Revised Second Edition

Higher Surveying

Dr A M Chandra

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PREFACE

The University of Roorkee, formerly known as Thomason College of Civil Engineering and now the Indian Institute of Technology, Roorkee, was established in 1847 as Roorkee College with the objective of producing engineers who would one day build the edifice of their motherland. Rightly, the Institute and more notably its Department of Civil Engineering can lay claim to be the cradle of engineering education and technology in our country.

The first step in engineering practice is surveying and the soundness of the former is dependent upon the reliability and accuracy of the latter. Therefore, it is imperative that a student of engineering has a good knowledge of surveying. With the above objective, the author delved into writing a book on the subject, knowing pretty well that many excellent books are available. The urge to write a book on this fascinating subject emanated from years of experience in teaching and its field applications in various types of situations. The author has already written a textbook *Plane Surveying* which is primarily aimed at the students who do not have any background of surveying.

This book, *Higher Surveying*, is in continuation to *Plane Surveying* and deals with the topics of higher surveying, its applications, and some of the modern developments in the field of geomatics. The book contains 11 chapters.

Chapter 1 describes the survey operations, triangulation and trilateration, for fixing control. Since the survey measurements contain errors, these have to be adjusted to determine the most probable values by adjustment methods. Chapter 2 thoroughly describes the various adjustment methods applied to survey measurements.

Chapters 3 through 6 have been devoted to distinct surveys such as topographic, hydrographic, construction, and route surveying including tunnel surveying. A detailed coverage on field astronomy has been presented in chapter 7. Chapter 8 deals with map projection.

Chapters 9 to 11 deal with recent advances in land surveying. Chapter 9 discusses principles of photogrammetry and photographic interpretation which make use of aerial photographs. Remote sensing technique which utilizes data acquires through satellites has been discussed in chapter 10.

Surveyors and engineers are now adopting modern surveying instrumentation and methods in their field operation. For processing of huge amount of data required for management and planning of natural resources, use of computers is becoming indispensable. Keeping these requirements in view, chapter 11 deals with modern systems in surveying and mapping. This chapter presents an overview of the latest techniques in the field of instrumentation and methodologies being employed in geomatics. It covers a range of instruments such as EDM, Total Station, laser based instruments, electronic field book, digital level, inertial positioning system, GPS, GIS, and automated photogrammetric system.

The author would like to thank the instrument manufacturers who generously furnished the photographs used in this book.

The author also wishes to place on record the invaluable support and unstrained encouragement received from his wife Archana Chandra, son Anshuman Chandra, and daughter Arushi Chandra during the preparation of the manuscripts.

The author also expresses his thanks to his colleagues and friends for extending valuable suggestions from time to time without which the book could not have been brought out in the present form.

Finally, the author wishes to record his gratitude to his Almamatter, the University of Roorkee and now the Indian Institute of Technology, Roorkee, which provided him with an intellectually stimulating environment and a working platform conducive to creativity.

Efforts will be made to rectify errors, if any, pointed out by readers, to whom the author shall be grateful. Suggestions for improvement are also welcome.

A.M. Chandra

CONTENTS

1

TRIANGULATION AND TRILATERATION

1.1 GENERAL

The horizontal positions of points is a network developed to provide accurate control for topographic mapping, charting lakes, rivers and ocean coast lines, and for the surveys required for the design and construction of public and private works of large extent. The horizontal positions of the points can be obtained in a number of different ways in addition to traversing. These methods are triangulation, trilateration, intersection, resection, and satellite positioning.

The method of surveying called *triangulation* is based on the trigonometric proposition that if one side and two angles of a triangle are known, the remaining sides can be computed. Furthermore, if the direction of one side is known, the directions of the remaining sides can be determined. A triangulation system consists of a series of joined or overlapping triangles in which an occasional side is measured and remaining sides are calculated from angles measured at the vertices of the triangles. The vertices of the triangles are known as *triangulation stations*. The side of the triangle whose length is predetermined, is called the *base line*. The lines of triangulation system form a network that ties together all the triangulation stations (Fig. 1.1).

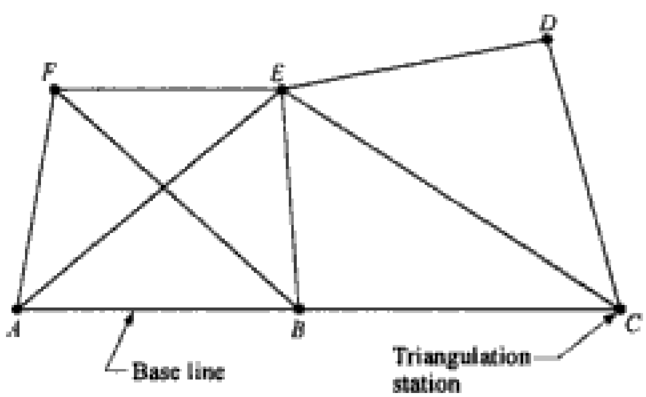

Fig. 1.1 Triangulation network

A *trilateration system* also consists of a series of joined or overlapping triangles. However, for trilateration the lengths of all the sides of the triangle are measured and few directions or angles are measured to establish azimuth. Trilateration has become feasible with the development of electronic distance measuring (EDM) equipment which has made possible the measurement of all lengths with high order of accuracy under almost all field conditions.

A combined triangulation and trilateration system consists of a network of triangles in which all the angles and all the lengths are measured. Such a combined system represents the strongest network for creating horizontal control.

Since a triangulation or trilateration system covers very large area, the curvature of the earth has to be taken into account. These surveys are, therefore, invariably geodetic. Triangulation surveys were first carried out by Snell, a Dutchman, in 1615.

Field procedures for the establishment of trilateration station are similar to the procedures used for triangulation, and therefore, henceforth in this chapter the term triangulation will only be used.

1.2 PRINCIPLE OF TRIANGULATION

Fig. 1.2 shows two interconnected triangles *ABC* and *BCD*. All the angles in both the triangles and the length *L* of the side *AB*, have been measured.

Also the azimuth θ of *AB* has been measured at the triangulation station *A*, whose coordinates (X_A, Y_A), are known.

The objective is to determine the coordinates of the triangulation stations *B*, *C*, and *D* by the method of triangulation. Let us first calculate the lengths of all the lines.

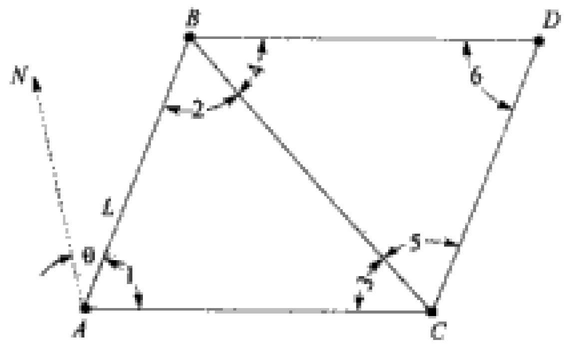

Fig. 1.2 Principle of triangulation

By sine rule in ΔABC, we have

$$\frac{AB}{\sin 3} = \frac{BC}{\sin 1} = \frac{CA}{\sin 2}$$

We have $AB = L = l_{AB}$

or $BC = \frac{L \sin 1}{\sin 3} = l_{BC}$

and $CA = \frac{L \sin 2}{\sin 3} = l_{CA}$

Now the side *BC* being known in ΔBCD, by sine rule, we have

$$\frac{BC}{\sin 6} = \frac{CD}{\sin 4} = \frac{BD}{\sin 5}$$

We have $BC = \frac{L \sin 1}{\sin 3} = l_{BC}$

or $CD = \left(\frac{L \sin 1}{\sin 3}\right)\frac{\sin 4}{\sin 6} = l_{CD}$

and $BC = \left(\frac{L \sin 1}{\sin 3}\right)\frac{\sin 5}{\sin 6} = l_{BD}$

Let us now calculate the azimuths of all the lines.

Azimuth of $AB = \theta = \theta_{AB}$

Azimuth of $AC = \theta + \angle 1 = \theta_{AC}$

Azimuth of $BC = \theta + 180° - \angle 2 = \theta_{BC}$

Azimuth of $BD = \theta + 180° - (\angle 2 + \angle 4) = \theta_{BD}$

Azimuth of $CD = \theta - \angle 2 + \angle 5 = \theta_{CD}$

From the known lengths of the sides and the azimuths, the consecutive coordinates can be computed as below.

Latitude of $AB = l_{AB} \cos\theta_{AB} = L_{AB}$

Departure of $AB = l_{AB} \sin\theta_{AB} = D_{AB}$

Latitude of $AC = l_{AC} \cos\theta_{AC} = L_{AC}$

Departure of $AC = l_{AC} \sin\theta_{AC} = D_{AC}$

Latitude of $BD = l_{BD} \cos\theta_{BD} = L_{BD}$

Departure of $BD = l_{BD} \sin\theta_{BD} = L_{BD}$

Latitude of $CD = l_{CD} \cos\theta_{CD} = L_{CD}$

Departure of $CD = l_{CD} \sin\theta_{CD} = D_{CD}$

The desired coordinates of the triangulation stations B, C, and D are as follows :

X-coordinate of B, $X_B = X_A + D_{AB}$

Y-coordinate of B, $Y_B = Y_B + L_{AB}$

X-coordinate of C, $X_C = X_A + D_{AC}$

Y-coordinate of C, $Y_C = Y_A + L_{AC}$

X-coordinate of D, $X_D = X_B + D_{BD}$

Y-coordinate of D, $Y_D = Y_B + L_{BD}$

It would be found that the length of side can be computed more than once following different routes, and therefore, to achieve a better accuracy, the mean of the computed lengths of a side is to be considered.

1.3 OBJECTIVE OF TRIANGULATION SURVEYS

The main objective of triangulation or trilateration surveys is to provide a number of stations whose relative and absolute positions, horizontal as well as vertical, are accurately established. More detailed location or engineering survey are then carried out from these stations.

The triangulation surveys are carried out

(*i*) to establish accurate control for plane and geodetic surveys of large areas, by terrestrial methods,
(*ii*) to establish accurate control for photogrammetric surveys of large areas,
(*iii*) to assist in the determination of the size and shape of the earth by making observations for latitude, longitude and gravity, and
(*iv*) to determine accurate locations of points in engineering works such as :
  (*a*) Fixing centre line and abutments of long bridges over large rivers.
  (*b*) Fixing centre line, terminal points, and shafts for long tunnels.
  (*c*) Transferring the control points across wide sea channels, large water bodies, etc.
  (*d*) Detection of crustal movements, etc.
  (*e*) Finding the direction of the movement of clouds.

1.4 CLASSIFICATION OF TRIANGULATION SYSTEM

Based on the extent and purpose of the survey, and consequently on the degree of accuracy desired, triangulation surveys are classified as *first-order* or *primary*, *second-order* or *secondary*, and *third-order* or *tertiary*. First-order triangulation is used to determine the shape and size of the earth or to cover a vast area like a whole country with control points to which a second-order triangulation system can be connected. A second-order triangulation system consists of a network within a first-order triangulation. It is used to cover areas of the order of a region, small country, or province. A third-order triangulation is a framework fixed within and connected to a second-order triangulation system. It serves the purpose of furnishing the immediate control for detailed engineering and location surveys.

Table 1.1 Triangulation system

S.No.	Characteristics	First-order triangulation	Second-order triangulation	Third-order triangulation
1.	Length of base lines	8 to 12 km	2 to 5 km	100 to 500 m
2.	Lengths of sides	16 to 150 km	10 to 25 km	2 to 10 km
3.	Average triangular error (after correction for spherical excess)	less than 1"	3"	12"
4.	Maximum station closure	not more than 3"	8"	15"
5.	Actual error of base	1 in 50,000	1 in 25,000	1 in 10,000
6.	Probable error of base	1 in 10,00,000	1 in 500,000	1 in 250,000
7.	Discrepancy between two measures (k is distance in kilometre)	$5\sqrt{k}$ mm	$10\sqrt{k}$ mm	$25\sqrt{k}$ mm
8.	Probable error of the computed distances	1 in 50,000 to 1 in 250,000	1 in 20,000 to 1 in 50,000	1 in 5,000 to 1 in 20,000
9.	Probable error in astronomical azimuth	0.5"	5"	10"

Table 1.1 presents the general specifications for the three types of triangulation systems.

1.5 TRIANGULATION FIGURES AND LAYOUTS

The basic figures used in triangulation networks are the triangle, braced or geodetic quadilateral, and the polygon with a central station (Fig. 1.3).

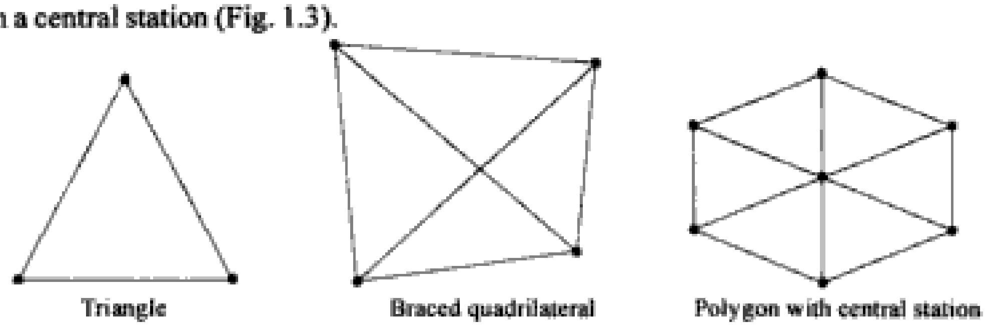

Fig. 1.3 Basic triangulation figures

The triangles in a triangulation system can be arranged in a number of ways. Some of the commonly used arrangements, also called *layouts*, are as follows :

1. Single chain of triangles
2. Double chain of triangles
3. Braced quadrilaterals
4. Centered triangles and polygons
5. A combination of above systems.

1.5.1 Single chain of triangles

When the control points are required to be established in a narrow strip of terrain such as a valley between ridges, a layout consisting of single chain of triangles is generally used as shown in Fig. 1.4. This system is rapid and economical due to its simplicity of sighting only four other stations, and does not involve observations of long diagonals. On the other hand, simple triangles of a triangulation system provide only one route through which distances can be computed, and hence, this system does not provide any check on the accuracy of observations. Check base lines and astronomical observations for azimuths have to be provided at frequent intervals to avoid excessive accumulation of errors in this layout.

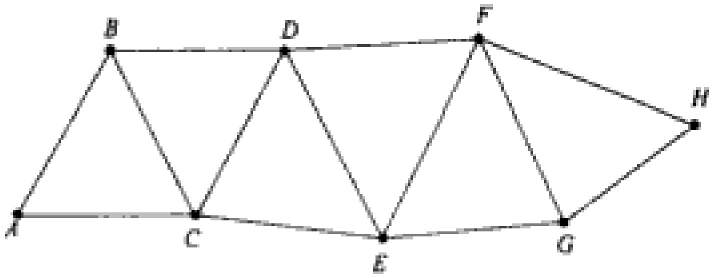

Fig. 1.4 Single of triangles

1.5.2 Double chain of triangles

A layout of double chain of triangles is shown in Fig. 1.5. This arrangement is used for covering the larger width of a belt. This system also has disadvantages of single chain of triangles system.

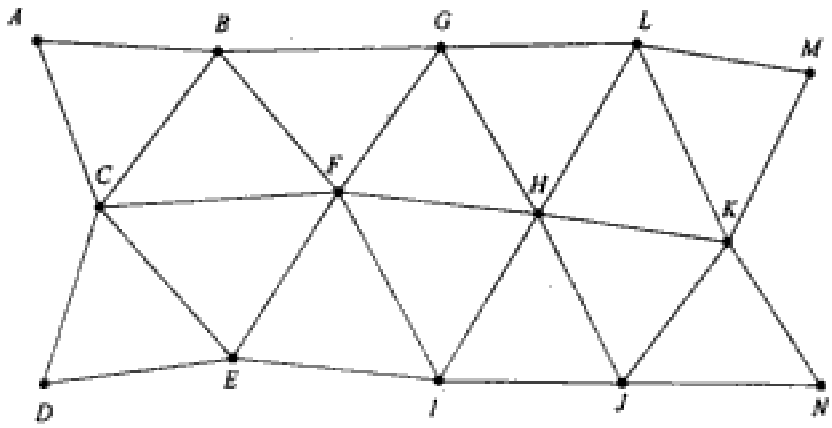

Fig. 1.5 Double chain of triangles

1.5.3 Braced quadrilaterals

A triangulation system consisting of figures containing four corner stations and observed diagonals shown in Fig. 1.6, is known as a layout of braced quadrilaterals. In fact, braced quadrilateral consists of overlapping triangles. This system is treated to be the strongest and the best arrangement of triangles, and it provides a means of computing the lengths of the sides using different combinations of sides and angles. Most of the triangulation systems use this arrangement.

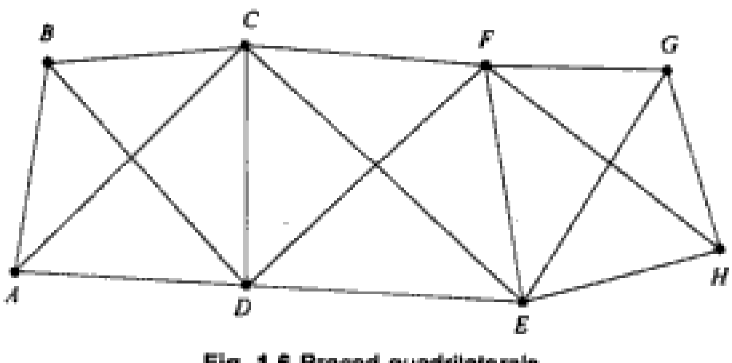

Fig. 1.6 Braced quadrilaterals

1.5.4 Centered triangles and polygons

A triangulation system which consists of figures containing interior stations in triangle and polygon as shown in Fig. 1.7, is known as centered triangles and polygons.

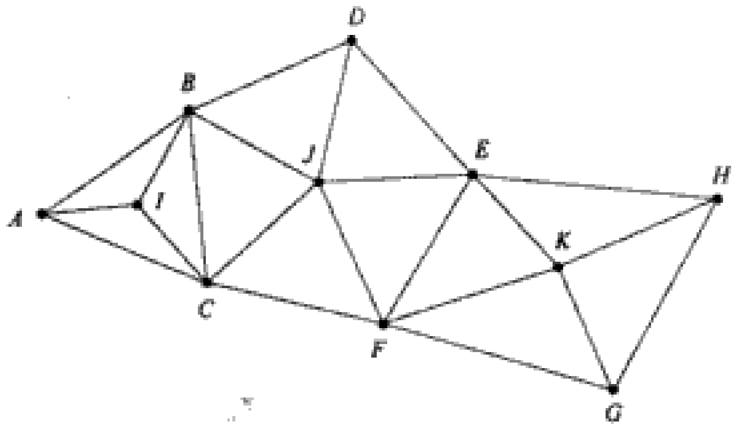

Fig. 1.7 Centered triangles and polygons

This layout in a triangulation system is generally used when vast area in all directions is required to be covered. The centered figures generally are quadrilaterals, pentagons, or hexagons with central stations. Though this system provides checks on the accuracy of the work, generally it is not as strong as the braced quadrilateral arrangement. Moreover, the progress of work is quite slow due to the fact that more settings of the instrument are required.

1.5.5 A combination of all above systems

Sometimes a combination of above systems may be used which may be according to the shape of the area and the accuracy requirements.

1.6 LAYOUT OF PRIMARY TRIANGULATION FOR LARGE COUNTRIES

The following two types of frameworks of primary triangulation are provided for a large country to cover the entire area.

1. Grid iron system
2. Central system.

1.6.1 Grid iron system

In this system, the primary triangulation is laid in series of chains of triangles, which usually runs roughly along meridians (north-south) and along perpendiculars to the meridians (east-west), throughout the country (Fig. 1.8). The distance between two such chains may vary from 150 to 250 km. The area between the parallel and perpendicular series of primary triangulation, are filled by the secondary and tertiary triangulation systems. Grid iron system has been adopted in India and other countries like Austria, Spain, France, etc.

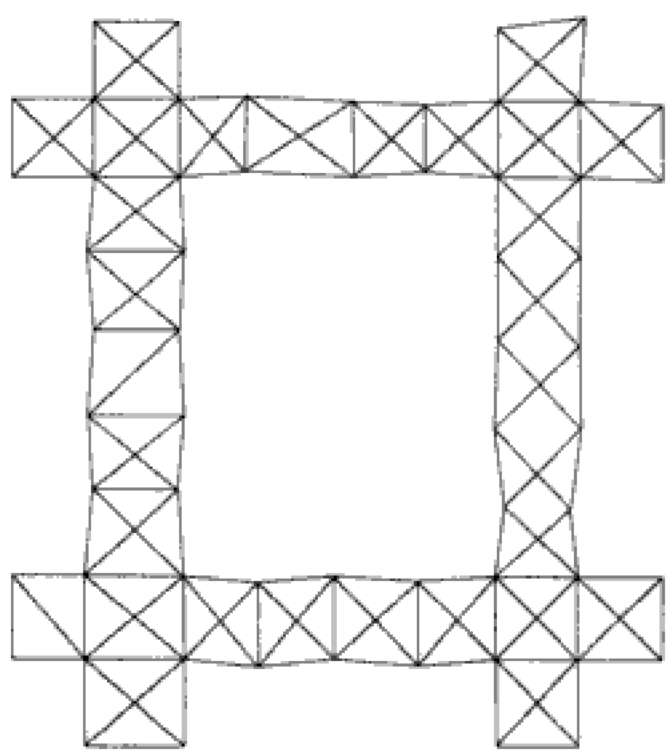

Fig. 1.8 Grid iron system of triangulation

1.6.2 Central system

In this system, the whole area is covered by a network of primary triangulation extending in all directions from the initial triangulation figure *ABC*, which is generally laid at the centre of the country (Fig. 1.9).

This system is generally used for the survey of an area of moderate extent. It has been adopted in United Kingdom and various other countries.

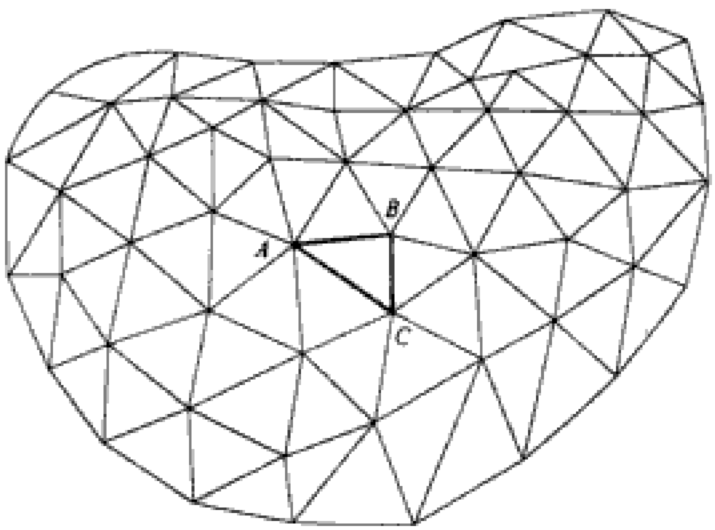

Fig. 1.9 Central system of triangulation

1.7 CRITERIA FOR SELECTION OF THE LAYOUT OF TRIANGLES

The under mentioned points should be considered while deciding and selecting a suitable layout of triangles.

1. Simple triangles should be preferably equilateral.
2. Braced quadrilaterals should be preferably approximate squares.
3. Centered polygons should be regular.
4. The arrangement should be such that the computations can be done through two or more independent routes.
5. The arrangement should be such that at least one route and preferably two routes form well-conditioned triangles.
6. No angle of the figure, opposite a known side should be small, whichever end of the series is used for computation.
7. Angles of simple triangles should not be less than 45°, and in the case of quadrilaterals, no angle should be less than 30°. In the case of centered polygons, no angle should be less than 40°.
8. The sides of the figures should be of comparable lengths. Very long lines and very short lines should be avoided.
9. The layout should be such that it requires least work to achieve maximum progress.
10. As far as possible, complex figures should not involve more than 12 conditions.

It may be noted that if a very small angle of a triangle does not fall opposite the known side it does not affect the accuracy of triangulation.

1.8 WELL-CONDITIONED TRIANGLES

The accuracy of a triangulation system is greatly affected by the arrangement of triangles in the layout and the magnitude of the angles in individual triangles. The triangles of such a shape, in which any error in angular measurement has a minimum effect upon the computed lengths, is known as *well-conditioned triangle.*

In any triangle of a triangulation system, the length of one side is generally obtained from computation of the adjacent triangle. The error in the other two sides if any, will affect the sides of the triangles whose computation is based upon their values. Due to accumulated errors, entire triangulation system is thus affected thereafter. To ensure that two sides of any triangle are equally affected, these should, therefore, be equal in length. This condition suggests that all the triangles must, therefore, be isoceles.

Let us consider an isosceles triangle ABC whose one side AB is of known length (Fig. 1.10). Let A, B, and C be the three angles of the triangle and a, b, and c are the three sides opposite to the angles, respectively.

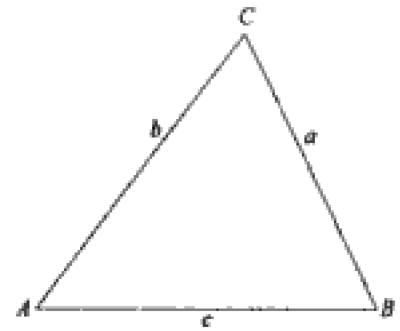

Fig. 1.10 Triangle in a triangulation system

As the triangle is isosceles, let the sides a and b be equal.

Applying sine rule to ΔABC, we have

$$\frac{a}{\sin A} = \frac{c}{\sin C} \qquad \text{...(1.1)}$$

or

$$a = c\,\frac{\sin A}{\sin C} \qquad \text{...(1.2)}$$

If an error of δA in the angle A, and δC in angle C introduce the errors δa_1 and δa_2, respectively, in the side a, then differentiating Eq. (1.2) partially, we get

$$\delta a_1 = c\,\frac{\cos A\,\delta A}{\sin C} \qquad \text{...(1.3)}$$

and

$$\delta a_2 = -c\,\frac{\sin A \cos C\,\delta C}{\sin^2 C} \qquad \text{...(1.4)}$$

Dividing Eq. (1.3) by Eq. (1.2), we get

$$\frac{\delta a_1}{a} = \delta A \cot A \qquad \text{...(1.5)}$$

Dividing Eq. (1.4) by Eq. (1.2), we get

$$\frac{\delta a_2}{a} = -\delta C \cot C \qquad \text{...(1.6)}$$

If $\delta A = \delta C = \pm\alpha$, is the probable error in the angles, then the probable errors in the side a are

$$\frac{\delta a}{a} = \pm\alpha\sqrt{\cot^2 A + \cot^2 C}$$

But $\quad C = 180° - (A + B)$

or $\quad = 180° - 2A$, $\quad A$ being equal to B.

Therefore

$$\frac{\delta a}{a} = \pm\alpha\sqrt{\cot^2 A + \cot^2 2A} \qquad \text{...(1.7)}$$

From Eq. (1.7), we find that, if $\frac{\delta a}{a}$ is to be minimum, $(\cot^2 A + \cot^2 2A)$ should be a minimum.

Differentiating $\cot^2 A + \cos^2 2A$ with respect to A, and equating to zero, we have

$$4\cos^4 A + 2\cos^2 A - 1 = 0 \qquad ...(1.8)$$

Solving Eq. (1.8), for $\cos A$, we get

$$A = 56°14' (approximately)$$

Hence, the best shape of an isoceles triangle is that triangle whose base angles are 56°14′ each. However, from practical considerations, an equilateral triangle may be treated as a well-conditional triangle. In actual practice, the triangles having an angle less than 30° or more than 120° should not be considered.

1.9 STRENGTH OF FIGURE

The strength of figure is a factor to be considered in establishing a triangulation system to maintain the computations within a desired degree of precision. It plays also an important role in deciding the layout of a triangulation system.

The U.S. Coast and Geodetic Surveys has developed a convenient method of evaluating the strength of a triangulation figure. It is based on the fact that computations in triangulation involve use of angles of triangle and length of one known side. The other two sides are computed by sine law. For a given change in the angles, the sine of small angles change more rapidly than those of large angles. This suggests that smaller angles less than 30° should not be used in the computation of triangulation. If, due to unavoidable circumstances, angles less than 30° are used, then it must be ensured that this is not opposite the side whose length is required to be computed for carrying forward the triangulation series.

The expression given by the U.S. Coast and Geodetic Surveys for evaluation of the strength of figure, is for the square of the probable error (L^2) that would occur in the sixth place of the logarithm of any side, if the computations are carried from a known side through a single chain of triangles after the net has been adjusted for the side and angle conditions. The expression for L^2 is

$$L^2 = \frac{4}{3} d^2 R \qquad ...(1.9)$$

where d is the probable error of an observed direction in seconds of arc, and R is a term which represents the shape of figure. It is given by

$$R = \frac{D - C}{D} \Sigma(\delta_A^2 + \delta_A \delta_B + \delta_B^2) \qquad ...(1.10)$$

where

D = the number of directions observed excluding the known side of the figure,

$\delta_A, \delta_B, \delta_C$ = the difference per second in the sixth place of logarithm of the sine of the distance angles A, B and C, respectively. (Distance angle is the angle in a triangle opposite to a side), and

C = the number of geometric conditions for side and angle to be satisfied in each figure. It is given by

$$C = (n' - S' + 1) + (n - 2S + 3) \qquad ...(1.11)$$

where

n = the total number of lines including the known side in a figure,

n' = the number of lines observed in both directions including the known side,

S = the total number of stations, and

S' = the number of stations occupied.

For the computation of the quantity $\Sigma(\delta_A^2 + \delta_A \delta_B + \delta_B^2)$ in Eq. (1.10), Table 1.2 may be used.

In any triangulation system more than one routes are possible for various stations. The strength of figure decided by the factor R alone determines the most appropriate route to adopt the best shaped triangulation net route. If the computed value of R is less, the strength of figure is more and *vice versa*.

Table 1.2 Values of $\delta_A^2 + \delta_A \delta_B + \delta_B^2$

	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
0																							
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	26	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

1.10 ACCURACY OF TRIANGULATION

Errors are inevitable and, therefore, inspite of all precautions the errors get accumulated. It is, therefore, essential to know the accuracy of the triangulation network achieved so that no appreciable error in plotting is introduced. The following formula for root mean square error may be used.

$$m = \sqrt{\frac{\Sigma E^2}{3n}} \qquad ...(1.12)$$

where m = the root mean square error of unadjusted horizontal angles in seconds of arc as obtained from the triangular errors,

ΣE = the sum of the squares of all the triangular errors in the triangulation series, and

n = the total number of triangles in the series.

It may be noted that

(*i*) all the triangles have been included in the computations,

(*ii*) all the four triangles of a braced quadrilateral have been included in the computations, and

(*iii*) if the average triangular error of the series is 8", probable error in latitudes and departures after a distance of 100 km, is approximately 8 m.

ILLUSTRATIVE EXAMPLES

Example 1.1 If the probable error of direction measurement is 1.20", compute the maximum value of R for the desired maximum probable error of (*i*) 1 in 20,000 and (*ii*) 1 in 10,000.

Solution: (*i*) L being the probable error of a logarithm, it represents the logarithm of the ratio of the true value and a value containing the probable error.

In this case L = the 6th place in $\log\left(1 \pm \frac{1}{20000}\right)$

= the 6th place in log ((1 ± 0.00005)

$\log(1 + 0.00005) = 0.0000217$

The 6th place in the log value = 21

Hence $L = \pm 21$

It is given that $d = 1.20''$

From Eq. (1.9), we have

$$L^2 = \frac{4}{3} d^2 R$$

$$R_{max} = \frac{3}{4}\frac{L^2}{d^2}$$

$$= \frac{3}{4} \times \frac{21^2}{1.20^2} = \mathbf{230.}$$

(*ii*) L = the 6th place in $\log\left(1 \pm \frac{1}{10000}\right)$

$\log(1 + 0.0001) = 0.0000434$

The 6th place in the log value = 43

Hence $L = \pm 43$

$$R_{max} = \frac{3}{4} \times \frac{43^2}{1.20^2} = \mathbf{963.}$$

Example 1.2 The probable error of direction measurement is 1″. Compute the maximum value of R if the maximum probable error is

(*i*) 1 in 25000

(*ii*) 1 in 5000.

Solution:

(*i*)
$$\log\left(1+\frac{1}{25000}\right) = 0.0000174$$

The 6th place in the log value $= 17$

Hence $L = \pm 17$

From Eq. (1.9), we get

$$R_{\max} = \frac{3L^2}{4d^2}$$

The value of d is given as 1″

$$R_{\max} = \frac{3\times 17^2}{4\times 1^2} = \mathbf{217}.$$

(*ii*)
$$\log\left(1+\frac{1}{50000}\right) = 0.0000086$$

The 6th place in the log value $= 9$

Hence $L = \pm 9$

$$R_{\max} = \frac{3\times 9^2}{4\times 1^2} = \mathbf{61}.$$

Example 1.3 Compute the value of $\frac{D-C}{D}$ for the following triangulation figures if all the stations have been occupied and all the lines have been observed in both directions :

(*i*) A single triangle

(*ii*) A braced quadrilateral

(*iii*) A four-sided central-point figure without diagonals

(*iv*) A four-sided central-point figure with one diagonal.

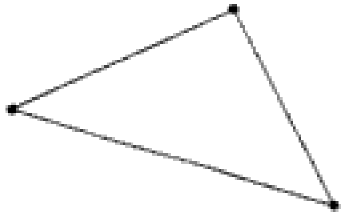

Fig. 1.11

Solution: (*i*) Single triangle (Fig. 1.11)

From Eq. (1.11), we have

$$C = (n' - S' + 1) + (n - 2S + 3)$$
$$n' = 3$$
$$n = 3$$
$$S = 3$$
$$S' = 3$$
$$C = (3 - 3 + 1) + (3 - 2\times 3 + 3) = 1$$

and D = the number of directions observed excluding the known side.

= 2 (total number of lines – 1)

$= 2 \times (3 - 1) = 4$

$$\frac{D-C}{D} = \frac{4-1}{4} = \mathbf{0.75}.$$

(*ii*) Braced quadrilateral (Fig. 1.12)

$$n = 6$$
$$n' = 6$$
$$S = 4$$
$$S' = 4$$
$$C' = (6 - 4 + 1) + (6 - 2 \times 4 + 3) = 4$$
$$D = 2 \times (6 - 1) = 10$$
$$\frac{D - C}{D} = \frac{10 - 4}{10} = \mathbf{0.6}.$$

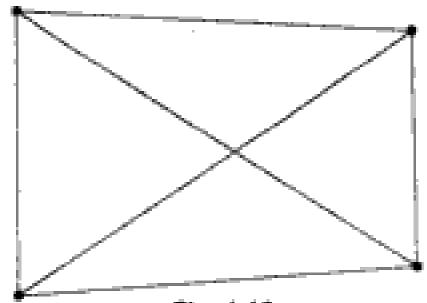

Fig. 1.12

(*iii*) Four-sided central-point figures without diagonals (Fig. 1.13)

$$n = 8$$
$$n' = 8$$
$$S = 5$$
$$S' = 5$$
$$C = (8 - 5 + 1) + (8 - 2 \times 5 + 3) = 5$$
$$D = 2 \times (8 - 1) = 14$$

Therefore
$$\frac{D - C}{D} = \frac{14 - 5}{14} = \mathbf{0.64}.$$

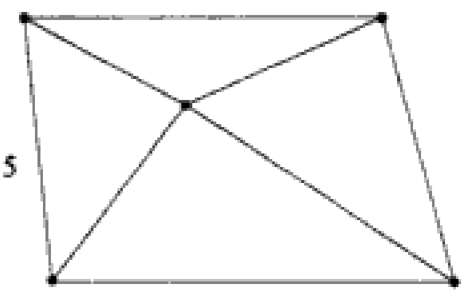

Fig. 1.13

(*iv*) Four-sided central-point figure with one diagonal. (Fig. 1.14)

$$n = 9$$
$$n' = 9$$
$$S = 5$$
$$S' = 5$$
$$C = (9 - 5 + 1) + (9 - 2 \times 5 + 3) = 7$$
$$D = 2 \times (9 - 1) = 16$$

Therefore
$$\frac{D - C}{D} = \frac{16 - 7}{16} = \mathbf{0.56}.$$

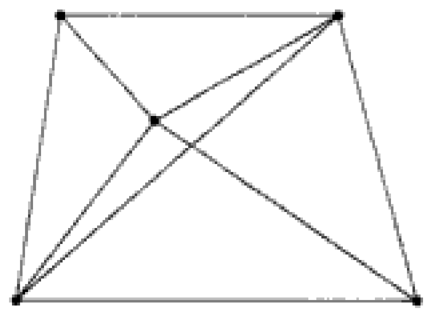

Fig. 1.14

Example 1.4 Compute the value of $\frac{D - C}{D}$ for the triangulation nets shown in Fig. 1.15 (*a – d*). The directions observed are shown by arrows.

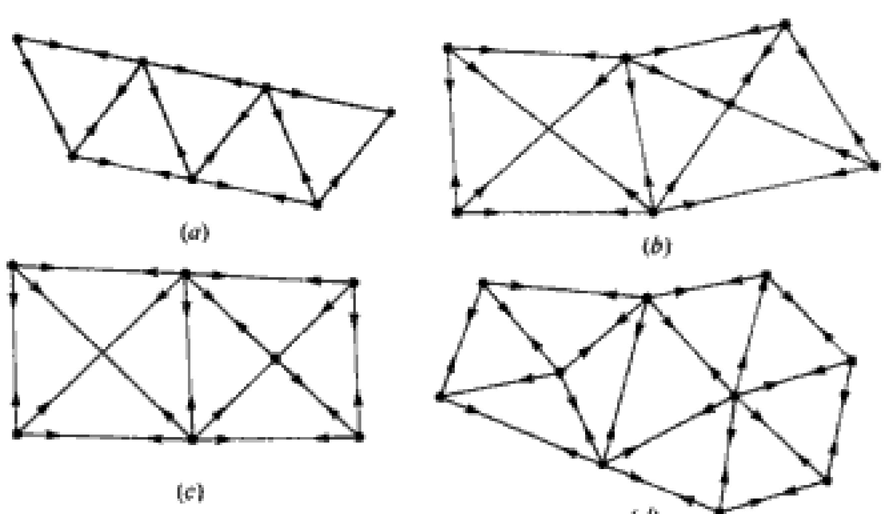

Fig. 1.15

Solution: (*i*) Fig. 1.15*a*

From Eq. (1.11), we have

$$C = (n' - S' + 1) + (n - 2S + 3)$$

n = the total number of lines
$= 11$

n' = the total number of lines observed in both directions
$= 9$

S = the total number of stations
$= 7$

S' = the total number of stations occupied
$= 6$

$$C = (9 - 6 + 1) + (11 - 2 \times 7 + 3) = 4$$

and D = the total number of directions observed excluding the known side
$= 2 \times (n' - 1)$ + number of lines observed in one direction
$= 2 \times (9 - 1) + 2 = 18$

Therefore $$\frac{D - C}{D} = \frac{18 - 4}{18} = \mathbf{0.78}.$$

(*ii*) Fig. 1.15*b*

$n = 13$
$n' = 11$
$S = 7$
$S' = 7$
$C = (11 - 7 + 1) + (13 - 2 \times 7 + 3) = 7$
$D = 2 \times (11 - 1) + 2 = 22$

Therefore $$\frac{D - C}{D} = \frac{22 - 7}{22} = \mathbf{0.68}.$$

(*iii*) Fig. 1.15*c*

$n = 13$
$n' = 11$
$S = 7$
$S' = 7$
$C = (11 - 7 + 1) + (13 - 2 \times 7 + 3) = 7$
$D = 2 \times (11 - 1) + 2 = 22$

Therefore $$\frac{D - C}{D} = \frac{22 - 7}{22} = \mathbf{0.68}.$$

(*iv*) Fig. 1.15*d*

$n = 19$
$n' = 19$
$S = 10$
$S' = 10$
$C = (19 - 10 + 1) + (19 - 2 \times 10 + 3) = 12$
$D = 2(19 - 1) + 0 = 36$

Therefore $$\frac{D - C}{D} = \frac{36 - 12}{36} = \mathbf{0.67}.$$

Example 1.5 Compute the strength of the figure *ABCD* for all the routes by which the length *CD* can be computed from the known side *AB*. Assume that all the stations were occupied.

Solution:

From Eq. (1.10), we have

$$R = \frac{D-C}{D}\Sigma(\delta_A^2 + \delta_A\delta_B + \delta_C^2)$$

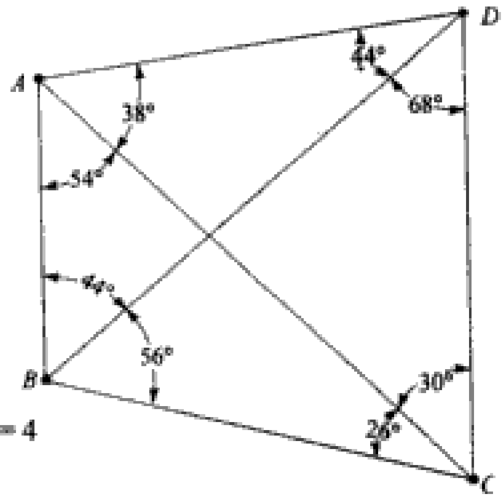

Fig. 1.16

For the given figure in Fig. 1.16, we have

$$n = 6$$
$$n' = 6$$
$$S = 4$$
$$S' = 4$$
$$D = 2 \times (n-1)$$
$$= 2 \times (6-1) = 10$$

Hence
$$C = (n' - S' + 1) + (n - 2S + 3)$$
$$= (6 - 4 + 1) + (6 - 2 \times 4 + 3) = 4$$

and
$$\frac{D-C}{D} = \frac{10-4}{10} = \mathbf{0.60}.$$

(*a*) Route-1, using Δ^s *ABC* and *ADC* with common side *AC*

For ΔABC the distance angles of *AB* and *AC* are 26° and 100° = 44° + 56°, respectively. From Table 1.2,

$$\delta_{100}^2 + \delta_{100}\delta_{26} + \delta_{26}^2 = 17$$

For ΔADC, the distance angles of *AC* and *DC* are 112° = (44° + 68°) and 38°, respectively,

$$\delta_{112}^2 + \delta_{112}\delta_{38} + \delta_{38}^2 = 6$$
$$R_1 = 0.6 \times (17 + 6) = 13.8 \simeq 14$$

(*b*) Route-2, using Δ^s *ABC* and *BCD* with common side *BC*

For ΔABC the distance angles of *AB* and *BC* are 26° and 54°, respectively,

$$\delta_{54}^2 + \delta_{54}\delta_{26} + \delta_{26}^2 = 27$$

For ΔBCD, the distance angle of *BC* and *CD* are 68° and 56°, respectively,

$$\delta_{68}^2 + \delta_{68}\delta_{56} + \delta_{56}^2 = 4$$
$$R_2 = 0.6 \times (27 + 4) = 18.6 \simeq 19$$

(*c*) Route-3, using Δ^s *ABD* and *ACD* with common side *AD*

From ΔABC the distance for both the sides *AB* and *AD* is 44°.

$$\delta_{44}^2 + \delta_{44}\delta_{44} + \delta_{44}^2 = 13$$

From ΔACD, the distance angles of *AD* and *CD* and 30° and 38°, respectively,

$$\delta_{38}^2 + \delta_{38}\delta_{30} + \delta_{30}^2 = 31$$
$$R_3 = 0.6 \times (13 + 31) = 26.4 \simeq 26$$

(*d*) Route-4, using Δ^s *ABD* and *BCD* with common side *BD*.

From ΔABD, the distance angles of *AB* and *DB* are 44° and 92° = (38° + 54°), respectively,

$$\delta_{92}^2 + \delta_{92}\delta_{38} + \delta_{38}^2 = 7$$

From ΔBCD, the distance angles of *BD* and *CD* are 56° = (30° + 26°) and 56°, respectively,

$$\delta_{56}^2\delta_{56}\delta_{56} + \delta_{56}^2 = 7$$

$$R_4 = 0.6 \times (7 + 7) = 8.4 \simeq 8$$

Since the lowest value of R represents the highest strength, the best route to compute the length of CD is Route-4, having $R_4 = 8$.

1.11 ROUTINE OF TRIANGULATION SURVEY

The routine of triangulation survey, broadly consists of

(*a*) field work, and (*b*) computations.

The field work of triangulation is divided into the following operations :

(*i*) Reconnaissance
(*ii*) Erection of signals and towers
(*iii*) Measurement of base line
(*iv*) Measurement of horizontal angles
(*v*) Measurement of vertical angles
(*vi*) Astronomical observations to determine the azimuth of the lines.

1.12 RECONNAISSANCE

Reconnaissance is the preliminary field inspection of the entire area to be covered by triangulation, and collection of relevant data. Since the basic principle of survey is working from whole to the part, reconnaissance is very important in all types of surveys. It requires great skill, experience and judgement. The accuracy and economy of triangulation greatly depends upon proper reconnaissance survey. It includes the following operations:

1. Examination of terrain to be surveyed.
2. Selection of suitable sites for measurement of base lines.
3. Selection of suitable positions for triangulation stations.
4. Determination of intervisibility of triangulation stations.
5. Selection of conspicuous well-defined natural points to be used as intersected points.
6. Collection of miscellaneous information regarding:
   (*a*) Access to various triangulation stations
   (*b*) Transport facilities
   (*c*) Availability of food, water, etc.
   (*d*) Availability of labour
   (*e*) Camping ground.

Reconnaissance may be effectively carried out if accurate topographical maps of the area are available. Help of aerial photographs and mosaics, if available, is also taken. If maps and aerial photographs are not available, a rapid preliminary reconnaissance is undertaken to ascertain the general location of possible schemes of triangulation suitable for the topography. Later on, main reconnaissance is done to examine these schemes. The main reconnaissance is a very rough triangulation. The plotting of the rough triangulation may be done by protracting the angles. The essential features of the topography are also sketched in. The final scheme is selected by studying the relative strengths and cost to various schemes.

For reconnaissance the following instruments are generally employed:

1. Small theodolite and sextant for measurement of angles.
2. Prismatic compass for measurement of bearings.
3. Steel tape.
4. Aneroid barometer for ascertaining elevations.
5. Heliotropes for ascertaining intervisibility.
6. Binocular.
7. Drawing instruments and material.
8. Guyed ladders, creepers, ropes, etc., for climbing trees.

1.12.1 Erection of signals and towers

A *signal* is a device erected to define the exact position of a triangulation station so that it can be observed from other stations whereas a *tower* is a structure over a station to support the instrument and the observer, and is provided when the station or the signal, or both are to be elevated.

Before deciding the type of signal to be used, the triangulation stations are selected. The selection of triangulation stations is based upon the following criteria.

Criteria for selection of triangulation stations

1. Triangulation stations should be intervisible. For this purpose the station points should be on the highest ground such as hill tops, house tops, etc.
2. Stations should be easily accessible with instruments.
3. Station should form well-conditioned triangles.
4. Stations should be so located that the lengths of sights are neither too small nor too long. Small sights cause errors of bisection and centering. Long sights too cause direction error as the signals become too indistinct for accurate bisection.
5. Stations should be at commanding positions so as to serve as control for subsidiary triangulation, and for possible extension of the main triangulation scheme.
6. Stations should be useful for providing intersected points and also for detail survey.
7. In wooded country, the stations should be selected such that the cost of clearing and cutting, and building towers, is minimum.
8. Grazing line of sights should be avoided, and no line of sight should pass over the industrial areas to avoid irregular atmospheric refraction.

Determination of intervisibility of triangulation stations

As stated above, triangulations stations should be chosen on high ground so that all relevant stations are intervisible. For small distances, intervisibility can be ascertained during reconnaissance by direct observation with the aid of binocular, contoured map of the area, plane mirrors or heliotropes using reflected sun rays from either station.

However, if the distance between stations is large, the intervisibility is ascertained by knowing the horizontal distance between the stations as under.

Case-I Invervisibility not obstructed by intervening ground

If the intervening ground does not obstruct the intervisibility, the distance of visible horizon from the station of known elevation is calculated from the following formula:

$$h = \frac{D^2}{2R}(1-2m) \qquad \text{...(1.13)}$$

where h = height of the station above datum,

D = distance of visible horizon,

R = earth's mean radius, and

m = mean coefficient of refraction taken as 0.07 for sights over land, and. 0.08 for sights over sea.

Substituting the values of m as 0.071 and R as 6370 km in Eq. (1.13), the value of h in metres is given by

$$h = 0.06735\ D^2 \qquad \text{...(1.14)}$$

where D is in kilometres.

In Fig. 1.17, the distance between two stations A and B of heights h_A and h_B, respectively, is D. If D_A and D_B are the distances of visible horizon from A and B, respectively, we have

$$D_A = \sqrt{\frac{h_A}{0.06735}} = 3.853\sqrt{h_A} \quad ...(1.15)$$

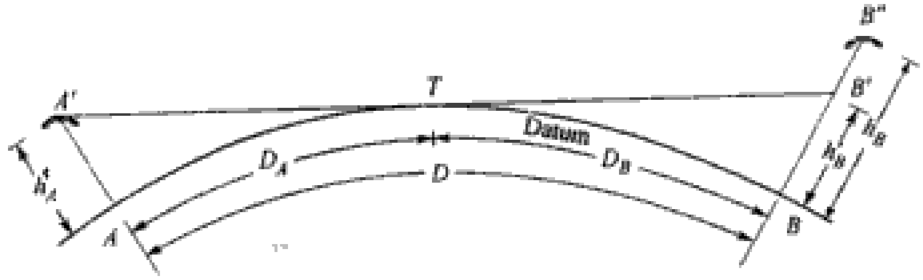

Fig. 1.17 Intervisibility not obstructed by intervening ground

We have $D = D_A + D_B$

or $D_B = D - D_A$

For the known distance of visible horizon D_B as above, the height of station B is computed. If the computed value is h'_B, then

$$h'_B = 0.06735\, D_B^2 \quad ...(1.16)$$

The computed value of height h'_B is compared with the known value h_B as below :

If $h_B \geq h'_B$, the station B will be visible from A, and

if $h_B < h'_B$, the station B will not be visible from A.

If B is not visible from A, $(h'_B - h_B)$ is the required amount of height of signal to be erected at B. While deciding the intervisibility of various stations, the line of sight should be taken at least 3 m above the point of tangency T of the earth's surface to avoid grazing rays.

Case-II Intervisibility obstructed by intervening ground

In Fig. 1.18, the intervening ground at C is obstructing the intervisibility between the stations A and B. From Eq. (1.15), we have

$$D_A = 3.853\sqrt{h_A} \quad ...(1.17)$$

The distance D_T of the peak C from the point of tangency T, is given by

$$D_T = D_A - D_C \quad ...(1.18)$$

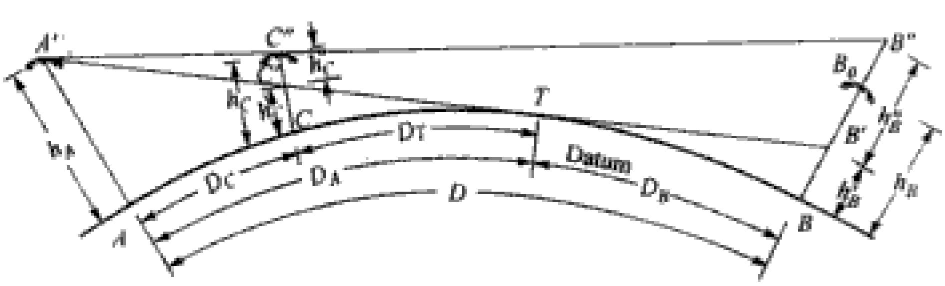

Fig. 1.18 Intervisibility obstructed by intervening ground

and $$h'_C = 0.06735\, D_T^{\,2} \quad \ldots(1.19)$$

$$h'_B = 0.06735\, D_B^{\,2} \quad \ldots(1.20)$$

If $h'_C > h_C$, the line of sight is clear of the obstruction, and it becomes Case-I discussed above. If $h'_C < h_C$ then the signal at B is to be raised. The amount of raising required at B is computed as below.

From similar $\Delta^s\, A'C'C''$ and $A'B'B''$ in Fig. 1.19, we get

$$\frac{h''_C}{D_C} = \frac{h''_B}{D}$$

or $$h''_B = \frac{D}{D_C}\, h''_C \quad \ldots(1.21)$$

where $h''_C = h_C - h'_C$.

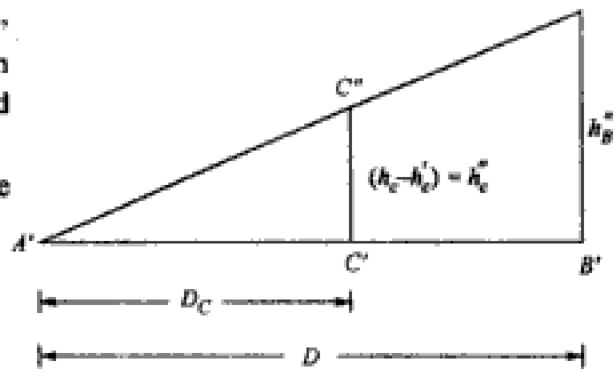

Fig. 1.19

The required height of signal above station B_0 is

$$B_0B'' = (BB' + B'B'') - BB_0$$
$$= (h'_B + h''_C) - h_B \quad \ldots(1.22)$$

Alternate method (Captain G.T. McCaw's method)

A comparison of elevations of the stations A and B (Fig. 1.20) decides whether the triangulation stations are intervisible or not. A direct solution suggested by Captain McCaw is known as Captain McCaw's method.

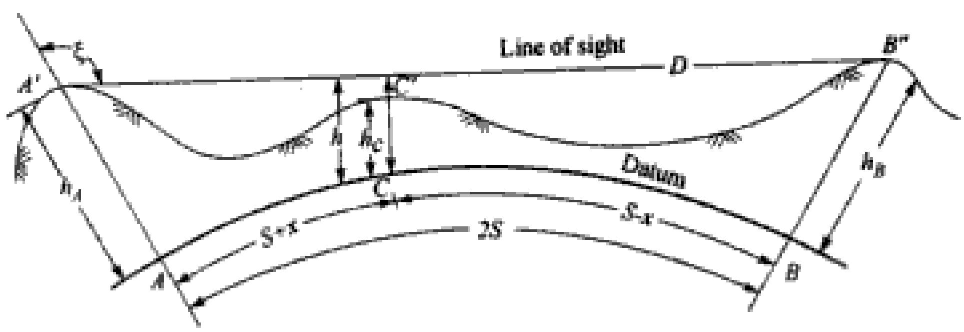

Fig. 1.20 Captain McCaw's method of ascertaining intervisibility

Let h_A = elevation of station A

h_B = elevation of station B

h_C = elevation of station C.

$2S$ = distance between A and B

$(S+x)$ = distance between A and C

$(S-x)$ = distance between C and B

h = elevation of the line of sight at C

ξ = zenith distance from A to B

= (90°-vertical angle).

From Captain McCaw's formula

$$h = \frac{1}{2}(h_B + h_A) + \frac{1}{2}(h_B - h_A)\frac{x}{S} - (S^2 - x^2)\operatorname{cosec}^2 \xi \frac{(1-2m)}{2R} \quad ...(1.23)$$

Practically in most of the cases, the zenith distance is very nearly equal to 90° and, therefore, the value of $\operatorname{cosec}^2 \xi$ may be taken approximately equal to unity.

However, for accurate calculations,

$$\operatorname{cosec}^2 \xi = 1 + \frac{(h_B - h_A)^2}{4S^2} \quad ...(1.24)$$

In Eq. (1.23), the value of $\left(\frac{1-2m}{2R}\right)$ is usually taken as 0.06735.

Therefore

$$h = \frac{1}{2}(h_B + h_A) + \frac{1}{2}(h_B - h_A)\frac{x}{S} - (S^2 - x^2) \times 0.06735 \quad ...(1.25)$$

If $h > h_C$, the line of sight is free of obstruction. In case $h < h_C$, the height of tower to raise the signal at B, is computed from Eqs. (1.21) and (1.22).

ILLUSTRATIVE EXAMPLES

Example 1.6 Two stations A and B, 80 km apart, have elevations 15 m and 270 m above mean sea level, respectively. Calculate the minimum height of the signal at B.

Solution: (Fig. 1.21)

It is given that

$$h_A = 15 \text{ m}$$
$$h_B = 270 \text{ m}$$
$$D = 80 \text{ km}$$

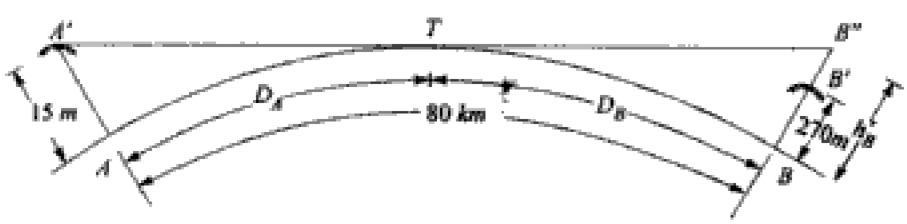

Fig. 1.21

From Eq. (1.15), we get

$$D_A = 3.853\sqrt{h_A} = 3.853 \times \sqrt{15} = 14.92 \text{ km}$$

We have

$$D_B = D - D_A$$

or

$$= 80 - 14.92$$
$$= 65.08 \text{ km}$$

Therefore

$$h'_B = 0.06735\ D_B^2$$
$$= 0.06735 \times 65.08^2 = 285.25 \text{ m}$$

Hence, since the elevation of B is 270 m, the height of signal required at B, is

$$= 285.25 - 270 = 15.25 \simeq \mathbf{15.5\ m}.$$

Example 1.7 There are two stations P and Q at elevations of 200 m and 995 m, respectively. The distance of Q from P is 105 km. If the elevation of a peak M at a distance of 38 km from P is 301 m, determine whether Q is visible from P or not. If not, what would be the height of scaffolding required at Q so that Q becomes visible from P?

Solution: (Fig. 1.22)

From Eq. (1.15), we get

$$PT = 3.853 \times \sqrt{200} = 54.45 \text{ km}$$

Therefore

$$MT = PT - PM$$
$$= 54.45 - 38 = 16.45 \text{ km}$$

Using Eq. (1.14) and the value of MT, we get

$$MM' = 0.06735 \times 16.45^2 = 18.23 \text{ m}$$

The distance of Q from the point of tangency T is

$$QT = 105 - 54.45 = 50.55 \text{ km}$$

Therefore

$$QQ' = 0.06735 \times 50.55^2 = 172.10 \text{ m}$$

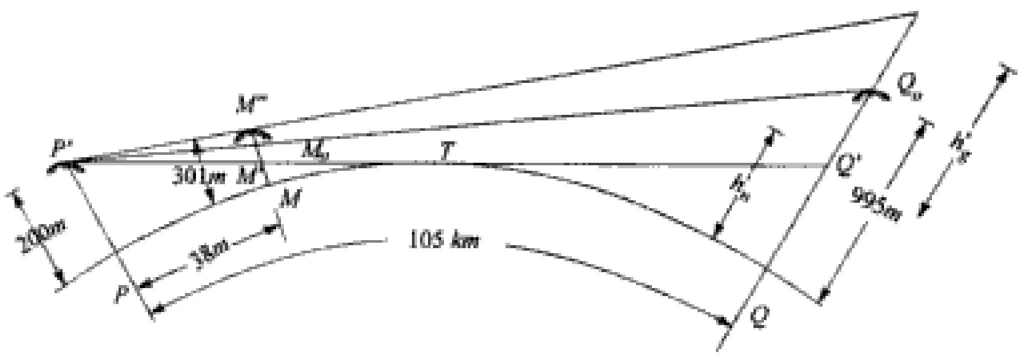

Fig. 1.22

From similar $\Delta^s P'M'M''$ and $P'Q'Q''$, we have

$$\frac{M'M''}{PM} = \frac{Q'Q''}{PQ}$$

$$Q'Q'' = \frac{PQ}{PM} M'M''$$

$$= \frac{PQ}{PM}(MM'' - MM')$$

$$= \frac{105}{38} \times (301 - 18.23) = 781.34 \text{ m}$$

We have

$$QQ'' = QQ' + Q'Q''$$
$$= 172.10 + 781.34 = 953.44 \text{ m}$$

As the elevation 995 m of Q is more than 953.44 m, the peak at M does not obstruct the line of sight.

Alternatively, from the similar $\Delta^s P'M'M_o$ and $PQ'Q_o$, we have

$$\frac{M'M_o}{PM} = \frac{Q'Q_o}{PQ}$$

or

$$M'M_o = \frac{PM}{PQ} Q'Q_o$$

$$= \frac{PM}{PQ}(QQ_o - QQ')$$

$$= \frac{38}{105} \times (995 - 172.10) = 297.81$$

The elevation of line of sight $P'Q_o$ at M is

$$MM_o = MM' + M'M_o$$

$$= 18.23 + 297.81 = 316.04.$$

Since the elevation of peak at M is 301 m, the line of sight is not obstructed by the peak and, therefore, no scaffolding is required at Q.

Example 1.8 Solve the problem given in Example 1.7 by Capt. McCaw's method.

Solution: (Fig. 1.22)

From Eq. (1.25), the elevation of line of sight at M joining the two stations is

$$h = \frac{1}{2}(h_Q + h_P) + \frac{1}{2}(h_Q - h_P)\frac{x}{S} - (S^2 - x^2) \times 0.06735$$

It is given that

$$h_P = 200 \text{ m}$$
$$h_Q = 995 \text{ m}$$
$$h_M = 301 \text{ m}$$
$$2S = 105 \text{ km or } S = 52.5 \text{ km}$$
$$S + x = 38 \text{ km or } x = -14.5 \text{ km}$$

Therefore

$$h = \frac{1}{2} \times (995 + 200) + \frac{1}{2} \times (995 - 200) \times \frac{(-14.5)}{52.5}$$
$$- (52.5^2 - 14.5^2) \times 0.06735$$
$$= \mathbf{316.24 \ m}.$$

The elevation of the line of sight $p'Q_0$ at M is 316.24 m, and the elevation of the peak is 301 m, therefore, the line of sight is clear of obstruction.

Example 1.9 In a triangulation survey, the altitudes of two proposed stations A and B, 100 km apart, are respectively 425 m and 750 m. The intervening ground situated at C, 60 km from A, has an elevation of 435 m. Ascertain if A and B are intervisible, and if necessary find by how much B should be raised so that the line of sight must nowhere be less than 3 m above the surface of the ground. Take R = 6400 km and m = 0.07.

Solution: (Fig. 1.20)

From the given data we have

$$h_A = 425 \text{ m}, \ h_B = 750 \text{ m}, \ h_C = 435 \text{ m}, R = 6400 \text{ km}, m = 0.07$$
$$2S = 100 \text{ km, or } S = 50 \text{ km}$$
$$S + x = 60 \text{ km or } x = 10 \text{ km}$$

Eq. (1.23) gives

$$h'_C = \frac{1}{2}(h_B + h_A) + \frac{1}{2}(h_B - h_A)\frac{x}{S} - (S^2 - x^2)\operatorname{cosec}^2 \xi \frac{(1 - 2m)}{2R}$$

Taking $\operatorname{cosec}^2 \xi = 1$, and substituting the values of the given data in the above equation, we have

$$h = \frac{1}{2} \times (705 + 425) + \frac{1}{2} \times (705 - 425) \times \frac{10}{50} - (50^2 - 10^2)$$
$$\times 1 \times \frac{(1 - 2 \times 0.07)}{2 \times 6400} \times 1000 = 431.75 \text{ m}$$

As the elevation of the line of sight at *C* is less than the elevation of *C*, the line of sight fails to clear *C* by

$$435 - 431.75 = 3.25\,\text{m}$$

To avoid grazing rays, the line of should be at least 3m above the ground. Therefore, the line of sight should be raised to 3.25 + 3 = 6.25 m at *C*.

Hence, the minimum height of signal to be erected at *B*

$$= \frac{6.25}{60} \times 100 = \mathbf{10.42\ m}.$$

Station Mark

The triangulation stations should be permanently marked on the ground so that the theodolite and signal may be centered accurately over them. The following points should be considered while marking the exact position of a triangulation station :

(*i*) The station should be marked on perfectly stable foundation or rock. The station mark on a large size rock is generally preferred so that the theodolite and observer can stand on it. Generally, a hole 10 to 15 cm deep is made in the rock and a copper or iron bolt is fixed with cement.

(*ii*) If no rock is available, a large stone is embeded about 1 m deep into the ground with a circle, and dot cut on it. A second stone with a circle and dot is placed vertically above the first stone.

(*iii*) A G.I. pipe of about 25 cm diameter driven vertically into ground up to a depth of one metre, also served as a good station mark.

(*iv*) The mark may be set on a concrete monument. The station should be marked with a copper or bronze tablet. The name of the station and the date on which it was set, should be stamped on the tablet.

(*v*) In earth, generally two marks are set, one about 75 cm below the surface of the ground, and the other extending a few centimeters above the surface of the ground. The underground mark may consist of a stone with a copper bolt in the centre, or a concrete monument with a tablet mark set on it (Fig. 1.23).

(*vi*) The station mark with a vertical pole placed centrally, should be covered with a conical heap of stones placed symmetrically. This arrangement of marking station, is known as placing a cairn (Fig. 1.27).

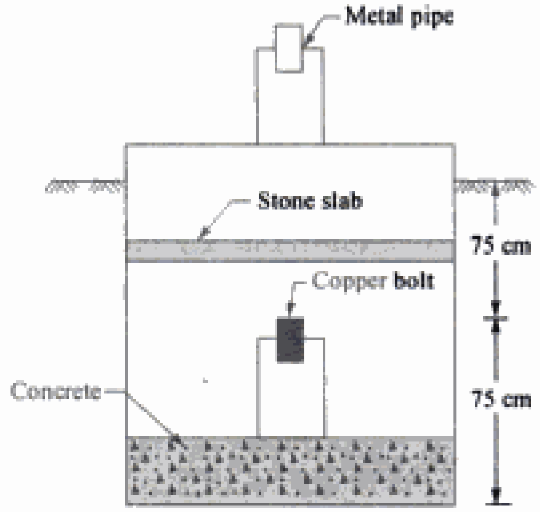

Fig. 1.23 Station mark

(*vii*) Three reference marks at some distances on fairly permanent features, should be established to locate the station mark, if it is disturbed or removed.

(*viii*) Surrounding the station mark a platform 3 m × 3 m × 0.5 m should be built up of earth.

1.13 SIGNALS

Signals are centered vertically over the station mark, and the observations are made to these signals from other stations. The accuracy of triangulation is entirely dependent on the degree of accuracy of centering the signals. Therefore, it is very essential that the signals are truly vertical, and centered over the station mark. Greatest care of centering the transit over the station mark will be useless, unless some degree of care in centering the signal is impressed upon.

A signal should fulfil the following requirements :

(*i*) It should be conspicuous and clearly visible against any background. To make the signal conspicuous, it should be kept at least 75 cm above the station mark.

(*ii*) It should be capable of being accurately centered over the station mark.

(*iii*) It should be suitable for accurate bisection from other stations.

(*iv*) It should be free from phase, or should exhibit little phase (*cf.*, Sec. 1.15).

1.13.1 Classification of signals

The signals may be classified as under :

(*i*) Non-luminous, opaque or daylight signals

(*ii*) Luminous signals.

(*i*) Non-luminous signals

Non-luminous signals are used during day time and for short distances. These are of various types, and the most commonly used are of following types.

(*a*) **Pole signal** (Fig. 1.24) : It consists of a round pole painted black and white in alternate strips, and is supported vertically over the station mark, generally on a tripod. Pole signals are suitable upto a distance of about 6 km.

(*b*) **Target signal** (Fig. 1.25): It consists of a pole carrying two squares or rectangular targets placed at right angles to each other. The targets are generally made of cloth stretched on wooden frames. Target signals are suitable upto a distance of 30 km.

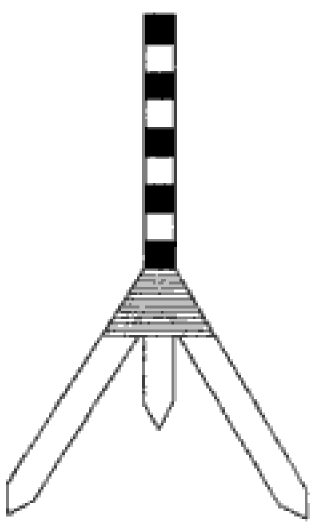

Fig. 1.24 Pole signal

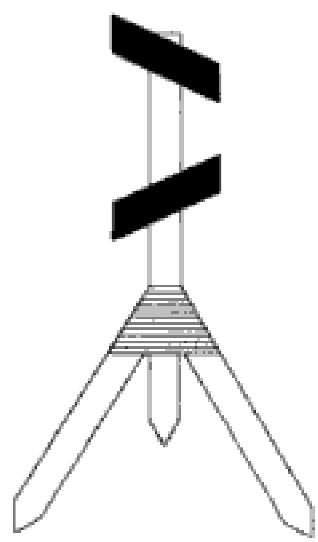

Fig. 1.25 Target signal

(*c*) **Pole and brush signal** (Fig. 1.26): It consists of a straight pole about 2.5 m long with a bunch of long grass tied symmetrically round the top making a cross. The signal is erected vertically over the station mark by heaping a pile of stones, upto 1.7 m round the pole. A rough coat of white wash is given to make it more conspicuous to be seen against black background. These signals are very useful, and must be erected over every station of observation during reconnaissance.

(*d*) **Stone cairn** (Fig. 1.27): A pile of stone heaped in a conical shape about 3 m high with a cross shape signal erected over the stone heap, is stone cairn. This white washed opaque signal is very useful if the background is dark.

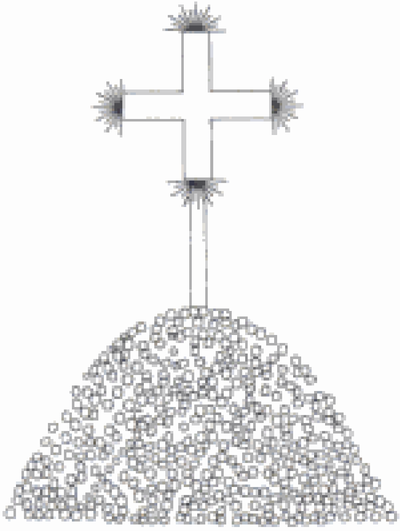

Fig. 1.26 Pole and brush signal

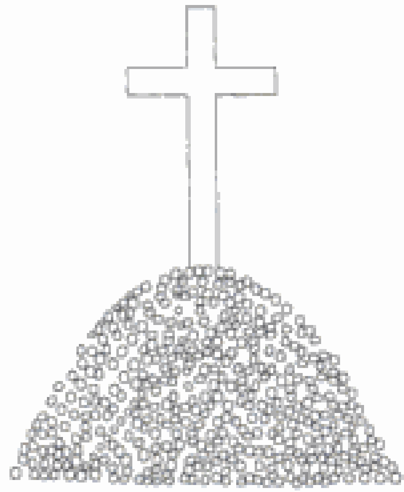

Fig. 1.27 Stone cairn

(*e*) **Beacons** (Fig. 1.28): It consists of red and white cloth tied round the three straight poles. The beacon can easily be centered over the station mark. It is very useful for making simultaneous observations.

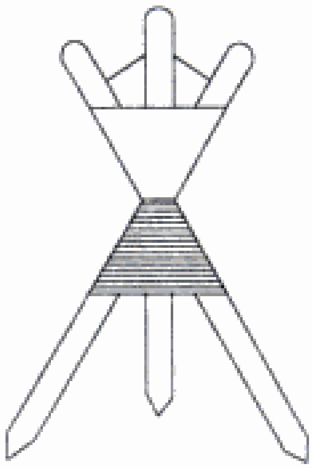

Fig. 1.28 Beacon

(*II*) Luminous signals

Luminous signals may be classified into two types :

(*i*) Sun signals
(*ii*) Night signals.

(*a*) **Sun signals** (Fig. 1.29): Sun signals reflect the rays of the sun towards the station of observation, and are also known as heliotropes. Such signals can be used only in day time in clear weather.

Heliotrope : It consists of a circular plane mirror with a small hole at its centre to reflect the sun rays, and a sight vane with an aperture carrying a cross-hairs. The circular mirror can be rotated horizontally as well as vertically through 360°. The heliotrope is centered over the station mark, and the line of sight is directed towards the station of observation. The sight vane is adjusted looking through the hole till the flashes given from the station of observation fall at the centre of the cross of the sight vane. Once this is achieved, the heliotrope is disturbed. Now the heliotrope frame carrying the mirror is rotated in such a way that the black shadow of the small central hole of the plane mirror falls exactly at the cross of the sight vane. By doing so, the reflected beam of rays will be seen at the station of observation. Due to motion of the sun, this small shadow also moves, and it should be constantly ensured that the shadow always remains at the cross till the observations are over.

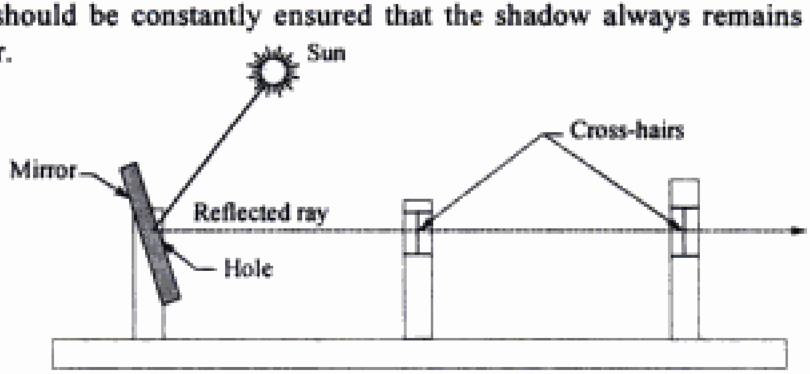

Fig. 1.29 Heliotrope

The heliotropes do not give better results compared to signals. These are useful when the signal station is in flat plane, and the station of observation is on elevated ground. When the distance between the stations exceed 30 km, the heliotropes become very useful.

(b) Night signals: When the observations are required to be made at night, the night signals of following types may be used.

1. Various forms of oil lamps with parabolic reflectors for sights less than 80 km.
2. Acetylene lamp designed by Capt. McCaw for sights more than 80 km.
3. Magnesium lamp with parabolic reflectors for long sights.
4. Drummond's light consisting of a small ball of lime placed at the focus of the parabolic reflector, and raised to a very high temperature by impinging on it a stream of oxygen.
5. Electric lamps.

1.14 TOWERS

A tower is erected at the triangulation station when the station or the signal or both are to be elevated to make the observations possible form other stations in case of problem of intervisibility. The height of tower depends upon the character of the terrain and the length of the sight.

The towers generally have two independent structures. The outer structure is for supporting the observer and the signal whereas the inner one is for supporting the instrument only. The two structures are made entirely independent of each other so that the movement of the observer does not disturb the instrument setting. The two towers may be made of masonary, timber or steel. For small heights, masonary towers are most suitable. Timber scaffolds are most commonly used, and have been constructed to heights over 50 m. Steel towers made of light sections are very portable, and can be easily erected and dismantled. Bilby towers patented by J.S. Bilby of the U.S. Coast and Geodetic Survey, are popular for heights ranging from 30 to 40 m. This tower weighing about 3 tonnes, can be easily erected by five persons in just 5 hrs. A schematic of such a tower is shown in Fig. 1.30.

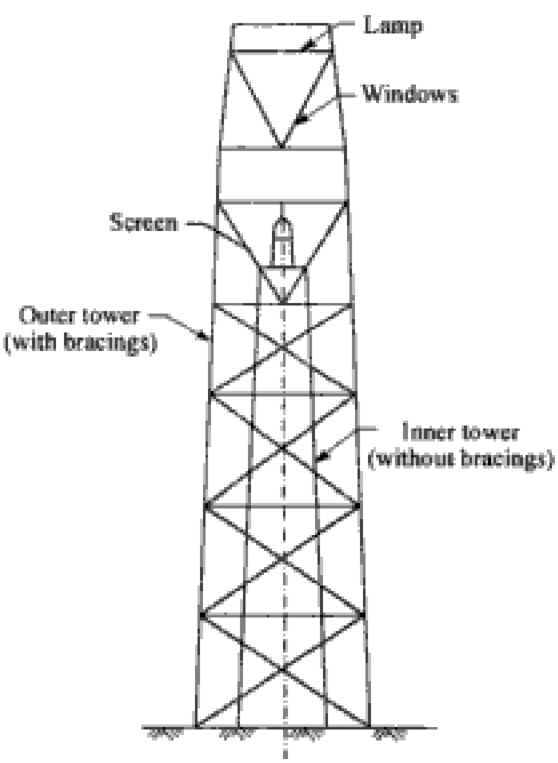

Fig. 1.30 Bilby tower

1.15 PHASE OF A SIGNAL

When cylindrical opaque signals are used, they require a correction in the observed horizontal angles due an error known as the *phase*. The cylindrical signal is partly illuminated by the sun, and the other part remains in shadow, and becomes invisible to the observer. While making the observations, the observer may bisect the bright portion or the bright line. Thus the signal is not bisected at the centre, and an error due to wrong bisection is introduced. It is, thus, the apparent displacement of the signal. The phase correction is thus necessary so that the observed horizontal angles may be reduced to that corresponding to the centre of the signal.

Depending upon the method of observation, phase correction is computed under the following two conditions.

(i) Observation made on bright portion

In Fig. 1.31, a cylindrical signal of radius r, is centered over the station P. The illuminated portion of the signal which the observer from O is able to see, is AB. The observer from the station O, makes two observations at A and B of the bright portion, AB. Let C be the midpoint of AB.

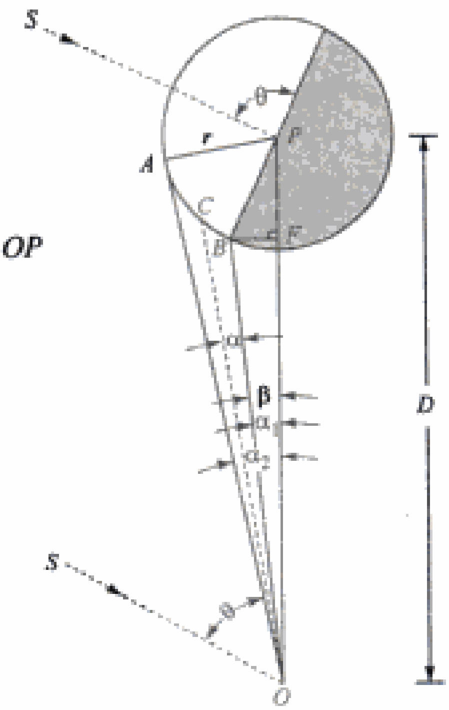

Fig. 1.31 Phase correction when observation made on the bright portion

Let θ = the angle between the sun and the line OP

α_1 and α_2 = the angles BOP and AOP, respectively

D = the horizontal distance OP

α = half of the angle AOB

$$= \frac{1}{2}(\alpha_2 - \alpha_1)$$

β = the phase correction

$$= \alpha_1 + \alpha = \alpha_1 + \frac{1}{2}(\alpha_2 - \alpha_1)$$

or

$$= \frac{1}{2}(\alpha_1 + \alpha_2) \qquad \text{...(1.26)}$$

From ΔOAP we get

$$\tan \alpha_2 = \frac{r}{D}$$

α_2 being small, we can write

$$\alpha_2 = \frac{r}{D} \text{ radians} \qquad \text{...(1.27)}$$

As the distance PF is very small compared to OP, OF may be taken as OP. Thus, from right angle ΔBFO, we get

$$\tan \alpha_1 = \frac{BF}{OF} = \frac{BF}{OP} = \frac{BF}{D} \qquad \text{...(1.28)}$$

From ΔPFB, we get

$$BF = r \sin (90 - \theta) = r \cos \theta$$

Substituting the value of BF in Eq. (1.28), we get

$$\tan \alpha_1 = \frac{r \cos\theta}{D}$$

α_1 being small, we can write

$$\alpha_1 = \frac{r\cos\theta}{D} \text{ radians} \qquad \text{...(1.29)}$$

Substituting the values of α_1 and α_2 in Eq. (1.26), we have

$$\beta = \frac{1}{2}\left(\frac{r}{D} + \frac{r\cos\theta}{D}\right) = \frac{r}{D}\left(\frac{1+\cos\theta}{2}\right)$$

$$= \frac{r}{D}\cos^2\frac{\theta}{2} \text{ radians} \qquad \text{...(1.30)}$$

$$= \frac{r}{D \sin 1''}\cos^2\frac{\theta}{2} \text{ seconds}$$

$$\beta = \frac{206265r}{D}\cos^2\frac{\theta}{2} \text{ seconds} \qquad \text{...(1.31)}$$

(ii) Observations made on the bright line

In this case, the bright line at C on the cylindrical signal of radius r, is sighted from O (Fig. 1.32).

Let CO = the reflected ray of the sun from the bright line at C

β = the phase correction

θ = the angle between the sun and the line OP

The rays of the sun are always parallel to each other, therefore, SC is parallel to S_1O.

$$\angle SCO = 180° - (\theta - \beta)$$

$$\angle PCO = 180° - \frac{1}{2}\angle SCO$$

or

$$= 180° - \frac{1}{2}[180° - (\theta - \beta)]$$

$$= 90° + \frac{1}{2}(\theta - \beta) \quad \text{... (1.32)}$$

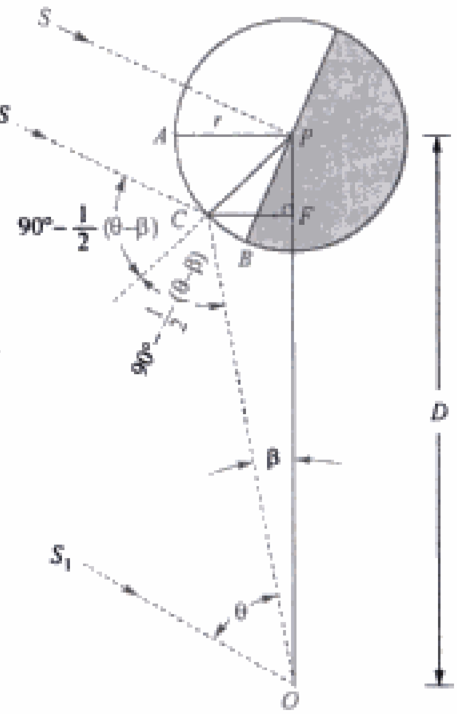

Fig. 1.32 Phase correction when observation made on the bright line

Therefore,

$$\angle CPO = 180° - (\beta + \angle PCO) \quad \text{... (1.33)}$$

Substituting the value of $\angle PCO$ from Eq. (1.32) in Eq. (1.33) and after simplification, we get

$$\angle CPO = 90° - \frac{1}{2}(\theta + \beta)$$

As β is very small compared to θ, it can be ignored,

Therefore

$$\angle CPO = 90° - \frac{1}{2}\theta$$

From the right angle ΔCFP, we have

$$\frac{CF}{CP} = \sin CPO = \sin\left(90° - \frac{1}{2}\theta\right)$$

or

$$CF = r\sin\left(90° - \frac{1}{2}\theta\right) \quad \text{... (1.34)}$$

From ΔCFO, we get

$$\tan\beta = \frac{CF}{OF} \quad \text{... (1.35)}$$

PF being very small compared to *OP*, *OF* may be taken as *OP*. Substituting the value of *CF* from Eq. (1.34) and taking *OF* equal to *D*, we get the Eq. (1.35) as

$$\tan\beta = \frac{r\sin\left(90° - \frac{1}{2}\theta\right)}{D}$$

or $$\beta = \frac{r\cos\frac{\theta}{2}}{D} \text{ radians}$$

$$\beta = \frac{206265r}{D}\cos\frac{\theta}{2} \text{ seconds} \quad \text{... (1.36)}$$

The phase correction β is applied to the observed horizontal angles in the following manner.

Let there be four stations S_1, S_2, P, and O as shown in (Fig. 1.33). The observer is at O, and the angles S_1OP and POS_2 have been measured from O as θ_1' and θ_2', respectively.

If the required corrected angles are θ_1 and θ_2, then

$$\theta_1 = \theta_1' + \beta$$

and $$\theta_2 = \theta_2' - \beta$$

when β is the phase correction.

While applying the corrections the directions of the phase correction, and the observed stations with respect to the line OP, must be noted carefully.

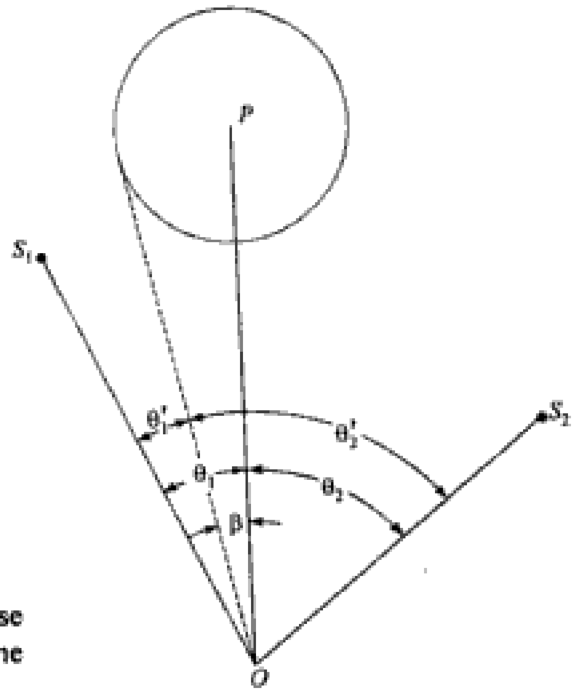

Fig. 1.33 Applying the phase correction to the measured horizontal angles

ILLUSTRATIVE EXAMPLES

Example 1.10 A cylindrical signal of diameter 4 m, was erected at station *B*. Observations were made on the signal from station *A*. Calculate the phase corrections when the observations were made

(*i*) on the bright portion, and

(*ii*) on the bright line.

Take the distance *AB* as 6950 m, and the bearings of the sun and the station *B* as 315° and 35°, respectively.

Solution: Given that θ = Bearing of sun – bearing of B

$$= 315° - 35° = 280°$$

$$r = \frac{\text{Diameter}}{2} = \frac{4}{2} = 2\text{ m}$$

$$D = 6950\text{ m}$$

(*i*) (Fig. 1.31)

From Eq. (1.31), the phase correction

$$\beta = \frac{206265r}{D}\cos^2\frac{\theta}{2} \text{ seconds}$$

$$= \frac{206265 \times 2}{6950} \times \cos^2\frac{280°}{2} = \mathbf{34.83\ seconds.}$$

(*ii*) (Fig. 1.32)

From Eq. (1.36), the phase correction

$$\beta = \frac{206265r}{D}\cos\frac{\theta}{2} \text{ seconds}$$

$$= \frac{206265 \times 2}{6950} \times \cos\frac{280°}{2} = \mathbf{45.47\ seconds.}$$

Example 1.11 The horizontal angle measured between two stations P and Q at station R, was 38°29′30″. The station Q is situated on the right of the line RP.

The diameter the cylindrical signal erected at station P, was 3 m and the distance between P and R was 5180 m. The bearing of the sun and the station P were measured as 60° and 15°, respectively. If the observations were made on the bright line, compute the correct horizontal angle PRQ.

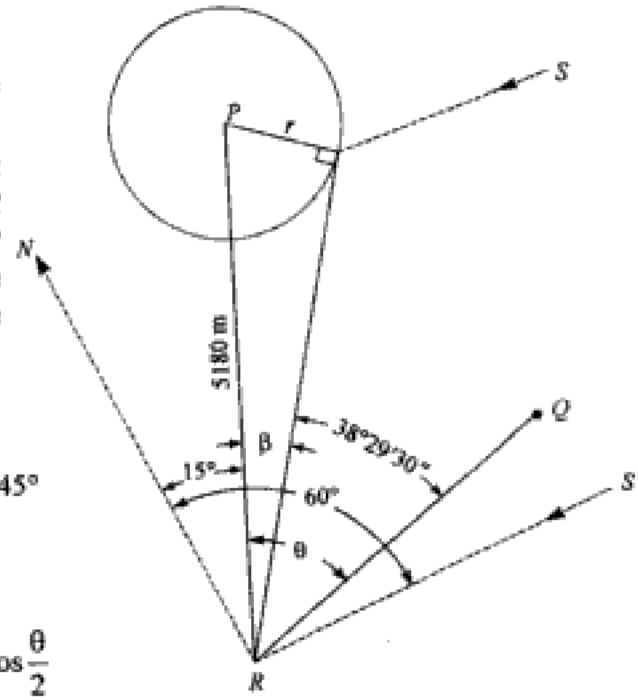

Fig. 1.34

Solution: (Fig. 1.34)

From the given data

$$\theta = 60° - 15° = 45°$$
$$D = 5180\,\text{m}$$
$$r = 1.5\,\text{m}$$

From Eq. (1.36), we get

$$\beta = \frac{206265r}{D}\cos\frac{\theta}{2}$$
$$= \frac{206265 \times 1.5}{5180}\cos\frac{45°}{2}$$
$$= 55.18 \text{ seconds}$$

The correct horizontal angle $PRQ = 38°29'30'' + \beta$

$= 38°29'30'' + 55.18'' = \mathbf{38°30'25.18''}$.

1.16 MEASUREMENT OF BASE LINE

The accuracy of an entire triangulation system depends on that attained in the measurement of the base line and, therefore, the measurement of base line forms the most important part of the triangulation operations. As base line forms the basis for computations of triangulation system it is laid down with great accuracy in its measurement and alignment. The length of the base line depends upon the grade of the triangulation. The length of the base is also determined by the desirability of securing strong figures in the base net. Ordinarily the longer base, the easier it will be found to secure strong figures.

