Ahmet Bindal

# Fundamentals of Computer Architecture and Design 

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Springer

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The Solutions Manual for instructors can be found at
http://www.springer.com/us/book/9783319258096
ISBN 978-3-319-25809-6
ISBN 978-3-319-25811-9 (eBook)
DOI 10.1007/978-3-319-25811-9
Library of Congress Control Number: 2016960285
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The registered company is Springer International Publishing AG
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For my mother who always showed me the right path...

## Preface

This book is written for young professionals and graduate students who have prior logic design background and want to learn how to use logic blocks to build complete systems from design specifications. My two-decade-long industry experience has taught me that engineers are "shape-oriented" people and that they tend to learn from charts and diagrams. Therefore, the teaching method I followed in this textbook caters this mind set: a lot of circuit schematics, block diagrams, timing diagrams, and examples supported by minimal text.

The book has eight chapters. The first three chapters give a complete review of the logic design principles since rest of the chapters significantly depend on this review. Chapter 1 concentrates on the combinational logic design. It describes basic logic gates, De Morgan's theorem, truth tables, and logic minimization. This chapter uses these key concepts in order to design mega cells, namely various types of adders and multipliers. Chapter 2 introduces sequential logic components, namely latches, flip-flops, registers, and counters. It introduces the concept of timing diagrams to explain the functionality of each logic block. The Moore and Mealy-type state machines, counter-decoder-type controllers, and the construction of simple memories are also explained in this chapter. Chapter 2 also illustrates the design process: how to develop architectural logic blocks using timing diagrams, and how to build a controller from a timing diagram to guide data flow. Chapter 3 focuses on the review of asynchronous logic design, which includes state definitions, primitive flow tables, and state minimization. Racing conditions in asynchronous designs, how to detect and correct them are also explained in this chapter. The chapter ends with designing an important asynchronous timing block: the C element (or the Mueller element), and it describes an asynchronous timing methodology that leads to a complete design using timing diagrams.

From Chapter 4 to Chapter 8, computer architecture-related topics are covered. Chapter 4 examines a very essential system element: system bus and communication protocols between system modules. This chapter defines the bus master and the bus slave concepts and examines their bus interfaces. Read and write bus cycles and protocols, bus handover and arbitration are also examined in this chapter. System memories, namely Static Random Access Memory (SRAM), Synchronous Dynamic Random Access Memory (SDRAM), Electrically-Erasable-Programmable-Read-Only-Memory (E²PROM) and Flash memory are examined in Chapter 5. This chapter also shows how to design bus interface for each memory type using timing diagrams and state machines. Chapter 6 is all about the design of a simple Reduced Instruction Set Computer (RISC) for central processing. The chapter starts with introducing a simple assembly instruction set and building individual hardware for each instruction. As other instructions are introduced to the design, techniques are shown how to integrate additional hardware to the existing CPU data-path to be able to execute multiple instructions. Fixed-point and floating-point Arithmetic Logic Units (ALU) are also studied in this chapter. Structural, data and program control hazards, and the required hardware to avoid them are shown. This chapter ends with the operation of various cache architectures, cache read and write protocols, and the functionality of write-through and write-back caches. The design of system peripherals, namely Direct Memory Access (DMA), interrupt controller, system timers, serial interface, display adapter and data controllers are covered in Chapter 7. The design methodology to construct data-paths with timing diagrams in Chapter 2 is closely followed in this chapter in order to design the bus interface for each peripheral. Chapter 8 describes the Field-Programmable Gate array (FPGA), and the fundamentals of data driven processors as special topics.

At the end of the book, there is a small appendix that introduces the Verilog language. Verilog is a widely used Hardware Design Language (HDL) to build and verify logic blocks, mega cells and systems. Interested readers are encouraged to go one step beyond and learn system Verilog to be able to verify large logic blocks.

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## About the Author



Ahmet Bindal received his M.S. and Ph.D. degrees in Electrical Engineering Department from the University of California, Los Angeles, CA. His doctoral thesis was the material characterization in High Electron Mobility (HEMT) GaAs transistors. During his graduate studies, he was a research associate and a technical consultant for Hughes Aircraft Co. In 1988, he joined the technical staff of IBM Research and Development Center in Fishkill, NY, where he worked as a device design and characterization engineer. He developed asymmetrical MOS transistors and ultrathin silicon on insulator (SOI) technologies for IBM. In 1993, he transferred to IBM at Rochester, MN, as a senior circuit design engineer to work on the floating-point unit for the AS-400 main frame processor. He continued his circuit design career at Intel Corporation in Santa Clara, CA, where he designed 16-bit packed multipliers and adders for the MMX unit in Pentium II processors. In 1996, he joined Philips Semiconductors in Sunnyvale, CA, where he was involved in the designs of instruction/data caches and various SRAM modules for the TriMedia processor. His involvement with VLSI architecture also started in Philips Semiconductors and led to the design of the Video-Out unit for the same processor. In 1998, he joined Cadence Design Systems as a VLSI architect and directed a team of engineers to design self-timed asynchronous processors. After approximately 20 years of industry work, he joined the computer engineering faculty at San Jose State University in 2002. His current research interests range from nano-scale electron devices to robotics.

Dr. Bindal has over 30 scientific journal and conference publications and 10 invention disclosures with IBM. He currently holds three US patents with IBM and one with Intel Corporation. On the light side of things, Dr. Bindal is a model aircraft builder and an avid windsurfer for more than 30 years.

## Review of Combinational Circuits

Logic gates are the essential elements in digital design, and ultimately constitute the building blocks for digital systems. A good understanding in designing complex logic blocks from primitive logic gates, and mastering the design tools and techniques that need to be incorporated in the design process is a requirement for the reader before moving to the details of computer architecture and design.

This chapter starts with defining the logic gates and the concept of truth table which then leads to the implementation of basic logic circuits. Later in the chapter, the concept of Karnaugh maps is introduced in order to minimize gate count, thereby completing the basic requirements of combinational logic design. Following the minimization techniques, various fundamental logic blocks such as multiplexers, encoders, decoders and one-bit adders are introduced so that they can be used to construct larger scale combinational logic circuits. The last section of this chapter is dedicated to the design of mega cells. These include different types of adders such as ripple-carry adder, carry-look-ahead adder, carry-select adder, and the combination of all three types depending on the goals of the design: gate count, circuit speed and power consumption. Subtractors, linear and barrel shifters, array and Booth multipliers constitute the remaining sections of this chapter.

It is vital for the reader to also invest time to learn a hardware design language such as Verilog while studying this chapter and the rest of the chapters in this book. A simulation platform incorporating Verilog and a set of tools that work with Verilog such as design synthesis, static timing analysis, and verification is an effective way to check if the intended design is correct or not. There is nothing more valuable than trying various design ideas on a professional design environment, and understanding what works and what does not while learning from your mistakes. An appendix introducing the basic principles of Verilog is included at the end of this book for reference.

### 1.1 Logic Gates

## AND gate

To understand how AND gate functions, assume that the output, OUT, in Fig. 1.1 is at logic 0 when both switches, A and B, are open. Unless both A and B close, the output stays at logic 0 .


Fig. 1.1 Switch representation of a two-input AND gate

A two-input AND gate functions similarly to the circuit in Fig. 1.1. If any of the two inputs, A or B , is at logic 0 in the AND gate in Fig. 1.2, the gate produces a logic 0 output at OUT. Both inputs of the gate must be equal to logic 1 in order to produce an output at logic 1 . This behavior is tabulated in Table 1.1, which is called a "truth table".


Fig. 1.2 Two-input AND gate symbol

Table 1.1 Two-input AND gate truth table

| $A$ | $B$ | OUT |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

The functional representation of the two-input AND gate is:
$\mathrm{OUT}=\mathrm{A} . \mathrm{B}$
Here, the symbol "." between inputs A and B represents the AND-function.

## OR gate

Now, assume a parallel connectivity between switches A and B as shown in Fig. 1.3. OUT becomes logic 1 if one of these switches closes; otherwise the output will stay at logic 0 .


Fig. 1.3 Switch representation of two-input OR gate

A two-input OR gate shown in Fig. 1.4 functions similarly to the circuit in Fig. 1.3. If any of the two inputs is at logic 1 , the gate produces an output, OUT, at logic 1. Both inputs must be equal to logic 0 in order to produce a logic 0 output. This behavior is tabulated in the truth table, Table 1.2.


Fig. 1.4 Two-input OR gate symbol

Table 1.2 Two-input OR gate truth table

| A | B | OUT |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

The functional representation of the two-input OR gate is:

$$
\mathrm{OUT}=\mathrm{A}+\mathrm{B}
$$

Here, the symbol "+" between inputs A and B signifies the OR-function.

## Exclusive OR gate

A two-input Exclusive OR gate, XOR gate, is shown in Fig. 1.5. The XOR gate produces a logic 0 output if both inputs are equal. Therefore, in many logic applications this gate is used to compare the input logic levels to see if they are equal. The functional behavior of the gate is tabulated in Table 1.3.


Fig. 1.5 Two-input XOR gate symbol
Table 1.3 Two-input XOR gate truth table

| A | B | OUT |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

The functional representation of the two-input XOR gate is:
OUT $=\mathrm{A} \oplus \mathrm{B}$
Here, the " $\oplus$ " symbol between inputs A and B signifies the XOR-function.

## Buffer

A buffer is a single input device whose output is logically equal to its input. The only use of this gate is to be able to supply enough current to the capacitive load created by a multitude of logic gates connected to its output. The logical representation of this gate is shown in Fig. 1.6.


Fig. 1.6 Buffer symbol

## Complementary Logic Gates

All basic logic gates need to have complemented forms in logic design. If a single input needs to be complemented, an inverter shown in Fig. 1.7 is used. The inverter truth table is shown in Table 1.4.


Fig. 1.7 Inverter symbol
Table 1.4 Inverter truth table

| IN | OUT |
| :---: | :---: |
| 0 | 1 |
| 1 | 0 |

The functional representation of the inverter is:
OUT $=\overline{\mathrm{IN}}$
Here, the "-" symbol on top of the input, IN, represents the complement-function.
The complemented form of two-input AND gate is called two-input NAND gate, where "N" signifies negation. The logic representation is shown in Fig. 1.8, where a circle at the output of the gate means complemented output. The truth table of this gate is shown in Table 1.5. Note that all output values in this table are exact opposites of the values given in Table 1.1.


Fig. 1.8 Two-input NAND gate symbol
Table 1.5 Two-input NAND gate truth table

| $A$ | $B$ | OUT |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

The functional representation of the two-input NAND gate is:

$$
\mathrm{OUT}=\overline{\mathrm{A} \cdot \mathrm{~B}}
$$

Similar to the NAND gate, the two-input OR and the two-input XOR gates have complemented configurations, called the two-input NOR and the two-input XNOR gates, respectively.

The symbolic representation and truth table of a two-input NOR gate is shown in Fig. 1.9 and Table 1.6, respectively. Again, all the outputs in Table 1.6 are the exact complements of the outputs in Table 1.2.


Fig. 1.9 Two-input NOR gate symbol
Table 1.6 Two-input NOR gate truth table

| A | B | OUT |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

The functional representation of the two-input NOR gate is:

$$
\mathrm{OUT}=\overline{\mathrm{A}+\mathrm{B}}
$$

The symbolic representation and truth table of a two-input XNOR gate is shown in Fig. 1.10 and Table 1.7, respectively. This gate, like its counterpart the two-input XOR gate, is often used to detect if input logic levels are equal.


Fig. 1.10 Two-input XNOR gate symbol
Table 1.7 Two-input XNOR gate truth table

| A | B | OUT |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

The functional representation of the two-input XNOR gate is:

$$
\mathrm{OUT}=\overline{\mathrm{A} \oplus \mathrm{~B}}
$$

## Tri-State Buffer and Inverter

It is often necessary to create an open circuit between the input and the output of a logic gate if the gate is not enabled. This need creates two more basic logic gates, the tri-state buffer and tri-state inverter.

The tri-state buffer is shown in Fig. 1.11. Its truth table in Table 1.8 indicates continuity between the input and the output terminals if the control input, EN, is at logic 1. When EN is lowered to logic 0 , an open circuit exists between the IN and the OUT terminals, which is defined as high impedance state (HiZ).


Fig. 1.11 Tri-state buffer symbol

Table 1.8 Tri-state buffer truth table

| EN | IN | OUT |
| :---: | :---: | :---: |
| 0 | 0 | HiZ |
| 0 | 1 | HiZ |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

The tri-state inverter is shown in Fig. 1.12 along with its truth table in Table 1.9. This gate behaves like an inverter when EN input is at logic 1. However, when EN is lowered to logic 0 , its output disconnects from its input.


Fig. 1.12 Tri-state inverter symbol

Table 1.9 Tri-state inverter truth table

| EN | IN | OUT |
| :---: | :---: | :---: |
| 0 | 0 | HiZ |
| 0 | 1 | HiZ |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

The control input, EN, to tri-state buffer and inverter can also be complemented in order to produce an active-low enabling scheme.

The tri-state buffer with the active-low enable input in Fig. 1.13 creates continuity when $\mathrm{EN}=0$.


Fig. 1.13 Tri-state buffer symbol with complemented enable input
The tri-state inverter with the active-low input in Fig. 1.14 also functions like an inverter when EN is at logic 0 , but its output becomes HiZ when EN is changed to logic 1 .


Fig. 1.14 Tri-state inverter symbol with complemented enable input

### 1.2 Boolean Algebra

It is essential to be able to reconfigure logic functions to suit our design goals. Logical reconfigurations may be as simple as regrouping the inputs to a single gate or complementing the inputs of several gates to reach a design objective.

The identity, commutative, associative, distributive laws and the DeMorgan's negation rules are used to perform logical manipulations. Table 1.10 tabulates these laws.

Table 1.10 Identity, commutative, associative, distributive and DeMorgan's rules

$$
\begin{aligned}
& A \cdot 1=A \\
& A \cdot 0=0 \\
& A \cdot A=A \\
& A \cdot \bar{A}=0 \\
& A+1=1 \\
& A+0=A \\
& A+A=A \\
& A+\bar{A}=1 \\
& \overline{\bar{A}}=A
\end{aligned}
$$

$$
A \cdot B=B \cdot A
$$

$$
A+B=B+A
$$

Commutative

| $A \cdot(B \cdot C)=(A \cdot B) \cdot C$ | Associative |
| :--- | :--- |
| $A+(B+C)=(A+B)+C$ |  |

$A \cdot(B+C)=A \cdot B+A \cdot C$
$A+B \cdot C=(A+B) \cdot(A+C) \quad$ Distributive

| $\overline{A \cdot B}$ | $=\bar{A}+\bar{B}$ |
| :--- | :--- |
| $\overline{A+B}=\bar{A} \cdot \bar{B} \quad$ DeMorgan's |  |

Example 1.1: Reduce $\mathrm{OUT}=\mathrm{A} \cdot \overline{\mathrm{B}} \cdot \mathrm{C}+\mathrm{A} . \mathrm{B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{B}}$ using algebraic rules.

$$
\begin{aligned}
\text { OUT } & =\mathrm{A} \cdot \overline{\mathrm{~B}} \cdot \mathrm{C}+\mathrm{A} \cdot \mathrm{~B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{~B}} \\
& =\mathrm{A} \cdot \mathrm{C} \cdot(\overline{\mathrm{~B}}+\mathrm{B})+\mathrm{A} \cdot \overline{\mathrm{~B}} \\
& =\mathrm{A} \cdot(\mathrm{C}+\overline{\mathrm{B}})
\end{aligned}
$$

Example 1.2: Reduce $\mathrm{OUT}=\mathrm{A}+\overline{\mathrm{A}} . \mathrm{B}$ using algebraic rules.

$$
\begin{aligned}
\text { OUT } & =\mathrm{A}+\overline{\mathrm{A}} \cdot \mathrm{~B} \\
& =(\mathrm{A}+\overline{\mathrm{A}}) \cdot(\mathrm{A}+\mathrm{B}) \\
& =\mathrm{A}+\mathrm{B}
\end{aligned}
$$

Example 1.3: Reduce $\mathrm{OUT}=\mathrm{A} . \mathrm{B}+\overline{\mathrm{A}} . \mathrm{C}+\mathrm{B} . \mathrm{C}$ using algebraic rules.

$$
\begin{aligned}
\text { OUT } & =\mathrm{A} \cdot \mathrm{~B}+\overline{\mathrm{A}} \cdot \mathrm{C}+\mathrm{B} \cdot \mathrm{C} \\
& =\mathrm{A} \cdot \mathrm{~B}+\overline{\mathrm{A}} \cdot \mathrm{C}+\mathrm{B} \cdot \mathrm{C} \cdot(\mathrm{~A}+\overline{\mathrm{A}}) \\
& =\mathrm{A} \cdot \mathrm{~B}+\overline{\mathrm{A}} \cdot \mathrm{C}+\mathrm{A} \cdot \mathrm{~B} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C} \\
& =\mathrm{A} \cdot \mathrm{~B} \cdot(1+\mathrm{C})+\overline{\mathrm{A}} \cdot \mathrm{C} \cdot(1+\mathrm{B}) \\
& =\mathrm{A} \cdot \mathrm{~B}+\overline{\mathrm{A}} \cdot \mathrm{C}
\end{aligned}
$$

Example 1.4: Reduce $\mathrm{OUT}=(\mathrm{A}+\mathrm{B}) \cdot(\overline{\mathrm{A}}+\mathrm{C})$ using algebraic rules.

$$
\begin{aligned}
\text { OUT } & =(\mathrm{A}+\mathrm{B}) \cdot(\overline{\mathrm{A}}+\mathrm{C}) \\
& =\mathrm{A} \cdot \overline{\mathrm{~A}}+\mathrm{A} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B}+\mathrm{B} \cdot \mathrm{C} \\
& =\mathrm{A} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B}+\mathrm{B} \cdot \mathrm{C} \\
& =\mathrm{A} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B}+\mathrm{B} \cdot \mathrm{C} \cdot(\mathrm{~A}+\overline{\mathrm{A}}) \\
& =\mathrm{A} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B}+\mathrm{A} \cdot \mathrm{~B} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C} \\
& =\mathrm{A} \cdot \mathrm{C} \cdot(1+\mathrm{B})+\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot(1+\mathrm{C}) \\
& =\mathrm{A} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \mathrm{~B}
\end{aligned}
$$

Example 1.5: Convert $\mathrm{OUT}=(\mathrm{A}+\mathrm{B}) . \overline{\mathrm{C} . \mathrm{D}}$ into an OR-combination of two-input AND gates using algebraic laws and DeMorgan's theorem.

$$
\begin{aligned}
\text { OUT } & =(\mathrm{A}+\mathrm{B}) \cdot \overline{\mathrm{C} \cdot \mathrm{D}} \\
& =(\mathrm{A}+\mathrm{B}) \cdot(\overline{\mathrm{C}}+\overline{\mathrm{D}}) \\
& =\mathrm{A} \cdot \overline{\mathrm{C}}+\mathrm{A} \cdot \overline{\mathrm{D}}+\mathrm{B} \cdot \overline{\mathrm{C}}+\mathrm{B} \cdot \overline{\mathrm{D}}
\end{aligned}
$$

Example 1.6: Convert OUT $=\mathrm{A} . \mathrm{B}+\mathrm{C} . \mathrm{D}$ into an AND-combination of two-input OR gates using algebraic laws and DeMorgan's theorem.

$$
\begin{aligned}
\text { OUT } & =\mathrm{A} \cdot \mathrm{~B}+\mathrm{C} \cdot \mathrm{D} \\
& =\overline{\overline{\mathrm{A} \cdot \mathrm{~B}+\mathrm{C} \cdot \mathrm{D}}} \\
& =(\overline{\overline{\mathrm{A}}+\overline{\mathrm{B}}) \cdot(\overline{\mathrm{C}}+\overline{\mathrm{D}}})
\end{aligned}
$$

### 1.3 Designing Combinational Logic Circuits Using Truth Tables

A combinational logic circuit is a cascaded form of basic logic gates without any feedback from the output to any of its inputs. The logic function is obtained from a truth table that specifies the complete functionality of the digital circuit.

Example 1.7: Using the truth table given in Table 1.11 determine the output function of the digital circuit.

Table 1.11 An arbitrary truth table with four inputs

| A | B | C | D | OUT |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

The output function can be expressed either as the OR combination of AND gates or the AND combination of OR gates.

If the output is expressed in terms of AND gates, all output entries that are equal to one in the truth table must be grouped together as a single OR gate.

$$
\begin{aligned}
\text { OUT }= & \overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D}+\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \mathrm{C} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \overline{\mathrm{C}} \cdot \mathrm{D} \\
& +\mathrm{A} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\mathrm{A} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D}+\mathrm{A} \cdot \overline{\mathrm{~B}} \cdot \mathrm{C} \cdot \overline{\mathrm{D}}
\end{aligned}
$$

This expression is called the Sum Of Products (SOP), and it contains seven terms each of which is called a "minterm". In the first minterm, each A, B, C and D input is complemented to produce $\mathrm{OUT}=1$ for the $\mathrm{A}=\mathrm{B}=\mathrm{C}=\mathrm{D}=0$ entry of the truth table. Each of the remaining six minterms also complies with producing OUT $=1$ for their respective input entries.

The resulting combinational circuit is is shown Fig 1.15.


Fig. 1.15 AND-OR logic representation of the truth table in Table 1.11

If the output function needs to be expressed in terms of OR gates, all the output entries that are equal to zero in the truth table must be grouped as a single AND gate.

$$
\begin{aligned}
\text { OUT }= & (\mathrm{A}+\mathrm{B}+\overline{\mathrm{C}}+\overline{\mathrm{D}}) \cdot(\mathrm{A}+\overline{\mathrm{B}}+\mathrm{C}+\mathrm{D}) \cdot(\mathrm{A}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\mathrm{D}) \\
& \cdot(\mathrm{A}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\overline{\mathrm{D}}) \cdot(\overline{\mathrm{A}}+\mathrm{B}+\overline{\mathrm{C}}+\overline{\mathrm{D}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\mathrm{C}+\mathrm{D}) \\
& \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\mathrm{C}+\overline{\mathrm{D}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\mathrm{D}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}}+\overline{\mathrm{D}})
\end{aligned}
$$

This expression is called the Product Of Sums (POS), and it contains nine terms each of which is called a "maxterm". The first maxterm produces OUT $=0$ for the $\mathrm{ABCD}=0011$ entry of the truth table. Since the output is formed with a nine-input AND gate, the values of the other maxterms do not matter to produce OUT $=0$. Each of the remaining eight maxterms generates OUT $=0$ for their corresponding truth table input entries.

The resulting combinational circuit is shown in Fig. 1.16.


Fig. 1.16 OR-AND logic representation of the truth table in Table 1.11

### 1.4 Combinational Logic Minimization—Karnaugh Maps

One of the most useful tools in logic design is the use of Karnaugh maps (K-map) to minimize combinational logic functions.

Minimization can be performed in two ways. To obtain the SOP form of a minimized logic function, logic 1 entries of the truth table must be grouped together in the K-map. To obtain the POS form of a minimized logic function, logic 0 entries of the truth table must be grouped together in the K-map.

Example 1.8: Using the truth table in Table 1.12, determine the minimized SOP and POS output functions. Prove them to be identical.

Table 1.12 An arbitrary truth table with three inputs

| A | B | C | OUT |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

The SOP function is formed by grouping logic 1s in Fig. 1.17 to obtain the minimized output function, OUT.


Fig. 1.17 K-map of the truth table in Table 1.12 to determine SOP
Grouping 1s takes place among neighboring boxes in the K-map where only one variable is allowed to change at a time. For instance, the first grouping of 1s combines the $\mathrm{ABC}=000$ and $\mathrm{ABC}=010$ entries as they are in the neighboring boxes. Only B changes from logic 0 to logic 1 while A and C stay constant at logic 0 . To obtain OUT $=1$, both A and C need to be complemented; this produces the first term, $\overline{\mathrm{A}} . \overline{\mathrm{C}}$, for the output function. Similarly, the second grouping of 1 s combines the neighboring boxes, $\mathrm{ABC}=000,001,100$ and 101, where both A and C change while B stays constant at logic 0 . To obtain OUT $=1, \mathrm{~B}$ needs to be complemented; this generates the second term, $\overline{\mathrm{B}}$, for the output function.

This means that either the term $\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}$ or $\overline{\mathrm{B}}$ makes OUT equal to logic 1 . Therefore, the minimized output function, OUT, in the SOP form is:

$$
\mathrm{OUT}=\overline{\mathrm{B}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}
$$

Grouping 0s produces the minimized POS output function as shown in Fig. 1.18.


Fig. 1.18 K-map of the truth table in Table 1.12 to determine POS
This time, the first grouping of 0 s combines the boxes, $\mathrm{ABC}=011$ and 111 , where A changes from logic 0 to logic 1 while $B$ and $C$ stay constant at logic 1 . This grouping targets OUT $=0$, which requires both B and C to be complemented. As a result, the first term of the output function, $\overline{\mathrm{B}}+\overline{\mathrm{C}}$, is generated. The second grouping combines $\mathrm{ABC}=110$ and 111 where $C$ changes from logic 0 to logic 1 while $A$ and $B$ stay at logic 1 . To obtain OUT $=0$, both $A$ and $B$ need to be complemented. Consequently, the second term, $\overline{\mathrm{A}}+\overline{\mathrm{B}}$, forms.

Therefore, either $\overline{\mathrm{B}}+\overline{\mathrm{C}}$ or $\overline{\mathrm{A}}+\overline{\mathrm{B}}$ should produce OUT $=0$, resulting the following POS function.

$$
\text { OUT }=(\overline{\mathrm{B}}+\overline{\mathrm{C}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}})
$$

To find out if the SOP and POS forms are identical to each other, we can manipulate the POS expression above using the algebraic rules given earlier.

$$
\begin{aligned}
\text { OUT } & =(\overline{\mathrm{B}}+\overline{\mathrm{C}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}) \\
& =\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{~B}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{C}} \\
& =\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}}+\overline{\mathrm{B}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{C}} \\
& =\overline{\mathrm{B}} \cdot(\overline{\mathrm{~A}}+1+\overline{\mathrm{C}})+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}} \\
& =\overline{\mathrm{B}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}
\end{aligned}
$$

This is the SOP form of the output function derived above.
Example 1.9: Using the truth table in Example 1.7 determine the minimized SOP and POS output functions.

To obtain the output function in SOP form, logic 1s in the K-map in Fig. 1.19 are grouped together as shown below.


Fig. 1.19 K-map of the truth table in Table 1.11 to determine SOP
The minimized output function contains only three minterms compared to seven minterms in Example 1.7. Also, the minterms are reduced to groups of two or three inputs instead of four.

OUT $=\overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}$
The resultant combinational circuit is shown in Fig. 1.20


Fig. 1.20 Minimized logic circuit in SOP form from the K-map in Fig. 1.19
Further minimization can be achieved algebraically, which then reduces the number of terms from three to two.

OUT $=\overline{\mathrm{B}} \cdot(\overline{\mathrm{C}}+\overline{\mathrm{D}})+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D}$
The corresponding combinational circuit is shown in Fig. 1.21.


Fig. 1.21 Logic circuit in Fig. 1.20 after algebraic minimizations are applied
To obtain the POS output function, logic 0s are grouped together as shown in Fig. 1.22.


Fig. 1.22 K-map of the truth table in Table 1.11 to determine POS
The minimized output function contains only three maxterms compared to nine in Example 1.7. Also, the maxterms are reduced to groups of two inputs instead of four.

$$
\mathrm{OUT}=(\overline{\mathrm{C}}+\overline{\mathrm{D}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}) \cdot(\overline{\mathrm{B}}+\mathrm{D})
$$

The resultant combinational circuit is shown in Fig. 1.23


Fig. 1.23 Minimized logic circuit in POS form from the K-map in Fig. 1.22

Example 1.10: Determine if the minimized SOP and POS output functions in Example 1.9 are identical to each other.

The POS expression for the OUT function in Example 1.9 can be re-written as follows:

$$
\begin{aligned}
\mathrm{OUT}= & (\overline{\mathrm{C}}+\overline{\mathrm{D}}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}) \cdot(\overline{\mathrm{B}}+\mathrm{D}) \\
= & (\overline{\mathrm{A}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}) \cdot(\overline{\mathrm{B}}+\mathrm{D}) \\
= & \overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}} \\
& +\overline{\mathrm{A}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D}+\overline{\mathrm{B}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D} \\
= & \overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{C}} \cdot \mathrm{D}
\end{aligned}
$$

The result is identical to the SOP expression given in Example 1.9.
Example 1.11: Determine the minimal SOP and POS forms of the output function, OUT, from the K-map in Fig. 1.24. Note that the " X " sign corresponds to a "don't care" condition that represents either logic 0 or logic 1 .


Fig. 1.24 An arbitrary K-map with "don't care" entries
For the SOP expression, logic 1s in the K-map in Fig. 1.25 are grouped. Boxes with "don't care" entries are used as logic 1s to achieve the minimal SOP expression.


Fig. 1.25 Grouping to determine SOP form for the K-map in Fig. 1.24
As a result, the SOP functional expression for OUT is:
OUT $=\mathrm{A} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \mathrm{B} \cdot \mathrm{C}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}$
For the POS expression, logic 0s in the K-map in Fig. 1.26 are grouped. Boxes with "don't care" symbols are used as logic 0s to achieve the minimal POS expression.


Fig. 1.26 Grouping to determine POS form for the K-map in Fig. 1.24
Therefore, the POS functional expression for OUT becomes:
OUT $=(\mathrm{C}+\overline{\mathrm{D}}) \cdot(\mathrm{B}+\overline{\mathrm{D}}) \cdot(\mathrm{A}+\overline{\mathrm{B}}+\mathrm{C}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}})$
To show that the SOP and POS expressions are identical, we use the algebraic manipulations in Table 1.10 on the POS expression in order to obtain the SOP expression.

$$
\begin{aligned}
\text { OUT } & =(\mathrm{C}+\overline{\mathrm{D}}) \cdot(\mathrm{B}+\overline{\mathrm{D}}) \cdot(\mathrm{A}+\overline{\mathrm{B}}+\mathrm{C}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{B}}+\overline{\mathrm{C}}) \\
& =(\mathrm{B} \cdot \mathrm{C}+\mathrm{C} \cdot \overline{\mathrm{D}}+\mathrm{B} \cdot \overline{\mathrm{D}}+\overline{\mathrm{D}}) \cdot(\mathrm{A} \cdot \overline{\mathrm{~B}}+\mathrm{A} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}}+\overline{\mathrm{B}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \mathrm{C}+\overline{\mathrm{B}} \cdot \mathrm{C}) \\
& =(\mathrm{B} \cdot \mathrm{C}+\overline{\mathrm{D}}) \cdot(\mathrm{A} \cdot \overline{\mathrm{C}}+\overline{\mathrm{A}} \cdot \mathrm{C}+\overline{\mathrm{B}}) \\
& =\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \mathrm{C} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}} \\
& =\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \mathrm{C} \cdot \overline{\mathrm{D}} \cdot(\mathrm{~B}+\overline{\mathrm{B}}) \\
& =\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C} \cdot \overline{\mathrm{D}}+\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \mathrm{C} \cdot \overline{\mathrm{D}} \\
& =\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C} \cdot(1+\overline{\mathrm{D}})+\mathrm{A} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}} \cdot(1+\overline{\mathrm{A}} \cdot \mathrm{C}) \\
& =\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \mathrm{C}+\mathrm{A} \cdot \overline{\mathrm{C}} \cdot \overline{\mathrm{D}}+\overline{\mathrm{B}} \cdot \overline{\mathrm{D}}
\end{aligned}
$$

This result is identical to the minimal SOP expression for the OUT function above.

### 1.5 Basic Logic Blocks

## 2-1 Multiplexer

The 2-1 multiplexer (MUX) is one of the most versatile logic elements in logic design. It is defined as follows:

$$
\mathrm{OUT}=\left[\begin{array}{l}
\mathrm{A} \text { if sel }=1 \\
\mathrm{~B} \text { else }
\end{array}\right.
$$

A functional diagram of the 2-1 MUX is given in Fig. 1.27. According to the functional description of this device when sel $=1$, input A passes through the device to become its output. When sel $=0$, input B passes through the device to become its output.


Fig. 1.27 2-1 MUX symbol

Table 1.13 2-1 MUX truth table

| sel | A | B | OUT |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

According to this functional definition, the truth table in Table 1.13 can be formed.
Now, let us transfer the output values from the truth table to the K-map in Fig. 1.28.


Fig. 1.28 2-1 MUX K-map
Grouping logic 1s in the K-map reveals the minimal output function of the 2-1 MUX in SOP form:
$\mathrm{OUT}=\mathrm{sel} . \mathrm{A}+\overline{\mathrm{sel}} . \mathrm{B}$
The corresponding combinational circuit is shown in Fig. 1.29.


Fig. 1.29 2-1 MUX logic circuit

## 4-1 Multiplexer

If we apply four inputs to a multiplexer instead of two, we form a 4-1 MUX whose functional description becomes as follows:

$$
\text { OUT }=\left[\begin{array}{l}
\text { A if sel1 }=0 \text { and sel2 }=0 \\
\text { B if sel1 }=0 \text { and sel2 }=1 \\
\text { C if sel1 }=1 \text { and sel2 }=0 \\
\text { D else }
\end{array}\right.
$$

According to this description, we can form a truth table and obtain the minimal SOP or POS expression for OUT. However, it is quite easy to decipher the SOP expression for OUT from the description above. The AND-combination of A, complemented sell and complemented sel2 inputs constitute the first minterm of our SOP. The second minterm should contain B, complemented sel1 and uncomplemented sel2 according to the description above. Similarly, the third minterm contains C, uncomplemented sel1 and complemented sel2. Finally, the last minterm contains D, uncomplemented sel1 and uncomplemented sel2 control inputs. Therefore, the SOP expression for the 4-1 MUX becomes equal to the logic expression below and is implemented in Fig. 1.30.

$$
\mathrm{OUT}=\overline{\operatorname{sel} 1} \cdot \overline{\operatorname{sel} 2} \cdot \mathrm{~A}+\overline{\operatorname{sel} 1} \cdot \operatorname{sel} 2 \cdot \mathrm{~B}+\operatorname{sel} 1 \cdot \overline{\operatorname{sel} 1} \cdot \mathrm{C}+\operatorname{sel} 1 \cdot \operatorname{sel} 2 \cdot \mathrm{D}
$$



Fig. 1.30 4-1 MUX logic circuit in SOP form

However, implementing a 4-1 MUX this way is not advantageous due to the amount of gate delays. A three-input AND gate is a series combination of a three-input NAND gate and an inverter. Similarly, a four-input OR requires a four-input NOR gate and an inverter. Therefore, we obtain a minimum of four gate delays instead of two according to this circuit.

Logic translations are possible to reduce the gate delay. The first stage of this process is to complement the outputs of all four three-input AND gates. This necessitates complementing the inputs to the four-input OR gate, and it results in a circuit in Fig. 1.31.


Fig. 1.31 Logic conversion of 4-1 MUX in Fig. 1.30

However, an OR gate with complemented inputs is equivalent to a NAND gate. Therefore, the circuit Fig. 1.32 becomes an optimal implementation to achieve the shortest gate delay for the 4-1 MUX because it contains only two gate delays instead of the earlier four.


Fig. 1.32 4-1 MUX logic circuit in NAND-NAND form

The symbolic diagram of the 4-1 MUX is shown in Fig. 1.33.


Fig. 1.33 4-1 MUX symbol

## Encoders

Encoders are the combinational logic blocks that receive $2^{\mathrm{N}}$ number of inputs and produce N number of encoded outputs.

Example 1.12: Generate an encoding logic from the truth table given in Table 1.14.
Table 1.14 An arbitrary encoder truth table with four inputs

| $\mathbf{I N} 1$ | IN 2 | IN 3 | IN 4 | OUT1 | OUT2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |

The K-maps in Fig. 1.34 group logic 1s, and produce the encoded SOP expressions for OUT1 and OUT2.


Fig. 1.34 K-map of the truth table in Table 1.14

$$
\begin{aligned}
\text { OUT1 }= & \mathrm{IN} 1 \cdot \mathrm{IN} 2+\overline{\mathrm{IN} 1} \cdot \overline{\mathrm{IN} 2} \cdot \mathrm{IN} 3 \cdot \overline{\mathrm{IN} 4}+\overline{\mathrm{IN} 1} \cdot \overline{\mathrm{IN} 2} \cdot \overline{\mathrm{IN} 3} \cdot \mathrm{IN} 4 \\
& +\mathrm{IN} 2 \cdot \overline{\mathrm{IN} 3} \cdot \overline{\mathrm{IN} 4}+\mathrm{IN} 2 \cdot \mathrm{IN} 3 \cdot \mathrm{IN} 4 \\
= & \mathrm{IN} 1 \cdot \mathrm{IN} 2+\overline{\mathrm{IN} 1} \cdot \overline{\mathrm{IN} 2} \cdot(\mathrm{IN} 3 \oplus \mathrm{IN} 4)+\mathrm{IN} 2 \cdot(\overline{\mathrm{IN} 3 \oplus \mathrm{IN} 4}) \\
\text { OUT2 }= & \overline{\mathrm{IN} 1} \cdot \overline{\mathrm{IN} 2}+\overline{\mathrm{IN} 2} \cdot \overline{\mathrm{IN} 3} \cdot \overline{\mathrm{IN} 4}+\overline{\mathrm{IN} 2} \cdot \mathrm{IN} 3 \cdot \mathrm{IN} 4+\mathrm{IN} 1 \cdot \mathrm{IN} 2 \cdot \overline{\mathrm{IN} 3} \cdot \mathrm{IN} 4 \\
& +\mathrm{IN} 1 \cdot \mathrm{IN} 2 \cdot \mathrm{IN} 3 \cdot \overline{\mathrm{IN} 4} \\
= & \overline{\mathrm{IN} 1} \cdot \overline{\mathrm{IN} 2}+\overline{\mathrm{IN} 2} \cdot(\overline{\mathrm{IN} 3 \oplus \mathrm{IN} 4})+\mathrm{IN} 1 \cdot \mathrm{IN} 2 \cdot(\mathrm{IN} 3 \oplus \mathrm{IN} 4)
\end{aligned}
$$

## Decoders

Decoders are the combinational logic blocks to decode encoded inputs. An ordinary decoder takes N inputs and produces $2^{\mathrm{N}}$ outputs.

Example 1.13: Design a line decoder in which an active-high enable signal activates one of the eight independent outputs according to the truth table in Table 1.15. When the enable signal is lowered to logic 0 , all eight outputs are disabled and stay at logic 0 .

Table 1.15 Truth table of a line decoder with three inputs with enable

| EN | $\operatorname{IN}[2]$ | $\operatorname{IN}[1]$ | $\operatorname{IN}[0]$ | OUT[7] | OUT[6] | OUT[5] | OUT[4] | OUT[3] | OUT[2] | OUT[1] | OUT[0] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

In this table, all outputs become logic 0 when the enable signal, EN , is at logic 0 . However, when EN $=1$, selective activation of the output starts. In this section, for each three-bit input entry, there is always one output equal to logic 1 . For example, when IN[2] = $\operatorname{IN}[1]=\operatorname{IN}[0]=0, \operatorname{OUT}[0]$ becomes active and equals to logic 1 while all the other outputs stay at logic $0 . \operatorname{IN}[2]=\operatorname{IN}[1]=\operatorname{IN}[0]=1$ activates OUT[7] and disables all other outputs.

We can produce the expression for each output from OUT[7] to OUT[0] simply by reading the input values from the truth table. The accompanying circuit is composed of eight AND gates, each with four inputs as shown in Fig. 1.35.

$$
\begin{aligned}
& \operatorname{OUT}[7]=\mathrm{EN} . \operatorname{IN}[2] . \operatorname{IN}[1] . \operatorname{IN}[0] \\
& \operatorname{OUT}[6]=\mathrm{EN} . \operatorname{IN}[2] . \operatorname{IN}[1] . \overline{\mathrm{IN}[0]} \\
& \mathrm{OUT}[5]=\mathrm{EN} . \operatorname{IN}[2] . \overline{\mathrm{IN}[1]} . \operatorname{IN}[0] \\
& \operatorname{OUT}[4]=\mathrm{EN} \cdot \operatorname{IN}[2] \cdot \overline{\mathrm{IN}[1]} \cdot \overline{\mathrm{IN}[0]} \\
& \operatorname{OUT}[3]=\mathrm{EN} . \overline{\mathrm{IN}[2]} . \mathrm{IN}[1] . \operatorname{IN}[0] \\
& \mathrm{OUT}[2]=\mathrm{EN} . \overline{\mathrm{IN}[2]} . \mathrm{IN}[1] . \overline{\mathrm{IN}[0]} \\
& \mathrm{OUT}[1]=\mathrm{EN} . \overline{\mathrm{IN}[2]} \cdot \overline{\mathrm{IN}[1]} . \operatorname{IN}[0] \\
& \operatorname{OUT}[0]=\mathrm{EN} . \overline{\mathrm{IN}[2]} \cdot \overline{\mathrm{IN}[1]} \cdot \overline{\mathrm{IN}[0]}
\end{aligned}
$$



Fig. 1.35 Logic circuit of a line decoder in Table 1.15

## One-Bit Full Adder

The one-bit full adder has three inputs: A, B, and carry-in (CIN), and it produces two outputs: sum (SUM) and carry-out (COUT). The symbolic representation of a full adder is shown in Fig. 1.36.


Fig. 1.36 One-bit full adder symbol
The one-bit full adder simply adds the contents of its two inputs, A and B, to CIN, and forms the truth table in Table 1.16.

Table 1.16 One-bit full adder truth table

| CIN | A | B | SUM COUT |  |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

We can obtain the minimized SOP expressions for SUM and COUT from the K-maps in Figs. 1.37 and 1.38 .


Fig. 1.37 SUM output of a one-bit full adder

Consequently,

$$
\begin{aligned}
\mathrm{SUM} & =\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \mathrm{CIN}+\overline{\mathrm{A}} \cdot \mathrm{~B} \cdot \overline{\mathrm{CIN}}+\mathrm{A} \cdot \mathrm{~B} \cdot \mathrm{CIN}+\mathrm{A} \cdot \overline{\mathrm{~B}} \cdot \overline{\mathrm{CIN}} \\
& =\mathrm{CIN} \cdot(\overline{\mathrm{~A}} \cdot \overline{\mathrm{~B}}+\mathrm{A} \cdot \mathrm{~B})+\overline{\mathrm{CIN}} \cdot(\overline{\mathrm{~A}} \cdot \mathrm{~B}+\mathrm{A} \cdot \overline{\mathrm{~B}}) \\
& =\mathrm{CIN} \cdot(\overline{\mathrm{~A} \oplus \mathrm{~B}})+\overline{\mathrm{CIN}} \cdot(\mathrm{~A} \oplus \mathrm{~B}) \\
& =\mathrm{A} \oplus \mathrm{~B} \oplus \mathrm{CIN}
\end{aligned}
$$



Fig. 1.38 COUT output of a one-bit full adder

Thus,

$$
\begin{aligned}
\mathrm{COUT} & =\mathrm{CIN} \cdot \mathrm{~B}+\mathrm{A} \cdot \mathrm{~B}+\mathrm{A} \cdot \mathrm{CIN} \\
& =\mathrm{CIN} \cdot(\mathrm{~A}+\mathrm{B})+\mathrm{A} \cdot \mathrm{~B}
\end{aligned}
$$

The resultant logic circuits for the SUM and COUT outputs are shown in Fig. 1.39.


Fig. 1.39 One-bit full adder logic circuit

## One-Bit Half Adder

The one-bit half adder has only two inputs, A and B with no CIN. The A and B inputs are added to generate the SUM and COUT outputs. The symbolic representation of half-adder is shown in Fig. 1.40.


Fig. 1.40 One-bit half-adder symbol
The truth table given in Table 1.17 describes the functionality of the half adder.

Table 1.17 One-bit half-adder truth table

| A | B | SUM COUT |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

From the truth table, the POS expressions for the SUM and COUT outputs can be written as:
$\mathrm{SUM}=\mathrm{A} \oplus \mathrm{B}$
$\mathrm{COUT}=\mathrm{A} . \mathrm{B}$
Therefore, we can produce the SUM and the COUT circuits as shown in Fig. 1.41.


Fig. 1.41 One-bit half adder logic circuit

### 1.6 Combinational Mega Cells

## Adders

One-bit full-adders can be cascaded serially to produce multiple-bit adder configurations. There are three basic adder types:

Ripple-Carry Adder
Carry-Look-Ahead Adder
Carry-Select Adder

However, hybrid topologies can also be formed by combining two or even three of these configurations. For the sake of simplicity, we will limit the number of bits to four and explain each topology in detail.

## Ripple-Carry Adder

The ripple-carry adder is a cascaded configuration of multiple one-bit full adders. The circuit topology of a four-bit ripple carry adder is shown in Fig. 1.42. In this figure, the carry-out output of a one-bit full adder is connected to the carry-in input of the next full adder to propagate the carry bit from one adder to the next.


Fig. 1.42 Four-bit ripple-carry adder
For the $0^{\text {th }}$ bit of this adder, we have:
$\mathrm{SUM}[0]=\mathrm{A}[0] \oplus \mathrm{B}[0] \oplus \mathrm{CIN}[0]$
$\operatorname{COUT}[0]=\operatorname{CIN}[1]=\mathrm{A}[0] . \mathrm{B}[0]+\mathrm{CIN}[0] .(\mathrm{A}[0]+\mathrm{B}[0])=\mathrm{G}[0]+\mathrm{P}[0] . \operatorname{CIN}[0]$
where,
$\mathrm{G}[0]=\mathrm{A}[0] . \mathrm{B}[0]$ as the zeroth order generation term
$\mathrm{P}[0]=\mathrm{A}[0]+\mathrm{B}[0]$ as the zeroth order propagation term

For the first bit:

$$
\begin{aligned}
\mathrm{SUM}[1]= & \mathrm{A}[1] \oplus \mathrm{B}[1] \oplus \mathrm{CIN}[1]=\mathrm{A}[1] \oplus \mathrm{B}[1] \oplus(\mathrm{G}[0]+\mathrm{P}[0] . \mathrm{CIN}[0]) \\
\operatorname{COUT}[1]= & \mathrm{CIN}[2]=\mathrm{G}[1]+\mathrm{P}[1] . \operatorname{CIN}[1] \\
= & \mathrm{G}[1]+\mathrm{P}[1] .(\mathrm{G}[0]+\mathrm{P}[0] . \mathrm{CIN}[0])=\mathrm{G}[1]+\mathrm{P}[1] . \mathrm{G}[0] \\
& +\mathrm{P}[1] . \mathrm{P}[0] . \operatorname{CIN}[0]
\end{aligned}
$$

where,
$\mathrm{G}[1]=\mathrm{A}[1] . \mathrm{B}[1]$ as the first order generation term
$\mathrm{P}[1]=\mathrm{A}[1]+\mathrm{B}[1]$ as the first order propagation term
For the second bit:

$$
\begin{aligned}
& \mathrm{SUM}[2]= \mathrm{A}[2] \oplus \mathrm{B}[2] \oplus \mathrm{CIN}[2]=\mathrm{A}[2] \oplus \mathrm{B}[2] \oplus\{\mathrm{G}[1]+\mathrm{P}[1] \cdot(\mathrm{G}[0] \\
&+\mathrm{P}[0] \cdot \mathrm{CIN}[0])\} \\
&= \mathrm{A}[2] \oplus \mathrm{B}[2] \oplus(\mathrm{G}[1]+\mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0]) \\
& \begin{aligned}
\mathrm{COUT}[2]= & \mathrm{CIN}[3]=\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{CIN}[2] \\
= & \mathrm{G}[2]+\mathrm{P}[2] \cdot\{\mathrm{G}[1]+\mathrm{P}[1] \cdot(\mathrm{G}[0]+\mathrm{P}[0] \cdot \mathrm{CIN}[0])\} \\
= & \mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] . \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0]
\end{aligned}
\end{aligned}
$$

where,
$\mathrm{G}[2]=\mathrm{A}[2] . \mathrm{B}[2]$ as the second order generation term
$\mathrm{P}[2]=\mathrm{A}[2]+\mathrm{B}[2]$ as the second order propagation term
And for the third bit:

$$
\begin{aligned}
\mathrm{SUM}[3]= & \mathrm{A}[3] \oplus \mathrm{B}[3] \oplus \mathrm{CIN}[3] \\
= & \mathrm{A}[3] \oplus \mathrm{B}[3] \oplus\{\mathrm{G}[2]+\mathrm{P}[2] \cdot\{\mathrm{G}[1]+\mathrm{P}[1] \cdot(\mathrm{G}[0]+\mathrm{P}[0] \cdot \mathrm{CIN}[0])\}\} \\
= & \mathrm{A}[3] \oplus \mathrm{B}[3] \oplus(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0] \\
& +\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])
\end{aligned}
$$

These functional expressions of the SUM and the COUT outputs also serve to estimate the maximum and the minimum propagation delays for each bit of this adder.

The circuit diagram in Fig. 1.43 shows the maximum delay path from bit 0 to bit 3. In this figure, the maximum gate delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ inputs to $\mathrm{SUM}[0]$ is $2 \mathrm{~T}_{\mathrm{XOR} 2}$, where $\mathrm{T}_{\mathrm{XOR} 2}$ is a single two-input XOR gate delay.

The maximum gate delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ to $\mathrm{COUT}[0]$ is $2 \mathrm{~T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{AND} 2}$ where $\mathrm{T}_{\mathrm{OR} 2}$ and $\mathrm{T}_{\mathrm{AND} 2}$ are the two-input OR gate and the two-input AND gate delays, respectively.

The gate delay from $\mathrm{A}[1]$ or $\mathrm{B}[1]$ to $\mathrm{SUM}[1]$ is still $2 \mathrm{~T}_{\mathrm{XOR} 2}$; however, the delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ to $\mathrm{SUM}[1]$ is $2 \mathrm{~T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{XOR} 2}$, which is more than $2 \mathrm{~T}_{\mathrm{XOR} 2}$ and must be considered the maximum gate delay for this particular bit position and for more significant bits.

The maximum gate delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ to $\mathrm{COUT}[1]$ is $3 \mathrm{~T}_{\mathrm{OR} 2}+2 \mathrm{~T}_{\mathrm{AND} 2}$. It may make more sense to expand the expression for COUT[1] as $\operatorname{COUT}[1]=\mathrm{G}[1]+\mathrm{P}[1]$. $\mathrm{G}[0]+\mathrm{P}[1] . \mathrm{P}[0] . \operatorname{CIN}[0]$, and figure out if the overall gate delay, $\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}$, is smaller compared to $3 \mathrm{~T}_{\mathrm{OR} 2}+2 \mathrm{~T}_{\mathrm{AND} 2}$. Here, $\mathrm{T}_{\mathrm{AND} 3}$ and $\mathrm{T}_{\mathrm{OR} 3}$ are the three-input AND and the three-input OR gate delays, respectively.


Fig. 1.43 Logic circuit of the four-bit adder with the maximum and minimum delays
The maximum gate delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ to $\mathrm{SUM}[2]$ is $3 \mathrm{~T}_{\mathrm{OR} 2}+2 \mathrm{~T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{XOR} 2}$. When the expression for $\operatorname{SUM}[2]$ is expanded as $\mathrm{SUM}[2]=\mathrm{A}[2] \oplus \mathrm{B}[2] \oplus(\mathrm{G}[1]+\mathrm{P}[1] . \mathrm{G}[0]$ $+\mathrm{P}[1] . \mathrm{P}[0] . \mathrm{CIN}[0])$, we see that this delay becomes $\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}+\mathrm{T}_{\mathrm{XOR} 2}$, and it may be smaller than the original delay if $\mathrm{T}_{\mathrm{AND} 3}<2 \mathrm{~T}_{\mathrm{AND} 2}$ and $\mathrm{T}_{\mathrm{OR} 3}<2 \mathrm{~T}_{\mathrm{OR} 2}$.

The maximum gate delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ to $\mathrm{COUT}[2]$ is $4 \mathrm{~T}_{\mathrm{OR} 2}+3 \mathrm{~T}_{\mathrm{AND} 2}$. When COUT[2] is expanded as COUT[2] $=\mathrm{G}[2]+\mathrm{P}[2] . \mathrm{G}[1]+\mathrm{P}[2] . \mathrm{P}[1] . \mathrm{G}[0]+\mathrm{P}[2]$. $\mathrm{P}[1] . \mathrm{P}[0] . \mathrm{CIN}[0]$, the maximum delay becomes $\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}$, and it may be smaller than the original delay if $\mathrm{T}_{\mathrm{AND} 4}<3 \mathrm{~T}_{\mathrm{AND} 2}$ and $\mathrm{T}_{\mathrm{OR} 4}<3 \mathrm{~T}_{\mathrm{OR} 2}$. Here, $\mathrm{T}_{\mathrm{AND} 4}$ and $\mathrm{T}_{\mathrm{OR} 4}$ are the four-input AND and the four-input OR gate delays, respectively.

Finally, the maximum delay from $\mathrm{A}[0]$ or $\mathrm{B}[0]$ to $\mathrm{SUM}[3]$ is $4 \mathrm{~T}_{\mathrm{OR} 2}+3 \mathrm{~T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{XOR} 2}$, which is also the maximum propagation delay for this adder. When the functional expression for $\operatorname{SUM}[3]$ is expanded as $\operatorname{SUM}[3]=\mathrm{A}[3] \oplus \mathrm{B}[3] \oplus(\mathrm{G}[2]+\mathrm{P}[2] . \mathrm{G}[1]+\mathrm{P}[2] . \mathrm{P}[1]$. $\mathrm{G}[0]+\mathrm{P}[2] . \mathrm{P}[1] . \mathrm{P}[0] . \mathrm{CIN}[0])$, the total propagation delay becomes $\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{AND} 4}+$ $\mathrm{T}_{\mathrm{OR} 4}+\mathrm{T}_{\mathrm{XOR} 2}$, and again it may be smaller compared to the original delay if $\mathrm{T}_{\text {AND4 }}<$ $3 \mathrm{~T}_{\mathrm{AND} 2}$ and $\mathrm{T}_{\mathrm{OR} 4}<3 \mathrm{~T}_{\mathrm{OR} 2}$.

## Carry-Look-Ahead Adder

The idea behind carry-look-ahead (CLA) adders is to create a topology where carry-in bits to all one-bit full adders are available simultaneously. A four-bit CLA circuit topology is shown in Fig. 1.44. In this figure, the SUM output of a more significant bit does not have to wait until the carry bit ripples from the least significant bit position, but it gets computed after some logic delay. In reality, all carry-in signals are generated by complex combinational logic blocks called CLA hook-ups as shown in Fig. 1.44. Each CLA block adds a certain propagation delay on top of the two-input XOR gate delay to produce a SUM output.


Fig. 1.44 A four-bit carry-look-ahead adder

The earlier SUM and CIN expressions derived for the ripple carry adder can be applied to the CLA adder to generate its functional equations.

Therefore,

$$
\begin{aligned}
& \mathrm{SUM}[0]=\mathrm{A}[0] \oplus \mathrm{B}[0] \oplus \mathrm{CIN}[0] \\
& \mathrm{SUM}[1]=\mathrm{A}[1] \oplus \mathrm{B}[1] \oplus \mathrm{CIN}[1] \\
& \mathrm{SUM}[2]=\mathrm{A}[2] \oplus \mathrm{B}[2] \oplus \mathrm{CIN}[2] \\
& \mathrm{SUM}[3]=\mathrm{A}[3] \oplus \mathrm{B}[3] \oplus \mathrm{CIN}[3]
\end{aligned}
$$

where,

$$
\begin{aligned}
& \mathrm{CIN}[1]=\mathrm{G}[0]+\mathrm{P}[0] \cdot \mathrm{CIN}[0] \\
& \mathrm{CIN}[2]=\mathrm{G}[1]+\mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0] \\
& \mathrm{CIN}[3]=\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0]
\end{aligned}
$$

Therefore, CIN[1] is generated by the COUT[0] function within the zeroth (the least significant) full adder bit. However, CIN[2] and CIN[3] have to be produced by separate logic blocks in order to provide the CIN signals for the second and the third (the most significant) full-adder bits.

According to Fig. 1.44, once a valid CIN[0] becomes available, it takes successively longer times to generate valid CIN inputs to produce higher order SUM outputs due to the increasing logic complexity in the CLA hook-ups.

Assume that $\mathrm{T}_{\text {SUM } 0}, \mathrm{~T}_{\text {SUM1 }}, \mathrm{T}_{\text {SUM } 2}$ and $\mathrm{T}_{\text {SUM } 3}$ are the propagation delays corresponding to the bits $0,1,2$ and 3 with respect to the CIN[0] signal. We can approximate $\mathrm{T}_{\text {SUM0 }}=$ $\mathrm{T}_{\mathrm{XOR} 2}$. To compute $\mathrm{T}_{\mathrm{SUM} 1}$, we need to examine the expression for CIN[1]. In this expression, $\mathrm{P}[0]$. CIN[0] produces a two-input AND gate delay, and G[0] + (P[0] . CIN[0]) produces a two-input OR gate delay to be added on top of $\mathrm{T}_{\mathrm{XOR} 2}$. Therefore, $\mathrm{T}_{\mathrm{SUM} 1}$ becomes $\mathrm{T}_{\mathrm{SUM} 1}=\mathrm{T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{XOR} 2}$. Similarly, the expressions for CIN[2] and CIN[3] produce $\mathrm{T}_{\mathrm{SUM} 2}=\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}+\mathrm{T}_{\mathrm{XOR} 2}$ and $\mathrm{T}_{\mathrm{SUM} 3}=\mathrm{T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}+\mathrm{T}_{\mathrm{XOR} 2}$, respectively.

The maximum propagation delay for this adder is, therefore, $\mathrm{T}_{\mathrm{SUM} 3}=\mathrm{T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}+$ $\mathrm{T}_{\mathrm{XOR} 2}$.

Despite the CLA adder's advantage of being faster than the ripple-carry adder, in most cases the extra CLA logic blocks make this adder topology occupy a larger chip area if the number of adder bits is above eight.

## Carry-Select Adder

Carry-Select Adders require two rows of identical adders. These adders can be as simple as two rows of ripple-carry adders or CLA adders depending on the design requirements. Fig. 1.45 shows the circuit topology of a four-bit carry-select adder composed of two rows of ripple carry adders.

In this figure, the full adder at the least significant bit position operates normally and generates a value for COUT[0]. As this value is generated the two one-bit full adders, one with $\operatorname{CINA}[1]=0$ and the other with $\operatorname{CINB}[1]=1$, simultaneously generate SUMA[1] and SUMB[1]. If COUT[0] becomes equal to one, SUMB[1] gets selected and becomes SUM[1]; otherwise, SUMA[1] becomes the SUM[1] output. Whichever value ends up being SUM[1], it is produced after a 2-1 MUX propagation delay.

However, we cannot say the same in generating SUM[2] and SUM[3] outputs in this figure. After producing SUM[1], carry ripples through both adders normally to generate SUM[2] and SUM[3]; hence, the speed advantage of having two rows of adders becomes
negligible. Therefore, we must be careful when employing a carry-select scheme before designing an adder, as this method practically doubles the chip area.


Fig. 1.45 A four-bit carry-select adder
Even though carry-select topology is ineffective in speeding up this particular four-bit adder, it may be advantageous if employed to an adder with greater number of bits in conjunction with another adder topology such as the CLA.

Example 1.14: Design a 32 -bit carry-look-ahead adder. Compute the worst-case propagation delay in the circuit.

We need to be careful in dealing with the CLA hook-ups when generating higher order terms because the complexity of these logic blocks can "grow" exponentially in size while they may only provide marginal speed gain when compared to ripple-carry scheme.

Therefore, the first step of the design process is to separate the adder into eight-bit segments with full CLA hook-ups. The proposed topology is shown in Fig. 1.46.


Fig. 1.46 A 32-bit carry-look-ahead topology
Each eight-bit CLA segment contains six CLA hook-ups from CLA0 to CLA5 as shown in Fig. 1.47.


Fig. 1.47 An eight-bit segment of the carry-look ahead adder in Fig. 1.46
CIN and SUM expressions from bit 0 through bit 7 are given below.
$\mathrm{SUM}[0]=\mathrm{A}[0] \oplus \mathrm{B}[0] \oplus \mathrm{CIN}[0]$
$\mathrm{SUM}[1]=\mathrm{A}[1] \oplus \mathrm{B}[1] \oplus \mathrm{CIN}[1]$
where,

$$
\mathrm{CIN}[1]=\mathrm{G}[0]+\mathrm{P}[0] . \operatorname{CIN}[0]
$$

$$
\mathrm{SUM}[2]=\mathrm{A}[2] \oplus \mathrm{B}[2] \oplus \mathrm{CIN}[2]
$$

where,

$$
\begin{aligned}
& \mathrm{CIN}[2]=\mathrm{G}[1]+\mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0] \\
& \mathrm{SUM}[3]=\mathrm{A}[3] \oplus \mathrm{B}[3] \oplus \mathrm{CIN}[3]
\end{aligned}
$$

where,

$$
\begin{aligned}
& \mathrm{CIN}[3]=\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0] \\
& \mathrm{SUM}[4]=\mathrm{A}[4] \oplus \mathrm{B}[4] \oplus \mathrm{CIN}[4]
\end{aligned}
$$

where,

$$
\begin{aligned}
\mathrm{CIN}[4]= & \mathrm{G}[3]+\mathrm{P}[3] \cdot(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0] \\
& +\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])
\end{aligned}
$$

$$
\mathrm{SUM}[5]=\mathrm{A}[5] \oplus \mathrm{B}[5] \oplus \mathrm{CIN}[5]
$$

where,

$$
\begin{aligned}
\mathrm{CIN}[5]= & \mathrm{G}[4]+\mathrm{P}[4] \cdot\{\mathrm{G}[3]+\mathrm{P}[3] \cdot(\mathrm{G}[2] \\
& +\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] . \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])\} \\
= & \mathrm{G}[4]+\mathrm{P}[4] \cdot \mathrm{G}[3]+\mathrm{P}[4] \cdot \mathrm{P}[3] \cdot(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1] \\
& +\mathrm{P}[2] . \mathrm{P}[1] . \mathrm{G}[0]+\mathrm{P}[2] . \mathrm{P}[1] . \mathrm{P}[0] . \mathrm{CIN}[0])
\end{aligned}
$$

$$
\mathrm{SUM}[6]=\mathrm{A}[6] \oplus \mathrm{B}[6] \oplus \mathrm{CIN}[6]
$$

where,

$$
\begin{aligned}
\mathrm{CIN}[6]= & \mathrm{G}[5]+\mathrm{P}[5] \cdot\{\mathrm{G}[4]+\mathrm{P}[4] \cdot \mathrm{G}[3]+\mathrm{P}[4] \cdot \mathrm{P}[3] \cdot(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1] \\
& +\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])\} \\
= & \mathrm{G}[5]+\mathrm{P}[5] \cdot \mathrm{G}[4]+\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{G}[3]+\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{P}[3] \cdot(\mathrm{G}[2] \\
& +\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])
\end{aligned}
$$

$$
\operatorname{SUM}[7]=\mathrm{A}[7] \oplus \mathrm{B}[7] \oplus \operatorname{CIN}[7]
$$

where,

$$
\begin{aligned}
\mathrm{CIN}[7]= & \mathrm{G}[6]+\mathrm{P}[6] \cdot\{\mathrm{G}[5]+\mathrm{P}[5] \cdot \mathrm{G}[4]+\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{G}[3] \\
& +\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{P}[3] \cdot(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0] \\
& +\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])\}
\end{aligned}
$$

And finally,

$$
\begin{aligned}
\operatorname{COUT}[7]=\mathrm{CIN}[8]= & \mathrm{G}[7]+\mathrm{P}[7] \cdot\{\mathrm{G}[6]+\mathrm{P}[6] \cdot\{\mathrm{G}[5]+\mathrm{P}[5] \cdot \mathrm{G}[4] \\
& +\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{G}[3]+\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{P}[3] \cdot(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1] \\
& +\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])\}\} \\
= & \mathrm{G}[7]+\mathrm{P}[7] \cdot \mathrm{G}[6]+\mathrm{P}[7] \cdot \mathrm{P}[6] \cdot\{\mathrm{G}[5]+\mathrm{P}[5] \cdot \mathrm{G}[4] \\
& +\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{G}[3]+\mathrm{P}[5] \cdot \mathrm{P}[4] \cdot \mathrm{P}[3] \cdot(\mathrm{G}[2]+\mathrm{P}[2] \cdot \mathrm{G}[1] \\
& +\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{G}[0]+\mathrm{P}[2] \cdot \mathrm{P}[1] \cdot \mathrm{P}[0] \cdot \mathrm{CIN}[0])\}
\end{aligned}
$$

In these derivations, particular attention was paid to limit the number of inputs to four in all AND and OR gates since larger gate inputs are counterproductive in reducing the overall propagation delay.

From these functional expressions, maximum propagation delays for SUM[7] and COUT[7] are estimated using the longest logic strings in Fig. 1.48.



Fig. 1.48 Propagation delay estimation of the eight-bit carry-look-ahead adder in Fig. 1.47
For SUM[7], the minterm, $\mathrm{P}[2] . \mathrm{P}[1]$. $\mathrm{P}[0]$. CIN[0], generates the first four-input AND gate. This is followed by a four-input OR gate whose minterms are G[2], P[2] . G[1], $\mathrm{P}[2]$. $\mathrm{P}[1]$. G[0], and P[2] . $\mathrm{P}[1]$. $\mathrm{P}[0]$. CIN[0]. The four-input OR gate is then cascaded by a four-input AND, a four-input OR, a two-input AND, a two-input OR and a two-input XOR-gates in successive order. The entire string creates a propagation delay of $\mathrm{T}_{\text {SUM } 7}=2$ $\left(\mathrm{T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+\mathrm{T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{XOR} 2}$ from CIN[0] to $\mathrm{SUM}[7]$.

For COUT[7], the longest propagation delay between CIN[0] and COUT[7] is $\mathrm{T}_{\text {COUT7 }}=2$ $\left(\mathrm{T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}$. The delays for the rest of the circuit in Fig. 1.46 become easy to determine since the longest propagation delays have already been evaluated.

The delay from CIN[0] to SUM[15], $\mathrm{T}_{\text {SUM15 }}$, simply becomes equal to the sum of $\mathrm{T}_{\text {COUT7 }}$ and $\mathrm{T}_{\text {SUM } 7 \text {. }}$ In other words, $\mathrm{T}_{\mathrm{SUM} 15}=4\left(\mathrm{~T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}+\mathrm{T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{OR} 2}+$ $\mathrm{T}_{\text {XOR2 } 2}$. Similarly, the delay from CIN[0] to COUT[15], $\mathrm{T}_{\text {COUT15 }}$, is equal to $\mathrm{T}_{\text {COUT15 }}=4$ $\left(\mathrm{T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+2\left(\mathrm{~T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}\right)$.

The remaining delays are evaluated in the same way and lead to the longest propagation delay in this circuit, $\mathrm{T}_{\text {SUM } 31}$, from $\operatorname{CIN}[0]$ to $\operatorname{SUM}[31]$.

Thus,

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{SUM} 31}= 3\left[2\left(\mathrm{~T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}\right]+2\left(\mathrm{~T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+\mathrm{T}_{\mathrm{AND} 2} \\
&+\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{XOR} 2} \\
& \text { or } \\
& \mathrm{T}_{\mathrm{SUM} 31}= 8\left(\mathrm{~T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+3\left(\mathrm{~T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}\right)+\mathrm{T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{OR} 2}+\mathrm{T}_{\mathrm{XOR} 2}
\end{aligned}
$$

Example 1.15: Design a 32-bit hybrid carry-select/carry-look-ahead adder. Compute the worst-case propagation delay in the circuit.

Large adders are where the carry-select scheme shines! This is a classical example in which the maximum propagation delay is reduced considerably compared to the CLA scheme examined in Example 1.14.

As mentioned earlier, a twin set of an adder configuration is required by the carry-select scheme. The adders can be ripple-carry, carry-look-ahead or the combination of the two.

In this example, the 32 -bit adder is again divided in eight-bit segments where each segment consists of a full CLA adder as shown in Fig. 1.49. The first segment, CLA-0, is a single unit which produces COUT[7] with full CLA hook-ups. The rest of the eight-bit segments are mirror images of each other, and they are either named A-segments (A1, A2 and A3) or B-segments (B1, B2 and B3).

As the CLA-0 generates a valid COUT[7], the CLA-A1 and CLA-B1 simultaneously generate COUTA[15] and COUTB[15]. When COUT[7] finally forms, it selects either COUTA[15] or COUTB[15] depending on its value. This segment produces COUT[15] and SUM[15:8].

COUT[15], on the other hand, is used to select between COUTA[23] and COUTB[23], both of which have already been formed when COUT[15] arrives at the 2-1 MUX as a control input. COUT[15] also selects between SUMA[23:16] and SUMB[23:16] to determine the correct SUM[23:16].

Similarly, COUT[23] is used as a control input to select between SUMA[31:24] and SUMB[31:24]. If there is a need for COUT[31], COUT[23] can also be used to determine the value of COUT[31].


Fig. 1.49 A 32-bit carry-look-ahead/carry-select adder
The maximum propagation delay for the 32-bit carry-select/CLA adder can be found using the logic string in Fig. 1.50. The first section of this string from CIN[0] to COUT[7] is identical to the eight-bit CLA carry delay in Fig. 1.48. In addition to the CLA delay, there are three cascaded MUX stages which correspond to the generation of COUT[15], COUT[23] and SUM[31].

Considering that a 2-1 MUX propagation delay consists of a two-input AND gate delay and a two-input OR gate delay, we obtain the maximum propagation delay for this 32-bit adder as follows:

$$
\mathrm{T}_{\mathrm{SUM} 31}=2\left(\mathrm{~T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)+\mathrm{T}_{\mathrm{AND} 3}+\mathrm{T}_{\mathrm{OR} 3}+3\left(\mathrm{~T}_{\mathrm{AND} 2}+\mathrm{T}_{\mathrm{OR} 2}\right)
$$



Fig. 1.50 Maximum delay propagation of the 32-bit adder in Fig. 1.49

Considering the maximum propagation delay in Example 1.14, this delay is shorter by at least $6\left(\mathrm{~T}_{\mathrm{AND} 4}+\mathrm{T}_{\mathrm{OR} 4}\right)$. Larger carry-select/carry-look-ahead adder schemes provide greater speed benefits at the cost of approximately doubling the adder area.

## Subtractors

Subtraction is performed by a technique called twos ( 2 s ) complement addition. Twos complement addition first requires complementing one of the adder inputs (1s complement) and then adding 1 to the least significant bit.

Example 1.16: Form -4 in four bits using 2 s complement.
-4 is formed by inverting all four bits of +4 (1s complement) and then adding one to it.
Therefore, the first step of this process is to represent +4 in four-bit binary format, which is 0100 . The second step is to determine its 1 s complement, which is 1011 . And the last step is to find its 2 s complement, which is $1011+0001=1100$.

Here, logic 0 signifies a positive sign, and logic 1 signifies a negative sign at the most significant bit position.

Example 1.17: Add +4 to -4 using 2 s complement.
The four bit binary format of +4 is 0100 . The 2 s complement of +4 is 1100 . If we add two numbers together, we obtain $0100+1100=10000$, where logic 1 at the overflow bit position is neglected due to the four-bit binary format. Therefore, the final result becomes $0000=0$ as expected.

Subtractors function according to the 2 s complement addition. We need to form the 1 s complement of the adder input to be subtracted and then add $\operatorname{CIN}[0]=1$ to the result to perform subtraction.

Figure 1.51 illustrates the topology of a 32 -bit subtractor where input B is complemented, and $\mathrm{CIN}[0]$ is tied to logic 1 to satisfy the 2 s complement addition requirements to produce A-B.


Fig. 1.51 A symbolic representation of a 32-bit subtractor

## Shifters

There are two types of shifters in logic design:
Linear shifters
Barrel shifters

## Linear Shifters

A linear shifter shifts its inputs by a number of bits to the right or to the left, and routes the result to its output.

Example 1.18: Design a four-bit linear shifter that shifts its inputs to the left by one bit and produces logic 0 at the least significant output bit when SHIFT $=1$. When SHIFT $=0$, the shifter routes each input directly to the corresponding output.

The logic diagram for this shifter is given in Fig. 1.52. In this figure, each input is connected to the port 0 terminal of the 2-1 MUX as well as the port 1 terminal of the next MUX at the higher bit position. Therefore, when SHIFT $=1$, logic $0, \operatorname{IN}[0], \operatorname{IN}[1]$, and $\operatorname{IN}[2]$ are routed through port 1 terminal of each 2-1 MUX and become OUT[0], OUT[1], OUT[2], and OUT[3], respectively. When SHIFT $=0$, each input goes through port 0 terminal of the corresponding 2-1 MUX and becomes the shifter output.


Fig. 1.52 Four-bit linear shifter

## Barrel Shifters

Barrel shifters rotate their inputs in either clockwise or counterclockwise direction by a number of bits and propagate them to their outputs.

Example 1.19: Design a four-bit barrel shifter that rotates its inputs in a clockwise direction by one bit when SHIFT $=1$. When SHIFT $=0$, the shifter routes each input directly to the corresponding output.

The logic diagram for this shifter is given in Fig. 1.53. The only difference between this circuit and the linear shifter in Fig. 1.52 is the removal of logic 0 from the least significant bit, and connecting this input to the IN[3] pin instead. Consequently, this leads to OUT[0] = $\operatorname{IN}[3]$, OUT[1] $=\operatorname{IN}[0]$, OUT[2] $=\operatorname{IN}[1]$ and OUT[3] $=\operatorname{IN}[2]$ when SHIFT $=1$, and OUT[0] $=\operatorname{IN}[0], \mathrm{OUT}[1]=\mathrm{IN}[1], \mathrm{OUT}[2]=\mathrm{IN}[2]$ and OUT[3] $=\mathrm{IN}[3]$ when $\mathrm{SHIFT}=0$.


Fig. 1.53 Four-bit barrel shifter
Example 1.20: Design a four-bit barrel shifter that rotates its inputs clockwise by one or two bits.

First, there must be three control inputs specifying "no shift", "shift 1 bit" and "shift 2 bits". This requires a two-bit control input, SHIFT[1:0], as shown in Table 1.18. All control inputs in this table are assigned arbitrarily. However, it makes sense to assign a "No shift" input to SHIFT[1:0] = 0, a "Shift 1 bit" input to SHIFT[1:0] = 1 and a "Shift 2 bits" input to SHIFT[1:0] $=2$ for the actual rotation amount.

Table 1.18 A four-bit barrel shifter truth table

| SHIFT[1] | SHIFT[0] | OPERATION | OUT[3] | OUT[2] | OUT[1] | OUT[0] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | No shift | IN[3] | $\mathrm{IN}[2]$ | $\mathrm{IN}[1]$ | IN[0] |
| 0 | 1 | Shift 1 bit | IN[2] | IN[1] | in[0] | IN[3] |
| 1 | 0 | Shift 2 bits | IN[1] | $\mathrm{IN}[0]$ | IN[3] | IN[2] |
| 1 | 1 | No shift | IN[3] | $\mathrm{IN}[2]$ | $\mathrm{IN}[1]$ | $\mathrm{IN}[0]$ |

According to this table, if there is no shift, each input bit is simply routed to its own output. If the "Shift 1 bit" input is active, then each input is routed to the neighboring output at the next significant bit position. In other words, IN[3] rotates clockwise and becomes OUT[0]. Similarly, IN[0], IN[1] and IN[2] shift one bit to the left and become OUT[1], OUT[2] and OUT[3], respectively. If the "Shift 2 bits" input becomes active, then each input is routed to the output of the neighboring bit which is two significant bits higher. This rotates all input bits twice before they are routed to the output, producing OUT[0] $=\operatorname{IN}[2]$, OUT[1] $=\operatorname{IN}[3]$, OUT[2] $=\operatorname{IN}[0]$ and OUT[3] $=\operatorname{IN}[1]$.

Therefore, using Table 1.18, we can conclude the logic diagram in Fig. 1.54.


Fig. 1.54 Logic diagram of the barrel shifter in Table 1.18
A more detailed view of Fig. 1.54 is given in Fig. 1.55.


Fig. 1.55 Logic circuit of the barrel shifter in Fig. 1.54

## Multipliers

There are two types of multipliers:
Array multiplier
Booth multiplier
An array multiplier is relatively simple to design, but it requires a large number of gates. A Booth multiplier, on the other hand, requires fewer gates but its implementation follows a rather lengthy algorithm.

## Array Multiplier

Similar to our everyday hand multiplication method, an array multiplier generates all partial products before summing each column in the partial product tree to obtain the result. This scheme is explained in Fig. 1.56 for a four-bit array multiplier.

|  |  |  |  | $\begin{array}{r} \mathrm{A}[3] \\ \times \quad \mathrm{B}[3] \\ \hline \end{array}$ | $\begin{aligned} & \mathrm{A}[2] \\ & \mathrm{B}[2] \end{aligned}$ | A[1] <br> B[1] | $\begin{aligned} & \mathrm{A}[0] \\ & \mathrm{B}[0] \end{aligned}$ | MULTIPLICAND MULTIPLIER |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | B[0].A[3] | B[0].A[2] | B[0].A[1] | B[0].A[0] | $0^{\text {th }}$ PARTIAL PRODUCT |
|  |  |  | B[1].A[3] | B[1].A[2] | B[1].A[1] | B[1].A[0] |  | $1{ }^{\text {st }}$ PARTIAL PRODUCT |
|  |  | B[2].A[3] | B[2].A[2] | B[2].A[1] | B[2].A[0] |  |  | $2{ }^{\text {nd }}$ PARTIAL PRODUCT |
| $\pm$ | B[3].A[3] | B[3].A[2] | B[3].A[1] | B[3].A[0] |  |  |  | $3{ }^{\text {rd }}$ PARTIAL PRODUCT |
| SUM[7] | SUM[6] | SUM[5] | SUM[4] | SUM[3] | SUM[2] | SUM[1] | SUM[0] | SUM OUTPUT |

Fig. $1.564 \times 4$ array multiplier algorithm
The rules of partial product generation are as follows:

1. The zeroth partial product aligns with multiplicand and multiplier bit columns.
2. Each partial product is shifted one bit to the left with respect to the previous one once it is created.
3. Each partial product is the exact replica of the multiplicand if the multiplier bit is one. Otherwise, it is deleted.

Example 1.21: Multiply 1101 and 1001 according to the rules of array multiplication.
Suppose 1101 is the multiplicand and 1001 is the multiplier. For a four-bit multiplier, four partial products must be formed. The bits that belong to each column of the partial product tree are then added successively while the resultant carry bits are propagated to more significant bit positions. This process is illustrated in Fig. 1.57.


Fig. 1.57 $4 \times 4$ array multiplier algorithm example
Example 1.22: Design the partial product tree for a four-bit array multiplier.
Following the convention in Fig. 1.56 and the rules of partial product generation for an array multiplier, we can implement the partial product tree as shown in Fig. 1.58.


Fig. 1.58 $4 \times 4$ array multiplier bit selector tree
In this figure, partial product elements of the zeroth partial product, B[0].A[3], B[0].A[2], $\mathrm{B}[0] . \mathrm{A}[1]$ and $\mathrm{B}[0] . \mathrm{A}[0]$, are replaced by $\mathrm{PP} 0[3: 0]$ for purposes of better illustration. Similarly, PP1[3:0], PP2[3:0] and PP3[3:0] are the new partial product outputs corresponding to the rows one, two and three.

Example 1.23: Design a full adder tree responsible for adding every partial product in the partial product tree for a four-bit array multiplier.

After generating the partial products, the next step in the design is to add the partial product elements column by column to generate the SUM outputs, SUM[7:0], while propagating carry bits to higher order columns. Following the naming convention in Fig. 1.58, all 16 partial
product elements are then fed to the carry-propagate adder in Fig. 1.59. The box outlined by dashed lines shows how the carry propagation takes place from one column to the next.


Fig. 1.59 An eight-bit propagate adder for the bit selector tree in Fig. 1.58

## Booth Multiplier

The Booth multiplier scheme halves the number of partial products using a lengthy algorithm given below.

Assume that the product of two binary integers, X and Y , forms $\mathrm{P}=\mathrm{X}$. Y , where X is a multiplicand and Y is a multiplier.

In binary form, Y is expressed in powers of two:

$$
\mathrm{Y}=\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}}
$$

where, the most significant bit, $\mathrm{y}_{\mathrm{n}-1}$, corresponds to the sign bit. When $\mathrm{y}_{\mathrm{n}-1}=0, \mathrm{Y}$ is considered a positive number, otherwise it is a negative number as mentioned earlier in the 2 s complement representation of integers.

In this section, we examine the Booth multiplication algorithm when Y is both a positive and a negative number.

CASE 1: $\mathrm{Y}>0$, thus $\mathrm{y}_{\mathrm{n}-1}=0$.
We can express a $\mathrm{k}^{\text {th }}$ term of Y as:

$$
\begin{aligned}
\mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}} & =\left(2 \mathrm{y}_{\mathrm{k}}-\mathrm{y}_{\mathrm{k}}\right) 2^{\mathrm{k}}=\left(2 \mathrm{y}_{\mathrm{k}}-1 / 22 \mathrm{y}_{\mathrm{k}}\right) 2^{\mathrm{k}} \\
& =\mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}+1}-2 \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}-1}
\end{aligned}
$$

Thus:

$$
\begin{aligned}
\mathrm{Y}= & \sum_{\mathrm{k}=0}^{\mathrm{n}-1} \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}}=\sum_{\mathrm{k}=0}^{\mathrm{n}-1}\left(\mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}+1}-2 \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}-1}\right) \\
= & \mathrm{y}_{\mathrm{n}-1} 2^{\mathrm{n}}-2 \mathrm{y}_{\mathrm{n}-1} 2^{\mathrm{n}-2} \\
& +\mathrm{y}_{\mathrm{n}-2} 2^{\mathrm{n}-2} \\
& +\mathrm{y}_{\mathrm{n}-3} 2^{\mathrm{n}-2}-2 \mathrm{y}_{\mathrm{n}-3} 2^{\mathrm{n}-4} \\
& +\mathrm{y}_{\mathrm{n}-4} 2^{\mathrm{n}-4}
\end{aligned}
$$

$$
\begin{aligned}
& +y_{3} 2^{4}-2 y_{3} 2^{2} \\
& +y_{2} 2^{2} \\
& +y_{1} 2^{2}-2 y_{1} 2^{0} \\
& +y_{0} 2^{0}
\end{aligned}
$$

Regrouping the terms of the same power yields:

$$
\begin{aligned}
\mathrm{Y}= & \mathrm{y}_{\mathrm{n}-1} 2^{\mathrm{n}}+2^{\mathrm{n}-2}\left(-2 \mathrm{y}_{\mathrm{n}-1}+\mathrm{y}_{\mathrm{n}-2}+\mathrm{y}_{\mathrm{n}-3}\right) \\
& +2^{\mathrm{n}-4}\left(-2 \mathrm{y}_{\mathrm{n}-3}+\mathrm{y}_{\mathrm{n}-4}+\mathrm{y}_{\mathrm{n}-5}\right) \\
& \cdot \\
& \cdot \\
& \cdot \\
& +2^{2}\left(-2 \mathrm{y}_{3}+\mathrm{y}_{2}+\mathrm{y}_{1}\right) \\
& +2^{0}\left(-2 \mathrm{y}_{1}+\mathrm{y}_{0}+\mathrm{y}_{-1}\right)
\end{aligned}
$$

But, $\mathrm{y}_{\mathrm{n}-1}=0$ and $\mathrm{y}_{-1}=0$ since $\mathrm{Y}>0$

$$
\begin{aligned}
\mathrm{Y}= & 2^{\mathrm{n}-2}\left(-2 \mathrm{y}_{\mathrm{n}-1}+\mathrm{y}_{\mathrm{n}-2}+\mathrm{y}_{\mathrm{n}-3}\right) \\
& +2^{\mathrm{n}-4}\left(-2 \mathrm{y}_{\mathrm{n}-3}+\mathrm{y}_{\mathrm{n}-4}+\mathrm{y}_{\mathrm{n}-5}\right)
\end{aligned}
$$

$$
\begin{aligned}
& +2^{2}\left(-2 \mathrm{y}_{3}+\mathrm{y}_{2}+\mathrm{y}_{1}\right) \\
& +2^{0}\left(-2 \mathrm{y}_{1}+\mathrm{y}_{0}+\mathrm{y}_{-1}\right)
\end{aligned}
$$

Now, let's define a new set of coefficients:

$$
\mathrm{z}_{\mathrm{k}}=-2 \mathrm{y}_{\mathrm{k}+1}+\mathrm{y}_{\mathrm{k}}+\mathrm{y}_{\mathrm{k}-1}
$$

Then:

$$
\mathrm{Y}=\sum_{\mathrm{k}=0}^{\mathrm{n}-2} \mathrm{z}_{\mathrm{k}} 2^{\mathrm{k}} \quad \text { where, } \mathrm{k}=0,2, \ldots,(\mathrm{n}-4),(\mathrm{n}-2)
$$

When X is multiplied by Y , one obtains:

$$
P=X . Y=\sum_{k=0}^{n-2}\left(z_{k} \cdot X\right) \cdot 2^{k}
$$

where, the number of partial products in the product term, P , is reduced by half.
Each $z_{k}=-2 y_{k+1}+y_{k}+y_{k-1}$ depends on the value of three adjacent bits, $y_{k+1}, y_{k}$ and $y_{k-1}$. This is tabulated in Table 1.19.

