frac{8}{\sqrt{8^2 + 9^2}} F_{AE} \right) (9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) - (0.9 \text{ kips})(40 \text{ ft}) = 0$$

$$F_{AE} = +17.46 \text{ kips}$$

$$F_{AE} = 17.46 \text{ kips } T \quad \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: -F_{EF} (9 \text{ ft}) - (1.8 \text{ kips})(12 \text{ ft}) - (1.8 \text{ kips})(26 \text{ ft}) - (0.9 \text{ kips})(40 \text{ ft}) = 0$$

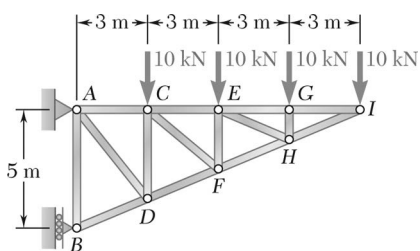
$$F_{EF} = -11.60 \text{ kips}$$

$$F_{EF} = 11.60 \text{ kips } C \quad \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: -F_{FJ} (8 \text{ ft}) - (0.9 \text{ kips})(8 \text{ ft}) - (1.8 \text{ kips})(20 \text{ ft}) - (1.8 \text{ kips})(34 \text{ ft}) - (0.9 \text{ kips})(48 \text{ ft}) = 0$$

$$F_{FJ} = -18.45 \text{ kips}$$

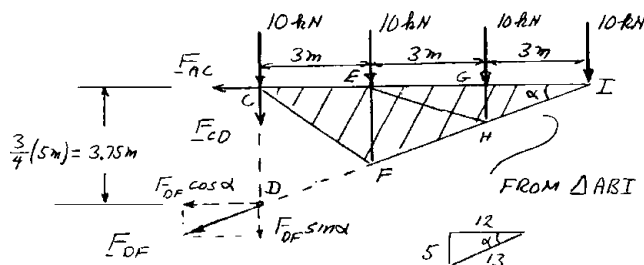
$$F_{FJ} = 18.45 \text{ kips } C \quad \blacktriangleleft$$



PROBLEM 6.53

Determine the force in members CD and DF of the truss shown.

SOLUTION



$$\tan \alpha = \frac{5}{12} \quad \alpha = 22.62^\circ$$

$$\sin \alpha = \frac{5}{13} \quad \cos \alpha = \frac{12}{13}$$

Member CD :

$$+\circlearrowleft \sum M_I = 0: F_{CD}(9 \text{ m}) + (10 \text{ kN})(9 \text{ m}) + (10 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(3 \text{ m}) = 0$$

$$F_{CD} = -20 \text{ kN}$$

$$F_{CD} = 20.0 \text{ kN} \quad C \blacktriangleleft$$

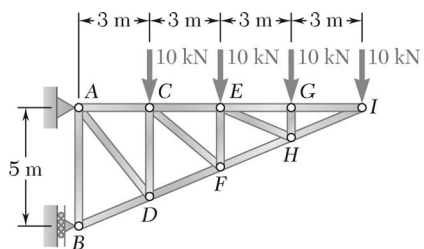
Member DF :

$$+\circlearrowleft \sum M_C = 0: (F_{DF} \cos \alpha)(3.75 \text{ m}) + (10 \text{ kN})(3 \text{ m}) + (10 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(9 \text{ m}) = 0$$

$$F_{DF} \cos \alpha = -48 \text{ kN}$$

$$F_{DF} \left(\frac{12}{13} \right) = -48 \text{ kN} \quad F_{DF} = -52.0 \text{ kN}$$

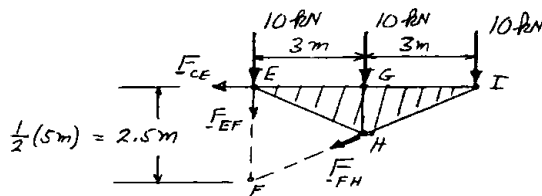
$$F_{DF} = 52.0 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.54

Determine the force in members CE and EF of the truss shown.

SOLUTION



Member CE :

$$+\circlearrowleft \sum M_F = 0: F_{CE}(2.5 \text{ m}) - (10 \text{ kN})(3 \text{ m}) - (10 \text{ kN})(6 \text{ m}) = 0$$

$$F_{CE} = +36 \text{ kN}$$

$$F_{CE} = 36.0 \text{ kN} \quad T \quad \blacktriangleleft$$

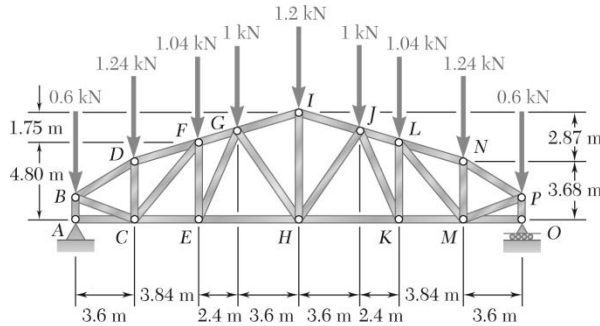
Member EF :

$$+\circlearrowleft \sum M_I = 0: F_{EF}(6 \text{ m}) + (10 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(3 \text{ m}) = 0$$

$$F_{EF} = -15 \text{ kN}$$

$$F_{EF} = 15.00 \text{ kN} \quad C \quad \blacktriangleleft$$

PROBLEM 6.55



The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members FG , EG , and EH .

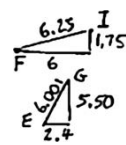
SOLUTION

Reactions at supports. Because of the symmetry of the loading,

$$A_x = 0, \quad A_y = O = \frac{1}{2} \text{ total load}$$

$$A = O = 4.48 \text{ kN} \uparrow \blacktriangleleft$$

We pass a section through members FG , EG , and EH , and use the free body shown.



$$\text{Slope } FG = \text{Slope } FI = \frac{1.75 \text{ m}}{6 \text{ m}}$$

$$\text{Slope } EG = \frac{5.50 \text{ m}}{2.4 \text{ m}}$$

$$\begin{aligned} + \curvearrowright \sum M_E = 0: & (0.6 \text{ kN})(7.44 \text{ m}) + (1.24 \text{ kN})(3.84 \text{ m}) \\ & - (4.48 \text{ kN})(7.44 \text{ m}) \\ & - \left(\frac{6}{6.25} F_{FG} \right) (4.80 \text{ m}) = 0 \end{aligned}$$

$$F_{FG} = -5.231 \text{ kN}$$

$$F_{FG} = 5.23 \text{ kN} \quad C \blacktriangleleft$$

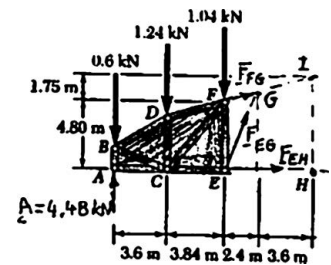
$$\begin{aligned} + \curvearrowright \sum M_G = 0: & F_{EH} (5.50 \text{ m}) + (0.6 \text{ kN})(9.84 \text{ m}) \\ & + (1.24 \text{ kN})(6.24 \text{ m}) + (1.04 \text{ kN})(2.4 \text{ m}) \\ & - (4.48 \text{ kN})(9.84 \text{ m}) = 0 \end{aligned}$$

$$F_{EH} = 5.08 \text{ kN} \quad T \blacktriangleleft$$

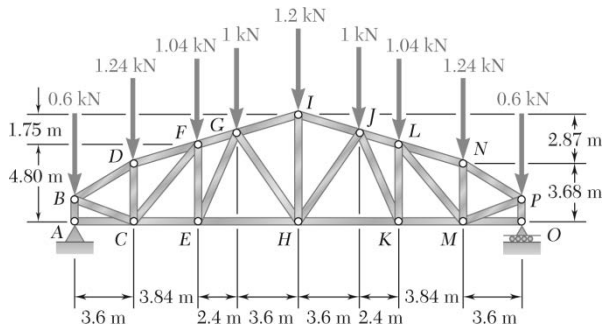
$$+ \uparrow \sum F_y = 0: \quad \frac{5.50}{6.001} F_{EG} + \frac{1.75}{6.25} (-5.231 \text{ kN}) + 4.48 \text{ kN} - 0.6 \text{ kN} - 1.24 \text{ kN} - 1.04 \text{ kN} = 0$$

$$F_{EG} = -0.1476 \text{ kN}$$

$$F_{EG} = 0.1476 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.56



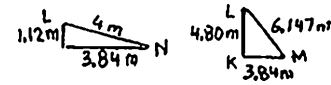
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members KM , LM , and LN .

SOLUTION

Because of symmetry of loading, $O = \frac{1}{2}$ load

$$O = 4.48 \text{ kN} \uparrow \leftarrow$$

We pass a section through KM , LM , LN , and use free body shown.



$$+\circlearrowright \sum M_M = 0: \left(\frac{3.84}{4} F_{LN} \right) (3.68 \text{ m}) + (4.48 \text{ kN} - 0.6 \text{ kN})(3.6 \text{ m}) = 0$$

$$F_{LN} = -3.954 \text{ kN}$$

$$F_{LN} = 3.95 \text{ kN} \quad C \leftarrow$$

$$+\circlearrowright \sum M_L = 0: -F_{KM} (4.80 \text{ m}) - (1.24 \text{ kN})(3.84 \text{ m}) + (4.48 \text{ kN} - 0.6 \text{ kN})(7.44 \text{ m}) = 0$$

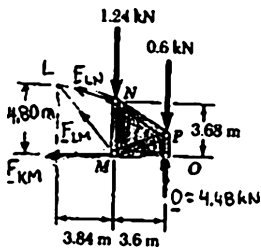
$$F_{KM} = +5.022 \text{ kN}$$

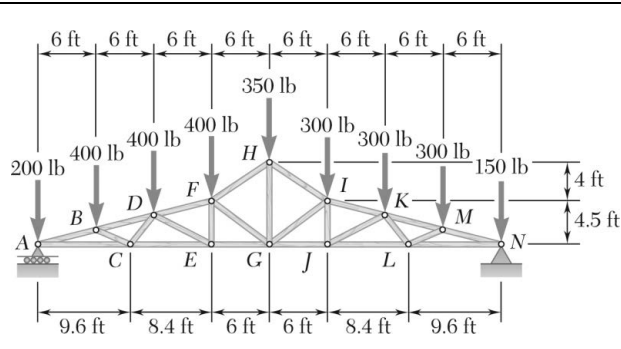
$$F_{KM} = 5.02 \text{ kN} \quad T \leftarrow$$

$$+\uparrow \sum F_y = 0: \frac{4.80}{6.147} F_{LM} + \frac{1.12}{4} (-3.954 \text{ kN}) - 1.24 \text{ kN} - 0.6 \text{ kN} + 4.48 \text{ kN} = 0$$

$$F_{LM} = -1.963 \text{ kN}$$

$$F_{LM} = 1.963 \text{ kN} \quad C \leftarrow$$



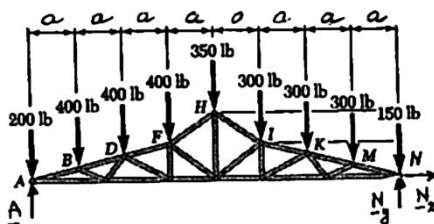


PROBLEM 6.57

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members DF , EF , and EG .

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: N_x = 0$$

$$+\curvearrowright \Sigma M_N = 0: (200 \text{ lb})(8a) + (400 \text{ lb})(7a + 6a + 5a) + (350 \text{ lb})(4a) + (300 \text{ lb})(3a + 2a + a) - A(8a) = 0$$

$$A = 1500 \text{ lb} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: 1500 \text{ lb} - 200 \text{ lb} - 3(400 \text{ lb}) - 350 \text{ lb} - 3(300 \text{ lb}) - 150 \text{ lb} + N_y = 0$$

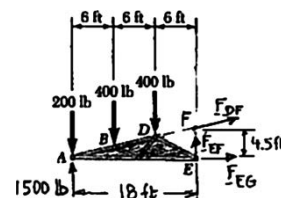
$$N_y = 1300 \text{ lb} \quad N = 1300 \text{ lb} \uparrow \triangleleft$$

We pass a section through DF , EF , and EG , and use the free body shown.

(We apply F_{DF} at F .)

$$+\curvearrowright \Sigma M_E = 0: (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft})$$

$$- \left(\frac{18}{\sqrt{18^2 + 4.5^2}} F_{DF} \right) (4.5 \text{ ft}) = 0$$



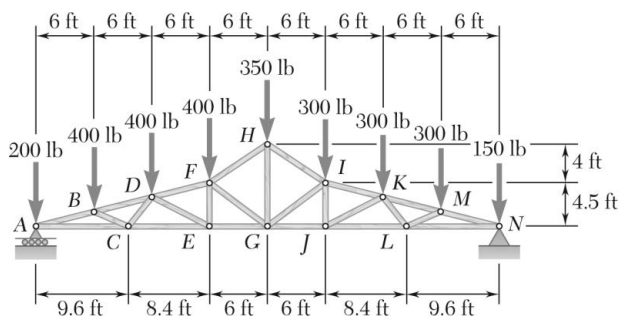
$$F_{DF} = -3711 \text{ lb} \quad F_{DF} = 3710 \text{ lb} \quad C \triangleleft$$

$$+\curvearrowright \Sigma M_A = 0: F_{EF}(18 \text{ ft}) - (400 \text{ lb})(6 \text{ ft}) - (400 \text{ lb})(12 \text{ ft}) = 0$$

$$F_{EF} = +400 \text{ lb} \quad F_{EF} = 400 \text{ lb} \quad T \triangleleft$$

$$+\curvearrowright \Sigma M_F = 0: F_{EG}(4.5 \text{ ft}) - (1500 \text{ lb})(18 \text{ ft}) + (200 \text{ lb})(18 \text{ ft}) + (400 \text{ lb})(12 \text{ ft}) + (400 \text{ lb})(6 \text{ ft}) = 0$$

$$F_{EG} = +3600 \text{ lb} \quad F_{EG} = 3600 \text{ lb} \quad T \triangleleft$$



PROBLEM 6.58

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members HI , GI , and GJ .

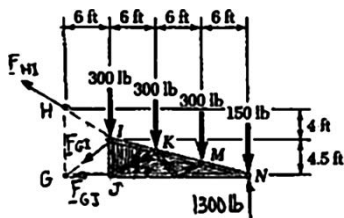
SOLUTION

See solution of Problem 6.57 for reactions:

$$A = 1500 \text{ lb } \uparrow, \quad N = 1300 \text{ lb } \uparrow \leftarrow$$

We pass a section through HI , GI , and GJ , and use the free body shown.

(We apply F_{HI} at H .)



$$\begin{aligned} +\curvearrowright \Sigma M_G = 0: & \left(\frac{6}{\sqrt{6^2 + 4^2}} F_{HI} \right) (8.5 \text{ ft}) + (1300 \text{ lb})(24 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) \\ & - (300 \text{ lb})(12 \text{ ft}) - (300 \text{ lb})(18 \text{ ft}) - (150 \text{ lb})(24 \text{ ft}) = 0 \end{aligned}$$

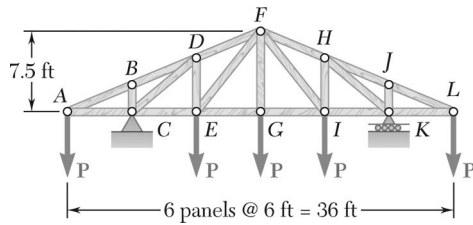
$$F_{HI} = -2375.4 \text{ lb} \quad F_{HI} = 2375 \text{ lb} \quad C \leftarrow$$

$$\begin{aligned} +\curvearrowright \Sigma M_I = 0: & (1300 \text{ lb})(18 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) - (300 \text{ lb})(12 \text{ ft}) \\ & - (150 \text{ lb})(18 \text{ ft}) - F_{GJ}(4.5 \text{ ft}) = 0 \end{aligned}$$

$$F_{GJ} = +3400 \text{ lb} \quad F_{GJ} = 3400 \text{ lb} \quad T \leftarrow$$

$$\begin{aligned} \pm \rightarrow \Sigma F_x = 0: & -\frac{4}{5} F_{GI} - \frac{6}{\sqrt{6^2 + 4^2}} (-2375.4 \text{ lb}) - 3400 \text{ lb} = 0 \end{aligned}$$

$$F_{GI} = -1779.4 \text{ lb} \quad F_{GI} = 1779 \text{ lb} \quad C \leftarrow$$



PROBLEM 6.59

Determine the force in members DE and DF of the truss shown when $P = 20$ kips.

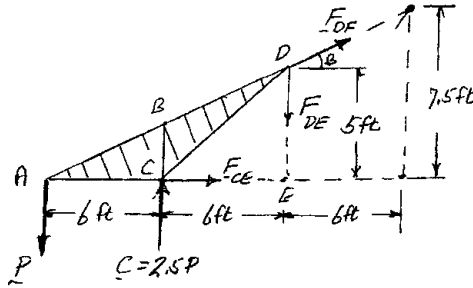
SOLUTION

Reactions:

$$C = K = 2.5P \uparrow$$

$$\tan \beta = \frac{7.5}{18}$$

$$\beta = 22.62^\circ$$



Member DE :

$$+\circlearrowleft \Sigma M_A = 0: (2.5P)(6 \text{ ft}) - F_{DE}(12 \text{ ft}) = 0$$

$$F_{DE} = +1.25P$$

For $P = 20$ kips,

$$F_{DE} = +1.25(20) = +25 \text{ kips}$$

$$F_{DE} = 25.0 \text{ kips } T \blacktriangleleft$$

Member DF :

$$+\circlearrowleft \Sigma M_E = 0: P(12 \text{ ft}) - (2.5P)(6 \text{ ft}) - F_{DF} \cos \beta(5 \text{ ft}) = 0$$

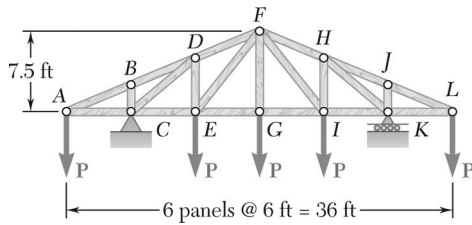
$$12P - 15P - F_{DF} \cos 22.62^\circ(5 \text{ ft}) = 0$$

$$F_{DF} = -0.65P$$

For $P = 20$ kips,

$$F_{DF} = -0.65(20) = -13 \text{ kips}$$

$$F_{DF} = 13.00 \text{ kips } C \blacktriangleleft$$



PROBLEM 6.60

Determine the force in members EG and EF of the truss shown when $P = 20$ kips.

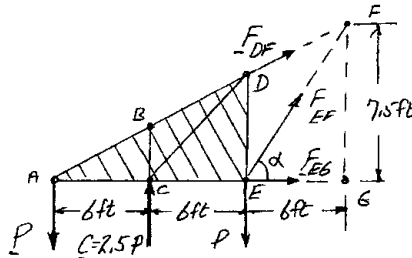
SOLUTION

Reactions:

$$C = K = 2.5P$$

$$\tan \alpha = \frac{7.5}{6}$$

$$\alpha = 51.34^\circ$$



Member EG :

$$+\circlearrowleft \sum M_F = 0: P(18 \text{ ft}) - 2.5P(12 \text{ ft}) - P(6 \text{ ft}) + F_{EG}(7.5 \text{ ft}) = 0$$

$$F_{EG} = +0.8P;$$

For $P = 20$ kips,

$$F_{EG} = 0.8(20) = +16 \text{ kips}$$

$$F_{EG} = 16.00 \text{ kips } T \blacktriangleleft$$

Member EF :

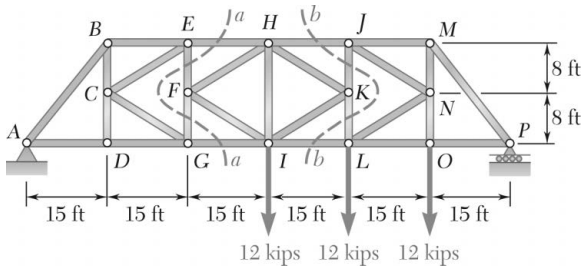
$$+\circlearrowleft \sum M_A = 0: 2.5P(6 \text{ ft}) - P(12 \text{ ft}) + F_{EF} \sin 51.34^\circ(12 \text{ ft}) = 0$$

$$F_{EF} = -0.320P;$$

For $P = 20$ kips,

$$F_{EF} = -0.320(20) = -6.4 \text{ kips}$$

$$F_{EF} = 6.40 \text{ kips } C \blacktriangleleft$$

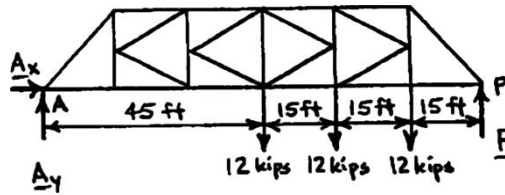


PROBLEM 6.61

Determine the force in members EH and GI of the truss shown. (Hint: Use section aa .)

SOLUTION

Reactions:

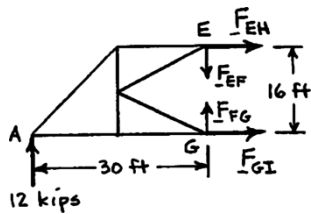


$$\Sigma F_x = 0: A_x = 0$$

$$+\curvearrowright \Sigma M_P = 0: 12(45) + 12(30) + 12(15) - A_y(90) = 0$$

$$A_y = 12 \text{ kips} \uparrow$$

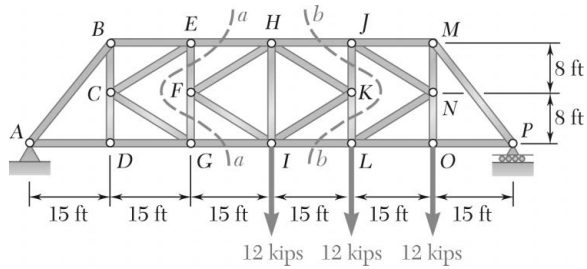
$$+\uparrow \Sigma F_y = 0: 12 - 12 - 12 - 12 + P = 0 \quad P = 24 \text{ kips} \uparrow$$



$$+\curvearrowright \Sigma M_G = 0: -(12 \text{ kips})(30 \text{ ft}) - F_{EH}(16 \text{ ft}) = 0$$

$$F_{EH} = -22.5 \text{ kips} \quad F_{EH} = 22.5 \text{ kips} \quad C \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: F_{GI} - 22.5 \text{ kips} = 0 \quad F_{GI} = 22.5 \text{ kips} \quad T \quad \blacktriangleleft$$

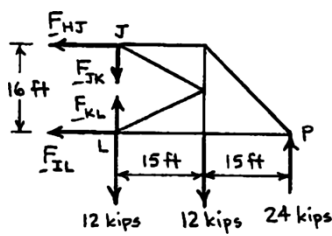


PROBLEM 6.62

Determine the force in members HJ and IL of the truss shown. (*Hint: Use section bb .*)

SOLUTION

See the solution to Problem 6.61 for free body diagram and analysis to determine the reactions at supports A and P .



$$A_x = 0; \quad A_y = 12.00 \text{ kips } \uparrow; \quad P = 24.0 \text{ kips } \uparrow$$

$$+\curvearrowright \Sigma M_L = 0: \quad F_{HJ}(16 \text{ ft}) - (12 \text{ kips})(15 \text{ ft}) + (24 \text{ kips})(30 \text{ ft}) = 0$$

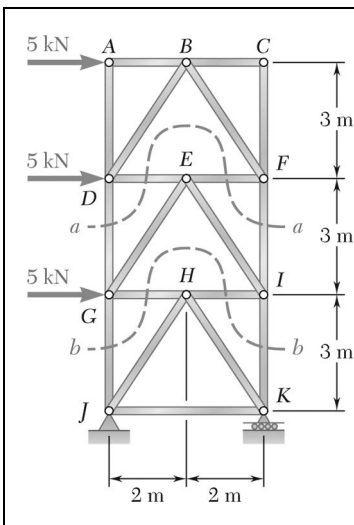
$$F_{HJ} = -33.75 \text{ kips}$$

$$F_{HJ} = 33.8 \text{ kips} \quad C \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad 33.75 \text{ kips} - F_{IL} = 0$$

$$F_{IL} = +33.75 \text{ kips}$$

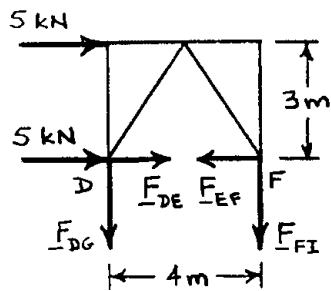
$$F_{IL} = 33.8 \text{ kips} \quad T \quad \blacktriangleleft$$



PROBLEM 6.63

Determine the force in members DG and FI of the truss shown. (*Hint: Use section aa .*)

SOLUTION



$$+\circlearrowleft \Sigma M_F = 0: F_{DG}(4 \text{ m}) - (5 \text{ kN})(3 \text{ m}) = 0$$

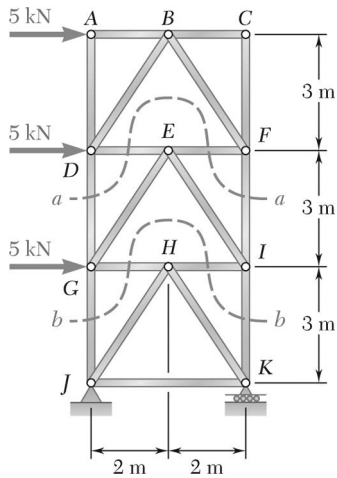
$$F_{DG} = +3.75 \text{ kN}$$

$$F_{DG} = 3.75 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -3.75 \text{ kN} - F_{FI} = 0$$

$$F_{FI} = -3.75 \text{ kN}$$

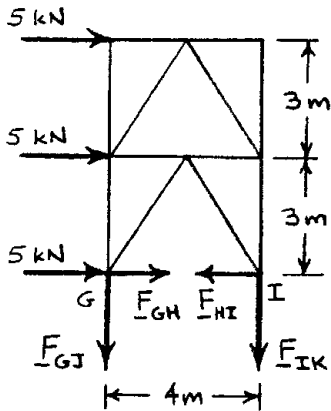
$$F_{FI} = 3.75 \text{ kN} \quad C \quad \blacktriangleleft$$



PROBLEM 6.64

Determine the force in members GJ and IK of the truss shown. (*Hint: Use section bb .*)

SOLUTION



$$+\curvearrowright \Sigma M_I = 0: F_{GJ}(4 \text{ m}) - (5 \text{ kN})(6 \text{ m}) - (5 \text{ kN})(3 \text{ m}) = 0$$

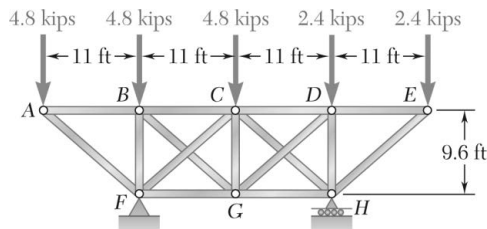
$$F_{GJ} = +11.25 \text{ kN}$$

$$F_{GJ} = 11.25 \text{ kN} \quad T \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -11.25 \text{ kN} - F_{IK} = 0$$

$$F_{IK} = -11.25 \text{ kN}$$

$$F_{IK} = 11.25 \text{ kN} \quad C \quad \blacktriangleleft$$

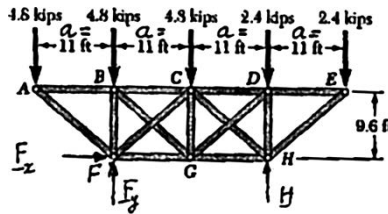


PROBLEM 6.65

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: F_x = 0$$

$$+\curvearrowright \Sigma M_H = 0: 4.8(3a) + 4.8(2a) + 4.8a - 2.4a - F_y(2a) = 0$$

$$F_y = +13.20 \text{ kips}$$

$$F = 13.20 \text{ kips} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: H + 13.20 \text{ kips} - 3(4.8 \text{ kips}) - 2(2.4 \text{ kips}) = 0$$

$$H = +6.00 \text{ kips}$$

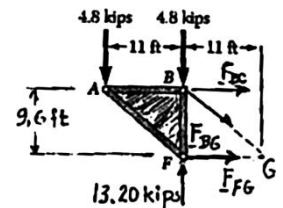
$$H = 6.00 \text{ kips} \uparrow \triangleleft$$

Free body: *ABF*:

We assume that counter *BG* is acting.

$$+\uparrow \Sigma F_y = 0: -\frac{9.6}{14.6} F_{BG} + 13.20 - 2(4.8) = 0$$

$$F_{BG} = +5.475$$



$$F_{BG} = 5.48 \text{ kips } T \triangleleft$$

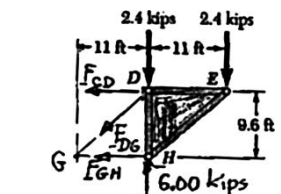
Since *BG* is in tension, our assumption was correct.

Free body: *DEH*:

We assume that counter *DG* is acting.

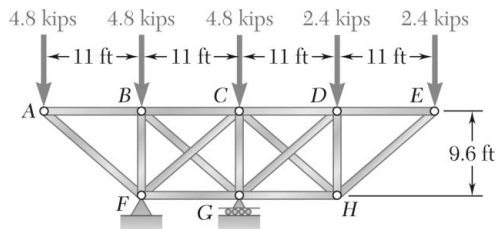
$$+\uparrow \Sigma F_y = 0: -\frac{9.6}{14.6} F_{DG} + 6.00 - 2(2.4) = 0$$

$$F_{DG} = +1.825$$



$$F_{DG} = 1.825 \text{ kips } T \triangleleft$$

Since *DG* is in tension, O.K.

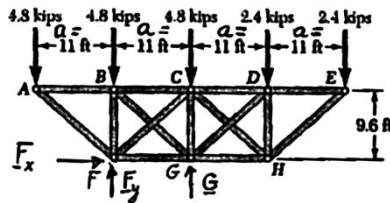


PROBLEM 6.66

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: F_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: -F_y a + 4.8(2a) + 4.8a - 2.4a - 2.4(2a) = 0$$

$$F_y = 7.20 \quad F = 7.20 \text{ kips} \quad \uparrow \triangleleft$$

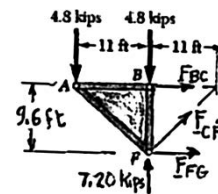
Free body: *ABF*:

We assume that counter *CF* is acting.

$$+\uparrow \Sigma F_y = 0: \frac{9.6}{14.6} F_{CF} + 7.20 - 2(4.8) = 0$$

$$F_{CF} = +3.65$$

$$F_{CF} = 3.65 \text{ kips} \quad T \quad \triangleleft$$



Since *CF* is in tension, O.K.

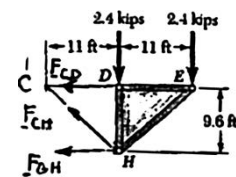
Free body: *DEH*:

We assume that counter *CH* is acting.

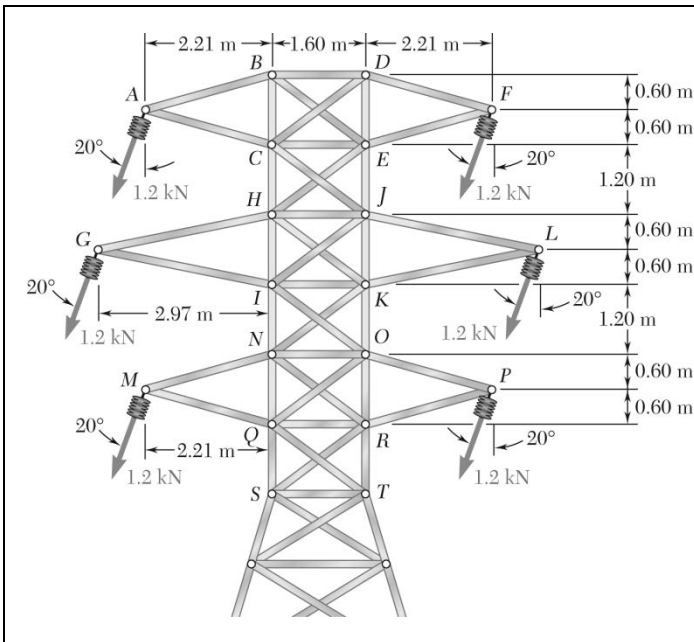
$$+\uparrow \Sigma F_y = 0: \frac{9.6}{14.6} F_{CH} - 2(2.4 \text{ kips}) = 0$$

$$F_{CH} = +7.30$$

$$F_{CH} = 7.30 \text{ kips} \quad T \quad \triangleleft$$



Since *CH* is in tension, O.K.



PROBLEM 6.67

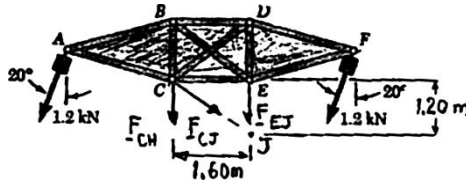
The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (a) which of the two counters listed below is acting, (b) the force in that counter.

Counters *CJ* and *HE*

SOLUTION

Free body: Portion *ABDFEC* of tower.

We assume that counter *CJ* is acting and show the forces exerted by that counter and by members *CH* and *EJ*.



$$\pm \rightarrow \Sigma F_x = 0: \quad \frac{4}{5} F_{CJ} - 2(1.2 \text{ kN}) \sin 20^\circ = 0 \quad F_{CJ} = +1.026 \text{ kN}$$

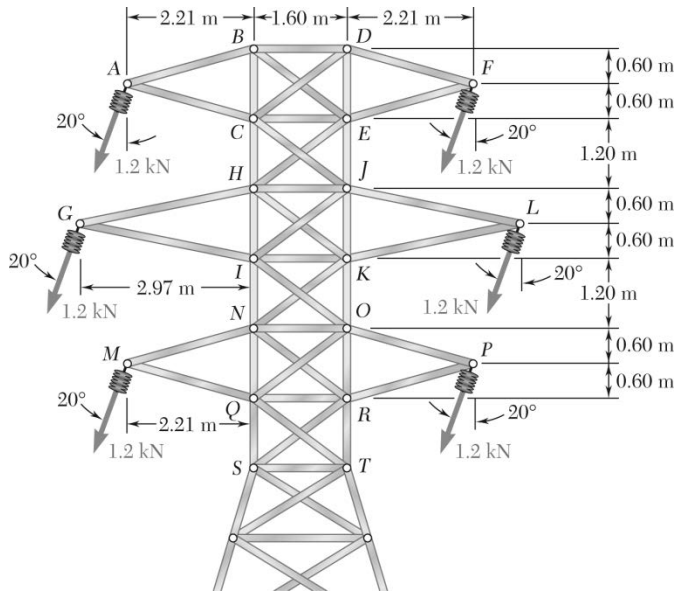
Since *CJ* is found to be in tension, our assumption was correct. Thus, the answers are

- (a) *CJ* ◀
 (b) 1.026 kN *T* ◀

PROBLEM 6.68

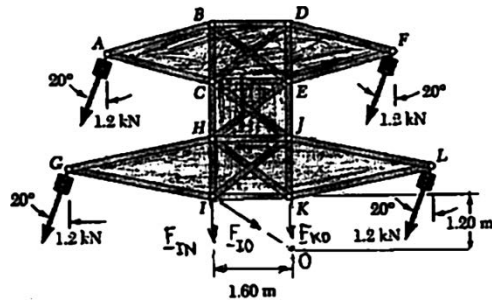
The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (a) which of the two counters listed below is acting, (b) the force in that counter.

Counters *IO* and *KN*



SOLUTION

Free body: Portion of tower shown.



We assume that counter *IO* is acting and show the forces exerted by that counter and by members *IN* and *KO*.

$$\pm \rightarrow \Sigma F_x = 0: \frac{4}{5} F_{IO} - 4(1.2 \text{ kN}) \sin 20^\circ = 0 \quad F_{IO} = +2.05 \text{ kN}$$

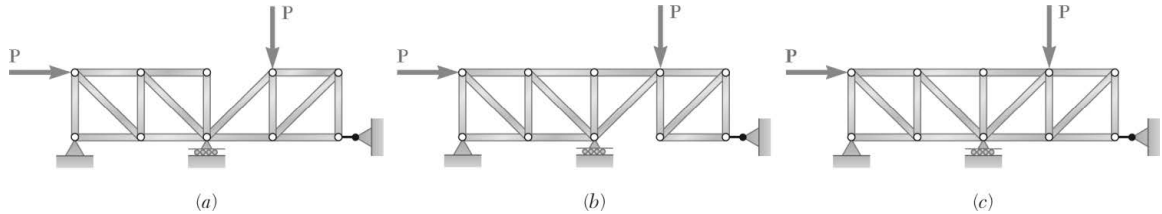
Since *IO* is found to be in tension, our assumption was correct. Thus, the answers are

(a) *IO* ◀

(b) 2.05 kN T ◀

PROBLEM 6.69

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a)

Number of members:

$$m = 16$$

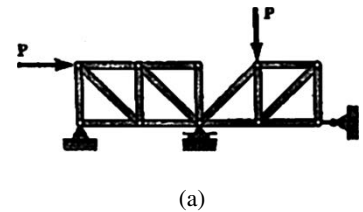
Number of joints:

$$n = 10$$

Reaction components:

$$r = 4$$

$$m + r = 20, \quad 2n = 20$$



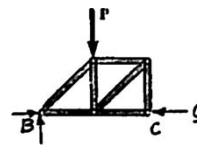
Thus,

$$m + r = 2n \triangleleft$$

To determine whether the structure is actually completely constrained and determinate, we must try to find the reactions at the supports. We divide the structure into two simple trusses and draw the free-body diagram of each truss.



This is a properly supported simple truss – O.K.



This is an improperly supported simple truss. (Reaction at C passes through B. Thus, Eq. $\sum M_B = 0$ cannot be satisfied.)

Structure is improperly constrained. ◀

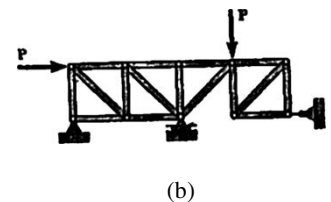
Structure (b)

$$m = 16$$

$$n = 10$$

$$r = 4$$

$$m + r = 20, \quad 2n = 20$$

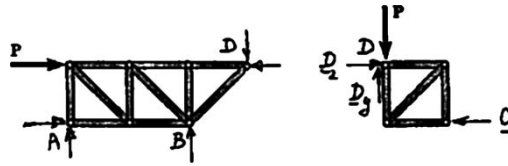


Thus,

$$m + r = 2n \triangleleft$$

PROBLEM 6.69 (Continued)

We must again try to find the reactions at the supports dividing the structure as shown.

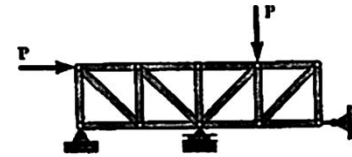


Both portions are simply supported simple trusses.

Structure is completely constrained and determinate. ◀

Structure (c)

$$\begin{aligned}
 m &= 17 \\
 n &= 10 \\
 r &= 4 \\
 m + r &= 21, \quad 2n = 20
 \end{aligned}$$



(c)

Thus,

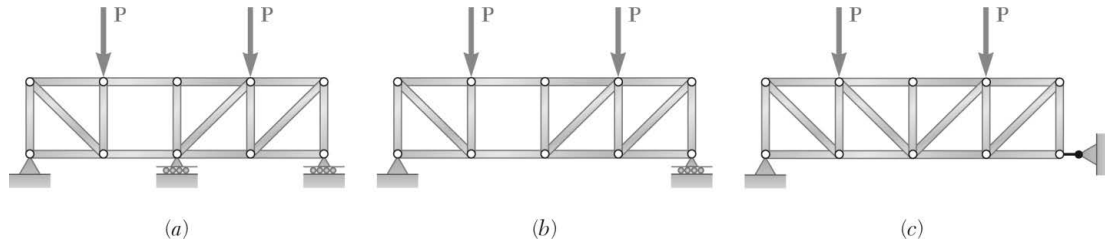
$$m + r > 2n \triangleleft$$

This is a simple truss with an extra support which causes reactions (and forces in members) to be indeterminate.

Structure is completely constrained and indeterminate. ◀

PROBLEM 6.70

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

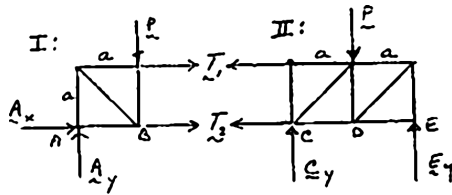


SOLUTION

Structure (a):

Nonsimple truss with $r = 4$, $m = 16$, $n = 10$,
so $m + r = 20 = 2n$, but we must examine further.

FBD Sections:



FBD I:	$\Sigma M_A = 0 \Rightarrow T_1$
II:	$\Sigma F_x = 0 \Rightarrow T_2$
I:	$\Sigma F_x = 0 \Rightarrow A_x$
I:	$\Sigma F_y = 0 \Rightarrow A_y$
II:	$\Sigma M_E = 0 \Rightarrow C_y$
II:	$\Sigma F_y = 0 \Rightarrow E_y$

Since each section is a simple truss with reactions determined,

structure is completely constrained and determinate. ◀

Structure (b):

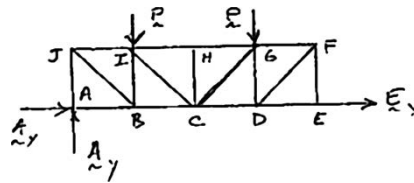
Nonsimple truss with $r = 3$, $m = 16$, $n = 10$,

so $m + r = 19 < 2n = 20$

Structure is partially constrained. ◀

PROBLEM 6.70 (Continued)

Structure (c):

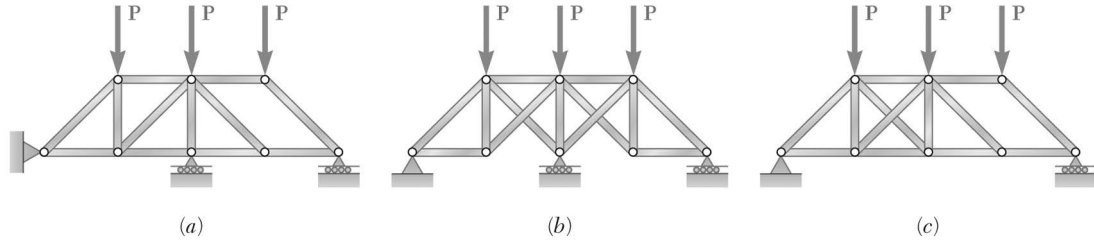


Simple truss with $r = 3$, $m = 17$, $n = 10$, $m + r = 20 = 2n$, but the horizontal reaction forces A_x and E_x are collinear and no equilibrium equation will resolve them, so the structure is improperly constrained and indeterminate.



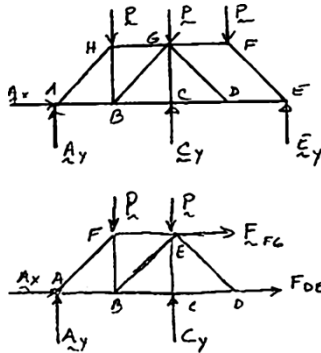
PROBLEM 6.71

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a):



Nonsimple truss with $r = 4$, $m = 12$, $n = 8$ so $r + m = 16 = 2n$.

Check for determinacy:

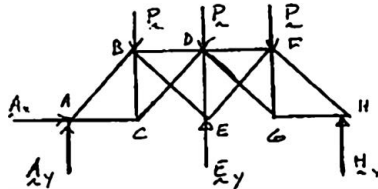
One can solve joint F for forces in EF , FG and then solve joint E for E_y and force in DE .

This leaves a simple truss $ABCDGH$ with

$$r = 3, m = 9, n = 6 \quad \text{so} \quad r + m = 12 = 2n$$

Structure is completely constrained and determinate. ◀

Structure (b):



Simple truss (start with ABC and add joints alphabetically to complete truss) with $r = 4$, $m = 13$, $n = 8$

so

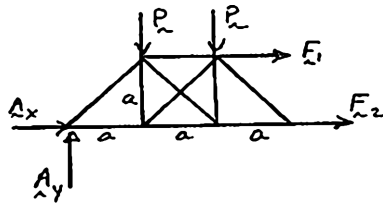
$$r + m = 17 > 2n = 16$$

Constrained but indeterminate ◀

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PROBLEM 6.71 (Continued)

Structure (c):



Nonsimple truss with $r = 3$, $m = 13$, $n = 8$ so $r + m = 16 = 2n$. To further examine, follow procedure in part (a) above to get truss at left.

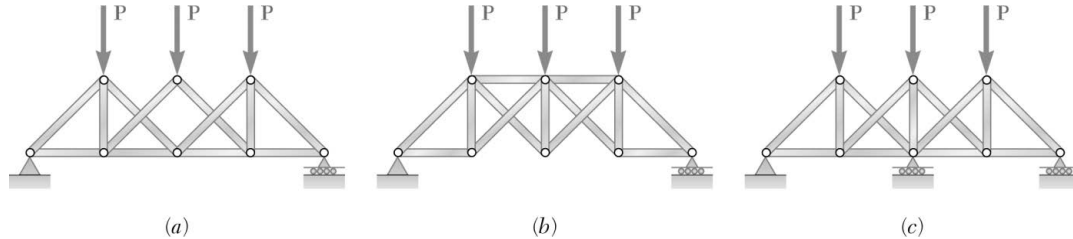
Since $F_1 \neq 0$ (from solution of joint F),

$\Sigma M_A = aF_1 \neq 0$ and there is no equilibrium.

Structure is improperly constrained. ◀

PROBLEM 6.72

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a)

Number of members:

$$m = 12$$

Number of joints:

$$n = 8$$

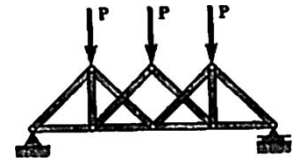
Reaction components:

$$r = 3$$

$$m + r = 15, \quad 2n = 16$$

Thus,

$$m + r < 2n$$



Structure is partially constrained. ◀

Structure (b)

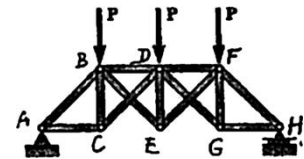
$$m = 13, \quad n = 8$$

$$r = 3$$

$$m + r = 16, \quad 2n = 16$$

Thus,

$$m + r = 2n$$



To verify that the structure is actually completely constrained and determinate, we observe that it is a simple truss (follow lettering to check this) and that it is simply supported by a pin-and-bracket and a roller. Thus,

structure is completely constrained and determinate. ◀

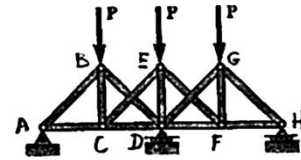
PROBLEM 6.72 (Continued)

Structure (c)

$$m = 13, \quad n = 8$$

$$r = 4$$

$$m + r = 17, \quad 2n = 16$$



Thus,

$$m + r > 2n$$

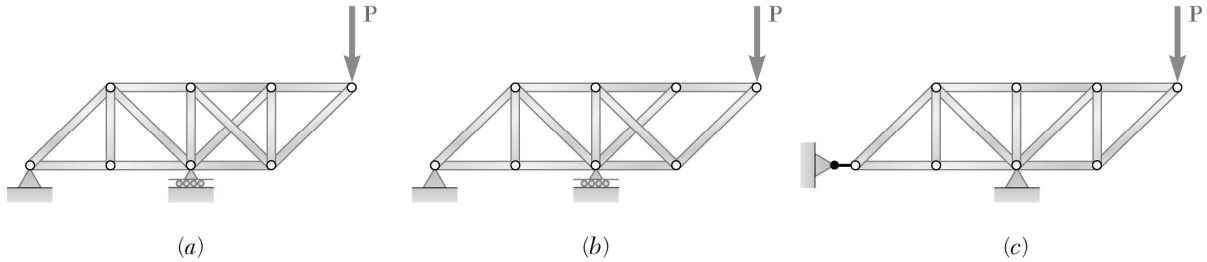


Structure is completely constrained and indeterminate. ◀

This result can be verified by observing that the structure is a simple truss (follow lettering to check this), therefore it is rigid, and that its supports involve four unknowns.

PROBLEM 6.73

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

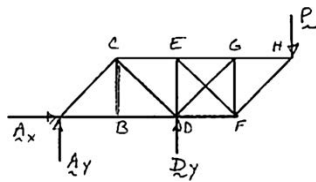


SOLUTION

Structure (a): Rigid truss with $r = 3$, $m = 14$, $n = 8$,
so $r + m = 17 > 2n = 16$

so completely constrained but indeterminate ◀

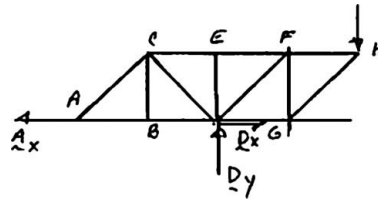
Structure (b): Simple truss (start with ABC and add joints alphabetically), with



$r = 3$, $m = 13$, $n = 8$, so $r + m = 16 = 2n$

so completely constrained and determinate ◀

Structure (c):

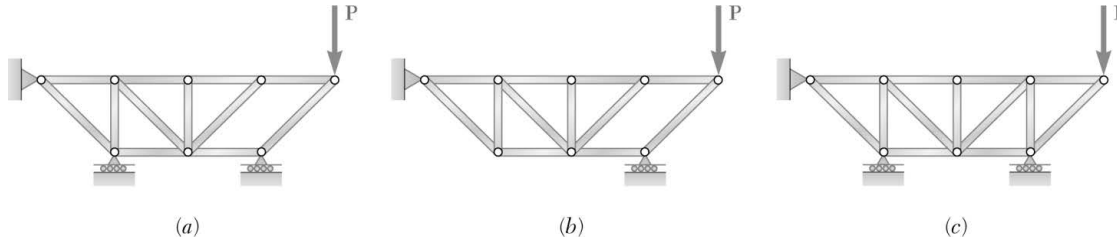


Simple truss with $r = 3$, $m = 13$, $n = 8$, so $r + m = 16 = 2n$, but horizontal reactions (A_x and D_x) are collinear, so cannot be resolved by any equilibrium equation.

Structure is improperly constrained. ◀

PROBLEM 6.74

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a):

No. of members

$$m = 12$$

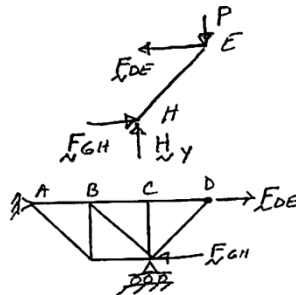
No. of joints

$$n = 8 \quad m + r = 16 = 2n$$

No. of reaction components

$$r = 4 \quad \text{unknowns} = \text{equations}$$

FBD of EH:



$$\Sigma M_H = 0 \rightarrow F_{DE}; \Sigma F_x = 0 \rightarrow F_{GH}; \Sigma F_y = 0 \rightarrow H_y$$

Then $ABCDGF$ is a simple truss and all forces can be determined.

This example is completely constrained and determinate. ◀

Structure (b):

No. of members

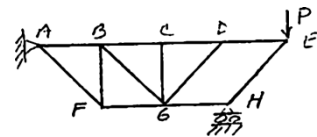
$$m = 12$$

No. of joints

$$n = 8 \quad m + r = 15 < 2n = 16$$

No. of reaction components

$$r = 3 \quad \text{unknowns} < \text{equations}$$



partially constrained ◀

Note: Quadrilateral $DEHG$ can collapse with joint D moving downward; in (a), the roller at F prevents this action.

PROBLEM 6.74 (Continued)

Structure (c):

No. of members

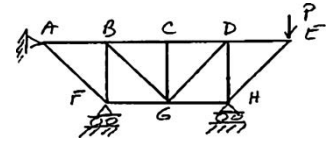
$$m = 13$$

No. of joints

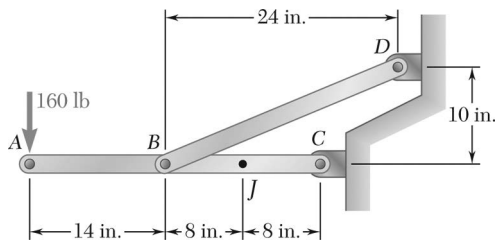
$$n = 8 \quad m + r = 17 > 2n = 16$$

No. of reaction components

$$r = 4 \quad \text{unknowns} > \text{equations}$$



completely constrained but indeterminate ◀



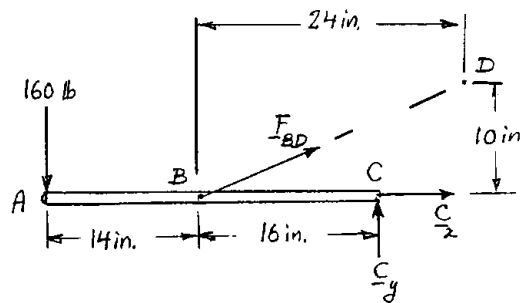
PROBLEM 6.75

Determine the force in member BD and the components of the reaction at C .

SOLUTION

We note that BD is a two-force member. The force it exerts on ABC , therefore, is directed along line BD .

Free body: ABC :



$$BD = \sqrt{(24)^2 + (10)^2} = 26 \text{ in.}$$

$$+\curvearrowright \Sigma M_C = 0: (160 \text{ lb})(30 \text{ in.}) - \left(\frac{10}{26} F_{BD}\right)(16 \text{ in.}) = 0$$

$$F_{BD} = +780 \text{ lb}$$

$$F_{BD} = 780 \text{ lb } T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma M_x = 0: C_x + \frac{24}{26}(780 \text{ lb}) = 0$$

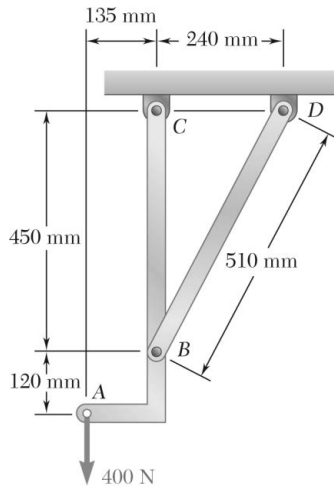
$$C_x = -720 \text{ lb}$$

$$C_x = 720 \text{ lb } \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: C_y - 160 \text{ lb} + \frac{10}{26}(780 \text{ lb}) = 0$$

$$C_y = -140.0 \text{ lb}$$

$$C_y = 140.0 \text{ lb } \downarrow \blacktriangleleft$$



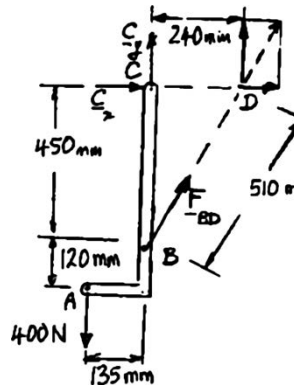
PROBLEM 6.76

Determine the force in member BD and the components of the reaction at C .

SOLUTION

We note that BD is a two-force member. The force it exerts on ABC , therefore, is directed along line BD .

Free body: ABC :



Attaching F_{BD} at D and resolving it into components, we write

$$+\circlearrowleft \Sigma M_C = 0: \quad (400 \text{ N})(135 \text{ mm}) + \left(\frac{450}{510} F_{BD}\right)(240 \text{ mm}) = 0$$

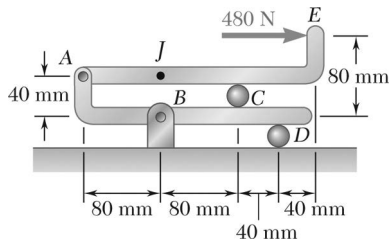
$$F_{BD} = -255 \text{ N} \quad F_{BD} = 255 \text{ N} \quad C \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad C_x + \frac{240}{510}(-255 \text{ N}) = 0$$

$$C_x = +120.0 \text{ N} \quad C_x = 120.0 \text{ N} \quad \rightarrow \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 400 \text{ N} + \frac{450}{510}(-255 \text{ N}) = 0$$

$$C_y = +625 \text{ N} \quad C_y = 625 \text{ N} \quad \uparrow \quad \blacktriangleleft$$

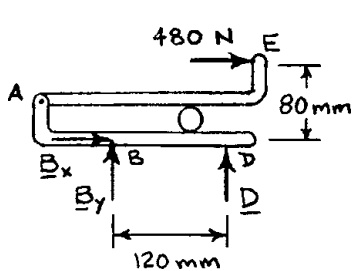


PROBLEM 6.77

Determine the components of all forces acting on member *ABCD* of the assembly shown.

SOLUTION

Free body: Entire assembly:



$$+\curvearrowright \Sigma M_B = 0: D(120 \text{ mm}) - (480 \text{ N})(80 \text{ mm}) = 0$$

$$D = 320 \text{ N} \uparrow \blacktriangleleft$$

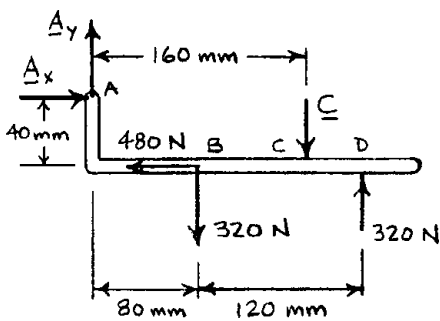
$$+\rightarrow \Sigma F_x = 0: B_x + 480 \text{ N} = 0$$

$$B_x = 480 \text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + 320 \text{ N} = 0$$

$$B_y = 320 \text{ N} \downarrow \blacktriangleleft$$

Free body: Member ABCD:



$$+\curvearrowright \Sigma M_A = 0: (320 \text{ N})(200 \text{ mm}) - C(160 \text{ mm}) - (320 \text{ N})(80 \text{ mm}) - (480 \text{ N})(40 \text{ mm}) = 0$$

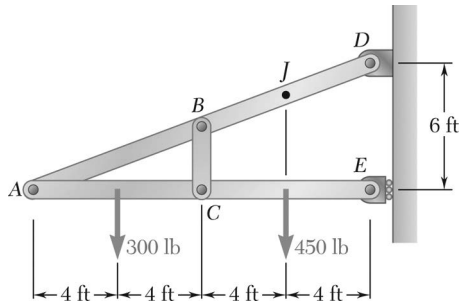
$$C = 120.0 \text{ N} \downarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: A_x - 480 \text{ N} = 0$$

$$A_x = 480 \text{ N} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A_y - 320 \text{ N} - 120 \text{ N} + 320 \text{ N} = 0$$

$$A_y = 120.0 \text{ N} \uparrow \blacktriangleleft$$

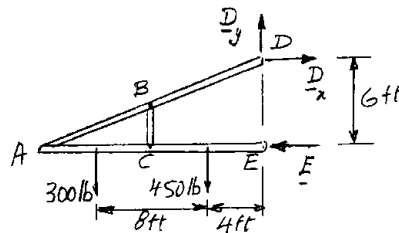


PROBLEM 6.78

Determine the components of all forces acting on member ABD of the frame shown.

SOLUTION

Free body: Entire frame:



$$+\curvearrowright \Sigma M_D = 0: (300 \text{ lb})(12 \text{ ft}) - (450 \text{ lb})(4 \text{ ft}) - E(6 \text{ ft}) = 0$$

$$E = +900 \text{ lb} \quad E = 900 \text{ lb} \leftarrow$$

$$\pm \rightarrow \Sigma F_x = 0: D_x - 900 \text{ lb} = 0$$

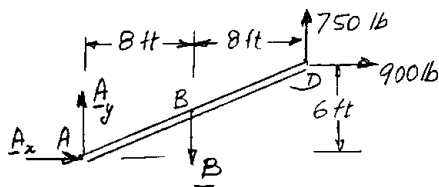
$$D_x = 900 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: D_y - 300 \text{ lb} - 450 \text{ lb} = 0$$

$$D_y = 750 \text{ lb} \uparrow$$

Free body: Member ABD :

We note that BC is a two-force member and that \mathbf{B} is directed along BC .



$$+\curvearrowright \Sigma M_A = 0: (750 \text{ lb})(16 \text{ ft}) - (900 \text{ lb})(6 \text{ ft}) - B(8 \text{ ft}) = 0$$

$$B = +825 \text{ lb} \quad B = 825 \text{ lb} \downarrow$$

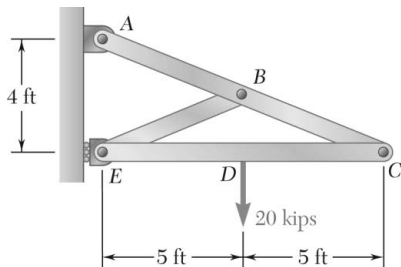
$$\pm \rightarrow \Sigma F_x = 0: A_x + 900 \text{ lb} = 0$$

$$A_x = -900 \text{ lb} \quad A_x = 900 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + 750 \text{ lb} - 825 \text{ lb} = 0$$

$$A_y = +75 \text{ lb} \quad A_y = 75.0 \text{ lb} \uparrow$$

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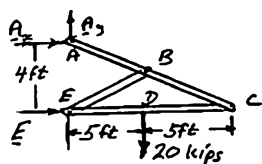


PROBLEM 6.79

For the frame and loading shown, determine the components of all forces acting on member ABC.

SOLUTION

Free body: Entire frame:



$$+\curvearrowright \Sigma M_E = 0: -A_x(4) - (20 \text{ kips})(5) = 0$$

$$A_x = -25 \text{ kips},$$

$$A_x = 25.0 \text{ kips} \leftarrow \blacktriangleleft$$

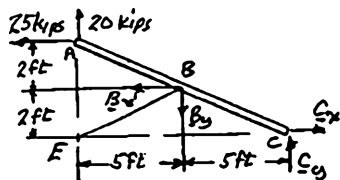
$$+\uparrow \Sigma F_y = 0: A_y - 20 \text{ kips} = 0$$

$$A_y = 20 \text{ kips}$$

$$A_y = 20.0 \text{ kips} \uparrow \blacktriangleleft$$

Free body: Member ABC:

Note: BE is a two-force member, thus **B** is directed along line BE and $B_y = \frac{2}{5} B_x$.



$$+\curvearrowright \Sigma M_C = 0: (25 \text{ kips})(4 \text{ ft}) - (20 \text{ kips})(10 \text{ ft}) + B_x(2 \text{ ft}) + B_y(5 \text{ ft}) = 0$$

$$-100 \text{ kip} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{2}{5} B_x(5 \text{ ft}) = 0$$

$$B_x = 25 \text{ kips}$$

$$B_x = 25.0 \text{ kips} \leftarrow \blacktriangleleft$$

$$B_y = \frac{2}{5}(B_x) = \frac{2}{5}(25) = 10 \text{ kips}$$

$$B_y = 10.00 \text{ kips} \downarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: C_x - 25 \text{ kips} - 25 \text{ kips} = 0$$

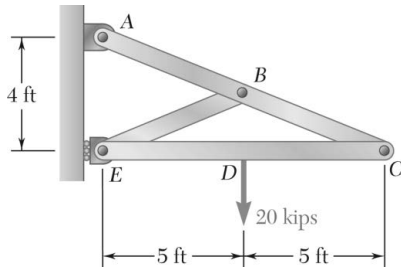
$$C_x = 50 \text{ kips}$$

$$C_x = 50.0 \text{ kips} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: C_y + 20 \text{ kips} - 10 \text{ kips} = 0$$

$$C_y = -10 \text{ kips}$$

$$C_y = 10.00 \text{ kips} \downarrow \blacktriangleleft$$



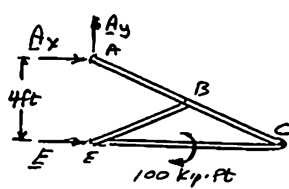
PROBLEM 6.80

Solve Problem 6.79 assuming that the 20-kip load is replaced by a clockwise couple of magnitude 100 kip · ft applied to member *EDC* at Point *D*.

PROBLEM 6.79 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame:



$$+\uparrow \Sigma F_y = 0: A_y = 0$$

$$+\curvearrowright \Sigma M_E = 0: -A_x(4 \text{ ft}) - 100 \text{ kip} \cdot \text{ft} = 0$$

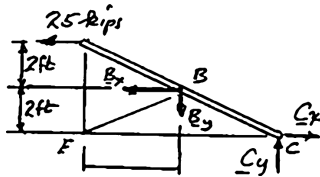
$$A_x = -25 \text{ kips}$$

$$A_x = 25.0 \text{ kips} \leftarrow$$

$$A = 25.0 \text{ kips} \leftarrow \blacktriangleleft$$

Free body: Member *ABC*:

Note: *BE* is a two-force member, thus **B** is directed along line *BE* and $B_y = \frac{2}{5} B_x$.



$$+\curvearrowright \Sigma M_C = 0: (25 \text{ kips})(4 \text{ ft}) + B_x(2 \text{ ft}) + B_y(5 \text{ ft}) = 0$$

$$100 \text{ kip} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{2}{5} B_x(5 \text{ ft}) = 0$$

$$B_x = -25 \text{ kips}$$

$$B_x = 25.0 \text{ kips} \rightarrow \blacktriangleleft$$

$$B_y = \frac{2}{5} B_x = \frac{2}{5}(-25) = -10 \text{ kips};$$

$$B_y = 10.00 \text{ kips} \uparrow \blacktriangleleft$$

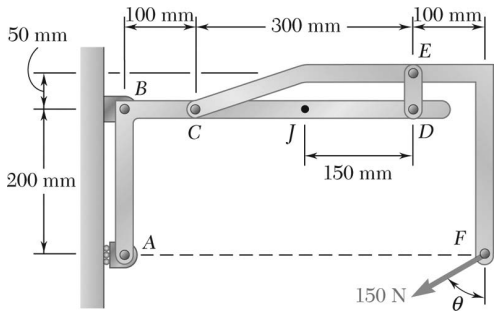
$$+\rightarrow \Sigma F_x = 0: -25 \text{ kips} + 25 \text{ kips} + C_x = 0 \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: +10 \text{ kips} + C_y = 0$$

$$C_y = -10 \text{ kips}$$

$$C_y = 10 \text{ kips} \downarrow$$

$$C = 10.00 \text{ kips} \downarrow \blacktriangleleft$$

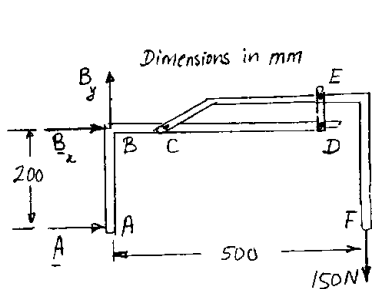


PROBLEM 6.81

Determine the components of all forces acting on member *ABCD* when $\theta = 0$.

SOLUTION

Free body: Entire assembly:



$$+\circlearrowleft \Sigma M_B = 0: A(200) - (150 \text{ N})(500) = 0$$

$$A = +375 \text{ N}$$

$$\mathbf{A} = 375 \text{ N} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: B_x + 375 \text{ N} = 0$$

$$B_x = -375 \text{ N}$$

$$\mathbf{B}_x = 375 \text{ N} \leftarrow \blacktriangleleft$$

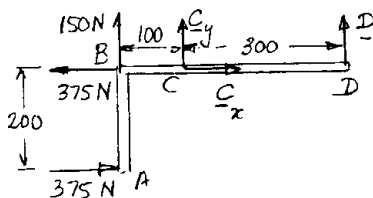
$$+\uparrow \Sigma F_y = 0: B_y - 150 \text{ N} = 0$$

$$B_y = +150 \text{ N}$$

$$\mathbf{B}_y = 150 \text{ N} \uparrow \blacktriangleleft$$

Free body: Member *ABCD*:

We note that *D* is directed along *DE*, since *DE* is a two-force member.



$$+\circlearrowleft \Sigma M_C = 0: D(300) - (150 \text{ N})(100) + (375 \text{ N})(200) = 0$$

$$D = -200 \text{ N}$$

$$\mathbf{D} = 200 \text{ N} \downarrow \blacktriangleleft$$

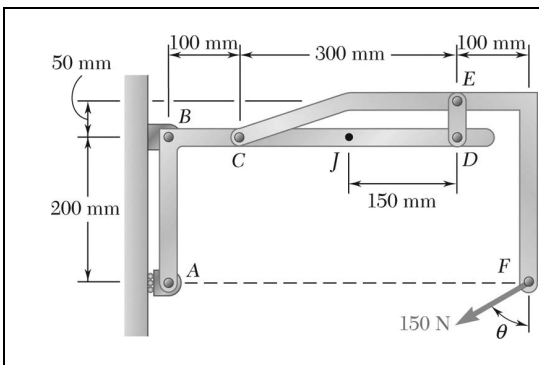
$$+\rightarrow \Sigma F_x = 0: C_x + 375 - 375 = 0$$

$$C_x = 0$$

$$+\uparrow \Sigma F_y = 0: C_y + 150 - 200 = 0$$

$$C_y = +50.0 \text{ N}$$

$$\mathbf{C} = 50.0 \text{ N} \uparrow \blacktriangleleft$$

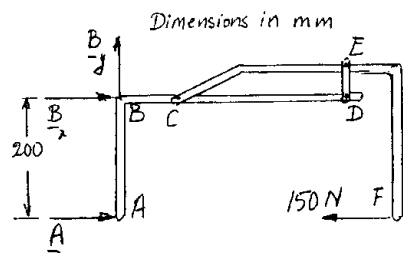


PROBLEM 6.82

Determine the components of all forces acting on member *ABCD* when $\theta = 90^\circ$.

SOLUTION

Free body: Entire assembly:



$$+\curvearrowright \Sigma M_B = 0: A(200) - (150 \text{ N})(200) = 0$$

$$A = +150.0 \text{ N} \quad \mathbf{A} = 150.0 \text{ N} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: B_x + 150 - 150 = 0$$

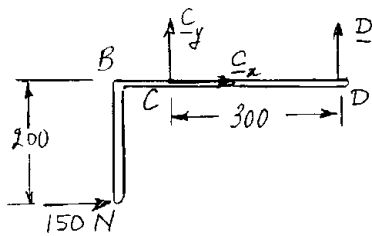
$$B_x = 0$$

$$+\uparrow \Sigma F_y = 0: B_y = 0$$

$$\mathbf{B} = 0 \blacktriangleleft$$

Free body: Member *ABCD*:

We note that *D* is directed along *DE*, since *DE* is a two-force member.



$$+\curvearrowright \Sigma M_C = 0: D(300) + (150 \text{ N})(200) = 0$$

$$D = -100.0 \text{ N} \quad \mathbf{D} = 100.0 \text{ N} \downarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: C_x + 150 \text{ N} = 0$$

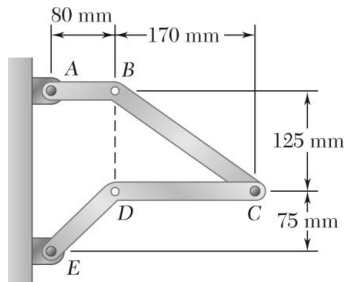
$$C_x = -150 \text{ N}$$

$$\mathbf{C}_x = 150.0 \text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: C_y - 100 \text{ N} = 0$$

$$C_y = +100.0 \text{ N}$$

$$\mathbf{C}_y = 100.0 \text{ N} \uparrow \blacktriangleleft$$



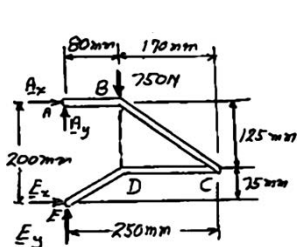
PROBLEM 6.83

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B , (b) at D .

SOLUTION

Free body: Entire frame:

The following analysis is valid for both parts (a) and (b) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: -(750 \text{ N})(80 \text{ mm}) - A_x(200 \text{ mm}) = 0$$

$$A_x = -300 \text{ N} \quad \mathbf{A}_x = 300 \text{ N} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: E_x - 300 \text{ N} = 0 \quad E_x = 300 \text{ N} \quad \mathbf{E}_x = 300 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + E_y - 750 \text{ N} = 0$$

(1)

(a) Load applied at B .

Free body: Member CE :

CE is a two-force member. Thus, the reaction at E must be directed along CE .

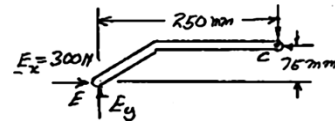
$$\frac{E_y}{300 \text{ N}} = \frac{75 \text{ mm}}{250 \text{ mm}} \quad E_y = 90 \text{ N} \uparrow$$

From Eq. (1): $A_y + 90 \text{ N} - 750 \text{ N} = 0 \quad A_y = 660 \text{ N} \uparrow$

Thus, reactions are

$$\mathbf{A}_x = 300 \text{ N} \leftarrow, \quad \mathbf{A}_y = 660 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 300 \text{ N} \rightarrow, \quad \mathbf{E}_y = 90.0 \text{ N} \uparrow \blacktriangleleft$$

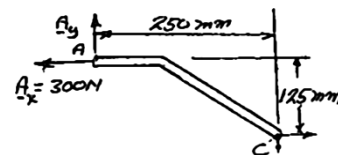


(b) Load applied at D .

Free body: Member AC :

AC is a two-force member. Thus, the reaction at A must be directed along AC .

$$\frac{A_y}{300 \text{ N}} = \frac{125 \text{ mm}}{250 \text{ mm}} \quad A_y = 150 \text{ N} \uparrow$$



PROBLEM 6.83 (Continued)

From Eq. (1): $A_y + E_y - 750 \text{ N} = 0$

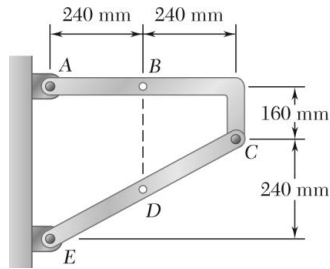
$$150 \text{ N} + E_y - 750 \text{ N} = 0$$

$$E_y = 600 \text{ N} \quad \mathbf{E}_y = 600 \text{ N} \uparrow$$

Thus, reactions are

$$\mathbf{A}_x = 300 \text{ N} \leftarrow, \quad \mathbf{A}_y = 150.0 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 300 \text{ N} \rightarrow, \quad \mathbf{E}_y = 600 \text{ N} \uparrow \blacktriangleleft$$



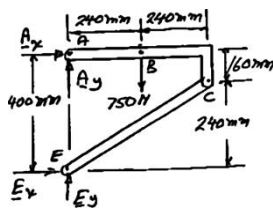
PROBLEM 6.84

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

Free-body: Entire frame:

The following analysis is valid for both parts (a) and (b) since position of load on its line of action is immaterial.



$$+\circlearrowleft \sum M_E = 0: -(750 \text{ N})(240 \text{ mm}) - A_x(400 \text{ mm}) = 0$$

$$A_x = -450 \text{ N} \quad \mathbf{A}_x = 450 \text{ N} \leftarrow$$

$$+\rightarrow \sum F_x = 0: E_x - 450 \text{ N} = 0 \quad E_x = 450 \text{ N} \quad \mathbf{E}_x = 450 \text{ N} \rightarrow$$

$$+\uparrow \sum F_y = 0: A_y + E_y - 750 \text{ N} = 0 \quad (1)$$

(a) Load applied at B.

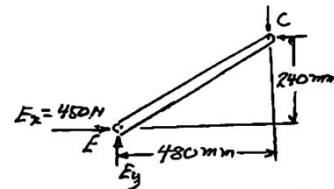
Free body: Member CE:

CE is a two-force member. Thus, the reaction at E must be directed along CE.

$$\frac{E_y}{450 \text{ N}} = \frac{240 \text{ mm}}{480 \text{ mm}}; \quad \mathbf{E}_y = 225 \text{ N} \uparrow$$

From Eq. (1): $A_y + 225 - 750 = 0; \quad \mathbf{A}_y = 525 \text{ N} \uparrow$

Thus, reactions are



$$\mathbf{A}_x = 450 \text{ N} \leftarrow, \quad \mathbf{A}_y = 525 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 450 \text{ N} \rightarrow, \quad \mathbf{E}_y = 225 \text{ N} \uparrow \blacktriangleleft$$

(b) Load applied at D.

Free body: Member AC:

AC is a two-force member. Thus, the reaction at A must be directed along AC.

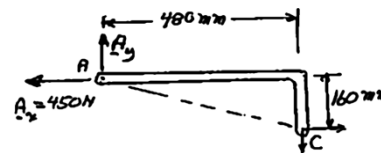
$$\frac{A_y}{450 \text{ N}} = \frac{160 \text{ mm}}{480 \text{ mm}} \quad \mathbf{A}_y = 150.0 \text{ N} \uparrow$$

From Eq. (1): $A_y + E_y - 750 \text{ N} = 0$

$$150 \text{ N} + E_y - 750 \text{ N} = 0$$

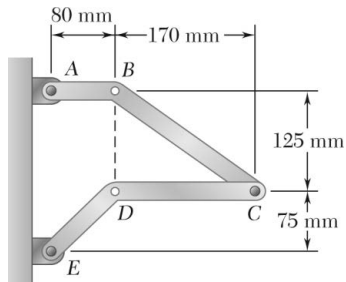
$$\mathbf{E}_y = 600 \text{ N} \quad \mathbf{E}_y = 600 \text{ N} \uparrow$$

Thus, reactions are



$$\mathbf{A}_x = 450 \text{ N} \leftarrow, \quad \mathbf{A}_y = 150.0 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 450 \text{ N} \rightarrow, \quad \mathbf{E}_y = 600 \text{ N} \uparrow \blacktriangleleft$$



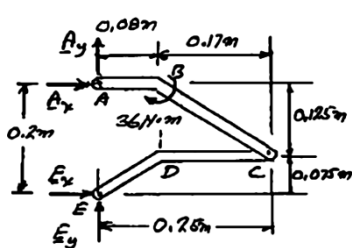
PROBLEM 6.85

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude $36 \text{ N} \cdot \text{m}$ applied (a) at B , (b) at D .

SOLUTION

Free body: Entire frame:

The following analysis is valid for both parts (a) and (b) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: -36 \text{ N} \cdot \text{m} - A_x(0.2 \text{ m}) = 0$$

$$A_x = -180 \text{ N} \quad A_x = 180 \text{ N} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: -180 \text{ N} + E_x = 0$$

$$E_x = 180 \text{ N} \quad E_x = 180 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + E_y = 0$$

(1)

(a) Couple applied at B .

Free body: Member CE :

AC is a two-force member. Thus, the reaction at E must be directed along EC .

$$\frac{E_y}{180 \text{ N}} = \frac{0.075 \text{ m}}{0.25 \text{ m}} \quad E_y = 54 \text{ N} \uparrow$$

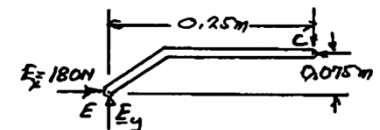
From Eq. (1): $A_y + 54 \text{ N} = 0$

$$A_y = -54 \text{ N} \quad A_y = 54.0 \text{ N} \downarrow$$

Thus, reactions are

$$A_x = 180.0 \text{ N} \leftarrow, \quad A_y = 54.0 \text{ N} \downarrow \blacktriangleleft$$

$$E_x = 180.0 \text{ N} \rightarrow, \quad E_y = 54.0 \text{ N} \uparrow \blacktriangleleft$$

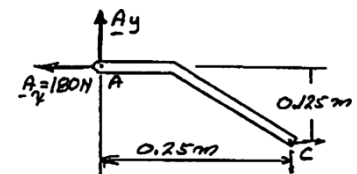


(b) Couple applied at D .

Free body: Member AC :

AC is a two-force member. Thus, the reaction at A must be directed along EC .

$$\frac{A_y}{180 \text{ N}} = \frac{0.125 \text{ m}}{0.25 \text{ m}} \quad A_y = 90 \text{ N} \uparrow$$



PROBLEM 6.85 (Continued)

From Eq. (1): $A_y + E_y = 0$

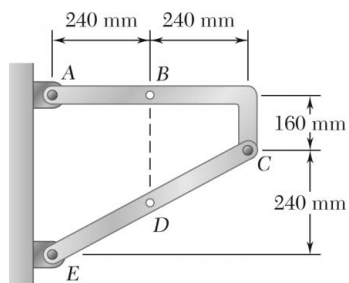
$$90 \text{ N} + E_y = 0$$

$$E_y = -90 \text{ N} \quad \mathbf{E}_y = 90 \text{ N} \downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 180.0 \text{ N} \leftarrow, \quad \mathbf{A}_y = 90.0 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = -180.0 \text{ N} \rightarrow, \quad \mathbf{E}_y = 90.0 \text{ N} \downarrow \blacktriangleleft$$



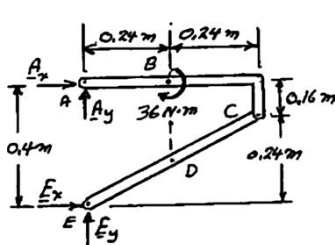
PROBLEM 6.86

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude $36 \text{ N} \cdot \text{m}$ applied (a) at B , (b) at D .

SOLUTION

Free body: Entire frame:

The following analysis is valid for both parts (a) and (b) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: -36 \text{ N} \cdot \text{m} - A_x(0.4 \text{ m}) = 0$$

$$A_x = -90 \text{ N} \quad A_x = 90.0 \text{ N} \leftarrow$$

$$\pm \rightarrow \Sigma F_x = 0: -90 + E_x = 0$$

$$E_x = 90 \text{ N} \quad E_x = 90.0 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: A_y + E_y = 0$$

(1)

(a) Couple applied at B .

Free body: Member CE :

AC is a two-force member. Thus, the reaction at E must be directed along EC .

$$\frac{E_y}{90 \text{ N}} = \frac{0.24 \text{ m}}{0.48 \text{ m}}; \quad E_y = 45.0 \text{ N} \uparrow$$

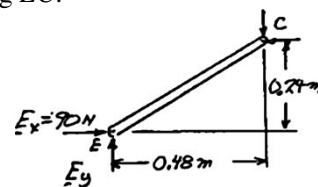
From Eq. (1): $A_y + 45 \text{ N} = 0$

$$A_y = -45 \text{ N} \quad A_y = 45.0 \text{ N} \downarrow$$

Thus, reactions are

$$A_x = 90.0 \text{ N} \leftarrow, \quad A_y = 45.0 \text{ N} \downarrow \blacktriangleleft$$

$$E_x = 90.0 \text{ N} \rightarrow, \quad E_y = 45.0 \text{ N} \uparrow \blacktriangleleft$$

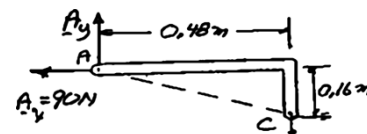


(b) Couple applied at D .

Free body: Member AC :

AC is a two-force member. Thus, the reaction at A must be directed along AC .

$$\frac{A_y}{90 \text{ N}} = \frac{0.16 \text{ m}}{0.48 \text{ m}}; \quad A_y = 30 \text{ N} \uparrow$$



PROBLEM 6.86 (Continued)

From Eq. (1):

$$A_y + E_y = 0$$

$$30 \text{ N} + E_y = 0$$

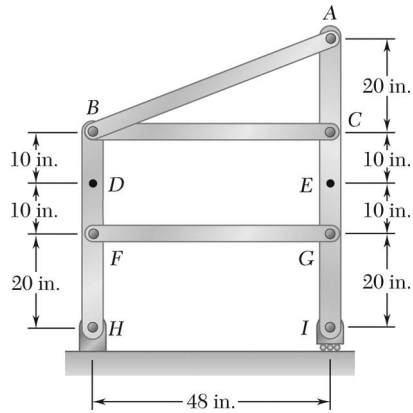
$$E_y = -30 \text{ N} \quad \mathbf{E}_y = 30 \text{ N} \downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 90.0 \text{ N} \leftarrow, \quad \mathbf{A}_y = 30.0 \text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{E}_x = -90.0 \text{ N} \rightarrow, \quad \mathbf{E}_y = 30.0 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 6.87

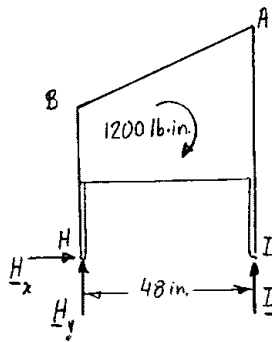


Determine all the forces exerted on member AI if the frame is loaded by a clockwise couple of magnitude $1200 \text{ lb} \cdot \text{in.}$ applied (a) at Point D , (b) at Point E .

SOLUTION

Free body: Entire frame:

Location of couple is immaterial.



$$+\circlearrowleft \Sigma M_H = 0: \quad I(48 \text{ in.}) - 1200 \text{ lb} \cdot \text{in.} = 0$$

$$I = +25.0 \text{ lb}$$

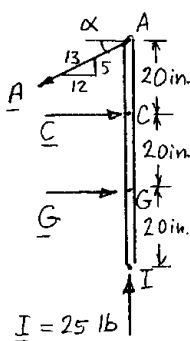
$$(a) \text{ and } (b) \quad \mathbf{I} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

We note that AB , BC , and FG are two-force members.

Free body: Member AI :

$$\tan \alpha = \frac{20}{48} = \frac{5}{12} \quad \alpha = 22.6^\circ$$

(a) Couple applied at D .



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25 \text{ lb} = 0$$

$$A = +65.0 \text{ lb}$$

$$\mathbf{A} = 65.0 \text{ lb} \nearrow 22.6^\circ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad \frac{12}{13}(65 \text{ lb})(40 \text{ in.}) - C(20 \text{ in.}) = 0$$

$$C = +120 \text{ lb}$$

$$\mathbf{C} = 120 \text{ lb} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad -\frac{12}{13}(65 \text{ lb}) + 120 \text{ lb} + G = 0$$

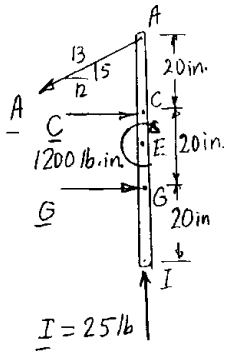
$$G = -60.0 \text{ lb}$$

$$\mathbf{G} = 60 \text{ lb} \leftarrow \blacktriangleleft$$

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PROBLEM 6.87 (Continued)

(b) Couple applied at E.



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25 \text{ lb} = 0$$

$$A = +65.0 \text{ lb}$$

$$A = 65.0 \text{ lb} \nearrow 22.6^\circ \blacktriangleleft$$

$$+\curvearrowright \Sigma M_G = 0: \quad \frac{12}{13}(65 \text{ lb}) + (40 \text{ in.}) - C(20 \text{ in.}) - 1200 \text{ lb} \cdot \text{in.} = 0$$

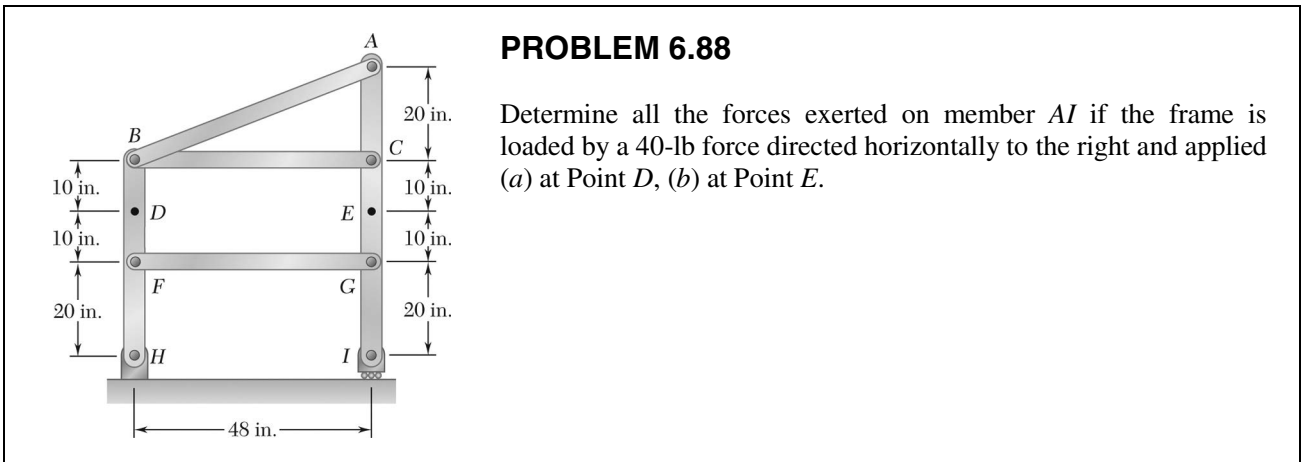
$$C = +60.0 \text{ lb}$$

$$C = 60.0 \text{ lb} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad -\frac{12}{13}(65 \text{ lb}) + 60 \text{ lb} + G = 0$$

$$G = 0 \blacktriangleleft$$

$$I = 25 \text{ lb}$$



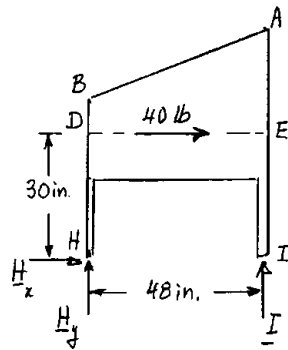
PROBLEM 6.88

Determine all the forces exerted on member *AI* if the frame is loaded by a 40-lb force directed horizontally to the right and applied (a) at Point *D*, (b) at Point *E*.

SOLUTION

Free body: Entire frame:

Location of 40-lb force on its line of action *DE* is immaterial.



$$+\circlearrowleft \Sigma M_H = 0: \quad I(48 \text{ in.}) - (40 \text{ lb})(30 \text{ in.}) = 0$$

$$I = +25.0 \text{ lb}$$

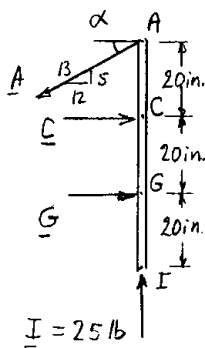
$$(a) \text{ and } (b) \quad I = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

We note that *AB*, *BC*, and *FG* are two-force members.

Free body: Member *AI*:

$$\tan \alpha = \frac{20}{48} = \frac{5}{12} \quad \alpha = 22.6^\circ$$

(a) Force applied at *D*.



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25 \text{ lb} = 0$$

$$A = +65.0 \text{ lb}$$

$$A = 65.0 \text{ lb} \nearrow 22.6^\circ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad \frac{12}{13}(65 \text{ lb})(40 \text{ in.}) - C(20 \text{ in.}) = 0$$

$$C = +120.0 \text{ lb}$$

$$C = 120.0 \text{ lb} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad -\frac{12}{13}(65 \text{ lb}) + 120 \text{ lb} + G = 0$$

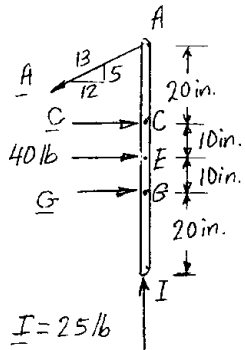
$$G = -60.0 \text{ lb}$$

$$G = 60.0 \text{ lb} \leftarrow \blacktriangleleft$$

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PROBLEM 6.88 (Continued)

(b) Force applied at E.



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25 \text{ lb} = 0$$

$$A = +65.0 \text{ lb}$$

$$A = 65.0 \text{ lb} \nearrow 22.6^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma M_G = 0: \quad \frac{12}{13}(65 \text{ lb})(40 \text{ in.}) - C(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

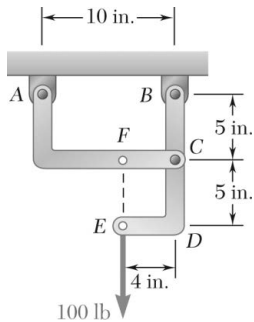
$$C = +100.0 \text{ lb}$$

$$C = 100.0 \text{ lb} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad -\frac{12}{13}(65 \text{ lb}) + 100 \text{ lb} + 40 \text{ lb} + G = 0$$

$$G = -80.0 \text{ lb}$$

$$G = 80.0 \text{ lb} \leftarrow \blacktriangleleft$$

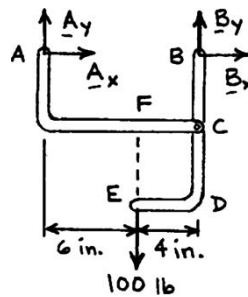


PROBLEM 6.89

Determine the components of the reactions at A and B, (a) if the 500-N load is applied as shown, (b) if the 500-N load is moved along its line of action and is applied at Point F.

SOLUTION

Free body: Entire frame:



Analysis is valid for either parts (a) or (b), since position of 100-lb load on its line of action is immaterial.

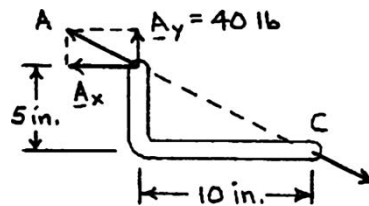
$$+\curvearrowright \Sigma M_A = 0: B_y(10) - (100 \text{ lb})(6) = 0 \quad B_y = +60 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: A_y + 60 - 100 = 0 \quad A_y = +40 \text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: A_x + B_x = 0 \quad (1)$$

(a) Load applied at E.

Free body: Member AC:



Since AC is a two-force member, the reaction at A must be directed along CA. We have

$$\frac{A_x}{10 \text{ in.}} = \frac{40 \text{ lb}}{5 \text{ in.}} \quad A_x = 80.0 \text{ lb} \leftarrow, \quad A_y = 40.0 \text{ lb} \uparrow \blacktriangleleft$$

From Eq. (1): $-80 + B_x = 0 \quad B_x = +80 \text{ lb}$

Thus, $B_x = 80.0 \text{ lb} \rightarrow, \quad B_y = 60.0 \text{ lb} \uparrow \blacktriangleleft$

PROBLEM 6.89 (Continued)

(b) Load applied at F .

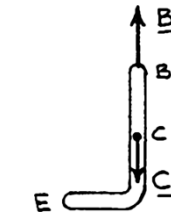
Free body: Member BCD :

Since BCD is a two-force member (with forces applied at B and C only), the reaction at B must be directed along CB . We have, therefore,

$$B_x = 0$$

The reaction at B is

$$\mathbf{B}_x = 0$$



$$\mathbf{B}_y = 60.0 \text{ lb } \uparrow \blacktriangleleft$$

From Eq. (1):

$$A_x + 0 = 0 \quad A_x = 0$$

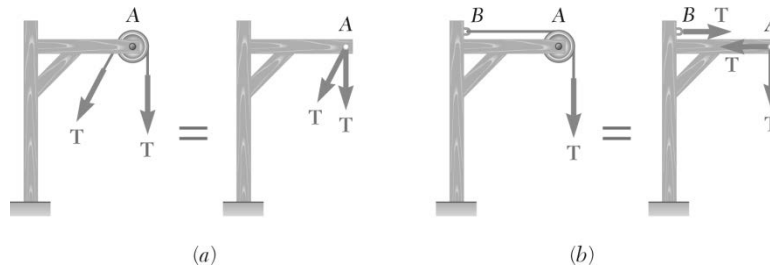
The reaction at A is

$$\mathbf{A}_x = 0$$

$$\mathbf{A}_y = 40.0 \text{ lb } \uparrow \blacktriangleleft$$

PROBLEM 6.90

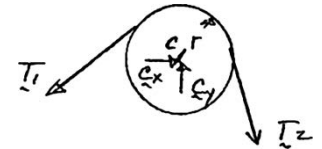
(a) Show that when a frame supports a pulley at A, an equivalent loading of the frame and of each of its component parts can be obtained by removing the pulley and applying at A two forces equal and parallel to the forces that the cable exerted on the pulley. (b) Show that if one end of the cable is attached to the frame at a Point B, a force of magnitude equal to the tension in the cable should also be applied at B.



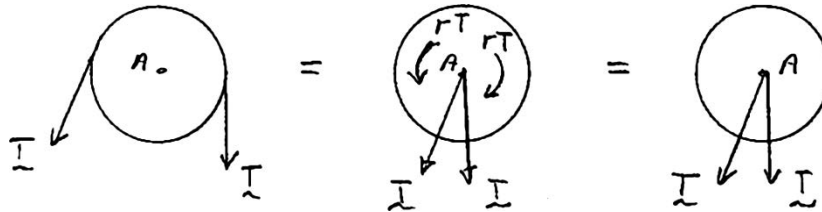
SOLUTION

First note that, when a cable or cord passes over a *frictionless, motionless* pulley, the tension is unchanged.

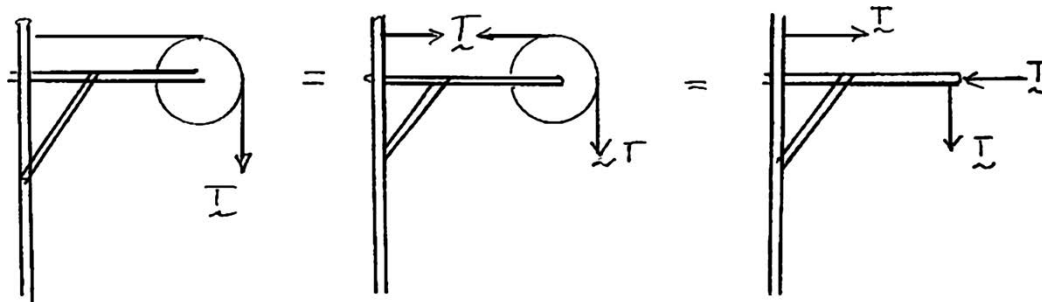
$$\left(+\Sigma M_C = 0: rT_1 - rT_2 = 0 \quad T_1 = T_2 \right)$$



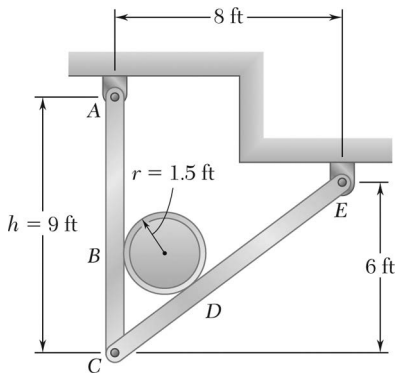
(a) Replace each force with an equivalent force-couple.



(b) Cut the cable and replace the forces on pulley with equivalent pair of forces at A as above.



PROBLEM 6.91

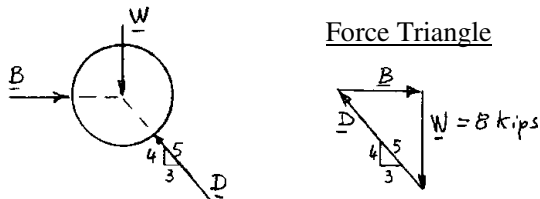


A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown. Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (a) of the reaction at E, (b) of the force exerted at C on member CDE.

SOLUTION

Free body: 16-ft length of pipe:

$$W = (500 \text{ lb/ft})(16 \text{ ft}) = 8 \text{ kips}$$



$$\frac{B}{3} = \frac{D}{5} = \frac{8 \text{ kips}}{4}$$

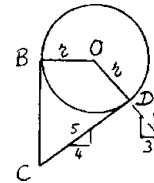
$$B = 6 \text{ kips} \quad D = 10 \text{ kips}$$

Determination of $CB = CD$.

We note that horizontal projection of $\overline{BO} + \overline{OD} =$ horizontal projection of \overline{CD}

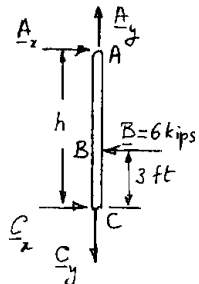
$$r + \frac{3}{5}r = \frac{4}{5}(CD)$$

$$CB = CD = \frac{8}{4}r = 2(1.5 \text{ ft}) = 3 \text{ ft}$$



Thus,

Free body: Member ABC:



$$+\circlearrowleft \Sigma M_A = 0: C_x h - (6 \text{ kips})(h - 3)$$

$$C_x = \frac{h - 3}{h}(6 \text{ kips}) \quad (1)$$

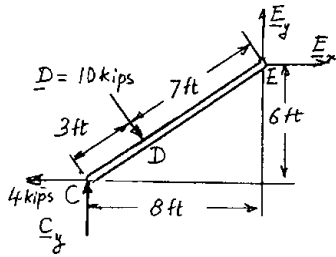
For $h = 9 \text{ ft}$,

$$C_x = \frac{9 - 3}{9}(6 \text{ kips}) = 4 \text{ kips}$$

PROBLEM 6.91 (Continued)

Free body: Member *CDE*:

From above, we have



$$+\curvearrowright \Sigma M_E = 0: (10 \text{ kips})(7 \text{ ft}) - (4 \text{ kips})(6 \text{ ft}) - C_y(8 \text{ ft}) = 0$$

$$C_y = +5.75 \text{ kips},$$

$$C_x = 4.00 \text{ kips} \leftarrow \blacktriangleleft$$

$$C_y = 5.75 \text{ kips} \uparrow \blacktriangleleft$$

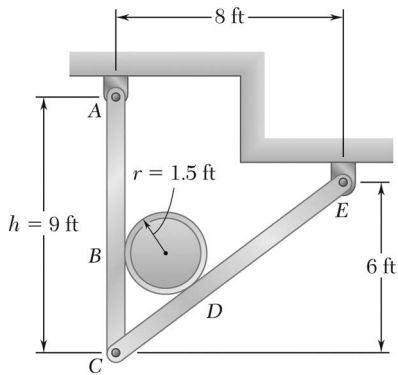
$$+\rightarrow \Sigma F_x = 0: -4 \text{ kips} + \frac{3}{5}(10 \text{ kips}) + E_x = 0$$

$$E_x = -2 \text{ kips},$$

$$E_x = 2.00 \text{ kips} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 5.75 \text{ kips} - \frac{4}{5}(10 \text{ kips}) + E_y = 0,$$

$$E_y = 2.25 \text{ kips} \uparrow \blacktriangleleft$$



PROBLEM 6.92

Solve Problem 6.91 for a frame where $h = 6$ ft.

PROBLEM 6.91 A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown. Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (a) of the reaction at E, (b) of the force exerted at C on member CDE.

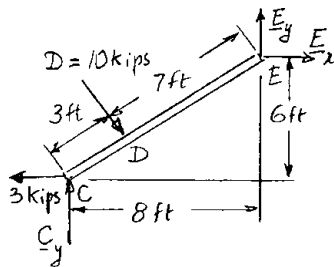
SOLUTION

See solution of Problem 6.91 for derivation of Eq. (1).

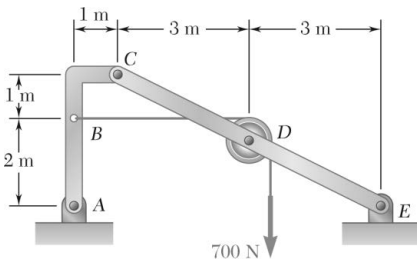
$$\text{For } h = 6 \text{ ft, } C_x = \frac{h-3}{h}(6 \text{ kips}) = \frac{6-3}{6} = 3 \text{ kips}$$

Free body: Member CDE:

From above, we have



$$\begin{aligned} C_x &= 3.00 \text{ kips } \leftarrow \blacktriangleleft \\ +\curvearrowright \Sigma M_E &= 0: (10 \text{ kips})(7 \text{ ft}) - (3 \text{ kips})(6 \text{ ft}) - C_y(8 \text{ ft}) = 0 \\ C_y &= +6.50 \text{ kips,} & C_y &= 6.50 \text{ kips } \uparrow \blacktriangleleft \\ +\rightarrow \Sigma F_x &= 0: -3 \text{ kips} + \frac{3}{5}(10 \text{ kips}) + E_x = 0 \\ E_x &= -3.00 \text{ kips,} & E_x &= 3.00 \text{ kips } \leftarrow \blacktriangleleft \\ +\uparrow \Sigma F_y &= 0: 6.5 \text{ kips} - \frac{4}{5}(10 \text{ kips}) + E_y = 0 \\ E_y &= 1.500 \text{ kips} & E_y &= 1.500 \text{ kips } \uparrow \blacktriangleleft \end{aligned}$$

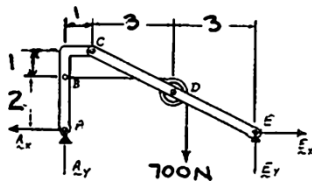


PROBLEM 6.93

Knowing that the pulley has a radius of 0.5 m, determine the components of the reactions at A and E.

SOLUTION

FBD Frame:



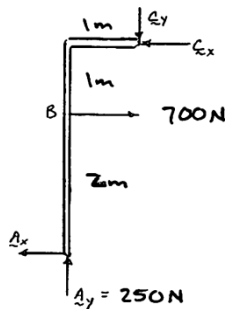
Dimensions in m

$$\curvearrowleft \Sigma M_A = 0: (7 \text{ m})E_y - (4.5 \text{ m})(700 \text{ N}) = 0 \quad E_y = 450 \text{ N} \uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: A_y - 700 \text{ N} + 450 \text{ N} = 0 \quad A_y = 250 \text{ N} \uparrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: A_x - E_x = 0 \quad A_x = E_x$$

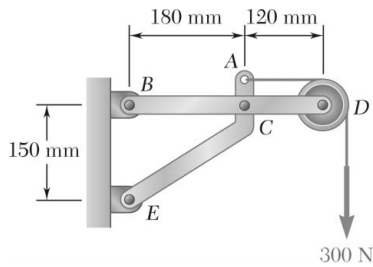
FBD Member ABC:



$$\curvearrowleft \Sigma M_C = 0: (1 \text{ m})(700 \text{ N}) - (1 \text{ m})(250 \text{ N}) - (3 \text{ m}) A_x = 0$$

$$A_x = 150.0 \text{ N} \leftarrow \blacktriangleleft$$

$$\text{so } E_x = 150.0 \text{ N} \rightarrow \blacktriangleleft$$

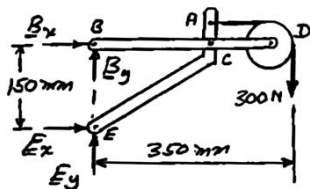


PROBLEM 6.94

Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at B and E .

SOLUTION

Free body: Entire assembly:



$$+\circlearrowleft \Sigma M_E = 0: -(300 \text{ N})(350 \text{ mm}) - B_x(150 \text{ mm}) = 0$$

$$B_x = -700 \text{ N} \quad \mathbf{B}_x = 700 \text{ N} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: -700 \text{ N} + E_x = 0$$

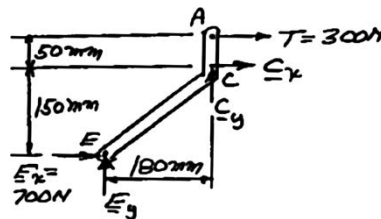
$$E_x = 700 \text{ N}$$

$$\mathbf{E}_x = 700 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: B_y + E_y - 300 \text{ N} = 0$$

(1)

Free body: Member ACE:



$$+\circlearrowleft \Sigma M_C = 0: (700 \text{ N})(150 \text{ mm}) - (300 \text{ N})(50 \text{ mm}) - E_y(180 \text{ mm}) = 0$$

$$E_y = 500 \text{ N}$$

$$\mathbf{E}_y = 500 \text{ N} \uparrow$$

From Eq. (1):

$$B_y + 500 \text{ N} - 300 \text{ N} = 0$$

$$B_y = -200 \text{ N}$$

$$\mathbf{B}_y = 200 \text{ N} \downarrow$$

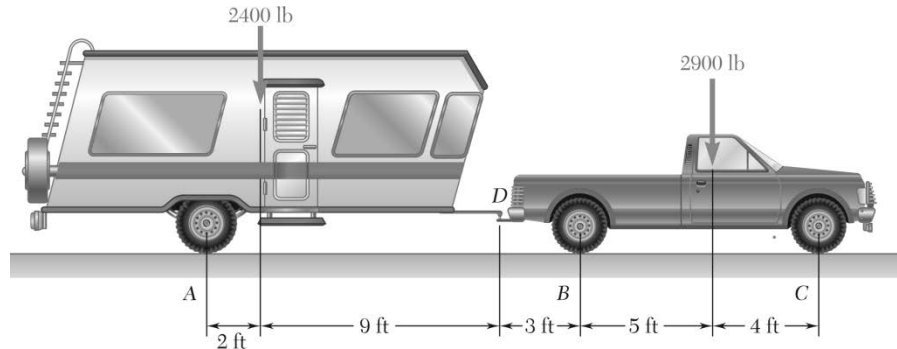
Thus, reactions are

$$\mathbf{B}_x = 700 \text{ N} \leftarrow, \quad \mathbf{B}_y = 200 \text{ N} \downarrow \blacktriangleleft$$

$$\mathbf{E}_x = 700 \text{ N} \rightarrow, \quad \mathbf{E}_y = 500 \text{ N} \uparrow \blacktriangleleft$$

PROBLEM 6.95

A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at D . Determine (a) the reactions at each of the six wheels when the truck and trailer are at rest, (b) the additional load on each of the truck wheels due to the trailer.



SOLUTION

(a) Free body: Trailer:

(We shall denote by A , B , C the reaction at one wheel.)

$$+\curvearrowright \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + D(11 \text{ ft}) = 0$$

$$D = 436.36 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 436.36 \text{ lb} = 0$$

$$A = 981.82 \text{ lb}$$

$$A = 982 \text{ lb} \quad \blacktriangleleft$$

Free body: Truck.

$$+\curvearrowright \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) - (2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = 732.83 \text{ lb}$$

$$C = 733 \text{ lb} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2900 \text{ lb} + 2(732.83 \text{ lb}) = 0$$

$$B = 935.35 \text{ lb}$$

$$B = 935 \text{ lb} \quad \blacktriangleleft$$

(b) Additional load on truck wheels.

Use free body diagram of truck without 2900 lb.

$$+\curvearrowright \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = -72.73 \text{ lb}$$

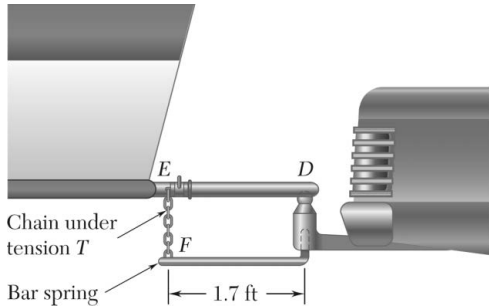
$$\Delta C = -72.7 \text{ lb} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2(72.73 \text{ lb}) = 0$$

$$B = 290.9 \text{ lb}$$

$$\Delta B = +291 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 6.96



In order to obtain a better weight distribution over the four wheels of the pickup truck of Problem 6.95, a compensating hitch of the type shown is used to attach the trailer to the truck. The hitch consists of two bar springs (only one is shown in the figure) that fit into bearings inside a support rigidly attached to the truck. The springs are also connected by chains to the trailer frame, and specially designed hooks make it possible to place both chains in tension. (a) Determine the tension T required in each of the two chains if the additional load due to the trailer is to be evenly distributed over the four wheels of the truck. (b) What are the resulting reactions at each of the six wheels of the trailer-truck combination?

PROBLEM 6.95 A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at D . Determine (a) the reactions at each of the six wheels when the truck and trailer are at rest, (b) the additional load on each of the truck wheels due to the trailer.

SOLUTION

(a) We shall first find the additional reaction Δ at each wheel due to the trailer.

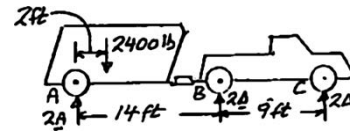
Free body diagram: (Same Δ at each truck wheel)

$$+\circlearrowleft \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + 2\Delta(14 \text{ ft}) + 2\Delta(23 \text{ ft}) = 0$$

$$\Delta = 64.86 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 4(64.86 \text{ lb}) = 0;$$

$$A = 1070 \text{ lb};$$



$$A = 1070 \text{ lb} \uparrow$$

Free body: Truck:

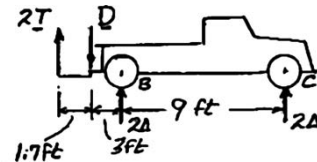
(Trailer loading only)

$$+\circlearrowleft \Sigma M_D = 0: \quad 2\Delta(12 \text{ ft}) + 2\Delta(3 \text{ ft}) - 2T(1.7 \text{ ft}) = 0$$

$$T = 8.824\Delta$$

$$= 8.824(64.86 \text{ lb})$$

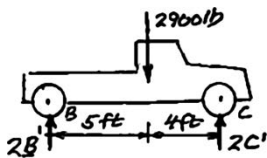
$$T = 572.3 \text{ lb}$$



$$T = 572 \text{ lb} \blacktriangleleft$$

Free body: Truck:

(Truck weight only)



$$+\circlearrowleft \Sigma M_B = 0: \quad -(2900 \text{ lb})(5 \text{ ft}) + 2C'(9 \text{ ft}) = 0$$

$$C' = 805.6 \text{ lb}$$

$$C' = 805.6 \text{ lb} \uparrow$$

PROBLEM 6.96 (Continued)

$$+\uparrow \Sigma F_y = 0: \quad 2B' - 2900 \text{ lb} + 2(805.6 \text{ lb}) = 0$$

$$B' = 644.4 \text{ lb}$$

$$\mathbf{B}' = 644.4 \text{ lb } \uparrow$$

Actual reactions:

$$B = B' + \Delta = 644.4 \text{ lb} + 64.86 = 709.2 \text{ lb}$$

$$\mathbf{B} = 709 \text{ lb } \uparrow \blacktriangleleft$$

$$C = C' + \Delta = 805.6 \text{ lb} + 64.86 = 870.46 \text{ lb}$$

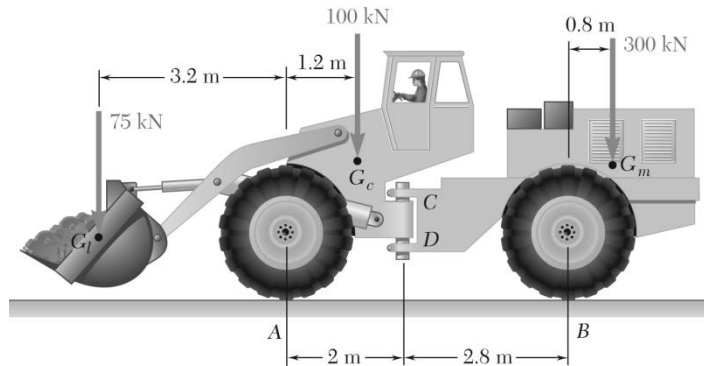
$$\mathbf{C} = 870 \text{ lb } \uparrow \blacktriangleleft$$

From part *a*:

$$\mathbf{A} = 1070 \text{ lb } \uparrow \blacktriangleleft$$

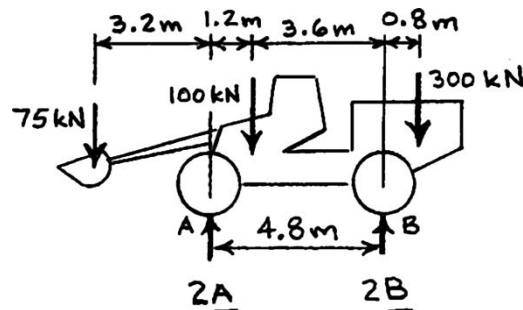
PROBLEM 6.97

The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m , while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l . Knowing that the machine is at rest with its brakes released, determine (a) the reactions at each of the four wheels, (b) the forces exerted on the motor unit at C and D .



SOLUTION

(a) Free body: Entire machine:



A = Reaction at each front wheel

B = Reaction at each rear wheel

$$+\curvearrowright \Sigma M_A = 0: \quad 75(3.2 \text{ m}) - 100(1.2 \text{ m}) + 2B(4.8 \text{ m}) - 300(5.6 \text{ m}) = 0$$

$$2B = 325 \text{ kN}$$

$$B = 162.5 \text{ kN} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 325 - 75 - 100 - 300 = 0$$

$$2A = 150 \text{ kN}$$

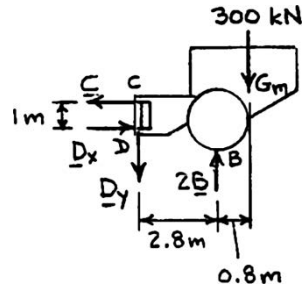
$$A = 75.0 \text{ kN} \uparrow \blacktriangleleft$$

PROBLEM 6.97 (Continued)

(b) Free body: Motor unit:

$$+\curvearrowright \Sigma M_D = 0: C(1 \text{ m}) + 2B(2.8 \text{ m}) - 300(3.6 \text{ m}) = 0$$

$$C = 1080 - 5.6B \quad (1)$$



Recalling $B = 162.5 \text{ kN}$, $C = 1080 - 5.6(162.5) = 170 \text{ kN}$

$$C = 170.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: D_x - 170 = 0$$

$$D_x = 170.0 \text{ kN} \rightarrow \blacktriangleleft$$

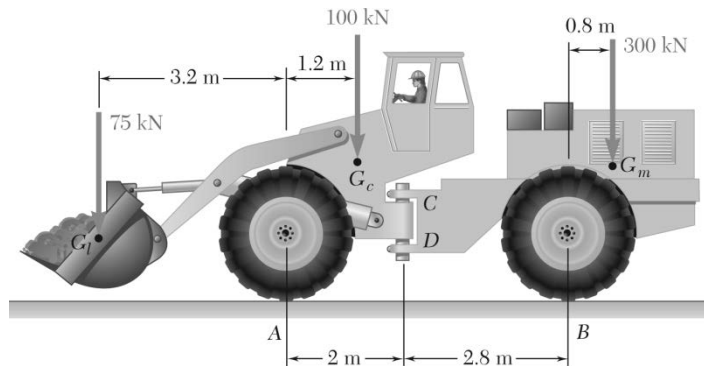
$$+\uparrow \Sigma F_y = 0: 2(162.5) - D_y - 300 = 0$$

$$D_y = 25.0 \text{ kN} \downarrow \blacktriangleleft$$

PROBLEM 6.98

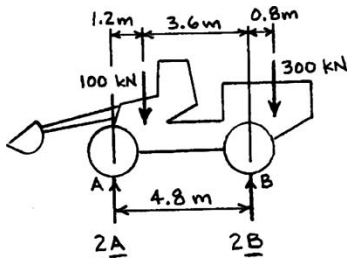
Solve Problem 6.97 assuming that the 75-kN load has been removed.

PROBLEM 6.97 The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m , while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l . Knowing that the machine is at rest with its brakes released, determine (a) the reactions at each of the four wheels, (b) the forces exerted on the motor unit at C and D .



SOLUTION

(a) Free body: Entire machine:



A = Reaction at each front wheel

B = Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: 2B(4.8 \text{ m}) - 100(1.2 \text{ m}) - 300(5.6 \text{ m}) = 0$$

$$2B = 375 \text{ kN}$$

$$B = 187.5 \text{ kN} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 2A + 375 - 100 - 300 = 0$$

$$2A = 25 \text{ kN}$$

$$A = 12.50 \text{ kN} \uparrow \blacktriangleleft$$

(b) Free body: Motor unit:

See solution of Problem 6.97 for free body diagram and derivation of Eq. (1). With $B = 187.5 \text{ kN}$, we have

$$C = 1080 - 5.6(187.5) = 30 \text{ kN}$$

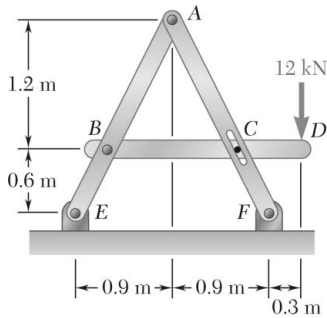
$$C = 30.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: D_x - 30 = 0$$

$$D_x = 30.0 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 2(187.5) - D_y - 300 = 0$$

$$D_y = 75.0 \text{ kN} \downarrow \blacktriangleleft$$



PROBLEM 6.99

For the frame and loading shown, determine the components of all forces acting on member ABE.

SOLUTION

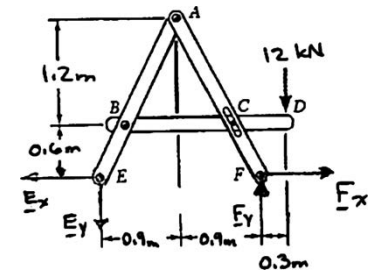
FBD Frame:

$$\left(\sum M_E = 0: (1.8 \text{ m})F_y - (2.1 \text{ m})(12 \text{ kN}) = 0\right.$$

$$F_y = 14.00 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0: -E_y + 14.00 \text{ kN} - 12 \text{ kN} = 0$$

$$E_y = 2 \text{ kN} \downarrow$$



$$E_y = 2.00 \text{ kN} \downarrow \blacktriangleleft$$

FBD member BCD:

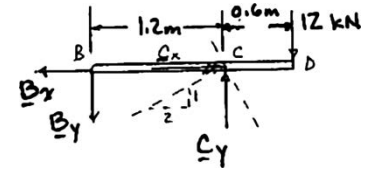
$$\left(\sum M_B = 0: (1.2 \text{ m})C_y - (12 \text{ kN})(1.8 \text{ m}) = 0 \quad C_y = 18.00 \text{ kN} \uparrow\right.$$

But C is \perp ACF, so $C_x = 2C_y$; $C_x = 36.0 \text{ kN} \rightarrow$

$$+\rightarrow \sum F_x = 0: -B_x + C_x = 0 \quad B_x = C_x = 36.0 \text{ kN}$$

$$B_x = 36.0 \text{ kN} \leftarrow \text{ on BCD}$$

$$+\uparrow \sum F_y = 0: -B_y + 18.00 \text{ kN} - 12 \text{ kN} = 0 \quad B_y = 6.00 \text{ kN} \downarrow \text{ on BCD}$$

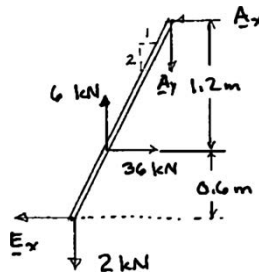


On ABE:

$$B_x = 36.0 \text{ kN} \rightarrow \blacktriangleleft$$

$$B_y = 6.00 \text{ kN} \uparrow \blacktriangleleft$$

FBD member ABE:



$$\left(\sum M_A = 0: (1.2 \text{ m})(36.0 \text{ kN}) - (0.6 \text{ m})(6.00 \text{ kN})\right. \\ \left.+ (0.9 \text{ m})(2.00 \text{ kN}) - (1.8 \text{ m})(E_x) = 0\right.$$

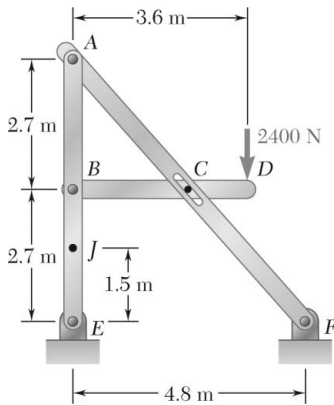
$$E_x = 23.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\rightarrow \sum F_x = 0: -23.0 \text{ kN} + 36.0 \text{ kN} - A_x = 0$$

$$A_x = 13.00 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \sum F_y = 0: -2.00 \text{ kN} + 6.00 \text{ kN} - A_y = 0$$

$$A_y = 4.00 \text{ kN} \downarrow \blacktriangleleft$$



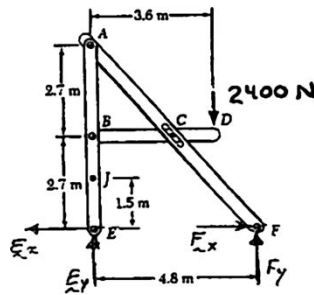
PROBLEM 6.100

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

PROBLEM 6.99 For the frame and loading shown, determine the components of all forces acting on member *ABE*.

SOLUTION

FBD Frame:



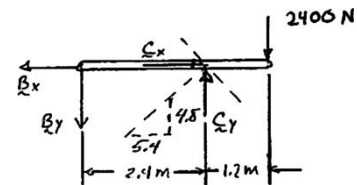
$$(+\Sigma M_F = 0: (1.2 \text{ m})(2400 \text{ N}) - (4.8 \text{ m})E_y = 0$$

$$E_y = 600 \text{ N} \uparrow \blacktriangleleft$$

FBD member BC:

$$C_y = \frac{4.8}{5.4} C_x = \frac{8}{9} C_x$$

$$(+\Sigma M_C = 0: (2.4 \text{ m})B_y - (1.2 \text{ m})(2400 \text{ N}) = 0 \quad B_y = 1200 \text{ N} \downarrow$$



$$B_y = 1200 \text{ N} \uparrow \blacktriangleleft$$

On *ABE*:

$$+\uparrow \Sigma F_y = 0: -1200 \text{ N} + C_y - 2400 \text{ N} = 0 \quad C_y = 3600 \text{ N} \uparrow$$

so

$$C_x = \frac{9}{8} C_y \quad C_x = 4050 \text{ N} \rightarrow$$

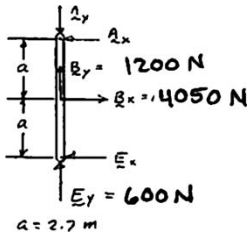
$$+\rightarrow \Sigma F_x = 0: -B_x + C_x = 0 \quad B_x = 4050 \text{ N} \leftarrow \text{ on BC}$$

On *ABE*:

$$B_x = 4050 \text{ N} \rightarrow \blacktriangleleft$$

PROBLEM 6.100 (Continued)

FBD member ABE:



$$\left(+ \sum M_A = 0: a(4050 \text{ N}) - 2aE_x = 0 \right.$$

$$E_x = 2025 \text{ N}$$

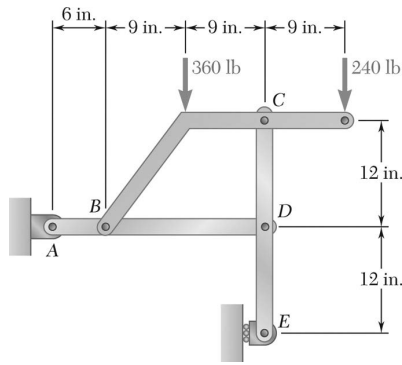
$$E_x = 2025 \text{ N} \leftarrow \blacktriangleleft$$

$$\left. + \rightarrow \sum F_x = 0: -A_x + (4050 - 2025) \text{ N} = 0 \right.$$

$$A_x = 2025 \text{ N} \leftarrow \blacktriangleleft$$

$$+ \uparrow \sum F_y = 0: 600 \text{ N} + 1200 \text{ N} - A_y = 0$$

$$A_y = 1800 \text{ N} \downarrow \blacktriangleleft$$

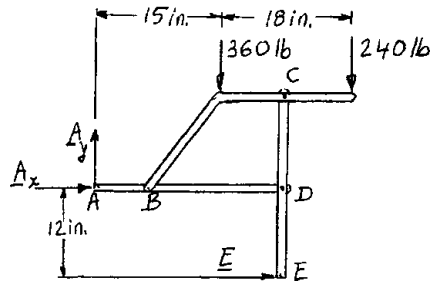


PROBLEM 6.101

For the frame and loading shown, determine the components of all forces acting on member ABD .

SOLUTION

Free body: Entire frame:



$$+\curvearrowright \Sigma M_A = 0: E(24 \text{ in.}) - (360 \text{ lb})(15 \text{ in.}) - (240 \text{ lb})(33 \text{ in.}) = 0$$

$$E = +1110 \text{ lb}$$

$$E = +1110 \text{ lb} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: A_x + 1110 \text{ lb} = 0$$

$$A_x = -1110 \text{ lb}$$

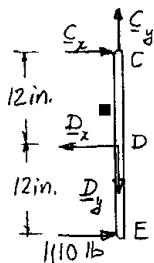
$$A_x = 1110 \text{ lb} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A_y - 360 \text{ lb} - 240 \text{ lb} = 0$$

$$A_y = +600 \text{ lb}$$

$$A_y = 600 \text{ lb} \uparrow \blacktriangleleft$$

Free body: Member CDE :



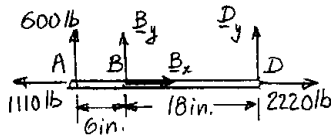
$$+\curvearrowright \Sigma M_C = 0: (1110 \text{ lb})(24 \text{ in.}) - D_x(12 \text{ in.}) = 0$$

$$D_x = +2220 \text{ lb}$$

PROBLEM 6.101 (Continued)

Free body: Member ABD:

From above:



$$+\curvearrowright \Sigma M_B = 0: D_y(18 \text{ in.}) - (600 \text{ lb})(6 \text{ in.}) = 0$$

$$D_y = +200 \text{ lb}$$

$$\mathbf{D}_y = 200 \text{ lb} \uparrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: B_x + 2220 \text{ lb} - 1110 \text{ lb} = 0$$

$$B_x = -1110 \text{ lb}$$

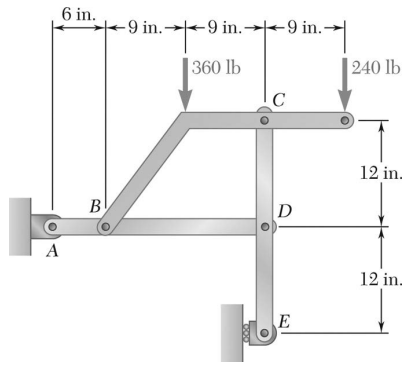
$$\mathbf{B}_x = 1110 \text{ lb} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + 200 \text{ lb} + 600 \text{ lb} = 0$$

$$B_y = -800 \text{ lb}$$

$$\mathbf{B}_y = 800 \text{ lb} \downarrow \blacktriangleleft$$

$$\mathbf{D}_x = 2220 \text{ lb} \rightarrow \blacktriangleleft$$



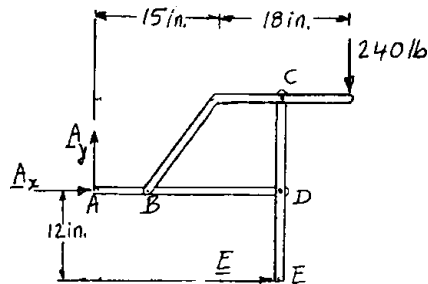
PROBLEM 6.102

Solve Problem 6.101 assuming that the 360-lb load has been removed.

PROBLEM 6.101 For the frame and loading shown, determine the components of all forces acting on member ABD .

SOLUTION

Free body diagram of entire frame.



$$+\curvearrowright \Sigma M_A = 0: E(12 \text{ in.}) - (240 \text{ lb})(33 \text{ in.}) = 0$$

$$E = +660 \text{ lb}$$

$$E = 660 \text{ lb} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: A_x + 660 \text{ lb} = 0$$

$$A_x = -660 \text{ lb}$$

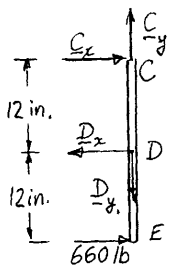
$$A_x = 660 \text{ lb} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A_y - 240 \text{ lb} = 0$$

$$A_y = +240 \text{ lb}$$

$$A_y = 240 \text{ lb} \uparrow \blacktriangleleft$$

Free body: Member CDE:



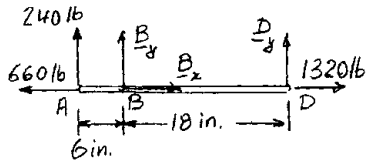
$$+\curvearrowright \Sigma M_C = 0: (660 \text{ lb})(24 \text{ in.}) - D_x(12 \text{ in.}) = 0$$

$$D_x = +1320 \text{ lb}$$

PROBLEM 6.102 (Continued)

Free body: Member ABD:

From above:



$$+\curvearrowright \Sigma M_B = 0: D_y(18 \text{ in.}) - (240 \text{ lb})(6 \text{ in.}) = 0$$

$$D_y = +80 \text{ lb}$$

$$\mathbf{D}_x = 1320 \text{ lb} \rightarrow \blacktriangleleft$$

$$\mathbf{D}_y = 80.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: B_x + 1320 \text{ lb} - 660 \text{ lb} = 0$$

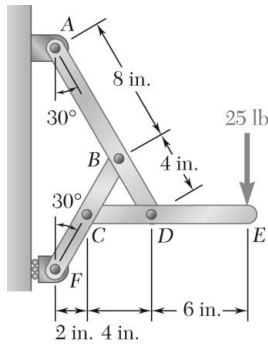
$$B_x = -660 \text{ lb}$$

$$\mathbf{B}_x = 660 \text{ lb} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + 80 \text{ lb} + 240 \text{ lb} = 0$$

$$B_y = -320 \text{ lb}$$

$$\mathbf{B}_y = 320 \text{ lb} \downarrow \blacktriangleleft$$

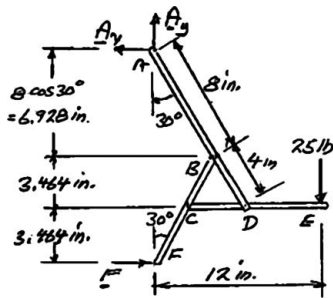


PROBLEM 6.103

For the frame and loading shown, determine the components of the forces acting on member CDE at C and D .

SOLUTION

Free body: Entire frame:



$$+\uparrow \Sigma M_y = 0: A_y - 25 \text{ lb} = 0$$

$$A_y = 25 \text{ lb}$$

$$A_y = 25 \text{ lb} \uparrow$$

$$+\curvearrowright \Sigma M_F = 0: A_x(6.928 + 2 \times 3.464) - (25 \text{ lb})(12 \text{ in.}) = 0$$

$$A_x = 21.651 \text{ lb}$$

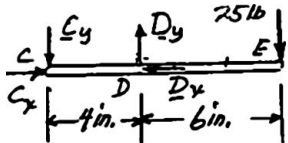
$$A_x = 21.65 \text{ lb} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: F - 21.651 \text{ lb} = 0$$

$$F = 21.651 \text{ lb}$$

$$F = 21.65 \text{ lb} \rightarrow$$

Free body: Member CDE :



$$+\curvearrowright \Sigma M_C = 0: D_y(4 \text{ in.}) - (25 \text{ lb})(10 \text{ in.}) = 0$$

$$D_y = +62.5 \text{ lb}$$

$$D_y = 62.5 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -C_y + 62.5 \text{ lb} - 25 \text{ lb} = 0$$

$$C_y = +37.5 \text{ lb}$$

$$C_y = 37.5 \text{ lb} \downarrow \blacktriangleleft$$

Free body: Member ABD :

$$+\curvearrowright \Sigma M_B = 0: D_x(3.464 \text{ in.}) + (21.65 \text{ lb})(6.928 \text{ in.})$$

$$- (25 \text{ lb})(4 \text{ in.}) - (62.5 \text{ lb})(2 \text{ in.}) = 0$$

$$D_x = +21.65 \text{ lb}$$

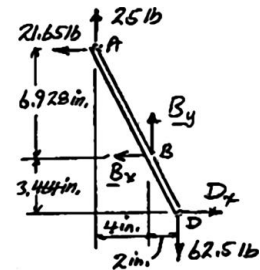
Return to free body: Member CDE :

From above:

$$D_x = +21.65 \text{ lb}$$

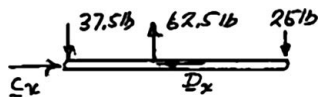
$$+\rightarrow \Sigma F_x = 0: C_x - 21.65 \text{ lb}$$

$$C_x = +21.65 \text{ lb}$$

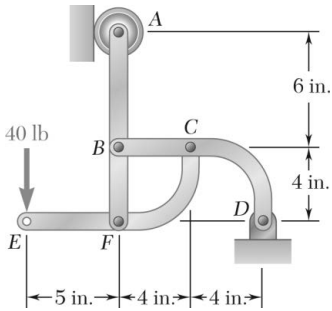


$$D_x = 21.7 \text{ lb} \leftarrow \blacktriangleleft$$

$$C_x = 21.7 \text{ lb} \rightarrow \blacktriangleleft$$



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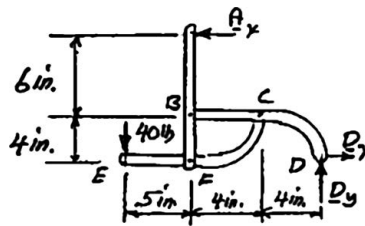


PROBLEM 6.104

For the frame and loading shown, determine the components of the forces acting on member CFE at C and F .

SOLUTION

Free body: Entire frame:



$$+\curvearrowright \Sigma M_D = 0: (40 \text{ lb})(13 \text{ in.}) + A_x(10 \text{ in.}) = 0$$

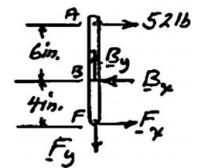
$$A_x = -52 \text{ lb,}$$

$$A_x = 52 \text{ lb} \rightarrow$$

Free body: Member ABF :

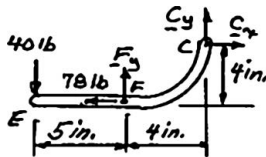
$$+\curvearrowright \Sigma M_B = 0: -(52 \text{ lb})(6 \text{ in.}) + F_x(4 \text{ in.}) = 0$$

$$F_x = +78 \text{ lb}$$



Free body: Member CFE :

From above:



$$+\curvearrowright \Sigma M_C = 0: (40 \text{ lb})(9 \text{ in.}) - (78 \text{ lb})(4 \text{ in.}) - F_y(4 \text{ in.}) = 0$$

$$F_y = +12 \text{ lb}$$

$$F_y = 12.00 \text{ lb} \uparrow \blacktriangleleft$$

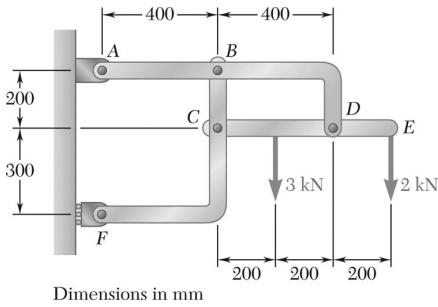
$$+\rightarrow \Sigma F_x = 0: C_x - 78 \text{ lb} = 0$$

$$C_x = +78 \text{ lb}$$

$$C_x = 78.0 \text{ lb} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -40 \text{ lb} + 12 \text{ lb} + C_y = 0; \quad C_y = +28 \text{ lb}$$

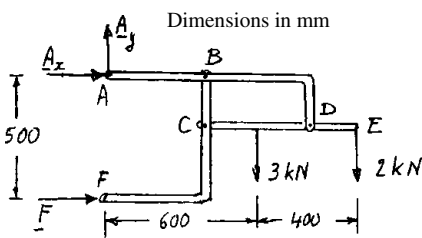
$$C_y = 28.0 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 6.105

For the frame and loading shown, determine the components of all forces acting on member ABD .

SOLUTION



Free body: Entire frame:

$$+\curvearrowright \Sigma M_A = 0: F(500) - (3 \text{ kN})(600) - (2 \text{ kN})(1000) = 0$$

$$F = +7.60 \text{ kN} \quad \mathbf{F} = 7.60 \text{ kN} \rightarrow$$

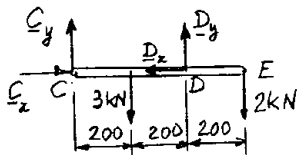
$$+\rightarrow \Sigma F_x = 0: A_x + 7.60 \text{ N} = 0,$$

$$A_x = -7.60 \text{ N} \quad \mathbf{A}_x = 7.60 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A_y - 3 \text{ kN} - 2 \text{ kN} = 0$$

$$A_y = +5 \text{ kN} \quad \mathbf{A}_y = 5.00 \text{ kN} \uparrow \blacktriangleleft$$

Free body: Member CDE:



$$+\curvearrowright \Sigma M_C = 0: D_y(400) - (3 \text{ kN})(200) - (2 \text{ kN})(600) = 0$$

$$D_y = +4.50 \text{ kN}$$

Free body: Member ABD:

From above: $\mathbf{D}_y = 4.50 \text{ kN} \downarrow \blacktriangleleft$

$$+\curvearrowright \Sigma M_B = 0: D_x(200) - (4.50 \text{ kN})(400) - (5 \text{ kN})(400) = 0$$

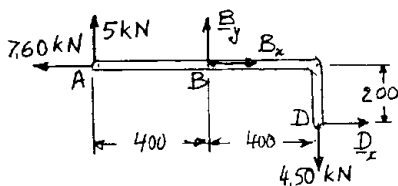
$$D_x = +19 \text{ kN} \quad \mathbf{D}_x = 19.00 \text{ kN} \rightarrow \blacktriangleleft$$

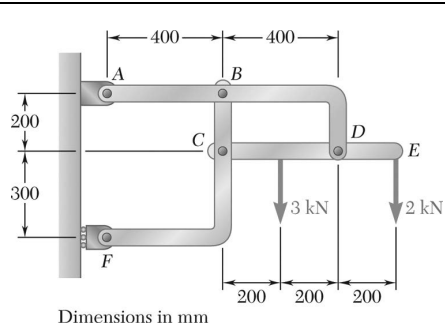
$$+\rightarrow \Sigma F_x = 0: B_x + 19 \text{ kN} - 7.60 \text{ kN} = 0$$

$$B_x = -11.40 \text{ kN} \quad \mathbf{B}_x = 11.40 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + 5 \text{ kN} - 4.50 \text{ kN} = 0$$

$$B_y = -0.50 \text{ kN} \quad \mathbf{B}_y = 0.500 \text{ kN} \downarrow \blacktriangleleft$$



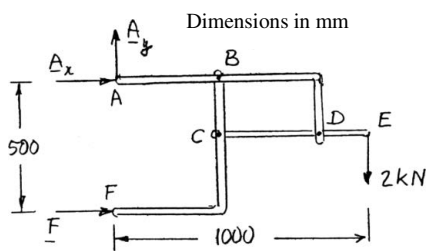


PROBLEM 6.106

Solve Problem 6.105 assuming that the 3-kN load has been removed.

PROBLEM 6.105 For the frame and loading shown, determine the components of all forces acting on member ABD .

SOLUTION



Free body: Entire frame:

$$+\curvearrowright \Sigma M_A = 0: F(500) - (2 \text{ kN})(1000) = 0$$

$$F = +4 \text{ kN}$$

$$\mathbf{F} = 4.00 \text{ kN} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: A_x + 4 \text{ kN} = 0,$$

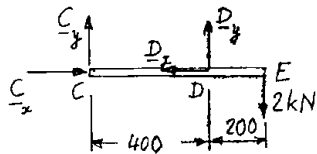
$$A_x = -4 \text{ kN}$$

$$\mathbf{A}_x = 4.00 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: A_y - 2 \text{ kN} = 0,$$

$$A_y = +2 \text{ kN}$$

$$\mathbf{A}_y = 2.00 \text{ kN} \uparrow \blacktriangleleft$$



Free body: Member CDE:

$$+\curvearrowright \Sigma M_C = 0: D_y(400) - (2 \text{ kN})(600) = 0$$

$$D_y = +3.00 \text{ kN}$$

Free body: Member ABD:

From above:

$$\mathbf{D}_y = 3.00 \text{ kN} \downarrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_B = 0: D_x(200) - (3 \text{ kN})(400) - (2 \text{ kN})(400) = 0$$

$$D_x = +10.00 \text{ kN}$$

$$\mathbf{D}_x = 10.00 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: B_x + 10 \text{ kN} - 4 \text{ kN} = 0$$

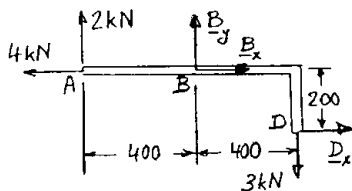
$$B_x = -6 \text{ kN}$$

$$\mathbf{B}_x = 6.00 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + 2 \text{ kN} - 3 \text{ kN} = 0$$

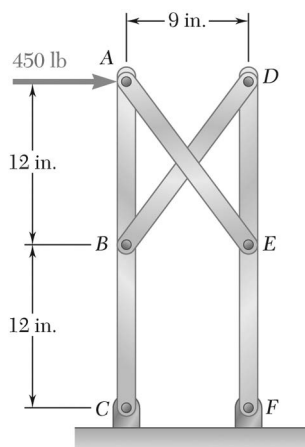
$$B_y = +1 \text{ kN}$$

$$\mathbf{B}_y = 1.000 \text{ kN} \uparrow \blacktriangleleft$$



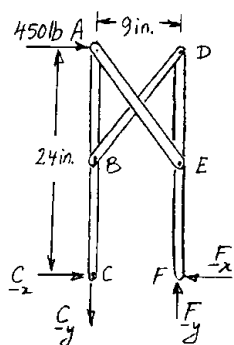
PROBLEM 6.107

Determine the reaction at F and the force in members AE and BD .



SOLUTION

Free body: Entire frame:

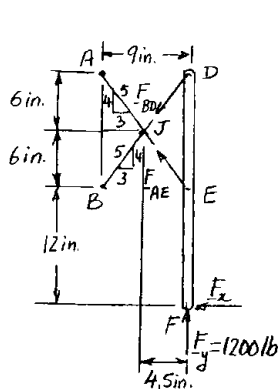


$$+\circlearrowleft \Sigma M_C = 0: F_y(9 \text{ in.}) - (450 \text{ lb})(24 \text{ in.}) = 0$$

$$F_y = 1200 \text{ lb}$$

$$F_y = 1200 \text{ lb } \uparrow$$

Free body: Member DEF :



$$+\circlearrowleft \Sigma M_J = 0: (1200 \text{ lb})(4.5 \text{ in.}) - F_x(18 \text{ in.}) = 0$$

$$F_x = 300 \text{ lb}$$

$$F_x = 300 \text{ lb } \leftarrow$$

$$+\circlearrowleft \Sigma M_D = 0: -F_x(24 \text{ in.}) - \left(\frac{3}{5}F_{AE}\right)(12 \text{ in.}) = 0$$

$$F_{AE} = -\frac{10}{3}F_x = -\frac{10}{3}(300 \text{ lb})$$

$$F_{AE} = -1000 \text{ lb}$$

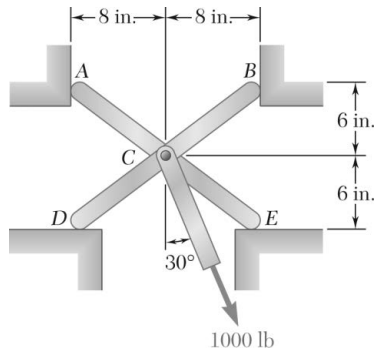
$$F_{AE} = 1000 \text{ lb } \leftarrow$$

$$+\uparrow \Sigma F_y = 0: 1200 \text{ lb} + \frac{4}{5}(-1000 \text{ lb}) - \frac{4}{5}F_{BD} = 0$$

$$F_{BD} = \frac{5}{4}(1200 \text{ lb}) - 1000 \text{ lb} = +500 \text{ lb}$$

$$F_{BD} = 500 \text{ lb } \leftarrow$$

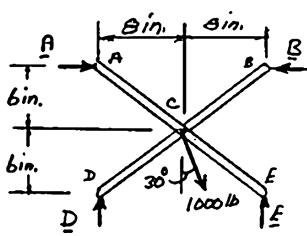
PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.



PROBLEM 6.108

For the frame and loading shown, determine the reactions at A, B, D, and E. Assume that the surface at each support is frictionless.

SOLUTION



Free body: Entire frame:

$$\begin{aligned} \rightarrow \Sigma F_x = 0: \quad A - B + (1000 \text{ lb}) \sin 30^\circ &= 0 \\ A - B + 500 &= 0 \end{aligned} \quad (1)$$

$$\begin{aligned} + \uparrow \Sigma F_y = 0: \quad D + E - (1000 \text{ lb}) \cos 30^\circ &= 0 \\ D + E - 866.03 &= 0 \end{aligned} \quad (2)$$

Free body: Member ACE:

$$\begin{aligned} + \curvearrowright \Sigma M_C = 0: \quad -A(6 \text{ in.}) + E(8 \text{ in.}) &= 0 \\ E &= \frac{3}{4}A \end{aligned} \quad (3)$$

Free body: Member BCD:

$$\begin{aligned} + \curvearrowright \Sigma M_C = 0: \quad -D(8 \text{ in.}) + B(6 \text{ in.}) &= 0 \\ D &= \frac{3}{4}B \end{aligned} \quad (4)$$

Substitute E and D from Eqs. (3) and (4) into Eq. (2):

$$\begin{aligned} -\frac{3}{4}A + \frac{3}{4}B - 866.06 &= 0 \\ A + B - 1154.71 &= 0 \end{aligned} \quad (5)$$

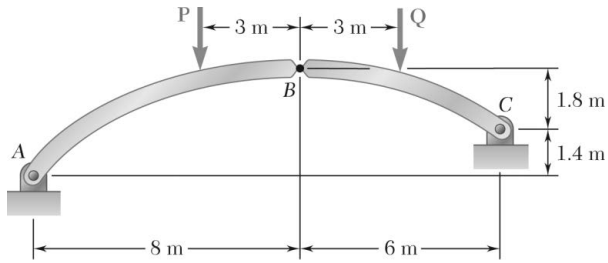
$$\text{From Eq. (1):} \quad A - B + 500 = 0 \quad (6)$$

$$\text{Eqs. (5) + (6):} \quad 2A - 654.71 = 0 \quad A = 327.4 \text{ lb} \quad \mathbf{A} = 327 \text{ lb} \rightarrow \blacktriangleleft$$

$$\text{Eqs. (5) - (6):} \quad 2B - 1654.71 = 0 \quad B = 827.4 \text{ lb} \quad \mathbf{B} = 827 \text{ lb} \leftarrow \blacktriangleleft$$

$$\text{From Eq. (4):} \quad D = \frac{3}{4}(827.4) \quad D = 620.5 \text{ lb} \quad \mathbf{D} = 621 \text{ lb} \uparrow \blacktriangleleft$$

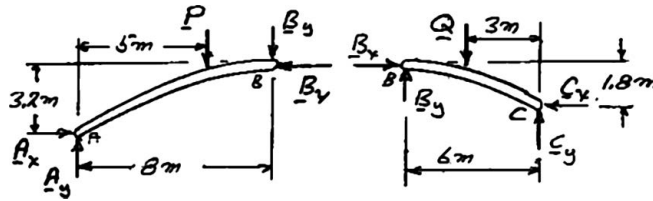
$$\text{From Eq. (3):} \quad E = \frac{3}{4}(327.4) \quad E = 245.5 \text{ lb} \quad \mathbf{E} = 246 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 6.109

The axis of the three-hinge arch ABC is a parabola with the vertex at B . Knowing that $P = 112$ kN and $Q = 140$ kN, determine (a) the components of the reaction at A , (b) the components of the force exerted at B on segment AB .

SOLUTION



Free body: Segment AB :

$$+\circlearrowleft \sum M_A = 0: B_x(3.2 \text{ m}) - B_y(8 \text{ m}) - P(5 \text{ m}) = 0 \quad (1)$$

$$0.75 \text{ (Eq. 1): } B_x(2.4 \text{ m}) - B_y(6 \text{ m}) - P(3.75 \text{ m}) = 0 \quad (2)$$

Free body: Segment BC :

$$+\circlearrowleft \sum M_C = 0: B_x(1.8 \text{ m}) + B_y(6 \text{ m}) - Q(3 \text{ m}) = 0 \quad (3)$$

$$\text{Add Eqs. (2) and (3): } 4.2B_x - 3.75P - 3Q = 0$$

$$B_x = (3.75P + 3Q)/4.2 \quad (4)$$

$$\text{From Eq. (1): } (3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$$

$$B_y = (-9P + 9.6Q)/33.6 \quad (5)$$

given that $P = 112$ kN and $Q = 140$ kN.

(a) Reaction at A .

Considering again AB as a free body,

$$+\rightarrow \sum F_x = 0: A_x - B_x = 0; \quad A_x = B_x = 200 \text{ kN} \quad A_x = 200 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \sum F_y = 0: A_y - P - B_y = 0$$

$$A_y - 112 \text{ kN} - 10 \text{ kN} = 0$$

$$A_y = +122 \text{ kN} \quad A_y = 122.0 \text{ kN} \uparrow \blacktriangleleft$$

PROBLEM 6.109 (Continued)

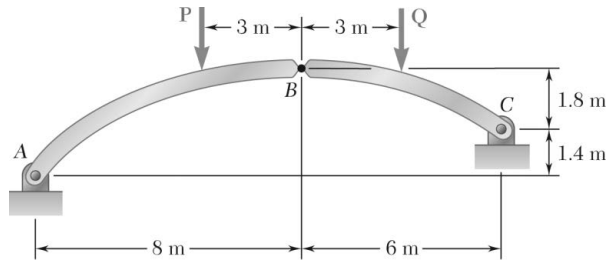
(b) Force exerted at B on AB .

From Eq. (4): $B_x = (3.75 \times 112 + 3 \times 140) / 4.2 = 200 \text{ kN}$

$B_x = 200 \text{ kN} \leftarrow \blacktriangleleft$

From Eq. (5): $B_y = (-9 \times 112 + 9.6 \times 140) / 33.6 = +10 \text{ kN}$

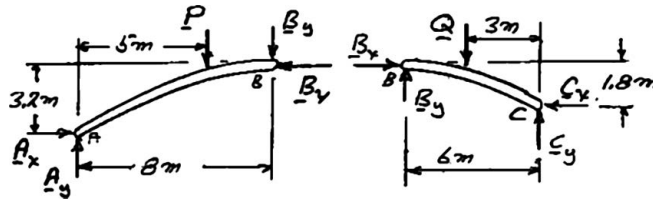
$B_y = 10.00 \text{ kN} \downarrow \blacktriangleleft$



PROBLEM 6.110

The axis of the three-hinge arch ABC is a parabola with the vertex at B . Knowing that $P = 140$ kN and $Q = 112$ kN, determine (a) the components of the reaction at A , (b) the components of the force exerted at B on segment AB .

SOLUTION



Free body: Segment AB :

$$+\circlearrowleft \sum M_A = 0: B_x(3.2 \text{ m}) - B_y(8 \text{ m}) - P(5 \text{ m}) = 0 \quad (1)$$

$$0.75 \text{ (Eq. 1): } B_x(2.4 \text{ m}) - B_y(6 \text{ m}) - P(3.75 \text{ m}) = 0 \quad (2)$$

Free body: Segment BC :

$$+\circlearrowleft \sum M_C = 0: B_x(1.8 \text{ m}) + B_y(6 \text{ m}) - Q(3 \text{ m}) = 0 \quad (3)$$

$$\text{Add Eqs. (2) and (3): } 4.2B_x - 3.75P - 3Q = 0$$

$$B_x = (3.75P + 3Q)/4.2 \quad (4)$$

$$\text{From Eq. (1): } (3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$$

$$B_y = (-9P + 9.6Q)/33.6 \quad (5)$$

given that $P = 140$ kN and $Q = 112$ kN.

(a) Reaction at A .

$$+\rightarrow \sum F_x = 0: A_x - B_x = 0; \quad A_x = B_x = 205 \text{ kN} \quad \mathbf{A_x = 205 \text{ kN} \rightarrow \blacktriangleleft}$$

$$+\uparrow \sum F_y = 0: A_y - P - B_y = 0$$

$$A_y - 140 \text{ kN} - (-5.5 \text{ kN}) = 0$$

$$\mathbf{A_y = 134.5 \text{ kN} \uparrow \blacktriangleleft}$$

PROBLEM 6.110 (Continued)

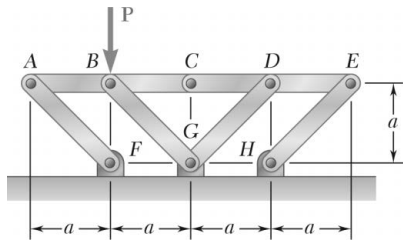
(b) Force exerted at B on AB.

From Eq. (4): $B_x = (3.75 \times 140 + 3 \times 112) / 4.2 = 205 \text{ kN}$

$B_x = 205 \text{ kN} \leftarrow \blacktriangleleft$

From Eq. (5): $B_y = (-9 \times 140 + 9.6 \times 112) / 33.6 = -5.5 \text{ kN}$

$B_y = 5.50 \text{ kN} \uparrow \blacktriangleleft$

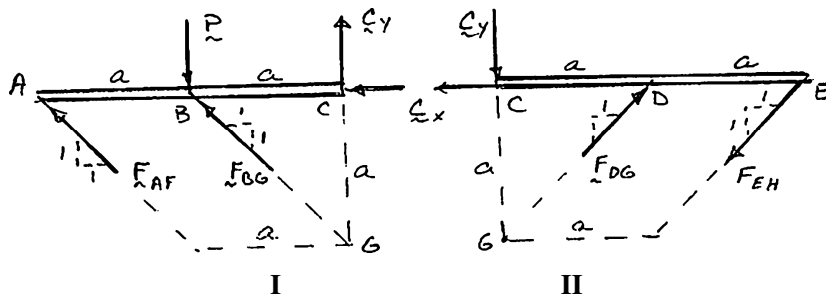


PROBLEM 6.111

Members ABC and CDE are pin-connected at C and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



FBD I: $\left(\sum M_B = 0: aC_y - a\frac{1}{\sqrt{2}}F_{AF} = 0 \quad F_{AF} = \sqrt{2}C_y\right)$

FBD II: $\left(\sum M_D = 0: aC_y - a\frac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{EH} = \sqrt{2}C_y\right)$

FBDs combined: $\left(\sum M_G = 0: aP - a\frac{1}{\sqrt{2}}F_{AF} - a\frac{1}{\sqrt{2}}F_{EH} = 0 \quad P = \frac{1}{\sqrt{2}}\sqrt{2}C_y + \frac{1}{\sqrt{2}}\sqrt{2}C_y\right)$

$$C_y = \frac{P}{2} \quad \text{so } F_{AF} = \frac{\sqrt{2}}{2}P \quad C \leftarrow$$

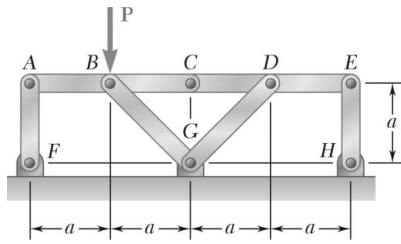
$$F_{EH} = \frac{\sqrt{2}}{2}P \quad T \leftarrow$$

FBD I: $\left(\sum F_y = 0: \frac{1}{\sqrt{2}}F_{AF} + \frac{1}{\sqrt{2}}F_{BG} - P + C_y = 0 \quad \frac{P}{2} + \frac{1}{\sqrt{2}}F_{BG} - P + \frac{P}{2} = 0\right)$

$$F_{BG} = 0 \quad \leftarrow$$

FBD II: $\left(\sum F_y = 0: -C_y + \frac{1}{\sqrt{2}}F_{DG} - \frac{1}{\sqrt{2}}F_{EH} = 0 \quad -\frac{P}{2} + \frac{1}{\sqrt{2}}F_{DG} - \frac{P}{2} = 0\right)$

$$F_{DG} = \sqrt{2}P \quad C \leftarrow$$

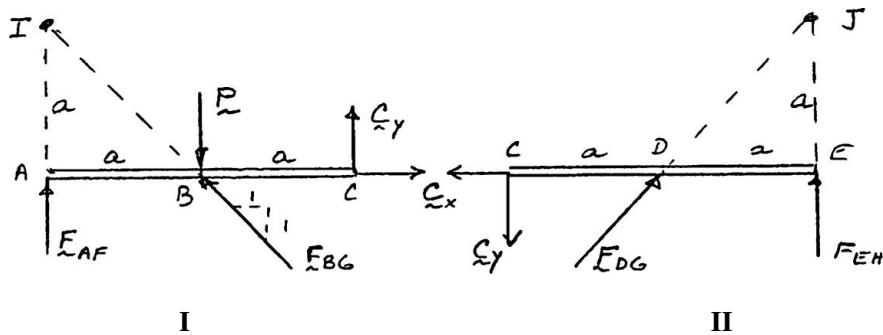


PROBLEM 6.112

Members ABC and CDE are pin-connected at C and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



FBD I: $\left(\sum M_I = 0: 2aC_y + aC_x - aP = 0 \quad 2C_y + C_x = P \right)$

FBD II: $\left(\sum M_J = 0: 2aC_y - aC_x = 0 \quad 2C_y - C_x = 0 \right)$

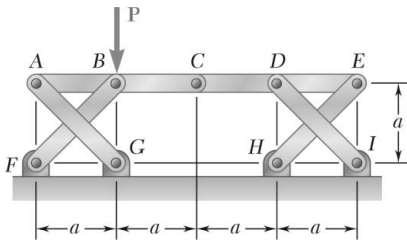
Solving, $C_x = \frac{P}{2}; C_y = \frac{P}{4}$ as shown.

FBD I: $\rightarrow \sum F_x = 0: -\frac{1}{\sqrt{2}}F_{BG} + C_x = 0 \quad F_{BG} = C_x\sqrt{2} \quad F_{BG} = \frac{P}{\sqrt{2}} \quad C \leftarrow$

$\uparrow \sum F_y = 0: F_{AF} - P + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}P\right) + \frac{P}{4} = 0 \quad F_{AF} = \frac{P}{4} \quad C \leftarrow$

FBD II: $\rightarrow \sum F_x = 0: -C_x + \frac{1}{\sqrt{2}}F_{DG} = 0 \quad F_{DG} = C_x\sqrt{2} \quad F_{DG} = \frac{P}{\sqrt{2}} \quad C \leftarrow$

$\uparrow \sum F_y = 0: -C_y + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}P\right) + F_{EH} = 0 \quad F_{EH} = \frac{P}{4} - \frac{P}{2} = -\frac{P}{4} \quad F_{EH} = \frac{P}{4} \quad T \leftarrow$

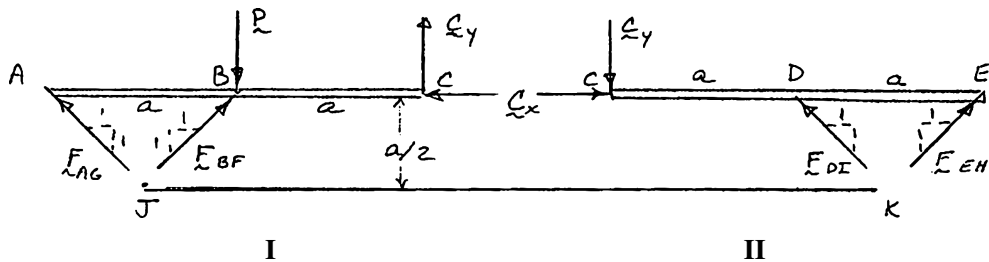


PROBLEM 6.113

Members ABC and CDE are pin-connected at C and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



From FBD I: $\left(\sum M_J = 0: \frac{a}{2} C_x + \frac{3a}{2} C_y - \frac{a}{2} P = 0 \quad C_x + 3C_y = P \right.$

FBD II: $\left(\sum M_K = 0: \frac{a}{2} C_x - \frac{3a}{2} C_y = 0 \quad C_x - 3C_y = 0 \right.$

Solving, $C_x = \frac{P}{2}; C_y = \frac{P}{6}$ as drawn.

FBD I: $\left(\sum M_B = 0: a C_y - a \frac{1}{\sqrt{2}} F_{AG} = 0 \quad F_{AG} = \sqrt{2} C_y = \frac{\sqrt{2}}{6} P \quad F_{AG} = \frac{\sqrt{2}}{6} P \quad C \leftarrow \right.$

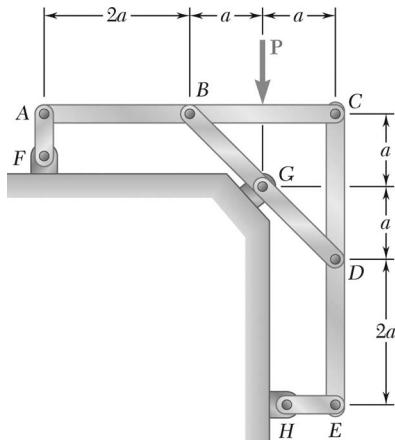
$\rightarrow \sum F_x = 0: -\frac{1}{\sqrt{2}} F_{AG} + \frac{1}{\sqrt{2}} F_{BF} - C_x = 0 \quad F_{BF} = F_{AG} + C_x \sqrt{2} = \frac{\sqrt{2}}{6} P + \frac{\sqrt{2}}{2} P$

$F_{BF} = \frac{2\sqrt{2}}{3} P \quad C \leftarrow$

FBD II: $\left(\sum M_D = 0: a \frac{1}{\sqrt{2}} F_{EH} + a C_y = 0 \quad F_{EH} = -\sqrt{2} C_y = -\frac{\sqrt{2}}{6} P \quad F_{EH} = \frac{\sqrt{2}}{6} P \quad T \leftarrow \right.$

$\rightarrow \sum F_x = 0: C_x - \frac{1}{\sqrt{2}} F_{DI} + \frac{1}{\sqrt{2}} F_{EH} = 0 \quad F_{DI} = F_{EH} + C_x \sqrt{2} = -\frac{\sqrt{2}}{6} P + \frac{\sqrt{2}}{2} P$

$F_{DI} = \frac{\sqrt{2}}{3} P \quad C \leftarrow$

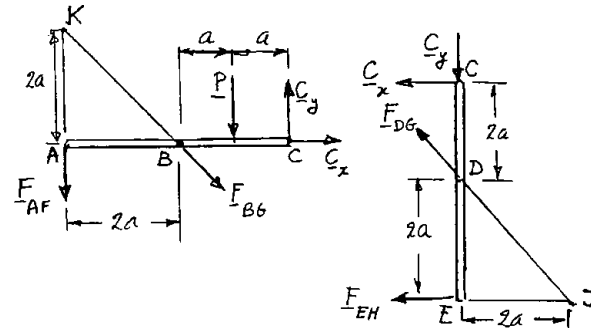


PROBLEM 6.114

Members ABC and CDE are pin-connected at C and supported by the four links AF , BG , DG , and EH . For the loading shown, determine the force in each link.

SOLUTION

We consider members ABC and CDE :



Free body: CDE :

$$+\curvearrowright \Sigma M_J = 0: C_x(4a) + C_y(2a) = 0 \quad C_y = -2C_x \quad (1)$$

Free body: ABC :

$$+\curvearrowright \Sigma M_K = 0: C_x(2a) + C_y(4a) - P(3a) = 0$$

Substituting for C_y from Eq. (1): $C_x(2a) - 2C_x(4a) - P(3a) = 0$

$$C_x = -\frac{1}{2}P \quad C_y = -2\left(-\frac{1}{2}P\right) = +P$$

$$\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{2}}F_{BG} + C_x = 0$$

$$F_{BG} = -\sqrt{2}C_x = -\sqrt{2}\left(-\frac{1}{2}P\right) = +\frac{P}{\sqrt{2}}, \quad F_{BG} = \frac{P}{\sqrt{2}} \quad T \quad \blacktriangleleft$$

PROBLEM 6.114 (Continued)

$$+\uparrow \Sigma F_y = 0: -F_{AF} - \frac{1}{\sqrt{2}} F_{BG} - P + C_y = 0$$

$$F_{AF} = -\frac{1}{\sqrt{2}} \frac{P}{\sqrt{2}} - P + P = -\frac{P}{2} \quad F_{AF} = \frac{P}{2} \quad C \quad \blacktriangleleft$$

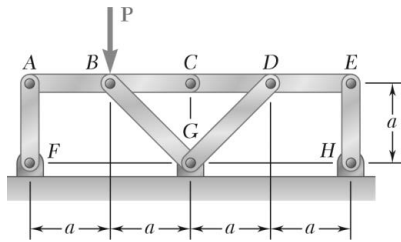
Free body: CDE:

$$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{2}} F_{DG} - C_y = 0$$

$$F_{DG} = \sqrt{2} C_y = +\sqrt{2} P \quad F_{DG} = \sqrt{2} P \quad T \quad \blacktriangleleft$$

$$\pm \rightarrow \Sigma F_x = 0: -F_{EH} - C_x - \frac{1}{\sqrt{2}} F_{DG} = 0$$

$$F_{EH} = -\left(-\frac{1}{2} P\right) - \frac{1}{\sqrt{2}} \sqrt{2} P = -\frac{P}{2} \quad F_{EH} = \frac{P}{2} \quad C \quad \blacktriangleleft$$



PROBLEM 6.115

Solve Problem 6.112 assuming that the force \mathbf{P} is replaced by a clockwise couple of moment \mathbf{M}_0 applied to member CDE at D .

PROBLEM 6.112 Members ABC and CDE are pin-connected at C and supported by four links. For the loading shown, determine the force in each link.

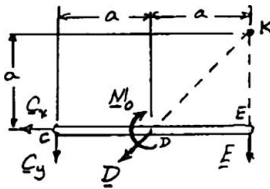
SOLUTION

Free body: Member ABC :

$$+\circlearrowleft \Sigma M_J = 0: C_y(2a) + C_x(a) = 0$$

$$C_x = -2C_y$$

Free body: Member CDE :



$$+\circlearrowleft \Sigma M_K = 0: C_y(2a) - C_x(a) - M_0 = 0$$

$$C_y(2a) - (-2C_y)(a) - M_0 = 0$$

$$C_x = -2C_y$$

$$\leftarrow \Sigma F_x = 0: \frac{D}{\sqrt{2}} + C_x = 0; \quad \frac{D}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$D = \frac{M_0}{\sqrt{2}a}$$

$$F_{DG} = \frac{M_0}{\sqrt{2}a} \quad T \quad \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: E(a) - C_y(a) + M_0 = 0$$

$$E(a) - \left(\frac{M_0}{4a}\right)(a) + M_0 = 0$$

$$E = -\frac{3M_0}{4a}$$

$$F_{EH} = \frac{3M_0}{4a} \quad C \quad \blacktriangleleft$$

Return to free body of ABC :

$$\leftarrow \Sigma F_x = 0: \frac{B}{\sqrt{2}} + C_x = 0; \quad \frac{B}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

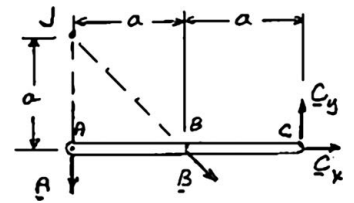
$$B = \frac{M_0}{\sqrt{2}a}$$

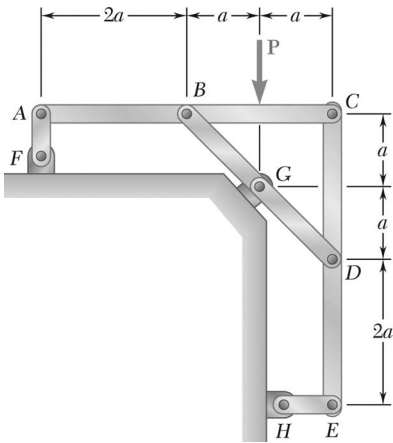
$$F_{BG} = \frac{M_0}{\sqrt{2}a} \quad T \quad \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: A(a) + C_y(a); \quad A(a) + \frac{M_0}{4a}(a) = 0$$

$$A = -\frac{M_0}{4a}$$

$$F_{AF} = \frac{M_0}{4a} \quad C \quad \blacktriangleleft$$





PROBLEM 6.116

Solve Problem 6.114 assuming that the force \mathbf{P} is replaced by a clockwise couple of moment \mathbf{M}_0 applied at the same point.

PROBLEM 6.114 Members ABC and CDE are pin-connected at C and supported by the four links AF , BG , DG , and EH . For the loading shown, determine the force in each link.

SOLUTION

Free body: CDE : (same as for Problem 6.114)

$$+\curvearrowright \Sigma M_J = 0: C_x(4a) + C_y(2a) = 0 \quad C_y = -2C_x \quad (1)$$

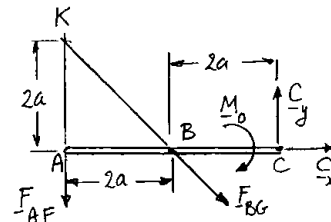
Free body: ABC :

$$+\curvearrowright \Sigma M_K = 0: C_x(2a) + C_y(4a) - M_0 = 0$$

Substituting for C_y from Eq. (1): $C_x(2a) - 2C_x(4a) - M_0 = 0$

$$C_x = -\frac{M_0}{6a}$$

$$C_y = -2\left(-\frac{M_0}{6a}\right) = +\frac{M_0}{3a}$$



$$+\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{2}} F_{BG} + C_x = 0$$

$$F_{BG} = -\sqrt{2}C_x = +\frac{\sqrt{2}M_0}{6a}$$

$$F_{BG} = \frac{\sqrt{2}M_0}{6a} \quad T \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: -F_{AF} - \frac{1}{\sqrt{2}} F_{BG} + C_y = 0$$

$$F_{AF} = -\frac{1}{\sqrt{2}} \frac{\sqrt{2}M_0}{6a} + \frac{M_0}{3a} = +\frac{M_0}{6a}$$

$$F_{AF} = \frac{M_0}{6a} \quad T \quad \blacktriangleleft$$

PROBLEM 6.116 (Continued)

Free body: CDE: (Use F.B. diagram of Problem 6.114.)

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}} F_{DG} - C_y = 0$$

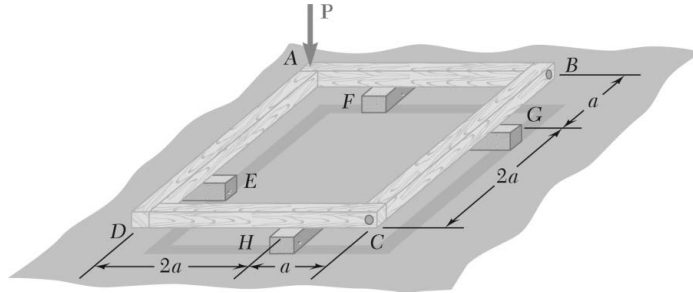
$$F_{DG} = \sqrt{2} C_y = + \frac{\sqrt{2} M_0}{3a} \qquad F_{DG} = \frac{\sqrt{2} M_0}{3a} \quad T \quad \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad -F_{EH} - C_x - \frac{1}{\sqrt{2}} F_{DG} = 0$$

$$F_{EH} = - \left(-\frac{M_0}{6a} \right) - \left(\frac{1}{\sqrt{2}} \right) \left(\frac{\sqrt{2} M_0}{3a} \right) = -\frac{M_0}{6a}, \quad F_{EH} = \frac{M_0}{6a} \quad C \quad \blacktriangleleft$$

PROBLEM 6.117

Four beams, each of length $3a$, are held together by single nails at A , B , C , and D . Each beam is attached to a support located at a distance a from an end of the beam as shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at E , F , G , and H .



SOLUTION

We shall draw the free body of each member. Force \mathbf{P} will be applied to member AFB . Starting with member AED , we shall express all forces in terms of reaction E .

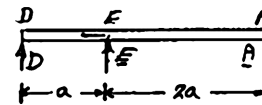
Member AFB :

$$+\curvearrowright \Sigma M_D = 0: A(3a) + E(a) = 0$$

$$A = -\frac{E}{3}$$

$$+\curvearrowright \Sigma M_A = 0: -D(3a) - E(2a) = 0$$

$$D = -\frac{2E}{3}$$



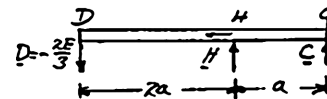
Member DHC :

$$+\curvearrowright \Sigma M_C = 0: \left(-\frac{2E}{3}\right)(3a) - H(a) = 0$$

$$H = -2E$$

$$+\curvearrowright \Sigma M_H = 0: \left(-\frac{2E}{3}\right)(2a) + C(a) = 0$$

$$C = +\frac{4E}{3}$$



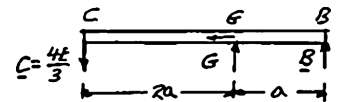
Member CGB :

$$+\curvearrowright \Sigma M_B = 0: +\left(\frac{4E}{3}\right)(3a) - G(a) = 0$$

$$G = +4E$$

$$+\curvearrowright \Sigma M_G = 0: +\left(\frac{4E}{3}\right)(2a) + B(a) = 0$$

$$B = -\frac{8E}{3}$$



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PROBLEM 6.117 (Continued)

Member AFB:

$$+\uparrow \Sigma F_y = 0: F - A - B - P = 0$$

$$F - \left(-\frac{E}{3}\right) - \left(-\frac{8E}{3}\right) - P = 0$$

$$F = P - 3E \tag{3}$$

$$+\curvearrowright \Sigma M_A = 0: F(a) - B(3a) = 0$$

$$(P - 3E)(a) - \left(-\frac{8E}{3}\right)(3a) = 0$$

$$P - 3E + 8E = 0; \quad E = -\frac{P}{5}$$

$$\mathbf{E} = \frac{P}{5} \downarrow \blacktriangleleft$$

Substitute $E = -\frac{P}{5}$ into Eqs. (1), (2), and (3).

$$H = -2E = -2\left(-\frac{P}{5}\right) \quad H = +\frac{2P}{5}$$

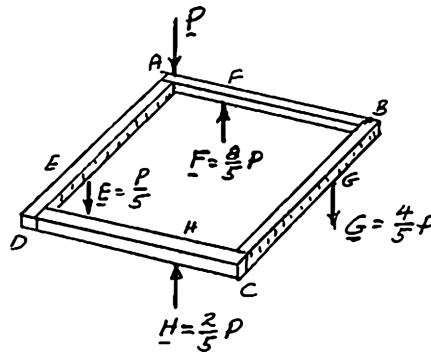
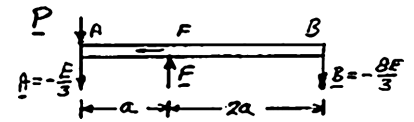
$$\mathbf{H} = \frac{2P}{5} \uparrow \blacktriangleleft$$

$$G = +4E = 4\left(-\frac{P}{5}\right) \quad G = -\frac{4P}{5}$$

$$\mathbf{G} = \frac{4P}{5} \downarrow \blacktriangleleft$$

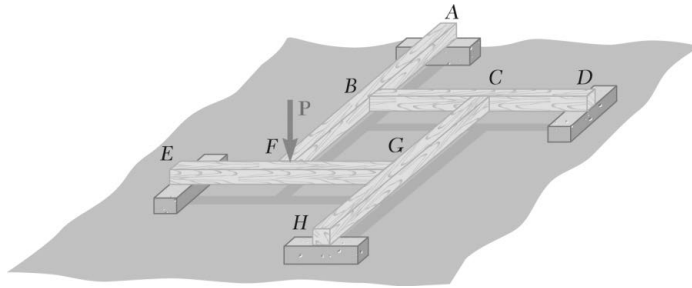
$$F = P - 3E = P - 3\left(-\frac{P}{5}\right) \quad F = +\frac{8P}{5}$$

$$\mathbf{F} = \frac{8P}{5} \uparrow \blacktriangleleft$$



PROBLEM 6.118

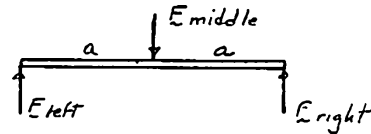
Four beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A , D , E , and H .



SOLUTION

Note that, if we assume P is applied to EG , each individual member FBD looks like

so
$$2F_{\text{left}} = 2F_{\text{right}} = F_{\text{middle}}$$



Labeling each interaction force with the letter corresponding to the joint of its application, we see that

$$B = 2A = 2F$$

$$C = 2B = 2D$$

$$G = 2C = 2H$$

$$P + F = 2G (= 4C = 8B = 16F) = 2E$$

From $P + F = 16F$,

$$F = \frac{P}{15}$$

$$\text{so } A = \frac{P}{15} \uparrow \blacktriangleleft$$

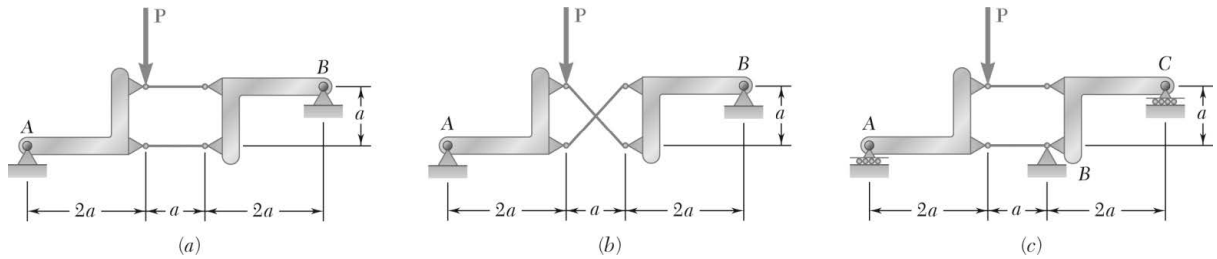
$$D = \frac{2P}{15} \uparrow \blacktriangleleft$$

$$H = \frac{4P}{15} \uparrow \blacktriangleleft$$

$$E = \frac{8P}{15} \uparrow \blacktriangleleft$$

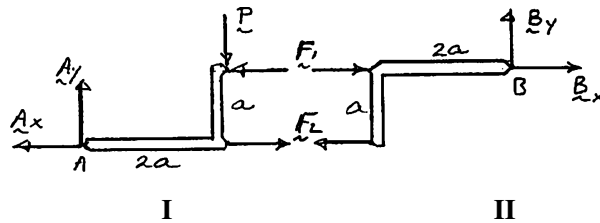
PROBLEM 6.119

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(a) Member FBDs:

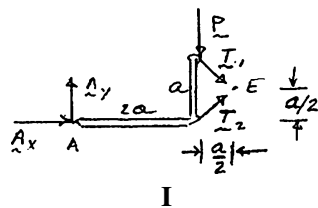


FBD I: $\left(\begin{aligned} \sum M_A = 0: & aF_1 - 2aP = 0 \quad F_1 = 2P; \\ \uparrow \sum F_y = 0: & A_y - P = 0 \quad A_y = P \uparrow \end{aligned} \right.$

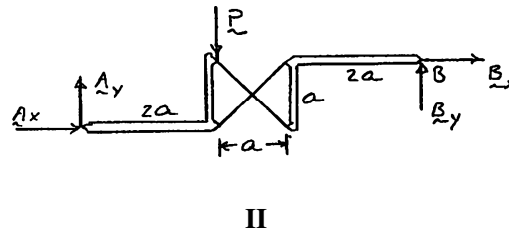
FBD II: $\left(\begin{aligned} \sum M_B = 0: & -aF_2 = 0 \quad F_2 = 0 \\ \rightarrow \sum F_x = 0: & B_x + F_1 = 0, \quad B_x = -F_1 = -2P \quad B_x = 2P \rightarrow \\ \uparrow \sum F_y = 0: & B_y = 0 \quad \text{so } \mathbf{B} = 2P \rightarrow \blacktriangleleft \end{aligned} \right.$

FBD I: $\rightarrow \sum F_x = 0: -A_x - F_1 + F_2 = 0 \quad A_x = F_2 - F_1 = 0 - 2P \quad A_x = 2P \rightarrow$
 so $\mathbf{A} = 2.24P \angle 26.6^\circ \blacktriangleleft$
 Frame is rigid. \blacktriangleleft

(b) FBD left:



FBD whole:



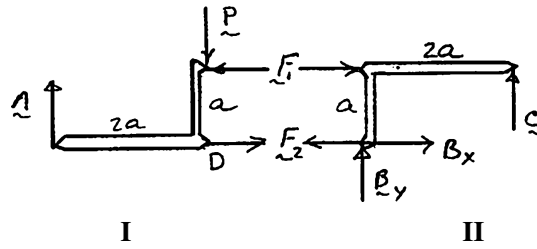
FBD I: $\left(\sum M_E = 0: \frac{a}{2}P + \frac{a}{2}A_x - \frac{5a}{2}A_y = 0 \quad A_x - 5A_y = -P \right.$

FBD II: $\left(\sum M_B = 0: 3aP + aA_x - 5aA_y = 0 \quad A_x - 5A_y = -3P \right.$

This is impossible unless $P = 0$. not rigid \blacktriangleleft

PROBLEM 6.119 (Continued)

(c) Member FBDs:



FBD I: $\Sigma F_y = 0: A - P = 0$ $A = P \uparrow \blacktriangleleft$

$\curvearrowleft \Sigma M_D = 0: aF_1 - 2aA = 0 \quad F_1 = 2P$

$\rightarrow \Sigma F_x = 0: F_2 - F_1 = 0 \quad F_2 = 2P$

FBD II: $\curvearrowleft \Sigma M_B = 0: 2aC - aF_1 = 0 \quad C = \frac{F_1}{2} = P$ $C = P \uparrow \blacktriangleleft$

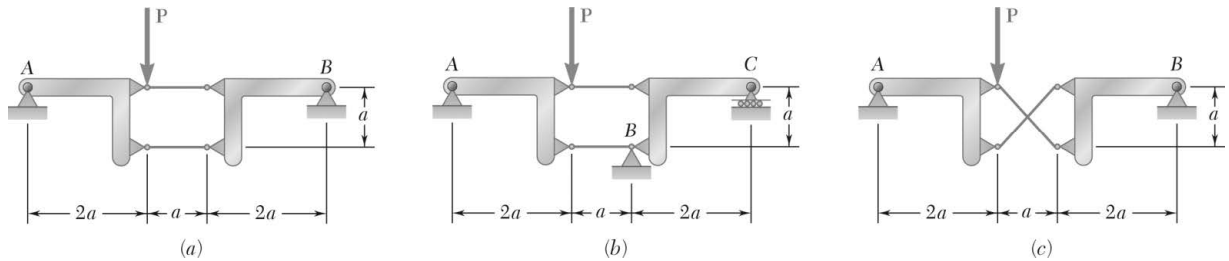
$\rightarrow \Sigma F_x = 0: F_1 - F_2 + B_x = 0 \quad B_x = P - P = 0$

$\uparrow \Sigma F_y = 0: B_y + C = 0 \quad B_y = -C = -P$ $B = P \blacktriangleleft$

Frame is rigid. \blacktriangleleft

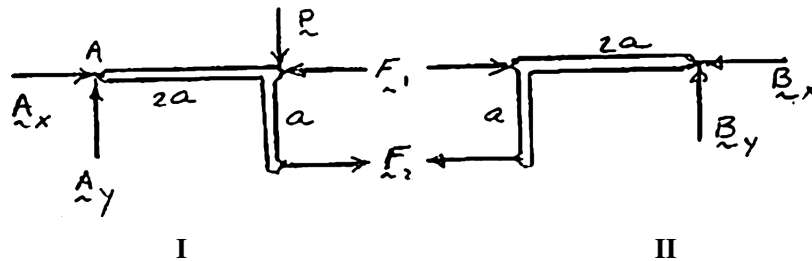
PROBLEM 6.120

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(a) Member FBDs:

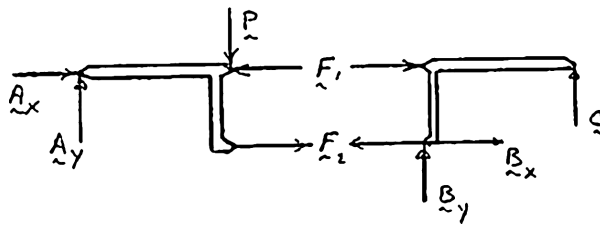


FBD II: $\uparrow \Sigma F_y = 0: B_y = 0$ $(\Sigma M_B = 0: aF_2 = 0 \quad F_2 = 0)$

FBD I: $(\Sigma M_A = 0: aF_2 - 2aP = 0, \text{ but } F_2 = 0)$

so $P = 0$ not rigid for $P \neq 0$ ◀

(b) Member FBDs:



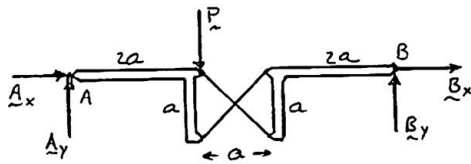
Note: Seven unknowns ($A_x, A_y, B_x, B_y, F_1, F_2, C$), but only six independent equations

System is statically indeterminate. ◀

System is, however, rigid. ◀

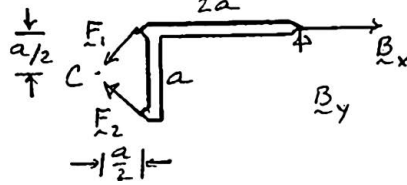
PROBLEM 6.120 (Continued)

(c) **FBD whole:**



I

FBD right:



II

FBD I: $\left(\sum M_A = 0: 5aB_y - 2aP = 0 \right.$

$\left. B_y = \frac{2}{5}P \uparrow \right)$

$\uparrow \sum F_y = 0: A_y - P + \frac{2}{5}P = 0$

$A_y = \frac{3}{5}P \uparrow$

FBD II: $\left(\sum M_C = 0: \frac{a}{2}B_x - \frac{5a}{2}B_y = 0 \right. B_x = 5B_y$

$\left. B_x = 2P \rightarrow \right)$

FBD I: $\rightarrow \sum F_x = 0: A_x + B_x = 0 \quad A_x = -B_x$

$A_x = 2P \leftarrow$

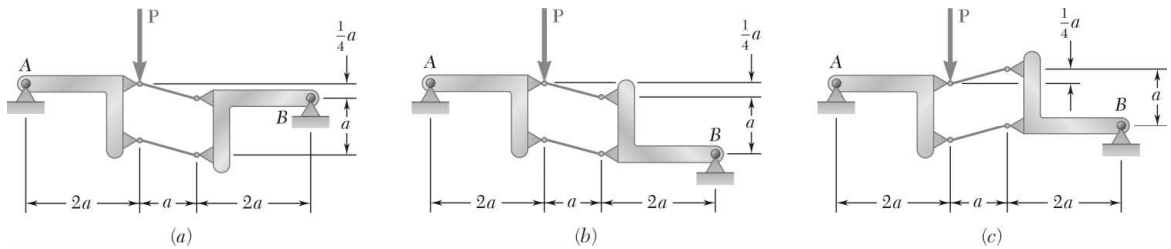
$A = 2.09P \searrow 16.70^\circ \blacktriangleleft$

$B = 2.04P \swarrow 11.31^\circ \blacktriangleleft$

System is rigid. \blacktriangleleft

PROBLEM 6.121

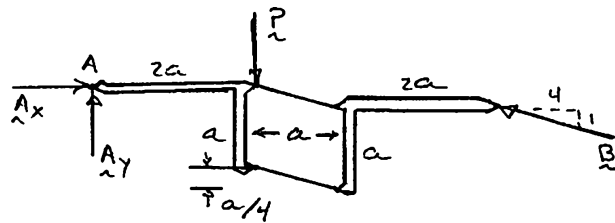
Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

Note: In all three cases, the right member has only three forces acting, two of which are parallel. Thus, the third force, at B, must be parallel to the link forces.

(a) **FBD whole:**



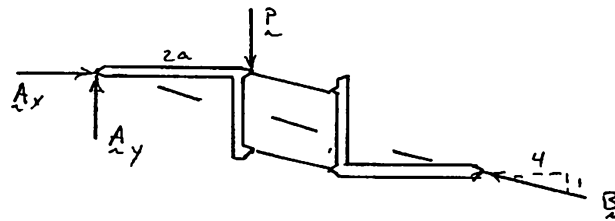
$$\left(\sum M_A = 0: -2aP - \frac{a}{4} \frac{4}{\sqrt{17}} B + 5a \frac{1}{\sqrt{17}} B = 0 \quad B = 2.06P \quad \searrow 14.04^\circ \quad \blacktriangleleft$$

$$\rightarrow \sum F_x = 0: A_x - \frac{4}{\sqrt{17}} B = 0 \quad A_x = 2P \quad \leftarrow$$

$$\uparrow \sum F_y = 0: A_y - P + \frac{1}{\sqrt{17}} B = 0 \quad A_y = \frac{P}{2} \quad \uparrow \quad A = 2.06P \quad \nearrow 14.04^\circ \quad \blacktriangleleft$$

rigid \blacktriangleleft

(b) **FBD whole:**



Since B passes through A, $\left(\sum M_A = 2aP = 0 \right)$ only if $P = 0$.

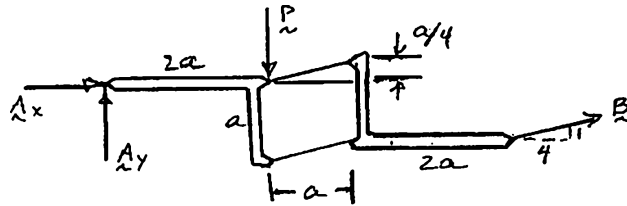
no equilibrium if $P \neq 0$

not rigid \blacktriangleleft

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PROBLEM 6.121 (Continued)

(c) FBD whole:



$$\left(\sum M_A = 0: 5a \frac{1}{\sqrt{17}} B + \frac{3a}{4} \frac{4}{\sqrt{17}} B - 2aP = 0 \quad B = \frac{\sqrt{17}}{4} P \right.$$

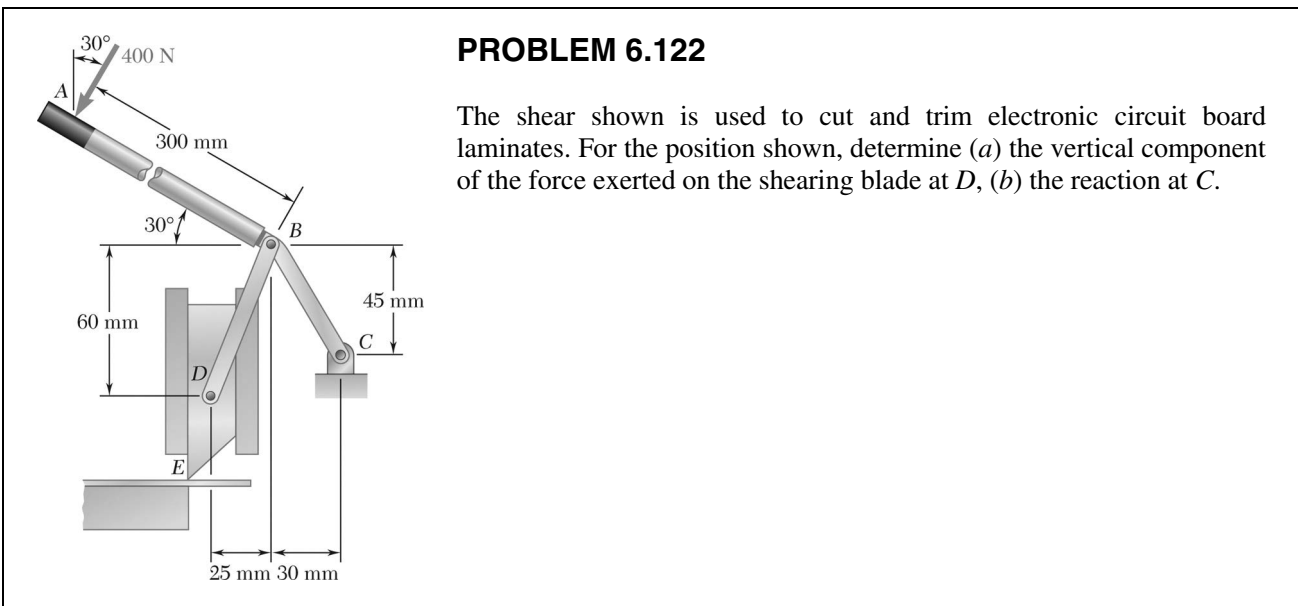
$$B = 1.031P \angle 14.04^\circ \blacktriangleleft$$

$$\rightarrow \sum F_x = 0: A_x + \frac{4}{\sqrt{17}} B = 0 \quad A_x = -P$$

$$\uparrow \sum F_y = 0: A_y - P + \frac{1}{\sqrt{17}} B = 0 \quad A_y = P - \frac{P}{4} = \frac{3P}{4}$$

$$A = 1.250P \searrow 36.9^\circ \blacktriangleleft$$

System is rigid. \blacktriangleleft



PROBLEM 6.122

The shear shown is used to cut and trim electronic circuit board laminates. For the position shown, determine (a) the vertical component of the force exerted on the shearing blade at D, (b) the reaction at C.

SOLUTION

We note that BD is a two-force member.

Free body: Member ABC: We have the components:

$$P_x = (400 \text{ N}) \sin 30^\circ = 200 \text{ N} \leftarrow$$

$$P_y = (400 \text{ N}) \cos 30^\circ = 346.41 \text{ N} \downarrow$$

$$(F_{BD})_x = \frac{25}{65} F_{BD} \rightarrow$$

$$(F_{BD})_y = \frac{60}{65} F_{BD} \uparrow$$

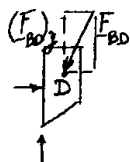
$$+\circlearrowleft \Sigma M_C = 0: (F_{BD})_x(45) + (F_{BD})_y(30) - P_x(45 + 300 \sin 30^\circ) - P_y(30 + 300 \cos 30^\circ) = 0$$

$$\left(\frac{25}{65} F_{BD} \right) (45) + \left(\frac{60}{65} F_{BD} \right) (30) = (200)(195) + (346.41)(289.81)$$

$$45 F_{BD} = 139.39 \times 10^3$$

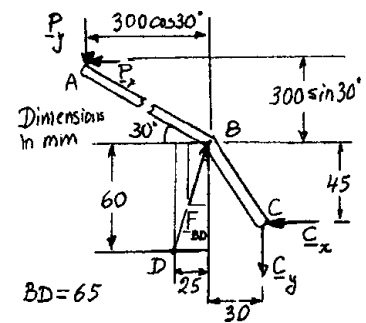
$$F_{BD} = 3097.6 \text{ N}$$

(a) Vertical component of force exerted on shearing blade at D.



$$(F_{BD})_y = \frac{60}{65} F_{BD} = \frac{60}{65} (3097.6 \text{ N}) = 2859.3 \text{ N}$$

$$(F_{BD})_y = 2860 \text{ N} \downarrow \blacktriangleleft$$



PROBLEM 6.122 (Continued)

(b) Returning to FB diagram of member ABC ,

$$\rightarrow \Sigma F_x = 0: (F_{BD})_x - P_x - C_x = 0$$

$$C_x = (F_{BD})_x - P_x = \frac{25}{65} F_{BD} - P_x$$

$$= \frac{25}{65} (3097.6) - 200$$

$$C_x = +991.39$$

$$C_x = 991.39 \text{ N } \leftarrow$$

$$+\uparrow \Sigma F_y = 0: (F_{BD})_y - P_y - C_y = 0$$

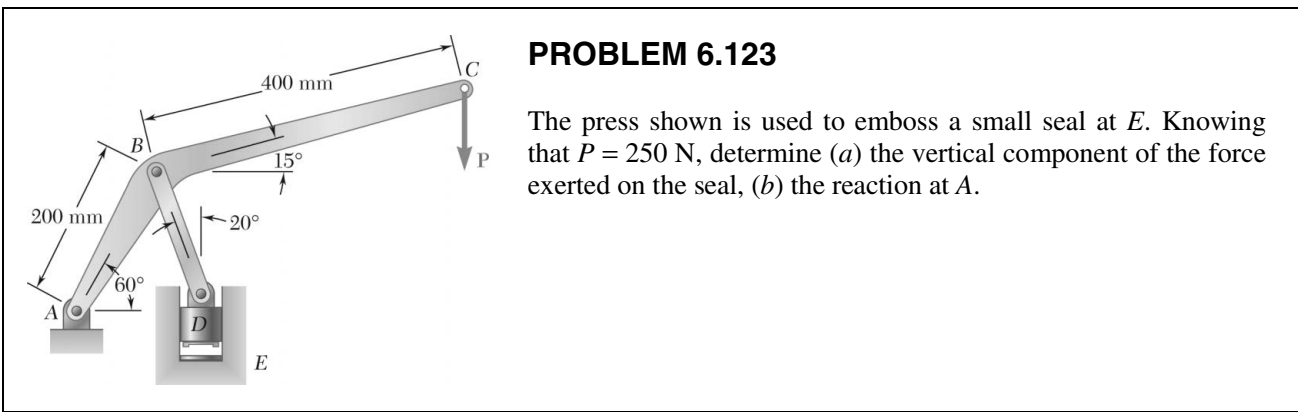
$$C_y = (F_{BD})_y - P_y = \frac{60}{65} F_{BD} - P_y = \frac{60}{65} (3097.6) - 346.41$$

$$C_y = +2512.9 \text{ N}$$

$$C_y = 2512.9 \text{ N } \downarrow$$

$$C = 2295 \text{ N}$$

$$C = 2700 \text{ N } \nearrow 68.5^\circ \blacktriangleleft$$



PROBLEM 6.123

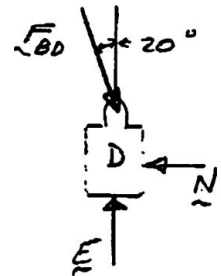
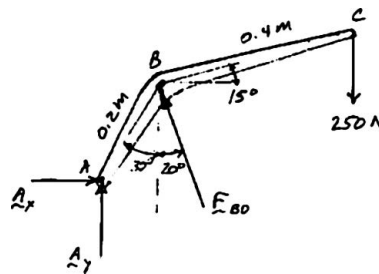
The press shown is used to emboss a small seal at *E*. Knowing that $P = 250 \text{ N}$, determine (a) the vertical component of the force exerted on the seal, (b) the reaction at *A*.

SOLUTION

FBD Stamp *D*:

$$\uparrow \Sigma F_y = 0: E - F_{BD} \cos 20^\circ = 0, E = F_{BD} \cos 20^\circ$$

FBD *ABC*:



$$\begin{aligned} \left(\Sigma M_A = 0: (0.2 \text{ m})(\sin 30^\circ)(F_{BD} \cos 20^\circ) + (0.2 \text{ m})(\cos 30^\circ)(F_{BD} \sin 20^\circ) \right. \\ \left. - [(0.2 \text{ m}) \sin 30^\circ + (0.4 \text{ m}) \cos 15^\circ](250 \text{ N}) = 0 \right. \end{aligned}$$

$$F_{BD} = 793.64 \text{ N} \quad C$$

and, from above,

$$E = (793.64 \text{ N}) \cos 20^\circ$$

$$(a) \quad E = 746 \text{ N} \downarrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: A_x - (793.64 \text{ N}) \sin 20^\circ = 0$$

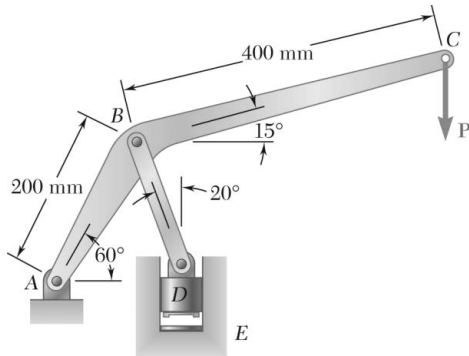
$$A_x = 271.44 \text{ N} \rightarrow$$

$$\uparrow \Sigma F_y = 0: A_y + (793.64 \text{ N}) \cos 20^\circ - 250 \text{ N} = 0$$

$$A_y = 495.78 \text{ N} \downarrow$$

$$\text{so } (b) \quad A = 565 \text{ N} \swarrow 61.3^\circ \blacktriangleleft$$

PROBLEM 6.124

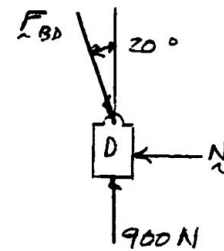


The press shown is used to emboss a small seal at *E*. Knowing that the vertical component of the force exerted on the seal must be 900 N, determine (a) the required vertical force *P*, (b) the corresponding reaction at *A*.

SOLUTION

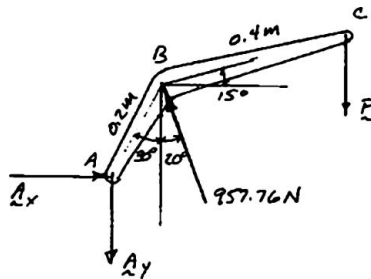
FBD Stamp *D*:

$$\uparrow \Sigma F_y = 0: \quad 900 \text{ N} - F_{BD} \cos 20^\circ = 0, \quad F_{BD} = 957.76 \text{ N} \quad C$$



(a)

FBD *ABC*:



$$\begin{aligned} \left(\Sigma M_A = 0: \right. & \quad [(0.2 \text{ m})(\sin 30^\circ)](957.76 \text{ N}) \cos 20^\circ + [(0.2 \text{ m})(\cos 30^\circ)](957.76 \text{ N}) \sin 20^\circ \\ & \quad - [(0.2 \text{ m}) \sin 30^\circ + (0.4 \text{ m}) \cos 15^\circ]P = 0 \end{aligned}$$

$$P = 301.70 \text{ N},$$

$$P = 302 \text{ N} \downarrow \blacktriangleleft$$

(b)

$$\rightarrow \Sigma F_x = 0: \quad A_x - (957.76 \text{ N}) \sin 20^\circ = 0$$

$$A_x = 327.57 \text{ N} \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + (957.76 \text{ N}) \cos 20^\circ - 301.70 \text{ N} = 0$$

$$A_y = 598.30 \text{ N} \downarrow$$

so

$$A = 682 \text{ N} \swarrow 61.3^\circ \blacktriangleleft$$

PROBLEM 6.125

Water pressure in the supply system exerts a downward force of 135 N on the vertical plug at A. Determine the tension in the fusible link *DE* and the force exerted on member *BCE* at B.

SOLUTION

Free body: Entire linkage:

$$+\uparrow \Sigma F_y = 0: C - 135 = 0$$

$$C = +135 \text{ N}$$

Free body: Member BCE:

$$+\rightarrow \Sigma F_x = 0: B_x = 0$$

$$+\curvearrowright \Sigma M_B = 0: (135 \text{ N})(6 \text{ mm}) - T_{DE}(10 \text{ mm}) = 0$$

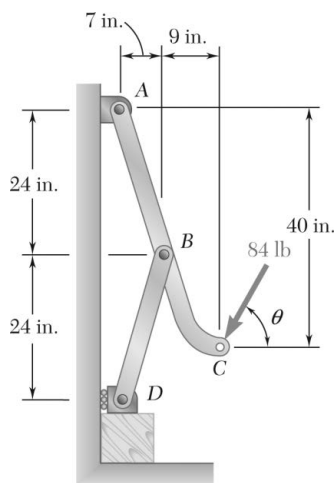
$$T_{DE} = 81.0 \text{ N} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 135 + 81 - B_y = 0$$

$$B_y = +216 \text{ N}$$

$$\mathbf{B} = 216 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 6.126



An 84-lb force is applied to the toggle vise at C . Knowing that $\theta = 90^\circ$, determine (a) the vertical force exerted on the block at D , (b) the force exerted on member ABC at B .

SOLUTION

We note that BD is a two-force member.

Free body: Member ABC :

We have

$$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$(F_{BD})_x = \frac{7}{25} F_{BD}, \quad (F_{BD})_y = \frac{24}{25} F_{BD}$$

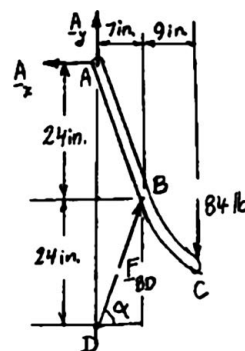
$$+\circlearrowleft \Sigma M_A = 0: (F_{BD})_x(24) + (F_{BD})_y(7) - 84(16) = 0$$

$$\left(\frac{7}{25} F_{BD}\right)(24) + \left(\frac{24}{25} F_{BD}\right)(7) = 84(16)$$

$$\frac{336}{25} F_{BD} = 1344$$

$$F_{BD} = 100 \text{ lb}$$

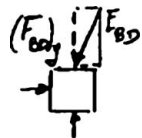
$$\tan \alpha = \frac{24}{7} \quad \alpha = 73.7^\circ$$



(b) Force exerted at B .

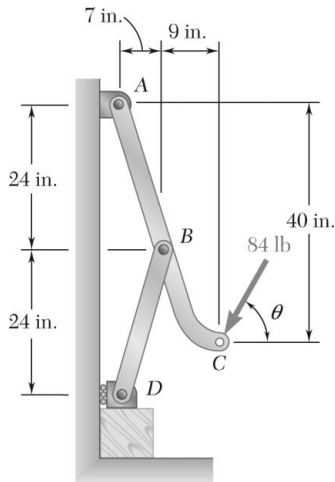
$$\mathbf{F}_{BD} = 100.0 \text{ lb} \nearrow 73.7^\circ \blacktriangleleft$$

(a) Vertical force exerted on block.



$$(F_{BD})_y = \frac{24}{25} F_{BD} = \frac{24}{25} (100 \text{ lb}) = 96 \text{ lb}$$

$$\mathbf{(F_{BD})_y} = 96.0 \text{ lb} \downarrow \blacktriangleleft$$



PROBLEM 6.127

Solve Problem 6.126 when $\theta = 0$.

PROBLEM 6.126 An 84-lb force is applied to the toggle vise at C. Knowing that $\theta = 90^\circ$, determine (a) the vertical force exerted on the block at D, (b) the force exerted on member ABC at B.

SOLUTION

We note that BD is a two-force member.

Free body: Member ABC:

We have

$$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$(F_{BD})_x = \frac{7}{25} F_{BD}, \quad (F_{BD})_y = \frac{24}{25} F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: (F_{BD})_x(24) + (F_{BD})_y(7) - 84(40) = 0$$

$$\left(\frac{7}{25} F_{BD}\right)(24) + \left(\frac{24}{25} F_{BD}\right)(7) = 84(40)$$

$$\frac{336}{25} F_{BD} = 3360$$

$$F_{BD} = 250 \text{ lb}$$

$$\tan \alpha = \frac{24}{7} \quad \alpha = 73.7^\circ$$

(b) Force exerted at B.

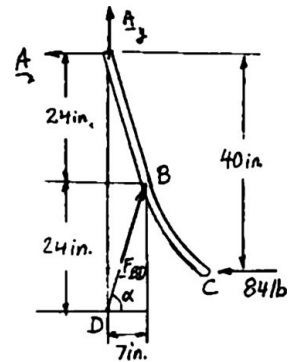
$$\mathbf{F}_{BD} = 250.0 \text{ lb} \nearrow 73.7^\circ \blacktriangleleft$$

(a) Vertical force exerted on block.

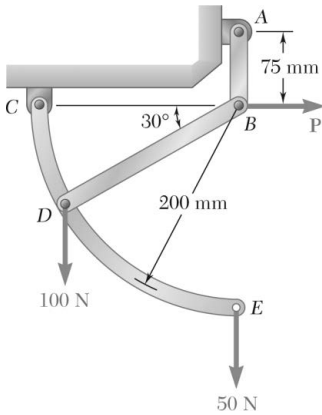
$$(F_{BD})_y = \frac{24}{25} F_{BD} = \frac{24}{25} (250 \text{ lb}) = 240 \text{ lb}$$



$$(F_{BD})_y = 240 \text{ lb} \downarrow \blacktriangleleft$$



PROBLEM 6.128



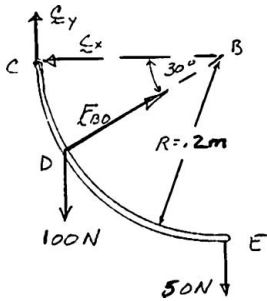
For the system and loading shown, determine (a) the force **P** required for equilibrium, (b) the corresponding force in member **BD**, (c) the corresponding reaction at **C**.

SOLUTION

Member FBDs:

FBD I:

I:



$$\left(\sum M_C = 0: R(F_{BD} \sin 30^\circ) - [R(1 - \cos 30^\circ)](100 \text{ N}) - R(50 \text{ N}) = 0 \right.$$

$$F_{BD} = 126.795 \text{ N} \quad (b) \quad F_{BD} = 126.8 \text{ N } T \quad \blacktriangleleft$$

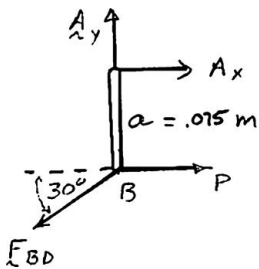
$$\rightarrow \sum F_x = 0: -C_x + (126.795 \text{ N}) \cos 30^\circ = 0 \quad C_x = 109.808 \text{ N } \leftarrow$$

$$\uparrow \sum F_y = 0: C_y + (126.795 \text{ N}) \sin 30^\circ - 100 \text{ N} - 50 \text{ N} = 0$$

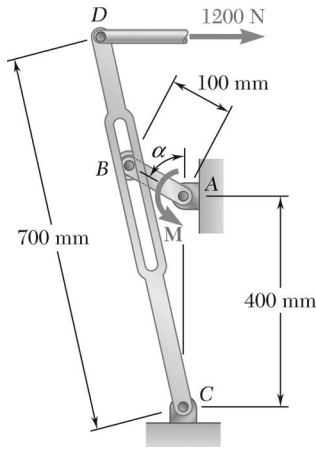
$$C_y = 86.603 \text{ N } \uparrow \quad \text{so } (c) \quad C = 139.8 \text{ N } \nearrow 38.3^\circ \quad \blacktriangleleft$$

II:

FBD II:



$$\left(\sum M_A = 0: aP - a[(126.795 \text{ N}) \cos 30^\circ] = 0 \right. \quad (a) \quad P = 109.8 \text{ N } \rightarrow \quad \blacktriangleleft$$



PROBLEM 6.129

The Whitworth mechanism shown is used to produce a quick-return motion of Point D. The block at B is pinned to the crank AB and is free to slide in a slot cut in member CD. Determine the couple **M** that must be applied to the crank AB to hold the mechanism in equilibrium when (a) $\alpha = 0$, (b) $\alpha = 30^\circ$.

SOLUTION

(a) Free body: Member CD:

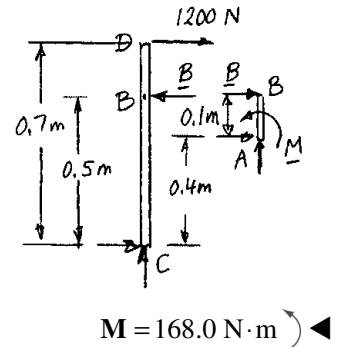
$$+\curvearrowright \Sigma M_C = 0: \quad B(0.5 \text{ m}) - (1200 \text{ N})(0.7 \text{ m}) = 0$$

$$B = 1680 \text{ N}$$

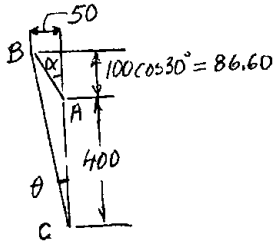
Free body: Crank AB:

$$+\curvearrowright \Sigma M_A = 0: \quad M - (1680 \text{ N})(0.1 \text{ m}) = 0$$

$$M = 168.0 \text{ N} \cdot \text{m}$$



(b) Geometry:



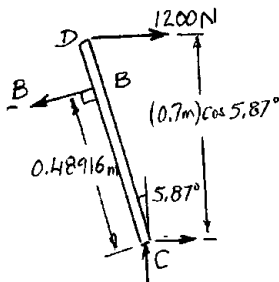
$$AB = 100 \text{ mm}, \quad \alpha = 30^\circ$$

$$\tan \theta = \frac{50}{486.6} \quad \theta = 5.87^\circ$$

$$BC = \sqrt{(50)^2 + (486.6)^2}$$

$$= 489.16 \text{ mm}$$

Free body: Member CD:

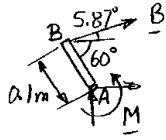


$$+\curvearrowright \Sigma M_C = 0: \quad B(0.48916) - (1200 \text{ N})(0.7) \cos 5.87^\circ = 0$$

$$B = 1708.2 \text{ N}$$

PROBLEM 6.129 (Continued)

Free body: Crank AB:

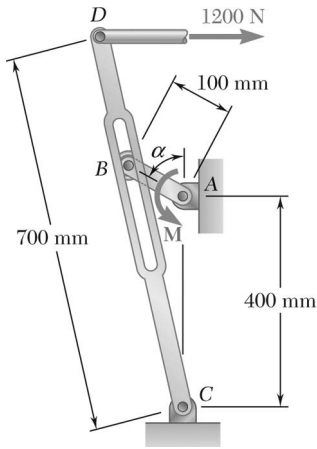


$$+\curvearrowright \Sigma M_A = 0: \quad M - (B \sin 65.87^\circ)(0.1 \text{ m}) = 0$$

$$M = (1708.2 \text{ N})(0.1 \text{ m}) \sin 65.87^\circ$$

$$M = 155.90 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 155.9 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$



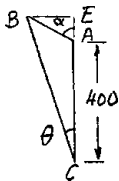
PROBLEM 6.130

Solve Problem 6.129 when (a) $\alpha = 60^\circ$, (b) $\alpha = 90^\circ$.

PROBLEM 6.129 The Whitworth mechanism shown is used to produce a quick-return motion of Point D . The block at B is pinned to the crank AB and is free to slide in a slot cut in member CD . Determine the couple M that must be applied to the crank AB to hold the mechanism in equilibrium when (a) $\alpha = 0^\circ$, (b) $\alpha = 30^\circ$.

SOLUTION

(a) Geometry:



$$AB = 100 \text{ mm}, \quad \alpha = 60^\circ$$

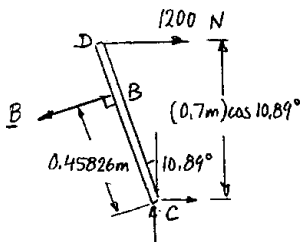
$$BE = 100 \cos 60^\circ = 86.60$$

$$CE = 400 + 100 \sin 60^\circ = 450$$

$$\tan \theta = \frac{86.60}{450} \quad \theta = 10.89^\circ$$

$$BC = \sqrt{(86.60)^2 + (450)^2} = 458.26 \text{ mm}$$

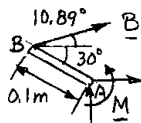
Free body: Member CD :



$$\rightarrow \Sigma M_C = 0: \quad B(0.45826 \text{ m}) - (1200 \text{ N})(0.7 \text{ m}) \cos 10.89^\circ = 0$$

$$B = 1800.0 \text{ N}$$

Free body: Crank AB :



$$\rightarrow \Sigma M_A = 0: \quad M - (B \sin 40.89^\circ)(0.1 \text{ m}) = 0$$

$$M = (1800.0 \text{ N})(0.1 \text{ m}) \sin 40.89^\circ$$

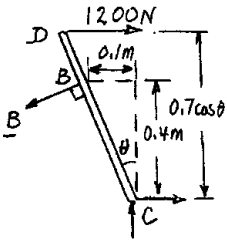
$$M = 117.83 \text{ N}\cdot\text{m}$$

$$\mathbf{M = 117.8 \text{ N}\cdot\text{m} \quad \curvearrowleft}$$

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PROBLEM 6.130 (Continued)

(b) Free body: Member CD:



$$\tan \theta = \frac{0.1 \text{ m}}{0.4 \text{ m}}$$

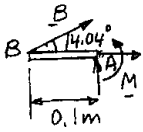
$$\theta = 14.04^\circ$$

$$BC = \sqrt{(0.1)^2 + (0.4)^2} = 0.41231 \text{ m}$$

$$+\curvearrowright \Sigma M_C = 0: \quad B(0.41231) - (1200 \text{ N})(0.7 \cos 14.04^\circ) = 0$$

$$B = 1976.4 \text{ N}$$

Free body: Crank AB:



$$+\curvearrowright \Sigma M_A = 0: \quad M - (B \sin 14.04^\circ)(0.1 \text{ m}) = 0$$

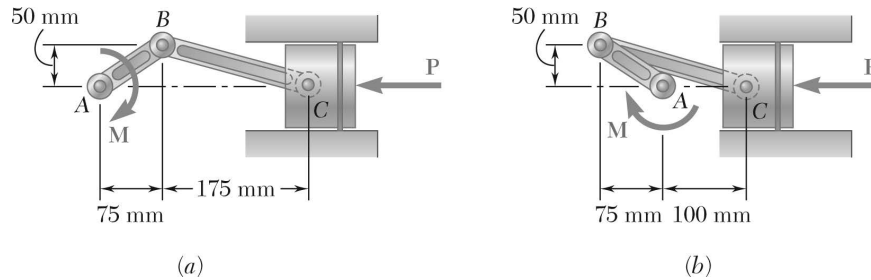
$$M = (1976.4 \text{ N})(0.1 \text{ m}) \sin 14.04^\circ$$

$$M = 47.948 \text{ N}\cdot\text{m}$$

$$M = 47.9 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$

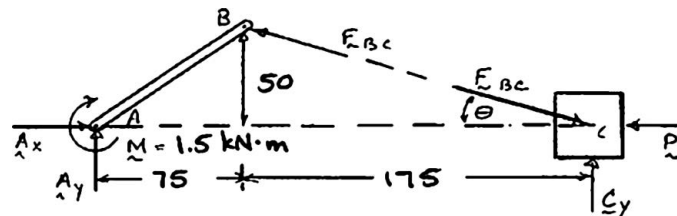
PROBLEM 6.131

A couple M of magnitude $1.5 \text{ kN} \cdot \text{m}$ is applied to the crank of the engine system shown. For each of the two positions shown, determine the force P required to hold the system in equilibrium.



SOLUTION

(a) FBDs:



Dimensions in mm

Note:

$$\tan \theta = \frac{50 \text{ mm}}{175 \text{ mm}}$$

$$= \frac{2}{7}$$

FBD whole:

$$\sum M_A = 0: (0.250 \text{ m})C_y - 1.5 \text{ kN} \cdot \text{m} = 0 \quad C_y = 6.00 \text{ kN}$$

FBD piston:

$$\sum F_y = 0: C_y - F_{BC} \sin \theta = 0 \quad F_{BC} = \frac{C_y}{\sin \theta} = \frac{6.00 \text{ kN}}{\sin \theta}$$

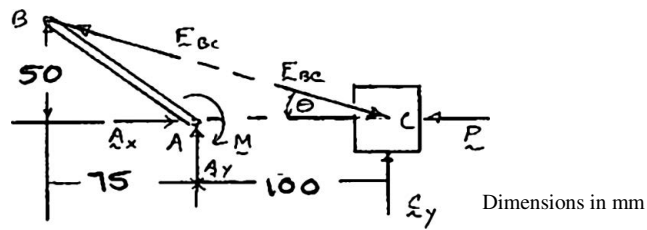
$$\sum F_x = 0: F_{BC} \cos \theta - P = 0$$

$$P = F_{BC} \cos \theta = \frac{6.00 \text{ kN}}{\tan \theta} = 7 \text{ kips}$$

$$P = 21.0 \text{ kN} \leftarrow \blacktriangleleft$$

PROBLEM 6.131 (Continued)

(b) **FBDs:**



Note: $\tan \theta = \frac{2}{7}$ as above

FBD whole: $\left(\sum M_A = 0: (0.100 \text{ m})C_y - 1.5 \text{ kN} \cdot \text{m} = 0 \quad C_y = 15 \text{ kN} \right.$

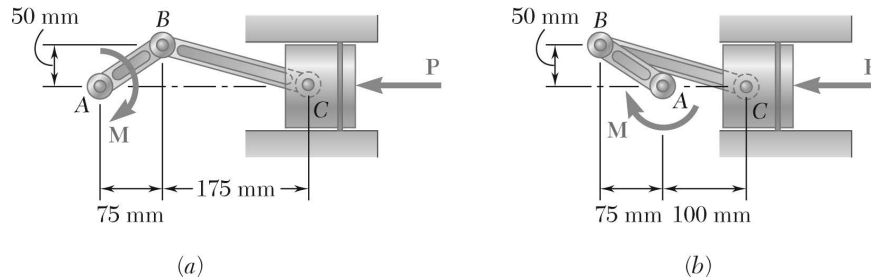
$$\sum F_y = 0: C_y - F_{BC} \sin \theta = 0 \quad F_{BC} = \frac{C_y}{\sin \theta}$$

$$\rightarrow \sum F_x = 0: F_{BC} \cos \theta - P = 0$$

$$P = F_{BC} \cos \theta = \frac{C_y}{\tan \theta} = \frac{15 \text{ kN}}{2/7} \quad \mathbf{P = 52.5 \text{ kN} \leftarrow \blacktriangleleft}$$

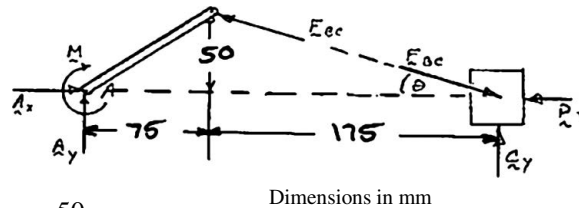
PROBLEM 6.132

A force P of magnitude 16 kN is applied to the piston of the engine system shown. For each of the two positions shown, determine the couple M required to hold the system in equilibrium.



SOLUTION

(a) **FBDs:**



Note:
$$\tan \theta = \frac{50 \text{ mm}}{175 \text{ mm}} = \frac{2}{7}$$

FBD piston:
$$\rightarrow \Sigma F_x = 0: F_{BC} \cos \theta - P = 0 \quad F_{BC} = \frac{P}{\cos \theta}$$

$$\uparrow \Sigma F_y = 0: C_y - F_{BC} \sin \theta = 0 \quad C_y = F_{BC} \sin \theta = P \tan \theta = \frac{2}{7} P$$

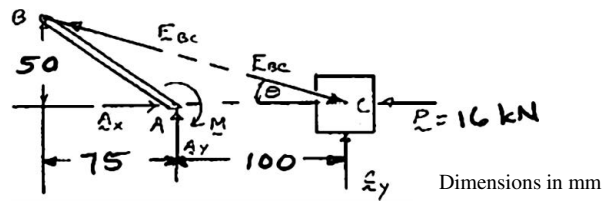
FBD whole:
$$\curvearrowleft \Sigma M_A = 0: (0.250 \text{ m})C_y - M = 0$$

$$M = (0.250 \text{ m}) \left(\frac{2}{7} \right) (16 \text{ kN}) = 1.14286 \text{ kN} \cdot \text{m}$$

$$\mathbf{M = 1143 \text{ N} \cdot \text{m} \curvearrowleft}$$

PROBLEM 6.132 (Continued)

(b) **FBDs:**

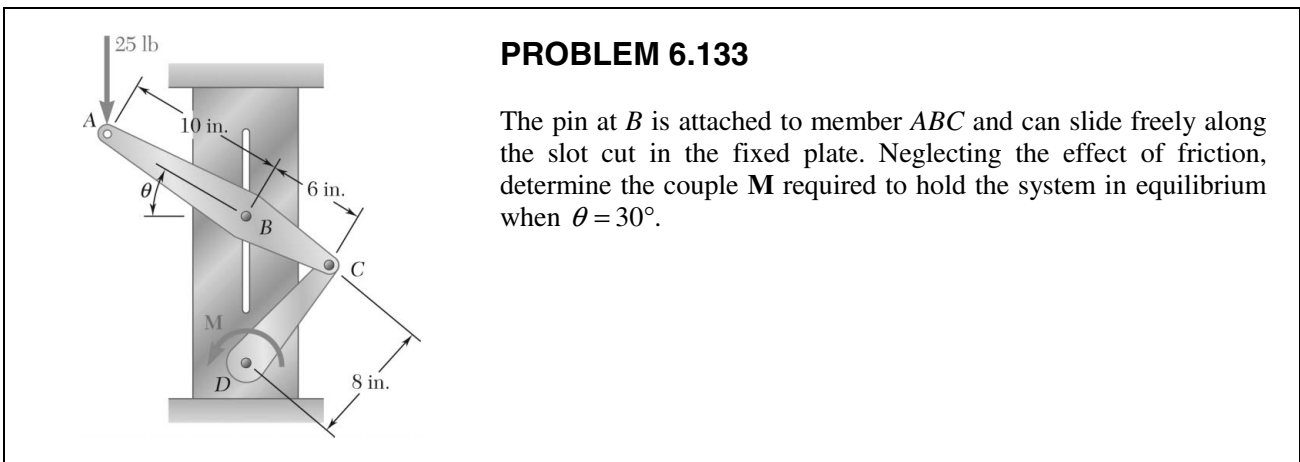


Note: $\tan \theta = \frac{2}{7}$ as above

FBD piston, as above: $C_y = P \tan \theta = \frac{2}{7} P$

FBD whole: $\left(\sum M_A = 0: (0.100 \text{ m}) C_y - M = 0 \quad M = (0.100 \text{ m}) \frac{2}{7} (16 \text{ kN}) \right.$

$$M = 0.45714 \text{ kN} \cdot \text{m} \quad \mathbf{M = 457 \text{ N} \cdot \text{m}} \left. \right) \blacktriangleleft$$



PROBLEM 6.133

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple M required to hold the system in equilibrium when $\theta = 30^\circ$.

SOLUTION

Free body: Member ABC:

$$\begin{aligned} +\curvearrowright \Sigma M_C = 0: & \quad (25 \text{ lb})(13.856 \text{ in.}) - B(3 \text{ in.}) = 0 \\ & \quad B = +115.47 \text{ lb} \end{aligned}$$

$$\begin{aligned} +\uparrow \Sigma F_y = 0: & \quad -25 \text{ lb} + C_y = 0 \\ & \quad C_y = +25 \text{ lb} \end{aligned}$$

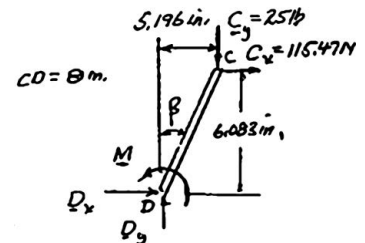
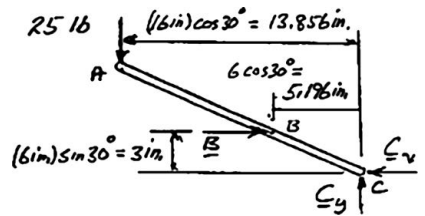
$$\begin{aligned} \pm \rightarrow \Sigma F_x = 0: & \quad 115.47 \text{ lb} - C_x = 0 \\ & \quad C_x = +115.47 \text{ lb} \end{aligned}$$

Free body: Member CD:

$$\beta = \sin^{-1} \frac{5.196}{8}; \quad \beta = 40.505^\circ$$

$$CD \cos \beta = (8 \text{ in.}) \cos 40.505^\circ = 6.083 \text{ in.}$$

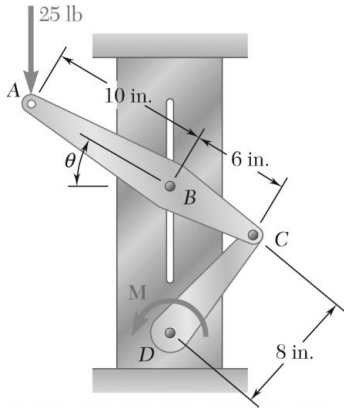
$$+\curvearrowright \Sigma M_D = 0: \quad M - (25 \text{ lb})(5.196 \text{ in.}) - (115.47 \text{ lb})(6.083 \text{ in.}) = 0$$



$$M = +832.3 \text{ lb} \cdot \text{in.}$$

$$M = 832 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

PROBLEM 6.134



The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple M required to hold the system in equilibrium when $\theta = 60^\circ$.

SOLUTION

Free body: Member ABC :

$$+\curvearrowright \Sigma M_C = 0: (25 \text{ lb})(8 \text{ in.}) - B(5.196 \text{ in.}) = 0$$

$$B = +38.49 \text{ lb}$$

$$\rightarrow \Sigma F_x = 0: 38.49 \text{ lb} - C_x = 0$$

$$C_x = +38.49 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: -25 \text{ lb} + C_y = 0$$

$$C_y = +25 \text{ lb}$$

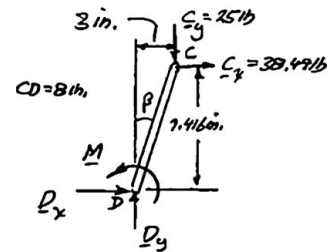
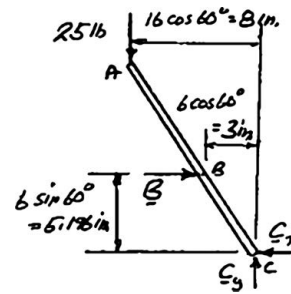
Free body: Member CD :

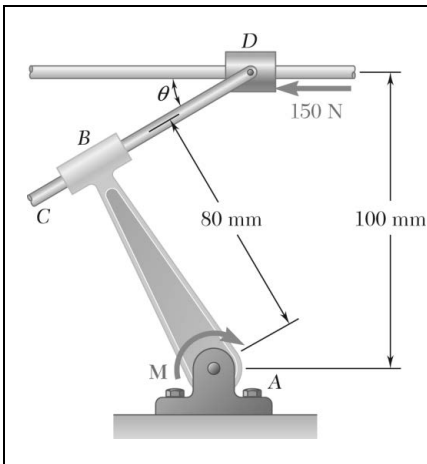
$$\beta = \sin^{-1} \frac{3}{8}; \quad \beta = 22.024^\circ$$

$$CD \cos \beta = (8 \text{ in.}) \cos 22.024^\circ = 7.416 \text{ in.}$$

$$+\curvearrowright \Sigma M_D = 0: M - (25 \text{ lb})(3 \text{ in.}) - (38.49 \text{ lb})(7.416 \text{ in.}) = 0$$

$$M = +360.4 \text{ lb} \cdot \text{in.} \quad M = 360 \text{ lb} \cdot \text{in.} \quad \curvearrowright \blacktriangleleft$$





PROBLEM 6.135

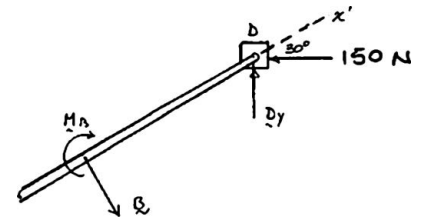
Rod CD is attached to the collar D and passes through a collar welded to end B of lever AB . Neglecting the effect of friction, determine the couple M required to hold the system in equilibrium when $\theta = 30^\circ$.

SOLUTION

FBD DC :

$$\sum F_x = 0: D_y \sin 30^\circ - (150 \text{ N}) \cos 30^\circ = 0$$

$$D_y = (150 \text{ N}) \cot 30^\circ = 259.81 \text{ N}$$



FBD machine:

$$\sum M_A = 0: (0.100 \text{ m})(150 \text{ N}) + d(259.81 \text{ N}) - M = 0$$

$$d = b - 0.040 \text{ m}$$

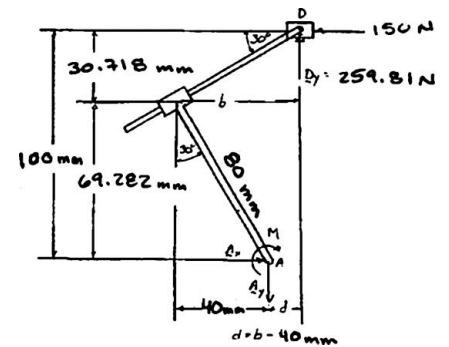
$$b = \frac{0.030718 \text{ m}}{\tan 30}$$

$$b = 0.053210 \text{ m}$$

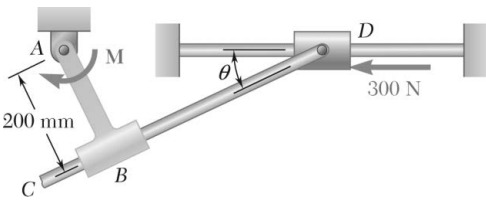
$$d = 0.0132100 \text{ m}$$

$$M = 18.4321 \text{ N} \cdot \text{m}$$

so



$$M = 18.43 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 6.136

Rod CD is attached to the collar D and passes through a collar welded to end B of lever AB . Neglecting the effect of friction, determine the couple M required to hold the system in equilibrium when $\theta = 30^\circ$.

SOLUTION

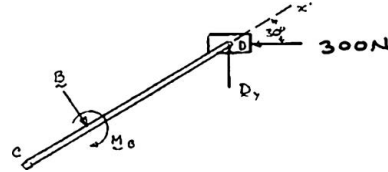
Note:

$$B \perp CD$$

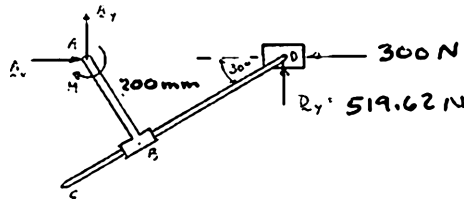
FBD DC :

$$\nearrow \Sigma F_{x'} = 0: D_y \sin 30^\circ - (300 \text{ N}) \cos 30^\circ = 0$$

$$D_y = \frac{300 \text{ N}}{\tan 30^\circ} = 519.62 \text{ N}$$



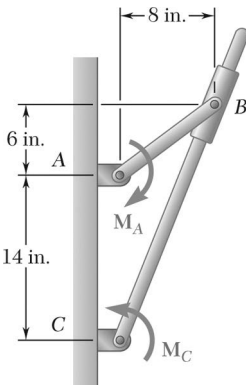
FBD machine:



$$\left(\Sigma M_A = 0: \frac{0.200 \text{ m}}{\sin 30^\circ} 519.62 \text{ N} - M = 0 \right.$$

$$M = 207.85 \text{ N} \cdot \text{m}$$

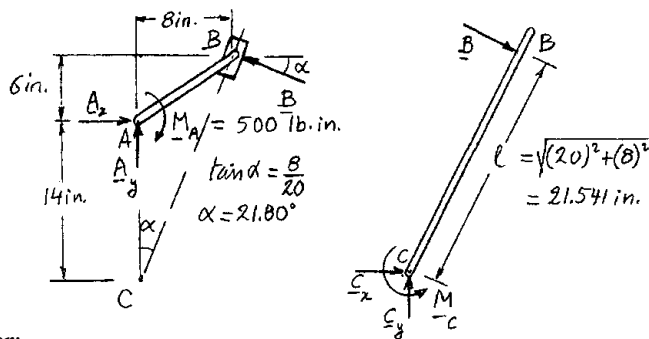
$$\mathbf{M = 208 \text{ N} \cdot \text{m}} \quad \blacktriangleleft$$



PROBLEM 6.137

Two rods are connected by a frictionless collar B . Knowing that the magnitude of the couple M_A is $500 \text{ lb} \cdot \text{in.}$, determine (a) the couple M_C required for equilibrium, (b) the corresponding components of the reaction at C .