The base is connected to the triangulation system through a base net. This connection may be made through a simple figure as shown in Fig. 1.35, or through a much more complicated figures discussed in the base line extension (Sec. 1.16.3).

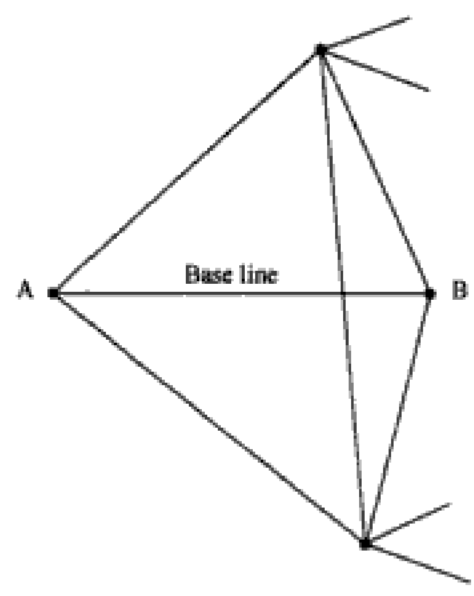

Fig. 1.35 Base net

Apart from main base line, several other check bases are also measured at some suitable intervals. In India, ten bases were measured, the length of nine bases vary from 6.4 to 7.8 miles, and that of the tenth base is 1.7 miles.

1.16.1 Selection of site for base line

Since the accuracy in the measurement of the base line depends upon the site conditions, the following points should be taken into consideration while selecting the site for a base line.

1. The site should be fairly level or gently undulating. If the ground is sloping, the slope should be uniform and gentle.
2. The site should be free from obstructions throughout the length of the base line.
3. The ground should be firm and smooth.
4. The two extremities of the base line should be intervisible.
5. The site should be such that well-conditioned triangles can be obtained while connecting extremities to the main triangulation stations.
6. The site should be such that a minimum length of the base line as specified, is available.

1.16.2 Equipment for base line measurement

Generally the following types of base measuring equipments are used :

1. Standardised tapes : These are used for measuring short bases in plain grounds.
2. Hunter's short base: It is used for measuring 80 m long base line and its extension is made by subtense method.
3. Tacheometric base measurements : It is used in undulating grounds for small bases (*cf.*, Chapter 8 of *Plane Surveying*).
4. Electronic distance measurement: This is used for fairly long distances and has been discussed in Chapter 11.

Standardised tapes : For measuring short bases in plain areas standardised tapes are generally used. After having measured the length, the correct length of the base is calculated by applying the required corrections. For details of corrections, refer to Chapter 3 of *Plane Surveying*. If the triangulation system is of extensive nature, the corrected lengths of the base is reduced to the mean sea level.

Hunter's short base : Dr. Hunter who was a Director of Survey of India, designed an equipment to measure the base line, which was named as Hunter's short base. It consists of four chains, each of 22 yards (20.117 m) linked together. There are 5 stands, three-intermediate two-legged stands, and two three-legged stands at ends (Fig. 1.36). A 1 kg weight is suspended at the end of an arm, so that the chains remain straight during observations. The correct length of the individual chains is supplied by the manufacturer or is determined in the laboratory. The lengths of the joints between two chains at intermediate supports, are measured directly with the help of a graduated scale. To obtain correct length between the centres of the targets, usual corrections such as temperature, sag, slope, etc., are applied.

To set up of the Hunter's short base the stand at the end *A* (marked in red colour) is centered on the ground mark and the target is fitted with a clip. The target *A* is made truly vertical so that the notch on its tip side is centered on the ground mark. The end of the base is hooked with the plate *A* and is spread carefully till its other end is reached. In between, at every joint of the chains, two-legged supports are fixed to carry the base. The end *B* (marked in green colour) is fixed to the *B* stand and the 1 kg weight is attached at the end of the lever. While fixing the end supports *A* and *B* it should be ensured that their third leg should face each other under the base. Approximate alignment of the base is the done by eye judgement.

For final alignment, a theodolite is set up exactly over the notch of the target *A*, levelled and centered accurately. The target at *B* is then bisected. All intermediate supports are set in line with the vertical cross-hair of the theodolite. At the end again ensure that all the intermediate supports and the target *B* are in one line.

In case the base is spread along undulating ground, slope correction is applied. To measure the slope angles of individual supports, a target is fixed to a long iron rod of such a length that it is as high above the tape at *A* as the trunion axis of the theodolite. The rod is held vertically at each support and the vertical angles for each support are read.

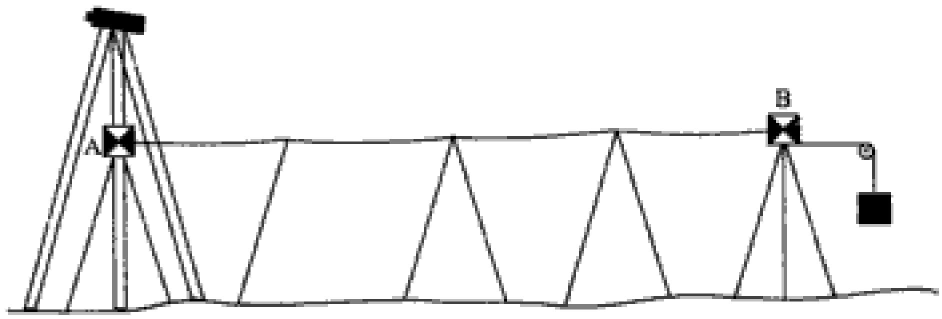

Fig. 1.36 Hunter's short base

ILLUSTRATIVE EXAMPLES

Example 1.12 A tape of standard length 20 m at 85° F was used to measure a base line. The measured distance was 882.10 m. The following being the slopes for the various segments of the line.

Segment	Slope
100 m	2°20′
150 m	4°12′
50 m	1°06′
200 m	7°45′
300 m	3°00′
82.10 m	5°10′

Find the true length of the base line if the mean temperature during measurement was 63°F. The coefficient of expansion of the tape material is 6.5×10^{-6} per °F.

Solution: (refer to Sec. 3.5 of *Plane Surveying*):

Correction for temperature

$$C_t = \alpha(t_m - t_0)L$$
$$= 6.5 \times 10^{-6} \times (63 - 65) \times 882.10$$
$$= 0.126 \text{ m (subtractive)}$$

Correction for slope

$$C_s = \Sigma[(1 - \cos\alpha)L]$$
$$= (1 - \cos 2°20') \times 100 + (1 - \cos 4°12') \times 150 + (1 - \cos 1°06') \times 50$$
$$+ (1 - \cos 7°48') \times 200 + (1 - \cos 3°00') \times 300 + (1 - \cos 5°10') \times 82.10$$
$$= 3.079 \text{ m (subtractive)}$$

$$\text{Total correction} = C_t + C_s$$
$$= 0.126 + 3.079$$
$$= 3.205 \text{ m (subtractive)}$$
$$\text{Corrected length} = 882.10 - 3.205$$
$$= \mathbf{878.895 \ m.}$$

Example 1.13. A base line was measured between two points A and B at an average elevation of 224.35 m. The corrected length after applying all correction was 149.3206 m. Reduce the length to mean sea level. Take earth's mean radius as 6367 km.

Solution: (Refer Sec. 3.5 of *Plane Surveying*):

The reduced length at mean seal level is

$$L' = \frac{R}{(R + h)} L$$

$$= \frac{6367}{6367 + \left(\frac{224.35}{1000}\right)} \times 149.3205$$

$$= \mathbf{149.3152\ m.}$$

1.16.3 Extension of base line

Usually the length of the base lines is much shorter than the average length of the sides of the triangles. This is mainly due to the following reasons:

(*a*) It is often not possible to get a suitable site for a longer base.

(*b*) Measurement of a long base line is difficult and expensive.

The extension of short base is done through forming a base net consisting of well-conditioned triangles. There are a great variety of the extension layouts but the following important points should be kept in mind in selecting the one.

(*i*) Small angles opposite the known sides must be avoided.

(*ii*) The length of the base line should be as long as possible.

(*iii*) The length of the base line should be comparable with the mean side length of the triangulation net.

(*iv*) A ratio of base length to the mean side length should be at least 0.5 so as to form well-conditioned triangles.

(*v*) The net should have sufficient redundant lines to provide three or four side equations within the figure.

(*vi*) Subject to the above, it should provide the quickest extension with the fewest stations.

There are two ways of connecting the selected base to the triangulation stations. There are

(*a*) extension by prolongation, and

(*b*) extension by double sighting.

(a) Extension by prolongation

Let up suppose that *AB* is a short base line (Fig. 1.37) which is required to be extended by four times. The following steps are involved to extend *AB*.

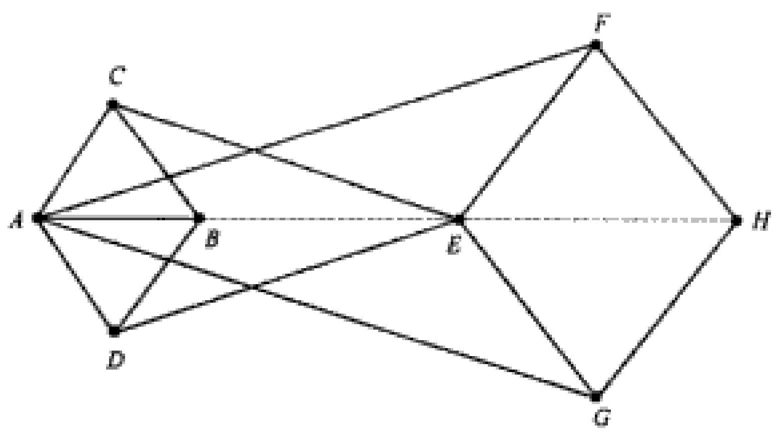

Fig. 1.37 Base extension by prolongation

(*i*) Select *C* and *D* two points on either side of *AB* such that the triangles *BAC* and *BAD* are well-conditioned.

(*ii*) Set up the theodolite over the station *A*, and prolong the line *AB* accurately to a point *E* which is visible from points *C* and *D*, ensuring that triangles *AEC* and *AED* are well-conditioned.

(*iii*) In triangle *ABC*, side *AB* is measured. The length of *AC* and *AD* are computed using the measured angles of the triangles *ABC* and *ABD*, respectively.

(*iv*) The length of *AE* is calculated using the measured angles of triangles *ACE* and *ADE*, and taking mean value.

(*v*) Length of *BE* is also computed in similar manner using the measured angles of the triangles *BEC* and *BDE*. The sum of lengths of *AB* and *BE* should agree with the length of *AE* obtained in step (*iv*).

(*vi*) If found necessary, the base can be extended to *H* in the similar way.

(*b*) Extension by double sighting

Let *AB* be the base line (Fig. 1.38). To extend the base to the length of side *EF*, following steps are involved.

(*i*) Chose intervisible points *C*, *D*, *E*, and *F*.

(*ii*) Measure all the angles marked in triangles *ABC* and *ABD*. The most probable values of these angles are found by the theory of least-squares discussed in Chapter 2.

(*iii*) Calculate the length of *CD* from these angles and the measured length *AB*, by applying the sine law to triangles *ACB* and *ADB* first, and then to triangles *ADC* and *BDC*.

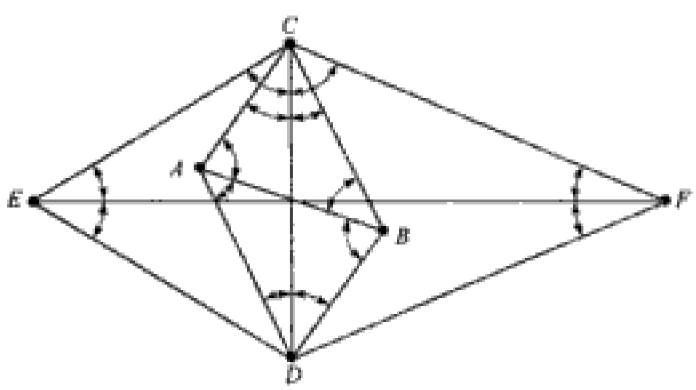

Fig. 1.38 Base extension by double sighting

(*iv*) The new base line *CD* can be further extended to the length *EF* following the same procedure as above. The line *EF* may from a side of the triangulation system.

If the base line *AB* is measured on a good site which is well located for extension and connection to the main triangulation system, the accuracy of the system is not much affected by the extension of the base line. In fact, in some cases, the accuracy may be higher than that of a longer base line measured over a poor terrain.

1.17 MEASUREMENT OF HORIZONTAL ANGLES

The instruments used for triangulation surveys, require great degree of precision. Horizontal angles are generally measured with an optical or electronic theodolite in primary and secondary triangulation. For tertiary triangulation generally transit or Engineer's transit having least count of 20" is used.

Various types of theodolities have been discussed in Sec. 4.4.5 of *Plane Surveying*. The salient features of the modern theodolities are as follow:

(*i*) These are small in dimension, and light in weight.

(*ii*) The graduations are engraved on glass circles, and are much finer.

(*iii*) The mean of two readings on the opposite sides of the circles can be read directly through an eyepiece, saving the observation time.

(*iv*) There is no necessity to adjust the micrometers.

(*v*) These are provided with optical plummet which makes possible accurate centering of the instrument even in high winds.
(*vi*) These are water proof and dust proof.
(*vii*) These are provided with electrical arrangement for illumination during nights if necessary.
(*viii*) Electronic theodolites directly display the value of the angle on *LCD* or *LED*.

1.17.1 Methods of observation of horizontal angles

The horizontal angles of a triangulation system can be observed by the following methods:

(*i*) Repetition method
(*ii*) Reiteration method.

The procedure of observation of the horizontal angles by the above methods has been discussed in Sec. 4.5 of *Plane Surveying*.

(*i*) Repetition method

For measuring an angle to the highest degree of precision, several sets of repetitions are usually taken. There are following two methods of taking a single set.

(*a*) In the first method, the angle is measured clockwise by 6 repetitions keeping the telescope normal. The first value of the angle is obtained by dividing the final reading by 6. The telescope is inverted, and the angle is measured again in anticlockwise direction by 6 repetitions. The second value of the angle is obtained by dividing the final reading by 6. The mean of the first and second values of the angle is the average value of the angle by first set.
For first-order work, five or six sets are usually required. The final value of the angle is the mean of the values obtained by different sets.

(*b*) In the second method, the angle is measured clockwise by six repetitions, the first three with telescope normal and the last three with telescope inverted. The first value of the angle is obtained by dividing the final reading by 6. Now without altering the reading obtained in the sixth repetition, the explement angle (i.e., 360°– the angle), is measured clockwise by six repetitions, the first three with telescope inverted and the last three with telescope normal. The final reading should theoretically be zero. If the final reading is not zero, the error is noted, and half of the error is distributed to the first value of the angle. The result is the corrected value of the angle by the first set. As many sets as desired are taken, and the mean of all the value of various sets, is the average value of the angle. For more accurate work and to eliminate the errors due to inaccurate graduations of the horizontal circle, the initial reading at the beginning of each set may not be set to zero but to different values. If n sets are required, the initial setting should be sucessively increased by $180°/n$. For example, for 6 sets the initial readings would be 0°, 30°, 60°, 90°, 120° and 150°, respectively.

(*ii*) Reiteration method or direction method

In the reiteration method, the triangulation signals are bisected successively, and a value is obtained for each direction in each of several rounds of observations. One of the triangulation stations which is likely to be always clearly visible may be selected as the initial station or reference station. The theodolites used for the measurement of angles for triangulation surveys, have more than one micrometer. One of the micrometer is set to 0° and with telescope normal, the initial station is bisected, and all the micrometers are read. Each of the successive stations are then bisected, and all the micrometers are read. The stations are then again bisected in the reverse direction, and all the micrometers are read after each bisection. Thus, two values are obtained for each angle when the telescope is normal. The telescope is then inverted, and the observations are repeated. This constitutes one set in which four value of each angle are obtained. The micrometer originally at 0° is now brought to a new reading equal to $360°/mn$ (where m is the number of micrometers and n is the number of sets), and a second set is observed in the same manner. The number of sets depends on the accuracy required. For first-order triangulation, sixteen such sets are required with a 1″ direction theodolite, while for second-order triangulation four, and for third-order triangulation two. With more refined instrument having finer graduations, however, six to eight sets are sufficient for the geodetic work.

1.18 MEASUREMENT OF VERTICAL ANGLES

Measurement of vertical angles is required to compute the elevation of the triangulation stations. The method of measurement of vertical angles is discussed in Sec. 4.5.4 of *Plane Surveying*.

1.19 ASTRONOMICAL OBSERVATIONS

To determine the azimuth of the initial side, intermediate sides, and the last side of the triangulation net, astronomical observations are made. For detailed procedure and methods of observation, refer to Chapter 7.

1.20 SOME EXTRA PRECAUTIONS IN TAKING OBSERVATIONS

To satisfy first-second, and third-order specifications as given in Table 1.1, care must be exercised. Observer must ensure the following:

1. The instrument and signals have been centred very carefully.
2. Phase in signals has been eliminated.
3. The instrument is protected form the heating effects of the sun and vibrations caused by wind.
4. The support for the instrument is adequately stable.
5. In case of adverse horizontal refraction, observations should be rescheduled to the time when the horizontal refraction is minimum.

Horizontal angles should be measured when the air is the clearest, and the lateral refraction is minimum. If the observations are planned for day hours, the best time in clear weather is from 6 AM to 9 AM and from 4 PM till sunset. In densely clouded weather satisfactory work can be done all day. The best time for measuring vertical angles is form 10 AM to 2 PM when the vertical refraction is the least variable.

First-order work is generally done at night, since observations at night using illuminated signals help in reducing bad atmospheric conditions, and optimum results can be obtained. Also working at night doubles the hours of working available during a day. Night operations are confined to period from sunset to midnight.

1.21 SATELLITE STATION AND REDUCTION TO CENTRE

To secure well-conditioned triangles or to have good visibility, objects such as chimneys, church spires, flat poles, towers, lighthouse, etc., are selected as triangulation stations. Such stations can be sighted from other stations but it is not possible to occupy the station directly below such excellent targets for making the observations by setting up the instrument over the station point. Also, signals are frequently blown out of position, and angles read on them have to be corrected to the true position of the triangulation station. Thus, there are two types of problems:

1. When the instrument is not set up over the true station, and
2. When the target is out of position.

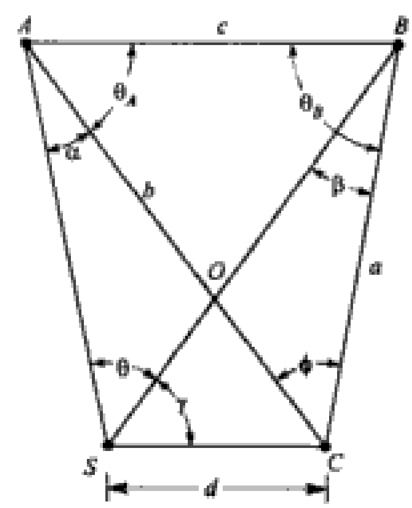

Fig. 1.39 Reduction to centre

In Fig. 1.39, *A*, *B*, and *C* are the three triangulation stations. It is not possible to place instrument at *C*. To solve this problem another station *S*, in the vicinity of *C*, is selected where the instrument can be set up, and from where all the three stations are visible for making the angle observations. Such station is known as *satellite station*. As the observations from *C* are not possible, the observations form *S* are made on *A*, *B*, and, *C* from *A* and *B* on *C*. From the observations made, the required angle *ACB* is calculated. This is known as *reduction to centre*.

In the other case, S is treated as the true station point, and the signal is considered to be shifted to the position C. This case may also be looked upon as a case of *eccentricity of signal.* Thus, the observations from S are made to the triangulation stations A and B, but from A and B the observations are made on the signal at the shifted position C. This causes errors in the measured values of the angles BAC and ABC.

Both the problems discussed above are solved by reduction to centre.

Let the measured

$$\angle BAC = \theta_A$$
$$\angle ABC = \theta_B$$
$$\angle ASB = \theta$$
$$\angle BSC = \gamma$$

Eccentric distance $SC = d$

The distance AB is known by computations form preceding triangle of the triangular net. Further, let

$$\angle SAC = \alpha$$
$$\angle SBC = \beta$$
$$\angle ACB = \phi$$
$$AB = c$$
$$AC = b$$
$$BC = a$$

As a first approximation in ΔABC the $\angle ACB$ may be taken as

$$= 180° - (\angle BAC + \angle ABC)$$

or $$\phi = 180° - (\theta_A + \theta_B) \quad \text{...(1.37)}$$

In the triangle ABC we have

$$\frac{c}{\sin\phi} = \frac{a}{\sin\theta_A} = \frac{b}{\sin\theta_B}$$

$$a = \frac{c.\sin\theta_A}{\sin\phi} \quad \text{...(1.38)}$$

and $$b = \frac{c.\sin\theta_B}{\sin\phi} \quad \text{...(1.39)}$$

Compute the values of a and b by substituting the value of ϕ obtained from Eq. (1.37) in Eqs. (1.38) and (1.39), respectively.

Now, from $\Delta^s SAC$ and SBC we have

$$\frac{d}{\sin\alpha} = \frac{b}{\sin(\theta+\gamma)}$$

$$\frac{d}{\sin\beta} = \frac{a}{\sin\gamma}$$

$$\sin\alpha = \frac{d\sin(\theta+\gamma)}{b}$$

$$\sin\beta = \frac{d\sin\gamma}{a}$$

As the satellite station S is chosen very close to the main station C, the angles α and β are extremely small. Therefore, taking $\sin\alpha = \alpha$, and $\sin\beta = \beta$ in radians, we get.

$$\alpha = \frac{d\sin(\theta+\gamma)}{b\sin 1''}$$

or $$= \frac{d \sin(\theta + \gamma)}{b} \times 206265 \text{ seconds} \quad ...(1.40)$$

and $$\beta = \frac{d \sin \gamma}{a} \times 206265 \text{ seconds} \quad ...(1.41)$$

In Eqs. (1.40) and (1.41), θ, γ, d, b and a are known quantities, therefore, the values of α and β can be computed. Now a more correct value of the angle $\angle ACB$ can be found.

We have

$$\angle AOB = \theta + \alpha = \phi + \beta$$

or $$\phi = \theta + \alpha - \beta \quad ...(1.42)$$

Eq. (1.42) gives the value of ϕ when the satellite station S is to the left of the main station C. In the general, the following four cases as shown in Fig. 1.40*a*, can occur depending on the field conditions.

Case I: S towards the left of C (Fig. 1.39)

$$\phi = \theta + \alpha - \beta$$

Case II: S towards the right of C (Fig. 1.40*b*), the position S_2.

$$\phi = \theta - \alpha + \beta \quad ...(1.43)$$

Case III: S inside the triangle ABC (Fig. 1.40*c*), the position S_3.

$$\phi = \theta - \alpha - \beta \quad ...(1.44)$$

Case IV: S outside the triangle ABC (Fig. 1.40*d*), the position S_4.

$$\phi = \theta + \alpha + \beta \quad ...(1.45)$$

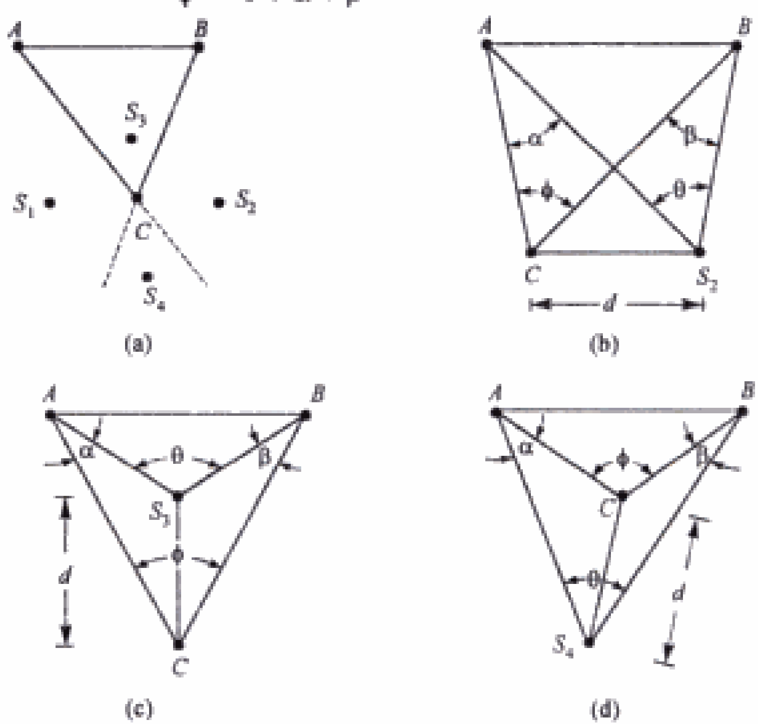

Fig. 1.40 Locations of satellite station with reference to triangulation stations C

1.22 ECCENTRICITY OF SIGNAL

When the signal is found shifted from its true position, the distance between the shifted signal and the station point d is measured. The corrections α and β to the observed angles BAC and ABC, respectively, are computed from Eqs. (1.40) and (1.41), and the corrected values of the angles are obtained as under (Fig. 1.39):

$$\text{Correct } \angle BAS = \theta_A + \alpha \quad ...(1.46)$$

$$\text{Correct } \angle ABS = \theta_B - \beta \quad ...(1.47)$$

For other cases shown in Fig. 1.40, one can easily find out the correct angles.

ILLUSTRATIVE EXAMPLES

Example 1.14 Directions were observed from a satellite station *S* 80 m from *C*, with following results: *A* (0°00′00″), *B* (72°50′44″) and *C* (290°22′00″). The approximate lengths of *AC* and *BC* are 17 km and 24.15 km, respectively. Compute the angle *ACB*.

Solution: (Fig. 1.41)

Let
$$\theta = 72°50'44''$$
$$\gamma = 360° - 299°22'00'' = 60°38'00''$$
$$\phi = \angle ACB$$
$$\delta_1 = \angle CBS$$
$$\delta_2 = \angle CAS$$
$$d = CS = 80 \text{ km}$$
$$a = AC = 17 \text{ km}$$
$$b = BC = 24.15 \text{ km}$$

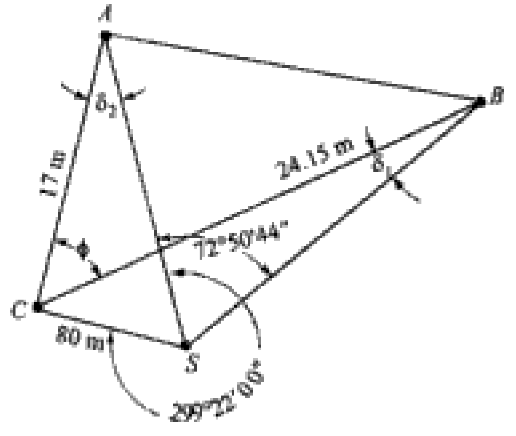

Fig. 1.41

From Eq. (1.43), we get
$$\phi = \theta - \delta_2 + \delta_1$$

From Eq. (1.40), we have
$$\delta_1 = d.\frac{\sin(\theta + \gamma)}{b} \times 206265 \text{ seconds}$$
$$= 80 \times \frac{\sin(70°50'44'' + 60°38'00'')}{24.15 \times 1000} \times 206265 \text{ seconds}$$
$$= 495.806 \text{ seconds}$$
$$= 0°8'15.81'' \text{seconds}$$

From Eq. (1.41), we have
$$\delta_2 = \frac{d\sin\gamma}{a}$$
$$= \frac{80 \times \sin 60°38'}{17 \times 1000} \times 206265 \text{ seconds}$$
$$= 845.93 \text{ seconds}$$
$$= 14'5.93''$$

Therefore
$$\phi = 72°50'44'' - 14'5.93'' + 8'15.81''$$
$$= \mathbf{72°44'53.88''}.$$

Example 1.15 From a satellite station *S*, 15 m from a triangulation station *A*, the angles measured to three stations are as follows:
$$\angle CSA = 35°12'55''$$
$$\angle BSC = 66°38'40''$$

The lengths of sides *AC* and *AB* are 5806 m and 1633 m, respectively. Calculate the angle *BAC*.

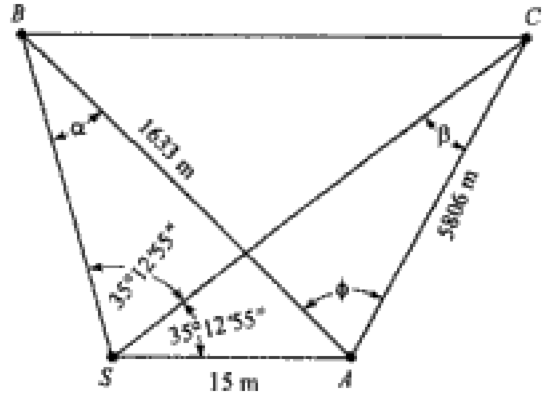

Fig. 1.42

Solution: (Fig. 1.42)

Let
$$\theta = 66°38'40''$$
$$\gamma = 35°12'55''$$
$$d = 15 \text{m}$$
$$a = 5806 \text{m}$$
$$b = 1633 \text{m}$$

From Eqs. (1.40) and (1.41), we have

$$\alpha = \frac{d \sin(\theta + \gamma)}{b} \times 206265 \text{ seconds}$$

$$= \frac{15 \times \sin(66°38'40'' + 35°12'55'')}{1633} \times 206265 \text{ seconds}$$

$$= 1854.21 \text{ seconds}$$

$$= 30'54.21''$$

and $$\beta = \frac{d \sin\gamma}{a} \times 206265 \text{ seconds}$$

$$= \frac{15 \times \sin 35°12'55''}{5806} \times 206265 \text{ seconds}$$

$$= 307.29 \text{ seconds}$$

$$= 5'7.29''$$

From Eq. (1.42), we have

$$\phi = \theta + \alpha - \beta$$

$$= 66°38'40'' + 30'54.21'' - 5'7.29''$$

$$= \mathbf{67°4'26.92''}.$$

Example 1.16 *S* is the satellite station to the triangulation station *A*. The bearings from *S* to *A*, *B*, and *C* are 169°, 201° and 268° respectively, and the lengths *SA*, *SB* and *SC* are 12 m, 1700 m and 2220 m respectively. Find the angle *BAC*.

Solution (Fig. 1.43):

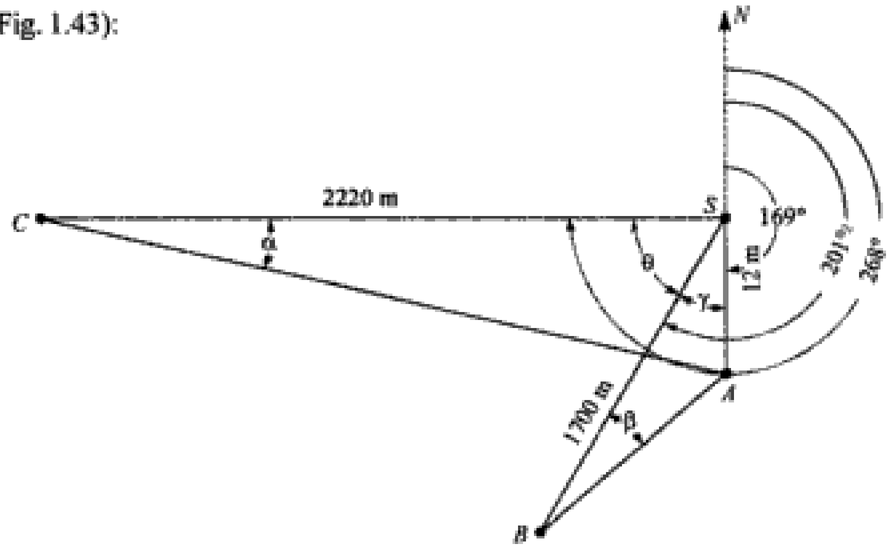

Fig. 1.43

The distance *SB* and *SC* are given, and *AB* and *AC* are required to be computed to get the $\angle BAC$. Since *SA* is very small compared to *SB* and *SC* or *AB* and *AC*, it may be assumed that *AB* and *AC* are nearly equal to *SB* and *SC*, respectively. From the given data, we have

$$\theta = 268° - 201° = 67°$$

$$\gamma = 201° - 169° = 32°$$

From Eq. (1.40), we get

$$\alpha = \frac{12 \times \sin(67° + 32°)}{2,220} \times 206265 \text{ seconds}$$

$$= 1,101.22 \text{ seconds}$$

$$= 18'21.22''$$

From Eq. (1.41), we have

$$\beta = \frac{12 \times \sin 32°}{1700} \times 206265 \text{ seconds}$$
$$= 771.56 \text{ seconds}$$
$$= 12'51.56''$$

From Eq. (1.43), we get

$$BAC = 67° - 12'\ 51.56'' + 18'21.22''$$
$$= \mathbf{67°05'\ 29.66''}.$$

Example 1.17 *A, B* and *C* are the stations in a minor triangulation survey. A satellite station *S* is set up near *C* such that *AC* and *BC* fall within triangle *ASB*. It is given that *AC* = 7.5 km, *BC* = 6.3 km, *CS* = 40.5 m, $\angle ASC = 62°10'40''$, and $\angle ASB = 75°15'15''$. Calculate angle *ACB*.

Solution: (Fig. 1.44) From the given data

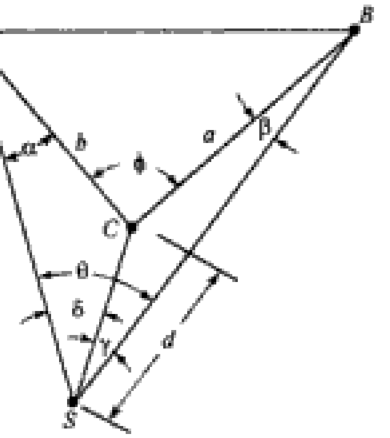

Fig. 1.44

$$\theta = 75°15'15''$$
$$\delta = 62°10'40''$$
$$\gamma = 75°15'15'' - 62°10'40'' = 13°04'35''$$
$$a = 6.3 \text{ km}$$
$$b = 7.5 \text{ km}$$
$$d = 40.5 \text{ m}$$

In Δ^s *ACS* and *BCS* by sine law, we get

$$\alpha = \frac{d \sin \delta}{b} \times 206265 \text{ seconds}$$
$$= \frac{40.5 \times \sin 62°10'40''}{7.5 \times 1000} \times 206265$$
$$= 985.07 \text{ seconds}$$
$$= 16'25.07''$$

and

$$\beta = \frac{d \sin \gamma}{a} \times 206265 \text{ seconds}$$
$$= \frac{40.5 \times \sin 13°04'35''}{6.3 \times 1000} \times 206265 \text{ seconds}$$
$$= 300.00 \text{ seconds}$$
$$= 5'00''$$

From Eq. (1.45), we have

$$\phi = \theta + \alpha + \beta$$
$$= 75°15'15'' + 16'25.07'' + 5'00''$$
$$= \mathbf{75°36'\ 40.07''}.$$

Example 1.18 At a satellite station *S*, 5.5 m from the main triangulation *A*, the following directions were observed:

$$\angle A = 00°00'00''$$
$$\angle B = 130°20'30''$$
$$\angle C = 233°25'05''$$
$$\angle D = 300°10'00''$$

The lengths *AB*, *AC*, and *AD* were computed to be 3200.7 m, 4120.5 m and 2996.6 m, respectively. Determine the directions *AB*, *AC*, and *AD*.

Solution: (Fig. 1.45)

Assuming *SA* as an arbitrary meridian, the correction to any direction is given by

$$\alpha = \frac{d \sin \theta}{D} \times 206265 \text{ seconds}$$

Where $d = AS$,

$D =$ the distance of the observed stations from the main triangulation station *A*, and

θ = the angle subtended at *S* by the observed station and main triangulation station *A*.

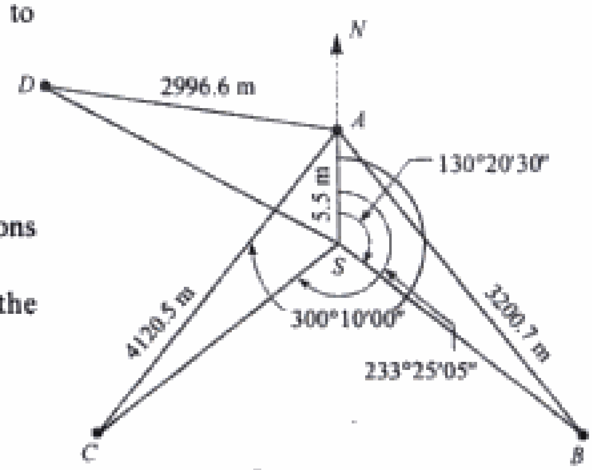

Fig. 1.45

(*i*) Correction to the direction of *B*

$$\alpha_B = \frac{5.5 \times \sin 130°20'30''}{3200.7} \times 206265$$

$$= 270.15 \text{ seconds} = \mathbf{4'30.15''}.$$

Direction of AB = direction of $SB + \alpha_B$

$$= 130°20'30'' + 4'30.15'' = \mathbf{130°25'00.15''}$$

(*ii*) Correction to the direction of *C*

$$\alpha_C = \frac{5.5 \times \sin 233°25'05''}{4120.5} \times 206265$$

$$= -221.08 \text{ seconds} = -3'41.08''$$

Direction of AC = Direction of $SC + \alpha_C$

$$= 233°25'05'' - 3'41.08'' = \mathbf{233°21'23.9''}.$$

(*iii*) Correction to the direction of *D*

$$\alpha_D = \frac{5.5 \times \sin 300°10'00''}{2996.6} \times 206265$$

$$= -327.31 \text{ seconds} = -5'27.31''$$

Direction AD = Direction of $SD + \alpha_D$

$$= 300°10'00'' - 5'27.31'' = \mathbf{300°04'32.69''}.$$

Problem 1.19. From a station A, angle measured to two triangulation stations *B* and *C* was $68°26'36''$. The angles measured at *B* and *C* on *A* were as $\angle CBA = 97°00'52''$ and $\angle BCA = 14°30'21''$. Later on it was detected that at the time of observations on *A* from *B* and *C*, the signal was shifted to *S* towards right of *A*, when looking towards *B* and *C*. To correct the angles measured at *B* and *C* the distance between the signal *S* and the true position of the station *A* was measured, and was found to be 1.4 m. Also the angle *CAS* was measured as 32°45′48″. The distances of *A* from *B* and *C* were also found to be 1441 m and 5678 m, *respectively*.

Determine the correct angles CBA and BCA.

Solution: (Fig. 1.46)

Since at the time of observations to *A* and *B* and *C*, the signal was at *S* in place of *A*, the given angles *CBA* and *BCA* are actually the angles *CBS* and *BCS*. Also, as the distance *AS* is very small compared to the distances *AB* and *AC*, the distances *SB* and *SC* may be taken equal to *AB* and *AC*, respectively. Thus

$$SB = AB = 1441 \text{ m}$$
$$SC = AC = 5678 \text{ m}$$

If the required corrections to the angles are α and β, from Eqs. (1.40) and (1.41).

We have

$$\alpha = \frac{1.4 \times \sin(68°26'36'' + 32°45'48'')}{1441} \times 206265 \text{ seconds}$$
$$= 196.58 \text{ seconds}$$
$$= 3'16.58''$$

and
$$\beta = \frac{1.4 \sin 32°45'48''}{5678} \times 206265 \text{ seconds}$$
$$= 27.52 \text{ seconds}$$

The correction angles *ABC* and *BCA* are

$$\angle ABC = \angle BCS + \alpha$$
$$= 97°00'52'' + 3'16.58''$$
$$= \mathbf{97°04'8.58''}.$$

and
$$\angle BCA = \angle BCS - \beta$$
$$= 14°30'21'' - 27.52''$$
$$= \mathbf{14°29'53.48''}.$$

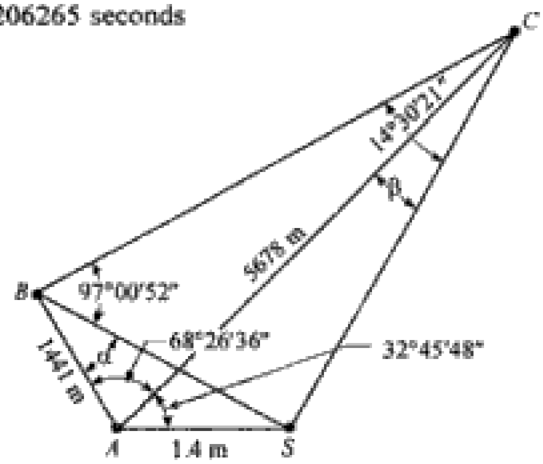

Fig. 1.46

1.23 LOCATION OF POINTS BY INTERSECTION AND RESECTION

It is possible to locate a point by observing directions from the points of known locations. For locating the points in this manner, it is not necessary to occupy it as the point can be located by the method of intersection. The points located by this method, are know as *intersected points.* In another case, it also possible to locate a point by taking directions to the points of known locations. In this method, the point to be located is occupied, and its location is obtained by the method of resection. Such points are known as *resected points.*

Generally the intersected points are located for subsequent use at the time of plane tabling to solve three point problem. Therefore, their locations are selected such that are visible from most of the places in the survey area. The resected points are additional stations which are established when the main triangulation has been completed, and it is found necessary to locate some additional stations for subsequent use as instrument stations as in topographic surveys. This problem also arises in hydrographic surveys where it is required to locate on plan the position of the observer in the boat.

1.23.1 Location by intersection

In Fig. 1.47, *A*, *B*, and *C* are the main triangulation stations whose locations are known. *P* is the intersected point whose location is to be determined. Observations are made on *P* from *A*, *B*, and *C*, and the angle α, β, and γ are measured. The lengths *a* and *b* of *AB* and *AC*, respectively, andthe angle β are known as the coordinates of *A*, *B*, and *C* are known. The angle β can also computed from the known coordinates of *A*, *B*, and *C* as a check on the observations. It is required to determine the distances *AP*, *BP* and *CP*, so that *P* can be plotted by intersection.

Let angles *APB* and *BPC* be θ and ϕ, respectively and *AP*, *BP* and *CP* be *x*, *y* and *z*, respectively.

From Δ's *ABP* and *BCP*, we get

$$\frac{y}{\sin\alpha} = \frac{a}{\sin\theta}$$

and
$$\frac{y}{\sin\gamma} = \frac{b}{\sin\phi}$$

or
$$y = \frac{a\sin\alpha}{\sin\theta} = \frac{b\sin\gamma}{\sin\phi} \qquad \text{...(1.48)}$$

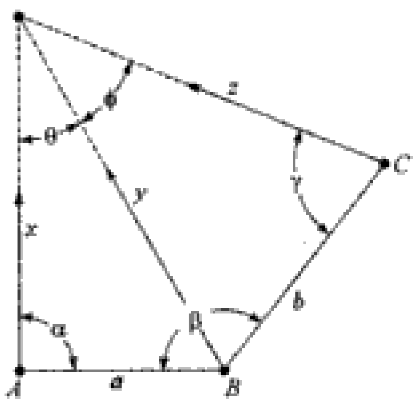

Fig. 1.47

Also for the quadrilateral $ABCP$, we have

$$\alpha + \beta + \gamma + \phi + \theta = 360° \quad ...(1.49)$$

We have two unknown θ and ϕ in two equations (1.48) and (1.49). The values of θ and ϕ can be found out by solving the two simultaneous equations and then x, y and z are computed by sine law.

1.23.2 Location by resection

In Fig. 1.48, A, B and C are the main triangulation stations and P is the respected point. The angles APB and BPC have been measured at P. The angle ABC and the distances AB and BC are known as the coordinates of A, B, and C are known.

Let
$$\angle BAP = \alpha$$
$$\angle ABC = \beta$$
$$\angle BCP = \gamma$$
$$\angle APB = \theta$$
$$\angle BPC = \phi$$
$$AB = a$$
$$BC = b$$
$$AP = x$$
$$BP = y$$
$$CP = z$$

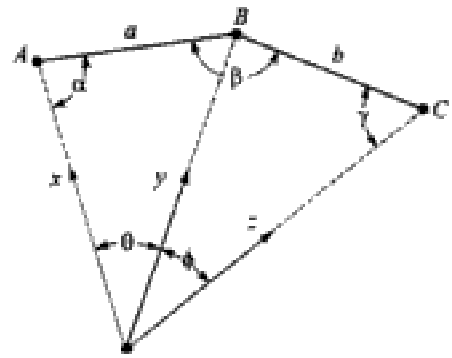

Fig. 1.48 Location of a point by resection.

In the quadrilateral $ABCP$ we have

$$\alpha + \gamma = 360° - (\beta + \theta + \phi) \quad ...(1.50)$$

From Δ^s ABP and BCP, we have

$$\frac{y}{\sin\alpha} = \frac{a}{\sin\theta}$$

and
$$\frac{y}{\sin\gamma} = \frac{b}{\sin\phi}$$

or
$$y = \frac{a\sin\alpha}{\sin\theta} = \frac{b\sin\gamma}{\sin\phi} \quad ...(1.51)$$

In this case the unknowns are α and γ. By solving Eqs. (1.50) and, 1.51) simultaneously, we get the value of α and γ. The value of x, y and z are found by sine law.

ILLUSTRATIVE EXAMPLES

Example 1.20 A point P is to be established by method of intersection. The following observations were made on P from three triangulation stations:

$\angle PAB = 65°27'48''$; $\angle PBA = 72°45'12''$; $\angle PBC = 67°33'24''$; $\angle PCB = 78°14'55''$.

If $AB = 2885$ m and $BC = 2539$ m, calculate the values of AP, BP, and CP.

Solution: (Fig. 1.49)

From Eq. (1.48), we get

$$\frac{a\sin\alpha}{\sin\theta} = \frac{b\sin\lambda}{\sin\phi}$$

or
$$\frac{2885 \times \sin 65°27'48''}{\sin\theta} = \frac{2539 \times \sin 78°14'55''}{\sin\phi}$$

or
$$\sin\phi = 0.9472\sin\theta \quad ...(a)$$

From Eq. (1.49), we get

$$\alpha + \beta + \gamma + \lambda + \phi + \theta = 360°$$

or $\theta = 360° - (\alpha + \beta + \gamma + \lambda) - \phi$

or $\theta = 360° - (65°27'48'' + 72°45'12'' + 67°33'24'' + 78°14'55'') - \phi$

or $\theta = 360° - 240°01'19'' - \phi$

$= 75°58'41'' - \phi$ ...(*b*)

Taking $\varepsilon = 75°58'41''$

$\theta = \varepsilon - \phi$

and from Eq. (*a*), we get

$$\sin\phi = 0.9472\sin(\varepsilon - \phi)$$
$$= 0.9472[\sin\varepsilon\cos\phi - \cos\varepsilon\sin\phi]$$

Dividing the above equation by $\sin\phi$, we get

$$1 = 0.9472[\sin\varepsilon\cot\phi - \cos\varepsilon]$$

or
$$\cot\phi = \frac{1}{\sin\varepsilon}\left(\frac{1}{0.9472} + \cos\varepsilon\right)$$
$$= \frac{1}{0.9472\sin\varepsilon} + \cot\varepsilon \quad ...(c)$$

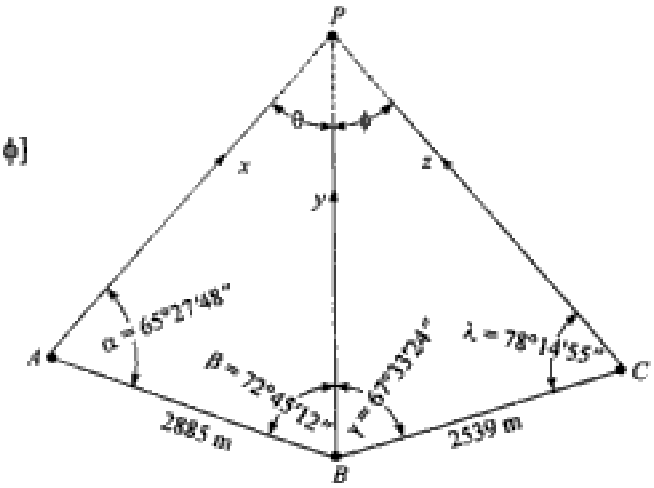

Fig. 1.49

Substituting the value of ε in Eq. (*c*), we get

$$\phi = 36°46'33.11''$$

Substituting the value of ϕ in Eq. (*b*), we get

$$\theta = 75°58'41'' - 36°46'33.11'' = 39°12'7.89''$$

Now for ΔAPB, we have

$$\frac{x}{\sin 72°45'12''} = \frac{2885}{\sin 39°12'7.89''}$$

or $x = 4359.22$ m

also
$$\frac{y}{\sin 65°27'24''} = \frac{2885}{\sin 39°12'7.89''}$$

or $y = 4152.26$ m

From ΔBPC we have

$$\frac{z}{\sin 67°33'24''} = \frac{2539}{\sin 36°46'33.11''}$$

or $z = 3919.73$ m

also
$$\frac{y}{\sin 78°14'55''} = \frac{2539}{\sin 36°46'33.11''}$$

or $y = 4152.06$ m

The difference between the value of *y* may be due to rounding off errors in computations and, therefore, mean of the two values may be adopted. As the two values of *y* obtained through different routes are almost same, it provides a check on the computations.

Thus

AP = 4359.22 m

BP = 4152.16 m

CP = 3919.73 m.

Example 1.21 To determine the location of point P, the following observations were made to three stations A, B, and C from P:

$$\angle APB = 123°49'00''$$
$$\angle BPC = 41°21'55''$$

The following data are known.

$$\angle ABC = 117°17'51''$$
$$AB = 4769.63 \text{ m}$$
$$BC = 5504.05 \text{ m}$$

Determine the lengths of PA, PB, and PC so that P can be fixed by method of resection.

Solution: (Fig. 1.50)

From the given data, we have

$$\theta = 123°49'00''$$
$$\phi = 41°21'55''$$
$$\beta = 117°17'51''$$
$$a = 4769.63 \text{ m}$$
$$b = 5504.05 \text{ m}$$

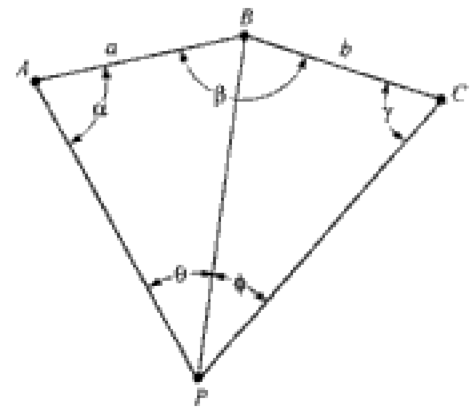

Fig. 1.50

From Eq. (1.50), we have

$$\alpha + \gamma = 360° - (\beta + \theta + \phi)$$
$$= 360° - (117°17'51'' + 123°49'00'' + 41°21'55'')$$
$$= 77°31'14'' \quad \text{...(a)}$$

$$\frac{a \sin\alpha}{\sin\theta} = \frac{b \sin\gamma}{\sin\phi}$$

or $$\frac{4769.63 \sin\alpha}{\sin 123°49'00''} = \frac{5504.05 \sin\gamma}{\sin 41°21'55''}$$

or $$\sin\alpha = 1.4508 \times \sin\gamma \quad \text{...(b)}$$

Substituting the value of γ from Eq. (a) in Eq. (b), we get

$$\sin\alpha = 1.4508 \sin(77°31'14'' - \alpha)$$

Taking $$\varepsilon = 77°31'14''$$

From Eq. (1.51), we have

$$\sin\alpha = 1.4508 \sin(\varepsilon - \alpha)$$
$$= 1.4508 (\sin\varepsilon \cos\alpha - \sin\alpha \cos\varepsilon)$$

Dividing by $\sin\alpha$, we have

or $$1 = 1.4508(\sin\varepsilon \cot\alpha - \cos\varepsilon)$$

$$\cot\alpha = \frac{1}{\sin\varepsilon}\left(\frac{1}{1.4508} + \cos\varepsilon\right)$$

or $$\tan\alpha = \frac{\sin\varepsilon}{\left(\frac{1}{1.4508} + \cos\varepsilon\right)}$$

Substituting the value of ε, we have

$$\tan\alpha = 1.07843$$

or $$\alpha = 47°09'33.93''$$

and $$\gamma = 77°31'14'' - \alpha = 30°21'40.07''$$

In ΔABP

$$\angle ABP = 180° - (\alpha + \theta)$$
$$= 180° - 47°09'33.93'' + 123°49'00'') = 9°01'26.07''$$

In ΔCBP

$$\angle CBP = 180° - (\gamma + \phi)$$
$$= 180° - (30°.21'40.07'' + 41°21'55'')$$
$$= 108°16'24.9''$$

Also $$\angle CBP = \beta - \angle ABP$$
$$= 117°17'51'' - 9°1'6.7'' = 108°16'24.3''$$

The two values of $\angle CBP$ obtained above are due to rounding off errors in computations, the mean of the two values may be adopted for further computations.

$$\angle CBP = \frac{1}{2}(108°16'30.9'' + 108°16'24.3'')$$
$$= 108°16'27.3''$$

Now, from ΔABP by sine rule we have

$$PA = \frac{4769.63 \times \sin 9°01'26.7''}{\sin 123°49'00''}$$
$$= 900.43 \text{ m}$$

and $$PB = \frac{4769.63 \times \sin 47°09'33.93''}{\sin 123°49'00''}$$
$$= 4209.70 \text{ m}$$

From ΔCBP by sine rule, we have

$$PC = \frac{5504.05 \times \sin 108°16'27.3''}{\sin 41°21'55''}$$
$$= 7908.61 \text{ m}$$

and $$PB = \frac{5504.05 \times \sin 30°21'40.07''}{\sin 41°21'55''}$$
$$= 4209.70 \text{ m}$$

Thus the values required are

$PA =$ **900.43 m**
$PB =$ **4209.48 m**
$PC =$ **7908.61 m.**

1.24 COMPUTATIONS IN TRIANGULATION

Following computations are required in a plane triangulation system:

1. Adjustment of observed angles.
2. Computation of lengths.
3. Computation of azimuths, latitudes and departures.
4. Computation of independent coordinates.

1.24.1 Adjustment of observed angles

The observed horizontal angles in a triangulations system must satisfy certain geometrical conditions of the figures and the stations. Therefore, before using them for computations they are adjusted by applying corrections, and their most probable values are determined. The methods of determination of most probable values are discussed in Chapter 2.

The station adjustment requires that sum of all the horizontal angles around a station should be equal to 360°. The conditions for figure adjustment depend upon the shape of the figure used in the triangulation system. For example, for plane triangle, the sum of the three angles should be 180° and for a braced quadrilateral, the sum of the eight angles should be 360°.

The computations discussed in the following sections are based on the assumption that all the observed angles have been adjusted.

1.24.2 Computation of lengths

The principle of computation of lengths has already been discussed in Sec. 1.2. Let us consider a braced quadrilateral *ABCD* shown in Fig. 1.51 in which the length of *AB* is known. To determine the positions of the forward stations *C* and *D*, the solution of two triangles is required. These two selected triangles should provide the strongest route. Let us assume that the strongest route is obtained by considering the triangles *CAB* and *CAD*. The length of the common side *AC* is determined from the triangle *CAB*, and then used to determine the lengths of *AD* and *CD*.

It is essential to check the accuracy of the field work by computing the lengths of the sides from alternate routes. The length of side *CD* can be computed again by considering the triangles *ABD* and *BDC* in which the common side is *BD*. The length of *BD* is computed from the triangle *ABD*, and then it is used to determine the length of *CD* from triangle *BDC*. However, the results of the strongest route are used for computing the length of the side *CD* (*cf.*, Example 1.5).

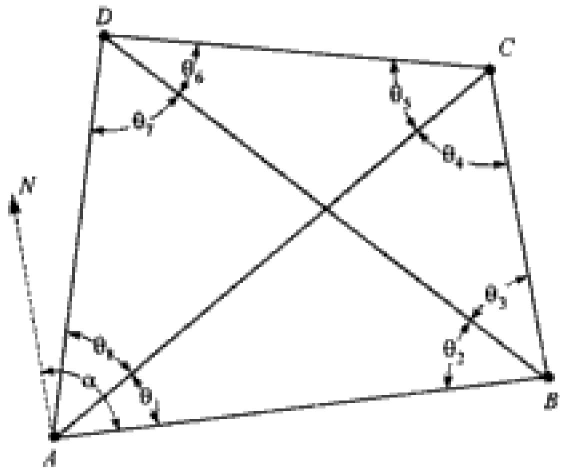

Fig. 1.51 Braced quadrilateral

1.24.3 Computations of azimuths, latitudes and departures

In Fig. 1.51, let the coordinates of *A* and *B* are known. Thus the length and azimuth of the line *AB* are known. From the known azimuth α of *AB*, azimuths of all the sides can be computed. For example,

$$\begin{aligned}\text{Azimuth of } AC &= \text{azimuth of } AB - \theta_1 \\ &= \alpha - \theta_1 \\ \text{Azimuth of } AD &= \text{azimuth of } AB - (\theta_1 + \theta_8) \\ &= \alpha - (\theta_1 + \theta_8) \\ \text{Azimuth of } BC &= \text{azimuth of } BA + (\theta_2 + \theta_3) \\ &= (\alpha + 180°) + (\theta_2 + \theta_3)\end{aligned}$$

The latitude and departure of the lines can be computed from their lengths and azimuths using the following relationships:

$$\text{Latitude} = L\cos\theta$$

$$\text{Departure} = L\sin\theta$$

Where L is the length of the line and θ is its azimuth.

1.24.4 Computations of independent coordinates

The independent coordinates of the triangulation stations are determined from the known coordinates of a station and the latitude and departure.

In Fig. 1.51 the coordinates of *A* and *B* have been assumed to be known and the latitudes and departures of all the sides have been computed. Then for example,

X-coordinates of D = X-coordinate of A + departure of AD
Y-coordinates of D = Y-coordinate of A + latitude of AD

The coordinates of D can also be computed using the coordinates of B. Thus for each point C and D, we get two sets of values of the coordinates. The two values should agree within permissible limits. The computation of coordinates of the triangulation station should be done using the strongest route of the triangles in the network. If coordinates of any station in the network are known, they provide check on the computations.

ILLUSTRATIVE EXAMPLES

Example 1.22 Fig. 1.52 shows a braced quadrilateral $ABCD$ which is a part of a triangulation system. The coordinates of A and B are (E1000, N1000) and (E2200, N1200), respectively. What are the coordinates of C and D?

Solution: (Fig. 1.52)

Let us first determine the strongest route considering that all the stations have been occupied. For such a braced quadrilateral, from Example 1.5, we have

$$\frac{D-C}{D} = 0.6$$

The value of R is computed using Eq. (1.10). The computed values of R for different possible routes are given Table 1.3.

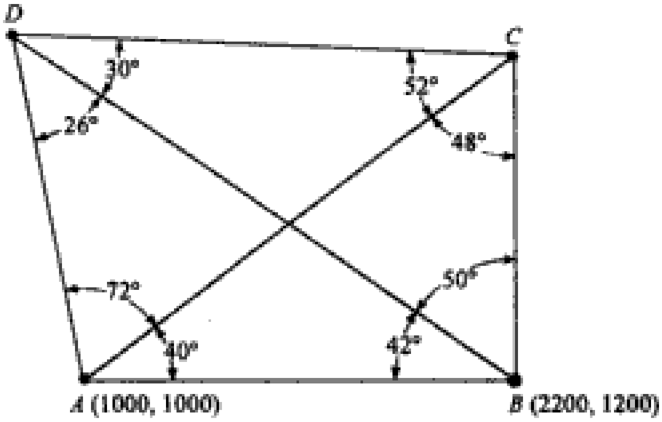

Fig. 1.52

Table. 1.3

Route No.	Triangles	Common side	Distance angles	$\Sigma(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\frac{D-C}{D}$	R
1	ABC BCD	BC	48°, 40° 30°, 50°	15 + 23 = 38	0.6	22.8
2	ABC ACD	AC	48°, 92° 56°, 72°	3 + 4 = 7	0.6	4.2
3	ADB ADC	AD	26°, 42° 52°, 72°	34 + 5 = 39	0.6	23.4
4	ADB DBC	BD	26°, 112° 100°, 50°	16 + 3 = 19	0.6	11.4

The lowest value of R is for route No. 2 hence the strongest route is through the $\Delta^s ABC$ and ACD
The length of AB is given by

$$AB = \sqrt{(2200-1000)^2 + (1200-1000)^2}$$
$$= 1216.55 \text{ m}$$

From ΔABC

$$\frac{AB}{\sin 48°} = \frac{BC}{\sin 40°} = \frac{AC}{\sin 92°}$$

$$BC = \frac{AB \sin 40°}{\sin 48°}$$

$$= \frac{1216.55 \times \sin 40°}{\sin 48°} = 1052.26 \text{ m}$$

$$AC = \frac{AB \sin 92°}{\sin 48°}$$

$$= \frac{1216.55 \times \sin 92°}{\sin 48°} = 1636.03 \text{ m}$$

From ΔACD

$$\frac{AC}{\sin 56°} = \frac{AD}{\sin 52°} = \frac{CD}{\sin 72°}$$

or

$$AD = \frac{AC \sin 52°}{\sin 56°}$$

$$= \frac{1636.06 \times \sin 52°}{\sin 56°} = 1555.10 \text{ m}$$

and

$$CD = \frac{AC \sin 72°}{\sin 56°}$$

$$= \frac{1636.03 \times \sin 72°}{\sin 56°} = 1876.82 \text{ m}$$

The azimuth of the line AB is given by

$$\tan\theta = \frac{\Delta D}{\Delta L}$$

Where θ is the azimuth of AB, and ΔD and ΔL are the differences in departures and latitudes of B and A, respectively.

$$\Delta D = 2200 - 1000 = 1200 \text{ m}$$

$$\Delta L = 1200 - 1000 = 200 \text{ m}$$

$$\tan\theta = \frac{1200}{200} = 6$$

$$\theta = \tan^{-1} 6$$

or

$$\theta = 80°32'16''$$

For line AB

$$\text{Azimuth} = 80°32'16''$$

$$\text{Latitude} = 1216.55 \times \cos 80°32'16'' = 200.00 \text{ m}$$

$$\text{Departure} = 1216.55 \times \sin 80°32'16'' = 1200.00 \text{ m}$$

For line AC

$$\text{Azimuth} = 80°32'16'' - 40° = 40°32'16''$$

$$\text{Latitude} = 1636.03 \times \cos 40°32'16'' = 1243.35 \text{ m}$$

$$\text{Departure} = 1636.03 \times \sin 40°32'16'' = 1063.34 \text{ m}$$

For line BC

$$\text{Azimuth} = 180° + 80°32'16'' + 92° = 352°32'16''$$

$$\text{Latitude} = 1052.26 \times \cos 352°32'16'' = 1043.35 \text{ m}$$

$$\text{Departure} = 1052.26 \times \sin 352°32'16'' = -136.66 \text{ m}$$

For line CD

$$\text{Azimuth} = 352°32'16'' - 180° + 100° = 272°32'16''$$

$$\text{Latitude} = 1876.82 \times \cos 272°32'16'' = 83.11 \text{ m}$$

$$\text{Departure} = 1876.82 \times \sin 272°32'16'' = -1874.98 \text{ m}$$

For line *AD*

$$\text{Azimuth} = 272°32'16'' - 180° + 56° + 180° = 328°32'16''$$
$$\text{Latitude} = 1555.10 \times \cos 328°32'16'' = 1326.46 \text{ m}$$
$$\text{Departure} = 1555.10 \times \sin 328°32'16'' = -811.64 \text{ m}$$

Independent coordinates

Point *C*

$$X\text{-coordinate} = X\text{-coordinate of } A + \text{departure of } AC$$
$$= 1000 + 1063.34 = 2063.34 \text{ m}$$
$$Y\text{-coordinate} = Y\text{-coordinate of } A + \text{latitude of } AC$$
$$= 1000 + 1243.35 = 2243.35 \text{ m}$$

Coordinate form an alternate route

$$X\text{-coordinate} = X\text{-coordinate of } B + \text{departure of } BC$$
$$= 2200 + (-136.66) = 2063.34 \text{ m}$$
$$Y\text{-coordinate} = Y\text{-coordinate of } B + \text{latitude of } BC$$
$$= 1200 + 1043.35 = 2243.35 \text{ m}$$

Point *D*

$$X\text{-coordinate} = X\text{-coordinate of } A + \text{departure of } AD$$
$$= 1000 + (-811.64) = 188.36 \text{ m}$$
$$Y\text{-coordinate} = Y\text{-coordinate of } A + \text{latitude of } AD$$
$$= 1000 + 1326.46 = 2326.46 \text{ m}$$

Coordinates from an alternate route

$$X\text{-coordinate} = X\text{-coordinate of } C + \text{departure of } CD$$
$$= 2063.34 + (-1874.98) = 188.36 \text{ m}$$
$$Y\text{-coordinate} = Y\text{-coordinate of } C + \text{latitude of } CD$$
$$= 2243.35 + 83.11 = 2326.46 \text{ m}$$

Hence the coordinates of *C* and *D* are **2063.34 m, 2243.35 m** and **188.36 m, 2326.46 m**, respectively.

Example 1.23 Following observations were made from two stations *R* and *S* to station *P* (*E*15000, *N*8000) and *Q*(*E*15600, *N*8800):

$$\angle PRS = 50°26'10''; \angle PSR = 30°00'00''; \angle PSQ = 15°00'00''$$

If the bearing of *RP* is 60°30′30″ and point *S* is roughly south-east of *R*, compute the distance *RS* and the coordinates of station *S*.

Solution: (Fig. 1.53)

If ΔE and ΔN are the differences in easting and northings of points *Q* and *P* then

$$\Delta E = 15600 - 15000 = 600 \text{ m}$$
$$\Delta N = 8800 - 8000 = 800 \text{ m}$$
$$PQ = \sqrt{\Delta E^2 + \Delta N^2}$$
$$= \sqrt{600^2 + 800^2} = 10000 \text{ m}$$

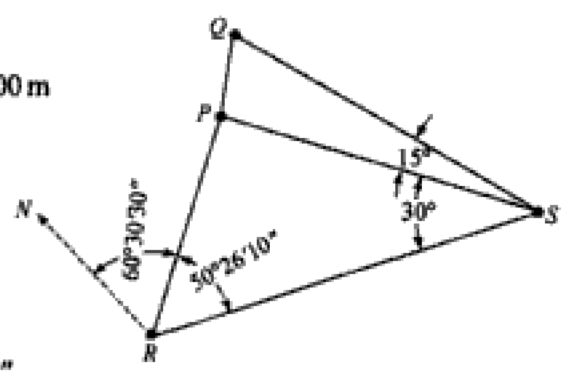

Fig. 1.53.

and if bearing of *PQ* is θ, then

$$\tan\theta = \frac{\Delta E}{\Delta N}$$

or

$$\theta = \tan^{-1}\frac{\Delta E}{\Delta N}$$
$$= \tan^{-1}\frac{600}{800} = 36°52'12''$$

$$\text{Bearing of } QP = 36°52'12'' + 180° = 216°52'12''$$

$$\text{Bearing of } RS = \text{Bearing of } RP + \angle PRS$$
$$= 60°30'30'' + 50°26'10'' = 110°56'40''$$

$$\text{Bearing of } SQ = \text{Bearing of } SR + \angle QSR$$
$$= 110°56'40'' + 180° + 45° = 335°56'40''$$

$$\text{Bearing of } QS = 335°56'40'' - 180° = 155°56'40''$$

$$\angle PQS = \text{Bearing of } QP - \text{Bearing of } QS$$
$$= 216°52'12'' - 155°56'40'' = 60°55'32''$$

From ΔPRS

$$\angle RPS = 180° - (50°26'10'' + 30°)$$
$$= 99°33'50''$$

From ΔQPS

$$\angle QPS = 180° - (60°55'32'' + 15°)$$
$$= 104°04'28''$$

From ΔQPS by sine rule we get

$$PS = \frac{PQ \sin PQS}{\sin QSP}$$

$$= \frac{1000 \times \sin 60°55'32''}{\sin 15°} = 3376.83 \text{ m}$$

From ΔPRS by the sine rule we get

$$RS = \frac{PS \sin RPS}{\sin PRS}$$

$$= \frac{3376.83 \times \sin 99°33'50''}{\sin 50°26'10''}$$

$$= \mathbf{4319.41 \text{ m}}.$$

$$\text{Bearing of } SP = \text{Bearing of } SR + \angle PSR$$
$$= 110°56'40'' + 180° + 30°$$
$$= 320°56'40''$$

$$\text{Latitude of } SP = 3376.83 \times \cos(350°56'40'')$$
$$= 2622.23 \text{ m}$$

$$\text{Departure of } SP = 3376.83 \times \sin(320°56'40'')$$
$$= -2127.65 \text{ m}$$

$$\text{Latitude of } PS = -2622.23 \text{ m}$$

$$\text{Departure of } PS = +2127.65 \text{ m}$$

$$X\text{-coordinate of } S = X\text{-coordinate of } P + \text{departure of } PS$$
$$= 15000 + 2127.65$$
$$= \mathbf{17127.65 \text{ m}}.$$

$$Y\text{-coordinate of } S = Y\text{-coordinate of } P + \text{latitudes of } PS$$
$$= 8000 + (-26.22.23)$$
$$= \mathbf{5377.77 \text{ m}}.$$

Example 1.24 To ascertain the verticality of a large chimney, two stations A and B were established for making observations on the chimney such that the chimney was on the left of line AB. Following theodolite observations were made from A and B on the centre points of top (T) and bottom (M) of the chimney:

$$\angle MAB = 59°28'14''$$
$$\angle TAB = 59°04'10''$$
$$\angle MBA = 55°18'16''$$
$$\angle TBA = 55°55'50''$$

The height of the chimney was measured as 60 m and the coordinates of A and B were assumed to be (E100, N180) and (E160, N100), respectively. Calculate the angle of the non-verticality of the chimney.

Solution: (Fig. 1.54)

Let
$$\Delta E = 160 - 100 = 60 \text{ m}$$
$$\Delta N = 180 - 100 = 80 \text{ m}$$
$$AB = \sqrt{60^2 + 80^2} = 100 \text{ m}$$

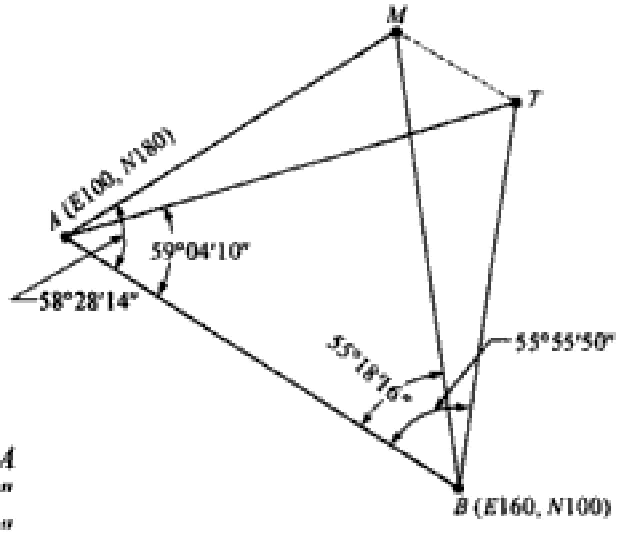

Fig. 1.54

Reduced bearing θ of AB will be given by

$$\theta = \tan^{-1}\frac{\Delta E}{\Delta N} = \tan^{-1}\frac{60}{80}$$
$$= S\,36°52'12''E$$
$$\text{W.C.B. of } AB = 180° - 36°52'12''$$
$$= 143°07'48''$$

and
$$\text{W.C.B. of } BA = 143°07'48'' + 180°$$
$$= 323°07'48''$$
$$\text{Bearing of } BM = \text{Bearing of } BA + \angle MBA$$
$$= 323°07'48'' + 55°18'16''$$
$$= 378°26'04'' = 18°26'04''$$
$$\text{Bearing of } BT = \text{Bearing of } BA + \angle TBA$$
$$= 323°07'48'' + 55°55'50''$$
$$= 379°03'38'' = 19°03'38''$$

From ΔAMB
$$\angle AMB = 180° - (59°28'14'' + 55°18'16'') = 65°13'30''$$

From ΔATB
$$\angle ATB = 180° - (59°04'10'' + 55°55'50'') = 65°00'00''$$

Now applying sine rule to ΔAMB
$$BM = \frac{AB \sin MAB}{\sin AMB}$$
$$= \frac{100 \times \sin 59°28'14''}{\sin 65°13'30''} = 94.87 \text{ m}$$

Applying sine rule to ΔATB
$$BT = \frac{AB \sin TBA}{\sin ATB}$$
$$= \frac{100 \times \sin 59°04'10''}{\sin 65°00'00''} = 94.65 \text{ m}$$

$$\text{Departure of } BM = 94.87 \sin 18°26'04'' = 30.00 \text{ m}$$
$$\text{Latitude of } BM = 94.87 \cos 18°26'04'' = 90.00 \text{ m}$$
$$X\text{-coordinate of } M = 160 + 30 = 190 \text{ m}$$
$$Y\text{-coordinate of } M = 100 + 90 = 190 \text{ m}$$
$$\text{Departure of } BT = 94.65 \times \sin 19°03'38'' = 30.910 \text{ m}$$
$$\text{Latitude of } BT = 94.65 \times \cos 19°03'38'' = 89.461 \text{ m}$$

X-coordinate of T = 160 + 30.910 = 190.910 m

Y-coordinate of T = 100 + 89.461 = 189.461 m

Difference in the X-coordinates of the centres

= 190.910 − 190.00 = 0.910 m

and difference in the Y-coordinates of the centres

= 190.00 − 189.461 = 0.539 m

$$\text{Eccentricity} = \sqrt{0.910^2 + 0.539^2}$$

$$= 1.059\text{m}$$

and angle of non-vertically

$$= \frac{1.058}{60} = 1°00'37''$$

$$\text{Bearing of eccentricity} = \tan^{-1}\frac{0.910}{0.539}$$

$$= S59°21'41''E.$$

1.25 TRILATERATION

In Sec. 1.0, the basic difference between the triangulation and trilateration has already been discussed. Though field procedures for establishing control points in both the systems are similar, still there are some essential points about the trilateration which a surveyor should know.

A triangulation system can be converted to a pure trilateration system by measuring the lengths of all the lines directly using the *EDM* equipment, without measuring any horizontal angle in the network. The network then can be computed and adjusted in order to obtain the coordinates of the stations. However, in order to maintain the accuracy of the azimuths of the lines in the trilateration net, astronomical observations are made at selected stations. These measured azimuths impose conditions that must be satisfied in the adjustment process.

Trilateration is frequently combined with triangulation in order to strengthen a net that may have serious deficiencies in geometric conditioning. Such a hybrid system imposes several rigid geometrical conditions that must be met in the adjustment of the net. However, in certain instances this rigidity is necessary. For example, the measurement of a very small displacements due to earthquake fault movement, must be made over fairly large areas. These measurement must be duplicated periodically with the same degree of reliability in order to reflect earth movement, and to avoid errors in measurement. The internal accuracy and reliability of such a network is greatly enhanced by a hybrid triangulation-trilateration system.

Before the lengths of the lines in the trilateration net can be used in any subsequent computations, their slope lengths determined by the instrument and corrected for atmospheric conditions, must be reduced to the corresponding sea-level distances. Just as the slope measurements discussed in Sec. 3.5 of *Plane Surveying* require auxiliary measurements to determine the slope corrections, so do the lines in the trilateration net. The auxiliary data are either reciprocal vertical or zenith angles measured at the two ends of each line, or the elevations of the two ends of each line.

1.25.1 Reduction of slope distances

There are two methods of reducing the slope distances measured using *EDM's* to the corresponding mean sea-level surface. These methods are

(*i*) Reduction by vertical angles, and

(*ii*) Reduction by station elevations.

1.25.2 Reduction of slope distance by vertical angles

Fig. 1.55. shows two stations *A* and *B* established by trilateration. α is the angles of elevation at *A* and β is the angle of depression at *B*. The angle υ is the refraction angle, assumed to be the same at both stations. For simplicity in computations, it is further assumed that the height of the instrument above the ground and the height of the signal above the ground at both stations, are same. Thus the axis signal correction is zero.

The angle subtended at the centre of the earth, *O* between the vertical lines through *A* and *B* is θ. The angle *OVB* is $(90° - \theta)$ as the angle *VBO* in triangle *VBO* is 90°. The distance *L* is measured along the refracted line (curved) form *A* and *B* by the *EDM*. This distance, within the range of the instrument, can be taken as the straight-line slope distance *AB*. The difference between the two distances in 100 km is less than 20 cm.

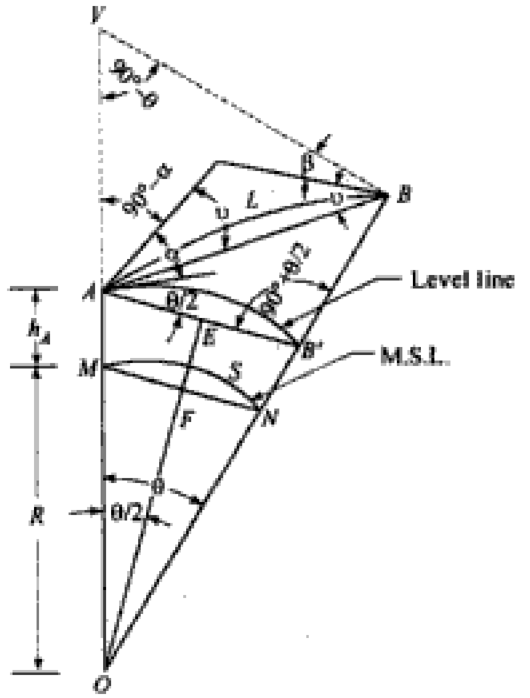

Fig. 1.55 Reduction to sea-level distance by vertical angles

In ΔVAB,

$$\angle VBA + \angle BAV + \angle AVB = 180°$$

$$(\beta + \upsilon) + \angle(90° - \alpha + \upsilon) + (90° - \theta) = 180°$$

$$2\upsilon = \alpha + \theta - \beta$$

$$\upsilon = \frac{\alpha}{2} + \frac{\theta}{2} - \frac{\beta}{2} \qquad ...(1.52)$$

In $\Delta BAB'$

$$\angle BAB' = \alpha - \upsilon + \frac{\theta}{2} \qquad ...(1.53)$$

Substituting the value of υ, we get

$$\angle BAB' = \alpha - \frac{1}{2}(\alpha + \theta - \beta) + \frac{\theta}{2}$$

or

$$\angle BAB' = \frac{\alpha + \beta}{2} \qquad ...(1.54)$$

Taking $\angle AB'B$ approximately equal to 90° in the triangle BAB', the distance AB' is very nearly given by

$$AB' \simeq AB \cos BAB' \qquad ...(1.55)$$

With approximate value of AB', the value θ can be obtained with sufficient accuracy from the triangle *AOE* by the relationship

$$\sin\frac{\theta}{2} = \frac{(AB'/2)}{R}$$

or

$$\sin\frac{\theta}{2} = \frac{AB'}{2R} \qquad ...(1.56)$$

where *R* is the radius of the earth for the latitude of the area in the direction of the line.

In $\Delta ABB'$

$$\angle ABB' = 90° - \frac{\theta}{2} - \angle BAB' \qquad ...(1.57)$$

The triangle ABB' can now be solved by the sine rule to give a more exact value of the distance AB'. Thus

$$AB' = AB \frac{\sin ABB'}{\sin(90° + \theta/2)} \qquad ...(1.58)$$

This distance MN is the length of the chord joining the points M and N which are the sea-level positions of the stations A and B, respectively. This distance is less than the chord length AB'. The distance MN is given by

$$MN = AB' - AB'\frac{h_A}{R} \qquad ...(1.59)$$

where h_A is the elevation of station A.

The sea-level length S is greater than the corresponding length of the chord MN and is given by

$$S = MN + \frac{MN^3}{24R^2} \qquad ...(1.60)$$

or

$$\delta l = S - MN = \frac{MN^3}{24R^2} \qquad ...(1.61)$$

The derivation of Eqs. (1.59) and (1.60) are given below.

Form similar $\Delta^s AB'O$ and MNO, we get

$$\frac{MN}{R} = \frac{AB'}{R + h_A}$$

or

$$MN = AB' \frac{R}{(R + h_A)}$$

$$= AB' \frac{1}{\left(1 + \frac{h_A}{R}\right)} = AB'\left(1 + \frac{h_A}{R}\right)^{-1}$$

$$= AB'\left(1 - \frac{h_A}{R} + \left(\frac{h_A}{R}\right)^2 - \ldots\right)$$

Neglecting the terms of higher powers, we have

$$MN = AB'\left(1 - \frac{h_A}{R}\right)$$

$$= AB' - AB'\frac{h_A}{R} \quad (Proved)$$

From ΔMOF

$$MF = MO \sin\frac{\theta}{2}$$

or

$$\frac{MN}{2} = R\sin\frac{\theta}{2}$$

or

$$MN = 2R\sin\frac{\theta}{2}$$

$$= 2R\left(\frac{\theta}{2} - \frac{\theta^3}{48} + \frac{\theta^5}{3840} - \ldots\right) \qquad ...(1.62)$$

also

$$\frac{S}{R} = \theta$$

or

$$S = 2R\left(\frac{\theta}{2}\right) \qquad ...(1.63)$$

Subtracting Eq. (1.62) from (1.63), and neglecting the terms of higher powers of θ, we have

$$S - MN = \frac{R\theta^3}{24} = \frac{(R\theta)^3}{24R^2}$$

or

$$S = MN + \frac{MN^3}{24R^2} \quad (Proved)$$

The difference δl is the amount to be added to the chord length to obtain the sea-level of the line. The value of δl is about 250 cm in a distance of 100 km, and decreases to 1 ppm of the measured length at about 30 km. Thus, for distances less than 30 km the chord-to-arc correction δl can be neglected.

1.25.3 Reduction of slope distance by station elevations

Sometimes, the elevations of the two ends *A* and *B* of the slope length *L* as shown in Fig. 1.56, are known and then the reduction to sea-level length can be made without making any further observations.

The vertical distance *BB'* is equal to the difference Δh in elevations of *A* and *B*. The distance *AB* is assumed to be the straight-line distance. Usually the slope of the line is small and, therefore, the difference between the slope distance *AB* and the horizontal distance *AC* can be taken as (*cf.*, Sec. 3.5 of *Plane Surveying*)

$$AB - AC = \frac{\Delta h^2}{2AB}$$

or

$$CD = \frac{\Delta h^2}{2AB} \quad ...(1.64)$$

From $\Delta CBB'$, we have

$$B'C = \Delta h \sin\frac{\theta}{2} \quad ...(1.65)$$

We have

$$AB' = AD - B'D$$

or

$$= AB - (B'C + CD)$$

$$= AB - \left(\Delta h \sin\frac{\theta}{2} + \frac{\Delta h^2}{2AB}\right) \quad ...(1.66)$$

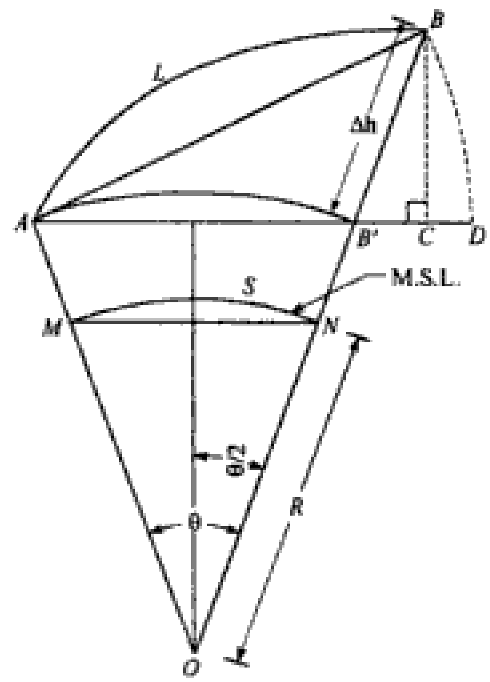

Fig. 1.56 Reduction of sea-level distance by station elevations

Now the value of *S* is calculated from Eq. (1.60) by substituting the value of *MN* from Eq. (1.59) and the value of *AB'* form Eq. (1.66).

1.25.4 Advantages and disadvantages of trilateration

The following are the advantages and disadvantages of trilateration:

Advantages

1. It is superior to triangulation in accuracy if properly executed.
2. It is generally more economical than triangulation.
3. The number of persons required to conduct trilateration is generally less than those required for triangulation.
4. It provides a scale control at all stages of trilateration which is only possible in triangulation by providing check bases.

Disadvantages

1. It has smaller number of internal checks as compared with triangulation.
2. It cannot be used in situations where precise angular observations are to be made to intersect reference objects.
3. To enhance the accuracy of triangulation system, accurate sampling of metrological conditions such as temperature, pressure and humidity, is required.
4. It requires expensive instrument *EDM*.

ILLUSTRATIVE EXAMPLES

Example 1.25. The slope distance *AB* in Fig. 1.55, corrected for meteorological conditions and instrument constants, is 23458.45 m. The vertical angle at *A* and *B* as measured are respectively, +3°02′05″ and – 3°12′55″. The elevation of *A* is 322.07 m. Determine the sea-level distance *MN*. (Take $R = 6370$ km).

Solution: (Fig. 1.55)

From Eq. (1.54), we have

$$\angle BAB' = \frac{\alpha + \beta}{2} = \frac{3°02'05'' + 3°12'55''}{2}$$
$$= 3°07'30''$$

From Eq. (1.56), we have

$$\sin\frac{\theta}{2} = \frac{AB'}{2R}$$

Substituting the value of AB' from Eq. (1.55), we get

$$\sin\frac{\theta}{2} = \frac{AB\cos BAB'}{2R}$$

As $\frac{\theta}{2}$ is very small, $\sin\frac{\theta}{2}$ can be taken equal to $\frac{\theta}{2}$

or
$$\frac{\theta}{2} = \frac{AB\cos BAB'}{2R}$$
$$= \frac{23458.45 \times \cos 3°07'30''}{2 \times 6370 \times 10^3}$$
$$= 1.8385845 \times 10^{-3} \text{ radians}$$
$$= 6'19.24''$$

In the triangle ABB'

$$\angle AB'B = 90° + \frac{\theta}{2}$$
$$= 90° + 6'19.24''$$
$$= 90°06'19.24''$$

$$\angle ABB' = 90° - \frac{\theta}{2} - \angle BAB'$$
$$= 90° - 6'19.24'' - 3°07'30''$$
$$= 86°46'10.76''$$

From Eq. (1.58), we get

$$AB' = AB\frac{\sin ABB'}{\sin\left(90° + \frac{\theta}{2}\right)}$$

$$= \frac{23458.45 \times \sin 86°46'10.76''}{\sin 90° + 6'19.24''}$$
$$= 23421.216 \text{ m}$$

From Eq. (1.59), we have

$$MN = AB' - AB' \frac{h_A}{R}$$
$$= 23421.216 \times \left(1 - \frac{322.07}{6370 \times 10^3}\right) = 23420.032 \text{ m}$$

The sea-level length from Eq. (1.60) is

$$S = MN + \frac{MN^3}{24R^2}$$
$$= 23420.032 + \frac{23420.032^3}{24 \times (16370 \times 10^3)^2}$$
$$= \mathbf{23420.045 \text{ m}}.$$

Example 1.26. The slope distance between two stations A and B of elevations 1572.25 m and 4260.46 m, respectively, corrected for meteorological conditions, is 33449.2150 m. Determine the sea-level distance. (Take R = 6370 km).

Solution: (Fig. 1.56)

From the given data, we have

$$\Delta h = 4260.46 - 1572.25$$
$$= 2688.21 \text{ m}$$

In the first approximation

$$AB' \simeq AC \simeq AD - CD \simeq AB - CD$$

Taking the value of *CDF* Eq. (1.64), we have

$$AB' = AB - \frac{\Delta h^2}{2AB}$$
$$= 33449.215 - \frac{2688.21^2}{2 \times 33449.215}$$
$$= 33341.19343$$

Form Eq. (1.56), we have

$$\sin\frac{\theta}{2} = \frac{AB'}{2R} = \frac{33341.19343}{2 \times 6370 \times 10^3}$$

or $$\frac{\theta}{2} = 2.6170482 \times 10^{-3} \text{ radians}$$

$$\frac{\theta}{2} = 8'59.81''$$

Now from Eq. (1.66), we get

$$AB' = AB - \left(\Delta h \sin\frac{\theta}{2} + \frac{\Delta h^2}{2AB}\right)$$
$$= 33449.215 - \left[2688.21 \times \sin 9'1.55'' + \frac{2688.21^2}{2 \times 33449.215}\right]$$
$$= 33334.13552 \text{ m}$$

From Eq. (1.59), we get

$$MN = AB' - \frac{AB' h_A}{R}$$

$$= 33334.13552 - \frac{33334.13552 \times 1572.25}{6370 \times 10^3}$$

$$= 33325.93063 \text{ m}$$

From Eq. (1.60), we get

$$S = MN + \frac{MN^3}{24R^2}$$

$$= 33325.90796 + \frac{33325.90796^3}{24 \times (6370 \times 10^3)^2}$$

$$= \mathbf{33325.96863 \text{ m}}.$$

PROBLEMS

1.1 What is a control point? What purpose does it serve? Differentiate between horizontal control and vertical control.

1.2 Discuss different methods for establishing a horizontal control.

1.3 What is triangulation? How is it different from traversing and trilateration?

1.4 Describe (*i*) triangulation system, (*ii*) trilateration system and cite examples of the application of the these types of surveys to practical problems.