Table 1.19 Booth encoder truth table

| $y_{k+1}$ | $y_{k}$ | $y_{k-1}$ | $z_{k}$ |
| :---: | :---: | :---: | ---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 2 |
| 1 | 0 | 0 | -2 |
| 1 | 0 | 1 | -1 |
| 1 | 1 | 0 | -1 |
| 1 | 1 | 1 | 0 |

Therefore, each partial product, $\left(\mathrm{z}_{\mathrm{k}} . \mathrm{X}\right)$, becomes one of the five different forms:
$\left(\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}\right)=0,+\mathrm{X},-\mathrm{X},+2 \mathrm{X},-2 \mathrm{X}$
These partial products can easily be obtained by the following methods:
For $\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}=0$, all multiplicand bits are replaced by 0 .
For $\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}=+\mathrm{X}$, all multiplicand bits are multiplied by one.
For, $\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}=+2 \mathrm{X}$, all multiplicand bits are shifted left by one bit.
For $\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}=-\mathrm{X}$, the multiplicand is 2 s complemented.
For $\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}=-2 \mathrm{X}$, all multiplicand bits are shifted left by one bit to form +2 X , and then 2 s complemented to form -2 X .
Now, the time has come to investigate when Y is negative.
CASE 2: $\mathrm{Y}<0$, thus $\mathrm{y}_{\mathrm{n}-1}=-1$
The first step is to sign-extend Y by one bit. Sign extension does not change the actual value of $Y$ but increases the terms from $n$ to $(\mathrm{n}+1)$. Thus:

$$
\mathrm{Y}=-2^{\mathrm{n}}+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}}
$$

Let us consider:
$\sum_{\mathrm{k}=0}^{\mathrm{n}-1} 2^{\mathrm{k}}=1+2+2^{2}+\ldots+2^{\mathrm{n}-1}=\mathrm{f}$
Then,
$\left(\mathrm{z}_{\mathrm{k}} \cdot \mathrm{X}\right)=0,+\mathrm{X},-\mathrm{X},+2 \mathrm{X},-2 \mathrm{X}$
Thus:
$\mathrm{f}=2 \mathrm{f}-\mathrm{f}=2^{\mathrm{n}}-1$
or

$$
2^{\mathrm{n}}=1+\mathrm{f}=1+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} 2^{\mathrm{k}}
$$

Substituting $-2^{\mathrm{n}}$ into Y yields:

$$
\mathrm{Y}=-2^{\mathrm{n}}+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}}=-1-\sum_{\mathrm{k}=0}^{\mathrm{n}-1} 2^{\mathrm{k}}+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}}=-1+\sum_{\mathrm{k}=0}^{\mathrm{n}-1}\left(\mathrm{y}_{\mathrm{k}}-1\right) 2^{\mathrm{k}}
$$

or
$-\mathrm{Y}=1+\sum_{\mathrm{k}=0}^{\mathrm{n}-1}\left(1-y_{\mathrm{k}}\right) 2^{\mathrm{k}}$
However, $\left(1-y_{k}\right)=1$ when $y_{k}=0$, and $\left(1-y_{k}\right)=0$ when $y_{k}=1$. That means:
$\left(1-y_{k}\right)=\overline{y_{k}}$
Thus:
$-\mathrm{Y}=1+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \overline{\overline{y_{k}}} 2^{\mathrm{k}}$
The same mathematical manipulation applied to $\mathrm{y}_{\mathrm{k}} 2^{\mathrm{k}}$ in CASE 1 can also be applied to $\overline{\mathrm{y}_{\mathrm{k}}} 2^{\mathrm{k}}$.
$\overline{\mathrm{y}_{\mathrm{k}}} 2^{\mathrm{k}}=\left(2 \overline{\mathrm{y}_{\mathrm{k}}}-\overline{\mathrm{y}_{\mathrm{k}}}\right) 2^{\mathrm{k}}=\overline{\mathrm{y}_{\mathrm{k}}} 2 \cdot 2^{\mathrm{k}}-\overline{\mathrm{y}_{\mathrm{k}}} 2 \cdot 1 / 2 \cdot 2^{\mathrm{k}}$

## Therefore,

$$
\begin{aligned}
\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \overline{\mathrm{y}_{\mathrm{k}}} 2^{\mathrm{k}} & =\sum_{\mathrm{k}=0}^{\mathrm{n}-1}\left(\overline{\mathrm{y}_{\mathrm{k}}} 2 \cdot 2^{\mathrm{k}}-\overline{\mathrm{y}_{\mathrm{k}}} 2 \cdot 1 / 2^{2} \cdot 2^{\mathrm{k}}\right) \\
& =\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \overline{\mathrm{y}_{\mathrm{k}}} 2^{\mathrm{k}+1}-\overline{\mathrm{y}_{\mathrm{k}}} 2 \cdot 2^{\mathrm{k}-1} \\
& =\overline{\mathrm{y}_{\mathrm{n}-1}} 2^{\mathrm{n}}-\overline{\mathrm{y}_{\mathrm{n}-1}} 2 \cdot 2^{\mathrm{n}-2} \\
& +\overline{\mathrm{y}_{\mathrm{n}-2}} 2^{\mathrm{n}-2} \\
& +\overline{\mathrm{y}_{\mathrm{n}-3}} 2^{\mathrm{n}-2}-\overline{\mathrm{y}_{\mathrm{n}-3}} 2 \cdot 2^{\mathrm{n}-4} \\
& +\overline{\mathrm{y}_{\mathrm{n}-4}} 2^{\mathrm{n}-4}
\end{aligned}
$$

$$
+\overline{y_{3}} 2^{4}-\overline{y_{3}} 2.2^{2}
$$

$$
+\overline{y_{2}} 2^{2}
$$

$$
+\overline{y_{1}} 2^{2}-\overline{y_{1}} 2.2^{0}
$$

$$
+2^{0} \overline{y_{0}}
$$

$$
=\overline{y_{n-1}} 2^{n}
$$

$$
+\left(-2 \overline{\mathrm{y}_{\mathrm{n}-1}}+\overline{\mathrm{y}_{\mathrm{n}-2}}+\overline{\mathrm{y}_{\mathrm{n}-3}}\right) 2^{\mathrm{n}-2}
$$

$$
+\left(-2 \overline{\mathrm{y}_{\mathrm{n}-3}}+\overline{\mathrm{y}_{\mathrm{n}-4}}+\overline{\mathrm{y}_{\mathrm{n}-5}}\right) 2^{\mathrm{n}-4}
$$

$$
\begin{aligned}
& +\left(-2 \overline{\mathrm{y}_{3}}+\overline{\mathrm{y}_{2}}+\overline{\mathrm{y}_{1}}\right) 2^{2} \\
& +\left(-2 \overline{\mathrm{y}_{1}}+\overline{\mathrm{y}_{0}}+\overline{\mathrm{y}_{-1}}\right) 2^{0}
\end{aligned}
$$

Here, $\overline{\mathrm{y}_{-1}}=0$.
Then,

$$
\begin{aligned}
-\mathrm{Y} & =1+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \overline{\mathrm{y}_{\mathrm{k}}} 2^{\mathrm{k}} \\
& =\overline{\mathrm{y}_{\mathrm{n}-1}} 2^{\mathrm{n}} \\
& +\left(-2 \overline{\mathrm{y}_{\mathrm{n}-1}}+\overline{\mathrm{y}_{\mathrm{n}-2}}+\overline{\mathrm{y}_{\mathrm{n}-3}}\right) 2^{\mathrm{n}-2} \\
& +\left(-2 \overline{\mathrm{y}_{\mathrm{n}-3}}+\overline{\mathrm{y}_{\mathrm{n}-4}}+\overline{\mathrm{y}_{\mathrm{n}-5}}\right) 2^{\mathrm{n}-4}
\end{aligned}
$$

$$
+\left(-2 \overline{\mathrm{y}_{3}}+\overline{\mathrm{y}_{2}}+\overline{\mathrm{y}_{1}}\right) 2^{2}
$$

$$
+\left(-2 \overline{y_{1}}+\overline{y_{0}}+\overline{y_{-1}}\right) 2^{0}+1
$$

However, the sign-extended term, $\overline{\mathrm{y}_{\mathrm{n}-1}} 2^{\mathrm{n}}=-1$
Thus,

$$
\begin{aligned}
-\mathrm{Y} & =1+\sum_{\mathrm{k}=0}^{\mathrm{n}-1} \overline{\mathrm{y}_{\mathrm{k}}} 2^{\mathrm{k}} \\
& =\left(-2 \overline{\mathrm{y}_{\mathrm{n}-1}}+\overline{\mathrm{y}_{\mathrm{n}-2}}+\overline{\mathrm{y}_{\mathrm{n}-3}}\right) 2^{\mathrm{n}-2} \\
& +\left(-2 \overline{\mathrm{y}_{\mathrm{n}-3}}+\overline{\mathrm{y}_{\mathrm{n}-4}}+\overline{\mathrm{y}_{\mathrm{n}-5}}\right) 2^{\mathrm{n}-4}
\end{aligned}
$$

$$
+\left(-2 \overline{y_{3}}+\overline{y_{2}}+\overline{y_{1}}\right) 2^{2}
$$

$$
+\left(-2 \overline{\mathrm{y}_{1}}+\overline{\mathrm{y}_{0}}+\overline{\mathrm{y}_{-1}}\right) 2^{0}
$$

Let
$\mathrm{w}_{\mathrm{k}}=-2 \overline{\mathrm{y}_{\mathrm{k}+1}}+\overline{\mathrm{y}_{\mathrm{k}}}+\overline{\mathrm{y}_{\mathrm{k}-1}}$
Then,
$-\mathrm{Y}=\sum_{\mathrm{k}=0}^{\mathrm{n}-2} \mathrm{w}_{\mathrm{k}} 2^{\mathrm{k}} \quad$ where, $\mathrm{k}=0,2, \ldots,(\mathrm{n}-4),(\mathrm{n}-2)$.
But,

$$
\begin{aligned}
\mathrm{wk}=-2 \overline{\mathrm{y}_{\mathrm{k}+1}}+\overline{\mathrm{y}_{\mathrm{k}}}+\overline{\mathrm{y}_{\mathrm{k}-1}} & =-2\left(1-\mathrm{y}_{\mathrm{k}+1}\right)+\left(1-\mathrm{y}_{\mathrm{k}}\right)+\left(1-\mathrm{y}_{\mathrm{k}-1}\right) \\
& =2 \mathrm{y}_{\mathrm{k}+1}-\mathrm{y}_{\mathrm{k}}-\mathrm{y}_{\mathrm{k}-1}=-\mathrm{z}_{\mathrm{k}}
\end{aligned}
$$

Then,
$-\mathrm{Y}=\sum_{\mathrm{k}=0}^{\mathrm{n}-2} \mathrm{w}_{\mathrm{k}} 2^{\mathrm{k}}=-\sum_{\mathrm{k}=0}^{\mathrm{n}-2} \mathrm{z}_{\mathrm{k}} 2^{\mathrm{k}}$
or
$\mathrm{Y}=\sum_{\mathrm{k}=0}^{\mathrm{n}-2} \mathrm{z}_{\mathrm{k}} 2^{\mathrm{k}}$ which is the same equation for $\mathrm{Y}>0$.
Therefore, for both positive and negative values of Y , we have:
$\mathrm{Y}=\sum_{\mathrm{k}=0}^{\mathrm{n}-2} \mathrm{z}_{\mathrm{k}} 2^{\mathrm{k}}$ where, $\mathrm{k}=0,2, \ldots,(\mathrm{n}-4),(\mathrm{n}-2)$.
where, $\mathrm{z}_{\mathrm{k}}=-2 \mathrm{y}_{\mathrm{k}+1}+\mathrm{y}_{\mathrm{k}}+\mathrm{y}_{\mathrm{k}-1}$
Example 1.24: Starting from the generation of its partial products, design an eight-bit
Booth multiplier.
The multiplier term, Y, for the eight-bit Booth multiplier follows the encoded expression:

$$
Y=\sum_{k=0}^{6} z_{k} 2^{k}=z_{0} 2^{0}+z_{2} 2^{2}+z_{4} 2^{4}+z_{6} 2^{6}
$$

Where, the encoded multiplier coefficients are:

$$
\begin{aligned}
& \mathrm{z}_{0}=-2 \mathrm{y}_{1}+\mathrm{y}_{0}+\mathrm{y}_{-1}=-2 \mathrm{y}_{1}+\mathrm{y}_{0} \\
& \mathrm{z}_{2}=-2 \mathrm{y}_{3}+\mathrm{y}_{2}+\mathrm{y}_{1} \\
& \mathrm{z}_{4}=-2 \mathrm{y}_{5}+\mathrm{y}_{4}+\mathrm{y}_{3} \\
& \mathrm{z}_{6}=-2 \mathrm{y}_{7}+\mathrm{y}_{6}+\mathrm{y}_{5}
\end{aligned}
$$

Thus, $\mathrm{P}=\mathrm{X} . \mathrm{Y}$ yields:

$$
P=2^{0}\left(X . z_{0}\right)+2^{2}\left(X . z_{2}\right)+2^{4}\left(X . z_{4}\right)+2^{6}\left(X . z_{6}\right)
$$

This reduces the number of partial products from eight to four as shown in Fig. 1.60. In this figure, $u 0, u 1, u 2$, and $u 3$ are added to the least significant bit position of each partial product to handle cases where the partial product becomes $-\mathrm{X},-2 \mathrm{X}$.


Fig. 1.60 Partial product tree of an eight-bit Booth multiplier
For cases $+\mathrm{X},+2 \mathrm{X}$ and 0 , all u-terms become equal to zero. All partial products are sign-extended and nine bits in length to be able to handle $\pm 2 \mathrm{X}$.

The calculation of the final product can be further simplified if the sign extension terms are eliminated. Let us add all the sign extension terms and form the term, SE, as shown below:

$$
\begin{aligned}
\mathrm{SE}= & \mathrm{a}_{8} \cdot\left(2^{15}+\ldots+2^{8}\right)+\mathrm{b}_{8} \cdot\left(2^{15}+\ldots 2^{10}\right)+\mathrm{c}_{8} \cdot\left(2^{15}+\ldots 2^{12}\right) \\
& +\mathrm{d}_{8} \cdot\left(2^{15}+2^{14}\right)
\end{aligned}
$$

But,

$$
\begin{aligned}
& 2^{15}+\ldots+2^{8}=2^{8} \cdot\left(2^{7}+\ldots+1\right)=2^{8} \cdot\left(2^{8}-1\right)=2^{16}-2^{8} \\
& 2^{15}+\ldots+2^{10}=2^{10} \cdot\left(2^{5}+\ldots+1\right)=2^{10} \cdot\left(2^{6}-1\right)=2^{16}-2^{10} \\
& 2^{15}+\ldots+2^{12}=2^{12} \cdot\left(2^{3}+\ldots+1\right)=2^{12} \cdot\left(2^{4}-1\right)=2^{16}-2^{12} \\
& 2^{15}+2^{14}=2^{14} \cdot(2+1)=2^{14} \cdot\left(2^{2}-1\right)=2^{16}-2^{14}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{SE} & =\mathrm{a}_{8} \cdot 2^{16}-\mathrm{a}_{8} .2^{8}+\mathrm{b}_{8} \cdot 2^{16}-\mathrm{b}_{8} \cdot 2^{10}+\mathrm{c}_{8} \cdot 2^{16}-\mathrm{c}_{8} \cdot 2^{12}+\mathrm{d}_{8} \cdot 2^{16}-\mathrm{d}_{8} \cdot 2^{14} \\
& =2^{16} .\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}\right)-\mathrm{a}_{8} .2^{8}-\mathrm{b}_{8} \cdot 2^{10}-\mathrm{c}_{8} \cdot 2^{12}-\mathrm{d}_{8} \cdot 2^{14}
\end{aligned}
$$

But,
$-\mathrm{a}_{8}=\overline{\mathrm{a}_{8}}-1$
$-\mathrm{b}_{8}=\overline{\mathrm{b}_{8}}-1$
$-\mathrm{c}_{8}=\overline{\mathrm{c}_{8}}-1$
$-\mathrm{d}_{8}=\overline{\mathrm{d}_{8}}-1$
Thus,

$$
\begin{aligned}
\mathrm{SE}= & 2^{16} \cdot\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}\right)+2^{8} \cdot\left(\overline{\mathrm{a}_{8}}-1\right)+2^{10} \cdot\left(\overline{\mathrm{~b}_{8}}-1\right)+2^{12} \cdot\left(\overline{\mathrm{c}_{8}}-1\right) \\
& +2^{14} \cdot\left(\overline{\mathrm{~d}_{8}}-1\right) \\
= & 2^{16}\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}\right)+2^{8} \cdot \overline{\mathrm{a}_{8}}+2^{10} \cdot \overline{\mathrm{~b}_{8}}+2^{12} \cdot \overline{\mathrm{c}_{8}} \\
& +2^{14} \cdot \overline{\mathrm{~d}_{8}}-2^{8}-2^{10}-2^{12}-2^{14} \\
\mathrm{SE}= & 2^{16} \cdot\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}\right)+2^{8} \cdot \overline{\mathrm{a}_{8}}+2^{10} \cdot \overline{\mathrm{~b}_{8}}+2^{12} \cdot \overline{\mathrm{c}_{8}}+2^{14} \cdot \overline{\mathrm{~d}_{8}} \\
& -2^{8}-2^{10}-2^{12}-2^{14}-2^{16}+2^{16}+(1-1) \\
= & 2^{16} \cdot\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}-1\right)+2^{8} \cdot \overline{8_{8}}+2^{10} \cdot \overline{\mathrm{~b}_{8}}+2^{12} \cdot \overline{\mathrm{c}_{8}}+2^{14} \cdot \overline{\mathrm{~d}_{8}} \\
& -2^{8}-2^{10}-2^{12}-2^{14}+\left(2^{16}-1\right)+1
\end{aligned}
$$

But,

$$
\begin{aligned}
2^{16}-1 & =\left(1+2+2^{2}+\ldots+2^{7}\right)+\left(2^{8}+2^{9}+\ldots+2^{15}\right) \\
& =\left(2^{8}-1\right)+\left(2^{8}+2^{9}+\ldots+2^{15}\right)
\end{aligned}
$$

Then,

$$
\begin{aligned}
\mathrm{SE}= & 2^{16} \cdot\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}-1\right)+2^{8} \cdot \overline{\mathrm{a}_{8}} \\
& +2^{10} \cdot \overline{\mathrm{~b}_{8}}+2^{12} \cdot \overline{\mathrm{c}_{8}}+2^{14} \cdot \overline{\mathrm{~d}_{8}} \\
& -2^{8}-2^{10}-2^{12}-2^{14}+1+\left(2^{8}-1\right) \\
& +\left(2^{8}+2^{9}+2^{10}+2^{11}+2^{12}+2^{13}+2^{14}+2^{15}\right)
\end{aligned}
$$

Regrouping the terms with the same power in SE yields:

$$
\begin{aligned}
\mathrm{SE}= & 2^{16} \cdot\left(\mathrm{a}_{8}+\mathrm{b}_{8}+\mathrm{c}_{8}+\mathrm{d}_{8}-1\right)+2^{8} \cdot\left(\overline{\mathrm{a}_{8}}+1\right)+2^{9}+2^{10} \cdot \overline{\mathrm{~b}_{8}}+2^{11} \\
& +2^{12} \cdot \overline{\mathrm{c}_{8}}+2^{13}+2^{14} \cdot \overline{\mathrm{~d}_{8}}+2^{15}
\end{aligned}
$$

But, the term, $2^{16}$, is unimportant since this is the overflow bit in the multiplier sum. Thus,

$$
\mathrm{SE}=2^{8} \cdot\left(\overline{\mathrm{a}_{8}}+1\right)+2^{9}+2^{10} \cdot \overline{\mathrm{~b}_{8}}+2^{11}+2^{12} \cdot \overline{\mathrm{c}_{8}}+2^{13}+2^{14} \cdot \overline{\mathrm{c}_{8}}+2^{15}
$$

Then the partial product tree in Fig. 1.60 simplifies and becomes Fig. 1.61.

|  | sign extension |  |  |  |  |  |  | $\begin{aligned} & \text { MSB } \\ & \stackrel{\square}{1} \end{aligned}$ | Mid bits |  |  |  |  |  |  | LSB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{z}_{0} \mathrm{X} \longrightarrow$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\overline{\mathrm{a}_{8}}$ | $\mathrm{a}_{7}$ | $\mathrm{a}_{6}$ | $\mathrm{a}_{5}$ | $\mathrm{a}_{4}$ | $\mathrm{a}_{3}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{0}$ |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathrm{u}_{0}$ |
| $\mathrm{z}_{2} \mathrm{X} \longrightarrow$ | 0 | 0 | 0 | 0 | 0 | $\overline{\mathrm{b}_{8}}$ | $\mathrm{b}_{7}$ | $\mathrm{b}_{6}$ | $\mathrm{b}_{5}$ | $\mathrm{b}_{4}$ | $\mathrm{b}_{3}$ | $\mathrm{b}_{2}$ | $\mathrm{b}_{1}$ | $\mathrm{b}_{0}$ | 0 | 0 |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathrm{u}_{1}$ | 0 | 0 |
| $\mathrm{z}_{4} \mathrm{X} \longrightarrow$ | 0 | 0 | 0 | $\overline{\mathrm{C}_{8}}$ | $\mathrm{C}_{7}$ | C6 | $\mathrm{C}_{5}$ | $\mathrm{C}_{4}$ | $\mathrm{C}_{3}$ | $\mathrm{C}_{2}$ | C 1 | $\mathrm{C}_{0}$ | 0 | 0 | 0 | 0 |
|  | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathrm{U}_{2}$ | 0 | 0 | 0 | 0 |
| $\mathrm{z}_{6} \mathrm{X} \longrightarrow$ | 0 | $\mathrm{d}_{8}$ | $\mathrm{d}_{7}$ | $\mathrm{d}_{6}$ | $\mathrm{d}_{5}$ | $\mathrm{d}_{4}$ | $\mathrm{d}_{3}$ | $\mathrm{d}_{2}$ | $\mathrm{d}_{1}$ | $\mathrm{d}_{0}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 十 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $\mathrm{u}_{3}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| SUM $\rightarrow$ | $\mathrm{S}_{15}$ | $\mathrm{S}_{14}$ | $\mathrm{s}_{13}$ | $\mathrm{S}_{12}$ | $\mathrm{S}_{11}$ | $\mathrm{S}_{10}$ | $\mathrm{S}_{9}$ | $\mathrm{S}_{8}$ | $\mathrm{S}_{7}$ | $\mathrm{S}_{6}$ | $\mathrm{S}_{5}$ | S | S | S |  | $\mathrm{S}_{0}$ |

Fig. 1.61 Partial product tree of an eight-bit Booth multiplier after minimization

In Fig. 1.60, $a_{i}=z_{0} \cdot x_{i}, b_{i}=z_{2} \cdot x_{i}, c_{i}=z_{4} \cdot x_{i}$, and $d_{i}=z_{6} \cdot x_{i}$ where $i=0,1,2, \ldots 7$, and $x_{i}$ represents each term in the multiplicand, X .

The complemented $\mathrm{a}_{8}, \mathrm{~b}_{8}, \mathrm{c}_{8}$ and $\mathrm{d}_{8}$ in Fig. 1.61 are the "reserved bits" in case of a one-bit left shift of the partial product.

Now, the time has come to implement the components of the eight-bit Booth multiplier: the encoder, the partial product tree and the full-adder tree.

The Booth encoder is a logic block that forms each partial product. Earlier, we obtained the Booth coefficient, $\mathrm{z}_{\mathrm{k}}=-2 \mathrm{y}_{\mathrm{k}+1}+\mathrm{y}_{\mathrm{k}}+\mathrm{y}_{\mathrm{k}-1}$, to aid the generation of each partial product in Fig. 1.61. In this expression, the multiplier bits, $y_{k+1}, y_{k}$ and $y_{k-1}$, from neighboring terms can be used as inputs to $\mathrm{z}_{\mathrm{k}}$ to obtain the encoder outputs. Table 1.19 is slightly modified to form the truth table for the Booth encoder as shown in Table 1.20.

Table 1.20 Modified Booth encoder truth table
Encoder inputs

| $\mathrm{y}_{\mathrm{k}+1}$ | $\mathrm{y}_{\mathrm{k}}$ | $\mathrm{y}_{\mathrm{k}-1}$ | $\mathrm{z}_{\mathrm{k}}$ | Encoder outputs |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | $\mathrm{ZERO}_{\mathrm{k}}=1$ |
| 0 | 0 | 1 | 1 | $\mathrm{P} 1_{\mathrm{k}}=1$ |
| 0 | 1 | 0 | 1 | $\mathrm{P} 1_{\mathrm{k}}=1$ |
| 0 | 1 | 1 | 2 | $\mathrm{P} 2_{\mathrm{k}}=1$ |
| 1 | 0 | 0 | -2 | $\mathrm{M} 2_{\mathrm{k}}=1$ |
| 1 | 0 | 1 | -1 | $\mathrm{M} 1_{\mathrm{k}}=1$ |
| 1 | 1 | 0 | -1 | $\mathrm{M} 1_{\mathrm{k}}=1$ |
| 1 | 1 | 1 | 0 | ZERO |
| k | $=1$ |  |  |  |

Following Table 1.20, the Booth encoder is implemented in Fig. 1.62. In this figure, $\mathrm{ZERO}_{\mathrm{k}}, \mathrm{P}_{\mathrm{k}}, \mathrm{M1}_{\mathrm{k}}, \mathrm{P} 2_{\mathrm{k}}$ and $\mathrm{M} 2_{\mathrm{k}}$ correspond to the Booth coefficients, $0,+1,-1,+2$ and -2 , to be multiplied with the multiplicand, respectively.

Each partial product in Fig. 1.61 contains a u-term, namely $u 0, u 1, u 2$ or u3, in case of a 2 s complement conversion of the partial product. Therefore, for $\mathrm{k}=0,1,2$ or $3 \mathrm{u}_{\mathrm{k}}$ becomes equal to one if $\mathrm{M} 1_{\mathrm{k}}$ or $\mathrm{M} 2_{\mathrm{k}}=1$ (if the multiplicand is multiplied by -1 ), else it is equal to zero (if the multiplicand is multiplied by zero or +1 ). The $u_{\mathrm{k}}$-terms are implemented as shown in Fig. 1.63.


Fig. 1.62 Booth encoder logic circuit


Fig. 1.63 Implementation of $u_{k}$

The LSBs, namely the terms, $a_{0}, b_{0}, c_{0}$ and $d_{0}$, in Fig. 1.61 become equal to the values listed in Table 1.21.

Table 1.21 Truth table of the LSB for Fig. 1.61

| Encoder output | LSB |
| :---: | :---: |
| $Z E R O_{k}=1$ | $a_{0}=b_{0}=c_{0}=d_{0}=0$ |
| $P 1_{k}=1$ | $a_{0}=b_{0}=c_{0}=d_{0}=x_{0}$ |
| $M 1_{k}=1$ | $a_{0}=b_{0}=c_{0}=d_{0}=\bar{x}_{0}$ |
| $P 2_{k}=1$ | $a_{0}=b_{0}=c_{0}=d_{0}=0$ |
| $M 2_{k}=1$ | $a_{0}=b_{0}=c_{0}=d_{0}=1$ |

According to this table, when the multiplicand is multiplied by zero, the LSB of the partial product becomes equal to zero. When the multiplicand is multiplied by +1 or -1 , the LSB is simply equal to the LSB of the multiplicand, $\mathrm{x}_{0}$, or its complement, respectively. Finally, when the multiplicand is multiplied by +2 or -2 , the partial product is shifted one bit to the left; the LSB becomes either zero or one depending on the sign. Therefore, the LSB of the partial product is generated using a 5-1 MUX as shown in Fig. 1.64.


Fig. 1.64 Logic diagram of the LSB for Fig. 1.61
The mid bits, $a_{1}$ to $\mathrm{a}_{7}, \mathrm{~b}_{1}$ to $\mathrm{b}_{7}, \mathrm{c}_{1}$ to $\mathrm{c}_{7}$, and $\mathrm{d}_{1}$ to $\mathrm{d}_{7}$, in Fig. 1.61 are generated according to Table 1.22. In this table, all mid bits of a partial product are equal to zero if the multiplicand is multiplied by zero. Mid bits become equal to the multiplicand bits or their complements depending on the multiplicand is multiplied by +1 or -1 , respectively.

Table 1.22 Truth table of the mid bits for Fig. 1.61

| Encoder output | Mid bits |  |
| :--- | :--- | :--- |
| $Z E R O_{k}=1$ | $a_{i}=b_{i}=c_{i}=d_{i}=0$ | where $i=1,2, \ldots 7$ |
| $P 1_{k}=1$ | $a_{i}=b_{i}=c_{i}=d_{i}=x_{i}$ | where $i=1,2, \ldots 7$ |
| $M 1_{k}=1$ | $a_{i}=b_{i}=c_{i}=d_{i}=\bar{x}_{i}$ | where $i=1,2, \ldots 7$ |
| $P 2_{k}=1$ | $a_{i}=b_{i}=c_{i}=d_{i}=x_{i-1}$ | where $i=1,2, \ldots 7$ |
| $M 2_{k}=1$ | $a_{i}=b_{i}=c_{i}=d_{i}=\overline{x_{i-1}}$ | where $i=1,2, \ldots 7$ |

When the multiplicand is multiplied by +2 or -2 , each term in the partial product is shifted one bit to the left. Therefore, each partial product bit becomes equal to the lesser significant multiplicand bit or its complement. Each mid bit in Table 1.22 can be implemented by a 5-1 MUX shown in Fig. 1.65.


Fig. 1.65 Logic diagram of the mid bits for Fig. 1.61
The MSBs, namely the terms, $\overline{\mathrm{a}}_{8}, \overline{\mathrm{~b}}_{8}, \overline{\mathrm{c}}_{8}$ or $\overline{\mathrm{d}}_{8}$, in Fig. 1.61 form according to Table 1.23.
Table 1.23 Truth table of the MSB for Fig. 1.61

| Encoder output | MSB |
| :--- | :---: |
| $\mathrm{ZERO}_{\mathrm{k}}=1$ | $\overline{\mathrm{a}}_{8}=\overline{\mathrm{b}}_{8}=\overline{\mathrm{c}}_{8}=\overline{\mathrm{d}}_{8}=0$ |
| $\mathrm{P}_{\mathrm{k}}=1$ | $\overline{\mathrm{a}}_{8}=\overline{\mathrm{b}}_{8}=\overline{\mathrm{c}}_{8}=\overline{\mathrm{d}}_{8}=\mathrm{x}_{7}$ |
| $\mathrm{M} 1_{\mathrm{k}}=1$ | $\overline{\mathrm{a}}_{8}=\overline{\mathrm{b}}_{8}=\overline{\mathrm{c}}_{8}=\overline{\mathrm{d}}_{8}=\overline{\mathrm{x}}_{7}$ |
| $\mathrm{P} 2_{\mathrm{k}}=1$ | $\overline{\mathrm{a}}_{8}=\overline{\mathrm{b}}_{8}=\overline{\mathrm{c}}_{8}=\overline{\mathrm{d}}_{8}=\mathrm{x}_{7}$ |
| $\mathrm{M} 2_{\mathrm{k}}=1$ | $\overline{\mathrm{a}}_{8}=\overline{\mathrm{b}}_{8}=\overline{\mathrm{c}}_{8}=\overline{\mathrm{d}}_{8}=\overline{\mathrm{x}}_{7}$ |

In this table, when the multiplicand is multiplied by zero, the MSB of the partial product becomes equal to zero. When the multiplicand is multiplied by +1 or -1 , the MSB is simply
equal to the sign-extended value of the most significant multiplicand bit, $\mathrm{x}_{7}$, or its complement, respectively. When the multiplicand is multiplied by +2 or -2 , the partial product shifts one bit to the left. Consequently, the MSB becomes equal to the most significant multiplicand bit or its complement, respectively. The MSB of a partial product can therefore be implemented by a 5-1 MUX as shown in Fig. 1.66.


Fig. 1.66 Logic diagram of the MSB for Fig. 1.61
When all of the components of the eight-bit Booth multiplier are integrated, we finally obtain the circuit topology in Fig. 1.67.

Example 1.25: Multiply $\mathrm{A}=10110101$ (multiplicand) by $\mathrm{B}=01110010$ (multiplier) using the Booth algorithm. The multiplier bits are as follows:

$$
\mathrm{y}_{0}=0, \mathrm{y}_{1}=1, \mathrm{y}_{2}=0, \mathrm{y}_{3}=0, \mathrm{y}_{4}=1, \mathrm{y}_{5}=1, \mathrm{y}_{6}=1, \mathrm{y}_{7}=0
$$

Therefore, the Booth coefficients become:

$$
\begin{aligned}
& \mathrm{z}_{0}=-2 \mathrm{y}_{1}+\mathrm{y}_{0}+\mathrm{y}_{-1}=-2 .(1)+0+0=-2 \\
& \mathrm{z}_{2}=-2 \mathrm{y}_{3}+\mathrm{y}_{2}+\mathrm{y}_{1}=-2 .(0)+0+1=+1 \\
& \mathrm{z}_{4}=-2 \mathrm{y}_{5}+\mathrm{y}_{4}+\mathrm{y}_{3}=-2 .(1)+1+0=-1 \\
& \mathrm{z}_{6}=-2 \mathrm{y}_{7}+\mathrm{y}_{6}+\mathrm{y}_{5}=-2 .(0)+1+1=+2
\end{aligned}
$$

According to these coefficients, the partial product tree forms as shown in Fig. 1.68.
In this figure, the zeroth partial product (top row) is generated by multiplying the multiplicand by -2 . This requires taking the 2 s complement of the multiplicand, and then shifting its contents one bit to the left. In other words, if the multiplicand is equal to 10110101, then its 2s complement becomes 01001011 . Shifting this value to the left by one bit reveals a nine-bit value of 010010110 . Since the sign bit of 01001011 is zero before any shifting takes place, sign extending 010010110 after the shift in a 16 -bit field yields 0000000010010110 .

Fig. 1.67 8x8 Booth multiplier

$\begin{array}{llllllllllllllll}1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0\end{array}$

Fig. 1.68 A numerical example of an $8 x 8$ Booth multiplier

The first partial product (second row from the top) is produced by multiplying the multiplicand by +1 . This simply replicates the multiplicand bits, 10110101, in the partial product. Since the sign bit of 10110101 is one, sign extending this value in a 14 -bit field yields 11111110110101 as the partial product. The second partial product (third row from the top) is formed by multiplying the multiplicand by -1 . This simply requires taking the 2 s complement of the multiplicand, which is 01001011 . Sign extending this value in a 12 -bit field yields 000001001011 as the partial product. The third partial product (last row) is obtained by multiplying the multiplicand by +2 , which shifts the multiplicand one bit to the left. Since the multiplicand is 10110101 , a nine-bit value of 101101010 is obtained after the shift. Sign extending this value within ten bits yields 1101101010 .

## Review Questions

1. Implement the following gates:
(a) Implement a two-input XOR gate using two-input NAND gates and inverters.
(b) Implement a two-input AND gate using two-input XNOR gates and inverters.
2. Simplify the equation below:

$$
\text { out }=(\overline{\mathrm{A}+\mathrm{B}}) \cdot(\overline{\overline{\mathrm{A}}+\overline{\mathrm{B}}})
$$

3. Simplify the equation below:

$$
\text { out }=(\overline{\mathrm{A}}+\mathrm{C}) \cdot(\overline{\mathrm{A}}+\overline{\mathrm{C}}) \cdot(\overline{\mathrm{A}+\mathrm{B}+\overline{\mathrm{C}}})
$$

4. Obtain the SOP and POS expressions for the following function:

$$
\text { out }=(\mathrm{A} \cdot \mathrm{~B}+\mathrm{C}) \cdot(\mathrm{B}+\mathrm{A} \cdot \mathrm{C})
$$

5. Implement the following function using NAND gates and inverters:

$$
\text { out }=\mathrm{A} \cdot \overline{\mathrm{C}}+\mathrm{B} \cdot \mathrm{C}+\overline{\mathrm{A}} \cdot \overline{\mathrm{~B}} \cdot \mathrm{D}
$$

6. Implement the following function using NOR gates and inverters:

$$
\text { out }=(\mathrm{A} \oplus \mathrm{~B}) \cdot(\overline{\mathrm{C} \oplus \mathrm{D}})
$$

7. Implement the following 2-1 multiplexer using AND and OR gates. Note that the function of this multiplexer must produce the following: If $\mathrm{En}=1$ then out $=\mathrm{A}$ else $($ when $\mathrm{En}=0)$ out $=\mathrm{B}$.

8. Implement the following 3-1 multiplexer using AND and OR gates.

Note that the function of this multiplexer must produce the following:
If $\mathrm{En}=2$ then out $=\mathrm{A}$; if $\mathrm{En}=1$ then out $=\mathrm{B}$ else $($ when $\mathrm{En}=0$ or $\mathrm{En}=3$ ) out $=\mathrm{C}$.

9. Implement a two-bit ripple-carry adder with inputs $\mathrm{A}[1: 0]$ and $\mathrm{B}[1: 0]$ and an output C [1:0] using one-bit half- and one-bit full-adders. Preserve the overflow bit at the output as C[2].
10. Implement a two-bit ripple-carry subtractor with inputs $A[1: 0]$ and $B[1: 0]$ and an output C [1:0] using one-bit half- and one-bit full-adders. Preserve the overflow bit at the output as $\mathrm{C}[2]$.
11. Implement a two-bit multiplier with inputs $\mathrm{A}[1: 0]$ and $\mathrm{B}[1: 0]$ and an output $\mathrm{C}[3: 0]$ using one-bit half- and one-bit full-adders.
12. Construct a four-bit comparator with inputs $A[3: 0]$ and $B[3: 0]$ using a subtractor. The comparator circuit should identify the following cases with active-high outputs:
$\mathrm{A}[3: 0]=\mathrm{B}[3: 0]$
$\mathrm{A}[3: 0]>\mathrm{B}[3: 0]$
$\mathrm{A}[3: 0]<\mathrm{B}[3: 0]$
13. Implement a two-bit decoder that produces four outputs.

When enabled the decoder generates the following outputs:
If in $[1: 0]=0$ then $\quad$ out $[3: 0]=1$
If in $[1: 0]=1$ then out $[3: 0]=2$
If in $[1: 0]=2$ then $\quad$ out $[3: 0]=4$
If in $[1: 0]=3$ then $\quad$ out $[3: 0]=8$

When disabled the out[3:0] always equals to zero regardless of the input value.
14. Design a 64-bit adder in ripple-carry form and compare it against carry-look-ahead, carry-select, and carry-look-ahead/carry-select hybrids in terms of speed and the number of gates which define the circuit area. Divide the 64-bit carry-look-ahead/carry-select hybrid into four-bit, eight-bit, 16-bit and 32-bit carry-look-ahead segments. Indicate which carry-look-ahead/carry-select hybrid produces the optimum design.
15. Implement a $4 \times 4$ Booth multiplier. Design the Booth encoders for the partial product tree and draw the entire schematic of the multiplier. Compare this implementation with the $4 \times 4$ array multiplier explained in this chapter. List the advantages of both designs in terms of speed and circuit area.

## Projects

1. Implement the 4-1 multiplexer in Fig. 1.30 and verify its functionality using Verilog.
2. Implement the encoder circuit in Table 1.14 and verify its functionality using Verilog.
3. Implement the decoder circuit in Table 1.15 and verify its functionality using Verilog.
4. Implement the four-bit Ripple-Carry Adder in Fig. 1.42 and verify its functionality using Verilog.
5. Implement the four-bit Carry-Look-Ahead Adder in Fig. 1.44 and verify its functionality using Verilog.
6. Implement the four-bit Carry-Select Adder in Fig. 1.45 and verify its functionality using Verilog.
7. Implement the four-bit Carry-Select/Carry-Look-Ahead Adder in Fig. 1.49 and verify its functionality using Verilog.
8. Implement the four-bit barrel shifter in Fig. 1.53 and verify its functionality using Verilog.
9. Implement the four-bit array multiplier in Fig. 1.56 and verify its functionality using Verilog.
10. Implement the eight-bit Booth multiplier in Fig. 1.67 and verify its functionality using Verilog.

## Review of Sequential Logic Circuits

The definition of clock and system timing are integral parts of a sequential digital circuit. Data in a digital system moves from one storage device to the next by the virtue of a system clock. During its travel, data is routed in and out of different combinational logic blocks, and becomes modified to satisfy a specific functionality.

This chapter is dedicated to reviewing the basics of memory devices that store data, and sequential circuits that use memory devices to operate. The chapter begins with the introduction of two basic memory elements, the latch and the flip-flop. It then explains how data travels between memory elements using timing diagrams, and how timing violations form as a result of unexpected combinational logic delays on the data path or in the clock line. Later in the chapter, the basic sequential building blocks such as registers, shift registers and counters are examined. Moore-type and Mealy-type state machines that control data movement are also studied; their advantages and disadvantages are compared against counter-decoder type controllers in various design tasks. The concept of block memory and how it is used in a digital system is introduced at the end of this chapter. The chapter concludes with a comprehensive example which demonstrates data transfer from one memory block to another, how to build a detailed data-path during the development of the design, and how to use timing diagrams to build a controller.

### 2.1 D Latch

The D Latch is the most basic memory element in logic design. It has a data input, D, a clock input, clock, and a data output, Q , as shown in the top portion of Fig. 2.1. It contains a tri-state inverter at its input stage followed by two back-to-back inverters connected in a loop configuration, which serves to store data.

The clock signal connected to the enable input of the tri-state inverter can be set either to active-high or active-low. In Fig. 2.1, the changes at the input transmit though the memory element, and become the output during the low phase of the clock. In contrast, the changes at the input are blocked during the high phase of the clock, and no data transmits to the output. Once the data is stored in the back-to-back inverter loop, it becomes stable and does not


Fig. 2.1 Logic and circuit diagrams of a D latch
change until different data is introduced at the input. The buffer at the output stage of the latch is used to drive multiple logic gate inputs.

The operation of the D latch is shown in Fig. 2.2. During the low phase of the clock, the tri-state inverter is enabled. The new data transmits through the tri-state inverter, overwrites the old data in the back-to-back inverter stage, and reaches the output. When the clock switches to its high phase, the input-output data transmission stops because the tri-state buffer is disabled and blocks any new data transfer. Therefore, if certain data needs to be retained in the latch, it needs to be stored some time before the rising edge of the clock. This time interval is called the set-up time, $\mathrm{t}_{\mathrm{s}}$, and it is approximately equal to the sum of delays through the tri-state inverter and the inverter in the memory element. At the high phase of the clock, the data stored in the latch can no longer change as shown in Fig. 2.2.


Fig. 2.2 Operation of D latch

### 2.2 Timing Methodology Using D Latches

Timing in logic systems is maintained by pipeline structures. A pipeline consists of combinational logic blocks bounded by memory elements as shown in the top portion of Fig. 2.3. The main purpose of pipelines is to process several data packets within the same clock cycle and maximize the data throughput.

To illustrate the concept of pipeline, latches are used as memory elements in the pipeline structure shown in Fig. 2.3. In every latch boundary, data propagates from one combinational logic stage to the next at the high and at the low phases of the clock.

The bottom part of Fig. 2.3 shows the timing diagram of a data transfer for a set of data packets ranging from D1 to D3 at the IN terminal. The first data packet, D1 ${ }^{0}$, retains its original value during the high phase of the clock (Cycle 1 H ) at the node A . D1 ${ }^{0}$ then propagates through the T 1 stage, and settles at the node B in a modified form, $\mathrm{D} 1^{1}$, sometime before the falling edge of the clock. Similarly, D $1^{1}$ at the node C retains its value during the low phase of the clock while its processed form, $\mathrm{D}^{2}$, propagates through the T 2 stage, and arrives at the node D before the rising edge of the clock. This data is processed further in the T3 stage, and transforms into D1 ${ }^{3}$ before it becomes available at the OUT terminal at the falling edge of the clock in Cycle 2L.

Similarly, the next two data packets, $\mathrm{D} 2^{0}$ and $\mathrm{D} 3^{0}$, are also fed into the pipeline at the subsequent negative clock edges. Both of these data propagate through the combinational logic stages, $\mathrm{T} 1, \mathrm{~T} 2$ and T 3 , and become available at the OUT terminal at the falling edge of Cycle 3L and Cycle 4L, respectively.

The total execution time for all three data packets takes four clock cycles according to the timing diagram in Fig. 2.3. If we were to remove all the latch boundaries between nodes A and F, and wait until all three data packets, D1, D2 and D3, were processed through the sum of the three combinational logic stages, T1, T2 and T3, the total execution time would have been $3 \times 1.5=4.5$ clock cycles as each combinational logic stage requires half a clock cycle to process data. Therefore, pipelining can be used advantageously to process data in a shorter amount of time and increase data throughput.


Fig. 2.3 Timing methodology using $D$ latches (" $X$ " marks correspond to changing data)

### 2.3 D Flip-Flop

The D flip-flop is another important timing element in logic design to maintain timely propagation of data from one combinational logic block to the next.

Similar to a latch, the flip-flop has also a data input, D, a clock input, clock, and a data output, Q , as shown in the top portion of Fig. 2.4.

The bottom part of Fig. 2.4 shows the circuit schematic of a typical flip-flop which contains two latches in series. The first latch has an active-low clock input, and it is called the master. The second latch has an active-high clock input, and it is called the slave. The
master accepts new data during the low phase of the clock, and transfers this data to the slave during the high phase of the clock.


Fig. 2.4 Logic and circuit diagrams of a D flip-flop
Figure 2.5 shows the timing attributes of a flip-flop. The set-up time, $\mathrm{t}_{\mathrm{s}}$, is the time interval for valid data to arrive and settle in the master latch before the rising edge of the clock. Hold time, $\mathrm{t}_{\mathrm{H}}$, is the time interval after the positive edge of the clock when valid data needs to be kept steady and unchanged. The data stored in the master latch propagates through the slave latch and becomes the flip-flop output some time after the rising edge of the clock, and it is called clock-to-q delay or $\mathrm{t}_{\mathrm{CLKQ}}$.


Fig. 2.5 Timing attributes of a D flip-flop
The operation of the flip-flop in two different phases of the clock is shown in Fig. 2.6. During the low phase of the clock, new data enters the master latch, and it is stored. This data cannot propagate beyond the master latch because the tri-state inverter in the slave latch acts as an open circuit during the low phase of the clock. The flip-flop output reveals only the old data stored in the slave latch. When the clock goes high, the new data stored in the master
latch transmits through the slave and arrives at the output. One can approximate values of $\mathrm{t}_{\mathrm{s}}$ and $\mathrm{t}_{\text {CLKQ }}$ using the existing gate delays in the flip-flop.


Fig. 2.6 Operation of D flip-flop

### 2.4 Timing Methodology Using D Flip-Flops

Data propagation through a pipeline with D flip-flops is shown in Fig. 2.7. The bottom part of Fig. 2.7 shows the timing diagram of a data transfer for a set of data packets ranging from D1 to D3 at the IN terminal.

The first data packet, $\mathrm{D} 1^{0}$, at the IN terminal has to be steady and valid during the set-up and hold periods of the flip-flop, but it is free to change during the remaining part of the clock period as shown by oscillatory lines. Once the clock goes high, the valid D1 ${ }^{0}$ starts to propagate through the combinational logic block of T1 and reaches the second flip-flop boundary. The processed data, $\mathrm{D} 1^{1}$, has to arrive at the second flip-flop input, B , no later than the set-up time of the flip-flop. Otherwise, the correct data cannot be latched. D $1^{1}$ propagates through the second (T2) and third (T3) combinational logic stages, and becomes D1 ${ }^{2}$ and D1 ${ }^{3}$, respectively, before exiting at the OUT terminal as shown in the timing diagram in Fig. 2.7.

The subsequent data packets, $\mathrm{D} 2^{0}$ and $\mathrm{D} 3^{0}$, are similarly fed into the pipeline stage from the IN terminal following $\mathrm{D} 1^{0}$. They are processed and modified by the $\mathrm{T} 1, \mathrm{~T} 2$ and T 3 combinational logic stages as they propagate through the pipeline, and emerge at the OUT terminal.

The total execution time for three input data packets, $\mathrm{D} 1^{0}, \mathrm{D} 2^{0}$ and $\mathrm{D} 3^{0}$, takes six clock cycles, including the initial three clock cycle build-up period before $\mathrm{D} 1^{3}$ emerges at the OUT terminal. If we were to remove all the flip-flop boundaries between the nodes A and F, and
wait for these three data packets to be processed without any pipeline structure, the total execution time would have been $3 \times 3=9$ clock cycles, assuming each T1, T2 or T3 logic stage imposes one clock cycle delay.

Once again, the pipelining technique considerably reduces the overall processing time and increase data throughput whether the timing methodology is latch-based or flip-flop-based.

The advantage of using latches as opposed to flip-flops is to be able to borrow time from neighboring stages. For example, the propagation delay in the T1 stage in Fig. 2.3 can be extended at the expense of shortening the propagation delay in the T2 stage. This flexibility does not exist in a flip-flop based design in Fig. 2.7.


Fig. 2.7 Timing methodology using D flip-flops (" $X$ " marks correspond to changing data)

### 2.5 Timing Violations

Although pipelining scheme helps reducing the overall data processing time, we still need to watch out possible timing violations because of the unexpected delays in the data-path and the clock network.

Therefore, this section examines the set-up and hold timing violations in flip-flop controlled pipelines, and proposes possible solutions to eliminate them.

Figure 2.8 shows a section of a pipeline where a combinational logic block with a propagation delay of $\mathrm{T}_{\mathrm{COMB}}$ is sandwiched between two flip-flop boundaries. At the rising edge of the clock, the valid data that meets the set-up and hold time requirements is introduced at the IN terminal. After $\mathrm{t}_{\mathrm{CLKQ}}$ delay, the data emerges at the node A and propagates through the combinational logic block as shown in the timing diagram. However, the data arrives at the node B too late and violates the allocated set-up time of the flip-flop. This is called the set-up violation. The amount of violation is dependent on the clock period and is calculated as follows:

Set-up violation $=\mathrm{t}_{\mathrm{S}}-\left[\mathrm{T}_{\mathrm{C}}-\left(\mathrm{t}_{\mathrm{CLKQ}}+\mathrm{T}_{\mathrm{COMB}}\right)\right]$


Fig. 2.8 Setup violation
Figure 2.9 describes the hold time violation where the clock shifts by $\mathrm{T}_{\text {CLK }}$ due to an unexpected delay in the clock line. In the timing diagram, the valid data is introduced to the pipeline at the IN terminal, and it arrives at the node B after a short delay equal to ( $\mathrm{t}_{\mathrm{CLKQ}}+$ $\mathrm{T}_{\text {COMB }}$ ). The shifted clock, on the other hand, creates a substantial set-up time slack equal to $\left(\mathrm{T}_{\mathrm{C}}+\mathrm{T}_{\mathrm{CLK}}-\mathrm{t}_{\mathrm{S}}-\mathrm{t}_{\text {CLKQ }}-\mathrm{T}_{\text {COMB }}\right.$ ), but it also produces a hold time violation at the delayed clock edge. The amount of violation is dependent on the clock delay and is calculated as follows:

Hold violation $=\left(\mathrm{T}_{\mathrm{CLK}}+\mathrm{t}_{\mathrm{H}}\right)-\left(\mathrm{t}_{\mathrm{CLKQ}}+\mathrm{T}_{\mathrm{COMB}}\right)$


Fig. 2.9 Hold violation
Set-up violations can be recovered simply by increasing the clock period, $\mathrm{T}_{\mathrm{C}}$. However, there is no easy way to fix hold violations as they need to be searched at each flip-flop input. When they are found, buffer delays are added to the combinational logic block, $\mathrm{T}_{\mathrm{COMB}}$, in order to avoid the violation.

The schematic in Fig. 2.10 examines the timing ramifications of two combinational logic blocks with different propagation delays merging into a single block. The data arrives at the node C much earlier than the node D as shown in the timing diagram. The data at the nodes C and D propagate through the last combinational block and arrive at the node E. This scenario creates minimum and maximum delay paths at the node E . We need to focus on the maximum path, $(\mathrm{T} 2+\mathrm{T} 3)$, when examining the possibility of a set-up violation and the minimum path, $(\mathrm{T} 1+\mathrm{T} 3)$, when examining the possibility of a hold violation at the next flip-flop boundary.


Fig. 2.10 A timing example combining two independent data-paths
To further illustrate the timing issues involving multiple combinational logic blocks, an example is given in Fig. 2.11 where two combinational logic blocks merge into a single block. The adder is bypassed with the inclusion of a 2-1 MUX which selects either the output of the adder or the bypass path, by a selector input, SEL.

The propagation delays of the inverter, $\mathrm{T}_{\mathrm{INV}}$, and the two-input NAND gate, $\mathrm{T}_{\text {NAND2 }}$, are given as 100 ps and 200 ps , respectively. The set-up, hold and clock-to-q delays are also given as $100 \mathrm{ps}, 0 \mathrm{ps}$ and 300 ps , respectively.


Fig. 2.11 An example with multiple propagation paths
Both the one-bit full adder and the 2-1 MUX are decomposed into basic logic gates, such as inverters and two-input NAND gates, as shown in Fig. 2.12. We obtain a total of seven propagation paths all merging at the node R. However, we only need to search for the maximum and the minimum delay paths to locate possible set-up and hold violations.

The maximum delay path consists of the inverter 1, and the series combination of four two-input NAND gates numbered as 1, 3, 4 and 6 shown in the schematic. This path results in a total delay of 900 ps . The minimum delay path, on the other hand, contains two two-input NAND gates numbered as 5 and 6, and it produces a delay of 400 ps. Placing these delays in the timing diagram in Fig. 2.13 yields a set-up slack of 100 ps at the node R when a clock period of 1400 ps is used. There is no need to investigate hold violations because there is no shift in the clock edge. However, if there were a shift in the clock line beyond $\mathrm{t}_{\text {CLKQ }}=300 \mathrm{ps}$, then we would have a hold violation, and it would require an additional combinational logic delay in the data-path to proportionally shift the valid data at the node R to compensate the hold violation. This, however, would also eliminate the 100 ps set-up slack in Fig. 2.13.


Fig. 2.12 Logic circuit of Fig. 2.11 showing maximum and minimum paths


Fig. 2.13 Timing diagram of the circuit in Fig. 2.12

### 2.6 Register

While the flip-flop holds data for only one clock cycle until new data arrives at the next clock edge, the register can hold the same data perpetually until the power is turned off.

Figure 2.14 shows the circuit diagram of a one-bit register composed of a flip-flop and a 2-1 MUX. The Write Enable pin, WE, is a selector input to the 2-1 MUX and transfers new data from the IN terminal to the flip-flop input when $\mathrm{WE}=1$. If the WE input is at logic 0 , any attempt to write new data to the register is blocked. The old data stored in the flip-flop simply circulates around the feedback loop from one clock cycle to the next.

The timing diagram at the bottom of Fig. 2.14 describes the operation of the one-bit register. The data at the IN terminal is blocked until the WE input becomes logic 1 in the middle of the second clock cycle. At this point, the new data is allowed to pass through the 2-1 MUX, and it renews the contents of the register at the beginning of the third clock cycle. The WE input transitions to logic 0 before the end of the third clock cycle, and causes the register output, OUT, to stay at logic 1 during the fourth clock cycle.


Fig. 2.14 One-bit register and a sample timing diagram
A 32-bit register shown in Fig. 2.15 is composed of 32 one-bit registers. All 32 registers have a common clock and WE input. Therefore, any new 32-bit data introduced at the register input changes the contents of the register at the rising edge of the clock if the WE input is set to logic 1 .


Fig. 2.15 32-bit register

### 2.7 Shift Register

The shift register is a particular version of an ordinary register, and it specializes in shifting data to the right or to the left according to the design needs.

Figure 2.16 shows the circuit schematic of a four-bit shift register that shifts serial data at the IN terminal to the left if enabled.

The operation of this shift register is explained in the timing diagram in Fig. 2.17. In cycle $1, \mathrm{SHIFT}=0$. Therefore, the change at the IN terminal during this cycle does not affect the register outputs. However, when the SHIFT input transitions to logic 1 in the middle of cycle 2, it allows $\mathrm{IN}=1$ to pass to the least significant output bit, OUT[0], at the beginning of the third clock cycle. From the middle of cycle 2 to the middle of cycle 13, SHIFT is kept at logic 1 . Therefore, any change at the IN node directly transmits to the OUT[0] node at the positive edge of each clock cycle. The other outputs, OUT[1], OUT[2] and OUT[3], produce delayed outputs one clock cycle apart from each other because the output of a lesser significant bit is connected to the input of a greater significant bit in the shift register.

When the SHIFT input becomes logic 0 from the middle of cycle 13 to cycle 17 , the shift register becomes impervious to any change at the IN terminal, and retains the old values from the beginning of cycle 13 to cycle 18 as seen in Fig. 2.17. From the middle of cycle 17 onwards, the SHIFT input becomes logic 1 again, and the shift register distributes all new data entries at the IN terminal to its outputs.


Fig. 2.16 Four-bit shift register


Fig. 2.17 A sample timing diagram of the four-bit shift register in Fig. 2.16

### 2.8 Counter

The counter is a special form of a register which is designed to count up (or down) at each rising edge of the clock.

The circuit schematic in Fig. 2.18 shows a typical 32-bit up-counter with two control inputs, COUNT and LOAD. The COUNT $=1$ entry enables the counter to count upwards at the rising edge of each clock cycle, and the LOAD $=1$ entry loads new data to the counter from its IN [31:0] terminal. Once loaded, the counter output, OUT[31:0], increments by one at the positive edge of each clock cycle until all the output bits become logic 1 . The next increment
automatically resets the counter output to logic 0 . When LOAD $=$ COUNT $=0$, the counter neither loads new data nor is able to count upwards; it stalls and repeats its old output value.

The sample timing diagram at the bottom of Fig. 2.18 illustrates its operation. Prior to the first clock edge, the LOAD input is at logic 1 which allows an input value, $\mathrm{IN}=3$, to be stored in the counter. This results in OUT[31:0] $=3$ at the positive edge of the first clock cycle. The LOAD $=0$ and COUNT $=1$ entries before the end of the first clock cycle start the up-count process, and the contents of the output, OUT[31:0] $=3$, is subsequently incremented by one. The result, $3+1=4$, passes through the C-port of the $3-1$ MUX and arrives at the flip-flop inputs. At the positive edge of the second clock cycle, this new value overwrites the old registered value, and the OUT[31:0] node becomes equal to 4 . In the next cycle, the counter goes through the same process and increments by one again. However, in the same cycle, the COUNT input also transitions to logic 0 , and turns on the I-port of the 3-1 MUX. This input value prevents any new data from entering the up-counter, and it keeps the old data in the following clock cycles. As a result, the counter output stops incrementing and stalls at the value of OUT[31:0] $=5$.


Fig. 2.18 A 32-bit counter and a sample timing diagram

### 2.9 Moore Machine

A state machine can be formed as soon as a flip-flop output is connected to a flip-flop input. Therefore, an overall state machine topology consists of flip-flops, feedback loops from flip-flop outputs to flip-flop inputs, and combinational logic blocks connected to flip-flop outputs and embedded in feedback loops.

Figure 2.19 shows the Moore-type state machine topology consisting of a flip-flop and a feedback loop. In this configuration, the feedback loop includes a combinational logic block that accepts both the flip-flop output and external inputs. If there are multiple flip-flops, the combination of all flip-flop outputs constitutes the "present" state of the machine. The combination of all flip-flop inputs is called the "next" state because at the positive edge of the clock these inputs become the flip-flop outputs, and form the present state. Flip-flop outputs are processed further by an additional combinational logic block, forming the present state outputs.

The basic state diagram of a Moore machine, therefore, includes the present state (PS) and the next state (NS) as shown in Fig. 2.19. The machine can transition from the PS to the NS if the required present state inputs are supplied. The outputs of the Moore machine are solely generated by the present state, and they emerge only from the current states as shown in the basic state diagram.


Fig. 2.19 Block diagram and state representation of Moore machine

The state diagram in Fig. 2.20 shows an example of a Moore-type machine with four states. Note that every state-to-state transition in the state diagram requires a valid present state input entry, and every node generates one present state output.

The state $0, \mathrm{~S} 0$, produces a present state output, $\mathrm{OUT}=1$, regardless of the value of the present state input, IN . When $\mathrm{IN}=1$, the state S 0 transitions to the next state S 1 . Otherwise, it circulates back to itself. The state S 1 produces OUT $=2$; its next state becomes S 1 if $\mathrm{IN}=$ 0 , or it becomes S 2 if $\mathrm{IN}=1$. The state S 2 also produces a present state output, $\mathrm{OUT}=3$, and transitions to the state S 3 if $\mathrm{IN}=1$. The state S 2 remains unchanged if $\mathrm{IN}=0$. In the fourth and the final state, the present state output from S3 becomes 4 . The machine stays in this state if IN stays at 0 ; otherwise, it goes back to the state S 1 .

The present state inputs and outputs of this Moore machine and its states can be tabulated in a table called the "state table" given in Fig. 2.21. In this table, the first column under the PS entry lists all the possible present states in the state diagram in Fig. 2.20. The middle two columns contain the next state entries for $\mathrm{IN}=0$ and $\mathrm{IN}=1$. The last column lists the present state outputs, one for each present state.


Fig. 2.20 State diagram of a Moore machine with four states

|  | $N S^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: |
| PS | $\stackrel{I N}{I}=0$ | $I N=1$ | OUT |
| S0 | S0 | S1 | 1 |
| S1 | S1 | S2 | 2 |
| S2 | S2 | S3 | 3 |
| S3 | S3 | S1 | 4 |

Fig. 2.21 State table of the Moore machine in Fig. 2.20

The binary state assignment is performed according to Fig. 2.22 where only one bit is changed between adjacent states.

| States | NS1 | NS0 |
| :---: | :---: | :---: |
| S0 | 0 | 0 |
| S1 | 0 | 1 |
| S2 | 1 | 1 |
| S3 | 1 | 0 |

Fig. 2.22 Bit representations of states S0, S1, S2 and S3
The binary form of the state table in Fig. 2.21 is reconstructed in Fig. 2.23 according to the state assignment in Fig. 2.22. This table is called the "transition table", and it includes the binary representation of the next state and the present state outputs.

| PS1 | PSO | $\mathrm{N}=0$ |  | $\mathrm{IN}=1$ |  | OUT2 | OUT1 | OUT0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | NS1 | NSO | NS1 | NSO |  |  |  |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |

Fig. 2.23 Transition table of the Moore machine in Fig. 2.20
Forming this machine's K-maps for the NS0, NS1, OUT0, OUT1 and OUT2 requires grouping all the input terms, PS1, PS0 and IN, according to the table in Fig. 2.23. The K-maps and their corresponding Sum of Products (SOP) expressions are shown in Fig. 2.24.


NS0 PS1 PS0


$$
\begin{aligned}
\mathrm{NS} 1 & =\mathrm{PS} 0 \cdot \mathrm{IN}+\mathrm{PS} 1 \cdot \overline{\mathrm{IN}} \\
\mathrm{NS} 0 & =\mathrm{PS} 0 \cdot \overline{\mathrm{IN}}+\overline{\mathrm{PS} 1 \cdot \mathrm{PS} 0}+\overline{\mathrm{PS} 0} \cdot \mathrm{IN} \\
& =(\mathrm{PS} 0 \oplus \mathrm{IN})+\overline{\mathrm{PS} 1} \cdot \mathrm{PS} 0 \\
\mathrm{OUT2} & =\mathrm{PS} 1 \cdot \overline{\mathrm{PS} 0} \\
\mathrm{OUT} 1 & =\overline{\mathrm{PS} 1} \cdot \mathrm{PS} 0+\mathrm{PS} 1 \cdot \mathrm{PS} 0=\mathrm{PS} 0 \\
\mathrm{OUT0} & =\overline{\mathrm{PS} 1} \cdot \overline{\mathrm{PS} 0}+\mathrm{PS} 1 \cdot \mathrm{PS} 0=\overline{\mathrm{PS} 0 \oplus \mathrm{PS} 0}
\end{aligned}
$$

Fig. 2.24 K-maps and SOP expressions for the Moore machine in Fig. 2.20
The next step is to generate the circuit diagram that produces all five outputs of the Moore machine according to these SOP expressions in Fig. 2.24. This circuit diagram is given in Fig. 2.25.

In order to generate this circuit, individual combinational logic blocks for NS0 and NS1 must be formed first in terms of PS0, PS1 and IN. Then, each NS0 and NS1 output is connected to the corresponding flip-flop input, producing the feedback loops for this state machine. The logic blocks for OUT0, OUT1 and OUT2 are generated directly from PS0 and PS1.


Fig. 2.25 Logic circuit of the Moore machine in Fig. 2.20

### 2.10 Mealy Machine

The Mealy machine shares the same circuit topology with the Moore machine. The machine configuration also contains flip-flops and feedback loops as shown in Fig. 2.26. However, the present state outputs are generated from the combinational logic block in the feedback loop rather than from the present states as in the Moore-type machines.

As a result of this topology, the basic state diagram of a Mealy machine includes the present state, the next state and the input condition that makes the state-to-state transition possible as shown in Fig. 2.26. The present state output(s) does not emerge from the present state; instead, it is a function of the present state input(s) and the present state.


Fig. 2.26 Block diagram and state representation of Mealy machine
The Mealy state diagram in Fig. 2.27 exhibits similar characteristics compared to the Moore state diagram in Fig. 2.20, and all the state names and the state-to-state transitions in this diagram are kept the same for comparison purposes. However, each arrow connecting one state to the next carries the value of the present state output as a function of the present state input and the present state as indicated in Fig. 2.26. As a result, the Mealy state table in Fig. 2.28 contains two separate columns that tabulate the values of NS and OUT for $\mathrm{IN}=0$ and $\mathrm{IN}=1$. The binary state assignment is the same as in Fig. 2.22, which results in a transition table in Fig. 2.29.


Fig. 2.27 State diagram of a Mealy machine with four states

|  | $N S$ |  | OUT |  |
| :---: | :---: | :---: | :---: | :---: |
| PS | $\stackrel{I N=0}{ }$ | $\mathbb{N N}=1$ | $I N=0$ | $I N=1$ |
| S0 | S0 | S1 | 1 | 2 |
| S1 | S1 | S2 | 2 | 3 |
| S2 | S2 | S3 | 3 | 4 |
| S3 | S3 | S1 | 4 | 2 |

Fig. 2.28 State table of the Mealy machine in Fig. 2.27

| PS1 | PSO | $\mathrm{IN}=0$ |  | l = $=1$ |  | $\mathrm{IN}=0$ |  |  | $\mathrm{IN}=1$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | NS1 | NSO | NS1 | NSO | OUT2 | OUT1 | OUTO | OUT2 | OUT1 | OUTO |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |

Fig. 2.29 Transition table of the Mealy machine in Fig. 2.27
The K-maps for NS0, NS1, OUT0, OUT1 and OUT2 are formed according to the table in Fig. 2.29 and shown in Fig. 2.30 with the corresponding SOP expressions. Figure 2.31 shows the circuit diagram of this machine according to the expressions in Fig. 2.30. The methodology used to construct this circuit diagram is identical to the methodology used in the circuit diagram for the Moore machine in Fig. 2.25.


$$
\begin{aligned}
& \text { NS1 }=\text { PS0.IN + PS1.IN } \\
& \mathrm{NS} 0=\mathrm{PS} 0 . \overline{\mathrm{IN}}+\overline{\mathrm{PS} 1} . \mathrm{PS} 0+\overline{\mathrm{PSO}} . \mathrm{IN} \\
& =(\mathrm{PSO} \oplus \mathrm{IN})+\overline{\mathrm{PS} 1} . \mathrm{PS} 0 \\
& \text { OUT2 }=\mathrm{PS} 1 \cdot \overline{\mathrm{PS} 0} \cdot \overline{\mathrm{IN}}+\mathrm{PS} 1 . \mathrm{PS} 0 . \mathrm{IN}=\mathrm{PS} 1 .(\overline{\mathrm{PSO} 0 \mathrm{IN})} \\
& \text { OUT1 }=(\mathrm{PS} 0 \oplus \mathrm{IN})+\overline{\mathrm{PS} 1} . \mathrm{PS} 0=\mathrm{NS} 0 \\
& \text { OUT0 }=\overline{\mathrm{PS} 1} \cdot \overline{\mathrm{PS} 0} \cdot \overline{\mathrm{IN}}+\overline{\mathrm{PS} 1} \cdot \mathrm{PS} 0 . \mathrm{IN}+\mathrm{PS} 1 . \mathrm{PS} 0 . \overline{\mathrm{IN}}=\overline{\mathrm{PS} 1} \cdot(\overline{\mathrm{PSO} \oplus \mathrm{IN}})+\mathrm{PS} 1 . \mathrm{PS} 0 . \overline{\mathrm{IN}}
\end{aligned}
$$

Fig. 2.30 K-maps and SOP expressions for the Mealy machine in Fig. 2.27


Fig. 2.31 Logic circuit of the Mealy machine in Fig. 2.27

### 2.11 Controller Design: Moore Machine Versus Counter-Decoder Scheme

Both Mealy and Moore-type state machines have practical implementation limits when it comes to design. A large ring-style state machine composed of N states such as in Fig. 2.32 may have multiple outputs attached to each state, making its implementation nearly impossible with conventional state machine implementation techniques. However, these types of designs are excellent candidates for the counter-decoder type of designs where each state in the state diagram is associated with a counter output value. Therefore, as the counter increments, present state outputs for each state can simply be generated by a set of decoders connected to the output of the counter.


Fig. 2.32 State diagram of a counter with N states
To illustrate this theory, a controller that generates the timing diagram in Fig. 2.33 will be implemented using both the Moore-type state machine and the counter-decoder approach.

From the timing diagram below, this state machine generates a single active-high output, Out $=1$, once in every 8 cycles as long as Stop $=0$. When Stop $=1$, however, the machine stalls and it retains its current state.


Fig. 2.33 Timing diagram of a state machine with a single input, Stop, and a single output
Once the state assignments are made for each clock cycle in Fig. 2.33, the state diagram for a Moore-type state machine emerges in Fig. 2.34.

The first and the second clock cycles in the timing diagram are assigned to the S 0 and the S 1 states, respectively. The third clock cycle is assigned to the S 2 state where Out $=1$. The fourth clock cycle corresponds to the S3 state. The machine stays in the S3 state as long as

Stop $=1$. This ranges from the fourth to the sixth clock cycle in the timing diagram. The state assignments from the seventh to the tenth clock cycle become the S4, S5, S6 and S7 states, respectively. The eleventh clock cycle returns to the S 0 state.


Fig. 2.34 Moore representation of the timing diagram in Fig. 2.33
Implementing the state diagram in Fig. 2.34 follows a lengthy process of producing state tables, transition tables, and K-maps, resulting in a total of four outputs (three flip-flop outputs due to eight states and one output for Out). However, using a counter-decoder approach minimizes this design task considerably and reveals a rather explicit circuit implementation.

When the timing diagram in Fig. 2.33 is redrawn to implement the counter-decoder design approach, it yields a simple three-bit counter which counts from zero to seven as shown in Fig. 2.35. The counter output, CountOut, is included in this figure to show the relationships between the state assignments, the input (Stop) and the output (Out). The figure also shows the clock cycle where the counter resets itself when its output reaches seven.


Fig. 2.35 Timing diagram of a three-bit counter with a single input, stop, and a single output

The first task for the design is to construct a three-bit up-counter as shown in Fig. 2.36. The counter in this figure is derived from a general counter topology, and it consists of a three-bit adder, three 2-1 MUXes and three flip-flops. A three-input AND gate is used as a decoder at the counter output to implement Out $=1$ when the CountOut node becomes 2 . Therefore, this method follows a simple, step-by-step design approach in producing the final circuit that does not require implicit logic design techniques.


Fig. 2.36 Counter-decoder representation of the timing diagram in Fig. 2.35

### 2.12 Memory

Small memory blocks can be assembled from one-bit registers in a variety of configurations as shown in Fig. 2.14. For example, a 32-bit wide, 16-bit deep memory block shown in Fig. 2.37 can be built by stacking 16 rows of 32 -bit registers on top of each other. Each 32-bit register contains tri-state buffers at its output to prevent logic contention during read as shown in Fig. 2.38.

The inputs to each column of the memory block in Fig. 2.37 are connected together to write data to a selected row. For example, the input terminal, IN[0], is connected to the $\operatorname{In}[0]$ pins of all 32 -bit registers between row 0 to row 15 to be able to write a single bit at a selected row. The same is true for the remaining inputs, IN[1] to IN[31].

Similarly, all outputs of each column in Fig. 2.37 are connected together to read data from the memory block. For example, the output pin, OUT[0], is connected to the Out[0] pin of every 32-bit register from row 0 to row 15 to be able to read one bit from a selected row. The same is true for the remaining output pins, OUT[1] through OUT[31].

Every row of the memory block in Fig. 2.37 is accessed by an individual Write Enable (WE) and Read Enable (RE) signal for writing or reading data, respectively.


Fig. 2.37 A $32 \times 16$ memory and the truth table of its address decoder


Fig. 2.38 A 32-bit register slice at every row of Fig. 2.37
In order to generate the WE inputs, WE[0] to WE[15], an address decoder is used. This decoder enables only one row while deactivating all the other rows using a four-bit address, Address[3:0], and a single WE input according to the truth table in Fig. 2.39. For example, a 32-bit data is written to row 0 if $\mathrm{WE}=1$ and Address[3:0] $=0000$ at the decoder input. However, $\mathrm{WE}=0$ blocks writing data to all rows of the memory block regardless of the input address as shown in the truth table in Fig. 2.40.

The RE inputs, RE[0] to RE[15], use address decoders similar to Figs. 2.39 and 2.40 to read a block of data from a selected row. The read operation is achieved with a valid input address and $\mathrm{RE}=1$ according to the truth table in Fig. 2.41. The $\mathrm{RE}=0$ entry disables reading data from any row regardless of the input address as shown in Fig. 2.42.

Therefore, a valid input address along with the RE and WE command inputs must be provided to the memory in order to perform a read or a write operation, respectively. The $\mathrm{WE}=0, \mathrm{RE}=1$ combination reads data from the selected row. Similarly, the WE $=1$ and $\mathrm{RE}=0$ combination writes data to a selected row. The $\mathrm{WE}=0$ and $\mathrm{RE}=0$ combination disables both reading and writing to the memory block. The control input entry, $\mathrm{WE}=1$ and $\mathrm{RE}=1$, is not allowed, and it should be interpreted as memory read.


Fig. 2.39 The address decoder for the $32 \times 16$ memory in Fig. 2.37 when $\mathrm{WE}=1$


Fig. 2.40 The address decoder for the $32 \times 16$ memory in Fig. 2.37 when $\mathrm{WE}=0$

| Address[3:0] | RE[15] RE[14] RE[13] |  |  | RE[2] RE[1] RE[0] |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 00000 | 0 | 0 | 0 | 0 | 0 | 1 |
| $\begin{array}{llll}0 & 0 & 0 & 1\end{array}$ | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 0 0 0 10 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\begin{array}{llll}1 & 1 & 1 & 0\end{array}$ | 0 | 1 | 0 | 0 | 0 | 0 |
| $\begin{array}{llll}1 & 1 & 1 & 1\end{array}$ | 1 | 0 | 0 | 0 | 0 | 0 |

Fig. 2.41 The address decoder for the $32 \times 16$ memory in Fig. 2.37 when $\mathrm{RE}=1$


Fig. 2.42 The address decoder for the $32 \times 16$ memory in Fig. 2.37 when $\mathrm{RE}=0$

### 2.13 A Design Example Using Sequential Logic and Memory

This design example combines the data-path and controller design concepts described in this chapter and in Chap. 1. It also introduces the use of important sequential logic blocks such as flip-flop, register, counter and memory in the same design.

Every design starts with gathering small or large logic blocks to meet the functional specifications of the design and to construct a data-path for proper data-flow. Once the data-path is set, then the precise data movements from one logic block to the next are described using timing diagrams. Any architectural change in the data-path should follow a corresponding change in the timing diagram or vice versa.

When the data-path design and the timing diagram fully associate with each other, and each describes identical data movements, the next step in the design process is to build the controller that governs the flow of data. To define the states of the controller, the clock periods that generate different sets of outputs are separated from each other and named individually as distinct states. Similarly, the clock periods revealing identical outputs are grouped together under the same state name. The controller can be Moore-type or Mealy-type state machine according to the design needs. The design methodology of building the data-path, timing diagram and controller shown here will be repeated in every design throughout this book, especially when designing peripherals for a computer system in Chap. 7.

The example design in this section reads two eight-bit data packets from an 8 x 8 source memory (memory A), processes them and stores the result in an 8 x 4 target memory (memory B). The processing part depends on the relative contents of each data packet: if the contents
of the first data packet are larger than the second, the contents of the data packets are added. Otherwise, they are subtracted from each other before the result is stored.

The block diagram in Fig. 2.43 demonstrates the data-path required for this memory-to-memory data transfer as described above. The timing diagram in Fig. 2.44 needs to accompany the data-flow in Fig. 2.43 since it depicts precise data movements and values in each clock cycle.

To be able to write data to a memory address in Fig. 2.37, a valid data and address must be available within the same clock cycle. In a similar fashion, data is read from the memory core a cycle after a valid address is introduced.

Initially, counter A generates the addresses, 0 to 7 , for memory A and writes the data packets, A0 to A7, through DataInA[7:0] port. This is shown in the timing diagram in Fig. 2.44 from clock cycles 1 through 8 . When this task is complete, counter A resets and reads the first data packet A 0 from $\mathrm{Addr} \mathrm{A}[2: 0]=0$ in clock cycle 9 . In the next clock cycle, A0 becomes available at DOut1, and the counter A increments by one. In cycle 11, AddrA [2:0] becomes 2, the data packet A1 is read from DOut1[7:0], and the data packet A0 transfers to DOut2[7:0]. In this cycle, the contents of the data packets A0 and A1 are compared with each other by subtracting A1 (at DOut1) from A0 (at DOut2). If the contents of A0 are less than A 1 , then the sign bit, Sign, of $(\mathrm{A} 0-\mathrm{A} 1)$ becomes negative. Sign $=1$ selects $(\mathrm{A} 0+\mathrm{A} 1)$ at $\mathrm{ADDOut}[7: 0]$ and routes this value to DataInB[7:0]. However, if the contents of A0 are greater than $\mathrm{A} 1,(\mathrm{~A} 0-\mathrm{A} 1)$ becomes positive. $\mathrm{Sign}=0$ selects $(\mathrm{A} 0-\mathrm{A} 1)$ and routes this value from SUBOut[7:0] to DataInB[7:0]. The result at DataInB[7:0] is written at $\mathrm{AddrB}[1: 0]=0$ of memory $B$ at the positive edge of clock cycle 12. In the same cycle, A1 is transferred to DOut2[7:0], and A2 becomes available at DOut1[7:0]. A comparison between A1 and A2 takes place, and either (A1 + A2) or (A1 - A2) is prompted to be written to memory B depending on the value of the Sign node. However, this is an unwarranted step in the data transfer process because the design requirement states that the comparison has to be done only once between two data packets from memory A. Since A1 is used in an earlier comparison with A0, A1 cannot be used in a subsequent comparison with A 2 , and neither $(\mathrm{A} 1+\mathrm{A} 2)$ nor ( $\mathrm{A} 1-\mathrm{A} 2$ ) should be written to memory B . The remaining clock cycles from 13 through 18 compare the values of A2 with A3, A4 with A5, and A6 with A7, and write the added or subtracted results into memory B. After clock cycle 19, all operations on this data-path suspend, the counters are reset and all writes to the memory core are disabled.


Fig. 2.43 Data-path of a memory transfer example

Fig. 2.44 Timing diagram for the memory transfer data-path in Fig. 2.43

To govern the data-flow in Fig. 2.44, a Moore-type state machine (or a counter-decoder-type controller) is used. A Mealy-type state machine for a controller design is usually avoided because present state inputs of this type of a state machine may change during the clock period and may cause jittery outputs to form.

The inclusion of the controller in Fig. 2.45 identifies the necessary control signals to be able to guide the data flow in Fig. 2.44 properly. These signals increment the counters A and B (with IncA and IncB), and enable writes to memory A or B (with WEA and WEB) when necessary. Thus, the timing diagram in Fig. 2.44 is expanded to include these control signals in Fig. 2.46, and this provides a complete picture of the data transfer process from memory A to memory B in contrast to the earlier timing diagram in Fig. 2.44.


Fig. 2.45 Compete block diagram of the memory transfer example with controller

The controller in Fig. 2.45 can be implemented either by a Moore-type state machine in Fig. 2.47 or by a counter-decoder-type design in Fig. 2.48.

In the Moore type design, the states from S1 through S18 are assigned to each clock cycle of the timing diagram in Fig. 2.46. The values of the present state outputs, WEA, IncA, WEB and IncB, in each clock cycle are read from the timing diagram and attached to each state in Fig. 2.47. The reset state, S 0 , is included in the Moore machine in case the data-path receives an external reset signal to interrupt an ongoing data transfer process. Whichever state the state machine may be in, a Reset $=1$ entry always forces the current state to transition back to the S 0 state at the positive edge of the clock. These transitions are not included in Fig. 2.47 for simplicity.

The counter-decoder style design in Fig. 2.48 consists of a five-bit counter and four decoders to generate WEA, IncA, WEB and IncB control signals. To show the operation of this design to generate WEA, for example, this particular decoder includes eight five-input AND gates, one for each clock cycle from cycle 1 to cycle 8 in order to keep WEA $=1$ in Fig. 2.46. The five-bit counter implicitly receives a reset signal from its output when it reaches clock cycle 18 , and resets counter A, counter B and the rest of the system in Fig. 2.45.

Fig. 2.46 The complete timing diagram for the memory transfer in Fig. 2.45


Fig. 2.47 Moore representation of the controller unit in Fig. 2.45


Fig. 2.48 Counter-decoder representation of the controller unit in Fig. 2.45

## Review Questions

1. Implement the following Moore machine:

2. Implement the following Moore machine using a timer. The timer is initiated when $\mathrm{In}=$ 1. With this input, the state machine goes to the A state and stays there for 10 cycles. In the tenth cycle, the state machine transitions to the B state and stays in this state for only one cycle before switching to the IDLE state. One implementation scheme is to construct a four-bit up-counter to generate the timer. When the counter output reaches 9 , the decoder at the output of the counter informs the state machine to switch from the A state to the B state.

3. The following truth table needs to be implemented using two-input NAND gates and inverters.

| A | B | C | Out |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 3 |
| 1 | 0 | 1 | 2 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

$\mathrm{T}_{\text {NAND }}$ (two-input NAND gate delay) $=500 \mathrm{ps}$
$\mathrm{T}_{\text {INV }}$ (inverter delay) $=500 \mathrm{ps}$
tclk-q (clock-to-q delay) $=200 \mathrm{ps}$
tsu $($ setup time $)=200 \mathrm{ps}$
th $($ hold time $)=300 \mathrm{ps}$
(a) Implement this truth table between two flip-flop boundaries.
(b) Find the maximum clock frequency using a timing diagram.
(c) Shift the clock by 500 ps at the receiving flip-flop boundary. Show whether or not there is a hold violation using a timing diagram.
4. A block diagram is given below:


Block A contains only two flip-flops. Block B contains a one-bit adder with SUM and COUT outputs connected to two flip-flops as shown below.