SOLUTION



(a) Free body: Rod AB & collar:

$$\begin{aligned}
 +\curvearrowright \Sigma M_A = 0: & \quad (B \cos \alpha)(6 \text{ in.}) + (B \sin \alpha)(8 \text{ in.}) - M_A = 0 \\
 & \quad B = (6 \cos 21.8^\circ + 8 \sin 21.8^\circ) - 500 = 0 \\
 & \quad B = 58.535 \text{ lb}
 \end{aligned}$$

Free body: Rod BC:

$$\begin{aligned}
 +\Sigma M_C = 0: & \quad M_C - B\ell = 0 \\
 M_C = B\ell = & \quad (58.535 \text{ lb})(21.541 \text{ in.}) = 1260.9 \text{ lb} \cdot \text{in.} \quad M_C = 1261 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft
 \end{aligned}$$

(b)

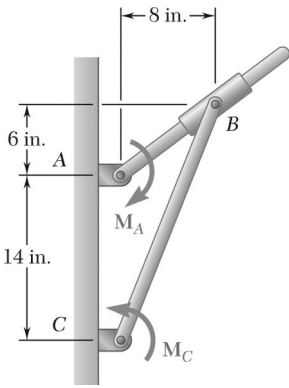
$$\begin{aligned}
 \pm \rightarrow \Sigma F_x = 0: & \quad C_x + B \cos \alpha = 0 \\
 C_x = -B \cos \alpha = & \quad -(58.535 \text{ lb}) \cos 21.8^\circ = -54.3 \text{ lb}
 \end{aligned}$$

$$C_x = 54.3 \text{ lb} \quad \blacktriangleleft$$

$$\begin{aligned}
 +\uparrow \Sigma F_y = 0: & \quad C_y - B \sin \alpha = 0 \\
 C_y = B \sin \alpha = & \quad (58.535 \text{ lb}) \sin 21.8^\circ = +21.7 \text{ lb}
 \end{aligned}$$

$$C_y = 21.7 \text{ lb} \quad \blacktriangleup$$

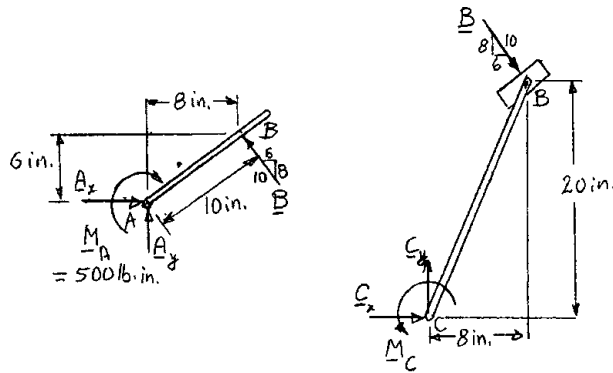
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PROBLEM 6.138

Two rods are connected by a frictionless collar B . Knowing that the magnitude of the couple M_A is $500 \text{ lb} \cdot \text{in.}$, determine (a) the couple M_C required for equilibrium, (b) the corresponding components of the reaction at C .

SOLUTION



(a) Free body: Rod AB:

$$+\curvearrowright \Sigma M_A = 0: B(10 \text{ in.}) - 500 \text{ lb} \cdot \text{in.} = 0$$

$$B = 50.0$$

Free body: Rod BC & collar:

$$+\curvearrowright \Sigma M_C = 0: M_C - (0.6B)(20 \text{ in.}) - (0.8B)(8 \text{ in.}) = 0$$

$$M_C - (30 \text{ lb})(20 \text{ in.}) - (40 \text{ lb})(8 \text{ in.}) = 0$$

$$M_C = 920 \text{ lb} \cdot \text{in.}$$

$$M_C = 920 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$

(b) $+\rightarrow \Sigma F_x = 0: C_x + 0.6B = 0$

$$C_x = -0.6B = -0.6(50.0 \text{ lb}) = -30.0 \text{ lb}$$

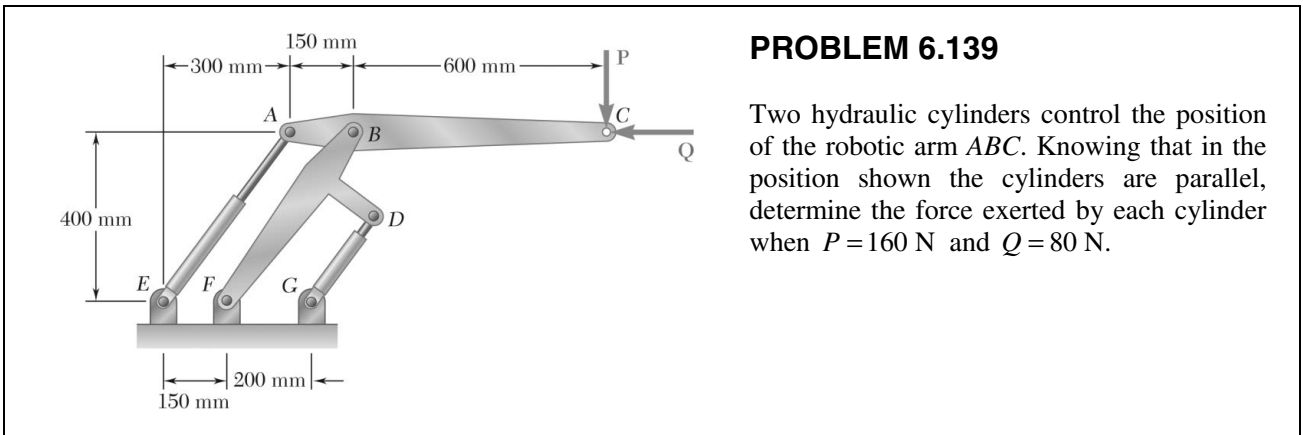
$$C_x = 30.0 \text{ lb} \leftarrow \blacktriangleleft$$

$+\uparrow \Sigma F_y = 0: C_y - 0.8B = 0$

$$C_y = 0.8B = 0.8(50.0 \text{ lb}) = +40.0 \text{ lb}$$

$$C_y = 40.0 \text{ lb} \uparrow \blacktriangleleft$$

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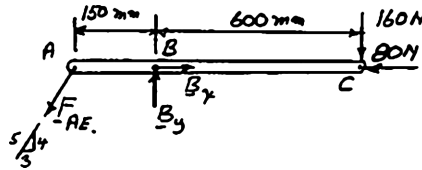


PROBLEM 6.139

Two hydraulic cylinders control the position of the robotic arm ABC. Knowing that in the position shown the cylinders are parallel, determine the force exerted by each cylinder when $P = 160 \text{ N}$ and $Q = 80 \text{ N}$.

SOLUTION

Free body: Member ABC:



$$+\curvearrowright \Sigma M_B = 0: \quad \frac{4}{5} F_{AE} (150 \text{ mm}) - (160 \text{ N})(600 \text{ mm}) = 0$$

$$F_{AE} = +800 \text{ N} \qquad F_{AE} = 800 \text{ N} \quad T \quad \blacktriangleleft$$

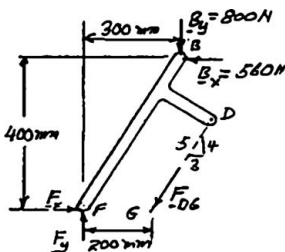
$$+\rightarrow \Sigma F_x = 0: \quad -\frac{3}{5} (800 \text{ N}) + B_x - 80 \text{ N} = 0$$

$$B_x = +560 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5} (800 \text{ N}) + B_y - 160 \text{ N} = 0$$

$$B_y = +800 \text{ N}$$

Free body: Member BDF:



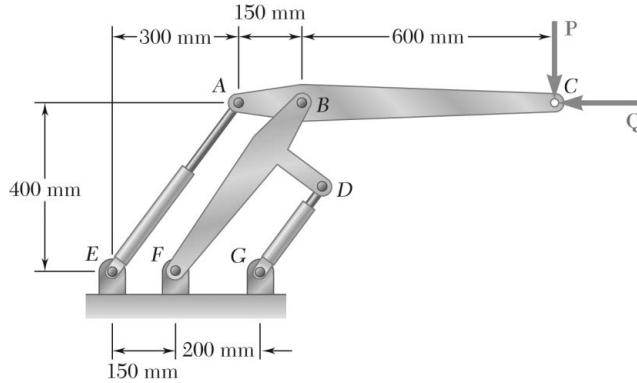
$$+\curvearrowright \Sigma M_F = 0: \quad (560 \text{ N})(400 \text{ mm}) - (800 \text{ N})(300 \text{ mm}) - \frac{4}{5} F_{DG} (200 \text{ mm}) = 0$$

$$F_{DG} = -100 \text{ N} \qquad F_{DG} = 100.0 \text{ N} \quad C \quad \blacktriangleleft$$

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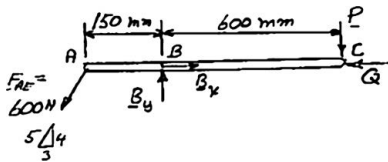
PROBLEM 6.140

Two hydraulic cylinders control the position of the robotic arm ABC . In the position shown, the cylinders are parallel and both are in tension. Knowing the $F_{AE} = 600 \text{ N}$ and $F_{DG} = 50 \text{ N}$, determine the forces \mathbf{P} and \mathbf{Q} applied at C to arm ABC .



SOLUTION

Free body: Member ABC :



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}(600 \text{ N})(150 \text{ mm}) - P(600 \text{ mm}) = 0$$

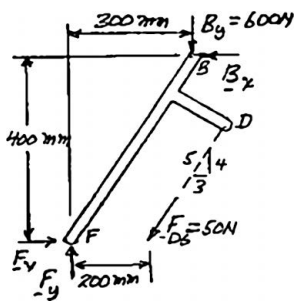
$$P = +120 \text{ N}$$

$$\mathbf{P} = 120.0 \text{ N} \downarrow \leftarrow$$

$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{4}{5}(600)(750 \text{ mm}) - B_y(600 \text{ mm}) = 0$$

$$B_y = +600 \text{ N}$$

Free body: Member BDF :



$$+\circlearrowleft \Sigma M_F = 0: \quad B_x(400 \text{ mm}) - (600 \text{ N})(300 \text{ mm})$$

$$- \frac{4}{5}(50 \text{ N})(200 \text{ mm}) = 0$$

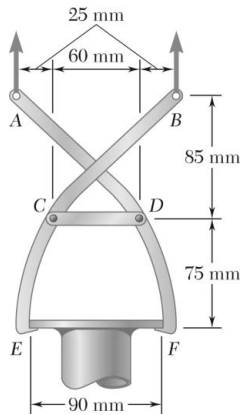
$$B_x = +470 \text{ N}$$

Return to free body: Member ABC :

$$\pm \Sigma F_x = 0: \quad -\frac{3}{5}(600 \text{ N}) + 470 \text{ N} - Q = 0$$

$$Q = +110 \text{ N}$$

$$\mathbf{Q} = 110.0 \text{ N} \leftarrow \leftarrow$$

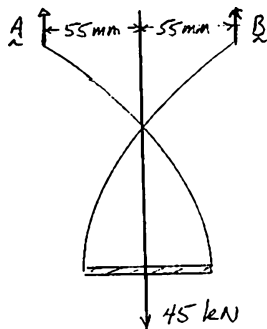


PROBLEM 6.141

The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the forces exerted at D and F on tong ADF .

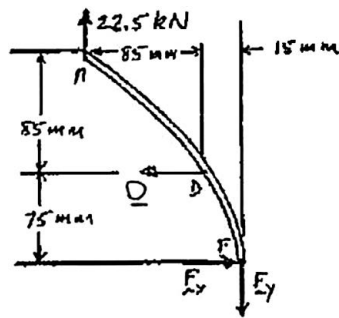
SOLUTION

FBD whole:



By symmetry, $A = B = 22.5 \text{ kN} \uparrow$

FBD ADF :



$$\left(\sum M_F = 0: (75 \text{ mm})D - (100 \text{ mm})(22.5 \text{ kN}) = 0 \right.$$

$$D = 30.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$\rightarrow \sum F_x = 0: F_x - D = 0$$

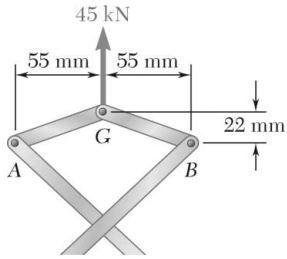
$$F_x = D = 30 \text{ kN}$$

$$\uparrow \sum F_y = 0: 22.5 \text{ kN} - F_y = 0$$

$$F_y = 22.5 \text{ kN}$$

so

$$F = 37.5 \text{ kN} \swarrow 36.9^\circ \blacktriangleleft$$



PROBLEM 6.142

If the toggle shown is added to the tongs of Problem 6.141 and a single vertical force is applied at G , determine the forces exerted at D and F on tong ADF .

SOLUTION

Free body: Toggle:

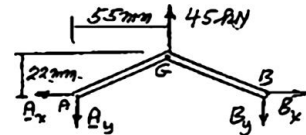
By symmetry,

$$A_y = \frac{1}{2}(45 \text{ kN}) = 22.5 \text{ kN}$$

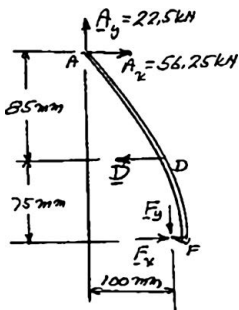
AG is a two-force member.

$$\frac{22.5 \text{ kN}}{22 \text{ mm}} = \frac{A_x}{55 \text{ mm}}$$

$$A_x = 56.25 \text{ kN}$$



Free body: Tong ADF :



$$+\uparrow \Sigma F_y = 0: 22.5 \text{ kN} - F_y = 0$$

$$F_y = +22.5 \text{ kN}$$

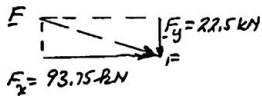
$$+\curvearrowright \Sigma M_F = 0: D(75 \text{ mm}) - (22.5 \text{ kN})(100 \text{ mm}) - (56.25 \text{ kN})(160 \text{ mm}) = 0$$

$$D = +150 \text{ kN}$$

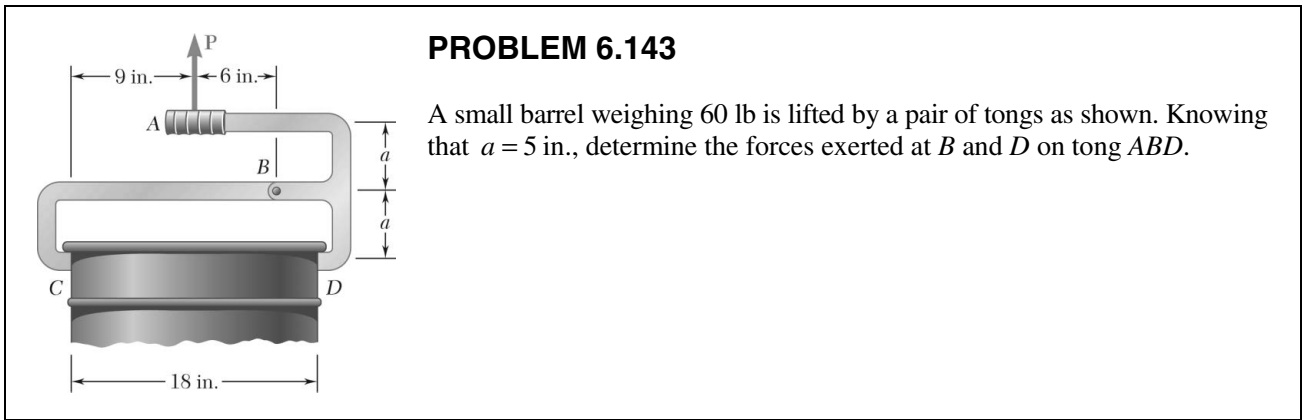
$$D = 150.0 \text{ kN} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: 56.25 \text{ kN} - 150 \text{ kN} + F_x = 0$$

$$F_x = 93.75 \text{ kN}$$



$$F = 96.4 \text{ kN} \searrow 13.50^\circ$$



SOLUTION

We note that BC is a two-force member.

Free body: Tong ABD :

$$\frac{B_x}{15} = \frac{B_y}{5} \quad B_x = 3B_y$$

$$+\curvearrowright \Sigma M_D = 0: \quad B_y(3 \text{ in.}) + 3B_y(5 \text{ in.}) - (60 \text{ lb})(9 \text{ in.}) = 0$$

$$B_y = 30 \text{ lb} \downarrow$$

$$B_x = 3B_y: \quad B_x = 90 \text{ lb} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: \quad -90 \text{ lb} + D_x = 0$$

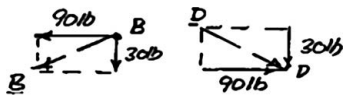
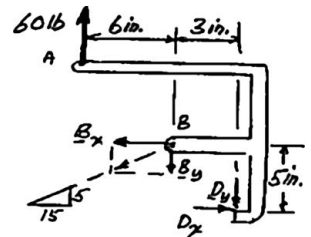
$$D_x = 90 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 60 \text{ lb} - 30 \text{ lb} - D_y = 0$$

$$D_y = 30 \text{ lb} \downarrow$$

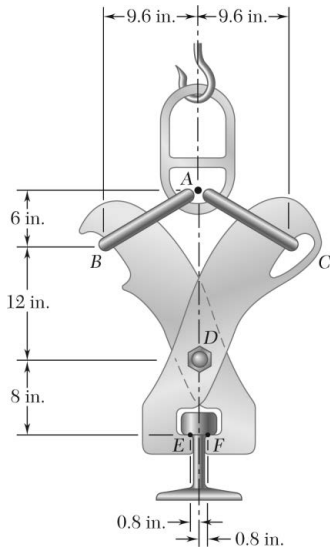
$$B = 94.9 \text{ lb} \nearrow 18.43^\circ \blacktriangleleft$$

$$D = 94.9 \text{ lb} \searrow 18.43^\circ \blacktriangleleft$$



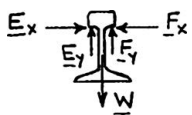
PROBLEM 6.144

A 39-ft length of railroad rail of weight 44 lb/ft is lifted by the tongs shown. Determine the forces exerted at D and F on tong BDF .



SOLUTION

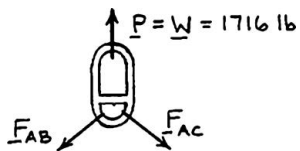
Free body: Rail:



$$W = (39 \text{ ft})(44 \text{ lb/ft}) = 1716 \text{ lb}$$

By symmetry,
$$E_y = F_y = \frac{1}{2}W = 858 \text{ lb}$$

Free body: Upper link:

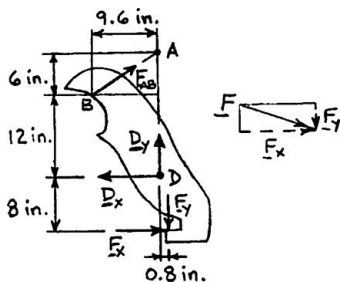


By symmetry,
$$(F_{AB})_y = (F_{AC})_y = \frac{1}{2}W = 858 \text{ lb}$$

Since AB is a two-force member,

$$\frac{(F_{AB})_x}{9.6} = \frac{(F_{AB})_y}{6} \quad (F_{AB})_x = \frac{9.6}{6}(858) = 1372.8 \text{ lb}$$

Free Body: Tong BDF :



$$+\curvearrowright \Sigma M_D = 0: \quad (\text{Attach } F_{AB} \text{ at } A.)$$

$$F_x(8) - (F_{AB})_x(18) - F_y(0.8) = 0$$

$$F_x(8) - (1372.8 \text{ lb})(18) - (858 \text{ lb})(0.8) = 0$$

$$F_x = +3174.6 \text{ lb}$$

$$\mathbf{F} = 3290 \text{ lb} \angle 15.12^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad -D_x + (F_{AB})_x + F_x = 0$$

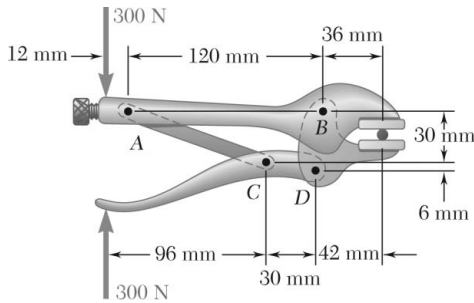
$$D_x = (F_{AB})_x + F_x = 1372.8 + 3174.6 = 4547.4 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + (F_{AB})_y - F_y = 0$$

$$D_y = 0$$

$$\mathbf{D} = 4550 \text{ lb} \leftarrow \blacktriangleleft$$

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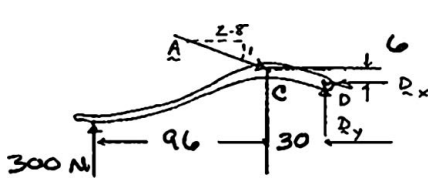
PROBLEM 6.145

Determine the magnitude of the gripping forces produced when two 300-N forces are applied as shown.

SOLUTION

We note that AC is a two-force member.

FBD handle CD :

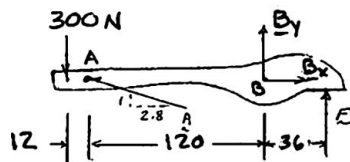


Dimensions in mm

$$\begin{aligned} \left(\sum M_D = 0: \right. & \quad -(126 \text{ mm})(300 \text{ N}) - (6 \text{ mm}) \frac{2.8}{\sqrt{8.84}} A \\ & \quad \left. + (30 \text{ mm}) \left(\frac{1}{\sqrt{8.84}} A \right) = 0 \right. \end{aligned}$$

$$A = 2863.6\sqrt{8.84} \text{ N}$$

FBD handle AB :

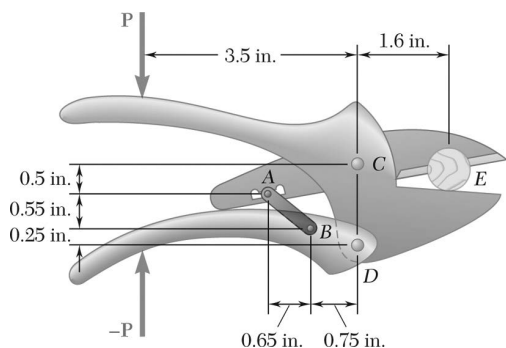


Dimensions in mm

$$\begin{aligned} \left(\sum M_B = 0: \right. & \quad (132 \text{ mm})(300 \text{ N}) - (120 \text{ mm}) \frac{1}{\sqrt{8.84}} (2863.6\sqrt{8.84} \text{ N}) \\ & \quad \left. + (36 \text{ mm})F = 0 \right. \end{aligned}$$

$$F = 8.45 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 6.146

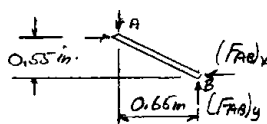


The compound-lever pruning shears shown can be adjusted by placing pin A at various ratchet positions on blade ACE . Knowing that 300-lb vertical forces are required to complete the pruning of a small branch, determine the magnitude P of the forces that must be applied to the handles when the shears are adjusted as shown.

SOLUTION

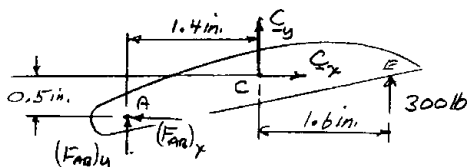
We note that AB is a two-force member.

$$\frac{(F_{AB})_x}{0.65 \text{ in.}} = \frac{(F_{AB})_y}{0.55 \text{ in.}}$$



$$(F_{AB})_y = \frac{11}{13}(F_{AB})_x \quad (1)$$

Free body: Blade ACE :



$$+\curvearrowright \Sigma M_C = 0: (300 \text{ lb})(1.6 \text{ in.}) - (F_{AB})_x(0.5 \text{ in.}) - (F_{AB})_y(1.4 \text{ in.}) = 0$$

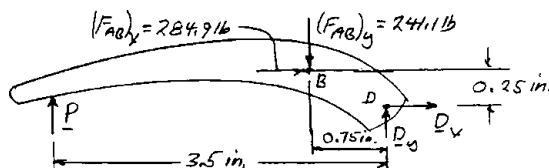
Use Eq. (1):

$$(F_{AB})_x(0.5 \text{ in.}) + \frac{11}{13}(F_{AB})_x(1.4 \text{ in.}) = 480 \text{ lb} \cdot \text{in.}$$

$$1.6846(F_{AB})_x = 480 \quad (F_{AB})_x = 284.9 \text{ lb}$$

$$(F_{AB})_y = \frac{11}{13}(284.9 \text{ lb}) \quad (F_{AB})_y = 241.1 \text{ lb}$$

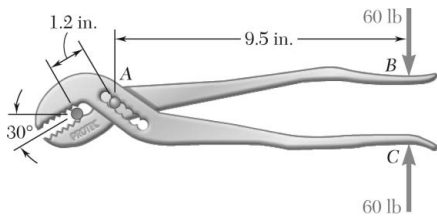
Free body: Lower handle:



$$+\curvearrowright \Sigma M_D = 0: (241.1 \text{ lb})(0.75 \text{ in.}) - (284.9 \text{ lb})(0.25 \text{ in.}) - P(3.5 \text{ in.}) = 0$$

$$P = 31.3 \text{ lb} \quad \blacktriangleleft$$

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PROBLEM 6.147

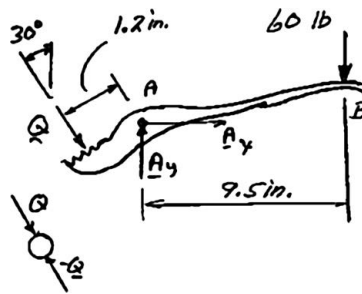
The pliers shown are used to grip a 0.3-in.-diameter rod. Knowing that two 60-lb forces are applied to the handles, determine (a) the magnitude of the forces exerted on the rod, (b) the force exerted by the pin at A on portion AB of the pliers.

SOLUTION

Free body: Portion AB:

$$(a) \quad +\curvearrowright \Sigma M_A = 0: \quad Q(1.2 \text{ in.}) - (60 \text{ lb})(9.5 \text{ in.}) = 0$$

$$Q = 475 \text{ lb} \quad \blacktriangleleft$$



$$(b) \quad +\rightarrow \Sigma F_x = 0: \quad Q(\sin 30^\circ) + A_x = 0$$

$$(475 \text{ lb})(\sin 30^\circ) + A_x = 0$$

$$A_x = -237.5 \text{ lb}$$

$$A_x = 237.5 \text{ lb} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -Q(\cos 30^\circ) + A_y - 60 \text{ lb} = 0$$

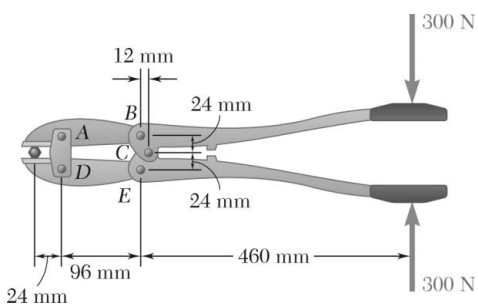
$$-(475 \text{ lb})(\cos 30^\circ) + A_y - 60 \text{ lb} = 0$$

$$A_y = +471.4 \text{ lb}$$

$$A_y = 471.4 \text{ lb} \quad \uparrow$$



$$A = 528 \text{ lb} \quad \searrow 63.3^\circ \quad \blacktriangleleft$$

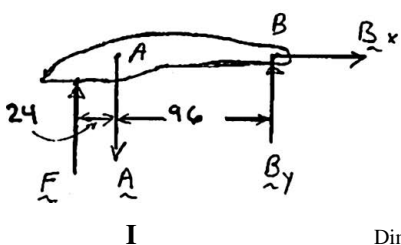


PROBLEM 6.148

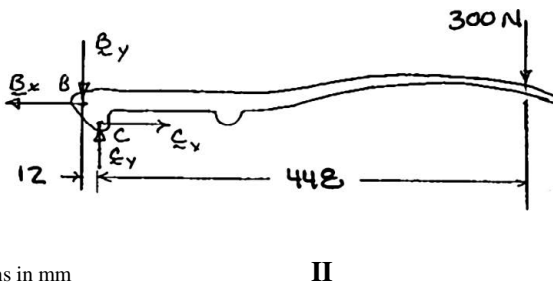
In using the bolt cutter shown, a worker applies two 300-N forces to the handles. Determine the magnitude of the forces exerted by the cutter on the bolt.

SOLUTION

FBD cutter AB:



FBD handle BC:



Dimensions in mm

$$\text{FBD I: } \rightarrow \Sigma F_x = 0: B_x = 0$$

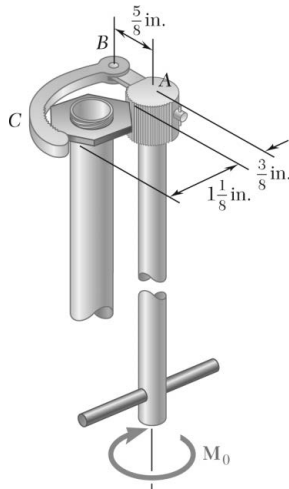
$$\text{FBD II: } \curvearrowleft \Sigma M_C = 0: (12 \text{ mm})B_y - (448 \text{ mm})300 \text{ N} = 0$$

$$B_y = 11,200.0 \text{ N}$$

Then

$$\text{FBD I: } \curvearrowleft \Sigma M_A = 0: (96 \text{ mm})B_y - (24 \text{ mm})F = 0 \quad F = 4B_y$$

$$F = 44,800 \text{ N} = 44.8 \text{ kN} \quad \blacktriangleleft$$



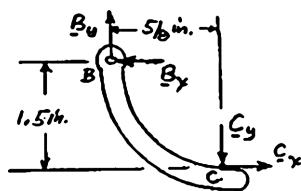
PROBLEM 6.149

The specialized plumbing wrench shown is used in confined areas (e.g., under a basin or sink). It consists essentially of a jaw BC pinned at B to a long rod. Knowing that the forces exerted on the nut are equivalent to a clockwise (when viewed from above) couple of magnitude $135 \text{ lb} \cdot \text{in.}$, determine (a) the magnitude of the force exerted by pin B on jaw BC , (b) the couple \mathbf{M}_0 that is applied to the wrench.

SOLUTION

Free body: Jaw BC :

This is a two-force member.



$$\frac{C_y}{1.5 \text{ in.}} = \frac{C_x}{\frac{5}{8} \text{ in.}} \quad C_y = 2.4 C_x$$

$$\Sigma F_x = 0: \quad B_x = C_x \quad (1)$$

$$\Sigma F_y = 0: \quad B_y = C_y = 2.4 C_x \quad (2)$$

Free body: Nut: $\Sigma F_x = 0: \quad C_x = D_x$

$$\Sigma M = 135 \text{ lb} \cdot \text{in.}$$

$$C_x (1.125 \text{ in.}) = 135 \text{ lb} \cdot \text{in.}$$

$$C_x = 120 \text{ lb}$$

(a) Eq. (1): $B_x = C_x = 120 \text{ lb}$

Eq. (2): $B_y = C_y = 2.4(120 \text{ lb}) = 288 \text{ lb}$

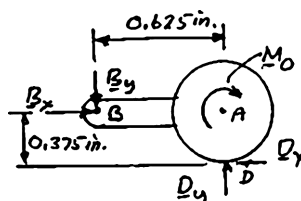
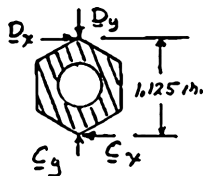
$$B = (B_x^2 + B_y^2)^{1/2} = (120^2 + 288^2)^{1/2} \quad B = 312 \text{ lb} \quad \blacktriangleleft$$

(b) Free body: Rod:

$$+\Sigma M_D = 0: \quad -M_0 + B_y(0.625 \text{ in.}) - B_x(0.375 \text{ in.}) = 0$$

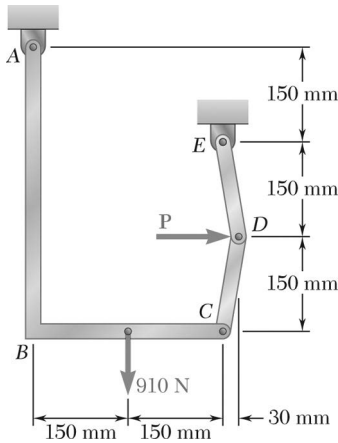
$$-M_0 + (288)(0.625) - (120)(0.375) = 0$$

$$\mathbf{M}_0 = 135.0 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$



PROBLEM 6.150

Determine the force \mathbf{P} that must be applied to the toggle CDE to maintain bracket ABC in the position shown.



SOLUTION

We note that CD and DE are two-force members.

Free body: Joint D :

$$\frac{(F_{CD})_x}{30} = \frac{(F_{CD})_y}{150} \quad (F_{CD})_y = 5(F_{CD})_x$$

Similarly,

$$(F_{DE})_y = 5(F_{DE})_x$$

$$+\uparrow \Sigma F_y = 0: (F_{DE})_y = (F_{CD})_y$$

It follows that

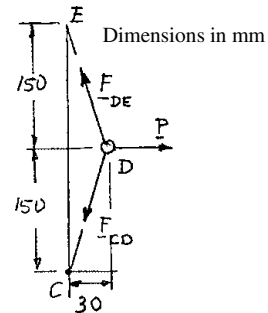
$$(F_{DE})_x = (F_{CD})_x$$

$$+\rightarrow \Sigma F_x = 0: P - (F_{DE})_x - (F_{CD})_x = 0$$

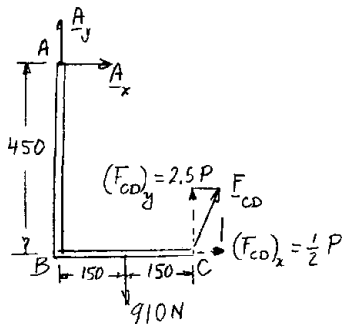
$$(F_{DE})_x = (F_{CD})_x = \frac{1}{2}P$$

Also,

$$(F_{DE})_y = (F_{CD})_y = 5\left(\frac{1}{2}P\right) = 2.5P$$



Free body: Member ABC :



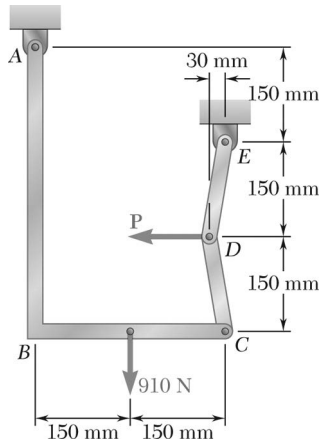
$$+\curvearrowright \Sigma M_A = 0: (2.5P)(300) + \left(\frac{1}{2}P\right)(450) - (910 \text{ N})(150) = 0$$

$$(750 + 225)P = (910 \text{ N})(150)$$

$$P = 140.0 \text{ N} \quad \blacktriangleleft$$

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PROBLEM 6.151



Determine the force P that must be applied to the toggle CDE to maintain bracket ABC in the position shown.

SOLUTION

We note that CD and DE are two-force members.

Free body: Joint D :

$$\frac{(F_{CD})_x}{30} = \frac{(F_{CD})_y}{150} \quad (F_{CD})_y = 5(F_{CD})_x$$

Similarly,

$$(F_{DE})_y = 5(F_{DE})_x$$

$$+\uparrow \Sigma F_y = 0: (F_{DE})_y = (F_{CD})_y$$

It follows that

$$(F_{DE})_x = (F_{CD})_x$$

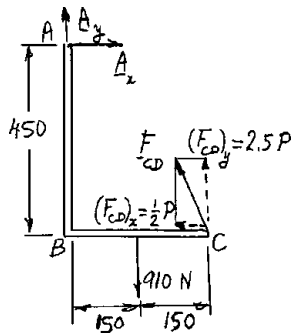
$$+\rightarrow \Sigma F_x = 0: (F_{DE})_x + (F_{CD})_x - P = 0$$

$$(F_{DE})_x = (F_{CD})_x = \frac{1}{2}P$$

Also,

$$(F_{DE})_y = (F_{CD})_y = 5\left(\frac{1}{2}P\right) = 2.5P$$

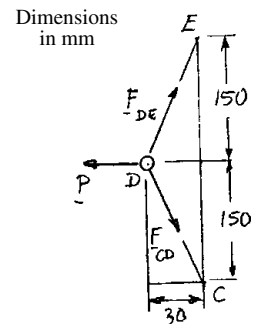
Free body: Member ABC :

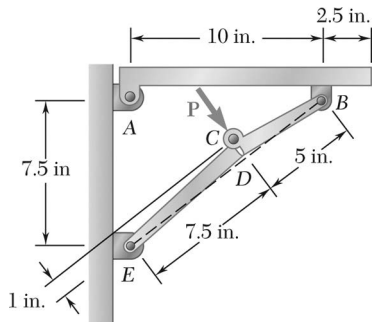


$$+\curvearrowright \Sigma M_A = 0: (2.5P)(300) - \left(\frac{1}{2}P\right)(450) - (910 \text{ N})(150) = 0$$

$$(750 - 225)P = (910 \text{ N})(150)$$

$$P = 260 \text{ N} \quad \blacktriangleleft$$



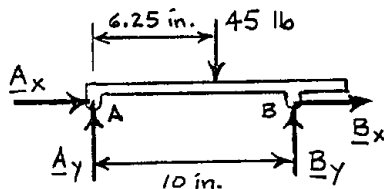


PROBLEM 6.152

A 45-lb shelf is held horizontally by a self-locking brace that consists of two parts EDC and CDB hinged at C and bearing against each other at D . Determine the force P required to release the brace.

SOLUTION

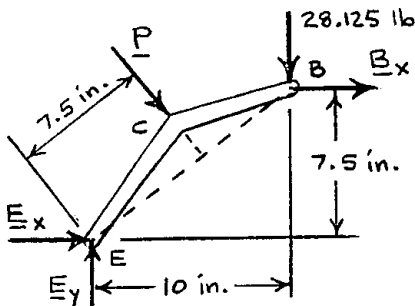
Free body: Shelf:



$$+\circlearrowleft \Sigma M_A = 0: B_y(10 \text{ in.}) - (45 \text{ lb})(6.25 \text{ in.}) = 0$$

$$B_y = 28.125 \text{ lb}$$

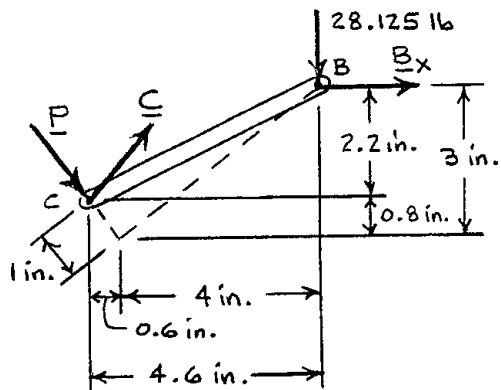
Free body: Portion ECB :



$$+\circlearrowleft \Sigma M_E = 0: -B_x(7.5 \text{ in.}) - P(7.5 \text{ in.}) - (28.125 \text{ lb})(10 \text{ in.}) = 0$$

$$B_x = -37.5 - P$$

Free body: Portion CDB :

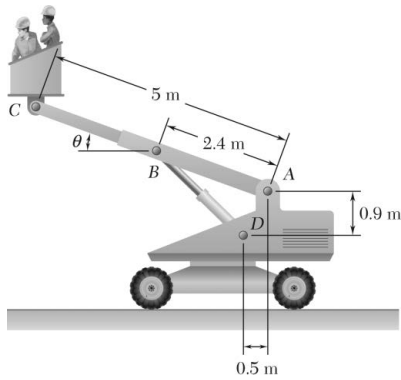


$$+\circlearrowleft \Sigma M_C = 0: -(28.125 \text{ lb})(4.6 \text{ in.}) - B_x(2.2 \text{ in.}) = 0$$

$$-(28.125 \text{ lb})(4.6 \text{ in.}) - (-37.5 - P)(2.2 \text{ in.}) = 0$$

$$P = 21.3 \text{ lb} \quad \mathbf{P} = 21.3 \text{ lb} \quad \swarrow \blacktriangleleft$$

PROBLEM 6.153



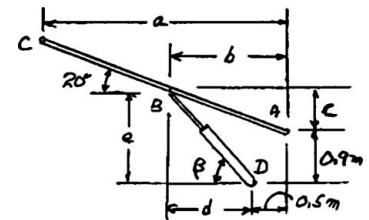
The telescoping arm ABC is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 200 kg and have a combined center of gravity located directly above C . For the position when $\theta = 20^\circ$, determine (a) the force exerted at B by the single hydraulic cylinder BD , (b) the force exerted on the supporting carriage at A .

SOLUTION

Geometry:

$$\begin{aligned} a &= (5 \text{ m}) \cos 20^\circ = 4.6985 \text{ m} \\ b &= (2.4 \text{ m}) \cos 20^\circ = 2.2553 \text{ m} \\ c &= (2.4 \text{ m}) \sin 20^\circ = 0.8208 \text{ m} \\ d &= b - 0.5 = 1.7553 \text{ m} \\ e &= c + 0.9 = 1.7208 \text{ m} \end{aligned}$$

$$\tan \beta = \frac{e}{d} = \frac{1.7208}{1.7553}; \quad \beta = 44.43^\circ$$



Free body: Arm ABC :

We note that BD is a two-force member.

$$W = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1.962 \text{ kN}$$

$$(a) \quad +\curvearrowright \Sigma M_A = 0: \quad (1.962 \text{ kN})(4.6985 \text{ m}) - F_{BD} \sin 44.43^\circ(2.2553 \text{ m}) + F_{BD} \cos 44.43^\circ(0.8208 \text{ m}) = 0$$

$$9.2185 - F_{BD}(0.9927) = 0: \quad F_{BD} = 9.2867 \text{ kN}$$

(b)

$$F_{BD} = 9.29 \text{ kN} \nearrow 44.4^\circ \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = 0: \quad A_x - F_{BD} \cos \beta = 0$$

$$A_x = (9.2867 \text{ kN}) \cos 44.43^\circ = 6.632 \text{ kN} \quad A_x = 6.632 \text{ kN} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1.962 \text{ kN} + F_{BD} \sin \beta = 0$$

$$A_y = 1.962 \text{ kN} - (9.2867 \text{ kN}) \sin 44.43^\circ = -4.539 \text{ kN}$$

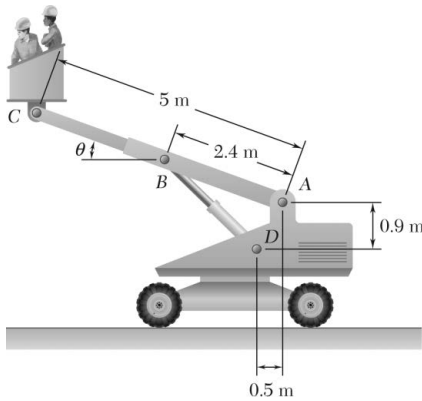


$$A_y = 4.539 \text{ kN} \downarrow$$

$$A = 8.04 \text{ kN} \searrow 34.4^\circ \blacktriangleleft$$

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PROBLEM 6.154



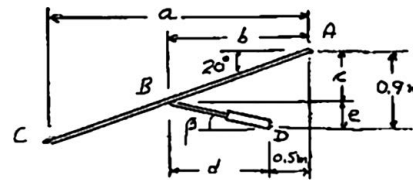
The telescoping arm ABC can be lowered until end C is close to the ground, so that workers can easily board the platform. For the position when $\theta = -20^\circ$, determine (a) the force exerted at B by the single hydraulic cylinder BD , (b) the force exerted on the supporting carriage at A .

SOLUTION

Geometry:

$$\begin{aligned} a &= (5 \text{ m}) \cos 20^\circ = 4.6985 \text{ m} \\ b &= (2.4 \text{ m}) \cos 20^\circ = 2.2552 \text{ m} \\ c &= (2.4 \text{ m}) \sin 20^\circ = 0.8208 \text{ m} \\ d &= b - 0.5 = 1.7553 \text{ m} \\ e &= 0.9 - c = 0.0792 \text{ m} \end{aligned}$$

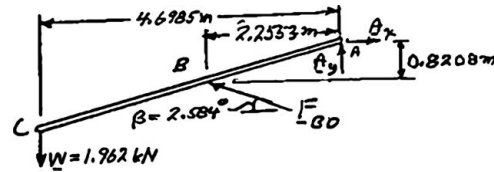
$$\tan \beta = \frac{e}{d} = \frac{0.0792}{1.7552}; \quad \beta = 2.584^\circ$$



Free body: Arm ABC :

We note that BD is a two-force member.

$$\begin{aligned} W &= (200 \text{ kg})(9.81 \text{ m/s}^2) \\ W &= 1962 \text{ N} = 1.962 \text{ kN} \end{aligned}$$

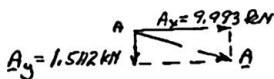


$$\begin{aligned} (a) \quad +\curvearrowright \Sigma M_A = 0: & (1.962 \text{ kN})(4.6985 \text{ m}) - F_{BD} \sin 2.584^\circ(2.2553 \text{ m}) - F_{BD} \cos 2.584^\circ(0.8208 \text{ m}) = 0 \\ & 9.2185 - F_{BD}(0.9216) = 0 \quad F_{BD} = 10.003 \text{ kN} \end{aligned}$$

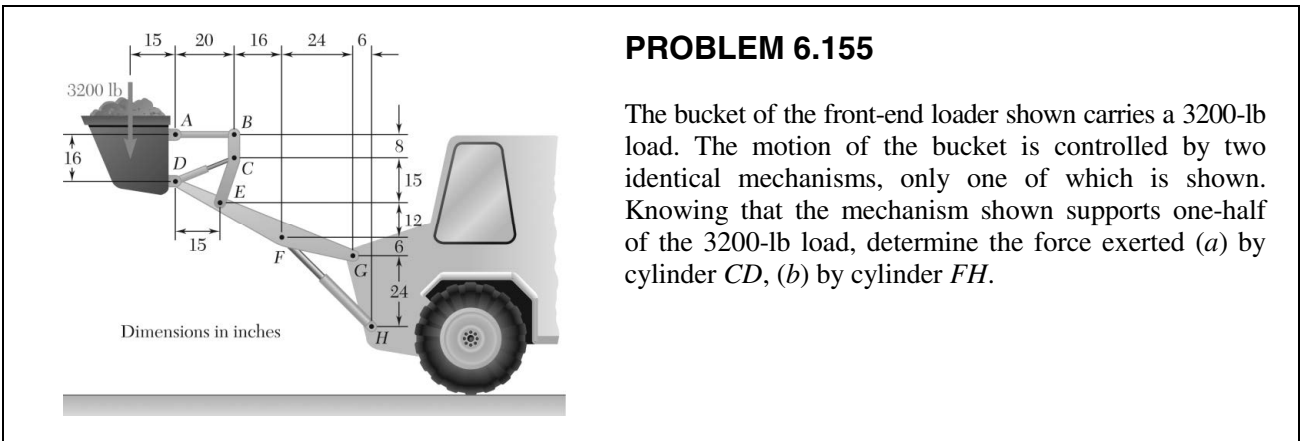
$$(b) \quad \mathbf{F}_{BD} = 10.00 \text{ kN} \nearrow 2.58^\circ \blacktriangleleft$$

$$\begin{aligned} +\rightarrow \Sigma F_x = 0: & A_x - F_{BD} \cos \beta = 0 \\ & A_x = (10.003 \text{ kN}) \cos 2.583^\circ = 9.993 \text{ kN} \quad \mathbf{A_x} = 9.993 \text{ kN} \rightarrow \end{aligned}$$

$$\begin{aligned} +\uparrow \Sigma F_y = 0: & A_y - 1.962 \text{ kN} + F_{BD} \sin \beta = 0 \\ & A_y = 1.962 \text{ kN} - (10.003 \text{ kN}) \sin 2.583^\circ = -1.5112 \text{ kN} \end{aligned}$$



$$\begin{aligned} \mathbf{A_y} &= 1.5112 \text{ kN} \downarrow \\ \mathbf{A} &= 10.11 \text{ kN} \searrow 8.60^\circ \blacktriangleleft \end{aligned}$$



PROBLEM 6.155

The bucket of the front-end loader shown carries a 3200-lb load. The motion of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism shown supports one-half of the 3200-lb load, determine the force exerted (a) by cylinder CD , (b) by cylinder FH .

SOLUTION

Free body: Bucket: (one mechanism)

$$+\curvearrowright \Sigma M_D = 0: (1600 \text{ lb})(15 \text{ in.}) - F_{AB}(16 \text{ in.}) = 0$$

$$F_{AB} = 1500 \text{ lb}$$

Note: There are two identical support mechanisms.

Free body: One arm BCE :

$$\tan \beta = \frac{8}{20}$$

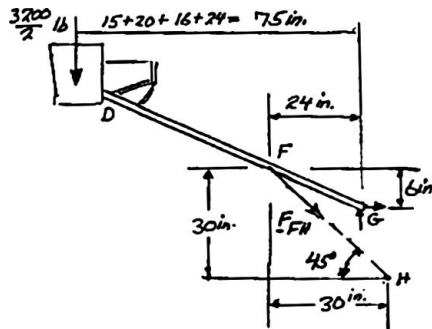
$$\beta = 21.8^\circ$$

$$+\curvearrowright \Sigma M_E = 0: (1500 \text{ lb})(23 \text{ in.}) + F_{CD} \cos 21.8^\circ(15 \text{ in.}) - F_{CD} \sin 21.8^\circ(5 \text{ in.}) = 0$$

$$F_{CD} = -2858 \text{ lb}$$

$$F_{CD} = 2.86 \text{ kips} \quad C \blacktriangleleft$$

Free body: Arm DFG :



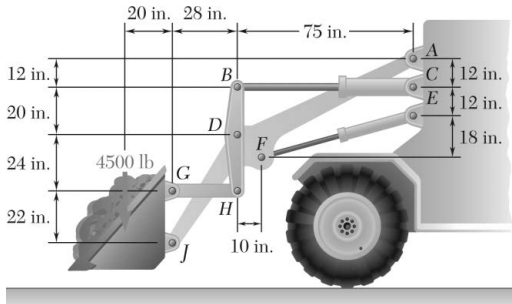
$$+\curvearrowright \Sigma M_G = 0: (1600 \text{ lb})(75 \text{ in.}) + F_{FH} \sin 45^\circ(24 \text{ in.}) - F_{FH} \cos 45^\circ(6 \text{ in.}) = 0$$

$$F_{FH} = -9.428 \text{ kips}$$

$$F_{FH} = 9.43 \text{ kips} \quad C \blacktriangleleft$$

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PROBLEM 6.156



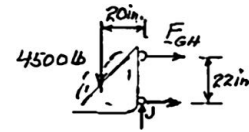
The motion of the bucket of the front-end loader shown is controlled by two arms and a linkage that are pin-connected at D . The arms are located symmetrically with respect to the central, vertical, and longitudinal plane of the loader; one arm AFJ and its control cylinder EF are shown. The single linkage $GHDB$ and its control cylinder BC are located in the plane of symmetry. For the position and loading shown, determine the force exerted (a) by cylinder BC , (b) by cylinder EF .

SOLUTION

Free body: Bucket

$$+\circlearrowleft \Sigma M_J = 0: (4500 \text{ lb})(20 \text{ in.}) - F_{GH}(22 \text{ in.}) = 0$$

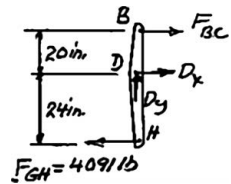
$$F_{GH} = 4091 \text{ lb}$$



Free body: Arm BDH

$$+\circlearrowleft \Sigma M_D = 0: -(4091 \text{ lb})(24 \text{ in.}) - F_{BC}(20 \text{ in.}) = 0$$

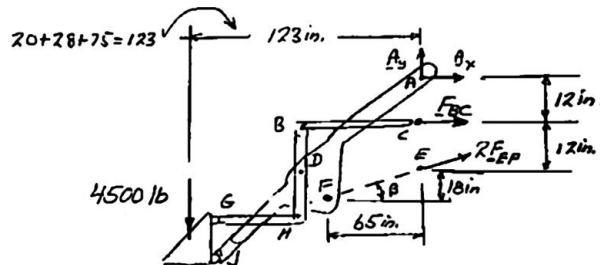
$$F_{BC} = -4909 \text{ lb}$$



$$F_{BC} = 4.91 \text{ kips} \quad C \blacktriangleleft$$

Free body: Entire mechanism

(Two arms and cylinders $AFJE$)



Note: Two arms thus $2F_{EF}$

$$\tan \beta = \frac{18 \text{ in.}}{65 \text{ in.}}$$

$$\beta = 15.48^\circ$$

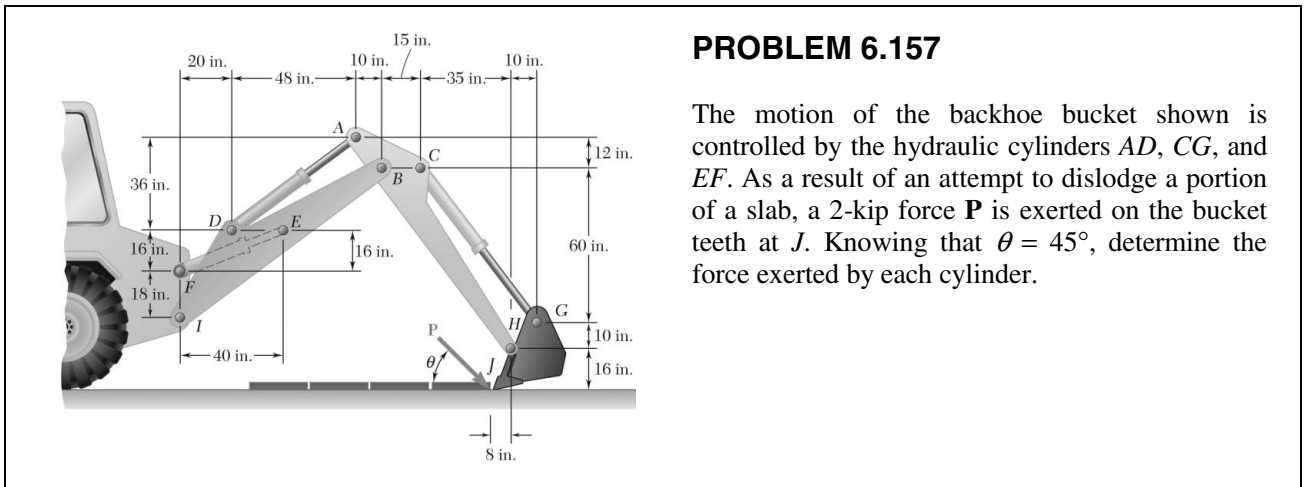
$$+\circlearrowleft \Sigma M_A = 0: (4500 \text{ lb})(123 \text{ in.}) + F_{BC}(12 \text{ in.}) + 2F_{EF} \cos \beta(24 \text{ in.}) = 0$$

$$(4500 \text{ lb})(123 \text{ in.}) - (4909 \text{ lb})(12 \text{ in.}) + 2F_{EF} \cos 15.48^\circ(24 \text{ in.}) = 0$$

$$F_{EF} = -10.690 \text{ lb}$$

$$F_{EF} = 10.69 \text{ kips} \quad C \blacktriangleleft$$

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PROBLEM 6.157

The motion of the backhoe bucket shown is controlled by the hydraulic cylinders AD , CG , and EF . As a result of an attempt to dislodge a portion of a slab, a 2-kip force \mathbf{P} is exerted on the bucket teeth at J . Knowing that $\theta = 45^\circ$, determine the force exerted by each cylinder.

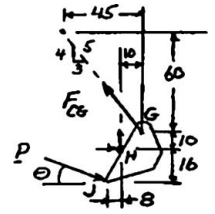
SOLUTION

Free body: Bucket:

$$+\circlearrowleft \Sigma M_H = 0: \quad (\text{Dimensions in inches})$$

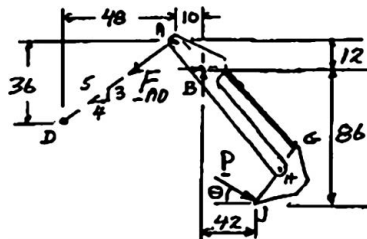
$$\frac{4}{5} F_{CG}(10) + \frac{3}{5} F_{CG}(10) + P \cos \theta(16) + P \sin \theta(8) = 0$$

$$F_{CG} = -\frac{P}{14}(16 \cos \theta + 8 \sin \theta) \quad (1)$$



Free body: Arm ABH and bucket:

(Dimensions in inches)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5} F_{AD}(12) + \frac{3}{5} F_{AD}(10) + P \cos \theta(86) - P \sin \theta(42) = 0$$

$$F_{AD} = -\frac{P}{15.6}(86 \cos \theta - 42 \sin \theta) \quad (2)$$

Free body: Bucket and arms $IEB + ABH$:

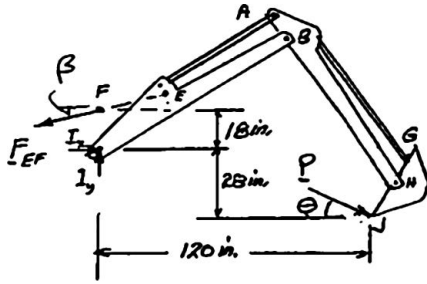
Geometry of cylinder EF :

$$\tan \beta = \frac{16 \text{ in.}}{40 \text{ in.}}$$

$$\beta = 21.801^\circ$$



PROBLEM 6.157 (Continued)



$$+\curvearrowright \Sigma M_I = 0: F_{EF} \cos \beta (18 \text{ in.}) + P \cos \theta (28 \text{ in.}) - P \sin \theta (120 \text{ in.}) = 0$$

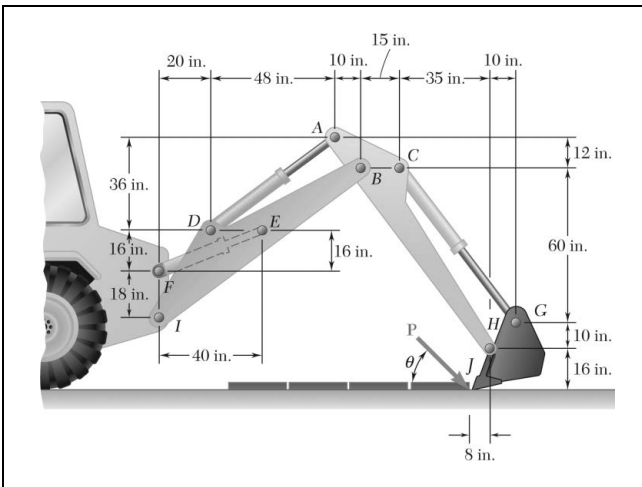
$$\begin{aligned} F_{EF} &= \frac{P(120 \sin \theta - 28 \cos \theta)}{\cos 21.8^\circ (18)} \\ &= \frac{P}{16.7126} (120 \sin \theta - 28 \cos \theta) \end{aligned} \quad (3)$$

For $P = 2$ kips, $\theta = 45^\circ$

From Eq. (1): $F_{CG} = -\frac{2}{14} (16 \cos 45^\circ + 8 \sin 45^\circ) = -2.42$ kips $F_{CG} = 2.42$ kips $C \blacktriangleleft$

From Eq. (2): $F_{AD} = -\frac{2}{15.6} (86 \cos 45^\circ - 42 \sin 45^\circ) = -3.99$ kips $F_{AD} = 3.99$ kips $C \blacktriangleleft$

From Eq. (3): $F_{EF} = \frac{2}{16.7126} (120 \sin 45^\circ - 28 \cos 45^\circ) = +7.79$ kips $F_{EF} = 7.79$ kips $T \blacktriangleleft$



PROBLEM 6.158

Solve Problem 6.157 assuming that the 2-kip force \mathbf{P} acts horizontally to the right ($\theta = 0$).

PROBLEM 6.157 The motion of the backhoe bucket shown is controlled by the hydraulic cylinders AD , CG , and EF . As a result of an attempt to dislodge a portion of a slab, a 2-kip force \mathbf{P} is exerted on the bucket teeth at J . Knowing that $\theta = 45^\circ$, determine the force exerted by each cylinder.

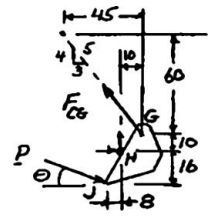
SOLUTION

Free body: Bucket:

$$+\circlearrowleft \Sigma M_H = 0: \quad (\text{Dimensions in inches})$$

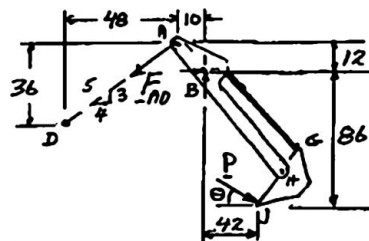
$$\frac{4}{5} F_{CG}(10) + \frac{3}{5} F_{CG}(10) + P \cos \theta(16) + P \sin \theta(8) = 0$$

$$F_{CG} = -\frac{P}{14}(16 \cos \theta + 8 \sin \theta) \quad (1)$$



Free body: Arm ABH and bucket:

(Dimensions in inches)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5} F_{AD}(12) + \frac{3}{5} F_{AD}(10) + P \cos \theta(86) - P \sin \theta(42) = 0$$

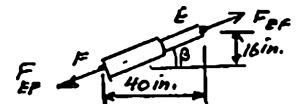
$$F_{AD} = -\frac{P}{15.6}(86 \cos \theta - 42 \sin \theta) \quad (2)$$

Free body: Bucket and arms $IEB + ABH$:

Geometry of cylinder EF :

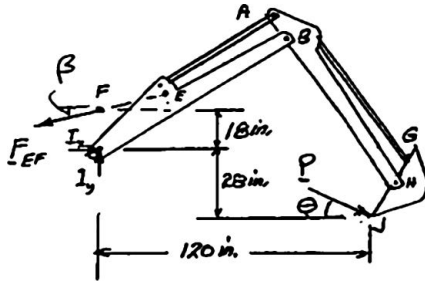
$$\tan \beta = \frac{16 \text{ in.}}{40 \text{ in.}}$$

$$\beta = 21.801^\circ$$



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PROBLEM 6.158 (Continued)



$$+\circlearrowleft \Sigma M_I = 0: F_{EF} \cos \beta (18 \text{ in.}) + P \cos \theta (28 \text{ in.}) - P \sin \theta (120 \text{ in.}) = 0$$

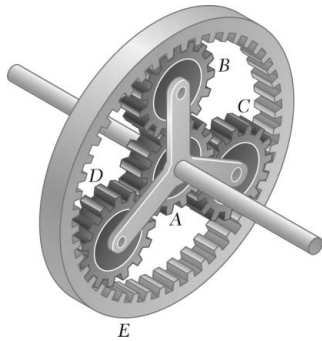
$$\begin{aligned} F_{EF} &= \frac{P(120 \sin \theta - 28 \cos \theta)}{\cos 21.8^\circ (18)} \\ &= \frac{P}{16.7126} (120 \sin \theta - 28 \cos \theta) \end{aligned} \quad (3)$$

For $P = 2$ kips, $\theta = 0$

From Eq. (1): $F_{CG} = -\frac{2}{14} (16 \cos 0 + 8 \sin 0) = -2.29$ kips $F_{CG} = 2.29$ kips C ◀

From Eq. (2): $F_{AD} = -\frac{2}{15.6} (86 \cos 0 - 42 \sin 0) = -11.03$ kips $F_{AD} = 11.03$ kips C ◀

From Eq. (3): $F_{EF} = \frac{2}{16.7126} (120 \sin 0 - 28 \cos 0) = -3.35$ kips $F_{EF} = 3.35$ kips C ◀

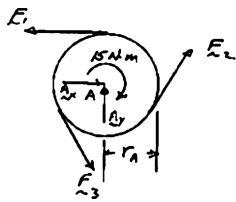


PROBLEM 6.159

In the planetary gear system shown, the radius of the central gear A is $a = 18$ mm, the radius of each planetary gear is b , and the radius of the outer gear E is $(a + 2b)$. A clockwise couple of magnitude $M_A = 10$ N · m is applied to the central gear A and a counterclockwise couple of magnitude $M_S = 50$ N · m is applied to the spider BCD . If the system is to be in equilibrium, determine (a) the required radius b of the planetary gears, (b) the magnitude M_E of the couple that must be applied to the outer gear E .

SOLUTION

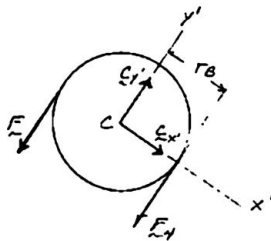
FBD Central Gear:



By symmetry, $F_1 = F_2 = F_3 = F$

$$\left(\sum M_A = 0: 3(r_A F) - 10 \text{ N} \cdot \text{m} = 0, \quad F = \frac{10}{3r_A} \text{ N} \cdot \text{m} \right.$$

FBD Gear C:

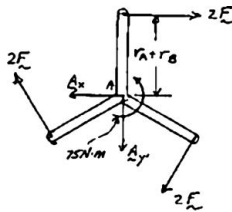


$$\left(\sum M_C = 0: r_B(F - F_4) = 0, \quad F_4 = F \right.$$

$$\left. \begin{aligned} \sum F_{x'} = 0: C_{x'} = 0 \\ \sum F_{y'} = 0: C_{y'} - 2F = 0, \quad C_{y'} = 2F \end{aligned} \right.$$

Gears B and D are analogous, each having a central force of $2F$.

FBD Spider:



$$\left(\sum M_A = 0: 50 \text{ N} \cdot \text{m} - 3(r_A + r_B)2F = 0 \right.$$

$$50 \text{ N} \cdot \text{m} - 3(r_A + r_B) \frac{20}{r_A} \text{ N} \cdot \text{m} = 0$$

$$\frac{r_A + r_B}{r_A} = 2.5 = 1 + \frac{r_B}{r_A},$$

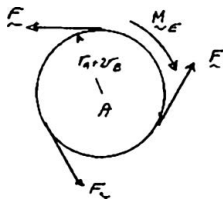
$$r_B = 1.5r_A$$

Since $r_A = 18$ mm,

(a)

$$r_B = 27.0 \text{ mm} \quad \blacktriangleleft$$

FBD Outer Gear:



$$\left(\sum M_A = 0: 3(r_A + 2r_B)F - M_E = 0 \right.$$

$$3(18 \text{ mm} + 54 \text{ mm}) \frac{10 \text{ N} \cdot \text{m}}{54 \text{ mm}} - M_E = 0$$

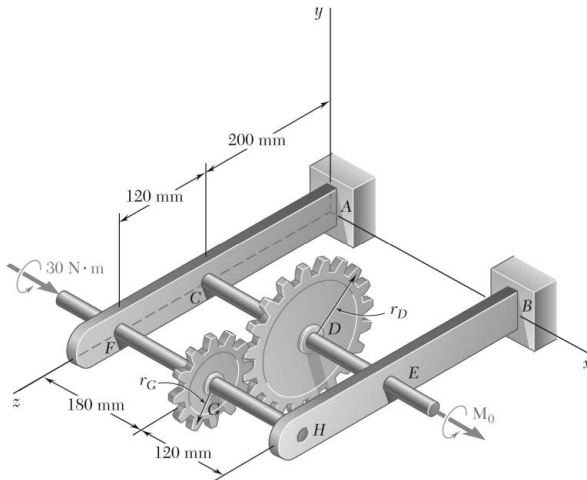
(b)

$$M_E = 40.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

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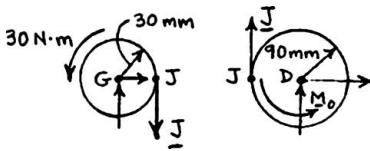
PROBLEM 6.160

The gears D and G are rigidly attached to shafts that are held by frictionless bearings. If $r_D = 90$ mm and $r_G = 30$ mm, determine (a) the couple \mathbf{M}_0 that must be applied for equilibrium, (b) the reactions at A and B .



SOLUTION

(a) Projections on yz plane.



Free body: Gear G :

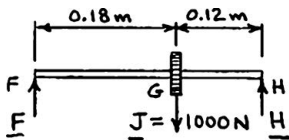
$$+\circlearrowleft \Sigma M_G = 0: 30 \text{ N} \cdot \text{m} - J(0.03 \text{ m}) = 0; \quad J = 1000 \text{ N}$$

Free body: Gear D :

$$+\circlearrowleft \Sigma M_D = 0: M_0 - (1000 \text{ N})(0.09 \text{ m}) = 0$$

$$M_0 = 90 \text{ N} \cdot \text{m} \quad \mathbf{M}_0 = (90.0 \text{ N} \cdot \text{m})\mathbf{i} \quad \blacktriangleleft$$

(b) Gear G and axle FH :



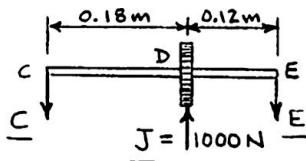
$$+\circlearrowleft \Sigma M_F = 0: H(0.3 \text{ m}) - (1000 \text{ N})(0.18 \text{ m}) = 0$$

$$H = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: F + 600 - 1000 = 0$$

$$F = 400 \text{ N}$$

Gear D and axle CE :



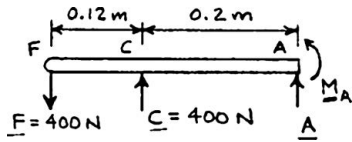
$$+\circlearrowleft \Sigma M_C = 0: (1000 \text{ N})(0.18 \text{ m}) - E(0.3 \text{ m}) = 0$$

$$E = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: 1000 - C - 600 = 0$$

$$C = 400 \text{ N}$$

PROBLEM 6.160 (Continued)

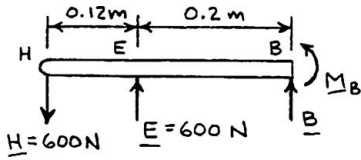


Free body: Bracket AE:

$$+\uparrow \Sigma F_y = 0: \quad A - 400 + 400 = 0 \quad \mathbf{A} = 0 \quad \blacktriangleleft$$

$$+\curvearrowright \Sigma M_A = 0: \quad M_A + (400 \text{ N})(0.32 \text{ m}) - (400 \text{ N})(0.2 \text{ m}) = 0$$

$$M_A = -48 \text{ N} \cdot \text{m} \quad \mathbf{M}_A = -(48.0 \text{ N} \cdot \text{m})\mathbf{i} \quad \blacktriangleleft$$



Free body: Bracket BH:

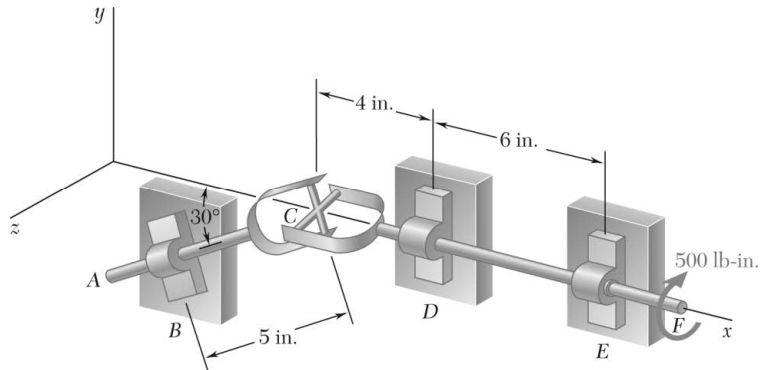
$$+\uparrow \Sigma F_y = 0: \quad B - 600 + 600 = 0 \quad \mathbf{B} = 0 \quad \blacktriangleleft$$

$$+\curvearrowright \Sigma M_B = 0: \quad M_B + (600 \text{ N})(0.32 \text{ m}) - (600 \text{ N})(0.2 \text{ m}) = 0$$

$$M_B = -72 \text{ N} \cdot \text{m} \quad \mathbf{M}_B = -(72.0 \text{ N} \cdot \text{m})\mathbf{i} \quad \blacktriangleleft$$

PROBLEM 6.161*

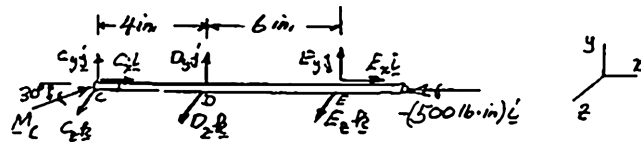
Two shafts AC and CF , which lie in the vertical xy plane, are connected by a universal joint at C . The bearings at B and D do not exert any axial force. A couple of magnitude $500 \text{ lb} \cdot \text{in.}$ (clockwise when viewed from the positive x -axis) is applied to shaft CF at F . At a time when the arm of the crosspiece attached to shaft CF is horizontal, determine (a) the magnitude of the couple that must be applied to shaft AC at A to maintain equilibrium, (b) the reactions at B , D , and E . (Hint: The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

We recall from Figure 4.10 that a universal joint exerts on members it connects a force of unknown direction and a couple about an axis perpendicular to the crosspiece.

Free body: Shaft DF :

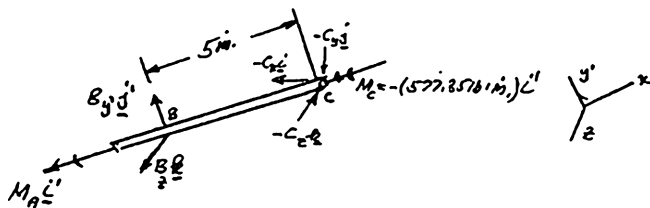


$$\Sigma M_x = 0: M_C \cos 30^\circ - 500 \text{ lb} \cdot \text{in.} = 0$$

$$M_C = 577.35 \text{ lb} \cdot \text{in.}$$

Free body: Shaft BC :

We use here x', y', z with x' along BC .



$$\Sigma M_C = 0: -M_R i' - (577.35 \text{ lb} \cdot \text{in.}) i' + (-5 \text{ in.}) i' \times (B_y j' + B_z k) = 0$$

PROBLEM 6.161* (Continued)

Equate coefficients of unit vectors to zero:

$$\begin{array}{llll}
 \mathbf{i}: & M_A - 577.35 \text{ lb} \cdot \text{in.} = 0 & & M_A = 577.35 \text{ lb} \cdot \text{in.} \\
 \mathbf{j}: & B_z = 0 & \left. \begin{array}{l} \\ \\ \end{array} \right\} & M_A = 577 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft \\
 \mathbf{k}: & B_y = 0 & \left. \begin{array}{l} \\ \\ \end{array} \right\} & \mathbf{B} = 0 \\
 & \Sigma \mathbf{F} = 0: B + C = 0, \text{ since } B = 0, & & \mathbf{C} = 0
 \end{array}$$

Return to free body of shaft DF .

$$\begin{aligned}
 \Sigma \mathbf{M}_D = 0 & \quad (\text{Note that } C = 0 \text{ and } M_C = 577.35 \text{ lb} \cdot \text{in.}) \\
 & (577.35 \text{ lb} \cdot \text{in.})(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}) - (500 \text{ lb} \cdot \text{in.})\mathbf{i} \\
 & + (6 \text{ in.})\mathbf{i} \times (E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k}) = 0 \\
 & (500 \text{ lb} \cdot \text{in.})\mathbf{i} + (288.68 \text{ lb} \cdot \text{in.})\mathbf{j} - (500 \text{ lb} \cdot \text{in.})\mathbf{i} \\
 & + (6 \text{ in.})E_y \mathbf{k} - (6 \text{ in.})E_z \mathbf{j} = 0
 \end{aligned}$$

Equate coefficients of unit vectors to zero:

$$\begin{array}{ll}
 \mathbf{j}: & 288.68 \text{ lb} \cdot \text{in.} - (6 \text{ in.})E_z = 0 \quad E_z = 48.1 \text{ lb} \\
 \mathbf{k}: & E_y = 0 \\
 & \Sigma \mathbf{F} = 0: \mathbf{C} + \mathbf{D} + \mathbf{E} = 0 \\
 & 0 + D_y \mathbf{j} + D_z \mathbf{k} + E_x \mathbf{i} + (48.1 \text{ lb})\mathbf{k} = 0 \\
 \mathbf{i}: & E_x = 0 \\
 \mathbf{j}: & D_y = 0 \\
 \mathbf{k}: & D_z + 48.1 \text{ lb} = 0 \quad D_z = -48.1 \text{ lb}
 \end{array}$$

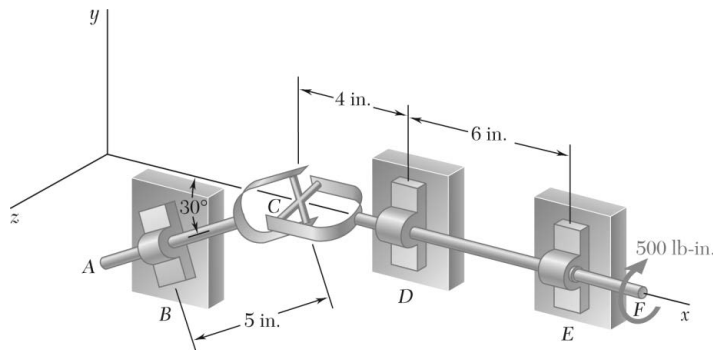
Reactions are:

$$\begin{array}{ll}
 & \mathbf{B} = 0 \quad \blacktriangleleft \\
 & \mathbf{D} = -(48.1 \text{ lb})\mathbf{k} \quad \blacktriangleleft \\
 & \mathbf{E} = (48.1 \text{ lb})\mathbf{k} \quad \blacktriangleleft
 \end{array}$$

PROBLEM 6.162*

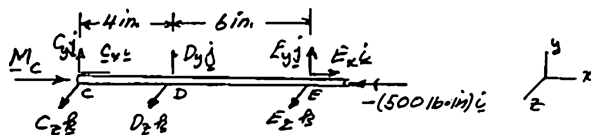
Solve Problem 6.161 assuming that the arm of the crosspiece attached to shaft CF is vertical.

PROBLEM 6.161 Two shafts AC and CF , which lie in the vertical xy plane, are connected by a universal joint at C . The bearings at B and D do not exert any axial force. A couple of magnitude $500 \text{ lb} \cdot \text{in.}$ (clockwise when viewed from the positive x -axis) is applied to shaft CF at F . At a time when the arm of the crosspiece attached to shaft CF is horizontal, determine (a) the magnitude of the couple that must be applied to shaft AC at A to maintain equilibrium, (b) the reactions at B , D , and E . (Hint: The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

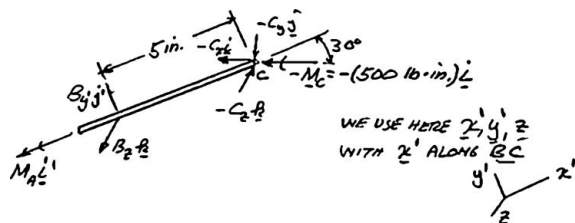
Free body: Shaft DF .



$$\Sigma M_x = 0: \quad M_C - 500 \text{ lb} \cdot \text{in.} = 0$$

$$M_C = 500 \text{ lb} \cdot \text{in.}$$

Free body: Shaft BC :



We resolve $-(520 \text{ lb} \cdot \text{in.})\mathbf{i}$ into components along x' and y' axes:

$$-\mathbf{M}_C = -(500 \text{ lb} \cdot \text{in.})(\cos 30^\circ \mathbf{i}' + \sin 30^\circ \mathbf{j}')$$

$$\Sigma \mathbf{M}_C = 0: \quad M_A \mathbf{i}' - (500 \text{ lb} \cdot \text{in.})(\cos 30^\circ \mathbf{i}' + \sin 30^\circ \mathbf{j}') + (5 \text{ in.})\mathbf{i}' \times (B_y \mathbf{j}' + B_z \mathbf{k}') = 0$$

$$M_A \mathbf{i}' - (433 \text{ lb} \cdot \text{in.})\mathbf{i}' - (250 \text{ lb} \cdot \text{in.})\mathbf{j}' + (5 \text{ in.})B_y \mathbf{k}' - (5 \text{ in.})B_z \mathbf{j}' = 0$$

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PROBLEM 6.162* (Continued)

Equate to zero coefficients of unit vectors:

$$\mathbf{i}': M_A - 433 \text{ lb}\cdot\text{in.} = 0 \qquad M_A = 433 \text{ lb}\cdot\text{in.} \quad \blacktriangleleft$$

$$\mathbf{j}': -250 \text{ lb}\cdot\text{in.} - (5 \text{ in.})B_z = 0 \qquad B_z = -50 \text{ lb}$$

$$\mathbf{k}: B_y = 0$$

Reactions at B.

$$\mathbf{B} = -(50 \text{ lb})\mathbf{k}$$

$$\Sigma \mathbf{F} = 0: \mathbf{B} - \mathbf{C} = 0$$

$$-(50 \text{ lb})\mathbf{k} - \mathbf{C} = 0$$

$$\mathbf{C} = -(50 \text{ lb})\mathbf{k}$$

Return to free body of shaft DF.

$$\begin{aligned} \Sigma \mathbf{M}_D = 0: & (6 \text{ in.})\mathbf{i} \times (E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) - (4 \text{ in.})\mathbf{i} \times (-50 \text{ lb})\mathbf{k} \\ & - (500 \text{ lb}\cdot\text{in.})\mathbf{i} + (500 \text{ lb}\cdot\text{in.})\mathbf{i} = 0 \\ & (6 \text{ in.})E_y\mathbf{k} - (6 \text{ in.})E_z\mathbf{j} - (200 \text{ lb}\cdot\text{in.})\mathbf{j} = 0 \end{aligned}$$

$$\mathbf{k}: E_y = 0$$

$$\mathbf{j}: -(6 \text{ in.})E_z - 200 \text{ lb}\cdot\text{in.} = 0 \qquad E_z = -33.3 \text{ lb}$$

$$\Sigma \mathbf{F} = 0: \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$$

$$-(50 \text{ lb})\mathbf{k} + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} - (33.3 \text{ lb})\mathbf{k} = 0$$

$$\mathbf{i}: E_x = 0$$

$$\mathbf{k}: -50 \text{ lb} - 33.3 \text{ lb} + D_z = 0$$

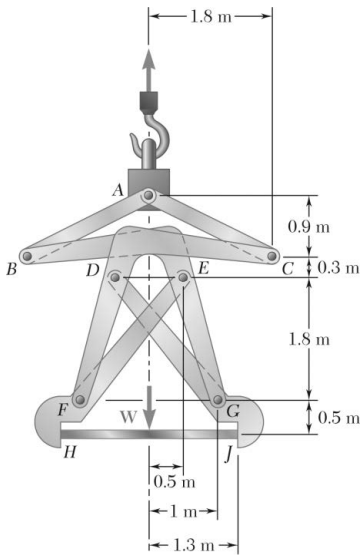
$$D_z = 83.3 \text{ lb}$$

Reactions are

$$\mathbf{B} = -(50 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{D} = (83.3 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{E} = -(33.3 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 6.163*

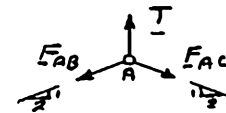
The large mechanical tongs shown are used to grab and lift a thick 7500-kg steel slab HJ . Knowing that slipping does not occur between the tong grips and the slab at H and J , determine the components of all forces acting on member EFH . (Hint: Consider the symmetry of the tongs to establish relationships between the components of the force acting at E on EFH and the components of the force acting at D on DGJ .)

SOLUTION

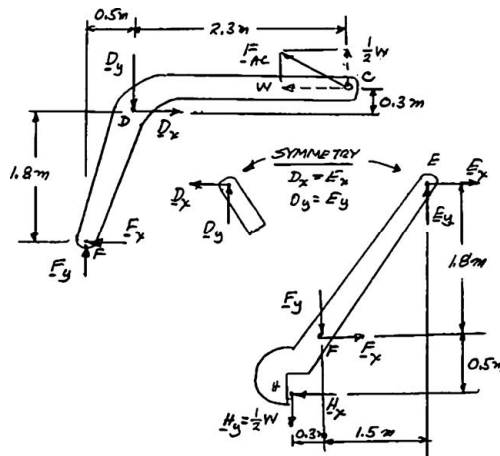
Free body: Pin A: $T = W = mg = (7500 \text{ kg})(9.81 \text{ m/s}^2) = 73.575 \text{ kN}$

$$\Sigma F_x = 0: (F_{AB})_x = (F_{AC})_x$$

$$\Sigma F_y = 0: (F_{AB})_y = (F_{AC})_y = \frac{1}{2}W$$



Also, $(F_{AC})_x = 2(F_{AC})_y = W$



Free body: Member CDE :

$$\curvearrowright \Sigma M_D = 0: W(0.3) + \frac{1}{2}W(2.3) - F_x(1.8) - F_y(0.5 \text{ m}) = 0$$

or
$$1.8F_x + 0.5F_y = 1.45W \quad (1)$$

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PROBLEM 6.163* (Continued)

$$\overset{+}{\rightarrow} \Sigma F_x = 0: D_x - F_x - W = 0$$

or
$$E_x - F_x = W \quad (2)$$

$$+\uparrow \Sigma F_y = 0: F_y - D_y + \frac{1}{2}W = 0$$

or
$$E_y - F_y = \frac{1}{2}W \quad (3)$$

Free body: Member EFH:

$$+\curvearrowright \Sigma M_E = 0: F_x(1.8) + F_y(1.5) - H_x(2.3) + \frac{1}{2}W(1.8 \text{ m}) = 0$$

or
$$1.8F_x + 1.5F_y = 2.3H_x - 0.9W \quad (4)$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: E_x + F_x - H_x = 0$$

or
$$E_x + F_x = H_x \quad (5)$$

Subtract Eq. (2) from Eq. (5):
$$2F_x = H_x - W \quad (6)$$

Subtract Eq. (4) from $3 \times (1)$:
$$3.6F_x = 5.25W - 2.3H_x \quad (7)$$

Add Eq. (7) to $2.3 \times (6)$:
$$8.2F_x = 2.95W \quad (8)$$

$$F_x = 0.35976W$$

Substitute from Eq. (8) into Eq. (1):

$$(1.8)(0.35976W) + 0.5F_y = 1.45W$$

$$0.5F_y = 1.45W - 0.64756W = 0.80244W$$

$$F_y = 1.6049W \quad (9)$$

Substitute from Eq. (8) into Eq. (2):
$$E_x - 0.35976W = W; \quad E_x = 1.35976W$$

Substitute from Eq. (9) into Eq. (3):
$$E_y - 1.6049W = \frac{1}{2}W \quad E_y = 2.1049W$$

From Eq. (5):
$$H_x = E_x + F_x = 1.35976W + 0.35976W = 1.71952W$$

Recall that
$$H_y = \frac{1}{2}W$$

PROBLEM 6.163* (Continued)

Since all expressions obtained are positive, all forces are directed as shown on the free-body diagrams.

Substitute

$$W = 73.575 \text{ kN}$$

$$\mathbf{E}_x = 100.0 \text{ kN} \rightarrow$$

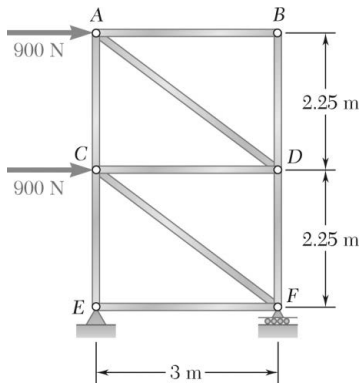
$$\mathbf{E}_y = 154.9 \text{ kN} \uparrow \blacktriangleleft$$

$$\mathbf{F}_x = 26.5 \text{ kN} \rightarrow$$

$$\mathbf{F}_y = 118.1 \text{ kN} \downarrow \blacktriangleleft$$

$$\mathbf{H}_x = 126.5 \text{ kN} \leftarrow$$

$$\mathbf{H}_y = 36.8 \text{ kN} \downarrow \blacktriangleleft$$

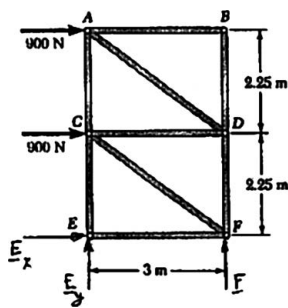


PROBLEM 6.164

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free Body: Truss:



$$+\circlearrowleft \Sigma M_E = 0: F(3 \text{ m}) - (900 \text{ N})(2.25 \text{ m}) - (900 \text{ N})(4.5 \text{ m}) = 0$$

$$F = 2025 \text{ N} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: E_x + 900 \text{ N} + 900 \text{ N} = 0$$

$$E_x = -1800 \text{ N} \quad E_x = 1800 \text{ N} \leftarrow$$

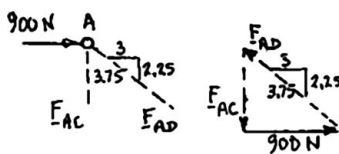
$$+\uparrow \Sigma F_y = 0: E_y + 2025 \text{ N} = 0$$

$$E_y = -2025 \text{ N} \quad E_y = 2025 \text{ N} \downarrow$$

We note that AB and BD are zero-force members.

$$F_{AB} = F_{BD} = 0 \quad \blacktriangleleft$$

Free body: Joint A:

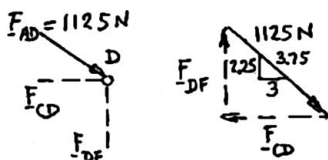


$$\frac{F_{AC}}{2.25} = \frac{F_{AD}}{3.75} = \frac{900 \text{ N}}{3}$$

$$F_{AC} = 675 \text{ N} \quad T \quad \blacktriangleleft$$

$$F_{AD} = 1125 \text{ N} \quad C \quad \blacktriangleleft$$

Free body: Joint D:

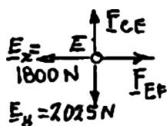


$$\frac{F_{CD}}{3} = \frac{F_{DE}}{2.25} = \frac{1125 \text{ N}}{3.75}$$

$$F_{CD} = 900 \text{ N} \quad T \quad \blacktriangleleft$$

$$F_{DF} = 675 \text{ N} \quad C \quad \blacktriangleleft$$

Free body: Joint E:



$$+\rightarrow \Sigma F_x = 0: F_{EF} - 1800 \text{ N} = 0$$

$$F_{EF} = 1800 \text{ N} \quad T \quad \blacktriangleleft$$

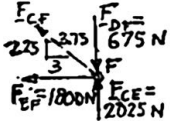
$$+\uparrow \Sigma F_y = 0: F_{CE} - 2025 \text{ N} = 0$$

$$F_{CE} = 2025 \text{ N} \quad T \quad \blacktriangleleft$$

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PROBLEM 6.164 (Continued)

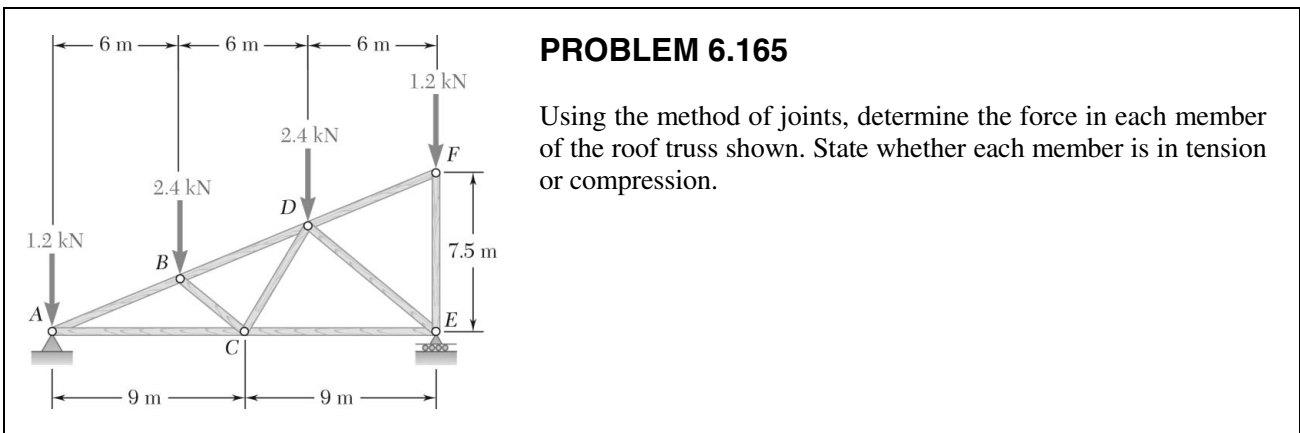
Free body: Joint F :



$$+\uparrow \Sigma F_y = 0: \frac{2.25}{3.75} F_{CF} + 2025 \text{ N} - 675 \text{ N} = 0$$

$$F_{CF} = -2250 \text{ N} \qquad F_{CF} = 2250 \text{ N} \quad C \blacktriangleleft$$

$$+\rightarrow \Sigma F_x = -\frac{3}{3.75} (-2250 \text{ N}) - 1800 \text{ N} = 0 \quad (\text{Checks})$$



PROBLEM 6.165

Using the method of joints, determine the force in each member of the roof truss shown. State whether each member is in tension or compression.

SOLUTION

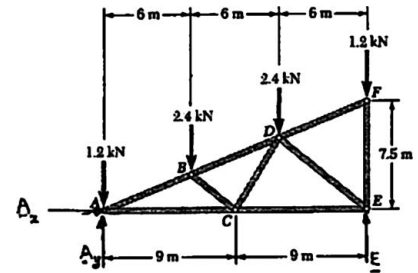
Free body: Truss:

$$\Sigma F_x = 0: A_x = 0$$

From symmetry of loading:

$$A_y = E = \frac{1}{2} \text{ total load}$$

$$A_y = E = 3.6 \text{ kN } \uparrow$$

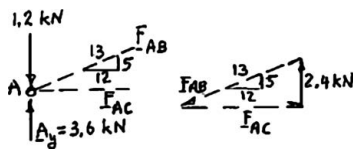


We note that *DF* is a zero-force member and that *EF* is aligned with the load. Thus,

$$F_{DF} = 0 \quad \blacktriangleleft$$

$$F_{EF} = 1.2 \text{ kN} \quad \blacktriangleleft$$

Free body: Joint A:

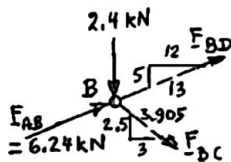


$$\frac{F_{AB}}{13} = \frac{F_{AC}}{12} = \frac{2.4 \text{ kN}}{5}$$

$$F_{AB} = 6.24 \text{ kN} \quad \blacktriangleleft$$

$$F_{AC} = 2.76 \text{ kN} \quad T \quad \blacktriangleleft$$

Free body: Joint B:



$$\rightarrow \Sigma F_x = 0: \frac{3}{3.905} F_{BC} + \frac{12}{13} F_{BD} + \frac{12}{13} (6.24 \text{ kN}) = 0 \quad (1)$$

$$\uparrow \Sigma F_y = 0: -\frac{2.5}{3.905} F_{BC} + \frac{5}{13} F_{BD} + \frac{5}{13} (6.24 \text{ kN}) - 2.4 \text{ kN} = 0 \quad (2)$$

PROBLEM 6.165 (Continued)

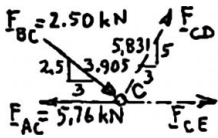
Multiply Eq. (1) by 2.5, Eq. (2) by 3, and add:

$$\frac{45}{13}F_{BD} + \frac{45}{13}(6.24 \text{ kN}) - 7.2 \text{ kN} = 0, \quad F_{BD} = -4.16 \text{ kN}, \quad F_{BD} = 4.16 \text{ kN} \quad C \leftarrow$$

Multiply Eq. (1) by 5, Eq. (2) by -12, and add:

$$\frac{45}{3.905}F_{BC} + 28.8 \text{ kN} = 0, \quad F_{BC} = -2.50 \text{ kN}, \quad F_{BC} = 2.50 \text{ kN} \quad C \leftarrow$$

Free body: Joint C:



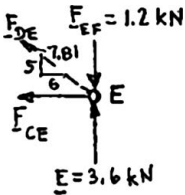
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{5.831}F_{CD} - \frac{2.5}{3.905}(2.50 \text{ kN}) = 0$$

$$F_{CD} = 1.867 \text{ kN} \quad T \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: \quad F_{CE} - 5.76 \text{ kN} + \frac{3}{3.905}(2.50 \text{ kN}) + \frac{3}{5.831}(1.867 \text{ kN}) = 0$$

$$F_{CE} = 2.88 \text{ kN} \quad T$$

Free body: Joint E:

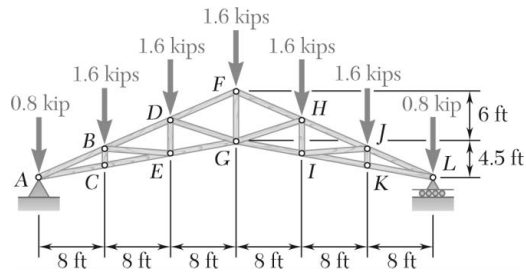


$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.81}F_{DE} + 3.6 \text{ kN} - 1.2 \text{ kN} = 0$$

$$F_{DE} = -3.75 \text{ kN} \quad F_{DE} = 3.75 \text{ kN} \quad C \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: \quad -F_{CE} - \frac{6}{7.81}(-3.75 \text{ kN}) = 0$$

$$F_{CE} = +2.88 \text{ kN} \quad F_{CE} = 2.88 \text{ kN} \quad T \quad (\text{Checks})$$



PROBLEM 6.166

A Howe scissor roof truss is loaded as shown. Determine the force in members DF , DG , and EG .

SOLUTION

Reactions at supports.

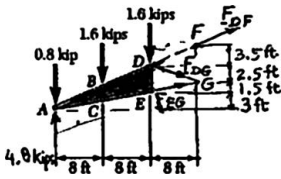
Because of symmetry of loading,

$$A_x = 0, \quad A_y = L = \frac{1}{2}(\text{total load}) = \frac{1}{2}(9.60 \text{ kips}) = 4.80 \text{ kips}$$

$$A = L = 4.80 \text{ kips} \uparrow \blacktriangleleft$$

We pass a section through members DF , DG , and EG , and use the free body shown.

We slide F_{DF} to apply it at F :



$$\begin{aligned} +\curvearrowright \Sigma M_G = 0: & (0.8 \text{ kip})(24 \text{ ft}) + (1.6 \text{ kips})(16 \text{ ft}) + (1.6 \text{ kips})(8 \text{ ft}) \\ & - (4.8 \text{ kips})(24 \text{ ft}) - \frac{8F_{DF}}{\sqrt{8^2 + 3.5^2}}(6 \text{ ft}) = 0 \end{aligned}$$

$$F_{DF} = -10.48 \text{ kips}, \quad F_{DF} = 10.48 \text{ kips} \quad C \blacktriangleleft$$

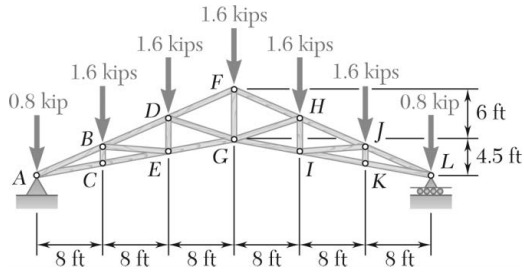
$$\begin{aligned} +\curvearrowright \Sigma M_A = 0: & -(1.6 \text{ kips})(8 \text{ ft}) - (1.6 \text{ kips})(16 \text{ ft}) \\ & - \frac{2.5F_{DG}}{\sqrt{8^2 + 2.5^2}}(16 \text{ ft}) - \frac{8F_{DG}}{\sqrt{8^2 + 2.5^2}}(7 \text{ ft}) = 0 \end{aligned}$$

$$F_{DG} = -3.35 \text{ kips}, \quad F_{DG} = 3.35 \text{ kips} \quad C \blacktriangleleft$$

$$+\curvearrowright \Sigma M_D = 0: (0.8 \text{ kips})(16 \text{ ft}) + (1.6 \text{ kips})(8 \text{ ft}) - (4.8 \text{ kips})(16 \text{ ft}) - \frac{8F_{EG}}{\sqrt{8^2 + 1.5^2}}(4 \text{ ft}) = 0$$

$$F_{EG} = +13.02 \text{ kips}, \quad F_{EG} = 13.02 \text{ kips} \quad T \blacktriangleleft$$

PROBLEM 6.167



A Howe scissor roof truss is loaded as shown. Determine the force in members GI , HI , and HJ .

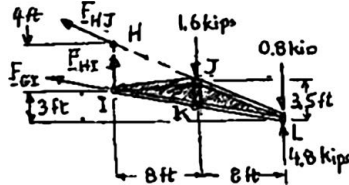
SOLUTION

Reactions at supports. Because of symmetry of loading,

$$\begin{aligned} A_x &= 0, & A_y &= L = \frac{1}{2}(\text{Total load}) \\ & & &= \frac{1}{2}(9.60 \text{ kips}) \\ & & &= 4.80 \text{ kips} \end{aligned}$$

$$A = L = 4.80 \text{ kips} \uparrow \leftarrow$$

We pass a section through members GI , HI , and HJ , and use the free body shown.



$$+\curvearrowright \Sigma M_H = 0: -\frac{16F_{GI}}{\sqrt{16^2 + 3^2}}(4 \text{ ft}) + (4.8 \text{ kips})(16 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) - (1.6 \text{ kips})(8 \text{ ft}) = 0$$

$$F_{GI} = +13.02 \text{ kips} \quad F_{GI} = 13.02 \text{ kips} \quad T \leftarrow$$

$$+\curvearrowright \Sigma M_L = 0: (1.6 \text{ kips})(8 \text{ ft}) - F_{HI}(16 \text{ ft}) = 0$$

$$F_{HI} = +0.800 \text{ kips}$$

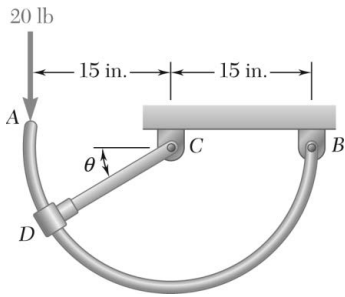
$$F_{HI} = 0.800 \text{ kips} \quad T \leftarrow$$

We slide F_{HG} to apply it at H .

$$+\curvearrowright \Sigma M_I = 0: \frac{8F_{HJ}}{\sqrt{8^2 + 3.5^2}}(4 \text{ ft}) + (4.8 \text{ kips})(16 \text{ ft}) - (1.6 \text{ kips})(8 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) = 0$$

$$F_{HJ} = -13.97 \text{ kips}$$

$$F_{HJ} = 13.97 \text{ kips} \quad C \leftarrow$$

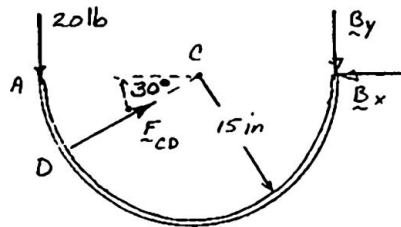


PROBLEM 6.168

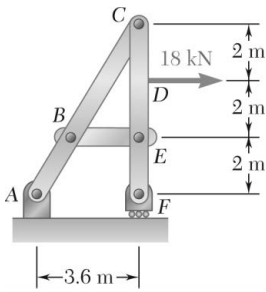
Rod CD is fitted with a collar at D that can be moved along rod AB , which is bent in the shape of an arc of circle. For the position when $\theta = 30^\circ$, determine (a) the force in rod CD , (b) the reaction at B .

SOLUTION

FBD:



$$\begin{aligned}
 (a) \quad & \left(\sum M_C = 0: (15 \text{ in.})(20 \text{ lb} - B_y) = 0 \right. && \mathbf{B}_y = 20 \text{ lb} \downarrow \\
 & \left. \uparrow \sum F_y = 0: -20 \text{ lb} + F_{CD} \sin 30^\circ - 20 \text{ lb} = 0 \right. && F_{CD} = 80.0 \text{ lb} \quad T \quad \blacktriangleleft \\
 (b) \quad & \rightarrow \sum F_x = 0: (80 \text{ lb}) \cos 30^\circ - B_x = 0 \\
 & && \mathbf{B}_x = 69.282 \text{ lb} \quad \leftarrow \\
 & && \text{so } \mathbf{B} = 72.1 \text{ lb} \nearrow 16.10^\circ \quad \blacktriangleleft
 \end{aligned}$$

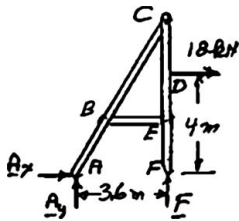


PROBLEM 6.169

For the frame and loading shown, determine the components of all forces acting on member ABC .

SOLUTION

Free body: Entire frame:



$$\rightarrow \Sigma F_x = 0: A_x + 18 \text{ kN} = 0$$

$$A_x = -18 \text{ kN}$$

$$A_x = 18.00 \text{ kN} \leftarrow$$

$$\curvearrowright \Sigma M_E = 0: -(18 \text{ kN})(4 \text{ m}) - A_y(3.6 \text{ m}) = 0$$

$$A_y = -20 \text{ kN}$$

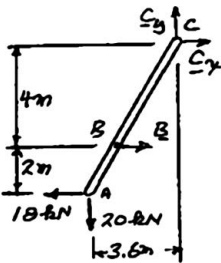
$$A_y = 20.0 \text{ kN} \downarrow$$

$$\uparrow \Sigma F_y = 0: -20 \text{ kN} + F = 0$$

$$F = +20 \text{ kN} \quad \mathbf{F} = 20 \text{ kN} \uparrow$$

Free body: Member ABC

Note: BE is a two-force member, thus \mathbf{B} is directed along line BE .



$$\curvearrowright \Sigma M_C = 0: B(4 \text{ m}) - (18 \text{ kN})(6 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) = 0$$

$$B = 9 \text{ kN}$$

$$\mathbf{B} = 9.00 \text{ kN} \rightarrow$$

$$\rightarrow \Sigma F_x = 0: C_x - 18 \text{ kN} + 9 \text{ kN} = 0$$

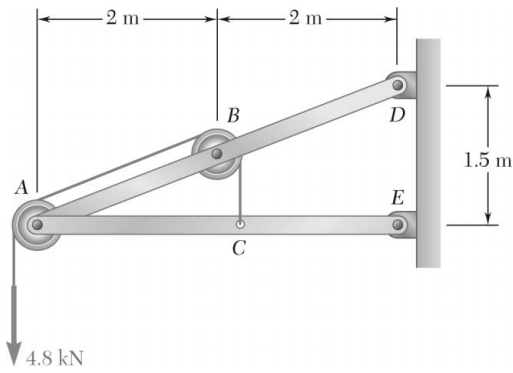
$$C_x = 9 \text{ kN}$$

$$C_x = 9.00 \text{ kN} \rightarrow$$

$$\uparrow \Sigma F_y = 0: C_y - 20 \text{ kN} = 0$$

$$C_y = 20 \text{ kN}$$

$$C_y = 20.0 \text{ kN} \uparrow$$

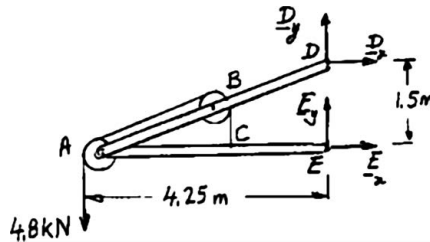


PROBLEM 6.170

Knowing that each pulley has a radius of 250 mm, determine the components of the reactions at D and E .

SOLUTION

Free body: Entire assembly:



$$+\curvearrowright \Sigma M_E = 0: (4.8 \text{ kN})(4.25 \text{ m}) - D_x(1.5 \text{ m}) = 0$$

$$D_x = +13.60 \text{ kN}$$

$$D_x = 13.60 \text{ kN} \rightarrow \blacktriangleleft$$

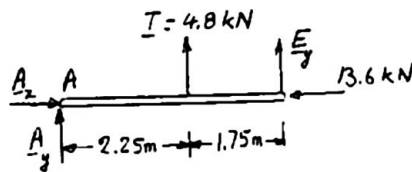
$$+\rightarrow \Sigma F_x = 0: E_x + 13.60 \text{ kN} = 0$$

$$E_x = -13.60 \text{ kN}$$

$$E_x = 13.60 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: D_y + E_y - 4.8 \text{ kN} = 0 \quad (1)$$

Free body: Member ACE:



$$+\curvearrowright \Sigma M_A = 0: (4.8 \text{ kN})(2.25 \text{ m}) + E_y(4 \text{ m}) = 0$$

$$E_y = -2.70 \text{ kN}$$

$$E_y = 2.70 \text{ kN} \downarrow \blacktriangleleft$$

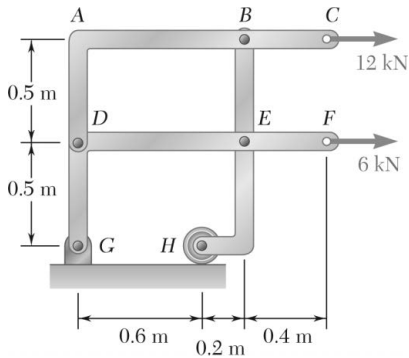
From Eq. (1):

$$D_y - 2.70 \text{ kN} - 4.80 \text{ kN} = 0$$

$$D_y = +7.50 \text{ kN}$$

$$D_y = 7.50 \text{ kN} \uparrow \blacktriangleleft$$

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PROBLEM 6.171

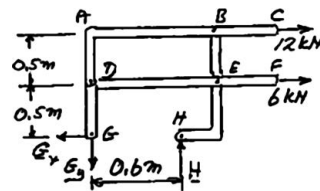
For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and *D*.

SOLUTION

Free body: Entire frame:

$$+\curvearrowright \Sigma M_G = 0: \quad H(0.6 \text{ m}) - (12 \text{ kN})(1 \text{ m}) - (6 \text{ kN})(0.5 \text{ m}) = 0$$

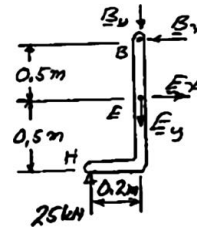
$$H = 25 \text{ kN} \quad \mathbf{H} = 25 \text{ kN} \uparrow$$



Free body: Member *BEH*:

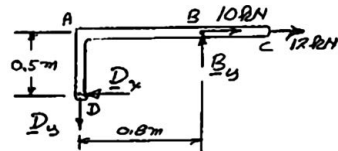
$$+\curvearrowright \Sigma M_F = 0: \quad B_x(0.5 \text{ m}) - (25 \text{ kN})(0.2 \text{ m}) = 0$$

$$B_x = +10 \text{ kN}$$



Free body: Member *DABC*:

From above:



$$+\curvearrowright \Sigma M_D = 0: \quad -B_y(0.8 \text{ m}) + (10 \text{ kN} + 12 \text{ kN})(0.5 \text{ m}) = 0$$

$$B_y = +13.75 \text{ kN} \quad \mathbf{B}_y = 13.75 \text{ kN} \uparrow \blacktriangleleft$$

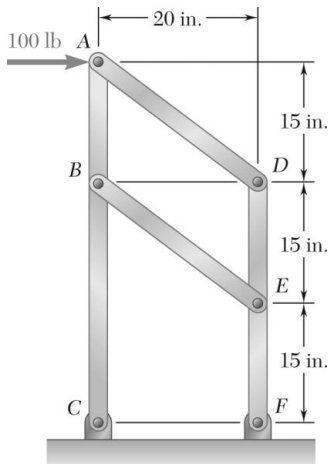
$$\pm \rightarrow \Sigma F_x = 0: \quad -D_x + 10 \text{ kN} + 12 \text{ kN} = 0$$

$$D_x = +22 \text{ kN} \quad \mathbf{D}_x = 22.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -D_y + 13.75 \text{ kN} = 0$$

$$D_y = +13.75 \text{ kN} \quad \mathbf{D}_y = 13.75 \text{ kN} \downarrow \blacktriangleleft$$

$$\mathbf{B}_x = 10.00 \text{ kN} \rightarrow \blacktriangleleft$$

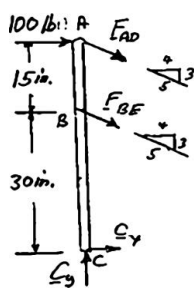


PROBLEM 6.172

For the frame and loading shown, determine (a) the reaction at C , (b) the force in member AD .

SOLUTION

Free body: Member ABC :



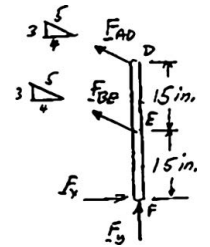
$$+\circlearrowleft \Sigma M_C = 0: \quad + (100 \text{ lb})(45 \text{ in.}) + \frac{4}{5} F_{AD}(45 \text{ in.}) + \frac{4}{5} F_{BE}(30 \text{ in.}) = 0$$

$$3F_{AD} + 2F_{BE} = -375 \text{ lb} \quad (1)$$

Free Body: Member DEF :

$$+\circlearrowleft \Sigma M_F = 0: \quad \frac{4}{5} F_{AD}(30 \text{ in.}) + \frac{4}{5} F_{BE}(15 \text{ in.}) = 0$$

$$F_{BE} = -2F_{AD}$$



$$(2)$$

(b) Substitute from Eq. (2) into Eq. (1):

$$3F_{AD} + 2(-2F_{AD}) = -375 \text{ lb}$$

$$F_{AD} = +375 \text{ lb}$$

$$F_{AD} = 375 \text{ lb} \quad T \quad \blacktriangleleft$$

From Eq. (2):

$$F_{BE} = -2F_{AD} = -2(375 \text{ lb})$$

$$F_{BE} = -750 \text{ lb}$$

$$F_{BE} = 750 \text{ lb} \quad C.$$

(a) Return to free body of member ABC .

$$\rightarrow \Sigma F_x = 0: \quad C_x + 100 \text{ lb} + \frac{4}{5} F_{AD} + \frac{4}{5} F_{BE} = 0$$

$$C_x + 100 + \frac{4}{5}(375) + \frac{4}{5}(-750) = 0$$

$$C_x = +200 \text{ lb} \quad C_x = 200 \text{ lb} \rightarrow$$

PROBLEM 6.172 (Continued)

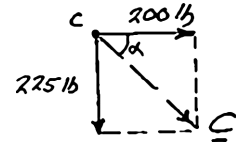
$$+\uparrow \Sigma F_y = 0: C_y - \frac{3}{5}F_{AD} - \frac{3}{5}F_{BF} = 0$$

$$C_y - \frac{3}{5}(375) - \frac{3}{5}(-750) = 0$$

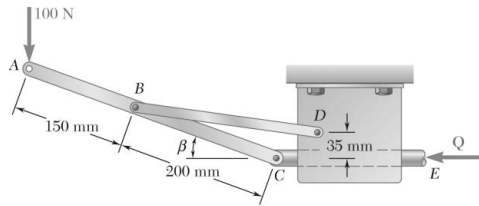
$$C_y = -225 \text{ lb} \quad C_y = 225 \text{ lb} \downarrow$$

$$\alpha = 48.37^\circ$$

$$C = 301.0 \text{ lb}$$



$$C = 301 \text{ lb} \nearrow 48.4^\circ \blacktriangleleft$$



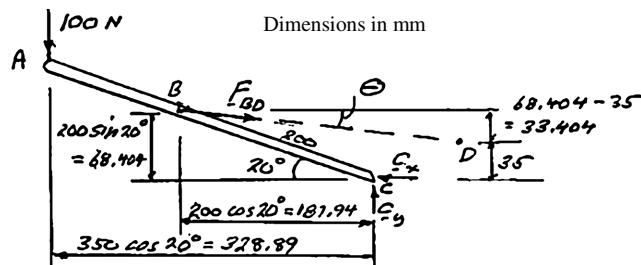
PROBLEM 6.173

The control rod CE passes through a horizontal hole in the body of the toggle system shown. Knowing that link BD is 250 mm long, determine the force Q required to hold the system in equilibrium when $\beta = 20^\circ$.

SOLUTION

We note that BD is a two-force member.

Free body: Member ABC :



Since $BD = 250$,

$$\theta = \sin^{-1} \frac{33.404}{250}; \quad \theta = 7.679^\circ$$

$$+\curvearrowright \Sigma M_C = 0: (F_{BD} \sin \theta)187.94 - (F_{BD} \cos \theta)68.404 + (100 \text{ N})328.89 = 0$$

$$F_{BD}[187.94 \sin 7.679^\circ - 68.404 \cos 7.679^\circ] = 32,889$$

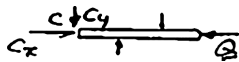
$$F_{BD} = 770.6 \text{ N}$$

$$+\rightarrow \Sigma F_x = 0: (770.6 \text{ N}) \cos 7.679^\circ = C_x = 0$$

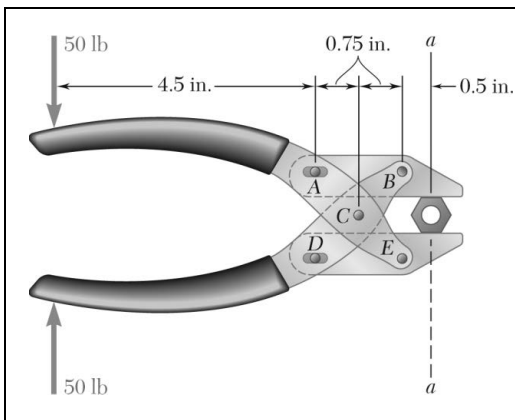
$$C_x = +763.7 \text{ N}$$

Member CQ :

$$\Sigma F_x = 0: Q = C_x = 763.7 \text{ N}$$



$$Q = 764 \text{ N} \leftarrow \blacktriangleleft$$

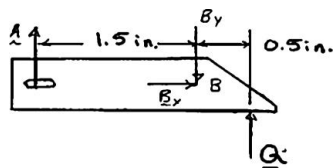


PROBLEM 6.174

Determine the magnitude of the gripping forces exerted along line aa on the nut when two 50-lb forces are applied to the handles as shown. Assume that pins A and D slide freely in slots cut in the jaws.

SOLUTION

FBD jaw AB :



$$\rightarrow \Sigma F_x = 0: B_x = 0$$

$$\curvearrowleft \Sigma M_B = 0: (0.5 \text{ in.})Q - (1.5 \text{ in.})A = 0$$

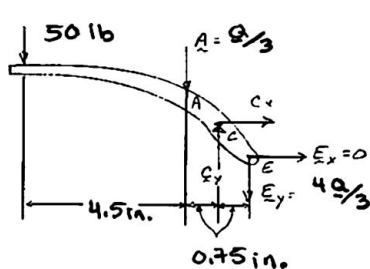
$$A = \frac{Q}{3}$$

$$\uparrow \Sigma F_y = 0: A + Q - B_y = 0$$

$$B_y = A + Q = \frac{4Q}{3}$$

FBD handle ACE :

By symmetry and FBD jaw DE ,



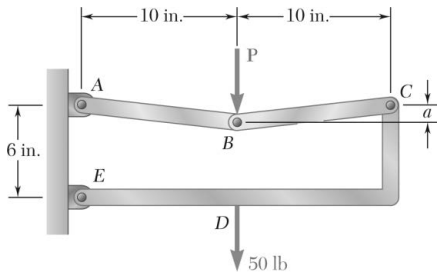
$$D = A = \frac{Q}{3}$$

$$E_x = B_x = 0$$

$$E_y = B_y = \frac{4Q}{3}$$

$$\curvearrowleft \Sigma M_C = 0: (5.25 \text{ in.})(50 \text{ lb}) + (0.75 \text{ in.})\frac{Q}{3} - (0.75 \text{ in.})\frac{4Q}{3} = 0$$

$$Q = 350 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 6.175

Knowing that the frame shown has a sag at B of $a = 1$ in., determine the force \mathbf{P} required to maintain equilibrium in the position shown.

SOLUTION

We note that AB and BC are two-force members.

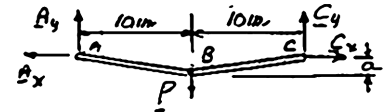
Free body: Toggle:

By symmetry,

$$C_y = \frac{P}{2}$$

$$\frac{C_x}{10 \text{ in.}} = \frac{C_y}{a}$$

$$C_x = \frac{10}{a} C_y = \frac{10}{a} \cdot \frac{P}{2} = \frac{5P}{a}$$

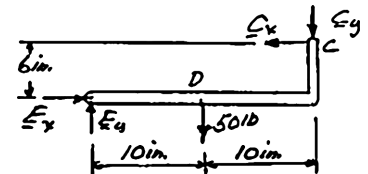


Free body: Member CDE:

$$+\circlearrowleft \Sigma M_E = 0: C_x(6 \text{ in.}) - C_y(20 \text{ in.}) - (50 \text{ lb})(10 \text{ in.}) = 0$$

$$\frac{5P}{a}(6) - \frac{P}{2}(20) = 500$$

$$P \left(\frac{30}{a} - 10 \right) = 500 \quad (1)$$



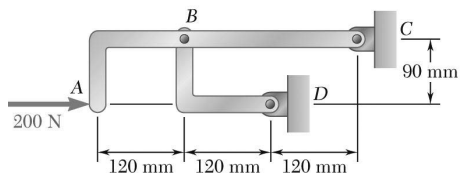
For

$$a = 1.0 \text{ in.}$$

$$P \left(\frac{30}{1} - 10 \right) = 500$$

$$20P = 500$$

$$\mathbf{P} = 25.0 \text{ lb} \downarrow \blacktriangleleft$$



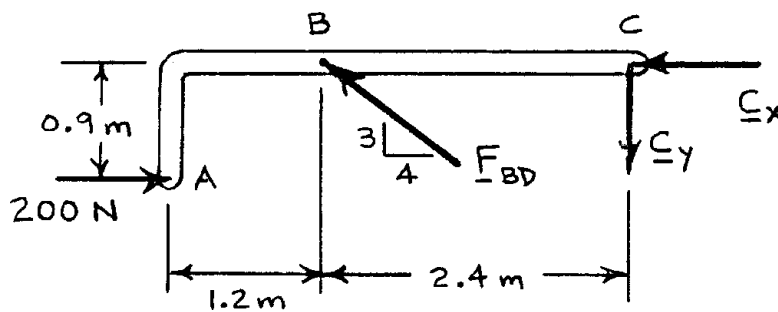
PROBLEM 6.F1

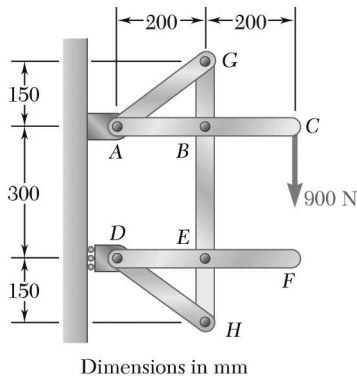
For the frame and loading shown, draw the free-body diagram(s) needed to determine the forces acting on member ABC at B and C .

SOLUTION

We note that BD is a two-force member.

Free body: ABC





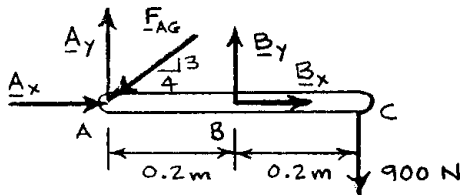
PROBLEM 6.F2

For the frame and loading shown, draw the free-body diagram(s) needed to determine all forces acting on member *GBEH*.

SOLUTION

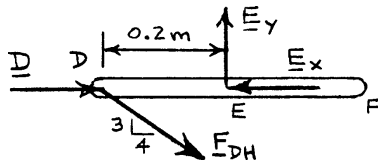
We note that *AG*, *DEF*, and *DH* are two-force members.

Free body: *ABC*



Note: Sum moments about A to determine B_y .

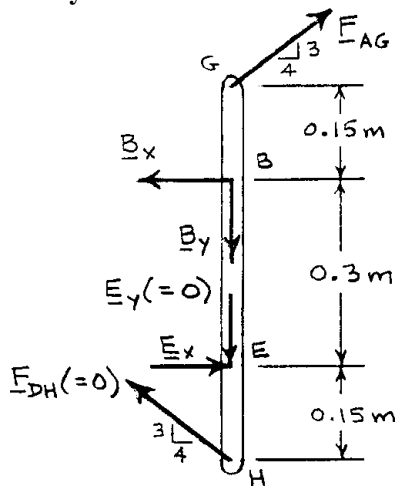
Free body: *DEF*



Note: Sum moments about E to determine F_{DH} (which becomes zero).

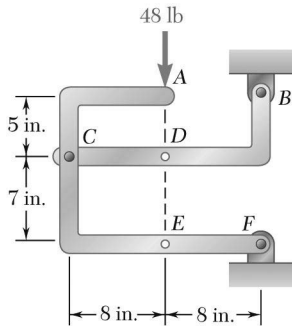
Sum forces in *y* to determine E_y (which also becomes zero).

Free body: *GBEH*



Note: With B_y , E_y , and F_{DH} previously determined, apply three equilibrium equations to determine three remaining forces on *GBEH*.

PROBLEM 6.F3

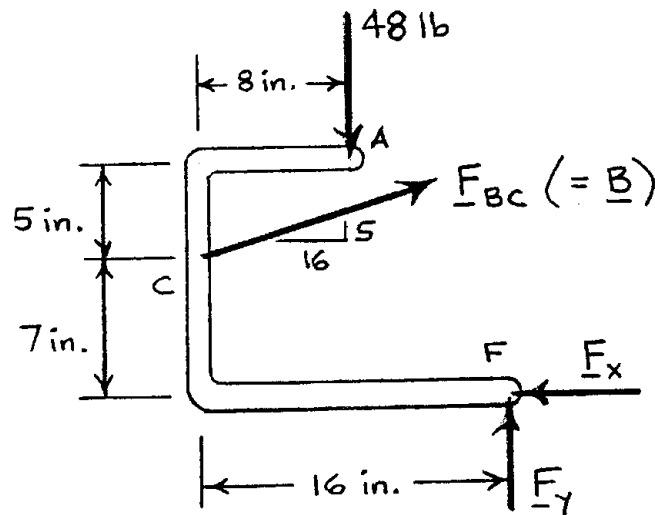


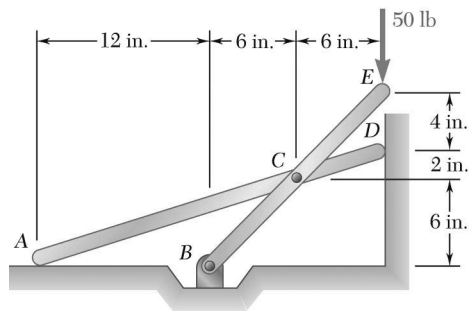
For the frame and loading shown, draw the free-body diagram(s) needed to determine the reactions at B and F .

SOLUTION

We note that BC is two-force member.

Free body: ACF



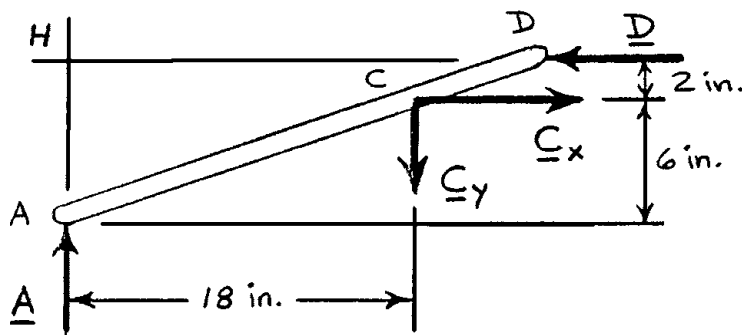


PROBLEM 6.F4

Knowing that the surfaces at A and D are frictionless, draw the free-body diagram(s) needed to determine the forces exerted at B and C on member BCE .

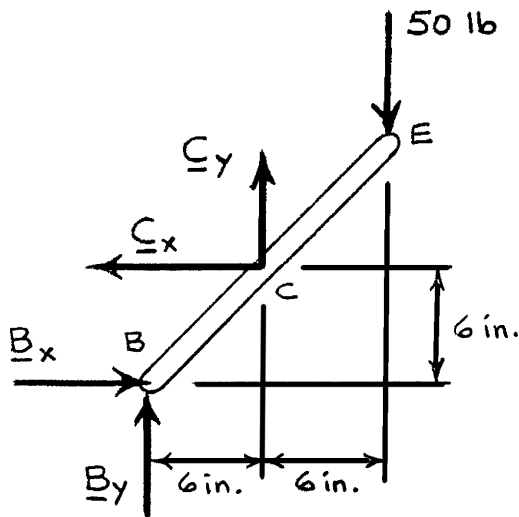
SOLUTION

Free body: ACD



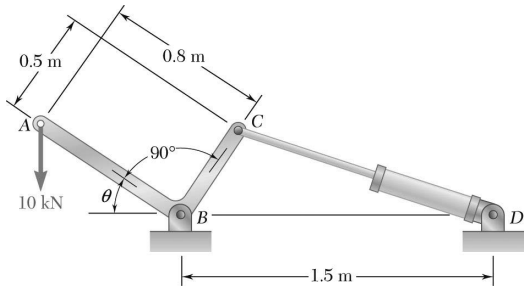
Note: Sum moments about H to relate C_x and C_y .

Free body: BCE



Note: Sum moments about B to relate C_x and C_y ; combine with relation from free body ACD to determine C_x and C_y ; finally, sum forces in x and y to determine B_x and B_y .

PROBLEM 6.F5

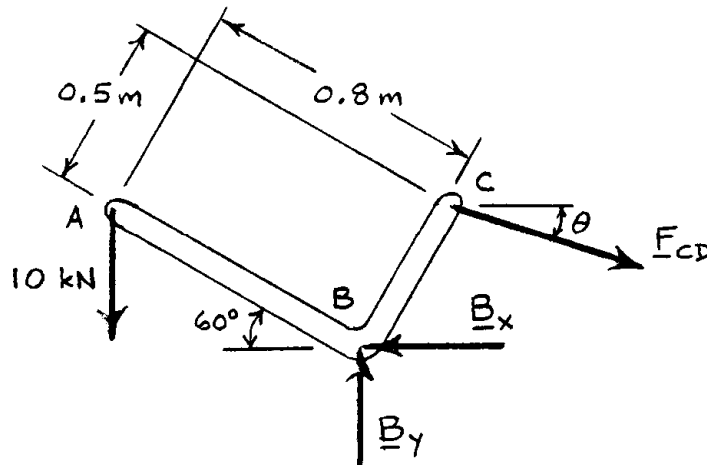


The position of member ABC is controlled by the hydraulic cylinder CD . Knowing that $\theta = 30^\circ$, draw the free-body diagram(s) needed to determine the force exerted by the hydraulic cylinder on pin C , and the reaction at B .

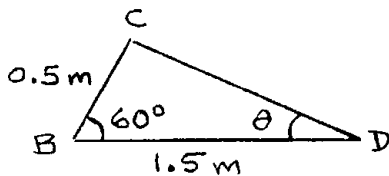
SOLUTION

We note that CD is a two-force member.

Free body: ABC



Note: To find θ , consider geometry of triangle BCD :

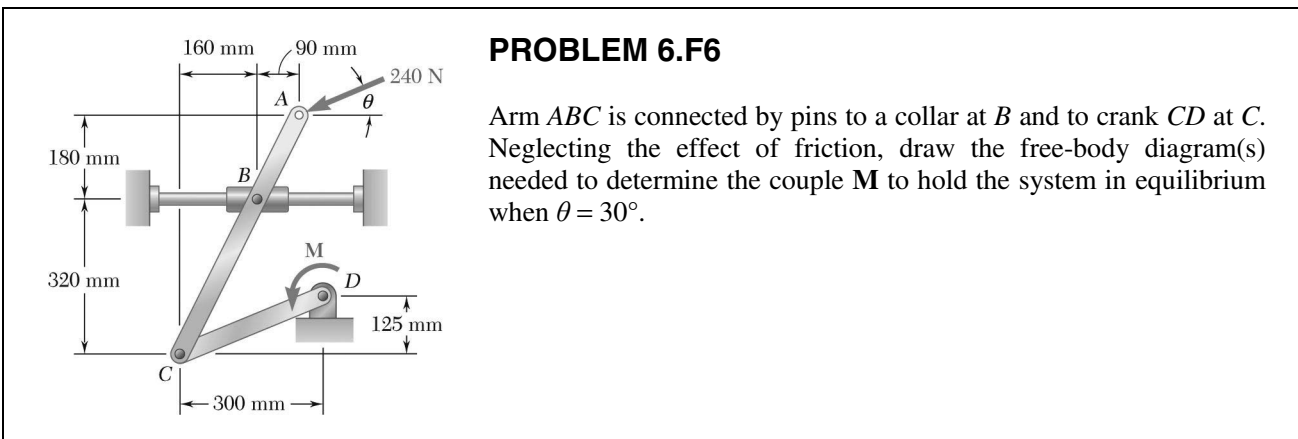


Law of cosines: $(CD)^2 = (0.5)^2 + (1.5)^2 - 2(0.5)(1.5)\cos 60^\circ$

$$CD = 1.32288 \text{ m}$$

Law of sines: $\frac{\sin \theta}{0.5 \text{ m}} = \frac{\sin 60^\circ}{1.32288 \text{ m}}$

$$\theta = 19.1066^\circ$$

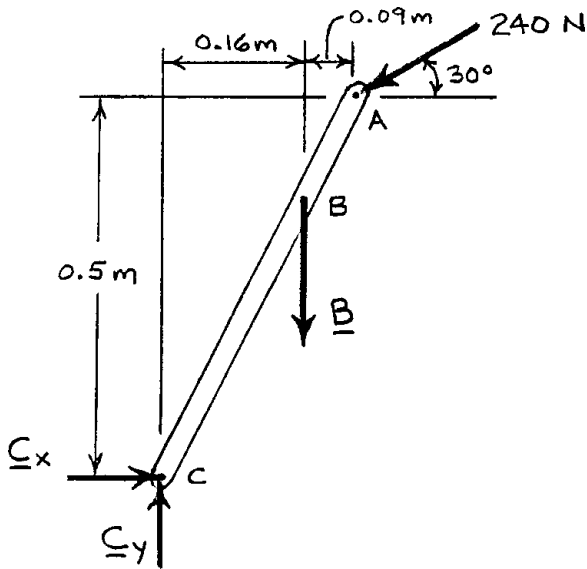


PROBLEM 6.F6

Arm *ABC* is connected by pins to a collar at *B* and to crank *CD* at *C*. Neglecting the effect of friction, draw the free-body diagram(s) needed to determine the couple *M* to hold the system in equilibrium when $\theta = 30^\circ$.

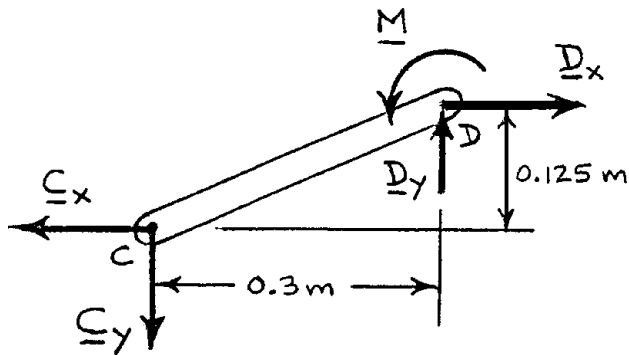
SOLUTION

Free body: *ABC*



Note: Sum forces in *x* to determine C_x ; sum moments about *B* to determine C_y .

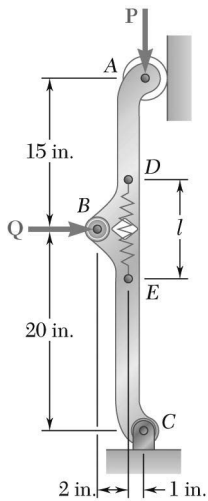
Free body: *CD*



Note: Sum moments about *D* to determine *M*.

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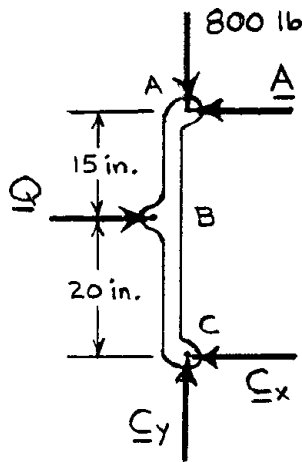
PROBLEM 6.F7



Since the brace shown must remain in position even when the magnitude of \mathbf{P} is very small, a single safety spring is attached at D and E . The spring DE has a constant of 50 lb/in. and an unstretched length of 7 in. Knowing that $l = 10$ in. and that the magnitude of \mathbf{P} is 800 lb, draw the free-body diagram(s) needed to determine the force \mathbf{Q} required to release the brace.

SOLUTION

Free body: ABC



Note: Sum moments about C to relate \mathbf{Q} to \mathbf{A} .

Spring force.

Unstretched length: $\ell_0 = 7$ in.

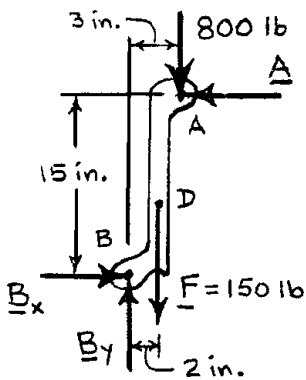
Stretched length: $\ell = 10$ in.

$k = 50$ lb/in.

$$F = kx = k(\ell - \ell_0)$$

$$= 50(10 - 7) = 150 \text{ lb}$$

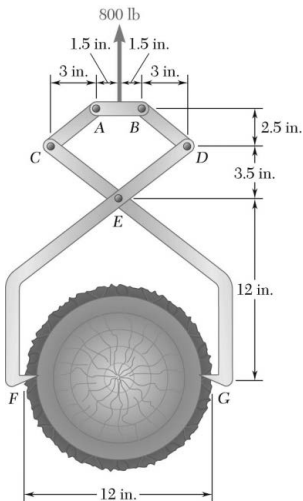
Free body: ADB



Note: Sum moments about B to determine \mathbf{A} ; use relation from free body ABC to determine \mathbf{Q} .

PROBLEM 6.F8

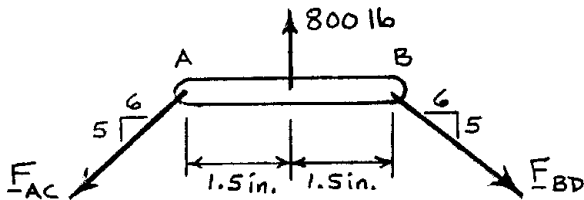
A log weighing 800 lb is lifted by a pair of tongs as shown. Draw the free-body diagram(s) needed to determine the forces exerted at E and F on tong DEF .



SOLUTION

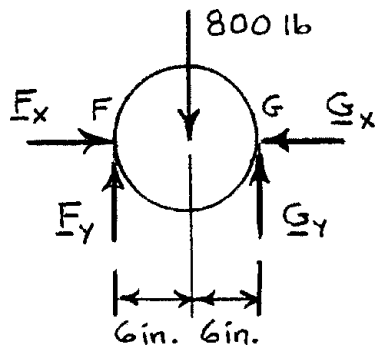
We note that AC and BD are two-force members.

Free body: AB



Note: Sum moments about A to determine F_{BD} .

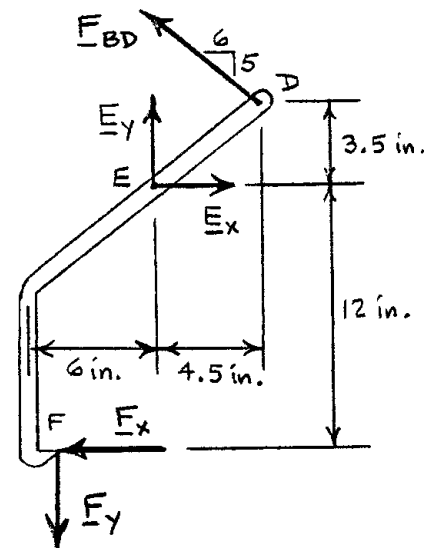
Free body: LOG



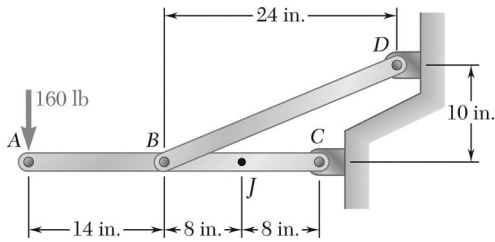
Note: Sum moments about G to determine F_y .

Free body: DEF

Note: Sum moments about E to find F_x ; sum forces in x and y to find E_x and E_y .



CHAPTER 7



PROBLEM 7.1

Determine the internal forces (axial force, shearing force, and bending moment) at Point J of the structure indicated.

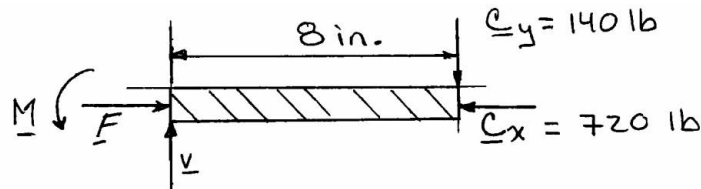
Frame and loading of Problem 6.75.

SOLUTION

From Problem 6.75: $C_x = 720 \text{ lb} \leftarrow$

$C_y = 140 \text{ lb} \downarrow$

FBD of JC :



$$+\rightarrow \Sigma F_x = 0: F - 720 \text{ lb} = 0$$

$$F = +720 \text{ lb}$$

$$\mathbf{F} = 720 \text{ lb} \rightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: V - 140 \text{ lb} = 0$$

$$V = 140 \text{ lb}$$

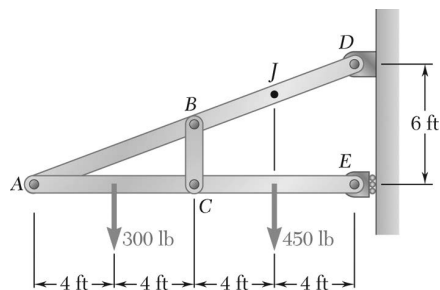
$$\mathbf{V} = 140.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_J = 0: M - (140 \text{ lb})(8 \text{ in.}) = 0$$

$$M = +1120 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 1120 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$

PROBLEM 7.2



Determine the internal forces (axial force, shearing force, and bending moment) at Point J of the structure indicated.

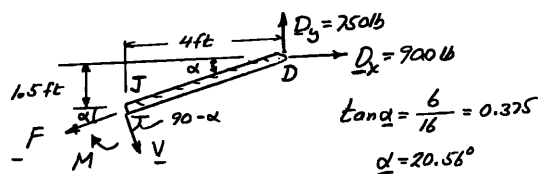
Frame and loading of Problem 6.78.

SOLUTION

From Problem 6.78: $D_x = 900 \text{ lb} \rightarrow$

$D_y = 750 \text{ lb} \uparrow$

FBD of JD :



$$+\curvearrowright \Sigma M_J = 0: -M + (750 \text{ lb})(4 \text{ ft}) - (900 \text{ lb})(1.5 \text{ ft}) = 0$$

$$M = +1650 \text{ lb} \cdot \text{ft}$$

$$\mathbf{M} = 1650 \text{ lb} \cdot \text{ft} \curvearrowright \blacktriangleleft$$

$$+\nearrow \Sigma F = 0: -V + (750 \text{ lb}) \cos 20.56^\circ - (900 \text{ lb}) \sin 20.56^\circ = 0$$

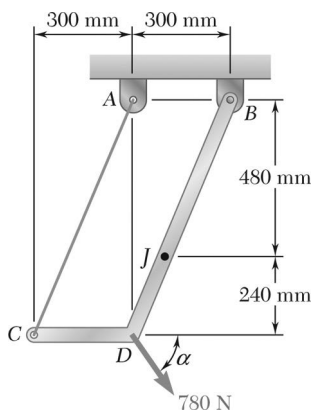
$$V = +386.2 \text{ lb}$$

$$\mathbf{V} = 386 \text{ lb} \searrow 69.4^\circ \blacktriangleleft$$

$$+\swarrow \Sigma F = 0: F - (750 \text{ lb}) \sin 20.56^\circ + (900 \text{ lb}) \cos 20.56^\circ = 0$$

$$F = +1106.1 \text{ lb}$$

$$\mathbf{F} = 1106 \text{ lb} \nearrow 20.6^\circ \blacktriangleleft$$

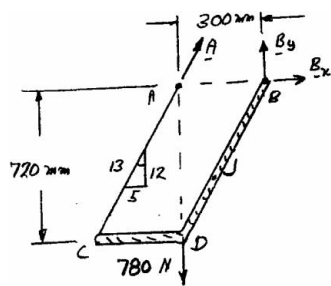


PROBLEM 7.3

Determine the internal forces at Point J when $\alpha = 90^\circ$.

SOLUTION

Reactions ($\alpha = 90^\circ$)



$$\Sigma M_A = 0: B_y = 0$$

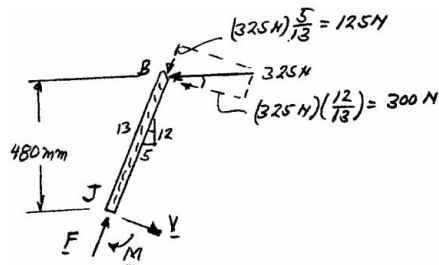
$$+\uparrow \Sigma F_y = 0: A \left(\frac{12}{13} \right) - 780 \text{ N} = 0$$

$$A = 845 \text{ N} \quad \mathbf{A} = 845 \text{ N} \nearrow$$

$$+\rightarrow \Sigma F_x = 0: (845 \text{ N}) \frac{5}{13} + B_x = 0$$

$$B_x = -325 \text{ N} \quad \mathbf{B}_x = 325 \text{ N} \leftarrow$$

FBD BJ:



$$+\nearrow \Sigma F = 0: 125 \text{ N} - F = 0$$

$$\mathbf{F} = 125.0 \text{ N} \nearrow 67.4^\circ \blacktriangleleft$$

$$+\searrow \Sigma F = 0: V - 300 \text{ N} = 0$$

$$\mathbf{V} = 300 \text{ N} \searrow 22.6^\circ \blacktriangleleft$$

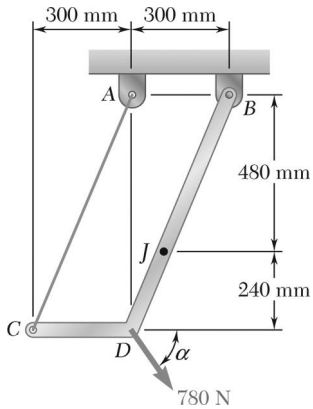
$$+\curvearrowright \Sigma M = 0: (325 \text{ N})(0.480 \text{ m}) - M = 0$$

$$M = +156 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 156.0 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$

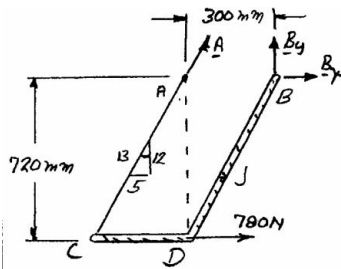
PROBLEM 7.4

Determine the internal forces at Point J when $\alpha = 0$.



SOLUTION

Reactions ($\alpha = 0$)



$$+\curvearrowright \Sigma M_A = 0: (780 \text{ N})(0.720 \text{ m}) + B_y(0.3 \text{ m}) = 0$$

$$B_y = -1872 \text{ N} \quad \mathbf{B}_y = 1872 \text{ N} \downarrow$$

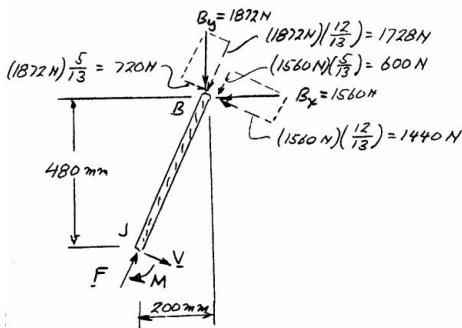
$$+\uparrow \Sigma F_y = 0: A \left(\frac{12}{13} \right) - 1872 \text{ N} = 0$$

$$\mathbf{A} = 2028 \text{ N} \nearrow$$

$$+\rightarrow \Sigma F_x = 0: (2028 \text{ N}) \left(\frac{5}{13} \right) + 780 \text{ N} + B_x = 0$$

$$B_x = -1560 \text{ N} \quad \mathbf{B}_x = 1560 \text{ N} \leftarrow$$

FBD BJ:



$$\swarrow \Sigma F = 0: 1728 \text{ N} + 600 \text{ N} - F = 0$$

$$F = +2328 \text{ N} \quad \mathbf{F} = 2330 \text{ N} \nearrow 67.4^\circ \blacktriangleleft$$

$$\searrow \Sigma F = 0: 720 \text{ N} - 1440 \text{ N} + V = 0$$

$$V = +720 \text{ N} \quad \mathbf{V} = 720 \text{ N} \searrow 22.6^\circ \blacktriangleleft$$

$$BJ = \sqrt{480^2 + 200^2} = 520 \text{ mm}$$

$$+\curvearrowright \Sigma M_J = 0: (1440 \text{ N})(0.520 \text{ m}) - (720 \text{ N})(0.520 \text{ m}) - M = 0$$

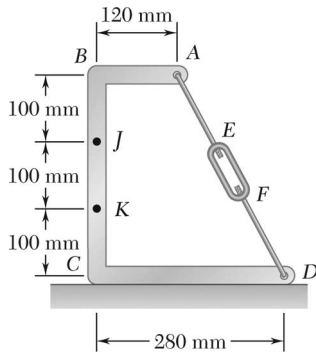
$$M = +374.4 \text{ N}\cdot\text{m} \quad \mathbf{M} = 374 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$

Alternate

Computation of M using $\mathbf{B}_x + \mathbf{B}_y$:

$$+\curvearrowright \Sigma M_J = 0: (1560 \text{ N})(0.48 \text{ m}) - (1872 \text{ N})(0.2 \text{ m}) - M = 0$$

$$M = +374.4 \text{ N}\cdot\text{m} \quad \mathbf{M} = 374 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$

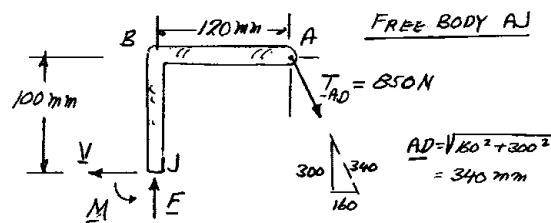


PROBLEM 7.5

Knowing that the turnbuckle has been tightened until the tension in wire AD is 850 N, determine the internal forces at point indicated:

Point J .

SOLUTION



$$\pm \rightarrow \Sigma F_x = 0: -V + \left(\frac{160}{340}\right)(850 \text{ N}) = 0$$

$$V = +400 \text{ N}$$

$$V = 400 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: F - \left(\frac{300}{340}\right)(850 \text{ N}) = 0$$

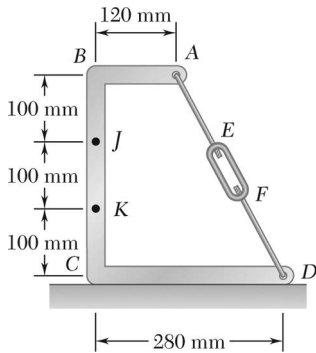
$$F = +750 \text{ N}$$

$$F = 750 \text{ N} \uparrow$$

$$+\curvearrowright \Sigma M_J = 0: M - \left(\frac{300}{340}\right)(850 \text{ N})(120 \text{ mm}) - \left(\frac{160}{340}\right)(850 \text{ N})(100 \text{ mm}) = 0$$

$$M = +130 \text{ N}\cdot\text{m}$$

$$M = 130 \text{ N}\cdot\text{m} \curvearrowright$$



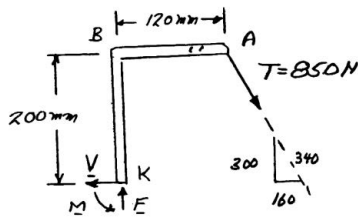
PROBLEM 7.6

Knowing that the turnbuckle has been tightened until the tension in wire AD is 850 N, determine the internal forces at point indicated:

Point K .

SOLUTION

Free body AK :



$$AD = \sqrt{160^2 + 300^2} = 340 \text{ mm}$$

On portion KBA :

$$+\rightarrow \Sigma F_x = 0: -V + \left(\frac{160}{340}\right)(850 \text{ N}) = 0$$

$$V = +400 \text{ N}$$

$$\mathbf{V} = 400 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: F - \left(\frac{300}{340}\right)(850 \text{ N}) = 0$$

$$F = +750 \text{ N}$$

$$\mathbf{F} = 750 \text{ N} \uparrow$$

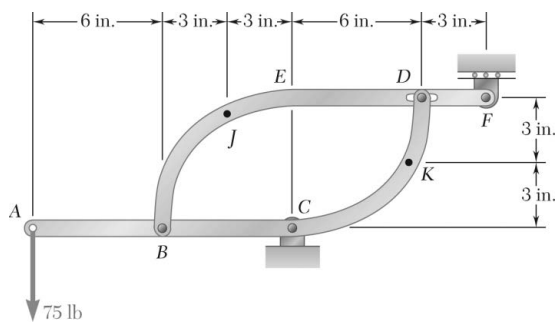
$$+\curvearrowright \Sigma M_J = 0: M - \left(\frac{300}{340}\right)(850 \text{ N})(120 \text{ mm}) - \left(\frac{160}{340}\right)(850 \text{ N})(200 \text{ mm}) = 0$$

$$M = +170 \text{ N}\cdot\text{m}$$

$$\mathbf{M} = 170.0 \text{ N}\cdot\text{m} \curvearrowright$$

Internal forces acting on KCD are equal and opposite

$$\mathbf{F} = 750 \text{ N} \downarrow, \mathbf{V} = 400 \text{ N} \rightarrow, \mathbf{M} = 170.0 \text{ N}\cdot\text{m} \curvearrowleft$$

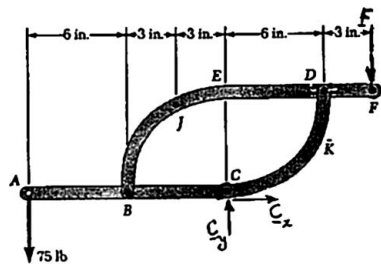


PROBLEM 7.7

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point J.

SOLUTION

Free body: Entire frame



$$+\curvearrowright \Sigma M_C = 0: (75 \text{ lb})(12 \text{ in.}) - F(9 \text{ in.}) = 0$$

$$\mathbf{F} = 100 \text{ lb} \downarrow \triangleleft$$

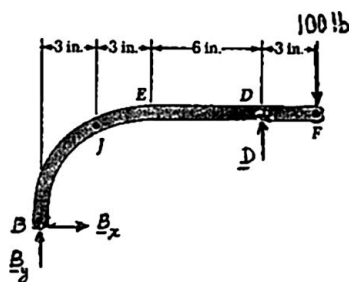
$$\pm \rightarrow \Sigma F_x = 0: C_x = 0$$

$$+\uparrow \Sigma F_y = 0: C_y - 75 \text{ lb} - 100 \text{ lb} = 0$$

$$C_y = +175 \text{ lb}$$

$$\mathbf{C} = 175 \text{ lb} \uparrow \triangleleft$$

Free body: Member BEDF



$$+\curvearrowright \Sigma M_B = 0: D(12 \text{ in.}) - (100 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{D} = 125 \text{ lb} \uparrow \triangleleft$$

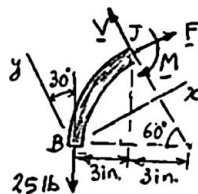
$$\pm \rightarrow \Sigma F_x = 0: B_x = 0$$

$$+\uparrow \Sigma F_y = 0: B_y + 125 \text{ lb} - 100 \text{ lb} = 0$$

$$B_y = -25 \text{ lb}$$

$$\mathbf{B} = 25 \text{ lb} \downarrow \triangleleft$$

Free body: BJ



$$\nearrow \Sigma F_x = 0: F - (25 \text{ lb}) \sin 30^\circ = 0$$

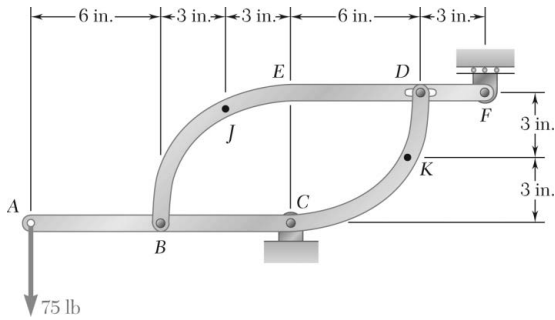
$$\mathbf{F} = 12.50 \text{ lb} \swarrow 30.0^\circ \triangleleft$$

$$+\searrow \Sigma F_y = 0: V - (25 \text{ lb}) \cos 30^\circ = 0$$

$$\mathbf{V} = 21.7 \text{ lb} \nearrow 60.0^\circ \triangleleft$$

$$+\curvearrowright \Sigma M_J = 0: -M + (25 \text{ lb})(3 \text{ in.}) = 0$$

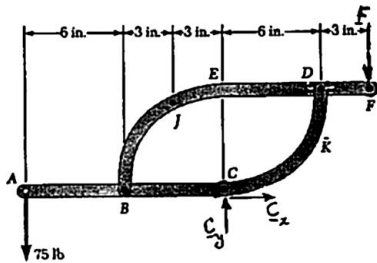
$$\mathbf{M} = 75.0 \text{ lb} \cdot \text{in.} \curvearrowright \triangleleft$$



PROBLEM 7.8

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point K.

SOLUTION



Free body: Entire frame

$$+\curvearrowright \Sigma M_C = 0: (75 \text{ lb})(12 \text{ in.}) - F(9 \text{ in.}) = 0$$

$$F = 100 \text{ lb} \downarrow \triangleleft$$

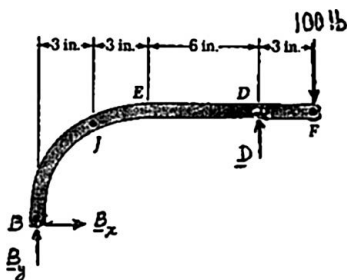
$$+\rightarrow \Sigma F_x = 0: C_x = 0$$

$$+\uparrow \Sigma F_y = 0: C_y - 75 \text{ lb} - 100 \text{ lb} = 0$$

$$C_y = +175 \text{ lb}$$

$$C = 175 \text{ lb} \uparrow \triangleleft$$

Free body: Member *BEDF*



$$+\curvearrowright \Sigma M_B = 0: D(12 \text{ in.}) - (100 \text{ lb})(15 \text{ in.}) = 0$$

$$D = 125 \text{ lb} \uparrow \triangleleft$$

$$+\rightarrow \Sigma F_x = 0: B_x = 0$$

$$+\uparrow \Sigma F_y = 0: B_y + 125 \text{ lb} - 100 \text{ lb} = 0$$

$$B_y = -25 \text{ lb}$$

$$B = 25 \text{ lb} \downarrow \triangleleft$$

Free body: *DK*

We found in Problem 7.11 that

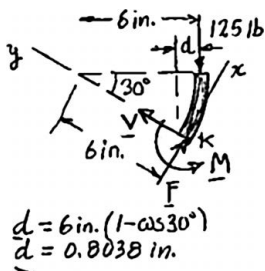
$$D = 125 \text{ lb} \uparrow \text{ on } BEDF.$$

Thus

$$D = 125 \text{ lb} \downarrow \text{ on } DK. \triangleleft$$

$$+\nearrow \Sigma F_x = 0: F - (125 \text{ lb}) \cos 30^\circ = 0$$

$$F = 108.3 \text{ lb} \nearrow 60.0^\circ \blacktriangleleft$$



PROBLEM 7.8 (Continued)

$$\nearrow \Sigma F_y = 0: V - (125 \text{ lb}) \sin 30^\circ = 0$$

$$V = 62.5 \text{ lb} \nearrow 30.0^\circ \blacktriangleleft$$

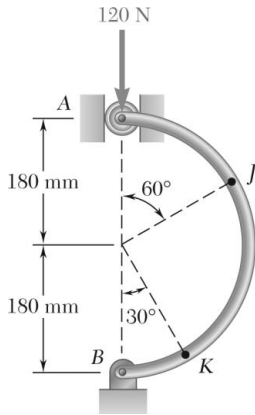
$$+\curvearrowright \Sigma M_K = 0: M - (125 \text{ lb})d = 0$$

$$\begin{aligned} M &= (125 \text{ lb})d = (125 \text{ lb})(0.8038 \text{ in.}) \\ &= 100.5 \text{ lb} \cdot \text{in.} \end{aligned}$$

$$M = 100.5 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$

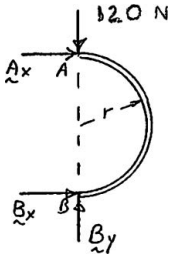
PROBLEM 7.9

A semicircular rod is loaded as shown. Determine the internal forces at Point J .



SOLUTION

FBD Rod:



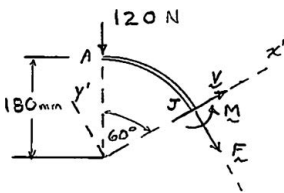
$$\left(\sum M_B = 0: A_x(2r) = 0 \right.$$

$$A_x = 0$$

$$\nearrow \sum F_{x'} = 0: V - (120 \text{ N}) \cos 60^\circ = 0$$

$$V = 60.0 \text{ N} \nearrow \blacktriangleleft$$

FBD AJ:



$$\searrow \sum F_{y'} = 0: F + (120 \text{ N}) \sin 60^\circ = 0$$

$$F = -103.923 \text{ N}$$

$$F = 103.9 \text{ N} \nwarrow \blacktriangleleft$$

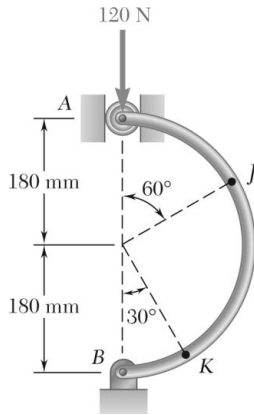
$$\left(\sum M_J = 0: M - [(0.180 \text{ m}) \sin 60^\circ](120 \text{ N}) = 0 \right.$$

$$M = 18.7061$$

$$M = 18.71 \curvearrowright \blacktriangleleft$$

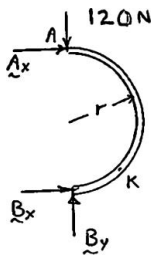
PROBLEM 7.10

A semicircular rod is loaded as shown. Determine the internal forces at Point K.



SOLUTION

FBD Rod:



$$\uparrow \Sigma F_y = 0: B_y - 120 \text{ N} = 0 \quad B_y = 120 \text{ N} \uparrow$$

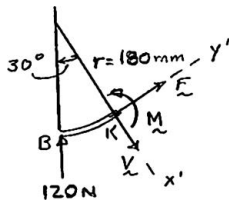
$$\curvearrowleft \Sigma M_A = 0: 2rB_x = 0 \quad B_x = 0$$

$$\searrow \Sigma F_{x'} = 0: V - (120 \text{ N}) \cos 30^\circ = 0$$

$$V = 103.923 \text{ N}$$

$$V = 103.9 \text{ N} \searrow \blacktriangleleft$$

FBD BK:



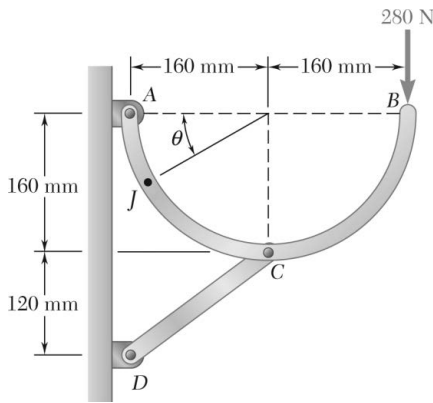
$$\nearrow \Sigma F_{y'} = 0: F + (120 \text{ N}) \sin 30^\circ = 0$$

$$F = -60 \text{ N}$$

$$F = 60.0 \text{ N} \nearrow \blacktriangleleft$$

$$\curvearrowleft \Sigma M_K = 0: M - [(0.180 \text{ m}) \sin 30^\circ](120 \text{ N}) = 0$$

$$M = 10.80 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$



PROBLEM 7.11

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30^\circ$.

SOLUTION

FBD AB:

$$\left(\sum M_A = 0: r \left(\frac{4}{5} C \right) + r \left(\frac{3}{5} C \right) - 2r(280 \text{ N}) = 0 \right.$$

$$C = 400 \text{ N} \nearrow$$

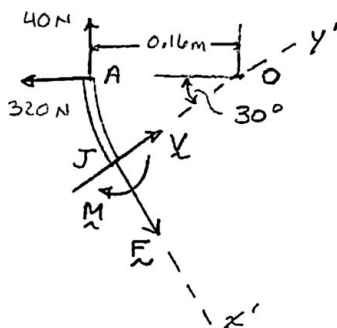
$$\rightarrow \sum F_x = 0: -A_x + \frac{4}{5}(400 \text{ N}) = 0$$

$$A_x = 320 \text{ N} \leftarrow$$

$$\uparrow \sum F_y = 0: A_y + \frac{3}{5}(400 \text{ N}) - 280 \text{ N} = 0$$

$$A_y = 40.0 \text{ N} \uparrow$$

FBD AJ:



$$\searrow \sum F_x = 0: F - (320 \text{ N}) \sin 30^\circ - (40.0 \text{ N}) \cos 30^\circ = 0$$

$$F = 194.641 \text{ N}$$

$$F = 194.6 \text{ N} \searrow 60.0^\circ \blacktriangleleft$$

$$\nearrow \sum F_y = 0: V - (320 \text{ N}) \cos 30^\circ + (40 \text{ N}) \sin 30^\circ = 0$$

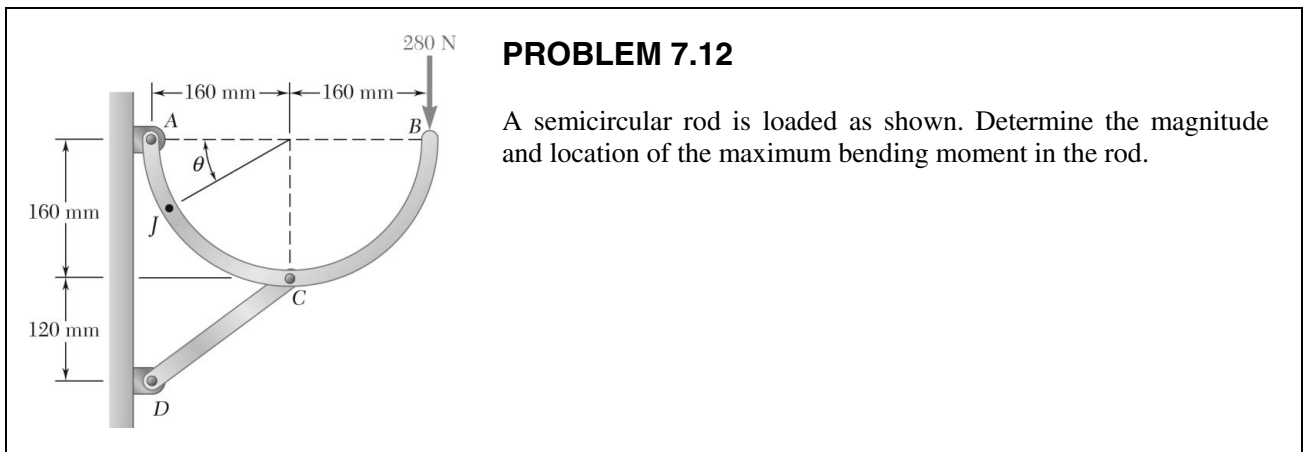
$$V = 257.13 \text{ N}$$

$$V = 257 \text{ N} \nearrow 30.0^\circ \blacktriangleleft$$

$$\left(\sum M_O = 0: (0.160 \text{ m})(194.641 \text{ N}) - (0.160 \text{ m})(40.0 \text{ N}) - M = 0 \right.$$

$$M = 24.743$$

$$M = 24.7 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$

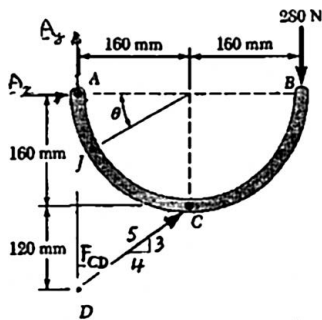


PROBLEM 7.12

A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION

Free body: Rod ACB



$$+\curvearrowright \Sigma M_A = 0: \left(\frac{4}{5} F_{CD}\right)(0.16 \text{ m}) + \left(\frac{3}{5} F_{CD}\right)(0.16 \text{ m}) - (280 \text{ N})(0.32 \text{ m}) = 0$$

$$F_{CD} = 400 \text{ N} \nearrow \triangleleft$$

$$+\rightarrow \Sigma F_x = 0: A_x + \frac{4}{5}(400 \text{ N}) = 0$$

$$A_x = -320 \text{ N}$$

$$A_x = 320 \text{ N} \leftarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(400 \text{ N}) - 280 \text{ N} = 0$$

$$A_y = +40.0 \text{ N}$$

$$A_y = 40.0 \text{ N} \uparrow \triangleleft$$

Free body: AJ (For $\theta < 90^\circ$)

$$+\curvearrowright \Sigma M_J = 0: (320 \text{ N})(0.16 \text{ m}) \sin \theta - (40.0 \text{ N})(0.16 \text{ m})(1 - \cos \theta) - M = 0$$

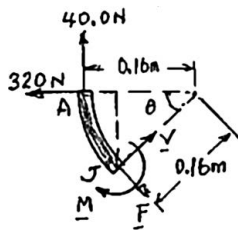
$$M = 51.2 \sin \theta + 6.4 \cos \theta - 6.4 \quad (1)$$

For maximum value between A and C:

$$\frac{dM}{d\theta} = 0: 51.2 \cos \theta - 6.4 \sin \theta = 0$$

$$\tan \theta = \frac{51.2}{6.4} = 8$$

$$\theta = 82.87^\circ \blacktriangleleft$$



Carrying into (1):

$$M = 51.2 \sin 82.87^\circ + 6.4 \cos 82.87^\circ - 6.4 = +45.20 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROBLEM 7.12 (Continued)

Free body: BJ (For $\theta > 90^\circ$)

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (280 \text{ N})(0.16 \text{ m})(1 - \cos \phi) = 0$$

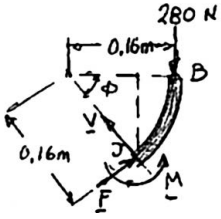
$$M = (44.8 \text{ N} \cdot \text{m})(1 - \cos \phi)$$

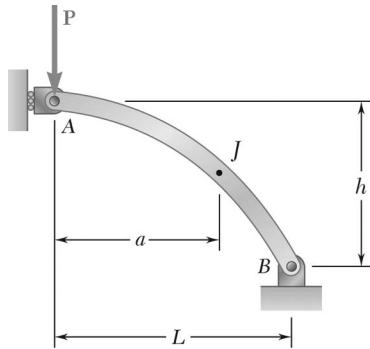
Largest value occurs for $\phi = 90^\circ$, that is, at C , and is

$$M_C = 44.8 \text{ N} \cdot \text{m} \quad \triangleleft$$

We conclude that

$$M_{\max} = 45.2 \text{ N} \cdot \text{m} \quad \text{for} \quad \theta = 82.9^\circ \quad \blacktriangleleft$$



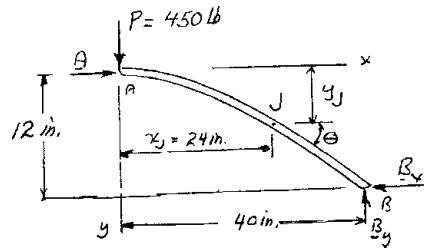


PROBLEM 7.13

The axis of the curved member AB is a parabola with vertex at A . If a vertical load P of magnitude 450 lb is applied at A , determine the internal forces at J when $h = 12$ in., $L = 40$ in., and $a = 24$ in.

SOLUTION

Free body AB



$$\uparrow \Sigma F_y = 0: -450 \text{ lb} + B_y = 0$$

$$B_y = 450 \text{ lb} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: B_x(12 \text{ in.}) - (450 \text{ lb})(40 \text{ in.}) = 0$$

$$B_x = 1500 \text{ lb} \leftarrow$$

$$\Sigma F_x = 0: A = 1500 \text{ lb} \rightarrow$$

Parabola: $y = kx^2$

At B : $12 \text{ in.} = k(40 \text{ in.})^2 \quad k = 0.0075$

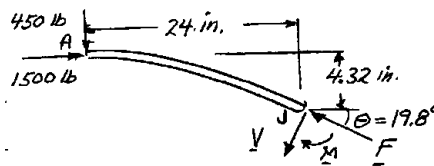
Equation of parabola: $y = 0.0075x^2$

$$\text{slope} = \frac{dy}{dx} = 0.015x$$

At J : $x_J = 24 \text{ in.} \quad y_J = 0.0075(24)^2 = 4.32 \text{ in.}$

$$\text{slope} = 0.015(24) = 0.36, \quad \tan \theta = 0.36, \quad \theta = 19.8^\circ$$

Free body AJ



$$+\circlearrowleft \Sigma M_J = 0: (450 \text{ lb})(24 \text{ in.}) - (1500 \text{ lb})(4.32 \text{ in.}) - M = 0$$

$$M = 4320 \text{ lb} \cdot \text{in.} \quad M = 4320 \text{ lb} \cdot \text{in.} \quad \leftarrow$$

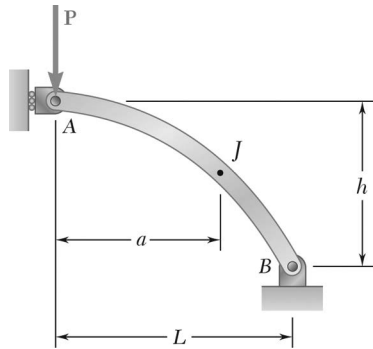
PROBLEM 7.13 (Continued)

$$+\nearrow \Sigma F = 0: F - (450 \text{ lb})\sin 19.8^\circ - (1500 \text{ lb})\cos 19.8^\circ = 0$$

$$F = +1563.8 \text{ lb} \quad \mathbf{F} = 1564 \text{ lb} \nearrow 19.8^\circ \blacktriangleleft$$

$$+\swarrow \Sigma F = 0: -V - (450 \text{ lb})\cos 19.8^\circ + (1500 \text{ lb})\sin 19.8^\circ = 0$$

$$V = +84.71 \text{ lb} \quad \mathbf{V} = 84.7 \text{ lb} \swarrow 70.2^\circ \blacktriangleleft$$



PROBLEM 7.14

Knowing that the axis of the curved member AB is a parabola with vertex at A , determine the magnitude and location of the maximum bending moment.

SOLUTION

Parabola

$$y = kx^2$$

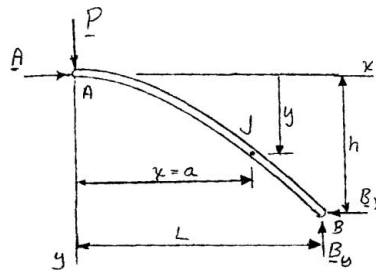
At B :

$$h = kL^2$$

$$k = h/L^2$$

Equation of parabola

$$y = hx^2/L^2$$

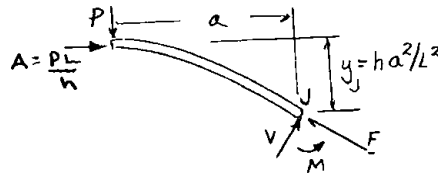


$$+\circlearrowleft \Sigma M_B = 0: P(L) - A(h) = 0$$

$$A = PL/h \rightarrow$$

Free body AJ

$$\text{At } J: x_J = a \quad y_J = ha^2/L^2$$



$$+\circlearrowleft \Sigma M_J = 0: P_a - (PL/h)(ha^2/L^2) + M = 0$$

$$M = P \left(\frac{a^2}{L} - a \right)$$

For maximum:

$$\frac{dM}{da} = P \left(\frac{2a}{L} - 1 \right) = 0$$

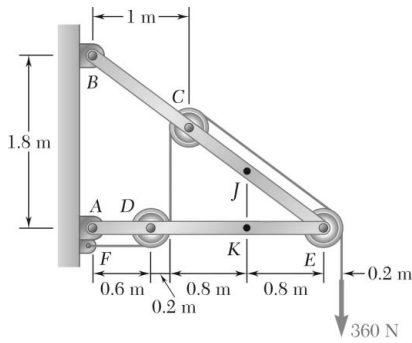
$$M_{\max} \text{ occurs at: } a = \frac{1}{2}L \blacktriangleleft$$

$$M_{\max} = P \left[\frac{(L/2)^2}{L} - \frac{L}{2} \right] = -\frac{PL}{4}$$

$$|M|_{\max} = \frac{1}{4}PL \blacktriangleleft$$

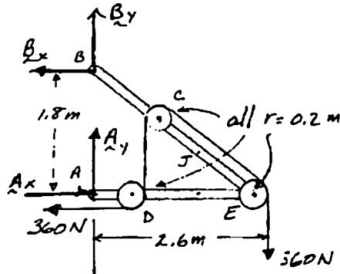
PROBLEM 7.15

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.



SOLUTION

FBD Frame with pulley and cord:

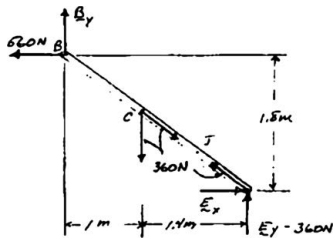


$$\begin{aligned} \sum M_A = 0: & (1.8 \text{ m})B_x - (2.6 \text{ m})(360 \text{ N}) \\ & - (0.2 \text{ m})(360 \text{ N}) = 0 \end{aligned}$$

$$B_x = 560 \text{ N} \leftarrow$$

FBD BE:

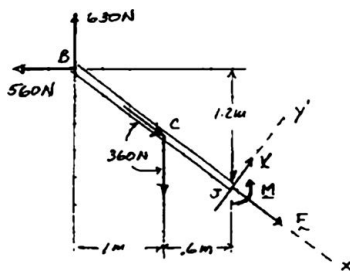
Note: Cord forces have been moved to pulley hub as per Problem 6.91.



$$\begin{aligned} \sum M_E = 0: & (1.4 \text{ m})(360 \text{ N}) + (1.8 \text{ m})(560 \text{ N}) \\ & - (2.4 \text{ m})B_y = 0 \end{aligned}$$

$$B_y = 630 \text{ N} \uparrow$$

FBD BJ:



$$\begin{aligned} \sum F_x = 0: & F + 360 \text{ N} - \frac{3}{5}(630 \text{ N} - 360 \text{ N}) \\ & - \frac{4}{5}(560 \text{ N}) = 0 \end{aligned}$$

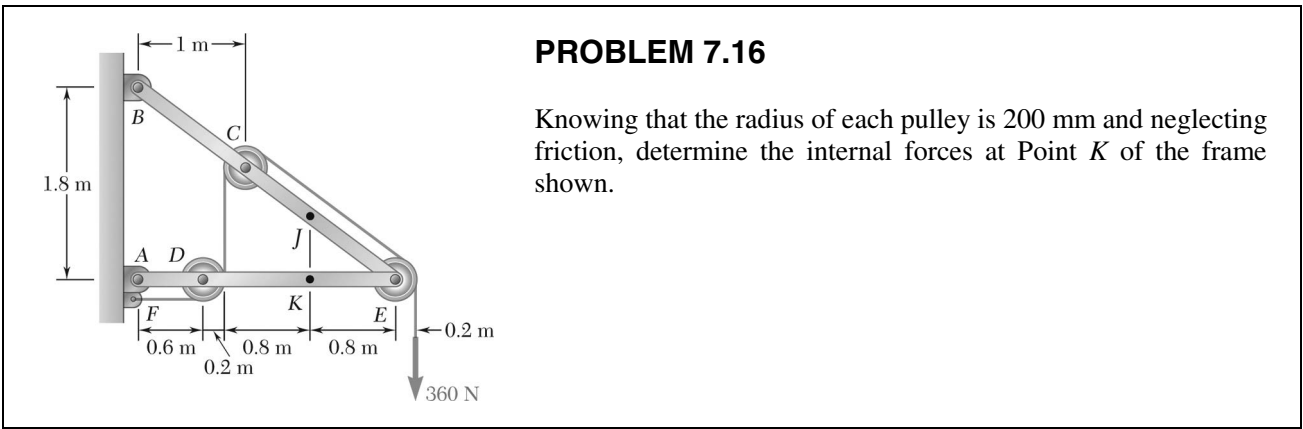
$$F = 250 \text{ N} \swarrow 36.9^\circ \leftarrow$$

$$\sum F_y = 0: V + \frac{4}{5}(630 \text{ N} - 360 \text{ N}) - \frac{3}{5}(560 \text{ N}) = 0$$

$$V = 120.0 \text{ N} \nearrow 53.1^\circ \leftarrow$$

$$\begin{aligned} \sum M_J = 0: & M + (0.6 \text{ m})(360 \text{ N}) + (1.2 \text{ m})(560 \text{ N}) \\ & - (1.6 \text{ m})(630 \text{ N}) = 0 \end{aligned}$$

$$M = 120.0 \text{ N} \cdot \text{m} \curvearrowright \leftarrow$$



SOLUTION

Free body: frame and pulleys

$$\begin{aligned}
 +\curvearrowright \Sigma M_A = 0: & \quad -B_x(1.8 \text{ m}) - (360 \text{ N})(0.2 \text{ m}) \\
 & \quad - (360 \text{ N})(2.6 \text{ m}) = 0 \\
 & \quad B_x = -560 \text{ N} \quad \mathbf{B}_x = 560 \text{ N} \leftarrow \triangleleft \\
 +\rightarrow \Sigma F_x = 0: & \quad A_x - 560 \text{ N} - 360 \text{ N} = 0 \\
 & \quad A_x = +920 \text{ N} \quad \mathbf{A}_x = +920 \text{ N} \rightarrow \triangleleft \\
 +\uparrow \Sigma F_y = 0: & \quad A_y + B_y - 360 \text{ N} = 0 \\
 & \quad A_y + B_y = 360 \text{ N} \quad (1)
 \end{aligned}$$

Free body: member *AE*

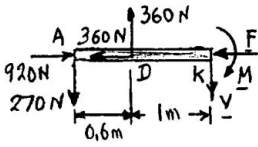
We recall that the forces applied to a pulley may be applied directly to the axle of the pulley.

$$\begin{aligned}
 +\curvearrowright \Sigma M_E = 0: & \quad -A_y(2.4 \text{ m}) - (360 \text{ N})(1.8 \text{ m}) = 0 \\
 & \quad A_y = -270 \text{ N} \quad \mathbf{A}_y = 270 \text{ N} \downarrow \triangleleft \\
 \text{From (1):} & \quad B_y = 360 \text{ N} + 270 \text{ N} \\
 & \quad B_y = 630 \text{ N} \quad \mathbf{B}_y = 630 \text{ N} \uparrow \triangleleft
 \end{aligned}$$

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PROBLEM 7.16 (Continued)

Free body: AK



$$\rightarrow \Sigma F_x = 0: 920 \text{ N} - 360 \text{ N} - F = 0$$

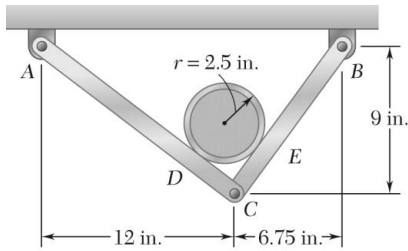
$$F = +560 \text{ N} \quad \mathbf{F} = 560 \text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: 360 \text{ N} - 270 \text{ N} - V = 0$$

$$V = +90.0 \text{ N} \quad \mathbf{V} = 90.0 \text{ N} \downarrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_K = 0: (270 \text{ N})(1.6 \text{ m}) - (360 \text{ N})(1 \text{ m}) - M = 0$$

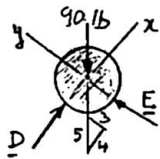
$$M = +72.0 \text{ N}\cdot\text{m} \quad \mathbf{M} = 72.0 \text{ N}\cdot\text{m} \curvearrowright \blacktriangleleft$$



PROBLEM 7.17

A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member AC.

SOLUTION



Free body: 10-ft section of pipe

$$+\nearrow \Sigma F_x = 0: D - \frac{4}{5}(90 \text{ lb}) = 0 \quad \mathbf{D} = 72 \text{ lb} \nearrow \triangleleft$$

$$+\nwarrow \Sigma F_y = 0: E - \frac{3}{5}(90 \text{ lb}) = 0 \quad \mathbf{E} = 54 \text{ lb} \nwarrow \triangleleft$$

Free body: Frame

$$+\curvearrowright \Sigma M_B = 0: -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

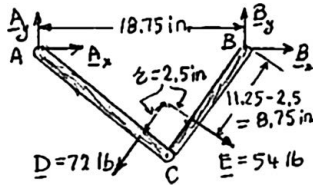
$$A_y = +34.8 \text{ lb} \quad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \quad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \triangleleft$$

$$+\rightarrow \Sigma F_x = 0: A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \quad (1)$$



Free body: Member AC

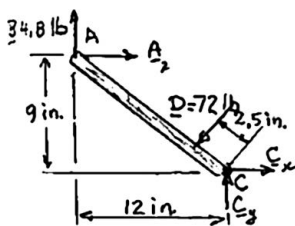
$$+\curvearrowright \Sigma M_C = 0: (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

$$A_x = -26.4 \text{ lb} \quad \mathbf{A}_x = 26.4 \text{ lb} \leftarrow \triangleleft$$

From (1):

$$B_x = -A_x = +26.4 \text{ lb}$$

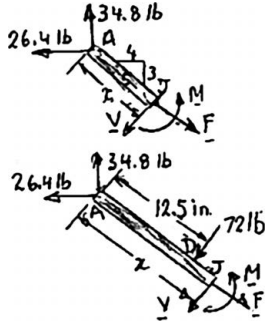
$$\mathbf{B}_x = 26.4 \text{ lb} \rightarrow \triangleleft$$



PROBLEM 7.17 (Continued)

Free body: Portion AJ

For $x \leq 12.5$ in. ($AJ \leq AD$):



$$+\circlearrowleft \Sigma M_J = 0: (26.4 \text{ lb}) \frac{3}{5}x - (34.8 \text{ lb}) \frac{4}{5}x + M = 0$$

$$M = 12x$$

$$M_{\max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

$$M_{\max} = 150.0 \text{ lb} \cdot \text{in. at } D \blacktriangleleft$$

For $x > 12.5$ in. ($AJ > AD$):

$$+\circlearrowleft \Sigma M_J = 0: (26.4 \text{ lb}) \frac{3}{5}x - (34.8 \text{ lb}) \frac{4}{5}x + (72 \text{ lb})(x - 12.5) + M = 0$$

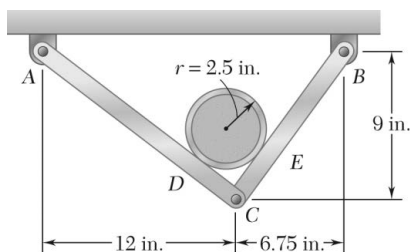
$$M = 900 - 60x$$

$$M_{\max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

Thus:

$$M_{\max} = 150.0 \text{ lb} \cdot \text{in. at } D \blacktriangleleft$$

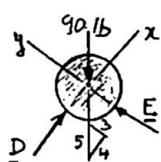
PROBLEM 7.18



For the frame of Problem 7.17, determine the magnitude and location of the maximum bending moment in member BC.

PROBLEM 7.17 A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member AC.

SOLUTION



Free body: 10-ft section of pipe

$$+\nearrow \Sigma F_x = 0: D - \frac{4}{5}(90 \text{ lb}) = 0 \quad \mathbf{D = 72 \text{ lb} \nearrow \triangleleft}$$

$$\nwarrow + \Sigma F_y = 0: E - \frac{3}{5}(90 \text{ lb}) = 0 \quad \mathbf{E = 54 \text{ lb} \nwarrow \triangleleft}$$

Free body: Frame

$$+\curvearrowright \Sigma M_B = 0: -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

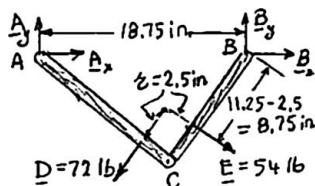
$$A_y = +34.8 \text{ lb} \quad \mathbf{A_y = 34.8 \text{ lb} \uparrow \triangleleft}$$

$$+\uparrow \Sigma F_y = 0: B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \quad \mathbf{B_y = 55.2 \text{ lb} \uparrow \triangleleft}$$

$$\rightarrow + \Sigma F_x = 0: A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \quad (1)$$



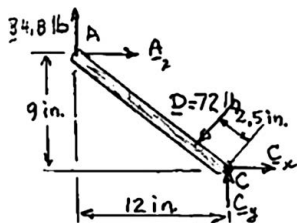
Free body: Member AC

$$+\curvearrowright \Sigma M_C = 0: (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

$$A_x = -26.4 \text{ lb} \quad \mathbf{A_x = 26.4 \text{ lb} \leftarrow \triangleleft}$$

From (1): $B_x = -A_x = +26.4 \text{ lb}$

$$\mathbf{B_x = 26.4 \text{ lb} \rightarrow \triangleleft}$$



PROBLEM 7.18 (Continued)

Free body: Portion BK

For $x \leq 8.75$ in. ($BK \leq BE$):

$$+\circlearrowleft \Sigma M_K = 0: (55.2 \text{ lb}) \frac{3}{5} x - (26.4 \text{ lb}) \frac{4}{5} x - M = 0$$

$$M = 12x$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for } x = 8.75 \text{ in.}$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{at } E \quad \blacktriangleleft$$

For $x > 8.75$ in. ($BK > BE$):

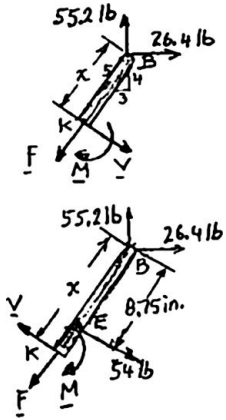
$$+\circlearrowleft \Sigma M_K = 0: (55.2 \text{ lb}) \frac{3}{5} x - (26.4 \text{ lb}) \frac{4}{5} x - (54 \text{ lb})(x - 8.75 \text{ in.}) - M = 0$$

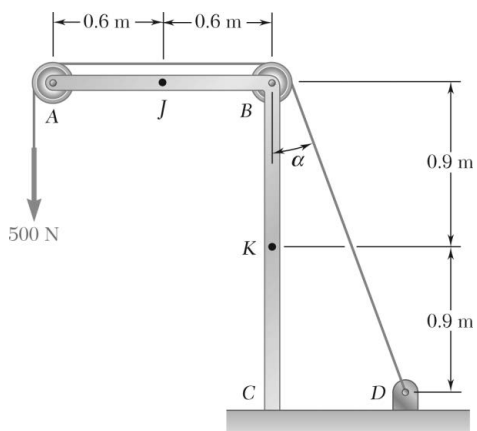
$$M = 472.5 - 42x$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for } x = 8.75 \text{ in.}$$

Thus

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{at } E \quad \blacktriangleleft$$





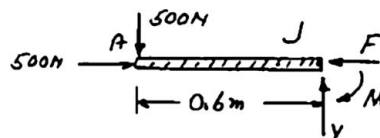
PROBLEM 7.19

Knowing that the radius of each pulley is 150 mm, that $\alpha = 20^\circ$, and neglecting friction, determine the internal forces at (a) Point J, (b) Point K.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins A and B. Radius does not enter computations: (cf. Problem 6.90)

(a) Free body: AJ



$$\pm \rightarrow \Sigma F_x = 0: 500 \text{ N} - F = 0$$

$$F = 500 \text{ N}$$

$$\mathbf{F} = 500 \text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: V - 500 \text{ N} = 0$$

$$V = 500 \text{ N}$$

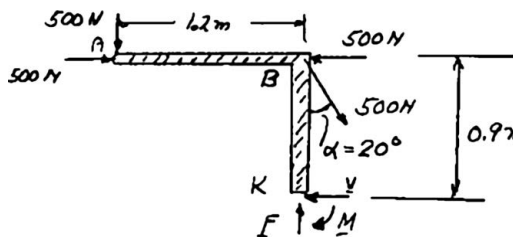
$$\mathbf{V} = 500 \text{ N} \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_J = 0: (500 \text{ N})(0.6 \text{ m}) = 0$$

$$M = 300 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 300 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$

(b) Free body: ABK



$$\pm \rightarrow \Sigma F_x = 0: 500 \text{ N} - 500 \text{ N} + (500 \text{ N}) \sin 20^\circ - V = 0$$

$$V = 171.01 \text{ N}$$

$$\mathbf{V} = 171.0 \text{ N} \leftarrow \blacktriangleleft$$

PROBLEM 7.19 (Continued)

$$+\uparrow \Sigma F_y = 0: -500 \text{ N} - (500 \text{ N}) \cos 20^\circ + F = 0$$

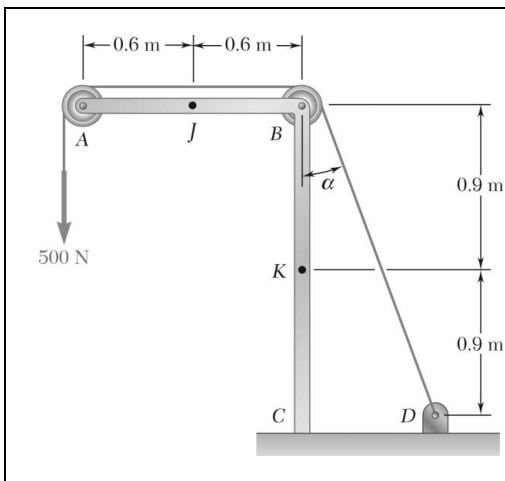
$$F = 969.8 \text{ N}$$

$$\mathbf{F} = 970 \text{ N } \uparrow \blacktriangleleft$$

$$+\curvearrowright \Sigma M_K = 0: (500 \text{ N})(1.2 \text{ m}) - (500 \text{ N}) \sin 20^\circ (0.9 \text{ m}) - M = 0$$

$$M = 446.1 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 446 \text{ N} \cdot \text{m } \curvearrowright \blacktriangleleft$$



PROBLEM 7.20

Knowing that the radius of each pulley is 150 mm, that $\alpha = 30^\circ$, and neglecting friction, determine the internal forces at (a) Point J, (b) Point K.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins A and B. Radius does not enter computations: (cf. Problem 6.90)

(a) Free body: AJ:

$$\rightarrow \Sigma F_x = 0: 500 \text{ N} - F = 0$$

$$F = 500 \text{ N}$$

$$F = 500 \text{ N} \leftarrow$$

$$\uparrow \Sigma F_y = 0: V - 500 \text{ N} = 0$$

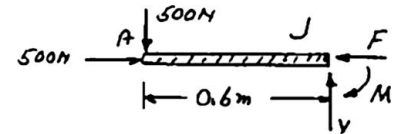
$$V = 500 \text{ N}$$

$$V = 500 \text{ N} \uparrow$$

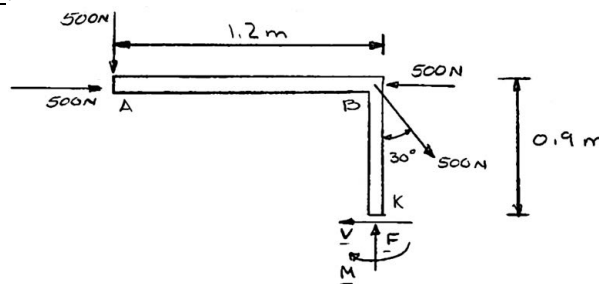
$$\curvearrowright \Sigma M_J = 0: (500 \text{ N})(0.6 \text{ m}) = 0$$

$$M = 300 \text{ N} \cdot \text{m}$$

$$M = 300 \text{ N} \cdot \text{m} \curvearrowright$$



(b) FBD: Portion ABK:



$$\rightarrow \Sigma F_x = 0: 500 \text{ N} - 500 \text{ N} + (500 \text{ N}) \sin 30^\circ - V$$

$$V = 250 \text{ N} \leftarrow$$

$$\uparrow \Sigma F_y = 0: -500 \text{ N} - (500 \text{ N}) \cos 30^\circ + F = 0$$

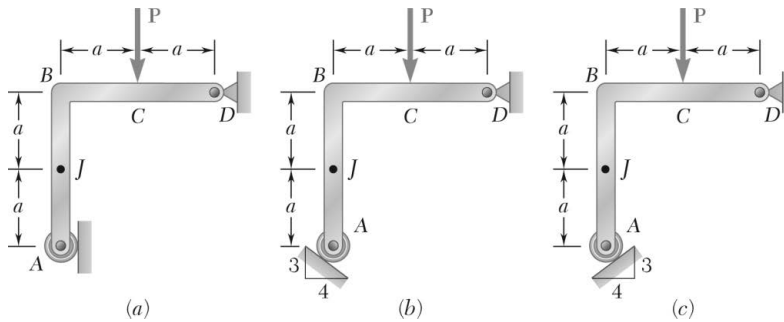
$$F = 933 \text{ N} \uparrow$$

$$\curvearrowright \Sigma M_K = 0: (500 \text{ N})(1.2 \text{ m}) - (500 \text{ N}) \sin 30^\circ (0.9 \text{ m}) - M = 0$$

$$M = 375 \text{ N} \cdot \text{m} \curvearrowright$$

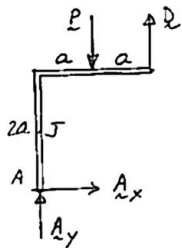
PROBLEM 7.21

A force P is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point J .



SOLUTION

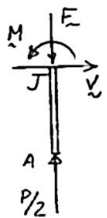
(a) **FBD Rod:**



$$\rightarrow \Sigma F_x = 0: \quad A_x = 0$$

$$\left(\Sigma M_D = 0: \quad aP - 2aA_y = 0 \quad A_y = \frac{P}{2} \right)$$

FBD AJ:



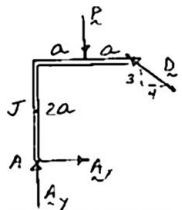
$$\rightarrow \Sigma F_x = 0: \quad V = 0 \quad \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{P}{2} - F = 0$$

$$F = \frac{P}{2} \quad \blacktriangleleft$$

$$\left(\Sigma M_J = 0: \quad M = 0 \quad \blacktriangleleft \right)$$

(b) **FBD Rod:**



$$\left(\Sigma M_A = 0: \quad 2a \left(\frac{4}{5} D \right) + 2a \left(\frac{3}{5} D \right) - aP = 0 \quad D = \frac{5P}{14} \right)$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{5} \frac{5P}{14} = 0$$

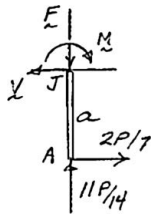
$$A_x = \frac{2P}{7}$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{3}{5} \frac{5P}{14} = 0$$

$$A_y = \frac{11P}{14}$$

PROBLEM 7.21 (Continued)

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \quad \frac{2}{7}P - V = 0$$

$$V = \frac{2P}{7} \leftarrow \blacktriangleleft$$

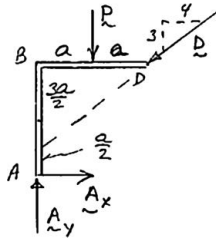
$$\uparrow \Sigma F_y = 0: \quad \frac{11P}{14} - F = 0$$

$$F = \frac{11P}{14} \downarrow \blacktriangleleft$$

$$\curvearrowleft \Sigma M_J = 0: \quad a \frac{2P}{7} - M = 0$$

$$M = \frac{2}{7}aP \curvearrowleft \blacktriangleleft$$

(c) **FBD Rod:**



$$\curvearrowleft \Sigma M_A = 0: \quad \frac{a}{2} \left(\frac{4D}{5} \right) - aP = 0$$

$$D = \frac{5P}{2}$$

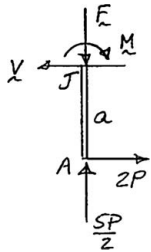
$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{5} \frac{5P}{2} = 0$$

$$A_x = 2P$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P - \frac{3}{5} \frac{5P}{2} = 0$$

$$A_y = \frac{5P}{2}$$

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \quad 2P - V = 0$$

$$V = 2P \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{5P}{2} - F = 0$$

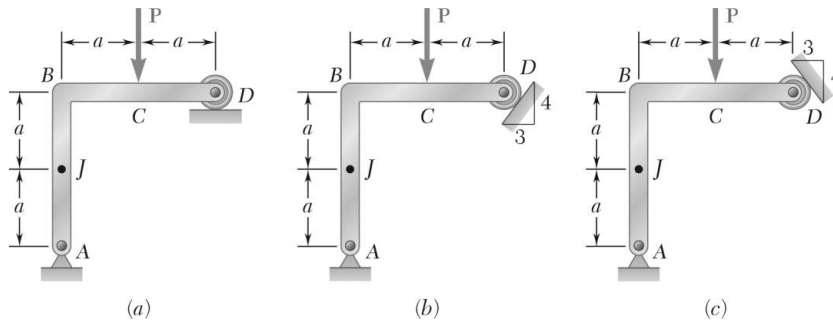
$$F = \frac{5P}{2} \downarrow \blacktriangleleft$$

$$\curvearrowleft \Sigma M_J = 0: \quad a(2P) - M = 0$$

$$M = 2aP \curvearrowleft \blacktriangleleft$$

PROBLEM 7.22

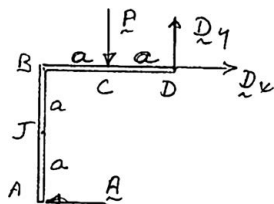
A force P is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point J .



SOLUTION

(a) **FBD Rod:**

$$\left(\sum M_D = 0: aP - 2aA = 0 \right.$$



$$A = \frac{P}{2} \leftarrow$$

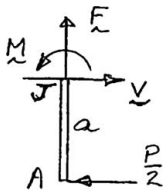
$$\rightarrow \sum F_x = 0: V - \frac{P}{2} = 0$$

$$V = \frac{P}{2} \rightarrow \blacktriangleleft$$

FBD AJ:

$$\uparrow \sum F_y = 0:$$

$$F = 0 \blacktriangleleft$$

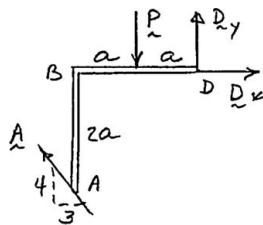


$$\left(\sum M_J = 0: M - a \frac{P}{2} = 0 \right.$$

$$M = \frac{aP}{2} \curvearrowright \blacktriangleleft$$

(b) **FBD Rod:**

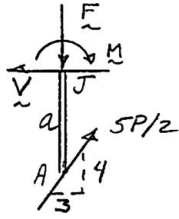
$$\left(\sum M_D = 0: aP - \frac{a}{2} \left(\frac{4}{5} A \right) = 0 \right.$$



$$A = \frac{5P}{2} \nearrow$$

PROBLEM 7.22 (Continued)

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \frac{3}{5} \frac{5P}{2} - V = 0$$

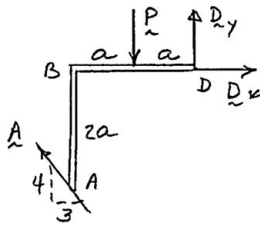
$$V = \frac{3P}{2} \leftarrow$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5} \frac{5P}{2} - F = 0$$

$$F = 2P \downarrow$$

$$M = \frac{3}{2} aP \curvearrowright$$

(c) **FBD Rod:**



$$\curvearrowleft \Sigma M_D = 0: aP - 2a \left(\frac{3}{5} A \right) - 2a \left(\frac{4}{5} A \right) = 0$$

$$A = \frac{5P}{14}$$

$$\rightarrow \Sigma F_x = 0: V - \left(\frac{3}{5} \frac{5P}{14} \right) = 0$$

$$V = \frac{3P}{14} \rightarrow$$

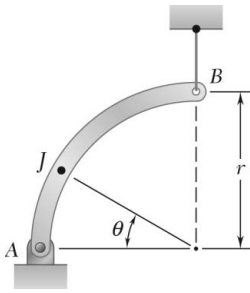
$$\uparrow \Sigma F_y = 0: \frac{4}{5} \frac{5P}{14} - F = 0$$

$$F = \frac{2P}{7} \downarrow$$

$$\curvearrowleft \Sigma M_J = 0: M - a \left(\frac{3}{5} \frac{5P}{14} \right) = 0$$

$$M = \frac{3}{14} aP \curvearrowright$$

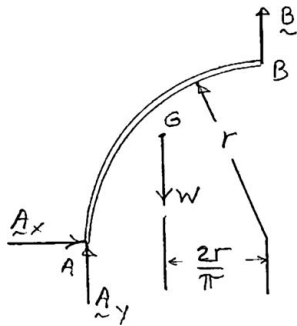
PROBLEM 7.23



A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30^\circ$.

SOLUTION

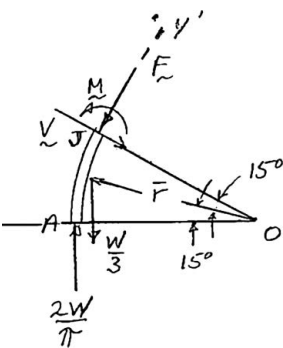
FBD Rod:



$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\curvearrowleft \Sigma M_B = 0: \frac{2r}{\pi} W - r A_y = 0 \quad A_y = \frac{2W}{\pi} \uparrow$$

FBD AJ:



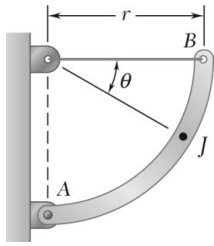
$$\alpha = 15^\circ, \text{ weight of segment} = W \frac{30^\circ}{90^\circ} = \frac{W}{3}$$

$$\bar{r} = \frac{r}{\alpha} \sin \alpha = \frac{r}{\frac{\pi}{12}} \sin 15^\circ = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \frac{2W}{\pi} \cos 30^\circ - \frac{W}{3} \cos 30^\circ - F = 0$$

$$F = \frac{W\sqrt{3}}{2} \left(\frac{2}{\pi} - \frac{1}{3} \right) \nearrow$$

$$\curvearrowleft \Sigma M_0 = M + r \left(F - \frac{2W}{\pi} \right) + \bar{r} \cos 15^\circ \frac{W}{3} = 0 \quad M = 0.0557Wr \curvearrowright \blacktriangleleft$$

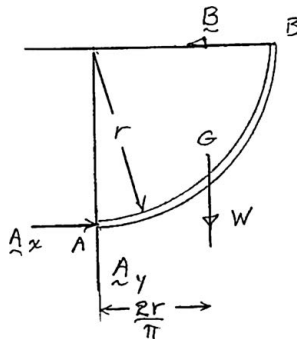


PROBLEM 7.24

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30^\circ$.

SOLUTION

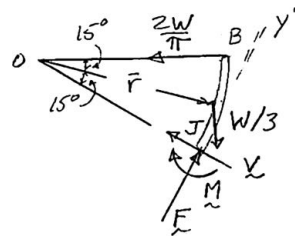
FBD Rod:



$$\left(\sum M_A = 0: rB - \frac{2r}{\pi}W = 0 \right.$$

$$B = \frac{2W}{\pi} \leftarrow$$

FBD BJ:



$$\alpha = 15^\circ = \frac{\pi}{12}$$

$$\bar{r} = \frac{r}{\frac{\pi}{12}} \sin 15^\circ = 0.98862r$$

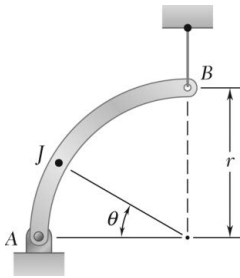
$$\text{Weight of segment} = W \frac{30^\circ}{90^\circ} = \frac{W}{3}$$

$$\nearrow \sum F_{y'} = 0: F - \frac{W}{3} \cos 30^\circ - \frac{2W}{\pi} \sin 30^\circ = 0$$

$$F = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi} \right) W \nearrow$$

$$\left(\sum M_0 = 0: rF - (\bar{r} \cos 15^\circ) \frac{W}{3} - M = 0 \right.$$

$$M = rW \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi} \right) - \left(0.98862 \frac{\cos 15^\circ}{3} \right) W r \quad \mathbf{M = 0.289Wr} \quad \blacktriangleleft$$



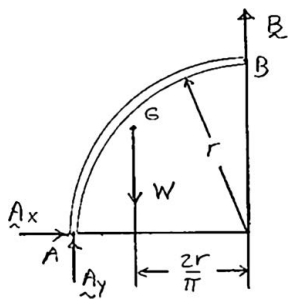
PROBLEM 7.25

For the rod of Problem 7.23, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.23 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30^\circ$.

SOLUTION

FBD Rod:



$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\curvearrowleft \Sigma M_B = 0: \frac{2r}{\pi}W - rA_y = 0 \quad A_y = \frac{2W}{\pi}$$

$$\alpha = \frac{\theta}{2}, \quad \bar{r} = \frac{r}{\alpha} \sin \alpha$$

$$\text{Weight of segment} = W \frac{2\alpha}{\frac{\pi}{2}} = \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_x = 0: -F - \frac{4\alpha}{\pi}W \cos 2\alpha + \frac{2W}{\pi} \cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1 - 2\alpha) \cos 2\alpha = \frac{2W}{\pi}(1 - \theta) \cos \theta$$

FBD AJ: $\curvearrowleft \Sigma M_O = 0: M + \left(F - \frac{2W}{\pi}\right)r + (\bar{r} \cos \alpha) \frac{4\alpha}{\pi}W = 0$

$$M = \frac{2W}{\pi}(1 + \theta \cos \theta - \cos \theta)r - \frac{4\alpha W}{\pi} \frac{r}{\alpha} \sin \alpha \cos \alpha$$

But, $\sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha = \frac{1}{2} \sin \theta$

so $M = \frac{2r}{\pi}W(1 - \cos \theta + \theta \cos \theta - \sin \theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin \theta - \theta \sin \theta + \cos \theta - \cos \theta) = 0$$

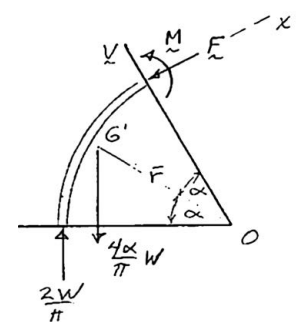
for $(1 - \theta) \sin \theta = 0$

$$\frac{dM}{d\theta} = 0 \quad \text{for } \theta = 0, 1, n\pi (n = 1, 2, \dots)$$

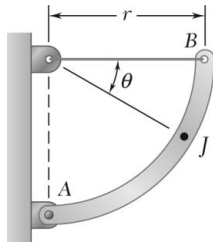
Only 0 and 1 in valid range

At $\theta = 0$ $M = 0$, at $\theta = 1$ rad

$$\text{at } \theta = 57.3^\circ$$



$$M = M_{\max} = 0.1009Wr \quad \blacktriangleleft$$



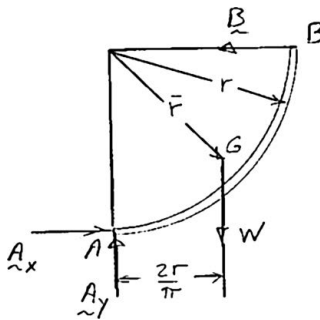
PROBLEM 7.26

For the rod of Problem 7.24, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.24 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30^\circ$.

SOLUTION

FBD Bar:



$$\left(\sum M_A = 0: rB - \frac{2r}{\pi} W = 0 \quad B = \frac{2W}{\pi} \leftarrow \right.$$

$$\alpha = \frac{\theta}{2} \quad \text{so} \quad 0 \leq \alpha \leq \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha} \sin \alpha$$

$$\text{Weight of segment} = W \frac{2\alpha}{\frac{\pi}{2}}$$

$$= \frac{4\alpha}{\pi} W$$

$$\nearrow \sum F_x = 0: F - \frac{4\alpha}{\pi} W \cos 2\alpha - \frac{2W}{\pi} \sin 2\alpha = 0$$

$$F = \frac{2W}{\pi} (\sin 2\alpha + 2\alpha \cos 2\alpha)$$

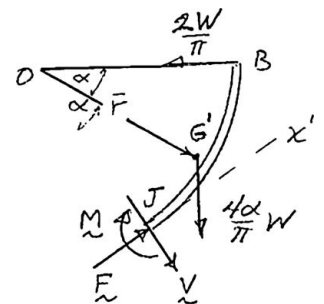
$$= \frac{2W}{\pi} (\sin \theta + \theta \cos \theta)$$

FBD BJ:

$$\left(\sum M_0 = 0: rF - (\bar{r} \cos \alpha) \frac{4\alpha}{\pi} W - M = 0 \right.$$

$$M = \frac{2}{\pi} Wr (\sin \theta + \theta \cos \theta) - \left(\frac{r}{\alpha} \sin \alpha \cos \alpha \right) \frac{4\alpha}{\pi} W$$

But, $\sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha = \frac{1}{2} \sin \theta$



PROBLEM 7.26 (Continued)

so
$$M = \frac{2Wr}{\pi}(\sin \theta + \theta \cos \theta - \sin \theta)$$

or
$$M = \frac{2}{\pi}Wr\theta \cos \theta$$

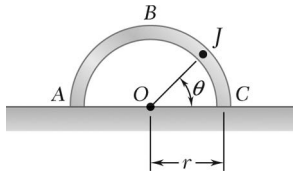
$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr(\cos \theta - \theta \sin \theta) = 0 \quad \text{at } \theta \tan \theta = 1$$

Solving numerically $\theta = 0.8603 \text{ rad}$

and
$$M = 0.357Wr \quad \left. \right) \blacktriangleleft$$

at $\theta = 49.3^\circ \quad \blacktriangleleft$

(Since $M = 0$ at both limits, this is the maximum)



PROBLEM 7.27

A half section of pipe rests on a frictionless horizontal surface as shown. If the half section of pipe has a mass of 9 kg and a diameter of 300 mm, determine the bending moment at Point J when $\theta = 90^\circ$.

SOLUTION

For half section $m = 9 \text{ kg}$

$$W = mg = (9)(9.81) = 88.29 \text{ N}$$

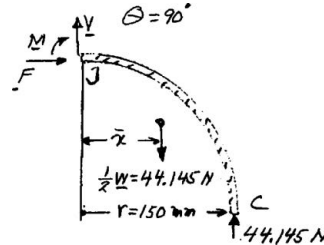
Portion JC:

$$\text{Weight} = \frac{1}{2}W = 44.145 \text{ N}$$

From Fig. 5.8B:

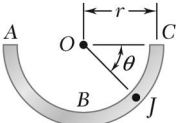
$$x = \frac{2r}{\pi} = \frac{2(150)}{\pi}$$

$$x = 95.49 \text{ mm}$$



$$+\curvearrowright \Sigma M_J = 0: (44.145 \text{ N})(0.15 \text{ m}) - (44.145 \text{ N})(0.09549 \text{ m}) - M = 0$$

$$M = +2.406 \text{ N}\cdot\text{m} \quad \mathbf{M = 2.41 \text{ N}\cdot\text{m}} \quad \blacktriangleleft$$



PROBLEM 7.28

A half section of pipe rests on a frictionless horizontal surface as shown. If the half section of pipe has a mass of 9 kg and a diameter of 300 mm, determine the bending moment at point J when $\theta = 90^\circ$.

SOLUTION

For half section $m = 9 \text{ kg}$

$$W = mg = (9 \text{ kg})(9.81 \text{ m/s}^2) = 88.29 \text{ N}$$

Free body JC

$$\text{Weight of portion } JC, \quad = \frac{1}{2}W = 44.145 \text{ N}$$

$$r = 150 \text{ mm}$$

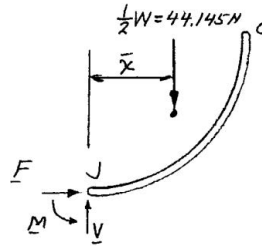
From Fig. 5.8B:

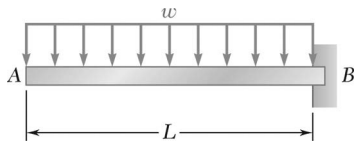
$$x = \frac{2r}{\pi} = \frac{2(150)}{\pi} = 95.49 \text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (44.145 \text{ N})(0.09549 \text{ m}) = 0$$

$$M = +4.2154 \text{ N}\cdot\text{m}$$

$$\mathbf{M} = 4.22 \text{ N}\cdot\text{m} \quad \curvearrowright \blacktriangleleft$$

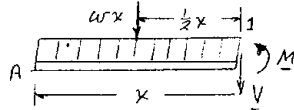
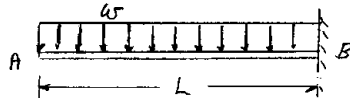




PROBLEM 7.29

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

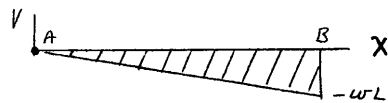


$$+\uparrow \Sigma F_y = 0: -wx - V = 0$$

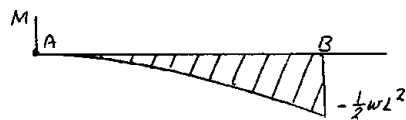
$$V = -wx$$

$$+\curvearrowright \Sigma M_1 = 0: wx\left(\frac{x}{2}\right) + M = 0$$

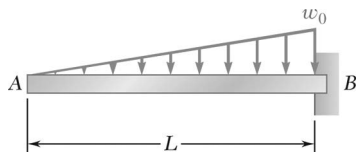
$$M = -\frac{1}{2}wx^2$$



$$|V|_{\max} = wL \quad \blacktriangleleft$$



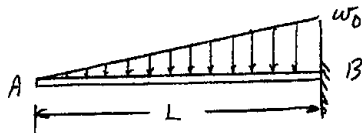
$$|M|_{\max} = \frac{1}{2}wL^2 \quad \blacktriangleleft$$



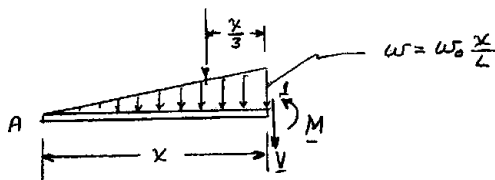
PROBLEM 7.30

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

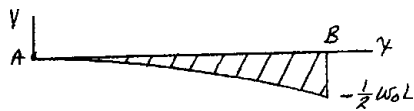


$$\frac{1}{2}wx = \frac{1}{2}\left(w_0 \frac{x}{L}\right)x = \frac{1}{2}w_0 \frac{x^2}{L} \quad \text{By similar } \Delta\text{'s}$$

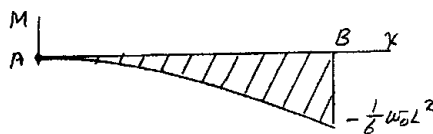


$$+\uparrow \Sigma F_y = 0: \quad -\frac{1}{2}w_0 \frac{x^2}{L} - V = 0 \quad V = -\frac{1}{2}w_0 \frac{x^2}{L}$$

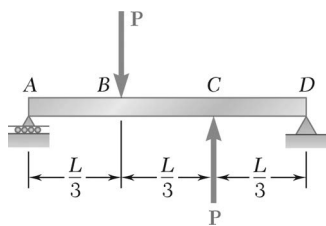
$$+\curvearrowright \Sigma F_y = 0: \quad \left(\frac{1}{2}w_0 \frac{x^2}{L}\right)\frac{x}{3} + M = 0 \quad M = -\frac{1}{6}w_0 \frac{x^3}{L}$$



$$|V|_{\max} = \frac{1}{2}w_0 L \quad \blacktriangleleft$$



$$|M|_{\max} = \frac{1}{6}w_0 L^2 \quad \blacktriangleleft$$

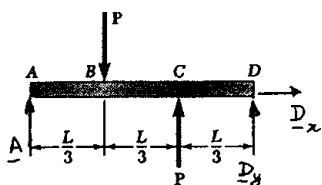


PROBLEM 7.31

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: entire beam



$$+\curvearrowright \Sigma M_D = 0: P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$$

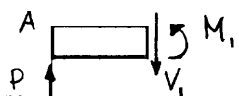
$$A = P/3 \uparrow$$

$$\Sigma F_x = 0: D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \frac{P}{3} - P + P + D_y = 0$$

$$D_y = -P/3 \quad D = P/3 \downarrow$$

- (a) Shear and bending moment. Since the loading consists of concentrated loads, the shear diagram is made of horizontal straight-line segments and the B. M. diagram is made of oblique straight-line segments.



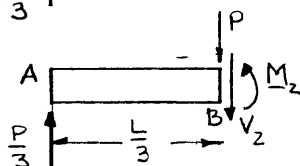
Just to the right of A:

$$+\uparrow \Sigma F_y = 0: -V_1 + \frac{P}{3} = 0$$

$$V_1 = +P/3 \triangleleft$$

$$+\curvearrowright \Sigma M_1 = 0: M_1 - \frac{P}{3}(0) = 0$$

$$M_1 = 0 \triangleleft$$



Just to the right of B:

$$+\uparrow \Sigma F_y = 0: -V_2 + \frac{P}{3} - P = 0,$$

$$V_2 = -2P/3 \triangleleft$$

$$+\curvearrowright \Sigma M_2 = 0: M_2 - \frac{P}{3}\left(\frac{L}{3}\right) + P(0) = 0$$

$$M_2 = +PL/9 \triangleleft$$

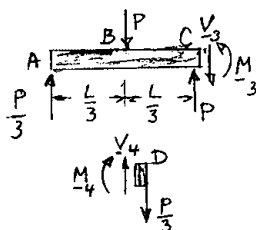
Just to the right of C:

$$+\uparrow \Sigma F_y = 0: \frac{P}{3} - P + P - V_3 = 0$$

$$V_3 = +P/3 \triangleleft$$

$$+\curvearrowright \Sigma M_3 = 0: M_3 - \frac{P}{3}\left(\frac{2L}{3}\right) + P\frac{L}{3} - P(0) = 0$$

$$M_3 = -PL/9 \triangleleft$$



PROBLEM 7.31 (Continued)

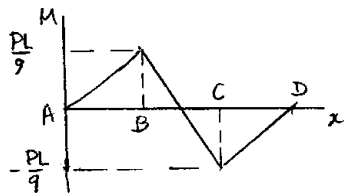
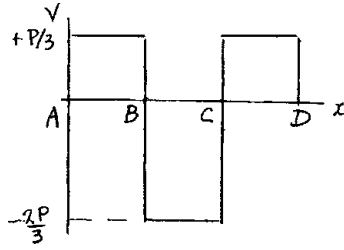
Just to the left of D :

$$+\uparrow \Sigma F_y = 0: V_4 - \frac{P}{3} = 0$$

$$V_4 = +\frac{P}{3} \triangleleft$$

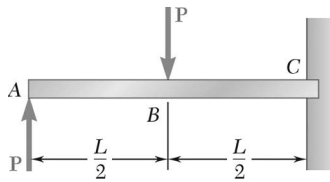
$$+\curvearrowright \Sigma M_4 = 0: -M_4 - \frac{P}{3}(0) = 0$$

$$M_4 = 0 \triangleleft$$



(b)

$$|V|_{\max} = 2P/3; |M|_{\max} = PL/9 \triangleleft$$



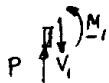
PROBLEM 7.32

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) Shear and bending moment.

Just to the right of A:



$$V_1 = +P;$$

$$M_1 = 0 \quad \triangleleft$$

Just to the right of B:



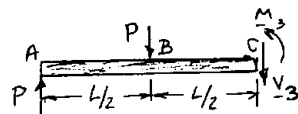
$$+\uparrow \Sigma F_y = 0: \quad P - P - V_2 = 0;$$

$$V_2 = 0 \quad \triangleleft$$

$$+\curvearrowright \Sigma M_2 = 0: \quad M_2 - P\left(\frac{L}{2}\right) = 0;$$

$$M_2 = +PL/2 \quad \triangleleft$$

Just to the left of C:

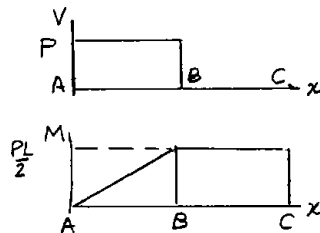


$$+\uparrow \Sigma F_y = 0: \quad P - P - V_3 = 0,$$

$$V_3 = 0 \quad \triangleleft$$

$$+\curvearrowright \Sigma M_3 = 0: \quad M_3 + P\left(\frac{L}{2}\right) - PL = 0,$$

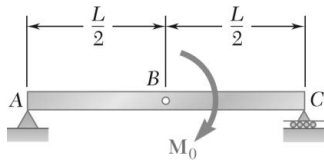
$$M_3 = +PL/2 \quad \triangleleft$$



(b)

$$|V|_{\max} = P \quad \triangleleft$$

$$|M|_{\max} = PL/2 \quad \triangleleft$$



PROBLEM 7.33

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) **FBD Beam:**

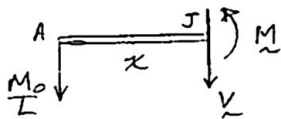
$$\left(\sum M_C = 0: LA_y - M_0 = 0 \right.$$

$$A_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \sum F_y = 0: -A_y + C = 0$$

$$C = \frac{M_0}{L} \uparrow$$

Along AB:



$$\uparrow \sum F_y = 0: -\frac{M_0}{L} - V = 0$$

$$V = -\frac{M_0}{L}$$

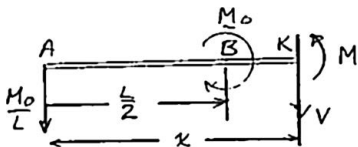
$$\left(\sum M_J = 0: x \frac{M_0}{L} + M = 0 \right.$$

$$M = -\frac{M_0}{L} x$$

Straight with

$$M = -\frac{M_0}{2} \text{ at } B$$

Along BC:



$$\uparrow \sum F_y = 0: -\frac{M_0}{L} - V = 0 \quad V = -\frac{M_0}{L}$$

$$\left(\sum M_K = 0: M + x \frac{M_0}{L} - M_0 = 0 \quad M = M_0 \left(1 - \frac{x}{L} \right) \right.$$

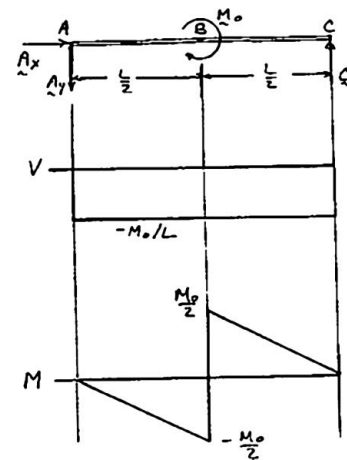
Straight with

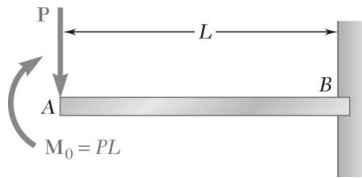
$$M = \frac{M_0}{2} \text{ at } B \quad M = 0 \text{ at } C$$

(b) From diagrams:

$$|V|_{\max} = M_0/L \quad \blacktriangleleft$$

$$|M|_{\max} = \frac{M_0}{2} \text{ at } B \quad \blacktriangleleft$$



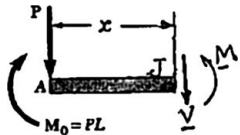


PROBLEM 7.34

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Portion AJ



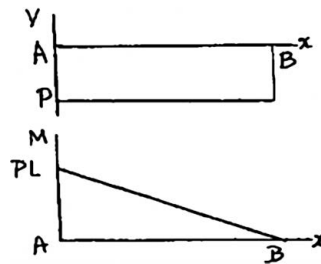
$$+\uparrow \Sigma F_y = 0: -P - V = 0$$

$$V = -P \quad \triangleleft$$

$$+\curvearrowright \Sigma M_J = 0: M + P_x - PL = 0$$

$$M = P(L - x) \quad \triangleleft$$

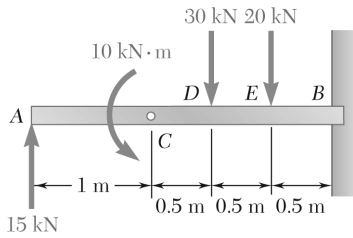
(a) The V and M diagrams are obtained by plotting the functions V and M .



(b)

$$|V|_{\max} = P \quad \triangleleft$$

$$|M|_{\max} = PL \quad \triangleleft$$



PROBLEM 7.35

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) Just to the right of A:

$$+\uparrow \Sigma F_y = 0 \quad V_1 = +15 \text{ kN} \quad M_1 = 0$$

Just to the left of C:

$$V_2 = +15 \text{ kN} \quad M_2 = +15 \text{ kN} \cdot \text{m}$$

Just to the right of C:

$$V_3 = +15 \text{ kN} \quad M_3 = +5 \text{ kN} \cdot \text{m}$$

Just to the right of D:

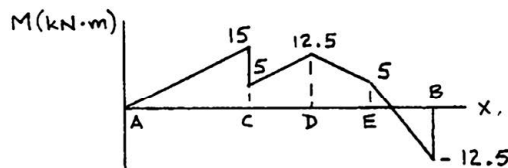
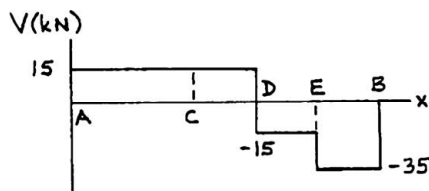
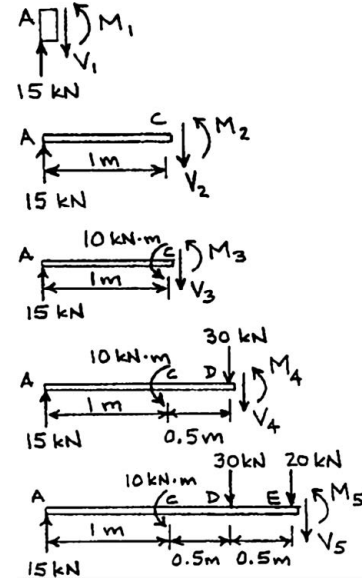
$$V_4 = -15 \text{ kN} \quad M_4 = +12.5 \text{ kN} \cdot \text{m}$$

Just to the right of E:

$$V_5 = -35 \text{ kN} \quad M_5 = +5 \text{ kN} \cdot \text{m}$$

At B:

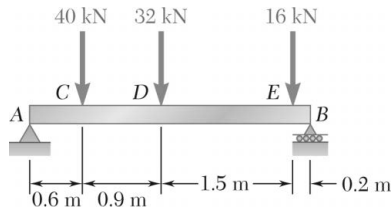
$$M_B = -12.5 \text{ kN} \cdot \text{m}$$



(b)

$$|V|_{\max} = 35.0 \text{ kN}$$

$$|M|_{\max} = 12.50 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

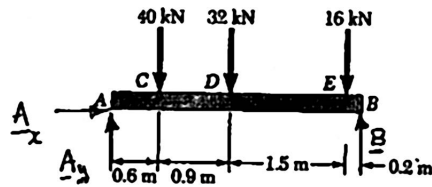


PROBLEM 7.36

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: B(3.2 \text{ m}) - (40 \text{ kN})(0.6 \text{ m}) - (32 \text{ kN})(1.5 \text{ m}) - (16 \text{ kN})(3 \text{ m}) = 0$$

$$B = +37.5 \text{ kN}$$

$$B = 37.5 \text{ kN} \uparrow \triangleleft$$

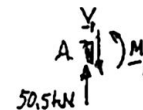
$$\Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y + 37.5 \text{ kN} - 40 \text{ kN} - 32 \text{ kN} - 16 \text{ kN} = 0$$

$$A_y = +50.5 \text{ kN}$$

$$A = 50.5 \text{ kN} \uparrow \triangleleft$$

(a) Shear and bending moment

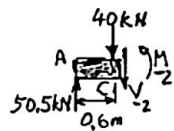


Just to the right of A:

$$V_1 = 50.5 \text{ kN}$$

$$M_1 = 0 \triangleleft$$

Just to the right of C:



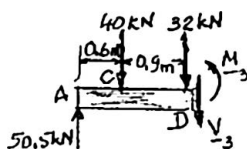
$$+\uparrow \Sigma F_y = 0: 50.5 \text{ kN} - 40 \text{ kN} - V_2 = 0$$

$$V_2 = +10.5 \text{ kN} \triangleleft$$

$$+\circlearrowleft \Sigma M_2 = 0: M_2 - (50.5 \text{ kN})(0.6 \text{ m}) = 0$$

$$M_2 = +30.3 \text{ kN} \cdot \text{m} \triangleleft$$

Just to the right of D:



$$+\uparrow \Sigma F_y = 0: 50.5 - 40 - 32 - V_3 = 0$$

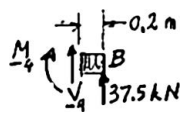
$$V_3 = -21.5 \text{ kN} \triangleleft$$

$$+\circlearrowleft \Sigma M_3 = 0: M_3 - (50.5)(1.5) + (40)(0.9) = 0$$

$$M_3 = +39.8 \text{ kN} \cdot \text{m} \triangleleft$$

PROBLEM 7.36 (Continued)

Just to the right of E:



$$+\uparrow \Sigma F_y = 0: V_4 + 37.5 = 0$$

$$V_4 = -37.5 \text{ kN} \triangleleft$$

$$+\curvearrowright \Sigma M_4 = 0: -M_4 + (37.5)(0.2) = 0$$

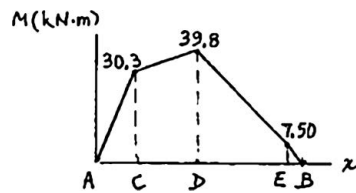
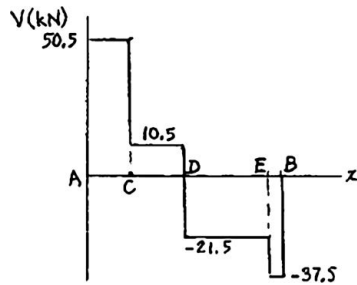
$$M_4 = +7.50 \text{ kN} \cdot \text{m} \triangleleft$$

At B:

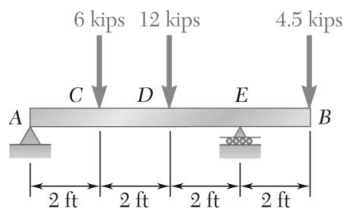
$$V_B = M_B = 0 \triangleleft$$

(b)

$$|V|_{\max} = 50.5 \text{ kN} \blacktriangleleft$$



$$|M|_{\max} = 39.8 \text{ kN} \cdot \text{m} \blacktriangleleft$$

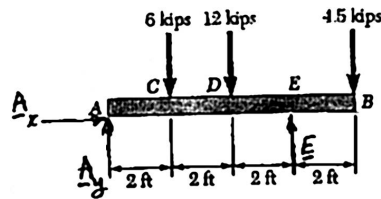


PROBLEM 7.37

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\curvearrowright \Sigma M_A = 0: E(6 \text{ ft}) - (6 \text{ kips})(2 \text{ ft}) - (12 \text{ kips})(4 \text{ ft}) - (4.5 \text{ kips})(8 \text{ ft}) = 0$$

$$E = +16 \text{ kips}$$

$$E = 16 \text{ kips} \uparrow \triangleleft$$

$$+\rightarrow \Sigma F_x = 0: A_x = 0$$

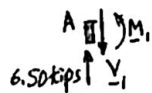
$$+\uparrow \Sigma F_y = 0: A_y + 16 \text{ kips} - 6 \text{ kips} - 12 \text{ kips} - 4.5 \text{ kips} = 0$$

$$A_y = +6.50 \text{ kips}$$

$$A = 6.50 \text{ kips} \uparrow \triangleleft$$

(a) Shear and bending moment

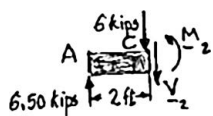
Just to the right of A:



$$V_1 = +6.50 \text{ kips} \quad M_1 = 0$$

\triangleleft

Just to the right of C:



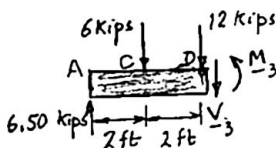
$$+\uparrow \Sigma F_y = 0: 6.50 \text{ kips} - 6 \text{ kips} - V_2 = 0$$

$$V_2 = +0.50 \text{ kips} \triangleleft$$

$$+\curvearrowright \Sigma M_2 = 0: M_2 - (6.50 \text{ kips})(2 \text{ ft}) = 0$$

$$M_2 = +13 \text{ kip} \cdot \text{ft} \triangleleft$$

Just to the right of D:



$$+\uparrow \Sigma F_y = 0: 6.50 - 6 - 12 - V_3 = 0$$

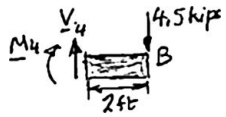
$$V_3 = +11.5 \text{ kips} \triangleleft$$

$$+\curvearrowright \Sigma M_3 = 0: M_3 - (6.50)(4) - (6)(2) = 0$$

$$M_3 = +14 \text{ kip} \cdot \text{ft} \triangleleft$$

PROBLEM 7.37 (Continued)

Just to the right of E :



$$+\uparrow \Sigma F_y = 0: V_4 - 4.5 = 0$$

$$V_4 = +4.5 \text{ kips} \triangleleft$$

$$+\curvearrowright \Sigma M_4 = 0: -M_4 - (4.5)(2) = 0$$

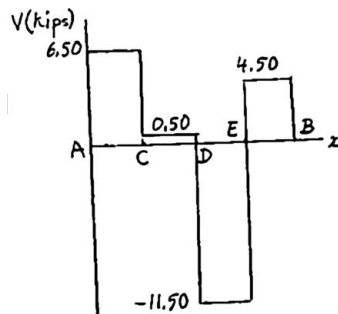
$$M_4 = -9 \text{ kip} \cdot \text{ft} \triangleleft$$

At B :

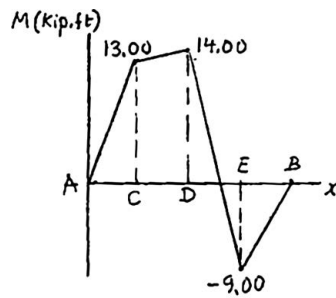
$$V_B = M_B = 0$$

\triangleleft

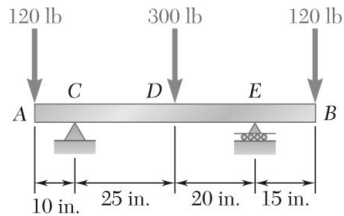
(b)



$$|V|_{\max} = 11.50 \text{ kips} \blacktriangleleft$$



$$|M|_{\max} = 14.00 \text{ kip} \cdot \text{ft} \blacktriangleleft$$

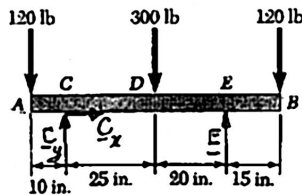


PROBLEM 7.38

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_C = 0: (120 \text{ lb})(10 \text{ in.}) - (300 \text{ lb})(25 \text{ in.}) + E(45 \text{ in.}) - (120 \text{ lb})(60 \text{ in.}) = 0$$

$$E = +300 \text{ lb}$$

$$E = 300 \text{ lb} \uparrow \triangleleft$$

$$\Sigma F_x = 0: C_x = 0$$

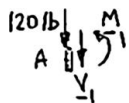
$$+\uparrow \Sigma F_y = 0: C_y + 300 \text{ lb} - 120 \text{ lb} - 300 \text{ lb} - 120 \text{ lb} = 0$$

$$C_y = +240 \text{ lb}$$

$$C = 240 \text{ lb} \uparrow \triangleleft$$

(a) Shear and bending moment

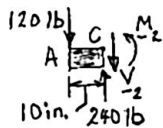
Just to the right of A:



$$+\uparrow \Sigma F_y = 0: -120 \text{ lb} - V_1 = 0$$

$$V_1 = -120 \text{ lb}, \quad M_1 = 0 \triangleleft$$

Just to the right of C:



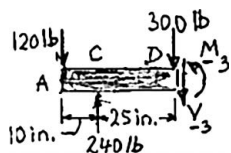
$$+\uparrow \Sigma F_y = 0: 240 \text{ lb} - 120 \text{ lb} - V_2 = 0$$

$$V_2 = +120 \text{ lb} \triangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: M_2 + (120 \text{ lb})(10 \text{ in.}) = 0$$

$$M_2 = -1200 \text{ lb} \cdot \text{in.} \triangleleft$$

Just to the right of D:



$$+\uparrow \Sigma F_y = 0: 240 - 120 - 300 - V_3 = 0$$

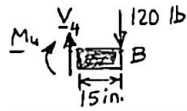
$$V_3 = -180 \text{ lb} \triangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: M_3 + (120)(35) - (240)(25) = 0,$$

$$M_3 = +1800 \text{ lb} \cdot \text{in.} \triangleleft$$

PROBLEM 7.38 (Continued)

Just to the right of *E*:



$$+\Sigma F_y = 0: V_4 - 120 \text{ lb} = 0$$

$$V_4 = +120 \text{ lb} \triangleleft$$

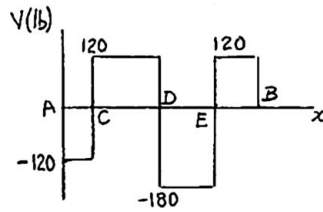
$$+\Sigma M_4 = 0: -M_4 - (120 \text{ lb})(15 \text{ in.}) = 0$$

$$M_4 = -1800 \text{ lb} \cdot \text{in.} \triangleleft$$

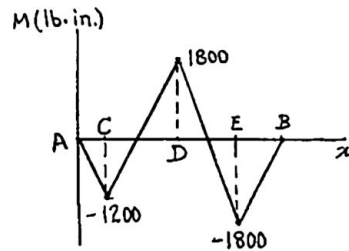
At *B*:

$$V_B = M_B = 0 \triangleleft$$

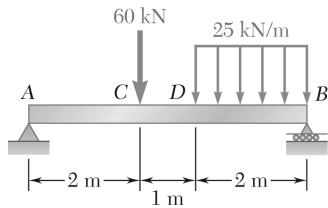
(b)



$$|V|_{\max} = 180.0 \text{ lb} \blacktriangleleft$$



$$|M|_{\max} = 1800 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

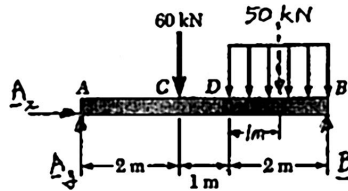


PROBLEM 7.39

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\curvearrowright \Sigma M_A = 0: B(5 \text{ m}) - (60 \text{ kN})(2 \text{ m}) - (50 \text{ kN})(4 \text{ m}) = 0$$

$$B = +64.0 \text{ kN}$$

$$B = 64.0 \text{ kN} \uparrow \triangleleft$$

$$\Sigma F_x = 0: A_x = 0$$

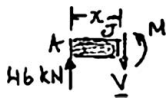
$$+\uparrow \Sigma F_y = 0: A_y + 64.0 \text{ kN} - 6.0 \text{ kN} - 50 \text{ kN} = 0$$

$$A_y = +46.0 \text{ kN}$$

$$A = 46.0 \text{ kN} \uparrow \triangleleft$$

(a) Shear and bending-moment diagrams.