1.5 Discuss the principle of triangulation surveys.

1.6 How the triangulation surveys are classified? Discuss how the triangulation systems of different orders are related to each other.

1.7 Discuss various geometrical figures used to extend triangulation, drawing sketches for each type. What figures are best adopted to precise work? Also discuss merits and demerits of each figure.

1.8 Describe grid iron system and central system of triangulation giving their suitability.

1.9 What is a well-conditional triangle? Prove that isoceles triangle is a well-conditioned triangle with two angles opposite to equal sides, approximately equal to 56°14'.

1.10 What is meant by strength of figure? How is it determined?

1.11 What is reconnaissance? Describe the process of reconnaissance for triangulation.

1.12 What are the functions of (*i*) signal and (*ii*) towers? Describe various types of signals and their suitability.

1.13 What are the factors that affect the selection of triangulation station?

1.14 What are the methods of checking intervisibility of stations? If two stations are not intervisible, what steps are needed to make them intervisible.

1.15 What are the requirements of a good signal which it should fulfil?

1.16 What do you understand by phase of a signal? Derive the formula for reducing the observations at the centre of the signal.

1.17 What are the points which have to be considered in selection of site for base line measurements?

1.18 What is the extension of base? How is it done?

1.19 What are the corrections which are required to be applied to the measured lengths of a base line?

1.20 what is meant by extension of a base line? Explain with neat sketches how a base line is extended in the field?

1.21 Discuss the measurement of horizontal angles in triangulation surveys by (*i*) Repetition method, (*ii*) Reiteration method.

1.22 What extra precautions one should take while making angle observations in triangulation?

1.23 What is a satellite station? Discuss the method of reduction of horizontal angles to centre.

1.24 What do you mean by eccentricity of signal? How it is corrected?

1.25 What are the intersected and resected points in triangulation? What purpose do they serve?

1.26 Describe the method of location of points by (*i*) intersection, and (*ii*) resection.

1.27 Explain the steps required in computation of independent coordinates of triangulation stations.

1.28 Discuss the methods of reducing the measured slope distance by *EDM's* to sea-level distances.

1.29 Discuss the advantages and disadvantages of trilateration.

1.30 If the probable error of direction measurement is 1.30", compute the maximum value of R for desired maximum probable error of (*i*) 1 in 10,000 (*ii*) 1 in 50,000.

1.31 Determine the value of $\frac{D-C}{D}$ for the figures given in Fig. 1.57. The narrow indicates the direction of measurement and the heavy line indicates the starting line.

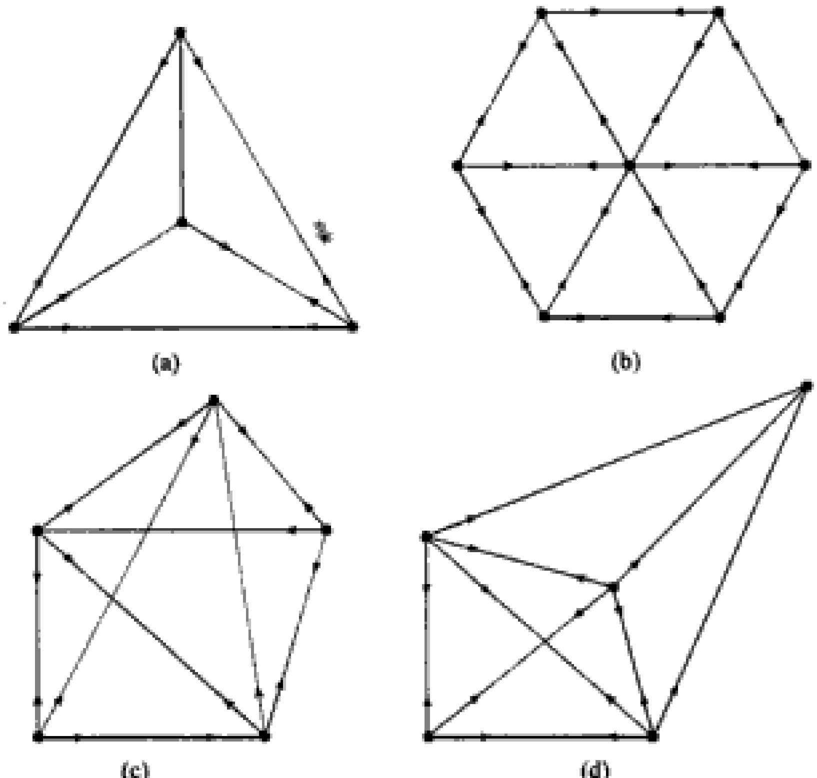

Fig. 1.57

1.32 There are two stations A and B at elevations of 200 m and 1000 m, respectively. The distance between A and B is 100 km. If the elevations of a peak P at a distance of 40 km from A is 300 m, show that stations A and B are intervisible.

1.33 The elevations of two proposed triangulation stations A and B, 100 km apart, are 140 m and 416 m above mean sea-level, respectively. The elevation of an intervening peak at C, 60 km from A, which is likely to obstruct the line of sight, is 150 m. Ascertain if A and B are intervisible and if not, find the height required for the scaffold at B so that the line of sight clears C by 3 m.

1.34 During the reconnaissance of a hilly part of the country for geodetic surveying, the following data were obtained regarding the profile of intervening ground between stations P and Q, the distance PQ being 120 km. The elevations above mean sea-level are P : 210 m, Q : 1050 m, L = 330 m and M = 557 m.

Peak L and M are situated in the line PQ such that PL = 50 km and PM = 80 km. Determine whether P and Q are intervisible, and if necessary, find the minimum height of the scaffolding at Q, assuming P as the ground station. The line of sight should clear the intervening peaks at least 3 *m*.

1.35 Two triangulation stations A and B are 40 km apart and have elevations of 178 m and 175 m, respectively. Find the minimum height of signal required at B so that the line of sight may not pass nearer the ground than 3 m. The intervening ground may be assumed to have a uniform elevation of 150 m.

1.36 Solve Problem No. 1.33 by Captain McCaw's method.

1.37 Compute the strength of a braced quadrilateral *ABCD* for each of the routes by which the length of *BD* can be computed from the known side *AC*. All the stations were occupied. All the angles measured are:

$$\angle ABD = 28°;\ \angle DBC = 36°;\ \angle BCA = 54°;\ \angle ACD = 58°$$

$$\angle CDB = 32°;\ \angle BDA = 23°;\ \angle DAC = 67°;\ \angle CAB = 62°$$

1.38 In a triangulation survey observations were made from an eccentric signal *E*, 12.25 m to the west of the main station *Q*, and the following angles were measured:

$$\angle QER = 72°25'32'' \text{ and } \angle REP = 54°32'20''$$

The stations *E* and *R* are to the opposite sides of the line *PQ*. Calculate the correct angle *PQR* if the lengths of *PQ* and *QR* are 5286.5 m and 4932.2 m, respectively.

1.39 From a satellite station *S*, 60 m from triangulation station *C*, the horizontal angles observed to other stations are as follows:

Instrument	Sight to	Horizontal angle
S	*A*	0°00′ 0″
	B	71°54′ 32″
	C	296°12′ 02″

The approximate lengths of *AC* and *BC* are 18024 m and 23761 m, respectively. Compute the angle subtended at station *C* by the other two stations *A* and *B*.

1.40 Observations were made from instrument station *A* to the signal at *B*. The sun makes an angle of 60° with the line *BA*. Calculate the phase correction if (*i*) the observation was made on the bright portion, and (*ii*) the observation was made on the bright line. The distance *AB* is 9460 m. The diameter of the signal is 12 cm.

1.41 A nominal distance of 30 m was set out with a steel tape from a mark on the top of one peg to a mark on the top of another. The tape was in catenary under a pull of 160 N and at temperature of 32 °C. The top of one peg was 0.532 m above the top of the another. Determine the horizontal distance between the marks on the two pegs. The tape was standardized in catenary under a pull of 130 N and at a temperature of 27 °C. Take

Mass of tape = 0.022 kg/m
Cross-sectional = 3.15 mm^2
Coefficient of linear expansion = 9×10^{-7} per °C
Young's modulus of elasticity = 1.5×10^5 N/mm^2.

1.42 A base line was measured in 4 bays and the following observation were taken:

Bay	Measured Length (m)	Temperature (°C)	Difference in level (m)	Tension (N)
1	29.8315	22.0	+0.064	185
2	29.8432	22.1	+0.338	185
3	29.8238	21.9	− 0.217	185
4	29.8841	22.2	− 0.212	185

Determine the correct length of the base at mean sea-level. The additional required data for the computations are as under:

Mass of tape = 0.025 kg/m
Cross sectional area = 2.55 mm^2
Coefficient of linear expansion = 9×10^{-7} per °C
Young's modulus of elasticity = 1.5×10^5 N/mm^2
Mean elevation of the base line = 150.65 m
The tape was standardized on flat ground at 25 °C under a pull of 90 N.
Take $g = 9.806$ m/sec^2 and $R = 6370$ km.

1.43 As shown in Fig. 1.58, five angles were measured about station *P*. Each set of measurements consisted of six repetitions with the telescope direct and six with the telescope reversed, and three sets of measurements were taken. The mean values of the angles are given below. Adjust the mean angles, and prepare an abstract of directions to the observed stations. Take the direction to station *A* as 0°00′ 00″.

θ_1 = 55°02′ 15.0″

θ_2 = 58°33′ 12.7″

θ_3 = 52°42′ 42.5″

θ_4 = 38°57′ 22.5″

θ_5 = 154°44′ 23.3″

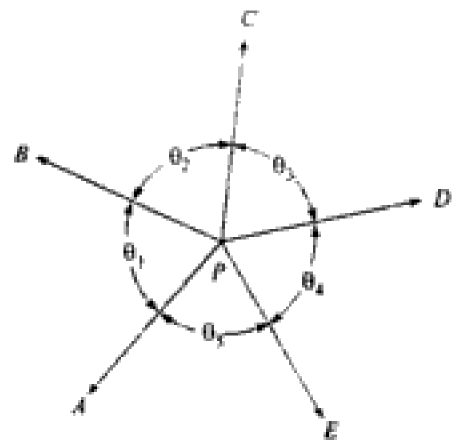

Fig. 1.58

1.44 The coordinates a point *P* are to be determined by making observations from two triangulation stations *A* and *B* whose coordinates are:

X_A = 17123.06 m, Y_A = 7532.68 m

X_B = 17222.08 m, Y_B = 7498.90 m

The measured angle ∠*PAB* and ∠*PBA* are 75°19′21″ and 38°39′50″, respectively. Compute the coordinates of *P*.

1.45 For establishing a station *P* from three triangulation stations *A*, *B* and *C* the following data were collected:

∠*APC* = 42°15′ 21″; *AC* = 314.153 m

∠*APB* = 37°14′ 15″ ; *AB* = 532.415 m

∠*BAC* = 122°35′ 18″

Determine the lengths of the lines *PA*, *PB* and *PC*.

2

ADJUSTMENT OF SURVEY MEASUREMENTS

2.1 GENERAL

Although measurement of a quantity is a single act, a typical survey measurement may involve several elementary operations, such as centering, pointing, setting and reading. In performing these operations and due to human limitations, imperfection in instrument, environmental changes or carelessness on the part of the observer, certain amount of error is bound to creep into the measurements. Hence, the measurements always contain errors. Since the measured quantities are used to calculate other quantities such as area, volume, elevation, slope, through relationships with the measured quantities, the errors in measured quantities get propagated into the calculated quantities.

The different types of errors, their sources and how they propagate have already been discussed in Chapter 2 of *Plane Surveying*. The errors in the measured quantities should be eliminated or minimised before they are used for computing other quantities.

After removing the blunders and systematic errors, the errors which remain in the measurements are residual, random or accidental errors. These errors are minimised or adjusted, and the adjusted quantity is known as the *most probable value* of a measured quantity. It is the most probable value of a measured quantity which is used for computing other quantities.

2.2 DEFINITIONS

The following definitions of some of the terms should be clearly understood in adjustment of the measurements.

True value: The value of a quanitity which is free from all the errors, is called the true value of quantity. Because it is not possible to eliminate all the errors completely from a measured quantity, the true value cannot be determined.

Observation: The measured numerical value of a quantity is known as observation. No measurement is made until something is observed. Accordingly, the terms measurement and observation are often used synonymously.

The observations may be classified as

(*i*) Direct observations

(*ii*) Indirect observations.

Direct observation: If the value of a quantity is measured directly, for example, measurement of an angle, the observation is said to be a direct observation.

Indirect observation: An observation is said to be indirect if the value of the quantity is deduced from the measurements of other quantities. For example, the value of the angle at the main triangulation station computed from the measured angles at the satellite station.

Observed value of a quantity: Observed value of a quantity is the value obtained from the observation after eliminating the mistakes and systematic errors. The observed values contain random or residual errors.

The observed value of a quantity may be classified as

(*i*) Independent quantity

(*ii*) Conditioned quantity.

Independent quantity: If the value of an observed quantity is independent of the value of other quantities, it is said to be an independent quantity. For example, reduced level of a point.

Conditioned quantity: If the value of an observed quantity is dependent upon the values of other quantities it is called a conditioned quantity. For example, the three angles A, B and C in a plane triangle are conditioned quantities since they are related by the condition equation $A + B + C = 180°$.

Most probable value: The most probable value of a quantity is the value which has more chances of being true than any other value.

True error: The difference between the true value and the observed value is known as true error. Thus,

$$\text{True error} = \text{Observed value} - \text{True value.}$$

Most probable error: It may be defined as the quantity which is subtracted from or added to the most probable value of a quantity. It fixes the bounded limits within which, the true value of the observed quantity may lie.

Residual error: The difference between the observed value of quantity and its most probable value, is called residual error, residual or variation. Thus,

$$\text{Residual error or residual} = \text{Observed value} - \text{most probable value.}$$

Observation equation: The relation between the observed quantities is known as an observation equation. For example, $\alpha + \beta = 67°39'15''$.

Conditioned equation: A conditioned equation is the equation expressing the relation existing between the several dependent quantities. For example, at a station if four angles θ_1, θ_2, θ_3 and θ_4 have been observed then $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360°$.

In a braced quadrilateral the sum of all the eight observed angles is 360°. Thus,

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8 = 360°.$$

Normal euqation: A normal equation is the one which is formed by multiplying each equation by the coefficient of the unknown whose normal equation is to be found, and by adding the equations thus formed. The number of normal equations is the same as the number of unknowns. The most probable values of the unknowns are found out by using by the normal equations.

2.3 WEIGHT OF OBSERVATIONS

In Sec. 2.6 of *Plane Surveying*, some indicators of precision of a measurement, such as standard deviation (σ), variance (σ^2), where introduced. A measurement of high precision has a small variance and conversely, a measurement of low precision has a large variance, i.e., the value of variance goes in opposite direction to that of the precision. There is another measure of precision that is directly related to the precision of a measurement, and does not behave in opposite manner as the variance. This measure is called the *weight* of an observation, and is always expressed in numbers. For any given observation, the higher the weight the higher is the precision and *vice versa.* Accordingly, the weight ω of a single observation is defined as a quantity that is inversely proportional to the variance (σ^2) of the observation, i.e.,

$$\omega = k/\sigma^2 \qquad ...(2.1)$$

Where k is a constant of proportionality. For an observation of unit weight, i.e., $\omega = 1$, k is equal to the variance of the observation.

$$1 = \frac{k}{\sigma_0^2} \text{ (for } \omega = 1, \sigma^2 = \sigma_0^2)$$

or $$k = \sigma_0^2 \quad ...(2.2)$$

Thus, $$\omega = \frac{\sigma_0^2}{\sigma^2} \quad ...(2.3)$$

The variance σ_0^2 of an observation of unit weight is referred to by several names, such as *variance factor*, *variance of unit weight*, and *reference variance*.

2.3.1 Laws of weights

The method of least squares for errors (*cf.*, Sec. 2.5) adopts the following laws of weight:

(*i*) The weight of the arithmetic mean of a number of observations of unit weight, is equal to the number of the observations.

(*ii*) The weight of the weighted arithmetic mean of a number of observations, is equal to the sum of the individual weights of observations.

(*iii*) The weight of the sum of the quantities added algebraically, is equal to the reciprocal of the sum of the reciprocals of the individual weights.

(*iv*) The weight of the product of any quantity multiplied by a constant, is equal to the weight of that quantity divided by the square of that constant.

(*v*) The weight of the quotient of any quantity divided by a constant, is equal to the weight of that quantity multiplied by the square of that constant.

(*vi*) The weight of an equation remains unchanged if all the signs of the equation are changed.

(*vii*) The weight of an equation remains unchanged if the equation is added to or subtracted from a constant.

(*viii*) If an equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of that equation.

2.3.2 Guidelines for assigning weightage to the field observations

The following guidelines are applied in assigning the weightage to the field observations:

(*i*) The weight of an angle varies directly to the number of observations.

(*ii*) The weights of the level lines vary inversely as the lengths of their routes.

(*iii*) If an angle is measured a large number of time-s, its weight is inversely proportionals to the square of the probable error.

(*iv*) The corrections to be applied to various observed quantities, are in inverse proportion to their weights.

ILLUSTRATIVE EXAMPLES

Example 2.1 An angle *A* was measured six times as below. If all the observations are of unit weight, what is the weight of the arithmetic mean of the angle?

∠*A*	Weight	∠*A*	Weight
42°25′10″	1	42°25′7″	1
42°25′8″	1	42°25′11″	1
42°25′9″	1	42°25′9″	1

Solution: The number of observations of unit weight, $n = 6$.

$$\text{Arithmetic mean} = 42°25' + \frac{1}{n}(10'' + 8'' + 9'' + 7'' + 11'' + 9'')$$

$$= 42°25'\frac{54''}{6}$$

$$= 42°25'9''$$

From rule (*i*), the weight of the arithmetic mean = n

$$= \mathbf{6}.$$

Example 2.2 An angle A was observed three times as given below with their respective weights. What is the weight of the weighted arithmetic mean of the angle?

$\angle A$	Weight
60°15′7″	1
60°15′10″	2
60°15′15″	3

Solution:

Weighted arithmetic mean of A

$$= 60°15' + \frac{7''\times 1 + 10''\times 2 + 15''\times 3}{1+2+3}$$

$$= 60°15'12''$$

Sum of the individual weights

$$= 1+2+3=6$$

From rule (*ii*) the weight of the weighted arithmetic mean of the angle

= sum of the individual weights

= **6.**

Example 2.3 Calculate the weights of $(\alpha+\beta)$ and $(\alpha-\beta)$ if the measured values and the weights of α and β, respectively, are:

$$\alpha = 44°55'30'' \text{ wt. } 3$$

$$\beta = 30°32'25'' \text{ wt. } 4$$

Solution:

$$\alpha+\beta = 77°27'55''$$

$$\alpha-\beta = 12°23'05''$$

From rule (*iii*), the weights of $(\alpha+\beta)$ and $(\alpha-\beta)$

$$= \frac{1}{\frac{1}{\omega_1}+\frac{1}{\omega_2}} = \frac{1}{\frac{1}{3}+\frac{1}{4}}$$

$$= \mathbf{\frac{12}{7}}.$$

Example 2.4 What is the weight of 3α if $\alpha = 35°25'40''$ and its weight is 4?

Solution: As α is multiplied by 3, the constant of multiplication C is 3 and weight ω of the observation α is 4.

From rule (*iv*) the weight of $(C.\alpha)$ is $\frac{\omega}{C^2}$

$$\text{Weight of } (3\alpha = 106°17'00'') = \frac{4}{3^2}$$

$$= \mathbf{\frac{4}{9}}.$$

Example 2.5 Compute the weight of $\frac{\alpha}{4}$ if $\alpha = 46°26'24''$ of weight 4.

Solution:

The constant C of division is 4 and weight ω of the observation is 4.

From rule (*v*), *the weihgt of* $\frac{\alpha}{C}$ *is* ωC^2

$$\text{Weight of} \quad \left(\frac{\alpha}{4} = 11°36'36''\right) = 4 \times 4^2$$
$$= \mathbf{64}.$$

Example 2.6 If weight of $\alpha + \beta = 72°30'55''$ is 3, what is the weight of $(-\alpha - \beta)$?

Solution:

From rule (*vi*) the weights of $(-\alpha - \beta)$ will remain the same.

Weight of $[(-\alpha - \beta) = -72°30'55''] = \mathbf{3}$.

Example 2.7 If weight of $(\alpha + \beta) = 55°15'30''$ *is* $\frac{2}{3}$, what will be the weights of $180° + (\alpha + \beta)$, $180° - (\alpha + \beta)$ and $(\alpha + \beta) - 180°$?

Solution:

*From rule (*vii*) the weights of* $180° + (\alpha + \beta)$, $180° - (\alpha + \beta)$, *and* $(\alpha + \beta) - 180°$ *will remain same as of* $(\alpha + \beta)$.

$$\text{Weight of } [\{180° + (\alpha + \beta)\} = 235°15'30''] = \frac{\mathbf{2}}{\mathbf{3}}.$$

$$\text{Weight of } [\{180° - (\alpha + \beta)\} = 124°44'30''] = \frac{\mathbf{2}}{\mathbf{3}}.$$

$$\text{Weight of } [\{(\alpha + \beta) - 180°\} = -124°44'30''] = \frac{\mathbf{2}}{\mathbf{3}}.$$

Example 2.8 Calculate the weight of the equation $\frac{3}{4}(\alpha + \beta)$ if weight of $(\alpha + \beta)$ is $\frac{3}{4}$. The observed value of $(\alpha + \beta)$ is $115°22'48''$.

Solution:

As the equations is being multiplied by its own weight, from rule (*viii*) the weight of $\{\omega(\alpha + \beta)\}$ will be $\frac{1}{\omega}$ if the ω is the weight of $(\alpha + \beta)$.

$$\text{Weight of } \left[\frac{3}{4}(\alpha + \beta) = 86°32°06''\right] = \frac{1}{\frac{3}{4}} = \frac{\mathbf{4}}{\mathbf{3}}.$$

2.4 THE CONCEPT OF ADJUSTMENT

The general relationships which relate the measurements to the other quantities of interest, constitute what is known as the *model*. Since almost all the relationships encountered in surveying computations are mathematical representations of underlying physical and geometric conditions, the model is frequently called the *mathematical model*.

Once a problem is stated, the underlying mathematical model is also specified. At the same time minimum number of elements or variables necessary to determine the model uniquely, should also be determined. For example, if the problem is to determine the size and shape of a triangle, the minimum number of elements may be

(*i*) one side and two internal angles,
(*ii*) two sides and one-angle, or
(*iii*) all the three sides.

It follows that the minimum number of the elements required to determine the size and shape of a triangle is three, and one of them must be a side. The three angles of a triangle alone can give the shape of the triangle but not the size.

Having determined the minimum number n_0 of elements, one should decide what measurements are to be made. If exactly one side and two internal angles are measured, the triangle will be uniquely fixed, and if any blunder or mistake is committed in any of the three measurements, it will remain undetected. Therefore, in practice, always more elements than required are measured. Thus if the total number of measurements taken is n then each observation in excess of the minimum number n_0, is called a reduntant measurement. The total number of redundant measurements is called redundancy or statistical degree of freedom, and is given by

$$r = n - n_0. \qquad ...(2.4)$$

Now, if the model is constructed using each subset of n_0 measurements of the n given measurements ($n > n_0$), a slightly different determination of the model is likely to exist from each subset of n_0 measurements. This interesting situation arises because of the unavoidable errors in the measurements, the n observations no longer fit the model exactly. This apparent inconsistency with the model is resolved through the replacement of the given observations l by another set of values which are the *estimates* $\hat{l}$ of the observations. The values of the estimates are determined such that they fit the model exactly.

Each estimated observation $\hat{l}_i$, can be looked upon as corrected observation. They are obtained by adding the corrections v_i to the respective measured values l_i, i.e.,

$$\hat{l}_i = l_i + v_i (i = 1, 2, 3, ..., n) \qquad ...(2.5)$$

Eq. (2.5) is equivalent to Eq. (2.2) of Chapter 2 of *Plane Surveying*, in which v is defined as the residual. In matrix notation each term of Eq. (2.5) is a column vector having n element, i.e.,

$$\begin{bmatrix} \hat{l}_1 \\ \hat{l}_2 \\ \hat{l}_3 \\ \vdots \\ \hat{l}_n \end{bmatrix} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ \vdots \\ l_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} \qquad ...(2.6)$$

There are a variety of criteria possible to determine as set of values of the residuals which would yield an optimum solution but the one which is most commonly used in surveying is that given by the principle of least squares. (*cf.*, Sec. 2.5)

The operation of finding the new set of estimates $\hat{l}$ according to some criterion is called *adjustment*.

2.5 LEAST SQUARES METHOD

In the preceeding section it has been indicated that it is a general practice in surveying to always have redundant observations as they help in detection of mistakes or blunders. Redundant observations require a method which can yield a unique solution of the model. Although, approximate methods, both graphical and computational, may be adequate for some limited cases, a more general and systematic procedure is needed for application to all situations. The *least squares* adjustment method is such a method.

Assuming that all the observations are uncorrelated and of equal precision then the least squares method of adjustment is based upon the following criterion:

"The sum of the squares of the residuals (observational) must be a minimum".

or $$\phi = v_1^2 + v_2^2 + v_3^2 + \ldots v_n^2 = \sum_{i=1}^{n} v_i^2 = \text{a minimum.} \qquad \ldots (2.7)$$

Thus, the above condition which the residuals have to satisfy, is in addition to the conditions which the adjusted observations should satisfy for a given model. The adjusted observations which satisfy a given model may be said to be that value of the quantity which has more chances of being true value than any other value and therefore, the adjusted value is the most probable value of the quantity.

If the observations are uncorrelated and of equal precision such as several measurements of length of a line, it can be proved by method of least squares that the arithmetic mean of such observations is the most probable value of the quantity.

Let $l_1, l_2, l_3, \ldots, l_n$ be the observations and $\hat{l}$ be the arithmetic mean of the observations then the residuals $v_1, v_2, v_3, \ldots v_n$ are given by

$$\begin{aligned} \hat{l} - l_1 &= v_1 \\ \hat{l} - l_2 &= v_2 \\ \hat{l} - l_3 &= v_3 \\ \vdots &= \vdots \\ \hat{l} - l_n &= v_n \end{aligned} \qquad \ldots (2.8)$$

or $$(\hat{l} - l_1)^2 + (\hat{l} - l_2)^2 + \ldots + (\hat{l} - l_n)^2 = \sum_{i=1}^{n} v_i^2 \qquad \ldots (2.9)$$

In order to make $\sum_{i=1}^{n} v_i^2$ a minimum in accordance to Eq. (2.7), differentiate Eq. (2.9) with respect to $\hat{l}$, and set the derivative equal to zero. Thus

$$\frac{d}{d\hat{l}}\left(\sum_{i=1}^{n} v_i^2\right) = 2(\hat{l} - l_1) + 2(\hat{l} - l_2) + \ldots + 2(\hat{l} - l_n) = 0 \qquad \ldots (2.10)$$

or $$(\hat{l} - l_1) + (\hat{l} - l_2) + \ldots + (\hat{l} - l_n) = 0$$

or $$n\hat{l} - (l_1 + l_2 + \ldots + l_n) = 0$$

$$n\hat{l} = l_1 + l_2 + \ldots + l_n$$

or $$\hat{l} = \frac{l_1 + l_2 + \ldots l_n}{n}$$

= arithmetic mean.

ILLUSTRATIVE EXAMPLES

Example 2.9 A distance is measured 6 times with the following results: $l_1 = 74.31$ m, $l_2 = 74.28$ m, $l_3 = 74.32$ m, $l_4 = 74.33$ m, $l_5 = 74.30$ m, and $l_6 = 74.31$ m.

Determine the most probable value of the distance by the method least squares.

Solution: The total number of observations $n = 6$.

The minimum number of observations required to determine the distance $n_0 = 1$.

Let the most probable value of the distance be $\hat{l}$. Carrying $\hat{l}$ as the unknown parameter estimate, the number of conditions will be $n' = r + 1$ where r is the redundancy. Thus

$$r = n - n_0 = 6 - 1 - 1 = 5$$

and $$n' = 5 + 1 = 6$$

The conditions are

$$\begin{aligned} l_1 + v_1 &= \hat{l} \\ l_2 + v_2 &= \hat{l} \\ l_3 + v_3 &= \hat{l} \\ l_4 + v_4 &= \hat{l} \\ l_5 + v_5 &= \hat{l} \\ l_6 + v_6 &= \hat{l} \end{aligned}$$

Another condition to be satisfied from the least squares principle, is

$$\begin{aligned} \phi &= v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2 = \text{a minimum} \\ &= (\hat{l} - l_1)^2 + (\hat{l} - l_2)^2 + (\hat{l} - l_3)^2 + (\hat{l} - l_4)^2 + (\hat{l} - l_5)^2 + (\hat{l} - l_6)^2 = \text{a minimum} \end{aligned}$$

or $$\frac{\partial \phi}{\partial \hat{l}} = 2(\hat{l} - l_1) + 2(\hat{l} - l_2) + 2(\hat{l} - l_3) + 2(\hat{l} - l_4) + 2(\hat{l} - l_5) + 2(\hat{l} - l_6) = 0$$

or $$6\hat{l} = l_1 + l_2 + l_3 + l_4 + l_5 + l_6$$

$$\begin{aligned} \hat{l} &= \frac{l_1 + l_2 + l_3 + l_4 + l_5 + l_6}{6} \\ &= \frac{74.31 + 74.28 + 74.32 + 74.33 + 74.30 + 74.31}{6} \\ &= \mathbf{74.31\ m.} \end{aligned}$$

Example 2.10 The three angles of a plane triangle are $\theta_1 = 52°33'$, $\theta_2 = 64°45'$, and $\theta_3 = 62°39'$. Determine the least squares estimate of the angles.

Solution:

Minimum number of angles required to fix the triangle $n_0 = 2$

Total number of observations $n = 3$

Redundancy $r = 3 - 2 = 1$

The condition to be satisfied for the triangle is

$$\theta_1 + \theta_2 + \theta_3 = 180°$$

or $$(\theta_1 + v_1) + (\theta_2 + v_2) + (\theta_3 + v_3) = 180°$$

or $$\begin{aligned} v_1 + v_2 + v_3 &= 180° - \theta_1 + \theta_2 + \theta_3) \\ &= 180° - (52°33' + 64°45' + 62°39') \\ &= 180° - 179°57' \\ &= 3' \end{aligned}$$

or $$v_3 = 3' - (v_1 + v_2)$$

From least squares principle

$$\begin{aligned} \phi &= v_1^2 + v_2^2 + v_3^2 \\ &= v_1^2 + v_2^2 + (3' - v_1 - v_2)^2 = \text{a minimum} \end{aligned}$$

$$\frac{\partial \phi}{\partial v_1} = 2v_1 + 2(3' - v_1 - v_2)(-1) = 0$$

$$\frac{\partial \phi}{\partial v_2} = 2v_2 + 2(3' - v_1 - v_2)(-1) = 0$$

or
$$2v_1 + v_2 = 3$$
$$v_1 + 2v_2 = 3$$

The solution of the above equations which are known as normal equations, yields.

$$v_1 = 1'$$
$$v_2 = 1'$$

and
$$v_3 = 3' - v_1 - v_2 = 1'$$

Therefore, the least squares estimates of the angles are

$$\hat{\theta}_1 = \theta_1 + v_1 = 52°33' + 1' = 52°34'$$
$$\hat{\theta}_2 = \theta_2 + v_2 = 64°45' + 1' = 64°46'$$
$$\hat{\theta}_3 = \theta_3 + v_3 = 62°39' + 1' = 62°40'$$

Sum = **180°00'**.

Example 2.11 In Fig. 2.1, the observed values of the distances *AB*, *BC*, *CD*, *AC*, and *BD* are as 50.000 m, 50.070 m, 50.050 m, 100.090 m, and 100.010 m, respectively.

Determine the adjusted value of AD assuming that all the observations are of equal reliability and uncorrelated.

Fig. 2.1.

Solution (Fig. 2.1):

Let *AB*, *BC*, *CD*, *AC*, and *BD* be x_1, x_2, x_3, x_4, and x_5, respectively. To determine the distance *AD*, *a* minimum of three distances x_1, x_2, and x_3 are required. Therefore, $n = 5$, $n_0 = 3$, and $r = 5 - 3 = 2$. The number of condition equations will be equal to r + number of unknowns, i.e., $2 + 3 = 5$. If $\hat{x}_1, \hat{x}_2$, and $\hat{x}_3$, be the adjusted values of x_1, x_2, and x_3 then

$$x_1 + v_1 = \hat{x}_1 \quad \text{or} \quad v_1 = \hat{x}_1 - x_1$$
$$x_2 + v_2 = \hat{x}_2 \quad \text{or} \quad v_2 = \hat{x}_2 - x_2$$
$$x_3 + v_3 = \hat{x}_3 \quad \text{or} \quad v_3 = \hat{x}_3 - x_3$$
$$x_4 + v_4 = \hat{x}_1 + \hat{x}_2 \quad \text{or} \quad v_4 = \hat{x}_1 + \hat{x}_2 - x_4$$
$$x_5 + v_5 = \hat{x}_2 + \hat{x}_3 \quad \text{or} \quad v_5 = \hat{x}_2 + \hat{x}_3 - x_5$$

From least squares principle

$$\phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 = \text{a minimum}$$
$$= (\hat{x}_1 - x_1)^2 + (\hat{x}_2 - x_2)^2 + (\hat{x}_3 - x_3)^2 + (\hat{x}_1 - \hat{x}_2 - x_4)^2 + (\hat{x}_2 - \hat{x}_3 - x_5)^2$$

Differentiating partially with respect to $\hat{x}_1, \hat{x}_2$, and $\hat{x}_3$, we get

$$\frac{\partial \phi}{\partial \hat{x}_1} = 2(\hat{x}_1 - x_1) + 2(\hat{x}_1 + \hat{x}_2 - x_4) = 0 \qquad \ldots(a)$$

$$\frac{\partial \phi}{\partial \hat{x}_2} = 2(\hat{x}_2 - x_2) + 2(\hat{x}_1 + \hat{x}_2 - x_4) + 2(\hat{x}_2 - \hat{x}_3 - x_5) = 0 \qquad \ldots(b)$$

$$\frac{\partial \phi}{\partial \hat{x}_3} = 2(\hat{x}_3 - x_3) + 2(\hat{x}_2 + \hat{x}_3 - x_5) = 0 \qquad \ldots(c)$$

Substituting the values of x_1, x_2, x_3 and x_3, in Eqs. (*a*), (*b*), and (*c*), and rearranging the terms of equations, we have

$$2\hat{x}_1 + \hat{x}_2 = 150.090 = d_1 \quad \text{...}(d)$$
$$\hat{x}_1 + 3\hat{x}_2 + \hat{x}_3 = 250.170 = d_2 \quad \text{...}(e)$$
$$\hat{x}_2 + 2\hat{x}_3 = 150.060 = d_3 \quad \text{...}(f)$$

The above three equations are normal equations. Solution of these normal equation gives the values of $\hat{x}_1, \hat{x}_2$, and $\hat{x}_3$ as below.

From Eq. (*d*)

$$\hat{x}_2 = d_1 - 2\hat{x}_1 \quad \text{...}(g)$$

From Eq. (*f*)

$$\hat{x}_3 = \frac{1}{2}(d_3 - d_1 + 2\hat{x}_1) \quad \text{...}(h)$$

Substituting the values of $\hat{x}_2$ and $\hat{x}_3$ in Eq. (*e*)

$$\hat{x}_1 = \frac{1}{8}(5d_1 - 2d_2 + d_3)$$
$$= \frac{1}{8}(5 \times 150.090 - 2 \times 250.170 + 150.060)$$
$$= 50.021 \text{ m}$$

Substituting the value of $\hat{x}_1$ in Eq. (*g*)

$$\hat{x}_2 = 50.048 \text{ m}$$

and in Eq. (*h*)

$$\hat{x}_3 = 50.006 \text{ m}$$

Thus, the adjusted distance

$$AD = \hat{x}_1 + \hat{x}_2 + \hat{x}_3 = 50.021 + 50.048 + 50.006$$
$$= \mathbf{150.075\ m.}$$

Example 2.12 Find the most probable values of the angles α and β from the following observations:

$$\alpha = 40°15'21.4''$$
$$\beta = 45°12'18.4''$$
$$\alpha + \beta = 85°27'45.2''.$$

Solution:

Let the most probable values of α and β be $\hat{\alpha}$ and $\hat{\beta}$, respectively then

$$\alpha + v_1 = \hat{\alpha} = 40°15'21.4'' + v_1$$
$$\beta + v_2 = \hat{\beta} = 45°12'18.4'' + v_2$$
$$(\alpha + \beta) + v_3 = \hat{\alpha} + \hat{\beta} = 85°27'45.2'' + v_3$$

or

$$v_1 = \hat{\alpha} - 40°15'21.4''$$
$$v_2 = \hat{\beta} - 40°12'18.4''$$
$$v_3 = \hat{\alpha} + \hat{\beta} - 85°27'45.2''$$

From laest squares theory

$$\phi = v_1^2 + v_2^2 + v_3^2 = \text{a minimum}$$
$$= (\hat{\alpha} - 40°15'21.4'')^2 + (\hat{\beta} - 45°12'18.4'')^2 + (\hat{\alpha} + \hat{\beta} - 85°27'45.2'')^2$$

Differentiating the above equation partially with respect to $\hat{\alpha}$ and $\hat{\beta}$, we have

$$\frac{\partial\phi}{\partial\hat{\alpha}} = 2(\hat{a} - 40°15'21.4'') + 2(\hat{\alpha} + \hat{\beta} - 85°27'45.2'') = 0$$

$$\frac{\partial\phi}{\partial\hat{\beta}} = (\hat{\beta} - 45°12'18.4'') + 2(\hat{\alpha} + \hat{\beta} - 85°27'45.2'') = 0$$

$$2\hat{\alpha} + \hat{\beta} = 125°43'6.6''$$

$$\hat{\alpha} + 2\hat{\beta} = 130°40'3.6''$$

Solving the above to equation, we get

$$\hat{\alpha} = \mathbf{40°15'23.2''}$$

$$\hat{\beta} = \mathbf{45°12'20.2''}.$$

Example 2.13 From the following data, determine the most probable values of a and b which are the slope and y-intercept in the equation $y - ax - b = 0$ of a line in a plane. Assume that the y-coordinates are the only observation, i.e., the x-coordinates are error-free constant.

Point	Coordinates	
	x (cm)	y (cm)
1	17.0	18.2
2	19.0	19.0
3	21.0	20.0

Solution:

Let the most probable value of a and b be $\hat{a}$ and $\hat{b}$. If the residuals in y_1, y_2, and y_3 are v_1, v_2, and v_3, respectively, we have

$$(v_1 + y_1) - \hat{a}x_1 - \hat{b} = 0$$

$$(v_2 + y_2) - \hat{a}x_2 - \hat{b} = 0$$

$$(v_3 + y_3) - \hat{a}x_3 - \hat{b} = 0$$

$$v_1 = 17.0\hat{a} + \hat{b} - 18.2$$

$$v_2 = 19.0\hat{a} + \hat{b} - 19.2$$

$$v_3 = 21.0\hat{a} + \hat{b} - 20.0$$

To get the most probable values

$$\phi = v_1^2 + v_2^2 + v_3^2 = \text{a minimum}$$

$$= (17.0\hat{a} + \hat{b} - 18.2)^2(19.0\hat{a} + \hat{b} - 19.0)^2 + (21.0\hat{a} + \hat{b} - 20.0)^2$$

Differentiating the above equation partially with respect to $\hat{a}$ and $\hat{b}$ we have

$$\frac{\partial\phi}{\partial\hat{a}} = 2(17.0\hat{a} + \hat{b} - 18.2)(17.0) + 2(19.0\hat{a} + \hat{b} - 19.0)(19.0)$$
$$+ 2(21.0\hat{a} + \hat{b} - 20.0)(21.0) = 0$$

$$\frac{\partial\phi}{\partial\hat{b}} = 2(17.0\hat{a} + \hat{b} - 18.2)(1) + 2(19.0\hat{a} + \hat{b} - 19.0)(1)$$
$$+ 2(21.0\hat{a} + \hat{b} - 20.0)(1) = 0$$

or $$2182\hat{a}+114\hat{b} = 2180.8$$

$$114\hat{a}+6\hat{b} = 114.4$$

From solution of the above two equations

$$\hat{a} = \mathbf{0.45}$$

$$\hat{b} = \mathbf{10.52\ cm.}$$

Example 2.14 The following angles shown in Fig. 2.2, were observed for a triangulation scheme. Determine the least squares estimates of the angles.

$$\theta_1 = 44°42'00''; \theta_2 = 46°00'00''$$

$$\theta_3 = 43°48'00''; \theta_4 = 44°31'12''$$

$$\theta_5 = 42°06'00''; \theta_6 = 48°52'48''$$

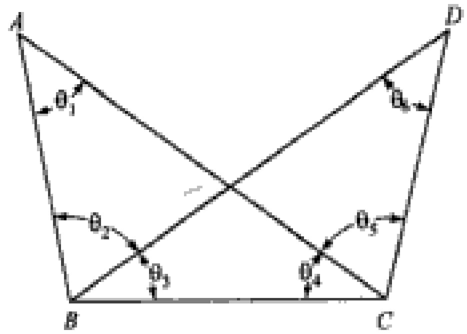

Fig. 2.2

Solution : (Fig. 2.2)

From the triangles *ABC* and *BDC*, we have

$$\theta_1+\theta_2+\theta_3+\theta_4 = 180°$$

$$\theta_3+\theta_4+\theta_5+\theta_6 = 180°$$

Let the least squares estimates of the angles be $\hat{\theta}_1, \hat{\theta}_2, ...\hat{\theta}_6$, and the residuals of the observed angles be $v_1, ... v_2, ..., v_6$, respectively.

From the above two conditions

$$v_1+v_2+v_3+v_4 = 180° - (\theta_1+\theta_2+\theta_3+\theta_4)$$
$$= 180° - 179°01'12''$$
$$= 58'48''$$
$$= 0.98° \quad ...(a)$$

and

$$v_3+v_4+v_5+v_6 = 180° - (\theta_3+\theta_4+\theta_5+\theta_6)$$
$$= 180° - 179°18'00''$$
$$= 42'00''$$
$$= 0.70° \quad ...(b)$$

From the least squares principle

$$\phi = v_1^2+v_2^2+v_3^2+v_4^2+v_5^2+v_6^2 = \text{a minimum}$$

Since there are only two condition equations and the six residuals, we can have four independent unknowns from Eqs. (*a*) and (*b*) as below.

$$v_1 = 0.98 - v_2 - v_3 - v_4$$

and

$$v_6 = 0.70 - v_3 - v_4 - v_5$$

$$\phi = (0.98 - v_3 - v_5 - v_4)^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + (0.70 - v_3 - v_4 - v_5)^2$$

For ϕ to be a minimum, we have

$$\frac{\partial\phi}{\partial v_2} = 2(0.98 - v_2 - v_3 - v_4)(-1) + 2v_2 = 0$$

$$\frac{\partial\phi}{\partial v_3} = 2(0.98 - v_2 - v_3 - v_4)(-1) + 2v_3 + 2(0.70 - v_3 - v_4 - v_5)(-1) = 0$$

$$\frac{\partial\phi}{\partial v_4} = 2(0.98 - v_2 - v_3 - v_4)(-1) + 2v_4 + 2(0.70 - v_3 - v_4 - v_5)(-1) = 0$$

$$\frac{\partial\phi}{\partial v_5} = 2v_5 + 2(0.70 - v_3 - v_4 - v_5)(-1) = 0$$

By clearing and rearranging, we have

$$2v_2 + v_3 + v_4 = 0.98 \quad \text{...}(c)$$
$$v_2 + 3v_3 + 2v_4 + v_5 = 1.68 \quad \text{...}(d)$$
$$v_2 + 2v_3 + 3v_4 + v_5 = 1.68 \quad \text{...}(e)$$
$$v_3 + v_4 + 2v_5 = 0.70 \quad \text{...}(f)$$

Subtracting Eq. (*d*) from Eq. (*e*), we get

$$-v_3 + v_4 = 0 \text{ or } v_3 = v_4$$

Replacing v_4 by v_3 in Eq. (*c*), (*d*) and (*f*) we have

$$v_2 + v_3 = 0.49 \quad \text{...}(g)$$
$$v_2 + 5v_3 + v_5 = 1.68 \quad \text{...}(h)$$
$$v_3 + v_5 = 0.35 \quad \text{...}(i)$$
$$v_5 = 0.35 - v_3$$

Substituting the value of v_5 in Eq. (*h*), we get

$$v_2 + 5v_3 + (0.35° - v_3) = 1.68$$

or

$$v_2 + 4v_3 = 1.33 \quad \text{...}(j)$$

From Eq. (g)

$$v_3 = 0.49 - v_2$$

Substituting the value of v_3 in Eq. (*j*) we get

$$v_2 + 4(0.49 - v_2) = 1.33$$
$$3v_2 = 1.96 - 1.33$$
$$v_2 = \frac{1.96 - 1.33}{3}$$
$$= \frac{0.63}{3}$$
$$= 0.21° = 12'36''$$
$$v_3 = 0.49 - 0.21$$
$$= 0.28° = v_4 = 16'48''$$
$$v_5 = 0.35 - 0.28$$
$$= 0.07° = 4'12''$$
$$v_1 = 0.98 - v_2 - v_3 - v_4$$
$$= 0.98 - 0.21 - 0.28 - 0.28$$
$$= 0.21° = 12'36''$$
$$v_6 = 0.70 - v_3 - v_4 - v_5$$
$$= 0.70 - 0.28 - 0.28 - 0.07$$
$$= 0.07° = 4'12''$$

Thus

$$\hat{\theta}_1 = \theta_1 + v_1 = 44°42'00'' + 12'36'' = \mathbf{44°54'36''}$$
$$\hat{\theta}_2 = \theta_2 + v_2 = 46°00'00'' + 12'36'' = \mathbf{46°12'36''}$$
$$\hat{\theta}_3 = \theta_3 + v_3 = 43°48'00'' + 16'48'' = \mathbf{44°04'48''}$$
$$\hat{\theta}_4 = \theta_4 + v_4 = 44°31'12'' + 16'48'' = \mathbf{44°48'00''}$$
$$\text{Sum} = 180°00'00''$$
$$\hat{\theta}_3 = \theta_3 + v_3 = \mathbf{44°04'48''}$$
$$\hat{\theta}_4 = \theta_4 + v_4 = \mathbf{44°48'00''}$$
$$\hat{\theta}_5 = \theta_5 + v_5 = 42°06'00'' + 4'12'' = \mathbf{42°10'12''}$$
$$\hat{\theta}_6 = \theta_6 + v_6 = 48°52'48'' + 4'12'' = \mathbf{48°57'00''}.$$
$$\text{Sum} = 180°00'00''.$$

Example 2.15 Fig. 2.3 depicts a level net. The observed differences in elevation are tabulated below.

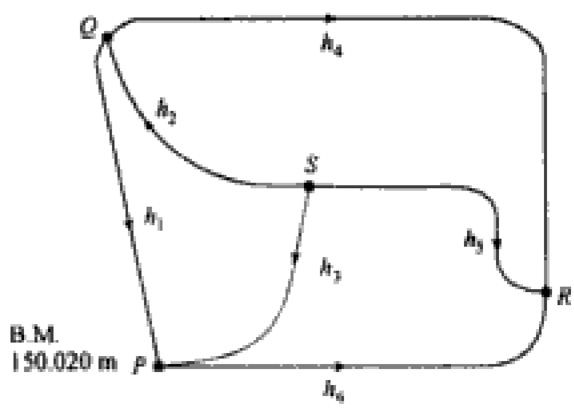

Fig. 2.3.

From (Lower point)	To (Higher point)	Diff. in elevation h (m)
Q	P	$h_1 = 6.226$
S	Q	$h_2 = 5.133$
S	P	$h_3 = 11.368$
Q	R	$h_4 = 23.521$
S	R	$h_5 = 28.639$
P	R	$h_6 = 17.275$

If all the observations are uncorrelated and of equal reliability, determine the most probable values of the elevations of Q, R, and S. The point P is a bench mark of 150.020 m elevation.

Solution: (Fig. 2.3)

Let the residuals be $v_1, v_2, ..., v_6$ for the six values of the differences in elevation. If the elevation of the point is designated by its own name, i.e., the elevation of P by P, then the six condition equations are

$$
\begin{aligned}
Q+\hat{h}_1-P &= Q+h_1+v_1-150.020=0\\
S+\hat{h}_2-Q &= S+h_2+v_2-Q=0\\
S+\hat{h}_3-P &= S+h_3+v_3-150.020=0\\
Q+\hat{h}_4-R &= Q+h_4+v_4-R=0\\
S+\hat{h}_5-R &= S+h_5+v_5-R=0\\
P+\hat{h}_6-R &= 150.020+h_6+v_6-R=0
\end{aligned}
$$

Substituting the observed values of h_1, h_2, etc. and rearranging, we get the condition equations as

$$
\begin{aligned}
v_1 &= 143.794-Q\\
v_2 &= Q-S-5.133\\
v_3 &= 138.652-S\\
v_4 &= R-Q-23.521\\
v_5 &= R-S-28.639\\
v_6 &= R-167.295
\end{aligned}
$$

From least squares criterion

$$
\begin{aligned}
\phi &= v_1^2+v_2^2+v_3^2+v_4^2+v_5^2+v_6^2 = \text{a minimum}\\
&= (143.794-Q)^2+(Q-S-5.133)^2+(138.652-S)^2\\
&\quad +(R-Q-23.521)^2+(R-S-28.639)^2+(R-167.295)^2
\end{aligned}
$$

To minmize ϕ

$$\frac{\partial \phi}{\partial Q} = -2\,(143.794 - Q) + 2\,(Q - S - 5.133) - 2\,(R - Q - 23.521) = 0$$

$$\frac{\partial \phi}{\partial R} = 2\,(R - Q - 23.521) + 2\,(R - S - 28.639) + 2\,(R - 167.295) = 0$$

$$\frac{\partial \phi}{\partial S} = -2\,(Q - S - 5.133) - 2\,(138.652 - S) - 2\,(R - S - 28.639) = 0$$

By clearing and collecting terms, we get

$$3Q - R - S = 125.406 \qquad \text{...}(a)$$

$$-Q + 3R - S = 219.455 \qquad \text{...}(b)$$

$$-Q - R + 3S = 104.880 \qquad \text{...}(c)$$

By subtracting Eqs. (*c*) from (*a*), we get

$$Q - S = 5.132 \qquad \text{...}(d)$$

By multiplying Eq. (*a*) by 3 and adding to Eq. (*b*)

$$2Q - S = 148.918 \text{ m} \qquad \text{...}(e)$$

By multiplying Eq. (*e*) by (– 1) and adding it to Eq. (*d*), we get

$$Q = 143.786 \text{ m}$$

and then from Eq. (*d*)

$$S = 143.786 - 5.132 = 138.654$$

Substituting the values of Q and S in Eq. (*a*) we get

$$R = 3 \times 143.786 - 138.654 - 125.406$$
$$= 167.298 \text{ m}$$

Therefore, the most probable values of the elevation of

$$\mathbf{Q = 143.786\ m}$$
$$\mathbf{R = 167.298\ m}$$
$$\mathbf{S = 138.654\ m.}$$

2.5.1 Adjustment of uncorrelated observations of unequal precision

The criterion for minimizing the sum of the squares of residuals of the observations of unequal precision or weights, is to minimize the weighted function as below.

$$\phi = \omega_1 v_1^2 + \omega_2 v_2^2 + \ldots.. + \omega_n v_n^2 = \text{a minimum}$$

$$= \sum_{i=1}^{n} \omega_i v_i^2 \qquad \text{...}(2.12)$$

The most probable value of such observations as measurement of length of a line, is the weighted arithmetic mean of the observations. Let us prove this by least squares criterion. Let the measurements be l_1, l_2, ..., l_n and the corresponding weights be $\omega_1, \omega_2, \ldots, \omega_n$, respectively. If $\hat{l}$ is the most probable value of the measured length then

			Weight
$\hat{l} - l_1$	=	v_1	ω_1
$\hat{l} - l_2$	=	v_2	ω_2
⋮	⋮	⋮	⋮
$\hat{l} - l_n$	=	v_n	ω_n

From Eq. (2.12)

$$\phi = \omega_1 v_1^2 + \omega_2 v_2^2 + \ldots + \omega_n v_n^2$$

$$= \omega_1(\hat{l}-l_1)^2 + \omega_2(\hat{l}-l_2) + ... + \omega_n(\hat{l}-l_n)^2 \quad ...(2.13)$$

To minimize Eq. (2.13)

$$\frac{\partial\phi}{\delta\hat{l}} = 2\omega_1(\hat{l}-l_1) + 2\omega_2(\hat{l}-l_2) + ... + 2\omega_n(\hat{l}-l_n) = 0$$

$$(\omega_1 + \omega_2 + ... + \omega_n)\hat{l} = \omega_1 l_1 + \omega_2 l_2 + ... + \omega_n l_n$$

$$\hat{l} = \frac{\omega_1 l_1 + \omega_2 l_2 + ... + \omega_n l_n}{\omega_1 + \omega_2 + ... + \omega_n}$$

= weighted arithmetic mean of the observations.

ILLUSTRATIVE EXAMPLES

Example 2.16 An angle *A* was measured by different observers and the folowing are the values:

Angle *A*	Weight
65.503°	3
65.497°	2
65.500°	2
65.506°	4
65.503°	2

Determine the most probable value of the angle.

Solution:

If the residuals of the observations are v_1, v_2, ... v_5 then from least squares criterion the quantity to be minimized is

$$\phi = \omega_1 v_1^2 + \omega_2 v_2^2 + \omega_3 v_3^2 + \omega_4 v_4^2 + \omega_5 v_5^2$$

If the most probable value of the angle is *A* then

$$\phi = 3(A-65.503)^2 + 2(A-65.497)^2 + 2(A-65.500)^2 + 4(A-65.506)^2 + 2(A-65.503)^2$$

To minimize *A*.

$$\frac{\partial\phi}{\partial A} = 2\times3(A-65.503) + 2\times2(A-65.497) + 2\times2(A-65.500) + 2\times4(A-65.506) + 2\times2(A-65.503) = 0$$

$$13A = 3\times65.503 + 2\times65.497 + 2\times65.500 + 4\times65.506 + 2\times65.503$$

$$= 851.533$$

$$A = \frac{851.533}{13}$$

$$= \mathbf{65.503°}.$$

Example 2.17 *Find the least squares estimate of the distance l from the following observations*

Distance (l)	Weight (w)
$2l$ = 292.500 m	1
$3l$ = 438.690 m	2
$4l$ = 585.140 m	3

Solution:

Let the residuals be v_1, v_2, and v_3 and the least squares estimate of the distance be $\hat{l}$ then

$$\begin{aligned}\phi &= \omega_1 l_1^2 + \omega_2 l_2^2 + \omega_3 l_3^2 \\ &= 1 \times (2\hat{l} - 292.500)^2 + 2 \times (3\hat{l} - 438.690)^2 + 3 \times (4\hat{l} - 585.140)^2\end{aligned}$$

To minimize ϕ

$$\begin{aligned}\frac{\partial \phi}{\partial \hat{l}} &= 1 \times (2\hat{l} - 292.500) \times 2 + 2 \times (3\hat{l} - 438.690) \times 3 + 3 \times (4\hat{l} - 585.140)^2 \times 4 = 0 \\ 70\hat{l} &= 10238.82 \\ &= \frac{10238.82}{70} \\ &= \mathbf{146.269\ m}.\end{aligned}$$

2.5.2 Adjustment of indirect observations

In the adjustment of the indirect observations by least squares principle the following may be noted:

(*i*) The condition equations include both observations and unknown parameters (and the constants whenever necessary).

(*ii*) The number of condition equations is equal to the number of observations.

(*iii*) Each condition equation contains only one observation with unit coefficient.

Let us take the example of condition Eq. (2.5) in measurement of a distance, and write it as

$$v_i - \hat{l} = -l_i$$

or

$$\begin{aligned}v_1 - \hat{l} &= -l_1 \\ v_2 - \hat{l} &= -l_2 \\ v_3 - \hat{l} &= -l_3\end{aligned}$$

In matrix notation the above equations can be written as

$$v + \boldsymbol{BX} = \boldsymbol{f} \qquad ...(2.14)$$

where

$$\text{Column vector } \boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\text{Column vector } \boldsymbol{B} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\text{Column vector } \boldsymbol{f} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

and row vector $\boldsymbol{X}$ is of unknown quantities.

Eq. (2.14) is the general form of the condition equations for the adjustment of indirect observations by least squares criterion.

Let there be n condition equations with m unknowns $x_1, x_2, ..., x_m$, then

$$\begin{aligned}
v_1 + a_{11}x_1 + a_{12}x_2 + ... + a_{1m}x_m &= d_1 - l_1 = f_1 \\
v_2 + a_{12}x_1 + a_{22}x_2 + ... + a_{2m}x_m &= d_2 - l_2 = f_2 \\
\vdots \quad \vdots \quad \vdots \qquad\quad \vdots \quad & \quad \vdots \quad\quad \vdots \\
v_n + a_{n1}x_1 + a_{n2}x_2 + ... + a_{nm}x_m &= d_n - l_n = f_n
\end{aligned} \qquad ...(2.15)$$

where, $v_1 v_2, ..., V_n$ = the residuals,

$a_{11}, a_{12}, ..., a_{mn}$ = the coefficients of the unknown quantities,

$x_1, x_2, ..., x_m$ = the unknown quantities,

$d_1, d_2, ..., d_n$ = the numerical constants (for linear condition equations),

$l_1, l_2, ..., l_n$ = the numerical values of the n observations, and

$f_1 f_2, ... f_n$ = the numerical constant terms on the right-hand side of equations.

Eq. (2.15) in matrix form is written as

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & ..... & a_{1m} \\ a_{21} & a_{22} & .... & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & ..... & a_{nm} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} \qquad ...(2.16)$$

In matrix notation Eq. (2.16) is

$$\underset{(n,1)}{\mathbf{v}} + \underset{(n,m)}{\mathbf{B}} - \underset{m,1}{\mathbf{x}} = \underset{(n,1)}{\mathbf{d}} - \underset{(n,1)}{\mathbf{l}} = \underset{(n,1)}{\mathbf{f}} \qquad ...(2.17)$$

From the least squares criterion what we get is a set of normal equations. These normal equations have the following properties:

(*i*) The number of normal equations is equal to the number of unknowns.

(*ii*) They are formed by multiplying each condition equation by the coefficient of the unknown quantities whose normal equation is to be found and by adding the equations thus found.

(*iii*) If the coefficients of the unknown quantities in the normal equations are written in matrix form, the elements of i^{th} row are equal to the elements of i^{th} column.

(*iv*) The matrix of coefficients of the unknown quantities in the normal equations, is a symmetric matrix.

Let us frame the normal equations directly for the following condition equations having three unknowns x_1, x_2, and x_3.

$$\begin{aligned}
a_1x_1 + b_1x_2 + c_1x_3 - d_1 &= 0 \\
a_2x_1 + b_2x_2 + c_2x_3 - d_3 &= 0 \\
\vdots \qquad \vdots \qquad \vdots \quad \vdots \quad & \vdots \\
a_nx_1 + b_nx_2 + c_nx_3 - d_n &= 0
\end{aligned}$$

From (*ii*) above, the normal equation in x_1

$$\{a_1(a_1x_1 + b_1x_2 + c_1x_3) - a_1d_1\} + \{a_2(a_2x_1 + b_2x_2 + c_2x_3) - a_2d_2\} + ...$$

$$+\{a_n(a_nx_1 + b_nx_2 + c_nx_3) - a_nd_n\} = 0$$

or

$$(\Sigma a^2)x_1 + (\Sigma ab)x_2 + (\Sigma ac)x_3 - \Sigma ad = 0 \qquad ...(2.18)$$

The normal equation in x_2

$$\{b_1(a_1x_1+b_1x_2+c_1x_3)-b_1d_1\}+\{b_2(a_2x_1+b_2x_2+c_2x_3)-b_2d_2\}+\ldots$$
$$+\{b_n(a_nx_1+b_nx_2+c_nx_3)-b_nd_n\}=0$$

or
$$(\Sigma ab)x_1+(\Sigma b^2)x_2+(\Sigma bc)x_3-\Sigma bd=0 \qquad \ldots(2.19)$$

The normal equation in x_3

$$\{c_1(a_1x_1+b_1x_2+c_1x_3)-c_1d_1\}+\{c_2(a_2x_1+b_2x_2+c_2x_3)-c_2d_2\}+\ldots$$
$$+\{c_n(a_nx+b_nx_2+c_nx_3)-c_nd_n\}=0$$

or
$$(\Sigma ac)x_1+(\Sigma bc)x_2+(\Sigma c^2)x_3-\Sigma cd=0 \qquad \ldots(2.20)$$

Writing again the Eqs. (2.18), (2.19), and (2.20) in matrix form, we get

$$\begin{bmatrix}\Sigma a^2 & \Sigma ab & \Sigma ac\\ \Sigma ab & \Sigma b^2 & \Sigma bc\\ \Sigma ac & \Sigma bc & \Sigma c^2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}-\begin{bmatrix}\Sigma ad\\ \Sigma bd\\ \Sigma cd\end{bmatrix}=0 \qquad \ldots(2.21)$$

In matrix notation the equations are

$$CX=D$$

where C = the coefficient matrix of normal equations,
X = the column vector of unknown quantities, and
D = the column vector of constants.

On examining the coefficient matrix in Eq. (2.21), we find that the elements of first row are same as of first column. Similarly the elements of second row are same as in second column, and so on. Also this matrix is symmetrical. Thus, the diagonal elements and the elements above or below the diagonals are only required to be computed. This saves computing time in the case of large system of equations having large number of unknowns.

When the condition equations are of different weights, the condition equations are also multiplied by their respective weights. This is explained in Example (2.18).

ILLUSTRATIVE EXAMPLE

Example 2.18 The following are the condition equations of different weights, involving the unknowns x, y, and z. Construct the normal equations for x, y, and z.

$$4x+2y+z-11=0 \qquad \text{weight}=3$$
$$3x+3y+2z-9=0 \qquad \text{weight}=2$$
$$5x+y+3z-16=0 \qquad \text{weight}=4$$

Solution:

To get the normal equation for x, multiply the first equation by its weight 3 and the coefficient 4 of x, the second equation by its weight 2 and the coefficient 3 of x, and the third equation by its weight 4 and the coefficient 5 of x, and add the equations obtained after multiplication. Thus,

$$(3\times4)(4x+2y+z-11)+(2\times3)(3x+3y+2z-9)+(4\times5)(5x+y+3z-16)=0 \qquad \ldots(a)$$

To get the normal equation for y, multiply the first equation by its weight 3 and the coefficient 3 of y, the second equation by its weight 2 and the coefficient 3 of y, and the third equation by its weight 4 and the coefficient 1 of y, and add the equations obtained after multiplication. Thus

$$(3\times2)(4x+2y+z-11)+(2\times3)(3x+3y+2z-9)+(4\times1)(5x+y+3z-16)=0 \qquad \ldots(b)$$

Similarly, the normal equation for y is obtained as below.

$$(3\times1)(4x+2y+z-11)+(2\times2)(3x+3y+2z-9)+(4\times3)(5x+y+3z-16)=0 \qquad \ldots(c)$$

Rearranging and clearing the terms of the Eqs. (*a*), (*b*), and (*c*), we get the normal equations as below.

$$\begin{aligned} 166x+62y+84z &= 506 \\ 62x+34y+30z &= 184 \\ 84x+30y+47z &= 261 \end{aligned} \qquad \text{...(d)}$$

Let us see that the above normal equations are obtained by following the procedure of least squares. If v_1, v_2, and v_3 are the residuals then

$$\begin{aligned} v_1 &= 4x+2y+z-11 \quad &\text{weight} = 3 \\ v_2 &= 3x+3y+2z-9 \quad &\text{weight} = 2 \\ v_3 &= 5x+y+3z-16 \quad &\text{weight} = 4 \end{aligned}$$

The quantity to be minimized is

$$\begin{aligned} \phi &= \omega_1 v_1^2 + \omega_2 v_2^2 + \omega_3 v_3^2 \\ &= 3\times(4x+2y+z-11)^2 + 2\times(3x+3y+2z-9)^2 \\ &\quad +4\times(5x+y+3z-16)^2 \end{aligned}$$

To minimize ϕ

$$\frac{\partial\phi}{\partial x} = (2\times3)(4x+2y+z-11)\times4 + (2\times2)(3x+3y+2z-9)\times3$$
$$+(2\times4)(5x+y+3z)\times5 = 0$$

or $(3\times4)(4x+2y+z-11)+(2\times3)(3x+3y+2z-9)+(4\times5)(5x+y+3z-16)=0$

$$\frac{\partial\phi}{\partial y} = (2\times3)(4x+2y+z-11)\times2 + (2\times2)(3x+3y+2z-9)\times3$$
$$+(2\times4)(5x+y+3z-16)\times1 = 0$$

or $(3\times2)(4x+2y+z-11)+(2\times3)(3x+3y+2z-9)+(4\times1)(5x+y+3z-16)=0$

$$\frac{\partial\phi}{\partial z} = (2\times3)(4x+2y+3-11)\times1 + (2\times2)(3x+3y+2z-9)\times2$$
$$+(2\times4)(5x+y+3z-16)\times3 = 0$$

or $(3\times1)(4x+2y+z-11)+(2\times2)(3x+3y+2z-9)+(4\times3)(5x+y+3z-16)=0$

The above three equations are same as the Eq. (*a*), (*b*), and (*c*) and, therefore, they will yield the normal equations given in Eq. (*d*).

2.6 DETERMINATION OF THE MOST PROBABLE VALUES

In this section the principle of least squares will be used to determine the least squares estimates or most probable values of the observed quantities in the following cases:

(*i*) Direct observations of equal weight
(*ii*) Direct observations of unequal weight
(*iii*) Indirect observations involving unknowns of equal weight
(*iv*) Indirect observations involving unknowns of unequal weight
(*v*) Observation equations accompanied by condition equations.

2.6.1 Direct observations of equal weight

The most probable value of the directly observed quantity of observation of equal weights is the arithmetic mean of the observations. See Example (2.9).

2.6.2 Direct observations of unequal weights

The most probable value of the directly observed quantity of observations of unequal weights is the weighted arithmetic mean of the observations. See Example (2.16).

2.6.3 Indirectly observed quantities involving unknowns of equal weights

In this case the most probable values of the quantities are expressed in terms of the observed quantities, and the least squares criterion is applied to minimize the residuals. See Examples (2.12), and (2.13).

2.6.4 Indirectly observed quantities involving unknowns of unequal weights

When the indirectly observed quantities are of unequal weights, the normal equations are formed considering the weights to get the most probable values. See Examples (2.17) and (2.18).

2.6.5 Observation equations accompanied by condition equations

When the observation equations are accompanied by one or more condition equations, the latter may be reduced to an observation equation which will eliminate one of the unknowns. The normal equations are then formed for the remaining unknowns to determine the most probable values. Examples (2.10) and (2.14) are the examples showing that how the condition equation is used to eliminate one of the unknowns.

There is also another method, known as the *method of correlates* to eliminate the condition equations. However, the use of condition equations in eliminating some of unknowns discussed above, is suitable for simple cases while the method of correlates is suitable for complicated cases.

2.6.6 Method of correlates

Correlates or correlatives are the unknown multipliers (independent constants) used for finding the most probable values of unknown parameters. The number of correlates is equal to the number of condition equations, excluding the one imposed by the least squares principle, i.e., the sum of squares of residuals is to be minimum.

Let at a station four angles A, B, C and D were measured closing the horizon. The weights of the angles is respectively $\omega_1, \omega_2, \omega_3$, and ω_4. If E is the residual error then

$$(A+B+C+D)-360^\circ = E$$

$$\text{total correction} = -E \qquad \dots(2.22)$$

If e_1, e_2, e_3, and e_4 are the corrections, and the most probable values of the angles be $\hat{A}, \hat{B}, \hat{C}$, and $\hat{D}$, respectively, then

$$\begin{aligned} A+e_1 &= \hat{A} \\ B+e_2 &= \hat{B} \\ C+e_3 &= \hat{C} \\ D+e_4 &= \hat{D} \end{aligned} \qquad \dots(2.23)$$

and $$A+B+C+D+(e_1+e_2+e_3+e_4) = 360^\circ$$

$$e_1+e_2+e_3+e_4 = 360^\circ-(A+B+C+D)$$

or $$e = e_1+e_2+e_3+e_4 = -E \qquad \dots(2.24)$$

From the least squares criterion

$$\phi = \omega_1 e_1^2+\omega_2 e_2^2+\omega_3 e_3^2+\omega_4 e_4^2 = \text{a minimum} \qquad \dots(2.25)$$

Thus we have two conditions Eqs. (2.24) and (2.25). Differentiating these equations, we get

$$\partial e = \partial e_1+\partial e_2+\partial e_3+\partial e_4 = 0 \qquad \dots(2.26)$$

$$\partial\phi = \omega_1 e_1\partial e_1+\omega_2 e_2\partial e_2+\omega_3 e_3\partial e_3+\omega_4 e_4\partial e_4 = 0 \qquad \dots(2.27)$$

Multiply Eq. (2.26) by a correlative $-\lambda$, and add the result to Eq. (2.27). Thus, we have

$$-\lambda(\partial e_1+\partial e_2+\partial e_3+\partial e_4)+\omega_1 e_1\partial e_1+\omega_2 e_2\partial e_2+\omega_3 e_3\partial e_3+\omega_4 e_4\partial e_4 = 0$$

or $$(\omega_1 e_1-\lambda)\partial e_1+(\omega_2 e_2-\lambda)\partial e_2+(\omega_3 e_3-\lambda)\partial e_3+(\omega_4 e_4-\lambda)\partial e_4 = 0 \qquad \dots(2.28)$$

Since $\partial e_1, \partial e_2, \partial e_3$ and ∂e_4 are definite quantities and are independent of each other, their coefficients must vanish independently. Thus

$$\begin{aligned}\omega_1 e_1 - \lambda = 0 \text{ or } e_1 &= \frac{\lambda}{\omega_1} \\ \omega_2 e_2 - \lambda = 0 \text{ or } e_2 &= \frac{\lambda}{\omega_2} \\ \omega_3 e_3 - \lambda = 0 \text{ or } e_3 &= \frac{\lambda}{\omega_3} \\ \omega_4 e_4 - \lambda = 0 \text{ or } e_4 &= \frac{\lambda}{\omega_4}\end{aligned} \qquad \text{...(2.29)}$$

Substituting the values of e_1, e_2, etc., in Eq. (2.24), we get

$$\frac{\lambda}{\omega_1} + \frac{\lambda}{\omega_2} + \frac{\lambda}{\omega_3} + \frac{\lambda}{\omega_4} = -E$$

or

$$\lambda = \frac{-E}{\dfrac{1}{\omega_1} + \dfrac{1}{\omega_2} + \dfrac{1}{\omega_3} + \dfrac{1}{\omega_4}} \qquad \text{...(2.30)}$$

From Eq. (2.30) we get the value of λ, and substituting the value of λ in Eq. (2.29) the values of e_1, e_2, e_3, and e_4 are computed. Substituting these values in Eq. (2.23), we get the most probable values $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$, respectively.

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Example 2.19 Adjust the angle of a triangle *ABC* from the following data. Use method of correlates.

$$\begin{aligned}\angle A &= 86°35'11.1'' \quad \text{weight} = 2 \\ \angle B &= 42°15'17.0'' \quad \text{weight} = 1 \\ \angle C &= 51°09'34.0'' \quad \text{weight} = 3\end{aligned}$$

Solution:

In a triangle sum of all the angles should be 180°. From observed angles, we have

$$(A + B + C) - 180° = \text{total error}$$

$$(86°35'11.1'' + 42°15'17.0'' + 51°09'34.0'') - 180° = 2.1''$$

$$\text{Total error} = 2.1''$$

$$\text{Total correction} = -2.1''$$

Let the corrections to *A*, *B*, and *C* be e_1, e_2, and e_3, respectively, then

$$e = e_1 + e_2 + e_3 = -2.1'' \qquad \text{...}(a)$$

Also from least squares principle,

$$\phi = \omega_1 e_1^2 + \omega_2 e_2^2 + \omega_3 e_3^2 = \text{a minimum} \qquad \text{...}(b)$$

Differentiating (*a*) and (*b*), we get

$$\partial e = \partial e_1 + \partial e_2 + \partial e_3 = 0 \qquad \text{...}(c)$$

$$\partial \phi = \omega_1 e_1 de_1 + \omega_2 e_2 \partial e_2 + \omega_3 e_3 \partial e_3 = 0 \qquad \text{...}(d)$$

Multiplying (c) by $-\lambda$, and adding the result to (d), we have

$$-\lambda(\partial e_1 + \partial e_2 + \partial e_3) + \omega_1 e_1 \partial e_1 + \omega_2 e_2 \partial e_2 + \omega_3 e_3 \partial e_3 = 0$$

or $$(\omega_1 e_1 - \lambda)\partial e_1 + (\omega_2 e_2 - \lambda)\partial e_2 + (\omega_3 e_3 - \lambda)\partial e_3 = 0$$

$$\omega_1 e_1 - \lambda = 0 \quad \text{or} \quad e_1 = \frac{\lambda}{\omega_1}$$

$$\omega_2 e_2 - \lambda = 0 \quad \text{or} \quad e_2 = \frac{\lambda}{\omega_2} \qquad \text{...}(e)$$

$$\omega_3 e_3 - \lambda = 0 \quad \text{or} \quad e_3 = \frac{\lambda}{\omega_3}$$

Substituting the values of e_1, e_2, and e_3 in (a), we get

$$\frac{\lambda}{\omega_1} + \frac{\lambda}{\omega_2} + \frac{\lambda}{\omega_3} = -2.1$$

or $$\lambda = \frac{-2.1}{\frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3}}$$

$$= \frac{-2.1}{\frac{1}{2} + \frac{1}{1} + \frac{1}{3}}$$

$$= -1.15''.$$

From Eq. (e)

$$e_1 = \frac{-1.15}{2} = -0.57''$$

$$e_2 = \frac{-1.15}{1} = -1.15''$$

$$e_3 = \frac{-1.15}{3} = -0.38$$

$$\text{Sum} = -2.10''$$

The most probable values of

$\angle A = 86°35'11.1'' - 0.57'' =$ **86°35'10.53"**

$\angle B = 42°15'17.0'' - 1.15'' =$ **42°15'15.85"**

$\angle C = 51°09'34.0'' - 0.38'' =$ **51°09'33.62"**

Sum = 180°00'00".

Example 2.20 The following round of angles was observed from a central station to the surrounding stations:

$\angle A = 95°43'22''$ weight = 2

$\angle B = 76°32'39''$ weight = 3

$\angle C = 103°13'44''$ weight = 2

$\angle D = 84°29'50''$ weight = 3

In addition to above, the angle ($A + B$) was observed separately as 172°16'06" with weight 2. Determine the least squares estimate of the angles A, B, C, and D.

Solution:

$$A+B+C+D = 359°59'35''$$

Total closing error $E_1 = 359°35' - 360°$

$= -25''$

Total correction $C_1 = +25''$

Also $(A+B) = A+B$

or Error $E_2 = (A+B) - A - B$

$= 172°16'06'' - 95°43'22'' - 76°32'39''$

$= +5''$

Correction $C_2 = -5''$

Let e_1, e_2, e_3, e_4 and e_5 be the corrections to the observed angles A, B, C, D, and $(A + B)$ then

$$e_1 + e_2 + e_3 + e_4 = 25'' \quad \text{...}(a)$$

$$e_5 - e_1 - e_2 = -5'' \quad \text{...}(b)$$

From the theory of least squares, we have

$$\phi = \omega_1 e_1^2 + \omega_2 e_2^2 + \omega_3 e_3^2 + \omega_4 e_4^2 + \omega_5 e_5^2 = \text{a minimum}$$

$$= 2e_1^2 + 3e_2^2 + 2e_3^2 + 3e_4^2 + 2e_5^2 \quad \text{...}(c)$$

Differentiating (a), (b) and (c), we get

$$\partial e_1 + \partial e_2 + \partial e_3 + \partial e_4 = 0 \quad \text{...}(d)$$

$$\partial e_5 - \partial e_1 - \partial e_2 = 0 \quad \text{...}(e)$$

$$\partial\phi = 2e_1\partial e_1 + 3e_2\partial e_2 + 2e_3\partial e_3 + 3e_4\partial e_4 + 2e_5\partial e_5 = 0 \quad \text{...}(f)$$

Multiplying (d) by $-\lambda_1$, (e) by $-\lambda_2$ and adding to (f), we get

$$-\lambda_1(\partial e_1 + \partial e_2 + \partial e_3 + \partial e_4) - \lambda_2(\partial e_5 - \partial e_1 - \partial e_2) + (2e_1\partial e_1 + 3e_2\partial e_2 + 2e_3\partial e_3 + 3e_4\partial e_4 + 2e_5\partial e_5) = 0$$

or $$(2e_1 - \lambda_1 + \lambda_2)\partial e_1 + (3e_2 - \lambda_1 + \lambda_2)\partial e_2 + (2e_3 - \lambda_1)\partial e_3 + (3e_4 - \lambda_1)\partial e_4 + (2e_5 - \lambda_2)\partial e_5 = 0 \quad \text{...}(g)$$

Since in (g) the coefficients of $\partial e_1, \partial e_2$, etc., must vanish independently, we get

$$2e_1 - \lambda_1 + \lambda_2 = 0 \quad \text{or } e_1 = \frac{\lambda_1 - \lambda_2}{2}$$

$$3e_2 - \lambda_1 + \lambda_2 = 0 \quad \text{or } e_2 = \frac{\lambda_1 - \lambda_2}{3}$$

$$2e_4 - \lambda_1 = 0 \quad \text{or } e_3 = \frac{\lambda_1}{2} \quad \text{...}(h)$$

$$3e_4 - \lambda_1 = 0 \quad \text{or } e_4 = \frac{\lambda_1}{3}$$

$$2e_5 - \lambda_2 = 0 \quad \text{or } e_5 = \frac{\lambda_2}{2}$$

Substituting the values of e_1, e_2, etc., in (a) and (b), we get

$$\frac{\lambda_1 - \lambda_2}{2} + \frac{\lambda_1 - \lambda_2}{3} + \frac{\lambda_1}{2} + \frac{\lambda_1}{3} = 25'' \quad \text{...}(i)$$

$$\frac{\lambda_2}{2} - \frac{\lambda_1 - \lambda_2}{2} - \frac{\lambda_1 - \lambda_2}{3} = -5'' \quad \text{...}(j)$$

Clearing and combining the terms in (i) and (j), we get

$$2\lambda_1 - \lambda_2 = 30 \quad \text{...}(k)$$

$$5\lambda_1 - 8\lambda_2 = 30 \quad \text{...}(l)$$

By solving the Eqs. (*k*) and (*l*), we get

$$\lambda_1 = \frac{210}{11}$$

$$\lambda_2 = \frac{90}{11}$$

Substituting the values of λ_1 and λ_2 in equation (*h*), we get

$$e_1 = \frac{210-90}{22} = 3.64''$$

$$e_2 = \frac{210-90}{33} = 5.45''$$

$$e_3 = \frac{210}{22} = 9.55''$$

$$e_4 = \frac{210}{33} = 6.36''$$

$$e_5 = \frac{90}{22} = 4.09''$$

Check: From (*a*)

$$3.64''+5.45''+9.55''+6.36'' = 25''$$

From (*b*)

$$4.09''-3.64''-5.45'' = -5''$$

Thus the least squares estimates of the angles are

$$A+e_1 = 95°43'22''+3.64'' = \mathbf{95°43'25.64''}$$
$$B+e_2 = 76°32'39''+5.45'' = \mathbf{76°32'44.45''}$$
$$C+e_3 = 103°13'44''+9.55'' = \mathbf{103°13'53.55''}$$
$$D+e_4 = 84°29'50''+6.36'' = \mathbf{84°29'56.36''}.$$
$$\text{Sum} = 360°00'00.00''.$$

Following check can also be applied.

$$(A+B)+e_5 = (A+e_1)+(B+e_2)$$
$$176°16'06''+4.09'' = 95°43'25.64''+76°32'44.45''$$
$$= 176°16'10''\text{(O.K.)}$$

Example 2.21 The following is the result of the levelling operations of a closed circuit *ABCDA*. The levelling was started from *A*.

Lower point	Higher point	Difference in elevation (m)	Weight
A	*B*	$h_1 = 8.164$	2
B	*C*	$h_2 = 6.284$	2
C	*D*	$h_3 = 5.626$	3
A	*D*	$h_4 = 19.964$	3

Determine the most probable values of the heights of *B*, *C*, and *D* above *A* by the method of correlates.

Solution:

In a closed level net

$$h_1 + h_2 + h_3 = h_4 \qquad \ldots(a)$$

Error of closure

$$= h_1 + h_2 + h_3 - h_4$$
$$= 20.074 - 19.964$$
$$= 0.11 \text{ m}$$

Let e_1, e_2, e_3, and e_4 be the corrections to h_1, h_2, h_3, and h_4 then we have

$$e_1 + e_2 + e_3 - e_4 = -0.11 \quad \text{...}(b)$$

Also from least squares criterion, the quantity to be minimized is

$$\phi = \omega_1 e_1^2 + \omega_2 e_2^2 + \omega_3 e_3^2 + \omega_4 e_4^2 \quad \text{...}(c)$$

Differentiating (*b*) and (*c*), we get

$$\partial e_1 + \partial e_2 + \partial e_3 - \partial e_4 = 0 \quad \text{...}(d)$$

$$\partial\phi = 2\omega_1 e_1 \partial e_1 + 2\omega_2 e_2 \partial e_2 + 2\omega_3 e_3 \partial e_3 + 2\omega_4 e_4 \partial e_4 = 0 \quad \text{...}(e)$$

Multiplying (*d*) by $-\lambda$ and adding the result to (*e*), we get

$$-\lambda(\partial e_1 + \partial e_2 + \partial e_3 - \partial e_4) + \omega_1 e_1 \partial e_1 + \omega_2 e_2 \partial e_2 + \omega_3 e_3 \partial e_3 + \omega_4 e_4 \partial e_4 = 0$$

or $$(\omega_1 e_1 - \lambda)\partial e_1 + (\omega_2 e_2 - \lambda)\partial e_2 + (\omega_3 e_3 - \lambda)\partial e_3 + (\omega_4 e_4 + \lambda)\partial e_4 = 0$$

Since the coefficients of $\partial e_1, \partial e_2$, etc., must vanish independently, we get

$$\left.\begin{aligned} \omega_1 e_1 - \lambda &= 0 \quad \text{or} \quad e_1 = \frac{\lambda}{2} \\ \omega_2 e_2 - \lambda &= 0 \quad \text{or} \quad e_2 = \frac{\lambda}{2} \\ \omega_3 e_3 - \lambda &= 0 \quad \text{or} \quad e_3 = \frac{\lambda}{3} \\ \omega_4 e_4 + \lambda &= 0 \quad \text{or} \quad e_4 = -\frac{\lambda}{3} \end{aligned}\right\} \quad \text{...}(f)$$

Substituting the values of e_1, e_2, e_3, and e_4 form (*f*) into (*b*), we get

$$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{3} + \frac{\lambda}{3} = -0.11$$

$$\lambda = -0.11 \times \frac{3}{5}$$

$$= -\frac{0.33}{5}$$

Hence from (*f*)

$$e_1 = \frac{1}{2} \times \left(-\frac{0.33}{5}\right) = -0.033 \text{ m}$$

$$e_2 = \frac{1}{2} \times \left(-\frac{0.33}{5}\right) = -0.033 \text{ m}$$

$$e_3 = \frac{1}{3} \times \left(-\frac{0.33}{5}\right) = -0.022 \text{ m}$$

$$e_4 = -\frac{1}{3} \times \left(-\frac{0.33}{5}\right) = +0.022 \text{ m}$$

Thus, the most probable values of the differences in elevations are

$$h_1 + e_1 = 8.164 - 0.033 = \mathbf{8.131\ m}$$

$$h_2 + e_2 = 6.284 - 0.033 = \mathbf{6.251\ m}$$

$$h_3 + e_3 = 5.626 - 0.022 = \mathbf{5.604\ m}$$

$$h_4 + e_4 = 19.964 + 0.022 = \mathbf{19.986\ m.}$$

Check: Substitute the above most probable values in (*a*) and check that L.H.S. value is equal to R.H.S. value, i.e.,

$$8.131 + 6.251 + 5.604 = 19.986 \text{ (O.K.)}$$

2.6.7 Alternative methods

There are a few alternative methods to determine the most probable values of the quantities and they have been discussed below.

Adjustment of closing error

Whenever measurements are taken in the field, it is always necessary to check for the closing error, if any. The closing error should be distributed to the observed quantities giving proper weightage to the observations. The weightage of the observation may be in the form of weight, probable error, number of observations or length of a level line. To distribute the errors based on the criterion of weightage, a few rules have already been given in Sec. 2.3.2. The following Examples (2.22 to 2.25) explain the distribution of closing error based on weightage of the observed quantities. The values of the quantities so obtained are treated as the most probable values.

Method of differences

Direct method of forming the normal equations for the observed quantities is suitable for simple cases. The method becomes laborious when the normal equations are in large numbers. The method of differences which involves the use of corrections, is employed so as to simplify the normal equations. A set of values is assumed for the most probable values of the unknown quantities, and the most probable corrections are determined by solving the normal equations for the corrections. The corrections so found are then added algebraically to the observed values to get the most probable values of the quantities. In this method, the observation equations are replaced by the equations in terms of the corrections to express the discrepancy between the observed results and those given by the assumed values. The method is explained in the Examples (2.26 and 2.27).

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Example 2.22 Determine the adjusted values of the following observed angles closing the horizon.

$$\angle A = 108°20'40'' \quad \text{weight} = 4$$
$$\angle B = 94°30'15'' \quad \text{weight} = 1$$
$$\angle C = 58°12'02'' \quad \text{weight} = 2$$
$$\angle D = 98°57'01'' \quad \text{weight} = 3$$

Solution:

$$A + B + C + D = 360°00'04''$$
$$\text{Total closing error} = 360°00'04'' - 360°$$
$$= +4''$$
$$\text{Total correction} = -4''$$

From rule (*iv*) in Sec. 2.3.2, the total correction to the angles is applied in inverse proportion to the weights of the angles. If c_1, c_2, c_3, and c_4 are the corrections to A, B, C, and D, respectively, then

$$c_1 + c_2 + c_3 + c_4 = -4'' \quad \text{...}(a)$$

and

$$c_1 : c_2 : c_3 : c_4 = \frac{1}{4} : \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$$
$$= 1 : 4 : 2 : \frac{4}{3} \quad \text{...}(b)$$

From (*b*)

$$c_2 = 4c_1$$
$$c_3 = 2c_1 \quad \text{...}(c)$$
$$c_4 = \frac{4}{3}c_1$$

Substituting the values of c_2, c_3 and c_4 in (a), we get

$$c_1 + 4c_1 + 2c_1 + \frac{4}{3}c_1 = -4''$$

or

$$c_1 = -0.48''$$

From (c), we get

$$c_2 = 4(-0.48) = -1.92''$$
$$c_3 = 2(-0.48) = -96''$$
$$c_4 = \frac{4}{3}(-0.48) = 0.64''$$

Hence the adjusted angles are

$A + c_1 = 108°20'46'' - 0.48'' =$ **108°20′45.52″**
$B + c_2 = 94°30'15'' - 1.92'' =$ **94°30′13.08″**
$C + c_3 = 58°12'02'' - 0.96'' =$ **58°12′1.04″**
$D + c_4 = 98°57'01'' - 0.64'' =$ **98°57′0.36″**.
Sum = 360°00′00.00″.

Example 2.23 An angle A was measured by different persons and the following values were obtained:

Angle	Number of observations
45°20′ 40″	2
46°20′ 20″	3
46°20′ 30″	3
46°20′ 50″	4
46°20′ 40″	3

What is the most probable value of the angle?

Solution:

In this case the number of observations are taken as the weight of an observation from rule (i) in Sec. 2.3.2. Hence the most probable value of the angle will be weighted arithmetic mean.

$$A = \frac{\omega_1 A_1 + \omega_2 A_2 + \omega_3 A_3 + \omega_4 A_4 + \omega_5 A_5}{\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5}$$

$$= \frac{2 \times 46°20'40'' + 3 \times 46°20'20'' + 3 \times 46°20'30'' + 4 \times 46°20'50'' + 3 \times 46°20'40''}{2+3+3+4+3}$$

= **46°20′ 36.67″**

Example 2.24 Three angles A, B, and C were observed at a station closing the horizon. The observed value along with their probable errors of measurements, are given below.

$$\angle A = 81°12'12'' \pm 3''$$
$$\angle B = 133°48'30'' \pm 2''$$
$$\angle C = 144°59'08'' \pm 4''$$

Determine the corrected values of the angles.