(a) Using the logic gates with propagation delays listed below, determine the setup time for A, B, and CIN with respect to clkshift.

(b) Assuming $\mathrm{T}=0 \mathrm{~ns}$ and $\mathrm{T}_{\mathrm{CLK}}$ (clock period) $=5 \mathrm{~ns}$, if data at $\mathrm{A}, \mathrm{B}$ and CIN become valid and stable 4 ns after the positive edge of clkshift, will there be any timing violations? Assume $\mathrm{t}_{\mathrm{H}}$ (hold time) $=3 \mathrm{~ns}$ for the flip-flop.
(c) How can you eliminate the timing violations? Show your calculations and draw a timing diagram with no timing violations.
5. A schematic is given below:

(a) If tsu $($ setup time $)=200 \mathrm{ps}$, th $($ hold time $)=200 \mathrm{ps}$ and tclk-q $($ clock-to-q delay $)=$ 300 ps for the flip-flop, and $\mathrm{TA}=1000 \mathrm{ps}, \mathrm{TB}=100 \mathrm{ps}$ for the internal logic blocks on the schematic, show if there is any timing violation or timing slack in a detailed timing diagram if $\mathrm{TC}=0 \mathrm{ps}$.
(b) What happens if $\mathrm{TC}=400 \mathrm{ps}$ ? Show it in a separate timing diagram.
6. The state diagram of a Moore machine is given below:


The assignment of the states $\mathrm{A}, \mathrm{B}$ and C are indicated as follows:

| states | $\mathrm{PS}[1]$ | $\mathrm{PS}[0]$ |
| :---: | :---: | :---: |
| A | 0 | 0 |
| B | 0 | 1 |
| C | 1 | 1 |

(a) Implement this state machine using inverters, two-input and three-input AND gates and two-input OR gates.
(b) Find the maximum operating frequency of the implementation in part (a) if the following timing assignments are applied:
tsu $($ setup time $)=100 \mathrm{ps}$, th $($ hold time $)=100 \mathrm{ps}$, tclk-q $($ clock-to-q delay $)=200 \mathrm{ps}$, $\mathrm{T}_{\text {INV }}$ (inverter delay) $=200 \mathrm{ps}, \mathrm{T}_{\text {AND } 2}$ (two-input AND gate delay) $=300 \mathrm{ps}, \mathrm{T}_{\text {AND }}$ (three-input AND gate delay) $=400 \mathrm{ps}, \mathrm{T}_{\mathrm{OR} 2}$ (two-input OR gate delay) $=400 \mathrm{ps}$.
7. Data is transferred from Memory Tx to Memory Rx starting from the address $0 x 00$ and ending at the address $0 x 0 \mathrm{~F}$ as shown below. Once a valid address is produced for Memory Tx, the data is read from this address at the next positive clock edge. On the other hand, data is written to Memory Rx at the positive edge of the clock when a valid address is available. The operating clock frequency of Memory Tx is twice the clock frequency of Memory Rx.

(a) Assuming address generators for Memory Tx and Memory Rx start generating valid addresses at the same positive clock edge, show which data is actually stored in Memory Rx using a timing diagram. Indicate all the address and data values for Memory Tx and Memory Rx in the timing diagram.
(b) Now, assume that the operating clock frequency of Memory Tx is four times higher than the clock frequency of Memory Rx, and an actual write to Memory Rx takes place at the negative edge of the clock when a valid address is present. Redraw the timing diagram indicating all address and data values transferred from Memory Tx to Memory Rx.
8. Serial data is transferred to program four eight-bit registers. The start of the transfer is indicated by a seven-bit sequence $=\{1010101\}$ immediately followed by the address of the register (two bits) and the data (eight bits). The transfer stops after programming the last register. After this point, all other incoming bits to the serial input are ignored. Design this interface by developing a data-path and a timing diagram simultaneously. Implement the state diagram. Can this controller be implemented by a counter-decoder scheme?

## Projects

1. Implement the one-bit register in Fig. 2.14 and verify its functionality using Verilog. Use timing attributes in the flip-flop and the multiplexer to create gate propagation delays. Change the clock frequency until set-up time violation is produced.
2. Implement the four-bit shift register in Fig. 2.16 and verify its functionality using Verilog. Use timing attributes for flip-flops and the multiplexers to create gate propagation delays. Examine the resulting timing diagram.
3. Implement the 32-bit counter in Fig. 2.18 and verify its functionality using Verilog.
4. Implement the four-state Moore-type state machine in Fig. 2.20 and verify its functionality using Verilog.
5. Implement the four-state Mealy-type state machine in Fig. 2.27 and verify its functionality using Verilog.
6. Implement the three-bit counter-decoder in Fig. 2.36 and verify its functionality using Verilog and examining the resulting timing diagram.
7. Implement the $32 \times 16$ memory block in Fig. 2.37 using Verilog. How can this memory be verified functionally?
8. Implement the memory-to-memory transfer circuit in Fig. 2.45 and verify its functionality using Verilog.

## Review of Asynchronous Logic Circuits

A digital system is often comprised of different time domains. Some domains work with clock, and data is sequentially transferred from one flip-flop (or latch) boundary to the next. In other domains, data is asynchronously processed and handled without the aid of a clock.

This chapter introduces asynchronous circuits that require no clock input. The complete design methodology is given in terms of state assignments from timing diagrams, construction of flow tables and gate minimization, which then leads to the implementation of fundamental mode circuits [1]. The chapter concludes with an asynchronous timing methodology with C (Mueller) elements that allows data propagation between logic blocks without any clock input.

### 3.1 S-R Latch

A common storage element in asynchronous circuits is a Set-Reset (S-R) latch. This circuit is composed of two NAND gates whose outputs are connected to their inputs as shown in Fig. 3.1.


Fig. 3.1 S-R latch
Initially, both $S$ and $R$ inputs may be at logic 0 , producing $\mathrm{Q}=\overline{\mathrm{Q}}=1$. If the S input transitions to logic 1 while $\mathrm{R}=0, \mathrm{Q}$ stays at logic 1 and $\overline{\mathrm{Q}}$ transitions to logic 0 . This state is called the set state of the S-R latch. If, on the other hand, the R input goes to logic 1 while the $S$ input stays at $0, \mathrm{Q}$ transitions to logic 0 and $\overline{\mathrm{Q}}$ stays at logic 1 . This state is called the
reset state. Simultaneously changing both the $S$ and $R$ inputs from logic 0 to logic 1 causes a racing condition. If NAND gate number 1 has a shorter gate delay than the NAND gate number $2, \overline{\mathrm{Q}}$ switches to logic 0 first, and forces Q to stay at logic 1. If NAND gate number 2 has a shorter gate delay, Q switches to logic 0 first. Therefore, simultaneously switching more than one input in asynchronous circuits creates unexpected outputs due to multiple racing paths in a circuit. The fundamental-mode design methodology corrects this problem by permitting only one input to change, and eliminates all unwanted transitions in the circuit.

### 3.2 Fundamental-Mode Circuit Topology

An asynchronous circuit requires no clock input to operate, and it is composed of a combinational logic block and a delay block. The delay block is a combinational logic circuit whose inputs constitute the present state. The outputs of the delay block are fed back to the inputs of the combinational logic to form the next state as shown in Fig. 3.2.


Fig. 3.2 Fundamental mode asynchronous circuit topology
Designing an asynchronous circuit requires to follow a certain procedure. The first task is to form a primitive state flow table tabulating all possible states and transitions that the asynchronous circuit can produce. This table must contain only one stable state per row to maintain the fundamental mode of operation. Similarly, another table, containing only the outputs of the circuit, is formed. An output table is also formed, and it includes every output change as the circuit makes a transition from one state to another. Minimization of state and output tables is achieved by implication tables prior to producing a final circuit. In asynchronous circuit design, it is common to create race conditions where circuit delays produce multiple simultaneous state transitions resulting in unwanted outputs. An effective method to eliminate racing conditions is to work on the minimized state table and remove such state transitions that cause the circuit to depart from its fundamental-mode of operation.

### 3.3 Fundamental-Mode Asynchronous Logic Circuits

In this section, an example will be used to present the entire process designing fundamental-mode asynchronous circuits from the creation of primitive flow tables to the removal of racing conditions.

The circuit in this example consists of two inputs, in1 and in2, and a single output. The output is at logic 0 whenever in $1=0$. The first change in in 2 produces out $=1$ while in1 $=1$. The output transitions back to logic 0 when in1 switches to logic 0 .

The timing diagram in Fig. 3.3 summarizes the behavior of this circuit. State assignments in the timing diagram follow the basic rule that requires the change in only one input per state. All stable states are numbered and circled.


Fig. 3.3 Timing diagram and state assignments
The state (1) is when in $1=\mathrm{in} 2=0$ and out $=0$. Any change in in 2 transitions the circuit to the state (2) with out $=0$. Additional ripples at in2 while in $1=0$ change the state of the circuit between the states (1) and (2). Switching in1 from logic 0 to logic 1 while in $2=1$ forms a new state, state (3), and produces out $=0$. The first change in in 2 while in1 $=1$ creates another new state, state (4), with out $=1$. As in 2 transitions back from logic 0 to logic 1 while in $1=1$, the state of the circuit changes to the state (5), but the output stays at out $=1$. Further ripples in in 2 while in $1=1$ create no change at the output, and the circuit ends up transitioning between the states (4) and (5). When in 1 transitions to logic 0 while in $2=1$, the state of the circuit switches back to the state (2). Finally, when in 2 also switches back to logic 0 , the state of the circuit becomes the state (1).

Now, let us assume that in 1 transitions to logic 1 first while in 2 is steady at logic 0 . This condition creates a new state, state (6), with an output value of out $=0$. When in 2 also transitions to logic 1 while in $1=1$, the state of the circuit changes from the state (6) to a new state, state (7), and out becomes logic 1 . As soon as in 2 goes back to logic 0 while in1 $=1$, the circuit also switches back to the state (4) but out stays at logic 1. Further ripples in in 2 while in $1=1$ simply change the state of the circuit between the states (4) and (7). As soon as
in1 transitions back to logic 0 , the circuit goes to the state (2). When in 2 also changes to logic 0 , the circuit goes back to the state (1).

Constructing primitive state table and output flow table is the direct extension of the timing diagram in Fig. 3.3. When transferring stable, circled states from the timing diagram to the primitive state table in Fig. 3.4, the fundamental-mode rule that enforces one stable state per row is strictly observed. The non-circled states in this table are considered "transitionary" states: the circuit momentarily stays in these states until it makes a permanent move to a stable state. For example, as in 2 transitions from logic 0 to logic 1 while in1 stays at logic 0 in the first row of Fig. 3.4, the state of the circuit changes from the stable state (1) to the transitionary state 2 . The circuit stays in this transitionary state only for a brief moment until it finally transitions to the stable state (2) in row 2 . Similarly, as in1 switches from logic 0 to logic 1 while in $2=1$, the circuit transitions from the state (2) to the state (3) through a transitionary state 3 in the second row. All simultaneous dual input transitions are forbidden because of the primary rule in the fundamental mode of operation. Therefore, a transition from in $1=$ in $2=0$ to in $1=$ in $2=1$ in the first row is not allowed. In this figure, the boxes marked with "-" indicate the forbidden transitions.

| in1 in2 |  |  |  |
| :---: | :---: | :---: | :---: |
| 00 | 11 |  |  |
| 1 | 01 | - | 6 |
| 1 | 2 | 3 | - |
| - | 2 | 3 | 4 |
| 1 | - | 5 | 4 |
| - | 2 | 5 | 4 |
| 1 | - | 7 | 6 |
| - | 2 | 7 | 4 |



Fig. 3.4 Primitive state (left) and output (right) flow tables for Fig. 3.3
Primitive state and output flow tables are usually integrated to produce a compact table as shown in Fig. 3.5. Furthermore, labeling the present and next states clarifies the state transitions and the output values during each transition. Figure 3.5 also separates the states that produce out $=0$ from the states that produce out $=1$. This design practice comes in handy during the minimization step when equivalence classes are formed.

| ps | in1 in2 |  |  |  | in1 in2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 01 | 11 | 10 |  | 01 | 11 | 10 |
| (1) | (1) | 2 | - | 6 | 0 |  |  |  |
| (2) | 1 | (2) | 3 | - |  | 0 |  |  |
| (3) | - | 2 | (3) | 4 |  |  | 0 |  |
| (6) | 1 | - | 7 | (6) |  |  |  | 0 |
| (4) | 1 | - | 5 | (4) |  |  |  | 1 |
| (5) | - | 2 | (5) | 4 |  |  | 1 |  |
| (7) | - | 2 | (7) | 4 |  |  | 1 |  |
|  | ns |  |  |  | out |  |  |  |

Fig. 3.5 Integrated primitive state and output flow tables
The first step towards state minimization is to form an implication table as shown in Fig. 3.6. This table includes all permitted, forbidden and "implied" states that can either go into the permitted or forbidden categories.


Fig. 3.6 The implication table for Fig. 3.5
Figure 3.6 replicates the present state column in Fig. 3.5 in its vertical axis with the exception of the state 1 (the first row in Fig. 3.5), and in its horizontal axis with the exception of state 7 (the last row in Fig. 3.5).

The box located at $(2,1)$ in Fig. 3.6 is checked because there are no other states involved in a transition from the state (1) to the state (2) in Fig. 3.5. The box at $(3,1)$ contains $4-6$, because the "implied" states 6 and 4 in the column, in 1 in $2=10$, in Fig. 3.5 must be traversed in order to go from the state (1) to the state (3). The columns, in1 in2 = 00 and 11 , contain forbidden transitions, and they cannot be used as implied states to allow a transition from the state (1) to the state (3). The only other column for a transition from the state (1) to
the state (3) is the column, in 1 in $2=01$, but it requires a transition from the state 2 to the state 2, which is not possible. The box at $(4,6)$ in Fig. 3.6 contains an " $X$ " mark because the outputs produced at the states (4) and (6) are different. The same applies to boxes at $(5,3)$ and $(7,3)$. The box at $(6,3)$ contains two implied transitions, 3-7 and 4-6, which correspond to the columns, in1 in2 = 11 and 10 , respectively.

We can further eliminate some of the implied state entries in the implication table of Fig. 3.6 if these states are "related" to the boxes that have already been crossed out. Figure 3.7 shows the new implication table after eliminations. In this table, the boxes that contain the transitionary states 4-6, 3-5 and 3-7 are safely crossed out since they are related to the boxes at $(4,6),(5,3)$ and $(7,3)$, respectively.


Fig. 3.7 The implication table after eliminations
Forming the equivalence class table is the next step for minimization. First, all states in the horizontal axis of the implication table in Fig. 3.7 are repeated backwards under the "States" column in the equivalence class table in Fig. 3.8.

Column 5 in Fig. 3.7 contains a checked box. This box corresponds to a joint state, (5-7), listed as an equivalence class in the first row of Fig. 3.8. Column 4 in Fig. 3.7 consists of a checked box at $(4,5)$ and a second box containing an implied state 5-7 at $(4,7)$. Therefore, the second row of Fig. 3.8 contains two new joint states, $(4,5)$ and $(4,7)$, as well as the earlier joint state, $(5,7)$, from the first row. Since in this row the joint states, $(5,7),(4,7)$ and $(4,5)$, overlap, they can be combined in a compact joint state $(4,5,7)$. Columns 6 and 3 in Fig. 3.7 have crossed-out boxes that contain no implied states. Therefore, the third and the fourth row of Fig. 3.8 do not have new state entries, but only a single combined state carried over from the second row. Column 2 in Fig. 3.7 has also crossed-out boxes except at $(2,3)$, and this produces a new joint state, (2, 3), in the fifth row of Fig. 3.8. Finally, column 1 in Fig. 3.7 contains two checked boxes at $(1,2)$ and $(1,6)$, which form two additional joint states, $(1,2)$ and $(1,6)$, in the last row of Fig. 3.8. Therefore, the final equivalence class list includes the joint states, $(4,5,7),(2,3)$ and $(1,6)$. The joint state $(1,2)$ is a shared state between $(2,3)$ and $(1,6)$, and it is absorbed in the final list.

| States | Equivalence Classes |
| :---: | :--- |
| 5 | $(5,7)$ |
| 4 | $(5,7)(4,7)(4,5)=(4,5,7)$ |
| 6 | $(4,5,7)$ |
| 3 | $(4,5,7)$ |
| 2 | $(4,5,7)(2,3)$ |
| 1 | $(4,5,7)(2,3)(1,2)(1,6)$ |
| Final List | $(4,5,7)(2,3)(1,6)$ |

Fig. 3.8 Equivalent class table
The final equivalence class list indicates the presence of only three states. Therefore, the same number of states must be present in the minimal state flow table in Fig. 3.9, where the joint states, $(1,6),(2,3)$ and $(4,5,7)$ are assigned to the states $\mathrm{A}, \mathrm{B}$ and C , respectively.

| ps | in1 in2 |  |  |  | in1 in2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
| $\begin{array}{r} (1,6)=A \\ (2,3)=B \\ (4,5,7)=C \end{array}$ | (A) | B | C | (A) | 0 | - | - | 0 |
|  | A | (B) | (B) | C | - | 0 | 0 | - |
| $(4,5,7)=C$ | A | B | (C) | (C) | - | - | 1 | 1 |
|  | ns |  |  |  |  |  |  |  |

Fig. 3.9 Minimized integrated primitive state and output flow tables
In Fig. 3.5, the present states (1) and (6) correspond to the states (1) and 1 when in 1 in $2=$ 00 . However, the states (1) and 1 now belong to the new assigned state A in Fig. 3.9. Therefore, the present state A transitions to a stable state A with out $=0$ when in1 in2 $=00$. Similarly, the present states (1) and (6) in Fig. 3.5 correspond to the states 2 and "-" when in 1 in2 $=01$, and produce no output. In the new table, this translates to a transition from the present state $A$ to the stable state $B$ when in $1 \mathrm{in} 2=01$. The other entries in the next state, ns , and the output, out, columns in Fig. 3.9 are formed in a similar manner.

The three states, A, B and C, in Fig. 3.9 require two next state bits, ns1 and ns2. The state assignments are shown in Fig. 3.10.

|  | ns1 | ns2 |
| :---: | :---: | :---: |
| A | 0 | 0 |
| B | 0 | 1 |
| C | 1 | 1 |

Fig. 3.10 State assignments
Employing the state assignment table in Fig. 3.10 in the combined minimal state and output flow table in Fig. 3.9 leads to the state and the output K-maps in Fig. 3.11. The crossed-out entries in this figure correspond to "don't care" conditions, and they are treated as either logic 0 or logic 1 to achieve the minimal Sum of Products (SOP) expression for each K-map.


$$
\begin{aligned}
& \mathrm{ns} 1=\text { in1.ps1 }+\mathrm{in} 1 . \overline{\mathrm{n} 2} . \mathrm{ps} 2+\mathrm{in} 1 . \mathrm{in} 2 . \overline{\mathrm{ps} 2} \\
& \mathrm{~ns} 2=\mathrm{in} 2+\mathrm{in} 1 . \mathrm{ps} 2 \\
& \mathrm{out}=\mathrm{in} 1 . \mathrm{ps} 1
\end{aligned}
$$



Fig. 3.11 K-maps for the next states, ns1 and ns2, and the output, out
Finally, the SOP expressions for ns1, ns2 and out in Fig. 3.11 generate the circuit diagram in Fig. 3.12. To draw the complete circuit, first combinational logic blocks for ns1, ns2 and out are formed using their SOP expressions. Then, each next state, ns1 and ns2, is connected to its corresponding present state, ps 1 and ps 2 , to complete the circuit diagram in Fig. 3.12.


Fig. 3.12 Fundamental mode asynchronous circuit for Fig. 3.3
Even though the circuit in Fig. 3.12 represents the required state behavior, it may not eliminate all possible racing conditions. When the state table in Fig. 3.9 is transformed into a state diagram in Fig. 3.13, the forward and backward transitions between the states A and C may induce racing conditions because two inputs change simultaneously.


Fig. 3.13 State diagram showing possible racing conditions
To prevent racing conditions, a fictitious fourth state is introduced between the states A and C in Fig. 3.13. This fourth state, $\alpha$, forms a bridge when going from the state A to the state C (or vice versa) and allows only one state input to change to prevent possible hazards as shown in Fig. 3.14.


Fig. 3.14 Reconfiguration of the state diagram to eliminate racing conditions
However, the inclusion of the new state, $\alpha$, necessitates reconfiguring the original state table in Fig. 3.9. The new state table contains the state $\alpha$ as a transitionary state in Fig. 3.15, and the transitions into this state do not produce any output.

| ps | in1 in2 |  |  |  |  | in1 in2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |  |  |
| a | a | b | $\alpha$ | a | 0 | - | - | 0 |  |  |
|  | b | a | b | b | c | - | 0 | 0 |  |  |
| c | $\alpha$ | b | C | C | - | - | 1 | 1 |  |  |
|  | $\alpha$ | $a$ | - | $c$ | - | - | - | - |  |  |

Fig. 3.15 Integrated state and output flow tables without racing conditions
When the hazard-free state and output K-maps are formed based on the flow table in Fig. 3.15, the resultant SOP expressions for ns1 and ns2 in Fig. 3.16 contain only an additional term with respect to the ones in Fig. 3.11. This is a small price to pay in the total gate count for the benefit of eliminating all racing conditions.


$$
\begin{aligned}
& \mathrm{ns} 1=\mathrm{in} 1 . \mathrm{ps} 1+\mathrm{in} 1 . \overline{\mathrm{n} 2} . \mathrm{ps} 2+\overline{\mathrm{in} 2} . \mathrm{ps} 1 . \mathrm{ps} 2+\mathrm{in} 1 . \mathrm{in} 2 . \overline{\mathrm{ps} 2} \\
& \mathrm{~ns} 2=\overline{\mathrm{in} 1} . \mathrm{in} 2+\mathrm{in} 1 . \mathrm{ps} 2+\mathrm{in} 1 . \mathrm{ps} 1 \\
& \mathrm{out}=\mathrm{in} 1 . \mathrm{ps} 1
\end{aligned}
$$

Fig. 3.16 Next state and output K-maps producing no racing conditions

### 3.4 Asynchronous Timing Methodology

Asynchronous data propagation through combinational logic blocks can be achieved using C-elements (Mueller elements) as shown in Fig. 3.17.

In this figure, combinational logic blocks, CL1, CL2 and CL3, are connected through flip-flops to propagate data. However, data propagation through the combinational logic does not have to be complete within a fixed clock period as in conventional sequential circuits. The C-elements in conjunction with inverting delay blocks, D1, D2 and D3, allow variable data propagation to take place for each stage.


Fig. 3.17 Asynchronous timing methodology using C-elements
As data propagates through a particular combinational logic block, the positive edge produced at the Cout terminal of a C-element also propagates through the corresponding delay circuit. When data reaches the next flip-flop boundary, the C-element in this stage also produces a positive Cout edge for the flip-flop to fetch the incoming data.

The details of variable data propagation in Fig. 3.17 are illustrated in the timing diagram in Fig. 3.18. In this figure, the C-element produces a positive edge at Cout1 and enables the flip-flop to dispatch data1 from its output after a clock-to-q delay. As this data propagates through the combinational logic block, CL1, the positive edge of Cout1 also travels through the delay block, D1, and reaches the next C-element to form a positive edge at Cout 2 to fetch the incoming data.

Data propagation in the second stage and the positive edge formation of Cout2 is identical to the first stage with the exception of longer propagation delays, CL2 and D2. The third stage presents a much smaller propagation delay, CL3, and requires a smaller delay element, D3.

Even though each combinational data-path delay in Fig. 3.18 is approximately twice as large as the propagation delay of its corresponding delay element, the flip-flop set-up time, tsu, must be taken into account to fine-tune the length of delay for each delay element.


Fig. 3.18 A timing diagram with variable clock lengths and stage delays
Figure 3.19 shows the detailed Input/Output timing diagram of the C-element. In the first stage of the data-path (CL1 in Fig. 3.17), a positive edge at Coutl travels though the inverting delay element, D1, and produces a negative edge at Cin2 for the next C-element. The C-element is designed such that the negative edge at its Cin input creates a negative edge from its F output. Therefore, the negative edge at the F1 node comes back to the first C-element as an input and prompts the first C-element to lower its Cout output, resulting in Cout $1=0$, and creating a pulse with a duration of D1. The negative edge at Cout1, on the other hand, travels through the inverting delay element, D1, the second time, and produces a
positive edge at Cin 2 . This positive edge, in turn, enables the second C -element to generate positive edges at F1 and Cout2 to latch the valid data at the data 2 port.

As the data propagates through the second stage, the sequence of timing events that took place between the first and second C-elements repeat once again between the second and the third C-elements that define the boundaries of CL2. This results in generating a positive pulse at the Cout2, two negative pulses at Cin3 and F2, and a positive Cout3 edge to be able to receive a new data at data3.


Fig. 3.19 C-element and delay-element input/output activity in Fig. 3.17
The vertical slicing in Fig. 3.19 helps to define all possible stable states in designing the C-element. Even though Fig. 3.19 only samples the inputs and the outputs of the second C-element, all C-elements in Fig. 3.17 yield identical results. Every stable state from the state (1) to the state (4) allows only one input change as the fundamental-mode rule in asynchronous design methodology. The state (1) is entered when Cin $2=\mathrm{F} 2=1$, and produces F1 = 1 and Cout $2=0$. As Cin2 transitions to logic 0 , the circuit goes into the state (2) where it yields F1 $=0$ and Cout $2=0$. The start of the pulse at Cout 2 defines the state (3)
where Cin 2 transitions back to logic 1, and both outputs, F1 and Cout2, change to logic 1 . The last state, state (4), emerges when F2 switches to logic 0 . This state also causes Cout 2 to change to logic 0, but retains F1 at logic 1. The transition of F2 to logic 1 prompts the C-element to go back to the state (1).

All possible state and output changes of the C-element in Fig. 3.19 are condensed in an integrated state and output flow table in Fig. 3.20. In this figure, the "?" mark indicates that the C-element never reaches these transitionary cases. The "-" mark again defines the forbidden states where the fundamental-mode design is violated.

|  |  |  | Cin2 |  |  | F2 | Cin2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 | 00 | 01 | 11 | 10 |
| (1) | - | ? | (1) | 2 |  |  | 10 |  |
| (2) | ? | - | 3 | (2) |  |  |  | 00 |
| (3) | - | 4 | (3) | ? |  |  | 11 |  |
| (4) | ? | (4) | 1 | - |  | 10 |  |  |
|  | ns |  |  |  | F1 Cout2 |  |  |  |

Fig. 3.20 Integrated primitive state and output flow tables for C-element
The state assignments for the four stable states in Fig. 3.20 are shown in Fig. 3.21. This is a vital step in the design since the states in Fig. 3.20 are still in symbolic form and have not yet been converted into binary values.

|  | ns1 | ns2 |
| :--- | :---: | :---: |
| 1 | 0 | 0 |
| $(2)$ | 0 | 1 |
| 3 | 1 | 1 |
| 4 | 1 | 0 |

Fig. 3.21 State assignments for Fig. 3.20
Since the fundamental-mode design rule of changing only one input between state transitions is fully observed, the state table in Fig. 3.22 shows no potential racing hazards in the current C-element design.


Fig. 3.22 Hazard-free state diagram of the C-element
Once the primitive flow table and the state assignments are complete, the next state and the output K-maps of the C-element can be constructed as shown in Fig. 3.23.

ns2
F2 Cin2


Cout2 F2 Cin2


$$
\begin{aligned}
& \mathrm{ns} 1=\mathrm{F} 2 . \operatorname{Cin} 2 . \mathrm{ps} 2+\overline{\mathrm{F} 2} . \mathrm{Cin} 2 . \mathrm{ps} 1=\operatorname{Cin} 2 .(\mathrm{F} 2 . \mathrm{ps} 2+\overline{\mathrm{F} 2} . \mathrm{ps} 1) \\
& \mathrm{ns} 2=\mathrm{F} 2 . \mathrm{Cin} 2 . \mathrm{ps} 2+\mathrm{F} 2 . \overline{\mathrm{Cin} 2} \cdot \overline{\mathrm{ps} 1}=\mathrm{F} 2 .(\mathrm{Cin} 2 . \mathrm{ps} 2+\overline{\mathrm{Cin} 2} \cdot \overline{\mathrm{ps}} 1) \\
& \text { F1 }=\operatorname{Cin} 2 . p s 1+F 2 . C i n 2=C i n 2 .(p s 1+F 2) \\
& \text { Cout2 }=\text { Cin2.ps1.ps2 }
\end{aligned}
$$

Fig. 3.23 Next state and output K-maps of the C-element

In this figure, the cases marked by "?" and "-" are directly transferred from the primitive flow table in Fig. 3.20. When generating the SOP expressions for the ns1, ns2, F1 and Cout2 outputs, the cases marked with "?" and "-" signs in Kmaps are deliberately excluded from the SOP expressions in Fig. 3.23. This ensures that unwanted state transitions and outputs do not take place in the final circuit in Fig. 3.24.


Fig. 3.24 C-element circuit according to the fundamental mode design rules

The input and output names of the second C-element in Fig. 3.17 are also changed for a generic C-element. According to Fig. 3.24, the inputs, Cin 2 and F2, have become Cin and Fin, and the outputs, F1 and Cout2, have become Fout and Cout of a generic C-element, respectively.

## Review Questions

1. An asynchronous circuit has two inputs, in 1 and in 2 , and an output, out. When in $1=1$, the first transition from logic 0 to logic 1 at in 2 generates out $=1$ in the waveform below. Output stays at logic 1 unless in1 goes back to logic 0 . The first transition from logic 1 to $\operatorname{logic} 0$ at in 2 switches the output back to logic 0 while in $1=0$.

A sample waveform is given below.


Define all possible states using the waveform above and form an integrated primitive state and output flow table. Form an associated implication table leading to the minimization of states and outputs. Design the resultant fundamental mode asynchronous circuit.
2. An asynchronous circuit has two inputs, in 1 and in 2 , and an output, out. When in $1=0$, the first transition at in 2 produces out $=0$ as shown in the waveform below. When in $1=1$, a transition from logic 0 to logic 1 at in2 increments the value of out by one. When out $=3$ and in $1=1$, an additional logic 0 to logic 1 transition at in 2 produces out $=0$.


Define the possible states from the waveform above, and form the primitive state and output flow tables. Define the resultant implication table to minimize the initial states and outputs. Design the resultant fundamental mode asynchronous circuit.
3. An asynchronous circuit has three inputs, in 1 , in 2 and in 3 , and an output, out. When all inputs are at logic 0 , the first logic 0 to logic 1 transition at any input causes the output to display the input ID. For example, a logic 0 to logic 1 transition at in1 while in $2=$ in $3=0$ produces out $=1$ because this value is the ID number of in1. Similarly, the first logic 0 to logic 1 transition at in 2 while in $1=\mathrm{in} 3=0$ produces out $=2$. Logic 0 to logic 1 transition at any input while one or more inputs are at logic 1 does not change the
output value. Similarly, logic 1 to logic 0 transition does not affect neither the state of the circuit nor the output value.
in1
in2
in3
out [1:0]


Form the primitive state, the output flow table(s) and the implication table to minimize the initial state and the output assignments from the waveform above. Design the fundamental mode asynchronous circuit.
4. The schematic below is a data-path of an asynchronous system controlled by the C-elements. The combinational delays are shown by T1 through T4 blocks, each of which has a single input and output. There are also junction delays, J1 through J4, which accept two or more inputs and generate a single output.
(a) Compute D1 and D2 in terms of combinational and junction delays, T1, T2, T3, T4, J1, J2, J3 and J4.
(b) Show the data-flow that includes the signals from Cin 1 to Cin 3 , and from Cout1 to Cout3 in a timing diagram. Assume the clock-to-q delay is equal to Tc in the timing diagram.

5. Data is transferred from a $32 \times 8$ source memory to a $32 \times 8$ destination memory as shown in the schematic below. When a 32-bit data is fetched from the source memory, the high (HI) and the low (LO) 16 bits are multiplied by an integer multiplier, and the product is delivered at a destination address. The initial values of the source and the destination addresses are zero and seven, respectively. During the data transfer the source address increments by one while the destination address decrements by one until all eight data packets in the source memory are processed. Assuming that the source and the destination memories are asynchronous in nature, and neither needs a clock input to read or write data, include the C-elements in the circuit schematic to make this data transfer possible. Assume Tacc is the access time to fetch data from the source memory, Twrite is the time interval to write data to the destination memory, and Taddr is the time interval to produce addresses from the address pointer 1 and the address pointer 2.


## Projects

1. Implement the S-R latch in Fig. 3.1 and verify its functionality using Verilog.
2. Implement the fundamental mode asynchronous circuit in Fig. 3.12 and verify its functionality using Verilog.
3. Implement the C-element in Fig. 3.24 and verify its functionality using Verilog.

## Reference

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## System Bus

A system bus is responsible for maintaining all communications between the Central Processing Unit (CPU), system peripherals and memories. The system bus operates with a certain bus protocol to exchange data between a bus master and a bus slave. The bus protocol ensures to isolate all other system devices from interfering the bus while the bus master exchanges data with a bus slave. Bus master initiates the data transfer, and sends or receives data from a slave device or a system memory. Bus slave, on the other hand, does not have the capability to start data transfer.

There are two types of bus architectures. Serial bus architecture is essentially composed of a single data line between a master and a slave where data bits are exchanged one bit at a time. A parallel bus, on the other hand, is comprised of many lines, and multitude of bits are sent or received all at once. In this chapter, we will describe several serial and parallel bus protocols and priority schemes.

### 4.1 Parallel Bus Architectures

There are two types of parallel bus architectures in a typical system: unidirectional bus and bidirectional bus. A unidirectional bus contains two separate paths for data: one that originates from a bus master and ends at a slave, and the other that starts from a slave and ends at the master. A bidirectional bus, on the other hand, shares the same data path which allows data to flow in both directions. However, this type of bus requires logic overhead and control complexity.

Figure 4.1 below describes a 32 -bit unidirectional bus architecture containing two bus masters and three slaves. In this figure, the two unidirectional data-paths are highlighted with thicker lines. The first path is the "write" path, which a bus master uses to write data to a slave. This path requires a Write Data (WData) port from every master and slave. The second path is the "read" path for reading data from a slave. This also requires a Read Data (RData) port from each master and slave. Both the bus master and the slave have address and control
ports that define the destination address, the direction of data transfer, the data bit width and the length of data.

All bus masters have to negotiate with a bus arbiter to gain the ownership of the bus before starting a data transfer. When there are pending requests from multiple bus masters, the arbiter decides which bus master should start the data transfer first according to a priority scheme and issues an acknowledgement to the bus master with the highest priority. Therefore, every bus master has a Request (Req) and Acknowledge (Ack) port to communicate with the arbiter. Once the permission is granted, the master sends out the address and control signals to the selected slave in the first bus cycle, and writes or reads data in the next cycle. The decoder (DEC) unit connected to the address bus generates an Enable (EN) signal to activate the selected slave. Every master and slave device has a Ready port that indicates if the selected slave is ready to transmit or receive data.


Fig. 4.1 A unidirectional bus structure with two bus masters and three slaves
A 32-bit bidirectional bus architecture is shown in Fig. 4.2. The number of masters and slaves are kept the same as in Fig. 4.1 for comparison purposes. The only difference between the two figures is the replacement of the unidirectional data bus in the earlier architecture with a bidirectional bus for reading and writing data. Tri-state buffers on data lines are essential for bidirectional bus architectures to isolate nonessential system devices from the
bus when data transfer takes place between a master and a slave. The address bus in Fig. 4.2 can be also integrated with the data bus to allow both address and data to be exchanged on the same bus. However, this scheme is much slower and requires extra control logic overhead to maintain proper data-flow and management.


Fig. 4.2 A bidirectional bus structure with two bus masters and three slaves

Figure 4.3 shows all the Input/Output (I/O) ports of a typical bus master. The Req and Ack ports directly communicate with the arbiter as mentioned earlier. Bus master uses the "Ready" port to determine if the slave is ready to transmit or receive data. The WData, RData and Address ports are used for writing and reading data, and specifying the slave address, respectively. The control signals, Status, Write, Size and Burst, describe the nature of the data transfer.


Fig. 4.3 Bus master interface
The Status port is a two-bit bus that describes the status of the bus master as shown in Table 4.1. According to this table, the bus master may initiate a new data transfer by issuing the START signal. If the master is in the midst of exchanging data with a slave, it issues the Continue (CONT) signal. The IDLE signal is used when the bus master finishes the data transfer. The bus master may also be in the midst of an internal operation while exchanging data with a slave. For this particular instance, the master may momentarily stall the data transfer by issuing the BUSY signal.

Table 4.1 Bus master Status control

| Status[1:0] | Bus Master Status |
| :---: | :--- |
| 0 | 0 |
| 0 | 1 |

The Write port, as its name implies, describes if the master is in the process of writing or reading data as shown in Table 4.2.

Table 4.2 Bus master Write control

| Write | Bus Master Write |
| :---: | :---: |
| 0 | Read |
| 1 | Write |

The Size port describes the data bit width during a transfer and is shown in Table 4.3. A bus master is allowed to transmit or receive data in eight bits (byte), 16 bits (half-word), 32 bits (word) or 64 bits (double-word), which cannot be changed during the transfer.

Table 4.3 Bus master Size control

| Size[1:0] | Number of bits |
| :---: | :---: |
| 0 | 0 |
| 0 | 1 |

The Burst port describes the number of data packets to be sent or received by the bus master according to Table 4.4. In this table, a bus master can transfer from one packet to over 32,000 packets of data in a single burst.

Table 4.4 Bus master Burst control

| Burst[3:0] |  |  |  | Number of data packets |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 2 |
| 0 | 0 | 1 | 0 | 4 |
| 0 | 0 | 1 | 1 | 8 |
| 0 | 1 | 0 | 0 | 16 |
| 0 | 1 | 0 | 1 | 32 |
| 0 | 1 | 1 | 0 | 64 |
| 0 | 1 | 1 | 1 | 128 |
| 1 | 0 | 0 | 0 | 256 |
| 1 | 0 | 0 | 1 | 512 |
| 1 | 0 | 1 | 0 | 1024 |
| 1 | 0 | 1 | 1 | 2048 |
| 1 | 1 | 0 | 0 | 4096 |
| 1 | 1 | 0 | 1 | 8192 |
| 1 | 1 | 1 | 0 | 16384 |
| 1 | 1 | 1 | 1 | 32768 |

Figure 4.4 shows the I/O ports of a typical bus slave. The Req and Ack ports are omitted since the slave is not authorized to initiate a data transfer. The Ready signal indicates if the slave is ready to transmit or receive data once the transfer is initiated by the bus master. The WData, RData and Address ports are used to write data, read data, and specify a destination address, respectively. The control inputs, Status, Write, Size and Burst, describe the nature of the transfer as mentioned above. The Enable (EN) input is produced by the address decoder, and based on the address generated by the bus master to activate a particular slave.


Fig. 4.4 Bus slave interface

### 4.2 Basic Write Transfer

From this point forward, we will be using timing diagram(s) as a standard tool to show the bus activity between a master and a slave.

The bus protocol for write describes how a bus master writes data to a slave on a unidirectional bus as shown in Fig. 4.5.


Fig. 4.5 Basic write transfer

In the first clock cycle, the bus master sends out the destination address and the control signals, A 1 and C 1 , to the slave regardless of the slave status. If the slave status is "Ready", the actual data packet, WData1, is sent in the second cycle along with the address and the control signals, A2 and C2, of the next data packet. The slave should be able to read WData1 at the positive edge of the second clock cycle if it is ready. However, there are instances where the slave may not be ready to receive or send data. As an example, the slave changes its status to "Not Ready" in the second cycle of Fig. 4.6. As soon as the slave's status is detected at the positive edge of the third clock cycle, the master stalls the write transfer. This means that the current data packet, WData2, and the next address and control signals, A3 and C3, are repeated as long as the slave keeps its Not Ready status. The normal data transfer resumes when the slave becomes Ready to receive the remaining data.


Fig. 4.6 Basic write transfer including the case Ready $=0$
Example 4.1: What happens to the write sequence when the slave changes its status frequently?

When the slave changes its status to Not Ready during a clock cycle, the master detects this change at the next positive clock edge, and holds the current data, the next address and the control signals until the slave becomes Ready again.

An example where the slave changes its status frequently is shown in Fig. 4.7. In this figure, the slave is Not Ready in the first cycle. Therefore, the first address and control packets, A 1 and C 1 , are prolonged, and no data is sent to the slave. When the slave produces
a Ready signal during the second cycle, the bus master produces the first data packet, WData1, and changes the address and control signals to A2 and C2 at the positive edge of the third cycle. However, the slave decides to change its status to Not Ready once again during the third and the fourth clock cycles. The master detects the status change at the positive edge of the fourth and fifth clock cycles and responds by not changing A2, C2 and WData1. The Ready signal in the fifth cycle prompts the master to produce A3, C3 and WData2 at the beginning of the sixth cycle. The master holds these values until the beginning of the eighth cycle when the slave changes its status to Ready again. At this point, the master sends the new A4, C4 and WData3.


Fig. 4.7 A basic write transfer with varying Ready signal

### 4.3 Basic Read Transfer

A basic read transfer is shown in Fig. 4.8. According to this figure, the slave produces data for the master anytime after it issues the Ready signal. Once the master detects the Ready signal at a positive clock edge, it produces the next address and the control signals for the slave in the same cycle, but reads the slave's response at the next positive clock edge.


Fig. 4.8 Basic read transfer

Example 4.2: What happens to the read sequence when the slave changes its status frequently?

Figure 4.9 shows an example where the slave changes its status frequently. In this figure, the slave is Not Ready prior to the first cycle. Therefore, the master holds the first address and the control packets, A1 and C1, until the slave becomes Ready. When the master detects a Ready signal at the positive edge of the third cycle, it responds by issuing a new set of address and control signals, A2 and C2, for the slave. Within the third cycle, the slave also issues RData1 for the master. The master is not able to read this data until it detects a Ready signal from the slave at the beginning of the sixth cycle.

The rest of the read transactions in Fig. 4.9 follow the same protocol described above. In other words, the master produces a new set of address and control signals every time it detects a Ready signal from the slave, and the slave issues a new data packet for the master after it becomes Ready.


Fig. 4.9 A basic read transfer with varying Ready signal

### 4.4 Bus Master Status Change

It is possible that the bus master may be intermittently busy during a data transfer. In the event the bus master is busy to carry out its own internal tasks, the bus protocol requires the bus master to hold the address, control and data values as long as it is busy.

Figure 4.10 illustrates an example where the bus master becomes Busy in clock cycles 2,9 and 10 while writing data into a slave. The master starts the data transfer by issuing Status $=$ Start, and promptly sends out the first address, A1. In the second cycle, the bus master becomes busy with its internal operations and issues a Status = Busy signal. As a result, it repeats the previous address, A1, but is unable to dispatch any data to the slave even though Ready $=1$ during this period. In the third cycle, all internal operations cease, and the bus master continues the normal data transfer by generating Status $=$ Cont signal. The master also detects that the slave is Ready at the positive edge of the third cycle and issues the second address, A2 along with the first write data, WD1. In the next cycle, the master repeats A2 and WD1 because the slave is not Ready at the positive edge of the fourth cycle. Despite the slave showing the Not Ready condition in cycle 7, the normal data transfer sequence continues until cycle 9 where the bus master changes its status to Busy again. This change, in turn, causes the bus master to extend the address, A5, and the data, WD4, into cycles 9 and 10 irrespective of the slave status.


Fig. 4.10 Bus master Status control change
Example 4.3: What happens to the data transfer when the master changes its status frequently?

Assume that the bus master transfers two half words (16-bit wide data packets) to the addresses $0 \times 20$ and $0 \times 22$ of a memory block followed by 4 words (32-bit wide data packets) to the addresses $0 \times 5 \mathrm{c}, 0 \mathrm{x} 60,0 \times 64$ and $0 \times 68$. The address map of this byte addressable memory is shown in Fig. 4.11 where the numbers in each box indicate individual byte addresses.


Fig. 4.11 A byte-addressable memory
During the data transfer, the master issues frequent Busy signals during cycles 2, 3, 4, 7 and 8 as shown in the timing diagram in Fig. 4.12. Note that the slave is continuously Ready from cycle 2 until the end of the data transfer.


Fig. 4.12 A write transfer example to the byte addressable memory in Fig. 4.11
The bus master starts transferring the first data packet by issuing Status $=$ Start, Burst $=$ Two (data packets), Size $=$ Half word, Write $=1$ and Address $=0 \times 20$ in the first cycle according to Tables 4.1, 4.2, 4.3 and 4.4. Since the slave is Ready at the beginning of the second cycle, the master prepares to dispatch the next address, $0 \times 22$, and the first write data, Data 20. However, in this cycle the master also becomes busy with internal operations until the beginning of the fifth cycle and issues a Busy signal as shown in Fig. 4.12. The Busy condition requires the bus master to repeat its control, address and data signals during this period. Therefore, when the master finally changes its status to Cont in the fifth cycle, it is able to send Data 20 in the same cycle, and Data 22 in the following cycle.

As soon as the first data transfer finishes, the master starts another write transfer in the sixth cycle by issuing Status $=$ Start, Burst $=$ Four (data packets), Size $=$ Word, Write $=1$ and Address $=0 \times 5$ C. Since the slave's status is Ready, the master prepares to issue the next address and data packets at the beginning of the seventh cycle. However, its internal operations interfere with this process once again until the beginning of the ninth cycle. The master issues a Busy signal, and repeats its control, address and data outputs. When the master finally changes its status to Cont in the ninth cycle, it delivers the second address, $0 x 60$, and the first data, Data 5C. In the tenth cycle, the next address, $0 x 64$, and data, Data 60, are issued, respectively. The master writes Data 64 in the eleventh cycle, and finishes the transfer by writing the last data, Data 68, in the twelfth cycle. In this cycle, the bus master changes its status to Idle, indicating the end of the data transfer.

### 4.5 Bus Master Handshake

Each bus master communicates with the arbiter using request-acknowledge signals, which form the basic "handshake" protocol. The master needing a data transfer issues a request signal, Req, requesting the ownership of the bus from the arbiter. The arbiter grants this request by an acknowledgement signal, Ack. If there is no ongoing data transfer, the acknowledgement is usually issued in the following cycle after the master generates a request. However, the Ack signal may not be generated many cycles after the Req signal is issued due to an existing data transfer.

Figure 4.13 shows the timing diagram of a handshake mechanism between a bus master and the arbiter before the bus ownership is granted to the master. The double " $\sim$ " sign on the Ack signal signifies that this signal is generated many cycles after the arbiter has received a request from the bus master. As soon as the master receives the Ack signal in the nth cycle, it changes its status to Start in the $(\mathrm{n}+1)$ th cycle, and sends out the first address, A1, regardless of the slave's status. If the slave is Ready, the master subsequently sends out the second address, A2, and the first data, WData1, in the following cycle.


Fig. 4.13 Bus master-arbiter handshake protocol

### 4.6 Arbiter

Bus arbitration is an essential part of bus management if there are more than one bus master requesting the ownership of the bus. The arbitration is either hardware-coded and implemented as a state machine or programmable and register-based.

Table 4.5 explains a hardware-coded bus arbitration mechanism between two bus masters. When there are no requests to the arbiter, no Ack is generated to either bus master. However, if two requests are issued at the same time, the acknowledge is issued to bus master 1 according to this table since bus master 1 is assumed to have higher priority than bus master 2 as shown in the last row.

Table 4.5 Bus arbitration table for two bus masters

From IDLE State $\rightarrow$| Req1 | Req2 | Ack1 | Ack2 |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

The Table 4.5 is implemented as a state machine in Fig. 4.14. In this figure, the shorthand representation of $\operatorname{Req}=($ Req1 Req2 $)$ corresponds to the bus master request inputs 1 and 2. Similarly, Ack $=($ Ack1 Ack2 $)$ corresponds to the acknowledge signals generated by the arbiter for bus masters 1 and 2 , respectively.


Fig. 4.14 Bus arbiter with two bus masters
The arbiter is normally in the IDLE state when there are no pending requests. If there are simultaneous requests from bus masters 1 and 2 , the arbiter moves from the IDLE state to the ACK1 state, generates Ack1 = 1 for bus master 1, and ignores the request from bus master 2 by Ack2 $=0$ according to Table 4.5. The inputs for this state-to-state transition are shown by
$\operatorname{Req}=(1 \mathrm{x})$, where Req1 $=1$ and Req2 $=\mathrm{x}$ (don't care). When bus master 1 terminates the data transfer by issuing Req1 $=0$, the arbiter either stays in the ACK1 state if there is another pending request from bus master 1 or moves back to the IDLE state if there are no requests. However, if the arbiter receives Req1 $=0$ and Req2 $=1$ while in the ACK1 state, it transitions to the ACK2 state, and issues Ack2 $=1$ to bus master 2.

In a similar fashion, the transition from the IDLE state to the ACK2 state requires Req2 $=1$ and Req1 $=0$. Once in the ACK2 state, the arbiter grants the usage of the bus to bus master 2 by issuing Ack2 $=1$ and Ack1 $=0$. When bus master 2 finishes the transfer by issuing Req2 $=0$, the arbiter either goes back to the IDLE state or transitions to the ACK1 state if Req1 $=1$. In case the higher priority bus master, bus master 1, requests the ownership of the bus by Req1 = 1 while the lower priority bus master is in the middle of a transfer, the arbiter stays in the ACK2 state as long as Req2 $=1$ from bus master 2, ensuring the data transfer is complete.

Example 4.4: Design a hardware-coded arbiter with three bus masters where bus master 1 has the highest priority followed by bus masters 2 and 3 .

According to this definition, the bus master priorities can be tabulated in Table 4.6.
Table 4.6 Bus arbitration table for three bus masters

| From IDLE State $\rightarrow$ Req1 | Req2 | Req3 | Ack1 | Ack2 | Ack3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 |

This table generates no acknowledge signal if there are no requests from any of the bus masters (the top row). If only bus master 3 requests the bus, Ack $3=1$ is generated for bus master 3 (the second row from the top). If there are two pending requests from bus masters 2 and 3 , Ack $2=1$ is issued for bus master 2 because it has higher priority than bus master 3 (fourth row from the top). If all three bus masters request the ownership of the bus, the arbiter grants the bus to bus master 1 by Ack1 = 1 because it has the highest priority with respect to the remaining bus masters (the last row).

The implementation of this priority table is shown in Fig. 4.15 as a state machine. The naming convention in representing request and acknowledge signals in Fig. 4.15 is the same as in Fig. 4.14. Therefore, Req = (Req1 Req2 Req3) corresponds to bus master requests 1, 2 and 3, and Ack $=($ Ack1 Ack2 Ack3) corresponds to the arbiter acknowledge signals for bus masters 1,2 and 3, respectively. Normally, the arbiter is in the IDLE state when there are no requests. If there are three simultaneous requests from bus masters 1,2 and 3 , the arbiter transitions to the

ACK1 state where it generates Ack1 = 1 since bus master 1 has the highest priority. When bus master 1 finishes the data transfer, the arbiter can either stay in the ACK1 state or transition to the ACK2 state or transition the ACK3 state depending on the requests from all three bus masters. If there are no pending requests, the arbiter goes back to the IDLE state.

The arbiter does not issue acknowledge signals to higher priority bus masters until an ongoing data transfer of a lower priority bus master is complete. For example, in the ACK3 state the requests from bus masters 1 and 2 are ignored as long as bus master 3 keeps its request high to continue transferring data.


Fig. 4.15 Bus arbiter with three bus masters

### 4.7 Bus Master Handover

The bus may be handed over to a different bus master if the current bus master lowers its request.

Figure 4.16 describes how this bus ownership takes place in a unidirectional bus. In this timing diagram, the current bus master, bus master 1, starts a new transfer by generating a Start signal a cycle after it receives Ack1 $=1$ from the arbiter. The write transfer continues until the eighth cycle when the bus master delivers its last address, A4. In cycle nine, the bus master delivers its last data, WA4, lowers its request, and changes its status to Idle, thus terminating the data transfer. At the positive edge of the tenth cycle, the arbiter detects Req1 $=0$ and Req2 $=1$,
and switches the bus ownership by issuing Ack1 $=0$ and Ack2 $=1$. The new master, bus master 2, starts a new transfer in the eleventh cycle and generates its first address, B1. The write transfer continues until the fourteenth cycle when bus master 2 delivers its last data, WB2.


Fig. 4.16 Bus master handover protocol

### 4.8 Serial Buses

Peripheral devices and external buffer memories that operate at low frequencies communicate with the processor using a serial bus.

There are currently two popular serial buses used in low-speed communication. The Serial Peripheral Interface (SPI) was introduced in 1979 by Motorola as an external microprocessor bus for the well-known Motorola 68000 microprocessor. The SPI bus normally requires four wires; however, wire count increments by one every time a peripheral device is added to the system. The second bus, Inter-Integrated Circuit ( $\mathrm{I}^{2} \mathrm{C}$ ), was developed by Philips in 1982 to connect Philips CPUs to peripheral chips in a TV set. This bus requires only two wires, but it is considerably slower compared to the SPI bus.

## Serial Peripheral Interface (SPI)

SPI is designed as a very straightforward serial bus. Four signals establish all the serial communication between a CPU and a peripheral device. The SPI clock signal, SCK, is
distributed to all the slaves in a system, and forces each peripheral to be synchronous with a single master. The Slave Select signal ( $\overline{\mathrm{SS}})$ is an active-low signal, and is used to enable a particular slave prior to data transfer. The Serial-Data-Out (SDO) or Master-Out-Slave-In (MOSI) port is what the master uses to send serial data to a slave. The Serial-Data-In (SDI) or Master-In-Slave-Out (MISO) port is what the master uses to read serial data from a slave.

Figure 4.17 shows the serial bus configuration between the bus master and a single slave. All SPI signals with the exception of SDI must be initiated by the bus master.


Fig. 4.17 SPI bus between a master and a single slave
When the bus master is connected to a multitude of slaves, it needs to generate an active-low Slave Select signal for each slave as shown in Fig. 4.18.


Fig. 4.18 SPI bus between a master and three slaves

SPI is considered a single-master serial communication protocol. This means that only one master is assigned to initiate and carry out all serial communications with slaves. When the SPI master wishes to send or request data from a slave, first it selects a particular slave by lowering the corresponding $\overline{\mathrm{SS}}$ signal to logic 0 , and then it produces a clock signal for the slave as shown in Fig. 4.19. Once the select and the clock signals are established, the master sends out serial data to the selected slave from its SDO port at the negative edge of SCK, and simultaneously samples slave data at its SDI port at the positive edge of SCK. According to the SPI protocol, the slave is capable of sending and receiving data except it cannot generate SCK.

In the following example in Fig. 4.19, the master sends out serial data, DataM1 (most significant bit) to DataM4 (least significant bit), from its SDO port at the negative edge of SCK, and samples slave data, DataS1 (most significant bit) to DataS4 (least significant bit), at its SDI port at the positive edge of SCK. The slave, on the other hand, can also dispatch serial data packets, DataS1 (most significant bit) to DataS4 (least significant bit), from its SDO port at the negative edge of SCK, and sample the master data, DataM1 (most significant bit) to DataM4 (least significant bit), at its SDI port at the positive edge of SCK.

SCK is initially at logic 0
Data release at the negative edge of SCK
Data fetch at the positive edge of SCK


Fig. 4.19 SPI bus protocol between a master and a single slave

There are four communication modes available for the SPI bus protocol. Each protocol is categorized according to the initial SCK level (the logic level at which SCK resides at steady state) and the data generation edge of the SCK. Each communication mode is shown in Fig. 4.20.

MODE 0 communication protocol assumes that the steady state level for SCK is at logic 0 . Each data bit is generated at the negative edge of SCK by the master (or the slave), and is sampled at the positive edge. A good example of the MODE 0 protocol is shown in Fig. 4.19.

MODE 1 still assumes the steady state level of SCK to be at logic 0, but the data generation takes place at the positive edge of SCK. Both the master and the slave read data at the negative edge in this mode.

MODE 2 switches the steady state level of SCK to logic 1. Data is released at the positive edge of SCK and is sampled at the negative edge as shown in Fig. 4.20.

MODE 3 also accepts the steady state level of SCK at logic 1. However, data is released at the negative edge of SCK and is sampled at the positive edge.

A master-slave pair must use the same mode during a data exchange. If multiple slaves are used, and each slave uses a different communication mode, the master has to reconfigure itself each time it communicates with a different slave.

SPI bus has neither an acknowledgement mechanism to confirm receipt of data nor it offers any other data-flow control. In reality, an SPI bus master has no knowledge if a physical slave exists on the other side of the bus, or the data it sends is properly received by the slave. Most SPI implementations pack eight bits of data in a clock burst. However, many variants of SPI today use 16 or even 32 clock cycles to send more data bits in a burst in order to gain speed.

MODE 0
SCK is initially at logic 0
Data release at the negative edge of SCK


MODE 1
SCK is initially at logic 0
Data release at the positive edge of SCK
SCK

DATA


MODE 2
SCK is initially at logic 1
Data release at the positive edge of SCK


MODE 3
SCK is initially at logic 1
Data release at the negative edge of SCK


Fig. 4.20 SPI bus protocol modes

## Inter Integrated Circuit $\left(\mathbf{I}^{2} \mathbf{C}\right.$ )

Inter Integrated Circuit $\left(\mathrm{I}^{2} \mathrm{C}\right)$ is a multi-master bus protocol that transmits and receives data between bus masters and slaves using only two signal lines, Serial Clock (SCL) and Serial Data (SDA).

Slave selection with slave select signals, address decoders or arbitrations is not necessary for this particular bus protocol. Any number of slaves and masters can be employed in an $\mathrm{I}^{2} \mathrm{C}$ bus using only two lines.

The data rate is commonly at 100 Kbps which is the standard mode. However, the bus can operate as fast as 400 Kbps or even at 3.4 Mbps at high speed mode.

Physically, the $\mathrm{I}^{2} \mathrm{C}$ bus consists of the two active wires, SDA and SCL, between a master and a slave device as shown in Fig. 4.21. The clock generation and data-flow are both bidirectional. This protocol assumes the device initiating the data transfer to be the bus master; all the other devices on the $\mathrm{I}^{2} \mathrm{C}$ bus are regarded as bus slaves.


Fig. 4.21 $\mathrm{I}^{2} \mathrm{C}$ architecture
In a typical $\mathrm{I}^{2} \mathrm{C}$ bus, both the bus master and the slave have two input ports, SCL In and SDA In, and two output ports, SCL Out and SDA Out, as shown in Fig. 4.21. When a master issues SCL Out = 1 (or SDA Out =1), the corresponding n-channel MOSFET turns on, and pulls the SCL line (or SDA line) to ground. When the master issues SCL Out $=0$ (or SDA Out = 0), it causes the corresponding n-channel transistor to turn off, resulting SCL (or SDA) afloat. However, neither SCL nor SDA is truly left floating in an undetermined voltage level. The pull-up resistor, Rpu, immediately lifts the floating line to the power supply voltage level, VDD. On the other side of the bus, the $\mathrm{I}^{2} \mathrm{C}$ slave detects the change at the SCL In (or SDA In) port, and determines the current bus value.

Each slave on the $I^{2} \mathrm{C}$ bus is defined by an address field of either seven bits or ten bits as shown in Fig. 4.22. Each data packet following the address is eight bits long. There are only four control signals that regulate the data flow: Start, Stop, Write/Read and Acknowledge.

The seven-bit and ten-bit versions of read and write data transfers are shown in Fig. 4.22. The top sequence in this figure explains how a bus master writes multiple bytes of data to a slave that uses a seven-bit address. The master begins the sequence by generating a Start bit. This acts as a "wake-up" call to all the slave devices and enables them to watch for the incoming address. This step is followed by a seven-bit long slave address. The bus master sends the most significant address bit first. The remaining address bits are released one bit at a time until the least significant bit. At this point, all slave devices compare the bus address just sent out with their own addresses. If the address does not match, the slave simply ignores the rest of the incoming bits on the SDA bus, and waits for the beginning of the next bus transfer. If the addresses match, however, the addressed slave waits for the next bit that indicates the type of the data transfer from the master. When the master sends out a Write bit, the slave responds with an acknowledge signal, SAck, by pulling the SDA line to ground. The master detects the acknowledge signal, and sends out the first eight-bit long data packet. The format for transmitting data bits is the same as the address: the most significant bit of the data packet is sent out first followed by the intermediate bits and the least significant bit. The slave produces another acknowledgement when all eight data bits are successfully received. The data delivery continues until the master completes sending all of its data packets. The transfer ends when the master generates a Stop signal.

The second entry in Fig. 4.22 shows the write transfer to a bus slave whose address is ten bits long. Following the Start bit, the bus master sends out a five-bit preamble, 11110, indicating that it is about to send out a ten-bit slave address. Next, the master sends out the two most significant address bits followed by the Write bit. When the delivery of all these entries is acknowledged by the slave(s) whose two most significant address bits match the ones sent by the master, the master sends out the remaining eight address bits. This is followed by another slave acknowledgement, and the master transmitting all of its data bytes to the designated slave. The data transfer completes when the bus master generates a Stop bit.

The third and the fourth entries in Fig. 4.22 show the seven-bit and ten-bit read sequences initiated by the bus master. In each sequence, after receiving the Start bit and the address from the master, the designated slave starts sending out data packets to the master. After successfully receiving the first data byte, the master responds to the slave with an acknowledge signal, MAck, after which the slave transmits the next byte. The transfer continues until the slave delivers all of its data bytes to the master. However, right before issuing a Stop bit, the master generates a no-acknowledgement bit, MNack, signaling the end of the transfer as shown in Fig. 4.22.

| Addr Mode $=7$ bits |  |  |  | Master response |  | Slave response |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| START | Slave Address | Write | SAck | Data1 | SAck | DataN | SAck | STOP |
| 1 bit | 7 bits | 1 bit | 1 bit | 8 bits | 1 bit | 8 bits | 1 bit | 1 bit |



| Master $=$ Read Addr Mode $=7$ bits |  |  |  | Master response |  | Slave response |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| START | Slave Address | Read | SAck | Data1 | MAck | DataN | MNack | STOP |
| 1 bit | 7 bits | 1 bit | 1 bit | 8 bits | 1 bit | 8 bits | 1 bit | 1 bit |


| Master $=$ Read |  | Addr Mode $=10$ bits |  | Master response |  | Slave response |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| START | 11110 | Slave Address | Read | SAck | Slave Address | SAck | Data1 | MAck | DataN | MNack | STOP |
| 1 bit | 5 bits | msb (2 bits) | 1 bit | 1 bit | Isb (8 bits) | 1 bit | 8 bits | 1 bit | 8 bits | 1 bit | 1 bit |

Fig. 4.22 $\mathrm{I}^{2} \mathrm{C}$ modes of operation
The Start and Stop signals are generated by the combination of SCL and SDA values as shown in Fig. 4.23. According to this figure, a Start signal is produced when the SDA line is pulled to ground by the bus master while $\mathrm{SCL}=1$. Similarly, a Stop signal is created when the bus master releases the SDA line while $\mathrm{SCL}=1$.


Fig. 4.23 $\mathrm{I}^{2} \mathrm{C}$ data stream start and stop conditions
Figure 4.24 shows when data is permitted to change, and when it needs to be steady. The $I^{2} \mathrm{C}$ protocol only allows data changes when SCL is at logic 0 . If the data on SDA changes while SCL is at logic 1 , this may be interpreted as a Start or a Stop condition depending on the data transition. Therefore, the data on SDA is not allowed to change as long as $\mathrm{SCL}=1$.


Fig. 4.24 $I^{2} \mathrm{C}$ data change conditions
Figure 4.25 explains the timing diagram in which the bus master writes two bytes of data to a slave with a seven-bit address. According to this figure, the write process starts with transitioning the value at the SDA to logic 0 while $\mathrm{SCL}=1$. Following the Start bit, the slave address bits are delivered sequentially from the most significant bit, SA[6], to the least significant bit, SA[0]. Each address bit is introduced to the SDA only when SCL is at logic 0 according to the $\mathrm{I}^{2} \mathrm{C}$ bus protocol shown in Fig. 4.24. The Write command and the subsequent slave acknowledgement are generated next. The data bits in Byte 0 and Byte 1 are also delivered to the SDA starting from the most significant data bit, $\mathrm{D}[7]$. The write sequence finishes with the SDA transitioning to logic 1 while $\mathrm{SCL}=1$.


Fig. 4.25 $\mathrm{I}^{2} \mathrm{C}$ write timing diagram
Figure 4.26 shows the timing diagram of reading two bytes of data from a slave. Following the Start bit and the slave address, the master issues the Read command by SDA $=1$. Subsequently, bytes of data are transferred from the slave to the master with the master acknowledging the delivery of each data byte. The transfer ends with the master not acknowledging the last byte of data, MNack, and generating the Stop bit.


Fig. 4.26 $I^{2} \mathrm{C}$ read timing diagram
Here comes the reason why the $\mathrm{I}^{2} \mathrm{C}$ bus protocol excels in maintaining flawless communication between any number of masters and slaves using only two physical wires. For example, what happens if two or more devices are simultaneously trying to write data on the SDA? At the electrical level, there is actually no contention between multiple devices trying to simultaneously enter a logic level on the bus. If a particular device tries to write logic 0 to the bus while the other issues logic 1, then the physical bus structure with pull-up resistors in Fig. 4.21 ensures that there will be no electrical short or power drainage; the bus actually transitions to logic 0 . In other words, in any conflict, logic 0 always wins!

This physical structure of the $\mathrm{I}^{2} \mathrm{C}$ bus also allows bus masters to be able to read values from the bus or write values onto the bus freely without any danger of collision. In case of a conflict between two masters (suppose one is trying to write logic 0 and the other logic 1), the master that tries to write logic 0 gains the use of the bus without even being aware of the conflict. Only the master that tries to write logic 1 will know that it has lost the bus access because it reads logic 0 from the bus while trying to write logic 1 . In most cases, this device will just delay its access to the bus, and try it later.

Moreover, this bus protocol also helps to deal with communication problems. Any device present on the bus listens to the bus activity, particularly the presence of Start and Stop bits. Potential bus masters on the $I^{2} \mathrm{C}$ bus detecting a Start signal will wait until they detect the Stop signal before attempting to access the bus. Similarly, unaddressed slaves go back to hibernation mode until the Stop bit is issued.

Similarly, the master-slave pair is aware of each other's presence by the active-low acknowledge bit after the delivery of each byte. If anything goes wrong, and the device sending the data does not receive any acknowledgment, the device sending the data simply issues a Stop bit to stop the data transfer and releases the bus.

An important element of the $\mathrm{I}^{2} \mathrm{C}$ communication is that the master device determines the clock speed in order to synchronize with the slave. If there are situations where an $\mathrm{I}^{2} \mathrm{C}$ slave device is not able to keep up with the master because the clock speed is too high, the master
can lower the frequency by a mechanism referred to as "clock stretching". According to this mechanism, an $\mathrm{I}^{2} \mathrm{C}$ slave device is allowed to hold the SCL at logic 0 if it needs the bus master to reduce the bus speed. The master is required to observe the SCL signal level at all times and proceeds with the data transfer until the line is no longer pulled to logic 0 by the slave.

All in all, both SPI and $\mathrm{I}^{2} \mathrm{C}$ offer good support for low-speed peripheral communication. SPI is faster and better suited for single bus master applications, and $\mathrm{I}^{2} \mathrm{C}$ is slower but better suited for multi-master applications. The two protocols offer the same level of robustness and have been equally successful among vendors producing Flash memories, Analog-to-Digital and Digital-to-Analog converters, real-time clocks, sensors, liquid crystal display controllers etc.

## Review Questions

1. A CPU reads three bursts of data from a 32 -bit wide byte-addressable memory in the following manner:

- It reads four bytes with the starting address $0 x F 0$,
- Immediately after the first transaction, the CPU reads two half-words from the starting address 0xF4,
- Immediately after the second transaction, it reads one word from the starting address 0xF8.

The contents of the memory are as follows:

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 0xCC | 0xDD | 0xEE | 0xFF |
| 0x88 | $0 \times 99$ | 0xAA | 0xBB |
| 0x44 | $0 \times 55$ | 0x66 | 0x77 |
| 0x00 | $0 \times 11$ | 0x22 | 0x33 |

The unidirectional bus protocol states that the data communication between a bus master and a slave requires generating the address and control signals in the first cycle, and the data in the second cycle. The bus master always issues a Start signal to indicate the start of a data transmission. After an initial address, the master changes its status to Cont to indicate the continuation of the data transfer. The bus master issues Idle to indicate the end of the data transfer or Busy to indicate its incapability to produce address and control signals (and data if applicable). Any time the bus master is Busy, it repeats its address and control signals (and data if applicable) from the previous clock cycle. Similarly, if a Ready signal is not generated by the slave, the bus master also repeats its address and control signals (and data if applicable) in the next clock cycle. At the end of a data transfer when the bus master finishes issuing new addresses, it transitions to the Idle state even though there may be a residual data still in the process of being read or written in the subsequent clock cycle(s).

Fill in the blanks of the following timing diagram to complete all three data read bursts in the order specified above.

2. The following bidirectional bus maintains the communication between a CPU and a memory.


The CPU has the Addr and the WData outputs to dispatch the address and the write-data, respectively. It also uses the RData output to receive data from the memory.

The memory, on the other hand, has the Addr input to receive address from the CPU, and a bidirectional Data port to receive and send data.

To validate the address and write-data, the EnAddr and EnWData signals are issued to the bus controller, respectively. To validate read-data, the memory dispatches the Ready signal to the controller. With all these inputs from the CPU and the memory, the controller generates the WE and RE signals for the memory to write and read data, and the EnRData signal for the CPU to validate the read data. The signals, SelAddr, SelWData and SelRData, are also generated by the bus controller to manage the timely distribution of the address, write-data and read-data using a single 32 -bit wide bidirectional bus as shown in the figure above.

The write process to the memory requires a valid address with data as shown below:


The read process requires a cycle delay to produce valid data from the memory once an address is issued. This is shown below:

(a) Since the bus protocol delivers data following a valid address, construct a timing diagram to write the data, W1 to A1, W2 to A2, W3 to A3 and W4 to A4. Without any delay, perform a read sequence to fetch the data packets, R1, R2, R3 and R4, from the memory addresses, A5, A6, A7 and A8, respectively. Plot a timing diagram, including the 32 -bit bus, Bus[31:0], and the control signals, WE, RE, SelAddr, SelWData and SelRData, for the write and read sequences.
(b) Design the CPU and the memory interfaces with the bidirectional bus such that these two data transfers are possible (note that these interfaces are not state machines).
3. A bus master writes four bytes of data to the following address locations of a 32-bit wide byte-addressable memory (slave) organized in a Little Endian format:

| Address | Data |
| :---: | :---: |
| 0x0D | $0 \times 11$ |
| $0 \times 11$ | 0x22 |
| $0 \times 15$ | $0 \times 33$ |
| 0x19 | 0x44 |

Following the write cycle, the same bus master reads data (words) from the following slave addresses:

| Address | Data |
| :---: | :---: |
| 0x3C | $0 \times A A B B C C D D ~$ |
| 0x40 | 0x55667788 |

(a) Describe the contents of the memory after the writing and reading cycles shown above become complete.
(b) If the bus master generates the first address during the first clock cycle and keeps generating new addresses every time the slave responds with a Ready signal, what will be the values of the address, control and data entries in the timing diagram below? Assume that the bus master does not produce any Status signal comprised of Start, Cont, Busy and Idle.

4. A bus master reads four data packets starting from the address, $0 x 00$, and ending at the address, $0 x 03$, from an eight-bit wide memory. Immediately after this transaction, the bus master writes $0 \times 00,0 \times 11,0 \times 22$ and $0 \times 33$ into the addresses $0 \times 04,0 \times 05,0 \times 06$ and $0 \times 07$ respectively. This memory contains the following data after this operation:

| 7 |  |
| :---: | :---: |
| 00 | 0xAA |
| 01 | 0xBB |
| 02 | 0xCC |
| 03 | 0xDD |
| 04 | 0x00 |
| 05 | 0x11 |
| 06 | 0x22 |
| 07 | 0x33 |

Assuming the unidirectional bus protocol is the same as described in question 1, fill in the blanks of the timing diagram below to accommodate each read and write transfer.

5. A bus master is connected to four memory blocks acting as bus slaves in a bidirectional bus where 32-bit address, write-data and read-data are sent or received on the same bus. The I/O ports of the bus master and the slaves are shown below:

| Bus Master |
| ---: |
| RData[31:0] |
| WData[31:0] |
| Addr[31:0] |
| EN |
| W/R |
| EnAddr |
| Ready |


| Bus Slave |
| :--- |
| Addr/Data[31:0] |
|  |
| EN |
| WE |
| Ready |

The bus master has separate read and write data ports to receive and transmit data, respectively. The Enable, and W/R ports enable the bus master to write data, i.e. EN = 1 and $\mathrm{W} / \mathrm{R}=1$. Similarly, $\mathrm{EN}=1$ and $\mathrm{W} / \mathrm{R}=0$ enable the bus master to read. Since address and data entries share the same bus, the bus master provides a third control signal, EnAddr, to enable the address. The bus master determines if the slave is ready to receive of transmit data through its Ready signal.

The slave, in contrast, has only one port for receiving address or data. $\mathrm{EN}=1$ and $\mathrm{WE}=1$ writes data to the slave. If data needs to be read from the slave, then $\mathrm{EN}=1$ and $\mathrm{WE}=0$ are used.

Draw the architectural diagram of such a system. Make sure to use the most significant address bits, Addr[31:30], to select one of the slaves to read or write data.
6. A 16-bit digital system with a unidirectional data and address bus is given below.


This system contains three bus masters, a Microprocessor Unit (MPU), Co-processor 1 and Co-processor 2. It also contains three slaves, Memory 1, Memory 2 and Memory 3.

A bus arbiter is responsible for prioritizing the ownership of the bus among the three bus masters. The MPU has the highest and Co-processor 2 has the lowest priority to use the bus. When the arbiter gives the ownership of the bus to a bus master, the bus master is free to exchange data with a slave as long as it keeps its request signal at logic 1 . When the bus master lowers its request signal after finishing a data transfer, the arbiter also lowers its grant to assign the bus to another bus master according to the priority list.

As long as a bus master owns the bus, it can send a 16 -bit write-data (WData) to the selected slave at a specific address (Addr). Similarly, the bus master can read data from the selected slave using a 16-bit read-data (RData) bus.

For the sake of simplicity, control signals on the schematic are not shown; however, each memory has the Read Enable (RE) and Write Enable (WE) ports to control data storage.
(a) With the description above, draw the state diagram of the bus arbiter.
(b) While MPU and Co-processor 1 remain at idle, Co-processor 2 requests two data transfers. The first data transfer is from the addresses, $0 x A B C 0$ and $0 x A B C 1$, of Memory 1 to the addresses, 0x0000 and 0x0001, of Memory 2, respectively. The second one is from the address, $0 x 0002$, of Memory 2 to the address, $0 x A B C 2$, of Memory 1.

The first one is from the addresses $0 x A B C 0$ and $0 x A B C 1$ of Memory 1 to the addresses 0x0000 and 0x0001 of Memory 2, respectively. The second one is from the address $0 x 0002$ of Memory 2 to the address $0 x A B C 2$ of Memory 1, respectively.

Note that Co-processor 2 and Memory 1 operate at a clock frequency twice as high as the clock frequency used in Memory 2. Both memories have a read latency (access time) of one clock cycle, i.e. data becomes available in the next clock cycle after issuing a valid address with $\mathrm{RE}=1$. Write happens within the same clock cycle when the address is valid and $\mathrm{WE}=1$.

Including the request (reqd), grant (grantd) and address (Addrd) from Co-processor 2, and the control signals (RE1, WE1, RE2, WE2) from each memory, create a timing diagram that shows data transfers between Memory 1 and Memory 2. If you prefer, use the timing diagram template below to make your entries.


7. The waveforms below describe serial transmission of data using two known bus protocols, $\mathrm{I}^{2} \mathrm{C}$ and SPI.
(a) A bus master writes data to an $\mathrm{I}^{2} \mathrm{C}$-compliant slave according to the timing diagram below. Assuming that the bus master uses the negative edge of SCLK to produce data on the SDA bus, determine the slave address and the data packets in binary format.

(b) Now, the bus master transmits the same data packets on the SPI bus. Using the timing diagram below, show the value of each data bit at the SDI terminal. Note that the bus master uses Mode0 convention and produces data at the negative edge of SCK.


## Projects

1. Implement the unidirectional bus with two bus masters and three slaves shown in Fig. 4.1 using Verilog. Make sure both of the bus masters are able to produce status signals, START, CONT, BUSY and IDLE to transfer data packets from one byte to two words (64 bits) on a 32-bit wide bus. Similarly, ensure that all the slaves are able to generate Ready signals compliant to the parallel bus protocol given in this chapter. Design the bus arbiter shown in Fig. 4.14. Verify each individual block, i.e. the bus master, the slave, the arbiter and the overall system functionality.
2. Implement the bidirectional bus with two bus masters and three slaves as shown in Fig. 4.2 using Verilog. Make sure that the bus masters and slaves are fully compliant to the parallel bus protocol given in this chapter. Design the bus arbiter shown in Fig. 4.14. Again, verify each individual block, i.e. the bus master, the slave, the arbiter and the entire bus system.
3. Implement the SPI bus with one bus master and three slaves as shown in Fig. 4.18 using Verilog. Verify the system functionality with timing diagrams.
4. Implement the $\mathrm{I}^{2} \mathrm{C}$ bus in seven-bit addressing mode in Fig. 4.22 using Verilog. Verify the system functionality with timing diagrams.

## Memory Circuits and Systems

Basic serial and parallel bus structures and different forms of data transfer between a bus master and a slave were explained in Chapter 4. Regardless of the bus architecture, the bus master is defined as the logic block that initiates the data transfer, and the slave is defined as the device that exchanges data with the master on demand. Both devices, however, may include a buffer memory. The master may have an internal memory that stores user programs or data, and the slave may have a large capacity system memory or a small buffer memory.

Depending on the read and write speed, capacity and permanence of data, system memories and peripheral buffers can be categorized into three different forms. If fast read and write times are desired, Static Random Access Memory (SRAM) is used despite its relatively large cell size compared to other types of memory. SRAM is commonly used to store small temporary data, and it is typically connected to a high speed parallel bus in a system. If large amounts of storage are required, but slow read and write speed can be tolerated, then Dynamic Random Access Memory (DRAM) should be the main memory type to be used. DRAMs are still connected to the high speed parallel bus, but typically operate with large bursts of data. A typical DRAM cell is much smaller than an SRAM cell with significantly lower power consumption. The main drawbacks of DRAM are high data read and write latencies, the complexity of memory control and management of data.

The permanence of data yet calls for a third memory type whose cell type consists of a double-gated Metal-Oxide-Semiconductor (MOS) transistor. Data is permanently stored in the floating gate of the device until it is overwritten. Electrically-Erasable-Programmable-Read-Only-Memory ( $\mathrm{E}^{2} \mathrm{PROM}$ ) or Flash memory fit into this type of device category. The advantage of this memory type is that it keeps the stored data even after the system power is turned off. However, this memory is the slowest compared to all other memory types, and it is subject to a limited number of read and write cycles. Its optimal usage is, therefore, to store permanent data for Built-In-Operating-Systems (BIOS), especially in hand-held devices where power consumption is critical. A typical computing system can contain one or all three types of memories depending on the usage and application software.

The basic functionality of SDRAM, E ${ }^{2}$ PROM and Flash memory blocks in this chapter is inspired from Toshiba memory datasheets [1-6]. The more recent serial Flash memory with SPI interface in this chapter is based on an Atmel Flash memory datasheet [7]. In each case, the functionality of the memory block has been substantially simplified (and modified) compared to the original datasheet in order to increase reader's comprehension of the subject matter. The purpose here is to show how each memory type operates in a system, covering only the basic modes of operation to train the reader rather than going into the details of the actual datasheets. The address, data and control timing constraints for each memory have also been simplified compared to the datasheets. This allows us to design the bus interface for each memory type with ease. For the sake of simplicity, we avoided duplicating the port names, exact timing requirements and functionality details that can be found in the actual datasheets. After reading this chapter, interested readers are encouraged to study the referenced datasheets prior to carrying out their design tasks.

### 5.1 Static Random Access Memory

Static Random Access Memory (SRAM) is one of the most fundamental memory blocks in digital design. Among all different types of memory, SRAM ranks the fastest; however, its large memory cell size limits its usage for a variety of applications.

A typical SRAM architecture shown in Fig. 5.1 is composed of four different blocks: the SRAM core, the address decoder, the sense amplifier and the internal SRAM controller. The memory core retains all immediate data. The sense amplifier amplifies the cell voltage to full logic levels during read. The address decoder generates all $2^{\mathrm{N}}$ Word Lines (WL) from an N -bit wide address. Finally, the controller generates self-timed pulses required during a read or write cycle.

Each SRAM cell is composed of two back-to-back inverters like the ones used in a latch, and two N-channel Metal Oxide Semiconductor (NMOS) pass-gate transistors to isolate the existing data in the cell or to allow new data into the cell as shown in Fig. 5.2. When data needs to be written to a cell, $\mathrm{WL}=1$ turns on both NMOS transistors, allowing the true and complementary bits of data to be simultaneously written from the Bit and Bitbar inputs. Suppose the node A is initially at logic 0 , and the node B at logic 1 but $\mathrm{WL}=0$. The logic level at WL turns off both NMOS transistors, and the latch becomes completely isolated from its surroundings. As a result, logic 0 level is contained in the cell. But, if $\mathrm{WL}=1, \mathrm{Bit}=1$ and Bitbar $=0$, this time the logic level at WL turn on both NMOS transistors, and the values at the Bit and the Bitbar nodes overwrite the existing logic levels at the nodes A and B, thereby changing the stored bit in the cell from logic 0 to logic 1 .

Similarly, if the data needs to be read from the cell, both NMOS transistors are turned on by $\mathrm{WL}=1$, and the small differential potential developed between the Bit and the Bitbar outputs are amplified by a sense amplifier to reach full logic levels at the SRAM output.


Fig. 5.1 A typical SRAM architecture with eight-bit address and 32-bit data


Fig. 5.2 SRAM memory cell
The data write sequence starts with EN $($ Enable $)=1$ and Write Enable $(W E)=1$. This combination precharges the Bit and the Bitbar nodes in the SRAM core to a preset voltage and prepares the memory for a write. When the precharge cycle is complete, the controller enables the address decoder by EnWL $=1$ as shown in Fig. 5.1. The decoder activates a single WL input out of 256 WLs according to the value provided at AddrIn[7:0]. Within the
same time period, the controller also produces WritePulse $=1$, which allows the valid data at DIn[31:0] to be written into the specified address.

Reading data from the SRAM core is performed by EN $=1$ and $\mathrm{WE}=0$. Similar to the write operation, the controller first precharges the SRAM core prior to reading data, and then turns on the address decoder. According to the address value at the AddrIn port, the WL input to a specific row is activated, and the data is read from each cell to the corresponding Bit and Bitbar nodes at this row. The sense amplifier amplifies the cell voltage to full logic levels and delivers the data to the DOut port.

The SRAM I/O timing can be synchronized with clock as shown in Figs. 5.3 and 5.4. In Fig. 5.3, when EN and WE inputs are raised to logic 1, SRAM goes into the write mode, and the valid data is written to a specified address at the next positive clock edge. In Fig. 5.4, when $\mathrm{EN}=1$ and $\mathrm{WE}=0$, SRAM is enabled and operates in the read mode. The core delivers the valid data sometime after the next positive edge of the clock.