From A to C:



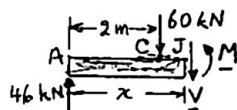
$$+\uparrow \Sigma F_y = 0: 46 - V = 0$$

$$V = +46 \text{ kN} \triangleleft$$

$$+\curvearrowright \Sigma M_y = 0: M - 46x = 0$$

$$M = (46x) \text{ kN} \cdot \text{m} \triangleleft$$

From C to D:



$$+\uparrow \Sigma F_y = 0: 46 - 60 - V = 0$$

$$V = -14 \text{ kN} \triangleleft$$

$$+\curvearrowright \Sigma M_j = 0: M - 46x + 60(x - 2) = 0$$

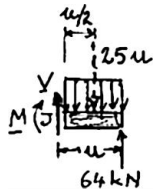
$$M = (120 - 14x) \text{ kN} \cdot \text{m}$$

For $x = 2 \text{ m}: M_C = +92.0 \text{ kN} \cdot \text{m} \triangleleft$

For $x = 3 \text{ m}: M_D = +78.0 \text{ kN} \cdot \text{m} \triangleleft$

PROBLEM 7.39 (Continued)

From *D* to *B*:



$$+\uparrow \Sigma F_y = 0: V + 64 - 25\mu = 0$$

$$V = (25\mu - 64)\text{kN}$$

$$+\curvearrowright \Sigma M_j = 0: 64\mu - (25\mu)\left(\frac{\mu}{2}\right) - M = 0$$

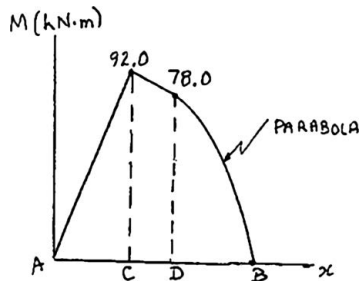
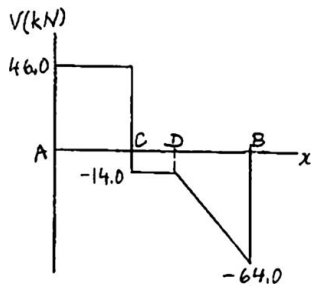
$$M = (64\mu - 12.5\mu^2)\text{kN} \cdot \text{m}$$

For $\mu = 0: V_B = -64 \text{ kN}$

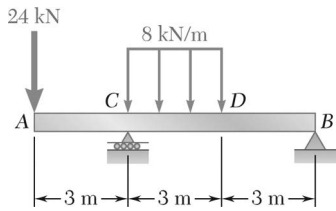
$M_B = 0 \triangleleft$

(b)

$|V|_{\max} = 64.0 \text{ kN} \triangleleft$



$|M|_{\max} = 92.0 \text{ kN} \cdot \text{m} \triangleleft$

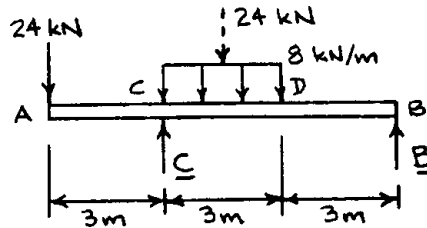


PROBLEM 7.40

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



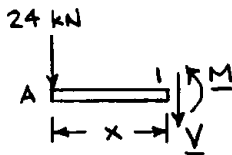
$$+\curvearrowright \Sigma M_B = 0: (24 \text{ kN})(9 \text{ m}) - C(6 \text{ m}) + (24 \text{ kN})(4.5 \text{ m}) = 0 \quad C = 54 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: 54 - 24 - 24 + B = 0$$

$$B = -6 \text{ kN}$$

$$B = 6 \text{ kN} \downarrow$$

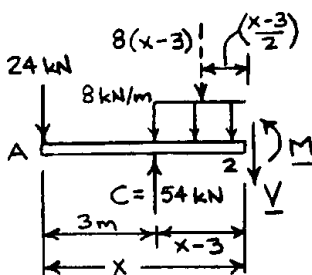
From A to C:



$$+\uparrow \Sigma F_y = 0: -24 - V = 0 \quad V = -24 \text{ kN}$$

$$+\curvearrowright \Sigma M_1 = 0: (24)(x) + M = 0 \quad M = (-24x) \text{ kN} \cdot \text{m}$$

From C to D:



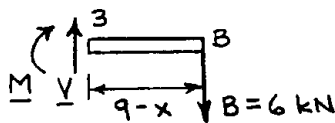
$$+\uparrow \Sigma F_y = 0: -24 - 8(x-3) - V + 54 = 0$$

$$V = (-8x + 54) \text{ kN}$$

$$+\curvearrowright \Sigma M_2 = 0: (24)(x) + 8(x-3)\left(\frac{x-3}{2}\right) - (54)(x-3) + M = 0$$

$$M = (-4x^2 + 54x - 198) \text{ kN} \cdot \text{m}$$

From D to B:



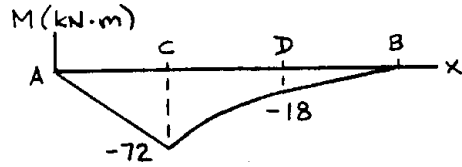
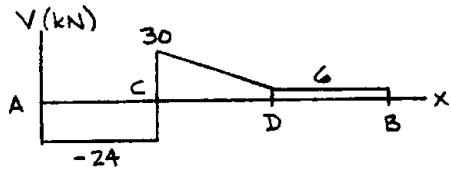
$$+\uparrow \Sigma F_y = 0: V - 6 = 0 \quad V = +6 \text{ kN}$$

$$+\curvearrowright \Sigma M_3 = 0: -M - (6)(9-x) = 0 \quad M = (6x - 54) \text{ kN} \cdot \text{m}$$

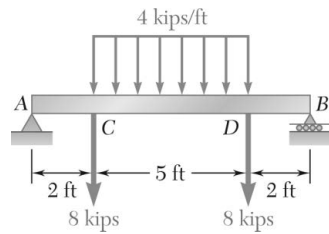
PROBLEM 7.40 (Continued)

(b)

$$|V|_{\max} = 30.0 \text{ kN} \blacktriangleleft$$



$$|M|_{\max} = 72.0 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 7.41

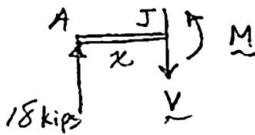
For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) By symmetry:

$$A_y = B = 8 \text{ kips} + \frac{1}{2}(4 \text{ kips})(5 \text{ ft}) \quad A_y = B = 18 \text{ kips} \uparrow$$

Along AC:

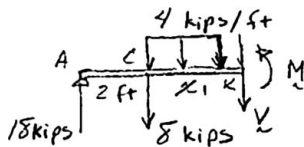


$$\uparrow \Sigma F_y = 0: 18 \text{ kips} - V = 0 \quad V = 18 \text{ kips}$$

$$\curvearrowleft \Sigma M_J = 0: M - x(18 \text{ kips}) \quad M = (18 \text{ kips})x$$

$$M = 36 \text{ kip} \cdot \text{ft} \text{ at } C(x = 2 \text{ ft})$$

Along CD:



$$\uparrow \Sigma F_y = 0: 18 \text{ kips} - 8 \text{ kips} - (4 \text{ kips/ft})x_1 - V = 0$$

$$V = 10 \text{ kips} - (4 \text{ kips/ft})x_1$$

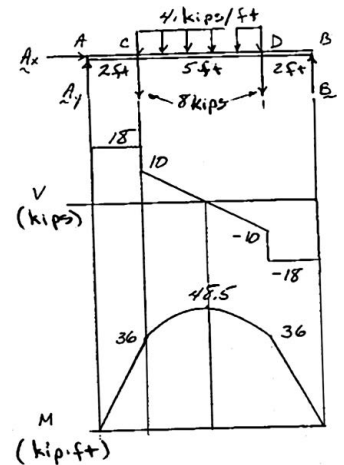
$$V = 0 \text{ at } x_1 = 2.5 \text{ ft (at center)}$$

$$\curvearrowleft \Sigma M_K = 0: M + \frac{x_1}{2}(4 \text{ kips/ft})x_1 + (8 \text{ kips})x_1 - (2 \text{ ft} + x_1)(18 \text{ kips}) = 0$$

$$M = 36 \text{ kip} \cdot \text{ft} + (10 \text{ kips/ft})x_1 - (2 \text{ kips/ft})x_1^2$$

$$M = 48.5 \text{ kip} \cdot \text{ft} \text{ at } x_1 = 2.5 \text{ ft}$$

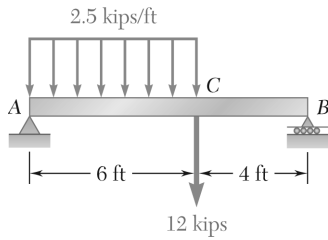
Complete diagram by symmetry



(b) From diagrams:

$$|V|_{\max} = 18.00 \text{ kips} \quad \blacktriangleleft$$

$$|M|_{\max} = 48.5 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

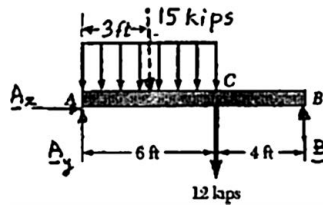


PROBLEM 7.42

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(10 \text{ ft}) - (15 \text{ kips})(3 \text{ ft}) - (12 \text{ kips})(6 \text{ ft}) = 0$$

$$B = +11.70 \text{ kips}$$

$$\mathbf{B} = 11.70 \text{ kips} \uparrow \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

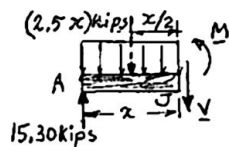
$$+\uparrow \Sigma F_y = 0: \quad A_y - 15 - 12 + 11.70 = 0$$

$$A_y = +15.30 \text{ kips}$$

$$\mathbf{A} = 15.30 \text{ kips} \uparrow \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad 15.30 - 2.5x - V = 0$$

$$V = (15.30 - 2.5x) \text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (2.5x)\left(\frac{x}{2}\right) - 15.30x = 0$$

$$M = 15.30x - 1.25x^2$$

For $x = 0$:

$$V_A = +15.30 \text{ kips}$$

$$M_A = 0 \triangleleft$$

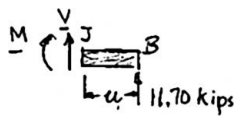
For $x = 6 \text{ ft}$:

$$V_C = +0.300 \text{ kip}$$

$$M_C = +46.8 \text{ kip} \cdot \text{ft} \triangleleft$$

PROBLEM 7.42 (Continued)

From C to B:



$$+\uparrow \Sigma F_y = 0: V + 11.70 = 0$$

$$V = -11.70 \text{ kips} \triangleleft$$

$$+\curvearrowright \Sigma M_J = 0: 11.70\mu - M = 0$$

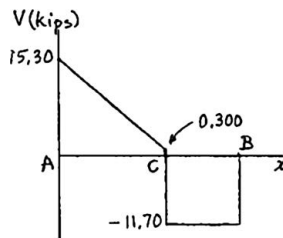
$$M = (11.70\mu) \text{ kip} \cdot \text{ft}$$

For $\mu = 4 \text{ ft}$:

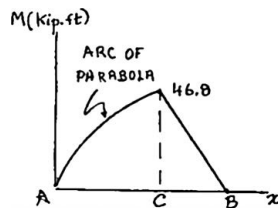
$$M_C = +46.8 \text{ kip} \cdot \text{ft} \triangleleft$$

For $\mu = 0$:

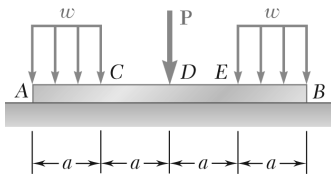
$$M_B = 0 \triangleleft$$



(b) $|V|_{\max} = 15.30 \text{ kips} \blacktriangleleft$



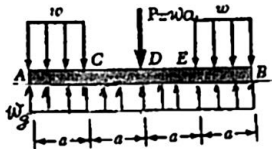
$$|M|_{\max} = 46.8 \text{ kip} \cdot \text{ft} \blacktriangleleft$$



PROBLEM 7.43

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - wa = 0$$

$$w_g = \frac{3}{4}w \quad \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wx - V = 0$$

$$V = -\frac{1}{4}wx$$

$$+\curvearrowright \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{3}{4}wx\right)\frac{x}{2} = 0$$

$$M = -\frac{1}{8}wx^2$$

For $x = 0$:

$$V_A = M_A = 0 \quad \triangleleft$$

For $x = a$: $V_C = -\frac{1}{4}wa$

$$M_C = -\frac{1}{8}wa^2 \quad \triangleleft$$

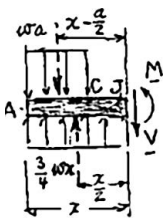
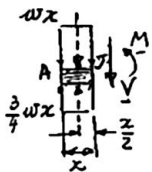
From C to D:

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wa - V = 0$$

$$V = \left(\frac{3}{4}x - a\right)w$$

$$+\curvearrowright \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{3}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{3}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \quad (1)$$

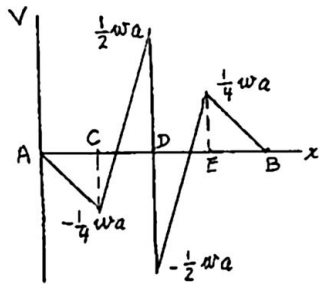


PROBLEM 7.43 (Continued)

For $x = a$: $V_C = -\frac{1}{4}wa$ $M_C = -\frac{1}{8}wa^2 \triangleleft$

For $x = 2a$: $V_D = +\frac{1}{2}wa$ $M_D = 0 \triangleleft$

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.



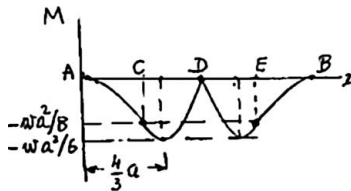
(b)

$|V|_{\max} = \frac{1}{2}wa \triangleleft$

To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a$$

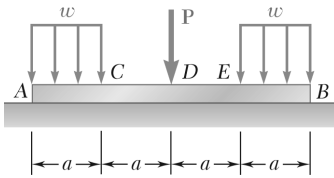
$$M = \frac{3}{8}w\left(\frac{4}{3}a\right)^2 - wa^2\left(\frac{4}{3} - \frac{1}{2}\right) = -\frac{wa^2}{6}$$



$|M|_{\max} = \frac{1}{6}wa^2 \triangleleft$

Bending-moment diagram consists of four distinct arcs of parabola.

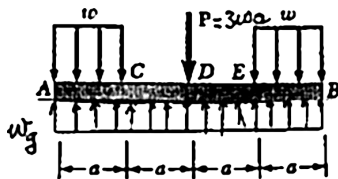
PROBLEM 7.44



Solve Problem 7.43 knowing that $P = 3wa$.

PROBLEM 7.43 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: w_g(4a) - 2wa - 3wa = 0$$

$$w_g = \frac{5}{4}w \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:

$$+\uparrow \Sigma F_y = 0: \frac{5}{4}wx - wx - V = 0$$

$$V = +\frac{1}{4}wx$$

$$+\curvearrowright \Sigma M_J = 0: M + (wx)\frac{x}{2} - \left(\frac{5}{4}wx\right)\frac{x}{2} = 0$$

$$M = +\frac{1}{8}wx^2$$

For $x = 0$:

$$V_A = M_A = 0 \triangleleft$$

For $x = a$: $V_C = +\frac{1}{4}wa$

$$M_C = +\frac{1}{8}wa^2 \triangleleft$$

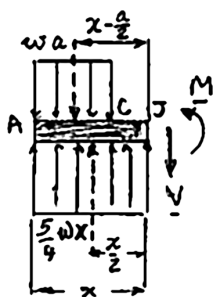
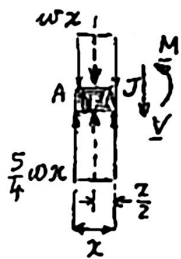
From C to D:

$$+\uparrow \Sigma F_y = 0: \frac{5}{4}wx - wa - V = 0$$

$$V = \left(\frac{5}{4}x - a\right)w$$

$$+\curvearrowright \Sigma M_J = 0: M + wa\left(x - \frac{a}{2}\right) - \frac{5}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{5}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \quad (1)$$

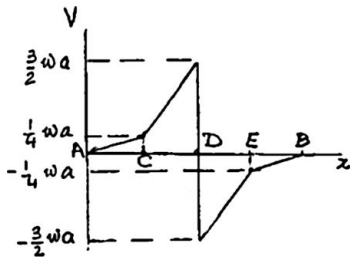


PROBLEM 7.44 (Continued)

For $x = a$: $V_C = +\frac{1}{4}wa$, $M_C = +\frac{1}{8}wa^2 \triangleleft$

For $x = 2a$: $V_D = +\frac{3}{2}wa$, $M_D = +wa^2 \triangleleft$

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.



(b)

$|V|_{\max} = \frac{3}{2}wa \triangleleft$

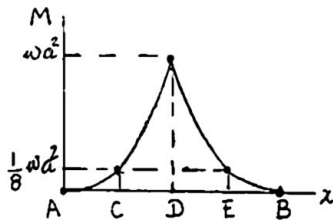
To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0$$

$$x = \frac{4}{5}a < a \quad (\text{outside portion } CD)$$

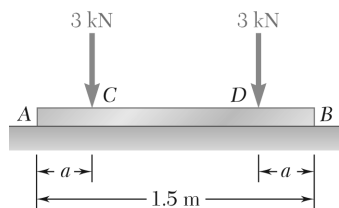
The maximum value of $|M|$ occurs at D :

$|M|_{\max} = wa^2 \triangleleft$



Bending-moment diagram consists of four distinct arcs of parabola.

PROBLEM 7.45



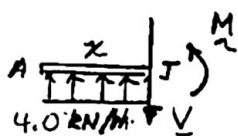
Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) **FBD Beam:** $\uparrow \Sigma F_y = 0: w(1.5 \text{ m}) - 2(3.0 \text{ kN}) = 0$

$$w = 4.0 \text{ kN/m}$$

Along AC:



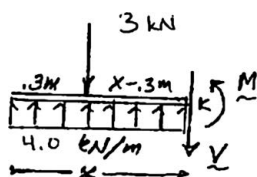
$$\uparrow \Sigma F_y = 0: (4.0 \text{ kN/m})x - V = 0$$

$$V = (4.0 \text{ kN/m})x$$

$$\left(\Sigma M_J = 0: M - \frac{x}{2}(4.0 \text{ kN/m})x = 0 \right.$$

$$M = (2.0 \text{ kN/m})x^2$$

Along CD:



$$\uparrow \Sigma F_y = 0: (4.0 \text{ kN/m})x - 3.0 \text{ kN} - V = 0$$

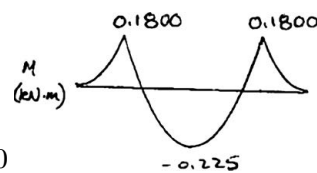
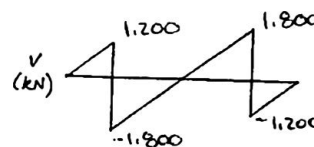
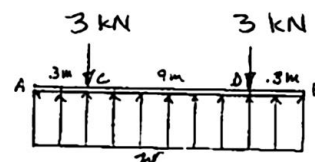
$$V = (4.0 \text{ kN/m})x - 3.0 \text{ kN}$$

$$\left(\Sigma M_K = 0: M + (x - 0.3 \text{ m})(3.0 \text{ kN}) - \frac{x}{2}(4.0 \text{ kN/m})x = 0 \right.$$

$$M = 0.9 \text{ kN} \cdot \text{m} - (3.0 \text{ kN})x + (2.0 \text{ kN/m})x^2$$

Note: $V = 0$ at $x = 0.75$ m, where $M = -0.225 \text{ kN} \cdot \text{m}$

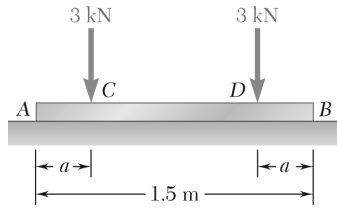
Complete diagrams using symmetry.



(b)

$$|V|_{\max} = 1.800 \text{ kN} \quad \blacktriangleleft$$

$$|M|_{\max} = 0.225 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$



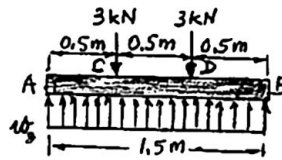
PROBLEM 7.46

Solve Problem 7.45 knowing that $a = 0.5$ m.

PROBLEM 7.45 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



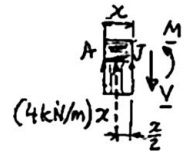
$$+\uparrow \Sigma F_y = 0: w_g (1.5 \text{ m}) - 3 \text{ kN} - 3 \text{ kN} = 0 \quad w_g = 4 \text{ kN/m} \quad \triangleleft$$

(a) Shear and bending moment

From A to C:

$$+\uparrow \Sigma F_y = 0: 4x - V = 0 \quad V = (4x) \text{ kN}$$

$$+\curvearrowright \Sigma M_J = 0: M - (4x) \frac{x}{2} = 0, \quad M = (2x^2) \text{ kN} \cdot \text{m}$$



For $x = 0$:

$$V_A = M_A = 0 \quad \triangleleft$$

For $x = 0.5$ m:

$$V_C = 2 \text{ kN},$$

$$M_C = 0.500 \text{ kN} \cdot \text{m} \quad \triangleleft$$

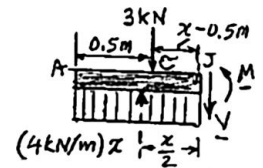
From C to D:

$$+\uparrow \Sigma F_y = 0: 4x - 3 \text{ kN} - V = 0$$

$$V = (4x - 3) \text{ kN}$$

$$+\curvearrowright \Sigma M_J = 0: M + (3 \text{ kN})(x - 0.5) - (4x) \frac{x}{2} = 0$$

$$M = (2x^2 - 3x + 1.5) \text{ kN} \cdot \text{m}$$



For $x = 0.5$ m:

$$V_C = -1.00 \text{ kN}, \quad M_C = 0.500 \text{ kN} \cdot \text{m} \quad \triangleleft$$

For $x = 0.75$ m:

$$V_C = 0, \quad M_C = 0.375 \text{ kN} \cdot \text{m} \quad \triangleleft$$

For $x = 1.0$ m:

$$V_C = 1.00 \text{ kN}, \quad M_C = 0.500 \text{ kN} \cdot \text{m} \quad \triangleleft$$

PROBLEM 7.46 (Continued)

From D to B :



$$+\uparrow \Sigma F_y = 0: V + 4\mu = 0 \quad V = -(4\mu) \text{ kN}$$

$$+\curvearrowright \Sigma M_J = 0: (4\mu) \frac{\mu}{2} - M = 0, \quad M = 2\mu^2$$

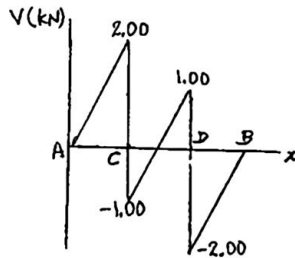
For $\mu = 0$:

$$V_B = M_B = 0 \quad \triangleleft$$

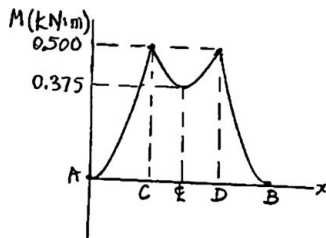
For $\mu = 0.5 \text{ m}$:

$$V_D = -2 \text{ kN},$$

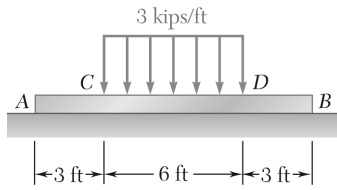
$$M_D = 0.500 \text{ kN} \cdot \text{m} \quad \triangleleft$$



$$(b) \quad |V|_{\max} = 2.00 \text{ kN} \quad \blacktriangleleft$$



$$|M|_{\max} = 0.500 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 7.47

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(12 \text{ ft}) - (3 \text{ kips/ft})(6 \text{ ft}) = 0$$

$$w_g = 1.5 \text{ kips/ft} \quad \triangleleft$$

(a) Shear and bending-moment diagrams from A to C:

$$+\uparrow \Sigma F_y = 0: \quad 1.5x - V = 0 \quad V = (1.5x) \text{ kips}$$

$$+\curvearrowright \Sigma M_J = 0: \quad M - (1.5x) \frac{x}{2} \quad M = (0.75x^2) \text{ kip} \cdot \text{ft}$$

For $x = 0$:

$$V_A = M_A = 0 \quad \triangleleft$$

For $x = 3 \text{ ft}$: $V_C = 4.5 \text{ kips}$,

$$M_C = 6.75 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

From C to D:

$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3(x-3) - V = 0$$

$$V = (9 - 1.5x) \text{ kips}$$

$$+\curvearrowright \Sigma M_J = 0: \quad M + 3(x-3) \frac{x-3}{2} - (1.5x) \frac{x}{2} = 0$$

$$M = [0.75x^2 - 1.5(x-3)^2] \text{ kip} \cdot \text{ft}$$

For $x = 3 \text{ ft}$: $V_C = 4.5 \text{ kips}$,

$$M_C = 6.75 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

For $x = 6 \text{ ft}$: $V_D = 0$,

$$M_D = 13.50 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

For $x = 9 \text{ ft}$: $V_D = -4.5 \text{ kips}$,

$$M_D = 6.75 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

At B:

$$V_B = M_B = 0 \quad \triangleleft$$

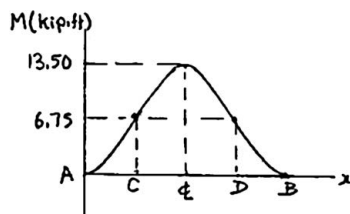
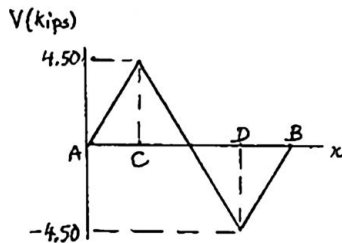
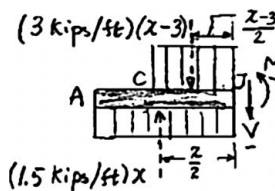
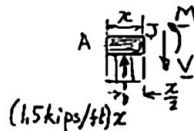
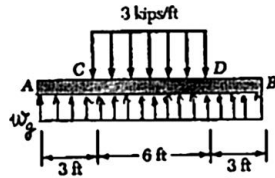
(b)

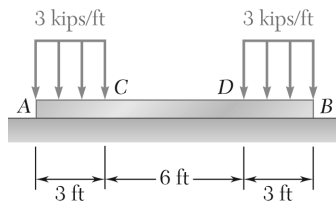
$$|V|_{\max} = 4.50 \text{ kips} \quad \blacktriangleleft$$

$$|M|_{\max} = 13.50 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

Bending-moment diagram consists of three distinct arcs of parabola, all located above the x axis.

Thus: $M \geq 0$ everywhere \blacktriangleleft



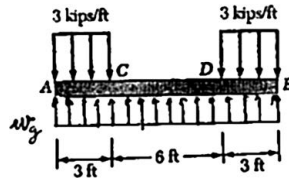


PROBLEM 7.48

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

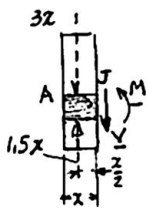
Free body: Entire beam



$$+\uparrow \Sigma F_y = 0: \quad w_g(12 \text{ ft}) - (3 \text{ kips/ft})(6 \text{ ft}) = 0 \quad w_g = 1.5 \text{ kips/ft} \quad \triangleleft$$

(a) Shear and bending-moment diagrams.

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3x - V = 0$$

$$V = (-1.5x) \text{ kips}$$

$$+\curvearrowright \Sigma M_J = 0: \quad M + (3x)\frac{x}{2} - (1.5x)\frac{x}{2} = 0$$

$$M = (-0.75x^2) \text{ kip} \cdot \text{ft}$$

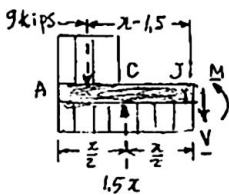
For $x = 0$:

$$V_A = M_A = 0 \quad \triangleleft$$

For $x = 3 \text{ ft}$:

$$V_C = -4.5 \text{ kips} \quad M_C = -6.75 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

From C to D:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x = 9 - V = 0, \quad V = (1.5x - 9) \text{ kips}$$

$$+\curvearrowright \Sigma M_J = 0: \quad M + 9(x - 1.5) - (1.5x)\frac{x}{2} = 0$$

$$M = 0.75x^2 - 9x + 13.5$$

For $x = 3 \text{ ft}$:

$$V_C = -4.5 \text{ kips}, \quad M_C = -6.75 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

For $x = 6 \text{ ft}$:

$$V_C = 0, \quad M_C = -13.50 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

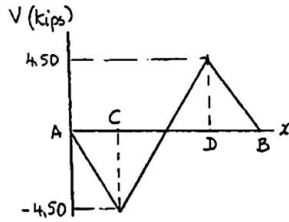
PROBLEM 7.48 (Continued)

For $x = 9$ ft:

$$V_D = 4.5 \text{ kips}, \quad M_D = -6.75 \text{ kip} \cdot \text{ft} \quad \triangleleft$$

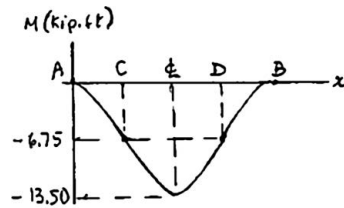
At B:

$$V_B = M_B = 0 \quad \triangleleft$$



(b) $|V|_{\max} = 4.50 \text{ kips} \quad \blacktriangleleft$

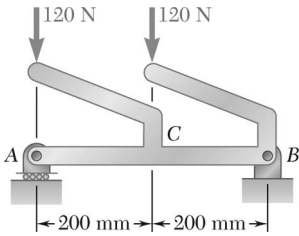
Bending-moment diagram consists of three distinct arcs of parabola.



$$|M|_{\max} = 13.50 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

Since entire diagram is below the x axis:

$$M \leq 0 \text{ everywhere} \quad \blacktriangleleft$$

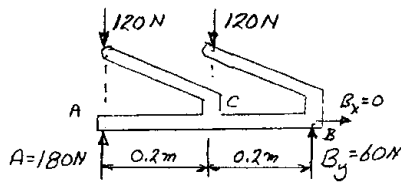


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam AB , and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Reactions:



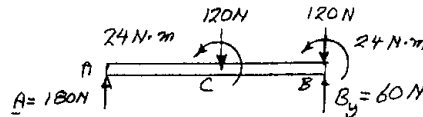
$$+\circlearrowleft \Sigma M_A = 0: B_y(0.4) - (120)(0.2) = 0$$

$$B_y = 60 \text{ N} \uparrow$$

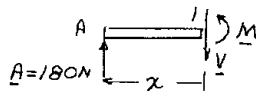
$$\Sigma F_x = 0: B_x = 0$$

$$\Sigma F_y = 0: A = 180 \text{ N} \uparrow$$

Equivalent loading on straight part of beam AB .



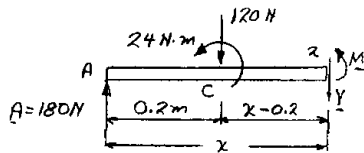
From A to C :



$$+\uparrow \Sigma F_y = 0: V = +180 \text{ N}$$

$$+\circlearrowleft \Sigma M_1 = 0: M = +180x$$

From C to B :

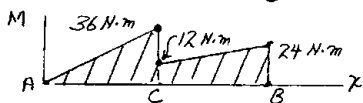
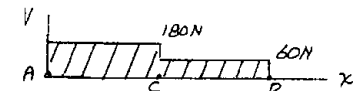


$$+\uparrow \Sigma F_y = 0: 180 - 120 - V = 0$$

$$V = 60 \text{ N}$$

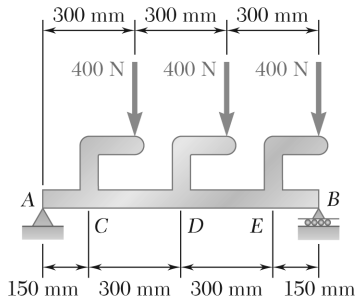
$$+\circlearrowleft \Sigma M_x = 0: -(180 \text{ N})(x) + 24 \text{ N} \cdot \text{m} + (120 \text{ N})(x - 0.2) + M = 0$$

$$M = +60x$$



$$|V|_{\max} = 180.0 \text{ N} \blacktriangleleft$$

$$|M|_{\max} = 36.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

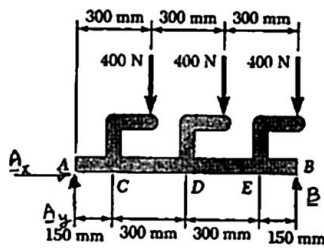


PROBLEM 7.50

Draw the shear and bending-moment diagrams for the beam AB , and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.9 \text{ m}) - (400 \text{ N})(0.3 \text{ m}) - (400 \text{ N})(0.6 \text{ m}) - (400 \text{ N})(0.9 \text{ m}) = 0$$

$$B = +800 \text{ N}$$

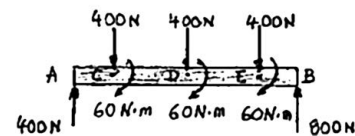
$$\mathbf{B} = 800 \text{ N} \uparrow \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800 \text{ N} - 3(400 \text{ N}) = 0$$

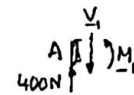
$$A_y = +400 \text{ N}$$

$$\mathbf{A} = 400 \text{ N} \uparrow \triangleleft$$



We replace the loads by equivalent force-couple systems at C , D , and E .

We consider successively the following F - B diagrams.



$$V_1 = +400 \text{ N}$$

$$V_5 = -400 \text{ N}$$

$$M_1 = 0$$

$$M_5 = +180 \text{ N} \cdot \text{m}$$

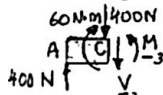


$$V_2 = +400 \text{ N}$$

$$V_6 = -400 \text{ N}$$

$$M_2 = +60 \text{ N} \cdot \text{m}$$

$$M_6 = +60 \text{ N} \cdot \text{m}$$

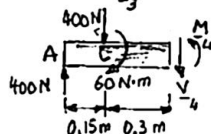


$$V_3 = 0$$

$$V_7 = -800 \text{ N}$$

$$M_3 = +120 \text{ N} \cdot \text{m}$$

$$M_7 = +120 \text{ N} \cdot \text{m}$$

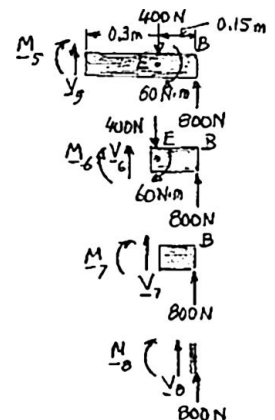


$$V_4 = 0$$

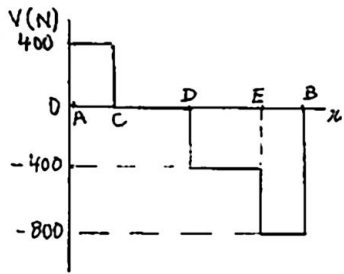
$$V_8 = -800 \text{ N}$$

$$M_4 = +120 \text{ N} \cdot \text{m}$$

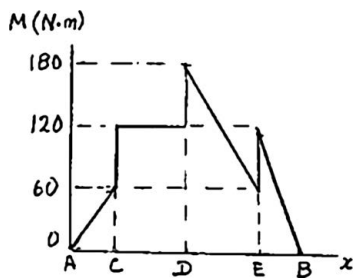
$$M_8 = 0$$



PROBLEM 7.50 (Continued)

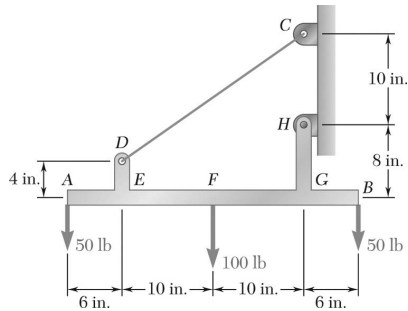


(b) $|V|_{\max} = 800 \text{ N} \blacktriangleleft$



$|M|_{\max} = 180.0 \text{ N} \cdot \text{m} \blacktriangleleft$

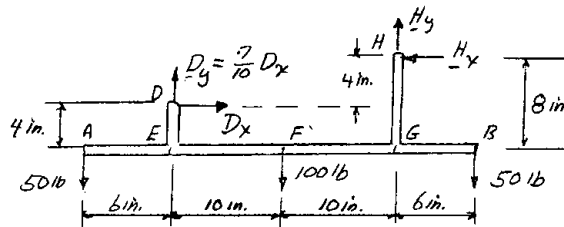
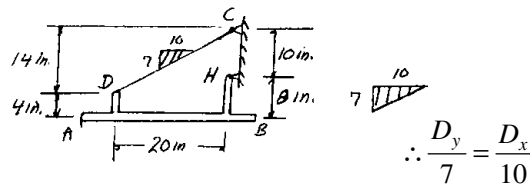
PROBLEM 7.51



Draw the shear and bending-moment diagrams for the beam AB , and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Slope of cable CD is



$$+\circlearrowleft \Sigma M_H = 0: (50 \text{ lb})(26 \text{ in.}) + D_x(4 \text{ in.}) - \left(\frac{7}{10} D_x\right)(20 \text{ in.}) + (100 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(6 \text{ in.}) = 0$$

$$2000 + (4 - 14)D_x = 0$$

$$D_x = 200 \text{ lb} \rightarrow$$

$$D_y = \frac{7}{10} D_x = \frac{7}{10} (200)$$

$$D_y = 140 \text{ lb} \uparrow$$

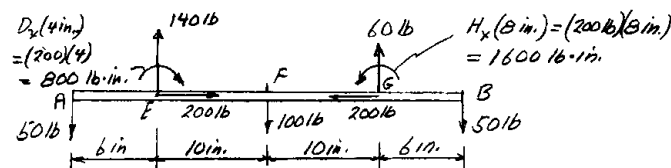
$$+\rightarrow \Sigma F_x = 0: 200 \text{ lb} - H_x = 0$$

$$H_x = 200 \text{ lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: 140 - 50 - 100 - 50 + H_y = 0$$

$$H_y = 60 \text{ lb} \uparrow$$

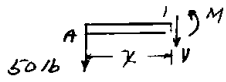
Equivalent loading on straight part of beam AB .



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PROBLEM 7.51 (Continued)

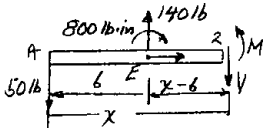
From A to E:



$$\sum F_y = 0: V = -50 \text{ lb}$$

$$\sum M_1 = 0: M = -50x$$

From E to F:



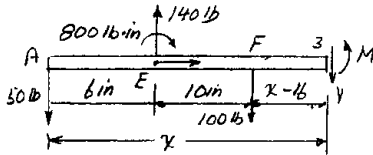
$$+\uparrow \sum F_y = 0: -50 + 140 - V = 0$$

$$V = +90 \text{ lb}$$

$$+\curvearrowright \sum M_2 = 0: (50 \text{ lb})x - (140 \text{ lb})(x - 6) - 800 \text{ lb} \cdot \text{in.} + M = 0$$

$$M = -40 + 90x$$

From F to G:



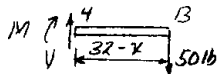
$$+\uparrow \sum F_y = 0: -50 + 140 - 100 - V = 0$$

$$V = -10 \text{ lb}$$

$$+\curvearrowright \sum M_3 = 0: 50x - 140(x - 6) + 100(x - 16) - 800 + M = 0$$

$$M = 1560 - 10x$$

From G to B:

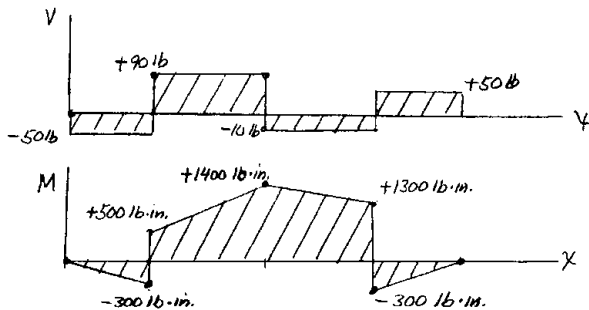


$$+\uparrow \sum F_y = 0: V - 50 = 0$$

$$V = 50 \text{ lb}$$

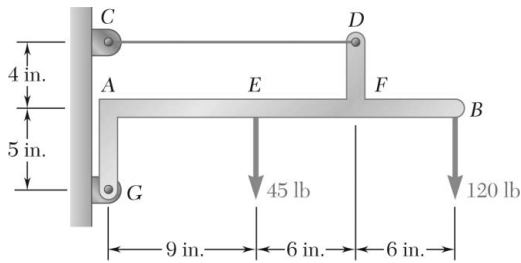
$$+\curvearrowright \sum M_4 = 0: -M - (50)(32 - x) = 0$$

$$M = -1600 + 50x$$



$$|V|_{\max} = 90.0 \text{ lb}; \quad |M|_{\max} = 1400 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

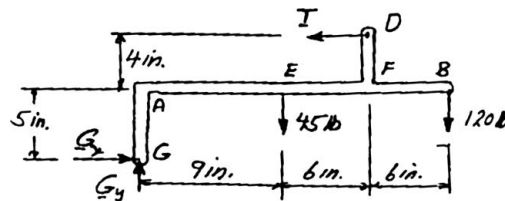
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PROBLEM 7.52

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

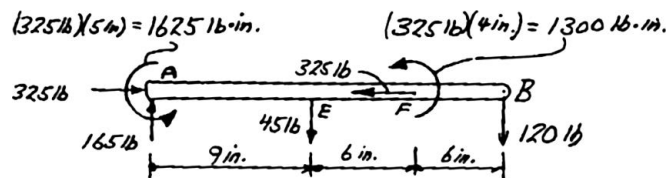


$$+\curvearrowright \Sigma F_G = 0: T(9 \text{ in.}) - (45 \text{ lb})(9 \text{ in.}) - (120 \text{ lb})(21 \text{ in.}) = 0 \quad T = 325 \text{ lb}$$

$$\pm \rightarrow \Sigma F_x = 0: -325 \text{ lb} + G_x = 0 \quad G_x = 325 \text{ lb} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: G_y - 45 \text{ lb} - 120 \text{ lb} = 0 \quad G_y = 165 \text{ lb} \uparrow$$

Equivalent loading on straight part of beam AB

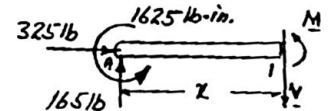


From A to E:

$$\Sigma F_y = 0: V = +165 \text{ lb}$$

$$+\curvearrowright \Sigma M_1 = 0: +1625 \text{ lb} \cdot \text{in.} - (165 \text{ lb})x + M = 0$$

$$M = -1625 + 165x$$



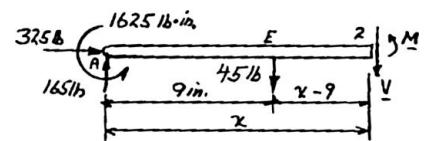
From E to F:

$$+\uparrow \Sigma F_y = 0: 165 - 45 - V = 0$$

$$V = +120 \text{ lb}$$

$$+\curvearrowright \Sigma M_2 = 0: +1625 \text{ lb} \cdot \text{in.} - (165 \text{ lb})x + (45 \text{ lb})(x - 9) + M = 0$$

$$M = -1220 + 120x$$



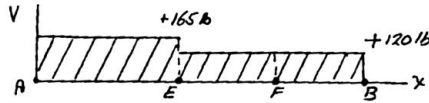
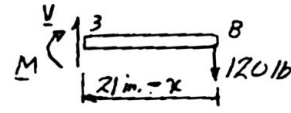
PROBLEM 7.52 (Continued)

From F to B:

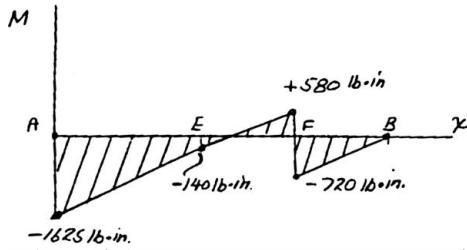
$$\Sigma F_y = 0 \quad V = +120 \text{ lb}$$

$$+\curvearrowright \Sigma M_3 = 0: \quad -(120)(21-x) - M = 0$$

$$M = -2520 + 120x$$

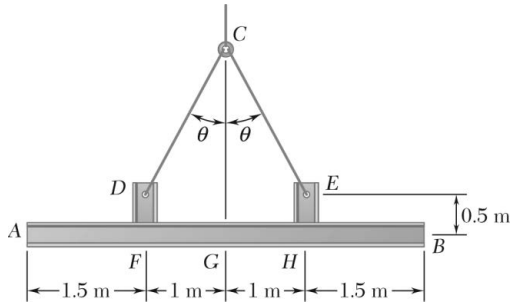


$$|V|_{\max} = 165.0 \text{ lb} \quad \blacktriangleleft$$



$$|M|_{\max} = 1625 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

PROBLEM 7.53



Two small channel sections DF and EH have been welded to the uniform beam AB of weight $W = 3 \text{ kN}$ to form the rigid structural member shown. This member is being lifted by two cables attached at D and E . Knowing that $\theta = 30^\circ$ and neglecting the weight of the channel sections, (a) draw the shear and bending-moment diagrams for beam AB , (b) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD Beam + channels:

(a) By symmetry:

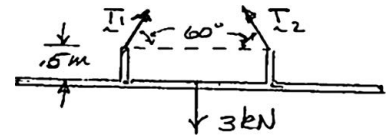
$$T_1 = T_2 = T$$

$$\uparrow \Sigma F_y = 0: 2T \sin 60^\circ - 3 \text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}} \text{ kN}$$

$$T_{1x} = \frac{3}{2\sqrt{3}}$$

$$T_{1y} = \frac{3}{2} \text{ kN}$$



FBD Beam:

$$M = (0.5 \text{ m}) \frac{3}{2\sqrt{3}} \text{ kN}$$

$$= 0.433 \text{ kN} \cdot \text{m}$$

With cable force replaced by equivalent force-couple system at F and G

Shear Diagram: V is piecewise linear

$$\left(\frac{dV}{dx} = -0.6 \text{ kN/m} \right) \text{ with } 1.5 \text{ kN}$$

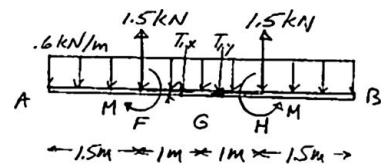
discontinuities at F and H .

$$V_{F^-} = -(0.6 \text{ kN/m})(1.5 \text{ m}) = 0.9 \text{ kN}$$

V increases by 1.5 kN to $+0.6 \text{ kN}$ at F^+

$$V_G = 0.6 \text{ kN} - (0.6 \text{ kN/m})(1 \text{ m}) = 0$$

Finish by invoking symmetry



PROBLEM 7.53 (Continued)

Moment diagram: M is piecewise parabolic

$$\left(\frac{dM}{dx} \text{ decreasing with } V \right)$$

with discontinuities of .433 kN at F and H .

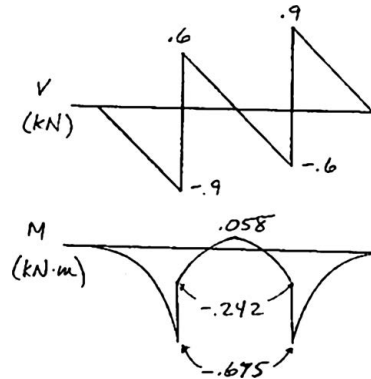
$$\begin{aligned} M_{F^-} &= -\frac{1}{2}(0.9 \text{ kN})(1.5 \text{ m}) \\ &= -0.675 \text{ kN} \cdot \text{m} \end{aligned}$$

M increases by 0.433 kN m to $-0.242 \text{ kN} \cdot \text{m}$ at F^+

$$\begin{aligned} M_G &= -0.242 \text{ kN} \cdot \text{m} + \frac{1}{2}(0.6 \text{ kN})(1 \text{ m}) \\ &= 0.058 \text{ kN} \cdot \text{m} \end{aligned}$$

Finish by invoking symmetry

(b)



$$|V|_{\max} = 900 \text{ N} \blacktriangleleft$$

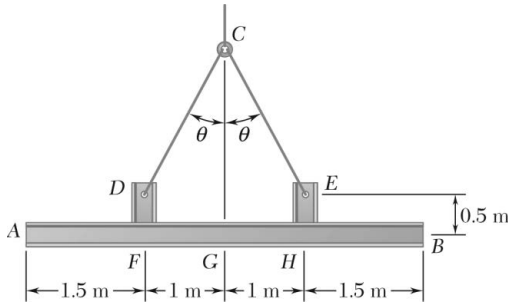
at F^- and G^+

$$|M|_{\max} = 675 \text{ N} \cdot \text{m} \blacktriangleleft$$

at F and G

PROBLEM 7.54

Solve Problem 7.53 when $\theta = 60^\circ$.



PROBLEM 7.53 Two small channel sections DF and EH have been welded to the uniform beam AB of weight $W = 3 \text{ kN}$ to form the rigid structural member shown. This member is being lifted by two cables attached at D and E . Knowing that $\theta = 30^\circ$ and neglecting the weight of the channel sections, (a) draw the shear and bending-moment diagrams for beam AB , (b) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

Free body: Beam and channels

From symmetry:

$$E_y = D_y$$

Thus:

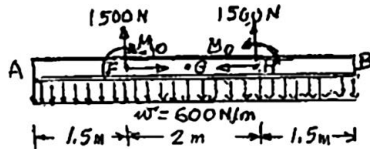
$$E_x = D_x = D_y \tan \theta \quad (1)$$

$$+\uparrow \Sigma F_y = 0: D_y + E_y - 3 \text{ kN} = 0 \quad D_y = E_y = 1.5 \text{ kN} \uparrow \triangleleft$$

From (1):

$$D_x = (1.5 \text{ kN}) \tan \theta \rightarrow \quad E = (1.5 \text{ kN}) \tan \theta \leftarrow \triangleleft$$

We replace the forces at D and E by equivalent force-couple systems at F and H , where



$$M_0 = (1.5 \text{ kN} \tan \theta)(0.5 \text{ m}) = (750 \text{ N} \cdot \text{m}) \tan \theta \quad (2)$$

We note that the weight of the beam per unit length is

$$w = \frac{W}{L} = \frac{3 \text{ kN}}{5 \text{ m}} = 0.6 \text{ kN/m} = 600 \text{ N/m}$$

(a) Shear and bending moment diagrams

From A to F:

$$+\uparrow \Sigma F_y = 0: -V - 600x = 0 \quad V = (-600x) \text{ N}$$

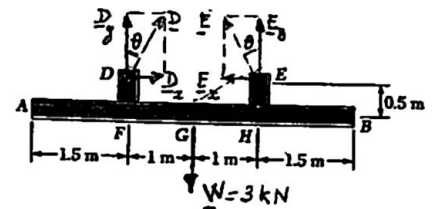
$$+\curvearrowright \Sigma M_J = 0: M + (600x) \frac{x}{2} = 0, \quad M = (-300x^2) \text{ N} \cdot \text{m}$$

For $x = 0$:

$$V_A = M_A = 0 \triangleleft$$

For $x = 1.5 \text{ m}$:

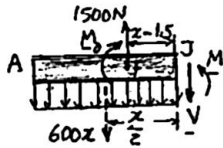
$$V_F = -900 \text{ N}, \quad M_F = -675 \text{ N} \cdot \text{m} \triangleleft$$



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PROBLEM 7.54 (Continued)

From *F* to *H*:



$$+\uparrow \Sigma F_y = 0: 1500 - 600x - V = 0$$

$$V = (1500 - 600x) \text{ N}$$

$$+\curvearrowright \Sigma M_J = 0: M + (600x) \frac{x}{2} - 1500(x - 1.5) - M_0 = 0$$

$$M = M_0 - 300x^2 + 1500(x - 1.5) \text{ N}\cdot\text{m}$$

For $x = 1.5$ m:

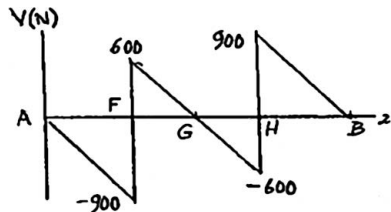
$$V_F = +600 \text{ N}, \quad M_F = (M_0 - 675) \text{ N}\cdot\text{m} \quad \triangleleft$$

For $x = 2.5$ m:

$$V_G = 0, \quad M_G = (M_0 - 375) \text{ N}\cdot\text{m} \quad \triangleleft$$

From *G* to *B*, The *V* and *M* diagrams will be obtained by symmetry,

$$(b) \quad |V|_{\max} = 900 \text{ N} \quad \blacktriangleleft$$



Making $\theta = 60^\circ$ in Eq. (2):

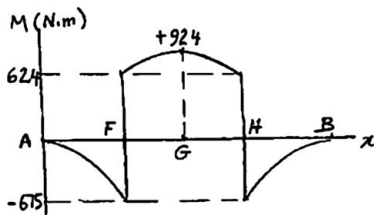
$$M_0 = 750 \tan 60^\circ = 1299 \text{ N}\cdot\text{m}$$

Thus, just to the right of *F*:

$$M = 1299 - 675 = 624 \text{ N}\cdot\text{m} \quad \triangleleft$$

and

$$M_G = 1299 - 375 = 924 \text{ N}\cdot\text{m} \quad \triangleleft$$

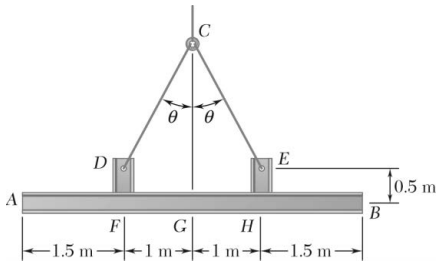


$$(b) \quad |V|_{\max} = 900 \text{ N} \quad \blacktriangleleft$$

$$|M|_{\max} = 924 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 7.55

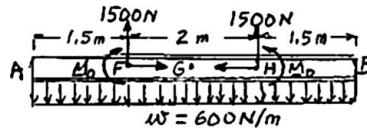
For the structural member of Problem 7.53, determine (a) the angle θ for which the maximum absolute value of the bending moment in beam AB is as small as possible, (b) the corresponding value of $|M|_{\max}$. (Hint: Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)



PROBLEM 7.53 Two small channel sections DF and EH have been welded to the uniform beam AB of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at D and E . Knowing that $\theta = 30^\circ$ and neglecting the weight of the channel sections, (a) draw the shear and bending-moment diagrams for beam AB , (b) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

See solution of Problem 7.50 for reduction of loading on beam AB to the following:

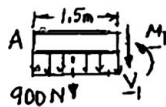


where

$$M_0 = (750 \text{ N} \cdot \text{m}) \tan \theta \quad \triangleleft$$

[Equation (2)]

The largest negative bending moment occurs Just to the left of F :

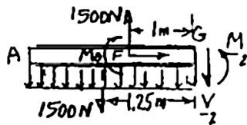


$$+\circlearrowleft \sum M_1 = 0: \quad M_1 + (900 \text{ N}) \left(\frac{1.5 \text{ m}}{2} \right) = 0$$

$$M_1 = -675 \text{ N} \cdot \text{m} \quad \triangleleft$$

The largest positive bending moment occurs

At G :



$$+\circlearrowleft \sum M_2 = 0: \quad M_2 - M_0 + (1500 \text{ N})(1.25 \text{ m} - 1 \text{ m}) = 0$$

$$M_2 = M_0 - 375 \text{ N} \cdot \text{m} \quad \triangleleft$$

Equating M_2 and $-M_1$:

$$M_0 - 375 = +675$$

$$M_0 = 1050 \text{ N} \cdot \text{m}$$

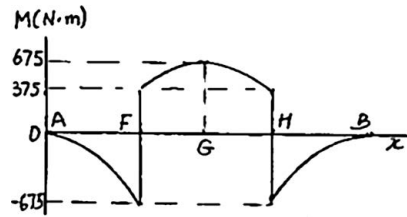
PROPRIETARY MATERIAL. © 2013 The McGraw-Hill Companies, Inc. All rights reserved. No part of this Manual may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

PROBLEM 7.55 (Continued)

(a) From Equation (2):

$$\tan \theta = \frac{1050}{750} = 1.400$$

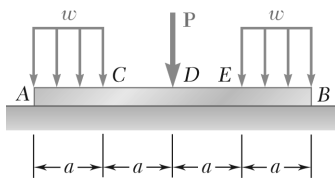
$$\theta = 54.5^\circ \blacktriangleleft$$



(b)

$$|M|_{\max} = 675 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROBLEM 7.56



For the beam of Problem 7.43, determine (a) the ratio $k = P/wa$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

PROBLEM 7.43 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - kwa = 0$$

$$w_g = \frac{w}{4}(2+k)$$

Setting

$$\frac{w_g}{w} = \alpha \quad (1)$$

We have

$$k = 4\alpha - 2 \quad (2)$$

Minimum value of B.M. For M to have negative values, we must have $w_g < w$. We verify that M will then be negative and keep decreasing in the portion AC of the beam. Therefore, M_{\min} will occur between C and D .

From C to D:

$$+\curvearrowright \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \alpha wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}w(\alpha x^2 - 2ax + a^2) \quad (3)$$

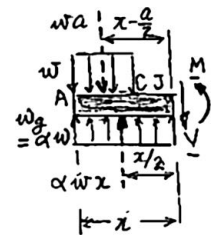
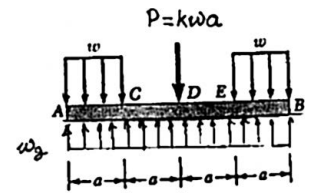
We differentiate and set $\frac{dM}{dx} = 0$:

$$\alpha x - a = 0 \quad x_{\min} = \frac{a}{\alpha} \quad (4)$$

Substituting in (3):

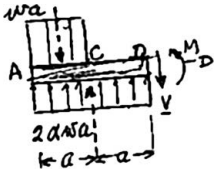
$$M_{\min} = \frac{1}{2}wa^2\left(\frac{1}{\alpha} - \frac{2}{\alpha} + 1\right)$$

$$M_{\min} = -wa^2\frac{1-\alpha}{2\alpha} \quad (5)$$



PROBLEM 7.56 (Continued)

Maximum value of bending moment occurs at D



$$+\curvearrowright \Sigma M_D = 0: M_D + wa \left(\frac{3a}{2} \right) - (2\alpha wa)a = 0$$

$$M_{\max} = M_D = wa^2 \left(2\alpha - \frac{3}{2} \right) \quad (6)$$

Equating $-M_{\min}$ and M_{\max} :

$$wa^2 \frac{1-\alpha}{2\alpha} = wa^2 \left(2\alpha - \frac{3}{2} \right)$$

$$4\alpha^2 - 2\alpha - 1 = 0$$

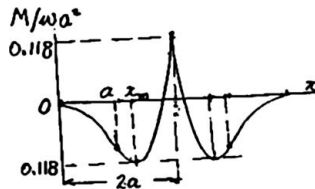
$$\alpha = \frac{2 + \sqrt{20}}{8}$$

$$\alpha = \frac{1 + \sqrt{5}}{4} = 0.809$$

(a) Substitute in (2):

$$k = 4(0.809) - 2$$

$$k = 1.236 \blacktriangleleft$$



(b) Substitute for α in (5):

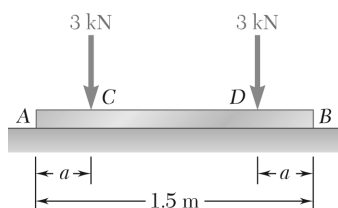
$$|M|_{\max} = -M_{\min} = -wa^2 \frac{1-0.809}{2(0.809)}$$

$$|M|_{\max} = 0.1180wa^2 \blacktriangleleft$$

Substitute for α in (4):

$$x_{\min} = \frac{a}{0.809} 1.236a \triangleleft$$

PROBLEM 7.57



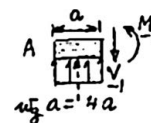
For the beam of Problem 7.45, determine (a) the distance a for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

PROBLEM 7.45 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Force per unit length exerted by ground:

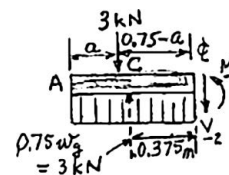
$$w_g = \frac{6 \text{ kN}}{1.5 \text{ m}} = 4 \text{ kN/m}$$



The largest positive bending moment occurs Just to the left of C:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 = (4a) \frac{a}{2} \qquad M_1 = 2a^2 \quad \triangleleft$$

The largest negative bending moment occurs



At the center line:

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 + 3(0.75 - a) - 3(0.375) = 0 \qquad M_2 = -(1.125 - 3a) \quad \triangleleft$$

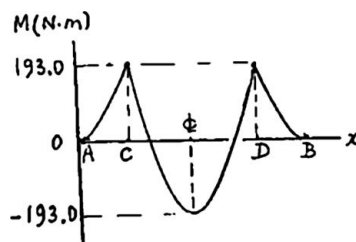
Equating M_1 and $-M_2$:

$$2a^2 = 1.125 - 3a$$

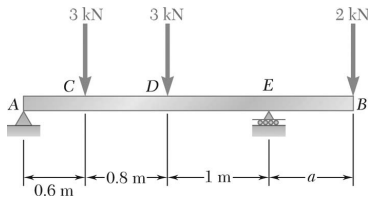
$$a^2 + 1.5a - 0.5625 = 0$$

(a) Solving the quadratic equation: $a = 0.31066$, $a = 0.311 \text{ m} \quad \blacktriangleleft$

(b) Substituting: $|M|_{\max} = M_1 = 2(0.31066)^2$ $|M|_{\max} = 193.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$



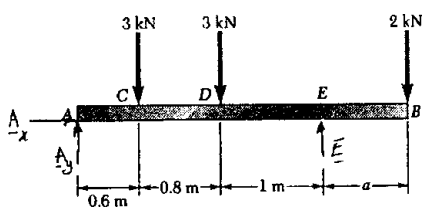
PROBLEM 7.58



For the beam and loading shown, determine (a) the distance a for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

Free body: Entire beam



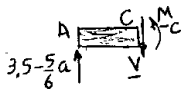
$$\Sigma F_x = 0: A_x = 0$$

$$+\circlearrowleft \Sigma M_E = 0: -A_y(2.4) + (3)(1.8) + 3(1) - (2)a = 0$$

$$A_y = 3.5 \text{ kN} - \frac{5}{6}a$$

$$A = 3.5 \text{ kN} - \frac{5}{6}a \quad \triangleleft$$

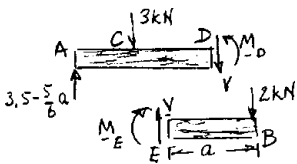
Free body: AC



$$+\circlearrowleft \Sigma M_C = 0: M_C - \left(3.5 - \frac{5}{6}a\right)(0.6 \text{ m}) = 0,$$

$$M_C = +2.1 - \frac{a}{2} \quad \triangleleft$$

Free body: AD



$$+\circlearrowleft \Sigma M_D = 0: M_D - \left(3.5 - \frac{5}{6}a\right)(1.4 \text{ m}) + (3 \text{ kN})(0.8 \text{ m}) = 0$$

$$M_D = +2.5 - \frac{7}{6}a \quad \triangleleft$$

Free body: EB

$$+\circlearrowleft \Sigma M_E = 0: -M_E - (2 \text{ kN})a = 0$$

$$M_E = -2a \quad \triangleleft$$

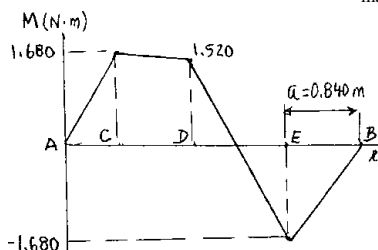
We shall assume that $M_C > M_D$ and, thus, that $M_{\max} = M_C$.

$$\text{We set } M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = |M_E| = 2.1 - \frac{a}{2} = 2a$$

$$a = 0.840 \text{ m} \quad \triangleleft$$

$$|M|_{\max} = M_C = |M_E| = 2a = 2(0.840)$$

$$|M|_{\max} = 1.680 \text{ N} \cdot \text{m} \quad \triangleleft$$



We must check our assumption.

$$M_D = 2.5 - \frac{7}{6}(0.840) = 1.520 \text{ N} \cdot \text{m}$$

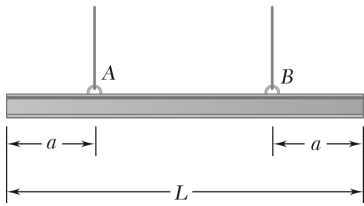
Thus, $M_C > M_D$, O.K.

The answers are

$$(a) \quad a = 0.840 \text{ m} \quad \triangleleft$$

$$(b) \quad |M|_{\max} = 1.680 \text{ N} \cdot \text{m} \quad \triangleleft$$

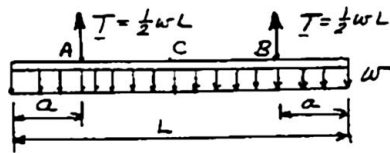
PROBLEM 7.59



A uniform beam is to be picked up by crane cables attached at A and B . Determine the distance a from the ends of the beam to the points where the cables should be attached if the maximum absolute value of the bending moment in the beam is to be as small as possible. (*Hint: Draw the bending-moment diagram in terms of a , L , and the weight w per unit length, and then equate the absolute values of the largest positive and negative bending moments obtained.*)

SOLUTION

$w =$ weight per unit length

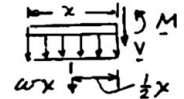


To the left of A:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M + wx \left(\frac{x}{2} \right) = 0$$

$$M = -\frac{1}{2}wx^2$$

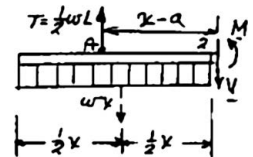
$$M_A = -\frac{1}{2}wa^2$$



Between A and B:

$$+\circlearrowleft \Sigma M_2 = 0: \quad M - \left(\frac{1}{2}wL \right)(x - a) + (wx) \left(\frac{1}{2}x \right) = 0$$

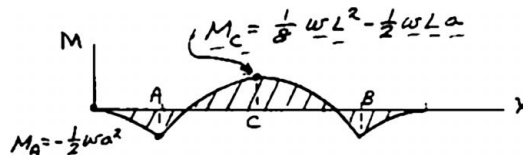
$$M = -\frac{1}{2}wx^2 + \frac{1}{2}wLx - \frac{1}{2}wLa$$



At center C:

$$x = \frac{L}{2}$$

$$M_C = -\frac{1}{2}w \left(\frac{L}{2} \right)^2 + \frac{1}{2}wL \left(\frac{L}{2} \right) - \frac{1}{2}wLa$$



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PROBLEM 7.59 (Continued)

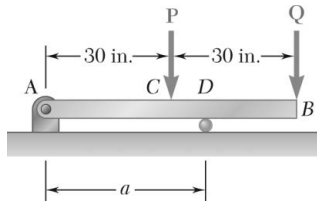
We set $|M_A| = |M_C|$: $\left| -\frac{1}{2}wa^2 \right| = \left| \frac{1}{8}wL^2 - \frac{1}{2}wLa \right| + \frac{1}{2}wa^2 = \frac{1}{8}wL^2 - \frac{1}{2}wLa$

$$a^2 + La - 0.25L^2 = 0$$

$$a = \frac{1}{2}(L \pm \sqrt{L^2 + L^2}) = \frac{1}{2}(\sqrt{2} - 1)L$$

$$M_{\max} = \frac{1}{2}w(0.207L)^2 = 0.0214wL^2$$

$$a = 0.207L \blacktriangleleft$$

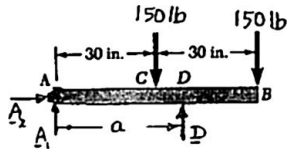


PROBLEM 7.60

Knowing that $P = Q = 150$ lb, determine (a) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (b) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

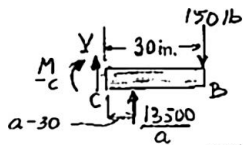
Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (150)(30) - (150)(60) = 0$$

$$D = \frac{13,500}{a} \quad \triangleleft$$

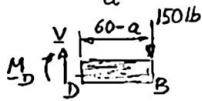
Free body: CB



$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{13,500}{a}(a-30) = 0$$

$$M_C = 9000 \left(1 - \frac{45}{a} \right) \quad \triangleleft$$

Free body: DB



$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60-a) = 0$$

$$M_D = -150(60-a) \quad \triangleleft$$

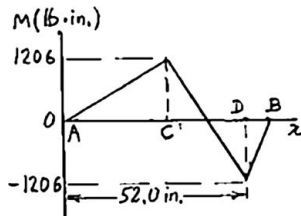
(a) We set

$$M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = -M_D: \quad 9000 \left(1 - \frac{45}{a} \right) = 150(60-a)$$

$$60 - \frac{2700}{a} = 60 - a$$

$$a^2 = 2700 \quad a = 51.96 \text{ in.}$$

$$a = 52.0 \text{ in.} \quad \blacktriangleleft$$

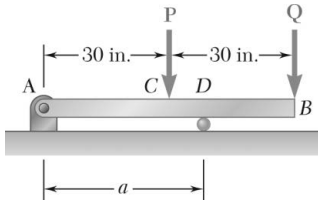


(b) $|M|_{\max} = -M_D = 150(60 - 51.96)$

$$|M|_{\max} = 1206 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

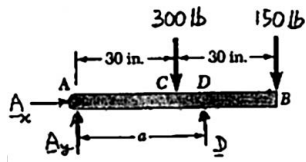
PROBLEM 7.61

Solve Problem 7.60 assuming that $P = 300$ lb and $Q = 150$ lb.



PROBLEM 7.60 Knowing that $P = Q = 150$ lb, determine (a) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (b) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

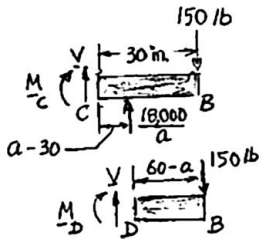


Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (300)(30) - (150)(60) = 0$$

$$D = \frac{18,000}{a} \triangleleft$$

Free body: CB



$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{18,000}{a}(a - 30) = 0$$

$$M_C = 13,500 \left(1 - \frac{40}{a}\right) \triangleleft$$

Free body: DB

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60 - a) = 0$$

$$M_D = -150(60 - a) \triangleleft$$

(a) We set

$$M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = -M_D: \quad 13,500 \left(1 - \frac{40}{a}\right) = 150(60 - a)$$

$$90 - \frac{3600}{a} = 60 - a$$

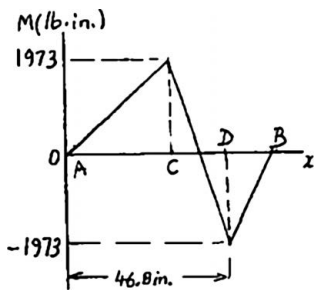
$$a^2 + 30a - 3600 = 0$$

$$a = \frac{-30 + \sqrt{15,300}}{2} = 46.847$$

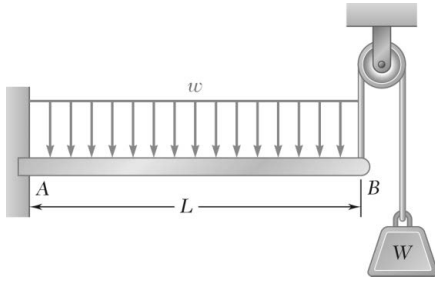
$$a = 46.8 \text{ in.} \triangleleft$$

(b) $|M|_{\max} = -M_D = 150(60 - 46.847)$

$$|M|_{\max} = 1973 \text{ lb} \cdot \text{in.} \triangleleft$$



PROBLEM 7.62*

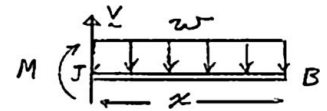


In order to reduce the bending moment in the cantilever beam AB , a cable and counterweight are permanently attached at end B . Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{\max}$. Consider (a) the case when the distributed load is permanently applied to the beam, (b) the more general case when the distributed load may either be applied or removed.

SOLUTION

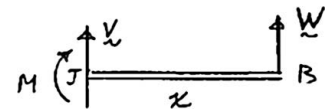
M due to distributed load:

$$\begin{aligned} \left(\sum M_J = 0: \quad -M - \frac{x}{2} wx = 0 \right. \\ \left. M = -\frac{1}{2} wx^2 \right. \end{aligned}$$



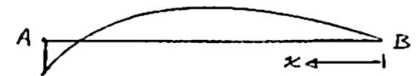
M due to counter weight:

$$\begin{aligned} \left(\sum M_J = 0: \quad -M + xw = 0 \right. \\ \left. M = wx \right. \end{aligned}$$



(a) **Both applied:**

$$\begin{aligned} M = Wx - \frac{w}{2} x^2 \\ \frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w} \end{aligned}$$



And here $M = \frac{W^2}{2w} > 0$ so M_{\max} ; M_{\min} must be at $x = L$

So $M_{\min} = WL - \frac{1}{2} wL^2$. For minimum $|M|_{\max}$ set $M_{\max} = -M_{\min}$,

$$\text{so } \frac{W^2}{2w} = -WL + \frac{1}{2} wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0$$

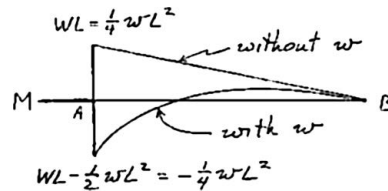
$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need+)} \qquad W = (\sqrt{2} - 1)wL = 0.414 wL \quad \blacktriangleleft$$

PROBLEM 7.62* (Continued)

(b) w may be removed

$$M_{\max} = \frac{W^2}{2w} = \frac{(\sqrt{2}-1)^2}{2} wL^2$$

$$M_{\max} = 0.0858 wL^2 \quad \blacktriangleleft$$



Without w ,

$$M = Wx$$

$$M_{\max} = WL \text{ at } A$$

With w (see Part a)

$$M = Wx - \frac{w}{2} x^2$$

$$M_{\max} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}$$

$$M_{\min} = WL - \frac{1}{2} wL^2 \text{ at } x = L$$

For minimum M_{\max} , set $M_{\max}(\text{no } w) = -M_{\min}(\text{with } w)$

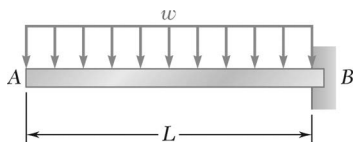
$$WL = -WL + \frac{1}{2} wL^2 \rightarrow W = \frac{1}{4} wL \rightarrow$$

$$M_{\max} = \frac{1}{4} wL^2 \quad \blacktriangleleft$$

With

$$W = \frac{1}{4} wL \quad \blacktriangleleft$$

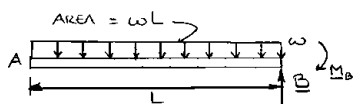
PROBLEM 7.63



Using the method of Section 7.6, solve Problem 7.29.

PROBLEM 7.29 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

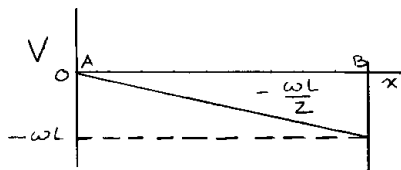


Free body: Entire beam

$$\uparrow \Sigma F_y = 0: \quad wL - B = 0$$

$$\mathbf{B} = wL \uparrow$$

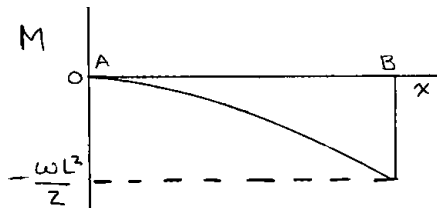
Shear diagram



$$\rightarrow \Sigma M_B = 0: \quad (wL) \left(\frac{L}{2} \right) - M_B = 0$$

$$\mathbf{M}_B = \frac{wL^2}{2} \curvearrowright$$

Moment diagram

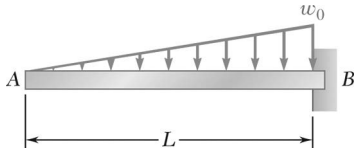


(b) $|V|_{\max} = wL;$

$$|M|_{\max} = \frac{wL^2}{2} \blacktriangleleft$$

PROBLEM 7.64

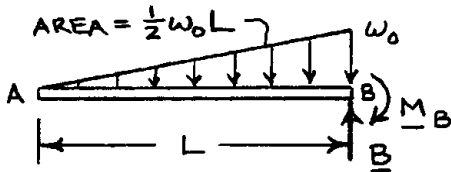
Using the method of Section 7.6, solve Problem 7.30.



PROBLEM 7.30 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

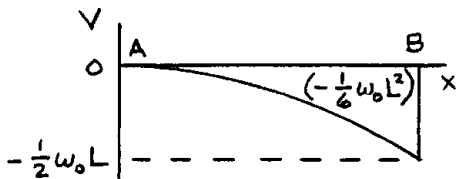
Free body: Entire beam



$$+\uparrow \Sigma F_y = 0: \quad B - \frac{1}{2}(w_0)(L) = 0$$

$$B = \frac{1}{2}w_0L \uparrow$$

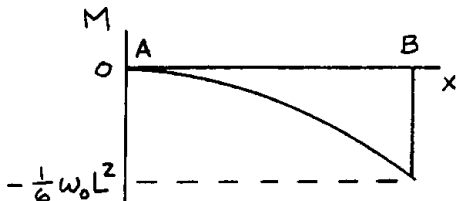
Shear diagram



$$+\curvearrowright \Sigma M_B = 0: \quad \frac{1}{2}(w_0)(L)\left(\frac{L}{3}\right) - M_B = 0$$

$$M_B = \frac{1}{6}w_0L^2 \curvearrowright$$

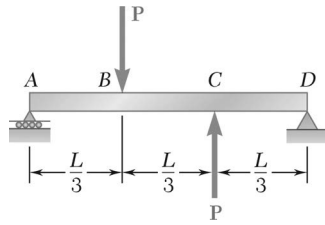
Moment diagram



(b)

$$|V|_{\max} = w_0L/2 \quad \blacktriangleleft$$

$$|M|_{\max} = w_0L^2/6 \quad \blacktriangleleft$$



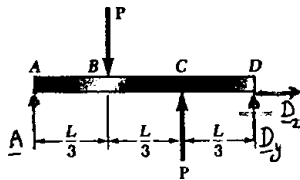
PROBLEM 7.65

Using the method of Section 7.6, solve Problem 7.31.

PROBLEM 7.31 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



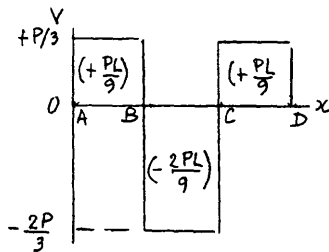
$$+\curvearrowright \Sigma M_D = 0: P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$$

$$A = P/3 \uparrow$$

Shear diagram

We note that

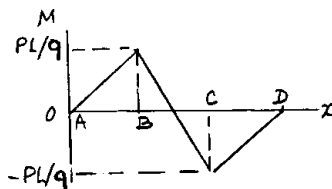
$$V_A = A = +P/3$$



$$|V|_{\max} = 2P/3 \quad \blacktriangleleft$$

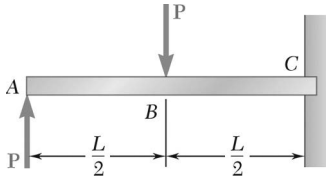
Bending diagram

We note that $M_A = 0$



$$|M|_{\max} = PL/9 \quad \blacktriangleleft$$

PROBLEM 7.66



Using the method of Section 7.6, solve Problem 7.32.

PROBLEM 7.32 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

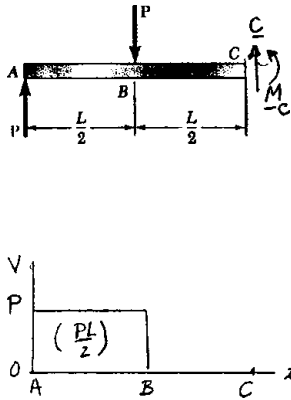
Free body: Entire beam

$$\Sigma F_y = 0: \quad C = 0$$

$$\Sigma M_C = 0: \quad M_C = \frac{1}{2} PL \curvearrowright$$

Shear diagram

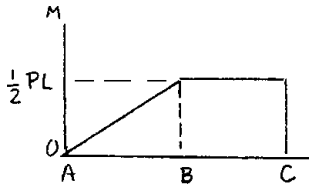
At A: $V_A = +P$



$$|V|_{\max} = P \quad \blacktriangleleft$$

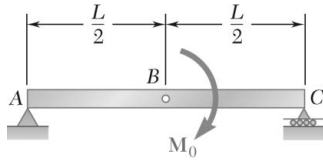
Moment diagram

At A: $M_A = 0$



$$|M|_{\max} = \frac{1}{2} PL \quad \blacktriangleleft$$

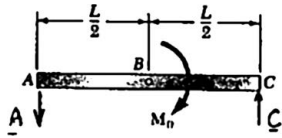
PROBLEM 7.67



Using the method of Section 7.6, solve Problem 7.33.

PROBLEM 7.33 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

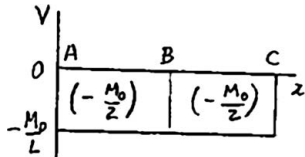


Free body: Entire beam

$$\Sigma F_y = 0: A = C$$

$$+\curvearrowright \Sigma M_C = 0: Al - M_0 = 0$$

$$A = C = \frac{M_0}{L}$$

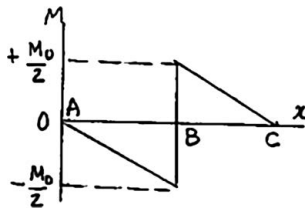


Shear diagram

At A:

$$V_A = -\frac{M_0}{L}$$

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$



Bending-moment diagram

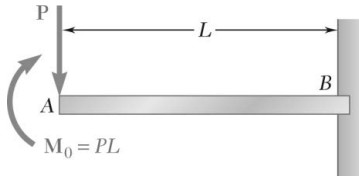
At A:

$$M_A = 0$$

At B, M increases by M_0 on account of applied couple.

$$|M|_{\max} = M_0/2 \blacktriangleleft$$

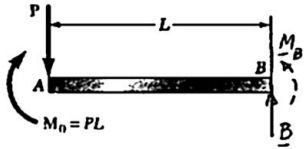
PROBLEM 7.68



Using the method of Section 7.6, solve Problem 7.34.

PROBLEM 7.34 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



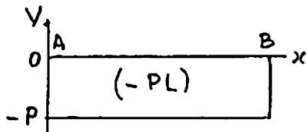
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: B - P = 0$$

$$B = P \uparrow$$

$$+\curvearrowright \Sigma M_B = 0: M_B - M_0 + PL = 0$$

$$M_B = 0$$

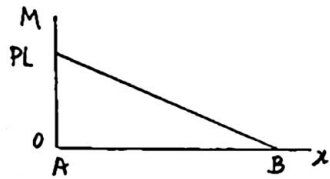


Shear diagram

At A:

$$V_A = -P$$

$$|V|_{\max} = P \blacktriangleleft$$

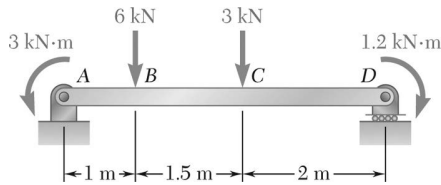


Bending-moment diagram

At A:

$$M_A = M_0 = PL$$

$$|M|_{\max} = PL \blacktriangleleft$$



PROBLEM 7.69

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

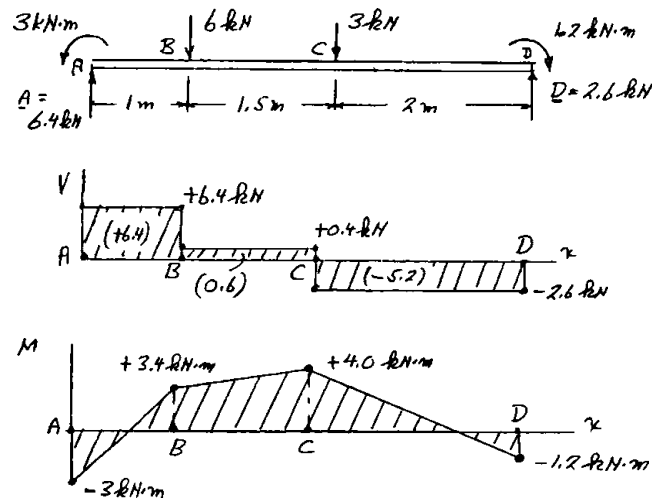
SOLUTION

Reactions

$$\Sigma F_x = 0 \quad A_x = 0$$

$$+\curvearrowright \Sigma M_D = 0: +3 \text{ kN} \cdot \text{m} + (6 \text{ kN})(3.5 \text{ m}) + (3 \text{ kN})(2 \text{ m}) - 1.2 \text{ kN} \cdot \text{m} - A_y(4.5 \text{ m}) = 0$$

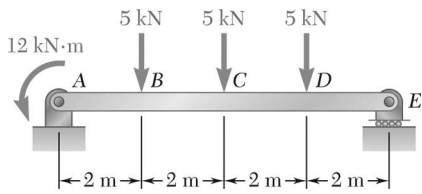
$$A_y = +6.4 \text{ kN} \quad A_y = +6.4 \text{ kN} \uparrow$$



(b)

$$|V|_{\max} = 6.40 \text{ kN};$$

$$|M|_{\max} = 4.00 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 7.70

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

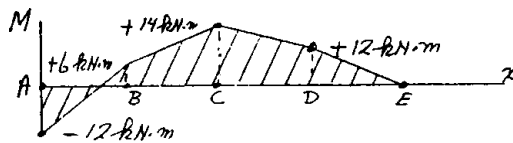
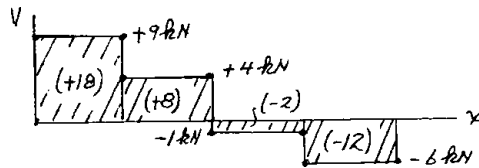
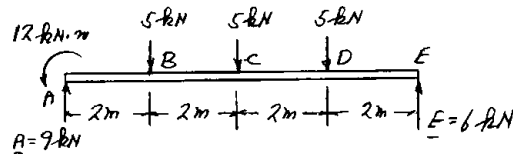
Reactions

$$+\circlearrowleft \Sigma M_A = 0: 12 \text{ kN} \cdot \text{m} - 3(5 \text{ kN})(4 \text{ m}) + E(8 \text{ m}) = 0$$

$$E = 6 \text{ kN} \uparrow$$

$$\Sigma F_y = 0$$

$$A = 9 \text{ kN} \uparrow$$

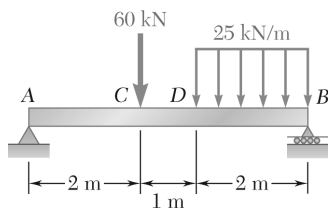


(b)

$$|V|_{\max} = 9.00 \text{ kN};$$

$$|M|_{\max} = 14.00 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

PROBLEM 7.71

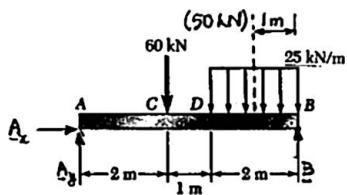


Using the method of Section 7.6, solve Problem 7.39.

PROBLEM 7.39 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: A_x = 0$$

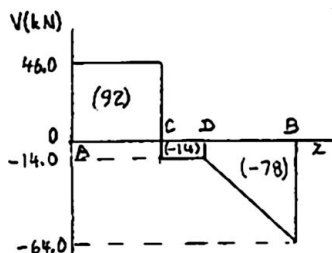
$$+\curvearrowright \Sigma M_B = 0: (60 \text{ kN})(3 \text{ m}) + (50 \text{ kN})(1 \text{ m}) - A_y(5 \text{ m}) = 0$$

$$A_y = +46.0 \text{ kN} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: B + 46.0 \text{ kN} - 60 \text{ kN} - 50 \text{ kN} = 0$$

$$B = +64.0 \text{ kN} \quad \triangleleft$$

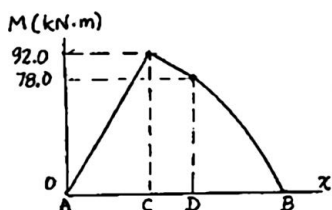
Shear diagram



At A: $V_A = A_y = +46.0 \text{ kN}$

$$|V|_{\max} = 64.0 \text{ kN} \quad \blacktriangleleft$$

Bending-moment diagram



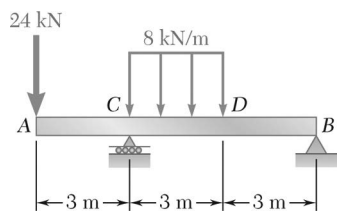
At A: $M_A = 0$

$$|M|_{\max} = 92.0 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

Parabola from D to B. Its slope at D is same as that of straight-line segment CD since V has no discontinuity at D.

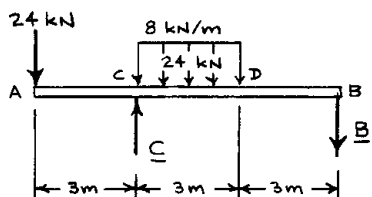
PROBLEM 7.72

Using the method of Section 7.6, solve Problem 7.40.



PROBLEM 7.40 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

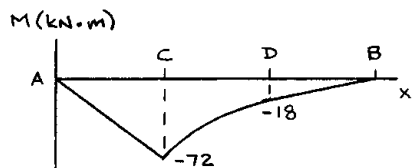
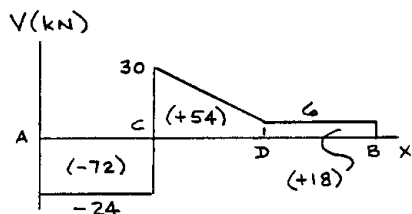
$$+\circlearrowleft \Sigma M_B = 0: (24 \text{ kN})(9 \text{ m}) - C(6 \text{ m}) + (24 \text{ kN})(4.5 \text{ m}) = 0$$

$$C = 54 \text{ kN} \uparrow$$

Shear diagram

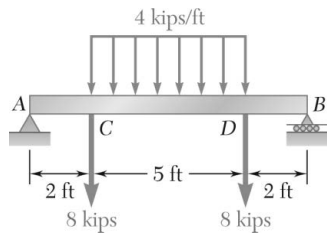
$$+\uparrow \Sigma F_y = 0: 54 - 24 - 24 - B = 0$$

$$B = 6 \text{ kN} \downarrow$$



(b) $|V|_{\max} = 30.0 \text{ kN};$

$|M|_{\max} = 72.0 \text{ kN} \cdot \text{m} \blacktriangleleft$

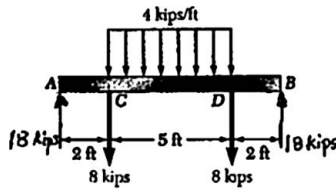


PROBLEM 7.73

Using the method of Section 7.6, solve Problem 7.41.

PROBLEM 7.41 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

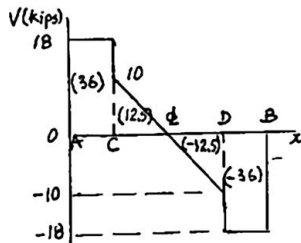


Reactions at supports.

Because of the symmetry:

$$A = B = \frac{1}{2}(8 + 8 + 4 \times 5) \text{ kips}$$

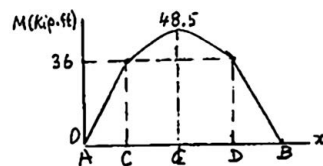
$$A = B = 18 \text{ kips} \uparrow \triangleleft$$



Shear diagram

At A: $V_A = +18 \text{ kips}$

$$|V|_{\max} = 18.00 \text{ kips} \triangleleft$$

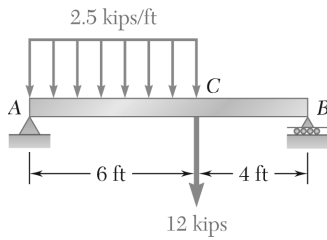


Bending-moment diagram

At A: $M_A = 0$

$$|M|_{\max} = 48.5 \text{ kip} \cdot \text{ft} \triangleleft$$

Discontinuities in slope at C and D, due to the discontinuities of V.



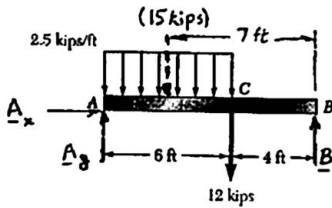
PROBLEM 7.74

Using the method of Section 7.6, solve Problem 7.42.

PROBLEM 7.42 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Beam



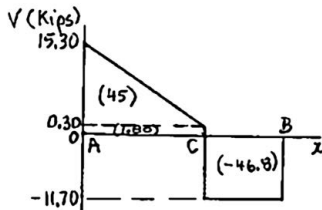
$$\Sigma F_x = 0: A_x = 0$$

$$+\curvearrowright \Sigma M_B = 0: (12 \text{ kips})(4 \text{ ft}) + (15 \text{ kips})(7 \text{ ft}) - A_y(10 \text{ ft}) = 0$$

$$A_y = +15.3 \text{ kips} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: B + 15.3 - 15 - 12 = 0$$

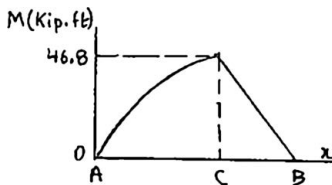
$$B = +11.7 \text{ kips} \quad \triangleleft$$



Shear diagram

At A: $V_A = A_y = 15.3 \text{ kips}$

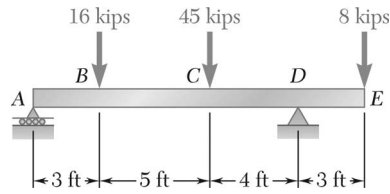
$$|V|_{\max} = 15.30 \text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = 0$

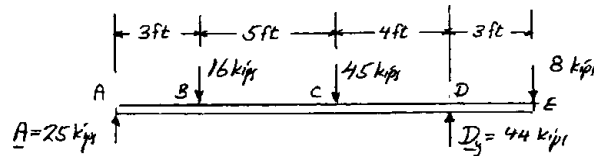
$$|M|_{\max} = 46.8 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$



PROBLEM 7.75

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Reactions

$$+\circlearrowleft \Sigma M_D = 0: (16)(9) + (45)(4) - (8)(3) - A(12) = 0$$

$$A = +25 \text{ kips}$$

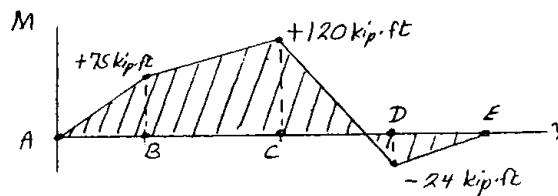
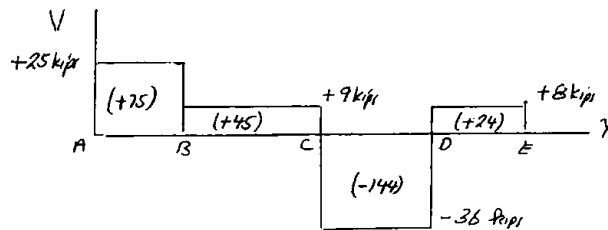
$$A = 25 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: 25 - 16 - 45 - 8 + D_y = 0$$

$$D_y = +44,$$

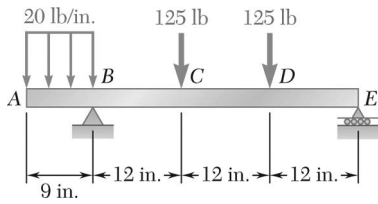
$$D_y = 44 \text{ kips} \uparrow$$

$$\Sigma F_x = 0: D_x = 0$$



(b) $|V|_{\max} = 36.0 \text{ kips};$

$|M|_{\max} = 120.0 \text{ kip} \cdot \text{ft} \blacktriangleleft$



PROBLEM 7.76

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Reactions

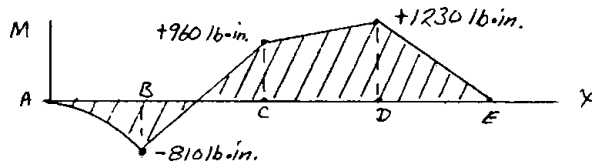
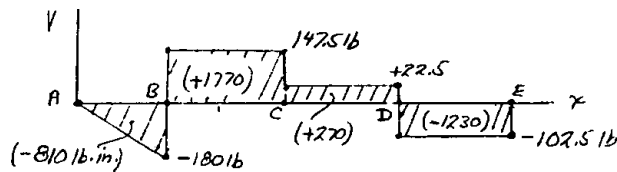
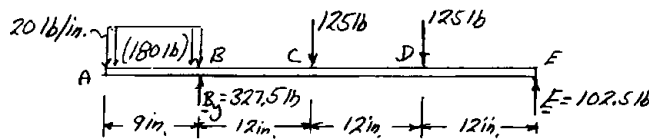
$$\Sigma F_x = 0: B_x = 0$$

$$+\curvearrowright \Sigma M_E = 0: (20 \text{ lb/in.})(9 \text{ in.})(40.5 \text{ in.}) + (125 \text{ lb.})(24 \text{ in.}) + (125 \text{ lb.})(12 \text{ in.}) - B(36 \text{ in.}) = 0$$

$$B_y = +327.5 \text{ lb} \quad \mathbf{B}_y = 327.5 \text{ lb} \uparrow$$

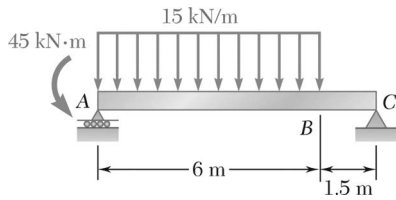
$$+\uparrow \Sigma F_y = 0: -(20 \text{ lb/in.})(9 \text{ in.}) - 125 \text{ lb} - 125 \text{ lb} + 327.5 \text{ lb} + E = 0$$

$$E = +102.5 \text{ lb} \quad \mathbf{E} = 102.5 \text{ lb} \uparrow$$



(b) $|V|_{\max} = 180.0 \text{ lb};$

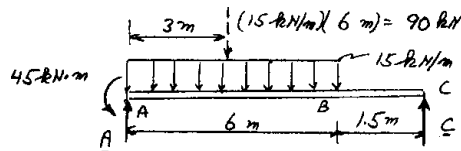
$|M|_{\max} = 1230 \text{ lb} \cdot \text{in.} \blacktriangleleft$



PROBLEM 7.77

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Reactions

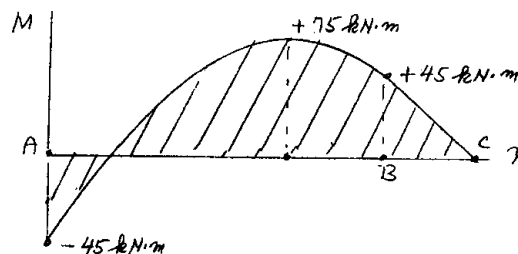
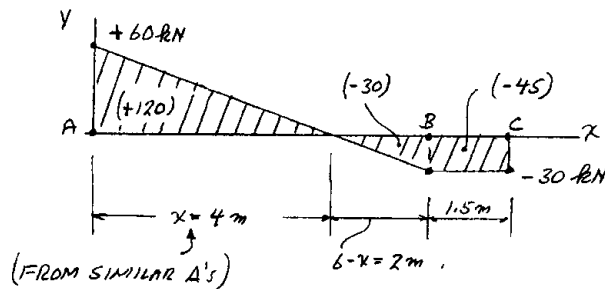
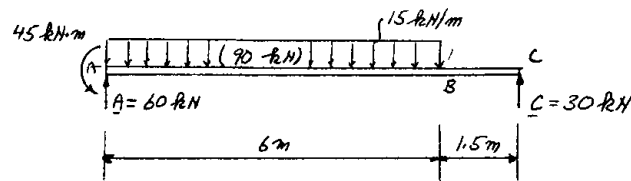
$$+\circlearrowleft \sum M_A = 0: +45 \text{ kN}\cdot\text{m} - (90 \text{ kN})(3 \text{ m}) + C(7.5 \text{ m}) = 0$$

$$C = +30 \text{ kN}$$

$$C = 30 \text{ kN} \uparrow$$

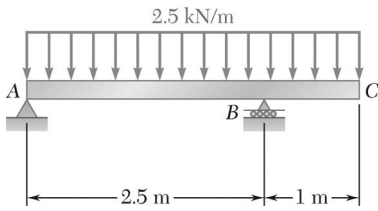
$$+\uparrow \sum F_y = 0: A - 90 \text{ kN} + 30 \text{ kN} = 0$$

$$A = 60 \text{ kN} \uparrow$$



(b)

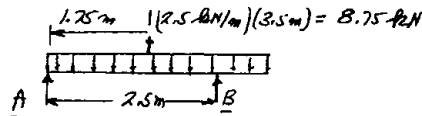
75.0 kN·m, 4.00 m from A ◀



PROBLEM 7.78

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

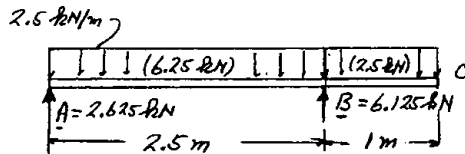


$$+\circlearrowleft \sum M_A = 0: (8.75)(1.75) - B(2.5) = 0$$

$$B = 6.125 \text{ kN} \uparrow$$

$$\sum F_y = 0:$$

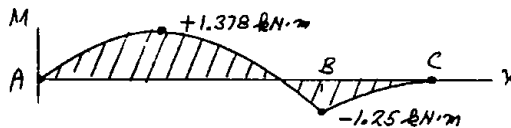
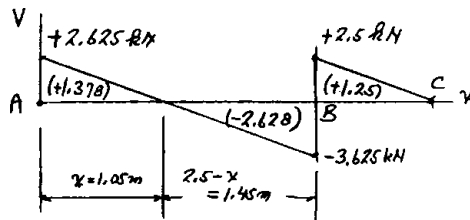
$$A = 2.625 \text{ kN} \uparrow$$



Similar Δ 's

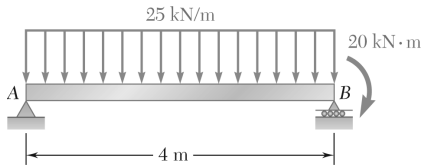
$$\frac{x}{2.625} = \frac{2.5 - x}{3.625} = \frac{2.5}{6.25}$$

Add numerators and denominators
 $x = 1.05 \text{ m}$



(b)

1.378 kN · m, 1.050 m from A ◀

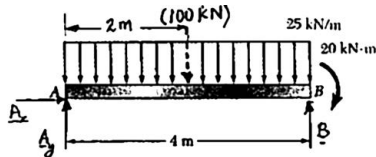


PROBLEM 7.79

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

Free body: Beam



$$+\circlearrowleft \sum M_A = 0: \quad B(4 \text{ m}) - (100 \text{ kN})(2 \text{ m}) - 20 \text{ kN} \cdot \text{m} = 0$$

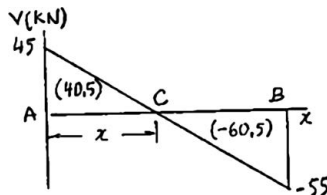
$$B = +55 \text{ kN} \quad \triangleleft$$

$$\sum F_x = 0: \quad A_x = 0$$

$$+\uparrow \sum F_y = 0: \quad A_y + 55 - 100 = 0$$

$$A_y = +45 \text{ kN} \quad \triangleleft$$

Shear diagram



At A: $V_A = A_y = +45 \text{ kN}$

To determine Point C where $V = 0$:

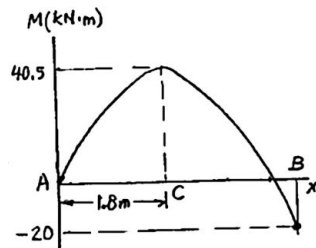
$$V_C - V_A = -wx$$

$$0 - 45 \text{ kN} = -(25 \text{ kN} \cdot \text{m})x$$

$$x = 1.8 \text{ m} \quad \triangleleft$$

We compute all areas bending-moment

Bending-moment diagram



At A: $M_A = 0$

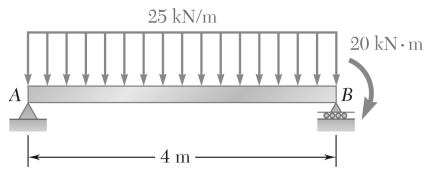
At B: $M_B = -20 \text{ kN} \cdot \text{m}$

$$|M|_{\max} = 40.5 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

$$1.800 \text{ m from A} \quad \blacktriangleleft$$

Single arc of parabola

PROBLEM 7.80

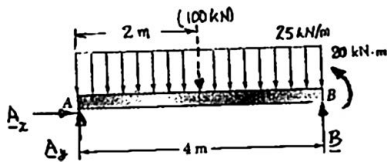


Solve Problem 7.79 assuming that the 20-kN · m couple applied at B is counterclockwise.

PROBLEM 7.79 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

Free body: Beam



$$+\circlearrowleft \Sigma M_A = 0: B(4 \text{ m}) - (100 \text{ kN})(2 \text{ m}) - 20 \text{ kN} \cdot \text{m} = 0$$

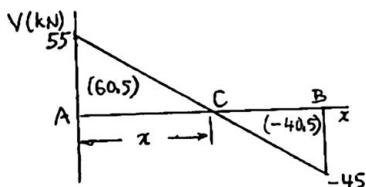
$$B = +45 \text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: A_x = 0$$

$$+\uparrow \Sigma F_y = 0: A_y + 45 - 100 = 0$$

$$A_y = +55 \text{ kN} \quad \triangleleft$$

Shear diagram



At A: $V_A = A_y = +55 \text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$

$$0 - 55 \text{ kN} = -(25 \text{ kN/m})x$$

$$x = 2.20 \text{ m} \quad \triangleleft$$

We compute all areas bending-moment

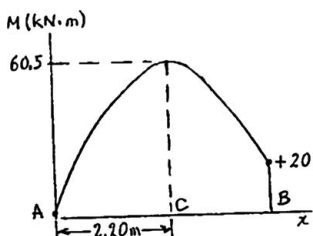
Bending-moment diagram

At A: $M_A = 0$

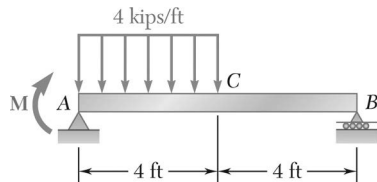
At B: $M_B = +20 \text{ kN} \cdot \text{m}$

$$|M|_{\text{max}} = 60.5 \text{ kN} \cdot \text{m} \quad \triangleleft$$

$$2.20 \text{ m from A} \quad \triangleleft$$



Single arc of parabola



PROBLEM 7.81

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (a) $M = 0$, (b) $M = 24 \text{ kip} \cdot \text{ft}$.

SOLUTION

Free body: Beam

$$\begin{aligned} \Sigma F_x = 0: \quad A_x &= 0 \\ \Sigma M_B = 0: \quad (16 \text{ kips})(6 \text{ ft}) - A_y(8 \text{ ft}) - M &= 0 \\ A_y &= 12 \text{ kips} - \frac{1}{8}M \quad (1) \triangleleft \\ \Sigma F_y = 0: \quad B + 12 - \frac{1}{8}M - 16 &= 0 \\ B &= 4 \text{ kips} + \frac{1}{8}M \quad (2) \triangleleft \end{aligned}$$

(a) $M = 0$:

Load diagram

Making $M = 0$ in. (1) and (2).

$$A_y = +12 \text{ kips}$$

$$B = +4 \text{ kips}$$

Shear diagram

$$V_A = A_y = +12 \text{ kips}$$

To determine Point D where $V = 0$:

$$\begin{aligned} V_D - V_A &= -wx \\ 0 - 12 \text{ kips} &= -(4 \text{ kips/ft})x \end{aligned}$$

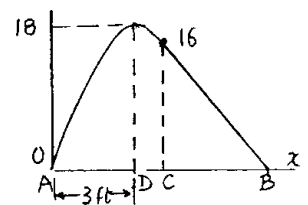
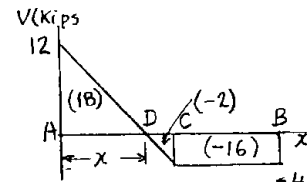
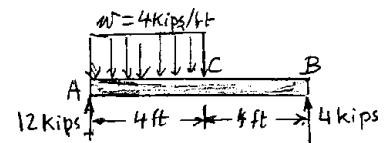
$$x = 3 \text{ ft} \triangleleft$$

We compute all areas

B. M. Diagram

$$\text{At A:} \quad M_A = 0$$

Parabola from A to C



$$|M|_{\max} = 18.00 \text{ kip} \cdot \text{ft}, \triangleleft$$

$$3.00 \text{ ft from A} \triangleleft$$

PROBLEM 7.81 (Continued)

(b) $M = 24 \text{ kip} \cdot \text{ft}$

Load diagram

Making $M = 24 \text{ kip} \cdot \text{ft}$ in (1) and (2)

$$A = 12 - \frac{1}{8}(24) = +9 \text{ kips}$$

$$B = 4 + \frac{1}{8}(24) = +7 \text{ kips}$$

Shear diagram

$$V_A = A_y = +9 \text{ kips}$$

To determine Point D where $V = 0$:

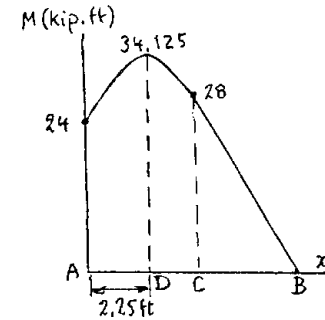
$$V_D = V_A - wx$$

$$0 - 9 \text{ kips} = -(4 \text{ kips/ft})x$$

$$x = 2.25 \text{ ft} \triangleleft$$

B. M. Diagram

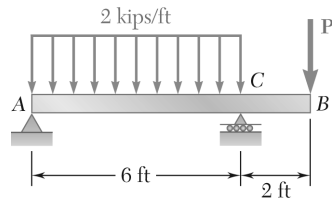
At A: $M_A = +24 \text{ kip} \cdot \text{ft}$



$$|M|_{\max} = 34.1 \text{ kip} \cdot \text{ft}, \triangleleft$$

$$2.25 \text{ ft from A} \triangleleft$$

Parabola from A to C.

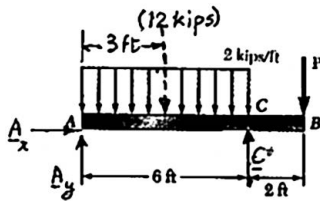


PROBLEM 7.82

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (a) $P = 6$ kips, (b) $P = 3$ kips.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: C(6 \text{ ft}) - (12 \text{ kips})(3 \text{ ft}) - P(8 \text{ ft}) = 0$$

$$C = 6 \text{ kips} + \frac{4}{3}P \quad (1) \triangleleft$$

$$\Sigma F_y = 0: A_y + \left(6 + \frac{4}{3}P\right) - 12 - P = 0$$

$$A_y = 6 \text{ kips} - \frac{1}{3}P \quad (2) \triangleleft$$

(a) $P = 6$ kips.

Load diagram

Substituting for P in Eqs. (2) and (1):

$$A_y = 6 - \frac{1}{3}(6) = 4 \text{ kips}$$

$$C = 6 + \frac{4}{3}(6) = 14 \text{ kips}$$

Shear diagram

$$V_A = A_y = +4 \text{ kips}$$

To determine Point D where $V = 0$:

$$\begin{aligned} V_D - V_A &= -wx \\ 0 - 4 \text{ kips} &= (2 \text{ kips/ft})x \end{aligned}$$

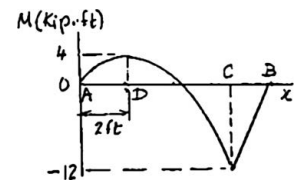
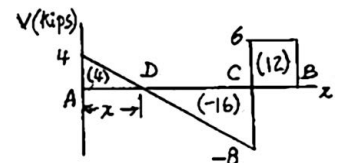
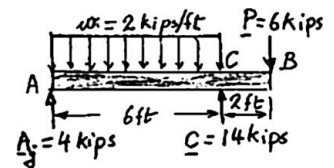
$$x = 2 \text{ ft} \triangleleft$$

We compute all areas

Bending-moment diagram

$$\text{At A: } M_A = 0$$

Parabola from A to C



$$|M|_{\max} = 12.00 \text{ kip}\cdot\text{ft, at C} \triangleleft$$

PROBLEM 7.82 (Continued)

(b) $P = 3$ kips

Load diagram

Substituting for P in Eqs. (2) and (1):

$$A = 6 - \frac{1}{3}(3) = 5 \text{ kips}$$

$$C = 6 + \frac{4}{3}(3) = 10 \text{ kips}$$

Shear diagram

$$V_A = A_y = +5 \text{ kips}$$

To determine D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - (5 \text{ kips}) = -(2 \text{ kips/ft})x$$

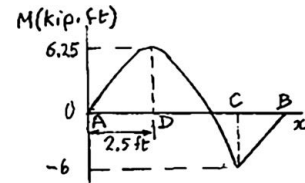
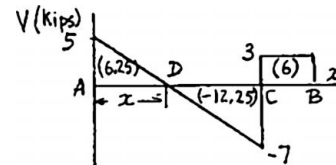
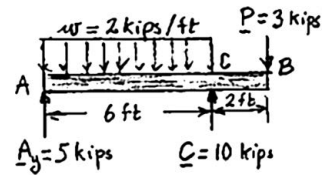
$$x = 2.5 \text{ ft} \quad \blacktriangleleft$$

We compute all areas

Bending-moment diagram

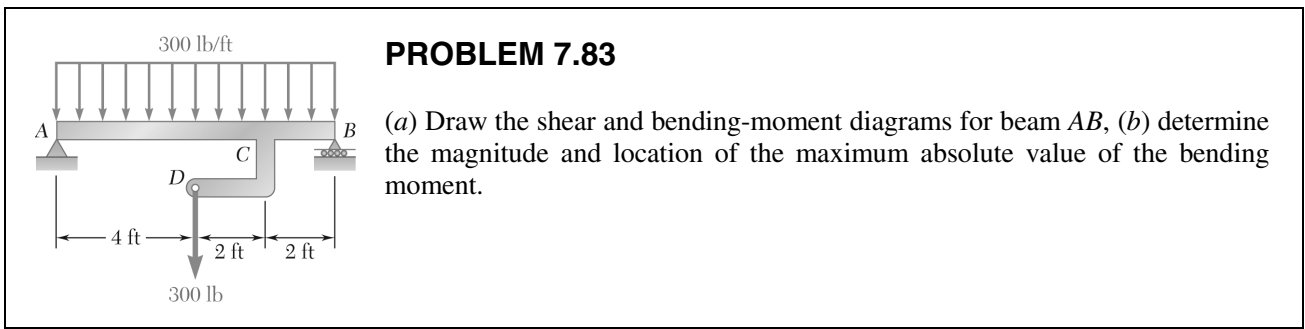
At A: $M_A = 0$

Parabola from A to C.



$$|M|_{\max} = 6.25 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

$$2.50 \text{ ft from A} \quad \blacktriangleleft$$



PROBLEM 7.83

(a) Draw the shear and bending-moment diagrams for beam AB, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

Reactions at supports
 Because of symmetry of load

$$A = B = \frac{1}{2}(300 \times 8 + 300)$$

$A = B = 1350 \text{ lb} \uparrow \triangleleft$

Load diagram for AB
 The 300-lb force at D is replaced by an equivalent force-couple system at C.

Shear diagram

At A: $V_A = A = 1350 \text{ lb}$

To determine Point E where $V = 0$:

$$V_E - V_A = -wx$$

$$0 - 1350 \text{ lb} = -(300 \text{ lb/ft})x$$

$x = 4.50 \text{ ft} \triangleleft$

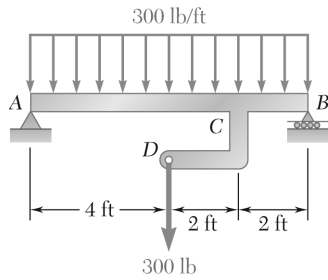
We compute all areas

Bending-moment diagram

At A: $M_A = 0$

Note 600 – lb · ft drop at C due to couple

$M|_{\text{max}} = 3040 \text{ lb} \cdot \text{ft} \triangleleft$
 4.50 ft from A \triangleleft



PROBLEM 7.84

Solve Problem 7.83 assuming that the 300-lb force applied at D is directed upward.

PROBLEM 7.83 (a) Draw the shear and bending-moment diagrams for beam AB , (b) determine the magnitude and location of the maximum absolute value of the bending moment.

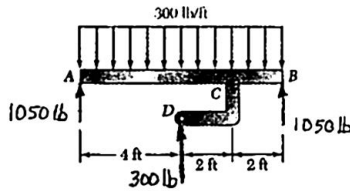
SOLUTION

Reactions at supports

Because of symmetry of load:

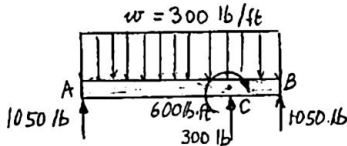
$$A = B = \frac{1}{2}(300 \times 8 - 300)$$

$$A = B = 1050 \text{ lb} \uparrow \triangleleft$$



Load diagram

The 300-lb force at D is replaced by an equivalent force-couple system at C .



Shear diagram

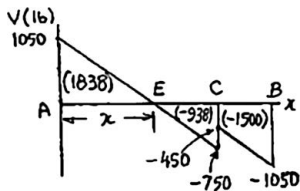
At A : $V_A = A = 1050 \text{ lb}$

To determine Point E where $V = 0$:

$$V_E - V_A = -wx$$

$$0 - 1050 \text{ lb} = -(300 \text{ lb/ft})x$$

$$x = 3.50 \text{ ft} \triangleleft$$



We compute all areas

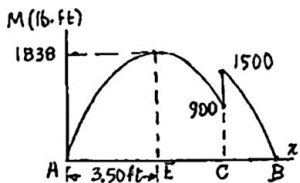
Bending-moment diagram

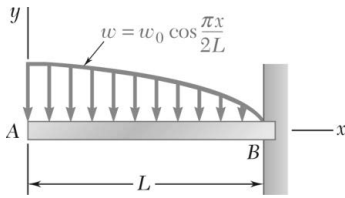
At A : $M_A = 0$

Note 600-lb·ft increase at C due to couple

$$|M|_{\max} = 1838 \text{ lb} \cdot \text{ft} \triangleleft$$

$$3.50 \text{ ft from } A \triangleleft$$

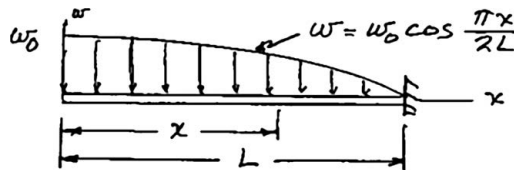




PROBLEM 7.85

For the beam and loading shown, (a) write the equations of the shear and bending-moment curves, (b) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\frac{dv}{dx} = -w = w_0 \cos \frac{\pi x}{2L}$$

$$V = -\int w dx = -w_0 \left(\frac{2L}{\pi} \right) \sin \frac{\pi x}{2L} + C_1 \quad (1)$$

$$\frac{dM}{dx} = V = -w_0 \left(\frac{2L}{\pi} \right) \sin \frac{\pi x}{2L} + C_1$$

$$M = \int V dx = +w_0 \left(\frac{2L}{\pi} \right)^2 \cos \frac{\pi x}{2L} + C_1 x + C_2 \quad (2)$$

Boundary conditions

At $x=0$: $V = C_1 = 0 \quad C_1 = 0$

At $x=L$: $M = +w_0 \left(\frac{2L}{\pi} \right)^2 \cos(0) + C_2 = 0$

$$C_2 = -w_0 \left(\frac{2L}{\pi} \right)^2$$

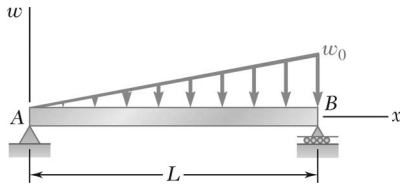
Eq. (1)

$$V = -w_0 \left(\frac{2L}{\pi} \right) \sin \frac{\pi x}{2L} \quad \blacktriangleleft$$

$$M = w_0 \left(\frac{2L}{\pi} \right)^2 \left(-1 + \cos \frac{\pi x}{2L} \right) \quad \blacktriangleleft$$

M_{\max} at $x=L$:

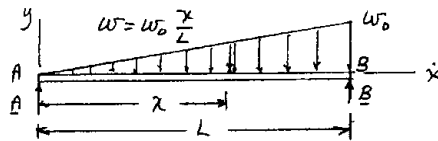
$$|M_{\max}| = w_0 \left(\frac{2L}{\pi} \right)^2 |-1 + 0| = \frac{4}{\pi^2} w_0 L^2 \quad \blacktriangleleft$$



PROBLEM 7.86

For the beam and loading shown, (a) write the equations of the shear and bending-moment curves, (b) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\text{Eq. (7.1): } \frac{dV}{dx} = -w = -w_0 \frac{x}{L}$$

$$V = \int -w_0 \frac{x}{L} dx = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 \quad (1)$$

$$\text{Eq. (7.3): } \frac{dM}{dx} = V = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1$$

$$M = \int \left(-\frac{1}{2} w_0 \frac{x^2}{L} + C_1 \right) dx = -\frac{1}{6} w_0 \frac{x^3}{L} + C_1 x + C_2 \quad (2)$$

(a) Boundary conditions

$$\text{At } x=0: \quad M=0=0+0+C_2 \quad C_2=0$$

$$x=L: \quad M=0=-\frac{1}{6} w_0 L^2 + C_1 L \quad C_1 = \frac{1}{6} w_0 L$$

Substituting C_1 and C_2 into (1) and (2):

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L \quad V = \frac{1}{2} w_0 L \left(\frac{1}{3} - \frac{x^2}{L^2} \right) \blacktriangleleft$$

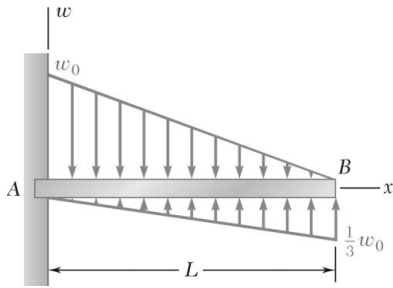
$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x \quad M = \frac{1}{6} w_0 L^2 \left(\frac{x}{L} - \frac{x^3}{L^3} \right) \blacktriangleleft$$

(b) Max moment occurs when $V=0$:

$$1 - 3 \frac{x^2}{L^2} = 0 \quad \frac{x}{L} = \frac{1}{\sqrt{3}} \quad x = 0.577L \quad \blacktriangleleft$$

$$M_{\max} = \frac{1}{6} w_0 L^2 \left[\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}} \right)^3 \right] \quad M_{\max} = 0.0642 w_0 L^2 \quad \blacktriangleleft$$

Note: At $x=0$, $A = V_A = \frac{1}{2} w_0 L \left(\frac{1}{3} \right) \quad A = \frac{1}{6} w_0 L \uparrow$



PROBLEM 7.87

For the beam and loading shown, (a) write the equations of the shear and bending-moment curves, (b) determine the magnitude and location of the maximum bending moment.

SOLUTION

(a) We note that at $B(x=L)$: $V_B = 0$, $M_B = 0$ (1)

Load: $w(x) = w_0 \left(1 - \frac{x}{L}\right) - \frac{1}{3} w_0 \left(\frac{x}{L}\right) = w_0 \left(1 - \frac{4x}{3L}\right)$

Shear: We use Eq. (7.2) between $C(x=x)$ and $B(x=L)$:

$$V_B - V_C = -\int_x^L w(x) dx \quad 0 - V(x) = -\int_x^L w(x) dx$$

$$V(x) = w_0 \int_x^L \left(1 - \frac{4x}{3L}\right) dx$$

$$= w_0 \left[x - \frac{2x^2}{3L} \right]_x^L = w_0 \left(L - \frac{2L}{3} - x + \frac{2x^2}{3L} \right)$$

$$V(x) = \frac{w_0}{3L} (2x^2 - 3Lx + L^2) \quad (2) \blacktriangleleft$$

Bending moment: We use to Eq. (7.4) between $C(x=x)$ and $B(x=L)$:

$$M_B - M_C = \int_x^L V(x) dx \quad 0 - M(x)$$

$$= \frac{w_0}{3L} \int_x^L (2x^2 - 3Lx + L^2) dx$$

$$M(x) = -\frac{w_0}{3L} \left[\frac{2}{3} x^3 - \frac{3}{2} Lx^2 + L^2 x \right]_x^L$$

$$= -\frac{w_0}{18L} [4x^3 - 9Lx^2 + 6L^2 x]_x^L$$

$$= -\frac{w_0}{18L} [(4L^3 - 9L^3 + 6L^3) - (4x^3 - 9Lx^2 + 6L^2 x)]$$

$$M(x) = \frac{w_0}{18L} (4x^3 - 9Lx^2 + 6L^2 x - L^3) \quad (3) \blacktriangleleft$$

PROBLEM 7.87 (Continued)

(b) Maximum bending moment

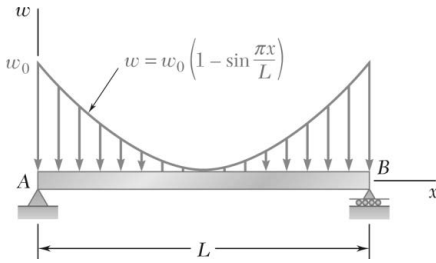
$$\frac{dM}{dx} = V = 0$$

Eq. (2): $2x^2 - 3Lx + L^2 = 0$

$$x = \frac{3 - \sqrt{9 - 8}}{4} L = \frac{L}{2}$$

Carrying into (3): $M_{\max} = \frac{w_0 L^2}{72}$, At $x = \frac{L}{2}$ ◀

Note: $|M|_{\max} = \frac{w_0 L^2}{18}$ At $x = 0$



PROBLEM 7.88

For the beam and loading shown, (a) write the equations of the shear and bending-moment curves, (b) determine the maximum bending moment.

SOLUTION

(a) Reactions at supports: $A = B = \frac{1}{2}W$, where $\frac{W}{L} = \text{Total load}$

$$\begin{aligned} W &= \int_0^L w dx = w_0 \int_0^L \left(1 - \sin \frac{\pi x}{L}\right) dx \\ &= w_0 \left[x + \frac{L}{\pi} \cos \frac{\pi x}{L} \right]_0^L \\ &= w_0 L \left(1 - \frac{2}{\pi}\right) \end{aligned}$$

Thus $V_A = A = \frac{1}{2}W = \frac{1}{2}w_0 L \left(1 - \frac{2}{\pi}\right)$

$$M_A = 0 \quad (1)$$

Load: $w(x) = w_0 \left(1 - \sin \frac{\pi x}{L}\right)$

Shear: From Eq. (7.2):

$$\begin{aligned} V(x) - V_A &= -\int_0^x w(x) dx \\ &= -w_0 \int_0^x \left(1 - \sin \frac{\pi x}{L}\right) dx \end{aligned}$$

Integrating and recalling first of Eqs. (1),

$$\begin{aligned} V(x) - \frac{1}{2}w_0 L \left(1 - \frac{2}{\pi}\right) &= -w_0 \left[x + \frac{L}{\pi} \cos \frac{\pi x}{L} \right]_0^x \\ V(x) &= \frac{1}{2}w_0 L \left(1 - \frac{2}{\pi}\right) - w_0 \left(2 + \frac{L}{\pi} \cos \frac{\pi x}{L}\right) + w_0 \frac{L}{\pi} \\ V(x) &= w_0 \left(\frac{L}{2} - x - \frac{L}{\pi} \cos \frac{\pi x}{L}\right) \end{aligned} \quad (2) \blacktriangleleft$$

PROBLEM 7.88 (Continued)

Bending moment: From Eq. (7.4) and recalling that $M_A = 0$.

$$\begin{aligned}M(x) - M_A &= \int_0^x V(x) dx \\&= w_0 \left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right)^2 \sin \frac{\pi x}{L} \right]_0^x \\M(x) &= \frac{1}{2} w_0 \left(Lx - x^2 - \frac{2L^2}{\pi^2} \sin \frac{\pi x}{L} \right) \quad (3) \blacktriangleleft\end{aligned}$$

(b) Maximum bending moment

$$\frac{dM}{dx} = V = 0.$$

This occurs at $x = \frac{L}{2}$ as we may check from (2):

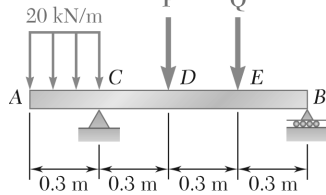
$$V\left(\frac{L}{2}\right) = w_0 \left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi} \cos \frac{\pi}{2} \right) = 0$$

From (3):

$$\begin{aligned}M\left(\frac{L}{2}\right) &= \frac{1}{2} w_0 \left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2} \sin \frac{\pi}{2} \right) \\&= \frac{1}{8} w_0 L^2 \left(1 - \frac{8}{\pi^2} \right) \\&= 0.0237 w_0 L^2\end{aligned}$$

$$M_{\max} = 0.0237 w_0 L^2, \quad \text{at } x = \frac{L}{2} \quad \blacktriangleleft$$

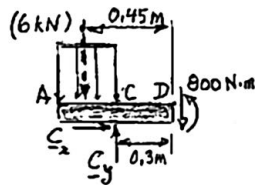
PROBLEM 7.89



The beam AB is subjected to the uniformly distributed load shown and to two unknown forces P and Q . Knowing that it has been experimentally determined that the bending moment is $+800 \text{ N}\cdot\text{m}$ at D and $+1300 \text{ N}\cdot\text{m}$ at E , (a) determine P and Q , (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(a) Free body: Portion AD

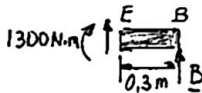


$$\Sigma F_x = 0: C_x = 0$$

$$+\curvearrowright \Sigma M_D = 0: -C_y(0.3 \text{ m}) + 0.800 \text{ kN}\cdot\text{m} + (6 \text{ kN})(0.45 \text{ m}) = 0$$

$$C_y = +11.667 \text{ kN} \quad C = 11.667 \text{ kN} \uparrow \triangleleft$$

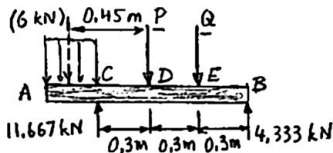
Free body: Portion EB



$$+\curvearrowright \Sigma M_E = 0: B(0.3 \text{ m}) - 1.300 \text{ kN}\cdot\text{m} = 0$$

$$B = 4.333 \text{ kN} \uparrow \triangleleft$$

Free body: Entire beam

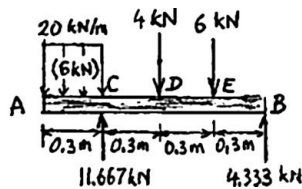


$$+\curvearrowright \Sigma M_D = 0: (6 \text{ kN})(0.45 \text{ m}) - (11.667 \text{ kN})(0.3 \text{ m}) - Q(0.3 \text{ m}) + (4.333 \text{ kN})(0.6 \text{ m}) = 0$$

$$Q = 6.00 \text{ kN} \downarrow \blacktriangleleft$$

$$+\uparrow \Sigma M_y = 0: 11.667 \text{ kN} + 4.333 \text{ kN} - 6 \text{ kN} - P - 6 \text{ kN} = 0$$

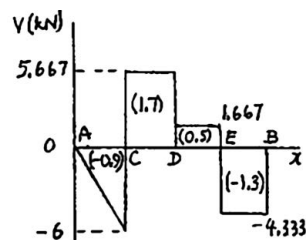
$$P = 4.00 \text{ kN} \downarrow \blacktriangleleft$$



Load diagram

(b) Shear diagram

At A: $V_A = 0$

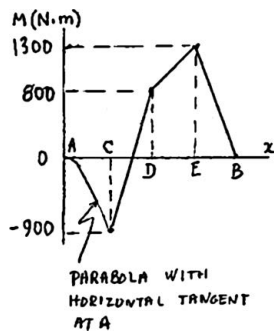


$$|V|_{\max} = 6 \text{ kN} \triangleleft$$

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PROBLEM 7.89 (Continued)

Bending-moment diagram



At A: $M_A = 0$

$$|M|_{\max} = 1300 \text{ N} \cdot \text{m} \triangleleft$$

We check that

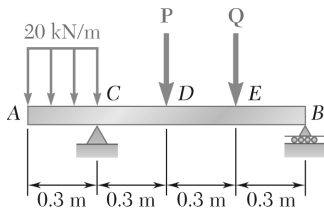
$$M_D = +800 \text{ N} \cdot \text{m} \quad \text{and} \quad M_E = +1300 \text{ N} \cdot \text{m}$$

As given:

At C:

$$M_C = -900 \text{ N} \cdot \text{m} \blacktriangleleft$$

PROBLEM 7.90

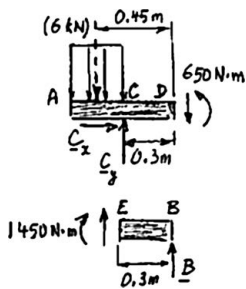


Solve Problem 7.89 assuming that the bending moment was found to be $+650 \text{ N}\cdot\text{m}$ at D and $+1450 \text{ N}\cdot\text{m}$ at E .

PROBLEM 7.89 The beam AB is subjected to the uniformly distributed load shown and to two unknown forces P and Q . Knowing that it has been experimentally determined that the bending moment is $+800 \text{ N}\cdot\text{m}$ at D and $+1300 \text{ N}\cdot\text{m}$ at E , (a) determine P and Q , (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(a) Free body: Portion AD



$$\Sigma F_x = 0: C_x = 0$$

$$+\curvearrowright \Sigma M_D = 0: -C(0.3 \text{ m}) + 0.650 \text{ kN}\cdot\text{m} + (6 \text{ kN})(0.45 \text{ m}) = 0$$

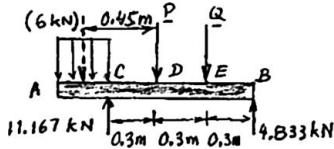
$$C_y = +11.167 \text{ kN} \quad C = 11.167 \text{ kN} \uparrow \triangleleft$$

Free body: Portion EB

$$+\curvearrowright \Sigma M_E = 0: B(0.3 \text{ m}) - 1.450 \text{ kN}\cdot\text{m} = 0$$

$$B = 4.833 \text{ kN} \uparrow \triangleleft$$

Free body: Entire beam



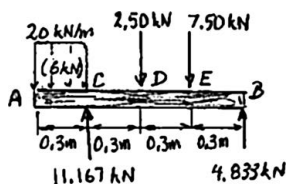
$$+\curvearrowright \Sigma M_D = 0: (6 \text{ kN})(0.45 \text{ m}) - (11.167 \text{ kN})(0.3 \text{ m}) - Q(0.3 \text{ m}) + (4.833 \text{ kN})(0.6 \text{ m}) = 0$$

$$Q = 7.50 \text{ kN} \downarrow \triangleleft$$

$$+\uparrow \Sigma M_y = 0: 11.167 \text{ kN} + 4.833 \text{ kN}$$

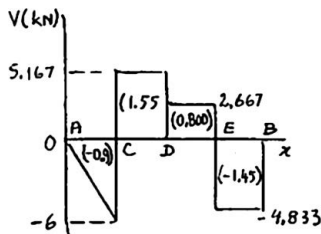
$$-6 \text{ kN} - P - 7.50 \text{ kN} = 0$$

$$P = 2.50 \text{ kN} \downarrow \triangleleft$$



Load diagram

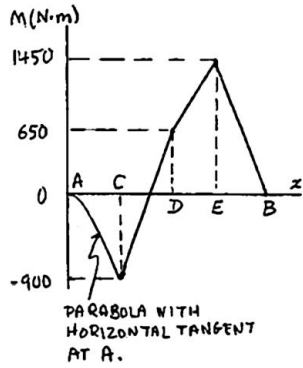
(b) Shear diagram



At A: $V_A = 0$

$$|V|_{\max} = 6 \text{ kN} \triangleleft$$

PROBLEM 7.90 (Continued)



Bending-moment diagram

At A: $M_A = 0$

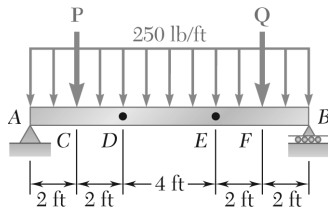
$$|M|_{\max} = 1450 \text{ N} \cdot \text{m} \triangleleft$$

We check that

$$M_D = +650 \text{ N} \cdot \text{m} \quad \text{and} \quad M_E = +1450 \text{ N} \cdot \text{m}$$

As given:

At C: $M_C = -900 \text{ N} \cdot \text{m} \blacktriangleleft$



PROBLEM 7.91*

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces P and Q . Knowing that it has been experimentally determined that the bending moment is $+6.10 \text{ kip}\cdot\text{ft}$ at D and $+5.50 \text{ kip}\cdot\text{ft}$ at E , (a) determine P and Q , (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

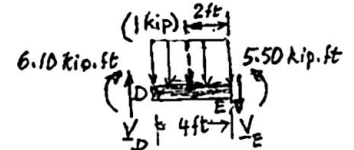
(a) Free body: Portion DE

$$+\circlearrowleft \sum M_E = 0: \quad 5.50 \text{ kip}\cdot\text{ft} - 6.10 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

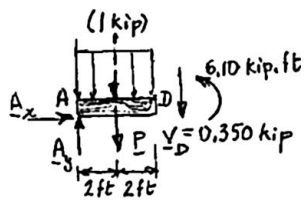
$$V_D = +0.350 \text{ kip}$$

$$+\uparrow \sum F_y = 0: \quad 0.350 \text{ kip} - 1 \text{ kip} - V_E = 0$$

$$V_E = -0.650 \text{ kip}$$



Free body: Portion AD



$$+\circlearrowleft \sum M_A = 0: \quad 6.10 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.350 \text{ kip})(4 \text{ ft}) = 0$$

$$P = 1.350 \text{ kips} \downarrow \blacktriangleleft$$

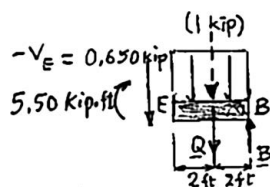
$$\sum F_x = 0: \quad A_x = 0$$

$$+\uparrow \sum F_y = 0: \quad A_y - 1 \text{ kip} - 1.350 \text{ kip} - 0.350 \text{ kip} = 0$$

$$A_y = +2.70 \text{ kips}$$

$$A = 2.70 \text{ kips} \uparrow \blacktriangleleft$$

Free body: Portion EB



$$+\circlearrowleft \sum M_B = 0: \quad (0.650 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 5.50 \text{ kip}\cdot\text{ft} = 0$$

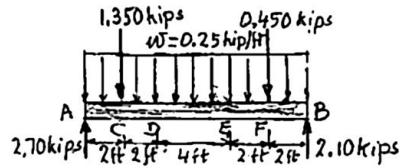
$$Q = 0.450 \text{ kip} \downarrow \blacktriangleleft$$

$$+\uparrow \sum F_y = 0: \quad B - 0.450 - 1 - 0.650 = 0$$

$$B = 2.10 \text{ kips} \uparrow \blacktriangleleft$$

PROBLEM 7.91* (Continued)

(b) Load diagram



Shear diagram

At A: $V_A = A = +2.70$ kips

To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$

$$0 - 0.85 \text{ kips} = -(0.25 \text{ kip/ft})\mu$$

$$\mu = 3.40 \text{ ft} \blacktriangleleft$$

We next compute all areas

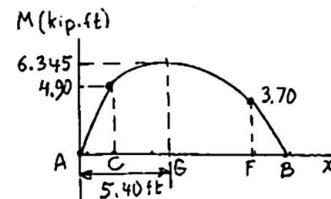
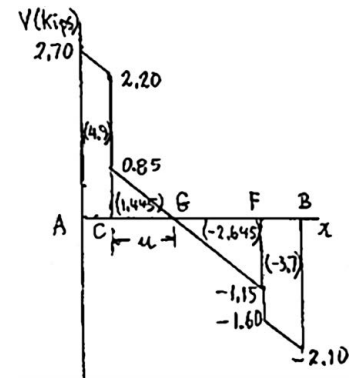
$$|V|_{\max} = 2.70 \text{ kips at A} \blacktriangleleft$$

Bending-moment diagram

At A: $M_A = 0$

Largest value occurs at G with

$$AG = 2 + 3.40 = 5.40 \text{ ft}$$

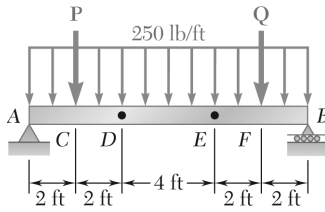


$$|M|_{\max} = 6.345 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

$$5.40 \text{ ft from A} \blacktriangleleft$$

Bending-moment diagram consists of 3 distinct arcs of parabolas.

PROBLEM 7.92*



Solve Problem 7.91 assuming that the bending moment was found to be +5.96 kip·ft at D and +6.84 kip·ft at E .

PROBLEM 7.91* The beam AB is subjected to the uniformly distributed load shown and to two unknown forces P and Q . Knowing that it has been experimentally determined that the bending moment is +6.10 kip·ft at D and +5.50 kip·ft at E , (a) determine P and Q , (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

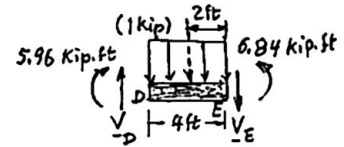
(a) Free body: Portion DE

$$+\circlearrowleft \sum M_E = 0: \quad 6.84 \text{ kip} \cdot \text{ft} - 5.96 \text{ kip} \cdot \text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

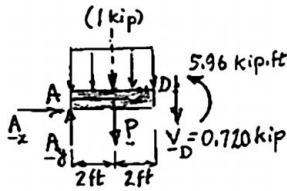
$$V_D = +0.720 \text{ kip}$$

$$+\uparrow \sum F_y = 0: \quad 0.720 \text{ kip} - 1 \text{ kip} - V_E = 0$$

$$V_E = -0.280 \text{ kip}$$



Free body: Portion AD



$$+\circlearrowleft \sum M_A = 0: \quad 5.96 \text{ kip} \cdot \text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.720 \text{ kip})(4 \text{ ft}) = 0$$

$$P = 0.540 \text{ kip} \quad \blacktriangleleft$$

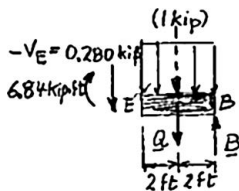
$$\sum F_x = 0: \quad A_x = 0$$

$$+\uparrow \sum F_y = 0: \quad A_y - 1 \text{ kip} - 0.540 \text{ kip} - 0.720 \text{ kip} = 0$$

$$A_y = +2.26 \text{ kips}$$

$$A = 2.26 \text{ kips} \quad \blacktriangleup$$

Free body: Portion EB



$$+\circlearrowleft \sum M_B = 0: \quad (0.280 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 6.84 \text{ kip} \cdot \text{ft} = 0$$

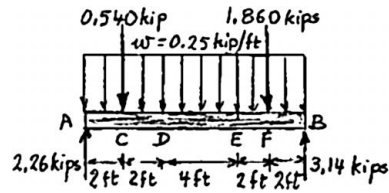
$$Q = 1.860 \text{ kips} \quad \blacktriangleleft$$

$$+\uparrow \sum F_y = 0: \quad B - 1.860 - 1 - 0.280 = 0$$

$$B = 3.14 \text{ kips} \quad \blacktriangleleft$$

PROBLEM 7.92* (Continued)

(b) Load diagram



Shear diagram

At A: $V_A = A = +2.26$ kips

To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$

$$0 - (1.22 \text{ kips}) = -(0.25 \text{ kip/ft})\mu$$

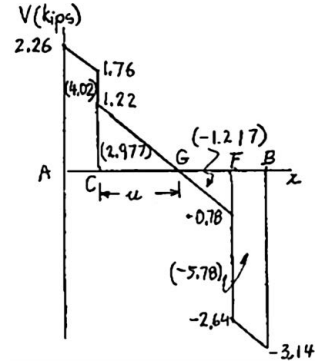
We next compute all areas

Bending-moment diagram

At A: $M_A = 0$

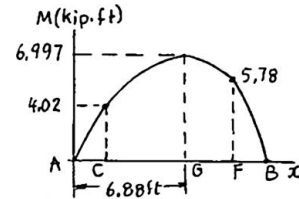
Largest value occurs at G with

$$AG = 2 + 4.88 = 6.88 \text{ ft}$$



$$\mu = 4.88 \text{ ft} \triangleleft$$

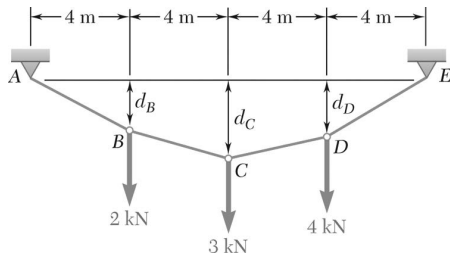
$$|V|_{\max} = 3.14 \text{ kips at B} \blacktriangleleft$$



$$|M|_{\max} = 6.997 \text{ kip}\cdot\text{ft} \blacktriangleleft$$

$$6.88 \text{ ft from A} \blacktriangleleft$$

Bending-moment diagram consists of 3 distinct arcs of parabolas.

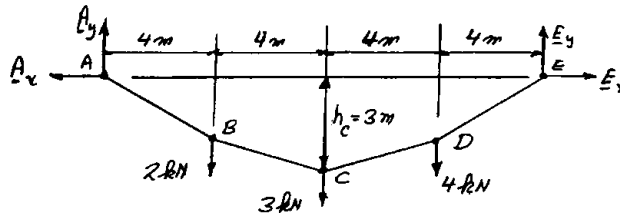


PROBLEM 7.93

Three loads are suspended as shown from the cable $ABCDE$. Knowing that $d_C = 3$ m, determine (a) the components of the reaction at E , (b) the maximum tension in the cable.

SOLUTION

(a)

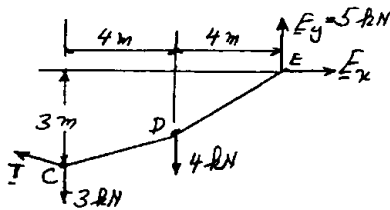


$$+\circlearrowleft \sum M_A = 0: E_y(16 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (3 \text{ kN})(8 \text{ m}) - (4 \text{ kN})(12 \text{ m}) = 0$$

$$E_y = +5 \text{ kN}$$

$$E_y = 5.00 \text{ kN} \uparrow \blacktriangleleft$$

Portion CDE :



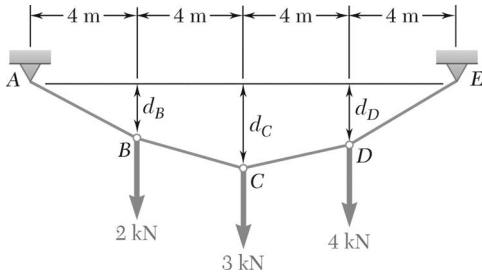
$$+\circlearrowleft \sum M_C = 0: (5 \text{ kN})(8 \text{ m}) - (4 \text{ kN})(4 \text{ m}) - E_x(3 \text{ m}) = 0$$

$$E_x = 8 \text{ kN} \quad E_x = 8.00 \text{ kN} \rightarrow \blacktriangleleft$$

(b) Maximum tension occurs in DE :

$$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{8^2 + 5^2}$$

$$T_m = 9.43 \text{ kN} \blacktriangleleft$$

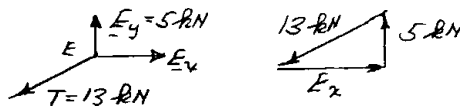


PROBLEM 7.94

Knowing that the maximum tension in cable $ABCDE$ is 13 kN, determine the distance d_C .

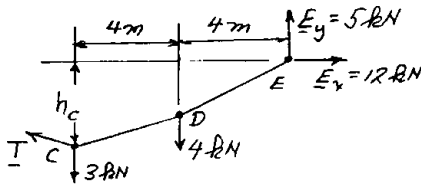
SOLUTION

Maximum tension of 13 kN occurs in DE . See solution of Problem 7.93 for the determination of $E_y = 5.00 \text{ kN} \uparrow$



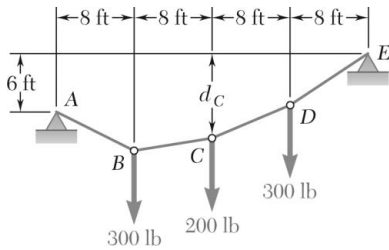
From force triangle $E_x^2 + 5^2 = 13^2$

Portion CDE : $E_x = 12 \text{ kN}$



$$+\circlearrowleft \sum M_C = 0: (5 \text{ kN})(8 \text{ m}) - (12 \text{ kN})h_C - (4 \text{ kN})(4 \text{ m}) = 0$$

$$h_C = 2.00 \text{ m} \quad \blacktriangleleft$$



PROBLEM 7.95

If $d_C = 8$ ft, determine (a) the reaction at A, (b) the reaction at E.

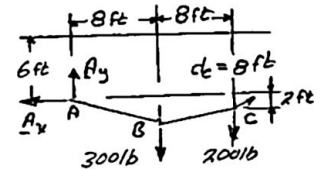
SOLUTION

Free body: Portion ABC

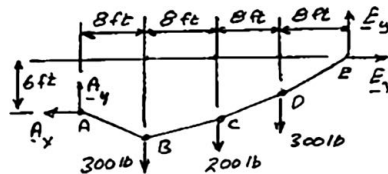
$$\begin{aligned} +\curvearrowright \Sigma M_C &= 0 \\ 2A_x - 16A_y + 300(8) &= 0 \end{aligned}$$

$$A_x = 8A_y - 1200$$

(1)



Free body: Entire cable



$$\begin{aligned} +\curvearrowright \Sigma M_E &= 0: +6A_x + 32A_y - (300 \text{ lb} + 200 \text{ lb} + 300 \text{ lb})16 \text{ ft} = 0 \\ 3A_x + 16A_y - 6400 &= 0 \end{aligned}$$

Substitute from Eq. (1):

$$3(8A_y - 1200) + 16A_y - 6400 = 0$$

$$A_y = 250 \text{ lb} \uparrow$$

Eq. (1)

$$A_x = 8(250) - 1200$$

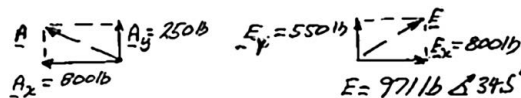
$$A_x = 800 \text{ lb} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: -A_x + E_x = 0 \quad -800 \text{ lb} + E_x = 0$$

$$E_x = 800 \text{ lb} \rightarrow$$

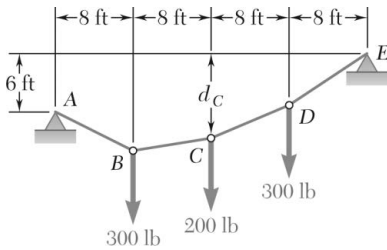
$$+\uparrow \Sigma F_y = 0: 250 + E_y - 300 - 200 - 300 = 0$$

$$E_y = 550 \text{ lb} \uparrow$$



(a) $A = 838 \text{ lb} \searrow 17.35^\circ \blacktriangleleft$

(b) $E = 971 \text{ lb} \nearrow 34.5^\circ \blacktriangleleft$

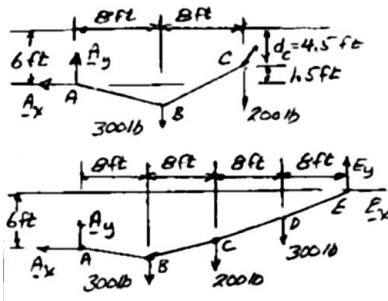


PROBLEM 7.96

If $d_C = 4.5$ ft, determine (a) the reaction at A, (b) the reaction at E.

SOLUTION

Free body: Portion ABC



$$\rightarrow \Sigma M_C = 0: -1.5A_x - 16A_y + 300 \times 8 = 0$$

$$A_x = \frac{(2400 - 16A_y)}{1.5} \quad (1)$$

Free body: Entire cable

$$\rightarrow \Sigma M_E = 0: +6A_x + 32A_y - (300 \text{ lb} + 200 \text{ lb} + 300 \text{ lb})16 \text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

Substitute from Eq. (1):

$$\frac{3(2400 - 16A_y)}{1.5} + 16A_y - 6400 = 0$$

$$A_y = -100 \text{ lb}$$

Thus A_y acts downward

$$A_y = 100 \text{ lb} \downarrow$$

Eq. (1)

$$A_x = \frac{(2400 - 16(-100))}{1.5} = 2667 \text{ lb}$$

$$A_x = 2667 \text{ lb} \leftarrow$$

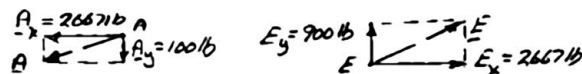
$$\rightarrow \Sigma F_x = 0: -A_x + E_x = 0 \quad -2667 + E_x = 0$$

$$E_x = 2667 \text{ lb} \rightarrow$$

$$\uparrow \Sigma F_y = 0: A_y + E_y - 300 - 200 - 300 = 0$$

$$-100 \text{ lb} + E_y - 800 \text{ lb} = 0$$

$$E_y = 900 \text{ lb} \uparrow$$



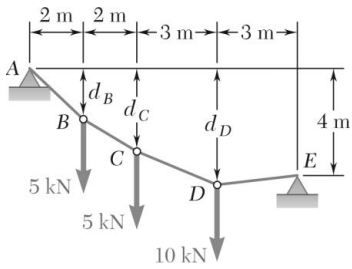
(a)

$$A = 2670 \text{ lb} \nearrow 2.10^\circ \leftarrow$$

(b)

$$E = 2810 \text{ lb} \nearrow 18.65^\circ \leftarrow$$

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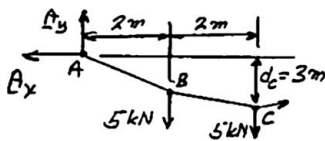


PROBLEM 7.97

Knowing that $d_C = 3$ m, determine (a) the distances d_B and d_D (b) the reaction at E.

SOLUTION

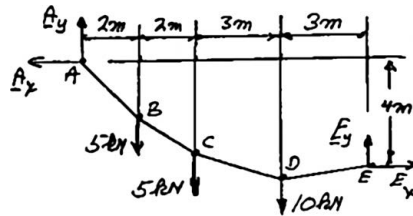
Free body: Portion ABC



$$+\circlearrowleft \Sigma M_C = 0: 3A_x - 4A_y + (5 \text{ kN})(2 \text{ m}) = 0$$

$$A_x = \frac{4}{3}A_y - \frac{10}{3} \quad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: 4A_x - 10A_y + (5 \text{ kN})(8 \text{ m}) + (5 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(3 \text{ m}) = 0$$

$$4A_x - 10A_y + 100 = 0$$

Substitute from Eq. (1):

$$4\left(\frac{4}{3}A_y - \frac{10}{3}\right) - 10A_y + 100 = 0$$

$$A_y = +18.571 \text{ kN} \quad A_y = 18.571 \text{ kN} \uparrow$$

$$\text{Eq. (1)} \quad A_x = \frac{4}{3}(18.571) - \frac{10}{3} = +21.429 \text{ kN} \quad A_x = 21.429 \text{ kN} \leftarrow$$

$$+\rightarrow \Sigma F_x = 0: -A_x + E_x = 0 \quad -21.429 + E_x = 0 \quad E_x = 21.429 \text{ kN} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: 18.571 \text{ kN} + E_y + 5 \text{ kN} + 5 \text{ kN} + 10 \text{ kN} = 0 \quad E_y = 1.429 \text{ kN} \uparrow$$

$$E_y = 1.429 \text{ kN} \uparrow \quad E_x = 21.429 \text{ kN} \rightarrow$$

(b)

$$E = 21.5 \text{ kN} \angle 3.81^\circ \nwarrow$$

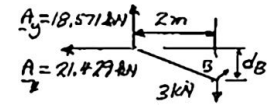
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PROBLEM 7.97 (Continued)

(a) Portion AB

$$+\circlearrowleft \Sigma M_B = 0: (18.571 \text{ kN})(2 \text{ m}) - (21.429 \text{ kN})d_B = 0$$

$$d_B = 1.733 \text{ m} \quad \blacktriangleleft$$



Portion DE

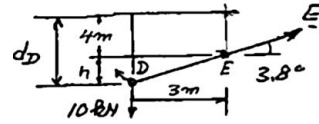
Geometry

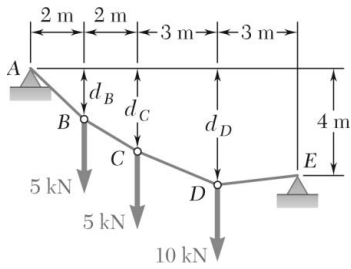
$$h = (3 \text{ m}) \tan 3.8^\circ$$

$$= 0.199 \text{ m}$$

$$d_D = 4 \text{ m} + 0.199 \text{ m}$$

$$d_D = 4.20 \text{ m} \quad \blacktriangleleft$$



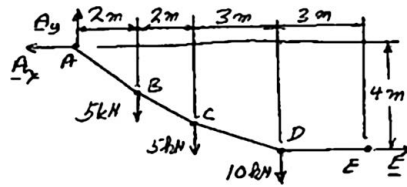


PROBLEM 7.98

Determine (a) distance d_C for which portion DE of the cable is horizontal, (b) the corresponding reactions at A and E .

SOLUTION

Free body: Entire cable



$$(b) \quad +\uparrow \Sigma F_y = 0: \quad A_y - 5 \text{ kN} - 5 \text{ kN} - 10 \text{ kN} = 0 \quad A_y = 20 \text{ kN} \uparrow$$

$$+\curvearrowright \Sigma M_A = 0: \quad E(4 \text{ m}) - (5 \text{ kN})(2 \text{ m}) - (5 \text{ kN})(4 \text{ m}) - (10 \text{ kN})(7 \text{ m}) = 0$$

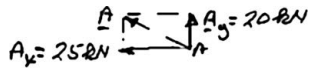
$$E = +25 \text{ kN}$$

$$E = 25.0 \text{ kN} \rightarrow \blacktriangleleft$$

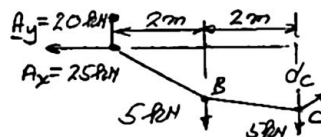
$$\Sigma F_x = 0: \quad -A_x + 25 \text{ kN} = 0$$

$$A_x = 25 \text{ kN} \leftarrow$$

$$A = 32.0 \text{ kN} \searrow 38.7^\circ \blacktriangleleft$$



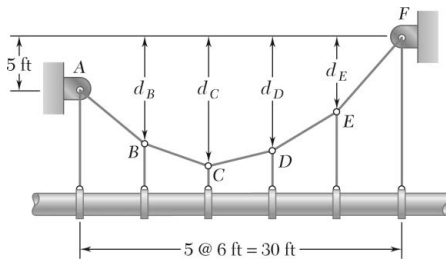
(a) Free body: Portion ABC



$$+\curvearrowright \Sigma M_C = 0: \quad (25 \text{ kN})d_C - (20 \text{ kN})(4 \text{ m}) + (5 \text{ kN})(2 \text{ m}) = 0$$

$$25d_C - 70 = 0$$

$$d_C = 2.80 \text{ m} \blacktriangleleft$$

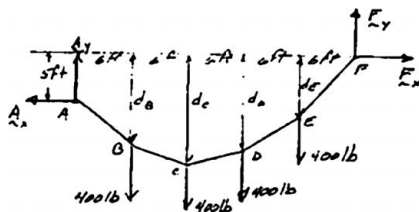


PROBLEM 7.99

An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (a) the maximum tension in the cable, (b) the distance d_D .

SOLUTION

FBD Cable: Hanger forces at A and F act on the supports, so A_y and F_y act on the cable.



$$\left(\sum M_F = 0: (6 \text{ ft} + 12 \text{ ft} + 18 \text{ ft} + 24 \text{ ft})(400 \text{ lb}) \right.$$

$$\left. - (30 \text{ ft})A_y - (5 \text{ ft})A_x = 0 \right.$$

$$A_x + 6A_y = 4800 \text{ lb} \quad (1)$$

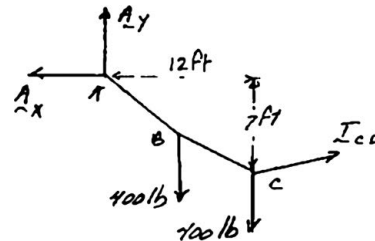
FBD ABC: $\left(\sum M_C = 0: (7 \text{ ft})A_x - (12 \text{ ft})A_y + (6 \text{ ft})(400 \text{ lb}) = 0 \right. \quad (2)$

Solving (1) and (2)

$$A_x = 800 \text{ lb} \leftarrow$$

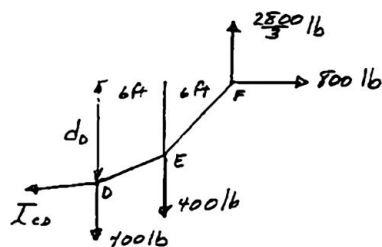
$$A_y = \frac{2000}{3} \text{ lb} \uparrow$$

From FBD Cable: $\rightarrow \sum F_x = 0: -800 \text{ lb} + F_x = 0$



$$F_x = 800 \text{ lb} \rightarrow$$

FBD DEF:



$$\uparrow \sum F_y = 0: \frac{2000}{3} \text{ lb} - 4(400 \text{ lb}) + F_y = 0$$

$$F_y = \frac{2800}{3} \text{ lb} \uparrow$$

Since $A_x = F_x$ and $F_y > A_y$,

$$T_{\max} = T_{EF} = \sqrt{(800 \text{ lb})^2 + \left(\frac{2800}{3} \text{ lb} \right)^2}$$

PROBLEM 7.99 (Continued)

(a) $T_{\max} = 1229.27 \text{ lb,}$

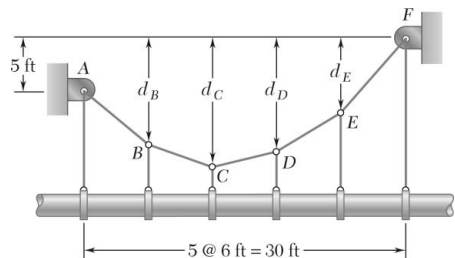
$T_{\max} = 1229 \text{ lb} \blacktriangleleft$

$$\left(\sum M_D = 0: (12 \text{ ft}) \left(\frac{2800}{3} \text{ lb} \right) - d_D (800 \text{ lb}) - (6 \text{ ft})(400 \text{ lb}) = 0 \right.$$

(b)

$d_D = 11.00 \text{ ft} \blacktriangleleft$

PROBLEM 7.100

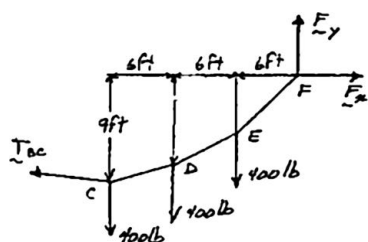


Solve Problem 7.99 assuming that $d_C = 9$ ft.

PROBLEM 7.99 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (a) the maximum tension in the cable, (b) the distance d_D .

SOLUTION

FBD CDEF:



$$\left(\sum M_C = 0: (18 \text{ ft})F_y - (9 \text{ ft})F_x - (6 \text{ ft} + 12 \text{ ft})(400 \text{ lb}) = 0 \right.$$

$$F_x - 2F_y = -800 \text{ lb} \quad (1)$$

FBD Cable:

$$\left(\sum M_A = 0: (30 \text{ ft})F_y - (5 \text{ ft})F_x \right.$$

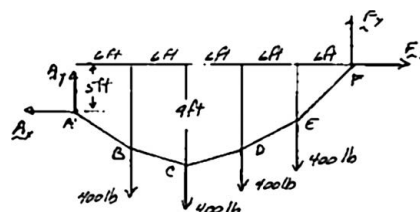
$$- (6 \text{ ft})(1 + 2 + 3 + 4)(400 \text{ lb}) = 0$$

$$F_x - 6F_y = -4800 \text{ lb} \quad (2)$$

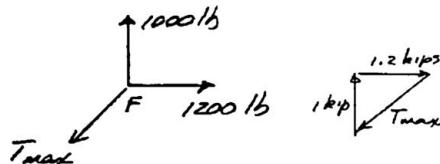
Solving (1) and (2),

$$F_x = 1200 \text{ lb} \rightarrow, \quad F_y = 1000 \text{ lb} \uparrow$$

$$\rightarrow \sum F_x = 0: -A_x + 1200 \text{ lb} = 0, \quad A_x = 1200 \text{ lb} \rightarrow$$



Point F:



$$\uparrow \sum F_y = 0: A_y + 1000 \text{ lb} - 4(400 \text{ lb}) = 0, \quad A_y = 600 \text{ lb} \uparrow$$

PROBLEM 7.100 (Continued)

Since

$$A_x = A_y \quad \text{and} \quad F_y > A_y, \quad T_{\max} = T_{EF}$$

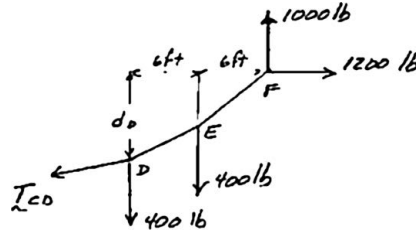
$$T_{\max} = \sqrt{(1 \text{ kip})^2 + (1.2 \text{ kips})^2}$$

(a)

$$T_{\max} = 1.562 \text{ kips} \quad \blacktriangleleft$$

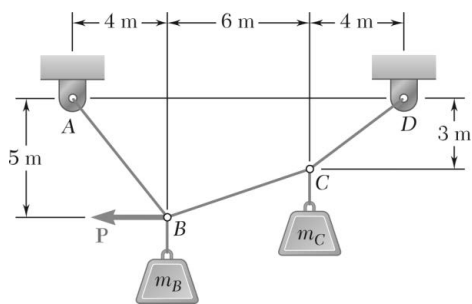
FBD DEF:

$$\left(\sum M_D = 0: (12 \text{ ft})(1000 \text{ lb}) - d_D(1200 \text{ lb}) \right. \\ \left. - (6 \text{ ft})(400 \text{ lb}) = 0 \right.$$



(b)

$$d_D = 8.00 \text{ ft} \quad \blacktriangleleft$$



PROBLEM 7.101

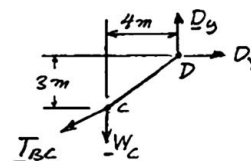
Knowing that $m_B = 70$ kg and $m_C = 25$ kg, determine the magnitude of the force \mathbf{P} required to maintain equilibrium.

SOLUTION

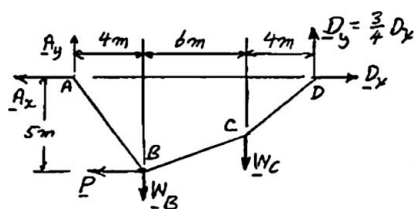
Free body: Portion CD

$$+\curvearrowright \Sigma M_C = 0: D_y(4 \text{ m}) - D_x(3 \text{ m}) = 0$$

$$D_y = \frac{3}{4} D_x$$

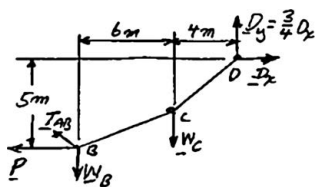


Free body: Entire cable



$$+\curvearrowright \Sigma M_A = 0: \frac{3}{4} D_x(14 \text{ m}) - W_B(4 \text{ m}) - W_C(10 \text{ m}) - P(5 \text{ m}) = 0 \quad (1)$$

Free body: Portion BCD



$$+\curvearrowright \Sigma M_B = 0: \frac{3}{4} D_x(10 \text{ m}) - D_x(5 \text{ m}) - W_C(6 \text{ m}) = 0$$

$$D_x = 2.4W_C \quad (2)$$

For

$$m_B = 70 \text{ kg} \quad m_C = 25 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2:$$

$$W_B = 70g \quad W_C = 25g$$

Eq. (2):

$$D_x = 2.4W_C = 2.4(25g) = 60g$$

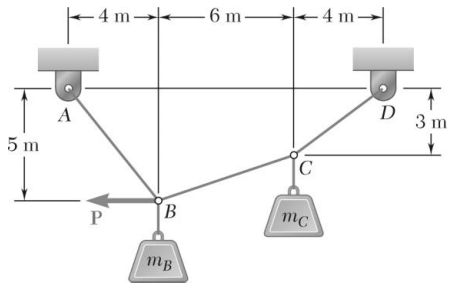
Eq. (1):

$$\frac{3}{4} 60g(14) - 70g(4) - 25g(10) - 5P = 0$$

$$100g - 5P = 0: \quad P = 20g$$

$$P = 20(9.81) = 196.2 \text{ N}$$

$$P = 196.2 \text{ N} \quad \blacktriangleleft$$



PROBLEM 7.102

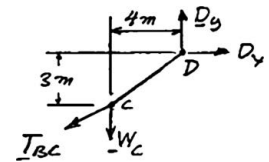
Knowing that $m_B = 18 \text{ kg}$ and $m_C = 10 \text{ kg}$, determine the magnitude of the force \mathbf{P} required to maintain equilibrium.

SOLUTION

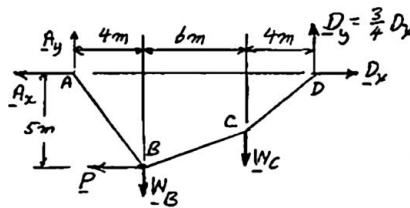
Free body: Portion CD

$$+\curvearrowright \Sigma M_C = 0: D_y(4 \text{ m}) - D_x(3 \text{ m}) = 0$$

$$D_y = \frac{3}{4} D_x$$

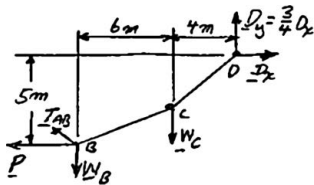


Free body: Entire cable



$$+\curvearrowright \Sigma M_A = 0: \frac{3}{4} D_x(14 \text{ m}) - W_B(4 \text{ m}) - W_C(10 \text{ m}) - P(5 \text{ m}) = 0 \quad (1)$$

Free body: Portion BCD



$$+\curvearrowright \Sigma M_B = 0: \frac{3}{4} D_x(10 \text{ m}) - D_x(5 \text{ m}) - W_C(6 \text{ m}) = 0$$

$$D_x = 2.4W_C \quad (2)$$

For

$$m_B = 18 \text{ kg} \quad m_C = 10 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2:$$

$$W_B = 18g \quad W_C = 10g$$

Eq. (2):

$$D_x = 2.4W_C = 2.4(10g) = 24g$$

Eq. (1):

$$\frac{3}{4} 24g(14) - (18g)(4) - (10g)(10) - 5P = 0$$

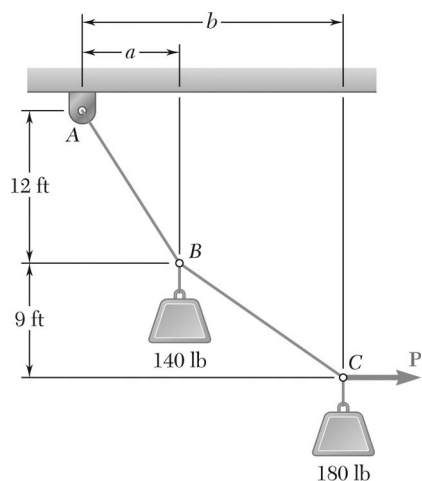
$$80g - 5P: P = 16g$$

$$P = 16(9.81) = 156.96 \text{ N}$$

$$P = 157.0 \text{ N} \quad \blacktriangleleft$$

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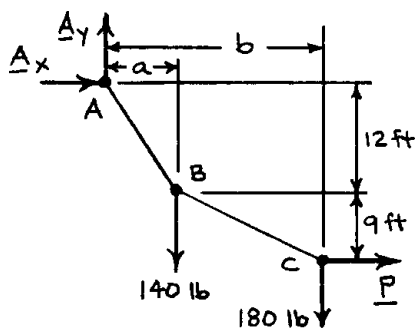
PROBLEM 7.103



Cable ABC supports two loads as shown. Knowing that $b = 21$ ft, determine (a) the required magnitude of the horizontal force P , (b) the corresponding distance a .

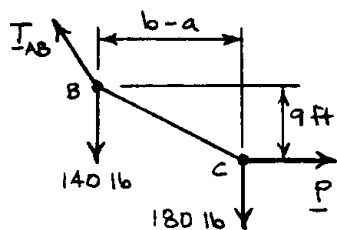
SOLUTION

Free body: ABC



$$+\circlearrowleft \Sigma M_A = 0: P(12 \text{ ft}) - (140 \text{ lb})a - (180 \text{ lb})b = 0 \quad (1)$$

Free body: BC



$$+\circlearrowleft \Sigma M_B = 0: P(9 \text{ ft}) - (180 \text{ lb})(b - a) = 0 \quad (2)$$

Data $b = 21$ ft

$$\text{Eq. (1): } 21P - 140a - 180(21) = 0$$

$$P = \frac{20}{3}a + 180 \quad (3)$$

$$\text{Eq. (2): } 9P - 180(21 - a) = 0$$

$$P = -20a + 420 \quad (4)$$

Equate (3) and (4) through P : $\frac{20}{3}a + 180 = -20a + 420$

$$(b) \quad a = 9.00 \text{ ft} \quad \blacktriangleleft$$

$$\text{Eq. (3): } P = \frac{20}{3}(9.00) + 180$$

$$(a) \quad P = 240 \text{ lb} \quad \blacktriangleleft$$

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PROBLEM 7.104

Cable ABC supports two loads as shown. Determine the distances a and b when a horizontal force \mathbf{P} of magnitude 200 lb is applied at A .

SOLUTION

Free body: ABC

$$+\curvearrowright \Sigma M_A = 0: P(12 \text{ ft}) - (140 \text{ lb})a - (180 \text{ lb})b = 0 \quad (1)$$

Free body: BC

$$+\curvearrowright \Sigma M_B = 0: P(9 \text{ ft}) - (180 \text{ lb})(b - a) = 0 \quad (2)$$

Data $P = 200 \text{ lb}$

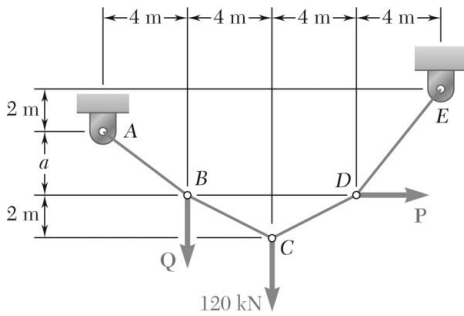
Eq. (1): $200(21) - 140a - 180b = 0$ (3)

Eq. (2): $200(9) + 180a - 180b = 0$ (4)

(3) - (4): $2400 - 320a = 0$ $a = 7.50 \text{ ft} \blacktriangleleft$

Eq. (2): $(200 \text{ lb})(9 \text{ ft}) - (180 \text{ lb})(b - 7.50 \text{ ft}) = 0$ $b = 17.50 \text{ ft} \blacktriangleleft$

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PROBLEM 7.105

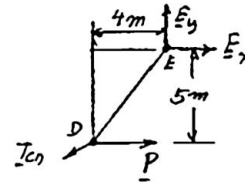
If $a = 3$ m, determine the magnitudes of P and Q required to maintain the cable in the shape shown.

SOLUTION

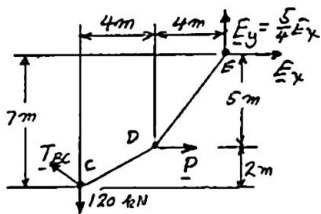
Free body: Portion DE

$$+\circlearrowleft \Sigma M_D = 0: E_y(4 \text{ m}) - E_x(5 \text{ m}) = 0$$

$$E_y = \frac{5}{4} E_x$$



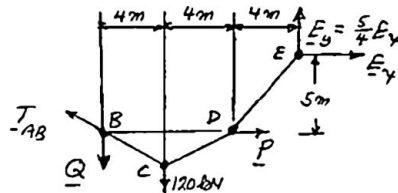
Free body: Portion CDE



$$+\circlearrowleft \Sigma M_C = 0: \frac{5}{4} E_x(8 \text{ m}) - E_x(7 \text{ m}) - P(2 \text{ m}) = 0$$

$$E_x = \frac{2}{3} P \quad (1)$$

Free body: Portion $BCDE$



$$+\circlearrowleft \Sigma M_B = 0: \frac{5}{4} E_x(12 \text{ m}) - E_x(5 \text{ m}) - (120 \text{ kN})(4 \text{ m}) = 0$$

$$10E_x - 480 = 0; \quad E_x = 48 \text{ kN}$$

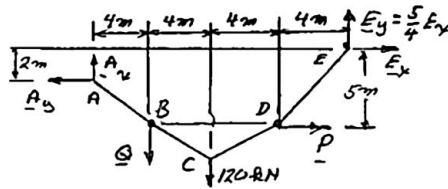
Eq. (1):

$$48 \text{ kN} = \frac{2}{3} P$$

$$P = 72.0 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 7.105 (Continued)

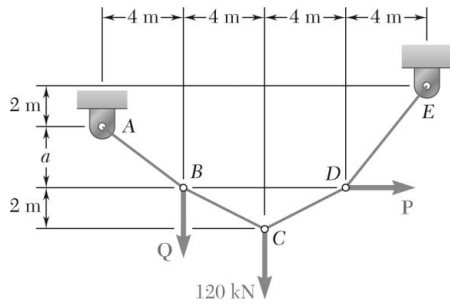
Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{5}{4} E_x (16 \text{ m}) - E_x (2 \text{ m}) + P(3 \text{ m}) - Q(4 \text{ m}) - (120 \text{ kN})(8 \text{ m}) = 0$$

$$(48 \text{ kN})(20 \text{ m} - 2 \text{ m}) + (72 \text{ kN})(3 \text{ m}) - Q(4 \text{ m}) - 960 \text{ kN} \cdot \text{m} = 0$$

$$4Q = 120 \quad Q = 30.0 \text{ kN} \quad \blacktriangleleft$$



PROBLEM 7.106

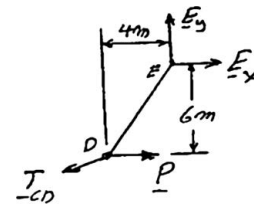
If $a = 4$ m, determine the magnitudes of P and Q required to maintain the cable in the shape shown.

SOLUTION

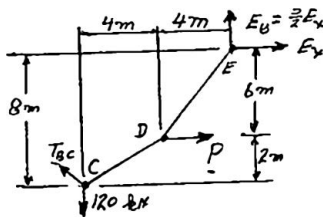
Free body: Portion DE

$$+\circlearrowleft \Sigma M_D = 0: E_y(4 \text{ m}) - E_x(6 \text{ m}) = 0$$

$$E_y = \frac{3}{2} E_x$$



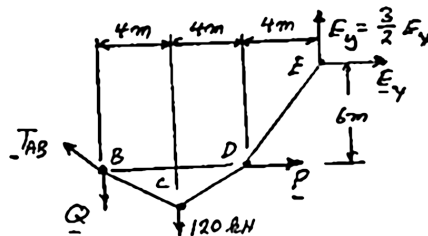
Free body: Portion CDE



$$+\circlearrowleft \Sigma M_C = 0: \frac{3}{2} E_x(8 \text{ m}) - E_x(8 \text{ m}) - P(2 \text{ m}) = 0$$

$$E_x = \frac{1}{2} P \quad (1)$$

Free body: Portion $BCDE$



$$+\circlearrowleft \Sigma M_B = 0: \frac{3}{2} E_x(12 \text{ m}) - E_x(6 \text{ m}) + (120 \text{ kN})(4 \text{ m}) = 0$$

$$12E_x = 480 \quad E_x = 40 \text{ kN}$$

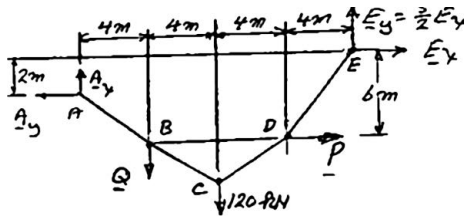
Eq (1):

$$E_x = \frac{1}{2} P; \quad 40 \text{ kN} = \frac{1}{2} P$$

$$P = 80.0 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 7.106 (Continued)

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{2} E_x (16 \text{ m}) - E_x (2 \text{ m}) + P(4 \text{ m}) - Q(4 \text{ m}) - (120 \text{ kN})(8 \text{ m}) = 0$$

$$(40 \text{ kN})(24 \text{ m} - 2 \text{ m}) + (80 \text{ kN})(4 \text{ m}) - Q(4 \text{ m}) - 960 \text{ kN} \cdot \text{m} = 0$$

$$4Q = 240$$

$$Q = 60.0 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 7.107

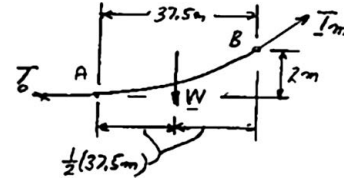
A transmission cable having a mass per unit length of 0.8 kg/m is strung between two insulators at the same elevation that are 75 m apart. Knowing that the sag of the cable is 2 m, determine (a) the maximum tension in the cable, (b) the length of the cable.

SOLUTION

$$w = (0.8 \text{ kg/m})(9.81 \text{ m/s}^2) = 7.848 \text{ N/m}$$

$$W = (7.848 \text{ N/m})(37.5 \text{ m})$$

$$W = 294.3 \text{ N}$$



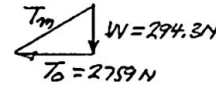
$$(a) \quad +\curvearrowright \Sigma M_B = 0: T_0(2 \text{ m}) - W\left(\frac{1}{2}37.5 \text{ m}\right) = 0$$

$$T_0(2 \text{ m}) - (294.3 \text{ N})\frac{1}{2}(37.5 \text{ m}) = 0$$

$$T_0 = 2759 \text{ N}$$

$$T_m^2 = (294.3 \text{ N})^2 + (2759 \text{ N})^2$$

$$T_m = 2770 \text{ N} \quad \blacktriangleleft$$



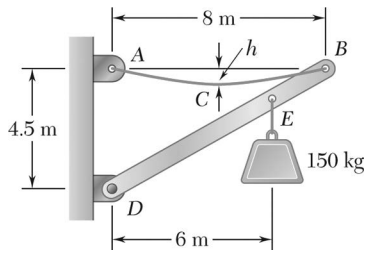
$$(b) \quad s_B = x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 + \dots \right]$$

$$= 37.5 \text{ m} \left[1 + \frac{2}{3} \left(\frac{2 \text{ m}}{37.5 \text{ m}} \right)^2 + \dots \right]$$

$$= 37.57 \text{ m}$$

$$\text{Length} = 2s_B = 2(37.57 \text{ m})$$

$$\text{Length} = 75.14 \text{ m} \quad \blacktriangleleft$$

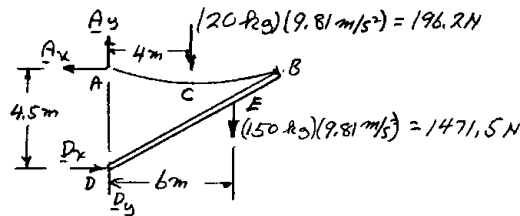


PROBLEM 7.108

The total mass of cable ACB is 20 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine (a) the sag h , (b) the slope of the cable at A .

SOLUTION

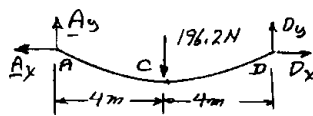
Free body: Entire frame



$$+\circlearrowleft \sum M_D = 0: A_x(4.5 \text{ m}) - (196.2 \text{ N})(4 \text{ m}) - (1471.5 \text{ N})(6 \text{ m}) = 0$$

$$A_x = 2136.4 \text{ N}$$

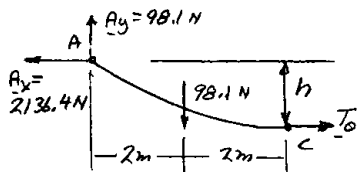
Free body: Entire cable



$$+\circlearrowleft \sum M_D = 0: A_y(8 \text{ m}) - (196.2 \text{ N})(4 \text{ m}) = 0$$

$$A_y = 98.1 \text{ N}$$

(a) Free body: Portion AC



$$\sum F_x = 0: T_0 = A_x = 2136.4 \text{ N}$$

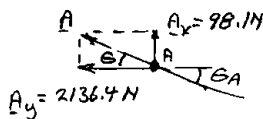
$$+\circlearrowleft \sum M_A = 0: T_0 h - (98.1 \text{ N})(2 \text{ m}) = 0$$

$$(2136.4 \text{ N})h - 196.2 \text{ N} \cdot \text{m} = 0$$

$$h = 0.09183 \text{ m}$$

$$h = 91.8 \text{ mm} \blacktriangleleft$$

(b)



$$\tan \theta_A = \frac{A_x}{A_y} = \frac{98.1 \text{ N}}{2136.4 \text{ N}}$$

$$\tan \theta_A = 0.045918$$

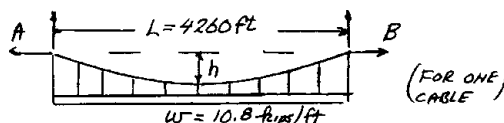
$$\theta_A = 2.629^\circ$$

$$\theta_A = 2.63^\circ \blacktriangleleft$$

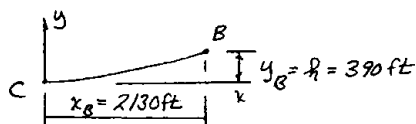
PROBLEM 7.109

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The uniform load supported by each cable is $w = 10.8$ kips/ft along the horizontal. Knowing that the span L is 4260 ft and that the sag h is 390 ft, determine (a) the maximum tension in each cable, (b) the length of each cable.

SOLUTION



(a)



At B:
$$y_B = \frac{wx_B^2}{2T_0}$$

$$390 \text{ ft} = \frac{(10.8 \text{ kips/ft})(2130 \text{ ft})^2}{2T_0} \quad T_0 = 62,819 \text{ kips}$$

$$\begin{aligned} T_m &= \sqrt{T_0^2 + w^2 x_B^2} \\ &= \sqrt{(62,819 \text{ kips})^2 + (10.8 \text{ kips/ft})^2 (2130 \text{ ft})^2} \\ &= \sqrt{62,819^2 + (23,004)^2} = 66,898 \text{ kips} \quad T_m = 66,900 \text{ kips} \quad \blacktriangleleft \end{aligned}$$

(b) Length of cable

$$\begin{aligned} s_B &= x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 - \frac{2}{5} \left(\frac{y_B}{x_B} \right)^4 \right] \\ &= (2130 \text{ ft}) \left[1 + \frac{2}{3} \left(\frac{390 \text{ ft}}{2130 \text{ ft}} \right)^2 - \frac{2}{5} \left(\frac{390 \text{ ft}}{2130 \text{ ft}} \right)^4 \right] = 2176.65 \text{ ft} \end{aligned}$$

Total length: $2s_B = 2(2176.65 \text{ ft}) = 4353.3 \text{ ft}$

$L = 4353 \text{ ft} \quad \blacktriangleleft$

PROBLEM 7.110

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allows for the effect of extreme temperature changes that cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

Eq. 7.10:

$$s_B = x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 - \frac{2}{5} \left(\frac{y_B}{x_B} \right)^4 + \dots \right]$$

Winter:

$$y_B = h = 386 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130) \left[1 + \frac{2}{3} \left(\frac{386}{2130} \right)^2 - \frac{2}{5} \left(\frac{386}{2130} \right)^4 + \dots \right] = 2175.715 \text{ ft}$$

Summer:

$$y_B = h = 394 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130) \left[1 + \frac{2}{3} \left(\frac{394}{2130} \right)^2 - \frac{2}{5} \left(\frac{394}{2130} \right)^4 + \dots \right] = 2177.59 \text{ ft}$$

$$\Delta = 2(\Delta s_B) = 2(2177.59 \text{ ft} - 2175.715 \text{ ft}) = 2(1.875 \text{ ft})$$

Change in length = 3.75 ft ◀

PROBLEM 7.111

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (a) the maximum tension in each cable, (b) the length of each cable.

SOLUTION

Eq. (7.8) Page 386:

At B :

$$y_B = \frac{wx_B^2}{2T_0}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(11.1 \text{ kip/ft})(2075 \text{ ft})^2}{2(464 \text{ ft})}$$

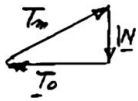
$$T_0 = 51.500 \text{ kips}$$

$$W = wx_B = (11.1 \text{ kips/ft})(2075 \text{ ft}) = 23.033 \text{ kips}$$

$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(51.500 \text{ kips})^2 + (23.033 \text{ kips})^2}$$

$$T_m = 56,400 \text{ kips} \quad \blacktriangleleft$$

(a)



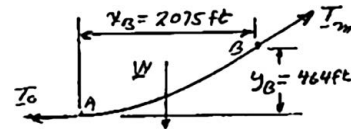
(b)

$$s_B = x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 - \frac{2}{5} \left(\frac{y_B}{x_B} \right)^4 + \dots \right] \quad \frac{y_B}{x_B} = \frac{464 \text{ ft}}{2075 \text{ ft}} = 0.22361$$

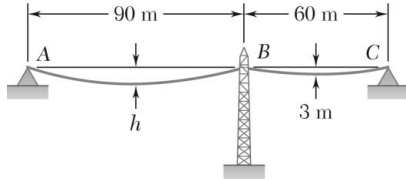
$$s_B = (2075 \text{ ft}) \left[1 + \frac{2}{3} (0.22361)^2 - \frac{2}{5} (0.22361)^4 + \dots \right] = 2142.1 \text{ ft}$$

$$\text{Length} = 2s_B = 2(2142.1 \text{ ft})$$

$$\text{Length} = 4284 \text{ ft} \quad \blacktriangleleft$$



PROBLEM 7.112



Two cables of the same gauge are attached to a transmission tower at B . Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at B is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m , determine (a) the required sag h , (b) the maximum tension in each cable.

SOLUTION

$$W = wx_B$$

$$+\circlearrowleft \Sigma M_B = 0: T_0 y_B - (wx_B) \frac{y_B}{2} = 0$$

(a) Horiz. comp. = $T_0 = \frac{wx_B^2}{2y_B}$

Cable AB: $x_B = 45 \text{ m}$

$$T_0 = \frac{w(45 \text{ m})^2}{2h}$$

Cable BC: $x_B = 30 \text{ m}, y_B = 3 \text{ m}$

$$T_0 = \frac{w(30 \text{ m})^2}{2(3 \text{ m})}$$

Equate $T_0 = T_0$ $\frac{w(45 \text{ m})^2}{2h} = \frac{w(30 \text{ m})^2}{2(3 \text{ m})}$

$$h = 6.75 \text{ m} \quad \blacktriangleleft$$

(b) $T_m^2 = T_0^2 + W^2$

Cable AB: $w = (0.4 \text{ kg/m})(9.81 \text{ m/s}^2) = 3.924 \text{ N/m}$

$$x_B = 45 \text{ m}, y_B = h = 6.75 \text{ m}$$

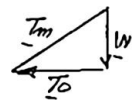
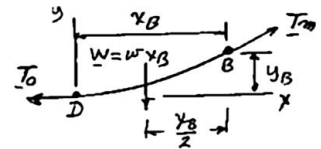
$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(45 \text{ m})^2}{2(6.75 \text{ m})} = 588.6 \text{ N}$$

$$W = wx_B = (3.924 \text{ N/m})(45 \text{ m}) = 176.58 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (176.58 \text{ N})^2$$

For AB:

$$T_m = 615 \text{ N} \quad \blacktriangleleft$$



PROBLEM 7.112 (Continued)

Cable BC:

$$x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(30 \text{ m})^2}{2(3 \text{ m})} = 588.6 \text{ N} \quad (\text{Checks})$$

$$W = wx_B = (3.924 \text{ N/m})(30 \text{ m}) = 117.72 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (117.72 \text{ N})^2$$

For BC:

$$T_m = 600 \text{ N} \quad \blacktriangleleft$$

PROBLEM 7.113

A 50.5-m length of wire having a mass per unit length of 0.75 kg/m is used to span a horizontal distance of 50 m. Determine (a) the approximate sag of the wire, (b) the maximum tension in the wire. [Hint: Use only the first two terms of Eq. (7.10).]

SOLUTION

First two terms of Eq. 7.10

$$(a) \quad s_B = \frac{1}{2}(50.5 \text{ m}) = 25.25 \text{ m},$$

$$x_B = \frac{1}{2}(50 \text{ m}) = 25 \text{ m}$$

$$y_B = h$$

$$s_B = x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 \right]$$

$$25.25 \text{ m} = 25 \text{ m} \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 \right]$$

$$\left(\frac{y_B}{x_B} \right)^2 = 0.01 \left(\frac{3}{2} \right)^2 = \sqrt{0.015}$$

$$\frac{y_B}{x_B} = 0.12247$$

$$\frac{h}{25 \text{ m}} = 0.12247$$

$$h = 3.0619 \text{ m}$$

$$h = 3.06 \text{ m} \quad \blacktriangleleft$$

(b) Free body: Portion CB

$$w = (0.75 \text{ kg/m})(9.81 \text{ m}) = 7.3575 \text{ N/m}$$

$$W = s_B w = (25.25 \text{ m})(7.3575 \text{ N/m})$$

$$W = 185.78 \text{ N}$$

$$+\curvearrowright \Sigma M_0 = 0: T_0(3.0619 \text{ m}) - (185.78 \text{ N})(12.5 \text{ m}) = 0$$

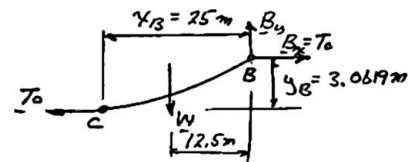
$$T_0 = 758.4 \text{ N}$$

$$B_x = T_0 = 758.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: B_y - 185.78 \text{ N} = 0 \quad B_y = 185.78 \text{ N}$$

$$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4 \text{ N})^2 + (185.78 \text{ N})^2}$$

$$T_m = 781 \text{ N} \quad \blacktriangleleft$$



PROBLEM 7.114

A cable of length $L + \Delta$ is suspended between two points that are at the same elevation and a distance L apart. (a) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ . (b) If $L = 100$ ft and $\Delta = 4$ ft, determine the approximate sag. [Hint: Use only the first two terms of Eq. (7.10).]

SOLUTION

Eq. 7.10

(First two terms)

$$(a) \quad s_B = x_B \left[1 + \frac{2}{3} \left(\frac{y_B}{x_B} \right)^2 \right]$$

$$x_B = L/2$$

$$s_B = \frac{1}{2}(L + \Delta)$$

$$y_B = h$$

$$\frac{1}{2}(L + \Delta) = \frac{L}{2} \left[1 + \frac{2}{3} \left(\frac{h}{L/2} \right)^2 \right]$$

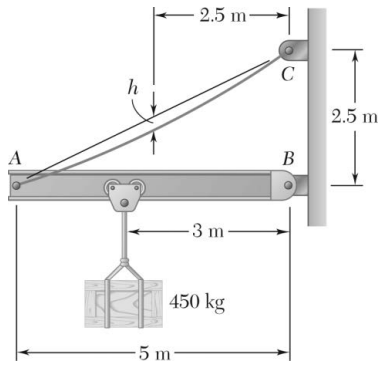
$$\frac{\Delta}{2} = \frac{4}{3} \frac{h^2}{L}; \quad h^2 = \frac{3}{8} L \Delta;$$

$$h = \sqrt{\frac{3}{8} L \Delta} \quad \blacktriangleleft$$

(b)

$$L = 100 \text{ ft}, \quad h = 4 \text{ ft}, \quad h = \sqrt{\frac{3}{8} (100)(4)};$$

$$h = 12.25 \text{ ft} \quad \blacktriangleleft$$



PROBLEM 7.115

The total mass of cable AC is 25 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag h and the slope of the cable at A and C.

SOLUTION

Cable: $m = 25 \text{ kg}$
 $W = 25(9.81)$
 $= 245.25 \text{ N}$

Block: $m = 450 \text{ kg}$
 $W = 4414.5 \text{ N}$

$$+\circlearrowleft \Sigma M_B = 0: (245.25)(2.5) + (4414.5)(3) - C_x(2.5) = 0$$

$$C_x = 5543 \text{ N}$$

$$\Sigma F_x = 0: A_x = C_x = 5543 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: C_y(5) - (5543)(2.5) - (245.25)(2.5) = 0$$

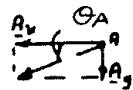
$$C_y = 2894 \text{ N} \uparrow$$

$$+\uparrow \Sigma F_y = 0: C_y - A_y - 245.25 \text{ N} = 0$$

$$2894 \text{ N} - A_y - 245.25 \text{ N} = 0 \quad A_y = 2649 \text{ N} \downarrow$$

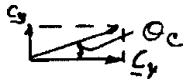
Point A: $\tan \theta_A = \frac{A_y}{A_x} = \frac{2649}{5543} = 0.4779;$

$$\theta_A = 25.5^\circ \blacktriangleleft$$

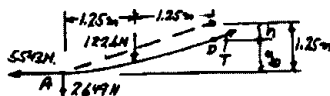


Point C: $\tan \theta_C = \frac{C_y}{C_x} = \frac{2894}{5543} = 0.5221;$

$$\theta_C = 27.6^\circ \blacktriangleleft$$



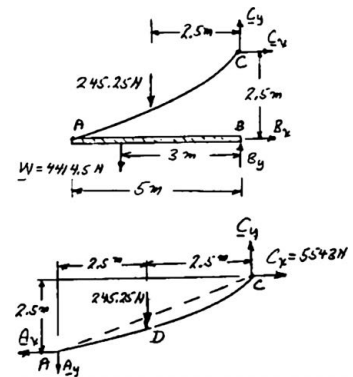
Free body: Half cable $W = (12.5 \text{ kg})g = 122.6$

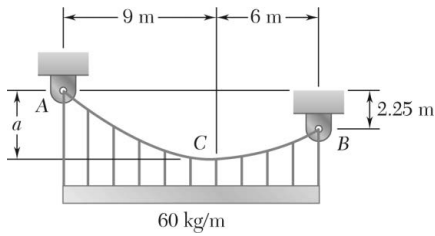


$$+\circlearrowleft \Sigma M_D = 0: (122.6 \text{ N})(1.25 \text{ m}) + (2649 \text{ N})(2.5 \text{ m}) - (5543 \text{ N})y_d = 0$$

$$y_d = 1.2224 \text{ m}; \text{ sag} = h = 1.25 \text{ m} - 1.2224 \text{ m}$$

$$h = 0.0276 \text{ m} = 27.6 \text{ mm} \blacktriangleleft$$





PROBLEM 7.116

Cable ACB supports a load uniformly distributed along the horizontal as shown. The lowest Point C is located 9 m to the right of A . Determine (a) the vertical distance a , (b) the length of the cable, (c) the components of the reaction at A .

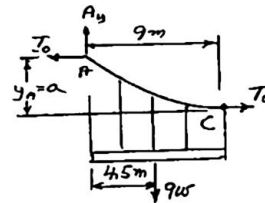
SOLUTION

Free body: Portion AC

$$+\uparrow \Sigma F_y = 0: A_y - 9w = 0$$

$$A_y = 9w \uparrow$$

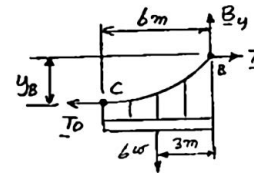
$$+\curvearrowright \Sigma M_A = 0: T_0 a - (9w)(4.5 \text{ m}) = 0 \quad (1)$$



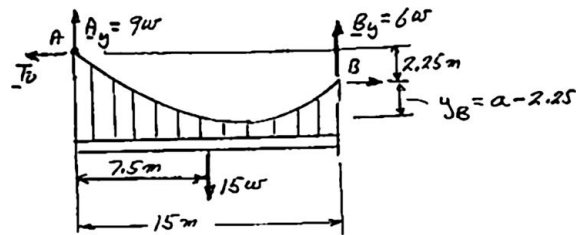
Free body: Portion CB

$$+\uparrow \Sigma F_y = 0: B_y - 6w = 0$$

$$B_y = 6w \uparrow$$



Free body: Entire cable



$$+\curvearrowright \Sigma M_A = 0: 15w(7.5 \text{ m}) - 6w(15 \text{ m}) - T_0(2.25 \text{ m}) = 0$$

$$(a) \quad T_0 = 10w$$

$$\text{Eq. (1):} \quad T_0 a - (9w)(4.5 \text{ m}) = 0$$

$$10wa = (9w)(4.5) = 0$$

$$a = 4.05 \text{ m} \quad \blacktriangleleft$$

PROBLEM 7.116 (Continued)

(b) Length = AC + CB

Portion AC: $x_A = 9 \text{ m}, \quad y_A = a = 4.05 \text{ m}; \quad \frac{y_A}{x_A} = \frac{4.05}{9} = 0.45$

$$s_{AC} = x_B \left[1 + \frac{2}{3} \left(\frac{y_A}{x_A} \right)^2 - \frac{2}{5} \left(\frac{y_B}{x_A} \right)^4 + \dots \right]$$

$$s_{AC} = 9 \text{ m} \left(1 + \frac{2}{3} 0.45^2 - \frac{2}{5} 0.45^4 + \dots \right) = 10.067 \text{ m}$$

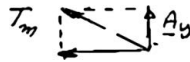
Portion CB: $x_B = 6 \text{ m}, \quad y_B = 4.05 - 2.25 = 1.8 \text{ m}; \quad \frac{y_B}{x_B} = 0.3$

$$s_{CB} = 6 \text{ m} \left(1 + \frac{2}{3} 0.3^2 - \frac{2}{5} 0.3^4 + \dots \right) = 6.341 \text{ m}$$

Total length = 10.067 m + 6.341 m

Total length = 16.41 m ◀

(c) Components of reaction at A.

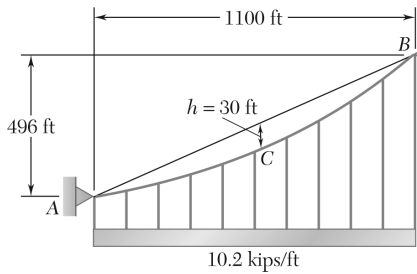


$$A_y = 9w = 9(60 \text{ kg/m})(9.81 \text{ m/s}^2) = 5297.4 \text{ N}$$

$$A_x = T_0 = 10w = 10(60 \text{ kg/m})(9.81 \text{ m/s}^2) = 5886 \text{ N}$$

$A_x = 5890 \text{ N} \leftarrow \blacktriangleleft$

$A_y = 5300 \text{ N} \uparrow \blacktriangleleft$

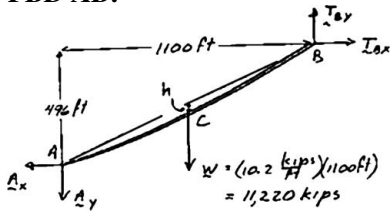


PROBLEM 7.117

Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (a) the maximum tension in each cable, (b) the slope at B .

SOLUTION

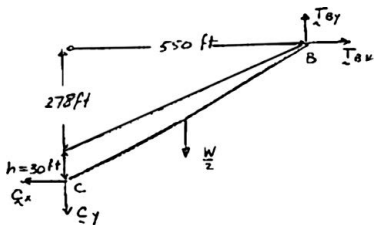
FBD AB:



$$\left(\sum M_A = 0: (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0 \right.$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \quad (1)$$

FBD CB:



$$\left(\sum M_C = 0: (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0 \right.$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \quad (2)$$

Solving (1) and (2)

$$T_{By} = 28,798 \text{ kips}$$

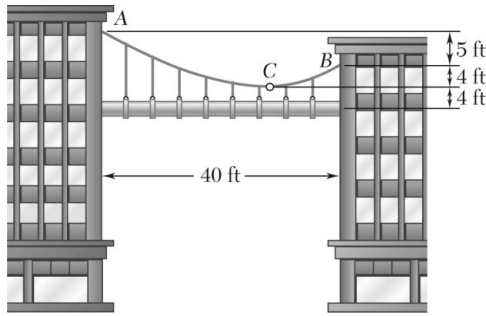
$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{\max} = T_B = \sqrt{T_{Bx}^2 + T_{By}^2} \quad \tan \theta_B = \frac{T_{By}}{T_{Bx}}$$

So that

$$(a) \quad T_{\max} = 58,900 \text{ kips} \quad \blacktriangleleft$$

$$(b) \quad \theta_B = 29.2^\circ \quad \blacktriangleleft$$



PROBLEM 7.118

A steam pipe weighting 45 lb/ft that passes between two buildings 40 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable system is equivalent to a uniformly distributed loading of 5 lb/ft, determine (a) the location of the lowest Point C of the cable, (b) the maximum tension in the cable.

SOLUTION

Note:

$$x_B - x_A = 40 \text{ ft}$$

or

$$x_A = x_B - 40 \text{ ft}$$

(a) Use Eq. 7.8

Point A:

$$y_A = \frac{wx_A^2}{2T_0}; \quad 9 = \frac{w(x_B - 40)^2}{2T_0} \quad (1)$$

Point B:

$$y_B = \frac{wx_B^2}{2T_0}; \quad 4 = \frac{wx_B^2}{2T_0} \quad (2)$$

Dividing (1) by (2):

$$\frac{9}{4} = \frac{(x_B - 40)^2}{x_B^2}; \quad x_B = 16 \text{ ft}$$

Point C is 16.00 ft to left of B ◀

(b) Maximum slope and thus T_{\max} is at A

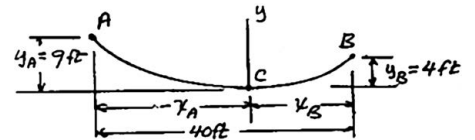
$$x_A = x_B - 40 = 16 - 40 = -24 \text{ ft}$$

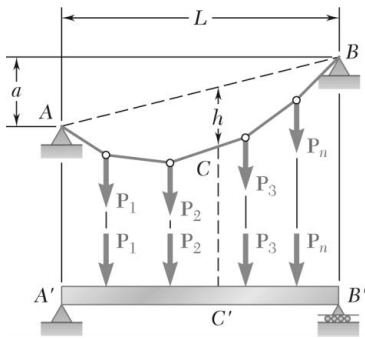
$$y_A = \frac{wx_A^2}{2T_0}; \quad 9 \text{ ft} = \frac{(50 \text{ lb/ft})(-24 \text{ ft})^2}{2T_0}; \quad T_0 = 1600 \text{ lb}$$

$$W_{AC} = (50 \text{ lb/ft})(24 \text{ ft}) = 1200 \text{ lb}$$

$$\begin{array}{l} T_{\max} = A \leftarrow \rightarrow A_y = W_{AC} = 1200 \text{ lb} \\ \leftarrow \rightarrow A_x = T_0 = 1600 \text{ lb} \end{array}$$

$$T_{\max} = 2000 \text{ lb} \quad \blacktriangleleft$$





PROBLEM 7.119*

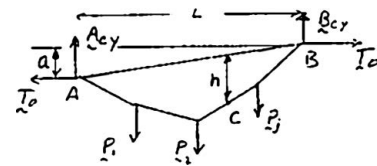
A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product $T_0 h$, where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between Point C and the chord joining the points of support A and B .

SOLUTION

$$\left(\sum M_B = 0: LA_{Cy} + aT_0 - \sum M_B^{\text{loads}} \right) = 0 \quad (1)$$

FBD Cable:

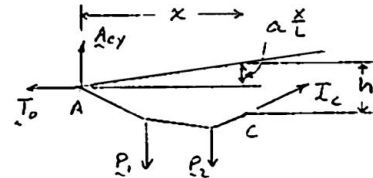
(Where $\sum M_B^{\text{loads}}$ includes all applied loads)



$$\left(\sum M_C = 0: xA_{Cy} - \left(h - a \frac{x}{L} \right) T_0 - \sum M_C^{\text{left}} \right) = 0 \quad (2)$$

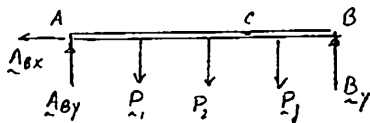
FBD AC:

(Where $\sum M_C^{\text{left}}$ includes all loads left of C)



$$\frac{x}{L}(1) - (2): hT_0 - \frac{x}{L} \sum M_B^{\text{loads}} + \sum M_C^{\text{left}} = 0 \quad (3)$$

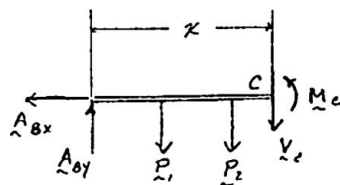
FBD Beam:



$$\left(\sum M_B = 0: LA_{By} - \sum M_B^{\text{loads}} \right) = 0 \quad (4)$$

$$\left(\sum M_C = 0: xA_{By} - \sum M_C^{\text{left}} - M_C \right) = 0 \quad (5)$$

FBD AC:

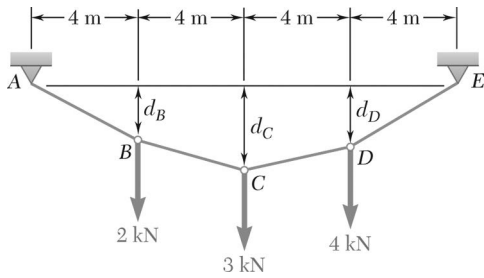


$$\frac{x}{L}(4) - (5): -\frac{x}{L} \sum M_B^{\text{loads}} + \sum M_C^{\text{left}} + M_C = 0 \quad (6)$$

Comparing (3) and (6)

$$M_C = hT_0 \quad \text{Q.E.D.}$$

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PROBLEM 7.120

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.94 Knowing that the maximum tension in cable $ABCDE$ is 13 kN, determine the distance d_C .

SOLUTION

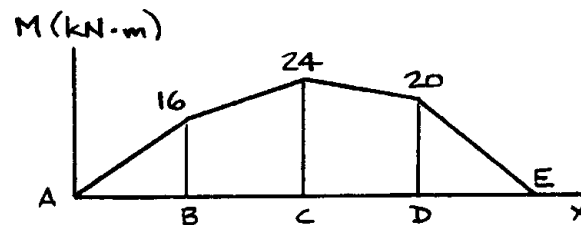
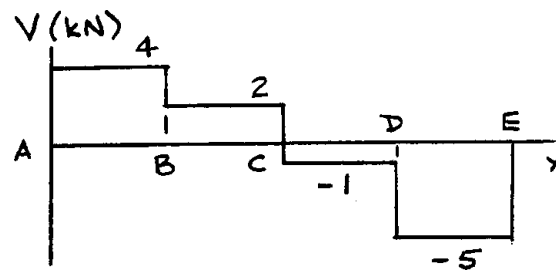
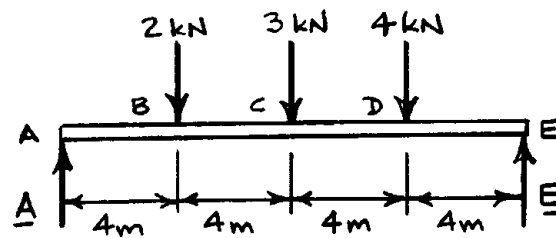
Free body: beam AE

$$+\circlearrowleft \Sigma M_E = 0: -A(16) + 2(12) + 3(8) + 4(4) = 0$$

$$A = 4 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: 4 - 2 - 3 - 4 + E = 0$$

$$E = 5 \text{ kN} \uparrow$$

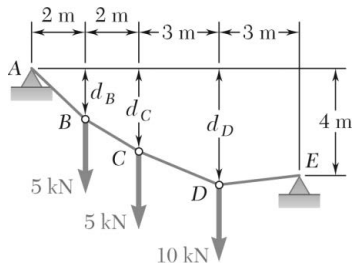


At E: $T_m^2 = T_0^2 + E^2$ $13^2 = T_0^2 + 5^2$ $T_0 = 12 \text{ kN}$

At C: $M_C = T_0 h_C$; $24 \text{ kN} \cdot \text{m} = (12 \text{ kN})h_C$

$$h_C = d_C = 2.00 \text{ m}$$

2.00 m ◀

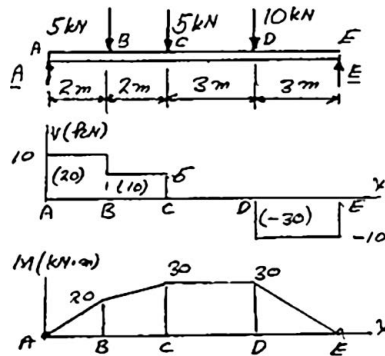


PROBLEM 7.121

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.97 (a) Knowing that $d_C = 3$ m, determine the distances d_B and d_D .

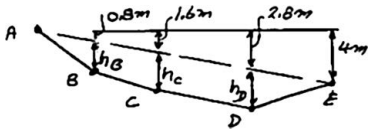
SOLUTION



$$+\circlearrowleft \sum M_B = 0: A(10 \text{ m}) - (5 \text{ kN})(8 \text{ m}) - (5 \text{ kN})(6 \text{ m}) - (10 \text{ kN})(3 \text{ m}) = 0$$

$$A = 10 \text{ kN}$$

Geometry:



$$d_C = 1.6 \text{ m} + h_C$$

$$3 \text{ m} = 1.6 \text{ m} + h_C$$

$$h_C = 1.4 \text{ m}$$

Since $M = T_0 h$, h is proportional to M , thus

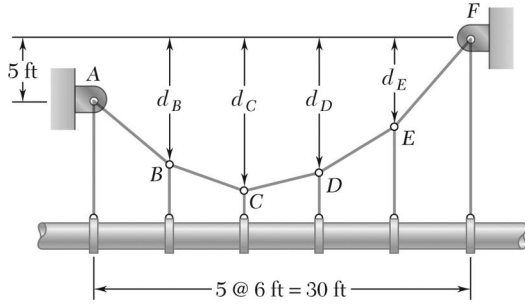
$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20 \text{ kN} \cdot \text{m}} = \frac{1.4 \text{ m}}{30 \text{ kN} \cdot \text{m}} = \frac{h_D}{30 \text{ kN} \cdot \text{m}}$$

$$h_B = 1.4 \left(\frac{20}{30} \right) = 0.9333 \text{ m} \quad \parallel \quad h_D = 1.4 \left(\frac{30}{30} \right) = 1.4 \text{ m}$$

$$d_B = 0.8 \text{ m} + 0.9333 \text{ m} \quad \parallel \quad d_D = 2.8 \text{ m} + 1.4 \text{ m}$$

$$d_B = 1.733 \text{ m} \quad \blacktriangleleft$$

$$d_D = 4.20 \text{ m} \quad \blacktriangleleft$$

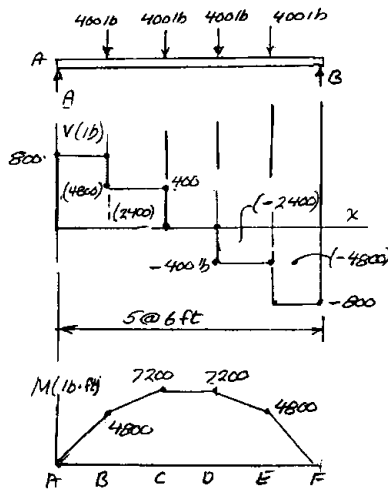


PROBLEM 7.122

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

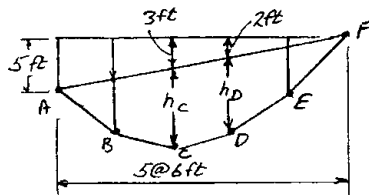
PROBLEM 7.99 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (a) the maximum tension in the cable.

SOLUTION



$$A = B = \frac{1}{2}(4 \times 400) = 800 \text{ lb}$$

Geometry



$$\begin{aligned} d_C &= h_C + 3 \text{ ft} \\ 12 \text{ ft} &= h_C + 3 \text{ ft} \\ h_C &= 9 \text{ ft} \\ d_D &= h_D + 2 \text{ ft} \end{aligned} \tag{1}$$

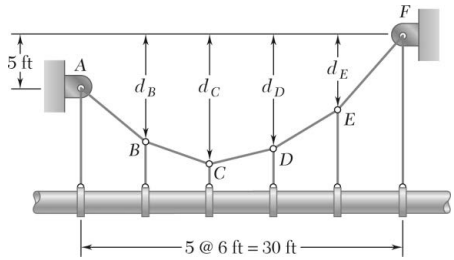
At C: $M_C = T_0 h_C$
 $7200 \text{ lb} \cdot \text{ft} = T_0 (9 \text{ ft}) \quad T_0 = 800 \text{ lb}$

At D: $M_D = T_0 h_D$
 $7200 \text{ lb} \cdot \text{ft} = (800 \text{ lb}) h_D \quad h_D = 9 \text{ ft}$

Eq. (1): $d_D = 9 \text{ ft} + 2 \text{ ft}$

$d_D = 11.00 \text{ ft} \blacktriangleleft$

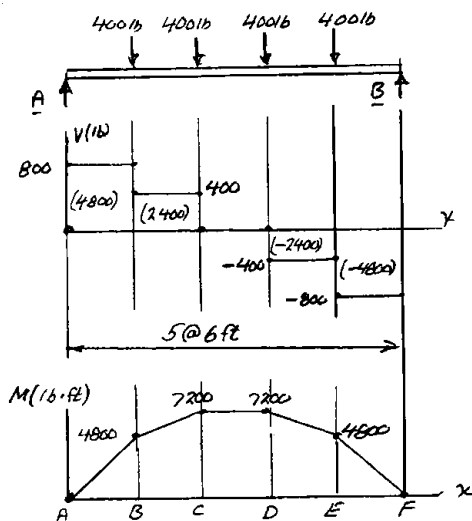
PROBLEM 7.123



Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.100 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 9$ ft, determine (b) the distance d_D .

SOLUTION



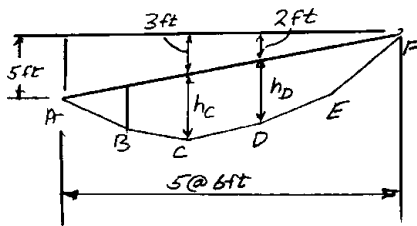
$$A = B = \frac{1}{2}(4 \times 400)$$

$$A = B = 800 \text{ lb}$$

At any point: $M = T_0 h$

We note that since $M_C = M_D$, we have $h_C = h_D$

Geometry



$$d_C = h_C + 3 \text{ ft}$$

$$9 \text{ ft} = h_C + 3 \text{ ft}$$

$$h_C = 6 \text{ ft}$$

and $h_D = 6 \text{ ft}$

$$d_D = h_D + 2 \text{ ft} = 6 \text{ ft} + 2 \text{ ft}$$

$$d_D = 8.00 \text{ ft} \blacktriangleleft$$

PROBLEM 7.124*

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:

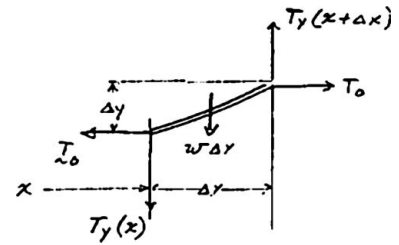
$$\uparrow \Sigma F_y = 0: T_y(x + \Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x + \Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0} \Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

So
$$\frac{\frac{dy}{dx} \Big|_{x+\Delta x} - \frac{dy}{dx} \Big|_x}{\Delta x} = \frac{w(x)}{T_0}$$

In $\lim_{\Delta x \rightarrow 0}$:
$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} \quad \text{Q.E.D.}$$

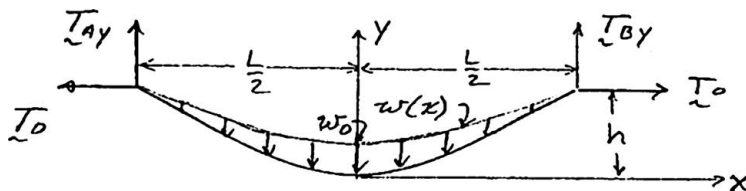


PROBLEM 7.125*

Using the property indicated in Problem 7.124, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x/L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION



$$w(x) = w_0 \cos \frac{\pi x}{L}$$

From Problem 7.124

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0} \cos \frac{\pi x}{L}$$

So
$$\frac{dy}{dx} = \frac{w_0 L}{T_0 \pi} \sin \frac{\pi x}{L} \quad \left(\text{using } \frac{dy}{dx} \Big|_0 = 0 \right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2} \left(1 - \cos \frac{\pi x}{L} \right) \quad [\text{using } y(0) = 0] \quad \blacktriangleleft$$

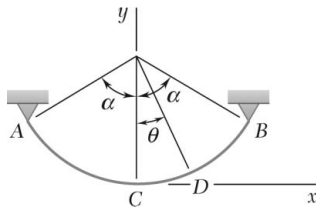
But
$$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2} \left(1 - \cos \frac{\pi}{2} \right) \quad \text{so} \quad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And
$$T_0 = T_{\min} \quad \text{so} \quad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \quad \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \frac{dy}{dx} \Big|_{x=L/2} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi} \quad T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi} \sqrt{1 + \left(\frac{L}{\pi h} \right)^2} \quad \blacktriangleleft$$

PROBLEM 7.126*



If the weight per unit length of the cable AB is $w_0/\cos^2 \theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Problem 7.124.)

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

Elemental Segment:

Load on segment*

$$w(x)dx = \frac{w_0}{\cos^2 \theta} ds$$

But

$$dx = \cos \theta ds, \quad \text{so} \quad w(x) = \frac{w_0}{\cos^3 \theta}$$

From Problem 7.119

$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0 \cos^3 \theta}$$

In general

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (\tan \theta) = \sec^2 \theta \frac{d\theta}{dx}$$

So

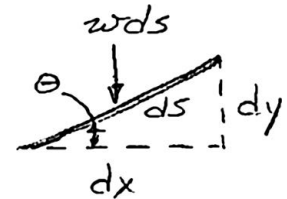
$$\frac{d\theta}{dx} = \frac{w_0}{T_0 \cos^3 \theta \sec^2 \theta} = \frac{w_0}{T_0 \cos \theta}$$

or

$$\frac{T_0}{w_0} \cos \theta d\theta = dx = r d\theta \cos \theta$$

Giving $r = \frac{T_0}{w_0} = \text{constant}$. So curve is circular arc Q.E.D.

*For large sag, it is not appropriate to approximate ds by dx .



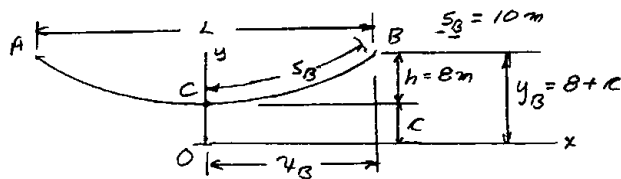
PROBLEM 7.127

A 20-m chain of mass 12 kg is suspended between two points at the same elevation. Knowing that the sag is 8 m, determine (a) the distance between the supports, (b) the maximum tension in the chain.

SOLUTION

$$\text{mass/meter} = (12 \text{ kg})/(20 \text{ m}) = 0.6 \text{ kg/m}$$

$$w = (0.6 \text{ kg/m})(9.81 \text{ m/s}^2) = 5.886 \text{ N/m}$$



Eq. 7.17:

$$y_B^2 - s_B^2 = c^2; \quad (8 + c)^2 - 10^2 = c^2$$
$$64 + 16c + c^2 - 100 = c^2$$
$$16c = 36 \quad c = 2.25 \text{ m}$$

Eq. 7.18:

$$T_m = wy_B = (5.886 \text{ N/m})(8 \text{ m} + 2.25 \text{ m})$$
$$T_m = 60.33 \text{ N}$$

$$T_m = 60.3 \text{ N} \quad \blacktriangleleft$$

Eq. 7.15:

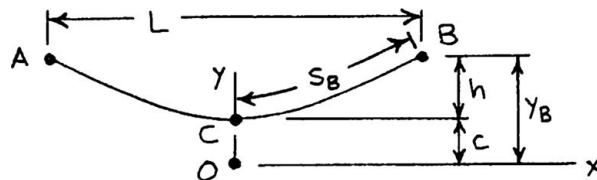
$$s_B = c \sinh \frac{x_B}{c}; \quad 10 = (2.25 \text{ m}) \sinh \frac{x_B}{c}$$
$$\sinh \frac{x_B}{c} = 4.444; \quad \frac{x_B}{c} = 2.197$$
$$x_B = 2.197(2.25 \text{ m}) = 4.944 \text{ m}; \quad L = 2x_B = 2(4.944 \text{ m}) = 9.888 \text{ m}$$

$$L = 9.89 \text{ m} \quad \blacktriangleleft$$

PROBLEM 7.128

A 600-ft-long aerial tramway cable having a weight per unit length of 3.0 lb/ft is suspended between two points at the same elevation. Knowing that the sag is 150 ft, find (a) the horizontal distance between the supports, (b) the maximum tension in the cable.

SOLUTION



Given:

$$\begin{aligned} \text{Length} &= 600 \text{ ft} \\ \text{Unit mass} &= 3.0 \text{ lb/ft} \\ h &= 150 \text{ ft} \end{aligned}$$

Then,

$$\begin{aligned} s_B &= 300 \text{ ft} \\ y_B &= h + c = 150 \text{ ft} + c \\ y_B^2 - s_B^2 &= c^2; \quad (150 + c)^2 - (300)^2 = c^2 \\ 150^2 + 300c + c^2 - 300^2 &= c^2 \\ c &= 225 \text{ ft} \end{aligned}$$

$$s_B = c \sinh \frac{x_B}{c}; \quad 300 = 225 \sinh \frac{x_B}{225}$$

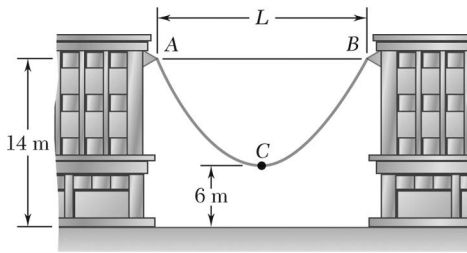
$$x_B = 247.28 \text{ ft}$$

$$\text{span} = L = 2x_B = 2(247.28 \text{ ft})$$

$$L = 495 \text{ ft} \quad \blacktriangleleft$$

$$T_m = wy_B = (3 \text{ lb/ft})(150 \text{ ft} + 225 \text{ ft})$$

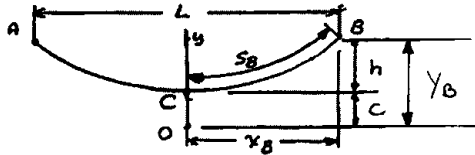
$$T_m = 1125 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 7.129

A 40-m cable is strung as shown between two buildings. The maximum tension is found to be 350 N, and the lowest point of the cable is observed to be 6 m above the ground. Determine (a) the horizontal distance between the buildings, (b) the total mass of the cable.

SOLUTION



$$s_B = 20 \text{ m}$$

$$T_m = 350 \text{ N}$$

$$h = 14 \text{ m} - 6 \text{ m} = 8 \text{ m}$$

$$y_B = h + c = 8 \text{ m} + c$$

Eq. 7.17:

$$y_B^2 - s_B^2 = c^2; \quad (8 + c)^2 - 20^2 = c^2$$

$$64 + 16c + c^2 - 400 = c^2$$

$$c = 21.0 \text{ m}$$

Eq. 7.15:

$$s_B = c \sinh \frac{x_B}{c}; \quad 20 = (21.0) \sinh \frac{x_B}{21.0}$$

(a) $x_B = 17.7933 \text{ m}; \quad L = 2x_B \quad L = 35.6 \text{ m} \blacktriangleleft$

(b) Eq. 7.18:

$$T_m = wy_B; \quad 350 \text{ N} = w(8 + 21.0)$$

$$w = 12.0690 \text{ N/m}$$

$$W = 2s_B w = (40 \text{ m})(12.0690 \text{ N/m})$$

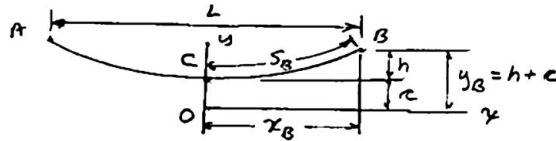
$$= 482.76 \text{ N}$$

$$m = \frac{W}{g} = \frac{482.76 \text{ N}}{9.81 \text{ m/s}^2} \quad \text{Total mass} = 49.2 \text{ kg} \blacktriangleleft$$

PROBLEM 7.130

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}$$

$$w = \left(\frac{4 \text{ lb}}{200 \text{ ft}} \right) = 0.02 \text{ lb/ft} \quad T_m = 16 \text{ m}$$

Eq. 7.18: $T_m = wy_B; \quad 16 \text{ lb} = (0.02 \text{ lb/ft})y_B; \quad y_B = 800 \text{ ft}$

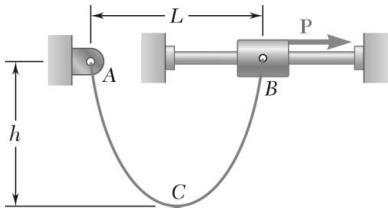
Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (800)^2 - (100)^2 = c^2; \quad c = 793.73 \text{ ft}$

Eq. 7.15: $s_B = c \sinh \frac{x_B}{c}; \quad 100 = 793.73 \sinh \frac{x_B}{c}$

$$\frac{x_B}{c} = 0.12566; \quad x_B = 99.737 \text{ ft}$$

$$L = 2x_B = 2(99.737 \text{ ft})$$

$$L = 199.5 \text{ ft} \quad \blacktriangleleft$$

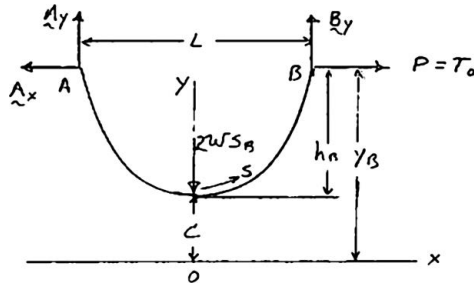


PROBLEM 7.131

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the force \mathbf{P} for which $h = 8$ m, (b) the corresponding span L .

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \quad \left(\text{so } s_B = \frac{20 \text{ m}}{2} = 10 \text{ m} \right)$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) \\ = 1.96200 \text{ N/m}$$

$$h_B = 8 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So

$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

$$c = \frac{(10 \text{ m})^2 - (8 \text{ m})^2}{2(8 \text{ m})} \\ = 2.250 \text{ m}$$

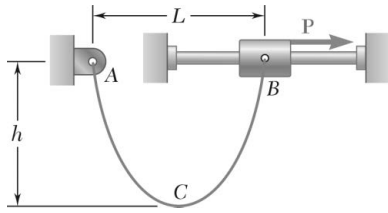
Now

$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} \\ = (2.250 \text{ m}) \sinh^{-1} \left(\frac{10 \text{ m}}{2.250 \text{ m}} \right)$$

$$x_B = 4.9438 \text{ m}$$

$$P = T_0 = wc = (1.96200 \text{ N/m})(2.250 \text{ m}) \quad (a) \quad \mathbf{P} = 4.41 \text{ N} \rightarrow \blacktriangleleft$$

$$L = 2x_B = 2(4.9438 \text{ m}) \quad (b) \quad L = 9.89 \text{ m} \blacktriangleleft$$

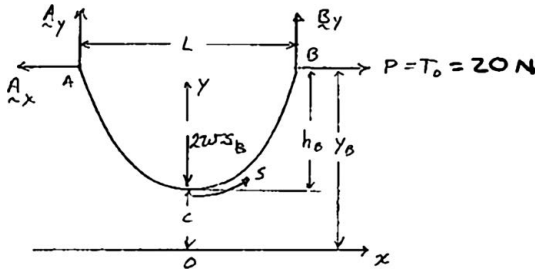


PROBLEM 7.132

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Knowing that the magnitude of the horizontal force applied to the collar is $P = 20$ N, determine (a) the sag h , (b) the span L .