Solution:

From rule (iii) in Sec. 2.3.2, the weight of an observation is inversely proportional to the square of the probable error. If the weights of the observations are ω_1, ω_2, and ω_3 then

$$\omega_1 = \frac{1}{3^2} = \frac{1}{9}$$

$$\omega_2 = \frac{1}{2^2} = \frac{1}{4}$$

$$\omega_3 = \frac{1}{4^4} = \frac{1}{16}$$

The total closing error $= A + B + C - 360°$

$= 359°59'50'' - 360°$

$= -10''$

Total correction $= +10''$

If c_1, c_2 and c_3 are the respective corrections to the angles then

$$c_1 + c_2 + c_3 = +10'' \quad \ldots (a)$$

and

$$c_1 : c_2 : c_3 = \frac{1}{\omega_1} : \frac{1}{\omega_2} : \frac{1}{\omega_3}$$

$$= 9 : 4 : 16$$

Therefore

$$c_2 = \frac{4}{9} c_1$$

$$c_3 = \frac{16}{9} c_1$$

Substituting the values of c_2 and c_3 in (*a*), we get

$$\left(1 + \frac{4}{9} + \frac{16}{9}\right) c_1 = 10''$$

$$c_1 = \frac{10 \times 9}{29}$$

$$= \frac{90}{29} = 3.10''$$

$$c_2 = \frac{4}{9} \times \frac{90}{29} = 1.38''$$

$$c_3 = \frac{16}{9} \times \frac{90}{29} = 5.52''$$

Therefore, the corrected values of the angles are

$A + c_1 = 81°12'12'' + 3.10'' =$ **81°12′15.10″**

$B + c_2 = 133°48'30'' + 1.38'' =$ **133°48′31.48″**

$C + c_3 = 144°59'08'' + 5.52'' =$ **144°59′13.52″.**

Sum $= 360°00'00.00''$.

Example 2.25 A line of levels is run from *A* (B.M. = 155.200 m) to *E* (B.M. = 153.845 m) connecting the intermediate points *B*, *C*, and *D* enroute. The observations made are as below:

Sections	*A* to *B*	*B* to *C*	*C* to *D*	*D* to *E*
Distance (km)	6	4	8	10
Observed difference in elevation (m)	– 1.455	+ 0.835	– 2.420	+ 1.835

Determine the most probable values of the elevations of *B*, *C*, and *D*.

Solution:

$$\begin{aligned}
\text{Elevation of } B &= 155.200 - 1.455 = 153.745 \text{ m} \\
\text{Elevation of } C &= 153.745 + 0.835 = 154.580 \text{ m} \\
\text{Elevation of } D &= 154.580 - 2.420 = 152.160 \text{ m} \\
\text{Elevation of } E &= 152.160 + 1.835 = 153.995 \text{ m} \\
\text{Total closing error} &= \text{observed elevation of } E - \text{B.M. value of } E \\
&= 153.995 - 153.845 \\
&= 0.150 \text{ m} \\
\text{Total correction} &= -0.150 \text{ m} \\
\text{Total distance} &= 6 + 4 + 8 + 10 = 28 \text{ km}
\end{aligned}$$

From rule (*ii*) in Sec. 2.3.2, we get

Correction to elevation of $B = \dfrac{6}{28} \times (-0.15) = -0.032$ m

Correction to elevation of $C = \dfrac{(6+4)}{28} \times (-0.15) = -0.054$ m

Correction to elevation of $D = \dfrac{(6+4+8)}{28} \times (-0.15) = -0.096$ m

Correction to elevation of $F = \dfrac{(6+4+8+10)}{28} \times (-0.15) = -0.15$ m

Therefore,

$$\begin{aligned}
\text{Elevation of } B &= 153.745 - 0.032 = \mathbf{153.713 \text{ m}} \\
\text{Elevation of } C &= 154.580 - 0.054 = \mathbf{154.526 \text{ m}} \\
\text{Elevation of } D &= 152.160 - 0.096 = \mathbf{152.064 \text{ m}} \\
\text{Elevation of } E &= 153.995 - 0.150 = \mathbf{153.845 \text{ m}}. \\
&= \text{B.M. value of } E.
\end{aligned}$$

Check:

Difference in elevation between

$$\begin{aligned}
A \text{ to } B &= 153.713 - 155.200 = -1.487 \text{ m} \\
B \text{ to } C &= 154.526 - 153.713 = +0.813 \text{ m} \\
C \text{ to } D &= 152.064 - 154.526 = -2.462 \text{ m} \\
D \text{ to } E &= 153.845 - 152.064 = +1.781 \text{ m} \\
&\text{Total} = 1.355 \text{ m}
\end{aligned}$$

Total of the differences in elevation

$$\begin{aligned}
&= \text{elevation of } E - \text{elevation of } A \\
&= 155.200 - 153.845 = 1.355 \text{ m}
\end{aligned}$$

ILLUSTRATIVE EXAMPLES

Example 2.26 Determine the most probable values of the angles *A*, *B*, and *C* from the following observed values by the method of differences.

$$\begin{aligned}
A &= 39°14°15.3'' \\
B &= 31°15'26.4'' \\
C &= 42°18'18.4'' \\
A + B &= 70°29'45.2'' \\
B + C &= 73°33'48.3''
\end{aligned}$$

Solution:

Let k_1, k_2 and k_3 be the corrections to the angles A, B, and C, respectively.

Then,

$$\begin{aligned} A &= 39°14'15.3'' + k_1 \\ B &= 31°15'26.4'' + k_2 \\ C &= 42°18'18.4'' + k_3 \\ A + B &= (39°14'15.3'' + 31°15'26.4'') + k_1 + k_2 \\ &= 70°29'41.7'' + k_1 + k_2 \\ B + C &= (31°15'26.4'' + 42°18'18.4'') + k_2 + k_3 \\ &= 73°33'44.8'' + k_2 + k_3 \end{aligned} \quad \ldots (a)$$

Equating equations (a) to respective observed values, we get

$$\begin{aligned} 39°14'1.3'' + k_1 &= 39°14'15.3'' \\ 31°15'26.4'' + k_2 &= 31°15'26.4'' \\ 42°18'18.4'' + k_3 &= 42°18'18.4'' \\ 70°29'41.7'' + k_1 + k_2 &= 70°29'45.2'' \\ 73°33'44.8'' + k_2 + k_3 &= 73°33'48.3'' \end{aligned} \quad \ldots (b)$$

The equations (b) reduce to

$$\begin{aligned} k_1 &= 0 \\ k_2 &= 0 \\ k_3 &= 0 \\ k_1 + k_2 &= 3.5 \\ k_2 + k_3 &= 3.5 \end{aligned} \quad \ldots (c)$$

The normal equation for k_1

$$\begin{aligned} k_1 &= 0 \\ k_1 + k_2 &= 3.5 \\ \hline 2k_1 + k_2 &= 3.5 \end{aligned} \quad \ldots (d)$$

The normal equation for k_2

$$\begin{aligned} k_2 &= 0 \\ k_1 + k_2 &= 3.5 \\ k_2 + k_3 &= 3.5 \\ \hline k_1 + 3k_2 + k_3 &= 7.0 \end{aligned} \quad \ldots (e)$$

The normal equation for k_3

$$\begin{aligned} k_3 &= 0 \\ k_2 + k_3 &= 3.5 \\ \hline k_2 + 2k_3 &= 3.5 \end{aligned} \quad \ldots (f)$$

The three normal equations (d), (e) and (f) are

$$\begin{aligned} 2k_1 + k_2 &= 3.5 \\ k_1 + 3k_2 + k_3 &= 7.0 \\ k_2 + 2k_3 &= 3.5 \end{aligned}$$

By solving these simultaneous equations, we get

$$\begin{aligned} k_1 &= 0.88'' \\ k_2 &= 1.75'' \\ k_3 &= 0.88'' \end{aligned}$$

Therefore, the most probable values are

$$\begin{aligned} A &= 39°14'15.3'' + 0.88'' = \mathbf{39°14'16.18''} \\ B &= 31°15'26.4'' + 1.75'' = \mathbf{31°15'28.15''} \\ C &= 42°18'18.4'' + 0.88'' = \mathbf{42°18'19.28''} \end{aligned}$$

Example 2.27 The following observations were made at one station:

Observed angles	Weight
$\alpha = 75°32'46.3''$	3
$\beta = 55°09'53.2''$	2
$\gamma = 108°09'28.8''$	2
$\alpha+\beta = 130°42'41.6''$	2
$\beta+\gamma = 163°19'22.5''$	1
$\alpha+\beta+\gamma = 238°52'9.8''$	1

What are the most probable values of $\alpha+\beta$ and γ ?

Solution:

Let k_1, k_2, and k_3 be the corrections to $\alpha+\beta$ and γ respectively. Therefore,

	Weight
$75°32'46.3''+k_1 = 75°32'46.3''$	3
$55°09'53.2''+k_2 = 55°09'53.2''$	2
$108°09'28.8''+k_3 = 108°09'28.8''$	2
$(75°32'46.3''+55°09'53.2'')+k_1+k_2 = 130°42'41.6''$	2
$(55°09'53.2''+108°09'28.8'')+k_2+k_3 = 163°19'22.5''$	1
$(75°32'46.3''+55°09'53.2''+108°09'28.8'')+k_1+k_2+k_3 = 238°52'9.8''$	1

...(*a*)

From equations (*a*), we get

	weight
$k_1 = 0$	3
$k_2 = 0$	2
$k_3 = 0$	2
$k_1+k_2 = 2.1$	2
$k_2+k_3 = 0.5$	1
$k_1+k_2+k_3 = 1.5$	1

...(*b*)

The normal equation for k_1

$$3k_1 = 0$$
$$2k_1+2k_2 = 4.2$$
$$k_1+k_2+k_3 = 1.5$$
$$6k_1+3k_2+k_3 = 5.7 \quad ...(c)$$

The normal equation for k_2

$$2k_2 = 0$$
$$2k_1+2k_2 = 4.2$$
$$k_2+k_3 = 0.5$$
$$k_1+k_2+k_3 = 1.5$$
$$3k_1+6k_2+2k_3 = 6.2 \quad ...(d)$$

The normal equation for k_3

$$2k_3 = 0$$
$$k_2+k_3 = 0.5$$
$$k_1+k_2+k_3 = 1.5$$
$$k_1+2k_2+4k_3 = 2.0 \quad ...(e)$$

The three normal equations (*c*), (*d*) and (*e*) are

$$6k_1 + 3k_2 + k_3 = 5.7$$
$$3k_1 + 6k_2 + 2k_3 = 6.2$$
$$k_1 + 2k_2 + 4k_3 = 2.0$$

Solution of the above simultaneous equations give the values

$$k_1 = 0.58''$$
$$k_2 = 0.75''$$
$$k_3 = -0.02''$$

Hence the most probable values of the angles are

$$\alpha + k_1 = 75°32'46.3'' + 0.58'' = \mathbf{75°32'46.88''}$$

$$\beta + k_2 = 55°09'53.2'' + 0.75'' = \mathbf{55°09'35.95''}$$

$$\gamma + k_3 = 108°09'28.8'' - 0.02'' = \mathbf{108°09'28.78''}.$$

2.7 TRIANGULATION ADJUSTMENTS

After the field work is completed, it is necessary to adjust the observed angles so as to satisfy the geometrical conditions of triangulation figures involved in a triangulation network. The most accurate method of adjustments is the least squares method. It requires that the entire triangulation system should be adjusted in one step as one mass. As it involves the adjustment of different types of figures with different geometric conditions, computation becomes extremely laborious in absence of digital computers with large memory. For convenience, the triangulation adjustment is divided into the following phases:

(*i*) Angle adjustment
(*ii*) Station adjustment
(*iii*) Figure adjustment.

2.7.1 Angle adjustment

In triangulation, each of the individual angles are measured several times. If all the observations are made under similar conditions by the same observer, they can be considered equally precise or of the same weight. Under different conditions of observations or when the observers differ, the measurements must be assigned weights on a sound basis. In absence of any other criterion, the rules for assigning the weightage given in Sec. 2.3.2 may be employed. A more scientific method for assigning the weightage is by using the *gauss' rule* given below:

$$\omega = \frac{n^2/2}{\Sigma(x-\bar{x})^2} \quad \text{...(2.31)}$$

where ω = the weight to be assigned to the observed quantity,
n = the number of observations,
x = the value of each observation, and
$\bar{x}$ = the arithmetic mean of observations.

If all the observations are of equal weight, the most probable value of the quantity is the arithmetic mean, and in the case of unequal weight, the most probable value is the weighted arithmetic mean (*cf.*, Sec. 2.5).

2.7.2 Station adjustment

Station adjustment involves the determination of the most probable values of two or more angles measured at a station so that they are geometrically consistent. The station adjustment is first carried out on all the stations in a figure before proceeding to the figure adjustment. There can be following three cases in station adjustments:

(*i*) Horizon closed with angles of equal weight
(*ii*) Horizon closed with angles of unequal weights
(*iii*) Several angles measured at a station individually and also in combination.

Horizon closed with angles of equal weight

When the angles are measured with equal weight at a station closing the horizion, the sum of all the angles should come to 360°. In case of any discrepancy, the corrections are applied equally to all the angles (*cf.*, Example 2.34).

Horizon closed with angles of unequal weights

When the angles measured at station closing the horizon, are of unequal weight, the most probable values are determined by distributing the closing error among the angles in inverse proportion to the weights (*cf.*, Example 2.22).

Several angles measured at a station individually and also in combination

When the angles are measured individually, and also in combination with horizon not being closed as shown in Fig. 2.4, they may be of equal precision or of unequal precision. In both the cases, the most probable values are determined by forming normal equations. The normal equations may be in terms of the angles (*cf.*, Example 2.12) or the corrections (*cf.*, Examples 2.26 and 2.27). The other method of determining the most probable values is the method of correlates (*cf.*, Example 2.20).

2.7.3 Figure adjustment

Figure adjustment is carried out to determine the most probable values of all the angles in a geometrical figure to satisfy the necessary geometrical relations involving one or more condition equations. Before taking up the figure adjustment it should be ensured that the station adjustment at all the stations in a figure, has been completed.

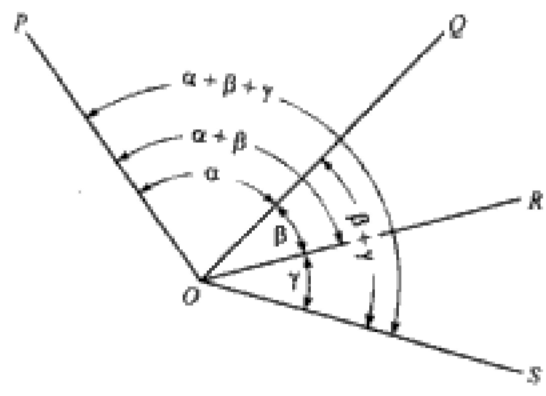


Fig. 2.4 Horizontal angles measured individually and in combination

The basic geometrical figures used in triangulation networks are triangles, geodetic quadrilaterals (barced quadrilaterals), polygons and polygons with central stations. The simplest polygon with a central station is a triangle.

It has been observed that even if the necessary geometrical conditions represented by the condition equations are satisfied, the outer sides may be discontinuous but parallel to their correct positions, leading to the figure that does not close (*cf.*, Fig. 2.10). This problem is rectified by imposing an additional condition known as *side condition*. However, such condition is relevant only for geodetic quadrilateral or polygon with central station, and not for a triangle.

The methods for the figure adjustment can be conventional or rational. The conventional method includes those of trial and error, and equal shifts. Whereas the rational method involves the use of least squares principle. Though the rational method is laborious, it is preferred in all important works as it has sound mathematical basis.

2.8 ADJUSTMENT OF A TRIANGLE

If the area of a triangle is small the triangle is treated as a plane triangle. But in case, the area of the triangle is very large, the effect of sphericity of the earth becomes significant, and the triangle is called a *geodetic triangle* or *a spherical triangle* (*cf.*, Fig. 2.5). If the length of the sides is less than 3 km, the triangle may be treated as a plane triangle.

2.8.1 Triangle (or plane triangle)

A triangle is a simplest geometric figure having three interior angles. In triangulation, all the three angles are measured independently. The sum of all the angles should be 180°. If the sum is not equal to 180°, the total error E is found, and the correction c (= − error) is applied to all the angles according to one of the following rules.

(*i*) If the angles are of equal weight, the correction is applied equally to the three angles (*cf.*, Example 2.28). Thus

$$c_1 = c_2 = c_3 = \frac{c}{3} \qquad \text{...(2.32)}$$

(*ii*) If the angles are of unequal weight, the correction to the angle is in inverse proportion to the weight of the angle (*cf.*, rule (*iv*) in Sec. 2.3.2 and Example 2.29). Thus

$$c_1 : c_2 : c_3 = \frac{1}{\omega_1} : \frac{1}{\omega_2} : \frac{1}{\omega_3} \qquad \text{...(2.33)}$$

(*iii*) If the probable errors (or standard errors) of the measurements are known, the correction to the angle is in direct proportion to the square of the probable error (or standard error) of the angle (*cf.*, rule (*iii*) in Sec. 2.3.2 and Example 2.30). Thus

$$c_1 : c_2 : c_3 = \sigma_1^2 : \sigma_2^2 : \sigma_3^2 \qquad \text{...(2.34)}$$

(*iv*) If the number of times (n) a measurement taken is known, the correction to the angle is in inverse proportion to the number of measurement (*c.f.*, rule (*i*) in Sec. 2.3.2). Thus

$$c_1 : c_2 : c_3 = \frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3} \qquad \text{...(2.35)}$$

(*v*) If the number of times (n) an observation taken is known, and also the residual errors (v) are known, the weight of measurement of the angle is determined by Gauss' rule (*cf.*, Eq. (2.31) and Example 2.31).

2.8.2 Geodetic triangle

In the case of a geodetic triangle (Fig. 2.5), the sum of the three angles is more than 180°, i.e., 180° + ε, where ε is called *spherical excess*. The spherical excess is the amount by which the theoretical sum of the three angles of a spherical triangle exceeds 180°. The value of the spherical excess is about 1″ for every 197 sq. km of area. The spherical excess ε, is calculated from the following formula:

$$\varepsilon = \frac{A_0 \times 180}{\pi R^2} \text{ degrees}$$

or

$$= \frac{A_0}{R^2 \sin 1''} \text{ seconds} \qquad \text{...(2.36)}$$

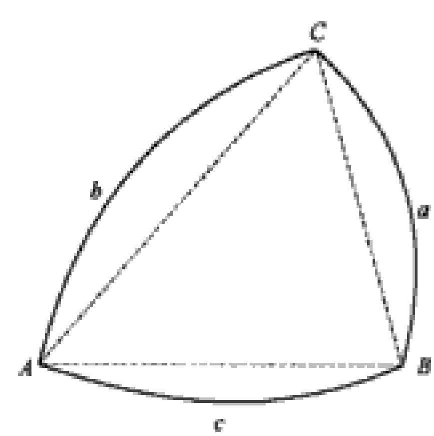


Fig. 2.5 Geodetic triangle or spherical triangle

where ε = the spherical excess

A_0 = the area of the triangle in square kilometre, and

R = the mean radius of the earth in kilometre ($\simeq$ 6370 km)

In Eq. (2.36), substituting the value of R as 6370 km, we get A_0 equal to 197 sq km for ε equal to 1″.

The sum of the three angles for a spherical triangle is given by

$$A + B + C = 180° + \varepsilon$$

Thus the total error in observation of the three angles is

$$e = (A + B + C) - (180° + \varepsilon)$$

The correction to the angles is applied as per rules given above for a plane triangle.

2.8.3 Determination of Spherical Excess

The determination of ε involves the area A_0 of the spherical triangle. The value of A_0 cannot be determined unless the angles A, B, and C are known. These angles, in turn, depend upon the spherical excess. Therefore, the following procedure is adopted to determine A_0 first and then ε.

1. Calculate A_0 assuming the spherical triangle as a plane triangle (shown by dashed lines in Fig. 2.5) as a first approximation. Thus

$$A_0 = \frac{1}{2} bc \sin A \qquad \text{... (2.37)}$$

From sine rule, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Therefore

$$b = \frac{a \sin B}{\sin A}$$

$$c = \frac{a \sin C}{\sin A}$$

Substituting the values of b and c in Eq. (2.37), we get

$$A_0 = \frac{1}{2} a^2 \frac{\sin B \sin C}{\sin A} \qquad \text{... (2.38)}$$

where a is the known side BC, and A, B, and C are the observed angles. The computed value of A_0 from Eq. (2.38) is substituted in Eq. (2.36) to get the value ε.

ILLUSTRATIVE EXAMPLES

Example 2.28 Adjust the following observed angles of a triangle:

$$\angle A = 64°12'40''$$
$$\angle B = 55°14'23''$$
$$\angle C = 60°33'21''.$$

Solution:

The three angles of a triangle should satisfy the following condition:

$$A + B + C = 180°$$

The total error = (A + B + C) − 180°

= (64°12′40″ + 55°14′23″ + 60°33′21″) − 180°

= 180°00′24″ − 180°

= +24″

The total corection = −24″

If the corrections to the individual angles are c_1, c_2, and c_3 then from Eq. (2.32), we get

$$c_1 = c_2 = c_3 = \frac{-24''}{3} = -8''$$

Therefore, the adjusted values of the angles are

$$A + c_1 = 64°12'40'' - 8' = \mathbf{64°12'32''}$$
$$B + c_2 = 55°14'23'' - 8' = \mathbf{55°14'15''}$$
$$C + c_3 = 60°33'21'' - 8' = \mathbf{60°33'13''}.$$
$$\text{Sum} = 180°00'00''.$$

Example 2.29 Adjust the angle of a triangle ABC from the following data:

$$\angle A = 64°12'40'' \quad \text{weight} = 3$$
$$\angle B = 55°14'23'' \quad \text{weight} = 2$$
$$\angle C = 60°33'21'' \quad \text{weight} = 1.$$

Solution:

The total correction (from Example 2.28)

$$c_1 + c_2 + c_3 = -24'' \quad ...(a)$$

From Eq. (2.33), the corrections to the angles will be

$$c_1 : c_2 : c_3 = \frac{1}{\omega_1} : \frac{1}{\omega_2} : \frac{1}{\omega_3}$$
$$= \frac{1}{3} : \frac{1}{2} : \frac{1}{1}$$
$$= 1 : \frac{3}{2} : 3$$

Thus

$$c_2 = \frac{3}{2} c_1$$
$$c_4 = 3\, c_1$$

Substituting the values of c_2 and c_3 in (*a*), we get

$$c_1 + \frac{3}{2} c_1 + 3c_1 = -24''$$

or

$$(2 + 3 + 6)\, c_1 = -24 \times 2 = -48''$$

Therefore,

$$c_1 = -\frac{48}{11} = -4.36''$$
$$c_2 = \frac{3}{2} \times \left(-\frac{48}{11}\right) = -6.55''$$
$$c_3 = 3 \times \left(-\frac{48}{11}\right) = -13.09''$$

Therefore, the adjusted values of the angles are

$$A + c_1 = 64°12'40'' - 4.36'' = \mathbf{64°12'35.64''}$$
$$B + c_2 = 55°14'23'' - 6.55'' = \mathbf{55°14'16.45''}$$
$$C + c_3 = 60°33'21'' - 13.09'' = \mathbf{60°33'7.91''}$$
$$\text{Sum} = 180°00'00.00''.$$

Example 2.30 Determine the most probable values of the angles *A*, *B*, and *C* of a triangle *ABC* from the following observed angles and the respective probable errors of measurements:

$$\angle A = 64°12'40'' \pm 3''$$
$$\angle B = 55°14'23'' \pm 2''$$
$$\angle C = 64°33'21'' \pm 3''.$$

Solution:

The total correction (From Example 2.28) is

$$c_1 + c_2 + c_3 = -24'' \qquad ...(a)$$

From Eq. (2.34), we have

$$c_1 : c_2 : c_3 = \sigma_1^2 : \sigma_2^2 : \sigma_3^2$$
$$= 3^2 : 2^2 : 4^2$$
$$= 9 : 4 : 16$$
$$= 1 : \frac{4}{9} : \frac{16}{9}$$

or
$$c_2 = \frac{4}{9} c_1$$
$$c_3 = \frac{16}{9} c_1$$

Substituting the values of c_2 and c_3 in (*a*), we get

$$c_1 + \frac{4}{9} c_1 + \frac{16}{9} c_1 = -24''$$
$$(9 + 4 + 16) c_1 = -24 \times 9$$

or
$$c_1 = -\frac{216}{29} = -7.45''$$

and
$$c_2 = \frac{4}{9} \times \left(-\frac{216}{29} \right) = -3.31''$$
$$c_3 = \frac{16}{9} \times \left(-\frac{216}{29} \right) = -13.24''$$

Therefore, the most probable values are

$$A + c_1 = 64°12'40'' - 7.45'' = \mathbf{64°12'32.55''}$$
$$B + c_2 = 55°14'23'' - 3.31'' = \mathbf{55°14'19.69''}$$
$$C + c_3 = 60°33'21'' - 13.24'' = \mathbf{60°33'7.76''}$$
$$\text{Sum} = 180°00'00.00''.$$

Example 2.31 The following observations were made for the three interior angles of a triangle *ABC*. What are the most probable values of the angles *A*, *B*, and *C*?

$$\angle A = 64°12'40'', 64°12'42'', 64°12'37'', 64°12'41''.$$
$$\text{Mean } \angle A = 64°12'40''$$
$$\angle B = 55°14'21'', 55°14'25'', 55°14'22'', 55°14'23'', 55°14'24''.$$
$$\text{Mean } \angle B = 55°14'23''$$
$$\angle C = 60°33'20'', 60°33'22'', 60°33'18'', 60°33'24'', 60°33'23'', 60°33'19''.$$
$$\text{Mean } \angle C = 60°33'21''.$$

Solution:

The mean values of the angles are same as the given angles in Example 2.28, therefore, the total correction to the angles is

$$c_1 + c_2 + c_3 = -24'' \qquad ...(a)$$

Since the observed values are known, and the number of observations made for each angle is also known, the weightage of the each angle can be found out from Gauss' rule given by Eq. (2.35).

$$\omega = \frac{n^2/2}{\Sigma(x - \bar{x})^2}$$

For $\angle A$

The number of observations $n = 4$

The sum of the square of residuals $\Sigma(x-\bar{x})^2$

$$= 0^2 + 2^2 + (-3)^2 + 1^2 = 14$$

Therefore $$\omega_1 = \frac{4^2/2}{14} = \frac{4}{7}$$

For $\angle B$

$$n = 5$$

$$\Sigma(x-\bar{x})^2 = (-2)^2 + 2^2 + (-1)^2 + 0^2 + 1^2 = 10$$

Therefore $$\omega_2 = \frac{5^2/2}{10} = \frac{25}{20}$$

For $\angle C$

$$n = 6$$

$$\Sigma(x-\bar{x})^2 = (-1)^2 + 1^2 + (-3)^2 + 3^2 + 2^2 + (-2)^2 = 28$$

Therefore $$\omega_3 = \frac{6^2/2}{28} = \frac{9}{14}$$

From Eq. (2.33), the corrections to the angles will be

$$c_1 : c_2 : c_3 = \frac{7}{4} : \frac{20}{25} : \frac{14}{9}$$

$$= 1 : \frac{80}{175} : \frac{56}{63}$$

Therefore $$c_2 = \frac{80}{175} c_1$$

$$c_3 = \frac{56}{63} c_1$$

From (*a*)

$$c_1 + \frac{80}{175} c_1 + \frac{56}{63} c_1 = -24''$$

$$c_1 = \frac{11025}{25865} \times (-24) = -10.23''$$

$$c_2 = \frac{80}{175} \times (-10.23) = -4.68''$$

$$c_3 = \frac{56}{63} \times (-10.23) = -0.09''$$

Therefore, the most probable values are

$$A + c_1 = 64°12'40'' - 10.23'' = \mathbf{64°12'29.77''}$$

$$B + c_2 = 55°14'23'' - 4.68'' = \mathbf{55°14'18.32''}$$

$$C + c_3 = 60°33'21'' - 9.09'' = \mathbf{60°33'11.91''}$$

$$\text{Sum} = 180°00'00.00''.$$

Example 2.32 The mean observed angles of a spherical triangle *ABC* are as follows:

$$\alpha = 55°18'24.45''\ \text{weight} = 1$$
$$\beta = 62°23'34.24''\ \text{weight} = 2$$
$$\gamma = 62°18'10.34''\ \text{weight} = 3$$

The length of the side *BC* opposite the angle α, was the also measured as 59035.6 m. If the mean radius of earth is 6370 km, calculate the most probable values of the spherical angles.

Solution:

Considering the spherical triangle as a plane triangle the total error is

$$= (\alpha + \beta + \gamma) - 180°$$
$$= (55°18'24.45'' + 62°23'34.24'' + 62°18'10.34'') - 180°$$
$$= 180°00'09.03'' - 180°$$
$$= 9.03''$$

Therefore, the sum of corrections c_1, c_2, and c_3 to the angles α, β, and γ, respectively, will be

$$c_1 + c_2 + c_3 = -9.03''$$

The correction to each angle (without considering the weights at present)

$$= \frac{1}{3} \times (-9.03)$$
$$= -3.01''$$

The corrected plane angles $(\alpha', \beta' \text{and}\ \gamma')$ will be

$$\alpha' = \alpha - 3.01'' = 55°18'24.44''$$
$$\beta' = \beta - 3.01'' = 62°23'31.23''$$
$$\gamma' = \gamma - 3.01'' = 62°18'07.33''$$
$$\text{Sum} = 180°00'00.00''$$

From Eq. (2.36), the spherical excess is given by

$$\varepsilon = \frac{A_0}{R^2 \sin 1''} \text{seconds}$$
$$= \frac{206265\, A_0}{6370^2}$$
$$= 50833 \times 10^{-3} A_0 \text{ seconds}$$

From Eq. (2.38), the area A_0 can be calculated. Thus

$$A_0 = \frac{1}{2} a^2 \frac{\sin \beta' \sin \gamma'}{\sin \alpha'}$$
$$= \frac{1}{2} \times 59035.6^2 \times \frac{\sin 62°23'31.23'' \times \sin 62°18'07.33''}{\sin 55°18'24.44''}$$
$$= 1662.8794 \text{ sq.km}$$

Therefore, $$\varepsilon = \frac{206265 \times 1662.8794}{6370^2}$$
$$= 8.45''$$

Thus the theoretical sum of the spherical angles should be

$$= 180° + \varepsilon$$
$$= 180° + 8.45'' = 180°00'08.45''$$

The observed sum of the angles $= 180°00'09.03''$

The total error $= 180°00'09.03'' - 180°00'08.45''$

$$= +0.58''$$

The total correction $= c_1 + c_2 + c_3 = -0.58''$ ... (*a*)

Considering weights of the observations, we get

$$c_1 : c_2 : c_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$$

Therefore
$$c_2 = \frac{1}{2}c_1$$
$$c_3 = \frac{1}{3}c_1$$

Substituting the values of C_2 and C_3 in (*a*), we get

$$c_1 + \frac{1}{2}c_1 + \frac{1}{3}c_1 = -0.58''$$
$$c_1 = \frac{6}{11}(-0.58) = -0.32''$$
$$c_2 = -0.16''$$
$$c_3 = -0.10''$$

Therefore, the most probable values of the spherical angles are

$$\alpha + c_1 = 55°18'24.45'' - 0.32'' = \mathbf{55°18'24.13''}$$
$$\beta + c_2 = 62°23'34.24'' - 0.16'' = \mathbf{62°23'34.08''}$$
$$\gamma + c_3 = 62°18'10.34'' - 0.10'' = \mathbf{62°18'10.24''}.$$
$$\text{Sum} = 180°00'08.45''.$$

2.8.4 Two connected triangles

Fig. 2.6 shows two connected triangles. These triangles form a quadrilateral with only one diagonal which is also observed. Let the observed angles be $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ and θ_6. In addition to these observations the angles *ABC* $(= \theta_2 + \theta_5 = \alpha)$ and *ADC* $(= \theta_3 + \theta_4 = \beta)$ are also measured.

Thus, the total number of angles measured are eight. Let us assume that these angles are corrected for station adjustment. These corrected angles must satisfy the following independent conditions:

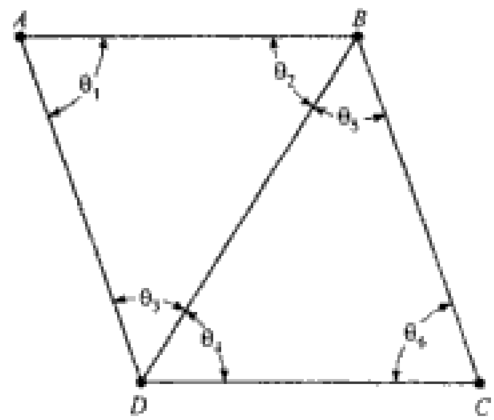


Fig. 2.6 Two connected triangles or quadrilateral with one diagonal

$$\theta_1 + \theta_2 + \theta_3 = 180°$$
$$\theta_4 + \theta_5 + \theta_6 = 180°$$
$$\theta_2 + \theta_5 = \alpha$$
$$\theta_3 + \theta_4 = \beta$$

There are eight unknowns and four equations, therefore, the four unknowns $\theta_1, \theta_6, \alpha$, and β may be expressed in terms of the dependent unknowns $\theta_2, \theta_3, \theta_4$, and θ_5. Thus

$$\theta_1 = 180° - \theta_2 + \theta_3$$
$$\theta_6 = 180° - \theta_4 + \theta_5$$
$$\alpha = \theta_2 + \theta_5$$
$$\beta = \theta_3 + \theta_4$$

From the above new observation equations, the normal equations are formed, and the most probable values of the angles can be determined by solving the normal equations (*c. f.*, Example 2.33).

2.8.5 Chain of triangles

Fig. 2.7 shows a triangulation network consisting a chain of triangles. The angles shown in the figure are adjusted for the closing errors at each station. In each triangle the angles would be adjusted for the triangular error. Thus,

$$\angle 1 + \angle 2 + \angle 3 = 180°$$
$$\angle 4 + \angle 5 + \angle 6 = 180°$$
$$\angle 7 + \angle 8 + \angle 9 = 180°$$
$$\angle 10 + \angle 11 + \angle 12 = 180°$$

The discrepancy or triangular error in each triangle is distributed equally among all the three angles if they are of equal weight. Otherwise, the error is distributed inversely proportional to the weights.

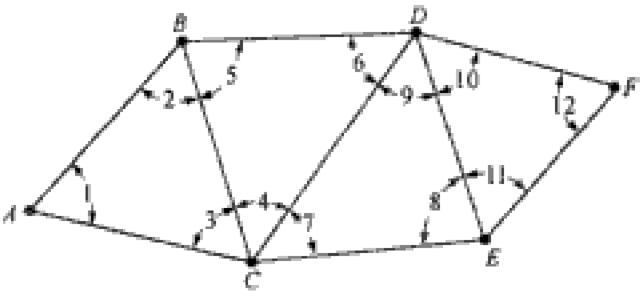


Fig. 2.7 Chain of triangles

A chain of triangles is the simplest network. It is not accurate, and therefore, not used for primary triangulation.

ILLUSTRATIVE EXAMPLES

Example 2.33 Fig. 2.8 shows two connected triangles *PQR* and *QRS*. The measured angles of equal weight, are given below. Adjust the angles.

θ_1	$= 62°18'30''$	θ_5	$= 68°12'33''$
θ_2	$= 49°29'00''$	θ_6	$= 52°28'35''$
θ_3	$= 61°33'35''$	θ_7	$= 128°16'30''$
θ_4	$= 65°57'55''$	θ_8	$= 1112°02'30''$

Solution: (Fig. 2.8)

The condition equations are

$$\theta_1 + \theta_2 + \theta_5 = 180°$$
$$\theta_3 + \theta_6 + \theta_6 = 180°$$
$$\theta_1 + \theta_4 = \theta_7$$
$$\theta_2 + \theta_3 = \theta_8 \qquad \text{... } (a)$$

Rewriting the condition equations (*a*) as below, we get

$$\theta_1 + \theta_2 = 180° - \theta_5$$
$$\theta_3 + \theta_4 = 180° - \theta_6 \qquad \text{... } (b)$$
$$\theta_1 + \theta_4 = \theta_7$$
$$\theta_2 + \theta_3 = \theta_8$$

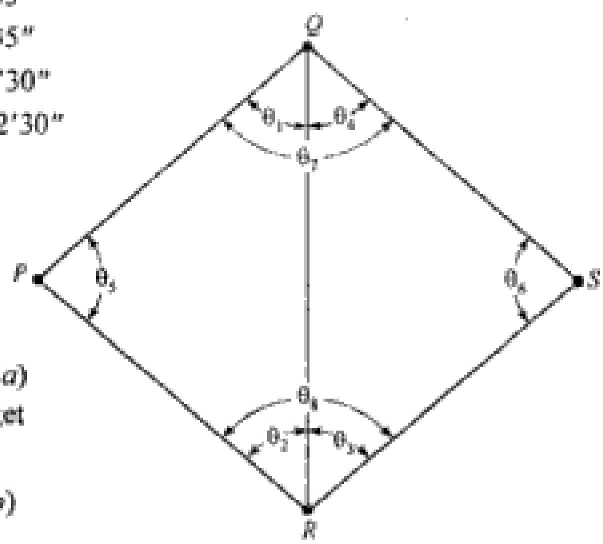


Fig. 2.8

If corrections to the angles $\theta_1, \theta_2, \theta_3$ and θ_4 be c_1, c_2, c_3, and c_4, respectively, then the equations (*b*) become

$$62°18'30'' + c_1 + 49°29'00'' + c_2 = 180° - 68°12'33''$$
$$61°33'35'' + c_3 + 65°57'55'' + c_4 = 180° - 52°28'35''$$
$$62°18'30'' + c_1 + 65°57'55'' + c_4 = 128°16'30'' \qquad \text{... } (c)$$
$$49°29'00'' + c_2 + 61°33'35'' + c_3 = 111°02'30''$$

or

$$c_1 + c_2 = -3''$$
$$c_3 + c_4 = -5''$$
$$c_1 + c_4 = -5'' \qquad \text{... } (d)$$
$$c_2 + c_3 = -5''$$

Also we have

$$\begin{aligned}\theta_1 + c_1 &= \theta_1\\ \theta_2 + c_2 &= \theta_2\\ \theta_3 + c_3 &= \theta_3\\ \theta_4 + c_4 &= \theta_4\end{aligned} \qquad \text{...}(e)$$

or

$$\begin{aligned}c_1 &= 0\\ c_2 &= 0\\ c_3 &= 0\\ c_4 &= 0\end{aligned}$$

Thus the observation equations and condition equations in terms of the corrections, are

$$\begin{aligned}c_1 &= 0\\ c_2 &= 0\\ c_3 &= 0\\ c_4 &= 0\\ c_1 + c_2 &= -3\\ c_3 + c_4 &= -5\\ c_1 + c_4 &= 5\\ c_2 + c_3 &= -5\end{aligned}$$

The normal equation for c_1

$$\begin{aligned}c_1 + (c_1 + c_2) + (c_1 + c_4) &= -3 + 5\\ 3c_1 + c_2 + c_4 &= 2\end{aligned}$$

The normal equation for c_2

$$\begin{aligned}c_2 + (c_1 + c_2) + (c_2 + c_3) &= -3 - 5\\ c_1 + 3c_2 + c_3 &= -8\end{aligned}$$

The normal equation for c_3

$$\begin{aligned}c_3 + (c_3 + c_4) + (c_2 + c_3) &= -5 - 5\\ c_2 + 3c_3 + c_4 &= -10\end{aligned}$$

The normal equation for c_4

$$\begin{aligned}c_4 + (c_3 + c_4) + (c_1 + c_4) &= 0\\ c_1 + c_3 + 3c_4 &= 0\end{aligned}$$

Writing the three normal equation below, we have

$$\begin{aligned}3c_1 + c_2 + c_4 &= 2\\ c_1 + 3c_2 + c_3 &= -8\\ c_2 + 3c_3 + c_4 &= -10\\ c_1 + c_3 + 3c_4 &= 0\end{aligned} \qquad \text{...}(f)$$

The solution of the simultaneous equations (f) is

$$\begin{aligned}c_1 &= 1.20''\\ c_2 &= -2.14''\\ c_3 &= -2.80''\\ c_4 &= 0.53''\end{aligned}$$

Therefore, the adjusted angles are:

$$\theta_1 + c_1 = 62°18'30'' + 1.20'' = \mathbf{62°18'31.20''}$$
$$\theta_2 + c_2 = 49°29'00'' - 2.14'' = \mathbf{49°28'57.86''}$$
$$\theta_3 + c_3 = 61°33'35'' - 2.80'' = \mathbf{61°33'32.20''}$$
$$\theta_4 + c_4 = 65°57'55'' + 0.53'' = \mathbf{65°57'55.53''}$$
$$\theta_5 = 180° - (\theta_1 + \theta_2) = \mathbf{68°12'30.94''}$$
$$\theta_6 = 180° - (\theta_3 + \theta_4) = \mathbf{52°28'32.27''}.$$

Example 2.34 Fig. 2.9 shows a chain of triangles. If the average observed values of the angles at each station are as follows, compute the adjusted values of the angles.

$\angle 1 = 100°25'22''$ $\quad \angle 8 = 49°02'21''$
$\angle 2 = 259°34'40''$ $\quad \angle 9 = 40°52'06''$
$\angle 3 = 38°42'35''$ $\quad \angle 10 = 209°24'24''$
$\angle 4 = 39°33'19''$ $\quad \angle 11 = 55°11'10''$
$\angle 5 = 58°29'05''$ $\quad \angle 12 = 95°24'23''$
$\angle 6 = 223°14'57''$ $\quad \angle 13 = 66°19'48''$
$\angle 7 = 274°05'36''$ $\quad \angle 14 = 293°40'06''$

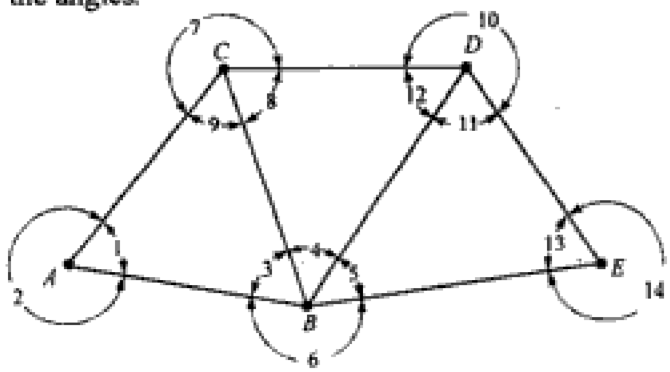


Fig. 2.9

Solution: (Fig. 2.9)

Station adjustment

At *A* $\quad \angle 1 + \angle 2 = 360°$, Error $= 360°00'02' - 360° = +2''$

Correction to each angle $= -\dfrac{2}{2} = -1''$

Corrected angles

$$\angle 1 = 100°25'22' - 1'' = 100°25'21'$$
$$\angle 2 = 259°34'40'' - 1'' = 259°34'39''$$
$$\text{Sum} = 360°00'00''.$$

At *B* $\quad \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$, Error $= 359°59'56'' - 360° = -4''$

Correction to each angle $= \dfrac{4''}{4} = +1''$

Corrected angles

$$\angle 3 = 38°42'35'' + 1'' = 38°42'36''$$
$$\angle 4 = 39°33'19'' + 1'' = 39°33'20''$$
$$\angle 5 = 58°29'05'' + 1'' = 58°29'06''$$
$$\angle 6 = 223°14'57'' + 1'' = 223°14'58''$$
$$\text{Sum} = 360°00'00''$$

At *C* $\quad \angle 7 + \angle 8 + \angle 9 = 360°$, Error $= 360°00'03'' - 360° = +3''$

Correction to each angle $= -\dfrac{3}{3} = -1''$

Corrected angles

$$\angle 7 = 274°05'36'' - 1'' = 274°05'35''$$
$$\angle 8 = 49°02'21'' - 1'' = 49°02'20''$$
$$\angle 9 = 40°42'06 - 1'' = 40°52'05''$$
$$\text{Sum} = 360°00'00''$$

At *D* $\quad \angle 10 + \angle 11 + \angle 12 = 360°$, Error $= 359°59'57'' - 360° = -3''$

Corrected angles

$$\angle 10 = 209°24'24'' + 1'' = 209°24'25''$$
$$\angle 11 = 55°11'10'' + 1'' = 55°11'11''$$
$$\angle 12 = 95°24'23'' + 1'' = 95°24'24''$$
$$\text{Sum} = 360°00'00''.$$

At E $\quad \angle 13 + \angle 14 = 360°$, Error $= 359°59'54'' - 360° = -6''$

Correction to each angle $= +\dfrac{6}{2} = +3''$

Corrected angles

$$\angle 13 = 66°19'48'' + 3'' = 66°19'51''$$
$$\angle 14 = 293°40'06'' + 3'' = 293°40'09''$$
$$\text{Sum} = 360°00'00''.$$

Figure adjustment

For ΔABC, $\angle 1 + \angle 3 + \angle 9 = 180°$, Error $= 180°00'02'' - 180° = +2''$

Correction to each angle $= -\dfrac{2}{3} = -0.67''$

Corrected angles

$$\angle 1 = 100°25'21'' - 0.69'' = \mathbf{100°25'20.33''}$$
$$\angle 3 = 38°42'36'' - 0.69'' = \mathbf{38°42'35.33''}$$
$$\angle 9 = 40°52'05'' - 0.69'' = \mathbf{40°52'04.34''}$$
$$\text{Sum} = 180°00'00.00''.$$

For ΔBCD, $\angle 4 + \angle 8 + \angle 12 = 180°$, Error $= 180°00'04'' - 180° = +4''$

Correction to each angle $= -\dfrac{4}{3} = -1.33''$

Corrected angles

$$\angle 4 = 39°33'20'' - 1.33'' = \mathbf{39°33'18.67''}$$
$$\angle 8 = 49°02'20'' - 1.33'' = \mathbf{45°02'18.66''}$$
$$\angle 12 = 95°24'24'' - 1.33'' = \mathbf{95°24'22.69''}$$
$$\text{Sum} = 180°00'00.00''.$$

For ΔBDE, $5 + 11 + 13 = 180°$, Error $= 180°00'08'' - 180 = +8''$

Correction to each angle $= -\dfrac{8}{3} = -2.67''$

Corrected angles

$$\angle 5 = 58°29'06'' - 2.67'' = \mathbf{58°29'03.34''}$$
$$\angle 11 = 55°11'11'' - 2.67'' = \mathbf{55°11'08.33''}$$
$$\angle 13 = 66°19'51'' - 2.67'' = \mathbf{66°19'48.33''}.$$
$$\text{Sum} = 180°00'00.00''.$$

The above corrected values of the angles will be out of adjustment for station adjustment, and require readjustment in a similar manner as above. After adjusting the angles again for closing error at the stations, the figure adjustment for triangular error for each triangle is carried out. The second set of the adjusted values are given below.

Corrected values after station adjustment

$$\begin{aligned}
\angle 1 &= 100°25'20.67'', & \angle 8 &= 45°02'19.33'' \\
\angle 2 &= 259°34'39.33'', & \angle 9 &= 40°52'05.00'' \\
\angle 3 &= 38°42'36.50'', & \angle 10 &= 209°24'26.33'' \\
\angle 4 &= 39°33'19.33'', & \angle 11 &= 55°11'09.67'' \\
\angle 5 &= 58°29'03.50'', & \angle 12 &= 95°24'24.00'' \\
\angle 6 &= 223°14'59.17'', & \angle 13 &= 66°19'49.67'' \\
\angle 7 &= 274°05'35.67'', & \angle 14 &= 293°40'10.33''
\end{aligned}$$

Adjusted values after figure adjustments:

$$\begin{aligned}
\angle 1 &= 100°25'19.95'', & \angle 4 &= 39°33'18.44'' \\
\angle 3 &= 38°42'35.77'', & \angle 8 &= 45°02'18.44'' \\
\angle 9 &= 40°52'04.28'', & \angle 12 &= 95°24'23.12'' \\
\text{Sum} &= 180°00'00.00'' & \text{Sum} &= 180°00'00.00''
\end{aligned}$$

$$\begin{aligned}
\angle 5 &= 58°29'02.56'' \\
\angle 11 &= 55°11'09.72'' \\
\angle 13 &= 66°19'48.72'' \\
\text{Sum} &= 180°00'00.00''.
\end{aligned}$$

After computing the second cycle, the process may be continued till the results converge.

2.9 ADJUSTMENT OF A GEODETIC QUADRILATERAL

A quadrilateral with interlacing diagonals shown in Fig. 2.10, is called a geodetic quadrilateral if there is no station at the intersection of the diagonals. It is also called a braced quadrilateral. It is the figure which is most widely used in triagulation systems.

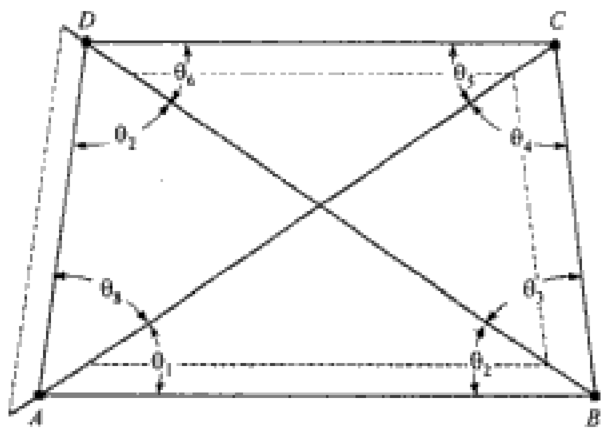


Fig. 2.10 Geodetic quadrilateral

In the geodetic quadrilateral all the eight angles $\theta_1, \theta_2, \ldots, \theta_8$ are measured. From the geometry of the figure, the following angle conditions must be satisfied:

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8 = 360°$$

$$\theta_1 + \theta_2 = \theta_5 + \theta_6 \qquad \ldots (2.39)$$

$$\theta_3 + \theta_4 = \theta_7 + \theta_8$$

Even if the above conditions have been satisfied, the quadrilateral may have the outer sides discontinuous, but parallel to their correct position, as indicated by the dashed lines. This emposes an additional condition, known as *side condition*, which is found from the fact that the product of the sines of the left-hand angles equals to that of the sines of the right-hand angles. By considering the logarithms of the sines, this condition becomes

$$\Sigma \log \sin \text{(left-hand angles)} = \Sigma \log \sin \text{(right-hand angles)} \quad \dots (2.40)$$

or $$\Sigma \log \sin \text{(odd angles)} = \Sigma \log \sin \text{(even angles)}$$

The observed angles may be adjusted by the rigorous method of least squares using correlates or by approximate method.

2.9.1 Rigorous method of least squares

Let $c_1, c_2, \ldots, c_8$ be the required corrections for the angles $\theta_1, \theta_2, \ldots, \theta_8$, respectively, shown in Fig. 2.10.

Again, let C_1, C_2, C_3, and C_4 be the total discrepancies given by each condition equations, i.e.,

$$360° - (\theta_1 + \theta_2 + \theta_3 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8) = C_1 \quad \dots (2.41)$$

$$(\theta_5 + \theta_6) - (\theta_1 + \theta_2) = C_2 \quad \dots (2.42)$$

$$(\theta_7 + \theta_8) - (\theta_3 + \theta_4) = C_3 \quad \dots (2.43)$$

Denoting the left-angles by L and the right-angles by R, and using a pocket calculator, the side condition gives

$$(\Sigma \log \sin L - \Sigma \log \sin R) \times 10^7 = C_4 \quad \dots (2.44)$$

Therefore,

$$c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 = C_1 \quad \dots (2.45)$$

$$c_1 + c_2 - c_5 - c_6 = C_2 \quad \dots (2.46)$$

$$c_3 + c_4 - c_7 - c_8 = C_3 \quad \dots (2.47)$$

$$c_1 f_1 - c_2 f_2 + c_3 f_3 - c_4 f_4 + c_5 f_5 - c_6 f_6 + c_7 f_7 - c_8 f_8 = C_4 \quad \dots (2.48)$$

where $f_1, f_2, \ldots, f_8$ are the log sin differences for 1″ in the values of the angles $\theta_1, \theta_2, \ldots, \theta_8$, respectively, multiplied by 10^7.

An additional condition to be satisfied from the least squares criterion, is

$$\phi = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 = \text{a minimum} \quad \dots (2.49)$$

Differentiating the Eqs. (2.45) to (2.49), we get

$$\partial c_1 + \partial c_2 + \partial c_3 + \partial c_4 + \partial c_5 + \partial c_6 + \partial c_7 + \partial c_8 = 0 \quad \dots (2.50)$$

$$\partial c_1 + \partial c_2 + \partial c_5 - \partial c_6 = 0 \quad \dots (2.51)$$

$$\partial c_3 + \partial c_4 - \partial c_7 - \partial c_8 = 0 \quad \dots (2.52)$$

$$f_1 \partial c_1 - f_2 \partial c_2 + f_3 \partial c_3 - f_4 \partial c_4 + f_5 \partial c_5 - f_6 \partial c_6 + f_7 \partial c_7 - f_8 \partial c_8 = 0 \quad \dots (2.53)$$

$$c_1 \partial c_1 + c_2 \partial c_2 + c_3 \partial c_3 + c_4 \partial c_4 + c_5 \partial c_5 + c_6 \partial c_6 + c_7 \partial c_7 + c_8 \partial c_8 = 0 \quad \dots (2.54)$$

Multiply Eqs. (2.50), (2.51), (2.52), and (2.53) respectively, by $-\lambda_1, -\lambda_2, -\lambda_3$, and $-\lambda_4$, and add to the Eq. (2.54). Thus

$$(c_1 - \lambda_1 - \lambda_2 - f_1\lambda_4)\partial c_1 + (c_2 - \lambda_1 - \lambda_2 + f_2\lambda_4)\partial c_2 + (c_3 - \lambda_1 - \lambda_3 - f_3\lambda_4)\partial c_3$$

$$+ (c_4 - \lambda_1 - \lambda_3 + f_4\lambda_4)\partial c_4 + (c_5 - \lambda_1 + \lambda_2 - f_5\lambda_4)\partial c_5 + (c_6 - \lambda_1 + \lambda_2 + f_6\lambda_4)\partial c_6$$

$$+ (c_7 - \lambda_1 + \lambda_3 - f_7\lambda_4)\partial c_7 + (c_8 - \lambda_1 + \lambda_3 + f_8\lambda_4)\partial c_8 = 0$$

Now equating the coefficients of $\partial c_1, \partial c_2, \ldots, \partial c_8$, equal to zero separately, we get

$$c_1 = \lambda_1 + \lambda_2 + f_1\lambda_4 \quad \text{...(2.55)}$$

$$c_2 = \lambda_1 + \lambda_2 + f_2\lambda_4 \quad \text{...(2.56)}$$

$$c_3 = \lambda_1 + \lambda_3 + f_3\lambda_4 \quad \text{...(2.57)}$$

$$c_4 = \lambda_1 + \lambda_3 - f_4\lambda_4 \quad \text{...(2.58)}$$

$$c_5 = \lambda_1 - \lambda_2 + f_5\lambda_4 \quad \text{...(2.59)}$$

$$c_6 = \lambda_1 - \lambda_2 - f_6\lambda_4 \quad \text{...(2.60)}$$

$$c_7 = \lambda_1 - \lambda_3 + f_7\lambda_4 \quad \text{...(2.61)}$$

$$c_8 = \lambda_1 - \lambda_3 - f_8\lambda_4 \quad \text{...(2.62)}$$

Substituting the values of $c_1, c_2, \ldots, c_8$ in Eqs. (2.45) to (2.48), we get

$$8\lambda_1 + (f_1 - f_2 + f_3 - f_4 + f_5 - f_6 + f_7 - f_8)\lambda_4 = C_1 \quad \text{...(2.63)}$$

$$4\lambda_2 + \{(f_1 - f_2) - (f_5 - f_6)\}\lambda_4 = C_2 \quad \text{...(2.64)}$$

$$4\lambda_3 + \{(f_3 - f_4) - (f_7 - f_8)\}\lambda_4 = C_3 \quad \text{...(2.65)}$$

$$\begin{aligned}&\{(f_1 - f_2) + (f_3 - f_4) + (f_5 - f_6) + (f_7 - f_8)\}\lambda_1 \\ &+ \{(f_1 - f_2) - (f_5 - f_6)\}\lambda_2 + (f_3 - f_4) - (f_7 - f_8)\}\lambda_3 \\ &+ (f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_5^2 + f_6^2 + f_7^2 + f_8^2)\lambda_4 = C_4\end{aligned} \quad \text{...(2.66)}$$

Let

$$F = f_1 - f_2 + f_3 - f_4 + f_5 - f_6 + f_7 - f_8$$

$$F^2 = f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_5^2 + f_6^2 + f_7^2 + f_8^2$$

$$F_{12} = f_1 - f_2$$

$$F_{34} = f_3 - f_4$$

$$F_{56} = f_5 - f_6$$

$$F_{78} = f_7 - f_8$$

Then the Eqs. (2.63) to (2.66) become

$$8\lambda_1 + F\lambda_4 = C_1 \quad \text{...(2.67)}$$

$$4\lambda_2 + (F_{12} - F_{56})\lambda_4 = C_2 \quad \text{...(2.68)}$$

$$4\lambda_3 + (F_{34} - F_{78})\lambda_4 = C_3 \quad \text{...(2.69)}$$

$$(F_{12} + F_{34} + F_{56} + F_{78})\lambda_1 + (F_{12} - F_{56})\lambda_2 + (F_{34} - F_{78})\lambda_3 + F^2\lambda_4 = C_4 \quad \text{...(2.70)}$$

Solving the Eqs. (2.67) to (2.70), the values of $\lambda_1, \lambda_2, \lambda_3$, and λ_4 are obtained and substituting the values $\lambda_1, \lambda_2, \lambda_3$, and λ_4 in Eqs. (2.55) to (2.62), the values of $c_1, c_2, \ldots, c_8$ are obtained. These values are finally used to get the most probable values of the angles.

2.9.2 Approximate Method

The approximate method of adjustment is used for less important works, and when the quadrilaterals are of modrate size. Though the method is not as accurate as the least squares method, it gives fairly satisfactory results for all practical purposes.

In this method, the three angle conditions given by Eq. (2.39) and the side condition given by Eq. (2.40), are only satisfied, and the condition of the least squares is not considered at all.

The following steps are involved for the adjustment of a braced quadrilateral by approximate method (Fig. 2.10). C_1, C_2, C_3 and C_4 are the same as defined in rigorous method.

1. Determine C_1, and distribute it equally to each angle so that sum of the angles becomes 360°.
2. Determine C_2, and distributes it equally to the four angles $\theta_1, \theta_2, \theta_5$, and θ_6 so that $\theta_1 + \theta_2 = \theta_5 + \theta_6$.
3. Determine C_3 and distribute it equally to the four angles $\theta_3, \theta_4, \theta_7$, and θ_8 so that $\theta_3 + \theta_4 = \theta_7 + \theta_8$.

4. Find the log sin of the angles obtained in steps 2 and 3. Find the difference of the sum of log sin left angles and right angles, ignoring the signs, to get the discrepancy δ i.e.,

$$\delta = (\Sigma \log \sin L - \Sigma \log \sin R) \times 10^7 \text{ (ignoring the signs)}$$

where L = left-hand angle

R = right-hand angle

5. Find the differences up to seventh place, $f_1, f_2, ..., f_8$ for 1″ for log sin θ_1, log sin θ_2, ..., log sin θ_8, multiplied by 10^7.
6. Correct the angles by applying the corrections computed from the following formula.

$$\text{Correction to angle } \theta_n = \frac{f_n \delta}{\Sigma f^2}$$

The sign of the correction is decided as under:

(*a*) If $\Sigma \log \sin L$ is greater than $\Sigma \log \sin R$, the correction to the left angles will be negative and positive for right angles.

(*b*) If $\Sigma \log \sin L$ is less than $\Sigma \log \sin R$, the correction to the left angles will be positive and negative for the right angles.

After completing the steps 4 to 6, the condition given by Eq. (2.40) is satisfied.

The illustrative Examples 2.35 and 2.36, explain the use of rigorous and approximate methods, respectively, of adjustment of a geodetic quadrilateral.

ILLUSTRATIVE EXAMPLES

Example 2.35 The following eight angles of a geodetic quadrilaterals *ABCD* were observed, and adjusted for the closing errors at four stations *A, B, C,* and *D*. Adjust the angles by least squares method. The angles are corrected for the spherical excess.

$\theta_1 = 44°31'30''$ $\theta_2 = 43°37'37''$

$\theta_3 = 37°46'21''$ $\theta_4 = 54°04'40''$

$\theta_5 = 47°04'03''$ $\theta_6 = 41°05'05''$

$\theta_7 = 50°29'27''$ $\theta_8 = 41°21'34''$

Solution: (Fig. 2.10)

Let the corrections to the eight angles be $c_1, c_2, ..., c_8$ then

$$c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 = 360° - \Sigma\theta$$
$$= 360° - 360°00'17''$$
$$= -17'' = C_1 \quad ...(a)$$

$$c_1 + c_2 - c_5 - c_6 = -(\theta_1 + \theta_2) + (\theta_5 + \theta_6)$$
$$= -(88°09'07'') + (88°09'08'')$$
$$= +1'' = C_2 \quad ...(b)$$

$$c_3 + c_4 - c_7 - c_8 = -(\theta_3 + \theta_4) + (\theta_7 + \theta_8)$$
$$= -(91°51'01'') + (91°51'01'')$$
$$= 0'' = C_3 \quad ...(c)$$

$$\Sigma \log \sin L = \log \sin \theta_1 + \log \sin \theta_3 + \log \sin \theta_5 + \log \sin \theta_7$$
$$= -0.1541455 - 0.2128742 - 0.1353960 - 0.1126512$$
$$= -0.6150669$$

$$\Sigma \log\sin R = \log\sin\theta_2 + \log\sin\theta_4 + \log\sin\theta_6 + \log\sin\theta_8$$
$$= -0.1611761 - 0.0916146 - 0.1823194 = -0.1799426$$
$$= -0.6150527$$
$$(\Sigma \log\sin L - \Sigma \log\sin R)\times 10^7 = (-0.6150669 + 0.6150527)\times 10^7$$
$$= -142$$

or $C_4 = -142$ (to seventh of decimal place)

$$f_1 = 21; \quad f_2 = 22$$
$$f_3 = 27; \quad f_4 = 15$$
$$f_5 = 20; \quad f_6 = 24$$
$$f_7 = 17; \quad f_8 = 24$$

Thus

$$f_1c_1 - f_2c_2 + f_3c_3 - f_4c_4 + f_5c_5 - f_7c_7 + f_8c_8$$
$$= 21c_1 - 22c_2 + 27c_3 - 15c_4 + 20c_5 - 24c_6 + 17c_7 - 24c_8 = -142 \qquad \text{...}(d)$$

From least squares criterion, we have

$$\phi = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 = \text{a minimum} \qquad \text{...}(e)$$

The procedure to determine the corrections in terms of correlates and the equations containing the correlates as unknowns, is same as discussed in rigorous method of adjustment of quadrilateral in Sec. 2.9.1, and they are directly reproduced here.

$$c_1 = \lambda_1 + \lambda_2 + f_1\lambda_4; \qquad c_2 = \lambda_1 + \lambda_2 - f_2\lambda_4$$
$$c_3 = \lambda_1 + \lambda_3 + f_3\lambda_4; \qquad c_4 = \lambda_1 + \lambda_3 - f_4\lambda_4$$
$$c_5 = \lambda_1 - \lambda_2 + f_5\lambda_4; \qquad c_6 = \lambda_1 - \lambda_2 - f_6\lambda_4$$
$$c_7 = \lambda_1 - \lambda_3 + f_7\lambda_4; \qquad c_8 = \lambda_1 - \lambda_3 - f_8\lambda_4$$

To find the values of f_1, f_2 etc., using a pocket calculator, the following procedure may be adopted. Let $\log\sin(\theta + 1'') = x$, and $\log\sin\theta = y$

then taking $x - y = 0.00000\,abc$

$$f = 0.00000\,abc \times 10^7$$

i.e. $= ab.c$ (to seventh place of decimal)

The following examples will make the determination of f more clear.

$$\log\sin 44°31'31'' = x = -0.1541433$$
$$\log\sin 44°31'30'' = y = -0.1541455$$

Therefore, $x - y = 0.0000022 \times 10^7$

$$f = 22$$

Using the above procedure the values of $f_1, f_2, \ldots f_8$, have been found, and are given above.

Let us compute the values of F, F^2, F_{12}, F_{34}, F_{56} and F_{78}, defined in rigorous method of quadrilateral adjustment (*cf.*, Sec. 2.9.1).

$$F = 21 - 22 + 27 - 15 + 20 - 24 + 17 - 24 = 0$$
$$F^2 = 21^2 + 22^2 + 27^2 + 15^2 + 20^2 + 24^2 + 17^2 + 24^2 = 3720$$
$$f_{12} = 21 - 22 = -1$$
$$f_{34} = 27 - 15 = 12$$
$$f_{56} = 20 - 24 = -4$$
$$f_{78} = 17 - 24 = -7$$

The Eqs. (2.67) to (2.70) after substituting the values of F, F^2, F_{12}, F_{34}, F_{56}, F_{78}, C_1, C_2, C_3, and C_4, we get as below:

$$\begin{aligned} 8\lambda_1 + 0\lambda_4 &= -17 \\ 4\lambda_2 + 3\lambda_4 &= 1 \\ 4\lambda_3 + 19\lambda_4 &= 0 \\ 0\lambda_1 + 3\lambda_2 + 19\lambda_3 + 3720\lambda_4 &= -142 \end{aligned} \qquad \ldots (a)$$

From the above equations we get

$$\lambda_1 = -\frac{17}{8} = -2.125$$

$$\lambda_2 = \frac{1-3\lambda_4}{4} = 0.25 - 0.75\lambda_4$$

$$\lambda_3 = -\frac{19}{4}\lambda_4 = -4.75\lambda_4$$

Substituting the values of λ_1, λ_2, and λ_3 in Eq. (*a*), we get

$$0\lambda_1 + [3\times(0.25 - 0.75\lambda_4) + 19\times(-4.75\lambda_4) + 3720\lambda_4 = -142$$

By arranging and clearing the terms, we get

$$\begin{aligned} \lambda_4 &= -\frac{142.75}{3627.5} = -0.039 \\ \lambda_1 &= -2.125 \\ \lambda_2 &= 0.25 - 0.75\times(-0.039) = +0.279 \\ \lambda_3 &= -4.75\times(-0.039) = +0.185 \end{aligned}$$

Thus,

$$\begin{aligned} c_1 &= -2.125 + 0.279 + 2\times(-0.039) &= -2.665'' \\ c_2 &= -2.125 + 0.279 - 22\times(-0.039) &= -0.988'' \\ c_3 &= -2.125 + 0.185 + 27\times(-0.039) &= -2.993'' \\ c_4 &= -2.125 + 0.185 - 15\times(-0.039) &= -1.355'' \\ c_5 &= -2.125 - 0.279 + 20\times(-0.039) &= -3.184'' \\ c_6 &= -2.125 - 0.279 - 24\times(-0.039) &= -1.468'' \\ c_7 &= -2.125 - 0.185 + 17\times(-0.039) &= -2.973'' \\ c_8 &= -2.125 - 0.185 - 24\times(-0.039) &= -1.374'' \end{aligned}$$

Check:

$$c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8$$
$$= -2.665 - 0.988 - 2.993 - 1.355 - 3.184 - 1.468 - 2.973 - 1.374 = -17''$$

$$c_1 + c_2 - c_5 - c_6 = -2.665 - 0.988 + 3.184 + 1.468 = +0.999'' = +1''$$

$$c_3 + c_4 - c_7 - c_8 = -2.993 - 1.355 - 2.973 - 1.374 = -0.001 = 0''$$

Thus, the most probable value of the angles are:

$$\begin{aligned} \theta_1 + c_1 &= 44°31'30'' - 2.665'' = \mathbf{44°31'27.33''} \\ \theta_2 + c_2 &= 43°37'37'' - 0.988'' = \mathbf{43°37'36.01''} \\ \theta_3 + c_3 &= 37°46'21'' - 2.993'' = \mathbf{37°46'18.01''} \\ \theta_4 + c_4 &= 54°04'40'' - 1.355'' = \mathbf{54°04'38.65''} \\ \theta_5 + c_5 &= 47°04'03'' - 3.184'' = \mathbf{47°03'59.82''} \\ \theta_6 + c_6 &= 41°05'05'' - 1.468'' = \mathbf{41°05'03.53''} \\ \theta_7 + c_7 &= 50°29'27'' - 2.973'' = \mathbf{50°29'24.03''} \\ \theta_8 + c_8 &= 41°21'34'' - 1.374'' = \mathbf{41°21'32.63''} \end{aligned}$$

Check:

$$\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 + \theta_7 + \theta_8 = 360°00'00''.$$

$$\theta_1 + \theta_2 - (\theta_5 + \theta_6) = 88°09'03.34'' - 88°09'03.35'' = 0$$

$$\theta_3 + \theta_4 - (\theta_7 + \theta_8) = 91°50'56.66'' - 91°50'56.66'' = 0$$

Example 2.36 Adjust the angles of the geodetic quadrilateral given in Example 2.35, by approximate method.

Solution: (Fig. 2.10)

The computations have been shown in Table 2.1. The values shown in the various columns have been computed as below.

Column - 1

Write $\theta_1, \theta_2, \theta_3, \ldots, \theta_8$.

Column - 2

Enter the values of the observed angles against $\theta_1, \theta_2, \ldots, \theta_8$.

Column - 3

The total correction for sum of angles is

$(360° - \Sigma\theta) = 360° - 360°00'17'' = -17''$

$$\text{Correction to each angle} = -\frac{17}{8} = -2.125''$$

Enter the corrections for each angle.

Column - 4

Enter the corrected angles after applying the correction $-2.125''$ to each angles. Check that $\Sigma\theta = 360°$.

Column - 5

Determine the corrections for opposite angles condition as below:

$$(\theta_1 + \theta_2) - (\theta_5 + \theta_6) = 88°09'2.75'' - 88°09'3.75'' = 1'' \text{ (ignoring the sign)}$$

$$\text{Correction to each angle} = \frac{1}{4} = 0.25''$$

Since $(\theta_1 + \theta_2)$ is less than $(\theta_5 + \theta_6)$ the correction to θ_1 and θ_2 will be $+0.25''$ each and to θ_5 and θ_6 $-0.25''$ each.

$$\theta_3 + \theta_4 - (\theta_7 + \theta_8) = 91°50'56.75'' - 91°50'56.75'' = 0.00'' \text{ (ignoring the sign)}$$

Since $(\theta_3 + \theta_4)$ is equal to $(\theta_7 + \theta_8)$, the correction to these angles will be zero.

Enter the above corrections for each angle.

Column - 6

Enter the corrected angles for opposite angles conditions.

Column - 7 and - 8

Enter the values of (log sin θ) for the values of θ in Column - 6, ignoring the sign.

Column - 9

Enter the values of f which is difference for 1'' of the angle for (log sin θ) at 7th place of decimal, i.e., the difference value $\times 10^7$.

Column - 10

Enter the values of f^2.

Column - 11

Enter the corrections for side equation. The corrections are computed as below:

$$\text{Correction} = \frac{f}{\Sigma f^2} \times \delta$$

$$\delta = (\Sigma \log\sin L - \Sigma \log\sin R) \times 10^7$$

$$= (0.6150850 - 0.6150710) \times 10^7 \text{ (ignoring the sign)}$$

$$= 140.$$

The correction to the left angles will be negative and to the right angles positive, since $\Sigma \log\sin L$ is greater than Σ log sin R.

Apply the corrections given in column - 11 to the angles given in Column - 6 and enter them in Column - 12. These are the final adjusted values of the angles. Check that sum of the final angles is 360°.

Table 2.1 Adjustment of quadrilateral by approximate method (Example 2.35)

θ	Observed angle	Correction for sum of all angle	Corrected angle	Correction for sum of opp. angles	Corrected angles	Log sine		f	f^2	Side equation correction	Adjusted angle
						Left angle	Right angle				
1	2	3	4	5	6	7	8	9	10	11	12
θ_1	44°31'30"	– 2.125"	44°31'27.875"	+ 0.25"	44°31'28.125"	0.1541495		22	484	– 0.828"	44°31'27.297"
θ_2	43°37'37"	– 2.125"	43°37'34.875"	+ 0.25"	43°37'35.125"		0.1611802	22	484	+0.828"	43°37'35.953"
θ_3	37°46'21"	– 2.125"	37°46'18.875"	0.00"	37°46'18.875"	0.2128800		27	729	– 1.016"	37°46'17.889"
θ_4	54°04'40"	– 2.125"	54°04'37.875"	0.00"	54°04'37.875"		0.0916179	15	225	+ 0.565"	54°04'38.440"
θ_5	47°04'03"	– 2.125"	47°04'00.875"	– 0.25"	47°04'00.625"	0.13454006		20	400	– 0.753"	47°03'59.872"
θ_6	41°05'05"	– 2.125"	41°05'02.875"	– 0.25"	41°05'02.625"		0.1823252	24	576	+ 0.903"	41°05'03.528"
θ_7	50°29'27"	– 2.125"	50°29'24.875"	0.00"	50°29'24.825"	0.1126549		17	289	– 0.640"	50°29'24.235"
θ_8	41°21'34"	– 2.125"	41°21'31.875"	0.00"	41°21'31.875"		0.1799477	24	576	+ 0.903"	41°21'32.778"
Σ	360°00'17"	17"	360°00'00"		360°00'00.000"	0.6150850	0.6150710		3720		360°00'00.000"

2.10 ADJUSTMENT OF A POLYGON WITH A CENTRAL STATION

The polygon can have any number of sides, the simplest being a triangle. Hence the adjustment of the following two cases will be discussed:

(*i*) Triangle with a central station.

(*ii*) Polygon having more than three sides and with a central station.

2.10.1 Triangle with a central station

Fig. 2.11 shows a triangle *ABC* with a central station *O*. The observed angles which are to be adjusted are $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8$, and θ_9. These angles have been corrected for station adjustment and spherical excess. The conditions for such figures to be satisfied, are:

$$\theta_1 + \theta_2 + \theta_7 = 180° \quad \text{...(2.71)}$$

$$\theta_3 + \theta_4 + \theta_8 = 180° \quad \text{...(2.72)}$$

$$\theta_5 + \theta_6 + \theta_9 = 180° \quad \text{...(2.73)}$$

$$\theta_7 + \theta_8 + \theta_9 = 360° \quad \text{...(2.74)}$$

$$\Sigma \log \sin L = \Sigma \log \sin R \quad \text{...(2.75)}$$

Let the corrections to the angles be $c_1, c_2, \ldots, c_9$, and the total corections from Eqs. (2.71) to (2.75) be C_1, C_2, C_3, C_4, and C_5, respectively, then

$$C_1 = 180° - (\theta_1 + \theta_2 + \theta_7) \quad \text{...(2.76)}$$

$$C_2 = 180° - (\theta_3 + \theta_4 + \theta_8) \quad \text{...(2.77)}$$

$$C_3 = 180° - (\theta_5 + \theta_6 + \theta_9) \quad \text{...(2.78)}$$

$$C_4 = 360° - (\theta_7 + \theta_8 + \theta_9) \quad \text{...(2.79)}$$

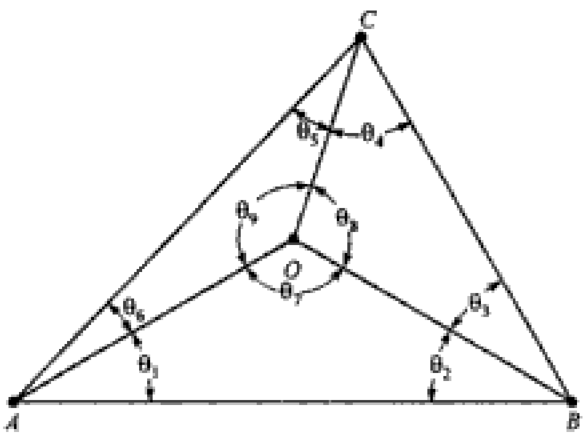


Fig. 2.11 Triangle with a central station

$$C_5 = (\Sigma \log \sin L - \Sigma \log \sin R) \times 10^7 \quad \text{...(2.80)}$$

and

$$c_1 + c_2 + c_7 = C_1 \quad \text{...(2.81)}$$

$$c_3 + c_4 + c_8 = C_2 \quad \text{...(2.82)}$$

$$c_5 + c_6 + c_9 = C_3 \quad \text{...(2.83)}$$

$$c_7 + c_8 + c_9 = C_4 \quad \text{...(2.84)}$$

$$f_1 c_1 - f_2 c_2 - f_3 c_3 - f_4 c_4 + f_5 c_5 - f_6 c_6 = C_5 \quad \text{...(2.85)}$$

Another condition from least squares criterion is

$$\phi = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2 = \text{a minimum} \quad \text{...(2.86)}$$

Differentiating Eqs. (2.81) to (2.86), we get

$$\partial c_1 + \partial c_2 + \partial c_7 = 0 \quad \text{...(2.87)}$$

$$\partial c_3 + \partial c_4 + \partial c_8 = 0 \quad \text{...(2.88)}$$

$$\partial c_5 + \partial c_6 + \partial c_9 = 0 \quad \text{...(2.89)}$$

$$\partial c_7 + \partial c_8 + \partial c_9 = 0 \quad \text{...(2.90)}$$

$$f_1 \partial c_1 - f_2 \partial c_2 - f_3 \partial c_3 - f_4 \partial c_4 - f_5 \partial c_5 - f_6 \partial c_6 = 0 \quad \text{...(2.91)}$$

$$c_1 \partial c_1 + c_2 \partial c_2 + c_3 \partial c_3 + c_4 \partial c_4 + c_5 \partial c_5 + c_6 \partial c_6 + c_7 \partial c_7 + c_8 \partial c_8 + c_9 \partial c_9 = 0 \quad \text{...(2.92)}$$

Multiplying Eqs. (2.87) to (2.91) by $-\lambda_1, -\lambda_2, -\lambda_3, -\lambda_4$ and $-\lambda_5$, respectively, adding to Eq. (2.92) and equating the coefficients of $\partial c_1, \partial c_2, \ldots, \partial c_9$, each equal to zero, we get

$$c_1 = \lambda_1 + f_1\lambda_5 \qquad c_6 = \lambda_3 - f_6\lambda_5$$
$$c_2 = \lambda_1 - f_2\lambda_5 \qquad c_7 = \lambda_1 + \lambda_4$$
$$c_3 = \lambda_2 + f_3\lambda_5 \qquad c_8 = \lambda_2 + \lambda_4 \qquad \text{...(2.93)}$$
$$c_4 = \lambda_2 - f_4\lambda_5 \qquad c_9 = \lambda_3 + \lambda_4$$
$$c_5 = \lambda_3 + f_5\lambda_5$$

Substituting the values of $c_1, c_2, \ldots, c_9$ in Eqs. (2.81) to (2.85), we get

$$3\lambda_1 + \lambda_4 + F_{12}\lambda_5 = C_1 \qquad \text{...(2.94)}$$
$$3\lambda_2 + \lambda_4 + F_{34}\lambda_5 = C_2 \qquad \text{...(2.95)}$$
$$3\lambda_3 + \lambda_4 + F_{56}\lambda_5 = C_3 \qquad \text{...(2.96)}$$
$$\lambda_1 + \lambda_2 + \lambda_3 + 3\lambda_4 = C_4 \qquad \text{...(2.97)}$$
$$F_{12}\lambda_1 + F_{34}\lambda_2 + F_{56}\lambda_3 + F^2\lambda_5 = C_5 \qquad \text{...(2.98)}$$

where
$$F_{12} = f_1 - f_2$$
$$F_{34} = f_3 - f_4$$
$$F_{56} = f_5 - f_6$$
$$F^2 = f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_5^2 + f_6^2$$

Solve the Eqs. (2.94) to (2.98) to get the values of $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and λ_5. Substitute these values in Eqs. (2.93), to determine the corrections $c_1, c_2, \ldots, c_9$.

Finally apply these corrections to the observed values of the angles to get their most probable values (*cf.*, Example 2.37).

2.10.2 Polygon with a central station

The method of adjustment of a polygon with a central station is same as that of a triangle with a central station. The only difference is that the number of the conditions of triangular error to be satisfied increases, and is equal to the number of sides in a polygon.

Let us consider the following two cases:

(*i*) Quadrilateral with a central station

(*ii*) Pentagon with a central station.

2.10.3 Quadrilateral with a central station

Fig. 2.12 shows a quadrilateral *ABCD* with a central station *O*. The measured angles are shown in the figure. The conditions to be satisfied are:

$$\theta_1 + \theta_2 + \theta_9 = 180°$$
$$\theta_3 + \theta_4 + \theta_{10} = 180°$$
$$\theta_5 + \theta_6 + \theta_{11} = 180°$$
$$\theta_7 + \theta_8 + \theta_{12} = 180°$$
$$\theta_9 + \theta_{10} + \theta_{11} + \theta_{12} = 360°$$
$$\Sigma \log \sin L = \Sigma \log \sin R$$

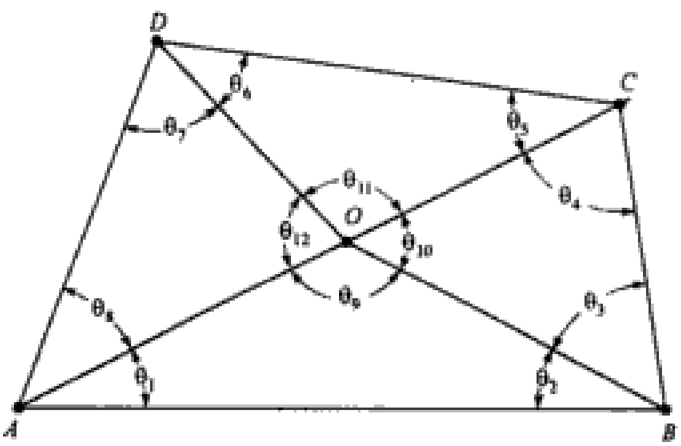


Fig. 2.12 Quadrilateral with a central station.

The total corrections will be

$$C_1 = 180° - (\theta_1 + \theta_2 + \theta_9)$$
$$C_2 = 180° - (\theta_3 + \theta_4 + \theta_{10})$$
$$C_3 = 180° - (\theta_5 + \theta_6 + \theta_{11})$$
$$C_4 = 180° - (\theta_7 + \theta_8 + \theta_{12})$$
$$C_5 = 360° - (\theta_9 + \theta_{10} + \theta_{11} + \theta_{12})$$
$$C_6 = (\Sigma \log \sin L - \Sigma \log \sin R) \times 10^7$$

The corrections $c_1, c_2, \ldots c_{12}$ to the angles are given by

$$c_1 + c_2 + c_9 = C_1 \quad \ldots(2.99)$$
$$c_3 + c_4 + c_{10} = C_2 \quad \ldots(2.100)$$
$$c_5 + c_6 + c_{11} = C_3 \quad \ldots(2.101)$$
$$c_7 + c_8 + c_{12} = C_4 \quad \ldots(2.102)$$
$$c_9 + c_{10} + c_{11} + c_{12} = C_5 \quad \ldots(2.103)$$
$$f_1c_1 - f_2c_2 + f_3c_3 - f_4c_4 + f_5c_5 - f_6c_6 + f_7c_7 - f_8c_8 = C_6 \quad \ldots(2.104)$$

The condition from least squares states that

$$\phi = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2 + c_{10}^2 + c_{11}^2 + c_{12}^2 = \text{a minimum} \quad \ldots(2.105)$$

Differentiating Eqs. (2.99) to (2.104), multiplying them by $-\lambda_1, -\lambda_2, -\lambda_3, -\lambda_4, -\lambda_5$, and $-\lambda_6$, and adding to the differential of Eq. (2.105), we get

$$\begin{aligned} c_1 &= \lambda_1 + f_1\lambda_6 & c_7 &= \lambda_4 + f_7\lambda_6 \\ c_2 &= \lambda_1 - f_2\lambda_6 & c_8 &= \lambda_4 - f_8\lambda_6 \\ c_3 &= \lambda_2 + f_3\lambda_6 & c_9 &= \lambda_1 + \lambda_5 \\ c_4 &= \lambda_2 - f_4\lambda_6 & c_{10} &= \lambda_2 + \lambda_5 \\ c_5 &= \lambda_3 + f_5\lambda_6 & c_{11} &= \lambda_3 + \lambda_5 \\ c_6 &= \lambda_3 - f_6\lambda_6 & c_{12} &= \lambda_4 + \lambda_5 \end{aligned} \quad \ldots(2.106)$$

Substituting the values of $c_1, c_2, \ldots c_{12}$ in Eqs. (2.99) to (2.104), we get the following equations:

$$\begin{aligned} 3\lambda_1 + \lambda_5 + F_{12}\lambda_6 &= C_1 \\ 3\lambda_2 + \lambda_5 + F_{34}\lambda_6 &= C_2 \\ 3\lambda_3 + \lambda_5 + F_{56}\lambda_6 &= C_3 \\ 3\lambda_4 + \lambda_5 + F_{78}\lambda_6 &= C_4 \\ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + 4\lambda_5 &= C_5 \\ F_{12}\lambda_1 + F_{34}\lambda_2 + F_{56}\lambda_3 + F_{78}\lambda_4 + F^2\lambda_6 &= C_6 \end{aligned} \quad \ldots(2.107)$$

where

$$\begin{aligned} F_{12} &= f_1 - f_2 & F_{78} &= f_7 - f_8 \\ F_{34} &= f_3 - f_4 & F^2 &= f_1^2 + f_2^2 + \ldots + f_8^2 \\ F_{56} &= f_5 - f_6 \end{aligned}$$

The above equations are solved simultaneously to determine the values of $\lambda_1, \lambda_2, \ldots \lambda_{12}$. From these values the corrections $c_1, c_2, \ldots c_{12}$ are determined to get the most probable values of the angles (*c.f.*, Example 2.38).

2.10.4 Pentagon with a central station

Fig. 1.13 shows a pentagon *ABCDE* with a central station *O*. The observed angles are shown in the figure. The conditions to be satisfied for the figure are

$$\begin{aligned} \theta_1 + \theta_2 + \theta_{11} &= 180^\circ \\ \theta_3 + \theta_4 + \theta_{12} &= 180^\circ \\ \theta_5 + \theta_6 + \theta_{13} &= 180^\circ \\ \theta_7 + \theta_8 + \theta_{14} &= 180^\circ \\ \theta_9 + \theta_{10} + \theta_{15} &= 180^\circ \\ \theta_{11} + \theta_{12} + \theta_{13} + \theta_{14} + \theta_{15} &= 360^\circ \\ \Sigma \log \sin L &= \Sigma \log \sin R \end{aligned}$$

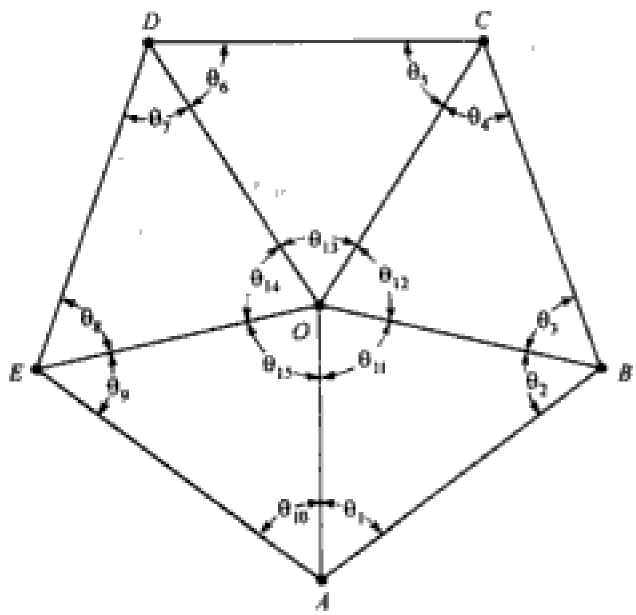


Fig. 2.13 Pentagon with a central station

The total corrections $C_1, C_2, C_3, C_4, C_5, C_6$, and C_7 are found from the above seven equations, and then the coorections to the individual angles $c_1, c_2, \ldots, c_{15}$ are written as

$$\begin{aligned} c_1 + c_2 + c_{11} &= C_1 \\ c_3 + c_4 + c_{12} &= C_2 \\ c_5 + c_6 + c_{13} &= C_3 \\ c_7 + c_8 + c_{14} &= C_4 \\ c_9 + c_{10} + c_{15} &= C_5 \\ c_{11} + c_{12} + c_{13} + c_{14} + c_{15} &= C_6 \\ f_1c_1 - f_2c_2 + f_3c_3 - f_4c_4 + f_5c_5 - f_6c_6 + f_7c_7 - f_8c_8 + f_9c_9 - f_{10}c_{10} &= C_7 \end{aligned} \quad \ldots (2.108)$$

From least squares principle

$$\phi = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2 + c_{10}^2 + c_{11}^2 + c_{12}^2 + c_{13}^2 + c_{14}^2 + c_{15}^2 = \text{a minimum}$$

Following the procedure adopted in adjustment of triangle or quadrilateral with central figures, discussed earlier, the corrections in terms of seven corelatives are as below:

$$\begin{aligned} c_1 &= \lambda_1 + f_1\lambda_7 & c_9 &= \lambda_5 + f_9\lambda_7 \\ c_2 &= \lambda_1 + f_2\lambda_7 & c_{10} &= \lambda_5 + f_{10}\lambda_7 \\ c_3 &= \lambda_2 + f_3\lambda_7 & c_{11} &= \lambda_1 + \lambda_6 \\ c_4 &= \lambda_2 - f_4\lambda_7 & c_{12} &= \lambda_2 + \lambda_6 \\ c_5 &= \lambda_3 + f_5\lambda_7 & c_{13} &= \lambda_3 + \lambda_6 \\ c_6 &= \lambda_3 - f_6\lambda_7 & c_{14} &= \lambda_4 + \lambda_6 \\ c_7 &= \lambda_4 + f_7\lambda_7 & c_{15} &= \lambda_5 + \lambda_6 \\ c_8 &= \lambda_4 - f_8\lambda_7 \end{aligned} \quad \ldots (2.109)$$

The equations containing the correlatives as unknown, are as under:

$$\begin{aligned} 3\lambda_1 + \lambda_6 + F_{12}\lambda_7 &= C_1 \\ 3\lambda_2 + \lambda_6 + F_{34}\lambda_7 &= C_2 \\ 3\lambda_3 + \lambda_6 + F_{56}\lambda_7 &= C_3 \\ 3\lambda_4 + \lambda_6 + F_{78}\lambda_7 &= C_4 \end{aligned} \quad \ldots (2.110)$$

$$3\lambda_5 + \lambda_6 + F_{910}\lambda_7 = C_5$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + 5\lambda_6 = C_6$$

$$F_{12}\lambda_1 + F_{34}\lambda_2 + F_{56}\lambda_3 + F_{78}\lambda_4 + F_{910}\lambda_5 + F^2\lambda_7 = C_7$$

where

$$F_{12} = f_1 - f_2 \qquad F_{78} = f_7 - f_8$$

$$F_{34} = f_3 - f_4 \qquad F_{910} = f_9 - f_{10}$$

$$F_{56} = f_5 - f_6 \qquad F^2 = f_1^2 + f_2^2 + ... + f_{15}^2$$

Remaining procedure to get the values of the most probable values of the angles is same as discussed in triangle or quadrilateral with a central station.