Fig. 5.3 SRAM I/O timing for write


Fig. 5.4 SRAM I/O timing for read

One of the important tasks to integrate an SRAM module to an existing system is to design its bus interface. Figure 5.5 shows the block diagram of such an implementation. The bus interface basically translates all bus control signals to SRAM control signals (and vice versa), but it seldom makes any modifications on address or data. In the unidirectional bus protocol described in Chapter 4, SRAM is considered to be a bus slave that exchanges data with the bus master on the basis of a Ready signal. As also mentioned in Chapter 4, a bus master has four control signals to indicate the data transfer. The Status signal indicates if the bus master is sending the first data packet (START) or is in the process of sending remaining data packets (CONT). The bus master may also send IDLE or BUSY signals to indicate if it has finished the current data transfer or busy with an internal task, respectively. The Write signal specifies if the bus master intends to write data to a slave or read from a slave. The


Fig. 5.5 SRAM bus interface block diagram

Burst signal designates the number of data packets in the transaction, and the Size signal defines the width of the data.

The timing diagram in Fig. 5.6 shows how to write four data packets, W1 to W4, to four consecutive SRAM addresses, A1 to A4, as an example to build the bus interface in Fig. 5.5. To initiate a write sequence, the bus master issues a valid address, Status = START and Write $=1$ in the first clock cycle of this timing diagram, and enables the bus interface for a


Fig. 5.6 SRAM bus interface timing diagram for write
write by producing an active-high Bus Interface Write Enable (BIWEn) signal. Upon receiving the BIWEn $=1$, the bus interface produces Ready $=1$ in the next cycle, and prompts the bus master to change the address and control signals in the third cycle. As the bus master changes its address from A1 to A2, it also sends its first data packet, W1, according to the unidirectional bus protocol explained in Chapter 4 . However, in order to write data into an SRAM address, a valid data must be available within the same cycle as the valid address as shown in Fig. 5.3. Therefore, a set of eight flip-flops are added to the write path prior to the AddrIn port in Fig. 5.5 so that the address, A1, is delayed for one clock cycle, and aligned with the current data, W 1 . The bus interface also produces $\mathrm{EN}=\mathrm{WE}=1$ in the third cycle so that W1 is written to A1 at the positive edge of the fourth clock cycle. The next write is accomplished in the same way: the SRAM address is delayed for one cycle in order to write W2 into A2 at the positive edge of the fifth cycle. In the sixth cycle, the bus interface keeps $\mathrm{EN}=\mathrm{WE}=1$ to be able to write W 3 to A 3 , but lowers the Ready signal to logic 0 so that the bus master stops incrementing the slave address and suspends the controls in the next cycle. In the seventh and final write cycle, the bus interface lowers both EN and WE to logic 0 but allows the last data, W 4 , to be written to A 4 at the positive edge of the clock.

The bus interface state diagram for write in Fig. 5.7 is developed as a result of the timing diagram in Fig. 5.6. The first state, Idle state, is the result of the bus interface waiting to receive BIWEn $=1$ from the bus master, which corresponds to the first clock cycle of the timing diagram in Fig. 5.6. The next state that follows the Idle state is the Standby state where the bus interface generates Ready $=1$. This state is one clock cycle long and represents the second clock cycle in the timing diagram. The Write state is the state during which the actual write sequence takes place: EN and WE are kept at logic 1 as long as the number of write addresses issued by the bus master is less than Burst length. This state corresponds to the third, fourth and fifth clock periods in the timing diagram. When the number of write addresses reaches the value of the Burst length, the bus interface goes to the Last Write stage and Ready signal becomes logic 0 . The bus master writes the final data packet to the last SRAM address at the positive edge of the seventh clock cycle before it enters the Idle state.

In order to initiate a read sequence, the bus master issues a valid SRAM address, Status = START and Write $=0$ signals in the first clock cycle of Fig. 5.8. This combination produces an active-high Bus Interface Read Enable, BIREn $=1$, which is interpreted as the bus master intending to read data from an SRAM address. Consequently, the bus interface generates $\mathrm{EN}=1$, $\mathrm{WE}=0$, Ready $=1$ in the second cycle. This fetches the first data, R1, from the SRAM address, B 1 , in the third cycle. The read transactions in the fourth and fifth cycles are identical to the third, and the bus master reads R2 and R3 from the addresses, B2 and B3, respectively. In the sixth cycle, the bus interface retains Ready $=1$ so that the bus master can still read the last data, R4, from the address, B4.


Fig. 5.7 SRAM bus interface for write

As in the write case, the read bus interface in Fig. 5.9 is also a direct consequence of the timing diagram in Fig. 5.8. The Idle state corresponds to the first clock cycle of the timing diagram in Fig. 5.8. As soon as BIREn $=1$ is generated, the bus interface transitions to the Standby state where it produces $\mathrm{EN}=1, \mathrm{WE}=0$ and Ready $=1$. The interface enters the Read state in the third cycle and produces the same outputs as before so that the bus master can read its first data, R1, and sends a new address in the next cycle. The interface stays in the Read state until the number of read addresses issued by the bus master is less than the Burst length. The Read state covers from the third to the fifth cycle in the timing diagram in Fig. 5.8. After the number of read addresses reaches the Burst length, the bus interface


Fig. 5.8 SRAM bus interface timing diagram for read
transitions to the Last Read state in cycle six where it continues to generate Ready $=1$. This is done so that the bus master is able to read the last data as mentioned earlier. The interface unconditionally goes back to the Idle state in the following cycle.


Fig. 5.9 SRAM bus interface for read

Increasing SRAM capacity necessitates employing extra address bits. In the example shown in Fig. 5.10, the SRAM capacity is increased from $32 \times 16$ bits to $32 \times 64$ bits by appending two extra address bits, Addr[5:4], which serves to access one of the four SRAM blocks. In this figure, even though Addr[3:0] points to the same address location for all four 32x16 SRAM blocks, Addr[5:4] in conjunction with EN enables only one of the four blocks. Furthermore, the data read from the selected block is routed through the 4-1 MUX using Addr[5:4] inputs. $\operatorname{Addr}[5: 4]=00$ selects the contents of DOut0 port and routes the data through port 0 of the 4-1 MUX to Out[31:0]. Similarly, $\operatorname{Addr}[5: 4]=01,10$ and 11 select ports 1,2 and 3 of the $4-1$ MUX, and route data from DOut1, DOut2 and DOut3 ports to Out[31:0], respectively.


Fig. 5.10 Increasing SRAM address space

### 5.2 Synchronous Dynamic Random Access Memory

Synchronous Dynamic Random Access Memory (SDRAM) is a variation of the older DRAM, and it constitutes the main memory of almost every computing system. Even though its capacity can be many orders of magnitude higher than SRAM, it lacks speed. Therefore, its usage is limited to storing large blocks of data when speed is not important.

An SDRAM module is composed of four blocks. The memory core is where data is stored. The row and column decoders locate the data. The sense amplifier amplifies the cell voltage during read. The controller manages all the read and write sequences.

The block diagram in Fig. 5.11 shows a typical 32-bit SDRAM architecture composed of four memory cores, called banks, accessible by a single bidirectional input/output port. Prior to operating the memory, the main internal functions, such as addressing mode, data latency and burst length, must be stored in the Address Mode Register. Once programmed, the active-low Row Address Strobe, $\overline{\mathrm{RAS}}$, Column Address Strobe, $\overline{\mathrm{CAS}}$, and Write Enable, $\overline{\mathrm{WE}}$, signals determine the functionality of the memory as shown in Table 5.1. The data from a selected bank can be masked by the Read/Write logic block at the DInOut port so that only part of the data can be fetched from the memory. This unit also serves blocking parts of the 32-bit data to be written to a bank.


Fig. 5.11 Typical SDRAM architecture

Table 5.1 SDRAM modes of operation

| $\overline{\mathrm{CS}}$ | $\overline{\mathrm{RAS}}$ | $\overline{\mathrm{CAS}}$ | $\overline{\mathrm{WE}}$ | OPERATION |
| :---: | :---: | :---: | :---: | :--- |
| 0 | 0 | 0 | 0 | Program Addr. Mode Register |
| 0 | 0 | 0 | 1 | Self Refresh |
| 0 | 0 | 1 | 0 | Precharge a Bank with BS[1:0] |
| 0 | 0 | 1 | 1 | Activate a Bank with BS[1:0] |
| 0 | 1 | 0 | 0 | Write into a Bank with BS[1:0] |
| 0 | 1 | 0 | 1 | Read from a Bank with BS[1:0] |
| 0 | 1 | 1 | 0 | Burst Stop |
| 0 | 1 | 1 | 1 | Reserved |
| 1 | X | X | X | SDRAM Deselect |

The SDRAM cell is a simple device composed of an NMOS transistor to control the data-flow in and out of the cell and a capacitor to store data as shown in Fig. 5.12. When new data needs to be written into the cell, the NMOS transistor is turned on by Control $=1$, and the data at the DIn/Out terminal overwrites the old data at the Cell node. Reading data from the cell, on the other hand, requires amplification of the cell voltage, thus activation of the
sense amplifier when the NMOS transistor is turned on. When data needs to be preserved, the NMOS transistor is simply turned off by Control $=0$. However, the charge on the cell capacitor slowly leaks through its insulator, resulting in a reduced cell voltage. Thus, an automatic or manual cell refresh cycle becomes mandatory during SDRAM operation to preserve the bit value in the cell.


Fig. 5.12 SDRAM memory cell
The first row of the truth table in Table 5.1 indicates how to program the internal Address Mode Register. At the positive edge of the clock, $\overline{\mathrm{CS}}, \overline{\mathrm{RAS}}, \overline{\mathrm{CAS}}$ and $\overline{\mathrm{WE}}$ signals are pulled low to logic 0 to program the Address Mode Register as shown in Fig. 5.13. In the program, the address bits, $\mathrm{A}[2: 0]$, define the data burst length as shown in Table 5.2. Burst length can range from one word of data to a full page, which is equal to the contents of the entire bank. The address bit, A[3], defines how the SDRAM address is incremented for each data packet. In sequential addressing mode, the starting address is incremented by one while the carry bit is eliminated according to the size of the burst length. In linear addressing mode, the address is incremented without eliminating the carry bit. The address bits, A[5:4], determine the data latency when reading takes place from the memory. Latency can range from two to five clock cycles depending on the need.


Fig. 5.13 Timing diagram for programming the address mode register

Table 5.2 Truth tables for programming the address mode register

| $A[2]$ | $A[1]$ | $A[0]$ | Burst Length |
| :---: | :---: | :---: | :--- |
| 0 | 0 | 0 | 1 Word |
| 0 | 0 | 1 | 2 Words |
| 0 | 1 | 0 | 4 Words |
| 0 | 1 | 1 | 8 Words |
| 1 | 0 | 0 | 16 Words |
| 1 | 0 | 1 | 32 Words |
| 1 | 1 | 0 | 64 Words |
| 1 | 1 | 1 | Full Page |


| $A[3]$ | Addressing Mode |
| :---: | :--- |
| 0 | Sequential |
| 1 | Linear |


| A[5] | A[4] | Latency |
| :---: | :---: | :---: |
| 0 | 0 | 2 |
| 0 | 1 | 3 |
| 1 | 0 | 4 |
| 1 | 1 | 5 |

The example in Table 5.3 shows the elimination of the carry bit in sequential addressing mode for burst lengths of 2,4 and 8 . In this example, if the starting address is 13 and the burst length is two words, the carry bit from the column $\mathrm{A}[0]$ is eliminated, resulting the next address to be 12 . In the same example, if the burst length is increased to four, this time the carry bit from the column $\mathrm{A}[1]$ is eliminated, and the address values following the starting address 13 become 14,15 and 12 . If the burst length becomes eight, the carry bit from the column $\mathrm{A}[2]$ is eliminated, and the address values of $14,15,8,9,10,11$ and 12 follow the initial address 13. Sequential addressing confines reading or writing of data within a predefined, circulatory memory space, convenient for specific software applications.

The linear addressing mode is a simplified version of the interleave addressing in various SDRAMs, and increments the SDRAM address linearly without eliminating the carry bit as shown in Table 5.4. In this example, if the starting address is 13 and the burst length is two, the next address will be 14. If the burst length is increased to four, the next three addresses following 13 will be 14,15 and 16 . In contrast to the sequential addressing mode, the linear addressing increments SDRAM address one bit at a time, not confining the data in a circulatory address space.

The second row of the operational truth table in Table 5.1 shows how to initiate a self refresh cycle as shown in Fig. 5.14. In self refresh, SDRAM automatically replenishes node voltage values at each cell because the charge across the cell capacitor leaks through its dielectric layer over time. The time duration between refresh cycles depends on the technology used, the quality of the oxide growth and the thickness of the dielectric used between capacitor plates.

Table 5.3 SDRAM sequential mode addressing for burst lengths of 2,4 and 8

Starting Address $=13$, Burst Length $=2$, Mode $=$ Sequential

| $A[9]$ | $A[8]$ | $A[7]$ | $A[6]$ | $A[5]$ | $A[4]$ | $A[3]$ | $A[2]$ | $A[1]$ | $A[0]$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | $=13$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | $=12$ |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

Starting Address $=13$, Burst Length $=4$, Mode $=$ Sequential


Starting Address $=13$, Burst Length $=8$, Mode $=$ Sequential

| A[9] | A[8] | A[7] | A[6] | A[5] | A[4] | A[3] | \| A[2] | A[1] | A[0] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | $1=13$ |
|  |  |  |  |  |  |  | I |  | + 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | $0=14$ |
|  |  |  |  |  |  |  | I |  | +1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | $1=15$ |
|  |  |  |  |  |  |  | I |  | + 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | $0=8$ |
|  |  |  | delete | the ca | ry bit |  |  |  | +1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | $1=9$ |
|  |  |  |  |  |  |  | I |  | + 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 10 | 1 | $0=10$ |
|  |  |  |  |  |  |  | I |  | +1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 10 | 1 | $1=11$ |
|  |  |  |  |  |  |  | I |  | +1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | $0=12$ |

Table 5.4 SDRAM linear addressing mode for burst lengths of 2 and 4
Starting Address $=13$, Burst Length $=2$ Mode $=$ Linear

| $\mathrm{A}[9]$ | $\mathrm{A}[8]$ | $\mathrm{A}[7]$ | $\mathrm{A}[6]$ | $\mathrm{A}[5]$ | $\mathrm{A}[4]$ | $\mathrm{A}[3]$ | $\mathrm{A}[2]$ | $\mathrm{A}[1]$ | $\mathrm{A}[0]$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | $=13$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | $=$ |

Starting Address $=13$, Burst Length $=4$, Mode $=$ Linear

| $\mathrm{A}[9]$ | $\mathrm{A}[8]$ | $\mathrm{A}[7]$ | $\mathrm{A}[6]$ | $\mathrm{A}[5]$ | $\mathrm{A}[4]$ | $\mathrm{A}[3]$ | $\mathrm{A}[2]$ | $\mathrm{A}[1]$ | $\mathrm{A}[0]$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | $=13$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | $=14$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | $=15$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | $=16$ |



Fig. 5.14 Timing diagram for self-refresh

Rows three to six in Table 5.1 define the read and write sequences in an SDRAM as shown in Fig. 5.15. In this figure, a read or a write sequence always starts with precharging all the rows and columns of the SDRAM core. This is followed by an activation cycle where the row address is generated. In the last cycle, the column address is generated, and the data is either written or read from the memory according to the control signals, $\overline{\mathrm{CS}}, \overline{\mathrm{RAS}}, \overline{\mathrm{CAS}}$ and $\overline{\mathrm{WE}}$.


Fig. 5.15 Write and read operation cycles
Prior to a read or a write, all the rows and columns of a bank must be precharged to a certain voltage level for a period of one clock cycle as shown in Fig. 5.16. During precharge, $\overline{\mathrm{CS}}, \overline{\mathrm{RAS}}$ and $\overline{\mathrm{WE}}$, must be lowered to logic 0 , and $\overline{\mathrm{CAS}}$ must be kept at logic 1 as shown in the third row of Table 5.1. The value of the precharge voltage can be anywhere between 0 V and the full supply voltage depending on the technology and the requirements of the circuit design. The activation cycle starts right after precharging a bank. The time interval between the precharge and activation cycles is called the precharge time period, $\mathrm{t}_{\text {PRE }}$, as shown in Fig. 5.16. The activation cycle is enabled by lowering $\overline{\mathrm{CS}}$ and $\overline{\mathrm{RAS}}$ to logic 0 , but keeping $\overline{\mathrm{CAS}}$ and $\overline{\mathrm{WE}}$ at logic 1 as shown in the fourth row of Table 5.1. Following the activation cycle, the next precharge period must not start until after a certain time period has elapsed for the same bank. This time interval is called the RAS time period, $\mathrm{t}_{\text {RAS }}$, as shown in Fig. 5.16.


Fig. 5.16 Bank precharge and activation cycles

The fifth row of Table 5.1 shows how to write to a selected bank when $\overline{\mathrm{CS}}=\overline{\mathrm{CAS}}=\overline{\mathrm{WE}}=0$ and $\overline{\operatorname{RAS}}=1$. The actual write takes place in the last phase of the write sequence in Fig. 5.15 following the precharge and activation cycles. To illustrate the write sequence in detail, an example illustrating a single write burst is given in Fig. 5.17. In this figure, the write cycle starts with precharging Bank 1. After $t=t_{\text {PRE }}$, the activation period starts and the row address is defined for the SDRAM. When the column address is generated after a time period of $t_{\text {CAS }}$, four data packets, $D(0)$ to $D(3)$, are written to SDRAM core in four consecutive clock cycles. Note that in this figure if the same bank is used for another write, a new time period, $\mathrm{t}_{\text {RAS }}$, needs to be placed between the bank activation cycle and the next precharge period.


Fig. 5.17 A single write cycle
The example in Fig. 5.18 shows two separate write sequences into two different banks. When writing takes place more than a single bank, interleaving one bank's precharge and activation time periods with respect to the other may result in a time-saving scenario where one write burst takes place immediately after the other, resulting in a shorter overall write period. In this figure, the interleaving technique results in writing four words to bank 1 immediately after writing four words to bank 0 without any cycle loss. Therefore, writing to two (or more) different banks as opposed to writing continuously to a single bank is a preferred scheme because this process eliminates all unnecessary waiting periods between precharge cycles. However, as the burst length involves a lot more than four words, the relative placement of the bank precharge cycle in the timing diagram becomes less important.


Fig. 5.18 Multiple write cycles to different banks ( $\mathrm{t}_{\mathrm{PRE}}=1$ cycle, $\mathrm{t}_{\mathrm{CAS}}=2$ cycles)
The sixth row of Table 5.1 shows how to initiate a read cycle from a selected bank. Reading words from SDRAM involves a latency period, and it needs to be programmed in the Address Mode Register. The example in Fig. 5.19 shows the start of a read burst after a latency period of three clock cycles once the read command and the address are given. A latency of three clock cycles means that the data becomes available at the output of SDRAM in the third clock cycle after the address is issued.

Read 8 WORDS from the Starting Address $=13$ in Sequential Mode with Latency $=3$


Fig. 5.19 Definition of latency during a read cycle

The example in Fig. 5.20 shows a single read sequence from bank 1. Until the read command and the column address are issued, the read and write sequences follow identical paths. However, after this point the read burst starts following a latency period. Similar to the write process, a certain $t_{\text {RAS }}$ period must elapse between read bursts before additional streams of data become available from the same bank. In this figure, $\mathrm{t}_{\text {WAIT }}$ corresponds to the waiting period between the last data packet read from the memory and the start of the next precharge period.


Fig. 5.20 Single read cycle
The example in Fig. 5.21 describes multiple reads from the same bank and assumes $\mathrm{t}_{\text {WAIT }}$ is equal to zero. This scenario produces a read burst of four words, $D(0)$ to $D(3)$, from bank 1 and causes the same bank to be precharged during the last data delivery. The second read burst from bank 1 follows the same pattern as the first one and delivers $\mathrm{D}(4)$ to $\mathrm{D}(7)$ after a programmed latency of two clock periods. If $\mathrm{t}_{\text {WAIT }}$ is different from zero, then the second precharge period should start right after the $\mathrm{t}_{\text {WAIT }}$ period as described in Fig. 5.20.

The interleaving technique used in reading data from two different banks in Fig. 5.22 is not any different from the one used when writing data to two different banks. As with the write case, the placement of the second precharge cycle in the timing diagram is important to achieve two consecutive read bursts, $\mathrm{D} 0(0)$ to $\mathrm{D} 0(3)$ from bank 0 and $\mathrm{D} 1(0)$ to $\mathrm{D} 1(3)$ from bank 1, without any cycle loss in between, accomplishing the shortest possible time to fetch data from SDRAM.


Fig. 5.21 Multiple read cycles from the same bank ( $\mathrm{t}_{\mathrm{PRE}}=1$ cycle, $\mathrm{t}_{\mathrm{CAS}}=2$ cycles, $\mathrm{t}_{\mathrm{WAIT}}=0$ cycle)


Fig. 5.22 Multiple read cycles from different banks $\left(\mathrm{t}_{\text {PRE }}=1\right.$ cycle, $\mathrm{t}_{\mathrm{CAS}}=2$ cycles $)$
The seventh row of Table 5.1 shows how to stop a read or a write burst. Figure 5.23 shows a single write sequence when the burst stop command is issued in the middle of a data
burst. Upon receiving this command, the selected bank goes into the standby mode and waits for the next precharge command.


Fig. 5.23 Burst stop during write
When the burst stop command is given in the middle of a read, the last data packet is still delivered at the clock edge following the burst stop command as shown in Fig. 5.24.


Fig. 5.24 Burst stop during read

The Input/Output data can be masked with the Read/Write Logic block in Fig. 5.11. The truth table in Table 5.5 lists all the possible cases of blocking and transmitting data to/from SDRAM core. When Mask[3:0] $=0000$, for example, no mask is applied to the output data, and all 32 bits are allowed to be written to the selected bank or read from it. The case, Mask [3:0] = 1111, on the other hand, blocks all four bytes of data, and allows no byte to be written or read from the selected address.

Table 5.5 Truth table for data output mask

| Mask [3] | Mask [2] | Mask [1] | Mask [0] | MASKED BITS |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | None |
| 0 | 0 | 0 | 1 | DInOut[7:0] |
| 0 | 0 | 1 | 0 | DInOut[15:8] |
| 0 | 0 | 1 | 1 | DInOut[15:0] |
| 0 | 1 | 0 | 0 | DInOut[23:16] |
| 0 | 1 | 0 | 1 | DInOut[23:16] and DInOut[7:0] |
| 0 | 1 | 1 | 0 | DInOut[23: 8] |
| 0 | 1 | 1 | 1 | DInOut[23:0] |
| 1 | 0 | 0 | 0 | DInOut[31:24] |
| 1 | 0 | 0 | 1 | DInOut[31:24] and DInOut[7:0] |
| 1 | 0 | 1 | 0 | DInOut[31:24] and DInOut[15:8] |
| 1 | 0 | 1 | 1 | DInOut[31:24] and DInOut[15:0] |
| 1 | 1 | 0 | 0 | DInOut[31:16] |
| 1 | 1 | 0 | 1 | DInOut[31:16] and DInOut[7:0] |
| 1 | 1 | 1 | 0 | DInOut[31:8] |
| 1 | 1 | 1 | 1 | DInOut[31:0] |

Figure 5.25 shows the data-path and the controller of the SDRAM bus interface. Prior to operating SDRAM, each ten-bit wide bus interface register containing the precharge ( $\mathrm{t}_{\text {PRE }}$ ), CAS ( $\mathrm{t}_{\mathrm{CAS}}$ ), burst ( $\mathrm{t}_{\mathrm{BURST}}$ ), latency ( $\mathrm{t}_{\mathrm{LAT}}$ ), and wait ( $\mathrm{t}_{\mathrm{WAIT}}$ ) periods must be programmed through a 10 -bit program bus. The precharge, CAS and wait registers contain the number of clock cycles to achieve the required waiting period. The burst register should store the number of data packets of the data transfer. Therefore, the value in this register must be identical to the value programmed in the Address Mode Register. The latency register specifies the number of clock cycles prior to reading the first data from an SDRAM address. The details of how the programming takes place prior to the normal SDRAM operation and the required hardware are omitted from Fig. 5.25 to avoid complexity. The Address Mode


Fig. 5.25 SDRAM bus interface block diagram

Register (or a set of registers defining basic SDRAM functionality) exists in many older SDRAMs. However, recent SDRAM modules omit the mode register completely and rely on the bus interface unit to store such information. To manage the precharge, CAS, burst and wait periods the memory controller is continuously interact with a down-counter in Fig. 5.25 since these periods are often many clock cycles long.

For normal SDRAM operation, the address bus, Addr[31:0], has been divided into several segments. In the example in Fig. 5.26, the four most significant bits of the SDRAM address, Addr[31:28], indicate the SDRAM chip identification, and it is used to activate the corresponding bus interface. Addr[21:20] is used to select the SDRAM bank, BS[1:0]. Addr[19:10] and $\operatorname{Addr}[9: 0]$ specify the row and column addresses, respectively.


Fig. 5.26 SDRAM bus interface address mapping
Figure 5.27 shows a typical SDRAM write sequence. In this timing diagram, all five SDRAM interface registers must be programmed prior to the IDLE/PROG clock cycle as mentioned earlier. The SDRAM write sequence starts with the system bus sending Status = START, Write $=1$ and the starting SDRAM address. These three signals cause the Bus Interface Write Enable signal, BIWEn, to transition to logic 1, which in turn, enables the bus interface for write in the first cycle of Fig. 5.27. Once enabled, the bus interface stores the starting SDRAM address in the Address Reg by StoreReg $=1$ and issues the precharge command by $\overline{\mathrm{CS}}=0, \overline{\mathrm{RAS}}=0, \overline{\mathrm{CAS}}=1$ and $\overline{\mathrm{WE}}=0$ for the selected bank. Within the same cycle, the counter is loaded with the precharge wait period, $\mathrm{t}_{\text {PRE }}$, by $\operatorname{Loadt}_{\text {PRE }}=1$ as shown in the timing diagram.

The Precharge wait period is calculated by multiplying the number of clock cycles by the clock period. The counter in this design is a down-counter. When its output value, CountOut, becomes one, the controller initiates the activation cycle for the selected SDRAM bank and dispatches the row address. The activation period starts with loading the value of $\mathrm{t}_{\mathrm{CAS}}$ into the


Fig. 5.27 Write cycle with SDRAM bus interface
down-counter by Loadt ${ }_{\text {CAS }}=1$. Within the same clock cycle, the row address, $\operatorname{Addr}[19: 10]$, is transferred from the Address Reg to the SDRAM through the R-port of the 3-1 MUX by SelRow $=1$. When the activation wait period expires, the controller uses the Loadt $_{\text {BURST }}$ input to load the length of the write burst (the number of data packets) to the down-counter and subsequently initiates the write sequence in the next cycle.

During the START WRITE period, the controller transfers the column address, Addr [9:0], from the Address Reg through the C-port of the 3-1 MUX to the A[9:0] port of the SDRAM by generating $\mathrm{SelCol}=1$. In the same cycle, the controller also generates $\overline{\mathrm{CS}}=0$, $\overline{\mathrm{RAS}}=1, \overline{\mathrm{CAS}}=0, \overline{\mathrm{WE}}=0$ and enables the tri-state buffer by EnWData $=1$ in order to write the first data packet, D0, to the SDRAM. To be able to write the remaining data packets, the controller issues Ready $=1$ from this point forward. When the sequence comes to the LOAD WAIT period (where the last write takes place), the controller lowers the Ready signal, but keeps the EnWData signal at logic 1 in order to write the last data packet, D3. This clock cycle also signifies the start of the wait period, $\mathrm{t}_{\mathrm{wart}}$. The controller issues Loadt $_{\text {WAIT }}=1$ to load $\mathrm{t}_{\text {WAIT }}$ into the down-counter if another write sequence needs to take place for the same bank.

The remaining control signals, Burst and Size, are omitted from the timing diagram for simplicity. During the entire data transfer process, Burst is set to four and Size is set to 32 in Fig. 5.27. For byte and half-word transfers, Size needs to be defined with masking in place as described in Table 5.5.

The state diagram of the controller for write is shown in Fig. 5.28. In this diagram, when the interface receives BIWEn $=1$, the controller transitions from the IDLE/PROG state, which corresponds to the first cycle of the timing diagram in Fig. 5.27, to the LOAD PRE state, which corresponds to the second clock cycle in the same timing diagram. In the LOAD PRE state, the controller resets $\overline{\mathrm{CS}}, \overline{\mathrm{RAS}}$ and $\overline{\mathrm{WE}}$, but sets $\overline{\mathrm{CAS}}$ for the selected bank to start the precharge process. In this state, two additional signals are generated: StoreReg $=1$ to store the bus address in the Address Reg, and Loadt ${ }_{\text {PRE }}=1$ to start the precharge wait period. In the next cycle, the controller transitions to the precharge wait state, $\mathrm{t}_{\text {PRE }}$. The controller remains in this state until CountOut $=1$. The controller then transitions to the LOAD CAS state where it activates the selected bank, issues SelRow $=1$ to transfer the row address to the SDRAM, and produces Loadt $_{\text {CAS }}=1$ to initiate the activation wait period. The next state, $\mathrm{t}_{\mathrm{CAS}}$, is another wait state where the controller waits until the activation period expires. Once this period is over, the controller first goes into the LOAD BURST state, and then to the START WRITE state to initiate writing data to the SDRAM. The latter corresponds to the state where the first data packet is written to the SDRAM core as mentioned earlier. The subsequent writes take place when the controller transitions to the WRITE state. The controller stays in this state until CountOut $=2$, which signifies one more data packet to be written to the SDRAM. The last data packet is finally written when the controller moves to the LOAD WAIT state. Before attempting another write process, the controller waits in the $\mathrm{t}_{\text {wart }}$ state until CountOut $=1$.

Note that all the state names in Fig. 5.28 and the cycle names on top of Fig. 5.27 are kept the same to make one-to-one correspondence between the timing diagram and the state diagram.


Fig. 5.28 SDRAM bus interface for write
The SDRAM read sequence also starts with the system bus sending Status = START, Write $=0$ and an initial SDRAM address. This combination sets the Bus Interface Read Enable signal, BIREn $=1$, to enable the bus interface to read data from the SDRAM core in the first cycle of the timing diagram in Fig. 5.29. The remainder of the read process is

Fig. 5.29 Read cycle with SDRAM bus interface
identical to the write process until the controller issues the read command during the START READ cycle, and sends the column address of the selected SDRAM bank. Since SDRAM data becomes available after a latency period, the controller must replicate this exact delay prior to a read burst and produce the control signals during and after the burst. For example, a cycle before the latency period expires, the controller needs to generate Loadt $_{\text {BURST }}=1$ to load the burst duration to the down-counter to be able to detect the beginning and the end of burst data. As a result, the controller can determine when to generate EnRData $=1$ for the tri-state buffer to read data packets, D0 to D3, from the SDRAM RData output. During the last data delivery, the controller issues Loadt ${ }_{\text {wAIT }}=1$ to load the value of $t_{\text {WAIT }}$ to the down-counter in the event the same bank is selected for another read.

The state diagram in Fig. 5.30 for the read sequence is a direct result of the timing diagram in Fig. 5.29. In this diagram, when the interface receives BIREn $=1$, the controller transitions from the IDLE state which corresponds to the first cycle in Fig. 5.29, to the LOAD PRE state which corresponds to the second clock cycle in the same timing diagram. In the LOAD PRE state, the precharge process is initiated by $\overline{\mathrm{CS}}=0, \overline{\mathrm{RAS}}=0, \overline{\mathrm{WE}}=0$ and $\overline{\mathrm{CAS}}=1$ for the selected bank. In this state, the controller stores the valid bus address in the Address Reg by StoreReg = 1, and loads the precharge wait period into the down-counter by Loadt $_{\text {PRE }}=1$. Then the controller moves to the $t_{\text {PRE }}$ and stays in this state until the precharge value in the down-counter expires. Next, the controller transitions to the LOAD CAS state where it activates the selected bank by $\overline{\mathrm{CS}}=0, \overline{\mathrm{RAS}}=0, \overline{\mathrm{CAS}}=1$ and $\overline{\mathrm{WE}}=1$, issues SelRow $=1$ to transfer the row address from the Address Reg to the SDRAM, and generates Loadt $_{\text {CAS }}=1$ to load the activation wait period to the down-counter. The CAS wait period corresponds to the $\mathrm{t}_{\text {CAS }}$ state in Fig. 5.30. When this period is over at CountOut $=1$, the controller transitions to the START READ state where it issues $\overline{\mathrm{CS}}=0, \overline{\mathrm{RAS}}=1, \overline{\mathrm{CAS}}=0$ and $\overline{\mathrm{WE}}=1$ to initiate the data read and produces $\mathrm{SelCol}=1$ to transfer the column address from Address Reg to the SDRAM address port. This state is followed by four individual latency states to select the programmed read latency period. Since the read latency in Fig. 5.29 is equal to two, the state machine traces through a single LAT WAIT state. In the LAT WAIT state, the controller issues Loadt ${ }_{\text {BURST }}=1$ and loads the value of the data burst, $\mathrm{t}_{\text {BURST }}$, to the down-counter. Following the latency states, the state machine transitions to the READ state where it stays until CountOut $=2$, signifying the end of the read burst. Here, it produces EnRData $=1$ to enable the data output buffer and Ready $=1$ to validate the read data. At the end of the burst period, the state machine moves to the LOAD WAIT state and issues Loadt $_{\text {WAIT }}=1$ to load the required wait period into the down-counter until the next precharge takes place. Subsequently, the state machine transitions to the $\mathrm{t}_{\text {WAIT }}$ state and stays there until the wait period is over.


Fig. 5.30 SDRAM bus interface for read

### 5.3 Electrically-Erasable-Programmable-Read-Only-Memory

Electrically-Erasable-Programmable-Read-Only-Memory ( $\mathrm{E}^{2} \mathrm{PROM}$ ) is historically considered the predecessor of Flash memory and also the slowest memory in a computing system. Its greatest advantage over the other memories is its ability to retain data after the system power is turned off due to the floating-gate MOS transistor in its memory core. Its relatively small size compared to electromechanical hard disks makes this device an ideal candidate to store Built-In-Operating-Systems (BIOS) especially for hand-held computing platforms.

A typical $E^{2}$ PROM memory is composed of multiple sectors, each of which contains multiple pages as shown in the example in Fig. 5.31. A single word in E2 PROM can be located by specifying its sector address, page address and row address. The sector address indicates which sector a particular word resides. The page address locates the specific page inside a sector. Finally, the row address points to the location of the data byte inside a page. There are five control signals in $E^{2}$ PROM to perform read, write or erase operations. The active-low Enable signal, $\overline{\mathrm{EN}}$, places a particular page in standby mode and prepares it for an upcoming operation. The active-low Command Enable signal, $\overline{\mathrm{CE}}$, is issued with a command code, such as read, write (program) or erase. The active-low Address Enable signal, $\overline{\mathrm{AE}}$, is


Fig. 5.31 A typical $E^{2}$ PROM organization
issued when an address is provided. Finally, the active-low Write Enable signal, $\overline{\mathrm{WE}}$, and the Read Enable signal, $\overline{\mathrm{RE}}$, are issued for writing and reading data, respectively.

Typical $E^{2}$ PROM architecture consists of a memory core, an address decoder, an output data buffer, status, address and command registers, and a control logic circuit as shown in Fig. 5.32. Prior to any operation, command and address registers are programmed. When the operation starts, the control logic enables the address decoder, the data buffer and the memory core using the active-high Enable Address (ENA), Enable Data Buffer (END), and Write Enable Core (WEC) or Read Enable Core (REC) signals depending on the operation. The address stored in the address register is decoded to point the location of data. If the read operation needs to be performed, the required data is retrieved from the $E^{2}$ PROM core and stored in the data buffer before it is delivered to the I/O bus. If the operation is a write (or program), the data is stored in the data buffer first before it is uploaded to the designated $E^{2}$ PROM address. In all cases, $\overline{\mathrm{EN}}$ needs to be at logic 0 to place $\mathrm{E}^{2}$ PROM into standby mode before starting an operation. The table in Fig. 5.33 describes all major operation modes. Hibernate mode disables the address decoder, memory core and data buffer to reduce power dissipation, and puts the device into sleep.


Fig. 5.32 A typical $E^{2}$ PROM architecture

| $\overline{\mathrm{EN}}$ | $\overline{\mathrm{WE}}$ | $\overline{\mathrm{RE}}$ |  |
| :---: | :---: | :---: | :--- |
| 0 | 1 | 1 | Standby |
| 0 | 0 | 1 | Write |
| 0 | 1 | 0 | Read |
| 1 | X | X | Hibernate |

Fig. 5.33 $E^{2}$ PROM major operation modes
The $E^{2}$ PROM cell shown in Fig. 5.34 is basically an N-channel MOS transistor with an additional floating gate layer sandwiched between its control gate terminal (Wordline) and the channel where the electronic conduction takes place. This device has also drain (Bitline) and source (Sourceline) terminals for connecting the cell to the neighboring circuitry.

To write logic 0 into the memory cell, a high voltage is applied between Wordline and Bitline terminals while the Sourceline node remains connected to ground. This configuration generates hot carriers in the transistor channel which tunnel through the gate oxide and reach the floating gate, raising the threshold voltage of the transistor. The raised threshold voltage prevents the programmed device to be turned on by the standard gate-source voltage used during normal circuit operations, and causes the value stored in the device to be interpreted as logic 0 . An unprogrammed device with no charge on the floating gate, on the other hand, exhibits low threshold voltage characteristics and can be turned on by the standard gate-source voltage, producing a channel current. In this state, the value stored in the device is interpreted as logic 1 .


Fig. 5.34 $\mathrm{E}^{2} \mathrm{PROM}$ cell
Figure 5.35 shows a typical command input sequence. There are four basic commands for this $E^{2}$ PROM example: read, write (program), page-erase and status register read. The write and program commands will be used interchangeably here and when describing the Flash memory since they mean the same operation. The operation sequence always starts with the command input followed by the address and data entries. To issue a command input, $\overline{\mathrm{EN}}$ is lowered to logic $0, \overline{\mathrm{AE}}$ is raised to logic 1 (because the entry is not an address), and $\overline{\mathrm{CE}}$ is lowered to logic 0 , indicating that the value on the I/O bus is a command input. Since the command input is written into the command register $\overline{\mathrm{WE}}$ is also lowered to logic 0 some
time after the negative edge of $\overline{\mathrm{CE}}$ signal. This delay is called the setup time $\left(\mathrm{t}_{\mathrm{s}}\right)$ as shown in Fig. 5.35. The low phase of $\overline{\mathrm{WE}}$ signal lasts for a period of $\mathrm{t}_{\mathrm{LO}}$, and transitions back to logic 1 some time before the positive edge of $\overline{\mathrm{CE}}$. This time interval is called the hold time, $\mathrm{t}_{\mathrm{H}}$. Prior to the positive edge of $\overline{\mathrm{WE}}$, a valid command input is issued, satisfying the data setup time, $\mathrm{t}_{\mathrm{DS}}$, and the data hold time, $\mathrm{t}_{\mathrm{DH}}$, as shown in Fig. 5.35.


Fig. 5.35 Command input timing diagram
The address input timing shown in Fig. 5.36 has the same principle as the command input timing described above: $\overline{\mathrm{EN}}$ needs to be at logic 0 to enable the device, $\overline{\mathrm{AE}}$ must be at logic 0 for the address entry, and $\overline{\mathrm{CE}}$ needs to be at logic 1 because this operation is not a command entry. During the low phase of $\overline{\mathrm{EN}}$ signal, $\overline{\mathrm{WE}}$ signal must be lowered to logic 0 twice to locate data in the $E^{2}$ PROM. The first time $\overline{\mathrm{WE}}=0$, an eight-bit row address is entered at the first positive edge of $\overline{\mathrm{WE}}$. This is followed by the combination of four-bit page address and four-bit sector address at the next $\overline{\mathrm{WE}}=0$. The $\overline{\mathrm{WE}}$ signal must be lowered to logic 0 after a period of $\mathrm{t}_{\mathrm{S}}$, following the negative edge of the $\overline{\mathrm{EN}}$ signal, and then back to logic 1 for a period of $\mathrm{t}_{\mathrm{H}}$ before the positive edge of $\overline{\mathrm{EN}}$. The $\overline{\mathrm{WE}}$ signal must also be at the low phase for a period of $\mathrm{t}_{\mathrm{LO}}$ and at the high phase for a period of $\mathrm{t}_{\mathrm{HI}}$ (or longer) during the address entry. Valid address values are issued at each positive edge of $\overline{\mathrm{WE}}$ within the $\mathrm{t}_{\mathrm{DS}}$ and $\mathrm{t}_{\mathrm{DH}}$ setup and hold time periods.


Fig. 5.36 Address input timing diagram
Figure 5.37 describes data entry sequence where $(M+1)$ number of data packets are written to the $\mathrm{E}^{2} \mathrm{PROM}$. During the entire write cycle $\overline{\mathrm{AE}}$ signal must be at logic 1, indicating that the operation is a data entry but not an address. Data packets are written at each positive edge of $\overline{\mathrm{WE}}$ signal.


Fig. 5.37 Data input (write or program) timing diagram
During a read the active-low control signals, $\overline{\mathrm{AE}}$ and $\overline{\mathrm{CE}}$, are kept at logic 1. The Read Enable signal, $\overline{\mathrm{RE}}$, enables the $\mathrm{E}^{2}$ PROM to read data from the memory core at each negative edge as shown in Fig. 5.38. The time delay between the negative edge of $\overline{\mathrm{RE}}$ and the actual
availability of data from the memory is called the access time, $\mathrm{t}_{\mathrm{A}}$, as shown in the same timing diagram. The $\overline{\mathrm{RE}}$ signal must have the specified $\mathrm{t}_{\mathrm{S}}, \mathrm{t}_{\mathrm{H}}, \mathrm{t}_{\mathrm{LO}}$ and $\mathrm{t}_{\mathrm{HI}}$ time periods to be able to read data from the memory core.


Fig. 5.38 Data output (read) timing diagram
Reading data from the Status Register is a two-step process. The first step involves entering the command input, Status Register Read, at the positive edge of the $\overline{\mathrm{WE}}$ signal. The contents of the register are subsequently read sometime $\left(\mathrm{t}_{\mathrm{A}}\right)$ after the negative edge of $\overline{\mathrm{RE}}$ as shown in Fig. 5.39. Note that $\overline{\mathrm{CE}}$ signal is initially kept at logic 0 when entering the command input, but raised to logic 1 when reading the contents of the Status Register.

A full-page write data entry consists of the combination of four tasks as shown in Fig. 5.40. The first task is entering the Write to Data Buffer command at the positive edge of $\overline{\mathrm{WE}}$ while keeping $\overline{\mathrm{CE}}$ at logic 0 . The second task is entering the page and sector addresses at the positive edge of $\overline{\mathrm{WE}}$ while $\overline{\mathrm{AE}}$ is at logic 0 . The third task is entering the full-page of data from $\mathrm{D}(0)$ to $\mathrm{D}(255)$ into the data buffer at each positive edge of $\overline{\mathrm{WE}}$ signal. Both $\overline{\mathrm{AE}}$ and $\overline{\mathrm{CE}}$ are kept at logic 1 during this phase. The last task is entering the Write to Core Memory command in order to transfer all 256 bytes of data from the data buffer to the memory core. The last cycle needs a relatively longer time period, $\mathrm{t}_{\text {WRITE }}$, to complete the full-page write.

The read operation is composed of three individual tasks similar to the write operation as shown in Fig. 5.41. The first task is entering the Read from Memory command at the positive edge of $\overline{\mathrm{WE}}$ while $\overline{\mathrm{CE}}$ is at logic 0 . The second step is entering the starting address by specifying the row, page and sector address values at each positive edge of $\overline{\mathrm{WE}}$ while $\overline{\mathrm{AE}}$ is at logic 0 . The third task is to read data from the memory core at each negative edge of $\overline{\mathrm{RE}}$ while $\overline{\mathrm{CE}}$ and $\overline{\mathrm{AE}}$ signals are at logic 1 .


Fig. 5.39 Timing diagram for reading status register


Fig. 5.40 Timing diagram for full-page write (program)


Fig. 5.41 Timing diagram for full-page read
A typical full-page erase is described in Fig. 5.42. In this figure, the Erase Full Page command is entered first at the positive edge of $\overline{\mathrm{WE}}$ while $\overline{\mathrm{CE}}$ is at logic 0 . The memory address,


Fig. 5.42 Timing diagram for full-page erase
composed of page and sector addresses, is entered next while $\overline{\mathrm{AE}}$ is at logic 0. The Erase Core Memory command is entered following the address while $\overline{\mathrm{CE}}$ is at logic 0 . Full-page erase time period, $\mathrm{t}_{\text {ERASE }}$, must be employed to complete the operation.

### 5.4 Flash Memory

Flash memory is the successor of the Electrically-Erasable-Programmable-Read-OnlyMemory ( $\mathrm{E}^{2} \mathrm{PROM}$ ), and as its predecessor it has the capability of retaining data after power is turned off. Therefore, it is ideal to use in hand-held computers, cell phones and other mobile platforms.

A typical Flash memory is composed of multiple sectors and pages as shown in Fig. 5.43. An eight-bit word can be located in a Flash memory by specifying the sector, the page and the row addresses. To be compatible with the $\mathrm{E}^{2} \mathrm{PROM}$ architecture example given in the previous section, this particular Flash memory also contains 16 sectors and 16 pages. Each page contains 256 bytes. The sector address constitutes the most significant four bits of the 16-bit Flash address, namely Addr[15:12]. Each page in a sector is addressed by Addr[11:8], and each byte in a page is addressed by $\operatorname{Addr}[7: 0]$. There are five main control signals in Flash memory to perform basic read, write (program), erase, protect and reset operations. Write and program commands are equivalent to each other, and used interchangeably


Fig. 5.43 Flash memory organization
throughout the manuscript when describing Flash memory operations. Many Flash datasheets use the term, program, to define writing a byte or a block of data to Flash memory.

The active-low Enable input, $\overline{\mathrm{EN}}$, activates a particular page in the Flash memory to prepare it for an upcoming operation. The active-low Read Enable input, $\overline{\mathrm{RE}}$, activates the Read/Write interface to read data from the memory. The active-low Write Enable input, $\overline{\mathrm{WE}}$, enables to write (program) data to the memory. The active-low Reset input, $\overline{\text { Reset }}$, is used for resetting the hardware. After this command, Flash memory automatically goes into the read mode.

Typical Flash memory architecture, much like the other memory structures we have examined earlier, consists of a memory core, address decoder, sense amplifier, data buffer and control logic as shown in Fig. 5.44. When a memory operation starts, the control logic enables the address decoder, the address register, and the appropriate data buffers in order to activate the read or the write data-path. The address in the address register is decoded to point the location of data in the memory core. If a read operation needs to be performed, the retrieved data is first stored in the data buffer, and then released to the bus. If the operation calls for a write, the data is stored in the data buffer first, and then directed to the designated address in the memory core. The standby mode neither writes to the memory nor reads from it. The hibernation mode disables the address decoder, memory core and data buffer to reduce power dissipation. The main Flash operation modes are tabulated in Fig. 5.45.


Fig. 5.44 Flash memory architecture

| $\overline{\mathrm{EN}}$ | $\overline{\mathrm{RE}}$ | $\overline{\mathrm{WE}}$ | $\overline{\text { reset }}$ | MODE |
| :---: | :---: | :---: | :---: | :--- |
| 0 | 0 | 1 | 1 | Read |
| 0 | 1 | 0 | 1 | Write |
| 0 | 1 | 1 | 1 | Standby |
| 1 | X | X | 1 | Hibernate |
| X | X | X | 0 | Hardware reset |

Fig. 5.45 Main modes of Flash memory
The Flash memory cell shown in Fig. 5.34 is the basic storage element in the Flash memory core. It is an N -channel MOS transistor with a floating gate whose sole purpose is to store electronic charge. The device needs high voltages well above the power supply voltage to create and transfer electrons back and forth to the floating gate according to the need. If there is no secondary high voltage supply for this job, the control logic in Fig. 5.44 may contain a charge pump circuit composed of a constant current source and a capacitor in order to obtain a higher DC voltage from the primary power supply for a short duration. As the constant current charges the capacitor, the voltage across the capacitor rises linearly with time, ultimately reaching a high DC potential to create electron-hole pairs near the Bitline contact (drain) of the device, ultimately causing the electrons to tunnel to the floating gate. The mechanism of electron tunneling to the floating gate requires time. Therefore, a write or erase operation may take many consecutive clock cycles compared to simple control operations such as suspend or resume.

Figure 5.46 shows the basic read operation provided that data has already been transferred from the memory core to the data buffer. Once a valid address is issued, data is produced at the I/O terminal some time after the falling edge of the Read Enable signal, $\overline{\mathrm{RE}}$. Data is held at the I/O port for a period of hold time, $\mathrm{t}_{\mathrm{h}}$, following the rising edge of $\overline{\mathrm{RE}}$ as shown in the timing diagram below. The actual read operation takes about four clock cycles as the entire data retrieval process from the memory core takes time. This involves sensing the voltage level at the Flash cell, amplifying this value using the sense amplifier, and propagating the data from the sense amplifier to the data buffer.

In contrast to read, the basic write operation follows the timing diagram of Fig. 5.47. In this figure, a valid address must be present at the address port when the Enable and the Write Enable signals, $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$, are both at logic 0 . Valid data satisfying the setup and hold times, $\mathrm{t}_{\mathrm{s}}$ and $\mathrm{t}_{\mathrm{h}}$, is subsequently written to the data buffer. The actual write process can take up to four clock cycles due to the data propagation from the I/O port to the data buffer, and then from the data buffer to the Flash cell.


Fig. 5.46 Basic read operation timing diagram


Fig. 5.47 Basic write (program) operation timing diagram
Disabling the I/O port for read or write, and therefore putting the device in standby mode requires $\overline{\mathrm{EN}}$ signal to be at logic 0 as shown in Fig. 5.48. The I/O port will float and show high impedance (Hi-Z).


Fig. 5.48 Basic standby operation
Hardware reset requires only lowering the $\overline{\text { Reset }}$ signal during the command cycle as shown in Fig. 5.49. The actual reset operation takes three bus cycles and resets the entire Flash memory.


Fig. 5.49 Basic hardware reset operation timing diagram

Basic Flash memory operations are tabulated in Fig. 5.50. In actuality, there are a lot more commands in commercially available Flash memories than what is shown in this table. This section considers only essential byte-size operations in a Flash memory. Word-size

| MAIN COMMANDS | CYCLE 1 |  | CYCLE 2 |  | CYCLE 3 |  | CYCLE 4 |  | CYCLE 5 |  | CYCLE 6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Addr | Data | Addr | Data | Addr | Data | Addr | Data |  |  |  |  |
| Read | 0x5555 | 0xAA | 0xAAAA | 0x55 | 0x5555 | 0x00 | Read Addr | Read Data |  |  |  |  |
| ID Read | 0x5555 | 0xAA | 0xAAAA | $0 \times 55$ | 0x5555 | 0x10 | Manuf. Addr | Manuf. Data | Device Addr | Device Data |  |  |
| Write | 0x5555 | 0xAA | 0xAAAA | 0x55 | 0x5555 | 0x20 | Write Addr | Write Data |  |  |  |  |
| Write suspend | Page Addr | 0x30 |  |  |  |  |  |  |  |  |  |  |
| Write resume | Page Addr | 0x40 |  |  |  |  |  |  |  |  |  |  |
| Chip erase | 0x5555 | 0xAA | 0xAAAA | $0 \times 55$ | 0x5555 | 0x50 | 0x5555 | 0xAA | 0xAAAA | $0 \times 55$ | 0x5555 | 0x60 |
| Page erase | 0x5555 | 0xAA | 0xAAAA | 0x55 | 0x5555 | 0x50 | 0x5555 | 0xAA | 0xAAAA | 0x55 | Page Addr | 0x70 |
| Page erase suspend | Page Addr | 0x80 |  |  |  |  |  |  |  |  |  |  |
| Page erase resume | Page Addr | 0x90 |  |  |  |  |  |  |  |  |  |  |
| Page protect | Page Addr | 0xA0 | Page Addr | 0xA0 | Page Addr | 0xA0 | Page Addr | Verif. Code |  |  |  |  |
| Fast write set | 0x5555 | 0xAA | 0xAAAA | 0x55 | 0x5555 | 0xB0 |  |  |  |  |  |  |
| Fast write | $0 \times X X X X$ | 0xC0 | Write Addr | Write Data |  |  |  |  |  |  |  |  |
| Fast write reset | 0xXXXX | 0xD0 | 0xXXXX | 0xE0 |  |  |  |  |  |  |  |  |
| Reset | 0x5555 | 0xAA | 0xAAAA | 0x55 | 0x5555 | 0xF0 |  |  |  |  |  |  |

Fig. 5.50 Flash memory commands with required clock cycles
operations, very specific Flash command sequences, such as hidden ROM programs, query and verification commands and boot protection processes are avoided in order to emphasize the core Flash memory operations for the reader. Address and data entries for each specific command in Fig. 5.50 are also modified compared to the actual datasheets to simplify the read, write (program) and erase sequences. The number of clock cycles, the address and data preamble values in each cycle, and the operational codes to perform read, write, page erase, chip erase, page protect, fast write and other modes of operation may be different from the actual datasheets.

The first task in Fig. 5.50 is the Flash memory read sequence which takes four clock cycles. The first three clock cycles of this sequence represents the waiting period to prepare the read path from the memory core. During this period, address and data values in the form of alternating combinations of 1 s and 0 s , such as $0 \times 5555 / 0 \mathrm{xAA}$ and then $0 \times \mathrm{xAAA} / 0 \mathrm{x} 55$, are introduced at the address and data ports of the Flash memory as shown in Fig. 5.51. Once the read command, $0 x 00$, is issued in the third clock cycle, a byte of data becomes available shortly after the negative edge of the $\overline{\mathrm{RE}}$ signal in the fourth and final clock cycle.


Fig. 5.51 Timing diagram for read operation

Figure 5.52 shows an example of the read operation which extracts the manufacturer's ID and device ID from the Flash memory. The first three clock cycles of this sequence are the same as the normal read operation, but with the exception of the ID read code, 0x10. The next two cycles deliver the manufacturer's ID and the device ID following the negative edge of the $\overline{\mathrm{RE}}$ signal.


Fig. 5.52 Timing diagram for ID read operation
There are basically two types of write (program) operations for the Flash memory: auto write (program) and fast write (program). Figure 5.53 explains the auto write sequence where the first three cycles are the same as the read sequence with the exception of the auto write command code, $0 \times 20$, in the third clock cycle. In the fourth cycle, a valid address and a data are entered to the device when $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$ are both lowered to logic 0 . The valid data is subsequently written to the specified address at the positive edge of $\overline{\mathrm{WE}}$. The data written to the Flash memory can be retrieved in the following cycle without going through a separate read sequence. This is called the auto write verification step, and the most recent written data becomes available at the I/O port as soon as $\overline{\mathrm{RE}}$ is lowered to logic 0 .


Fig. 5.53 Timing diagram for write (program) operation
The write sequence can be suspended or resumed depending on the need. Both operations take only one clock cycle with the appropriate suspend and resume codes as shown in Figs. 5.54 and 5.55 , respectively. The write suspend and resume codes can be read in the second cycle as a verification step.


Fig. 5.54 Timing diagram for write (program) suspend operation


Fig. 5.55 Timing diagram for write (program) resume operation
The erase operation can be applied either to the entire chip or to a particular page. Both sequences take six clock cycles because of the lengthy nature of erase process. In the auto chip erase operation, the first three and the last three cycles are almost identical except the two new codes, $0 \times 50$ and $0 \times 60$, are introduced in the third and in the sixth cycles as shown in Fig. 5.56.


Chip erase command
Fig. 5.56 Timing diagram for chip erase operation

The first five clock cycles of the page erase operation are identical to the chip erase as shown in Fig. 5.57. The page address to be erased is supplied with the page erase command, $0 x 70$, in the sixth cycle.


Fig. 5.57 Timing diagram for page erase operation
A certain Flash memory page can be protected from being overwritten or erased by issuing a page protect operation. This is a three-cycle operation as shown in Fig. 5.58. In all three cycles, the page address and the page protect code, 0 x 0 A , have to be specified.


Fig. 5.58 Timing diagram for page protect operation

If faster writing speed is required from the Flash memory, the fast write (program) sequence can be used. This sequence is composed of three parts: fast write set, fast write and fast write reset. The fast write set and reset codes are entered at the beginning and at the end of a write sequence. Figure 5.59 shows the timing diagram for the fast write set sequence where the set code, $0 \times \mathrm{xB} 0$, is entered in the third clock cycle.


Fig. 5.59 Timing diagram for fast write (program) set operation
The timing diagram for the fast write is a two-cycle sequence as shown in Fig. 5.60. In the first cycle, the fast write code, $0 \mathrm{xC0}$, is entered. In the second cycle, a valid address/data pair is entered at the positive edge of $\overline{\mathrm{WE}}$.


Fig. 5.60 Timing diagram for fast write (program) operation

The fast write reset sequence shown in Fig. 5.61 is also a two-cycle process with two fast write termination codes, 0 xD 0 and 0 xE 0 , entered in two consecutive clock cycles.


Fig. 5.61 Timing diagram for fast write (program) reset operation

Device reset can be initiated either by $\overline{\text { Reset }}$ input in Fig. 5.48 or by entering the reset code, $0 x F 0$, in the third clock cycle of Fig. 5.62.


Fig. 5.62 Timing diagram for Reset operation

Both reading and writing (programming) data are cyclic processes. This means that a loop has to be established in the user program to generate a series of memory addresses to read or write data.

Figure 5.63 shows the auto write (program) flow chart where a loop is created to generate the next write address. Each box in the flow chart corresponds to a clock cycle. The first three boxes of the flow chart prepare the memory core for a write operation. The preparation period terminates with the auto write code, 0x20, as mentioned earlier in Fig. 5.53. After entering the first write address and data, the memory address is incremented in the fourth cycle. The same process repeats itself prior to issuing the next address and data. When the final address is reached, the write process simply terminates.


Fig. 5.63 Flow chart for write (program)

The sequence of events is a little different for the fast write (program) in Fig. 5.64. The fast write phase starts with the three-cycle long fast write set sequence followed by the two cycle long fast write sequence. The memory address keeps incrementing until the last data byte is written to the core. The fast write process ends with the two-cycle long fast write reset sequence.

Auto write and fast write processes can be interrupted or resumed by issuing one-cycle long suspend and resume commands anytime during the write process.


Fig. 5.64 Flow chart for fast write (program)

In the following sections, we will demonstrate how to design three individual $\mathrm{I}^{2} \mathrm{C}$ bus interfaces with Flash memory to perform read, write and erase operations. In each design, we will assume only one mode of operation to simplify the design process. After studying each design example, the reader is encouraged to design a single interface that integrates all three operations.

## Design Example 1: <br> $I^{2} \mathbf{C}$ Fast Write (Program) Interface for Flash Memory

The following design example constructs only the $I^{2} C$ fast write (program) interface for a Flash memory that has parallel address and data ports as shown in Fig. 5.44 using a modified seven-bit address mode. No read, auto write, erase, page protect, reset or other modes are included in this design for the sake of simplicity.

Before dealing with the design details and methodology, it may be prudent to review the timing diagram of $\mathrm{I}^{2} \mathrm{C}$ write sequence using the seven-bit addressing mode. Although Fig. 5.65 includes only one byte of data, it describes the entire write protocol for the seven-bit address mode in Fig. 4.22. This diagram also includes the start and the stop conditions in Fig. 4.23, and when data (or address) is allowed to change in Fig. 4.24. After generating the start condition, the bus master delivers a seven-bit slave address, starting from the most significant bit, A6. The address sequence is followed by the write bit at logic 0 . Once the slave receives the seven-bit address and the write command, it produces an acknowledgment, ACK, by lowering the SDA bus to logic 0 . The master detects the ACK signal, and sends out an eight-bit data starting from the most significant bit, D7. Once the entire byte is received, the slave responds with another ACK. More data packets follow the same routine until the master generates the stop condition.


Fig. 5.65 A simple timing diagram for $\mathrm{I}^{2} \mathrm{C}$ write (program) using seven-bit address mode

In Fig. 5.65, the names that appear on top of each SCL cycle describe a distinct state. If a state machine needs to be constructed from this timing diagram, we simply assign an independent state that corresponds to each name in Fig. 5.65 and produce a state diagram in Fig. 5.66. In this diagram, the start condition activates the state machine, which goes through the address and the command sequences before the data. As long as the state machine does
not detect any stop condition, it constantly traces the data states D7 to D0. However, when there is a stop condition, the state machine goes to the IDLE state and waits for another start condition to emerge.


Fig. 5.66 The state diagram for the simple $\mathrm{I}^{2} \mathrm{C}$ write (program) in Fig. 5.65
Even though this example only shows the fast write interface, it sets up a solid foundation of how to design any typical $I^{2} \mathrm{C}$ interface between a bus master and a Flash memory. The first step of the design process is to create a rough interface block diagram showing all the major I/O ports between the Flash memory and the $\mathrm{I}^{2} \mathrm{C}$ bus as shown in Fig. 5.67. For the fast write sequence, the address and data packets are serially transferred to the interface through the SDA port. However, the Flash memory needs the address and data fields all at once. Therefore, the write operation requires the interface to perform serial-to-parallel conversion of incoming data. The interface also needs to produce two control signals, $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$, for the fast write sequence, and the control signal, EnDataOut, for writing an eight-bit data to a designated Flash memory address.


Fig. 5.67 Simplified page diagram of the $I^{2} C$ fast write (program) interface
Figure 5.68 shows the architectural block diagram of the Flash memory interface for the fast write operation. As mentioned earlier when designing SRAM and SDRAM memory interfaces, creating a complete data-path for an interface is not a single-step process. The design methodology requires building a simple data-path with all of its functional units and a corresponding timing diagram showing the flow of data in each clock cycle, the start and stop conditions, address and data formations. However, as more detail is added to the architecture, the initial timing diagram also becomes more complex to match the architecture. Figures 5.69, 5.70 and 5.71 show a set of timing diagrams related to the architecture in Fig. 5.68. These diagrams describe the start and stop conditions, preamble sequence, address and data formations, repacking and delivery of serial address and data in a systematic manner.

Once the cycle-by-cycle nature of address and data entries are accurately described in the timing diagram, the control signals responsible for routing the address and data can be added to the diagram. The final step of the design process is to assign distinct states to each clock cycle in the timing diagram that contains different sets of control signals in order to generate a Moore-type state machine.

To start, we need to include four functional units in Fig. 5.68 to be able to handle a simple fast write operation. The first functional unit is an eight-bit shift register whose sole purpose is to convert the incoming serial data from the SDA port into a parallel form. If the fast-write process requires an authentication step prior to data exchange, the first eight-bit packet coming to this interface must be delivered to the device ID register, which is considered the second functional unit. The second and the third eight-bit data packets arriving at the interface belong to the most and the least significant bytes of the 16 -bit starting Flash memory address, respectively, and they are stored in the address counter. The address counter, which constitutes the third functional unit, uses this initial address to generate subsequent addresses for programming the Flash memory. All eight-bit data that follows the address entry is routed directly to the data port of the Flash memory. There are also several fixed-value registers connected to the inputs of the address and data MUXes in Fig. 5.68. These registers contain
the preamble data for setting and resetting the fast write modes for the Flash memory prior to address and data sequences. The write controller, which is considered to be the fourth functional unit, generates all the control signals necessary for storing data, incrementing the address, and routing the address and data to the output ports of the interface. The host processor delivers all address, control and data signals to the interface at the negative edge of the SCL, which requires all the registers to operate at the positive edge of SCL in Fig. 5.68.


Fig. 5.68 $\mathrm{I}^{2} \mathrm{C}$ fast write (program) interface data-path

The START condition in Fig. 5.69 is produced by the bus master in cycle 1 by lowering the SDA signal to logic 0 while keeping the SCL signal at logic 1 . Once the START condition is detected, the serial data on the SDA port is transferred to an eight-bit shift register which converts this data into a parallel form before sending it to different registers in Fig. 5.68. In clock cycles 2 to 8 , the seven-bit Flash memory ID is loaded to the shift register starting from the most significant bit if device authentication is required prior to data transmission. In cycle 9, the bus master sends the write bit, W, stored in the shift register. In cycle 10, a number of events take place simultaneously. First, the write bit at the least significant bit position of the shift register activates the write controller. Second, the device ID and the write bit are transferred from the shift register to a special device ID register. Third, the Flash memory interface produces the first acknowledge signal, ACK, by EnSDAOut $=1$, thus lowering the SDA bus to logic 0 . Finally, the interface sends the first preamble which consists of the address, 0x5555, and the data, 0xAA, to the Flash memory and lowers $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$ to logic 0 as the first step of the fast write set.

In cycle 11, the most significant bit of the 16-bit initial Flash memory address, Add15, is received from the SDA bus and stored in the shift register. This cycle is also considered a hold period for the $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$ signals. In cycle 12, the second most significant address bit, Add14, is stored in the shift register. In this cycle, the second preamble that contains the address, 0xAAAA, and the data, $0 \times 55$, are sent to the Flash memory as the second step of the fast write set. In cycle 14 , the third address and command preamble, $0 \times 5555$ and $0 x B 0$, are sent to the Flash memory, completing the fast write set sequence. The fast write sequence starts at cycle 16 where the fast write command, $0 x C 0$, is sent to the Flash memory. In cycle 19 , the most significant byte of the 16 -bit starting Flash memory address, StAddMSB, is transferred from the shift register to the address register which resides inside the address counter. In this cycle, the interface also generates the second ACK signal by EnSDAOut $=1$.

From cycles 20 to 27 in Fig. 5.70, the least significant byte of the starting Flash address, StAddLSB, is received by the shift register. In cycle 28, this byte is transferred to the least significant byte of the address register in order to form the 16-bit starting Flash address. In this cycle, the interface generates the third ACK signal. From cycles 29 to 36, the first set of data bits starting from the most significant bit, DF7, to the least significant bit, DF0, are received by the shift register. In cycle 37, the first eight-bit data packet, Data0, is transferred to the Flash memory through its bidirectional I/O port. The tri-state buffers in Fig. 5.68 need to be enabled by the control signal, EnDataOut, to be able write this data packet to the Flash memory.

Cycles 38 to 45 in Fig. 5.71 are used to store the second eight-bit data packet in the shift register. Cycle 46 transfers this data packet, Data1, to the I/O port and generates an ACK signal for receiving the second data byte from the bus master. If the STOP condition is detected during the next clock cycle, the fast write process halts. The write controller goes into the fast write reset mode and asynchronously produces selAux $=1$ to engage the auxiliary clock, SCLAux, instead of using the main SCL clock, SCLMaster, generated by the bus

Fig. 5.69 $I^{2} \mathrm{C}$ fast write (program) set sequence with device ID and memory address



Fig. 5.71 $\mathrm{I}^{2} \mathrm{C}$ fast write (program) reset sequence
master. This is because the Flash memory needs two more preambles that contain 0xD0 and $0 x E 0$ commands to complete the fast write reset sequence. Therefore, starting from cycle 48, SCL resumes with three more cycles. In cycle 48 , the first preamble that contains $0 x \mathrm{DD} 0$, and in cycle 50 the second preamble that contains $0 x E 0$ are sent to the Flash memory by setting selD0 and then selE0 to logic 1, respectively. In the next cycle, selAux becomes logic 0 , and SCL switches back to the SCLMaster input which permanently stays at logic 0 .

The Moore machine in Fig. 5.72 implements the write controller in Fig. 5.68. At the onset of the START condition, the controller wakes up and goes into the device ID retrieval mode. From cycles 2 to 8 in Fig. 5.69, the serial device ID is received by the shift register on the SDA bus. These cycles correspond to the states DAdd6 to DAdd0 in Fig. 5.72 where the shift signal is constantly kept at logic 1 , and writing data to the Flash memory is disabled. In cycle 9 , the write bit is also stored in the shift register. This corresponds to the W state in the state machine. In cycle 10, numerous events take place simultaneously. First, shifting serial data into the shift register stops by shift $=0$. Second, the seven-bit device ID and the write bit are delivered to the device ID register by LoadDevID $=1$. Third, the first address and data preamble, $0 \times 5555$ and 0 xAA , is delivered to the Flash memory through port 1 of the address MUX by sel5555 = 1 and port 1 of the data MUX by selAA $=1$. Fourth, the control signals, $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$, are lowered to logic 0 in order to write the address and data preamble to the Flash memory. Finally, an ACK signal is generated by EnSDAOut $=1$. This cycle corresponds to the DevID ACK state of the write controller.

In cycle 11, shifting data resumes, and the shift register receives the most significant bit of the initial Flash address, Add15, while in the Add15 state. In cycle 12, which corresponds to the Add14 state, the second most significant address bit, Add14, is latched in the shift register by shift $=1$. In the same cycle, the second address and data preamble, $0 x A A A A$ and $0 \times 55$, is delivered to the Flash memory through port 2 of the address MUX by selAAAA $=1$ and port 2 of the data MUX by sel55 $=1$. The control signals, $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$, are also lowered to logic 0 in order to write this preamble to the Flash memory. In cycle 13, Add13 is stored in the shift register. This cycle corresponds to the Add13 state. In cycle 14, the third address and data preamble, $0 \times 5555$ and $0 \times B 0$, is written to the Flash memory through port 1 of the address MUX by sel5555 = 1 and port 3 of the data MUX by selB $0=1$. In this cycle, the control signals, $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$, are lowered to logic 0 in order to write the last address and data preambles. This cycle corresponds to the Add12 state. Cycle 15 designates the end of the fast write set cycle, and corresponds to the Add11 state when the address bit, Add11, is loaded to the shift register.


Fig. 5.72 $\mathrm{I}^{2} \mathrm{C}$ fast write (program) interface controller

Cycle 16 enters the fast write command mode and writes the address and data preambles, $0 x \mathrm{XXXX}$ and 0 xC 0 , through port 4 of the data MUX by selC $0=1$. In this cycle, $\overline{\mathrm{EN}}$ and $\overline{\mathrm{WE}}$ signals are lowered to logic 0 to accommodate the write operation, and Add10 is latched in the shift register. This clock cycle corresponds to the Add10 state. Storing the higher byte of the initial Flash address becomes complete by the end of cycle 18. In cycle 19, the higher byte of the initial address is transferred to the address register by LoadAddMSB $=1$, and an acknowledge signal is generated by EnSDAOut $=1$. This cycle corresponds to the AddMSB ACK state in the state diagram. Similar events take place when storing the least significant byte of the starting address in the shift register. These states are marked as Add7 to Add0 in the state diagram, and correspond to the cycles 20 though 27, respectively. The cycle 28 , which corresponds to the AddLSB ACK state, generates the third acknowledge for the bus master by EnSDAOut = 1, and transfers the least significant byte of the starting Flash memory address to the address register by LoadAddLSB $=1$.

From cycles 29 to 36, the first set of data bits are delivered to the shift register starting from the most significant data bit, DF7. This sequence is shown as the states DF7 to DF0 in the state diagram. In cycle 37, an acknowledgement is sent to the bus master by EnSDAOut = 1 shown as the Data ACK state. During this period, the initial 16-bit address and eight-bit data are delivered to the Flash memory through port 0 of the address MUX by selAddr $=1$ and port 0 of the data MUX by selData $=1$. Tri-state buffer at the I/O port is also enabled by EnDataOut $=1$. The second data byte is received during cycles 38 to 45 , which correspond to the states D7 to D0, respectively. Cycle 45 is also the cycle to increment the Flash memory address by issuing IncrAddr $=1$. In Cycle 46, the write controller goes into the Data ACK state once again and issues an acknowledgement for receiving the second data packet by EnSDAOut $=1$. In this cycle, the second data packet is delivered to the incremented Flash memory address, StAdd +1 , by selAddr $=1$, selData $=1$ and EnDataOut $=1$. As long as the STOP condition is not detected, data packets are delivered to the Flash memory at each incremented address. However, if the bus master issues a STOP condition, the auxiliary SCL generator, SCLAux, is asynchronously enabled within the same cycle by selAux $=1$. The write controller goes into the fast write reset mode in the next clock cycle and keeps the auxiliary SCL generator enabled by selAux $=1$. For the next three clock cycles, the $0 x \mathrm{D} 0$ and 0 xE 0 command codes, corresponding to the 0 xD 0 and 0 xE 0 states in the state diagram, are delivered to the Flash memory through port 5 of the data MUX by selD0 $=1$ and port 6 of the data MUX by selE0 $=1$.

## Design Example 2:

## $I^{2} \mathbf{C}$ Read Interface for Flash Memory

The following design example constructs only the $\mathrm{I}^{2} \mathrm{C}$ read interface for a Flash memory that has parallel address and data ports shown in Fig. 5.44 using a modified seven-bit address mode. No other modes are included in this design except the read.

The timing diagram for $\mathrm{I}^{2} \mathrm{C}$ read sequence is given in Fig. 5.73 where each eight-bit data packet is serially read from a slave after issuing an initial seven-bit address. The address sent by the bus master requires an acknowledgment (ACK) from the slave. In contrast, data packets sent by the slave require the master's acknowledgment. If the bus master chooses not to acknowledge the receipt of data (NACK), the data transfer stops in the next cycle. Figure 5.74 shows the sequence of events taking place in Fig. 5.73 in the form of a state diagram where the logic level in Master ACK/NACK state determines the continuation or the end of the data transfer.


Fig. 5.73 A simple timing diagram for the $\mathrm{I}^{2} \mathrm{C}$ read operation using seven-bit address mode
The Flash memory read sequence described in Fig. 5.75 is a four-cycle process as mentioned earlier in Fig. 5.50. The bus master sends the address and data preambles, $0 \times 5555 / 0 x A A$ and $0 x A A A A / 0 x 55$, in the first two cycles. This is followed by the 0x5555/0x00 preamble containing the read command code in the third cycle. All three cycles can be considered a preparation period for a read operation which takes place in the fourth cycle. Following the read operation, the address is incremented either by one or a predefined value according to the Flash memory address generation protocol before the next data read sequence takes place.

To read data from the Flash memory, the address and command entries are serially sent by the host processor to the $\mathrm{I}^{2} \mathrm{C}$ interface through the SDA port. The Flash memory requires a


Fig. 5.74 The state diagram for the simple $\mathrm{I}^{2} \mathrm{C}$ read operation in Fig. 5.73

16-bit address all at once in order to read an eight-bit data, and this necessitates an interface to perform both serial-to-parallel and parallel-to serial conversions. The interface has to produce three active-low control signals, $\overline{\mathrm{EN}}, \overline{\mathrm{WE}}$ and $\overline{\mathrm{RE}}$, to be able to read data from the Flash memory. It also needs to produce the control signals, EnDataIn and EnDataOut, to route the incoming and outgoing data.


Fig. 5.75 Flow chart for the read sequence
Figure 5.76 shows the architectural diagram of the Flash memory read interface. Figures 5.77, 5.78 and 5.79 show the timing diagrams related to this architecture. These waveforms describe a complete picture of the preamble formation, device ID creation, address generation and serializing the read data from the Flash memory.

The architecture in Fig. 5.76 still contains four functional units as in the fast write (program) data-path. The first functional unit is an eight-bit shift register which identifies the serial address, command and data boundaries, and converts the serial data on the SDA bus into a parallel form and the parallel data from the Flash memory into a serial form. The second functional unit stores the device ID if the Flash memory requires an identification process prior to data exchange, and the command bit. The third unit stores the initial 16-bit Flash memory address and generates the subsequent memory addresses using an up-counter. The fourth unit is the read controller, which is responsible for storing the device ID, the command bit, forming and incrementing the initial address, and handling the proper data-flow that complies with the timing diagrams in Figs. 5.77, 5.78 and 5.79. There are also several fixed-value registers, each of which contains the address and data preambles for retrieval of data from the Flash memory. The host processor dispatches all address and
control signals at the negative edge of the SCL clock in order to read data packets from the Flash memory, and therefore it requires all the registers in Fig. 5.76 to operate at the positive edge of the SCL clock.


Fig. 5.76 $I^{2} \mathrm{C}$ read interface data-path


[^0]
Fig. 5.78 $I^{2} C$ read sequence with the MSB of starting memory address and data formation


| SCL |
| :--- |
| SDA |
| ShiftRegOut |
| ShiftRegln |
| Addr |
| AddrCountOut |
| I/O[7:0] |
| EN |
| WE |
| RE |
| Shiftln |
| ShiftOut |
| LoadShift |
| LoadDevID |
| LoadAddMSB |
| IncrAddr |
| selStartAddr |
| selAddr |
| sel5555 |
| selAAAA |
| sel55 |
| selAA |
| sel00 |
| EnDataOut |
| EnDataln |
| SDAIn |
| SDAOut |
| EnSDAOut |

Fig. 5.79 $\mathrm{I}^{2} \mathrm{C}$ read sequence emphasizing the end of read cycle

Figures 5.77, 5.78 and 5.79 describe the complete picture of reading data from the Flash memory. Figure 5.77 shows the device ID and the command bit formations followed by the generation of the most significant byte of the initial Flash memory address. Figure 5.78 describes the formation of the least significant byte of the initial Flash memory address and the first data byte read from the memory. Figure 5.79 includes two additional bytes of data sent to the bus master and the termination of data transfer.

The bus master initiates the data transfer by issuing the START condition in Fig. 5.77. Between cycles 1 and 8, the bus master sends the device ID followed by the read command on the SDA bus, both of which are serially loaded to an eight-bit shift register by ShiftIn $=1$. These cycles are represented by the states DAdd6 to DAdd0 followed by the R state, which corresponds to the read command, in the state diagram in Fig. 5.80. In cycle 9, the Flash memory interface responds to the bus master with an acknowledgement by issuing EnSDAOut $=1$, but pauses shifting data by ShiftIn $=0$. In the same cycle, the interface also transfers data from the shift register to the device ID register by LoadDevID $=1$. This cycle corresponds to the first slave-acknowledgement state, DevID SACK, in Fig. 5.80. In cycle 10, the interface starts sending the preamble to the Flash memory for a read operation. In this cycle, the first address and data preamble, 0x5555/0xAA, is fetched from the fixed-data registers, 0x5555 and 0xAA in Fig. 5.76. This preamble is subsequently sent to the address port of the device through port 1 of the address MUX by sel5555 = 1, and to the data port of the device through port 1 of the data MUX by selAA $=1$. In this cycle, the bus master also sends the most significant bit of the 16 -bit Flash memory address, Add15, on the SDA bus.

In cycle 12 , the interface sends the second address and data preamble, $0 x \mathrm{AAAA} / 0 x 55$, through port 2 of the address MUX by selAAAA $=1$ and port 0 of the data MUX by sel55 $=1$. In cycle 14 , the last address and data preamble, $0 \times 5555 / 0 \times 00$, containing the read command, is sent to the Flash memory address and data ports. The cycles 10 to 17 correspond to storing the most significant byte of the starting Flash memory address, Add15 to Add8, in the shift register by ShiftIn $=1$. In cycle 18 , the interface sends an acknowledgement to the bus master by EnSDAOut $=1$ to indicate that it has received the higher byte of the starting Flash memory address. Within the same cycle, this higher byte is stored in the 16-bit address register that resides in the address counter by LoadAddMSB $=1$. This cycle represents the second slave-acknowledgement state, AddLSB SACK, in Fig. 5.80.

In cycles 19 to 26, the bus master sends the least significant byte of the starting Flash memory address by ShiftIn $=1$. These cycles correspond to the states Add7 to Add0 in Fig. 5.80, respectively. Cycle 27 constitutes the third slave-acknowledgement state, AddLSB SACK, in Fig. 5.80. There are numerous events that take place during this clock cycle, and they are all inter-related. The first event concatenates the least significant byte of the starting Flash memory address in the shift register with the most significant byte in the Addr MSB register to form the complete 16-bit starting Flash memory address. This address is subsequently sent to the $\operatorname{Addr}[15: 0]$ terminal of the Flash memory through port 1 of the address register MUX by selStartAddr $=1$ and port 0 of the address MUX by selAddr $=1$.


Fig. $5.80 \mathrm{I}^{2} \mathrm{C}$ read interface controller

The second event lowers $\overline{\mathrm{EN}}$ and $\overline{\mathrm{RE}}$ control signals to logic 0 and produces EnDataIn $=1$ in order to fetch the first data byte from the Flash memory, Data0, since the read preamble has already been sent between cycles 10 and 14 . The third event stores Data0 in the shift register through its ShiftRegIn port by LoadShift $=1$. Finally, the last event sends an acknowledgement signal to the bus master by EnSDAOut = 1, signifying the least significant byte of the starting address has been received so that the bus master can start receiving serial data on the SDA bus in the next cycle.

In cycle 28, the starting address, which could not be registered due to time limitations in the earlier cycle, is now registered in the address register, and the address counter output, AddrCountOut, becomes equal to the starting address, StartAdd. In the same cycle, the most significant bit of Data0, D7, becomes available on the SDA bus by SDAOut $=1$. Starting from cycle 30, the read preamble associated with the second data is sent to the Flash memory. The read preamble could have been issued as early as cycle 28 or 29 since the address counter still held StartAdd at the AddrCountOut node during these periods. In cycle 35, the interface increments the starting address by IncrAddr $=1$ and uses port 2 of the address register MUX to feed through the result. Until the beginning of cycle 36, all eight bits of Data0, D7 to D0, are serially sent to the bus master by $\mathrm{SDAOut}=1$. Therefore, cycles 28 to 35 correspond to the states D7 to D0 in Fig. 5.80, respectively. In cycle 36, while the bus master acknowledges the reception of Data0 by MasterAck $=1$, and thereby lowering the SDA bus to logic 0 , the interface sends the incremented Flash memory address, StAdd + 1, to the Addr[15:0] terminal through port 0 of the address counter MUX by selAddr $=1$. In the same cycle, the interface lowers $\overline{\mathrm{EN}}$ and $\overline{\mathrm{RE}}$ signals to logic 0 , fetches Data1 from the I/O port of the Flash memory by EnDataIn $=1$, and stores this value in the shift register by LoadShift $=1$. This particular cycle corresponds to the master-acknowledge state, MACK, in Fig. 5.80, where the state machine continues fetching data from the Flash memory.

Cycles 37 to 44 and cycles 46 to 53 contain identical events to the ones between cycles 28 and 35 . They both correspond to the states D7 to D0 in Fig. 5.80. In cycle 54, the SDA bus transitions to logic 1 because the bus master decides not to issue any more acknowledgements by MasterAck $=0$. Even though the current address increments at this point, the interface neither lowers the $\overline{\mathrm{EN}}$ nor lowers the $\overline{\mathrm{RE}}$ signal to logic 0 as shown by the master-no-acknowledge state, MNACK. Therefore, no data reading takes place from the Flash memory. In the next cycle, the bus master terminates the SCL activity and issues the STOP condition, signifying the end of data transfer.

## Design Example 3:

## $\mathbf{I}^{\mathbf{2}} \mathbf{C}$ Page Erase Interface for Flash Memory

The following design example constructs only the $\mathrm{I}^{2} \mathrm{C}$ page erase interface for a Flash memory that has parallel address and data ports as shown in Fig. 5.44. Using a modified seven-bit address mode. No other Flash memory mode is implemented in this design except the erase.

The Flash memory page erase is a six-cycle sequence as described earlier in Fig. 5.50. The flow chart for this process is shown in Fig. 5.81. In the first five cycles, the bus master sends fixed address/data combinations to the Flash memory as a preamble to prepare the memory to erase a block of data at a specified memory location. The page erase command is the $0 \times 50$ entry in the third cycle followed by the page address and the second erase command, $0 x 70$, in the sixth cycle to initiate the process.


Fig. 5.81 Flow chart for page erase

Figure 5.82 shows the data-path for the $\mathrm{I}^{2} \mathrm{C}$ page erase interface. The shift register acquires the device ID (if the Flash memory requires any type of device authentication prior to page erase) and the page address from the SDA bus, and transfers them to the device ID register and the page address register, respectively. There are also address and data registers that store only the fixed values, and they are routed to the address and data ports of the Flash memory in order to produce the correct preamble and page erase commands in Fig. 5.81.


Fig. 5.82 $I^{2} \mathrm{C}$ page erase interface data-path

The page erase process is described in the timing diagrams of Figs. 5.83, 5.84 and 5.85. The process starts with the bus master generating the START condition in Fig. 5.83. In cycles 2 to 9 , the bus master sends the seven-bit device ID and the write bit, starting with the most significant device ID bit, DAdd6. Even though the write bit is considered a command bit, it does not have any significance in the page erase preamble. The bus master sends this bit only to comply with the $\mathrm{I}^{2} \mathrm{C}$ protocol. All these bits are temporarily stored in the shift register and correspond to the states DAdd6 to W in the state diagram in Fig. 5.86. In cycle 10, the interface generates an acknowledgement, ACK, to signify that it has received the first eight bits from the bus master by EnSDAOut = 1, and transfers the device ID stored in the shift register to the device ID register by LoadDevID $=1$. This cycle corresponds to the DevID ACK state in Fig. 5.86.