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m}, \quad w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$P = T_0 = wc \quad c = \frac{P}{w}$$

$$c = \frac{20 \text{ N}}{1.9620 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \quad s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$h^2 + 2(10.1937 \text{ m})h - 100 \text{ m}^2 = 0$$

$$h = 4.0861 \text{ m}$$

(a)

$$h = 4.09 \text{ m} \quad \blacktriangleleft$$

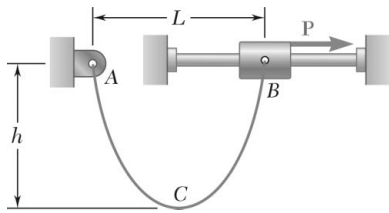
$$s_B = c \sinh \frac{x_A}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (10.1937 \text{ m}) \sinh^{-1} \left(\frac{10 \text{ m}}{10.1937 \text{ m}} \right)$$

$$= 8.8468 \text{ m}$$

$$L = 2x_B = 2(8.8468 \text{ m})$$

(b)

$$L = 17.69 \text{ m} \quad \blacktriangleleft$$

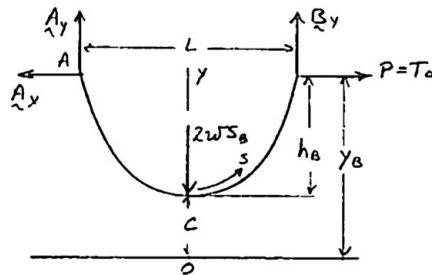


PROBLEM 7.133

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the sag h for which $L = 15$ m, (b) the corresponding force P .

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \rightarrow s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$L = 15 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{L}{c}$$

$$10 \text{ m} = c \sinh \frac{7.5 \text{ m}}{c}$$

Solving numerically:

$$c = 5.5504 \text{ m}$$

$$y_B = c \cosh \left(\frac{x_B}{c} \right) = (5.5504) \cosh \left(\frac{7.5}{5.5504} \right)$$

$$y_B = 11.4371 \text{ m}$$

$$h_B = y_B - c = 11.4371 \text{ m} - 5.5504 \text{ m}$$

$$(a) \quad h_B = 5.89 \text{ m} \quad \blacktriangleleft$$

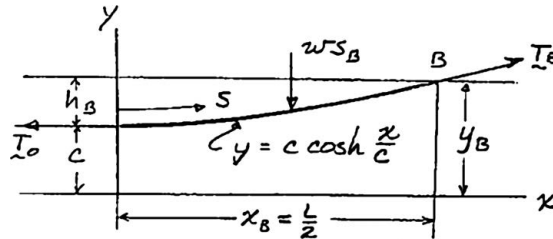
$$P = wc = (1.96200 \text{ N/m})(5.5504 \text{ m})$$

$$(b) \quad \mathbf{P} = 10.89 \text{ N} \rightarrow \blacktriangleleft$$

PROBLEM 7.134

Determine the sag of a 30-ft chain that is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{30 \text{ ft}}{2} = 15 \text{ ft} \quad L = 20 \text{ ft}$$

$$x_B = \frac{L}{2} = 10 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}$$

$$15 \text{ ft} = c \sinh \frac{10 \text{ ft}}{c}$$

Solving numerically:

$$c = 6.1647 \text{ ft}$$

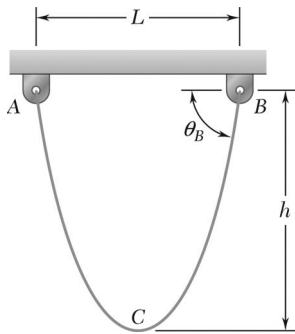
$$y_B = c \cosh \frac{x_B}{c}$$

$$= (6.1647 \text{ ft}) \cosh \frac{10 \text{ ft}}{6.1647 \text{ ft}}$$

$$= 16.2174 \text{ ft}$$

$$h_B = y_B - c = 16.2174 \text{ ft} - 6.1647 \text{ ft}$$

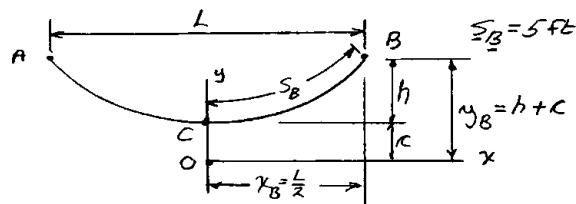
$$h_B = 10.05 \text{ ft} \quad \blacktriangleleft$$



PROBLEM 7.135

A 10-ft rope is attached to two supports A and B as shown. Determine (a) the span of the rope for which the span is equal to the sag, (b) the corresponding angle θ_B .

SOLUTION



Note: Since $L = h$,

$$x_B = \frac{L}{2} = \frac{h}{2}$$

Eq. 7.16:

$$y_B = \cosh \frac{x_B}{c}$$

$$h + c = c \cosh \frac{h/2}{c}$$

$$\frac{h}{c} + 1 = \cosh \left(\frac{1}{2} \frac{h}{c} \right)$$

Solve for h/c :

$$\frac{h}{c} = 4.933$$

Eq. 7.16:

$$y = c \cosh \frac{x}{c}$$

$$\frac{dy}{dx} = \sinh \frac{x}{c}$$

At B:

$$\tan \theta_B = \left. \frac{dy}{dx} \right|_B = \sinh \frac{x_B}{c}$$

Substitute

$$x_B = \frac{h}{2}: \quad \tan \theta_B = \sinh \left(\frac{1}{2} \frac{h}{c} \right) = \sinh \left(\frac{1}{2} \times 4.933 \right)$$

$$\tan \theta_B = 5.848$$

$$\theta_B = 80.3^\circ \blacktriangleleft$$

PROBLEM 7.135 (Continued)

Eq. 7.17:

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \left(\frac{1}{2} \frac{h}{c} \right)$$

$$5 \text{ ft} = c \sinh \left(\frac{1}{2} \times 4.933 \right)$$

$$5 \text{ ft} = c(5.848)$$

$$c = 0.855$$

Recall that

$$\frac{h}{c} = 4.933$$

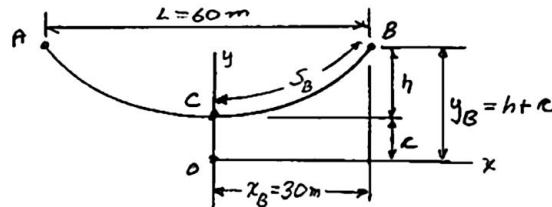
$$h = 4.933(0.855) = 4.218$$

$$h = 4.22 \text{ ft} \blacktriangleleft$$

PROBLEM 7.136

A 90-m wire is suspended between two points at the same elevation that are 60 m apart. Knowing that the maximum tension is 300 N, determine (a) the sag of the wire, (b) the total mass of the wire.

SOLUTION



$$s_B = 45 \text{ m}$$

Eq. 7.17:

$$s_B = c \sinh \frac{x_B}{c}$$

$$45 = c \sinh \frac{30}{c}; \quad c = 18.494 \text{ m}$$

Eq. 7.16:

$$y_B = c \cosh \frac{x_B}{c}$$

$$y_B = (18.494) \cosh \frac{30}{18.494}$$

$$y_B = 48.652 \text{ m}$$

$$y_B = h + c$$

$$48.652 = h + 18.494$$

$$h = 30.158 \text{ m}$$

$$h = 30.2 \text{ m} \quad \blacktriangleleft$$

Eq. 7.18:

$$T_m = wy_B$$

$$300 \text{ N} = w(48.652 \text{ m})$$

$$w = 6.166 \text{ N/m}$$

Total weight of cable

$$W = w(\text{Length})$$

$$= (6.166 \text{ N/m})(90 \text{ m})$$

$$= 554.96 \text{ N}$$

Total mass of cable

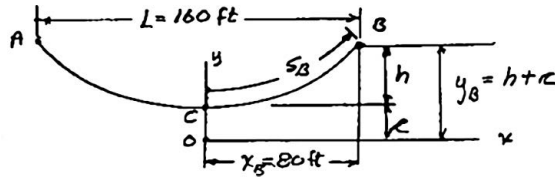
$$m = \frac{W}{g} = \frac{554.96 \text{ N}}{9.81 \text{ m/s}^2} = 56.57 \text{ kg}$$

$$m = 56.6 \text{ kg} \quad \blacktriangleleft$$

PROBLEM 7.137

A cable weighing 2 lb/ft is suspended between two points at the same elevation that are 160 ft apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 400 lb.

SOLUTION



Eq. 7.18: $T_m = wy_B; 400 \text{ lb} = (2 \text{ lb/ft})y_B; y_B = 200 \text{ ft}$

Eq. 7.16: $y_B = c \cosh \frac{x_B}{c}$
 $200 \text{ ft} = c \cosh \frac{80 \text{ ft}}{c}$

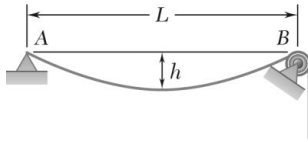
Solve for c : $c = 182.148 \text{ ft}$ and $c = 31.592 \text{ ft}$

$$y_B = h + c; h = y_B - c$$

For $c = 182.148 \text{ ft}; h = 200 - 182.147 = 17.852 \text{ ft} \triangleleft$

For $c = 31.592 \text{ ft}; h = 200 - 31.592 = 168.408 \text{ ft} \triangleleft$

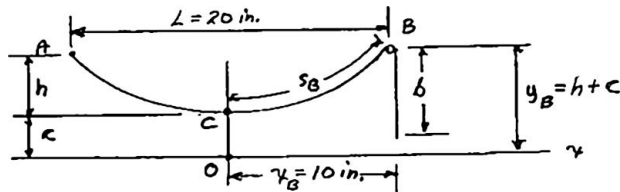
For $T_m \leq 400 \text{ lb};$ smallest $h = 17.85 \text{ ft} \blacktriangleleft$



PROBLEM 7.138

A uniform cord 50 in. long passes over a pulley at B and is attached to a pin support at A . Knowing that $L = 20$ in. and neglecting the effect of friction, determine the smaller of the two values of h for which the cord is in equilibrium.

SOLUTION



Length of overhang: $b = 50 \text{ in.} - 2s_B$

Weight of overhang equals max. tension

$$T_m = T_B = wb = w(50 \text{ in.} - 2s_B)$$

Eq. 7.15: $s_B = c \sinh \frac{x_B}{c}$

Eq. 7.16: $y_B = c \cosh \frac{x_B}{c}$

Eq. 7.18: $T_m = wy_B$
 $w(50 \text{ in.} - 2s_B) = wy_B$

$$w \left(50 \text{ in.} - 2c \sinh \frac{x_B}{c} \right) = wc \cosh \frac{x_B}{c}$$

$$x_B = 10: \quad 50 - 2c \sinh \frac{10}{c} = c \cosh \frac{10}{c}$$

Solve by trial and error: $c = 5.549 \text{ in.}$ and $c = 27.742 \text{ in.}$

For $c = 5.549 \text{ in.}$ $y_B = (5.549 \text{ in.}) \cosh \frac{10 \text{ in.}}{5.549 \text{ in.}} = 17.277 \text{ in.}$

$$y_B = h + c; \quad 17.277 \text{ in.} = h + 5.549 \text{ in.}$$

$$h = 11.728 \text{ in.}$$

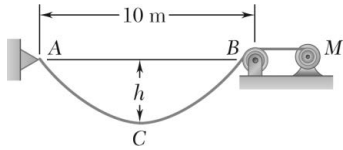
$$h = 11.73 \text{ in.} \quad \blacktriangleleft$$

For $c = 27.742 \text{ in.}$ $y_B = (27.742 \text{ in.}) \cosh \frac{10 \text{ in.}}{27.742 \text{ in.}} = 29.564 \text{ in.}$

$$y_B = h + c; \quad 29.564 \text{ in.} = h + 27.742 \text{ in.}$$

$$h = 1.8219 \text{ in.}$$

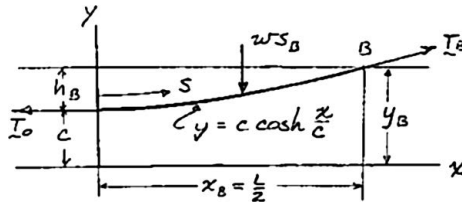
$$h = 1.822 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 7.139

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m , determine the maximum tension in the cable when $h = 5 \text{ m}$.

SOLUTION



$$w = 0.4 \text{ kg/m} \quad L = 10 \text{ m} \quad h_B = 5 \text{ m}$$

$$y_B = c \cosh \frac{x_B}{c}$$

$$h_B + c = c \cosh \frac{L}{2c}$$

$$5 \text{ m} = c \left(\cosh \frac{5 \text{ m}}{c} - 1 \right)$$

Solving numerically:

$$c = 3.0938 \text{ m}$$

$$y_B = h_B + c = 5 \text{ m} + 3.0938 \text{ m}$$

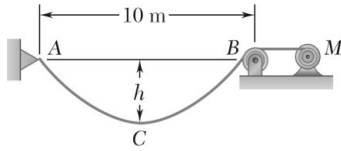
$$= 8.0938 \text{ m}$$

$$T_{\max} = T_B = w y_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(8.0938 \text{ m})$$

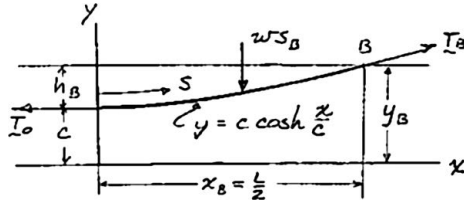
$$T_{\max} = 31.8 \text{ N} \quad \blacktriangleleft$$

PROBLEM 7.140



A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m , determine the maximum tension in the cable when $h = 3 \text{ m}$.

SOLUTION



$$w = 0.4 \text{ kg/m}, \quad L = 10 \text{ m}, \quad h_B = 3 \text{ m}$$

$$y_B = h_B + c = c \cosh \frac{x_B}{c} = c \cosh \frac{L}{2c}$$

$$3 \text{ m} = c \left(c \cosh \frac{5 \text{ m}}{c} - 1 \right)$$

Solving numerically:

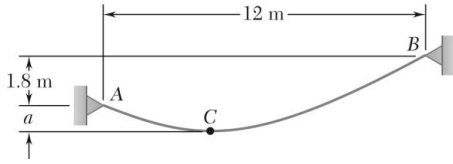
$$c = 4.5945 \text{ m}$$

$$y_B = h_B + c = 3 \text{ m} + 4.5945 \text{ m} \\ = 7.5945 \text{ m}$$

$$T_{\max} = T_B = wy_B \\ = (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(7.5945 \text{ m})$$

$$T_{\max} = 29.8 \text{ N} \quad \blacktriangleleft$$

PROBLEM 7.141

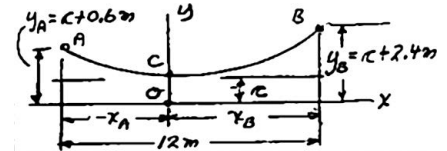


The cable ACB has a mass per unit length of 0.45 kg/m . Knowing that the lowest point of the cable is located at a distance $a = 0.6 \text{ m}$ below the support A , determine (a) the location of the lowest Point C , (b) the maximum tension in the cable.

SOLUTION

Note: $x_B - x_A = 12 \text{ m}$

or, $-x_A = 12 \text{ m} - x_B$



Point A: $y_A = c \cosh \frac{-x_A}{c}; \quad c + 0.6 = c \cosh \frac{12 - x_B}{c}$ (1)

Point B: $y_B = c \cosh \frac{x_B}{c}; \quad c + 2.4 = c \cosh \frac{x_B}{c}$ (2)

From (1): $\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1} \left(\frac{c + 0.6}{c} \right)$ (3)

From (2): $\frac{x_B}{c} = \cosh^{-1} \left(\frac{c + 2.4}{c} \right)$ (4)

Add (3) + (4): $\frac{12}{c} = \cosh^{-1} \left(\frac{c + 0.6}{c} \right) + \cosh^{-1} \left(\frac{c + 2.4}{c} \right)$

Solve by trial and error: $c = 13.6214 \text{ m}$

Eq. (2): $13.6214 + 2.4 = 13.6214 \cosh \frac{x_B}{c}$

$$\cosh \frac{x_B}{c} = 1.1762; \quad \frac{x_B}{c} = 0.58523$$

$$x_B = 0.58523(13.6214 \text{ m}) = 7.9717 \text{ m}$$

Point C is 7.97 m to left of B ◀

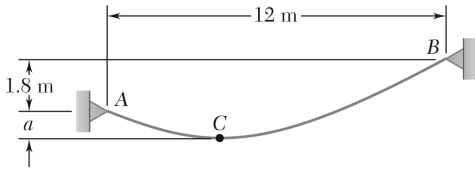
$$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214 \text{ m}$$

Eq. 7.18: $T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(16.0214 \text{ m})$

$$T_m = 70.726 \text{ N}$$

$$T_m = 70.7 \text{ N} \quad \blacktriangleleft$$

PROBLEM 7.142



The cable ACB has a mass per unit length of 0.45 kg/m . Knowing that the lowest point of the cable is located at a distance $a = 2 \text{ m}$ below the support A , determine (a) the location of the lowest Point C , (b) the maximum tension in the cable.

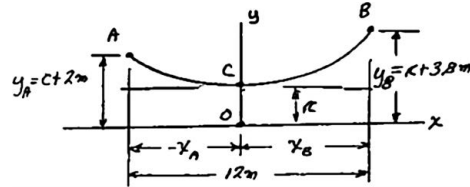
SOLUTION

Note:

$$x_B - x_A = 12 \text{ m}$$

or

$$-x_A = 12 \text{ m} - x_B$$



Point A:

$$y_A = c \cosh \frac{-x_A}{c}; \quad c + 2 = c \cosh \frac{12 - x_B}{c} \quad (1)$$

Point B:

$$y_B = c \cosh \frac{x_B}{c}; \quad c + 3.8 = c \cosh \frac{x_B}{c} \quad (2)$$

From (1):

$$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1} \left(\frac{c + 2}{c} \right) \quad (3)$$

From (2):

$$\frac{x_B}{c} = \cosh^{-1} \left(\frac{c + 3.8}{c} \right) \quad (4)$$

Add (3) + (4):

$$\frac{12}{c} = \cosh^{-1} \left(\frac{c + 2}{c} \right) + \cosh^{-1} \left(\frac{c + 3.8}{c} \right)$$

Solve by trial and error:

$$c = 6.8154 \text{ m}$$

Eq. (2):

$$6.8154 \text{ m} + 3.8 \text{ m} = (6.8154 \text{ m}) \cosh \frac{x_B}{c}$$

$$\cosh \frac{x_B}{c} = 1.5576 \quad \frac{x_B}{c} = 1.0122$$

$$x_B = 1.0122(6.8154 \text{ m}) = 6.899 \text{ m}$$

Point C is 6.90 m to left of B ◀

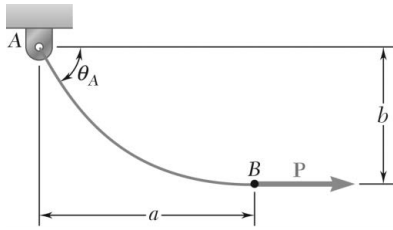
$$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154 \text{ m}$$

Eq. (7.18):

$$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(10.6154 \text{ m})$$

$$T_m = 46.86 \text{ N}$$

$$T_m = 46.9 \text{ N} \quad \blacktriangleleft$$



PROBLEM 7.143

A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force P applied at B . Knowing that $P = 180$ lb and $\theta_A = 60^\circ$, determine (a) the location of Point B , (b) the length of the cable.

SOLUTION

Eq. 7.18:

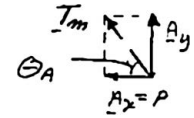
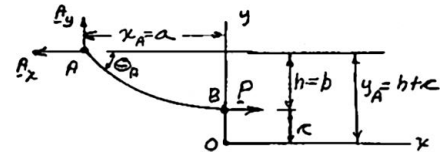
$$T_0 = P = cw$$

$$c = \frac{P}{w} = \frac{180 \text{ lb}}{3 \text{ lb/ft}} \quad c = 60 \text{ ft}$$

At A :

$$T_m = \frac{P}{\cos 60^\circ}$$

$$= \frac{cw}{0.5} = 2cw$$



(a) Eq. 7.18:

$$T_m = w(h+c)$$

$$2cw = w(h+c)$$

$$2c = h+c \quad h = b = c$$

$$b = 60.0 \text{ ft} \quad \blacktriangleleft$$

Eq. 7.16:

$$y_A = c \cosh \frac{x_A}{c}$$

$$h+c = c \cosh \frac{x_A}{c}$$

$$(60 \text{ ft} + 60 \text{ ft}) = (60 \text{ ft}) \cosh \frac{x_A}{60}$$

$$\cosh \frac{x_A}{60 \text{ m}} = 2 \quad \frac{x_A}{60 \text{ m}} = 1.3170$$

$$x_A = 79.02 \text{ ft}$$

$$a = 79.0 \text{ ft} \quad \blacktriangleleft$$

(b) Eq. 7.15:

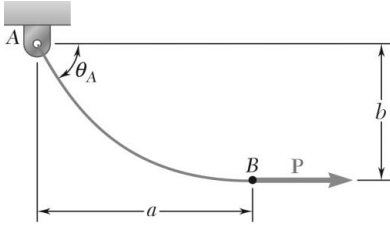
$$s_A = c \sinh \frac{x_B}{c} = (60 \text{ ft}) \sinh \frac{79.02 \text{ ft}}{60 \text{ ft}}$$

$$s_A = 103.92 \text{ ft}$$

$$\text{length} = s_A$$

$$s_A = 103.9 \text{ ft} \quad \blacktriangleleft$$

PROBLEM 7.144



A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force P applied at B . Knowing that $P = 150$ lb and $\theta_A = 60^\circ$, determine (a) the location of Point B , (b) the length of the cable.

SOLUTION

Eq. 7.18:

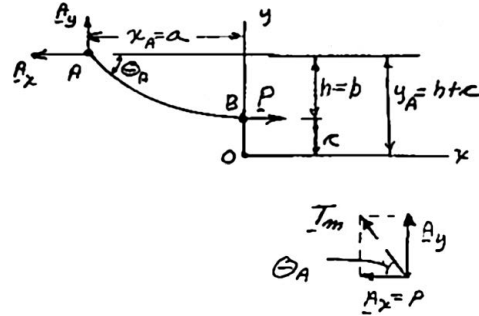
$$T_0 = P = cw$$

$$c = \frac{P}{w} = \frac{150 \text{ lb}}{3 \text{ lb/ft}} = 50 \text{ ft}$$

At A :

$$T_m = \frac{P}{\cos 60^\circ}$$

$$= \frac{cw}{0.5} = 2cw$$



(a) Eq. 7.18:

$$T_m = w(h + c)$$

$$2cw = w(h + c)$$

$$2c = h + c \quad h = c = b$$

$$b = 50.0 \text{ ft} \quad \blacktriangleleft$$

Eq. 7.16:

$$y_A = c \cosh \frac{x_A}{c}$$

$$h + c = c \cosh \frac{x_A}{c}$$

$$(50 \text{ ft} + 50 \text{ ft}) = (50 \text{ ft}) \cosh \frac{x_A}{c}$$

$$\cosh \frac{x_A}{c} = 2 \quad \frac{x_A}{c} = 1.3170$$

$$x_A = 1.3170(50 \text{ ft}) = 65.85 \text{ ft}$$

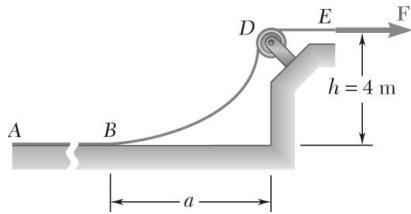
$$a = 65.8 \text{ ft} \quad \blacktriangleleft$$

(b) Eq. 7.15:

$$s_A = c \sinh \frac{x_A}{c} = (50 \text{ ft}) \sinh \frac{65.85 \text{ ft}}{50 \text{ ft}}$$

$$s_A = 86.6 \text{ ft}$$

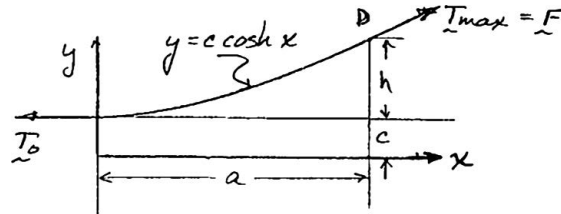
$$\text{length} = s_A = 86.6 \text{ ft} \quad \blacktriangleleft$$



PROBLEM 7.145

To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m , determine the force \mathbf{F} when $a = 3.6 \text{ m}$.

SOLUTION



$$x_D = a = 3.6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c \cosh \frac{x_D}{c}$$

$$h + c = c \cosh \frac{a}{c}$$

$$4 \text{ m} = c \left(\cosh \frac{3.6 \text{ m}}{c} - 1 \right)$$

Solving numerically:

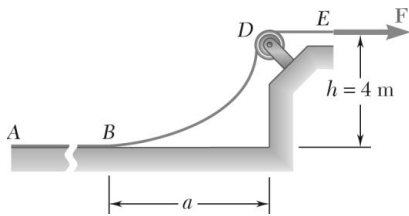
$$c = 2.0712 \text{ m}$$

Then

$$y_B = h + c = 4 \text{ m} + 2.0712 \text{ m} = 6.0712 \text{ m}$$

$$F = T_{\max} = wy_B = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(6.0712 \text{ m})$$

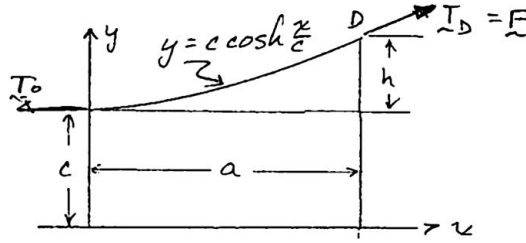
$$\mathbf{F} = 119.1 \text{ N} \rightarrow \blacktriangleleft$$



PROBLEM 7.146

To the left of Point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force *F* when $a = 6$ m.

SOLUTION



$$x_D = a = 6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c \cosh \frac{x_D}{c}$$

$$h + c = c \cosh \frac{a}{c}$$

$$4 \text{ m} = c \left(\cosh \frac{6 \text{ m}}{c} - 1 \right)$$

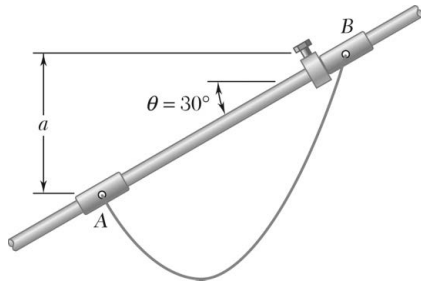
Solving numerically:

$$c = 5.054 \text{ m}$$

$$y_B = h + c = 4 \text{ m} + 5.054 \text{ m} = 9.054 \text{ m}$$

$$F = T_D = wy_D = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(9.054 \text{ m})$$

$$\mathbf{F = 177.6 \text{ N} \rightarrow \blacktriangleleft}$$

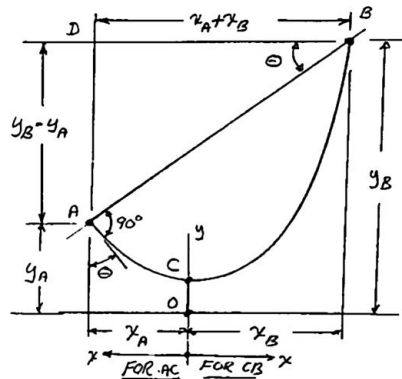


PROBLEM 7.147*

The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a .

SOLUTION

Collar at A : Since $\mu = 0$, cable \perp rod



Point A :

$$y = c \cosh \frac{x}{c}; \quad \frac{dy}{dx} = \sinh \frac{x}{c}$$

$$\tan \theta = \left| \frac{dy}{dx} \right|_A = \sinh \frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90^\circ - \theta))$$

$$x_A = c \sinh(\tan(90^\circ - \theta)) \quad (1)$$

Length of cable = 10 ft

$$10 \text{ ft} = AC + CB$$

$$10 = c \sinh \frac{x_A}{c} + c \sinh \frac{x_B}{c}$$

$$\sinh \frac{x_B}{c} = \frac{10}{c} - \sinh \frac{x_A}{c}$$

$$x_B = c \sinh^{-1} \left[\frac{10}{c} - \sinh \frac{x_A}{c} \right] \quad (2)$$

$$y_A = c \cosh \frac{x_A}{c} \quad y_B = c \cosh \frac{x_B}{c} \quad (3)$$

In $\triangle ABD$:

$$\tan \theta = \frac{y_B - y_A}{x_B + x_A} \quad (4)$$

PROBLEM 7.147* (Continued)

Method of solution:

For given value of θ , choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c , calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A , x_B , y_A , y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ .

For $\theta = 30^\circ$

Result of trial and error procedure:

$$c = 1.803 \text{ ft}$$

$$x_A = 2.3745 \text{ ft}$$

$$x_B = 3.6937 \text{ ft}$$

$$y_A = 3.606 \text{ ft}$$

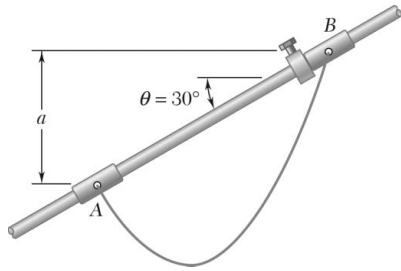
$$y_B = 7.109 \text{ ft}$$

$$a = y_B - y_A$$

$$= 7.109 \text{ ft} - 3.606 \text{ ft}$$

$$= 3.503 \text{ ft}$$

$$a = 3.50 \text{ ft} \blacktriangleleft$$



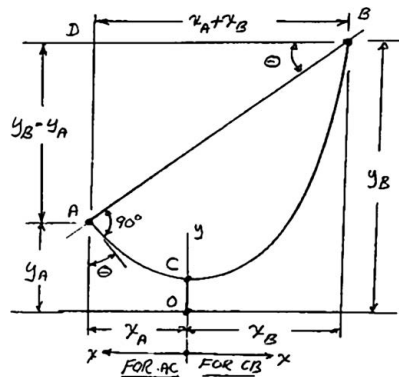
PROBLEM 7.148*

Solve Problem 7.147 assuming that the angle θ formed by the rod and the horizontal is 45° .

PROBLEM 7.147 The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a .

SOLUTION

Collar at A: Since $\mu = 0$, cable \perp rod



Point A:

$$y = c \cosh \frac{x}{c}; \quad \frac{dy}{dx} = \sinh \frac{x}{c}$$

$$\tan \theta = \left| \frac{dy}{dx} \right|_A = \sinh \frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90^\circ - \theta))$$

$$x_A = c \sinh(\tan(90^\circ - \theta)) \quad (1)$$

Length of cable = 10 ft

$$10 \text{ ft} = AC + CB$$

$$10 = c \sinh \frac{x_A}{c} + c \sinh \frac{x_B}{c}$$

$$\sinh \frac{x_B}{c} = \frac{10}{c} - \sinh \frac{x_A}{c}$$

$$x_B = c \sinh^{-1} \left[\frac{10}{c} - \sinh \frac{x_A}{c} \right] \quad (2)$$

$$y_A = c \cosh \frac{x_A}{c} \quad y_B = c \cosh \frac{x_B}{c} \quad (3)$$

PROBLEM 7.148* (Continued)

In $\triangle ABD$:
$$\tan \theta = \frac{y_B - y_A}{x_B + x_A} \quad (4)$$

Method of solution:

For given value of θ , choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c , calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A , x_B , y_A , y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ .

For $\theta = 45^\circ$

Result of trial and error procedure:

$$c = 1.8652 \text{ ft}$$

$$x_A = 1.644 \text{ ft}$$

$$x_B = 4.064 \text{ ft}$$

$$y_A = 2.638 \text{ ft}$$

$$y_B = 8.346 \text{ ft}$$

$$a = y_B - y_A$$

$$= 8.346 \text{ ft} - 2.638 \text{ ft}$$

$$= 5.708 \text{ ft}$$

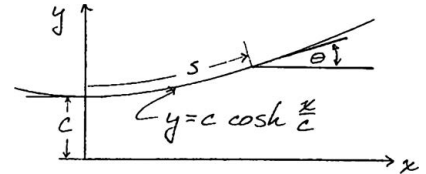
$$a = 5.71 \text{ ft} \blacktriangleleft$$

PROBLEM 7.149

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (a) $s = c \tan \theta$,
(b) $y = c \sec \theta$.

SOLUTION

(a)
$$\tan \theta = \frac{dy}{dx} = \sinh \frac{x}{c}$$
$$s = c \sinh \frac{x}{c} = c \tan \theta \quad \text{Q.E.D.}$$



(b) Also
$$y^2 = s^2 + c^2 (\cosh^2 x = \sinh^2 x + 1)$$

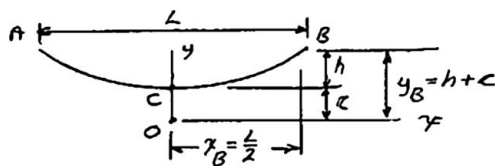
so
$$y^2 = c^2 (\tan^2 \theta + 1) c^2 \sec^2 \theta$$

and
$$y = c \sec \theta \quad \text{Q.E.D.}$$

PROBLEM 7.150*

(a) Determine the maximum allowable horizontal span for a uniform cable of weight per unit length w if the tension in the cable is not to exceed a given value T_m . (b) Using the result of part a, determine the maximum span of a steel wire for which $w = 0.25$ lb/ft and $T_m = 8000$ lb.

SOLUTION



$$\begin{aligned}
 (a) \quad T_m &= wy_B \\
 &= wc \cosh \frac{x_B}{c} \\
 &= wx_B \left(\frac{1}{\frac{x_B}{c}} \right) \cosh \frac{x_B}{c}
 \end{aligned}$$

We shall find ratio $\left(\frac{x_B}{c}\right)$ for when T_m is minimum

$$\frac{dT_m}{d\left(\frac{x_B}{c}\right)} = wx_B \left[\frac{1}{\frac{x_B}{c}} \sinh \frac{x_B}{c} - \left(\frac{1}{\frac{x_B}{c}}\right)^2 \cosh \frac{x_B}{c} \right] = 0$$

$$\begin{aligned}
 \frac{\sinh \frac{x_B}{c}}{\cosh \frac{x_B}{c}} &= \frac{1}{\frac{x_B}{c}} \\
 \tanh \frac{x_B}{c} &= \frac{c}{x_B}
 \end{aligned}$$

$$\text{Solve by trial and error for: } \frac{x_B}{c} = 1.200 \quad (1)$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh(1.200): \quad \frac{s_B}{c} = 1.509$$

$$\begin{aligned}
 \text{Eq. 7.17: } \quad y_B^2 - s_B^2 &= c^2 \\
 y_B^2 &= c^2 \left[1 + \left(\frac{s_B}{c}\right)^2 \right] = c^2(1 + 1.509^2) \\
 y_B &= 1.810c
 \end{aligned}$$

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PROBLEM 7.150* (Continued)

Eq. 7.18:

$$\begin{aligned}T_m &= wy_B \\ &= 1.810wc \\ c &= \frac{T_m}{1.810w}\end{aligned}$$

Eq. (1):

$$x_B = 1.200c = 1.200 \frac{T_m}{1.810w} = 0.6630 \frac{T_m}{w}$$

Span:

$$L = 2x_B = 2(0.6630) \frac{T_m}{w}$$

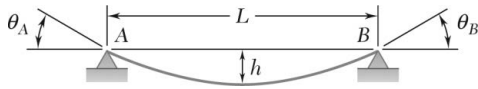
$$L = 1.326 \frac{T_m}{w} \blacktriangleleft$$

(b) For $w = 0.25 \text{ lb/ft}$ and $T_m = 8000 \text{ lb}$

$$\begin{aligned}L &= 1.326 \frac{8000 \text{ lb}}{0.25 \text{ lb/ft}} \\ &= 42,432 \text{ ft}\end{aligned}$$

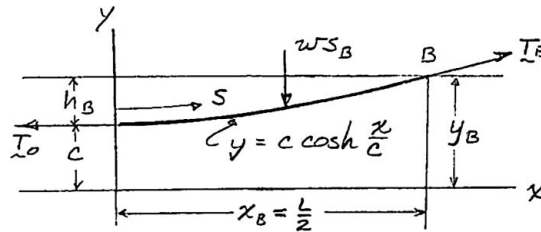
$$L = 8.04 \text{ miles} \blacktriangleleft$$

PROBLEM 7.151*



A cable has a mass per unit length of 3 kg/m and is supported as shown. Knowing that the span L is 6 m, determine the *two* values of the sag h for which the maximum tension is 350 N.

SOLUTION



$$y_{\max} = c \cosh \frac{L}{2c} = h + c$$

$$w = (3 \text{ kg/m})(9.81 \text{ m/s}^2) = 29.43 \text{ N/m}$$

$$T_{\max} = w y_{\max}$$

$$y_{\max} = \frac{T_{\max}}{w}$$

$$y_{\max} = \frac{350 \text{ N}}{29.43 \text{ N/m}} = 11.893 \text{ m}$$

$$c \cosh \frac{3 \text{ m}}{c} = 11.893 \text{ m}$$

Solving numerically:

$$c_1 = 0.9241 \text{ m}$$

$$c_2 = 11.499 \text{ m}$$

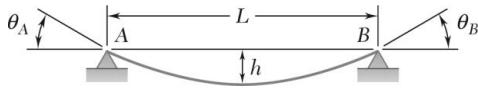
$$h = y_{\max} - c$$

$$h_1 = 11.893 \text{ m} - 0.9241 \text{ m}$$

$$h_1 = 10.97 \text{ m} \blacktriangleleft$$

$$h_2 = 11.893 \text{ m} - 11.499 \text{ m}$$

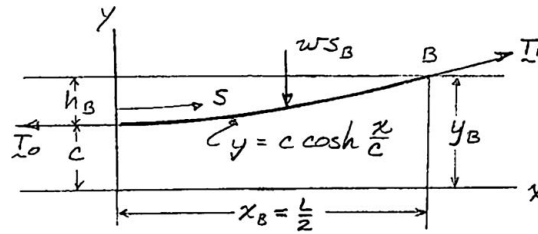
$$h_2 = 0.394 \text{ m} \blacktriangleleft$$



PROBLEM 7.152*

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable AB.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c \cosh \frac{L}{2c} = 2c \sinh \frac{L}{2c}$$

$$\tanh \frac{L}{2c} = \frac{1}{2}$$

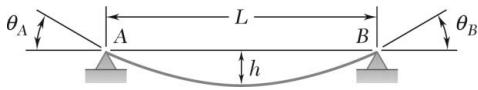
$$\frac{L}{2c} = \tanh^{-1} \frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh \frac{L}{2c} - 1 = 0.154701$$

$$\frac{h_B}{L} = \frac{\frac{h_B}{c}}{2\left(\frac{L}{2c}\right)} = \frac{0.5(0.154701)}{0.549306} = 0.14081$$

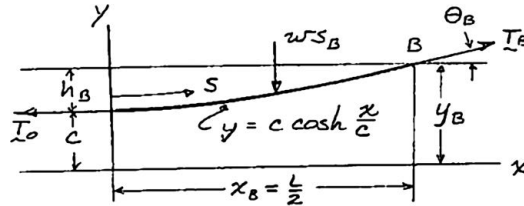
$$\frac{h_B}{L} = 0.1408 \quad \blacktriangleleft$$

PROBLEM 7.153*



A cable of weight per unit length w is suspended between two points at the same elevation that are a distance L apart. Determine (a) the sag-to-span ratio for which the maximum tension is as small as possible, (b) the corresponding values of θ_B and T_m .

SOLUTION



$$(a) \quad T_{\max} = wy_B = wc \cosh \frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w \left(\cosh \frac{L}{2c} - \frac{L}{2c} \sinh \frac{L}{2c} \right)$$

$$\text{For } \min T_{\max}, \quad \frac{dT_{\max}}{dc} = 0$$

$$\tanh \frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh \frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2} \frac{h}{c} \left(\frac{2c}{L} \right) \right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$$\frac{h}{L} = 0.338 \quad \blacktriangleleft$$

$$(b) \quad T_0 = wc \quad T_{\max} = wc \cosh \frac{L}{2c} \quad \frac{T_{\max}}{T_0} = \cosh \frac{L}{2c} = \frac{y_B}{c}$$

$$\text{But} \quad T_0 = T_{\max} \cos \theta_B \quad \frac{T_{\max}}{T_0} = \sec \theta_B$$

$$\text{So} \quad \theta_B = \sec^{-1} \left(\frac{y_B}{c} \right) = \sec^{-1}(1.8102) = 56.46^\circ$$

$$\theta_B = 56.5^\circ \quad \blacktriangleleft$$

$$T_{\max} = wy_B = w \frac{y_B}{c} \left(\frac{2c}{L} \right) \left(\frac{L}{2} \right) = w(1.8102) \frac{L}{2(1.1997)}$$

$$T_{\max} = 0.755wL \quad \blacktriangleleft$$

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PROBLEM 7.154

Determine the internal forces at Point *J* of the structure shown.

SOLUTION

FBD ABC:

$$\left(\sum M_D = 0: (0.375 \text{ m})(400 \text{ N}) - (0.24 \text{ m})C_y = 0 \right.$$

$$C_y = 625 \text{ N} \uparrow$$

$$\left(\sum M_B = 0: -(0.45 \text{ m})C_x + (0.135 \text{ m})(400 \text{ N}) = 0 \right.$$

$$C_x = 120 \text{ N} \rightarrow$$

FBD CJ:

$$\uparrow \sum F_y = 0: 625 \text{ N} - F = 0 \quad \mathbf{F = 625 \text{ N} \downarrow \blacktriangleleft}$$

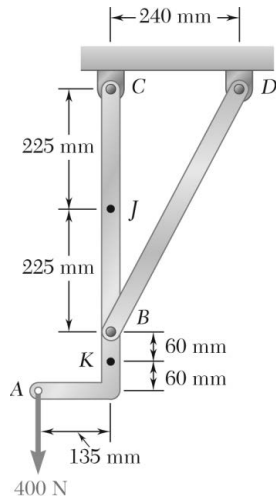
$$\rightarrow \sum F_x = 0: 120 \text{ N} - V = 0 \quad \mathbf{V = 120.0 \text{ N} \leftarrow \blacktriangleleft}$$

$$\left(\sum M_J = 0: M - (0.225 \text{ m})(120 \text{ N}) = 0 \right.$$

$$\mathbf{M = 27.0 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft}$$

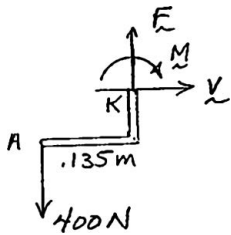
PROBLEM 7.155

Determine the internal forces at Point K of the structure shown.



SOLUTION

FBD AK:



$$\rightarrow \Sigma F_x = 0: V = 0$$

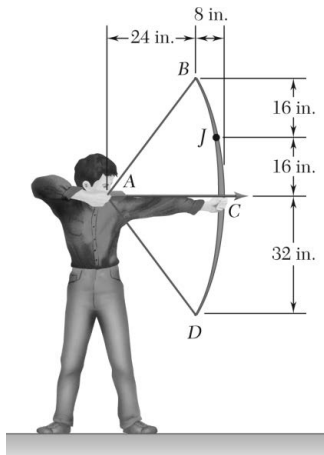
$$V = 0 \quad \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: F - 400 \text{ N} = 0$$

$$F = 400 \text{ N} \quad \uparrow \blacktriangleleft$$

$$\curvearrowleft \Sigma M_K = 0: (0.135 \text{ m})(400 \text{ N}) - M = 0$$

$$M = 54.0 \text{ N} \cdot \text{m} \quad \curvearrowleft \blacktriangleleft$$



PROBLEM 7.156

An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point J .

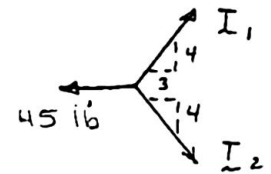
SOLUTION

FBD Point A:

By symmetry

$$T_1 = T_2$$

$$\rightarrow \Sigma F_x = 0: 2\left(\frac{3}{5}T_1\right) - 45 \text{ lb} = 0 \quad T_1 = T_2 = 37.5 \text{ lb}$$



Curve CJB is parabolic: $x = ay^2$

FBD BJ:

At B : $x = 8 \text{ in.}$

$$y = 32 \text{ in.}$$

$$a = \frac{8 \text{ in.}}{(32 \text{ in.})^2} = \frac{1}{128 \text{ in.}}$$

$$x = \frac{y^2}{128}$$

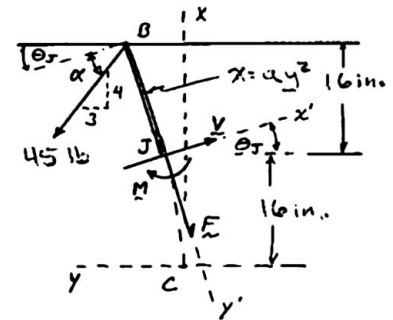
$$\text{Slope of parabola} = \tan \theta = \frac{dx}{dy} = \frac{2y}{128} = \frac{y}{64}$$

$$\text{At } J: \quad \theta_J = \tan^{-1} \left[\frac{16}{64} \right] = 14.036^\circ$$

$$\text{So} \quad \alpha = \tan^{-1} \frac{4}{3} - 14.036^\circ = 39.094^\circ$$

$$\nearrow \Sigma F_x = 0: V - (37.5 \text{ lb}) \cos(39.094^\circ) = 0$$

$$V = 29.1 \text{ lb} \quad \nearrow 14.04^\circ \quad \blacktriangleleft$$



PROBLEM 7.156 (Continued)

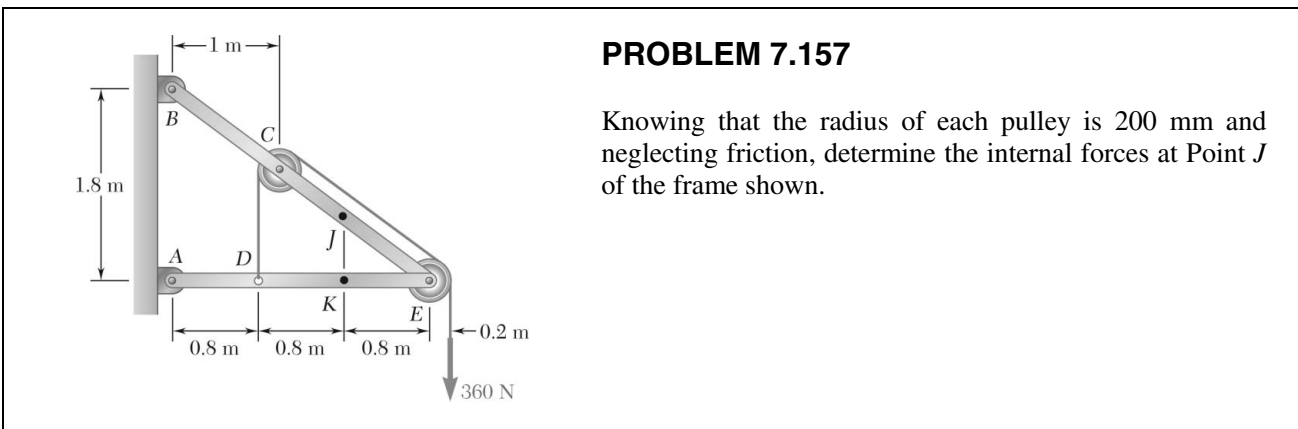
$$\searrow \Sigma F_y = 0: F + (37.5 \text{ lb}) \sin (39.094^\circ) = 0$$

$$F = -23.647$$

$$\mathbf{F} = 23.6 \text{ lb } \nearrow 76.0^\circ \blacktriangleleft$$

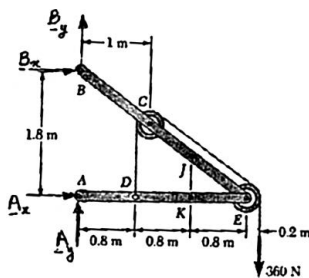
$$\curvearrowleft \Sigma M_J = 0: M + (16 \text{ in.}) \left[\frac{3}{5} (37.5 \text{ lb}) \right] + [(8 - 2) \text{ in.}] \left[\frac{4}{5} (37.5 \text{ lb}) \right] = 0$$

$$\mathbf{M} = 540 \text{ lb} \cdot \text{in.} \curvearrowright \blacktriangleleft$$



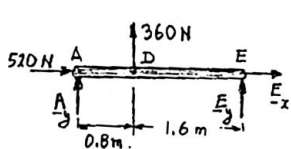
SOLUTION

Free body: Frame and pulleys



$$\begin{aligned}
 +\curvearrowright \Sigma M_A = 0: & \quad -B_x(1.8 \text{ m}) - (360 \text{ N})(2.6 \text{ m}) = 0 \\
 & \quad B_x = -520 \text{ N} \qquad \qquad \qquad \mathbf{B}_x = 520 \text{ N} \leftarrow \triangleleft \\
 +\rightarrow \Sigma F_x = 0: & \quad A_x - 520 \text{ N} = 0 \\
 & \quad A_x = +520 \text{ N} \qquad \qquad \qquad \mathbf{A}_x = 520 \text{ N} \rightarrow \triangleleft \\
 +\uparrow \Sigma F_y = 0: & \quad A_y + B_y - 360 \text{ N} = 0 \\
 & \quad A_y + B_y = 360 \text{ N} \qquad \qquad \qquad (1)
 \end{aligned}$$

Free body: Member *AE*



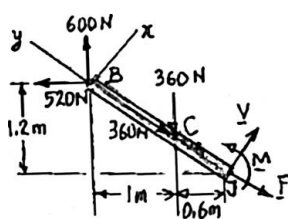
$$\begin{aligned}
 +\curvearrowright \Sigma M_E = 0: & \quad -A_y(2.4 \text{ m}) - (360 \text{ N})(1.6 \text{ m}) = 0 \\
 & \quad A_y = -240 \text{ N} \qquad \qquad \qquad \mathbf{A}_y = 240 \text{ N} \downarrow \triangleleft
 \end{aligned}$$

From (1):

$$\begin{aligned}
 B_y &= 360 \text{ N} + 240 \text{ N} \\
 B_y &= +600 \text{ N} \qquad \qquad \qquad \mathbf{B}_y = 600 \text{ N} \uparrow \triangleleft
 \end{aligned}$$

Free body: *BJ*

We recall that the forces applied to a pulley may be applied directly to its axle.



$$\begin{aligned}
 +\swarrow \Sigma F_y = 0: & \quad \frac{3}{5}(600 \text{ N}) + \frac{4}{5}(520 \text{ N}) \\
 & \quad -360 \text{ N} - \frac{3}{5}(360 \text{ N}) - F = 0 \\
 & \quad F = +200 \text{ N} \qquad \qquad \qquad \mathbf{F} = 200 \text{ N} \swarrow 36.9^\circ \triangleleft
 \end{aligned}$$

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PROBLEM 7.157 (Continued)

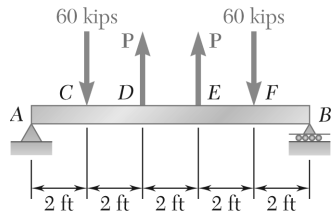
$$+\nearrow \Sigma F_x = 0: \frac{4}{5}(600 \text{ N}) - \frac{3}{5}(520 \text{ N}) - \frac{4}{5}(360 \text{ N}) + V = 0$$

$$V = +120.0 \text{ N} \quad \mathbf{V} = 120.0 \text{ N} \nearrow 53.1^\circ \blacktriangleleft$$

$$+\curvearrowright \Sigma M_J = 0: (520 \text{ N})(1.2 \text{ m}) - (600 \text{ N})(1.6 \text{ m}) + (360 \text{ N})(0.6 \text{ m}) + M = 0$$

$$M = +120.0 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 120.0 \text{ N} \cdot \text{m} \curvearrowright \blacktriangleleft$$



PROBLEM 7.158

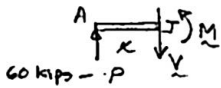
For the beam shown, determine (a) the magnitude P of the two upward forces for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding value of $|M|_{\max}$.

SOLUTION

By symmetry: $A_y = B = 60 \text{ kips} - P$

Along AC:

$$\begin{aligned} \sum M_J = 0: \quad M - x(60 \text{ kips} - P) &= 0 \\ M &= (60 \text{ kips} - P)x \\ M &= 120 \text{ kip} \cdot \text{ft} - (2 \text{ ft})P \quad \text{at } x = 2 \text{ ft} \end{aligned}$$



Along CD:

$$\begin{aligned} \sum M_K = 0: \quad M + (x - 2 \text{ ft})(60 \text{ kips}) - x(60 \text{ kips} - P) &= 0 \\ M &= 120 \text{ kip} \cdot \text{ft} - Px \\ M &= 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P \quad \text{at } x = 4 \text{ ft} \end{aligned}$$

Along DE:

$$\begin{aligned} \sum M_L = 0: \quad M - (x - 4 \text{ ft})P + (x - 2 \text{ ft})(60 \text{ kips}) \\ - x(60 \text{ kips} - P) &= 0 \\ M &= 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P \quad (\text{const}) \end{aligned}$$

Complete diagram by symmetry

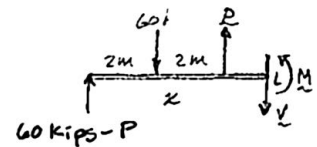
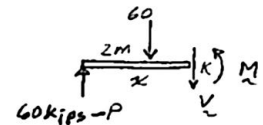
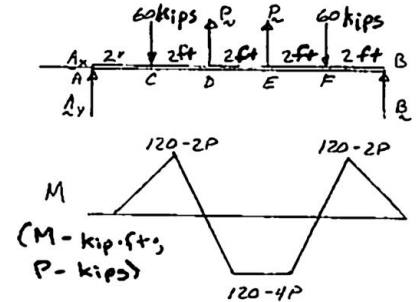
For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

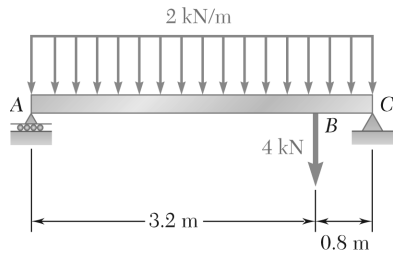
$$120 \text{ kip} \cdot \text{ft} - (2 \text{ ft})P = -[120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P]$$

(a) $P = 40.0 \text{ kips}$ ◀

$$M_{\min} = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P$$

(b) $|M|_{\max} = 40.0 \text{ kip} \cdot \text{ft}$ ◀





PROBLEM 7.159

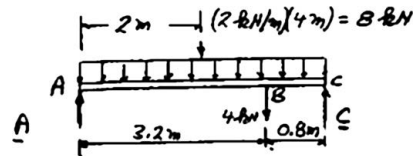
For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

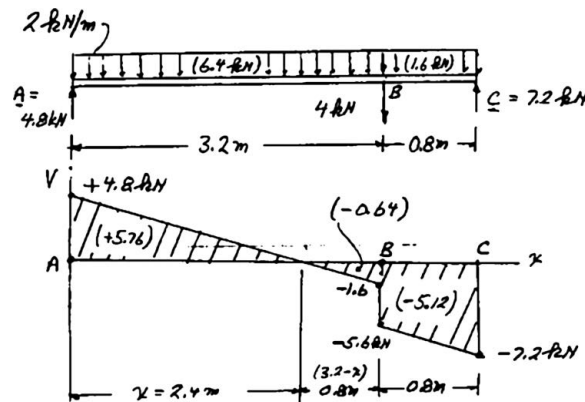
$$+\circlearrowleft \Sigma M_A = 0: (8)(2) + (4)(3.2) - 4C = 0$$

$$C = 7.2 \text{ kN} \uparrow$$

$$\Sigma F_y = 0: A = 4.8 \text{ kN} \uparrow$$



(a) Shear diagram



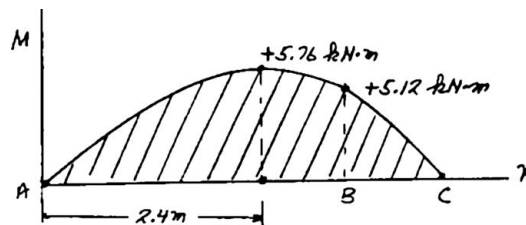
Similar Triangles:

$$\frac{x}{4.8} = \frac{3.2 - x}{1.6} = \frac{3.2}{6.4}; \quad x = 2.4 \text{ m}$$

↑

Add num. & den.

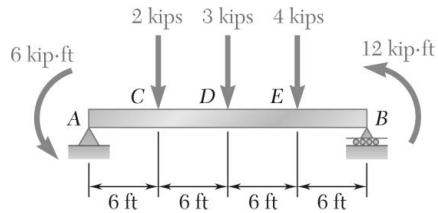
Bending-moment diagram



(b)

$$|V|_{\max} = 7.20 \text{ kN} \blacktriangleleft$$

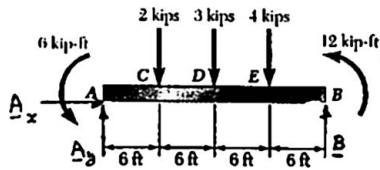
$$|M|_{\max} = 5.76 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 7.160

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

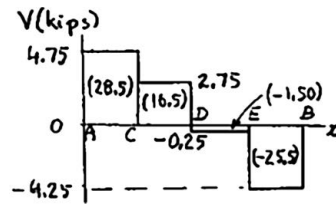


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: 6 \text{ kip} \cdot \text{ft} + 12 \text{ kip} \cdot \text{ft} + (2 \text{ kips})(18 \text{ ft}) + (3 \text{ kips})(12 \text{ ft}) + (4 \text{ kips})(6 \text{ ft}) - A_y(24 \text{ ft}) = 0$$

$$A_y = +4.75 \text{ kips} \quad \blacktriangleleft$$

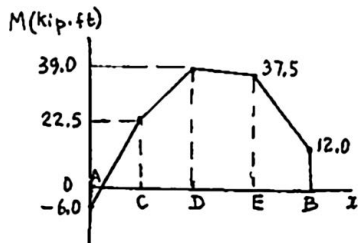
$$\Sigma F_x = 0: A_x = 0$$



Shear diagram

At A: $V_A = A_y = +4.75 \text{ kips}$

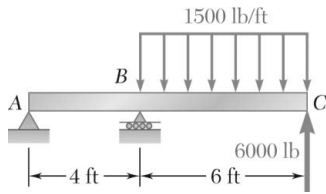
$$|V|_{\max} = 4.75 \text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = -6 \text{ kip} \cdot \text{ft}$

$$|M|_{\max} = 39.0 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$



PROBLEM 7.161

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: (6 \text{ kips})(10 \text{ ft}) - (9 \text{ kips})(7 \text{ ft}) + 8(4 \text{ ft}) = 0$$

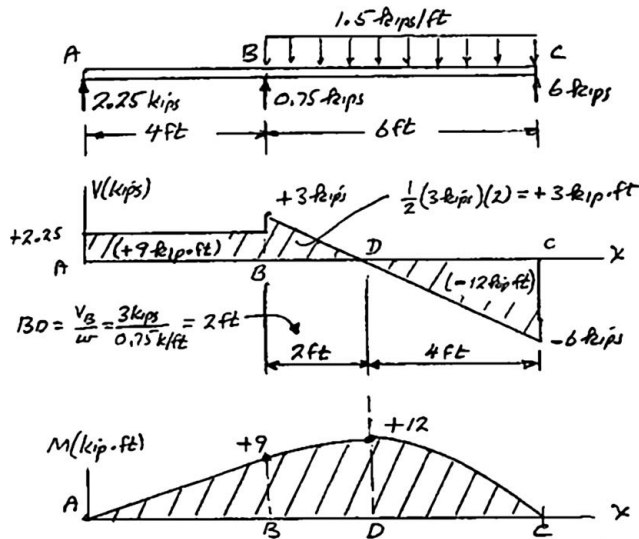
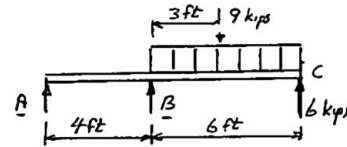
$$B = +0.75 \text{ kips}$$

$$\mathbf{B} = 0.75 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: A + 0.75 \text{ kips} - 9 \text{ kips} + 6 \text{ kips} = 0$$

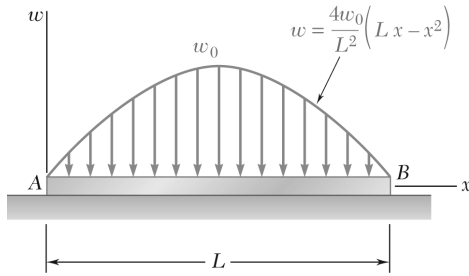
$$A = +2.25 \text{ kips}$$

$$\mathbf{A} = 2.25 \text{ kips} \uparrow$$



$$M_{\max} = 12.00 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$

6.00 ft from A



PROBLEM 7.162

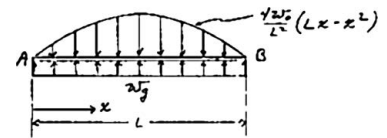
The beam AB , which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (a) write the equations of the shear and bending-moment curves, (b) determine the maximum bending moment.

SOLUTION

(a) $\uparrow \Sigma F_y = 0: w_g L - \int_0^L \frac{4w_0}{L^2} (Lx - x^2) dx = 0$

$$w_g L = \frac{4w_0}{L^2} \left(\frac{1}{2} LL^2 - \frac{1}{3} L^3 \right) = \frac{2}{3} w_0 L$$

$$w_g = \frac{2w_0}{3}$$



Define

$$\xi = \frac{x}{L} \text{ so } d\xi = \frac{dx}{L} \rightarrow \text{net load } w = 4w_0 \left[\frac{x}{L} - \left(\frac{x}{L} \right)^2 \right] - \frac{2}{3} w_0$$

or

$$w = 4w_0 \left(-\frac{1}{6} + \xi - \xi^2 \right)$$

$$V = V(0) - \int_0^\xi 4w_0 L \left(-\frac{1}{6} + \xi - \xi^2 \right) d\xi$$

$$= 0 + 4w_0 L \left(\frac{1}{6} \xi + \frac{1}{2} \xi^2 - \frac{1}{3} \xi^3 \right) \quad V = \frac{2}{3} w_0 L (\xi - 3\xi^2 + 2\xi^3) \quad \blacktriangleleft$$

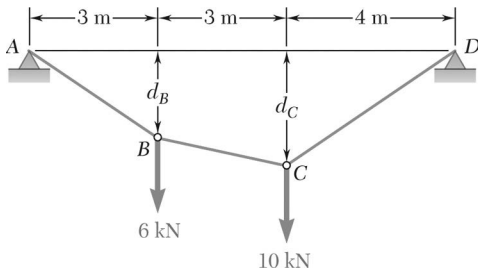
$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3} w_0 L^2 \int_0^\xi (\xi - 3\xi^2 + 2\xi^3) d\xi$$

$$= \frac{2}{3} w_0 L^2 \left(\frac{1}{2} \xi^2 - \xi^3 + \frac{1}{2} \xi^4 \right) = \frac{1}{3} w_0 L^2 (\xi^2 - 2\xi^3 + \xi^4) \quad \blacktriangleleft$$

(b) Max M occurs where $V = 0 \rightarrow 1 - 3\xi + 2\xi^2 = 0 \rightarrow \xi = \frac{1}{2}$

$$M \left(\xi = \frac{1}{2} \right) = \frac{1}{3} w_0 L^2 \left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16} \right) = \frac{w_0 L^2}{48}$$

$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam } \quad \blacktriangleleft$$

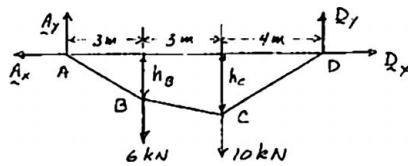


PROBLEM 7.163

Two loads are suspended as shown from the cable $ABCD$. Knowing that $d_B = 1.8$ m, determine (a) the distance d_C , (b) the components of the reaction at D , (c) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\rightarrow \Sigma F_x = 0: -A_x + D_x = 0 \quad A_x = D_x$$

$$\curvearrowleft \Sigma M_A = 0: (10 \text{ m})D_y - (6 \text{ m})(10 \text{ kN}) - (3 \text{ m})(6 \text{ kN}) = 0$$

$$D_y = 7.8 \text{ kN} \uparrow$$

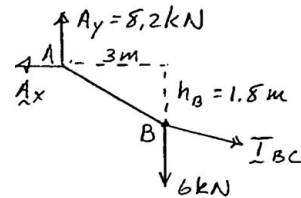
$$\uparrow \Sigma F_y = 0: A_y - 6 \text{ kN} - 10 \text{ kN} + 7.8 \text{ kN} = 0$$

$$A_y = 8.2 \text{ kN} \uparrow$$

FBD AB:

$$\curvearrowleft \Sigma M_B = 0: (1.8 \text{ m})A_x - (3 \text{ m})(8.2 \text{ kN}) = 0$$

$$A_x = \frac{41}{3} \text{ kN} \leftarrow$$



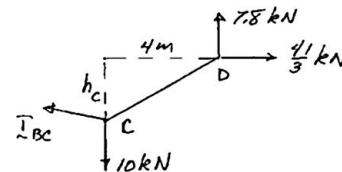
From above

$$D_x = A_x = \frac{41}{3} \text{ kN}$$

FBD CD:

$$\curvearrowleft \Sigma M_C = 0: (4 \text{ m})(7.8 \text{ kN}) - d_C \left(\frac{41}{3} \text{ kN} \right) = 0$$

$$d_C = 2.283 \text{ m}$$



(a)

$$d_C = 2.28 \text{ m} \blacktriangleleft$$

(b)

$$D_x = 13.67 \text{ kN} \rightarrow \blacktriangleleft$$

$$D_y = 7.80 \text{ kN} \uparrow \blacktriangleleft$$

Since $A_x = D_x$ and $A_y > D_y$, max T is T_{AB}

$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3} \text{ kN} \right)^2 + (8.2 \text{ kN})^2}$$

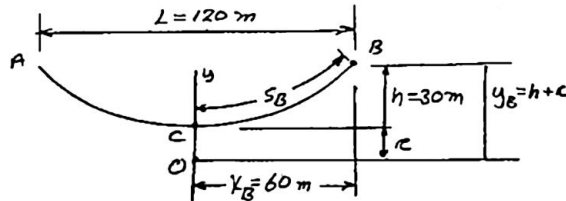
(c)

$$T_{\max} = 15.94 \text{ kN} \blacktriangleleft$$

PROBLEM 7.164

A wire having a mass per unit length of 0.65 kg/m is suspended from two supports at the same elevation that are 120 m apart. If the sag is 30 m, determine (a) the total length of the wire, (b) the maximum tension in the wire.

SOLUTION



Eq. 7.16:
$$y_B = c \cosh \frac{x_B}{c}$$

$$30\text{ m} + c = c \cosh \frac{60}{c}$$

Solve by trial and error:
$$c = 64.459\text{ m}$$

Eq. 7.15:
$$s_B = c \sinh \frac{x_B}{c}$$

$$s_B = (64.456\text{ m}) \sinh \frac{60\text{ m}}{64.459\text{ m}}$$

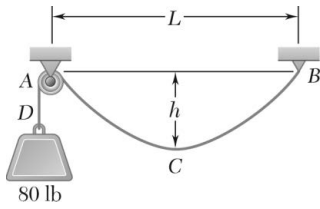
$$s_B = 69.0478\text{ m}$$

$$\text{Length} = 2s_B = 2(69.0478\text{ m}) = 138.0956\text{ m} \quad L = 138.1\text{ m} \blacktriangleleft$$

Eq. 7.18:
$$T_m = wy_B = w(h + c)$$

$$= (0.65\text{ kg/m})(9.81\text{ m/s}^2)(30\text{ m} + 64.459\text{ m})$$

$$T_m = 602.32\text{ N} \quad T_m = 602\text{ N} \blacktriangleleft$$



PROBLEM 7.165

A counterweight D is attached to a cable that passes over a small pulley at A and is attached to a support at B . Knowing that $L = 45$ ft and $h = 15$ ft, determine (a) the length of the cable from A to B , (b) the weight per unit length of the cable. Neglect the weight of the cable from A to D .

SOLUTION

Given:

$$L = 45 \text{ ft}$$

$$h = 15 \text{ ft}$$

$$T_A = 80 \text{ lb}$$

$$x_B = 22.5 \text{ ft}$$

By symmetry:

$$T_B = T_A = T_m = 80 \text{ lb}$$

We have

$$y_B = c \cosh \frac{x_B}{c} = c \cosh \frac{22.5}{c}$$

and

$$y_B = h + c = 15 + c$$

Then

$$c \cosh \frac{22.5}{c} = 15 + c$$

or

$$\cosh \frac{22.5}{c} = \frac{15}{c} + 1$$

Solve by trial for c :

$$c = 18.9525 \text{ ft}$$

(a)

$$\begin{aligned} s_B &= c \sinh \frac{x_B}{c} \\ &= (18.9525 \text{ ft}) \sinh \frac{22.5}{18.9525} \\ &= 28.170 \text{ ft} \end{aligned}$$

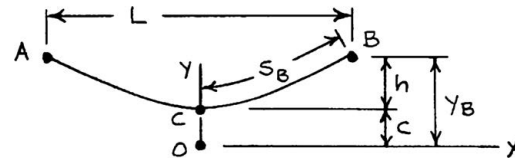
$$\text{Length} = 2s_B = 2(28.170 \text{ ft}) = 56.3 \text{ ft} \quad \blacktriangleleft$$

(b)

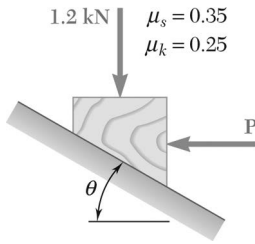
$$T_m = wy_B = w(h + c)$$

$$80 \text{ lb} = w(15 \text{ ft} + 18.9525 \text{ ft})$$

$$w = 2.36 \text{ lb/ft} \quad \blacktriangleleft$$



CHAPTER 8



PROBLEM 8.1

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25^\circ$ and $P = 750 \text{ N}$.

SOLUTION

Assume equilibrium:

$$\sum F_x = 0: F + (1200 \text{ N}) \sin 25^\circ - (750 \text{ N}) \cos 25^\circ = 0$$

$$F = +172.6 \text{ N} \qquad \mathbf{F} = 172.6 \text{ N} \searrow$$

$$\sum F_y = 0: N - (1200 \text{ N}) \cos 25^\circ - (750 \text{ N}) \sin 25^\circ = 0$$

$$N = 1404.5 \text{ N}$$

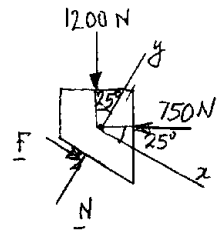
Maximum friction force: $F_m = \mu_s N = 0.35(1404.5 \text{ N}) = 491.6 \text{ N}$

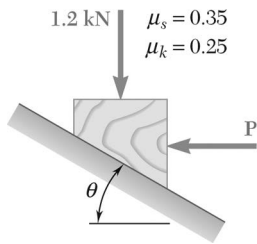
Since $F < F_m$,

block is in equilibrium ◀

Friction force:

$$\mathbf{F} = 172.6 \text{ N} \searrow 25.0^\circ \quad \blacktriangleleft$$



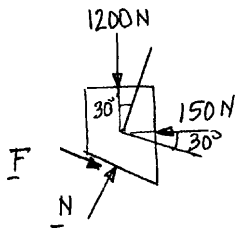


PROBLEM 8.2

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30^\circ$ and $P = 150 \text{ N}$.

SOLUTION

Assume equilibrium:



$$\swarrow \Sigma F_x = 0: F + (1200 \text{ N}) \sin 30^\circ - (150 \text{ N}) \cos 30^\circ = 0$$

$$F = -470.1 \text{ N} \qquad \mathbf{F} = 470.1 \text{ N} \nearrow$$

$$\nearrow \Sigma F_y = 0: N - (1200 \text{ N}) \cos 30^\circ - (150 \text{ N}) \sin 30^\circ = 0$$

$$N = 1114.2 \text{ N}$$

(a) Maximum friction force: $F_m = \mu_s N$
 $= 0.35(1114.2 \text{ N})$
 $= 390.0 \text{ N}$

Since \mathbf{F} is \nearrow and $F > F_m$, block moves down \blacktriangleleft

(b) Actual friction force: $F = F_k = \mu_k N = 0.25(1114.2 \text{ N}) = 279 \text{ N}$ $\mathbf{F} = 279 \text{ N} \searrow 30.0^\circ$ \blacktriangleleft

PROBLEM 8.3

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 100$ lb.

SOLUTION

Assume equilibrium:

$$\begin{aligned} \sum F_x = 0: & F + (45 \text{ lb}) \sin 30^\circ - (100 \text{ lb}) \cos 40^\circ = 0 \\ & F = +54.0 \text{ lb} \\ \sum F_y = 0: & N - (45 \text{ lb}) \cos 30^\circ - (100 \text{ lb}) \sin 40^\circ = 0 \\ & N = 103.2 \text{ lb} \end{aligned}$$

(a) Maximum friction force:

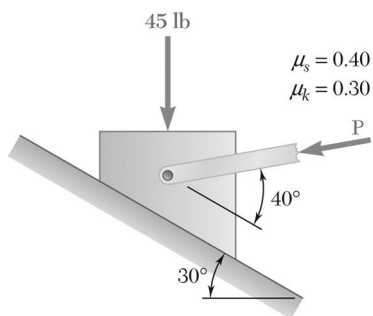
$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(103.2 \text{ lb}) \\ &= 41.30 \text{ lb} \end{aligned}$$

We note that $F > F_m$. Thus, block moves up ◀

(b) Actual friction force:

$$F = F_k = \mu_k N = 0.30(103.2 \text{ lb}) = 30.97 \text{ lb}, \quad \mathbf{F = 31.0 \text{ lb} } \swarrow 30.0^\circ \text{ } \blacktriangleleft$$

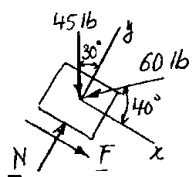
PROBLEM 8.4



Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 60$ lb.

SOLUTION

Assume equilibrium:



$$\rightarrow \Sigma F_x = 0: F + (45 \text{ lb}) \sin 30^\circ - (60 \text{ lb}) \cos 40^\circ = 0$$

$$F = +23.46 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: N - (45 \text{ lb}) \cos 30^\circ - (60 \text{ lb}) \sin 40^\circ = 0$$

$$N = 77.54 \text{ lb}$$

(a) Maximum friction force:

$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(77.54 \text{ lb}) \\ &= 31.02 \text{ lb} \end{aligned}$$

We check that $F < F_m$. Thus,

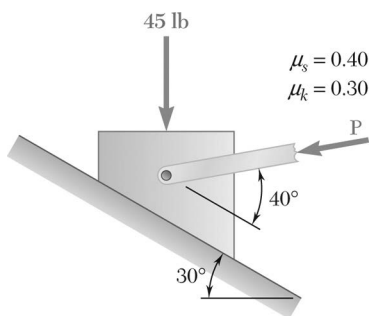
block is in equilibrium ◀

(b) Thus

$$F = +23.46 \text{ lb}$$

$$\mathbf{F} = 23.5 \text{ lb} \searrow 30.0^\circ \quad \blacktriangleleft$$

Note: We have $F_k = \mu_k N = 0.30(77.54) = 23.26$ lb. Thus $F > F_k$. If block originally in motion, it will keep moving with $F_k = 23.26$ lb.



PROBLEM 8.5

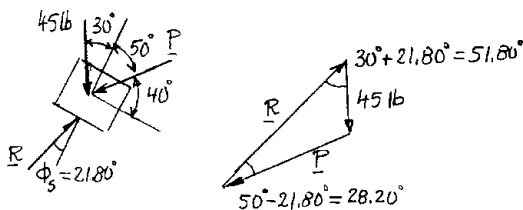
Determine the smallest value of P required to (a) start the block up the incline, (b) keep it moving up, (c) prevent it from moving down.

SOLUTION

(a) To start block up the incline:

$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40 = 21.80^\circ$$



From force triangle:

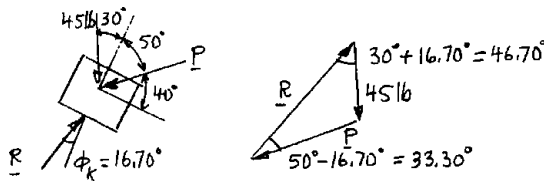
$$\frac{P}{\sin 51.80^\circ} = \frac{45 \text{ lb}}{\sin 28.20^\circ}$$

$$P = 74.8 \text{ lb} \quad \blacktriangleleft$$

(b) To keep block moving up:

$$\mu_k = 0.30$$

$$\phi_k = \tan^{-1} 0.30 = 16.70^\circ$$



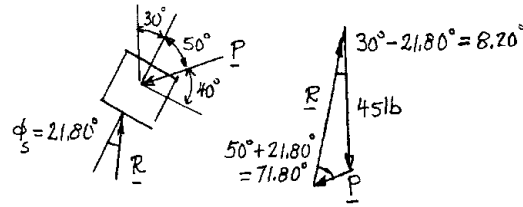
From force triangle:

$$\frac{P}{\sin 46.70^\circ} = \frac{45 \text{ lb}}{\sin 33.30^\circ}$$

$$P = 59.7 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 8.5 (Continued)

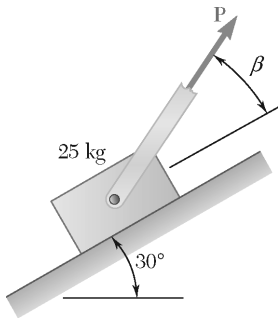
(c) To prevent block from moving down:



From force triangle:

$$\frac{P}{\sin 8.20^\circ} = \frac{45 \text{ lb}}{\sin 71.80^\circ}$$

$$P = 6.76 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 8.6

Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25$, determine (a) the smallest value of P required to start the block moving up the incline, (b) the corresponding value of β .

SOLUTION

FBD block (Impending motion up)

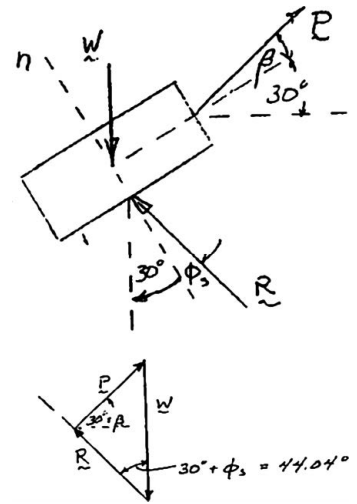
$$\begin{aligned} W &= mg \\ &= (25 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 245.25 \text{ N} \end{aligned}$$

$$\begin{aligned} \phi_s &= \tan^{-1} \mu_s \\ &= \tan^{-1}(0.25) \\ &= 14.04^\circ \end{aligned}$$

(a) (Note: For minimum P , $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$.)

Then

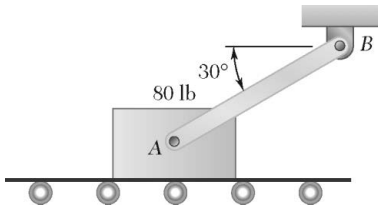
$$\begin{aligned} P &= W \sin(30^\circ + \phi_s) \\ &= (245.25 \text{ N}) \sin 44.04^\circ \end{aligned}$$



$$P_{\min} = 170.5 \text{ N} \quad \blacktriangleleft$$

(b) We have $\beta = \phi_s$

$$\beta = 14.04^\circ \quad \blacktriangleleft$$



PROBLEM 8.7

The 80-lb block is attached to link AB and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force \mathbf{P} that should be applied to the belt to maintain its motion (a) to the right, (b) to the left.

SOLUTION

We note that link AB is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31^\circ$

(a) Belt moves to right

Free body: Block

Force triangle:

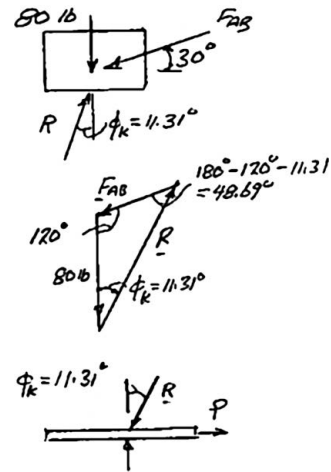
$$\frac{R}{\sin 120^\circ} = \frac{80 \text{ lb}}{\sin 48.69^\circ}$$

$$R = 92.23 \text{ lb}$$

Free body: Belt

$$\rightarrow \Sigma F_x = 0: P - (92.23 \text{ lb}) \sin 11.31^\circ$$

$$P = 18.089 \text{ lb}$$



$$P = 18.09 \text{ lb} \rightarrow$$

(b) Belt moves to left

Free body: Block

Force triangle:

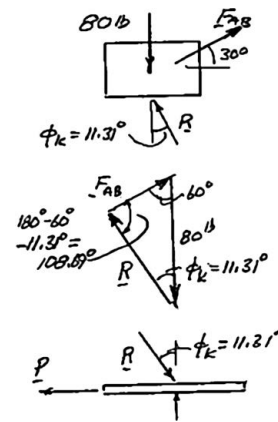
$$\frac{R}{\sin 60^\circ} = \frac{80 \text{ lb}}{\sin 108.69^\circ}$$

$$R = 73.139 \text{ lb}$$

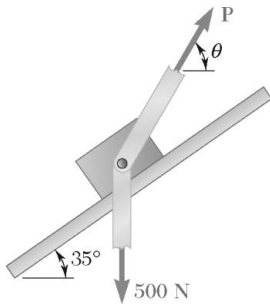
Free body: Belt

$$\rightarrow \Sigma F_x = 0: (73.139 \text{ lb}) \sin 11.31^\circ - P = 0$$

$$P = 14.344 \text{ lb}$$



$$P = 14.34 \text{ lb} \leftarrow$$



PROBLEM 8.8

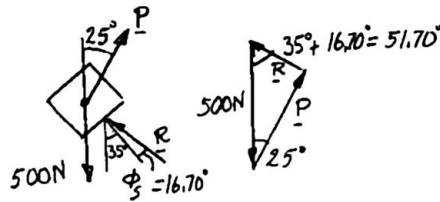
The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65^\circ$, determine the smallest value of P required (a) to start the block moving up the rail, (b) to keep it from moving down.

SOLUTION

(a) To start block up the rail:

$$\mu_s = 0.30$$

$$\phi_s = \tan^{-1} 0.30 = 16.70^\circ$$

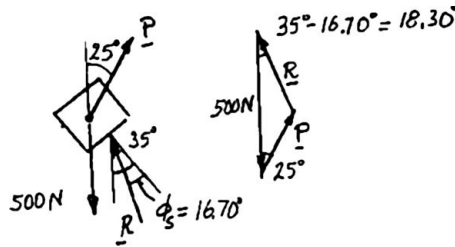


Force triangle:

$$\frac{P}{\sin 51.70^\circ} = \frac{500 \text{ N}}{\sin (180^\circ - 25^\circ - 51.70^\circ)}$$

$$P = 403 \text{ N} \quad \blacktriangleleft$$

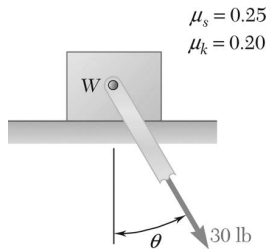
(b) To prevent block from moving down:



Force triangle:

$$\frac{P}{\sin 18.30^\circ} = \frac{500 \text{ N}}{\sin (180^\circ - 25^\circ - 18.30^\circ)}$$

$$P = 229 \text{ N} \quad \blacktriangleleft$$

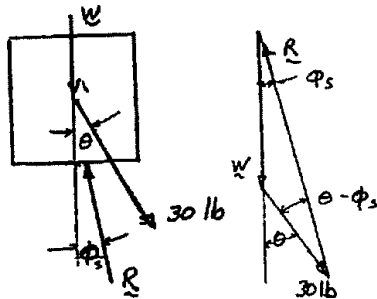


PROBLEM 8.9

Considering only values of θ less than 90° , determine the smallest value of θ required to start the block moving to the right when (a) $W = 75$ lb, (b) $W = 100$ lb.

SOLUTION

FBD block (Motion impending):



$$\phi_s = \tan^{-1} \mu_s = 14.036^\circ$$

$$\frac{30 \text{ lb}}{\sin \phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

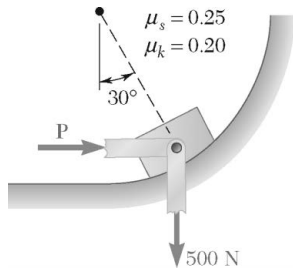
$$\sin(\theta - \phi_s) = \frac{W \sin 14.036^\circ}{30 \text{ lb}}$$

or

$$\sin(\theta - 14.036^\circ) = \frac{W}{123.695 \text{ lb}}$$

(a) $W = 75 \text{ lb: } \theta = 14.036^\circ + \sin^{-1} \frac{75 \text{ lb}}{123.695 \text{ lb}} \quad \theta = 51.4^\circ \blacktriangleleft$

(b) $W = 100 \text{ lb: } \theta = 14.036^\circ + \sin^{-1} \frac{100 \text{ lb}}{123.695 \text{ lb}} \quad \theta = 68.0^\circ \blacktriangleleft$



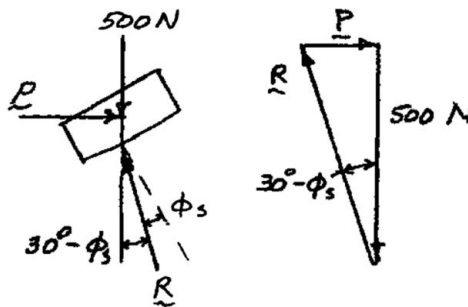
PROBLEM 8.10

Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block:

(Impending motion down):

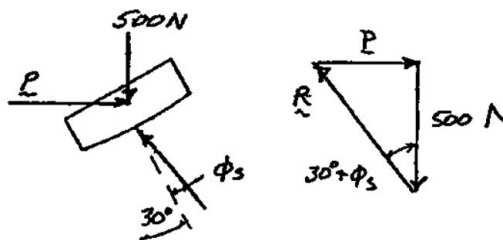


$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.25$$

$$P = (500 \text{ N}) \tan(30^\circ - \tan^{-1} 0.25)$$

$$= 143.03 \text{ N}$$

(Impending motion up):

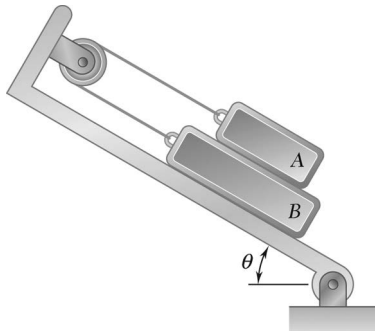


$$P = (500 \text{ N}) \tan(30^\circ + \tan^{-1} 0.25)$$

$$= 483.46 \text{ N}$$

Equilibrium is maintained for

$$143.0 \text{ N} \leq P \leq 483 \text{ N} \quad \blacktriangleleft$$



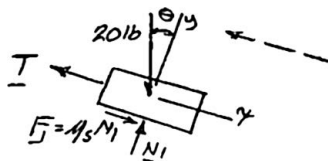
PROBLEM 8.11

The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ between $A + B$

Free body: Block A



Impending motion:

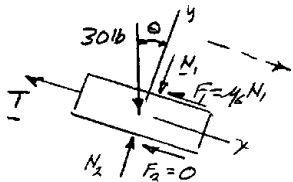
$$\Sigma F_y = 0: N_1 = 20 \cos \theta$$

$$\Sigma F_x = 0: T - 20 \sin \theta - \mu_s N_1 = 0$$

$$T = 20 \sin \theta + 0.15(20 \cos \theta)$$

$$T = 20 \sin \theta + 3 \cos \theta \quad (1)$$

Free body: Block B



Impending motion:

$$\Sigma F_x = 0: T - 30 \sin \theta + \mu_s N_1 = 0$$

$$T = 30 \sin \theta - \mu_s N_1$$

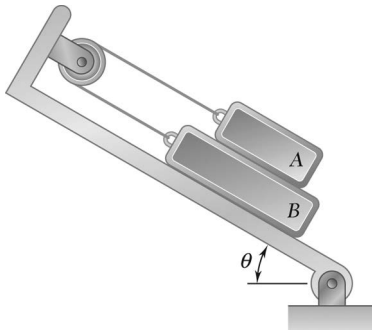
$$= 30 \sin \theta - 0.15(20 \cos \theta)$$

$$T = 30 \sin \theta - 3 \cos \theta \quad (2)$$

Eq. (1)-Eq. (2): $20 \sin \theta + 3 \cos \theta - 30 \sin \theta + 3 \cos \theta = 0$

$$6 \cos \theta = 10 \sin \theta: \tan \theta = \frac{6}{10}; \theta = 30.96^\circ$$

$$\theta = 31.0^\circ \blacktriangleleft$$



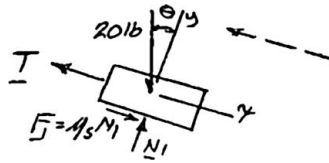
PROBLEM 8.12

The 20-lb block A and the 30-lb block B are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

Free body: Block A



Impending motion:

$$\Sigma F_y = 0: N_1 = 20 \cos \theta$$

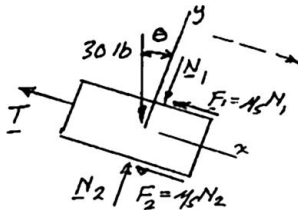
$$\Sigma F_x = 0: T - 20 \sin \theta - \mu_s N_1 = 0$$

$$T = 20 \sin \theta + 0.15(20 \cos \theta)$$

$$T = 20 \sin \theta + 3 \cos \theta$$

(1)

Free body: Block B



Impending motion:

$$\Sigma F_y = 0: N_2 - 30 \cos \theta - N_1 = 0$$

$$N_2 = 30 \cos \theta + 20 \cos \theta = 50 \cos \theta$$

$$F_2 = \mu_s N_2 = 0.15(50 \cos \theta) = 7.5 \cos \theta$$

$$\Sigma F_x = 0: T - 30 \sin \theta + \mu_s N_1 + \mu_s N_2 = 0$$

$$T = 30 \sin \theta - 0.15(20 \cos \theta) - 0.15(50 \cos \theta)$$

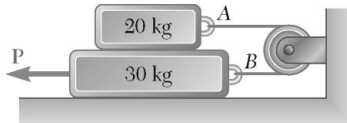
$$T = 30 \sin \theta - 3 \cos \theta - 7.5 \cos \theta$$

(2)

Eq. (1) subtracted by Eq. (2): $20 \sin \theta + 3 \cos \theta - 30 \sin \theta + 3 \cos \theta + 7.5 \cos \theta = 0$

$$13.5 \cos \theta = 10 \sin \theta, \quad \tan \theta = \frac{13.5}{10}$$

$$\theta = 53.5^\circ \blacktriangleleft$$



PROBLEM 8.13

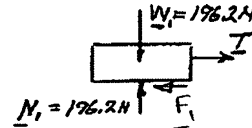
The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force \mathbf{P} required to start the 30-kg block moving if cable AB (a) is attached as shown, (b) is removed.

SOLUTION

(a) Free body: 20-kg block

$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$



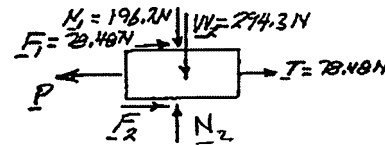
$$\pm \rightarrow \Sigma F = 0: T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block

$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$



$$\leftarrow \pm \Sigma F = 0: P - F_1 - F_2 - T = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} + 78.48 \text{ N} = 353.2 \text{ N}$$

$$\mathbf{P} = 353 \text{ N} \leftarrow$$

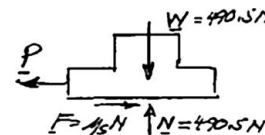
(b) Free body: Both blocks

Blocks move together

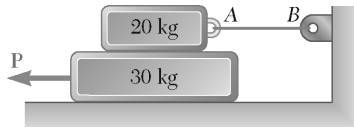
$$W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$$

$$\leftarrow \pm \Sigma F = 0: P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$



$$\mathbf{P} = 196.2 \text{ N} \leftarrow$$



PROBLEM 8.14

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force \mathbf{P} required to start the 30-kg block moving if cable AB (a) is attached as shown, (b) is removed.

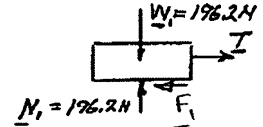
SOLUTION

(a) Free body: 20-kg block

$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$

$$\pm \rightarrow \Sigma F = 0: T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$



Free body: 30-kg block

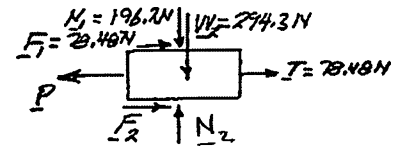
$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

$$\pm \rightarrow \Sigma F = 0: P - F_1 - F_2 = 0$$

$$P = 78.48 \text{ N} + 196.2 \text{ N} = 274.7 \text{ N}$$



$$\mathbf{P} = 275 \text{ N} \leftarrow$$

(b) Free body: Both blocks

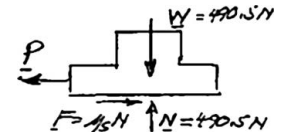
Blocks move together

$$W = (50 \text{ kg})(9.81 \text{ m/s}^2)$$

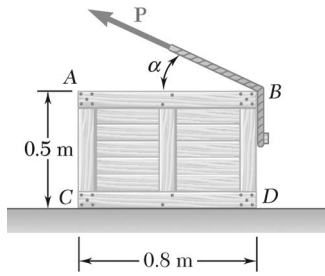
$$= 490.5 \text{ N}$$

$$\pm \rightarrow \Sigma F = 0: P - F = 0$$

$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$



$$\mathbf{P} = 196.2 \text{ N} \leftarrow$$



PROBLEM 8.15

A 40-kg packing crate must be moved to the left along the floor without tipping. Knowing that the coefficient of static friction between the crate and the floor is 0.35, determine (a) the largest allowable value of α , (b) the corresponding magnitude of the force \mathbf{P} .

SOLUTION

(a) Free-body diagram

If the crate is about to tip about C , contact between crate and ground is only at C and the reaction \mathbf{R} is applied at C . As the crate is about to slide, \mathbf{R} must form with the vertical an angle

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.35 = 19.29^\circ$$

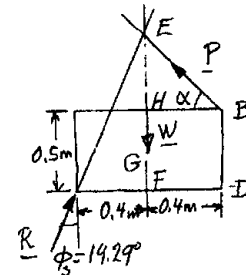
Since the crate is a 3-force body, \mathbf{P} must pass through E where \mathbf{R} and \mathbf{W} intersect.

$$EF = \frac{CF}{\tan \theta_s} = \frac{0.4 \text{ m}}{0.35} = 1.1429 \text{ m}$$

$$EH = EF - HF = 1.1429 - 0.5 = 0.6429 \text{ m}$$

$$\tan \alpha = \frac{EH}{HB} = \frac{0.6429 \text{ m}}{0.4 \text{ m}}$$

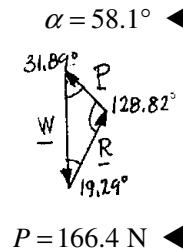
$$\alpha = 58.11^\circ$$



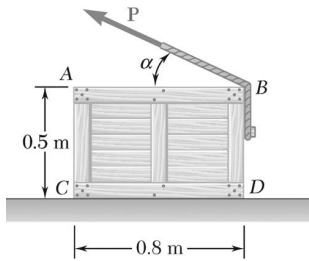
(b) Force Triangle

$$\frac{P}{\sin 19.29^\circ} = \frac{W}{\sin 128.82^\circ} \quad P = 0.424 W$$

$$P = 0.424(40 \text{ kg})(9.81 \text{ m/s}^2),$$



Note: After the crate starts moving, μ_s should be replaced by the lower value μ_k . This will yield a larger value of α .



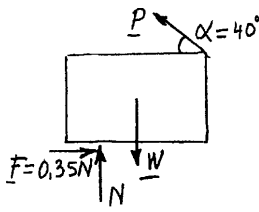
PROBLEM 8.16

A 40-kg packing crate is pulled by a rope as shown. The coefficient of static friction between the crate and the floor is 0.35. If $\alpha = 40^\circ$, determine (a) the magnitude of the force \mathbf{P} required to move the crate, (b) whether the crate will slide or tip.

SOLUTION

Force P for which sliding is impending

(We assume that crate does not tip)



$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: N - W + P \sin 40^\circ = 0$$

$$N = W - P \sin 40^\circ \quad (1)$$

$$+\rightarrow \Sigma F_x = 0: 0.35 N - P \cos 40^\circ = 0$$

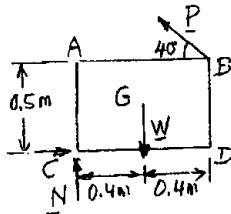
Substitute for N from Eq. (1):

$$0.35(W - P \sin 40^\circ) - P \cos 40^\circ = 0$$

$$P = \frac{0.35W}{0.35 \sin 40^\circ + \cos 40^\circ} \quad P = 0.3532W \quad \blacktriangleleft$$

Force P for which crate rotates about C

(We assume that crate does not slide)



$$+\curvearrowright \Sigma M_C = 0: (P \sin 40^\circ)(0.8 \text{ m}) + (P \cos 40^\circ)(0.5 \text{ m})$$

$$-W(0.4 \text{ m}) = 0$$

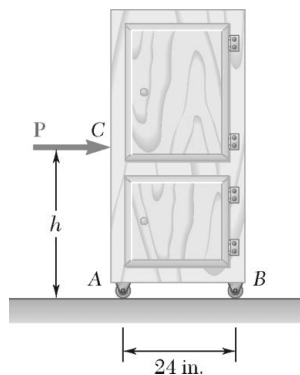
$$P = \frac{0.4W}{0.8 \sin 40^\circ + 0.5 \cos 40^\circ} = 0.4458W \quad \blacktriangleleft$$

Crate will first slide \blacktriangleleft

$$P = 0.3532(392.4 \text{ N})$$

$$P = 138.6 \text{ N} \quad \blacktriangleleft$$

PROBLEM 8.17



A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 32$ in., determine the magnitude of the force \mathbf{P} required to move the cabinet to the right (a) if all casters are locked, (b) if the casters at B are locked and the casters at A are free to rotate, (c) if the casters at A are locked and the casters at B are free to rotate.

SOLUTION

FBD cabinet: Note: for tipping,

$$N_A = F_A = 0$$

$$\left(\sum M_B = 0: (12 \text{ in.})W - (32 \text{ in.})P_{\text{tip}} = 0 \right.$$

$$P_{\text{tip}} = 2.66667$$

(a) All casters locked. Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$\uparrow \sum F_y = 0: N_A + N_B - W = 0$$

$$N_A + N_B = W$$

So $F_A + F_B = \mu_s W$

$$\rightarrow \sum F_x = 0: P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb})$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(b) Casters at A free, so

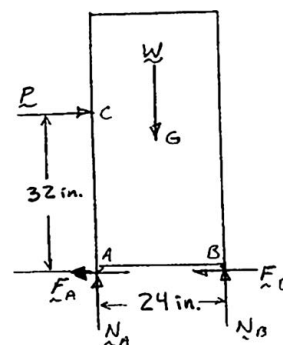
$$F_A = 0$$

Impending slip:

$$F_B = \mu_s N_B$$

$$\rightarrow \sum F_x = 0: P - F_B = 0$$

$$P = F_B = \mu_s N_B \quad N_B = \frac{P}{\mu_s}$$



$$W = 120 \text{ lb}$$

$$\mu_s = 0.3$$

or $\mathbf{P} = 36.0 \text{ lb} \rightarrow \blacktriangleleft$

PROBLEM 8.17 (Continued)

$$\left(\sum M_A = 0: (32 \text{ in.})P + (12 \text{ in.})W - (24 \text{ in.})N_B = 0 \right.$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \quad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$P = 0.25(120 \text{ lb})$$

$$\text{or } \mathbf{P = 30.0 \text{ lb}} \rightarrow \blacktriangleleft$$

(c) Casters at B free, so

$$F_B = 0$$

Impending slip:

$$F_A = \mu_s N_A$$

$$\rightarrow \sum F_x = 0: P - F_A = 0 \quad P = F_A = \mu_s N_A$$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

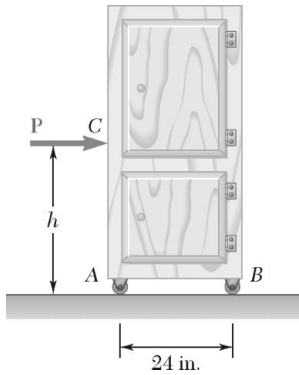
$$\left(\sum M_B = 0: (12 \text{ in.})W - (32 \text{ in.})P - (24 \text{ in.})N_A = 0 \right.$$

$$3W - 8P - 6\frac{P}{0.3} = 0$$

$$P = 0.107143W = 12.8572$$

$$(P < P_{\text{tip}} \quad \text{OK})$$

$$\mathbf{P = 12.86 \text{ lb}} \rightarrow \blacktriangleleft$$



PROBLEM 8.18

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both *A* and *B* are locked, determine (a) the force **P** required to move the cabinet to the right, (b) the largest allowable value of *h* if the cabinet is not to tip over.

SOLUTION

FBD cabinet:

$$(a) \quad \uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W$$

Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

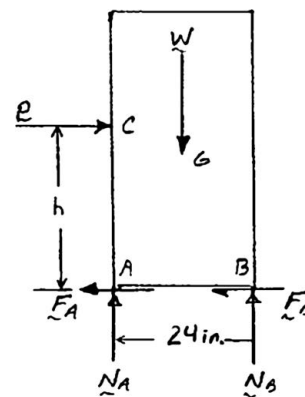
So

$$F_A + F_B = \mu_s W$$

$$\rightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb}) = 141.26 \text{ N}$$



$$W = 120 \text{ lb}$$

$$\mu_s = 0.3$$

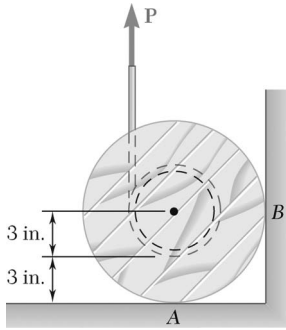
$$\mathbf{P} = 36.0 \text{ lb} \rightarrow \blacktriangleleft$$

$$(b) \quad \text{For tipping,} \quad N_A = F_A = 0$$

$$\curvearrowleft \Sigma M_B = 0: \quad hP - (12 \text{ in.})W = 0$$

$$h_{\max} = (12 \text{ in.}) \frac{W}{P} = (12 \text{ in.}) \frac{1}{\mu_s} = \frac{12 \text{ in.}}{0.3}$$

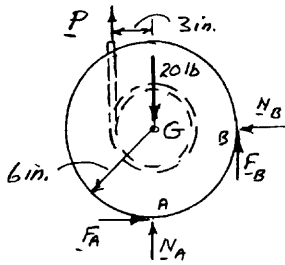
$$h_{\max} = 40.0 \text{ in.} \blacktriangleleft$$



PROBLEM 8.19

Wire is being drawn at a constant rate from a spool by applying a vertical force \mathbf{P} to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both A and B are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force \mathbf{P} .

SOLUTION



Since spool is rotating

$$F_A = \mu_k N_A \quad F_B = \mu_k N_B$$

$$+\curvearrowright \Sigma M_G = 0: P(3 \text{ in.}) - F_A(6 \text{ in.}) - F_B(6 \text{ in.}) = 0$$

$$3P - 6\mu_k(N_A + N_B) = 0 \quad (1)$$

$$\pm \rightarrow \Sigma F_x = 0: F_A - N_B = 0$$

$$N_B = \mu_k N_A \quad (2)$$

$$+\uparrow \Sigma F_y = 0: P + N_A + F_B - 20 \text{ lb} = 0$$

$$P + N_A + \mu_k N_B - 20 = 0$$

$$P + N_A + \mu_k N_A - 20 = 0$$

Substitute for N_B from (2):

$$N_A = \frac{20 - P}{1 + \mu_k^2} \quad (3)$$

Substitute from (2) into (1):

$$3P - 6\mu_k(N_A + \mu_k N_A) = 0$$

$$N_A = \frac{1}{2} \frac{P}{\mu_k(1 + \mu_k)} \quad (4)$$

(3)=(4):

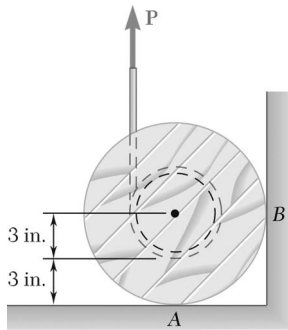
$$\frac{20 - P}{1 + \mu_k^2} = \frac{P}{2(\mu_k + \mu_k^2)}$$

Substitute $\mu_k = 0.30$:

$$\frac{20 - P}{1 + (0.3)^2} = \frac{P}{2(0.3)(1.03)}$$

$$20 - P = 1.3974P; \quad 2.3974P = 20;$$

$$P = 8.34 \text{ lb} \quad \blacktriangleleft$$

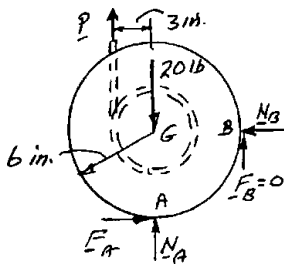


PROBLEM 8.20

Solve Problem 8.19 assuming that the coefficients of friction at B are zero.

PROBLEM 8.19 Wire is being drawn at a constant rate from a spool by applying a vertical force \mathbf{P} to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both A and B are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force \mathbf{P} .

SOLUTION



Since spool is rotating

$$F_A = \mu_k N_A$$

$$+\curvearrowright \Sigma M_G = 0: P(3 \text{ in.}) - F_A(6 \text{ in.}) = 0$$

$$P = 2F_A = 2\mu_k N_A \quad (1)$$

$$+\uparrow \Sigma F_y = 20: P - 20 \text{ lb} + N_A = 0$$

$$N_A = 20 - P \quad (2)$$

Substitute for N_A from (2) into (1) $P = 2\mu_k(20 - P)$

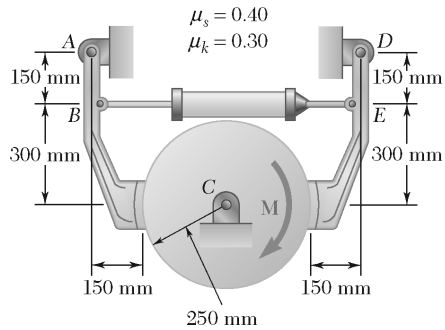
Substitute $\mu_k = 0.30$: $P = 2(0.3)(20 - P)$

$$1.667P = 20 - P$$

$$2.667P = 20$$

$$P = 7.50 \text{ lb}$$

$$P = 7.50 \text{ lb} \quad \blacktriangleleft$$

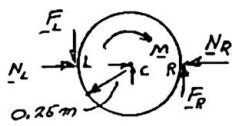


PROBLEM 8.21

The hydraulic cylinder shown exerts a force of 3 kN directed to the right on Point B and to the left on Point E. Determine the magnitude of the couple M required to rotate the drum clockwise at a constant speed.

SOLUTION

Free body: Drum



$$+\circlearrowleft \Sigma M_C = 0: \quad M - (0.25 \text{ m})(F_L + F_R) = 0$$

$$M = (0.25 \text{ m})(F_L + F_R) \quad (1)$$

Since drum is rotating

$$F_L = \mu_k N_L = 0.3N_L$$

$$F_R = \mu_k N_R = 0.3N_R$$

Free body: Left arm ABL

$$+\circlearrowleft \Sigma M_A = 0: \quad (3 \text{ kN})(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.45 \text{ kN} \cdot \text{m} + (0.3N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.405N_L = 0.45$$

$$N_L = 1.111 \text{ kN}$$

$$F_L = 0.3N_L = 0.3(1.111 \text{ kN})$$

$$= 0.3333 \text{ kN} \quad (2)$$

Free body: Right arm DER

$$+\circlearrowleft \Sigma M_D = 0: \quad (3 \text{ kN})(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.45 \text{ kN} \cdot \text{m} - (0.3N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.495N_R = 0.45$$

$$N_R = 0.9091 \text{ kN}$$

$$F_R = \mu_k N_R = 0.3(0.9091 \text{ kN})$$

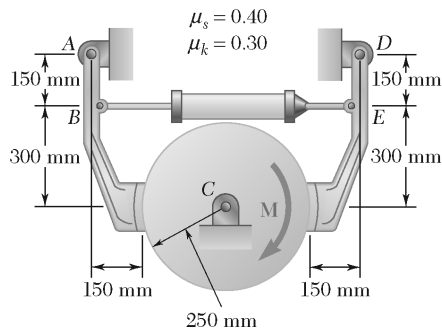
$$= 0.2727 \text{ kN} \quad (3)$$

Substitute for F_L and F_R into (1):

$$M = (0.25 \text{ m})(0.333 \text{ kN} + 0.2727 \text{ kN})$$

$$M = 0.1515 \text{ kN} \cdot \text{m}$$

$$M = 151.5 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

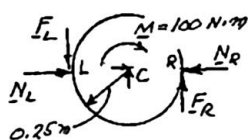


PROBLEM 8.22

A couple M of magnitude $100 \text{ N} \cdot \text{m}$ is applied to the drum as shown. Determine the smallest force that must be exerted by the hydraulic cylinder on joints B and E if the drum is not to rotate.

SOLUTION

Free body: Drum



$$+\circlearrowleft \Sigma M_C = 0: 100 \text{ N} \cdot \text{m} - (0.25 \text{ m})(F_L + F_R) = 0$$

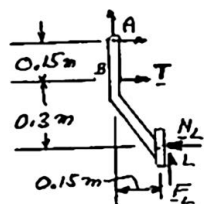
$$F_L + F_R = 400 \text{ N} \quad (1)$$

Since motion impends

$$F_L = \mu_s N_L = 0.4 N_L$$

$$F_R = \mu_s N_R = 0.4 N_R$$

Free body: Left arm ABL



$$+\circlearrowleft \Sigma M_A = 0: T(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

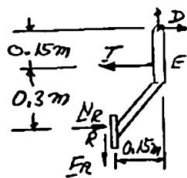
$$0.15T + (0.4N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.39N_L = 0.15T; \quad N_L = 0.38462T$$

$$F_L = 0.4N_L = 0.4(0.38462T)$$

$$F_L = 0.15385T \quad (2)$$

Free body: Right arm DER



$$+\circlearrowleft \Sigma M_D = 0: T(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.15T - (0.4N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.51N_R = 0.15T; \quad N_R = 0.29412T$$

$$F_R = 0.4N_R = 0.4(0.29412T)$$

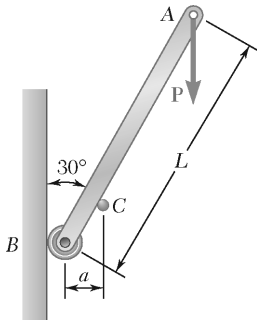
$$F_R = 0.11765T \quad (3)$$

Substitute for F_L and F_R into Eq. (1):

$$0.15385T + 0.11765T = 400$$

$$T = 1473.3 \text{ N}$$

$$T = 1.473 \text{ kN} \quad \blacktriangleleft$$



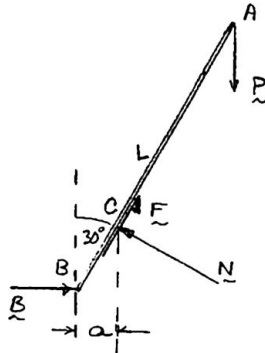
PROBLEM 8.23

A slender rod of length L is lodged between peg C and the vertical wall and supports a load P at end A . Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:

Free-body diagram: For motion of B impending upward:



$$+\curvearrowright \Sigma M_B = 0: \quad PL \sin \theta - N_C \left(\frac{a}{\sin \theta} \right) = 0$$

$$N_C = \frac{PL}{a} \sin^2 \theta \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C \sin \theta - \mu_s N_C \cos \theta - P = 0$$

$$N_C (\sin \theta - \mu \cos \theta) = P$$

Substitute for N_C from Eq. (1), and solve for a/L .

$$\frac{a}{L} = \sin^2 \theta (\sin \theta - \mu_s \cos \theta) \quad (2)$$

For $\theta = 30^\circ$ and $\mu_s = 0.15$:

$$\frac{a}{L} = \sin^2 30^\circ (\sin 30^\circ - 0.15 \cos 30^\circ)$$

$$\frac{a}{L} = 0.092524 \quad \frac{L}{a} = 10.808$$

For motion of B impending downward, reverse sense of friction force F_C . To do this we make

$\mu_s = -0.15$ in Eq. (2).

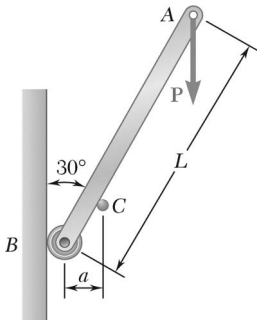
Eq. (2):

$$\frac{a}{L} = \sin^2 30^\circ (\sin 30^\circ - (-0.15) \cos 30^\circ)$$

$$\frac{a}{L} = 0.15748 \quad \frac{L}{a} = 6.350$$

Range of values of L/a for equilibrium:

$$6.35 \leq \frac{L}{a} \leq 10.81 \quad \blacktriangleleft$$



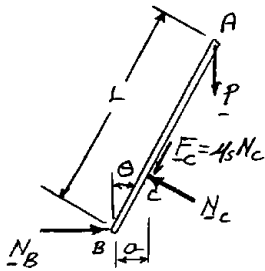
PROBLEM 8.24

Solve Problem 8.23 assuming that the coefficient of static friction between the peg and the rod is 0.60.

PROBLEM 8.23 A slender rod of length L is lodged between peg C and the vertical wall and supports a load P at end A . Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

Free-body diagram: For motion of B impending upward



$$+\curvearrowright \Sigma M_B = 0: \quad PL \sin \theta - N_C \left(\frac{a}{\sin \theta} \right) = 0$$

$$N_C = \frac{PL}{a} \sin^2 \theta \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C \sin \theta - \mu_s N_C \cos \theta - P = 0$$

$$N_C (\sin \theta - \mu_s \cos \theta) = P$$

Substitute for N_C from (1), and solve for $\frac{a}{L}$

$$\frac{a}{L} = \sin^2 \theta (\sin \theta - \mu_s \cos \theta)$$

For $\theta = 30^\circ$ and $\mu_s = 0.60$:

$$\frac{a}{L} = \sin^2 30^\circ (\sin 30^\circ - 0.60 \cos 30^\circ) \quad (2)$$

$$\frac{a}{L} = -0.0049 < 0$$

Thus, slipping of B upward does not occur for motion of B impending downward, reverse sense of friction force F_C . To do this we make $\mu_C = -0.60$ in Eq. (2).

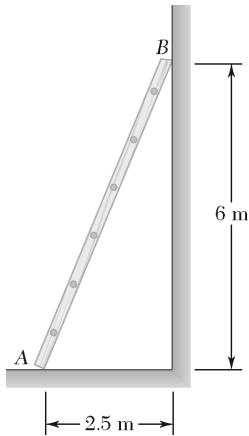
$$\frac{a}{L} = \sin^2 30^\circ (\sin 30^\circ - (-0.60) \cos 30^\circ)$$

$$\frac{a}{L} = 0.2459 \quad \frac{L}{a} = 3.923$$

Range of L/a for equilibrium:

$$L/a \geq 3.92 \quad \blacktriangleleft$$

PROBLEM 8.25



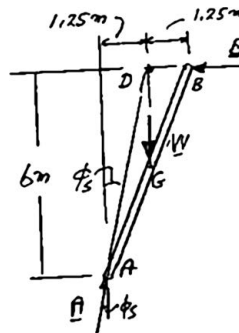
A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at B , determine the smallest value of μ_s at A for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Three-force body.

Line of action of \mathbf{A} must pass through D , where \mathbf{W} and \mathbf{B} intersect.

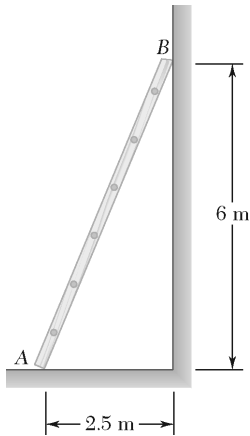


At A :

$$\mu_s = \tan \phi_s = \frac{1.25 \text{ m}}{6 \text{ m}} = 0.2083$$

$$\mu_s = 0.208 \quad \blacktriangleleft$$

PROBLEM 8.26



A 6.5-m ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B , determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Free body: Ladder

Motion impending:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$+\circlearrowleft \Sigma M_A = 0: W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \quad (1)$$

$$+\uparrow \Sigma F_y = 0: N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$

$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \quad (2)$$

$$+\rightarrow \Sigma F_x = 0: \mu_s N_A - N_B = 0$$

Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$

$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$

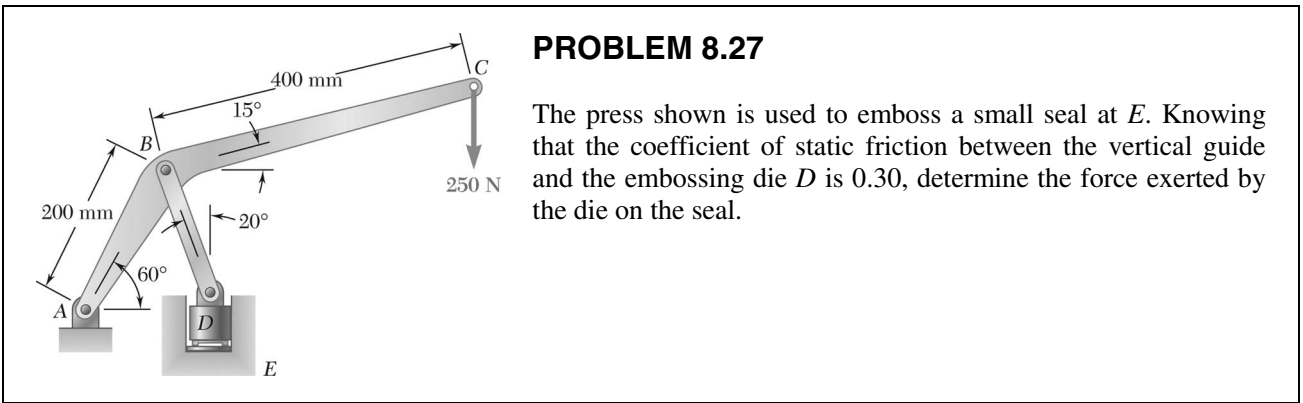
$$\mu_s = 0.2$$

and

$$\mu_s = -5 \quad (\text{Discard})$$

$$\mu_s = 0.200 \quad \blacktriangleleft$$

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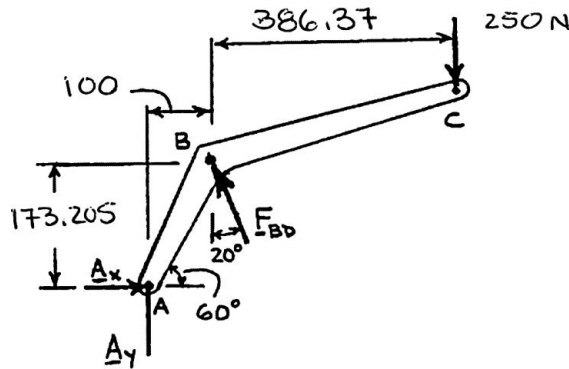


PROBLEM 8.27

The press shown is used to emboss a small seal at *E*. Knowing that the coefficient of static friction between the vertical guide and the embossing die *D* is 0.30, determine the force exerted by the die on the seal.

SOLUTION

Free body: Member ABC

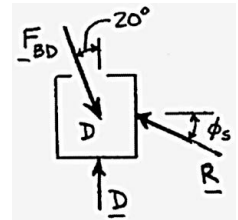


Dimensions in mm

$$\begin{aligned}
 +\curvearrowright \Sigma M_A = 0: & F_{BD} \cos 20^\circ(100) + F_{BD} \sin 20^\circ(173.205) \\
 & - (250 \text{ N})(100 + 386.37) = 0 \\
 & F_{BD} = 793.64 \text{ N}
 \end{aligned}$$

Free body: Die D

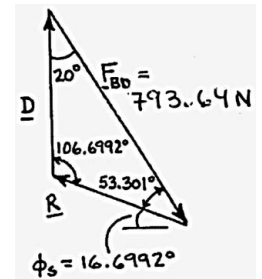
$$\begin{aligned}
 \phi_s &= \tan^{-1} \mu_s \\
 &= \tan^{-1} 0.3 \\
 &= 16.6992^\circ
 \end{aligned}$$



Force triangle:

$$\frac{D}{\sin 53.301^\circ} = \frac{793.64 \text{ N}}{\sin 106.6992^\circ}$$

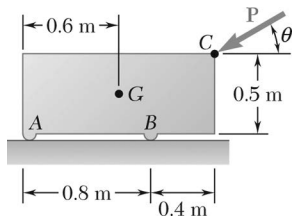
$$D = 664.35 \text{ N}$$



On seal:

$$\mathbf{D} = 664 \text{ N} \downarrow \leftarrow$$

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PROBLEM 8.28

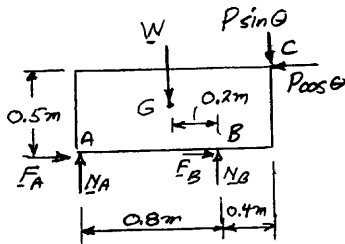
The machine base shown has a mass of 75 kg and is fitted with skids at A and B . The coefficient of static friction between the skids and the floor is 0.30. If a force P of magnitude 500 N is applied at corner C , determine the range of values of θ for which the base will not move.

SOLUTION

Free-body: Machine base

$$m = (75 \text{ kg})(9.81 \text{ m/s}^2) = 735.75 \text{ N}$$

Assume sliding impends



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: N_A + N_B - W - P \sin \theta = 0$$

$$(N_A + N_B) = W + P \sin \theta \quad (1)$$

$$+\rightarrow \Sigma F_x = 0: F_A + F_B - P \cos \theta = 0$$

$$\mu_s (N_A + N_B) = P \cos \theta = 0 \quad (2)$$

$$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \mu_s = \frac{P \cos \theta}{W + P \sin \theta}$$

$$\mu_s W + \mu_s P \sin \theta = P \cos \theta$$

$$0.30(735.75 \text{ N}) + 0.30(500 \text{ N}) \sin \theta = 500 \cos \theta$$

$$500 \cos \theta - 150 \sin \theta = 220.73$$

$$\text{Solve for } \theta: \quad \theta = 48.28^\circ$$

Assume tipping about B impends: $\therefore N_A = 0$

$$+\curvearrowright \Sigma M_B = 0: P \sin \theta (0.4 \text{ m}) - P \cos \theta (0.5 \text{ m}) - W (0.2 \text{ m}) = 0$$

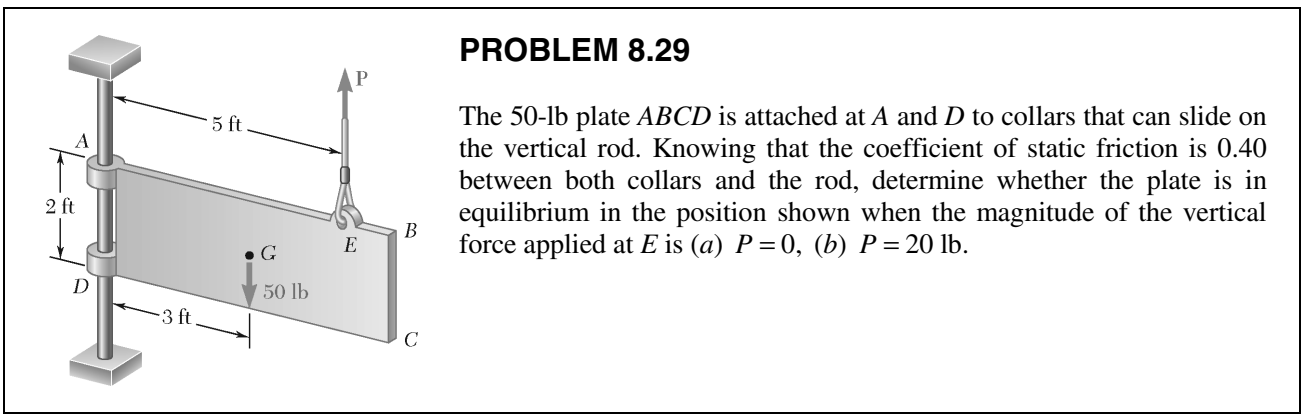
$$500 \sin \theta (0.4) - 500 \cos \theta (0.5) - 735.75 (0.2 \text{ m}) = 0$$

$$200 \sin \theta - 250 \cos \theta = 147.15$$

$$\text{Solve for } \theta: \quad \theta = 78.70^\circ$$

Range for no motion:

$$48.3^\circ \leq \theta \leq 78.7^\circ \quad \blacktriangleleft$$



PROBLEM 8.29

The 50-lb plate $ABCD$ is attached at A and D to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (a) $P = 0$, (b) $P = 20$ lb.

SOLUTION

(a) $P = 0$

$$+\circlearrowleft \Sigma M_D = 0: N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) = 0$$

$$N_A = 75 \text{ lb}$$

$$\Sigma F_x = 0: N_D = N_A = 75 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: F_A + F_D - 50 \text{ lb} = 0$$

$$F_A + F_D = 50 \text{ lb}$$

But:

$$(F_A)_m = \mu_s N_A = 0.40(75 \text{ lb}) = 30 \text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.40(75 \text{ lb}) = 30 \text{ lb}$$

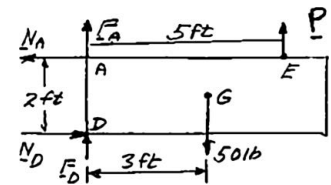
Thus:

$$(F_A)_m + (F_D)_m = 60 \text{ lb}$$

and

$$(F_A)_m + (F_D)_m > F_A + F_D$$

Plate is in equilibrium ◀



(b) $P = 20$ lb

$$+\circlearrowleft \Sigma M_D = 0: N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) + (20 \text{ lb})(5 \text{ ft}) = 0$$

$$N_A = 25 \text{ lb}$$

$$\Sigma F_x = 0: N_D = N_A = 25 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: F_A + F_D - 50 \text{ lb} + 20 \text{ lb} = 0$$

$$F_A + F_D = 30 \text{ lb}$$

But:

$$(F_A)_m = \mu_s N_A = 0.4(25 \text{ lb}) = 10 \text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.4(25 \text{ lb}) = 10 \text{ lb}$$

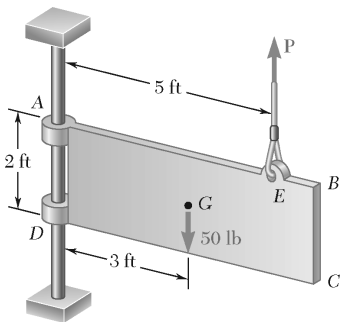
Thus:

$$(F_A)_m + (F_D)_m = 20 \text{ lb}$$

and

$$F_A + F_D > (F_A)_m + (F_D)_m$$

Plate moves downward ◀



PROBLEM 8.30

In Problem 8.29, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

PROBLEM 8.29 The 50-lb plate $ABCD$ is attached at A and D to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (a) $P = 0$, (b) $P = 20$ lb.

SOLUTION

We shall consider the following two cases:

(1) $0 < P < 30$ lb

$$+\curvearrowright \Sigma M_D = 0: N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) + P(5 \text{ ft}) = 0$$

$$N_A = 75 \text{ lb} - 2.5P$$

(Note: $N_A \geq 0$ and directed \leftarrow for $P \leq 30$ lb as assumed here)

$$\Sigma F_x = 0: N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

But:

$$\begin{aligned} (F_A)_m &= (F_D)_m = \mu_s N_A \\ &= 0.40(75 - 2.5P) \\ &= 30 - P \end{aligned}$$

Plate moves \downarrow if:

$$F_A + F_D > (F_A)_m + (F_D)_m$$

or

$$50 - P > (30 - P) + (30 - P)$$

$$P > 10 \text{ lb} \triangleleft$$

(2) $30 \text{ lb} < P < 50$ lb

$$+\curvearrowright \Sigma M_D = 0: -N_A(2 \text{ ft}) - (50 \text{ lb})(3 \text{ ft}) + P(5 \text{ ft}) = 0$$

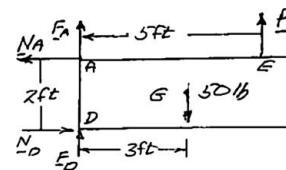
$$N_A = 2.5P - 75$$

(Note: $N_A > 0$ and directed \rightarrow for $P > 30$ lb as assumed)

$$\Sigma F_x = 0: N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$



PROBLEM 8.30 (Continued)

But:

$$\begin{aligned}(F_A)_m &= (F_D)_m = \mu_s N_A \\ &= 0.40(2.5P - 75) \\ &= P - 30 \text{ lb}\end{aligned}$$

Plate moves ↓ if:

$$F_A + F_D > (F_A)_m + (F_D)_m$$

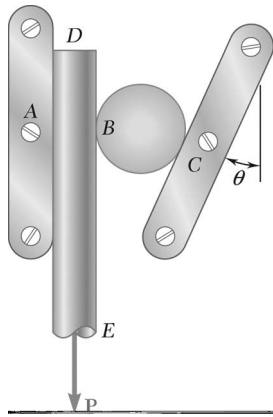
$$50 - P > (P - 30) + (P - 30)$$

$$P < \frac{110}{3} = 36.7 \text{ lb} \triangleleft$$

Thus, plate moves downward for:

$$10.00 \text{ lb} < P < 36.7 \text{ lb} \blacktriangleleft$$

(Note: For $P > 50$ lb, plate is in equilibrium)



PROBLEM 8.31

A rod DE and a small cylinder are placed between two guides as shown. The rod is not to slip downward, however large the force \mathbf{P} may be; i.e., the arrangement is said to be self-locking. Neglecting the weight of the cylinder, determine the minimum allowable coefficients of static friction at A , B , and C .

SOLUTION

Free body: Cylinder

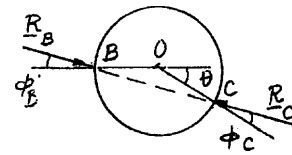
Since cylinder is a two-force body, \mathbf{R}_B and \mathbf{R}_C have the same line of action. Thus $\phi_B = \phi_C$:

From triangle OBC :

$$\phi_B + \phi_C = \theta$$

Thus:

$$\phi_B = \phi_C = \frac{\theta}{2}$$



For no sliding, we must have $\tan \phi_B \leq (\mu_B)_s$, $\tan \phi_C \leq (\mu_C)_s$

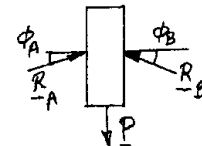
Therefore: $(\mu_B)_s \geq \tan \frac{\theta}{2}$,

$$(\mu_C)_s \geq \tan \frac{\theta}{2} \blacktriangleleft$$

We also note that R_B and R_C are indeterminate.

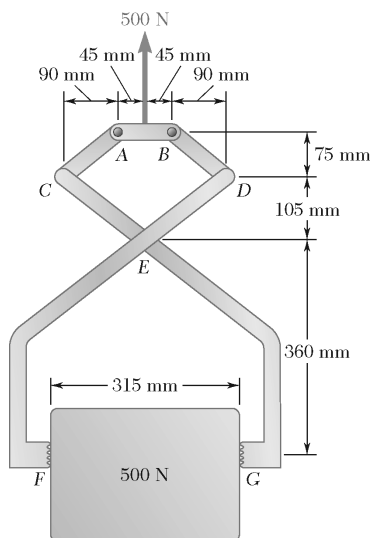
Free body: Rod

Since R_B is indeterminate, it may be as large as necessary to satisfy equation $\Sigma F_y = 0$, no matter how large \mathbf{P} is or how small ϕ_A is. Therefore



$$(\mu_A)_s \text{ may have any value } \blacktriangleleft$$

PROBLEM 8.32



A 500-N concrete block is to be lifted by the pair of tongs shown. Determine the smallest allowable value of the coefficient of static friction between the block and the tongs at F and G .

SOLUTION

Free body: Members CA, AB, BD

By symmetry:

$$C_y = D_y = \frac{1}{2}(500) = 250 \text{ N}$$

Since CA is a two-force member,

$$\frac{C_x}{90 \text{ mm}} = \frac{C_y}{75 \text{ mm}} = \frac{250 \text{ N}}{75 \text{ mm}}$$

$$C_x = 300 \text{ N}$$

$$\Sigma F_x = 0: D_x = C_x$$

$$D_x = 300 \text{ N}$$

Free body: Tong DEF

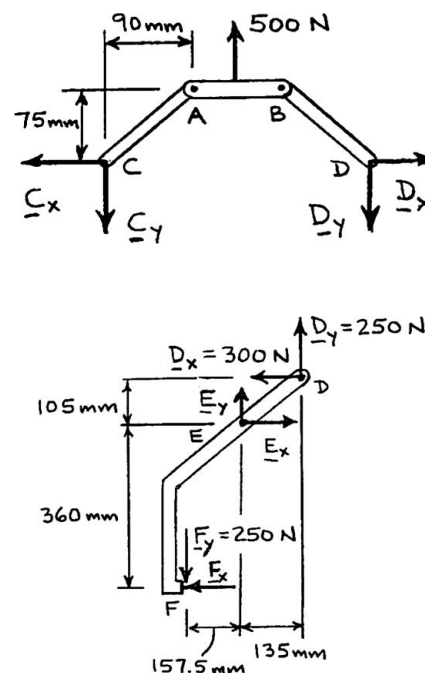
$$\begin{aligned} +\curvearrowright \Sigma M_E = 0: & (300 \text{ N})(105 \text{ mm}) + (250 \text{ N})(135 \text{ mm}) \\ & + (250 \text{ N})(157.5 \text{ mm}) - F_x(360 \text{ mm}) = 0 \end{aligned}$$

$$F_x = +290.625 \text{ N}$$

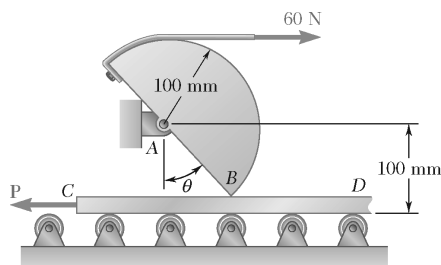
Minimum value of μ_s :

$$\mu_s = \frac{F_y}{F_x} = \frac{250 \text{ N}}{290.625 \text{ N}}$$

$$\mu_s = 0.860 \quad \blacktriangleleft$$



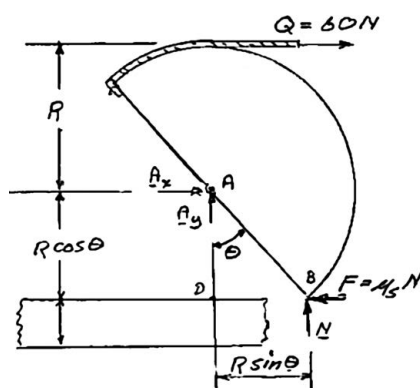
PROBLEM 8.33



The 100-mm-radius cam shown is used to control the motion of the plate CD . Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (a) the force \mathbf{P} required to maintain the motion of the plate, knowing that the plate is 20 mm thick, (b) the largest thickness of the plate for which the mechanism is self-locking (i.e., for which the plate cannot be moved however large the force \mathbf{P} may be).

SOLUTION

Free body: Cam



Impending motion:

$$F = \mu_s N$$

$$+\circlearrowleft \sum M_A = 0: QR - NR \sin \theta + (\mu_s N)R \cos \theta = 0$$

$$N = \frac{Q}{\sin \theta - \mu_s \cos \theta} \quad (1)$$

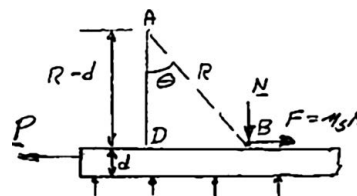
Free body: Plate

$$\sum F_x = 0 \quad P = \mu_s N \quad (2)$$

Geometry in $\triangle ABD$ with $R = 100$ mm and $d = 20$ mm

$$\begin{aligned} \cos \theta &= \frac{R - d}{R} \\ &= \frac{80 \text{ mm}}{100 \text{ mm}} \\ &= 0.8 \end{aligned}$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} = 0.6$$



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PROBLEM 8.33 (Continued)

(a) Eq. (1) using

$$Q = 60 \text{ N} \quad \text{and} \quad \mu_s = 0.45$$

$$N = \frac{60 \text{ N}}{0.6 - (0.45)(0.8)}$$
$$= \frac{60}{0.24} = 250 \text{ N}$$

Eq. (2)

$$P = \mu_s N = (0.45)(250 \text{ N})$$

$$P = 112.5 \text{ N} \quad \blacktriangleleft$$

(b) For $P = \infty$, $N = \infty$. Denominator is zero in Eq. (1).

$$\sin \theta - \mu_s \cos \theta = 0$$

$$\tan \theta = \mu_s = 0.45$$

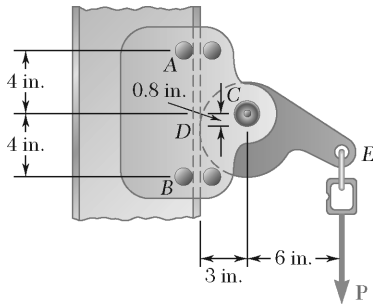
$$\theta = 24.23^\circ$$

$$\cos \theta = \frac{R - d}{R}$$

$$\cos 24.23 = \frac{100 - d}{100}$$

$$d = 8.81 \text{ mm} \quad \blacktriangleleft$$

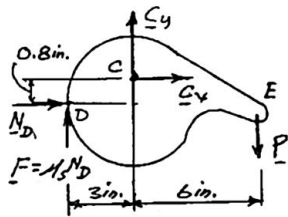
PROBLEM 8.34



A safety device used by workers climbing ladders fixed to high structures consists of a rail attached to the ladder and a sleeve that can slide on the flange of the rail. A chain connects the worker's belt to the end of an eccentric cam that can be rotated about an axle attached to the sleeve at C. Determine the smallest allowable common value of the coefficient of static friction between the flange of the rail, the pins at A and B, and the eccentric cam if the sleeve is not to slide down when the chain is pulled vertically downward.

SOLUTION

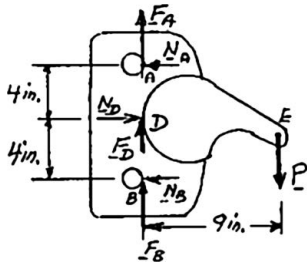
Free body: Cam



$$+\circlearrowleft \Sigma M_C = 0: N_D(0.8 \text{ in.}) - \mu_s N_D(3 \text{ in.}) - P(6 \text{ in.}) = 0$$

$$N_D = \frac{6P}{0.8 - 3\mu_s} \quad (1)$$

Free body: Sleeve and cam



$$+\rightarrow \Sigma F_x = 0: N_D - N_A - N_B = 0$$

$$N_A + N_B = N_D \quad (2)$$

$$+\uparrow \Sigma F_y = 0: F_A + F_B + F_D - P = 0$$

or

$$\mu_s(N_A + N_B + N_D) = P \quad (3)$$

Substitute from Eq. (2) into Eq. (3):

$$\mu_s(2N_D) = P \quad N_D = \frac{P}{2\mu_s} \quad (4)$$

Equate expressions for N_D from Eq. (1) and Eq. (4):

$$\frac{P}{2\mu_s} = \frac{6P}{0.8 - 3\mu_s}; \quad 0.8 - 3\mu_s = 12\mu_s$$

$$\mu_s = \frac{0.8}{15} \quad \mu_s = 0.0533 \quad \blacktriangleleft$$

(Note: To verify that contact at pins A and B takes places as assumed, we shall check that $N_A > 0$ and $N_B = 0$.)

PROBLEM 8.34 (Continued)

From Eq. (4):
$$N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

From free body of cam and sleeve:

$$+\curvearrowright \Sigma M_B = 0: N_A(8 \text{ in.}) - N_D(4 \text{ in.}) - P(9 \text{ in.}) = 0$$

$$8N_A = (9.375P)(4) + 9P$$

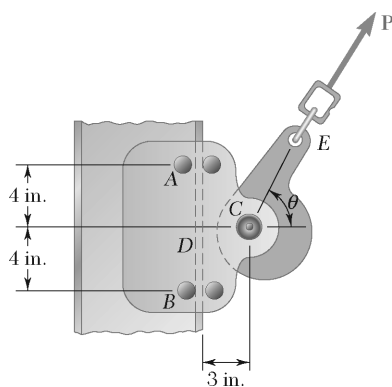
$$N_A = 5.8125P > 0 \quad \text{OK}$$

From Eq. (2):

$$N_A + N_B = N_D$$

$$5.8125P + N_B = 9.375P$$

$$N_B = 3.5625P > 0 \quad \text{OK}$$



PROBLEM 8.35

To be of practical use, the safety sleeve described in Problem 8.34 must be free to slide along the rail when pulled upward. Determine the largest allowable value of the coefficient of static friction between the flange of the rail and the pins at A and B if the sleeve is to be free to slide when pulled as shown in the figure, assuming (a) $\theta = 60^\circ$, (b) $\theta = 50^\circ$, (c) $\theta = 40^\circ$.

SOLUTION

Note the cam is a two-force member.

Free body: Sleeve

We assume contact between rail and pins as shown.

$$+\curvearrowright \Sigma M_C = 0: F_A(3 \text{ in.}) + F_B(3 \text{ in.}) - N_A(4 \text{ in.}) - N_B(4 \text{ in.}) = 0$$

But

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

We find

$$3\mu_s(N_A + N_B) - 4(N_A + N_B) = 0$$

$$\mu_s = \frac{4}{3} = 1.33333$$

We now verify that our assumption was correct.

$$+\rightarrow \Sigma F_x = 0: N_A - N_B + P \cos \theta = 0$$

$$N_B - N_A = P \cos \theta \quad (1)$$

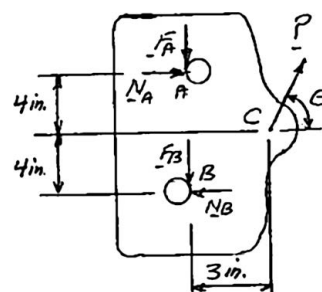
$$+\uparrow \Sigma F_y = 0: -F_A - F_B + P \sin \theta = 0$$

$$\mu_s N_A + \mu_s N_B = P \sin \theta$$

$$N_A + N_B = \frac{P \sin \theta}{\mu_s} \quad (2)$$

Add Eqs. (1) and (2):

$$2N_B = P \left(\cos \theta + \frac{\sin \theta}{\mu_s} \right) > 0 \quad \text{OK}$$



PROBLEM 8.35 (Continued)

Subtract Eq. (1) from Eq. (2):

$$2N_A = P \left(\frac{\sin \theta}{\mu_s} - \cos \theta \right)$$

$N_A > 0$ only if

$$\frac{\sin \theta}{\mu_s} - \cos \theta > 0$$

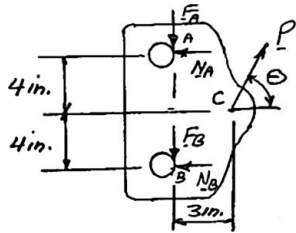
$$\tan \theta > \mu_s = 1.33333$$

$$\theta = 53.130^\circ$$

(a) For case (a): Condition is satisfied, contact takes place as shown. Answer is correct.

$$\mu_s = 1.333 \quad \blacktriangleleft$$

But for (b) and (c): $\theta < 53.130^\circ$ and our assumption is wrong, N_A is directed to left.



$$\begin{aligned} \rightarrow \Sigma F_x = 0: & \quad -N_A - N_B + P \cos \theta = 0 \\ & \quad N_A + N_B = P \cos \theta \end{aligned} \quad (3)$$

$$\begin{aligned} \uparrow \Sigma F_y = 0: & \quad -F_A - F_B + P \sin \theta = 0 \\ & \quad \mu_s (N_A + N_B) = P \sin \theta \end{aligned} \quad (4)$$

Divide Eq. (4) by Eq. (3):

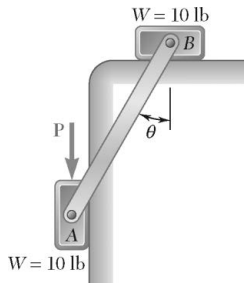
$$\mu_s = \tan \theta \quad (5)$$

(b) We make $\theta = 50^\circ$ in Eq. (5):

$$\mu_s = \tan 50^\circ \quad \mu_s = 1.192 \quad \blacktriangleleft$$

(c) We make $\theta = 40^\circ$ in Eq. (5):

$$\mu_s = \tan 40^\circ \quad \mu_s = 0.839 \quad \blacktriangleleft$$



PROBLEM 8.36

Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30^\circ$ with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD block B:

(a) Since $P = 2.69$ lb to initiate motion,

equilibrium exists with $P = 0$ ◀

(b) For P_{\max} , motion impends at both surfaces:

Block B: $\uparrow \Sigma F_y = 0: N_B - 10 \text{ lb} - F_{AB} \cos 30^\circ = 0$

$$N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2} F_{AB} \quad (1)$$

Impending motion: $F_B = \mu_s N_B = 0.3 N_B$

$$\rightarrow \Sigma F_x = 0: F_B - F_{AB} \sin 30^\circ = 0$$

$$F_{AB} = 2F_B = 0.6 N_B \quad (2)$$

Solving Eqs. (1) and (2): $N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2} (0.6 N_B) = 20.8166 \text{ lb}$

FBD block A:

Then $F_{AB} = 0.6 N_B = 12.4900 \text{ lb}$

Block A: $\rightarrow \Sigma F_x = 0: F_{AB} \sin 30^\circ - N_A = 0$

$$N_A = \frac{1}{2} F_{AB} = \frac{1}{2} (12.4900 \text{ lb}) = 6.2450 \text{ lb}$$

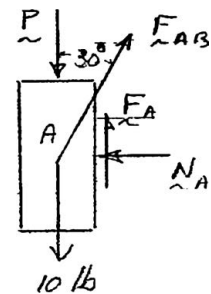
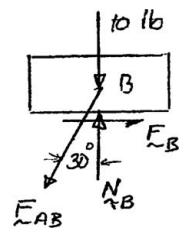
Impending motion: $F_A = \mu_s N_A = 0.3(6.2450 \text{ lb}) = 1.8735 \text{ lb}$

$$\uparrow \Sigma F_y = 0: F_A + F_{AB} \cos 30^\circ - P - 10 \text{ lb} = 0$$

$$P = F_A + \frac{\sqrt{3}}{2} F_{AB} - 10 \text{ lb}$$

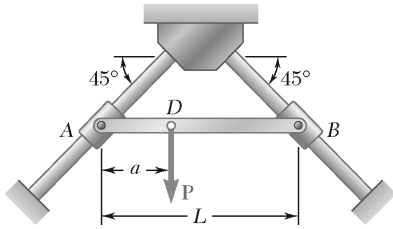
$$= 1.8735 \text{ lb} + \frac{\sqrt{3}}{2} (12.4900 \text{ lb}) - 10 \text{ lb}$$

$$= 2.69 \text{ lb}$$



$P = 2.69$ lb ◀

PROBLEM 8.37



Bar AB is attached to collars that can slide on the inclined rods shown. A force \mathbf{P} is applied at Point D located at a distance a from end A . Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio a/L for which equilibrium is maintained.

SOLUTION

FBD bar and collars:

Impending motion:

$$\begin{aligned}\phi_s &= \tan^{-1} \mu_s \\ &= \tan^{-1} 0.3 \\ &= 16.6992^\circ\end{aligned}$$

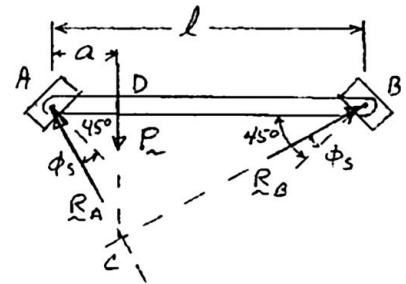
Neglect weights: 3-force FBD and $\angle ACB = 90^\circ$

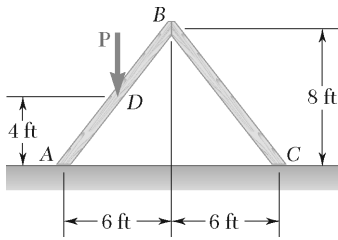
so

$$\begin{aligned}AC &= \frac{a}{\cos(45^\circ + \phi_s)} \\ &= l \sin(45^\circ - \phi_s)\end{aligned}$$

$$\frac{a}{l} = \sin(45^\circ - 16.6992^\circ) \cos(45^\circ + 16.6992^\circ)$$

$$\frac{a}{l} = 0.225 \quad \blacktriangleleft$$



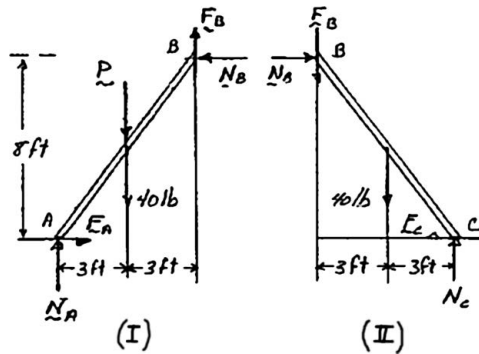


PROBLEM 8.38

Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (a) the largest magnitude of the force P for which equilibrium will be maintained, (b) the surface at which motion will impend.

SOLUTION

Board FBDs:



Assume impending motion at C, so

$$F_C = \mu_s N_C = 0.4N_C$$

FBD II:

$$\begin{aligned} \left(\sum M_B = 0: \right. & (6 \text{ ft})N_C - (8 \text{ ft})F_C - (3 \text{ ft})(40 \text{ lb}) = 0 \\ & [6 \text{ ft} - 0.4(8 \text{ ft})]N_C = (3 \text{ ft})(40 \text{ lb}) \end{aligned}$$

or

$$N_C = 42.857 \text{ lb}$$

and

$$F_C = 0.4N_C = 17.143 \text{ lb}$$

$$\rightarrow \sum F_x = 0: N_B - F_C = 0$$

$$N_B = F_C = 17.143 \text{ lb}$$

$$\uparrow \sum M_y = 0: -F_B - 40 \text{ lb} + N_C = 0$$

$$F_B = N_C - 40 \text{ lb} = 2.857 \text{ lb}$$

Check for motion at B:

$$\frac{F_B}{N_B} = \frac{2.857 \text{ lb}}{17.143 \text{ lb}} = 0.167 < \mu_s, \text{ OK, no motion.}$$

FBD I:

$$\left(\sum M_A = 0: \right. (8 \text{ ft})N_B + (6 \text{ ft})F_B - (3 \text{ ft})(P + 40 \text{ lb}) = 0$$

$$\begin{aligned} P &= \frac{(8 \text{ ft})(17.143 \text{ lb}) + (6 \text{ ft})(2.857 \text{ lb})}{3 \text{ ft}} - 40 \text{ lb} \\ &= 11.429 \text{ lb} \end{aligned}$$

PROBLEM 8.38 (Continued)

Check for slip at A (unlikely because of P):

$$\rightarrow \Sigma F_x = 0: F_A - N_B = 0 \quad \text{or} \quad F_A = N_B = 17.143 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: N_A - P - 40 \text{ lb} + F_B = 0$$

or

$$\begin{aligned} N_A &= 11.429 \text{ lb} + 40 \text{ lb} - 2.857 \text{ lb} \\ &= 48.572 \text{ lb} \end{aligned}$$

Then

$$\frac{F_A}{N_A} = \frac{17.143 \text{ lb}}{48.572 \text{ lb}} = 0.353 < \mu_s$$

OK, no slip \Rightarrow assumption is correct.

Therefore

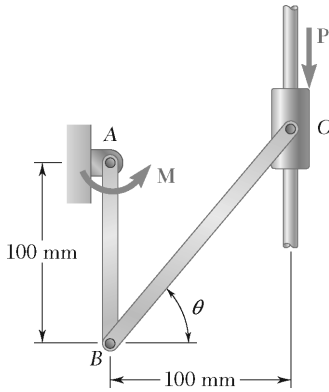
(a)

$$P_{\max} = 11.43 \text{ lb} \quad \blacktriangleleft$$

(b)

Motion impends at C \blacktriangleleft

PROBLEM 8.39



Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50^\circ$ and $M = 20 \text{ N}\cdot\text{m}$.

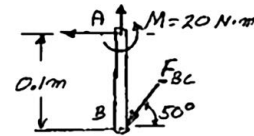
SOLUTION

Free body member AB:

BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad 20 \text{ N}\cdot\text{m} - F_{BC} \cos 50^\circ (0.1 \text{ m}) = 0$$

$$F_{BC} = 311.145 \text{ N}$$

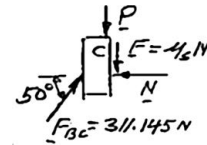


Motion of C impending upward:

$$+\rightarrow \Sigma F_x = 0: \quad (311.145 \text{ N}) \cos 50^\circ - N = 0$$

$$N = 200 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145 \text{ N}) \sin 50^\circ - P - (0.35)(200 \text{ N}) = 0$$



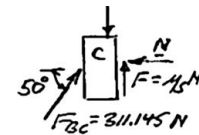
$$P = 168.351 \text{ N} \quad \triangleleft$$

Motion of C impending downward:

$$+\rightarrow \Sigma F_x = 0: \quad (311.145 \text{ N}) \cos 50^\circ - N = 0$$

$$N = 200 \text{ N}$$

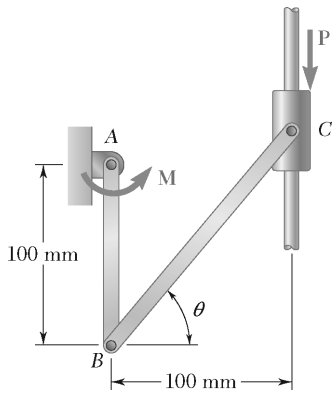
$$+\uparrow \Sigma F_y = 0: \quad (311.145 \text{ N}) \sin 50^\circ - P + (0.35)(200 \text{ N}) = 0$$



$$P = 308.35 \text{ N} \quad \triangleleft$$

Range of P:

$$168.4 \text{ N} \leq P \leq 308 \text{ N} \quad \blacktriangleleft$$



PROBLEM 8.40

Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of M for which equilibrium is maintained when $\theta = 60^\circ$ and $P = 200 \text{ N}$.

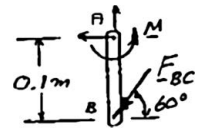
SOLUTION

Free body member AB:

BC is a two-force member.

$$+\curvearrowright \Sigma M_A = 0: M - F_{BC} \cos 60^\circ (0.1 \text{ m}) = 0$$

$$M = 0.05 F_{BC} \quad (1)$$



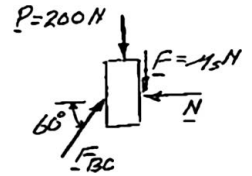
Motion of C impending upward:

$$+\rightarrow \Sigma F_x = 0: F_{BC} \cos 60^\circ - N = 0$$

$$N = 0.5 F_{BC}$$

$$+\uparrow \Sigma F_y = 0: F_{BC} \sin 60^\circ - 200 \text{ N} - (0.40)(0.5 F_{BC}) = 0$$

$$F_{BC} = 300.29 \text{ N}$$



Eq. (1): $M = 0.05(300.29)$

$$M = 15.014 \text{ N} \cdot \text{m} \quad \triangleleft$$

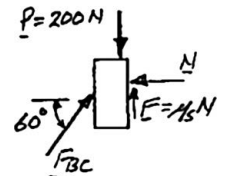
Motion of C impending downward:

$$+\rightarrow \Sigma F_x = 0: F_{BC} \cos 60^\circ - N = 0$$

$$N = 0.5 F_{BC}$$

$$+\uparrow \Sigma F_y = 0: F_{BC} \sin 60^\circ - 200 \text{ N} + (0.40)(0.5 F_{BC}) = 0$$

$$F_{BC} = 187.613 \text{ N}$$



Eq. (1): $M = 0.05(187.613)$

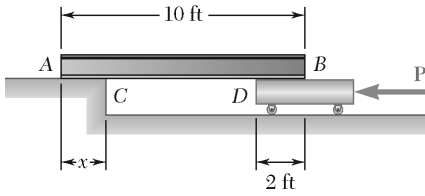
$$M = 9.381 \text{ N} \cdot \text{m} \quad \triangleleft$$

Range of M :

$$9.38 \text{ N} \cdot \text{m} \leq M \leq 15.01 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

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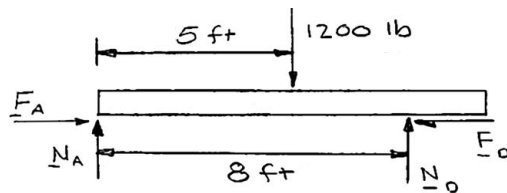
PROBLEM 8.41



A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force \mathbf{P} is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force \mathbf{P} required to start moving the beam. (*Hint: The beam is supported at A and D.*)

SOLUTION

FBD beam:



$$+\curvearrowright \Sigma M_A = 0: N_D(8\text{ ft}) - (1200\text{ lb})(5\text{ ft}) = 0$$

$$N_D = 750\text{ lb} \uparrow$$

$$+\uparrow \Sigma F_y = 0: N_A - 1200 + 750 = 0$$

$$N_A = 450\text{ lb} \uparrow$$

$$(F_A)_m = \mu_s N_A = 0.3(450) = 135.0\text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.3(750) = 225\text{ lb}$$

Since $(F_A)_m < (F_D)_m$, sliding first impends at A with

$$F_A = (F_A)_m = 135\text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: F_A - F_D = 0$$

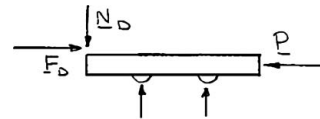
$$F_D = F_A = 135.0\text{ lb}$$

FBD dolly:

From *FBD* of dolly:

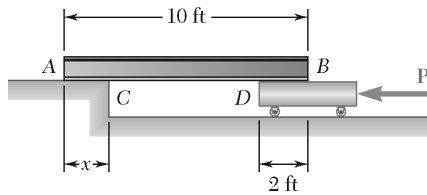
$$+\rightarrow \Sigma F_x = 0: F_D - P = 0$$

$$P = F_D = 135.0\text{ lb}$$



$$P = 135.0\text{ lb} \leftarrow$$

PROBLEM 8.42



(a) Show that the beam of Problem 8.41 *cannot* be moved if the top surface of the dolly is slightly *lower* than the platform. (b) Show that the beam *can* be moved if two 175-lb workers stand on the beam at B and determine how far to the left the beam can be moved.

PROBLEM 8.41 A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force \mathbf{P} is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force \mathbf{P} required to start moving the beam. (*Hint*: The beam is supported at A and D.)

SOLUTION

(a) Beam alone

$$+\curvearrowright \Sigma M_C = 0: N_B(8 \text{ ft}) - (1200 \text{ lb})(3 \text{ ft}) = 0$$

$$N_B = 450 \text{ lb} \uparrow$$

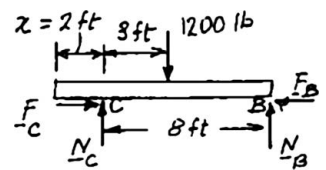
$$+\uparrow F_y = 0: N_C + 450 - 1200 = 0$$

$$N_C = 750 \text{ lb} \uparrow$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225 \text{ lb}$$

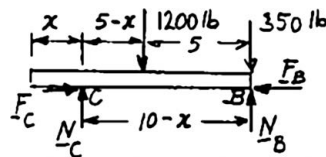
$$(F_B)_m = \mu_s N_B = 0.3(450) = 135 \text{ lb}$$

Since $(F_B)_m < (F_C)_m$, sliding first impends at B, and



Beam cannot be moved ◀

(b) Beam with workers standing at B



$$+\curvearrowright \Sigma M_C = 0: N_B(10 - x) - (1200)(5 - x) - 350(10 - x) = 0$$

$$N_B = \frac{9500 - 1550x}{10 - x}$$

$$+\curvearrowright \Sigma M_B = 0: (1200)(5) - N_C(10 - x) = 0$$

$$N_C = \frac{6000}{10 - x}$$

PROBLEM 8.42 (Continued)

Check that beam starts moving for $x = 2$ ft:

$$\begin{aligned}\text{For } x = 2 \text{ ft:} \quad N_B &= \frac{9500 - 1550(2)}{10 - 2} = 800 \text{ lb} \\ N_C &= \frac{6000}{10 - 2} = 750 \text{ lb} \\ (F_C)_m &= \mu_s N_C = 0.3(750) = 225 \text{ lb} \\ (F_B)_m &= \mu_s N_B = 0.3(800) = 240 \text{ lb}\end{aligned}$$

Since $(F_C)_m < (F_B)_m$, sliding first impends at C ,

Beam moves ◀

How far does beam move?

Beam will stop moving when

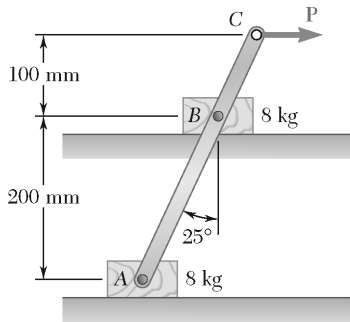
$$F_C = (F_B)_m$$

$$\text{But} \quad F_C = \mu_k N_C = 0.25 \frac{6000}{10 - x} = \frac{1500}{10 - x}$$

$$\text{and} \quad (F_B)_m = \mu_s N_B = 0.30 \frac{9500 - 1550x}{10 - x} = \frac{2850 - 465x}{10 - x}$$

$$\text{Setting } F_C = (F_B)_m: \quad 1500 = 2850 - 465x \quad x = 2.90 \text{ ft} \quad \blacktriangleleft$$

(Note: We have assumed that, once started, motion is continuous and uniform (no acceleration).)

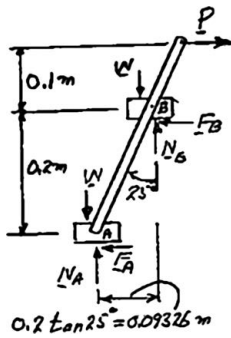


PROBLEM 8.43

Two 8-kg blocks *A* and *B* resting on shelves are connected by a rod of negligible mass. Knowing that the magnitude of a horizontal force **P** applied at *C* is slowly increased from zero, determine the value of *P* for which motion occurs, and what that motion is, when the coefficient of static friction between all surfaces is (a) $\mu_s = 0.40$, (b) $\mu_s = 0.50$.

SOLUTION

(a) $\mu_s = 0.40$: Assume blocks slide to right.



$$W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.48 \text{ N}$$

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: N_A + N_B - 2W = 0$$

$$N_A + N_B = 2W$$

$$+\rightarrow \Sigma F_x = 0: P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s(N_A + N_B) = \mu_s(2W) \quad (1)$$

$$P = 0.40(2)(78.48 \text{ N}) = 62.78 \text{ N}$$

$$+\curvearrowright \Sigma M_B = 0: P(0.1 \text{ m}) - (N_A - W)(0.09326 \text{ m}) + F_A(0.2 \text{ m}) = 0$$

$$(62.78)(0.1) - (N_A - 78.48)(0.09326) + (0.4)(N_B)(0.2) = 0$$

$$0.17326N_A = 1.041$$

$$N_A = 6.01 \text{ N} > 0 \quad \text{OK}$$

System slides: $P = 62.8 \text{ N}$ ◀

(b) $\mu_s = 0.50$: See part *a*.

Eq. (1):

$$P = 0.5(2)(78.48 \text{ N}) = 78.48 \text{ N}$$

$$+\curvearrowright \Sigma M_B = 0: P(0.1 \text{ m}) + (N_A - W)(0.09326 \text{ m}) + F_A(0.2 \text{ m}) = 0$$

$$(78.48)(0.1) + (N_A - 78.48)(0.09326) + (0.5)N_A(0.2) = 0$$

$$0.19326N_A = -0.529$$

$$N_A = -2.73 \text{ N} < 0 \quad \text{uplift, rotation about } B$$

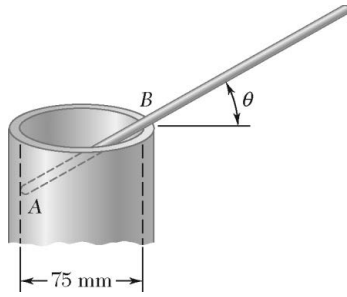
PROBLEM 8.43 (Continued)

For $N_A = 0$:

$$\curvearrowright \Sigma M_B = 0: P(0.1 \text{ m}) - W(0.09326 \text{ m}) = 0$$

$$P = (78.48 \text{ N})(0.09326 \text{ m}) / (0.1) = 73.19$$

System rotates about B: $P = 73.2 \text{ N}$ ◀

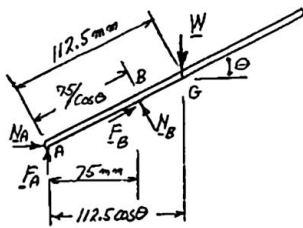


PROBLEM 8.44

A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends down at A and to left at B.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$+\rightarrow \Sigma F_x = 0: \quad N_A - N_B \sin \theta + F_B \cos \theta = 0$$

$$N_A - N_B \sin \theta + \mu_s N_B \cos \theta = 0$$

$$N_A = N_B (\sin \theta - \mu_s \cos \theta) \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + N_B \cos \theta + F_B \sin \theta - W = 0$$

$$\mu_s N_A + N_B \cos \theta + \mu_s N_B \sin \theta - W = 0 \quad (2)$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$\mu_s N_B (\sin \theta - \mu_s \cos \theta) + N_B \cos \theta + \mu_s N_B \sin \theta - W = 0$$

$$N_B = \frac{W}{(1 - \mu_s^2) \cos \theta + 2\mu_s \sin \theta} = \frac{W}{(1 - 0.2^2) \cos \theta + 2(0.2) \sin \theta} \quad (3)$$

$$+\curvearrowright \Sigma M_A = 0: \quad N_B \left(\frac{75}{\cos \theta} \right) - W(112.5 \cos \theta) = 0$$

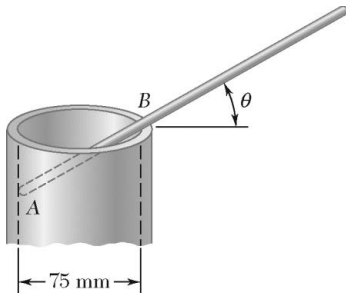
Substitute for N_B from Eq. (3), cancel W , and simplify to find

$$9.6 \cos^3 \theta + 4 \sin \theta \cos^2 \theta - 6.6667 = 0$$

$$\cos^3 \theta (2.4 + \tan \theta) = 1.6667$$

Solve by trial & error:

$$\theta = 35.8^\circ \quad \blacktriangleleft$$



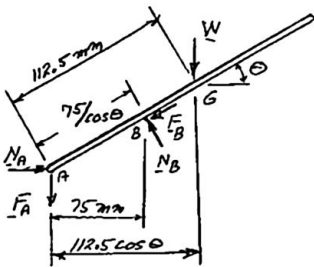
PROBLEM 8.45

In Problem 8.44, determine the smallest value of θ for which the rod will not fall out the pipe.

PROBLEM 8.44 A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends up at A and right at B.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - N_B \sin \theta - F_B \cos \theta = 0$$

$$N_A - N_B \sin \theta - \mu_s N_B \cos \theta = 0$$

$$N_A = N_B (\sin \theta + \mu_s \cos \theta) \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A + N_B \cos \theta - F_B \sin \theta - W = 0$$

$$-\mu_s N_A + N_B \cos \theta - \mu_s N_B \sin \theta - W = 0 \quad (2)$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$-\mu_s N_B (\sin \theta + \mu_s \cos \theta) + N_B \cos \theta - \mu_s N_B \sin \theta - W = 0$$

$$N_B = \frac{W}{(1 - \mu_s^2) \cos \theta - 2\mu_s \sin \theta} = \frac{W}{(1 - 0.2^2) \cos \theta - 2(0.2) \sin \theta} \quad (3)$$

$$+\curvearrowright \Sigma M_A = 0: \quad N_B \left(\frac{75}{\cos \theta} \right) - W(112.5 \cos \theta) = 0$$

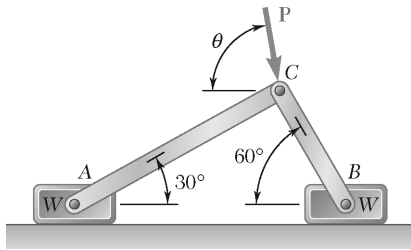
Substitute for N_B from Eq. (3), cancel W , and simplify to find

$$9.6 \cos^3 \theta - 4 \sin \theta \cos^2 \theta - 6.6667 = 0$$

$$\cos^3 \theta (2.4 - \tan \theta) = 1.6667$$

Solve by trial + error:

$$\theta = 20.5^\circ \quad \blacktriangleleft$$

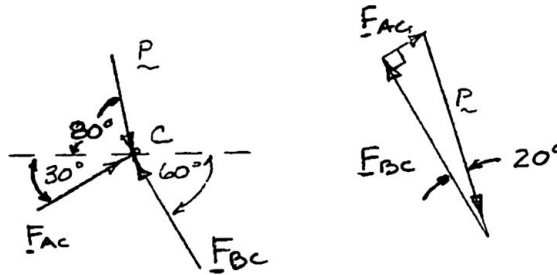


PROBLEM 8.46

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B , each of weight W . Knowing that $\theta = 80^\circ$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C :



$$F_{AC} = P \sin 20^\circ = 0.34202P$$

$$F_{BC} = P \cos 20^\circ = 0.93969P$$

$$\uparrow \Sigma F_y = 0: N_A - W - F_{AC} \sin 30^\circ = 0$$

or

$$N_A = W + 0.34202P \sin 30^\circ = W + 0.171010P$$

FBD block A :

$$\rightarrow \Sigma F_x = 0: F_A - F_{AC} \cos 30^\circ = 0$$

or

$$F_A = 0.34202P \cos 30^\circ = 0.29620P$$

For impending motion at A :

$$F_A = \mu_s N_A$$

Then

$$N_A = \frac{F_A}{\mu_s}: W + 0.171010P = \frac{0.29620}{0.3}P$$

or

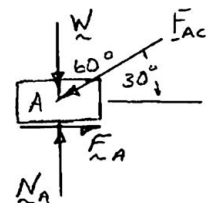
$$P = 1.22500W$$

$$\uparrow \Sigma F_y = 0: N_B - W - F_{BC} \cos 30^\circ = 0$$

$$N_B = W + 0.93969P \cos 30^\circ = W + 0.81380P$$

$$\rightarrow \Sigma F_x = 0: F_{BC} \sin 30^\circ - F_B = 0$$

$$F_B = 0.93969P \sin 30^\circ = 0.46985P$$



PROBLEM 8.46 (Continued)

FBD block B:

For impending motion at B:

$$F_B = \mu_s N_B$$

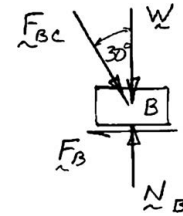
Then

$$N_B = \frac{F_B}{\mu_s}: W + 0.81380P = \frac{0.46985P}{0.3}$$

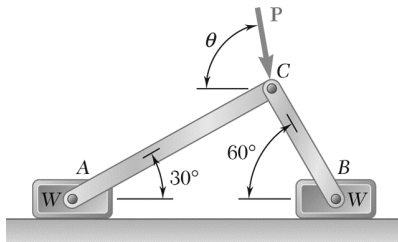
or

$$P = 1.32914W$$

Thus, maximum P for equilibrium



$$P_{\max} = 1.225W \blacktriangleleft$$



PROBLEM 8.47

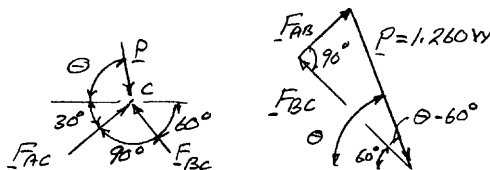
Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B , each of weight W . Knowing that $P = 1.260W$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30 , determine the range of values of θ , between 0 and 180° , for which equilibrium is maintained.

SOLUTION

AC and BC are two-force members

Free body: Joint C

Force triangle:



From Force triangle:

$$F_{AB} = P \sin(\theta - 60^\circ) = 1.26W \sin(\theta - 60^\circ) \quad (1)$$

$$F_{BC} = P \cos(\theta - 60^\circ) = 1.26W \cos(\theta - 60^\circ) \quad (2)$$

We shall, in turn, seek θ corresponding to impending motion of each block

For motion of A impending to left

from solution of Prob. 8.46: $F_{AC} = 0.419W$

$$\text{Eq. (1):} \quad F_{AC} = 0.419W = 1.26W \sin(\theta - 60^\circ)$$

$$\sin(\theta - 60^\circ) = 0.33254$$

$$\theta - 60^\circ = 19.423^\circ$$

$$\theta = 79.42^\circ \quad \triangleleft$$

For motion of B impending to right.

from solution of Prob. 8.46: $F_{BC} = 1.249W$

$$\text{Eq. (2):} \quad F_{BC} = 1.249W = 1.26W \cos(\theta - 60^\circ)$$

$$\cos(\theta - 60^\circ) = 0.99127$$

$$\theta - 60^\circ = \pm 7.58^\circ$$

$$\theta - 60^\circ = +7.58^\circ$$

$$\theta = 67.6^\circ \quad \triangleleft$$

$$\theta - 60^\circ = -7.58^\circ$$

$$\theta = 52.4^\circ \quad \triangleleft$$

PROBLEM 8.47 (Continued)

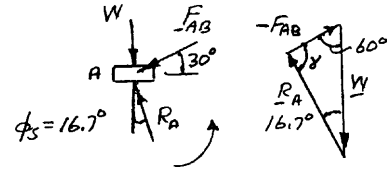
For motion of A impending to right

$$\gamma = 180^\circ - 60^\circ - 16.7^\circ = 103.3^\circ$$

Law of sines:

$$\frac{-F_{AB}}{\sin 16.7^\circ} = \frac{W}{\sin 103.3^\circ}$$

$$F_{AB} = -0.29528W$$



Note: Direction of $+F_{AB}$ is kept same as in free body of Joint C.

Eq. (1): $F_{AB} = -0.29528W = 1.26W \sin(\theta - 60^\circ)$

$$\sin(\theta - 60^\circ) = -0.23435$$

$$(\theta - 60^\circ) = -13.553^\circ$$

$$\theta = 46.4^\circ \triangleleft$$

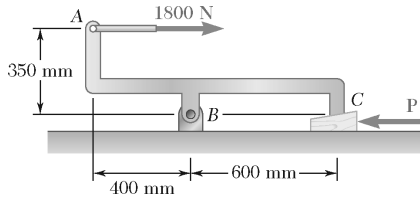
Summary:

A moves to right	No motion	B moves to right	No motion	A moves to left
46.4°	52.4°	67.6°	79.4°	θ

No motion for:

$$46.4^\circ \leq \theta \leq 52.4^\circ \text{ and } 67.6^\circ \leq \theta \leq 79.4^\circ$$





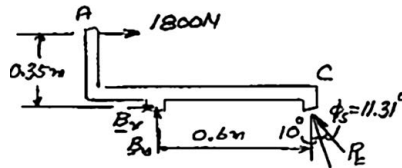
PROBLEM 8.48

The machine part ABC is supported by a frictionless hinge at B and a 10° wedge at C . Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20 , determine (a) the force P required to move the wedge, (b) the components of the corresponding reaction at B .

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$$

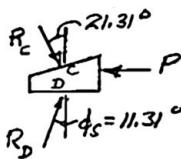
Free body: Part ABC



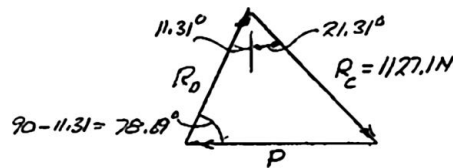
$$+\circlearrowleft \Sigma M_B = 0 \quad (1800 \text{ N})(0.35 \text{ m}) - R_C \cos 21.31^\circ (0.6 \text{ m}) = 0$$

$$R_C = 1127.1 \text{ N}$$

Free body: Wedge



Force triangle:



(a) Law of sines:

$$\frac{P}{\sin(11.31^\circ + 21.31^\circ)} = \frac{1127.1 \text{ N}}{\sin 78.69^\circ}$$

$$P = 619.6 \text{ N}$$

$$\mathbf{P} = 620 \text{ N} \leftarrow \blacktriangleleft$$

(b) Return to part ABC :

$$+\rightarrow \Sigma F_x = 0: \quad B_x + 1800 \text{ N} - R_C \sin 21.31^\circ = 0$$

$$B_x + 1800 \text{ N} - (1127.1 \text{ N}) \sin 21.31^\circ$$

$$B_x = -1390.4 \text{ N}$$

$$\mathbf{B}_x = 1390 \text{ N} \leftarrow \blacktriangleleft$$

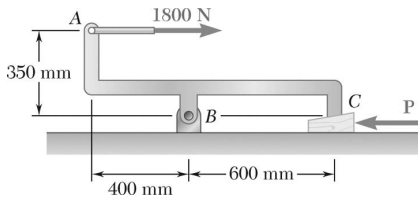
$$+\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 21.31^\circ = 0$$

$$B_y + (1127.1 \text{ N}) \cos 21.31^\circ = 0$$

$$B_y = -1050 \text{ N}$$

$$\mathbf{B}_y = 1050 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 8.49



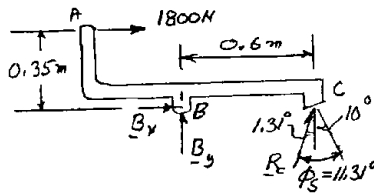
Solve Problem 8.48 assuming that the force **P** is directed to the right.

PROBLEM 8.48 The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (a) the force **P** required to move the wedge, (b) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$$

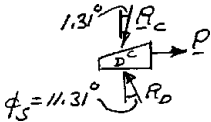
Free body: Part *ABC*



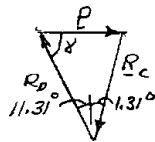
$$+\curvearrowright \Sigma M_B = 0: (1800 \text{ N})(0.35 \text{ m}) - R_C \cos 1.31^\circ (0.6 \text{ m}) = 0$$

$$R_C = 1050.3 \text{ N}$$

Free body: Wedge



Force triangle:



$$\gamma = 90^\circ - 11.31^\circ = 78.69^\circ$$

(a) Law of sines:

$$\frac{P}{\sin (11.31^\circ + 1.31^\circ)} = \frac{1050.3 \text{ N}}{\sin 78.69^\circ}$$

$$P = 234 \text{ N}$$

$$\mathbf{P} = 234 \text{ N} \rightarrow \blacktriangleleft$$

(b) Return to Part *ABC*:

$$+\rightarrow \Sigma F_x = 0: B_x + 1800 \text{ N} + R_C \sin 1.31^\circ = 0$$

$$B_x + 1800 \text{ N} + (1050.3 \text{ N}) \sin 1.31^\circ = 0$$

$$B_x = -1824 \text{ N}$$

$$\mathbf{B}_x = 1824 \text{ N} \leftarrow \blacktriangleleft$$

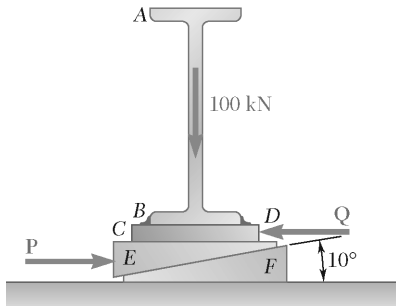
$$+\uparrow \Sigma F_y = 0: B_y + R_C \cos 1.31^\circ = 0$$

$$B_y + (1050.3 \text{ N}) \cos 1.31^\circ = 0$$

$$B_y = -1050 \text{ N}$$

$$\mathbf{B}_y = 1050 \text{ N} \downarrow \blacktriangleleft$$

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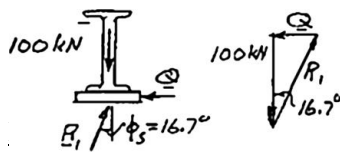


PROBLEM 8.50

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force *Q*, determine (a) the force *P* required to raise the beam, (b) the corresponding force *Q*.

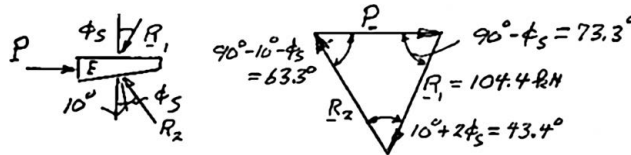
SOLUTION

Free body: Beam and plate *CD*



$$R_1 = \frac{(100 \text{ kN})}{\cos 16.7^\circ}$$

$$R_1 = 104.4 \text{ kN}$$



$$(a) \quad \frac{P}{\sin 43.4^\circ} = \frac{104.4 \text{ kN}}{\sin 63.3^\circ} \quad \mathbf{P = 80.3 \text{ kN} \rightarrow \blacktriangleleft}$$

$$(b) \quad \phi_s = \tan^{-1} 0.3 = 16.7^\circ$$

$$Q = (100 \text{ kN}) \tan 16.7^\circ \quad \mathbf{Q = 30 \text{ kN} \leftarrow \blacktriangleleft}$$

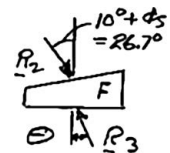
Free body: Wedge *F*

(To check that it does not move.)

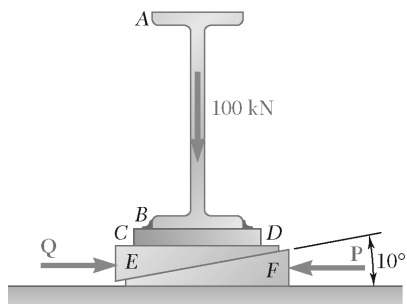
Since wedge *F* is a two-force body, R_2 and R_3 are colinear

Thus $\theta = 26.7^\circ$

But $\phi_{\text{concrete}} = \tan^{-1} 0.6 = 31.0^\circ > \theta$ OK



PROBLEM 8.51

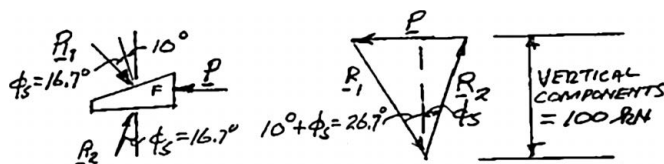


The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges E and F . The base plate CD has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN . The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force Q , determine (a) the force P required to raise the beam, (b) the corresponding force Q .

SOLUTION

Free body: Wedge F

$$\phi_s = \tan^{-1} 0.30 = 16.7^\circ$$



(a)

$$P = (100\text{ kN}) \tan 26.7^\circ + (100\text{ kN}) \tan \phi_s$$

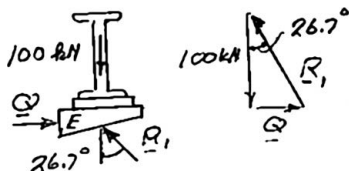
$$P = 50.29\text{ kN} + 30\text{ kN}$$

$$P = 80.29\text{ kN}$$

$$P = 80.3\text{ kN} \leftarrow \blacktriangleleft$$

$$R_1 = \frac{(100\text{ kN})}{\cos 26.7^\circ} = 111.94\text{ kN}$$

Free body: Beam, plate, and wedge E

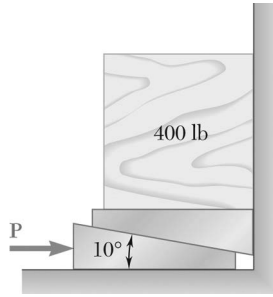


(b)

$$Q = W \tan 26.7^\circ = (100\text{ kN}) \tan 26.7^\circ$$

$$Q = 50.29\text{ kN}$$

$$Q = 50.3\text{ kN} \rightarrow \blacktriangleright$$



PROBLEM 8.52

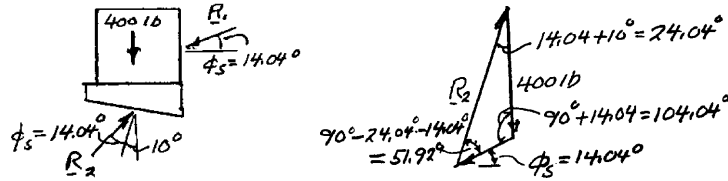
Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force P that should be applied as shown to one of the wedges.

SOLUTION

Free body: Block and top wedge

$$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$$

Force triangle

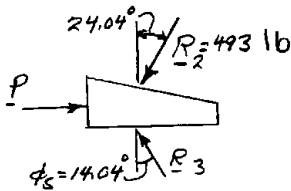


Law of sines:

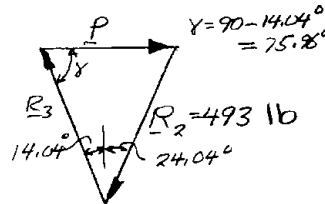
$$\frac{R_2}{\sin 104.04^\circ} = \frac{400 \text{ lb}}{\sin 51.92^\circ}$$

$$R_2 = 493 \text{ lb}$$

Free body: Lower wedge



Force triangle



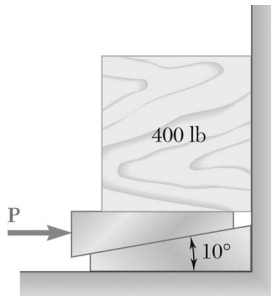
Law of sine:

$$\frac{P}{\sin(14.04^\circ + 24.04^\circ)} = \frac{493 \text{ lb}}{\sin 75.96^\circ}$$

$$P = 313.4 \text{ N}$$

$$P = 313 \text{ lb} \rightarrow \blacktriangleleft$$

PROBLEM 8.53



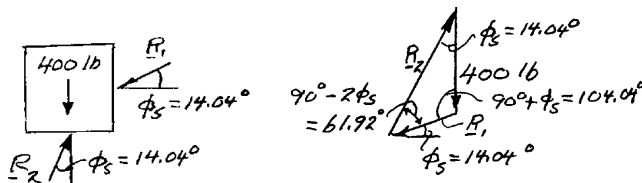
Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force P that should be applied as shown to one of the wedges.

SOLUTION

Free body: Block

$$\phi_s = \tan 0.25 = 14.04^\circ$$

Force triangle



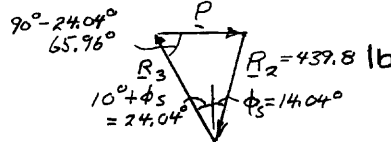
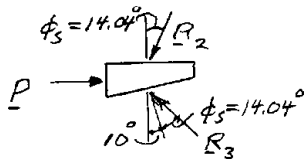
Law of sines:

$$\frac{R_2}{\sin 104.04^\circ} = \frac{400 \text{ lb}}{\sin 61.92^\circ}$$

$$R_2 = 439.8 \text{ lb}$$

Free body: Wedge

Force triangle

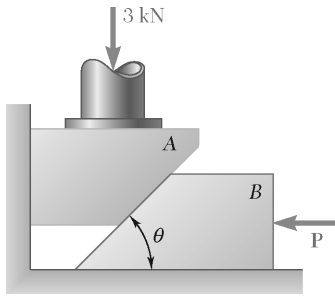


Law of sines:

$$\frac{P}{\sin(24.04^\circ + 14.04^\circ)} = \frac{439.8 \text{ lb}}{\sin 65.96^\circ}$$

$$P = 297.0 \text{ lb}$$

$$P = 297 \text{ lb} \rightarrow \blacktriangleleft$$



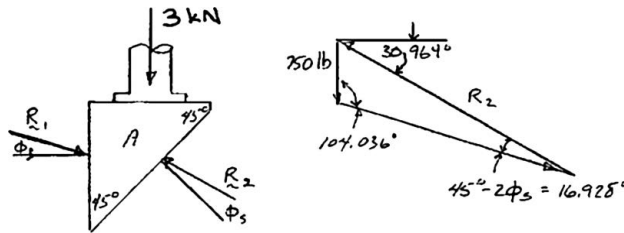
PROBLEM 8.54

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45^\circ$, determine the smallest force P required to raise block A.

SOLUTION

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.25 = 14.036^\circ$$

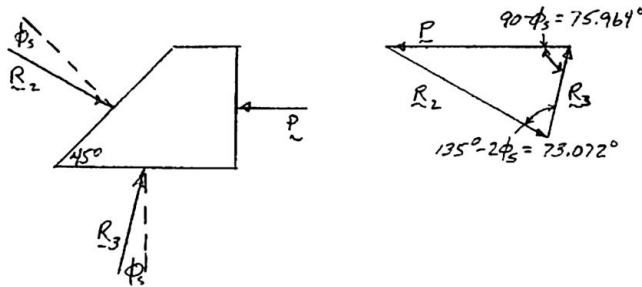
FBD block A:



$$\frac{R_2}{\sin 104.036^\circ} = \frac{3 \text{ kN}}{\sin 16.928^\circ}$$

$$R_2 = 10.0000 \text{ kN}$$

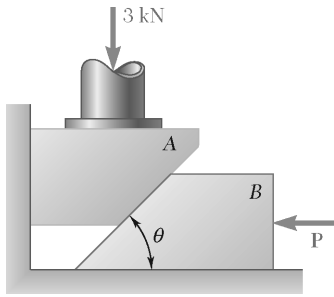
FBD wedge B:



$$\frac{P}{\sin 73.072^\circ} = \frac{10.0000 \text{ kN}}{\sin 75.964^\circ}$$

$$P = 9.8611 \text{ kN}$$

$$P = 9.86 \text{ kN} \leftarrow \blacktriangleleft$$



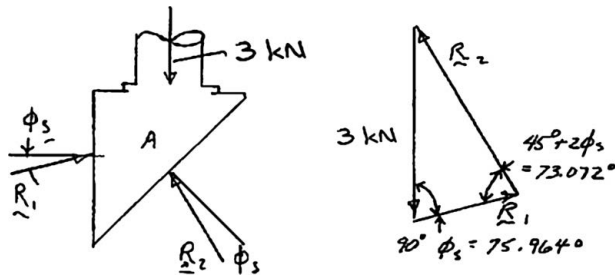
PROBLEM 8.55

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45^\circ$, determine the smallest force P for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.25 = 14.036^\circ$$

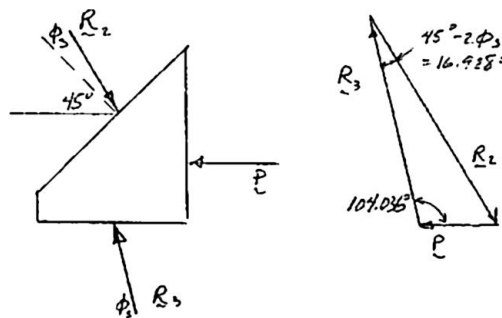
FBD block A:



$$\frac{R_2}{\sin(75.964^\circ)} = \frac{3 \text{ kN}}{\sin(73.072^\circ)}$$

$$R_2 = 3.0420 \text{ kN}$$

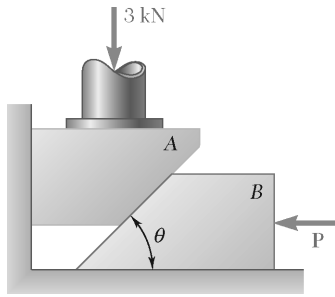
FBD wedge B:



$$\frac{P}{\sin 16.928^\circ} = \frac{3.0420 \text{ kN}}{\sin 104.036^\circ}$$

$$P = 0.91300 \text{ kN}$$

$$P = 913 \text{ N} \leftarrow \blacktriangleleft$$



PROBLEM 8.56

Block A supports a pipe column and rests as shown on wedge B. The coefficient of static friction at all surfaces of contact is 0.25. If $P = 0$, determine (a) the angle θ for which sliding is impending, (b) the corresponding force exerted on the block by the vertical wall.

SOLUTION

Free body: Wedge B

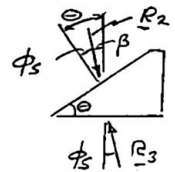
$$\phi_s = \tan^{-1} 0.25 = 14.04^\circ$$

- (a) Since wedge is a two-force body, R_2 and R_3 must be equal and opposite. Therefore, they form equal angles with vertical

$$\beta = \phi_s$$

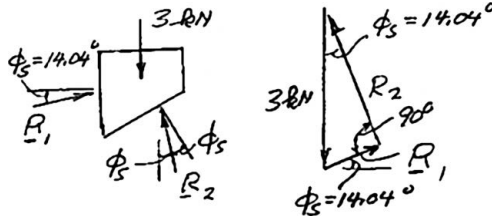
and

$$\begin{aligned} \theta - \phi_s &= \phi_s \\ \theta &= 2\phi_s = 2(14.04^\circ) \end{aligned}$$



$$\theta = 28.1^\circ \quad \blacktriangleleft$$

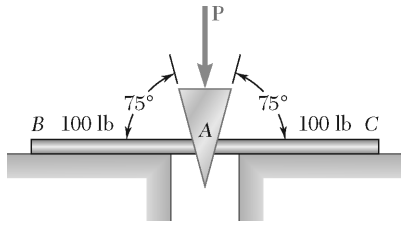
Free body: Block A



$$R_1 = (3 \text{ kN}) \sin 14.04^\circ = 0.7278 \text{ kN}$$

- (b) Force exerted by wall:

$$R_1 = 728 \text{ N} \quad \nearrow 14.04^\circ \quad \blacktriangleleft$$



PROBLEM 8.57

A wedge A of negligible weight is to be driven between two 100-lb plates B and C . The coefficient of static friction between all surfaces of contact is 0.35. Determine the magnitude of the force \mathbf{P} required to start moving the wedge (a) if the plates are equally free to move, (b) if plate C is securely bolted to the surface.

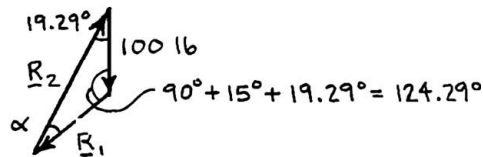
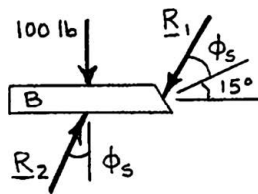
SOLUTION

(a) With plates equally free to move

Free body: Plate B

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.35 = 19.2900^\circ$$

Force triangle:



$$\alpha = 180^\circ - 124.29^\circ - 19.29^\circ = 36.42^\circ$$

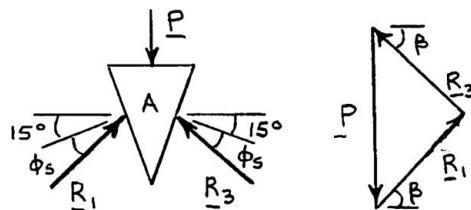
Law of sines:

$$\frac{R_1}{\sin 19.29^\circ} = \frac{100 \text{ lb}}{\sin 36.42^\circ}$$

$$R_1 = 55.643 \text{ lb}$$

Free body: Wedge A

Force triangle:



By symmetry,

$$R_3 = R_1 = 55.643 \text{ lb}$$

$$\beta = 19.29^\circ + 15^\circ = 34.29^\circ$$

Then

$$P = 2R_1 \sin \beta$$

or

$$P = 2(55.643) \sin 34.29^\circ$$

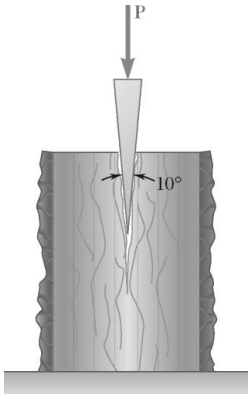
$$P = 62.7 \text{ lb} \quad \blacktriangleleft$$

(b) With plate C bolted

The free body diagrams of plate B and wedge A (the only members to move) are same as above.

Answer is thus the same.

$$P = 62.7 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 8.58

A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force \mathbf{P} of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion \downarrow):

$$\begin{aligned}\phi_s &= \tan^{-1} \mu_s \\ &= \tan^{-1} 0.35 \\ &= 19.29^\circ\end{aligned}$$

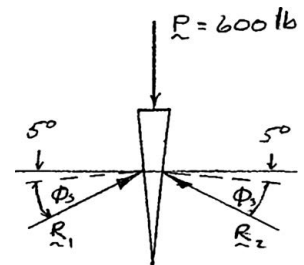
By symmetry:

$$R_1 = R_2$$

$$\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(5^\circ + \phi_s) - 600 \text{ lb} = 0$$

or

$$R_1 = R_2 = \frac{300 \text{ lb}}{\sin(5^\circ + 19.29^\circ)} = 729.30 \text{ lb}$$

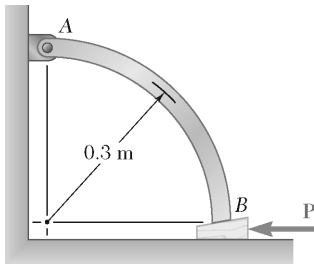


When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

$$\begin{aligned}R_{1x} &= R_{2x} \\ &= R_1 \cos(5^\circ + \phi_s) \\ &= (729.30 \text{ lb}) \cos(5^\circ + 19.29^\circ)\end{aligned}$$

$$R_{1x} = R_{2x} = 665 \text{ lb} \quad \blacktriangleleft$$

(Note that $\phi_s > 5^\circ$, so wedge is self-locking.)

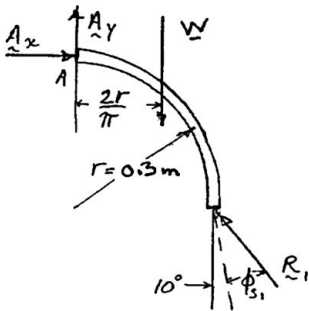


PROBLEM 8.59

A 10° wedge is to be forced under end B of the 5-kg rod AB . Knowing that the coefficient of static friction is 0.40 between the wedge and the rod and 0.20 between the wedge and the floor, determine the smallest force \mathbf{P} required to raise end B of the rod.

SOLUTION

FBD AB:



$$W = mg$$

$$W = (5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W = 49.050 \text{ N}$$

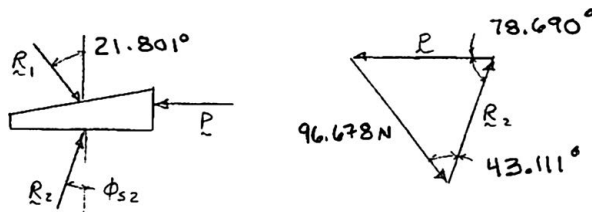
$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.40 = 21.801^\circ$$

$$\left(\sum M_A = 0: rR_1 \cos(10^\circ + 21.801^\circ) - rR_1 \sin(10^\circ + 21.801^\circ)\right)$$

$$-\frac{2r}{\pi}(49.050 \text{ N}) = 0$$

$$R_1 = 96.678 \text{ N}$$

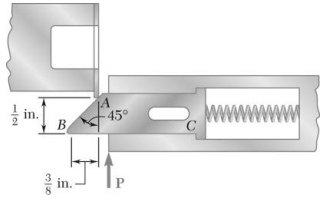
FBD wedge:



$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.20 = 11.3099$$

$$\frac{P}{\sin(43.111^\circ)} = \frac{96.678 \text{ N}}{\sin 78.690^\circ}$$

$$\mathbf{P} = 67.4 \text{ N} \leftarrow \blacktriangleleft$$



PROBLEM 8.60

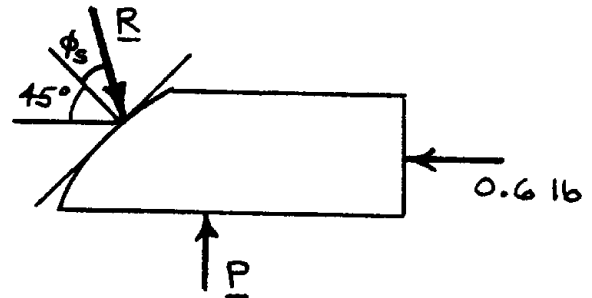
The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force \mathbf{P} required to start closing the door.

SOLUTION

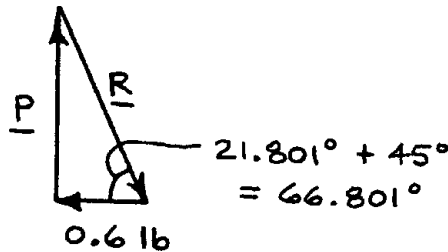
Free body: Bolt

$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40 \\ = 21.801^\circ$$



Force triangle:



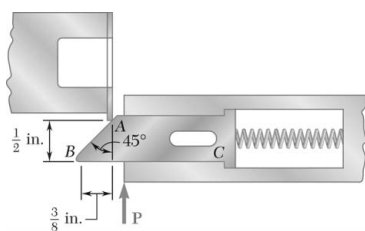
From force triangle,

$$P = (0.6 \text{ lb}) \tan 66.801^\circ$$

$$P = 1.39997 \text{ lb}$$

$$P = 1.400 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 8.61



In Problem 8.60, determine the angle that the face of the bolt should form with the line BC if the force \mathbf{P} required to close the door is to be the same for both the position shown and the position when B is almost at the strike plate.

PROBLEM 8.60 The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force \mathbf{P} required to start closing the door.

SOLUTION

For position shown in figure:

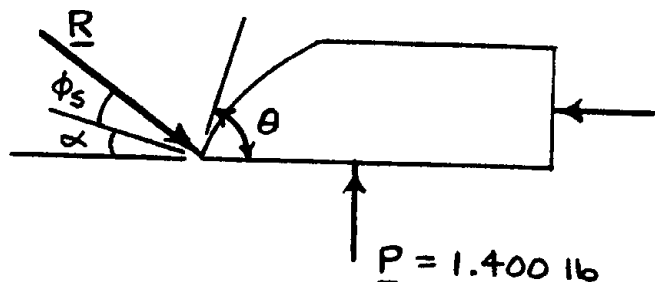
From Prob. 8.60: $P = 1.400 \text{ lb}$

For position when B reaches strike plate:

Free body: Bolt

$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40 = 21.801^\circ$$

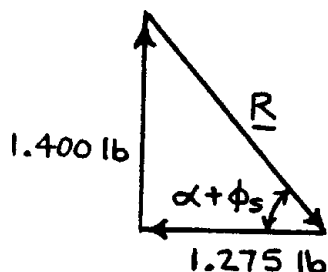


$$F = 0.6 \text{ lb} + kx$$

$$= 0.6 \text{ lb} + (1.8 \text{ lb/in.}) \left(\frac{3}{8} \text{ in.} \right)$$

$$= 1.275 \text{ lb}$$

Force triangle:



From force triangle,

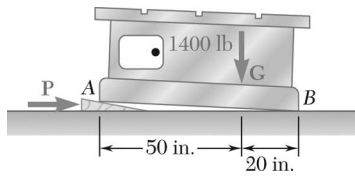
$$\tan(\alpha - \phi_s) = \frac{1.400 \text{ lb}}{1.275 \text{ lb}} = 1.09804$$

$$\alpha + \phi_s = 47.675^\circ$$

$$\alpha = 47.675^\circ - 21.801^\circ = 25.874^\circ$$

$$\theta = 180^\circ - 90^\circ - \alpha = 90^\circ - \alpha = 90^\circ - 25.874^\circ$$

$$\theta = 64.1^\circ \blacktriangleleft$$

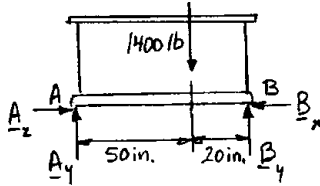


PROBLEM 8.62

A 5° wedge is to be forced under a 1400-lb machine base at A. Knowing that the coefficient of static friction at all surfaces is 0.20, (a) determine the force \mathbf{P} required to move the wedge, (b) indicate whether the machine base will move.

SOLUTION

Free body: Machine base



$$+\curvearrowright \Sigma M_B = 0: (1400 \text{ lb})(20 \text{ in}) - A_y(70 \text{ in}) = 0$$

$$A_y = 400 \text{ lb}$$

$$+\rightarrow \Sigma F_x = 0: A_y + B_y - 1400 \text{ lb} = 0$$

$$B_y = 1400 - 400 = 1000 \text{ lb}$$

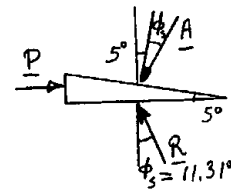
Free body: Wedge

(Assume machine will not move)

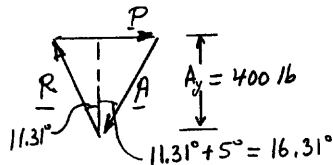
$$\mu_s = 0.20, \quad \phi_s = \tan 0.20 = 11.31^\circ$$

We know that

$$A_y = 400 \text{ lb}$$



Force triangle:



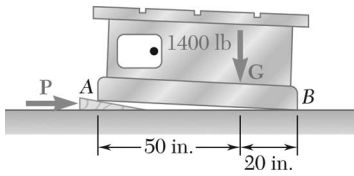
(a) $P = (400 \text{ lb})(\tan 11.31^\circ + \tan 16.31^\circ) = 197.0 \text{ lb}$ $\mathbf{P} = 197.0 \text{ lb} \rightarrow \blacktriangleleft$

(b) Total maximum friction force at A and B:

$$F_m = \mu_c W = 0.20(1400 \text{ lb}) = 280 \text{ lb}$$

Since $P < F_m$: $\mathbf{Machine\ will\ not\ move} \blacktriangleleft$

PROBLEM 8.63



Solve Problem 8.62 assuming that the wedge is to be forced under the machine base at B instead of A .

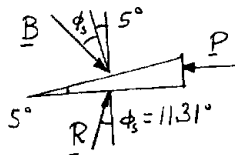
PROBLEM 8.62 A 5° wedge is to be forced under a 1400-lb machine base at A . Knowing that the coefficient of static friction at all surfaces is 0.20, (a) determine the force P required to move the wedge, (b) indicate whether the machine base will move.

SOLUTION

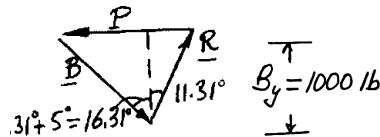
See solution to Prob. 8.62 for F.B.D. of machine base and determination of $B_y = 1000$ lb.

Free body: Wedge (Assume machine will not move)

$$\mu_s = 0.20 \quad \phi_s = \tan^{-1} 0.20 = 11.31^\circ$$



Force triangle:



$$P = (1000 \text{ lb})(\tan 11.31^\circ + \tan 16.31^\circ) = 493 \text{ lb}$$

Total maximum friction force at A and B :

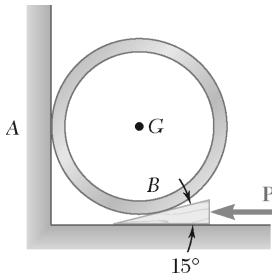
$$F_m = \mu_s W = 0.20(1400 \text{ lb}) = 280 \text{ lb} < 493 \text{ lb}$$

(b) Since $P > F_m$,

machine will move with wedge ◀

(a) We then have $P = F_m$,

$P = 280 \text{ lb}$ ◀◀



PROBLEM 8.64

A 15° wedge is forced under a 50-kg pipe as shown. The coefficient of static friction at all surfaces is 0.20. (a) Show that slipping will occur between the pipe and the vertical wall. (b) Determine the force **P** required to move the wedge.

SOLUTION

Free body: Pipe

$$+\circlearrowleft \Sigma M_B = 0: \quad Wr \sin \theta + F_A r(1 + \sin \theta) - N_A r \cos \theta = 0$$

Assume slipping at A:

$$F_A = \mu_s N_A$$

$$N_A \cos \theta - \mu_s N_A (1 + \sin \theta) = W \sin \theta$$

$$N_A = \frac{W \sin \theta}{\cos \theta - \mu_s (1 + \sin \theta)}$$

$$N_A = \frac{W \sin 15^\circ}{\cos 15^\circ - (0.20)(1 + \sin 15^\circ)}$$

$$= 0.36241W$$

$$+\nearrow \Sigma F_x = 0: \quad -F_B - W \sin \theta - F_A \sin \theta + N_A \cos \theta = 0$$

$$F_B = N_A \cos \theta - \mu_s N_A \sin \theta - W \sin \theta$$

$$F_B = (0.36241W) \cos 15^\circ - 0.20(0.36241W) \sin 15^\circ - W \sin 15^\circ$$

$$F_B = 0.072482W$$

$$+\searrow \Sigma F_y = 0: \quad N_B - W \cos \theta - F_A \cos \theta - N_A \sin \theta = 0$$

$$N_B = N_A \sin \theta + \mu_s N_A \cos \theta + W \cos \theta$$

$$N_B = (0.36241W) \sin 15^\circ + 0.20(0.36241W) \cos 15^\circ + W \cos 15^\circ$$

$$N_B = 1.12974W$$

Maximum available:

$$F_B = \mu_s N_B = 0.22595W$$

(a) We note that $F_B < F_{\max}$

No slip at B ◀

(b)

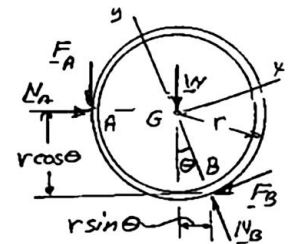
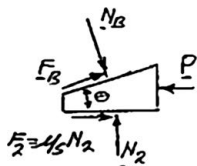
Free body: Wedge

$$+\uparrow \Sigma F_y = 0: \quad N_2 - N_B \cos \theta + F_B \sin \theta = 0$$

$$N_2 = N_B \cos \theta - F_B \sin \theta$$

$$N_2 = (1.12974W) \cos 15^\circ - (0.07248W) \sin 15^\circ$$

$$N_2 = 1.07249W$$



PROBLEM 8.64 (Continued)

$$\rightarrow \Sigma F_x = 0: F_B \cos \theta + N_B \sin \theta + \mu_s N_2 - P = 0$$

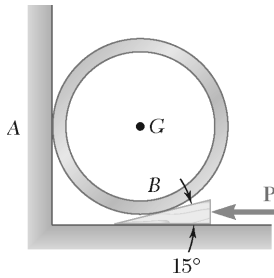
$$P = F_B \cos \theta + N_B \sin \theta + \mu_s N_2$$

$$P = (0.07248W) \cos 15^\circ + (1.12974W) \sin 15^\circ + 0.2(1.07249W)$$

$$P = 0.5769W$$

$$W = mg: P = 0.5769(50 \text{ kg})(9.81 \text{ m/s}^2)$$

$$\mathbf{P} = 283 \text{ N} \leftarrow \blacktriangleleft$$



PROBLEM 8.65

A 15° wedge is forced under a 50-kg pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping will occur at A.

SOLUTION

Free body: Pipe

$$+\circlearrowleft \Sigma M_A = 0: N_B r \cos \theta - \mu_B N_B r - (\mu_B N_B \sin \theta) r - W r = 0$$

$$N_B = \frac{W}{\cos \theta - \mu_B (1 + \sin \theta)}$$

$$N_B = \frac{W}{\cos 15^\circ - 0.2(1 + \sin 15^\circ)}$$

$$N_B = 1.4002W$$

$$+\rightarrow \Sigma F_x = 0: N_A - N_B \sin \theta - \mu_B N_B \cos \theta = 0$$

$$N_A = N_B (\sin \theta + \mu_B \cos \theta)$$

$$= (1.4002W)(\sin 15^\circ + 0.2 \times \cos 15^\circ)$$

$$N_A = 0.63293W$$

$$+\uparrow \Sigma F_y = 0: -F_A - W + N_B \cos \theta - \mu_B N_B \sin \theta = 0$$

$$F_A = N_B (\cos \theta - \mu_B \sin \theta) - W$$

$$F_A = (1.4002W)(\cos 15^\circ - 0.2 \times \sin 15^\circ) - W$$

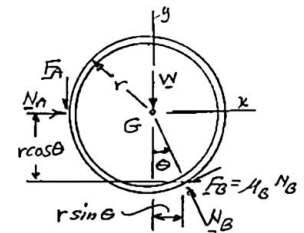
$$F_A = 0.28001W$$

For slipping at A:

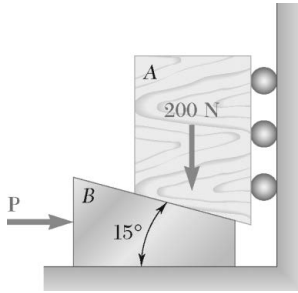
$$F_A = \mu_A N_A$$

$$\mu_A = \frac{F_A}{N_A} = \frac{0.28001W}{0.63293W}$$

$$\mu_A = 0.442 \quad \blacktriangleleft$$



PROBLEM 8.66*

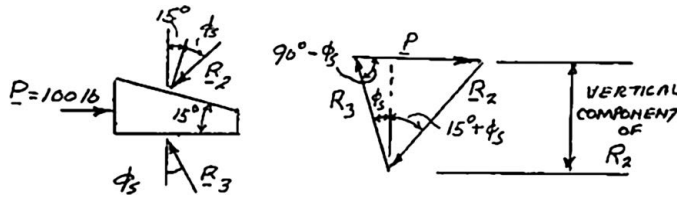


A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For $P = 100$ N, determine the value of μ_s for which motion is impending. (*Hint: Solve the equation obtained by trial and error.*)

SOLUTION

Free body: Wedge

Force triangle:



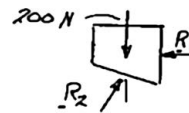
Law of sines:

$$\frac{R_2}{\sin(90^\circ - \phi_s)} = \frac{P}{\sin(15^\circ + 2\phi_s)}$$

$$R_2 = P \frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} \quad (1)$$

Free body: Block

$$\Sigma F_y = 0$$



Vertical component of R_2 is 200 N

Return to force triangle of wedge. Note $P = 100$ N

$$100 \text{ N} = (200 \text{ N}) \tan \phi + (200 \text{ N}) \tan(15^\circ + \phi_s)$$

$$0.5 = \tan \phi + \tan(15^\circ + \phi_s)$$

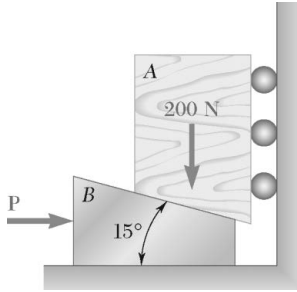
Solve by trial and error

$$\phi_s = 6.292$$

$$\mu_s = \tan \phi_s = \tan 6.292^\circ$$

$$\mu_s = 0.1103 \quad \blacktriangleleft$$

PROBLEM 8.67*



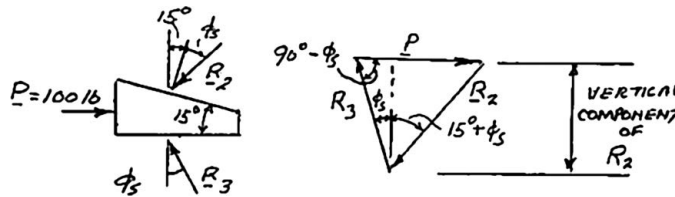
Solve Problem 8.66 assuming that the rollers are removed and that μ_s is the coefficient of friction at all surfaces of contact.

PROBLEM 8.66* A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For $P = 100$ N, determine the value of μ_s for which motion is impending. (Hint: Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:



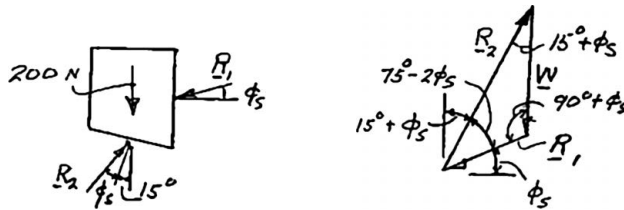
Law of sines:

$$\frac{R_2}{\sin(90^\circ - \phi_s)} = \frac{P}{\sin(15^\circ + 2\phi_s)}$$

$$R_2 = P \frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} \quad (1)$$

Free body: Block (Rollers removed)

Force triangle:



Law of sines:

$$\frac{R_2}{\sin(90^\circ + \phi_s)} = \frac{W}{\sin(75^\circ - 2\phi_s)}$$

$$R_2 = W \frac{\sin(90^\circ + \phi_s)}{\sin(75^\circ - 2\phi_s)} \quad (2)$$

PROBLEM 8.67* (Continued)

Equate R_2 from Eq. (1) and Eq. (2):

$$P \frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} = W \frac{\sin(90^\circ + \phi_s)}{\sin(75^\circ - 2\phi_s)}$$

$$P = 100 \text{ lb}$$

$$W = 200 \text{ N}$$

$$0.5 = \frac{\sin(90^\circ + \phi_s) \sin(15^\circ + 2\phi_s)}{\sin(75^\circ - 2\phi_s) \sin(90^\circ - \phi_s)}$$

Solve by trial and error:

$$\phi_s = 5.784^\circ$$

$$\mu_s = \tan \phi_s = \tan 5.784^\circ$$

$$\mu_s = 0.1013 \quad \blacktriangleleft$$

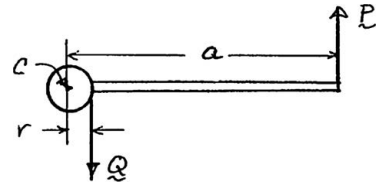
PROBLEM 8.68

Derive the following formulas relating the load W and the force P exerted on the handle of the jack discussed in Section 8.6. (a) $P = (Wr/a) \tan(\theta + \phi_s)$, to raise the load; (b) $P = (Wr/a) \tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (c) $P = (Wr/a) \tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

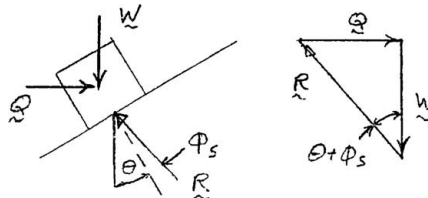
FBD jack handle:

See Section 8.6. $(\sum M_C = 0: aP - rQ = 0 \text{ or } P = \frac{r}{a}Q$



FBD block on incline:

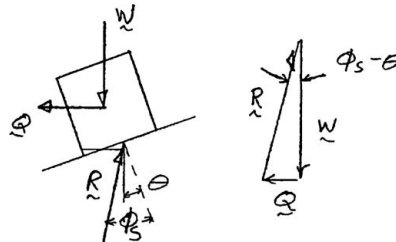
(a) Raising load



$$Q = W \tan(\theta + \phi_s)$$

$$P = \frac{r}{a} W \tan(\theta + \phi_s) \quad \blacktriangleleft$$

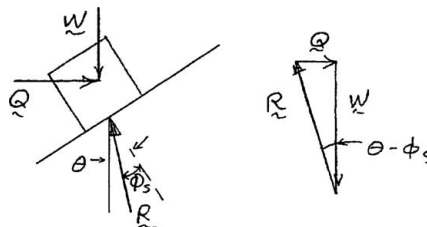
(b) Lowering load if screw is self-locking (i.e., if $\phi_s > \theta$)



$$Q = W \tan(\phi_s - \theta)$$

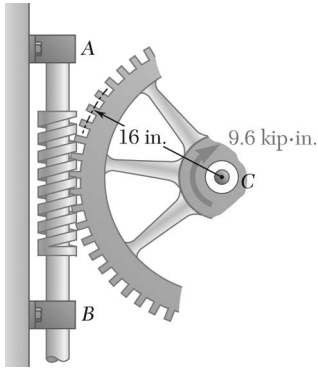
$$P = \frac{r}{a} W \tan(\phi_s - \theta) \quad \blacktriangleleft$$

(c) Holding load if screw is not self-locking (i.e., if $\phi_s < \theta$)



$$Q = W \tan(\theta - \phi_s)$$

$$P = \frac{r}{a} W \tan(\theta - \phi_s) \quad \blacktriangleleft$$



PROBLEM 8.69

The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft AB in order to rotate the large gear counterclockwise. Neglect friction in the bearings at A , B , and C .

SOLUTION

Free body: Large gear

$$\begin{aligned} +\curvearrowright \Sigma M_C = 0: \quad W(16 \text{ in.}) - 9.6 \text{ kip} \cdot \text{in.} &= 0 \\ W &= 0.6 \text{ kip} = 600 \text{ lb} \end{aligned}$$

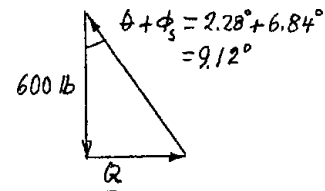
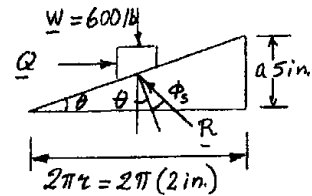
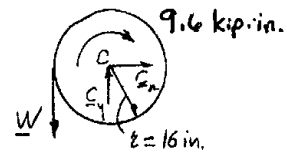
Block-and-Incline analysis of worm gear

$$\begin{aligned} \tan \theta &= \frac{0.5 \text{ in.}}{2\pi(2 \text{ in.})} = 0.039789 \\ \theta &= 2.28^\circ \end{aligned}$$

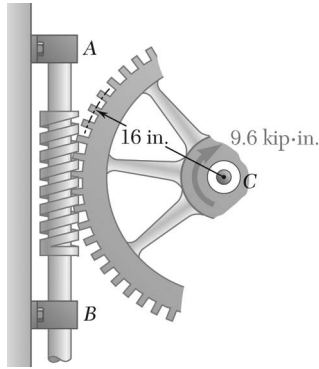
$$\mu_s = 0.12, \quad \phi_s = \tan^{-1} 0.12 = 6.84^\circ$$

$$\begin{aligned} Q &= (600 \text{ lb}) \tan 9.12^\circ \\ &= 96.32 \text{ lb} \end{aligned}$$

$$\begin{aligned} \text{Torque} &= Q_r = (96.32 \text{ lb})(2 \text{ in.}) \\ &= 192.6 \text{ lb} \cdot \text{in.} \end{aligned}$$



$$\text{Torque} = 16.05 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$



PROBLEM 8.70

In Problem 8.69, determine the couple that must be applied to shaft AB in order to rotate the large gear clockwise.

PROBLEM 8.69 The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft AB in order to rotate the large gear counterclockwise. Neglect friction in the bearings at A , B , and C .

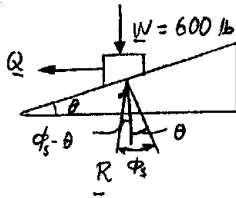
SOLUTION

Free body: Large gear

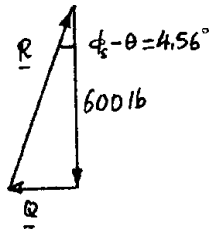
See solution to Prob. 8.69. We find $W = 600$ lb

Block-and-incline analysis of worm gear

From Prob. 8.69 we have $\theta = 2.28^\circ$ and $\phi_s = 6.84^\circ$. Since $\theta < \phi_s$, gear is self-locking and a torque must be applied to it to rotate large gear clockwise.



$$\phi_s - \theta = 6.84^\circ - 2.28^\circ = 4.56^\circ$$



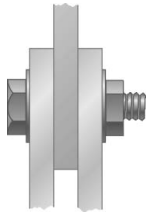
$$Q = (600 \text{ lb}) \tan 4.56^\circ$$

$$= 47.85 \text{ lb}$$

$$\text{Torque} = Q_r = (47.85 \text{ lb})(2 \text{ in.})$$

$$= 95.70 \text{ lb} \cdot \text{in.}$$

$$\text{Torque} = 7.98 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$

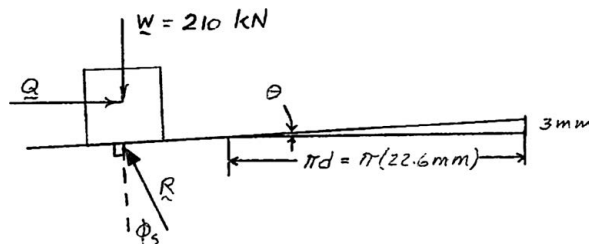


PROBLEM 8.71

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

FBD block on incline:



$$\theta = \tan^{-1} \frac{3 \text{ mm}}{(22.6 \text{ mm})\pi}$$

$$= 2.4195^\circ$$

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.40$$

$$\phi_s = 21.801^\circ$$

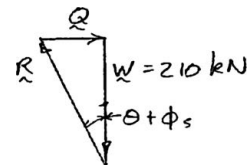
$$Q = (210 \text{ kN}) \tan (21.801^\circ + 2.4195^\circ)$$

$$Q = 94.468 \text{ kN}$$

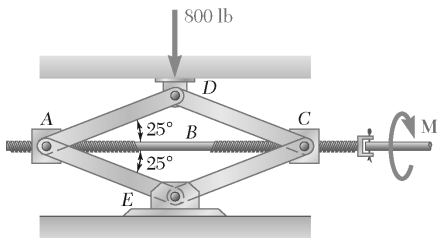
$$\text{Torque} = \frac{d}{2} Q$$

$$= \frac{22.6 \text{ mm}}{2} (94.468 \text{ kN})$$

$$= 1067.49 \text{ N} \cdot \text{m}$$



$$\text{Torque} = 1068 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 8.72

The position of the automobile jack shown is controlled by a screw ABC that is single-threaded at each end (right-handed thread at A , left-handed thread at C). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple M that must be applied to raise the automobile.

SOLUTION

Free body: Parts A, D, C, E

Two-force members

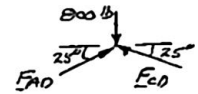
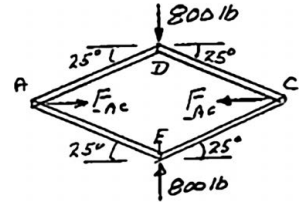
Joint D :

Symmetry:

$$F_{AD} = F_{CD}$$

$$\uparrow \Sigma F_y = 0: 2F_{CD} \sin 25^\circ - 800 \text{ lb} = 0$$

$$F_{CD} = 946.5 \text{ lb}$$



Joint C :

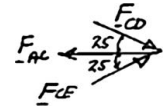
Symmetry:

$$F_{CE} = F_{CD}$$

$$\rightarrow \Sigma F_x = 0: 2F_{CD} \cos 25^\circ - F_{AC} = 0$$

$$F_{AC} = 2(946.5 \text{ lb}) \cos 25^\circ$$

$$F_{AC} = 1715.6 \text{ lb}$$



Block-and-incline analysis of one screw:

$$\tan \theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$$

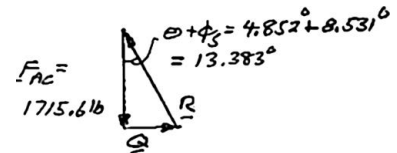
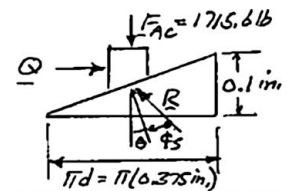
$$\theta = 4.852^\circ$$

$$\phi_s = \tan^{-1} 0.15$$

$$= 8.531^\circ$$

$$Q = (1715.6 \text{ lb}) \tan 13.383^\circ$$

$$Q = 408.2 \text{ lb}$$



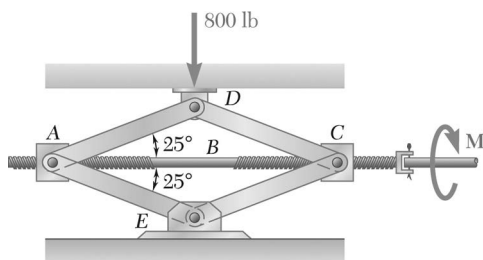
But, we have *two* screws:

$$\text{Torque} = 2Qr = 2(408.2 \text{ lb}) \left(\frac{0.375 \text{ in.}}{2} \right)$$

$$\text{Torque} = 153.1 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

PROBLEM 8.73

For the jack of Problem 8.72, determine the magnitude of the couple M that must be applied to lower the automobile.



PROBLEM 8.72 The position of the automobile jack shown is controlled by a screw ABC that is single-threaded at each end (right-handed thread at A , left-handed thread at C). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple M that must be applied to raise the automobile.

SOLUTION

Free body: Parts A, D, C, E

Two-force members

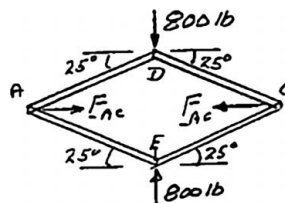
Joint D:

Symmetry:

$$F_{AD} = F_{CD}$$

$$\uparrow \Sigma F_y = 0: 2F_{CD} \sin 25^\circ - 800 \text{ lb} = 0$$

$$F_{CD} = 946.5 \text{ lb}$$



Joint C:

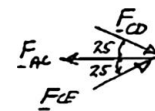
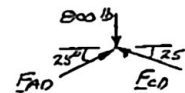
Symmetry:

$$F_{CE} = F_{CD}$$

$$\pm \Sigma F_x = 0: 2F_{CD} \cos 25^\circ - F_{AC} = 0$$

$$F_{AC} = 2(946.5 \text{ lb}) \cos 25^\circ;$$

$$F_{AC} = 1715.6 \text{ lb}$$



Block-and-incline analysis of one screw:

$$\tan \theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$$

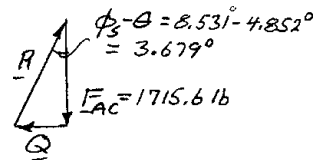
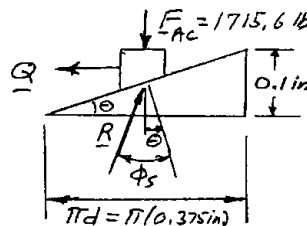
$$\theta = 4.852^\circ$$

$$\begin{aligned} \phi_s &= \tan^{-1} 0.15 \\ &= 8.531^\circ \end{aligned}$$

Since $\phi_s > \theta$, the screw is self-locking

$$Q = (1715.6 \text{ lb}) \tan 3.679^\circ$$

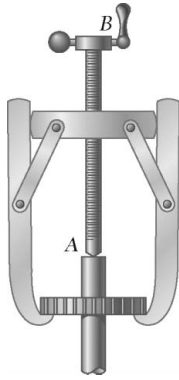
$$Q = 110.3 \text{ lb}$$



For two screws:

$$\text{Torque} = 2(110.3 \text{ lb}) \frac{1}{2}(0.375 \text{ in.})$$

$$\text{Torque} = 41.4 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

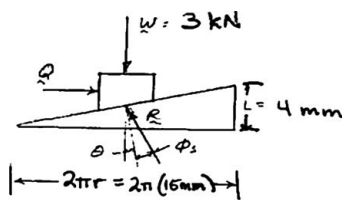


PROBLEM 8.74

In the gear-pulling assembly shown the square-threaded screw AB has a mean radius of 15 mm and a lead of 4 mm. Knowing that the coefficient of static friction is 0.10, determine the couple that must be applied to the screw in order to produce a force of 3 kN on the gear. Neglect friction at end A of the screw.

SOLUTION

Block/Incline:



$$\theta = \tan^{-1} \frac{4 \text{ mm}}{30\pi \text{ mm}}$$

$$= 2.4302^\circ$$

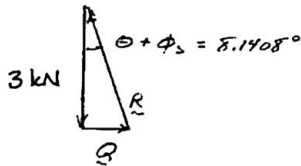
$$\phi_s = \tan^{-1} \mu_s$$

$$= \tan^{-1}(0.10)$$

$$= 5.7106^\circ$$

$$Q = (3000 \text{ N}) \tan(8.1408^\circ)$$

$$= 429.14 \text{ N}$$



$$\text{Couple} = rQ$$

$$= (0.015 \text{ m})(429.14 \text{ N})$$

$$= 6.4371 \text{ N} \cdot \text{m}$$

$$M = 6.44 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

PROBLEM 8.75

The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod *A* has a right-handed thread and rod *B* has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.



SOLUTION

To draw rods together:

Screw at A

$$\tan \theta = \frac{2 \text{ mm}}{2\pi(6 \text{ mm})}$$

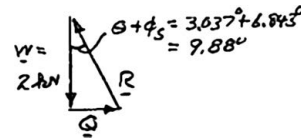
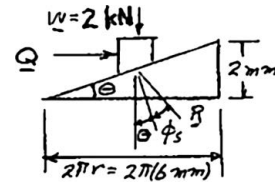
$$\theta = 3.037^\circ$$

$$\phi_s = \tan^{-1} 0.12 = 6.843^\circ$$

$$Q = (2 \text{ kN}) \tan 9.88^\circ = 348.3 \text{ N}$$

$$\begin{aligned} \text{Torque at A} &= Qr \\ &= (348.3 \text{ N})(0.006 \text{ m}) \\ &= 2.0898 \text{ N} \cdot \text{m} \end{aligned}$$

Same torque required at *B*



Total torque = 4.18 N · m ◀

PROBLEM 8.76

Assuming that in Problem 8.75 a right-handed thread is used on *both* rods *A* and *B*, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

PROBLEM 8.75 The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod *A* has a right-handed thread and rod *B* has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.



SOLUTION

From the solution to Problem 8.70,

$$\text{Torque at } A = 2.09 \text{ N} \cdot \text{m}$$

Screw at *B*: Loosening

$$\theta = 3.037^\circ$$

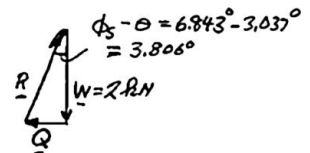
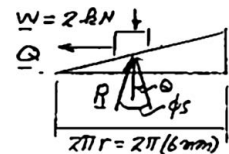
$$\phi_s = 6.843^\circ$$

$$Q = (2 \text{ kN}) \tan 3.806^\circ \\ = 133.1 \text{ N}$$

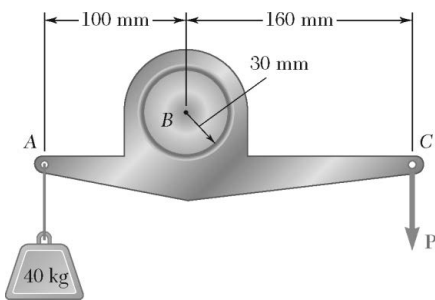
$$\text{Torque at } B = Qr$$

$$= (133.1 \text{ N})(0.006 \text{ m}) \\ = 0.79860 \text{ N} \cdot \text{m}$$

$$\text{Total torque} = 2.0848 \text{ N} \cdot \text{m} + 0.79860 \text{ N} \cdot \text{m}$$



$$\text{Total torque} = 2.89 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 8.77

A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force \mathbf{P} of magnitude 275 N will just start the lever rotating clockwise, determine (a) the coefficient of static friction between the shaft and the lever, (b) the smallest force \mathbf{P} for which the lever does not start rotating counterclockwise.

SOLUTION

(a) Impending motion ↷

$$W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

$$+\curvearrowright \Sigma M_D = 0: P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

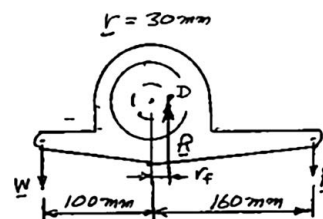
$$r_f = \frac{(160 \text{ mm})(275 \text{ N}) - (100 \text{ mm})(392.4 \text{ N})}{275 \text{ N} + 392.4 \text{ N}}$$

$$r_f = 7.132 \text{ mm}$$

$$r_f = r \sin \phi_s = r \mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132 \text{ mm}}{30 \text{ mm}} = 0.2377$$

$$\mu_s = 0.238 \quad \blacktriangleleft$$



(b) Impending motion ↶

$$r_f = r \sin \phi_s = r \mu_s$$

$$= (30 \text{ mm})(0.2377)$$

$$r_f = 7.132 \text{ mm}$$

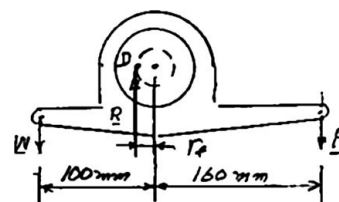
$$+\curvearrowright \Sigma M_D = 0: P(160 + r_f) - W(100 - r_f) = 0$$

$$P = W \frac{100 - r_f}{160 + r_f}$$

$$P = (392.4 \text{ N}) \frac{100 \text{ mm} - 7.132 \text{ mm}}{160 \text{ mm} + 7.132 \text{ mm}}$$

$$P = 218.04 \text{ N}$$

$$P = 218 \text{ N} \quad \blacktriangleleft$$



PROBLEM 8.78

A hot-metal ladle and its contents weigh 130 kips. Knowing that the coefficient of static friction between the hooks and the pinion is 0.30, determine the tension in cable AB required to start tipping the ladle.