ILLUSTRATIVE EXAMPLES

Examples 2.37 Fig. 2.11 shows a triangle *ABC* with a central station *O*. The mean observed values corrected for station adjustments and spherical excess are as follows :

$$\angle 1 = 24°29'24'' \qquad \angle 2 = 27°30'56''$$

$$\angle 3 = 32°03'20'' \qquad \angle 4 = 28°07'02''$$

$$\angle 5 = 33°42'44'' \qquad \angle 6 = 34°06'33''$$

$$\angle 7 = 127°59'34'' \qquad \angle 8 = 119°49'44''$$

$$\angle 9 = 112°10'46''$$

Assuming all the observations of equal weight, determine the most probable values of the angles.

Solution: (Fig. 2.11)

The angles conditions are

$$\angle 1 + \angle 2 + \angle 7 = 180°$$

$$\angle 2 + \angle 4 + \angle 8 = 180°$$

$$\angle 5 + \angle 6 + \angle 9 = 180°$$

$$\angle 7 + \angle 8 + \angle 9 = 360°$$

The side condition is

$$\Sigma \log \sin L = \Sigma \log \sin R$$

If the corrections to the individual angles are $c_1, c_2, ..., c_9$ and the total corrections to satisfy the above conditions are C_1, C_2, C_3, C_4, and C_5 then

$$\begin{aligned} C_1 &= 180° - (\angle 1 + \angle 2 + \angle 7) \\ &= 180° - (24°29'24'' + 27°30'56'' + 127°59'34'') \\ &= 180° - 179°59'54'' \\ &= +6'' \end{aligned}$$

$$\begin{aligned} C_2 &= 180° - (\angle 3 + \angle 4 + \angle 8) \\ &= 180° - (32°03'20'' + 28°07'02'' + 119°49'44'') \\ &= 180° - 180°00'06'' \\ &= -6'' \end{aligned}$$

$$\begin{aligned} C_3 &= 180° - (\angle 5 + \angle 6 + \angle 9) \\ &= 180° - (33°42'44'' + 34°06'33'' + 112°10'46'') \\ &= 180° - 180°00'03'' \\ &= -3'' \end{aligned}$$

$$C_4 = 360° - (\angle 7 + \angle 8 + \angle 9)$$
$$= 360° - (127°59'34'' + 119°49'44'' + 112°10'46'')$$
$$= 360° - 360°00'04''$$
$$= -4''$$

$$C_5 = (\Sigma \log \sin L - \Sigma \log \sin R) \times 10^7$$
$$= (-0.9132464 + 0.9133058) \times 10^7$$
$$= 594$$

$$f_1c_1 - f_2c_2 + f_3c_3 - f_4c_4 + f_5c_5 - f_6c_6 = C_5$$

or $$46c_1 - 40c_2 + 34c_3 - 39c_4 + 32c_5 - 31c_6 = 594$$

From least squares condition we have

$$\phi = c_1^2 + c_2^2 + c_3^2 + c_4^2 + c_5^2 + c_6^2 + c_7^2 + c_8^2 + c_9^2 = \text{a minimum}$$

From Eqs. (2.93), we have

$$\begin{aligned} c_1 &= \lambda_1 + f_1\lambda_5 & c_7 &= \lambda_1 + \lambda_4 \\ c_2 &= \lambda_1 + f_2\lambda_5 & c_8 &= \lambda_2 + \lambda_4 \\ c_3 &= \lambda_2 + f_3\lambda_5 & c_9 &= \lambda_3 + \lambda_4 \\ c_4 &= \lambda_2 - f_4\lambda_5 \\ c_5 &= \lambda_3 + f_5\lambda_5 \\ c_6 &= \lambda_3 - f_6\lambda_5 \end{aligned} \qquad \ldots(a)$$

From Eqs. (2.94) to (2.98), we get

$$\begin{aligned} 3\lambda_1 + \lambda_4 + F_{12}\lambda_5 &= C_1 \\ 3\lambda_2 + \lambda_4 + F_{34}\lambda_5 &= C_2 \\ 3\lambda_3 + \lambda_4 + F_{56}\lambda_5 &= C_3 \\ \lambda_1 + \lambda_2 + \lambda_3 + 3\lambda_4 &= C_4 \\ F_{12}\lambda_1 + F_{34}\lambda_2 + F_{56}\lambda_3 + F^2\lambda_5 &= C_5 \end{aligned} \qquad \ldots(b)$$

$$F_{12} = f_1 - f_2 = 46 - 40 = 6$$
$$F_{34} = f_3 - f_4 = 34 - 39 = -5$$
$$F_{56} = f_5 - f_6 = 32 - 31 = 1$$
$$F^2 = 46^o + 40^2 + 34^2 + 39^2 + 32^2 + 31^2 = 8378$$

Substituting the values of $F_{12}, F_{34}, F_{56}, F^2, C_1, C_2, C_3, C_4$, and C_5 in (*a*), we get

$$3\lambda_1 + \lambda_4 + 6\lambda_5 = 6 \qquad \ldots(b)$$
$$3\lambda_2 + \lambda_4 + 5\lambda_5 = -6 \qquad \ldots(c)$$
$$3\lambda_3 + \lambda_4 + \lambda_5 = -3 \qquad \ldots(d)$$
$$\lambda_1 + \lambda_2 + \lambda_3 + 3\lambda_4 = -4 \qquad \ldots(e)$$
$$6\lambda_1 - 5\lambda_2 + \lambda_3 + 8378\lambda_5 = 594 \qquad \ldots(f)$$

Adding (*b*), (*c*) and (*d*), we get

$$3(\lambda_1 + \lambda_2 + \lambda_3) + 3\lambda_4 + (6 - 5 + 1)\lambda_5 = 6 - 6 - 3$$
$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \frac{2}{3}\lambda_5 = -1$$

or $$\lambda_1 + \lambda_2 + \lambda_3 = -1 - \left(\lambda_4 + \frac{2}{3}\lambda_5\right) \qquad \ldots(g)$$

Substituting the values of $(\lambda_1+\lambda_2+\lambda_3)$ in (*e*) from (*g*), we get

$$-1-\left(\lambda_4+\frac{2}{3}\lambda_5\right)+3\lambda_4 = -4$$

$$2\lambda_4-\frac{2}{3}\lambda_5 = -3$$

$$\lambda_4 = \frac{1}{3}\lambda_5-1.5 \qquad \ldots(h)$$

Substituting the values of λ_4 from (*h*), in (*b*), (*c*) and (*d*), we get

$$3\lambda_1+\left(\frac{1}{3}\lambda_5-1.5\right)+6\lambda_5 = 6$$

$$3\lambda_2+\left(\frac{1}{3}\lambda_5-1.5\right)-5\lambda_5 = -6$$

$$3\lambda_3+\left(\frac{1}{3}\lambda_5-1.5\right)-\lambda_5 = -3$$

$$3\lambda_1+\frac{19}{3}\lambda_5 = 7.5$$

$$3\lambda_2+\frac{14}{3}\lambda_5 = -4.5$$

$$3\lambda_3+\frac{4}{3}\lambda_5 = -1.5$$

$$\lambda_1 = 2.5-2.111\,\lambda_5 \qquad \ldots(i)$$

$$\lambda_2 = -1.5+1.556\,\lambda_5 \qquad \ldots(j)$$

$$\lambda_3 = -0.5-0.444\,\lambda_5 \qquad \ldots(k)$$

Substituting the values of $\lambda_4, \lambda_1, \lambda_2,$ and λ_3 from (*h*) to (*k*) in (*f*), we get

$$6\times(2.5-2.111\,\lambda_5)-5\times(-1.5+1.556\,\lambda_5)+(-0.5-0.444\,\lambda_5)+8378\,\lambda_5 = 594$$

or

$$8357.11\,\lambda_5 = 572$$

$$\lambda_5 = +0.068$$

$$\lambda_1 = +2.356$$

$$\lambda_2 = -1.394$$

$$\lambda_3 = -0.530$$

$$\lambda_4 = -1.477$$

Substituting the values of $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 in (*a*), we get

$$c_1 = +5.484$$

$$c_2 = -0.364$$

$$c_3 = +0.918$$

$$c_4 = -4.046$$

$$c_5 = +1.646$$

$$c_6 = -2.638$$

$$c_7 = +0.879$$

$$c_8 = -2.871$$

$$c_9 = -2.007$$

Checks:

$$c_1 + c_2 + c_7 = 5.484 - 0.364 + 0.879 = +6''(O.K)$$
$$c_3 + c_4 + c_8 = 0.918 - 4.046 - 2.871 = -6''(O.K)$$
$$c_5 + c_6 + c_9 = 1.646 - 2.638 - 2.007 = -3''(O.K)$$
$$c_7 + c_8 + c_9 = 0.879 - 2.871 - 2.007 = -4''(O.K)$$

Apply the above corrections to the observed angles, we get the most probable values of the angles as below:

$$\theta_1 + c_1 = 24°29'24'' + 5.484'' = \mathbf{24°29'29.48''}$$
$$\theta_2 + c_2 = 27°30'56'' - 0.364'' = \mathbf{27°31'00.64''}$$
$$\theta_7 + c_7 = 127°59'34'' + 0.879'' = \mathbf{127°59'34.88''}.$$
$$\text{Sum} = 180°00'00.00''.$$
$$\theta_3 + c_3 = 32°03'20'' + 0.918'' = \mathbf{32°03'20.92''}$$
$$\theta_4 + c_4 = 28°07'02'' - 4.046'' = \mathbf{28°06'57.95''}$$
$$\theta_8 + c_8 = 119°49'44'' - 2.871'' = \mathbf{119°49'41.13''}.$$
$$\text{Sum} = 180°00'00.00''.$$
$$\theta_5 + c_5 = 33°42'44'' + 1.646'' = \mathbf{33°42'45.65''}$$
$$\theta_6 + c_6 = 34°06'33'' - 2.638'' = \mathbf{34°06'30.36''}$$
$$\theta_9 + c_9 = 112°10'46'' - 2.007'' = \mathbf{112°10'43.99''}.$$
$$\text{Sum} = 180°00'00.00''.$$
$$\theta_7 + c_7 = \mathbf{127°59'34.88''}$$
$$\theta_8 + c_8 = \mathbf{119°49'41.13''}$$
$$\theta_9 + c_9 = \mathbf{112°10'43.99''}$$
$$\text{Sum} = 360°00'00.00''.$$

Example 2.38 The following are the measured values of the angles of a quadrilateral *ABCD* (Fig. 2.12) with a central station *O*.

$$\theta_1 = 29°17'00'' \qquad \theta_2 = 28°42'00''$$
$$\theta_3 = 62°59'49'' \qquad \theta_4 = 56°28'01''$$
$$\theta_5 = 29°32'06'' \qquad \theta_6 = 32°03'54''$$
$$\theta_7 = 59°56'06'' \qquad \theta_8 = 61°00'54''$$
$$\theta_9 = 122°00'55'' \qquad \theta_{10} = 60°32'05''$$
$$\theta_{11} = 118°23'50'' \qquad \theta_{12} = 59°03'10''$$

Assuming the angles corrected for station adjustment, spherical excess and of same reliability, determine the adjusted values of the angles.

Solution: (Fig. 2.12)

Refer to the adjustment of a quadrilateral with a central station in Sec. 2.10.3.

The angle conditions to be satisfied are

$$\theta_1 + \theta_2 + \theta_9 = 180° \text{ or } C_1 = 180° - (\theta_1 + \theta_2 + \theta_9) = +5''$$
$$\theta_3 + \theta_4 + \theta_{10} = 180° \text{ or } C_2 = 180° - (\theta_3 + \theta_4 + \theta_{10}) = +5''$$
$$\theta_5 + \theta_6 + \theta_{11} = 180° \text{ or } C_3 = 180° - (\theta_5 + \theta_6 + \theta_{11}) = +10''$$
$$\theta_7 + \theta_8 + \theta_{12} = 180° \text{ or } C_4 = 180° - (\theta_7 + \theta_8 + \theta_{12}) = -10''$$
$$\theta_9 + \theta_{10} + \theta_{11} + \theta_{12} = 360° \text{ or } C_5 = 360° - (\theta_9 + \theta_{10} + \theta_{11} + \theta_{12}) = -0.0''$$

From side condition, we have

$$C_6 = (\Sigma \log\sin L - \Sigma \log\sin R)\times 10^7$$
$$= (-0.7306545 + 0.7307365)\times 10^7$$
$$= 820$$

$$f_1 = 38, \quad f_2 = 38 \quad f_3 = 11, \quad f_4 = 14$$

$$f_5 = 37 \quad f_6 = 34 \quad f_7 = 12, \quad f_8 = 12$$

From the side condition, we get

$$f_1c_1 - f_2c_2 + f_3c_3 - f_4c_4 + f_5c_5 - f_6c_6 + f_7c_7 - f_8c_8 = C_6$$
$$38c_1 - 38c_2 + 11c_3 - 14c_4 + 37c_5 - 34c_6 + 12c_7 - 12c_8 = 820$$

Eqs. (2.107) in $\lambda_1, \lambda_2, ..., \lambda_6$ are

$$3\lambda_1 + \lambda_5 + F_{12}\lambda_6 = 5 \quad ...(a)$$

$$3\lambda_2 + \lambda_5 + F_{34}\lambda_6 = 5 \quad ...(b)$$

$$3\lambda_3 + \lambda_5 + F_{56}\lambda_6 = 10 \quad ...(c)$$

$$3\lambda_4 + \lambda_5 + F_{78}\lambda_6 = -10 \quad ...(d)$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + 4\lambda_5 = 0 \quad ...(e)$$

$$F_{12}\lambda_1 + F_{34}\lambda_2 + F_{56}\lambda_3 + F_{78}\lambda_4 + F^2\lambda_6 = 820 \quad ...(f)$$

$$F_{12} = 38 - 38 = 0, \quad F_{34} = 11 - 14 = -3$$
$$F_{56} = 37 - 34 = 3, \quad F_{78} = 12 - 12 = 0$$
$$F^2 = 6018$$

Adding Eqs. (*a*) to (*d*), we get

$$3(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) + 4\lambda_5 + (F_{12} + F_{56} + F_{78})\lambda_6 = 10$$

$$3(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) + 4\lambda_5 + 0.0\lambda_6 = 10$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = \frac{10}{3} - \frac{4\lambda_5}{3} = 3.333 - 1.333\ \lambda_5 \quad ...(g)$$

Substituting the value of $(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$ from (*g*) in (*e*) are get

$$3.333 - 1.333\ \lambda_5 + 4\lambda_5 = 0$$

$$2.667\lambda_5 = -3.333$$

$$\lambda_5 = -1.250$$

Substituting the value of λ_5 in Eqs. (*a*) to (*d*), we get

$$3\lambda_1 = 5 + 1.250 - 0 \times \lambda_6 = 6.250$$

$$\lambda_1 = \frac{6.250}{3} = 2.083$$

$$3\lambda_2 = 5 + 1.250 + 3\lambda_6$$

$$\lambda_2 = 2.083 + \lambda_6$$

$$3\lambda_3 = 10 + 1.250 - 3\lambda_6$$

$$\lambda_3 = 3.750 - \lambda_6$$

$$3\lambda_4 = -10 + 1.250 - 0 \times \lambda_6$$

$$\lambda_4 = -2.917$$

Substituting the values of $\lambda_1, \lambda_2, \lambda_3$ and λ_4 in (*f*), we get

$$0\times 2.083+(-3)\times(2.083+\lambda_6)+3\times(3.750-\lambda_6)+0\times(-2.917)+6018\lambda_6=820$$

$$6012\lambda_6 = 815$$

$$\lambda_6 = 0.136$$

$$\lambda_2 = 2.083+0.136=2.91$$

$$\lambda_3 = 3.750-0.136=3.164$$

Thus

$$\lambda_1 = 2.083, \qquad \lambda_4 = -2.917$$

$$\lambda_2 = 2.219, \qquad \lambda_5 = -1.250$$

$$\lambda_3 = 3.614, \qquad \lambda_6 = +0.136$$

From Eqs. (2.106), we get

$$c_1 = \lambda_1 + f_1\lambda_6 = 2.083+38\times 0.136 = 7.251''$$

$$c_2 = \lambda_1 + f_2\lambda_6 = 2.083-38\times 0.136 = -3.085''$$

$$c_3 = \lambda_2 + f_3\lambda_6 = 2.219+11\times 0.136 = 3.715''$$

$$c_4 = \lambda_2 - f_4\lambda_6 = 2.219-14\times 0.136 = 0.315''$$

$$c_5 = \lambda_3 + f_5\lambda_6 = 3.614+37\times 0.136 = 8.646''$$

$$c_6 = \lambda_3 - f_6\lambda_6 = 3.614-34\times 0.136 = -1.010''$$

$$c_7 = \lambda_4 + f_7\lambda_6 = -2.917+12\times 0.136 = -1.285''$$

$$c_8 = \lambda_4 - f_8\lambda_6 = -2.917-12\times 0.126 = -4.549''$$

$$c_9 = \lambda_1 + \lambda_5 = 2.083-1.250 = 0.833''$$

$$c_{10} = \lambda_2 + \lambda_5 = 2.219-1.250 = 0.969''$$

$$c_{11} = \lambda_3 + \lambda_5 = 3.614-1.250 = 2.364''$$

$$c_{12} = \lambda_4 + \lambda_5 = -2.917-1.250 = -4.167''$$

Applying the corrections to the respective angles we get the most probable values of the angles as under:

$$\theta_1 + c_1 = \mathbf{29°17'07.25''}$$

$$\theta_2 + c_2 = \mathbf{28°41'56.92''}$$

$$\theta_9 + c_9 = \mathbf{122°00'55.83''}.$$

$$\text{Sum} = 180°00'00.00''$$

$$\theta_3 + c_3 = \mathbf{62°59'52.71''}$$

$$\theta_4 + c_4 = \mathbf{56°28'01.31''}$$

$$\theta_{10} + c_{10} = \mathbf{60°32'05.97''}.$$

$$\text{Sum} = 180°00'00.00''$$

$$\theta_5 + c_5 = \mathbf{29°32'14.65''}$$

$$\theta_6 + c_6 = \mathbf{32°03'52.99''}$$

$$\theta_{11} + c_{11} = \mathbf{118°23'52.36''}.$$

$$\text{Sum} = 180°00'00.00''$$

$$\theta_7 + c_7 = \mathbf{59°56'04.72''}$$

$$\theta_8 + c_8 = \mathbf{61°00'49.45''}$$

$$\theta_{12} + c_{12} = \mathbf{59°03'05.83''}.$$

$$\text{Sum} = 180°00'00.00''$$

$$\theta_9 + c_9 = \mathbf{122°00'55.83''}$$

$$\theta_{10} + c_{10} = \mathbf{60°32'05.98''}$$

$$\theta_{11} + c_{11} = \mathbf{118°23'52.36''}$$

$$\theta_{12} + c_{12} = \mathbf{59°03'05.83''}.$$

$$\text{Sum} = 360°00'00.00''$$

PROBLEMS

2.1 What do you understand by Adjustment of Survey Measurements? Explain which type of errors is adjusted and why others are not adjusted.

2.2 Differentiate between

(*a*) direct observation and indirect observation,

(*b*) independent quantity and conditional quantity,

(*c*) observation equation, condition equation, and normal equation.

2.3 Explain the significane of weights in survey measurements. What are the rules for assigning the weightage to the survey measurements?

2.4 What are redundant observations? Why is it necessary to have redundant observations? Explain with the help of a suitable example.

2.5 Discuss least squares principle of adjustment of observations.

2.6 Prove the following by least squares criterion:

(*i*) If the observations of a quantity are uncorrelated and of equal precision, the most probable value of the quantity is the arithmetic mean of the observations.

(*ii*) If the observations of a quantity are uncorrelated and of different weights, the most probable value of the quantity is the weighted arithmetic mean of the observations.

2.7 Distinguish between the angle adjustment, station adjustment, and figure adjustment.

2.8 What do you understand by Angle Condition and Side Condition? Take a suitable example to explain these conditions.

2.9 What is spherical excess? Discuss its determination.

2.10 If an angle A was measured six times and the results are as below:

$\angle A$	Weight	$\angle A$	Weight
45°30′ 07″	1	45°30′ 10″	1
56°30′ 10″	1	45°30′ 09″	1
56°30′ 08″	1	45°30′ 10″	1

What is the weight of the weighted arithmetic mean of the observations?

2.11 If an angle A was measured six times, and the results are as below:

$\angle A$	Weight	$\angle A$	Weight
65°31′ 10″	2	65°31′ 10″	3
65°31′ 06″	3	65°31′ 09″	4
65°31′ 08″	2	65°31′ 10″	2

What is the weight of the weighted arithmetic mean of the observations?

2.12 Determine the weight of $(\alpha + \beta)$ if

$$\alpha = 32°24'20'' \quad \text{weight} = 2$$
$$\beta = 20°30'10'' \quad \text{weight} = 4.$$

2.13 If $\alpha = 60°33'30''$, weight = 6, what is the weight of

(*i*) 3α, and (*ii*) $\frac{\alpha}{3}$.

2.14 If weight of the equation $\alpha + \beta = 55°25'45''$ is $\frac{2}{5}$, compute the weight of $\frac{2}{5}(\alpha + \beta)$.

2.15 An angle was measured 6 time by the same observer under similar conditions and the following values were recorded:

1. 32°44′ 30″   4. 32°44′ 40″
2. 32°44′ 10″   5. 32°44′ 20″
3. 32°44′ 50″   6. 32°44′ 30″

Determine the most probable value of the angle by least squares method.

2.16 If the weights of the observations given in Prob. 2.15 are as below, determine the most probable value of the angles using the least squares criterion.

Observation	Weight	Observation	Weight
1	3	4	2
2	1	5	4
3	1	6	3

2.17 Determine the most probable value of the angle α from the follownig equations:

$2\alpha = 46°22'12''$ weight = 1
$3\alpha = 69°33'20''$ weight = 2
$4\alpha = 92°44'21''$ weight = 3.

2.18 The following observations refer to the values of the angles α, β, and γ at a, triangulation station.

$\alpha = 22°16'26.3''$
$\beta = 34°40'31.2''$
$\gamma = 56°56'55.8''$.

Determine the most probable values of the angles if $\alpha + \beta = \gamma$.

2.19 Find the least squares estimate of the angles *P*, *Q* and *R* from the following measured values of the angles at a station:

$P = 38°12'26.5''$
$Q = 32°45'13.2''$
$P + Q = 70°57'38.6''$
$P + Q + R = 126°28'00.6''$
$Q + R = 88°15'37.8''$.

2.20 The distances shown in Fig. 2.14 are measured. All measurements are uncorrelated and have the same precision. The measured distances are as below:

$l_1 = 100.010$ m, $l_3 = 200.070$ m.
$l_2 = 200.050$ m, $l_4 = 300.090$ m.

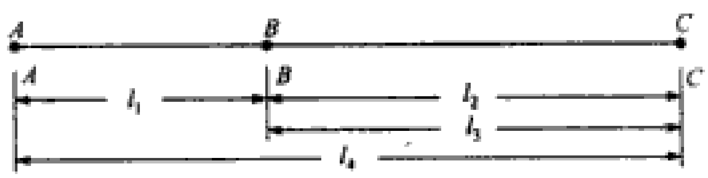


Fig. 2.14.

Use the principle of least squares to determine the adjusted distance between *A* and *C*.

2.21 Use the principle of least squares to estimate the parameters of the straight line, $y = ax + b$, that fits the following data:

x	1	2	3	4	5
y	9.60	8.85	8.05	7.50	7.15

Assume the x-coordinates are error-free constants and the y-coordinates are uncorrelated observations of equal precision.

2.22 Fig. 2.15 shows a level net connecting three bench marks, *A*, *B*, and *C*. The observed differences in elevation are: $h_1 = 20.410$ m, $h_2 = 10.100$ m, $h_3 = 10.300$ m, and $h_4 = 10.315$ m. The arrows indicate the directions of higher elevation. All observations are uncorrelated and have equal precision. Determine the adjusted values of the four elevation differences using the principle of least squares.

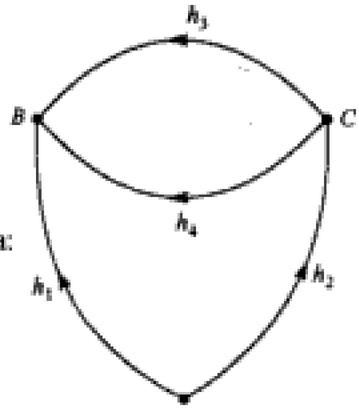


Fig. 215

2.23 (*a*) Adjust the angles *A*, *B*, and *C* of a triangle *ABC* from the following data:

$A = 62°25'30''$
$B = 60°45'15''$
$C = 56°49'20''$.

(*b*) Adjust the above angles if the weights of the angles *A*, *B*, and *C* and 2, 3 and 4, respectively.

2.24 The angles *A*, *B* and *C* of a triangle *ABC* have been observed several times with the following results:

∠A	∠B	∠C
58°51′30″	70°47′24″	50°21′12″
58°51′28″	70°47′22″	50°21′13″
58°51′26″	70°47′25″	50°21′13″
58°51′32″	70°47′23″	50°21′10″
58°51′27″	70°47′26″	
58°51′31″		

Assign weights to the angles by Gauss' rule and determine their least squares estimate.

2.25 Adjust the angles of the triangle *ABC* in Prob. 2.23, by method of correlates.

2.26 The measured values of the angles of the two connected triangles shown in Fig. 2.16 are as below:

$A = 68°12'24''$ $C_1 = 62°18'40''$
$B = 52°28'46''$ $C_2 = 65°57'51''$
$C = 128°16'30''$ $D_1 = 49°28'59''$
$D = 111°02'25''$ $D_2 = 61°33'28''$.

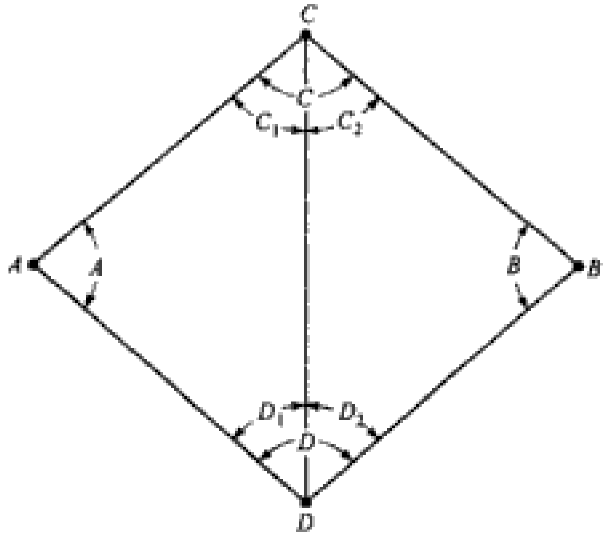


Fig. 2.16

Find the adjusted values of the angles.

2.27 The following are the observed values of eight angles of a quadrilateral shown in Fig. 2.17. The values are corrected for spherical excess and are of equal precision. Determine the most probable values of the angles.

θ_1 = 71°26′ 03.59″ θ_2 = 53°39′ 54.60″

θ_3 = 31°18′ 10.53″ θ_4 = 23°35′ 52.03″

θ_5 = 89°40′ 10.42″ θ_6 = 35°25′ 47.08″

θ_7 = 14°18′ 02.87″ θ_8 = 40°36′ 00.15″.

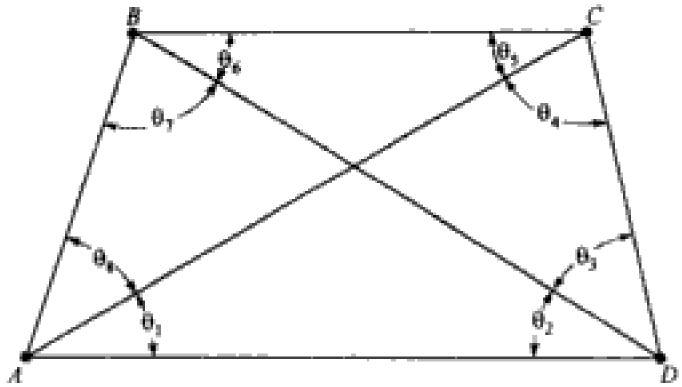


Fig. 2.17

2.28 Solve Prob. 2.27 by approximate method.

2.29 Fig. 2.11 shows a triangle *ABC* with a central station *O*. The mean observed angles are as follows:

∠1 = 24°29′ 24″ ∠2 = 27°30′ 56″

∠3 = 32°03′ 20″ ∠4 = 28°07′ 02″

∠5 = 33°42′ 44″ ∠6 = 34°06′ 33″

∠7 = 127°59′ 34″ ∠8 = 119°49′ 44″

∠9 = 112°10′ 46″.

2.30 The following are the observed values of the angles of a quadrilateral *ABCD* with the central station *O* shown in Fig. 2.12.

θ_1 = 29°17′ 00″ θ_2 = 28°42′ 00″

θ_3 = 60°59′ 49″ θ_4 = 56°28′ 01″

θ_5 = 29°32′ 06″ θ_6 = 32°03′ 54″

θ_7 = 59°56′ 06″ θ_8 = 61°00′ 54″

θ_9 = 122°00′ 55″ θ_{10} = 60°32′ 05″

θ_{11} = 118°23′ 50″ θ_{12} = 59°03′ 10″

Determine the most probable values of the angles by approximate method.

3

TOPOGRAPHIC SURVEYING

3.1 GENERAL

Topographic surveying is the process of determining the location of selected ground natural and man-made features of a given area, and of determining the configuration of the terrain. The location of the features is referred to as *planimetry,* and the configuration of the ground is referred to as *topography* or *hyposography.* The graphical portrayal of the features by conventional symbols is a *topographic map.* Such a map shows both the horizontal distances between the features and their elevations above a given datum. The representation of the difference in elevation is called *relief.*

The purpose of the topographic survey is to gather data necessary for the construction of topographic maps which are very essential for the planning and designing of the most engineering projects such as location of highways, railways, design of irrigation and drainage systems, the development of hydroelectric power, layout of city planning and industrial plants, and landscape architecture. Topographic maps are also helpful for directing military operations during a war.

A topographic survey consists of (1) establishing over the area to be mapped a system of horizontal and vertical control, which consists of selected stations connected by measurements of high precision; and (2) locating the details, including selected ground points, by measurements of lower precision from the control stations.

Topographic surveying is accomplished by ground methods requiring the use of the theodolite, plane table and alidade, level, hand level, tape, and levelling staff in various combinations. Total station EDM's are used to advantage in topographic surveying. The vast majority of topographic mapping is accomplished by aerial photogrammetric methods, or by some combination of ground survey methods and aerial photogrammatric methods.

3.2 SCALE OF TOPOGRAPHIC MAPS

Since a topographic map is a representation of a portion of the surface of the earth, the distance between any two points shown on the map must have a known definite ratio to the corresponding distance between the points on the ground. This ratio is known as the *scale* of the map (*cf.*, Sec. 1.7 of *Plane Surveying*). The topographic maps fall roughly into three classes according to the map scale employed as follows :

1. Large-scale maps: 1 in 1000 or larger
2. Intermediate-scale maps: 1 in 1000 to 1 in 10,000
3. Small-scale maps: 1 in 10,000 or smaller.

The scale to which a map is plotted depends primarily on the purpose of the map. For reconnaissance, a small-scale maps are adequate, but for detailed planning the large-scale maps would be required. The scale is usually decided even before commencing the field work. The scale of the map should be such that the given area can be plotted within the desired limits of the map, and the features are clearly shown on the map. The cost of the map increases as the scale of the map becomes larger. Therefore, the scale of a topographic map is decided on the basis of the features to be shown, the nature of the terrain, and the cost of survey.

3.3 PRECISION REQUIRED FOR TOPOGRAPHIC SURVEYS

If only plotting requirement is considered, the survey details need only be plotted at a precision level consistent with standard plotting precision. A good draftsman usually has a plotting accuracy of 0.25 mm. If the scale of the map is 1 in 500, the distance can be measured to the nearest 12.5 cm. However, if the scales are 1 in 1000 and 1 in 10,000, distances should be measured to the nearest 0.25 m and 2.5 m, respectively. For smaller-scale plans, the precision in measurements can be relaxed even further, but the field methods for large-scale plans should be more precise.

In addition to providing plotting data, topographic surveys also provide the design engineer with field dimensions that must be considered for related construction design. For example, the topographic survey must include the locations and elevations of all connecting pipe inverts for design of an extension to an existing storm sewer. These values are more precisely determined (0.005 m) because of design requirements.

In this regard the following points should be considered with respect to levels of precision:

1. Some detail can be precisely defined and located – for example, building corners, railways tracks, bridge beam seats, sewer and culvert inverts, and the like.
2. Some detail can be located with only moderate precision – for example, large single trees, manhole covers, walkways, and the like.
3. Some detail cannot be precisely defined or located – for example, stream banks, edges of gravel roads, limits of wooded areas, rock outcrops, top/bottom of slope, and the like.

Since most natural features are themselves not precisely defined, topographic surveys in areas having only natural features – for example, stream, or watercourse surveys, site development surveys, large-scale mapping surveys, and the like – can be accomplished by using relatively imprecise survey methods such as stadia surveying (*cf.*, Chapter 8 of *Plane Surveying*).

3.4 METHODS OF REPRESENTING TOPOGRAPHY OR RELIEF

The following systems of representing the relief on a topographic maps are in use:

1. Hachures or hill shading
2. Contour lines
3. Form lines
4. Tinting.

The system used for showing the relief on a map must fulfil the following two purposes:

(*i*) The user of the map should be able to interpret the map as a model of the ground.
(*ii*) It should be able to furnish definite information regarding the elevations of points shown on the map.

Hachures are a series of short lines drawn in the direction of the slope. For steep slope the lines are heavy and closely spaced. For a gentle slope they are fine and widely spaced. Hachures give a general impression of the configuration of the ground, but they do not give the actual elevations of the ground surface.

A *Contour line* or *contour* is an imaginary line that passes through points of same elevation. Since the contour lines on a map are drawn in their true horizontal positions with respect to the ground surface, a topographic map containing contour lines shows not only the elevations of the points on the ground, but also the slope of the various topographic features, such as hills, valleys, ridges, and overhanging cliffs.

The configuration of the ground and the elevations of points are most commonly represented by means of contour lines because contours give a maximum amount of information without obscuring the essential detail portrayed on the map. Chapter 7 of *Plane Surveying* on contouring, illustrates the principles of contours an their characteristics, etc. The readers are advised to refer to this chapter for the details about the contours.

Form lines are used in the maps intended for purposes of navigation to show peaks and hilltops along the coast lines. Form lines resemble contours, but are not drawn with the same degree of accuracy. All points on a form line are supposed to have the same elevation, but not enough points are actually located to conform to the standard of accuracy required for contour lines.

Elevation may also be indicated by *tinting* on the maps intended for special purposes such as for aeronautical charts, and for maps accompanying reports on some engineering projects. The area lying between two selected contours, is coloured by one tint, that between two other contours by another tint, and so on. The areas to be flooded by the construction of dams of different heights, for example, might be shown in different tints.

3.5 PLANNING THE SURVEY

Among the factors that govern the choice of the field methods to be employed in the compilation of a topographic map are

1. the scale of the map,
2. the contour interval,
3. the type of terrain,
4. the nature the project,
5. the equipment available,
6. the required accuracy,
7. the type of existing control,
8. the extent of the area to be mapped, and
9. the economic considerations.

In general, surveys for detailed maps should be made by more refined methods than surveys for maps of a general character. For example,

1. The earthwork estimates to be made from a topographic map must be determined from a map which represents the ground surface much more accurately in both horizontal and vertical dimensions than one to be used in estimating the storage capacity of a reservoir.
2. A survey for bridge site should be more detailed and more accurate in the immediate vicinity of the river crossing than in areas remote therefrom.
3. It is more difficult to maintain a desired precision in the relative location of the points over a large area than over a small area. Control measurements for large area should be more precise than those for a small area.
4. When the errors in the field measurements are considered not to be greater than the errors in plotting, the former are unimportant. The ratio between the field errors and plotting errors, should be roughly one to three.

The area to be mapped for highway or railway location and design is generally in the form of a strip with a width varying from 30 m to perhaps more than 300 m. The control lines for such surveys, are the sides of a traverse established by the method of traversing (*cf.*, Chapter 9 of *Plane Surveying*) and which have been stationed and profiled by the method of levelling (*cf.*, Chapter 6 of *Plane Surveying*). The method of locating of topography most commonly employed for this purpose, is the cross-section method (*cf.*, Sec 6.4.10 of *Plane Surveying*).

For engineering study of the works involving drainage, irrigation, or water impounding or to prepare an accurate map of an area having little relief, each contour line must be carefully located in its correct horizontal position on the map by following it along the ground by trace contour method or direct method of contouring (*cf.*, Sec. 7.5.1. of *Plane Surveying*). When an area of limited extent is moderately rolling, and has many constant slopes, points forming a grid are located on the ground, and the elevations of the grid points are determined (*cf.*, Sec. 7.5.2. of *Plane Surveying*).

If the area to be mapped is rather extensive, the contour lines are located by determining the elevations of selected points from which the positions of points on the contours are determined by interpolation (*cf.*, Sec. 7.6 of *Plane Surveying*).

Contour interval: Contour interval is a fixed vertical distance between the contours plotted on the map. The choice of suitable contour interval depends upon (1) the nature of the terrain, (2) the scale of the map, (3) the purpose and extent of the survey, and the time and expense. Considering these aspects, the suggested contour interval for specific purposes is given in Tables 7.1 and 7.2 of *Plane Surveying*.

The smaller the contour interval, the more refined should be the field methods for locating the contours.

3.6 ESTABLISHMENT OF CONTROL

Control consists of providing (1) *horizontal control*, in which planimetric positions of specified control points are located by trilateration, triangulation, intersection, resection, or traversing, and (2) *vertical control*, in which elevations are established on specified bench marks located throughout the area to be mapped. The horizontal control provides the skeleton of the survey which is later filled in with details. For an extensive survey, generally a primary system of control is established first by more precise measurements, and then it is extended by a secondary system of control by less precise measurements. For small areas only one control system is necessary, corresponding in precision to the secondary control used for larger areas. The required precision of horizontal control depends on the scale of the map and the size of the tract.

3.6.1 Horizontal control

The methods for establishing horizontal control are triangulation, trilateration, traversing, aerial photogrammetric methods, intertial and doppler positioning systems, and the global position system (GPS).

For relatively large topographic surveys, primary and secondary control are established by triangulation and trilateration (*cf.*, Chapter 1). These methods are also employed in areas of smaller extent when field conditions are appropriate (hilly, urban, or rugged mountainous regions).

Traversing with a total station (a theodolite equipped with EDM instrument) can also be used for establishing primary and secondary control (*cf.*, Chapter 9 of *Plane Surveying*).

When the area is large and scale of mapping is small, establishment of horizontal control can be performed by aerial photogrammetric methods. However, the method requires a basic framework of horizontal control points which is established by triangulation and/or trilateration or by inertial, Doppler, or GPS methods.

When the extent of the area is very large, it is most appropriate to establish primary control by inertial, Doppler or GPS methods. These methods can cover inaccessible regions or the regions requiring conduct of survey governed by special conditions. A description of these modern positioning systems is given in Chapter 11.

3.6.2 Vertical control

The purpose of vertical control is to establish bench marks at suitable places all over the area to serve as points of reference for the levelling operations when locating details, and to help in subsequent construction works.

The method of direct differential levelling (*cf.*, Sec. 6.4.1 of *Plane Surveying*) is usually employed for establishing the vertical control but for small areas or in rough country, the vertical control is frequently established by trigonometric levelling (*cf.*, Sec. 6.5 of *Plane Surveying*).

Initially, a first-order level route is established in the area to be mapped. A second-order route is then tied to the first-order route. Normally, the second-order level route follows the traverse lines, and the traverse stations are used as bench marks.

The maximum closing error which is permissible for first-order and second order vertical control is given in Table 6.3 of *Plane Surveying*.

3.7 HORIZONTAL AND VERTICAL CONTROL BY THREE-DIMENSIONAL TRAVERSE

Vertical control may be established concurrently with horizontal control by stadia tacheometry (*cf.*, Chapter 8 of *Plane Surveying*). In this method, which is called as three-dimensional traversing, the elevation and horizontal position of each station are determined using the method of stadia tacheometry. The procedure consists of observing directions by azimuth method, and distances by stadia. The differences in elevation are determined by observing vertical or zenith angles, or stadia arc readings and stadia interval.

The method of establishing control by stadia method, is suitable for small area where the contour interval exceeds 1m, and a relative accuracy of less than 1 in 500 or 600 is satisfactory. For achieving a higher accuracy, total station or electronic tacheometer (*cf.*, Chapter 11) should be used.

3.8 INSTRUMENTS FOR LOCATION OF DETAILS

Location of details which is determination of horizontal and vertical positions of selected ground points, is done after establishing the horizontal and vertical controls. The instruments which are normally employed for location of details are as under:

1. Theodolite and tacheometer
2. Plane table with alidade
3. Level with staff or clinometer
4. Total station or electronic tacheometer
5. Aerial camera (for photogrammetric methods).

Generally, the theodolite and tacheometer are most suited for rough terrain or where ground vegetation restricts the visibility in plane-table survey. These instruments are very useful when several definite points are required to be located.

In an open country where visibility is not a problem, a plane table with an alidade is used with an advantage. Generally, plane-table survey is performed for small-scale mapping of an area of limited extent. Sometimes a tacheometer is used with plane table to measure the horizontal and vertical distances.

A level is used to determine the elevations of the points whose horizontal positions has already been located. A clinometer is also used with plane table for determining the horizontal and vertical locations of the points (*cf.*, Sec. 10.13 of *Plane Surveying*)

Total station and electronic tacheometer give directions, horizontal distances, and elevations of various points, and can be used for topographic survey with an advantage (*cf.*, Chapter 11).

An aerial camera is used to acquire aerial photograph of the area to be mapped using photogrammetric methods. Photogrammetric methods are extremely useful for preparing topographic map of areas of large extent (*cf.*, Chapter 9).

3.9 LOCATION OF DETAILS

Location of details start after the required horizontal and vertical controls have been established. The four principal methods for acquiring topographic detail in the field, are the following:

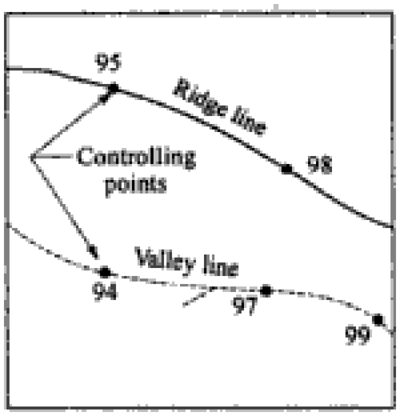
(*i*) Controlling-point method
(*ii*) Cross-profile or cross-section method
(*iii*) Checker-board method
(*iv*) Trace-contour method.

The above methods are analogous to the systems of points used to plot contours described in Chapter 7 of *Plane Surveying*. A combination of these methods may be used for a given survey with the aim of locating maximum number of points with minimum time and effort.

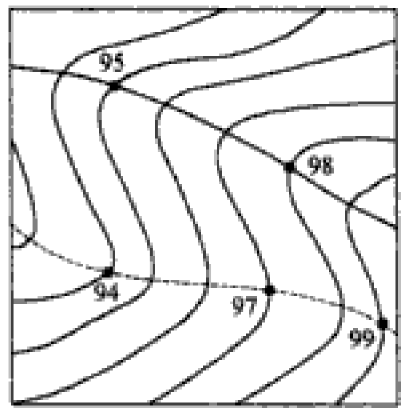
3.9.1 Controlling-point method

In this method a few controlling points at selected locations are established by determining their positions and elevations. These controlling points subsequently help as guide points in drawing the contours. The method has universal application and takes time to master it. The method is economical for small-scale mapping of large areas.

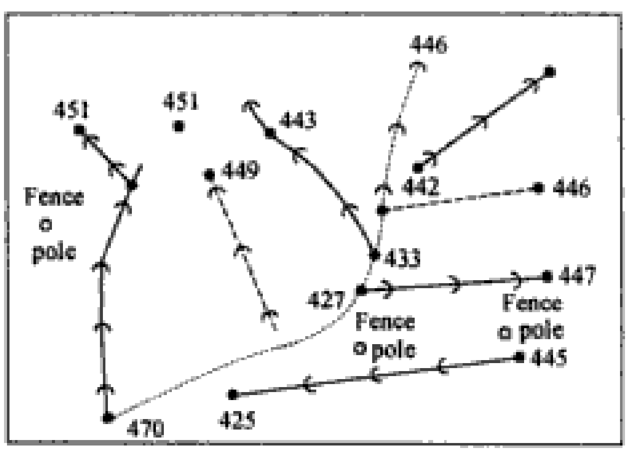
The controlling points can be located by the method of stadia tacheometry using the primary or secondary control. The location of details and elevation of desired points are determined using the established controls points in the area. Field sketches should always be drawn to supplement the observed data. When details are numerous, it is convenient to use a drawing board near the instrument station, to plot the details on a drawing sheet on small scale. The drawings help in drawing the contours.



(a)



(b)



(c)

Fig. 3.1 (a) (b) (c) Controlling-point method

Fig. 3.1a shows the controlling points established along a ridge line and a valley line. Fig. 3.1b shows the use of controlling points in drawing the contours. Other examples of locating the contours by controlling-point method are shown in Fig. 3.1 c and d.

The location of control points and plotting of details can also be done by plane tabling in which a plane table and telescopic alidade are used.

3.9.2 Cross-profile method

In this method a base line or centre-line is selected, and the ground points are taken on lines at right angles to the centre-line. The method is mostly used in route surveying for highways and railways.

The control points are established along the traverse lines at a suitable interval, say 10 m to 20 m depending upon the nature of the terrain, contour interval, and purpose of survey. A theodolite and a steel tape are usually employed to establish the horizontal control. To determine the elevations of the control points, the method of direct differential levelling may be used.

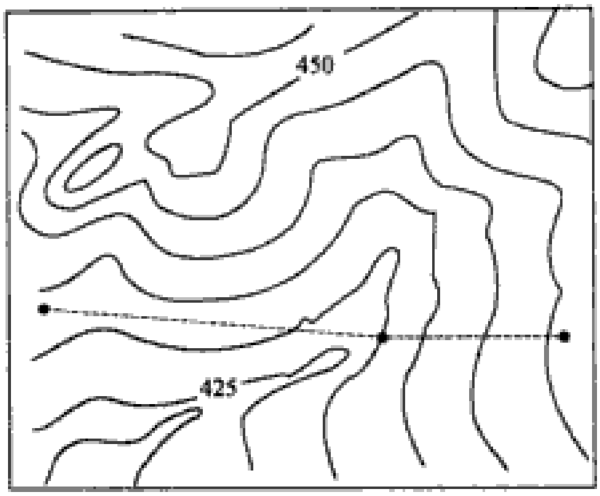


Fig. 3.1 (d) Controlling-point method

The location of details can be done using combination of theodolite, tacheometer, level or hand level, plane table and tape.

Fig. 3.2 shows the cross-section lines marked with the points 1, 2, 3, etc., where the spot levels are taken. From these spot levels the contours are interpolated.

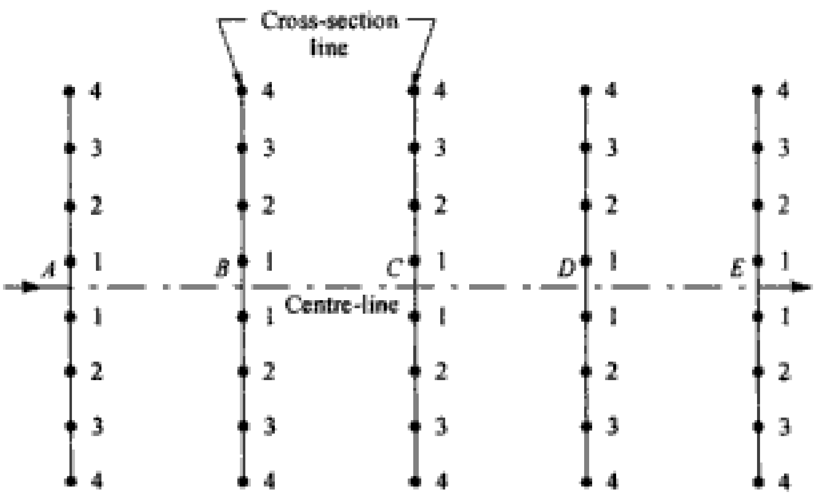


Fig. 3.2 Cross-profile method

3.9.3 Checker-board method

In this method a grid is formed, and the spot levels at the grid points are determined. The contours are interpolated from the spot levels taken at these grid points. The method is suitable for large-scale topographic surveys of areas of limited extent where the topography is fairly regular.

The method consists of running a traverse around the area, and establishing the corner points. The area is then divided into squares or rectangles of uniform size, size depending upon the type of the terrain and the accuracy desired (Fig. 3.3). A line of levels is run around the traverse to determine the elevation of corner points. The elevations of the ground at each grid point are determined by levelling using the known elevation of the corner points. The location of the features such as buildings, roads, fences, traces, etc., is done by measurements from the grid points.

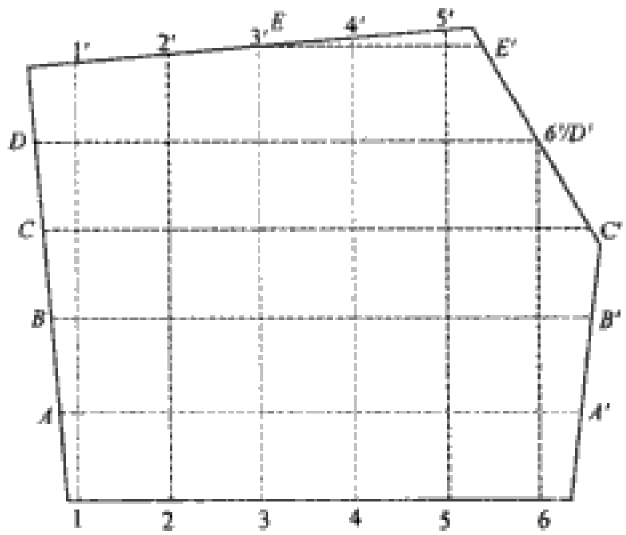


Fig. 3.3 Checker-board method

3.9.4 Trace-contour method

By trace-contour method, the contour is physically traced on the ground by determining the locations of the points on a particular contour line. The method is generally used for large-scale maps where the ground is quite regular (*cf.*, Sec. 7.5.1. of *Plane Surveying*). Generally, a plane table with a telescopic alidade is used to trace the contour in the field.

3.10 ELECTRONIC POSITIONING SYSTEMS FOR TOPOGRAPHIC SURVEYS

Since the horizontal distance and difference in elevation are the basic quantities involved in most of the methods for topographic surveys, the instruments such as total station and electronic tacheometer are ideally suited for gathering data for topographic surveys, particularly by the controlling-point method. The procedures for use of such instruments for a topographic survey by the controlling-point method are similar the those suggested for acquiring topographic details by transit or self-reducing tacheometer. The major differences occur in the operation of the instruments and methods for recording the data (*cf.*, Chapter 11).

3.11 USE OF DTM AND DEM IN TOPOGRAPHIC SURVEYS

The digital terrain model (DTM) or digital elevation model (DEM) is a dense network of points which defines the configuration of the terrain stored in the memory of a computer. DTM or DEM may be processed by a computer software which permits plotting planimetric detail and contours to produce a topographic map.

PROBLEMS

3.1 What is topographic surveying? What are its main uses?

3.2 Describe the major steps involved in performing a topographic survey.

3.3 Differentiate among large -, intermediate -, and small - scale topographic surveys.

3.4 What are the factors which affect the choice of

1. Scale of a map, and
2. Contour interval?

3.5 What factors govern the choice of surveying methods used in topographic surveying.

3.6 Write a short note on the precision required for topographic surveying.

3.7 What are the different methods of representing relief on topographic maps? What are the advantages of representing relief by contours over other methods?

3.8 Discuss the various methods appropriate for establishing horizontal and vertical control for topographic mapping, and the conditions that favour each method.

3.9 What sort of equipment could be required for running a three-dimensional traverse to establish horizontal and vertical control for a topographic survey?

3.10 List the principal methods for acquiring topographic details.

3.11 Discuss in brief the controlling-point method of locating details. What are its advantages and disadvantages?

3.12 The cross-profile method of locating details is best suited to what type of topographic survey?

3.13 Briefly discuss the Checker-board method of locating the topographic details. How would you fix the grid points?

3.14 What are the advantages and disadvantages of locating details by the trace-contour method?

3.15 Describe how electronic positioning systems can be used for locating topographic details.

3.16 Discuss how the digital elevation model (DEM) is useful for topographic surveys.

3.17 Fig. 3.4 shows the controlling points obtained by plane table and alidade. The elevations of the controlling points were determined by differential levelling. Trace the figure and sketch 10-m contour lines. Number and broaden every fifth contour lines including the 600-m line.

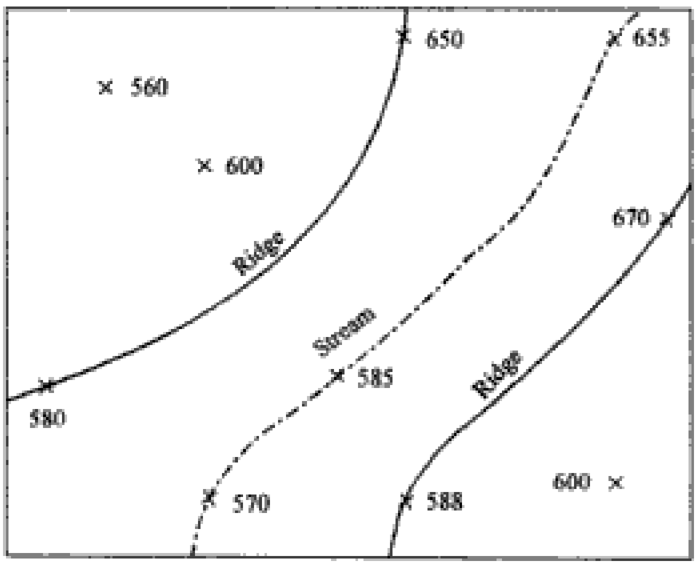


Fig. 3.4

4

HYDROGRAPHIC SURVEYING

4.1 GENERAL

A hydrographic survey is one whose principal purpose is to secure information concerning the physical features of water areas. Such information is essential for the preparation of modern nautical charts on which are shown available depths, improved channels, breakwaters, piers, aids to navigation harbour facilities, shoals, menaces to navigation, magnetic declinations, sailing courses, and other details of concern to mariners. Also, a hydrographic survey may deal with various subaqueous investigations which are conducted to secure information needed for the construction, development, and improvement of port facilities to obtain data necessary for the design of piers and subaqueous structures, to determine the loss in capacity of lakes or reservoirs because of silting, and to ascertain the quantity of dredged material.

Hydrographic surveying has obvious application in the fields of dock and harbour engineering and coast protection works. There are many more applications in civil engineering projects involving irrigation, water supply, water power, flood control, river works, land reclamation, and sewage disposal. The measurement of discharges of rivers and streams is also one of the activities of hydrographic surveys.

The most common uses of the hydrographic surveys can be listed as:

1. Measurement of tides
2. Determination of bed depths
3. Determination of scour, silting, and irregularities of the bed
4. Determination of shore lines
5. Establishment of the mean-seal level
6. Preparation of navigation charts
7. Measurement of discharge of rivers and streams
8. Determination of direction of currents to locate sewer fall
9. Providing help in the planning of projects like bridges, dams, reservoirs, and harbours.

4.2 METHOD OF HYDROGRAPHIC SURVEYS

To locate the subaqueous details on hydrographic charts, a number of suitably special control points are established along the shore line. The depth of water at various points forming a network, is determined by making soundings from stationery boats. The locations of the soundings can be determined by making observations either from the boat to the control points or from the control points to a target fixed in the boat.

Before locating the details in hydrographic survey, the following preliminary steps are required to be carried out:

1. Reconnaissance
2. Establishing horizontal control
3. Establishing vertical control.

4.2.1 Reconnaissance

Although the principal operation of the hydrographic survey is the obtaining the hydrography or making the soundings. This phase of the project cannot be performed until certain preliminary steps are taken. The first of these is a careful reconnaissance of the area to be surveyed in order to select the most-expeditious manner of performing the survey, and to plan all operations so that the survey is satisfactorily completed in accordance with the requirements and specifications governing such work. The use of aerial photographs can be of considerable help in this preliminary study.

4.2.2 Establishment of horizontal control

The horizontal control is a framework by which land and marine features are held in their true relative positions. It usually consists of a series of connected lines whose lengths and azimuths are determined accurately. Triangulation and, to a lesser extent, traverse are most commonly excuted to provide horizontal control. For rough works, a tacheometric traverse or a plane-table traverse may be run. However, for more precise control, a theodolite and a tape traverse is usually run. For extended surveys, where great precision is required, horizontal control is provided by triangulation.

As the general topography, vegetation, type and size of water body railways, and highways of the coast line are the factors which decide the character of control, no definite rules can be given for providing horizontal control for hydrographic surveys. However, the following guide lines may help in selection of the type of control.

1. If the water body is more than one kilometre wide, it is advisable to run two traverses along each shore, connecting to each other by frequent tie lines.
2. If the water body is narrow, i.e., rivers, inlets, etc., and shore conditions are favourable it is advisable to run a traverse line only along one of the banks.
3. If the shore lines are obscured by vegetation it is advisable to adopt triangulation system.
4. For large lakes and ocean shore lines, the horizontal control consists of a network of connected triangulation system on shore.

These triangulation systems are usually, supplemented by traverses run along the shore and connected to the primary triangulation system. A combined triangulation and traverse system is shown in Fig. 4.1.

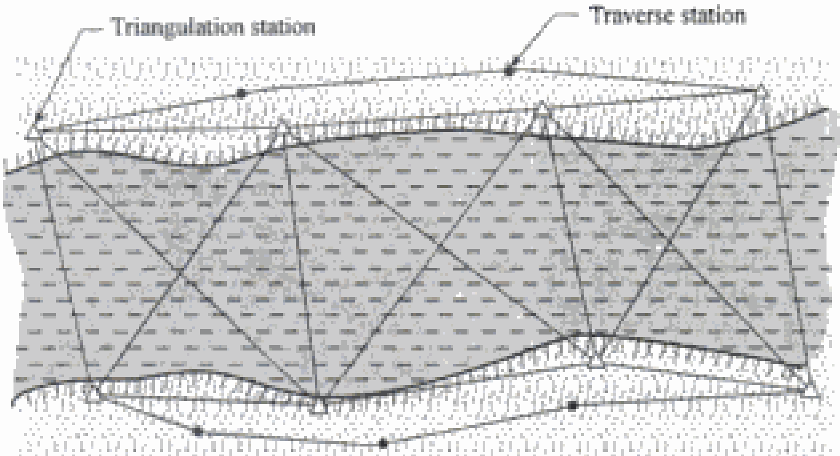


Fig. 4.1 A combined triangulation and traverse system

Previously established control is very important asset in any hydrographic survey. Every effort should be made to obtain and utilize data from earlier surveys.

4.2.3 Establishment of vertical control

Before sounding operations are begun it is essential to establish the vertical control in order that the stage or elevation of the water surface can be known when the soundings are obtained. Vertical control data are also needed for the limited topography shown on all nautical charts.

A number of bench marks are established near the shore line at close spacing to serve as vertical control. The bench marks are used for setting and checking the levels of gauges to which the soundings refer to.

4.3 MEAN SEA-LEVEL

The elevation of the points generally refer to the mean sea-level (M.S.L.) which is considered as datum for all types of surveys. The mean sea-level is determined by making observations extended over a period of 19 years.

The procedure for establishing the mean sea-level as datum, is as below:

1. A gauge is set at a location where it is protected from rough waves and where the water level is not affected by local conditions.
2. The gauge should be located at a low level in sufficient depth of water so that the gauge reading can be taken even at low tide.
3. The zero of the gauge is referred to a bench mark located on shore.
4. The readings of the elevation of water surface are taken continuously for 19 years and the mean of all the readings is the mean sea-level (M.S.L.).
5. The gauge reading corresponding to the mean sea-level is transferred to a permanent point on the shore which serves as a bench mark for determining the elevations of other points with reference to the datum, i.e., M.S.L.

The observations extending over one lunar month give results quite close to the average of 19 years. These results are sufficiently accurate for use in most of the hydrographic surveys.

4.4 SHORE-LINE SURVEYS

Shore-line surveys are conducted for one or more purposes like delineation of shore lines, determination of high and low-water lines, and the location of prominent points on the shore to which the horizontal positions of soundings may be referred. The work involved may be for horizontal or vertical control.

Normal methods of chain survey and plane-table survey are adopted to survey irregularities in the shore line. Points which are clearly visible from the water surface, are selected as reference points such as churchs, temple, light houses, wind mills, transmission pylons, etc., and their locations plotted easily.

Determination of high-water line may be easily judged roughly from the marks on permanent rocks. But to locate the line accurately, method of contouring may be adopted in which a number of points are established at the time of high tide. The line connecting these points so obtained, is the required high-water line. The determination of low-water line is usually done by interpolation from sounding since the low-water line does not remain exposed for a longer time.

4.5 TIDES

Tides are periodical variations in the level of the surface of a large body of water like a sea or an ocean. These variations are mainly due to the attraction of the sun and the moon. Tides have a bearing on a number of aspects of hydrographic surveys.

4.5.1 Tide gauges

Gauges used to measure the height of tidal water over the areas to be surveyed are called *tide gauges.* These instruments measure vertical movements of tides. It is desirable that the site for a gauge must have deep water and shelter from storms. The most commonly used tide gauges are classified as under:

1. Non-registering type of tide gauges
2. Self-registering type of tide gauges.

The non-registering type of tide gauge requires the attention of the observer for taking readings, whereas self-registering type automatically records the readings.

The non-registering gauges are of the following three types:

1. Staff gauge
2. Flat gauge
3. Weight gauge.

Staff gauge: The staff gauge is the simplest type of tide gauge. It consists of a graduated board, 150 to 250 mm wide and 100 mm thick, fixed in a vertical position. It is marked in metres and decimetres from bottom upward. The board is made long enough to cover sufficient height so that the highest and lowest tides may be recorded. The zero graduation is generally below the lowest water level so that all readings are positive. The reduced level of the zero mark is obtained by levelling. The staff gauge is fixed vertically at the site of observation (Fig. 4.2).

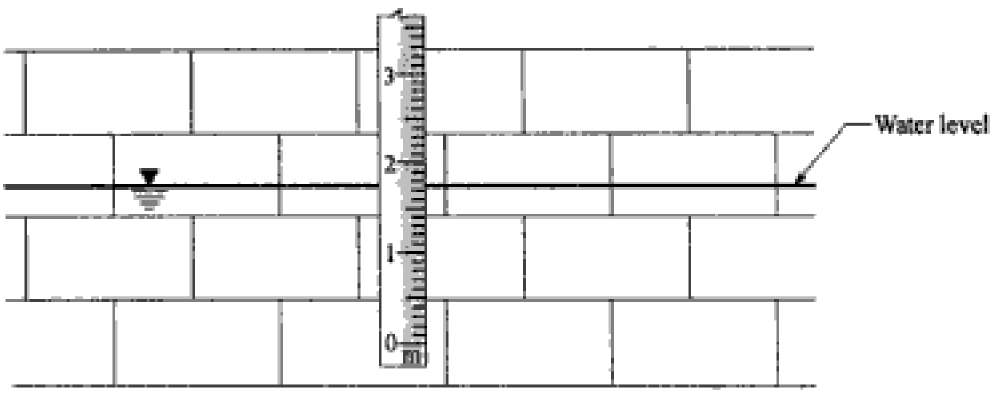


Fig. 4.2 Staff gauge

The staff gauge is read by noting down the readings of crests and troughs of several waves. The two values are recorded and the average value is taken as the water level. The staff gauges are difficult to read when the intensity of tides is high.

Float gauge: The float gauge is designed to overcome the difficulty in reading a staff gauge when the intensity of tides is high, and the variations of water level is more. It consists of a float to which a graduated vertical staff is attached. The float and the staff are enclosed in a stilling well which is generally made of pipe or wooden boards having cross-section of 300 mm × 300 mm with some orifices (holes) in the bottom and the sides of the well to permit entry of the water into the well (Fig. 4.3). The reading is taken against an index mark through a slit window.

Weight gauge: The weight gauge consists a weight attached to brass chain or wire. The chain passes over a pulley, and is laid horizontal along the side of a graduated scale (Fig. 4.4). The weight is lowered to touch the water surface, and the reading is taken on the graduated scale against an index attached to the chain.

Before making use of the weight gauge, it is calibrated. The reduced level of water surface corresponding to the zero reading of the gauge is determined by differential levelling. The foot of the levelling staff is kept against the bottom of the weight when the index of the chain is against zero reading, and a foresight is taken with a levelling instrument.

Although the length of the chain has to match the range of water levels, the graduated scale need not be unduly long, as a second or third index can be attached to the chain at suitable interval.

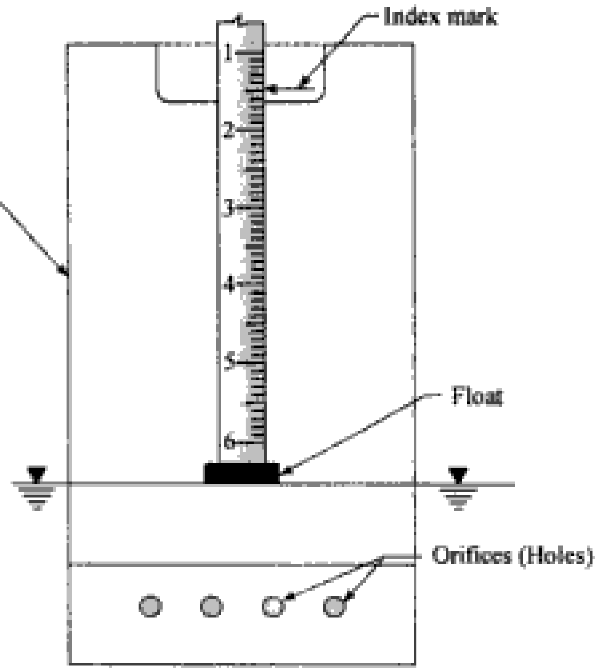


Fig. 4.3 Float gauge

When the gauge is not in use, it is hooked to the board to which the graduated scale is attached.

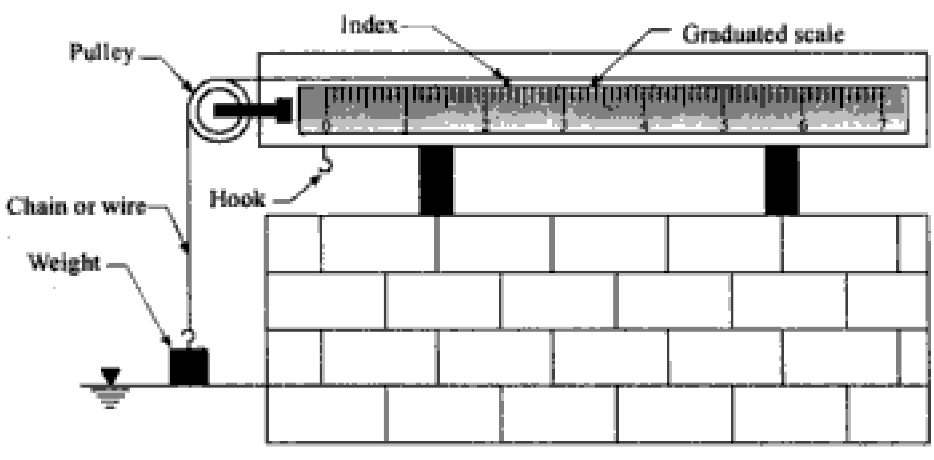


Fig. 4.4 Weight gauge

Self-registering gauges: The self-registering tide gauges automatically record the variation of water level with time. It consists of a float protected from wind and waves. The vertical movement of the weight is transferred through a float wheel and gearing to a stylus (or a pen) which traces a curve on a sheet of paper (graph paper) wound round a drum attached to a clockwork machine. The clockwork machine rotates the drum at a constant speed. The time is indicated on the horizontal axis of the graph, and the water level is indicated on the vertical axis by the stylus. The gauge gives a graphical record of the movement of the float with time. The stylus wire is kept under constant tension by two counterweights (Fig. 4.5). Sometimes the self-registering tide gauges stop working. They require frequent visual checking by an attendant. They are usually housed in a well, constructed under a building in order to minimize the effect of wind and other disturbances.

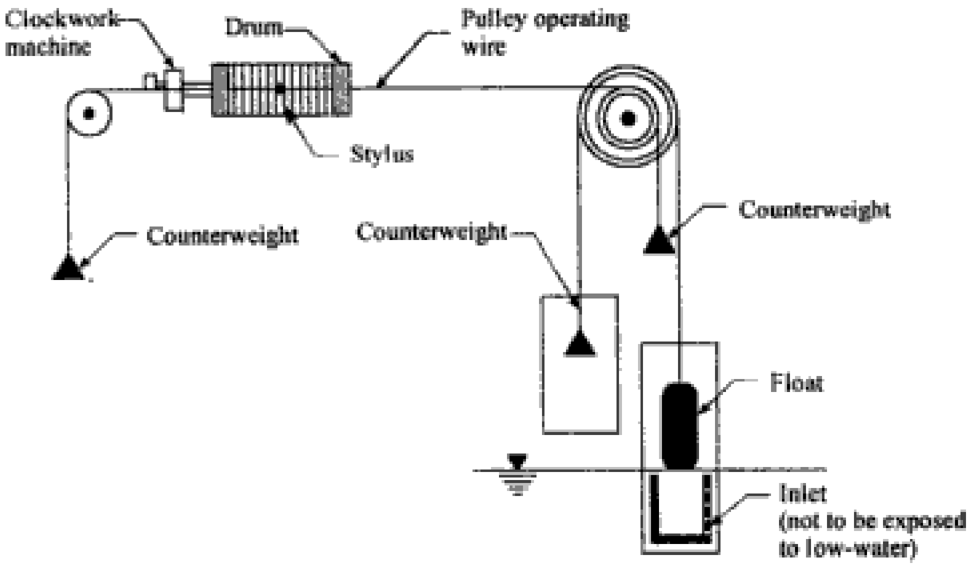


Fig. 4.5 Self-registering tide gauge

4.6 SOUNDING

One of the main objectives of hydrographic surveys is determination of general topography of the bottom of water bodies. The process of determinating depths below the water surface is called *sounding*. Sounding is analogous to levelling on land, and therefore, the reduced level of any point on the bottom of a water body is obtained by subtracting the sounding from the mean sea-level.

Sounding, are required for the following purposes:

1. Preparation of accurate charts for navigation.
2. Determination of the quantities of the material to be dredged.
3. Location of the areas from where material to be dredged, and where to be dumped.
4. Obtaining information for the design of breakwaters, wharves, sea-wells, etc.

4.7 EQUIPMENT FOR SOUNDING

The essential equipment and instrument employed for taking the sounding may be grouped as below:

1. Shore signals and buoys
2. Sounding equipment
3. Angle measuring instruments.

4.7.1 Shore signal and buoys

Shore signals are required to mark the range lines. The line joining two or three signals in a straight line, usually perpendicular to shore line, constitutes the *range line* along which sounding are taken. These are also used for making angular observations from the sounding boats. Signals are made sufficiently conspicuous to make them visible from considerable distances in the sea. They are either 10 cm × 10 cm masts painted white, and firmly braced at the bottom, or 2.5 cm × 2.5 cm poles fitted with iron shoes. Different range lines are distinguished by flags of different colours, attached to the mast or pole of the signal.

A *buoy* is a float made of light wood or hallow air tight vessel properly weighted at the bottom, and is anchored in a vertical position by means of guy wires. A hole is bored through the vertical axis of the buoy to accommodate a flag. In deep waters, the range lines are marked by a signal at shore and buoys in water.

4.7.2 Sounding equipment

Sounding boat: The sounding operation is generally carried out from a flat-bottom boat of low draft. The boats of large size equipped with a motor or launches, are used for sounding in sea. These boats are generally provided with opening, called wells through which soundings are taken (Fig. 4.6). In smaller boats, sounding platforms extended far enough over the sides, are provided so that the sounding line or sounding poles to the bottom of the water body, does not strike the boat.

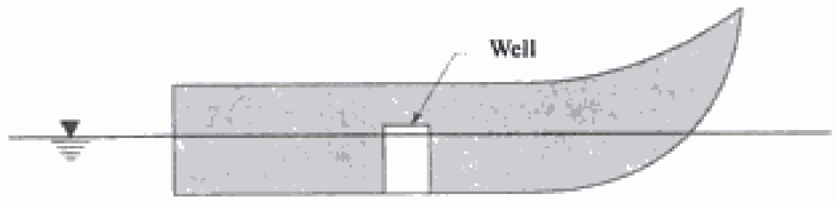


Fig. 4.6 Sounding boat

Sounding pole or rod: These are made of strong well seasoned timber usually 5 to 10 cm in diameter and 5 to 8 m in length. The sounding rods consist of two or three lengths screwed together so that unnecessary length may be removed when not required for soundings in shallow water. A lead shoe of sufficient weight is fitted at the bottom to keep the rod vertical in flowing water, and to avoid sinking in mud or sand. The graduations on the rod are marked from bottom upwards. Thus the readings on the rod corresponding to the water surface, is directly the depth of the water (Fig. 4.7).

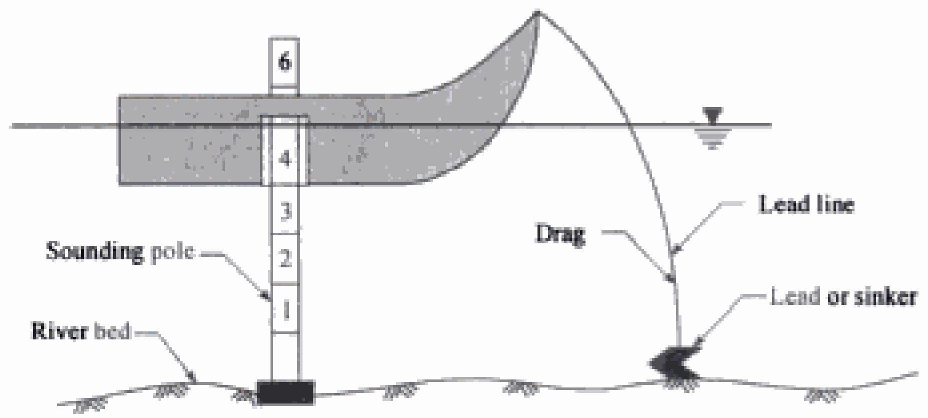


Fig. 4.7 Sounding pole and lead line

Lead line: A lead line is a graduated roap made of hemp or chain attached to the lead or sinker. (Fig. 4.7). Chains of brass are better than hemp lead lines because they maintain their length. To minimise elongation of hemp lead lines due to wetting, the line is first stretched when wet and then dried. The process of wetting and drying is repeated till the stretch becomes negligible. The line is then soaked in water and graduated by tags of cloth or leather at every metre intervals.

The mass of the lead is generally between 5 to 10 kg, depending upon the strength of current and the depth of water. To know the types of soil of the sub-marine surface, leads are sometimes provided with recesses in their bottom for lifting tallow. A correction is required to be applied to the measured length to get the true depth when using the lead line in deep and swift-flowing water. Due to the drag, the measured length will be greater than the true depth (Fig. 4.7).

Weddell's sounding machine: When there is a lot of sounding work, some form of sounding machine attached to sounding line is used. Weddell's hand driven machine consists of a malleable cost iron casing carrying on a spindle a gun-metal barrel. A lead weight (about 8 kg.) carried at the end of a flexible wire cord attached to the barrel, can be lowered at a desired rate, the speed of the drum being controlled by a brake. The spindle is connected through gears to two reading dials. The outer dial records the depth in metres and the inner records in centimetres. The lead may be raised or lowered by a winding handle, and may be suspended at any height by means of pawl and ratchet. A standard machine is designed to measure maximum depths up to 30 to 40 m. The machines is bolted over the well of the sounding boat.

Fathometer (Echo-sounding): A fathometer is an echo-sounding instrument used to determine the depth of oceans indirectly. It works on the principle of recording time of travel by sound waves. Knowing the time of travel from a point on the surface of the water to the bottom of the ocean and back, and the velocity of sound waves, the depth can be calculated. As shown in Fig. 4.8, the sound waves are transmitted from A, and the reflected waves from the bed are received at B. The total time of travel of the waves from A to C and C to B is recorded. The depth D can be calculated knowing the velocity of the sound and the distance AB.

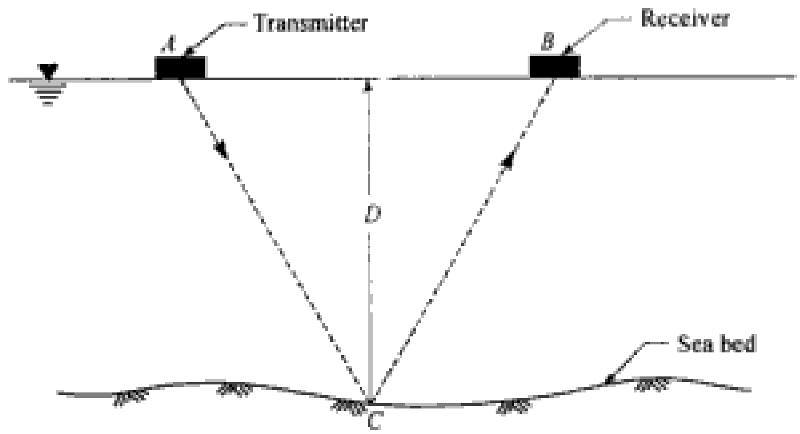


Fig. 4.8 Echo-Sounding

The fathometer gives truly vertical and accurate depths. It is more sensitive than a lead line. It gives a continuous profile by recording the measurements on a drum. The soundings can be made with greater speed. The rocks underlying soft oils are also recorded. Since the velocity of sound waves varies with the density of water, adjustments can be made to read the depth of water of any type.

4.7.3 Angle measuring instruments

Most common instruments for measuring angles are theodolite, poismatic compass, and sextant. The theodolite and prismatic compass are not suitable for angle measurements from sounding boats due to unstability of rowing boats. On the other hand, a sextant has been found to be most suitable for measuring angle in any plane. Navigators and surveyors measure angles from a sounding boat by sextant only. However, when the observations are made from the shore, theodolite and prismatic compass are used.

The sextant used in hydrographic surveying is known as the sounding sextant. It slightly differs in construction from the astronomical sextant, but the method of measuring angles is same (*cf.*, Sec. 4.12 of *Plane Surveying*).

The following points may be kept in view when using a sextant for angle measurements :

1. The angle measured with a sextant is an oblique unless three points sighted lie in horizontal plane.
2. The size of the angles and the length of sights affect the precision of angle measurement.
3. A sextant is not recommended for angle measurements when the angles are less than 15° and the sights are less than 300 m.
4. Vertical angles can also be measured with a sextant in a manner similar to that of horizontal angles.

4.8 LOCATING THE SOUNDINGS

The points where the soundings have been made in the water body, are required to be located for plotting on the drawing sheet. Depending upon the places of observation points from where the locations are made, the methods may be classified as:

1. Observations from the shore
2. Observations from the sounding boat
3. Observations from both the shore and the sounding boat.

4.8.1 Observations from the shore

The points where soundings have been made, can be located entirely by making observations from the shore. Some of the common method are discussed below.

Location by transit and stadia or tacheometer

In this method, the shore line is divided by a number of range lines *A'A, B'B, C'C*, etc., at equal intervals (Fig. 4.9). The instrument is set at *A, B, C*, etc., and signals are erected at *A', B', C'*, etc., for ranging. The boatman rows the boat in a direction *A'A* which is perpendicular to the line *AD*. When the boat is at the point a_1, the sounding is taken, and also at the same time stadia readings are recorded. The boat is then rowed to the point a_2 along the same range line, and the sounding as well as stadia readings are recorded. Similar observations are made for other points on the range line *A'A* till stadia rod is clearly visible from *A*. The soundings and the stadia readings are recorded for other range lines also in a similar manner.

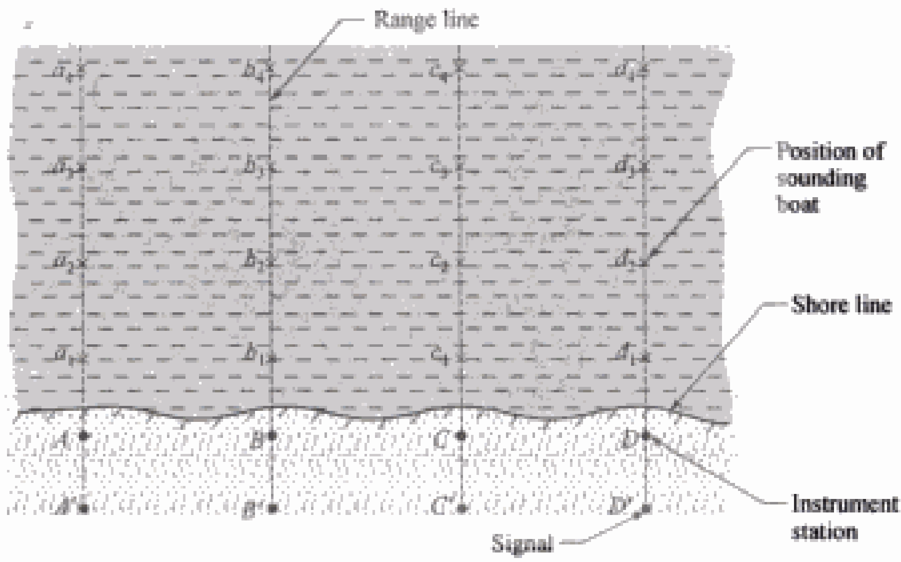


Fig. 4.9 Transit and staida method

The following points should be noted while locating the points from the shore:

1. The theodolite should be set up close to the water surface to take horizontal sights, and avoid measurement of vertical angles.
2. Though the method is rapid and sufficiently accurate it is suitable only in shallow and smooth water.
3. The method is unsuitable for long sights.

Location by range and one angle from the shore

In this method also the shore line is divided by a number of range lines *A'A, B'B, C'C*, etc. at equal intervals, and the instrument stations *A, B, C*, etc., and the signal stations *A', B', C'*, etc., are fixed. When the sounding is taken at a_1, which is on the range line *AA'*, *the angle* θ is also observed from *B* using a theodolite (Fig. 4.10). Knowing the distance *AB* the distance Aa_1 is calculated.

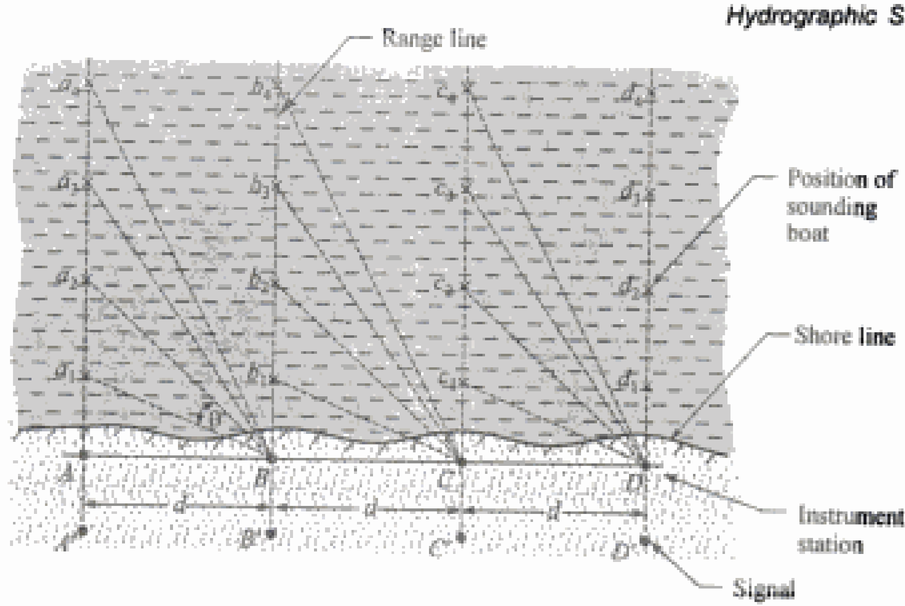


Fig. 4.10 Range and one angle from shore

Let $AB = BC = CD = d$, then from the right angled triangle BAa_1, we have

$$Aa_1 = d \tan \theta \qquad \text{...(4.1)}$$

In a similar manner the distances of points on other range lines can be determined. The time of all soundings are recorded. Every tenth sounding is usually located by observing angles and the intermediate soundings are located by interpolation. The method is quit convenient and gives reasonable accuracy.

Location by two angles from the shore line

This method is used when it is not possible to keep the sounding boat on a fixed range line or where the shore topography does not permit laying out a system of intersecting lines. The method requires two theodolites setup at two stations A and B on the shore line, the distance between them being known (Fig. 4.11). The instrument stations A and B should be selected in such a way that they provide good intervisibility and intersections.

Let the sounding boat be at P. When the sounding is taken, the boatman gives a signal to the observers at A and B, and the angles α_1 and α_2 are measured. If the distance AB is d, the distances x and y for P can be computed from the following expressions.

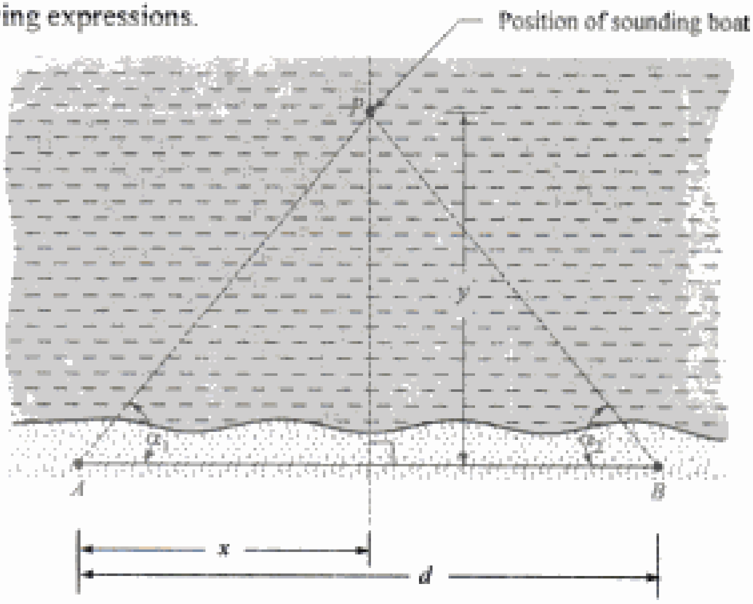


Fig. 4.11 Two angles from shore line

$$x = \frac{d \tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \quad ...(4.2)$$

and

$$y = \frac{d \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 + \tan \alpha_2} \quad ...(4.3)$$

The time at which sounding is taken should be noted. If the bearing of the line AB is known, the bearings of the lines AP and BP can be computed.

The method gives good results if the angle of intersection APB is nearly 90°. The disadvantages of the method are that it requires two instruments, and it requires much time in occupying new stations of observations.

4.8.2 Observations from the sounding boat

The method requires observations taken from the sounding boat with a sextant at the time of sounding. The location of the point where sounding is taken, can be fixed by any one of the following methods:

1. Range and one angle from the boat
2. Two angles from the boat.

Location by range and one angle from the boat

In this method the stations A, A', B, B', C, C', etc., are fixed, and signals are erected on these stations to facilitate in ranging (Fig. 4.12). These stations are fixed in a similar manner as in transit and stadia method. The distance d between the stations is measured.

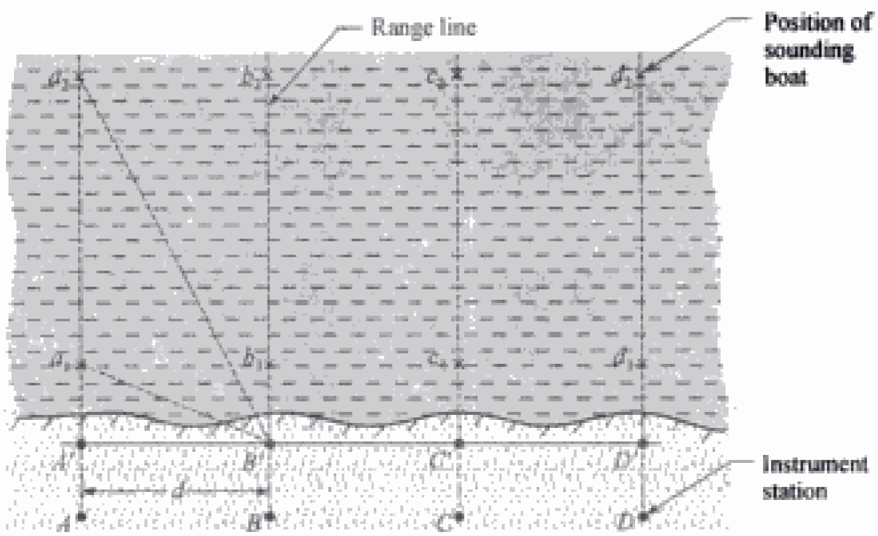


Fig. 4.12 Range and one angle from the boat

For location of soundings at points on the range line AA', the observations are made on the signal B. The boat is rowed along the range line AA' to the sounding point a_1, and the angle Aa_1B is measured with the sextant, and time of observation is noted. The required distance Aa_1 is given by

$$Aa_1 = AB \cot \alpha_1$$

$$= d \cot \alpha_1 \quad ...(4.4)$$

where angle Aa_1B is equal to α_1.