From cycles 11 to 18, which correspond to the states Add15 to Add8 in the state diagram, the bus master sends the most significant byte of the Flash memory page address to the interface. These bits are received by the shift register and immediately transferred to the page address register in cycle 19 by LoadBlkMSB $=1$. In this cycle, the interface also sends a second acknowledgment to the bus master by EnSDAOut $=1$, which is represented by the AddMSB ACK state in the state diagram.

From cycles 20 to 27, the interface receives the least significant byte of the page address. It stores this byte in cycle 28 by LoadBlkLSB $=1$, and sends a third acknowledgement to the bus master by EnSDAOut $=1$. These events are shown by the states Add7 to Add0 and the state AddLSB ACK in the state diagram, respectively. Starting in cycle 29 , the complete page address becomes available at the Addr[15:0] terminal in Fig. 5.82 even though the page erase process has not been initiated. This cycle is also the starting point for the bus master to send the erase command, 0x50, to the Flash memory interface. Without this step, the interface will not be able to recognize if the ongoing process is actually about erasing a block of data.

From cycle 29 to 36 that correspond to the states $0 \times 50-0$ to $0 \times 50-7$, the interface receives all eight bits of the command code, $0 \times 50$, in the shift register. Then in cycle 37 , it generates the fourth acknowledgment by EnSDAOut = 1, and transfers the contents of the shift register, $0 \times 50$, to the command register by $\mathrm{LoadCom}=1$. Later on, the interface uses this value to be able to generate the correct preamble for the page erase operation. Cycle 37 corresponds to the $0 \times 50$ ACK state in the state diagram. While the bus master sends the second command code, $0 \times 70$, from cycles 38 to 45 to initiate the page erase, the interface, now aware of the page erase operation, sends the first address and data preamble, 0x5555/0xAA, to the Flash memory in cycle 39. In this cycle, the fixed register value, $0 \times 5555$, is routed through port 1 of the address MUX by sel5555 $=1$. The fixed register data, $0 x A A$, is also sent to the I/O[7:0] port through port 0 of the data MUX by selAA $=1$ and EnDataOut $=1$. In cycle 41 , the second address and data preamble, $0 \times \mathrm{AAAA} / 0 \times 55$, is sent. This is followed by sending the third preamble (including the first page erase command), $0 \times 5555 / 0 \times 50$, in cycle 43 , and then the fourth preamble, $0 x 5555 / 0 x A A$, in cycle 45 . The interface pauses for one cycle after dispatching each address and data combination to comply with the Flash memory protocol of writing data.

Fig. 5.83 $\mathrm{I}^{2} \mathrm{C}$ page erase sequence with device ID and the LSB of page address

Fig. 5.84 $\mathrm{I}^{2} \mathrm{C}$ page erase sequence with the MSB of page address and the erase command

Fig. 5.85 $\mathrm{I}^{2} \mathrm{C}$ page erase preamble and the page erase command


Fig. $5.86 \mathrm{I}^{2} \mathrm{C}$ page erase interface controller

Cycles 38 to 45 are represented by the states $0 \times 70-0$ to $0 \times 70-7$ in the state diagram, respectively. The interface sends the fifth acknowledgment to the bus master in cycle 46 by EnSDAOut $=1$ while in the $0 \times 70$ ACK state. In cycle 47 , the interface sends the fifth address and data preamble, $0 \times \mathrm{AAAA} / 0 \mathrm{x} 55$, and finally in cycle 49 , it sends the page address with the second page erase command, $0 \times 70$, to erase the entire block of data. Cycles 47 to 49 are represented by the DontCare-0, DontCare-1 and DontCare-2 states in Fig. 5.86, respectively.

An architecture combining the read and the fast write interfaces can be implemented by a data-path shown in Fig. 5.87. A shift register can be used to receive the incoming device address and command bit from the SDA bus, which is subsequently is stored in an auxiliary register as shown in this figure. The seven-bit device ID field can be used to activate one of the maximum 128 Flash memory chips. The command bit, R or W , is used to enable either the read interface or the fast write interface depending on its value. The Flash memory address residing in the shift register is then forwarded to the eight-bit shift register in either the read or the fast write interface to prepare the Flash memory for a data transfer.


Fig. 5.87 $\mathrm{I}^{2} \mathrm{C}$ read and fast write (program) interface topologies
The reader should be quite familiar with all three $\mathrm{I}^{2} \mathrm{C}$ interface designs shown above to be able to integrate them in one interface to achieve a complete design.

### 5.5 Serial Flash Memory

Recent Flash memory chips already include $\mathrm{I}^{2} \mathrm{C}$ or SPI interfaces to interact with a host processor or another bus master. The user does not have to deal with preambles, waiting periods or other complexities of the memory, but simply write an $\mathrm{I}^{2} \mathrm{C}$ or SPI-compliant embedded program to initiate a read, write or erase operation with the Flash memory.

This section examines the operation of a typical Flash memory with an SPI interface. Figure 5.88 shows the basic internal architecture of the Flash memory where an external active-low Slave Select control signal, $\overline{\mathrm{SS}}$, is applied to enable the memory. The clock is supplied through the SCK port. The serial data comes into the memory through the Serial-Data-In (SDI) port and departs from the Serial-Data-Out (SDO) port. Once a serial address is retrieved from the SPI bus, it is stored in the address register. The address decoder uses the contents of the address register to access the Flash memory core and read the data to an internal data buffer. The serial data is subsequently delivered to the bus master through the SDO port. If the operation is a write, the bus master sends serial data to the SDI port, which is then transferred to an internal data buffer, and subsequently to the memory core.


Fig. 5.88 Serial Flash memory architecture with SPI interface
Figure 5.89 describes a typical partitioning scheme of a 1 MB memory core that requires a 20-bit address for each byte of data. To be consistent with the E ${ }^{2}$ PROM and Flash memory organizations discussed in previous sections, the entire memory block in this example is divided into sixteen 64 KB sectors. Each sector is subdivided into sixteen 4 KB blocks, and each block is further subdivided into 16 pages. Each page contains 256 bytes, any of which is accessible through the SPI bus. Figure 5.90 shows the detailed address mapping of Block 0 in Sector 0 to further illustrate the internal memory organization.


Fig. 5.89 A serial Flash core memory organization: 16 sectors and 16 blocks in each sector




Address mapping of Block 0 of Sector 0
Fig. 5.90 Memory organization of block 0 of sector 0 : 16 pages, 256 bytes per page
Figure 5.91 shows nine basic modes of operation for this Flash memory. Some of the modes in this table are further divided into sub-modes according to the complexity of the main mode. For example, in the write (program) mode, the opcode $0 \times 20$ assumes to write between 1 and 255 bytes into the memory core while the opcode $0 \times 23$ writes 64 KB of data into a sector. Similarly, the erase mode can be configured to erase a page, a block or the entire chip. The protect operation prevents overwriting to a sector or the chip. The write enable feature is a security measure for the serial Flash memory, and it is used prior to an actual write or an erase operation. Once the write enable command is issued, any kind of data alteration in the memory core becomes possible. Both the write enable and protect features are registered
in the status register and can be read on demand. The status register also indicates whether the device is busy, such as in the middle of a write or read operation, write enable is engaged or not, or which sector is protected. The Flash memory can be placed into a long term hibernation mode to save power. The modes in Fig. 5.91 are at minimum compared to a typical serial Flash memory to emphasize only the primary modes of operation. The opcode value for each mode is also randomly selected. Actual serial Flash memory datasheets contain many more operational modes with different opcode values assigned to each mode.

| FLASH MEMORY COMMANDS |  | OPCODES |
| :--- | :--- | :---: |
| Read | Write Byte (1-255) | $0 \times 10$ |
|  | Write Page | $0 \times 20$ |
|  | Write Block | $0 \times 21$ |
|  | Write Sector | $0 \times 22$ |
| Erase | Erase Page | $0 \times 23$ |
|  | Erase Block | $0 \times 30$ |
|  | Erase Chip | $0 \times 31$ |
| Protect/Unprotect | Protect Sector | $0 \times 32$ |
|  | Unprotect Sector | $0 \times 40$ |
|  | Protect Chip | $0 \times 41$ |
|  | Unprotect Chip | $0 \times 42$ |
| Write Enable |  | $0 \times 43$ |
| Write Disable |  | $0 \times 55$ |
| Read Status Register |  | $0 \times 66$ |
| Hibernate |  | $0 \times 77$ |
| Wake up |  | $0 \times 88$ |

Fig. 5.91 Main serial Flash memory commands

This particular Flash memory operates in both mode 0 (SCK is initially at logic 0 ) and mode 3 (SCK is initially at logic 1) of the SPI protocol. However, most of the timing diagrams in this section will refer to mode 0 when explaining different commands in Fig. 5.91.

Figures 5.92 and 5.93 explain the basic write protocols in mode 0 and mode 3, respectively. Once $\overline{\mathrm{SS}}$ signal is lowered to logic 0 , data bits at SDI port can be written into the Flash memory's data buffer at the positive edge of SCK. The data transaction stops when $\overline{\mathrm{SS}}$ is raised to logic 1 . The entire data buffer is subsequently transferred to the memory core within the write period, $\mathrm{t}_{\text {WRITE }}$.

Mode 0 - Write Protocol


Fig. 5.92 Serial Flash memory mode 0 SPI write (program) protocol


Fig. 5.93 Serial Flash memory mode 3 SPI write (program) protocol
Similarly, Figs. 5.94 and 5.95 explain the basic read protocols in mode 0 and mode 3, respectively. When the $\overline{\mathrm{SS}}$ signal is lowered to logic 0 , data is delivered from the memory core to the data buffer, and then from the data buffer to the SDO terminal at the negative edge of each SCK cycle. The memory access is equal to tread $^{\text {with respect to the negative edge of }}$ SCK. When the $\overline{\mathrm{SS}}$ signal is raised to logic 1, SCK is no longer allowed to change, and the read process terminates.

In both write and read operations, the most significant data bit is delivered first, and the least significant data bit is delivered last.


Fig. 5.94 Serial Flash memory mode 0 SPI read protocol

Mode 3 - Read Protocol


Fig. 5.95 Serial Flash memory mode 3 SPI read protocol

A typical Flash memory byte read is shown in Fig. 5.96. The process starts with sending the opcode, 0x10, corresponding to a read operation according to the table in Fig. 5.91. A 20-bit address follows the opcode with the most significant address bit, A19, first, and the least significant address bit, A0, last. The first data bit, D7 (also the most significant bit of data), is delivered to the SDO terminal at the negative edge of SCK according to Fig. 5.96. The remaining seven bits of data are sequentially delivered at each negative edge of SCK until the SCK signal stabilizes at logic 0 , and the $\overline{\mathrm{SS}}$ signal transitions to logic 1 .


Fig. 5.96 Serial Flash memory byte read in mode 0

Once a starting 20-bit address is issued, a number of bytes, ranging from one byte to the contents of the entire memory, can be read from the SDO port as long as the $\overline{\mathrm{SS}}$ signal is kept at logic 0 , and the SCK activity is present. Terminating SCK and raising $\overline{\mathrm{SS}}$ to logic 1 ceases the read process as shown in Fig. 5.97.


Fig. 5.97 Serial Flash memory read burst in mode 0
The Flash memory write (program) mode has four sub modes. In the byte write mode, bytes ranging between 1 and 255 can be written to a page following the opcode, $0 \times 20$, and a 20-bit memory address as shown in Fig. 5.98. After data is written to the last address of the page, subsequent bytes at SDI terminal are considered invalid and will be ignored even though there may still be SCK activity and/or $\overline{\text { SS }}$ may still be at logic 0 . In some serial Flash memory chips, excess data is not ignored but written to the memory core starting from the first address of the page (address looping).


Fig. 5.98 Serial Flash memory write burst ( 1 to 255 bytes) in mode 0

Figure 5.99 describes the page write mode. After issuing the write page opcode, $0 \times 21$, and a 20 -bit page address, 256 bytes of data are sequentially written into the memory core starting from the top of the page. Any data beyond 256 bytes will be ignored by the device. It is vital that the 20-bit starting address aligns with the first address of the page. For example, if page 0 of block 0 in sector 0 needs to be accessed to write data, the starting address has to be $0 x 00000$ according to Fig. 5.90. Similarly, the starting address has to be $0 x 00100$ for page 1 or $0 x 00 \mathrm{~F} 00$ for page 15 if the contents of either page need to be written.

Writing to a block or a sector is not any different from writing to a page. In both instances, the starting 20-bit address needs to align with the topmost address of the block or the sector. For example, writing a 4 KB of data to block 0 of sector 0 requires the starting address to be 0x00000. Similarly, the starting address of block 1 of sector 0 has to be $0 x 01000$ if 4 KB data needs to be written to this block.


Fig. 5.99 Serial Flash memory page write in mode 0
Erase can be performed on a page, a block or the entire chip according to the table in Fig. 5.91. The page erase requires the opcode, $0 \times 30$, followed by the topmost address of the page as shown in Fig. 5.100. Erasing the entire chip only requires the opcode, 0x32, as shown in Fig. 5.101.


Fig. 5.100 Serial Flash memory page erase in mode 0


Fig. 5.101 Serial Flash memory chip erase in mode 0
Accidentally altering the contents of the Flash memory is a non-reversible process. Therefore, many manufacturers formulate a security measure, such as a write enable command, prior to a write or an erase operation. The write enable command requires issuing an opcode, $0 \times 55$, according to Fig. 5.91, and it is implemented in Fig. 5.102. This code changes the write enable bit in the status register which then enables the Flash memory for write or erase. For example, in Fig. 5.103 the write enable opcode, 0x55, is issued prior to the write byte opcode, $0 \times 20$, to allow any number of bytes to be written to a page. If the write enable opcode is omitted prior to a byte, a page, a block or a sector write, the data delivered to the memory core becomes invalid and is ignored.


Fig. 5.102 Serial Flash memory write enable operation in mode 0


Fig. 5.103 Serial Flash memory write (program) burst (1 to 255 bytes) followed by write enable

Protecting a sector or the entire chip is also a vital security measure for the Flash memory. For example, if a Flash memory contains BIOS data in specific sectors, accidentally accessing these sectors for write or erase becomes fatal. Therefore, such accesses need to be prevented at all costs. The opcode, $0 \times 40$, is issued with a specific sector address to protect the data in this sector as shown in Fig. 5.104. However, as with the write and erase modes, the write enable opcode, $0 x 55$, must accompany the sector protect opcode, $0 \times 40$, to make the sector protect a valid entry as shown in the timing diagram in Fig. 5.105.


Fig. 5.104 Serial Flash memory protect sector operation in mode 0


Fig. 5.105 Serial Flash memory write enable operation followed by protect sector in mode 0

The user may reverse the write enable status of the device by issuing a write disable command, 0x66, as shown in the timing diagram in Fig. 5.106.

Status register constitutes an important part of the Flash memory programming. For this particular Flash memory, there are four entries in the status register that contain vital operational information as shown in Fig. 5.107. The SP0, SP1 and SP2 bits identify which


Fig. 5.106 Serial Flash memory write disable operation in mode 0

| 7 |  | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reserved |  |  |  | SP2 | SP1 | SP0 | WEL | WIP |
| $\begin{aligned} & \text { WIP }=\text { Write } \operatorname{In} \text { Progress = } \begin{array}{l} \text { Device is busy with write } \\ 0 \end{array} \\ & \text { Device is not busy with write } \end{aligned}$ |  |  |  |  |  |  |  |  |
| $\begin{aligned} \text { WEL }=\text { Write Enable Status }= & 1 \text { Write Enable is active } \\ 0 & \text { Write Enable is inactive } \end{aligned}$ |  |  |  |  |  |  |  |  |
| SP2 | SP1 | SP0 |  |  |  |  |  |  |
| 0 | 0 | 0 | No sector is protected |  |  |  |  |  |
| 0 | 0 | 1 | Address $0 \times 00000$ to 0x0FFFF is protected |  |  |  |  |  |
| 0 | 1 | 0 | Address $0 \times 00000$ to $0 \times 1 \mathrm{FFFF}$ is protected |  |  |  |  |  |
| 0 | 1 | 1 | Address $0 \times 00000$ to $0 \times 3 F F F F$ is protected |  |  |  |  |  |
| 1 | 0 | 0 | Address $0 \times 00000$ to $0 \times 7 \mathrm{FFFF}$ is protected |  |  |  |  |  |
| 1 | 0 | 1 | Address 0x00000 to 0xFFFFF is protected |  |  |  |  |  |
| 1 | 1 | 0 | Address $0 \times 00000$ to 0xFFFFFF is protected |  |  |  |  |  |
| 1 | 1 | 1 | Address $0 \times 00000$ to 0xFFFFF is protected |  |  |  |  |  |

Fig. 5.107 Serial Flash memory status register
sectors are protected. The Write Enable Latch bit, WEL, signifies if the device has already been write-enabled or not. The Write-In-Progress, WIP, bit defines if the device is busy with a write process.

The user can access the contents of the status register at any time by issuing the read status register command, 0x77, as shown in Fig. 5.108. After executing this command, the contents of the status register become available at the SDO port.

The user can also place the Flash memory into the sleep mode to conserve power by issuing hibernate opcode, $0 \times 88$, as shown in Fig. 5.109. The hibernation mode can be reversed by issuing the wake-up opcode, 0x99, as shown in Fig. 5.91.


Fig. 5.108 Serial Flash memory status register read operation in mode 0


Fig. 5.109 Serial Flash memory chip hibernate operation in mode 0

## Review Questions

1. An SDRAM is composed of two16-bit wide banks, bank 0 and bank 1, as shown below.


The truth table below defines the precharge, activate and read cycles.

| Operation | $\overline{\mathrm{CS}}$ | $\overline{\mathrm{RAS}}$ | $\overline{\mathrm{CAS}}$ | $\overline{\mathrm{WE}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Precharge | 0 | 0 | 1 | 0 |
| Activate | 0 | 0 | 1 | 1 |
| Read | 0 | 1 | 0 | 1 |

Each 16-bit SDRAM address is composed of two parts: the most significant byte corresponds to the row address, and the least significant byte corresponds to the column address as shown below.
Address $=\{$ Row Address, Column Address $\}$
The precharge wait period is two clock cycles between the positive edge of the precharge cycle and the positive edge of the activate cycle. Similarly, the CAS wait period is two cycles between the positive edge of the activate cycle and the positive edge of the read command. The read burst from the specified address starts after a latency of two cycles. The waiting period between the last data packet and the precharge cycle is also two cycles if the read repeats from the same bank.
(a) Show the two read sequences in sequential addressing mode from Bank 0. Each burst contains four data packets: the first burst is from the address, $0 x \mathrm{AB} 03$, and the next from the address, $0 x \mathrm{AB} 07$.
(b) Show the two read sequences in sequential addressing mode from different banks with no delay in between. Each burst contains four data packets: the first burst is from the Bank 0 with the starting address, $0 x A B 03$, and the next from Bank 1 with the address, 0xCD06.
2. A Flash memory is composed of two byte-addressable sectors. It has an eight-bit bidirectional I/O port for reading and writing data, and a 16-bit unidirectional address port. The upper eight bits of the address field are allocated for the sector address and the lower eight bits for the program address. The three active-low inputs, $\overline{\mathrm{EN}}, \overline{\mathrm{RE}}$ and $\overline{\mathrm{WE}}$, control the Flash memory according to the following chart:

| Operations | $\overline{\mathrm{EN}}$ | $\overline{\mathrm{RE}}$ | $\overline{\mathrm{WE}}$ |
| :---: | :---: | :---: | :---: |
| Read | 0 | 0 | 1 |
| Write | 0 | 1 | 0 |
| Standby | 0 | 1 | 1 |
| Off | 1 | x | x |

The initial data contents in this memory are shown below:

Sector 0

|  | 0 |  |
| :---: | :---: | :---: |
| 0xEE | 0xFF | 0x00 |
| 0xCC | 0xDD | $0 \times 02$ |
| 0xAA | $0 \times B B$ | 0x04 |
| 0x88 | 0x99 | $0 \times 06$ |

Sector 1

|  | 0 |  |
| :---: | :---: | :---: |
| 0x34 | $0 \times 12$ | 0x00 |
| 0x78 | 0x56 | 0x02 |
| 0xBC | 0x9A | 0x04 |
| 0xF0 | $0 \times D E$ | $0 \times 06$ |

The Flash command chart is as follows:

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Addr | Data | Addr | Data | Addr | Data |
| Sector protect | word | 0x5555 | 0xAA | 0xAAAA | 0x55 | Sector Addr | 0x01 |
| Fast write | word | 0x5555 | 0xAA | 0xAAAA | 0x02 | Write Addr | Write Data |
|  | byte | 0xAAAA |  | 0x5555 |  | Write Addr | Write Data |
| Read | word | $0 \times 5555$ | 0xAA | 0xAAAA | 0x03 | Read Addr | Read Data |
|  | byte | 0xAAAA |  | 0x5555 |  | Read Addr | Read Data |

In this diagram, the sector protect is a three-cycle sequence where the sector protect code, $0 x 01$, is provided in the third cycle along with the sector address.
Both the fast write and the read processes are initially three cycles. However, once the process starts, additional reads or writes are reduced to two-cycle operations as shown in the flow chart below. According to this chart, for each additional data to be read or written, the command code must be employed in the second cycle, and the address/data combination in the third cycle.
The flow chart for the fast write and read is as follows:

(a) Perform protect sector 1 . Show the timing diagram with control inputs, address and data.
(b) Perform fast write to sector 0 with four bytes of data, $0 \times 11,0 \times 22,0 \times 33,0 \times 44$. Start from the address, $0 x 04$, and increment the address to write each byte.
(c) Read four bytes from sector 0 at the addresses, $0 x 00,0 x 02,0 x 04$ and $0 x 06$. Show the timing diagram with control inputs, address and data.
3. Two reads need to be accomplished from a 16-bit wide SDRAM organized in four banks with the data shown below.


Each SDRAM address is composed of an eight-bit wide row address, RA[7:0], and an eight-bit wide column address, CA[7:0], as in the following format: SDRAM Address $=\{$ RA[7:0], CA[7:0] $\}$ where the row address occupies higher bits. To control SDRAM, the following controls are supplied:

| Operation | $\overline{\mathrm{CS}}$ | $\overline{\mathrm{RAS}}$ | $\overline{\mathrm{CAS}}$ | $\overline{\mathrm{WE}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Precharge | 0 | 0 | 1 | 0 |
| Activate | 0 | 0 | 1 | 1 |
| Read | 0 | 1 | 0 | 1 |

The wait period between the precharge and the activate cycles is one clock cycle. Similarly, the wait period between the activate and the read cycles is one clock cycle. The precharge cycle for the next read operation takes place after the last data is read out from SDRAM. $\operatorname{BS}[1: 0]=0$ selects Bank0, BS[1:0] $=1$ selects Bank1, BS[1:0] $=2$ selects Bank2, and $\mathrm{BS}[1: 0]=3$ selects Bank3 in the timing diagrams.
(a) Assuming that the mode register is pre-programmed in sequential mode addressing with a burst length of four and a CAS latency of two, construct a timing diagram to show the two reads from the SDRAM addresses, 0xAA00 and 0xAA04. Start from the precharge cycle to accomplish each read.
(b) With the same mode register contents in part (a), construct a timing diagram such that the two reads from the SDRAM addresses, 0xAA00 and $0 x B B 02$, take place in the shortest possible time. Again start from the precharge cycle to accomplish each read.
(c) With the same mode register contents in part (a), accomplish one read from the SDRAM address, 0xCC00, with a burst length of two, and one read from the SDRAM address, 0xDD02, with a burst length of eight. Start from the precharge cycle to accomplish each read.
4. Subsequent write and read operations are performed on an SDRAM that consists of two banks. Both banks have eight-bit wide I/O data ports.
The first SDRAM operation is a write operation that writes $0 \times 11$ to the starting address of $0 x A B$ in bank 0 . This is followed by writing the data values, $0 \mathrm{xEE}, 0 \mathrm{x} 00$ and 0 xFF , to bank 0 in sequential mode.
The read operation takes place from bank 1 without any interruption. This means that the first read-data is delivered to the data bus immediately after the last data, 0 xFF , has been written. The first read-address is defined as $0 x 12$. Four data packets are read from this starting address in sequential mode with a latency of two cycles.
Both write and read operations require $t_{\text {PRE }}=t_{\text {CAS }}=1$ cycle.
Construct a timing diagram with control, address and data values to achieve these two consecutive operations. Assume all initial data values in Bank 0 are 0x00. Make sure to mark each precharge, activate, write and read cycle on the timing diagram. Indicate where latency happens.


| 7 |  |
| :---: | :---: |
| 0x88 | Addr[0x10] |
| 0x99 | Addr[0x11] |
| $0 \times A A$ | Addr[0x12] |
| $0 \times B B$ | Addr[0x13] |
| 0xCC | Addr[0x14] |
| 0xDD | Addr[0x15] |
| 0xEE | Addr[0x16] |
| 0xFF | Addr[0x17] |
| 0x00 | Addr[0x18] |
| $0 \times 11$ | Addr[0x19] |
| $0 \times 22$ | Addr[0x1A] |
| 0x33 | Addr[0x1B] |
| 0x44 | Addr[0x1C] |
| 0x55 | Addr[0x1D] |
| 0x66 | Addr[0x1E] |
| 0x77 | Addr[0x1F] |
| BANK 1 |  |

5. An $E^{2}$ PROM memory is organized in four sectors. There are eight rows in each sector but no pages. The existing data in this memory is shown below.

| 3 | 0 |
| :---: | :---: |
| 7 | $0 \times 7$ |
|  | $0 \times 6$ |
|  | $0 \times 5$ |
| 4 | $0 \times 4$ |
| 3 | $0 \times 3$ |
| 2 | $0 \times 2$ |
| 1 | $0 \times 1$ |
| 0 | $0 \times 0$ |



| 3 | 0 |
| :---: | :---: |
| 7 | $0 \times 0$ |
|  | $0 \times 1$ |
|  | $0 \times 2$ |
| 4 | $0 \times 3$ |
| 3 | $0 \times 4$ |
| 2 | $0 \times 5$ |
| 1 | $0 \times 6$ |
| 0 | $0 \times 7$ |
|  | Sector 2 |



The command truth table is given below.

| Function | Command code |
| :--- | :---: |
| SR Read | $0 \times 0$ |
| Read | $0 \times 1$ |
| Write buffer | $0 \times 2$ |
| Write core | $0 \times 3$ |

The memory has five control pins:
$\overline{\mathrm{EN}}$ is an active-low signal that activates the sector
AE is an active-high signal that accepts address
CE is an active-high signal that enables command function
$\overline{\mathrm{WE}}$ is an active-low signal that enables write
$\overline{\mathrm{RE}}$ is an active-low signal that enables read
Writing to the memory takes place at the rising edge of $\overline{\mathrm{WE}}$. At the falling edge of $\overline{\mathrm{RE}}$, reads take place from the memory. The write sequence starts with the command function followed by the address and then the data. The read sequence follows a similar fashion: it starts with the read command, then the address and then the data. Assume all AE, $\overline{\mathrm{WE}}$ and $\overline{\mathrm{RE}}$ set-up times are 0 s . The setup and hold times for command, address and data are all different from 0 s . It takes $\mathrm{t}_{\text {WRITE }}$ amount of time to transfer data from the buffer to the memory core.
(a) Draw a timing diagram to read data from the row address $=0$ and the sector address $=3$.
(b) Draw a timing diagram to write $0 x A, 0 x B, 0 x C, 0 x D, 0 x E, 0 x F, 0 x 7,0 x 6$ starting from the row address $=2$ and the sector address $=2$ in the following manner: the first data, $0 x A$, to the row address 2 ; the second data, 0 xB , to the row address 0 x 3 and so forth. Draw the contents of the memory after the write sequence is complete.
6. A Flash memory block has an eight-bit address, and executes all reads and writes on an eight-bit bidirectional data bus. The Flash memory write sequence contains a preamble, a write command, and an address/data combination as shown in the flow chart below. Once the write command is issued, the address/data combination is generated continuously until the last write takes place. The sequence ends with the same preamble that starts the write.


In the read sequence, the bus master starts fetching data once the preamble and the read command are issued. The sequence has the same exit preamble as shown below.


START and DONE do not have any significance in timing diagrams other than that they indicate the start and the end of the sequence, respectively.
The preamble, write and read commands are issued with the hexadecimal values shown in the truth table below.

| COMMANDS | Address | Data |
| :---: | :---: | :---: |
| Preamble | FF | 00 |
| Write com | AA | FF |
| Read com | BB | FF |

The state of the Flash memory before any read or write operation is shown below. The leftmost column in this figure shows the Flash memory address in hexadecimal.


The bus master produces three data transmissions for the Flash memory. In the first transmission, four data packets are written to the Flash memory as shown below.

| Packet no | Address | Data |
| :---: | :---: | :---: |
| 1 | F8 | 00 |
| 2 | F9 | 11 |
| 3 | FA | 22 |
| 4 | FB | 33 |

In the second transmission, the bus master reads two data packets from the following addresses below.

| Packet no | Address | Data |
| :---: | :---: | :---: |
| 1 | FA |  |
| 2 | FB |  |

In the third transmission, the bus master reads two more data packets from the following addresses.

| Packet no | Address | Data |
| :---: | :---: | :---: |
| 1 | FE |  |
| 2 | FF |  |

Construct a timing diagram, including the address, the active-low $\overline{\mathrm{WE}}$ and $\overline{\mathrm{RE}}$ signals, and the data. Note that the Flash memory requires a hold period which coincides with the high phase of $\overline{\mathrm{EN}}$ signal. However, in the low phase, when the Flash memory is active, the device either writes or reads depending on the value of the $\overline{\mathrm{WE}}$ and $\overline{\mathrm{RE}}$ signals, respectively.
7. A serial on-chip SPI bus described in Chapter 4 is used to program an SDRAM register file that consists of five registers (see the SDRAM bus interface architecture).
Assume that each register in the register file has an eight-bit long address. Data in each register is also assumed to be eight bits long.
The Wait register receives the number of clock periods which is equivalent to $t_{\text {WAIT }}$, the Latency register to $t_{\text {LAT }}$, the Burst register to $t_{\text {BURST }}$, the CAS register to $t_{\text {CAS }}$, and the Precharge register to $\mathrm{t}_{\text {PRE }}$.
The SDI port of the SDRAM programming interface receives an eight-bit address (starting with the most significant bit) followed by an eight-bit data (again starting with the most significant bit) at the positive edge of SCK until all five registers are programmed while $\overline{\mathrm{SS}}=0$. Once the programming is finished, the $\overline{\mathrm{SS}}$ node pulls back to logic 1.
Design the interface between the SPI bus and the register file. Make sure to show each SPI-compliant I/O port (such as SCK, SDI, $\overline{\mathrm{SS}}$ etc.), the internal address, data and control signals of the interface on the timing diagram. The functionality of the interface must be the same in both the timing diagram and the data-path.
Start building the timing diagram that includes only the address and the data. Then form the corresponding data-path that matches the timing diagram. Increase the complexity of the design by including the control signals in the timing diagram, guiding the data flow. Lastly, draw the state diagram of the Moore type controller for the interface.

## Projects

1. Implement and verify the SRAM bus interface unit described in Fig. 5.5 with the unidirectional bus designed in Chapter 4. Use Verilog as the hardware design language for the module implementation and functional verification. Make sure the interface complies with the timing diagrams shown in Figs. 5.6 and 5.8 and includes a controller unit as shown in Figs. 5.7 and 5.9.
2. Implement and verify the SDRAM bus interface unit described in Fig. 5.25 with the unidirectional bus designed in Chapter 4. Use Verilog as the hardware design language for the module implementation and functional verification. Make sure the interface complies with the timing diagrams shown in Figs. 5.27 and 5.29 and includes a controller unit as shown in Figs. 5.28 and 5.30. Produce the hardware to program the SDRAM register file. Assume a serial bus such as SPI or $\mathrm{I}^{2} \mathrm{C}$ to distribute the program data to the registers.
3. Implement and verify the $I^{2} C$ fast write interface in the first design example of Chapter 5 using Verilog. Make sure your design is consistent with the state machine shown in Fig. 5.72.
4. Implement and verify the $\mathrm{I}^{2} \mathrm{C}$ read interface in the second design example using Verilog. Make sure to be consistent with the state machine shown in Fig. 5.80.
5. Combine the read and the fast write interfaces into a single unit. Design and verify the complete interface using Verilog.

Note that for projects 3 through 5, write a behavioral Verilog code that mimics the bus master in order to send data on the $\mathrm{I}^{2} \mathrm{C}$ bus.

## References

1. Toshiba datasheet TC59S6416/08/04BFT/BFTL-80, -10 Synchronous Dynamic RAM
2. Toshiba datasheet TC58DVM72A1FT00/TC58DVM72F1FT00 128Mbit E²PROM
3. Toshiba datasheet TC58256AFT $256 \mathrm{Mbit} \mathrm{E}^{2} \mathrm{PROM}$
4. Toshiba datasheet TC58FVT004/B004FT-85, -10, -12 4MBit CMOS Flash memory
5. Toshiba datasheet TC58FVT400/B400F/FT-85, -10, -12 4MBit CMOS Flash memory
6. Toshiba datasheet TC58FVT641/B641FT/XB-70, -10 64MBit CMOS Flash memory
7. Atmel datasheet AT26DF161 16Mbit serial data Flash memory

## Central Processing Unit

This chapter describes a basic Central Processing Unit (CPU) that operates with a Reduced Instruction Set (RISC) [1, 2, 3, 4]. The chapter is divided into four parts.

In the first part, fixed-point Arithmetic Logic Unit (ALU) instructions are described. This section first develops a dedicated hardware (data-path) to execute a single RISC instruction, and then groups several data-paths together to be able to execute variety of user programs. In each step of this process, the instruction field is dissected into several segments as the instruction flows through the data-path, and the necessary hardware is formed to execute the instruction and generate an output.

The second part of this chapter explains the IEEE single and double-precision floating-point formats, leading to the designs of floating-point adder and multiplier. These designs are then integrated with the fixed-point hardware to obtain a RISC CPU capable of executing both fixed-point and floating-point arithmetic instructions.

In the third part, structural, data and program control hazards in CPU pipelines are discussed. This section shows how to generate additional hardware (forwarding loops) to overcome various hazards [4].

The last section of this chapter is devoted to explaining different types of cache memory architectures, their operation and design trade-offs.

### 6.1 RISC Instruction Formats

In a RISC CPU, all instructions include an Operation Code (OPC) field which instructs the processor what to do with the rest of the fields in the instruction, and when to activate different hardware components in the CPU to be able to execute the instruction. The OPC field is followed by one or more operand fields. Each field either corresponds to a register address in the Register File (RF) or contains immediate user data to process the instruction.

There are three types of instructions in a RISC CPU: register-to-register-type, immediate-type and jump-type.

A register-to-register-type instruction contains an OPC followed by three operands: two source register addresses and one destination register address pointing the RF, namely RS1, RS2 and RD. The format of this instruction is shown below.

OPC RS1, RS2, RD
This type of instruction fetches the contents of the first and second source registers, $\operatorname{Reg}[\mathrm{RS} 1]$ and $\operatorname{Reg}[\mathrm{RS} 2]$, from the RF, processes them according to the OPC, and writes the result to the destination register, $\operatorname{Reg}[\mathrm{RD}]$ in the RF. This operation is described below.
$\operatorname{Reg}[\mathrm{RS} 1](\mathrm{OPC}) \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
An immediate-type instruction contains an OPC followed by three operands: one source register address, RS , one destination register address, RD, and an immediate data as shown below.

## OPC RS, RD, Imm Value

This type of instruction combines the contents of the source register, Reg[RS], with a sign-extended immediate value according to the OPC, and writes the result to the destination register, $\operatorname{Reg}[R D]$, in the RF. This operation is shown below.
$\operatorname{Reg}[\mathrm{RS}]$ (OPC) Immediate Value $\rightarrow \operatorname{Reg}[R D]$
The jump-type instruction contains an OPC followed by an immediate value shown below.

OPC Imm Value
This type of instruction uses the immediate field to modify the contents of the Program Counter (PC) for the instruction memory. The operation of this instruction is given below. Immediate Value $\rightarrow$ PC

All three instruction types fit in a 32-bit wide instruction memory as shown Fig. 6.1. In this figure, the numbers on top of each field correspond to the bit positions of the instruction memory, defining the borders of the OPC or a particular operand field.

Register-to-Register Type

| 31 | 2625 |  | 1615 |  |
| :--- | :--- | :--- | :--- | :--- |
| OPC | RS1 | RS2 | RD | Not Used |

Immediate Type

| 31 | 2625 |  | 212015 |  | RD | Immediate Value/Not Used |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |

Jump Type
31 26 25

| OPC | Immediate Value/Not Used |
| :---: | :---: | :---: |

Fig. 6.1 Instruction field formats

### 6.2 CPU Data-Path

A modern RISC CPU is composed of small and large size memories, such as Register File (RF), instruction and data memories, and an ALU to execute an instruction. A Program Counter (PC) generates an address for the instruction memory as shown in Fig. 6.2. Each instruction is fetched from this memory and separated into OPC and operand fields. The OPC field guides the data-flow through the rest of the CPU. The operand field contains either a number of RF addresses or the user data or the combination of the two. Once the source and destination RF addresses become available at the output of the instruction memory, the corresponding data is read from the RF and processed in the ALU according to the OPC. The read function from the RF is achieved by simply disabling the write process to the RF or Write Enable $(\mathrm{WE})=0$. The processed data in the ALU is subsequently written back to a destination address in the RF by $\mathrm{WE}=1$. On the other hand, if a particular instruction needs to fetch data from the data memory instead of the RF, first the ALU calculates the effective memory address for the data memory. When data becomes available at the output of the data memory, then the OPC decoder writes this data back to a destination address in the RF by $\mathrm{WE}=1$. Sometimes, instructions contain a user-defined immediate value. This is separated from the rest of the operand fields and combined with the contents of a source register, $\operatorname{Reg}[R S]$, in the ALU. The processed data is written back to the RF by $\mathrm{WE}=1$.


Fig. 6.2 A non-pipelined CPU
Instructions can be executed in RISC CPUs in two ways. In a non-pipelined CPU architecture, instructions are fetched from the instruction memory and processed through the remaining four stages of the CPU in Fig. 6.2 before the CPU takes the next instruction. This is shown in Fig. 6.3. In this figure, IF, RF, A, DM and WB represent the Instruction Fetch, Register File access, ALU, Data Memory and Write-Back stages, respectively.

| NONPIPELINED | I |  |  | cycle |  |  |  |  | $\begin{gathered} \text { cycle } \\ 2 \end{gathered}$ |  |  |  |  | $\begin{gathered} \text { cycle } \\ 3 \end{gathered}$ |  | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Instruction 1 | 1 | IF | RF | A | DM | WB |  |  |  |  |  |  |  |  |  | I |
| Instruction 2 | 1 |  |  |  |  |  | IF | RF | A | DM | WB |  |  |  |  | I |
| Instruction 3 | 1 |  |  |  |  |  |  |  |  |  |  | IF | RF | A | DM | WB ! |

Fig. 6.3 A non-pipelined CPU timing table
Non-pipelined structures are inefficient in terms of data throughput but require lower clock frequencies to operate. The CPU becomes more efficient in terms of throughput if the architecture in Fig. 6.2 is subdivided into smaller functional stages where each stage is separated from its neighboring stage by a flip-flop boundary that stores data (or address) only for one clock cycle as shown in Fig. 6.4. In this figure, the CPU data-path consists of five stages where individual tasks are executed in each stage within one clock cycle. According to this scheme, the clock frequency becomes five times higher compared to the architecture in Fig. 6.2.

The first stage of this new pipeline in Fig. 6.4 is the instruction memory access. In this stage, each program instruction is fetched from the instruction memory and stored in the instruction register at the first flip-flop boundary. This architecture supports a word-addressable instruction memory, and therefore requires the PC to increment by one.


Fig. 6.4 A pipelined five-stage CPU
The next pipeline stage is the RF stage where the instruction OPC is separated from its operands. The OPC is decoded in order to generate control signals to route the address and data in the rest of the CPU. Operand fields are either source register addresses to access the data in the RF or immediate data supplied by the user as mentioned earlier. If the operand corresponds to an RF address, the data fetched from this address is loaded to the register that resides at the second flip-flop boundary. If the operand is an immediate data, it is sign extended to 32 bits before it is loaded to a register in the second flip-flop boundary.

The third stage of the CPU pipeline is the ALU stage. The data from the source registers in the RF or the immediate data are processed in this stage according to the OPC and loaded to the register at the third flip-flop boundary.

The fourth stage is the data memory stage. This stage either calculates an effective address for the data memory or bypasses the data memory. If the instruction calls for loading or storing data, the ALU calculates the data memory address to access its contents. Otherwise, the ALU result simply bypasses the data memory and stored in a register at the fourth flip-flop boundary.

The last stage of the CPU pipeline is the write-back stage. In this stage, data is either routed from the output of the data memory or from the bypass path to a designated destination address in the RF.

A pipelined RISC CPU's efficiency and speed are shown in the timing table in Fig. 6.5. This figure extends to 15 high frequency clock cycles, which is the equivalent to three low frequency clock cycles in Fig. 6.3. The number of completed instructions in this new pipeline is almost 12, which is four times larger than the number of instructions executed in a non-pipelined CPU in Fig. 6.3. The difference between non-pipelined and pipelined CPU efficiency only gets better as the number of instructions increases.

| PIPELINED |  | cycle |  | cycle |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | 1 | 2 | 1 | 3 | 1 | 4 | 1 | 5 | 1 | 6 | 1 | 7 | 1 | 8 | 1 | 9 | 1 | 10 | 1 | 11 | 1 | 12 | 1 | 13 | \| 14 | 1 | 15 | 1 | 1 |
| Instruction 1 | 1 | IF | I | RF | I | A | 1 | DM |  | WB | I |  | I |  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | I | 1 |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | I |  | 1 |  | 1 |  | 1 |  | I |  | I |  | 1 | 1 |  | 1 | 1 |
| Instruction 2 | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  |  |  | 1 |  |  |  | I |  |  |  | 1 | 1 |  |  | 1 |
|  | 1 |  | 1 | IF | 1 | RF | 1 | A | 1 | DM | 1 | WB | I |  | I |  | 1 |  | 1 |  |  |  | I |  | I |  | 1 | 1 |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | I |  |  |  | 1 |  |  |  | 1 | 1 |  |  | 1 |
| Instruction 3 | 1 |  | 1 |  | 1 | IF | 1 | RF | 1 | A | 1 | DM | I | WB | 1 |  | I |  | 1 |  | I |  | I |  | 1 |  | 1 | 1 |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 | 1 |  |  | 1 |
| Instruction 4 | 1 |  | 1 |  | 1 |  | 1 | IF | 1 | RF | 1 | A | 1 | DM | 1 | WB |  |  | 1 |  |  |  |  |  |  |  | 1 | 1 |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 | F | 1 |  | 1 | A | I |  | 1 |  |  |  |  |  |  |  | I |  | I |  | 1 | I |  | 1 | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  | 1 |  | 1 |  |  |  | I |  | \| |  | 1 | 1 |  |  | 1 |
| Instruction 5 | 1 |  | 1 |  | 1 |  | 1 |  | 1 | IF | I | RF | I | A | 1 | DM | 1 | WB | 1 |  |  |  | I |  | I |  | 1 | 1 |  | 1 | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  |  |  | 1 | 1 |  |  | 1 |
| Instruction 6 | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 | IF | I | RF | 1 | A | 1 | DM |  | WB |  |  | , |  | , |  | 1 | I |  | 1 | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 | I |  |  | 1 |
| Instruction 7 | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  |  |  |  |  | 1 | 1 |  |  | 1 |
|  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I | IF | 1 | RF | 1 | A | 1 | DM |  | WB |  |  | I |  | 1 | 1 |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 | I |  |  | 1 |
| Instruction 8 | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 | IF | 1 | RF | 1 | A | 1 | DM | 1 | WB |  |  | 1 | 1 |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 | 1 |  |  | 1 |
| Instruction 9 | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  | 1 | IF |  | RF | 1 | A | 1 | DM |  | WB | 1 | 1 |  | 1 | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  |  |  | 1 | 1 |  | 1 | 1 |
| Instruction 10 | 1 |  | 1 |  | I |  | I |  | 1 |  | 1 |  | I |  |  |  |  |  | I |  |  |  | 1 |  |  |  | , | , |  |  | 1 |
|  | I |  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  |  |  |  |  |  | IF | 1 | RF | 1 | A | \| | DM | , WB | । |  |  | 1 |
|  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | I |  |  |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 | 1 |  |  | 1 |
| Instruction 11 | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  | 1 |  | 1 |  | 1 | IF | 1 | RF | 1 | A | 1 DM | 1 | WB |  | 1 |
|  | 1 |  | 1 |  | I |  | 1 |  | 1 |  | 1 |  | I |  | 1 |  |  |  | 1 |  | 1 |  | 1 |  | , |  | , | I |  |  | 1 |
| Instruction 12 | I |  | I |  | 1 |  | 1 |  | 1 |  | 1 |  | I |  | I |  | I |  | 1 |  | 1 |  |  | IF |  | RF | 1 A | 1 | DM |  | I WB |

Fig. 6.5 A pipelined CPU timing table
The next section of this chapter examines the hardware requirements of register-to-register-type, immediate-type and jump-type RISC instructions.

### 6.3 Fixed-Point Register-to-Register Type ALU Instructions

Fixed-point register-to-register type ALU instructions interact with the ALU only. The most fundamental instruction in this category is the Add (ADD) instruction that contains the ADD opcode, two source register addresses, RS1 and RS2, and a destination register address, RD, as shown below.

ADD RS1, RS2, RD
This instruction fetches data from the source addresses, RS1 and RS2, adds them, and returns the result to the destination address RD according to the equation below.
$\operatorname{Reg}[\mathrm{RS} 1]+\operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
The field format of this instruction in the instruction memory is shown in Fig. 6.6. The numbers on top of each field represent the OPC and operand boundaries.

| 31 | 2625 |  | 1615 |  |
| :--- | :--- | :--- | :--- | :--- |
| ADD | RS1 | RS2 | RD | Not Used |

Fig. 6.6 Fixed-point ADD instruction field format
The required hardware for the ADD instruction is shown in Fig. 6.7. In this figure, the PC generates an address for the instruction memory and loads the contents of the ADD instruction to the instruction register at the end of the first clock cycle. Since the instruction contains two source register addresses, RS1 and RS2, the contents of these registers, $\operatorname{Reg}[\mathrm{RS} 1]$ and $\operatorname{Reg}[\mathrm{RS} 2]$, are read from the RF and stored at the second flip-flop boundary. As mentioned earlier, the read function is achieved by disabling the write process to the RF or $\mathrm{WE}=0$ during this cycle. In the third clock cycle, the ALU adds Reg[RS1] and Reg [RS2], and stores the result at the third flip-flop boundary. In the fourth clock cycle, the ALU result is written back to the destination register address, RD, in the RF. In this cycle, WE = 1 is generated by the OPC decoder to enable the write. No processing is done to RD, which propagates from one stage to another without any modification to point where the processed ALU result needs to go in the RF.


Fig. 6.7 ADD instruction data-path

Since the ADD instruction does not require any data to be stored or fetched from the data memory, the data memory stage is omitted from this data-path which reduces the number of stages from five to four.

Similar to the ADD instruction, the Subtract (SUB) instruction subtracts the 32-bit data at RS2 from the 32-bit data at RS1 and returns the result to RD. The instruction and its operational equation are shown below.

SUB RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1]-\operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
The field format of the SUB instruction in the instruction memory is the same as the ADD instruction in Fig. 6.6 except the ADD OPC is replaced by SUB. This instruction also follows the same data-path as the ADD instruction except the OPC selects the subtractor in the ALU instead of the adder.

The fixed-point Multiplication (MUL) instruction requires four operands as shown below. This instruction multiplies the contents of RS1 and RS2 and generates a 64-bit result. The lower and upper 32 bits of the result in curly brackets are written to RD1 and RD2, respectively.

MUL RS1, RS2, RD1, RD2
$\operatorname{Reg}[\mathrm{RS} 1] * \operatorname{Reg}[\mathrm{RS} 2] \rightarrow\{\operatorname{Reg}[\mathrm{RD} 2], \operatorname{Reg}[\mathrm{RD} 1]\}$
The field format of this instruction in the instruction memory is shown in Fig. 6.8.

| 31 | 2625 | 212016 |  | 1110 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| MUL | RS1 | RS2 | RD2 | RD1 | Not Used |

Fig. 6.8 Fixed-point MUL instruction field format

There are two ways to generate a data-path for this particular instruction. The first method executes this instruction using four pipeline stages and requires two RF write-back ports as shown in Fig. 6.9. In this figure, the lower 32 bits of the multiplication result, MUL[31:0], are written to the address RD1 while the higher 32 bits, MUL[63:32], are written to the address RD2 in the RF. In the write-back cycle, $\mathrm{WE}=1$ is generated by the OPC decoder to enable the two simultaneous writes.


Fig. 6.9 Data-path for fixed-point multiplication using two write-back ports

The second method writes the multiplication result back to the RF in two successive clock cycles instead of one but does not require the RF to have two write-back ports. In this scheme, the 64-bit multiplication result is divided between two distinct paths in the ALU as shown in Fig. 6.10. The lower 32 bits, MUL[31:0], are written back into the RF at the end of fourth cycle while the higher 32 bits, MUL[63:32], are stored at an additional flip-flop boundary. The higher 32 bits are subsequently written back to the RF at the end of the fifth cycle.


Fig. 6.10 Data-path for fixed-point multiplication using a single write-back port (WE signal to RF is not shown for clarity)

The And (AND) instruction bitwise "ANDs" the contents of RS1 and RS2, and returns the result to the address RD in the RF. Again in the fourth cycle, WE $=1$ is generated by the OPC decoder to enable the write. The operational equation of this instruction contains the "\&" sign to indicate that this is an AND operation. The field format of this instruction is shown in Fig. 6.11.

| 31 | 2625 | 2120 |  | 1615 |
| :--- | :--- | :--- | :--- | :--- |
| AND | RS1 | RS2 | RD | Not Used |

Fig. 6.11 Fixed-point AND instruction field format

AND RS1,RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1] \& \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
The AND instruction data-path in Fig. 6.12 is identical to the ADD or SUB instruction data-paths except for the ALU which requires 32 sets of two-input AND gates to carry out the instruction.


Fig. 6.12 AND instruction data-path
The Or (OR), Exclusive Or (XOR), Nand (NAND), Nor (NOR) and Exclusive Nor (XNOR) instructions have identical instruction formats except the opcode field. These operations are shown below.

OR RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1] \mid \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$

XOR RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1]{ }^{\wedge} \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$

NAND RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1] \sim \& \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$

NOR RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1] \sim \mid \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$

XNOR RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1] \sim^{\wedge} \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$

Here, "|" and " $\wedge$ " signs refer to the OR and XOR operations, respectively. The " $\sim$ " sign corresponds to negation and generates a complemented value. Therefore, " $\sim \& ", " \sim \mid "$ and " $\sim \wedge$ " operations denote the bitwise-NAND, NOR and XNOR, respectively.

The OR, XOR, NAND, NOR and XNOR instructions follow the same, four-stage data-path as the AND instruction in Fig. 6.12. However, each logical instruction requires different types of logic gates in the ALU stage, and the OPC field selects which to use.

Another important register-to-register type instruction is the shift instruction. The Shift Left (SL) instruction shifts the contents of RS1 to the left by the amount stored at the address RS2, and returns the result to RD. The format and the operation of this instruction are shown below. The " $\ll$ " sign indicates left-shift operation. This instruction's field format is similar to the previous register-to-register-type instructions as shown in Fig. 6.13.

| 31 | 2625 | 2120 |  | 161510 |  | Not Used |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: |

Fig. 6.13 Fixed-point Shift-Left (SL) instruction field format
SLRS1, RS2, RD
$\operatorname{Reg}[\operatorname{RS} 1] \ll \operatorname{Reg}[\operatorname{RS} 2] \rightarrow \operatorname{Reg}[R D]$
The Shift Right (SR) instruction is similar to the SL instruction except the contents of RS1 are shifted to the right by the amount indicated in RS2. The " $>$ " sign corresponds to the SR operation.

SR RS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1]>\operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
Both the SL and SR instructions require linear shifters in the ALU. These units are large combinational logic blocks that are predominantly made out of multiplexers as examined in Chapter 1. Both of these instructions follow the same data-path as any other register-register-type instructions with three operands. Figure 6.14 shows the combined data-path for the SL and SR instructions. The ALU stage contains both a left and a right linear shifter. The first input, $\operatorname{Reg}[R S 1]$, represents the value to be shifted to the right or to the left. The second input, $\operatorname{Reg}[R S 2]$, specifies the amount to be shifted in number of bits. Even though the ALU executes both the left and right-shifted versions of $\operatorname{Reg}[\mathrm{RS} 1]$ simultaneously, only one result is selected by the OPC and written back to the RF. If the instruction is a SR instruction, then OPC selects port 0 of the 2-1 MUX, and the SR result is written to the RF. Otherwise, the OPC selects port 1 , and the SL result is written to the RF.


Fig. 6.14 SL and SR instruction data-paths

Both the SL and SR instructions require a four-stage CPU pipeline. In the write-back cycle, $\mathrm{WE}=1$ is generated by the OPC decoder to enable the write.

As an example, we can combine the individual data-paths for ADD, SUB, AND, NAND, OR, NOR, XOR, XNOR, SL and SR instructions in a single CPU to execute a user program. The architecture in Fig. 6.15 shows eight individual functional units in the ALU followed by an 8-1 MUX to select the desired ALU output. In this figure, there is only one adder, and it is able to execute a two's complement addition to perform subtraction. The left and right linear shifters are also combined in a single unit as in Fig. 6.14. The SL or SR opcode selects the output of either the left shifter or the right shifter for the destination register. The outputs of all logical units are selected by an 8-1 MUX and forwarded to the RF.
ADD, SUB selects S/A port,
AND, NAND, OR, NOR, XOR,

Fig. 6.15 Combined register-to-register ALU instruction data-paths

The next category of register-register-type instructions are the Set instructions used in decision-making situations where two source register values are compared against each other prior to a branch instruction.

The Set-Greater-than-or-Equal (SGE) instruction below describes setting the contents of RD to $0 x 00000001$ if the contents of RS 1 are found to be greater than or equal to the contents of RS2. The data in RS1 and RS2 registers are considered unsigned integers. If the comparison fails, then the contents of RD are set to $0 \times 00000000$. The field format of this instruction is given in Fig. 6.16.

| 31 | 2625 |  | 2120 |  |
| :--- | :--- | :--- | :--- | :--- |
| SGE | RS1 | RS2 | RD | Not Used |

Fig. 6.16 Fixed-point Set-Greater-than-or-Equal (SGE) instruction field format
SGERS1, RS2, RD
If $\operatorname{Reg}[R S 1] \geq \operatorname{Reg}[R S 2]$ then $1 \rightarrow \operatorname{Reg}[R D]$ else $0 \rightarrow \operatorname{Reg}[R D]$
The data-path for the SGE instruction in Fig. 6.17 tests if the contents of RS1 are greater than or equal to the contents of RS2 using a subtractor in the ALU. Again, the data in RS1 and RS2 registers are considered unsigned integers. To perform this test, $\operatorname{Reg}[\mathrm{RS} 1]$ is subtracted from Reg[RS2], and the sign bit of the result is used to make the decision. If Reg [RS1] is greater than or equal to $\operatorname{Reg}[R S 2]$, the sign bit becomes zero. The complemented


Fig. 6.17 SGE instruction data-path (WE signal to RF is not shown for clarity)
sign bit is then forwarded to the $2-1 \mathrm{MUX}$ at the write-back port of the RF to store $0 x 00000001$ in the destination register. If the subtraction yields a negative number, the complemented sign bit selects $0 x 00000000$ to be stored in the RD.

The Set-Greater-Than (SGT) instruction is another instruction that tests if Reg[RS1] is greater than Reg[RS2]. If the comparison is successful, the instruction stores 0x00000001 in the RD. Otherwise, the instruction stores $0 x 00000000$ as described below.

## SGTRS1, RS2, RD

If $\operatorname{Reg}[R S 1]>\operatorname{Reg}[R S 2]$ then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$
The data-path of the SGT instruction is shown in Fig. 6.18. In this figure, two tests are performed in the ALU stage. The first test checks if $\operatorname{Reg}[\mathrm{RS} 1]$ is greater than or equal to Reg [RS2] using the sign bit of the subtractor as was done for the SGE instruction. The second test checks if $\operatorname{Reg}[\mathrm{RS} 1]$ is not equal to Reg[RS2] using 32 sets of two-input XNOR gates followed by a single 32 -input NAND gate. Both of these tests form the input to a two-input AND gate in the ALU, which then produces the selector input for the 2-1 MUX in the write-back stage to test the SGT condition. If $\operatorname{Reg}[R S 1]>\operatorname{Reg}[R S 2]$, then port 1 of the 2-1 MUX is selected to store $0 x 00000001$ in RD. Otherwise, port 0 is selected to store 0x00000000.

Similar to the SGE and SGT instructions, there are four other set instructions that compare the contents of the two source registers in a variety of different ways to set or reset the destination register. The Set-Less-than-or-Equal (SLE), Set-Less-Than (SLT), Set-Equal (SEQ) and Set-Not-Equal (SNE) instructions and how they operate in the CPU are listed below.

SLE RS1, RS2, RD
If $\operatorname{Reg}[\mathrm{RS} 1] \leq \operatorname{Reg}[\mathrm{RS} 2]$ then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SLTRS1, RS2, RD
If $\operatorname{Reg}[\operatorname{RS} 1]<\operatorname{Reg}[R S 2]$ then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SEQRS1, RS2, RD
If $\operatorname{Reg}[\mathrm{RS} 1]=\operatorname{Reg}[\mathrm{RS} 2]$ then $1 \rightarrow \operatorname{Reg}[\mathrm{RD}] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SNE RS1, RS2, RD
If $\operatorname{Reg}[\operatorname{RS} 1] \neq \operatorname{Reg}[\mathrm{RS} 2]$ then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$
All Set instructions require four clock cycles to write the result to the RF like all the other register-to-register-type instructions.

Fig. 6.18 SGT instruction data-path (WE signal to RF is not shown for clarity)

### 6.4 Fixed-Point Immediate Type ALU Instructions

Immediate ALU instructions allow user data to be included in the instruction. However, these instructions still fetch register data from the RF to be combined with the user data.

The Add-Immediate instruction (ADDI) adds the contents of RS to the user-supplied 16 -bit immediate value and returns the result to RD in the RF. This instruction and how it operates in the CPU are shown below. The field format of this instruction in the instruction memory is given in Fig. 6.19.

| 31 | 2625 |  | 212015 |  | Immediate Value |
| :--- | :--- | :--- | :--- | :---: | :---: |
| ADDI | RS | RD | 0 |  |  |

Fig. 6.19 Fixed-point ADD Immediate (ADDI) instruction field format

## ADDI RS, RD, Imm Value

$\operatorname{Reg}[\mathrm{RS}]+$ Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$
The ADDI instruction data-path is shown in Fig. 6.20. In this figure, the contents of the instruction are transferred from the instruction memory to the instruction register at the end


Fig. 6.20 ADDI instruction data-path
of the first clock cycle. In the second clock cycle, the 16-bit immediate value in the instruction is sign extended to 32 bits while the contents of RS are fetched from the RF. In the third clock cycle, the two values are added in the ALU. At the end of the fourth cycle, the processed data is written back to the RF at the address RD. Therefore the ADDI instruction requires only four clock cycles to execute.

The Subtract-Immediate instruction (SUBI) behaves similarly to the ADDI, but it subtracts the immediate value from the contents of RS, and returns the result to RD as illustrated below.

SUBI RS, RD, Imm Value
$\operatorname{Reg}[R S]$ - Immediate Value $\rightarrow \operatorname{Reg}[R D]$
There are also immediate logical instructions that operate with the user data. The AND-Immediate (ANDI) instruction, for example, bitwise ANDs the contents of RS with the immediate value and returns the result to RD as shown below. The field format of this instruction is given in Fig. 6.21.

| 31 | 2625 |  | 212015 |  | Immediate Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ANDI | RS | RD |  |  |  |

Fig. 6.21 Fixed-point AND Immediate (ANDI) instruction field format

ANDI RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}]$ \& Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$
The ANDI instruction uses a similar data-path to the ADDI instruction, but replaces the fixed-point adder with 32 two-input AND gates as shown in Fig. 6.22. The contents of RS and the sign-extended immediate value are combined using the AND gates, and the result is returned to RD in the RF.

Similar to the ANDI instruction, the ORI, XORI, NANDI, NORI, and XNORI instructions operate with the immediate data and follow the same data-path as the ANDI instruction. These instructions and how they operate in the CPU are listed below.The Or (OR), Exclusive Or (XOR), Nand (NAND), Nor (NOR) and Exclusive Nor (XNOR) instructions have identical instruction formats except the opcode field. These operations are shown below.

ORI RS, RD, Imm Value
$\operatorname{Reg}[R S] \mid$ Immediate Value $\rightarrow \operatorname{Reg}[R D]$

XORI RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}]$ ^ Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$

NANDI RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}] \sim \&$ Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$


Fig. 6.22 ANDI instruction data-path
NORI RS, RD, Imm Value
$\operatorname{Reg}[R S] \sim \mid$ Immediate Value $\rightarrow \operatorname{Reg}[R D]$

XNORI RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}] \sim^{\wedge}$ Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$

All logical immediate instructions require four cycles to form the result.
The Shift Left Immediate (SLI) and Shift Right Immediate (SRI) instructions use the same functional units as the SL and SR instructions in the ALU.

The SLI instruction fetches the contents of RS from the RF, shifts it to the left by the user-defined immediate value, and stores the result in RD as indicated below. Figure 6.23 shows the field format of this instruction.

| 31 | 2625 | 212015 |  |
| :--- | :--- | :--- | :--- |
| SLI | RS | RD | Immediate Value |

Fig. 6.23 Fixed-point SL Immediate (SLI) instruction field format

SLI RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}] \ll$ Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$
Similar to the SLI instruction, the SRI instruction shifts the contents of RS to the right by an amount equal to the immediate value, and stores the result in RD as shown below.

SRI RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}] \gg$ Immediate Value $\rightarrow \operatorname{Reg}[\mathrm{RD}]$
The SLI and SRI instruction data-paths in Fig. 6.24 still require the left and right linear shifters in the ALU stage. In these instructions, one shifter input receives the immediate data that specifies the number of bits to shift to the left or to the right while the other input receives the contents of RS.


Fig. 6.24 SLI and SRI instruction data-paths
Figure 6.25 combines all the immediate ALU instructions examined so far in one schematic. These include the ADDI, SUBI, SLI, SRI, ANDI, ORI, XORI, NANDI, NORI and XNORI instructions with their sign-extended input. The port selection process at the ALU MUX is as follows: if the OPC is ADDI or SUBI, the adder/subtractor output is routed through the S/A port; for the SLI and SRI OPCs, the shifter outputs are routed through the SH port; for the remaining OPCs, the processed data is routed through the corresponding MUX port carrying the same name as the OPC and becomes the ALU output.

Fig. 6.25 Combined Immediate ALU instruction data-paths

The set-immediate instructions compare the contents of RS with an immediate value for setting or resetting the register RD.

The Set-Greater-than-or-Equal-Immediate (SGEI) instruction sets the contents of RD if the instruction finds the contents of RS to be greater than or equal to the immediate value. This instruction and how it operates in the CPU is shown below. The field format is given in Fig. 6.26.

| 31 | 2625 |  | 1615 |  | Immediate Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SGEI | RS | RD | 0 |  |  |

Fig. 6.26 Fixed-point SGE Immediate (SGEI) instruction field format

## SGEI RS, RD, Imm Value

If $\operatorname{Reg}[R S] \geq$ Immediate Value then $1 \rightarrow \operatorname{Reg}[R D]$ else $0 \rightarrow \operatorname{Reg}[R D]$
The SGEI instruction data-path in Fig. 6.27 tests the relative magnitudes of $\operatorname{Reg}[R S]$ and the sign-extended immediate value to make a decision about the contents of RD. If $\operatorname{Reg}[\mathrm{RS}]$ is larger than the immediate value or equal to it, the sign bit of the ALU result becomes zero, which in turn, stores $0 x 00000001$ into RD. Otherwise, RD becomes $0 x 00000000$.


Fig. 6.27 SGEI instruction data-path (WE signal to RF is not shown for clarity)

Similarly, the instruction format for the Set-Greater-Than-Immediate (SGTI), Set-Less-than-or-Equal-Immediate (SLEI), Set-Less-Than-Immediate (SLTI), Set-Equal-Immediate (SEQI) and Set-Not-Equal-Immediate (SNEI) instructions and how they operate in the CPU are given below.

SGTI RS, RD, Imm Value
If $\operatorname{Reg}[R S]>$ Immediate Value then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SLEI RS, RD, Imm Value
If $\operatorname{Reg}[\mathrm{RS}] \leq$ Immediate Value then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SLTI RS, RD, Imm Value
If $\operatorname{Reg}[R S]<$ Immediate Value then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SEQI RS, RD, Imm Value
If $\operatorname{Reg}[R S]=$ Immediate Value then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$

SNEI RS, RD, Imm Value
If $\operatorname{Reg}[R S] \neq$ Immediate Value then $1 \rightarrow \operatorname{Reg}[R D] \quad$ else $0 \rightarrow \operatorname{Reg}[R D]$
All Set Immediate instructions require four clock cycles to store the result in RD.

### 6.5 Data Movement Instructions

The first data movement instruction that relocates data from the data memory to a register in the RF is the Load (LOAD) instruction. This instruction first adds the contents of RS to a user-defined immediate value to form an effective data memory address. It then fetches the data at this address and moves it to a destination register, RD, in the RF. This instruction and how it operates in the CPU is shown below. The term, Reg[RS] + Imm Value, defines the effective data memory address, and mem $\{\operatorname{Reg}[\mathrm{RS}]+\operatorname{Imm}$ Value $\}$ corresponds to the data at this address. The field format of this instruction while it is in the instruction memory is given in Fig. 6.28.

| 31 | 2625 |  |  |
| :--- | :--- | :--- | :--- |
| LOAD | RS | RD | Immediate Value |

Fig. 6.28 Fixed-point LOAD instruction field format

LOAD RS, RD, Imm Value
$\operatorname{mem}\{\operatorname{Reg}[R S]+$ Immediate Value $\} \rightarrow \operatorname{Reg}[R D]$
The LOAD instruction data-path in Fig. 6.29 adds the contents of RS to the sign-extended immediate value and uses this sum as an address for the data memory. The OPC selects the


Fig. 6.29 LOAD instruction data-path
adder in the ALU to calculate the effective data memory address and enables the data memory for read. The data read from the memory is subsequently written back to the RF at the address RD by $\mathrm{WE}=1$. This instruction requires five clock cycles to complete, and it traces through all five stages of the CPU.

The Store (STORE) instruction moves data in the opposite direction of the LOAD instruction. This instruction uses the contents of RD and the immediate value to form a data memory address, and moves the contents of RS to this address as described below. Figure 6.30 shows this instruction's field format.

| 31 | 2625 |  | 212015 |  | Immediate Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STORE | RS | RD | 0 |  |  |

Fig. 6.30 Fixed-point STORE instruction field format

STORE RS, RD, Imm Value
$\operatorname{Reg}[\mathrm{RS}] \rightarrow \operatorname{mem}\{\operatorname{Reg}[\mathrm{RD}]+$ Immediate Value $\}$
The data-path for the STORE instruction is shown in Fig. 6.31. In this figure, the OPC selects the adder in the ALU to perform an add operation between the sign-extended immediate value and Reg[RD]. The OPC then enables the data memory to write the contents of RS at this calculated address. The STORE instruction needs only four clock cycles to complete.

The Move (MOVE) instruction does not interact with the data memory. Nevertheless, it moves data from one register to another in the RF. The instruction and how it operates in the CPU are given below. The bit field format for the MOVE instruction is shown in Fig. 6.32.

MOVERS, RD
$\operatorname{Reg}[\mathrm{RS}] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
The Move Immediate (MOVEI) instruction moves an immediate value to a destination register, RD, in the RF as shown below. The bit field format of this instruction is given in Fig. 6.33.

Fig. 6.31 STORE instruction data-path

| 31 | 2625 |  | 1615 |  | Not Used |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MOVE | RS | RD | 0 |  |  |

Fig. 6.32 Fixed-point MOVE instruction field format

| 31 | 2620 |  | 1615 |  | Immediate Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MOVEI | Not Used | RD |  |  |  |

Fig. 6.33 Fixed-point MOVEI instruction field format
MOVEI Imm Value, RD
Immediate Value $\rightarrow \operatorname{Reg}[R D]$
The schematic in Fig. 6.34 combines the data-paths of all data movement instructions discussed above except MOVEI. The data memory address is selected between $\{\operatorname{Reg}[\mathrm{RS}]+$ $\operatorname{Imm}\}$ and $\{\operatorname{Reg}[R D]+\operatorname{Imm}\}$ by a $2-1$ MUX at the input of the adder depending on the OPC. Another 2-1 MUX at the write-back stage selects between the contents of RS for the MOVE instruction and the contents of data memory for the LOAD instruction before the result is written back to the RF.

### 6.6 Program Control Instructions

To be able to make decisions in a program, we need program control instructions.
The Branch (BRA) instruction is one of the most essential instructions in a program as it controls direction of the program flow. This instruction first compares the contents of RS with a five-bit RS value specified by the user. If the comparison is successful, the BRA instruction redirects the program to fetch an instruction from a different PC address as shown below. This new PC address is calculated by incrementing the old PC address by an immediate value specified by the user. If the comparison is not successful, the program skips the next instruction, but it executes the instruction after next. This instruction's field format is shown in Fig. 6.35.

```
BRA RS, RS Value, Imm Value
If \(\operatorname{Reg}[\mathrm{RS}]=\mathrm{RS}\) Value then \(\mathrm{PC}+\) Immediate Value \(\rightarrow \mathrm{PC}\)
else \(\mathrm{PC}+2 \rightarrow \mathrm{PC}\)
```

The BRA instruction usually depends on a previously stored data value in the RF as a result of a Set or Set-Immediate instruction. To be able to carry out the BRA instruction, first the contents of RS is compared with the RS Value. The comparator in Fig. 6.36 is composed

Fig. 6.34 Data movement instruction data-paths (MOVE, LOAD and STORE)

| 31 | 2625 |  | 1615 |  | Immediate Value |
| :--- | :--- | :--- | :--- | :---: | :---: |
| BRA | RS | RS Value | 0 |  |  |

Fig. 6.35 Fixed-point BRANCH instruction field format
of 32 two-input XNOR gates to perform a bitwise comparison between $\operatorname{Reg}[\mathrm{RS}]$ and the unsigned (positive) RS Value. All 32 XNOR outputs are then fed to a 32-input AND gate to make a decision for the new PC value. If the comparison is successful, the current PC value is replaced with ( $\mathrm{PC}+\mathrm{Imm}$ Value). If the comparison is unsuccessful, however, the PC value is incremented by $(\mathrm{PC}+2)$. The reason for $(\mathrm{PC}+2)$ is because the PC increments by two by the time a new value forms at the input of the PC. In actuality, the instruction following the BRA instruction at $(\mathrm{PC}+1)$ already enters the pipeline. However, this instruction needs to be independent of the branch instruction. To be able to execute this instruction (following the BRA instruction) and to remove the pending hazard, the compiler either inserts a No Operation (NOP) instruction right after the BRA instruction or finds an unrelated instruction in the program and inserts it at the $(\mathrm{PC}+1)$ location.


Fig. 6.36 BRA instruction data-path
Unconditional decisions in the program do not need a comparison. The programmer can simply change the flow of the program by using a jump-type instruction.

The first unconditional jump-type instruction is the Jump (JUMP) instruction which simply replaces the current PC value with an immediate value as shown below. Its field format is shown in Fig. 6.37.


Fig. 6.37 Fixed-point JUMP instruction field format

JUMP Imm Value
Immediate Value $\rightarrow$ PC

The data-path for the JUMP instruction is shown in Fig. 6.38. In this data-path, the 26-bit jump value is an unsigned integer extended to 32 bits before being forwarded to the PC. However, the PC value has already incremented once by the time the immediate value from the instruction register replaces the current value at the input of the PC. In the next clock cycle, this immediate value transmits to the output of the PC , pointing a different instruction memory location, instead of $(\mathrm{PC}+2)$. Therefore, this instruction also creates a control hazard when the CPU fetches an instruction at $(\mathrm{PC}+1)$ and requires compiler's intervention to remove the hazard.


Fig. 6.38 JUMP instruction data-path

The second unconditional jump-type instruction is the Jump Register (JREG) instruction, which is similar to the JUMP instruction, but uses the contents of RS to replace the current PC value as shown below. Figure 6.39 describes this instruction's bit field format.

| 31 | 2625 |  |
| :--- | :--- | :--- |
| JREG | RS | Not Used |

Fig. 6.39 Describes this instruction's bit field format

## JREG RS

$\operatorname{Reg}[\mathrm{RS}] \rightarrow \mathrm{PC}$
The data-path for the JREG instruction is shown in Fig. 6.40. This instruction reads the contents of RS and loads it to the PC as the jump value. The program control hazard also exists for this instruction. The hazard can be removed either by inserting a NOP instruction or another unrelated instruction in the program after the JREG instruction.


Fig. 6.40 JREG instruction data-path

The Jump-And-Link (JAL) instruction is the third unconditional jump-type instruction, and it requires two steps to operate. In the first step, the PC address, ( $\mathrm{PC}+2$ ), following the JAL instruction is stored in the register R31. In the second step, the current PC value is replaced with an immediate value as shown below. The instruction's bit field format is given in Fig. 6.41.

| 31 | 26 |
| :---: | :---: |
| JAL | Immediate Value |

Fig. 6.41 Fixed-point Jump-And-Link (JAL) instruction field format

JAL Imm Value
$(\mathrm{PC}+2) \rightarrow \mathrm{Reg}[\mathrm{R} 31]$ followed by Immediate Value $\rightarrow \mathrm{PC}$
The last unconditional jump-type instruction is the Jump-And-Link Register (JALR) instruction. This instruction also requires a two-step process. In the first step, the PC address, (PC + 2), is stored in the register R31. In the second step, the PC is loaded with the contents of RS as shown below. The field format of this instruction is shown in Fig. 6.42.

| 31 | 2625 |  |
| :--- | :--- | :--- |
| JALR | RS | Not Used |

Fig. 6.42 Fixed-point Jump-And-Link Register (JALR) instruction field format

## JALR RS

$$
(\mathrm{PC}+2) \rightarrow \operatorname{Reg}[\mathrm{R} 31] \text { followed by } \operatorname{Reg}[\mathrm{RS}] \rightarrow \mathrm{PC}
$$

The Return instruction (RET) works with the JAL or JALR instruction. It retrieves the old program address stored in the register R31, and replaces the current PC value with the contents of R31 in order to go back to the original program location as described below. This instruction's bit field format is given in Fig. 6.43.


Fig. 6.43 Fixed-point Return (RET) instruction field format

RET
$\operatorname{Reg}[\mathrm{R} 31] \rightarrow \mathrm{PC}$

### 6.7 Design Example I: A Fixed-Point CPU with Four Instructions

This first design example explains how to construct a single CPU data-path that executes the ADD, LOAD, STORE and MOVE instructions.

The first step of the design process is to start with a single instruction in the instruction list and build its data-path. Each new instruction brings new hardware requirements to the design, and they are added incrementally to the existing data-path. Once the data-path reaches its final form and is able to execute a set of instructions, the second step is to build the OPC decoders to control each pipeline stage and guide the data.

In this design, we start building the data-path for the ADD instruction as shown in Fig. 6.7. The next step is to introduce additional hardware for the LOAD instruction. To accommodate this requirement, the first modification is to place a $2-1$ MUX in the ALU stage to select between the immediate value required by the LOAD instruction and Reg [RS2] required by the ADD instruction as shown in Fig. 6.44. The second modification is to add a bypass path in the data memory stage so that the result of the adder is either guided through this bypass path if the OPC is ADD, or it is used as an effective address for the data memory if the OPC is LOAD. Finally, a third modification is to place a $2-1$ MUX in the write-back stage to select either the contents of the data memory (from the LOAD instruction) or the adder output (from the ADD instruction) before writing the result back to the destination register, RD.

When the STORE instruction is introduced as a third instruction, it prompts a change in the calculation of the data memory address from $\{\operatorname{Reg}[R S]+\operatorname{Imm}$ Value $\}$ to $\{\operatorname{Reg}[R D]+\operatorname{Imm}$ Value\}. This change requires a secondary 2-1 MUX to be placed in the ALU stage to guide the effective memory address to the AIn port of the data memory, and an additional path to transfer Reg[RS] to the DIn port of the data memory as shown in Fig. 6.45. The modifications for the STORE instruction, however, should not alter the existing data-paths for the ADD and LOAD instructions. If the ADD and LOAD instruction data-paths are individually traced after adding the hardware to support the STORE instruction, both $\operatorname{Reg}[\mathrm{RS} 1]+\operatorname{Reg}[\mathrm{RS} 2]$ (from the ADD instruction) and mem\{Reg[RS] + Imm Value $\}$ (from the LOAD instruction) paths should still be available to write the result back to a destination register in the RF.

Introducing the MOVE instruction requires another write-back path to be integrated with the three existing write-back paths in the architecture. Therefore, a 3-1 MUX needs to placed in the write-back stage to pass the contents of RS to the RF as shown in Fig. 6.46. While the MOVE and LOAD instructions use port 0 and port 1 of the 3-1 MUX, respectively, the rest of the instructions use port 2 of this MUX to write data back to the RF.

Fig. 6.44 CPU data-path with ADD and LOAD instructions (WE signal to the RF is omitted for clarity)

Fig. 6.45 CPU data-path with ADD, LOAD and STORE instructions (WE signal to the RF is omitted for clarity)


The controller for each stage of the data-path is OPC-dependent and completely combinational as shown in Fig. 6.47. Since the OPC propagates from one stage to another with the data, it can effectively be used as a control input to guide data and to activate the required hardware in each stage. Four instructions need only two input bits stemming from the instruction register, IR[27:26], to design the OPC decoder in Fig. 6.47. The more significant four OPC bits, IR[31:28], are considered zero for an instruction set of four.

| OPC | $\mathrm{IR}[27]$ | $\mathrm{IR}[26]$ |
| :--- | :---: | :---: |
| ADD | 0 | 0 |
| LOAD | 0 | 1 |
| STORE | 1 | 0 |
| MOVE | 1 | 1 |



Fig. 6.47 OPC table for ADD, LOAD, STORE and MOVE instructions and the control circuitry

To generate the ADD selector input, both $\operatorname{IR}[27]$ and $\operatorname{IR}[26]$ are complemented and then ANDed according to the table in Fig. 6.47. The selector inputs for LOAD, STORE and MOVE instructions are also generated using the same OPC table.

The STORE selector input is connected to Write Enable (WE) bit of the data memory since the STORE instruction is the only instruction that writes data to the data memory. All other instructions write back the results to the RF , and therefore should produce a WE signal to enable the RF for write. However, this signal is not shown in Figs. 6.44, 6.45 and 6.46 to avoid complexity.

### 6.8 Design Example II: A Fixed-Point CPU with Eight Instructions

The design methodology used in this example is the same as in the previous example. First, as additional instructions are introduced to the design, new hardware for each instruction should be incrementally added to the existing data-path. Second, an OPC truth table should be constructed from the instruction set. Third, controller outputs should be generated from the OPC truth table to guide the data in each CPU stage.

In this example, the instruction set consists of the ADD, LOAD, STORE, MOVE, SLI, SRI, JUMP and BRA instructions.

We start with the ADD instruction data-path given in Fig. 6.7 to form the base platform. The SLI and SRI instructions are implemented next. Both of these instructions require left and right linear shifters in the ALU stage where each shifter can individually be selected by the SLI or SRI inputs as shown in Fig. 6.48. These instructions also require a bypass path in the RF stage that connects IR[15:0] to the ALU input as explained earlier.

When the LOAD instruction is introduced as the fourth instruction, the existing 32-bit adder in the ALU is used to calculate the effective address for the data memory. The 2-1 MUX in the write-back stage is replaced by a 3-1 MUX to be able to write the contents of the data memory back to the RF.

The STORE instruction is the fifth instruction added to this design. This instruction requires two separate paths to calculate the data memory address and to write the contents of RS to the data memory. The STORE instruction also necessitates a secondary 2-1 MUX in the ALU stage so that an immediate value is added to the contents of RD instead of RS.

The MOVE instruction is the sixth instruction which requires a path to write the contents of RS to the RF, bypassing both the ALU and the data memory, and using port 0 of the 3-1 MUX in the write-back stage.

The BRA instruction is the seventh instruction in this set and requires a path to compare $\operatorname{Reg}[R S]$ with RS Value, IR[20:16]. The bitwise comparison is done by 32 two-input XNOR gates followed by a single 32-input AND gate, and produces a single bit that selects between (PC + Imm Value) and (PC +2). The selected target address is loaded to the PC to redirect the program to a different PC address. This instruction also requires a special 32-bit adder in the RF stage to calculate ( $\mathrm{PC}+\mathrm{Imm}$ Value) as shown earlier.

Fig. 6.48 CPU data-path with ADD, LOAD, STORE, MOVE, SLI, SRI, JUMP and BRA instructions (WE signal to the RF and Data Memory are omitted for clarity)

The JUMP instruction simply forwards the jump value, IR[25:0], extended to 32 bits to the PC. It requires a second 2-1 MUX in the RF stage that selects this jump value when the OPC is JUMP. The ALU adder is selected by the ADD, LOAD and STORE instructions. However, this selection is not shown in Fig. 6.48 to avoid complexity.

Once the data-path for all eight instructions is complete, the OPC truth table in Fig. 6.49 is formed. Since there are eight instructions in this set, only IR[28:26] are used for designing the OPC decoder. The upper three bits of the OPC field become equal to zero.


Fig. 6.49 OPC table for ADD, LOAD, STORE, MOVE, SLI, SRI, JUMP and BRA instructions and the control circuitry

To generate the ADD selector input (used in the RF and ALU stages), the first row of the OPC table is implemented. This requires IR[28], IR[27] and IR[26] to be complemented and ANDed. The LOAD, STORE, MOVE, SLI, SRI, JUMP and BRA selector inputs are also generated similarly using the same OPC table. The ALU stage requires the ADD, LOAD, SLI and SRI selector inputs to be ORed to select Reg[RS] for the adder input. Similarly, SLI and SRI inputs are ORed to select the shifter outputs. The STORE selector input is connected to WE input of the data memory since the STORE instruction is the only instruction in the instruction set that writes data to the data memory. The WE for the RF is omitted to avoid complexity.

### 6.9 Floating-Point Instructions

This RISC instruction set contains two floating-point (FP) instructions: Floating-Point Add (ADDF) and Floating-Point Multiply (MULF). Both instructions use the IEEE single precision floating-point format which defines the most significant bit to be the sign, the next eight most significant bits to be the exponent and the least 23 significant bits to be the fraction.

The ADDF instruction adds two single precision floating-point numbers at the registers, RS1 and RS2, and returns the result to the register RD as described below. The bit field format of this instruction is given in Fig. 6.50.

| 31 | 2625 |  | 1615 |  |
| :--- | :--- | :--- | :--- | :--- |
| ADDF | RS1 | RS2 | RD | Not Used |

Fig. 6.50 Floating-point Add (ADDF) instruction field format
ADDFRS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1]+\operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$
The MULF instruction multiplies two floating-point numbers in registers RS1 and RS2, and returns the result to the register RD as shown below. This instruction's bit field format is described in Fig. 6.51.

| 312625 | 2120 |  | 1615 |  |
| :--- | :--- | :--- | :--- | :--- |
| MULF | RS1 | RS2 | RD | Not Used |

Fig. 6.51 Floating-point Multiply (MULF) instruction field format

MULFRS1, RS2, RD
$\operatorname{Reg}[\mathrm{RS} 1] * \operatorname{Reg}[\mathrm{RS} 2] \rightarrow \operatorname{Reg}[\mathrm{RD}]$

### 6.10 Floating-Point

Single and double precision floating-point bit field formats are conveniently used in many modern CPU platforms because the sum of sign, exponent and fraction bits fit into a 32-bit or a 64-bit wide bus.