SOLUTION

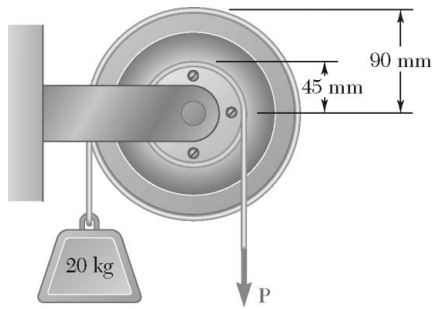
Free body: Ladle

$\sin \phi_s \approx \tan \phi_s = \mu_s = 0.30$
 $r_{\text{bearing}} = 8 \text{ in.}$
 $r_f = r_{\text{bearing}} \sin \phi_s = (8 \text{ in.}) (0.30) = 2.4 \text{ in.}$

\mathbf{R} is tangent to friction circle at A .

$\begin{aligned} +\curvearrowright \Sigma M_A = 0: & T(64 \text{ in.} + r_f) - (130 \text{ kips})r_f = 0 \\ & T(64 + 2.4) - (130)(2.4) = 0 \end{aligned}$

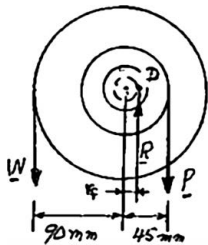
$T = 4.70 \text{ kips} \blacktriangleleft$



PROBLEM 8.79

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION



$$+\curvearrowright \Sigma M_D = 0: \quad P(45 - r_f) - W(90 + r_f) = 0$$

$$P = W \frac{90 + r_f}{45 - r_f}$$

$$= (196.2 \text{ N}) \frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 449.82 \text{ N}$$

$$P = 450 \text{ N} \quad \blacktriangleleft$$

PROBLEM 8.80

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

Find **P** required to start raising load

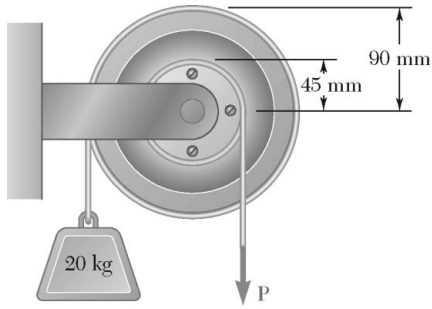
$$+\curvearrowright \Sigma M_D = 0: \quad P(45 - r_f) - W(90 - r_f) = 0$$

$$P = W \frac{90 - r_f}{45 - r_f}$$

$$= (196.2 \text{ N}) \frac{90 \text{ mm} - 4 \text{ mm}}{45 \text{ mm} - 4 \text{ mm}}$$

$$P = 411.54 \text{ N} \qquad P = 412 \text{ N} \quad \blacktriangleleft$$

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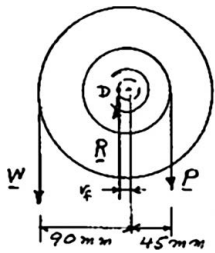


PROBLEM 8.81

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



$$+\curvearrowright \Sigma M_D = 0: \quad P(45 + r_f) - W(90 - r_f) = 0$$

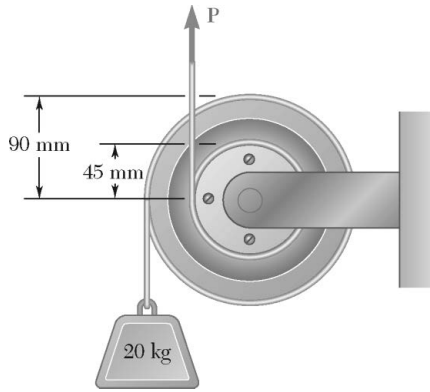
$$P = W \frac{90 - r_f}{45 + r_f}$$

$$= (196.2 \text{ N}) \frac{90 \text{ mm} - 4 \text{ mm}}{45 \text{ mm} + 4 \text{ mm}}$$

$$P = 344.35 \text{ N}$$

$$P = 344 \text{ N} \quad \blacktriangleleft$$

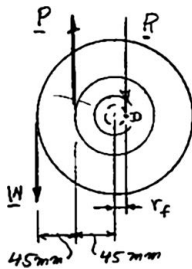
PROBLEM 8.82



The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force P required to maintain equilibrium.

SOLUTION

Find smallest P to maintain equilibrium



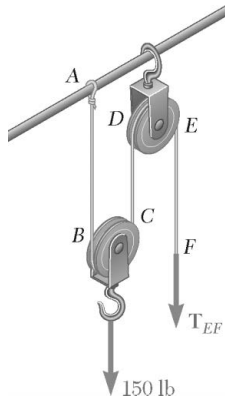
$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 + r_f) = 0$$

$$P = W \frac{90 + r_f}{45 + r_f}$$

$$= (196.2 \text{ N}) \frac{90 \text{ mm} + 4 \text{ mm}}{45 \text{ mm} + 4 \text{ mm}}$$

$$P = 376.38 \text{ N}$$

$$P = 376 \text{ N} \quad \blacktriangleleft$$



PROBLEM 8.83

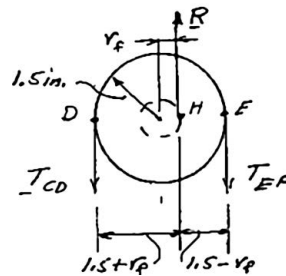
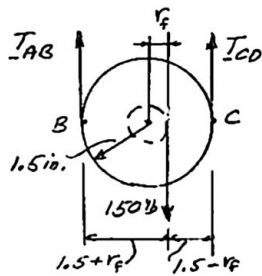
The block and tackle shown are used to raise a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

SOLUTION

For each pulley:

Axle diameter = 0.5 in.

$$r_f = r \sin \phi_s \approx \mu_s r = 0.20 \left(\frac{0.5 \text{ in.}}{2} \right) = 0.05 \text{ in.}$$



Pulley BC:

$$+\circlearrowleft \Sigma M_B = 0: T_{CD}(3 \text{ in.}) - (150 \text{ lb})(1.5 \text{ in.} + r_f) = 0$$

$$T_{CD} = \frac{1}{3}(150 \text{ lb})(1.5 \text{ in.} + 0.05 \text{ in.})$$

$$T_{CD} = 77.5 \text{ lb} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: T_{AB} + 77.5 \text{ lb} - 150 \text{ lb} = 0$$

$$T_{AB} = 72.5 \text{ lb} \quad \blacktriangleleft$$

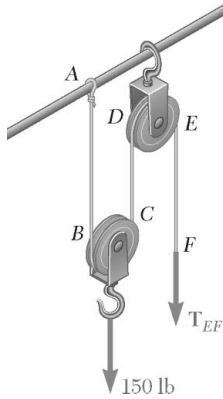
Pulley DE:

$$+\circlearrowleft \Sigma M_B = 0: T_{CD}(1.5 + r_f) - T_{EF}(1.5 - r_f) = 0$$

$$T_{EF} = T_{CD} \frac{1.5 + r_f}{1.5 - r_f}$$

$$= (77.5 \text{ lb}) \frac{1.5 \text{ in.} + 0.05 \text{ in.}}{1.5 \text{ in.} - 0.05 \text{ in.}}$$

$$T_{EF} = 82.8 \text{ lb} \quad \blacktriangleleft$$



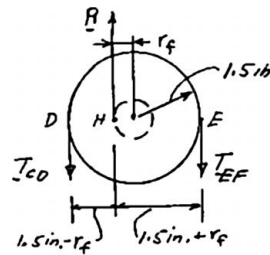
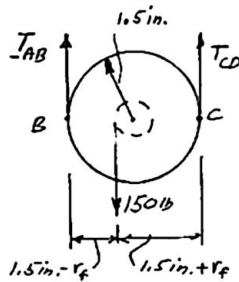
PROBLEM 8.84

The block and tackle shown are used to lower a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

For each pulley:

$$r_f = r\mu_s = \left(\frac{0.5 \text{ in.}}{2}\right)0.2 = 0.05 \text{ in.}$$



Pulley BC:

$$+\circlearrowleft \Sigma M_B = 0: T_{CD}(3 \text{ in.}) - (150 \text{ lb})(1.5 \text{ in.} - r_f) = 0$$

$$T_{CD} = \frac{(150 \text{ lb})(1.5 \text{ in.} - 0.05 \text{ in.})}{3 \text{ in.}}$$

$$T_{CD} = 72.5 \text{ lb} \quad \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: T_{AB} + 72.5 \text{ lb} - 150 \text{ lb} = 0$$

$$T_{AB} = 77.5 \text{ lb} \quad \blacktriangleleft$$

Pulley DE:

$$T_{CD}(1.5 \text{ in.} - r_f) - T_{EF}(1.5 \text{ in.} + r_f) = 0$$

$$T_{EF} = T_{CD} \frac{1.5 \text{ in.} - r_f}{1.5 \text{ in.} + r_f}$$

$$= (72.5 \text{ lb}) \frac{1.5 \text{ in.} - 0.05 \text{ in.}}{1.5 \text{ in.} + 0.05 \text{ in.}}$$

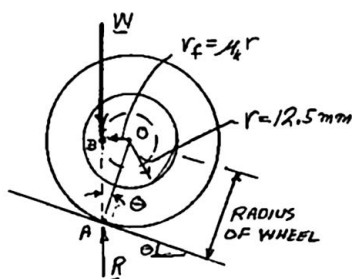
$$T_{EF} = 67.8 \text{ lb} \quad \blacktriangleleft$$

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PROBLEM 8.85

A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION



$$\tan \theta = \frac{2}{100} = 0.02$$

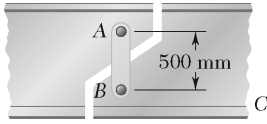
Since a scooter rolls at constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.

$$\begin{aligned} r_f &= \mu_k r = (0.10)(12.5 \text{ mm}) \\ &= 1.25 \text{ mm} \end{aligned}$$

$$\begin{aligned} OA &= \frac{OB}{\tan \theta} = \frac{r_f}{\tan \theta} = \frac{1.25 \text{ mm}}{0.02} \\ &= 62.5 \text{ mm} \end{aligned}$$

$$\text{Diameter of wheel} = 2(OA) = 125.0 \text{ mm} \blacktriangleleft$$

PROBLEM 8.86

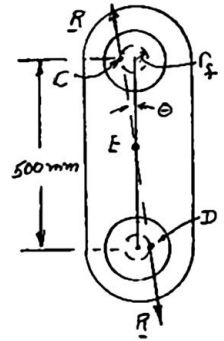


The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 60-mm-diameter pins A and B the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by BC on the link is 200 kN, determine (a) the horizontal force that should be exerted on beam BC to just move the link, (b) the angle that the resulting force exerted by beam BC on the link will form with the vertical.

SOLUTION

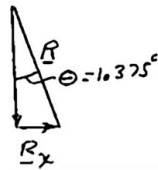
Bearing:

$$\begin{aligned} r &= 30 \text{ mm} \\ r_f &= \mu_s r \\ &= 0.20(30 \text{ mm}) \\ &= 6 \text{ mm} \end{aligned}$$



Resultant forces \mathbf{R} must be tangent to friction circles at Points C and D .

(a)

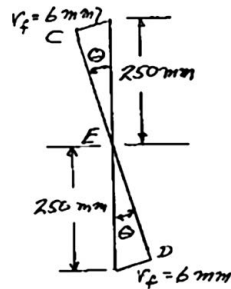


$$R_y = \text{Vertical component} = 200 \text{ kN}$$

$$\begin{aligned} R_x &= R_y \tan \theta \\ &= (200 \text{ kN}) \tan 1.375^\circ \\ &= 4.80 \text{ kN} \end{aligned}$$

Horizontal force = 4.80 kN ◀

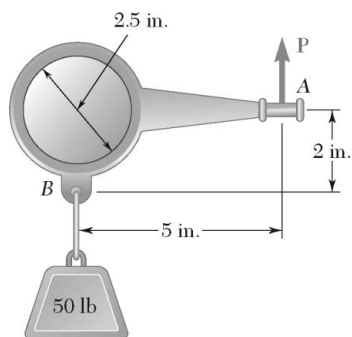
(b)



$$\sin \theta = \frac{6 \text{ mm}}{250 \text{ mm}}$$

$$\sin \theta = 0.024$$

$\theta = 1.375^\circ$ ◀



PROBLEM 8.87

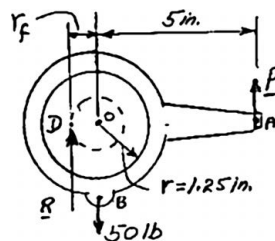
A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force \mathbf{P} required to start the lever rotating counterclockwise.

SOLUTION

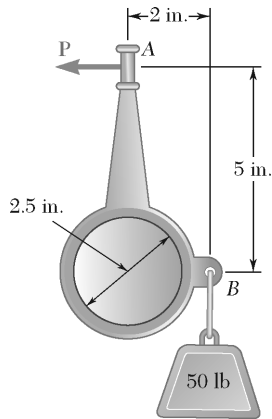
$$\begin{aligned} r_f &= \mu_s r \\ &= 0.15(1.25 \text{ in.}) \\ &= 0.1875 \text{ in.} \end{aligned}$$

$$+\curvearrowright \Sigma M_D = 0: \quad P(5 \text{ in.} + r_f) - (50 \text{ lb})r_f = 0$$

$$\begin{aligned} P &= \frac{50(0.1875)}{5.1875} \\ &= 1.807 \text{ lb} \end{aligned}$$



$$\mathbf{P} = 1.807 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 8.88

A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force \mathbf{P} required to start the lever rotating counterclockwise.

SOLUTION

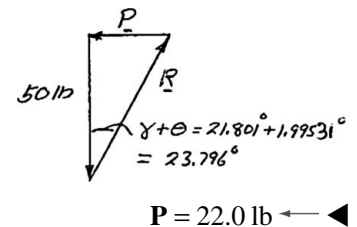
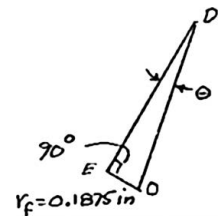
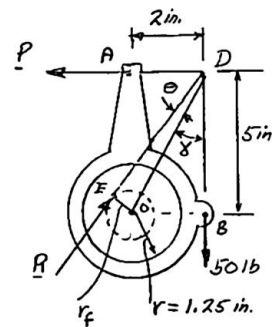
$$\begin{aligned}
 r_f &= \mu_s r \\
 &= 0.15(1.25 \text{ in.}) \\
 r_f &= 0.1875 \text{ in.} \\
 \tan \gamma &= \frac{2 \text{ in.}}{5 \text{ in.}} \\
 \gamma &= 21.801^\circ
 \end{aligned}$$

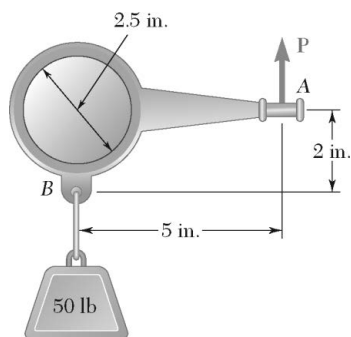
In $\triangle EOD$:

$$\begin{aligned}
 OD &= \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2} \\
 &= 5.3852 \text{ in.} \\
 \sin \theta &= \frac{OE}{OD} = \frac{r_f}{OD} \\
 &= \frac{0.1875 \text{ in.}}{5.3852 \text{ in.}} \\
 \theta &= 1.99531^\circ
 \end{aligned}$$

Force triangle:

$$\begin{aligned}
 P &= (50 \text{ lb}) \tan(\gamma + \theta) \\
 &= (50 \text{ lb}) \tan 23.796^\circ \\
 &= 22.049 \text{ lb}
 \end{aligned}$$

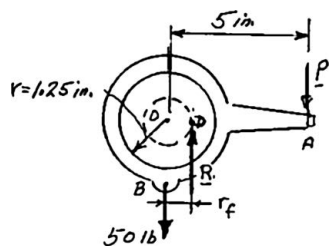




PROBLEM 8.89

A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force \mathbf{P} required to start the lever rotating clockwise.

SOLUTION



$$r_f = \mu_s r$$

$$= 0.15(1.25 \text{ in.})$$

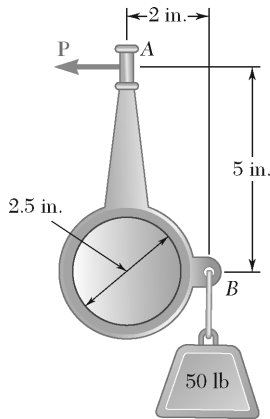
$$r_f = 0.1875 \text{ in.}$$

$$+\curvearrowright \Sigma M_D = 0: P(5 \text{ in.} - r_f) - (50 \text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5 - 0.1875}$$

$$= 1.948 \text{ lb}$$

$$\mathbf{P} = 1.948 \text{ lb} \downarrow \blacktriangleleft$$



PROBLEM 8.90

A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force P required to start the lever rotating clockwise.

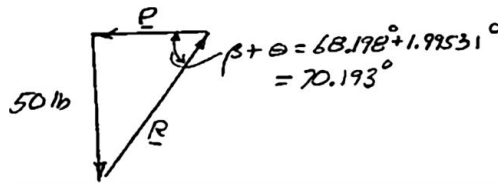
SOLUTION

$$\begin{aligned} r_f &= \mu_s r \\ &= 0.15(1.25 \text{ in.}) \\ &= 0.1875 \text{ in.} \\ \tan \beta &= \frac{5 \text{ in.}}{2 \text{ in.}} \\ \beta &= 68.198^\circ \end{aligned}$$

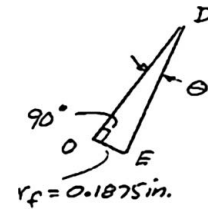
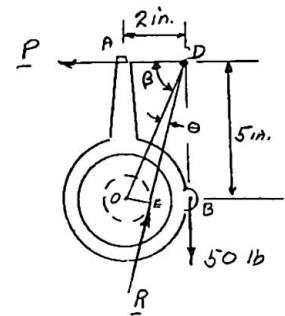
In $\triangle EOD$:

$$\begin{aligned} OD &= \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2} \\ OD &= 5.3852 \text{ in.} \\ \sin \theta &= \frac{OE}{OD} = \frac{0.1875 \text{ in.}}{5.3852 \text{ in.}} \\ \theta &= 1.99531^\circ \end{aligned}$$

Force triangle:



$$P = \frac{50}{\tan(\beta + \theta)} = \frac{50 \text{ lb}}{\tan 70.193^\circ}$$



$$P = 18.01 \text{ lb} \leftarrow \blacktriangleleft$$

PROBLEM 8.91

A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (a) to start the car moving, (b) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

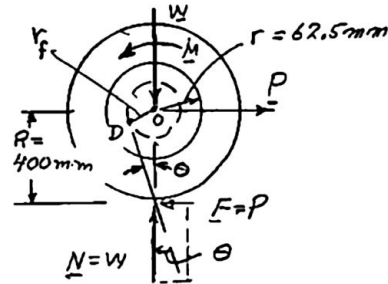
$$r_f = \mu r; \quad R = 400 \text{ mm}$$

$$\sin \theta = \tan \theta = \frac{r_f}{R} = \frac{\mu r}{R}$$

$$P = W \tan \theta = W \frac{\mu r}{R}$$

$$P = W \mu \frac{62.5 \text{ mm}}{400 \text{ mm}}$$

$$= 0.15625 W \mu$$



For one wheel:

$$W = \frac{1}{8} (30 \text{ Mg})(9.81 \text{ m/s}^2)$$

$$= \frac{1}{8} (294.3 \text{ kN})$$

For eight wheels of railroad car:

$$\Sigma P = 8(0.15625) \frac{1}{8} (294.3 \text{ kN}) \mu$$

$$= (45.984 \mu) \text{ kN}$$

(a) To start motion:

$$\mu_s = 0.020$$

$$\Sigma P = (45.984)(0.020)$$

$$= 0.9197 \text{ kN}$$

$$\Sigma P = 920 \text{ N} \quad \blacktriangleleft$$

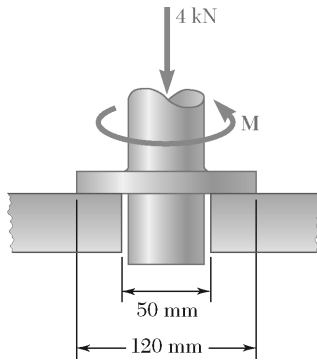
(b) To maintain motion:

$$\mu_k = 0.015$$

$$\Sigma P = (45.984)(0.015)$$

$$= 0.6897 \text{ kN}$$

$$\Sigma P = 690 \text{ N} \quad \blacktriangleleft$$



PROBLEM 8.92

Knowing that a couple of magnitude $30 \text{ N} \cdot \text{m}$ is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

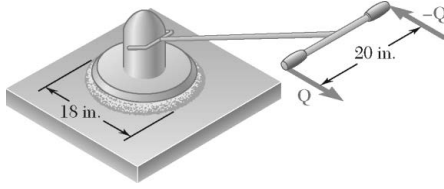
For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3} \mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30 \text{ N} \cdot \text{m} = \frac{2}{3} \mu_s (4000 \text{ N}) \frac{(0.06 \text{ m})^3 - (0.025 \text{ m})^3}{(0.06 \text{ m})^2 - (0.025 \text{ m})^2}$$

$$\mu_s = 0.1670 \quad \blacktriangleleft$$



PROBLEM 8.93

A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

See Figure 8.12 and Eq. (8.9).

Using:

$$R = 9 \text{ in.}$$

$$P = 50 \text{ lb}$$

and

$$\mu_k = 0.25$$

$$\begin{aligned} M &= \frac{2}{3} \mu_k PR = \frac{2}{3} (0.25)(50 \text{ lb})(9 \text{ in.}) \\ &= 75 \text{ lb} \cdot \text{in.} \end{aligned}$$

$\Sigma M_y = 0$ yields:

$$M = Q(20 \text{ in.})$$

$$75 \text{ lb} \cdot \text{in.} = Q(20 \text{ in.})$$

$$Q = 3.75 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 8.94*

The frictional resistance of a thrust bearing decreases as the shaft and bearing surfaces wear out. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r , show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by Eq. (8.9) for a new bearing.

SOLUTION

Using Figure 8.12, we assume

$$\Delta N = \frac{k}{r} \Delta A: \quad \Delta A = r \Delta \theta \Delta r$$

$$\Delta N = \frac{k}{r} r \Delta \theta \Delta r = k \Delta \theta \Delta r$$

We write

$$P = \Sigma \Delta N \quad \text{or} \quad P = \int dN$$

$$P = \int_0^{2\pi} \int_0^R k \Delta \theta \Delta r = 2\pi R k; \quad k = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P \Delta \theta \Delta r}{2\pi R}$$

$$\Delta M = r \Delta F = r \mu_k \Delta N = r \mu_k \frac{P \Delta \theta \Delta r}{2\pi R}$$

$$M = \int_0^{2\pi} \int_0^R \frac{\mu_k P}{2\pi R} r dr d\theta = \frac{2\pi \mu_k P}{2\pi R} \cdot \frac{R^2}{2} = \frac{1}{2} \mu_k P R$$

From Eq. (8.9) for a new bearing,

$$M_{\text{new}} = \frac{2}{3} \mu_k P R$$

Thus,

$$\frac{M}{M_{\text{new}}} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$$

$$M = 0.75 M_{\text{new}} \quad \blacktriangleleft$$

PROBLEM 8.95*

Assuming that bearings wear out as indicated in Problem 8.94, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is

$$M = \frac{1}{2} \mu_k P (R_1 + R_2)$$

where P = magnitude of the total axial force

R_1, R_2 = inner and outer radii of collar

SOLUTION

Let normal force on ΔA be ΔN , and $\frac{\Delta N}{\Delta A} = \frac{k}{r}$.

As in the text,

$$\Delta F = \mu \Delta N \quad \Delta M = r \Delta F$$

The total normal force P is

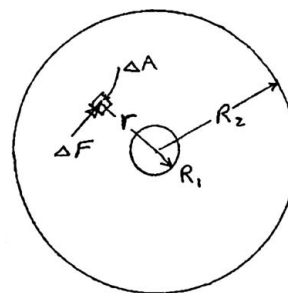
$$P = \lim_{\Delta A \rightarrow 0} \Sigma \Delta N = \int_0^{2\pi} \left(\int_{R_1}^{R_2} \frac{k}{r} r dr \right) d\theta$$

$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k (R_2 - R_1) \quad \text{or} \quad k = \frac{P}{2\pi(R_2 - R_1)}$$

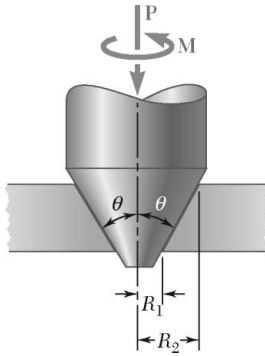
Total couple:

$$M_{\text{worn}} = \lim_{\Delta A \rightarrow 0} \Sigma \Delta M = \int_0^{2\pi} \left(\int_{R_1}^{R_2} r \mu \frac{k}{r} r dr \right) d\theta$$

$$M_{\text{worn}} = 2\pi \mu k \int_{R_1}^{R_2} (r dr) = \pi \mu k (R_2^2 - R_1^2) = \frac{\pi \mu P (R_2^2 - R_1^2)}{2\pi (R_2 - R_1)}$$



$$M_{\text{worn}} = \frac{1}{2} \mu P (R_2 + R_1) \quad \blacktriangleleft$$



PROBLEM 8.96*

Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3} \frac{\mu_k P}{\sin \theta} \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

SOLUTION

Let normal force on ΔA be ΔN and $\frac{\Delta N}{\Delta A} = k$.

So $\Delta N = k \Delta A$ $\Delta A = r \Delta s \Delta \phi$ $\Delta s = \frac{\Delta r}{\sin \theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation.

$$\Delta F_y = \Delta N \sin \theta = kr \Delta r \Delta \phi$$

Total vertical force:

$$P = \lim_{\Delta A \rightarrow 0} \sum \Delta F_y$$

$$P = \int_0^{2\pi} \left(\int_{R_1}^{R_2} kr dr \right) d\phi = 2\pi k \int_{R_1}^{R_2} r dr$$

$$P = \pi k (R_2^2 - R_1^2) \quad \text{or} \quad k = \frac{P}{\pi (R_2^2 - R_1^2)}$$

Friction force:

$$\Delta F = \mu \Delta N = \mu k \Delta A$$

Moment:

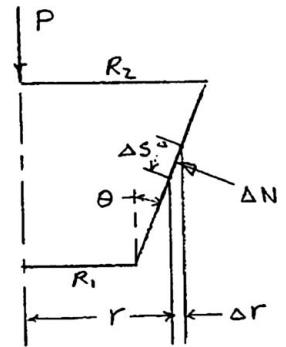
$$\Delta M = r \Delta F = r \mu k r \frac{\Delta r}{\sin \theta} \Delta \phi$$

Total couple:

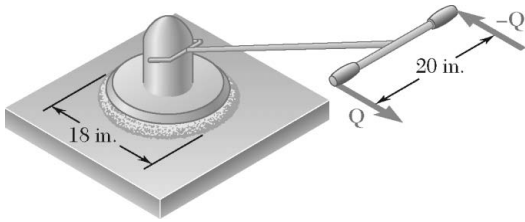
$$M = \lim_{\Delta A \rightarrow 0} \sum \Delta M = \int_0^{2\pi} \left(\int_{R_1}^{R_2} \frac{\mu k}{\sin \theta} r^2 dr \right) d\phi$$

$$M = 2\pi \frac{\mu k}{\sin \theta} \int_{R_1}^{R_2} r^2 dr = \frac{2}{3} \frac{\pi \mu}{\sin \theta} \frac{P}{\pi (R_2^2 - R_1^2)} (R_2^3 - R_1^3)$$

$$M = \frac{2}{3} \frac{\mu P}{\sin \theta} \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \quad \blacktriangleleft$$



PROBLEM 8.97



Solve Problem 8.93 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

PROBLEM 8.93 A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Let normal force on ΔA be ΔN and $\frac{\Delta N}{\Delta A} = k \left(1 - \frac{r}{R} \right)$.

$$\Delta F = \mu \Delta N = \mu k \left(1 - \frac{r}{R} \right) \Delta A = \mu k \left(1 - \frac{r}{R} \right) r \Delta r \Delta \theta$$

$$P = \lim_{\Delta A \rightarrow 0} \Sigma \Delta N = \int_0^{2\pi} \left[\int_0^R k \left(1 - \frac{r}{R} \right) r dr \right] d\theta$$

$$P = 2\pi k \int_0^R \left(1 - \frac{r}{R} \right) r dr = 2\pi k \left(\frac{R^2}{2} - \frac{R^3}{3R} \right)$$

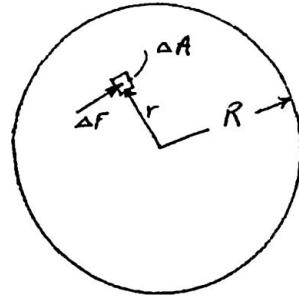
$$P = \frac{1}{3} \pi k R^2 \quad \text{or} \quad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A \rightarrow 0} \Sigma r \Delta F = \int_0^{2\pi} \left[\int_0^R r \mu k \left(1 - \frac{r}{R} \right) r dr \right] d\theta$$

$$= 2\pi \mu k \int_0^R \left(r^2 - \frac{r^3}{R} \right) dr$$

$$= 2\pi \mu k \left(\frac{R^3}{3} - \frac{R^4}{4R} \right) = \frac{1}{6} \pi \mu k R^3$$

$$= \frac{\pi \mu}{6} \frac{3P}{\pi R^2} R^3 = \frac{1}{2} \mu P R$$



where

$$\mu = \mu_k = 0.25 \quad R = 9 \text{ in.}$$

PROBLEM 8.97 (Continued)

$$P = W = 50 \text{ lb}$$

Then

$$\begin{aligned} M &= \frac{1}{2}(0.25)(50 \text{ lb})(9 \text{ in.}) \\ &= 56.250 \text{ lb}\cdot\text{in.} \end{aligned}$$

Finally

$$Q = \frac{M}{d} = \frac{56.250 \text{ lb}\cdot\text{in.}}{20 \text{ in.}}$$

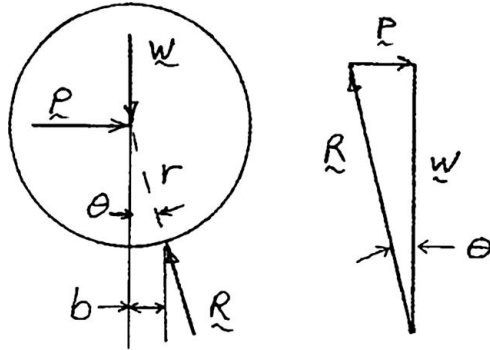
$$Q = 2.81 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 8.98

Determine the horizontal force required to move a 2500-lb automobile with 23-in.-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 0.05 in.

SOLUTION

FBD wheel:



$$r = 11.5 \text{ in.}$$

$$b = 0.05 \text{ in.}$$

$$\theta = \sin^{-1} \frac{b}{r}$$

$$P = W \tan \theta = W \tan \left(\sin^{-1} \frac{b}{r} \right) \text{ for each wheel, so for total}$$

$$P = 2500 \text{ lb} \tan \left(\sin^{-1} \frac{0.05}{11.5} \right)$$

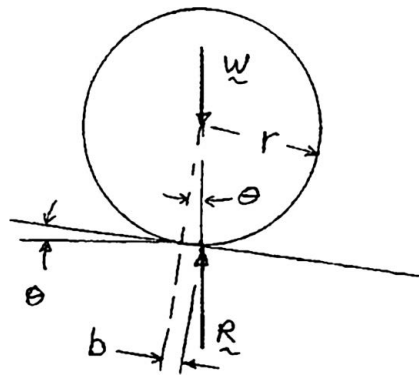
$$P = 10.87 \text{ lb} \blacktriangleleft$$

PROBLEM 8.99

Knowing that a 6-in.-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

FBD disk:

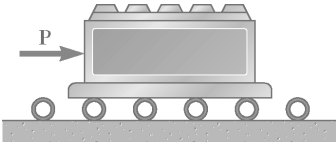


$$\tan \theta = \text{slope} = 0.02$$

$$b = r \tan \theta = (3 \text{ in.})(0.02)$$

$$b = 0.0600 \text{ in.} \blacktriangleleft$$

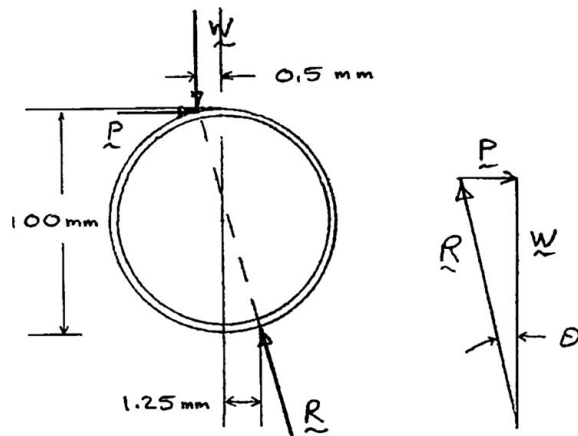
PROBLEM 8.100



A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force P required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$\begin{aligned} W &= mg \\ &= (900 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 8829.0 \text{ N} \end{aligned}$$

$$\begin{aligned} \theta &= \sin^{-1} \frac{.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}} \\ &= 1.00273^\circ \end{aligned}$$

$P = W \tan \theta$ for each pipe, so also for total

$$P = (8829.0 \text{ N}) \tan (1.00273^\circ)$$

$$P = 154.4 \text{ N} \quad \blacktriangleleft$$

PROBLEM 8.101

Solve Problem 8.85 including the effect of a coefficient of rolling resistance of 1.75 mm.

PROBLEM 8.85 A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

Since the scooter rolls at a constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.

a = Radius of wheel

$$\tan \theta = \frac{2}{100} = 0.02$$

Since b and r_f are small compared to a ,

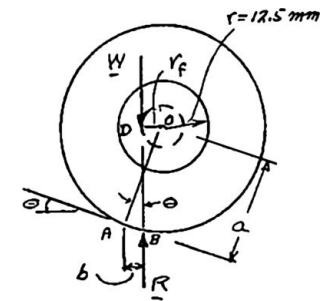
$$\tan \theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

Data:

$$\mu_k = 0.10, \quad b = 1.75 \text{ mm}, \quad r = 12.5 \text{ mm}$$

$$\frac{(0.10)(12.5 \text{ mm}) + 1.75 \text{ mm}}{a} = 0.02$$

$$a = 150 \text{ mm}$$



$$\text{Diameter} = 2a = 300 \text{ mm} \quad \blacktriangleleft$$

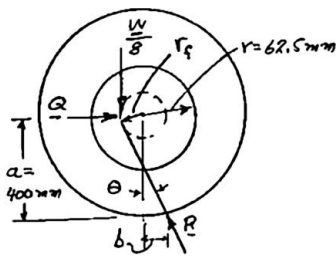
PROBLEM 8.102

Solve Problem 8.91 including the effect of a coefficient of rolling resistance of 0.5 mm.

PROBLEM 8.91 A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (a) to start the car moving, (b) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

For one wheel:



$$r_f = \mu r$$

$$\tan \theta \approx \sin \theta \approx \frac{r_f + b}{a}$$

$$\tan \theta = \frac{\mu r + b}{a}$$

$$Q = \frac{W}{8} \tan \theta = \frac{W}{8} \frac{\mu r + b}{a}$$

For eight wheels of car:

$$P = W \frac{\mu r + b}{a}$$

$$W = mg = (30 \text{ Mg})(9.81 \text{ m/s}^2) = 294.3 \text{ kN}$$

$$a = 400 \text{ mm}, \quad r = 62.5 \text{ mm}, \quad b = 0.5 \text{ mm}$$

(a) To start motion:

$$\mu = \mu_s = 0.02$$

$$P = (294.3 \text{ kN}) \frac{(0.020)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

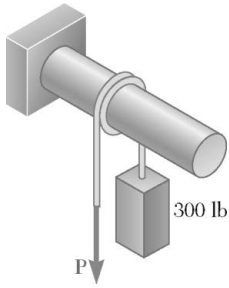
$$P = 1.288 \text{ kN} \quad \blacktriangleleft$$

(b) To maintain constant speed

$$\mu = \mu_k = 0.015$$

$$P = (294.3 \text{ kN}) \frac{(0.015)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

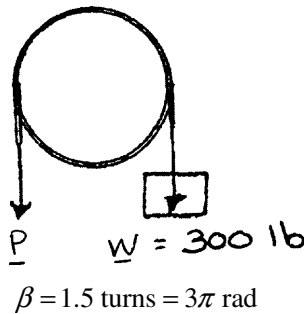
$$P = 1.058 \text{ kN} \quad \blacktriangleleft$$



PROBLEM 8.103

A 300-lb block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



For impending motion of W up,

$$P = We^{\mu_s\beta} = (300 \text{ lb})e^{(0.15)3\pi}$$

$$= 1233.36 \text{ lb}$$

For impending motion of W down,

$$P = We^{-\mu_s\beta} = (300 \text{ lb})e^{-(0.15)3\pi}$$

$$= 72.971 \text{ lb}$$

For equilibrium,

$$73.0 \text{ lb} \leq P \leq 1233 \text{ lb} \quad \blacktriangleleft$$

PROBLEM 8.104

A hawser is wrapped two full turns around a bollard. By exerting an 80-lb force on the free end of the hawser, a dockworker can resist a force of 5000 lb on the other end of the hawser. Determine (a) the coefficient of static friction between the hawser and the bollard, (b) the number of times the hawser should be wrapped around the bollard if a 20,000-lb force is to be resisted by the same 80-lb force.

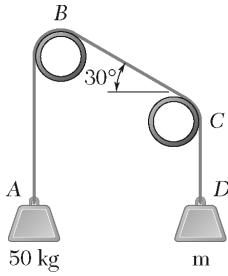
SOLUTION

(a)

$$\beta = 2 \text{ turns} = 2(2\pi) = 4\pi$$
$$T_1 = 80 \text{ lb}, \quad T_2 = 5000 \text{ lb}$$
$$\ln \frac{T_2}{T_1} = \mu_s \beta$$
$$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{4\pi} \ln \frac{5000 \text{ lb}}{80 \text{ lb}}$$
$$\mu_s = \frac{1}{4\pi} \ln 62.5 = \frac{4.1351}{4\pi} \qquad \mu_s = 0.329 \blacktriangleleft$$

(b)

$$T_1 = 80 \text{ lb}, \quad T_2 = 20,000 \text{ lb}, \quad \mu_s = 0.329$$
$$\ln \frac{T_2}{T_1} = \mu_s \beta$$
$$\beta = \frac{1}{\mu} \ln \frac{T_2}{T_1} = \frac{1}{0.329} \ln \frac{20,000 \text{ lb}}{80 \text{ lb}}$$
$$\beta = \frac{1}{0.329} \ln(250) = \frac{5.5215}{0.329} = 16.783$$
$$\text{Number of turns} = \frac{16.783}{2\pi} \qquad \text{Number of turns} = 2.67 \blacktriangleleft$$



PROBLEM 8.105

A rope $ABCD$ is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (a) the smallest value of the mass m for which equilibrium is possible, (b) the corresponding tension in portion BC of the rope.

SOLUTION

We apply Eq. (8.14) to pipe B and pipe C .

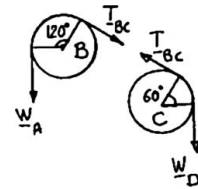
$$\frac{T_2}{T_1} = e^{\mu_s \beta} \quad (8.14)$$

Pipe B:

$$T_2 = W_A, \quad T_1 = T_{BC}$$

$$\mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$$

$$\frac{W_A}{T_{BC}} = e^{0.25(2\pi/3)} = e^{\pi/6} \quad (1)$$



Pipe C:

$$T_2 = T_{BC}, \quad T_1 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$$

$$\frac{T_{BC}}{W_D} = e^{0.25(\pi/3)} = e^{\pi/12} \quad (2)$$

(a) Multiplying Eq. (1) by Eq. (2):

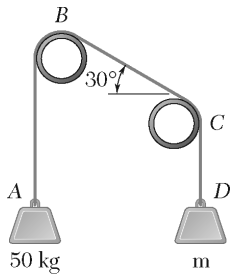
$$\frac{W_A}{W_D} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_D = \frac{W_A}{2.193} \quad m = \frac{W_D}{g} = \frac{\frac{W_A}{g}}{2.193} = \frac{m_A}{2.193} = \frac{50 \text{ kg}}{2.193}$$

$$m = 22.8 \text{ kg} \quad \blacktriangleleft$$

(b) From Eq. (1):

$$T_{BC} = \frac{W_A}{e^{\pi/6}} = \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{1.688} = 291 \text{ N} \quad \blacktriangleleft$$



PROBLEM 8.106

A rope $ABCD$ is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (a) the largest value of the mass m for which equilibrium is possible, (b) the corresponding tension in portion BC of the rope.

SOLUTION

See FB diagrams of Problem 8.105. We apply Eq. (8.14) to pipes B and C .

Pipe B: $T_1 = W_A, \quad T_2 = T_{BC}, \quad \mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}; \quad \frac{T_{BC}}{W_A} = e^{0.25(2\pi/3)} = e^{\pi/6} \quad (1)$$

Pipe C: $T_1 = T_{BC}, \quad T_2 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$

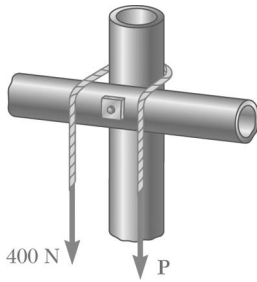
$$\frac{T_2}{T_1} = e^{\mu_s \beta}; \quad \frac{W_D}{T_{BC}} = e^{0.25(\pi/3)} = e^{\pi/12} \quad (2)$$

(a) Multiply Eq. (1) by Eq. (2):

$$\frac{W_D}{W_A} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_D = 2.193W_A \quad m = 2.193m_A = 2.193(50 \text{ kg}) \quad m = 109.7 \text{ kg} \quad \blacktriangleleft$$

(b) From Eq. (1): $T_{BC} = W_A e^{\pi/6} = (50 \text{ kg})(9.81 \text{ m/s}^2)(1.688) = 828 \text{ N} \quad \blacktriangleleft$



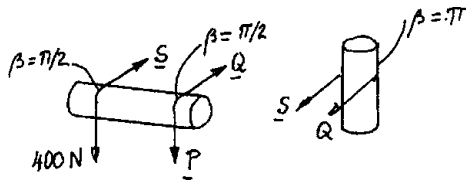
PROBLEM 8.107

Knowing that the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of P for which equilibrium is maintained.

SOLUTION

Horizontal pipe $\mu_s = \mu_h$

Vertical pipe $\mu_s = \mu_v$



For motion of P to be impending downward:

$$\frac{P}{Q} = e^{\mu_h(\pi/2)}, \quad \frac{Q}{S} = e^{\mu_v\pi}, \quad \frac{S}{400 \text{ N}} = e^{\mu_h(\pi/2)}$$

Multiply equation member by member:

$$\frac{P}{Q} \frac{Q}{S} \frac{S}{400} = e^{(\mu_h + 2\mu_v + \mu_h)(\pi/2)}$$

or:
$$\frac{P}{400 \text{ N}} = e^{(\mu_h + \mu_v)\pi} \quad (1)$$

For motion of P to be impending upward, we find in a similar way

$$\frac{P}{400 \text{ N}} = e^{-(\mu_h + \mu_v)\pi} \quad (2)$$

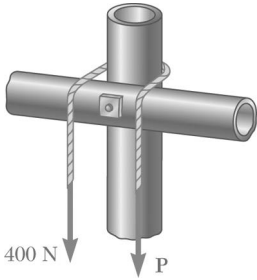
Given data: $\mu_h = 0.25, \mu_v = 0.20$

From (1): $P = (400 \text{ N})e^{0.45\pi} = 1644 \text{ N}$

From (2): $P = (400 \text{ N})e^{-0.45\pi} = 97.3 \text{ N}$

Range for equilibrium:

$$97.3 \text{ N} \leq P \leq 1644 \text{ N} \quad \blacktriangleleft$$



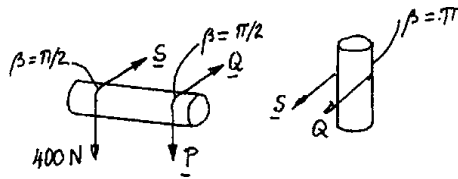
PROBLEM 8.108

Knowing that the coefficient of static friction is 0.30 between the rope and the horizontal pipe and that the smallest value of P for which equilibrium is maintained is 80 N, determine (a) the largest value of P for which equilibrium is maintained, (b) the coefficient of static friction between the rope and the vertical pipe.

SOLUTION

Horizontal pipe: $\mu_s = \mu_h$

Vertical pipe: $\mu_s = \mu_v$



For motion of P to be impending downward:

$$\frac{P}{Q} = e^{\mu_h(\pi/2)}, \quad \frac{Q}{S} = e^{\mu_v\pi}, \quad \frac{S}{400 \text{ N}} = e^{\mu_h(\pi/2)}$$

Multiply equation member by member:

$$\frac{P}{Q} \frac{Q}{S} \frac{S}{400} = e^{(\mu_h + 2\mu_v + \mu_h)(\pi/2)}$$

or:
$$\frac{P}{400 \text{ N}} = e^{(\mu_h + \mu_v)\pi} \quad (1)$$

For motion of P to be impending upward, we find in a similar way

$$\frac{P}{400 \text{ N}} = e^{-(\mu_h + \mu_v)\pi} \quad (2)$$

Setting $P = P_{\max}$ in Eq. (1), $P = 80 \text{ N}$ in Eq. (2) and $\mu_h = 0.30$ in both, we get

$$\frac{P_{\max}}{400 \text{ N}} = e^{(0.30 + \mu_v)\pi} \quad (1'); \quad \frac{80 \text{ N}}{400 \text{ N}} = e^{-(0.30 + \mu_v)\pi} \quad (2')$$

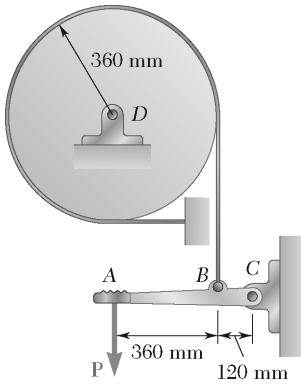
(a) Multiplying (1') by (2'):

$$\frac{P_{\max}}{400} \cdot \frac{80}{400} = e^0 = 1 \quad P_{\max} = 2000 \text{ N} \blacktriangleleft$$

(b) From (2'): $(0.30 + \mu_v)\pi = \ln \frac{400}{80} = \ln 5 = 1.60944$

$$0.30 + \mu_v = 0.512 \quad \mu_v = 0.212 \blacktriangleleft$$

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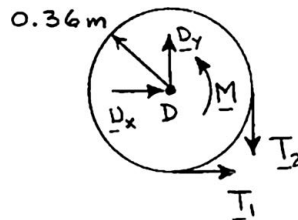


PROBLEM 8.109

A band brake is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. Determine the magnitude of the couple being applied to the flywheel, knowing that $P = 45 \text{ N}$ and that the flywheel is rotating counterclockwise at a constant speed.

SOLUTION

Free body: Cylinder



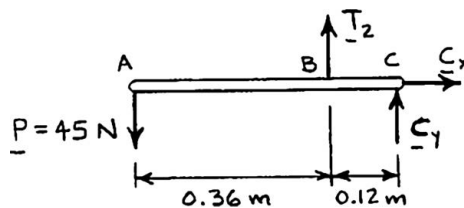
Since slipping of band relative to cylinder is clockwise, T_1 and T_2 are located as shown.

From free body: Lever ABC

$$+\curvearrowright \Sigma M_C = 0: (45 \text{ N})(0.48 \text{ m}) - T_2(0.12 \text{ m}) = 0$$

$$T_2 = 180 \text{ N}$$

Free body: Lever ABC



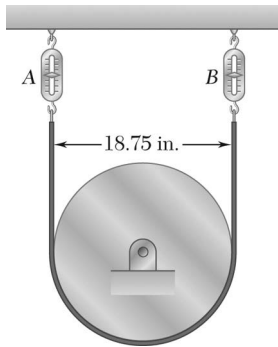
From free body: Cylinder

Using Eq. (8.14) with $\mu_k = 0.25$ and $\beta = 270^\circ = \frac{3\pi}{2}$ rad:

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{(0.25)(3\pi/2)} = e^{3\pi/8}$$

$$T_1 = \frac{T_2}{e^{3\pi/8}} = \frac{180 \text{ N}}{3.2482} = 55.415 \text{ N}$$

$$+\curvearrowright \Sigma M_D = 0: (55.415 \text{ N})(0.36 \text{ m}) - (180 \text{ N})(0.36 \text{ m}) + M = 0 \quad \mathbf{M = 44.9 \text{ N} \cdot \text{m}} \quad \blacktriangleleft$$



PROBLEM 8.110

The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring scale is 14 lb. If a 105-lb · in. couple must be applied to the flywheel to keep it rotating clockwise at a constant speed, determine (a) the reading of each scale at that time, (b) the coefficient of kinetic friction. Assume that the length of the belt does not change.

SOLUTION

- (a) Since the length of the belt is constant, the spring in scale B will increase in length by δ and the spring in scale A will decrease by the same amount. Thus, the sum of the readings in scales A and B remains constant:

$$T_A + T_B = 14 \text{ lb} + 14 \text{ lb} \qquad T_A + T_B = 28 \text{ lb} \quad (1)$$

On the other hand, the sum of the moments of T_A and T_B about axle must be equal to moment of couple:

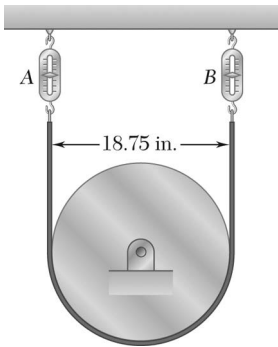
$$(T_B - T_A)(9.375 \text{ in.}) = 105 \text{ lb} \cdot \text{in.}, \qquad T_B - T_A = 11.2 \text{ lb} \quad (2)$$

Solving (1) and (2) simultaneously

$$T_A = 8.40 \text{ lb}; \qquad T_B = 19.60 \text{ lb} \quad \blacktriangleleft$$

- (b) Apply Eq. (8.13) with $T_2 = T_B$, $T_1 = T_A$, $\beta = 180^\circ = \pi$ rad.

$$\ln \frac{T_2}{T_1} = \mu_s \beta: \qquad \mu_s \pi = \ln \frac{19.60}{8.40} = 0.84730 \qquad \mu_s = 0.270 \quad \blacktriangleleft$$



PROBLEM 8.111

The setup shown is used to measure the output of a small turbine. The coefficient of kinetic friction is 0.20 and the reading of each spring scale is 16 lb when the flywheel is at rest. Determine (a) the reading of each scale when the flywheel is rotating clockwise at a constant speed, (b) the couple that must be applied to the flywheel. Assume that the length of the belt does not change.

SOLUTION

- (a) Since the length of the belt is constant, the spring in scale B will increase in length by δ and the spring in scale A will decrease by the same amount. Thus, the sum of the readings in scales A and B remains constant:

$$T_A + T_B = 16 \text{ lb} + 16 \text{ lb} \qquad T_A + T_B = 32 \text{ lb} \quad (1)$$

We now apply Eq. (8.14) with

$$T_1 = T_A, \quad T_2 = T_B, \quad \mu_k = 0.20, \quad \beta = 180^\circ = \pi \text{ rad.}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta}; \quad \frac{T_B}{T_A} = e^{0.20\pi} = 1.87446$$

$$T_B = 1.87446 T_A \quad (2)$$

Substituting (2) into (1):

$$T_A + 1.87446 T_A = 32 \text{ lb}$$

$$T_A = 11.1325 \text{ lb} \qquad T_A = 11.13 \text{ lb} \quad \blacktriangleleft$$

From (1): $T_B = 32 \text{ lb} - 11.1325 \text{ lb}$

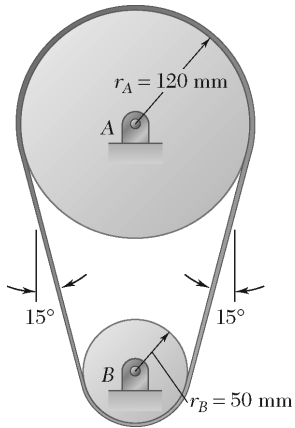
$$T_B = 20.868 \text{ lb} \qquad T_B = 20.9 \text{ lb} \quad \blacktriangleleft$$

- (b) Couple applied to flywheel:

$$\begin{aligned} M &= (T_B - T_A)r \\ &= (20.868 \text{ lb} - 11.1325 \text{ lb})(9.375 \text{ in.}) \end{aligned}$$

$$M = 91.3 \text{ lb} \cdot \text{in.} \quad \blacktriangleright \blacktriangleleft$$

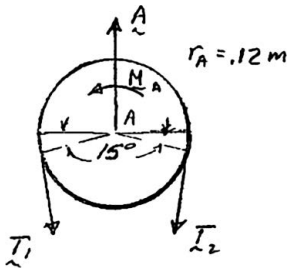
PROBLEM 8.112



A flat belt is used to transmit a couple from drum B to drum A . Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum A .

SOLUTION

FBD's drums:



$$\beta_A = 180^\circ + 30^\circ = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

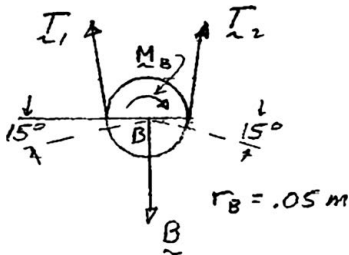
$$\beta_B = 180^\circ - 30^\circ = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on B (friction coefficients being equal)

So

$$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

$$450 \text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914 \text{ N}$$

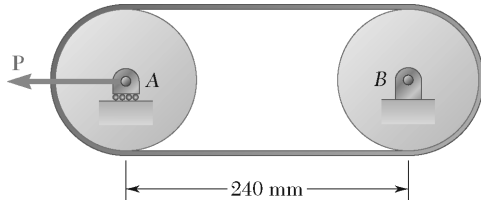


$$\left(\sum M_A = 0: \quad M_A + (0.12 \text{ m})(T_1 - T_2) = 0 \right.$$

$$M_A = (0.12 \text{ m})(450 \text{ N} - 157.914 \text{ N}) = 35.05 \text{ N} \cdot \text{m}$$

$$M_A = 35.1 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

PROBLEM 8.113



A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900 \text{ N}$ is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest couple that can be transmitted, (b) the corresponding maximum value of the tension in the belt.

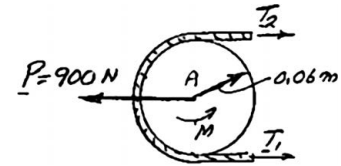
SOLUTION

Drum A:

$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{(0.35)\pi}$$

$$T_2 = 3.0028 T_1$$

$$\beta = 180^\circ = \pi \text{ radians}$$



(a) Torque: $\overset{+}{\curvearrowright} \Sigma M_A = 0: M - (675.15 \text{ N})(0.06 \text{ m}) + (224.84 \text{ N})(0.06 \text{ m})$

$$M = 27.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

(b) $\overset{+}{\rightarrow} \Sigma F_x = 0: T_1 + T_2 - 900 \text{ N} = 0$

$$T_1 + 3.0028 T_1 - 900 \text{ N} = 0$$

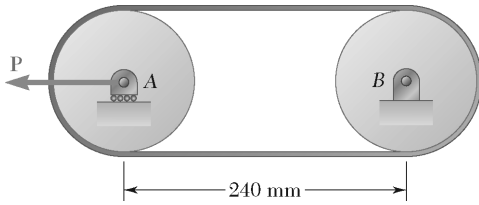
$$4.00282 T_1 = 900$$

$$T_1 = 224.841 \text{ N}$$

$$T_2 = 3.0028(224.841 \text{ N}) = 675.15 \text{ N}$$

$$T_{\max} = 675 \text{ N} \quad \blacktriangleleft$$

PROBLEM 8.114

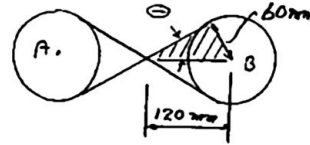
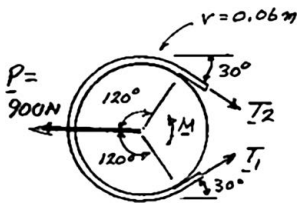


Solve Problem 8.113 assuming that the belt is looped around the pulleys in a figure eight.

PROBLEM 8.113 A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest couple that can be transmitted, (b) the corresponding maximum value of the tension in the belt.

SOLUTION

Drum A:



$$\beta = 240^\circ = 240^\circ \frac{\pi}{180^\circ} = \frac{4}{3}\pi$$

$$\sin \theta = \frac{60}{120} = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(4/3\pi)}$$

$$T_2 = 4.3322T_1$$

(a) Torque: $\sum M_B = 0: M - (844.3 \text{ N})(0.06 \text{ m}) + (194.9 \text{ N})(0.06 \text{ m}) = 0$

$$M = 39.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

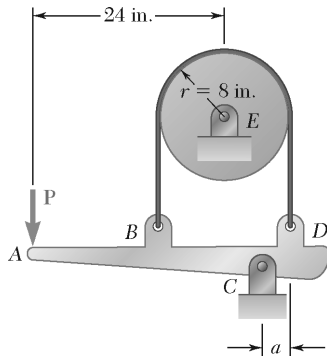
(b) $\sum F_x = 0: (T_1 + T_2) \cos 30^\circ - 900 \text{ N}$

$$(T_1 + 4.3322T_1) \cos 30^\circ = 900$$

$$T_1 = 194.90 \text{ N}$$

$$T_2 = 4.3322(194.90 \text{ N}) = 844.3 \text{ N}$$

$$T_{\max} = 844 \text{ N} \quad \blacktriangleleft$$



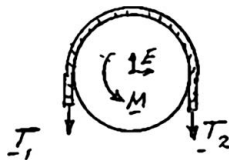
PROBLEM 8.115

The speed of the brake drum shown is controlled by a belt attached to the control bar AD . A force \mathbf{P} of magnitude 25 lb is applied to the control bar at A . Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that $a = 4$ in., and that the drum is rotating at a constant speed (a) counterclockwise, (b) clockwise.

SOLUTION

(a) Counterclockwise rotation

Free body: Drum



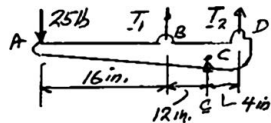
$$r = 8 \text{ in.} \quad \beta = 180^\circ = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933T_1$$

Free body: Control bar

$$+\circlearrowleft \Sigma M_C = 0: T_1(12 \text{ in.}) - T_2(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$



$$T_1(12) - 2.1933T_1(4) - 700 = 0$$

$$T_1 = 216.93 \text{ lb}$$

$$T_2 = 2.1933(216.93 \text{ lb}) = 475.80 \text{ lb}$$

Return to free body of drum

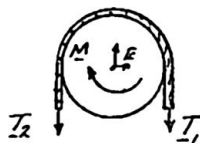
$$+\circlearrowleft \Sigma M_E = 0: M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (216.96 \text{ lb})(8 \text{ in.}) - (475.80 \text{ lb})(8 \text{ in.}) = 0$$

$$M = 2070.9 \text{ lb} \cdot \text{in.}$$

$$M = 2070 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

(b) Clockwise rotation



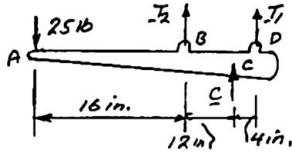
$$r = 8 \text{ in.} \quad \beta = \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933T_1$$

PROBLEM 8.115 (Continued)

Free body: Control rod



$$+\circlearrowleft \Sigma M_C = 0: T_2(12 \text{ in.}) - T_1(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$

$$2.1933T_1(12) - T_1(4) - 700 = 0$$

$$T_1 = 31.363 \text{ lb}$$

$$T_2 = 2.1933(31.363 \text{ lb})$$

$$T_2 = 68.788 \text{ lb}$$

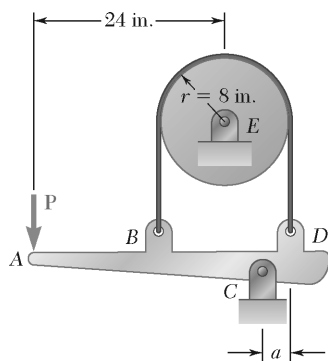
Return to free body of drum

$$+\circlearrowleft \Sigma M_E = 0: M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (31.363 \text{ lb})(8 \text{ in.}) - (68.788 \text{ lb})(8 \text{ in.}) = 0$$

$$M = 299.4 \text{ lb} \cdot \text{in.}$$

$$M = 299 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$



PROBLEM 8.116

The speed of the brake drum shown is controlled by a belt attached to the control bar AD . Knowing that $a = 4$ in., determine the maximum value of the coefficient of static friction for which the brake is not self-locking when the drum rotates counterclockwise.

SOLUTION

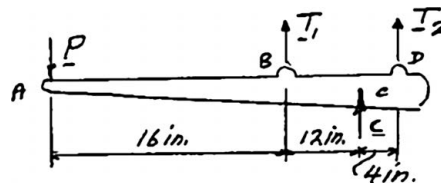


$$r = 8 \text{ in.}, \quad \beta = 180^\circ = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{\mu_s \pi}$$

$$T_2 = e^{\mu_s \pi} T_1$$

Free body: Control rod



$$\sum M_C = 0: \quad P(28 \text{ in.}) - T_1(12 \text{ in.}) + T_2(4 \text{ in.}) = 0$$

$$28P - 12T_1 + e^{\mu_s \pi} T_1(4) = 0$$

For self-locking brake:

$$P = 0$$

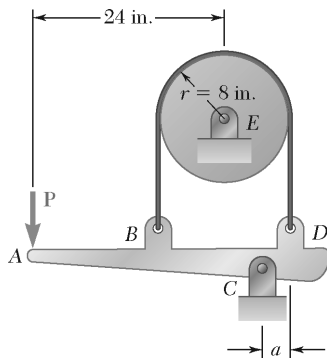
$$12T_1 = 4T_1 e^{\mu_s \pi}$$

$$e^{\mu_s \pi} = 3$$

$$\mu_s \pi = \ln 3 = 1.0986$$

$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

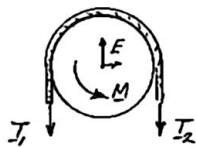
$$\mu_s = 0.350 \quad \blacktriangleleft$$



PROBLEM 8.117

The speed of the brake drum shown is controlled by a belt attached to the control bar AD . Knowing that the coefficient of static friction is 0.30 and that the brake drum is rotating counterclockwise, determine the minimum value of a for which the brake is not self-locking.

SOLUTION

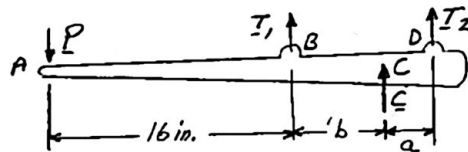


$$r = 8 \text{ in.}, \quad \beta = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663 T_1$$

Free body: Control rod



$$b = 16 \text{ in.} - a$$

$$+\circlearrowleft \Sigma M_C = 0: \quad P(16 \text{ in.} + b) - T_1 b + T_2 a = 0$$

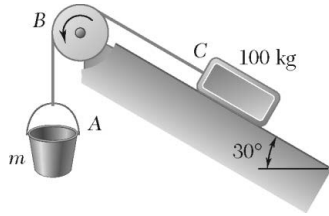
For brake to be self-locking, $P = 0$

$$T_2 a = T_1 b; \quad 2.5663 T_1 a = T_1 (16 - a)$$

$$2.5663 a = 16 - a$$

$$3.5663 a = 16$$

$$a = 4.49 \text{ in.} \quad \blacktriangleleft$$

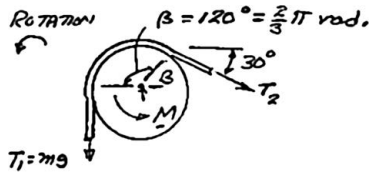


PROBLEM 8.118

Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (a) remain at rest, (b) start moving up the incline, (c) continue moving up the incline at a constant speed.

SOLUTION

Free body: Drum



$$\frac{T_2}{T_1} = e^{\mu \beta}$$

$$T_2 = T_1 e^{2\mu\pi/3} \quad (1)$$

(a) Smallest m for block C to remain at rest

Cable slips on drum.

$$\text{Eq. (1) with } \mu_k = 0.25; \quad T_2 = mg e^{2(0.25)\pi/3} = 1.6881mg$$

Block C: At rest, motion impending ↘

$$\begin{aligned} +\nearrow \Sigma F = 0: \quad N - m_C g \cos 30^\circ \\ N = m_C g \cos 30^\circ \\ F = \mu_s N = 0.35 m_C g \cos 30^\circ \\ m_C = 100 \text{ kg} \end{aligned}$$

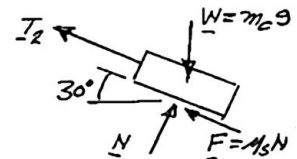
$$+\searrow \Sigma F = 0: \quad T_2 + F - m_C g \sin 30^\circ = 0$$

$$1.6881mg + 0.35m_C g \cos 30^\circ - m_C g \sin 30^\circ = 0$$

$$1.6881m = 0.19689m_C$$

$$m = 0.11663m_C = 0.11663(100 \text{ kg});$$

$$m = 11.66 \text{ kg} \quad \blacktriangleleft$$



(b) Smallest m to start block moving up

No slipping at both drum and block: $\mu_s = 0.35$

$$\text{Eq. (1):} \quad T_2 = mg e^{2(0.35)\pi/3} = 2.0814mg$$

PROBLEM 8.118 (Continued)

Block C:

Motion impending ↘

$$m_C = 100 \text{ kg}$$

$$+\nearrow \Sigma F = 0: N - m_C g \cos 30^\circ$$

$$N = m_C g \cos 30^\circ$$

$$F = \mu_s N = 0.35 m_C g \cos 30^\circ$$

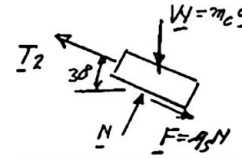
$$+\searrow \Sigma F = 0: T_2 - F - m_C g \sin 30^\circ = 0$$

$$2.0814 m g - 0.35 m_C g \cos 30^\circ - m_C g \sin 30^\circ = 0$$

$$2.0814 m = 0.80311 m_C$$

$$m = 0.38585 m_C = 0.38585(100 \text{ kg})$$

$$m = 38.6 \text{ kg} \blacktriangleleft$$



(c) Smallest m to keep block moving up drum: No slipping: $\mu_s = 0.35$

Eq. (1) with $\mu_s = 0.35$

$$T_2 = m g^{2\mu_s \pi / 3} = m g e^{2(0.35)\pi / 3}$$

$$T_2 = 2.0814 m g$$

Block C: Moving up plane, thus $\mu_k = 0.25$

Motion up ↗

$$+\nearrow \Sigma F = 0: N - m_C g \cos 30^\circ = 0$$

$$N = m_C g \cos 30^\circ$$

$$F = \mu_k N = 0.25 m_C g \cos 30^\circ$$

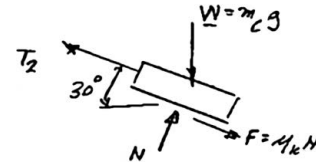
$$+\searrow \Sigma F = 0: T_2 - F - m_C g \sin 30^\circ = 0$$

$$2.0814 m g - 0.25 m_C g \cos 30^\circ - m_C g \sin 30^\circ = 0$$

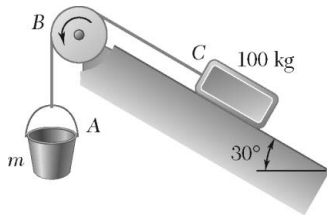
$$2.0814 m = 0.71651 m_C$$

$$m = 0.34424 m_C = 0.34424(100 \text{ kg})$$

$$m = 34.4 \text{ kg} \blacktriangleleft$$



PROBLEM 8.119



Solve Problem 8.118 assuming that drum B is frozen and cannot rotate.

PROBLEM 8.118 Bucket A and block C are connected by a cable that passes over drum B . Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass m of the bucket and its contents for which block C will (a) remain at rest, (b) start moving up the incline, (c) continue moving up the incline at a constant speed.

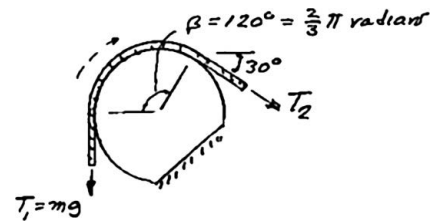
SOLUTION

(a) Block C remains at rest: Motion impends ↘

Drum:

$$\frac{T_2}{mg} = e^{\mu_k \beta} = e^{0.35(2\pi/3)}$$

$$T_2 = 2.0814mg$$



Block C: Motion impends ↘

$$\nearrow \Sigma F = 0: N - m_C g \cos 30^\circ = 0$$

$$N = m_C g \cos 30^\circ$$

$$F = \mu_s N = 0.35 m_C g \cos 30^\circ$$

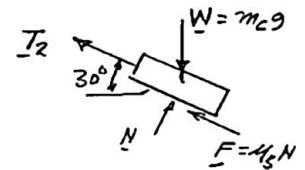
$$+\searrow \Sigma F = 0: T_2 + F - m_C g \sin 30^\circ = 0$$

$$2.0814mg + 0.35 m_C g \cos 30^\circ - m_C g \sin 30^\circ = 0$$

$$2.0814mg = 0.19689m_C$$

$$m = 0.09459m_C = 0.09459(100 \text{ kg})$$

$$m = 9.46 \text{ kg} \blacktriangleleft$$



(b) Block C: Starts moving up $\mu_s = 0.35$

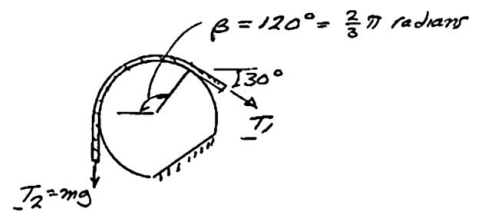
Drum: Impending motion of cable ↶

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{mg}{T_1} = e^{0.35(2/3\pi)}$$

$$T_1 = \frac{mg}{2.0814}$$

$$= 0.48045mg$$

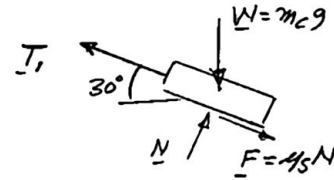


PROBLEM 8.119 (Continued)

Block C: Motion impends ↘

$$\begin{aligned}
 +\nearrow \Sigma F = 0: & \quad N - m_C g \cos 30^\circ \\
 & \quad N = m_C g \cos 30^\circ \\
 & \quad F = \mu_s N = 0.35 m_C g \cos 30^\circ
 \end{aligned}$$

$$\begin{aligned}
 +\searrow \Sigma F = 0: & \quad T_1 - F - m_C g \sin 30^\circ = 0 \\
 & \quad 0.48045 m g - 0.35 m_C g \cos 30^\circ - 0.5 m_C g = 0 \\
 & \quad 0.48045 m = 0.80311 m_C \\
 & \quad m = 1.67158 m_C = 1.67158(100 \text{ kg})
 \end{aligned}$$



$$m = 167.2 \text{ kg} \blacktriangleleft$$

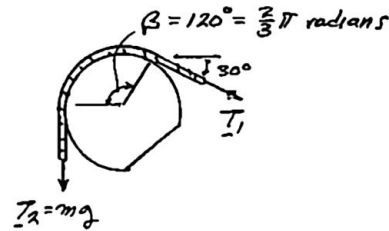
(c) Smallest m to keep block moving ↘

Drum: Motion of cable ↙

$$\begin{aligned}
 \mu_k &= 0.25 \\
 \frac{T_2}{T_1} &= e^{\mu_k \beta} = e^{0.25(2/3\pi)}
 \end{aligned}$$

$$\frac{mg}{T_1} = 1.6881$$

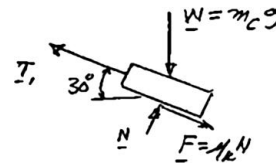
$$T_1 = \frac{mg}{1.6881} = 0.59238 mg$$



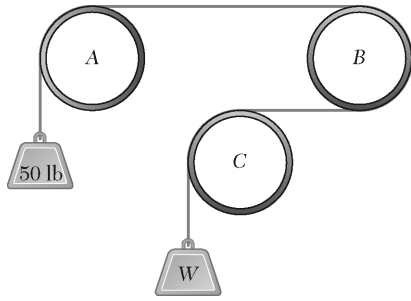
Block C: Block moves ↘

$$\begin{aligned}
 +\nearrow \Sigma F = 0: & \quad N - m_C g \cos 30^\circ = 0 \\
 & \quad N = m_C g \cos 30^\circ \\
 & \quad F = \mu_k N = 0.25 m_C g \cos 30^\circ
 \end{aligned}$$

$$\begin{aligned}
 +\searrow \Sigma F = 0: & \quad T_1 - F - m_C g \sin 30^\circ = 0 \\
 & \quad 0.59238 m g - 0.25 m_C g \cos 30^\circ - 0.5 m_C g = 0 \\
 & \quad 0.59238 m = 0.71651 m_C \\
 & \quad m = 1.20954 m_C = 1.20954(100 \text{ kg})
 \end{aligned}$$



$$m = 121.0 \text{ kg} \blacktriangleleft$$

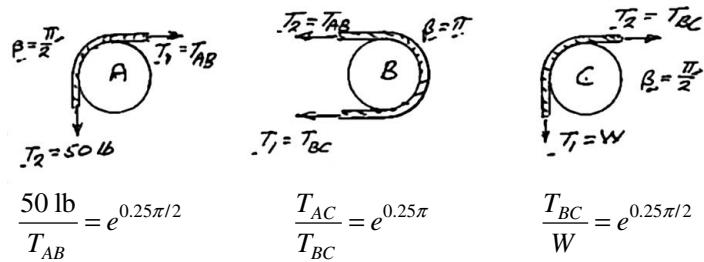


PROBLEM 8.120

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (a) the smallest weight W for which equilibrium is maintained, (b) the largest weight W that can be raised if pipe B is slowly rotated counterclockwise while pipes A and C remain fixed.

SOLUTION

(a) $\mu = \mu_s = 0.25$ at all pipes.



$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.25\pi/2}$$

$$\frac{T_{AC}}{T_{BC}} = e^{0.25\pi}$$

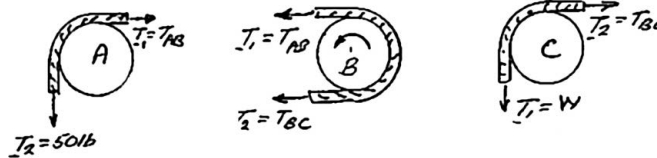
$$\frac{T_{BC}}{W} = e^{0.25\pi/2}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/8} \cdot e^{\pi/4} \cdot e^{\pi/8} = e^{\pi/8 + \pi/4 + \pi/8} = e^{\pi/2} = 4.8105$$

$$\frac{50 \text{ lb}}{W} = 4.8015; \quad W = 10.394 \text{ lb}$$

$$W = 10.39 \text{ lb} \quad \blacktriangleleft$$

(b) Pipe B rotated $\beta = \frac{\pi}{2}; \mu = \mu_k$ $\beta = \pi; \mu = \mu_s$ $\beta = \frac{\pi}{2}; \mu = \mu_k$



$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.2\pi/2}$$

$$\frac{T_{BC}}{T_{AB}} = e^{0.25\pi}$$

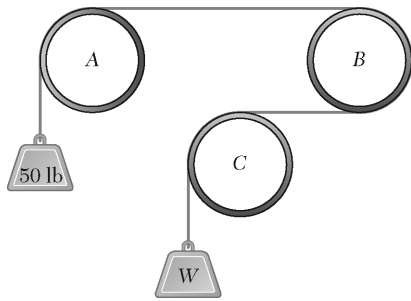
$$\frac{T_{BC}}{W} = e^{0.2\pi/2}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{-\pi/4} \cdot e^{\pi/10} = e^{\pi/10 - \pi/4 + \pi/10} = e^{-\pi/20} = 0.85464$$

$$\frac{50 \text{ lb}}{W} = 0.85464$$

$$W = \frac{50 \text{ lb}}{0.85464} = 58.504 \text{ lb}$$

$$W = 58.5 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 8.121

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (a) if only pipe A is rotated counterclockwise, (b) if only pipe C is rotated clockwise.

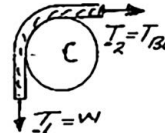
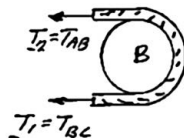
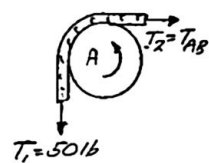
SOLUTION

(a) Pipe A rotates

$$\beta = \frac{\pi}{2}; \mu = \mu_s$$

$$\beta = \pi; \mu = \mu_k$$

$$\beta = \frac{\pi}{2}; \mu = \mu_k$$



$$\frac{T_{AB}}{50 \text{ lb}} = e^{0.25\pi/2}$$

$$\frac{T_{AB}}{T_{BC}} = e^{0.2\pi}$$

$$\frac{T_{BC}}{W} = e^{0.2\pi/2}$$

$$\frac{T_{AB}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\pi/8} \cdot e^{-\pi/5} \cdot e^{-\pi/10}$$

$$= e^{\pi(1/8 - 1/5 - 1/10)} = e^{-7\pi/40} = 0.57708$$

$$\frac{W}{50 \text{ lb}} = 0.57708; \quad W = 28.854 \text{ lb}$$

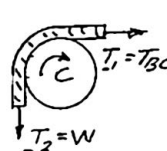
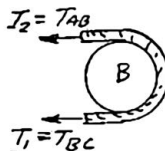
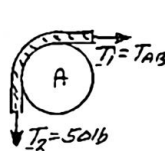
$$W = 28.9 \text{ lb} \quad \blacktriangleleft$$

(b) Pipe C rotates

$$\beta = \frac{\pi}{2}; \mu = \mu_k$$

$$\beta = \pi; \mu = \mu_k$$

$$\beta = \frac{\pi}{2}; \mu = \mu_s$$



$$\frac{50 \text{ lb}}{T_{AB}} = e^{0.2\pi/2}$$

$$\frac{T_{AB}}{T_{BC}} = e^{0.2\pi}$$

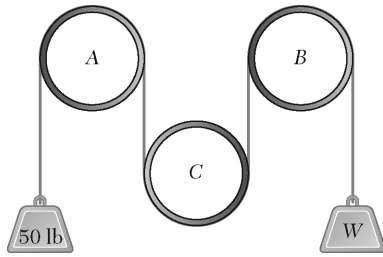
$$\frac{W}{T_{BC}} = e^{0.25\pi/2}$$

$$\frac{50 \text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{\pi/5} \cdot e^{-\pi/8} = e^{7\pi/40} = 0.57708$$

$$\frac{50 \text{ lb}}{W} = 0.57708$$

$$W = 28.854 \text{ lb}$$

$$W = 28.9 \text{ lb} \quad \blacktriangleleft$$

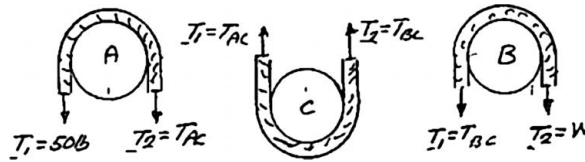


PROBLEM 8.122

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (a) the smallest weight W for which equilibrium is maintained, (b) the largest weight W that can be raised if pipe B is slowly rotated counterclockwise while pipes A and C remain fixed.

SOLUTION

(a) Smallest W for equilibrium $B = \pi, \mu = \mu_s$



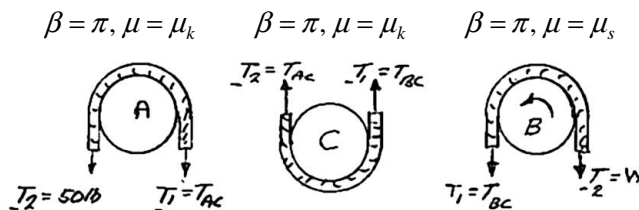
$$\frac{T_{AC}}{50 \text{ lb}} = e^{0.25\pi} \quad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \quad \frac{W}{T_{BC}} = e^{0.25\pi}$$

$$\frac{T_{AC}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{\pi/4} \cdot e^{\pi/4} = e^{3\pi/4} = 10.551$$

$$\frac{W}{50 \text{ lb}} = 10.551; \quad W = 4.739 \text{ lb}$$

$$W = 4.74 \text{ lb} \quad \blacktriangleleft$$

(b) Largest W which can be raised by pipe B rotated



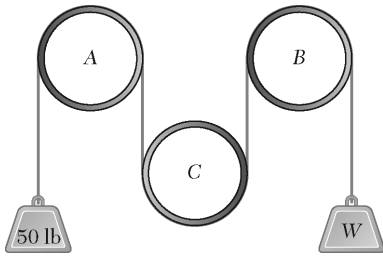
$$\frac{50 \text{ lb}}{T_{AC}} = e^{0.2\pi} \quad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \quad \frac{W}{T_{BC}} = e^{0.25\pi}$$

$$\frac{50 \text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{\pi/5} \cdot e^{-\pi/4} = e^{\pi(1/5+1/5-1/4)}$$

$$= e^{3\pi/20} = 1.602$$

$$\frac{50 \text{ lb}}{W} = 1.602; \quad W = \frac{50 \text{ lb}}{1.602} = 31.21 \text{ lb}$$

$$W = 31.2 \text{ lb} \quad \blacktriangleleft$$



PROBLEM 8.123

A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (a) if only pipe A is rotated counterclockwise, (b) if only pipe C is rotated clockwise.

SOLUTION

(a) Pipe A rotates

$$\beta = \pi, \mu = \mu_s \quad \beta = \pi, \mu = \mu_k \quad \beta = \pi, \mu = \mu_k$$

$$\frac{T_{AC}}{50 \text{ lb}} = e^{0.25\pi} \quad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \quad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{T_{AC}}{50 \text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{-\pi/5} \cdot e^{-\pi/5}$$

$$= e^{\pi(1/4 - 1/5 - 1/5)} = e^{-3\pi/20} = 0.62423$$

$$\frac{W}{50 \text{ lb}} = 0.62423; \quad W = 31.21 \text{ lb}$$

$$W = 31.2 \text{ lb} \quad \blacktriangleleft$$

(b) Pipe C rotates

$$\beta = \pi, \mu = \mu_k \quad \beta = \pi, \mu = \mu_s \quad \beta = \pi, \mu = \mu_k$$

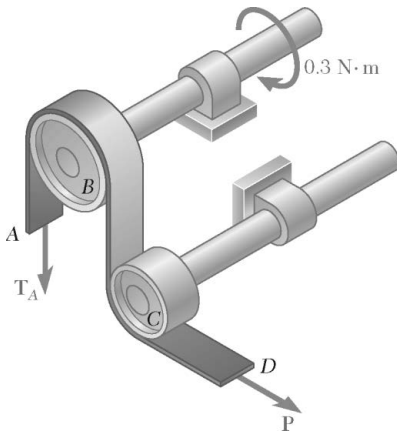
$$\frac{50 \text{ lb}}{T_{AC}} = e^{0.2\pi} \quad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \quad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{50 \text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{-\pi/4} \cdot e^{\pi/5} = e^{\pi(1/5 - 1/4 + 1/5)} = e^{3\pi/20}$$

$$\frac{50 \text{ lb}}{W} = e^{3\pi/20} = 1.602$$

$$W = \frac{50 \text{ lb}}{1.602} = 31.21 \text{ lb}$$

$$W = 31.2 \text{ lb} \quad \blacktriangleleft$$

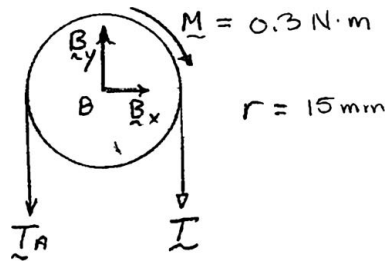


PROBLEM 8.124

A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C . Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\left(\sum M_B = 0: r(T_A - T) - M = 0 \right.$$

$$T_A - T = \frac{M}{r} = \frac{300 \text{ N} \cdot \text{mm}}{20 \text{ mm}} = 15.0000 \text{ N}$$

Impending slipping:

$$T_A = T e^{\mu_s \beta} = T e^{0.4\pi}$$

So

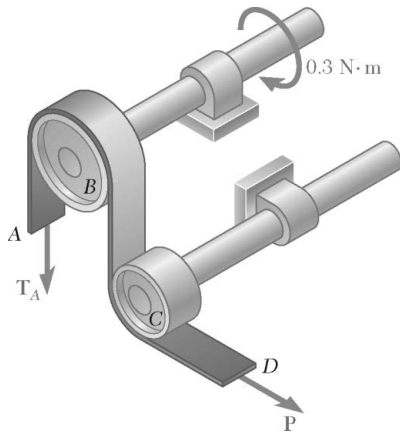
$$T(e^{0.4\pi} - 1) = 15.0000 \text{ N}$$

or

$$T = 5.9676 \text{ N}$$

If C is free to rotate, $P = T$

$$P = 5.97 \text{ N} \quad \blacktriangleleft$$



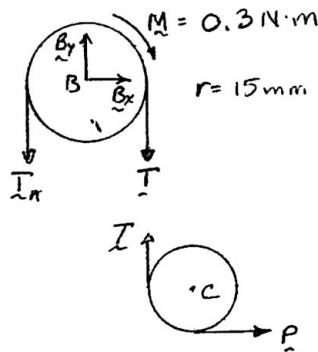
PROBLEM 8.125

Solve Problem 8.124 assuming that the idler drum C is frozen and cannot rotate.

PROBLEM 8.124 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C . Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\left(\sum M_B = 0: r(T_A - T) - M = 0 \right.$$

$$T_A - T = \frac{M}{r} = 300 \text{ N} \cdot \text{mm} = 15.0000 \text{ N}$$

Impending slipping:

$$T_A = T e^{\mu_s \beta} = T e^{0.4\pi}$$

So $(e^{0.4\pi} - 1)T = 15.000 \text{ N}$

or $T = 5.9676 \text{ N}$

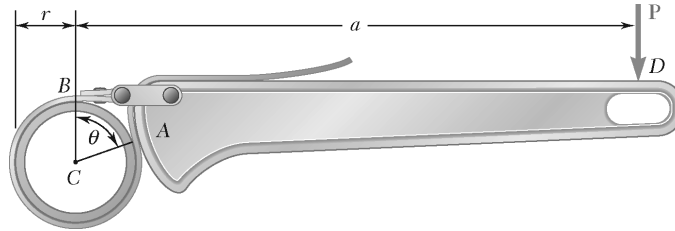
If C is fixed, the tape must slip (

So $P = T e^{\mu_k \beta_c} = (5.9676 \text{ N}) e^{0.3\pi/2} = 9.5600 \text{ N}$

$P = 9.56 \text{ N} \blacktriangleleft$

PROBLEM 8.126

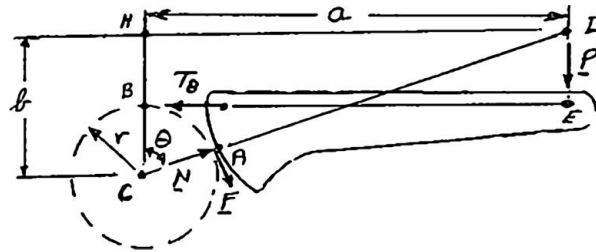
The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65^\circ$.



SOLUTION

For wrench to be self-locking ($P = 0$), the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop “belt friction” between strap and pipe.

Free body: Wrench handle



Geometry In $\triangle CDH$:

$$CH = \frac{a}{\tan \theta}$$

$$CD = \frac{a}{\sin \theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan \theta} - r$$

$$AD = CD - CA = \frac{a}{\sin \theta} - r$$

On wrench handle

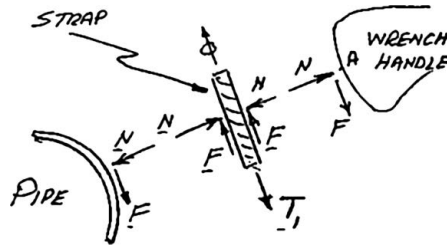
$$+\circlearrowleft \Sigma M_D = 0: T_B(DE) - F(AD) = 0$$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\frac{a}{\sin \theta} - r}{\frac{a}{\tan \theta} - r} \quad (1)$$

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PROBLEM 8.126 (Continued)

Free body: Strap at Point A



$$\begin{aligned} \sum F = 0: \quad T_1 - 2F &= 0 \\ T_1 &= 2F \end{aligned} \quad (2)$$

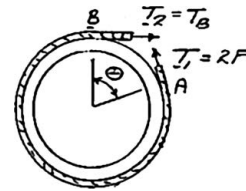
Pipe and strap

$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):

$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \quad (3)$$



Return to free body of wrench handle

$$\sum F_x = 0: \quad N \sin \theta + F \cos \theta - T_B = 0$$

$$\frac{N}{F} \sin \theta = \frac{T_B}{F} - \cos \theta$$

Since $F = \mu_s N$, we have

$$\frac{1}{\mu_s} \sin \theta = \frac{T_B}{F} - \cos \theta$$

or

$$\mu_s = \frac{\sin \theta}{\frac{T_B}{F} - \cos \theta} \quad (4)$$

(Note: For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $a = 200 \text{ mm}$, $r = 30 \text{ mm}$, $\theta = 65^\circ$

$$\begin{aligned} \text{Eq. (1):} \quad \frac{T_B}{F} &= \frac{\frac{200 \text{ mm}}{\sin 65^\circ} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 65^\circ} - 30 \text{ mm}} \\ &= \frac{190.676 \text{ mm}}{63.262 \text{ mm}} = 3.0141 \end{aligned}$$

$$\beta = 2\pi - \theta = 2\pi - 65^\circ \frac{\pi}{180^\circ} = 5.1487 \text{ radians}$$

PROBLEM 8.126 (Continued)

Eq. (3):
$$\begin{aligned}\mu_s &= \frac{1}{5.1487 \text{ rad}} \ln \frac{3.0141}{2} \\ &= \frac{0.41015}{5.1487} \\ &= 0.0797\end{aligned}$$
 ◁

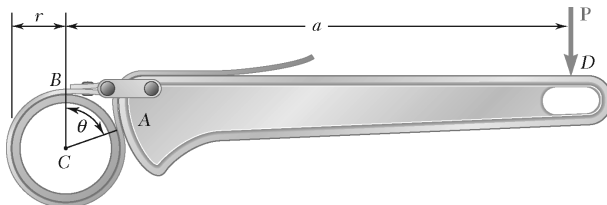
Eq. (4):
$$\begin{aligned}\mu_s &= \frac{\sin 65^\circ}{3.0141 - \cos 65^\circ} \\ &= \frac{0.90631}{2.1595} \\ &= 0.3497\end{aligned}$$
 ◁

We choose the larger value: $\mu_s = 0.350$ ◀

PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75^\circ$.

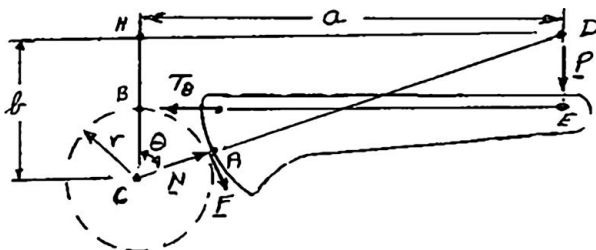
PROBLEM 8.126 The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65^\circ$.



SOLUTION

For wrench to be self-locking ($P = 0$), the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop “belt friction” between strap and pipe.

Free body: Wrench handle



Geometry In $\triangle CDH$:

$$CH = \frac{a}{\tan \theta}$$

$$CD = \frac{a}{\sin \theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan \theta} - r$$

$$AD = CD - CA = \frac{a}{\sin \theta} - r$$

On wrench handle

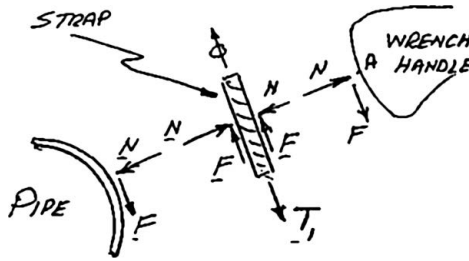
$$+\circlearrowleft \Sigma M_D = 0: T_B(DE) - F(AD) = 0$$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\frac{a}{\sin \theta} - r}{\frac{a}{\tan \theta} - r} \quad (1)$$

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PROBLEM 8.127 (Continued)

Free body: Strap at Point A



$$+\searrow \Sigma F = 0: T_1 - 2F = 0$$

$$T_1 = 2F$$

(2)

Pipe and strap

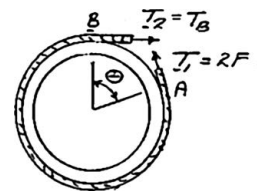
$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):

$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F}$$

(3)



Return to free body of wrench handle

$$+\rightarrow \Sigma F_x = 0: N \sin \theta + F \cos \theta - T_B = 0$$

$$\frac{N}{F} \sin \theta = \frac{T_B}{F} - \cos \theta$$

Since $F = \mu_s N$, we have

$$\frac{1}{\mu_s} \sin \theta = \frac{T_B}{F} - \cos \theta$$

or

$$\mu_s = \frac{\sin \theta}{\frac{T_B}{F} - \cos \theta}$$

(4)

(Note: For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For

$$a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 75^\circ$$

Eq. (1):

$$\begin{aligned} \frac{T_B}{F} &= \frac{\frac{200 \text{ mm}}{\sin 75^\circ} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 75^\circ} - 30 \text{ mm}} \\ &= \frac{177.055 \text{ mm}}{23.590 \text{ mm}} = 7.5056 \end{aligned}$$

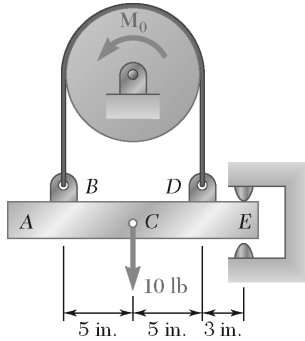
$$\beta = 2\pi - \theta = 2\pi - 75^\circ \frac{\pi}{180^\circ} = 4.9742$$

PROBLEM 8.127 (Continued)

Eq. (3):
$$\begin{aligned}\mu_s &= \frac{1}{4.9742 \text{ rad}} \ln \frac{7.5056}{2} \\ &= \frac{1.3225}{4.9742} \\ &= 0.2659\end{aligned}$$
 ◁

Eq. (4):
$$\begin{aligned}\mu_s &= \frac{\sin 75^\circ}{7.5056 - \cos 75^\circ} \\ &= \frac{0.96953}{7.2468} \\ &= 0.1333\end{aligned}$$
 ◁

We choose the larger value: $\mu_s = 0.266$ ◀



PROBLEM 8.128

The 10-lb bar AE is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end E of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (a) the largest counterclockwise couple M_0 that can be applied to the drum if slipping is not to occur, (b) the corresponding force exerted on end E of the bar.

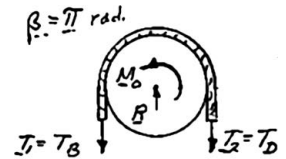
SOLUTION

Drum: Slipping impends

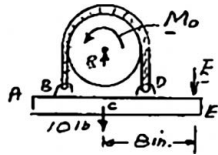
$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663T_B$$



(a) Free-body: Drum and bar



$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8 \text{ in.}) = 0$$

$$M_0 = (3.78649 \text{ lb})(8 \text{ in.}) \\ = 30.27 \text{ lb} \cdot \text{in.}$$

$$M_0 = 30.3 \text{ lb} \cdot \text{in.} \quad \swarrow \blacktriangleleft$$

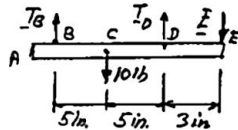
(b) Bar AE :

$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 10 \text{ lb} = 0$$

$$T_B + 2.5663T_B - E - 10 \text{ lb} = 0$$

$$3.5663T_B - E - 10 \text{ lb} = 0$$

$$E = 3.5663T_B - 10 \text{ lb} \quad (1)$$



$$+\circlearrowleft \Sigma M_D = 0: \quad E(3 \text{ in.}) - (10 \text{ lb})(5 \text{ in.}) + T_B(10 \text{ in.}) = 0$$

$$(3.5663T_B - 10 \text{ lb})(3 \text{ in.}) - 50 \text{ lb} \cdot \text{in.} + T_B(10 \text{ in.}) = 0$$

$$20.699T_B = 80 \quad T_B = 3.8649 \text{ lb}$$

Eq. (1):

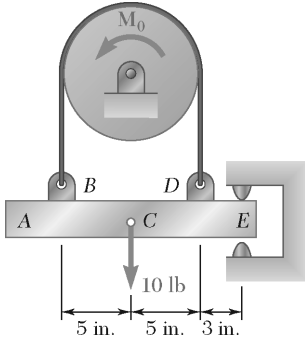
$$E = 3.5663(3.8649 \text{ lb}) - 10 \text{ lb}$$

$$E = +3.78347 \text{ lb}$$

$$E = 3.78 \text{ lb} \quad \downarrow \blacktriangleleft$$

PROBLEM 8.129

Solve Problem 8.128 assuming that a clockwise couple M_0 is applied to the drum.



PROBLEM 8.128 The 10-lb bar AE is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end E of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (a) the largest counterclockwise couple M_0 that can be applied to the drum if slipping is not to occur, (b) the corresponding force exerted on end E of the bar.

SOLUTION

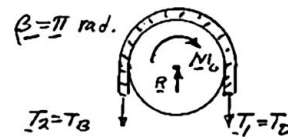
Drum: Slipping impends

$$\mu_s = 0.30$$

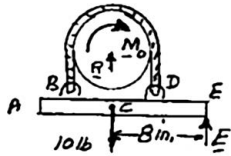
$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{T_B}{T_D} = e^{0.30\pi} = 2.5663$$

$$T_B = 2.5663T_D$$



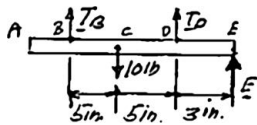
(a) Free body: Drum and bar



$$\begin{aligned} +\curvearrowright \Sigma M_C = 0: & M_0 - E(8 \text{ in.}) = 0 \\ & M_0 = (2.1538 \text{ lb})(8 \text{ in.}) \end{aligned}$$

$$M_0 = 17.23 \text{ lb} \cdot \text{in.} \quad \curvearrowleft$$

(b) Bar AE :



$$\begin{aligned} +\uparrow \Sigma F_y = 0: & T_B + T_D + E - 10 \text{ lb} = 0 \\ & = 2.5663T_D + T_D + E - 10 \text{ lb} \end{aligned}$$

$$E = -3.5663T_D + 10 \text{ lb} \quad (1)$$

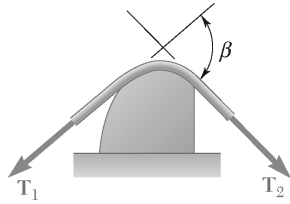
$$+\curvearrowright \Sigma M_B = 0: T_D(10 \text{ in.}) - (10 \text{ lb})(5 \text{ in.}) + E(13 \text{ in.}) = 0$$

$$\begin{aligned} T_D(10 \text{ in.}) - 50 \text{ lb} \cdot \text{in.} + (-3.5663T_D + 10 \text{ lb})(13 \text{ in.}) &= 0 \\ -36.362T_D + 80 \text{ lb} \cdot \text{in.} &= 0; \quad T_D = 2.200 \text{ lb} \end{aligned}$$

Eq. (1):

$$E = -3.5633(2.200 \text{ lb}) + 10 \text{ lb}$$

$$E = +2.1538 \text{ lb} \quad \uparrow \quad \curvearrowleft$$



PROBLEM 8.130

Prove that Eqs. (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION

$$\uparrow \Sigma F_n = 0: \Delta N - [T + (T + \Delta T)] \sin \frac{\Delta \theta}{2} = 0$$

or
$$\Delta N = (2T + \Delta T) \sin \frac{\Delta \theta}{2}$$

$$\rightarrow \Sigma F_t = 0: [(T + \Delta T) - T] \cos \frac{\Delta \theta}{2} - \Delta F = 0$$

or
$$\Delta F = \Delta T \cos \frac{\Delta \theta}{2}$$

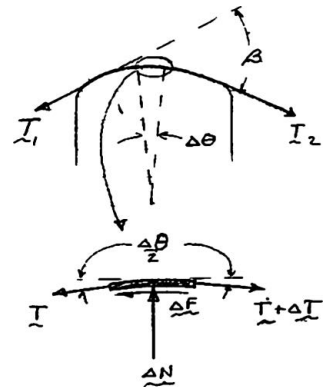
Impending slipping:
$$\Delta F = \mu_s \Delta N$$

So
$$\Delta T \cos \frac{\Delta \theta}{2} = \mu_s 2T \sin \frac{\Delta \theta}{2} + \mu_s \Delta T \frac{\sin \Delta \theta}{2}$$

In limit as $\Delta \theta \rightarrow 0: dT = \mu_s T d\theta$ or $\frac{dT}{T} = \mu_s d\theta$

So
$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\beta \mu_s d\theta$$

and
$$\ln \frac{T_2}{T_1} = \mu_s \beta$$
 or $T_2 = T_1 e^{\mu_s \beta} \blacktriangleleft$



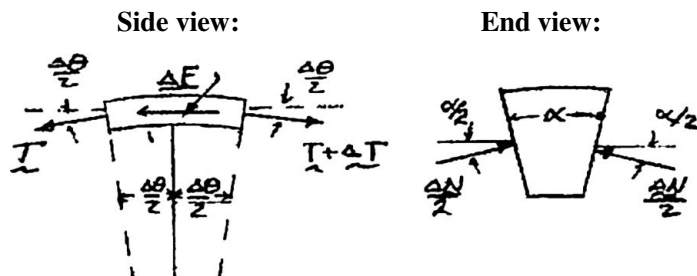
(Note: Nothing above depends on the shape of the surface, except it is assumed to be a smooth curve.)

PROBLEM 8.131

Complete the derivation of Eq. (8.15), which relates the tension in both parts of a V belt.

SOLUTION

Small belt section:



$$\uparrow \Sigma F_y = 0: 2 \frac{\Delta N}{2} \sin \frac{\alpha}{2} - [T + (T + \Delta T)] \sin \frac{\Delta \theta}{2} = 0$$

$$\rightarrow \Sigma F_x = 0: [(T + \Delta T) - T] \cos \frac{\Delta \theta}{2} - \Delta F = 0$$

Impending slipping:

$$\Delta F = \mu_s \Delta N \Rightarrow \Delta T \cos \frac{\Delta \theta}{2} = \mu_s \frac{2T + \Delta T}{\sin \frac{\alpha}{2}} \sin \frac{\Delta \theta}{2}$$

In limit as

$$\Delta \theta \rightarrow 0: dT = \frac{\mu_s T d\theta}{\sin \frac{\alpha}{2}} \quad \text{or} \quad \frac{dT}{T} = \frac{\mu_s}{\sin \frac{\alpha}{2}} d\theta$$

so

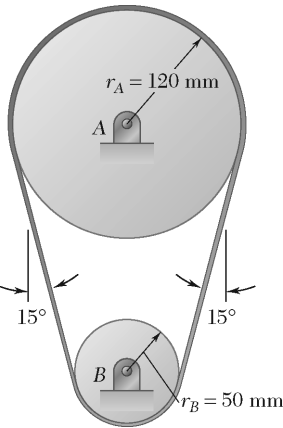
$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu_s}{\sin \frac{\alpha}{2}} \int_0^\beta d\theta$$

or

$$\ln \frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin \frac{\alpha}{2}}$$

or

$$T_2 = T_1 e^{\mu_s \beta / \sin \frac{\alpha}{2}} \blacktriangleleft$$



PROBLEM 8.132

Solve Problem 8.112 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36^\circ$. (The angle α is as shown in Figure 8.15a.)

PROBLEM 8.112 A flat belt is used to transmit a couple from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum A.

SOLUTION

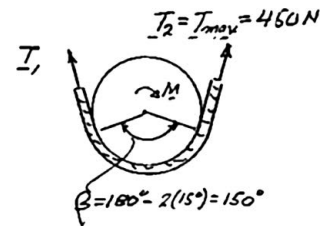
Since β is smaller for pulley B. The belt will slip first at B.

$$\beta = 150^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) = \frac{5}{6} \pi \text{ rad}$$

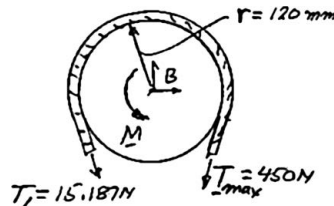
$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin \frac{\alpha}{2}}$$

$$\frac{450 \text{ N}}{T_1} = e^{(0.4) \frac{5}{6} \pi / \sin 18^\circ} = e^{3.389}$$

$$\frac{450 \text{ N}}{T_1} = 29.63; \quad T_1 = 15.187 \text{ N}$$



Torque on pulley A:

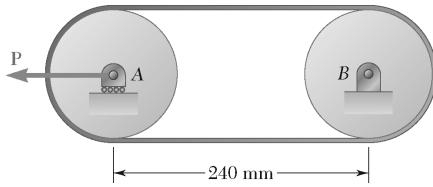


$$\begin{aligned} + \curvearrowright \Sigma M_B = 0: \quad & M - (T_{\max} - T_1)(0.12 \text{ m}) = 0 \\ & M - (450 \text{ N} - 15.187 \text{ N})(0.12 \text{ m}) = 0 \\ & M = 52.18 \text{ N} \cdot \text{m} \end{aligned}$$

$$M = 52.2 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

PROBLEM 8.133

Solve Problem 8.113 assuming that the flat belt and pulleys are replaced by a V belt and V pulleys with $\alpha = 36^\circ$. (The angle α is as shown in Figure 8.15a.)



PROBLEM 8.113 A flat belt is used to transmit a couple from pulley A to pulley B. The radius of each pulley is 60 mm, and a force of magnitude $P = 900 \text{ N}$ is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest couple that can be transmitted, (b) the corresponding maximum value of the tension in the belt.

SOLUTION

Pulley A:

$$\beta = \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin \frac{\alpha}{2}}$$

$$\frac{T_2}{T_1} = e^{0.35\pi / \sin 18^\circ}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1T_1$$

$$\rightarrow \Sigma F_x = 0: T_1 + T_2 - 900 \text{ N} = 0$$

$$T_1 + 35.1T_1 - 900 \text{ N} = 0$$

$$T_1 = 24.93 \text{ N}$$

$$T_2 = 35.1(24.93 \text{ N}) = 875.03 \text{ N}$$

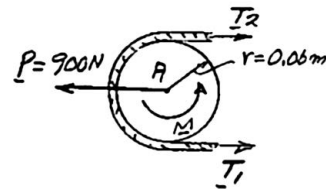
$$\curvearrowright \Sigma M_A = 0: M - T_2(0.06 \text{ m}) + T_1(0.06 \text{ m}) = 0$$

$$M - (875.03 \text{ N})(0.06 \text{ m}) + (24.93 \text{ N})(0.06 \text{ m}) = 0$$

$$M = 51.0 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

$$T_{\max} = T_2$$

$$T_{\max} = 875 \text{ N} \quad \blacktriangleleft$$



PROBLEM 8.134

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35^\circ$ and $P = 200\text{ N}$.

SOLUTION

Assume equilibrium:

$$+\nearrow \Sigma F_y = 0: N - (800\text{ N}) \cos 25^\circ + (200\text{ N}) \sin 10^\circ = 0$$

$$N = 690.3\text{ N} \qquad \mathbf{N} = 690.3\text{ N} \uparrow$$

$$+\searrow \Sigma F_x = 0: -F + (800\text{ N}) \sin 25^\circ - (200\text{ N}) \cos 10^\circ = 0$$

$$F = 141.13\text{ N} \qquad \mathbf{F} = 141.13\text{ N} \nearrow$$

Maximum friction force:

$$F_m = \mu_s N$$

$$= (0.20)(690.3\text{ N})$$

$$= 138.06\text{ N}$$

Since $F > F_m$, Block moves down $\searrow \blacktriangleleft$

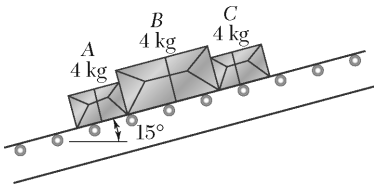
Friction force:

$$F = \mu_k N$$

$$= (0.15)(690.3\text{ N})$$

$$= 103.547\text{ N} \qquad \mathbf{F} = 103.5\text{ N} \searrow \blacktriangleleft$$

PROBLEM 8.135



Three 4-kg packages A, B, and C are placed on a conveyor belt that is at rest. Between the belt and both packages A and C the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package B and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

Consider C by itself: Assume equilibrium

$$+\nearrow \Sigma F_y = 0: N_C - W \cos 15^\circ = 0$$

$$N_C = W \cos 15^\circ = 0.966W$$

$$+\nearrow \Sigma F_x = 0: F_C - W \sin 15^\circ = 0$$

$$F_C = W \sin 15^\circ = 0.259W$$

But

$$\begin{aligned} F_m &= \mu_s N_C \\ &= 0.30(0.966W) \\ &= 0.290W \end{aligned}$$

Thus, $F_C < F_m$

Package C does not move ◀

$$\begin{aligned} F_C &= 0.259W \\ &= 0.259(4 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 10.16 \text{ N} \end{aligned}$$

$$F_C = 10.16 \text{ N} \nearrow \blacktriangleleft$$

Consider B by itself: Assume equilibrium. We find,

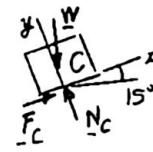
$$\begin{aligned} F_B &= 0.259W \\ N_B &= 0.966W \end{aligned}$$

But

$$\begin{aligned} F_m &= \mu_s N_B \\ &= 0.10(0.966W) \\ &= 0.0966W \end{aligned}$$

Thus, $F_B > F_m$.

Package B would move if alone ◀



PROBLEM 8.135 (Continued)

Consider A and B together: Assume equilibrium

$$F_A = F_B = 0.259W$$

$$N_A = N_B = 0.966W$$

$$F_A + F_B = 2(0.259W) = 0.518W$$

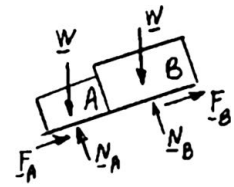
$$(F_A)_m + (F_B)_m = 0.3N_A + 0.1N_B = 0.386W$$

Thus,

$$F_A + F_B > (F_A)_m + (F_B)_m$$

$$F_A = \mu_k N_A = 0.2(0.966)(4)(9.81)$$

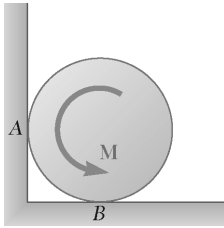
$$F_B = \mu_k N_B = 0.08(0.966)(4)(9.81)$$



A and B move ◀

$$F_A = 7.58 \text{ N} \nearrow \blacktriangleleft$$

$$F_B = 3.03 \text{ N} \nearrow \blacktriangleleft$$



PROBLEM 8.136

The cylinder shown is of weight W and radius r . Express in terms W and r the magnitude of the largest couple M that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (a) zero at A and 0.30 at B , (b) 0.25 at A and 0.30 at B .

SOLUTION

FBD cylinder:

For maximum M , motion impends at both A and B

$$F_A = \mu_A N_A$$

$$F_B = \mu_B N_B$$

$$\rightarrow \Sigma F_x = 0: N_A - F_B = 0$$

$$N_A = F_B = \mu_B N_B$$

$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: N_B + F_A - W = 0$$

$$N_B(1 + \mu_A \mu_B) = W$$

or

$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and

$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

$$\left(\Sigma M_C = 0: M - r(F_A + F_B) = 0 \right.$$

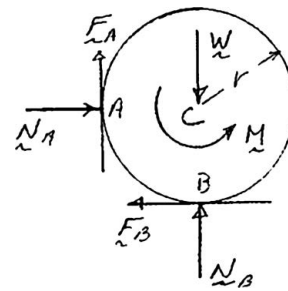
$$M = W r \mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

(a) For $\mu_A = 0$ and $\mu_B = 0.30$:

$$M = 0.300Wr \blacktriangleleft$$

(b) For $\mu_A = 0.25$ and $\mu_B = 0.30$:

$$M = 0.349Wr \blacktriangleleft$$



PROBLEM 8.137

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC . Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (a) the largest value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION

Free-body diagram

Three-force body. Line of action of \mathbf{R} must pass through D , where \mathbf{T} and \mathbf{R} intersect.

Motion impends:

$$\tan \phi_s = 0.4$$

$$\phi_s = 21.8^\circ$$

(a) Since $BG = GA$, it follows that $BD = DC$ and AD bisects $\angle BAC$

$$\frac{\theta}{2} + \phi_s = 90^\circ$$

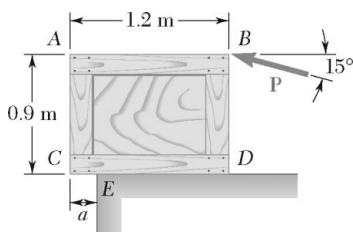
$$\frac{\theta}{2} + 21.8^\circ = 90^\circ$$

$$\theta = 136.4^\circ \quad \blacktriangleleft$$

(b) Force triangle (right triangle):

$$T = W \cos 21.8^\circ$$

$$T = 0.928W \quad \blacktriangleleft$$



PROBLEM 8.138

A worker slowly moves a 50-kg crate to the left along a loading dock by applying a force \mathbf{P} at corner B as shown. Knowing that the crate starts to tip about the edge E of the loading dock when $a = 200$ mm, determine (a) the coefficient of kinetic friction between the crate and the loading dock, (b) the corresponding magnitude P of the force.

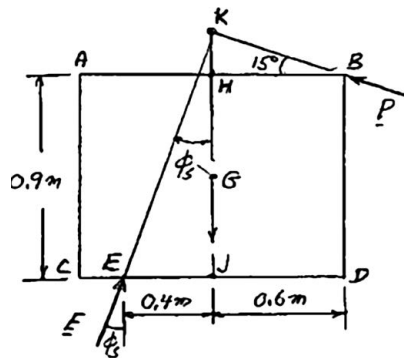
SOLUTION

Free body: Crate Three-force body.

Reaction E must pass through K where \mathbf{P} and \mathbf{W} intersect.

Geometry:

(a) $HK = (0.6 \text{ m}) \tan 15^\circ = 0.16077 \text{ m}$



$$JK = 0.9 \text{ m} + HK = 1.06077 \text{ m}$$

$$\tan \phi_s = \frac{0.4 \text{ m}}{1.06077 \text{ m}} = 0.37708$$

$$\mu_s = \tan \phi_s = 0.377 \quad \blacktriangleleft$$

$$\phi_s = 20.66^\circ$$

Force triangle:

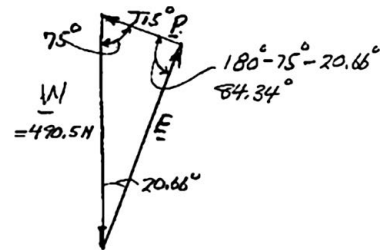
(b) $W = (50 \text{ kg})(9.81 \text{ m/s}^2) = 490.5 \text{ N}$

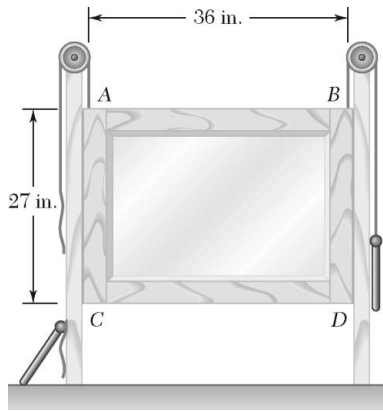
Law of sines:

$$\frac{P}{\sin 20.66^\circ} = \frac{490.5 \text{ N}}{\sin 84.34^\circ}$$

$$P = 173.91 \text{ N}$$

$$P = 173.9 \text{ N} \quad \blacktriangleleft$$





PROBLEM 8.139

A window sash weighing 10 lb is normally supported by two 5-lb sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller than the frame and will bind only at Points A and D.)

SOLUTION

FBD window:

$$T = 5 \text{ lb}$$

$$\begin{aligned} \rightarrow \Sigma F_x = 0: \quad N_A - N_D &= 0 \\ N_A &= N_D \end{aligned}$$

Impending motion:

$$F_A = \mu_s N_A$$

$$F_D = \mu_s N_D$$

$$\curvearrowleft \Sigma M_D = 0: \quad (18 \text{ in.})W - (27 \text{ in.})N_A - (36 \text{ in.})F_A = 0$$

$$W = 10 \text{ lb}$$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

$$F_A + F_D = W - T = \frac{W}{2}$$

Now

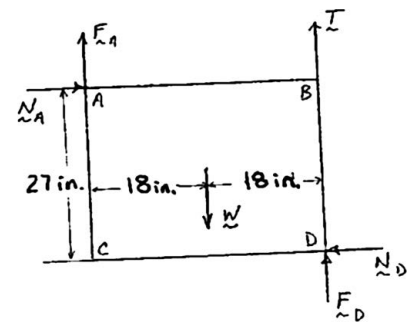
$$\begin{aligned} F_A + F_D &= \mu_s (N_A + N_D) \\ &= 2\mu_s N_A \end{aligned}$$

Then

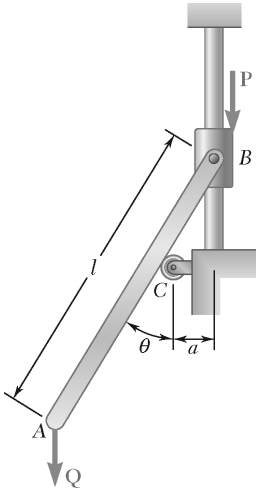
$$\frac{W}{2} = 2\mu_s \frac{2W}{3 + 4\mu_s}$$

or

$$\mu_s = 0.750 \quad \blacktriangleleft$$



PROBLEM 8.140



The slender rod AB of length $l = 600$ mm is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 80$ mm from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 100$ N and $\theta = 30^\circ$.

SOLUTION

For motion of collar at B impending upward:

$$F = \mu_s N \downarrow$$

$$+\curvearrowright \Sigma M_B = 0: \quad Ql \sin \theta - \frac{Ca}{\sin \theta} = 0$$

$$C = Q \left(\frac{l}{a} \right) \sin^2 \theta$$

$$\Sigma F_x = 0: \quad N = C \cos \theta = Q \left(\frac{l}{a} \right) \sin^2 \theta \cos \theta$$

$$+\downarrow \Sigma F_y = 0: \quad P + Q - C \sin \theta - \mu_s N = 0$$

$$P + Q - Q \left(\frac{l}{a} \right) \sin^3 \theta - \mu_s Q \left(\frac{l}{a} \right) \sin^2 \theta \cos \theta = 0$$

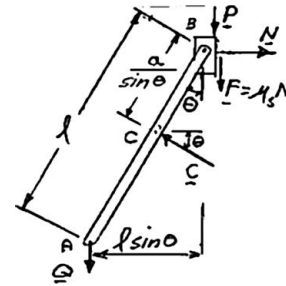
$$P = Q \left[\frac{l}{a} \sin^2 \theta (\sin \theta - \mu_s \cos \theta) - 1 \right] \quad (1)$$

Substitute data:

$$P = (100 \text{ N}) \left[\frac{600 \text{ mm}}{80 \text{ mm}} \sin^2 30^\circ (\sin 30^\circ - 0.25 \cos 30^\circ) - 1 \right]$$

$$P = -46.84 \text{ N} \quad (P \text{ is directed } \uparrow)$$

$$P = -46.8 \text{ N} \quad \triangleleft$$



PROBLEM 8.140 (Continued)

For motion of collar, impending downward:

$$F = \mu_s N \uparrow$$

In Eq. (1) we substitute $-\mu_s$ for μ_s .

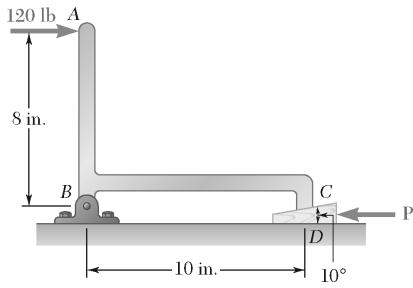
$$P = Q \left[\frac{l}{a} \sin^2 \theta (\sin \theta + \mu_s \cos \theta) - 1 \right]$$

$$P = (100 \text{ N}) \left[\frac{600 \text{ mm}}{80 \text{ mm}} \sin^2 30^\circ (\sin 30^\circ + 0.25 \cos \theta) - 1 \right]$$

$$P = +34.34 \text{ N} \triangleleft$$

For equilibrium:

$$-46.8 \text{ N} \leq P \leq 34.3 \text{ N} \blacktriangleleft$$



PROBLEM 8.141

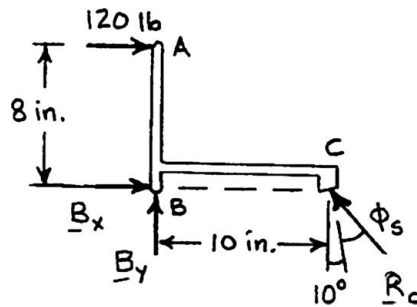
The machine part ABC is supported by a frictionless hinge at B and a 10° wedge at C . Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (a) the force \mathbf{P} required to move the wedge to the left, (b) the components of the corresponding reaction at B .

SOLUTION

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.20 = 11.3099^\circ$$

Free body: ABC

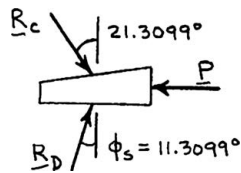


$$10^\circ + 11.3099^\circ = 21.3099^\circ$$

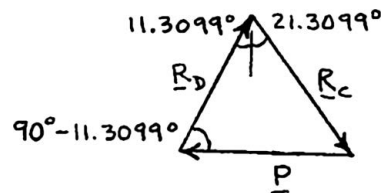
$$+\circlearrowleft \Sigma M_B = 0: (R_C \cos 21.3099^\circ)(10) - (120 \text{ lb})(8) = 0$$

$$R_C = 103.045 \text{ lb}$$

Free body: Wedge



Force triangle:



PROBLEM 8.141 (Continued)

Law of sines:

$$\frac{P}{\sin 32.6198^\circ} = \frac{(R_C = 103.045 \text{ lb})}{\sin 78.690^\circ}$$

(a)

$$\mathbf{P} = 56.6 \text{ lb} \leftarrow \blacktriangleleft$$

(b) Returning to free body of ABC:

$$+\rightarrow \Sigma F_x = 0: B_x + 120 - (103.045) \sin 21.3099^\circ = 0$$

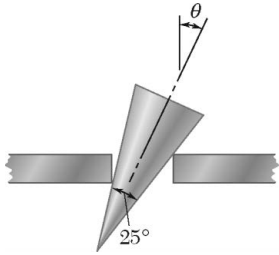
$$B_x = -82.552 \text{ lb}$$

$$\mathbf{B}_x = 82.6 \text{ lb} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: B_y + (103.045) \cos 21.3099^\circ = 0$$

$$B_y = -96.000 \text{ lb}$$

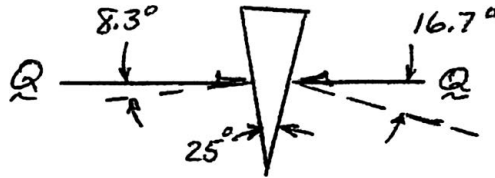
$$\mathbf{B}_y = 96.0 \text{ lb} \downarrow \blacktriangleleft$$



PROBLEM 8.142

A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (a) if $\mu_s = 0.20$, (b) if $\mu_s = 0.30$.

SOLUTION



As the plates are moved, the angle θ will decrease.

(a) $\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.2 = 11.31^\circ$.

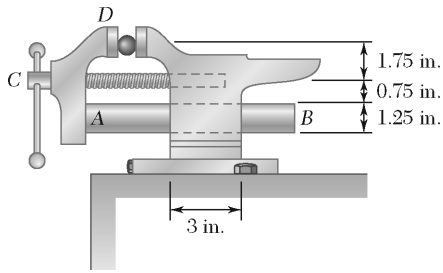
As θ decrease, the minimum angle at the contact approaches

$12.5^\circ > \phi_s = 11.31^\circ$, so the wedge will slide up and out from the slot. ◀

(b) $\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.3 = 16.70^\circ$.

As θ decreases, the angle at one contact reaches 16.7° . (At this

time the angle at the other contact is $25^\circ - 16.7^\circ = 8.3^\circ < \phi_s$). The wedge binds in the slot. ◀



PROBLEM 8.143

In the machinist's vise shown, the movable jaw D is rigidly attached to the tongue AB that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 0.75 in. and a pitch of 0.25 in. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 1 kip.

SOLUTION

Free body: Jaw D and tongue AB

P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: N_H - N_J = 0 \quad N_J = N_H = N$$

For final tightening,

$$F_H = F_J = \mu_s N = 0.25 N$$

$$+\rightarrow \Sigma F_x = 0: W - P - 2(0.25 N) = 0$$

$$N = 2(W - P) \quad (1)$$

$$+\curvearrowright \Sigma M_H = 0: P(3.75) - W(2) - N(3) + (0.25 N)(1.25) = 0$$

$$3.75P - 2W - 2.6875 N = 0 \quad (2)$$

Substitute Eq. (1) into Eq. (2): $3.75P - 2W - 2.6875[2(W - P)] = 0$

$$7.375W = 9.125P = 9.125(1 \text{ kip})$$

$$W = 1.23729 \text{ kips}$$

Block-and-incline analysis of screw:

$$\tan \phi_s = \mu_s = 0.25$$

$$\phi_s = 14.0362^\circ$$

$$\tan \theta = \frac{0.25 \text{ in.}}{\pi(0.75 \text{ in.})}$$

$$\theta = 6.0566^\circ$$

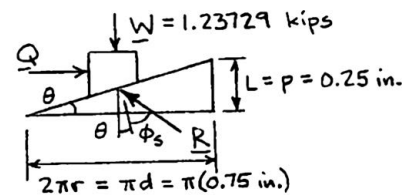
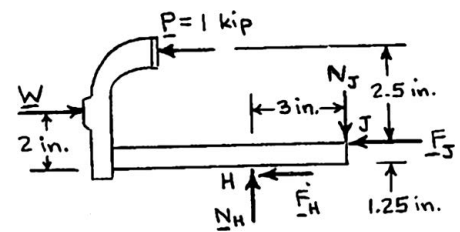
$$\theta + \phi_s = 20.093^\circ$$

$$Q = (1.23729 \text{ kips}) \tan 20.093^\circ$$

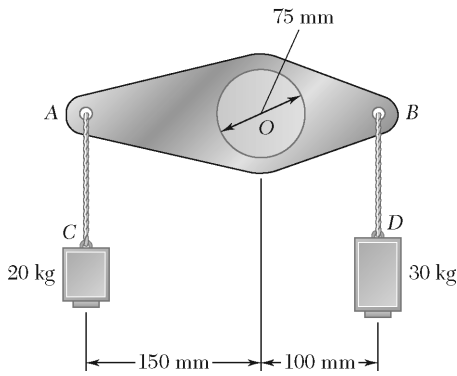
$$= 0.45261 \text{ kip}$$

$$T = Qr = (452.61 \text{ lb}) \left(\frac{0.75 \text{ in.}}{2} \right)$$

$$T = 169.7 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$



PROBLEM 8.144



A lever of negligible weight is loosely fitted onto a 75-mm-diameter fixed shaft. It is observed that the lever will just start rotating if a 3-kg mass is added at C. Determine the coefficient of static friction between the shaft and the lever.

SOLUTION

$$+\curvearrowright \Sigma M_O = 0: W_C(150) - W_D(100) - Rr_f = 0$$

But

$$W_C = (23 \text{ kg})(9.81 \text{ m/s}^2)$$

$$W_D = (30 \text{ kg})(9.81 \text{ m/s}^2)$$

$$R = W_C + W_D = (53 \text{ kg})(9.81)$$

Thus, after dividing by 9.81,

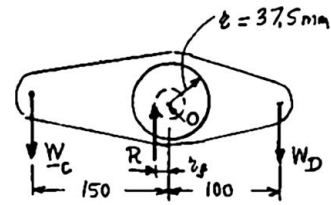
$$23(150) - 30(100) - 53 r_f = 0$$

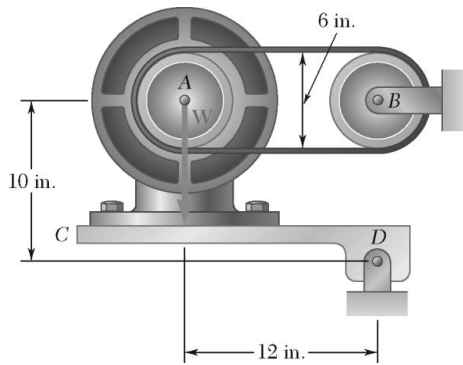
$$r_f = 8.49 \text{ mm}$$

But

$$\mu_s \approx \frac{r_f}{r} = \frac{8.49 \text{ mm}}{37.5 \text{ mm}}$$

$$\mu_s \approx 0.226 \quad \blacktriangleleft$$



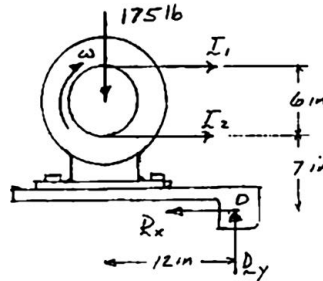


PROBLEM 8.145

In the pivoted motor mount shown the weight W of the 175-lb motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums A and B is 0.40, and neglecting the weight of platform CD , determine the largest couple that can be transmitted to drum B when the drive drum A is rotating clockwise.

SOLUTION

FBD motor and mount:



Impending belt slip: cw rotation

$$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.40\pi} = 3.5136 T_1$$

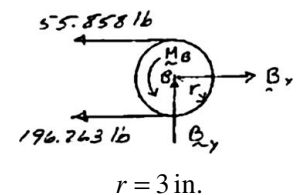
$$\left(\sum M_D = 0: (12 \text{ in.})(175 \text{ lb}) - (7 \text{ in.})T_2 - (13 \text{ in.})T_1 = 0 \right.$$

$$2100 \text{ lb} = [(7 \text{ in.})(3.5136) + 13 \text{ in.}]T_1$$

$$T_1 = 55.858 \text{ lb}, \quad T_2 = 3.5136 T_1 = 196.263 \text{ lb}$$

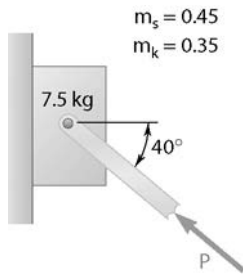
FBD drum at B :

$$\left(\sum M_B = 0: M_B - (3 \text{ in.})(196.263 \text{ lb} - 55.858 \text{ lb}) = 0 \right.$$



$$r = 3 \text{ in.}$$

$$M_B = 421 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$



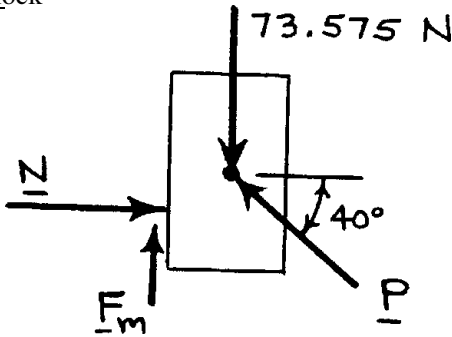
PROBLEM 8.F1

Draw the free-body diagram needed to determine the smallest force **P** for which equilibrium of the 7.5-kg block is maintained.

SOLUTION

$$W = (7.5 \text{ kg})(9.81 \text{ m/s}^2) = 73.575 \text{ N}$$

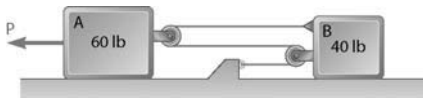
Free body: Block



Since we seek smallest P , motion impends downward, with

$$F_m = \mu_s N = 0.45 N$$

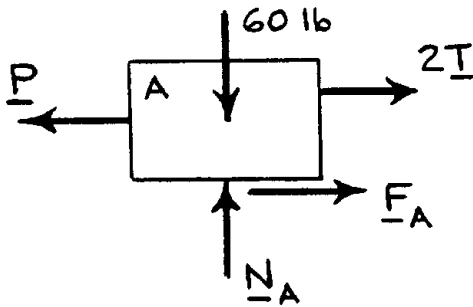
PROBLEM 8.F2



Two blocks A and B are connected by a cable as shown. Knowing that the coefficient of static friction at all surfaces of contact is 0.30 and neglecting the friction of the pulleys, draw the free-body diagrams needed to determine the smallest force P required to move the blocks.

SOLUTION

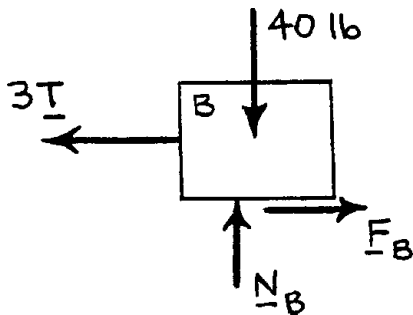
Free body: Block A



Motion impends to left, with

$$F_A = \mu_s N_A = 0.30 N_A$$

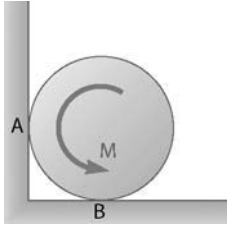
Free body: Block B



Motion impends to left, with

$$F_B = \mu_s N_B = 0.30 N_B$$

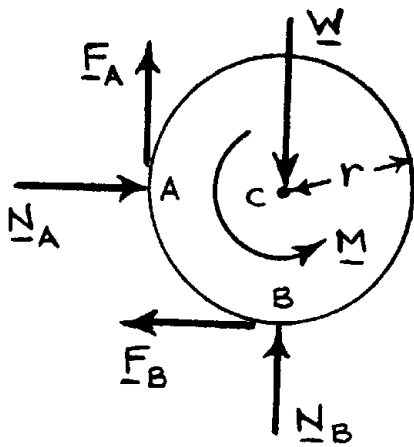
PROBLEM 8.F3



The cylinder shown is of weight W and radius r , and the coefficient of static friction μ_s is the same at A and B . Draw the free-body diagram needed to determine the largest couple M that can be applied to the cylinder if it is not to rotate.

SOLUTION

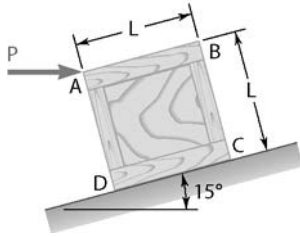
Free body: Cylinder



For maximum M , motion impends at both A and B , with

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$



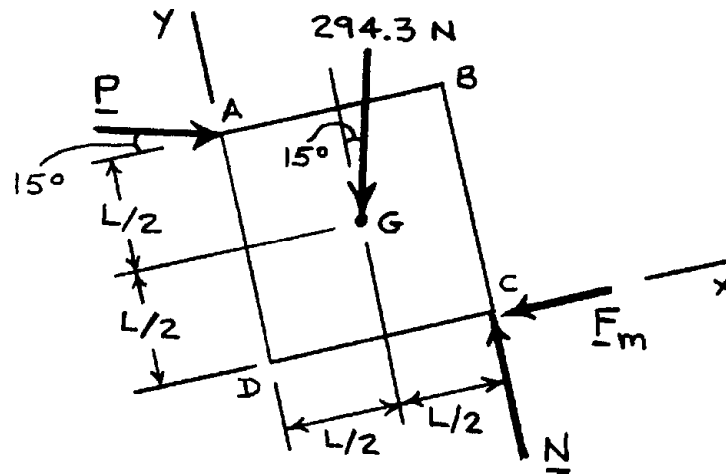
PROBLEM 8.F4

A uniform crate of mass 30 kg must be moved up along the 15° incline without tipping. Knowing that the force \mathbf{P} is horizontal, draw the free-body diagram needed to determine the largest allowable coefficient of static friction between the crate and the incline, and the corresponding force \mathbf{P} .

SOLUTION

$$W = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$

Free body: Crate



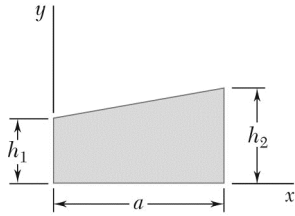
For impending tip, N must act at C .

For impending slip,

$$F_m = \mu_{\max} N$$

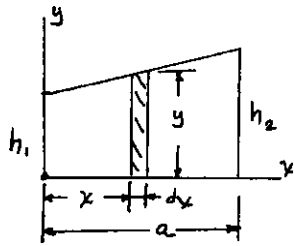
CHAPTER 9

PROBLEM 9.1



Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

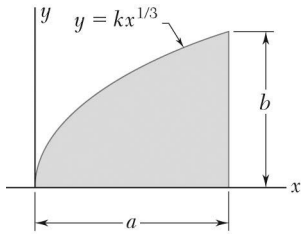


$$y = h_1 + (h_2 - h_1)\frac{x}{a}; dA = ydx$$

$$dI_y = x^2 dA = x^2 \left[h_1 + (h_2 - h_1)\frac{x}{a} \right] dx$$

$$\begin{aligned} I_y &= \int_0^a \left[h_1 x^2 + \frac{h_2 - h_1}{a} x^3 \right] dx \\ &= h_1 \frac{a^3}{3} + \frac{h_2 - h_1}{a} \frac{a^4}{4} \\ &= \frac{h_1 a^3}{12} + \frac{h_2 a^3}{4} \end{aligned}$$

$$I_y = \frac{a^3}{12}(h_1 + 3h_2) \blacktriangleleft$$



PROBLEM 9.2

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

For $x = a$:

$$y = kx^{1/3}$$

$$b = ka^{1/3}$$

$$k = b/a^{1/3}$$

Thus:

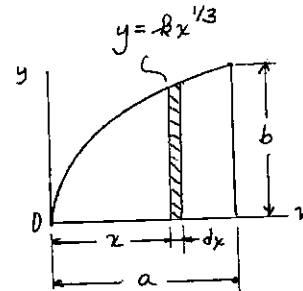
$$y = \frac{b}{a^{1/3}}x^{1/3}$$

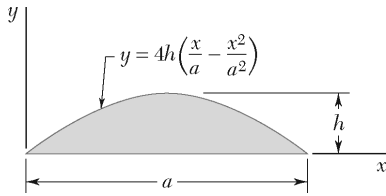
$$dI_y = x^2 dA = x^2 y dx$$

$$dI_y = x^2 \frac{b}{a^{1/3}} x^{1/3} dx = \frac{b}{a^{1/3}} x^{7/3} dx$$

$$I_y = \int dI_y = \frac{b}{a^{1/3}} \int_0^a x^{7/3} dx = \frac{b}{a^{1/3}} \left(\frac{3}{10} a^{10/3} \right)$$

$$I_y = \frac{3}{10} a^3 b \quad \blacktriangleleft$$

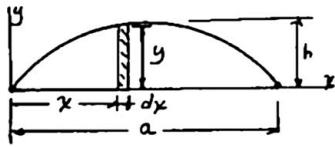




PROBLEM 9.3

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



$$y = 4h \left(\frac{x}{a} - \frac{x^2}{a^2} \right)$$

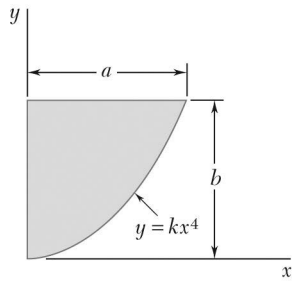
$$dA = y dx$$

$$dI_y = x^2 dA = 4hx^2 \left(\frac{x}{a} - \frac{x^2}{a^2} \right) dx$$

$$I_y = 4h \int_0^a \left(\frac{x^3}{a} - \frac{x^4}{a^2} \right) dx$$

$$I_y = 4h \left[\frac{x^4}{4a} - \frac{x^5}{5a^2} \right]_0^a = 4h \left(\frac{a^3}{4} - \frac{a^3}{5} \right)$$

$$I_y = \frac{1}{5} ha^3 \blacktriangleleft$$



PROBLEM 9.4

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

For $x = a$:

$$y = kx^4$$

$$b = ka^4$$

$$k = \frac{b}{a^4}$$

Thus:

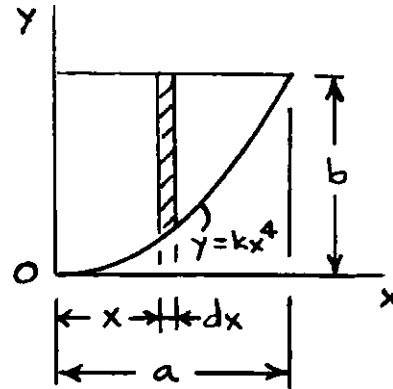
$$y = \frac{b}{a^4}x^4$$

$$dA = (b - y)dx$$

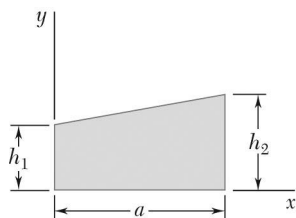
$$dI_y = x^2 dA = x^2(b - y)dx$$

$$= x^2 \left(b - \frac{b}{a^4}x^4 \right) dx$$

$$I_y = \int dI_y = \int_0^a \left(bx^2 - \frac{b}{a^4}x^6 \right) dx = \frac{1}{3}a^3b - \frac{1}{7}a^3b \quad I_y = 4a^3b/21 \quad \blacktriangleleft$$

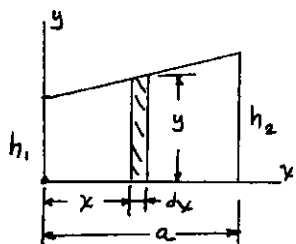


PROBLEM 9.5



Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



$$y = h_1 + (h_2 - h_1)\frac{x}{a} \quad dI_x = \frac{1}{3}y^3 dx$$

$$I_x = \int dI_y = \frac{1}{3} \int_0^a \left[h_1 + (h_2 - h_1)\frac{x}{a} \right]^3 dx$$

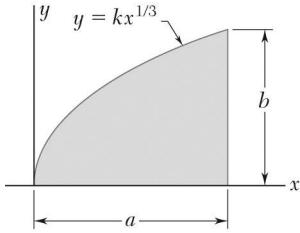
$$= \frac{1}{12} \left[h_1 + (h_2 - h_1)\frac{x}{a} \right]^4 \left(\frac{a}{h_2 - h_1} \right) \Big|_0^a$$

$$= \frac{a}{12(h_2 - h_1)} (h_2^4 - h_1^4) = \frac{a}{12} \cdot \frac{(h_2^2 + h_1^2)(h_2 + h_1)(h_2 - h_1)}{h_2 - h_1}$$

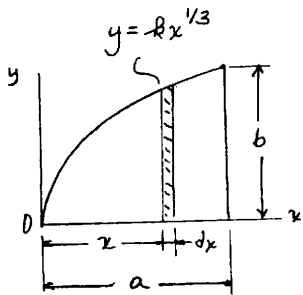
$$I_x = \frac{a}{12} (h_1^2 + h_2^2)(h_1 + h_2) \quad \blacktriangleleft$$

PROBLEM 9.6

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.



SOLUTION

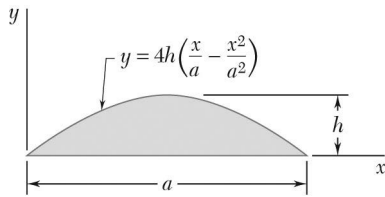


$$y = \frac{b}{a^{1/3}} x^{1/3}$$

$$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3} \left(\frac{b}{a^{1/3}} x^{1/3} \right)^3 dx = \frac{1}{3} \frac{b^3}{a} x dx$$

$$I_x = \int dI_x = \int_0^a \frac{1}{3} \frac{b^3}{a} x dx = \frac{1}{3} \frac{b^3}{a} \frac{a^2}{2}$$

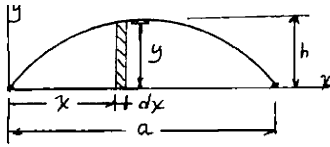
$$I_x = \frac{1}{6} ab^3 \blacktriangleleft$$



PROBLEM 9.7

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

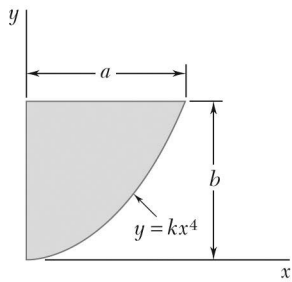


$$y = 4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

$$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3} \left[4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right) \right]^3 dx$$

$$\begin{aligned} I_x &= \int dI_x = \frac{64h^3}{3} \int_0^a \left(\frac{x^3}{a^3} - 3\frac{x^4}{a^4} + 3\frac{x^5}{a^5} - \frac{x^6}{a^6} \right) dx \\ &= \frac{64h^3}{3} \left[\frac{1}{4} \frac{x^4}{a^3} - \frac{3}{5} \frac{x^5}{a^4} + \frac{1}{2} \frac{x^6}{a^5} - \frac{1}{7} \frac{x^7}{a^6} \right]_0^a \\ &= \frac{64h^3}{3} a \left(\frac{1}{4} - \frac{3}{5} + \frac{1}{2} - \frac{1}{7} \right) = \frac{64h^3}{3} a \left(\frac{3}{420} \right) \end{aligned}$$

$$I_x = \frac{16}{105} ah^3 \blacktriangleleft$$

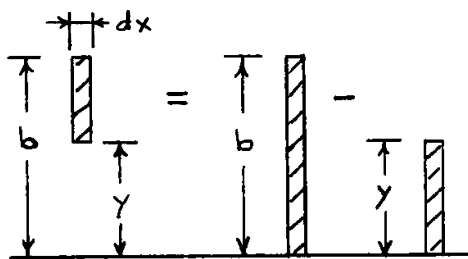


PROBLEM 9.8

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

See figure of solution of Problem 9.4.



$$y = \frac{b}{a^4} x^4$$

$$dI_x = \frac{1}{3} b^3 dx - \frac{1}{3} y^3 dx = \frac{1}{3} b^3 dx - \frac{1}{3} \frac{b^3}{a^{12}} x^{12} dx$$

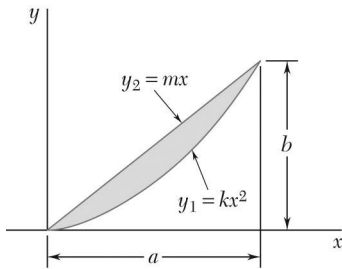
$$I_x = \int dI_x = \int_0^a \left(\frac{1}{3} b^3 - \frac{1}{3} \frac{b^3}{a^{12}} x^{12} \right) dx$$

$$= \frac{1}{3} b^3 a - \frac{1}{3} \frac{b^3}{a^{12}} \frac{a^{13}}{13}$$

$$= \left(\frac{1}{3} - \frac{1}{39} \right) ab^3$$

$$= \left(\frac{13}{39} - \frac{1}{39} \right) ab^3 = \frac{12}{39} ab^3$$

$$I_x = 4ab^3/13 \blacktriangleleft$$



PROBLEM 9.9

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

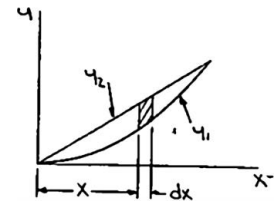
SOLUTION

At

$$x_1 = a, \quad y_1 = y_2 = b$$

$$y_1: \quad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$



Then

$$y_1 = \frac{b}{a^2} x^2$$

$$y_2 = \frac{b}{a} x$$

Now

$$\begin{aligned} dI_x &= \frac{1}{3} (y_2^3 - y_1^3) dx \\ &= \frac{1}{3} \left(\frac{b^3}{a^3} x^3 - \frac{b^3}{a^6} x^6 \right) dx \end{aligned}$$

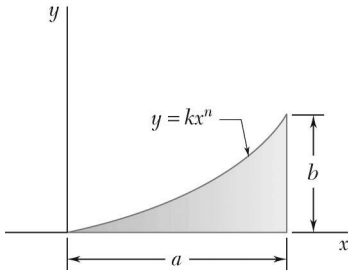
Then

$$\begin{aligned} I_x &= \int dI_x = \int_0^a \frac{b^3}{3} \left(\frac{1}{a^3} x^3 - \frac{1}{a^6} x^6 \right) dx \\ &= \frac{b^3}{3} \left[\frac{1}{4a^3} x^4 - \frac{1}{7a^6} x^7 \right]_0^a \end{aligned}$$

$$\text{or} \quad I_x = \frac{1}{28} ab^3 \quad \blacktriangleleft$$

PROBLEM 9.10

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.



SOLUTION

For $x = a$:

$$y = kx^n$$

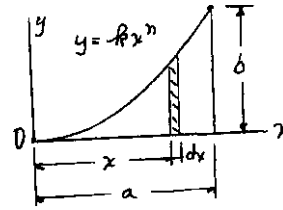
$$b = ka^n$$

$$k = b/a^n$$

Thus:

$$y = \frac{b}{a^n} x^n$$

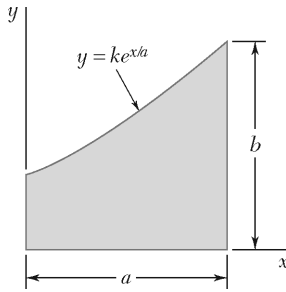
$$y = \frac{b}{a^n} x^n$$



$$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3} \frac{b^3}{a^{3n}} x^{3n} dx$$

$$I_x = \int dI_x = \frac{b^3}{3a^{3n}} \int_0^a x^{3n} dx = \frac{b^3}{3a^{3n}} \frac{a^{3n+1}}{(3n+1)}$$

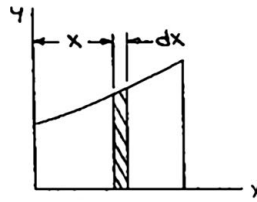
$$I_x = \frac{ab^3}{3(3n+1)} \blacktriangleleft$$



PROBLEM 9.11

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $x = a$, $y = b$:

$$b = ke^{a/a} \quad \text{or} \quad k = \frac{b}{e}$$

Then

$$y = \frac{b}{e} e^{x/a} = be^{x/a-1}$$

Now

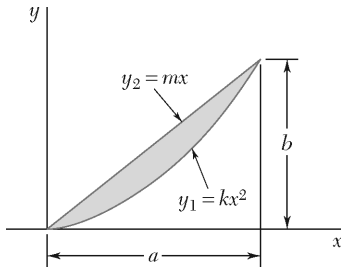
$$\begin{aligned} dI_x &= \frac{1}{3} y^3 dx = \frac{1}{3} (be^{x/a-1})^3 dx \\ &= \frac{1}{3} b^3 e^{3(x/a-1)} dx \end{aligned}$$

Then

$$\begin{aligned} I_x &= \int dI_x = \int_0^a \frac{1}{3} b^3 e^{3(x/a-1)} dx = \frac{b^3}{3} \left[\frac{a}{3} e^{3(x/a-1)} \right]_0^a \\ &= \frac{1}{9} ab^3 (1 - e^{-3}) \quad \text{or} \quad I_x = 0.1056ab^3 \quad \blacktriangleleft \end{aligned}$$

PROBLEM 9.12

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.



SOLUTION

At $x_1 = a, \quad y_1 = y_2 = b$

$$y_1: \quad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$

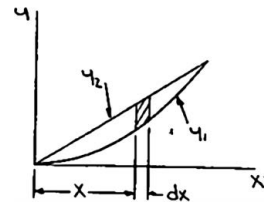
Then $y_1 = \frac{b}{a^2}x^2$

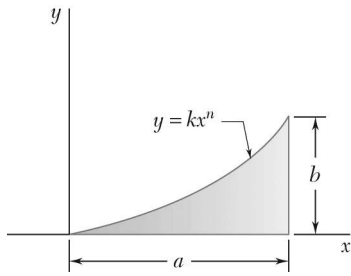
$$y_2 = \frac{b}{a}x$$

Now $dI_y = x^2 dA = x^2[(y_2 - y_1)dx] = x^2 \left[\left(\frac{b}{a}x - \frac{b}{a^2}x^2 \right) dx \right]$

Then
$$I_y = \int dI_y = \int_0^a b \left(\frac{1}{a}x^3 - \frac{1}{a^2}x^4 \right) dx$$
$$= b \left[\frac{1}{4a}x^4 - \frac{1}{5a^2}x^5 \right]_0^a$$

$$\text{or} \quad I_y = \frac{1}{20}a^3b \quad \blacktriangleleft$$





PROBLEM 9.13

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

For $x = a$:

$$y = kx^n$$

$$b = ka^n$$

$$k = b/a^n$$

Thus:

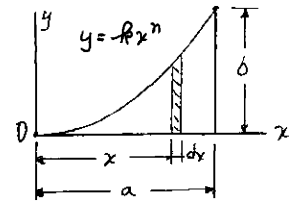
$$y = \frac{b}{a^n} x^n$$

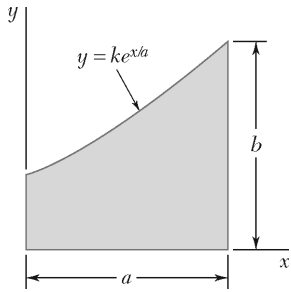
$$dI_y = x^2 dA = x^2 y dx$$

$$dI_y = x^2 \frac{b}{a^n} x^n dx = \frac{b}{a^n} x^{n+2} dx$$

$$I_y = \int dI_y = \frac{b}{a^n} \int_0^a x^{n+2} dx = \frac{b}{a^n} \frac{a^{n+3}}{n+3}$$

$$I_y = \frac{a^3 b}{n+3} \blacktriangleleft$$





PROBLEM 9.14

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

At $x = a$, $y = b$:

$$b = ke^{a/a}$$

or

$$k = \frac{b}{e}$$

Then

$$y = \frac{b}{e}e^{x/a} = be^{x/a-1}$$

Now

$$\begin{aligned} dI_y &= x^2 dA = x^2 (y dx) \\ &= x^2 (be^{x/a-1} dx) \end{aligned}$$

Then

$$I_y = \int dI_y = \int_0^a bx^2 e^{x/a-1} dx$$

Now use integration by parts with

$$\begin{aligned} u &= x^2 & dv &= e^{x/a-1} dx \\ du &= 2x dx & v &= ae^{x/a-1} \end{aligned}$$

Then

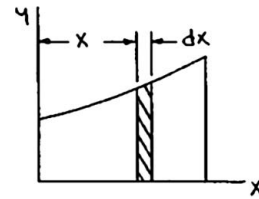
$$\begin{aligned} \int_0^a x^2 e^{x/a-1} dx &= \left[x^2 ae^{x/a-1} \right]_0^a - \int_0^a (ae^{x/a-1}) 2x dx \\ &= a^3 - 2a \int_0^a xe^{x/a-1} dx \end{aligned}$$

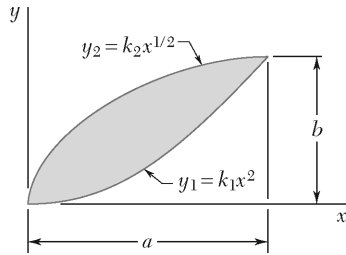
Using integration by parts with

$$\begin{aligned} u &= x & dv &= e^{x/a-1} dx \\ du &= dx & v &= ae^{x/a-1} \end{aligned}$$

Then

$$\begin{aligned} I_y &= b \left\{ a^3 - 2a \left[(xae^{x/a-1}) \Big|_0^a - \int_0^a (ae^{x/a-1}) dx \right] \right\} \\ &= b \left\{ a^3 - 2a \left[a^2 - (a^2 e^{x/a-1}) \Big|_0^a \right] \right\} \\ &= b \left\{ a^3 - 2a \left[a^2 - (a^2 - a^2 e^{-1}) \right] \right\} \quad \text{or } I_y = 0.264a^3b \quad \blacktriangleleft \end{aligned}$$





PROBLEM 9.15

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION

For

$$\begin{aligned}
 y_1 &= k_1 x^2 & y_2 &= k_2 x^{1/2} \\
 x=0 & \text{ and } & y_1 &= y_2 = b \\
 b &= k_1 a^2 & b &= k_2 a^{1/2} \\
 k_1 &= \frac{b}{a^2} & k_2 &= \frac{b}{a^{1/2}}
 \end{aligned}$$

Thus,

$$y_1 = \frac{b}{a^2} x^2 \quad y_2 = \frac{b}{a^{1/2}} x^{1/2}$$

$$dA = (y_2 - y_1) dx$$

$$A = \int_0^a \left[\frac{b}{a^{1/2}} x^{1/2} - \frac{b}{a^2} x^2 \right] dx$$

$$A = \frac{2ba^{3/2}}{3} - \frac{ba^3}{3a^2}$$

$$A = \frac{1}{3} ab$$

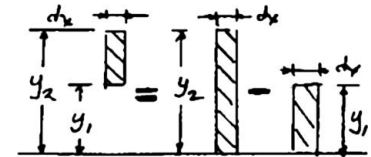
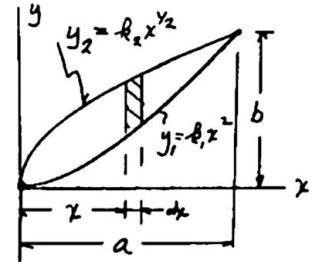
$$\begin{aligned}
 dI_x &= \frac{1}{3} y_2^3 dx - \frac{1}{3} y_1^3 dx \\
 &= \frac{1}{3} \frac{b^3}{a^{3/2}} x^{3/2} dx - \frac{1}{3} \frac{b^3}{a^6} x^6 dx
 \end{aligned}$$

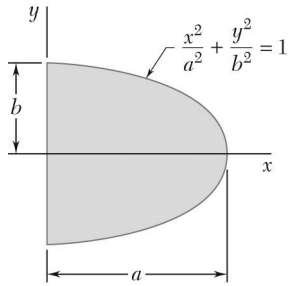
$$I_x = \int dI_x = \frac{b^3}{3a^{3/2}} \int_0^a x^{3/2} dx - \frac{b^3}{3a^6} \int_0^a x^6 dx$$

$$= \frac{b^3}{3a^{3/2}} \frac{a^{5/2}}{\left(\frac{5}{2}\right)} - \frac{b^3}{3a^6} \frac{a^7}{7} = \left(\frac{2}{15} - \frac{1}{21} \right) ab^3 \quad I_x = \frac{3}{35} ab^3 \quad \blacktriangleleft$$

$$k_x^2 = \frac{I_x}{A} = \frac{\left(\frac{3}{35} ab^3\right)}{\frac{ab}{b}}$$

$$k_x = b \sqrt{\frac{9}{35}} \quad \blacktriangleleft$$





PROBLEM 9.16

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION

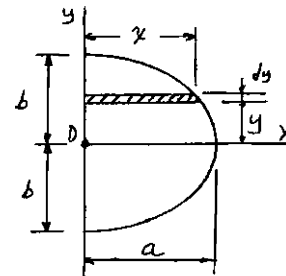
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x = a\sqrt{1 - \frac{y^2}{b^2}}$$

$$dA = xdy$$

$$dI_x = y^2 dA = y^2 x dy$$

$$I_x = \int dI_x = \int_{-b}^b xy^2 dy = a \int_{-b}^b y^2 \sqrt{1 - \frac{y^2}{b^2}} dy$$



Set:

$$y = b \sin \theta \quad dy = b \cos \theta d\theta$$

$$I_x = a \int_{-\pi/2}^{\pi/2} b^2 \sin^2 \theta \sqrt{1 - \sin^2 \theta} b \cos \theta d\theta$$

$$= ab^3 \int_{-\pi/2}^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta = ab^3 \int_{-\pi/2}^{\pi/2} \frac{1}{4} \sin^2 2\theta d\theta$$

$$= \frac{1}{4} ab^3 \int_{-\pi/2}^{\pi/2} \frac{1}{2} (1 - \cos 4\theta) d\theta = \frac{1}{8} ab^3 \left[\theta - \frac{1}{4} \sin 4\theta \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{8} ab^3 \left[\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right] = \frac{\pi}{8} ab^2$$

$$I_x = \frac{1}{8} \pi ab^3 \blacktriangleleft$$

$$A = \int dA = \int_{-b}^b x dy = a \int_{-b}^b \sqrt{1 - \frac{y^2}{b^2}} dy = a \int_{-\pi/2}^{\pi/2} \sqrt{1 - \sin^2 \theta} b \cos \theta d\theta$$

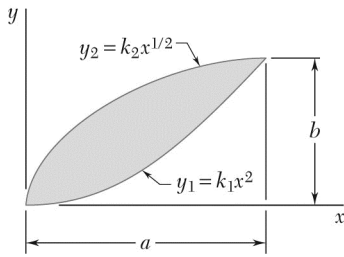
$$= ab \int_{-\pi/2}^{\pi/2} \cos^2 \theta d\theta = ab \int_{-\pi/2}^{\pi/2} \frac{1}{2} (1 + \cos 2\theta) d\theta$$

$$= ab \left[\theta + \frac{1}{2} \sin 2\theta \right]_{-\pi/2}^{\pi/2} = \frac{ab}{2} \left[\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right] = \frac{1}{2} \pi ab$$

$$I_x = k_x^2 A \quad k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{8} \pi ab^3}{\frac{1}{2} \pi ab} = \frac{1}{4} b^2$$

$$k_x = \frac{1}{2} b \blacktriangleleft$$

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PROBLEM 9.17

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION

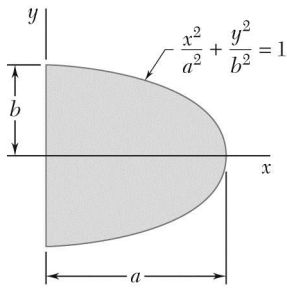
See figure of solution on Problem 9.15.

$$A = \frac{1}{3}ab \quad dI_y = x^2 dA = x^2(y_2 - y_1)dx$$

$$I_y = \int_0^a x^2 \left(\frac{b}{a^{1/2}} x^{1/2} - \frac{b}{a^2} x^2 \right) dx = \frac{b}{a^{1/2}} \int_0^a x^{5/2} dx - \frac{b}{a^2} \int_0^a x^4 dx$$

$$I_y = \frac{b}{a^{1/2}} \cdot \frac{b^{7/2}}{\left(\frac{7}{2}\right)} - \frac{b}{a^2} \cdot \frac{a^5}{5} = \left(\frac{2}{7} - \frac{1}{5}\right) a^3 b \quad I_y = \frac{3}{35} a^3 b \quad \blacktriangleleft$$

$$k_y^2 = \frac{I_y}{A} = \frac{\left(\frac{3}{35} a^3 b\right)}{\frac{ab}{3}} \quad k_y = a \sqrt{\frac{9}{35}} \quad \blacktriangleleft$$



PROBLEM 9.18

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION

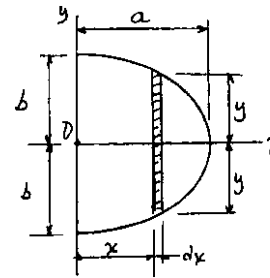
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$$dA = 2ydx$$

$$dI_y = x^2 dA = 2x^2 y dx$$

$$I_y = \int dI_y = \int_0^a 2x^2 y dx = 2b \int_0^a x^2 \sqrt{1 - \frac{x^2}{a^2}} dx$$



Set:

$$x = a \sin \theta \quad dx = a \cos \theta d\theta$$

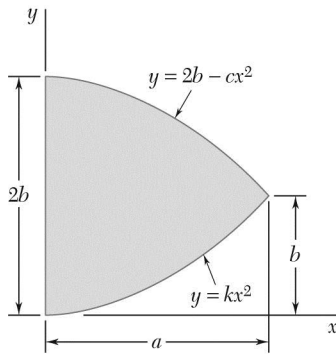
$$\begin{aligned} I_y &= 2b \int_0^{\pi/2} a^2 \sin^2 \theta \sqrt{1 - \sin^2 \theta} \cdot a \cos \theta d\theta \\ &= 2a^3 b \int_0^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta = 2a^3 b \int_0^{\pi/2} \frac{1}{4} \sin^2 2\theta d\theta \\ &= \frac{1}{2} a^3 b \int_0^{\pi/2} \frac{1}{2} (1 - \cos 4\theta) d\theta = \frac{1}{4} a^3 b \left[\theta - \frac{1}{4} \sin 4\theta \right]_0^{\pi/2} \\ &= \frac{1}{4} a^3 b \left[\frac{\pi}{2} - 0 \right] = \frac{\pi}{8} a^3 b \end{aligned}$$

$$I_y = \frac{1}{8} \pi a^3 b \quad \blacktriangleleft$$

From solution of Problem 9.16: $A = \frac{1}{2} \pi ab$

Thus:

$$I_y = k_y^2 A \quad k_y^2 = \frac{I_y}{A} = \frac{\frac{1}{8} \pi a^3 b}{\frac{1}{2} \pi ab} = \frac{1}{4} a^2 \quad k_y = \frac{1}{2} a \quad \blacktriangleleft$$



PROBLEM 9.19

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION

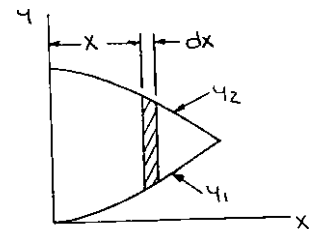
At $x = a$, $y_1 = y_2 = b$:

$$y_1: b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: b = 2b - ca^2 \quad \text{or} \quad c = \frac{b}{a^2}$$

Then

$$y_1 = \frac{b}{a^2}x^2 \quad y_2 = b\left(2 - \frac{x^2}{a^2}\right)$$



Now

$$dA = (y_2 - y_1)dx = \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2 \right] dx = \frac{2b}{a^2}(a^2 - x^2)dx$$

Then

$$A = \int dA = \int_0^a \frac{2b}{a^2}(a^2 - x^2)dx = \frac{2b}{a^2} \left[a^2x - \frac{1}{3}x^3 \right]_0^a = \frac{4}{3}ab$$

Now

$$\begin{aligned} dI_x &= \left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3 \right) dx = \frac{1}{3} \left\{ \left[b\left(2 - \frac{x^2}{a^2}\right) \right]^3 - \left[\frac{b}{a^2}x^2 \right]^3 \right\} dx \\ &= \frac{1}{3} \frac{b^3}{a^6} (8a^6 - 12a^4x^2 + 6a^2x^4 - x^6 - x^6) dx \\ &= \frac{2}{3} \frac{b^3}{a^6} (4a^6 - 6a^4x^2 + 3a^2x^4 - x^6) dx \end{aligned}$$

Then

$$\begin{aligned} I_x &= \int dI_x = \int_0^a \frac{2}{3} \frac{b^3}{a^6} (4a^6 - 6a^4x^2 + 3a^2x^4 - x^6) dx \\ &= \frac{2}{3} \frac{b^3}{a^6} \left[4a^6x - 2a^4x^3 + \frac{3}{5}a^6x^5 - \frac{1}{7}x^7 \right]_0^a \\ &= \frac{172}{105} ab^3 \end{aligned}$$

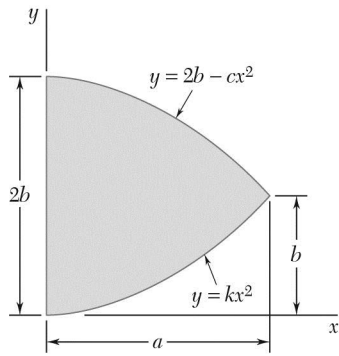
$$\text{or } I_x = 1.638ab^3 \quad \blacktriangleleft$$

and

$$k_x^2 = \frac{I_x}{A} = \frac{\frac{172}{105} ab^3}{\frac{4}{3} ab} = \frac{43}{35} b^2$$

$$\text{or } k_x = 1.108b \quad \blacktriangleleft$$

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PROBLEM 9.20

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION

At $x = a$, $y_1 = y_2 = b$:

$$y_1: b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: b = 2b - ca^2 \quad \text{or} \quad c = \frac{b}{a^2}$$

Then

$$y_1 = \frac{b}{a^2}x^2$$

$$y_2 = b\left(2 - \frac{x^2}{a^2}\right)$$

Now

$$dA = (y_2 - y_1)dx$$

$$= \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2 \right] dx$$

$$= \frac{2b}{a^2}(a^2 - x^2) dx$$

Then

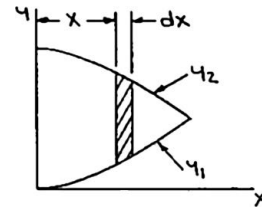
$$A = \int dA = \int_0^a \frac{2b}{a^2}(a^2 - x^2) dx$$

$$= \frac{2b}{a^2} \left[a^2x - \frac{1}{3}x^3 \right]_0^a$$

$$= \frac{4}{3}ab$$

Now

$$dI_y = x^2 dA = x^2 \left[\frac{2b}{a^2}(a^2 - x^2) dx \right]$$



PROBLEM 9.20 (Continued)

Then

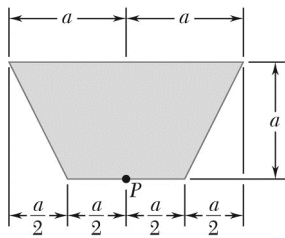
$$\begin{aligned} I_y &= \int dI_y = \int_0^a \frac{2b}{a^2} x^2 (a^2 - x^2) dx \\ &= \frac{2b}{a^2} \left[\frac{1}{3} a^2 x^3 - \frac{1}{5} x^5 \right]_0^a \end{aligned}$$

or $I_y = \frac{4}{15} a^3 b \blacktriangleleft$

and

$$k_y^2 = \frac{I_y}{A} = \frac{\frac{4}{15} a^3 b}{\frac{4}{3} ab} = \frac{1}{5} a^2$$

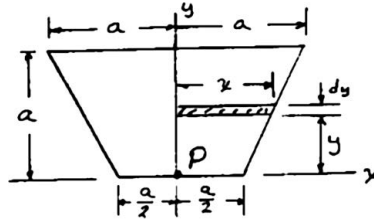
or $k_y = \frac{a}{\sqrt{5}} \blacktriangleleft$



PROBLEM 9.21

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point P .

SOLUTION



$$x = \frac{a}{2} + \frac{a}{2} \frac{y}{a} = \frac{1}{2}(a + y)$$

$$dA = x dy = \frac{1}{2}(a + y) dy$$

$$\frac{1}{2}A = \int_0^a dA = \int_0^a \frac{1}{2}(a + y) dy = \frac{1}{2} \left[ay + \frac{y^2}{2} \right]_0^a = \frac{1}{2} \left(a^2 + \frac{a^2}{2} \right) = \frac{3}{4}a^2 \quad A = \frac{3}{2}a^2 \triangleleft$$

$$\begin{aligned} \frac{1}{2}I_x &= \int y^2 dA = \int_0^a y^2 \frac{1}{2}(a + y) dy = \frac{1}{2} \int_0^a (ay^2 + y^3) dy \\ &= \frac{1}{2} \left[a \frac{y^3}{3} + \frac{y^4}{4} \right]_0^a = \frac{1}{2} \left(\frac{1}{3} + \frac{1}{4} \right) a^4 = \frac{1}{2} \frac{7}{12} a^4 \quad I_x = \frac{7}{12} a^4 \triangleleft \end{aligned}$$

$$\frac{1}{2}I_y = \int_0^a \frac{1}{3} x^3 dy = \frac{1}{3} \int_0^a \left(\frac{1}{2}(a + y) \right)^3 dy$$

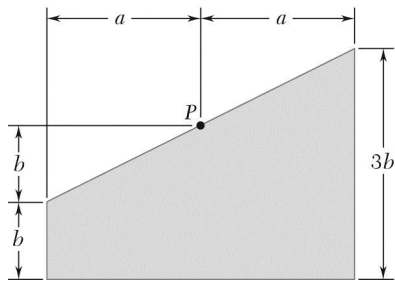
$$\frac{1}{2}I_y = \frac{1}{24} \int_0^a (a + y)^3 dy = \frac{1}{24} \left[\frac{1}{4}(a + y)^4 \right]_0^a = \frac{1}{96} [(2a)^4 - a^4] = \frac{15}{96} a^4$$

$$\frac{1}{2}I_y = \frac{5}{32} a^4 \quad I_y = \frac{5}{16} a^4 \triangleleft$$

From Eq. (9.4): $J_O = I_x + I_y = \frac{7}{12} a^4 + \frac{5}{16} a^4 = \left(\frac{28 + 15}{48} \right) a^4 \quad J_O = \frac{43}{48} a^4 \blacktriangleleft$

$$J_O = k_O^2 A \quad k_O^2 = \frac{J_O}{A} = \frac{\frac{43}{48} a^4}{\frac{3}{2} a^2} = \frac{43}{72} a^2 \quad k_O = a \sqrt{\frac{43}{72}} \quad k_O = 0.773a \blacktriangleleft$$

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PROBLEM 9.22

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point P .

SOLUTION

By observation $y = \frac{b}{a}x$

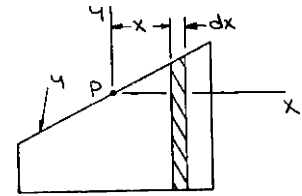
First note $dA = (y + 2b)dx$
 $= \frac{b}{a}(x + 2a)dx$

Now $dI_x = \left(\frac{1}{3}y_{\text{top}}^3 - \frac{1}{3}y_{\text{bottom}}^3 \right) dx$
 $= \frac{1}{3} \left[\left(\frac{b}{a}x \right)^3 - (-2b)^3 \right] dx$
 $= \frac{1}{3} \frac{b^3}{a^3} (x^3 + 8a^3) dx$

Then $I_x = \int dI_x = \int_{-a}^a \frac{1}{3} \frac{b^3}{a^3} (x^3 + 8a^3) dx$
 $= \frac{1}{3} \frac{b^3}{a^3} \left[\frac{1}{4}x^4 + 8a^3x \right]_{-a}^a$
 $= \frac{1}{3} \frac{b^3}{a^3} \left\{ \left[\frac{1}{4}(a)^4 + 8a^3(a) \right] - \left[\frac{1}{4}(-a)^4 + 8a^3(-a) \right] \right\} = \frac{16}{3} ab^3$

Also $dI_y = x^2 dA = x^2 \left[\frac{b}{a}(x + 2a) dx \right]$

Then $I_y = \int dI_y = \int_{-a}^a \frac{b}{a} x^2 (x + 2a) dx$
 $= \frac{b}{a} \left[\frac{1}{4}x^4 + \frac{2}{3}ax^3 \right]_{-a}^a$
 $= \frac{b}{a} \left\{ \left[\frac{1}{4}(a)^4 + \frac{2}{3}a(a)^3 \right] - \left[\frac{1}{4}(-a)^4 + \frac{2}{3}a(-a)^3 \right] \right\} = \frac{4}{3} a^3 b$



PROBLEM 9.22 (Continued)

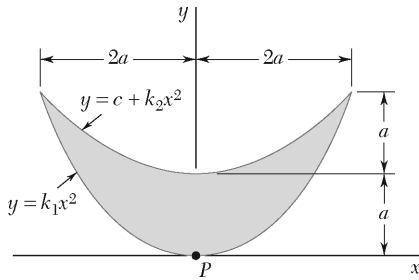
Now
$$J_P = I_x + I_y = \frac{16}{3}ab^3 + \frac{4}{3}a^3b$$

$$J_P = \frac{4}{3}ab(a^2 + 4b^2) \blacktriangleleft$$

and
$$k_P^2 = \frac{J_P}{A} = \frac{\frac{4}{3}ab(a^2 + 4b^2)}{(2a)(3b) - \frac{1}{2}(2a)(2b)}$$

$$= \frac{1}{3}(a^2 + 4b^2)$$

or
$$k_P = \sqrt{\frac{a^2 + 4b^2}{3}} \blacktriangleleft$$



PROBLEM 9.23

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point P .

SOLUTION

y_1 : At $x = 2a$, $y = 2a$:

$$2a = k_1(2a)^2 \quad \text{or} \quad k_1 = \frac{1}{2a}$$

y_2 : At $x = 0$, $y = a$:

$$a = c$$

At $x = 2a$, $y = 2a$:

$$2a = a + k_2(2a)^2 \quad \text{or} \quad k_2 = \frac{1}{4a}$$

Then

$$\begin{aligned} y_1 &= \frac{1}{2a}x^2 & y_2 &= a + \frac{1}{4a}x^2 \\ & & &= \frac{1}{4a}(4a^2 + x^2) \end{aligned}$$

Now

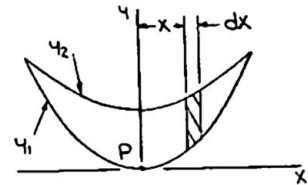
$$\begin{aligned} dA &= (y_2 - y_1)dx = \left[\frac{1}{4a}(4a^2 + x^2) - \frac{1}{2a}x^2 \right] dx \\ &= \frac{1}{4a}(4a^2 - x^2)dx \end{aligned}$$

Then

$$A = \int dA = 2 \int_0^{2a} \frac{1}{4a}(4a^2 - x^2)dx = \frac{1}{2a} \left[4a^2x - \frac{1}{3}x^3 \right]_0^{2a} = \frac{8}{3}a^2$$

Now

$$\begin{aligned} dI_x &= \left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3 \right) dx = \frac{1}{3} \left\{ \left[\frac{1}{4a}(4a^2 + x^2) \right]^3 - \left[\frac{1}{2a}x^2 \right]^3 \right\} dx \\ &= \frac{1}{3} \left[\frac{1}{64a^3} (64a^6 + 48a^4x^2 + 12a^2x^4 + x^6) - \frac{1}{8a^3}x^6 \right] dx \\ &= \frac{1}{192a^3} (64a^6 + 48a^4x^2 + 12a^2x^4 - 7x^6) dx \end{aligned}$$



PROBLEM 9.23 (Continued)

Then

$$\begin{aligned}
 I_x &= \int dI_x = 2 \int_0^{2a} \frac{1}{192a^3} (64a^6 + 48a^4x^2 + 12a^2x^4 - 7x^6) dx \\
 &= \frac{1}{96a^3} \left[64a^6x + 16a^4x^3 + \frac{12}{5}a^2x^5 - x^7 \right]_0^{2a} \\
 &= \frac{1}{96a^3} \left[64a^6(2a) + 16a^4(2a)^3 + \frac{12}{5}a^2(2a)^5 - (2a)^7 \right] \\
 &= \frac{1}{96}a^4 \left(128 + 128 + \frac{12}{5} \times 32 - 128 \right) = \frac{32}{15}a^4
 \end{aligned}$$

Also

$$dI_y = x^2 dA = x^2 \left[\frac{1}{4a} (4a^2 - x^2) dx \right]$$

Then

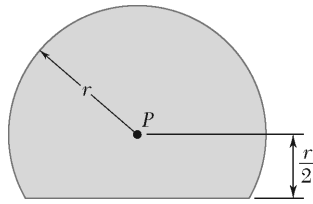
$$\begin{aligned}
 I_y &= \int dI_y = 2 \int_0^{2a} \frac{1}{4a} x^2 (4a^2 - x^2) dx = \frac{1}{2a} \left[\frac{4}{3}a^2x^3 - \frac{1}{5}x^5 \right]_0^{2a} \\
 &= \frac{1}{2a} \left[\frac{4}{3}a^2(2a)^3 - \frac{1}{5}(2a)^5 \right] = \frac{32}{2}a^4 \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{32}{15}a^4
 \end{aligned}$$

Now

$$J_P = I_x + I_y = \frac{32}{15}a^4 + \frac{32}{15}a^4 \quad \text{or} \quad J_P = \frac{64}{15}a^4 \quad \blacktriangleleft$$

and

$$k_P^2 = \frac{J_P}{A} = \frac{\frac{64}{15}a^4}{\frac{8}{3}a^2} = \frac{8}{5}a^2 \quad \text{or} \quad k_P = 1.265a \quad \blacktriangleleft$$



PROBLEM 9.24

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point P.

SOLUTION

The equation of the circle is

$$x^2 + y^2 = r^2$$

So that

$$x = \sqrt{r^2 - y^2}$$

Now

$$dA = xdy = \sqrt{r^2 - y^2} dy$$

Then

$$A = \int dA = 2 \int_{-r/2}^r \sqrt{r^2 - y^2} dy$$

Let

$$y = r \sin \theta; \quad dy = r \cos \theta d\theta$$

Then

$$\begin{aligned} A &= 2 \int_{-\pi/6}^{\pi/2} \sqrt{r^2 - (r \sin \theta)^2} r \cos \theta d\theta \\ &= 2 \int_{-\pi/6}^{\pi/2} r^2 \cos^2 \theta d\theta = 2r^2 \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_{-\pi/6}^{\pi/2} \\ &= 2r^2 \left[\frac{\pi}{2} - \left(\frac{-\pi}{6} + \frac{\sin -\frac{\pi}{3}}{4} \right) \right] = 2r^2 \left(\frac{\pi}{3} + \frac{\sqrt{3}}{8} \right) \\ &= 2.5274r^2 \end{aligned}$$

Now

$$dI_x = y^2 dA = y^2 (\sqrt{r^2 - y^2} dy)$$

Then

$$I_x = \int dI_x = 2 \int_{-r/2}^r y^2 \sqrt{r^2 - y^2} dy$$

Let

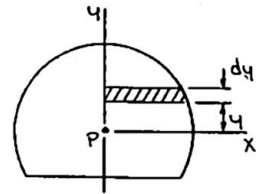
$$y = r \sin \theta; \quad dy = r \cos \theta d\theta$$

Then

$$\begin{aligned} I_x &= 2 \int_{-\pi/6}^{\pi/2} (r \sin \theta)^2 \sqrt{r^2 - (r \sin \theta)^2} r \cos \theta d\theta \\ &= 2 \int_{-\pi/6}^{\pi/2} r^2 \sin^2 \theta (r \cos \theta) r \cos \theta d\theta \end{aligned}$$

Now

$$\sin 2\theta = 2 \sin \theta \cos \theta \Rightarrow \sin^2 \theta \cos^2 \theta = \frac{1}{4} \sin 2\theta$$



PROBLEM 9.24 (Continued)

Then

$$\begin{aligned}
 I_x &= 2 \int_{-\pi/6}^{\pi/2} r^4 \left(\frac{1}{4} \sin^2 2\theta \right) d\theta = \frac{r^4}{2} \left[\frac{\theta}{2} - \frac{\sin 4\theta}{8} \right]_{-\pi/6}^{\pi/2} \\
 &= \frac{r^4}{2} \left[\frac{\pi}{2} - \left(\frac{\pi}{6} - \frac{\sin -\frac{2\pi}{3}}{8} \right) \right] \\
 &= \frac{r^4}{2} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{16} \right)
 \end{aligned}$$

Also

$$dI_y = \frac{1}{3} x^3 dy = \frac{1}{3} (\sqrt{r^2 - y^2})^3 dy$$

Then

$$I_y = \int dI_y = 2 \int_{-r/2}^r \frac{1}{3} (r^2 - y^2)^{3/2} dy$$

Let

$$y = r \sin \theta; \quad dy = r \cos \theta d\theta$$

Then

$$I_y = \frac{2}{3} \int_{-\pi/6}^{\pi/2} [r^2 - (r \sin \theta)^2]^{3/2} r \cos \theta d\theta$$

$$I_y = \frac{2}{3} \int_{-\pi/6}^{\pi/2} (r^3 \cos^3 \theta) r \cos \theta d\theta$$

Now

$$\cos^4 \theta = \cos^2 \theta (1 - \sin^2 \theta) = \cos^2 \theta - \frac{1}{4} \sin^2 2\theta$$

Then

$$\begin{aligned}
 I_y &= \frac{2}{3} \int_{-\pi/6}^{\pi/2} r^4 \left(\cos^2 \theta - \frac{1}{4} \sin^2 2\theta \right) d\theta \\
 &= \frac{2}{3} r^4 \left[\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right) - \frac{1}{4} \left(\frac{\theta}{2} - \frac{\sin 4\theta}{8} \right) \right]_{-\pi/6}^{\pi/2} \\
 &= \frac{2}{3} r^4 \left\{ \left[\frac{\pi}{2} - \frac{1}{4} \left(\frac{\pi}{2} \right) \right] - \left[\frac{-\pi}{6} + \frac{\sin -\frac{\pi}{3}}{4} - \frac{1}{4} \left(\frac{-\pi}{6} - \frac{\sin -\frac{2\pi}{3}}{8} \right) \right] \right\} \\
 &= \frac{2}{3} r^4 \left[\frac{\pi}{4} - \frac{\pi}{16} + \frac{\pi}{12} + \frac{1}{4} \left(\frac{\sqrt{3}}{2} \right) - \frac{\pi}{48} + \frac{1}{32} \left(\frac{\sqrt{3}}{2} \right) \right] \\
 &= \frac{2}{3} r^4 \left(\frac{\pi}{4} + \frac{9\sqrt{3}}{64} \right)
 \end{aligned}$$

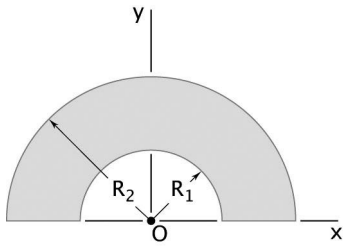
PROBLEM 9.24 (Continued)

Now

$$J_P = I_x + I_y = \frac{r^4}{2} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{16} \right) + \frac{2}{3} r^4 \left(\frac{\pi}{4} + \frac{9\sqrt{3}}{64} \right)$$
$$= r^4 \left(\frac{\pi}{3} + \frac{\sqrt{3}}{16} \right) = 1.15545r^4 \quad \text{or } J_P = 1.155r^4 \blacktriangleleft$$

and

$$k_P^2 = \frac{J_P}{A} = \frac{1.15545r^4}{2.5274r^2} \quad \text{or } k_P = 0.676r \blacktriangleleft$$



PROBLEM 9.25

(a) Determine by direct integration the polar moment of inertia of the semiannular area shown with respect to Point O . (b) Using the result of part a, determine the moments of inertia of the given area with respect to the x and y axes.

SOLUTION

(a) By definition

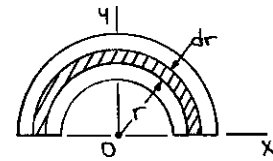
$$dJ_O = r^2 dA = r^2 (\pi r dr)$$

$$= \pi r^3 dr$$

Then

$$J_O = \int dJ_O = \int_{R_1}^{R_2} \pi r^3 dr$$

$$= \left[\frac{1}{4} \pi r^4 \right]_{R_1}^{R_2}$$



$$\text{or } J_O = \frac{\pi}{4} (R_2^4 - R_1^4) \blacktriangleleft$$

(b) First note that symmetry implies

$$(I_x)_1 = (I_y)_1 \quad (I_x)_2 = (I_y)_2$$

Also have

$$I_x = (I_x)_1 + (I_x)_2$$

and

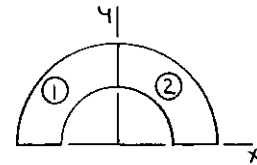
$$I_y = (I_y)_1 + (I_y)_2 = (I_x)_1 + (I_x)_2 = I_x$$

Now

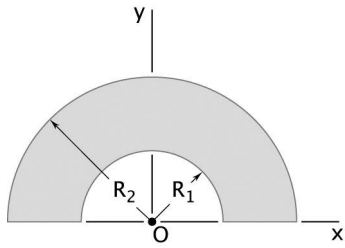
$$J_O = I_x + I_y \quad I_x = I_y$$

$$= 2I_x$$

$$\therefore I_x = I_y = \frac{1}{2} J_O$$



$$\text{or } I_x = I_y = \frac{\pi}{8} (R_2^4 - R_1^4) \blacktriangleleft$$



PROBLEM 9.26

(a) Show that the polar radius of gyration k_O of the semiannular area shown is approximately equal to the mean radius $R_m = (R_1 + R_2)/2$ for small values of the thickness $t = R_2 - R_1$. (b) Determine the percentage error introduced by using R_m in place of k_O for the following values of t/R_m : 1 , $\frac{1}{2}$, and $\frac{1}{10}$.

SOLUTION

(a) From the solution to Problem 9.25 have

$$J_O = \frac{\pi}{4} (R_2^4 - R_1^4)$$

Now

$$k_O^2 = \frac{J_O}{A} = \frac{\frac{\pi}{4} (R_2^4 - R_1^4)}{\frac{\pi}{2} (R_2^2 - R_1^2)} = \frac{1}{2} \frac{(R_2^2 - R_1^2)(R_2^2 + R_1^2)}{R_2^2 - R_1^2}$$

$$= \frac{1}{2} (R_2^2 + R_1^2) \quad (1)$$

Now

$$R_m = \frac{1}{2} (R_1 + R_2) \quad t = R_2 - R_1$$

Then

$$R_1 = R_m - \frac{1}{2}t \quad R_2 = R_m + \frac{1}{2}t$$

Substituting into Eq. (1),

$$k_O^2 = \frac{1}{2} \left[\left(R_m + \frac{1}{2}t \right)^2 + \left(R_m - \frac{1}{2}t \right)^2 \right]$$

$$= \frac{1}{2} \left(R_m^2 + R_m t + \frac{1}{4}t^2 + R_m^2 - R_m t + \frac{1}{4}t^2 \right)$$

$$= R_m^2 + \frac{1}{4}t^2 \quad \left(k_O = \sqrt{R_m^2 + \frac{1}{4}t^2} \right)$$

If $t \ll R_1, R_2$

Then $t \ll R_m$

So that $k_O^2 \approx R_m^2$ or $k_O \approx R_m \triangleleft$

PROBLEM 9.26 (Continued)

(b) The percentage error, % error, is given by

$$\begin{aligned}\% \text{ error} &= \frac{R_m - k_O}{k_O} \times 100\% \\ &= \frac{R_m - \sqrt{R_m^2 + \frac{1}{4}t^2}}{\sqrt{R_m^2 + \frac{1}{4}t^2}} \times 100\% \\ &= \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} \frac{t}{R_m}\right)^2}} - 1 \right) \times 100\%\end{aligned}$$

Then, for

$$\frac{t}{R_m} = 1: \quad \% \text{ error} = \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} + 1\right)^2}} - 1 \right) \times 100\%$$

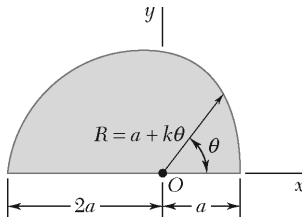
or % error = -10.56% ◀

$$\frac{t}{R_m} = \frac{1}{2}: \quad \% \text{ error} = \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} \times \frac{1}{2}\right)^2}} - 1 \right) \times 100\%$$

or % error = -2.99% ◀

$$\frac{t}{R_m} = \frac{1}{10}: \quad \% \text{ error} = \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} \times \frac{1}{10}\right)^2}} - 1 \right) \times 100\%$$

or % error = -0.1248% ◀



PROBLEM 9.27

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point O .

SOLUTION

At $\theta = \pi$, $R = 2a$:

$$2a = a + k(\pi)$$

or

$$k = \frac{a}{\pi}$$

Then

$$R = a + \frac{a}{\pi}\theta = a\left(1 + \frac{\theta}{\pi}\right)$$

Now

$$dA = (dr)(r d\theta) \\ = r dr d\theta$$

Then

$$A = \int dA = \int_0^{\pi} \int_0^{a(1+\theta/\pi)} r dr d\theta = \int_0^{\pi} \left[\frac{1}{2} r^2 \right]_0^{a(1+\theta/\pi)} d\theta$$

$$A = \int_0^{\pi} \frac{1}{2} a^2 \left(1 + \frac{\theta}{\pi}\right)^2 d\theta = \frac{1}{2} a^2 \left[\frac{\pi}{3} \left(1 + \frac{\theta}{\pi}\right)^3 \right]_0^{\pi} \\ = \frac{1}{6} \pi a^2 \left[\left(1 + \frac{\pi}{\pi}\right)^3 - (1)^3 \right] = \frac{7}{6} \pi a^2$$

Now

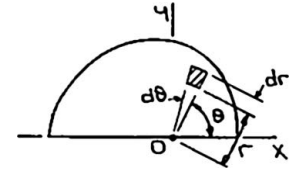
$$dJ_O = r^2 dA = r^2 (r dr d\theta)$$

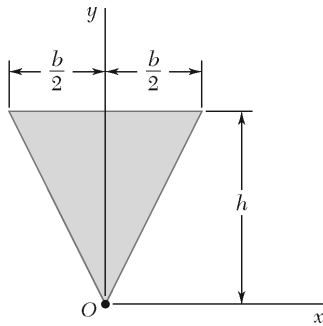
Then

$$J_O = \int dJ_O = \int_0^{\pi} \int_0^{a(1+\theta/\pi)} r^3 dr d\theta \\ = \int_0^{\pi} \left[\frac{1}{4} r^4 \right]_0^{a(1+\theta/\pi)} d\theta = \int_0^{\pi} \frac{1}{4} a^4 \left(1 + \frac{\theta}{\pi}\right)^4 d\theta \\ = \frac{1}{4} a^4 \left[\frac{\pi}{5} \left(1 + \frac{\theta}{\pi}\right)^5 \right]_0^{\pi} = \frac{1}{20} \pi a^4 \left[\left(1 + \frac{\pi}{\pi}\right)^5 - (1)^5 \right] \quad \text{or } J_O = \frac{31}{20} \pi a^4 \blacktriangleleft$$

and

$$k_O^2 = \frac{J_O}{A} = \frac{\frac{31}{20} \pi a^4}{\frac{7}{6} \pi a^2} = \frac{93}{70} a^2 \quad \text{or } k_O = 1.153a \blacktriangleleft$$





PROBLEM 9.28

Determine the polar moment of inertia and the polar radius of gyration of the isosceles triangle shown with respect to Point O .

SOLUTION

By observation:

$$y = \frac{h}{b}x$$

or

$$x = \frac{b}{2h}y$$

Now

$$dA = xdy = \left(\frac{b}{2h}y\right)dy$$

and

$$dI_x = y^2 dA = \frac{b}{2h}y^3 dy$$

Then

$$\begin{aligned} I_x &= \int dI_x = 2 \int_0^h \frac{b}{2h} y^3 dy \\ &= \frac{b}{h} \frac{y^4}{4} \Big|_0^h = \frac{1}{4}bh^3 \end{aligned}$$

From above:

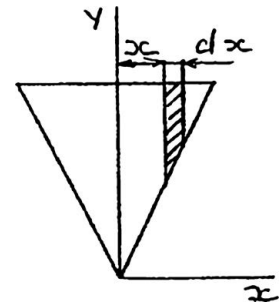
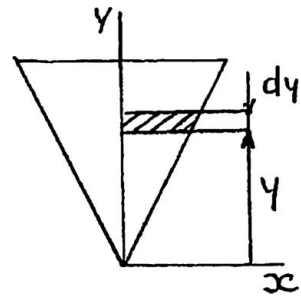
$$y = \frac{2h}{b}x$$

Now

$$\begin{aligned} dA &= (h - y)dx = \left(h - \frac{2h}{b}x\right)dx \\ &= \frac{h}{b}(b - 2x)dx \end{aligned}$$

and

$$dI_y = x^2 dA = x^2 \frac{h}{b}(b - 2x)dx$$



PROBLEM 9.28 (Continued)

Then

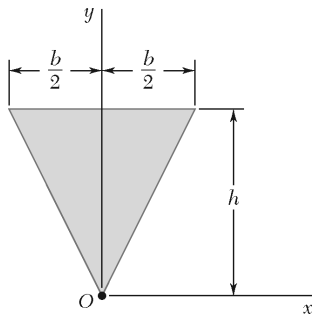
$$\begin{aligned} I_y &= \int dI_y = 2 \int_0^{b/2} \frac{h}{b} x^2 (b - 2x) dx \\ &= 2 \frac{h}{b} \left[\frac{1}{3} b x^3 - \frac{1}{2} x^4 \right]_0^{b/2} \\ &= 2 \frac{h}{b} \left[\frac{b}{3} \left(\frac{b}{2} \right)^3 - \frac{1}{2} \left(\frac{b}{2} \right)^4 \right] = \frac{1}{48} b^3 h \end{aligned}$$

Now

$$J_O = I_x + I_y = \frac{1}{4} b h^3 + \frac{1}{48} b^3 h \quad \text{or} \quad J_O = \frac{b h}{48} (12 h^2 + b^2) \quad \blacktriangleleft$$

and

$$k_O^2 = \frac{J_O}{A} = \frac{\frac{b h}{48} (12 h^2 + b^2)}{\frac{1}{2} b h} = \frac{1}{24} (12 h^2 + b^2) \quad \text{or} \quad k_O = \frac{\sqrt{12 h^2 + b^2}}{24} \quad \blacktriangleleft$$



PROBLEM 9.29*

Using the polar moment of inertia of the isosceles triangle of Problem 9.28, show that the centroidal polar moment of inertia of a circular area of radius r is $\pi r^4/2$. (*Hint: As a circular area is divided into an increasing number of equal circular sectors, what is the approximate shape of each circular sector?*)

PROBLEM 9.28 Determine the polar moment of inertia and the polar radius of gyration of the isosceles triangle shown with respect to Point O .

SOLUTION

First the circular area is divided into an increasing number of identical circular sectors. The sectors can be approximated by isosceles triangles. For a large number of sectors the approximate dimensions of one of the isosceles triangles are as shown.

For an isosceles triangle (see Problem 9.28):

$$J_O = \frac{bh}{48}(12h^2 + b^2)$$

Then with

$$b = r\Delta\theta \quad \text{and} \quad h = r$$

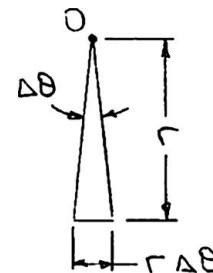
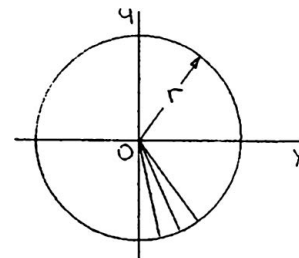
$$\begin{aligned} (\Delta J_O)_{\text{sector}} &\simeq \frac{1}{48}(r\Delta\theta)(r)[12r^2 + (r\Delta\theta)^2] \\ &= \frac{1}{48}r^4\Delta\theta[12 + (\Delta\theta)^2] \end{aligned}$$

Now

$$\begin{aligned} \frac{dJ_{O\text{sector}}}{d\theta} &= \lim_{\Delta\theta \rightarrow 0} \left(\frac{\Delta J_{O\text{sector}}}{\Delta\theta} \right) = \lim_{\Delta\theta \rightarrow 0} \left\{ \frac{1}{48}r^4[12 + (\Delta\theta)^2] \right\} \\ &= \frac{1}{4}r^4 \end{aligned}$$

Then

$$(J_O)_{\text{circle}} = \int dJ_{O\text{sector}} = \int_0^{2\pi} \frac{1}{4}r^4 d\theta = \frac{1}{4}r^4[\theta]_0^{2\pi}$$



or $(J_O)_{\text{circle}} = \frac{\pi}{2}r^4 \blacktriangleleft$

PROBLEM 9.30*

Prove that the centroidal polar moment of inertia of a given area A cannot be smaller than $A^2/2\pi$. (*Hint: Compare the moment of inertia of the given area with the moment of inertia of a circle that has the same area and the same centroid.*)

SOLUTION

From the solution to sample Problem 9.2, the centroidal polar moment of inertia of a circular area is

$$(J_C)_{\text{cir}} = \frac{\pi}{2} r^4$$

The area of the circle is

$$A_{\text{cir}} = \pi r^2$$

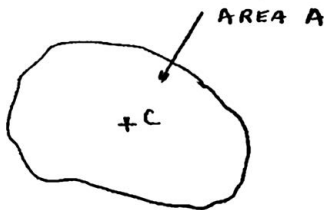
So that

$$[J_C(A)]_{\text{cir}} = \frac{A^2}{2\pi}$$

Two methods of solution will be presented. However, both methods depend upon the observation that as a given element of area dA is moved closer to some Point C . The value of J_C will be decreased ($J_C = \int r^2 dA$; as r decreases, so must J_C).

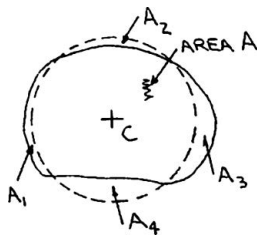
Solution 1

Imagine taking the area A and drawing it into a thin strip of negligible width and of sufficient length so that its area is equal to A . To minimize the value of $(J_C)_A$, the area would have to be distributed as closely as possible about C . This is accomplished by winding the strip into a tightly wound roll with C as its center; any voids in the roll would place the corresponding area farther from C than is necessary, thus increasing the value of $(J_C)_A$. (The process is analogous to rewinding a length of tape back into a roll.) Since the shape of the roll is circular, with the centroid of its area at C , it follows that



$$(J_C)_A \geq \frac{A^2}{2\pi} \text{ Q.E.D. } \blacktriangleleft$$

where the equality applies when the original area is circular.



PROBLEM 9.30* (Continued)

Solution 2

Consider an area A with its centroid at Point C and a circular area of area A with its center (and centroid) at Point C . Without loss of generality, assume that

$$A_1 = A_2 \quad A_3 = A_4$$

It then follows that

$$(J_C)_A = (J_C)_{\text{cir}} + [J_C(A_1) - J_C(A_2) + J_C(A_3) - J_C(A_4)]$$

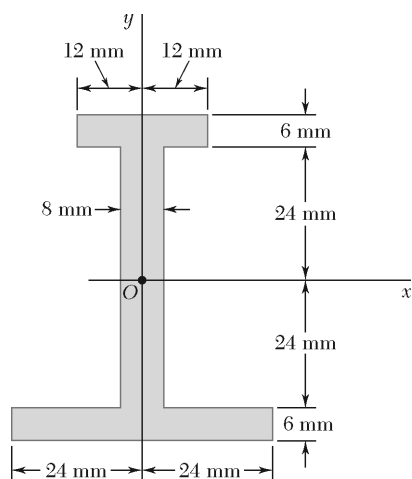
Now observe that

$$J_C(A_1) - J_C(A_2) \geq 0$$

$$J_C(A_3) - J_C(A_4) \geq 0$$

since as a given area is moved farther away from C its polar moment of inertia with respect to C must increase.

$$(J_C)_A \geq (J_C)_{\text{cir}} \quad \text{or} \quad (J_C)_A \geq \frac{A^2}{2\pi} \text{ Q.E.D. } \blacktriangleleft$$



PROBLEM 9.31

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

SOLUTION

First note that

$$\begin{aligned}
 A &= A_1 + A_2 + A_3 \\
 &= [(24)(6) + (8)(48) + (48)(6)] \text{ mm}^2 \\
 &= (144 + 384 + 288) \text{ mm}^2 \\
 &= 816 \text{ mm}^2
 \end{aligned}$$

Now

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

where

$$\begin{aligned}
 (I_x)_1 &= \frac{1}{12}(24 \text{ mm})(6 \text{ mm})^3 + (144 \text{ mm}^2)(27 \text{ mm})^2 \\
 &= (432 + 104,976) \text{ mm}^4 \\
 &= 105,408 \text{ mm}^4
 \end{aligned}$$

$$(I_x)_2 = \frac{1}{12}(8 \text{ mm})(48 \text{ mm})^3 = 73,728 \text{ mm}^4$$

$$\begin{aligned}
 (I_x)_3 &= \frac{1}{12}(48 \text{ mm})(6 \text{ mm})^3 + (288 \text{ mm}^2)(27 \text{ mm})^2 \\
 &= (864 + 209,952) \text{ mm}^4 = 210,816 \text{ mm}^4
 \end{aligned}$$

Then

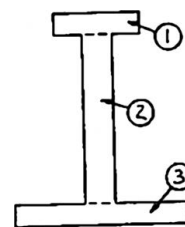
$$\begin{aligned}
 I_x &= (105,408 + 73,728 + 210,816) \text{ mm}^4 \\
 &= 389,952 \text{ mm}^4
 \end{aligned}$$

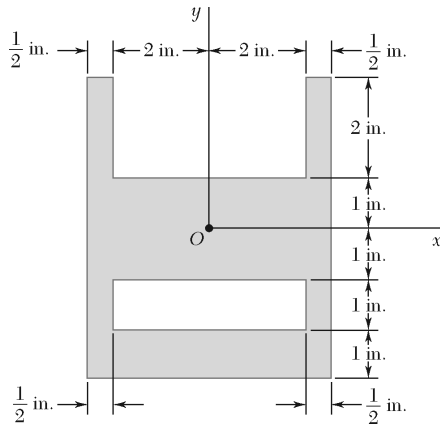
$$\text{or } I_x = 390 \times 10^3 \text{ mm}^4 \blacktriangleleft$$

and

$$k_x^2 = \frac{I_x}{A} = \frac{389,952 \text{ mm}^4}{816 \text{ mm}^2}$$

$$\text{or } k_x = 21.9 \text{ mm} \blacktriangleleft$$





PROBLEM 9.32

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

SOLUTION

First note that

$$\begin{aligned}
 A &= A_1 - A_2 - A_3 \\
 &= [(5)(6) - (4)(2) - (4)(1)] \text{ in}^2 \\
 &= (30 - 8 - 4) \text{ in}^2 \\
 &= 18 \text{ in}^2
 \end{aligned}$$

Now

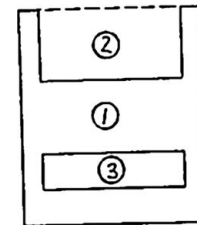
$$I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$

where

$$(I_x)_1 = \frac{1}{12} (5 \text{ in.})(6 \text{ in.})^3 = 90 \text{ in}^4$$

$$\begin{aligned}
 (I_x)_2 &= \frac{1}{12} (4 \text{ in.})(2 \text{ in.})^3 + (8 \text{ in}^2)(2 \text{ in.})^2 \\
 &= 34 \frac{2}{3} \text{ in}^4
 \end{aligned}$$

$$\begin{aligned}
 (I_x)_3 &= \frac{1}{12} (4 \text{ in.})(1 \text{ in.})^3 + (4 \text{ in}^2) \left(\frac{3}{2} \text{ in.} \right)^2 \\
 &= 9 \frac{1}{3} \text{ in}^4
 \end{aligned}$$



Then

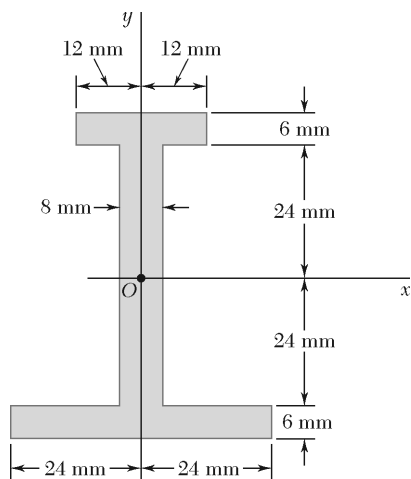
$$I_x = \left(90 - 34 \frac{2}{3} - 9 \frac{1}{3} \right) \text{ in}^4$$

$$\text{or } I_x = 46.0 \text{ in}^4 \blacktriangleleft$$

and

$$k_x^2 = \frac{I_x}{A} = \frac{46.0 \text{ in}^4}{18 \text{ in}^2}$$

$$\text{or } k_x = 1.599 \text{ in.} \blacktriangleleft$$



PROBLEM 9.33

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.

SOLUTION

First note that

$$\begin{aligned}
 A &= A_1 + A_2 + A_3 \\
 &= [(24 \times 6) + (8)(48) + (48)(6)] \text{ mm}^2 \\
 &= (144 + 384 + 288) \text{ mm}^2 \\
 &= 816 \text{ mm}^2
 \end{aligned}$$

Now

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

where

$$(I_y)_1 = \frac{1}{12} (6 \text{ mm})(24 \text{ mm})^3 = 6912 \text{ mm}^4$$

$$(I_y)_2 = \frac{1}{12} (48 \text{ mm})(8 \text{ mm})^3 = 2048 \text{ mm}^4$$

$$(I_y)_3 = \frac{1}{12} (6 \text{ mm})(48 \text{ mm})^3 = 55,296 \text{ mm}^4$$

Then

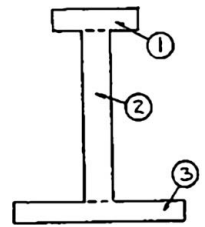
$$I_y = (6912 + 2048 + 55,296) \text{ mm}^4 = 64,256 \text{ mm}^4$$

$$\text{or } I_y = 64.3 \times 10^3 \text{ mm}^4 \blacktriangleleft$$

and

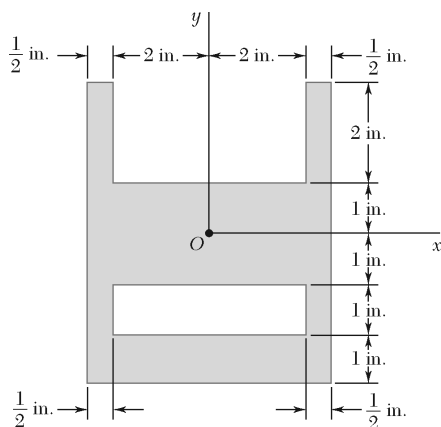
$$k_y^2 = \frac{I_y}{A} = \frac{64,256 \text{ mm}^4}{816 \text{ mm}^2}$$

$$\text{or } k_y = 8.87 \text{ mm} \blacktriangleleft$$



PROBLEM 9.34

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.



SOLUTION

First note that

$$\begin{aligned} A &= A_1 - A_2 - A_3 \\ &= [(5)(6) - (4)(2) - (4)(1)] \text{ in}^2 \\ &= (30 - 8 - 4) \text{ in}^2 \\ &= 18 \text{ in}^2 \end{aligned}$$

Now

$$I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$$

where

$$(I_y)_1 = \frac{1}{12} (6 \text{ in.})(5 \text{ in.})^3 = 62.5 \text{ in}^4$$

$$(I_y)_2 = \frac{1}{12} (2 \text{ in.})(4 \text{ in.})^3 = 10\frac{2}{3} \text{ in}^4$$

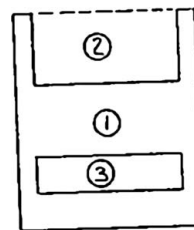
$$(I_y)_3 = \frac{1}{12} (1 \text{ in.})(4 \text{ in.})^3 = 5\frac{1}{3} \text{ in}^4$$

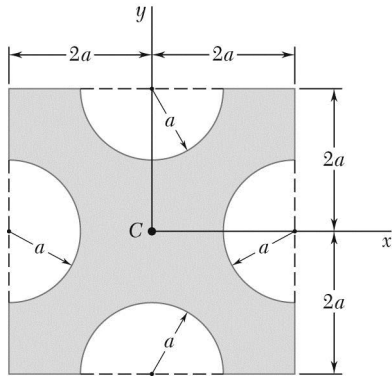
Then

$$I_y = \left(62.5 - 10\frac{2}{3} - 5\frac{1}{3} \right) \text{ in}^4 \quad \text{or} \quad I_y = 46.5 \text{ in}^4 \quad \blacktriangleleft$$

and

$$k_y^2 = \frac{I_y}{A} = \frac{46.5 \text{ in}^4}{18 \text{ in}^2} \quad \text{or} \quad k_y = 1.607 \text{ in.} \quad \blacktriangleleft$$





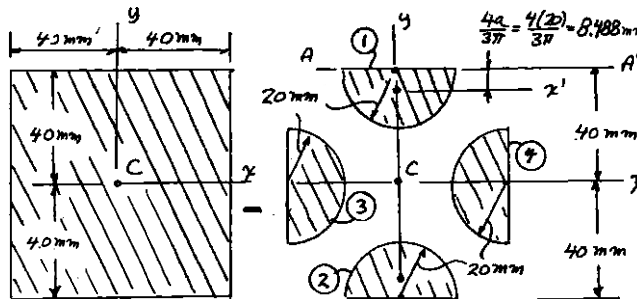
PROBLEM 9.35

Determine the moments of inertia of the shaded area shown with respect to the x and y axes when $a = 20$ mm.

SOLUTION

By symmetry: $I_x = I_y$

Given area = square - 4(semicircles)



Square
$$I_x = \frac{1}{12} (80 \text{ mm})^4 = 3.413 \times 10^6 \text{ mm}^4$$

Semicircle 1:
$$I_{AA'} = \frac{\pi}{8} (20 \text{ mm})^4 = 62.83 \times 10^3 \text{ mm}^4$$

$$I_{AA'} = \bar{I}_{x'} + Ad^2$$

$$62.83 \times 10^3 \text{ mm}^4 = \bar{I}_{x'} + \frac{\pi}{2} (20 \text{ mm})^2 (8.488 \text{ mm})^2$$

$$\bar{I}_{x'} = 17.56 \times 10^3 \text{ mm}^4$$

$$I_x = \bar{I}_{x'} + A(40 \text{ mm} - 8.488 \text{ mm})^2$$

$$= 17.56 \times 10^3 \text{ mm}^4 + \frac{\pi}{2} (20 \text{ mm})^2 (31.512 \text{ mm})^2$$

$$I_x = 641.5 \times 10^3 \text{ mm}^4$$

PROBLEM 9.35 (Continued)

Semicircle 2: Same as 1: $I_x = 641.5 \times 10^3 \text{ mm}^4$

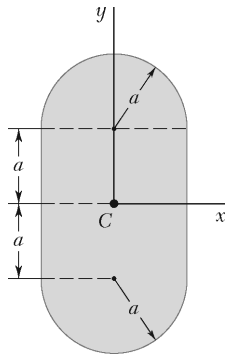
Semicircles 3 and 4 are equivalent to one 20-mm circle when computing I_x

$$I_x = \frac{\pi}{4} (20 \text{ mm})^4 = 125.66 \times 10^3 \text{ mm}^4$$

Entire area = square - 4 (semicircles)

$$\begin{aligned} \bar{I}_x &= 3.413 \times 10^6 \text{ mm}^4 - 2(641.5 \times 10^3 \text{ mm}^4) - 125.66 \times 10^3 \text{ mm}^4 \\ &= 2.0047 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\bar{I}_x = \bar{I}_y = 2.00 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.36

Determine the moments of inertia of the shaded area shown with respect to the x and y axes when $a = 20$ mm.

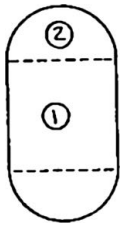
SOLUTION

We have

$$I_x = (I_x)_1 + 2(I_x)_2$$

where

$$\begin{aligned} (I_x)_1 &= \frac{1}{12}(40 \text{ mm})(40 \text{ mm})^3 \\ &= 213.33 \times 10^3 \text{ mm}^4 \end{aligned}$$



$$\begin{aligned} (I_x)_2 &= \left[\frac{\pi}{8}(20 \text{ mm})^4 - \frac{\pi}{2}(20 \text{ mm})^2 \left(\frac{4 \times 20}{3\pi} \text{ mm} \right)^2 \right] \\ &\quad + \frac{\pi}{2}(20 \text{ mm})^2 \left[\left(\frac{4 \times 20}{3\pi} + 20 \right) \text{ mm} \right]^2 \\ &= 527.49 \times 10^3 \text{ mm}^4 \end{aligned}$$

Then

$$I_x = [213.33 + 2(527.49)] \times 10^3 \text{ mm}^4$$

$$\text{or } I_x = 1.268 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

Also

$$I_y = (I_y)_1 + 2(I_y)_2$$

where

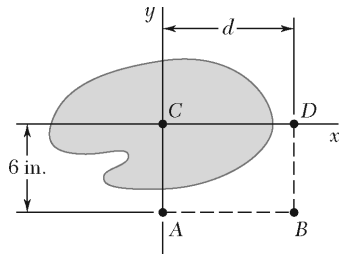
$$(I_y)_1 = \frac{1}{12}(40 \text{ mm})(40 \text{ mm})^3 = 213.33 \times 10^3 \text{ mm}^4$$

$$(I_y)_2 = \frac{\pi}{8}(20 \text{ mm})^4 = 62.83 \times 10^3 \text{ mm}^4$$

Then

$$I_y = [213.33 + 2(62.83)] \times 10^3 \text{ mm}^4$$

$$\text{or } I_y = 339 \times 10^3 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.37

The shaded area is equal to 50 in^2 . Determine its centroidal moments of inertia \bar{I}_x and \bar{I}_y , knowing that $\bar{I}_y = 2\bar{I}_x$ and that the polar moment of inertia of the area about Point A is $J_A = 2250 \text{ in}^4$.

SOLUTION

Given:

$$A = 50 \text{ in}^2 \quad \bar{I}_y = 2\bar{I}_x, \quad J_A = 2250 \text{ in}^4$$

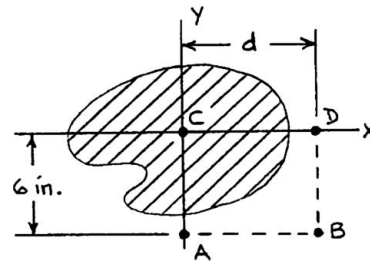
$$J_A = \bar{J}_C + A(6 \text{ in.})^2$$

$$2250 \text{ in}^4 = \bar{J}_C + (50 \text{ in}^2)(6 \text{ in.})^2$$

$$\bar{J}_C = 450 \text{ in}^4$$

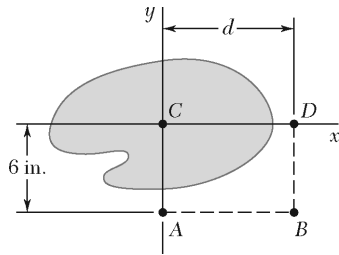
$$\bar{J}_C = \bar{I}_x + \bar{I}_y \quad \text{with} \quad \bar{I}_y = 2\bar{I}_x$$

$$450 \text{ in}^4 = \bar{I}_x + 2\bar{I}_x$$



$$\bar{I}_x = 150.0 \text{ in}^4 \quad \blacktriangleleft$$

$$\bar{I}_y = 2\bar{I}_x = 300 \text{ in}^4 \quad \blacktriangleleft$$



PROBLEM 9.38

The polar moments of inertia of the shaded area with respect to Points A , B , and D are, respectively, $J_A = 2880 \text{ in}^4$, $J_B = 6720 \text{ in}^4$, and $J_D = 4560 \text{ in}^4$. Determine the shaded area, its centroidal moment of inertia \bar{J}_C , and the distance d from C to D .

SOLUTION

See figure at solution of Problem 9.39.

Given: $J_A = 2880 \text{ in}^4$, $J_B = 6720 \text{ in}^4$, $J_D = 4560 \text{ in}^4$

$$J_B = \bar{J}_C + A(CB)^2; \quad 6720 \text{ in}^4 = \bar{J}_C + A(6^2 + d^2) \quad (1)$$

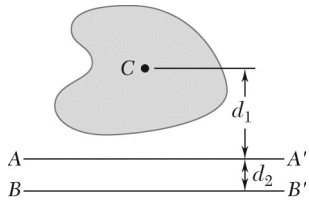
$$J_D = \bar{J}_C + A(CD)^2; \quad 4560 \text{ in}^4 = \bar{J}_C + Ad^2 \quad (2)$$

Eq. (1) subtracted by Eq. (2): $J_B - J_D = 2160 \text{ in}^4 = A(6)^2$ $A = 60.0 \text{ in}^2 \blacktriangleleft$

$$J_A = \bar{J}_C + A(AC)^2; \quad 2880 \text{ in}^4 = \bar{J}_C + (60 \text{ in}^2)(6 \text{ in.})^2$$
 $\bar{J}_C = 720 \text{ in}^4 \blacktriangleleft$

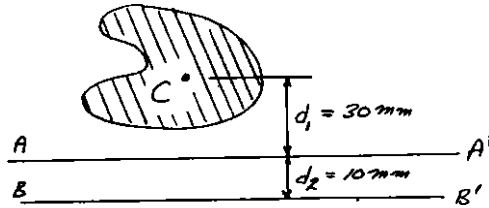
Eq. (2): $4560 \text{ in}^4 = 720 \text{ in}^4 + (60 \text{ in}^2)d^2$ $d = 8.00 \text{ in.} \blacktriangleleft$

PROBLEM 9.39



Determine the shaded area and its moment of inertia with respect to the centroidal axis parallel to AA' , knowing that $d_1 = 30 \text{ mm}$ and $d_2 = 10 \text{ mm}$, and that the moments of inertia with respect to AA' and BB' are $4.1 \times 10^6 \text{ mm}^4$ and $6.9 \times 10^6 \text{ mm}^4$, respectively.

SOLUTION



$$I_{AA'} = 4.1 \times 10^6 \text{ mm}^4 = \bar{I} + A(30 \text{ mm})^2 \quad (1)$$

$$I_{BB'} = 6.9 \times 10^6 \text{ mm}^4 = \bar{I} + A(40 \text{ mm})^2$$

$$I_{BB'} - I_{AA'} = (6.9 - 4.1) \times 10^6 = A(40^2 - 30^2)$$

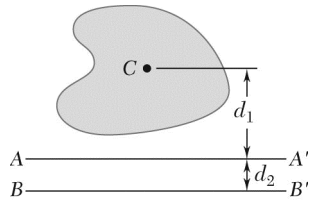
$$2.8 \times 10^6 = A(700)$$

$$A = 4000 \text{ mm}^2 \quad \blacktriangleleft$$

Eq. (1):

$$4.1 \times 10^6 = \bar{I} + (4000)(30)^2$$

$$\bar{I} = 500 \times 10^3 \text{ mm}^4 \quad \blacktriangleleft$$



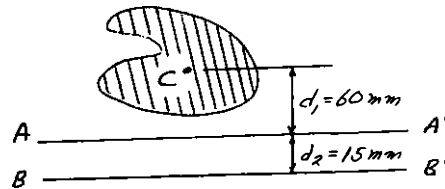
PROBLEM 9.40

Knowing that the shaded area is equal to 7500 mm^2 and that its moment of inertia with respect to AA' is $31 \times 10^6 \text{ mm}^4$, determine its moment of inertia with respect to BB' , for $d_1 = 60 \text{ mm}$ and $d_2 = 15 \text{ mm}$.

SOLUTION

Given:

$$A = 7500 \text{ mm}^2$$

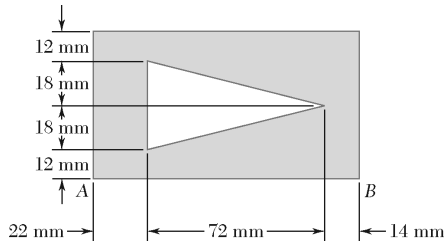


$$I_{AA'} = \bar{I} + Ad_1^2; \quad 31 \times 10^6 \text{ mm}^2 = \bar{I} + (7500 \text{ mm}^2)(60 \text{ mm})^2$$

$$\bar{I} = 4 \times 10^6 \text{ mm}^4$$

$$\begin{aligned} I_{BB'} &= \bar{I} + Ad^2 = 4 \times 10^6 \text{ mm}^4 + (7500 \text{ mm}^2)(60 \text{ mm} + 15 \text{ mm})^2 \\ &= 4 \times 10^6 + 7500(75)^2 = 46.188 \times 10^6 \text{ mm}^4 \end{aligned}$$

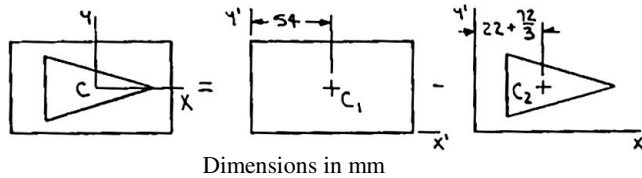
$$I_{BB'} = 46.2 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$



PROBLEM 9.41

Determine the moments of inertia \bar{I}_x and \bar{I}_y of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB .

SOLUTION



First locate centroid C of the area.

Symmetry implies $\bar{Y} = 30$ mm.

	A, mm^2	\bar{x}, mm	$\bar{x}A, \text{mm}^3$
1	$108 \times 60 = 6480$	54	349,920
2	$-\frac{1}{2} \times 72 \times 36 = -1296$	46	-59,616
Σ	5184		290,304

Then $\bar{X} \Sigma A = \Sigma \bar{x}A$: $\bar{X}(5184 \text{ mm}^2) = 290,304 \text{ mm}^3$

or $\bar{X} = 56.0$ mm

Now $\bar{I}_x = (I_x)_1 - (I_x)_2$

where $(I_x)_1 = \frac{1}{12}(108 \text{ mm})(60 \text{ mm})^3 = 1.944 \times 10^6 \text{ mm}^4$

$$(I_x)_2 = 2 \left[\frac{1}{36}(72 \text{ mm})(18 \text{ mm})^3 + \left(\frac{1}{2} \times 72 \text{ mm} \times 18 \text{ mm} \right) (6 \text{ mm})^2 \right]$$

$$= 2(11,664 + 23,328) \text{ mm}^4 = 69.984 \times 10^3 \text{ mm}^4$$

$[(I_x)_2$ is obtained by dividing A_2 into \Rightarrow]

Then $\bar{I}_x = (1.944 - 0.069984) \times 10^6 \text{ mm}^4$

or $\bar{I}_x = 1.874 \times 10^6 \text{ mm}^4 \blacktriangleleft$

PROBLEM 9.41 (Continued)

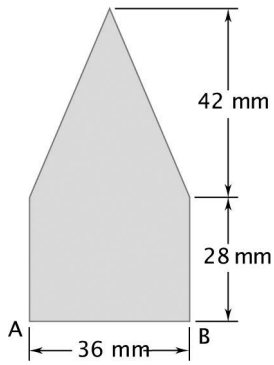
Also
$$\bar{I}_y = (I_y)_1 - (I_y)_2$$

where
$$(I_y)_1 = \frac{1}{12}(60 \text{ mm})(108 \text{ mm})^3 + (6480 \text{ mm}^2)[(56.54) \text{ mm}]^2$$
$$= (6,298,560 + 25,920) \text{ mm}^4 = 6.324 \times 10^6 \text{ mm}^4$$

$$(I_y)_2 = \frac{1}{36}(36 \text{ mm})(72 \text{ mm})^3 + (1296 \text{ mm}^2)[(56 - 46) \text{ mm}]^2$$
$$= (373,248 + 129,600) \text{ mm}^4 = 0.502 \times 10^6 \text{ mm}^4$$

Then
$$\bar{I}_y = (6.324 - 0.502)10^6 \text{ mm}^4$$

or
$$\bar{I}_y = 5.82 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.42

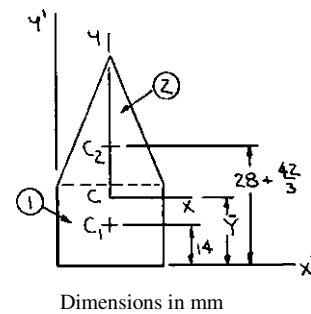
Determine the moments of inertia \bar{I}_x and \bar{I}_y of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB.

SOLUTION

First locate C of the area.

Symmetry implies $\bar{X} = 18$ mm.

	A, mm^2	\bar{y}, mm	$\bar{y}A, \text{mm}^3$
1	$36 \times 28 = 1008$	14	14,112
2	$\frac{1}{2} \times 36 \times 42 = 756$	42	31,752
Σ	1764		45,864



Then $\bar{Y} \Sigma A = \Sigma \bar{y}A: \bar{Y}(1764 \text{ mm}^2) = 45,864 \text{ mm}^3$
or $\bar{Y} = 26.0 \text{ mm}$

Now $\bar{I}_x = (I_x)_1 + (I_x)_2$

where $(I_x)_1 = \frac{1}{12}(36 \text{ mm})(28 \text{ mm})^3 + (1008 \text{ mm}^2)[(26 - 14) \text{ mm}]^2$
 $= (65,856 + 145,152) \text{ mm}^4 = 211.008 \times 10^3 \text{ mm}^4$

$(I_x)_2 = \frac{1}{36}(36 \text{ mm})(42 \text{ mm})^3 + (756 \text{ mm}^2)[(42 - 26) \text{ mm}]^2$
 $= (74,088 + 193,536) \text{ mm}^4 = 267.624 \times 10^3 \text{ mm}^4$

Then $\bar{I}_x = (211.008 + 267.624) \times 10^3 \text{ mm}^4$


or $\bar{I}_x = 479 \times 10^3 \text{ mm}^4 \blacktriangleleft$

PROBLEM 9.42 (Continued)

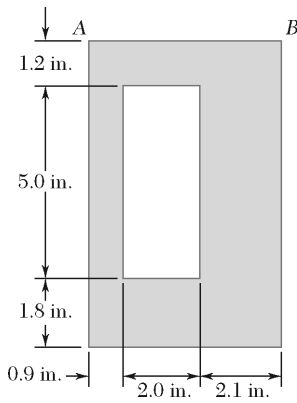
Also
$$\bar{I}_y = (I_y)_1 + (I_y)_2$$

where
$$(I_y)_1 = \frac{1}{12}(28 \text{ mm})(36 \text{ mm})^3 = 108.864 \times 10^3 \text{ mm}^4$$

$$(I_y)_2 = 2 \left[\frac{1}{36}(42 \text{ mm})(18 \text{ mm})^3 + \left(\frac{1}{2} \times 18 \text{ mm} \times 42 \text{ mm} \right) (6 \text{ mm})^2 \right]$$
$$= 2(6804 + 13,608) \text{ mm}^4 = 40.824 \times 10^3 \text{ mm}^4$$

$[(I_y)_2$ is obtained by dividing A_2 into ]

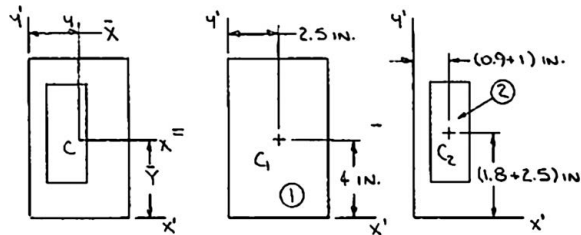
Then
$$\bar{I}_y = (108.864 + 40.824 \times 10^3) \text{ mm}^4 \quad \text{or} \quad \bar{I}_y = 149.7 \times 10^3 \text{ mm}^4 \quad \blacktriangleleft$$



PROBLEM 9.43

Determine the moments of inertia \bar{I}_x and \bar{I}_y of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB .

SOLUTION



First locate centroid C of the area.

	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$5 \times 8 = 40$	2.5	4	100	160
2	$-2 \times 5 = -10$	1.9	4.3	-19	-43
Σ	30			81	117

Then $\bar{X} \Sigma A = \Sigma \bar{x}A: \bar{X}(30 \text{ in}^2) = 81 \text{ in}^3$

or $\bar{X} = 2.70 \text{ in.}$

and $\bar{Y} \Sigma A = \Sigma \bar{y}A: \bar{Y}(30 \text{ in}^2) = 117 \text{ in}^3$

or $\bar{Y} = 3.90 \text{ in.}$

Now $\bar{I}_x = (I_x)_1 - (I_x)_2$

where $(I_x)_1 = \frac{1}{12}(5 \text{ in.})(8 \text{ in.})^3 + (40 \text{ in}^2)[(4 - 3.9) \text{ in.}]^2$
 $= (213.33 + 0.4) \text{ in}^4 = 213.73 \text{ in}^4$

$(I_x)_2 = \frac{1}{12}(2 \text{ in.})(5 \text{ in.})^3 + (10 \text{ in}^2)[(4.3 - 3.9) \text{ in.}]^2$
 $= (20.83 + 1.60) = 22.43 \text{ in}^4$

PROBLEM 9.43 (Continued)

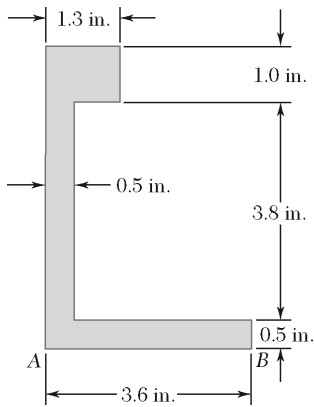
Then $\bar{I}_x = (213.73 - 22.43) \text{ in}^4$ or $\bar{I}_x = 191.3 \text{ in}^4 \blacktriangleleft$

Also $\bar{I}_y = (I_y)_1 - (I_y)_2$

where $(I_y)_1 = \frac{1}{12}(8 \text{ in.})(5 \text{ in.})^3 + (40 \text{ in}^2)[(2.7 - 2.5) \text{ in.}]^2$
 $= (83.333 + 1.6) \text{ in}^4 = 84.933 \text{ in}^4$

$$(I_y)_2 = \frac{1}{12}(5 \text{ in.})(2 \text{ in.})^3 + (10 \text{ in}^2)[(2.7 - 1.9) \text{ in.}]^2$$
$$= (3.333 + 6.4) \text{ in}^4 = 9.733 \text{ in}^4$$

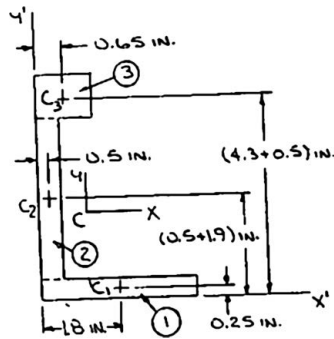
Then $\bar{I}_y = (84.933 - 9.733) \text{ in}^4$ or $\bar{I}_y = 75.2 \text{ in}^4 \blacktriangleleft$



PROBLEM 9.44

Determine the moments of inertia \bar{I}_x and \bar{I}_y of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB .

SOLUTION



First locate centroid C of the area.

	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$3.6 \times 0.5 = 1.8$	1.8	0.25	3.24	0.45
2	$0.5 \times 3.8 = 1.9$	0.25	2.4	0.475	4.56
3	$1.3 \times 1 = 1.3$	0.65	4.8	0.845	6.24
Σ	5.0			4.560	11.25

Then $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(5 \text{ in}^2) = 4.560 \text{ in}^3$

or $\bar{X} = 0.912 \text{ in.}$

and $\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(5 \text{ in}^2) = 11.25 \text{ in}^3$

or $\bar{Y} = 2.25 \text{ in.}$

PROBLEM 9.44 (Continued)

Now
$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

where
$$(I_x)_1 = \frac{1}{12}(3.6 \text{ in.})(0.5 \text{ in.})^3 + (1.8 \text{ in}^2)[(2.25 - 0.25) \text{ in.}]^2$$

$$= (0.0375 + 7.20) \text{ in}^4 = 7.2375 \text{ in}^4$$

$$(I_x)_2 = \frac{1}{12}(0.5 \text{ in.})(3.8 \text{ in.})^3 + (1.9 \text{ in}^2)[(2.4 - 2.25) \text{ in.}]^2$$

$$= (2.2863 + 0.0428) \text{ in}^4 = 2.3291 \text{ in}^4$$

$$(I_x)_3 = \frac{1}{12}(1.3 \text{ in.})(1 \text{ in.})^3 + (1.3 \text{ in}^2)[(4.8 - 2.25) \text{ in.}]^2$$

$$= (0.1083 + 8.4533) \text{ in}^4 = 8.5616 \text{ in}^4$$

Then
$$\bar{I}_x = (7.2375 + 2.3291 + 8.5616) \text{ in}^4 = 18.1282 \text{ in}^4$$

or
$$\bar{I}_x = 18.13 \text{ in}^4 \blacktriangleleft$$

Also
$$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

where
$$(I_y)_1 = \frac{1}{12}(0.5 \text{ in.})(3.6 \text{ in.})^3 + (1.8 \text{ in}^2)[(1.8 - 0.912) \text{ in.}]^2$$

$$= (1.9440 + 1.4194) \text{ in}^4 = 3.3634 \text{ in}^4$$

$$(I_y)_2 = \frac{1}{12}(3.8 \text{ in.})(0.5 \text{ in.})^3 + (1.9 \text{ in}^2)[(0.912 - 0.25) \text{ in.}]^2$$

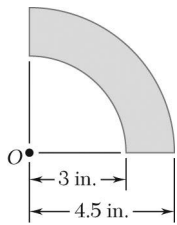
$$= (0.0396 + 0.8327) \text{ in}^4 = 0.8723 \text{ in}^4$$

$$(I_y)_3 = \frac{1}{12}(1 \text{ in.})(1.3 \text{ in.})^3 + (1.3 \text{ in}^2)[(0.912 - 0.65) \text{ in.}]^2$$

$$= (0.1831 + 0.0892) \text{ in}^4 = 0.2723 \text{ in}^4$$

Then
$$\bar{I}_y = (3.3634 + 0.8723 + 0.2723) \text{ in}^4 = 4.5080 \text{ in}^4$$

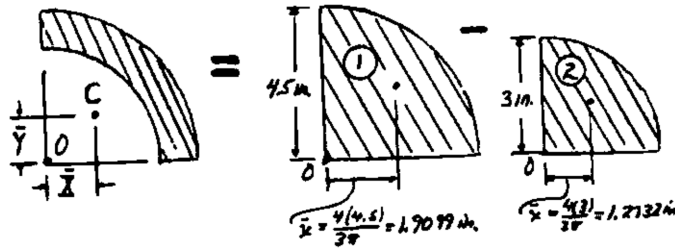
or
$$\bar{I}_y = 4.51 \text{ in}^4 \blacktriangleleft$$



PROBLEM 9.45

Determine the polar moment of inertia of the area shown with respect to (a) Point O , (b) the centroid of the area.