Similarly, the points a_2, a_3, etc., are located. The procedure is repeated for location of soundings on other range lines.

The method being not accurate is rarely used in practice. The only advantage of the method is the better control over the field work as the angles are measured from the sounding boat.

Location by two angles from the boat

In this method, three prominent shore signals *A*, *B*, and *C* as shown in Fig. 4.13, whose coordinates are known, are selected. At the time of sounding at *P* the angles α_1 and α_2 are measured simultaneously with a sextant from the sounding boat. The coordinates of *P* can be calculated as explained in Sec.1.23.2. Before taking observations it should be ensured that the point where soundings are taken, do not fall on the circumference of the circle passing through *A*, *B*, and *C* to avoid failure of fix. The failure of fix can be avoided by checking that $\alpha_1 + \alpha_2 + \phi \neq 180°$. The precision of the method is poor when *P* is near the circumference of the circle passing through *A*, *B*, and *C*. If *A*, *B*, and *C* are in a straight line or if *B* is nearer to *P* than *A* and *C* then location is strong unless one of the angles α_1 and α_2 is very small. If the sights *PA*, *PB*, and *PC* are extremely long, the angles α_1 and α_2 will be small and the location will be weak.

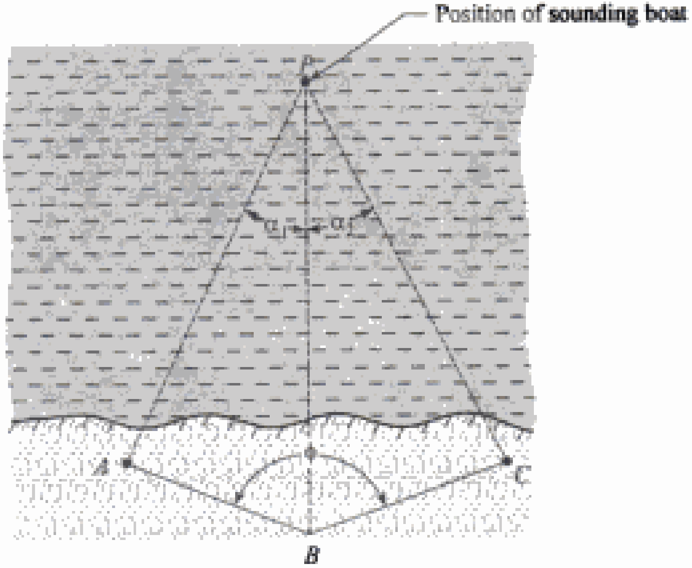


Fig. 4.13 Two angles from the boat

4.8.3. Observations from both the shore and the sounding boat

For fixing the location of soundings by making observations from both the shore and the boat, the following methods are commonly employed:

1. Location by range and time interval
2. Location by intersecting range lines
3. Location by one angle from the shore and one angle from the boat
4. Location by stretched wire across a river.

Location by range and time interval

In this method a number of range lines *A'A*, *B'B*, *C'C*, etc., as shown in Fig. 4.9, are set up at convenient intervals. The sounding boat is rowed from the shore along the range line A'A and the sounding at a_1 is taken after the boat has attained a constant speed, and the time of sounding is also recorded. The boat is rowed along the range line *A'A* and the soundings are taken at regular interval of time. The distances of the sounding points are calculated from the speed of the boat and the interval of time between the succeeding soundings.

The method is employed where accurate results are not required. It is used in still water for short distances. The method is useful for obtaining the locations of intermediate soundings between two soundings located by other methods.

Location by intersecting range lines

The method is generally employed to determine the changes in the topography of the bottom of a water body due to silting or scouring, or to determine the quantity of material removed by dredging. To determine these changes it is necessary to repeat the soundings at the same point.

In this method the ranges *A'A*, *B'B*, *C'C*, etc., are fixed permanently by means of the signals *A*, *A'*, *B*, *B'*, *C*, *C'*, etc. (Fig. 4.14). The sounding boat is rowed along the range line *A'A* for taking sounding when a_1 is also in range with *B* and *C'*, i.e., a_1 is the intersection of the lines *A'A* and *C'B* produced. Similarly, the a_2 is located by the intersection of ranges *A'A* and *D'C*. In a similar manner locations on the other ranges *B'B*, *C'C*, etc., are determined.

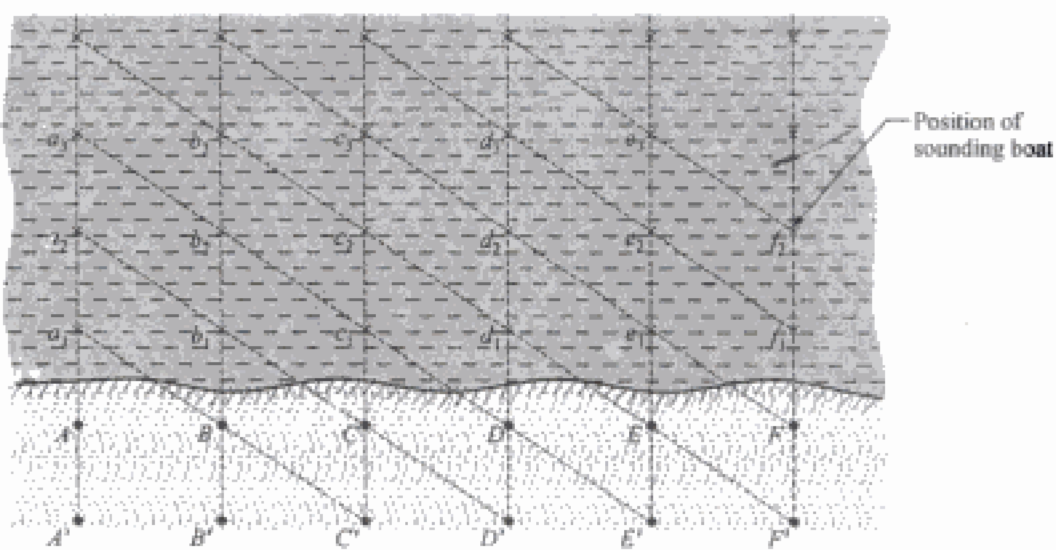


Fig. 4.14 Intersecting ranges

The accuracy of the method depends upon the distance between the intersecting ranges. The longer the distance, the more the accuracy.

Location by one angle from the shore and another from the boat

The method requires use of a theodolite for measuring angle from the shore, and a sextant for measuring the angle from the boat. On the shore two points *A* and *B* are selected (Fig. 4.15). Let the point *A* be the theodolite station and point *B* be the shore signal or any other prominent point. The distance *AB* is measured, and let it be *d*. To locate the sounding point *P*, the angles α and β are measured with the theodolite at *A* and with the sextant at *P*, respectively. The coordinates *x* and *y* of *P* are computed from the following relations.

$$x = \frac{d\sin(\alpha+\beta)}{\sin\beta}\cos\alpha \qquad \text{...(4.5)}$$

and

$$y = \frac{d\sin(\alpha+\beta)}{\sin\beta}\sin\alpha \qquad \text{...(4.6)}$$

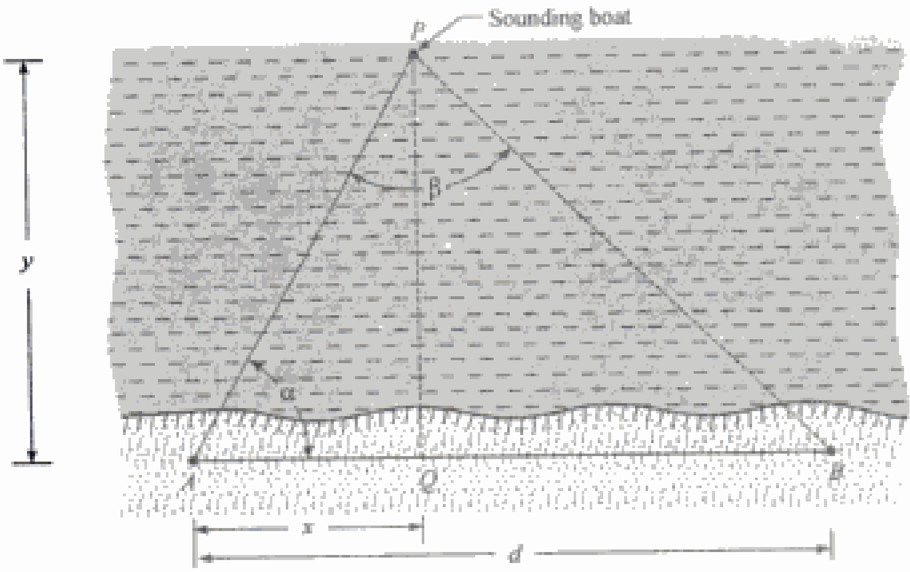


Fig. 4.15 One angle from the shore and another from the boat

Location by stretched wire across a river

This method may be employed when the river is of narrow width, and it is possible to stretch a wire between two points fixed on opposite banks. The width is divided into a number of segments by attaching metal tags or cloth pieces (Fig. 4.16). The distances of each tag are measured from a reference tag, called zero tag. The zero tag may be defined by a plumb bob. The boat is rowed to the points under each tag, and soundings are taken.

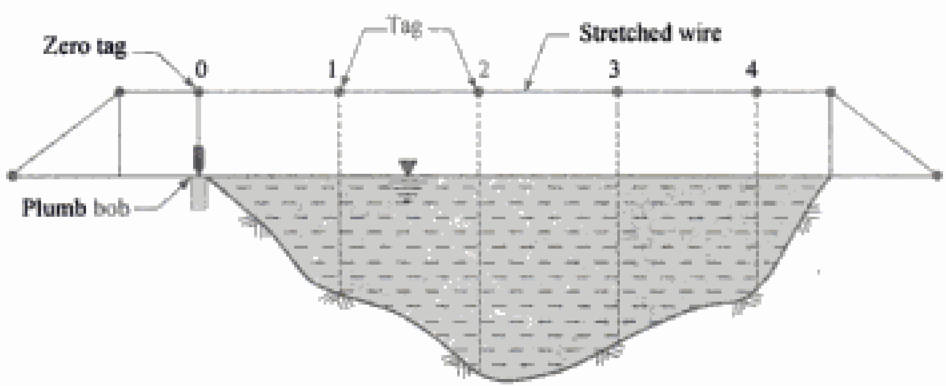


Fig. 4.16 Distances along stretched wire

This method is most accurate, but most expensive. This method is also used to survey the depth contours, submarine contours, fathoms or barthymetric contours of the river bed. As shown in Fig. 4.17, a theodolite traverse *ABCD* is run along one of the banks, and a number points *A*, *B*, *C*, *D*, etc., are established at 30 to 40 m apart. The soundings are taken by stretching wires across the river preferably in a direction perpendicular to the traverse lines.

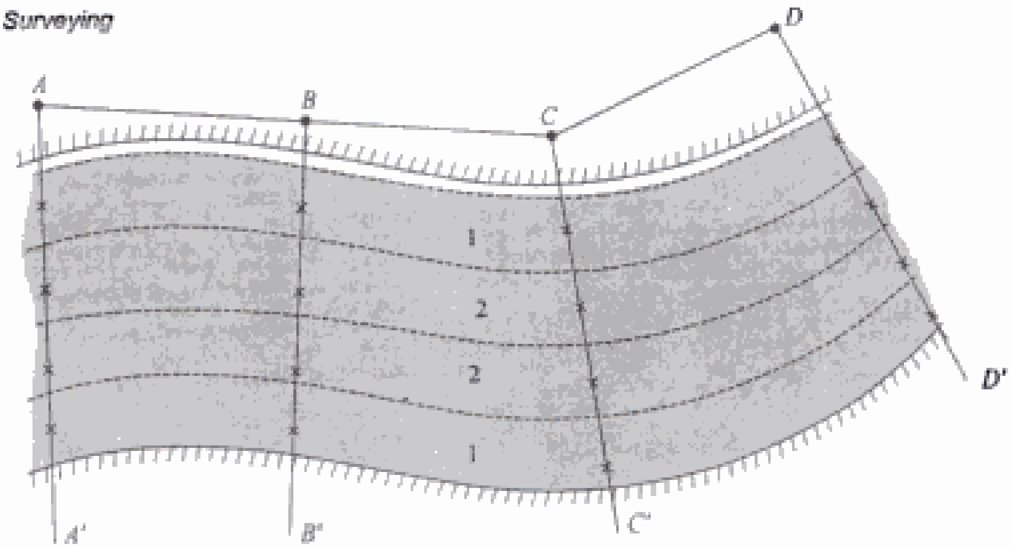


Fig. 4.17 Submarine or depth contours of a river bed

The depths of the points determined by soundings, are plotted on desired scale, and submarine contours are interpolated as in the case of contouring by spot levels.

4.9 REDUCTION OF SOUNDINGS

In tidal rivers and oceans, an appropriate safe route is required for navigation of vessels during low tides. For this a common datum of reduction of soundings, mean low water of spring tides (M.L.W.S. or L.W.O.S.T.) is commonly adopted. M.L.W.S. or L.W.O.S.T. is the lowest level of water in tidal areas. Since the soundings refer to existing water level at the time of observations, they are required to be reduced to the assumed datum, i.e., M.L.W.S. This is made possible by taking gauge readings at the time of soundings. The tidal gauge readings are recorded at every 5 minutes. For any sounding between five minutes of time, gauge readings can be obtained by interpolation. The soundings are reduced to M.L.W.S. by applying corrections to the soundings. The difference of levels between the gauge reading, i.e., actual water level at the time of soundings and M.L.W.S., is the required correction to be applied to the soundings. The correction may be positive or negative depending upon whether the actual water level is greater than the gauge reading or less then the gauge reading.

Let us assume that (Fig. 4.18)

G = the gauge reading at M.L.W.S.,
h = the gauge reading at the instant of sounding,
s_1, s_2, s_3, etc. = the soundings at locations 1, 2, 3, etc., and
c_s = the corrections to the soundings.

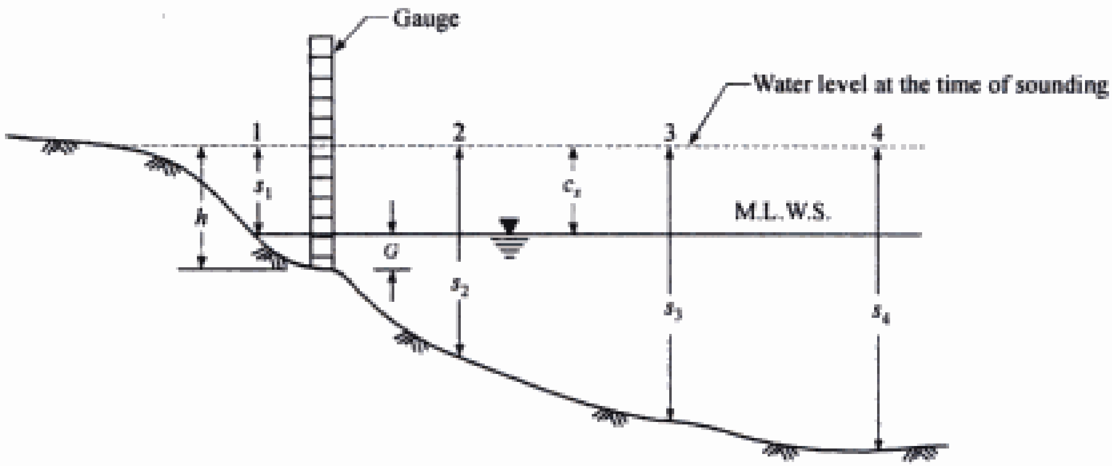


Fig. 4.18 Reduction of soundings.

then c_s *is given by*

$$c_s = G - h \qquad \text{...(4.7)}$$

and the reduced sounding is given by

$$S = s - c_s \qquad \text{...(4.8)}$$

where s is the sounding at any point.

Thus, the reduced sounding at the points 1, 2, 3, etc., are

$$S_1 = s_1 + c_s$$
$$S_2 = s_2 + c_s$$
$$S_3 = s_3 + c_s$$

The M.L.W.S. can be reduced to means sea-level by determining the level of the zero mark of the gauge by differential levelling.

ILLUSTRATIVE EXAMPLES

Example 4.1 Following observations were made from two stations A and B, the distance AB being 3.2 km, and the bearing AB to a sounding boat P being 50°40′:

$$\angle ABP = 30°50'; \ \angle BAP = 60°10'$$

Determine the coordinates of P if the coordinates of A are

$$x = 1000 \text{ m}; \ y = 800 \text{ m}.$$

Solution: (Fig. 4.19)

In ΔAPB, we have

$$\angle APB = 180° - (\angle BAP + \angle ABQ)$$
$$= 180° - (60°\,10' + 30°50')$$
$$= 89°$$

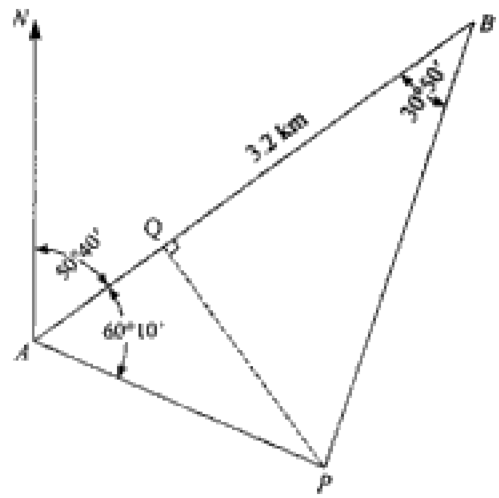


Fig. 4.19

By sine rule for ΔAPB, we have

$$\frac{AP}{\sin 30°50'} = \frac{BP}{\sin 60°10'} = \frac{3.2}{\sin 89°}$$

Therefore,
$$AP = \frac{3200 \times \sin 30°50'}{\sin 89°} = 1640.38 \text{ m}.$$

$$BP = \frac{3200 \times \sin 60°10'}{\sin 89°} = 2776.35 \text{ m}.$$

$$W.C.B \text{ of } AP = \text{W.C.B. of } AB + 60°10'$$
$$= 50°40' + 60°10'$$
$$= 110°50'$$

Therefore,
$$R.B. \text{ of } AP = 180° - 110°50'$$
$$= 69°10'$$

$$\text{Latitude of } AP = AP \cos 69°\,10' = 1640.38 \times \cos 69°10'$$
$$= 583.40 \text{ m}.$$

$$\text{Departure of } AP = AP \sin 69°\,10' = 1640.38 \times \sin 69°10'$$
$$= 1533.13 \text{ m}.$$

Therefore,
$$x\text{-coordinate of } P = x\text{-coordinate of } A + \text{departure of } P$$
$$= 1000 + 1533.13 = \mathbf{2533.13}.$$

and
$$y\text{-coordinate of } P = y\text{-coordinate of } A - \text{latitude of } P$$
$$= 800 - 583.40 = \mathbf{216.60}.$$

Example 4.2 The following soundings were taken when the gauge reading was 4.65 m:

1.5 m, 3.6 m, 5.7 m, 8.8 m, 10.3 m.

If the gauge reading at the lowest water of spring tide (L.W.O.S.T.) is 3.10 m, reduce the soundings to L.W.O.S.T.

Solution:

It is given that

$$G = 3.10 \text{ m}$$
$$h = 4.65 \text{ m}$$

From Eq. (4.7), the correction to the soundings is given as

$$c_s = G - h = 3.10 - 4.65 = -1.55 \text{ m}$$

The reduced soundings from Eq. (4.8), are

$$S_1 = 1.5 - 1.55 = -0.05 \text{ m}$$
$$S_2 = 3.6 - 1.55 = 2.05 \text{ m}$$
$$S_3 = 5.7 - 1.55 = 4.15 \text{ m}$$
$$S_4 = 8.8 - 1.55 = 7.25 \text{ m}$$
$$S_5 = 10.3 - 1.55 = 8.75 \text{ m}$$

The reduced sounding at point 1 being negative indicates that at the time of lowest tide of spring this point would be exposed to air by 0.05 m.

4.10 PLOTTING OF LOCATIONS OF SOUNDINGS

The method of plotting the locations of soundings depends upon the method employed for the location of sounding.

If the soundings have been made along the range lines, the position of signals and range lines are plotted on the drawing sheet. The locations of the soundings are plotted on the respective range lines by measuring the corresponding distances. When the locations of the soundings are determined by two angles from the shore, the lines are drawn at the corresponding angles, and the points of intersection of the lines are the desired locations. If the method of range and one angle form the boat has been used, the range line is plotted, and the soundings are plotted by measuring the calculated distances from the shore signals. For the location by measuring two angles from the boat, the sounding points are plotted by determining the positions discussed in Sec. 1.23.2. Alternatively, an instrument called *station pointer* or *three-arm protractor* may be used to plot the locations of the soundings. The use of station pointer is discussed below.

A station pointer consists of three arms and a circle (Fig. 4.20). The two outer arms A and C can be rotated with the help of tangent screws about the centre. The middle arm B is fixed such that its fiducial edge corresponds to zero of the circle. The circle is graduated in both directions from 0° to 360°. With the help of verier and tangent screw, the movable arms can be set to read angles upto 1 minute accurately.

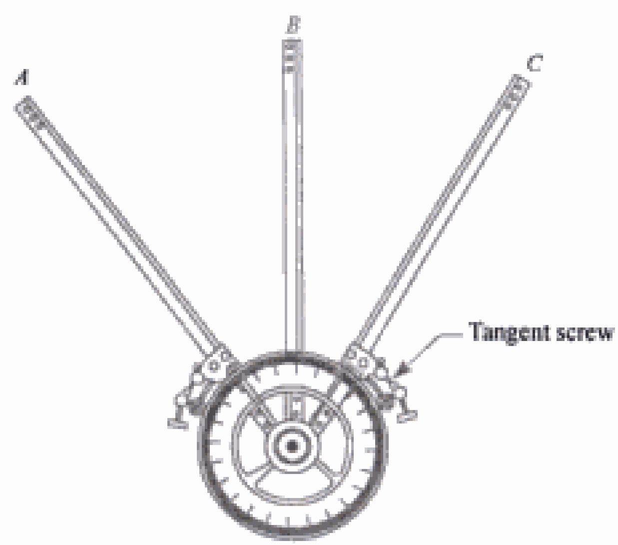


Fig. 4.20 Station pointer

For locating a sounding point P, the movable arms are set to read the angles α_1 and α_2 nearest to 1′. The station pointer is laid on the plan on which the positions of three known shore stations A, B, and C are already plotted. The station pointer is then laid on the plan, and moved until the bevelled edges of the three arms simultaneously pass through A, B, and C. When the pointer is in that position, the centre of the station pointer is pricked to obtain the location of P.

The location of a point using three points of known locations is the three point problem discussed in plane-table survey (*cf.*, Sec. 10.7.4 of *Plane Surveying*). Using a station pointer for solution of three point problem, is a mechanical process of the solution. It is a convenient and widely used method in practice. It gives accuracy of plot approximately the same as that of the sextant with which the angles are measured in the field. The extra lengths of the arm pieces are also supplied by the manufacturers with instrument to extend the arms if necessary.

4.11 THE CAPACITY OF A RESERVOIR OR LAKE

To determine the capacity of a reservoir or a lake, sounding data with their plan positions are required to plot depth contours on the plan. The area of each contour is found by planimeter, and the volume can be calculated by the prismoidal formula. The volume can also be found by multiplying the average depth with average area when accurate results are not required.

Alternatively, the area of each vertical section along the range lines from sounding is determined, and the volume of the reservoir or lake is obtained by the end-area method or the prismoidal formula.

The volume of dredged material can be determined by comparing the volumes before and after dredging, or by comparing the depths to the bed before and after dredging, and computing the difference of volumes.

4.12 STREAM GAUGING (RIVER SURVEYING)

Stream gauging is a process of measurement of discharge of streams and rivers. It is required by the engineer in connection with problems of water supply, irrigation, and power development. To obtain accurate knowledge of stream discharge, regular observations are made over a period of years. These observations include measurements of the maximum and minimum flow, and their duration.

Discharge may be defined as the rate at which water in stream flows past a given section. The unit of discharge commonly used is cubic metres per second or cumecs (m^3/s), and the unit for the velocity is metre per second (m/s).

The following methods are generally used to measure stream discharge:

1. Area-velocity method
2. Weir method
3. Chemical method.

4.12.1 Area-velocity method

In this method, the area is measured at various cross-sections of the stream by sounding in water or by ordinary levelling in dry beds and the velocity is measured either by using floats or current meter, or by calculation from the surface slope.

To obtain reliable results, the site at which the measurements for discharge are going to be made, should be such that the stream is nearly straight for some distance on either side. Also the bed should be free of obstructions, uniform in shape and character.

Velocity measurement using floats

Since the velocity of the stream varies throughout its cross-section in a manner which depends on the shape of the section, the depth of the water, and the roughness of the bed, there are different types of floats giving different accuracy of measurements. The minimum velocity occurs at the bed owing to friction, and the maximum occurs a little below the surface and near the middle. Fig. 4.21 shows a typical velocity distribution in a vertical section.

The distribution of surface or mean velocity in a section from one side of the stream to another, depends on the shape of the cross-section. If the bed is perfectly regular the variation is parabolic, with the maximum value occurring at midstream.

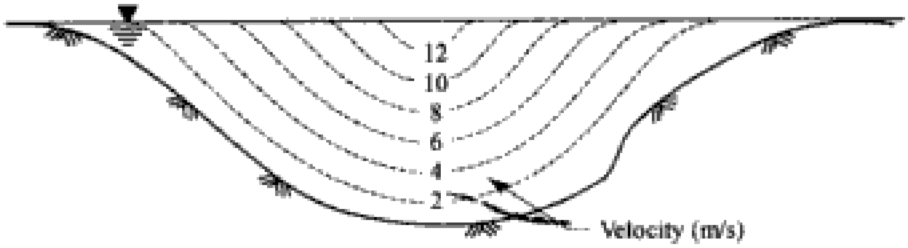


Fig. 4.21 Velocity distribution

Floats used for velocity measurements are of the following type:

(*i*) Surface floats
(*ii*) Sub-surface floats
(*iii*) Rod floats.

Surface floats: The surface floats are used to measure the surface velocity of the water (Fig. 4.22*a*). They are light in weight, and their shape is such that they offer least resistance to floating debris and eddy currents. Surface floats are not suitable in windy conditions.

The surface floats give the surface velocity, and to get the average velocity the surface velocity is multiplied by 0.85 to 0.9.

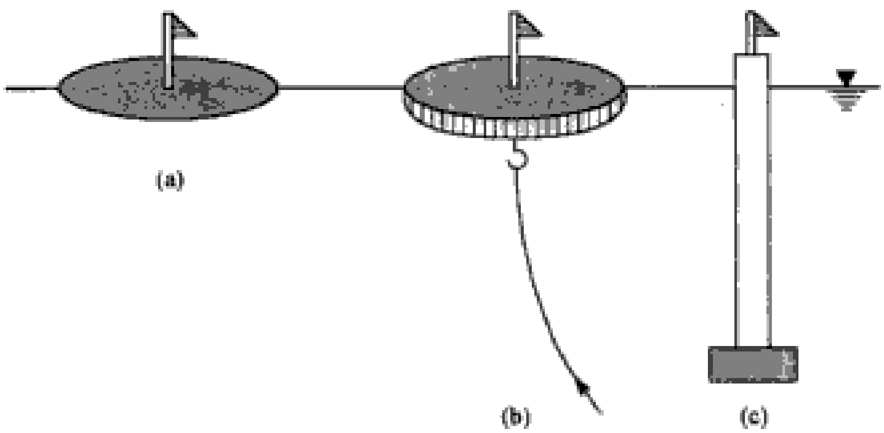


Fig. 4.22 (a) Surface float; (b) Sub-surface float; (c) Rod float

Sub-surface float: A sub-surface float also called a *double float* consists of a small float to which is attached a second float slightly heavier than water by a light and strong cord of adjustable length (Fig. 4.22b). The second float which is submerged, may be a hallow cylinder or a kite which offers resistance equally in all directions. It is assumed that the sub-surface floats move at a velocity of the water at the depth of the cylinder or kite. The length of the cord is adjusted so that the float directly gives mean velocity.

Sub-surface floats give better results than the surface floats because they are not much affected by wind and surface eddies.

Rod float: Rod floats are usually wooden rods or hallow tubes of copper or brass of 3 to 5 cm diameter, 2 to 6 m long, and weighted at the bottom so that they float vertically with just a small length exposed above water (Fig. 4.22c). The length of the rod should be adjusted to clear obstructions in the bed of streams. This can be ensured by taking the length of the float as $9/10^{th}$ of the depth. Velocities obtained by the rod floats are slightly greater than the actual mean velocity because of the clearance required at the bottom to avoid fouling. The percentage error varies with the depth of immersion of the rod and also with the shape of the channel.

The rod floats are unsuitable when the bed of the stream is irregular since the clearance becomes excessive over part of the run. The following formula given by Francis for rectangular channels, may be used to determine the mean velocity V_m.

$$V_m = V_r\left[1.012 - 0.116\sqrt{\frac{c}{d}}\right] \quad \text{...(4.9)}$$

where V_r = the velocity of the rod.

and c = the clearance, and

d = the depth of the stream.

Rod floats give good results for streams with smooth beds such as artificial channels.

Velocity measurement using a float with one theodolite

This method is used when only one theodolite is available.

In this method two stations *A* and *B* are fixed about 200 m apart on the shore, and the distances *AB* and *AC* are measured accurately (Fig. 4.23). Then two parallel sections *AP* and *BQ* are established. The theodolite is set up at a convenient point *C* on the base line *AB*. The float is released from a point about 50 m upstream of the section *AP*. The angle α is measured when the float crosses the section *AP* at *P* and then angle β when it crosses the section *BQ* at *Q*. The time is also recorded for the float positions *P* and *Q*.

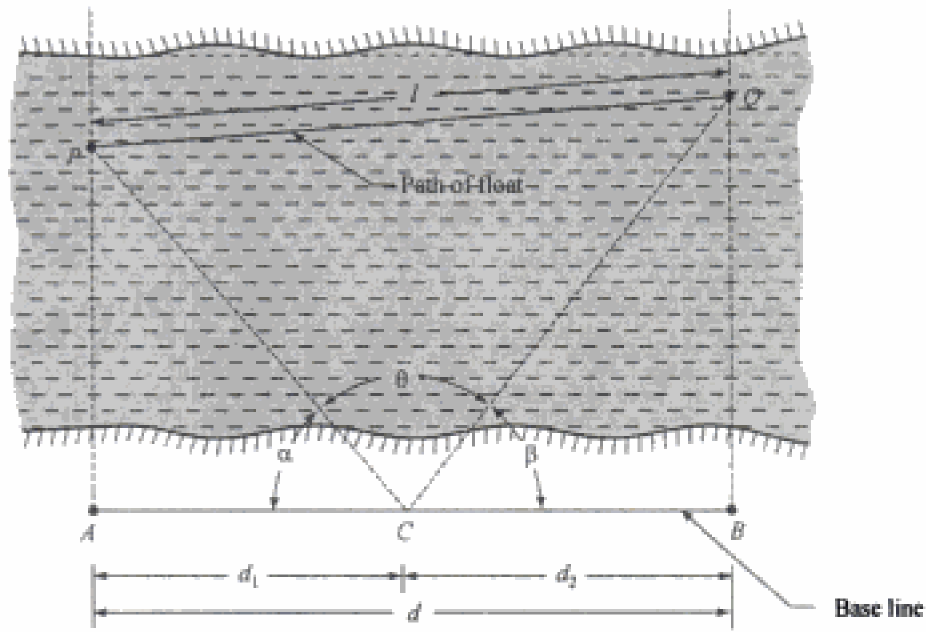


Fig. 4.23 Velocity measurement using a float with one theodolite

The distance l travelled in the time t is computed as under:

$$\theta = 180° - (\alpha + \beta)$$

$$d_2 = d - d_1$$

$$PC = d_1 \sec \alpha$$

$$QC = d_2 \sec \beta$$

$$l = \sqrt{PC^2 + QC^2 - 2PC\,QC\cos\theta} \quad \text{...(4.10)}$$

The velocity v is given by

$$V = \frac{l}{t}$$

where t is the time taken by the float in travelling the distance *PQ*.

Velocity measurement using a float with two theodolites

When two theodolites are used to measure the velocity, the shore stations *A* and *B* shown in Fig. 4.24, are established about 100 m apart, and the distance *AB* is measured accurately. Two parallel sections *AC* and *BD* are established at right angles to the base line *AB*. The float is released from a point about 50 m upstream of the section *AC*. As the float crosses the section *AC* at *P*, the instrument at *A* calls tick. The time is recorded, and the angle α is read from the instrument at *B*. In a similar manner, the angle β is read from *A* when the float crosses the section *BD* at *Q*, and the time is also recorded.

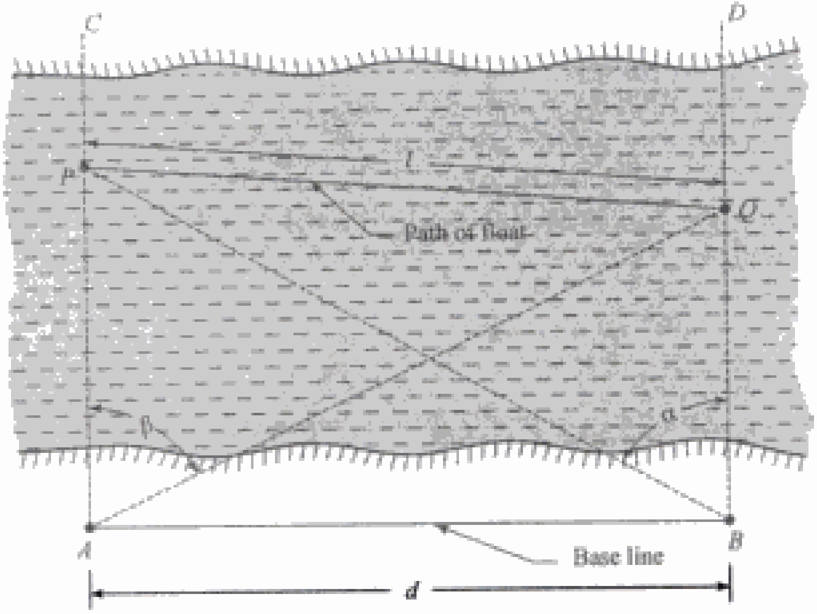


Fig. 4.24 Velocity measurement using a float with two theodites

The distance *l* travelled in time *t* is calculated from the measured base *d* and the angles α and β. The mean velocity is computed by dividing *l* by *t*.

Velocity measurement using a current meter

The velocity of the flow of water can be measured directly by means of a *current meter*. These are various types of current meters. A simple one designed by Gurley is described below (Fig. 4.25).

A Gurley's current meter consists of a number of conical vanes mounted on a vertical pivot. When the current meter is lowered into the water, the conical vanes start rotating with the flow of water. By means of an electrical arrangement the number of revolution can be recorded on a dial placed in a boat or on the shore. A rudder is provided with the meter to keep it in position and to dampen the possible twist of the entire system. After a pause of 30 revolutions an electrical contact is made which starts ringing a bell which continues ringing for the next 20 revolutions but remains silent for the next 30 revolutions. The number of revolution per second can be calibrated against the velocity at different depths, and a working formula of the kind, $v = cn + b$, where v is the velocity of the flow of water, n is the number of revolution, and c and b are the instrument constants which can be determined. In order to measure the velocity, the meter is suspended from a boat or from some convenient position like a bridge by means of a rod, chain, rope, or tape which is graduated. Thus lowering the rod or chain the velocity of water at different depths can be recoded. In order to ascertain the discharge fairly accurately, the product of the area of any particular strip and the mean velocity, will give the discharge of that vertical strip, and the summation of all such vertical strips over the entire width will give the total discharge of the stream. From a large number of experiments, it has been found that the mean velocity is approximately the velocity at 0.6 times the depth.

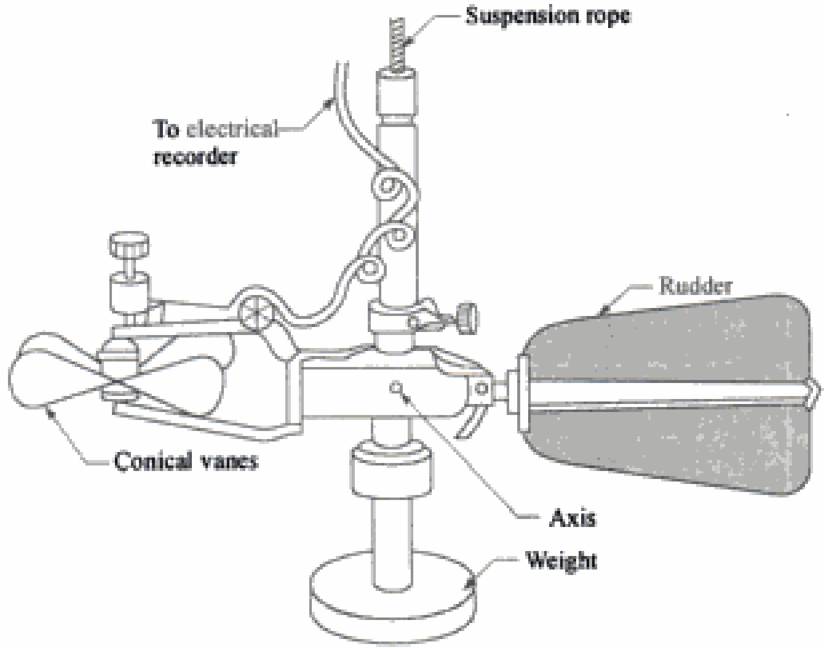


Fig. 4.25 Gurley's current meter

Mean velocity from the surface slope

It this method, the longitudinal slope of the water surface is measured, and the mean velocity V_m of the stream, is obtained from Manning's formula:

$$V_m = \frac{l}{n} R^{2/3} S^{1/2} \quad \text{...(4.11)}$$

$$R = \frac{A}{P} \quad \text{...(4.12)}$$

$$S = \frac{h}{L} \quad \text{...(4.13)}$$

where R = the hydraulic radius,
A = the area of the section,
S = the water surface slope,
p = the wetted perimeter,
h = the difference of water levels at the ends of the reach,
L = the length of the reach, and
n = the coefficient of rugosity or Manning's coefficient.

The discharge is given by

$$Q = V_m A \quad \text{...(4.14)}$$

The coefficient of rugosity n, depends on the roughness of the stream bed. For canals it varies from 0.020 to 0.035 with increasing roughness. It is taken to be 0.025 for rivers with smooth, sandy, or gravel beds, but increases to 0.050 for weedy beds. In using the formula, a straight reach of the river with as nearly a uniform cross-section as possible should be chosen. However, the method is inferior to techniques in which the velocity is directly measured.

Velocity measurement by Pitot-static tube

Fig. 4.26 shows a Pitot-static tube, also known as *Prandtl tube*. It is a combination of a total head tube and a static head tube. The difference in the total head and the static head is the velocity head $\frac{v^2}{2g}$, and it is read on a differential manometer suitably connected to the Pitot-static tube.

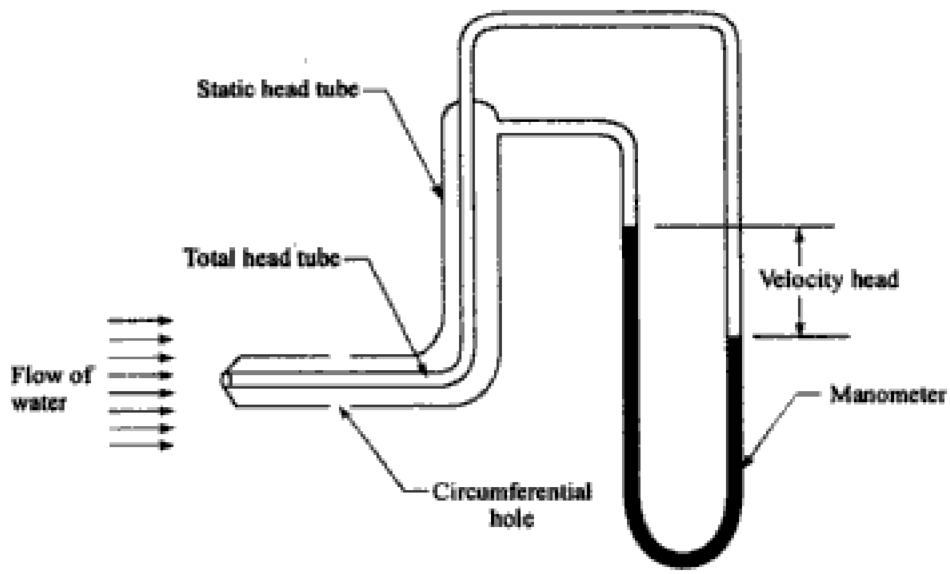


Fig. 4.26 Pitot-static tube (or Prandtl tube)

4.12.2 Weir method

A *weir* or *notch* is an artificial opening built across a stream through which the stream flows in a definite form. The discharge of the stream is calculated from the known dimensions of the weir. The weir method of measuring discharge is suitable for accurate gauging of small streams. The method becomes expensive for large rivers. An overflow dam can be used as a weir for measuring the discharge of the rivers if it is of sufficient height so that back water below the dam does not interfere with the free-flow over it.

The lower surface of the weir is called *crest*. A weir may be sharp-crested or broad-crested, the conditions of flow being different for each. The shape of the opening through which water flows can be rectangular, triangular, trapezoidal, or stepped (Fig. 4.27).

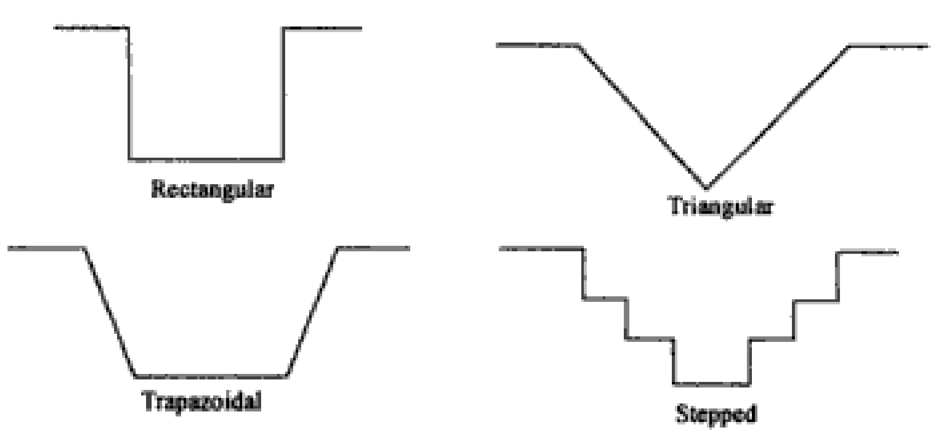


Fig. 4.27 Forms of weirs

The most commonly used weir for stream gauging is of rectangular form. The triangular or *V*-shaped weir is suitable for accurate gauging of small streams. A trapezoidal weir with 1:4 side slopes proposed by Cippolette, compensates the end contractions. The stepped weirs yield good results in both dry weather and floods.

The site for locating a weir is so chosen that the approach velocity of the water is as uniform as possible. For this, the stream or channel should have straight reach upstream from the weir. The head of flow over the weir has to be carefully measured at a minimum distance of three times the head from the weir on the upstream side to obtain the value at sensibly still water.

For different types of weirs, the formulae given below, are used to determine the discharge.

Rectangular weir

$$Q = \frac{2}{3} C_d \sqrt{2g} L H^{3/2} \quad \text{...(4.15)}$$

where Q = the discharge,
C_d = the coefficient of discharge,
L = the length of the weir,
H = the head over the weir, and
g = the acceleration due to gravity.

Triangular weir

$$Q = \frac{8}{15} C_d \sqrt{2g} H^{5/2} \tan\frac{\theta}{2} \quad \text{...(4.16)}$$

where θ = the angle of triangular weir.

For commonly used right-angled notch

$$Q = 1.418 H^{2.5} \quad \text{...(4.17)}$$

For the details of measurement of discharge using weirs, the reader may refer to any standard textbook on hydraulics and fluid mechanics.

4.12.3 Chemical method

The chemical methods are based on the analysis of chemicals added into the flowing water. The following two methods are commonly used:

1. Salt-velocity method
2. Salt-dilution method.

Salt-velocity method

In this method, a chemical (salt) in introduced into the river, and the discharge is determined from the concentration of the chemical at various sections. Introduction of salt into water increases the electrical conductivity of water which can be measured with the help of electrodes. The sets of electrodes are kept at a known distance apart in the river, one set is on the upstream end of the reach and the other at the downstream end. These electrodes are connected to a galvenometer to determine the changes in electrical conductivity of the water with respect to time.

For a water with no salt, the graph between conductivity and time is generally horizontal under normal conditions of flow. When the salt is added into the river on the upstream of electrodes, a sudden jump is shown in the graph. Later on, after sometime, when the salt reaches the downstream electrodes, a jump occurs again in the graph. The time of transit is the time between the centres of the two recorded peaks. Dividing the volume of water between the two electrode stations, by the transit time, gives the discharge.

Salt-dilution method

In this method, a fairly concentrated salt solution is introduced into the river water at a known uniform rate. The water downstream is analysed before and after the salt is introduced. Then the ratio of the percentage of chemical found to that in the solution must equal the ratio of the volume of solution introduced per second to the discharge of the stream.

Let P be the percentage of chemical in the solution, p the percentage of chemical found in water samples downstream, Q the discharge of the river, and q the flow of the solution. Then the rate at which the chemical is added, is $\frac{Pq}{100}$ and its dialution in the water sample is

$$\frac{Pq}{100Q} = \frac{P}{100}$$

or

$$Q = \frac{Pq}{p} \qquad \text{...(4.17)}$$

The success of the method depends upon many factors, such as efficiency of injecting the solution of proper concentration at a uniform rate across the stream over sufficient length so that the dilution becomes uniform on the downstream, and efficiency in analysing the concentration of the solution in water on the downstream.

ILLUSTRATIVE EXAMPLES

Example 4.3 In order to fix the location of a sounding, the following angles were measured with a sextant from the position of the boat O to three signals P, Q, and R on the shore:

$$\angle PQR = 108°40'; \angle POQ = 38°20'; \angle QOR = 33°00'.$$

The distances PQ and QR were obtained from a map, as 750 m and 820 m, respectively. What do you deduce from the above data?

Solution: (Fig. 4.28):

From the given data, we have

$$\angle PQR + \angle POQ + \angle QOR = 108°40' + 38°20' + 33°00' = 180°$$

Thus $\alpha + \beta = 360° - 180° = 180°$

This shows that $\angle Q + \angle O = \alpha + \beta$ which can only happen if the points A, B, C, and O lie on the circumference of a circle. Therefore, the problem is indeterminate.

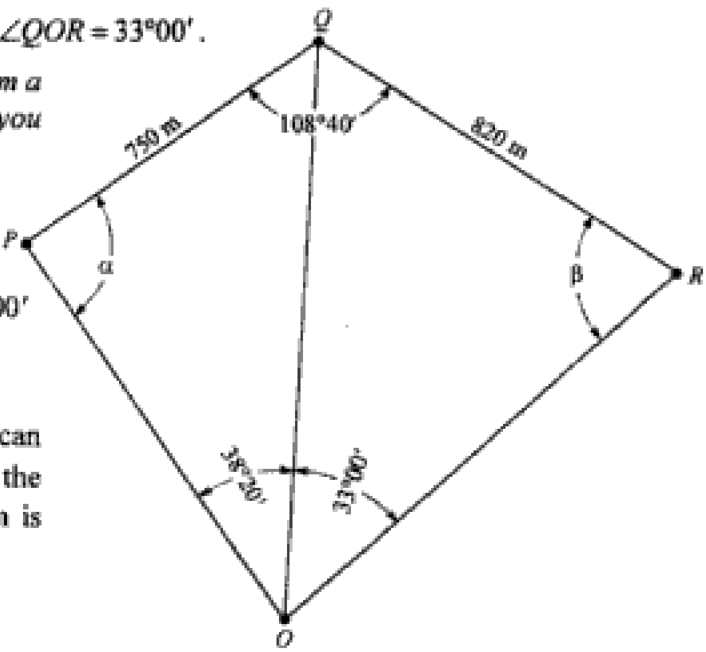


Fig. 4.28

Example 4.4 An observer taking soundings from a boat wished to locate his position P. He measures an angle to two stations A and B, AP being at right angles to AB. If the measured angle APB is 29° and the distance AB is 550 m, calculate the distance of P from A.

Solution: (Fig. 4.29)

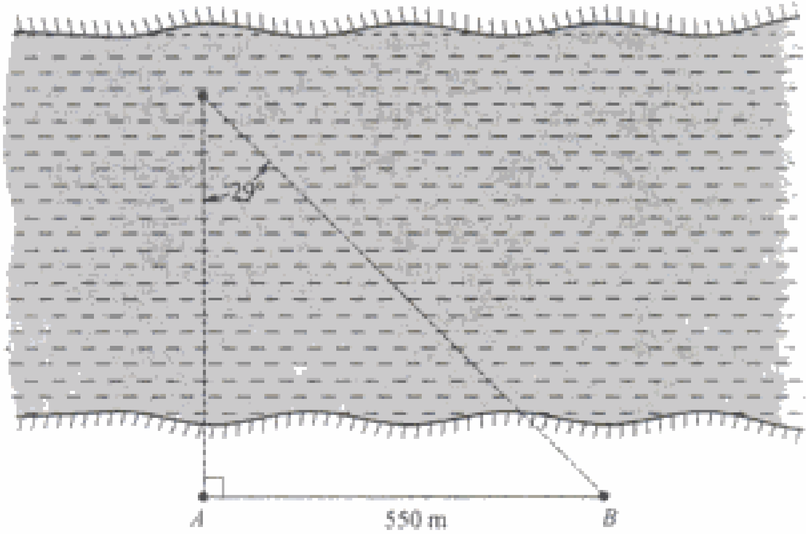


Fig. 4.29

From the right angled ΔPAB, we have

$$\begin{aligned} AP &= AB \cot APB \\ &= 550 \times \cot 29° \\ &= \mathbf{992.23\ m}. \end{aligned}$$

Example 4.5 Two cross-sections *AP* and *BQ*, each perpendicular to a base line *AB*, 220 m in length, are established to measure the velocity of flow of water in a stream. When the float was on the sections *AP* and *PB*, following angles was observed from a point *C* on the base line *AB*, 75 m from *A*:

$$\angle ACP = 62°24'00''; \angle QCB = 48°40'20''$$

If the time taken by the float to travel the distance *PQ* is 2 minutes and 10 seconds, what is the velocity of water?

Solution: (Fig. 4.23):

From the right angled Δ's *PAC* and *QBC*, we have

$$\begin{aligned} PC &= AC \sec \alpha = 75 \times \sec 62°24'00'' = 161.88\ \text{m} \\ QC &= CB \sec \beta = (220 - 75) \sec 48°40'20'' = 219.58\ \text{m} \end{aligned}$$

In ΔPCQ, We have

$$\begin{aligned} \angle PCQ &= \theta = 180° - (\alpha + \beta) \\ &= 180° - (62°24'00'' + 48°40'20'') \\ &= 68°55'40'' \end{aligned}$$

and

$$\begin{aligned} PQ &= \sqrt{PC^2 + QC^2 - 2PC\ QC \cos\theta} \\ &= \sqrt{161.88^2 + 219.58^2 - 2 \times 161.88 \times 219.58 \times \cos 68°55'40''} \\ &= 221.04\ \text{m} \end{aligned}$$

Therefore, the velocity of water

$$\begin{aligned} V &= \frac{PQ}{\text{time}} = \frac{221.04}{2^m 10^S} \\ &= \mathbf{1.70\ m/s.} \end{aligned}$$

Example 4.6 To measure the velocity of water using two theodolites, two cross-sections *AP* and *BQ*, each perpendicular to a base line *AB*, 250 m in length, were established. When the float was on the sections *AP* and *BQ*, the following angles were observed at the ends of the base :

$$\angle ABP = 44°45'30''; \ \angle BAQ = 43°58'20''$$

If the time taken by the float to travel from P to Q is 1 minute and 26 seconds, calculate the velocity of water.

Solution: (Fig. 4.24):

From the right angled Δ's *PAB* and *ABQ*, we get

$$AP = d \tan ABP = 250 \times \tan 44°45'30'' = 247.90 \text{ m}$$

$$BQ = d \tan BAQ = 250 \times \tan 43°58'20'' = 241.19 \text{ m}$$

Let the bearing of *PA* be 180°. Then

bearing of $AB = 180° + 90° + 180° = 90°$

and bearing of $BQ = 90° + 90° - 180° = 0°$

Let us assume the coordinates of *P* as (0, 0), then

latitude of $A = PA \cos 0° = 247.90 \times \cos 0° = 247.90$ m

departure of $A = PA \sin 0° = 247.90 \times \sin 0° = 0.00$ m

latitude of $B = AB \cos 90° = 250 \times \cos 90° = 0.00$ m

departure of $B = AB \sin 90° = 250 \times \sin 90° = 250.00$ m

latitude of $Q = BQ \cos 0° = 241.19 \times \cos 0° = 241.19$ m

departure of $Q = BQ \sin 0° = 241.19 \times \sin 0° = 0.00$ m

The total latitude of $Q = 0.00 - 247.90 + 0.00 + 241.19$

or $L = -6.71$ m

and the total departure of $Q = 0.00 + 0.00 + 250.00 + 0.00$

or $D = 250$ m

Therefore $PQ = \sqrt{L^2 + D^2} = \sqrt{(-6.71)^2 + 250^2}$

$= 250.09$ m

Thus the velocity

$$V = \frac{PQ}{\text{time}} = \frac{250.09}{86}$$

$= \mathbf{2.91}$ **m/s.**

Example 4.7 To determine the discharge of a stream, the readings were obtained with a current meter in four segments. Each segment has width of 5m. The readings taken at the depths and recorded in meter of flow for 20 second, are given in the following table, where *D* is the depth of water at the centre of each segment. Compute the discharge of the stream.

Segment No.	1	2	3	4
Depth of current meter (m) *D*	4.7 m	6.5 m	7.0 m	6.0 m
1	30	31	32	30
2	32	40	39	37
3	28	34	36	32
4	24	28	28	27
5	—	22	25	24
6	—	—	20	—
7	—	—	—	—

Solution: (4.30)

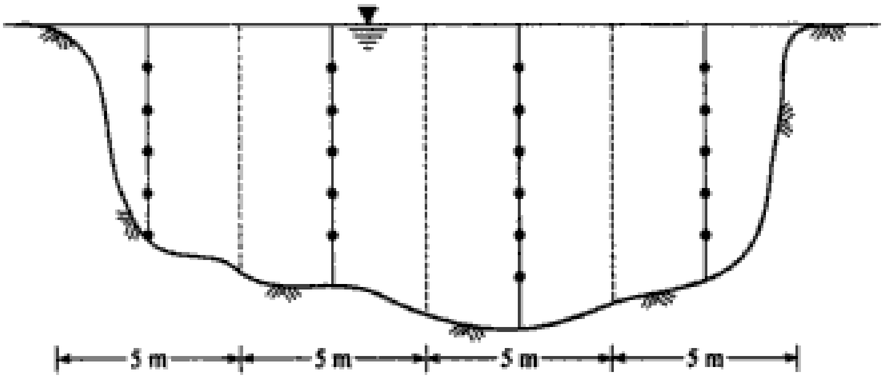


Fig. 4.30

The rate of discharge per segment = Area of cross-sector × mean velocity/sec. Discharge at

Segment 1 $= (5 \times 4.7) \times \dfrac{(30+32+28+24)}{4} \times \dfrac{1}{20}$

$= 33.49 \text{ m}^3/\text{s}$

Segment 2 $= (5 \times 6.5) \times \dfrac{(31+40+34+28+22)}{5} \times \dfrac{1}{2}$

$= 50.38 \text{ m}^3/\text{s}$

Segment 3 $= (5 \times 7) \times \dfrac{(32+39+36+28+25+20)}{6} \times \dfrac{1}{20}$

$= 52.50 \text{ m}^3/\text{s}$

Segment 4 $= (5 \times 6) \times \dfrac{(30+37+32+27+24)}{5} \times \dfrac{1}{20}$

$= 45.00 \text{ m}^3/\text{s}$

Total discharge from all the segments

$= 33.49 + 50.38 + 52.50 + 45.00$

$= \mathbf{181.37 \text{ m}^3/\text{s}}$.

Example 4.8 Compute the discharge of stream corresponding to a head of 0.74 m for a 90° V-notch.

Solution: (Fig. 4.31):

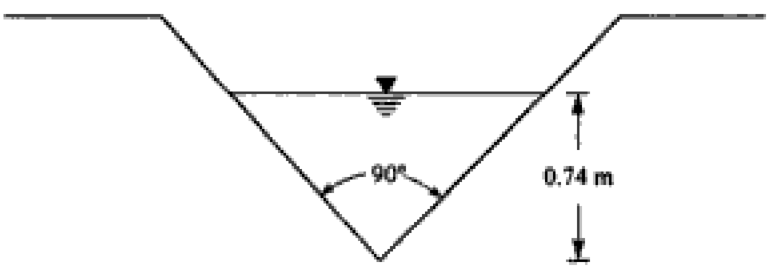


Fig. 4.31

From Eq. (4.17), we get the discharge as

$$Q = 1.418\, H^{2.5}$$
$$= 1.418 \times 0.74^{2.5}$$
$$= \mathbf{0.668 \text{ m}^3/\text{s}}.$$

Example 4.10 In a stream, the following observations were made:

Cross-sectional area = 425 m^2
Wetted perimeter = 140 m
Length of the reach = 180 m
Difference in water level at the two ends of the reach = 1.4 m
Determine the discharge taking Manning's rugosity equal to 0.025.

Solution:

From Eq. (4.12), the hydraulic radius is given by

$$R = \frac{A}{p} = \frac{425}{140} = 3.04 \text{ m}$$

From Eq. (4.13), the slope is given by

$$S = \frac{h}{L} = \frac{1.4}{180} = 0.0078$$

From Eq. (4.11), the mean velocity

$$V_m = \frac{1}{n} R^{2/3} S^{1/2} = \frac{1}{0.025} \times 3.04^{2/3} \times 0.0078^{1/2}$$
$$= 7.41 \text{ m/s}$$

From Eq. (4.14), the discharge is

$$Q = V_m \times A = 7.41 \times 425$$
$$= \mathbf{3149.25\ m^3/s.}$$

PROBLEMS

4.1 What do you understand by hydrographic surveying? What are its common uses?

4.2 Define hydrographic surveying. What are various operations conducted in hydrographic surveying?

4.3 Discuss various operations required for conducting a hydrographic survey.

4.4 Discuss the methods of fixing horizontal and vertical control for a hydrographic survey.

4.5 What do you understand by shore-line surveys? What is its purpose?

4.6 What is a tide gauge? Differentiate between a non-registering type gauge and a self-registering type gauge. Describe the to working of
- (*a*) a float gauge, and
- (*b*) a weight gauge.

4.7 Explain the working of a self-registering type tide gauge. What are its advantages and disadvantages?

4.8 What are soundings? Discuss various methods of taking soundings.

4.9 Discuss various methods of locating soundings. Also discuss their merits and demerits.

4.10 What do you mean by reduction of soundings? Explain the method of reduction of soundings.

4.11 Write short notes on the following:
- (*a*) Sub-marine contours
- (*b*) Echo-sounding
- (*c*) Buoy
- (*d*) Lead line
- (*e*) Station pointer.

4.12 What is stream gauging? Briefly discuss principles of various methods employed for stream gauging.

4.13 Discuss various types of floats employed for stream gauging giving their merits and demerits.

4.14 Discuss velocity measurement of water in a stream using two theodolites and a float.

4.15 Draw a neat sketch of a current meter and name all its parts. Explain its working and uses.

4.16 Write short notes on

(*a*) Weir method of discharge measurement.

(*b*) Chemical method of discharge measurement.

4.17 Determine the constants a and b in the equation $v = aN + b$ used for measuring the discharge of a stream by current meter from the following data:

v (velocity) m/s	1.55	3.2
N(r.p.s.)	2.60	5.0

Calculate the velocity for $N = 6.8$.

4.18 The following depths of soundings were taken at the centre of segments of 8 m across a river 64 m wide:

4.10, 4.55, 5.40, 5.80, 6.66, 6.38, 5.00, 3.40 m.

If the mean velocity observed for each are 1.9, 2.55, 3.08, 3.34, 3.62, 3.40, 2.50, 1.70 m/s, respectively, determine the discharge of the river.

4.19 In order to locate the position of P of a boat, observations were made with a sextant to three shore signals A, B, and C. The angles APB and BPC were found to be 50°56′ and 27°30′, respectively. From the plan, AB and AC were found to be 450 m and 220 m, while the angle ABC was measured as 163°18′. Determine the distances of P from A, B, and C.

4.20 Determine the discharge in a river from the following data:

Area of segment m²	23	35	42	37	26	20
Velocity of 0.7 d (m/s)	2.7	2.8	2.9	2.85	2.78	2.65
Velocity of 0.3 d (m/s)	2.4	2.5	2.65	2.78	2.50	2.45

4.21 Calculate the discharge through a 90°V-notch when the depth of water above the vertex is 0.38 m.

4.22 Determine the discharge of a river from the following data:

Area of cross-section	= 400 m²
Welted perimeter	= 210 m
Slope is	= 1 in 210
Manning coefficient N	= 0.030.

5

CONSTRUCTION SURVEYING

5.1 GENERAL

Construction surveying deals with the transfer of design dimensions from the engineering drawings to the ground to build a project in correct position and with proper relationship between its component parts. Thus, it is primarily concerned with the establishment of certain lines and grades that guide and control construction operations. Generally, the construction surveying deals with the setting out works on the ground.

The basic principles of surveying involved in the practice of construction surveying have been given in detail in the book *Plane Surveying*. However, the project engineer has to assess the field problem as intelligently as possible, and derive a solution combining various methods of surveying that will be sufficiently accurate and economical.

The project engineers must ensure that the structure is located in the correct position in accordance with the plan. It should be of the correct size in correct position and at the correct level.

Common types of construction surveying include setting out or staking out of grades or slopes, building, pipe lines, sewers, highways, railways, culverts, bridges, tunnels, and so on. The construction surveying, thus, generally consists of the following operations:

1. Establishing a system of control stations for making measurements for setting out.
2. Establishing lines and levels as needed during the construction.
3. To verify the location of completed parts of the structure by making measurements.

It would not be possible to discuss all types of setting out works due to the limited scope of the text. A few of the more common and simple cases are discussed in Sec. 5.4 and the engineer or surveyor should use his ingenuity to apply the principles and methods discussed here for other cases.

5.2 HORIZONTAL AND VERTICAL CONTROL

To set out a structure correctly in all the three dimensions and in the correct relative position, a number of control stations are established in the project area. The horizontal and vertical distances are measured from the control stations for setting out.

5.2.1 Horizontal control

For large and important structures like dam, and bridge, a number of secondary control points are established between triangulation stations, by traversing (Fig. 5.1). The secondary control points are established near the structure so that the horizontal distances to the salient points of the structure can be conveniently measured, but sufficiently away so that they are not disturbed during construction operations. In the process of establishing these control points the well known principle of 'working from the whole to the part' must be observed.

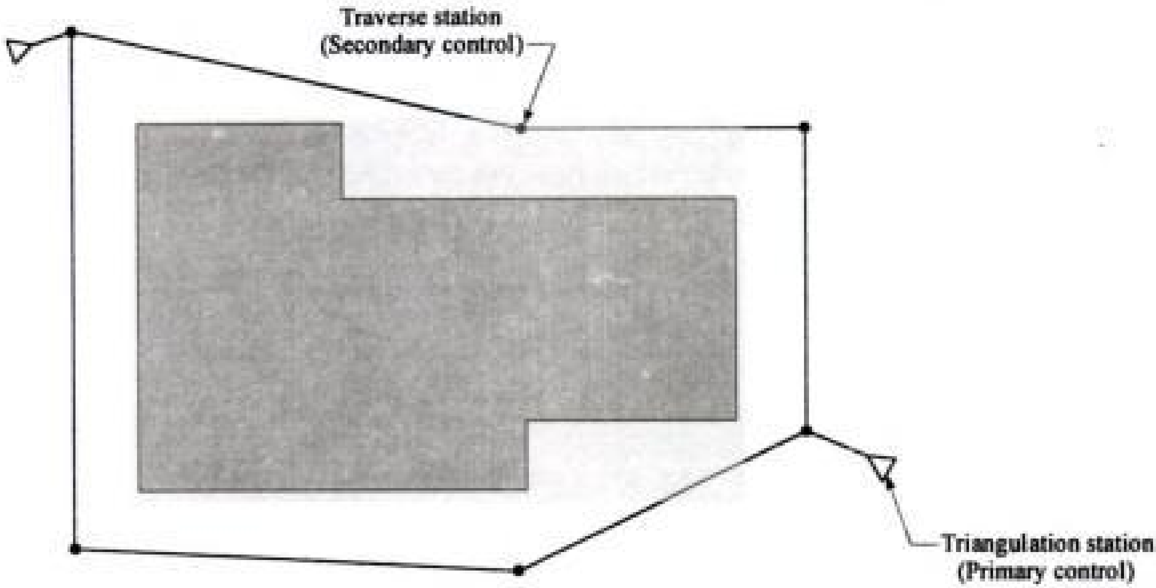


Fig. 5.1 Horizontal control

In order to increase the accuracy in measurements, a base line or two mutually perpendicular base lines may be established using the control points (Fig. 5.2).

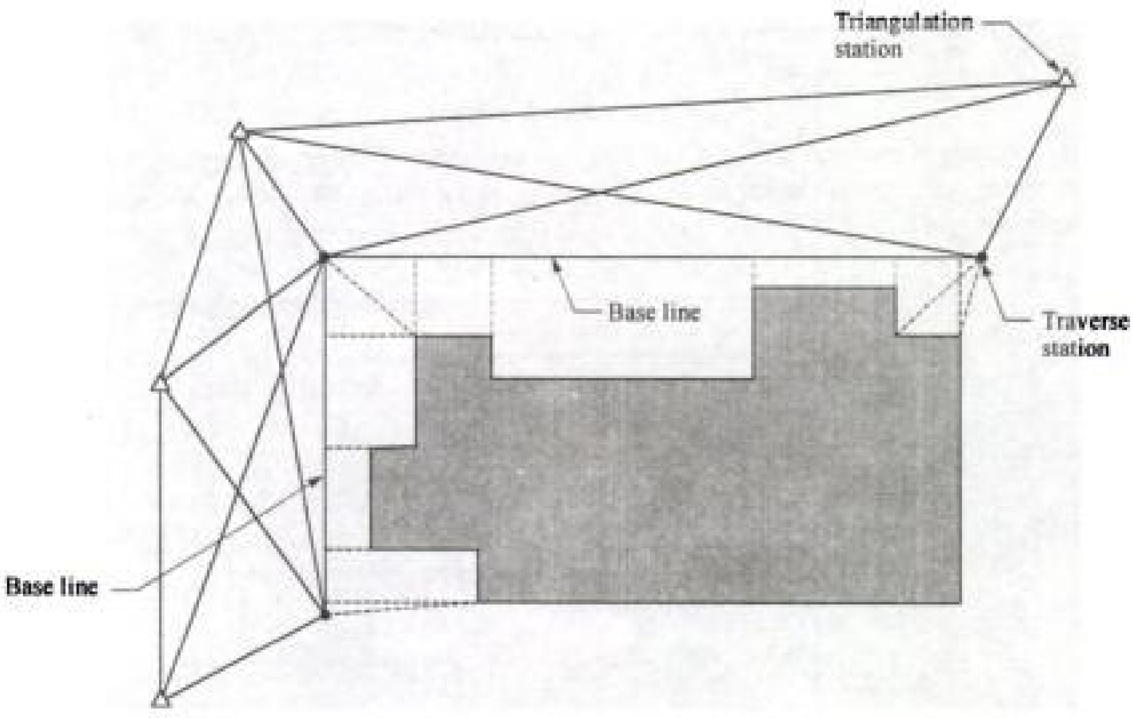


Fig. 5.2 Base lines

Reference grid

The following types of reference grids are used for accurate setting out works of large structures:

(*i*) Survey grid
(*ii*) Site grid
(*iii*) Structural grid
(*iv*) Secondary grid.

Survey grid is the one which is drawn on the survey plan. It consists of the traverse stations established during the land surveying of the area. The traverse stations form the control points of the grid. The *site grid* is the one which is used for actual setting out of the salient points of the structure (Fig. 5.3). The points of the site grid are marked with wooden or steel pegs set in concrete. When the survey grid has insufficient number of control points or when the control points are located quite far off, a separate site grid may be established.

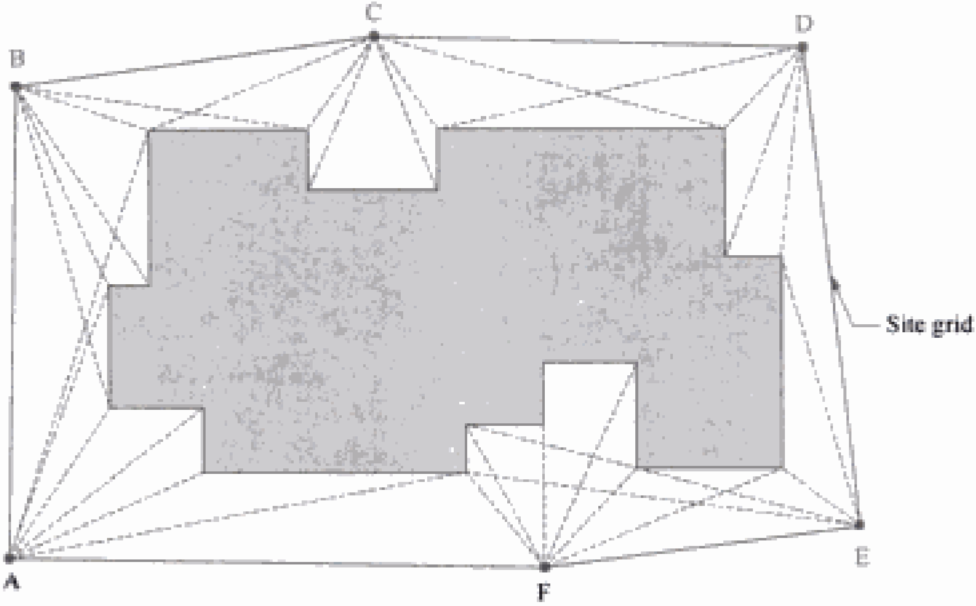


Fig. 5.3 Base grid

When the structural components of a building for example columns, are large in number and are so positioned that these component cannot be set out conveniently with sufficient accuracy, with reference to the site grid, another grid called *structural grid,* is used (Fig. 5.4).

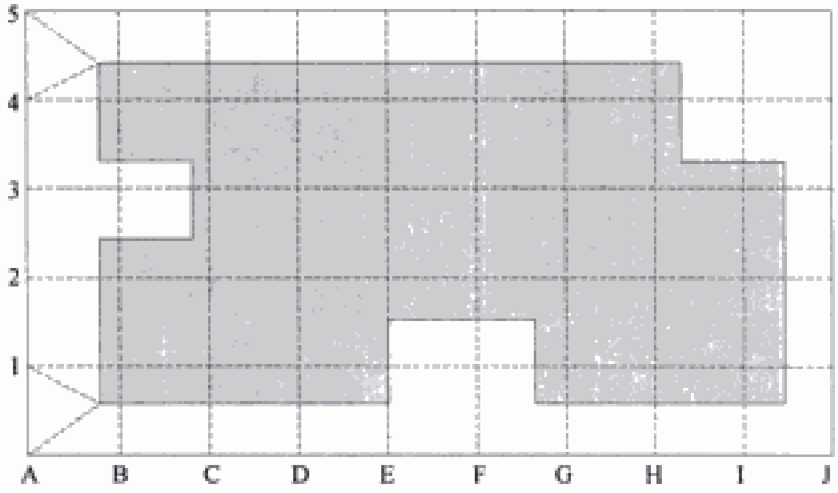


Fig. 5.4 Structural grid

The structural grid has points at a closer spacing, and it is set out inside the site grid from the grid points.

The *secondary grid* is established inside the structural grid, to establish the internal details of building which are not visible directly from the structural grid.

5.2.2 Vertical Control

The vertical control is provided by establishing the reference bench marks of known elevation relative to some specified datum. The levels of the various points on the structure are determined from these bench marks.

All the levels at the site are normally reduced to a nearby bench mark, usually called *master bench mark* (M.B.M). If no such bench mark exists in the area, the master bench mark should be established by running levels from the nearby bench mark. This master bench mark is used to establish a number of *transferred bench marks* or *temporary bench marks* (T.B.M.), with an accuracy of levelling within ± 0.010 m. The positions of the temporary bench marks should preferably be fixed during the initial site reconnaissance. As for as possible, permanent existing features should be used as temporary bench marks. Each temporary bench mark is referenced by a number or letter on the site plan, and should preferably be related to the master bench mark. All bench marks should be properly protected and their levels should be regularly checked. The distance between any two adjacent temporary bench marks should not be normally greater than 100 m.

5.2.3 Control stations

The control points are to be so constructed and protected that they are not disturbed during the course of construction. The control stations required for long duration should be established in concrete or masonery pillars. The metal plates or bolts are set in the top of the concrete block, usually of size 600 mm × 600 mm, on which the station position is punched or etched. (Fig. 5.5a). The control stations which are required for relatively short duration, may be in the form of wooden pegs of size 50 mm × 50 mm × 300 mm, driven directly into the ground (Fig. 5.5b).

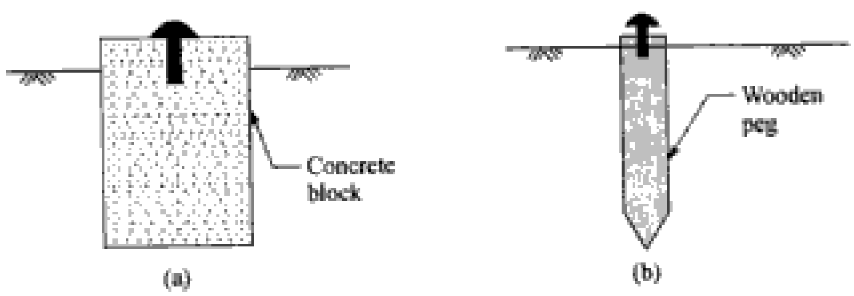


Fig. 5.5 Control stations

The control stations should be numbered and referenced using some suitable method so that they can be re-established if disturbed.

5.3 POSITIONING OF A STRUCTURE

The next operation after establishing the control, is to locate the salient points such as corners of the structure, on the ground. Some of the commonly employed methods are as below:

1. Offsets from the existing detail
2. Offsets from the site grid
3. Offsets from the base line.

When a single building of comparatively small size is to be located, its corners may be fixed by running a line between corners of existing building, and taking offsets from this line. If an existing building is not available, the salient points of the structure to be constructed, are coordinated in terms of site grid or base line. This can be achieved by the following methods.

By polar coordinates

In this method, the distance and bearing of each salient point are observed from at least three site grid points (Fig. 5.6a). Each point is located by the intersection of two lines, and checked by the third line.

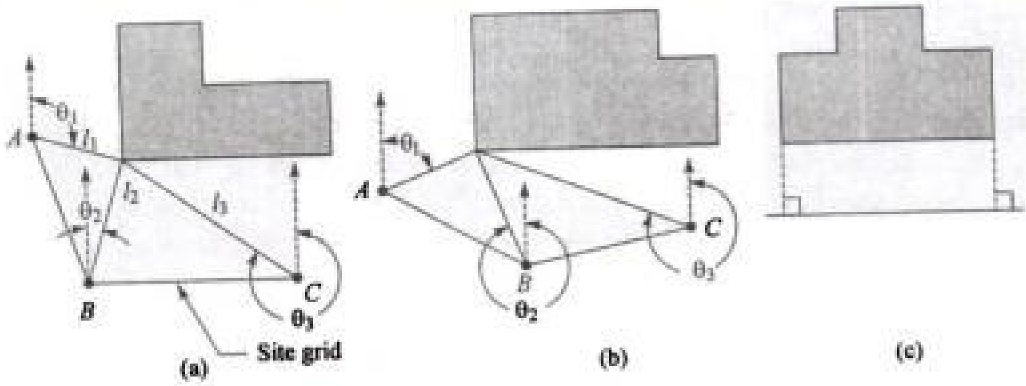


Fig. 5.6 Positioning of a structure.

By intersection

In this method, the bearing of each salient point is calculated with reference to three grid points. The salient point is located by intersection with the help of two theodolites set at two grid points (Fig. 5.6b). The location is checked from the third grid point.

By offsets from the base line

In this method, a suitable base line is selected, and the offsets from the base line to the salient points are measured for the setting out of the salient points (Fig. 5.6c).

Offset pegs

In Fig. 5.6, it has been illustrated that the corners of a structure can be set out by polar measurements from the points of site grid. If the corner pegs *A*, *B*, *C*, and *D* (Fig. 5.7) are driven in the ground, they will get dislocated during the excavation of the foundations. To avoid this problem, extra pegs called offset pegs, a_1, a_2, b_1, b_2., etc, are located on the lines near the sides of the structure but offset back from the true corner points. The offset distance should be sufficiently large so that the offset pegs are not disturbed during excavation.

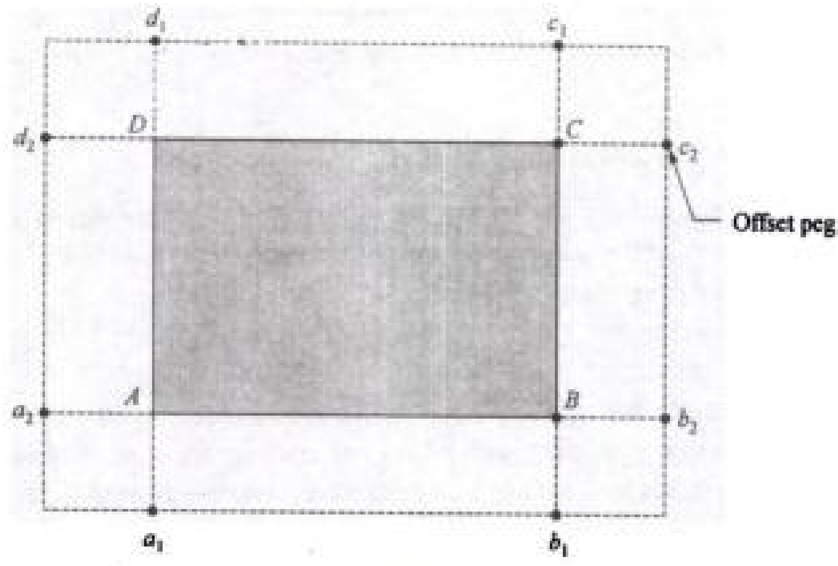


Fig. 5.7 Offset pegs.

5.4 SETTING OUT WORKS

The process of laying out engineering projects by placing pegs or marks at the site of work is known as setting out works. The main objective is to define the outlines of excavation and levels of the work for guidance of the workers so that the construction may proceed with reference to the already prepared plan. It helps in minimising the cost of digging foundation trenches by accurately defining the outlines of excavation stakes. Since in setting out work, the data are transferred from plan to the ground it is, in a sense, reverse of surveying.

In this section, the setting out of same of the common structures are dealt with.

5.4.1 Setting out a building

The foundation plan of the building (Fig. 5.8) is usually supplied or it can be prepared from the given plan of the building (Fig. 5.9).

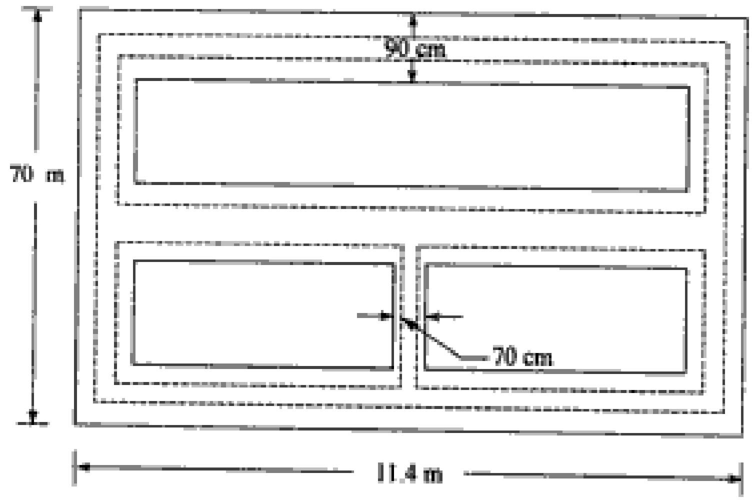


Fig. 5.8 Foundation plan

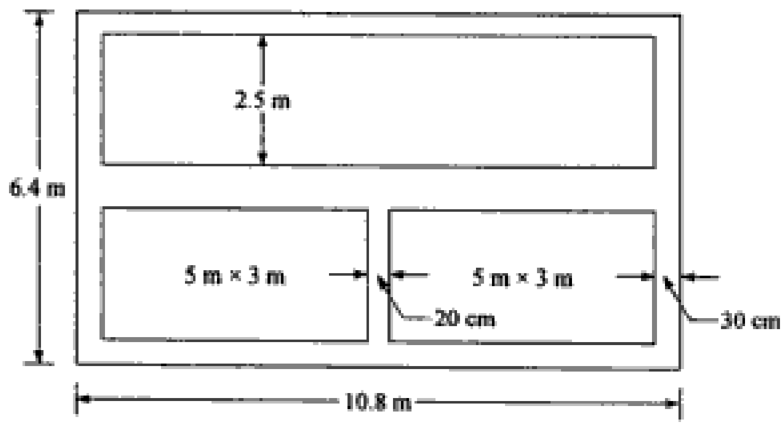


Fig. 5.9 Wall plan

After preparing the foundation plan, the setting out of the building can be done by the following methods.

1. Setting out by circumscribing rectangle
2. Setting out by centre-line rectangels.

Setting out by circumscribing rectangle

A reference rectangle as shown in Fig. 5.10, set outside the limits of excavation, 2 to 5 m from the building line, is known as *circumscribing rectangle*. The reference pegs *A, B, C*, and *D* fixed at the corners of the rectangle, remain undisturbed during excavation.

The coordinates of all the corners 1, 2, 3, 4, etc., with reference to the sides of reference rectangle are calculated and tabulated.

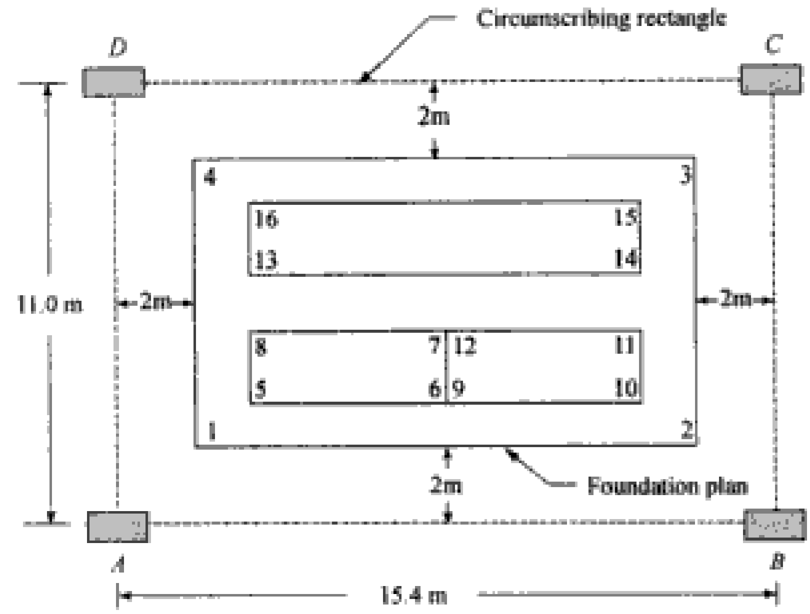


Fig. 5.10 Setting out by circumscribing rectangle

The field procedure is that the two stakes *A* and *B* are accurately driven at the required distance apart, and the centers of the stakes are marked by driving wire nails. A cord is stretched between *A* and *B*. At *A* a perpendicular *AD* to *AB* is set out with a tape by 3, 4, 5, triangle method. A stake at *D* is driven measuring *AD* equal to the desired distance. Check the diagonal *DB*. The stake *C* is also set following the similar procedure. Check the distance *CD* which should be equal to *AB*. The corners of the foundation plan are fixed now by measuring the respected calculated coordinates from the sides of the reference rectangle. The locations of the corner points are marked by driving pegs. A cord is passed round the periphery of the rectangles (1, 2, 3, 4), (5, 6, 7, 8), (9, 10, 11, 12), and (13, 14, 15, 16), and the outline of the foundation is marked by spreading lime along these lines.

Setting out by centre-line rectangle

In this method, the reference rectangle *ABCD* is formed by centre lines of the outside walls of the building (Fig. 5.11). The corners of the building are located by measuring their coordinates with reference to the sides of the centre-line rectangle. The stakes put in at *A*, *B*, *C*, and *D* will get disturbed during excavation, therefore, *reference stakes* are established on the prolongation of sides of this rectangle where they remain undisturbed. The reference stakes are generally established at least 2 m away from the building line.

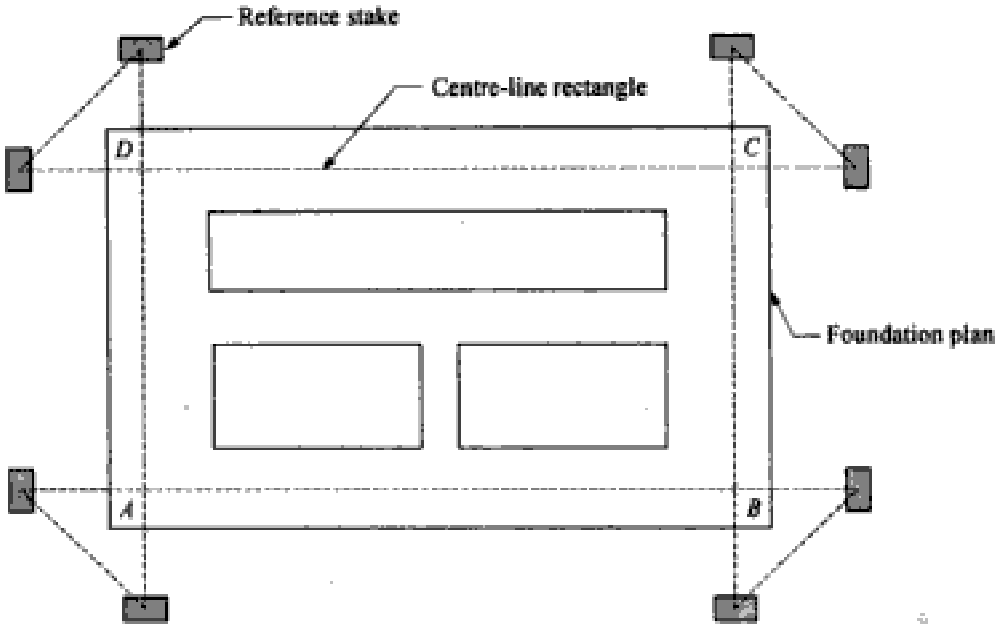


Fig. 5.11 Setting out by centre-line rectangle

In case of large and important works a theodolite should be used for setting out right angles.

Reference pillars

For large and important structures, *reference pillars* of masonry or concrete are constructed near each corner point (Fig. 5.12). The pillars are usually 20 cm to 30 cm thick and about 15 cm wider than the width of the foundation trench. The top of the pillars is usually set at plinth level. Nails or bolts are embedded in the top of these pillars to indicate the centre line and outer lines of the foundation trench.

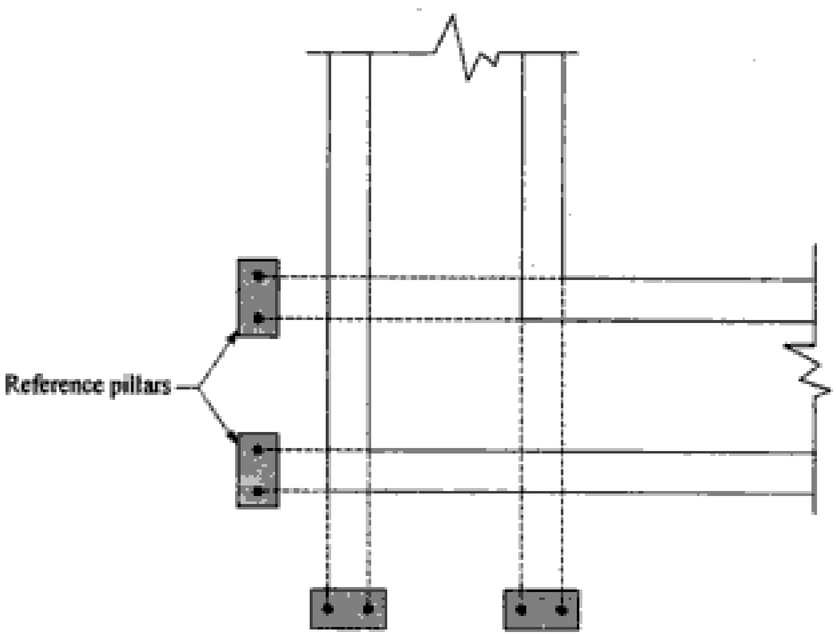


Fig. 5.12 Reference pillars

Batter boards: For accurate layout batter boards, also known as *profile boards* are used. A batter board consists of a horizontal board fixed to the top of vertical posts (Fig. 5.13). In foundation layout, the batter board used, consists of three posts forming a right angle in the plan.