An IEEE single-precision (SP) floating-point number shown in Fig. 6.52 has an eight-bit field for the exponent and 23-bit field for the fraction. The most significant bit constitutes the sign bit for the fraction.


Fig. 6.52 IEEE single-precision floating-point format
Mathematically, a single-precision floating-point number is expressed as follows:
Single - precision FP number $=(-1)^{\mathrm{S}}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{23} \times 2^{-23}\right) 2^{\text {REXP }}$

Here, $s=1$ if the fraction is negative, else the fraction is positive. F1 to F23 are the 23 fraction bits that range from the most significant to the least significant bit positions of the fraction field in Fig. 6.52, respectively.

Since the exponent field does not possess a sign bit, a biasing system is employed to distinguish the negative exponents from the positive exponents. The biased exponent (EXP) field shown in Fig. 6.52 is the combination of the real exponent (REXP) and the BIAS as shown below.
$\mathrm{EXP}=\mathrm{REXP}+\mathrm{BIAS}$
The BIAS is calculated by substituting the lowest and the highest eight-bit numbers in the EXP field to determine the most negative and the most positive real exponent values. Therefore, when the most negative real exponent, REXP, equals to -MAX, the expression becomes:
$0=-\mathrm{MAX}+$ BIAS
Similarly, when the most positive exponent, REXP, becomes equal to +MAX, the expression becomes:
$255=$ MAX + BIAS
Hence substituting MAX = BIAS into $255=$ MAX + BIAS yields:

Example 6.1: Represent $-0.75_{10}$ as a single-precision floating-point number.
$-0.75_{10}=-0.11_{2}=(-1)^{\mathrm{S}}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{23} \times 2^{-23}\right) 2^{\text {REXP }}$
$-1.1 \times 2^{-1}=(-1)^{\mathrm{S}}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{23} \times 2^{-23}\right) 2^{\text {REXP }}$

Thus,

$$
\begin{aligned}
& \mathrm{s}=1 \\
& \mathrm{~F} 1=1 \\
& \mathrm{~F} 2 \text { through } \mathrm{F} 23=0 \\
& \mathrm{REXP}=-1=\mathrm{EXP}-\mathrm{BIAS}=\mathrm{EXP}-127 \\
& \mathrm{EXP}=127-1=126
\end{aligned}
$$

Filling the fraction and the exponent fields in Fig. 6.53 yields:


Fig. 6.53 Single-precision floating-point number in Example 6.1

Example 6.2: Represent $-527.5_{10}$ as a single-precision floating-point number.
$-527.5_{10}=(-1)^{1}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{23} \times 2^{-23}\right) 2^{\text {REXP }}$
The closest real exponent to 527.5 is $512=2^{9}$. Thus, $-527.5_{10}=(-1)^{1}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{23} \times 2^{-23}\right) 2^{9}$
where,
$\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{23} \times 2^{-23}\right)=527.5 / 512=1.03027_{10}$ $\approx 1.00001_{2}=1.03125_{10}$

Error in the fraction $=\frac{1.03125-1.03027}{1.03125}=0.01 \%$

The biased exponent is calculated as follows:
$\operatorname{REXP}=9=\mathrm{EXP}-\mathrm{BIAS}=\mathrm{EXP}-127$
$\mathrm{EXP}=127+9=136$
After entering the fraction and exponent fields into Fig. 6.52, the single-precision floating-point number for $-527.5_{10}$ produces the format in Fig. 6.54.


Fig. 6.54 Single-precision floating-point number in Example 6.2
Example 6.3: Convert the single-precision floating-point number in Fig. 6.55 into a decimal number.


Fig. 6.55 Single-precision floating-point number in Example 6.3
Here, $\mathrm{s}=1$ corresponds to a negative fraction. The fraction and biased exponent fields yield 0.25 and 129 , respectively. Thus,

REXP $=129-127=2$
The decimal number $=(-1)^{1}(1+0.25) 2^{2}=-1.25 \times 4=-5$

Example 6.4: Convert the single-precision floating-point number in Fig. 6.56 into a decimal number.


Fig. 6.56 Single-precision floating-point number in Example 6.4

Here, $\mathrm{s}=0$ corresponds to a positive fraction. The fraction field yields approximately 1. The biased exponent produces EXP $=255$. Therefore,

REXP $=255-127=128$
The decimal number $=(-1)^{0}(1+1) 2^{128}=2 \times 10^{38}$
The IEEE double-precision (DP) floating-point number in Fig. 6.57 has 11 bits for the exponent and 52 bits for the fraction for better accuracy. Once again, the most significant bit corresponds to the sign bit for the fraction.


Fig. 6.57 IEEE double-precision floating-point format
The double-precision floating-point number is expressed as follows:
Double - precision FP number $=(-1)^{\mathrm{S}}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{52} \times 2^{-52}\right) 2^{\mathrm{REXP}}$

Here, $\mathrm{s}=1$ corresponds to a negative, and $\mathrm{s}=0$ corresponds to a positive fraction. Bits F1 to F52 are the 52 fraction bits from the most significant bit position to the least significant bit position, respectively.

The bias system used in single-precision floating-point numbers can be applied to the double-precision exponents to distinguish between the negative and the positive exponents. Thus,
$\mathrm{EXP}=\mathrm{REXP}+\mathrm{BIAS}$
Here, EXP is the 11-bit field in Fig. 6.57, and REXP is the real exponent. The BIAS is calculated by substituting the lowest and highest biased exponent in the equation, respectively.

Therefore, for the most negative exponent the equation becomes:
$0=-\mathrm{MAX}+$ BIAS
Similarly, for the most positive exponent the equation becomes:
$2047=$ MAX + BIAS
Substituting MAX = BIAS into $2047=$ MAX + BIAS yields BIAS $=1023$ for double-precision floating-point numbers.

Example 6.5: Represent $-0.75_{10}$ as a double-precision floating-point number.

$$
\begin{aligned}
-0.75_{10} & =-0.11_{2}=(-1)^{\mathrm{S}}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{52} \times 2^{-52}\right) 2^{\mathrm{REXP}} \\
-1.1 \times 2^{-1} & =(-1)^{\mathrm{S}}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{52} \times 2^{-52}\right) 2^{\text {REXP }}
\end{aligned}
$$

Thus,
$\mathrm{s}=1$
$\mathrm{F} 1=1$
F 2 through F52 $=0$
REXP $=-1=\mathrm{EXP}-\mathrm{BIAS}=\mathrm{EXP}-1023$
$\mathrm{EXP}=1023-1=1022$
Therefore, entering the fraction and the exponent fields in Fig. 6.58 yields:


Fig. 6.58 Double-precision floating-point in Example 6.5

Example 6.6: Represent $4.0_{10}$ as a double-precision floating-point number.
$4_{10}=(-1)^{0}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{52} \times 2^{-52}\right) 2^{\text {REXP }}$
The closest exponent to 4 is $4=2^{2}$. Thus,
$4_{10}=(-1)^{0}\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{52} \times 2^{-52}\right) 2^{2}$
where,
$\left(1+\mathrm{F}_{1} \times 2^{-1}+\mathrm{F}_{2} \times 2^{-2}+\mathrm{F}_{3} \times 2^{-3}+\cdots+\mathrm{F}_{52} \times 2^{-52}\right)=1.0_{2}$
Therefore,
F1 through F52 $=0$
REXP $=2=\mathrm{EXP}-\mathrm{BIAS}=\mathrm{EXP}-1023$
$\mathrm{EXP}=1025$
Thus, entering the fraction and exponent fields in Fig. 6.59 yields:


Fig. 6.59 Double-precision floating-point in Example 6.6
Example 6.7: Convert the double-precision floating-point number in Fig. 6.60 into a decimal number.


Fig. 6.60 Double-precision floating-point in Example 6.7
Here, $s=1$ corresponds to a negative fraction. The fraction and biased exponent fields yield 1 and 0 , respectively. Therefore,

REXP $=0-1023=-1023$
Thus, the decimal number $=(-1)^{1}(1+1) 2^{-1023}=-2 \times 10^{-308}$.

### 6.11 Floating-Point Adder

Floating-point addition requires equating the exponents before adding the fractions. There are two ways to equate exponents. The first method is to shift the fraction of the floating-point number with greater exponent to the left until both exponents become equal. The second method is to shift the fraction of the floating-point number with lesser exponent to the right until the exponents are equal.

The floating-point adder in Fig. 6.61 implements the second method. The first step of this method is to determine which of the two floating-point numbers has a smaller exponent. This leads to subtracting the two exponents from each other and examining the sign bit of the result. In the schematic in Fig. 6.61, the second number's exponent, EXP2, is subtracted from the first number's exponent, EXP1, to obtain the difference, $\triangle$ EXP = EXP1 - EXP2. If the sign bit of the difference, $\triangle \mathrm{EXP}$, becomes zero, it indicates EXP1 is larger than or equal to EXP2. Therefore, the fraction of the second number, FRAC2, is shifted to the right by $\Delta$ EXP before adding the fractions. If, on the other hand, the sign bit of $\triangle \mathrm{EXP}$ becomes one, FRAC1 is shifted to the right by $\triangle \mathrm{EXP}$ before adding FRAC1 to FRAC2.


Fig. 6.61 A floating-point adder implementation

The numerical example in Fig. 6.61 describes how to process the exponent and the fraction fields of two floating-point numbers to be added. In this figure, 126, 1.000, 125 and -1.110 are assigned to the EXP1, FRAC1, EXP2 and FRAC2 fields, respectively. Initially, EXP2 is subtracted from EXP1, yielding $\triangle \mathrm{EXP}=+1$. The sign of $\triangle \mathrm{EXP}$ becomes zero, and therefore the larger exponent, EXP1 $=126$, is routed to the adder that calculates the final exponent. The zero value of the sign bit also selects the fraction field of the larger exponent, FRAC1 $=1.000$, and forwards it to the secondary adder that computes the fraction field.

FRAC2 $=-1.110$, on the other hand, is directed to be the input of the right shifter, which shifts this value by $\triangle \mathrm{EXP}=1$ and produces -0.111 . This shifted fraction, -0.111 , is then added to $\mathrm{FRAC} 1=1.000$, producing 0.001 while the exponent stays at 126 . The normalization mechanism takes place next, and shifts the result, 0.001 , to the left until the leading one in the 0.001 field is detected. The normalizer output becomes 1.000 , but the shifted amount, -3 , is added to the current exponent, 126 , yielding 123 at the output of the adder that computes the exponent.

The normalized fraction goes through a rounding process and truncates the fraction field to 23 bits. The result is stored in a 23-bit register at the output of the floating-point adder. The output of the exponent adder is similarly stored in an eight-bit register along with the sign bit for further processing in the CPU.

This floating-point adder also deals with overflow and underflow conditions in case the exponent values become higher than 255 or smaller than 0 , both of which generate exceptions for the CPU.

### 6.12 Floating-Point Multiplier

The processing complexity of comparing two exponent fields in the floating-point adder does not take place in the floating-point multiplier. The algorithm used in multiplying two floating-point numbers simply multiplies the fractions and adds the exponents.

The floating-point multiplier architecture in Fig. 6.62 computes the fraction and exponent fields with a numerical example. In this example, 126, 1.000, 125 and 1.110 values are assigned to the EXP1, FRAC1, EXP2 and FRAC2 fields of the multiplier, respectively.

The first step of the multiplication process is to calculate the real exponent values of the two floating-point numbers. Therefore, both EXP1 $=126$ and EXP2 $=125$ are subtracted from the single-precision bias, 127 , yielding -1 and -2 , respectively. The real exponents are then added, producing -3 , which becomes the input of an adder that increments or decrements the real exponent after multiplication on the fractions is performed. The fractions, FRAC1 $=1.000$ and $\operatorname{FRAC} 2=1.110$, are multiplied and produce 1110000 . The floating point is assigned immediately after locating the leading one in the 1110000 field which


Fig. 6.62 A floating-point multiplier implementation
results in 1.11000 . This result is subsequently truncated to 1.110 . Since no normalization needs to be performed on 1.110 , this step effectively produces a zero increment/decrement value, and the fraction is stored in the output register after rounding. The real exponent, -3 , on the other hand, is added to the bias, 127, and the result is stored in the exponent register. Sign bits of the two floating-point numbers are also XORed and stored.

As in the floating-point adder, the underflow and overflow conditions in the exponent field of the floating-point multiplier cause the CPU to generate exceptions.

### 6.13 A RISC CPU with Fixed and Floating-Point Units

The floating-point adder, FP Adder, and the floating-point multiplier, FP Multiplier, can be included in the existing fixed-point data-path as shown in Fig. 6.63. The register file outputs are connected to the fixed-point ALU as well as the floating-point adder and the multiplier.


Fig. 6.63 CPU with fixed and floating-point data-paths

The OPC field for the ADDF instruction selects the floating-point adder result in the ALU, and routes it to port 0 terminal of the 3-1 MUX at the write-back stage. Similarly, the result from the floating-point multiplier is directed to port 0 of the 3-1 MUX if the OPC is MULF. Port 1 of the 3-1 MUX is dedicated to the LOAD instruction, and port 2 to the remainder of the OPCs in the instruction set.

One can also employ a secondary RF for only storing floating-point numbers since the formats of fixed and floating-point numbers are different from each other. However, this approach creates additional complexity in routing the RF outputs to the appropriate ALUs and requires additional hardware. Therefore, it is avoided in this design.

### 6.14 Structural Hazards

When instructions are fetched from the instruction memory and introduced to the CPU pipeline, they follow each other one clock cycle apart as shown in Fig. 6.64. In this figure, when the first instruction transitions to the RF stage in cycle 2 , the second instruction starts its instruction fetch cycle. In cycle 3, the first instruction starts the ALU stage, the second instruction enters the RF stage, and the third instruction goes into the instruction fetch stage.

| CYCLES | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| INSTRUCTION 1 | I | R | A | D | W |  |  |  |  |
| INSTRUCTION 2 |  | I | R | A | D | W |  |  |  |
| INSTRUCTION 3 |  |  | I | R | A | D | W |  |  |
| INSTRUCTION 4 |  |  |  | I | R | A | D | W |  |
| INSTRUCTION 5 |  |  |  |  | I | R | A | D | W |

Fig. 6.64 Structural hazards in a five-stage CPU

The fourth cycle in Fig. 6.64 should be viewed with a particular importance because it creates a structural hazard. In this cycle, the first instruction accesses the data memory while the fourth instruction is fetched from the instruction memory. If there is only one memory block with a single port (to read instructions and to store data), this configuration will create a structural hazard because the first and the fourth instructions will try to access the memory in the same cycle. This is the primary reason to have separate instruction and data memories in a RISC CPU.

Cycles 5 and 6 creates another type of structural hazard. In both of these cycles, data has to be read from the RF and written to the RF in the same cycle. If the data read from the RF depends on the data written to the RF, this condition will create a hazard because the write needs to take place before the read in order to produce correct results. Therefore, RF should be designed such that all writes have to be done at the high phase of the clock and all reads at the low phase.

### 6.15 Data Hazards

Data hazards create a situation where the required data is unavailable when it is needed by an instruction. There are four known data hazards in this architecture. The examples below illustrate each data hazard type and propose solutions in the CPU architecture to avoid them.

The first type of data hazard is shown in the example in Fig. 6.65. In this example, the ADD instruction adds the contents of R1 and R2, and then writes the result to a destination register, RD. The second, third and fourth instructions require the contents of RD to proceed. The only hazard-free case here is the data exchange between the ADD and the OR instructions since the RF permits data to be written during the high phase of the clock and read during the low phase. However, the SUB and the AND instructions try to fetch the contents of RD before they become available. Therefore, executing any of these instructions would produce two separate hazards. In order to circumvent this problem, a technique called "data-forwarding" is applied to the CPU pipeline. This method requires a special data route in the CPU data-path so that partially processed data is immediately transferred from a particular pipeline stage to the next when it is needed. Figure 6.66 shows the two forwarding paths to remove the data hazards associated with the SUB and the AND instructions. The first path transfers data from the ALU output to the ALU input when the SUB instruction needs the ADD instruction's ALU result to proceed. The second path transfers data from the output of the data memory to the ALU input when the AND instruction needs the ADD instruction's ALU output.


Fig. 6.65 Data hazard: a register-type instruction followed by other register-type instruction(s)


Fig. 6.66 Forwarding paths to remove the data hazards caused by a register-type instruction followed by other register-type instruction(s)

Figure 6.67 shows the first forwarding path from the ALU output to the ALU input to remove the data hazard caused by the ADD-SUB instruction pair in Fig. 6.65. In this figure, the source address of the SUB instruction is compared against the destination address of the ADD instruction. If there is a match, then the ALU output to ALU input forwarding path(s) is activated by selecting port 1 of the 2-1 MUX at the input of the ALU.


Fig. 6.67 ALU output to ALU input forwarding path
The second forwarding path shown in Fig. 6.68 feeds back the output of the data memory stage to the input of the ALU and removes the data hazard caused by the ADD-AND instruction pair in Fig. 6.65. Again, the source address of the AND instruction is compared against the destination address of the ADD instruction. The path that connects the data


Fig. 6.68 Data memory bypass output to ALU input forwarding path
memory output at the end of the bypass path to the ALU input is activated by selecting port 1 of the 2-1 MUX if there is a match.

Another type of data hazard is shown in Fig. 6.69. This hazard originates from an instruction that requires the contents of the data memory as a source operand. A forwarding path that connects the output of the data memory to the input of the ALU may not be sufficient to remove this data hazard as shown in Fig. 6.70. However, if a No Operation (NOP) instruction is inserted between the LOAD and ADD instructions, the one cycle delay created by this instruction can avoid this hazard as shown in Fig. 6.71. With the NOP instruction in place, the LOAD instruction can now forward the contents of the data memory as a source operand for the ADD instruction as it enters the ALU stage.


Fig. 6.69 Data hazard created by the LOAD instruction followed by a register-type instruction

```
LOAD R1,RD, Imm
ADD RD, R2, R3
```

I R A D D W
1 R $A$ W

Fig. 6.70 A forwarding path to remove the data hazard caused by the LOAD instruction followed by a register-type instruction


Fig. 6.71 A forwarding path and a NOP instruction to remove the data hazard caused by the LOAD instruction followed by a register-type instruction

Figure 6.72 shows the hardware implementation to remove the particular data hazard in Fig. 6.69. The data memory output to the ALU input path is activated by implementing a logic block that compares the destination address of the LOAD instruction with the source address of the ADD instruction, and enables port 1 of the 2-1 MUX if they are equal.


Fig. 6.72 Data memory output to ALU input forwarding path

The final data hazard shown in Fig. 6.73 stems from a LOAD instruction followed by a STORE instruction. Here, the destination address of the LOAD instruction is the same as the source address of the STORE instruction. This results in a situation where the data written back to the RF has to be stored in the data memory within the same clock cycle. A forwarding path that connects the output of the data memory to the input of the data memory removes this hazard as shown in Figs. 6.74 and 6.75. Once again, we need a comparator that compares the contents of the destination address of the LOAD instruction against the source address of the STORE instruction to activate this forwarding path.


Fig. 6.73 Data hazard: a STORE instruction followed by a register-type instruction


Fig. 6.74 A forwarding path to remove the data hazard caused by a STORE instruction followed by a register-type instruction


Fig. 6.75 Data memory output to data memory input forwarding path

Figure 6.76 shows the combination of all four forwarding paths to remove data hazards from the CPU pipeline. The selector inputs in this figure originate from individual comparators that compare the destination address of an instruction with the source address of a subsequent instruction.


Fig. 6.76 CPU schematic containing all data hazard corrections

### 6.16 Program Control Hazards

Branch and jump instructions also create hazards. In the program shown in Fig. 6.77, the earliest time for the BRA instruction to produce a branch target address is when this instruction is at the RF stage. Therefore, the instruction following the BRA instruction cannot be fetched from the instruction memory in the next clock cycle but it requires one cycle delay. This delay can be implemented either by inserting an unrelated instruction to branch or using a NOP instruction in the "branch delay slot" as shown in Fig. 6.78.


Fig. 6.77 Control hazard: a BRA instruction



Fig. 6.78 Removal of BRA control-hazard using an unrelated instruction in the program or a NOP instruction

Similar to the BRA instruction, the jump-type (JUMP, JREG, JAL and JALR) instructions also create control hazards since they can only define the address of the next instruction when they are in the RF stage as shown in Fig. 6.79. Inserting a NOP instruction or an unrelated instruction to jump in the "jump delay slot" removes the pending control hazard as shown in Fig. 6.80.

Even though the jump-type and branch-type instructions update the contents of the PC, the branch instruction requires a comparator and a special adder in the RF stage to calculate the target address.


Fig. 6.79 Control hazard: JUMP and JREG instructions
JUMP Imm
Delay slot
ADD R2, R3, R4


Fig. 6.80 Removal of JUMP and JREG control-hazards using an unrelated instruction in the program or a NOP instruction

### 6.17 Handling Hazards in a Five-Stage RISC CPU: An Example

This example shows the use of forwarding paths to avoid data and control hazards on an instruction chart. The use of the NOP instruction is also shown when forwarding paths become insufficient to remove a particular hazard.

The flow chart in Fig. 6.81 shows a small user program and the contents of the instruction memory before the program is executed. Data A in the flow chart is read from the memory address 100. Data Y, Z and W are stored at the memory addresses 200, 201 and 202, respectively.


Fig. 6.81 Flow-chart of an example program and data memory contents

After Y is stored, the flow chart comes to a decision box where the value of A is compared against one. At this point, the program splits into two where each branch performs calculations to determine the values of Z and W before they are stored in the data memory.

The instruction, LOAD R0, R1, 100, in Fig. 6.82 adds the contents of R0, whose bits are hard-wired to logic 0 due to the architectural specifications, to 100 to calculate the data memory address. It then fetches A from the data memory address 100, and writes this value to the RF address R1.

The SLI R1, R2, 1 instruction shifts the contents of R1 one bit to the left to produce 2A, and writes the result, $\mathrm{Y}=2 \mathrm{~A}$, to R2. The ADD R1, R2, R3 and SRI R1, R4, 1 instructions compute the values 3 A and 0.5 A , respectively. Both values will be used later in the program.

The BRA R1, 1, 5 instruction compares the contents of R1, which currently holds A, with the RS Value $=1$. If the comparison is successful, the program branches off to fetch the next instruction at the instruction memory location, $\mathrm{PC}=4+5=9$, to execute the SUBI R3, R5, 1 that computes $Z=3 A-1$. Otherwise, the program fetches the next instruction, ADDI R3, $\mathrm{R} 5,1$ at $\mathrm{PC}=4+2=6$ to compute $\mathrm{Z}=3 \mathrm{~A}+1$.

| PC | Instruction | Comments |
| :---: | :--- | :--- |
| 0 | LOAD R0, R1, 100 | $\mathrm{A} \rightarrow$ Reg [R1] |
| 1 | SLI R1, R2, 1 | $2 \mathrm{~A} \rightarrow$ Reg [R2] |
| 2 | ADD R1, R2, R3 | $3 \mathrm{~A} \rightarrow$ Reg [R3] |
| 3 | SRI R1, R4, 1 | $0.5 \mathrm{~A} \rightarrow$ Reg [R4] |
| 4 | BRA R1, 1, 5 | $\mathrm{If} \mathrm{A}=1$ then PC + 5 $\rightarrow \mathrm{PC}$ |
| 5 | STORE R2, R0, 200 | $2 \mathrm{~A} \rightarrow$ mem [200] |
| 6 | ADDI R3, R5, 1 | $\mathrm{Z}=3 \mathrm{~A}+1 \rightarrow \mathrm{Reg}$ [R5] |
| 7 | SUBI R4, R6, 1 | $\mathrm{W}=0.5 \mathrm{~A}-1 \rightarrow \mathrm{Reg}$ [R6] |
| 8 | JUMP 11 | $11 \rightarrow \mathrm{PC}$ |
| 9 | SUBI R3, R5, 1 | $\mathrm{Z}=3 \mathrm{~A}-1 \rightarrow \mathrm{Reg}$ [R5] |
| 10 | ADDI R4, R6, 1 | $\mathrm{W}=0.5 \mathrm{~A}+1 \rightarrow \mathrm{Reg}$ [R6] |
| 11 | STORE R5, R0, 201 | $\mathrm{Z} \rightarrow \mathrm{mem}$ [201] |
| 12 | STORE R6, R0, 202 | $\mathrm{W} \rightarrow$ mem [202] |

Fig. 6.82 Instruction memory contents of the example and explanation of each instruction

The STORE R2, R0, 200 instruction is an unrelated instruction to the branch. Therefore, it is used in the branch delay slot following the branch instruction. This instruction stores the contents of R2, which contains 2A, to the data memory location 200 in Fig. 6.81.

The JUMP 11 instruction at $\mathrm{PC}=8$ changes the value of the PC with an immediate value of 11. Therefore, the program skips both the SUBI and ADDI instructions following the JUMP instruction to execute the STORE R5, R0, 201 and STORE R6, R0, 202 instructions. As a result, the values of Z and W are stored at the data memory addresses 201 and 202, respectively, regardless of the branch outcome.

Figure 6.83 shows the instruction chart of the program in Fig. 6.82. The LOAD R0, R1, 100 instruction causes a data hazard as explained in Fig. 6.69, but it is corrected by a combination of a NOP instruction and a forwarding path (from the data memory stage (D) of the LOAD instruction to the ALU stage (A) of the SLI instruction).

The ADD R1, R2, R3 instruction also requires data forwarding from the A stage of the SLI instruction to the A stage of the ADD instruction.


Fig. 6.83 Five-stage CPU instructional chart
The BRA R1, 1, 5 instruction computes the branch target while it is in the RF (R) stage. An instruction unrelated to the branch instruction, such as STORE R2, R0, 200 is used in the branch delay slot so that the CPU has enough time to fetch ADDI R3, R5, 1 if the branch comparison is unsuccessful or SUBI R3, R5, 1 if the comparison is successful.

From this point forward, the program follows two separate instruction charts. The first chart shows the case where the branch comparison is unsuccessful. This chart contains a JUMP instruction followed by a NOP instruction used in the jump delay slot. The second chart follows the case where the branch comparison is successful, and contains two forwarding paths required by the STORE instructions. Both of these paths forward data from the D stage of SUBI R3, R5, 1 and ADDI R4, R6, 1 to the A stage of STORE R5, R0, 201 and STORE R6, R0, 202 to avoid data hazards, respectively.

The entire program takes 15 clock cycles to complete if the branch is successful and 17 cycles if unsuccessful.

Can the program in Fig. 6.82 be executed more efficiently in a shorter amount of time and with fewer forwarding paths if the number of pipeline stages is reduced? To answer this question, two additional CPU pipelines are implemented: one with four pipeline stages and the other with three.

### 6.18 Handling Hazards in a Four-Stage RISC CPU

Figure 6.84 shows a four-stage RISC CPU where the ALU and the data memory stages are combined in a single stage.


Fig. 6.84 Four-stage CPU data-path

If the instructional chart is reconstructed to execute the program in Fig. 6.82 in a four-stage CPU, it will produce a chart in Fig. 6.85. The first observation in this figure is that there are fewer NOP instructions. For example, the NOP instruction that follows LOAD R0, R1, 100 is eliminated because the memory contents become available during the combined ALU/Data Memory stage (AD). Also, the forwarding paths from SUBI R3, R5, 1 to STORE R5, R0, 201 and from ADDI R4, R6, 1 to STORE R6, R0, 202 in Fig. 6.83 are eliminated in Fig. 6.85 because the write-back stage (W) of SUBI R3, R5, 1 lines up with the RF access stage (R) of STORE R5, R0, 201, and similarly the W stage of ADDI R4, R6, 1 lines up with the R stage of STORE R6, R0, 202 in this new CPU pipeline.


Fig. 6.85 Four-stage CPU instructional chart

Branch and jump-related hazards still exist in the chart in Fig. 6.85. They are removed either by inserting NOP instructions or unrelated instructions to branch and jump in the corresponding delay slots.

The new CPU pipeline executes the program in a shorter time: 13 clock cycles if the branch is successful and 15 cycles if unsuccessful.

### 6.19 Handling Hazards in a Three-Stage RISC CPU

Can there be a continuing improvement in the program efficiency and the overall execution time if the number of pipeline stages is reduced further? To answer this question, the CPU pipeline in Fig. 6.84 is repartitioned into three stages where the ALU, the data memory and the write-back stage are grouped together to form a single stage as shown in Fig. 6.86.


Fig. 6.86 Three-stage CPU data-path

When the instructional chart is reconstructed for the three-stage CPU in Fig. 6.86, we see an immediate improvement: the absence of all forwarding paths. The new chart is shown in Fig. 6.87. However, the combined ALU, Data Memory and Write-Back stages (AW) create a practical engineering problem: calculating the data memory address, accessing the data memory and then writing the results back to the RF may not fit in half a clock cycle. Therefore, a NOP instruction is used after LOAD R0, R1, 100 to allow a full clock cycle delay to complete the write-back to the RF before SLI R1, R2, 1 accesses this data.

For the SLI R1, R2, 1 and ADD R1, R2, R3 instruction pair, half a cycle may be sufficient to shift the contents of R1 and store the result into R2 before the ADD instruction accesses this data. Therefore, no NOP instruction is inserted between the SLI and ADD instructions.

Branch and jump-related delay slots still cannot be avoided in Fig. 6.87. STORE R2, R0, 200 is inserted in the branch delay slot, and a NOP instruction is used as the jump delay in Fig. 6.87 just as in Figs. 6.83 and 6.85.

The three-stage CPU executes the program in 13 clock cycles if the branch is successful and 15 cycles if unsuccessful. Therefore, there is no gain in speed compared to the four-stage CPU. However, this may not be true for larger programs if they contain more instructions to access the data memory.


Fig. 6.87 Three-stage CPU instructional chart

### 6.20 Multi-cycle ALU and Related Data Hazards

Not all ALU operations can be executed within one clock cycle. Depending on the propagation delay of the functional modules, the ALU stage may take up several cycles especially if it includes a floating-point adder or multiplier.

To show the potential data hazards in a multi-cycle ALU, a RISC CPU containing both fixed and floating-point ALUs is used in Fig. 6.88. In this example, the fixed-point ALU is assumed to execute data in a single clock cycle, whereas the floating-point add and multiply units require three and five clock cycles to process data, respectively.


Fig. 6.88 Multi-cycle ALU of a five-stage CPU
The program below is executed in the CPU pipeline in Fig. 6.88, and deliberately designed to contain data dependencies. All matching source and destination register addresses are shown in bold letters.

LOAD R1, R2, 0

MULF R2, R3, R4
ADDF R4, R5, R6

STORE R6, R7, 0
The instructional chart for a five-stage CPU in Fig. 6.89 requires the combination of a forwarding path from the D stage of LOAD R1, R2, 0 to the A stage of MULF R2, R3, R4 and a NOP instruction in order to remove the pending data hazard.

But, the more importantly this chart employs four NOP instructions following the MULF instruction to provide the required delay for ADDF R4, R5, R6 to add the contents of R5 to the forwarded data from the floating-point multiplier output. Similarly, STORE R6, R7, 0 needs the combination of a NOP instruction and a forwarding path from the A3 stage of the ADDF instruction to calculate the data memory address. The entire program takes 14 clock cycles to complete.


Fig. 6.89 Instructional chart of a program with data dependencies

The chart in Fig. 6.90 is derived from the instruction chart in Fig. 6.89. This new chart uses different RF addresses, and produces no data dependencies. Consequently, the program is executed in 10 clock cycles instead of 14 due to the absence of NOP instructions.

```
LOAD R1, R2, 0 I R A D W
MULF R3, R4, R5 I N N M 1 M2 M3 M4 M5 D D W
ADDF R6, R7, R8 I R A 1 A2 A3 D W
STORE R9, R10, 0 I R A D W
```

Fig. 6.90 Instructional chart of a program with no data dependencies

Figure 6.91 alters the number of ALU cycles for the floating-point add from three to two clock cycles to investigate the outcome. In this figure, even though the D stage of the ADDF instruction aligns with the D stage of the STORE instruction, neither path interferes with each other since the ADDF instruction uses the data memory bypass path while the STORE instruction accesses the data memory with an address calculated by the fixed-point adder. Therefore, this situation does not create a structural hazard for the CPU.


Fig. 6.91 Instructional chart of a program with no data dependencies: The MULF instruction still requires five cycles but the ADDF instruction is reduced from three to two cycles

Similarly, the W stages of the ADDF and STORE instructions do not create a structural hazard because the ADDF instruction uses the RF write-back path while the STORE instruction terminates at the data memory stage.

Figure 6.92 examines the case where the ALU stage of the MULF instruction is reduced from five to three clock cycles, and there are still no data dependencies among instructions. In this case, both the MULF and ADDF instructions are forced to use the same data memory bypass and write-back paths. Therefore, the ADDF instruction must be delayed for one cycle


Fig. 6.92 Instructional chart of a program with no data dependencies: MULF is reduced to three cycles, but ADDF is still two cycles
with respect to the MULF instruction to ensure that these instructions are able to use the same paths in different clock cycles.

The D stage alignment between the ADDF and STORE instructions does not impose any structural hazard because the ADDF instruction uses the data memory bypass path while the STORE instruction uses a data memory address calculated by the fixed-point adder to store data.

### 6.21 Cache Topologies

Cache is a local memory to the CPU where the temporary blocks of data is kept until it is permanently stored in the system memory. No other bus master but the CPU is allowed to access the cache memory.

There are three types of cache architectures in modern CPUs: fully-associative, set-associative and direct-mapped.

Fully-associative cache protocol allows a block of data to be written (or read) anywhere in the cache as shown in Fig. 6.93. In this type of cache architecture, a block of data is searched in the entire cache before it is read. The range of cache memory addresses to place a block of data is called a set. In Fig. 6.93, the entire cache containing N number of blocks belongs to a single set.


Fig. 6.93 Fully-associative cache topology

Set-associative cache protocol, in contrast, allows a block of data to be written (or read) only to a limited set of addresses in the cache. The top figure of Fig. 6.94 shows a two-way set-associative cache where a block of data from the main memory is written only to two
possible cache addresses, which defines a set. Conversely, when data needs to be read from a two-way set-associative cache, data is searched only within a given set. Therefore, the time to search and locate data is reduced by a factor of N/2 in a two-way set associative cache compared to a fully-associative cache containing N number of blocks.


Fig. 6.94 Two-way and four-way set-associative cache topologies

Similar to the two-way set-associative cache, a block of data is searched in four possible addresses in a set when the CPU issues a cache read in a four-way set-associative cache as shown at the bottom part of Fig. 6.94. If data needs to be written to the cache, only four possible cache addresses in a set are considered according to the cache protocol.

The third cache type is the direct-mapped cache as shown in Fig. 6.95. Its organization is similar to an SRAM, and it maintains a one-to-one addressing scheme with the main memory. In other words, a block of data in the main memory can only be written to a specific location in the cache memory or vice versa.


Fig. 6.95 Direct-mapped cache topology
All stored data blocks in the cache memory are tagged. Consequently, all data transactions between the cache and the main memory (or the CPU) require validation of the tag field before performing a cache read or write operation. Physically, tag fields are stored in a different memory block in the same cache structure. However, both the cache and tag memories retain one-to-one association with each other as shown in Fig. 6.96. In addition, the tag memory comes with valid bits. Each bit specifies if a block of data residing in the cache has an identical twin in the main memory or not. A valid bit $=0$ means that the contents of the main memory have not been updated with the block of data residing in the cache at a certain address. When updating is complete and there is complete data coherency between the cache and main memories, the valid bit becomes logic 1 .

| Valid Bits |
| :---: |
| $\qquad$V Cache Memory Tag Memory <br> V Block 0 Tag 0 <br> V Block 1 Tag 1 <br> V Block 2 Tag 2 <br>  Block 3 Tag 3 <br>  $\vdots$ $\vdots$ <br>  $\vdots$ $\vdots$ <br>  $\vdots$ $\vdots$ <br>  $\vdots$ $\vdots$ <br>    <br> V Block (N-2) Tag (N-2) <br> V Block (N-1) Tag (N-1) |

## CPU ADDRESS:

BLOCK ADDRESS


Fig. 6.96 Cache structure
The CPU address that references the cache memory consists of three separate fields as shown at the bottom section of Fig. 6.96: tag, index and block offset. The index field specifies the set address where the cache block resides. Since a block may contain many words, block offset selects the word in the block. Therefore, before a cache-related operation takes place, the tag field in a CPU address is compared against all the tag fields in a given set. If the tag comparison is successful, the block of data at the specified set location is transferred out of the cache memory or vice versa.

### 6.22 Cache Write and Read Structures

The cache write operation starts with comparing the tag field of the CPU address with all the tag fields of a referenced set in the tag memory. If the comparison is successful, this creates a hit signal, and prompts the CPU to write data to the specified set (and block offset) location in the cache memory.

Figure 6.97 shows the cache write operation to a 32-bit, four-way set-associative cache. This cache contains 22-bit tag fields, 256 sets due to the eight-bit index field, and four words

CPU ADDRESS:
BLOCK ADDRESS

| TAG | INDEX | BLOCK OFFSET |
| :---: | :---: | :---: |
| 22 bits | 8 bits | 2 bits |

in a block due to the two-bit block offset field. The write process starts with identifying the set address using the eight-bit index field. All four tag fields at this set address are individually compared against the CPU tag using XNOR-gates as shown at the output stage of the tag memory in Fig. 6.97. If one of the tags at the set address compares successfully with the CPU tag, it creates a hit signal for the CPU to write a block of data to the corresponding set address. The CPU data is routed through the tri-state buffers placed at the input stage of the cache memory, and written to the designated address via the index field and the block offset field.

The cache read operation is similar to the cache write operation except that the cache and the tag memories are accessed simultaneously to shorten the cache read access period.

Figure 6.98 shows the same four-way set-associative cache structure shown in Fig. 6.97. With the eight-bit index field defining the set address, four tag blocks from the tag memory and four data blocks from the cache memory are read out simultaneously. Each tag is individually compared with the 22-bit tag field in the CPU address, and hit signals are generated using four XNOR gates. These hit signals are subsequently used as selector inputs for the 4-1 MUX at the output of the cache memory to select one of the cache blocks. The word from the chosen block is selected by the two-bit block offset field and given to the CPU.

### 6.23 A Direct-Mapped Cache Example

Even though cache read/write protocols and the related hardware have been explained in the earlier sections of this chapter, the example shown in Fig. 6.99 further clarifies the operation of a direct-mapped cache.

The cache structure in this example consists of eight sets due to the three-bit index field in the CPU address with no block offsets. Therefore, each data block contains a single five-bit word with a two-bit tag field.

Assume that the CPU issues cache reads from the following addresses: 10101, 10010, $10101,01010,10000,10110,10000,10111$ and 01111 . When the cache is first turned on, its tag, valid bits and data fields are all zero. The index column in Fig. 6.99 is not an actual part of cache memory. Its sole purpose is simply to indicate a set address.

When the CPU issues the first read from the address 10101, the tag memory contents 00 at the set address 101 are compared against the tag field contents of 10 in the CPU address. Since the two values are different from each other, the cache controller issues a cache miss, fetches the data, mem (10101), from the main memory address of 10101, delivers this data to the CPU, and stores the same data in the cache. It also updates the tag contents with 10 , and issues valid bit $=1$.

Next, the CPU issues the second read from the address 10010. This time, the tag memory contents 00 at the set address 010 are compared with the tag field of 10 in the CPU address. The comparison fails and produces a miss. The cache controller fetches the data, mem (10010), from the main memory, writes this data at the set address 010 of the cache memory, delivers the same data to the CPU, updates the tag memory with 10 , and produces valid bit $=1$.

SET 0
SET 1
SET 254
SET 255
CPU Tag

CPU ADDRESS:
BLOCK ADDRESS

Fig. 6.98 Cache read operation

| CPU Address: | TAG | INDEX | BOFF |
| :---: | :---: | :---: | :---: |
|  | 2 bits 3 bits | 0 bits |  |

(1) Initial state of the cache

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 00 | 0 | 0 |
| 001 | 00 | 0 | 0 |
| 010 | 00 | 0 | 0 |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 00 | 0 | 0 |
| 110 | 00 | 0 | 0 |
| 111 | 00 | 0 | 0 |

(3) After the MISS at address 10010

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 00 | 0 | 0 |
| 001 | 00 | 0 | 0 |
| 010 | 10 | 1 | mem (10010) |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 00 | 0 | 0 |
| 111 | 00 | 0 | 0 |

(5) After the MISS at address 01010

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 00 | 0 | 0 |
| 001 | 00 | 0 | 0 |
| 010 | 01 | 1 | mem (01010) |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 00 | 0 | 0 |
| 111 | 00 | 0 | 0 |

(7) After the MISS at address 10110

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 10 | 1 | mem (10000) |
| 001 | 00 | 0 | 0 |
| 010 | 01 | 1 | mem (01010) |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 10 | 1 | mem (10110) |
| 111 | 00 | 0 | 0 |

(9) After the MISS at address 10100

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 10 | 1 | mem (10000) |
| 001 | 00 | 0 | 0 |
| 010 | 01 | 1 | mem (01010) |
| 011 | 00 | 0 | 0 |
| 100 | 10 | 1 | mem (10100) |
| 101 | 10 | 1 | mem (10101) |
| 110 | 10 | 1 | mem (10110) |
| 111 | 00 | 0 | 0 |

(2) After the MISS at address 10101

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 00 | 0 | 0 |
| 001 | 00 | 0 | 0 |
| 010 | 00 | 0 | 0 |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 00 | 0 | 0 |
| 111 | 00 | 0 | 0 |

(4) After the HIT at address 10101

| INDEX | TAG | $V$ | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 00 | 0 | 0 |
| 001 | 00 | 0 | 0 |
| 010 | 10 | 1 | mem (10010) |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 00 | 0 | 0 |
| 111 | 00 | 0 | 0 |

(6) After the MISS at address 10000

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 10 | 1 | mem (10000) |
| 001 | 00 | 0 | 0 |
| 010 | 01 | 1 | mem (01010) |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 00 | 0 | 0 |
| 111 | 00 | 0 | 0 |

(8) After the HIT at address 10000

| INDEX | TAG | $V$ | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 10 | 1 | mem (10000) |
| 001 | 00 | 0 | 0 |
| 010 | 01 | 1 | mem (01010) |
| 011 | 00 | 0 | 0 |
| 100 | 00 | 0 | 0 |
| 101 | 10 | 1 | mem (10101) |
| 110 | 10 | 1 | mem (10110) |
| 111 | 00 | 0 | 0 |

(10) After the MISS at address 11111

| INDEX | TAG | V | DATA |
| :---: | :---: | :---: | :---: |
| 000 | 10 | 1 | mem (10000) |
| 001 | 00 | 0 | 0 |
| 010 | 01 | 1 | mem (01010) |
| 011 | 00 | 0 | 0 |
| 100 | 10 | 1 | mem (10100) |
| 101 | 10 | 1 | mem (10101) |
| 110 | 10 | 1 | mem (10110) |
| 111 | 11 | 1 | mem (11111) |

Fig. 6.99 A direct-mapped cache operation

The CPU reissues another read from the address 10101. The tag memory contents of 10 at the set address of 101 compare successfully with the CPU tag field of 10 . As a result, the cache controller issues a hit. The data, mem (10101), at the set address 101 is transferred directly from the cache memory to the CPU.

The next CPU address 01010 accesses the set 010 in the tag memory, but finds the tag memory contents of 10 at this address is different from the tag field contents of 01 in the CPU address. Therefore, the cache controller issues a miss, fetches mem (01010) from the main memory, and delivers this data to the CPU and the cache memory. It also updates the tag contents with 01 , and issues valid bit $=1$.

The fifth address 10000 creates another miss because the tag memory contents of 00 at the set address of 000 do not compare with the tag contents of 10 at the CPU address. The cache controller transfers mem (10000) from the main memory to the set address 000 of the cache, delivers the same data to the CPU, updates the tag contents with 10 , and produces valid bit $=1$.

When the CPU issues the sixth address 10110, the cache controller finds the tag memory contents of 00 at the set address of 110 to be different from the tag field contents of 10 in the CPU address, and issues a miss. Consequently, the cache controller fetches mem (10110) from the main memory, delivers it to the CPU and the cache memory. It also updates the tag memory with 10 , and assigns valid bit $=1$.

Next, the CPU reissues the address 10000. Since the CPU and the tag memory contents match, this creates a cache hit. The cache controller simply delivers mem (10000) from the set address 000 of the cache to the CPU.

The next address, 10100, creates another cache miss. The cache controller updates the set address contents with mem (10100) and delivers the same data to the CPU. It also updates the tag memory with a value of 10 and issues valid bit $=1$.

When the last CPU address, 11111, is issued, the cache controller finds the tag field contents of 11 in the CPU address to be different from the tag memory contents of 00 at the set address of 111 . It issues a miss and delivers mem (11111) to both the CPU and the cache. It updates the tag memory with 11 and assigns valid bit $=1$.

### 6.24 Write-Through and Write-Back Cache Structures in Set-Associative Caches

It is very common to see two types of cache structures when analyzing set-associative caches: write-through caches and write-back caches.

In write-through caches, the cache controller maintains data coherency between the cache and the main memory before starting a new task.

In write-back caches, the wait time for data coherency is an essence. For example, if the transaction is a read, but the main memory bus is busy with another data transfer, the cache controller temporarily stores the data from the cache in a write-back buffer instead of waiting to write it to the main memory, and starts a new task. When the bus arbiter grants the bus
access, the cache controller resumes transferring this data from the write-back buffer to the main memory.

### 6.25 A Two-Way Set-Associative Write-Through Cache Example

The example in Fig. 6.100 shows 14 different transactions between a two-way set-associative write-through cache and a CPU. The initial contents of the main memory are shown in the same figure. Each transaction is specified by a CPU address, the type of transaction and data.

The CPU address in this example consists of six bits: the most significant four bits define the tag address; the least significant two bits indicate the set address as shown in Fig. 6.101.


MAIN MEMORY
(Word Addressable)

Fig. 6.100 A two-way set-associative write-through cache pending transactions and initial data memory contents

CPU Address | TAG |  |
| :---: | :---: |
|  | 4 bits |



End of $6^{\text {th }}$ transaction:

|  | WAY 1 | WAY 0 |
| :--- | :---: | :---: |
| Set 0 | $0 \times 0 \mathrm{E}$ | $0 \times 2 \mathrm{~A}$ |
| Set 1 | $0 \times 0 \mathrm{~A}$ | $0 \times 1 \mathrm{~A}$ |
| Set 2 |  |  |
| Set 3 | $0 \times 0 \mathrm{~F}$ | $0 \times 3 \mathrm{~A}$ |
|  |  |  |

End of $10^{\text {th }}$ transaction:

|  | WAY 1 | WAY 0 |
| :--- | :---: | :---: |
| Set 0 | $0 \times 3 \mathrm{~B}$ | $0 \times 2 \mathrm{~B}$ |
| Set 1 | $0 \times 0 \mathrm{~A}$ | $0 \times 1 \mathrm{~A}$ |
| Set 2 | $0 \times 0 \mathrm{~B}$ | $0 \times 1 \mathrm{~B}$ |
| Set 3 | $0 \times 0 \mathrm{~F}$ | $0 \times 3 \mathrm{~A}$ |
|  |  |  |


|  | V | WAY 1 | V | WAY 0 |
| :---: | :---: | :---: | :---: | :---: |
| Set 0 | 1 | 1000 | 1 | 1100 |
| Set | 1 | 0101 | 1 | 1100 |
| Set 2 | 1 | 0001 | 1 | 0011 |
| Set 3 | 1 | 1000 | 1 | 0100 |
|  |  |  |  |  |

End of $12^{\text {th }}$ transaction:

|  | WAY 1 | WAY 0 |
| :--- | :---: | :---: |
| Set 0 | $0 \times 1 \mathrm{C}$ | $0 \times 2 \mathrm{~B}$ |
| Set 1 | $0 \times 0 \mathrm{~A}$ | $0 \times 1 \mathrm{~A}$ |
| Set 2 | $0 \times 0 \mathrm{~B}$ | $0 \times 1 \mathrm{~B}$ |
| Set 3 | $0 \times 0 \mathrm{~F}$ | $0 \times 3 \mathrm{~A}$ |
|  |  |  |


|  | V | WAY 1 | V | WAY 0 |
| :---: | :---: | :---: | :---: | :---: |
| Set 0 | 1 | 1001 | 1 | 1100 |
| Set | 1 | 0101 | 1 | 1100 |
| Set 2 | 1 | 0001 | 1 | 0011 |
| Set 3 | 1 | 1000 | 1 | 0100 |
|  |  |  |  |  |

End of $14^{\text {th }}$ transaction:

|  | WAY 1 | WAY 0 |
| :--- | :---: | :---: |
| Set 0 | $0 \times 1 \mathrm{C}$ | $0 \times 3 \mathrm{~F}$ |
| Set 1 | $0 \times 0 \mathrm{~A}$ | $0 \times 1 \mathrm{~A}$ |
| Set 2 | $0 \times 0 \mathrm{~B}$ | $0 \times 1 \mathrm{~B}$ |
| Set 3 | $0 \times 0 \mathrm{~F}$ | $0 \times 3 \mathrm{~A}$ |
|  |  |  |


|  | V | WAY 1 | V | WAY 0 |
| :---: | :---: | :---: | :---: | :---: |
| Set 0 | 1 | 1001 | 1 | 0000 |
| Set 1 | 1 | 0101 | 1 | 1100 |
| Set 2 | 1 | 0001 | 1 | 0011 |
| Set 3 | 1 | 1000 | 1 | 0100 |
|  |  |  |  |  |


| 000000 | 0x3F |
| :---: | :---: |
|  | - |
| 000110 | $0 \times 00 \rightarrow 0 \times 0 B$ |
|  | - |
| 001110 | $0 \times 00 \rightarrow 0 \times 1 \mathrm{~B}$ |
|  | - |
| 010011 | 0x3A |
|  | - |
| 010101 | 0x0A |
|  | - |
| 100000 | $0 \times 0 \mathrm{E} \rightarrow 0 \times 3 \mathrm{~B}$ |
|  | - |
| 100011 | 0x0F |
|  | - |
| 100100 | $0 \times 00 \rightarrow 0 \times 0 \mathrm{C} \rightarrow 0 \times 1 \mathrm{C}$ |
|  | - |
| 110000 | $0 \times 2 \mathrm{~A} \rightarrow 0 \times 2 \mathrm{~B}$ |
| 110001 | $0 \times 1 \mathrm{~A}$ |
|  | - |
| 111100 | $0 \times 00 \rightarrow 0 \times 2 \mathrm{C}$ |

Fig. 6.101 A two-way set-associative write-through cache, tag and data memory contents after the sixth, tenth, twelfth and fourteenth transactions

There are no bits for block offset, which indicates every block consists of a single word with six bits of data. Since this is a two-way set-associative cache, the memory is organized as two adjacent data blocks for every set. The tag memory also consists of two adjacent tag fields with valid bits for each set as shown in Fig. 6.101.

The data from the main memory can either go to the most significant (WAY 1) or the least significant (WAY 0) block position of the two-way set-associative cache. Therefore, a data replacement policy must be defined when designing a set-associative cache architecture. In this example, let us assume that the data replacement policy dictates the old data with smaller number of memory references (the total number of reads and writes) to be replaced. If the number of memory references at either cache blocks is equal to each other, the block of data at the most significant cache position is replaced with the new data.

The first transaction reads from the memory address of 010101. In this transaction, the cache controller compares the tag field contents of 0101 in the CPU address with the tag memory contents of 0000 at the set address 01 , and issues a miss. Next, the cache controller fetches $0 x 0 \mathrm{~A}$ from the main memory address of 010101, and delivers it to the CPU. Since the number of memory references at the most and the least significant cache positions are both zero at this point, the cache controller places 0x0A at the most significant cache position as the result of the data replacement policy. It also updates the tag memory with 0101, and assigns valid bit $=1$.

The next five read transactions result in five consecutive cache misses. By the end of the sixth transaction, six new data entries from the main memory are written to both the cache and the tag memories as shown in Fig. 6.101.

The seventh CPU transaction is a write. The CPU issues to write 0 x 0 B to the main memory address of 000110 . As for the read operations, the cache controller compares the tag memory contents of 0000 at the set address of 10 with the tag field contents of 0001 in the CPU address, and issues a miss. The data, 0x0B, is written to both the main memory address of 000110 and the most significant cache position at the set address of 10 . The tag memory is also updated with 0001 , and valid bit $=1$.

In the eight transaction, the cache controller again issues a miss for the set address of 10 because the tag field comparison fails. Consequently, the cache controller stores the CPU data, $0 \times 1 \mathrm{~B}$, in the main memory address of 001110 , and also writes this data to the least significant cache position at the set address of 10 . The tag memory is updated with 0011 and valid bit $=1$.

The next write compares the tag field entry of 1100 in the CPU address with all the tag memory entries at the set address of 00 . Since the least significant tag memory contents are identical to the CPU tag entry, the cache controller issues a hit, but still replaces the contents of the main memory at the address of 110000 and the contents of the cache memory at the set address of 00 with $0 \times 2 \mathrm{~B}$. This is because the architecture of this cache is a write-through which requires the cache and the main memory contents to be the same before the cache controller starts a new task. Therefore, there is really no difference between a cache write
miss and cache write hit when it comes to updating the cache and the main memory contents. In both cases, the cache controller has to wait until the current data transaction ends before updating the main memory. However, the tag memory contents require no updating following a cache write hit.

The tenth transaction is another CPU write which results in a cache hit. Like the previous write transaction, the cache controller has to replace the contents of the main memory at the address of 100000 and the contents of the cache memory at the set address of 00 with $0 x 3 \mathrm{~B}$. At the end of the tenth transaction, the cache and the tag memory contents are shown in Fig. 6.101 with two memory references for the set address of 00 , and one memory reference for the remaining set addresses.

The eleventh transaction creates a cache miss and causes the cache controller to replace the data 0x3B at the most significant cache position with the CPU data of 0x0C at the set address of 00 . The most significant tag entry, 1000 , is also replaced with 1001 at the same set address.

The twelfth transaction creates a cache hit because the CPU tag field, 1001, compares successfully with the tag memory contents at the set address of 00 . However, the cache controller replaces $0 \times 0 \mathrm{C}$ at the main memory address of 100100 and at the set address of 00 with the new CPU data 0x1C.

In the thirteenth transaction, the CPU address of 111100 causes a cache miss. The CPU data, $0 \times 2 \mathrm{C}$, is written both to the main memory address, 111100 , and the least significant cache position at the set address of 00 per data replacement policy. The least significant tag memory contents at the set address of 00 are also updated with 1111.

The fourteenth transaction is a memory read and causes another miss. The cache controller delivers 0x3F from the main memory location of 000000 to the CPU and writes the same data to the least significant cache position at the set address of 00 .

### 6.26 A Two-Way Set-Associative Write-Back Cache Example

The example in Fig. 6.102 shows the transactions between a CPU and a two-way set-associative write-back cache. The initial contents of the main memory and the cache are shown in the same figure. The CPU address in this example has eight bits. The most significant four bits are reserved for the tag field. The two-bit index field indicates that there are four sets in the cache memory. The two-bit block offset field signifies that there are four eight-bit wide words in each block. Therefore, the cache memory consists of two adjacent blocks at WAY 1 and WAY 0 positions, each containing four words. The tag memory has also two adjacent tag entries with valid bits. Each tag represents a block in the cache memory. The data replacement policy for this example assumes to replace the block of data in the cache memory with the least number of memory references. If the number of memory references at each block is the same, then the policy replaces the old data at the least significant block position.

## CPU Address

| TAG | INDEX | BOFF |
| :---: | :---: | :---: |
| 4 bits | 2 bits 2 bits |  |

Two-Way Set-Associative Tag Memory


Two-Way Set-Associative Cache Memory


Set 0
Set 1
Set 2
Set 3


Write-Back Buffer
Block 1
Block 0


| Transaction No | CPU Address | Read/Write | CPU Data |
| :---: | :---: | :---: | :---: |
| 1 | 10001000 | Read | - |
| 2 | 10000000 | Write | 0xE0 |
|  | 10000001 | Write | $0 x E 1$ |
|  | 10000010 | Write | 0xE2 |
| 3 | 10000011 | Write | 0xE3 |
|  | 10000000 | Write | 0xF0 |
|  | 10000001 | Write | 0xF1 |
|  | 10000010 | Write | $0 x F 2$ |
| 4 | 10000011 | Write | 0xF3 |
|  | 11000000 | Read | - |

MAIN MEMORY (Word Addressable)

|  |  |
| :---: | :---: |
| 10001000 | 0xA0 |
| 10001001 | $0 \times \mathrm{A} 1$ |
| 10001010 | 0xA2 |
| 10001011 | 0xA3 |
|  |  |
| 10101000 | 0xB0 |
| 10101001 | 0xB1 |
| 10101010 | 0xB2 |
| 10101011 | 0xB3 |
|  |  |
| 10111000 | 0xC0 |
| 10111001 | $0 \times C 1$ |
| 10111010 | $0 \times C 2$ |
| 10111011 | 0xC3 |
|  |  |
| 11000000 | 0xD0 |
| 11000001 | 0xD1 |
| 11000010 | 0xD2 |
| 11000011 | 0xD3 |
|  | : |

Fig. 6.102 A two-way set-associative write-back cache pending transactions and initial data memory

Since this is a write-back cache, its architecture enables the cache controller to store the CPU data temporarily in a buffer to be written to the main memory at a later time. Every time a block of data is written to this buffer, a dirty bit is attached to its tag, designating that this block is waiting to be written to the main memory. Therefore, the cache coherency mechanism that presides over write-through caches is not valid for write-back caches. Instead, the dirty bit attached to each tag entry determines whether or not a block of data exists in both the cache and the main memories.

Initially, identical data resides both in the main memory and the cache as shown in Fig. 6.102, and therefore valid bits at the set address of 10 in the tag memory become equal to one. Since the write-back buffer cache architecture does not offer any data waiting period for the main memory, the dirty bits at the set address 10 are equal to zero.

The first CPU transaction is to read data from the memory address 10001000. The cache controller compares the tag entry 1000 in the CPU address with all the tags at the set address 10 and issues a miss. It then transfers the block of data from the main memory address $10001000(0 x A 0,0 x A 1,0 x A 2$ and $0 x A 3)$ to replace the old block $(0 x B 0,0 x B 1,0 x B 2$ and $0 x B 3$ ) at the least significant cache position. The cache controller also updates the corresponding tag contents with 1000 as shown at the top portion of Fig. 6.103.

The next CPU transaction is a write, which results in a cache miss. Consequently, the cache controller writes the contents of the CPU data that consists of $0 \times \mathrm{xE} 0,0 \mathrm{xE} 1,0 \mathrm{xE} 2$ and $0 x E 3$ to the least significant cache position at the set address 00 , and updates the tag memory contents with 1000 . However, during this transaction the data bus happens to be busy. Therefore, the cache controller stores this block in the write-back buffer, assigns dirty bit $=1$ and valid bit $=0$, and starts the next transaction. When the bus access is granted, the cache controller resumes transferring this data from the write-back buffer to the main memory, and assigns dirty bit $=0$ and valid bit $=1$.

The third CPU transaction is a write and results in a cache hit. The cache controller simply writes the new block of data, $0 \mathrm{xF} 0,0 \mathrm{xF} 1,0 \mathrm{xF} 2$ and 0 xF 3 , to the write-back buffer and to the least significant block position of the cache memory, replacing the old block, 0xE0, 0xE1, $0 x E 2$ and $0 x E 3$. Since this transaction does not require transferring the old block from the write-back buffer to the main memory, write-back cache scheme creates a distinct speed advantage compared to the write-through scheme.

The last CPU transaction is a memory read that results in a miss. Consequently, the data block, $0 \mathrm{xD} 0,0 \mathrm{xD} 1,0 \mathrm{xD} 2$ and 0 xD 3 , that resides at the main memory address 11000000 is transferred to the most significant block position of the cache memory at the set address 00 since this position has zero memory references compared to the least significant block position. The tag memory contents are updated with 1100 , valid bit $=1$ and the dirty bit $=0$ at this set address.


Write-Back Buffer


## AFTER THE 2nd TRANSACTION

Two-Way Set-Associative Cache Memory

|  | WAY 1 |  |  |  | WAY 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 10 | 01 | 00 | 11 | 10 | 01 | 00 |
| Set 0 |  |  |  |  | 0xE3 | 0xE2 | 0xE1 | 0xE0 |
| Set 1 |  |  |  |  |  |  |  |  |
| Set 2 | 0xC3 | 0xC2 | 0xC1 | 0xC0 | 0xA3 | 0xA2 | 0xA1 | 0xA0 |
| Set 3 |  |  |  |  |  |  |  |  |

Write-Back Buffer


## AFTER THE 3rd TRANSACTION

Two-Way Set-Associative Cache Memory


Write-Back Buffer


AFTER THE 4th TRANSACTION
Two-Way Set-Associative Cache Memory

|  | WAY 1 |  |  |  | WAY 0 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11 | 10 | 01 | 00 | 11 | 10 | 01 | 00 |
| Set 0 | 0xD3 | 0xD2 | 0xD1 | 0xD0 | 0xF3 | 0xF2 | 0xF1 | 0xF0 |
| Set 1 |  |  |  |  |  |  |  |  |
| Set 2 | 0xC3 | 0xC2 | 0xC1 | 0xC0 | 0xA3 | 0xA2 | 0xA1 | 0xA0 |
| Set 3 |  |  |  |  |  |  |  |  |

Write-Back Buffer


Two-Way Set-Associative Tag Memory


Two-Way Set-Associative Tag Memory

|  | WAY 1 |  |  | WAY 0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | D | V | TAG | D | V | TAG |
| Set 0 |  |  |  | 1 | 0 | 1000 |
| Set 1 |  |  |  |  |  |  |
| Set 2 | 0 | 1 | 1011 | 0 | 1 | 1000 |
| Set 3 |  |  |  |  |  |  |

Two-Way Set-Associative Tag Memory

|  | WAY 1 |  |  | WAY 0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | D | V | TAG | D | V | TAG |
| Set 0 |  |  |  | 1 | 0 | 1000 |
| Set 1 |  |  |  |  |  |  |
| Set 2 | 0 | 1 | 1011 | 0 | 1 | 1000 |
| Set 3 |  |  |  |  |  |  |

Two-Way Set-Associative Tag Memory

|  | WAY 1 |  |  | WAY 0 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | D | V | TAG | D | V | TAG |
| Set 0 | 0 | 1 | 1100 | 1 | 0 | 1000 |
| Set 1 |  |  |  |  |  |  |
| Set 2 | 0 | 1 | 1011 | 0 | 1 | 1000 |
| Set 3 |  |  |  |  |  |  |

Fig. 6.103 A two-way set-associative write-back cache with tag and write-back buffers

## Review Questions

1. A 32-bit RISC CPU organized in Big Endian format has three pipeline stages to execute the following two instructions:

ADDI RS, RD, Imm Value $\operatorname{Reg}[R S]+\operatorname{Imm}$ Value $\rightarrow \operatorname{Reg}[R D]$
XOR RS1, RS2, RD $\quad \operatorname{Reg}[R S 1]+\operatorname{Reg}[R S 2] \rightarrow \operatorname{Reg}[R D]$
Draw the detailed ALU and the CPU schematic that executes these two instructions. Label all interconnections, bus widths and control signals.
2. An eight-bit CPU interacts with a two-way set-associative write-through cache organized in Little Endian format. The top row of this cache corresponds to set 0 . The CPU address has the following format:

| Tag | Index | Block offset |
| :---: | :---: | :---: |
| 3 bits | 3 bits | 2 bits |

Data replacement policy in case of a miss is as follows:
(i) Tags with invalid bits are replaced.
(ii) The least significant tag is replaced if the number of references is the same when valid bit $=1$ (every cache read or write is considered a reference).
(a) Draw the block diagram of the tag (with valid bits) and cache memories. Calculate how many bits are in each memory.
(b) Draw the block diagram and the contents of the cache and the tag memories at the end of the fifth, eleventh and thirteenth transactions according to the transaction list below:

| Transaction no. | CPU Address | Data | Write/Read |
| :---: | :---: | :---: | :---: |
| 1 | 10100110 | $0 \times 11$ | W |
| 2 | 00111001 | $0 \times 22$ | W |
| 3 | 11110100 | $0 \times 33$ | W |
| 4 | 10001111 | $0 \times 44$ | W |
| 5 | 00100110 | $0 \times 55$ | W |
| 6 | 00111011 | $0 \times 66$ | W |
| 7 | 00111001 | $0 \times 77$ | W |
| 8 | 11001111 | $0 \times 88$ | W |
| 9 | 01101101 | $0 \times 99$ | W |
| 10 | 00010100 | $0 \times \mathrm{AA}$ | W |
| 11 | 10100110 | $?$ | R |
| 12 | 00100111 | $?$ | R |
| 13 | 00101111 | $?$ | R |

3. The following specification is given for implementing a 32-bit RISC processor that executes the integer multiply-add (MADD) and add (ADD) instructions:
(i) Data, a, b, c and d are read from the DOut1, DOut2, DOut3 and DOut4 ports of the 32-bit RF at the same time.
(ii) There are four stages in the processor. The ALU consists of two stages.
(iii) Multiplication is the first ALU stage for the MADD instruction between a and $b$, and between c and d . It takes one clock cycle to produce results which are eventually written to the DinH (for higher 32 data bits) and DinL (for lower 32 data bits) ports of the RF simultaneously. This stage can be bypassed if addition is performed between a and c.
(iv) Addition is the second ALU stage, and it also takes one clock cycle to produce results.
(v) For MADD instruction, RS1 is the first source address that contains a, RS2 is the second source address that contains b, RS3 is the third source address that contains c, and RS4 is the fourth source address that contains d. RD1 is the first destination address that stores the lower 32 bits of the result, and RD2 is the second destination address that stores the higher 32 bits. For the ADD instruction, RS1 is the first source address that contains a, RS3 is the second address that contains c , and RD1 is the destination address that stores the result.
(a) Draw the instruction bit field format of these two instructions, indicating the opcode and operand fields.
(b) Draw the architectural diagram of the processor that executes the ADD and the MADD instructions, indicating all the necessary hardware such as the required memories, the RF, the detailed ALU with all the port names and bit widths. Show how the opcode decoder enables multiplexers and other hardware in each stage.
4. The area under $y=x$ is calculated until the area equals to 18 square units.


The incremental area is calculated in the flow chart given below.

(a) Assuming $\operatorname{Reg}[\mathrm{R} 0]=0$, write a program using the instruction set given in Chapter 6. Make comments next to each instruction in the program.
(b) Form an instruction chart (histogram) for this program and show all the data dependencies that require forwarding loops in the RISC processor. Stall the pipeline using the NOP instruction if necessary. Consider the branch or jump delay penalty to be 1 cycle.
5. A RISC CPU computes the following:

$$
X=2 A^{2}+1
$$

A is located at the data cache address, 100 , and X needs to be stored at the address, 200. All instructions take one cycle except the multiply, which takes three cycles. The RF contains only R0 and R1. The contents of R0 are 0 .
Make sure to have only 16-bit values in the source registers, RS1 and RS2, in order to avoid the overflow condition in the destination register, RD, when the MUL instruction is used.
(a) Write an assembly code to compute and store the value of X. Make sure to write comments next to each instruction to keep track of the register values.
(b) Rewrite the assembly code with an instruction chart. Indicate all the forwarding loops and the stalls caused by the NOP instructions on this chart.
6. Design a four-way set-associative cache for an eight-bit CPU. The cache is organized in Little Endian format. It has four sets, and each block in the set contains two words. The replacement policy on a cache miss is as follows:
(i) An entire block of data is exchanged between the CPU and the cache
(ii) The least significant block is replaced
(iii) The block with the fewest number of memory references is replaced (a memory reference corresponds to each memory-read or memory-write cycle)
Below is the list of CPU transactions:

| Transaction no. | CPU Address | Data | Write/Read |
| :---: | :---: | :---: | :---: |
| 1 | 10000010 | $?$ | R |
| 2 | 10000110 | $?$ | R |
| 3 | 01111101 | $?$ | R |
| 4 | 00100011 | $?$ | R |
| 5 | 00001010 | $?$ | R |
| 6 | 00001101 | $?$ | R |
| 7 | 10001001 | $?$ | R |
| 8 | 10001010 | $?$ | R |
| 9 | 10000000 | $0 \times \mathrm{AB}$ | W |
| 10 | 10000001 | $0 \times \mathrm{CD}$ | W |
|  | 00001010 | $0 \times \mathrm{AB}$ | W |
| 11 | 00001011 | $0 \times \mathrm{CD}$ | W |
|  | 11011010 | $0 \times \mathrm{AB}$ | W |
| 12 | 11011011 | $0 \times \mathrm{CD}$ | W |
|  | 11111011 | $?$ | R |

The contents of the main memory before any transaction are shown below:

| 00001010 | 7 | 0 |
| :---: | :---: | :---: |
|  | 0x33 |  |
| 00001011 | 0x44 |  |
| 00001100 | 0x55 |  |
| 00001101 | 0x66 |  |
| 00100010 | - |  |
|  | $0 \times 11$ |  |
| 00100011 | 0x22 |  |
|  | : |  |
| $\begin{aligned} & 01111100 \\ & 01111101 \end{aligned}$ | 0xEE |  |
|  | 0xFF |  |
|  | : |  |
| 10000000 | 0x00 |  |
| 10000001 | 0x00 |  |
| $\begin{aligned} & 10000010 \\ & 10000011 \end{aligned}$ | 0xAA |  |
|  | 0xBB |  |
|  | $:$ |  |
| 10000110 | 0xCC |  |
| 10000111 | 0xDD |  |
| 10001000 | 0x77 |  |
| 10001001 | 0x88 |  |
| 10001010 | 0x00 |  |
| 10001011 | 0x99 |  |
|  | $:$ |  |
| 11011010 | 0x00 |  |
| 11011011 | 0x00 |  |
|  | $:$ |  |
| 11111010 | $0 \times A B$ |  |
| 11111011 | $0 \times C D$ |  |

(a) Draw the block diagram of the cache and tag memories. Show the field format of the CPU address in terms of tag, index and block offset.
(b) Show the contents of the cache and the tag memories after the eighth, tenth and twelfth transactions. Update the main memory contents if there is any change.
7. A 32-bit, five-stage RISC CPU organized in Little Endian format executes the flow chart below. The CPU contains a register file with 32 registers where $\operatorname{Reg}[\mathrm{R} 0]=0$. The integer values, $\mathrm{SUM}=0, \mathrm{i}=1$ and the compare value of 100 , are stored at the data memory locations 100,101 and 102 , respectively. The final SUM needs to be stored at the data memory address, 200.

(a) Write an assembly program using the following instruction set. Accompany each instruction in the program with register data and comments.

| Instruction Set | Instruction Definition |
| :--- | :--- |

(b) Draw the CPU schematic that executes the instructions in the flow chart above.
8. The function, $\mathrm{Y}=\frac{5(\mathrm{~A}-\mathrm{B})}{32}$, needs to be executed using the instruction set below.

| Instruction Set | Instruction Definition |
| :--- | :--- |

A is located at the memory address 100 .
$B$ is located at the memory address 101 .
Y needs to be stored at the memory address 102.
$\operatorname{Reg}[R 0]=0$.
(a) Write a program to compute Y .
(b) If the LOAD and STORE instructions require two cycles to access the data cache, rewrite the program to accommodate this requirement. Show all the forwarding loops and include all the necessary NOPs in the instruction chart.
(c) Indicate the minimum number of clock cycles to execute the program in part (b).
9. A 32-bit CPU organized in Big Endian format has 32 general purpose registers (R0 is also a general purpose register and its contents are not zero). This CPU executes the following flow chart:


The instruction set and the bit-field format for each instruction is shown below.

| LOAD Imm Value, RD | mem (Imm Value) $\rightarrow$ Reg[RD] | $\begin{array}{llllll}0 & 56 & 1011 & 1516\end{array}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | LOAD | NU | RD | Imm Value |  |  |
| INVERT RS, RD |  | $\begin{array}{lllll}0 & 56 & 1011 & 1516\end{array}$ |  |  |  |  |  |
|  | $\mathrm{Reg}[\mathrm{RS}] \rightarrow \mathrm{Reg}[\mathrm{RD}]$ | INVERT | RS | RD | NU |  |  |
|  | lower upper | 0 56 1011 1516 2021 2526 31 |  |  |  |  |  |
| MUL RS1, RS2, RD1, RD2 | $\mathrm{Reg}[\mathrm{RS} 1]^{*} \mathrm{Reg}[\mathrm{RS} 2] \rightarrow \mathrm{Reg}[\mathrm{RD} 1]$, Reg[RD2] | MUL | RS1 | RS2 | RD1 | RD2 | NU |
| ADD RS1, RS2, RD | $\mathrm{Reg}[\mathrm{RS} 1]+\mathrm{Reg}[\mathrm{RS} 2] \rightarrow \mathrm{Reg}[\mathrm{RD}]$ |  |  |  |  |  |  |
|  |  | ADD | RS1 | RS2 | RD |  |  |
|  |  | 05 | 10 | 15 |  |  | 31 |
| STORE RS, Imm Value | $\mathrm{Reg}[\mathrm{RS}] \rightarrow$ mem\{Imm Value $\}$ | STORE | RS | NU |  | m Val |  |

The CPU maintains the following rules:
(i) Every instruction is executed in a different number of clock cycles
(ii) No NOP instruction is allowed
(iii) LOAD does not have an ALU cycle but requires two data memory cycles
(iv) INVERT does not have a data memory cycle but requires one ALU cycle
(v) MUL does not have a data memory cycle but requires three ALU cycles
(vi) ADD does not have a data memory cycle but requires two ALU cycles
(vii) STORE does not have an ALU cycle but requires one data memory cycle

Write a program and construct its instruction chart to execute the flow chart above. Show all the necessary forwarding loops and possible data hazards in the instruction chart. Show the cases in which there may be structural hazards and indicate how to prevent them.
10. An eight-bit CPU has a four-way set-associative, write-back data cache organized in Little Endian format (the most significant bit is at bit position 7 and the least significant bit is at bit position 0 ).
The cache has four sets, and each block contains two eight-bit words. In the case of a cache miss the cache controller replaces the least significant, least used block.
The write-back buffer stores four rows of block data in case of a cache miss. When all four rows are filled with data, the cache controller transfers the contents of the entire write-back buffer to the main memory.
CPU issues the following addresses to read and write data:

| Transaction No | CPU address | Data | Write/Read |
| :---: | :---: | :---: | :---: |
| 1 | 10000101 | $?$ | R |
| 2 | 11000100 | $?$ | R |
| 3 | 01000101 | $?$ | R |
| 4 | 01100100 | $?$ | R |
| 5 | 01110010 | $?$ | R |
| 6 | 10100011 | $?$ | R |
| 7 | 11100010 | $?$ | R |
| 8 | 11110011 | $?$ | R |
| 9 | 00100010 | $?$ | R |
| 10 | 00110101 | $?$ | R |
| 11 | 01000101 | $?$ | R |
| 12 | 11110010 | $?$ | R |
| 13 | 00001010 | $0 x C C$ | W |
|  | 00001011 | $0 x D \mathrm{P}$ |  |
| 14 | 00010010 | $0 x E E$ | W |
| 15 | 00010011 | $0 x F F$ |  |
| 16 | 00001100 | $0 x A A$ | W |
|  | 00001101 | $0 \times B B$ |  |
|  | 00011100 | $0 \times 55$ | W |
|  |  | $0 \times 66$ |  |

Main memory contents are as follows prior to the 16 transactions listed above.

|  |  |
| :---: | :---: |
| 00100010 | 0xEE |
| 00100011 | 0xFF |
|  |  |
| 00110100 | 0xAA |
| 00110101 | 0xBB |
|  |  |
| 01000100 | 0x55 |
| 01000101 | 0x66 |
|  |  |
| 01100100 | 0x77 |
| 01100101 | 0x88 |
|  |  |
| 01110010 | 0x12 |
| 01110011 | 0x34 |
|  |  |
| 10000100 | 0x11 |
| 10000101 | 0x22 |
|  |  |
| 10100010 | 0x56 |
| 10100011 | 0x78 |
|  |  |
| 11000100 | 0x33 |
| 11000101 | 0x44 |
|  |  |
| 11100010 | 0xAA |
| 11100011 | 0xBB |
|  |  |
| 11110010 | 0xCC |
| 11110011 | 0xDD |
|  |  |

(a) Draw the block diagrams of the cache and tag memories in Little Endian format, making sure to attach the dirty and valid bits to each tag block. Indicate how many bits are in each memory. Note that in this memory architecture, the top row of the cache contains set 0 , and the bottom row contains set 3 .
(b) Draw the block diagrams of the cache and tag memories at end of the eighth transaction. Show the address, control, data and cache hit/miss entries.
(c) Draw the block diagrams of the cache and tag memories at the end of the twelfth transaction. Show the address, control, data and cache hit/miss entries.
(d) Draw the block diagrams of the cache and tag memories, and the contents of the write-back buffer at the end of sixteenth transaction. Show the address, control, data and cache hit/miss entries.
11. The following instruction set needs to be executed in a 32-bit RISC CPU organized in Little Endian format. The CPU has three pipeline stages where the ALU and write-back stages are combined. The CPU is capable of executing the integer (ADDI, SLI and SRI) and floating-point (ADDF and MULF) instructions. The CPU stores the fixed and floating-point numbers in two separate register files, each containing 32 registers.
In the instruction set below, RS and RD are defined as the source and the destination addresses for the fixed-point registers. Similarly, FS1, FS2 and FD are the source and the destination addresses for the floating-point registers. Show a detailed data-path of this CPU, indicating all internal bus widths and port names. Include only the necessary functional units.


## Projects

1. Implement a 32-bit four-stage RISC CPU that executes only the ADD instruction in Fig. 6.7 using Verilog. Verify the data and the control signals at the output ports of the instruction memory, RF, ALU and write-back stages.
2. Implement the ADD, SUB, AND, NAND, OR, NOR, XOR, XNOR, SL and SR instructions in a 32-bit four-stage RISC CPU shown in Fig. 6.15, and perform a complete system verification using Verilog.
3. Implement a 32-bit five-stage RISC CPU that executes the LOAD, STORE, MOVE and MOVEI instructions using Verilog. Verify the data and the control signals at the output ports of the instruction memory, RF, ALU, data memory and write-back stages.
4. Implement a 32-bit four-stage RISC CPU that executes only the BRA instruction shown in Fig. 6.36 using Verilog. Verify the data and the control signals at the output ports of the instruction memory and RF stages.
5. Implement and verify the 32-bit floating-point adder in Fig. 6.61 using Verilog. Verify the validity of data at the outputs of every major stage using timing diagrams and perform functional verification for the entire adder.
6. Implement and verify the 32 -bit floating-point multiplier in Fig. 6.62 using Verilog. Verify the validity of data at the outputs of every major stage using timing diagrams, and perform functional verification for the entire multiplier. Use behavioral Verilog to mimic the exponent adder and the integer multiplier.

## References

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## System Peripherals

When the host processor executes a user program, it either exchanges data with system memories such as SRAM, SDRAM or Flash, or communicates with system peripherals to perform various tasks.

A conventional computing system may consists of one or more CPU cores, co-processors such as hardware accelerators to perform specialized tasks, a Direct Memory Access (DMA) controller to do routine data transfers from one memory to another, a display adaptor to support user screen, and an interrupt controller. In most cases, data converters to convert external analog signals into digital form or digital signals into analog form, timers to control the length of an event, and transceivers in charge of serially transmitting and receiving peripheral data are interrupt-driven and connected to the interrupt controller. The interrupt controller, on the other hand, manages all event-driven or program-driven tasks through a series of Interrupt Service Routines (ISR) that reside inside the program memory.

### 7.1 Overall System Arcitecture

A basic system architecture containing essential bus masters and slaves is shown in Fig. 7.1. In this figure, the CPU is a bus master that executes user programs. The Direct Memory Access (DMA) is another bus master in charge of transferring data between different system memories. Bus slaves are generally the system memories such as SRAM, SDRAM and Flash memory. However, other system devices that reside on the high speed bus such as the display adaptor or peripheral buffer memories connected to the low speed I/O bus are also considered bus slaves.

The display adaptor is considered an essential high-speed peripheral that displays the results of a running program or application on the screen. Because of its bandwidth, this unit is usually connected to the parallel port of the CPU. However, there are times when the
display adaptor can also be connected to the low speed I/O bus. This choice very much depends on how often the monitor needs to be used when running an application program.

Each type of memory mentioned in Chapter 5 serves a different purpose in a system. SRAM usually holds immediate data generated by the CPU or stores temporary data during a DMA transfer. Larger blocks of data are stored in SDRAM since this memory is many orders of magnitude larger in capacity compared to SRAM. Flash memory usually stores permanent data such as the Built-In Operating System (BIOS).

A bus adaptor translates commands, address and data signals between the parallel bus which operates at a high clock frequency and the low speed I/O bus which operates at a much lower clock frequency.

Sensors, electro-mechanical devices, human interface devices etc. that reside outside the main system commonly use SPI and $\mathrm{I}^{2} \mathrm{C}$ bus protocols, and considered I/O devices. Ultimately, they are all connected to the interrupt controller and memory-mapped due to their capability to store incoming or outgoing data.

The system can also be connected to other systems (or CPUs) with a network adaptor. The simplest connection protocol is Ethernet where many systems are serially connected to the same bus.


Fig. 7.1 A typical system architecture

### 7.2 Direct Memory Access Controller

The CPU assigns routine memory-to-memory data transfer operations to the Direct Memory Access (DMA) controller. Most of these transfers take place between two system memories or between the buffer memory of a peripheral device and a system memory. This section
shows how to design the basic architecture of a DMA controller that transfers data from a source to a destination memory.

The DMA interface in Fig. 7.2 shows the I/O port description of a typical DMA controller. In this figure, the DMA controller interacts with the CPU through handshake signals, ReqM and AckM. When the CPU initiates a DMA data transfer, it issues a request, ReqM, to the DMA controller. If the controller is not busy with another transfer, it then generates a request, ReqD, to the bus arbiter to use the bus. When the arbiter acknowledges the request by AckD, then the controller informs the CPU that it is ready to initiate the transfer by AckM, and at the same time it sends out its first address and control signals to the source memory. While the source memory is delivering data, the DMA controller issues the address and control signals for the destination memory within the same clock cycle. In order to accomplish this task, a direct data channel must exist between the source and the destination memories. This new configuration modifies the original bus structure in Fig. 5.1 in which all bus masters are assumed to have individual write data ports to be able to write data directly to a slave.


Fig. 7.2 The block diagram including a DMA, source and destination memories

Figure 7.3 shows a typical data transfer between the source and the destination memory until the last data packet, D4, is transmitted. The sequence starts with the DMA controller issuing Status $=$ START and AddrS $=$ AS1, indicating the beginning of the data transfer and the first source memory address, respectively. Since both memories are ready (ReadyS = ReadyD $=1$ ) after the first clock cycle, the controller continues the data transfer by issuing Status $=$ CONT, AddrS $=$ AS2 (the second source memory address), and AddrD $=$ AD1 (the first destination memory address) in the second cycle. In this cycle, the source memory also delivers the first data, D1, to the destination memory. The same process takes place in the third cycle, during which the DMA controller generates AddrS $=\mathrm{AS} 3, \mathrm{AddrD}=\mathrm{AD} 2$, and writes D2 to the destination memory. In the fifth clock cycle, as the DMA controller generates the last destination memory address, $\mathrm{AddrD}=\mathrm{AD} 4$, and writes the last data, D 4 , to AD4, it also changes its status to IDLE, indicating the end of the data transfer.