SOLUTION



Section	Area, in ²	\bar{x} , in.	$\bar{x}\bar{A}$, in ³
1	$\frac{\pi}{4}(4.5)^2 = 15.904$	1.9099	30.375
2	$-\frac{\pi}{4}(3)^2 = -7.069$	1.2732	-9.00
Σ	8.835		21.375

Then $\bar{x}A = \Sigma \bar{x}\bar{A}$: $\bar{X}(8.835 \text{ in}^2) = 21.375 \text{ in}^3$

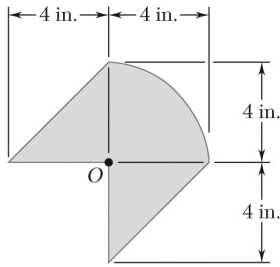
or $\bar{X} = 2.419 \text{ in.}$

Then $J_O = \frac{\pi}{8}(4.5 \text{ in.})^4 - \frac{\pi}{8}(3 \text{ in.})^4 = 129.22 \text{ in}^4$ $J_O = 129.2 \text{ in}^4 \blacktriangleleft$

$\overline{OC} = \sqrt{2}\bar{X} = \sqrt{2}(2.419 \text{ in.}) = 3.421 \text{ in.}$

$J_O = \bar{J}_C + A(\overline{OC})^2$:

$129.22 \text{ in}^4 = \bar{J}_C + (8.835 \text{ in.})(3.421 \text{ in.})^2$ $\bar{J}_C = 25.8 \text{ in}^4 \blacktriangleleft$



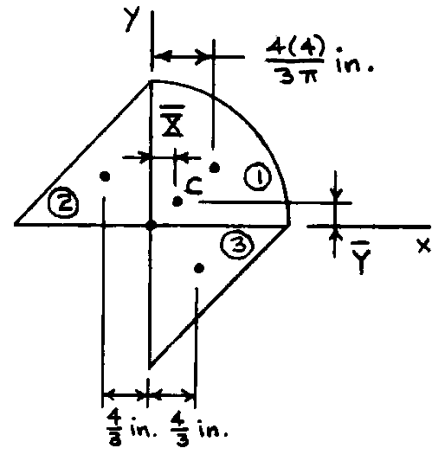
PROBLEM 9.46

Determine the polar moment of inertia of the area shown with respect to (a) Point O , (b) the centroid of the area.

SOLUTION

Determination of centroid C of entire section:

Section	Area, in ²	\bar{x} , in.	$\bar{x}A$, in ³
1	$\frac{\pi}{4}(4)^2 = 4\pi$	$\frac{16}{3\pi}$	21.333
2	$\frac{1}{2}(4)(4) = 8$	$-\frac{4}{3}$	-10.6667
3	$\frac{1}{2}(4)(4) = 8$	$\frac{4}{3}$	10.6667
Σ	28.566		21.333



$$\bar{X}\Sigma A = \Sigma \bar{x}A: \bar{X}(28.566 \text{ in}^2) = 21.333 \text{ in}^3$$

$$\bar{X} = 0.74680 \text{ in.}; \text{ by symmetry, } \bar{Y} = \bar{X} = 0.74680 \text{ in.}$$

(a) For section ①:
$$I_x = \frac{1}{4} \left(\frac{1}{4} \pi r^4 \right) = \frac{1}{16} \pi (4)^4 = 50.265 \text{ in}^4$$

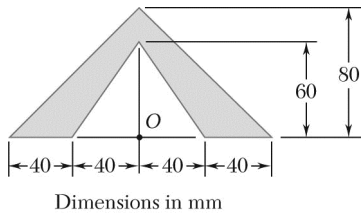
For sections ②, ③:
$$I_x = \frac{1}{12} bh^3 = \frac{1}{12} (4)(4)^3 = 21.333 \text{ in}^4$$

For total area,
$$I_x = 50.265 + 2(21.333) = 92.931 \text{ in}^4$$

By symmetry,
$$I_y = I_x; \quad J_O = I_x + I_y = 185.862 \text{ in}^4 \quad J_O = 185.9 \text{ in}^4 \blacktriangleleft$$

(b) Parallel axis theorem:
$$J_C = J_O - A(OC)^2 = J_O - A(\bar{X}^2 + \bar{Y}^2)$$

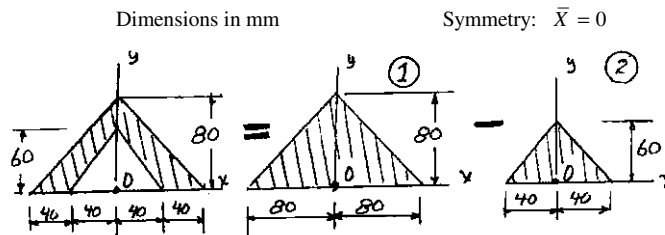
$$J_C = 185.862 - (28.566)(0.74680^2 + 0.74680^2) \quad J_C = 154.0 \text{ in}^4 \blacktriangleleft$$



PROBLEM 9.47

Determine the polar moment of inertia of the area shown with respect to (a) Point O , (b) the centroid of the area.

SOLUTION



Determination of centroid C of entire section

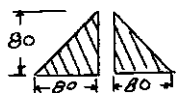
$$\begin{aligned}\bar{Y}\Sigma A &= \Sigma \bar{y}A \\ \bar{Y}(4000 \text{ mm}^2) &= 122.67 \times 10^3 \text{ mm}^3 \\ \bar{Y} &= 30.667 \text{ mm}\end{aligned}$$

	Area mm^2	\bar{y} , mm	$\bar{y}A$, mm^3
1	$\frac{1}{2}(160)(80) = 6400$	$\frac{80}{3}$	170.67×10^3
2	$-\frac{1}{2}(80)(60) = -2400$	20	-48×10^3
Σ	4000		122.67×10^3

(a) Polar moment of inertia J_O :

Section ①:
$$I_x = \frac{1}{12}(160)(80)^3 = 6.8267 \times 10^6 \text{ mm}^4$$

For I_y consider the following two triangles



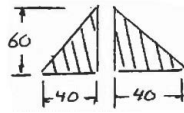
$$I_y = 2 \left[\frac{1}{12}(80)(80)^3 \right] = 6.8267 \times 10^6 \text{ mm}^4$$

$$J_O = I_x + I_y = (6.8267 + 6.8267)10^6 = 13.653 \times 10^6 \text{ mm}^4$$

PROBLEM 9.47 (Continued)

(b) Section ②: $I_x = \frac{1}{12}(80)(60)^3 = 1.44 \times 10^6 \text{ mm}^4$

For I_y consider the following two triangles



$$I_y = 2 \left[\frac{1}{12}(60)(40)^3 \right] = 0.640 \times 10^6 \text{ mm}^4$$

$$J_O = I_x + I_y = (1.44 + 0.640)10^6 = 2.08 \times 10^6 \text{ mm}^4$$

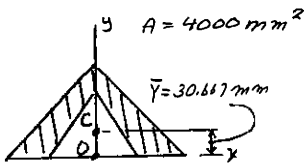
Entire section

$$J_O = (J_O)_1 - (J_O)_2 = 13.653 \times 10^6 - 2.08 \times 10^6$$

$$J_O = 11.573 \times 10^6 \text{ mm}^4$$

$$J_O = 11.57 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

(c) Polar moment of inertia J_O of entire area

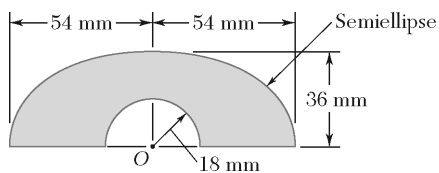


$$J_O = J_C + A\bar{Y}^2$$

$$11.573 \times 10^6 = J_C + (4000)(30.667)^2$$

$$J_C = 7.811 \times 10^6 \text{ mm}^4$$

$$J_C = 7.81 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

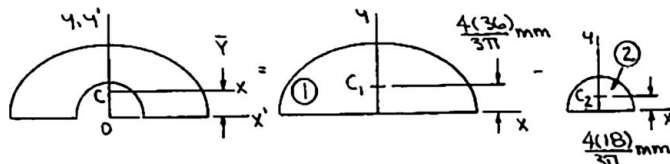


PROBLEM 9.48

Determine the polar moment of inertia of the area shown with respect to (a) Point O , (b) the centroid of the area.

SOLUTION

First locate centroid C of the area.



	A, mm^2	\bar{y}, mm	$\bar{y}A, \text{mm}^3$
1	$\frac{\pi}{2}(54)(36) = 3053.6$	$\frac{48}{\pi} = 15.2789$	46,656
2	$-\frac{\pi}{2}(18)^2 = -508.9$	$\frac{24}{\pi} = 7.6394$	-3888
Σ	2544.7		42,768

Then

$$\bar{Y}\Sigma A = \Sigma \bar{y}A: \bar{Y}(2544.7 \text{ mm}^2) = 42,768 \text{ mm}^3$$

$$\text{or } \bar{Y} = 16.8067 \text{ mm} \blacktriangleleft$$

(a)

$$J_O = (J_O)_1 - (J_O)_2$$

$$= \frac{\pi}{8}(54 \text{ mm})(36 \text{ mm})[(54 \text{ mm})^2 + (36 \text{ mm})^2] - \frac{\pi}{4}(18 \text{ mm})^4$$

$$= (3.2155 \times 10^6 - 0.0824 \times 10^6) \text{ mm}^4$$

$$= 3.1331 \times 10^6 \text{ mm}^4$$

$$\text{or } J_O = 3.13 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

(b)

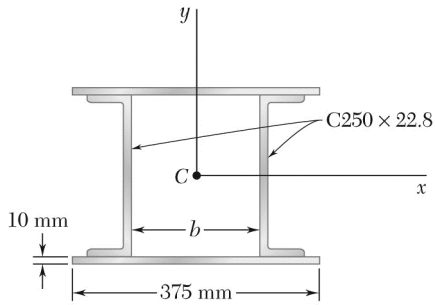
$$J_O = \bar{J}_C + A(\bar{Y})^2$$

or

$$\bar{J}_C = 3.1331 \times 10^6 \text{ mm}^4 - (2544.7 \text{ mm}^2)(16.8067 \text{ mm})^2$$

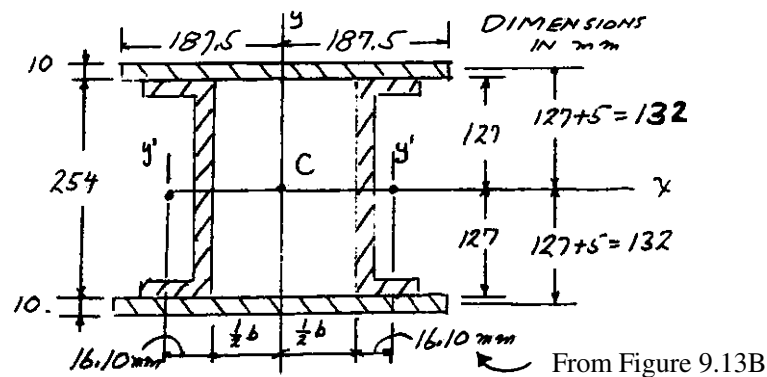
$$\text{or } \bar{J}_C = 2.41 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

PROBLEM 9.49

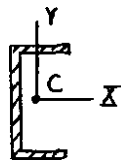


Two channels and two plates are used to form the column section shown. For $b = 200$ mm, determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION



For C250 \times 22.8



$$A = 2890 \text{ mm}^2$$

$$I_x = 28.0 \times 10^6 \text{ mm}^4$$

$$I_y = 0.945 \times 10^6 \text{ mm}^4$$

Total area $A = 2[2890 \text{ mm}^4 + (10 \text{ mm})(375 \text{ mm})] = 13.28 \times 10^3 \text{ mm}^2$

Given $b = 200$ mm:
$$\bar{I}_x = 2[28.0 \times 10^6 \text{ mm}^4] + 2 \left[\frac{1}{12} (375 \text{ mm})(10 \text{ mm})^3 + (375 \text{ mm})(10 \text{ mm})(132 \text{ mm})^2 \right]$$

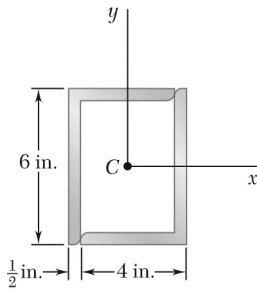
$$= 186.743 \times 10^6 \qquad \bar{I}_x = 186.7 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{k}_x^2 = \frac{\bar{I}_x}{A} = \frac{186.743 \times 10^6}{13.28 \times 10^3} = 14.0620 \times 10^3 \text{ mm}^2 \qquad \bar{k}_x = 118.6 \text{ mm} \blacktriangleleft$$

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PROBLEM 9.49 (Continued)

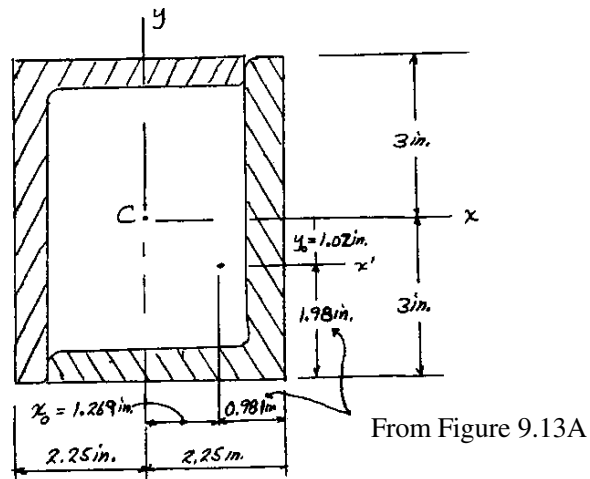
$$\begin{aligned}
 \bar{I}_y &= \Sigma(\bar{I}_y + Ad^2) = 2[\bar{I}_y + Ad^2] + 2\left[\frac{1}{12}(10 \text{ mm})(375 \text{ mm})^3\right] \\
 &= 2\left[0.945 \times 10^6 \text{ mm}^4 + (2890 \text{ mm}^2)\left(\frac{200 \text{ mm}}{2} + 16.10 \text{ mm}\right)^2\right] + 87.891 \times 10^6 \text{ mm}^4 \\
 &= 2[0.945 \times 10^6 + 38.955 \times 10^6] + 87.891 \times 10^6 \\
 &= 167.691 \times 10^6 \text{ mm}^4 \qquad \qquad \qquad \bar{I}_y = 167.7 \times 10^6 \text{ mm}^4 \blacktriangleleft \\
 k_y^2 &= \frac{I_y}{A} = \frac{167.691 \times 10^6}{13.28 \times 10^3} = 12.6273 \times 10^3 \qquad \qquad \qquad \bar{k}_y = 112.4 \text{ mm} \blacktriangleleft
 \end{aligned}$$



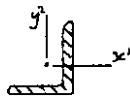
PROBLEM 9.50

Two $L6 \times 4 \times \frac{1}{2}$ -in. angles are welded together to form the section shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION



From Figure 9.13A: Area = $A = 4.75 \text{ in}^2$



$$I_{x'} = 17.3 \text{ in}^4$$

$$I_{y'} = 6.22 \text{ in}^4$$

$$\bar{I}_x = 2[\bar{I}_{x'} + Ay_0^2] = 2[17.3 \text{ in}^4 + (4.75 \text{ in}^2)(1.02 \text{ in.})^2] = 44.484 \text{ in}^4$$

$$\bar{I}_x = 44.5 \text{ in}^4 \quad \blacktriangleleft$$

Total area = $2(4.75) = 9.50 \text{ in}^2$

$$\bar{k}_x^2 = \frac{\bar{I}_x}{\text{area}} = \frac{44.484 \text{ in}^4}{9.50 \text{ in}^2} = 4.6825 \text{ in}^2$$

$$\bar{k}_x = 2.16 \text{ in.} \quad \blacktriangleleft$$

$$\begin{aligned} \bar{I}_y &= 2[\bar{I}_{y'} + Ax_0^2] = 2[6.22 \text{ in}^4 + (4.75 \text{ in}^2)(1.269 \text{ in.})^2] \\ &= 27.738 \text{ in}^4 \end{aligned}$$

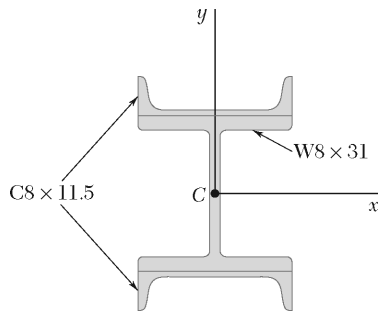
$$\bar{I}_y = 27.7 \text{ in}^4 \quad \blacktriangleleft$$

Total area = $2(4.75) = 9.50 \text{ in}^2$

$$\bar{k}_y^2 = \frac{\bar{I}_y}{\text{area}} = \frac{27.738 \text{ in}^4}{9.50 \text{ in}^2} = 2.9198$$

$$\bar{k}_y = 1.709 \text{ in} \quad \blacktriangleleft$$

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PROBLEM 9.51

Two channels are welded to a rolled W section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION

W section:

$$A = 9.12 \text{ in}^2$$

$$\bar{I}_x = 110 \text{ in}^4$$

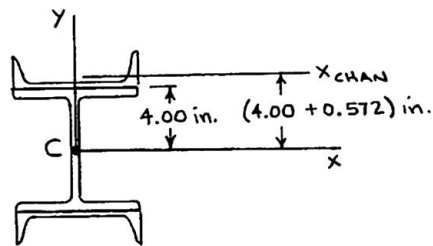
$$\bar{I}_y = 37.1 \text{ in}^4$$

Channel:

$$A = 3.37 \text{ in}^2$$

$$\bar{I}_x = 1.31 \text{ in}^4$$

$$\bar{I}_y = 32.5 \text{ in}^4$$



$$\begin{aligned} A_{\text{total}} &= A_W + 2A_{\text{chan}} \\ &= 9.12 + 2(3.37) = 15.86 \text{ in}^2 \end{aligned}$$

Now

$$\bar{I}_x = (\bar{I}_x)_W + 2(I_x)_{\text{chan}}$$

where

$$\begin{aligned} (I_x)_{\text{chan}} &= \bar{I}_{x_{\text{chan}}} + Ad^2 \\ &= 1.31 \text{ in}^4 + (3.37 \text{ in}^2)(4.572 \text{ in.})^2 = 71.754 \text{ in}^4 \end{aligned}$$

Then

$$\bar{I}_x = (110 + 2 \times 71.754) \text{ in}^4 = 253.51 \text{ in}^4 \quad \bar{I}_x = 254 \text{ in}^4 \quad \blacktriangleleft$$

and

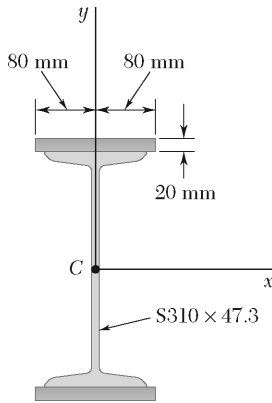
$$\bar{k}_x^2 = \frac{\bar{I}_x}{A_{\text{total}}} = \frac{253.51 \text{ in}^4}{15.86 \text{ in}^2} \quad \bar{k}_x = 4.00 \text{ in.} \quad \blacktriangleleft$$

Also

$$\begin{aligned} \bar{I}_y &= (\bar{I}_y)_W + 2(\bar{I}_y)_{\text{chan}} \\ &= (37.1 + 2 \times 32.5) \text{ in}^4 = 102.1 \text{ in}^4 \quad \bar{I}_y = 102.1 \text{ in}^4 \quad \blacktriangleleft \end{aligned}$$

and

$$\bar{k}_y^2 = \frac{\bar{I}_y}{A_{\text{total}}} = \frac{102.1 \text{ in}^4}{15.86 \text{ in}^2} \quad \bar{k}_y = 2.54 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 9.52

Two 20-mm steel plates are welded to a rolled S section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION

S section:

$$A = 6010 \text{ mm}^2$$

$$\bar{I}_x = 90.3 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 3.88 \times 10^6 \text{ mm}^4$$

Note:

$$\begin{aligned} A_{\text{total}} &= A_S + 2A_{\text{plate}} \\ &= 6010 \text{ mm}^2 + 2(160 \text{ mm})(20 \text{ mm}) \\ &= 12,410 \text{ mm}^2 \end{aligned}$$

Now

$$\bar{I}_x = (\bar{I}_x)_S + 2(I_x)_{\text{plate}}$$

where

$$\begin{aligned} (I_x)_{\text{plate}} &= \bar{I}_{x_{\text{plate}}} + Ad^2 \\ &= \frac{1}{12}(160 \text{ mm})(20 \text{ mm})^3 + (3200 \text{ mm}^2)[(152.5 + 10) \text{ mm}]^2 \\ &= 84.6067 \times 10^6 \text{ mm}^4 \end{aligned}$$

Then

$$\begin{aligned} \bar{I}_x &= (90.3 + 2 \times 84.6067) \times 10^6 \text{ mm}^4 \\ &= 259.5134 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\text{or } \bar{I}_x = 260 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

and

$$\bar{k}_x^2 = \frac{\bar{I}_x}{A_{\text{total}}} = \frac{259.5134 \times 10^6 \text{ mm}^4}{12410 \text{ mm}^2}$$

$$\text{or } \bar{k}_x = 144.6 \text{ mm} \blacktriangleleft$$

Also

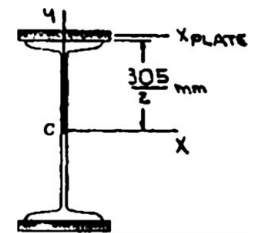
$$\begin{aligned} \bar{I}_y &= (\bar{I}_y)_S + 2(\bar{I}_y)_{\text{plate}} \\ &= 3.88 \times 10^6 \text{ mm}^4 + 2 \left[\frac{1}{12}(20 \text{ mm})(160 \text{ mm})^3 \right] \\ &= 17.5333 \times 10^6 \text{ mm}^4 \end{aligned}$$

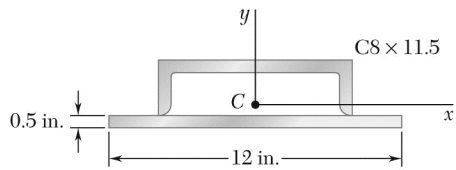
$$\text{or } \bar{I}_y = 17.53 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

and

$$\bar{k}_y^2 = \frac{\bar{I}_y}{A_{\text{total}}} = \frac{17.5333 \times 10^6 \text{ mm}^4}{12,410 \text{ mm}^2}$$

$$\text{or } \bar{k}_y = 37.6 \text{ mm} \blacktriangleleft$$

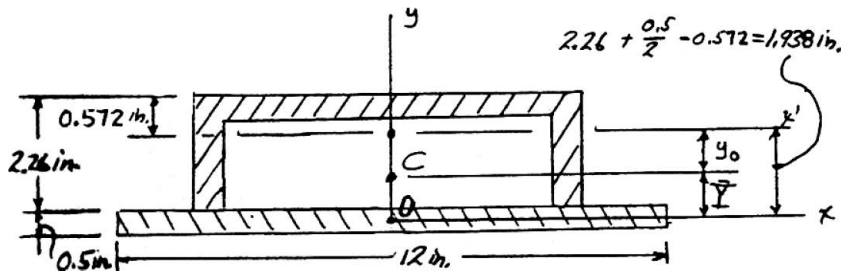




PROBLEM 9.53

A channel and a plate are welded together as shown to form a section that is symmetrical with respect to the y axis. Determine the moments of inertia of the combined section with respect to its centroidal x and y axes.

SOLUTION



From Figure 9.13B

For $C8 \times 11.5$:

(Note change of axes)

Location of centroid

$$\bar{Y} \Sigma A = \Sigma \bar{y} A: \quad \bar{Y} [3.7 \text{ in}^2 + (12 \text{ in.})(0.5 \text{ in.})] = (3.37 \text{ in}^2)(1.938 \text{ in.})$$

$$\bar{Y} = 0.69702 \text{ in.}$$

Moment of inertia with respect to x axis.

Plate:

$$I_x = \bar{I}_x + A\bar{Y}^2 = \frac{1}{12}(12 \text{ in.})(0.5 \text{ in.})^3 + (12 \text{ in.})(0.5 \text{ in.})(0.69702 \text{ in.})^2$$

$$= 0.125 + 2.9150 = 3.0400 \text{ in}^4$$

Channel:

$$I_x = \bar{I}_x + A\bar{Y}_0^2 = 1.31 \text{ in}^4 + (3.37 \text{ in.})(1.938 \text{ in.} - 0.69702 \text{ in.})^2$$

$$= 1.31 + 5.1899 = 6.4999 \text{ in}^4$$

Entire section: $I_x = 3.0400 \text{ in}^4 + 6.4999 \text{ in}^4 = 9.5399 \text{ in}^4$ $\bar{I}_x = 9.54 \text{ in}^4 \blacktriangleleft$

Moment of inertia with respect to y axis.

Plate:

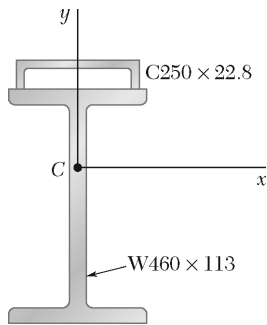
$$\bar{I}_y = \frac{1}{12}(0.5 \text{ in.})(12 \text{ in.})^3 = 72 \text{ in}^4$$

Channel:

$$\bar{I}_y = 32.5 \text{ in}^4$$

Entire section: $\bar{I}_y = 72 \text{ in}^4 + 32.5 \text{ in}^4 = 104.5 \text{ in}^4$ $\bar{I}_y = 104.5 \text{ in}^4 \blacktriangleleft$

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PROBLEM 9.54

The strength of the rolled W section shown is increased by welding a channel to its upper flange. Determine the moments of inertia of the combined section with respect to its centroidal x and y axes.

SOLUTION

W section:

$$A = 14,400 \text{ mm}^2$$

$$\bar{I}_x = 554 \times 10^6 \text{ mm}^4$$

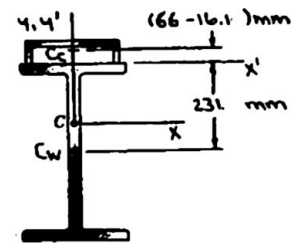
$$\bar{I}_y = 63.3 \times 10^6 \text{ mm}^4$$

Channel:

$$A = 2890 \text{ mm}^2$$

$$\bar{I}_x = 0.945 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 28.0 \times 10^6 \text{ mm}^4$$



First locate centroid C of the section.

	$A, \text{ mm}^2$	$\bar{y}, \text{ mm}$	$\bar{y}A, \text{ mm}^3$
W Section	14,400	-231	-33,26,400
Channel	2,890	49.9	1,44,211
Σ	17,290		-31,82,189

Then

$$\bar{Y} \Sigma A = \Sigma \bar{y} A: \bar{Y}(17,290 \text{ mm}^2) = -3,182,189 \text{ mm}^3$$

or

$$\bar{Y} = -184.047 \text{ mm}$$

Now

$$\bar{I}_x = (I_x)_W + (I_x)_C$$

where

$$(I_x)_W = \bar{I}_x + Ad^2$$

$$= 554 \times 10^6 \text{ mm}^4 + (14,400 \text{ mm}^2)(231 - 184.047)^2 \text{ mm}^2$$

$$= 585.75 \times 10^6 \text{ mm}^4$$

$$(I_x)_C = \bar{I}_x - Ad^2$$

$$= 0.945 \times 10^6 \text{ mm}^4 + (2,890 \text{ mm}^2)(49.9 + 184.047)^2 \text{ mm}^2$$

$$= 159.12 \times 10^6 \text{ mm}^4$$

PROBLEM 9.54 (Continued)

Then

$$\bar{I}_x = (585.75 + 159.12) \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 745 \times 10^6 \text{ mm}^4 \blacktriangleleft$

also

$$\begin{aligned}\bar{I}_y &= (I_y)_W + (I_y)_C \\ &= (63.3 + 28.0) \times 10^6 \text{ mm}^4\end{aligned}$$

or $\bar{I}_y = 91.3 \times 10^6 \text{ mm}^4 \blacktriangleleft$

PROBLEM 9.55

Two L76 × 76 × 6.4-mm angles are welded to a C250 × 22.8 channel. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the web of the channel.

SOLUTION

Angle: $A = 929 \text{ mm}^2$
 $\bar{I}_x = \bar{I}_y = 0.512 \times 10^6 \text{ mm}^4$

Channel: $A = 2890 \text{ mm}^2$
 $\bar{I}_x = 0.945 \times 10^6 \text{ mm}^4$
 $\bar{I}_y = 28.0 \times 10^6 \text{ mm}^4$

First locate centroid C of the section

	$A, \text{ mm}^2$	$\bar{y}, \text{ mm}$	$\bar{y}A, \text{ mm}^3$
Angle	$2(929) = 1858$	21.2	39,389.6
Channel	2890	-16.1	-46,529
Σ	4748		-7139.4

Then $\bar{Y} \Sigma A = \Sigma \bar{y} A: \bar{Y}(4748 \text{ mm}^2) = -7139.4 \text{ mm}^3$
or $\bar{Y} = -1.50366 \text{ mm}$

Now $\bar{I}_x = 2(I_x)_L + (I_x)_C$
where $(I_x)_L = \bar{I}_x + Ad^2 = 0.512 \times 10^6 \text{ mm}^4 + (929 \text{ mm}^2)[(21.2 + 1.50366) \text{ mm}]^2$
 $= 0.990859 \times 10^6 \text{ mm}^4$
 $(I_x)_C = \bar{I}_x + Ad^2 = 0.949 \times 10^6 \text{ mm}^4 + (2890 \text{ mm}^2)[(16.1 - 1.50366) \text{ mm}]^2$
 $= 1.56472 \times 10^6 \text{ mm}^4$

Then $\bar{I}_x = [2(0.990859) + 1.56472 \times 10^6] \text{ mm}^4$
or $\bar{I}_x = 3.55 \times 10^6 \text{ mm}^4 \blacktriangleleft$

PROBLEM 9.55 (Continued)

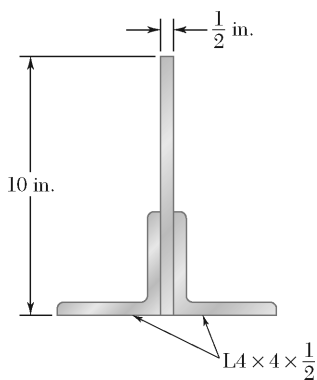
Also
$$\bar{I}_y = 2(I_y)_L + (I_y)_C$$

where
$$(I_y)_L = \bar{I}_y + Ad^2 = 0.512 \times 10^6 \text{ mm}^4 + (929 \text{ mm}^2)[(127 - 21.2) \text{ mm}]^2$$
$$= 10.9109 \times 10^6 \text{ mm}^4$$

$$(I_y)_C = \bar{I}_y$$

Then
$$\bar{I}_y = [2(10.9109) + 28.0] \times 10^6 \text{ mm}^4$$

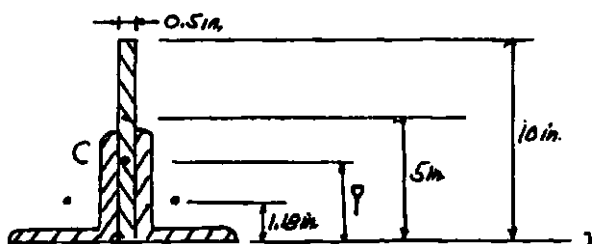
or
$$\bar{I}_y = 49.8 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.56

Two $L4 \times 4 \times \frac{1}{2}$ -in. angles are welded to a steel plate as shown. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the plate.

SOLUTION



For $4 \times 4 \times \frac{1}{2}$ -in. angle:

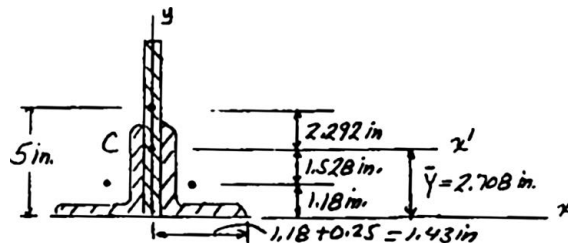
$$A = 3.75 \text{ in}^2, \quad \bar{I}_x = \bar{I}_y = 5.52 \text{ in}^4$$

$$\bar{Y}A = \Sigma \bar{y}A$$

$$\bar{Y}(12.5 \text{ in}^2) = 33.85 \text{ in}^3$$

$$\bar{Y} = 2.708 \text{ in.}$$

Section	Area, in^2	\bar{y} in.	$\bar{y}A$, in^3
Plate	$(0.5)(10) = 5$	5	25
Two angles	$2(3.75) = 7.5$	1.18	8.85
Σ	12.5		33.85

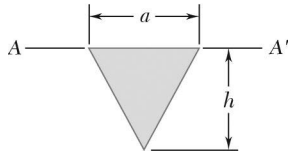


PROBLEM 9.56 (Continued)

Entire section:

$$\begin{aligned}\bar{I}_x &= \Sigma(\bar{I}_{x'} + Ad^2) = \left[\frac{1}{12}(0.5)(10)^3 + (0.5)(10)(2.292)^2 \right] + 2[5.52 + (3.75)(1.528)^2] \\ &= 41.667 + 26.266 + 1604 + 17.511 = 96.48 \text{ in}^4 \qquad \bar{I}_x = 96.5 \text{ in}^4 \blacktriangleleft\end{aligned}$$

$$\begin{aligned}\bar{I}_y &= \frac{1}{12}(10)(0.5)^3 + 2[5.52 + (3.75)(1.43)^2] \\ &= 0.104 + 11.04 + 15.367 = 26.51 \text{ in}^4 \qquad \bar{I}_y = 26.5 \text{ in}^4 \blacktriangleleft\end{aligned}$$



PROBLEM 9.57

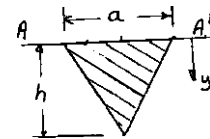
The panel shown forms the end of a trough that is filled with water to the line AA' . Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

From section 9.2: $R = \gamma \int y dA$, $M_{AA'} = \gamma \int y^2 dA$

Let y_P = distance of center of pressure from AA' . We must have:

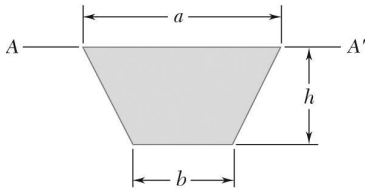
$$Ry_P = M_{AA'}: \quad y_P = \frac{M_{AA'}}{R} = \frac{\gamma \int y^2 dA}{\gamma \int y dA} = \frac{I_{AA'}}{\bar{y} A} \quad (1)$$



For triangular panel:

$$I_{AA'} = \frac{1}{12} ah^3 \quad \bar{y} = \frac{1}{3} h \quad A = \frac{1}{2} ah$$

$$y_P = \frac{I_{AA'}}{\bar{y} A} = \frac{\frac{1}{12} ah^3}{\left(\frac{1}{3} h\right) \left(\frac{1}{2} ah\right)} \quad y_P = \frac{1}{2} h \quad \blacktriangleleft$$

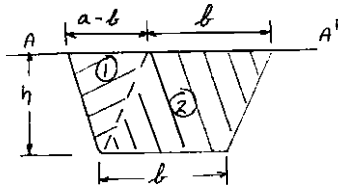


PROBLEM 9.58

The panel shown forms the end of a trough that is filled with water to the line AA' . Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

See solution of Problem 9.57 for derivation of Eq. (1):



$$y_P = \frac{I_{AA'}}{\bar{y}A} \quad (1)$$

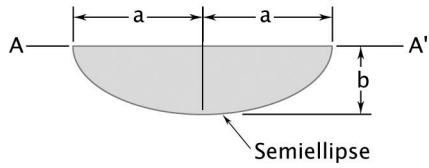
Divide trapezoid as shown:

$$\begin{aligned} I_{AA'} &= \frac{1}{12}(a-b)h^3 + \frac{1}{3}bh^3 \\ &= \frac{1}{2}ah^3 + \frac{1}{4}bh^3 \end{aligned}$$

$$\bar{y}A = y_1A_1 + y_2A_2 = \frac{1}{3}h \left[\frac{1}{2}(a-b)h \right] + \frac{1}{2}h(bh) = \frac{1}{6}ah^2 + \frac{1}{3}bh^2$$

$$y_P = \frac{I_{AA'}}{\bar{y}A} = \frac{\frac{1}{2}ah^3 + \frac{1}{4}bh^3}{\frac{1}{6}ah^2 + \frac{1}{3}bh^2}$$

$$y_P = \frac{a+3b}{2a+4b}h \quad \blacktriangleleft$$



PROBLEM 9.59

The panel shown forms the end of a trough that is filled with water to the line AA' . Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

Using the equation developed on page 475 of the text have:

$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

For a semi ellipse:

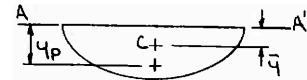
$$I_{AA'} = \frac{\pi}{8} ab^3$$

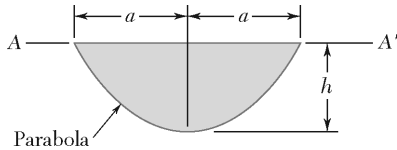
$$\bar{y} = \frac{4b}{3\pi} \quad A = \frac{\pi}{2} ab$$

Then

$$y_P = \frac{\frac{\pi}{8} ab^3}{\frac{4b}{3\pi} \times \frac{\pi}{2} ab} \text{ or}$$

$$\text{or } y_P = \frac{3\pi}{16} b \quad \blacktriangleleft$$





PROBLEM 9.60*

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

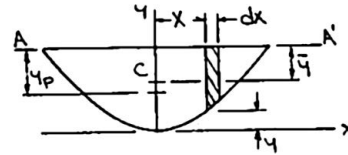
SOLUTION

Using the equation developed on page 491 of the text:

$$y_p = \frac{I_{AA'}}{\bar{y}A}$$

For a parabola:

$$\bar{y} = \frac{2}{5}h \quad A = \frac{4}{3}ah$$



Now

$$dI_{AA'} = \frac{1}{3}(h-y)^3 dx$$

By observation

$$y = \frac{h}{a^2}x^2$$

So that

$$\begin{aligned} dI_{AA'} &= \frac{1}{3} \left(h - \frac{h}{a^2}x^2 \right)^3 dx = \frac{1}{3} \frac{h^3}{a^6} (a^2 - x^2)^3 dx \\ &= \frac{1}{3} \frac{h}{a^6} (a^6 - 3a^4x^2 + 3a^2x^4 - x^6) dx \end{aligned}$$

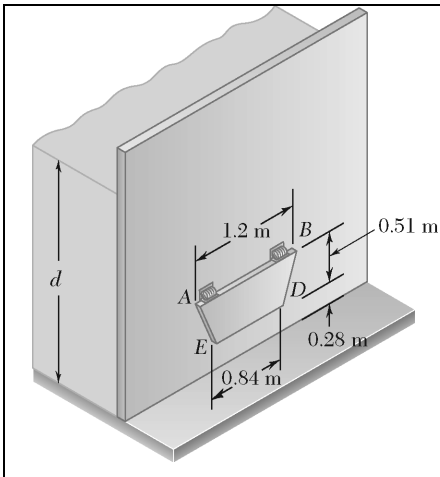
Then

$$\begin{aligned} I_{AA'} &= 2 \int_0^a \frac{1}{3} \frac{h^3}{a^6} (a^6 - 3a^4x^2 + 3a^2x^4 - x^6) dx \\ &= \frac{2}{3} \frac{h^3}{a^6} \left[a^6x - a^4x^3 + \frac{3}{5}a^2x^5 - \frac{1}{7}x^7 \right]_0^a \\ &= \frac{32}{105} ah^3 \end{aligned}$$

Finally,

$$y_p = \frac{\frac{32}{105} ah^3}{\frac{2}{5}h \times \frac{4}{3}ah}$$

or $y_p = \frac{4}{7}h \blacktriangleleft$



PROBLEM 9.61

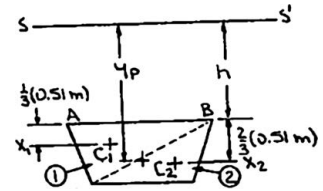
A vertical trapezoidal gate that is used as an automatic valve is held shut by two springs attached to hinges located along edge AB . Knowing that each spring exerts a couple of magnitude $1470 \text{ N} \cdot \text{m}$, determine the depth d of water for which the gate will open.

SOLUTION

From section 9.2:

$$R = \gamma \bar{y}A \quad y_P = \frac{I_{SS'}}{\bar{y}A}$$

where R is the resultant of the hydrostatic forces acting on the gate and y_P is the depth to the point of application of R . Now



$$\begin{aligned} \bar{y}A &= \Sigma \bar{y}A = [(h + 0.17) \text{ m}] \left(\frac{1}{2} \times 1.2 \text{ m} \times 0.51 \text{ m} \right) + [(h + 0.34) \text{ m}] \left(\frac{1}{2} \times 0.84 \times 0.51 \text{ m} \right) \\ &= (0.5202h + 0.124848) \text{ m}^3 \end{aligned}$$

Recalling that $\gamma = \rho g$, we have

$$\begin{aligned} R &= (10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2)(0.5202h + 0.124848) \text{ m}^3 \\ &= 5103.162(h + 0.24) \text{ N} \end{aligned}$$

Also

$$I_{SS'} = (I_{SS'})_1 + (I_{SS'})_2$$

where

$$\begin{aligned} (I_{SS'})_1 &= I_x + Ad^2 \\ &= \frac{1}{36}(1.2 \text{ m})(0.51 \text{ m})^3 + \left(\frac{1}{2} \times 1.2 \text{ m} \times 0.51 \text{ m} \right) [(h + 0.17) \text{ m}]^2 \\ &= [0.0044217 + 0.306(h + 0.17)^2] \text{ m}^4 \\ &= (0.306h^2 + 0.10404h + 0.0132651) \text{ m}^4 \end{aligned}$$

$$\begin{aligned} (I_{SS'})_2 &= \bar{I}_{x_2} + Ad^2 \\ &= \frac{1}{36}(0.84 \text{ m})(0.51 \text{ m})^3 + \left(\frac{1}{2} \times 0.84 \text{ m} \times 0.51 \text{ m} \right) [(h + 0.34) \text{ m}]^2 \\ &= [0.0030952 + 0.2142(h + 0.34)^2] \text{ m}^4 \\ &= (0.2142h^2 + 0.145656h + 0.0278567) \text{ m}^4 \end{aligned}$$

PROBLEM 9.61 (Continued)

Then
$$I_{SS'} = (I_{SS'})_1 + (I_{SS'})_2$$
$$= (0.5202h^2 + 0.249696h + 0.0411218) \text{ m}^4$$

and
$$y_P = \frac{(0.5202h^2 + 0.244696h + 0.0411218) \text{ m}^4}{(0.5202h + 0.124848) \text{ m}^3}$$
$$= \frac{h^2 + 0.48h + 0.07905}{h + 0.24} \text{ m}$$

For the gate to open, require that

$$\Sigma M_{AB}: M_{\text{open}} = (y_P - h)R$$

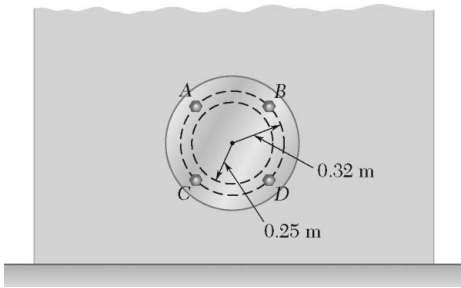
Substituting
$$2940 \text{ N} \cdot \text{m} = \left(\frac{h^2 + 0.48h + 0.07905}{h + 0.24} - h \right) \text{ m} \times 5103.162(h + 0.24) \text{ N}$$

or
$$5103.162(0.24h + 0.07905) = 2940$$

or
$$h = 2.0711 \text{ m}$$

Then
$$d = (2.0711 + 0.79) \text{ m}$$

or
$$d = 2.86 \text{ m} \blacktriangleleft$$



PROBLEM 9.62

The cover for a 0.5-m-diameter access hole in a water storage tank is attached to the tank with four equally spaced bolts as shown. Determine the additional force on each bolt due to the water pressure when the center of the cover is located 1.4 m below the water surface.

SOLUTION

From section 9.2:

$$R = \gamma \bar{y} A \quad y_P = \frac{I_{AA'}}{\bar{y} A}$$

where R is the resultant of the hydrostatic forces acting on the cover and y_P is the depth to the point of application of R .

Recalling that $\gamma = p \cdot y$, we have

$$\begin{aligned} R &= (10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2)(1.4 \text{ m})[\pi(0.25 \text{ m})^2] \\ &= 2696.67 \text{ N} \end{aligned}$$

Also

$$\begin{aligned} I_{AA'} &= \bar{I}_x + A\bar{y}^2 = \frac{\pi}{4}(0.25 \text{ m})^4 + [\pi(0.25 \text{ m})^2](1.4 \text{ m})^2 \\ &= 0.387913 \text{ m}^4 \end{aligned}$$

Then

$$y_P = \frac{0.387913 \text{ m}^4}{(1.4 \text{ m})[\pi(0.25 \text{ m})^2]} = 1.41116 \text{ m}$$

Now note that symmetry implies

$$F_A = F_B \quad F_C = F_D$$

Next consider the free-body of the cover.

$$\begin{aligned} \text{We have } \quad +\curvearrowright \Sigma M_{CD} = 0: & [2(0.32 \text{ m}) \sin 45^\circ](2F_A) \\ & - [0.32 \sin 45^\circ - (1.41116 - 1.4)] \text{ m} \\ & \times (2696.67 \text{ N}) = 0 \end{aligned}$$

or

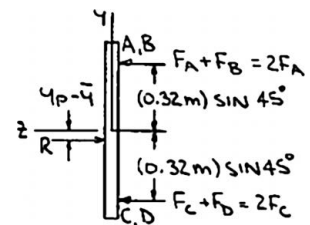
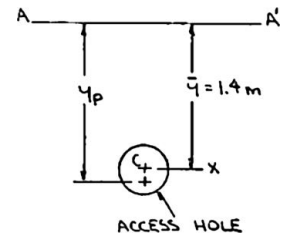
$$F_A = 640.92 \text{ N}$$

$$\text{Then } \quad \leftarrow \Sigma F_z = 0: \quad 2(640.92 \text{ N}) + 2F_C - 2696.67 \text{ N} = 0$$

or

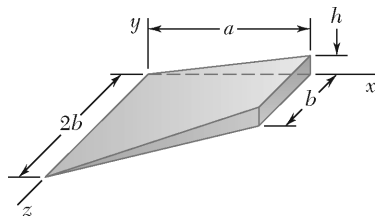
$$F_C = 707.42 \text{ N}$$

and



$$F_A = F_B = 641 \text{ N} \quad \blacktriangleleft$$

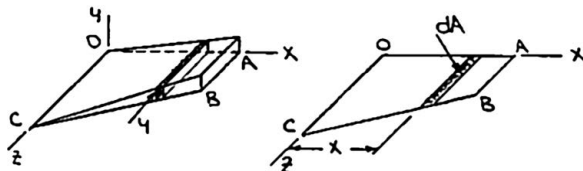
$$F_C = F_D = 707 \text{ N} \quad \blacktriangleleft$$



PROBLEM 9.63*

Determine the x coordinate of the centroid of the volume shown. (Hint: The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



First note that

$$y = \frac{h}{a}x$$

Now

$$\bar{x} \int dV = \int \bar{x}_{EL} dV$$

where

$$\bar{x}_{EL} = x \quad dV = y dA = \left(\frac{h}{a}x\right) dA$$

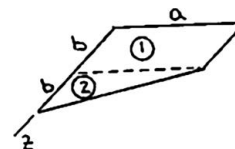
Then

$$\bar{x} = \frac{\int x \left(\frac{h}{a}x dA\right)}{\int \frac{h}{a}x dA} = \frac{\int x^2 dA}{\int x dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

where $(I_z)_A$ and $(\bar{x}A)_A$ pertain to area.

$OABC$: $(I_z)_A$ is the moment of inertia of the area with respect to the z axis, \bar{x}_A is the x coordinate of the centroid of the area, and A is the area of $OABC$. Then

$$\begin{aligned} (I_z)_A &= (I_z)_{A_1} + (I_z)_{A_2} \\ &= \frac{1}{3}(b)(a)^3 + \frac{1}{12}(b)(a)^3 \\ &= \frac{5}{12}a^3b \end{aligned}$$



and

$$\begin{aligned} (\bar{x}A)_A &= \Sigma \bar{x}A \\ &= \left[\left(\frac{a}{2}\right)(a \times b) \right] + \left[\left(\frac{a}{3}\right) \left(\frac{1}{2} \times a \times b\right) \right] \\ &= \frac{2}{3}a^2b \end{aligned}$$

PROBLEM 9.63* (Continued)

Finally,
$$\bar{x} = \frac{\frac{5}{12}a^3b}{\frac{2}{3}a^2b}$$

or
$$\bar{x} = \frac{5}{8}a \blacktriangleleft$$

Analogy with hydrostatic pressure on a submerged plate:

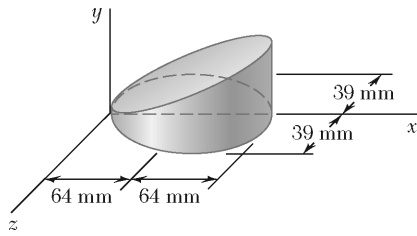
Recalling that $P = \gamma y$, it follows that the following analogies can be established.

Height $y \sim P$

$$\begin{aligned}dV &= ydA \sim pdA = dF \\xdV &= x(ydA) \sim ydF = dM\end{aligned}$$

Recalling that
$$y_P = \frac{M_x}{R} \left(= \frac{\int dM}{\int dF} \right)$$

It can then be concluded that $x \sim y_P$



PROBLEM 9.64*

Determine the x coordinate of the centroid of the volume shown; this volume was obtained by intersecting an elliptic cylinder with an oblique plane. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION

Following the “Hint,” it can be shown that (see solution to Problem 9.63)

$$\bar{x} = \frac{(I_z)_A}{(\bar{x}A)_A} x$$

where $(I_z)_A$ and $(\bar{x}A)_A$ are the moment of inertia and the first moment of the area, respectively, of the elliptical area of the base of the volume with respect to the z axis. Then

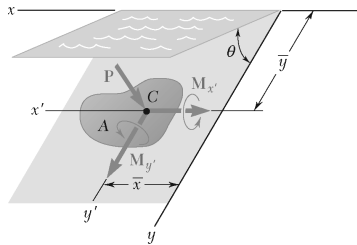
$$\begin{aligned} (I_z)_A &= \bar{I}_z + Ad^2 \\ &= \frac{\pi}{4} (39 \text{ mm})(64 \text{ mm})^3 + [\pi(64 \text{ mm})(39 \text{ mm})](64 \text{ mm})^2 \\ &= 12.779520\pi \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} (\bar{x}A)_A &= (64 \text{ mm})[\pi(64 \text{ mm})(39 \text{ mm})] \\ &= 0.159744\pi \times 10^6 \text{ mm}^3 \end{aligned}$$

Finally

$$\bar{x} = \frac{12.779520\pi \times 10^6 \text{ mm}^4}{0.159744\pi \times 10^6 \text{ mm}^3}$$

$$\text{or } \bar{x} = 80.0 \text{ mm} \blacktriangleleft$$



PROBLEM 9.65*

Show that the system of hydrostatic forces acting on a submerged plane area A can be reduced to a force \mathbf{P} at the centroid C of the area and two couples. The force \mathbf{P} is perpendicular to the area and is of magnitude $P = \gamma A \bar{y} \sin \theta$, where γ is the specific weight of the liquid, and the couples are $\mathbf{M}_{x'} = (\gamma \bar{I}_{x'} \sin \theta) \mathbf{i}$ and $\mathbf{M}_{y'} = (\gamma \bar{I}_{y'} \sin \theta) \mathbf{j}$, where $\bar{I}_{x'y'} = \int x'y' dA$ (see section 9.8). Note that the couples are independent of the depth at which the area is submerged.

SOLUTION

The pressure p at an arbitrary depth $(y \sin \theta)$ is

$$p = \gamma(y \sin \theta)$$

so that the hydrostatic force dF exerted on an infinitesimal area dA is

$$dF = (\gamma y \sin \theta) dA$$

Equivalence of the force \mathbf{P} and the system of infinitesimal forces dF requires

$$\Sigma F: P = \int dF = \int \gamma y \sin \theta dA = \gamma \sin \theta \int y dA$$

$$\text{or } P = \gamma A \bar{y} \sin \theta \quad \blacktriangleleft$$

Equivalence of the force and couple $(\mathbf{P}, \mathbf{M}_{x'} + \mathbf{M}_{y'})$ and the system of infinitesimal hydrostatic forces requires

$$\Sigma M_x: -\bar{y}P - M_{x'} = \int (-y dF)$$

$$\begin{aligned} \text{Now} \quad -\int y dF &= -\int y(\gamma y \sin \theta) dA = -\gamma \sin \theta \int y^2 dA \\ &= -(\gamma \sin \theta) I_x \end{aligned}$$

$$\text{Then} \quad -\bar{y}P - M_{x'} = -(\gamma \sin \theta) I_x$$

$$\begin{aligned} \text{or} \quad M_{x'} &= (\gamma \sin \theta) I_x - \bar{y}(\gamma A \bar{y} \sin \theta) \\ &= \gamma \sin \theta (I_x - A \bar{y}^2) \end{aligned}$$

$$\text{or } M_{x'} = \gamma \bar{I}_{x'} \sin \theta \quad \blacktriangleleft$$

$$\Sigma M_y: \bar{x}P + M_{y'} = \int x dF$$

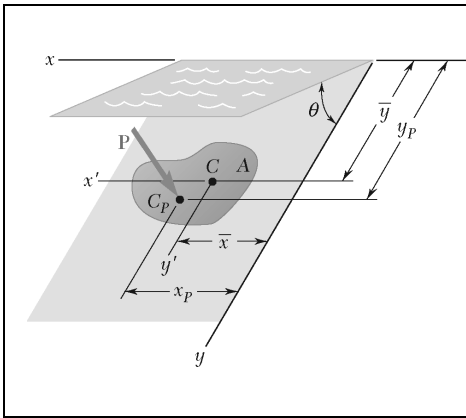
$$\begin{aligned} \text{Now} \quad \int x dF &= \int x(\gamma y \sin \theta) dA = \gamma \sin \theta \int xy dA \\ &= (\gamma \sin \theta) I_{xy} \end{aligned} \quad \text{(Equation 9.12)}$$

PROBLEM 9.65* (Continued)

Then $\bar{x}P + M_{y'} = (\gamma \sin \theta) I_{xy}$

or
$$M_{y'} = (\gamma \sin \theta) I_{xy} - \bar{x}(\gamma A \bar{y} \sin \theta)$$
$$= \gamma \sin \theta (I_{xy} - A \bar{x} \bar{y})$$

or $M_{y'} = \gamma \bar{I}_{x'y'} \sin \theta \blacktriangleleft$



PROBLEM 9.66*

Show that the resultant of the hydrostatic forces acting on a submerged plane area A is a force \mathbf{P} perpendicular to the area and of magnitude $P = \gamma A \bar{y} \sin \theta = \bar{p}A$, where γ is the specific weight of the liquid and \bar{p} is the pressure at the centroid C of the area. Show that \mathbf{P} is applied at a Point C_p , called the center of pressure, whose coordinates are $x_p = I_{xy}/A\bar{y}$ and $y_p = I_x/A\bar{y}$, where $I_{xy} = \int xy dA$ (see section 9.8). Show also that the difference of ordinates $y_p - \bar{y}$ is equal to \bar{k}_x^2/\bar{y} and thus depends upon the depth at which the area is submerged.

SOLUTION

The pressure P at an arbitrary depth $(y \sin \theta)$ is

$$P = \gamma(y \sin \theta)$$

so that the hydrostatic force dP exerted on an infinitesimal area dA is

$$dP = (\gamma y \sin \theta) dA$$

The magnitude \mathbf{P} of the resultant force acting on the plane area is then

$$\begin{aligned} P &= \int dP = \int \gamma y \sin \theta dA = \gamma \sin \theta \int y dA \\ &= \gamma \sin \theta (\bar{y}A) \end{aligned}$$

Now

$$\bar{p} = \gamma \bar{y} \sin \theta \qquad P = \bar{p}A \quad \blacktriangleleft$$

Next observe that the resultant \mathbf{P} is equivalent to the system of infinitesimal forces $d\mathbf{P}$. Equivalence then requires

$$\Sigma M_x: -y_p P = -\int y dP$$

Now

$$\begin{aligned} \int y dP &= \int y(\gamma y \sin \theta) dA = \gamma \sin \theta \int y^2 dA \\ &= (\gamma \sin \theta) I_x \end{aligned}$$

Then

$$y_p P = (\gamma \sin \theta) I_x$$

or

$$y_p = \frac{(\gamma \sin \theta) I_x}{\gamma \sin \theta (\bar{y}A)}$$

$$\text{or} \quad y_p = \frac{I_x}{A\bar{y}} \quad \blacktriangleleft$$

PROBLEM 9.66* (Continued)

$$\Sigma M_y: x_p P = \int x dP$$

Now
$$\int x dP = \int x(\gamma y \sin \theta) dA = \gamma \sin \theta \int xy dA$$

$$= (\gamma \sin \theta) I_{xy} \quad \text{(Equation 9.12)}$$

Then
$$x_p P = (\gamma \sin \theta) I_{xy}$$

or
$$x_p = \frac{(\gamma \sin \theta) I_{xy}}{\gamma \sin \theta (\bar{y} A)}$$

or
$$x_p = \frac{I_{xy}}{A \bar{y}} \blacktriangleleft$$

Now
$$I_x = \bar{I}_{x'} + A \bar{y}^2$$

From above
$$I_x = (A \bar{y}) y_p$$

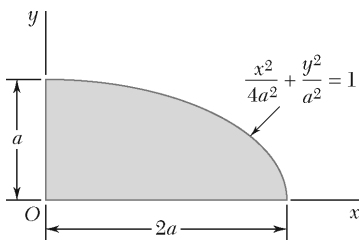
By definition
$$\bar{I}_{x'} = \bar{k}_{x'}^2 A$$

Substituting
$$(A \bar{y}) y_p = \bar{k}_{x'}^2 A + A \bar{y}^2$$

Rearranging yields
$$y_p - \bar{y} = \frac{\bar{k}_{x'}^2}{\bar{y}} \blacktriangleleft$$

Although $\bar{k}_{x'}$ is not a function of the depth of the area (it depends only on the shape of A), \bar{y} is dependent on the depth.

$$(y_p - \bar{y}) = f(\text{depth})$$



PROBLEM 9.67

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION

First note

$$y = a\sqrt{1 - \frac{x^2}{4a^2}}$$

$$= \frac{1}{2}\sqrt{4a^2 - x^2}$$

We have

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where

$$d\bar{I}_{x'y'} = 0 \quad (\text{symmetry}) \quad \bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{4}\sqrt{4a^2 - x^2}$$

$$dA = ydx = \frac{1}{2}\sqrt{4a^2 - x^2}dx$$

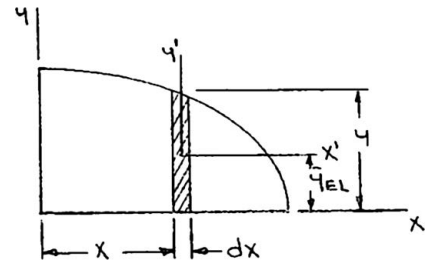
Then

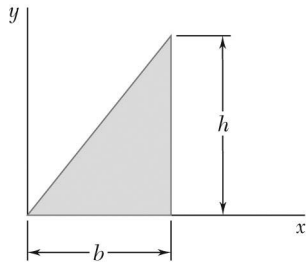
$$I_{xy} = \int dI_{xy} = \int_0^{2a} x \left(\frac{1}{4}\sqrt{4a^2 - x^2} \right) \left(\frac{1}{2}\sqrt{4a^2 - x^2} \right) dx$$

$$= \frac{1}{8} \int_0^{2a} (4a^2x - x^3) dx = \frac{1}{8} \left[2a^2x^2 - \frac{1}{4}x^4 \right]_0^{2a}$$

$$= \frac{a^4}{8} \left[2(2)^2 - \frac{1}{4}(2)^4 \right]$$

or $I_{xy} = \frac{1}{2}a^4 \blacktriangleleft$





PROBLEM 9.68

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

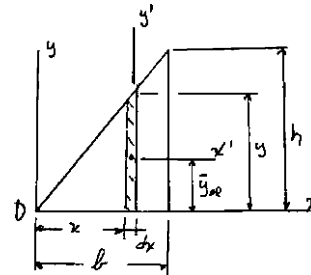
But

$$d\bar{I}_{x'y'} = 0 \quad (\text{by symmetry}) \quad \bar{x}_{EL} = x \quad \bar{y}_{EL} = \frac{1}{2}y$$

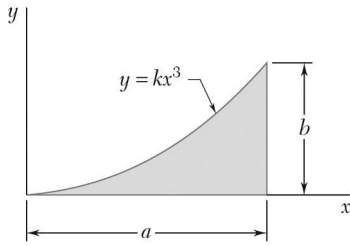
$$dA = yd_y$$

$$\frac{y}{x} = \frac{h}{b} \quad y = \frac{h}{b}x$$

$$\begin{aligned} I_{xy} &= \int dI_{xy} = \int_0^b x \left(\frac{1}{2}y \right) y dx = \frac{1}{2} \int_0^b x \left(\frac{h}{b}x \right)^2 dx \\ &= \frac{1}{2} \frac{h^2}{b^2} \int_0^b x^3 dx = \frac{1}{2} \frac{h^2}{b^2} \frac{b^4}{4} \end{aligned}$$



$$I_{xy} = \frac{1}{8}b^2h^2 \blacktriangleleft$$



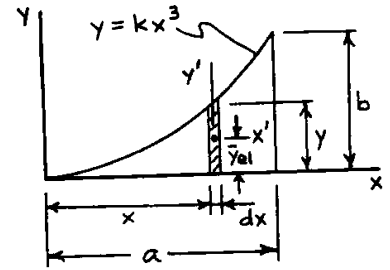
PROBLEM 9.69

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION

For $x = a, \quad b = ka^3 \quad \text{or} \quad k = \frac{b}{a^3}$

Thus, $y = \frac{b}{a^3} x^3$

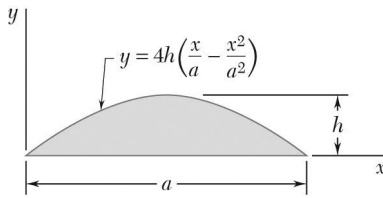


$$d\bar{I}_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA, \quad \text{But} \quad dI_{x'y'} = 0 \quad (\text{by symmetry})$$

with $\bar{x}_{EL} = x \quad \bar{y}_{EL} = \frac{y}{2} \quad dA = ydx$

$$\begin{aligned} I_{xy} &= \int dI_{xy} = \int_0^a x \left(\frac{y}{2} \right) y dx = \frac{1}{2} \int_0^a x \left(\frac{b}{a^3} x^3 \right)^2 dx \\ &= \frac{1}{2} \frac{b^2}{a^6} \left[\frac{x^8}{8} \right]_0^a \end{aligned}$$

$$I_{xy} = a^2 b^2 / 16 \quad \blacktriangleleft$$



PROBLEM 9.70

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION

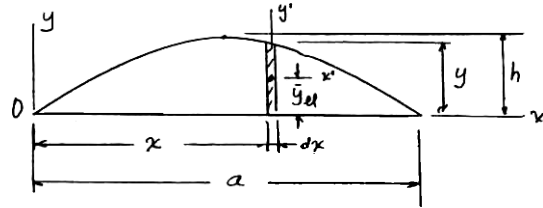
$$y = 4h \left(\frac{x}{a} - \frac{x^2}{a^2} \right)$$

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

But

$$d\bar{I}_{x'y'} = 0 \quad (\text{by symmetry})$$

$$\bar{x}_{EL} = x \quad \bar{y}_{EL} = \frac{1}{2}y \quad dA = ydx$$



$$I_{xy} = \int dI_{xy} = \int_0^a x \left(\frac{1}{2}y \right) y dx = \frac{1}{2} \int_0^a 16h^2 x \left(\frac{x}{a} - \frac{x^2}{a^2} \right) dx$$

$$= 8h^2 \int_0^a \left[\frac{x^3}{a^2} - 2\frac{x^4}{a^3} + \frac{x^5}{a^4} \right] dx = 8h^2 \left[\frac{x^4}{4a^2} - \frac{2x^5}{5a^3} + \frac{x^6}{6a^4} \right]_0^a$$

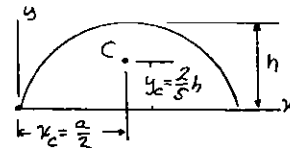
$$= 8h^2 a^2 \left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6} \right) = 8h^2 a^2 \left(\frac{15 - 24 + 10}{60} \right)$$

$$I_{xy} = \frac{2}{15} a^2 h^2 \quad \blacktriangleleft$$

Check: Apply parallel axis theorem to area see Figure 5.8:

$$\text{Area} = \frac{2}{3} ah$$

$$\text{by symmetry } I_{x'y'} = 0$$



$$I_{xy} = I_{x'y'} + x_c y_c A = 0 + \left(\frac{a}{2} \right) \left(\frac{2}{5} h \right) \left(\frac{2}{3} ah \right) = \frac{2}{15} a^2 h^2$$

PROBLEM 9.71

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION

We have
$$\bar{I}_{xy} = (I_{xy})_1 + (\bar{I}_{xy})_2 + (I_{xy})_3$$

Now symmetry implies
$$(\bar{I}_{xy})_2 = 0$$

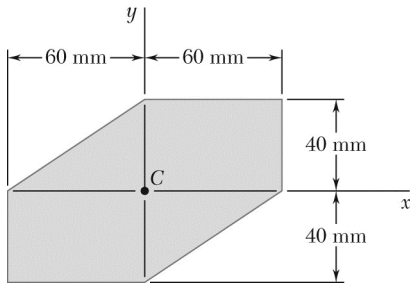
and for the other rectangles
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

where
$$I_{x'y'} = 0 \quad (\text{symmetry})$$

Thus
$$\bar{I}_{xy} = (\bar{x}\bar{y}A)_1 + (\bar{x}\bar{y}A)_3$$

	$A, \text{ mm}^2$	$\bar{x}, \text{ mm}$	$\bar{y}, \text{ mm}$	$\bar{x}\bar{y}A, \text{ mm}^4$
1	$10 \times 80 = 800$	-55	20	-880,000
3	$10 \times 80 = 800$	55	-20	-880,000
Σ				-1,760,000

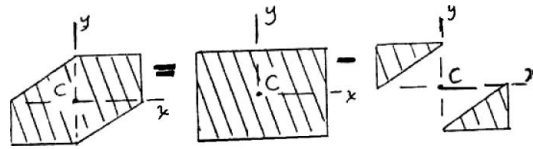
$\bar{I}_{xy} = -1.760 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$



PROBLEM 9.72

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

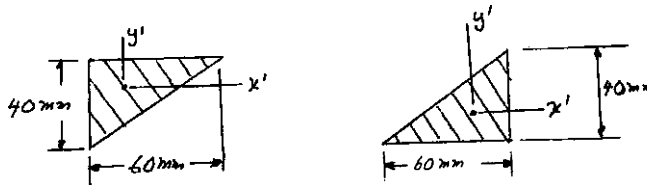
SOLUTION



Given area = Rectangle - (Two triangles)

For rectangle $I_{xy} = 0$, by symmetry

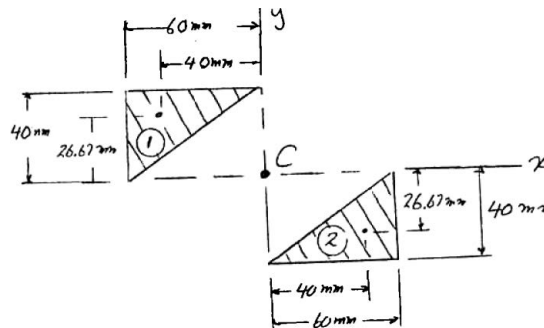
For each triangle:



Compare these triangles with the triangle of sample Problem 9.6, where $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2$.

For orientation of axes of this problem,

$$\bar{I}_{x'y'} = +\frac{1}{72}b^2h^2 = \frac{1}{72}(60 \text{ mm})^2(40 \text{ mm})^2 = +80 \times 10^3 \text{ mm}^4$$



PROBLEM 9.72 (Continued)

	Area, mm ²	\bar{x} , mm	\bar{y} , mm	$\bar{x}\bar{y}A$, mm ⁴
1	$\frac{1}{2}(60)(40) = 1200$	-40	+26.67	-1.280×10^6
2	$\frac{1}{2}(60)(40) = 1200$	+40	-26.67	-1.280×10^6

$$\Sigma \bar{x} \bar{y} A = -2.56 \times 10^6 \text{ mm}^4$$

For two triangles:

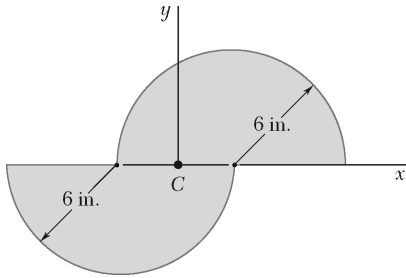
$$\begin{aligned} I_{xy} &= \Sigma(\bar{I}_{x'y'} + \bar{x} \bar{y} A) = +2(80 \times 10^3) - 2.56 \times 10^6 \\ &= -2.40 \times 10^6 \text{ mm}^4 \end{aligned}$$

Since we must subtract triangles,

$$I_{xy} = +2.40 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

PROBLEM 9.73

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.



SOLUTION

We have

$$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$$

For each semicircle

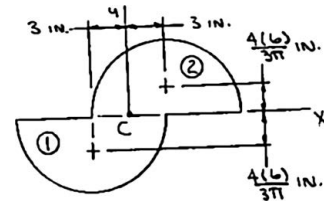
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

and

$$I_{x'y'} = 0 \quad (\text{symmetry})$$

Thus

$$\bar{I}_{xy} = \Sigma \bar{x}\bar{y}A$$



	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}\bar{y}A$
1	$\frac{\pi}{2}(6)^2 = 18\pi$	-3	$-\frac{8}{\pi}$	432
2	$\frac{\pi}{2}(6)^2 = 18\pi$	3	$\frac{8}{\pi}$	432
Σ				864

$$\bar{I}_{xy} = 864 \text{ in}^4 \quad \blacktriangleleft$$

PROBLEM 9.74

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION

We have
$$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$$

For each rectangle
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

and
$$\bar{I}_{x'y'} = 0 \quad (\text{symmetry})$$

Thus
$$\bar{I}_{xy} = \Sigma \bar{x}\bar{y}A$$

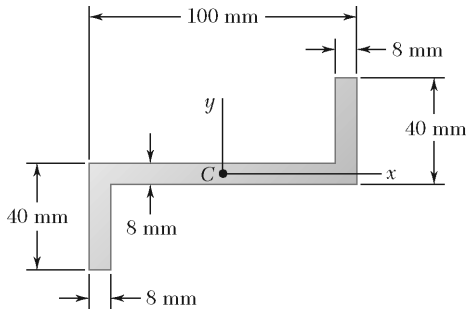
	$A, \text{ in}^2$	$\bar{x}, \text{ in.}$	$\bar{y}, \text{ in.}$	$\bar{x}\bar{y}A, \text{ in}^4$
1	$3 \times 0.25 = 0.75$	-0.520	0.362	-0.141180
2	$0.25 \times 1.75 = 0.4375$	0.855	-0.638	-0.238652
Σ				-0.379832

$\bar{I}_{xy} = -0.380 \text{ in}^4 \blacktriangleleft$

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PROBLEM 9.75

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.



SOLUTION

We have

$$I_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$$

Now symmetry implies

$$(\bar{I}_{xy})_1 = 0$$

and for the other rectangles

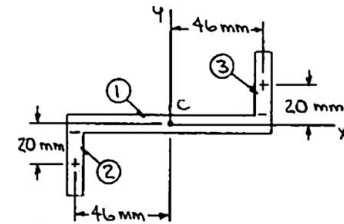
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

where

$$\bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

Thus

$$\bar{I}_{xy} = (\bar{x}\bar{y}A)_2 + (\bar{x}\bar{y}A)_3$$

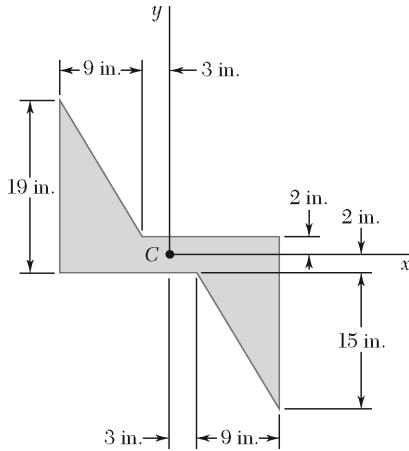


	A, mm^2	\bar{X}, mm	\bar{y}, mm	$\bar{x}\bar{y}A, \text{mm}^4$
2	$8 \times 32 = 256$	-46	-20	235,520
3	$8 \times 32 = 256$	46	20	235,520
Σ				471,040

$$\bar{I}_{xy} = 471 \times 10^3 \text{ mm}^4 \quad \blacktriangleleft$$

PROBLEM 9.76

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.



SOLUTION

We have

$$\bar{I}_{xy} = (\bar{I}_{xy})_1 + (\bar{I}_{xy})_2 + (\bar{I}_{xy})_3$$

Now, symmetry implies

$$(\bar{I}_{xy})_1 = 0$$

and for each triangle

$$I_{xy} = \bar{I}_{x'y'} + \bar{x} \bar{y} A$$

where, using the results of Sample Problem 9.6, $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2$ for both triangles. Note that the sign of $\bar{I}_{x'y'}$ is unchanged because the angles of rotation are 0° and 180° for triangles 2 and 3, respectively.

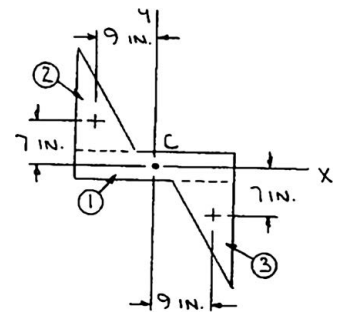
Now

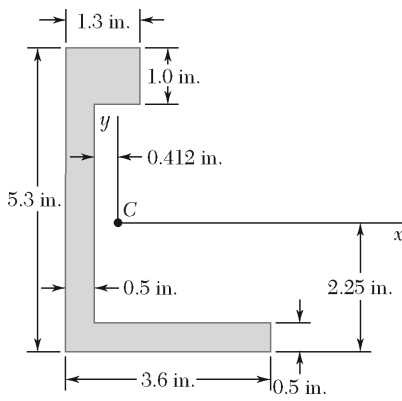
	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x} \bar{y} A, \text{in}^4$
2	$\frac{1}{2}(9)(15) = 67.5$	-9	7	-4252.5
3	$\frac{1}{2}(9)(15) = 67.5$	9	-7	-4252.5
Σ				-8505

Then

$$\begin{aligned} \bar{I}_{xy} &= 2 \left[-\frac{1}{72}(9 \text{ in.})^2 (15 \text{ in.})^2 \right] - 8505 \text{ in}^4 \\ &= -9011.25 \text{ in}^4 \end{aligned}$$

or $\bar{I}_{xy} = -9010 \text{ in}^4 \blacktriangleleft$





PROBLEM 9.77

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION

We have

$$\bar{I}_{xy} = (\bar{I}_{xy})_1 + (\bar{I}_{xy})_2 + (\bar{I}_{xy})_3$$

For each rectangle

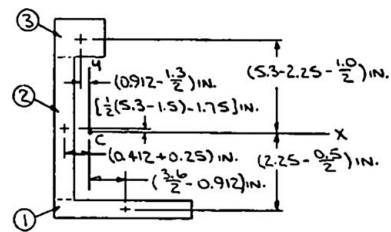
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

and

$$\bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

Thus

$$I_{xy} = \Sigma \bar{x}\bar{y}A$$

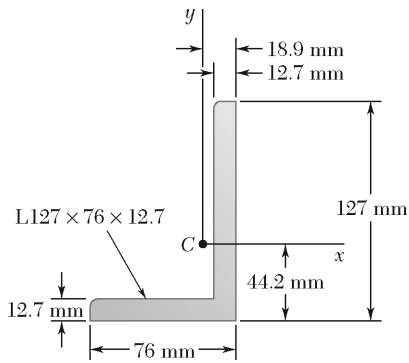


	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}\bar{y}A, \text{in}^4$
1	$3.6 \times 0.5 = 1.8$	0.888	-2.00	-3.196
2	$0.5 \times 3.8 = 1.9$	-0.662	0.15	-0.18867
3	$1.3 \times 1.0 = 1.3$	-0.262	2.55	-0.86853
Σ				-4.25320

$$\bar{I}_{xy} = -4.25 \text{ in}^4 \blacktriangleleft$$

PROBLEM 9.78

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.



SOLUTION

We have

$$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$$

For each rectangle

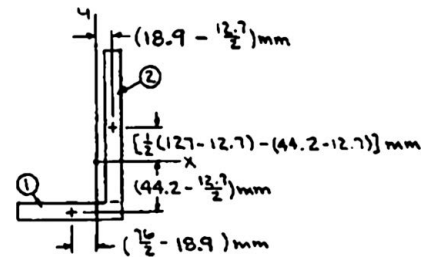
$$I_{xy} = \bar{I}_{x'y'} + \bar{x} \bar{y} A$$

and

$$\bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

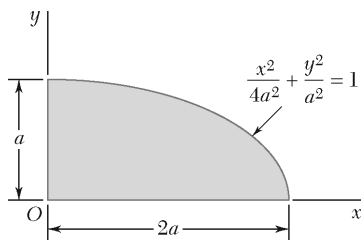
Thus

$$I_{xy} = \Sigma \bar{x} \bar{y} A$$



	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x} \bar{y} A, \text{mm}^4$
1	$76 \times 12.7 = 965.2$	-19.1	-37.85	697,777
2	$12.7 \times (127 - 12.7) = 1451.61$	12.55	25.65	467,284
Σ				1,165,061

$$\bar{I}_{xy} = 1.165 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$



PROBLEM 9.79

Determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the x and y axes about O (a) through 45° counterclockwise, (b) through 30° clockwise.

SOLUTION

From Figure 9.12:

$$I_x = \frac{\pi}{16}(2a)(a)^3$$

$$= \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{16}(2a)^3(a)$$

$$= \frac{\pi}{2}a^4$$

From Problem 9.67:

$$I_{xy} = \frac{1}{2}a^4$$

First note

$$\frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4$$

$$\frac{1}{2}(I_x - I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right) = -\frac{3}{16}\pi a^4$$

Now use Equations (9.18), (9.19), and (9.20).

Equation (9.18):

$$I_{x'} = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 2\theta - \frac{1}{2}a^4 \sin 2\theta$$

Equation (9.19):

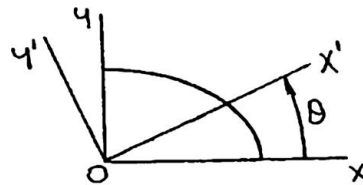
$$I_{y'} = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos 2\theta + \frac{1}{2}a^4 \sin 2\theta$$

Equation (9.20):

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta$$

$$= -\frac{3}{16}\pi a^4 \sin 2\theta + \frac{1}{2}a^4 \cos 2\theta$$



PROBLEM 9.79 (Continued)(a) $\theta = +45^\circ$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 90^\circ - \frac{1}{2}a^4 \sin 90^\circ$$

or $I_{x'} = 0.482a^4 \blacktriangleleft$

$$I_{y'} = \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos 90^\circ + \frac{1}{2}a^4$$

or $I_{y'} = 1.482a^4 \blacktriangleleft$

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin 90^\circ + \frac{1}{2}a^4 \cos 90^\circ$$

or $I_{x'y'} = -0.589a^4 \blacktriangleleft$

(b) $\theta = -30^\circ$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos(-60^\circ) - \frac{1}{2}a^4 \sin(-60^\circ)$$

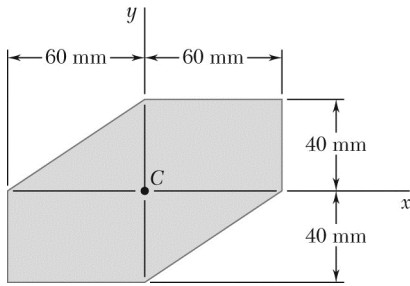
or $I_{x'} = 1.120a^4 \blacktriangleleft$

$$I_{y'} = \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos(-60^\circ) + \frac{1}{2}a^4 \sin(-60^\circ)$$

or $I_{y'} = 0.843a^4 \blacktriangleleft$

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin(-60^\circ) + \frac{1}{2}a^4 \cos(-60^\circ)$$

or $I_{x'y'} = 0.760a^4 \blacktriangleleft$

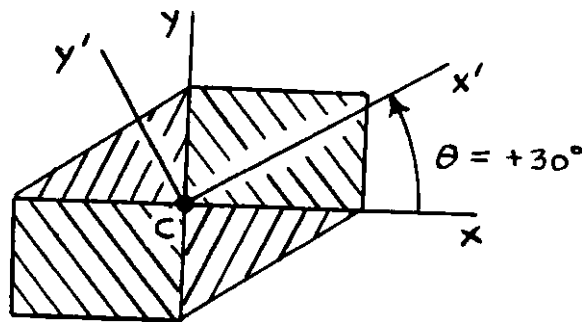


PROBLEM 9.80

Determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the x and y axes 30° counterclockwise.

SOLUTION

From Problem 9.72: $\bar{I}_{xy} = 2.40 \times 10^6 \text{ mm}^4$



Now, with two rectangles and two triangles:

$$\bar{I}_x = 2 \left[\frac{1}{3} (60 \text{ mm})(40 \text{ mm})^3 \right] + 2 \left[\frac{1}{12} (60 \text{ mm})(40 \text{ mm})^3 \right]$$

$$\bar{I}_x = 3.20 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 2 \left[\frac{1}{3} (40 \text{ mm})(60 \text{ mm})^3 \right] + 2 \left[\frac{1}{12} (40 \text{ mm})(60 \text{ mm})^3 \right] = 7.20 \times 10^6 \text{ mm}^4$$

Now $\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.20 + 7.20) \times 10^6 = 5.20 \times 10^6 \text{ mm}^4$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(3.20 - 7.20) \times 10^6 = -2.00 \times 10^6 \text{ mm}^4$$

Using Eqs. (9.18), (9.19), (9.20):

Eq. (9.18):
$$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta - \bar{I}_{xy} \sin 2\theta$$

$$= [5.20 + (-2.00) \cos 60^\circ - (2.40) \sin 60^\circ] \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 2.12 \times 10^6 \text{ mm}^4 \blacktriangleleft$

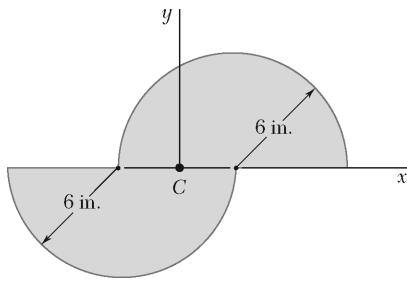
PROBLEM 9.80 (Continued)

Eq. (9.19):
$$\begin{aligned}\bar{I}_{y'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta + \bar{I}_{xy} \sin 2\theta \\ &= [5.20 - (-2.00) \cos 60^\circ + (2.40) \sin 60^\circ] \times 10^6 \text{ mm}^4\end{aligned}$$

or $\bar{I}_{y'} = 8.28 \times 10^6 \text{ mm}^4 \blacktriangleleft$

Eq. (9.20):
$$\begin{aligned}\bar{I}_{x'y'} &= \frac{1}{2}(\bar{I}_x - \bar{I}_y) \sin 2\theta + \bar{I}_{xy} \cos 2\theta \\ &= [(-2.00) \sin 60^\circ + (2.40) \cos 60^\circ] \times 10^6 \text{ mm}^4\end{aligned}$$

or $\bar{I}_{x'y'} = -0.532 \times 10^6 \text{ mm}^4 \blacktriangleleft$



PROBLEM 9.81

Determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the x and y axes 60° counterclockwise.

SOLUTION

From Problem 9.73:

$$\bar{I}_{xy} = 864 \text{ in}^4$$

Now

$$\bar{I}_x = (I_x)_1 + (I_x)_2$$

where

$$\begin{aligned} (I_x)_1 = (I_x)_2 &= \frac{\pi}{8} (6 \text{ in.})^4 \\ &= 162\pi \text{ in}^4 \end{aligned}$$

Then

$$\bar{I}_x = 2(162\pi \text{ in}^4) = 324\pi \text{ in}^4$$

Also

$$\bar{I}_y = (I_y)_1 + (I_y)_2$$

where

$$(I_y)_1 = (I_y)_2 = \frac{\pi}{8} (6 \text{ in.})^4 + \left[\frac{\pi}{2} (6 \text{ in.})^2 \right] (3 \text{ in.})^2 = 324\pi \text{ in}^4$$

Then

$$\bar{I}_y = 2(324\pi \text{ in}^4) = 648\pi \text{ in}^4$$

Now

$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 486\pi \text{ in}^4$$

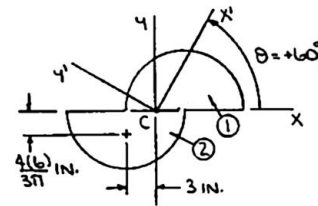
$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(324\pi - 648\pi) = -162\pi \text{ in}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):

$$\begin{aligned} \bar{I}_{x'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta - \bar{I}_{xy} \sin 2\theta \\ &= [486\pi + (-162\pi) \cos 120^\circ - 864 \sin 120^\circ] \text{ in}^4 \end{aligned}$$

$$\text{or } \bar{I}_{x'} = 1033 \text{ in}^4 \blacktriangleleft$$



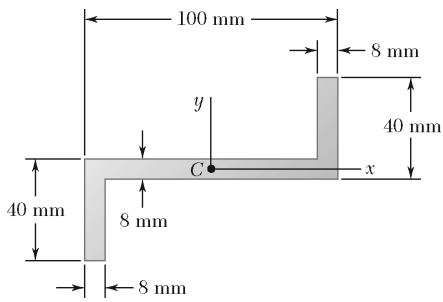
PROBLEM 9.81 (Continued)

Eq. (9.19):
$$\begin{aligned}\bar{I}_{y'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta \\ &= [486\pi - (-162\pi)\cos 120^\circ + 864\sin 120^\circ]\text{in}^4\end{aligned}$$

or $\bar{I}_{y'} = 2020\text{in}^4 \blacktriangleleft$

Eq. (9.20):
$$\begin{aligned}\bar{I}_{x'y'} &= \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta \\ &= [(-162\pi)\sin 120^\circ + 864\cos 120^\circ]\text{in}^4\end{aligned}$$

or $\bar{I}_{x'y'} = -873\text{in}^4 \blacktriangleleft$



PROBLEM 9.82

Determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION

From Problem 9.75:

$$\bar{I}_{xy} = 471,040 \text{ mm}^4$$

Now

$$\bar{I}_x = (\bar{I}_x)_1 + (I_x)_2 + (I_x)_3$$

where

$$(\bar{I}_x)_1 = \frac{1}{12}(100 \text{ mm})(8 \text{ mm})^3$$

$$= 4266.67 \text{ mm}^4$$

$$(I_x)_2 = (I_x)_3 = \frac{1}{12}(8 \text{ mm})(32 \text{ mm})^3 + [(8 \text{ mm})(32 \text{ mm})](20 \text{ mm})^2$$

$$= 124,245.33$$

Then

$$\bar{I}_x = [4266.67 + 2(124,245.33)] \text{ mm}^4 = 252,757 \text{ mm}^4$$

Also

$$\bar{I}_y = (\bar{I}_y)_1 + (I_y)_2 + (I_y)_3$$

where

$$(\bar{I}_y)_1 = \frac{1}{12}(8 \text{ mm})(100 \text{ mm})^3 = 666,666.7 \text{ mm}^4$$

$$(\bar{I}_y)_2 = (I_y)_3 = \frac{1}{12}(32 \text{ mm})(8 \text{ mm})^3 + [(8 \text{ mm})(32 \text{ mm})](46 \text{ mm})^2$$

$$= 543,061.3 \text{ mm}^4$$

Then

$$\bar{I}_y = [666,666.7 + 2(543,061.3)] \text{ mm}^4 = 1,752,789 \text{ mm}^4$$

Now

$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(252,757 + 1,752,789) \text{ mm}^4 = 1,002,773 \text{ mm}^4$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(252,757 - 1,752,789) \text{ mm}^4 = -750,016 \text{ mm}^4$$

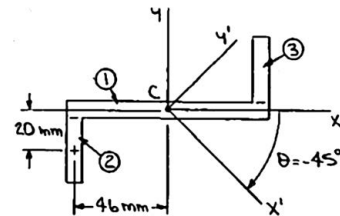
Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):

$$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta - \bar{I}_{xy} \sin 2\theta$$

$$= [1,002,773 + (-750,016) \cos(-90^\circ) - 471,040 \sin(-90^\circ)]$$

$$\text{or } \bar{I}_{x'} = 1.474 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



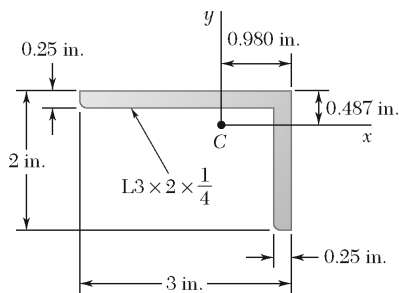
PROBLEM 9.82 (Continued)

Eq. (9.19):
$$\begin{aligned}\bar{I}_{y'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta + \bar{I}_{xy} \sin 2\theta \\ &= [1,002,773 - (-750,016) \cos(-90^\circ) + 471,040 \sin(-90^\circ)]\end{aligned}$$

or $\bar{I}_{y'} = 0.532 \times 10^6 \text{ mm}^4 \blacktriangleleft$

Eq. (9.20):
$$\begin{aligned}\bar{I}_{x'y'} &= \frac{1}{2}(\bar{I}_x - \bar{I}_y) \sin 2\theta + \bar{I}_{xy} \cos 2\theta \\ &= [(-750,016) \sin(-90^\circ) + 471,040 \cos(-90^\circ)]\end{aligned}$$

or $\bar{I}_{x'y'} = 0.750 \times 10^6 \text{ mm}^4 \blacktriangleleft$



PROBLEM 9.83

Determine the moments of inertia and the product of inertia of the $L3 \times 2 \times \frac{1}{4}$ -in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes 30° clockwise.

SOLUTION

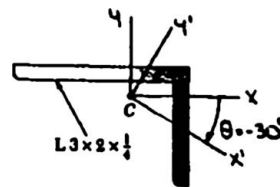
From Figure 9.13:

$$\bar{I}_x = 0.390 \text{ in}^4$$

$$\bar{I}_y = 1.09 \text{ in}^4$$

From Problem 9.74:

$$\bar{I}_{xy} = -0.37983 \text{ in}^4$$



Now

$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.390 + 1.09) \text{ in}^4 = 0.740 \text{ in}^4$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(0.390 - 1.09) \text{ in}^4 = -0.350 \text{ in}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):

$$\begin{aligned} \bar{I}_{x'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta - \bar{I}_{xy} \sin 2\theta \\ &= [0.740 + (-0.350) \cos(-60^\circ) - (-0.37983) \sin(-60^\circ)] \end{aligned}$$

$$\text{or } \bar{I}_{x'} = 0.236 \text{ in}^4 \blacktriangleleft$$

Eq. (9.19):

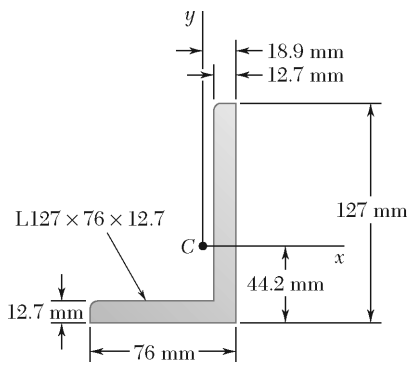
$$\begin{aligned} \bar{I}_{y'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta + \bar{I}_{xy} \sin 2\theta \\ &= [0.740 - (-0.350) \cos(-60^\circ) + (-0.37983) \sin(-60^\circ)] \end{aligned}$$

$$\text{or } \bar{I}_{y'} = 1.244 \text{ in}^4 \blacktriangleleft$$

Eq. (9.20):

$$\begin{aligned} \bar{I}_{x'y'} &= \frac{1}{2}(\bar{I}_x - \bar{I}_y) \sin 2\theta + \bar{I}_{xy} \cos 2\theta \\ &= [(-0.350) \sin(-60^\circ) + (-0.37983) \cos(-60^\circ)] \end{aligned}$$

$$\text{or } \bar{I}_{x'y'} = 0.1132 \text{ in}^4 \blacktriangleleft$$



PROBLEM 9.84

Determine the moments of inertia and the product of inertia of the L127×76×12.7-mm angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the x and y axes 45° counterclockwise.

SOLUTION

From Figure 9.13:

$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$$

From Problem 9.78:

$$\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$$

Now

$$\begin{aligned} \frac{1}{2}(\bar{I}_x + \bar{I}_y) &= \frac{1}{2}(3.93 + 1.06) \times 10^6 \text{ mm}^4 \\ &= 2.495 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(3.93 - 1.06) \times 10^6 \text{ mm}^4 = 1.435 \times 10^6 \text{ mm}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):

$$\begin{aligned} \bar{I}_{x'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta - \bar{I}_{xy} \sin 2\theta \\ &= [2.495 + 1.435 \cos 90^\circ - 1.165061 \sin 90^\circ] \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\text{or } \bar{I}_{x'} = 1.330 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

Eq. (9.19):

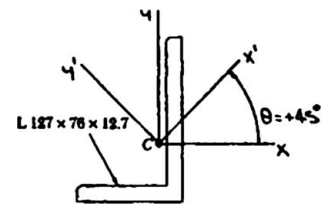
$$\begin{aligned} \bar{I}_{y'} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y) \cos 2\theta + \bar{I}_{xy} \sin 2\theta \\ &= [2.495 - 1.435 \cos 90^\circ + 1.165061 \sin 90^\circ] \times 10^6 \text{ mm}^4 \end{aligned}$$

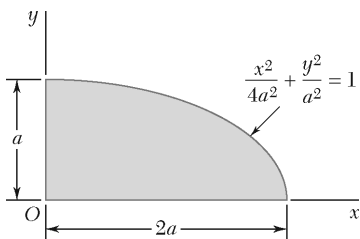
$$\text{or } \bar{I}_{y'} = 3.66 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

Eq. (9.20):

$$\begin{aligned} \bar{I}_{x'y'} &= \frac{1}{2}(\bar{I}_x - \bar{I}_y) \sin 2\theta + \bar{I}_{xy} \cos 2\theta \\ &= [(1.435 \sin 90^\circ + 1.165061 \cos 90^\circ)] \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\text{or } \bar{I}_{x'y'} = 1.435 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$





PROBLEM 9.85

For the quarter ellipse of Problem 9.67, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79:

$$I_x = \frac{\pi}{8}a^4 \quad I_y = \frac{\pi}{2}a^4$$

Problem 9.67:

$$I_{xy} = \frac{1}{2}a^4$$

Now Eq. (9.25):

$$\begin{aligned} \tan 2\theta_m &= -\frac{2I_{xy}}{I_x - I_y} = -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4} \\ &= \frac{8}{3\pi} = 0.84883 \end{aligned}$$

Then

$$2\theta_m = 40.326^\circ \quad \text{and} \quad 220.326^\circ$$

$$\text{or} \quad \theta_m = 20.2^\circ \quad \text{and} \quad 110.2^\circ \quad \blacktriangleleft$$

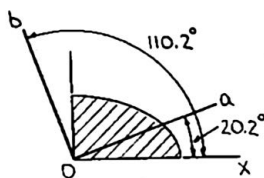
Also Eq. (9.27):

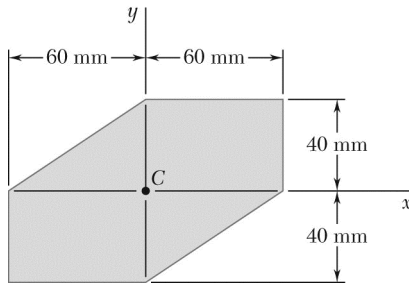
$$\begin{aligned} I_{\max, \min} &= \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \\ &= \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) \\ &\quad \pm \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2} \\ &= (0.981,748 \pm 0.772,644)a^4 \end{aligned}$$

$$\text{or} \quad I_{\max} = 1.754a^4 \quad \blacktriangleleft$$

$$\text{and} \quad I_{\min} = 0.209a^4 \quad \blacktriangleleft$$

By inspection, the a axis corresponds to I_{\min} and the b axis corresponds to I_{\max} .





PROBLEM 9.86

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.72.

SOLUTION

From Problem 9.80: $\bar{I}_x = 3.20 \times 10^6 \text{ mm}^4$

$$\bar{I}_y = 7.20 \times 10^6 \text{ mm}^4$$

From Problem 9.72: $\bar{I}_{xy} = 2.40 \times 10^6 \text{ mm}^4$

Now Eq. (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(2.40 \times 10^6)}{(3.20 - 7.20) \times 10^6} = 1.200$$

Then
$$2\theta_m = 50.194^\circ \quad \text{and} \quad 230.194^\circ$$

or $\theta_m = 25.1^\circ \text{ and } 115.1^\circ \blacktriangleleft$

Also Eq. (9.27):
$$\bar{I}_{\max, \min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

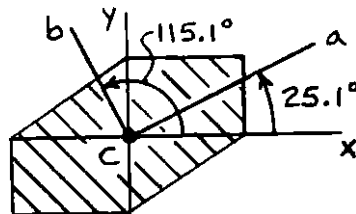
Then
$$\bar{I}_{\max, \min} = \left[\frac{3.20 + 7.20}{2} \pm \sqrt{\left(\frac{3.20 - 7.20}{2}\right)^2 + (2.40)^2} \right] \times 10^6 \text{ mm}^4$$

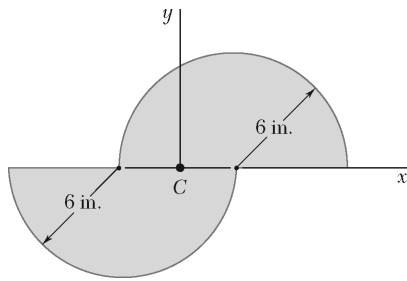
$$= (5.20 \pm 3.1241) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 8.32 \times 10^6 \text{ mm}^4 \blacktriangleleft$

and $\bar{I}_{\min} = 2.08 \times 10^6 \text{ mm}^4 \blacktriangleleft$

By inspection, the a axis corresponds to \bar{I}_{\min} and the b axis corresponds to \bar{I}_{\max} .





PROBLEM 9.87

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.73.

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in}^4$ $\bar{I}_y = 648\pi \text{ in}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in}^4$

Now Eq. (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(864)}{324\pi - 648\pi}$$

$$= 1.69765$$

Then $2\theta_m = 59.500^\circ$ and 239.500°

or $\theta_m = 29.7^\circ$ and 119.7° ◀

Also Eq. (9.27):
$$\bar{I}_{\max, \min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

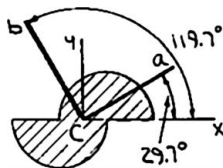
Then
$$\bar{I}_{\max, \min} = \frac{324\pi + 648\pi}{2} \pm \sqrt{\left(\frac{324\pi - 648\pi}{2}\right)^2 + 864^2}$$

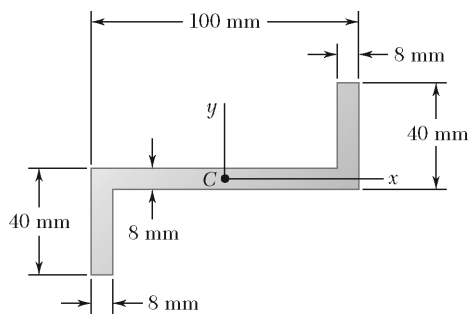
$$= (1526.81 \pm 1002.75) \text{ in}^4$$

or $\bar{I}_{\max} = 2530 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 524 \text{ in}^4$ ◀

By inspection, the a axis corresponds to \bar{I}_{\min} and the b axis corresponds to \bar{I}_{\max} .





PROBLEM 9.88

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.75.

SOLUTION

From Problem 9.82: $\bar{I}_x = 252,757 \text{ mm}^4$

$$\bar{I}_y = 1,752,789 \text{ mm}^4$$

Problem 9.75: $\bar{I}_{xy} = 471,040 \text{ mm}^4$

Now Eq. (9.25):

$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(471,040)}{252,757 - 1,752,789} \\ &= 0.62804 \end{aligned}$$

Then $2\theta_m = 32.130^\circ$ and 212.130°

or $\theta_m = 16.07^\circ$ and 106.1° ◀

Also Eq. (9.27):

$$\bar{I}_{\max, \min} = \frac{\bar{I}_x - \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

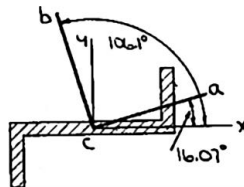
Then

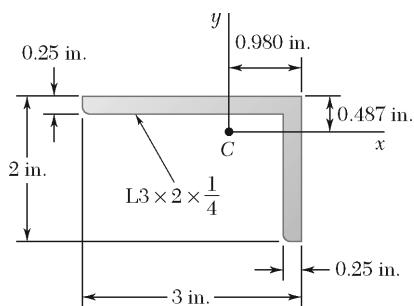
$$\begin{aligned} \bar{I}_{\max, \min} &= \frac{252,757 + 1,752,789}{2} \pm \sqrt{\left(\frac{252,757 - 1,752,789}{2}\right)^2 + 471,040^2} \\ &= (1,002,773 \pm 885,665) \text{ mm}^4 \end{aligned}$$

or $\bar{I}_{\max} = 1.888 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.1171 \times 10^6 \text{ mm}^4$ ◀

By inspection, the a axis corresponds to \bar{I}_{\min} and the b axis corresponds to \bar{I}_{\max} .





PROBLEM 9.89

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The $L3 \times 2 \times \frac{1}{4}$ -in. angle cross section of Problem 9.74.

SOLUTION

From Problem 9.83: $\bar{I}_x = 0.390 \text{ in}^4$ $\bar{I}_y = 1.09 \text{ in}^4$

Problem 9.74: $\bar{I}_{xy} = -0.37983 \text{ in}^4$

Now Eq. (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-0.37983)}{0.390 - 1.09}$$

$$= -1.08523$$

Then $2\theta_m = -47.341^\circ$ and 132.659°

or $\theta_m = -23.7^\circ$ and 66.3° ◀

Also Eq. (9.27):
$$\bar{I}_{\max, \min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

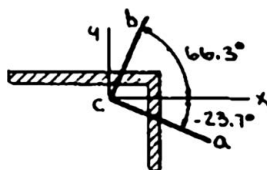
Then
$$\bar{I}_{\max, \min} = \frac{0.390 + 1.09}{2} \pm \sqrt{\left(\frac{0.390 - 1.09}{2}\right)^2 + (-0.37983)^2}$$

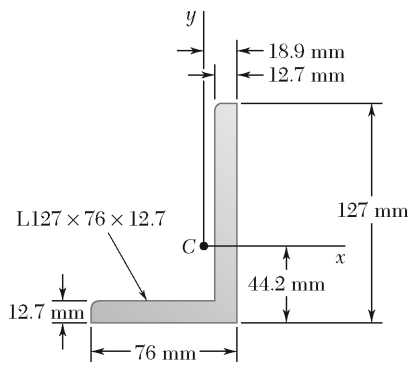
$$= (0.740 \pm 0.51650)^2 \text{ in}^4$$

or $\bar{I}_{\max} = 1.257 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 0.224 \text{ in}^4$ ◀

By inspection, the a axis corresponds to \bar{I}_{\min} and the b axis corresponds to \bar{I}_{\max} .





PROBLEM 9.90

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The L127 × 76 × 12.7-mm angle cross section of Problem 9.78.

SOLUTION

From Problem 9.84:

$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$$

Problem 9.78:

$$\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$$

Now Eq. (9.25):

$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6} \\ &= -0.81189 \end{aligned}$$

Then

$$2\theta_m = -39.073^\circ \quad \text{and} \quad 140.927^\circ$$

$$\text{or} \quad \theta_m = -19.54^\circ \quad \text{and} \quad 70.5^\circ \quad \blacktriangleleft$$

Also Eq. (9.27):

$$\bar{I}_{\max, \min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

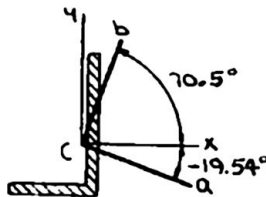
Then

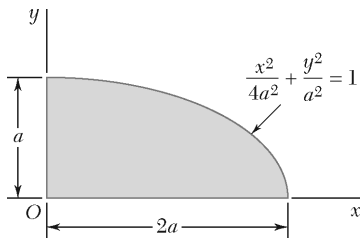
$$\begin{aligned} \bar{I}_{\max, \min} &= \left[\frac{3.93 + 1.06}{2} \pm \sqrt{\left(\frac{3.93 - 1.06}{2}\right)^2 + 1.165061^2} \right] \times 10^6 \text{ mm}^4 \\ &= (2.495 \pm 1.84840) \times 10^6 \text{ mm}^4 \end{aligned}$$

$$\text{or} \quad \bar{I}_{\max} = 4.34 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

$$\text{and} \quad \bar{I}_{\min} = 0.647 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

By inspection, the a axis corresponds to \bar{I}_{\max} and the b axis corresponds to \bar{I}_{\min} .





PROBLEM 9.91

Using Mohr's circle, determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the x and y axes about O (a) through 45° counterclockwise, (b) through 30° clockwise.

SOLUTION

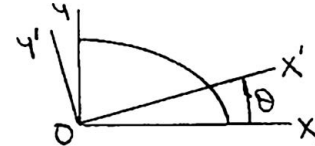
From Problem 9.79:

$$I_x = \frac{\pi}{8} a^4$$

$$I_y = \frac{\pi}{2} a^4$$

Problem 9.67:

$$I_{xy} = \frac{1}{2} a^4$$



The Mohr's circle is defined by the diameter XY , where

$$X\left(\frac{\pi}{8} a^4, \frac{1}{2} a^4\right) \text{ and } Y\left(\frac{\pi}{2} a^4, -\frac{1}{2} a^4\right)$$

Now

$$I_{ave} = \frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8} a^4 + \frac{\pi}{2} a^4\right) = \frac{5}{16} \pi a^4 = 0.98175 a^4$$

and

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8} a^4 - \frac{\pi}{2} a^4\right)\right]^2 + \left(\frac{1}{2} a^4\right)^2}$$

$$= 0.77264 a^4$$

The Mohr's circle is then drawn as shown.

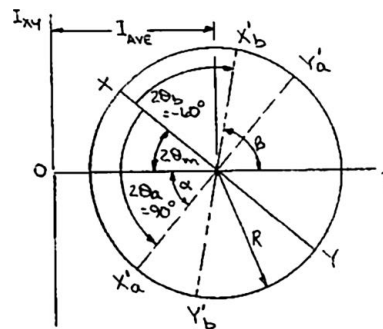
$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2} a^4\right)}{\frac{\pi}{8} a^4 - \frac{\pi}{2} a^4}$$

$$= 0.84883$$

or

$$2\theta_m = 40.326^\circ$$



PROBLEM 9.91 (Continued)

Then

$$\begin{aligned}\alpha &= 90^\circ - 40.326^\circ \\ &= 49.674^\circ\end{aligned}$$

$$\begin{aligned}\beta &= 180^\circ - (40.326^\circ + 60^\circ) \\ &= 79.674^\circ\end{aligned}$$

(a) $\theta = +45^\circ$:

$$I_{x'} = I_{\text{ave}} - R \cos \alpha = 0.98175a^4 - 0.77264a^4 \cos 49.674^\circ$$

$$\text{or } I_{x'} = 0.482a^4 \blacktriangleleft$$

$$I_{y'} = I_{\text{ave}} + R \cos \alpha = 0.98175a^4 + 0.77264a^4 \cos 49.674^\circ$$

$$\text{or } I_{y'} = 1.482a^4 \blacktriangleleft$$

$$I_{x'y'} = -R \sin \alpha = -0.77264a^4 \sin 49.674^\circ$$

$$\text{or } I_{x'y'} = -0.589a^4 \blacktriangleleft$$

(b) $\theta = -30^\circ$:

$$I_{x'} = I_{\text{ave}} + R \cos \beta = 0.98175a^4 + 0.77264a^4 \cos 79.674^\circ$$

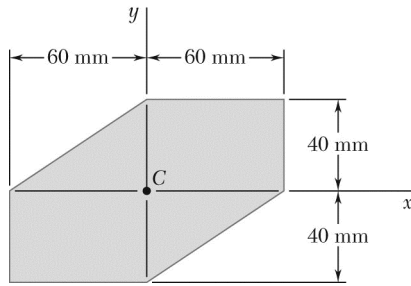
$$\text{or } I_{x'} = 1.120a^4 \blacktriangleleft$$

$$I_{y'} = I_{\text{ave}} - R \cos \beta = 0.98175a^4 - 0.77264a^4 \cos 79.674^\circ$$

$$\text{or } I_{y'} = 0.843a^4 \blacktriangleleft$$

$$I_{x'y'} = R \sin \beta = 0.77264a^4 \sin 79.674^\circ$$

$$\text{or } I_{x'y'} = 0.760a^4 \blacktriangleleft$$



PROBLEM 9.92

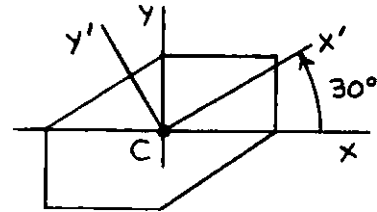
Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the x and y axes 30° counterclockwise.

SOLUTION

From Problem 9.80: $\bar{I}_x = 3.20 \times 10^6 \text{ mm}^4$

$$\bar{I}_y = 7.20 \times 10^6 \text{ mm}^4$$

From Problem 9.72: $\bar{I}_{xy} = 2.40 \times 10^6 \text{ mm}^4$



The Mohr's circle is defined by the diameter XY , where $X(3.20 \times 10^6, 2.40 \times 10^6)$ and $Y(7.20 \times 10^6, -2.40 \times 10^6)$.

Now
$$I_{ave} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.20 + 7.20) \times 10^6 = 5.20 \times 10^6 \text{ mm}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \left\{ \sqrt{\left[\frac{1}{2}(3.20 - 7.20)\right]^2 + (2.40)^2} \right\} \times 10^6 \text{ mm}^4$$

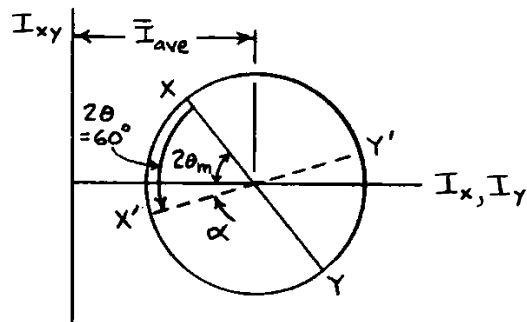
$$= 3.1241 \times 10^6 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.

$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(2.40 \times 10^6)}{(3.20 - 7.20) \times 10^6} \\ &= 1.200 \end{aligned}$$

or $2\theta_m = 50.1944^\circ$

Then $\alpha = 60^\circ - 50.1944^\circ = 9.8056^\circ$



PROBLEM 9.92 (Continued)

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R \cos \alpha = (5.20 - 3.1241 \cos 9.8056^\circ) \times 10^6$$

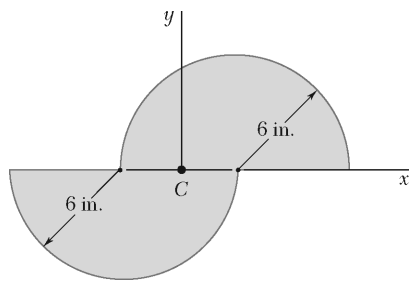
$$\text{or } \bar{I}_{x'} = 2.12 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R \cos \alpha = (5.20 + 3.1241 \cos 9.8056^\circ) \times 10^6$$

$$\text{or } \bar{I}_{y'} = 8.28 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = -R \sin \alpha = -(3.1241 \times 10^6) \sin 9.8056^\circ$$

$$\text{or } \bar{I}_{x'y'} = -0.532 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.93

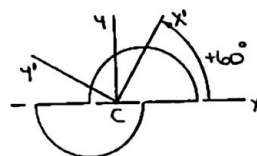
Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the x and y axes 60° counterclockwise.

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in}^4$

$$\bar{I}_y = 648\pi \text{ in}^4$$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in}^4$



The Mohr's circle is defined by the diameter XY , where $X(324\pi, 864)$ and $Y(648\pi, -864)$.

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 1526.81 \text{ in}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(324\pi - 648\pi)\right]^2 + 864^2}$$

$$= 1002.75 \text{ in}^4$$

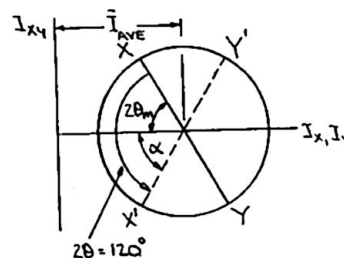
The Mohr's circle is then drawn as shown.

$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(864)}{324\pi - 648\pi} \\ &= 1.69765 \end{aligned}$$

or $2\theta_m = 59.500^\circ$

Then
$$\alpha = 120^\circ - 59.500^\circ$$

$$= 60.500^\circ$$



PROBLEM 9.93 (Continued)

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R \cos \alpha = 1526.81 - 1002.75 \cos 60.500^\circ$$

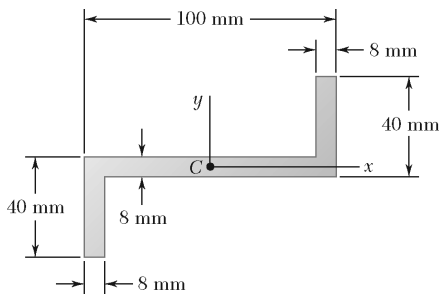
$$\text{or } \bar{I}_{x'} = 1033 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_y = \bar{I}_{\text{ave}} + R \cos \alpha = 1526.81 + 1002.75 \cos 60.500^\circ$$

$$\text{or } \bar{I}_y = 2020 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = -R \sin \alpha = -1002.75 \sin 60.500^\circ$$

$$\text{or } \bar{I}_{x'y'} = -873 \text{ in}^4 \blacktriangleleft$$



PROBLEM 9.94

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION

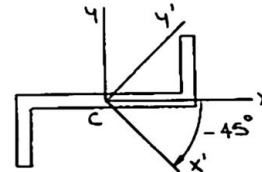
From Problem 9.82:

$$\bar{I}_x = 252,757 \text{ mm}^4$$

$$\bar{I}_y = 1,752,789 \text{ mm}^4$$

Problem 9.75:

$$\bar{I}_{xy} = 471,040 \text{ mm}^4$$



The Mohr's circle is defined by the diameter XY , where $X (252,757; 471,040)$ and $Y (1,752,789; -471,040)$.

Now

$$\begin{aligned} \bar{I}_{\text{ave}} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(252,757 + 1,752,789) \\ &= 1,002,773 \text{ mm}^4 \end{aligned}$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} \\ &= \sqrt{\left[\frac{1}{2}(252,757 - 1,752,789)\right]^2 + 471,040^2} \\ &= 885,665 \text{ mm}^4 \end{aligned}$$

The Mohr's circle is then drawn as shown.

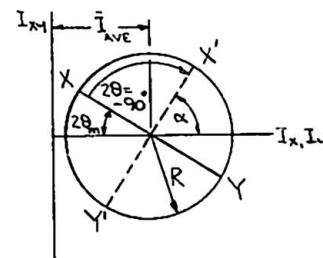
$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(471,040)}{252,757 - 1,752,789} \\ &= 0.62804 \end{aligned}$$

or

$$2\theta_m = 32.130^\circ$$

Then

$$\begin{aligned} \alpha &= 180^\circ - (32.130 + 90^\circ) \\ &= 57.870^\circ \end{aligned}$$



PROBLEM 9.94 (Continued)

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R \cos \alpha = 1,002,773 + 885,665 \cos 57.870^\circ$$

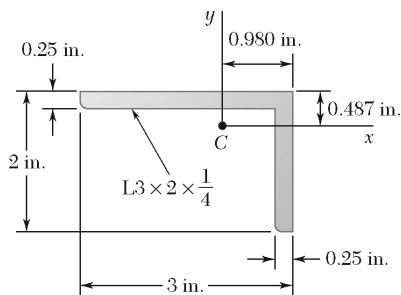
$$\text{or } \bar{I}_{x'} = 1.474 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R \cos \alpha = 1,002,773 - 885,665 \cos 57.870^\circ$$

$$\text{or } \bar{I}_{y'} = 0.532 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = R \sin \alpha = 885,665 \sin 57.870^\circ$$

$$\text{or } \bar{I}_{x'y'} = 0.750 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.95

Using Mohr's circle, determine the moments of inertia and the product of inertia of the $L3 \times 2 \times \frac{1}{4}$ -in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes 30° clockwise.

SOLUTION

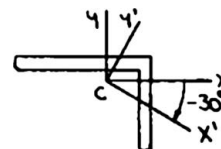
From Problem 9.83:

$$\bar{I}_x = 0.390 \text{ in}^4$$

$$\bar{I}_y = 1.09 \text{ in}^4$$

Problem 9.74:

$$\bar{I}_{xy} = -0.37983 \text{ in}^4$$



The Mohr's circle is defined by the diameter XY , where $X(0.390, -0.37983)$ and $Y(1.09, 0.37983)$.

Now

$$\begin{aligned} \bar{I}_{ave} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) \\ &= \frac{1}{2}(0.390 + 1.09) \\ &= 0.740 \text{ in}^4 \end{aligned}$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} \\ &= \sqrt{\left[\frac{1}{2}(0.390 - 1.09)\right]^2 + (-0.37983)^2} \\ &= 0.51650 \text{ in}^4 \end{aligned}$$

The Mohr's circle is then drawn as shown.

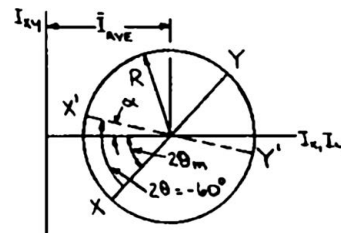
$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(-0.37983)}{0.390 - 1.09} \\ &= -1.08523 \end{aligned}$$

or

$$2\theta_m = -47.341^\circ$$

Then

$$\alpha = 60^\circ - 47.341^\circ = 12.659^\circ$$



PROBLEM 9.95 (Continued)

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R \cos \alpha = 0.740 - 0.51650 \cos 12.659^\circ$$

$$\text{or } \bar{I}_{x'} = 0.236 \text{ in}^4 \blacktriangleleft$$

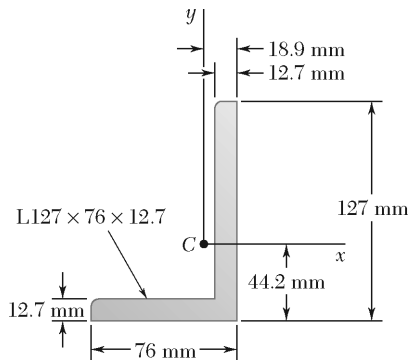
$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R \cos \alpha = 0.740 + 0.51650 \cos 12.659^\circ$$

$$\text{or } \bar{I}_{y'} = 1.244 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = R \sin \alpha = 0.51650 \sin 12.659^\circ$$

$$\text{or } \bar{I}_{x'y'} = 0.1132 \text{ in}^4 \blacktriangleleft$$

PROBLEM 9.96



Using Mohr's circle, determine the moments of inertia and the product of inertia of the L127 × 76 × 12.7-mm angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the x and y axes 45° counterclockwise.

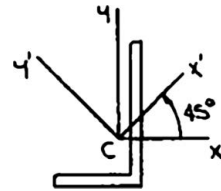
SOLUTION

From Problem 9.84: $\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$

$$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$$

Problem 9.78: $\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$

The Mohr's circle is defined by the diameter XY , where $X(3.93 \times 10^6, 1.165061 \times 10^6)$, $Y(1.06 \times 10^6, -1.165061 \times 10^6)$.



Now
$$\bar{I}_{ave} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.93 + 1.06) \times 10^6 = 2.495 \times 10^6 \text{ mm}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \left\{ \left[\frac{1}{2}(3.93 - 1.06) \right]^2 + 1.165061^2 \right\} \times 10^6 \text{ mm}^4$$

$$= 1.84840 \times 10^6 \text{ mm}^4$$

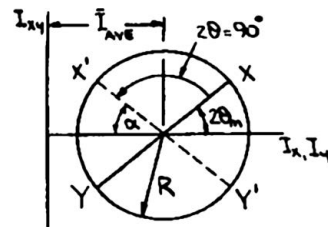
The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6}$$

$$= -0.81189$$

or
$$2\theta_m = -39.073^\circ$$



PROBLEM 9.96 (Continued)

Then
$$\alpha = 180^\circ - (39.073^\circ + 90^\circ)$$
$$= 50.927^\circ$$

Then
$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R \cos \alpha = (2.495 - 1.84840 \cos 50.927^\circ) \times 10^6$$

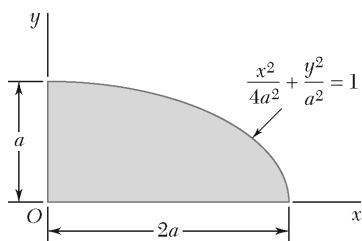
or
$$\bar{I}_{x'} = 1.330 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R \cos \alpha = (2.495 + 1.84840 \cos 50.927^\circ) \times 10^6$$

or
$$\bar{I}_{y'} = 3.66 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = R \sin \alpha = (1.84840 \times 10^6) \sin 50.927^\circ$$

or
$$\bar{I}_{x'y'} = 1.435 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.97

For the quarter ellipse of Problem 9.67, use Mohr's circle to determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79:

$$I_x = \frac{\pi}{8}a^4 \quad I_y = \frac{\pi}{2}a^4$$

Problem 9.67:

$$I_{xy} = \frac{1}{2}a^4$$

The Mohr's circle is defined by the diameter XY , where

$$X\left(\frac{\pi}{8}a^4, \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4, -\frac{1}{2}a^4\right)$$

Now

$$I_{\text{ave}} = \frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = 0.98175a^4$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(I_x - I_y)\right]^2 + I_{xy}^2} \\ &= \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2} \\ &= 0.77264a^4 \end{aligned}$$

The Mohr's circle is then drawn as shown.

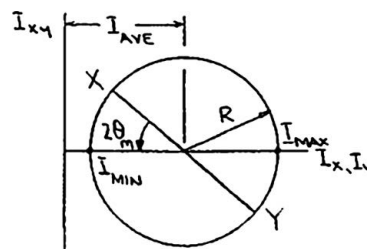
$$\begin{aligned} \tan 2\theta_m &= -\frac{2I_{xy}}{I_x - I_y} \\ &= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4} \\ &= 0.84883 \end{aligned}$$

or

$$2\theta_m = 40.326^\circ$$

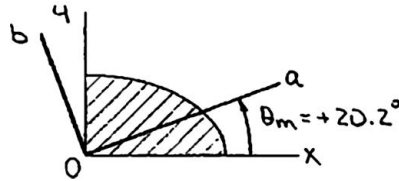
and

$$\theta_m = 20.2^\circ$$



PROBLEM 9.97 (Continued)

The principal axes are obtained by rotating the xy axes through 20.2° counterclockwise about O .



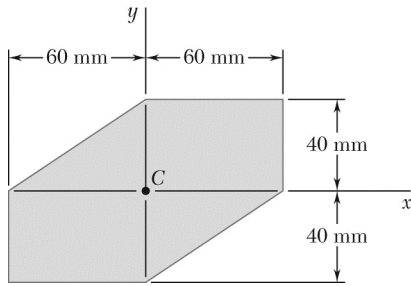
Now

$$I_{\max, \min} = I_{\text{ave}} \pm R = 0.98175a^4 \pm 0.77264a^4$$

$$\text{or } I_{\max} = 1.754a^4$$

$$\text{and } I_{\min} = 0.209a^4$$

From the Mohr's circle it is seen that the a axis corresponds to I_{\min} and the b axis corresponds to I_{\max} .



PROBLEM 9.98

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.72.

SOLUTION

From Problem 9.80: $\bar{I}_x = 3.20 \times 10^6 \text{ mm}^4$

$$\bar{I}_y = 7.20 \times 10^6 \text{ mm}^4$$

From Problem 9.72: $\bar{I}_{xy} = 2.40 \times 10^6 \text{ mm}^4$

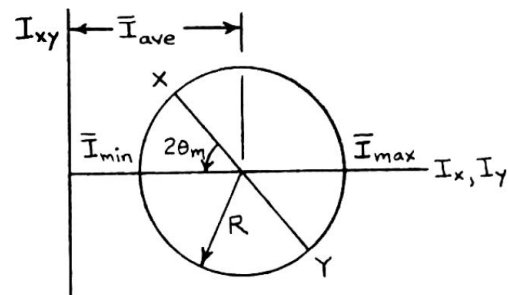
The Mohr's circle is defined by the diameter XY , where $X(3.20 \times 10^6, 2.40 \times 10^6)$ and $Y(7.20 \times 10^6, -2.40 \times 10^6)$.

Now

$$\begin{aligned} \bar{I}_{ave} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) \\ &= \frac{1}{2}(3.20 + 7.20) \times 10^6 \text{ mm}^4 \\ &= 5.20 \times 10^6 \text{ mm}^4 \end{aligned}$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} \\ &= \left\{ \sqrt{\left[\frac{1}{2}(3.20 - 7.20)\right]^2 + (2.40)^2} \right\} \times 10^6 \text{ mm}^4 \\ &= 3.1241 \times 10^6 \text{ mm}^4 \end{aligned}$$



The Mohr's circle is then drawn as shown.

$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(2.40 \times 10^6)}{(3.20 - 7.20) \times 10^6} = 1.200 \end{aligned}$$

or $2\theta_m = 50.194^\circ$

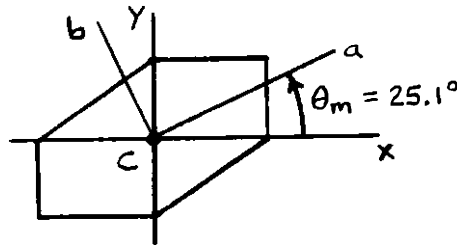
and $\theta_m = 25.097^\circ$

PROBLEM 9.98 (Continued)

The principal axes are obtained by rotating the xy axes through

25.1° counterclockwise ◀

about C .



Now

$$\bar{I}_{\max, \min} = \bar{I}_{\text{ave}} \pm R = (5.20 + 3.1241) \times 10^6$$

or $\bar{I}_{\max} = 8.32 \times 10^6 \text{ mm}^4$ ◀

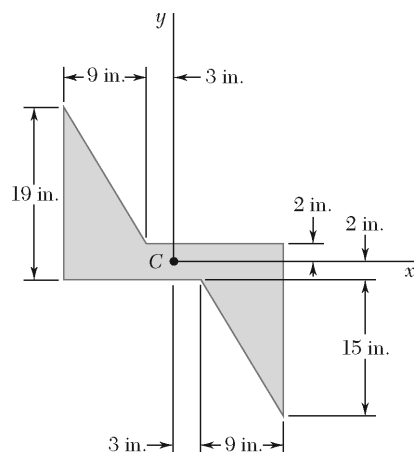
and $\bar{I}_{\min} = 2.08 \times 10^6 \text{ mm}^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to \bar{I}_{\min} and the b axis corresponds to \bar{I}_{\max} .

PROBLEM 9.99

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.76.



SOLUTION

From Problem 9.76:

$$\bar{I}_{xy} = -9011.25 \text{ in}^4$$

Now

$$\bar{I}_x = (\bar{I}_x)_1 + (I_x)_2 + (I_x)_3$$

where

$$(\bar{I}_x)_1 = \frac{1}{12} (24 \text{ in.})(4 \text{ in.})^3 = 128 \text{ in}^4$$

$$\begin{aligned} (I_x)_2 = (I_x)_3 &= \frac{1}{36} (9 \text{ in.})(15 \text{ in.})^3 + \left[\frac{1}{2} (9 \text{ in.})(15 \text{ in.}) \right] (7 \text{ in.})^2 \\ &= 4151.25 \text{ in}^4 \end{aligned}$$

Then

$$\begin{aligned} \bar{I}_x &= [128 + 2(4151.25)] \text{ in}^4 \\ &= 8430.5 \text{ in}^4 \end{aligned}$$

Also

$$\bar{I}_y = (\bar{I}_y)_1 + (I_y)_2 + (I_y)_3$$

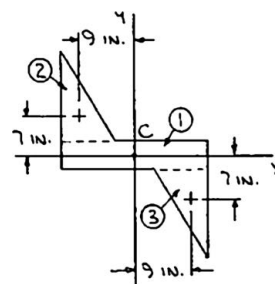
where

$$(\bar{I}_y)_1 = \frac{1}{12} (4 \text{ in.})(24 \text{ in.})^3 = 4608 \text{ in}^4$$

$$\begin{aligned} (I_y)_2 = (I_y)_3 &= \frac{1}{36} (15 \text{ in.})(9 \text{ in.})^3 + \left[\frac{1}{2} (9 \text{ in.})(15 \text{ in.}) \right] (9 \text{ in.})^2 \\ &= 5771.25 \text{ in}^4 \end{aligned}$$

Then

$$\bar{I}_y = [4608 + 2(5771.25)] \text{ in}^4 = 16150.5 \text{ in}^4$$



PROBLEM 9.99 (Continued)

The Mohr's circle is defined by the diameter XY , where $X(8430.5, -9011.25)$ and $Y(16150.5, 9011.25)$.

Now
$$\bar{I}_{ave} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(8430.5 + 16150.5) = 12290.5 \text{ in}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}(8430.5 - 16150.5)\right]^2 + (-9011.25)^2}$$

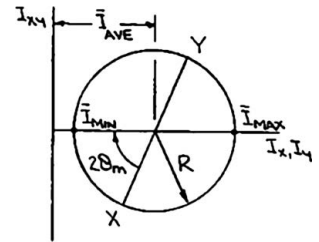
$$= 9803.17 \text{ in}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-9011.25)}{8430.5 - 16150.5}$$

$$= -2.33452$$

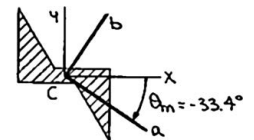


or
$$2\theta_m = -66.812^\circ$$

and
$$\theta_m = -33.4^\circ$$

The principal axes are obtained by rotating the xy axes through

33.4° clockwise ◀
about C .

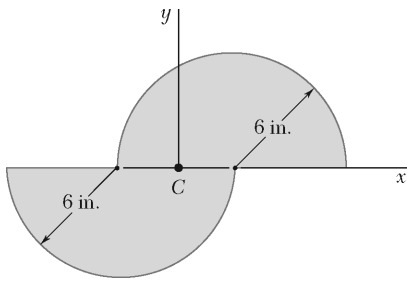


Now
$$\bar{I}_{max, min} = \bar{I}_{ave} \pm R = 12290.5 \pm 9803.17$$

or
$$\bar{I}_{max} = 22.1 \times 10^3 \text{ in}^4 \blacktriangleleft$$

and
$$\bar{I}_{min} = 2490 \text{ in}^4 \blacktriangleleft$$

From the Mohr's circle it is seen that the a axis corresponds to \bar{I}_{min} and the b axis corresponds to \bar{I}_{max} .



PROBLEM 9.100

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.73

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in}^4$ $\bar{I}_y = 648\pi \text{ in}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in}^4$

The Mohr's circle is defined by the diameter XY , where $X(324\pi, 864)$ and $Y(648\pi, -864)$.

Now $\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 1526.81 \text{ in}^4$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}(324\pi - 648\pi)\right]^2 + 864^2}$$

$$= 1002.75 \text{ in}^4$$

The Mohr's circle is then drawn as shown.

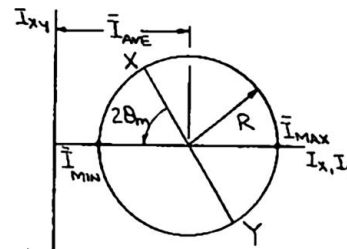
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(864)}{324\pi - 648\pi}$$

$$= 1.69765$$

or $2\theta_m = 59.4998^\circ$

and $\theta_m = 29.7^\circ$



The principal axes are obtained by rotating the xy axes through

29.7° counterclockwise ◀

about C .

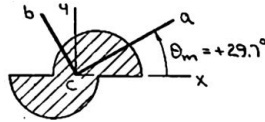
PROBLEM 9.100 (Continued)

Now

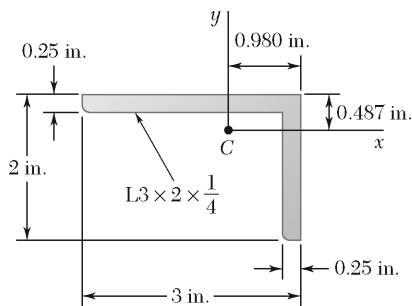
$$\bar{I}_{\max, \min} = \bar{I}_{\text{ave}} \pm R = 1526.81 \pm 1002.75$$

or $I_{\max} = 2530 \text{ in}^4 \blacktriangleleft$

and $\bar{I}_{\min} = 524 \text{ in}^4 \blacktriangleleft$



From the Mohr's circle it is seen that the a axis corresponds to I_{\min} and the b axis corresponds to \bar{I}_{\max} .



PROBLEM 9.101

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.74.

SOLUTION

From Problem 9.83:

$$\bar{I}_x = 0.390 \text{ in}^4$$

$$\bar{I}_y = 1.09 \text{ in}^4$$

Problem 9.74:

$$\bar{I}_{xy} = -0.37983 \text{ in}^4$$

The Mohr's circle is defined by the diameter XY , where $X(0.390, -0.37983)$ and $Y(1.09, 0.37983)$.

Now

$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.390 + 1.09) = 0.740 \text{ in}^4$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} \\ &= \sqrt{\left[\frac{1}{2}(0.390 - 1.09)\right]^2 + (-0.37983)^2} \\ &= 0.51650 \text{ in}^4 \end{aligned}$$

The Mohr's circle is then drawn as shown.

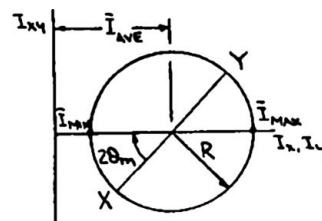
$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(-0.37983)}{0.390 - 1.09} \\ &= -1.08523 \end{aligned}$$

Then

$$2\theta_m = -47.341^\circ$$

and

$$\theta_m = -23.7^\circ$$



The principal axes are obtained by rotating the xy axes through

23.7° clockwise ◀

about C .

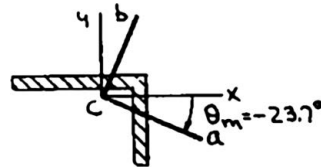
PROBLEM 9.101 (Continued)

Now

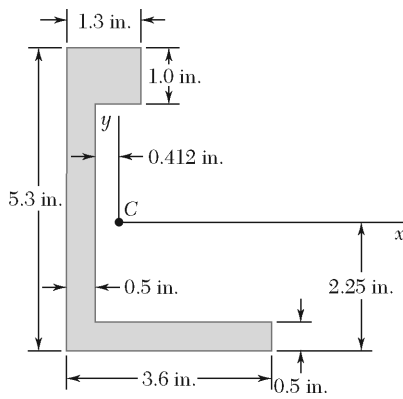
$$\bar{I}_{\max, \min} = \bar{I}_{\text{ave}} \pm R = 0.740 \pm 0.51650$$

$$\text{or } \bar{I}_{\max} = 1.257 \text{ in}^4 \blacktriangleleft$$

$$\text{and } \bar{I}_{\min} = 0.224 \text{ in}^4 \blacktriangleleft$$



From the Mohr's circle it is seen that the a axis corresponds to I_{\min} and the b axis corresponds to I_{\max} .



PROBLEM 9.102

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.77

(The moments of inertia \bar{I}_x and \bar{I}_y of the area of Problem 9.102 were determined in Problem 9.44).

SOLUTION

From Problem 9.44: $\bar{I}_x = 18.1282 \text{ in}^4$

$$\bar{I}_y = 4.5080 \text{ in}^4$$

Problem 9.77: $\bar{I}_{xy} = -4.25320 \text{ in}^4$

The Mohr's circle is defined by the diameter XY , where $X(18.1282, -4.25320)$ and $Y(4.5080, 4.25320)$.

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(18.1282 + 4.5080) = 11.3181 \text{ in}^4$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} \\ &= \sqrt{\left[\frac{1}{2}(18.1282 - 4.5080)\right]^2 + (-4.25320)^2} \\ &= 8.02915 \text{ in}^4 \end{aligned}$$

The Mohr's circle is then drawn as shown.

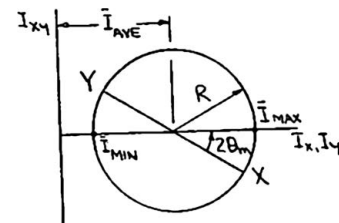
$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(-4.25320)}{18.1282 - 4.5080} \\ &= 0.62454 \end{aligned}$$

or

$$2\theta_m = 31.986^\circ$$

and

$$\theta_m = 15.99^\circ$$



The principal axes are obtained by rotating the xy axes through

15.99° counterclockwise ◀
about C .

PROBLEM 9.102 (Continued)

Now

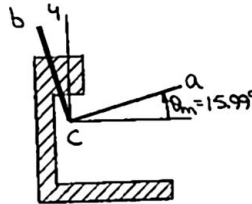
$$\bar{I}_{\max, \min} = \bar{I}_{\text{ave}} \pm R = 11.3181 \pm 8.02915$$

or

$$\bar{I}_{\max} = 19.35 \text{ in}^4 \quad \blacktriangleleft$$

and

$$\bar{I}_{\min} = 3.29 \text{ in}^4 \quad \blacktriangleleft$$



From the Mohr's circle it is seen that the a axis corresponds to I_{\max} and the b axis corresponds to I_{\min} .

PROBLEM 9.103

The moments and product of inertia of an $L4 \times 3 \times \frac{1}{4}$ -in. angle cross section with respect to two rectangular axes x and y through C are, respectively, $\bar{I}_x = 1.33 \text{ in}^4$, $\bar{I}_y = 2.75 \text{ in}^4$, and $\bar{I}_{xy} \leq 0$, with the minimum value of the moment of inertia of the area with respect to any axis through C being $I_{\min} = 0.692 \text{ in}^4$. Using Mohr's circle, determine (a) the product of inertia \bar{I}_{xy} of the area, (b) the orientation of the principal axes, (c) the value of \bar{I}_{\max} .

SOLUTION

(Note: A review of a table of rolled-steel shapes reveals that the given values of \bar{I}_x and \bar{I}_y are obtained when the 4-in. leg of the angle is parallel to the x axis. Further, for $\bar{I}_{xy} < 0$, the angle must be oriented as shown.)

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(1.33 + 2.75) = 2.040 \text{ in}^4$$

and
$$\bar{I}_{\min} = \bar{I}_{\text{ave}} - R \quad \text{or} \quad R = 2.040 - 0.692$$

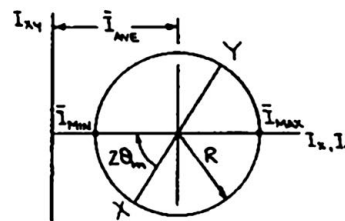
$$= 1.348 \text{ in}^4$$



Using \bar{I}_{ave} and R , the Mohr's circle is then drawn as shown; note that for the diameter XY , $X(1.33, \bar{I}_{xy})$ and $Y(2.75, |\bar{I}_{xy}|)$.

(a) We have
$$R^2 = \left[\frac{1}{2}(\bar{I}_x - \bar{I}_y) \right]^2 + \bar{I}_{xy}^2$$

or
$$\bar{I}_{xy}^2 = 1.348^2 - \left[\frac{1}{2}(1.33 - 2.75) \right]^2$$



Solving for \bar{I}_{xy} and taking the negative root (since $\bar{I}_{xy} < 0$) yields $\bar{I}_{xy} = -1.14586 \text{ in}^4$.

$$\bar{I}_{xy} = -1.146 \text{ in}^4 \quad \blacktriangleleft$$

(b) We have
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-1.14586)}{1.33 - 2.75}$$

$$= -1.61389$$

or
$$2\theta_m = -58.217^\circ \quad \theta_m = -29.1^\circ$$

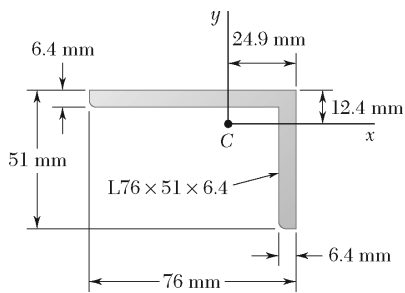
The principal axes are obtained by rotating the xy axes through

29.1° clockwise \blacktriangleleft

about C .

(c) We have
$$\bar{I}_{\max} = \bar{I}_{\text{ave}} + R = 2.040 + 1.348$$

$$\text{or} \quad \bar{I}_{\max} = 3.39 \text{ in}^4 \quad \blacktriangleleft$$



PROBLEM 9.104

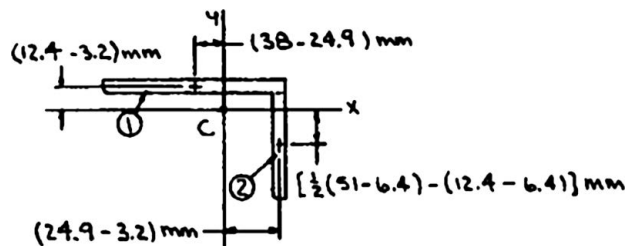
Using Mohr's circle, determine for the cross section of the rolled steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Figure 9.13.)

SOLUTION

From Figure 9.13B:

$$\bar{I}_x = 0.162 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.454 \times 10^6 \text{ mm}^4$$



We have
$$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$$

For each rectangle
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

and
$$\bar{I}_{x'y'} = 0 \quad (\text{symmetry}) \quad I_{xy} = \Sigma \bar{x}\bar{y}A$$

	A, mm^2	\bar{x}, mm	\bar{y}, mm	$\bar{x}\bar{y}A, \text{mm}^4$
1	$76 \times 6.4 = 486.4$	-13.1	9.2	-58620.93
2	$6.4 \times (51 - 6.4) = 285.44$	21.7	-16.3	-100962.98
Σ				-159583.91

$$\bar{I}_{xy} = -159584 \text{ mm}^4$$

The Mohr's circle is defined by the diameter XY where $X(0.162 \times 10^6, -0.159584 \times 10^6)$ and $Y(0.454 \times 10^6, 0.159584 \times 10^6)$

Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.162 + 0.454) \times 10^6$$

$$= 0.3080 \times 10^6 \text{ mm}^4$$

PROBLEM 9.104 (Continued)

and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + I_{xy}^2} = \left\{ \sqrt{\left[\frac{1}{2}(0.162 - 0.454)\right]^2 + (-0.159584)^2} \right\} \times 10^6$$

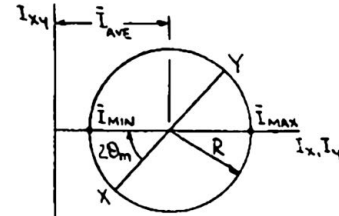
$$= 0.21629 \times 10^6 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-0.159584 \times 10^6)}{(0.162 - 0.454) \times 10^6}$$

$$= -1.09304$$



or

$$2\theta_m = -47.545$$

and

$$\theta_m = -23.8^\circ$$

The principal axes are obtained by rotating the xy axes through

23.8° clockwise ◀

About C.

Now

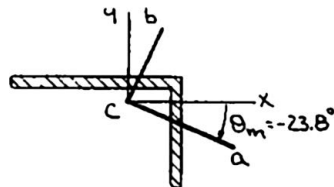
$$\bar{I}_{\max, \min} = \bar{I}_{ave} \pm R = (0.3080 \pm 0.21629) \times 10^6$$

or

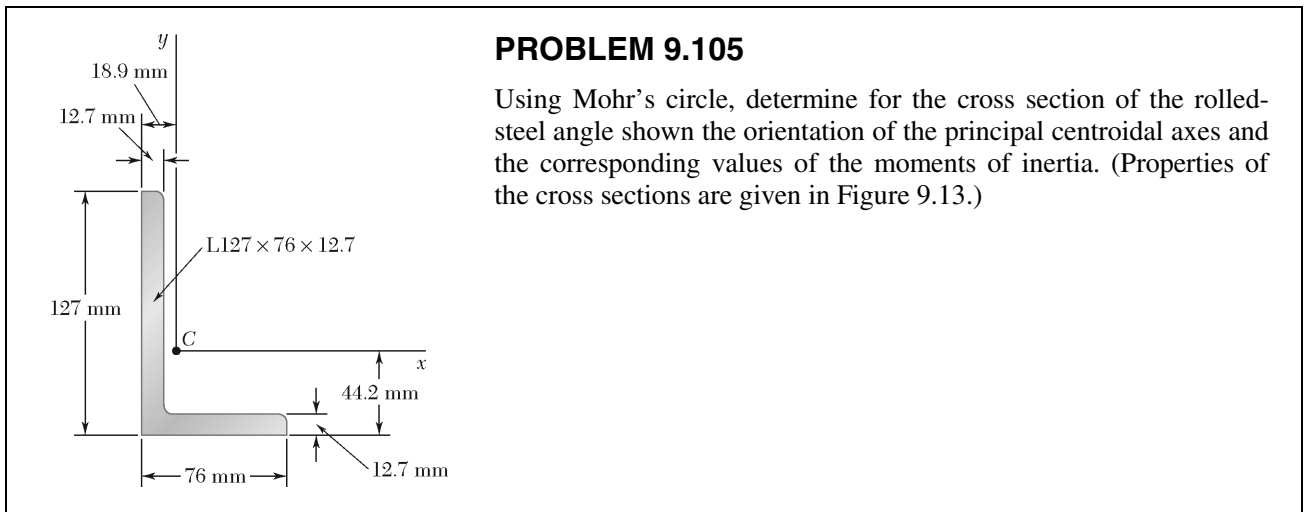
$$\bar{I}_{\max} = 0.524 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

and

$$\bar{I}_{\min} = 0.0917 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$



From the Mohr's circle it is seen that the a axis corresponds to \bar{I}_{\min} and the b axis corresponds to \bar{I}_{\max} .



PROBLEM 9.105

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Figure 9.13.)

SOLUTION

From Figure 9.13B:

$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$$

Problem 9.7B:

$$\bar{I}_{xy} = -1.165061 \times 10^6 \text{ mm}^4$$

(Note that the figure of Problem 9.105 is obtained by replacing x with $-x$ in the figure of Problem 9.78; thus the change in sign of \bar{I}_{xy} .)

The Mohr's circle is defined by the diameter XY , where $X(3.93 \times 10^6, -1.165061 \times 10^6)$ and $Y(1.06 \times 10^6, 1.165061 \times 10^6)$.

Now

$$I_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y)$$

$$= \frac{1}{2}(3.93 + 1.06) \times 10^6$$

$$= 2.495 \times 10^6 \text{ mm}^4$$

and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y) \right]^2 + \bar{I}_{xy}^2}$$

$$= \left\{ \sqrt{\left[\frac{1}{2}(3.93 - 1.06) \right]^2 + (-1.165061)^2} \right\} \times 10^6$$

$$= 1.84840 \times 10^6 \text{ mm}^4$$

PROBLEM 9.105 (Continued)

The Mohr's circle is then drawn as shown.

$$\begin{aligned} \tan 2\theta_m &= -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} \\ &= -\frac{2(-1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6} \\ &= 0.81189 \end{aligned}$$

or

$$2\theta_m = 39.073^\circ$$

and

$$\theta_m = 19.54^\circ$$

The principal axes are obtained by rotating the xy axes through

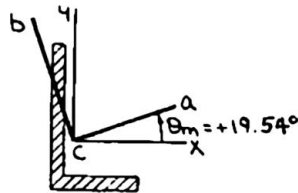
19.54° counterclockwise ◀
about C .

Now

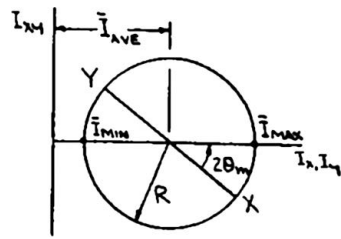
$$\bar{I}_{\max, \min} = \bar{I}_{\text{ave}} \pm R = (2.495 \pm 1.84840) \times 10^6$$

or $\bar{I}_{\max} = 4.34 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.647 \times 10^6 \text{ mm}^4$ ◀



From the Mohr's circle it is seen that the a axis corresponds to \bar{I}_{\max} and the b axis corresponds to \bar{I}_{\min} .



PROBLEM 9.106*

For a given area the moments of inertia with respect to two rectangular centroidal x and y axes are $\bar{I}_x = 1200 \text{ in}^4$ and $\bar{I}_y = 300 \text{ in}^4$, respectively. Knowing that after rotating the x and y axes about the centroid 30° counterclockwise, the moment of inertia relative to the rotated x axis is 1450 in^4 , use Mohr's circle to determine (a) the orientation of the principal axes, (b) the principal centroidal moments of inertia.

SOLUTION

We have
$$\bar{I}_{ave} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(1200 + 300) = 750 \text{ in}^4$$

Now observe that $\bar{I}_x > \bar{I}_{ave}$, $\bar{I}_{x'} > \bar{I}_x$, and $2\theta = +60^\circ$. This is possible only if $\bar{I}_{xy} < 0$. Therefore, assume $\bar{I}_{xy} < 0$ and (for convenience) $\bar{I}_{x'y'} > 0$. Mohr's circle is then drawn as shown.

We have
$$2\theta_m + \alpha = 60^\circ$$

Now using $\triangle ABD$:
$$R = \frac{\bar{I}_x - \bar{I}_{ave}}{\cos 2\theta_m} = \frac{1200 - 750}{\cos 2\theta_m} = \frac{450}{\cos 2\theta_m} \text{ (in}^4\text{)}$$

Using $\triangle AEF$:
$$R = \frac{\bar{I}_{x'} - \bar{I}_{ave}}{\cos \alpha} = \frac{1450 - 750}{\cos \alpha} = \frac{700}{\cos \alpha} \text{ (in}^4\text{)}$$

Then
$$\frac{450}{\cos 2\theta_m} = \frac{700}{\cos \alpha} \quad \alpha = 60^\circ - 2\theta_m$$

or
$$9 \cos(60^\circ - 2\theta_m) = 14 \cos 2\theta_m$$

Expanding:
$$9(\cos 60^\circ \cos 2\theta_m + \sin 60^\circ \sin 2\theta_m) = 14 \cos 2\theta_m$$

or
$$\tan 2\theta_m = \frac{14 - 9 \cos 60^\circ}{9 \sin 60^\circ} = 1.21885$$

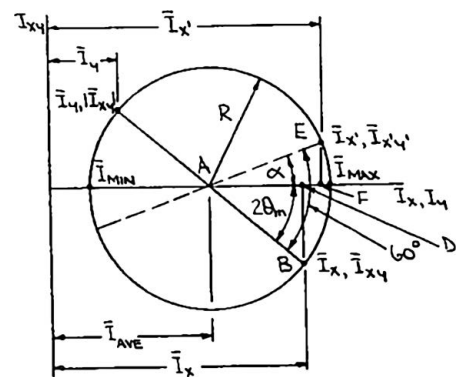
or
$$2\theta_m = 50.633^\circ \quad \text{and} \quad \theta_m = 25.3^\circ$$

(Note: $2\theta_m < 60^\circ$ implies assumption $\bar{I}_{x'y'} > 0$ is correct.)

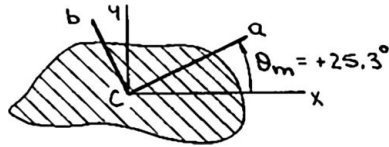
Finally,
$$R = \frac{450}{\cos 50.633^\circ} = 709.46 \text{ in}^4$$

(a) From the Mohr's circle it is seen that the principal axes are obtained by rotating the given centroidal x and y axes through θ_m about the centroid C or

25.3° counterclockwise ◀



PROBLEM 9.106* (Continued)



(b) We have

$$\bar{I}_{\max, \min} = \bar{I}_{\text{ave}} \pm R = 750 \pm 709.46$$

or $\bar{I}_{\max} = 1459 \text{ in}^4 \blacktriangleleft$

and $\bar{I}_{\min} = 40.5 \text{ in}^4 \blacktriangleleft$

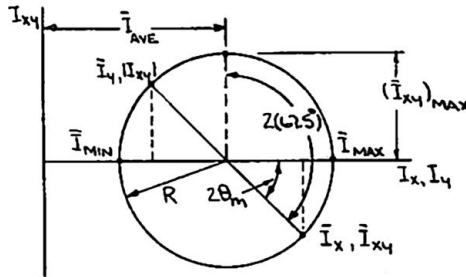
From the Mohr's circle it is seen that the a axis corresponds to \bar{I}_{\max} and the b axis corresponds to \bar{I}_{\min} .

PROBLEM 9.107

It is known that for a given area $\bar{I}_y = 48 \times 10^6 \text{ mm}^4$ and $\bar{I}_{xy} = -20 \times 10^6 \text{ mm}^4$, where the x and y axes are rectangular centroidal axes. If the axis corresponding to the maximum product of inertia is obtained by rotating the x axis 67.5° counterclockwise about C , use Mohr's circle to determine (a) the moment of inertia \bar{I}_x of the area, (b) the principal centroidal moments of inertia.

SOLUTION

First assume $\bar{I}_x > \bar{I}_y$ and then draw the Mohr's circle as shown. (Note: Assuming $\bar{I}_x < \bar{I}_y$ is not consistent with the requirement that the axis corresponding to $(\bar{I}_{xy})_{\max}$ is obtained after rotating the x axis through 67.5° CCW.)



From the Mohr's circle we have

$$2\theta_m = 2(67.5^\circ) - 90^\circ = 45^\circ$$

(a) From the Mohr's circle we have

$$\bar{I}_x = \bar{I}_y + 2 \frac{|I_{xy}|}{\tan 2\theta_m} = 48 \times 10^6 + 2 \frac{20 \times 10^6}{\tan 45^\circ}$$

$$\text{or } \bar{I}_x = 88.0 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

(b) We have

$$\begin{aligned} \bar{I}_{\text{ave}} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(88.0 + 48) \times 10^6 \\ &= 68.0 \times 10^6 \text{ mm}^4 \end{aligned}$$

and

$$R = \frac{|I_{xy}|}{\sin 2\theta_m} = \frac{20 \times 10^6}{\sin 45^\circ} = 28.284 \times 10^6 \text{ mm}^4$$

Now

$$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (68.0 \pm 28.284) \times 10^6$$

$$\text{or } \bar{I}_{\text{max}} = 96.3 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

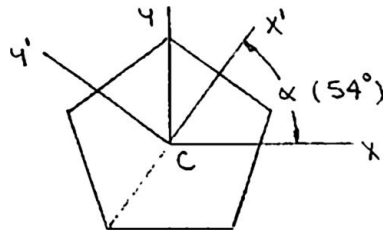
$$\text{and } \bar{I}_{\text{min}} = 39.7 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

PROBLEM 9.108

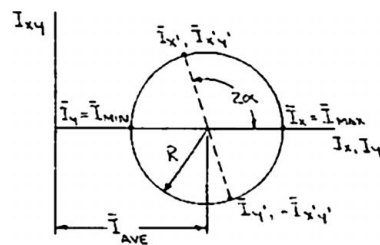
Using Mohr's circle, show that for any regular polygon (such as a pentagon) (a) the moment of inertia with respect to every axis through the centroid is the same, (b) the product of inertia with respect to every pair of rectangular axes through the centroid is zero.

SOLUTION

Consider the regular pentagon shown, with centroidal axes x and y .



Because the y axis is an axis of symmetry, it follows that $\bar{I}_{xy} = 0$. Since $\bar{I}_{xy} = 0$, the x and y axes must be principal axes. Assuming $\bar{I}_x = \bar{I}_{\max}$ and $\bar{I}_y = \bar{I}_{\min}$, the Mohr's circle is then drawn as shown.



Now rotate the coordinate axes through an angle α as shown; the resulting moments of inertia, $\bar{I}_{x'}$ and $\bar{I}_{y'}$, and product of inertia, $\bar{I}_{x'y'}$, are indicated on the Mohr's circle. However, the x' axis is an axis of symmetry, which implies $\bar{I}_{x'y'} = 0$. For this to be possible on the Mohr's circle, the radius R must be equal to zero (thus, the circle degenerates into a point). With $R = 0$, it immediately follows that

(a) $\bar{I}_x = \bar{I}_y = \bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_{\text{ave}}$ (for all moments of inertia with respect to an axis through C) ◀

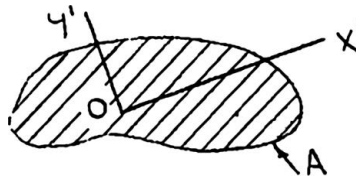
(b) $\bar{I}_{xy} = \bar{I}_{x'y'} = 0$ (for all products of inertia with respect to all pairs of rectangular axes with origin at C) ◀

PROBLEM 9.109

Using Mohr's circle, prove that the expression $I_x I_y - I_{x'y'}^2$ is independent of the orientation of the x' and y' axes, where I_x , I_y , and $I_{x'y'}$ represent the moments and product of inertia, respectively, of a given area with respect to a pair of rectangular axes x' and y' through a given Point O . Also show that the given expression is equal to the square of the length of the tangent drawn from the origin of the coordinate system to Mohr's circle.

SOLUTION

First observe that for a given area A and origin O of a rectangular coordinate system, the values of I_{ave} and R are the same for all orientations of the coordinate axes. Shown below is a Mohr's circle, with the moments of inertia, I_x and I_y , and the product of inertia, $I_{x'y'}$, having been computed for an arbitrary orientation of the $x'y'$ axes.

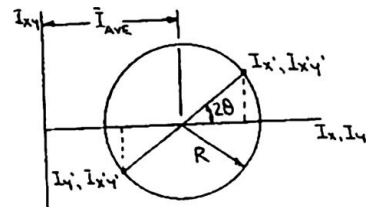


From the Mohr's circle

$$I_{x'} = I_{ave} + R \cos 2\theta$$

$$I_{y'} = I_{ave} - R \cos 2\theta$$

$$I_{x'y'} = R \sin 2\theta$$



Then, forming the expression

$$I_{x'} I_{y'} - I_{x'y'}^2$$

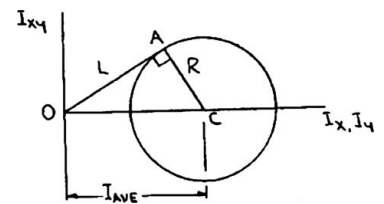
$$\begin{aligned} I_{x'} I_{y'} - I_{x'y'}^2 &= (I_{ave} + R \cos 2\theta)(I_{ave} - R \cos 2\theta) - (R \sin 2\theta)^2 \\ &= (I_{ave}^2 - R^2 \cos^2 2\theta) - (R^2 \sin^2 2\theta) \\ &= I_{ave}^2 - R^2 \quad \text{which is a constant} \end{aligned}$$

$I_{x'} I_{y'} - I_{x'y'}^2$ is independent of the orientation of the coordinate axes Q.E.D. ◀

Shown is a Mohr's circle, with line \overline{OA} , of length L , the required tangent.

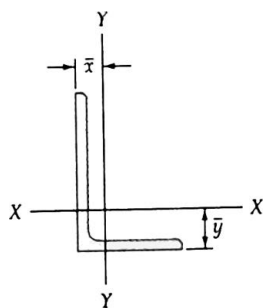
Noting that $\angle OAC$ is a right angle, it follows that

$$L^2 = I_{ave}^2 - R^2$$



or $L^2 = I_{x'} I_{y'} - I_{x'y'}^2$ Q.E.D. ◀

PROBLEM 9.110



Using the invariance property established in the preceding problem, express the product of inertia I_{xy} of an area A with respect to a pair of rectangular axes through O in terms of the moments of inertia I_x and I_y of A and the principal moments of inertia I_{\min} and I_{\max} of A about O . Use the formula obtained to calculate the product of inertia I_{xy} of the $L3 \times 2 \times \frac{1}{4}$ -in. angle cross section shown in Figure 9.13A, knowing that its maximum moment of inertia is 1.257 in^4 .

SOLUTION

Consider the following two sets of moments and products of inertia, which correspond to two different orientations of the coordinate axes whose origin is at Point O .

Case 1: $I_{x'} = I_x, \quad I_{y'} = I_y, \quad I_{x'y'} = I_{xy}$

Case 2: $I_{x'} = I_{\max}, \quad I_{y'} = I_{\min}, \quad I_{x'y'} = 0$

The invariance property then requires

$$I_x I_y - I_{xy}^2 = I_{\max} I_{\min} \quad \text{or} \quad I_{xy} = \pm \sqrt{I_x I_y - I_{\max} I_{\min}} \quad \blacktriangleleft$$

From Figure 9.13A:

$$\bar{I}_x = 1.09 \text{ in}^4$$

$$\bar{I}_y = 0.390 \text{ in}^4$$

Using Eq. (9.21):

$$\bar{I}_x + \bar{I}_y = \bar{I}_{\max} + \bar{I}_{\min}$$

Substituting

$$1.09 + 0.390 = 1.257 + \bar{I}_{\min}$$

or

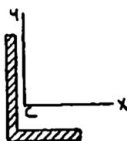
$$\bar{I}_{\min} = 0.223 \text{ in}^4$$

Then

$$\begin{aligned} \bar{I}_{xy} &= \sqrt{(1.09)(0.390) - (1.257)(0.223)} \\ &= \pm 0.381 \text{ in}^4 \end{aligned}$$

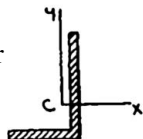
The two roots correspond to the following two orientations of the cross section.

For

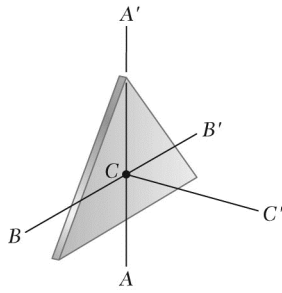


$$\bar{I}_{xy} = -0.381 \text{ in}^4 \quad \blacktriangleleft$$

and for



$$\bar{I}_{xy} = 0.381 \text{ in}^4 \quad \blacktriangleleft$$



PROBLEM 9.111

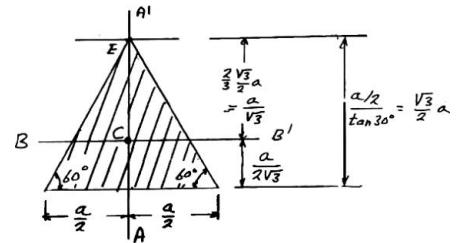
A thin plate of mass m is cut in the shape of an equilateral triangle of side a . Determine the mass moment of inertia of the plate with respect to (a) the centroidal axes AA' and BB' , (b) the centroidal axis CC' that is perpendicular to the plate.

SOLUTION

$$\text{Area} = \frac{1}{2} a \left(\frac{\sqrt{3}}{2} a \right) = \frac{\sqrt{3}}{4} a^2$$

$$\text{Mass} = m = \rho V = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$



(a) Axis AA' :

$$I_{AA', \text{area}} = 2 \left[\frac{1}{12} \left(\frac{\sqrt{3}}{2} a \right) \left(\frac{a}{2} \right)^3 \right] = \frac{\sqrt{3}}{96} a^4$$

$$I_{AA', \text{mass}} = \frac{m}{A} I_{AA', \text{area}} = \frac{\frac{m}{\sqrt{3}}}{4} a^2 \left(\frac{\sqrt{3}}{96} a^4 \right) = \frac{1}{24} ma^2 \quad \blacktriangleleft$$

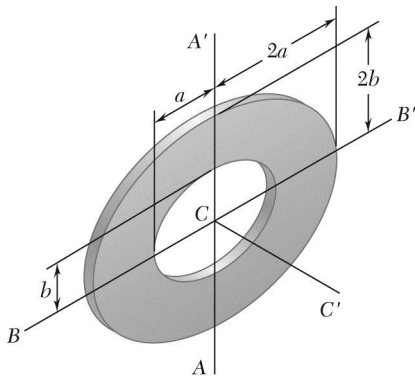
Axis BB' :

$$I_{BB', \text{area}} = \frac{1}{36} a \left(\frac{\sqrt{3}}{2} a \right)^3 = \frac{\sqrt{3}}{96} a^4 \quad (\text{we check that } I_{AA} = I_{BB})$$

$$I_{BB', \text{mass}} = \frac{m}{A} I_{BB', \text{area}} = \frac{\frac{m}{\sqrt{3}}}{4} a^2 \left(\frac{\sqrt{3}}{96} a^4 \right) = \frac{1}{24} ma^2 \quad \blacktriangleleft$$

(b) Eq. (9.38):

$$I_{CC'} = I_{AA'} + I_{BB'} = 2 \left(\frac{1}{24} ma^2 \right) \quad I_{CC'} = \frac{1}{12} ma^2 \quad \blacktriangleleft$$



PROBLEM 9.112

The elliptical ring shown was cut from a thin, uniform plate. Denoting the mass of the ring by m , determine its mass moment of inertia with respect to (a) the centroidal axis BB' , (b) the centroidal axis CC' that is perpendicular to the plane of the ring.

SOLUTION

First note

$$\begin{aligned} \text{Mass} = m &= \rho V = \rho t A \\ &= \rho t \times \eta [(2a)(2b) - ab] \\ &= 3\pi \rho t ab \end{aligned}$$

Also

$$\begin{aligned} I_{\text{mass}} &= \rho t I_{\text{area}} \\ &= \frac{m}{3\pi ab} I_{\text{area}} \end{aligned}$$

(a) Using Figure 9.12,

$$\begin{aligned} I_{BB', \text{area}} &= \frac{\pi}{4} [(2a)(2b)^3 - ab^3] \\ &= \frac{15}{4} \pi ab^3 \end{aligned}$$

Then

$$I_{BB', \text{mass}} = \frac{m}{3\pi ab} \times \frac{15}{4} \pi ab^3$$

$$\text{or } I_{BB'} = \frac{5}{4} mb^2 \quad \blacktriangleleft$$

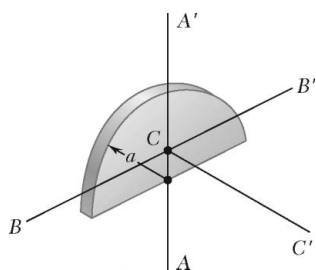
(b) Using Figure 9.12 and symmetry, we can conclude that

$$I_{AA', \text{mass}} = \frac{5}{4} ma^2$$

Now

$$\begin{aligned} I_{CC', \text{mass}} &= I_{AA', \text{mass}} + I_{BB', \text{mass}} \\ &= \frac{5}{4} ma^2 + \frac{5}{4} mb^2 \end{aligned}$$

$$\text{or } I_{CC'} = \frac{5}{4} m(a^2 + b^2) \quad \blacktriangleleft$$



PROBLEM 9.113

A thin semicircular plate has a radius a and a mass m . Determine the mass moment of inertia of the plate with respect to (a) the centroidal axis BB' , (b) the centroidal axis CC' that is perpendicular to the plate.

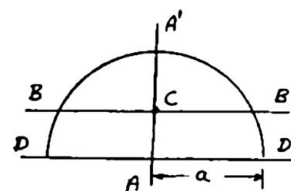
SOLUTION

$$\text{mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

Area:

$$A = \frac{1}{2} \pi a^2$$



$$I_{AA', \text{area}} = I_{DD', \text{area}} = \frac{1}{2} \left(\frac{\pi}{4} a^4 \right) = \frac{1}{8} \pi a^4$$

$$I_{AA', \text{mass}} = I_{DD', \text{mass}} = \frac{m}{A} I_{AA', \text{area}} = \frac{m}{\frac{1}{2} \pi a^2} \left(\frac{1}{8} \pi a^4 \right) = \frac{1}{4} m a^2$$

$$(a) \quad I_{BB'} = I_{DD'} - m(AC)^2 = \frac{1}{4} m a^2 - m \left(\frac{4a}{3\pi} \right)^2$$

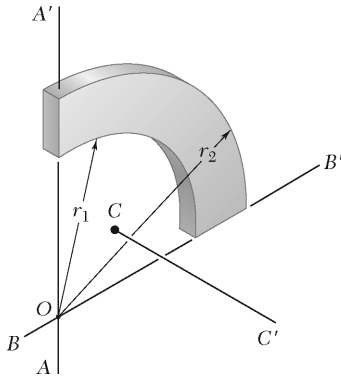
$$= (0.25 - 0.1801) m a^2$$

$$I_{BB'} = 0.0699 m a^2 \quad \blacktriangleleft$$

$$(b) \quad \text{Eq. (9.38):} \quad I_{CC'} = I_{AA'} + I_{BB'} = \frac{1}{4} m a^2 + 0.0699 m a^2$$

$$I_{CC'} = 0.320 m a^2 \quad \blacktriangleleft$$

PROBLEM 9.114



The quarter ring shown has a mass m and was cut from a thin, uniform plate. Knowing that $r_1 = \frac{3}{4} r_2$, determine the mass moment of inertia of the quarter ring with respect to (a) the axis AA' , (b) the centroidal axis CC' that is perpendicular to the plane of the quarter ring.

SOLUTION

First note

$$\begin{aligned} \text{mass} = m &= \rho V = \rho t A \\ &= \rho t \frac{\pi}{4} (r_2^2 - r_1^2) \end{aligned}$$

Also

$$\begin{aligned} I_{\text{mass}} &= \rho t I_{\text{area}} \\ &= \frac{m}{\frac{\pi}{4} (r_2^2 - r_1^2)} I_{\text{area}} \end{aligned}$$

(a) Using Figure 9.12,

$$I_{AA', \text{area}} = \frac{\pi}{16} (r_2^4 - r_1^4)$$

Then

$$\begin{aligned} I_{AA', \text{mass}} &= \frac{m}{\frac{\pi}{4} (r_2^2 - r_1^2)} \times \frac{\pi}{16} (r_2^4 - r_1^4) \\ &= \frac{m}{4} (r_2^2 + r_1^2) \\ &= \frac{m}{4} \left[r_2^2 + \left(\frac{3}{4} r_2 \right)^2 \right] \end{aligned}$$

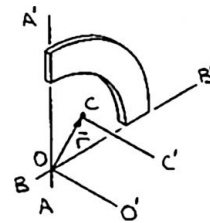
$$\text{or } I_{AA'} = \frac{25}{64} m r_2^2 \quad \blacktriangleleft$$

(b) Symmetry implies

$$I_{BB', \text{mass}} = I_{AA', \text{mass}}$$

Then

$$\begin{aligned} I_{DD'} &= I_{AA'} + I_{BB'} \\ &= 2 \left(\frac{25}{64} m r_2^2 \right) \\ &= \frac{25}{32} m r_2^2 \end{aligned}$$



PROBLEM 9.114 (Continued)

Now locate centroid C .

$$\bar{X} \Sigma A = \Sigma \bar{x} A$$

or

$$\bar{X} \left(\frac{\pi}{4} r_2^2 - \frac{\pi}{4} r_1^2 \right) = \frac{4r_2}{3\pi} \left(\frac{\pi}{4} r_2^2 \right) - \frac{4r_1}{3\pi} \left(\frac{\pi}{4} r_1^2 \right)$$

or

$$\bar{X} = \frac{4}{3\pi} \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$$

Now

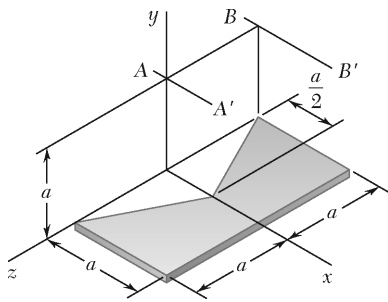
$$\begin{aligned} \bar{r} &= \bar{X} \sqrt{2} \\ &= \frac{4\sqrt{2}}{3\pi} \frac{r_2^3 - \left(\frac{3}{4}r_2\right)^3}{r_2^2 - \left(\frac{3}{4}r_2\right)^2} \\ &= \frac{37\sqrt{2}}{21\pi} r_2 \end{aligned}$$

Finally,

$$I_{DD'} = I_{CC'} + m\bar{r}^2$$

or

$$\frac{25}{32} mr_2^2 = I_{CC'} + m \left(\frac{37\sqrt{2}}{21\pi} r_2 \right)^2 \quad \text{or} \quad I_{CC'} = 0.1522mr_2^2 \blacktriangleleft$$



PROBLEM 9.115

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m , determine its mass moment of inertia with respect to (a) the x axis, (b) the y axis.

SOLUTION

First note

$$\begin{aligned} \text{mass} = m &= \rho V = \rho t A \\ &= \rho t \left[(2a)(a) - \frac{1}{2}(2a) \left(\frac{a}{2} \right) \right] \\ &= \frac{3}{2} \rho t a^2 \end{aligned}$$

Also

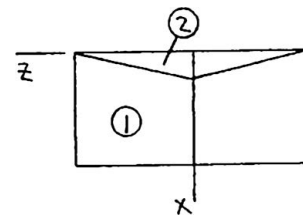
$$\begin{aligned} I_{\text{mass}} &= \rho t I_{\text{area}} \\ &= \frac{2m}{3a^2} I_{\text{area}} \end{aligned}$$

(a) Now

$$\begin{aligned} \bar{I}_{x,\text{area}} &= (I_x)_{1,\text{area}} - 2(I_x)_{2,\text{area}} \\ &= \frac{1}{12}(a)(2a)^3 - 2 \left[\frac{1}{12} \left(\frac{a}{2} \right) (a)^3 \right] \\ &= \frac{7}{12} a^4 \end{aligned}$$

Then

$$\bar{I}_{x,\text{mass}} = \frac{2m}{3a^2} \times \frac{7}{12} a^4$$



$$\text{or } \bar{I}_x = \frac{7}{18} m a^2 \blacktriangleleft$$

(b) We have

$$\begin{aligned} \bar{I}_{z,\text{area}} &= (I_z)_{1,\text{area}} - 2(I_z)_{2,\text{area}} \\ &= \frac{1}{3}(2a)(a)^3 - 2 \left[\frac{1}{12}(a) \left(\frac{a}{2} \right)^3 \right] \\ &= \frac{31}{48} a^4 \end{aligned}$$

Then

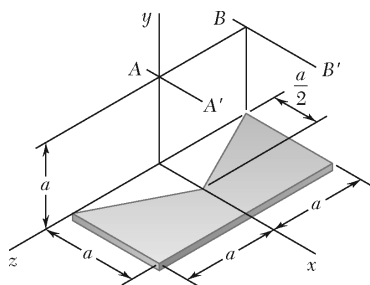
$$\begin{aligned} I_{z,\text{mass}} &= \frac{2m}{3a^2} \times \frac{31}{48} a^4 \\ &= \frac{31}{72} m a^2 \end{aligned}$$

PROBLEM 9.115 (Continued)

Finally,

$$\begin{aligned} I_{y,\text{mass}} &= \bar{I}_{x,\text{mass}} + I_{z,\text{mass}} \\ &= \frac{7}{18}ma^2 + \frac{31}{72}ma^2 \\ &= \frac{59}{72}ma^2 \end{aligned}$$

or $I_y = 0.819ma^2 \blacktriangleleft$



PROBLEM 9.116

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m , determine its mass moment of inertia with respect to (a) the axis AA' , (b) the axis BB' , where the AA' and BB' axes are parallel to the x axis and lie in a plane parallel to and at a distance a above the xz plane.

SOLUTION

First note that the x axis is a centroidal axis so that

$$I = \bar{I}_{x,\text{mass}} + md^2$$

and that from the solution to Problem 9.115,

$$\bar{I}_{x,\text{mass}} = \frac{7}{18}ma^2$$

(a) We have

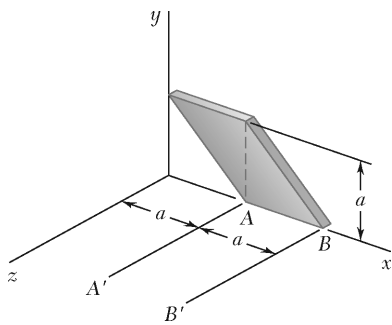
$$I_{AA',\text{mass}} = \frac{7}{18}ma^2 + m(a)^2$$

$$\text{or } I_{AA'} = 1.389ma^2 \blacktriangleleft$$

(b) We have

$$I_{BB',\text{mass}} = \frac{7}{18}ma^2 + m(a\sqrt{2})^2$$

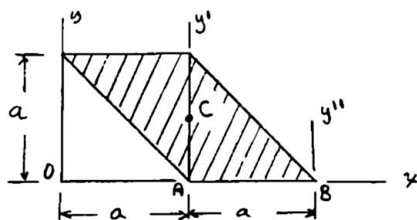
$$\text{or } I_{BB'} = 2.39ma^2 \blacktriangleleft$$



PROBLEM 9.117

A thin plate of mass m was cut in the shape of a parallelogram as shown. Determine the mass moment of inertia of the plate with respect to (a) the x axis, (b) the axis BB' , which is perpendicular to the plate.

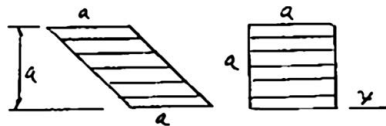
SOLUTION



$$\text{mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

- (a) Consider parallelogram as made of horizontal strips and slide strips to form a square since distance from each strip to x axis is unchanged.



$$I_{x,\text{area}} = \frac{1}{3} a^4$$

$$I_{x,\text{mass}} = \frac{m}{A} I_{x,\text{area}} = \frac{m}{a^2} \left(\frac{1}{3} a^4 \right)$$

$$I_x = \frac{1}{3} m a^2 \quad \blacktriangleleft$$

- (b) For centroidal axis y' :

$$\bar{I}_{y',\text{area}} = 2 \left[\frac{1}{12} a^4 \right] = \frac{1}{6} a^4$$

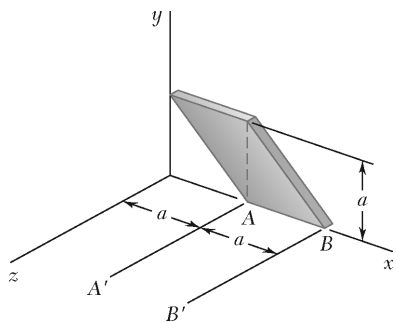
$$\bar{I}_{y',\text{mass}} = \frac{m}{A} \bar{I}_{y',\text{area}} = \frac{m}{a^2} \left(\frac{1}{6} a^4 \right) = \frac{1}{6} m a^2$$

$$I_{y''} = \bar{I}_{y'} + m a^2 = \frac{1}{6} m a^2 + m a^2 = \frac{7}{6} m a^2$$

For axis $BB' \perp$ to plate, Eq. (9.38):

$$I_{BB'} = I_x + I_{y''} = \frac{1}{3} m a^2 + \frac{7}{6} m a^2$$

$$I_{BB'} = \frac{3}{2} m a^2 \quad \blacktriangleleft$$



PROBLEM 9.118

A thin plate of mass m was cut in the shape of a parallelogram as shown. Determine the mass moment of inertia of the plate with respect to (a) the y axis, (b) the axis AA' , which is perpendicular to the plate.

SOLUTION

See sketch of solution of Problem 9.117.

(a) From part b of solution of Problem 9.117:

$$\bar{I}_{y'} = \frac{1}{6}ma^2$$

$$I_y = \bar{I}_{y'} + ma^2 = \frac{1}{6}ma^2 + ma^2$$

$$I_y = \frac{7}{6}ma^2 \quad \blacktriangleleft$$

(b) From solution of Problem 9.115:

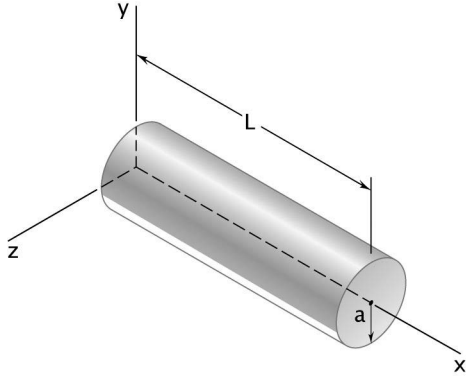
$$\bar{I}_{y'} = \frac{1}{6}ma^2 \quad \text{and} \quad I_x = \frac{1}{3}ma^2$$

Eq. (9.38):

$$I_{AA'} = \bar{I}_{y'} + I_x$$

$$= \frac{1}{6}ma^2 + \frac{1}{3}ma^2$$

$$I_{AA'} = \frac{1}{2}ma^2 \quad \blacktriangleleft$$



PROBLEM 9.119

Determine by direct integration the mass moment of inertia with respect to the z axis of the right circular cylinder shown, assuming that it has a uniform density and a mass m .

SOLUTION

For the cylinder: $m = \rho V = \rho \pi a^2 L$

For the element shown: $dm = \rho \pi a^2 dx$

$$= \frac{m}{L} dx$$

and

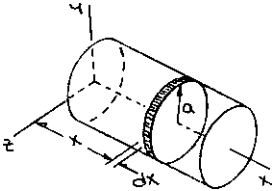
$$dI_z = dI_z + x^2 dm$$

$$= \frac{1}{4} a^2 dm + x^2 dm$$

Then

$$I_z = \int dI_z = \int_0^L \left(\frac{1}{4} a^2 + x^2 \right) \left(\frac{m}{L} dx \right)$$

$$= \frac{m}{L} \left[\frac{1}{4} a^2 x + \frac{1}{3} x^3 \right]_0^L$$



$$I_z = \frac{1}{12} m(3a^2 + 4L^2) \blacktriangleleft$$



PROBLEM 9.120

The area shown is revolved about the x axis to form a homogeneous solid of revolution of mass m . Using direct integration, express the mass moment of inertia of the solid with respect to the x axis in terms of m and h .

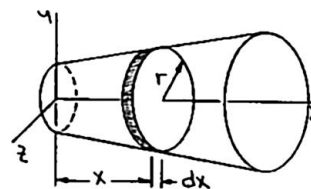
SOLUTION

We have
$$y = \frac{2h-h}{a}x + h$$

so that
$$r = \frac{h}{a}(x+a)$$

For the element shown:

$$\begin{aligned} dm &= \rho\pi r^2 dx & dI_x &= \frac{1}{2}r^2 dm \\ &= \rho\pi \left[\frac{h}{a}(x+a) \right]^2 dx \end{aligned}$$



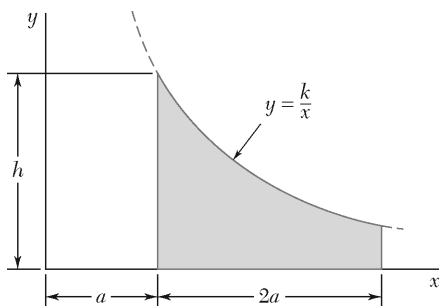
Then
$$\begin{aligned} m &= \int dm = \int_0^a \rho\pi \frac{h^2}{a^2}(x+a)^2 dx = \frac{1}{3}\rho\pi \frac{h^2}{a^2}[(x+a)^3]_0^a \\ &= \frac{1}{3}\rho\pi \frac{h^2}{a^2}(8a^3 - a^3) = \frac{7}{3}\rho\pi ah^2 \end{aligned}$$

Now
$$\begin{aligned} I_x &= \int dI_x = \int_0^a \frac{1}{2}r^2(\rho\pi r^2 dx) = \frac{1}{2}\rho\pi \int_0^a \left[\frac{h}{a}(x+a) \right]^4 dx \\ &= \frac{1}{2}\rho\pi \times \frac{1}{5} \frac{h^4}{a^4} [(x+a)^5]_0^a = \frac{1}{10}\rho\pi \frac{h^4}{a^4} (32a^5 - a^5) \\ &= \frac{31}{10}\rho\pi ah^4 \end{aligned}$$

From above:
$$\rho\pi ah^2 = \frac{3}{7}m$$

Then
$$I_x = \frac{31}{10} \left(\frac{3}{7}m \right) h^2 = \frac{93}{70}mh^2$$

or
$$I_x = 1.329 mh^2 \blacktriangleleft$$



PROBLEM 9.121

The area shown is revolved about the x axis to form a homogeneous solid of revolution of mass m . Determine by direct integration the mass moment of inertia of the solid with respect to (a) the x axis, (b) the y axis. Express your answers in terms of m and the dimensions of the solid.

SOLUTION

We have at $(a, h): h = \frac{k}{a}$

or $k = ah$

For the element shown:

$$r = a$$

$$dm = \rho \pi r^2 dx$$

$$= \rho \pi \left(\frac{ah}{x} \right)^2 dx$$

Then

$$\begin{aligned} m &= \int dm = \int_a^{3a} \rho \pi \left(\frac{ah}{x} \right)^2 dx \\ &= \rho \pi a^2 h^2 \left[-\frac{1}{x} \right]_a^{3a} \\ &= \rho \pi a^2 h^2 \left[-\frac{1}{3a} - \left(-\frac{1}{a} \right) \right] = \frac{2}{3} \rho \pi a h^2 \end{aligned}$$

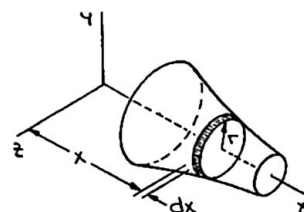
(a) For the element:

$$dI_x = \frac{1}{2} r^2 dm = \frac{1}{2} \rho \pi r^4 dx$$

Then

$$\begin{aligned} I_x &= \int dI_x = \int_a^{3a} \frac{1}{2} \rho \pi \left(\frac{ah}{x} \right)^4 dx = \frac{1}{2} \rho \pi a^4 h^4 \left[-\frac{1}{3x^3} \right]_a^{3a} \\ &= -\frac{1}{6} \rho \pi a^4 h^4 \left[\left(\frac{1}{3a} \right)^3 - \left(\frac{1}{a} \right)^3 \right] = \frac{1}{6} \times \frac{26}{27} \rho \pi a h^4 \\ &= \frac{1}{6} \times \frac{2}{3} \rho \pi a h^2 \times \frac{13}{9} h^2 = \frac{13}{54} m h^2 \end{aligned}$$

$$\text{or } I_x = 0.241 m h^2 \blacktriangleleft$$



PROBLEM 9.121 (Continued)

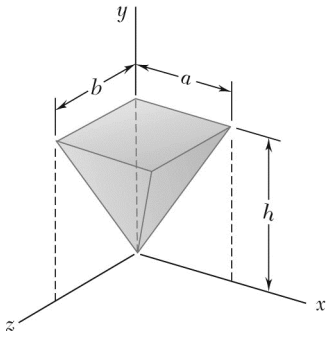
(b) For the element:

$$\begin{aligned} dI_y &= dI_y + x^2 dm \\ &= \frac{1}{4} r^2 dm + x^2 dm \end{aligned}$$

Then

$$\begin{aligned} I_y &= \int dI_y = \int_a^{3a} \left[\frac{1}{4} \left(\frac{ah}{x} \right)^2 + x^2 \right] \rho \pi \left(\frac{ah}{x} \right)^2 dx \\ &= \rho \pi a^2 h^2 \int_a^{3a} \left(\frac{1}{4} \frac{a^2 h^2}{x^4} + 1 \right) dx = \rho \pi a^2 h^2 \left[-\frac{1}{12} \frac{a^2 h^2}{x^3} + x \right]_a^{3a} \\ &= \left(\frac{3}{2} m \right) a \left\{ \left[-\frac{1}{12} \frac{a^2 h^2}{(3a)^3} + 3a \right] - \left[-\frac{1}{12} \frac{a^2 h^2}{(a)^3} + a \right] \right\} \\ &= \frac{3}{2} ma \left(\frac{1}{12} \times \frac{26}{27} \frac{h^2}{a} + 2a \right) = m \left(\frac{13}{108} h^2 + 3a^2 \right) \end{aligned}$$

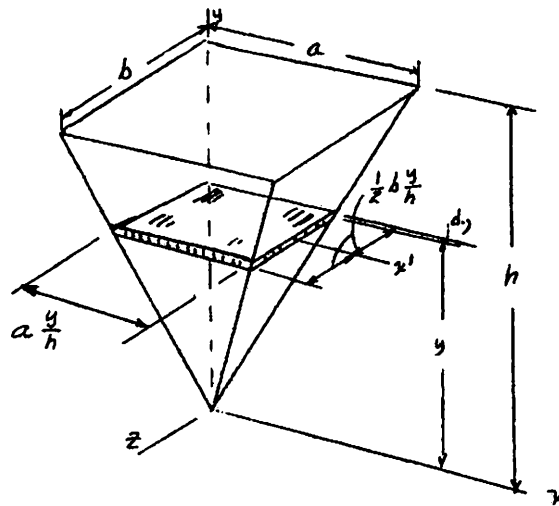
or $I_y = m(3a^2 + 0.1204h^2) \blacktriangleleft$



PROBLEM 9.122

Determine by direct integration the mass moment of inertia with respect to the x axis of the pyramid shown, assuming that it has a uniform density and a mass m .

SOLUTION



For element shown:

$$dm = \rho dV = \rho \left(a \frac{y}{h} \right) \left(b \frac{y}{h} \right) dy = \rho \frac{ab}{h^2} y^2 dy$$

$$dI_{x'} = \frac{1}{12} \left(b \frac{y}{h} \right)^2 dm = \frac{1}{12} \frac{b^2}{h^2} y^2 \left(\rho \frac{ab}{h^2} \right) y^2 dy = \frac{1}{12} \rho \frac{ab^3}{h^4} y^4 dy$$

Parallel-axis theorem

$$dI_x = dI_{x'} + d^2 dm \quad \text{where} \quad d^2 = y^2 + \left(\frac{1}{2} b \frac{y}{h} \right)^2$$

$$dI_x = \frac{1}{12} \rho \frac{ab^3}{h^4} y^4 dy + \left[y^2 + \frac{1}{4} \frac{b^2}{h^2} y^2 \right] \rho \frac{ab}{h^2} y^2 dy$$

$$= \left(\frac{\rho ab^3}{3 h^4} + \rho \frac{ab}{h^2} \right) y^4 dy$$

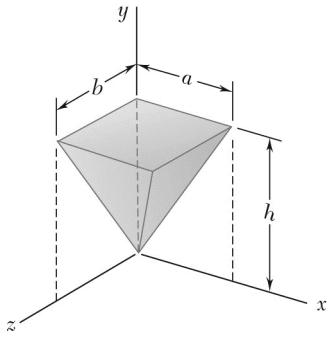
$$I_x = \int dI_x = \left(\frac{\rho ab^3}{3 h^4} + \rho \frac{ab}{h^2} \right) \int_0^h y^4 dy = \frac{\rho ab^3 h}{15} + \frac{\rho ab h^3}{5}$$

PROBLEM 9.122 (Continued)

For pyramid, $m = \rho v = \frac{1}{3} \rho abh$

Thus: $I_x = \left(\frac{1}{3} \rho abh \right) \left(\frac{b^2}{5} + \frac{3h^2}{5} \right)$

$$I_x = \frac{1}{5} m(b^2 + 3h^2) \blacktriangleleft$$



PROBLEM 9.123

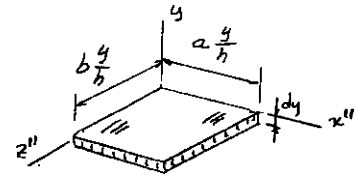
Determine by direct integration the mass moment of inertia with respect to the x axis of the pyramid shown, assuming that it has a uniform density and a mass m .

SOLUTION

See figure of solution of Problem 9.122 for element shown

$$dm = \rho \left(b \frac{y}{h} \right) \left(a \frac{y}{h} \right) dy$$

$$dm = \rho \frac{ab}{h^2} y^2 dy$$



For thin plate:

$$dI_y = dI_{x''} + dI_{z''}$$

$$\begin{aligned} dI_y &= \frac{1}{3} \left(b \frac{y}{h} \right)^2 dm + \frac{1}{3} \left(a \frac{y}{h} \right)^2 dm = \frac{1}{3h^2} (b^2 + a^2) y^2 dm \\ &= \frac{1}{3h^2} (a^2 + b^2) y^2 \left(\rho \frac{ab}{h^2} y^2 dy \right) = \frac{\rho ab}{3h^4} (a^2 + b^2) y^4 dy \end{aligned}$$

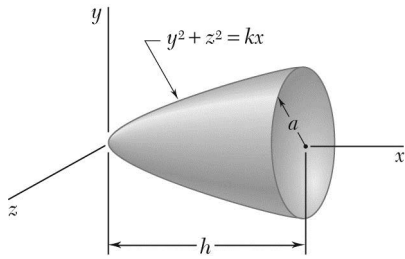
$$I_y = \int dI_y = \frac{\rho ab}{3h^4} (a^2 + b^2) \int_0^h y^4 dy = \frac{\rho ab}{15} (a^2 + b^2) h$$

For pyramid,

$$m = \rho v = \frac{1}{3} \rho abh$$

$$I_y = \left(\frac{1}{3} \rho abh \right) \frac{1}{5} (a^2 + b^2)$$

$$I_y = \frac{1}{5} m (a^2 + b^2) \blacktriangleleft$$



PROBLEM 9.124

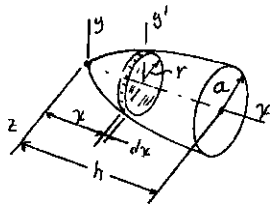
Determine by direct integration the mass moment of inertia with respect to the y axis of the paraboloid shown, assuming that it has a uniform density and a mass m .

SOLUTION

$$r^2 = y^2 + z^2 = kx:$$

$$\text{at } x = h, r = a; \quad a^2 = kh; \quad k = \frac{a^2}{h}$$

$$\text{Thus: } r^2 = \frac{a^2}{h} x$$



$$dm = \rho \pi r^2 dx = \rho \pi \frac{a^2}{h} x dx$$

$$m = \int_0^h dm = \rho \pi \frac{a^2}{h} \int_0^h x dx; \quad m = \frac{1}{2} \rho \pi a^2 h$$

$$dI_y = dI_{y'} + mx^2 = \frac{1}{4} r^2 dm + x^2 dm = \left(\frac{1}{4} \frac{a^2}{h} x + x^2 \right) dm$$

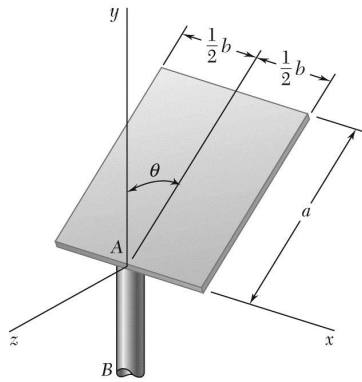
$$\begin{aligned} I_y &= \int_0^h I_y = \int_0^h \left(\frac{1}{4} \frac{a^2}{h} x + x^2 \right) \rho \pi \frac{a^2}{h} x dx = \rho \pi \frac{a^2}{h} \int_0^h \left(\frac{1}{4} \frac{a^2}{h} x^2 + x^3 \right) dx \\ &= \rho \pi \frac{a^2}{h} \left(\frac{1}{4} \frac{a^2}{h} \frac{h^3}{3} + \frac{h^4}{4} \right) \end{aligned}$$

$$I_y = \frac{1}{12} \rho \pi a^2 h (a^2 + 3h^2) \triangleleft$$

$$\text{Recall: } m = \frac{1}{2} \rho \pi a^2 h;$$

$$I_y = \left(\frac{1}{2} \rho \pi a^2 h \right) \left(\frac{1}{6} \right) (a^2 + 3h^2)$$

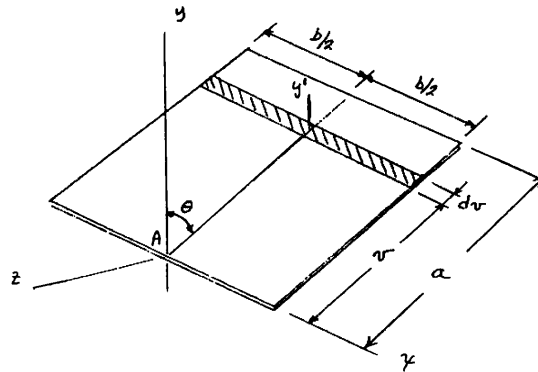
$$I_y = \frac{1}{6} m (a^2 + 3h^2) \blacktriangleleft$$



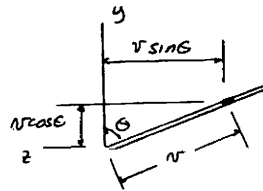
PROBLEM 9.125

A thin rectangular plate of mass m is welded to a vertical shaft AB as shown. Knowing that the plate forms an angle θ with the y axis, determine by direct integration the mass moment of inertia of the plate with respect to (a) the y axis, (b) the z axis.

SOLUTION



Projection on yz plane



Mass of plate: $m = \rho tab$

(a) For element shown:

$$dm = \rho btdv$$

$$d\bar{I}_y = d\bar{I}_{y'} + (v \sin \theta)^2 dm = \frac{1}{12} b^2 dm + v^2 \sin^2 \theta dm$$

$$= \left(\frac{1}{12} b^2 + v^2 \sin^2 \theta \right) \rho btdv$$

$$I_y = \int dI_y = \rho bt \int_0^a \left(\frac{1}{12} b^2 + v^2 \sin^2 \theta \right) dv$$

$$= \rho bt \left[\frac{1}{12} b^2 v + \frac{v^3}{3} \sin^2 \theta \right]_0^a = \rho bt \left(\frac{1}{12} ab^2 + \frac{a^3}{3} \sin^2 \theta \right)$$

$$= (\rho tab) \frac{1}{12} (b^2 + 4a^2 \sin^2 \theta)$$

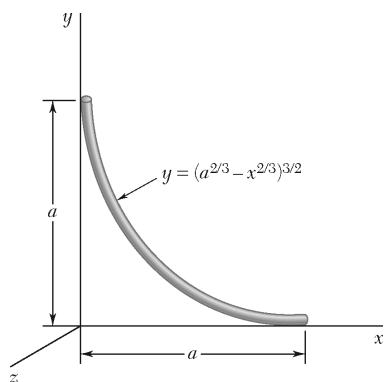
$$I_y = \frac{1}{12} m (b^2 + 4a^2 \sin^2 \theta) \blacktriangleleft$$

PROBLEM 9.125 (Continued)

(b)

$$\begin{aligned}dI_z &= d\bar{I}_z + (v \cos \theta)^2 dm = \frac{1}{12} b^2 dm + v^2 \cos^2 \theta dm \\&= \left(\frac{1}{12} b^2 + v^2 \cos^2 \theta \right) \rho b t dv \\I_z &= \int dI_z = \rho b t \int_0^a \left(\frac{1}{12} b^2 + v^2 \cos^2 \theta \right) dv \\&= \rho b t \left[\frac{1}{12} b^2 v + \frac{v^3}{3} \cos^2 \theta \right]_0^a = \rho b t \left(\frac{1}{12} a b^2 + \frac{a^3}{3} \cos^2 \theta \right) \\&= (\rho t a b) \frac{1}{12} (b^2 + 4a^2 \cos^2 \theta)\end{aligned}$$

$$I_z = \frac{1}{12} m (b^2 + 4a^2 \cos^2 \theta) \blacktriangleleft$$



PROBLEM 9.126*

A thin steel wire is bent into the shape shown. Denoting the mass per unit length of the wire by m' , determine by direct integration the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION

First note

$$\frac{dy}{dx} = -x^{-1/3} (a^{2/3} - x^{2/3})^{1/2}$$

Then

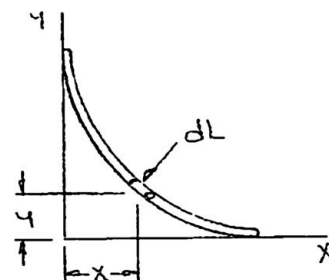
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-2/3} (a^{2/3} - x^{2/3})$$

$$= \left(\frac{a}{x}\right)^{2/3}$$

For the element shown:

$$dm = m' dL = m' \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= m' \left(\frac{a}{x}\right)^{1/3} dx$$



Then

$$m = \int dm = \int_0^a m' \frac{a^{1/3}}{x^{1/3}} dx = \frac{3}{2} m' a^{1/3} [x^{2/3}]_0^a = \frac{3}{2} m' a$$

Now

$$I_x = \int y^2 dm = \int_0^a (a^{2/3} - x^{2/3})^3 \left(m' \frac{a^{1/3}}{x^{1/3}} dx \right)$$

$$= m' a^{1/3} \int_0^a \left(\frac{a^2}{x^{1/3}} - 3a^{4/3} x^{1/3} + 3a^{2/3} x - x^{5/3} \right) dx$$

$$= m' a^{1/3} \left[\frac{3}{2} a^2 x^{2/3} - \frac{9}{4} a^{4/3} x^{4/3} + \frac{3}{2} a^{2/3} x^2 - \frac{3}{8} x^{8/3} \right]_0^a$$

$$= m' a^3 \left(\frac{3}{2} - \frac{9}{4} + \frac{3}{2} - \frac{3}{8} \right) = \frac{3}{8} m' a^3$$

$$\text{or } I_x = \frac{1}{4} ma^2 \blacktriangleleft$$

Symmetry implies

$$I_y = \frac{1}{4} ma^2 \blacktriangleleft$$

PROBLEM 9.126* (Continued)

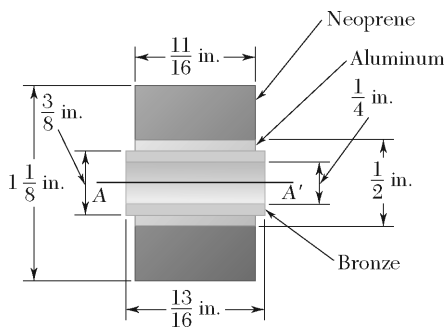
Alternative solution:

$$\begin{aligned} I_y &= \int x^2 dm = \int_0^a x^2 \left(m' \frac{a^{1/3}}{x^{1/3}} dx \right) = m' a^{1/3} \int_0^a x^{5/3} dx \\ &= m' a^{1/3} \times \frac{3}{8} \left[x^{8/3} \right]_0^a = \frac{3}{8} m' a^3 \\ &= \frac{1}{4} ma^2 \end{aligned}$$

Also

$$I_z = \int (x^2 + y^2) dm = I_y + I_x$$

or $I_z = \frac{1}{2} ma^2 \blacktriangleleft$



PROBLEM 9.127

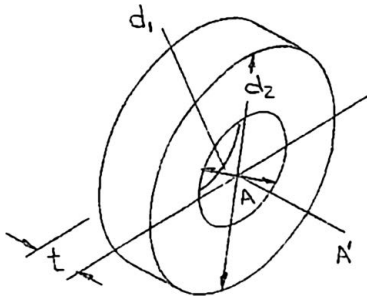
Shown is the cross section of an idler roller. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA' . (The specific weight of bronze is 0.310 lb/in^3 ; of aluminum, 0.100 lb/in^3 ; and of neoprene, 0.0452 lb/in^3 .)

SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4} (d_2^2 - d_1^2) = \frac{\pi}{4} \rho t (d_2^2 - d_1^2)$$

and, using Figure 9.28, that



$$\begin{aligned} I_{AA'} &= \frac{1}{2} m_2 \left(\frac{d_2}{2} \right)^2 - \frac{1}{2} m_1 \left(\frac{d_1}{2} \right)^2 \\ &= \frac{1}{8} \left[\left(\rho t \times \frac{\pi}{4} d_2^2 \right) d_2^2 - \left(\rho t \times \frac{\pi}{4} d_1^2 \right) d_1^2 \right] \\ &= \frac{1}{8} \left(\frac{\pi}{4} \rho t \right) (d_2^4 - d_1^4) \\ &= \frac{1}{8} \left(\frac{\pi}{4} \rho t \right) (d_2^2 - d_1^2) (d_2^2 + d_1^2) \\ &= \frac{1}{8} m (d_1^2 + d_2^2) \end{aligned}$$

Now treat the roller as three concentric rings and, working from the bronze outward, we have

$$\begin{aligned} m &= \frac{\pi}{4} \times \frac{1}{32.2} \text{ ft/s}^2 \left\{ (0.310 \text{ lb/in}^3) \left(\frac{13}{16} \text{ in.} \right) \left[\left(\frac{3}{8} \right)^2 - \left(\frac{1}{4} \right)^2 \right] \text{ in}^2 \right. \\ &\quad + (0.100 \text{ lb/in}^3) \left(\frac{11}{16} \text{ in.} \right) \left[\left(\frac{1}{2} \right)^2 - \left(\frac{3}{8} \right)^2 \right] \text{ in}^2 \\ &\quad \left. + (0.0452 \text{ lb/in}^3) \left(\frac{11}{16} \text{ in.} \right) \left[\left(1\frac{1}{8} \right)^2 - \left(\frac{1}{2} \right)^2 \right] \text{ in}^2 \right\} \\ &= (479.96 + 183.41 + 769.80) \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft} \\ &= 1.4332 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft} \end{aligned}$$

PROBLEM 9.127 (Continued)

and

$$\begin{aligned} I_{AA'} &= \frac{1}{8} \left\{ (479.96) \left[\left(\frac{1}{4} \right)^2 + \left(\frac{3}{8} \right)^2 \right] + (183.41) \left[\left(\frac{3}{8} \right)^2 + \left(\frac{1}{2} \right)^2 \right] \right. \\ &\quad \left. + (769.80) \left[\left(\frac{1}{8} \right)^2 + \left(\frac{1}{2} \right)^2 \right] \right\} \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft} \times \text{in}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} \\ &= (84.628 + 62.191 + 1012.78) \times 10^{-9} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= 1.15960 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

$$\text{or } I_{AA'} = 1.160 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$

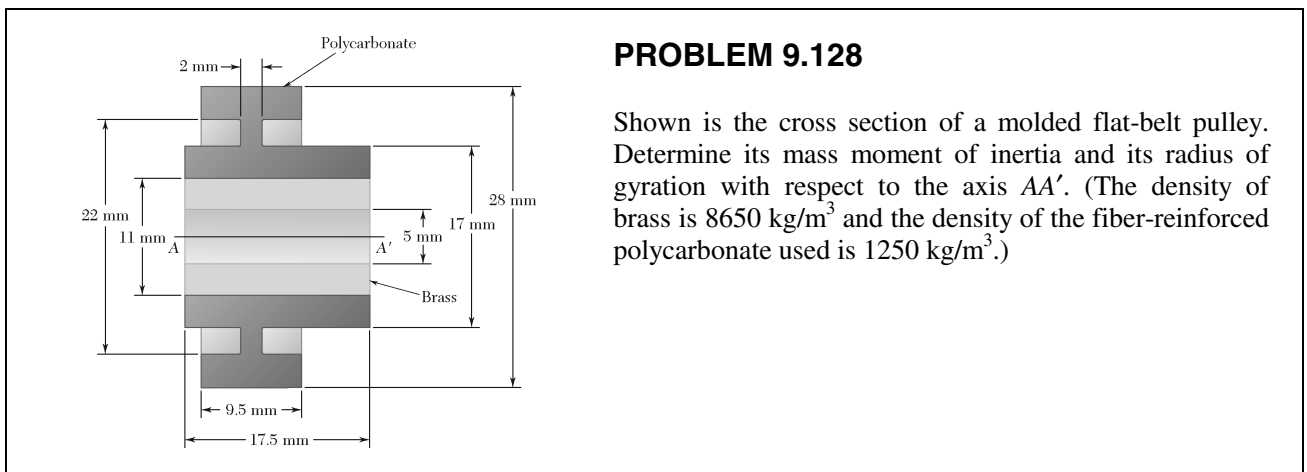
Now

$$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.15960 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2}{1.4332 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}} = 809.09 \times 10^{-6} \text{ ft}^2$$

Then

$$k_{AA'} = 28.445 \times 10^{-3} \text{ ft}$$

$$\text{or } k_{AA'} = 0.341 \text{ in. } \blacktriangleleft$$



PROBLEM 9.128

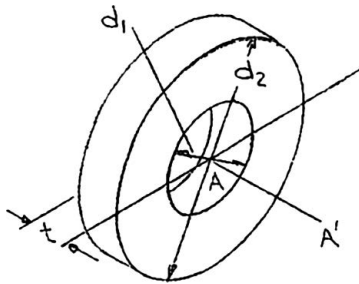
Shown is the cross section of a molded flat-belt pulley. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA' . (The density of brass is 8650 kg/m^3 and the density of the fiber-reinforced polycarbonate used is 1250 kg/m^3 .)

SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4} (d_2^2 - d_1^2)$$

and, using Figure 9.28, that



$$\begin{aligned} I_{AA'} &= \frac{1}{2} m_2 \left(\frac{d_2}{2} \right)^2 - \frac{1}{2} m_1 \left(\frac{d_1}{2} \right)^2 \\ &= \frac{1}{8} \left[\left(\rho t \times \frac{\pi}{4} d_2^2 \right) d_2^2 - \left(\rho t \times \frac{\pi}{4} d_1^2 \right) d_1^2 \right] \\ &= \frac{1}{8} \left(\frac{\pi}{4} \rho t \right) (d_2^4 - d_1^4) \\ &= \frac{1}{8} \left(\frac{\pi}{4} \rho t \right) (d_2^2 - d_1^2) (d_2^2 + d_1^2) \\ &= \frac{1}{8} m (d_1^2 + d_2^2) \end{aligned}$$

Now treat the pulley as four concentric rings and, working from the brass outward, we have

$$\begin{aligned} m &= \frac{\pi}{4} \left\{ 8650 \text{ kg/m}^3 \times (0.0175 \text{ m}) \times (0.011^2 - 0.005^2) \text{ m}^2 \right. \\ &\quad + 1250 \text{ kg/m}^3 [(0.0175 \text{ m}) \times (0.017^2 - 0.011^2) \text{ m}^2 \\ &\quad + (0.002 \text{ m}) \times (0.022^2 - 0.017^2) \text{ m}^2 \\ &\quad \left. + (0.0095 \text{ m}) \times (0.028^2 - 0.022^2) \text{ m}^2 \right\} \\ &= (11.4134 + 2.8863 + 0.38288 + 2.7980) \times 10^{-3} \text{ kg} \\ &= 17.4806 \times 10^{-3} \text{ kg} \end{aligned}$$

PROBLEM 9.128 (Continued)

and

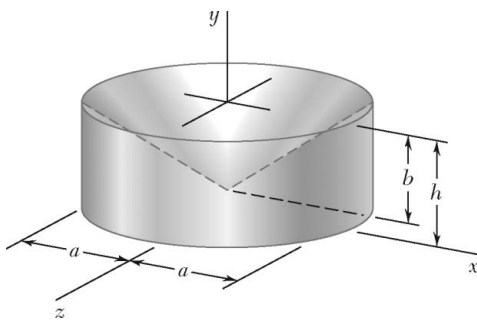
$$\begin{aligned} I_{AA'} &= \frac{1}{8}[(11.4134)(0.005^2 + 0.011^2) + (2.8863)(0.011^2 + 0.017^2) \\ &\quad + (0.38288)(0.017^2 + 0.022^2) \\ &\quad + (2.7980)(0.022^2 + 0.028^2)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ &= (208.29 + 147.92 + 37.00 + 443.48) \times 10^{-9} \text{ kg} \cdot \text{m}^2 \\ &= 836.69 \times 10^{-9} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

or $I_{AA'} = 837 \times 10^{-9} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

Now

$$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{836.69 \times 10^{-9} \text{ kg} \cdot \text{m}^2}{17.4806 \times 10^{-3} \text{ kg}} = 47.864 \times 10^{-6} \text{ m}^2$$

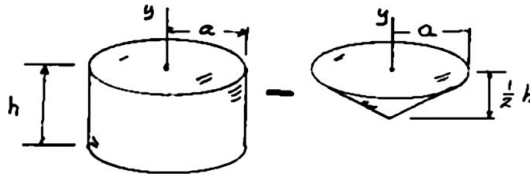
or $k_{AA'} = 6.92 \text{ mm} \blacktriangleleft$



PROBLEM 9.129

The machine part shown is formed by machining a conical surface into a circular cylinder. For $b = \frac{1}{2}h$, determine the mass moment of inertia and the radius of gyration of the machine part with respect to the y axis.

SOLUTION



Mass:

$$m_{\text{cyl}} = \rho\pi a^2 h$$

$$m_{\text{cone}} = \frac{1}{3}\rho\pi a^2 \frac{h}{2} = \frac{1}{6}\rho\pi a^2 h$$

$$I_y: I_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2$$

$$= \frac{1}{2}\rho\pi a^4 h$$

$$I_{\text{cone}} = \frac{3}{10}m_{\text{cone}}a^2$$

$$= \frac{1}{20}\rho\pi a^4 h$$

For entire machine part:

$$m = m_{\text{cyl}} - m_{\text{cone}} = \rho\pi a^2 h - \frac{1}{6}\rho\pi a^2 h = \frac{5}{6}\rho\pi a^2 h$$

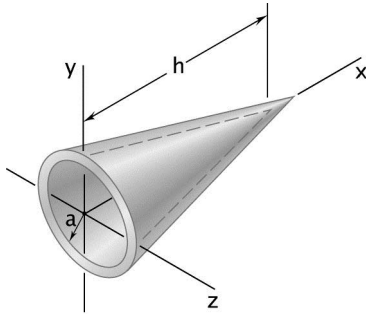
$$I_y = I_{\text{cyl}} - I_{\text{cone}} = \frac{1}{2}\rho\pi a^4 h - \frac{1}{20}\rho\pi a^4 h = \frac{9}{20}\rho\pi a^4 h$$

or

$$I_y = \left(\frac{5}{6}\rho\pi a^2 h\right)\left(\frac{6}{5}\right)\left(\frac{9}{20}\right)a^2 \quad I_y = \frac{27}{50}ma^2 \blacktriangleleft$$

Then

$$k_y^2 = \frac{I}{m} = \frac{27}{50}a^2 \quad k_y = 0.735a \blacktriangleleft$$



PROBLEM 9.130

Given the dimensions and the mass m of the thin conical shell shown, determine the mass moment of inertia and the radius of gyration of the shell with respect to the x axis. (*Hint:* Assume that the shell was formed by removing a cone with a circular base of radius a from a cone with a circular base of radius $a + t$, where t is the thickness of the wall. In the resulting expressions, neglect terms containing t^2 , t^3 , etc. Do not forget to account for the difference in the heights of the two cones.)

SOLUTION

First note

$$\frac{h}{a+t} = \frac{h'}{a}$$

or

$$h' = \frac{h}{a}(a+t)$$

For a cone of height H whose base has a radius r , have

$$I_x = \frac{3}{10}mr^2$$

$$m = \rho V = \rho \times \frac{\pi}{3}r^2H$$

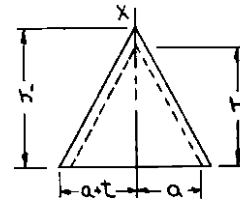
Then

$$\begin{aligned} I_x &= \frac{3}{10} \left(\frac{\pi}{3} \rho r^2 H \right) r^2 \\ &= \frac{\pi}{10} \rho r^4 H \end{aligned}$$

Now following the hint have

$$\begin{aligned} m_{\text{shell}} &= m_{\text{outer}} - m_{\text{inner}} = \frac{\pi}{3} \rho [(a+t)^2 h' - a^2 h] \\ &= \frac{\pi}{3} \rho \left[(a+t)^2 = \frac{b}{a}(a+t) - a^2 h \right] \\ &= \frac{\pi}{3} \rho a^2 h \left[\left(1 + \frac{t}{a}\right)^3 - 1 \right] = \frac{\pi}{3} \rho a^2 h \left(1 + 3\frac{t}{a} + \dots - 1 \right) \end{aligned}$$

Neglecting the t^2 and t^3 terms obtain $m_{\text{shell}} = \pi \rho a h t$



PROBLEM 9.130 (Continued)

Also

$$\begin{aligned}(I_x)_{\text{shell}} &= (I_x)_{\text{outer}} - (I_x)_{\text{inner}} \\ &= \frac{\pi}{10} \rho [(a+t)^4 h' - a^4 h] \\ &= \frac{\pi}{10} \rho \left[(a+t)^4 \times \frac{h}{a} (a+t) - a^4 h \right] \\ &= \frac{\pi}{10} \rho a^4 h \left[\left(1 + \frac{t}{a}\right)^5 - 1 \right] = \frac{\pi}{10} \rho a^4 h \left(1 + 5 \frac{t}{a} + \dots - 1 \right)\end{aligned}$$

Neglecting t^2 and higher order terms, obtain

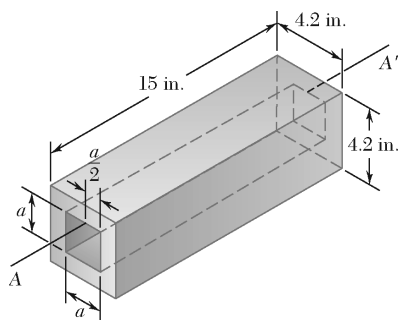
$$(I_x)_{\text{shell}} = \frac{\pi}{2} \rho a^3 h t$$

Now

$$k_x^2 = \frac{I_x}{m} = \frac{\frac{1}{2} m a^2}{m}$$

or $I_x = \frac{1}{2} m a^2 \blacktriangleleft$

or $k_x = \frac{a}{\sqrt{2}} \blacktriangleleft$



PROBLEM 9.131

A square hole is centered in and extends through the aluminum machine component shown. Determine (a) the value of a for which the mass moment of inertia of the component with respect to the axis AA' , which bisects the top surface of the hole, is maximum, (b) the corresponding values of the mass moment of inertia and the radius of gyration with respect to the axis AA' . (The specific weight of aluminum is 0.100 lb/in^3 .)

SOLUTION

First note

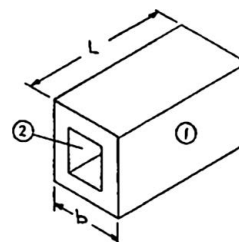
$$m_1 = \rho V_1 = \rho b^2 L$$

and

$$m_2 = \rho V_2 = \rho a^2 L$$

(a) Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned} I_{AA'} &= (I_{AA'})_1 - (I_{AA'})_2 \\ &= \left[\frac{1}{12} m_1 (b^2 + b^2) + m_1 \left(\frac{a}{2} \right)^2 \right] \\ &\quad - \left[\frac{1}{12} m_2 (a^2 + a^2) + m_2 \left(\frac{a}{2} \right)^2 \right] \\ &= (\rho b^2 L) \left(\frac{1}{6} b^2 + \frac{1}{4} a^2 \right) - (\rho a^2 L) \left(\frac{5}{12} a^2 \right) \\ &= \frac{\rho L}{12} (2b^4 + 3b^2 a^2 - 5a^4) \end{aligned}$$



Then

$$\frac{dI_{AA'}}{da} = \frac{\rho L}{12} (6b^2 a - 20a^3) = 0$$

or

$$a = 0 \quad \text{and} \quad a = b \sqrt{\frac{3}{10}}$$

Also

$$\frac{d^2 I_{AA'}}{da^2} = \frac{\rho L}{12} (6b^2 - 60a^2) = \frac{1}{2} \rho L (b^2 - 10a^2)$$

Now for $a = 0$,

$$\frac{d^2 I_{AA'}}{da^2} > 0$$

and for $a = b \sqrt{\frac{3}{10}}$,

$$\frac{d^2 I_{AA'}}{da^2} < 0$$

PROBLEM 9.131 (Continued)

$(I_{AA'})_{\max}$ occurs when $a = b\sqrt{\frac{3}{10}}$

Then $a = (4.2 \text{ in.})\sqrt{\frac{3}{10}}$

or $a = 2.30 \text{ in.} \blacktriangleleft$

(b) From part a:

$$\begin{aligned} (I_{AA'})_{\max} &= \frac{\rho L}{12} \left[2b^4 + 3b^2 \left(b\sqrt{\frac{3}{10}} \right)^2 - 5 \left(b\sqrt{\frac{3}{10}} \right)^4 \right] = \frac{49}{240} \rho L b^4 \\ &= \frac{49}{240} \frac{\gamma_{AL}}{g} L b^4 = \frac{49}{240} \times \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (15 \text{ in.})(4.2 \text{ in.})^4 \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \end{aligned}$$

or $(I_{AA'})_{\max} = 20.6 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$

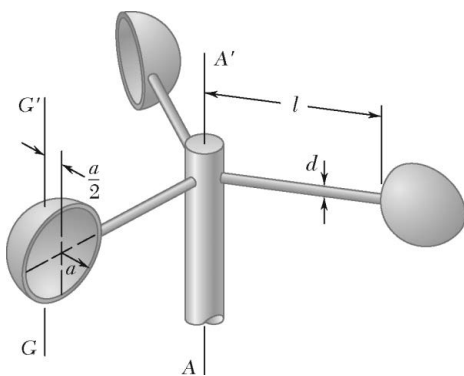
Now $k_{AA'}^2 = \frac{(I_{AA'})_{\max}}{m}$

where

$$\begin{aligned} m &= m_1 - m_2 = \rho L(b^2 - a^2) \\ &= \rho L \left[b^2 - \left(b\sqrt{\frac{3}{10}} \right)^2 \right] \\ &= \frac{7}{10} \rho L b^2 \end{aligned}$$

Then $k_{AA'}^2 = \frac{\frac{49}{240} \rho L b^4}{\frac{7}{10} \rho L b^2} = \frac{7}{24} b^2 = \frac{7}{24} (4.2 \text{ in.})^2$

or $k_{AA'} = 2.27 \text{ in.} \blacktriangleleft$



PROBLEM 9.132

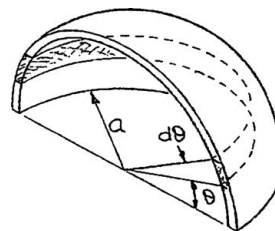
The cups and the arms of an anemometer are fabricated from a material of density ρ . Knowing that the mass moment of inertia of a thin, hemispherical shell of mass m and thickness t with respect to its centroidal axis GG' is $5ma^2/12$, determine (a) the mass moment of inertia of the anemometer with respect to the axis AA' , (b) the ratio of a to l for which the centroidal moment of inertia of the cups is equal to 1 percent of the moment of inertia of the cups with respect to the axis AA' .

SOLUTION

(a) First note
$$m_{\text{arm}} = \rho V_{\text{arm}} = \rho \times \frac{\pi}{4} d^2 l$$

and
$$dm_{\text{cup}} = \rho dV_{\text{cup}} = \rho [(2\pi a \cos \theta)(t)(ad\theta)]$$

Then
$$m_{\text{cup}} = \int dm_{\text{cup}} = \int_0^{\pi/2} 2\pi \rho a^2 t \cos \theta d\theta = 2\pi \rho a^2 t [\sin \theta]_0^{\pi/2} = 2\pi \rho a^2 t$$



Now
$$(I_{AA'})_{\text{anem}} = (I_{AA'})_{\text{cups}} + (I_{AA'})_{\text{arms}}$$

Using the parallel-axis theorem and assuming the arms are slender rods, we have

$$\begin{aligned} (I_{AA'})_{\text{anem}} &= 3 \left[(I_{GG'})_{\text{cup}} + m_{\text{cup}} d_{AG}^2 \right] + 3 \left[\bar{I}_{\text{arm}} + m_{\text{arm}} d_{AG_{\text{arm}}}^2 \right] \\ &= 3 \left\{ \frac{5}{12} m_{\text{cup}} a^2 + m_{\text{cup}} \left[(l+a)^2 + \left(\frac{a}{2} \right)^2 \right] \right\} + 3 \left[\frac{1}{2} m_{\text{arm}} l^2 + m_{\text{arm}} \left(\frac{l}{2} \right)^2 \right] \\ &= 3 m_{\text{cup}} \left(\frac{5}{3} a^2 + 2la + l^2 \right) + m_{\text{arm}} l^2 \\ &= 3(2\pi \rho a^2 t) \left(\frac{5}{3} a^2 + 2la + l^2 \right) + \left(\frac{\pi}{4} \rho d^2 l \right) (l^2) \end{aligned}$$

$$\text{or } (I_{AA'})_{\text{anem}} = \pi \rho l^2 \left[6a^2 t \left(\frac{5}{3} \frac{a^2}{l^2} + 2 \frac{a}{l} + 1 \right) + \frac{d^2 l}{4} \right] \blacktriangleleft$$

PROBLEM 9.132 (Continued)

(b) We have
$$\frac{(I_{GG'})_{\text{cup}}}{(I_{AA'})_{\text{cup}}} = 0.01$$

or
$$\frac{5}{12}m_{\text{cup}}a^2 = 0.01m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) \text{ (from part a)}$$

Now let $\zeta = \frac{a}{l}$.

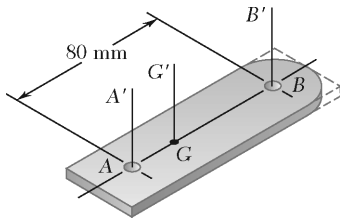
Then
$$5\zeta^2 = 0.12\left(\frac{5}{3}\zeta^2 + 2\zeta + 1\right)$$

or
$$40\zeta^2 - 2\zeta - 1 = 0$$

Then
$$\zeta = \frac{2 \pm \sqrt{(-2)^2 - 4(40)(-1)}}{2(40)}$$

or
$$\zeta = 0.1851 \text{ and } \zeta = -0.1351$$

$$\frac{a}{l} = 0.1851 \blacktriangleleft$$



PROBLEM 9.133

After a period of use, one of the blades of a shredder has been worn to the shape shown and is of mass 0.18 kg. Knowing that the mass moments of inertia of the blade with respect to the AA' and BB' axes are $0.320 \text{ g} \cdot \text{m}^2$ and $0.680 \text{ g} \cdot \text{m}^2$, respectively, determine (a) the location of the centroidal axis GG' , (b) the radius of gyration with respect to axis GG' .

SOLUTION

(a) We have

$$d_B = (0.08 - d_A) \text{ m}$$

and, using the parallel axis

$$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

$$I_{BB'} = \bar{I}_{GG'} + md_B^2$$

Then

$$\begin{aligned} I_{BB'} - I_{AA'} &= m(d_B^2 - d_A^2) \\ &= m[(0.08 - d_A)^2 - d_A^2] \\ &= m(0.0064 - 0.16d_A) \end{aligned}$$

Substituting:

$$(0.68 - 0.32) \times 10^{-3} \text{ kg} \cdot \text{m}^2 = 0.18 \text{ kg}(0.0064 - 0.16d_A) \text{ m}^2$$

$$\text{or } d_A = 27.5 \text{ mm} \blacktriangleleft$$

to the right of A

(b) We have

$$I_{AA'} = I_{GG'} + md_A^2$$

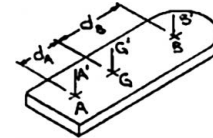
or

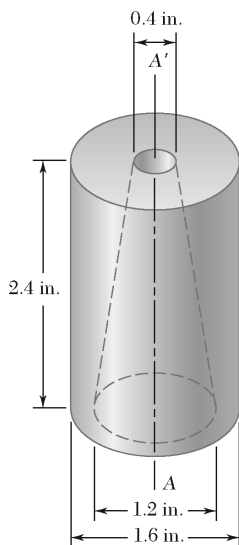
$$\begin{aligned} I_{GG'} &= 0.32 \times 10^{-3} \text{ kg} \cdot \text{m}^2 - 0.18 \text{ kg} \cdot (0.0275 \text{ m})^2 \\ &= 0.183875 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

Then

$$k_{GG'}^2 = \frac{I_{GG'}}{m} = \frac{0.183875 \times 10^{-3} \text{ kg} \cdot \text{m}^2}{0.18 \text{ kg}} = 1.02153 \times 10^{-3} \text{ m}^2$$

$$\text{or } k_{GG'} = 32.0 \text{ mm} \blacktriangleleft$$



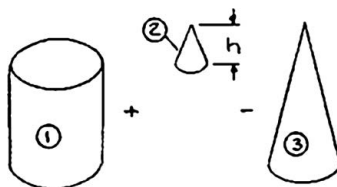


PROBLEM 9.134

Determine the mass moment of inertia of the 0.9-lb machine component shown with respect to the axis AA' .

SOLUTION

First note that the given shape can be formed adding a small cone to a cylinder and then removing a larger cone as indicated.



Now
$$\frac{h}{0.4} = \frac{h + 2.4}{1.2} \quad \text{or} \quad h = 1.2 \text{ in.}$$

The weight of the body is given by

$$W = mg = g(m_1 + m_2 - m_3) = \rho g(V_1 + V_2 - V_3)$$

or
$$0.9 \text{ lb} = \rho \times 32.2 \text{ ft/s}^2$$

$$\left[\pi(0.8)^2(2.4) + \frac{\pi}{3}(0.2)^2(1.2) - \frac{\pi}{3}(0.6)^2(3.6) \right] \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3$$

$$= \rho \times 32.2 \text{ ft/s}^2 (2.79253 + 0.02909 - 0.78540) \times 10^{-3} \text{ ft}^3$$

or
$$\rho = 13.7266 \text{ lb} \cdot \text{s}^2/\text{ft}^4$$

Then
$$m_1 = (13.7266)(2.79253 \times 10^{-3}) = 0.038332 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = (13.7266)(0.02909 \times 10^{-3}) = 0.000399 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = (13.7266)(0.78540 \times 10^{-3}) = 0.010781 \text{ lb} \cdot \text{s}^2/\text{ft}$$

PROBLEM 9.134 (Continued)

Finally, using Figure 9.28, we have

$$\begin{aligned} I_{AA'} &= (I_{AA'})_1 + (I_{AA'})_2 - (I_{AA'})_3 \\ &= \frac{1}{2} m_1 a_1^2 + \frac{3}{10} m_2 a_2^2 - \frac{3}{10} m_3 a_3^2 \\ &= \left[\frac{1}{2} (0.038332)(0.8)^2 + \frac{3}{10} (0.000399)(0.2)^2 - \frac{3}{10} (0.010781)(0.6)^2 \right] (\text{lb} \cdot \text{s}^2/\text{ft}) \times \text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &= (85.1822 + 0.0333 - 8.0858) \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

$$\text{or } I_{AA'} = 77.1 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$

To the instructor:

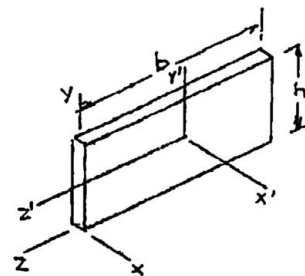
The following formulas for the mass moment of inertia of thin plates and a half cylindrical shell are derived at this time for use in the solutions of Problems 9.135 through 9.140.

Thin rectangular plate

$$\begin{aligned} (I_x)_m &= (\bar{I}_{x'})_m + md^2 \\ &= \frac{1}{12}m(b^2 + h^2) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right] \\ &= \frac{1}{3}m(b^2 + h^2) \end{aligned}$$

$$\begin{aligned} (I_y)_m &= (\bar{I}_{y'})_m + md^2 \\ &= \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{3}mb^2 \end{aligned}$$

$$\begin{aligned} I_z &= (\bar{I}_{z'})_m + md^2 \\ &= \frac{1}{12}mh^2 + m\left(\frac{h}{2}\right)^2 = \frac{1}{3}mh^2 \end{aligned}$$



Thin triangular plate

We have

$$m = \rho V = \rho\left(\frac{1}{2}bht\right)$$

and

$$\bar{I}_{z,\text{area}} = \frac{1}{36}bh^3$$

Then

$$\begin{aligned} \bar{I}_{z,\text{mass}} &= \rho t I_{z,\text{area}} \\ &= \rho t \times \frac{1}{36}bh^3 \\ &= \frac{1}{18}mh^2 \end{aligned}$$

Similarly,

$$\bar{I}_{y,\text{mass}} = \frac{1}{18}mb^2$$

Now

$$\bar{I}_{x,\text{mass}} = \bar{I}_{y,\text{mass}} + \bar{I}_{z,\text{mass}} = \frac{1}{18}m(b^2 + h^2)$$

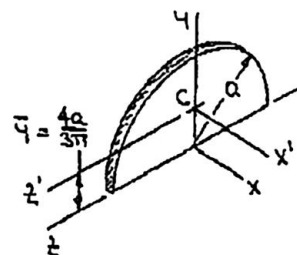
Thin semicircular plate

We have

$$m = \rho V = \rho\left(\frac{\pi}{2}a^2t\right)$$

and

$$\bar{I}_{y,\text{area}} = I_{z,\text{area}} = \frac{\pi}{8}a^4$$



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(Continued)

Then

$$\begin{aligned}\bar{I}_{y,\text{mass}} = I_{z,\text{mass}} &= \rho t \bar{I}_{y,\text{area}} \\ &= \rho t \times \frac{\pi}{8} a^4 \\ &= \frac{1}{4} m a^2\end{aligned}$$

Now

$$I_{x,\text{mass}} = \bar{I}_{y,\text{mass}} + I_{z,\text{mass}} = \frac{1}{2} m a^2$$

Also

$$I_{x,\text{mass}} = \bar{I}_{x',\text{mass}} + m \bar{y}^2 \quad \text{or} \quad \bar{I}_{x',\text{mass}} = m \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) a^2$$

and

$$I_{z,\text{mass}} = \bar{I}_{z',\text{mass}} + m \bar{z}^2 \quad \text{or} \quad \bar{I}_{z',\text{mass}} = m \left(\frac{1}{4} - \frac{16}{9\pi^2} \right) a^2$$

$$\bar{y} = \bar{z} = \frac{4a}{3\pi}$$

Thin Quarter-Circular Plate

We have

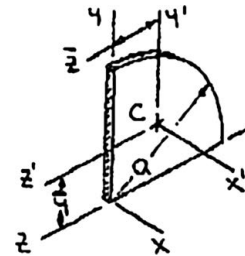
$$m = \rho V = \rho \left(\frac{\pi}{4} a^2 t \right)$$

and

$$I_{y,\text{area}} = I_{z,\text{area}} = \frac{\pi}{16} a^4$$

Then

$$\begin{aligned}I_{y,\text{mass}} = I_{z,\text{mass}} &= \rho t I_{y,\text{area}} \\ &= \rho t \times \frac{\pi}{16} a^4 \\ &= \frac{1}{4} m a^2\end{aligned}$$



Now

$$I_{x,\text{mass}} = I_{y,\text{mass}} + I_{z,\text{mass}} = \frac{1}{2} m a^2$$

Also

$$I_{x,\text{mass}} = \bar{I}_{x',\text{mass}} + m(\bar{y}^2 + \bar{z}^2)$$

or

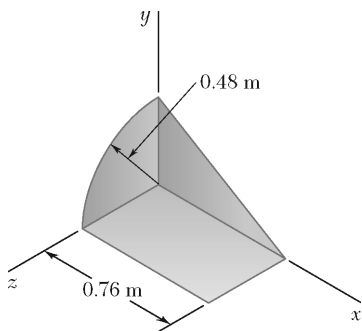
$$\bar{I}_{x',\text{mass}} = m \left(\frac{1}{2} - \frac{32}{9\pi^2} \right) a^2$$

and

$$I_{y,\text{mass}} = I_{y',\text{mass}} + m \bar{z}^2$$

or

$$\bar{I}_{y',\text{mass}} = m \left(\frac{1}{4} - \frac{16}{9\pi^2} \right) a^2$$



PROBLEM 9.135

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have

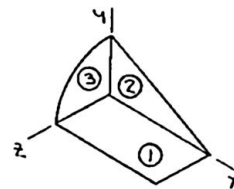
$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.76 \times 0.48) \text{ m}^2 \\ = 5.72736 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{1}{2} \times 0.76 \times 0.48 \right) \text{ m}^2 \\ = 2.86368 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{\pi}{4} \times 0.48^2 \right) \text{ m}^2 \\ = 2.84101 \text{ kg}$$



Using Figure 9.28 for component 1 and the equations derived above for components 2 and 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 \\ = \left[\frac{1}{12} (5.72736 \text{ kg})(0.48 \text{ m})^2 + (5.72736 \text{ kg}) \left(\frac{0.48}{2} \text{ m} \right)^2 \right] \\ + \left[\frac{1}{18} (2.86368 \text{ kg})(0.48 \text{ m})^2 + (2.86368 \text{ kg}) \left(\frac{0.48}{3} \text{ m} \right)^2 \right] + \left[\frac{1}{2} (2.84101 \text{ kg})(0.48 \text{ m})^2 \right] \\ = [(0.109965 + 0.329896) + (0.036655 + 0.073310) + (0.327284)] \text{ kg} \cdot \text{m}^2 \\ = (0.439861 + 0.109965 + 0.327284) \text{ kg} \cdot \text{m}^2$$

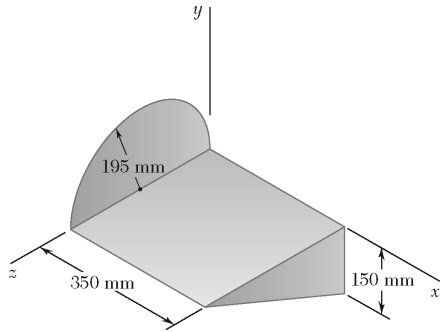
$$\text{or } I_x = 0.877 \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.135 (Continued)

$$\begin{aligned} I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 \\ &= \left\{ \frac{1}{12} (5.72736 \text{ kg})(0.76^2 + 0.48^2) \text{ m}^2 + (5.72736 \text{ kg}) \left[\left(\frac{0.76}{2} \right)^2 + \left(\frac{0.48}{2} \right)^2 \right] \text{ m}^2 \right\} \\ &\quad + \left[\frac{1}{18} (2.86368 \text{ kg})(0.76 \text{ m})^2 + (2.86368 \text{ kg}) \left(\frac{0.76}{3} \text{ m} \right)^2 \right] + \left[\frac{1}{4} (2.84101 \text{ kg})(0.48 \text{ m})^2 \right] \\ &= [(0.385642 + 1.156927) + (0.091892 + 0.183785) + (0.163642)] \text{ kg} \cdot \text{m}^2 \\ &= (1.542590 + 0.275677 + 0.163642) \text{ kg} \cdot \text{m}^2 \\ &\qquad\qquad\qquad \text{or } I_y = 1.982 \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$

$$\begin{aligned} I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 \\ &= \left[\frac{1}{12} (5.72736 \text{ kg})(0.76 \text{ m})^2 + (5.72736 \text{ kg}) \left(\frac{0.76}{2} \text{ m} \right)^2 \right] \\ &\quad + \left\{ \frac{1}{18} (2.86368 \text{ kg})(0.76^2 + 0.48^2) \text{ m}^2 + (2.86368 \text{ kg}) \left[\left(\frac{0.76}{3} \right)^2 + \left(\frac{0.48}{3} \right)^2 \right] \text{ m}^2 \right\} \\ &\quad + \left[\frac{1}{4} (2.84101 \text{ kg})(0.48 \text{ m})^2 \right] \\ I_z &= [(0.275677 + 0.827031) + (0.128548 + 0.257095) \\ &\quad + (0.163642)] \text{ kg} \cdot \text{m}^2 \\ &= (1.102708 + 0.385643 + 0.163642) \text{ kg} \cdot \text{m}^2 \\ &\qquad\qquad\qquad \text{or } I_z = 1.652 \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$

PROBLEM 9.136



A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have

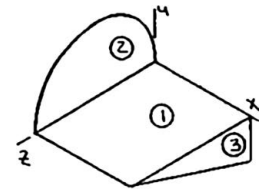
$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$\begin{aligned} m_1 &= (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.35 \times 0.39) \text{ m}^2 \\ &= 2.14305 \text{ kg} \end{aligned}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{\pi}{2} \times 0.195^2 \right) \text{ m}^2 = 0.93775 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{1}{2} \times 0.39 \times 0.15 \right) \text{ m}^2 = 0.45923 \text{ kg}$$



Using Figure 9.28 for component 1 and the equations derived above for components 2 and 3, we have

$$\begin{aligned} I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 \\ &= \left[\frac{1}{12} (2.14305 \text{ kg})(0.39 \text{ m})^2 + (2.14305 \text{ kg}) \left(\frac{0.39}{2} \text{ m} \right)^2 \right] \\ &\quad + \left\{ \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) (0.93775 \text{ kg})(0.195 \text{ m})^2 + (0.93775 \text{ kg}) \left[\left(\frac{4 \times 0.195}{3\pi} \right)^2 + (0.195)^2 \right] \text{ m}^2 \right\} \\ &\quad + \left\{ \frac{1}{18} (0.45923 \text{ kg}) [(0.39)^2 + (0.15)^2] \text{ m}^2 + (0.45923 \text{ kg}) \left[\left(\frac{0.39}{3} \right)^2 + \left(\frac{0.15}{3} \right)^2 \right] \text{ m}^2 \right\} \\ &= [(0.027163 + 0.081489) + (0.011406 + 0.042081) + (0.004455 + 0.008909)] \text{ kg} \cdot \text{m}^2 \\ &= (0.108652 + 0.053487 + 0.013364) \text{ kg} \cdot \text{m}^2 \\ &= 0.175503 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_x = 175.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

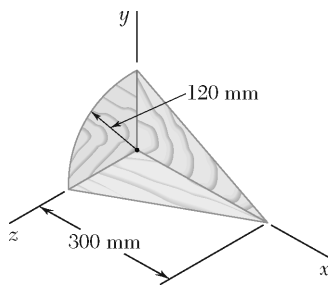
PROBLEM 9.136 (Continued)

$$\begin{aligned}
 I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 \\
 &= \left\{ \frac{1}{12} (2.14305 \text{ kg}) [(0.35)^2 + (0.39)^2] \text{ m}^2 + (2.14305 \text{ kg}) \left[\left(\frac{0.35}{2} \right)^2 + \left(\frac{0.39}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &\quad + \left[\frac{1}{4} (0.93775 \text{ kg}) (0.195 \text{ m})^2 + (0.93775 \text{ kg}) (0.195 \text{ m})^2 \right] \\
 &\quad + \left\{ \frac{1}{18} (0.45923 \text{ kg}) (0.39 \text{ m})^2 + (0.45923 \text{ kg}) \left[(0.35)^2 + \left(\frac{0.39}{3} \right)^2 \right] \text{ m}^2 \right\} \\
 &= [(0.049040 + 0.147120) + (0.008914 + 0.035658) \\
 &\quad + (0.003880 + 0.064017)] \text{ kg} \cdot \text{m}^2 \\
 &= (0.196160 + 0.044572 + 0.067897) \text{ kg} \cdot \text{m}^2 \\
 &= 0.308629 \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

or $I_y = 309 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

$$\begin{aligned}
 I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 \\
 &= \left[\frac{1}{12} (2.14305 \text{ kg}) (0.35 \text{ m})^2 + (2.14305 \text{ kg}) \left(\frac{0.35}{2} \text{ m} \right)^2 \right] \\
 &\quad + \left[\frac{1}{4} (0.93775 \text{ kg}) (0.195 \text{ m})^2 \right] \\
 &\quad + \left\{ \frac{1}{18} (0.45923 \text{ kg}) (0.15 \text{ m})^2 + (0.45923 \text{ kg}) \left[(0.35)^2 + \left(\frac{0.15}{3} \right)^2 \right] \text{ m}^2 \right\} \\
 &= [(0.021877 + 0.065631) + 0.008914] + (0.000574 + 0.057404) \text{ kg} \cdot \text{m}^2 \\
 &= (0.087508 + 0.008914 + 0.057978) \text{ kg} \cdot \text{m}^2 \\
 &= 0.154400 \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

or $I_z = 154.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$



PROBLEM 9.137

A subassembly for a model airplane is fabricated from three pieces of 1.5-mm plywood. Neglecting the mass of the adhesive used to assemble the three pieces, determine the mass moment of inertia of the subassembly with respect to each of the coordinate axes. (The density of the plywood is 780 kg/m^3 .)