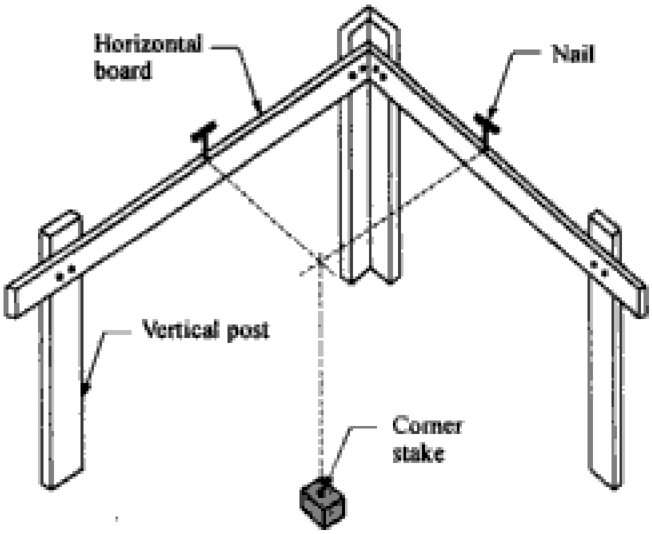


Fig. 5.13 Batter boards

The posts, which are long stakes, are driven at each corner of the building as offset pegs. Horizontal boards are then nailed to the top of the posts at the desired elevation using a levelling instrument.

Nails are driven on the top surface of the horizontal board to indicate the position of the centre lines. The corner of the building is located by stretching cords from the nails on the boards at the opposite corners of the wall shown in Fig. 5.14.

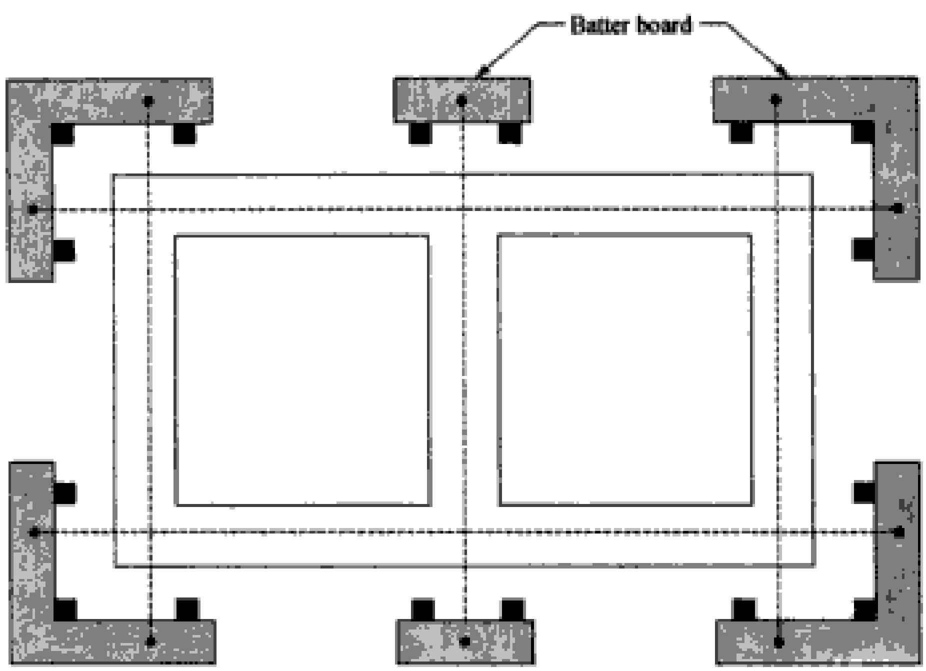


Fig. 5.14 Batter board method of setting out a building

Reference stakes should be fixed outside the batter boards so that if the batter boards are disturbed, they may be refixed correctly. A theodolite should be used for setting out right angles. The bench marks are also established around the site in the convenient locations.

5.4.2 Setting out a culvert

Setting out culverts involves the location of corners of the headwalls (or the components analogous to abutments in bridges) and wing walls at the foundation by means of their coordinates with reference to the centre-line of a road, railway or a stream crossed. The centre lines of road and stream which intersect each other at right angles are taken as axes of coordinates, their origin being the intersection point.

The coordinates of different points are found from the foundation plan of the culvert, and indicated in tabular form.

In Fig. 5.15a, *PQ* and *RS* are the centre-lines of the road and the stream, respectively, passing through *O*, which happens to be the centre of the culvert. These centre-lines are staked out by setting a theodolite at *O*. On these lines the points *A*, *B*, *C*, ..., *a*, *b*, *c*, ..., etc., are marked at the required distances from *O* by stretching the cords along *PQ* and RS. Arrows are fixed at all these points. The point 1 is set out by measuring the coordinates X_1 and Y_1 from *A* and *a*, respectively, with the help of two tapes, and the point is marked with a peg. Similarly, other points are also fixed, and marked with the pegs. Then a cord is passed around the periphery of the head wall and wing walls as 1, 2, 3, 4, ..., 8, and the outline of the foundation is marked with lime and nicking, i.e., cutting a narrow trench along the line. In a similar manner, the corners of other head wall and wing walls are fixed, and outline is marked. The levels of all the pegs are determined, and depth and quantities of, excavation are computed.

If the wing walls are curved as shown in Fig. 5.15b, the points on the curve may be set out by offsets to the chords 1-2 and 3-4.

The skew culverts may be set out in the same way except that the centre-line of the culvert is set out at the appropriate skew angle with respect to that of the centre-line of the stream.

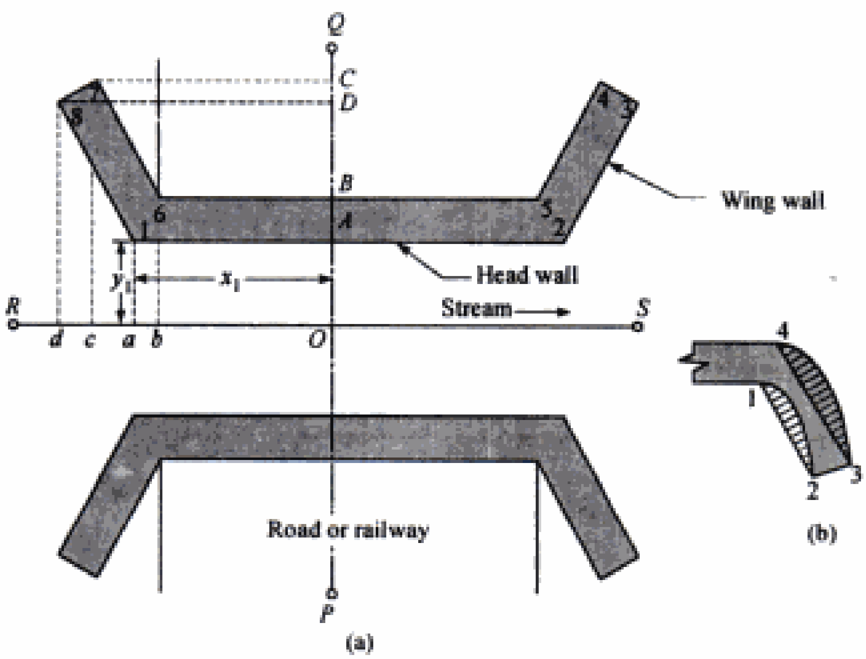


Fig. 5.15 Setting out a culvert

5.4.3 Setting out a bridge

The setting out of a culvert is quite simple because there is only one span and the flow of water is less. Even if the flow of water is more it can be easily diverted. But in the case of bridges and dams, the flow of water being more it can not be diverted, and also the span of the bridge or dam being very long, accurate setting out is difficult from the centre of the bridge.

While setting out the bridges, following two problems are encountered:

1. To determine the accurate distance between the end points of the bridge.
2. To locate the central points of piers of the bridge.

Determination of the distance between the end points

If the bridge is short, the length may be measured directly with a standardised tape. Necessary corrections are applied to the measured length to obtain the absolute length of the span of the bridge. But in the case of long bridges, the span is usually determined by method of triangulation.

Let A and B be the two end points of the bridge, situated on the centre-line of the bridge. Any one of the following two methods can be adopted to determine the length AB.

Method-I (Fig. 5.16)

Two lines AC and BD are set out perpendicular to AB at A and B, respectively, using a theodolite, and the distances AC and BD are measured accurately. The angle θ_1 and θ_2 are measured at C and D, respectively, by the method of repetition making observations on both the faces. The length AB is computed as

$$l_1 = AC \tan \theta_1$$

and $$l_2 = BD \tan \theta_2$$

The mean of l_1 and l_2 is taken as the length of AB.

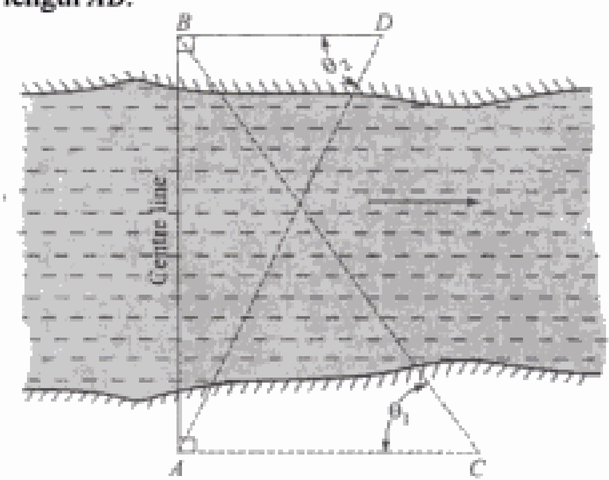


Fig. 5.16 Length of the bridge by Method-I

Method-II (Fig. 5.17)

Two base lines AD, and BC, approximately at right angles to AB, are set out along both the banks of the river, and their lengths are measured accurately. The eight angles of the braced quadrilateral $ABCD$ are measured, and the quadrilateral is adjusted (*cf.*, Sec. 2.9). The length of BC is determined by computations, using the known length AD and the adjusted angles, and compared with its measured value. If the discrepancy is less than 1 in 5000, the computed length of AB is accepted. If the discrepancy is more, the observations are repeated.

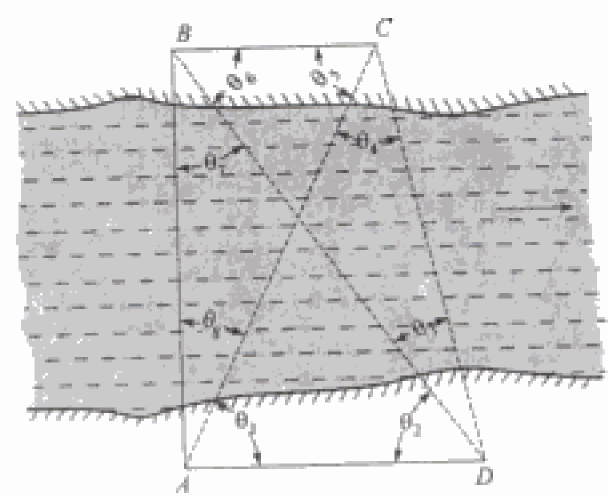


Fig. 5.17 Length of bridge by Method-II

Location of the central points of piers

After obtaining the length of centre-line, AB, the inter distances AP_1, P_1P_2, and P_2B, where P_1 and P_2 are the centre positions of the piers, are calculated. To locate the positions of P_1 and P_2, one of the following methods may be used.

Method-I (Fig. 5.18)

In the first method, the angles θ_1, θ_2, θ_3 and θ_4 are calculated from the known distances as below:

$$\theta_1 = \tan^{-1}\frac{AP_1}{AD}$$

$$\theta_2 = \tan^{-1}\frac{AP_1}{AD}$$

$$\theta_3 = \tan^{-1}\frac{BP_2}{BC}$$

$$\theta_4 = \tan^{-1}\frac{BP_1}{BC}$$

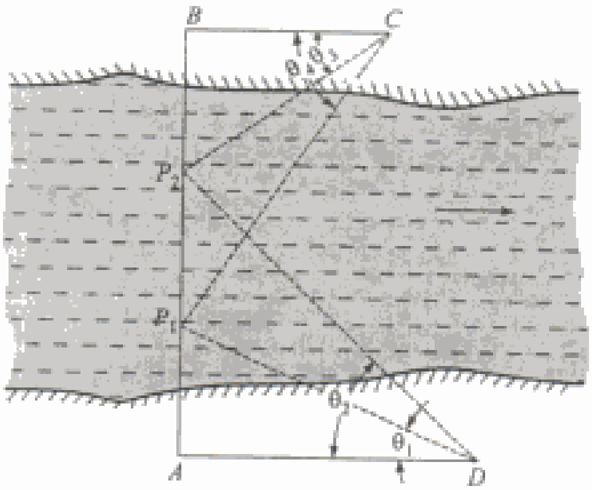


Fig. 5.18 Locating the centres of piers by Method-I

Two theodolites are set up at A and D. The instrument at A is directed to B, and the instrument at D is directed to A. The angle θ_1 is set at D, and by simultaneous observations, the position P_1 is located. By setting the angle θ_2, P_2 is located in the similar manner.

The positions P_2 and P_1 are checked by setting up the instrument at C, and setting the angles θ_3 and θ_4, respectively. If the line of sights pass through P_2 and P_1, the locations are correct.

Reference points are established in line of DP_1, DP_2, CP_1, and CP_2 for reference during construction.

Methods-II (Fig. 5.19)

In this method, the base lines at A and B perpendicular to AB are extended on both upstream and downstream sides. The distances $A1$ and $B1$ are set off on the base lines equal to AP_1 (= BP_2), and $A2$ and $B2$ equal to AP_2 (=BP_1). Since the angles at points 1 and 2 will be equal to 45°, the point P_1 is located by making simultaneous observations from points 1 on one base line and checking from the other base line by making simultanuous observations for P_1 from points 2. The intersection of the two sights 1-2 is the location of the centre of the pier. The point P_2 is established in the similar manner.

The method is fairly accurate and simple, but is suitable only when the perpendicular base lines on both sides of the centre-line of the bridge are possible.

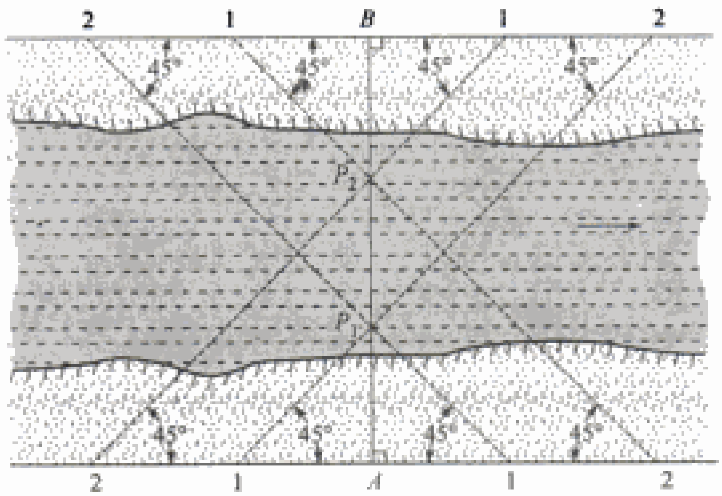


Fig. 5.19 Locating the centre points of piers by Method-II

5.4.4 Setting out slope of earthwork

This consists in setting out stakes at a given slope or grade to indicate how much cutting or filling is required to be done to bring the surface of the ground to a given grade. After marking the grade or formation line on the profile, the formation levels are determined for each station. From the formation level and the height of instrument, the staff readings required to set the stakes at given grade may be obtained by subtracting the formation level from the height of instrument. For example, if a point *A* is to be established on given grade and the formation level at *A* is 46.450 m then the reading on the staff placed on the top of the peg (Fig. 5.20) will be

$$= \text{H.I.} - 46.450$$

Taking the height of instrument (H.I.) as 48.805 m, the staff reading is

$$= 48.805 - 46.450 = 2.355 \text{ m}.$$

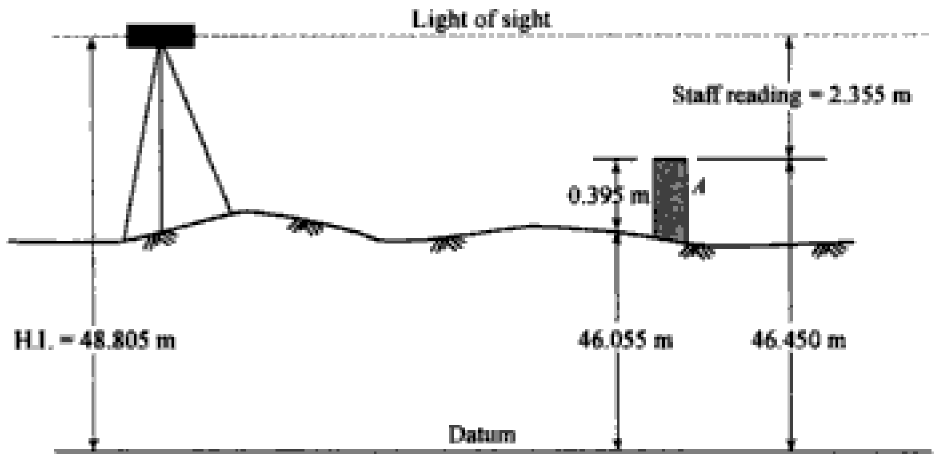


Fig. 5.20 Establishing a point on the given grade

The peg can be raised or lowered to give the desired reading. If the ground level at *A* is 46.055 m, the top of the peg will be 46.450 – 46.055 = 0.395 m above the ground.

Similarly other points are established on the given grade.

The above method is suitable only if a few points are to be established on a given grade, but if the number of points is fairly large such as in the case of a road, railway, canal, sewer, etc., the grades are set out by theodolite, Abney's level, tilting level. To set out the points in vertical direction, the help of following rods and boards are taken:

1. Boning rods or travellers
2. Sight rails
3. Slope rails
4. Profile boards or batter boards.

Boning rods or travellers: A boning rod is a T-shaped rod as shown in Fig. 5.21a. Boning rods are made in sets of three, and may consist of three T-shaped rods, each of equal size and shape, or two rods identical to each other and a third one consisting of a longer rod with a movable rod placed at right angles to the longer rod (Fig. 5.21b). The third rod is called a *traveller* or a *travelling rod.* The longer rod has graduations, and the movable rod can be clamped at any desired height on the graduated rod with the help of a clamping screw.

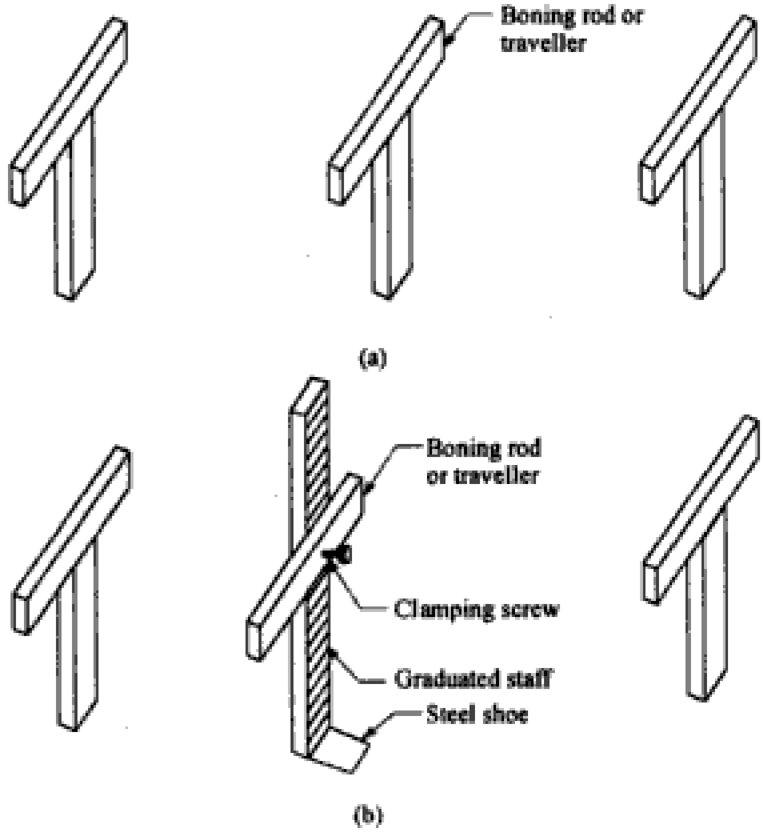


Fig. 5.21 Boning rod or travellor

Sight rails: A sight rail consists of a horizontal cross-piece nailed to a single upright or a pair of uprights driven into the ground. The upper edge of the cross-piece is set to a convenient height above the required plane of the structure to enable a man to conveniently align his eye with the upper edge.

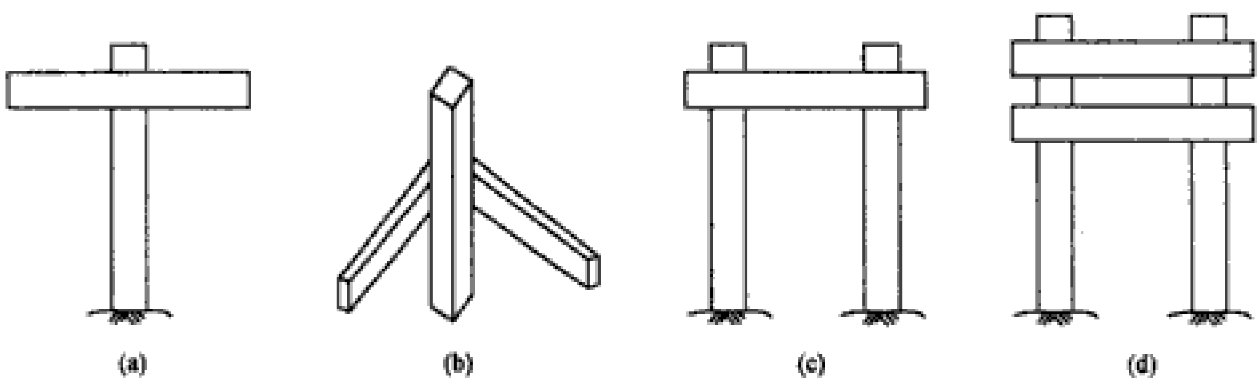


Fig. 5.22 Sight rails

Various forms of sight rails are shown in Fig. 5.22. The sight rail shown in Fig. 5.22a, is used for road works, footings, and small diameter pipes, while at the corners of buildings sight rail shown in Fig. 5.22b is used. For trenches and large diameter pipes, sight rail shown in Fig. 5.22c, is used. A stepped sight rail or double sight rail shown in Fig. 5.22d, is used in highly undulating or falling ground.

Slope rails: Slope rails are used for controlling the side slopes in embankment and in cutting. These consist of two vertical posts with a sloping board (Fig. 5.23).

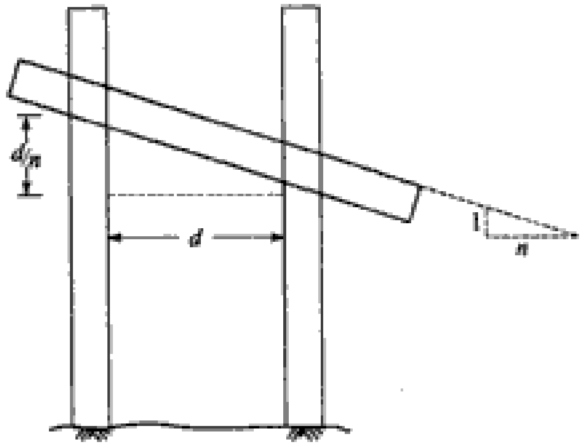


Fig. 5.23 Slope rail

Profile boards or batter boards: These are similar to sight rails, but are used to define the corners or sides of a building. They have been already discussed in Sec. 5.4.1. (Fig. 5.13).

Setting out grades by a theodolite

This method is employed to establish grade stakes when the grade is uniform for a considerable distance. The line of sight parallel to the grade is established with a theodolite, and a grade rod or levelling staff is used to establish the grade at the desired level for all points.

Two points A and B shown in Fig. 5.24, are fixed at the required grade. The theodolite is set up at A with line of sight set at the desired grade. The height of instrument is measured from the top of the stake A and let it be h. A target is fixed on the grade rod at height h, and the rod is placed at the stake B. The telescope is directed towards grade rod at B, and the grade rod is raised or lowered until the target is bisected. The instrument is clamped. The stake at B is fixed, touching the bottom of the grade rod. The intermediate stakes C, D, E, etc., are established by observing the same staff reading, or by fixing the target at the required height on the grade rod.

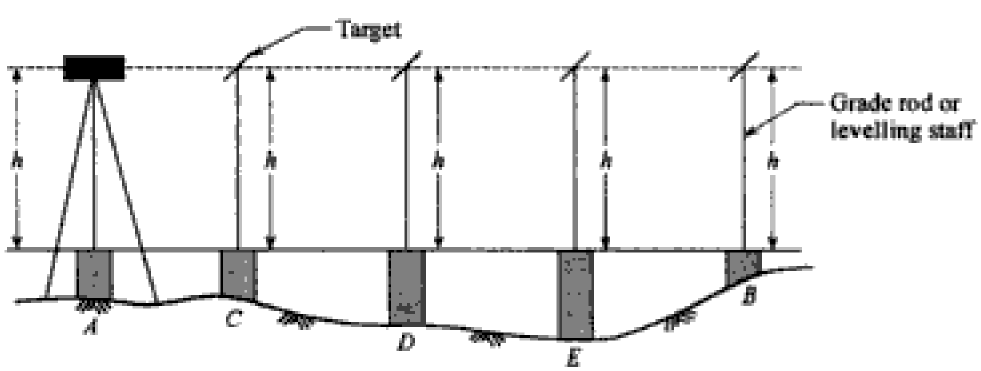


Fig. 5.24 Fixing a line at the required grade

Setting out grades for sewers and pipe lines

The sight rails and boning rods are used to set the proper gradient between two points on the alignment of sewers, pipe lines, roads or railways, etc. The sight rails are placed at regular intervals of distances along the centre-line (Fig. 5.25) and at each change of gradient and direction. The top of each sight rail is kept exactly horizontal, and is accurately set to the desired elevation. The line joining the top edges of the two consecutive sight rails is exactly at the same gradient as that of the centre-line of the work. A cord of string is stretched between the rails driven at the top of each sight rails. The length of the boning rod is determined by taking difference between the elevations of the top of the sight rail and of the actual centre-line of the work. The boning rods are placed with their tops touching the string, and the line joining their bottom points is parallel to the string, and thus has the desired gradient.

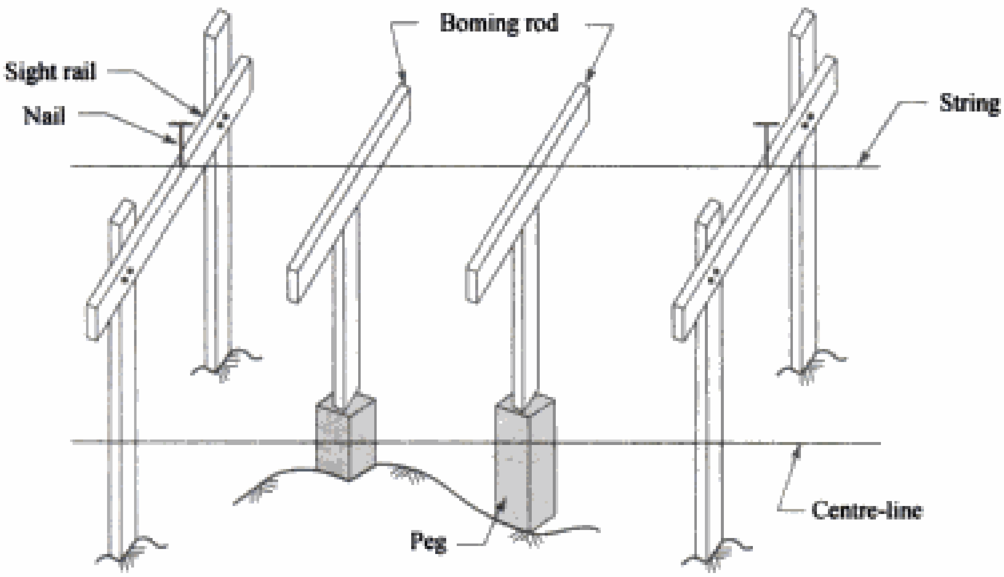
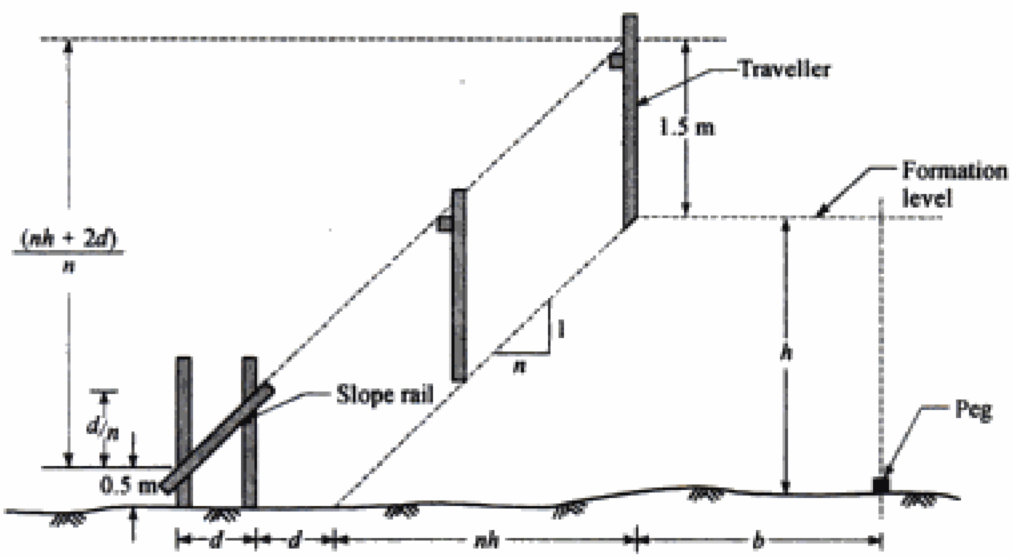


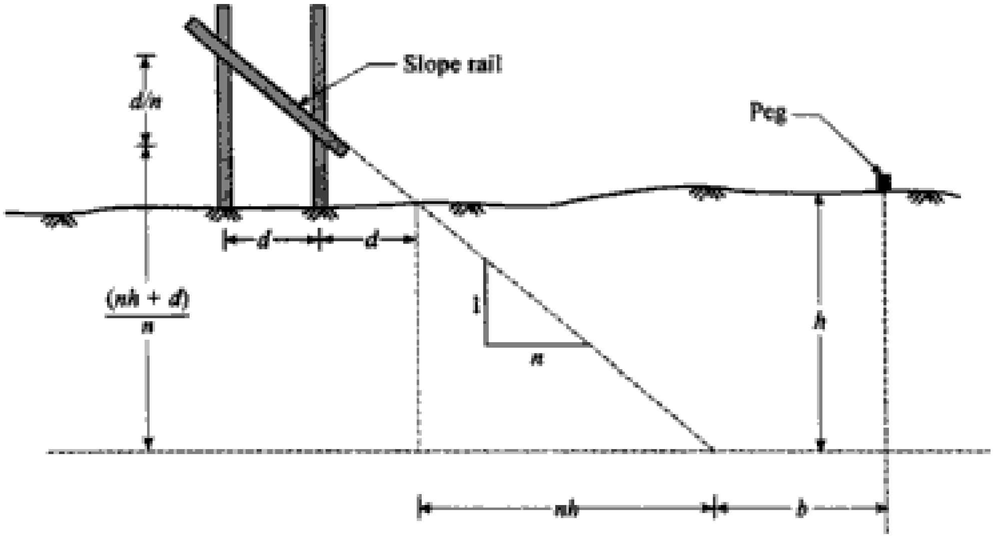
Fig. 5.25 Setting out grade with sight rails and boning rods for sewers

Setting out slopes in embankment and cutting

A slope rail and a traveller of say 1.5 m height, are used to control the slope during filling operations in embankments (Fig. 5.26a). The slope rail is set to the desired slope, and is placed at some distance from the toe of the slope to prevent it from disturbance during the earthwork operations. Fig. 5.26b shows the use of slope rail in cutting.



(a)



(b)

Fig. 5.26 Setting out slopes using slope rails

5.5 GRADE AND ALIGNMENT BY A LASER DEVICE

The basic idea in the application of laser devices (*cf.*, Chapter 11) to construction layout is that the laser plane replaces the horizontal line of sight of the engineer's level, and the laser beam replaces the string line. The laser can be projected for long distances, and is clearly visible as an illuminated spot on a target. The plane generated by laser or its beam can be located in space at any intermediate point by using a small plastic target or an electronic laser detector which automatically seeks the laser plane or beam when it is within a specified range. Consequently, no operator is required for the instrument while obtaining a reading. Other advantage of the laser is that it is not affected by wind or rain. However, it is unable to penetrate fog.

5.6 TUNNEL SURVEYS

Tunnels provide the shortest route in a highway or a railway. They are also used for the transportation of water for irrigation and power generation. A careful topographical study of the area is done for the selection of the most economical and safe alignment for a proposed tunnel.

Tunnel surveys consist of the following operations:

1. Surface surveys or staking out the alignment of the tunnel
2. Underground surveys
3. Transferring the surface alignment underground through a shaft
4. Transferring the levels underground.

Tunnel surveys require a high degree of precision, as it may not be possible to check the work until the tunnel is practically through. The permissible errors for alignment are of the order of 20 mm to 30 mm, and for levelling they are 2 mm to 20 mm, which are quite small compared to the permissible errors in ordinary ground surveys.

5.6.1 Surface surveys and tunnel alignment

After making reconnaissance of the area, two or three preliminary alignments are selected, and the final alignment is selected out of these considering feasibility and economy. The final alignment is staked out on the surface of the ground, and exact length of the tunnel is determined.

Short tunnels are generally driven from one end only. Long tunnles are usually driven from both the ends, as well as from one or more shafts. If there is only one ridge or peak from which both the ends of the tunnel are visible alignment between the ends may be done by the method of reciprocal ranging (*cf.*, Sec. 3.4.1 of *Plane Surveying*) or by the method of balancing-in (*cf.*, Sec. 4.6.5 of *Plane Surveying*). The ends of the tunnel and the proposed locations of the shafts are marked in a permanent manner preferably on metal plates. The alignment is extended on either side, and at least two stations are fixed beyond the ends on each side, so that the alignment may be re-established if necessary (Fig. 5.27).

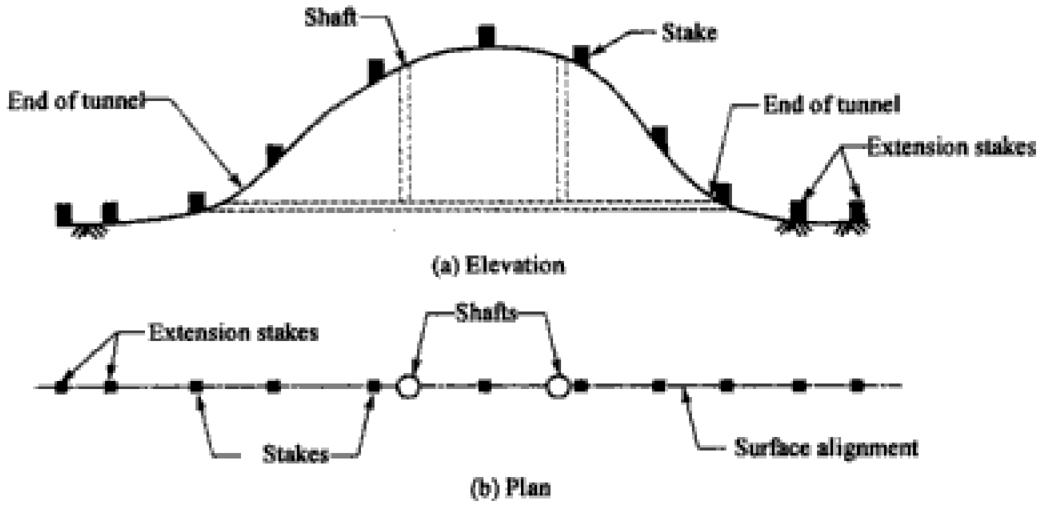


Fig. 5.27 Surface alignment of a tunnel

After marking the alignment on the ground, the exact horizontal distance between the two ends or terminals is measured, and corrections are applied if necessary. If any obstacle is found which does not allow the direct measurements of length of the tunnel and bearing of its axis then the techniques discussed in Sec. 9.10 of *Plane Surveying*, may be used.

In the case of comparatively very long tunnels and curved tunnels, an accurate network of ground control points, is provided on either side of the hill as well as on the top of the hill by the method of triangulation of first-order. Two intervisible points, about 1 km apart and in the area free from the disturbance due to blasting of rocks during construction of the tunnel, are preserved near the two terminals of the tunnel.

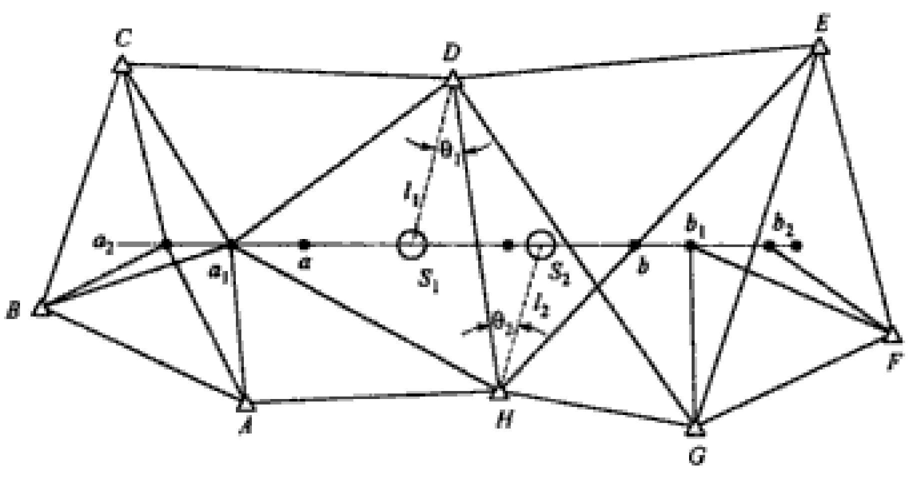


Fig. 5.28 Ground control for a tunnel

In Fig. 5.28, let the main triangulation stations be A, B, C, etc. The two terminals of the tunnel are a and b. The reference points a_1 and a_2 are situated on the extention of the centre-line ab near the terminal a, and the reference points b_1 and b_2 near the terminal b. S_1 and S_2 are the locations of the two shafts. Cartesian coordinates of the triangulation stations as well as those of reference points are computed. The elevation of each station is determined accurately by differential or trignometrical levelling.

The location of the shaft is fixed by measuring the distance from the triangulation station and the angle from one of the sides of the triangulation scheme as shown in Fig. 5.28.

5.6.2 Underground surveys

Underground surveys are conducted while driving a tunnel or later for maintenance works. Special instruments and procedures are required to overcome the problems posed by constraints of work space and headroom, and also due to inadequate ventilation and lighting. The signals used underground are illuminated for better sighting. These may be a plumb line seen against a white background of oiled paper and illuminated from behind by a lamp. Sometimes, a mining transit is used in place of the ordinary transit. Underground surveys may also be required for checking and correcting the alignment of the tunnel, the locations of the shafts, and so on.

5.6.3 Transferring the surface alignment through a shaft

Transferring the surface alignment through a vertical shaft is a difficult operation in view of the small size of the shaft (3 to 5 m diameter).

To transfer the surface alignment shown in Fig. 5.29, two timber beams P and Q are placed across the shaft opening at approximately right angles to the alignment. Metal plates are fixed on top of the timbers roughly in line. On the metal plates two points are marked which lie on the alignment using a theodolite and lining-in method. At these points, holes are drilled, and long piano wires are suspended through them and heavy plumb bobs p and q of about 5 kg in weight are attached to their lower ends. The plumb bobs are immersed in oil filled containers to keep them taut and plumb, and free from oscillations. The line joining the wires at the bottom gives the alignment.

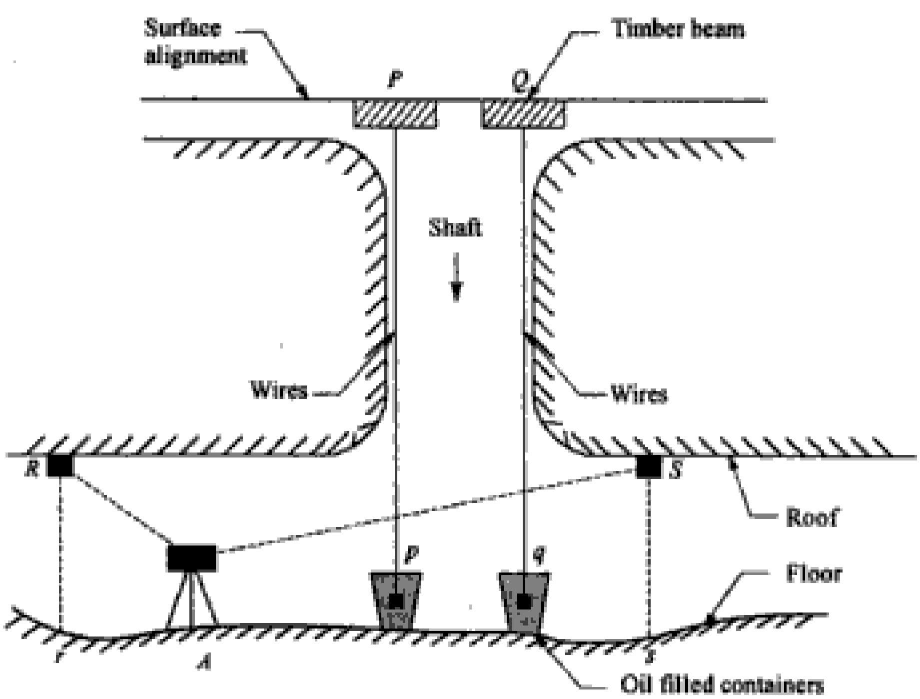


Fig. 5.29 Transferring surface alignment underground

In order to mark the alignment accurately, a theodolite is set-up in the tunnel at A in line with both the wires. Two marks R and S, along the back sight and fore sight directions, respectively, and fixed permanently on the roof by driving spikes. The line RS is the transferred surface alignment.

An alternative method of transferring the alignment is *Weisbach triangle* method. By this method the theodolite may be set-up in line with lines as described below.

Let p and q be the plan positions of the wires in the tunnel (Fig. 5.30). A theodolite, reading directly one second, is set up at A', approximately in line with p and q. The angle $pA'q$ is measured by the method of repetition, and the lengths of sides of triangle $pA'q$ are also measured correct up to millimetre. The angle pqA' is calculated by applying sin rule to the triangle pqA' as below.

$$\sin qpA' = \frac{qA' \sin pA'q}{pq}$$

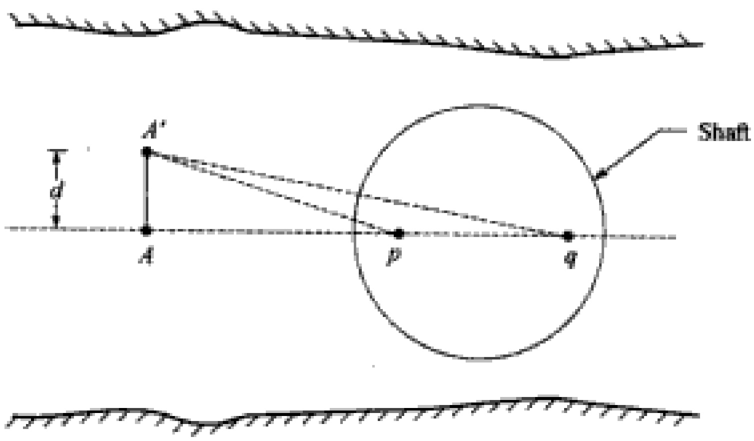


Fig. 5.30 Weisbach triangle

Now, the perpendicular distance d of A' from the line pq produced, can be calculated from the following expression.

$$d = pA' \frac{qA'}{pq} \sin pA'q$$

The points p and q are joined by a fine thread, and a perpendicular $A'A$ equal to d in length, is dropped from A' on the thread. The location A is the required point on the line qp produced which may be occupied by the theodolite for fixing the points R and S shown in Fig. 5.29.

5.6.4 Transferring the levels underground

Levelling along the surface alignment is performed in the usual manner. A longitudinal section of the whole surface alignment is obtained, if possible, and bench marks are established at the ends of the tunnel and near the shaft.

A steel tape is suspended from the cross beam PQ vertically through the shaft with a weight attached to its zero end (Fig. 5.31). The tape is held at a full metre mark at the point A of known elevation on the cross beam.

A level is set-up close to the tape in the tunnel, and a reading is taken at the bottom end of the tape ensuring proper focussing of the telescope.

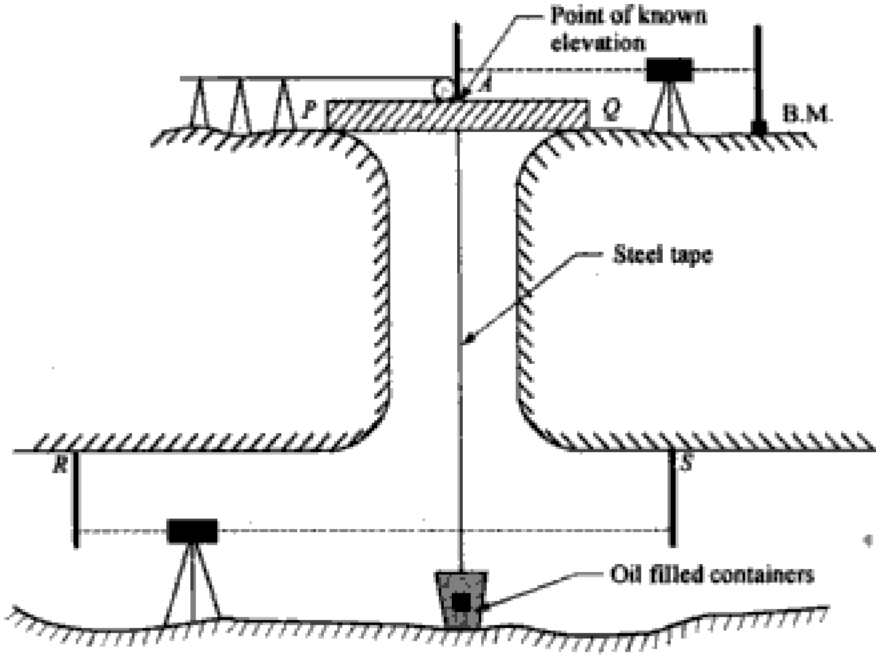


Fig. 5.31 Transferring levels underground

A nail is fixed in a solid timber at the elevation of the line of sight, and its elevation is established. This serves as a bench mark for further extension of level net in the tunnel. Two points *R* and *S* are also established in the roof of the tunnel, and their elevations are established to get the elevations of other points in the tunnel in case the bench mark established on the timber is disturbed.

PROBLEMS

5.1 What do you understand by construction surveys and why are they done? What are the main operations which are performed for construction surveys?

5.2 Write a short note on horizontal and vertical control in construction surveys.

5.3 What are different types of grids used in setting out works? Discuss each briefly.

5.4 Discuss the various methods of positioning a structure.

5.5 Describe the methods of giving layout of a building.

5.6 Write a brief note on setting out a building using circumscribing rectangle.

5.7 Explain the method of staking out a building by centre-line rectangle.

5.8 Discuss the use of the following in setting out works:

(*i*) Reference pillars

(*ii*) Batter boards.

5.9 How would you set out a culvert in the field? Explain the method with neat sketches.

5.10 How the setting out of a bridge is different from that of a culvert? Describe the methods of determination of distance between the end points of a long bridge.

5.11 Describe the methods of locating the piers of a bridge.

5.12 Describe the following with neat sketches and their uses:

(*i*) Boning rods or travellers

(*ii*) Sight rails

(*iii*) Slope rails

(*iv*) Profile boards.

5.13 Explain the operations involved in staking out water supply and sewer lines true to alignment and grade.

5.14 Describe the use of slope stakes in setting out slopes in embankment and cutting.

5.15 Explain the method of fixing alignment of a tunnel on the ground surface.

5.16 How are the surface surveys are transferred underground?

5.17 Explain the use of 'Weisbach triangle' in tunnel surveys.

5.18 Describe the method of transferring the surface levels underground in tunnel operations.

5.19 Describe the procedure of setting out a two-stories rectangular building with 200 mm thick load bearing outer walls all around. The foundation width is 750 mm, and the outer dimensions above plinth level are 1200 mm × 1800 mm. Draw the foundation plan of the outer wall showing all dimensions (not to scale).

In the above plan, show the positions of different pegs required for setting out the building.

6

ROUTE SURVEYING

6.1 GENERAL

The term 'route surveying', in a very general sense, may be used for the surveys carried along a comparatively narrow strip of territory for the location, design and construction of any route of transportation such as highways, railways, canals, sewers, water lines, power lines, telegraph lines, telephone lines, cableways, belt conveyors, and similar routes. Route surveying is conducted for the purpose of selecting the best route between two termini or end stations, and to establish the horizontal and vertical alignments of the selected route.

A comprehensive route survey consists of the following:

1. Reconnaissance
2. Preliminary survey
3. Location survey
4. Construction survey.

Surveys of some type are required for practically all phases of route alignment, planning, design, and construction work. Although the details of surveying methods on a particular project depend upon the nature of the project, the topography and many other factors, the general procedure being similar for all types of routes. Most of the discussions in this chapter are confined to route surveying for highways, and railways. However, the same methods with a few minor modifications may be used for other types of routes.

6.2 RECONNAISSANCE

A reconnaissance is a rapid and rough survey in which a thorough examination of the area through which the proposed survey line is to run, is conducted to ascertain the best routes and the approximate cost of the project. The reconnaissance survey is the key to the project and therefore, it must be done under the supervision of an experienced engineer who should be unbiased, resourceful and gifted with aptitude of engineering projects and having wide power of observation for the present as well as for the future requirements.

The first step in reconnaissance survey is to collect the available map and aerial photographs of the area. Various possible routes are marked on the map keeping the general topography of the area in the mind. The area under consideration is examined in detail in order to assess the feasibility and relative merits of all the possible routes.

In the second step of reconnaissance, approximate elevation and distances are measured. The elevations are determined by an altimeter or a clinometer. The distances are found by pacing. The directions and angles are measured by a magnetic compass. If necessary, a stadia survey of the area can be conducted. From all these measurements, the possible gradients and the probable necessary lengths are determined for each alternative route. The relative costs of the various routes are worked out, and the best route is selected for the more accurate preliminary survey.

The reconnaissance also includes the collection of information about the type of soil, geological structures, drainage patterns, ecology, land usage, etc. These information help in selecting the best possible route as they provide a general impression of the terrain.

At the end, a reconnaissance report is prepared which includes the following information:

(*i*) Accurate topography of the country with a short description of the topography.
(*ii*) Obligatory points, towns, bridges, highways, railways, river crossings, etc., on the routes.
(*iii*) Geological characteristics of the soils of the area and land use.
(*iv*) Width of the waterway required for each river and other drainages.
(*v*) Maximum discharge and high flood level of the rivers.
(*vi*) Availability of building materials, labour, machinery, etc.
(*vii*) Total length of each route and approximate gradients.
(*viii*) Probable radii of the horizontal curves.
(*ix*) Amount of expected earthwork and cost of construction of each route.
(*x*) A critical analysis of the various routes with economic analysis and conclusions, and justification for the selected best route.

A route may have three locations: (*i*) valley location, (*ii*) cross-country location, and (*iii*) ridge-line location. In the case of valley location, the route follows the valleys and the drainage lines, and has few excessive grades. There is often danger of washouts and floods. A number of bridges may be required to cross the tributaries. The reconnaissance of the route should include the entire valley since it often happens that a more advantageous location is achieved by crossing the valley at strategic points. In the case of cross-country location, the line is located in opposition to the drainage. Such a line crosses the ridges very often, and will have steep grade. The construction costs along such a line may also be excessive. Locations along ridges are relatively free of drainage problems and major drainage structures. However, since ridges are seldom straight, considerable curvature may have to be employed in such a location. Also steep grades are encountered when the location drops into valleys, or when the ridge is regained.

6.3 PRELIMINARY SURVEY

A preliminary survey is a detailed survey of a narrow belt of the country through which the proposed line is expected to run. The purpose of such surveys is to prepare an accurate topographic map of the selected belt of the country to arrive at close estimate of the cost of the line. Preliminary surveys also help in the preparation of construction plans.

The selected belt or strip should be of sufficient widths to accommodate any unexpected variation in the location of the route. Usually, the width of the strip is taken as 100-200 m for highways and 400-500 m for railways. The width of the strip also depends upon the character of the terrain through which the route has to pass.

The field work in preliminary survey usually consists of traversing by theodolite along the selected routes, and the distances, heights and angles between the traverse stations are measured accurately in order to prepare a topographic map of the selected narrow strip. The longitudinal section and cross-sections are also run along the strip. The preliminary survey should be quite accurate so that the selection of the final alignment can be made. The instruments generally employed for preliminary survey are

(*i*) theodolite, (*ii*) compass, (*iii*) engineer's level, (*iv*) hand level or Abney level or any clinometer, (*v*) levelling staff, (*vi*) chain and tape, (*vii*) plane table, (*viii*) substance bar, and miscellaneous equipment like ranging rods, pegs, etc.

On small projects, the entire field work for preliminary survey is usually done by a single party. However, for large projects, the field work is conducted under control of location engineer, and may be divided into the following parties:

(*i*) Transit party
(*ii*) Level party
(*iii*) Topographic party.

6.3.1 Transit party

A transit party consist of a transitman, two chainmen, a flagman, a clearing squad for clearing the line and setting stakes, and a recorded for important projects. Before starting preliminary survey, the following information should be collected:

1. Location of the bench mark near the route, if any.
2. Location of the secondary control, if any.
3. Mode of transportation to access the site.
4. Ownership of the land to be trespassed for obtaining permission, if required.
5. Availability of local labour.

The survey work of the transit party is to conduct open traversing with a theodolite along the selected route. Usually the method of deflection angles is employed in the case of highways, and the method of back angles in the case of railways. The azimuths of the first and the last lines of the traverse are determined by astronical observations. For long traverses, azimuths are observed at about 20 km intervals. The plane table can, sometimes, be used to advantage particularly in rough country which is comparatively free from obstructions such as trees, bushes, etc. Stadia method which is rapid and economical but less precise, can also be employed.

The transit party should make note of the following:

1. Checking of bearings of a few intermediate lines, preferably after 10 to 15 km.
2. Recording and marking of turning or intersection points.
3. Fixing of stakes at the intersections with roads, railways, canals, streams, etc.
4. Taking observations on prominent objects such as chimneys, spines, temples, mosques, pylons, etc., from at least three stations.
5. Connecting the traverse to the existing control, if possible.
6. Plotting of each day's work to detect gross mistakes and omissions.
7. Marking every 30 m stations as full station and the intermediate station as plus station.

6.3.2 Level party

A level party consists of a level man and one or two staffmen. The party follows the transit party to run longitudinal sections along the traverse lines. It also establishes bench marks along the route at regular and convenient places, giving their descriptions in the notes. The elevations of the ground at all stakes, intersections with roads, railways, streams, etc., and on the points of change in slope, are determined in the usual way. The bed levels, depth of water, and high flood levels of the streams crossing the route, should also be determined. It is a practice to read the staff to the nearest 0.01 m at full stations and the points of change in slope, and up to 0.005 m at transit points, transit stations and bench marks. As far as possible, the line of levels should be checked by taking observations on the existing bench marks.

At the end of day's work, the longitudinal section along the traverse line is plotted on the following scales:

Project	Horizontal scale	Vertical scale
Highways	1:1000	1:100
Railways	1:2000	1:200

6.3.3 Topographic party

A topographic party consists of a surveyor, two staffmen, and two chainmen. The topographic party follows the level party. Its main job is to prepare the topographic map of the narrow belt along the proposed route. The natural and artificial details, rivers, streams, roads, railways, canals, villages, property lines, etc., are plotted. The contours are also plotted at suitable contour interval. The party also collects the information regarding the character of land, cultivation, excavation, rock type, etc. Cross-sections at right angles to the traverse lines are also taken, normally at spacing of 30 m. However the spacing may be reduced to 5 m for hilly areas, and increased to 100 m or 80 for smooth country.

Knowing the coordinates of the traverse stations and other intersected points, their location can be easily plotted on the scale say 1 : 1000. Using a plane table the details are plotted either by the method of intersection or radiation. Stadia method can be employed for the measurement of horizontal distances and vertical distances required for contouring. The contour interval may be kept as 0.5 to 1.0 m for flat ground and 2.0 m for steep ground. The stadia method is rapid and also economical but not so precise. It is also not suitable for forest areas.

6.4 LOCATION SURVEY

Location survey is carried out in two stages: (*i*) Office location, and (*ii*) field location.

6.4.1 Office location

Marking of the selected final alignment including the horizontal curves at the desired locations on the map of the selected strip, is known as *paper location*. The final alignment may be anywhere in the strip and in most favourable position. The longitudinal section or profile is drawn for the final alignment from the contours on the map, and the grade line is marked on the profile in pencil.

The following points should be considered while selecting the final alignment.

(*i*) Minimum gradient
(*ii*) Minimum curvature
(*iii*) Equalization (balancing) of earthwork
(*iv*) Heavy earthwork
(*v*) Minimum number of expensive bridges
(*vi*) Minimum number of retaining and breast walls
(*vii*) Suitable crossings for rivers.

The new line of the route and its corresponding profile are further studied to improve the alignment, if possible. Repeated trials are made till a well-balanced location line is obtained. To avoid excessive erasing of the pencil lines, it is advisable to use needles and thread for various alignments, and the only final alignment and grade line need be drawn in pencil on the map and the profile, respectively.

Some of the specifications needed for making the paper location, are given below:

Ruling gradient

The ruling gradient along the straight portion of the railways, depends upon the maximum load of train, the least speed of climbing, the design speed of the locomotive, and its weight. The recommended ruling gradients on railways are 1 in 125 to 1 in 200 in plain country and 1 in 40 to 1 in 80 in hilly country. For highways, ruling gradient is usually 1 in 20 to 1 in 25. The ruling gradient is further compensated for curves, and varies according to degree of curves and the gauge in the case of railways.

Degree of curves and their compensation for railways

Maximum permissible degree of curves and their compensation employed in India are given below:

Gauge	Maximum degree	Compensation %
B.G.	10°	0.04
M.G.	16°	0.03
N.G.	40°	0.02

Radius for highway curves

The recommended minimum radius of curve for highway are as under:

Country	Radius of Curve (m)
Undulating	30
Hilly	15
Hilly steep	11 (for hair pin)

6.4.2 Field location

Field location is the process of transferring the paper location of the final alignment of a route to the ground, i.e., it is setting out the paper location on the ground. Sometimes, minor changes in the alignment are made in the field location, if necessary.

The central-line of the final alignment of the route is set out on the ground by the following methods:

1. By intersection between the paper location of the alignment and the traverse.
2. By scaling the positions of various points on the map and transferring them to the ground.

The procedure most commonly used to locate the alignment on the ground, is given below.

The positions of the various points to be transferred on the ground are scaled off from the preliminary map. For this, perpendicular offsets may be taken from the traverse line. The chainage of the intersections of the line with traverse may be ascertained, and if necessary angles and distances may also be used. The angle of intersection at the intersection of the adjoining tangents produced, is carefully measured. From the degree of curve and angle of intersection, the necessary data are computed for setting out the simple curves. Stakes are driven at every 30 m interval, and hubs are set out at all important stations such as theodolite stations, intersection points, points of tangencies, etc., giving their proper reference in the field book. Profile levelling is then run on the located alignment on the field. The profile is drawn, gradient lines are marked, and vertical curves are shown on the profile. A critical study is made of the profile obtained with that obtained in the preliminary survey, and minor modifications in gradients, etc., are made to fit the ground, if necessary.

After the line has been finally located on the ground, it is plotted both in plan and in profile. The line is called *final location* and the map is called *final location map*. The final location map should show all important features in the vicinity of line, all points where hubs are set, all bench marks, and the boundaries of private properties. The names of the owners of the private properties are also collected to facilitate in acquisition of the land for the right of way.

The approximate quantities of the earthwork is calculated by drawing cross-sections of the located lines on the preliminary map, using contours. It is advisable to run the cross-sections again after the final location.

6.5 CONSTRUCTION SURVEY

The purpose of the construction survey is to re-establish points, lines and grades on the ground for construction of the project. It is essentially setting out the details of the route on the ground. It also consists in setting out culverts, bridges, aqueducts, syphons, etc., and in carrying on such other surveying as may be needed for the purpose of construction.

A project construction engineer is entrusted with the entire job which may consist of the following:

(*i*) Visiting the located line, and checking the location stakes.
(*ii*) Resetting the missing or disturbed stakes from the field notes.
(*iii*) Checking the levels of various points, and establishing additional bench marks.
(*iv*) Setting out additional stakes at the intersections of the tangents to be used to restake the final location when the location stakes are disturbed during construction.
(*v*) Setting out the side slope stakes for earthwork as well as grade stakes.
(*vi*) Setting out the stakes for structures such culverts, bridge abutments, etc.
(*vii*) Setting out the curves (simple, transition and vertical).
(*viii*) Setting out the sides of barrow pits.
(*ix*) Taking final cross-sections for ascertaining the quantity of earthwork.
(*x*) Determining the waterway for each bridge.
(*xi*) Measuring the finished work at regular intervals for regular payment of the contractors.

Setting out works have been discussed in Chapter 5.

PROBLEMS

6.1 What is route surveying? Discuss briefly the various stages of route surveying.

6.2 What do you mean by reconnaissance? What data are normally collected in reconnaissance?

6.3 Discuss in brief preliminary survey required for a route.

6.4 Describe various surveys required before fixing final alignment of a highway to connect two cities.

6.5 Write short notes on the following:

(*i*) Paper location
(*ii*) Field location.

6.6 Enumerate the jobs entrusted to a project construction engineer for the construction of a highway.

7

FIELD ASTRONOMY

7.1 GENERAL

The engineer or surveyor needs a knowledge of field astronomy for the determination of absolute locations of points and directions of lines on the surface of the earth. The absolute location of a point is defined in terms of latitude and longitude whereas the direction is fixed in terms of azimuth or bearing with the true meridian. These quantities are determined by making observations on the celestial bodies such as stars and sun, using the principles of astronomy. In plane surveying, a very high degree of precision is not required in the determination of azimuth. In most cases, it would be sufficient if the azimuth is established with a degree of precision at least equal to that of the instrument used for measurement of angles. The determination of the latitude and longitude is done with great precision and it involves use of instruments of very high precision. Most azimuth methods in use require the instant at which the observation is made and the latitude of the place of observations. It is thus frequently necessary to determine the time and latitude by observation before the azimuth can be determined.

The purpose of this chapter is to present in condensed form the methods of observation and computation which can be used when the survey is being conducted with an ordinary engineer's transit or theodolite.

7.2 EARTH

The various celestial bodies are considered to be located on a sphere of infinite radius. The centre of this sphere is the centre of the earth. The shape of the earth is like the shape of an *oblate-spheroid* or *ellipsoid of revolution. The* diameter of the earth along the polar axis is smaller than that along the equator, and hence it is flattened at the poles. The flattening f is about 1/293, and it is expressed as

$$f = \frac{a-b}{a} \quad \text{...(7.1)}$$

where a and b are the semi-major axis or *equatorial radius* and semi-minor axis or *polar radius* of the ellipsoid, respectively. According to Hayford the values of a and b are 63,78,388 m and 63,56,912 m.

The points where the polar axis intersects the surface of the earth are called the *geographical* or *terrestrial north pole* and *south pole* (Fig. 7.1). The earth revolves about its polar axis on an average once in 24 hours from west to east. To an observer on the earth, the sun and stars appear to revolve from east to west.

The intersection of the surface of the ellipsoid or earth with any plane perpendicular to the polar axis is a circle. If such a plane passes through the centre of the earth, the intersection is called a *great circle* and for any other such planes, the intersection is called *small circle*. The distance on the arc of a great circle corresponding to an angle of 1 minute subtended by the arc at the centre of the earth, is equal to one *nautical mile*. Thus,

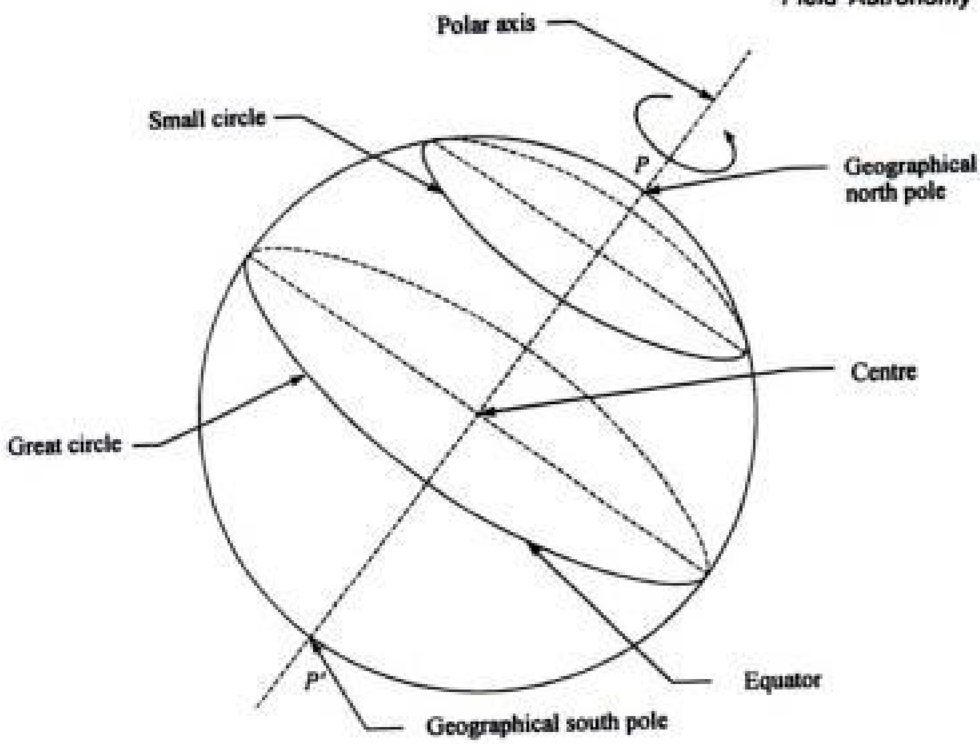


Fig. 7.1 The earth

$$1 \text{ nautical mile} = \frac{\text{Circumference of the great circle of the earth}}{360° \times 60'}$$

Taking the mean radius of the earth approximately equal to 6370 km,

$$1 \text{ nautical mile} = \frac{2\pi \times 6370}{360° \times 60'} = 1.853 \text{ km.}$$

The earth has been divided into certain zones depending upon the parallel of latitudes (θ) above and below the equator as shown in Fig. 7.2.

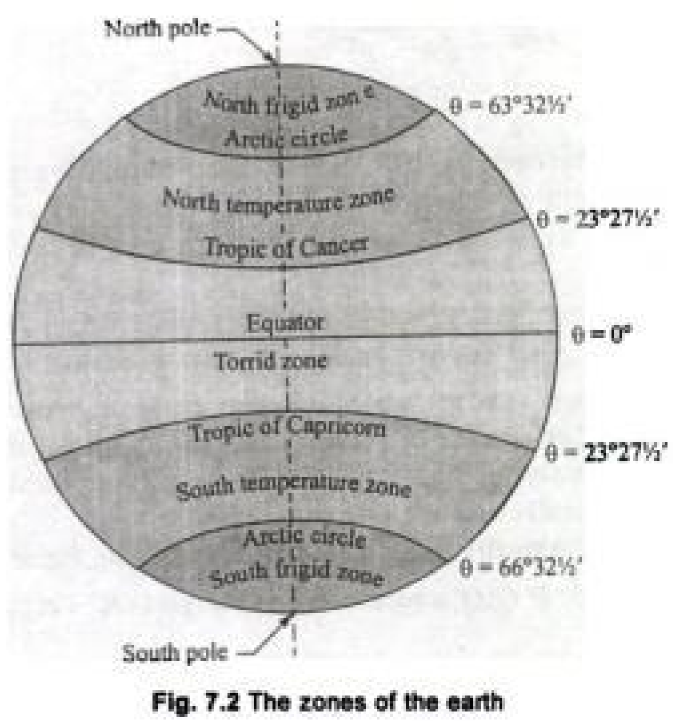


Fig. 7.2 The zones of the earth

7.3 A SPHERE AND ITS PROPERTIES

The celestial bodies appear to lie on the surface of a very large sphere which appears to move around the earth. To understand the real and apparent motions of the celestial bodies, a surveyor must be familiar with the geometry of sphere and spherical triangle.

A sphere is a solid bounded by a surface whose every point is equidistant from a fixed point called the centre of the sphere. A sphere has the following properties (Fig. 7.3):

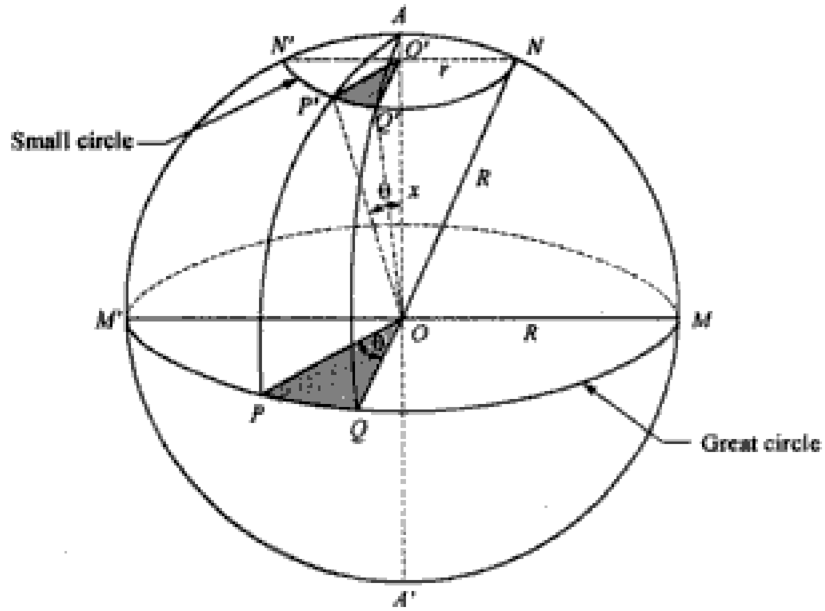


Fig. 7.3 Properties of a sphere

(*i*) A section of a sphere by any plane is a circle whose radius is inversely proportional to the perpendicular distance of the plane from the centre of the sphere. If the centres of the sphere and the circle are O and O', respectively, and the perpendicular distance of the plane of the circle from O is OO', then the radius of the circle is given by

$$O'P' = \sqrt{R^2 - x^2} \quad ...(7.2)$$

where R = the radius of the sphere

x = the perpendicular distance OO'.

(*ii*) The shortest distance between any two points on the surface of the sphere is along an arc of great circle passing through them. There can be one and only one such arc.

(*iii*) The length of an arc of a great circle is equal to the angle which it subtends at the centre of a sphere of a unit radius.

Let PQ be an arc of the great circle $M'PQMM'$ and θ be the angle which it subtends at the centre O then

$$PQ = R\theta = \theta \text{ (for a unit sphere } R = 1) \quad ...(7.3)$$

(*ii*) The arc of a small circle is equal to the corresponding arc of the great circle multiplied by either the cosine of the distance between the two circles or the sine of the angular radius of the small circle. The distance of any point on the small circle from its nearer pole, is called the *angular radius* or the *polar distance* of the small circle.

The lines AP and AQ meet the small circle at P' and Q', respectively. Since the planes of the two circles are parallel to each other the triangle $P'O'Q'$ is parallel to the triangle POQ. Let the angle $P'OO'$ be ϕ, and the radius of the small circle be r.

Since the angle $P'O'O$ is a right angle,

$$\sin P'OO' = \frac{P'O'}{P'O}$$

or

$$\sin \phi = \frac{r}{R}$$

Also,

$$\frac{\text{arc } P'Q'}{\text{arc } PQ} = \frac{r}{R} = \sin \phi$$

$$P'Q' = PQ \sin \phi \quad \text{...(7.4)}$$

Since the arc AP' lies on a great circle passing through A and A', from Eq. (7.3), we have

$$AP' = \phi$$

Therefore Eq. (7.4) becomes

$$P'Q' = PQ \sin AP' \quad \text{...(7.5)}$$

But

$$PP' = QQ' = 90° - \phi$$

$$\phi = AP' = 90° - PP'$$

Thus Eq. (7.5) is

$$P'Q' = PQ \sin (90° - PP')$$

or

$$P'Q' = PQ \cos PP' \quad \text{...(7.6)}$$

7.4 A SPHERICAL TRIANGLE AND ITS PROPERTIES

A triangle formed on the surface of a sphere by intersection of three great circles, is a *spherical triangle*. The angle formed by intersection of two great circles is called the *spherical angle*. It may be also defined by the plane angle between tangents to the great circles at their point of intersection. In Fig. 7.4, AB, BC and CA are the arcs of three great circles of the same sphere, and they form a spherical triangle ABC. The angle θ shown in Fig. 7.4b, is the spherical angle.

A spherical triangle consists of six elements. These are its three sides a, b, and c, and three angles A, B, and C shown in Fig. 7.4b. All the six elements of a spherical triangle are expressed as angles.

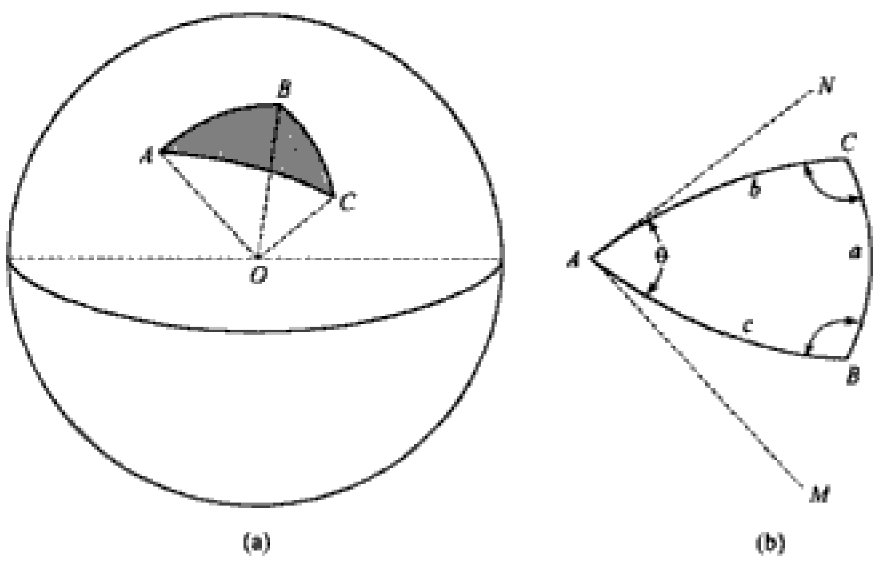


Fig. 7.4 Spherical triangle and spherical angle

Celestial sphere: The imaginary sphere on which heavenly bodies such as stars, sun, moon, etc., appear to lie, is known as the celestial sphere. These heavenly bodies are at varying distances from us. But in astronomy we are concerned with their relative directions only rather than their distances from the observer. It is convenient to visualize the stars as distributed over the surface of an imaginary spherical sky having its centre at the position of the observer. Since the radius of the earth is very small compared to the radius of the imaginary sphere, called celestial sphere, the centre of the sphere may be taken at the centre of the earth, and the earth is represented as a point.

Zenith and nadir: The point on the celestial sphere vertically above the observer's station is called the zenith (Z), and the point vertically below is called the nadir (Z').

Celestial horizon: The plane of the great circle traced upon the celestial sphere which is perpendicular to the zenith-nadir line and passing through the centre of the earth, is called the celestial horizon.

Visible horizon: The small circle of the earth obtained by visual rays passing through the point of observation, is known as the visible horizon. The radius depends upon the altitude of the point of observation. The radius increases with the increase in the altitude of the point of observation.

Sensible horizon: The small circle obtained by passing a plane through the observer's station tangential to the earth's surface and perpendicular to the zenith-nadir line at the point of observation, is called the sensible horizon.

Celestial equator: The great circle of the celestial sphere, formed by the intersection of a plane perpendicular to the axis of rotation of the earth with the celestial sphere, is called celestial equator.

Terestrial equator: It is the great circle of the earth, the plane of which is perpendicular to the axis of rotation of the earth.

Celestial poles: The celestial poles are the points on the celestial sphere where the earth's axis of rotation when prolonged intersects the celestial sphere.

Terrestrial poles: The terrestrial poles are the points on the surface of the earth where the earth's axis of rotation when produced intersects the surface of the earth (*cf.*, Sec 7.2).

Vertical circles: The great circles of the celestial sphere passing through the zenith and nadir points, are the vertical circles.

Observer's meridian: The vertical circle which passes through observer's zenith and nadir, and the poles is called the observer's meridian.

Prime vertical: The vertical circle whose plane is perpendicular to the plane of the observer's meridian and which contains east and west points of the horizon, is known as the prime vertical.

North and south points: The projections of the elevated north and depressed south poles on the horizon are known as north and south points, respectively.

East and west points: The points of intersections of the prime vertical with the horizon are known as east and west points. These points may also obtained by the intersections of the equator and horizon.

Ecliptic: The great circle of the celestial sphere which sun appears to describe with earth as centre in the course of one year, is called the ecliptic. The plane of the ecliptic is inclined to the plane of the equator at an angle of about 23°27′ but subjected a diminution of about 5″ in a century. The angle between the planes of the ecliptic and the equator is called the *obliquity* (Fig. 7.8).

Equinoctial points: The points of intersection of the ecliptic with the equator are called the equinoctial points. The declination (defined in Sec 7.6.2) of the sun is zero at the equinoctial points. The *vernal equinox* or the *first point of Aries* (♈) is the point in which the sun's declination changes from south to north, and marks the commencement of spring (Fig. 7.8). It occurs on March 21st every year. It is a fixed point on the celestial sphere. The *autumnal equinox* or the *first point of Libra* (♎) is the point which the sun's declination is zero and it changes from north to south, and marks the commencements of autumn. It occurs on September 23rd every year. Both the equinocatial points are six months apart in time.

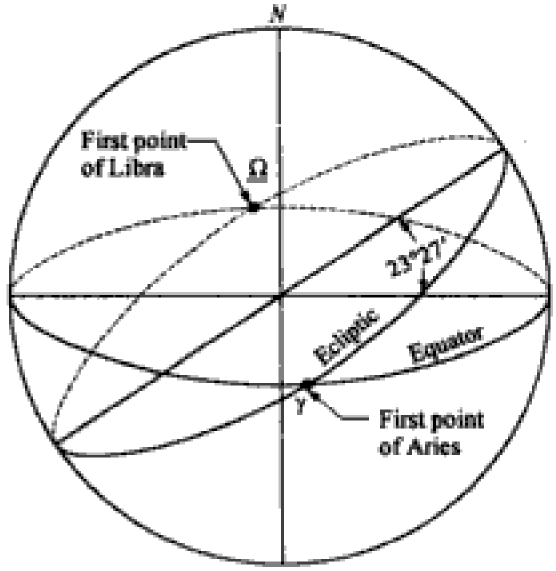


Fig. 7.8 The ecliptic

7.6 ASTRONOMICAL COORDINATE SYSTEMS

To locate the position of a heavenly body it is sufficient to measure its angular distances along arcs of two great circles intersecting each other at right angles. One of these great circles is called the *primary circle of reference* and the other the *secondary circle of references.* In Fig. 7.9, let O be the origin of the coordinate system. The position of a point M can be specified with reference to the plane OAB and the line OA. If a plane perpendicular to the plane OAB is passed through OM, it will intersect the latter in the line OB. The arcs AB and BM, or the two angles AOB and BOM, form the basis of the two coordinates to specify the location of M.

In field astronomy, the following coordinate systems are used to define the position of a celestial body:

1. The horizon system
2. The independent equatorial system
3. The dependent equatorial system
4. The celestial latitude and longitude system.

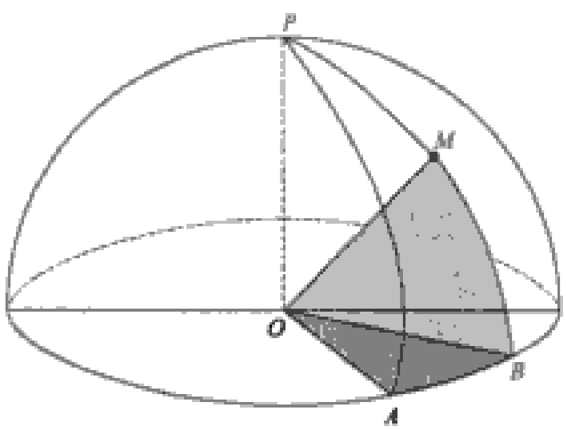


Fig. 7.9 Circles of reference

7.6.1 The horizon system

In the horizon system, the horizon is the plane of reference. Since the horizon depends upon the position of the observer, the horizon system is dependent on the observer's position. In this system, the azimuth and the altitude of a celestial body are the coordinates and, therefore, this system is also known as the *altitude and azimuth system*.

In Fig. 7.10 the two great circles of reference are the horizon SWNBE and the observer's meridian SZN, the former being the primary circle and the latter the secondary circle. *M* is a heavenly body (shown in the eastern part of the celestial sphere). *Z* is the observer's zenith and *P* is the celestial pole. ZMB is the arc of the great circle passing through Z and *M*, and is at right angles to the horizon. The first coordinate the angle NOB between the observer's meridian and the vertical circle through *M*, is the *azimuth* (*A*). The azimuth is also expressed as the angular distance *NB*, measured along the horizon from the observer's meridian to the foot of the vertical circle through the point. The azimuth is always measured from north either eastwards or westwards in northern hemisphere, and from south either eastward or westward in southern hemisphere.

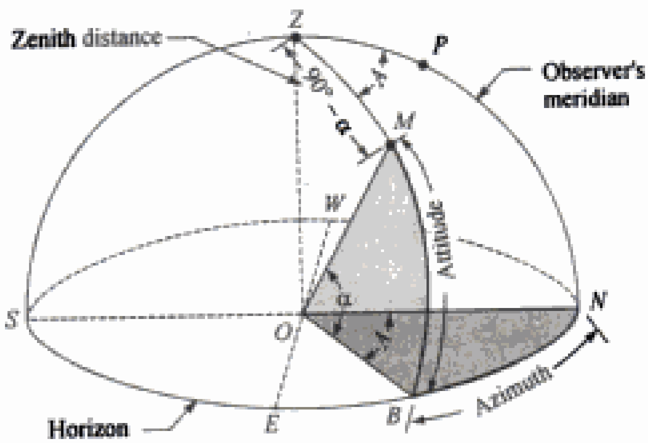


Fig. 7.10 The horizon system of coordinates

The other coordinate of *M* is its angular distance *BM* measured on the vertical circle through *M*, above or below the horizon and it is called the *altitude* (α).

The angular distance *ZM* on the vertical circle through *M*, is called the *zenith distance*. The position of a body may also be specified in terms of zenith distance and azimuth. The zenith distance is the complement of the altitude, thus

Zenith distance (z) $= 90° - \alpha$.

7.6.2 The independent equatorial system

In this system, the two great circles of reference are the equatorial circle and the declination circle, the former being the primary circle and the latter the secondary circle of reference. The two coordinates in this system are the right ascension and the declination.

In Fig. 7.11, the angular distance *AB*, along the arc of the celestial equator measured from the first point of Aries (♈) as the point of reference towards east up to the declination circle passing through the body *M* is called the *right ascension* (R.A.). It is also the angle measured eastward at the celestial pole, between the hour circle through the first point of aries and the declination circle through *M*. Since the motion of the celestial bodies is from east to west, the R.A. is measured in a direction opposite to the motion of the celestial bodies. It may be measured in degrees, minutes and seconds of arc or in hours, minutes and second of time.

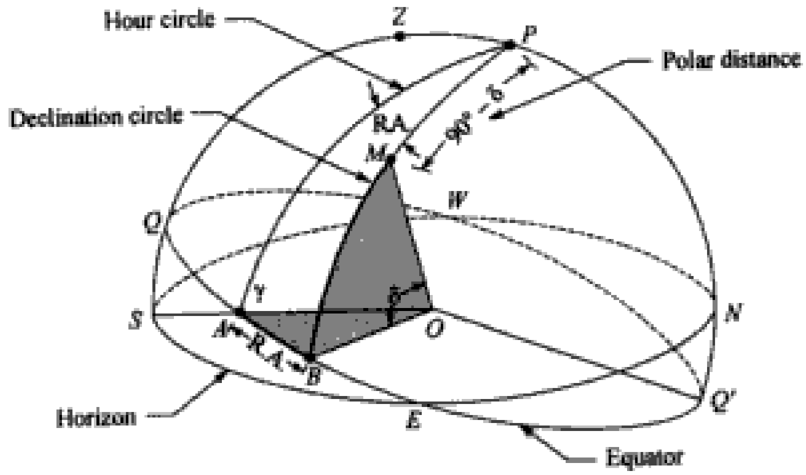


Fig. 7.11 The independent equatorial system

The other coordinate the *declination* (δ) is the angular distance *BM* of the body from the equator measured along the arc of the declination circle. The declination is positive when the body is north of equator, and negative when it is to south.

The complement *PM* of the declination is known as the *polar distance* (p), thus

$$\text{Polar distance } (p) = 90° - \delta .$$

7.6.3 The dependent equatorial system

In this system, the two great circles of references are the horizon and the declination circle through the celestial body, the former being the primary circle and the latter the secondary circle of reference.

The two coordinates in this system are the hour angle and the declination. The *hour angle* (*H*) is the angular distance SA along the arc of the horizon measured from the observer's meredian to the declination circle passing through the celestial body *M* (Fig. 7.12a). It is also measured as the angle subtended at the pole *P* between the observer's meridian and the declination circle of the body. In the northern hemisphere, the hour angle is measured from the south and towards west up to the declination circle, and its value varies from 0° to 360°.

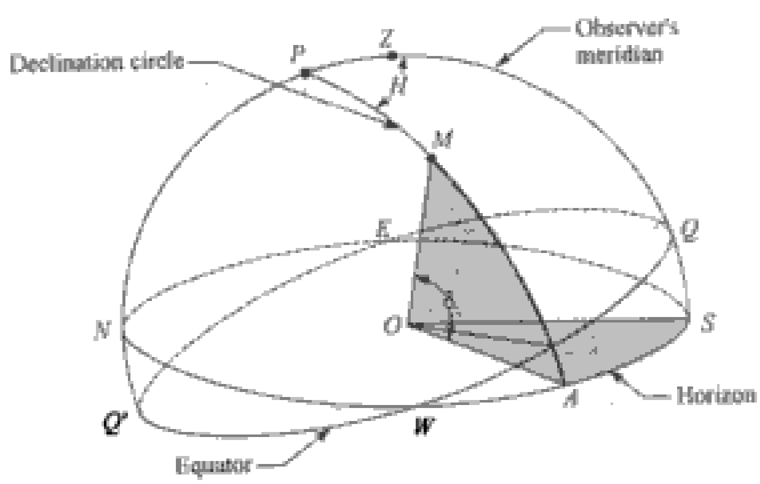


Fig. 7.12 (a) The dependent equatorial system

The following may be noted (Fig. 7.12*b* and *c*):

(*i*) In northern sphere, the hour angle of a celestial body is always measured from the south and towards west, and its value lies between 0° to 360°.

motion of the earth they change from time to time. The equator, ecliptic, the first point of Aries, and the poles of the equator are common to all the observers. The pole in the northern sphere can be easily fixed by knowing the latitude of the point of observation and consequently, the equator can be imagined with fair accuracy. But, it is difficult to imagine the positions of the first point of Aries and first point of Libra at any instant as they move on the equator due to diurnal motion. It is comparatively more difficult to imagine the ecliptic.

In view of the above, it is evident that it is not easy to locate celestial bodies particularly stars, in the sky by their right-ascension and declination or by latitude and longitude. There is no instrument available which can directly measure the right ascension and declination. On the other hand, the azimuth and altitude of a celestial body can be measured directly in the field with a theodolite. If the azimuth and hour angle are known, the right ascension and declination can be computed from the solution of the astronomical triangle *PZM* (Fig. 7.14). This requires that the instant of time at which the body was in a certain position should be determined so that the hour angle can be found.