Fig. 7.3 Timing diagram showing a DMA-assisted data transfer

Any time one of the Ready signals from the source or the destination memories transitions to logic 0 , the DMA controller stalls the data transfer by repeating the address and the control signals as long as ReadyS or ReadyD is at logic 0 . Figure 7.4 shows a typical data transfer in which the destination memory is busy in the third cycle, and prompts the DMA controller to repeat AddrS $=\mathrm{AS} 3$ and $\mathrm{AddrD}=\mathrm{AD} 2$ in the next cycle. The DMA controller stalls the bus again in the sixth cycle when it detects the source memory to be busy in the fifth cycle.


Fig. 7.4 DMA-assisted data transfer with varying Ready signals from memories
A basic DMA controller is shown in Fig. 7.5. There are three modules in this architecture. The first module is the DMA register file that stores the initial and incremental source address values, InitAddS, StepAddrS. The same register file contains four other registers. The InitAddrD and the StepAddrD registers store the initial and the incremental destination addresses. The Size and the Burst registers store the data width and the burst length, respectively. A program data bus is used to store all six register entries before regular operations take place.

The second section of the DMA controller manages the handshake between the CPU and the bus arbiter. This section also provides the internal signals to the DMA data-path to guide the data.

In the third section, the DMA data-path produces source and destination addresses, AddrS and AddrD, and the bus master control signals, Status, Size, Burst, WED (write-enable for the destination memory) and RES (the read-enable for the source memory).

To be able to implement this architecture, three elements need to be examined in the design phase simultaneously: a timing diagram describing an entire data transfer process including the stall periods, a data-path that fully complies with the timing diagram, and a control logic that manages the timely flow of data on the data-path.

As pointed out in previous chapters, the design always starts with forming a timing diagram that describes the complete data-flow in a logic block. The timing diagram generally includes a single clock (multiple clock domains or asynchronous event signals are also common but not relevant for a basic DMA design), address, data and control signals with respect to this clock. In order to generate an accurate timing diagram, a data-path must be concurrently developed. The data-path generally consists of registers and logic gates. However, it can also contain mega cells such as complex arithmetic units or memories. As the design develops and more details are added to the original data-path, the corresponding


Fig. 7.5 A typical DMA architecture
timing diagram becomes more complex to accommodate the changes in the hardware. The design of the controller to govern the data flow is the last step in the design process. This step does not start until every internal and external signal is determined, and the complete block functionality, including all corner cases of the data-flow, is explicitly depicted on the timing diagram.

A detailed timing diagram describing a typical DMA transfer is shown in Fig. 7.6. As we mentioned earlier in the memory-to-memory example in Chapter 2, this diagram is also developed in two phases. In the first phase, the main DMA signals, namely the handshake signals with the CPU and the arbiter (ReqM, AckM, ReqD and AckD), the source and the destination memory addresses (AddrS and AddrD), the data (Data), the bus master control signals (Status, Burst, Size, WriteD and ReadS), and the slave response signals (ReadyS and ReadyD) are included in the timing diagram. In the second phase, all internal control signals that support the address and data movement in the timing diagram are brought into the picture. This section also includes the control signals for an internal down-counter to keep track of the number of data packets transferred from one memory to the next.


Fig. 7.6 Detailed timing diagram of a DMA transfer
In Fig. 7.6, the CPU initiates a DMA-assisted data transfer by issuing a request to the DMA controller, ReqM, in clock cycle 1. This request enables the DMA controller to generate a subsequent request, ReqD , to the arbiter in order to use the system bus in cycle 2 . An acknowledgment from the arbiter, AckD, may come in the third cycle or many cycles later depending on the bus traffic and the other pending requests from higher priority bus masters. However, as soon as the DMA controller receives the acknowledgement from the arbiter, it notifies the CPU by issuing an acknowledgement signal, AckM, in cycle 4. This cycle also prepares the DMA for an upcoming data transfer by setting the SetAddrS and SetSTART signals to logic 1. That way, the first source memory address, AS1, can be fetched from the InitAddrS register and delivered to the AddrS port in Fig. 7.7, and similarly


Fig. 7.7 Internal DMA architecture showing its data-path and controller
the START code can be produced at the Status port in cycle 5. The port selection guide for each 3-1 MUX in Fig. 7.7 is shown in Table 7.1. In cycle 5, the first data read command from AS1 is issued by ReadS $=1$. This cycle also sets the control signals, SetAddrD, IncrAddrS and SetCONT, to logic 1, so that the first destination memory address can be retrieved from the InitAddrD register and transferred to the destination address port, AddrD, the second source memory address, (InitAddrS + StepAddrS), can be formed at the AddrS output, and the status code can be changed from START to CONT in the next clock cycle. Still in cycle 5, the down-counter, responsible for counting the number of data packets to be delivered to the destination memory, is set with an initial value from the burst register by SetCount $=1$. Therefore, when clock cycle 6 starts, the second source memory address, AS2, is formed at the AddrS port along with the first destination address, AD1, at the AddrD port. The Status output indicates CONT code specifying the ongoing data transfer. In this cycle, the first data, D1, is transferred from the AS1 address to the AD1 address with an active-high WriteD signal and subsequently written in the destination memory. The CountOut output also shows the initial value from the burst register, defining the number of data packets to be written to the destination memory. This cycle sets the control signals, IncrAddrS, IncrAddrD and DecCount, to logic 1 in order to prepare the next source and destination addresses, and to decrement the CountOut by one in the next cycle.

Table 7.1 3-1 MUX port assignments in Fig. 7.7

| INPUT | PORT | INPUT | PORT | INPUT | PORT |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SetAddrS = 1 | set | SetAddrD = 1 | set | SetCount = 1 | set |
| $\text { IAS }\left[\begin{array}{l} \text { ReadyS = 1 } \\ \text { ReadyD = 1 } \\ \text { IncrAddrS = } \end{array}\right.$ | incr | $\operatorname{IAD}\left[\begin{array}{l} \text { ReadyS = } \\ \text { ReadyD }=1 \\ \text { IncrAddrD }=1 \end{array}\right.$ | incr | DC $\left[\begin{array}{l}\text { ReadyS }=1 \\ \text { Ready }=1 \\ \text { DecCount }=1\end{array}\right.$ | decr |
| ELSE | stall | ELSE | stall | ELSE | stall |

The routine data transfer continues until either the source or the destination Ready signal transitions to logic 0 . When this happens, the entire data transfer stalls, disengaging an ongoing write process at the destination memory. In a stall, the previous source and destination address values repeat themselves at the AddrS and AddrD ports; the down-counter stops, displaying the remaining number of data packets to be written to the destination memory.

The data transfer resumes when the ReadyS and ReadyD signals become logic 1. CountOut $=2$ defines the end of the data transfer. In this cycle, the IncrAddrS signal also transitions to logic 0 , indicating that there will be no more new source address generation at the AddS port. Similarly, the SetIDLE signal goes to logic 1, changing the bus master status code from CONT to IDLE in the next cycle. When CountOut $=1$ in the following cycle, the last data is written to the destination address. From this point forward, the DMA handshakes with the CPU and the arbiter to terminate the data transfer. ReqD $=0$ forces the arbiter to lower the acknowledge signal, AckD. AckM $=0$ prompts the CPU to lower its DMA request signal, ReqM.

The controller design is a direct outcome of the timing diagram in Fig. 7.6. The first step in the controller design is to assign a name to each clock cycle in the timing diagram that produces a different set of outputs from the previous clock cycle. In other words, each clock cycle that produces a different set of outputs has to be labeled with a new state in the controller state diagram.

Cycle 1 is named as the IDLE state, producing ReqD $=0$ and $\mathrm{AckM}=0$ in Fig. 7.8. When ReqM $=1$ is received in cycle 1 , ReqD switches from logic 0 to logic 1 in cycle 2 ,


Fig. 7.8 DMA controller state diagram
producing a new state, REQD. ReqD $=1$, on the other hand, prompts $\mathrm{AckD}=1$ in cycle 3 which creates the ACKD state in this cycle. Cycle 4 also creates a new state, SET SRC, because a different set of control signals $(\operatorname{AckM}=1$, SetAddrS $=1$ and SetSTART $=1$ ) emerges in this cycle. The next cycle generates a new set of outputs (ReadS = SetAddrD $=$ IncrAddrS $=$ SetCONT $=$ SetCount $=1$ ) compared to the previous clock cycles, and therefore it is labeled as the SET DEST state. Between cycles 6 and 13, the controller outputs remain the same. Therefore, all these cycles can be grouped together under the same state name, INCR. In cycle 14, the IncrAddrS signal transitions to logic 0 and the SetIDLE signal transitions to logic 1 . This new set causes the creation of a new state, END SRC. Cycle 15 changes the controller output values once more with respect to the previous cycle, and it is labeled as the END DEST state. In cycle 16, the down-counter output finally reaches zero, and all the controller outputs except the AckM and ReqD signals become logic 0 . Therefore, this cycle is named as the END COUNT state. The controller lowers the AckM and ReqD signals to logic 0, which creates a new state, END REQD in cycle 17 . In the next cycle, $\mathrm{AckM}=\mathrm{ReqD}=0$ prompts the CPU and the arbiter to lower ReqM and AckD signals to logic 0, respectively, and the controller transitions to the IDLE state.

### 7.3 Interrupt Controller

There are numerous events that may interrupt the normal flow of program execution. External events are created by I/O devices that need specific utility programs because they may have data ready for the CPU or require data from the CPU. There are also internal events within the CPU that result from errors encountered when executing user programs, such as divide-by-zero or overflow conditions, which create exceptions.

There are four types of interrupts according to their priority. The interrupt for resetting the CPU takes the precedence over all other interrupts because when reset occurs, the data in each CPU register needs to be preserved in a special memory in order to be restored later on. Internal interrupts take the second priority after the CPU reset. These interrupts generally originate from errors encountered in user programs or may result from breakpoints installed in a user program. Software interrupts take the third place in the priority list. These interrupts are actually vectored subroutine calls that stem from software emulation routines. Floating-point division produces one such example. Hardware interrupts are placed last in the priority list. Even though prioritizing hardware interrupts is completely user-programmable, the operating system may also manage the hardware priority list and communicate with a specific device though device drivers.

In this chapter, we will examine the sequence of events that take place to handle a hardware interrupt, and design a simple interrupt controller interface that serves up to 256 external I/O devices.

The interrupt process begins when one or more I/O devices submit interrupt requests to the interrupt interface in Fig. 7.9. In this figure, the interrupt interface is designed to handle up to 256 interrupt inputs, INTR0 through INTR255. Interrupt controller is a programmable state machine that prioritizes all pending interrupts, selects a highest priority device according to a priority list, and communicates with the CPU using the INTR output as shown in Fig. 7.9. When the acknowledgement signal, INTA, is received from the CPU, the interrupt interface transmits the device ID, INTRID, causing the interrupt on an eight-bit wide bus.


Fig. 7.9 Interrupt interface input/output description
The interrupt ID in Fig. 7.9 matches the interrupt number at the input of the interface, and ranges between 0 and 255 . Each interrupt ID correlates to a specific address in the Interrupt Address Table, IAT, in Fig. 7.10. Each address stored in IAT points the starting address of a particular Interrupt Service Routine (ISR) residing in the instruction memory. Therefore, when the interrupt interface generates an INTRID and accesses a specific memory address in the IAT, the contents at this address is immediately loaded to the Program Counter (PC). This prompts the CPU to pause executing the normal user program, and jump to the starting ISR address in the instruction memory to execute the corresponding ISR instructions. This basically translates to jumping from Instr3 of the user program to Intr1 of the ISR in the example in Fig. 7.10. While in the ISR, a return instruction, IntrRET, indicates the end of interrupt service routine. At this point, the program returns to ARET to execute the rest of the user program.

To convert the block diagram in Fig. 7.10 into a detailed data-path, each step of the interrupt service routine outlined above should be translated into a timing diagram. Creating a timing diagram, on the other hand, is usually accomplished in two steps. The first step defines all primary, bus-level signals such as data and address and includes them in the timing diagram. The second step generates the necessary control signals to manage the data-flow.

To achieve the first step, let us consider the signals, INTRx (INTR0 to INTR255), INTR, INTA, INTRID, DOutIAT and PCOut, in Fig. 7.10 and in the timing diagram in Fig. 7.11. The signals, INTRx, INTR and INTA, are not bus-level signals; but, they are considered as


Fig. 7.10 A block diagram describing an interrupt
the primary I/O signals that indicate the start of an interrupt. Therefore, they will be grouped together with the other bus-level signals to show the complete interrupt sequence. In Fig. 7.11, an I/O device issues an interrupt, $\operatorname{INTRx}=1$, to the interrupt interface in clock cycle 1. In response, the interrupt interface generates INTR $=1$ for the Interrupt Control Unit (ICU) in charge of the handshake signals in cycle 2 . In cycle 3 , the ICU generates INTA $=1$, and prompts the interrupt interface to transmit an eight-bit INTRID to the IAT in the following cycle. As mentioned earlier, the INTRID signal is also an address for the IAT. The data stored at this address is actually the starting address of an ISR in the instruction memory. Therefore, the interrupt controller simply reads the memory contents at the address, INTRID, from the DOutIAT port and waits for the interrupt service routine to begin in cycle 5 . In the

Fig. 7.11 Detailed timing diagram of an interrupt sequence
mean time, at the beginning of cycle 3 when the ICU is aware of a pending interrupt by means of the INTR signal, it immediately stalls the CPU pipeline by stopping the PC from incrementing. However, the PC has already incremented to A3 at this point, and there are uncompleted instructions from the A1 and A2 addresses in the CPU pipeline. We know that from the onset of PC address generation to the end of the write-back cycle, a normal instruction takes four clock cycles to complete (latency $=4$ ) according to the simplified CPU data-path in Fig. 7.12. Therefore, the PC output stays at the A3 address until the end of cycle 6 when the instructions, Instr1, Instr2 and Instr3, are completely flushed out of the CPU pipeline and written back to the CPU's register file (RF). However, the interaction with the CPU pipeline adds the RF block and all its corresponding data paths to the existing block diagram in Fig. 7.10, transforming the interrupt controller into a more detailed architecture in Fig. 7.13.


Fig. 7.12 A four-stage CPU employed in the interrupt sequence in Fig. 7.7

After the last write-back is completed in cycle 6, the PC is incremented by one, and the return address, ARET, is stored in a special register, R31, in the RF as shown in cycle 7. This step assumes that there is no Jump-and-Link (JAL) or Jump-and-Link-Register (JALR) instruction in the CPU instruction set because the return address will simply be overwritten by one of these instructions. Before the CPU starts executing the interrupt service routine, it copies the contents of the entire register file into a temporary memory identical to the register file. This step is called "context switching", and it is omitted from the timing diagram in

Fig. 7.13 Interrupt interface data-path producing the timing diagram in Fig. 7.11

Fig. 7.11 to maintain simplicity. Only after the contents of all 32 registers in the register file are stored, the interrupt controller starts executing the instructions in the interrupt service subroutine, which starts in cycle 8 . In this cycle, the first interrupt instruction address, I1, at the output of the IAT is loaded to the PC. Once loaded to the CPU data-path, it takes four clock cycles to execute the first interrupt instruction due to the CPU's write-back latency. The remaining interrupt instructions are similarly fetched from the instruction memory addresses, I 2 to IN , executed, and written back to the RF until the end of cycle $(\mathrm{N}+3)$. In cycle $(N+4)$, the register R31 is accessed. In the following cycle, the return address, ARET, is fetched from R31. This cycle also completes the interrupt service routine and prompts the ICU to transfer the control over to the CPU to execute the remaining user instructions. In cycle $(\mathrm{N}+6)$, the ICU lowers the INTA signal to logic 0 , and the interrupt controller loads the value of the ARET to the PC. In response to the INTA, the interrupt interface also lowers the INTR signal to logic 0 and invalidates the INTRID in cycle $(\mathrm{N}+7$ ). New interrupt arbitration will take place in cycle $(\mathrm{N}+8)$ to service the next interrupt.

In the previous paragraphs, we only explained how the address and data bus values changed once an interrupt signal is received from the interrupt interface. Now, we are ready to explain the second part of the timing diagram that includes the control signals to manage the data-flow. After examining the detailed data-path in Fig. 7.13, the control signals can be grouped into three categories. The first group supports the PC input control and contains the StallPC, SelIAT and SelRF signals. The StallPC signal simply routes the output of the PC, PCOut, to its input through the P-port of the 4-1 MUX to stall the PC. The SelIAT signal routes the output of the IAT, DOutIAT, to the input of the PC through the I-port of the 4-1 MUX to load an interrupt address. The SelRF signal enables the R-port, and connects the output of the RF, DOutRF, to the input of the PC to load the return address, ARET, once the interrupt service is over. If none of these control signals are generated, then the PC increments through the C-port. The second group controls the address and data inputs to the RF and consists of the SelR31 and SelPC inputs. The SelR31 input selects the register R31 to be the address for the RF at the AddrRF port. The SelPC input selects the contents of the PC to be the data for the RF at the DInRF port. The third group controls the read and the write enable signals, REIAT and WEIAT, for the IAT, respectively. Although writing into the IAT does not take place during a routine interrupt service, it will be used to reprogram a new set of ISR addresses in the IAT.

All three groups of controls manage the proper data flow in Fig. 7.13. The StallPC signal transitions to logic 1 at the beginning of cycle 3 and stays there until cycle 6 to stop the PC from incrementing so that the CPU completes writing the instructions, Instr1, Instr2 and Instr3, back to the RF. Because of these write-backs, the write-enable signal for the RF, WERF, is also kept at logic 1 from cycles 4 to 6 . The read-enable signal for the IAT, REIAT, is kept high in cycle 4 because the first interrupt instruction address, I1, needs to be fetched from the IAT following a valid INTRID. Cycle 7 is a special cycle to load the register R31 with the program return address, ARET. Therefore, the signals, SelR31, SelPC and WERF,
all become logic 1 during this cycle. The SelIAT signal is also kept at logic 1 during cycle 7 in order to load the first interrupt address, I1, to the PC in cycle 8 . The WERF signal is kept at logic 1 from cycle 11 to cycle $(\mathrm{N}+3)$ to be able to complete all interrupt-related write-backs to the RF. The StallPC signal is kept at logic 1 from cycle $(N+1)$ to cycle $(\mathrm{N}+4)$ to stall the value of PCOut at IRET. Cycle $(\mathrm{N}+5)$ is dedicated to retrieving the program return address, ARET, from the RF. Therefore, the SelRF signal is kept at logic 1 in this cycle to load the PC with the contents of ARET in the following cycle. The WERF signal transitions to logic 1 in cycle $(\mathrm{N}+9)$ in order to write the result of the instruction, InstrRET, back to the RF.

Figure 7.14 shows the resultant state diagram for the interrupt controller. Its design is solely based on the values of the control signals from the timing diagram in Fig. 7.11. The name of each state in the state machine comes from the labels on top of the timing diagram in


Fig. 7.14 Interrupt controller state diagram

Fig. 7.11. The machine starts with the IDLE state where there is no INTR signal. Therefore, this state generates INTA $=0$. When a valid INTR is received, the state machine transitions to the INTA state and produces two outputs, INTA $=1$ and $\operatorname{StallPC}=1$. This state corresponds to cycle 3 of the timing diagram. As INTR $=1$ continues, the machine goes though the INTRID, INTR ADDR, LAST WB and STORE ARET states, which correspond to cycles $4,5,6$ and 7 , respectively. These are the preparation states prior to an ISR. The FETCH I1 state indicates the first interrupt instruction fetch, which corresponds to cycle 8. The interrupt controller goes though the FETCH I2 and FETCH I3 states where it fetches the second and third interrupt instructions. These are indicated in cycles 9 and 10, respectively. A cycle later the machine enters the INTR WB state where it starts writing the results of interrupt instructions back to the RF. The interrupt controller stays at this state until the interrupt address reaches its last value, IN. When the last interrupt address is fetched, the machine transitions to the INTR END state where it performs three additional interrupt write-backs, and stalls the PC at IRET until the last interrupt write-back, WBIN, completes. This state continues during cycles $(\mathrm{N}+1)$, $(\mathrm{N}+2)$ and $(\mathrm{N}+3)$ in the timing diagram. Following the last interrupt write-back, the interrupt controller prepares the system to finish the current interrupt service before receiving another interrupt. The closing states are the ACCESS R31, LOAD ARET and FETCH ARET states, which correspond to cycles $(\mathrm{N}+4),(\mathrm{N}+5)$ and $(\mathrm{N}+6)$ in the timing diagram, respectively. The interrupt controller goes back to the IDLE state in cycle $(\mathrm{N}+7)$ where the INTRID becomes no longer valid.

However, a crucial problem arises when implementing this state machine. The interrupt controller needs to know the end of an ISR. Somehow the number of instructions in the interrupt service routine must be determined in advance in order to continue the state transitions in the state machine. The states, INTR WB and INTR END, are the examples of this problem. The interrupt controller needs to stay in the INTR WB state from the first to the ( $\mathrm{N}-4$ )th interrupt write-backs, and similarly in the INTR END state from the $(\mathrm{N}-3)$ th to the Nth interrupt write-backs during an ISR. Since the number of instructions varies in an ISR program, this state machine's implementation becomes impossible for the ICU design, necessitating a change in the original timing diagram, which will affect the data-path in Fig. 7.13 and the state diagram in Fig. 7.14.

Figure 7.15 shows a slightly modified version of the interrupt controller data-path to circumvent this problem. In this figure, a decoder is added between the output of the instruction register, IROut, and the interrupt controller in order to detect the return opcode in the IntrRET instruction at the last interrupt address, IRET. This, however, creates an additional input, DetIRET, for the ICU. Therefore, when the return opcode is decoded in cycle $(\mathrm{N}+2)$ in the new timing diagram in Fig. 7.16, the DetIRET signal becomes logic 1 and prompts the ICU to make preparations to end the current ISR. In this figure, the StallPC signal is also lowered to logic 0 between cycles $(\mathrm{N}+1)$ and $(\mathrm{N}+4)$ because not stalling the PC during this interval will simply generate invalid addresses at the PCOut port, and this is

Fig. 7.15 Modified interrupt interface data-path

Fig. 7.16 Timing diagram of the modified interrupt interface in Fig. 7.15
not critical for the operation of the interrupt controller since the program return address, ARET, becomes a valid address at PCOut in cycle ( $\mathrm{N}+6$ ).

This modification leads to a number of changes in the ICU's state diagram shown in Fig. 7.17. After transitioning to the INTR WB state in cycle 11, the state machine stays in this state until it detects DetIRET $=1$. This input forces the ICU to move the WBIN state to complete the last interrupt write-back. At this point, the machine goes through three more states, ACCESS R31, LOAD ARET and FETCH ARET, to load the user program return address back to the PC in order to resume the original program.


Fig. 7.17 Modified interrupt controller state diagram

### 7.4 Serial Transmitter and Receiver Interface

There are times when the CPU needs to use its serial $\mathrm{I}^{2} \mathrm{C}$ or SPI interface in order to communicate with an I/O device. The serial interface consists of a transmitter to send serial data to an I/O device, and a receiver to receive serial data from the same device on a one-bit bus. The following section describes the basic structure of a transceiver composed of a transmitter and a receiver to handle one-bit serial data.

## Transmitter

Figure 7.18 shows the data-path of a transmitter where an incoming 32-bit data from the CPU is received at the TXIn[31:0] port, and stored in one of the two buffers before being serially sent out from the TXOut port. Each buffer is essentially a shift register. Once a 32-bit data packet is loaded into a shift register, the bits start shifting from the least significant bit position to the most significant bit position until all 32 bits are sent out.


Fig. 7.18 Transmitter data-path

This design uses a dual buffer scheme to overcome moderate waiting periods to access system's main memory. If the waiting period takes too long, the transmitter architecture may require more than two buffers to sustain continuous stream of serial data from TXOut. A single buffer may also be sufficient for the transmitter provided that there should not be any waiting period to access the main memory. Once the main memory is accessed, storing a 32-bit data in one of the transmitter buffers takes only a cycle. Streaming all 32 bits from a particular buffer, on the other hand, takes 32 consecutive clock cycles. The clock used to send serial data may also be a slower clock depending on the design constraints. Therefore, the transmitter uses all 32 clock periods to request, wait and receive data to its secondary buffer while it streams bits out of the first buffer. When the first buffer becomes empty, the transmitter immediately starts streaming data out of its secondary buffer while the first buffer is being filled.

PassTxBuf input in Fig. 7.18 is a control signal for the 2-1 MUX that determines when to switch buffer outputs. LoadTxBuf0 and LoadTXBuf1 inputs load data from TXIn[31:0] to the first and the second buffers, respectively. ShiftTXBuf0 and ShiftTXBuf1 inputs control the beginning and the end of the serial data shift from buffer 0 and buffer1, respectively. The CSTX is a Chip-Select input port for the transmitter, and it stays at logic 1 as long as the system uses the transmitter.

The timing diagram in Fig. 7.19 shows how data is stored and streamed out of the data buffers. It is constructed while the transmitter data-path in Fig. 7.18 is being developed. Once again, the top part of this diagram shows the bus-level data signals that describe the data-flow while the bottom part contains the control signals that govern this data-flow.

The transmitter wakes up when it receives an active-high CSTX signal from the CPU in cycle 1 . In cycles 2 and 3, the 32-bit data packets, Buf0 and Buf1, fill the first and second transmitter buffers, TXBuf0 and TXBuf1, respectively. Once the first buffer is full in cycle 2, single bits start coming out of the least significant bit position of the buffer (the shifting mechanism in the buffer can also be configured such that single bits start emerging from the most significant bit position instead) in cycle 3. The first bit that comes out of TXBuf0 in cycle 3 is Bit0 which is the least significant bit of the data packet. This is followed by Bit1 through Bit31 between cycles 4 to 34, respectively. When the first buffer becomes empty, the transmitter immediately switches to its second buffer and starts streaming bits from TXBuf1. In the mean time, the transmitter fills TXBuf0 with a new 32-bit of data in cycle 35 as long as there is no bus traffic. Emptying the second buffer takes until cycle 66 when Bit31 is sent out from the TXOut terminal. When TXBuf1 is empty, the transmitter starts streaming out data from TXBuf0 in cycle 67 while filling TXBuf1. The process of filling one buffer while streaming bits out of the second continues as long as the data stored in the main memory is fully exhausted. Figure 7.19 shows all 32-bit data packets from the main memory immediately available when the transmitter switches from its empty buffer to its full buffer, and ignores any delay associated with accessing the main memory. In reality, when the transmitter starts streaming data out of the full buffer, it immediately generates an interrupt for its

Fig. 7.19 Transmitter timing diagram
empty buffer to fetch data from the main memory. The waiting period is 31 clock cycles. In the 32 nd clock cycle, the empty buffer must be full. Otherwise, the transmitter stalls, and no new data can be transmitted.

The control signals that govern the data-flow in Fig. 7.18 constitute the second part of the timing diagram. Once the active-high CSTX signal is received, the LoadTXBuf0 signal transitions to logic 1 to load the first 32-bit data packet, Buf0, to TXBuf0 in cycle 2. In cycle 3, the second 32-bit data packet, Buf1, is loaded to TXBuf1, which requires the LoadTXBuf1 signal to be at logic 1. From cycle 3 until cycle 33, TXBuf0 works as a shift register to stream bits 0 to 31 . Therefore, the ShiftTXBuf0 signal stays at logic 1 during this period. In cycle 34, the last bit is shifted out of TXBuf0, and therefore, all data-flow controls for this buffer transition to logic 0 . In cycle 35, the LoadTXBuf0 signal transitions to logic 1 to fill the empty buffer, TXBuf0. The ShiftTXBuf1 signal also transitions to logic 1 in the same cycle to shift bits from 0 to 31 until cycle 66 . Cycle 35 is also the time to switch the buffer outputs. Therefore, the PassTXBuf signal becomes logic 1 in this cycle until cycle 67 when all the data in TXBuf1 is streamed out. Cycle 66 is the cycle to deliver the last bit out of TXBuf1. Cycles 67 through 98 are exact replicas of cycles 3 through 34 where TXBuf1 is filled while bits are shifted out of TXBuf0. The ValidTXData signal validates bits as they are streamed out of the transmitter. Therefore, this signal stays at logic 1 from cycle 3 until the last transmitter bit.

Figure 7.20 shows the state diagram of the controller unit that loads and shifts data in each buffer, to switch buffer outputs, and to validate bits out of the transmitter. The state names in this figure follow the names depicted at the top of the timing diagram in Fig. 7.19. When there is no activity in CSTX signal, the state machine stays in the IDLE state. When CSTX $=1$, the controller transitions to the PRELOAD BUF0 state where it produces LoadTXBuf0 $=1$ to load TXBuf0, and LoadTXCount $=1$ to load the TXcounter in Fig. 7.18 with the value of 31 . This state is associated with cycle 2 in the timing diagram. Note that the TXCounter is a five-bit counter which is used to detect the end of the serial data stream, and it is essential for the controller to be able to make a transition to the next state. As long as CSTX $=1$, the state machine transitions to the LOAD BUF1 state where it produces LoadTXBuf1 $=$ ShiftTXBuf0 $=$ ValidTXData $=1$. This state corresponds to cycle 3 in the timing diagram. This state is also the beginning of the count-down stage where the TXCounter output starts decrementing from 31 towards 0 by DecTXCount $=1$. Next, the state machine goes to the EMPTY BUF0 state, and stays there as long as the output of the TXCounter, TXCount, is greater than one. This state corresponds to cycles 4 through 33 in the timing diagram. When TXCount $=1$, the controller goes to the EMPTY Last Bit BUF0 state where Bit31 is shifted out of TXBuf0. This state is equivalent to cycle 34 in the timing diagram when the TXCounter is reloaded with the value of 31 to start another count-down. As long as CSTX $=1$, the state machine first transitions to the LOAD BUF0 state in cycle 35, and then to the EMPTY BUF1 state as the TXCounter decrements towards 1 from cycles 36 to 65 . When TXCount $=1$, the state machine transitions to the EMPTY Last Bit BUF1


Fig. 7.20 Transmitter controller state diagram
state in cycle 66 when the last bit of TXBuf1 is shifted out. From cycle 67 onwards, the state machine goes back to the LOAD BUF1 state, and traces through the previous five states since the control outputs generated in each state are identical to the ones in the timing diagram in each clock cycle. The state diagram in Fig. 7.20 does not show the transitions from an arbitrary state to the IDLE state when CSTX $=0$ to improve readability. The case when the transmitter exhausts all of its valid data from both of its buffers and forced to stall is not shown in Fig. 7.20 either. If heavy bus traffic is expected, the reader should employ an additional STALL state in case new data is not yet loaded to TXBuf0 or TXBuf1 before the state machine transitions to the LOAD BUF0 or LOAD BUF1 states, respectively.

## Receiver

Figure 7.21 shows the data-path of a receiver where incoming data bits are serially received by the RX buffer, RXBuf, at the RXIn port before being packed as 32-bit data packets and


Fig. 7.21 Receiver data-path
sent to the CPU from the RXOut[31:0] port. This architecture can also be accomplished with multiple buffers in case the receive clock frequency becomes much higher than the processor clock frequency.

Figure 7.22 summarizes the operation of the receiver in a timing diagram. Once the receiver is activated by CSRX $=1$ in cycle 1 , any incoming data bit is ignored until the ValidRXDIn signal transitions to logic 1 . In other words, this external signal validates the data bit at the RXIn port, and indicates when to start latching data bits into the RX buffer. As a result, Bit0 is stored in RXBuf in cycle 2 and Bit31 in cycle 33. In cycle 34, a new Bit0 is fetched for the receive buffer. This cycle is also the time period to pack all 32 bits, and send them out of the RXOut[31:0] port. The 32-bit data is accompanied by the ValidRXDOut signal for validation. In order to determine which clock cycle the ValidRXDOut signal transitions to logic 1, a five-bit counter is used. This counter starts decrementing as soon as the DecRXCount or the LatchRXData signal goes to logic 1 . When the counter reaches 0 , the RX Controller produces ValidRXDOut $=1$ in the following cycle to validate the 32-bit word at the RXOut[31:0] port. From cycles 34 to 67 , the receiver keeps latching new valid bits into RXBuf. As an example, there may be a period where the serial bit stream may not be valid (ValidRXDIn $=0$ ) such as in cycles 38 and 39 . During this period, both the latching action at the RXIn port and the count-down mechanism at the RX Counter should stop immediately by LatchRXData $=0$ and $\operatorname{DecRXCount}=0$, respectively. The normal receiver operation resumes as soon as the ValidRXDIn signal transitions to logic 1 in cycle 40.


The RX controller is a simple state machine with three states as shown in Fig. 7.23. The IDLE state is the state when CSRX $=0$ or ValidRXDIn $=0$. Even when the CSRX signal goes to logic 1, the controller stays in this state as long as ValidRXDIn $=0$. This translates to cycles 1 and 2 in Fig. 7.22. When the CSRX and ValidRXDIn signals both go to logic 1, the controller moves to the COUNT DOWN state to fill RXBuf. The controller stays in this state until the RXCount signal reaches 0 . This state covers the cycles from 3 to 33 in the timing diagram. In the next cycle, the state machine goes to the VALID DOUT state where it stays for only one clock cycle and produces ValidRXDOut $=1$ for the 32-bit data at the RXOut[31:0] port. Following the VALID DOUT state, the controller goes back to the COUNT DOWN state where it starts filling the receive buffer again. As long as valid bits arrive at the RXIn port, the state machine rotates between the COUNT DOWN and the VALID DOUT states in Fig. 7.23. When CSRX $=0$ or ValidRXDIn $=0$, the state machine transitions either from the COUNT DOWN state or from the VALID DOUT state to the IDLE state. These transitions are omitted in Fig. 7.23 to maintain simplicity.


Fig. 7.23 Receiver controller state diagram

### 7.5 Timers

Every digital system contains programmable timers to handle a multitude of tasks. If an external event needs to be monitored, it calls for a timer. Periodic internal system tasks are also managed by timers. Timers can also be used to generate square waveforms or pulses
with adjustable pulse widths such as Pulse Width Modulation (PWM) signals to control output devices or perform periodic tasks.

A basic system timer is shown in Fig. 7.24 [1]. This timer essentially consists of a counter, a compare unit and two registers. The first register stores the entire timer period after which the counter receives an automatic reset, and the other divides the clock frequency of the counter. The period and the divide by N registers are fully programmable. The compare unit is simply a subtractor which subtracts the counter output from the period value. As the counter starts incrementing from zero and ultimately reaches the value stored in the period register, the output of the subtactor and its sign bit become all zero. These bits are ultimately decoded by the compare unit to produce logic 1 at the timer output and reset the counter as shown in this figure.



Fig. 7.24 Simplified timer block diagram

The basic philosophy used in a timer block diagram can be modified into different forms. However, all modifications still end up with a counter, a register set and a comparator for the intended functionality. The comparison can be achieved by a digital block that may produce one or multiple timer outputs, or by a subtractor/decoder scheme as in Fig. 7.24.

The following section presents many different forms of timers, each of which can still be modified and converted into various other forms that can produce additional features and functionality. The basic timers in this section are configured to produce a one-time pulse (one-shot timer), a periodic waveform with adjustable duty cycle (rate generator), a square waveform with fully programmable period (square wave generator) and a step function with adjustable delay (interrupt generator). There are subtle differences and incremental enhancements from one timer circuit to another, but in the end each timer uses a counter, a register and a comparator as pointed out before.

## One-Shot Timer

The one-shot timer, as its name suggests, generates a single, non-repetitive pulse whose pulse width is programmed by the user. Figure 7.25 shows the micro-architecture of a typical one-shot timer.

Before the operation starts, the pulse width for the one-shot timer is stored in the OneShot register via a program bus. Once programmed, the data in this register is routed through the L-port of the 3-1 MUX by LoadOneShot $=1$. This provides an initial value for the down-counter. As a numerical example, assume that the OneShot register is programmed with a value of four, which produces OneShotOut $=4$ during cycle 1 in the timing diagram in Fig. 7.26. In cycle 2, the RData[31:0] node becomes 4. Since this value is different from 0 , the decoder placed at the RData[31:0] node (a 32-input AND gate with an inverter) produces logic 1 at the OneShot output. This, in turn, activates the D-port of the 3-1 MUX and routes the decremented RData value, $($ RData -1$)=3$, to the input of the down-counter. In cycle 3 , the RData[31:0] node becomes 3, and the OneShot output stays at logic 1, keeping the D-port of the 3-1 MUX active. The decremented RData value, (RData -1 ) $=2$, is fed back to the timer input once again. The D-port of the 3-1 MUX stays active until cycle 6 when the RData[31:0] node becomes zero. From this point forward, the Idle-port of the 3-1 MUX becomes active, and the timer output becomes zero. The timer stays in this state until it is reprogrammed with a new value.

## Rate Generator

The rate generator is another type of timer which periodically generates single pulses separated by a programmable time duration. Once a desired rate is stored in the RateGen register via program bus, it is routed through the L-port of the 2-1 MUX to the input of the timer by LoadRateGen $=1$ as shown in Fig. 7.27. This step is shown in cycle 1 of the timing diagram


Fig. 7.25 Block diagram of the One-Shot timer


Fig. 7.26 Timing diagram of the One-Shot timer (OneShot register in Fig. 7.25 is programmed with a value of 4 as an example)
in Fig. 7.28 with RateGenOut $=4$ as an example. In cycle 2, the RData[31:0] node becomes four, and the RateGen output becomes zero. Because neither the RateGen port nor the LoadRateGen input is at logic 1, the D-port of the 2-1 MUX becomes automatically active to allow the decremented RData, $($ RData -1$)=3$, to become the input of the down-counter. In cycle 3, RData[31:0] becomes equal to three, which keeps the D-port still active because RateGen $=0$. The decremented RData, $($ RData -1$)=2$, is routed once again to the input of the timer. This path stays active until RData[31:0] $=1$. At this point, RateGen output becomes equal to one, and selects the L-port of the 2-1 MUX. The value in the RateGen register is loaded to the input of the down-counter. This is shown in cycle 5 of the timing diagram. In cycle 6, RData[31:0] becomes four, and the RateGen output becomes zero. The cycles 7 through 9 are the exact replicas of the cycles 3 through 5. The RData node keeps decrementing until it becomes equal to one, at which point the RateGen output also becomes one. Therefore, periodic single pulses are generated once in every four consecutive cycles at the RateGen output once the RateGen register is programmed with a value of four.


Fig. 7.27 Block diagram of the Rate Generator


Fig. 7.28 Timing diagram of the Rate Generator (RateGen register in Fig. 7.27 is programmed with a value of 4 as an example)

## Square Wave Generator

Square waveforms can also be generated by the timer as shown in Fig. 7.29. The pulse duration of the square wave is initially stored in the SqWave register through a program bus. Once the programming is finished, LoadSqWave $=1$ loads the value to the Sqwave register through the L-port of the 2-1 MUX to the input of the down counter. This is shown in cycle 1 of the timing diagram in Fig. 7.30 with SqWaveOut $=3$ as a numerical example. In cycle 2, RData[31:0] becomes three and RateOut becomes zero since the 32-input AND gate can only produce logic 1 when RData[31:0] = . The decremented RData value, (RData - 1) $=2$, is routed through the active D-port of the 2-1 MUX to the input of the down-counter. In cycle 3, the RData[31:0] node becomes two, but the RateOut node still stays at zero. The decremented RData value, $(\operatorname{RData}-1)=1$, is routed to the input of the down-counter once again. When RData[31:0] becomes equal to one in cycle 4, the 32-input AND gate produces RateOut $=1$, and activates the L-port of the 2-1 MUX. As a result, the down-counter is reloaded with the value in the SqW ave register. From this point forward, the circuit repeats the same pattern, producing a pulse in every three cycles at the RateOut node. Although the rate generator and the square wave circuits look identical, the square wave generator contains an additional state machine whose clock is controlled by the RateOut node. Therefore, at every positive edge of the RateOut signal, the value at the SqWave port alternates. If the SqWave output is initially assumed to produce logic 0 between cycles 1 and 3, the positive edge of the RateOut signal in cycle 4 switches the value of the SqWave output from logic 0 to logic 1. Similarly, the RateOut pulse in cycle 7 changes the value of the SqWave output


Fig. 7.29 Block diagram of the Square Wave Generator


Fig. 7.30 Timing diagram of the Square Wave Generator (SqWave register in Fig. 7.29 is programmed with a value of 3 as an example)
back to logic 0 . Therefore, the circuit in Fig. 7.29 creates a square waveform whose frequency is fully programmable by the SqW ave register.

## Interrupt Generator

One of the most useful timer functions is to have a timer generate a predetermined interrupt signal for the system. If an external event needs to be observed at a specific time or if periodic sampling needs to be employed for an event, the system must have the means to generate an interrupt. It achieves this by using the circuit in Fig. 7.31. This circuit is composed of a basic down-counter whose output is configured to make a transition from logic 0 to logic 1 when the count-down reaches zero.

As with the other timers, the count-down period is programmed in the GenInt register using a program bus as shown in Fig. 7.31. Once programming is finished, the L-port of the 3-1


Fig. 7.31 Block diagram of the Interrupt Generator

MUX is activated by LoadGenInt $=1$ to allow the value in the GenInt register to be loaded to the down-counter. As a numerical example, assume that the GenInt register is programmed with a value of four, which produces GenIntOut $=4$ during cycle 1 in the timing diagram in Fig. 7.32. In cycle 2, the contents of the down-counter input are transferred to the RData[31:0] node. Therefore, RData[31:0] $=4$ produces logic 1 at the Decrement node, which activates the D-port of the 3-1 MUX, and allows the decremented value of RData, $($ RData -1$)=3$, to be loaded to the input of the down-counter. In cycle 3 , RData[31:0] $=3$, but the Decrement node stays at logic 1, keeping the D-port active for the rest of the count-down process. When the RData[31:0] node finally reaches one in cycle 5, the Set node in Fig. 7.31 transitions to logic 1, and turns on the S-port of the second 3-1 MUX. In cycle 6, the GenInt output transitions from logic 0 to logic 1 after four cycles of count-down. Because the LoadGenInt input and the Decrement node are both at logic 0 , the Idle port of the 3-1 MUX automatically becomes active. As a result, $\mathrm{RData}[31: 0]=0$ keeps circulating back to the timer input until another value is loaded to the GenInt register. The GenInt output needs to be reset by the active-high Reset signal in Fig. 7.31 in order to generate another interrupt signal.


Fig. 7.32 Timing diagram of the Interrupt Generator (GenInt register in Fig. 7.31 is programmed with a value of 4 as an example)

### 7.6 Display Adaptor

The display is one of the most crucial peripherals in a system because it establishes a link between the user and the system. The format in all modern LCD or LED displays is composed of an active image area surrounded by vertical and horizontal blanking regions [2]. The size of the blanking regions is adjusted according to the response time of a particular display. Slower displays require larger vertical and horizontal blanking regions to sync with the active image
which has a frame rate between 30 and 60 frames per second. Pixels of an active image are fetched from the system memory and displayed on a non-interlaced screen following the vertical and the horizontal blanking sections as shown in Fig. 7.33.


Fig. 7.33 Non-interlaced display format with an active image of 1024 pixels by 1024 lines
Vertical and horizontal blanking areas are made out of black pixels. On the other hand, each pixel in the active image is composed of eight-bit wide Red (R), Green (G) and Blue (B) components. The basic operation consists of fetching a 24 -bit pixel from the system memory and placing it in the active image area one at a time. Usually each display has a frame buffer to store pixels from the system memory. When the data in the frame buffer is exhausted, the display controller requests another block of data to be transferred from the system memory. As the system complexity increases, bus traffic between the main memory and the CPU (and the system peripherals) increases proportionally. As a result, the display unit may have to wait before the next block of data arrives at its buffer. However, this is not an acceptable solution since this situation also creates choppy images for the user. Therefore, dual, quadruple or even higher frame buffer techniques are used to maintain continuous stream of images to be displayed on the monitor without any interruption. A dual frame buffer implementation is shown in Fig. 7.34 where image data is displayed from one frame buffer while the other buffer is being filled.

Prior to image processing, the Horizontal Blank register, Vertical Blank register, Active Image Pixel register and Active Image Line register are programmed using a separate program bus as shown in Fig. 7.34. The program bus can be a serial bus because the processing speed is not critical when the system programs the control, address and data registers prior to normal operation. The reader should refer to the design examples and review questions in Chapter 5 to devise a serial interface to program these registers. The data stored in these four registers define the normal operational parameters of the display unit.


Fig. 7.34 The data-path of the display unit

The image data, on the other hand, is continuously fed to the display buffers, Buf0 and Buf1, by a 32 -bit wide system bus, WData[31:0], as shown in Fig. 7.34. Even though the bus width is 32 bits, only the lower 24 bits are used in this architecture to transfer pixels to each buffer. The display controller first places the incoming pixels from the system bus to both Buf0 and Buf1, and then transfers pixels from one of these buffers to the image frame. The controller also generates two timing attributes, SycnHB and SyncVB, to indicate the start of
the horizontal blanking and the start of the vertical blanking sections of the frame, respectively. This is done in order to synchronize the display adaptor with the monitor.

To illustrate the operation of the display unit, a five pixel wide, nine line tall active image (the white area in Fig. 7.35) is considered as an example. This image is surrounded by two lines of vertical blanking and three pixels of horizontal blanking (shaded area) in the same figure. The numbers in each box represent a pixel component whether it belongs to the active image or the blanking sections. Therefore, the component numbers, 0,1 and 2 , constitute the first blank pixel in the vertical blanking section whereas the numbers, 57, 58 and 59, correspond to the $\mathrm{R}, \mathrm{G}$ and B components of the first pixel in the active image area. The blank pixels have no values and equal to $0 x 00$ as shown in Fig. 7.34. The active image pixels, however, are fetched from one of the image buffers in Fig. 7.34 and placed in the active frame in ascending order. For example, the component 57 is fetched first and placed at the upper left corner of the image frame, and the component 263 is fetched last and placed at the lower right corner.


Fig. 7.35 An example of an image frame composed of active image and blanking components

The display unit is activated by CSDisplay $=1$ as shown in cycle 1 of the timing diagram in Fig. 7.36. This figure shows data-path signals in the upper rows and control signals in the lower rows. Once active, the first component of the blanking pixel $0,0 \times 00$, arrives at the frame in cycle 2 . Within the same cycle, the SycnVB signal becomes logic 1, indicating the start of the vertical blanking for the frame. In cycles 3 and 4, the other two components of the blanking pixel 0 arrive at the frame. The first line of the vertical blanking completes in

Fig. 7.36 Timing diagram for the data-path in Fig. 7.34
cycle 25 . The second blanking line follows the same pattern as the first one: the first component of the blanking pixel 0 arrives at the frame in cycle 26 , and the last component of the blanking pixel 7 arrives in cycle 49 . Cycle 49 is also the cycle that resets the line counter by ResetLine $=1$ because the next line starts the active image. In cycle 50 , SycnHB becomes logic 1 , signifying the start of horizontal blanking. From cycles 50 to 58, the first line of horizontal blanking is formed prior to the active image. Cycle 58 also defines the border between the horizontal blanking and the active image. In this cycle, the read enable signal for Buf0, RE0, becomes logic 1 in order to read the first image pixel from this buffer. Blanking pixels are supplied to the frame from the BK-port of the 3-1 MUX as shown in Fig. 7.34, and delivered to the frame through FrameIn input.

The timing diagram in Fig. 7.37 is the continuation of Fig. 7.36 and focuses on the active image pixel delivery. The R-component of the first image pixel comes into the image frame in cycle 59 followed by the G and the B components in cycles 60 and 61 , respectively. Since the image data comes from the first display buffer, Buf0, the port assignment in the 3-1 MUX must be changed from port BK to port B0 by SelBuf0 $=1$ from cycle 59 onwards. Therefore, after the first image pixel is delivered to the frame, the address pointer for buffer 0 , Addr0, is incremented by one in cycle 61 to be able to fetch the R-component of the next pixel from Buf0 in cycle 62 (one cycle memory read latency). From cycles 59 to 73 , the first line of the active image is delivered to the frame by incrementing Addr0 from 0 to 4 . Cycle 73 also indicates the start of the second horizontal blanking line. In this cycle, RE0 transitions to logic 0, and Addr0 stops incrementing because pixel flow from Buf0 needs to be interrupted in order to start delivering blank pixels to the frame. To accommodate this, the port assignment in the 3-1 MUX is changed from port B0 to port BK. After completing the delivery of horizontal blanking pixels, the second line of the active image is delivered between cycles 83 and 97 . Addr0 is incremented from 5 to 9 during this period to fetch pixels from Buf0 and form the second active image line. The rest of the image is delivered to the frame by the end of cycle 265 . In cycle 266, a new frame is formed with $\mathrm{SyncVB}=1$. As in the previous frame, pixels that constitute the vertical blanking are followed by pixels that form the horizontal blanking and the active image. The new active image pixels are delivered from Buf1. In this example, both Buf0 and Buf1 are assumed to contain only 45 pixels, and therefore each buffer has $3 \times 45=135$ bytes of image data as opposed to the architecture in Fig. 7.34 that contains $1024 \times 1024=1,048,576$ active image pixels in each buffer.

Figure 7.38 shows the display controller design to manage the data-flow in Fig. 7.34. The state machine in Fig. 7.38 is a Moore-type composed of a string of states, each responsible for delivering blank or active image pixels to the frame. The state machine needs to keep track of the pixel and line numbers in the frame, and be able to define the boundaries between the blanking and the active image regions. Therefore, its functionality largely depends on the pixel and the line counter values in Fig. 7.34.

The state machine stays in the IDLE state until it is externally activated by CSDisplay $=1$, which corresponds to cycle 1 of the timing diagram in Fig. 7.36. Once

Fig. 7.37 Continuation of the timing diagram in Fig. 7.36
IDLE $\xrightarrow{\text { CSDisplay }=1} \xrightarrow{\text { STARTO }}\left[\begin{array}{l}\text { SeliBlank }=1 \\ \text { Buft } 1 \text { mply }=1 \\ \text { SyncVB }=1\end{array}\right.$

Fig. 7.38 Display adaptor controller for Buf0
activated, the R-component of the first blank pixel enters the frame through the BK-port of the 3-1 MUX, and prompts SelBlank $=1$ in cycle 2 . This refers to the START0 state in Fig. 7.38. In cycle 3, the G-component of the first blank pixel is delivered to the frame. This is shown as the VB0-G state in the state diagram. In this state, SelBlank $=1$ in order to transmit the G-component of the first blank pixel to the frame. The B-component of the first blank pixel arrives in cycle 4, which corresponds to the VB0-B state. During this cycle, SelBlank $=1$ and IncPx $=1$ to increment the pixel counter by one. At this point, the controller checks if the end of the first vertical blanking line has been reached by forming PxOut $=($ HBOut + AIPOut -2$)$. Here, PxOut corresponds to the output of the pixel counter, HBOut corresponds to the number of horizontal blanking pixels in the Horizontal Blanking register, and AIPOut corresponds to the number of active image pixels in the Active Image Pixel register in Fig. 7.34. If the end has not been reached, the machine keeps circling around the VB0-R, VB0-G and VB0-B states until PxOut becomes equal to (HBOut + AIPOut -2 ). This period translates from cycles 5 to 22 in the timing diagram in Fig. 7.36. During this period, every time the B-component of a blank pixel is delivered to the frame, the pixel counter increments by one. When the end point is detected, the state machine goes to the Reset VB0-R state where it delivers the R-component of the last blanking pixel that belongs to the first blanking line. This state corresponds to cycle 23 in the timing diagram. The controller delivers the remaining G and B-components of the last blanking pixel in cycles 24 and 25, which translate to the Reset VB0-G and Reset VB0-B states, respectively. In cycle 25 , the pixel counter is reset by ResetPx $=1$, and the line counter is incremented by IncLine $=1$ in order to produce the next vertical blanking line. However, the contents of the Vertical Blanking register, VBOut, needs to be checked prior to the start of the next vertical blanking line in case this register is programmed to have only one vertical blanking line. Therefore, while in the Reset VB0-G state, the line counter output, LineOut, is compared against (VBOut - 1). If the line counter output is less than (VBOut - 1), then the state machine first goes to the Reset VB0-B state and then back to the VB0-R state in order to generate another blanking line as described in cycles 26 to 49 in the timing diagram. If LineOut is equal to (VBOut - 1), the state machine goes to the Switch0 VB-to-HB state where it generates SelBlank $=1$, ResetPx $=1$ and ResetLine $=1$ in order to terminate the vertical blanking and start the first line of the horizontal blanking.

Cycle 50 starts the beginning of horizontal blanking region, and delivers the R-component of the first horizontal blanking pixel to the frame. This cycle translates to the Sync HB0-A state because $\mathrm{SyncHB}=1$ is also generated in this state. The state machine moves through the HB0-G and HB0-B states in cycles 51 and 52, and checks if the end of the horizontal blanking region has been reached by comparing the PxOut with (HBOut - 2). If PxOut $<$ (HBOut - 2), then more R, G and B blanking pixel components are brought into the frame
through the BK-port of the 3-1 MUX. However, if PxOut $=($ HBOut -2$)$, then the state machine enters the Reset HB0-R state to deliver the R-component of the last horizontal blanking pixel in cycle 56 as this condition indicates the end of horizontal blanking. In cycles 57 and 58, the machine traverses through the Reset HB0-G and the Reset HB0-B states. The latter state resets the pixel counter by ResetPx $=1$, and enables Buf0 by RE0 $=1$ to start reading active image pixels.

In cycle 59 , the state machine enters the R0 state to deliver the R-component of the first active image pixel from Buf0. In this state, port 0 of the 3-1 MUX at the output of Buf0 becomes active by $\operatorname{SelR} 0=1$, and port B 0 of the $3-1$ frame MUX becomes active by SelBuf0 $=1$. The read enable input for Buf0 also stays at logic 1 by RE0 $=1$. In cycle 60 , the state machine goes to the G0 state where it delivers the G-component of the first active image pixel to the frame. This cycle requires SelG0 $=1$ to activate port 1 of the 3-1 MUX at the output of Buf0 while keeping SelBuf0 $=1$ and $\mathrm{RE} 0=1$. Addr0 is also incremented in this state by IncAddr0 $=1$. In cycle 61 , the controller reaches the B 0 state where it increments the pixel counter by $\operatorname{IncPx}=1$, selects port 2 of the 3-1 MUX at the output of Buf0 by $\operatorname{SelB} 0=1$, and maintains both $\operatorname{SelBuf0}=1$ and $\mathrm{RE} 0=1$. In this state, the controller checks if the end of active image has been reached by comparing PxOut against (AIPOut - 2). If the controller finds PxOut < (AIPOut - 2), it goes back to the R0 state to retrieve more image pixels from Buf0. This scenario corresponds to cycles 62 to 70 of the timing diagram in Fig. 7.37. If the controller finds PxOut $=($ AIPOut -2$)$, it moves to the Reset R0 state in cycle 71 to deliver the last R -component of the image pixel, and generates $\operatorname{SelR} 0=1$, SelBuf0 $=1$ and RE0 $=1$. The state machine then moves to the Reset G0 state in cycle 72 and the Reset B0 state in cycle 73. In the Reset B0 state, the controller resets the pixel counter by ResetPx = 1, increments the line counter by IncLine $=1$, selects port 2 of the 3-1 Buf0 MUX by SelB0 $=1$, and keeps SelBuf0 $=1$. While in this state, the controller checks to see if the active image is more than a single line or not, and compares the output of the line counter, LineOut, against (AILOut - 2). If the controller finds that LineOut < (AILOut 2), it first moves to the Sync HB0 state to generate SyncHB $=1$, and then back to the HB0-G state to start fetching horizontal blanking pixels for the next line. However, if the controller finds that LineOut $=($ AILOut -2$)$, it realizes that it will be processing the last line of the current frame. First, it goes to the Sync HB0-B state and generates SyncHB = 1, and then to the Last HB0-G state to deliver the G-component of a horizontal blanking pixel.

The states from Last HB0-R to Reset Last-B0 are the exact replicas of the states from HB0-R to Reset B0 except that in the Reset Last-G0 state, Buf0 address pointer is reset by ResetAddr0 $=1$, and in the Reset Last-B0 state, the line counter is reset by ResetLine $=1$. Once the controller exhausts all the active image pixels in Buf0, it switches to Buf1 to construct the next image frame as shown in the state diagram of Fig. 7.39. In this figure, the

Fig. 7.39 Display adaptor controller for Buf1
states controlling the blanking and the image pixel delivery from Buf1 is exactly the same as in Buf0. Once all the pixels are delivered to the frame from Buf1, the state machine in Fig. 7.39 hands over the control of the display adaptor to the state machine in Fig. 7.38.

### 7.7 Data Converters

## Analog-to-Digital Converter

All analog domains interface with digital systems through Analog-to-Digital Converters (ADC). In Fig. 7.40, an analog signal from a sensor is amplified to a certain level before sampling takes place in a sample-and-hold circuit inside the ADC. The sampled analog signal is then converted into digital form and directed to the CPU for processing according to an embedded program.


Fig. 7.40 Typical Analog-to-Digital and Digital-to-Analog Converter data-paths

The signal resolution is an important factor to consider in an ADC design. It simply means dividing a sampled analog signal by $2^{\mathrm{N}}$ number of voltage levels where N represents the number of bits in the ADC. The second important consideration is the range of analog values an ADC can capture and process.

Figure 7.41 describes the ADC resolution in a numerical example where an analog signal changes between 0 V and 5 V . The bit resolution is only three bits, and therefore the ADC uses $2^{3}=8$ levels to identify the value of an analog signal at the ADC input. For example, an analog signal of 2.501 V is identified by a digital output of 100 . If the analog signal increases to 3.124 V , the digital output that represents this voltage value still stays at 100 . In other words, in a three-bit ADC there is no difference between 2.501 V and 3.124 V in terms of their digital representation. The 0.625 V step size is the natural occurring error in a three-bit ADC, and it can be reduced only if the number of bits in the ADC is increased. In general, increasing the number of ADC bits by one halves the error. Therefore, designing a four-bit ADC instead of a three-bit ADC reduces the quantization error by 0.3125 V .


Fig. 7.41 Input-output description of a three-bit ADC
The reference voltage of an ADC is generally determined by the maximum voltage level of the analog signal, and it is used to calculate the step size. In this three-bit ADC example in Fig. 7.41, the reference voltage is 5 V because the amplified analog voltage at the input of the ADC is limited not go beyond 5 V .

ADC samples non-periodic analog signals in regular time intervals as described in Fig. 7.42. The time interval between sampling points is called the sampling period. The sampling period is adjusted according to the processing speed of the ADC in order to generate accurate digital outputs.

Once sampled, the analog voltage at the input of an ADC is held steady throughout the sampling period while the conversion takes place as shown in Fig. 7.43. The shape of the converted signal may be quite different from the original analog signal due to the ADC resolution and the time duration between samples. In a three-bit ADC , sampling takes place


Fig. 7.42 Sampling a continuous analog signal


Fig. 7.43 Sampling period, hold concept and regeneration of an analog signal
in 0.625 V increments. Therefore, each sampling point becomes subject to a dynamic quantization error which changes between 0 V and 0.3125 V . For example, a three-bit ADC samples 3.4 V according to its closest sampling level of 3.125 V , and produces a 0.275 V error. Arbitrary signals that change with a frequency faster than the sampling frequency are subject to much larger dynamic errors. When converted back to their analog form, these signals show large deviations from their original shapes.

A basic sample-and-hold circuit consists of an NMOS transistor and a capacitor as shown in Fig. 7.44. The control input simply turns on the N-channel MOSFET for a short period of time, called the sampling width, during which the analog voltage level at the input is stored on the capacitor. When the transistor is turned off, this analog value is held constant until the next sampling point.


Fig. 7.44 A typical sample-and-hold circuit

## Flash ADC

The simplest ADC is the flash-type as shown in Fig. 7.45. This three-bit ADC contains $2^{3}=8$ operational amplifiers. The analog signal is applied to all eight positive input terminals. The reference voltage is distributed to each negative input terminal via a voltage divider circuit. Each operational amplifier acts as a differential amplifier and amplifies the difference between a continuously changing analog signal and the portion of the reference voltage.

Figure 7.46 describes the operation of the three-bit flash ADC and its encoder in a truth table. When the analog voltage is less than or equal to 0.625 V , only Out[0] becomes logic 1 , all other outputs from Out[1] to Out[7] become logic 0 . When the analog signal exceeds 0.625 V but less than 1.25 V , only Out[0] and Out[1] become logic 1, and again all others become logic 0 . Higher analog voltages at the input successively produce more logic 1 levels


Fig. 7.45 Typical three-bit flash ADC schematic
as shown in Fig. 7.46. An encoder is placed at the output stage of all operational amplifiers to transform the voltage levels at Out[7:0] into a three-bit digital output, DOut[2:0]. The digital output is subject to a maximum error of 0.625 V because only three bits are used for conversion.

| Analog Input | Out[7] | Out[6] | Out[5] | Out[4] | Out[3] | Out[2] | Out[1] Out[0] | DOut[2] | DOut[1] | DOut[0] |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0.625>\mathrm{V}_{\mathbb{I N}}>0.000$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| $1.250>\mathrm{V}_{\mathbb{I N}}>0.625$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| $1.875>\mathrm{V}_{\mathbb{I N}}>1.250$ | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| $2.500>\mathrm{V}_{\mathbb{I N}}>1.875$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| $3.125>\mathrm{V}_{\mathbb{I N}}>2.500$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| $3.750>\mathrm{V}_{\mathbb{I N}}>3.125$ | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| $4.375>\mathrm{V}_{\mathbb{I}}>3.750$ | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| $5.000>\mathrm{V}_{\mathbb{I N}}>4.375$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Fig. 7.46 Three-bit flash ADC truth table describing its operation

## Ramp ADC

The ramp ADC uses only a single operation amplifier, but it employs an up-counter and a Digital-to-Analog Converter (DAC) in a loop structure as shown in Fig. 7.47. The digital output is obtained from the C[3:0] terminals, and progressively generates a final output within several clock periods.


Fig. 7.47 Typical four-bit ramp ADC schematic

The top portion of Fig. 7.48 describes the output voltage assignments of a four-bit ramp ADC using two different types of number-rounding schemes. The down-rounding scheme assigns a lower analog value to each digital output compared to the up-rounding scheme. For example, the down-rounding scheme produces a digital output of $\mathrm{C}[3: 0]=0100$ for analog voltages between 1.0937 V and 1.4062 V applied to its input. If the up-rounding scheme is used, the same digital output becomes equivalent to an analog voltage anywhere between 1.4062 V and 1.7187 V.

The middle table in Fig. 7.48 shows how the conversion takes place if the down-rounding mechanism is used in an example. Prior to its operation, the four-bit up-counter is reset and produces $\mathrm{C}[3: 0]=0000$. Assuming an analog voltage of 2 V is applied to the input, which must be kept constant until the conversion is complete, $\mathrm{C}[3: 0]=0000$ causes the DAC output, $\mathrm{DAC}_{\text {Out }}$, to produce 0 V according to the down-rounding scheme. Since this value is less than 2 V at the sample/hold circuit output, $\mathrm{SH}_{\mathrm{OUT}}$, the output of the differential amplifier, INCR, transitions to the positive supply potential of the operational amplifier, $+\mathrm{V}_{\mathrm{CC}}=5 \mathrm{~V}$, which prompts the four-bit counter to increment to $\mathrm{C}[3: 0]=0001$. Consequently, the DAC generates $\mathrm{DAC}_{\text {Out }}=0.3125 \mathrm{~V}$ according to the truth table in Fig. 7.48. However, this value is still less than $\mathrm{SH}_{\mathrm{Out}}=2 \mathrm{~V}$. Therefore, the differential amplifier produces another INCR $=5 \mathrm{~V}$ which prompts the counter to increment again to C $[3: 0]=0010$. Up-counting continues until $\mathrm{C}[3: 0]=0111$ or $\mathrm{DAC}_{\text {Out }}=2.1875 \mathrm{~V}$. Since this last voltage is greater than $\mathrm{SH}_{\mathrm{OUT}}=2 \mathrm{~V}$, the differential amplifier output switches back to its negative supply voltage, $-\mathrm{V}_{\mathrm{CC}}=0 \mathrm{~V}$, and stops the up-counter from incrementing further. The digital output stays steady at $\mathrm{C}[3: 0]=0111$ from this point forward, representing 2 V analog voltage with a dynamic error of 0.1875 V .

The table at the bottom part of Fig. 7.48 represents the conversion steps if the up-rounding mechanism is used in this ADC. External reset still produces $\mathrm{C}[3: 0]=0000$ initially. However, the DAC output starts the conversion with an increased amount of 0.3125 V instead of 0 V . The counter increments until $\mathrm{C}[3: 0]=0110$, and produces 2.1875 V at the $\mathrm{DAC}_{\text {Out }}$ node. At this value, INCR becomes 0 V , and the up-counter stops incrementing further. As a result, $\mathrm{C}[3: 0]=0110$ becomes the final ADC output for 2 V .

## Successive Approximation ADC

The third type ADC is based on the successive approximation technique to estimate the value of the analog voltage. This converter type is a trade-off between the flash-type and the ramp-type ADC in terms of speed and the number of components used in the circuit. As a

| Step Size $=5 / 2^{4}=0.3125 \mathrm{~V}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C[3] | C[2] | $\mathrm{C}[1]$ | $\mathrm{C}[0]$ | Down-Round $(\mathrm{V})$ | Up-Round $(\mathrm{V})$ |
| 0 | 0 | 0 | 0 | 0.0000 | 0.3125 |
| 0 | 0 | 0 | 1 | 0.3125 | 0.6250 |
| 0 | 0 | 1 | 0 | 0.6250 | 0.9375 |
| 0 | 0 | 1 | 1 | 0.9375 | 1.2500 |
| 0 | 1 | 0 | 0 | 1.2500 | 1.5625 |
| 0 | 1 | 0 | 1 | 1.5625 | 1.8750 |
| 0 | 1 | 1 | 0 | 1.8750 | 2.1875 |
| 0 | 1 | 1 | 1 | 2.1875 | 2.5000 |
| 1 | 0 | 0 | 0 | 2.5000 | 2.8125 |
| 1 | 0 | 0 | 1 | 2.8125 | 3.1250 |
| 1 | 0 | 1 | 0 | 3.1250 | 3.4375 |
| 1 | 0 | 1 | 1 | 3.4375 | 3.7500 |
| 1 | 1 | 0 | 0 | 3.7500 | 4.0625 |
| 1 | 1 | 0 | 1 | 4.0625 | 4.3750 |
| 1 | 1 | 1 | 0 | 4.3750 | 4.6875 |
| 1 | 1 | 1 | 1 | 4.6875 | 5.0000 |

Analog Input $=2 \mathrm{~V}$ with Down-Rounding Mechanism

| Step | C[3] | C[2] | C[1] | C[0] | DACout $\left.^{2} \mathrm{~V}\right)$ | INCR(V) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 0.0000 | 5.0 |
| 2 | 0 | 0 | 0 | 1 | 0.3125 | 5.0 |
| 3 | 0 | 0 | 1 | 0 | 0.6250 | 5.0 |
| 4 | 0 | 0 | 1 | 1 | 0.9375 | 5.0 |
| 5 | 0 | 1 | 0 | 0 | 1.2500 | 5.0 |
| 6 | 0 | 1 | 0 | 1 | 1.5625 | 5.0 |
| 7 | 0 | 1 | 1 | 0 | 1.8750 | 5.0 |
| 8 | 0 | 1 | 1 | 1 | 2.1875 | 0.0 |
|  |  |  |  |  | 4 |  |

Final output with quantization error of 0.3125 V
Analog Input $=2 \mathrm{~V}$ with Up-Rounding Mechanism

| Step | C[3] | C[2] | C[1] | C[0] | DACout(V) | INCR(V) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 0.3125 | 5.0 |
| 2 | 0 | 0 | 0 | 1 | 0.6250 | 5.0 |
| 3 | 0 | 0 | 1 | 0 | 0.9375 | 5.0 |
| 4 | 0 | 0 | 1 | 1 | 1.2500 | 5.0 |
| 5 | 0 | 1 | 0 | 0 | 1.5625 | 5.0 |
| 6 | 0 | 1 | 0 | 1 | 1.8750 | 5.0 |
| 7 | 0 | 1 | 1 | 0 | 2.1875 | 0.0 |
|  |  |  |  |  | 4 |  |

Final output with quantization error of 0.3125 V

Fig. 7.48 Four-bit ramp ADC truth table describing its operation
numerical example, a typical four-bit successive approximation ADC schematic is shown in Fig. 7.49. In this figure, the up-counter in the ramp ADC is replaced by a control logic which successively transforms an analog input into a digital output by a trial and error method. The output is obtained at the C[3:0] terminal.


Fig. 7.49 Typical four-bit successive approximation ADC schematic
The top portion of Fig. 7.50 shows the truth table to operate a four-bit successive approximation ADC. Two numerical examples in this figure illustrate the down-rounding and up-rounding schemes used during conversion.

The first example in Fig. 7.50 illustrates the down rounding mechanism in a four-bit successive approximation ADC. In this example, an analog voltage of 3.5 V is applied to the analog input of the ADC. An external reset starts the converter at $\mathrm{C}[3: 0]=1000$, which is considered a mid point between $\mathrm{C}[3: 0]=0000$, representing the minimum analog input of 0 V , and $\mathrm{C}[3: 0]=1111$, representing the maximum analog input of 5 V for this ADC. For C [3:0] $=1000$, the DAC generates an initial analog voltage of 2.5 V at the DAC ${ }_{\text {Out }}$ node. Since this value is less than the sampled analog voltage of 3.5 V at the $\mathrm{SH}_{\text {Out }}$ node, the operational amplifier produces $\mathrm{IN}=5 \mathrm{~V}$, and prompts the control logic to try a slightly

| Step Size $=5 / 2^{4}=0.3125 \mathrm{~V}$ |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| C[3] | $\mathrm{C}[2]$ | $\mathrm{C}[1]$ | $\mathrm{C}[0]$ | Down-Round $(\mathrm{V})$ | Up-Round $(\mathrm{V})$ |
| 0 | 0 | 0 | 0 | 0.0000 | 0.3125 |
| 0 | 0 | 0 | 1 | 0.3125 | 0.6250 |
| 0 | 0 | 1 | 0 | 0.6250 | 0.9375 |
| 0 | 0 | 1 | 1 | 0.9375 | 1.2500 |
| 0 | 1 | 0 | 0 | 1.2500 | 1.5625 |
| 0 | 1 | 0 | 1 | 1.5625 | 1.8750 |
| 0 | 1 | 1 | 0 | 1.8750 | 2.1875 |
| 0 | 1 | 1 | 1 | 2.1875 | 2.5000 |
| 1 | 0 | 0 | 0 | 2.5000 | 2.8125 |
| 1 | 0 | 0 | 1 | 2.8125 | 3.1250 |
| 1 | 0 | 1 | 0 | 3.1250 | 3.4375 |
| 1 | 0 | 1 | 1 | 3.4375 | 3.7500 |
| 1 | 1 | 0 | 0 | 3.7500 | 4.0625 |
| 1 | 1 | 0 | 1 | 4.0625 | 4.3750 |
| 1 | 1 | 1 | 0 | 4.3750 | 4.6875 |
| 1 | 1 | 1 | 1 | 4.6875 | 5.0000 |

Analog Input $=3.5 \mathrm{~V}$ with Down-Rounding Mechanism START with (5.0/2) $=2.5$

| Step | C[3] | C[2] | C[1] | $\mathrm{C}[0]$ | DACout(V) | Control Logic In |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 0 | 0 | 2.5000 | $3.5>2.5000$ |
|  |  |  |  |  |  | Thus try $2.5+(2.5 / 2)=3.75$ |
| 2 | 1 | 1 | 0 | 0 | 3.7500 | $3.5<3.7500$ |
|  |  |  |  |  |  | Thus try $2.5+(2.5 / 4)=3.125$ |
| 3 | 1 | 0 | 1 | 0 | 3.1250 | $3.5>3.1250$ |
|  |  |  |  |  |  | Thus try $2.5+(2.5 / 4)+(2.5 / 8)=3.4375$ |
| 4 | 1 | 0 | 1 | 1 | 3.4375 | $3.5>3.4375$ |
|  |  |  |  |  | 4 | Stop at 3.4375 since there is no resolution under (2.5/8) |

Final output with quantization error of 0.3125 V

Analog Input $=3.5 \mathrm{~V}$ with Up-Rounding Mechanism $\quad$ START with $(5.0 / 2)=2.8125$

| Step | C[3] | C[2] | C[1] | C[0] | DACout $(\mathrm{V})$ | Control Logic In |
| :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 1 | 1 | 0 | 0 | 0 | 2.8125 | $3.5>2.8125$ <br> Thus try $2.8125+(2.5 / 2)=4.0625$ |
| 2 | 1 | 1 | 0 | 0 | 4.0625 | $3.5<4.0625$ <br> Thus try $2.8125+(2.5 / 4)=3.4375$ |
| 3 | 1 | 0 | 1 | 0 | 3.4375 | $3.5>3.4375$ |
|  |  |  |  |  |  | Stop at 3.4375 since there is no resolution under $(2.5 / 8)$ |

Final output with quantization error of 0.3125 V

Fig. 7.50 Four-bit successive approximation ADC truth table describing down-rounding and up-rounding approximation techniques
higher digital output. As a result, the control logic produces $\mathrm{C}[3: 0]=1100$ as its first trial, which is equivalent to a midway point between $\mathrm{C}[3: 0]=1000$ and 1111 . DAC Out becomes $2.5+(2.5 / 2)=3.75 \mathrm{~V}$. But, this new voltage is larger than $\mathrm{SH}_{\mathrm{Out}}=3.5 \mathrm{~V}$, and the operational amplifier produces $\mathrm{IN}=0 \mathrm{~V}$ in return. The drop at the IN node is an indication to the control logic that its initial attempt of $\mathrm{C}[3: 0]=1100$ was too large, and it must lower its output. This time, the control logic tries $\mathrm{C}[3: 0]=1010$, which is between $\mathrm{C}[3: 0]=1000$ and 1100 , and produces a DAC output, $\mathrm{DAC}_{\text {OUT }}=2.5+(2.5 / 4)=3.125 \mathrm{~V}$. This value, in turn, generates $\mathrm{IN}=5 \mathrm{~V}$, and prompts the control logic to try a slightly higher output between $\mathrm{C}[3: 0]=1010$ and 1100. In the third round, the control logic produces $\mathrm{C}[3: 0]=1011$. DAC OUt node becomes $2.5+(2.5 / 4)+(2.5 / 8)=3.4375 \mathrm{~V}$ and generates $\mathrm{IN}=5 \mathrm{~V}$. This new input suggests that the control logic should try a slightly higher output in the next round, producing $2.5+(2.5 / 4)+(2.5 / 8)+(2.5 / 16)=3.5937 \mathrm{~V}$ at DAC Out . However, $2.5 / 8=$ 0.3125 V is the resolution limit for this four-bit ADC , and as a result, the controller stalls at $\mathrm{C}[3: 0]=1011$, revealing $\mathrm{DAC}_{\text {OUT }}=3.4375 \mathrm{~V}$. This voltage differs from $\mathrm{SH}_{\text {OUT }}=3.5 \mathrm{~V}$ by only 0.0625 V .

The second example in Fig. 7.50 explains the successive approximation technique if the up-rounding scheme is employed. The conversion again starts at $\mathrm{C}[3: 0]=1000$, but with an incremented value of $2.5+0.3125=2.8125 \mathrm{~V}$ at the DAC output. Since this voltage is below $\mathrm{SH}_{\text {OUT }}=3.5 \mathrm{~V}$, IN node becomes 5 V and prompts the control logic to produce a larger digital output. The control logic responds to this with a digital output of $\mathrm{C}[3: 0]=1100$, which corresponds to $2.8125+(2.5 / 2)=4.0625 \mathrm{~V}$ at the $\mathrm{DAC}_{\text {Out }}$ node. As a result, IN node becomes 0 V , and forces the control logic to lower its digital output. This time, the control logic tries $\mathrm{C}[3: 0]=1010$ which is equivalent to $2.8125+(2.5 / 4)=3.4375$ at the $\mathrm{DAC}_{\text {OUT }}$ node. Due to the resolution limit of this four-bit ADC, this step also becomes the end of successive approximation.

The control circuit of the four-bit ADC with down-rounding scheme is shown in Fig. 7.51. In this figure, the approximation process starts at the midpoint, $\mathrm{C}[3: 0]=1000$, corresponding to $\mathrm{DAC}_{\mathrm{OUT}}=2.5 \mathrm{~V}$ according to the table in Fig. 7.50. When the external reset is removed, and the difference between $\mathrm{SH}_{\text {OUT }}$ and $\mathrm{DAC}_{\text {OUt }}(\Delta \mathrm{v})$ is found to be greater than the 0.3125 V step size, the control logic either goes to the state 1.25 and produces $\mathrm{C}[3: 0]=0100$ (equivalent to 1.25 V ), or to the state 3.75 and produces $\mathrm{C}[3: 0]=1100$ (equivalent to 3.75 V ) in the first step of the successive approximation. This decision depends on the value of the control logic input, IN. If IN $=0$, which translates to the analog input to be less than 2.5 V , the next state becomes the state 1.25 . However, if $\mathrm{IN}=1$, the analog input is considered to be greater than 2.5 V , and the next state becomes the state 3.75 . If $\Delta \mathrm{v}$ is less than 0.3125 V , on the other hand, the control logic cannot proceed further due to its resolution limit, and moves to the

Fig. 7.51 Four-bit successive approximation control circuit
state DONE. In the second step of successive approximation, the state 1.25 either transitions to the state 0.625 or to the state 1.875 , depending on the value at the IN node. Similar transitions take place from the state 3.75 to either the state 3.125 or the state 4.375 , again depending on the value of the IN node. After this point, the state machine performs one last approximation to estimate the value of analog input voltage, and reaches the DONE state with an output value as shown in Fig. 7.51.

### 7.8 Digital-to-Analog Converter (DAC)

The most common DAC utilizes the weighted summation method of digital inputs. A three-bit DAC with a weighted binary adder is shown in Fig. 7.52 as an example.


Fig. 7.52 Three-bit DAC schematic with weighted binary adder
This circuit is composed of two parts. The first part adds all three binary input bits, IN[2] (the most significant bit), IN[1] and IN[0] (the least significant bit), and produces an output, $\mathrm{ADD}_{\text {OUt }}=-(0.5 \mathrm{IN}[2]+0.25 \mathrm{IN}[1]+0.125 \mathrm{IN}[0])$ according to the equation in Fig. 7.53. The second part is an analog inverter which forms OUT $=-$ ADD $_{\text {Out }}$.

Therefore, the circuit in Fig. 7.52 generates OUT $=0.5 \operatorname{IN}[2]+0.25 \mathrm{IN}[1]+0.125 \mathrm{IN}[0]$, where each binary value at $\operatorname{IN}[2: 0]$ input is multiplied by the coefficients, $2^{-1}, 2^{-2}$ and $2^{-3}$, before they are added to produce an output. For example, the combination of $\operatorname{IN}[2]=1$, $\operatorname{IN}[1]=0$ and $\operatorname{IN}[0]=1$, with +5 V and 0 V logic levels generates $\mathrm{OUT}=2.5+0.625=$ 3.125 V . All the other analog outputs in Fig. 7.53 can be generated using the equation in this figure with a maximum error of 0.625 V .

$$
\begin{aligned}
& A D D_{\text {OUT }}=-\frac{R}{2 R} \operatorname{IN}[2]-\frac{R}{4 R} \operatorname{IN}[1]-\frac{R}{8 R} \operatorname{IN}[0] \\
& =-0.5 \mathrm{IN}[2]-0.25 \mathrm{IN}[1]-0.125 \mathrm{IN}[0] \\
& \text { OUT }=- \text { ADD }_{\text {OUT }}=0.5 \mathrm{IN}[2]+0.25 \mathrm{IN}[1]+0.125 \mathrm{IN}[0] \\
& \text { Example: } \\
& \operatorname{IN}[2]=1, \operatorname{IN}[1]=0, \operatorname{IN}[0]=1 \text { with }+5 \mathrm{~V} / 0 \mathrm{~V} \text { logic levels } \\
& \mathrm{ADD}_{\text {OUT }}=-2.5-0-0.625=-3.125 \mathrm{~V} \\
& \text { OUT }=+3.125 \mathrm{~V} \text { with } 0.625 \mathrm{~V} \text { quantization error }
\end{aligned}
$$

Fig. 7.53 Three-bit DAC operation with weighted binary adder

## Review Questions

1. A DMA controller transfers four words of data, D1, D2, D3 and D4, from SRAM 1 (source memory) to SRAM 2 (destination memory) on a 32-bit wide bidirectional bus as shown below. D1, D2, D3 and D4 are fetched from addresses, AS1, AS2, AS3 and AS4, in the source memory and placed at the AD1, AD2, AD3 and AD4 addresses in the destination memory, respectively. Since each SRAM memory has a single I/O port, address and data cannot be transferred in the same clock cycle. Therefore, for the read operation, data becomes available at the SRAM I/O port a cycle after a valid address is presented. For the write operation, data has to be present at the SRAM port a cycle after the valid address. The active-high RE and WE signals enable the SRAM for read or write operations when a valid address is available.

The DMA controller has two programming ports to program the initial address and the incremented address values. It uses ProgAddrS and ProgIncrS control inputs to initialize and increment the source address, and the ProgAddrD and ProgIncrD inputs to initialize and increment the destination memory address. The active-high StartS produces the first address at AddrOutS. The IncrS input increments addresses based on the initial address. Similarly, the StartD input produces the first address at the AddrOutD port, and the IncrD increments addresses based on the initial address.
The Capture1 and Capture2 inputs capture data from the bus temporarily and store it in the Buf1 and Buf2 data buffers, respectively. All active-high enable inputs enable the tri-state buffers when they are at logic 1. Otherwise, there will be no connection between the address pointers and the bus or between the memories and the bus.

(a) Draw the schematic for the source and destination address pointers.
(b) Form a timing diagram to transfer four words of data from SRAM1 to SRAM2.
2. The waveforms shown below are generated by two different timers.
(a) A rate generating timer produces an active-high pulse in every 128 cycles as shown below. Design this timer and draw its schematic.

(b) A one-shot timer produces a continuous pulse 100 cycles after the start signal as shown below. Design this timer and draw its schematic.

3. An interrupt controller managing four hardware interrupts is connected to a 16 -bit CPU, which consists of a 16-bit wide instruction memory, Imem, a data memory, Dmem, a program counter for the Imem, PC, a program counter for the data memory, DC, the data registers A and B , and a controller. This schematic is shown below.

The interrupt protocol in this schematic is as follows:
Step 1: The active-high interrupt, INTR, is generated by the interrupt controller to inform the CPU about the presence of an interrupt. The interrupt controller must have a request from an external device with an interrupt ID before generating INTR.
Step 2: The CPU acknowledges receiving an INTR with an active-high interrupt acknowledge, INTA.


Step 3: The interrupt controller produces an interrupt ID on the 16-bit bi-directional data bus.
Step 4: The interrupt ID is loaded to the PC to access the interrupt service routine (ISR) address.
Step 5: The ISR address is loaded to the PC.
Step 6: One of the four interrupt service instructions is fetched from the instruction memory for a particular interrupt. The steps are as follows:

Step 6a: The first interrupt instruction specifies the data memory address to be loaded to the DC.
Step 6b: The second instruction contains the value of A to be loaded to the A register.
Step 6c: The third instruction contains the value of B to be loaded to the B register.
Step 6d: The fourth instruction provides either the contents of the A register or the contents of B register or the added results of both registers to be loaded to the data memory at a DC address. This value will later be used in the program (the related hardware is not shown in the schematic above).

Step 7: When the four-cycle interrupt service is complete, the CPU lowers its INTA signal. In response, the interrupt controller lowers its INTR signal a cycle after the INTA input transitions to logic 0 , and waits for the next interrupt.