SOLUTION

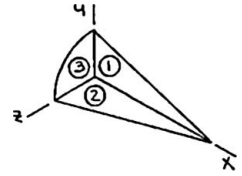
First compute the mass of each component. We have

$$m = \rho V = \rho t A$$

Then

$$\begin{aligned} m_1 = m_2 &= (780 \text{ kg/m}^3)(0.0015 \text{ m}) \left(\frac{1}{2} \times 0.3 \times 0.12 \right) \text{ m}^2 \\ &= 21.0600 \times 10^{-3} \text{ kg} \end{aligned}$$

$$m_3 = (780 \text{ kg/m}^3)(0.0015 \text{ m}) \left(\frac{\pi}{4} \times 0.12^2 \right) \text{ m}^2 = 13.2324 \times 10^{-3} \text{ kg}$$



Using the equations derived above and the parallel-axis theorem, we have

$$(I_x)_1 = (I_x)_2$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= 2 \left[\frac{1}{18} (21.0600 \times 10^{-3} \text{ kg})(0.12 \text{ m})^2 + (21.0600 \times 10^{-3} \text{ kg}) \left(\frac{0.12}{3} \text{ m} \right)^2 \right]$$

$$+ \left[\frac{1}{2} (13.2324 \times 10^{-3} \text{ kg})(0.12 \text{ m})^2 \right]$$

$$= [2(16.8480 + 33.6960) + (95.2733)] \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

$$= [2(50.5440) + (95.2733)] \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

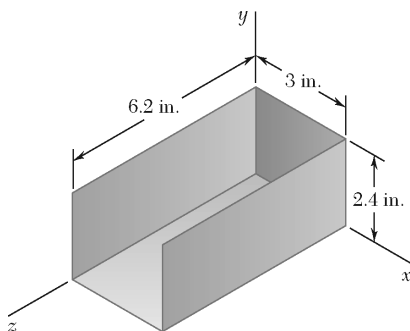
$$\text{or } I_x = 196.4 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.137 (Continued)

$$\begin{aligned}
I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 \\
&= \left[\frac{1}{18} (21.0600 \times 10^{-3} \text{ kg})(0.3 \text{ m})^2 + (21.0600 \text{ kg}) \left(\frac{0.3}{3} \text{ m} \right)^2 \right] \\
&\quad + \left\{ \frac{1}{18} (21.0600 \times 10^{-3} \text{ kg}) [(0.3)^2 + (0.12)^2] \text{ m}^2 \right. \\
&\quad \left. + (21.0600 \times 10^{-3} \text{ kg}) \left[\left(\frac{0.3}{3} \right)^2 + \left(\frac{0.12}{3} \right)^2 \right] \text{ m}^2 \right\} \\
&\quad + \left[\frac{1}{4} (13.2324 \times 10^{-3} \text{ kg})(0.12 \text{ m})^2 \right] \\
&= [(105.300 + 210.600) + (122.148 + 244.296) \\
&\quad + (47.637)] \times 10^{-6} \text{ kg} \cdot \text{m}^2 \\
&= (315.900 + 366.444 + 47.637) \times 10^{-6} \text{ kg} \cdot \text{m}^2 \\
&\qquad\qquad\qquad \text{or} \qquad I_y = 730 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \blacktriangleleft
\end{aligned}$$

Symmetry implies $I_y = I_z$

$$I_z = 730 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$



PROBLEM 9.138

The cover for an electronic device is formed from sheet aluminum that is 0.05 in. thick. Determine the mass moment of inertia of the cover with respect to each of the coordinate axes. (The specific weight of aluminum is 0.100 lb/in³.)

SOLUTION

First compute the mass of each component. We have

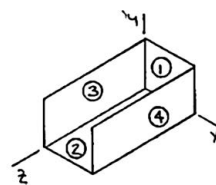
$$m = \rho V = \frac{\gamma}{g} tA$$

Then

$$m_1 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (3 \times 2.4) \text{ in}^2 = 1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (3 \times 6.2) \text{ in}^2 = 2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = m_4 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (2.4 \times 6.2) \text{ in}^2 = 2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$



Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned} (I_x)_3 &= (I_x)_4 \\ I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 \\ &= \left[\frac{1}{12} (1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (2.4 \text{ in.})^2 \right. \\ &\quad \left. + (1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{2.4}{2} \text{ in.} \right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &\quad + \left[\frac{1}{12} (2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (6.2 \text{ in.})^2 \right. \\ &\quad \left. + (2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{6.2}{2} \text{ in.} \right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &\quad + 2 \left\{ \frac{1}{12} (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) [(2.4)^2 + (6.2)^2] \text{ in}^2 \right. \\ &\quad \left. + (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[\left(\frac{2.4}{2} \right)^2 + \left(\frac{6.2}{2} \right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \end{aligned}$$

PROBLEM 9.138 (Continued)

$$\begin{aligned}
 &= [(3.7267 + 11.1801) + (64.2491 + 192.7472) \\
 &\quad + 2(59.1011 + 177.3034)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 &= [14.9068 + 256.9963 + 2(236.4045)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2
 \end{aligned}$$

$$\text{or } I_x = 745 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$

$$\begin{aligned}
 I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 \\
 &= \left[\frac{1}{12} (1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (3 \text{ in.})^2 \right. \\
 &\quad \left. + (1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{3}{2} \text{ in.} \right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ \frac{1}{12} (2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) [(3)^2 + (6.2)^2] \text{ in}^2 \right. \\
 &\quad \left. + (2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[\left(\frac{3}{2} \right)^2 + \left(\frac{6.2}{2} \right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left[\frac{1}{12} (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (6.2 \text{ in.})^2 \right. \\
 &\quad \left. + (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{6.2}{2} \text{ in.} \right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ \frac{1}{12} (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (6.2 \text{ in.})^2 \right. \\
 &\quad \left. + (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[(3)^2 + \left(\frac{6.2}{2} \right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &= [(5.8230 + 17.4689) + (79.2918 + 237.8754) + (51.3993 + 154.1978) \\
 &\quad + (51.3993 + 298.6078)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 &= (23.2919 + 317.1672 + 205.5971 + 350.0071) \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2
 \end{aligned}$$

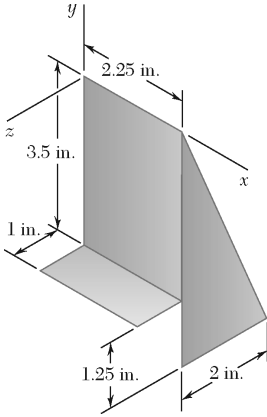
$$\text{or } I_y = 896 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$

PROBLEM 9.138 (Continued)

$$\begin{aligned}
 I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 \\
 &= \left\{ \frac{1}{12} (1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) [(3)^2 + (2.4)^2] \text{ in}^2 \right. \\
 &\quad + (1.11801 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[\left(\frac{3}{2} \right)^2 + \left(\frac{2.4}{2} \right)^2 \right] \text{ in}^2 \left. \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left[\frac{1}{12} (2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (3 \text{ in.})^2 \right. \\
 &\quad \left. + (2.88820 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{3}{2} \text{ in.} \right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left[\frac{1}{12} (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (2.4 \text{ in.})^2 \right. \\
 &\quad \left. + (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{2.4}{2} \text{ in.} \right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ \frac{1}{12} (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (2.4 \text{ in.})^2 \right. \\
 &\quad \left. + (2.31056 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[(3)^2 + \left(\frac{2.4}{2} \right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &= [(9.5497 + 28.6490) + (15.0427 + 45.1281) \\
 &\quad + (7.7019 + 23.1056) + (7.7019 + 167.5156)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 &= (38.1987 + 60.1708 + 30.8075 + 175.2175) \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 &\quad \text{or} \quad I_z = 304 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft
 \end{aligned}$$

PROBLEM 9.139

A framing anchor is formed of 0.05-in.-thick galvanized steel. Determine the mass moment of inertia of the anchor with respect to each of the coordinate axes. (The specific weight of galvanized steel is 470 lb/ft³.)



SOLUTION

First compute the mass of each component. We have

$$m = \rho V = \frac{\gamma_{G.S.}}{g} t A$$

Then

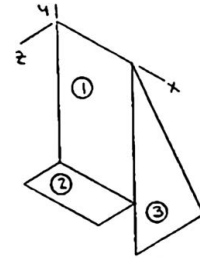
$$\begin{aligned} m_1 &= \frac{470 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (2.25 \times 3.5) \text{ in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 \\ &= 3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$

$$\begin{aligned} m_2 &= \frac{470 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times (2.25 \times 1) \text{ in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 \\ &= 950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$

$$\begin{aligned} m_3 &= \frac{470 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.05 \text{ in.} \times \left(\frac{1}{2} \times 2 \times 4.75\right) \text{ in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 \\ &= 2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for component 3, we have

$$\begin{aligned} I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 \\ &= \left[\frac{1}{12} (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}) (3.5 \text{ in.})^2 \right. \\ &\quad \left. + (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{3.5}{2} \text{ in.}\right)^2 \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &\quad + \left\{ \frac{1}{12} (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}) (1 \text{ in.})^2 \right. \\ &\quad \left. + (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[(3.5)^2 + \left(\frac{1}{2}\right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \end{aligned}$$



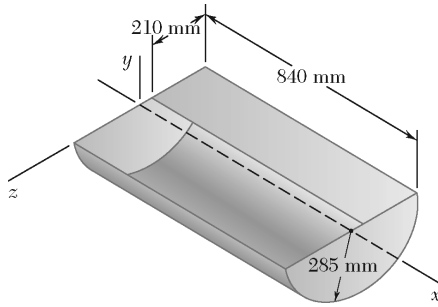
PROBLEM 9.139 (Continued)

$$\begin{aligned}
 & + \left\{ \frac{1}{18} (2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(4.75)^2 + (2)^2] \text{ in}^2 \right. \\
 & + (2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[\left(\frac{2}{3} \times 4.75 \right)^2 + \left(\frac{1}{3} \times 2 \right)^2 \right] \text{ in}^2 \left. \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 & = [(23.578 + 70.735) + (0.550 + 82.490) \\
 & \quad + (20.559 + 145.894)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 & = (94.313 + 83.040 + 166.453) \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 & \qquad \qquad \qquad \text{or} \qquad I_x = 344 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 I_y & = (I_y)_1 + (I_y)_2 + (I_y)_3 \\
 & = \left[\frac{1}{12} (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) (2.25 \text{ in.})^2 \right. \\
 & \quad + (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left(\frac{2.25}{2} \text{ in.} \right)^2 \left. \right] \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 & \quad + \left\{ \frac{1}{12} (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(2.25)^2 + (1)^2] \text{ in}^2 \right. \\
 & \quad + (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[\left(\frac{2.25}{2} \right)^2 + \left(\frac{1}{2} \right)^2 \right] \text{ in}^2 \left. \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 & \quad + \left\{ \frac{1}{18} (2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) (2 \text{ in.})^2 \right. \\
 & \quad + (2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(2.25)^2 + \left(\frac{1}{3} \times 2 \right)^2 \right] \text{ in}^2 \left. \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 & = [(9.744 + 29.232) + (3.334 + 10.002) \\
 & \quad + (3.096 + 76.720)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 & = (38.976 + 13.336 + 79.816) \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 & \qquad \qquad \qquad \text{or} \qquad I_y = 132.1 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft
 \end{aligned}$$

PROBLEM 9.139 (Continued)

$$\begin{aligned} I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 \\ &= \left\{ \frac{1}{12} (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(2.25)^2 + (3.5)^2] \text{ in}^2 \right. \\ &\quad \left. + (3325.97 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[\left(\frac{2.25}{2} \right)^2 + \left(\frac{3.5}{2} \right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &\quad + \left\{ \frac{1}{12} (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) (2.25 \text{ in.})^2 \right. \\ &\quad \left. + (950.28 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[\left(\frac{2.25}{2} \right)^2 + (3.5)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &\quad + \left\{ \frac{1}{18} (2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) (4.75 \text{ in.})^2 \right. \\ &\quad \left. + (2006.14 \times 10^{-6} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(2.25)^2 + \left(\frac{2}{3} \times 4.75 \right)^2 \right] \text{ in}^2 \right\} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &= [(33.322 + 99.967) + (2.784 + 89.192) \\ &\quad + (17.463 + 210.231)] \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= (133.289 + 91.976 + 227.694) \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &\quad \text{or } I_z = 453 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft \end{aligned}$$



PROBLEM 9.140*

A farmer constructs a trough by welding a rectangular piece of 2-mm-thick sheet steel to half of a steel drum. Knowing that the density of steel is 7850 kg/m^3 and that the thickness of the walls of the drum is 1.8 mm, determine the mass moment of inertia of the trough with respect to each of the coordinate axes. Neglect the mass of the welds.

SOLUTION

First compute the mass of each component. We have

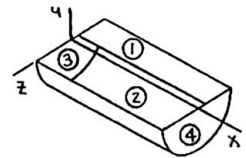
$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$\begin{aligned} m_1 &= (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.84 \times 0.21) \text{ m}^2 \\ &= 2.76948 \text{ kg} \end{aligned}$$

$$\begin{aligned} m_2 &= (7850 \text{ kg/m}^3)(0.0018 \text{ m})(\pi \times 0.285 \times 0.84) \text{ m}^2 \\ &= 10.62713 \text{ kg} \end{aligned}$$

$$\begin{aligned} m_3 &= m_4 = (7850 \text{ kg/m}^3)(0.0018 \text{ m})\left(\frac{\pi}{2} \times 0.285^2\right) \text{ m}^2 \\ &= 1.80282 \text{ kg} \end{aligned}$$



Using Figure 9.28 for component 1 and the equations derived above for components 2 through 4, we have

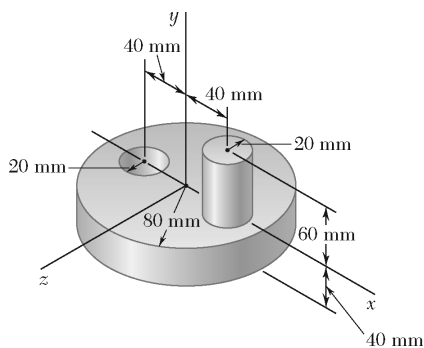
$$\begin{aligned} (I_x)_3 &= (I_x)_4 \\ I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 \\ &= \left[\frac{1}{12} (2.76948 \text{ kg})(0.21 \text{ m})^2 + (2.76948 \text{ kg}) \left(0.285 - \frac{0.21}{2} \right)^2 \text{ m}^2 \right] \\ &\quad + [(10.62713 \text{ kg})(0.285 \text{ m})^2] + 2 \left[\frac{1}{2} (1.80282 \text{ kg})(0.285 \text{ m})^2 \right] \\ &= [(0.01018 + 0.08973) + (0.86319) + 2(0.07322)] \text{ kg} \cdot \text{m}^2 \\ &= [(0.09991 + 0.86319 + 2(0.07322))] \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_x = 1.110 \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.140* (Continued)

$$\begin{aligned}
 I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 \\
 &= \left\{ \frac{1}{12} (2.76948 \text{ kg}) [(0.84)^2 + (0.21)^2] \text{ m}^2 \right. \\
 &\quad \left. + (2.76948 \text{ kg}) \left[\left(\frac{0.84}{2} \right)^2 + \left(0.285 - \frac{0.21}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &\quad + \left\{ \frac{1}{12} (10.62713 \text{ kg}) [(0.84)^2 + 6(0.285)^2] \text{ m}^2 + (10.62713 \text{ kg}) \left(\frac{0.84}{2} \text{ m} \right)^2 \right\} \\
 &\quad + \left[\frac{1}{4} (1.80282 \text{ kg}) (0.285 \text{ m})^2 \right] \\
 &\quad + \left[\frac{1}{4} (1.80282 \text{ kg}) (0.285 \text{ m})^2 + (1.80282 \text{ kg}) (0.84 \text{ m})^2 \right] \\
 &= [(0.17302 + 0.57827) + (1.05647 + 1.87463) \\
 &\quad + (0.03661) + (0.03661 + 1.27207)] \text{ kg} \cdot \text{m}^2 \\
 &= (0.75129 + 2.93110 + 0.03661 + 1.30868) \text{ kg} \cdot \text{m}^2 \\
 &\qquad\qquad\qquad \text{or } I_y = 5.03 \text{ kg} \cdot \text{m}^2 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 \\
 &= \left[\frac{1}{12} (2.76948 \text{ kg}) (0.84 \text{ m})^2 + (2.76948 \text{ kg}) \left(\frac{0.84}{2} \text{ m} \right)^2 \right] \\
 &\quad + \left\{ \frac{1}{12} (10.62713 \text{ kg}) [(0.84)^2 + 6(0.285)^2] \text{ m}^2 + (10.62713 \text{ kg}) \left(\frac{0.84}{2} \text{ m} \right)^2 \right\} \\
 &\quad + \left[\frac{1}{4} (1.80282 \text{ kg}) (0.285 \text{ m})^2 \right] \\
 &\quad + \left[\frac{1}{4} (1.80282 \text{ kg}) (0.285 \text{ m})^2 + (1.80282 \text{ kg}) (0.84 \text{ m})^2 \right] \\
 &= [(0.16285 + 0.48854) + (1.05647 + 1.87463) \\
 &\quad + (0.03661) + (0.03661 + 1.27207)] \text{ kg} \cdot \text{m}^2 \\
 &= (0.65139 + 2.93110 + 0.03661 + 1.30868) \text{ kg} \cdot \text{m}^2 \\
 &\qquad\qquad\qquad \text{or } I_z = 4.93 \text{ kg} \cdot \text{m}^2 \blacktriangleleft
 \end{aligned}$$



PROBLEM 9.141

The machine element shown is fabricated from steel. Determine the mass moment of inertia of the assembly with respect to (a) the x axis, (b) the y axis, (c) the z axis. (The density of steel is 7850 kg/m^3 .)

SOLUTION

First compute the mass of each component. We have

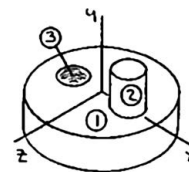
$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(\pi(0.08 \text{ m})^2(0.04 \text{ m})) \\ = 6.31334 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)[\pi(0.02 \text{ m})^2(0.06 \text{ m})] = 0.59188 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)[\pi(0.02 \text{ m})^2(0.04 \text{ m})] = 0.39458 \text{ kg}$$



Using Figure 9.28 and the parallel-axis theorem, we have

$$(a) \quad I_x = (I_x)_1 + (I_x)_2 - (I_x)_3 \\ = \left\{ \frac{1}{12} (6.31334 \text{ kg}) [3(0.08)^2 + (0.04)^2] \text{ m}^2 + (6.31334 \text{ kg})(0.02 \text{ m})^2 \right\} \\ + \left\{ \frac{1}{12} (0.59188 \text{ kg}) [3(0.02)^2 + (0.06)^2] \text{ m}^2 + (0.59188 \text{ kg})(0.03 \text{ m})^2 \right\} \\ - \left\{ \frac{1}{12} (0.39458 \text{ kg}) [3(0.02)^2 + (0.04)^2] \text{ m}^2 + (0.39458 \text{ kg})(0.02 \text{ m})^2 \right\} \\ = [(10.94312 + 2.52534) + (0.23675 + 0.53269) \\ - (0.09207 + 0.15783)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ = (13.46846 + 0.76944 - 0.24990) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ = 13.98800 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\text{or } I_x = 13.99 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.141 (Continued)

$$\begin{aligned}
 (b) \quad I_y &= (I_y)_1 + (I_y)_2 - (I_y)_3 \\
 &= \left[\frac{1}{2}(6.31334 \text{ kg})(0.08 \text{ m})^2 \right] \\
 &\quad + \left[\frac{1}{2}(0.59188 \text{ kg})(0.02 \text{ m})^2 + (0.59188 \text{ kg})(0.04 \text{ m})^2 \right] \\
 &\quad - \left[\frac{1}{2}(0.39458 \text{ kg})(0.02 \text{ m})^2 + (0.39458 \text{ kg})(0.04 \text{ m})^2 \right] \\
 &= [(20.20269) + (0.11838 + 0.94701) \\
 &\quad - (0.07892 + 0.63133)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= (20.20269 + 1.06539 - 0.71025) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= 20.55783 \times 10^{-3} \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

or $I_y = 20.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

$$\begin{aligned}
 (c) \quad I_z &= (I_z)_1 + (I_z)_2 - (I_z)_3 \\
 &= \left\{ \frac{1}{12}(6.31334 \text{ kg})[3(0.08)^2 + (0.04)^2] \text{ m}^2 + (6.31334 \text{ kg})(0.02 \text{ m})^2 \right\} \\
 &\quad + \left\{ \frac{1}{12}(0.59188 \text{ kg})[3(0.02)^2 + (0.06)^2] \text{ m}^2 + (0.59188 \text{ kg})[(0.04)^2 + (0.03)^2] \text{ m}^2 \right\} \\
 &\quad - \left\{ \frac{1}{12}(0.39458 \text{ kg})[3(0.02)^2 + (0.04)^2] \text{ m}^2 + (0.39458 \text{ kg})[(0.04)^2 + (0.02)^2] \text{ m}^2 \right\} \\
 &= [(10.94312 + 2.52534) + (0.23675 + 1.47970) \\
 &\quad - (0.09207 + 0.78916)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= (13.46846 + 1.71645 - 0.88123) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= 14.30368 \times 10^{-3} \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

or $I_z = 14.30 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

To the Instructor:

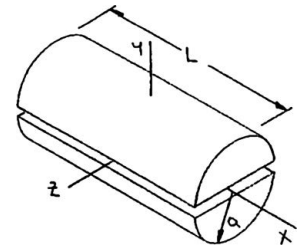
The following formulas for the mass of inertia of a semicylinder are derived at this time for use in the solutions of Problems 9.142 through 9.145.

From Figure 9.28:

Cylinder

$$(I_x)_{\text{cyl}} = \frac{1}{2} m_{\text{cyl}} a^2$$

$$(I_y)_{\text{cyl}} = (I_z)_{\text{cyl}} = \frac{1}{12} m_{\text{cyl}} (3a^2 + L^2)$$



Symmetry and the definition of the mass moment of inertia ($I = \int r^2 dm$) imply

$$(I)_{\text{semicylinder}} = \frac{1}{2} (I)_{\text{cylinder}}$$

$$(I_x)_{\text{sc}} = \frac{1}{2} \left(\frac{1}{2} m_{\text{cyl}} a^2 \right)$$

and

$$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{2} \left[\frac{1}{12} m_{\text{cyl}} (3a^2 + L^2) \right]$$

However,

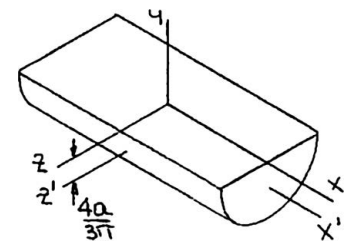
$$m_{\text{sc}} = \frac{1}{2} m_{\text{cyl}}$$

Thus,

$$(I_x)_{\text{sc}} = \frac{1}{2} m_{\text{sc}} a^2$$

and

$$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{12} m_{\text{sc}} (3a^2 + L^2)$$



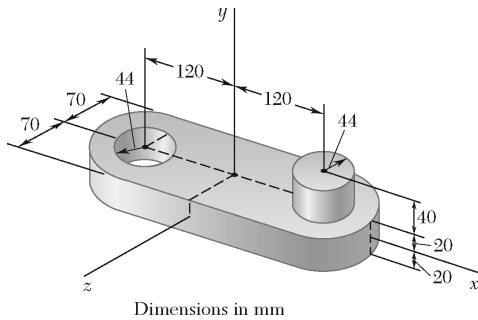
Also, using the parallel axis theorem find

$$\bar{I}_{x'} = m_{\text{sc}} \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) a^2$$

$$\bar{I}_{z'} = m_{\text{sc}} \left[\left(\frac{1}{4} - \frac{16}{9\pi^2} \right) a^2 + \frac{1}{12} L^2 \right]$$

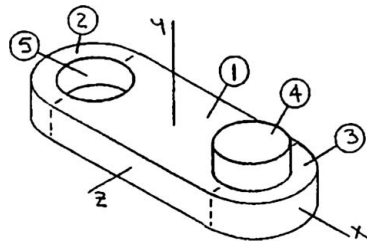
where x' and z' are centroidal axes.

PROBLEM 9.142



Determine the mass moments of inertia and the radii of gyration of the steel machine element shown with respect to the x and y axes. (The density of steel is 7850 kg/m^3 .)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V$$

Then

$$\begin{aligned} m_1 &= (7850 \text{ kg/m}^3)(0.24 \times 0.04 \times 0.14) \text{ m}^3 \\ &= 10.5504 \text{ kg} \end{aligned}$$

$$m_2 = m_3 = (7850 \text{ kg/m}^3) \left[\frac{\pi}{2} (0.07)^2 \times 0.04 \right] \text{ m}^3 = 2.41683 \text{ kg}$$

$$m_4 = m_5 = (7850 \text{ kg/m}^3) [\pi (0.044)^2 \times (0.04)] \text{ m}^3 = 1.90979 \text{ kg}$$

Using Figure 9.28 for components 1, 4, and 5 and the equations derived above (before the solution to Problem 9.144) for a semicylinder, we have

$$\begin{aligned} I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 - (I_x)_5 \quad \text{where} \quad (I_x)_2 = (I_x)_3 \\ &= \left[\frac{1}{12} (10.5504 \text{ kg})(0.04^2 + 0.14^2) \text{ m}^2 \right] + 2 \left\{ \frac{1}{12} (2.41683 \text{ kg}) [3(0.07 \text{ m})^2 + (0.04 \text{ m})^2] \right\} \\ &\quad + \left\{ \frac{1}{12} (1.90979 \text{ kg}) [3(0.044 \text{ m})^2 + (0.04 \text{ m})^2] + (1.90979 \text{ kg})(0.04 \text{ m})^2 \right\} \\ &\quad - \left\{ \frac{1}{12} (1.90979 \text{ kg}) [3(0.044 \text{ m})^2 + (0.04 \text{ m})^2] \right\} \\ &= [(0.0186390) + 2(0.0032829) + (0.0011790 + 0.0030557) - (0.0011790)] \text{ kg} \cdot \text{m}^2 \\ &= 0.0282605 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or} \quad I_x = 28.3 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.142 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 - (I_y)_5$$

where

$$(I_y)_2 = (I_y)_3 \quad (I_y)_4 = |(I_y)_5|$$

Then

$$\begin{aligned} I_y &= \left[\frac{1}{12} (10.5504 \text{ kg})(0.24^2 + 0.14^2) \text{ m}^2 \right] \\ &\quad + 2 \left[(2.41683 \text{ kg}) \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) (0.07 \text{ m}^2) + (2.41683 \text{ kg}) \left(0.12 + \frac{4 \times 0.07}{3\pi} \right)^2 \text{ m}^2 \right] \\ &= [(0.0678742) + 2(0.0037881 + 0.0541678)] \text{ kg} \cdot \text{m}^2 \\ &= 0.1837860 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_y = 183.8 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

Also

$$\begin{aligned} m &= m_1 + m_2 + m_3 + m_4 - m_5 \text{ where } m_2 = m_3, \quad m_4 = |m_5| \\ &= (10.5504 + 2 \times 2.41683) \text{ kg} = 15.38406 \text{ kg} \end{aligned}$$

Then

$$k_x^2 = \frac{I_x}{m} = \frac{0.0282605 \text{ kg} \cdot \text{m}^2}{15.38406 \text{ kg}}$$

$$\text{or } k_x = 42.9 \text{ mm} \blacktriangleleft$$

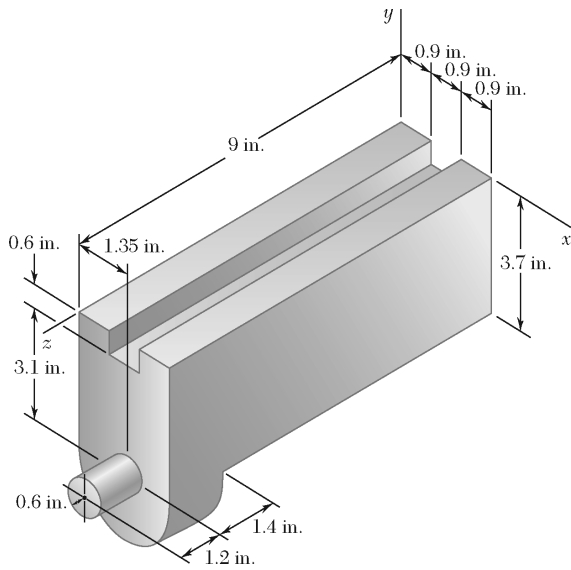
and

$$k_y^2 = \frac{I_y}{m} = \frac{0.1837860 \text{ kg} \cdot \text{m}^2}{15.38406 \text{ kg}}$$

$$\text{or } k_y = 109.3 \text{ mm} \blacktriangleleft$$

PROBLEM 9.143

Determine the mass moment of inertia of the steel machine element shown with respect to the y axis. (The specific weight of steel is 490 lb/ft^3 .)



SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} V$$

Then

$$m_1 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (2.7 \times 3.7 \times 9) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3$$

$$= 791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}$$

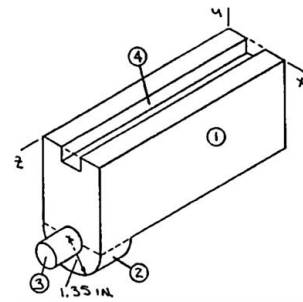
$$m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left(\frac{\pi}{2} \times 1.35^2 \times 1.4 \right) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3$$

$$= 35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}$$

$$m_3 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (\pi \times 0.6^2 \times 1.2) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3$$

$$= 11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}$$

$$m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (0.9 \times 0.6 \times 9) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 = 42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}$$



The mass moments of inertia are now computed using Figure 9.28 (components 1, 3, and 4) and the equations derived above (component 2).

PROBLEM 9.143 (Continued)

Find: I_y

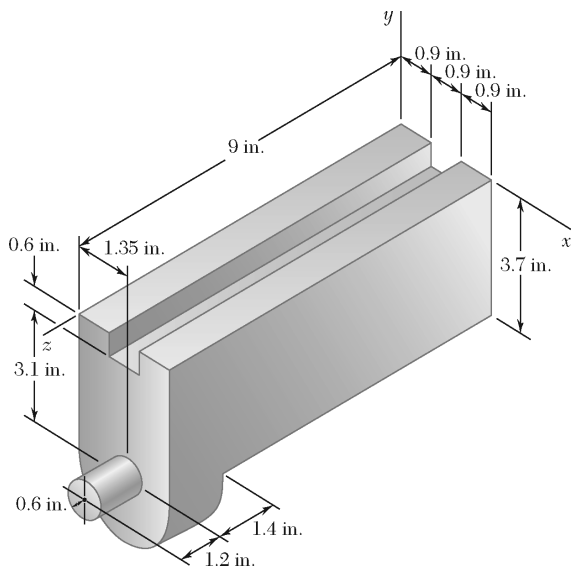
We have

$$\begin{aligned}
 I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 - (I_y)_4 \\
 &= \left\{ \frac{1}{12} (791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(2.7)^2 + (9)^2] \text{ in}^2 \right. \\
 &\quad \left. + (791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[\left(\frac{2.7}{2} \right)^2 + \left(\frac{9}{2} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ \frac{1}{12} (35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [3(1.35)^2 + (1.4)^2] \text{ in}^2 \right. \\
 &\quad \left. + (35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(1.35)^2 + \left(9 - \frac{1.4}{2} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ \frac{1}{12} (11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [3(0.6)^2 + (1.2)^2] \text{ in}^2 \right. \\
 &\quad \left. + (11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(1.35)^2 + \left(9 + \frac{1.2}{2} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad - \left\{ \frac{1}{12} (42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(0.9)^2 + (9)^2] \text{ in}^2 \right. \\
 &\quad \left. + (42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(1.35)^2 + \left(\frac{9}{2} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &= [(40.4550 + 121.3650) + (0.1517 + 17.3319) \\
 &\quad + (0.0174 + 7.8005) - (2.0263 + 6.5603)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 &= (161.8200 + 17.4836 + 7.8179 - 8.5866) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2
 \end{aligned}$$

or $I_y = 0.1785 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$

PROBLEM 9.144

Determine the mass moment of inertia of the steel machine element shown with respect to the z axis. (The specific weight of steel is 490 lb/ft^3 .)



SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} V$$

Then

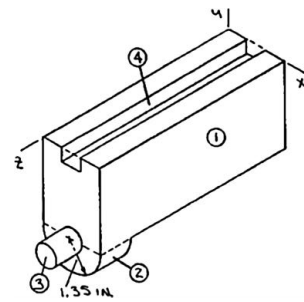
$$\begin{aligned} m_1 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (2.7 \times 3.7 \times 9) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft} \end{aligned}$$

$$\begin{aligned} m_2 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left(\frac{\pi}{2} \times 1.35^2 \times 1.4 \right) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft} \end{aligned}$$

$$\begin{aligned} m_3 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (\pi \times 0.6^2 \times 1.2) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft} \end{aligned}$$

$$m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (0.9 \times 0.6 \times 9) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 = 42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}$$

The mass moments of inertia are now computed using Figure 9.28 (components 1, 3, and 4) and the equations derived above (component 2).



PROBLEM 9.144 (Continued)

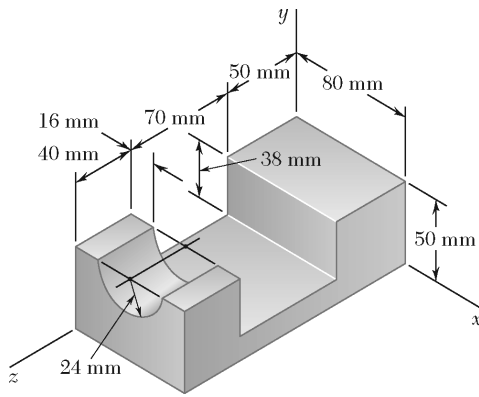
Find: I_z

We have

$$\begin{aligned}
 I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4 \\
 &= \left\{ \frac{1}{12} (791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(2.7)^2 + (3.7)^2] \text{ in}^2 \right. \\
 &\quad \left. + (791.780 \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[\left(\frac{2.7}{2} \right)^2 + \left(\frac{3.7}{2} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ (35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) (1.35 \text{ in.})^2 \right. \\
 &\quad \left. + (35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(1.35)^2 + \left(3.7 + \frac{4 \times 1.35}{3\pi} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad + \left\{ \frac{1}{12} (11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) (0.6 \text{ in.})^2 \right. \\
 &\quad \left. + (11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(1.35)^2 + (3.7)^2] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 &\quad - \left\{ \frac{1}{12} (42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) [(0.9)^2 + (0.6)^2] \text{ in}^2 \right. \\
 &\quad \left. + (42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2 / \text{ft}) \left[(1.35)^2 + \left(\frac{0.6}{2} \right)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\
 I_z &= [(9.6132 + 28.8395) + (0.1429 + 4.9219) \\
 &\quad + (0.0149 + 1.2875) - (0.0290 + 0.5684)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\
 &= (38.4527 + 5.0648 + 1.3024 - 0.5974) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2
 \end{aligned}$$

or $I_z = 0.0442 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$

PROBLEM 9.145



Determine the mass moment of inertia of the steel fixture shown with respect to (a) the x axis, (b) the y axis, (c) the z axis. (The density of steel is 7850 kg/m^3 .)

SOLUTION

First compute the mass of each component. We have

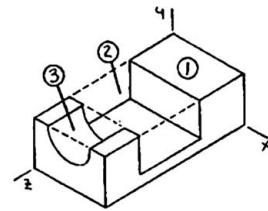
$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.05 \times 0.160) \text{ m}^3 \\ = 5.02400 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.038 \times 0.07) \text{ m}^3 = 1.67048 \text{ kg}$$

$$m_3 = 7850 \text{ kg/m}^3 \times \left(\frac{\pi}{2} \times 0.024^2 \times 0.04 \right) \text{ m}^3 = 0.28410 \text{ kg}$$



Using Figure 9.28 for components 1 and 2 and the equations derived above for component 3, we have

$$(a) \quad I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$

$$= \left\{ \frac{1}{12} (5.02400 \text{ kg}) [(0.05)^2 + (0.16)^2] \text{ m}^2 + (5.02400 \text{ kg}) \left[\left(\frac{0.05}{2} \right)^2 + \left(\frac{0.16}{2} \right)^2 \right] \text{ m}^2 \right\} \\ - \left\{ \frac{1}{12} (1.67048 \text{ kg}) [(0.038)^2 + (0.07)^2] \text{ m}^2 + (1.67048 \text{ kg}) \left[\left(0.05 - \frac{0.038}{2} \right)^2 + \left(0.05 + \frac{0.07}{2} \right)^2 \right] \text{ m}^2 \right\} \\ - \left\{ (0.28410 \text{ kg}) \left[\left(\frac{1}{4} - \frac{16}{9\pi^2} \right) (0.024)^2 + \frac{1}{12} (0.04)^2 \right] \text{ m}^2 \right. \\ \left. + (0.28410 \text{ kg}) \left[\left(0.05 - \frac{4 \times 0.024}{3\pi} \right)^2 + \left(0.16 - \frac{0.04}{2} \right)^2 \right] \text{ m}^2 \right\} \\ = [(11.7645 + 35.2936) - (0.8831 + 13.6745) - (0.0493 + 6.0187)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ = (47.0581 - 14.5576 - 6.0680) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ = 26.4325 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \text{or} \quad I_x = 26.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.145 (Continued)

$$\begin{aligned}
 (b) \quad I_y &= (I_y)_1 - (I_y)_2 - (I_y)_3 \\
 &= \left\{ \frac{1}{12} (5.02400 \text{ kg}) [(0.08)^2 + (0.16)^2] \text{ m}^2 + (5.02400 \text{ kg}) \left[\left(\frac{0.08}{2} \right)^2 + \left(\frac{0.16}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &\quad - \left\{ \frac{1}{12} (1.67048 \text{ kg}) [(0.08)^2 + (0.07)^2] \text{ m}^2 + (1.67048 \text{ kg}) \left[\left(\frac{0.08}{2} \right)^2 + \left(0.05 + \frac{0.07}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &\quad - \left\{ \frac{1}{12} (0.28410 \text{ kg}) [3(0.024)^2 + (0.04)^2] \text{ m}^2 + (0.28410 \text{ kg}) \left[\left(\frac{0.08}{2} \right)^2 + \left(0.16 - \frac{0.04}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &= [(13.3973 + 40.1920) - (1.5730 + 14.7420) - (0.0788 + 6.0229)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= (53.5893 - 16.3150 - 6.1017) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= 31.1726 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or } I_y = 31.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad I_z &= (I_z)_1 - (I_z)_2 - (I_z)_3 \\
 &= \left\{ \frac{1}{12} (5.02400 \text{ kg}) [(0.08)^2 + (0.05)^2] \text{ m}^2 + (5.02400 \text{ kg}) \left[\left(\frac{0.08}{2} \right)^2 + \left(\frac{0.05}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &\quad - \left\{ \frac{1}{12} (1.67048 \text{ kg}) [(0.08)^2 + (0.038)^2] \text{ m}^2 + (1.67048 \text{ kg}) \left[\left(\frac{0.08}{2} \right)^2 + \left(0.05 - \frac{0.038}{2} \right)^2 \right] \text{ m}^2 \right\} \\
 &\quad - \left\{ (0.28410 \text{ kg}) \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) (0.024 \text{ m})^2 + (0.28410 \text{ kg}) \left[\left(\frac{0.08}{2} \right)^2 + \left(0.05 - \frac{4 \times 0.024}{3\pi} \right)^2 \right] \text{ m}^2 \right\} \\
 &= [(3.7261 + 11.1784) - (1.0919 + 4.2781) - (0.0523 + 0.9049)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= (14.9045 - 5.3700 - 0.9572) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= 8.5773 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or } I_z = 8.58 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft
 \end{aligned}$$

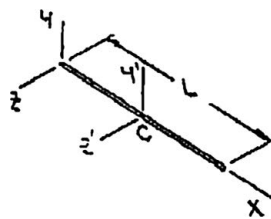
To the instructor:

The following formulas for the mass moment of inertia of wires are derived or summarized at this time for use in the solutions of Problems 9.146 through 9.148.

Slender Rod

$$I_x = 0 \quad \bar{I}_{y'} = \bar{I}_{z'} = \frac{1}{12}mL^2 \text{ (Figure 9.28)}$$

$$I_y = I_z = \frac{1}{3}mL^2 \text{ (Sample Problem 9.9)}$$



Circle

We have

$$\bar{I}_y = \int r^2 dm = ma^2$$

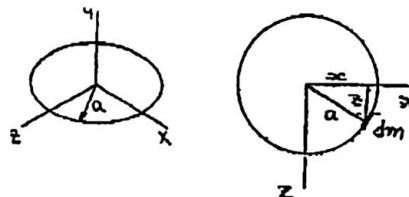
Now

$$\bar{I}_y = \bar{I}_x + \bar{I}_z$$

And symmetry implies

$$\bar{I}_x = \bar{I}_z$$

$$\bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2$$



Semicircle

Following the above arguments for a circle, We have

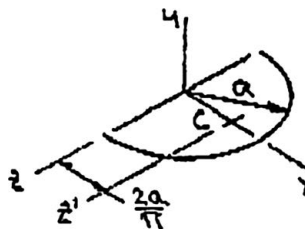
$$\bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2 \quad I_y = ma^2$$

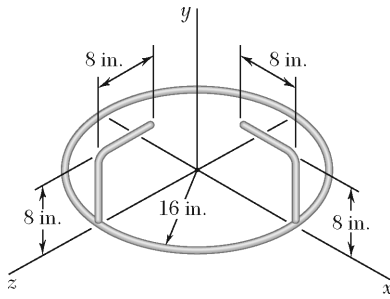
Using the parallel-axis theorem

$$I_z = \bar{I}_z + m\bar{x}^2 \quad x = \frac{2a}{\pi}$$

or

$$I_{z'} = m\left(\frac{1}{2} - \frac{4}{\pi^2}\right)a^2$$





PROBLEM 9.146

Aluminum wire with a weight per unit length of 0.033 lb/ft is used to form the circle and the straight members of the figure shown. Determine the mass moment of inertia of the assembly with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g} \left(\frac{W}{L} \right)_{AL} L$$

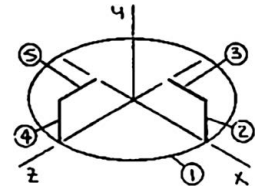
Then

$$m_1 = \frac{1}{32.2 \text{ ft/s}^2} \times 0.033 \text{ lb/ft} \times (2\pi \times 16 \text{ in.}) \times \frac{1 \text{ ft}}{12 \text{ in.}}$$

$$= 8.5857 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = m_3 = m_4 = m_5 = \frac{1}{32.2 \text{ ft/s}^2} \times 0.033 \text{ lb/ft} \times 8 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}}$$

$$= 0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}$$



Using the equations given above and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5$$

$$= \left[\frac{1}{2} (8.5857 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (16 \text{ in.})^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$+ \left[\frac{1}{3} (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}) (8 \text{ in.})^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$+ [0 + (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}) (8 \text{ in.})^2] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$+ \left[\frac{1}{12} (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}) (8 \text{ in.})^2 + (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}) (4^2 + 16^2) \text{ in}^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$+ \left[\frac{1}{12} (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}) (8 \text{ in.})^2 + (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft}) (8^2 + 12^2) \text{ in}^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$= [(7.6315) + (0.1012) + (0.3036) + (0.0253 + 1.2905) + (0.0253 + 0.9868)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= (7.6315 + 0.1012 + 0.3036) + 1.3158 + 1.0121 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= 10.3642 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \text{or} \quad I_x = 10.36 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$

PROBLEM 9.146 (Continued)

$$(I_y)_2 = (I_y)_4, \quad (I_y)_3 = (I_y)_5$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5$$

$$= [(8.5857 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(16 \text{ in.})^2] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ 2[0 + (0.6832 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(16 \text{ in.})^2] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ 2\left[\frac{1}{12}(0.6832 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(8 \text{ in.})^2 + (0.6832 \text{ lb} \cdot \text{s}^2/\text{ft})(12 \text{ in.})^2\right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= [(15.2635) + 2(1.2146) + 2(0.0253 + 0.6832)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= [15.2635 + 2(1.2146) + 2(0.7085)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

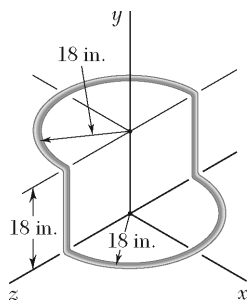
$$= 19.1097 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\text{or } I_y = 19.11 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$

Symmetry implies

$$I_x = I_z$$

$$I_z = 10.36 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$



PROBLEM 9.147

The figure shown is formed of $\frac{1}{8}$ -in.-diameter steel wire. Knowing that the specific weight of the steel is 490 lb/ft^3 , determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION

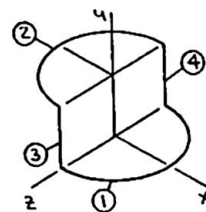
First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} AL$$

Then

$$\begin{aligned} m_1 = m_2 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi \left(\frac{1}{8} \text{ in.} \right)^2}{4} \right] \times (\pi \times 18 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$

$$\begin{aligned} m_3 = m_4 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} = \left[\frac{\pi \left(\frac{1}{8} \text{ in.} \right)^2}{4} \right] \times 18 \text{ in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 1.9453 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$



Using the equations given above and the parallel-axis theorem, we have

$$(I_x)_3 = (I_x)_4$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4$$

$$= \left[\frac{1}{2} (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (18 \text{ in.})^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$+ \left[\frac{1}{2} (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (18 \text{ in.})^2 + (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (18 \text{ in.})^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$+ 2 \left[\frac{1}{12} (1.9453 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (18 \text{ in.})^2 + (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) (9^2 + 18^2) \text{ in}^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2$$

$$= [(6.8751) + (6.8751 + 13.1502) + 2(0.3647 + 5.4712)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= [6.8751 + 20.6252 + 2(5.8359)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= 39.1721 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\text{or } I_x = 0.0392 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$

PROBLEM 9.147 (Continued)

$$(I_y)_1 = (I_y)_2, \quad (I_y)_3 = (I_y)_4$$

$$\begin{aligned} I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 \\ &= 2[(6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &\quad + 2[(0 + 1.9453 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &= [2(13.7502) + 2(4.3769)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= 36.2542 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

or $I_y = 0.0363 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$

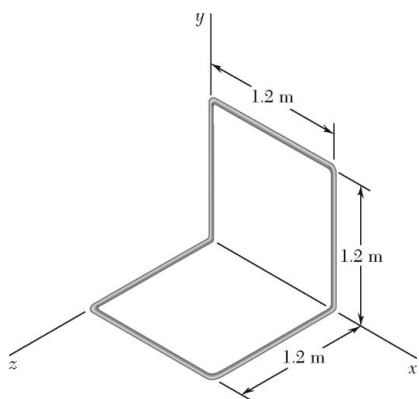
$$(I_z)_3 = (I_z)_4$$

$$\begin{aligned} I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 \\ &= \left[\frac{1}{2}(6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2\right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &\quad + \left\{ (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(\frac{1}{2} - \frac{4}{\pi^2}\right) (18 \text{ in.})^2 \right. \\ &\quad \left. + (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left[\left(\frac{2 \times 18}{\pi}\right)^2 + (18)^2 \right] \text{ in}^2 \right\} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &\quad + 2 \left[\frac{1}{3}(1.9453 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})(18 \text{ in.})^2 \right] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2 \\ &= [(6.8751) + (1.3024 + 19.3229) + 2(1.4590)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= [6.8751 + 20.6253 + 2(1.4590)] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= 30.4184 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

or $I_z = 0.0304 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$

PROBLEM 9.148

A homogeneous wire with a mass per unit length of 0.056 kg/m is used to form the figure shown. Determine the mass moment of inertia of the wire with respect to each of the coordinate axes.



SOLUTION

First compute the mass m of each component. We have

$$\begin{aligned} m &= (m/L)L \\ &= 0.056 \text{ kg/m} \times 1.2 \text{ m} \\ &= 0.0672 \text{ kg} \end{aligned}$$

Using the equations given above and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5 + (I_x)_6$$

Now

$$(I_x)_1 = (I_x)_3 = (I_x)_4 = (I_x)_6 \quad \text{and} \quad (I_x)_2 = (I_x)_5$$

Then

$$\begin{aligned} I_x &= 4 \left[\frac{1}{3} (0.0672 \text{ kg})(1.2 \text{ m})^2 \right] + 2[0 + (0.0672 \text{ kg})(1.2 \text{ m})^2] \\ &= [4(0.03226) + 2(0.09677)] \text{ kg} \cdot \text{m}^2 \\ &= 0.32258 \text{ kg} \cdot \text{m}^2 \quad \text{or} \quad I_x = 0.323 \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft \end{aligned}$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5 + (I_y)_6$$

Now

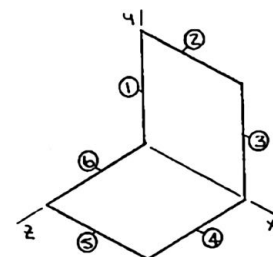
$$(I_y)_1 = 0, \quad (I_y)_2 = (I_y)_6, \quad \text{and} \quad (I_y)_4 = (I_y)_5$$

Then

$$\begin{aligned} I_y &= 2 \left[\frac{1}{3} (0.0672 \text{ kg})(1.2 \text{ m})^2 \right] + [0 + (0.0672 \text{ kg})(1.2 \text{ m})^2] \\ &\quad + 2 \left[\frac{1}{12} (0.0672 \text{ kg})(1.2 \text{ m})^2 + (0.0672 \text{ kg})(1.2^2 + 0.6^2) \text{ m}^2 \right] \\ &= [2(0.03226) + (0.09677) + 2(0.00806 + 0.12096)] \text{ kg} \cdot \text{m}^2 \\ &= [2(0.03226) + (0.09677) + 2(0.12902)] \text{ kg} \cdot \text{m}^2 \\ &= 0.41933 \text{ kg} \cdot \text{m}^2 \quad \text{or} \quad I_y = 0.419 \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft \end{aligned}$$

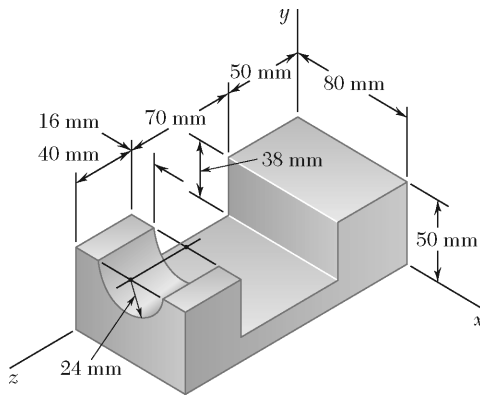
Symmetry implies

$$I_y = I_z \quad \text{or} \quad I_z = 0.419 \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$



PROBLEM 9.149

Determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the steel fixture shown. (The density of steel is 7850 kg/m^3 .)



SOLUTION

First compute the mass of each component. We have

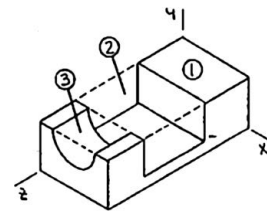
$$m = \rho V$$

Then

$$m_1 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.05 \times 0.16) \text{ m}^3 = 5.02400 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.038 \times 0.07) \text{ m}^3 = 1.67048 \text{ kg}$$

$$m_3 = 7850 \text{ kg/m}^3 \times \left(\frac{\pi}{2} \times 0.024^2 \times 0.04 \right) \text{ m}^3 = 0.28410 \text{ kg}$$



Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{y}_2 = \left(0.05 - \frac{0.038}{2} \right) \text{ m} = 0.031 \text{ m}$$

$$\bar{y}_3 = \left(0.05 - \frac{4 \times 0.024}{3\pi} \right) \text{ m} = 0.039814 \text{ m}$$

and then

	m , kg	\bar{x} , m	\bar{y} , m	\bar{z} , m	$m\bar{x}\bar{y}$, $\text{kg} \cdot \text{m}^2$	$m\bar{y}\bar{z}$, $\text{kg} \cdot \text{m}^2$	$m\bar{z}\bar{x}$, $\text{kg} \cdot \text{m}^2$
1	5.02400	0.04	0.025	0.08	5.0240×10^{-3}	10.0480×10^{-3}	16.0768×10^{-3}
2	1.67048	0.04	0.031	0.085	2.0714×10^{-3}	4.4017×10^{-3}	5.6796×10^{-3}
3	0.28410	0.04	0.039814	0.14	0.4524×10^{-3}	1.5836×10^{-3}	1.5910×10^{-3}

Finally,

$$I_{xy} = (I_{xy})_1 - (I_{xy})_2 - (I_{xy})_3 = [(0 + 5.0240) - (0 + 2.0714) - (0 + 0.4524)] \times 10^{-3}$$

$$= 2.5002 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\text{or } I_{xy} = 2.50 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.149 (Continued)

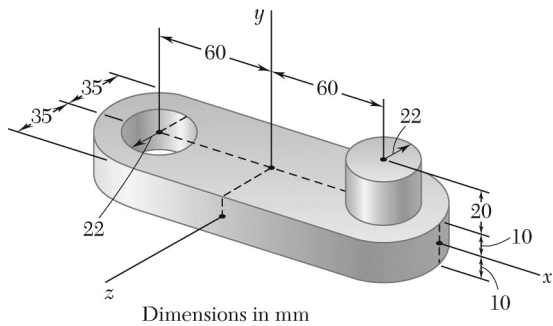
$$\begin{aligned} I_{yz} &= (I_{yz})_1 - (I_{yz})_2 - (I_{yz})_3 = [(0 + 10.0480) - (0 + 4.4017) - (0 + 1.5836)] \times 10^{-3} \\ &= 4.0627 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_{yz} = 4.06 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

$$\begin{aligned} I_{zx} &= (I_{zx})_1 - (I_{zx})_2 - (I_{zx})_3 = [(0 + 16.0768) - (0 + 5.6796) - (0 + 1.5910)] \times 10^{-3} \\ &= 8.8062 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_{zx} = 8.81 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.150



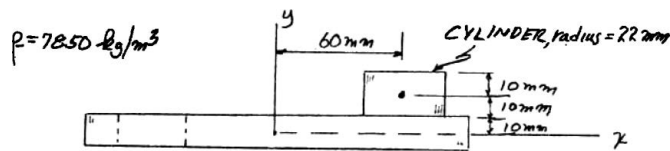
Determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the steel machine element shown. (The density of steel is 7850 kg/m^3 .)

SOLUTION

Since the machine element is symmetrical with respect to the xy plane, $I_{yz} = I_{zx} = 0$

Also, $I_{xy} = 0$

for all components except the cylinder, since these are symmetrical with respect to the xz plane.

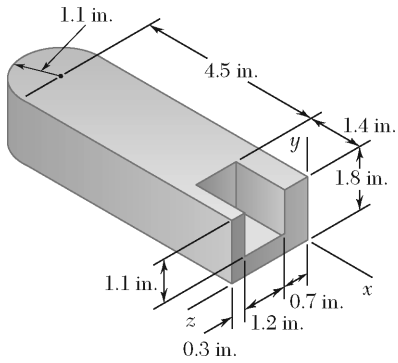


For the cylinder

$$m = \rho V = (7850 \text{ kg/m}^3)\pi(0.022 \text{ m})^2(0.02 \text{ m}) = 0.2387 \text{ kg}$$

$$I'_{xy} = \bar{I}_{x'y'} + m\bar{x}\bar{y} = 0 + (0.2387 \text{ kg})(0.06 \text{ m})(0.02 \text{ m}) = +286.4 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

$$I_{xy} = 286 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$



PROBLEM 9.151

Determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the cast aluminum machine component shown. (The specific weight of aluminum is 0.100 lb/in^3)

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{AL}} V = \frac{\gamma_{\text{AL}}}{g} V$$

Then

$$m_1 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (5.9 \times 1.8 \times 2.2) \text{ in}^3$$

$$= 72.5590 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2} (1.1)^2 \times 1.8 \right] \text{ in}^3$$

$$= 10.6248 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (1.4 \times 1.1 \times 1.2) \text{ in}^3$$

$$= 5.7391 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

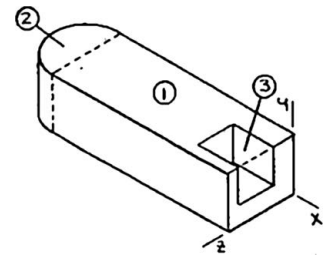
Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{x}_2 = - \left(5.9 + \frac{4 \times 1.1}{3\pi} \right) \text{ in.}$$

$$= -6.36685 \text{ in.}$$

$$\bar{y}_3 = \left(1.8 - \frac{1.1}{2} \right) \text{ in.}$$

$$= 1.25 \text{ in.}$$



PROBLEM 9.151 (Continued)

and then

	$m, \text{lb} \cdot \text{s}^2/\text{ft}$	\bar{x}, ft	\bar{y}, ft	\bar{z}, ft	$m\bar{x}\bar{y}, \text{lb} \cdot \text{ft} \cdot \text{s}^2$	$m\bar{y}\bar{z}, \text{lb} \cdot \text{ft} \cdot \text{s}^2$	$m\bar{z}\bar{x}, \text{lb} \cdot \text{ft} \cdot \text{s}^2$
1	72.5590×10^{-3}	$-\frac{2.95}{12}$	$\frac{0.9}{12}$	$\frac{1.1}{12}$	-1.33781×10^{-3}	0.49884×10^{-3}	-1.63510×10^{-3}
2	10.6248×10^{-3}	$-\frac{6.36685}{12}$	$\frac{0.9}{12}$	$\frac{1.1}{12}$	-0.42279×10^{-3}	0.07305×10^{-3}	-0.51674×10^{-3}
3	5.7391×10^{-3}	$-\frac{0.7}{12}$	$\frac{1.25}{12}$	$\frac{1.3}{12}$	-0.03487×10^{-3}	0.06476×10^{-3}	-0.03627×10^{-3}

Finally,

$$I_{xy} = (I_{xy})_1 + (I_{xy})_2 - (I_{xy})_3$$

$$= [(0 - 1.33781) + (0 - 0.42279) - (0 - 0.03487)] \times 10^{-3}$$

$$\text{or } I_{xy} = -1.726 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$

$$I_{yz} = (I_{yz})_1 + (I_{yz})_2 - (I_{yz})_3$$

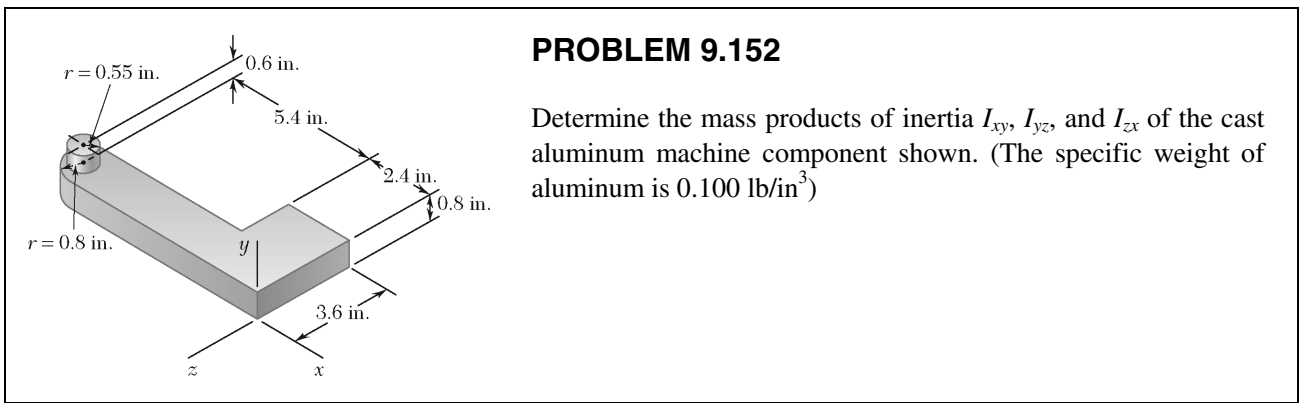
$$= [(0 + 0.49884) + (0 + 0.07305) - (0 + 0.06476)] \times 10^{-3}$$

$$\text{or } I_{yz} = 0.507 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$

$$I_{zx} = (I_{zx})_1 + (I_{zx})_2 - (I_{zx})_3$$

$$= [(0 - 1.63510) + (0 - 0.51674) - (0 - 0.3627)] \times 10^{-3}$$

$$\text{or } I_{zx} = -2.12 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$



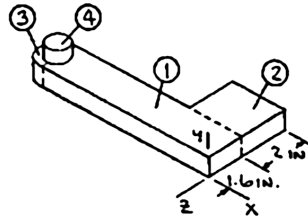
PROBLEM 9.152

Determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the cast aluminum machine component shown. (The specific weight of aluminum is 0.100 lb/in^3)

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{AL} V = \frac{\gamma_{AL}}{g} V$$



Then

$$m_1 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (7.8 \times 0.8 \times 1.6) \text{ in}^3$$

$$= 31.0062 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (2.4 \times 0.8 \times 2) \text{ in}^3$$

$$= 11.9255 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2} (0.8)^2 \times 0.8 \right] \text{ in}^3 = 2.4977 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_4 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times [\pi (0.55)^2 \times 0.6] \text{ in}^3 = 1.7708 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{x}_3 = - \left(7.8 + \frac{4 \times 0.8}{3\pi} \right) \text{ in.} = -8.13953 \text{ in.}$$

PROBLEM 9.152 (Continued)

and then

	$m, \text{lb} \cdot \text{s}^2/\text{ft}$	\bar{x}, ft	\bar{y}, ft	\bar{z}, ft	$m\bar{x}\bar{y}, \text{lb} \cdot \text{ft} \cdot \text{s}^2$	$m\bar{y}\bar{z}, \text{lb} \cdot \text{ft} \cdot \text{s}^2$	$m\bar{z}\bar{x}, \text{lb} \cdot \text{ft} \cdot \text{s}^2$
1	31.0062×10^{-3}	$-\frac{3.9}{12}$	$\frac{0.4}{12}$	$-\frac{0.8}{12}$	-335.901×10^{-6}	-68.903×10^{-6}	671.801×10^{-6}
2	11.9255×10^{-3}	$-\frac{1.2}{12}$	$\frac{0.4}{12}$	$-\frac{2.6}{12}$	-39.752×10^{-6}	-86.129×10^{-6}	258.386×10^{-6}
3	2.4977×10^{-3}	$-\frac{8.13953}{12}$	$\frac{0.4}{12}$	$-\frac{0.8}{12}$	-56.473×10^{-6}	-5.550×10^{-6}	112.945×10^{-6}
4	1.7708×10^{-3}	$-\frac{7.8}{12}$	$\frac{1.1}{12}$	$-\frac{0.8}{12}$	-105.511×10^{-6}	-10.822×10^{-6}	76.735×10^{-6}
Σ					-537.637×10^{-6}	-171.404×10^{-6}	1119.867×10^{-6}

Then

$$I_{xy} = \Sigma(\overset{0}{\cancel{I_{x'y'}}} + m\bar{x}\bar{y}) \quad \text{or} \quad I_{xy} = -538 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$

$$I_{yz} = \Sigma(\overset{0}{\cancel{I_{y'z'}}} + m\bar{y}\bar{z}) \quad \text{or} \quad I_{yz} = -171.4 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$

$$I_{zx} = \Sigma(\overset{0}{\cancel{I_{z'x'}}} + m\bar{z}\bar{x}) \quad \text{or} \quad I_{zx} = 1120 \times 10^{-6} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft$$

PROBLEM 9.153

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the component.

SOLUTION

First compute the mass of each component.

We have
$$m = \rho_{\text{ST}}V = \rho_{\text{ST}}tA$$

Then

$$m_1 = (7850 \text{ kg/m}^3)[(0.002)(0.4)(0.45)]\text{m}^3 = 2.8260 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \left[(0.002) \left(\frac{1}{2} \times 0.45 \times 0.18 \right) \right] \text{m}^3 = 0.63585 \text{ kg}$$

Now observe that

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = (\bar{I}_{z'x'})_1 = 0$$

$$(\bar{I}_{y'z'})_2 = (\bar{I}_{z'x'})_2 = 0$$

From Sample Problem 9.6:
$$(\bar{I}_{x'y'})_{2,\text{area}} = -\frac{1}{72}b_2^2h_2^2$$

Then
$$(\bar{I}_{x'y'})_2 = \rho_{\text{ST}}t(I_{x'y'})_{2,\text{area}} = \rho_{\text{ST}}t \left(-\frac{1}{72}b_2^2h_2^2 \right) = -\frac{1}{36}m_2b_2h_2$$

Also
$$\bar{x}_1 = \bar{y}_1 = \bar{z}_1 = 0 \quad \bar{x}_2 = \left(-0.225 + \frac{0.45}{3} \right) \text{m} = -0.075 \text{ m}$$

PROBLEM 9.153 (Continued)

Finally,

$$I_{xy} = \Sigma(\bar{I}_{xy} + m\bar{x}\bar{y}) = (0 + 0) + \left[-\frac{1}{36}(0.63585 \text{ kg})(0.45 \text{ m})(0.18 \text{ m}) \right. \\ \left. + (0.63585 \text{ kg})(-0.075 \text{ m})\left(\frac{0.18 \text{ m}}{3}\right) \right] \\ = (-1.43066 \times 10^{-3} - 2.8613 \times 10^{-3}) \text{ kg} \cdot \text{m}^2$$

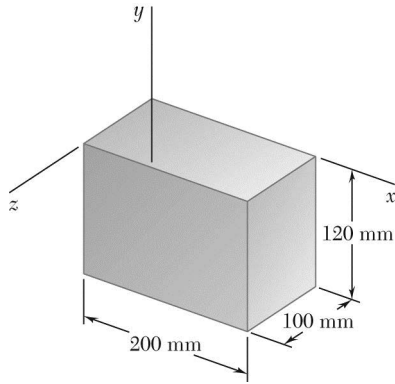
or $I_{xy} = -4.29 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

and $I_{yz} = \Sigma(\bar{I}_{y'z'} + m\bar{y}\bar{z}) = (0 + 0) + (0 + 0) = 0$

or $I_{yz} = 0 \blacktriangleleft$

$$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\bar{z}\bar{x}) = (0 + 0) + (0 + 0) = 0$$

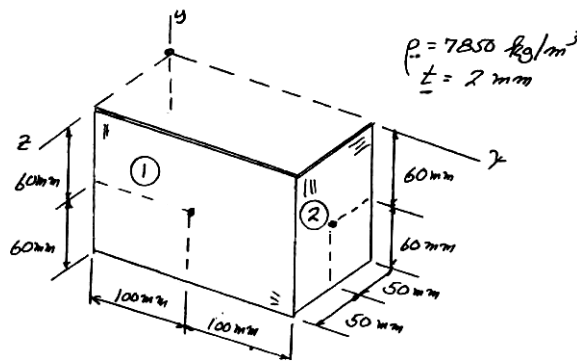
or $I_{zx} = 0 \blacktriangleleft$



PROBLEM 9.154

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the component.

SOLUTION



$$m_1 = \rho V = 7850 \text{ kg/m}^3 (0.120 \text{ m})(0.200 \text{ m})(0.002 \text{ m}) = 0.3768 \text{ kg}$$

$$m_2 = \rho V = 7850 \text{ kg/m}^3 (0.120 \text{ m})(0.100 \text{ m})(0.002 \text{ m}) = 0.1884 \text{ kg}$$

For each panel the centroidal product of inertia is zero with respect to each pair of coordinate axes.

	$m, \text{ kg}$	$\bar{x}, \text{ m}$	$\bar{y}, \text{ m}$	$\bar{z}, \text{ m}$	$m\bar{x}\bar{y}$	$m\bar{y}\bar{z}$	$m\bar{z}\bar{x}$
①	0.3768	0.1	-0.06	0.1	-2.261×10^{-3}	-2.261×10^{-3}	$+3.768 \times 10^{-3}$
②	0.1884	0.2	-0.06	0.05	-2.261×10^{-3}	-0.565×10^{-3}	$+1.884 \times 10^{-3}$
Σ					-4.522×10^{-3}	-2.826×10^{-3}	$+5.652 \times 10^{-3}$

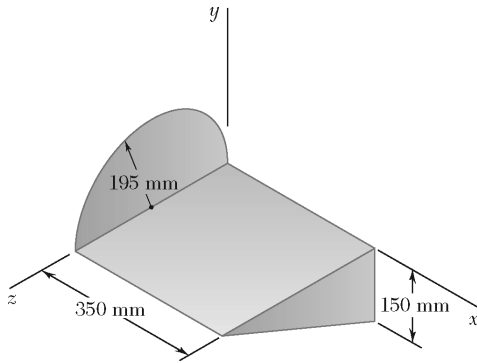
$$I_{xy} = -4.52 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$I_{yz} = -2.83 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$I_{zx} = +5.65 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.155

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the component.



SOLUTION

First compute the mass of each component. We have

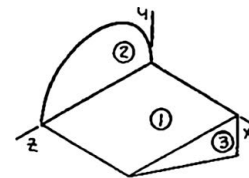
$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$\begin{aligned} m_1 &= (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.35 \times 0.39) \text{ m}^2 \\ &= 2.14305 \text{ kg} \end{aligned}$$

$$\begin{aligned} m_2 &= (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{\pi}{2} \times 0.195^2 \right) \text{ m}^2 \\ &= 0.93775 \text{ kg} \end{aligned}$$

$$\begin{aligned} m_3 &= (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{1}{2} \times 0.39 \times 0.15 \right) \text{ m}^2 \\ &= 0.45923 \text{ kg} \end{aligned}$$



Now observe that because of symmetry the centroidal products of inertia of components 1 and 2 are zero and

$$(\bar{I}_{y'z'})_3 = (\bar{I}_{z'x'})_3 = 0$$

Also

$$(\bar{I}_{y'z'})_{3,\text{mass}} = \rho_{\text{ST}} t (\bar{I}_{y'z'})_{3,\text{area}}$$

Using the results of Sample Problem 9.6 and noting that the orientation of the axes corresponds to a 90° rotation, we have

$$(\bar{I}_{y'z'})_{3,\text{area}} = \frac{1}{72} b_3^2 h_3^2$$

Then

$$(I_{y'z'})_3 = \rho_{\text{ST}} t \left(\frac{1}{72} b_3^2 h_3^2 \right) = \frac{1}{36} m_3 b_3 h_3$$

Also

$$\begin{aligned} \bar{y}_1 &= \bar{x}_2 = 0 \\ \bar{y}_2 &= \frac{4 \times 0.195}{3\pi} \text{ m} = 0.082761 \text{ m} \end{aligned}$$

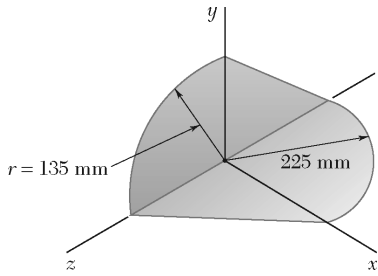
PROBLEM 9.155 (Continued)

Finally,

$$\begin{aligned} I_{xy} &= \Sigma(\bar{I}_{x'y'} + m\bar{x}\bar{y}) \\ &= (0+0) + (0+0) + \left[0 + (0.45923 \text{ kg})(0.35 \text{ m}) \left(-\frac{0.15}{3} \text{ m} \right) \right] \\ &= -8.0365 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or } I_{xy} = -8.04 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$

$$\begin{aligned} I_{yz} &= \Sigma(\bar{I}_{y'z'} + m\bar{y}\bar{z}) \\ &= (0+0) + [0 + (0.93775 \text{ kg})(0.082761 \text{ m})(0.195 \text{ m})] \\ &\quad + \left[\frac{1}{36} (0.45923 \text{ kg})(0.39 \text{ m})(0.15 \text{ m}) + (0.45923 \text{ kg}) \left(-\frac{0.15}{3} \text{ m} \right) \left(\frac{0.39}{3} \text{ m} \right) \right] \\ &= [(15.1338) + (0.7462 - 2.9850)] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ &= (15.1338 - 2.2388) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ &= 12.8950 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or } I_{yz} = 12.90 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$

$$\begin{aligned} I_{zx} &= \Sigma(\bar{I}_{z'x'} + m\bar{z}\bar{x}) \\ &= [0 + (2.14305 \text{ kg})(0.175 \text{ m})(0.195 \text{ m})] + (0+0) \\ &\quad + \left[0 + (0.45923 \text{ kg}) \left(\frac{0.39}{3} \text{ m} \right) (0.35 \text{ m}) \right] \\ &= (73.1316 + 20.8950) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ &= 94.0266 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or } I_{zx} = 94.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$



PROBLEM 9.156

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the component.

SOLUTION

First compute the mass of each component. We have

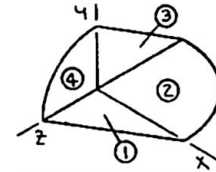
$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$m_1 = m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{1}{2} \times 0.225 \times 0.135 \right) \text{m}^2 = 0.23844 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{\pi}{4} \times 0.225^2 \right) \text{m}^2 = 0.62424 \text{ kg}$$

$$m_4 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \left(\frac{\pi}{4} \times 0.135^2 \right) \text{m}^2 = 0.22473 \text{ kg}$$



Now observe that the following centroidal products of inertia are zero because of symmetry.

$$\begin{aligned} (\bar{I}_{x'y'})_1 &= (\bar{I}_{y'z'})_1 = 0 & (\bar{I}_{x'y'})_2 &= (\bar{I}_{y'z'})_2 = 0 \\ (\bar{I}_{x'y'})_3 &= (\bar{I}_{z'x'})_3 = 0 & (\bar{I}_{x'y'})_4 &= (\bar{I}_{z'x'})_4 = 0 \end{aligned}$$

Also $\bar{y}_1 = \bar{y}_2 = 0$ $\bar{x}_3 = \bar{x}_4 = 0$

Now $I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\bar{y})$

so that

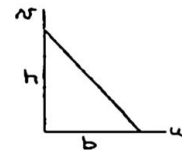
$$I_{xy} = 0 \quad \blacktriangleleft$$

Using the results of Sample Problem 9.6, we have

$$I_{uv,\text{area}} = \frac{1}{24} b^2 h^2$$

Now

$$\begin{aligned} I_{uv,\text{mass}} &= \rho_{\text{ST}} t I_{uv,\text{area}} \\ &= \rho_{\text{ST}} t \left(\frac{1}{24} b^2 h^2 \right) \\ &= \frac{1}{12} m b h \end{aligned}$$



Thus,

$$(I_{zx})_1 = \frac{1}{12} m_1 b_1 h_1$$

PROBLEM 9.156 (Continued)

While
$$(I_{yz})_3 = -\frac{1}{12}m_3b_3h_3$$

because of a 90° rotation of the coordinate axes.

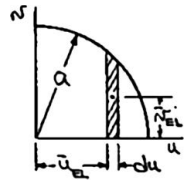
To determine I_{uv} for a quarter circle, we have

$$dI_{uv} = d\bar{I}_{u'v'} + u_{EL}v_{EL}dm$$

Where

$$\bar{u}_{EL} = u \quad \bar{v}_{EL} = \frac{1}{2}v = \frac{1}{2}\sqrt{a^2 - u^2}$$

$$dm = \rho_{ST}t dA = \rho_{ST}tv du = \rho_{ST}t\sqrt{a^2 - u^2} du$$



Then

$$\begin{aligned} I_{uv} &= \int dI_{uv} = \int_0^a (u) \left(\frac{1}{2}\sqrt{a^2 - u^2} \right) (\rho_{ST}t\sqrt{a^2 - u^2} du) \\ &= \frac{1}{2}\rho_{ST}t \int_0^a u(a^2 - u^2) du \\ &= \frac{1}{2}\rho_{ST}t \left[\frac{1}{2}a^2u^2 - \frac{1}{4}u^4 \right]_0^a = \frac{1}{8}\rho_{ST}t a^4 = \frac{1}{2\pi}ma^4 \end{aligned}$$

Thus

$$(I_{zx})_2 = -\frac{1}{2\pi}m_2a_2^2$$

because of a 90° rotation of the coordinate axes. Also

$$(I_{yz})_4 = \frac{1}{2\pi}m_4a_4^2$$

Finally,

$$\begin{aligned} I_{yz} &= \Sigma(I_{yz}) = [(\bar{I}_{y'z'}) + m_1\bar{y}_1\bar{z}_1] + [(\bar{I}_{y'z'}) + m_2\bar{y}_2\bar{z}_2] + (I_{yz})_3 + (I_{yz})_4 \\ &= \left[-\frac{1}{12}(0.23844 \text{ kg})(0.225 \text{ m})(0.135 \text{ m}) \right] + \left[\frac{1}{2\pi}(0.22473 \text{ kg})(0.135 \text{ m})^2 \right] \\ &= (-0.60355 + 0.65185) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

or $I_{yz} = 48.3 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

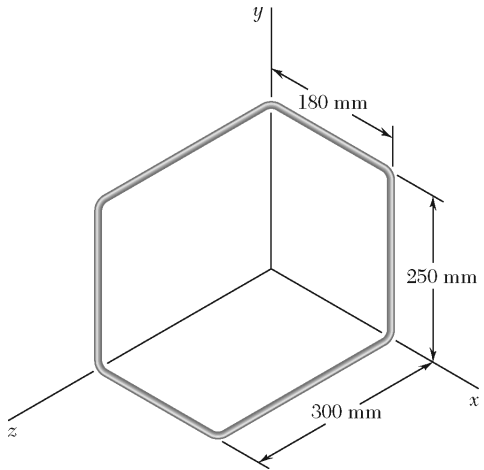
$$\begin{aligned} I_{zx} &= \Sigma(I_{zx}) = (I_{zx})_1 + (I_{zx})_2 + [(\bar{I}_{z'x'}) + m_3\bar{z}_3\bar{x}_3] + [(\bar{I}_{z'x'}) + m_4\bar{z}_4\bar{x}_4] \\ &= \left[\frac{1}{12}(0.23844 \text{ kg})(0.225 \text{ m})(0.135 \text{ m}) \right] + \left[-\frac{1}{2\pi}(0.62424 \text{ kg})(0.225 \text{ m})^2 \right] \\ &= (0.60355 - 5.02964) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

or $I_{zx} = -4.43 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$

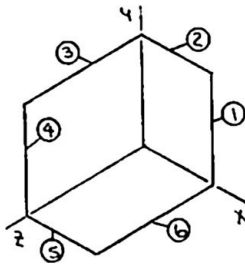
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PROBLEM 9.157

The figure shown is formed of 1.5-mm-diameter aluminum wire. Knowing that the density of aluminum is 2800 kg/m^3 , determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the wire figure.



SOLUTION



First compute the mass of each component. We have

$$m = \rho_{AL} V = \rho_{AL} AL$$

Then

$$\begin{aligned} m_1 = m_4 &= (2800 \text{ kg/m}^3) \left[\frac{\pi}{4} (0.0015 \text{ m})^2 \right] (0.25 \text{ m}) \\ &= 1.23700 \times 10^{-3} \text{ kg} \end{aligned}$$

$$\begin{aligned} m_2 = m_5 &= (2800 \text{ kg/m}^3) \left[\frac{\pi}{4} (0.0015 \text{ m})^2 \right] (0.18 \text{ m}) \\ &= 0.89064 \times 10^{-3} \text{ kg} \end{aligned}$$

$$\begin{aligned} m_3 = m_6 &= (2800 \text{ kg/m}^3) \left[\frac{\pi}{4} (0.0015 \text{ m})^2 \right] (0.3 \text{ m}) \\ &= 1.48440 \times 10^{-3} \text{ kg} \end{aligned}$$

PROBLEM 9.157 (Continued)

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

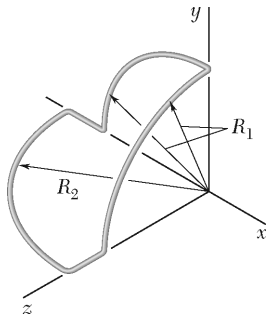
	$m, \text{ kg}$	$\bar{x}, \text{ m}$	$\bar{y}, \text{ m}$	$\bar{z}, \text{ m}$	$m\bar{x}\bar{y}, \text{ kg} \cdot \text{m}^2$	$m\bar{y}\bar{z}, \text{ kg} \cdot \text{m}^2$	$m\bar{z}\bar{x}, \text{ kg} \cdot \text{m}^2$
1	1.23700×10^{-3}	0.18	0.125	0	27.8325×10^{-6}	0	0
2	0.89064×10^{-3}	0.09	0.25	0	20.0394×10^{-6}	0	0
3	1.48440×10^{-3}	0	0.25	0.15	0	55.6650×10^{-6}	0
4	1.23700×10^{-3}	0	0.125	0.3	0	46.3875×10^{-6}	0
5	0.89064×10^{-3}	0.09	0	0.3	0	0	24.0473×10^{-6}
6	1.48440×10^{-3}	0.18	0	0.15	0	0	40.0788×10^{-6}
Σ					47.8719×10^{-6}	102.0525×10^{-6}	64.1261×10^{-6}

Then

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\bar{y}) \quad \text{or} \quad I_{xy} = 47.9 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$I_{yz} = \Sigma(\bar{I}_{y'z'} + m\bar{y}\bar{z}) \quad \text{or} \quad I_{yz} = 102.1 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\bar{z}\bar{x}) \quad \text{or} \quad I_{zx} = 64.1 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$



PROBLEM 9.158

Thin aluminum wire of uniform diameter is used to form the figure shown. Denoting by m' the mass per unit length of the wire, determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the wire figure.

SOLUTION

First compute the mass of each component. We have

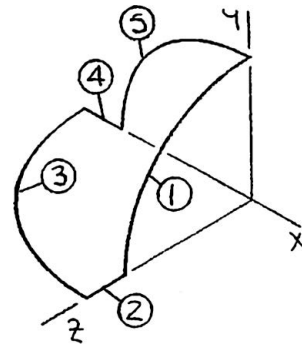
$$m = \left(\frac{m}{L}\right)L = m'L$$

Then

$$m_1 = m_5 = m' \left(\frac{\pi}{2} R_1\right) = \frac{\pi}{2} m' R_1$$

$$m_2 = m_4 = m'(R_2 - R_1)$$

$$m_3 = m' \left(\frac{\pi}{2} R_2\right) = \frac{\pi}{2} m' R_2$$



Now observe that because of symmetry the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of components 2 and 4 are zero and

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{z'x'})_1 = 0 \quad (\bar{I}_{x'y'})_3 = (\bar{I}_{y'z'})_3 = 0$$

$$(\bar{I}_{y'z'})_5 = (\bar{I}_{z'x'})_5 = 0$$

Also

$$\bar{x}_1 = \bar{x}_2 = 0 \quad \bar{y}_2 = \bar{y}_3 = \bar{y}_4 = 0 \quad \bar{z}_4 = \bar{z}_5 = 0$$

Using the parallel-axis theorem [Equations (9.47)], it follows that $I_{xy} = I_{yz} = I_{zx}$ for components 2 and 4.

To determine I_{uv} for one quarter of a circular arc, we have $dI_{uv} = uvdm$

where

$$u = a \cos \theta \quad v = a \sin \theta$$

and

$$dm = \rho dV = \rho[A(a d\theta)]$$

where A is the cross-sectional area of the wire. Now

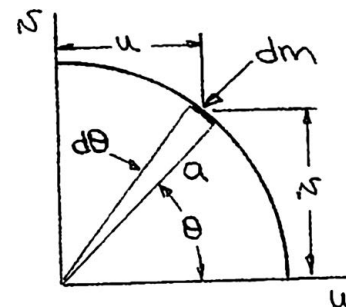
$$m = m' \left(\frac{\pi}{2} a\right) = \rho A \left(\frac{\pi}{2} a\right)$$

so that

$$dm = m' a d\theta$$

and

$$\begin{aligned} dI_{uv} &= (a \cos \theta)(a \sin \theta)(m' a d\theta) \\ &= m' a^3 \sin \theta \cos \theta d\theta \end{aligned}$$



PROBLEM 9.158 (Continued)

Then

$$I_{uv} = \int dI_{uv} = \int_0^{\pi/2} m'a^3 \sin \theta \cos \theta d\theta$$

$$= m'a^3 \left[\frac{1}{2} \sin^2 \theta \right]_0^{\pi/2} = \frac{1}{2} m'a^3$$

Thus,

$$(I_{yz})_1 = \frac{1}{2} m'R_1^3$$

and

$$(I_{zx})_3 = -\frac{1}{2} m'R_2^3 \quad (I_{xy})_5 = -\frac{1}{2} m'R_1^3$$

because of 90° rotations of the coordinate axes. Finally,

$$I_{xy} = \Sigma(I_{xy}) = [(\bar{I}_{x'y'})_1 + m_1 \bar{x}_1 \bar{y}_1] + [(\bar{I}_{x'y'})_3 + m_3 \bar{x}_3 \bar{y}_3] + (I_{xy})_5$$

or $I_{xy} = -\frac{1}{2} m'R_1^3 \blacktriangleleft$

$$I_{yz} = \Sigma(I_{yz}) = (I_{yz})_1 + [(\bar{I}_{y'z'})_3 + m_3 \bar{y}_3 \bar{z}_3] + [(\bar{I}_{y'z'})_5 + m_5 \bar{y}_5 \bar{z}_5]$$

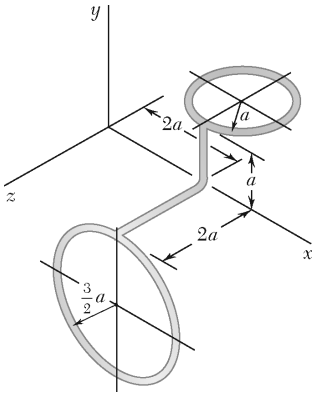
or $I_{yz} = \frac{1}{2} m'R_1^3 \blacktriangleleft$

$$I_{zx} = \Sigma(I_{zx}) = [(\bar{I}_{z'x'})_1 + m_1 \bar{z}_1 \bar{x}_1] + (I_{zx})_3 + [(\bar{I}_{z'x'})_5 + m_5 \bar{z}_5 \bar{x}_5]$$

or $I_{zx} = -\frac{1}{2} m'R_2^3 \blacktriangleleft$

PROBLEM 9.159

Brass wire with a weight per unit length w is used to form the figure shown. Determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the wire figure.



SOLUTION

First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g} wL$$

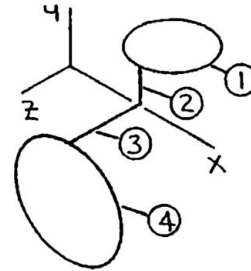
Then

$$m_1 = \frac{W}{g} (2\pi \times a) = 2\pi \frac{w}{g} a$$

$$m_2 = \frac{w}{g} (a) = \frac{w}{g} a$$

$$m_3 = \frac{w}{g} (2a) = 2 \frac{w}{g} a$$

$$m_4 = \frac{w}{g} \left(2\pi \times \frac{3}{2} a \right) = 3\pi \frac{w}{g} a$$



Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m	\bar{x}	\bar{y}	\bar{z}	$m\bar{x}\bar{y}$	$m\bar{y}\bar{z}$	$m\bar{z}\bar{x}$
1	$2\pi \frac{w}{g} a$	$2a$	a	$-a$	$4\pi \frac{w}{g} a^3$	$-2\pi \frac{w}{g} a^3$	$-4\pi \frac{w}{g} a^3$
2	$\frac{w}{g} a$	$2a$	$\frac{1}{2} a$	0	$\frac{w}{g} a^3$	0	0
3	$2 \frac{w}{g} a$	$2a$	0	a	0	0	$4 \frac{w}{g} a^3$
4	$3\pi \frac{w}{g} a$	$2a$	$-\frac{3}{2} a$	$2a$	$-9\pi \frac{w}{g} a^3$	$-9\pi \frac{w}{g} a^3$	$12\pi \frac{w}{g} a^3$
Σ					$\frac{w}{g} (1 - 5\pi) a^3$	$-11\pi \frac{w}{g} a^3$	$4 \frac{w}{g} (1 + 2\pi) a^3$

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PROBLEM 9.159 (Continued)

Then

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\bar{y})$$

or $I_{xy} = \frac{w}{g} a^3 (1 - 5\pi) \blacktriangleleft$

$$I_{yz} = \Sigma(\bar{I}_{y'z'} + m\bar{y}\bar{z})$$

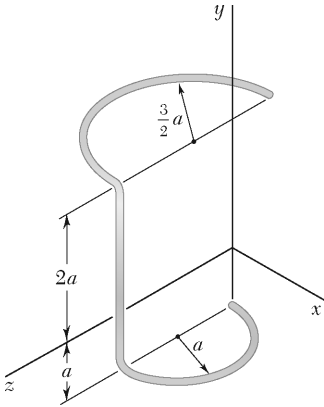
or $I_{yz} = -11\pi \frac{w}{g} a^3 \blacktriangleleft$

$$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\bar{z}\bar{x})$$

or $I_{zx} = 4 \frac{w}{g} a^3 (1 + 2\pi) \blacktriangleleft$

PROBLEM 9.160

Brass wire with a weight per unit length w is used to form the figure shown. Determine the mass products of inertia I_{xy} , I_{yz} , and I_{zx} of the wire figure.



SOLUTION

First compute the mass of each component. We have

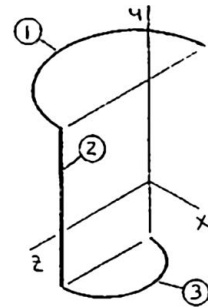
$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then

$$m_1 = \frac{W}{g} \left(\pi \times \frac{3}{2} a \right) = \frac{3}{2} \pi \frac{w}{g} a$$

$$m_2 = \frac{w}{g} (3a) = 3 \frac{w}{g} a$$

$$m_3 = \frac{w}{g} (\pi \times a) = \pi \frac{w}{g} a$$



Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m	\bar{x}	\bar{y}	\bar{z}	$m\bar{x}\bar{y}$	$m\bar{y}\bar{z}$	$m\bar{z}\bar{x}$
1	$\frac{3}{2} \pi \frac{w}{g} a$	$-\frac{2}{\pi} \left(\frac{3}{2} a \right)$	$2a$	$\frac{1}{2} a$	$-9 \frac{w}{g} a^3$	$\frac{3}{2} \pi \frac{w}{g} a^3$	$-\frac{9}{4} \frac{w}{g} a^3$
2	$3 \frac{w}{g} a$	0	$\frac{1}{2} a$	$2a$	0	$3 \frac{w}{g} a^3$	0
3	$\pi \frac{w}{g} a$	$\frac{2}{\pi} (a)$	$-a$	a	$-2 \frac{w}{g} a^3$	$-\pi \frac{w}{g} a^3$	$2 \frac{w}{g} a^3$
Σ					$-11 \frac{w}{g} a^3$	$\frac{w}{g} \left(\frac{\pi}{2} + 3 \right) a^3$	$-\frac{1}{4} \frac{w}{g} a^3$

PROBLEM 9.160 (Continued)

Then

$$I_{xy} = \Sigma(\bar{J}_{x'y'} + m\bar{x}\bar{y})$$

or $I_{xy} = -11\frac{w}{g}a^3 \blacktriangleleft$

$$I_{yz} = \Sigma(\bar{J}_{y'z'} + m\bar{y}\bar{z})$$

or $I_{yz} = \frac{1}{2}\frac{w}{g}a^3(\pi + 6) \blacktriangleleft$

$$I_{zx} = \Sigma(\bar{J}_{z'x'} + m\bar{z}\bar{x})$$

or $I_{zx} = -\frac{1}{4}\frac{w}{g}a^3 \blacktriangleleft$

PROBLEM 9.161

Complete the derivation of Eqs. (9.47), which express the parallel-axis theorem for mass products of inertia.

SOLUTION

We have
$$I_{xy} = \int xy dm \quad I_{yz} = \int yz dm \quad I_{zx} = \int zx dm \quad (9.45)$$

and
$$x = x' + \bar{x} \quad y = y' + \bar{y} \quad z = z' + \bar{z} \quad (9.31)$$

Consider
$$I_{xy} = \int xy dm$$

Substituting for x and for y

$$\begin{aligned} I_{xy} &= \int (x' + \bar{x})(y' + \bar{y}) dm \\ &= \int x'y' dm + \bar{y} \int x' dm + \bar{x} \int y' dm + \bar{x} \bar{y} \int dm \end{aligned}$$

By definition
$$\bar{I}_{x'y'} = \int x'y' dm$$

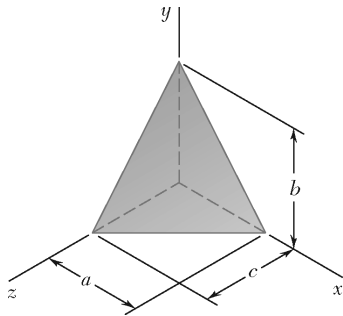
and
$$\begin{aligned} \int x' dm &= m\bar{x}' \\ \int y' dm &= m\bar{y}' \end{aligned}$$

However, the origin of the primed coordinate system coincides with the mass center G , so that

$$\bar{x}' = \bar{y}' = 0$$

$$I_{xy} = \bar{I}_{x'y'} + m\bar{x}\bar{y} \quad \text{Q.E.D.} \quad \blacktriangleleft$$

The expressions for I_{yz} and I_{zx} are obtained in a similar manner.



PROBLEM 9.162

For the homogeneous tetrahedron of mass m shown, (a) determine by direct integration the mass product of inertia I_{zx} , (b) deduce I_{yz} and I_{xy} from the result obtained in part a.

SOLUTION

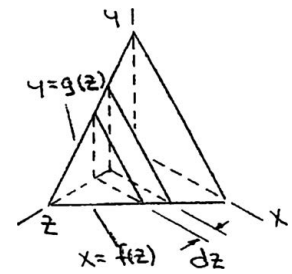
(a) First divide the tetrahedron into a series of thin vertical slices of thickness dz as shown.

Now
$$x = -\frac{a}{c}z + a = a\left(1 - \frac{z}{c}\right)$$

and
$$y = -\frac{b}{c}z + b = b\left(1 - \frac{z}{c}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho \left(\frac{1}{2} xy dz \right) = \frac{1}{2} \rho ab \left(1 - \frac{z}{c} \right)^2 dz$$



Then
$$m = \int dm = \int_0^c \frac{1}{2} \rho ab \left(1 - \frac{z}{c} \right)^2 dz$$

$$= \frac{1}{2} \rho ab \left[\left(-\frac{c}{3} \right) \left(1 - \frac{z}{c} \right)^3 \right]_0^c = \frac{1}{6} \rho abc$$

Now
$$dI_{zx} = d\bar{I}_{z'x'} + \bar{z}_{EL} \bar{x}_{EL} dm$$

where
$$d\bar{I}_{z'x'} = 0 \quad (\text{symmetry})$$

and
$$\bar{z}_{EL} = z \quad \bar{x}_{EL} = \frac{1}{3}x = \frac{1}{3}a\left(1 - \frac{z}{c}\right)$$

Then
$$I_{zx} = \int dI_{zx} = \int_0^c z \left[\frac{1}{3}a\left(1 - \frac{z}{c}\right) \right] \left[\frac{1}{2} \rho ab \left(1 - \frac{z}{c} \right)^2 dz \right]$$

$$= \frac{1}{6} \rho a^2 b \int_0^c \left(z - 3\frac{z^2}{c} + 3\frac{z^3}{c^2} - \frac{z^4}{c^3} \right) dz$$

$$= \frac{m}{c} a \left[\frac{1}{2} z^2 - \frac{z^3}{c} + \frac{3}{4} \frac{z^4}{c^2} - \frac{1}{5} \frac{z^5}{c^3} \right]_0^c$$

or
$$I_{zx} = \frac{1}{20} mac \quad \blacktriangleleft$$

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PROBLEM 9.162 (Continued)

- (b) Because of the symmetry of the body, I_{xy} and I_{yz} can be deduced by considering the circular permutation of (x, y, z) and (a, b, c) .

Thus,

$$I_{xy} = \frac{1}{20} mab \quad \blacktriangleleft$$

$$I_{yz} = \frac{1}{20} mbc \quad \blacktriangleleft$$

Alternative solution for part a:

First divide the tetrahedron into a series of thin horizontal slices of thickness dy as shown.

Now
$$x = -\frac{a}{b}y + a = a\left(1 - \frac{y}{b}\right)$$

and
$$z = -\frac{c}{b}y + c = c\left(1 - \frac{y}{b}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho \left(\frac{1}{2} xz dy \right) = \frac{1}{2} \rho ac \left(1 - \frac{y}{b} \right)^2 dy$$

Now
$$dI_{zx} = \rho t dI_{zx, \text{area}}$$

where
$$t = dy$$

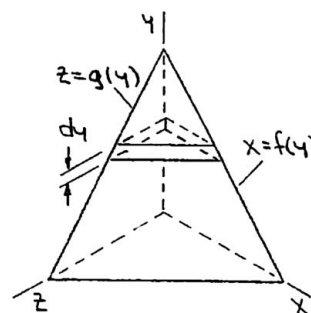
and
$$dI_{zx, \text{area}} = \frac{1}{24} x^2 z^2$$
 from the results of Sample Problem 9.6.

Then
$$dI_{zx} = \rho(dy) \left\{ \frac{1}{24} \left[a \left(1 - \frac{y}{b} \right)^2 \right] \left[c \left(1 - \frac{y}{b} \right) \right]^2 \right\}$$

$$= \frac{1}{24} \rho a^2 c^2 \left(1 - \frac{y}{b} \right)^4 dy = \frac{1}{4} \frac{m}{b} ac \left(1 - \frac{y}{b} \right)^4 dy$$

Finally,
$$I_{zx} = \int dI_{zx} = \int_0^b \frac{1}{4} \frac{m}{b} ac \left(1 - \frac{y}{b} \right)^4 dy$$

$$= \frac{1}{4} \frac{m}{b} ac \left[\left(-\frac{b}{5} \right) \left(1 - \frac{y}{b} \right)^5 \right]_0^b \quad \text{or} \quad I_{zx} = \frac{1}{20} mac \quad \blacktriangleleft$$



PROBLEM 9.162 (Continued)

Alternative solution for part *a*:

The equation of the included face of the tetrahedron is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

so that

$$y = b \left(1 - \frac{x}{a} - \frac{z}{c} \right)$$

For an infinitesimal element of sides dx , dy , and dz :

$$dm = \rho dV = \rho dy dx dz$$

From part *a*

$$x = a \left(1 - \frac{z}{c} \right)$$

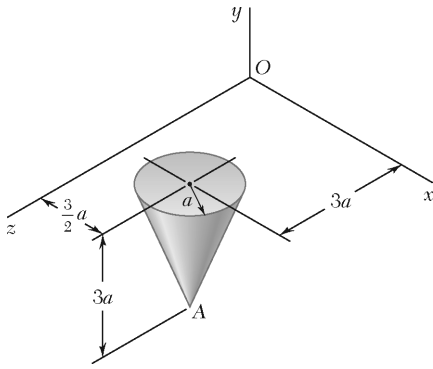
Now

$$\begin{aligned} I_{zx} &= \int z x dm = \int_0^c \int_0^{a(1-z/c)} \int_0^{b(1-x/a-z/c)} z x (\rho dy dx dz) \\ &= \rho \int_0^c \int_0^{a(1-z/c)} z x \left[b \left(1 - \frac{x}{a} - \frac{z}{c} \right) \right] dx dz \\ &= \rho b \int_0^c z \left[\frac{1}{2} x^2 - \frac{1}{3} \frac{x^3}{a} - \frac{1}{2} \frac{z}{c} x^2 \right]_0^{a(1-z/c)} dz \\ &= \rho b \int_0^c z \left[\frac{1}{2} a^2 \left(1 - \frac{z}{c} \right)^2 - \frac{1}{3a} a^3 \left(1 - \frac{z}{c} \right)^3 - \frac{1}{2} \frac{z}{c} a^2 \left(1 - \frac{z}{c} \right)^2 \right] dz \\ &= \rho b \int_0^c \frac{1}{6} a^2 z \left(1 - \frac{z}{c} \right)^3 dz \\ &= \frac{1}{6} \rho a^2 b \int_0^c \left(z - 3 \frac{z^2}{c} + 3 \frac{z^3}{c^2} - \frac{z^4}{c^3} \right) dz \\ &= \frac{m}{c} a \left[\frac{1}{2} z^2 - \frac{z^3}{c} + \frac{3}{4} \frac{z^4}{c^2} - \frac{1}{5} \frac{z^5}{c^3} \right]_0^c \end{aligned}$$

or $I_{zx} = \frac{1}{20} mac \blacktriangleleft$

PROBLEM 9.163

The homogeneous circular cone shown has a mass m . Determine the mass moment of inertia of the cone with respect to the line joining the origin O and Point A .



SOLUTION

First note that

$$d_{OA} = \sqrt{\left(\frac{3}{2}a\right)^2 + (-3a)^2 + (3a)^2} = \frac{9}{2}a$$

Then

$$\lambda_{OA} = \frac{1}{\frac{9}{2}a} \left(\frac{3}{2}a\mathbf{i} - 3a\mathbf{j} + 3a\mathbf{k} \right) = \frac{1}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

For a rectangular coordinate system with origin at Point A and axes aligned with the given x, y, z axes, we have (using Figure 9.28)

$$I_x = I_z = \frac{3}{5}m \left[\frac{1}{4}a^2 + (3a)^2 \right] \quad I_y = \frac{3}{10}ma^2$$

$$= \frac{111}{20}ma^2$$

Also, symmetry implies

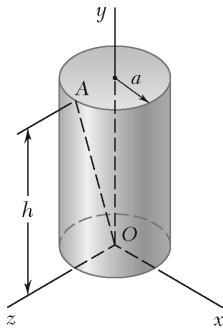
$$I_{xy} = I_{yz} = I_{zx} = 0$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2$$

$$= \frac{111}{20}ma^2 \left(\frac{1}{3}\right)^2 + \frac{3}{10}ma^2 \left(-\frac{2}{3}\right)^2 + \frac{111}{20}ma^2 \left(\frac{2}{3}\right)^2$$

$$= \frac{193}{60}ma^2 \quad \text{or } I_{OA} = 3.22ma^2 \blacktriangleleft$$



PROBLEM 9.164

The homogeneous circular cylinder shown has a mass m . Determine the mass moment of inertia of the cylinder with respect to the line joining the origin O and Point A that is located on the perimeter of the top surface of the cylinder.

SOLUTION

From Figure 9.28:

$$I_y = \frac{1}{2}ma^2$$

and using the parallel-axis theorem

$$I_x = I_z = \frac{1}{12}m(3a^2 + h^2) + m\left(\frac{h}{2}\right)^2 = \frac{1}{12}m(3a^2 + 4h^2)$$

Symmetry implies

$$I_{xy} = I_{yz} = I_{zx} = 0$$

For convenience, let Point A lie in the yz plane. Then

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}(h\mathbf{j} + a\mathbf{k})$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

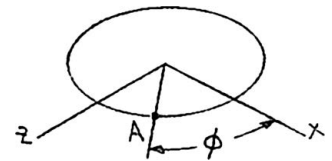
$$\begin{aligned} I_{OA} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 \\ &= \frac{1}{2}ma^2 \left(\frac{h}{\sqrt{h^2 + a^2}} \right)^2 + \frac{1}{12}m(3a^2 + 4h^2) \left(\frac{a}{\sqrt{h^2 + a^2}} \right)^2 \end{aligned}$$

$$\text{or } I_{OA} = \frac{1}{12}ma^2 \frac{10h^2 + 3a^2}{h^2 + a^2} \blacktriangleleft$$

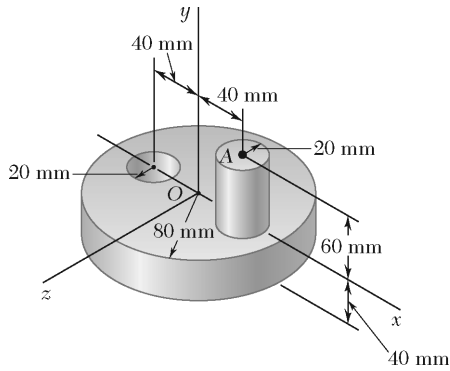
Note: For Point A located at an arbitrary point on the perimeter of the top surface, λ_{OA} is given by

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}(a \cos \phi \mathbf{i} + h\mathbf{j} + a \sin \phi \mathbf{k})$$

which results in the same expression for I_{OA} .

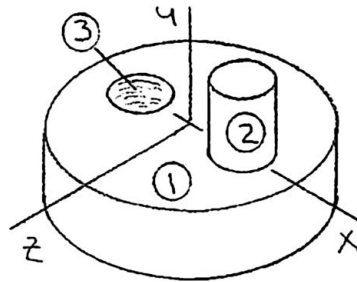


PROBLEM 9.165



Shown is the machine element of Problem 9.141. Determine its mass moment of inertia with respect to the line joining the origin O and Point A .

SOLUTION



First compute the mass of each component.

We have
$$m = \rho_{\text{ST}} V = \frac{0.284 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} V = (0.008819 \text{ lb} \cdot \text{s}^2 / \text{ft} \cdot \text{in}^3) V$$

Then

$$m_1 = (7850 \text{ kg/m}^3) [\pi (0.08 \text{ m})^2 (0.04 \text{ m})] = 6.31334 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3) [\pi (0.02 \text{ m})^2 (0.06 \text{ m})] = 0.59188 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3) [\pi (0.02 \text{ m})^2 (0.04 \text{ m})] = 0.39458 \text{ kg}$$

Symmetry implies

$$I_{yz} = I_{zx} = 0 \quad (I_{xy})_1 = 0$$

and

$$(\bar{I}_{x'y'})_2 = (\bar{I}_{x'y'})_3 = 0$$

Now

$$\begin{aligned} I_{xy} &= \Sigma (\bar{I}_{x'y'} + m\bar{x}\bar{y}) = m_2 \bar{x}_2 \bar{y}_2 - m_3 \bar{x}_3 \bar{y}_3 \\ &= [0.59188 \text{ kg} (0.04 \text{ m})(0.03 \text{ m})] - [0.39458 \text{ kg} (-0.04 \text{ m})(-0.02 \text{ m})] \\ &= (0.71026 - 0.31566) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ &= 0.39460 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

PROBLEM 9.165 (Continued)

From the solution to Problem 9.141, we have

$$I_x = 13.98800 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_y = 20.55783 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_z = 14.30368 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

By observation

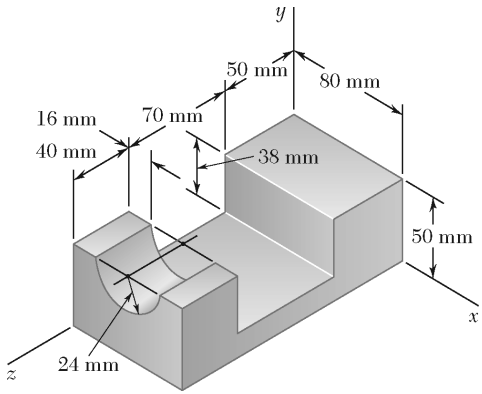
$$\lambda_{OA} = \frac{1}{\sqrt{13}}(2\mathbf{i} + 3\mathbf{j})$$

Substituting into Eq. (9.46)

$$\begin{aligned} I_{OA} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x \\ &= \left[(13.98800) \left(\frac{2}{\sqrt{13}} \right)^2 + (20.55783) \left(\frac{3}{\sqrt{13}} \right)^2 \right. \\ &\quad \left. - 2(0.39460) \left(\frac{2}{\sqrt{13}} \right) \left(\frac{2}{\sqrt{13}} \right) \right] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\ &= (4.30400 + 14.23234 - 0.36425) \times 10^{-3} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_{OA} = 18.17 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.166



Determine the mass moment of inertia of the steel fixture of Problems 9.145 and 9.149 with respect to the axis through the origin that forms equal angles with the x , y , and z axes.

SOLUTION

From the solutions to Problems 9.145 and 9.149, we have

Problem 9.145:

$$I_x = 26.4325 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_y = 31.1726 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_z = 8.5773 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Problem 9.149:

$$I_{xy} = 2.5002 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{yz} = 4.0627 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{zx} = 8.8062 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

From the problem statement it follows that

$$\lambda_x = \lambda_y = \lambda_z$$

Now

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \Rightarrow 3\lambda_x^2 = 1$$

or

$$\lambda_x = \lambda_y = \lambda_z = \frac{1}{\sqrt{3}}$$

Substituting into Eq. (9.46)

$$I_{OL} = I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x$$

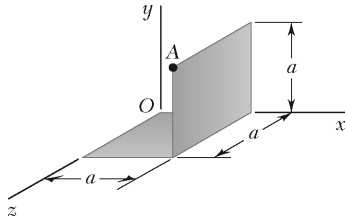
Noting that

$$\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x \lambda_y = \lambda_y \lambda_z = \lambda_z \lambda_x = \frac{1}{3}$$

We have

$$I_{OL} = \frac{1}{3} [26.4325 + 31.1726 + 8.5773 - 2(2.5002 + 4.0627 + 8.8062)] \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\text{or } I_{OL} = 11.81 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$



PROBLEM 9.167

The thin bent plate shown is of uniform density and weight W . Determine its mass moment of inertia with respect to the line joining the origin O and Point A .

SOLUTION

First note that

$$m_1 = m_2 = \frac{1}{2} \frac{W}{g}$$

and that

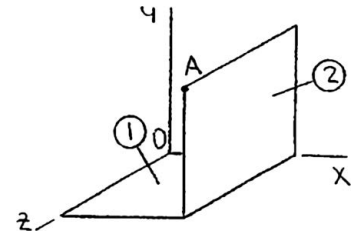
$$\lambda_{OA} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned} I_x &= (I_x)_1 + (I_x)_2 \\ &= \left[\frac{1}{12} \left(\frac{1}{2} \frac{W}{g} \right) a^2 + \frac{1}{2} \frac{W}{g} \left(\frac{a}{2} \right)^2 \right] \\ &\quad + \left\{ \frac{1}{12} \left(\frac{1}{2} \frac{W}{g} \right) (a^2 + a^2) + \frac{1}{2} \frac{W}{g} \left[\left(\frac{a}{2} \right)^2 + \left(\frac{a}{2} \right)^2 \right] \right\} \\ &= \frac{1}{2} \frac{W}{g} \left[\left(\frac{1}{12} + \frac{1}{4} \right) a^2 + \left(\frac{1}{6} + \frac{1}{2} \right) a^2 \right] = \frac{1}{2} \frac{W}{g} a^2 \end{aligned}$$

$$\begin{aligned} I_y &= (I_y)_1 + (I_y)_2 \\ &= \left\{ \frac{1}{12} \left(\frac{1}{2} \frac{W}{g} \right) (a^2 + a^2) + \frac{1}{2} \frac{W}{g} \left[\left(\frac{a}{2} \right)^2 + \left(\frac{a}{2} \right)^2 \right] \right\} \\ &\quad + \left\{ \frac{1}{12} \left(\frac{1}{2} \frac{W}{g} \right) a^2 + \frac{1}{2} \frac{W}{g} \left[(a)^2 + \left(\frac{a}{2} \right)^2 \right] \right\} \\ &= \frac{1}{2} \frac{W}{g} \left[\left(\frac{1}{6} + \frac{1}{2} \right) a^2 + \left(\frac{1}{12} + \frac{5}{4} \right) a^2 \right] = \frac{W}{g} a^2 \end{aligned}$$

$$\begin{aligned} I_z &= (I_z)_1 + (I_z)_2 \\ &= \left[\frac{1}{12} \left(\frac{1}{2} \frac{W}{g} \right) a^2 + \frac{1}{2} \frac{W}{g} \left(\frac{a}{2} \right)^2 \right] \\ &\quad + \left\{ \frac{1}{12} \left(\frac{1}{2} \frac{W}{g} \right) a^2 + \frac{1}{2} \frac{W}{g} \left[(a)^2 + \left(\frac{a}{2} \right)^2 \right] \right\} \\ &= \frac{1}{2} \frac{W}{g} \left[\left(\frac{1}{12} + \frac{1}{4} \right) a^2 + \left(\frac{1}{12} + \frac{5}{4} \right) a^2 \right] = \frac{5}{6} \frac{W}{g} a^2 \end{aligned}$$



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PROBLEM 9.167 (Continued)

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of both components are zero because of symmetry. Also, $\bar{y}_1 = 0$

Then

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\bar{y}) = m_2\bar{x}_2\bar{y}_2 = \frac{1}{2} \frac{W}{g} (a) \left(\frac{a}{2} \right) = \frac{1}{4} \frac{W}{g} a^2$$

$$I_{yz} = \Sigma(\bar{I}_{y'z'} + m\bar{y}\bar{z}) = m_2\bar{y}_2\bar{z}_2 = \frac{1}{2} \frac{W}{g} \left(\frac{a}{2} \right) \left(\frac{a}{2} \right) = \frac{1}{8} \frac{W}{g} a^2$$

$$I_{zx} = \Sigma(\bar{I}_{z'x'} + m\bar{z}\bar{x}) = m_1\bar{z}_1\bar{x}_1 + m_2\bar{z}_2\bar{x}_2$$

$$= \frac{1}{2} \frac{W}{g} \left(\frac{a}{2} \right) \left(\frac{a}{2} \right) + \frac{1}{2} \frac{W}{g} \left(\frac{a}{2} \right) (a) = \frac{3}{8} \frac{W}{g} a^2$$

Substituting into Equation (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

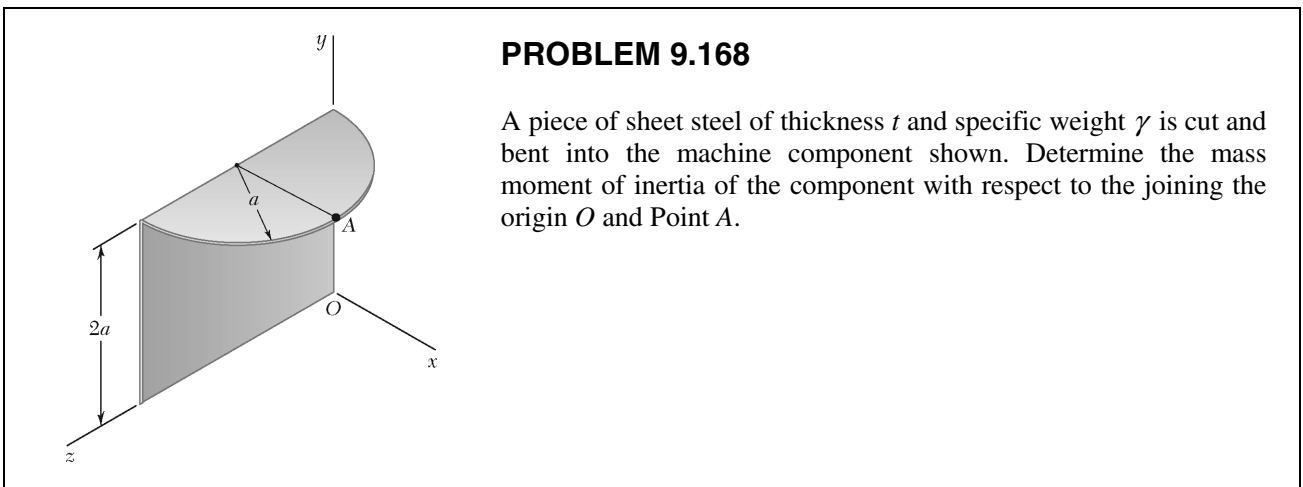
Noting that

$$\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \frac{1}{3}$$

We have

$$I_{OA} = \frac{1}{3} \left[\frac{1}{2} \frac{W}{g} a^2 + \frac{W}{g} a^2 + \frac{5}{6} \frac{W}{g} a^2 - 2 \left(\frac{1}{4} \frac{W}{g} a^2 + \frac{1}{8} \frac{W}{g} a^2 + \frac{3}{8} \frac{W}{g} a^2 \right) \right]$$

$$= \frac{1}{3} \left[\frac{14}{6} - 2 \left(\frac{3}{4} \right) \right] \frac{W}{g} a^2 \qquad \text{or} \qquad I_{OA} = \frac{5}{18} \frac{W}{g} a^2 \quad \blacktriangleleft$$



PROBLEM 9.168

A piece of sheet steel of thickness t and specific weight γ is cut and bent into the machine component shown. Determine the mass moment of inertia of the component with respect to the joining the origin O and Point A .

SOLUTION

First note that

$$\lambda_{OA} = \frac{1}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

Next compute the mass of each component. We have

$$m = \rho V = \frac{\gamma}{g}(tA)$$

Then

$$m_1 = \frac{\gamma}{g}t(2a \times 2a) = 4\frac{\gamma t}{g}a^2$$

$$m_2 = \frac{\gamma}{g}t\left(\frac{\pi}{2} \times a^2\right) = \frac{\pi}{2}\frac{\gamma t}{g}a^2$$

Using Figure 9.28 for component 1 and the equations derived above (following the solution to Problem 9.134) for a semicircular plate for component 2, we have

$$\begin{aligned}
 I_x &= (I_x)_1 + (I_x)_2 \\
 &= \left\{ \frac{1}{12} \left(4\frac{\gamma t}{g}a^2 \right) [(2a)^2 + (2a)^2] + 4\frac{\gamma t}{g}a^2(a^2 + a^2) \right\} \\
 &\quad + \left\{ \frac{1}{4} \left(\frac{\pi}{2}\frac{\gamma t}{g}a^2 \right) a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2[(2a)^2 + (a)^2] \right\} \\
 &= 4\frac{\gamma t}{g}a^2 \left(\frac{2}{3} + 2 \right) a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2 \left(\frac{1}{4} + 5 \right) a^2 \\
 &= 18.91335 \frac{\gamma t}{g} a^4
 \end{aligned}$$

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PROBLEM 9.168 (Continued)

$$\begin{aligned}
 I_y &= (I_y)_1 + (I_y)_2 \\
 &= \left[\frac{1}{12} \left(4 \frac{\gamma t}{g} a^2 \right) (2a)^2 + 4 \frac{\gamma t}{g} a^2 (a^2) \right] \\
 &\quad + \left\{ \frac{\pi \gamma t}{2 g} a^2 \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) a^2 + \frac{\pi \gamma t}{2 g} a^2 \left[\left(\frac{4a}{3\pi} \right)^2 + (a)^2 \right] \right\} \\
 &= 4 \frac{\gamma t}{g} a^2 \left(\frac{1}{3} + 1 \right) a^2 + \frac{\pi \gamma t}{2 g} a^2 \left(\frac{1}{2} - \frac{16}{9\pi^2} + \frac{16}{9\pi^2} + 1 \right) a^2 \\
 &= 7.68953 \frac{\gamma t}{g} a^4
 \end{aligned}$$

$$\begin{aligned}
 I_z &= (I_z)_1 + (I_z)_2 \\
 &= \left[\frac{1}{12} \left(4 \frac{\gamma t}{g} a^2 \right) (2a)^2 + 4 \frac{\gamma t}{g} a^2 (a^2) \right] \\
 &\quad + \left\{ \frac{\pi \gamma t}{2 g} a^2 \left(\frac{1}{4} - \frac{16}{9\pi^2} \right) a^2 + \frac{\pi \gamma t}{2 g} a^2 \left[\left(\frac{4a}{3\pi} \right)^2 + (2a)^2 \right] \right\} \\
 &= 4 \frac{\gamma t}{g} a^2 \left(\frac{1}{3} + 1 \right) a^2 + \frac{\pi \gamma t}{2 g} a^2 \left(\frac{1}{4} - \frac{16}{9\pi^2} + \frac{16}{9\pi^2} + 4 \right) a^2 \\
 &= 12.00922 \frac{\gamma t}{g} a^4
 \end{aligned}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of both components are zero because of symmetry. Also $\bar{x}_1 = 0$.

Then

$$\begin{aligned}
 I_{xy} &= \Sigma (\bar{I}_{x'y'} + m\bar{x}\bar{y}) = m_2 \bar{x}_2 \bar{y}_2 = \frac{\pi \gamma t}{2 g} a^2 \left(\frac{4a}{3\pi} \right) (2a) \\
 &= 1.33333 \frac{\gamma t}{g} a^4
 \end{aligned}$$

$$\begin{aligned}
 I_{yz} &= \Sigma (\bar{I}_{y'z'} + m\bar{y}\bar{z}) = m_1 \bar{y}_1 \bar{z}_1 + m_2 \bar{y}_2 \bar{z}_2 \\
 &= 4 \frac{\gamma t}{g} a^2 (a)(a) + \frac{\pi \gamma t}{2 g} a^2 (2a)(a) \\
 &= 7.14159 \frac{\gamma t}{g} a^4
 \end{aligned}$$

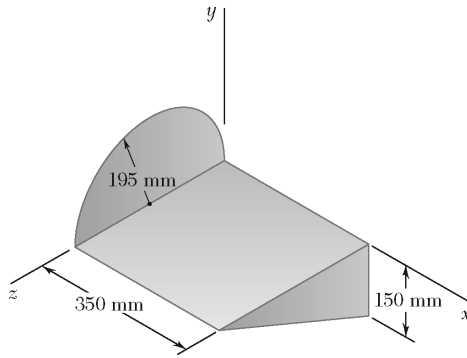
$$\begin{aligned}
 I_{zx} &= \Sigma (\bar{I}_{z'x'} + m\bar{z}\bar{x}) = m_2 \bar{z}_2 \bar{x}_2 = \frac{\pi \gamma t}{2 g} a^2 (a) \left(\frac{4a}{3\pi} \right) \\
 &= 0.66667 \frac{\gamma t}{g} a^4
 \end{aligned}$$

PROBLEM 9.168 (Continued)

Substituting into Eq. (9.46)

$$\begin{aligned}
I_{OA} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x \\
&= 18.91335 \frac{\gamma t}{g} a^4 \left(\frac{1}{\sqrt{6}} \right)^2 + 7.68953 \frac{\gamma t}{g} a^4 \left(\frac{2}{\sqrt{6}} \right)^2 \\
&\quad + 12.00922 \frac{\gamma t}{g} a^4 \left(\frac{1}{\sqrt{6}} \right)^2 - 2 \left(1.33333 \frac{\gamma t}{g} a^4 \right) \left(\frac{1}{\sqrt{6}} \right) \left(\frac{2}{\sqrt{6}} \right) \\
&\quad - 2 \left(7.14159 \frac{\gamma t}{g} a^4 \right) \left(\frac{2}{\sqrt{6}} \right) \left(\frac{2}{\sqrt{6}} \right) - 2 \left(0.66667 \frac{\gamma t}{g} a^4 \right) \left(\frac{1}{\sqrt{6}} \right) \left(\frac{1}{\sqrt{6}} \right) \\
&= (3.15223 + 5.12635 + 2.00154 - 0.88889 - 4.76106 - 0.22222) \frac{\gamma t}{g} a^4
\end{aligned}$$

$$\text{or } I_{OA} = 4.41 \frac{\gamma t}{g} a^4 \blacktriangleleft$$



PROBLEM 9.169

Determine the mass moment of inertia of the machine component of Problems 9.136 and 9.155 with respect to the axis through the origin characterized by the unit vector $\lambda = (-4\mathbf{i} + 8\mathbf{j} + \mathbf{k})/9$.

SOLUTION

From the solutions to Problems 9.136 and 9.155. We have

Problem 9.136:

$$I_x = 175.503 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_y = 308.629 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_z = 154.400 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Problem 9.155:

$$I_{xy} = -8.0365 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{yz} = 12.8950 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_{zx} = 94.0266 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

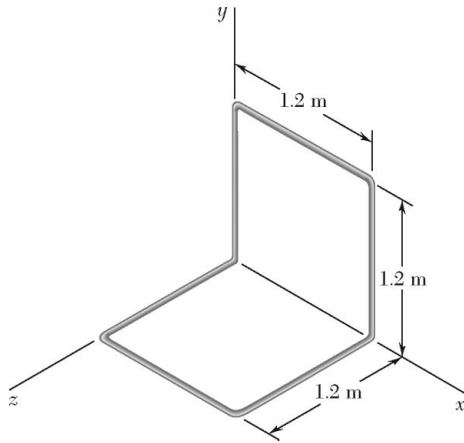
Substituting into Eq. (9.46)

$$\begin{aligned}
 I_{OL} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x \\
 &= \left[175.503 \left(-\frac{4}{9} \right)^2 + 308.629 \left(\frac{8}{9} \right)^2 + 154.400 \left(\frac{1}{9} \right)^2 \right. \\
 &\quad \left. - 2(-8.0365) \left(-\frac{4}{9} \right) \left(\frac{8}{9} \right) - 2(12.8950) \left(\frac{8}{9} \right) \left(\frac{1}{9} \right) \right. \\
 &\quad \left. - 2(94.0266) \left(\frac{1}{9} \right) \left(-\frac{4}{9} \right) \right] \times 10^{-3} \text{ kg} \cdot \text{m}^2 \\
 &= (34.6673 + 243.855 + 1.906 - 6.350 \\
 &\quad - 2.547 + 9.287) \times 10^{-3} \text{ kg} \cdot \text{m}^2
 \end{aligned}$$

$$\text{or } I_{OL} = 281 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.170

For the wire figure of Problem 9.148, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\lambda = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.



SOLUTION

First compute the mass of each component. We have

$$m = \left(\frac{m}{L}\right)L = 0.056 \text{ kg/m} \times 1.2 \text{ m} \\ = 0.0672 \text{ kg}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, for each component are zero because of symmetry.

Also

$$\bar{x}_1 = \bar{x}_6 = 0 \quad \bar{y}_4 = \bar{y}_5 = \bar{y}_6 = 0 \quad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$$

Then

$$I_{xy} = \Sigma(\bar{J}_{x'y'}^0 + m\bar{x}\bar{y}) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3 \\ = (0.0672 \text{ kg})(0.6 \text{ m})(1.2 \text{ m}) + (0.0672 \text{ kg})(1.2 \text{ m})(0.6 \text{ m}) \\ = 0.096768 \text{ kg} \cdot \text{m}^2$$

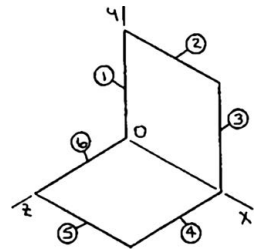
$$I_{yz} = \Sigma(\bar{J}_{y'z'}^0 + m\bar{y}\bar{z}) = 0$$

$$I_{zx} = \Sigma(\bar{J}_{z'x'}^0 + m\bar{z}\bar{x}) = m_4\bar{z}_4\bar{x}_4 + m_5\bar{z}_5\bar{x}_5 \\ = (0.0672 \text{ kg})(0.6 \text{ m})(1.2 \text{ m}) + (0.0672 \text{ kg})(1.2 \text{ m})(0.6 \text{ m}) \\ = 0.096768 \text{ kg} \cdot \text{m}^2$$

From the solution to Problem 9.148, we have

$$I_x = 0.32258 \text{ kg} \cdot \text{m}^2$$

$$I_y = I_z = 0.41933 \text{ kg} \cdot \text{m}^2$$



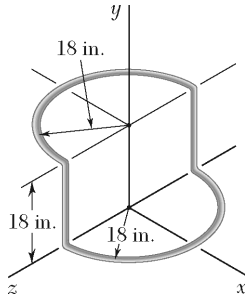
PROBLEM 9.170 (Continued)

Substituting into Eq. (9.46)

$$\begin{aligned} I_{OL} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x \\ &= \left[0.32258 \left(-\frac{3}{7} \right)^2 + 0.41933 \left(-\frac{6}{7} \right)^2 + 0.41933 \left(\frac{2}{7} \right)^2 \right. \\ &\quad \left. - 2(0.096768) \left(-\frac{3}{7} \right) \left(-\frac{6}{7} \right) - 2(0.096768) \left(\frac{2}{7} \right) \left(-\frac{3}{7} \right) \right] \text{kg} \cdot \text{m}^2 \end{aligned}$$

$$I_{OL} = (0.059249 + 0.30808 + 0.034231 - 0.071095 + 0.023698) \text{kg} \cdot \text{m}^2$$

$$\text{or } I_{OL} = 0.354 \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$



PROBLEM 9.171

For the wire figure of Problem 9.147, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\lambda = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION

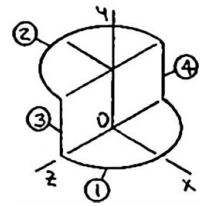
First compute the mass of each component. We have

$$m = \rho_{ST} V = \frac{\gamma_{ST}}{g} AL$$

Then

$$\begin{aligned} m_1 = m_2 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4} \left(\frac{1}{8} \text{ in.} \right)^2 \right] \times (\pi \times 18 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$

$$\begin{aligned} m_3 = m_4 &= \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4} \left(\frac{1}{8} \text{ in.} \right)^2 \right] \times 18 \text{ in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3 \\ &= 1.9453 \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$



Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, for each component are zero because of symmetry.

$$\text{Also} \quad \bar{x}_3 = \bar{x}_4 = 0 \quad \bar{y}_1 = 0 \quad \bar{z}_1 = \bar{z}_2 = 0$$

Then

$$\begin{aligned} I_{xy} &= \Sigma (\bar{J}_{x'y'}^0 + m\bar{x}\bar{y}) = m_2 \bar{x}_2 \bar{y}_2 \\ &= (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}) \left(-\frac{2 \times 18}{\pi} \text{ in.} \right) (18 \text{ in.}) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^2 \\ &= -8.75480 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

$$I_{yz} = \Sigma (\bar{J}_{y'z'}^0 + m\bar{y}\bar{z}) = m_3 \bar{y}_3 \bar{z}_3 + m_4 \bar{y}_4 \bar{z}_4$$

Now

$$m_3 = m_4, \quad \bar{y}_3 = \bar{y}_4, \quad \bar{z}_4 = -\bar{z}_3 \quad I_{yz} = 0$$

$$I_{zx} = \Sigma (\bar{J}_{z'x'}^0 + m\bar{z}\bar{x}) \quad \text{or} \quad I_{zx} = 0$$

PROBLEM 9.171 (Continued)

From the solution to Problem 9.147, we have

$$I_x = 39.1721 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

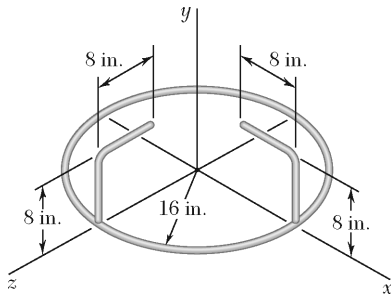
$$I_y = 36.2542 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$I_z = 30.4184 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Substituting into Eq. (9.46)

$$\begin{aligned} I_{OL} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x \\ &= \left[39.1721 \left(-\frac{3}{7} \right)^2 + 36.2542 \left(-\frac{6}{7} \right)^2 + 30.4184 \left(\frac{2}{7} \right)^2 \right. \\ &\quad \left. - 2(-8.75480) \left(-\frac{3}{7} \right) \left(-\frac{6}{7} \right) \right] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= (7.19488 + 26.6357 + 2.48313 + 6.43210) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

$$\text{or } I_{OL} = 0.0427 \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \blacktriangleleft$$



PROBLEM 9.172

For the wire figure of Problem 9.146, determine the mass moment of inertia of the figure with respect to the z axis through the origin characterized by the unit vector $\lambda = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION

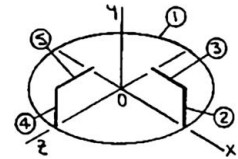
First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g} (W/L)_{AL} L$$

Then

$$\begin{aligned} m_1 &= \frac{1}{32.2 \text{ ft/s}^2} (0.033 \text{ lb/ft})(2\pi \times 16 \text{ in.}) \times \frac{1 \text{ ft}}{12 \text{ in.}} \\ &= 8.5857 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft} \end{aligned}$$

$$\begin{aligned} m_2 = m_3 = m_4 = m_5 &= \frac{1}{32.2 \text{ ft/s}^2} (0.033 \text{ lb/ft})(8 \text{ in.}) \times \frac{1 \text{ ft}}{12 \text{ in.}} \\ &= 0.6832 \times 10^{-3} \text{ lb} \cdot \text{ft/s}^2 \end{aligned}$$



Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Also

$$\bar{x}_1 = \bar{x}_4 = \bar{x}_5 = 0 \quad \bar{y}_1 = 0 \quad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$$

Then

$$\begin{aligned} I_{xy} &= \Sigma(\bar{J}_{x'y'} + m\bar{x}\bar{y}) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3 \\ &= 0.6832 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft} \left(\frac{16}{12} \text{ ft} \right) \left(\frac{4}{12} \text{ ft} \right) \\ &\quad + 0.6832 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft} \left(\frac{12}{12} \text{ ft} \right) \left(\frac{8}{12} \text{ ft} \right) \\ &= (0.30364 + 0.45547) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= 0.75911 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \end{aligned}$$

Symmetry implies

$$I_{yz} = I_{xy} \quad I_{yz} = 0.75911 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$I_{zx} = \Sigma(\bar{J}_{z'x'} + m\bar{z}\bar{x}) = 0$$

PROBLEM 9.172 (Continued)

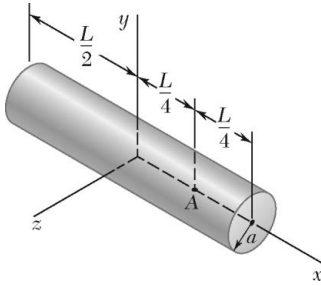
From the solution to Problem 9.146, we have

$$I_x = I_z = 10.3642 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$I_y = 19.1097 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Substituting into Eq. (9.46)

$$\begin{aligned} I_{OL} &= I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy} \lambda_x \lambda_y - 2I_{yz} \lambda_y \lambda_z - 2I_{zx} \lambda_z \lambda_x \\ &= \left[10.3642 \left(-\frac{3}{7} \right)^2 + 19.1097 \left(-\frac{6}{7} \right)^2 + 10.3642 \left(\frac{2}{7} \right)^2 \right. \\ &\quad \left. - 2(0.75911) \left(-\frac{3}{7} \right) \left(-\frac{6}{7} \right) - 2(0.75911) \left(-\frac{6}{7} \right) \left(\frac{2}{7} \right) \right] \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &= (1.90663 + 14.03978 + 0.84606 - 0.55771 + 0.37181) \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \\ &\quad \text{or } I_{OL} = 16.61 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2 \quad \blacktriangleleft \end{aligned}$$



PROBLEM 9.173

For the homogeneous circular cylinder shown, of radius a and length L , determine the value of the ratio a/L for which the ellipsoid of inertia of the cylinder is a sphere when computed (a) at the centroid of the cylinder, (b) at Point A.

SOLUTION

(a) From Figure 9.28:

$$\bar{I}_x = \frac{1}{2}ma^2$$

$$\bar{I}_y = \bar{I}_z = \frac{1}{12}m(3a^2 + L^2)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \quad \frac{1}{2}ma^2 x^2 + \frac{1}{12}m(3a^2 + L^2)y^2 + \frac{1}{12}m(3a^2 + L^2)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore

$$\frac{1}{2}ma^2 = \frac{1}{12}m(3a^2 + L^2) \quad \text{or} \quad \frac{a}{L} = \frac{1}{\sqrt{3}} \blacktriangleleft$$

(b) Using Figure 9.28 and the parallel-axis theorem, we have

$$I_{x'} = \frac{1}{2}ma^2$$

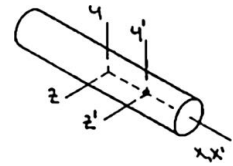
$$I_{y'} = I_{z'} = \frac{1}{12}m(3a^2 + L^2) + m\left(\frac{L}{4}\right)^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right)$$

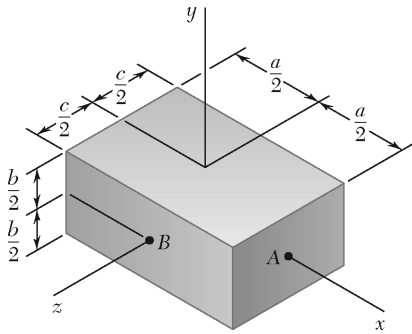
Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part a it then immediately follows that

$$\frac{1}{2}ma^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right) \quad \text{or} \quad \frac{a}{L} = \sqrt{\frac{7}{12}} \blacktriangleleft$$





PROBLEM 9.174

For the rectangular prism shown, determine the values of the ratios b/a and c/a so that the ellipsoid of inertia of the prism is a sphere when computed (a) at Point A, (b) at Point B.

SOLUTION

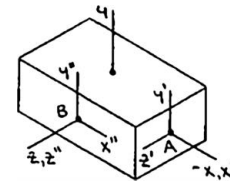
- (a) Using Figure 9.28 and the parallel-axis theorem, we have at Point A

$$I_{x'} = \frac{1}{12}m(b^2 + c^2)$$

$$I_{y'} = \frac{1}{12}m(a^2 + c^2) + m\left(\frac{a}{2}\right)^2$$

$$= \frac{1}{12}m(4a^2 + c^2)$$

$$I_{z'} = \frac{1}{12}m(a^2 + b^2) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m(4a^2 + b^2)$$



Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_{x'}x^2 + I_{y'}y^2 + I_{z'}z^2 = 1: \quad \frac{1}{12}m(b^2 + c^2)x^2 + \frac{1}{12}m(4a^2 + c^2)y^2 + \frac{1}{12}m(4a^2 + b^2)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore

$$\frac{1}{12}m(b^2 + c^2) = \frac{1}{12}m(4a^2 + c^2)$$

$$= \frac{1}{12}m(4a^2 + b^2)$$

Then $b^2 + c^2 = 4a^2 + c^2$ or $\frac{b}{a} = 2 \blacktriangleleft$

and $b^2 + c^2 = 4a^2 + b^2$ or $\frac{c}{a} = 2 \blacktriangleleft$

PROBLEM 9.174 (Continued)

(b) Using Figure 9.28 and the parallel-axis theorem, we have at Point B

$$I_{x'} = \frac{1}{12}m(b^2 + c^2) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m(b^2 + 4c^2)$$

$$I_{y'} = \frac{1}{12}m(a^2 + c^2) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m(a^2 + 4c^2)$$

$$I_{z'} = \frac{1}{12}m(a^2 + b^2)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From part a it then immediately follows that

$$\frac{1}{12}m(b^2 + 4c^2) = \frac{1}{12}m(a^2 + 4c^2) = \frac{1}{12}m(a^2 + b^2)$$

Then

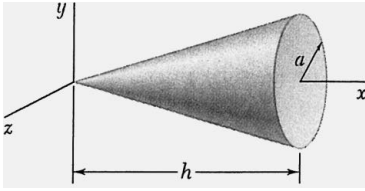
$$b^2 + 4c^2 = a^2 + 4c^2$$

or $\frac{b}{a} = 1 \blacktriangleleft$

and

$$b^2 + 4c^2 = a^2 + b^2$$

or $\frac{c}{a} = \frac{1}{2} \blacktriangleleft$



PROBLEM 9.175

For the right circular cone of Sample Problem 9.11, determine the value of the ratio a/h for which the ellipsoid of inertia of the cone is a sphere when computed (a) at the apex of the cone, (b) at the center of the base of the cone.

SOLUTION

(a) From Sample Problem 9.11, we have at the apex A

$$I_x = \frac{3}{10} ma^2$$

$$I_y = I_z = \frac{3}{5} m \left(\frac{1}{4} a^2 + h^2 \right)$$

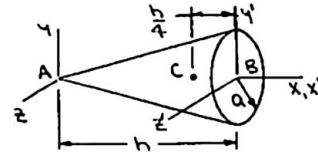
Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \quad \frac{3}{10} ma^2 x^2 + \frac{3}{5} m \left(\frac{1}{4} a^2 + h^2 \right) y^2 + \frac{3}{5} m \left(\frac{1}{4} a^2 + h^2 \right) z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{3}{10} ma^2 = \frac{3}{5} m \left(\frac{1}{4} a^2 + h^2 \right) \quad \text{or} \quad \frac{a}{h} = 2 \quad \blacktriangleleft$$



(b) From Sample Problem 9.11, we have

$$I_{x'} = \frac{3}{10} ma^2$$

and at the centroid C

$$I_{y''} = \frac{3}{20} m \left(a^2 + \frac{1}{4} h^2 \right)$$

Then
$$I_{y'} = I_{z'} = \frac{3}{20} m \left(a^2 + \frac{1}{4} h^2 \right) + m \left(\frac{h}{4} \right)^2 = \frac{1}{20} m (3a^2 + 2h^2)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part a it then immediately follows that

$$\frac{3}{10} ma^2 = \frac{1}{20} m (3a^2 + 2h^2) \quad \text{or} \quad \frac{a}{h} = \sqrt{\frac{2}{3}} \quad \blacktriangleleft$$

PROBLEM 9.176

Given an arbitrary body and three rectangular axes x , y , and z , prove that the mass moment of inertia of the body with respect to any one of the three axes cannot be larger than the sum of the mass moments of inertia of the body with respect to the other two axes. That is, prove that the inequality $I_x \leq I_y + I_z$ and the two similar inequalities are satisfied. Further, prove that $I_y \geq \frac{1}{2}I_x$ if the body is a homogeneous solid of revolution, where x is the axis of revolution and y is a transverse axis.

SOLUTION

(i) To prove $I_y + I_z \geq I_x$

By definition
$$I_y = \int (z^2 + x^2) dm$$

$$I_z = \int (x^2 + y^2) dm$$

Then
$$\begin{aligned} I_y + I_z &= \int (z^2 + x^2) dm + \int (x^2 + y^2) dm \\ &= \int (y^2 + z^2) dm + 2 \int x^2 dm \end{aligned}$$

Now
$$\int (y^2 + z^2) dm = I_x \quad \text{and} \quad \int x^2 dm \geq 0$$

$$I_y + I_z \geq I_x \quad \text{Q.E.D.}$$

The proofs of the other two inequalities follow similar steps.

(ii) If the x axis is the axis of revolution, then

$$I_y = I_z$$

and from part (i)
$$I_y + I_z \geq I_x$$

or
$$2I_y \geq I_x$$

or
$$I_y \geq \frac{1}{2}I_x \quad \text{Q.E.D.}$$

PROBLEM 9.177

Consider a cube of mass m and side a . (a) Show that the ellipsoid of inertia at the center of the cube is a sphere, and use this property to determine the moment of inertia of the cube with respect to one of its diagonals. (b) Show that the ellipsoid of inertia at one of the corners of the cube is an ellipsoid of revolution, and determine the principal moments of inertia of the cube at that point.

SOLUTION

(a) At the center of the cube have (using Figure 9.28)

$$I_x = I_y = I_z = \frac{1}{12}m(a^2 + a^2) = \frac{1}{6}ma^2$$

Now observe that symmetry implies

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is

$$\left(\frac{1}{6}ma^2\right)x^2 + \left(\frac{1}{6}ma^2\right)y^2 + \left(\frac{1}{6}ma^2\right)z^2 = 1$$

$$\text{or } x^2 + y^2 + z^2 = \frac{6}{ma^2} (= R^2) \quad \blacktriangleleft$$

which is the equation of a sphere.

Since the ellipsoid of inertia is a sphere, the moment of inertia with respect to any axis OL through the center O of the cube must always

$$\text{be the same } \left(R = \frac{1}{\sqrt{I_{OL}}}\right).$$

$$I_{OL} = \frac{1}{6}ma^2 \quad \blacktriangleleft$$

(b) The above sketch of the cube is the view seen if the line of sight is along the diagonal that passes through corner A . For a rectangular coordinate system at A and with one of the coordinate axes aligned with the diagonal, an ellipsoid of inertia at A could be constructed. If the cube is then rotated 120° about the diagonal, the mass distribution will remain unchanged. Thus, the ellipsoid will also remain unchanged after it is rotated. As noted at the end of Section 9.17, this is possible only if the ellipsoid is an ellipsoid of revolution, where the diagonal is both the axis of revolution and a principal axis.

It then follows that

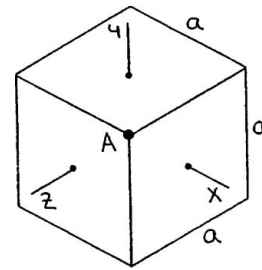
$$I_x = I_{OL} = \frac{1}{6}ma^2 \quad \blacktriangleleft$$

In addition, for an ellipsoid of revolution, the two transverse principal moments of inertia are equal and any axis perpendicular to the axis of revolution is a principal axis. Then, applying the parallel-axis theorem between the center of the cube and corner A for any perpendicular axis

$$I_{y'} = I_{z'} = \frac{1}{6}ma^2 + m\left(\frac{\sqrt{3}}{2}a\right)^2$$

$$\text{or } I_{y'} = I_{z'} = \frac{11}{12}ma^2 \quad \blacktriangleleft$$

(Note: Part b can also be solved using the method of Section 9.18.)



PROBLEM 9.177 (Continued)

First note that at corner A

$$I_x = I_y = I_z = \frac{2}{3}ma^2$$

$$I_{xy} = I_{yz} = I_{zx} = \frac{1}{4}ma^2$$

Substituting into Equation (9.56) yields

$$k^3 - 2ma^2k^2 + \frac{55}{48}m^2a^6k - \frac{121}{864}m^3a^9 = 0$$

For which the roots are

$$k_1 = \frac{1}{6}ma^2$$

$$k_2 = k_3 = \frac{11}{12}ma^2$$

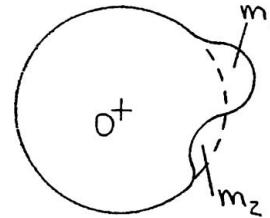
PROBLEM 9.178

Given a homogeneous body of mass m and of arbitrary shape and three rectangular axes x , y , and z with origin at O , prove that the sum $I_x + I_y + I_z$ of the mass moments of inertia of the body cannot be smaller than the similar sum computed for a sphere of the same mass and the same material centered at O . Further, using the result of Problem 9.176, prove that if the body is a solid of revolution, where x is the axis of revolution, its mass moment of inertia I_y about a transverse axis y cannot be smaller than $3ma^2/10$, where a is the radius of the sphere of the same mass and the same material.

SOLUTION

(i) Using Equation (9.30), we have

$$\begin{aligned} I_x + I_y + I_z &= \int (y^2 + z^2) dm + \int (z^2 + x^2) dm + \int (x^2 + y^2) dm \\ &= 2 \int (x^2 + y^2 + z^2) dm \\ &= 2 \int r^2 dm \end{aligned}$$



where r is the distance from the origin O to the element of mass dm . Now assume that the given body can be formed by adding and subtracting appropriate volumes V_1 and V_2 from a sphere of mass m and radius a which is centered at O ; it then follows that

$$m_1 = m_2 \quad (m_{\text{body}} = m_{\text{sphere}} = m)$$

Then

$$\begin{aligned} (I_x + I_y + I_z)_{\text{body}} &= (I_x + I_y + I_z)_{\text{sphere}} + (I_x + I_y + I_z)_{V_1} \\ &\quad - (I_x + I_y + I_z)_{V_2} \end{aligned}$$

or

$$(I_x + I_y + I_z)_{\text{body}} = (I_x + I_y + I_z)_{\text{sphere}} + 2 \int_{m_1} r^2 dm - 2 \int_{m_2} r^2 dm$$

Now, $m_1 = m_2$ and $r_1 \geq r_2$ for all elements of mass dm in volumes 1 and 2.

$$\int_{m_1} r^2 dm - \int_{m_2} r^2 dm \geq 0$$

so that

$$(I_x + I_y + I_z)_{\text{body}} \geq (I_x + I_y + I_z)_{\text{sphere}} \quad \text{Q.E.D.}$$

PROBLEM 9.178 (Continued)

(ii) First note from Figure 9.28 that for a sphere

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$

Thus $(I_x + I_y + I_z)_{\text{sphere}} = \frac{6}{5}ma^2$

For a solid of revolution, where the x axis is the axis of revolution, we have

$$I_y = I_z$$

Then, using the results of part (i)

$$(I_x + 2I_y)_{\text{body}} \geq \frac{6}{5}ma^2$$

From Problem 9.178 we have $I_y \geq \frac{1}{2}I_x$

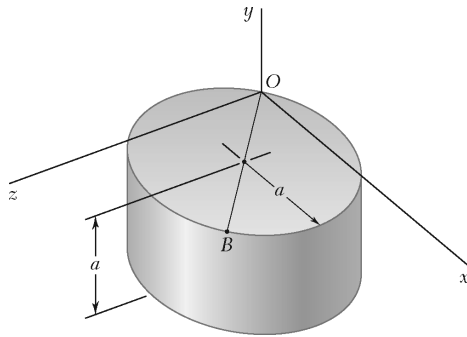
or $(2I_y - I_x)_{\text{body}} \geq 0$

Adding the last two inequalities yields

$$(4I_y)_{\text{body}} \geq \frac{6}{5}ma^2$$

or

$$(I_y)_{\text{body}} \geq \frac{3}{10}ma^2 \quad \text{Q.E.D.}$$



PROBLEM 9.179*

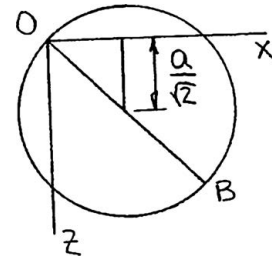
The homogeneous circular cylinder shown has a mass m , and the diameter OB of its top surface forms 45° angles with the x and z axes. (a) Determine the principal mass moments of inertia of the cylinder at the origin O . (b) Compute the angles that the principal axes of inertia at O form with the coordinate axes. (c) Sketch the cylinder, and show the orientation of the principal axes of inertia relative to the x , y , and z axes.

SOLUTION

- (a) First compute the moments of inertia using Figure 9.28 and the parallel-axis theorem.

$$I_x = I_z = \frac{1}{12}m(3a^2 + a^2) + m\left[\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2\right] = \frac{13}{12}ma^2$$

$$I_y = \frac{1}{2}ma^2 + m(a)^2 = \frac{3}{2}ma^2$$



Next observe that the centroidal products of inertia are zero because of symmetry. Then

$$I_{xy} = \bar{I}_{x'y'} + m\bar{x}\bar{y} = m\left(\frac{a}{\sqrt{2}}\right)\left(-\frac{a}{2}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{yz} = \bar{I}_{y'z'} + m\bar{y}\bar{z} = m\left(-\frac{a}{2}\right)\left(\frac{a}{\sqrt{2}}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{zx} = \bar{I}_{z'x'} + m\bar{z}\bar{x} = m\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{1}{2}ma^2$$

Substituting into Equation (9.56)

$$\begin{aligned} K^3 - \left(\frac{13}{12} + \frac{3}{2} + \frac{13}{12}\right)ma^2K^2 \\ + \left[\left(\frac{13}{12} \times \frac{3}{2}\right) + \left(\frac{3}{2} \times \frac{13}{12}\right) + \left(\frac{13}{12} \times \frac{13}{12}\right) - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2\right](ma^2)^2K \\ - \left[\left(\frac{13}{12} \times \frac{3}{2} \times \frac{13}{12}\right) - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right) - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\right] \\ - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - 2\left(-\frac{1}{2\sqrt{2}}\right)\left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{2}\right)\right](ma^2)^3 = 0 \end{aligned}$$

PROBLEM 9.179* (Continued)

Simplifying and letting $K = ma^2\zeta$ yields

$$\zeta^3 - \frac{11}{3}\zeta^2 + \frac{565}{144}\zeta - \frac{95}{96} = 0$$

Solving yields

$$\zeta_1 = 0.363383 \quad \zeta_2 = \frac{19}{12} \quad \zeta_3 = 1.71995$$

The principal moments of inertia are then

$$K_1 = 0.363ma^2 \quad \blacktriangleleft$$

$$K_2 = 1.583ma^2 \quad \blacktriangleleft$$

$$K_3 = 1.720ma^2 \quad \blacktriangleleft$$

- (b) To determine the direction cosines λ_x , λ_y , λ_z of each principal axis, we use two of the equations of Equations (9.54) and (9.57).

Thus,

$$(I_x - K)\lambda_x - I_{xy}\lambda_y - I_{zx}\lambda_z = 0 \quad (9.54a)$$

$$-I_{zx}\lambda_x - I_{yz}\lambda_y + (I_z - K)\lambda_z = 0 \quad (9.54c)$$

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \quad (9.57)$$

(Note: Since $I_{xy} = I_{yz}$, Equations (9.54a) and (9.54c) were chosen to simplify the “elimination” of λ_y during the solution process.)

Substituting for the moments and products of inertia in Equations (9.54a) and (9.54c)

$$\left(\frac{13}{12}ma^2 - K\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y - \left(\frac{1}{2}ma^2\right)\lambda_z = 0$$

$$-\left(\frac{1}{2}ma^2\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y + \left(\frac{13}{12}ma^2 - K\right)\lambda_z = 0$$

or
$$\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_z = 0 \quad (i)$$

and
$$-\frac{1}{2}\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y + \left(\frac{13}{12} - \zeta\right)\lambda_z = 0 \quad (ii)$$

Observe that these equations will be identical, so that one will need to be replaced, if

$$\frac{13}{12} - \zeta = -\frac{1}{2} \quad \text{or} \quad \zeta = \frac{19}{12}$$

PROBLEM 9.179* (Continued)

Thus, a third independent equation will be needed when the direction cosines associated with K_2 are determined. Then for K_1 and K_3

Eq. (i) through Eq. (ii):
$$\left[\frac{13}{12} - \zeta - \left(-\frac{1}{2} \right) \right] \lambda_x + \left[-\frac{1}{2} - \left(\frac{13}{12} - \zeta \right) \right] \lambda_z = 0$$

or
$$\lambda_z = \lambda_x$$

Substituting into Eq. (i):
$$\left(\frac{13}{12} - \zeta \right) \lambda_x + \frac{1}{2\sqrt{2}} \lambda_y - \frac{1}{2} \lambda_x = 0$$

or
$$\lambda_y = 2\sqrt{2} \left(\zeta - \frac{7}{12} \right) \lambda_x$$

Substituting into Equation (9.57):

$$\lambda_x^2 + \left[2\sqrt{2} \left(\zeta - \frac{7}{12} \right) \lambda_x \right]^2 + (\lambda_x)^2 = 1$$

or
$$\left[2 + 8 \left(\zeta - \frac{7}{12} \right)^2 \right] \lambda_x^2 = 1 \tag{iii}$$

K₁: Substituting the value of ζ_1 into Eq. (iii):

$$\left[2 + 8 \left(0.363383 - \frac{7}{12} \right)^2 \right] (\lambda_x)_1^2 = 1$$

or
$$(\lambda_x)_1 = (\lambda_z)_1 = 0.647249$$

and then
$$\begin{aligned} (\lambda_y)_1 &= 2\sqrt{2} \left(0.363383 - \frac{7}{12} \right) (0.647249) \\ &= -0.402662 \end{aligned}$$

$$(\theta_x)_1 = (\theta_z)_1 = 49.7^\circ \quad (\theta_y)_1 = 113.7^\circ \blacktriangleleft$$

K₃: Substituting the value of ζ_3 into Eq. (iii):

$$\left[2 + 8 \left(1.71995 - \frac{7}{12} \right)^2 \right] (\lambda_x)_3^2 = 1$$

or
$$(\lambda_x)_3 = (\lambda_z)_3 = 0.284726$$

and then
$$\begin{aligned} (\lambda_y)_3 &= 2\sqrt{2} \left(1.71995 - \frac{7}{12} \right) (0.284726) \\ &= 0.915348 \end{aligned}$$

$$(\theta_x)_3 = (\theta_z)_3 = 73.5^\circ \quad (\theta_y)_3 = 23.7^\circ \blacktriangleleft$$

PROBLEM 9.179* (Continued)

K_2 : For this case, the set of equations to be solved consists of Equations (9.54a), (9.54b), and (9.57).
Now

$$-I_{xy}\lambda_x + (I_y - K)\lambda_y - I_{yz}\lambda_z = 0 \quad (9.54b)$$

Substituting for the moments and products of inertia.

$$-\left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_x + \left(\frac{3}{2}ma^2 - K\right)\lambda_y - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_z = 0$$

or

$$\frac{1}{2\sqrt{2}}\lambda_x + \left(\frac{3}{2} - \xi\right)\lambda_y + \frac{1}{2\sqrt{2}}\lambda_z = 0 \quad (iv)$$

Substituting the value of ξ_2 into Eqs. (i) and (iv):

$$\left(\frac{13}{12} - \frac{19}{12}\right)(\lambda_x)_2 + \frac{1}{2\sqrt{2}}(\lambda_y)_2 - \frac{1}{2}(\lambda_z)_2 = 0$$

$$\frac{1}{2\sqrt{2}}(\lambda_x)_2 + \left(\frac{3}{2} - \frac{19}{12}\right)(\lambda_y)_2 + \frac{1}{2\sqrt{2}}(\lambda_z)_2 = 0$$

or

$$-(\lambda_x)_2 + \frac{1}{\sqrt{2}}(\lambda_y)_2 - (\lambda_z)_2 = 0$$

and

$$(\lambda_x)_2 - \frac{\sqrt{2}}{6}(\lambda_y)_2 + (\lambda_z)_2 = 0$$

Adding yields

$$(\lambda_y)_2 = 0$$

and then

$$(\lambda_y)_2 = -(\lambda_x)_2$$

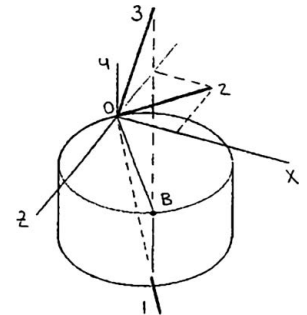
Substituting into Equation (9.57)

$$(\lambda_x)_2^2 + (\lambda_y)_2^2 + (-\lambda_x)_2^2 = 1$$

or

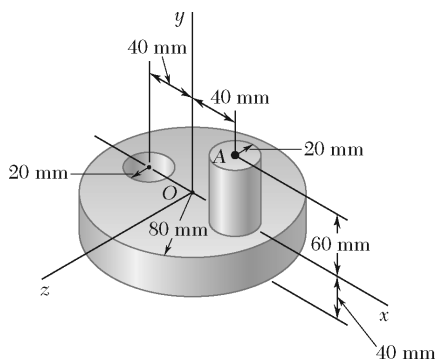
$$(\lambda_x)_2 = \frac{1}{\sqrt{2}} \quad \text{and} \quad (\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$

$$(\theta_x)_2 = 45.0^\circ \quad (\theta_y)_2 = 90.0^\circ \quad (\theta_z)_2 = 135.0^\circ \quad \blacktriangleleft$$



- (c) Principal axes 1 and 3 lie in the vertical plane of symmetry passing through Points O and B . Principal axis 2 lies in the xz plane.

PROBLEM 9.180



For the component described in Problem 9.165, determine (a) the principal mass moments of inertia at the origin, (b) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the x , y , and z axes.

SOLUTION

(a) From the solutions to Problems 9.141 and 9.165 we have

Problem 9.141:

$$I_x = 13.98800 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_y = 20.55783 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_z = 14.30368 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Problem 9.165:

$$I_{yz} = I_{zx} = 0$$

$$I_{xy} = 0.39460 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Eq. (9.55) then becomes

$$\begin{vmatrix} I_x - K & -I_{xy} & 0 \\ -I_{xy} & I_y - K & 0 \\ 0 & 0 & I_z - K \end{vmatrix} = 0 \quad \text{or} \quad (I_x - K)(I_y - K)(I_z - K) - (I_z - K)I_{xy}^2 = 0$$

Thus

$$I_z - K = 0 \quad I_x I_y - (I_x + I_y)K + K^2 - I_{xy}^2 = 0$$

Substituting:

$$K_1 = 14.30368 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\text{or} \quad K_1 = 14.30 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

and $(13.98800 \times 10^{-3})(20.55783 \times 10^{-3}) - (13.98800 + 20.55783)(10^{-3})K + K^2 - (0.39460 \times 10^{-3})^2 = 0$

or $K^2 - (34.54583 \times 10^{-3})K + 287.4072 \times 10^{-6} = 0$

Solving yields $K_2 = 13.96438 \times 10^{-3} \text{ kg} \cdot \text{m}^2$

$$\text{or} \quad K_2 = 13.96 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

and $K_3 = 20.58145 \times 10^{-3} \text{ kg} \cdot \text{m}^2$

$$\text{or} \quad K_3 = 20.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

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PROBLEM 9.180 (Continued)

(b) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, use two of the equations of Eq. (9.54) and Eq. (9.57). Then

K_1 : Begin with Eqs. (9.54a) and (9.54b) with $I_{yz} = I_{zx} = 0$.

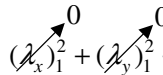
$$\begin{aligned} (I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 &= 0 \\ -I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 &= 0 \end{aligned}$$

Substituting:

$$\begin{aligned} [(13.98800 - 14.30368) \times 10^{-3}](\lambda_x)_1 - (0.39460 \times 10^{-3})(\lambda_y)_1 &= 0 \\ -(0.39460 \times 10^{-3})(\lambda_x)_1 + [(20.55783 - 14.30368) \times 10^{-3}](\lambda_y)_1 &= 0 \end{aligned}$$

Adding yields

$$(\lambda_x)_1 = (\lambda_y)_1 = 0$$

Then using Eq. (9.57)  $(\lambda_x)_1^2 + (\lambda_y)_1^2 + (\lambda_z)_1^2 = 1$

or $(\lambda_z)_1 = 1$

$$(\theta_x)_1 = 90.0^\circ \quad (\theta_y)_1 = 90.0^\circ \quad (\theta_z)_1 = 0^\circ \quad \blacktriangleleft$$

K_2 : Begin with Eqs. (9.54b) and (9.54c) with $I_{yz} = I_{zx} = 0$.


$$\begin{aligned} -I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 &= 0 & \text{(i)} \\ (I_z - K_2)(\lambda_z)_2 &= 0 \end{aligned}$$

Now $I_z \neq K_2 \Rightarrow (\lambda_z)_2 = 0$

Substituting into Eq. (i):

$$-(0.39460 \times 10^{-3})(\lambda_x)_2 + [(20.55783 - 13.96438) \times 10^{-3}](\lambda_y)_2 = 0$$

or $(\lambda_y)_2 = 0.059847(\lambda_x)_2$

Using Eq. (9.57):  $(\lambda_x)_2^2 + [0.05984](\lambda_x)_2^2 + (\lambda_z)_2^2 = 1$

or $(\lambda_x)_2 = 0.998214$

and $(\lambda_y)_2 = 0.059740$

$$(\theta_x)_2 = 3.4^\circ \quad (\theta_y)_2 = 86.6^\circ \quad (\theta_z)_2 = 90.0^\circ \quad \blacktriangleleft$$

K_3 : Begin with Eqs. (9.54b) and (9.54c) with $I_{yz} = I_{zx} = 0$.

$$\begin{aligned} -I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 &= 0 & \text{(ii)} \\ (I_z - K_3)(\lambda_z)_3 &= 0 \end{aligned}$$

PROBLEM 9.180 (Continued)

Now $I_z \neq K_3 \Rightarrow (\lambda_z)_3 = 0$

Substituting into Eq. (ii):

$$-(0.39460 \times 10^{-3})(\lambda_x)_3 + [(20.55783 - 20.58145) \times 10^{-3}](\lambda_y)_3 = 0$$

or $(\lambda_y)_3 = -16.70618(\lambda_x)_3$

Using Eq. (9.57): $(\lambda_x)_3^2 + [-16.70618(\lambda_x)_3]^2 + (\lambda_z)_3^2 = 1$

or $(\lambda_x)_3 = -0.059751$ (axes right-handed set \Rightarrow "-")

and $(\lambda_y)_3 = 0.998211$

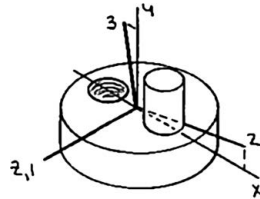
$$(\theta_x)_3 = 93.4^\circ \quad (\theta_y)_3 = 3.43^\circ \quad (\theta_z)_3 = 90.0^\circ \quad \blacktriangleleft$$

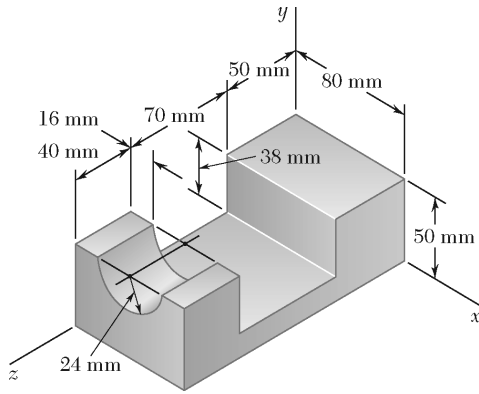
Note: Since the principal axes are orthogonal and $(\theta_z)_2 = (\theta_z)_3 = 90^\circ$, it follows that

$$|(\lambda_x)_2| = |(\lambda_y)_3| \quad |(\lambda_y)_2| = |(\lambda_z)_3|$$

The differences in the above values are due to round-off errors.

- (c) Principal axis 1 coincides with the z axis, while principal axes 2 and 3 lie in the xy plane.





PROBLEM 9.181*

For the component described in Problems 9.145 and 9.149, determine (a) the principal mass moments of inertia at the origin, (b) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the x , y , and z axes.

SOLUTION

(a) From the solutions to Problems 9.145 and 9.149, we have

$$\text{Problem 9.145: } I_x = 26.4325 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \text{Problem 9.149: } I_{xy} = 2.5002 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_y = 31.1726 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad I_{yz} = 4.0627 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$I_z = 8.5773 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad I_{zx} = 8.8062 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Substituting into Eq. (9.56):

$$\begin{aligned} K^3 - [(26.4325 + 31.1726 + 8.5773)(10^{-3})]K^2 \\ + [(26.4325)(31.1726) + (31.1726)(8.5773) + (8.5773)(26.4325) \\ - (2.5002)^2 - (4.0627)^2 - (8.8062)^2](10^{-6})K \\ - [(26.4325)(31.1726)(8.5773) - (26.4325)(4.0627)^2 \\ - (31.1726)(8.8062)^2 - (8.5773)(2.5002)^2 \\ - 2(2.5002)(4.0627)(8.8062)](10^{-9}) = 0 \end{aligned}$$

$$\text{or} \quad K^3 - (66.1824 \times 10^{-3})K^2 + (1217.76 \times 10^{-6})K - (3981.23 \times 10^{-9}) = 0$$

Solving yields

$$K_1 = 4.1443 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \text{or} \quad K_1 = 4.14 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$K_2 = 29.7840 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \text{or} \quad K_2 = 29.8 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$K_3 = 32.2541 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \text{or} \quad K_3 = 32.3 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

PROBLEM 9.181* (Continued)

(b) To determine the direction cosines λ_x , λ_y , λ_z of each principal axis, use two of the equations of Eqs. (9.54) and Eq. (9.57). Then

K_1 : Begin with Eqs. (9.54a) and (9.54b).

$$\begin{aligned}(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 &= 0 \\ I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 &= 0\end{aligned}$$

Substituting:

$$\begin{aligned}[(26.4325 - 4.1443)(10^{-3})](\lambda_x)_1 - (2.5002 \times 10^{-3})(\lambda_y)_1 - (8.8062 \times 10^{-3})(\lambda_z)_1 &= 0 \\ -(2.5002 \times 10^{-3})(\lambda_x)_1 + [(31.1726 - 4.1443)(10^{-3})](\lambda_y)_1 - (4.0627 \times 10^{-3})(\lambda_z)_1 &= 0\end{aligned}$$

Simplifying

$$\begin{aligned}8.9146(\lambda_x)_1 - (\lambda_y)_1 - 3.5222(\lambda_z)_1 &= 0 \\ -0.0925(\lambda_x)_1 + (\lambda_y)_1 - 0.1503(\lambda_z)_1 &= 0\end{aligned}$$

Adding and solving for $(\lambda_z)_1$:

$$(\lambda_z)_1 = 2.4022(\lambda_x)_1$$

and then

$$\begin{aligned}(\lambda_y)_1 &= [8.9146 - 3.5222(2.4022)](\lambda_x)_1 \\ &= 0.45357(\lambda_x)_1\end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [0.45357(\lambda_x)_1]^2 + [2.4022(\lambda_x)_1]^2 = 1$$

or

$$(\lambda_x)_1 = 0.37861$$

and

$$(\lambda_y)_1 = 0.17173$$

$$(\lambda_z)_1 = 0.90950$$

$$(\theta_x)_1 = 67.8^\circ \quad (\theta_y)_1 = 80.1^\circ \quad (\theta_z)_1 = 24.6^\circ \quad \blacktriangleleft$$

K_2 : Begin with Eqs. (9.54a) and (9.54b).

$$\begin{aligned}(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 &= 0 \\ -I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 &= 0\end{aligned}$$

Substituting:

$$\begin{aligned}[(26.4325 - 29.7840)(10^{-3})](\lambda_x)_2 - (2.5002 \times 10^{-3})(\lambda_y)_2 - (8.8062 \times 10^{-3})(\lambda_z)_2 &= 0 \\ -(2.5002 \times 10^{-3})(\lambda_x)_2 + [(31.1726 - 29.7840)(10^{-3})](\lambda_y)_2 - (4.0627 \times 10^{-3})(\lambda_z)_2 &= 0\end{aligned}$$

PROBLEM 9.181* (Continued)

Simplifying

$$\begin{aligned} -1.3405(\lambda_x)_2 - (\lambda_y)_2 - 3.5222(\lambda_z)_2 &= 0 \\ -1.8005(\lambda_x)_2 + (\lambda_y)_2 - 2.9258(\lambda_z)_2 &= 0 \end{aligned}$$

Adding and solving for $(\lambda_z)_2$:

$$(\lambda_z)_2 = -0.48713(\lambda_x)_2$$

and then

$$\begin{aligned} (\lambda_y)_2 &= [-1.3405 - 3.5222(-0.48713)](\lambda_x)_2 \\ &= 0.37527(\lambda_x)_2 \end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [0.37527(\lambda_x)_2]^2 + [-0.48713(\lambda_x)_2]^2 = 1$$

or

$$(\lambda_x)_2 = 0.85184$$

and

$$(\lambda_y)_2 = 0.31967 \qquad (\lambda_z)_2 = -0.41496$$

$$(\theta_x)_1 = 31.6^\circ \quad (\theta_y)_2 = 71.4^\circ \quad (\theta_z)_2 = 114.5^\circ \quad \blacktriangleleft$$

K_3 : Begin with Eqs. (9.54a) and (9.54b).

$$\begin{aligned} (I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 &= 0 \\ -I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_z)_3 &= 0 \end{aligned}$$

Substituting:

$$\begin{aligned} [(26.4325 - 32.2541)(10^{-3})](\lambda_x)_3 - (2.5002 \times 10^{-3})(\lambda_y)_3 - (8.8062 \times 10^{-3})(\lambda_z)_3 &= 0 \\ -(2.5002 \times 10^{-3})(\lambda_x)_3 + [(31.1726 - 32.2541)(10^{-3})](\lambda_y)_3 - (4.0627 \times 10^{-3})(\lambda_z)_3 &= 0 \end{aligned}$$

Simplifying

$$\begin{aligned} -2.3285(\lambda_x)_3 - (\lambda_y)_3 - 3.5222(\lambda_z)_3 &= 0 \\ 2.3118(\lambda_x)_3 + (\lambda_y)_3 + 3.7565(\lambda_z)_3 &= 0 \end{aligned}$$

Adding and solving for $(\lambda_z)_3$:

$$(\lambda_z)_3 = 0.071276(\lambda_x)_3$$

and then

$$\begin{aligned} (\lambda_y)_3 &= [-2.3285 - 3.5222(0.071276)](\lambda_x)_3 \\ &= -2.5795(\lambda_x)_3 \end{aligned}$$

PROBLEM 9.181* (Continued)

Now substitute into Eq. (9.57):

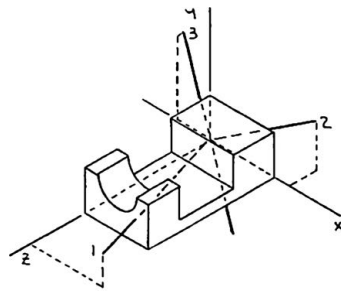
$$(\lambda_x)_3^2 + [-2.5795(\lambda_x)_3]^2 + [0.071276(\lambda_x)_3]^2 = 1 \quad (i)$$

or $(\lambda_x)_3 = 0.36134$

and $(\lambda_y)_3 = 0.93208$ $(\lambda_z)_3 = 0.025755$

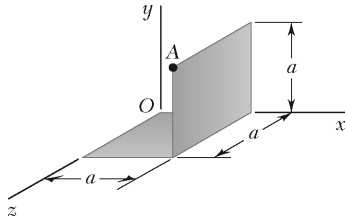
$$(\theta_x)_3 = 68.8^\circ \quad (\theta_y)_3 = 158.8^\circ \quad (\theta_z)_3 = 88.5^\circ \quad \blacktriangleleft$$

- (c) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.36134$.



Then

$$(\theta_x)_3 = 111.2^\circ \quad (\theta_y)_3 = 21.2^\circ \quad (\theta_z)_3 = 91.5^\circ \quad \blacktriangleleft$$



PROBLEM 9.182*

For the component described in Problem 9.167, determine (a) the principal mass moments of inertia at the origin, (b) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the x , y , and z axes.

SOLUTION

(a) From the solution of Problem 9.167, we have

$$I_x = \frac{1}{2} \frac{W}{g} a^2 \quad I_{xy} = \frac{1}{4} \frac{W}{g} a^2$$

$$I_y = \frac{W}{g} a^2 \quad I_{yz} = \frac{1}{8} \frac{W}{g} a^2$$

$$I_z = \frac{5}{6} \frac{W}{g} a^2 \quad I_{zx} = \frac{3}{8} \frac{W}{g} a^2$$

Substituting into Eq. (9.56):

$$\begin{aligned} K^3 - \left[\left(\frac{1}{2} + 1 + \frac{5}{6} \right) \left(\frac{W}{g} a^2 \right) \right] K^2 \\ + \left[\left(\frac{1}{2} \right) (1) + (1) \left(\frac{5}{6} \right) + \left(\frac{5}{6} \right) \left(\frac{1}{2} \right) - \left(\frac{1}{4} \right)^2 - \left(\frac{1}{8} \right)^2 - \left(\frac{3}{8} \right)^2 \right] \left(\frac{W}{g} a^2 \right)^2 K \\ - \left[\left(\frac{1}{2} \right) (1) \left(\frac{5}{6} \right) - \left(\frac{1}{2} \right) \left(\frac{1}{8} \right)^2 - (1) \left(\frac{3}{8} \right)^2 - \left(\frac{5}{6} \right) \left(\frac{1}{4} \right)^2 - 2 \left(\frac{1}{4} \right) \left(\frac{1}{8} \right) \left(\frac{3}{8} \right) \right] \left(\frac{W}{g} a^2 \right)^3 = 0 \end{aligned}$$

Simplifying and letting $K = \frac{W}{g} a^2 K$ yields

$$K^3 - 2.33333K^2 + 1.53125K - 0.192708 = 0$$

Solving yields

$$K_1 = 0.163917 \quad K_2 = 1.05402 \quad K_3 = 1.11539$$

The principal moments of inertia are then

$$K_1 = 0.1639 \frac{W}{g} a^2 \quad \blacktriangleleft$$

$$K_2 = 1.054 \frac{W}{g} a^2 \quad \blacktriangleleft$$

$$K_3 = 1.115 \frac{W}{g} a^2 \quad \blacktriangleleft$$

PROBLEM 9.182* (Continued)

(b) To determine the direction cosines λ_x , λ_y , λ_z of each principal axis, use two of the equations of Eq. (9.54) and Eq. (9.57). Then

K_1 : Begin with Eqs. (9.54a) and (9.54b).

$$\begin{aligned}(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 &= 0 \\ -I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 &= 0\end{aligned}$$

Substituting

$$\begin{aligned}\left[\left(\frac{1}{2} - 0.163917\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_1 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_1 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 &= 0 \\ -\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_1 + \left[1 - 0.163917\right]\left(\frac{W}{g}a^2\right)(\lambda_y)_1 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 &= 0\end{aligned}$$

Simplifying

$$\begin{aligned}1.34433(\lambda_x)_1 - (\lambda_y)_1 - 1.5(\lambda_z)_1 &= 0 \\ -0.299013(\lambda_x)_1 + (\lambda_y)_1 - 0.149507(\lambda_z)_1 &= 0\end{aligned}$$

Adding and solving for $(\lambda_z)_1$:

$$(\lambda_z)_1 = 0.633715(\lambda_x)_1$$

and then

$$\begin{aligned}(\lambda_y)_1 &= [1.34433 - 1.5(0.633715)](\lambda_x)_1 \\ &= 0.393758(\lambda_x)_1\end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [0.393758(\lambda_x)_1]^2 + [0.633715(\lambda_x)_1]^2 = 1$$

or

$$(\lambda_x)_1 = 0.801504$$

and

$$(\lambda_y)_1 = 0.315599$$

$$(\lambda_z)_1 = 0.507925$$

$$(\theta_x)_1 = 36.7^\circ \quad (\theta_y)_1 = 71.6^\circ \quad (\theta_z)_1 = 59.5^\circ \quad \blacktriangleleft$$

K_2 : Begin with Eqs. (9.54a) and (9.54b):

$$\begin{aligned}(I_x - k_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 &= 0 \\ -I_{xy}(\lambda_x)_2 + (I_y - k_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 &= 0\end{aligned}$$

PROBLEM 9.182* (Continued)

Substituting

$$\left[\left(\frac{1}{2} - 1.05402 \right) \left(\frac{W}{g} a^2 \right) \right] (\lambda_x)_2 - \left(\frac{1}{4} \frac{W}{g} a^2 \right) (\lambda_y)_2 - \left(\frac{3}{8} \frac{W}{g} a^2 \right) (\lambda_z)_2 = 0$$

$$- \left(\frac{1}{4} \frac{W}{g} a^2 \right) (\lambda_x)_2 + \left[(1 - 1.05402) \left(\frac{W}{g} a^2 \right) \right] (\lambda_y)_2 - \left(\frac{1}{8} \frac{W}{g} a^2 \right) (\lambda_z)_2 = 0$$

Simplifying

$$-2.21608(\lambda_x)_2 - (\lambda_y)_2 - 1.5(\lambda_z)_2 = 0$$

$$4.62792(\lambda_x)_2 + (\lambda_y)_2 + 2.31396(\lambda_z)_2 = 0$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2 = -2.96309(\lambda_x)_2$$

and then

$$(\lambda_y)_2 = [-2.21608 - 1.5(-2.96309)](\lambda_x)_2$$

$$= 2.22856(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [2.22856(\lambda_x)_2]^2 + [-2.96309(\lambda_x)_2]^2 = 1$$

or

$$(\lambda_x)_2 = 0.260410$$

and

$$(\lambda_y)_2 = 0.580339$$

$$(\lambda_z)_2 = -0.771618$$

$$(\theta_x)_2 = 74.9^\circ \quad (\theta_y)_2 = 54.5^\circ \quad (\theta_z)_2 = 140.5^\circ \quad \blacktriangleleft$$

K_3 : Begin with Eqs. (9.54a) and (9.54b):

$$(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 = 0$$

$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_z)_3 = 0$$

Substituting

$$\left[\left(\frac{1}{2} - 1.11539 \right) \left(\frac{W}{g} a^2 \right) \right] (\lambda_x)_3 - \left(\frac{1}{4} \frac{W}{g} a^2 \right) (\lambda_y)_3 - \left(\frac{3}{8} \frac{W}{g} a^2 \right) (\lambda_z)_3 = 0$$

$$- \left(\frac{1}{4} \frac{W}{g} a^2 \right) (\lambda_x)_3 + \left[(1 - 1.11539) \left(\frac{W}{g} a^2 \right) \right] (\lambda_y)_3 - \left(\frac{1}{8} \frac{W}{g} a^2 \right) (\lambda_z)_3 = 0$$

Simplifying

$$-2.46156(\lambda_x)_3 - (\lambda_y)_3 - 1.5(\lambda_z)_3 = 0$$

$$2.16657(\lambda_x)_3 + (\lambda_y)_3 + 1.08328(\lambda_z)_3 = 0$$

PROBLEM 9.182* (Continued)

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = -0.707885(\lambda_x)_3$$

and then

$$\begin{aligned} (\lambda_y)_3 &= [-2.46156 - 1.5(-0.707885)](\lambda_x)_3 \\ &= -1.39973(\lambda_x)_3 \end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_3^2 + [-1.39973(\lambda_x)_3]^2 + [-0.707885(\lambda_x)_3]^2 = 1 \quad (i)$$

or

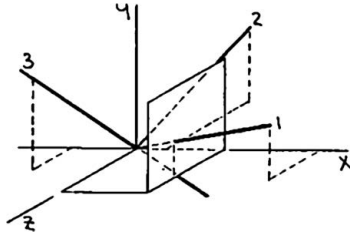
$$(\lambda_x)_3 = 0.537577$$

and

$$(\lambda_y)_3 = -0.752463 \quad (\lambda_z)_3 = -0.380543$$

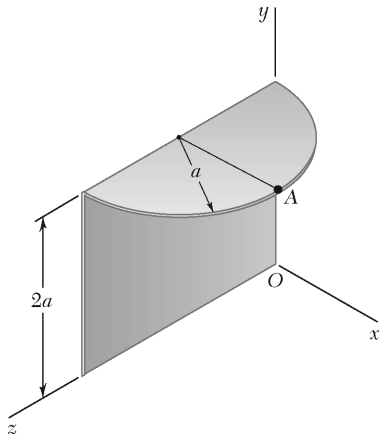
$$(\theta_x)_3 = 57.5^\circ \quad (\theta_y)_3 = 138.8^\circ \quad (\theta_z)_3 = 112.4^\circ \quad \blacktriangleleft$$

- (c) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.537577$.



Then

$$(\theta_x)_3 = 122.5^\circ \quad (\theta_y)_3 = 41.2^\circ \quad (\theta_z)_3 = 67.6^\circ \quad \blacktriangleleft$$



PROBLEM 9.183*

For the component described in Problem 9.168, determine (a) the principal mass moments of inertia at the origin, (b) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the x , y , and z axes.

SOLUTION

(a) From the solution to Problem 9.168, we have

$$I_x = 18.91335 \frac{\gamma t}{g} a^4$$

$$I_{xy} = 1.33333 \frac{\gamma t}{g} a^4$$

$$I_y = 7.68953 \frac{\gamma t}{g} a^4$$

$$I_{yz} = 7.14159 \frac{\gamma t}{g} a^4$$

$$I_z = 12.00922 \frac{\gamma t}{g} a^4$$

$$I_{zx} = 0.66667 \frac{\gamma t}{g} a^4$$

Substituting into Eq. (9.56):

$$\begin{aligned} & K^3 - \left[(18.91335 + 7.68953 + 12.00922) \left(\frac{\gamma t}{g} a^4 \right) \right] K^2 \\ & + [(18.91335)(7.68953) + (7.68953)(12.00922) + (12.00922)(18.91335) \\ & - (1.33333)^2 - (7.14159)^2 + (0.66667)^2] \left(\frac{\gamma t}{g} a^4 \right)^2 K \\ & - [(18.91335)(7.68953)(12.00922) - (18.91335)(7.14159)^2 \\ & - (7.68953)(0.66667)^2 - (12.00922)(1.33333)^2 \\ & - 2(1.33333)(7.14159)(0.66667)] \left(\frac{\gamma t}{g} a^4 \right)^3 = 0 \end{aligned}$$

Simplifying and letting

$$K = \frac{\gamma t}{g} a^4 K \quad \text{yields}$$

$$K^3 - 38.61210K^2 + 411.69009K - 744.47027 = 0$$

PROBLEM 9.183* (Continued)

Solving yields

$$K_1 = 2.25890 \quad K_2 = 17.27274 \quad K_3 = 19.08046$$

The principal moments of inertia are then

$$K_1 = 2.26 \frac{\gamma t}{g} a^4 \quad \blacktriangleleft$$

$$K_2 = 17.27 \frac{\gamma t}{g} a^4 \quad \blacktriangleleft$$

$$K_3 = 19.08 \frac{\gamma t}{g} a^4 \quad \blacktriangleleft$$

- (b) To determine the direction cosines λ_x , λ_y , λ_z of each principal axis, use two of the equations of Eq. (9.54) and Eq. (9.57). Then

K_1 : Begin with Eqs. (9.54a) and (9.54b):

$$\begin{aligned} (I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 &= 0 \\ -I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 &= 0 \end{aligned}$$

Substituting

$$\begin{aligned} \left[(18.91335 - 2.25890) \left(\frac{\gamma t}{g} a^4 \right) \right] (\lambda_x)_1 - \left(1.33333 \frac{\gamma t}{g} a^4 \right) (\lambda_y)_1 - \left(0.66667 \frac{\gamma t}{g} a^4 \right) (\lambda_z)_1 &= 0 \\ - \left(1.33333 \frac{\gamma t}{g} a^4 \right) (\lambda_x)_1 + \left[(7.68953 - 2.25890) \left(\frac{\gamma t}{g} a^4 \right) \right] (\lambda_y)_1 - \left(7.14159 \frac{\gamma t}{g} a^4 \right) (\lambda_z)_1 &= 0 \end{aligned}$$

Simplifying

$$\begin{aligned} 12.49087(\lambda_x)_1 - (\lambda_y)_1 - 0.5(\lambda_z)_1 &= 0 \\ -0.24552(\lambda_x)_1 + (\lambda_y)_1 - 1.31506(\lambda_z)_1 &= 0 \end{aligned}$$

Adding and solving for $(\lambda_z)_1$

$$(\lambda_z)_1 = 6.74653(\lambda_x)_1$$

and then

$$\begin{aligned} (\lambda_y)_1 &= [12.49087 - (0.5)(6.74653)](\lambda_x)_1 \\ &= 9.11761(\lambda_x)_1 \end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [9.11761(\lambda_x)_1]^2 + [6.74653(\lambda_x)_1]^2 = 1$$

or

$$(\lambda_x)_1 = 0.087825$$

and

$$(\lambda_y)_1 = 0.80075 \quad (\lambda_z)_1 = 0.59251$$

$$(\theta_x)_1 = 85.0^\circ \quad (\theta_y)_1 = 36.8^\circ \quad (\theta_z)_1 = 53.7^\circ \quad \blacktriangleleft$$

PROBLEM 9.183* (Continued)

K_2 : Begin with Eqs. (9.54a) and (9.54b):

$$\begin{aligned}(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 &= 0 \\ -I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 &= 0\end{aligned}$$

Substituting

$$\begin{aligned}[(18.91335 - 17.27274)\left(\frac{\gamma t}{g} a^4\right)(\lambda_x)_2 - (1.33333\frac{\gamma t}{g} a^4)(\lambda_y)_2 - (0.66667\frac{\gamma t}{g} a^4)(\lambda_z)_2 &= 0 \\ -\left(1.33333\frac{\gamma t}{g} a^4\right)(\lambda_x)_2 + \left[(7.68953 - 17.27274)\left(\frac{\gamma t}{g} a^4\right)\right](\lambda_y)_2 - \left(7.14159\frac{\gamma t}{g} a^4\right)(\lambda_z)_2 &= 0\end{aligned}$$

Simplifying

$$\begin{aligned}1.23046(\lambda_x)_2 - (\lambda_y)_2 - 0.5(\lambda_z)_2 &= 0 \\ 0.13913(\lambda_x)_2 + (\lambda_y)_2 + 0.74522(\lambda_z)_2 &= 0\end{aligned}$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2 = -5.58515(\lambda_x)_2$$

and then

$$\begin{aligned}(\lambda_y)_2 &= [1.23046 - (0.5)(-5.58515)](\lambda_x)_2 \\ &= 4.02304 (\lambda_x)_2\end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [4.02304(\lambda_x)_2]^2 + [-5.58515(\lambda_x)_2]^2 = 1$$

or

$$(\lambda_x)_2 = 0.14377$$

and

$$(\lambda_y)_2 = 0.57839 \quad (\lambda_z)_2 = -0.80298$$

$$(\theta_x)_2 = 81.7^\circ \quad (\theta_y)_2 = 54.7^\circ \quad (\theta_z)_2 = 143.4^\circ \quad \blacktriangleleft$$

K_3 : Begin with Eqs. (9.54a) and (9.54b):

$$\begin{aligned}(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 &= 0 \\ -I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_z)_3 &= 0\end{aligned}$$

Substituting

$$\begin{aligned}[(18.91335 - 19.08046)\left(\frac{\gamma t}{g} a^4\right)(\lambda_x)_3 - (1.33333\frac{\gamma t}{g} a^4)(\lambda_y)_3 - (0.66667\frac{\gamma t}{g} a^4)(\lambda_z)_3 &= 0 \\ -\left(1.33333\frac{\gamma t}{g} a^4\right)(\lambda_x)_3 + \left[(7.68953 - 19.08046)\left(\frac{\gamma t}{g} a^4\right)\right](\lambda_y)_3 - \left(7.14159\frac{\gamma t}{g} a^4\right)(\lambda_z)_3 &= 0\end{aligned}$$

PROBLEM 9.183* (Continued)

Simplifying

$$\begin{aligned} -0.12533(\lambda_x)_3 - (\lambda_y)_3 - 0.5(\lambda_z)_3 &= 0 \\ 0.11705(\lambda_x)_3 + (\lambda_y)_3 + 0.62695(\lambda_z)_3 &= 0 \end{aligned}$$

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = 0.06522(\lambda_x)_3$$

and then

$$\begin{aligned} (\lambda_y)_3 &= [-0.12533 - (0.5)(0.06522)](\lambda_x)_3 \\ &= -0.15794(\lambda_x)_3 \end{aligned}$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_3^2 + [-0.15794(\lambda_x)_3]^2 + [0.06522(\lambda_x)_3]^2 = 1 \quad (i)$$

or

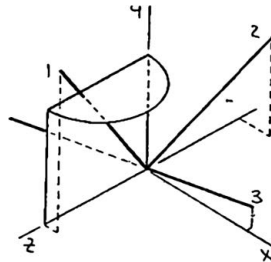
$$(\lambda_x)_3 = 0.98571$$

and

$$(\lambda_y)_3 = -0.15568 \quad (\lambda_z)_3 = 0.06429$$

$$(\theta_x)_3 = 9.7^\circ \quad (\theta_y)_3 = 99.0^\circ \quad (\theta_z)_3 = 86.3^\circ \quad \blacktriangleleft$$

- (c) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.98571$.



Then

$$(\theta_x)_3 = 170.3^\circ \quad (\theta_y)_3 = 81.0^\circ \quad (\theta_z)_3 = 93.7^\circ \quad \blacktriangleleft$$

PROBLEM 9.184*

For the component described in Problems 9.148 and 9.170, determine (a) the principal mass moments of inertia at the origin, (b) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the x , y , and z axes.

SOLUTION

(a) From the solutions to Problems 9.148 and 9.170. We have

$$I_x = 0.32258 \text{ kg} \cdot \text{m}^2 \qquad I_y = I_z = 0.41933 \text{ kg} \cdot \text{m}^2$$

$$I_{xy} = I_{zx} = 0.096768 \text{ kg} \cdot \text{m}^2 \qquad I_{yz} = 0$$

Substituting into Eq. (9.56) and using

$$I_y = I_z \qquad I_{xy} = I_{zx} \qquad I_{yz} = 0$$

$$K^3 - [0.32258 + 2(0.41933)]K^2 + [2(0.32258)(0.41933) + (0.41933)^2 - 2(0.096768)^2]K - [(0.32258)(0.41933)^2 - 2(0.41933)(0.096768)^2] = 0$$

Simplifying

$$K^3 - 1.16124K^2 + 0.42764K - 0.048869 = 0$$

Solving yields

$$K_1 = 0.22583 \text{ kg} \cdot \text{m}^2 \qquad \text{or} \qquad K_1 = 0.226 \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$K_2 = 0.41920 \text{ kg} \cdot \text{m}^2 \qquad \text{or} \qquad K_2 = 0.419 \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

$$K_3 = 0.51621 \text{ kg} \cdot \text{m}^2 \qquad \text{or} \qquad K_3 = 0.516 \text{ kg} \cdot \text{m}^2 \quad \blacktriangleleft$$

(b) To determine the direction cosines λ_x , λ_y , λ_z of each principal axis, use two of the equations of Eqs. (9.54) and (9.57). Then

K_1 : Begin with Eqs. (9.54b) and (9.54c):

$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - I_{yz}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_1 - I_{yz}(\lambda_y)_1 + (I_z - K_1)(\lambda_z)_3 = 0$$

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PROBLEM 9.184* (Continued)

Substituting

$$\begin{aligned} -(0.096768)(\lambda_x)_1 + (0.41933 - 0.22583)(\lambda_y)_1 &= 0 \\ -(0.096768)(\lambda_x)_1 + (0.41933 - 0.22583)(\lambda_z)_1 &= 0 \end{aligned}$$

Simplifying yields

$$(\lambda_y)_1 = (\lambda_z)_1 = 0.50009(\lambda_x)_1$$

Now substitute into Eq. (9.54):

$$(\lambda_x)_1^2 + 2[0.50009(\lambda_x)_1]^2 = 1$$

or

$$(\lambda_x)_1 = 0.81645$$

and

$$(\lambda_y)_1 = (\lambda_z)_1 = 0.40830$$

$$(\theta_x)_1 = 35.3^\circ \quad (\theta_y)_1 = (\theta_z)_1 = 65.9^\circ \quad \blacktriangleleft$$

K_2 : Begin with Eqs. (9.54a) and (9.54b)

$$\begin{aligned} (I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{xz}(\lambda_z)_2 &= 0 \\ -I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 &= 0 \end{aligned}$$

Substituting

$$(0.32258 - 0.41920)(\lambda_x)_2 - (0.096768)(\lambda_y)_2 - (0.096768)(\lambda_z)_2 = 0 \quad \text{(i)}$$

$$-(0.096768)(\lambda_x)_2 + (0.41933 - 0.41920)(\lambda_y)_2 = 0 \quad \text{(ii)}$$

Eq. (ii) $\Rightarrow (\lambda_x)_2 = 0$

and then Eq. (i) $\Rightarrow (\lambda_z)_2 = -(\lambda_y)_2$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + (\lambda_y)_2^2 + [-(\lambda_y)_2]^2 = 1$$

or

$$(\lambda_y)_2 = \frac{1}{\sqrt{2}}$$

and

$$(\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$

$$(\theta_x)_2 = 90.0^\circ \quad (\theta_y)_2 = 45.0^\circ \quad (\theta_z)_2 = 135.0^\circ \quad \blacktriangleleft$$

PROBLEM 9.184* (Continued)

K_3 : Begin with Eqs. (9.54b) and (9.54c):

$$I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 + I_{yz}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_3 - I_{yz}(\lambda_y)_3 + (I_z - K_3)(\lambda_z)_3 = 0$$

Substituting

$$-(0.096768)(\lambda_x)_3 + (0.41933 - 0.51621)(\lambda_y)_3 = 0$$

$$-(0.096768)(\lambda_x)_3 + (0.41933 - 0.51621)(\lambda_z)_3 = 0$$

Simplifying yields

$$(\lambda_y)_3 = (\lambda_z)_3 = -(\lambda_x)_3$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_3^2 + 2[-(\lambda_x)_3]^2 = 1 \quad (i)$$

or

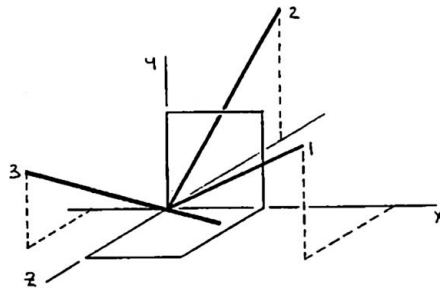
$$(\lambda_x)_3 = \frac{1}{\sqrt{3}}$$

and

$$(\lambda_y)_3 = (\lambda_z)_3 = -\frac{1}{\sqrt{3}}$$

$$(\theta_x)_3 = 54.7^\circ \quad (\theta_y)_3 = (\theta_z)_3 = 125.3^\circ \quad \blacktriangleleft$$

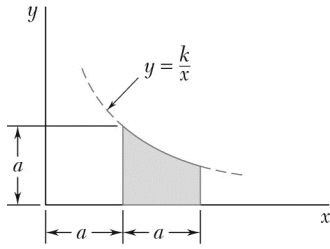
(c)



Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen;

That is,
$$(\lambda_x)_3 = -\frac{1}{\sqrt{3}}$$

Then
$$(\theta_x)_3 = 125.3^\circ \quad (\theta_y)_3 = (\theta_z)_3 = 54.7^\circ \quad \blacktriangleleft$$



PROBLEM 9.185

Determine by direct integration the moments of inertia of the shaded area with respect to the x and y axes.

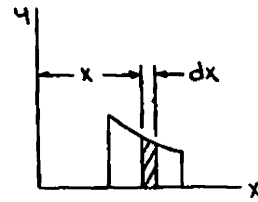
SOLUTION

At $x = a$, $y = a$:
$$a = \frac{k}{a} \quad \text{or} \quad k = a^2$$

Then
$$y = \frac{a^2}{x}$$

Now
$$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3} \left(\frac{a^2}{x} \right)^3 dx$$

$$= \frac{1}{3} \frac{a^6}{x^3} dx$$



Then
$$I_x = \int dI_x = \int_a^{2a} \frac{1}{3} \frac{a^6}{x^3} dx = \frac{1}{3} a^6 \left[-\frac{1}{2} \frac{1}{x^2} \right]_a^{2a}$$

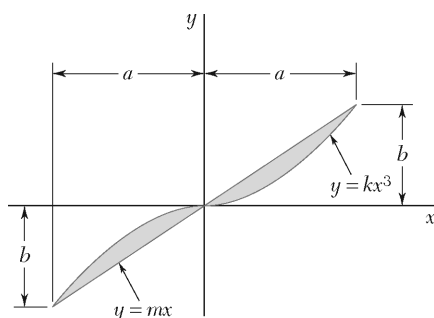
$$= -\frac{1}{6} a^6 \left[\frac{1}{(2a)^2} - \frac{1}{(a)^2} \right] \quad \text{or} \quad I_x = \frac{1}{8} a^4 \blacktriangleleft$$

Now
$$dI_y = x^2 \quad dA = x^2 (y dx)$$

$$= x^2 \left(\frac{a^2}{x} dx \right) = a^2 x dx$$

Then
$$I_y = \int dI_y = \int_a^{2a} a^2 x dx = a^2 \left[\frac{1}{2} x^2 \right]_a^{2a} = \frac{a^2}{2} [(2a)^2 - (a)^2]$$

$$\text{or} \quad I_y = \frac{3}{2} a^4 \blacktriangleleft$$



PROBLEM 9.186

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION

At $x = a$, $y_1 = y_2 = b$:

$$y_1: b = ka^3 \quad \text{or} \quad k = \frac{b}{a^3}$$

$$y_2: b = ma \quad \text{or} \quad m = \frac{b}{a}$$

Then

$$y_1 = \frac{b}{a^3}x^3$$

$$y_2 = \frac{b}{a}x$$

Now

$$dA = (y_2 - y_1)dx = \left(\frac{b}{a}x - \frac{b}{a^3}x^3 \right) dx$$

Then

$$\begin{aligned} A &= \int dA = 2 \int_0^a \frac{b}{a} \left(x - \frac{1}{a^2}x^3 \right) dx \\ &= 2 \frac{b}{a} \left[\frac{1}{2}x^2 - \frac{1}{4a^2}x^4 \right]_0^a = \frac{1}{2}ab \end{aligned}$$

Now

$$dI_y = x^2 dA = x^2 \left[\left(\frac{b}{a}x - \frac{b}{a^3}x^3 \right) dx \right]$$

Then

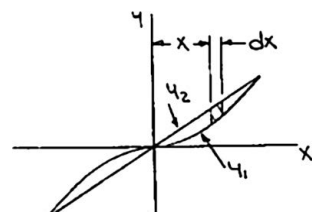
$$\begin{aligned} I_y &= \int dI_y = 2 \int_0^a \frac{b}{a} x^2 \left(x - \frac{1}{a^2}x^3 \right) dx \\ &= 2 \frac{b}{a} \left[\frac{1}{4}x^4 - \frac{1}{6} \frac{1}{a^2}x^6 \right]_0^a \end{aligned}$$

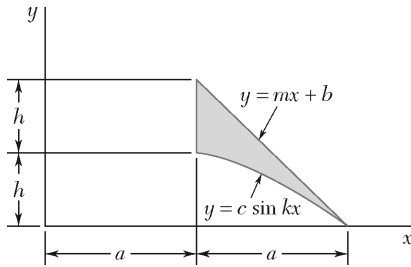
$$\text{or} \quad I_y = \frac{1}{6}a^3b \quad \blacktriangleleft$$

and

$$k_y^2 = \frac{I_y}{A} = \frac{\frac{1}{6}a^3b}{\frac{1}{2}ab} = \frac{1}{3}a^2$$

$$\text{or} \quad k_y = \frac{a}{\sqrt{3}} \quad \blacktriangleleft$$





PROBLEM 9.187

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION

$$y_1: \text{ At } x=2a, \quad y=0: \quad 0 = c \sin k(2a)$$

$$2ak = \pi \quad \text{or} \quad k = \frac{\pi}{2a}$$

$$\text{At } x=a, \quad y=h: \quad h = c \sin \frac{\pi}{2a}(a) \quad \text{or} \quad c = h$$

$$y_2: \text{ At } x=a, \quad y=2h: \quad 2h = ma - b$$

$$\text{At } x=2a, \quad y=0: \quad 0 = m(2a) + b$$

Solving yields

$$m = -\frac{2h}{a}, \quad b = 4h$$

Then

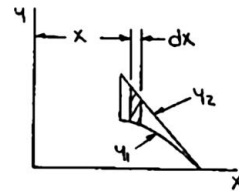
$$y_1 = h \sin \frac{\pi}{2a} x \quad y_2 = -\frac{2h}{a} x + 4h \\ = \frac{2h}{a} (-x + 2a)$$

Now

$$dA = (y_2 - y_1) dx = \left[\frac{2h}{a} (-x + 2a) - h \sin \frac{\pi}{2a} x \right] dx$$

Then

$$A = \int dA = \int_a^{2a} h \left[\frac{2}{a} (-x + 2a) - \sin \frac{\pi}{2a} x \right] dx \\ = h \left[-\frac{1}{a} (-x + 2a)^2 + \frac{2a}{\pi} \cos \frac{\pi}{2a} x \right]_a^{2a} \\ = h \left[\left(-\frac{2a}{\pi} \right) + \frac{1}{a} (-a + 2a)^2 \right] = ah \left(1 - \frac{2}{\pi} \right) \\ = 0.36338ah$$



PROBLEM 9.187 (Continued)

Find: I_x and k_x

We have

$$dI_x = \left(\frac{1}{3} y_2 - \frac{1}{3} y_1 \right) dx = \frac{1}{3} \left\{ \left[\frac{2h}{a} (-x + 2a) \right]^3 - \left(h \sin \frac{\pi}{2a} x \right)^3 \right\} dx$$

$$= \frac{h^3}{3} \left[\frac{8}{a^3} (-x + 2a)^3 - \sin^3 \frac{\pi}{2a} x \right]$$

Then

$$I_x = \int dI_x = \int_a^{2a} \frac{h^3}{3} \left[\frac{8}{a^3} (-x + 2a)^3 - \sin^3 \frac{\pi}{2a} x \right] dx$$

Now

$$\sin^3 \theta = \sin \theta (1 - \cos^2 \theta) = \sin \theta - \sin \theta \cos^2 \theta$$

Then

$$I_x = \frac{h^3}{3} \int_a^{2a} \left[\frac{8}{a^3} (-x + 2a)^3 - \left(\sin \frac{\pi}{2a} x - \sin \frac{\pi}{2a} x \cos^2 \frac{\pi}{2a} x \right) \right] dx$$

$$= \frac{h^3}{3} \left[-\frac{2}{a^3} (-x + 2a)^4 + \frac{2a}{\pi} \cos \frac{\pi}{2a} x - \frac{2a}{3\pi} \cos^3 \frac{\pi}{2a} x \right]_a^{2a}$$

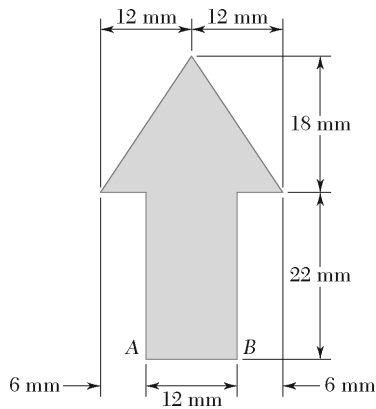
$$= \frac{h^3}{3} \left[\left(-\frac{2a}{\pi} + \frac{2a}{3\pi} \right) + \frac{2}{a^3} (-a + 2a)^4 \right]$$

$$= \frac{2}{3} ah^3 \left(1 - \frac{2}{3\pi} \right)$$

$$I_x = 0.52520ah^3 \qquad \text{or} \quad I_x = 0.525ah^3 \quad \blacktriangleleft$$

and

$$k_x^2 = \frac{I_x}{A} = \frac{0.52520ah^3}{0.36338ah} \qquad \text{or} \quad k_x = 1.202h \quad \blacktriangleleft$$



PROBLEM 9.188

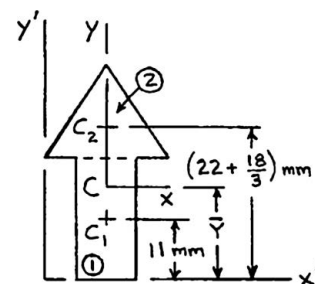
Determine the moments of inertia \bar{I}_x and \bar{I}_y of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB .

SOLUTION

First locate C of the area:

Symmetry implies $\bar{X} = 12$ mm.

	A, mm^2	\bar{y}, mm	$\bar{y}A, \text{mm}^3$
1	$12 \times 22 = 264$	11	2904
2	$\frac{1}{2}(24)(18) = 216$	28	6048
Σ	480		8952



Then $\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(480 \text{ mm}^2) = 8952 \text{ mm}^3$

$$\bar{Y} = 18.65 \text{ mm}$$

Now $\bar{I}_x = (I_x)_1 + (I_x)_2$

where $(I_x)_1 = \frac{1}{12}(12 \text{ mm})(22 \text{ mm})^3 + (264 \text{ mm}^2)[(18.65 - 11) \text{ mm}]^2$
 $= 26,098 \text{ mm}^4$

$$(I_x)_2 = \frac{1}{36}(24 \text{ mm})(18 \text{ mm})^3 + (216 \text{ mm}^2)[(28 - 18.65) \text{ mm}]^2$$

$$= 22,771 \text{ mm}^4$$

Then $\bar{I}_x = (26.098 + 22.771) \times 10^3 \text{ mm}^4$


or $\bar{I}_x = 48.9 \times 10^3 \text{ mm}^4 \blacktriangleleft$

PROBLEM 9.188 (Continued)

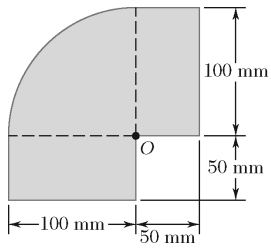
Also
$$\bar{I}_y = (I_y)_1 + (I_y)_2$$

where
$$(I_y)_1 = \frac{1}{12}(22 \text{ mm})(12 \text{ mm})^3 = 3168 \text{ mm}^4$$

$$(I_y)_2 = 2 \left[\frac{1}{36}(18 \text{ mm})(12 \text{ mm})^3 + \left(\frac{1}{2} \times 18 \text{ mm} \times 12 \text{ mm} \right) (4 \text{ mm})^2 \right]$$
$$= 5184 \text{ mm}^4$$

[(I_y)₂ is obtained by dividing A_2 into ]

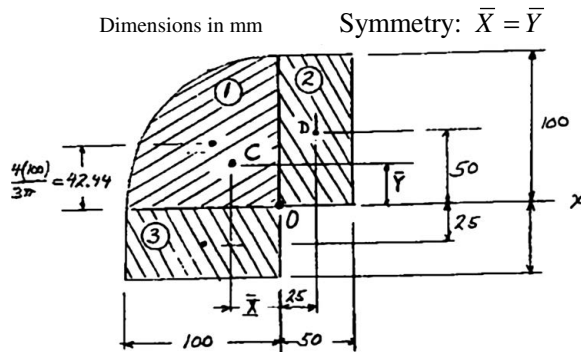
Then
$$\bar{I}_y = (3168 + 5184) \text{ mm}^4 \qquad \text{or} \qquad \bar{I}_y = 8.35 \times 10^3 \text{ mm}^4 \quad \blacktriangleleft$$



PROBLEM 9.189

Determine the polar moment of inertia of the area shown with respect to (a) Point O , (b) the centroid of the area.

SOLUTION



Determination of centroid C of entire section.

Section	Area, mm^2	\bar{y} , mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{4}(100)^2 = 7.854 \times 10^3$	42.44	333.3×10^3
2	$(50)(100) = 5 \times 10^3$	50	250×10^3
3	$(100)(50) = 5 \times 10^3$	-25	-125×10^3
Σ	17.854×10^3		458.3×10^3

$$\bar{Y} \Sigma A = \Sigma \bar{y}A: \bar{Y}(17.854 \times 10^3 \text{ mm}^2) = 458.3 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = 25.67 \text{ mm} \quad \bar{X} = \bar{Y} = 25.67 \text{ mm}$$

Distance O to C : $\overline{OC} = \sqrt{2}\bar{Y} = \sqrt{2}(25.67) = 36.30 \text{ mm}$

(a) Section 1: $J_O = \frac{\pi}{8}(100)^4 = 39.27 \times 10^6 \text{ mm}^4$

Section 2: $J_O = J + A(\overline{OD})^2 = \frac{1}{12}(50)(100)[50^2 + 100^2] + (50)(100) \left[\left(\frac{50}{2} \right)^2 + \left(\frac{100}{2} \right)^2 \right]$

$$J_O = 5.208 \times 10^6 + 15.625 \times 10^6 = 20.83 \times 10^6 \text{ mm}^4$$

PROBLEM 9.189 (Continued)

Section 3: Same as Section 2; $J_O = 20.83 \times 10^6 \text{ mm}^4$

Entire section:

$$\begin{aligned} J_O &= 39.27 \times 10^6 + 2(20.83 \times 10^6) \\ &= 80.94 \times 10^6 \end{aligned}$$

$$J_O = 80.9 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

(b) Recall that,

$$\overline{OC} = 36.30 \text{ mm} \quad \text{and} \quad A = 17.854 \times 10^3 \text{ mm}^2$$

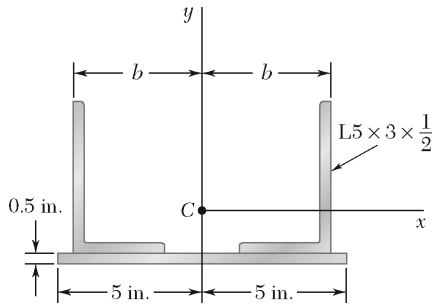
$$J_O = \bar{J}_C + A(\overline{OC})^2$$

$$80.94 \times 10^6 \text{ mm}^4 = \bar{J}_C + (17.854 \times 10^3 \text{ mm}^2)(36.30 \text{ mm})^2$$

$$\bar{J}_C = 57.41 \times 10^6 \text{ mm}^4$$

$$\bar{J}_C = 57.4 \times 10^6 \text{ mm}^4 \quad \blacktriangleleft$$

PROBLEM 9.190

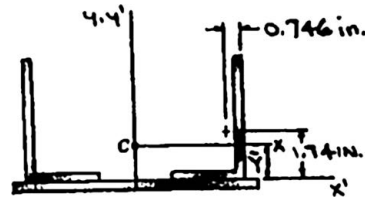


Two $L5 \times 3 \times \frac{1}{2}$ -in. angles are welded to a $\frac{1}{2}$ -in. steel plate. Determine the distance b and the centroidal moments of inertia \bar{I}_x and \bar{I}_y of the combined section, knowing that $\bar{I}_y = 4\bar{I}_x$.

SOLUTION

Angle: $A = 3.75 \text{ in}^2$
 $\bar{I}_x = 9.43 \text{ in}^4$
 $\bar{I}_y = 2.55 \text{ in}^4$

First locate centroid C of the section.



	Area, in^2	\bar{y} , in.	$\bar{y}A$, in^3
Angle	$2(3.75) = 7.50$	1.74	13.05
Plate	$(10)(0.5) = 5$	-0.25	-1.25
Σ	12.50		11.80

Then $\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(12.50 \text{ in}^2) = 11.80 \text{ in}^3$

or $\bar{Y} = 0.944 \text{ in.}$

Now $\bar{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$

where $(I_x)_{\text{angle}} = I_x + Ad^2 = 9.43 \text{ in}^4 + (3.75 \text{ in}^2)[(1.74 - 0.944) \text{ in.}]^2$
 $= 11.8061 \text{ in}^4$

$$(I_x)_{\text{plate}} = \bar{I}_x + Ad^2 = \frac{1}{12}(10 \text{ in.})(0.5 \text{ in.})^3 + (5 \text{ in}^2)[(0.25 + 0.944) \text{ in.}]^2$$

$$= 7.2323 \text{ in}^4$$

Then $\bar{I}_x = [2(11.8061) + 7.2323] \text{ in}^4 = 30.8445 \text{ in}^4$

or $\bar{I}_x = 30.8 \text{ in}^4 \blacktriangleleft$

PROBLEM 9.190 (Continued)

We have

$$\bar{I}_y = 4\bar{I}_x = 4(30.8445 \text{ in}^4) = 123.378 \text{ in}^4$$

$$\text{or } \bar{I}_y = 123.4 \text{ in}^4 \blacktriangleleft$$

Now

$$\bar{I}_y = 2(I_y)_{\text{angle}} + (\bar{I}_y)_{\text{plate}}$$

where

$$(\bar{I}_y)_{\text{angle}} = \bar{I}_y + Ad^2 = 2.55 \text{ in}^4 + (3.75 \text{ in}^2)[(b - 0.746) \text{ in.}]^2$$

$$(\bar{I}_y)_{\text{plate}} = \frac{1}{12}(0.5 \text{ in.})(10 \text{ in.})^3 = 41.6667 \text{ in}^4$$

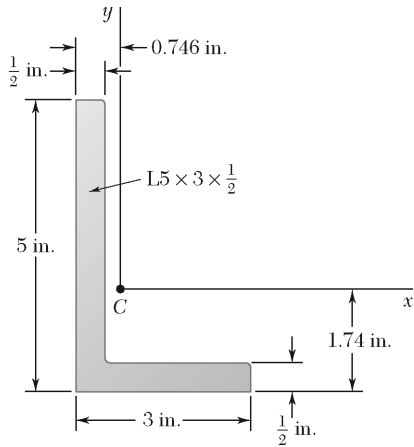
Then

$$123.378 \text{ in}^4 = 2[2.55 + 3.75(b - 0.746)^2] \text{ in}^4 + 41.6667 \text{ in}^4$$

$$\text{or } b = 3.94 \text{ in. } \blacktriangleleft$$

PROBLEM 9.191

Using the parallel-axis theorem, determine the product of inertia of the $L5 \times 3 \times \frac{1}{2}$ -in. angle cross section shown with respect to the centroidal x and y axes.



SOLUTION

We have

$$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$$

For each rectangle:

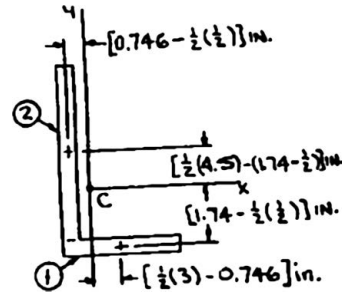
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$$

and

$$\bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

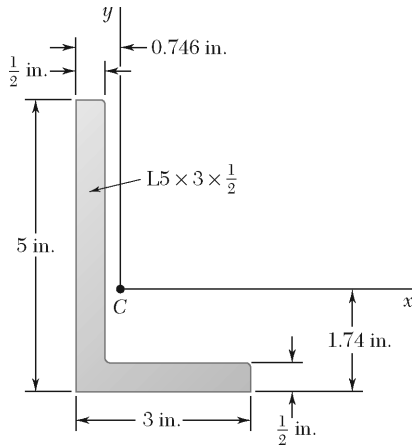
Thus

$$I_{xy} = \Sigma \bar{x}\bar{y}A$$



	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}\bar{y}A, \text{in}^4$
1	$3 \times \frac{1}{2} = 1.5$	0.754	-1.49	-1.68519
2	$4.5 \times \frac{1}{2} = 2.25$	-0.496	1.01	-1.12716
Σ				-2.81235

$$\bar{I}_{xy} = -2.81 \text{ in}^4 \blacktriangleleft$$



PROBLEM 9.192

For the $L5 \times 3 \times \frac{1}{2}$ -in. angle cross section shown, use Mohr's circle to determine (a) the moments of inertia and the product of inertia with respect to new centroidal axes obtained by rotating the x and y axes 30° clockwise, (b) the orientation of the principal axes through the centroid and the corresponding values of the moments of inertia.

SOLUTION

From Figure 9.13a:

$$\bar{I}_x = 9.43 \text{ in}^4 \quad \bar{I}_y = 2.55 \text{ in}^4$$

From the solution to Problem 9.191

$$\bar{I}_{xy} = -2.81235 \text{ in}^4$$

The Mohr's circle is defined by the diameter XY , where

$$X(9.43 - 2.81235), \quad Y(2.55, 2.81235)$$

Now

$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(9.43 + 2.55) = 5.99 \text{ in}^4$$

and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(9.43 - 2.55)\right]^2 + (-2.81235)^2}$$

$$= 4.4433 \text{ in}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-2.81235)}{9.43 - 2.55}$$

$$= 0.81754$$

or

$$2\theta_m = 39.267^\circ$$

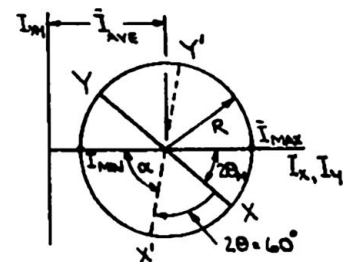
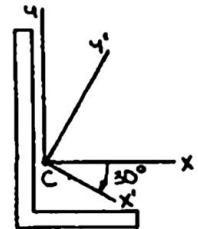
and

$$\theta_m = 19.6335^\circ$$

Then

$$\alpha = 180^\circ - (39.267^\circ + 60^\circ)$$

$$= 80.733^\circ$$



PROBLEM 9.192 (Continued)

(a) We have

$$\bar{I}_{x'} = \bar{I}_{ave} - R \cos \alpha = 5.99 - 4.4433 \cos 80.733^\circ$$

$$\text{or } \bar{I}_{x'} = 5.27 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_{y'} = \bar{I}_{ave} + R \cos \alpha = 5.99 + 4.4433 \cos 80.733^\circ$$

$$\text{or } \bar{I}_{y'} = 6.71 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_{x'y'} = -R \sin \alpha = -4.4433 \sin 80.733^\circ$$

$$\text{or } \bar{I}_{x'y'} = -4.39 \text{ in}^4 \blacktriangleleft$$

(b)

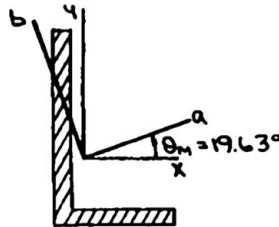
First observe that the principal axes are obtained by rotating the xy axes through 19.63° counterclockwise \blacktriangleleft about C .

Now

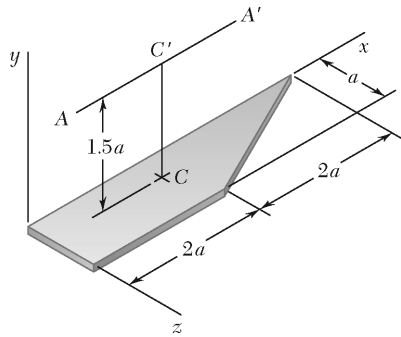
$$\bar{I}_{\max, \min} = \bar{I}_{ave} \pm R = 5.99 \pm 4.4433$$

$$\text{or } \bar{I}_{\max} = 10.43 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_{\min} = 1.547 \text{ in}^4 \blacktriangleleft$$



From the Mohr's circle it is seen that the a axis corresponds to \bar{I}_{\max} and the b axis corresponds to \bar{I}_{\min} .



PROBLEM 9.193

A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (a) the x axis, (b) the y axis.

SOLUTION

First note

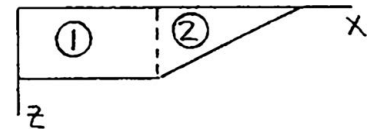
$$\begin{aligned} \text{mass} = m &= \rho V = \rho t A \\ &= \rho t \left[(2a)(a) + \frac{1}{2}(2a)(a) \right] = 3\rho t a^2 \end{aligned}$$

Also

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{3a^2} I_{\text{area}}$$

(a) Now

$$\begin{aligned} I_{x,\text{area}} &= (I_x)_{1,\text{area}} + (I_x)_{2,\text{area}} \\ &= \frac{1}{3}(2a)(a)^3 + \frac{1}{12}(2a)(a)^3 \\ &= \frac{5}{6}a^4 \end{aligned}$$



Then

$$I_{x,\text{mass}} = \frac{m}{3a^2} \times \frac{5}{6}a^4 \quad \text{or} \quad I_{x,\text{mass}} = \frac{5}{18}ma^2 \quad \blacktriangleleft$$

(b) We have

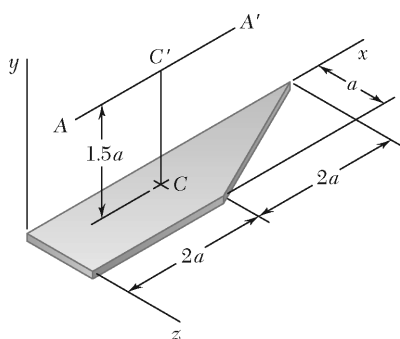
$$\begin{aligned} I_{z,\text{area}} &= (I_z)_{1,\text{area}} + (I_z)_{2,\text{area}} \\ &= \left[\frac{1}{3}(a)(2a)^3 \right] + \left[\frac{1}{36}(a)(2a)^3 + \frac{1}{2}(2a)(a) \left(2a + \frac{1}{3} \times 2a \right)^2 \right] = 10a^4 \end{aligned}$$

Then

$$I_{x,\text{mass}} = \frac{m}{3a^2} \times 10a^4 = \frac{10}{3}ma^2$$

Finally,

$$\begin{aligned} I_{y,\text{mass}} &= I_{x,\text{mass}} + I_{z,\text{mass}} \\ &= \frac{5}{18}ma^2 + \frac{10}{3}ma^2 \\ &= \frac{65}{18}ma^2 \quad \text{or} \quad I_{y,\text{mass}} = 3.61ma^2 \quad \blacktriangleleft \end{aligned}$$



PROBLEM 9.194

A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (a) the centroidal axis CC' that is perpendicular to the plate, (b) the axis AA' that is parallel to the x axis and is located at a distance $1.5a$ from the plate.

SOLUTION

First locate the centroid C .

$$\bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(2a^2 + a^2) = a(2a^2) + \left(2a + \frac{1}{3} \times 2a\right)(a^2)$$

or
$$\bar{X} = \frac{14}{9}a$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A: \quad \bar{Z}(2a^2 + a^2) = \left(\frac{1}{2}a\right)(2a^2) + \left(\frac{1}{3}a\right)(a^2)$$

or
$$\bar{Z} = \frac{4}{9}a$$

(a) We have
$$I_{y, \text{mass}} = \bar{I}_{CC', \text{mass}} + m(\bar{X}^2 + \bar{Z}^2)$$

From the solution to Problem 9.117:

$$I_{y, \text{mass}} = \frac{65}{18}ma^2$$

Then
$$\bar{I}_{cc', \text{mass}} = \frac{65}{18}ma^2 - m \left[\left(\frac{14}{9}a\right)^2 + \left(\frac{4}{9}a\right)^2 \right]$$

or
$$\bar{I}_{cc'} = 0.994ma^2 \quad \blacktriangleleft$$

(b) We have
$$I_{x, \text{mass}} = \bar{I}_{BB', \text{mass}} + m(\bar{Z})^2$$

and
$$I_{AA', \text{mass}} = \bar{I}_{BB', \text{mass}} + m(1.5a)^2$$

Then
$$I_{AA', \text{mass}} = I_{x, \text{mass}} + m \left[(1.5a)^2 - \left(\frac{4}{9}a\right)^2 \right]$$

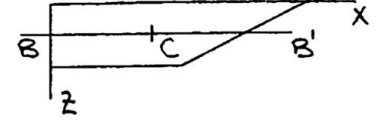
PROBLEM 9.194 (Continued)

From the solution to Problem 9.193:

$$I_{x,\text{mass}} = \frac{5}{18}ma^2$$

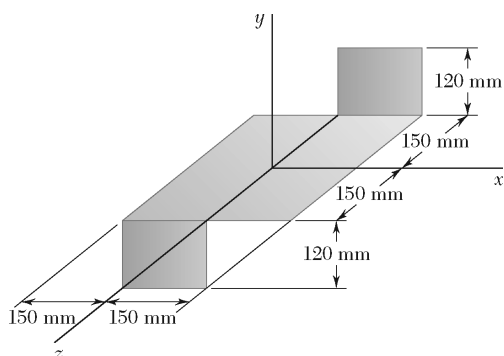
Then

$$I_{AA',\text{mass}} = \frac{5}{18}ma^2 + m \left[(1.5a)^2 - \left(\frac{4}{9}a \right)^2 \right]$$



or $I_{AA'} = 2.33ma^2$ ◀

PROBLEM 9.195



A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3 , determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$\begin{aligned} m_1 &= (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.3 \text{ m})^2 \\ &= 1.413 \text{ kg} \end{aligned}$$

$$\begin{aligned} m_2 &= m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m}) \times (0.15 \times 0.12) \text{ m}^2 \\ &= 0.2826 \text{ kg} \end{aligned}$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned} I_x &= (I_x)_1 + 2(I_x)_2 \\ &= \left[\frac{1}{12} (1.413 \text{ kg})(0.3 \text{ m})^2 \right] \\ &\quad + 2 \left[\frac{1}{12} (0.2826 \text{ kg})(0.12 \text{ m})^2 + (0.2826 \text{ kg})(0.15^2 + 0.06^2) \text{ m}^2 \right] \\ &= [(0.0105975) + 2(0.0003391 + 0.0073759)] \text{ kg} \cdot \text{m}^2 \\ &= [(0.0105975) + 2(0.0077150)] \text{ kg} \cdot \text{m}^2 \\ &\quad \text{or } I_x = 26.0 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$

$$\begin{aligned} I_y &= (I_y)_1 + 2(I_y)_2 \\ &= \left[\frac{1}{12} (1.413 \text{ kg})(0.3^2 + 0.3^2) \text{ m}^2 \right] \\ &\quad + 2 \left[\frac{1}{12} (0.2826 \text{ kg})(0.15 \text{ m})^2 + (0.2826 \text{ kg})(0.075^2 + 0.15^2) \text{ m}^2 \right] \\ &= [(0.0211950) + 2(0.0005299 + 0.0079481)] \text{ kg} \cdot \text{m}^2 \\ &= [(0.0211950) + 2(0.0084780)] \text{ kg} \cdot \text{m}^2 \\ &\quad \text{or } I_y = 38.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft \end{aligned}$$

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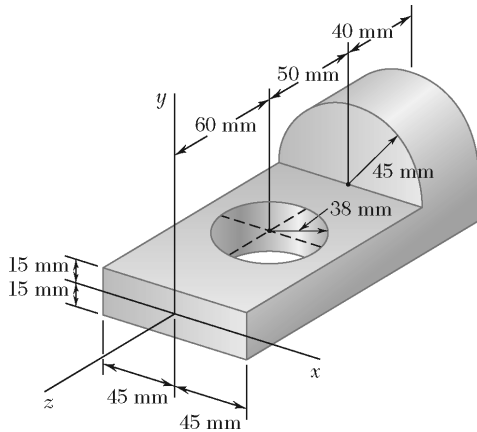
PROBLEM 9.195 (Continued)

$$\begin{aligned} I_z &= (I_z)_1 + 2(I_z)_2 \\ &= \left[\frac{1}{12} (1.413 \text{ kg})(0.3 \text{ m})^2 \right] \\ &\quad + 2 \left[\frac{1}{12} (0.2826 \text{ kg})(0.15^2 + 0.12^2) \text{ m}^2 + (0.2826 \text{ kg})(0.075^2 + 0.06^2) \text{ m}^2 \right] \\ &= [(0.0105975) + 2(0.0008690 + 0.0026070)] \text{ kg} \cdot \text{m}^2 \\ &= [(0.0105975) + 2(0.0034760)] \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_z = 17.55 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.196

Determine the mass moment of inertia and the radius of gyration of the steel machine element shown with respect to the x axis. (The density of steel is 7850 kg/m^3 .)