Note: Imem or Dmem have SRAM configurations. Storing data at a valid address is achieved by WE $=1$ within the same clock cycle. Reading data from a memory address is achieved by $\mathrm{RE}=1$ with a latency of two clock cycles (data is not read at the beginning of the next clock cycle, but the one after next).
(a) If the priority scheme in the interrupt controller is such that device 0 has the highest priority and device 3 has the lowest priority, design this controller with the I/O port description shown on the schematic. Note that this controller can support only hardware interrupts.
(b) Show a timing diagram outlining the complete interrupt service from step 1 to step 7 above.
(c) Show the state diagram of the controller, including all the inputs to operate the instruction and data memories, registers etc.
4. A display unit works on a unidirectional bus that transmits 24-bit video pixels. Each pixel is comprised of eight-bit wide $\mathrm{R}, \mathrm{G}$ and B components which occupy the least significant three bytes of the write-data bus (the most significant byte is always $0 \times 00$ ). The write-bus fills a video buffer at a frequency of 2f. Similarly, the frequency of fetching data from a buffer to fill a video frame is f (buffer emptying rate). Assume the horizontal and the vertical blanking sections of the video frame are both zero.
The display unit is the highest priority peripheral on the bus because of the fact that it requires a minimum rate of 30 frames/sec to process and display data. However, other
peripherals in the system also use the same bus in order to send or receive data between video bursts.
Each timing diagram below contains three vital entries for a frame buffer. The top row indicates the ID number of a flag associated with an empty buffer. The middle row indicates the ID number of an empty buffer. The bottom row shows the ID number of a full buffer. The flag is a direct input to the CPU, and points out which buffer in the display unit needs to be filled. Note that all buffers are considered full before data transactions start. Each square in the timing diagram corresponds to filling or emptying an entire buffer. Since filling a buffer takes half the time of emptying it, the number of squares doubles at the bottom row relative to the middle row.
Suppose you have the flexibility of using a two buffer, a three buffer or a four buffer system in the video unit. The key consideration in this design is to have enough frame buffers in the system so that a continuous stream of data can be supplied to the display unit while other peripherals use the bus.
Once full, define the length of the data burst from each buffer in the timing tables below. Mark each entry with buffer numbers, and use the letters E or F to indicate whether each buffer is empty or full, respectively. The video unit empties buffers in the following order. In a two-buffer system, buffer 1 empties first, buffer 2 empties second. In a three-buffer system, buffer 1 empties first, buffer 2 second, and buffer 3 third. In a four-buffer system, buffer 1 empties first, buffer 2 second, buffer 3 third, and buffer 4 fourth. Indicate the flag number for each empty buffer inside the circle.
2. Buffer system:

3. Buffer system:


## 4. Buffer system:


5. A three-bit successive approximation $A D C$ is given below. A Sample-Hold circuit (S/H) samples varying analog voltages at the Analog Input port in periodic time intervals and directs them to the operational amplifier.


Use the empty timing diagram below and fill the blanks for Analog Input $=0.5 \mathrm{~V}$ with down-rounding mechanism.

6. A three-bit ramp ADC below operates with $+\mathrm{V}_{\mathrm{CC}}=3 \mathrm{~V}$ and $-\mathrm{V}_{\mathrm{CC}}=0 \mathrm{~V}$. This ADC is designed to take any analog input between 0 V and 3 V .

(a) Assume that the DAC has an up-rounding scheme to generate analog outputs from digital inputs. Apply 1.2 V to the Analog Input port, and draw the timing diagram that contains the clock, Reset, counter output (COUNT ${ }_{\text {OUT }}$ ), DAC output (DAC OUT ) and operational amplifier output (INCR). Show what happens to the timing diagram when the active-high Reset signal transitions to logic 1 after the ADC produces the desired digital output.
(b) Assume that the rounding scheme has changed from up-rounding to down rounding.
(c) Assume that the DAC rounding scheme is changed from up-rounding to down-rounding scheme. Apply 1.2 V to the Analog Input and generate the timing diagram with the input and output signals listed above. Show what happens to the timing diagram when the Reset signal transitions to logic 1 after the ADC produces the desired digital output. Do you see any issues with the operation of this circuit?
(d) Now apply 2.9 V to the Analog Input port and generate the timing diagram with the input and output signals listed above. Do you see any issues in the operation of this circuit?
7. The following circuit shows the block diagram of a simple transmitter-receiver. The transmission protocol starts with the transmitter sending the request signal, Req, to the receiver. The receiver acknowledges the request by producing an acknowledgement signal, Ack, and starts reading data from the transmitter at the next positive edge of the clock following the Ack signal. Once the data transmission ends, the transmitter keeps sending the last data packet on the Data bus.

(a) Assume the transmitter sends out three consecutive data packets, D0, D1, D2. Complete the timing diagram below for the Ack and Data signals. Show which data the receiver actually receives.

(b) In this case, the transmitter sends out three consecutive data packets, D0, D1, D2, again, but uses a slower clock. Complete the timing diagram below for the Ack and Data signals, and show which data the receiver acquires.

8. A black-and-white display supports 256 shades of gray, ranging from white to black. The physical display frame has 100 pixels in the x-direction and 100 lines in the $y$-direction, and it requires a frame rate of 100 frames $/ \mathrm{sec}$. The display needs no horizontal or vertical blanking pixels to synch with the display adaptor. The clock frequency of the display adaptor is set to 1 MHz . The display adaptor is connected to an eight-bit wide high-speed bus operating at 10 MHz to receive data. Once the adaptor recognizes that one of its buffers is empty, it immediately sends a request signal to the bus arbiter to own the bus. The acknowledgement from the bus arbiter requires 18 ms delay due to heavy bus traffic. Once the acknowledgment is received, the display adaptor fills all of its buffers. Each buffer contains exact number of pixels to fill only one frame.
(a) With the timing specs defined above, determine the number of buffers that need to be used in this system. Draw a timing table that shows how these buffers are periodically emptied and filled following an 18 ms bus waiting period. Note that this is not a timing diagram that includes the frame clock, bus clock or propagation of data.
(b) Draw an architectural data-path of the display adaptor including the buffers, the gray-scale frame and the related hardware (counters, multiplexers, controller etc.). Make sure to generate all the internal I/O signals of the controller to operate the arbiter and maintain the proper data flow in the display adaptor.
9. A display adaptor has an overlay feature where an overlay image is mapped over the active image as long as the overlay image area is smaller than the active image area. The system neither requires any blanking space nor needs a dual buffering scheme.
Assume the pixels in the image and overlay buffers are not separated into RGB components, but rather integrated into single pixels when they are taken out of these buffers to feed the frame. There is a certain synchronization mechanism between the image and the overlay address counters, and also between the pixel and line counters. As soon as a pixel is fetched from the image or overlay buffer, it is placed in the display frame with the aid of the pixel and the line counter.
There are no write-enable controls for the buffers as these buffers will not be replenished with new pixels once exhausted. The image is displayed only once. The read-enables for both buffers are also kept at logic 1 until the buffers are empty. Therefore, the only mechanism that aids the pixels out of these buffers and moves them to the display frame is incrementing the address counters and switching the selector inputs at the 2-1 buffer MUX. After removing the external reset, the operation of the display unit starts as shown in the timing diagram.


(a) Build the register file. Indicate the programmable values in each register to support the operation of this unit.
(b) Fill in each blank space that corresponds to the numbers in the timing diagram (if there are no numbers, ignore the entries), and complete the control signals, selImage (to select the image buffer), selOL (to select the overlay buffer), IncrPix (to increment the pixel counter), ResetPix (to reset the pixel counter to 0 ), IncrLine (to increment the line counter) and IncrOL (to increment the overlay counter).
Use the following notation to represent the data when retrieving them from the data buffers:
For the Image buffer, Data $=\mathrm{IM}$ [Image Buffer Address]
For the Overlay buffer, Data $=$ OL [OL Buffer Address]
10. An interrupt controller interfaced with a three-stage RISC CPU is shown below. Once an external interrupt (INTR0 to INTR15) is generated, the interrupt interface selects the highest priority interrupt and generates a single interrupt (INTR) bit for the Interrupt Control Unit (ICU). The ICU acknowledges the interrupt with an interrupt acknowledge signal (INTA), which prompts the interface to send a four-bit interrupt ID (INTRID) to the Interrupt Address Table (IAT). A 32-bit interrupt address is then produced from the IAT which causes the program counter (PC) to jump and execute an Interrupt Service Routine (ISR) program in the instruction memory. Before the ISR is executed, the remains of the original program in the CPU pipeline have to be to be executed and stored in the register file (RF). Also, the address of the next instruction in the user program is stored in R31 in the RF. Upon the completion of a particular ISR, the program returns to its original location by retrieving the address stored in R31 and executes the rest of the user program.
In this particular case, $\operatorname{INTR} 0=1$, as the highest priority interrupt, prompts the interrupt interface to generate INTRID $=0$. This is the beginning of a four instruction long ISR0 cycle that starts at the address 100 as shown in the instruction memory.
Assuming there are a total of seven instructions in the user program, each instruction written back to the RF is labeled as WUi. For example, the results of Ins1 are written back to the RF as WU1, the results of Ins2 as WU2 etc. as shown in the timing diagram. Similarly, each ISR instruction written back to the RF is labeled as WIi. For example, the results of Intr1 are written back to the RF as WI1, the results of Intr2 as WI2 etc. Also, once the PC generates a value, each instruction produces an RF output as DUi. For example, Ins1 produces DU1, Ins2 produces DU2 etc. Similarly, each interrupt instruction produces an RF output as DIi. For example, Intr1 produces DI1, Intr2 produces DI2 etc.
Based on the inputs in the preceding paragraphs, fill the blanks in the timing diagram below, and indicate when each write takes place to the RF with a little arrow.


11. Design an SPI (see Chapter 4 on serial bus) interface using an integrated transmitter and receiver shown below. The eight-bit Shift Register at the interface transmits one-bit data from the SDO port at the negative edge of SCK, and simultaneously receives one-bit data from the SDI port at the following positive edge. The transmit data is first loaded to the SPI Register using an eight-bit system bus. Subsequently, the contents of the SPI Register are loaded to the Tx Buffer if the buffer is empty and then to the Shift Register when it requires new data. In a similar fashion, when the Shift Register acquires new eight new bits through its SDI port, it transfers its contents to Rx Buffer first and then to the SPI Register. When the transmit function is desired, the received data is considered junk data. When the receive function is desired, the transmit data is, in turn, assumed junk. The two flags, TxF and RxF, update the SPI status register and indicate if the TxBuffer and the RxBuffer are empty or not.
Design the SPI data-path and the controller using timing diagrams. The SCK applied to the shift register is assumed to be a slower clock, and it has a period of eight system clock periods. The designer should feel free to alter the design and add additional hardware or signals when necessary to conclude a different implementation.


## Projects

1. Implement and verify the DMA in Fig. 7.7 that supports two identical $32 \times 64$ SRAM memories using Verilog. The DMA should produce a timing diagram similar to the one shown in Fig. 7.6 and include a controller in Fig. 7.8.
2. Implement and verify the interrupt controller in Fig. 7.15 that supports 256 hardware interrupts using Verilog. Include the hardware for context switching, i.e. transferring the contents of the entire register file to a temporary buffer prior to executing interrupt service routine instructions.
3. Implement the one-shot timer in Fig. 7.24 using Verilog. Produce its timing diagram as shown in Fig. 7.25.
4. Implement the rate generator in Fig. 7.26 using Verilog. Produce its timing diagram as shown in Fig. 7.27.
5. Implement and verify the display adaptor unit in Fig. 7.34 that supports a screen with eight pixels, two blanking lines and nine active image lines as shown in Fig. 7.35.

## References

1. Microchip Technologies, dsPIC33FJ128MCX02/X04 datasheet, Timer1, pp 195-204
2. Philips TM1000 PCI media processor preliminary data book (1997) Chapter 7, pp 1-19

## Special Topics

This chapter introduces two core topics that may be part of a computing system. The first topic is a brief introduction to programmable logic. The second topic is the analysis of a basic data-driven processor that operates with arrival of new data.

When it comes to prototyping an application-specific digital block, the first thing that comes to mind is the Field-Programmable-Gate-Array (FPGA) platform. This platform is flexible enough to implement any combinatorial, sequential or asynchronous logic with ease. Using programmable logic, we can create mega cells such as ALU blocks or simple memories, logic blocks that perform specific functions, processors, even a System on Chip (SoC) platform using a Hardware Design Language (HDL).

The second topic in this chapter describes a data-driven architecture that works with a cluster of simple processors. Each processor in the cluster is designed to carry out specific task(s), and each becomes active when valid data arrives from a neighboring processor. In a data-driven system, either an individual processor carries out a specific task and transfers the result to the next processor or every processor in the cluster simultaneously execute many different tasks all at once to produce a single result.

### 8.1 Field-Programmable-Gate Array

The basic idea behind the Field-Programmable-Gate-Array (FPGA) architecture is the use of Look-Up-Tables (LUT) [1, 2]. A typical three-input LUT in Fig. 8.1 contains eight registers to store bits, an 8-1 MUX to select one of the eight register outputs, and a flip-flop at the output of the 8-1 MUX to implement sequential logic. The programming phase consists of serially distributing the desired bit values to all eight registers through the ProgIn port when the Prog input is set to logic 1. This is achieved by using all eight LUT registers in a shift register configuration, and shifting an eight-bit data from $\operatorname{Bit}[0]$ to $\operatorname{Bit}[7]$ at each positive edge of the clock. The bottom register at the Bit[7] position has another output, ProgOut,


Fig. 8.1 Three-input look-up-table (LUT) block diagram
connected to the ProgIn input of another LUT such that every LUT on the FPGA chip can be serially programmed using a single wire to save wiring space.

In normal operation, bits stored in the LUT registers constitute the output values of the truth table in Table 8.1. The inputs of the truth table, on the other hand, are the selectors of the 8-1 MUX, from LUTIn[0] to LUTIn[2], in Fig. 8.1. Therefore, any arbitrary truth table can be produced simply by programming the LUT registers, needing no other conventional logic gate. For example, when LUTIn[2] $=\operatorname{LUTIn}[1]=\operatorname{LUTIn}[0]=0$ in Fig. 8.1, the 8-1 MUX routes the value stored in Bit[0] to its output. Similarly, LUTIn[2] $=\operatorname{LUTIn}[1]=\operatorname{LUTIn}[0]=1$ combination routes Bit[7] to the output. The 2-1 MUX is used to bypass the flip-flop output if a combinational logic implementation is preferred.

The number of registers in a LUT is determined by the number of outputs in the truth table. For example, if there are three inputs in the truth table, this combination generates $2^{3}=8$ possible outputs. Therefore, the LUT must contain eight registers. In general, N inputs require $2^{\mathrm{N}}$ registers in a LUT.

In summary, in order to implement a logic function using FPGA, the inputs of a logic function (truth table) must be applied to the MUX selectors, and the outputs must be stored in the LUT registers according to Table 8.1.

To demonstrate how a combinational logic block is implemented in an FPGA platform, we will design a four-bit Ripple-Carry-Adder (RCA) as shown in Fig. 8.2. The circuit consists of four full adders all connected serially to propagate the carry bit from right to left. The sum outputs, from SUM0 to SUM3, and the carry-out ports, from Cout0 to Cout3, have to be generated by programming in the LUT registers.

Figure 8.3 describes how a full adder sum output is stored in a three-input LUT. This process is the same to generate each sum output, from SUM0 to SUM3. In this figure, the LUT output value at the first row (logic 0 ) is stored in the Bit[0] position, and the last output entry at the last row (logic 1) is stored in the Bit[7] position. This bit arrangement in the LUT

Table 8.1 Three-input LUT truth table (when bypass port is set to 1 )

| LUTIn[2] | LUTIn[1] | LUTIn[0] | LUTOut |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | $\operatorname{Bit}[0]$ |
| 0 | 0 | 1 | $\operatorname{Bit}[1]$ |
| 0 | 1 | 0 | $\operatorname{Bit}[2]$ |
| 0 | 1 | 1 | $\operatorname{Bit}[3]$ |
| 1 | 0 | 0 | $\operatorname{Bit}[4]$ |
| 1 | 0 | 1 | $\operatorname{Bit}[5]$ |
| 1 | 1 | 0 | $\operatorname{Bit}[6]$ |
| 1 | 1 | 1 | $\operatorname{Bit}[7]$ |



$$
\begin{aligned}
& \text { SUM0 }=\mathrm{A} 0 \oplus \mathrm{~B} 0 \oplus \mathrm{Cin} 0 \\
& \text { SUM1 }=\mathrm{A} 1 \oplus \mathrm{~B} 1 \oplus \mathrm{Cin} 1 \\
& \text { SUM2 }=\mathrm{A} 2 \oplus \mathrm{~B} 2 \oplus \mathrm{Cin} 2 \\
& \text { SUM3 }=\mathrm{A} 3 \oplus \mathrm{~B} 3 \oplus \mathrm{Cin} 3
\end{aligned}
$$

$$
\text { Cout0 }=\mathrm{A} 0 . \mathrm{BO}+\mathrm{Cin} 0 .(\mathrm{A} 0+\mathrm{BO})
$$

$$
\text { Cout1 = A1.B1 + Cin1. }(\mathrm{A} 1+\mathrm{B} 1)
$$

$$
\text { Cout2 }=\mathrm{A} 2 . \mathrm{B} 2+\mathrm{Cin} 2 .(\mathrm{A} 2+\mathrm{B} 2)
$$

$$
\text { Cout } 3=A 3 \cdot B 3+C i n 3 .(A 3+B 3)
$$

Fig. 8.2 Four-bit ripple-carry-adder
registers implements the SUM function if Cin, A and B are applied to the 8-1 MUX as selector inputs. The bypass input at the output 2-1 MUX must also be set to logic 1 to bypass the flip-flop stage since this design is not a sequential circuit.

The Cout function of the full adder is implemented similarly as shown in Fig. 8.4. The Cout function in the last column of the truth table is programmed in the LUT registers while Cin, A and B are applied to the $8-1$ MUX as selector inputs. The bypass bit is also set to logic 1 to bypass the flip-flop since the implementation is purely combinational.

Figure 8.5 shows the FPGA implementation of the four-bit RCA in Fig. 8.2 after the programming phase is complete. In this design, each FPGA cell, called a cluster, is assumed to contain two LUTs. While A0, A1, A2, A3, B0, B1, B2, B3 and Cin0 are external input pins for the four-bit RCA, Cin1, Cin2 and Cin3 inputs are all internally generated from Cout0, Cout1 and Cout2 function blocks, and routed between clusters to maintain interconnectivity. All the bypass inputs, from bypass-Cout0 to bypass-SUM3, have to be at logic 1 and stored in a separate LUT during programming phase.


Fig. 8.3 Programming the full adder SUM output with a three-input LUT


Fig. 8.4 Programming the full adder Cout output with a three-input LUT


Fig. 8.5 Four-bit ripple-carry-adder data-path in FPGA

A commercial FPGA cluster contains a multitude of multiplexers connected to the LUT inputs to accomplish the maximum flexibility in logic configuration [3]. Figure 8.6 shows one such cluster configuration that contains two LUTs, each with three inputs. A cluster configured this way is able to achieve maximum networking capability with other clusters in addition to implementing many types of combinational and sequential logic circuits. Programming these LUTs is achieved simply by connecting the ProgOut port of one LUT to the ProgIn port of the neighboring LUT, and feeding the serial data from the ProgIn port as shown in the figure.


Fig. 8.6 A commercial FPGA cluster containing two LUTs per cluster

The detailed schematic in Fig. 8.7 shows the FPGA implementation of Cluster 0 in Fig. 8.5 but uses a commercial FPGA architecture in Fig. 8.6. Each 3-1 MUX selector input from SelOut0[0] to SelOut1[5] is stored in a 12-bit shift register (LUT2) in Fig. 8.7.


Fig. 8.7 Implementing SUM0 and Cout0 using a single cluster containing two LUTs

In this figure, $\operatorname{SelOut1[i]}=0$ and $\operatorname{SelOut} 0[\mathrm{i}]=1$ combination selects port 1 of the 3-1MUX where i changes from 0 to 5 . Similarly, SelOut1[i] $=1$ and $\operatorname{SelOut0[i]~}=0$ combination selects port 2 . When SelOut1[i] $=\operatorname{SelOut} 0[\mathrm{i}]=0$, the default port 0 is selected, and an external input becomes one of the selector inputs for the 8-1 LUT MUX. The bypass pins, bypass0 and bypass1, are also stored in a two-bit shift register (LUT3).

A Moore or Mealy type state machine can also be implemented using an FPGA platforms. The state diagram in Fig. 8.8 produces the transition table in Table 8.2, which includes two next state outputs, NS0 and NS1, two present state inputs, PS0 and PS1, an external input, IN, and a present state output, OUT[2:0], to produce integer values between one and four. For state assignments only one bit is allowed to change between neighboring states, i.e. $\mathrm{S} 0=00, \mathrm{~S} 1=01, \mathrm{~S} 2=11$ and $\mathrm{S} 3=10$. The resultant circuit in Fig. 8.9 shows the locations of the present and next states, the input and the output.


Fig. 8.8 A Moore machine
Programming the NS0 function in Table 8.2 in a three-input LUT configuration is explained in Fig. 8.10. In this figure, the NS0 column is distributed among eight LUT registers, storing the first bit of NS0 in a LUT register at the Bit[0] position, and the last bit of NS0 in a register at the Bit[7] position. The inputs, PS0, PS1 and IN, that generate NS0 are connected to the 8-1 MUX selector pins, LUTIn[0], LUTIn[1] and LUTIn[2], respectively. Since the bypass input pin is set to logic 0 , the NS0 node becomes the input of the flip-flop, and the PS0 node becomes the output.

Table 8.2 The transition table for the Moore machine in Fig. 8.8

| IN | PS1 | PS0 | NS1 | NS0 | OUT[2] | OUT[1] | OUT[0] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |



Fig. 8.9 The circuit diagram for the Moore machine in Fig. 8.8


Fig. 8.10 Programming the PS0/NS0 output with a three-input LUT

The NS1 functionality is implemented in a similar fashion as shown in Fig. 8.11. The NS1 column in Table 8.2 is stored in the LUT registers. PS0, PS1 and IN inputs are connected to LUTIn[0], LUTIn[1] and LUTIn[2] MUX selector pins, respectively. The bypass pin is set to logic 0 in order to form NS1 node at the input of the flip-flop and PS1 node at the output.


Fig. 8.11 Programming the PS1/NS1 output with a three-input LUT

The OUT[0], OUT[1] and OUT[2] outputs are also programmed in the LUT registers according to Table 8.2, and shown in Figs. 8.12, 8.13 and 8.14, respectively. However, the bypass input in each case must be set to logic 1 in order to bypass the flip-flop stage since these outputs are completely combinational and do not require any clock in their paths.


Fig. 8.12 Programming the OUT[0] output with a three-input LUT
$\left.\begin{array}{|ccc|c|}\hline \text { IN } & \text { PS1 } & \text { PSO } & \text { OUT[1] } \\ \hline 0 & 0 & 0 & \left(\begin{array}{c}0 \\ 0\end{array}\right. \\ 0 & 1 & \left(\begin{array}{c}1 \\ 0\end{array}\right. & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1\end{array}\right)$


Fig. 8.13 Programming the OUT[1] output with a three-input LUT


Fig. 8.14 Programming the OUT[2] output with a three-input LUT

Figure 8.15 describes the implementation of the Moore machine in Fig. 8.9 after programming each LUT in three different clusters. Cluster 0 generates NS0 and NS1 functions implicitly but also produces PS0 and PS1 outputs. Cluster 1 and Cluster 2 implement OUT [2:0]. The IN port is the only external input that goes to all three clusters to maintain the logic functionality. All the bypass inputs from bypass-PS0 to bypass-OUT2 are stored in a separate LUT.


Fig. 8.15 Moore state machine data-path in FPGA
The schematic in Fig. 8.16 shows the implementation of Cluster 0 in Fig. 8.15 in a commercial FPGA platform in Fig. 8.6. In this schematic, all 3-1 MUX selector inputs from SelOut0[0] to SelOut1[5], are stored in LUT2. Similarly, the bypass inputs, bypass0 and bypass1, are stored in LUT3 to be used during normal operation.


Fig. 8.16 Implementing PS0 and PS1 using a single cluster containing two LUTs

### 8.2 Data-Driven Processors

## Data-Flow Graphs

Programming data-driven processors is achieved by data-flow graphs [4]. Each graph consists of a group of functional nodes and communication paths, connecting the nodes. Each node in the flow-graph executes two incoming data tokens. When they arrive, the node produces an output operand according to the function defined in the node. Therefore, a node function can simply be defined by an instruction. Each instruction representing a functional node contains an operation code (OPC), input operand(s) (OPER), and output operand node addresses. All input-output paths among functional nodes are connected with directed communication paths to guide the flow of data. With this picture in mind, operands that flow into a functional node are executed according to the node's operation code. Once executed in the node, new operands form and flow out of the node to other nodes. A simple example is given in Fig. 8.17. In this example, the operands, $\mathrm{OPER}_{\mathrm{S} 1}$ and $\mathrm{OPER}_{\mathrm{S} 2}$, are executed by the node's operation code, $\mathrm{OPC}_{\mathrm{S}}$, when they arrive at the functional node, $\mathrm{N}_{\mathrm{S}}$. After the execution, new operands form and flow out of the node to two new destination nodes, $\mathrm{N}_{\mathrm{D} 0}$ and $\mathrm{N}_{\mathrm{D} 1}$, where they meet with two other operands, $\mathrm{OPER}_{\mathrm{D} 0}$ and $\mathrm{OPER}_{\mathrm{D} 1}$, respectively. The data-flow program stops when all operands are executed.


Fig. 8.17 Sample flow graph for a data-driven machine
The parallel nature of data-flow architecture makes parallel processing tasks quite achievable in data-driven machines. While conventional processors are set to be maximally serial to minimize hardware, data-driven architectures can be maximally parallel with up to one
processor per operation to maximize performance. For the example in Fig. 8.17, two different data-driven processors can be used simultaneously to perform $\mathrm{OPC}_{\mathrm{D} 0}$ and $\mathrm{OPC}_{\mathrm{D} 1}$ following the operation at the node $\mathrm{N}_{\mathrm{S}}$. Multi-processor platforms formed by a group of conventional processors may have limitations to achieve certain parallel processing tasks. In contrast, data-driven processors can time-share the processing load of several functional nodes, and therefore reduce the hardware requirement to implement a data-flow graph.

## Data-Flow Node Types

The types of data-flow nodes used in this architecture are classified according to the number of input operands fed into the functional nodes. Figure 8.18 illustrates this classification.



Fig. 8.18 Functional node types
Functional nodes that accept no inputs are nodes with constants (CONS). The contents of this node type do not change during programming. Functional nodes that accept one input are the unitary (UNIT) nodes. This node type transforms an operand as soon as it arrives at its input. Invert (negate), Set and Reset are the unitary nodes in this architecture that require a single input operand. Functional nodes that accept two input operands are the combinatorial (COMB) nodes. This node type executes incoming operands when they are both valid at the input. The operation of some combinatorial functional nodes, such as subtract and shift, depends on the relative placement of the input operands. This is called the operand polarity. It is reflected in the instruction format by defining input operands as left or right input operands. In this architecture, operands with changing data values are directed to the left side of a functional node, whereas constant operands or operands used as control signals are placed on the right side of the functional node in a data-flow diagram.

While operands emanating from functional nodes are forwarded only to one destination address in the earlier data-flow diagrams, this architecture offers the flexibility where the same operand can be forwarded to two different destination addresses. This unique feature saves the number of nodes used in the data-flow diagram as well as increases execution speed of the program.

## Basic Data-Flow Program Structures

There are three basic programming structures in data-driven architectures when constructing data-flow diagrams: sequential, conditional and recursive. Each structure is illustrated in Fig. 8.19.

Sequential programming constructs imply that data-flow is unidirectional, from one functional node to the next without any loops or paths related to a condition. The simple data-flow diagram in Fig. 8.17 is one such example of the sequential programming structure. Another example composed of multi-layer functional nodes is given in the top figure of Fig. 8.19.

The conditional programming structure consists of a functional node that accepts a conditional input besides a data input. If the condition is satisfied, a valid operand at the data input becomes a valid operand at the output. If the condition is not satisfied, the operand at the output retains its old value. The conditional input enters the node from the right side since it is considered to be a control input as mentioned earlier. Gate and Compare instructions are considered conditional since the data-flow produced at the output of the node depends on whether the condition is satisfied or not. The output operand does not change its value until the condition is satisfied. An example of this type is shown in the middle figure of Fig. 8.19.

The recursive programming structure contains looping constructs in the form of a feedback path from one node to another as shown in the bottom figure of Fig. 8.19. The number of iterations in a loop continues until all input operands are exhausted. The loop can be broken to allow a conditional node if needed, otherwise the loop is activated on the arrival of new operands either from the right side or from the left side of the node.

A simple example in Fig. 8.20 combines all the programming structures mentioned above. This example calculates the area under a straight line, $\mathrm{Y}=(\mathrm{X}-1)$, from $\mathrm{X}=2$ to $X=3$. The increment in the $x$-axis is defined to be DeltaX, which is equal to 0.1 in the flow chart.

The flow chart in Fig. 8.20 has been transformed into a data-flow graph shown in Fig. 8.21. All the nodes with constants in the data-flow graph in Fig. 8.21 are zero-input nodes, which accept no operands. The only unitary functional node is the one with the SET operation code. Upon the arrival of an operand to its single input, this node generates logic 1 at its output. Otherwise, its output stays at logic 0 .

There are two types of two-input functional nodes in the same data-flow diagram. The majority of these nodes are conditional type: they either wait for a condition to arrive (GATE) or they have a permanent condition attached at their right port in terms of a constant. Greater-Than-Or-Equal-To (GE), Less-Than (LT), Add-with-Constant (ADC), and Multiply-with-Constant (MULC) functional nodes belong to the latter category.

The rest of the two-input nodes are sequential such as Add node (ADD) where polarity information is not important for data execution.


Fig. 8.19 Data-flow programming structures


Fig. 8.20 Flow chart integrating the area under $Y=(X-1)$
During the programming phase, the initial values of Xstart, Ystart and SUMstart are stored at the proper arcs in Fig. 8.21. When program execution starts, Xstart $=2$ is added to a floating-point constant, $\mathrm{C}=0.1$, at the nodes 0 and 1 , generating the first value of X . While X is compared against $\mathrm{C}=3$ at the nodes 3 and 4 , it is also directed to the node 2 for a Gate operation, and added to $\mathrm{C}=-1$ at the node 5, producing Y. Subsequently, Y is directed to the node 6 to be added to Ystart $=1$, and to the node 7 for another Gate operation. The output of the node 6 multiplies with $\mathrm{C}=0.05$ at the node 8 , producing the first incremental area value, and it is directed to the node 9 to be added with SUMstart $=0$. The output of the node 9 , SUM, is then forwarded to the nodes 10 and 11 for two other Gate operations. Depending on comparisons at the nodes 3 and 4, the SUM output will either be forwarded outside of the processor or forwarded to the node 12 in order to set this node. If the node is set, Gate operations at the nodes 2, 7 and 11 take place, replacing the old values of Xstart, Ystart and SUMstart with X, Y and SUM, respectively. Iterations continue until the node 10 becomes active and the result is delivered to the user.


Fig. 8.21 Data-flow graph integrating the area under $Y=(X-1)$

## Input Flags

An input operand to a functional node contains an operand flag to indicate whether or not the data processing is complete. A high flag implies that the input operand is valid and ready to be processed. The input operand flag goes to logic 0 as soon as the functional node processes the input operand.

## Nodal Networks

Data-flow diagrams in this architecture can be structured in three different ways. The first is a direct connection among functional nodes: data flows from one node to another freely in an unobstructed fashion as shown at the top left corner of Fig. 8.22. The only control mechanism for processing data at each functional node is that both input operand flags must be at logic 1 . The second and the third nodal networks use programmable routers to send operands from the source to the destination nodes. The simple router at the top right corner of Fig. 8.22 uses a local network to connect a group of functional nodes and thereby creates a cluster. Note that the inputs to a functional node in a cluster can come from any functional node in this network. This decision is made by a simple arbitration scheme in the router, which dictates that any node in the process of generating a new input operand for itself has priority over the other nodes in a cluster. In other words, if a neighboring node produces an input operand for a particular node in a cluster while this particular node is in the process of generating an input operand for itself, the arbiter stalls any data processing in the neighboring node until the self-operand generation is complete. The bottom structure in Fig. 8.22



Fig. 8.22 Data-flow graphs without router, and with local and hierarchical cluster networks
illustrates the hierarchical organization of clusters where an inter-cluster network manages many local cluster networks. While each cluster arbiter manages its own individual cluster, all cluster-to-cluster communication is maintained by a separate inter-cluster arbiter.

## Processor Design Overview

The processor implements the node functionality by reading the node instruction from the memory, executing it, and writing the result back to the two destination node addresses specified in the instruction. In order to implement this sequence, each processor needs to have a memory, an ALU and a controller. The memory contains all nodal instructions. Each nodal instruction consists of two input operands with their valid flags, an operation code and the two destination node addresses where the results are sent as shown in Fig. 8.23.

| 17 | 1 | 17 | 1 | 5 | 1 | 6 | 1 | 1 | 6 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OPER ${ }_{\text {L }}$ | $\mathrm{F}_{\mathrm{L}}$ | OPER ${ }_{\text {R }}$ | $\mathrm{F}_{\mathrm{R}}$ | OPC | $\mathrm{V}_{0}$ | $\mathrm{N}_{0}$ | $\mathrm{P}_{0}$ | $\mathrm{V}_{1}$ | $\mathrm{N}_{1}$ | $\mathrm{P}_{1}$ |
| L |  |  |  |  |  |  |  |  |  |  |
| Left Operand |  |  | Right |  | Destination Destination |  |  |  |  |  |
|  |  |  | Address 0 Address 1 |  |  |  |  |  |
| Flag |  |  |  |  |  |  |  |  |  |  |

Fig. 8.23 Instruction format

Consider a processor implementing a single node. Initially, assume the processor is at idle. When both operand flags in the instruction become valid, the controller starts. In the first step, the controller generates a nodal address for the instruction that resides in the memory. In the second step, the controller fetches the input operands and operation code from this instruction and forwards them to the ALU. The ALU combines the input operands and generates an output operand, which is directed to the first destination address. In the third step, the controller writes the same output operand to the second destination address, sets the operand flag at the first destination address, and clears input operand flags at the source address. In the fourth and final step, the controller sets the operand flag at the second destination address.

If the processor needs to execute more than one node, then each nodal address in the flow graph must be mapped to a physical address in the memory. This approach automatically transfers the left and the right nodal operands, the operation code and the destination addresses of a particular node from the flow graph to an instruction in the memory. However, during this process each operand flag is stored in a separate tag memory to allow the controller to continuously search for valid operand flags. If the controller finds a node with valid left and right operand flags, it sends the corresponding operands to the ALU for execution. After the operands are processed and sent to the destination addresses in the instruction, the controller points the next node to be processed. If there is no other node with
valid operand flags, the controller stalls the processor until an instruction with valid operand flags emerges in the instruction memory.

The processing efficiency and speed in the processor can be increased by pipelining. After an instruction is executed, sending the ALU result to a destination address can be overlapped with tasks such as generating an address or fetching a different instruction. This can be achieved using a dual port memory.

## Instruction Format

In this architecture, the instruction format contains two 17-bit input operands, $\mathrm{OPER}_{\mathrm{L}}$ and $\mathrm{OPER}_{\mathrm{R}}$, two flags to validate the operands, $\mathrm{F}_{\mathrm{L}}$ and $\mathrm{F}_{\mathrm{R}}$, one five-bit operation code, OPC, and two eight-bit destination address fields. Each destination address is composed of a valid bit, V, a six-bit node number, N, and a left-right polarity bit, P. The valid bits in the destination address fields, $\mathrm{V}_{0}$ and $\mathrm{V}_{1}$ indicate the validity of the corresponding nodal address. This instruction format is shown in Fig. 8.23.

## Architecture and Operation

Figure 8.24 shows a simplified block diagram of a processor executing the simple program in Fig. 8.17. Each node number in the data-flow graph in Fig. 8.17 corresponds to an address in the instruction memory in Fig. 8.24.

The program execution starts when the controller detects an instruction with valid left and right operand flags, such as the one at the memory location $\mathrm{N}_{\mathrm{S}}$ with $\mathrm{F}_{\mathrm{L}}=\mathrm{F}_{\mathrm{R}}=1$. The controller reads out the instruction and sends the operands, $\mathrm{OPER}_{\mathrm{S} 1}$ and $\mathrm{OPER}_{\mathrm{S} 2}$, and the operation code, $\mathrm{OPC}_{\mathrm{S}}$, to the ALU. The ALU executes the operands according to the $\mathrm{OPC}_{\mathrm{S}}$, and the controller clears both operand flags of the instruction, underlining the completion of this instruction. The controller subsequently sends the ALU result to the first and second destination addresses, $\mathrm{N}_{\mathrm{D} 0}$ and $\mathrm{N}_{\mathrm{D} 1}$, as data tokens as shown in Fig. 8.24. When the result is delivered to a valid destination address, the controller automatically sets the operand flag to specify the validity of data for further processing. For example, the right operand flag at $\mathrm{N}_{\mathrm{D} 0}$ is set when $\mathrm{OPER}_{\mathrm{A}}=\mathrm{OPER}_{\mathrm{S} 1}$ $\left(\mathrm{OPC}_{\mathrm{S}}\right) \mathrm{OPER}_{\mathrm{S} 2}$ is delivered to this address. Similarly, the controller sets the left operand flag at $\mathrm{N}_{\mathrm{D} 1}$ when it delivers the same ALU result to this address in the next cycle.

After executing the instruction at $\mathrm{N}_{\mathrm{S}}$, the controller detects the next instruction with valid operand flags at the memory location $\mathrm{N}_{\mathrm{D} 0}$. Once again, the controller extracts the operation code, $\mathrm{OPC}_{\mathrm{D} 0}$, and the operands, $\mathrm{OPER}_{\mathrm{D} 0}$ and $\mathrm{OPER}_{\mathrm{A}}$, from the instruction and sends them to the ALU for execution. Subsequently, the controller clears the operand flags at $\mathrm{N}_{\mathrm{D} 0}$ and sends the ALU result as a left operand to the memory address $\mathrm{N}_{\mathrm{K}}$. This is shown as the third data token in Fig. 8.24. The controller performs the same set of tasks for the instruction located at the address $\mathrm{N}_{\mathrm{D} 1}$ and forms the fourth data token. The program execution stops when the controller can no longer find a pair of valid operand flags in the tag memory.


Fig. 8.24 Processor architecture executing the program in Fig. 8.17

## Implementation

Figure 8.25 shows the implemented instruction field format. Besides the operation code and the input operand fields, each destination address in Fig. 8.23 is now expanded to contain an additional seven-bit processor ID, ProcID, and a cluster ID, ClusID, to allow multiple processor communication in a local network. In this architecture, we have the flexibility of choosing a network ranging from 128 processors in a single cluster to two processors per cluster for 64 different clusters. The presence of ClusID and ProcID enables independent but simultaneous networking activities to take place among clusters and processors. In other


Fig. 8.25 Instruction format in the implementation
words, the source processor can write the same ALU result to two different destination processors within the same cluster or in different clusters.

## Processor Micro-architecture

The simplified data-driven processor architecture shown in Fig. 8.24 is implemented in Fig. 8.26. One of the essential elements in Fig. 8.26 is the presence of a dual-port RAM. While the controller fetches an instruction from the first data port, it writes the ALU result of another instruction to the second port to increase processor performance and programming efficiency.


Fig. 8.26 Processor data-path and micro-architecture
In this architecture, all operand flags are stored in a separate tag memory in the processor. The left and right operand flags at each tag address are AND-gated and connected to the node address generator as inputs. When the operand flags that belong to a specific tag memory address become valid, the address generator uses this tag address as a read address, RAddress, to read the corresponding instruction from the dual-port RAM. If there is more than one set of valid flags in the tag memory, the node address generator selects a pair of flags with the lowest nodal address value, and delivers this address to the memory. When there are no more valid flags, the address generator produces a NoSelect signal for the
network arbiter which, in turn, stalls the processor. From this moment on, the processor suspends all its activity and waits for new operands to be delivered to the instruction memory.

When a processor interacts with other processors in a network, it is quite possible that it may receive a hold request from the network arbiter while attempting to write into a neighboring processor. This hold signal is generated because the neighboring processor may be busy processing data for itself or writing data to another processor as mentioned earlier. When the source processor receives a hold signal, it stalls all its processing functions and retains its internal status and output data values until the hold is removed. Therefore, RAddress is also stored in the node retainer to preserve the node address, $\mathrm{N}_{\mathrm{S}}$, and the source operand polarity, $\mathrm{P}_{\mathrm{S}}$, for the tag memory in case the network arbiter issues a hold.

Once the instruction at RAddress is read from the RData port of the dual-port memory, its right and left operands are routed to the ALU along with the operation code for execution. The source operands are also stored in the operand retainer in case the program execution is put on a momentary hold by the network arbiter. Both of the destination addresses, WAddr0 and WAddr1, are buffered in the write address register and used alternately to deliver the processed data from the Out terminal to destination processors. The write address register also keeps the old write addresses for the duration of hold.

The processed data either produced locally or from other processors in a network is eventually written to the dual-port RAM through the WData port. The destination address at the WAddress port simply accompanies the newly arrived data, and it is directed to the tag memory to update the corresponding operand flags.

## Processor Programming

Prior to the program execution, instructions are loaded to the dual-port RAM through the WData port in Fig. 8.26. While the program is loaded to the memory, operand flags of each instruction are also stored in the tag memory.

## Inter-processor Arbiter and Router

In a data-driven architecture, the organization of processors in a network is hierarchical. A group of processors form a local cluster, in which each individual processor owns a processor ID, ProcID, to communicate with other processors using a simple arbitration protocol.

A local cluster has also an identification number, ClusID, in a network. Only one processor in a cluster can communicate with another processor in a different cluster at a given time.

The inter-processor router connects each processor's destination address and data output to other processors address and data ports with a massive multiplexing network as shown in Fig. 8.27. The arbiter is designed to give address and data transfer privileges to a single processor while issuing a hold to all lower-priority processors in a cluster. There are two general rules observed in the arbiter's priority scheme. The first rule states that if a processor


Fig. 8.27 Inter-processor arbiter and router
issues a write to itself, it has the highest priority over the other processors in a cluster. The second rule is that if two or more processors issue write requests simultaneously to a processor at idle, the highest priority among these processors belongs to the one with the lowest ProcID.

## Review Questions

1. The following sum of products (SOP) function is given:
out $=A C+A B \bar{C}+\bar{B}$
Implement this function using three-input LUTs only.
2. The following product of sums (POS) function is given:
out $=(\mathrm{A}+\mathrm{B}) \cdot(\overline{\mathrm{B}}+\overline{\mathrm{C}}) \cdot(\overline{\mathrm{A}}+\mathrm{B}+\mathrm{C})$
Implement this function using two-input LUTs only.
3. The following truth table needs to be implemented in FPGA.

| $A$ | $B$ | $C$ | out |
| :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Assume the three-input LUT configuration is given below:

(a) Implement the truth table only with three-input LUTs.
(b) Implement the same truth table only with four-input LUTs.
(c) Implement the same truth table only with two-input LUTs.
4. Implement the 3-1 multiplexer below using three-input LUTs.

The definition of the MUX is given below:
If $\operatorname{sel}[1: 0]=00$ or 11 then out $=\mathrm{a}$
If $\operatorname{sel}[1: 0]=01$ then out $=b$
If $\operatorname{sel}[1: 0]=10$ then out $=\mathrm{c}$

5. A simple Moore state machine that consists of two states is given below. After constructing the state and transitional tables, draw the circuit diagram of this state machine using minimal number of logic gates and flip-flops.
Once the logic diagram is finalized, implement the state machine in FPGA using two-input LUTs and three LUTs in a cluster. Draw the architectural diagram of this FPGA platform, including interconnections.
Assign the values $\mathrm{A}=0$ and $\mathrm{B}=1$ to the state machine below.

6. A three-bit counter is given below. The Hold input activates port 1 of the 2-1 MUX to retain the output value. Otherwise, the counter keeps incrementing by one.


Assume that the three-bit adder is simply a ripple-carry adder composed of three full-adders (FA) with sum (Sum) and carry-out (Cout) outputs as shown below.


Sum $[\mathrm{i}]=\mathrm{A}[\mathrm{i}] \oplus \mathrm{B}[\mathrm{i}] \oplus \operatorname{Cin}[\mathrm{i}]$
$\operatorname{Cout}[\mathrm{i}]=\mathrm{A}[\mathrm{i}] . \mathrm{B}[\mathrm{i}]+\mathrm{Cin}[\mathrm{i}-1] .(\mathrm{A}[\mathrm{i}]+\mathrm{B}[\mathrm{i}])$
Implement this circuit using three-input LUTs in FPGA platform.

## Projects

1. Implement and verify the three-input LUT in Fig. 8.1 using Verilog.
2. Implement the four-bit ripple-carry adder in Fig. 8.2 using Verilog. Use three-input LUTs from project 1 to create an FPGA implementation of the adder in Fig. 8.5. Perform functional verification on the entire circuit.
3. Implement the Moore state machine in Fig. 8.8 and the corresponding logic diagram in Fig. 8.9 using Verilog. Use three-input LUTs from project 1 to create an FPGA implementation of the state machine in Fig. 8.15. Perform functional verification on the entire circuit.

## References

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## Appendix: An Introduction to Verilog Hardware Design Language

## A.1. Module Definition

A digital system in Verilog is defined in terms of modules as shown in Fig. A.1. Each module has inputs and outputs with different bit widths. Modules can be integrated to form bigger digital blocks as shown in Fig. A.2. In this figure, the top module contains four smaller modules, each of which has inputs and outputs. They are interconnected with each other to produce much larger system functionality with a new set of external input and output signals.


Fig. A. 1 A typical module in Verilog


Fig. A. 2 Module integration to form a much larger system

When creating a Verilog code to represent a digital block such as in Fig. A.3, the module name is written first. The input and output names are written in parentheses next to the module name as shown below. Their order is not important. The module statement is followed by separate input-output (I/O) statements and the description of the module.


Fig. A. 3 A Verilog module, blockX, with in[3:0] and out
module blockX (out, in);
output out;
input [3:0] in;
/* module description here */
endmodule
For a more specific example let us write a small Verilog code for the two-input AND gate in Fig. A.4.

```
module andgate (out, in1, in2);
```

output out;
input in1, in2;
/* module description here */
endmodule


Fig. A. 4 Two-input AND gate

Now, let us integrate blockX in Fig. A. 3 with the two-input AND gate in Fig. A. 4 to form a larger digital block as shown in Fig. A.5.


Fig. A. 5 A Verilog module integrating blockX in Fig. A. 3 and the two-input AND gate in Fig. A. 4

This new module, called blockY, has different I/O names due to the naming convention used in the schematic in Fig. A.5. As a preference, the I/O names in the module definition statement and in individual I/O statements list the outputs first and the inputs second. However, this is not a requirement but a choice.

In addition, both the blockX and the two-input AND gate need to be instantiated inside the blockY. This is achieved by using the "dotted" convention as shown below. Here, the individual module I/O names are written outside the parentheses; the I/O names used in the top module are written inside the parentheses. The names, i1 and i2, are the instantiation names of the modules, andgate and blockX, respectively.

```
module blockY (out1, out2, a, b, c);
output out1, out2;
input [3:0] a;
input b, c;
andgate i1 (.out(out2),
        .in1(b),
        .in2(c));
blockX i2 (.out(out1),
        .in[0](a[0]),
        .in[1](a[1]),
        .in[2](a[2]),
        .in[3](a[3]));
```

/* the rest of the code here */
endmodule

Basic logic gates (or Verilog primitives) do not need instantiations. The two-input AND gate in Fig. A. 4 could have been written without any "dotted" instantiation in the top module above.

The Verilog primitives are the following gates:
AND, NAND, OR, NOR, XOR, XNOR, BUF, NOT.
Here, BUF corresponds to a buffer, and NOT corresponds to an inverter.
Example: Let us implement one-bit full adder using Verilog.
In a full adder, the sum and carry-out functions are shown as:

```
sum =a }\oplus\textrm{b}\oplus\textrm{cin
cout = a b b + cin. (a + b)
```

In the functional equations above, cin corresponds to the carry-in input and cout corresponds to the carry-out output of the full adder. The terms, $a$ and $b$, are the two inputs to the full-adder.

The schematic to produce sum and carry-out functions are shown in Fig. A.6. The intermediate nodes are named as node1, node2, node3 and node4 as interconnecting nodes.


Fig. A. 6 Hardware implementation of the full adder
The Verilog code below contains no "dotted" instantiations. This style of Verilog code is called structural where only primitive gates are used.

```
module fulladder (sum, cout, a, b, cin);
output sum, cout;
input a, b, cin;
/* structural style module description */
xor (node1, a, b);
xor (sum, node1, cin);
and (node2, a, b);
or (node3, a, b);
and (node4, cin, node3);
or (cout, node2, node4);
```

endmodule

After the Verilog code is written, a test fixture needs to be produced to verify the module. For our example of the one-bit full adder, Fig. A. 7 illustrates the concept of building the test fixture.


Fig. A. 7 Test fixture formation to verify the functionality of the full adder

In this figure, the outputs of the full adder must be declared as the inputs to the test fixture using the wire statement. The outputs of the test fixture are the inputs for the full adder, and they should generate all the verification vectors needed to test the full adder thoroughly. For each test input, the outputs of the test fixture must remain unchanged until a new set of outputs are produced. Therefore, each output has a memory, and uses the reg statement in Verilog. Also, the test fixture neither contains any I/O names in the module definition nor has any input and output statements. Thus, the general structure of the test fixture becomes as follows:

```
module testfixture;
/* all test fixture outputs are declared as reg statements */
/* all test fixture inputs are declared as wire statements */
/* module instantiation here */
/* verification vectors here */
/* display results here */
endmodule
```

The verification vectors are executed only once using the initial statement in Verilog. For a combinational logic such as a full adder, these vectors are simply the inputs of the truth table. The outputs, on the other hand, need to be either displayed or stored in a file in order to be compared against the "expected" outputs. Therefore, for the one-bit full adder test fixture shown in Fig. A.7, the Verilog code becomes as follows:

```
module testfixture;
/* all test fixture outputs are declared as reg statements */
reg in1, in2, carryin;
/* all test fixture inputs are declared as wire statements */
wire sum, carryout;
/* module instantiation here */
fulladder i1 (.sum(sum),
    .cout(carryout),
    .a(in1),
    .b(in2),
    .cin(carryin));
    /* verification vectors here are executed only once due to initial statement */
    initial
    begin
                carryin = 0; in1 = 0; in2 = 0;
        #10
                        in2 = 1;
    #10 in1 = 1; in2 = 0;
        #10
                                    in2 = 1;
        #10 carryin = 1; in1 = 0; in2 = 0;
        #10 in2 = 1;
        #10 in1 = 1; in2 = 0;
        #10 in2 = 1;
        end
        /* display results here */
        endmodule
```

The "\#" sign used in almost every statement inside the initial block indicates a delay function. For example, the code waits for 10 time units after the first line $(\operatorname{cin}=0 ;$ in $1=0$; in $2=0 ;$ ) is executed before the value of in 2 is changed from logic 0 to logic 1 in the second line. Since in1, in2 and cin are all reg statements, their values are retained until changed.

Monitoring the results are achieved by the \$time and \$monitor statements. \$time displays the current simulation time. \$monitor displays the variable value whenever the variable value changes.

For example, \$monitor (\$time, output, input1, input2); statement displays simulation time and the values of output, input1 and input2. On the other hand, \$monitor (\$time,"'output = \%h input $=\% \mathrm{~b}$ ", out, in $\backslash n$ ); statement displays the simulation time, leaves a space between the simulation time and "output" because of the extra comma, displays "output" in hexadecimal format and "input" as binary format. After each set of simulation time, input and output, a new line starts for the second set due to carriage return, "\n", entry. One can also use " 1 " to insert a tab between the terms to achieve separation.

All Verilog files are stored with the ".v" extension after the file name, such as fulladder.v. When executing the Verilog command to simulate multiple files, including the test fixture, the command line should include the top module last and all the remaining modules inside the top module first.

In this full adder example, we need to write fulladder.v first and then testfixture.v because the test fixture contains the full adder module. Therefore, the command line becomes:
verilog fulladder.v testfixture.v
The notation for a comment in Verilog is identical to the ones used in C-programming. For a single line comment, a pair of slashes, "//", is used. For a multiple line comment, the comment starts with a slash and a star, "/*", and ends with a star and a slash, "**".

## A.2. Numbers in Verilog

The numbers in Verilog are represented by three distinct terms:

## size `base value

Note that the tick mark, "", attached to the BASE entry is not apostrophe.
As an example, 32`h3C represents a 32-bit hexadecimal number, 0x0000003C. 8`b1 represents an eight-bit binary number, 00000001 . Any time the size entry is omitted, the size defaults to 32 bits. For example, `h8A corresponds to \(0 x 0000008\) A. The base entry to represent high impedance or floating wire is " \(z\) ". For example, 8 " bz means an eight-bit bus with all its eight bits are floating or zzzzzzzz. Similarly, don't care bits are shown by "x". For example, 4`bx means four wires either at logic 0 or logic 1 , and represents xxxx.

## A.3. Time Directives for Compiler

To mimic propagation delays in Verilog simulation, a timing directive is used. The directive, 'timescale, is preceded by a tick mark to point out that the command is for the compiler to delay the execution of a Verilog statement if the statement starts with the pound mark, "\#".

The `timescale directive is the first line in a Verilog code. The statement does not end with a semicolon and contains two entries separated by a " $/$ " sign. The first entry corresponds to the actual delay. The second entry represents the simulation resolution.

For example, ‘timescale $10 \mathrm{~ns} / 100 \mathrm{ps}$ means each time the compiler sees a " "\#" sign in a Verilog statement, it delays the execution of the statement by multiplies of 10 ns , depending on the number that follows the "\#" sign. The resolution is 100 ps . Therefore, for a delay of 10 ns , the simulation accuracy is $1: 100$.

Let us consider the following structural Verilog code as an example.

```
`timescale 1 ns / 100 ps
module inverter (out, in);
output out;
input in;
not #2 (out, in); //2 ns delay between input and output
endmodule
```

This is a module that represents a single inverter. The propagation delay in the inverter is 2 ns because the number following the "\#" sign in the "not" gate is 2 , and this number is multiplied by 1 ns in the timescale directive. The simulation resolution is 100 ps . Therefore, there is 1:20 accuracy when generating a delay function for the inverter.

Other common compiler directives are the `define and `include statements, and neither ends with a semicolon. The `define statement defines a variable for the Verilog code. For example, in the following Verilog code a variable called inv_delay corresponds a delay of 30 ps with 1 ps simulation resolution.

```
`timescale 10ps / 1ps
`define inv_delay #3
module inverter (out, in);
output out;
input in;
not inv_delay (out, in); //30ps delay between input and output
endmodule
```

The `include statement fetches smaller modules from various directories and includes them into a bigger module for simulation. For example, the following Verilog code brings the full_adder.v module located in the verilog_modules directory to be used in bigger_module.v. `include "verilog_modules/full_adder.v"
‘timescale 10ps / 1ps
`define inv_delay \#3
module bigger_module (out1, out2, in1, in2, in3);
endmodule

## A.4. Parameters

Parameters are used to replace numbers for enhancing the readability of Verilog code. The parameter statement is not a compiler directive. Therefore, it is ended with a semicolon. The statement has only one entry, which attaches a name to a number.

```
parameter name = value;
```

Assume the following example:

```
module alu (out,in1,in2);
output [31:0] out;
input [31:0] in1,in2;
endmodule
```

The value, 31, can be replaced by the name, BUS, using the parameter statement to enhance readability of the Verilog code. Thus,

| module | alu (out, in1, in2); |
| :--- | :--- |
| parameter | BUS=31; |
| output | $[B U S: 0]$ out; |
| input | $[B U S: 0]$ in1, in2; |

endmodule

## A.5. Basics of Structural Verilog Modeling

Structural modeling was introduced in the earlier sections and described how to use basic logic gates in a Verilog code. Structural Verilog eliminates the dotted convention when instantiating Verilog primitives. The logic gates supported by Verilog are:

| AND | NAND |
| :--- | :--- |
| OR | NOR |
| XOR | XNOR |
| BUF | NOT |

For example, a three-input NOR gate with inputs, in 1 , in 2 and in3, is represented by: nor (out, in1, in2, in3);

As another example, a buffer with an input, in, and an output, out, is written as: buf (out, in);

In each structural statement, the output of the logic gate is listed first followed by the inputs.

Tri-state buffers and inverters are represented by conditional structural statements. The statement bufif1 represents a tri-state buffer with an active-high enable as shown in Fig. A.8.


Fig. A. 8 Tri-state buffer with active-high enable

This gate behaves like a buffer when enable $=1$, and becomes an open circuit when enable $=0$. The structural Verilog statement for the tri-state buffer becomes:
bufif1 (out, in, enable);
A tri-state buffer with an active-low enable signal shown in Fig. A. 9 makes this logic gate behave like a buffer when enable $=0$, and an open-circuit when enable $=1$.


Fig. A. 9 Tri-state buffer with active-low enable
The structural Verilog statement for this gate uses a bufif0 statement:
bufif0 (out, in, enable);
Tri-state inverters use the same active-high or active-low enable signals. To represent a tri-state inverter with an active-high signal in Fig. A.10, the notif1 statement is used.
notif1 (out, in, enable);


Fig. A. 10 Tri-state inverter with active-high enable
The tri-state inverter with an active-low enable signal in Fig. A. 11 uses the notif0 statement as shown below.
notif0 (out, in, enable);


Fig. A. 11 Tri-state inverter with active-low enable

## A.6. Behavioral Modeling

There are two types of procedural blocks in behavioral Verilog coding. The first one is called the "initial" statement. Each Verilog statement included in the initial statement is executed only once. The procedural block below shows the general form of the initial statement. The statements in an initial block are enveloped with the "begin" and "end" clauses. The initial statement may or may not come with a condition(s) listed in parentheses after the symbol "@". If the initial statement comes with a condition, the statement is executed when the condition occurs. Otherwise, the program omits the initial statement.

```
initial @ (condition)
    begin
    statement 1; // all statements within the initial statement are executed only once
    statement 2;
    end
```

As an example, let us consider a test fixture that verifies the functionality of a module with two inputs, in1 and in2, and receives three outputs, $\mathrm{a}, \mathrm{b}$ and c , from the module. The test vectors applied to this module need to be executed only once. The form of the initial statement will be as follows:

```
module test;
reg input1, input2;
wire outa, outb, outc;
testmodule i1 (.in1(input1),
                .in2(input2),
        .a(outa),
        .b(outb),
        .c(outc))
initial
    begin
        input1 = 0; input2 = 0;
    #10 input2 = 1;
    #10 input1 = 1; input2 = 0;
    #10 input2 = 1;
    end
```

endmodule

The second type of procedural block is the "always" statement. This statement may also come with a condition in parentheses. Unlike the initial statement, the always statement is executed repeatedly. If the always statement comes with a condition, the execution of statements within the always statement takes place only when the condition is encountered. Otherwise, the program skips over the always statement. The general form of the always statement is shown below.

```
always @ (condition)
    begin
    statement 1; // all statements within the always statement are executed repeatedly
    statement 2;
    end
```

An example is to implement a flip-flop with two inputs, d and clock, and two outputs, q and qbar.

```
timescale 10ps / 1ps
module flip_flop (q, qbar, d, clock);
output q, qbar; // qbar is the inverted output, q
input clock, d;
reg q, qbar;
always @ (posedge clock)
    begin
    #2 q=d;
    #1 qbar = ~d;
    end
```

endmodule

In the Verilog code above, the logical value at the flip-flop output, q , becomes equal to the logical value at the input, $\mathrm{d}, 20 \mathrm{ps}$ after the rising edge of the clock. The logical value at the qbar output waits for the completion of the first statement, and becomes equal to the inverted $d$ input 10 ps after the first statement. This waiting period from one statement to the next arises because these two statements are the blocking type. In other words, when an assignment uses the " $=$ " sign in a statement inside the procedural block, the statement becomes a blocking statement, which blocks the execution of the next statement until it is executed first. The reg statement is also added to this program because values or other variables (inputs in this case) are assigned to the outputs, q and qbar , inside the always procedural block.

The same Verilog code can be rewritten as:

```
`timescale 10ps / 1ps
module flip_flop (q, qbar, d, clock);
output q, qbar;
input clock, d;
reg q, qbar;
always @ (posedge clock)
    begin
    #2 q <= d;
    #3 qbar <= ~d;
    end
```

endmodule

In this program, the first and the second statements in the always block become non-blocking type due to the "<=" sign, and they are executed simultaneously. Therefore, the output, q , becomes equal to the input, $\mathrm{d}, 20 \mathrm{~ns}$ after the positive edge of the clock. Similarly, the output, qbar, becomes equal to the inverted input, $\sim \mathrm{d}, 30 \mathrm{ps}$ after the positive edge of the clock.

More than one condition can be included in an initial or always procedural block. For example, a flip-flop with an asynchronous active-low reset input can be modeled as follows:

```
`timescale 10ps/1ps
module flip_flop (q, qbar, d, clock, reset);
output q, qbar;
input d, clock, reset;
reg q, qbar;
always @ (posedge clock or negedge reset)
        begin
        if (reset == 0)
            begin
            #2 q <= 0;
            #2 qbar <= 1;
            end
        else
            begin
            #4 q <= d;
            #5 qbar <= ~d;
            end
        end
endmodule
```

If the program encounters an active-low reset before the positive edge of clock, both q and qbar outputs become logic 020 ps after the negative edge of reset. Otherwise, without any reset, the flip-flop operates normally, and the output, q, becomes equal to the input, $\mathrm{d}, 40 \mathrm{ps}$ after the positive edge of the clock, and the output, qbar, becomes equal to the inverted input, $\sim \mathrm{d}, 50 \mathrm{ps}$ after the positive edge of the clock due to the non-blocking nature of these assignments.

## A.7. Arithmetic and Logical Operators in Verilog

There are two types of operators used in Verilog: arithmetic and logical. The arithmetic operators simply add, subtract, multiply or divide the variables used in a program. The symbol for each operation is given below:

| Add | + |
| :--- | :---: |
| Subtract | - |
| Multiply | * |
| Divide | / |

The logical operators execute all the logic functions, comparisons, bit shifting and concatenation. The symbol for each operation is given below:

| Bitwise AND | $\&$ |
| :--- | :--- |
| Bitwise NAND | $\sim \&$ |
| Bitwise OR | $\mid$ |
| Bitwise NOR | $\sim \mid$ |
| Bitwise XOR | $\wedge$ |
| Bitwise XNOR | $\sim \wedge$ |
| Bitwise NOT | $\sim$ |
| Less than | $<$ |
| Greater than | $>=$ |
| Greater than or equal | $>=$ |
| Less than or equal | $<=$ |
| Equal | $==$ |
| Logical left shift | $\ll$ |
| Logical right shift | $\gg$ |
| Conditional | $?:$ |
| Concatenation | $\}$ |

For example, if four-bit variables, X and Y , are equal to 0110 and 1011, respectively, the logical operations on X and Y become as follows:

$$
\begin{array}{ll}
\sim Y & =0100 \\
X \& Y & =0010 \\
X \sim \mid Y & =0000
\end{array}
$$

As another example, let us assume $A=4$ 'b1101 and $B=8^{\prime} b 01110101$. Shifting B one bit to the left becomes:

```
B<< 1 = 11101010
A & (B<< 1) = 00001101 & 11101010=00001000
```


## A.8. Conditional Statement

Conditional statements are equivalent to the "if-then" statements in C-programming. They follow the same format in C-language but enveloped between the begin and end clauses.

The simplest form of a conditional statement is given below:

```
begin
if (condition1)
    if (condition2)
                if (condition3)
                statement 1;
                else
                statement 2;
            else
            statement 3;
else
statement 4;
end
```

Conditions may also be combined using logical operators. For example, the Verilog code below AND-gates all three conditions, condition 1, condition 2 and condition 3, and produces a single condition for the if-clause.

```
begin
```

if (condition $1 \& \&$ condition $2 \& \&$ condition 3 )
statement 1;
else
statement 2;
end

The condition can include many operators as shown in the example below.

```
begin
if \((a>0)\)
    if ( \(x<=0\) )
    \(y=1 ;\)
    else \(\quad / / x>0\) is implied
        y != 2;
else if ( \(a==0\) )
            if ( \(x<=0\) )
        \(y=3 ;\)
            else \(\quad / / x>0\) is implied
        y !=4;
else // no if statement, thus a \(<0\) is implied
            if ( \(x<=0\) )
        \(y=5 ;\)
        else \(\quad / / x>0\) is implied
        y ! = 6;
```

end

## A.9. Case Statement

Conditional statements can be written using a "case" statement. The example below implements an 8-1 multiplexer in Fig. A. 12 with a case statement.


Fig. A. 12 8-1 MUX

The case statement for this MUX can be written as follows:

```
`define sel_value0 3'b000
`define sel_value1 3'b001
`define sel_value2 3'b010
`define sel_value3 3'b011
`define sel_value4 3'b100
`define sel_value5 3'b101
`define sel_value6 3'b110
`define sel_value7 3'b111
module mux (out, sel, in0,in1, in2, in3, in4, in5, in6, in7);
output out;
input in0, in1, in2, in3, in4, in5, in6, in7;
input [2:0] sel;
reg out;
always @ (sel or in0 or in1 or in2 or in3 or in4 or in5 or in6 or in7)
begin
    case (sel)
    `sel_value0 : out = in0;
    `sel_value1: out = in1;
    `sel_value2 : out = in2;
    `sel_value7 : out = in7;
    default: out = in0;
    endcase
end
endmodule
```

In this code, the case statement is executed if any of the inputs, in0 to in7, or the select input, sel [2:0], changes. Once inside the case statement, the output of the MUX, out, becomes equal to one of the MUX inputs according to the input select signal, sel_value0 to sel_value7. The output of the MUX, out, also needs to be declared with a reg statement because this variable is declared inside the procedural block, and assigned with different input values, in0 to in7. The default statement inside the case statement declares the value of the output if none of the other cases apply. The case statement is enclosed between the case and the endcase clauses. Since the case statement is in a procedural block it also needs the begin and the end statements enveloping the case statement.

A simple example is the implementation of a 4-1 MUX in Fig. A.13.
module mux (out, sel, in0, in1, in2, in3);
output out;
input in0, in1, in2, in3;
input [1:0] sel;
reg out;
always @ (sel or in0 or in1 or in2 or in3)
begin
case (sel)
2'b00: out = in0;
2'b01: out = in1;
2'b10 : out = in2;
2'b11 : out = in3;
default: out = in 0 ;
endcase
end
endmodule


Fig. A. 13 4-1 MUX
The case statement can also be used to implement an Arithmetic Logic Unit (ALU) as shown in Fig. A. 14 because the ALU output is accompanied by a multiplexer.

```
`define XOR 2'b00
`define SHIFT 2'b01
`define ADD 2'b10
`define SUB 2'b11
module alu (out, opcode, a, b);
output [7:0] out;
input [7:0] a, b;
input [1:0] opcode;
reg [7:0] out;
always @ (opcode or a or b)
begin
        case (opcode)
        `XOR: out =a^b;
        `SHIFT:out = a << b;
        `ADD: out =a + b;
        SUB: out =a-b;
        default: out = a + b;
        endcase
```

end
endmodule


Fig. A. 14 A simple ALU

## A.10. Looping Statements

There are two useful looping statements in Verilog, the "for" statement and the "while" statement. Both statements have to be included in a procedural block.

In the for-loop example given below, the variable, $i$, starts from 0 , and stops at 10 , incrementing by 1 . The variable, $j$, is defined in terms of the variable, $i$. The array that follows determines $\mathrm{a}[\mathrm{j}]$ in terms of $\mathrm{a}[\mathrm{i}]$. Thus,

```
for (i = 0; i <= 10; i ++)
    begin
    j= i + 1;
    a[j] = a[i] + 1;
    end
```

The while-loop waits for the occurrence of an event. When the event takes place, the statements in the while-loop are executed. In the example below, a for-loop is engaged, and the variable, $\mathrm{x}[\mathrm{i}]$, is determined in terms of $\mathrm{a}[\mathrm{i}]$ and $\mathrm{b}[\mathrm{i}]$ as long as the variable, sum, is not equal to 0 .

```
while (sum != 0)
    begin
        for (i=0; i < 10; i++)
        begin
        x[i] = a[i] - b[i];
        end
    end
```


## A.11. State Machine Implentations

There are two types of state machines: Mealy-type and Moore-type. Both types can easily be implemented in Verilog using case statements.

The present state of a state machine is defined by flip-flop outputs. The next state is defined by flip-flop inputs because at the positive edge of clock the next state becomes the present state.

## A.12. Mealy Machine

The present state outputs of the Mealy machine stems from the present state and the present state inputs. Therefore, if the present state inputs change during the clock period, this change affects the present state outputs and the next state instantaneously as shown in Fig. A.15.


Fig. A. 15 Block diagram and state representation of Mealy machine

The example in Fig. A. 16 shows a Mealy-type state machine with four states. When implementing this state machine in Verilog, it is best to divide the overall circuit topology into two sections as shown in Fig. A.17.

The first section is purely combinational. This section's inputs stem from the present state and the present state inputs as shown in Fig. A.17. The outputs form the next state and the present state outputs. Since both the outputs and the next state are functions of the inputs and the present state, this section can conveniently be implemented with a multiplexer. The second section constitutes a sequential logic circuit with flip-flop inputs and outputs. This section, as we will see below, is implemented with an always statement.


Fig. A. 16 State diagram of a Mealy machine with four states (reset not shown for simplicity)


Fig. A. 17 Segmentation of the Mealy machine (with asynchronous reset)
The Verilog code below first implements the combinational logic section of the state machine then the sequential part. The numeric values corresponding to each of the four present states in Fig. A. 16 are assigned to the parameters, s0, s1, s2 and s3, using the parameter statement because this simplifies the observation of the input and the output values at a particular state.

The combinational part of the state machine is implemented by a case statement inside an always procedural block, and executed if one of the multiplexer inputs, in and pstate, changes. Here, the input, in, corresponds the only input, IN, in Fig. A.16, and the input, pstate, corresponds to the present state. If the pstate input is assumed to be the selector input to a multiplexer, the case statement then lists all possible combinations of the multiplexer output as a function of pstate. The default in the case statement always corresponds to the initial state of the state machine. In each case statement, the output assignments are always non-blocking type because in real hardware the outputs are produced concurrently and independent of each other.

The sequential part of the state machine is implemented by an always statement. However, this statement becomes active only at the positive edge of the clock and the negative edge of the reset rather than static values of these two signals.
// Mealy machine with asynchronous reset module mealy (out, reset, in, clock);
output [2:0] out;
input reset, in, clock;
reg [2:0] out;
reg [1:0] nstate, pstate;
parameter $s 0=2$ 'b00, $s 1=2$ 'b01, $s 2=2$ 'b10, $s 3=2$ 'b11;
always @ (in or pstate)
begin
case (pstate)
s0: begin
if (in ==0)
begin out <=1;
nstate <= s0;
end
else
begin
out <= 2;
nstate <= s1;
end
end
s1: begin
if (in == 0 )
begin
out <= 2;
nstate <= s1;
end
else
begin
out <= 3;
nstate <= s2;
end
end
s2: begin
if (in ==0)
begin
out <= 3;
nstate <= s2;
end

```
            else
                    begin
                out < = 4;
                nstate <= s3;
                end
        end
        s3: begin
        if (in ==0)
            begin
            out <= 4;
            nstate <= s3;
            end
        else
            begin
            out < = 2;
            nstate <= s1;
            end
        end
        default: begin
        out <=1;
        nstate <= s0;
        end
        endcase
end
always @ (posedge clock or negedge reset)
begin
        if (reset == 0)
        pstate <= s0;
        else
        pstate <= nstate;
end
endmodule
```


## A.13. Moore Machine

Implementing the Moore machine is not any different from the Mealy machine except the formation of present state outputs. Figure A. 18 shows the present state outputs of the Moore machine to be only a function of the present state, and independent of any present state inputs.


Fig. A. 18 Block diagram and state representation of Moore machine
If we consider a four-state Moore machine in Fig. A.19, its implementation in Verilog requires combining the two combinational logic sections of the circuit in Fig. A. 20 with a case statement, and implementing the sequential part with an always statement.

Implementing the combinational logic parts of the state machine is accomplished by a multiplexer. The present state input, IN, and the present state, pstate, constitute the inputs to this multiplexer. Implementing the sequential section of the state machine, on the other hand, requires the next state of the state machine to be flip-flop inputs and the present state to be flip-flop outputs.


Fig. A. 19 State diagram of a Moore machine with four states (reset not shown for simplicity)


Fig. A. 20 Segmentation of the Moore machine (with asynchronous reset)
The program below assigns numeric values to all four states with a parameter statement. The first always statement includes a case statement to show what happens to the multiplexer outputs if one of the selector inputs changes. Again, all multiplexer outputs form concurrently. Therefore, all output assignments are defined to be non-blocking type. As opposed to the Mealy machine, the present state outputs of the Moore machine are solely generated from the present state. Therefore, for each state from s0 to s3 the output assignments are written first, independent of any present state input.

The sequential part is implemented by an always statement which includes the edge dependency of the clock and the reset. This statement is executed only if the indicated edges of these two inputs take place. Otherwise, it is ignored.
// Moore machine with asynchronous reset
module moore_async (out, reset, in, clock);
output [2:0] out;
input reset, in, clock;
reg [2:0] out;
reg [1:0] nstate, pstate;
parameter $\quad s 0=2^{\prime} b 00, s 1=2 \prime b 01, s 2=2^{\prime} b 10, s 3=2 \prime b 11$;
always @ (in or pstate)
begin
case (pstate)
s0: begin
out < $=1$;
if (in == 1)
nstate <= s1;
else
nstate <= s0;
end
s1: begin
out <= 2;
if (in == 1)
nstate <= s2;
else
nstate <= s1;
end
s2: begin
out < $=3$;
if (in == 1)
nstate <= s3;
else
nstate <= s2;
end
s3: begin
out <= 4;
if (in == 1 )
nstate <= s1;
else
nstate <= s3;
end
default: begin
out <= 1 ;
nstate <= s0;
end
endcase
end

```
always @ (posedge clock or negedge reset)
    begin
    if \((\) reset \(==0)\)
        pstate <=s0;
    else
        pstate <= nstate;
    end
endmodule
```

The Verilog code below implements the Moore machine with a synchronous reset. This time, reset is not an isolated input to the flip-flops as in Fig. A.20, but instead it is applied to the combinational logic block along with the other present state inputs.
//Moore machine with synchronous reset module more_sync (out, reset, in, clock);

| output <br> input | $[2: 0]$ out; |
| :--- | :--- |
| reg | reset, in, clock; |
| reg | $[2: 0]$ out; |
| parameter | $[1: 0]$ nstate, pstate; |
| s0 02 'b00, s1 $=2 ' b 01, s 2=2 ' b 10, ~ s 3=2 ' b 11 ; ~$ |  |

```
always @ (in or reset or pstate)
begin
    case (pstate)
    s0: begin
    out <= 1;
        if (reset == 0 )
        nstate <=s0;
            else
                begin
                if (in == 1)
                nstate <= s1;
                else
                nstate <= s0;
            end
            end
        s1: begin
    out <= 2;
    if (reset == 0)
        nstate <= s0;
    else
        begin
        if (in == 1)
            nstate <= s2;
            else
                nstate <= s1;
            end
    end
```

```
    s2: begin
    out <= 3;
        if (reset == 0)
        nstate <= s0;
    else
        begin
        if (in == 1)
                nstate <= s3;
        else
                nstate <= s2;
            end
        end
s3: begin
    out <= 4;
    if (reset == 0)
        nstate <= sO;
    else
        begin
        if (in == 1)
                nstate <= s1;
        else
                nstate <= s3;
        end
    end
default: begin
    out <= 1;
    nstate <= s0;
    end
endcase
end
always @ (posedge clock)
    begin
    pstate <= nstate;
    end
endmodule
```


## A.14. Principles of Register-Transfer-Logic Type Coding

The Register-Transfer-Logic (RTL) style of Verilog coding inherits many C-program constructs and implements the intended hardware with ease. Although structural or behavioral Verilog coding may be necessary for certain types of logic blocks, RTL is still the most common coding style to build hardware.

## A.15. Wire Assignment

The first statement in RTL style coding is the wire statement. This statement is either accompanied by an assign statement or declared by itself, and it resides outside the procedural block. In the first example below, the inputs, a and b, form an XOR gate with an output, out. Separate wire and assign statements are used to implement the XOR gate.
wire out;
assign out $=\mathrm{a}^{\wedge} \mathrm{b}$;
However, the two statements can be combined to form a single wire statement.
wire out $=a^{\wedge} \mathrm{b}$;
If the implementation requires multiple wires in the form of a bus, then the statements for the node, out, can be written as follows:
wire [7:0] out;
assign out $=\mathrm{a} \wedge \mathrm{b}$;
or
wire [7:0] out $=\mathrm{a} \wedge \mathrm{b}$;

## A.16. Conditional Operator

Another useful RTL construct is the conditional operator. Assume a tri-state buffer in Fig. A. 21.


Fig. A. 21 Tri-state buffer implemented by conditional operator

The wire statement that includes the conditional operator can be written as follows:

```
wire out;
assign out = enable ? in : 1'bz;
```

The "?" in the above statement signifies the condition for the input, enable, to be equal to logic 1 or not. If enable is logic 0 , then the output, out, becomes an open circuit. Since out is only one bit, the high impedance state is shown as 1 'bz.

The wire and assign statements can also be combined to produce a single statement.
wire out $=$ enable $?$ in : 1 'bz;
Another example is a 3-1 MUX shown in Fig. A.22.


Fig. A. 22 A 3-1 MUX implemented by conditional operator

```
wire out;
assign out = (sel == 2'b00) ? a : (sel == 2'b01) ? b : c;
```

In this statement, if sel $[1: 0]=00$ then the output, out, becomes a. If sel $[1: 0]=01$ then out becomes $b$. For all the other values of sel, out becomes $c$.

The same statement can also be written without the assign statement as:

```
wire out = (sel == 2'b00) ? a : (sel == 2'b01) ? b : c;
```

If the 3-1 MUX accepts bus inputs to produce a bus output as in Fig. A.23, then the wire statement has to be modified to include the bus width.


Fig. A. 23 An eight-input 3-1 MUX implemented by conditional operator
wire [7:0] out = (sel == 2'b00) ? a : (sel == 2'b01) ? b:c;
The bus width of the inputs, a, b, c and sel, should be declared in the input statements prior to the wire statement as shown below.

```
input [7:0] a, b, c;
input [1:0] sel;
wire [7:0] out = (sel == 2'b00) ? a : (sel == 2'b01) ? b : c;
```


## A.17. Memory Declaration

Memory is declared by a reg statement in Verilog. Assume an SRAM-like memory with ( $\mathrm{x}+1$ ) number of bits and $(y+1)$ number of rows as shown in Fig. A.24. This memory is declared as follows:
reg [x:0] fifo [y:0];
Here, "fifo" is the name of the memory with a dimension of $[\mathrm{x}: 0]$ by $[\mathrm{y}: 0]$.


Fig. A. $24(x+1)$ wide $(y+1)$ deep memory

## A.18. Memory Addressing

Following the memory declaration, the memory addressing should be specified to locate a specific data in the memory.

Assume an eight-bit wide memory with 16 rows in Fig. A.25. In order to access the most significant bit (msb) and the least significant bit (lsb) of this memory at any row, an address needs to be formed in the Verilog code.

```
input [3:0] address;
reg [7:0] mem [15:0];
reg [7:0] row;
reg msb, lsb;
row = mem [address];
msb = row [7];
lsb = row [0];
```

In this example, the reg statement, reg [7:0] mem [15:0], declares a $8 \times 16$ memory with a name, mem.


Fig. A. 25 An $8 \times 16$ memory
If the memory address is externally supplied to the memory, this input needs to be declared in the input statement. Each row is declared with a second reg statement, reg [7:0] row. Once the row declaration is finished, the most and the least significant bits are then declared in the third reg statement, reg msb, lsb.

Therefore, each bit in an arbitrary row can be accessed by the statement, row = mem [address], in the Verilog program above. To access the most significant bit and the least significant bit are accomplished by msb = row [7] and lsb = row [0], respectively.

## A.19. Memory Modeling

Different types of memory require different styles of memory modeling. The simple SRAM memory shown in Fig. A. 26 has a single bidirectional data port and operates with a clock. The data is written to an arbitrary row or read from an arbitrary row at the positive edge of the clock signal once the memory address is specified.


Fig. A. 26 An $8 \times 16$ single port, bidirectional memory

Since the data port is bidirectional, this port needs to be declared as an inout statement. Therefore, the Verilog program can be written as follows:

```
module mem (data, address, WE, RE, clock);
inout [7:0] data;
input [3:0] address;
input WE, RE, clock;
reg [7:0] SRAM [15:0];
reg [7:0] data;
reg [1:0] read_write_state;
always @ (posedge clock)
begin
    read_write_state = {WE, RE}; // curly brackets are for concatenating WE and RE
    case (read_write_state)
    2'b00: data = 8'bz;
    2'b01: data = SRAM [address];
    2'b10: SRAM [address] = data;
    2'b11: $display ("error"); // WE and RE cannot be at logic 1 simultaneously
    default: data = SRAM [address]; // SRAM needs to be in the read mode when idling
    encase
end
endmodule
```

In the example above, the case statement is formed in a procedural block because both the read and the write cycles take place at the positive edge of the clock. When neither RE nor WE is at logic 1 , the bidirectional data port must be at a high impedance state, data $=8^{\prime} \mathrm{bz}$. When a read takes place, data is read out from a specified memory address and directed to the bidirectional bus, data [7:0]. When a write takes place, data from the bidirectional bus is written to a specified address, SRAM [address]. When both WE and RE are at logic 1, this should be indicated as an error.

A set of if statements can also be used to replace the case statement except the case statement is more compact, and it includes all possible cases to model a memory.

The following example models a unidirectional, byte addressable memory shown in Fig. A. 27 with two data ports. The input port, DataIn [31:0], is byte-addressable and therefore contains four bytes. The output port, DataOut, is not byte-addressable, and it is used to read all 32 bits of data. Thus,

```
module mem (DataOut, Dataln, address, clock, ByteEn, WE, RE);
output [31:0] DataOut;
input [31:0] Dataln;
input [3:0] address, ByteEn;
input clock, RE, WE;
reg [31:0] SRAM [15:0];
reg [31:0] temp;
always @ (posedge clock)
begin
if (WE == 1 && RE == 0)
    begin
    casex (ByteEn)
    4'b0000: temp [31:0] = 32'bz;
    4'b0001: temp [7:0] = Dataln [7:0];
    4'b0010: temp [15:8] = Dataln [15:8];
    4'b0011: temp [15:0] = Dataln [15:0];
    4'b0100: temp [23:16] = Dataln [23:16];
    4'b0101: begin
                                    temp [23:16] = Dataln [23:16];
        temp [7:0] = Dataln [7:0];
        end
    4'b0110: temp [23:8] = Dataln [23:8];
    4'b1111: temp [31:0] = Dataln [31:0];
    defaul: begin
                                    temp [31:0] = 32'bx;
                $display ("no bytes are enabled");
                end
        endcase
        SRAM [address] = temp;
        end
else if (RE == 1&& WE == 0)
        DataOut = SRAM [address];
else if (RE == 0 && WE == 0)
        DataOut = 32'bz;
else
        display ("Error - RE and WE are enabled");
end
endmodule
```



Fig. A. 27 32x16 dual port, unidirectional memory

## A.20. A Few Words about Functional Verification

Functional verification is a very critical step in logic design and needs to cover every possible corner case and input combination to a Verilog module. However, when the circuit is not purely combinational but contains sequential components, the difficulty of functional verification increases. A proper process in this case is to isolate the sequential sections from the combinational sections of the circuit and verify each section individually before verifying the entire system.

When a combinational circuit goes through a formal functional verification step, the best method is to apply the inputs of the entire truth table as test vectors to the module, store the circuit's response in an output file, and then compare this output file with the one that contains the expected outputs (the ouputs of the truth table) as shown in Fig. A.28.

However, if the circuit is sequential, each state-to-state transition needs to be examined in the state machine when the inputs to the state change. Furthermore, the outputs from each state need to match the expected output values.

It may be sufficient to do functional check using timing diagrams if the size of the circuit is small. However, for bigger circuits, including many combinational and sequential modules, the verification process is applied to each individual module, and then to the entire system. Both types of verification are essentially a file matching process as shown in Fig. A. 28.


Fig. A. 28 Formal functional verification process

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[^0]:    Fig. 5.77 $\mathrm{I}^{2} \mathrm{C}$ read sequence with device ID and the LSB of starting memory address