SOLUTION

First compute the mass of each component. We have

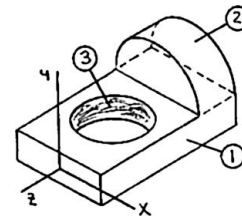
$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(0.09 \times 0.03 \times 0.15) \text{ m}^3 \\ = 3.17925 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3) \left[\frac{\pi}{2} (0.045)^2 \times 0.04 \right] \text{ m}^3 \\ = 0.998791 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3) [\pi (0.038)^2 \times 0.03] \text{ m}^3 = 1.06834 \text{ kg}$$



Using Figure 9.28 for components 1 and 3 and the equation derived above (before the solution to Problem 9.142) for a semicylinder, we have

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3 \\ = \left[\frac{1}{12} (3.17925 \text{ kg})(0.03^2 + 0.15^2) \text{ m}^2 + (3.17925 \text{ kg})(0.075 \text{ m})^2 \right] \\ + \left\{ (0.998791 \text{ kg}) \left[\left(\frac{1}{4} - \frac{16}{9\pi^2} \right) (0.045 \text{ m})^2 + \frac{1}{12} (0.04 \text{ m})^2 \right] \right. \\ \left. + (0.998791 \text{ kg}) \left[(0.13)^2 + \left(\frac{4 \times 0.045}{3\pi} + 0.015 \right)^2 \right] \text{ m}^2 \right\} \\ - \left\{ \frac{1}{12} (1.06834 \text{ kg}) [3(0.038 \text{ m})^2 + (0.03 \text{ m})^2] + (1.06834 \text{ kg})(0.06 \text{ m})^2 \right\}$$

PROBLEM 9.196 (Continued)

$$\begin{aligned} &= [(0.0061995 + 0.0178833) + (0.0002745 + 0.0180409) \\ &\quad - (0.0004658 + 0.0038460)] \text{ kg} \cdot \text{m}^2 \\ &= (0.0240828 + 0.0183154 - 0.0043118) \text{ kg} \cdot \text{m}^2 \\ &= 0.0380864 \text{ kg} \cdot \text{m}^2 \end{aligned}$$

$$\text{or } I_x = 38.1 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

Now

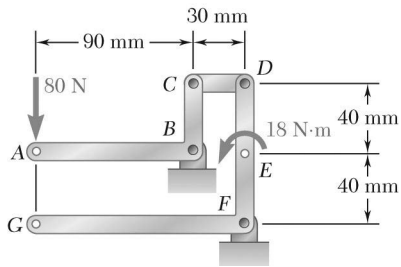
$$\begin{aligned} m &= m_1 + m_2 - m_3 = (3.17925 + 0.998791 - 1.06834) \text{ kg} \\ &= 3.10970 \text{ kg} \end{aligned}$$

and

$$k_x^2 = \frac{I_x}{m} = \frac{0.0380864 \text{ kg} \cdot \text{m}^2}{3.10970 \text{ kg}}$$

$$\text{or } k_x = 110.7 \text{ mm} \blacktriangleleft$$

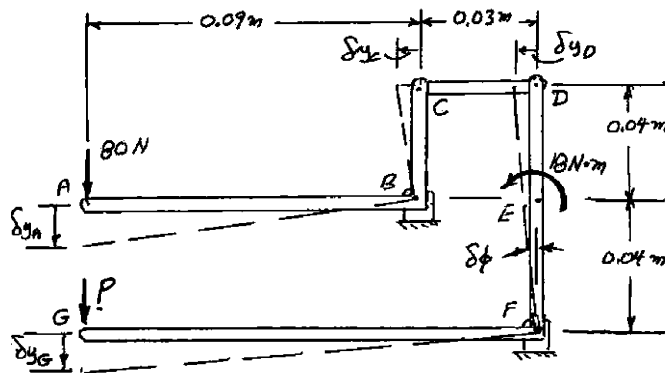
CHAPTER 10



PROBLEM 10.1

Determine the vertical force P that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{0.04}{0.09} \delta y_A = \frac{4}{9} \delta y_A \leftarrow, \quad \delta y_D = \delta y_C = \frac{4}{9} \delta y_A \leftarrow$$

$$\delta y_G = \frac{0.12 \text{ m}}{0.08 \text{ m}} \delta y_D = 1.5 \left(\frac{4}{9} \delta y_A \right) = \frac{2}{3} \delta y_A \downarrow$$

$$\delta \phi = \frac{\delta y_D}{0.08} = \frac{4}{9} \delta y_A / 0.08 = \frac{4}{0.72} \delta y_A = \frac{50}{9} \delta y_A \curvearrowright$$

Virtual Work:

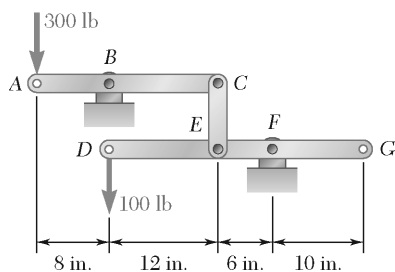
$$\delta U = 0: \quad (80 \text{ N}) \delta y_A + (18 \text{ N} \cdot \text{m}) \delta \phi + P \delta y_G = 0$$

$$80 \delta y_A + 18 \left(\frac{50}{9} \delta y_A \right) + P \left(\frac{2}{3} \delta y_A \right) = 0$$

$$80 + 100 + \frac{2}{3} P = 0$$

$$P = -270 \text{ N}$$

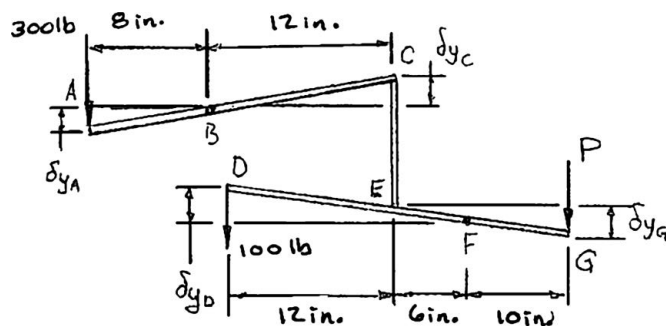
$$P = 270 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 10.2

Determine the vertical force P that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



Assuming $\delta y_A \downarrow$

it follows

$$\delta y_C = \frac{12}{8} \delta y_A = 1.5 \delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5 \delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6} \delta y_A = 3(1.5 \delta y_A) = 4.5 \delta y_A \uparrow$$

$$\delta y_G = \frac{10}{6} \delta y_A = \frac{10}{6} (1.5 \delta y_A) = 2.5 \delta y_A \downarrow$$

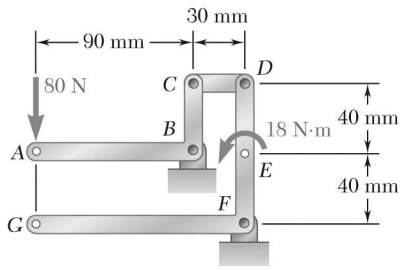
Then, by virtual work

$$\delta U = 0: (300 \text{ lb})\delta y_A - (100 \text{ lb})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

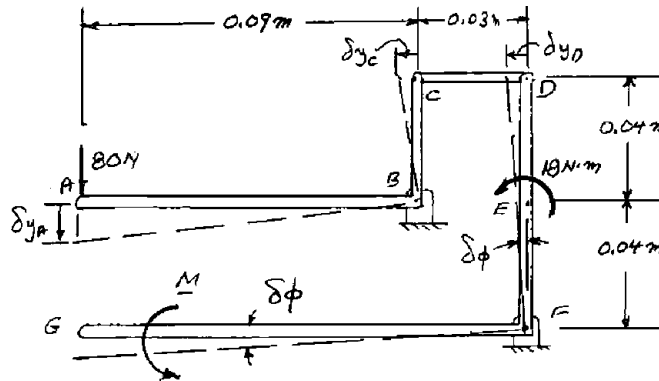
$$P = 60.0 \text{ lb} \quad \mathbf{P = 60.0 \text{ lb} \downarrow \blacktriangleleft}$$



PROBLEM 10.3

Determine the couple M that must be applied to member $DEFG$ to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{0.04}{0.09} \delta y_A = \frac{4}{9} \delta y_A \leftarrow, \quad \delta y_D = \delta y_C = \frac{4}{9} \delta y_A \leftarrow$$

$$\delta \phi = \frac{\delta y_C}{0.08} = \frac{4}{9} \delta y_A / 0.08 = \frac{4}{0.72} \delta y_A = \frac{50}{9} \delta y_A \rightarrow$$

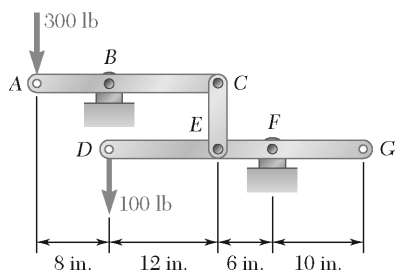
Virtual Work:

$$\delta U = 0: \quad (80 \text{ N}) \delta y_A + (18 \text{ N} \cdot \text{m}) \delta \phi + M \delta \phi = 0$$

$$80 \delta y_A + 18 \left(\frac{50}{9} \delta y_A \right) + M \left(\frac{50}{9} \delta y_A \right) = 0$$

$$80 + 100 + \frac{50}{9} M = 0$$

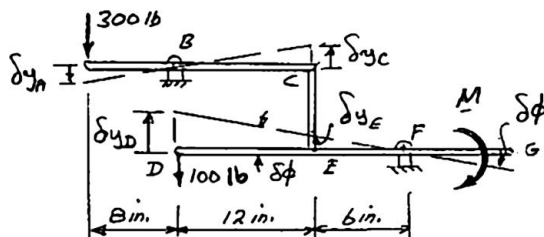
$$M = -32.4 \text{ N} \cdot \text{m} \quad \mathbf{M} = 32.4 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$



PROBLEM 10.4

Determine the couple M that must be applied to member $DEFG$ to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{12}{8} \delta y_A = 1.5 \delta y_A \uparrow, \quad \delta y_E = \delta y_C = 1.5 \delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6} \delta y_E = 3(1.5 \delta y_A) = 4.5 \delta y_A \uparrow$$

$$\delta \phi = \frac{\delta y_E}{6} = \frac{1.5 \delta y_A}{6} = \frac{1}{4} \delta y_A \curvearrowright$$

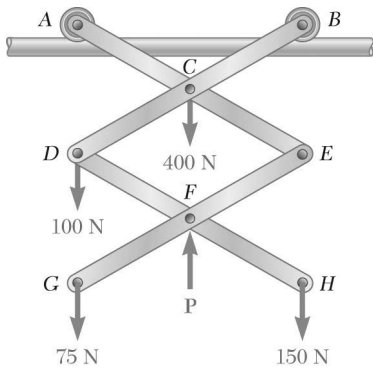
Virtual Work:

$$\delta U = 0: \quad (300 \text{ lb})\delta y_A - (100 \text{ lb})\delta y_D + M \delta \phi = 0$$

$$300 \delta y_A - 100(1.5 \delta y_A) + M \left(\frac{1}{4} \delta y_A \right) = 0$$

$$300 - 450 + \frac{1}{4} M = 0$$

$$M = +600 \text{ lb} \cdot \text{in.} \quad \mathbf{M = 600 \text{ lb} \cdot \text{in.}} \quad \blacktriangleleft$$



PROBLEM 10.5

Determine the force **P** required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at **A** and **B** roll freely on the horizontal rod.

SOLUTION

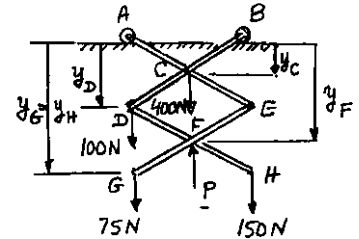
Using y_C as independent variable:

$$y_D = 2y_C \quad \delta y_D = 2\delta y_C$$

$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$

$$y_G = y_H = 4y_C$$

$$\delta y_G = \delta y_H = 4\delta y_C$$



Virtual Work:

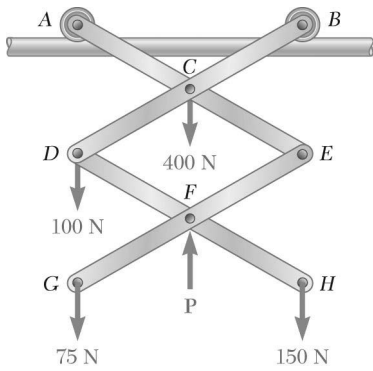
$$\delta U = (400 \text{ N})\delta y_C + (100 \text{ N})\delta y_D - P\delta y_F + (75 \text{ N})\delta y_G + (150 \text{ N})\delta y_H = 0$$

$$400\delta y_C + 100(2\delta y_C) - P(3\delta y_C) + (75 + 150)(4\delta y_C) = 0$$

$$3P = 400 + 200 + 900$$

$$P = +500 \text{ N}$$

$$P = 500 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 10.6

Solve Problem 10.5 assuming that the vertical force **P** is applied at Point *E*.

PROBLEM 10.5 Determine the force **P** required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at *A* and *B* roll freely on the horizontal rod.

SOLUTION

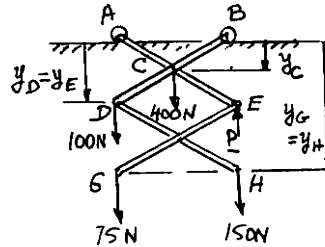
Using y_C as independent variable:

$$y_D = y_E = 2y_C$$

$$\delta y_D = \delta y_E = 2\delta y_C$$

$$y_G = y_H = 4y_C$$

$$\delta y_G = \delta y_H = 4\delta y_C$$



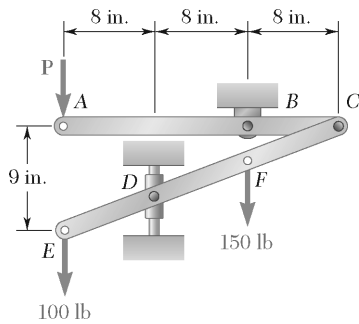
Virtual Work:

$$\delta U = (400 \text{ N})\delta y_C + (100 \text{ N})\delta y_D - P\delta y_E + (75 \text{ N})\delta y_G + (150 \text{ N})\delta y_H = 0$$

$$400\delta y_C + 100(2\delta y_C) - P(2\delta y_C) + (75 + 150)(4\delta y_C) = 0$$

$$2P = 400 + 200 + 900 \qquad P = +750 \text{ N}$$

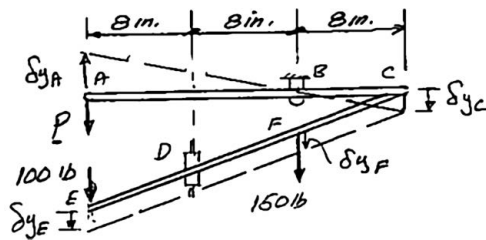
$$P = 750 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 10.7

The two-bar linkage shown is supported by a pin and bracket at B and a collar at D that slides freely on a vertical rod. Determine the force P required to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \uparrow$:

$$\delta y_C = \frac{8 \text{ in.}}{16 \text{ in.}} \delta y_A; \quad \delta y_C = \frac{1}{2} \delta y_A \downarrow$$

Since bar CD move in translation

$$\delta y_E = \delta y_F = \delta y_C$$

or

$$\delta y_E = \delta y_F = \frac{1}{2} \delta y_A \downarrow$$

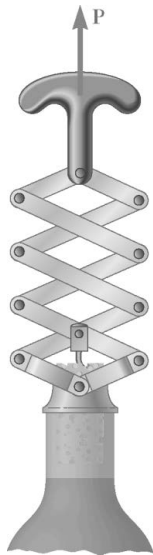
Virtual Work:

$$\delta U = 0: \quad -P \delta y_A + (100 \text{ lb}) \delta y_E + (150 \text{ lb}) \delta y_F = 0$$

$$-P \delta y_A + 100 \left(\frac{1}{2} \delta y_A \right) + 150 \left(\frac{1}{2} \delta y_A \right) = 0$$

$$P = 125 \text{ lb}$$

$$P = 125.0 \text{ lb} \downarrow \blacktriangleleft$$



PROBLEM 10.8

Knowing that the maximum friction force exerted by the bottle on the cork is 60 lb, determine (a) the force \mathbf{P} that must be applied to the corkscrew to open the bottle, (b) the maximum force exerted by the base of the corkscrew on the top of the bottle.

SOLUTION

From sketch

$$y_A = 4y_C$$

Thus

$$\delta y_A = 4\delta y_C$$

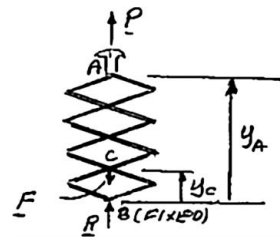
(a) Virtual Work:

$$\delta U = 0: P\delta y_A - F\delta y_C = 0$$

$$P(4\delta y_C) - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

$$F = 60 \text{ lb}: P = \frac{1}{4}(60 \text{ lb}) = 15 \text{ lb}$$

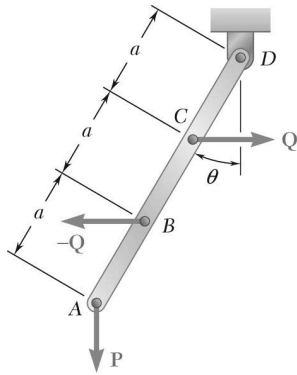


$$\mathbf{P} = 15.00 \text{ lb} \uparrow \blacktriangleleft$$

(b) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0 \quad R + P - F = 0; \quad R + 15 \text{ lb} - 60 \text{ lb} = 0$$

$$\text{On corkscrew: } \mathbf{R} = 45 \text{ lb} \uparrow \quad \text{On bottle: } \mathbf{R} = 45.0 \text{ lb} \downarrow \blacktriangleleft$$



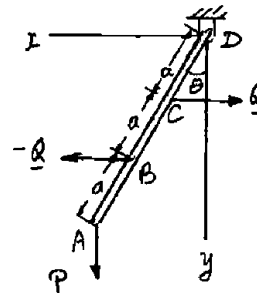
PROBLEM 10.9

Rod AD is acted upon by a vertical force P at end A , and by two equal and opposite horizontal forces of magnitude Q at points B and C . Derive an expression for the magnitude Q of the horizontal forces required for equilibrium.

SOLUTION

We have

$$\begin{aligned} x_C &= a \sin \theta \\ \delta x_C &= a \cos \theta \delta \theta \\ x_B &= 2a \sin \theta \\ \delta x_B &= 2a \cos \theta \delta \theta \\ y_A &= 3a \cos \theta \\ \delta y_A &= -3a \sin \theta \delta \theta \end{aligned}$$



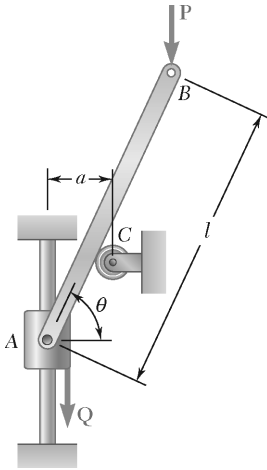
Virtual Work: We note that P tends to increase y_A and $-Q$ tends to increase x_B , while Q tends to decrease x_C . Therefore

$$\begin{aligned} \delta U &= P \delta y_A + Q \delta x_B - Q \delta x_C = 0 \\ &= P(-3a \sin \theta \delta \theta) + Q(2a \cos \theta \delta \theta) - Q(a \cos \theta \delta \theta) = 0 \\ Q \cos \theta &= 3P \sin \theta' \end{aligned}$$

$$Q = 3P \tan \theta \quad \blacktriangleleft$$

PROBLEM 10.10

The slender rod AB is attached to a collar A and rests on a small wheel at C . Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force Q required to maintain the equilibrium of the rod.



SOLUTION

For $\triangle AA'C$:

$$A'C = a \tan \theta$$

$$y_A = -(A'C) = -a \tan \theta$$

$$\delta y_A = -\frac{a}{\cos^2 \theta} \delta \theta$$

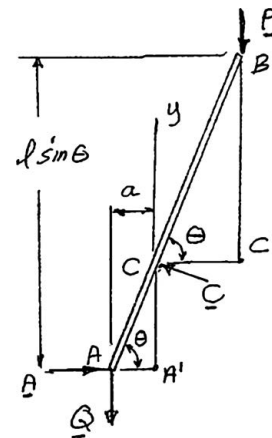
For $\triangle CC'B$:

$$BC' = l \sin \theta - A'C$$

$$= l \sin \theta - a \tan \theta$$

$$y_B = BC' = l \sin \theta - a \tan \theta$$

$$\delta y_B = l \cos \theta \delta \theta - \frac{a}{\cos^2 \theta} \delta \theta$$



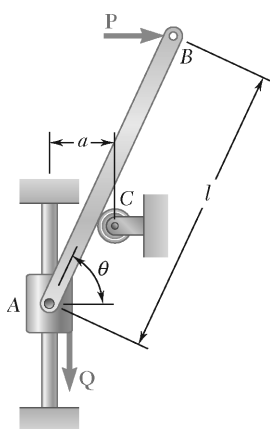
Virtual Work:

$$\delta U = 0: Q \delta y_A - P \delta y_B = 0$$

$$-Q \left(-\frac{a}{\cos^2 \theta} \right) \delta \theta - P \left(l \cos \theta - \frac{a}{\cos^2 \theta} \right) \delta \theta = 0$$

$$Q \left(\frac{a}{\cos^2 \theta} \right) = P \left(l \cos \theta - \frac{a}{\cos^2 \theta} \right)$$

$$Q = P \left(\frac{l}{a} \cos^3 \theta - 1 \right) \blacktriangleleft$$



PROBLEM 10.11

The slender rod AB is attached to a collar A and rests on a small wheel at C . Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force Q required to maintain the equilibrium of the rod.

SOLUTION

For $\triangle AA'C$:

$$A'C = a \tan \theta$$

$$y_A = -(A'C) = -a \tan \theta$$

$$\delta y_A = -\frac{a}{\cos^2 \theta} \delta \theta$$

For $\triangle BB'C$:

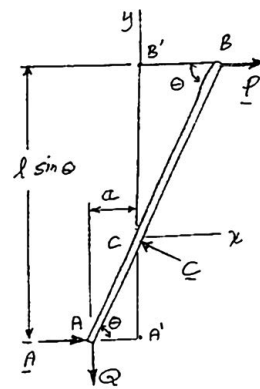
$$B'C = l \sin \theta - A'C$$

$$= l \sin \theta - a \tan \theta$$

$$BB' = \frac{B'C}{\tan \theta} = \frac{l \sin \theta - a \tan \theta}{\tan \theta}$$

$$x_B = BB' = l \cos \theta - a$$

$$\delta x_B = -l \sin \theta \delta \theta$$



Virtual Work:

$$\delta U = 0: P \delta x_B - Q \delta y_A = 0$$

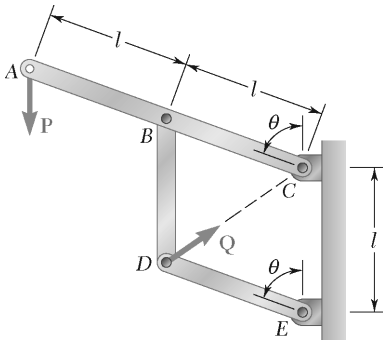
$$P(-l \sin \theta \delta \theta) - Q \left(-\frac{a}{\cos^2 \theta} \delta \theta \right) = 0$$

or

$$Pl \sin \theta \cos^2 \theta = Qa$$

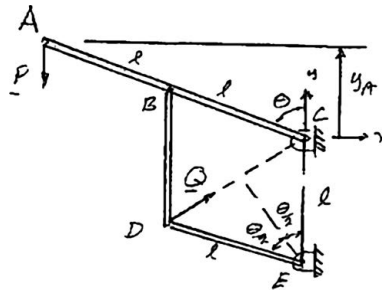
$$Q = P \frac{l}{a} \sin \theta \cos^2 \theta \quad \blacktriangleleft$$

PROBLEM 10.12



Knowing that the line of action of the force Q passes through Point C , derive an expression for the magnitude of Q required to maintain equilibrium.

SOLUTION



We have

$$y_A = 2l \cos \theta; \quad \delta y_A = -2l \sin \theta \delta \theta$$

$$CD = 2l \sin \frac{\theta}{2}; \quad \delta(CD) = l \cos \frac{\theta}{2} \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad -P \delta y_A - Q \delta(CD) = 0$$

$$-P(-2l \sin \theta \delta \theta) - Q \left(l \cos \frac{\theta}{2} \delta \theta \right) = 0$$

$$Q = 2P \frac{\sin \theta}{\cos(\theta/2)} \quad \blacktriangleleft$$

PROBLEM 10.13

Solve Problem 10.12 assuming that the force **P** applied at Point A acts horizontally to the left.

PROBLEM 10.12 Knowing that the line of action of the force **Q** passes through Point C, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION

We have

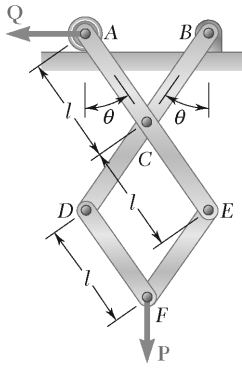
Virtual Work:

$$x_A = 2l \cos \theta; \quad \delta x_A = -2l \sin \theta \delta \theta$$

$$CD = 2l \sin \frac{\theta}{2}; \quad \delta(CD) = l \cos \frac{\theta}{2} \delta \theta$$

$$\delta U = 0: \quad P \delta x_A - Q \delta(CD) = 0$$

$$P(2l \cos \theta \delta \theta) - Q \left(l \cos \frac{\theta}{2} \delta \theta \right) = 0 \qquad Q = 2P \frac{\cos \theta}{\cos(\theta/2)} \blacktriangleleft$$



PROBLEM 10.14

The mechanism shown is acted upon by the force **P**; derive an expression for the magnitude of the force **Q** required to maintain equilibrium.

SOLUTION

Virtual Work:

We have

$$x_A = 2l \sin \theta$$

$$\delta x_A = 2l \cos \theta \delta \theta$$

and

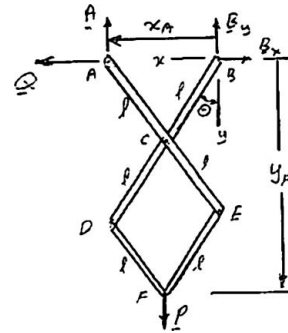
$$y_F = 3l \cos \theta$$

$$\delta y_F = -3l \sin \theta \delta \theta$$

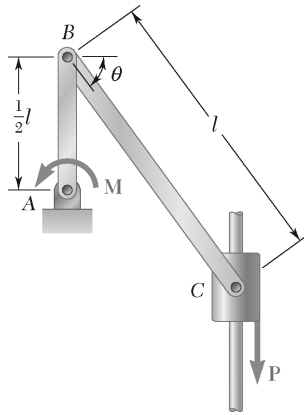
Virtual Work:

$$\delta U = 0: \quad Q \delta x_A + P \delta y_F = 0$$

$$Q(2l \cos \theta \delta \theta) + P(-3l \sin \theta \delta \theta) = 0$$



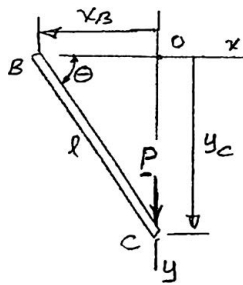
$$Q = \frac{3}{2} P \tan \theta \quad \blacktriangleleft$$



PROBLEM 10.15

Derive an expression for the magnitude of the couple M required to maintain the equilibrium of the linkage shown.

SOLUTION



We have $x_B = l \cos \theta$

$$\delta x_B = -l \sin \theta \delta \theta \quad (1)$$

$$y_C = l \sin \theta$$

$$\delta y_C = l \cos \theta \delta \theta$$

Now

$$\delta x_B = \frac{1}{2} l \delta \theta$$

Substituting from Equation (1)

$$-l \sin \theta \delta \theta = \frac{1}{2} l \delta \theta$$

or

$$\delta \theta = -2 \sin \theta \delta \theta$$

Virtual Work:

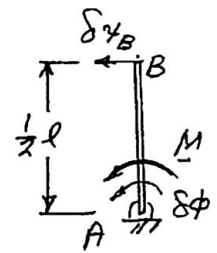
$$\delta U = 0: \quad M \delta \theta + P \delta y_C = 0$$

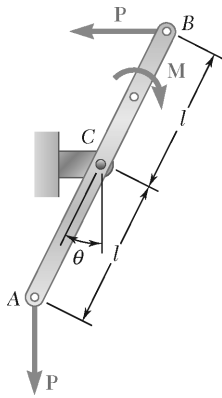
$$M (-2 \sin \theta \delta \theta) + P (l \cos \theta \delta \theta) = 0$$

or

$$M = \frac{1}{2} Pl \frac{\cos \theta}{\sin \theta}$$

$$M = \frac{Pl}{2 \tan \theta} \blacktriangleleft$$





PROBLEM 10.16

Derive an expression for the magnitude of the couple M required to maintain the equilibrium of the linkage shown.

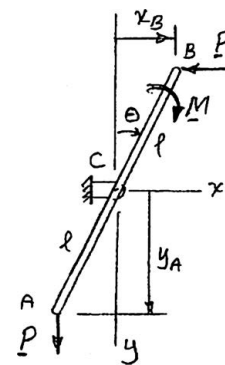
SOLUTION

We have

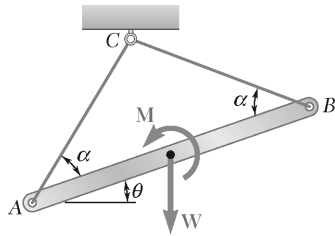
$$\begin{aligned}x_B &= l \sin \theta \\ \delta x_B &= l \cos \theta \delta \theta \\ y_A &= l \cos \theta \\ \delta y_A &= -l \sin \theta \delta \theta\end{aligned}$$

Virtual Work:

$$\begin{aligned}\delta U = 0: \quad M \delta \theta - P \delta x_B + P \delta y_A &= 0 \\ M \delta \theta - P(l \cos \theta \delta \theta) + P(-l \sin \theta \delta \theta) &= 0\end{aligned}$$



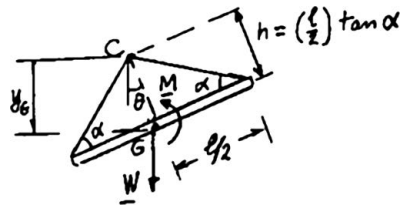
$$M = Pl(\sin \theta + \cos \theta) \quad \blacktriangleleft$$



PROBLEM 10.17

A uniform rod AB of length l and weight W is suspended from two cords AC and BC of equal length. Derive an expression for the magnitude of the couple M required to maintain equilibrium of the rod in the position shown.

SOLUTION



$$y_G = h \cos \theta = \frac{1}{2} l \tan \alpha \cos \theta$$

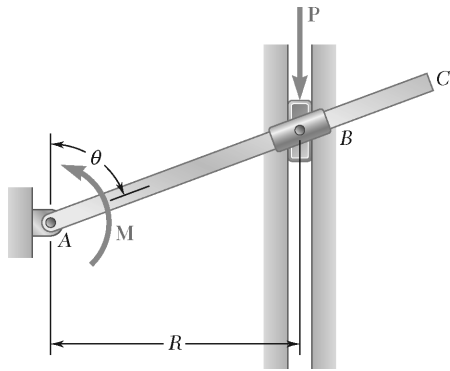
$$\delta y_G = -\frac{1}{2} l \tan \alpha \sin \theta \delta \theta$$

Virtual Work:

$$\delta U = W \delta y_G + M \delta \theta = 0$$

$$W \left(-\frac{1}{2} l \tan \alpha \sin \theta \delta \theta \right) + M \delta \theta = 0$$

$$M = \frac{1}{2} W l \tan \alpha \sin \theta$$



PROBLEM 10.18

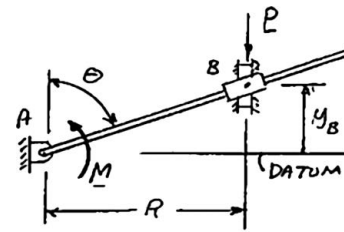
Collar B can slide along rod AC and is attached by a pin to a block that can slide in the vertical slot shown. Derive an expression for the magnitude of the couple M required to maintain equilibrium.

SOLUTION

$$y_B = \frac{R}{\tan(90^\circ - \theta)}$$

$$\delta y_B = \frac{-R\delta\theta}{\cos^2(90^\circ - \theta)}$$

$$\delta y_B = \frac{-R\delta\theta}{\sin^2 \theta}$$



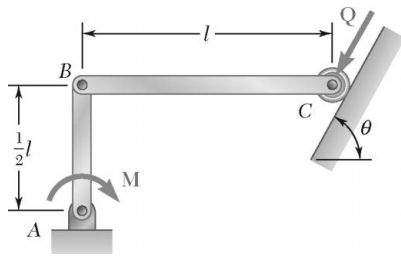
Virtual Work:

$$\delta U = 0: \quad \delta U = -M \delta\theta - P \delta y_B = 0$$

$$-M \delta\theta + PR \frac{1}{\sin^2 \theta} \delta\theta = 0$$

$$M = \frac{PR}{\sin^2 \theta}$$

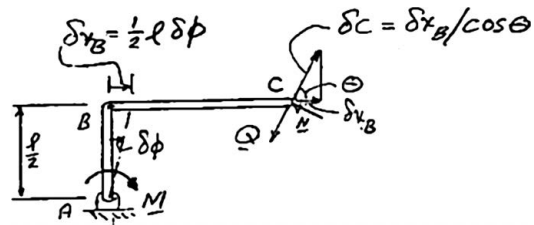
$$M = PR \csc^2 \theta \quad \blacktriangleleft$$



PROBLEM 10.19

For the linkage shown, determine the couple M required for equilibrium when $l = 1.8$ ft, $Q = 40$ lb, and $\theta = 65^\circ$.

SOLUTION



$$\delta C = \frac{\frac{1}{2} l \delta \phi}{\cos \theta}$$

Virtual Work:

$$\delta U = 0: \quad M \delta \phi - Q \delta C = 0$$

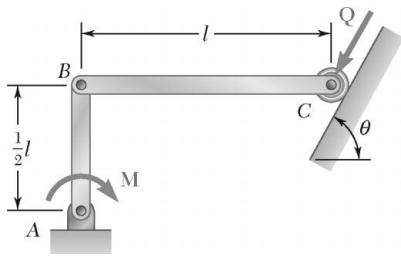
$$M \delta \phi - Q \left(\frac{1}{2} \frac{l}{\cos \theta} \right) \delta \phi = 0$$

$$M = \frac{1}{2} \frac{Ql}{\cos \theta}$$

Data:

$$M = \frac{1}{2} \frac{(40 \text{ lb})(1.8 \text{ ft})}{\cos 65^\circ} = 85.18 \text{ lb} \cdot \text{ft}$$

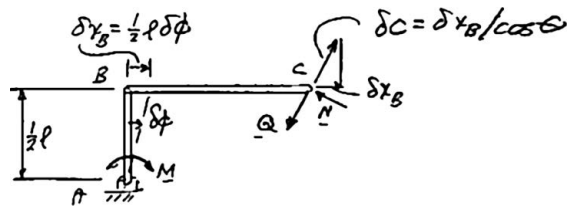
$$M = 85.2 \text{ lb} \cdot \text{ft} \quad \blacktriangleleft$$



PROBLEM 10.20

For the linkage shown, determine the force Q required for equilibrium when $l = 18$ in., $M = 600$ lb·in., and $\theta = 70^\circ$.

SOLUTION



$$\delta C = \frac{1}{2} \frac{l \delta \phi}{\cos \theta}$$

Virtual Work:

$$\delta U = 0: \quad M \delta \phi - Q \delta C = 0$$

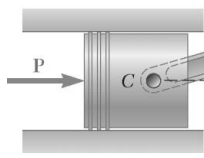
$$M \delta \phi - Q \left(\frac{1}{2} \frac{l}{\cos \theta} \right) \delta \phi = 0$$

$$Q = \frac{2M \cos \theta}{l}$$

Data:

$$Q = \frac{2(600 \text{ lb} \cdot \text{in.}) \cos 70^\circ}{18 \text{ in.}} = 22.801 \text{ lb}$$

$$Q = 22.8 \text{ lb} \nearrow 70.0^\circ \blacktriangleleft$$

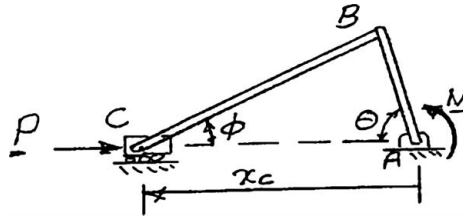


PROBLEM 10.21

A 4-kN force P is applied as shown to the piston of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the couple M required to maintain the equilibrium of the system when (a) $\theta = 30^\circ$, (b) $\theta = 150^\circ$.

SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin \phi}{AB} = \frac{\sin \theta}{BC}$$

$$\sin \phi = \frac{AB}{BC} \sin \theta \quad (1)$$

Now

$$x_c = AB \cos \theta + BC \cos \phi$$

$$\delta x_c = -AB \sin \theta \delta \theta - BC \sin \phi \delta \phi \quad (2)$$

Now, from Equation (1)

$$\cos \phi \delta \phi = \frac{AB}{BC} \cos \theta \delta \theta$$

or

$$\delta \phi = \frac{AB \cos \theta}{BC \cos \phi} \delta \theta \quad (3)$$

From Equation (2)

$$\delta x_c = -AB \sin \theta \delta \theta - BC \sin \phi \left(\frac{AB \cos \theta}{BC \cos \phi} \delta \theta \right)$$

or

$$\delta x_c = -\frac{AB}{\cos \phi} (\sin \theta \cos \phi + \sin \phi \cos \theta) \delta \theta$$

Then

$$\delta x_c = -\frac{AB \sin(\theta + \phi)}{\cos \phi} \delta \theta$$

PROBLEM 10.21 (Continued)

Virtual Work:

$$\delta U = 0: -P\delta x_C - M\delta\theta = 0$$

$$-P\left[-\frac{AB\sin(\theta + \phi)}{\cos\phi}\delta\theta\right] - M\delta\theta = 0$$

Thus,
$$M = AB\frac{\sin(\theta + \phi)}{\cos\phi}P \quad (4)$$

(a)
$$P = 4 \text{ kN}, \quad \theta = 30^\circ$$

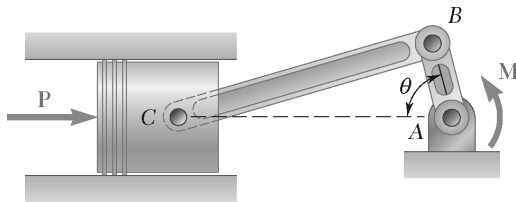
Eq. (1):
$$\sin\phi = \frac{50 \text{ mm}}{200 \text{ mm}}\sin 30^\circ \quad \phi = 7.181^\circ$$

Eq. (4):
$$M = (0.05 \text{ m})\frac{\sin(30^\circ + 7.181^\circ)}{\cos 7.181^\circ}(4 \text{ kN}) \quad \mathbf{M} = 121.8 \text{ N}\cdot\text{m} \curvearrowleft$$

(b)
$$P = 4 \text{ kN}, \quad \theta = 150^\circ$$

Eq. (1):
$$\sin\phi = \frac{50 \text{ mm}}{200 \text{ mm}}\sin 160^\circ \quad \phi = 7.181^\circ$$

Eq. (4):
$$M = (0.05 \text{ m})\frac{\sin(150^\circ + 7.181^\circ)}{\cos 7.181^\circ}(4 \text{ kN}) \quad \mathbf{M} = 78.2 \text{ N}\cdot\text{m} \curvearrowleft$$

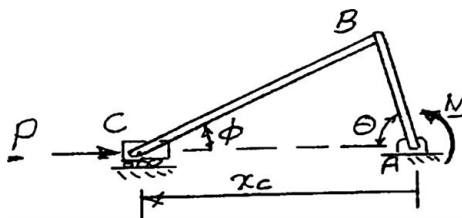


PROBLEM 10.22

A couple M of magnitude $100 \text{ N} \cdot \text{m}$ is applied as shown to the crank of the engine system. Knowing that $AB = 50 \text{ mm}$ and $BC = 200 \text{ mm}$, determine the force P required to maintain the equilibrium of the system when (a) $\theta = 60^\circ$, (b) $\theta = 120^\circ$.

SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin \phi}{AB} = \frac{\sin \theta}{BC}$$

$$\sin \phi = \frac{AB}{BC} \sin \theta \quad (1)$$

Now

$$x_c = AB \cos \theta + BC \cos \phi$$

$$\delta x_c = -AB \sin \theta \delta \theta - BC \sin \phi \delta \phi \quad (2)$$

Now, from Equation (1)

$$\cos \phi \delta \phi = \frac{AB}{BC} \cos \theta \delta \theta$$

or

$$\delta \phi = \frac{AB \cos \theta}{BC \cos \phi} \delta \theta \quad (3)$$

From Equation (2)

$$\delta x_c = -AB \sin \theta \delta \theta - BC \sin \phi \left(\frac{AB \cos \theta}{BC \cos \phi} \delta \theta \right)$$

or

$$\delta x_c = -\frac{AB}{\cos \phi} (\sin \theta \cos \phi + \sin \phi \cos \theta) \delta \theta$$

Then

$$\delta x_c = -\frac{AB \sin(\theta + \phi)}{\cos \phi} \delta \theta$$

PROBLEM 10.22 (Continued)

Virtual Work:

$$\delta U = 0: -P\delta x_C - M\delta\theta = 0$$

$$-P\left[-\frac{AB\sin(\theta + \phi)}{\cos\phi}\delta\theta\right] - M\delta\theta = 0$$

Thus,
$$M = AB\frac{\sin(\theta + \phi)}{\cos\phi}P \quad (4)$$

(a)
$$M = 100 \text{ N} \cdot \text{m}, \quad \theta = 60^\circ$$

Eq. (1):
$$\sin\phi = \frac{50 \text{ mm}}{200 \text{ mm}}\sin 60^\circ \quad \phi = 12.504^\circ$$

Eq. (4):
$$100 \text{ N} \cdot \text{m} = (0.05 \text{ m})\frac{\sin(60^\circ + 12.504^\circ)}{\cos 12.504^\circ}P$$

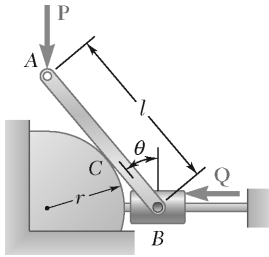
$$P = 2047 \text{ N} \quad \mathbf{P} = 2.05 \text{ kN} \rightarrow \blacktriangleleft$$

(b)
$$M = 100 \text{ N} \cdot \text{m}, \quad \theta = 120^\circ$$

Eq. (1):
$$\sin\phi = \frac{50 \text{ mm}}{200 \text{ mm}}\sin 120^\circ \quad \phi = 12.504^\circ$$

Eq. (4):
$$100 \text{ N} \cdot \text{m} = (0.05 \text{ m})\frac{\sin(120^\circ + 12.504^\circ)}{\cos 12.504^\circ}P$$

$$P = 2649 \text{ N} \quad \mathbf{P} = 2.65 \text{ kN} \rightarrow \blacktriangleleft$$



PROBLEM 10.23

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r . Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N.

SOLUTION

Geometry

$$OC = r$$

$$\cos \theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos \theta}$$

$$\delta x_B = \frac{r \sin \theta}{\cos^2 \theta} \delta \theta$$

$$y_A = l \cos \theta$$

$$\delta y_A = -l \sin \theta \delta \theta$$

Virtual Work:

$$\delta U = 0: P(-\delta y_A) - Q \delta x_B = 0$$

$$Pl \sin \theta \delta \theta - Q \frac{r \sin \theta}{\cos^2 \theta} \delta \theta = 0$$

$$\cos^2 \theta = \frac{Qr}{Pl} \quad (1)$$

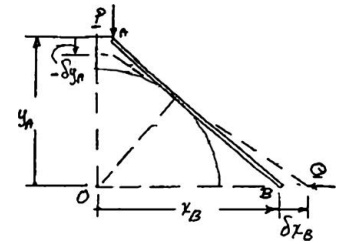
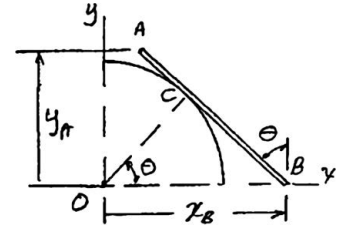
Then, with $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N

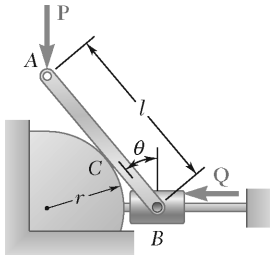
$$\cos^2 \theta = \frac{(80 \text{ N})(60 \text{ mm})}{(40 \text{ N})(200 \text{ mm})} = 0.6$$

or

$$\theta = 39.231^\circ$$

$$\theta = 39.2^\circ \blacktriangleleft$$





PROBLEM 10.24

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r . Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb.

SOLUTION

Geometry

$$OC = r$$

$$\cos \theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos \theta}$$

$$\delta x_B = \frac{r \sin \theta}{\cos^2 \theta} \delta \theta$$

$$y_A = l \cos \theta; \quad \delta y_A = -l \sin \theta \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q \delta x_B = 0$$

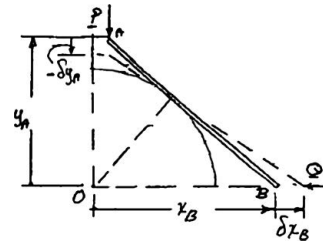
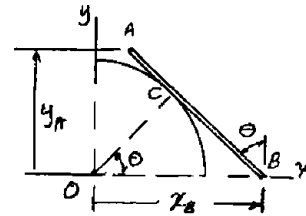
$$Pl \sin \theta \delta \theta - Q \frac{r \sin \theta}{\cos^2 \theta} \delta \theta = 0$$

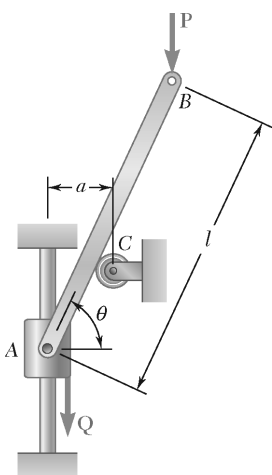
$$\cos^2 \theta = \frac{Qr}{Pl} \quad (1)$$

Then, with $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb

Eq. (1) becomes:
$$\cos^2 \theta = \frac{(150 \text{ lb})(5 \text{ in.})}{(75 \text{ lb})(14 \text{ in.})} = 0.71429$$

$$\theta = 32.3^\circ \quad \blacktriangleleft$$





PROBLEM 10.25

Determine the value of θ corresponding to the equilibrium position of the rod of Problem 10.10 when $l = 30$ in., $a = 5$ in., $P = 25$ lb, and $Q = 40$ lb.

PROBLEM 10.10 The slender rod AB is attached to a collar A and rests on a small wheel at C . Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force Q required to maintain the equilibrium of the rod.

SOLUTION

For $\triangle AA'C$:

$$A'C = a \tan \theta$$

$$y_A = -(A'C) = -a \tan \theta$$

$$\delta y_A = -\frac{a}{\cos^2 \theta} \delta \theta$$

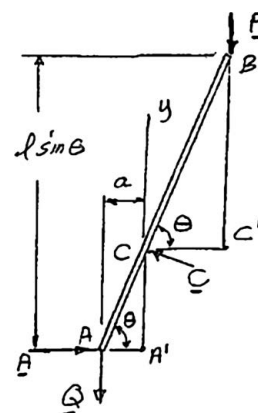
For $\triangle CC'B$:

$$BC' = l \sin \theta - A'C$$

$$= l \sin \theta - a \tan \theta$$

$$y_B = BC' = l \sin \theta - a \tan \theta$$

$$\delta y_B = l \cos \theta \delta \theta - \frac{a}{\cos^2 \theta} \delta \theta$$



Virtual Work:

$$\delta U = 0: \quad -Q \delta y_A - P \delta y_B = 0$$

$$-Q \left(-\frac{a}{\cos^2 \theta} \right) \delta \theta - P \left(l \cos \theta - \frac{a}{\cos^2 \theta} \right) \delta \theta = 0$$

$$Q \left(\frac{a}{\cos^2 \theta} \right) = P \left(l \cos \theta - \frac{a}{\cos^2 \theta} \right)$$

or

$$Q = P \left(\frac{l}{a} \cos^3 \theta - 1 \right)$$

PROBLEM 10.25 (Continued)

with

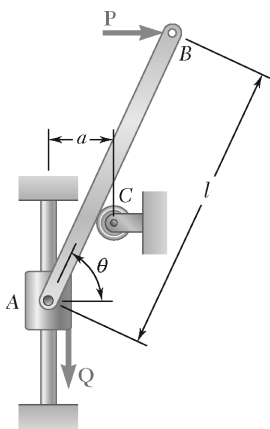
$$l = 30 \text{ in.}, a = 5 \text{ in.}, P = 25 \text{ lb, and } Q = 40 \text{ lb}$$

$$(40 \text{ lb}) = (25 \text{ lb}) \left(\frac{30 \text{ in.}}{5 \text{ in.}} \cos^3 \theta - 1 \right)$$

or

$$\cos^3 \theta = 0.4333$$

$$\theta = 40.8^\circ \blacktriangleleft$$



PROBLEM 10.26

Determine the values of θ corresponding to the equilibrium position of the rod of Problem 10.11 when $l = 600$ mm, $a = 100$ mm, $P = 50$ N, and $Q = 90$ N.

PROBLEM 10.11 The slender rod AB is attached to a collar A and rests on a small wheel at C . Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force Q required to maintain the equilibrium of the rod.

SOLUTION

For $\triangle AA'C$:

$$A'C = a \tan \theta$$

$$y_A = -(A'C) = -a \tan \theta$$

$$\delta y_A = -\frac{a}{\cos^2 \theta} \delta \theta$$

For $\triangle BB'C$:

$$B'C = l \sin \theta - A'C$$

$$= l \sin \theta - a \tan \theta$$

$$BB' = \frac{B'C}{\tan \theta} = \frac{l \sin \theta - a \tan \theta}{\tan \theta}$$

$$x_B = BB' = l \cos \theta - a$$

$$\delta x_B = -l \sin \theta \delta \theta$$

Virtual Work:

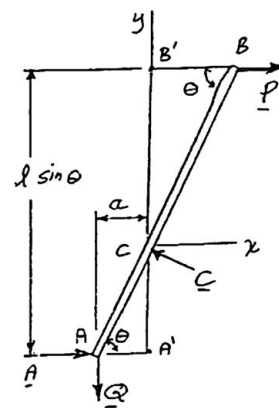
$$\delta U = 0: P \delta x_B - Q \delta y_A = 0$$

$$P(-l \sin \theta \delta \theta) - Q \left(-\frac{a}{\cos^2 \theta} \delta \theta \right) = 0$$

$$Pl \sin \theta \cos^2 \theta = Qa$$

or

$$Q = P \frac{l}{a} \sin \theta \cos^2 \theta$$



PROBLEM 10.26 (Continued)

with

$$l = 600 \text{ mm}, a = 100 \text{ mm}, P = 50 \text{ N}, \text{ and } Q = 90 \text{ N}$$

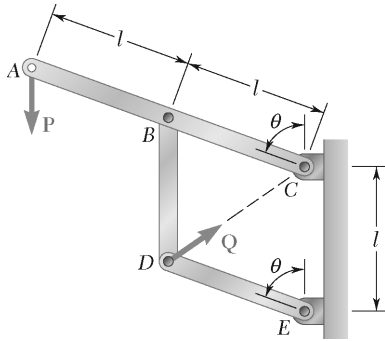
$$90 \text{ N} = (50 \text{ N}) \frac{600 \text{ mm}}{100 \text{ mm}} \sin \theta \cos^2 \theta$$

or

$$\sin \theta \cos^2 \theta = 0.300$$

Solving numerically

$$\theta = 19.81^\circ \text{ and } 51.9^\circ \blacktriangleleft$$



PROBLEM 10.27

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.12 when $P = 80 \text{ N}$ and $Q = 100 \text{ N}$.

PROBLEM 10.12 Knowing that the line of action of the force Q passes through Point C , derive an expression for the magnitude of Q required to maintain equilibrium.

SOLUTION

From geometry

$$y_A = 2l \cos \theta, \quad \delta y_A = -2l \sin \theta \delta \theta$$

$$CD = 2l \sin \frac{\theta}{2}, \quad \delta(CD) = l \cos \frac{\theta}{2} \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad -P \delta y_A - Q \delta(CD) = 0$$

$$-P(-2l \sin \theta \delta \theta) - Q \left(l \cos \frac{\theta}{2} \delta \theta \right) = 0$$

or

$$Q = 2P \frac{\sin \theta}{\cos \left(\frac{\theta}{2} \right)}$$

with

$$P = 80 \text{ N}, \quad Q = 100 \text{ N}$$

$$(100 \text{ N}) = 2(80 \text{ N}) \frac{\sin \theta}{\cos \left(\frac{\theta}{2} \right)}$$

$$\frac{\sin \theta}{\cos \left(\frac{\theta}{2} \right)} = 0.625$$

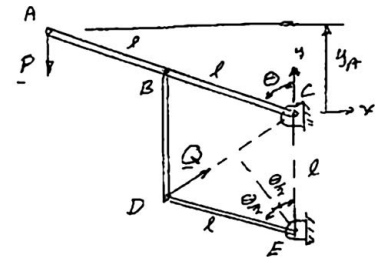
or

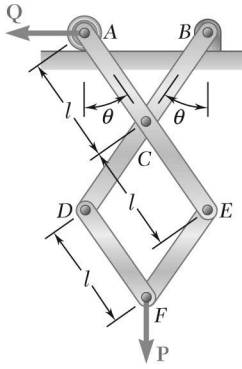
$$\frac{2 \sin \left(\frac{\theta}{2} \right) \cos \left(\frac{\theta}{2} \right)}{\cos \left(\frac{\theta}{2} \right)} = 0.625$$

$$\theta = 36.42^\circ$$

$$\theta = 36.4^\circ \blacktriangleleft$$

(Additional solutions discarded as not applicable are $\theta = \pm 180^\circ$)





PROBLEM 10.28

Determine the value of θ corresponding to the equilibrium position of the mechanism of Prob. 10.14 when $P = 270 \text{ N}$ and $Q = 960 \text{ N}$.

SOLUTION

Virtual Work:

$$x_A = 2l \sin \theta, \quad \delta x_A = 2l \cos \theta \delta \theta$$

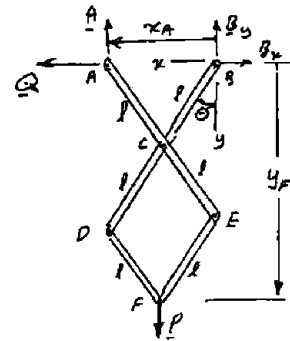
$$y_F = 3l \cos \theta, \quad \delta y_F = -3l \sin \theta \delta \theta$$

$$\delta U = 0: \quad Q \delta x_A + P \delta y_F = 0$$

$$\delta U = 0: \quad Q \delta \theta x_A + P \delta y_F = 0$$

$$Q(2l \cos \theta \delta \theta) + P(-3l \sin \theta \delta \theta) = 0$$

$$Q = \frac{3}{2} P \tan \theta$$

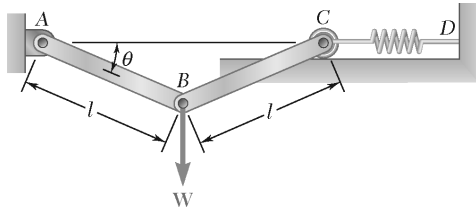


Data:

$$P = 270 \text{ N}, \quad Q = 960 \text{ N}$$

$$(960 \text{ N}) = \frac{3}{2} (270 \text{ N}) \tan \theta$$

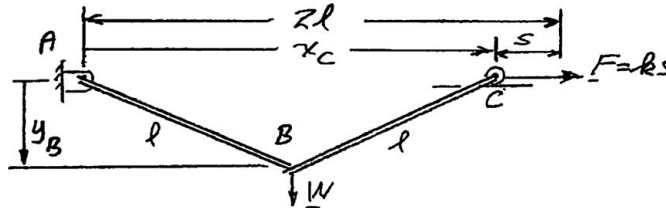
$$\theta = 67.1^\circ \quad \blacktriangleleft$$



PROBLEM 10.29

A load W of magnitude 600 N is applied to the linkage at B . The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = 2l \cos \theta \quad \delta x_C = -2l \sin \theta \delta \theta$$

$$y_B = l \sin \theta \quad \delta y_B = l \cos \theta \delta \theta$$

$$F = ks = k(2l - x_C) = 2kl(1 - \cos \theta)$$

Virtual Work:

$$\delta U = 0: \quad F \delta x_C + W \delta y_B = 0$$

$$2kl(1 - \cos \theta)(-2l \sin \theta \delta \theta) + W(l \cos \theta \delta \theta) = 0$$

$$4kl^2(1 - \cos \theta) \sin \theta = Wl \cos \theta$$

or

$$(1 - \cos \theta) \tan \theta = \frac{W}{4kl}$$

Given:

$$l = 0.3 \text{ m}, \quad W = 600 \text{ N}, \quad k = 2500 \text{ N/m}$$

Then

$$(1 - \cos \theta) \tan \theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

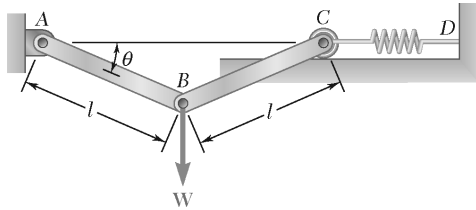
or

$$(1 - \cos \theta) \tan \theta = 0.2$$

Solving numerically

$$\theta = 40.22^\circ$$

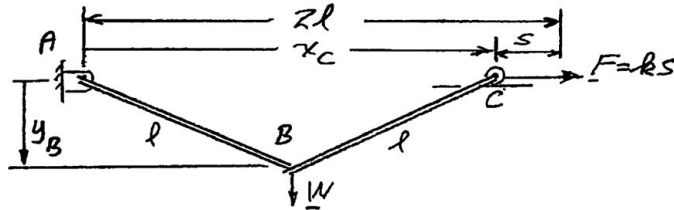
$$\theta = 40.2^\circ \quad \blacktriangleleft$$



PROBLEM 10.30

A vertical load W is applied to the linkage at B . The constant of the spring is k , and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, derive an equation in θ , W , l , and k that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$x_C = 2l \cos \theta \quad \delta x_C = -2l \sin \theta \delta \theta$$

$$y_B = l \sin \theta \quad \delta y_B = l \cos \theta \delta \theta$$

$$F = ks = k(2l - x_C) = 2kl(1 - \cos \theta)$$

Virtual Work:

$$\delta U = 0: \quad F \delta x_C + W \delta y_B = 0$$

$$2kl(1 - \cos \theta)(-2l \sin \theta \delta \theta) + W(l \cos \theta \delta \theta) = 0$$

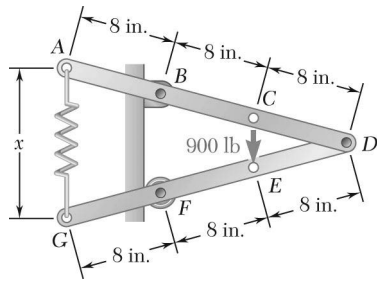
$$4kl^2(1 - \cos \theta) \sin \theta = Wl \cos \theta$$

or

$$(1 - \cos \theta) \tan \theta = \frac{W}{4kl}$$

From above

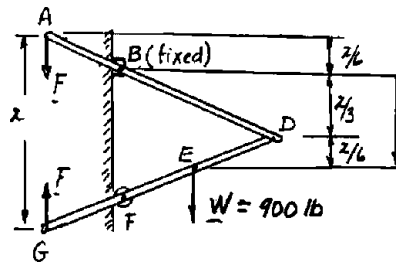
$$(1 - \cos \theta) \tan \theta = \frac{W}{4kl} \quad \blacktriangleleft$$



PROBLEM 10.31

Two bars AD and DG are connected by a pin at D and by a spring AG . Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at E as shown.

SOLUTION



$$y_E = \frac{x}{3} + \frac{x}{6} = \frac{x}{2} \quad \delta y_E = \frac{1}{2} \delta x$$

$$F = ks = (125 \text{ lb/in.})(x - 12 \text{ in.})$$

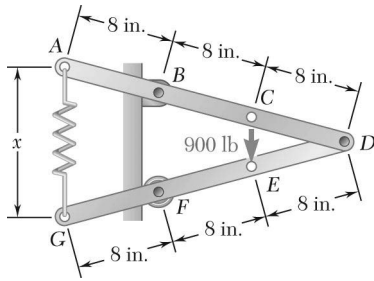
Virtual Work:

$$\delta U = 0: \quad F \delta x + W \delta y_E = 0$$

$$-(125)(x - 12) \delta x + (900) \left(\frac{1}{2} \delta x \right) = 0$$

$$-125x + 1500 + 450 = 0$$

$$x = 15.60 \text{ in.} \quad \blacktriangleleft$$

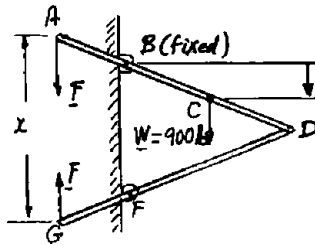


PROBLEM 10.32

Solve Problem 10.31 assuming that the 900-lb vertical force is applied at C instead of E.

PROBLEM 10.31 Two bars AD and DG are connected by a pin at D and by a spring AG. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at E as shown.

SOLUTION



$$y_C = \frac{1}{6}x \quad \delta y_C = \frac{1}{6}\delta x$$

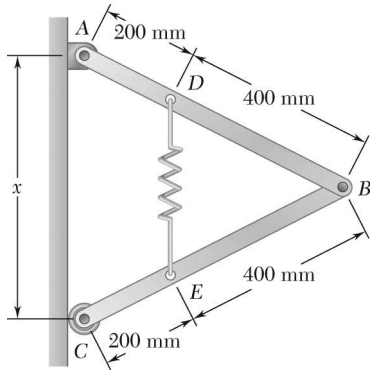
$$F = ks = (125 \text{ lb/in.})(x - 12 \text{ in.})$$

Virtual Work:

$$\delta U = -F\delta x + W\delta y_C = 125(x - 12)\delta x + 900\left(\frac{1}{6}\delta x\right) = 0$$

$$-125x + 1500 + 150 = 0$$

$$x = 13.20 \text{ in.} \quad \blacktriangleleft$$



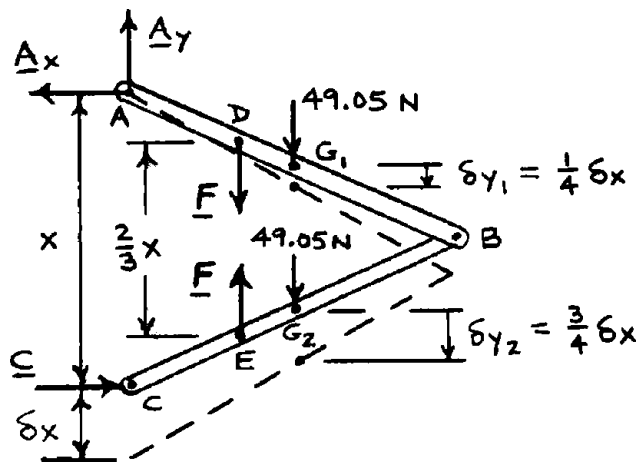
PROBLEM 10.33

Two 5-kg bars AB and BC are connected by a pin at B and by a spring DE . Knowing that the spring is 150 mm long when unstretched and that the constant of the spring is 1 kN/m, determine the value of x corresponding to equilibrium.

SOLUTION

First note:

$$W_{\text{bar}} = (5 \text{ kg})(9.81 \text{ m/s}^2) = 49.05 \text{ N}$$



During the virtual displacement, Points D and E move apart a distance $\delta(DE) = \frac{2}{3}\delta x$ and the total work done by the forces exerted at D and E is $-F\left(\frac{2}{3}\delta x\right)$

$$\delta U = 0: \quad -F\left(\frac{2}{3}\delta x\right) + 49.05 \text{ N}\left(\frac{1}{4}\delta x\right) + 49.05 \text{ N}\left(\frac{3}{4}\delta x\right) = 0$$

$$F = 73.575 \text{ N}$$

For $F = 73.575 \text{ N}$, elongation of spring is

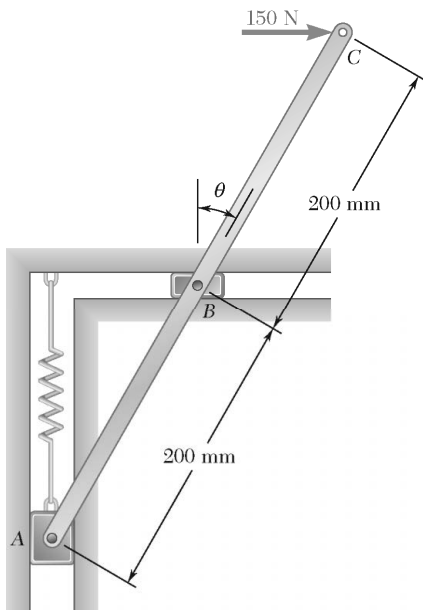
$$\frac{F}{k} = \frac{73.575 \text{ N}}{1000 \text{ N/m}} = 73.575 \times 10^{-3} \text{ m}$$

$$= 73.575 \text{ mm}$$

Since undeformed length of spring is 150 mm, total length is

$$DE = \frac{2}{3}x = 150 \text{ mm} + 73.575 \text{ mm}$$

$$x = 355 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 10.34

Rod ABC is attached to blocks A and B that can move freely in the guides shown. The constant of the spring attached at A is $k = 3 \text{ kN/m}$, and the spring is unstretched when the rod is vertical. For the loading shown, determine the value of θ corresponding to equilibrium.

SOLUTION

$$x_C = (0.4 \text{ m}) \sin \theta$$

$$\delta x_C = 0.4 \cos \theta \delta \theta$$

$$y_A = (0.2 \text{ m}) \cos \theta$$

$$\delta y_A = -0.2 \sin \theta \delta \theta$$

Spring:

Unstretched length = 0.2 m

$$F = k(0.2 \text{ m} - y_A) = k(0.2 - 0.2 \cos \theta)$$

$$= (300 \text{ N/m})(0.2)(1 - \cos \theta)$$

$$F = 600(1 - \cos \theta)$$

Virtual Work:

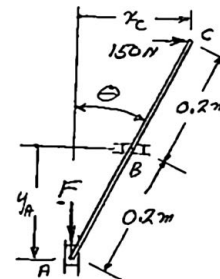
$$\delta U = 0: (150 \text{ N}) \delta x_C + F \delta y_A = 0$$

$$150(0.4 \cos \theta \delta \theta) + 600(1 - \cos \theta)(-0.2 \sin \theta \delta \theta) = 0$$

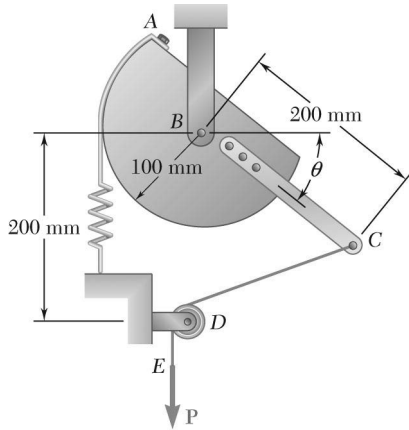
$$\frac{150(0.4)}{600(0.2)} = \frac{1}{2}; \quad \frac{1}{2} = (1 - \cos \theta) \tan \theta$$

Solve by trial and error:

$$\theta = 57.2^\circ \blacktriangleleft$$



PROBLEM 10.35



A vertical force P of magnitude 150 N is applied to end E of cable CDE , which passes over a small pulley D and is attached to the mechanism at C . The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium.

SOLUTION

$$l = BC = 0.2 \text{ m}$$

$$r = 0.1 \text{ m}$$

$$\angle CBD = 90^\circ - \theta$$

$$v = 2l \sin\left(45^\circ - \frac{\theta}{2}\right)$$

$$\delta v = -l \cos\left(45^\circ - \frac{\theta}{2}\right) \delta\theta$$

$$s = r\theta \quad \delta s = r\delta\theta$$

$$F = ks = kr\theta$$

$$\delta U = 0: \quad -P\delta v - F\delta s = 0$$

$$-P \left[-l \cos\left(45^\circ - \frac{\theta}{2}\right) \right] \delta\theta - kr\theta(r\delta\theta) = 0$$

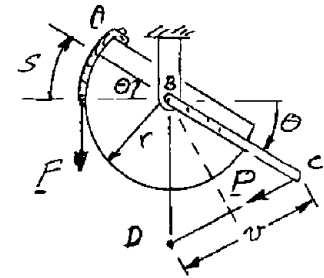
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45^\circ - \frac{\theta}{2}\right)}$$

$$\frac{Pl}{kr^2} = \frac{(150 \text{ N})(0.2 \text{ m})}{(4000 \text{ N/m})(0.1 \text{ m})^2} = 0.75$$

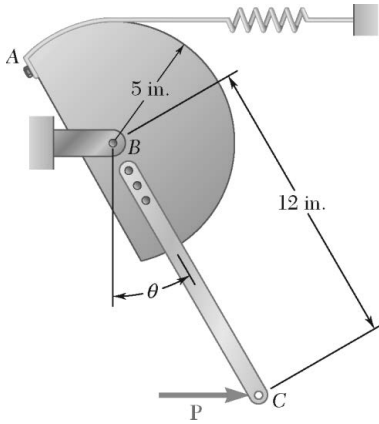
$$0.75 = \frac{\theta}{\cos\left(45^\circ - \frac{\theta}{2}\right)}$$

$$\theta = 0.67623 \text{ rad} = 38.745^\circ$$

$$\theta = 38.7^\circ \blacktriangleleft$$



PROBLEM 10.36



A horizontal force \mathbf{P} of magnitude 40 lb is applied to the mechanism at C. The constant of the spring is $k = 9 \text{ lb/in.}$, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism, determine the value of θ corresponding to equilibrium.

SOLUTION

$$s = r\theta$$

$$\delta s = r\delta\theta$$

Spring is unstretched at $\theta = 0^\circ$

$$F_{SP} = ks = kr\theta$$

$$x_C = l \sin \theta$$

$$\delta x_C = l \cos \theta \delta\theta$$

Virtual Work:

$$\delta U = 0: P\delta x_C - F_{SP}\delta s = 0$$

$$P(l \cos \theta \delta\theta) - kr\theta(r\delta\theta) = 0$$

or

$$\frac{Pl}{kr^2} = \frac{\theta}{\cos \theta}$$

Thus

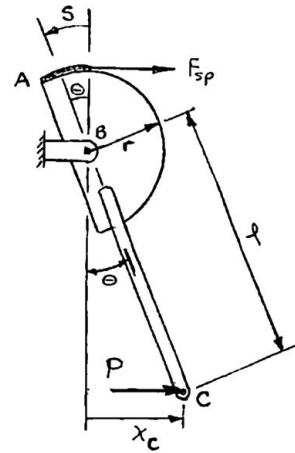
$$\frac{(40 \text{ lb})(12 \text{ in.})}{(9 \text{ lb/in.})(5 \text{ in.})^2} = \frac{\theta}{\cos \theta}$$

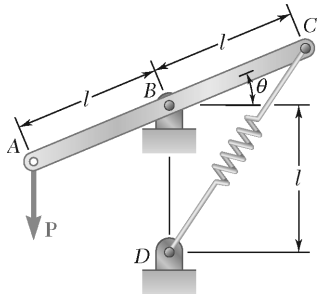
or

$$\frac{\theta}{\cos \theta} = 2.1333$$

$$\theta = 1.054 \text{ rad} = 60.39^\circ$$

$$\theta = 60.4^\circ \blacktriangleleft$$





PROBLEM 10.37

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 300 \text{ N}, \quad l = 400 \text{ mm}, \quad k = 5 \text{ kN/m.}$$

SOLUTION

$$y_A = l \sin \theta$$

$$\delta y_A = l \cos \theta \delta \theta$$

Spring:

$$v = CD$$

Unstretched when

$$\theta = 0$$

so that

$$v_0 = \sqrt{2}l$$

For θ :

$$v = 2l \sin \left(\frac{90^\circ + \theta}{2} \right)$$

$$\delta v = l \cos \left(45^\circ + \frac{\theta}{2} \right) \delta \theta$$

Stretched length:

$$s = v - v_0 = 2l \sin \left(45^\circ + \frac{\theta}{2} \right) - \sqrt{2}l$$

Then

$$F = ks = kl \left[2 \sin \left(45^\circ + \frac{\theta}{2} \right) - \sqrt{2} \right]$$

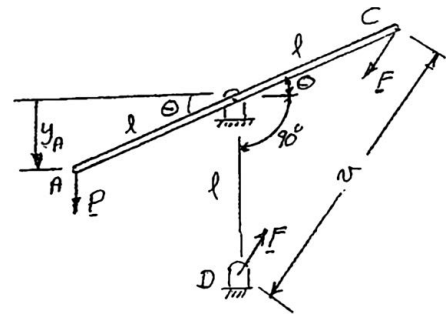
Virtual Work:

$$\delta U = 0: \quad P \delta y_A - F \delta v = 0$$

$$Pl \cos \theta \delta \theta - kl \left[2 \sin \left(45^\circ + \frac{\theta}{2} \right) - \sqrt{2} \right] l \cos \left(45^\circ + \frac{\theta}{2} \right) \delta \theta = 0$$

or

$$\begin{aligned} \frac{P}{kl} &= \frac{1}{\cos \theta} \left[2 \sin \left(45^\circ + \frac{\theta}{2} \right) \cos \left(45^\circ + \frac{\theta}{2} \right) - \sqrt{2} \cos \left(45^\circ + \frac{\theta}{2} \right) \right] \\ &= \frac{1}{\cos \theta} \left[2 \sin \left(45^\circ + \frac{\theta}{2} \right) \cos \left(45^\circ + \frac{\theta}{2} \right) \cos \theta - \sqrt{2} \cos \left(45^\circ + \frac{\theta}{2} \right) \right] \\ &= 1 - \sqrt{2} \frac{\cos \left(45^\circ + \frac{\theta}{2} \right)}{\cos \theta} \end{aligned}$$



PROBLEM 10.37 (Continued)

Now, with $P = 300 \text{ N}$, $l = 400 \text{ mm}$, and $k = 5 \text{ kN/m}$

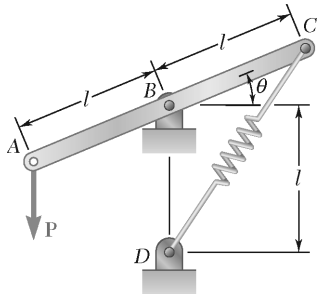
$$\frac{(300 \text{ N})}{(5000 \text{ N/m})(0.4 \text{ m})} = 1 - \sqrt{2} \frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta}$$

or

$$\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} = 0.60104$$

Solving numerically

$$\theta = 22.6^\circ \blacktriangleleft$$



PROBLEM 10.38

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 75 \text{ lb}, \quad l = 15 \text{ in.}, \quad k = 20 \text{ lb/in.}$$

SOLUTION

From the analysis of Problem 10.37, we have

$$\frac{P}{kl} = 1 - \sqrt{2} \frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta}$$

with $P = 75 \text{ lb}$, $l = 15 \text{ in.}$ and $k = 20 \text{ lb/in.}$

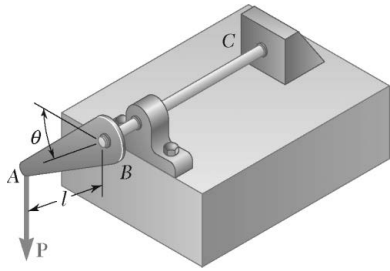
$$\frac{(75 \text{ lb})}{(20 \text{ lb/in.})(15 \text{ in.})} = 1 - \sqrt{2} \frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta}$$

or

$$\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} = 0.53033$$

Solving numerically

$$\theta = 51.1^\circ \blacktriangleleft$$



PROBLEM 10.39

The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C . The torsional spring constant of the shaft BC is K ; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100 \text{ N}$, $l = 250 \text{ mm}$, and $K = 12.5 \text{ N} \cdot \text{m/rad}$.

SOLUTION

We have

$$y_A = l \sin \theta$$

$$\delta y_A = l \cos \theta \delta \theta$$

Virtual Work:

$$\delta U = 0: P \delta y_A - M \delta \theta = 0$$

$$Pl \cos \theta \delta \theta - K \theta \delta \theta = 0$$

or

$$\frac{\theta}{\cos \theta} = \frac{Pl}{K} \quad (1)$$

with

$$P = 100 \text{ N}, \quad l = 250 \text{ mm} \quad \text{and} \quad K = 12.5 \text{ N} \cdot \text{m/rad}$$

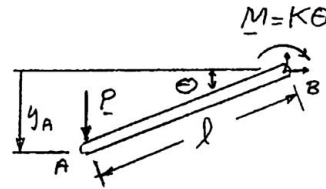
$$\frac{\theta}{\cos \theta} = \frac{(100 \text{ N})(0.250 \text{ m})}{12.5 \text{ N} \cdot \text{m/rad}}$$

or

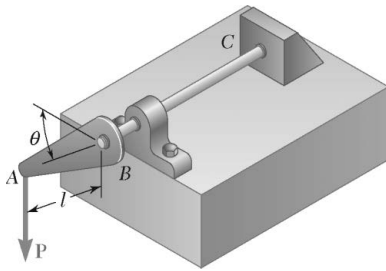
$$\frac{\theta}{\cos \theta} = 2.0000$$

Solving numerically

$$\theta = 59.0^\circ \quad \blacktriangleleft$$



PROBLEM 10.40



Solve Problem 10.39 assuming that $P = 350 \text{ N}$, $l = 250 \text{ mm}$, and $K = 12.5 \text{ N} \cdot \text{m/rad}$. Obtain answers in each of the following quadrants: $0 < \theta < 90^\circ$, $270^\circ < \theta < 360^\circ$, $360^\circ < \theta < 450^\circ$.

PROBLEM 10.39 The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C . The torsional spring constant of the shaft BC is K ; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100 \text{ N}$, $l = 250 \text{ mm}$, and $K = 12.5 \text{ N} \cdot \text{m/rad}$.

SOLUTION

Using Equation (1) of Problem 10.39 and

$$P = 350 \text{ N}, \quad l = 250 \text{ mm} \quad \text{and} \quad K = 12.5 \text{ N} \cdot \text{m/rad}$$

We have

$$\frac{\theta}{\cos \theta} = \frac{(350 \text{ N})(0.250 \text{ m})}{12.5 \text{ N} \cdot \text{m/rad}}$$

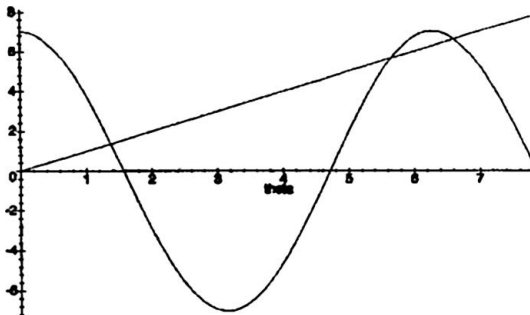
or

$$\frac{\theta}{\cos \theta} = 7 \quad \text{or} \quad \theta = 7 \cos \theta \quad (1)$$

The solutions to this equation can be shown graphically using any appropriate graphing tool, such as Maple, with the command: `plot ({theta, 7 * cos(theta)}, t = 0.5 * Pi/2);`

Thus, we plot $y = \theta$ and $y = 7 \cos \theta$ in the range

$$0 \leq \theta \leq \frac{5\pi}{2}$$



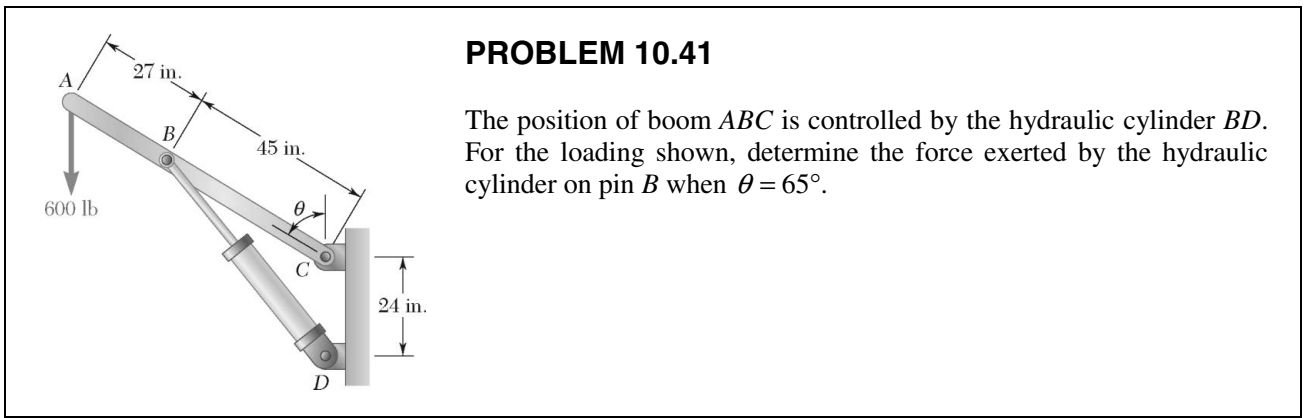
PROBLEM 10.40 (Continued)

We observe that there are three points of intersection, which implies that Equation (1) has three roots in the specified range of θ .

$$0 \leq \theta \leq 90^\circ \left(\frac{\pi}{2} \right); \quad \theta = 1.37333 \text{ rad}, \quad \theta = 78.69^\circ \quad \theta = 78.7^\circ \blacktriangleleft$$

$$270 \leq \theta \leq 360^\circ \left(\frac{3\pi}{2} \leq \theta \leq 2\pi \right); \quad \theta = 5.65222 \text{ rad}, \quad \theta = 323.85^\circ \quad \theta = 324^\circ \blacktriangleleft$$

$$360 \leq \theta \leq 450^\circ \left(2\pi \leq \theta \leq \frac{5\pi}{2} \right); \quad \theta = 6.61597 \text{ rad}, \quad \theta = 379.07^\circ \quad \theta = 379^\circ \blacktriangleleft$$

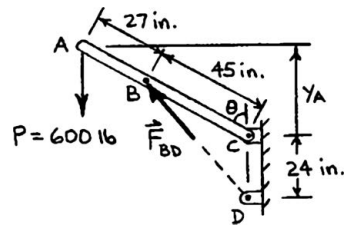


PROBLEM 10.41

The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* when $\theta = 65^\circ$.

SOLUTION

We have



$$y_A = (72 \text{ in.}) \cos \theta \quad \delta y_A = -72 \sin \theta \delta \theta$$

$$(BD)^2 = (BC)^2 + (CD)^2 - 2(BC)(CD) \cos(180^\circ - \theta)$$

$$= (45)^2 + (24)^2 + 2(45)(24) \cos \theta$$

$$(BD)^2 = 2601 + 2160 \cos \theta \tag{1}$$

Differentiating: $2(BD)\delta(BD) = -2160 \sin \theta \delta \theta$

$$\delta(BD) = -\frac{1080}{BD} \sin \theta \delta \theta \tag{2}$$

Virtual Work: Noting that **P** tends to decrease y_A and F_{BD} tends to increase BD , write

$$\delta U = -P \delta y_A + F_{BD} \delta(BD) = 0$$

$$-P(-72 \sin \theta \delta \theta) + F_{BD} \left(-\frac{1080}{BD} \sin \theta \delta \theta \right) = 0$$

$$F_{BD} = \frac{1}{15} (BD) P$$

or, since

$$P = 600 \text{ lb: } F_{BD} = \frac{600}{15} (BD) = (40 \text{ lb})(BD) \tag{3}$$

Making $\theta = 65^\circ$ in Eq. (1), we have

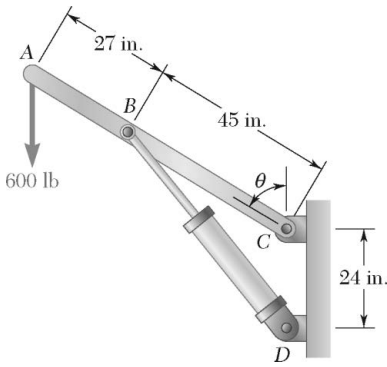
$$(BD)^2 = 2601 + 2160 \cos 65^\circ = 3513.9$$

$$BD = 59.278$$

Carrying into Eq. (3):

$$F_{BD} = (40 \text{ lb})(59.278) = 2371.1 \text{ lb} \qquad F_{BD} = 2370 \text{ lb} \blacktriangleleft$$

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PROBLEM 10.42

The position of boom ABC is controlled by the hydraulic cylinder BD . For the loading shown, (a) express the force exerted by the hydraulic cylinder on pin B as a function of the length BD , (b) determine the smallest possible value of the angle θ if the maximum force that the cylinder can exert on pin B is 2.5 kips.

SOLUTION

(a) See solution of Problem 10.41 for the derivation of Eq. (3):

$$F_{BD} = (40 \text{ lb})(BD) \quad \blacktriangleleft$$

(b) For $(F_{BD})_{\max} = 2.5 \text{ kips} = 2500 \text{ lb}$, we have

$$2500 \text{ lb} = (40 \text{ lb})(BD)$$

$$BD = 62.5$$

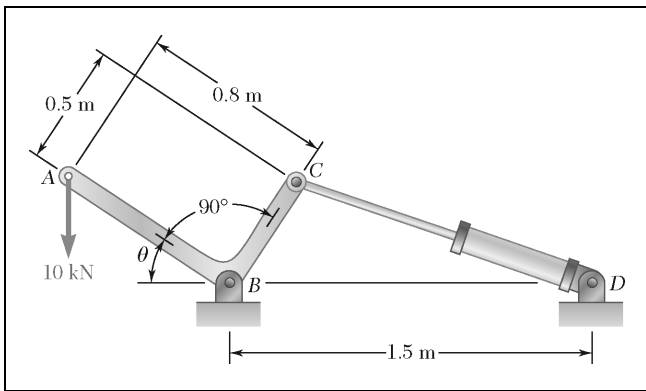
Carrying this value into Eq. (1) of Problem 10.41, write

$$(BD)^2 = 2601 + 2160 \cos \theta$$

$$(62.5)^2 = 2601 + 2160 \cos \theta$$

$$\cos \theta = 0.60428$$

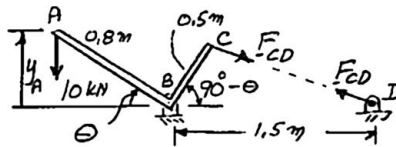
$$\theta = 52.8^\circ \quad \blacktriangleleft$$



PROBLEM 10.43

The position of member ABC is controlled by the hydraulic cylinder CD . For the loading shown, determine the force exerted by the hydraulic cylinder on pin C when $\theta = 55^\circ$.

SOLUTION



$$\begin{aligned}
 y_A &= (0.8 \text{ m}) \sin \theta \\
 \delta y_A &= 0.8 \cos \theta \delta \theta \\
 CD^2 &= BC^2 + BD^2 - 2(BC)(BD) \cos(90^\circ - \theta) \\
 CD^2 &= 0.5^2 + 1.5^2 - 2(0.5)(1.5) \sin \theta \\
 CD^2 &= 2.5 - 1.5 \sin \theta \tag{1}
 \end{aligned}$$

$$2(CD)(\delta_{CD}) = -1.5 \cos \theta \delta \theta \quad \delta_{CD} = -\frac{3 \cos \theta}{4CD} \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad -(10 \text{ kN}) \delta y_A - F_{CD} \delta_{CD} = 0$$

$$-10(0.8 \cos \theta \delta \theta) - F_{CD} \left(-\frac{3 \cos \theta}{4CD} \delta \theta \right) = 0$$

$$F_{CD} = \frac{32}{3} CD \tag{2}$$

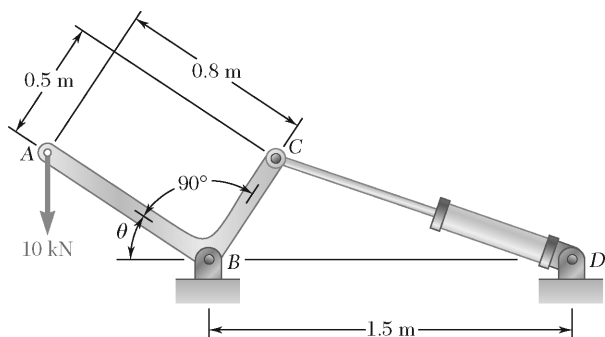
For $\theta = 55^\circ$:

Eq. (1): $CD^2 = 2.5 - 1.5 \sin 55^\circ = 1.2713; \quad CD = 1.1275 \text{ m}$

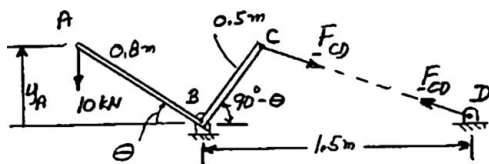
Eq. (2): $F_{CD} = \frac{32}{3} CD = \frac{32}{3} (1.1275) = 12.027 \text{ kN} \quad \mathbf{F}_{CD} = 12.03 \text{ kN} \quad \swarrow \blacktriangleleft$

PROBLEM 10.44

The position of member ABC is controlled by the hydraulic cylinder CD . Determine the angle θ knowing that the hydraulic cylinder exerts a 15-kN force on pin C .



SOLUTION



$$\begin{aligned}
 y_A &= (0.8 \text{ m}) \sin \theta \\
 \delta y_A &= 0.8 \cos \theta \delta \theta \\
 CD^2 &= BC^2 + BD^2 - 2(BC)(BD) \cos(90^\circ - \theta) \\
 CD^2 &= 0.5^2 + 1.5^2 - 2(0.5)(1.5) \sin \theta \\
 CD^2 &= 2.5 - 1.5 \sin \theta \tag{1}
 \end{aligned}$$

$$2(CD)(\delta_{CD}) = -1.5 \cos \theta \delta \theta; \quad \delta_{CD} = -\frac{3 \cos \theta}{4CD} \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad -(10 \text{ kN}) \delta y_A - F_{CD} \delta_{CD} = 0$$

$$-10(0.8 \cos \theta \delta \theta) - F_{CD} \left(-\frac{3 \cos \theta}{4CD} \delta \theta \right) = 0$$

$$F_{CD} = \frac{32}{3} CD \tag{2}$$

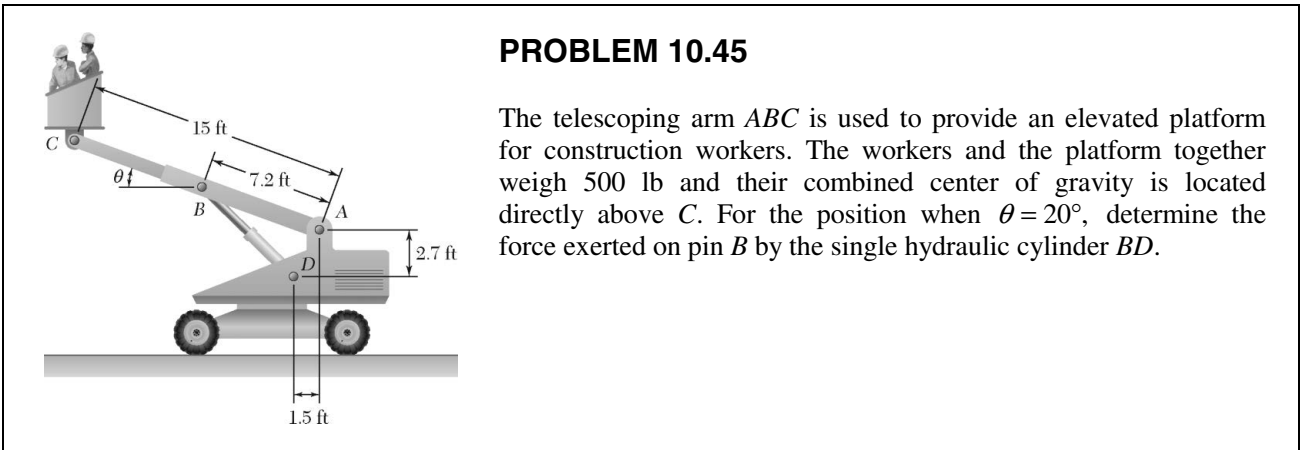
For $F_{CD} = 15 \text{ kN}$:

$$\text{Eq. (2):} \quad 15 \text{ kN} = \frac{32}{3} CD; \quad CD = \frac{45}{32} = 1.40625 \text{ m}$$

$$\text{Eq. (1):} \quad (1.40625)^2 = 2.5 - 1.5 \sin \theta; \quad \sin \theta = 0.34831$$

$$\theta = 20.38^\circ$$

$$\theta = 20.4^\circ \blacktriangleleft$$



PROBLEM 10.45

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20^\circ$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION

In $\triangle ADE$:

$$\tan \alpha = \frac{AE}{DE} = \frac{2.7 \text{ ft}}{1.5 \text{ ft}}$$

$$\alpha = 60.945^\circ$$

$$AD = \frac{2.7 \text{ ft}}{\sin 60.945^\circ} = 3.0887 \text{ m}$$

From the geometry:

$$y_C = (15 \text{ ft}) \sin \theta$$

$$\delta y_C = (15 \text{ ft}) \cos \theta \delta \theta$$

Then, in triangle *BAD*: Angle $BAD = \alpha + \theta$

Law of cosines:

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD) \cos(\alpha + \theta)$$

or

$$BD^2 = (7.2 \text{ ft})^2 + (3.0887 \text{ ft})^2 - 2(7.2 \text{ ft})(3.0887 \text{ ft}) \cos(\alpha + \theta)$$

$$BD^2 = 61.380 \text{ ft}^2 - (44.477 \cos(\alpha + \theta)) \text{ ft}^2 \tag{1}$$

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PROBLEM 10.45 (Continued)

And then

$$2(BD)(\delta BD) = (44.477 \sin(\alpha + \theta))\delta\theta$$

$$\delta BD = \frac{44.477 \sin(\alpha + \theta)}{2(BD)} \delta\theta$$

Virtual Work:

$$\delta U = 0: -P\delta y_C + F_{BD}\delta BD = 0$$

Substituting

$$-(500 \text{ lb})(15 \text{ ft}) \cos \theta \delta\theta + F_{BD} \left[\frac{(44.477 \text{ ft}^2) \sin(\alpha + \theta)}{2(BD)} \delta\theta \right] = 0$$

or

$$F_{BD} = \left[337.25 \frac{\cos \theta}{\sin(\alpha + \theta)} BD \right] \text{ lb/ft} \quad (2)$$

Now, with $\theta = 20^\circ$ and $\alpha = 60.945^\circ$

Equation (1):

$$BD^2 = 61.380 - 44.477 \cos(60.945^\circ + 20^\circ)$$

$$BD^2 = 54.380$$

$$BD = 7.3743 \text{ ft}$$

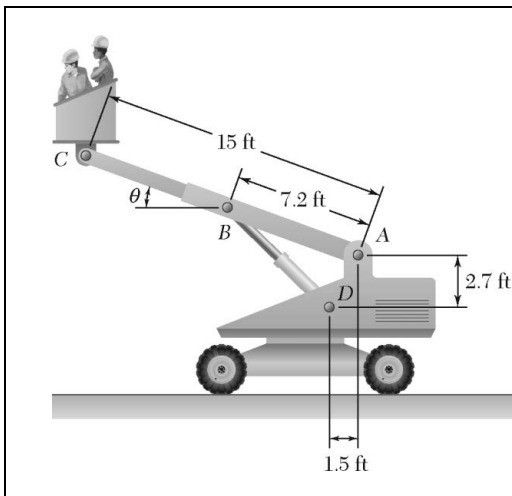
Equation (2):

$$F_{BD} = \left[337.25 \frac{\cos 20^\circ}{\sin(60.945^\circ + 20^\circ)} (7.3743 \text{ ft}) \right] \text{ lb/ft}$$

or

$$F_{BD} = 2366 \text{ lb}$$

$$\mathbf{F}_{BD} = 2370 \text{ lb} \swarrow \blacktriangleleft$$



PROBLEM 10.46

Solve Problem 10.45 assuming that the workers are lowered to a point near the ground so that $\theta = -20^\circ$.

PROBLEM 10.45 The telescoping arm ABC is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above C . For the position when $\theta = 20^\circ$, determine the force exerted on pin B by the single hydraulic cylinder BD .

SOLUTION

Using the figure and analysis of Problem 10.45, including Equations (1) and (2), and with $\theta = -20^\circ$, we have

Equation (1): $BD^2 = 61.380 - 44.477 \cos(60.945^\circ - 20^\circ)$

$$BD^2 = 27.785$$

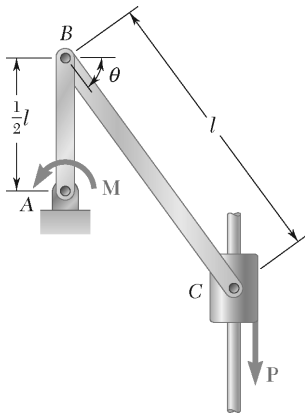
$$BD = 5.2711 \text{ ft}$$

Equation (2): $F_{BD} = 337.25 \frac{\cos(-20^\circ)}{\sin(60.945^\circ - 20^\circ)} (5.2711)$

$$F_{BD} = 2549 \text{ lb}$$

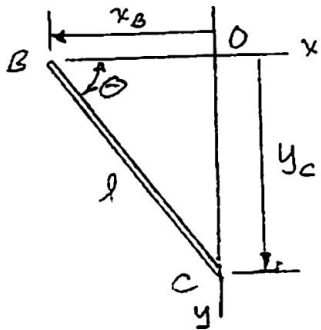
$$\text{or } \mathbf{F}_{BD} = 2550 \text{ lb} \swarrow \blacktriangleleft$$

PROBLEM 10.47



Denoting by μ_s the coefficient of static friction between collar C and the vertical rod, derive an expression for the magnitude of the largest couple M for which equilibrium is maintained in the position shown. Explain what happens if $\mu_s \geq \tan \theta$.

SOLUTION



Member BC : We have

$$x_B = l \cos \theta$$

$$\delta x_B = -l \sin \theta \delta \theta \quad (1)$$

and

$$y_C = l \sin \theta$$

$$\delta y_C = l \cos \theta \delta \theta \quad (2)$$

Member AB : We have

$$\delta x_B = \frac{1}{2} l \delta \phi$$

Substituting from Equation (1),

$$-l \sin \theta \delta \theta = \frac{1}{2} l \delta \phi$$

or

$$\delta \phi = -2 \sin \theta \delta \theta \quad (3)$$

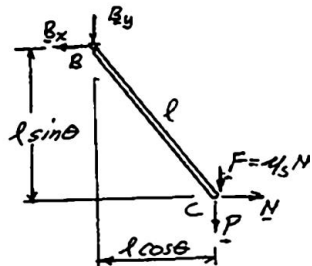
Free body of rod BC

For M_{\max} , motion of collar C impends upward

$$+\circlearrowleft \Sigma M_B = 0: \quad N l \sin \theta - (P + \mu_s N)(l \cos \theta) = 0$$

$$N \tan \theta - \mu_s N = P$$

$$N = \frac{P}{\tan \theta - \mu_s}$$



PROBLEM 10.47 (Continued)

Virtual Work:

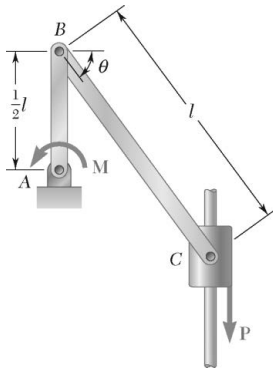
$$\delta U = 0: M \delta\phi + (P + \mu_s N) \delta y_C = 0$$

$$M(-2 \sin \theta \delta\theta) + (P + \mu_s N) l \cos \theta \delta\theta = 0$$

$$M_{\max} = \frac{(P + \mu_s N)}{2 \tan \theta} l = \frac{P + \mu_s \frac{P}{\tan \theta - \mu_s}}{2 \tan \theta} l$$

$$\text{or } M_{\max} = \frac{Pl}{2(\tan \theta - \mu_s)} \blacktriangleleft$$

If $\mu_s = \tan \theta$, $M = \infty$, system becomes *self-locking*.



PROBLEM 10.48

Knowing that the coefficient of static friction between collar C and the vertical rod is 0.40, determine the magnitude of the largest and smallest couple \mathbf{M} for which equilibrium is maintained in the position shown, when $\theta = 35^\circ$, $l = 600$ mm, and $P = 300$ N.

SOLUTION

From the analysis of Problem 10.50, we have

$$M_{\max} = \frac{Pl}{2(\tan \theta + \mu_s)}$$

With $\theta = 35^\circ$, $l = 0.6$ m, $P = 300$ N

$$\begin{aligned} M_{\max} &= \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35^\circ - 0.4)} \\ &= 299.80 \text{ N} \cdot \text{m} \end{aligned}$$

$$M_{\max} = 300 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

For M_{\min} , motion of C impends downward and F acts upward. The equations of Problem 10.50 can still be used if we replace μ_s by $-\mu_s$. Then

$$M_{\min} = \frac{Pl}{2(\tan \theta + \mu_s)}$$

Substituting,

$$\begin{aligned} M_{\min} &= \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35^\circ + 0.4)} \\ &= 81.803 \text{ N} \cdot \text{m} \end{aligned}$$

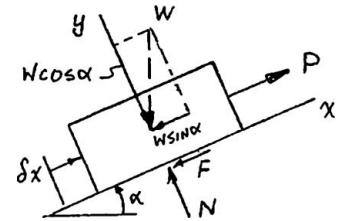
$$M_{\min} = 81.8 \text{ N} \cdot \text{m} \quad \blacktriangleleft$$

PROBLEM 10.49

A block of weight W is pulled up a plane forming an angle α with the horizontal by a force \mathbf{P} directed along the plane. If μ is the coefficient of friction between the block and the plane, derive an expression for the mechanical efficiency of the system. Show that the mechanical efficiency cannot exceed $\frac{1}{2}$ if the block is to remain in place when the force \mathbf{P} is removed.

SOLUTION

$$\begin{aligned} \text{Input work} &= P\delta x \\ \text{Output work} &= (W \sin \alpha)\delta x \end{aligned}$$



Efficiency:

$$\eta = \frac{W \sin \alpha \delta x}{P \delta x} \quad \text{or} \quad \eta = \frac{W \sin \alpha}{P} \quad (1)$$

$$+\nearrow \Sigma F_x = 0: \quad P - F - W \sin \alpha = 0 \quad \text{or} \quad P = W \sin \alpha + F \quad (2)$$

$$+\searrow \Sigma F_y = 0: \quad N - W \cos \alpha = 0 \quad \text{or} \quad N = W \cos \alpha$$

$$F = \mu N = \mu W \cos \alpha$$

Equation (2):
$$P = W \sin \alpha + \mu W \cos \alpha = W (\sin \alpha + \mu \cos \alpha)$$

Equation (1):
$$\eta = \frac{W \sin \alpha}{W (\sin \alpha + \mu \cos \alpha)} \quad \text{or} \quad \eta = \frac{1}{1 + \mu \cot \alpha} \quad \blacktriangleleft$$

If block is to remain in place when $P = 0$, we know (see Chapter 8) that $\phi_s \geq \alpha$ or, since

$$\mu = \tan \phi_s, \quad \mu \geq \tan \alpha$$

Multiply by $\cot \alpha$:
$$\mu \cot \alpha \geq \tan \alpha \cot \alpha = 1$$

Add 1 to each side:
$$1 + \mu \cot \alpha \geq 2$$

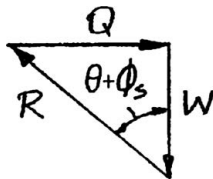
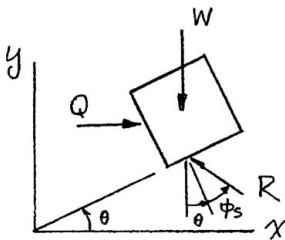
Recalling the expression for η , we find
$$\eta \geq \frac{1}{2} \quad \blacktriangleleft$$

PROBLEM 10.50

Derive an expression for the mechanical efficiency of the jack discussed in Section 8.6. Show that if the jack is to be self-locking, the mechanical efficiency cannot exceed $\frac{1}{2}$.

SOLUTION

Recall Figure 8.9a. Draw force triangle



$$Q = W \tan(\theta + \phi_s)$$

$$y = x \tan \theta \text{ so that } \delta y = \delta x \tan \theta$$

$$\text{Input work} = Q \delta x = W \tan(\theta + \phi_s) \delta x$$

$$\text{Output work} = W \delta y = W (\delta x) \tan \theta$$

Efficiency:

$$\eta = \frac{W \tan \theta \delta x}{W \tan(\theta + \phi_s) \delta x};$$

$$\eta = \frac{\tan \theta}{\tan(\theta + \phi_s)} \blacktriangleleft$$

From Page 432, we know the jack is self-locking if

$$\phi \geq \theta$$

Then

$$\theta + \phi_s \geq 2\theta$$

so that

$$\tan(\theta + \phi_s) \geq \tan 2\theta$$

From above

$$\eta = \frac{\tan \theta}{\tan(\theta + \phi_s)}$$

It then follows that

$$\eta \leq \frac{\tan \theta}{\tan 2\theta}$$

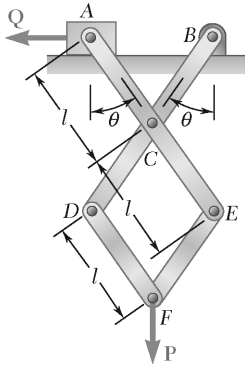
But

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Then

$$\eta \leq \frac{\tan \theta (1 - \tan^2 \theta)}{2 \tan \theta} = \frac{1 - \tan^2 \theta}{2}$$

$$\eta \leq \frac{1}{2} \blacktriangleleft$$



PROBLEM 10.51

Denoting by μ_s the coefficient of static friction between the block attached to rod ACE and the horizontal surface, derive expressions in terms of P , μ_s , and θ for the largest and smallest magnitude of the force Q for which equilibrium is maintained.

SOLUTION

For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: -x_A + \frac{x_A}{2}P = 0 \quad \text{or} \quad A = \frac{P}{2} \uparrow$$

Then:

$$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2} \mu_s P$$

Now

$$x_A = 2l \sin \theta$$

$$\delta x_A = 2l \cos \theta \delta \theta$$

and

$$y_F = 3l \cos \theta$$

$$\delta y_F = -3l \sin \theta \delta \theta$$

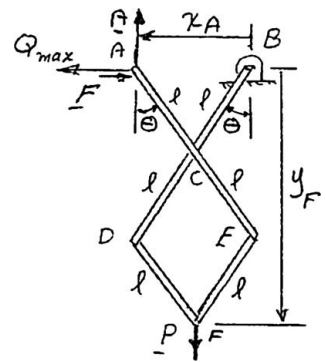
Virtual Work:

$$\delta U = 0: (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

$$\left(Q_{\max} - \frac{1}{2} \mu_s P \right) (2l \cos \theta \delta \theta) + P(-3l \sin \theta \delta \theta) = 0$$

or

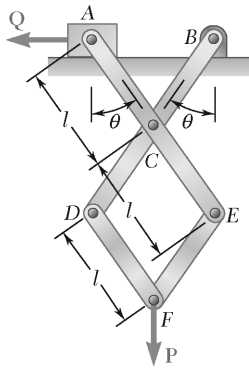
$$Q_{\max} = \frac{3}{2} P \tan \theta + \frac{1}{2} \mu_s P$$



$$Q_{\max} = \frac{P}{2} (3 \tan \theta + \mu_s) \quad \blacktriangleleft$$

For Q_{\min} , motion of A impends to the right and F acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2} (3 \tan \theta - \mu_s) \quad \blacktriangleleft$$



PROBLEM 10.52

Knowing that the coefficient of static friction between the block attached to rod ACE and the horizontal surface is 0.15, determine the magnitude of the largest and smallest force Q for which equilibrium is maintained when $\theta = 30^\circ$, $l = 0.2$ m, and $P = 40$ N.

SOLUTION

Using the results of Problem 10.48 with

$$\theta = 30^\circ$$

$$l = 0.2 \text{ m}$$

$$P = 40 \text{ N, and } \mu_s = 0.15$$

We have

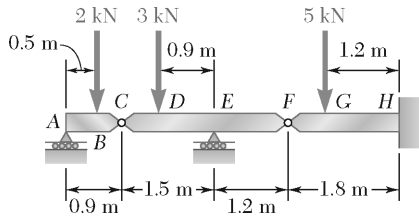
$$\begin{aligned} Q_{\max} &= \frac{P}{2}(3 \tan \theta + \mu_s) \\ &= \frac{(40 \text{ N})}{2}(3 \tan 30^\circ + 0.15) \\ &= 37.64 \text{ N} \end{aligned}$$

$$Q_{\max} = 37.6 \text{ N} \blacktriangleleft$$

and

$$\begin{aligned} Q_{\min} &= \frac{P}{2}(3 \tan \theta - \mu_s) \\ &= \frac{(40 \text{ N})}{2}(3 \tan 30^\circ - 0.15) \\ &= 31.64 \text{ N} \end{aligned}$$

$$Q_{\min} = 31.6 \text{ N} \blacktriangleleft$$

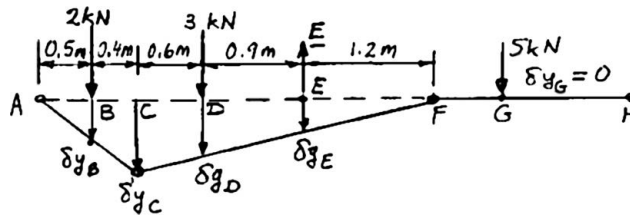


PROBLEM 10.53

Using the method of virtual work, determine the reaction at E .

SOLUTION

We release the support at E and assume a virtual displacement δy_E for Point E .



From similar triangles:

$$\delta y_D = \frac{2.1}{1.2} \delta y_E = 1.75 \delta y_E$$

$$\delta y_C = \frac{2.7}{1.2} \delta y_E = 2.25 \delta y_E$$

$$\delta y_B = \frac{0.5}{0.9} \delta y_C = \frac{0.5}{0.9} (2.25 \delta y_E) = 1.25 \delta y_E$$

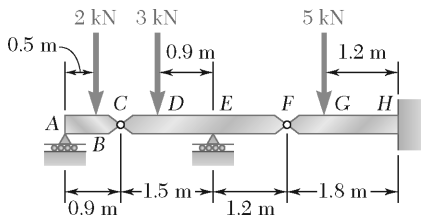
Virtual Work:

$$\delta U = (2 \text{ kN}) \delta y_B + (3 \text{ kN}) \delta y_D - E \delta y_E = 0$$

$$2(1.25 \delta y_E) + 3(1.75 \delta y_E) - E \delta y_E = 0$$

$$E = +7.75 \text{ kN}$$

$$\mathbf{E} = 7.75 \text{ kN} \uparrow \blacktriangleleft$$

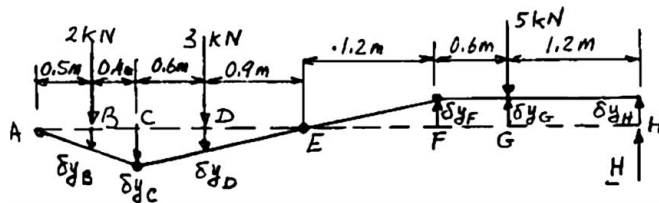


PROBLEM 10.54

Using the method of virtual work, determine separately the force and couple representing the reaction at H .

SOLUTION

Force at H . We give a vertical virtual displacement δy_H to Point H , keeping member FH horizontal.



From the geometry of the diagram:

$$\delta y_F = \delta y_G = \delta y_H$$

$$\delta y_D = \frac{0.9}{1.2} \delta y_F = \frac{0.9}{1.2} \delta y_H = 0.75 \delta y_H$$

$$\delta y_C = \frac{1.5}{0.9} \delta y_D = \frac{1.5}{0.9} (0.75 \delta y_H) = 1.25 \delta y_H$$

$$\delta y_B = \frac{0.5}{0.9} \delta y_C = \frac{0.5}{0.9} (1.25 \delta y_H) = 0.69444 \delta y_H$$

Virtual Work:

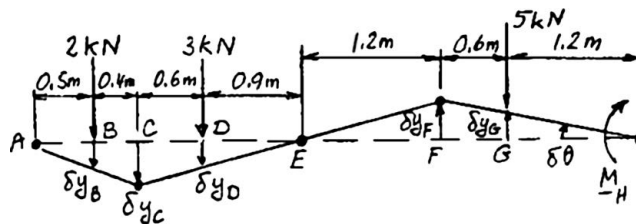
$$\delta U = (2 \text{ kN}) \delta y_B + (3 \text{ kN}) \delta y_D - (5 \text{ kN}) \delta y_G + H \delta y_H = 0$$

$$2(0.69444 \delta y_H) + 3(0.75 \delta y_H) - 5 \delta y_H + H \delta y_H = 0$$

$$H = +1.3611 \text{ kN}$$

$$\mathbf{H} = 1.361 \text{ kN} \uparrow \leftarrow$$

Couple at H . We rotate beam FH through $\delta \theta$ about Point H .



PROBLEM 10.54 (Continued)

From the geometry of the diagram:

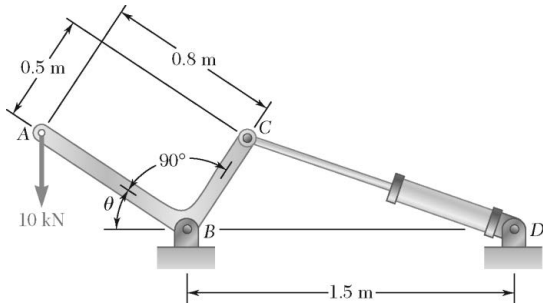
$$\begin{aligned}\delta y_G &= 1.2\delta\theta & \delta y_F &= 1.8\delta\theta \\ \delta y_D &= \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}(1.8\delta\theta) = 1.35\delta\theta \\ \delta y_C &= \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(1.35\delta\theta) = 2.25\delta\theta \\ \delta y_B &= \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta\theta) = 1.25\delta\theta\end{aligned}$$

Virtual Work:

$$\begin{aligned}\delta U &= (2 \text{ kN})\delta y_B + (3 \text{ kN})\delta y_D - (5 \text{ kN})\delta y_G + M_H\delta\theta = 0 \\ &2(1.25\delta\theta) + 3(1.35\delta\theta) - 5(1.2\delta\theta) + M_H\delta\theta = 0\end{aligned}$$

$$M_H = -0.550 \text{ kN} \cdot \text{m} \qquad \mathbf{M}_H = 550 \text{ N} \cdot \text{m} \curvearrowleft$$

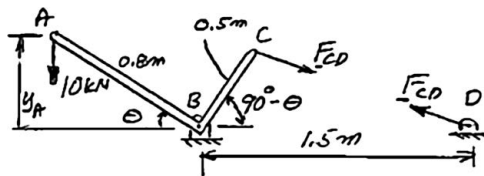
PROBLEM 10.55



Referring to Problem 10.43 and using the value found for the force exerted by the hydraulic cylinder CD , determine the change in the length of CD required to raise the 10-kN load by 15 mm.

PROBLEM 10.43 The position of member ABC is controlled by the hydraulic cylinder CD . For the loading shown, determine the force exerted by the hydraulic cylinder on pin C when $\theta = 55^\circ$.

SOLUTION



Virtual Work: Assume both δy_A and δ_{CD} increase

$$\delta U = 0: \quad -(10 \text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$$

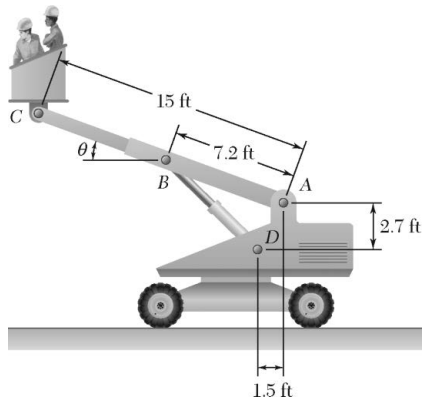
Substitute: $\delta y_A = 15 \text{ mm}$ and $F_{CD} = 12.03 \text{ kN}$

$$-(10 \text{ kN})(15 \text{ mm}) - (12.03 \text{ kN})\delta_{CD} = 0$$

$$\delta_{CD} = -12.47 \text{ mm}$$

The negative sign indicates that CD shortened

$$\delta_{CD} = 12.47 \text{ mm shorter} \quad \blacktriangleleft$$

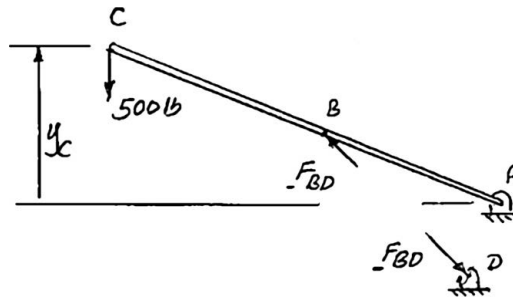


PROBLEM 10.56

Referring to Problem 10.45 and using the value found for the force exerted by the hydraulic cylinder BD , determine the change in the length of BD required to raise the platform attached at C by 2.5 in.

PROBLEM 10.45 The telescoping arm ABC is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above C . For the position when $\theta = 20^\circ$, determine the force exerted on pin B by the single hydraulic cylinder BD .

SOLUTION



Virtual Work: Assume both δy_C and δ_{BD} increase

$$\delta U = 0: \quad -(500 \text{ lb})\delta y_C + F_{BD}\delta_{BD} = 0$$

Substitute:

$$\delta y_C = 2.5 \text{ in.} \quad \text{and} \quad F_{BD} = 2370 \text{ lb}$$

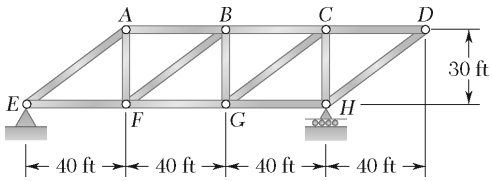
$$-(500 \text{ lb})(2.5 \text{ in.}) + (2370 \text{ lb})\delta_{BD} = 0$$

$$\delta_{BD} = +0.527 \text{ in.}$$

The positive sign indicates that cylinder BD increases in length

$$\delta_{BD} = 0.527 \text{ in. longer} \quad \blacktriangleleft$$

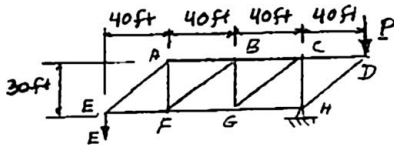
PROBLEM 10.57



Determine the vertical movement of joint D if the length of member BF is increased by 1.5 in. (*Hint: Apply a vertical load at joint D , and, using the methods of Chapter 6, compute the force exerted by member BF on joints B and F . Then apply the method of virtual work for a virtual displacement resulting in the specified increase in length of member BF . This method should be used only for small changes in the lengths of members.*)

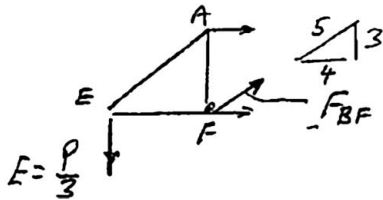
SOLUTION

Apply vertical load P at D .



$$+\circlearrowleft \Sigma M_H = 0: \quad -P(40 \text{ ft}) + E(120 \text{ ft}) = 0$$

$$E = \frac{P}{3} \downarrow$$



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5} F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9} P$$

Virtual Work:

We remove member BF and replace it with forces \mathbf{F}_{BF} and $-\mathbf{F}_{BF}$ at pins F and B , respectively. Denoting the virtual displacements of Points B and F as $\delta \mathbf{r}_B$ and $\delta \mathbf{r}_F$, respectively, and noting that \mathbf{P} and δD have the same direction, we have

Virtual Work:

$$\delta U = 0: \quad P \delta D + \mathbf{F}_{BF} \cdot \delta \mathbf{r}_F + (-\mathbf{F}_{BF}) \cdot \delta \mathbf{r}_B = 0$$

$$P \delta D + F_{BF} \delta r_F \cos \theta_F - F_{BF} \delta r_B \cos \theta_B = 0$$

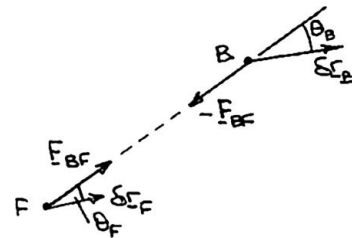
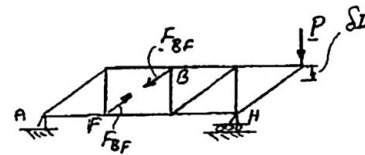
$$P \delta D - F_{BF} (\delta r_B \cos \theta_B - \delta r_F \cos \theta_F) = 0$$

where $(\delta r_B \cos \theta_B - \delta r_F \cos \theta_F) = \delta_{BF}$, which is the change in length of member BF . Thus,

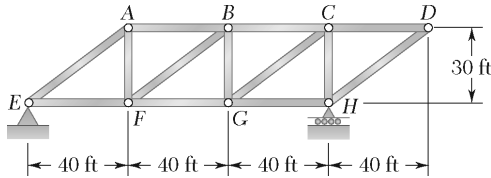
$$P \delta D - F_{BF} \delta_{BF} = 0$$

$$P \delta D - \left(\frac{5}{9} P \right) (1.5 \text{ in.}) = 0$$

$$\delta D = 0.833 \text{ in.}$$



$$\delta D = 0.833 \text{ in.} \downarrow \blacktriangleleft$$

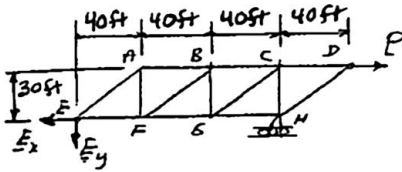


PROBLEM 10.58

Determine the horizontal movement of joint D if the length of member BF is increased by 1.5 in. (See the hint for Problem 10.57.)

SOLUTION

Apply horizontal load P at D .

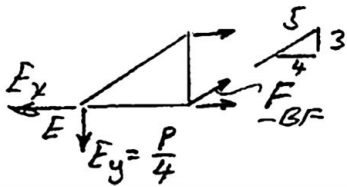


$$+\circlearrowleft \Sigma M_H = 0: P(30 \text{ ft}) - E_y(120 \text{ ft}) = 0$$

$$E_y = \frac{P}{4} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \frac{3}{5} F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12} P$$



We remove member BF and replace it with forces \mathbf{F}_{BF} and $-\mathbf{F}_{BF}$ at pins F and B , respectively. Denoting the virtual displacements of Points B and F as $\delta \mathbf{r}_B$ and $\delta \mathbf{r}_F$, respectively, and noting that \mathbf{P} and δD have the same direction, we have

Virtual Work:

$$\delta U = 0: P \delta D + F_{BF} \cdot \delta \mathbf{r}_F + (-F_{BF}) \cdot \delta \mathbf{r}_B = 0$$

$$P \delta D + F_{BF} \delta r_F \cos \theta_F - F_{BF} \delta r_B \cos \theta_B = 0$$

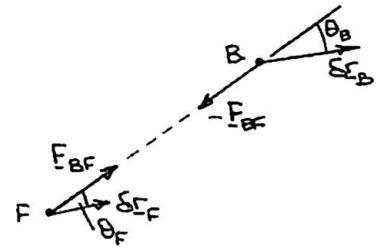
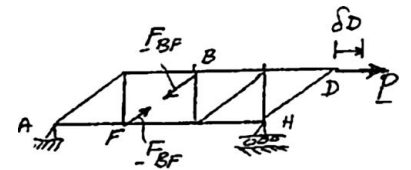
$$P \delta D - F_{BF} (\delta r_B \cos \theta_B - \delta r_F \cos \theta_F) = 0$$

where $(\delta r_B \cos \theta_B - \delta r_F \cos \theta_F) = \delta_{BF}$, which is the change in length of member BF . Thus,

$$P \delta D - F_{BF} \delta_{BF} = 0$$

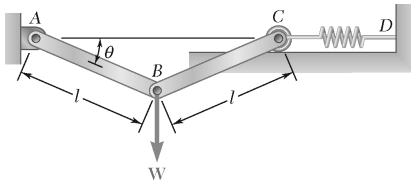
$$P \delta D - \left(\frac{5}{12} P \right) (1.5 \text{ in.}) = 0$$

$$\delta D = 0.625 \text{ in.}$$



$$\delta D = 0.625 \text{ in.} \rightarrow \blacktriangleleft$$

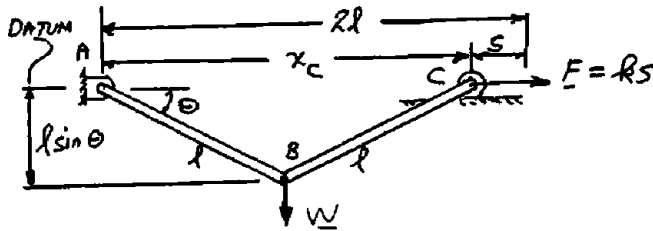
PROBLEM 10.59



Using the method of Section 10.8, solve Problem 10.29.

PROBLEM 10.29 A load W of magnitude 600 N is applied to the linkage at B . The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$W = 600 \text{ N}$$

$$l = 0.3 \text{ m, and } k = 2500 \text{ N/m}$$

We have

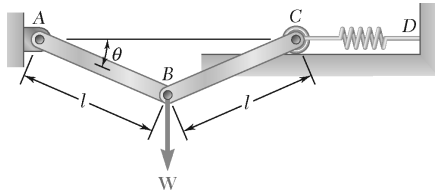
$$(l - \cos \theta) \tan \theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

$$= 0.2$$

Solving numerically

$$\theta = 40.2^\circ \blacktriangleleft$$

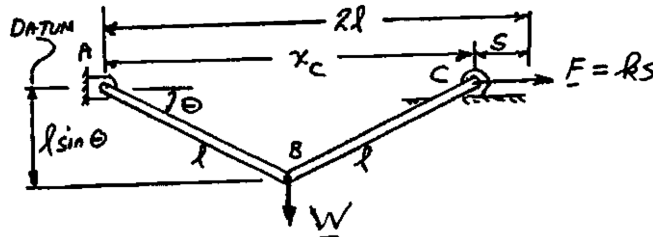
PROBLEM 10.60



Using the method of Section 10.8, solve Problem 10.30.

PROBLEM 10.30 A vertical load W is applied to the linkage at B . The constant of the spring is k , and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, derive an equation in θ , W , l , and k that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$V = \frac{1}{2} k s^2 + W y_B$$

$$V = \frac{1}{2} k (2l - x_C)^2 + W y_B$$

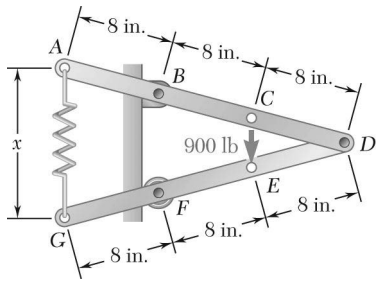
$$x_C = 2l \cos \theta \quad \text{and} \quad y_B = -l \sin \theta$$

Thus

$$V = \frac{1}{2} k (2l - 2l \cos \theta)^2 - W l \sin \theta$$

$$= 2kl^2 (1 - \cos \theta)^2 - W l \sin \theta$$

$$\frac{dV}{d\theta} = 2kl^2 2(1 - \cos \theta) \sin \theta - W l \cos \theta = 0 \quad (1 - \cos \theta) \tan \theta = \frac{W}{4kl} \quad \blacktriangleleft$$



PROBLEM 10.61

Using the method of Section 10.8, solve Problem 10.31.

PROBLEM 10.31 Two bars AD and DG are connected by a pin at D and by a spring AG . Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at E as shown.

SOLUTION

$$V = \frac{1}{2}ks^2 + Wy_E$$

But

$$s = x - 12 \text{ in.}$$

and

$$y_E = -\frac{x}{3} - \frac{x}{6}$$

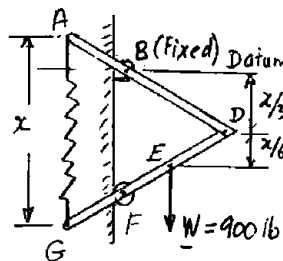
$$= -\frac{x}{2}$$

Thus

$$V = \frac{1}{2}k(x-12)^2 - W\left(\frac{x}{2}\right)$$

$$\frac{dV}{dx} = k(x-12) - \frac{1}{2}W = 0$$

$$x = 12 + \frac{W}{2k} = 12 \text{ in.} + \frac{900 \text{ lb}}{2(125 \text{ lb/in.})}$$



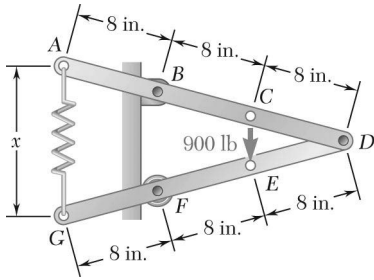
$$x = 15.60 \text{ in.} \blacktriangleleft$$

PROBLEM 10.62

Using the method of Section 10.8, solve Problem 10.32.

PROBLEM 10.32 Solve Problem 10.31 assuming that the 900-lb vertical force is applied at C instead of E .

PROBLEM 10.31 Two bars AD and DG are connected by a pin at D and by a spring AG . Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at E as shown.



SOLUTION

$$V = \frac{1}{2}ks^2 + Wy_C$$

But

$$s = x - 12 \text{ in.}$$

and

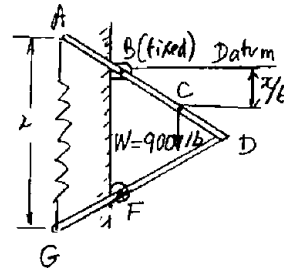
$$y_E = -\frac{x}{6}$$

Thus

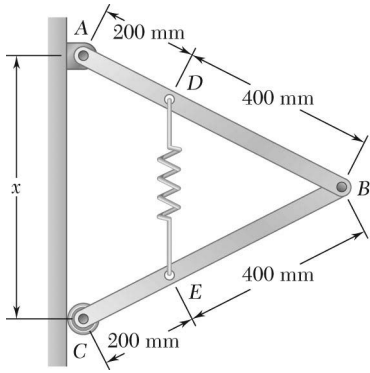
$$V = \frac{1}{2}k(x - 12)^2 - \frac{1}{6}Wx$$

$$\frac{dV}{dx} = k(x - 12) - \frac{1}{6}W = 0$$

$$x = 12 + \frac{W}{6k} = 12 \text{ in.} + \frac{900 \text{ lb}}{6(125 \text{ lb/in.})}$$



$$x = 13.20 \text{ in.} \blacktriangleleft$$



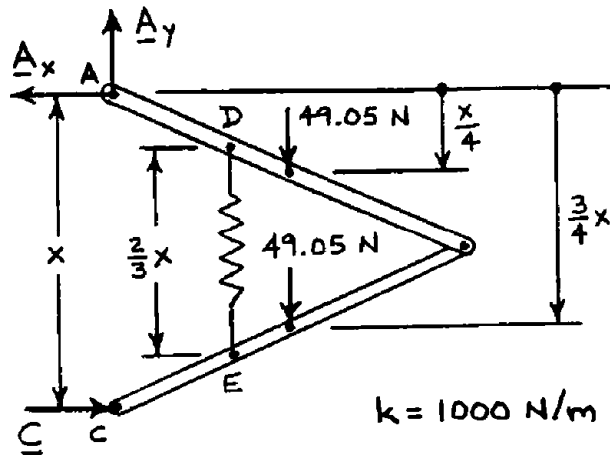
PROBLEM 10.63

Using the method of Section 10.8, solve Problem 10.33.

PROBLEM 10.33 Two 5-kg bars AB and BC are connected by a pin at B and by a spring DE . Knowing that the spring is 150 mm long when unstretched and that the constant of the spring is 1 kN/m, determine the value of x corresponding to equilibrium.

SOLUTION

First note: $W_{\text{bar}} = (5 \text{ kg})(9.81 \text{ m/s}^2) = 49.05 \text{ N}$



Since unstretched length of spring is 150 mm, or 0.15 m, we have

$$\Delta_s = \frac{2}{3}x - 0.15$$

$$V = \frac{1}{2}k\Delta_s^2 - (49.05 \text{ N})\frac{x}{4} - (49.05 \text{ N})\frac{3x}{4}$$

$$V = \frac{1}{2}(1000 \text{ N/m})\left(\frac{2}{3}x - 0.15\right)^2 - 12.2625x - 36.7875x$$

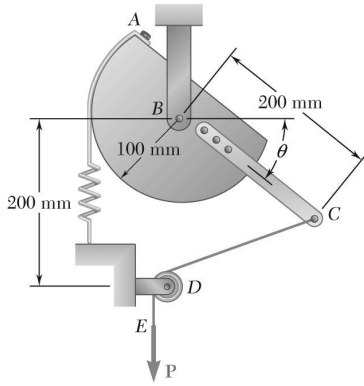
$$\frac{dV}{dx} = 1000\left(\frac{2}{3}x - 0.15\right)\frac{2}{3} - 12.2625 - 36.7875 = 0$$

$$x = 0.335 \text{ m}$$

$$x = 335 \text{ mm} \blacktriangleleft$$

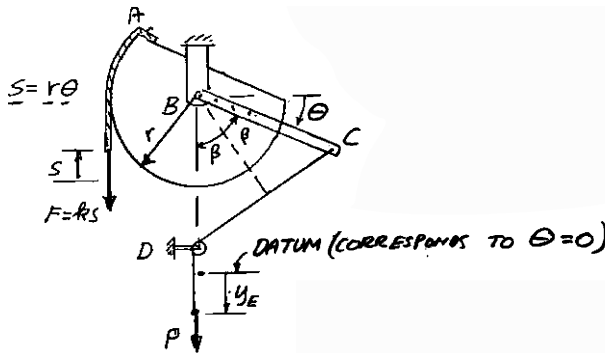
PROBLEM 10.64

Using the method of Section 10.8, solve Problem 10.35.



PROBLEM 10.35 A vertical force \mathbf{P} of magnitude 150 N is applied to end E of cable CDE , which passes over a small pulley D and is attached to the mechanism at C . The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium.

SOLUTION



$$\beta = \frac{1}{2}(90^\circ - \theta) = 45^\circ - \frac{\theta}{2}$$

$$BC = BD = l$$

$$CD = 2l \sin \beta = 2l \sin \left(45^\circ - \frac{\theta}{2} \right)$$

$$\text{For } \theta = 0: (CD)_0 = 2l \sin 45^\circ = \sqrt{2}l$$

$$y_E = (CD)_0 - CD$$

$$= \sqrt{2}l - 2l \sin \left(45^\circ - \frac{\theta}{2} \right)$$

Potential energy:

$$V = \frac{1}{2}ks^2 - Py_E$$

$$V = \frac{1}{2}k(r\theta)^2 - P \left[\sqrt{2}l - 2l \sin \left(45^\circ - \frac{\theta}{2} \right) \right]$$

$$\frac{dV}{d\theta} = kr^2\theta + 2Pl \cos \left(45^\circ - \frac{\theta}{2} \right) \left(-\frac{1}{2} \right) = 0$$

$$\frac{Pl}{kr^2} = \frac{\theta}{\cos \left(45^\circ - \frac{\theta}{2} \right)}$$

$$\frac{Pl}{kr^2} = \frac{(150 \text{ N})(0.2 \text{ m})}{(4000 \text{ N/m})(0.1 \text{ m})^2} = 0.75$$

$$0.75 = \frac{\theta}{\cos \left(45^\circ - \frac{\theta}{2} \right)}$$

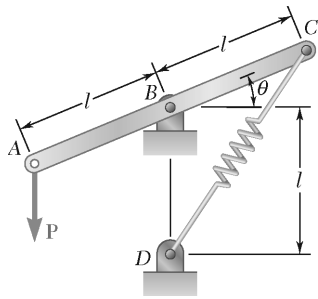
Solve by trial and error:

$$\theta = 38.745^\circ$$

$$\theta = 0.67623 \text{ rad}$$

$$\theta = 38.7^\circ \blacktriangleleft$$

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PROBLEM 10.65

Using the method of Section 10.8, solve Problem 10.37.

PROBLEM 10.37 and 10.38 Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

PROBLEM 10.37 $P = 300 \text{ N}$, $l = 400 \text{ mm}$, $k = 5 \text{ kN/m}$.

SOLUTION

Spring $v = 2l \sin\left(\frac{90^\circ + \theta}{2}\right)$

$$v = 2l \sin\left(45^\circ + \frac{\theta}{2}\right)$$

Unstretched ($\theta = 0$) $v_0 = 2l \sin 45^\circ = \sqrt{2}l$

Deflection of spring $s = v - v_0 = 2l \sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}l$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2 \left[2 \sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2} \right]^2 + P(-l \sin \theta)$$

$$\frac{dV}{d\theta} = kl^2 \left[2 \sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2} \right] \cos\left(45^\circ + \frac{\theta}{2}\right) - Pl \cos \theta = 0$$

$$\left[2 \sin\left(45^\circ + \frac{\theta}{2}\right) \cos\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2} \cos\left(45^\circ + \frac{\theta}{2}\right) \right] = \frac{P}{kl} \cos \theta$$

$$\cos \theta - \sqrt{2} \cos\left(45^\circ + \frac{\theta}{2}\right) = \frac{P}{kl} \cos \theta$$

Divide each member by $\cos \theta$ $1 - \sqrt{2} \frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} = \frac{P}{kl}$

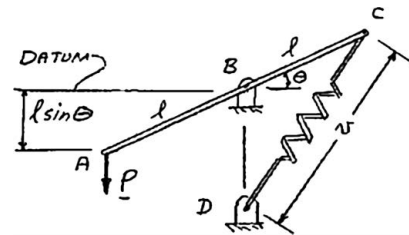
Then with $P = 300 \text{ N}$, $l = 0.4 \text{ m}$ and $k = 5000 \text{ N/m}$

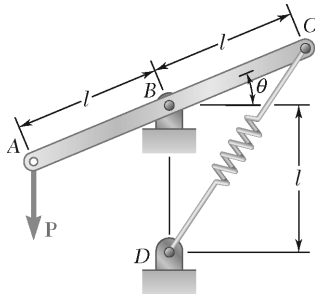
$$1 - \sqrt{2} \frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} = \frac{300 \text{ N}}{(5000 \text{ N/m})(0.4 \text{ m})} = 0.15$$

or $\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} = 0.60104$

Solving numerically

$$\theta = 22.6^\circ \blacktriangleleft$$





PROBLEM 10.66

Using the method of Section 10.8, solve Problem 10.38.

PROBLEM 10.37 and 10.38 Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

PROBLEM 10.38 $P = 75 \text{ lb}$, $l = 15 \text{ in.}$, $k = 20 \text{ lb/in.}$

SOLUTION

Using the results of Problem 10.65 with $P = 75 \text{ lb}$, $l = 15 \text{ in.}$ and $k = 20 \text{ lb/in.}$, we have

$$\begin{aligned}
 1 - \sqrt{2} \frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} &= \frac{P}{kl} \\
 &= \frac{75 \text{ lb}}{(20 \text{ lb/in.})(15 \text{ in.})} \\
 &= 0.25
 \end{aligned}$$

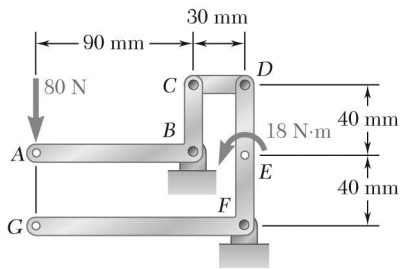
or

$$\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos \theta} = 0.53033$$

Solving numerically

$$\theta = 51.058^\circ$$

$$\theta = 51.1^\circ \blacktriangleleft$$

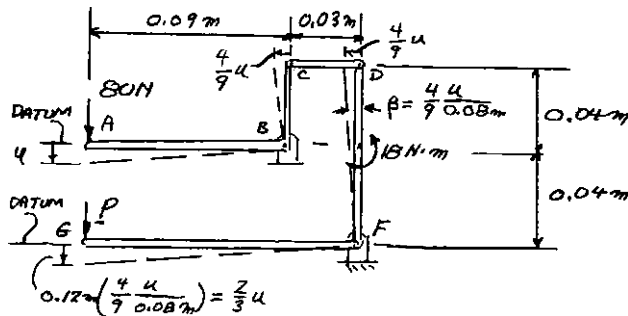


PROBLEM 10.67

Show that equilibrium is neutral in Problem 10.1.

PROBLEM 10.1 Determine the vertical force \mathbf{P} that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



We have

$$y_A = -u, \quad y_G = -\frac{2}{3}u, \quad \beta = \frac{4u}{0.72}$$

$$V = (80 \text{ N})y_A + P(y_G) - (18 \text{ N} \cdot \text{m})\beta$$

$$V = 80(-u) + P\left(-\frac{2}{3}u\right) - (18)\frac{4u}{0.72}$$

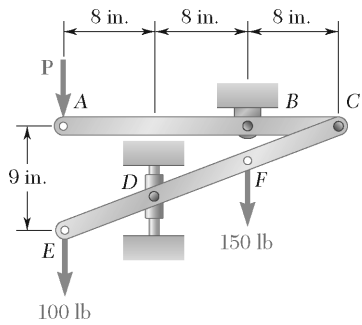
$$\frac{dV}{du} = -80 - \frac{2}{3}P - 100 = 0$$

$$P = 270 \text{ N}$$

$$P = 270 \text{ N} \uparrow \blacktriangleleft$$

Substituting $P = 270 \text{ N}$ in the expression for V , we have $V = 0$. Thus V is constant

and equilibrium is neutral \blacktriangleleft

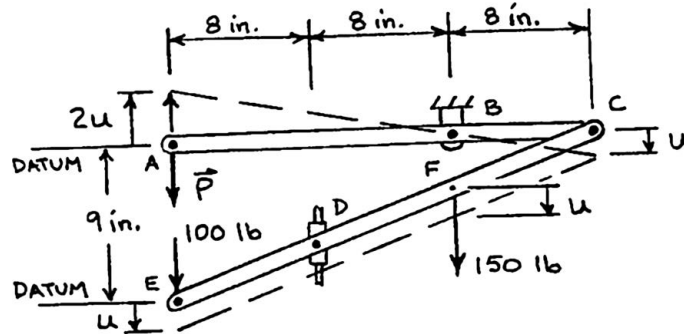


PROBLEM 10.68

Show that equilibrium is neutral in Problem 10.7.

PROBLEM 10.7 The two-bar linkage shown is supported by a pin and bracket at B and a collar at D that slides freely on a vertical rod. Determine the force P required to maintain the equilibrium of the linkage.

SOLUTION



$$y_A = 2u$$

$$y_E = -u$$

$$y_F = -u$$

$$V = Py_A + (100 \text{ lb})y_E + (150 \text{ lb})y_F$$

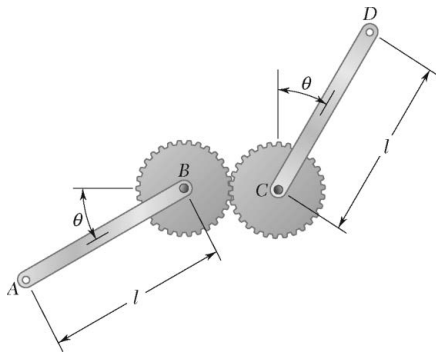
$$V = P(2u) + (100 \text{ lb})(-u) + (150 \text{ lb})(-u)$$

$$\frac{dV}{du} = 2P - 100 - 150 = 0$$

$$P = 125 \text{ lb}$$

Now, substitute $P = 125 \text{ lb}$ in expression for V , making $V = 0$. Thus, V is constant and

equilibrium is neutral. ◀



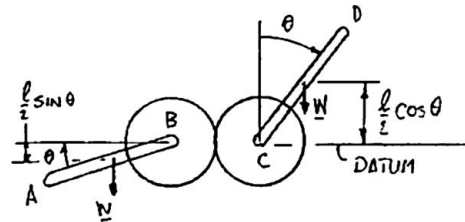
PROBLEM 10.69

Two uniform rods, each of mass m , are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Potential energy

$$\begin{aligned}
 V &= W \left(-\frac{l}{2} \sin \theta \right) + W \left(\frac{l}{2} \cos \theta \right) \quad W = mg \\
 &= W \frac{l}{2} (\cos \theta - \sin \theta) \\
 \frac{dV}{d\theta} &= \frac{Wl}{2} (-\sin \theta - \cos \theta) \\
 \frac{d^2V}{d\theta^2} &= \frac{Wl}{2} (\sin \theta - \cos \theta)
 \end{aligned}$$



For equilibrium:

$$\frac{dV}{d\theta} = 0: \quad \sin \theta = -\cos \theta$$

or

$$\tan \theta = -1$$

Thus

$$\theta = -45.0^\circ \quad \text{and} \quad \theta = 135.0^\circ$$

Stability:

At $\theta = -45.0^\circ$:

$$\begin{aligned}
 \frac{d^2V}{d\theta^2} &= \frac{Wl}{2} [\sin(-45^\circ) - \cos 45^\circ] \\
 &= \frac{Wl}{2} \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) < 0
 \end{aligned}$$

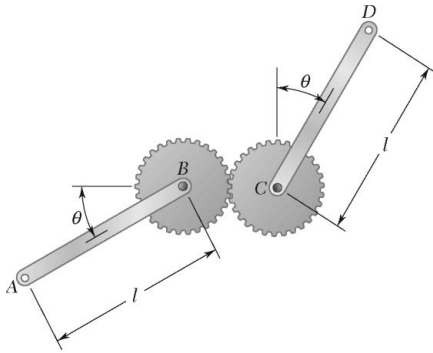
$\theta = -45.0^\circ$, Unstable ◀

At $\theta = 135.0^\circ$:

$$\begin{aligned}
 \frac{d^2V}{d\theta^2} &= \frac{Wl}{2} (\sin 135^\circ - \cos 135^\circ) \\
 &= \frac{Wl}{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) > 0
 \end{aligned}$$

$\theta = 135.0^\circ$, Stable ◀

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PROBLEM 10.70

Two uniform rods, AB and CD , are attached to gears of equal radii as shown. Knowing that $W_{AB} = 8 \text{ lb}$ and $W_{CD} = 4 \text{ lb}$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Potential energy

$$V = (3.5 \text{ kg} \times 9.81 \text{ m/s}^2) \left(-\frac{l}{2} \sin \theta \right) + (1.75 \text{ kg} \times 9.81 \text{ m/s}^2) \left(\frac{l}{2} \cos \theta \right)$$

$$= (8.5838 \text{ N})l(-2 \sin \theta + \cos \theta)$$

$$\frac{dV}{d\theta} = (8.5838 \text{ N})l(-2 \cos \theta - \sin \theta)$$

$$\frac{d^2V}{d\theta^2} = (8.5838 \text{ N})l(2 \sin \theta - \cos \theta)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: \quad -2 \cos \theta - \sin \theta = 0$$

or

$$\tan \theta = -2$$

Thus

$$\theta = -63.4^\circ \quad \text{and} \quad 116.6^\circ$$

Stability:

At $\theta = -63.4^\circ$:

$$\frac{d^2V}{d\theta^2} = (8.5838 \text{ N})l[2 \sin(-63.4^\circ) - \cos(-63.4^\circ)]$$

$$= (8.5838 \text{ N})l(-1.788 - 0.448) < 0$$

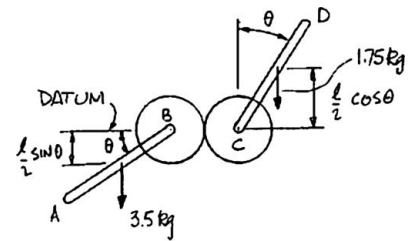
$\theta = -63.4^\circ$, Unstable ◀

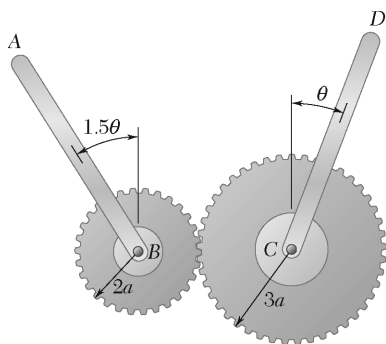
At $\theta = 116.6^\circ$:

$$\frac{d^2V}{d\theta^2} = (8.5838 \text{ N})l[2 \sin(116.6^\circ) - \cos(116.6^\circ)]$$

$$= (8.5838 \text{ N})l(1.788 + 0.447) > 0$$

$\theta = 116.6^\circ$, Stable ◀

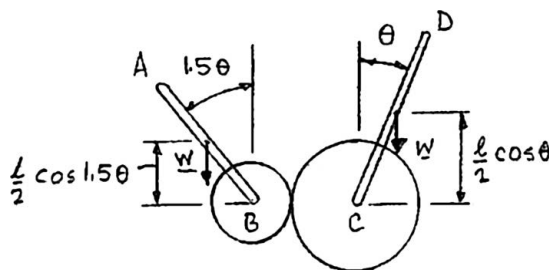




PROBLEM 10.71

Two uniform rods, each of mass m and length l , are attached to gears as shown. For the range $0 \leq \theta \leq 180^\circ$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential energy

$$V = W \left(\frac{l}{2} \cos 1.5\theta \right) + W \left(\frac{l}{2} \cos \theta \right) \quad W = mg$$

$$\begin{aligned} \frac{dV}{d\theta} &= \frac{Wl}{2} (-1.5 \sin 1.5\theta) + \frac{Wl}{2} (-\sin \theta) \\ &= -\frac{Wl}{2} (1.5 \sin 1.5\theta + \sin \theta) \end{aligned}$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2} (2.25 \cos 1.5\theta + \cos \theta)$$

For equilibrium

$$\frac{dV}{d\theta} = 0: \quad 1.5 \sin 1.5\theta + \sin \theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than 180° can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8^\circ$$

Now

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2} (2.25 \cos 1.5\theta + \cos \theta)$$

At $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2} (2.25 \cos 0^\circ + \cos 0^\circ)$$

$$= -\frac{Wl}{2} (3.25) (< 0)$$

$\theta = 0$, Unstable ◀

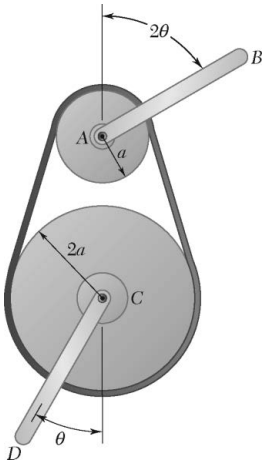
PROBLEM 10.71 (Continued)

At $\theta = 137.8^\circ$:

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2} [2.25 \cos(1.5 \times 137.8^\circ) + \cos 137.8^\circ]$$

$$= \frac{Wl}{2} (2.75) (> 0)$$

$\theta = 137.8^\circ$, Stable ◀



PROBLEM 10.72

Two uniform rods, each of mass m and length l , are attached to drums that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the drums, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

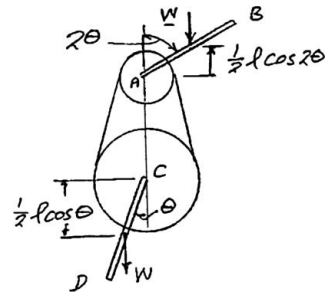
SOLUTION

$$W = mg$$

$$V = W \left(\frac{l}{2} \cos 2\theta \right) - W \left(\frac{l}{2} \cos \theta \right)$$

$$\frac{dV}{d\theta} = W \frac{l}{2} (-2 \sin 2\theta + \sin \theta)$$

$$\frac{d^2V}{d\theta^2} = W \frac{l}{2} (-4 \cos 2\theta - \cos \theta)$$



Equilibrium:

$$\frac{dV}{d\theta} = 0: \quad \frac{Wl}{2} (-2 \sin 2\theta + \sin \theta) = 0$$

or

$$\sin \theta (-4 \cos \theta + 1) = 0$$

Solving

$$\theta = 0, 75.5^\circ, 180^\circ, \text{ and } 284^\circ$$

Stability:

$$\frac{d^2V}{d\theta^2} = W \frac{l}{2} (-4 \cos 2\theta - \cos \theta)$$

At $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = W \frac{l}{2} (-4 - 1) < 0$$

$\theta = 0$, Unstable ◀

At $\theta = 75.5^\circ$:

$$\frac{d^2V}{d\theta^2} = W \frac{l}{2} (-4(-.874) - .25) > 0$$

$\theta = 75.5^\circ$, Stable ◀

At $\theta = 180^\circ$:

$$\frac{d^2V}{d\theta^2} = W \frac{l}{2} (-4 + 1) < 0$$

$\theta = 180.0^\circ$, Unstable ◀

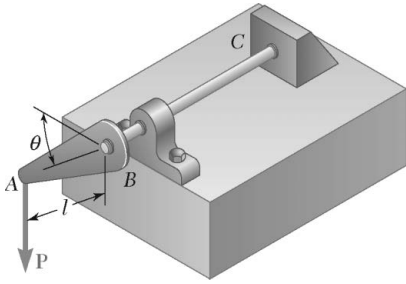
At $\theta = 284^\circ$:

$$\frac{d^2V}{d\theta^2} = W \frac{l}{2} (-4(-.874) - .25) > 0$$

$\theta = 284^\circ$, Stable ◀

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PROBLEM 10.73



Using the method of Section 10.8, solve Problem 10.39. Determine whether the equilibrium is stable, unstable, or neutral. (*Hint:* The potential energy corresponding to the couple exerted by a torsion spring is $\frac{1}{2}K\theta^2$, where K is the torsional spring constant and θ is the angle of twist.)

PROBLEM 10.39 The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C . The torsional spring constant of the shaft BC is K ; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100 \text{ N}$, $l = 250 \text{ mm}$, and $K = 12.5 \text{ N} \cdot \text{m/rad}$.

SOLUTION

Potential energy

$$V = \frac{1}{2}K\theta^2 - Pl \sin \theta$$

$$\frac{dV}{d\theta} = K\theta - Pl \cos \theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl \sin \theta$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: \quad \cos \theta = \frac{K}{Pl} \theta$$

For

$$P = 100 \text{ N}, \quad l = 0.25 \text{ m}, \quad K = 12.5 \text{ N} \cdot \text{m/rad}$$

$$\begin{aligned} \cos \theta &= \frac{12.5 \text{ N} \cdot \text{m/rad}}{(100)(0.25 \text{ m})} \theta \\ &= 0.500\theta \end{aligned}$$

Solving numerically, we obtain

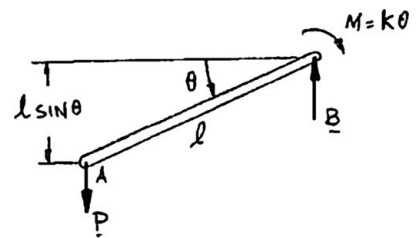
$$\theta = 1.02967 \text{ rad} = 59.000^\circ$$

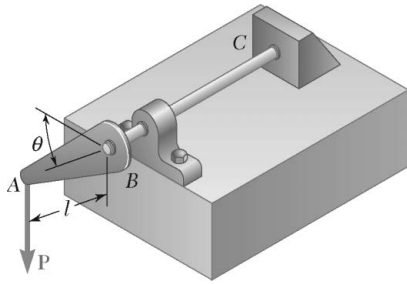
$$\theta = 59.0^\circ \quad \blacktriangleleft$$

Stability

$$\frac{d^2V}{d\theta^2} = (12.5 \text{ N} \cdot \text{m/rad}) + (100 \text{ N})(0.25 \text{ m}) \sin 59.0^\circ > 0$$

Stable \blacktriangleleft





PROBLEM 10.74

In Problem 10.40, determine whether each of the positions of equilibrium is stable, unstable, or neutral. (See hint for Problem 10.73.)

PROBLEM 10.40 Solve Problem 10.39 assuming that $P = 350 \text{ N}$, $l = 250 \text{ mm}$, and $K = 12.5 \text{ N} \cdot \text{m/rad}$. Obtain answers in each of the following quadrants: $0 < \theta < 90^\circ$, $270^\circ < \theta < 360^\circ$, $360^\circ < \theta < 450^\circ$.

SOLUTION

Potential energy $V = \frac{1}{2} K \theta^2 - Pl \sin \theta$

$$\frac{dV}{d\theta} = K\theta - Pl \cos \theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl \sin \theta$$

Equilibrium $\frac{dV}{d\theta} = 0: \cos \theta = \frac{K}{Pl} \theta$

For $P = 350 \text{ N}$, $l = 0.250 \text{ m}$ and $K = 12.5 \text{ N} \cdot \text{m/rad}$

$$\cos \theta = \frac{12.5 \text{ N} \cdot \text{m/rad}}{(350 \text{ N})(0.250 \text{ m})} \theta$$

or $\cos \theta = \frac{\theta}{7}$

Solving numerically $\theta = 1.37333 \text{ rad}$, 5.652 rad , and 6.616 rad

or $\theta = 78.7^\circ$, 323.8° , 379.1°

Stability at $\theta = 78.7^\circ$: $\frac{d^2V}{d\theta^2} = (12.5 \text{ N} \cdot \text{m/rad}) + (350 \text{ N})(0.250 \text{ m}) \sin 78.7^\circ$

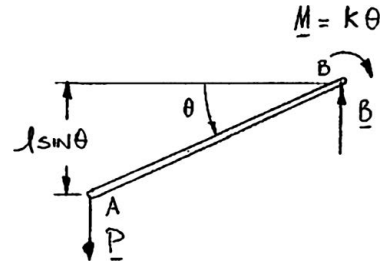
$$= 98.304 > 0 \quad \theta = 78.7^\circ, \text{ Stable} \blacktriangleleft$$

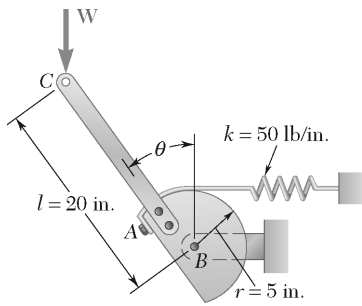
At $\theta = 323.8^\circ$: $\frac{d^2V}{d\theta^2} = (12.5 \text{ N} \cdot \text{m/rad}) + (350 \text{ N})(0.250 \text{ m}) \sin 323.8^\circ$

$$= -39.178 \text{ N} \cdot \text{m} < 0 \quad \theta = 324^\circ, \text{ Unstable} \blacktriangleleft$$

At At $\theta = 379.1^\circ$: $\frac{d^2V}{d\theta^2} = (12.5 \text{ N} \cdot \text{m/rad}) + (350 \text{ N})(0.250 \text{ m}) \sin 379.1^\circ$

$$= 44.132 \text{ N} \cdot \text{m} > 0 \quad \theta = 379^\circ, \text{ Stable} \blacktriangleleft$$





PROBLEM 10.75

A load W of magnitude 100 lb is applied to the mechanism at C . Knowing that the spring is unstretched when $\theta = 15^\circ$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

We have

$$y_C = l \cos \theta$$

$$V = \frac{1}{2} k [r(\theta - \theta_0)]^2 + W y_C \quad \theta_0 = 15^\circ = \frac{\pi}{12} \text{ rad}$$

$$= \frac{1}{2} k r^2 (\theta - \theta_0)^2 + W l \cos \theta$$

$$\frac{dV}{d\theta} = k r^2 (\theta - \theta_0) - W l \sin \theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad k r^2 (\theta - \theta_0) - w l \sin \theta = 0 \quad (1)$$

with

$$W = 100 \text{ lb}, \quad R = 50 \text{ lb./in.}, \quad l = 20 \text{ in.}, \quad \text{and} \quad r = 5 \text{ in.}$$

$$(50 \text{ lb./in.})(25 \text{ in.}^2) \left(\theta - \frac{\pi}{12} \right) - (100 \text{ lb})(20 \text{ in.}) \sin \theta = 0$$

or

$$0.625\theta - \sin \theta = 0.16362$$

Solving numerically

$$\theta = 1.8145 \text{ rad} = 103.97^\circ$$

$$\theta = 104.0^\circ \quad \blacktriangleleft$$

Stability

$$\frac{d^2V}{d\theta^2} = k r^2 - W l \cos \theta \quad (2)$$

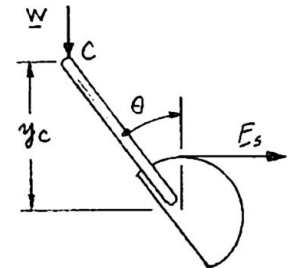
or

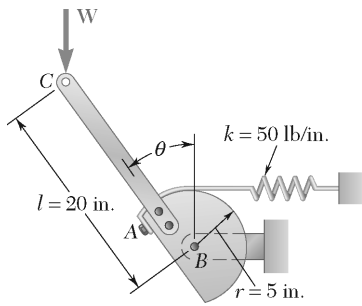
$$= 1250 - 2000 \cos \theta$$

For $\theta = 104.0^\circ$:

$$= 1734 \text{ in.} \cdot \text{lb} > 0$$

Stable \blacktriangleleft





PROBLEM 10.76

A load W of magnitude 100 lb is applied to the mechanism at C . Knowing that the spring is unstretched when $\theta = 30^\circ$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

Using the solution of Problem 10.75, particularly Equation (1), with 15° replaced by $30^\circ \left(\frac{\pi}{6} \text{ rad} \right)$:

$$\text{For equilibrium} \quad kr^2 \left(\theta - \frac{\pi}{6} \right) - Wl \sin \theta = 0$$

With $k = 50 \text{ lb/in.}$, $W = 100 \text{ lb}$, $r = 5 \text{ in.}$, and $l = 20 \text{ in.}$

$$(50 \text{ lb/in.})(25 \text{ in.}^2) \left(\theta - \frac{\pi}{6} \right) - (100 \text{ lb})(20 \text{ in.}) \sin \theta = 0$$

$$\text{or} \quad 1250\theta - 654.5 - 200 \sin \theta = 0$$

$$\text{Solving numerically,} \quad \theta = 1.9870 \text{ rad} = 113.8^\circ$$

$$\theta = 113.8^\circ \blacktriangleleft$$

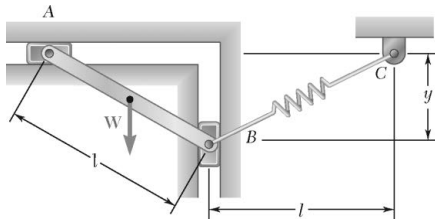
Stability: Equation (2), Problem 75:

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl \cos \theta$$

$$\text{or} \quad = 1250 - 2000 \cos \theta$$

$$\text{For } \theta = 113.8^\circ: \quad = 2057 \text{ in.} \cdot \text{lb} > 0$$

$$\text{Stable} \blacktriangleleft$$



PROBLEM 10.77

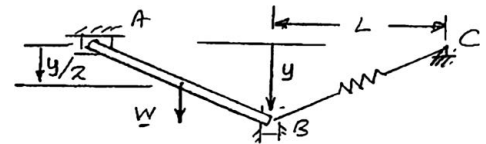
A slender rod AB , of weight W , is attached to two blocks A and B that can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $W = 80 \text{ N}$, $l = 500 \text{ mm}$, and $k = 600 \text{ N/m}$.

SOLUTION

Deflection of spring = s , where

$$s = \sqrt{l^2 + y^2} - l$$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 + y^2}}$$



Potential energy:

$$V = \frac{1}{2}ks^2 - W \frac{y}{2}$$

$$\frac{dV}{dy} = ks \frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k \left(\sqrt{l^2 + y^2} - l \right) \frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k \left(1 - \frac{l}{\sqrt{l^2 + y^2}} \right) y - \frac{1}{2}W$$

Equilibrium

$$\frac{dV}{dy} = 0: \left(1 - \frac{l}{\sqrt{l^2 + y^2}} \right) y = \frac{1}{2} \frac{W}{k}$$

Now

$$W = 80 \text{ N}, \quad l = 0.500 \text{ m}, \quad \text{and} \quad k = 600 \text{ N/m}$$

Then

$$\left(1 - \frac{0.500 \text{ m}}{\sqrt{(0.500)^2 + y^2}} \right) y = \frac{1}{2} \frac{(80 \text{ N})}{(600 \text{ N/m})}$$

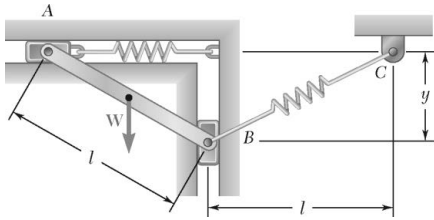
or

$$\left(1 - \frac{0.500}{\sqrt{0.25 + y^2}} \right) y = 0.066667$$

Solving numerically,

$$y = 0.357 \text{ m}$$

$$y = 357 \text{ mm} \quad \blacktriangleleft$$



PROBLEM 10.78

A slender rod AB , of weight W , is attached to two blocks A and B that can move freely in the guides shown. Knowing that both springs are unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $W = 80 \text{ N}$, $l = 550 \text{ mm}$, and $k = 600 \text{ N/m}$.

SOLUTION

Spring deflections

$$S_{AD} = l - \sqrt{l^2 - y^2}$$

$$S_{BC} = \sqrt{l^2 + y^2} - l$$

$$V = \frac{1}{2}kS_{AD}^2 + \frac{1}{2}kS_{BC}^2 - W\frac{y}{2}$$

$$V = \frac{1}{2}k\left(l - \sqrt{l^2 - y^2}\right)^2 + \frac{1}{2}k\left(\sqrt{l^2 + y^2} - l\right)^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = k\left(l - \sqrt{l^2 - y^2}\right)\left(\frac{y}{\sqrt{l^2 - y^2}}\right) + k\left(\sqrt{l^2 + y^2} - l\right)\left(\frac{y}{\sqrt{l^2 + y^2}}\right) - \frac{W}{2}$$

$$\frac{dV}{dy} = 0: \left[\left(\frac{l}{\sqrt{l^2 - y^2}} - 1\right) + \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right) \right] y = \frac{W}{2k}$$

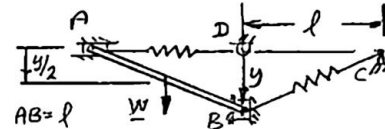
Data: $W = 80 \text{ N}$, $l = 0.5 \text{ m}$, $k = 600 \text{ N/m}$

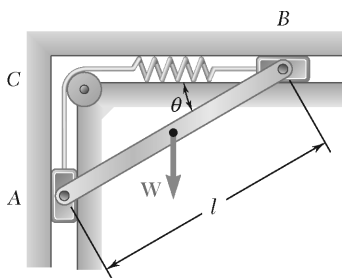
$$\left[\frac{0.5}{\sqrt{(0.5)^2 - y^2}} - \frac{0.5}{\sqrt{(0.5)^2 + y^2}} \right] y = \frac{80}{2(1200)} = 0.066667$$

Solve by trial and error:

$$y = 0.252 \text{ m}$$

$$y = 252 \text{ mm} \quad \blacktriangleleft$$

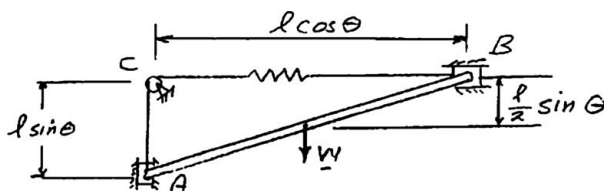




PROBLEM 10.79

A slender rod AB , of weight W , is attached to two blocks A and B that can move freely in the guides shown. The constant of the spring is k , and the spring is unstretched when AB is horizontal. Neglecting the weight of the blocks, derive an equation in θ , W , l , and k that must be satisfied when the rod is in equilibrium.

SOLUTION



Elongation of spring:

$$s = l \sin \theta + l \cos \theta - l$$

$$s = l(\sin \theta + \cos \theta - 1)$$

Potential energy:

$$V = \frac{1}{2}ks^2 - W \frac{1}{2} \sin \theta \quad W = mg$$

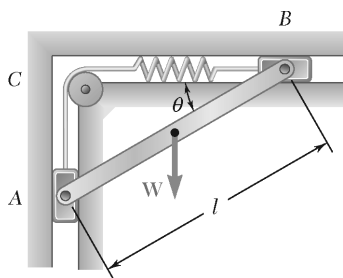
$$= \frac{1}{2}kl^2(\sin \theta + \cos \theta - 1)^2 - mg \frac{l}{2} \sin \theta$$

$$\frac{dV}{d\theta} = kl^2(\sin \theta + \cos \theta - 1)(\cos \theta - \sin \theta) - \frac{1}{2}mgl \cos \theta \quad (1)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: (\sin \theta + \cos \theta - 1)(\cos \theta - \sin \theta) - \frac{mg}{2kl} \cos \theta = 0$$

$$\text{or } \cos \theta \left[(\sin \theta + \cos \theta - 1)(1 - \tan \theta) - \frac{mg}{2kl} \right] = 0 \quad \blacktriangleleft$$



PROBLEM 10.80

A slender rod AB , of weight W , is attached to two blocks A and B that can move freely in the guides shown. Knowing that the spring is unstretched when AB is horizontal, determine three values of θ corresponding to equilibrium when $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the results of Problem 10.79, particularly the condition of equilibrium

$$\cos \theta \left[(\sin \theta + \cos \theta - 1)(1 - \tan \theta) - \frac{mg}{2kl} \right] = 0$$

Now, with $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in.

$$\frac{W}{2kl} = \frac{300 \text{ lb}}{(16 \text{ in.})(75 \text{ lb/in.})} = 0.25$$

Thus: $\cos \theta [(\sin \theta + \cos \theta - 1)(1 - \tan \theta) - 0.25] = 0$

$$\cos \theta = 0 \quad \text{and} \quad (\sin \theta + \cos \theta - 1)(1 - \tan \theta) = 0.25$$

First equation yields $\theta = 90^\circ$. Solving the second equation by trial, we find $\theta = 9.39^\circ$ and 34.16°

Values of θ for equilibrium are

$$\theta = 9.39^\circ, 34.2^\circ, \text{ and } 90.0^\circ$$

Stability: we differentiate Eq. (1).

$$\begin{aligned} \frac{d^2 y}{ds^2} &= kl^2 [(\cos \theta - \sin \theta)(\cos \theta - \sin \theta) + (\sin \theta + \cos \theta - 1)(-\sin \theta - \cos \theta)] + \frac{1}{2} wl \sin \theta \\ &= kl^2 \left[\cos^2 \theta + \sin^2 \theta - 2 \cos \theta \sin \theta - \sin^2 \theta - \cos^2 \theta - 2 \cos \theta \sin \theta + \sin \theta + \cos \theta + \frac{W}{2kl} \sin \theta \right] \\ &= kl^2 \left[\left(1 + \frac{W}{2kl} \right) \sin \theta + \cos \theta - 2 \sin 2\theta \right] \end{aligned}$$

$$\frac{d^2 V}{d\theta^2} = kl^2 (1.25 \sin \theta + \cos \theta - 2 \sin 2\theta)$$

$$\theta = 9.39^\circ: \quad \frac{d^2 V}{d\theta^2} = kl^2 (1.25 \sin 9.4 + \cos 9.4 - 2 \sin 18.8)$$

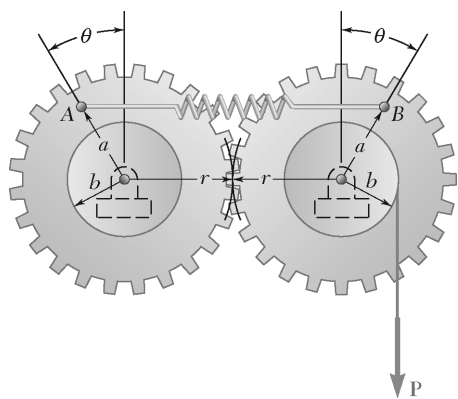
$$= kl^2 (+0.55) < 0$$

Stable ◀

PROBLEM 10.80 (Continued)

$$\begin{aligned}\theta = 34.2^\circ: \quad \frac{d^2v}{d\theta^2} &= kl^2(1.25 \sin 34.2^\circ + \cos 34.3^\circ - 2 \sin 68.4^\circ) \\ &= kl^2(-0.33) < 0 \quad \text{Unstable} \blacktriangleleft\end{aligned}$$

$$\begin{aligned}\theta = 90.0^\circ: \quad \frac{d^2V}{d\theta^2} &= kl^2(1.25 \sin 90^\circ + \cos 90^\circ - 2 \sin 180^\circ) \\ &= kl^2(1.25) > 0 \quad \text{Stable} \blacktriangleleft\end{aligned}$$



PROBLEM 10.81

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, determine two values of the angle θ corresponding to equilibrium when $P = 30$ lb, $a = 4$ in., $b = 3$ in., $r = 6$ in., and $k = 5$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Elongation of spring

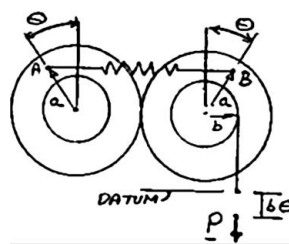
$$s = 2(a \sin \theta) = 2a \sin \theta$$

$$V = \frac{1}{2}ks^2 - Pb\theta$$

$$= \frac{1}{2}k(2a \sin \theta)^2 - Pb\theta$$

$$\frac{dV}{d\theta} = 4ka^2 \sin \theta \cos \theta - Pb$$

$$= 2ka^2 \sin 2\theta - Pb \quad (1)$$



Equilibrium

$$\frac{dV}{d\theta} = 0: \quad \sin 2\theta = \frac{Pb}{2ka^2}$$

$$\sin 2\theta = \frac{(30 \text{ lb})(3 \text{ in.})}{2(5 \text{ lb/in.})(4 \text{ in.})^2}; \quad \sin 2\theta = 0.5625$$

$$2\theta = 34.229^\circ \quad \text{and} \quad 145.771^\circ$$

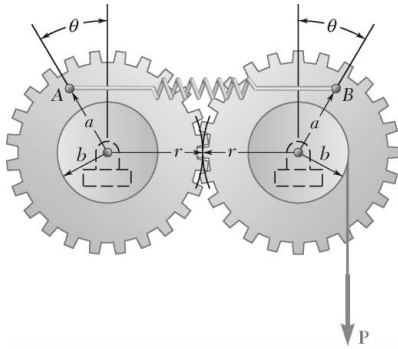
$$\theta = 17.11^\circ \quad \text{and} \quad 72.9^\circ \quad \blacktriangleleft$$

Stability: We differentiate Eq. (1)

$$\frac{d^2V}{d\theta^2} = 4ka^2 \cos 2\theta$$

$$\theta = 17.11^\circ: \quad \frac{d^2V}{d\theta^2} = 4ka^2 \cos 34.2^\circ = 4ka^2(0.83) > 0 \quad \text{Stable} \quad \blacktriangleleft$$

$$\theta = 72.9^\circ: \quad \frac{d^2V}{d\theta^2} = 4ka^2 \cos 145.8^\circ = 4ka^2(-0.83) < 0 \quad \text{Unstable} \quad \blacktriangleleft$$



PROBLEM 10.82

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, and given that $a = 60$ mm, $b = 45$ mm, $r = 90$ mm, and $k = 6$ kN/m, determine (a) the range of values of P for which a position of equilibrium exists, (b) two values of θ corresponding to equilibrium if the value of P is equal to half the upper limit of the range found in part a.

SOLUTION

Elongation of spring

$$s = 2(a \sin \theta) = 2a \sin \theta$$

Potential energy

$$V = \frac{1}{2}ks^2 - Pb\theta = \frac{1}{2}k(2a \sin \theta)^2 - Pb\theta$$

$$V = 2ka^2 \sin^2 \theta - Pb\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad \frac{dV}{d\theta} = 4ka^2 \sin \theta \cos \theta - Pb = 0 \quad (1)$$

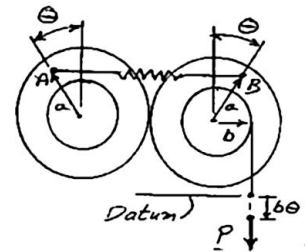
$$= 2ka^2 \sin 2\theta - Pb = 0$$

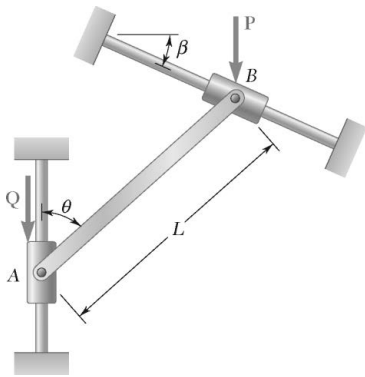
$$\sin 2\theta = \frac{Pb}{2ka^2}; \quad \text{For } P_{\max}; \quad \frac{P_{\max}b}{2ka^2} = 1$$

$$(a) \quad \frac{P_{\max}(0.045 \text{ m})}{2(6000 \text{ N/m})(0.06 \text{ m})^2} = 1 \quad P_{\max} = 960 \text{ N} \blacktriangleleft$$

$$(b) \quad \text{For } P = \frac{1}{2}P_{\max}, \quad \sin 2\theta = \frac{1}{2}; \quad 2\theta = 30^\circ \quad \text{and} \quad 150^\circ$$

$$\theta = 15.00^\circ \quad \text{and} \quad 75.0^\circ \blacktriangleleft$$

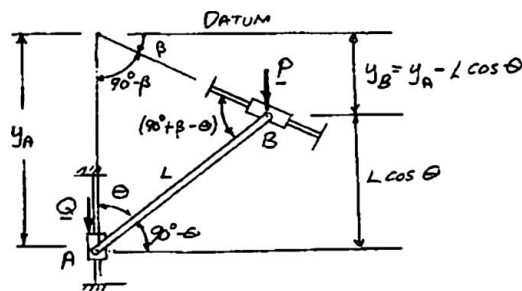




PROBLEM 10.83

A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30^\circ$ and $P = Q = 400$ N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION



Law of Sines

$$\frac{y_A}{\sin(90^\circ + \beta - \theta)} = \frac{L}{\sin(90^\circ - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{L}{\cos \beta}$$

or

$$y_A = L \frac{\cos(\theta - \beta)}{\cos \beta}$$

From the figure:

$$y_B = L \frac{\cos(\theta - \beta)}{\cos \beta} - L \cos \theta$$

Potential Energy:

$$V = -Py_B - Qy_A = -P \left[L \frac{\cos(\theta - \beta)}{\cos \beta} - L \cos \theta \right] - QL \frac{\cos(\theta - \beta)}{\cos \beta}$$

$$\frac{dV}{d\theta} = -PL \left[-\frac{\sin(\theta - \beta)}{\cos \beta} + \sin \theta \right] + QL \frac{\sin(\theta - \beta)}{\cos \beta}$$

$$= L(P + Q) \frac{\sin(\theta - \beta)}{\cos \beta} - PL \sin \theta$$

PROBLEM 10.83 (Continued)

Equilibrium $\frac{dV}{d\theta} = 0: L(P+Q)\frac{\sin(\theta-\beta)}{\cos\beta} - PL\sin\theta = 0$

or $(P+Q)\sin(\theta-\beta) = P\sin\theta\cos\beta$

$$(P+Q)(\sin\theta\cos\beta - \cos\theta\sin\beta) = P\sin\theta\cos\beta$$

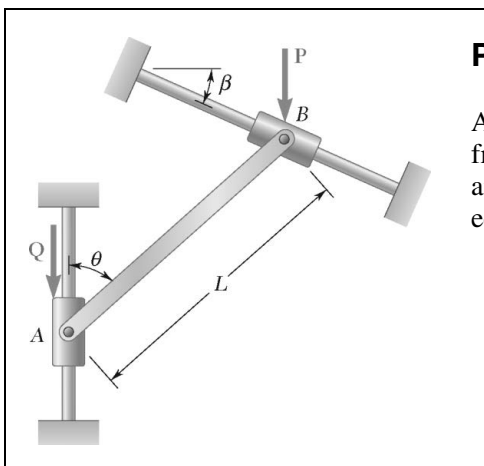
or $-(P+Q)\cos\theta\sin\beta + Q\sin\theta\cos\beta = 0$

$$-\frac{P+Q}{Q}\frac{\sin\beta}{\cos\beta} + \frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = \frac{P+Q}{Q}\tan\beta \quad (2)$$

With $P = Q = 400 \text{ N}, \quad \beta = 30^\circ$

$$\tan\theta = \frac{800 \text{ N}}{400 \text{ N}}\tan 30^\circ = 1.1547 \quad \theta = 49.1^\circ \blacktriangleleft$$



PROBLEM 10.84

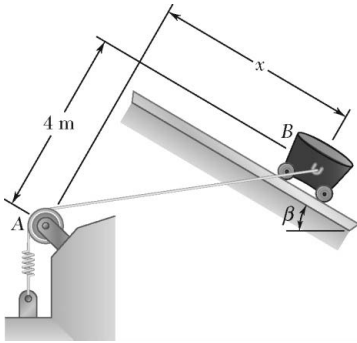
A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30^\circ$, $P = 100\text{ N}$, and $Q = 25\text{ N}$, determine the value of the angle θ corresponding to equilibrium.

SOLUTION

Using Equation (2) of Problem 10.83, with $P = 100\text{ N}$, $Q = 25\text{ N}$, and $\beta = 30^\circ$, we have

$$\begin{aligned}\tan \theta &= \frac{(100\text{ N})(25\text{ N})}{(25\text{ N})} \tan 30^\circ \\ &= 57.735 \\ \theta &= 89.007^\circ\end{aligned}$$

$$\theta = 89.0^\circ \blacktriangleleft$$

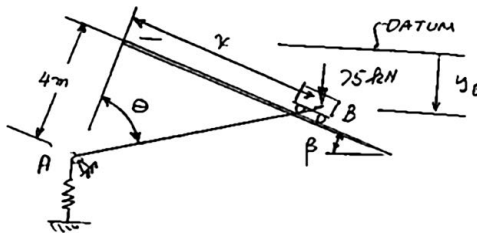


PROBLEM 10.85

Cart B , which weighs 75 kN , rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m , and the spring is unstretched when $x=0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 30^\circ$.

SOLUTION



$$x = (4 \text{ m}) \tan \theta \quad (1)$$

$$y_B = x \sin \beta = 4 \tan \theta \sin \beta$$

$$AC = (4 \text{ m}) \cos \theta$$

For $x = 0$,

$$(AC)_0 = 4 \text{ m}$$

Stretch of spring.

$$s = AC - (AC)_0 = \frac{4}{\cos \theta} - 4 = 4 \left(\frac{1}{\cos \theta} - 1 \right)$$

$$V = \frac{1}{2} ks^2 - (75 \text{ kN})y_B$$

$$= \frac{1}{2} (5 \text{ kN/m}) 16 \left(\frac{1}{\cos \theta} - 1 \right)^2 - (75 \text{ kN}) 4 \tan \theta \sin \beta$$

$$\frac{dV}{d\theta} = 80 \left(\frac{1}{\cos \theta} - 1 \right) \frac{\sin \theta}{\cos^2 \theta} - 300 \frac{\sin \beta}{\cos^2 \theta}$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \left(\frac{1}{\cos \theta} - 1 \right) \sin \theta = 3.75 \sin \beta \quad (2)$$

Given:

$$\beta = 30^\circ, \quad \sin \theta = 0.5$$

$$\text{Eq. (2):} \quad \left(\frac{1}{\cos \theta} - 1 \right) \sin \theta = 3.75(0.5) = 1.875$$

Solve by trial and error:

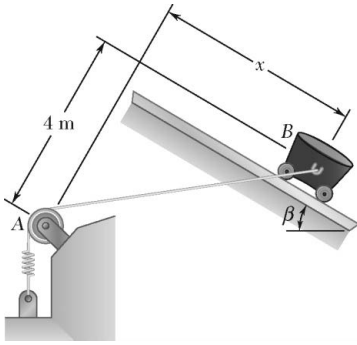
$$\theta = 70.46^\circ$$

Eq. (1):

$$x = (4 \text{ m}) \tan 70.46^\circ$$

$$x = 11.27 \text{ m} \quad \blacktriangleleft$$

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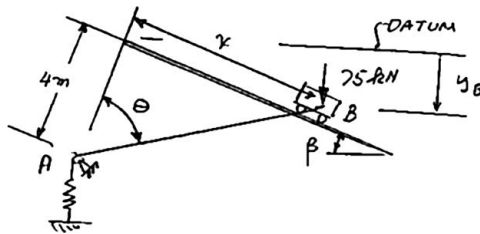


PROBLEM 10.86

Cart B , which weighs 75 kN , rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m , and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 60^\circ$.

SOLUTION



$$x = (4 \text{ m}) \tan \theta \quad (1)$$

$$y_B = x \sin \beta = 4 \tan \theta \sin \beta$$

$$AC = (4 \text{ m}) \cos \theta$$

For $x = 0$,

$$(AC)_0 = 4 \text{ m}$$

Stretch of spring:

$$s = AC - (AC)_0 = \frac{4}{\cos \theta} - 4 = 4 \left(\frac{1}{\cos \theta} - 1 \right)$$

$$V = \frac{1}{2} ks^2 - (75 \text{ kN})y_B$$

$$= \frac{1}{2} (5 \text{ kN/m}) 16 \left(\frac{1}{\cos \theta} - 1 \right)^2 - (75 \text{ kN}) 4 \tan \theta \sin \beta$$

$$\frac{dV}{d\theta} = 80 \left(\frac{1}{\cos \theta} - 1 \right) \frac{\sin \theta}{\cos^2 \theta} - 300 \frac{\sin \beta}{\cos^2 \theta}$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \left(\frac{1}{\cos \theta} - 1 \right) \sin \theta = 3.75 \sin \beta \quad (2)$$

Given:

$$\beta = 60^\circ, \sin \theta = 0.86603$$

$$\text{Eq. (2):} \quad \left(\frac{1}{\cos \theta} - 1 \right) \sin \theta = 3.75(0.86603) = 3.2476$$

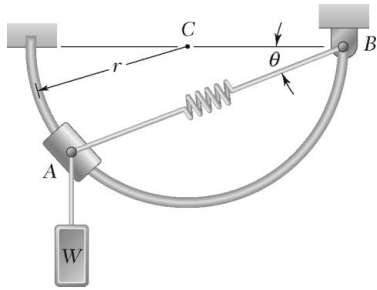
Solve by trial and error:

$$\theta = 76.67^\circ$$

Eq. (1):

$$x = (4 \text{ m}) \tan 26.67^\circ$$

$$x = 16.88 \text{ m} \quad \blacktriangleleft$$



PROBLEM 10.87

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r , determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION

Stretch of spring

$$s = AB - r$$

$$s = 2(r \cos \theta) - r$$

$$s = r(2 \cos \theta - 1)$$

Potential energy:

$$V = \frac{1}{2}ks^2 - Wr \sin 2\theta \quad W = mg$$

$$V = \frac{1}{2}kr^2(2 \cos \theta - 1)^2 - Wr \sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2 \cos \theta - 1)2 \sin \theta - 2Wr \cos 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: -kr^2(2 \cos \theta - 1) \sin \theta - 2Wr \cos 2\theta = 0$$

$$\frac{(2 \cos \theta - 1) \sin \theta}{\cos 2\theta} = -\frac{W}{kr}$$

Now

$$\frac{W}{kr} = \frac{(50 \text{ lb})}{(15 \text{ lb/in.})(9 \text{ in.})} = 0.37037$$

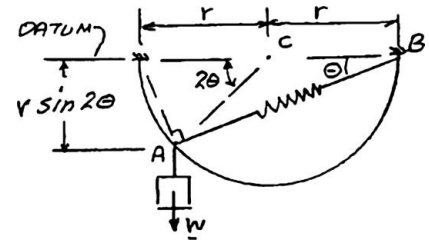
Then

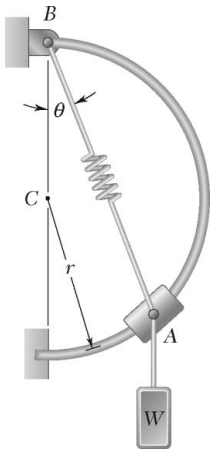
$$\frac{(2 \cos \theta - 1) \sin \theta}{\cos 2\theta} = -0.37037$$

Solving numerically,

$$\theta = 0.95637 \text{ rad} = 54.8^\circ$$

$$\theta = 54.8^\circ \blacktriangleleft$$





PROBLEM 10.88

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r , determine the value of θ corresponding to equilibrium when $W = 50 \text{ lb}$, $r = 9 \text{ in.}$, and $k = 15 \text{ lb/in.}$

SOLUTION

Stretch of spring

$$s = AB - r = 2(r \cos \theta) - r$$

$$s = r(2 \cos \theta - 1)$$

$$V = \frac{1}{2}ks^2 - Wr \cos 2\theta$$

$$= \frac{1}{2}kr^2(2 \cos \theta - 1)^2 - Wr \cos 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2 \cos \theta - 1)2 \sin \theta + 2Wr \sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: -kr^2(2 \cos \theta - 1) \sin \theta + Wr \sin 2\theta = 0$$

$$-kr^2(2 \cos \theta - 1) \sin \theta + Wr(2 \sin \theta \cos \theta) = 0$$

or

$$\frac{(2 \cos \theta - 1) \sin \theta}{2 \cos \theta} = \frac{W}{kr}$$

Now

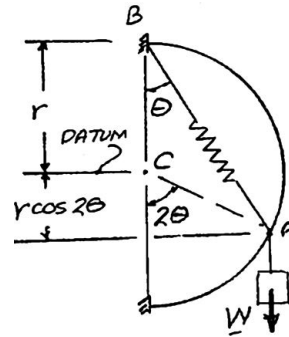
$$\frac{W}{kr} = \frac{(50 \text{ lb})}{(15 \text{ lb/in.})(9 \text{ in.})} = 0.37037$$

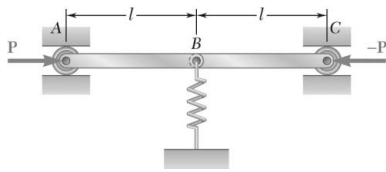
Then

$$\frac{2 \cos \theta - 1}{2 \cos \theta} = 0.37037$$

Solving

$$\theta = 37.4^\circ \blacktriangleleft$$

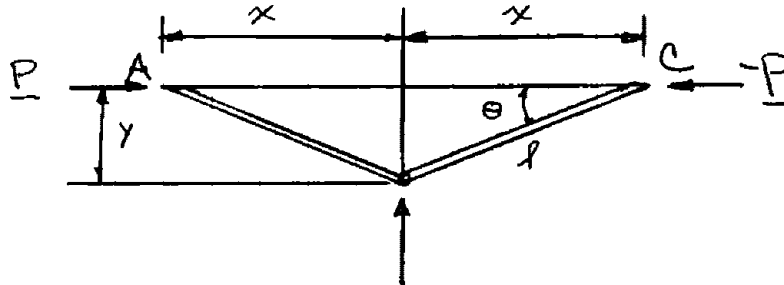




PROBLEM 10.89

Two bars AB and BC of negligible weight are attached to a single spring of constant k that is unstretched when the bars are horizontal. Determine the range of values of the magnitude P of two equal and opposite forces \mathbf{P} and $-\mathbf{P}$ for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$x = l \cos \theta$$

$$y = l \sin \theta$$

$$V = 2Px + \frac{1}{2}ky^2$$

$$v = 2Pl \cos \theta + \frac{1}{2}kl^2 \sin^2 \theta$$

$$\frac{dV}{d\theta} = -2Pl \sin \theta + kl^2 \sin \theta \cos \theta$$

$$= -2Pl \sin \theta + \frac{1}{2}kl^2 \sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl \cos \theta + kl^2 \cos 2\theta \quad (1)$$

For equilibrium position $\theta = 0$ to be stable

$$\frac{d^2V}{d\theta^2} = -2Pl + kl^2 > 0 \quad (2)$$

$$P < \frac{1}{2}kl \quad \blacktriangleleft$$

PROBLEM 10.89 (Continued)

Note: For $P = \frac{1}{2}kl$, we have $\frac{d^2V}{d\theta^2} = 0$ and we must determine which is the first derivative to be $\neq 0$.

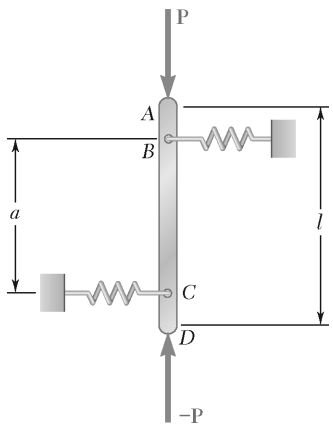
Differentiating Eq. (1):

$$\frac{d^3V}{d\theta^3} = +2Pl \sin \theta - 2kl^2 \sin 2\theta = 0 \text{ for } \theta = 0$$

$$\frac{d^4V}{d\theta^4} = 2Pl \cos \theta - 4kl^2 \cos 2\theta = 2Pl - 4kl^2 \text{ for } \theta = 0$$

But $P = \frac{1}{2}kl$. Thus $\frac{d^4V}{d\theta^4} = kl^2 - 4kl^2 < 0$ and we conclude that the equilibrium is unstable for $P = \frac{1}{2}kl$.

The sign $<$ in Eq. (2) is thus correct.



PROBLEM 10.90

A vertical bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Determine the range of values of the magnitude P of two equal and opposite vertical forces \mathbf{P} and $-\mathbf{P}$ for which the equilibrium position is stable if (a) $AB = CD$, (b) $AB = 2CD$.

SOLUTION

For both (a) and (b): Since \mathbf{P} and $-\mathbf{P}$ are vertical, they form a couple of moment

$$M_P = +Pl \sin \theta$$

The forces \mathbf{F} and $-\mathbf{F}$ exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa \cos \theta$$

We have

$$\begin{aligned} dU &= M_P d\theta + M_F d\theta \\ &= (Pl \sin \theta - Fa \cos \theta) d\theta \end{aligned}$$

but

$$F = ks = k \left(\frac{1}{2} a \sin \theta \right)$$

Thus,

$$dU = \left(Pl \sin \theta - \frac{1}{2} ka^2 \sin \theta \cos \theta \right) d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl \sin \theta d\theta + \frac{1}{4} ka^2 \sin 2\theta d\theta$$

or

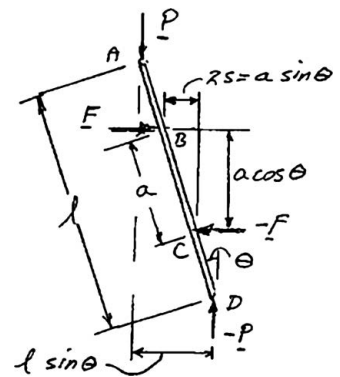
$$\frac{dV}{d\theta} = -Pl \sin \theta + \frac{1}{4} ka^2 \sin 2\theta$$

and

$$\frac{d^2V}{d\theta^2} = -Pl \cos \theta + \frac{1}{2} ka^2 \cos 2\theta \quad (1)$$

For $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2} ka^2$$



PROBLEM 10.90 (Continued)

For Stability: $\frac{d^2V}{d\theta^2} > 0, \quad -Pl + \frac{1}{2}ka^2 > 0$

or (for Parts *a* and *b*)

$$P < \frac{ka^2}{2l} \quad \blacktriangleleft$$

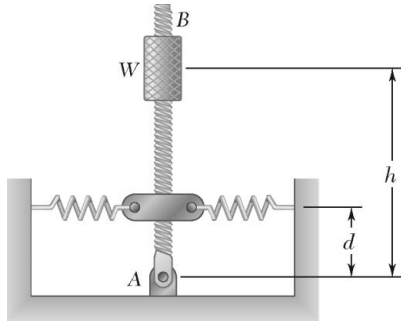
Note: To check that equilibrium is unstable for $P = \frac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl \sin \theta - ka^2 \sin 2\theta = 0, \quad \text{for } \theta = 0,$$

$$\frac{d^4V}{d\theta^4} = Pl \cos \theta - 2ka^2 \cos 2\theta$$

For $\theta = 0$ $\frac{d^4V}{d\theta^4} = Pl - 2ka^2 = \frac{ka^2}{2} - 2ka^2 < 0$

Thus, equilibrium is unstable when $P = \frac{ka^2}{2l}$



PROBLEM 10.91

Rod AB is attached to a hinge at A and to two springs, each of constant k . If $h = 25$ in., $d = 12$ in., and $W = 80$ lb, determine the range of values of k for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

We have

$$x_C = d \sin \theta \quad y_B = h \cos \theta$$

Potential Energy:

$$\begin{aligned} V &= 2 \left(\frac{1}{2} k x_C^2 + W y_B \right) \\ &= k d^2 \sin^2 \theta + W h \cos \theta \end{aligned}$$

Then

$$\begin{aligned} \frac{dV}{d\theta} &= 2 k d^2 \sin \theta \cos \theta - W h \sin \theta \\ &= k d^2 \sin 2\theta - W h \sin \theta \end{aligned}$$

and

$$\frac{d^2V}{d\theta^2} = 2 k d^2 \cos 2\theta - W h \cos \theta \quad (1)$$

For equilibrium position $\theta = 0$ to be stable, we must have

$$\frac{d^2V}{d\theta^2} = 2 k d^2 - W h > 0$$

or

$$k d^2 > \frac{1}{2} W h \quad (2)$$

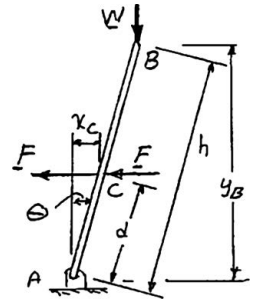
Note: For $k d^2 = \frac{1}{2} W h$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write

$$\frac{d^3V}{d\theta^3} = -4 k d^2 \sin 2\theta + W h \sin \theta = 0 \quad \text{for } \theta = 0$$

$$\frac{d^4V}{d\theta^4} = -8 k d^2 \cos 2\theta + W h \cos \theta$$

For $\theta = 0$:

$$\frac{d^4V}{d\theta^4} = -8 k d^2 + W h$$



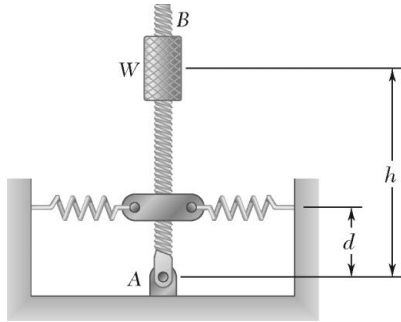
PROBLEM 10.91 (Continued)

Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

With $W = 80 \text{ lb}$, $h = 25 \text{ in.}$, and $d = 12 \text{ in.}$

Equation (2) gives $k(12 \text{ in.})^2 > \frac{1}{2}(80 \text{ lb})(25 \text{ in.})$

or $k > 6.944 \text{ lb/in.}$ $k > 6.94 \text{ lb/in.} \blacktriangleleft$



PROBLEM 10.92

Rod AB is attached to a hinge at A and to two springs, each of constant k . If $h = 45$ in., $k = 6$ lb/in., and $W = 60$ lb, determine the smallest distance d for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Using Equation (2) of Problem 10.91 with

$$h = 45 \text{ in.}, \quad k = 6 \text{ lb/in.}, \quad \text{and} \quad W = 60 \text{ lb}$$

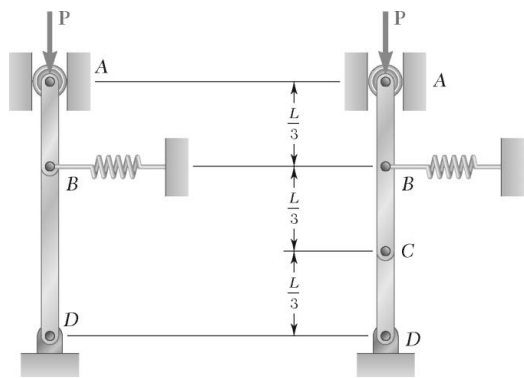
$$(6 \text{ lb/in.})d^2 > \frac{1}{2}(60 \text{ lb})(45 \text{ in.})$$

or

$$d^2 > 225 \text{ in.}^2$$

$$d > 15.0000 \text{ in.}$$

$$\text{smallest } d = 15.00 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 10.93

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION

$$s = \frac{L}{3} \sin \phi = \frac{2L}{3} \sin \theta$$

For small values of ϕ and θ

$$\phi = 2\theta$$

$$V = P \left(\frac{L}{3} \cos \phi + \frac{2L}{3} \cos \theta \right) + \frac{1}{2} k s^2$$

$$V = \frac{PL}{3} (\cos 2\theta + 2 \cos \theta) + \frac{1}{2} k \left(\frac{2L}{3} \sin \theta \right)^2$$

$$\begin{aligned} \frac{dV}{d\theta} &= \frac{PL}{3} (-2 \sin 2\theta - 2 \sin \theta) + \frac{2}{9} k L^2 \sin \theta \cos \theta \\ &= -\frac{PL}{3} (2 \sin 2\theta + 2 \sin \theta) + \frac{2}{9} k L^2 \sin 2\theta \end{aligned}$$

$$\frac{d^2V}{d\theta^2} = -\frac{PL}{3} (4 \cos 2\theta + 2 \cos \theta) + \frac{4}{9} k L^2 \cos 2\theta$$

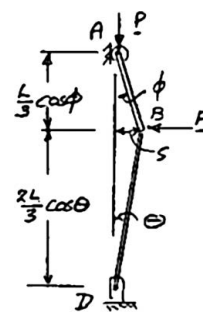
when $\theta = 0$:

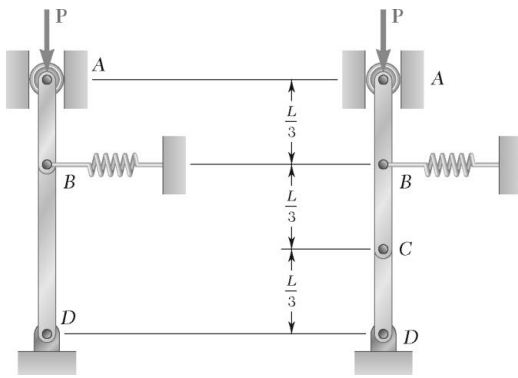
$$\frac{d^2V}{d\theta^2} = -\frac{6PL}{3} + \frac{4}{9} k L^2$$

For stability:

$$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{4}{9} k L^2 > 0$$

$$P < \frac{2}{9} k L \quad \blacktriangleleft$$





PROBLEM 10.94

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION

$$a = \frac{2L}{3} \sin \theta = \frac{L}{3} \sin \phi$$

For small values of ϕ and θ

$$\phi = 2\theta$$

$$s = \frac{L}{3} \sin \theta$$

$$V = P \left(\frac{2L}{3} \cos \theta + \frac{L}{3} \cos \phi \right) + \frac{1}{2} ks^2$$

$$= \frac{PL}{3} (2 \cos \theta + \cos 2\theta) + \frac{1}{2} k \left(\frac{L}{3} \sin \theta \right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3} (-2 \sin \theta - 2 \sin 2\theta) + \frac{kL^2}{9} \sin \theta \cos \theta$$

$$\frac{dV}{d\theta} = -\frac{2PL}{3} (\sin \theta + \sin 2\theta) + \frac{kL^2}{18} \sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{2PL}{3} (\cos \theta + 2 \cos 2\theta) + \frac{kL^2}{9} \cos 2\theta$$

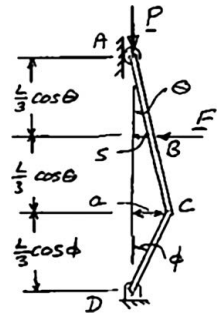
when $\theta = 0$:

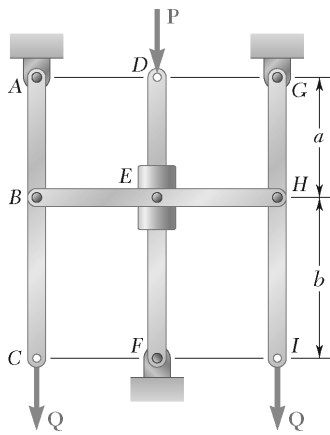
$$\frac{d^2V}{d\theta^2} = -2PL + \frac{kL^2}{9}$$

For stability:

$$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{kL^2}{9} > 0$$

$$P < \frac{1}{18} kL \quad \blacktriangleleft$$

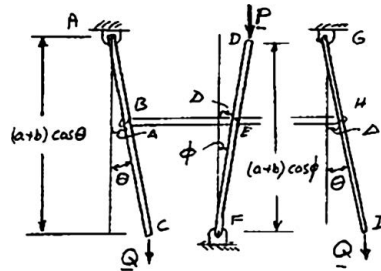




PROBLEM 10.95

The horizontal bar BEH is connected to three vertical bars. The collar at E can slide freely on bar DF . Determine the range of values of Q for which the equilibrium of the system is stable in the position shown when $a = 24$ in., $b = 20$ in., and $P = 150$ lb.

SOLUTION



First note

$$A = a \sin \theta = b \sin \phi$$

For small values of θ and ϕ :

$$a\theta = b\phi$$

or

$$\phi = \frac{a}{b}\theta$$

$$V = P(a+b)\cos\phi - 2Q(a+b)\cos\theta$$

$$= (a+b) \left[P \cos\left(\frac{a}{b}\theta\right) - 2Q \cos\theta \right]$$

$$\frac{dV}{d\theta} = (a+b) \left[-\frac{a}{b} P \sin\left(\frac{a}{b}\theta\right) + 2Q \sin\theta \right]$$

$$\frac{d^2V}{d\theta^2} = (a+b) \left[-\frac{a^2}{b^2} P \cos\left(\frac{a}{b}\theta\right) + 2Q \cos\theta \right]$$

when $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = (a+b) \left(-\frac{a^2}{b^2} P + 2Q \right)$$

PROBLEM 10.95 (Continued)

Stability: $\frac{d^2V}{d\theta^2} > 0: -\frac{a^2}{b^2}P + 2Q > 0$

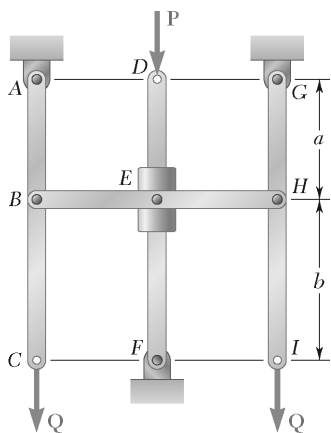
$$P < 2\frac{b^2}{a^2}Q \quad (1)$$

or $Q > \frac{a^2}{2b^2}P \quad (2)$

with $P = 150 \text{ lb}$, $a = 24 \text{ in.}$, and $b = 20 \text{ in.}$

Equation (1): $Q > \frac{(24 \text{ in.})^2}{2(20 \text{ in.})^2}(150 \text{ lb}) = 108.000 \text{ lb}$

For stability $Q > 108.0 \text{ lb} \blacktriangleleft$



PROBLEM 10.96

The horizontal bar BEH is connected to three vertical bars. The collar at E can slide freely on bar DF . Determine the range of values of P for which the equilibrium of the system is stable in the position shown when $a = 150$ mm, $b = 200$ mm, and $Q = 45$ N.

SOLUTION

Using Equation (2) of Problem 10.95 with

$$Q = 45 \text{ N}, \quad a = 150 \text{ mm}, \quad \text{and} \quad b = 200 \text{ mm}$$

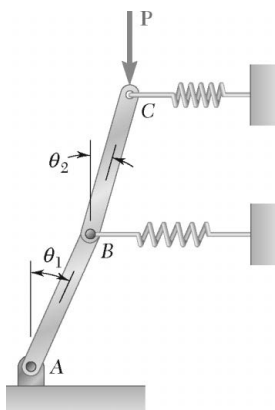
Equation (2)

$$P < 2 \frac{(200 \text{ mm})^2}{(150 \text{ mm})^2} (45 \text{ N})$$

$$= 160.000 \text{ N}$$

For stability

$$P < 160.0 \text{ N} \quad \blacktriangleleft$$



PROBLEM 10.97*

Bars AB and BC , each of length l and of negligible weight, are attached to two springs, each of constant k . The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of P for which the equilibrium position is stable.

SOLUTION

We have

$$x_B = l \sin \theta_1$$

$$x_C = l \sin \theta_1 + l \sin \theta_2$$

$$y_C = l \cos \theta_1 + l \cos \theta_2$$

$$V = Py_C + \frac{1}{2} kx_B^2 + \frac{1}{2} kx_C^2$$

or
$$V = Pl(\cos \theta_1 + \cos \theta_2) + \frac{1}{2} kl^2 [\sin^2 \theta_1 + (\sin \theta_1 + \sin \theta_2)^2]$$

For small values of θ_1 and θ_2 :

$$\sin \theta_1 \approx \theta_1, \quad \sin \theta_2 \approx \theta_2, \quad \cos \theta_1 \approx 1 - \frac{1}{2} \theta_1^2, \quad \cos \theta_2 \approx 1 - \frac{1}{2} \theta_2^2$$

Then

$$V = Pl \left(1 - \frac{\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2} \right) + \frac{1}{2} kl^2 [\theta_1^2 + (\theta_1 + \theta_2)^2]$$

and

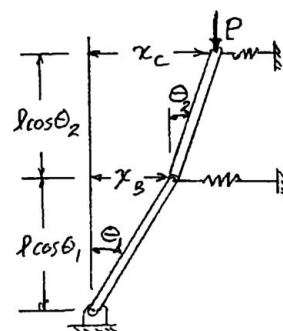
$$\frac{\partial V}{\partial \theta_1} = -Pl\theta_1 + kl^2[\theta_1 + (\theta_1 + \theta_2)]$$

$$\frac{\partial V}{\partial \theta_2} = -Pl\theta_2 + kl^2(\theta_1 + \theta_2)$$

$$\frac{\partial^2 V}{\partial \theta_1^2} = -Pl + 2kl^2 \quad \frac{\partial^2 V}{\partial \theta_2^2} = -Pl + kl^2$$

$$\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = kl^2$$

Stability: Conditions for stability (see Page 583).



PROBLEM 10.97* (Continued)

For $\theta_1 = \theta_2 = 0$: $\frac{\partial V}{\partial \theta_1} = \frac{\partial V}{\partial \theta_2} = 0$ (condition satisfied)

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} \right)^2 - \frac{\partial^2 V}{\partial \theta_1^2} \frac{\partial^2 V}{\partial \theta_2^2} < 0$$

Substituting $(kl^2)^2 - (-Pl + 2kl^2)(-Pl + kl) < 0$
 $k^2l^4 - P^2l^2 + 3Pkl^3 - 2k^2l^4 < 0$
 $P^2 - 3klP + k^2l^2 > 0$

Solving $P < \frac{3 - \sqrt{5}}{2}kl$ or $P > \frac{3 + \sqrt{5}}{2}kl$

or $P < 0.382kl$ or $P > 2.62kl$

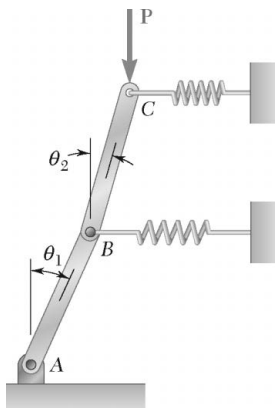
$$\frac{\partial^2 V}{\partial \theta_1^2} > 0: -Pl + 2kl^2 > 0$$

or $P < \frac{1}{2}kl$

$$\frac{\partial^2 V}{\partial \theta_2^2} > 0: -Pl + kl^2 > 0$$

or $P < kl$

Therefore, all conditions for stable equilibrium are satisfied when $0 \leq P < 0.382kl$ ◀



PROBLEM 10.98*

Solve Problem 10.97 knowing that $l = 800$ mm and $k = 2.5$ kN/m.

PROBLEM 10.97* Bars AB and BC , each of length l and of negligible weight, are attached to two springs, each of constant k . The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of P for which the equilibrium position is stable.

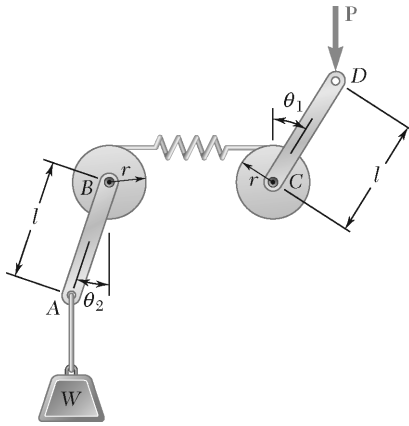
SOLUTION

From the analysis of Problem 10.97 with

$$l = 800 \text{ mm} \quad \text{and} \quad k = 2.5 \text{ kN/m}$$

$$P < 0.382kl = 0.382(2500 \text{ N/m})(0.8 \text{ m}) = 764 \text{ N}$$

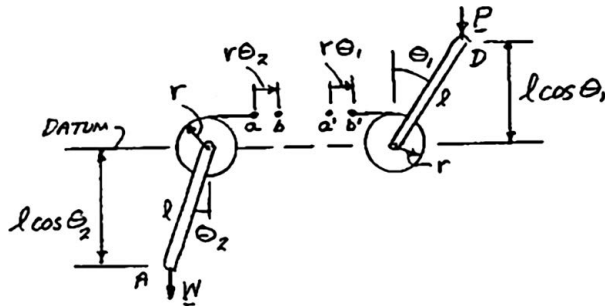
$$P < 764 \text{ N} \quad \blacktriangleleft$$



PROBLEM 10.99*

Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k . Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION



Left end of spring moves from a to b . Right end of spring moves from a' to b' . Elongation of spring

$$s = a'b' - ab = r\theta_1 - r\theta_2 = r(\theta_1 - \theta_2)$$

$$\begin{aligned} V &= \frac{1}{2}ks^2 + pl \cos \theta_1 - wl \cos \theta_2 \\ &= \frac{1}{2}kr^2(\theta_1 - \theta_2)^2 + pl \cos \theta_1 - wl \cos \theta_2 \end{aligned}$$

$$\frac{\partial V}{\partial \theta_1} = kr^2(\theta_1 - \theta_2) - pl \sin \theta_1$$

$$\frac{\partial V}{\partial \theta_2} = -kr^2(\theta_1 - \theta_2) + wl \sin \theta_2$$

$$\frac{\partial^2 V}{\partial \theta_1^2} = kr^2 - pl \cos \theta_1$$

$$\frac{\partial^2 V}{\partial \theta_2^2} = kr^2 + wl \cos \theta_2$$

$$\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = -kr^2$$

PROBLEM 10.99* (Continued)

For $\theta_1 = \theta_2 = 0$: $\frac{\partial^2 v}{\partial \theta_1^2} = kr^2 - pl$, $\frac{\partial^2 v}{\partial \theta_2^2} = +kr^2 + wl$, $\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2} = -kr^2$

Conditions for stability (see Page 583)

$$\left(\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2} \right)^2 - \frac{\partial^2 v}{\partial \theta_1^2} \cdot \frac{\partial^2 v}{\partial \theta_2^2} < 0$$

$$(kr^2)^2 - (kr^2 - pl)(kr^2 + wl) < 0$$

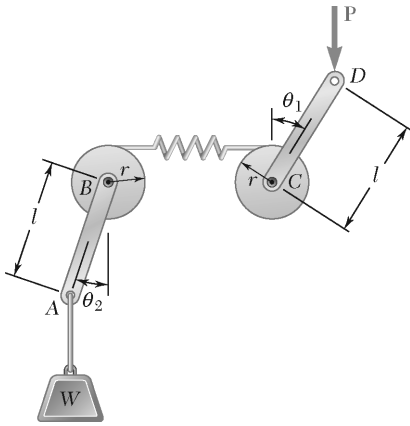
$$pl(kr^2 + wl) - kr^2 wl < 0$$

$$P < \frac{wkr^2}{kr^2 + wl}; \quad P < \frac{kr^2}{l} \left(\frac{W}{\frac{kr^2}{l} + w} \right) \quad \triangleleft$$

$$\frac{\partial^2 v}{\partial \theta_1^2} > 0: \quad kr^2 - pl > 0; \quad P < \frac{kr^2}{l} \quad \triangleleft$$

We choose:

$$P < \frac{kr^2}{l} \left(\frac{W}{\frac{kr^2}{l} + W} \right) \quad \blacktriangleleft$$



PROBLEM 10.100*

Solve Problem 10.99 knowing that $k = 20 \text{ lb/in.}$, $r = 3 \text{ in.}$, $l = 6 \text{ in.}$, and (a) $W = 15 \text{ lb}$, (b) $W = 60 \text{ lb}$.

PROBLEM 10.99* Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k . Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION

$$k = 20 \text{ lb/in.}$$

$$r = 3 \text{ in.}$$

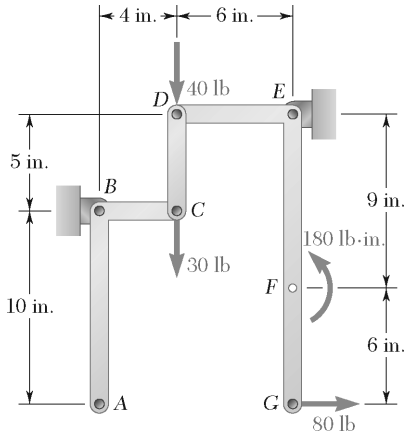
$$l = 6 \text{ in.}$$

$$\frac{kr^2}{l} = \frac{(20 \text{ lb/in.})(3 \text{ in.})^2}{6 \text{ in.}} = 30 \text{ lb}$$

$$(a) \quad W = 15 \text{ lb: } P < (30 \text{ lb}) \frac{15 \text{ lb}}{(30 \text{ lb}) + (15 \text{ lb})} \quad P < 10.00 \text{ lb} \blacktriangleleft$$

$$(b) \quad W = 60 \text{ lb: } P < (30 \text{ lb}) \frac{60 \text{ lb}}{(30 \text{ lb}) + (60 \text{ lb})} \quad P < 20.0 \text{ lb} \blacktriangleleft$$

PROBLEM 10.101



Determine the horizontal force **P** that must be applied at A to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta$ ↻

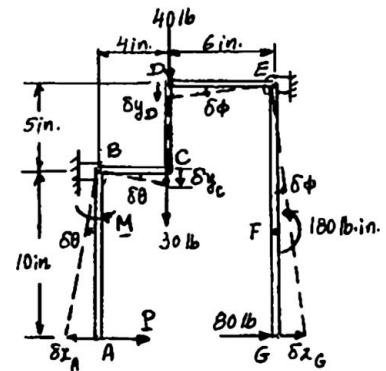
$$\delta x_A = 10\delta\theta \leftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \curvearrowright$$

$$\begin{aligned} \delta x_G &= 15\delta\phi \\ &= 15\left(\frac{2}{3}\delta\theta\right) \\ &= 10\delta\theta \rightarrow \end{aligned}$$



Virtual Work: We shall assume that a force **P** and a couple **M** are applied to member *ABC* as shown.

$$\delta U = -P\delta x_A - M\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

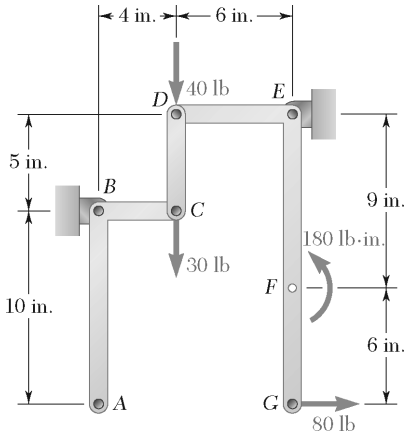
$$(10 \text{ in.})P + M = 1200 \text{ lb} \cdot \text{in.} \quad (1)$$

Making $M = 0$ in Eq. (1):

$$P = +120.0 \text{ lb}$$

$$\mathbf{P} = 120.0 \text{ lb} \rightarrow \blacktriangleleft$$

PROBLEM 10.102



Determine the couple M that must be applied to member ABC to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta$ \curvearrowright

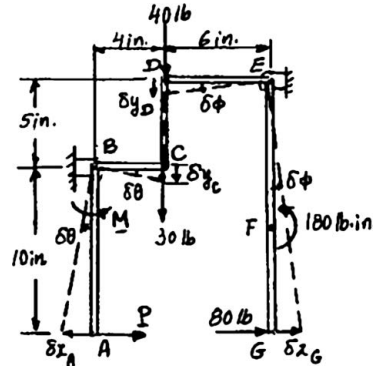
$$\delta x_A = 10\delta\theta \leftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \curvearrowright$$

$$\begin{aligned} \delta x_G &= 15\delta\phi \\ &= 15\left(\frac{2}{3}\delta\theta\right) \\ &= 10\delta\theta \rightarrow \end{aligned}$$



Virtual Work: We shall assume that a force P and a couple M are applied to member ABC as shown.

$$\delta U = -P\delta x_A - M\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

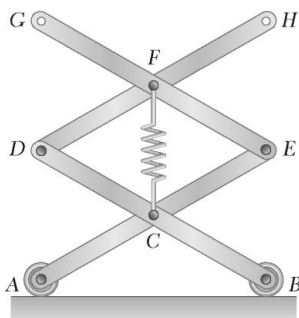
$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10 \text{ in.})P + M = 1200 \text{ lb} \cdot \text{in.} \quad (1)$$

Now from Eq. (1) for

$$P = 0$$

$$M = 1200 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$



PROBLEM 10.103

A spring of constant 15 kN/m connects Points C and F of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring and the vertical motion of Point G when a vertical downward 120-N force is applied (a) at Point C , (b) at Points C and H .

SOLUTION

$$y_G = 4y_C$$

$$y_H = 4y_C \quad \delta y_H = 4\delta y_C$$

$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$

$$y_E = 2y_C \quad \delta y_E = 2\delta y_C$$

For spring:

$$\Delta = y_F - y_C$$

Q = Force in spring (assumed in tension)

$$Q = +k\Delta = k(y_F - y_C) = k(3y_C - y_C) = 2ky_C \quad (1)$$

(a) $C = 120 \text{ N}, \quad E = F = H = 0$

Virtual Work:

$$\delta U = 0: \quad -(120 \text{ N})\delta y_C + Q\delta y_C - Q\delta y_F = 0$$

$$-120\delta y_C + Q\delta y_C - Q(3\delta y_C) = 0$$

$$Q = -60 \text{ N} \quad Q = 60.0 \text{ N} \quad C \blacktriangleleft$$

Eq. (1): $Q = 2ky_C, \quad -60 \text{ N} = 2(15 \text{ kN/m})y_C, \quad y_C = -2 \text{ mm}$

At Point G : $y_G = 4y_C = 4(-2 \text{ mm}) = -8 \text{ mm} \quad y_G = 8.00 \text{ mm} \downarrow \blacktriangleleft$

(b) $C = H = 120 \text{ N}, \quad E = F = 0$

Virtual Work:

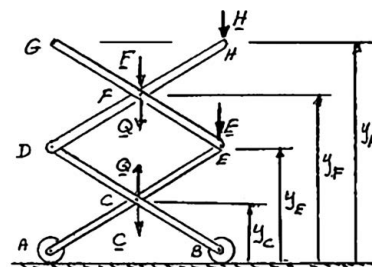
$$\delta U = 0: \quad -(120 \text{ N})\delta y_C - (120 \text{ N})\delta y_H + Q\delta y_C - Q\delta y_F = 0$$

$$-120\delta y_C - 120(4\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$

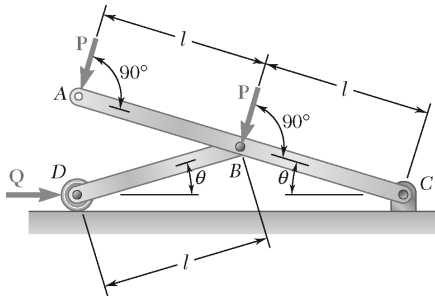
$$Q = -300 \text{ N} \quad Q = 300 \text{ N} \quad C \blacktriangleleft$$

Eq. (1): $Q = 2ky_C \quad -300 \text{ N} = 2(15 \text{ kN/m})y_C, \quad y_C = -10 \text{ mm}$

At Point G : $y_G = 4y_C = 4(-10 \text{ mm}) = -40 \text{ mm} \quad y_G = 40.0 \text{ mm} \downarrow \blacktriangleleft$

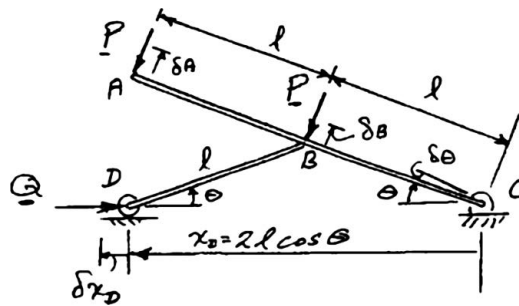


PROBLEM 10.104



Derive an expression for the magnitude of the force Q required to maintain the equilibrium of the mechanism shown.

SOLUTION



We have

$$x_D = 2l \cos \theta \quad \text{so that} \quad \delta x_D = -2l \sin \theta \delta \theta$$

$$\delta A = 2l \delta \theta$$

$$\delta B = l \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad -Q \delta x_D - P \delta A - P \delta B = 0$$

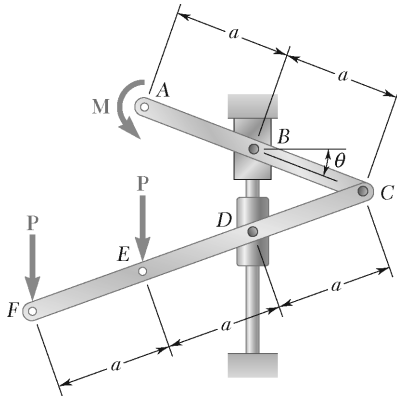
$$-Q(-2l \sin \theta \delta \theta) - P(2l \delta \theta) - P(l \delta \theta) = 0$$

$$2Ql \sin \theta - 3Pl = 0$$

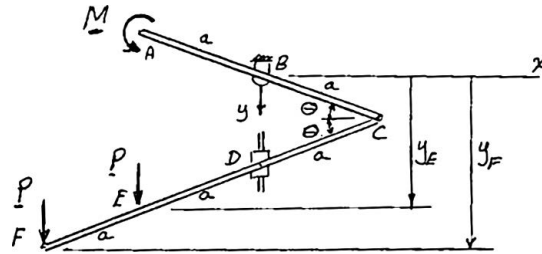
$$Q = \frac{3}{2} \frac{P}{\sin \theta} \quad \blacktriangleleft$$

PROBLEM 10.105

Derive an expression for the magnitude of the couple M required to maintain the equilibrium of the linkage shown.



SOLUTION



$$y_E = 3a \sin \theta \quad \delta y_E = 3a \cos \theta \delta \theta$$

$$y_F = 4a \sin \theta \quad \delta y_F = 4a \cos \theta \delta \theta$$

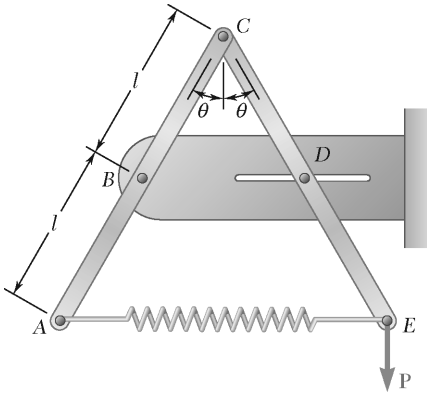
Virtual Work:

$$\delta U = 0: \quad -M \delta \theta + P \delta y_E + P \delta y_F = 0$$

$$-M \delta \theta + P(3a \cos \theta \delta \theta) + P(4a \cos \theta \delta \theta) = 0$$

$$M = 7Pa \cos \theta \quad \blacktriangleleft$$

PROBLEM 10.106



Two rods AC and CE are connected by a pin at C and by a spring AE . The constant of the spring is k , and the spring is unstretched when $\theta = 30^\circ$. For the loading shown, derive an equation in P , θ , l , and k that must be satisfied when the system is in equilibrium.

SOLUTION

$$y_E = l \cos \theta$$

$$\delta y_E = -l \sin \theta \delta \theta$$

Spring:

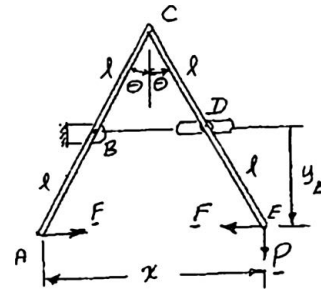
Unstretched length = $2l$

$$x = 2(2l \sin \theta) = 4l \sin \theta$$

$$\delta x = 4l \cos \theta \delta \theta$$

$$F = k(x - 2l)$$

$$F = k(4l \sin \theta - 2l)$$



Virtual Work:

$$\delta U = 0: \quad P \delta y_E - F \delta x = 0$$

$$P(-l \sin \theta \delta \theta) - k(4l \sin \theta - 2l)(4l \cos \theta \delta \theta) = 0$$

$$-P \sin \theta - 8kl(2 \sin \theta - 1) \cos \theta = 0$$

or

$$\frac{P}{8kl} = (1 - 2 \sin \theta) \frac{\cos \theta}{\sin \theta}$$

$$\frac{P}{8kl} = \frac{1 - 2 \sin \theta}{\tan \theta} \quad \blacktriangleleft$$

PROBLEM 10.107

A force \mathbf{P} of magnitude 240 N is applied to end E of cable CDE , which passes under pulley D and is attached to the mechanism at C . Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium. The constant of the spring is $k = 4 \text{ kN/m}$, and the spring is unstretched when $\theta = 90^\circ$.

SOLUTION

$$s = r \left(\frac{\pi}{2} - \theta \right)$$

$$\delta s = -r \delta \theta$$

$$F = ks = kr \left(\frac{\pi}{2} - \theta \right)$$

$$CD = 2l \sin \frac{\theta}{2}$$

$$\delta(CD) = 2l \cos \frac{\theta}{2} \left(\frac{1}{2} \delta \theta \right)$$

$$= l \cos \frac{\theta}{2} \delta \theta$$

Virtual Work:

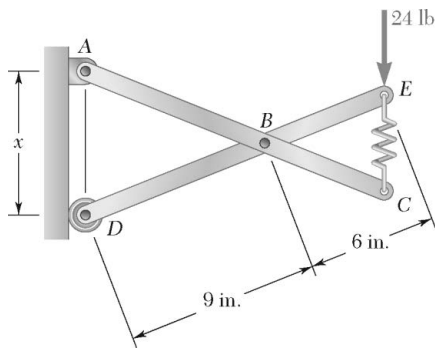
Since \mathbf{F} tends to decrease s and \mathbf{P} tends to decrease CD , we have

$$\delta U = -F \delta s - P \delta(CD) = 0$$

$$-kr \left(\frac{\pi}{2} - \theta \right) (-r \delta \theta) - P \left(l \cos \frac{\theta}{2} \delta \theta \right) = 0$$

$$\frac{\frac{\pi}{2} - \theta}{\cos \frac{\theta}{2}} = \frac{pl}{kr^2} = \frac{(240 \text{ N})(0.3 \text{ m})}{(4000 \text{ N/m})(0.12 \text{ m})^2} = 1.25$$

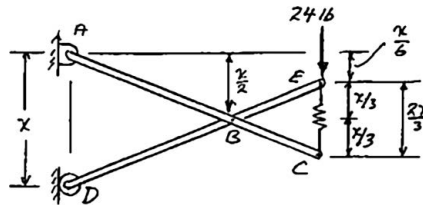
Solving by trial and error: $\theta = 0.33868 \text{ rad}$ $\theta = 19.40^\circ$ ◀



PROBLEM 10.108

Two identical rods ABC and DBE are connected by a pin at B and by a spring CE . Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance x corresponding to equilibrium when a 24-lb load is applied at E as shown.

SOLUTION



Deformation of spring

$$s = EC - 4 \text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24 \text{ lb})\frac{x}{6} = \frac{1}{2}(8 \text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 4x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 4$$

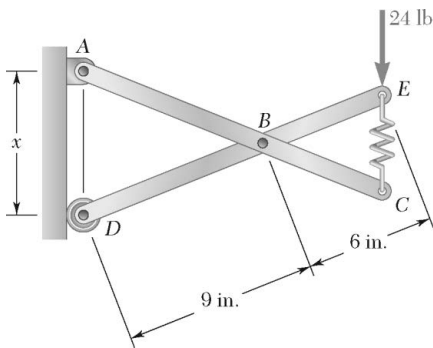
Equilibrium:

$$\frac{dV}{dx} = 0 \quad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 4 = 0$$

$$\frac{2x}{3} - 4 = 4\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + \frac{3}{4}$$

$$x = 7.13 \text{ in.} \quad \blacktriangleleft$$

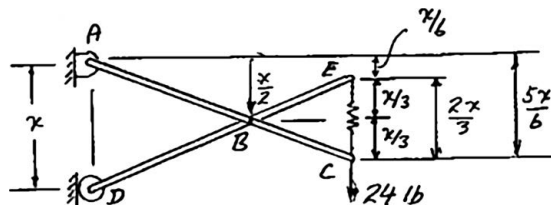


PROBLEM 10.109

Solve Problem 10.108 assuming that the 24-lb load is applied at C instead of E.

PROBLEM 10.108 Two identical rods ABC and DBE are connected by a pin at B and by a spring CE . Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance x corresponding to equilibrium when a 24-lb load is applied at E as shown.

SOLUTION



Deformation of spring

$$s = EC - 4 \text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24 \text{ lb})\frac{5x}{6} = \frac{1}{2}(8 \text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 20x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 20$$

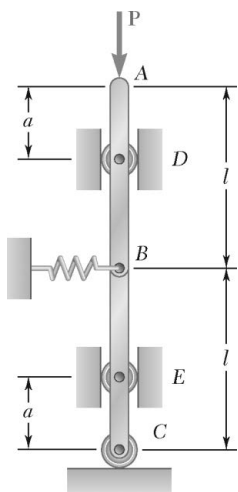
Equilibrium:

$$\frac{dV}{dx} = 0 \quad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 20 = 0$$

$$\frac{2x}{3} - 4 = 20\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + 3.75$$

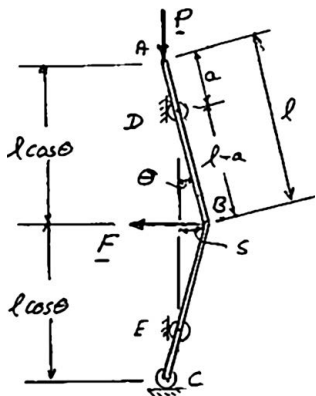
$$x = 11.63 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 10.110

Two bars AB and BC are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$s = (l - a) \sin \theta$$

$$\begin{aligned} V &= P(2l \cos \theta) + \frac{1}{2}ks^2 \\ &= 2Pl \cos \theta + \frac{1}{2}k(l - a)^2 \sin^2 \theta \end{aligned}$$

$$\frac{dV}{d\theta} = -2Pl \sin \theta + k(l - a)^2 \sin \theta \cos \theta$$

$$= -2Pl \sin \theta + \frac{1}{2}k(l - a)^2 \sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl \cos \theta + k(l - a)^2 \cos 2\theta \quad (1)$$

when

$$\theta = 0: \quad \frac{d^2V}{d\theta^2} = -2Pl + k(l - a)^2$$

Stability:

$$\frac{d^2V}{d\theta^2} > 0: \quad -2Pl + k(l - a)^2 > 0 \quad P < \frac{k(l - a)^2}{2l} \blacktriangleleft$$

To check whether equilibrium is unstable for $P = \frac{k(l - a)^2}{2l}$, we differentiate

Eq. (1) twice:

$$\frac{d^3V}{d\theta^3} = 2Pl \sin \theta - 2k(l - a)^2 \sin 2\theta = 0, \quad \text{For } \theta = 0$$

$$\frac{d^4V}{d\theta^4} = 2Pl \cos \theta - 4k(l - a)^2 \cos 2\theta$$

PROBLEM 10.110 (Continued)

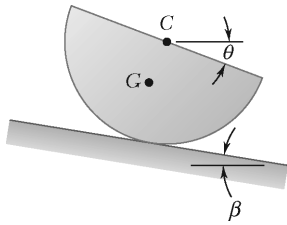
For $G = 0$ and

$$P = \frac{k(l-a)^2}{2l}$$

$$\begin{aligned}\frac{d^4V}{d\theta^4} &= 2Pl - 4k(l-a)^2 \\ &= k(l-a)^3 - 4k(l-a)^2 < 0\end{aligned}$$

Thus equilibrium is unstable for

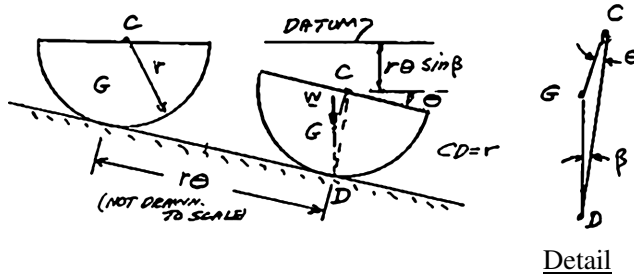
$$P = \frac{k(l-a)^2}{2l}$$



PROBLEM 10.111

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine the angle θ corresponding to equilibrium when $\beta = 10^\circ$.

SOLUTION



$$CG = \frac{3}{8}r$$

$$V = W(-r\theta \sin \beta - (CG) \cos \theta)$$

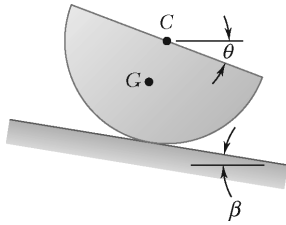
$$\frac{dV}{d\theta} = W \left(-r \sin \beta + \frac{3}{8}r \sin \theta \right)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0, \quad -\sin \beta + \frac{3}{8} \sin \theta = 0$$

$$\sin \beta = \frac{3}{8} \sin \theta \tag{1}$$

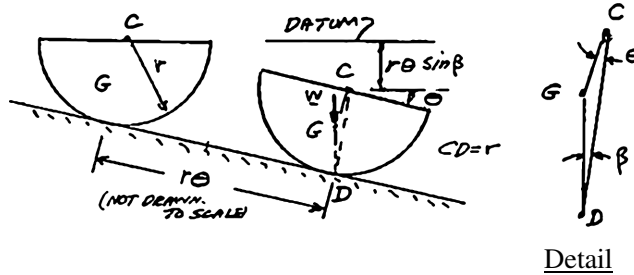
For $\beta = 10^\circ$ $\sin 10^\circ = \frac{3}{8} \sin \theta$ $\sin \theta = 0.46306, \quad \theta = 27.6^\circ \blacktriangleleft$



PROBLEM 10.112

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine (a) the largest angle β for which a position of equilibrium exists, (b) the angle θ corresponding to equilibrium when the angle β is equal to half the value found in part a.

SOLUTION



$$CG = \frac{3}{8}r$$

$$V = W(-r\theta \sin \beta - (CG) \cos \theta)$$

$$\frac{dV}{d\theta} = W\left(-r \sin \beta + \frac{3}{8}r \sin \theta\right)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0, \quad -\sin \beta + \frac{3}{8} \sin \theta = 0$$

$$\sin \beta = \frac{3}{8} \sin \theta \quad (1)$$

(a) For β_{\max} , $\theta = 90^\circ$

$$\text{Eq. (1)} \quad \sin \beta_{\max} = \frac{3}{8} \sin 90^\circ, \quad \sin \beta_{\max} = \frac{3}{8} = 22.02^\circ \quad \beta_{\max} = 22.0^\circ \blacktriangleleft$$

(b) When $\beta = \frac{1}{2} \beta_{\max} = 11.01^\circ$

$$\text{Eq. (1)} \quad \sin 11.01^\circ = \frac{3}{8} \sin \theta; \quad \sin \theta = 0.5093 \quad \theta = 30.6^\circ \blacktriangleleft$$

Note: We can also use $\triangle CGD$ and law of sines to derive Eq. (1).

$$\frac{\sin \beta}{CG} = \frac{\sin \theta}{CD}; \quad \sin \beta = \frac{CG}{CD} \sin \theta; \quad \sin \beta = \frac{3}{8} \sin \theta$$