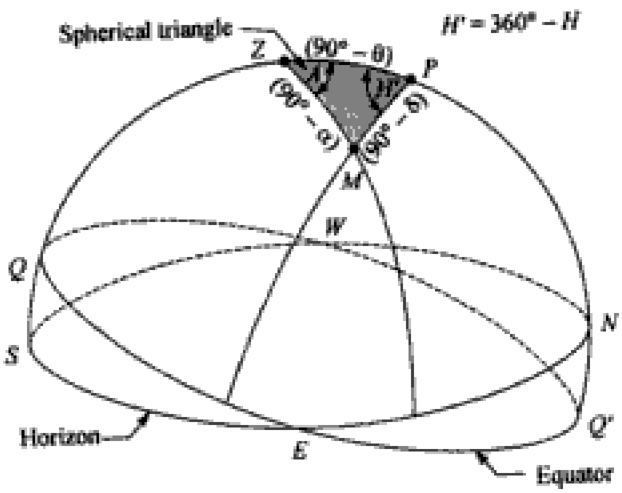


Fig. 7.14 Relation between different coordinate systems

The publication of star catalogues, almanacs or ephemerides in which the position of celestial bodies are referred to, requires a system of coordinates which is independent of the observer's position. The right ascension and declination system is such a system which does not depend upon the observer's position. The right ascension and declination can be determined if the hour angle and azimuth of the body are known. Therefore, the altitude and azimuth system, and the hour angle and declination system of coordinates are required for the publication of the star cataougues in which the coordinates are given in terms of the right ascension and declination system. Thus all the three systems are required to locate the position of the celestial bodies in a system which is independent of observer's position.

7.7 TERRESTRIAL COORDINATE SYSTEM

In order to locate the position of the points on the surface of the earth, the system of coordinates used is called *terrestrial coordinate system*. In this system the two coordinates are the terrestrial latitude and longitude.

For convenience, the earth is assumed to be spherical in shape as shown in Fig. 7.15. The two great circles of reference are the terrestrial equator and the terrestrial meridian. The *terrestrial equator* is the great circle on the surface of the earth whose plane is perpendicular to the earth's axis of rotation. The other great circle on the surface of the earth whose plane contains the earth's axis is the *terrestrial meridian*. The fixed meridian which has been universally adopted, passes through Greenwich and it is called as *standard meridian* or *Greenwich meridian*.

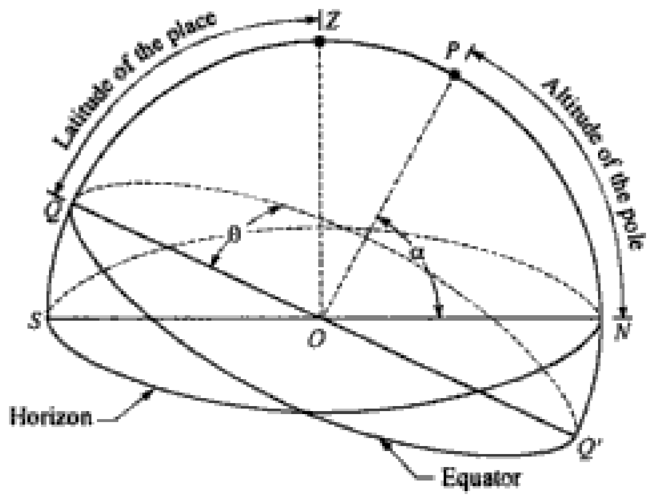


Fig. 7.17 Relation between the altitude of the pole and latitude of the place

Hence, the altitude of the evaluated pole is always equal to the latitude of the place of observation.

7.8.2 Relationship between the latitude, declination and altitude of a celestial body

Fig. 7.18, shows the three positions M_1, M_2, and M_3 of a celestial body in northern hemisphere. In the figure the declination δ_1 and the coaltitude $(90° - \alpha_1)$ are shown only for the position M_1.

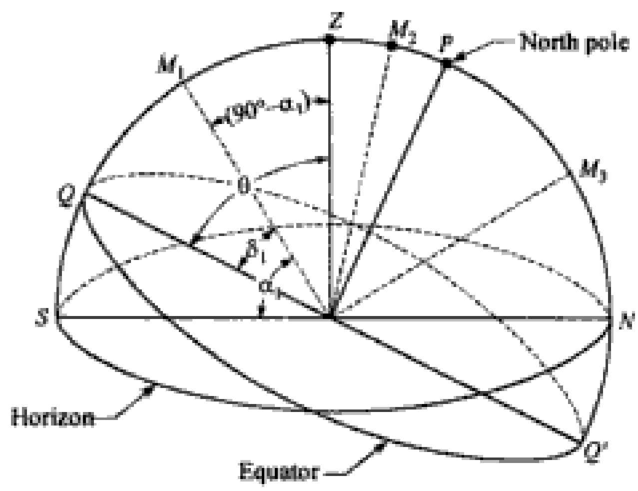


Fig. 7.18 Relation between the latitude, declination, and altitude of a celestial body

(*i*) For the position M_1 of a celestial body

Declination $= QM_1 = \delta_1$

Altitude $= SM_1 = \alpha_1$

Coaltitude or zenith distance $= M_1Z = 90° - \alpha_1 = z_1$

Latitude of the observer's position $= QZ = \theta$

From the figure, we have

$$QZ = QM_1 + M_1Z$$

or $$\theta = \delta_1 + (90° - \alpha_1)$$

or $$= \delta_1 + z_1$$

and taking

$$a = 90° - \delta$$
$$b = 90° - \theta$$
$$c = 90° - \alpha$$

we get

$$\cos A = \frac{\cos(90° - \delta) - \cos(90° - \theta)\cos(90° - \alpha)}{\sin(90° - \theta)\sin(90° - \alpha)}$$
$$= \frac{\sin\delta - \sin\theta\,\sin\alpha}{\cos\theta\,\cos\alpha}$$

or
$$\cos A = \frac{\sin\delta}{\cos\theta\,\cos\alpha} - \tan\theta\,\tan\alpha \qquad \text{...(7.33a)}$$

when the celestial body is in the eastern hemisphere (Fig. 7.20a), and

$$\cos A' = \frac{\sin\delta}{\cos\theta\,\cos\alpha} - \tan\theta\,\tan\alpha \qquad \text{...(7.33b)}$$

when in the western hemisphere (Fig. 7.20b)
Similarly,

$$\cos H' = \frac{\sin\alpha}{\cos\theta\,\cos\delta} - \tan\theta\,\tan\alpha \qquad \text{...(7.34a)}$$

when the celestial body is in the eastern hemisphere (Fig. 7.20a), and

$$\cos(H) = \frac{\sin\alpha}{\cos\theta\,\cos\delta} - \tan\theta\,\tan\alpha \qquad \text{...(7.34b)}$$

when in the western hemisphere (Fig. 7.20b)

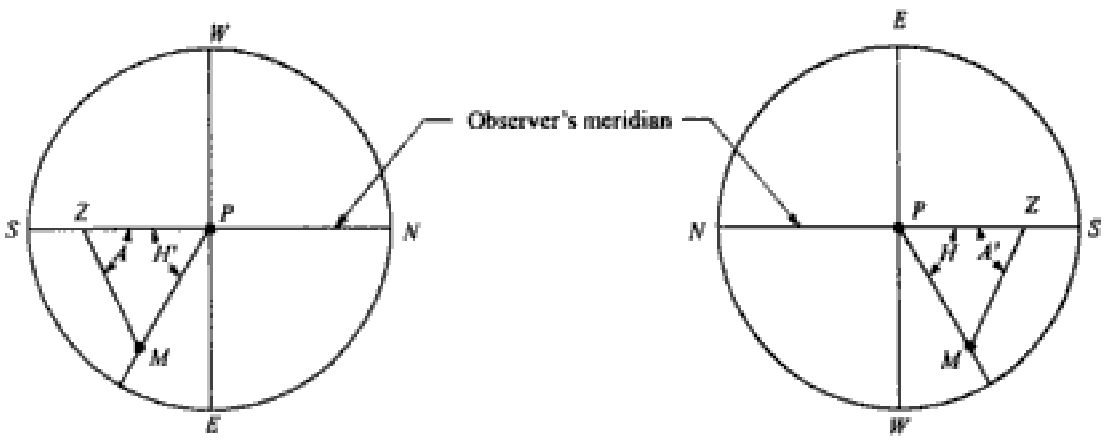


Fig. 7.20 Celestial body in different hemispheres

The following sign conventions should be clearly understood in using the above formulae:

(*i*) Azimuth is always measured from the north either eastwards or westwards in northern hemisphere and from the south either eastwards or westwards in southern hemisphere.

(*ii*) Hour angle is measured westwards (in clockwise direction) from the upper branch of the meridian from Q along the equator. If it is measured eastward (in anticlockwise direction) as H', then

$$H = 360° - H'$$

(*iii*) While ascertaining the clockwise and anticlockwise directions, one should imagine himself to be at the top of the celestial sphere.

ILLUSTRATIVE EXAMPLES

Example 7.1 What is the hour angle and declination of a star if the latitude of the place of observation is 49°10' N, the azimuth and altitude of the star are 50°30' W and 27°56', respectively.

Solution: (Fig. 7.21):

It is given that

$$\theta = 49°10'\text{N}$$
$$A' = 50°30'\text{W}$$
$$\alpha = 27°56'$$

Here $A' = 360° - A$

Therefore, Colatitude $= 90° - \theta = 90° - 49°10' = 40°50'$

Colatitude $= 90° - \alpha = 90° - 27°56' = 62°04'$

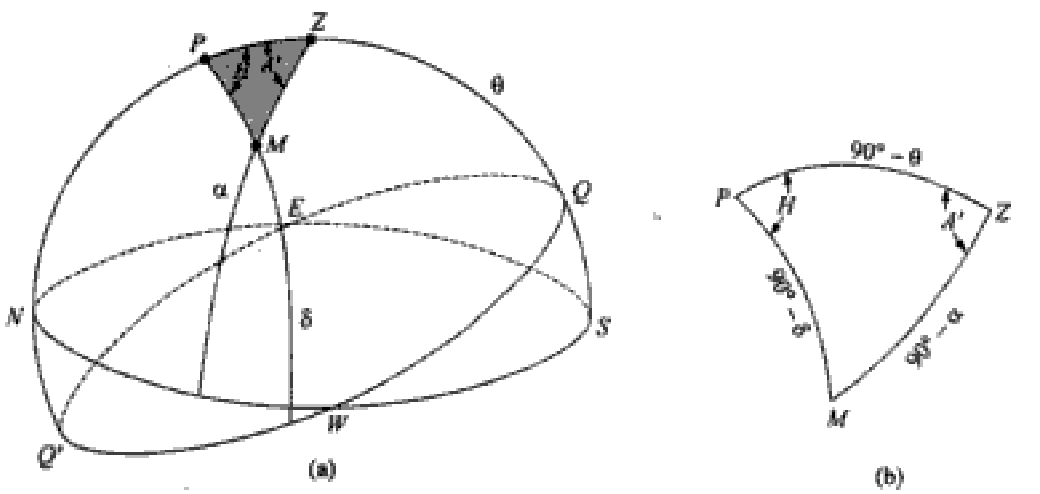


Fig. 7.21

From Eq. (7.8), we get

$$\cos(90° - \delta) = \cos(90° - \alpha)\cos(90 - \theta) + \sin(90° - \alpha) \times \sin(90° - \theta)\cos A'$$

or $\sin\delta = \cos 62°04 \cos 40°50' + \sin 62°04' \times \sin 40°50' \cos 50°30'$

or $= 0.721882$

$$\delta = \mathbf{46°12'36.39''}.$$

Alternatively, Eq. (7.33b) may be used to get the value of δ.

From Eq. (7.34b), we get

$$\cos(H) = \frac{\sin\alpha}{\cos\theta\cos\delta} - \tan\theta\tan\delta$$

$$= \frac{\sin 27°56'}{\cos 49°10' \cos 46°12'36.39''} - \tan 49°10' \tan 46°12'36.39''$$

$$= -0.171814$$

or $H = \mathbf{99°53'35''.98}$.

Solution: (Fig. 7.23):

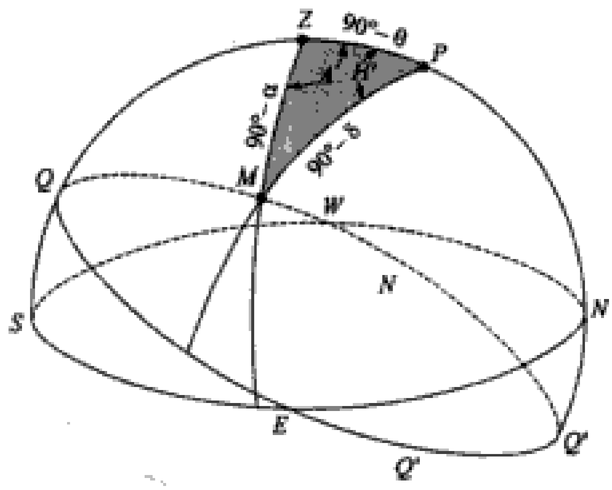


Fig. 7.23

Since the hour angle is greater than 180° the star is in the eastern hemisphere.

Therefore,

$$H' = 360° - H = 360° - 330° = 30°$$

Codeclination $\quad 90° - \delta = 90° - (-12°20') = 102°20'$

Colatitude $\quad 90° - \theta = 90° - 46° = 44°$

Since in deriving Eq. (7.34a), it is assumed that δ is less than 90°, this equation should not be used to compute α.

From Eq. (7.8), we have

$$\cos(90° - \alpha) = \cos(90° - \theta).\cos(90° - \delta) + \sin(90° - \theta)\sin(90° - \delta)\cos H'$$

$$= \cos 44° \times \cos 102°20' + \sin 44° \times \sin 102°20' \times \cos 30°$$

$$= 0.434058$$

or $\quad 90° - \alpha = 64°16'28.71''$

or $\quad \alpha = 90° - 64°16'28.71''$

Altitude = **25°43′31.29″**

Again from Eq. (7.8), we get

$$\cos(90° - \delta) = \cos(90° - \alpha)\cos(90° - \theta) + \sin(90° - \alpha)\sin(90° - \theta)\cos A$$

or $$\cos A = \frac{\cos(90° - \delta) - \cos(90° - \alpha)\cos(90° - \theta)}{\sin(90° - \alpha)\sin(90° - \theta)}$$

$$= \frac{\cos 102°20' - \cos 64°16'28.71'' \times \cos 44°}{\sin 64°16'28.71'' \times \sin 44°}$$

$$= -0.840249$$

or $\quad A = 147°\,09'59.06''$

or $\quad$ Azimuth = **147°09′57.06″** (*East*).

Example 7.4 Determine the shortest distance between the points A and B from the following data:

Point	Latitude (θ)	Longitude (φ)
A	20°20′N	50°50′E
B	17°10′N	53°20′E

Mean earth's radius $R = 6370$ km.

Also find the direction of the line AB on the great circle route.

Solution: (Fig. 7.24):

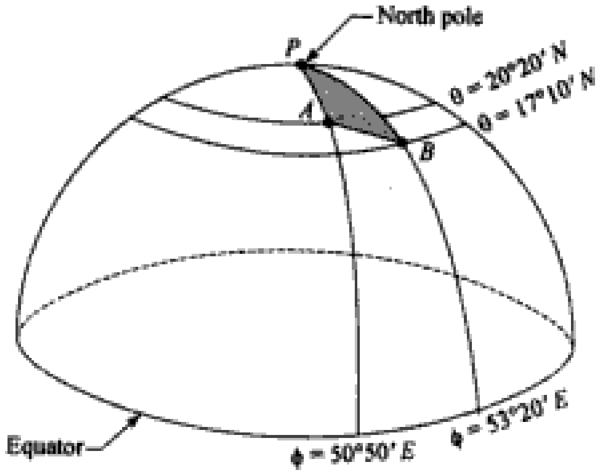


Fig. 7.24

The shortest distance between A and B is the length of the arc AB along the great circle passing through the points.

In ΔAPB, we have

$$\angle APB = 53°20' - 50°50' = 2°30'$$
$$\text{arc } AP = 90° - 20°20' = 69°40'$$
$$\text{arc } BP = 90° - 17°10' = 72°50'$$

From Eq. (7.8), we have

$$\cos AB = \cos PA \cos PB + \sin PA \sin PB \cos APB$$
$$= \cos 69°40' \times \cos 72°50' + \sin 69°40' \times \sin 72°50' \times \cos 2°30'$$
$$= 0.997620$$

or $$AB = 3°57'12.52''$$

or $$= \frac{2\pi \times 6370}{360°} \times 3.953479°$$

$$= \mathbf{439.54\ km.}$$

The direction of AB is given by the angle PAB. In ΔPAB, the angle A is given by Eq. (7.16) and (7.17). Therefore,

$$\tan\frac{A+B}{2} = \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2}$$

$$= \frac{\cos\dfrac{72°50' - 69°40'}{2}}{\cos\dfrac{72°50' + 69°40'}{2}} \times \cot\frac{2°30'}{2}$$

or $= 142.520935$

or $$\frac{A+B}{2} = 89°35'52.76'' \qquad \text{...}(a)$$

and $$\tan\frac{A-B}{2} = \frac{\sin\dfrac{a-b}{2}}{\sin\dfrac{a+b}{2}} \cot\frac{C}{2}$$

$$= \frac{\sin\dfrac{72°50' - 69°40'}{2}}{\sin\dfrac{72°50' + 69°40'}{2}} \times \cot\frac{2°30'}{2}$$

$$= 1.337273$$

$$\frac{A-B}{2} = 53°12'40.39'' \qquad \text{...}(b)$$

By adding Eqs. (*a*) and (*b*), we get

$$A = 89°35'52.76'' + 53°12'40.39''$$
$$= 142°48'33.1''$$

The direction of B from A = **$S\,37°11'26.9''$**.

Example 7.5 A straight line *AB* is run due east of *A*, the latitude of *A* being 48° N. A surveyor travels due north from *B* so as to reach the parallel of latitude of 48°N at *C*. Determine the angle *ABC* at which the surveyor must set out. Also compute the distance *BC*. Take the length of the line *AB* as 370 nautical miles.

Solution: (Fig. 7.25):

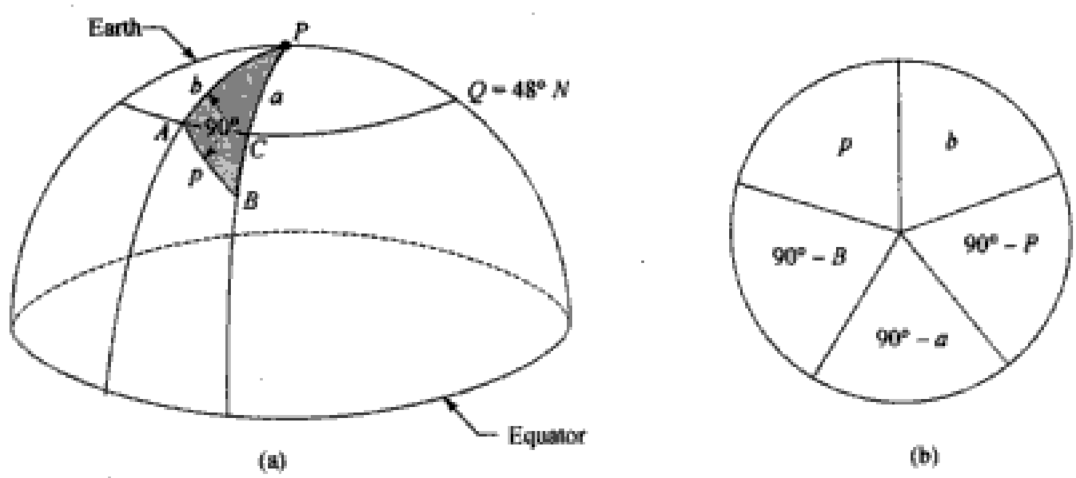


Fig. 7.25

The spherical triangle *PAB* is a right angle triangle at *A*. In this triangle

$$\angle A = 90°$$
$$PA = b = 90° - 48° = 42°$$
$$AB = p = 370 \text{ nautical miles} = 370 \text{ minutes} = 6°10' \text{ at the centre}$$
$$PB = a$$

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(*a*) For M_1 when $\delta > \theta$

The zenith distance at upper culmination,

$$ZU_1 = ZP - ZU_1$$
$$= (90° - \theta) - (90° - \delta)$$

or
$$ZU_1 = \delta - \theta \quad \text{...(7.42)}$$

The zenith distance at lower culmination,

$$ZL_1 = ZP + PL_1$$
$$= (90° - \theta) + (90° - \delta)$$
$$= 180° - (\theta + \delta) \quad \text{...(7.43)}$$

(*b*) For M_2 when $\delta < \theta$

The zenith distance at upper culmination,

$$ZU_2 = PU_2 - PZ$$
$$= (90° - \delta) - (90° - \theta)$$
$$= \theta - \delta \quad \text{...(7.44)}$$

The zenith distance at the lower culmination,

$$ZL_2 = ZP + PL_2$$
$$= (90° - \theta) + (90° - \delta)$$
$$= 180° - (\theta + \delta) \quad \text{[Same as the Eq. (7.43)]} \quad \text{...(7.45)}$$

7.10.5 Circumpolar star

Those stars which are always above the horizon, and which do not, therefore, set are called the *circumpolar stars*. A circumpolar star appears to describe a circle about the pole. The number of circumpolar stars which can be observed increase as the altitude of the observer increases. For the observer at the equator, the number of circumpolar star is zero, whereas at the pole, at the stars are circumpolar.

In Fig. 7.30, M_1 is a circumpolar star having its path A_1B_1 which is always above the horizon HH', where as the star M_2 having its path A_2B_2 is always below the horizon. Therefore, M_1 never sets and M_2 never rises.

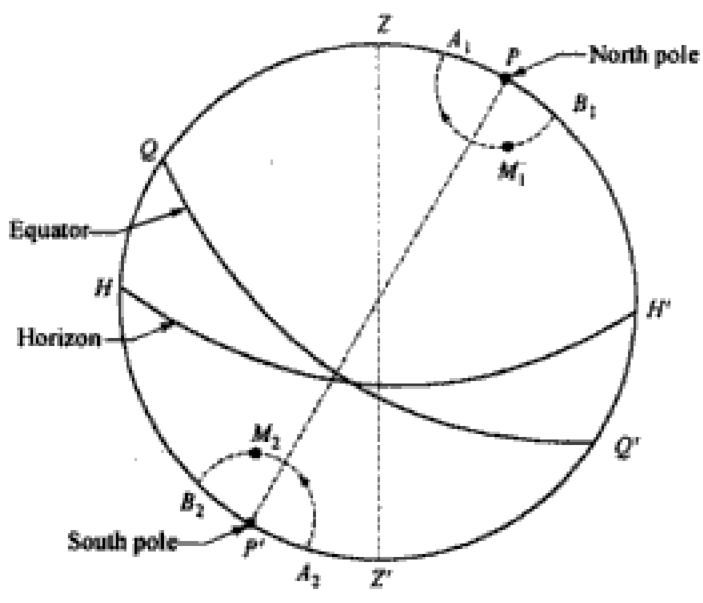

Fig. 7.30 Circumpolar stars

For a circumpolar star which does not set, the distance PB_1 from the pole should be less than the distance PH' of the pole from the horizon.

Example 7.8 What are the zenith distance and altitude of a star at its lower culmination if its declination is 86°25' and latitude of the place of observation is 45°45'?

Solution: (Fig. 7.29):

The given data are

$$\delta = 86°25'$$
$$\theta = 45°45'$$

$\therefore$ Colatitude $(90° - \theta) = 90° - 45°45' = 44°15'$

Since $\delta > 90° - \theta$, the star is a circumpolar star and does not set.

Also $\delta > \theta$, therefore from Eq. (7.43) for lower culmination, we get

$$\text{Zenith distance} = 180° - (\delta + \theta)$$
$$= 180° - (86°25' + 45°45')$$
$$= 180° - 132°10' = \mathbf{47°50'}$$

$$\therefore \quad \text{Altitude } \alpha = 90° - \text{zenith distance}$$
$$= 90° - 47°50' = \mathbf{42°10'}$$

Example 7.9 Find the azimuth, altitude, and the hour angle of the star if its declination at eastern elongation was 80°51' 18" N. The latitude of the place of observation is 32°38' 36" N.

Solution: (Fig. 7.26):

Given that

$$\delta = 80°51'18''\text{N}$$
$$\theta = 32°38'36''\text{N}$$

From Eq. (7.35), the azimuth is given by

$$\sin A = \cos\delta \sec\theta$$
$$= \cos 80°51'18'' \times \sec 32°38'36''$$
$$= 0.188747$$

or $\quad$ Azimuth $A = \mathbf{10°52'46.84''}$.

From Eq. (7.36), the altitude is given by

$$\sin\alpha = \sin\theta \ \text{cosec}\ \delta$$
$$= \sin 32°38'36'' \times \text{cosec}\ 80°51'18''$$
$$= \mathbf{0.546352}$$

or $\quad$ Altitude $\alpha = \mathbf{33°07'1.65''}$.

From Eq. (7.37), the hour angle is given by

$$\cos H' = \tan\theta \cot\delta$$
$$= \tan 32°38'36'' \times \cot 80°51'18''$$
$$= 0.103122$$

or $\quad H' = 84°04'51.59''$

Hour angle $\quad H = 360° - 84°04'51.59''$

$$= \mathbf{275°55'8.41''}.$$

Example 7.10 The upper transit of a star having declination of 54°40' N, is at the zenith of the place of observation. Find the altitude of the star at its lower transit.

The zenith distance at the lower transit

$$= ZL_1 = ZP + PL_1$$
$$= (90° - \theta) + (90° - \delta)$$
$$= 180° - (\delta + \theta)$$

The altitude of the star at its lower transit

$$20°20' = 90° - ZL_1 = 90° - \{180° - (\delta + \theta)\}$$
$$\delta + \theta = 90° + 22°20' = 112°20' \quad \text{...}(b)$$

Solution of Eq. (*a*) and (*b*) gives

$$\delta = \mathbf{67°05'}$$
$$\theta = \mathbf{45°15'}$$

Example 7.12 Find the altitude and azimuth of a star from the following data:

Declination of the star = 21°40′N
Hour angle of the star = 41°10′
Latitude of the place = 54° N

Solution: (Fig. 7.22):

The hour angle of the star is 41°10′ and since it is measured towards west, the star is in the western hemisphere.

Given that

$$\delta = 21°40'N$$
$$H = 41°10'$$
$$\theta = 54°N$$

Codeclation $(90° - \delta) = 90° - 21°40' = 68°20'$

Colatitude $(90° - \theta) = 90° - 54° = 36°$

In the astronomical triangle *MPZ*, from Eq. (7.8), we get

$$\cos(90° - \alpha) = \cos(90° - \delta)\cos(90° - \theta) + \sin(90° - \delta)\sin(90° - \theta)\cos(H)$$
$$= \cos 68°20' \times \cos 36° + \sin 68°20' \times \sin 36° \times \cos 41°10'$$
$$= 0.70991$$
$$90° - \alpha = 44°46'20.66''$$

or
$$\text{Altitude } \alpha = 90° - 44°46'19.19''$$
$$= \mathbf{45°13'39.34''}.$$

From Eq. (7.9), we get

$$\cos A' = \frac{\cos(90° - \delta) - \cos(90° - \alpha)\cos(90° - \theta)}{\sin(90° - \alpha)\sin(90° - \theta)}$$
$$= \frac{\cos 68°20' - \cos 44°46'20.66'' \times \cos 36°}{\sin 44°46'20.66'' \times \sin 36°}$$
$$= -0.495599$$

Since the value of $\cos A'$ is negative, the angle A lies between 90° and 180°. Therefore,

$$\cos(180° - A) = -\cos A' = 0.495599$$

or
$$180° - A = 60°17'50.38''$$

or
$$A = 180° - 60°17'50.38''$$
$$= \mathbf{119°42'09.62''\ W}.$$

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From Eq. (7.8), we get

$$\cos(90°-\delta) = \cos(90°-\alpha)\cos(90°-\theta) + \sin(90°-\alpha)\sin(90°-\theta)\cos A$$

$$= \cos 67°30' \times \cos 40° + \sin 67°30' \times \sin 44° \times \cos 142°$$

$$= -0.174814$$

or $\cos\{180° - (90° - \delta)\} = -\cos(90° - \delta) = 0.174814$

or $90° + \delta = 79°55'55.75''$

or $\delta = 79°55'55.75 - 90°$

or $= -10°04'04.25''$

or Declination δ = **10°04′04.25″ S.**

From Eq. (7.9), we get

$$\cos H' = \frac{\cos(90°-\alpha) - \cos(90°-\delta).\cos(90°-\theta)}{\sin(90°-\delta).\sin(90°-\theta)}$$

Codeclination $(90° - \delta) = 90° - (-10°04'04.25'')$

$$= 100°04'04.25''$$

Thus, $$\cos H' = \frac{\cos 67°30' - \cos 100°04'04.25'' \times \cos 40°}{\sin 100°04'04.25'' \times \cos 40°}$$

$$= 0.816254$$

or $H' = 35°17'18.31''$

or Hour angle $H = 360° - H'$

$$= 360° - 35°17'18.31''$$

= **324°42′41.69″**.

Example 7.16 What is the sun's azimuth and hour angle at sunset at the place whose latitude is 48°20′ N when the declination of the sun is

(*i*) 25°15′N
(*ii*) 25°15′S

Solution: (Fig. 7.33):

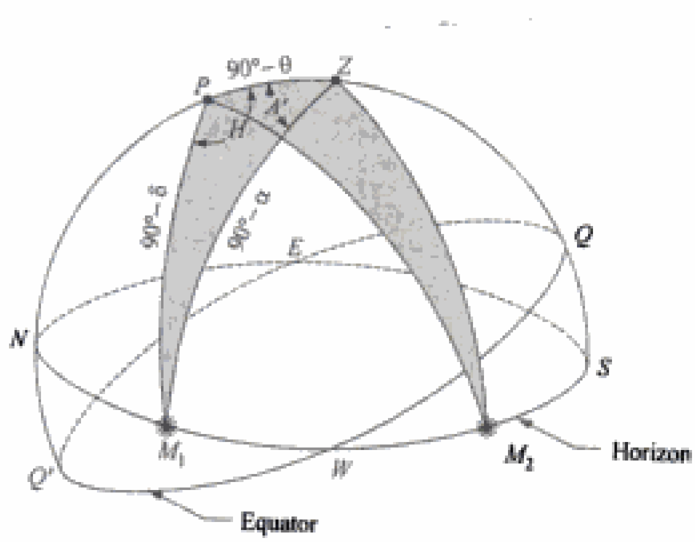

Fig. 7.33

When the sun is at sunset, its altitude is zero,

i.e. $ZM_1 = ZM_2 = 90° - 0° = 90°$

(*i*) Considering the astronomical triangle ZM_1P, we get

$$ZP = 90° - \theta = 90° - 40°20' = 49°40'$$

$$PM_1 = 90° - \delta = 90° - 25°15' = 64°45'$$

$$ZM_1 = 90° - \alpha = 90° - 0° = 90°$$

From Eq. (7.9), we get

$$\cos A' = \frac{\cos(90° - \delta) - \cos 90°.\cos(90 - \theta)}{\sin 90°.\sin(90 - \theta)} \qquad ...(a)$$

$$= \frac{\cos 64°45' - \cos 90° \times \cos 49°40'}{\sin 90° \times \sin 49°40'}$$

$$= 0.559587$$

$\therefore$ Azimuth A' = **55°58' 21.9 W.**

From Eq. (7.8), we have

$$\cos(90° - \alpha) = \cos(90° - \theta).\cos(90° - \delta) + \sin(90° - \theta).\sin(90° - \delta).\cos(H)$$

or $\cos 90° = \cos 49°40' \times \cos 64°45' + \sin 49°40' \times \sin 64°45' \times \cos(H)$

or $\cos(H) = -\cot 49°40' \times \cot 64°45'$ ...(*b*)

$= -0.400444$

or $\cos(180° - H) = -\cos(H) = 0.400444$

$180° - H = 66°23'38.67''$

or H = **113°36′21.3″**.

$$\text{Hence, the hour angle of the sun} = \frac{113.605917°}{15} = 7^h\, 34^m\, 25^s$$

(*ii*) Considering the astronomical triangle ZM_2P, we get

$$ZP = 90° - \theta = 90° - 40°20' = 49°40'$$

$$PM_2 = 90° - \delta = 90° - (-25°15') = 115°15'$$

From Eq. (a), we have

$$\cos A' = \frac{\cos(90 - \delta) - \cos 90° \times \cos(90° - \theta)}{\sin 90° \times \sin(90° - \theta)}$$

$$= \frac{\cos 115°15'}{\sin 49°40'}$$

$$= -0.559587$$

or $\cos(180° - A') = \cos A' = 0.559587$

or $180° - A' = 55°58'21.9''$

or Azimuth A' = **124°01′38.1″**.

From Eq. (b), we have

$$\cos(H) = -\cot 49°40' \times \cot 115°15'$$

$$= -(-0.400444)$$

$$= 0.400444$$

or H = **66°23′38.67″**.

$$\text{Hence, the hour angle of the sun} = \frac{66.394075°}{15} = 4^h\, 25^m\, 35^s.$$

Example 7.17 Determine the azimuth and hour angle of the sun at sunrise for a place in latitude 40°30′ S when the declination of the sun is 25°15′ N.

Solution: (Fig. 7.34):

When the sun is at sunrise, it is on the horizon and its altitude $\alpha = 0$, i.e. the zenith distance = $90° - \alpha = 90°$. Considering the astronomical triangle $Z'MP'$, we have

$$Z'P' = 90° - \theta = 90° - 40°30' = 49°30'$$
$$P'M = 90° - \delta = 90° - (-25°15') = 115°15'$$
$$Z'M = 90° - \alpha = 90° - 0° = 90°$$

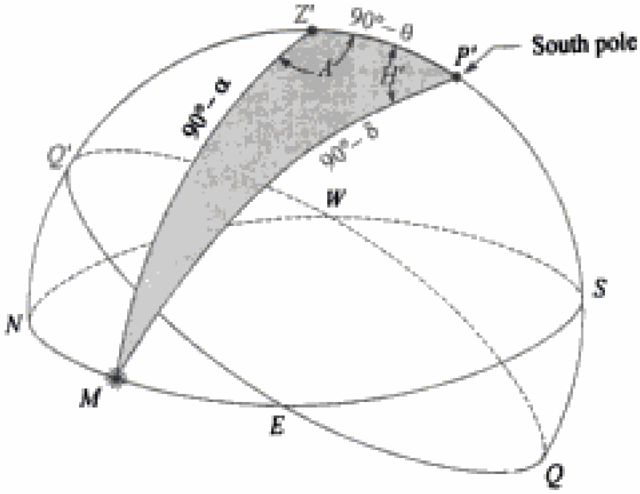

Fig. 7.34

From Eq. (7.9), we get

$$\cos A = \frac{\cos(90° - \delta) - \cos(90° - \alpha)\cos(90° - \theta)}{\sin(90 - \alpha)\sin(90° - \theta)}$$
$$= \frac{\cos 115°15' - \cos 90° \times \cos 49°30'}{\sin 90° \times \sin 49°30'}$$
$$= \frac{\cos 115°15'}{\sin 49°30'}$$
$$= -0.560975$$

or $$\cos(180° - A) = -\cos A = 0.560975$$
$$180° - A = 55°52'36.29''$$

or Azimuth of the sun A = **124°07′23.71″ E.**

From Eq. (7.9), we have

$$\cos H' = \frac{\cos(90° - \alpha) - \cos(90° - \delta)\cos(90° - \theta)}{\sin(90° - \delta)\sin(90° - \theta)}$$
$$= \frac{\cos 90° - \cos 115°15' \times \cos 49°30'}{\sin 115°15' \times \sin 49°30'}$$
$$= -\cot 115°15' \times \cot 49°30'$$
$$= -(-0.402811)$$
$$= 0.402811$$

or $$H' = 66°14'45.6''$$

A *tropical year* is the time period taken by the sun apparently to make a complete circuit of the eclipitic. In one tropical year there are 366.2422 revolutions of the earth. The vernal equinox also makes 366.2422 revolutions about the earth. But because of the slower motion of the sun during the same period, it travels through a total hour angle of 360° or 24 hours less than that traversed by the vernal equinox. Hence during a tropical year the sun apparently makes 365.2422 revolutions about the earth.

Time is the interval which lapses, between any two instances. A *solar day* is the time taken by one apparent revolution of the sun about the earth. A *sidereal day* is the time interval taken by the vernal equinox in its one apparent revolution about the earth. Since the period of 366.2422 sidereal days is equal to 365.2422 solar days, a sidereal day is shorter than the solar day. Thus, we have

$$365.2422 \text{ solar days} = 366.2422 \text{ sidereal days}$$

or $$1 \text{ solar day} = 1.0027379 \text{ sidereal day}$$

or $$1 \text{ sidereal day} = 0.9972696 \text{ solar day.}$$

7.11.1 Units of time

There are following systems for the measurement of time:

1. Sidereal time
2. Solar apparent time
3. Mean solar time
4. Standard time
5. Universal time
6. Atomic time
7. Astronomical and civil time.

Sidereal time

Since the earth rotates on its axis from west to east, all heavenly bodies such as the sun and the fixed stars appear to revolve from east to west in clockwise direction. Such motion of the heavenly bodies is called *apparent motion*. Considering the earth that it revolves on its axis with absolute regular speed, the star appear to complete one revolution round the celestial pole as centre, in constant interval of time, crossing the observer's meridian twice each day. The sidereal day is one of the principal unit of time in astronomy.

A sidereal day is the interval of time between successive upper transits of the vernal equinox (♈). It begins at the instant when the vernal equinox records $0^h\ 0^m\ 0^s$. At any other instant the sidereal time will be the hour angle of ♈ reckoned westward from 0^h to 24^h. A sidereal day has 24 hours, each hour has 60 minutes and each minute has 60 seconds.

Local sidereal time (L.S.T.)

The time interval ellapsed since the transit of the vernal equinox (♈) over the meridian of the place, is the *local sidereal time*. It is the therefore, a measure of the angle through which the earth has rotated since the equinox was on the meridian.

Fig. 7.36a shows the celestial sphere in plan. The observer's meridian is *PZ*. When the vernal equinox is on the meridian at *Z'* the local sidereal time is 0 hour. When the vernal equinox has moved to ♈ from the observer's meridian by 120°, the local sidereal time is

$$= \frac{120°}{360°} \times 24 = 8 \text{ hour}$$

In Fig. 7.36a, *M*, is a star whose right ascension and hour angle are the angles (♈) *PM'* measured anticlockwise from ♈ and *Z'PM'* measured clockwise from the south side, respectively.

Mean solar time

Since the sun's apparent motion round the earth is not uniform, a fictitious sun is assumed to move at a uniform rate along the equator so as to have a solar day of uniform duration and to overcome the difficulty of recording the variation of apparent solar time. The fictitious sun is called the *mean sun* since it is assumed that the motion of the mean sun is the average of that of the true sun in right ascension. The start and arrival of the mean sun and the true sun are assumed to be the same at the vernal equinox.

The interval of time between two successive lower transits of the mean sun is known as the *mean solar day*. The duration of a mean solar day is the average of all the apparent solar days of the year. The rate of increase of right ascesion of the true sun is not uniform whereas that of the mean sun is assumed to be constant throughout the year.

The instant when the mean sun crosses the meridian at its upper transit, is known as the *local mean noon* (L.M.N.). The mean time at any other instant given by the hour angle of the mean sun reckoned westward from 0 to 24 hours, is known as the *local mean time* (L.M.T.). The zero hour of the mean solar day is at the local mean mid-night (L.M.M.). The mean solar day begins at the mid might and completes at the next mid-night. The difference in local mean time of two places, is always equal to the difference of their longitudes.

Standard time

The local mean time of every place is different as the local mean time at any meridian is reckoned from the lower transit of the mean sun at the meridian of the place. For convenience and to avoid confusion arising from the use of different local mean times within a country, it is the usual practice to use the local mean time of a central meridian of a country as the standard time for the entire country. Some vast countries have more than one standard time. The standard meridian of a country is generally selected such that it lies an exact number of hours from Greenwich. Of course, India is an exception to this as its standard meridian is 5½ hours (longitude 82°30′) east of Greenwich. It passes some where near Allahabad. All watches and clocks keep the standard time of the country, irrespective of their locations. Thus although the local mean time of Bombay and Calcutta are quite different, the standard time of these two places are the same.

As the sun rises in the east, the local mean day commences earlier at the places in the east. The difference between the local mean time of a place and the standard time is equal to the difference of the longitude of the place and that of the standard meridian.

Therefore,

$$\text{local mean time} = \text{standard time} \pm \text{difference of longitude} \quad \text{...(7.53)}$$

In Eq. (7.53), plus sign is used if the place is east of the standard meridian and negative for the places west of the standard meridian. For the places east of the standard meridian, the local mean time will be ahead of the standard time and for those west of the stand meridian, it will be behind the standard time. Thus, the local mean time of Calcutta will be ahead of the standard time and that of the Bombay, it will be behind the standard time.

Universal time

Since the standard time of a country depends upon the standard meridian adopted for the country, the standard time of different countries is different. For astronomical purposes and scientific investigations, it is required to have a common standard time for all the countries. For this, a standard meridian of Greenwich in U.K. is taken for the standard time which is the universally adopted standard time and it is called the *universal time* (U.T.) or *Greenwich mean time* (G.M.T.)

Table 7.1 gives the standard time of some of the countries corresponding to 12 hours at Greenwich, i.e., at the time of Greenwich mean noon (G.M.N.).

Table 7.1 Standard time at the time of G.M.N.

Countries	Standard longitude	Hours	Standard time
Germany	15° E	1 hr	13-00 hr
India	82°½ E	5 ½ hr	17-30 hr
Western Australia	120° E	8 hr	20-00 hr
Central zones of U.S.A.	90° W	6 hr	6-00 hr

Atomic time

In recent times atomic clocks have been developed which are so accurate that they easily reveal irregularities in the earth's rotational velocity, i.e., in sidereal time. International Atomic Time (I.A.T.) is thus now the most uniform kind of time available to the world. Its rate is quite independent of that of any form of astronomically determined time. The Syste'me International (SI) unit of time is the atomic second which was defined in 1967 by the 13th General Conference of Weights and Measures as the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium atom 133.

Coordinated universal time (U.T.C.) is kept at the same rate as I.A.T. but is displaced by an integral number of seconds from it; (I.A.T. —U.T.C.) = + 19 second exactly in January 1980, and gaining at about 1 second per year. U.T.C. is broadcast by radio time-signal stations.

Astronomical and civil time

The astronomers count the mean solar day beginning at mid-night from 0 hour to 24 hours. However, for civilian use, the day is divided into two halves, each having zero at the starting. In this system mid-night to noon is called *ante meridian* (A.M.). and noon to mid-night is called *post-meridian* (P.M.).

7.11.2 Equation of time

The difference between the apparent solar time and the mean solar time at any instant is known as the *equation of time*. The values of equation of time at 0 hour (mid-night) at Greenwich are tabulated in the Nautical Almanac for every day of the year. The equation of time is thus treated as a correction which may be applied to mean time to obtain apparent time. it may be written as

$$\text{Equation of time} = \text{Apparent solar time} - \text{Mean solar time} \quad \ldots (7.54)$$

The equation of time is positive when the apparent solar time is greater than the mean solar time. In Fig. 7.37, the positions of the true sun S and the mean sun M, show that the local apparent time is greater than the local mean time and the equation of time is positive. On the other hand, the equation of time is negative if the local apparent time is less than the local mean time.

When the equation of time is positive, to get the apparent solar time, the equation of time should be added to mean solar time. For example, at 0^h Greenwich mean time on 15th October 1949, the equation of time is + $13^m\ 12^s$. This means that the apparent time at 0^h mean time is $0^h\ 13^m\ 12^s$. In other words, the true sun is $13^m\ 12^s$ ahead of the mean sun. Similarly, when the equation of time is negative, the apparent solar time is obtained by subtracting

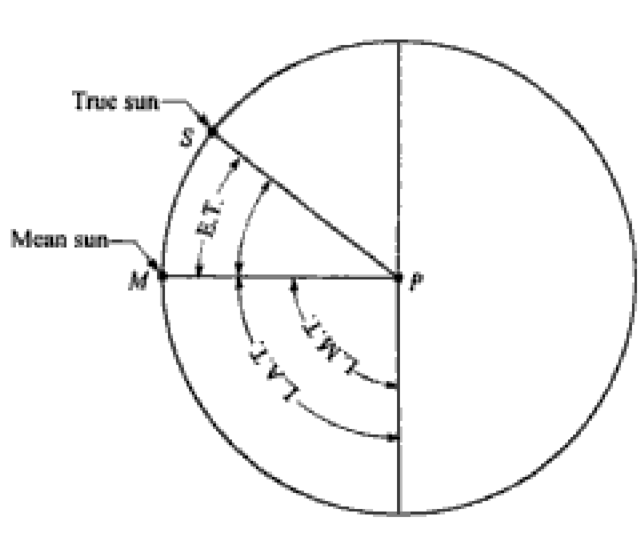

Fig. 7.37 Equation of time

the equation of time from the mean solar time. For example, at 0^h Greenwich mean time on 18th January 1949, the equation of time is $-10^m 47^s$. This means that the apparent time at 0^h mean time is $23^h 49^m 13^s$ on January 17. In other words, the true sun is behind the mean sun at that time.

The main causes of the existence of the equation of time and variations in it, are the following:

(*i*) The movement of the earth around the sun is along an ellipse and not along a circle. Hence its motion is not uniform and varies with its distance from the sun.

(*ii*) The movement of the true sun is along the ecliptic which does not correspond to the movement of the mean sun assumed to be moving along the equator.

In the following sections the abbreviations used are as under:

Local sidereal time	L.S.T.
Greenwich sidereal time	G.S.T.
Greenwich apparent noon	G.A.N.
Greenwich apparent mid-night	G.A.M
Greenwich apparent time	G.A.T
Greenwich mean time	G.M.T.
Greenwich mean mid-night	G.M.M.
Greenwich mean noon	G.M.N.
Local mean time	L.M.T.
Local mean mid-night (0hr)	L.M.M.
Local mean noon (12 hr)	L.M.N.
Local apparent time	L.A.T.
Local apparent mid-night (0hr)	L.A.M.
Local apparent noon (12 hr)	L.A.N.
Indian standard time	I.S.T.
Universal time	U.T.
Equation of time	E.T.
Sidereal interval	S.I.
Nautical Almanae	N.A.

ILLUSTRATIVE EXAMPLES

Example 7.18 Convert the following difference in longitudes into interval of time:

(*i*) 42°22′32″

(*ii*) 172°20′52″

Solution:

We know that

$$
\begin{aligned}
15° \text{ of longitude} &= 1 \text{ hour}\\
1° \text{ of longitude} &= 4 \text{ minutes}\\
15' \text{ of longitude} &= 1 \text{ minute}\\
1' \text{ of longitude} &= 4 \text{ seconds}\\
15'' \text{ of longitude} &= 1 \text{ second}
\end{aligned}
$$

Therefore,

(*i*)
$$42° = \frac{42}{15}h = 2^h 48^m 0^s$$

$$22' = \frac{22}{15}m = 0^h 1^m 28^s$$

Example 7.21 Find the equation of time of 12^h G.M.T. on July 1, 1951 from the following data:

(*i*) ET at G.M.M. on July 1, 1951 = $-3^m28.41^s$

(*ii*) Change between the value for 0^h July 1, and that for 0^h July 2 = -11.82^s

Solution:

Since the change in ET for 24 hours is -11.82^s, the change in ET for 12 hours will be

$$= \frac{-11.82}{24} \times 12 = -5.91^s$$

Therefore,

$$\text{ET at } 12^h \text{ G.M.T} = -3^m 28.41^s - 5.91^s$$
$$= \mathbf{-3^m\ 34.32^s}.$$

Example 7.22 Determine the G.A.T. corresponding to 0 hour G.M.T.

(*i*) On a day if ET = $+8^m 2.6^s$

(*ii*) On another day if ET = $-12^m 24.2^s$

Solution:

From Eq. (7.54), we have

$$\text{ET} = \text{G.A.T.} - \text{G.M.T.}$$

or

$$\text{G.A.T.} = \text{G.M.T.} + \text{ET}$$

(*i*) $= 0^h + 8^m 2.6^s = \mathbf{0^h\ 8^m\ 2.6^s}$.

(*ii*) $= 0^h + (-12^m 24.2^s) = \mathbf{23^h\ 35^m\ 59.8^s}$ of previous day.

Example 7.23 Find the equation of time at 15 hours G.M.T. on April 2, 1991 from the following data:

(*i*) ET at G.M.M. on April 2, 1991 is $-3^m 51.4^s$

(*ii*) ET at G.M.M. on April 3, 1991 is $-3^m 33.5^s$

Solution:

Change of ET in 24 hours $= -3^m 33.5^s - (-3^m 51.4^s)$
$= +17.9^s$

Change in ET in 15 hours $= +\frac{17.9}{24} \times 15 = +11.19^s$

ET at 15 hours G.M.T. $= -3^m 51.4^s + 11.19^s$
$= \mathbf{-3^m\ 40.21^s}$.

Example 7.24 Find the G.A.T. on February 18, 1961, when the G.M.T. is $8^h 30^m$ A.M. The equation of time of G.M.N. on February 18, 1961 is $-12^m 08^s$ and increasing at the rate of 1.5^s per hour.

Solution:

$8^h 30^m$ A.M. occurs $3^h 30^m$ before the noon.

$$\text{Change in ET in } 3^h 30^m = 1.5^s \times 3^h 30^m$$
$$= +5.25^s$$

Since the ET is increasing after G.M.N., its value will be less than $12^m 08^s$ before noon. Thus,

$$\text{ET at } 8^h 30^m \text{ A.M.} = -12^m 08^s + 5.25^s = -12^m 2.75^s$$

From Eq. (7.54), we have

$$\text{G.A.T.} = \text{G.M.T.} + \text{ET}$$
$$= 8^h 30^m + (-12^m 2.75^s)$$
$$= \mathbf{8^h\ 17^m\ 57.25^s}.$$

7.11.14 L.S.T. of elongation of a star

From Eq. (7.47) we find that the local sidereal time is sum of the local hour angle of the star and its right ascension. Thus, to get the L.S.T. of elongation of the star the westerly hour angle is added to the right ascension of the star at elongation and the easterly hour angle is subtracted from the right ascension of the star at its elongation. In case the result is more than 24^h, deduct 24^h from the result and if the result is negative, add 24^h to the result.

ILLUSTRATIVE EXAMPLES

Example 7.25 Convert $8^h20^m35^s$ sidereal time interval to mean solar time interval.

Solution:

The given sidereal interval $= 8^h\,20^m\,35^s$

$= 8.3430556^h$

The total retardation in 8.3430556^h sidereal time interval at the rate of 9.8296^s per hour will be

$= 9.8296 \times 8.3430556$

$= 82.01^s$

We know that

Mean solar time interval = Sidereal time interval + retardation

$= 8^h20^m35^s + (-82.01^s)$

$= \mathbf{8^h\,19^m\,12.99^s}$.

Example 7.26 Convert the $6^h\,40^m\,30^s$ mean solar time interval to sidereal time interval.

Solution:

The given mean time interval $= 6^h40^m30^s$

$= 6.6750000^h$

The total acceleration in 6.6750000^h mean time interval at the rate of 9.8296^s per hour will be $= 65.61^s$

We known that

Sidereal time interval = Mean time interval + acceleration

$= 6^h40^m30^s + 65.79^s$

$= \mathbf{6^h\,41^m\,35.79^s}$.

Example 7.27 The standard meridian of time in India is 82°30′ E. Find the local mean time of the places having longitudes

(*i*) 25° E, and (*ii*) 25° W. The standard time at any instant is $18^h20^m8^s$.

Solution:

(*i*) Longitude of standard meridian = 82°30′E

Longitude of the place = 25° E

Difference in longitudes = 57°30′

$$57°30' = 57.5° = \frac{57.5}{15} \text{ hours } = 3^h\,50^m$$

Since the longitude of the place is less than that of the standard meridian, the place is to the west of the standard meridian, therefore, L.M.T. at the place will be less than the standard time.

Thus,

L.M.T. = Standard time – difference in longitude

$= 18^h20^m8^s - 3^h50^m$

$= 14^h30^m8^s$ past mid-night

$= \mathbf{2^h\,30^m\,8^s}$ **P.M.**

Therefore, L.M.T. = G.M.T. + longitude (E)

$= 17^h34^m15^s + 4^h41^m20^s$

$= 22^h15^m35^s$ past mid-night

= $10^h\ 15^m\ 35^s$ P.M. on July 4, 1966.

(*b*) Longitude of the place = 70°20′W = 70.3333333 W

$$\frac{70.3333333}{15} \text{ hours} = 4^h41^m20^s$$

Since the place is to the west of Greenwich the local time will be lesser than the standard time.

The given G.M.T. = $17^h34^m15^s$ past mid-night

Therefore, L.M.T. = G.M.T. – longitude (W)

$= 17^h34^m15^s - 4^h41^m20^s$

$= 12^h52^m55^s$ past mid-night

= $0^h\ 52^m\ 55^s$ P.M. on July 4, 1966.

(*c*) Longitude of the place = 112° 38′30″E = 112.6416667 E

$$\frac{112.6416667}{15} \text{ hours} = 7^h\ 30^m\ 34^s$$

Since the place is to the east of Greenwich, the local time will be more than the Greenwich time.

The given G.M.T. = $17^h34^m15^s$ past mid-night

Therefore, L.M.T. = G.M.T. + longitude (E)

$= 17^h34^m15^m + 7^h30^m34^s$

$= 25^h4^m49^s$ past mid-night of July 4, 1966.

$= 25^h4^m49^s - 24^h$

= $1^h\ 4^m\ 49^s$ A.M. on July 5, 1966.

Example 7.30 Determine the L.A.T. at the place of observation having longitude 64°12′ E, corresponding to L.M.T. $12^h22^m40^s$. The equation of time at G.M.N. being $4^m55.20^s$ additive to the mean time, and decreasing at the rate of 0.30^s per hour.

Solution:

From Eq. (7.54), we have

ET = Apparent solar time – Mean solar time ... (*a*)

It is given that

ET = **$4^m\ 55.20^s$** at G.M.N. additive to the mean time and decreasing at the rate of 0.30^s per hour.

To find the ET at the given L.M.T., first corresponding G.M.T. is to be calculated and converted to G.A.T. and from this G.A.T., L.A.T. can be determined.

Longitude of the place = 64°12′E = 64.20° E

$$\frac{64.20}{15} \text{ lower} = 4^h16^m48^s$$

L.M.T. of observation = $12^h22^m40^s$

Therefore, G.M.T. of observation = L.M.T. – longitude (E)

$= 12^h22^m40^s - 4^h16^m48^s$

$= 8^h5^m52^s$

The mean time interval before G.M.N.

$= 12^h - 8^h5'52'' = 3^h54^m8^s$

It is given that ET decreases at the rate of 0.30^s per hour after G.M.N., therefore, it will have more value for any time instant before G.M.N.

Thus,

Increase for $3^h43^m8^s (= 3.9022222^h)$ at the rate of 0.30^s per hour

$$= 3.9022222 \times 0.30 = 1.17^s$$

$$\text{ET at G.M.N.} = 4^m\,55.20^s$$

$$\text{ET at observation} = 4^m55.20^s + 1.17^s = 4^m56.37^s$$

Now from Eq. (*a*), we have

$$\text{G.A.T.} = \text{G.M.T.} + \text{ET}$$

$$= 8^h5^m52^s + 4^m56.37^s = 8^h10^m48.37^s$$

Therefore,

$$\text{L.A.T. of observation} = \text{G.A.T. of observation} + \text{longitude (E)}$$

$$= 8^h10^m48.37^s + 4^h16^m48^s$$

$$= \mathbf{12^h\,27^m36.37^s}.$$

Example 7.31 From the following data, find the L.M.T. of observation of a place having longitude 22°30′ W:

$$\text{L.A.T. of observation} = 12^h18^m36^s$$

ET at G.M.N. = *$4^m\,50.27^s$* additive to apparent time and increasing at the rate of *0.20^s* per hour.

Solution:

$$\text{Longitude of the place} = 22°30'\text{W} = 22.50°\text{ W}$$

$$\frac{22.50}{15}\text{ hours} = 1^h30^m0^s\text{ W}$$

Since the place of observation is to the west of Greenwich, G.A.T. will be more than the L.A.T. and, therefore,

$$\text{G.A.T.} = \text{L.A.T.} + \text{longitude (W)}$$

$$= 12^h18^m36^s + 1^h30^m0^s = 13^h48^m36^s$$

Given that

ET at G.M.N. = $4^m\,50.27^s$

$$\text{The time interval after G.M.N.} = 13^h48^m36^s - 12^h$$

$$= 1^h48^m36^s$$

$$= 1.81^h.$$

Increase for 1.81^h at the rate of 0.20^s per hour

$$= (1.81 \times 0.20) = 0.36^s$$

Therefore,

$$\text{ET at the time of observation} = 4^m50.27^s + 0.36^s$$

$$= 4^m50.63^s$$

$$\text{G.M.T. of observation} = \text{G.A.T.} + \text{ET}$$

$$= 13^h48^m36^s + 4^m50.63^s$$

$$= 13^h53^m26.63^s$$

$$\text{L.M.T.} = \text{G.M.T.} - \text{longitude (W)}$$

$$= 13^h53^m26.63^s - 1^h30^m0^s$$

$$= \mathbf{12^h\,23^m\,26.63^s}.$$

Now L.S.T. at L.M.T. $= \text{L.S.T. at L.M.N.} + \text{S.I. since L.M.N.}$

$= 7^h35^m23.67^s + 7^h31^m13.93^s$

$= 15^h6^m37.59^s$

$= \mathbf{3^h\, 6^m\, 37.59^s}$ **P.M.**

Solution of this problem by the alternative method is given below:

L.M.T. $= 7^h30^m$ P.M. $= 19^h30^m$

G.M.T. $= \text{L.M.T.} - \text{longitude (E)}$

$= 19^h30^m - 4^h42^m$

$= 14^h48^m$

Mean time interval since G.M.N. $= 14^h48^m - 12^h$

$= 2^h48^m = 2.8^h$

S.I. $=$ Mean time interval + acceleration @ 9.8567^s per hour

$= 2^h48^m + (9.8567 \times 2.8)^s$

$= 2^h48^m + 27.7^s$

$= 2^h48^m27.6^s$ since G.M.N.

G.S.T. at G.M.N. $= 7^h36^m10^s$

G.S.T. at the given instant $=$ G.S.T. at G.M.N. + S.I.

$= 7^h36^m10^s + 2^h48^m27.6^s$

$= 10^h24^m37.6^s$

L.S.T. of L.M.T. $=$ G.S.T + longitude (E)

$= 10^h24^m37.6^s + 4^h42^m$

$= 15^h6^m37.6^s$

$= \mathbf{3^h\, 6^m\, 37.6^s}$ P.M.

Example 7.34 Find the L.M.T. corresponding to the L.S.T. $16^h\ 24^m\ 10^s$ at place having longitude $110°30'30''$ W, given that G.S.T. at G.M.M is $10^h\ 10^m\ 10^s$ on that day.

Solution:

$$\text{Longitude} = 110°30'30''\text{ W} = 110.5083333°\text{ W} = \frac{110.5083333}{15}\text{ hours}$$

$= 7^h22^m2^s$ W

$= 7.3672222^h$ W

For the place having west longitude, we have

L.S.T. at L.M.M. $=$ G.S.T at G.M.M. $+ 9.8565^s \times$ longitude (W)

$= 10^h10^m10^s + (9.8565 \times 7.3672222)^s$

$= 10^h10^m10^s + 1^m\, 12.62^s$

$= 10^h11^m\, 22.62^s$

S.I. since L.M.M. $=$ L.S.T. $-$ L.S.T. at L.M.M.

$= 16^h24^m10^s - 10^h11^m22.62^s$

$= 6^h12^m47.38^s = 6.2131611^h$

Now mean time interval = S.I. + retardation @ 9.8296^s per hour

$= 6^h12^m47.38^s - (9.8296 \times 6.2131611)^s$

$= 6^h12^m47.68^s - 1^m1.07^s$

$= 6^h11^m46.61^s$ since L.M.M.

or L.M.T. $= \mathbf{6^h\ 11^m\ 46.61^s}$

Solution of this problem by the alternative method is given below:

G.S.T. at the instant = L.S.T. + longitude (W)

$= 16^h24^m10^s + 7^h22^m2^s$

$= 23^h46^m12^s$

S.I. since G.M.M. = G.S.T. at instant − G.S.T. at G.M.M.

$= 23^h46^m12^s - 10^h10^m10^s$

$= 13^h36^m2^s$

$= 13.6005556^h$

Mean time interval = S.I. + retardation @ 9.8296^s per hour of S.I.

$= 13^h36^m2^s - (9.8296 \times 13.6005556)^s$

$= 13^h36^m2^s - 2^m13.69^s$

or G.M.T. $= 13^h33^m48.31^s$

Now L.M.T. = G.M.T. − longitude (W)

$= 13^h33^m48.31^s - 7^h22^m2^s$

$= \mathbf{6^h\ 11^m\ 46.31^s}$.

Example 7.35 Find the local mean time of upper and following lower transit at a place in longitude 168°30′45″ W of a star whose right ascension is $20^h10^m15^s$, if the G.S.T. of previous G.M.N. is $11^h15^m30^s$.

Solution:

$$\text{Longitude} = 168°30'45''\text{ W} = 168.5125°\text{ W} = \frac{168.5125}{15}\text{ hour}$$

$= 11^h14^m3^s\text{ W} = 11.2341667^h\text{ W}$

Right ascension (R.A.) of the star = L.S.T.

L.S.T. of L.M.N. = G.S.T. of G.M.N. + @ 9.8567^h per hour of longitude

$= 11^h15^m30^s + (9.8567 \times 11.2341667)^s$

$= 11^h15^m30^s + 1^m50.73^s$

$= 11^h17^m20.73^s$

S.I. since L.M.N. = R.A. of the star − L.S.T. of L.M.N.

= L.S.T. − L.S.T. of L.M.N.

$= 20^h10^m15^s - 11^h17^m20.73^s$

$= 8^h52^m54.27^s = 8.8817417$

Mean time interval = S.I. + retardation @ 9.8296^s per hour of S.I.

$= 8^h52^m54.27^s - (9.8296 \times 8.8817417)^s$

$= 8^h52^m54.27 - 1^m27.3^s$

$= 8^h51^m26.97^s$ since L.M.N.

or L.M.T. of upper transit $= \mathbf{8^h\ 51^m\ 26.97^s}$ **P.M.**

Since the lower transit of the star will occur after 12 sidereal hours, let us find the mean time corresponding to 12 sidereal hours.

Example 7.42 If the G.S.T. of G.M.N. is 13^h45^m30 , find the H.A. of a star of R.A. $22^h30^m20^s$ at a place in longitude 125°15′ W at 2.15 A.M. G.M.T. the same day.

Solution: We know that

L.S.T. = (R.A. + H) of the star.

Let us first find the L.S.T. corresponding to the given L.M.T., given the G.S.T. of G.M.N.

$$\text{Longitude} = 125°15' \text{W} = 125.25° \text{W} = \frac{125.25}{15} \text{ hours} = 8^h21^m = 8.35^h$$

Since the place is to the west of Greenwich the acceleration at the rate 9.8567^s per hour of longitude should be added to the G.S.T. of G.M.N. to get the L.S.T. of L.M.N. Thus,

L.S.T. of L.M.N. = G.S.T. of G.M.N. + acceleration @ 9.8567^s per hour of longitude (W)

$= 13^h45^m30^s + (9.8567 \times 8.35)^s$

$= 13^h46^m52.3^s$

L.M.T. of the event = G.M.T. – longitude (W)

$= 2^h15^m - 8^h21^m$

$= -6^h6^m$

$= 24^h - 6^h6^m$ of previous day

$= 17^h54^m$ of previous day

Mean time interval = L.M.N. on the day of given G.S.T. of G.M.N.

– L.M.T. between the event of the previous day and the L.M.N.

$= 12^h - 17^h54^m$

$= -5^h54^m$

$= 24^h - 5^h54^m$

$= 18^h6^m = 18.10^h$

S.I. between the event and L.M.N. = Mean time interval between the event and L.M.N. + acceleration @ 9.8567^s per hour of mean time interval

$= 18^h6^m (9.8567 \times 18.10)^s$

$= 18^h8^m58.4^s$ before L.M.N.

L.S.T. of event = L.S.T. of L.M.N. – S.I.

$= 13^h46^m52.3^s - 18^h8^m58.4^s$

$= -4^h22^m6.1^s$

$= 24^h - 4^h22^m6.1^s$

$= 19^h37^m53.9^s$

Now, H = L.S.T. – R.A.

$= 19^h37^m53.9^s - 22^h30^m20^s$

$= -2^h52^m26.1^s$ (– ive H)

$= 24^h - 2^h52^m26.1^s$ (+ive H.A.)

$= \mathbf{21^h07^m33.9^s}$.

Example 7.43 If the G.S.T. at G.M.M. on August 12, 1950 in a place in longitude 70°30′ W is $21^h14^m28.42^s$, find

(*i*) the R.A. of the meridian of the place, and

(*ii*) the R.A. of the mean sun at 5^h45^m A.M. on August 12, 1950.

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(*ii*) If the observed angle is the angle of elevation, the correction is positive when the left-hand end of the axis is higher and negative when the right-hand end is higher.

(*iii*) If the observed angle is the angle of depression, the correction is positive when the right-hand end of the axis is higher and negative when the left-hand end is higher.

(*iv*) The horizontal circle reading for each direction should be corrected and then the horizontal angle should be obtained by subtraction.

Therefore, if the correct angle *AOB* at *O* is required to be obtained, a correction ($b \tan \alpha_1$) is applied to the direction *OA* and a correction (b and α_2) to the direction *OB*. Then the correct horizontal angle will be

$$\angle AOB = \text{Corrected direction } OA - \text{corrected direction OB} \qquad \ldots (7.73)$$

7.12.2 Astronomical corrections

Some or all of the following corrections may be applied to the observed altitudes and azimuths of the celestial bodies, to deduce their correct altitudes at the time of observation, and correct azimuth of the lines observed.

1. Correction for refraction
2. Correction for parallax
3. Correction for semi-diameter
4. Correction for dip of the horizon.

The correction for refraction is required for the observations on both stars and the sun. Correction for parallax and for semi-diameter are only for the sun observations. Correction for dip of the horizon is applied to the observations made with a nautical sextant.

(a) Correction for refraction

The earth is surrounded by atmospheric layers of gases of varying density, decreasing with increase of distance from the earth (Fig. 7.38). When a ray of light emanating from a celestial body *S* passes through the atmosphere of the earth, it is bent downwards, and the body appears to be nearer to the zenith than it actually is, consequently, the celestial bodies appear to be at higher altitude above the observer's horizon than they actually are, i.e., at *S'*. This result into greater measured altitude than the actual altitude. The angle of deviation of the ray from its original direction on intering the earth's atmosphere to its direction at the surface of the earth is called the refraction angle. The refraction correction which is equal to the refraction angle, is given by

$$C_r = -58''. \cot\alpha' \qquad \ldots (7.74)$$

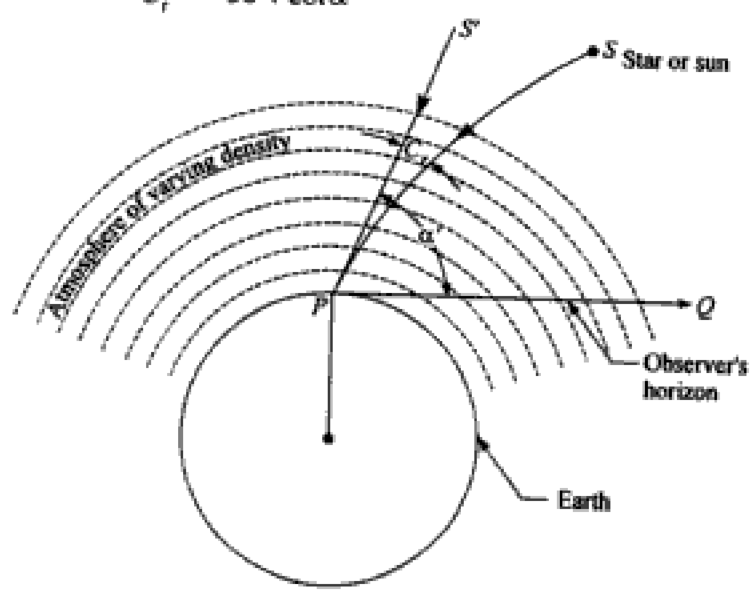


Fig. 7.38 Correction for refraction

Where α' is the apparent altitude. For the values of α' less than 20°, Eq. (7.74) does not give good results. C_r is always subtractive.

The magnitude of the refraction correction depends upon the temperature, barometric pressure of the atmosphere, and the altitude of the celestial body. It does not depend upon the distance of the body from the earth and therefore, it is same for the sun and stars.

The refraction correction for low altitudes is uncertain and hence observation for precise work should never be taken on a celestial body which is nearer to the horizon.

It may be noted that the refraction does not affect the horizontal angles and azimuth, it affects only the altitude.

(b) Correction for parallax

As the stars are assumed to be projected on a celestial sphere of infinite radius, their altitude above the sensible horizon and above the horizon passing through the centre of the earth are practically the same (in Fig. 7.39, α' and α, respectively). The sun being comparatively nearer, its altitude when measured at any point on the surface of the earth (topocentre) differs from that if deduced at the centre of the earth (geocentre).

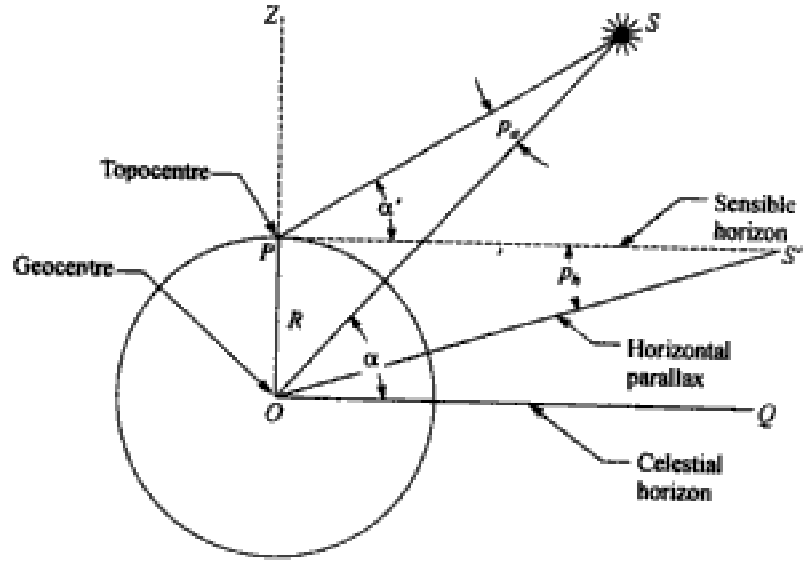


Fig. 7.39 Parallax correction for the sun

The apparent change in the direction of a body when viewed from different positions, is called the *parallax*. The parallax in altitude is due to the difference in the direction of a celestial body as seen from the geocentre and topocentre. The stars being very far, the parallax correction is insignificant when the observations are made on them. However, in the case of sun or moon, the parallax is significant and parallax correction as given below, should be applied to the observed altitude.

$$p_a = p_h . \cos\alpha' \quad ...(7.75)$$

where p_a = the parallax correction.
p_h = sun's horizontal parallax
α = the true altitude of the sun corrected for parallax
α' = the observed altitude of the sun.

The sun's horizontal parallax varies inversely with its distance OS' from the geocentre. Its value is approximately 8.95" early in January and 8.66" early in July. Taking the mean value as 8.8", Eq. (7.75) will be

$$p_a = +\,8.8'' \cos\alpha' \quad ...(7.76)$$

The parallax correction is always positive since α is always greater than α'. It is maximum when the sun is at the horizon ($\alpha' = 0$).

ILLUSTRATIVE EXAMPLES

Example 7.44 Find the correct vertical angle and the index error of the instrument from the following face left and face right readings, respectively:

(*i*) 15°22′20″ (*ii*) 15°22′44″

Solution:

Given that

$$\alpha_1 = 15°22'20''$$

$$\alpha_2 = 15°22'44''$$

$$\text{Correct vertical angle } \alpha = \frac{\alpha_1 + \alpha_2}{2}$$

$$= 15°22' + \frac{(20'' + 44'')}{2}$$

$$= \mathbf{15°22'32''}.$$

From Eq. (7.69), the index error is

$$e = \frac{\alpha_2 - \alpha_1}{2} = \frac{15°22'44'' - 15°22'20''}{2}$$

$$= \mathbf{12''}.$$

Also, $e = \alpha - \alpha_1 = 15°22'32'' - 15°22'20''$

$= 12'$ for face left observations

and $e = \alpha - \alpha_2 = 15°22'32'' - 15°22'44''$

$= -12''$ for face right observations.

Example 7.45 While making observations for altitude on both the faces, the readings of the ends of babble were as below:

$O_1 = 12$, $O_2 = 18$

$E_1 = 18$, $E_2 = 10$

If the angular value of one division of the bubble is 10″, find the correction for the altitude bubble error.

Solution:

Given that

$$\Sigma O = O_1 + O_2 = 12 + 18 + 30$$

$$\Sigma E = E_1 + E_2 = 18 + 10 = 28$$

$$v = 10''$$

$$n = 4 \text{ for both face observations}$$

From Eq. (7.70), we get the correction for the altitude bubble error

$$C = \frac{(\Sigma O - \Sigma E)}{n} . v \text{ seconds}$$

$$= \frac{(30 - 28)}{4} \times 10$$

$$= \mathbf{5''}.$$

Example 7.46 While taking the following observations to the points *A* and *B* from *O*, it was found that the instrument was in perfect adjustment except that its trinion axis was not exactly perpendicular to the vertical axis.

Point	Azimuth	Vertical angle	Striding level readings			
			I-position		II-position	
			l_1	r_1	l_2	r_2
A	38°34′20″	+ 11°31′15″	13	9	14	8
B	124°55′12″	− 4°5′25″	12	9	14	7

Determine the correct horizontal angle *AOB*. Take the value of one division of the bubble as 10″.

Solution:

From Eq. (7.72), we get

$$b = \frac{(\Sigma l - \Sigma r)}{4}.v \text{ seconds}$$

$$= \frac{(13+14)-(9+8)}{4} \times 10$$

$$= 25'' \text{ for observations to } A$$

and

$$b = \frac{(12+14)-(9+7)}{4} \times 10$$

$$= 25'' \text{ for observations to } B$$

From Eq. (7.71) for observations to *A*, we have

$$t_A = b.\tan\alpha$$

$$= 25 \times \tan(11°31'15'') = +5.1''$$

and for *B*

$$t_B = 25 \times \tan(-4°5'25'') = -1.79''$$

t is positive for angle of elevation and negative for angle of depression.

The corrected azimuth for *A*

$$= 38°34'20'' + 5.1'' = 38°34'25.1''$$

The corrected azimuth for *B*

$$= 124°55'12'' - 1.79'' = 124°55'10.21''$$

Therefore,

$$\text{Correct horizontal angle} = 124°55'10.21'' - 38°34'25.1''$$

$$= \mathbf{86°20'45.11''}.$$

Example 7.47 Determine the true altitude of the sun's centre if the apparent altitude observed at its lower limb is 58°22′33″, given that the sun's horizontal parallax is 7″ and diameter is 30′42″.

Solution:

From Eq. (7.74), we have

Correction for refraction $C_r = 58''\cot\alpha'$

$$= -58'' \times \cot(58°22'33'')$$

$$= 35.72''(\text{negative})$$

From Eq. (7.76), we have

$$\text{Parallax } p_a = p_h \cos\alpha'$$

$$= 7'' \times \cos(58°22'33'')$$

$$= 3.67''(\text{positive})$$

$$\text{Correction or semi-diameter} = \frac{1}{2}.\gamma$$

$$= \frac{1}{2} \times 30'42''$$

$$= 15'21''$$

The correction for semi-diameter is positive since the observation has been made on the lower limb. Therefore, the total correction

$$= -35.72'' + 3.67'' + 15'21''$$
$$= 14'48.95''$$

Thus,

$$\text{Correct altitude} = 58°22'33'' + 14'48.95''$$
$$= \mathbf{58°37'21.95''}.$$

7.13 SOLAR AND STELLAR OBSERVATIONS

The observations made on the sun are called the *solar observations* whereas those on star are called the *stellar observations*. If very high degree of precision is not required the stellar observations may be made like the solar observations. The stellar observations are made for achieving higher degree of precision. In stellar observations, refind instruments and methods are used and special care is taken to eliminate all systematic errors.

While making solar observations the sun should never be sighted with naked eye directly through telescope as it may cause serious injury to the eye. A piece of coloured or smoked glass should be used between the eye and the eyepiece. Generally the instruments are equipped with a sun glass which may be attached to the eyepiece before making solar observations. The instruments are also provided with a diagonal or prismatic eyepiece which is attached to telescope for making solar observations when the sun's altitude is high.

Stellar observations do not have the above problems. On the other hand, since the stellar observations are made during night, artificial illumination is required to make the cross hairs visible. Some instruments are equipped with a reflector sleeve which can be slipped over the objective like a sunshade. A flash light is then held to one side of the reflector to illuminate the cross hairs. Modern optical theodolites are equipped with built in illumination system for illumination of the scales and the cross hairs.

There are a large number of stars in the sky and out of these, a suitable star is selected which is quite favourable for the precise determination of azimuth, latitude or longitude. While selecting a particular star, the following considerations should be made.

For determination of azimuth

(*i*) by measuring altitude, the star should be far enough above the horizon to reduce the uncertain refraction. The star should be far east or far west of the meridian so as to form a strong astronomical triangle.

(*ii*) by measuring hour angle, a circumpolar star should be selected. The azimuth of a circumpolar star changes very slowly. Hence an error in hour angle will have less effect on the computed azimuth.

For determination of latitude

This requires the altitude of the star at culmination. The stars should have a fairly high altitude so that the uncertainty of the refraction correction is small. Moreover, the rate of apparent movement of the star should be small to have a series of observations without any appreciable change in its altitude. The stars which are near the pole satisfy the above conditions.

For determination of longitude or time

The most suited stars for determination of longitude or time, are those which are near the celestial equator as they apparently move rapidly and therefore, more accurate results are obtained.

Star charts show the various constellation of stars and should be used to identify the stars. It is not necessary to distinguish a star from among the neighbouring stars. The published directions and altitude of a particular star are set off on the theodolite with sufficient precision and the star is brought into the field of view at a given time.

(b) Solar observations

If the observations are made on the sun when at upper transit, its hour angle is zero and the L.A.T. is 12 hrs. The transit of the sun is observed by noting the chronometer time at the instants when the right and left limbs of the sun cross the vertical hair of the theodolite. The mean of the readings gives the mean time at the local apparent noon. The G.M.T. of G.A.N. for the given date is obtained from the Nautical Almanac and then L.M.T. of L.A.N. is calculated. This L.M.T. of L.A.N. is compared with the chronometer time at the time of the observation to get the chronometer error.

Since it is impracticable to secure that the line of sight of the instrument lies exactly in the plane of meridian, the method of meridian observation of star or the sun, is not very much used. The method requires the following corrections to the observed time:

(i) Azimuth correction $= e. \sin z. \sec \delta$ ...(7.83)

(ii) Level correction $= b. \cos z. \sec \delta$ ...(7.84)

(iii) Collimation correction $= c. \sec \delta$ ...(7.85)

where e = the error of azimuth in seconds of time (positive if the line of sight is too far east when the telescope is pointed south, and negative if the line of sight is too far west).

b = the inclination of the horizontal axis in seconds of arc (positive when the left end of the horizontal axis of the instrument is higher, and negative when lower).

c = the error of collination in seconds of time (positive when the line of sight is to the east of the meridian and negative when to the west).

z = the zenith distance.

δ = the declination.

7.14.2 By ex-meridian observation of a star or the sun

The method of ex-meridian observation of a star or the sun for the determination of time, is the most convenient and suitable for surveyors. In this method the altitude of the celestial body is observed when it is out of the meridian. The time of the actual observation is recorded from the chronometer. The hour angle is computed by the solution of the astronomical triangle (Fig. 7.44).

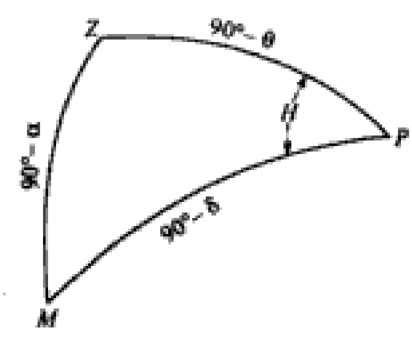


Fig. 7.44 Determination of hour angle from the astronomical triangle

This method requires that the latitude of the place of the observation should be either known or determined separately.

(a) Stellar observations

When the observations are made on a star, the L.S.T. is obtained by adding the westerly hour angle to the R.A. of the star. The L.S.T. is converted into the L.M.T. and then the chronometer error is determined by observing the mean solar time. The R.A. and declination of the star are found from the Nautical Almanac.

In the astronomical triangle ZPM (Fig. 7.44) the known sides are

$$ZP = (90° - \theta) = c \text{ (say)}$$

$$PM = (90° - \delta) = p$$

$$ZM = (90° - \alpha) = z$$

and $\angle ZPM = H$

Taking $s = \frac{1}{2}(c + p + z)$, the H can be computed by any one of the following formulae:

$$\tan\frac{H}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin s \sin(s-z)}} \quad \text{...(7.86)}$$

$$\sin\frac{H}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin c \sin p}} \quad \text{...(7.87)}$$

$$\cos\frac{H}{2} = \sqrt{\frac{\sin s \sin(s-z)}{\sin c \sin p}} \quad \text{...(7.88)}$$

$$\cos H = \frac{\sin\alpha - \sin\theta\sin\delta}{\cos\theta\cos\delta} \quad \text{...(7.89)}$$

Since the H.A. of the star is near to 0° or 90°, the tangent formula [Eq. (7.86)], gives the best results.

Due to uncertainties in the refraction for low altitudes, the star observed should have an altitude of at least 15°. The observed altitude should be corrected for refraction.

When the star is on or near the prime vertical, its altitude changes very rapidly. If the observations are made at this time, the results are more accurate. The influence of error in observation of altitude is a minimum when the star is actually on the prime vertical. To minimize the errors of observation, several altitudes of the star are observed in quick succession and the chronometer time of each observation is recorded. Half of the observations are taken with face left and half with face right. If all the observations are completed in a short time (say, 10 minutes), the mean chronometer time may be taken as the time for the mean altitude. More accurate results can be obtained by making observations on two stars, one east and the other west of the meridian, thus eliminating the instrumental errors.

When the star is on the prime vertical the hour angle is given by

$$\cos H = \frac{\tan\delta}{\tan\theta} \quad \text{..(7.90)}$$

Knowing the H (in degrees), the L.S.T. is obtained as below:

$$\text{L.S.T.} = \text{R.A.} \pm \frac{H}{15} \quad \text{...(7.91)}$$

Take plus sign when the star is to the west of the meridian and minus when it is to the east. Knowing the G.S.T. of G.M.M. or G.M.N. the L.S.T. can be converted to L.M.T., and the error of the chronometer keeping the mean solar time can be determined.

(b) Solar observations

When the observations are made on the sun, the procedure of taking observation is the same as for a star. In the case of the sun, the altitude of the lower limb of the sun is observed with the telescope normal, and then the altitude of the upper limb is observed with the telescope inverted. The chronometer time at the instant of each observation is noted. The balancing is affected by measuring a succession of altitudes both in the morning and afternoon, the most suitable timings being between 8 and 9 A.M. and between 3 and 4 P.M. Each set consists of a minimum number of four observations both face observations of upper limb and both face observations of lower limb. If the sun is not very near the meridian and the observations are completed in a few minutes of time (say, 10 minutes), the mean of the observed altitudes may be assumed to correspond to the mean of the observed times, thus neglecting the curvature of the path of the sun. The correction for index error, refraction, and parallax should be applied to the mean altitude. If the observations have been made on only one limb, the correction for the semi-diameter should also be applied.

The hour angle of the sun is computed using the Eq. (7.86) when the sun is east of meridian, and then for the sun to the west of meridian.

L.A.T. of observation since L.A.N. $= \dfrac{H}{15}$

When the sun is to the east of meridian, L.A.T. of observation

$$\text{since L.A.N.} = 24^h - \frac{H}{15}$$

$$\text{and since L.A.M.} = 12^h - \frac{H}{15}$$

If the sun is to the west of meridian, L.A.T. of observation

$$\text{since L.A.N.} = \frac{H}{15}$$

$$\text{and since L.A.M.} = 12^h + \frac{H}{15}$$

Now from the L.A.T. obtained above, L.M.T. can be calculated as explained earlier.

For recording of the field observations the form given in Table 7.2 may be used.

Table 7.2 Time determination by observations on a star

Star observed	Face	Vertical angle																
		A			B		Mean of A and B			Mean angle			Time			Mean of time		
		°	′	″	′	″	°	′	″	°	′	″	h	m	s	h	m	s
	L																	
	R																	
	R																	
	L																	

7.14.3 By equal altitude of a star or the sun

This is a very simple method of determining time and is generally used when accurate direction of the observer's meridian is not known and accurate result is required.

(a) Stellar observations

In this method a star is observed when it is at the same altitude on opposite sides of the meridian. The mean of the two chronometer times, is the time at which the star transits the observer's meridian. When the star crosses the meridian, its hour angle is zero and the L.S.T. is equal to the right ascension of the star.

The method does not require the determination of altitude of the star and therefore, no correction for refraction is required. The observations must be made when the star is near the prime vertical so that its altitude change rapidly.

The L.S.T. obtained above is converted to the L.M.T. which is then compared with the mean time of the chronometer, to obtain the chronometer error.

ILLUSTRATIVE EXAMPLES

Example 7.48 The time of transit of a star having R.A. as $10^h42^m17.25^s$, recorded with a chronometer keeping standard time of 5^h30^m E was $9^h12^m25^s$ P.M. the longitude of the place of observation is $4^h45^m E$. If G.S.T. at G.M.M. on the day of observation is $14^h12^m24^s$ determine the chronometer error.

Solution:

Longitude $= 4^h45^m \text{ E} = 4.75^h \text{ E}$

To convert G.S.T. of G.M.M. into L.S.T. of L.M.M., the loss in the sidereal time at the rate of 9.8567^s per hour of longitude

$= 9.8567 \times 4.75 = 46.82^s$

or retardation $= 46.82^s$

L.S.T. of L.M.M. = G.S.T. of G.M.M. + retardation

$= 14^h12^m24^s - 46.82^s$

$= 14^h11^m37.18^s$

The L.S.T. of observation = R.A. of the star

$= 10^h42^m17.25^s$

S.I. = L.S.T. of observation – L.S.T of L.M.M.

$= (10^h42^m17.25^s - 14\ 11^m37.18^s) + 24^h$

$= 20^h30^m40.07^s$

$= 20.5111304^h$

Mean time interval since L.M.M. = S.I. + retardation @ 9.8296^s per hour of S.I.

$= 20^h30^m40.07^s - 9.8296 \times 20.5111304$

$= 20^h27^m18.51^s$

Standard time shown by chronometer

$= 9^h12^m25^s$ P.M.

$= (12^h + 9^h12^m25^s)$ since L.M.M.

$= 21^h12^m25^s$ since L.M.M.

Local time of chronometer

= standard time shown by chronometer – difference of longitude

$= 21^h12^m25^s - (5^h30^m - 4^h45^m)$

$= 20^h27^m25^s$

Since the place of observation is at 45^m to the west of the standard meridian, the chronometer error

$= 20^h27^m25^s - 20^h27^m18.51^s$

$=$ **6.49^s** (fast).

Example 7.49 Following observations were made to determine the chronometer error at a place having latitude of 30°36′20″N and longitude 73°16′14″E:

Mean corrected altitude of the sun = 32°42′35″

Mean chronometer time of observation = $15^h57^m36^s$

Declination of the sun at the time of observation = 16°24′45″N

G.M.T. of G.A.N. $= 11^h52^m23.4^s$

Determine the correct chronometer error if the chronometer is known to be 3^m fast on L.M.T.

Solution:

$$\text{Longitude} = 73°16'14''\text{E} = \frac{73.2705556}{15} = 4^h53^m4.93^s$$

$$\text{Zenith distance} = z = 90° - \alpha = 90° - 32°42'35'' = 57°17'25''$$

$$\text{Colatitude} = c = 90° - \theta = 90° - 30°36'20'' = 59°23'40''$$

$$\text{Polar distance} = p = 90° - \delta = 90° - 16°24'45'' = 73°35'15''$$

$$z + c + p = 2s = 190°16'20''$$

or $$s = 95°8'10''$$

From Eq. (7.86), we have

$$\tan\frac{H}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin s \sin(s-z)}}$$

$$s - c = 95°8'10'' - 59°23'40'' = 35°44'30''$$

$$s - p = 95°8'10'' - 73°35'15'' = 21°32'55''$$

$$s - z = 95°8'10'' - 57°17'25'' = 37°50'45''$$

$$\tan\frac{H}{2} = \sqrt{\frac{\sin 35°44'30'' \sin 21°32'55''}{\sin 95°8'10'' \sin 37°50'45''}}$$

$$= 0.5925333$$

or $$H = 61°17'46.71''$$

$$= 4^h5^m11.11^s$$

$$\text{L.A.T.} = 12^h + \text{H} = 12^h + 4^h5^m11.11^s = 16^h5^m11.11^s$$

$$\text{G.A.T.} = \text{L.A.T.} - \text{longitude}$$

$$= 16^h5^m11.11^s - 4^h53^m4.93^s$$

Given that $$= 11^h12^m6.18^s$$

$$\text{G.M.T. of G.A.N.} = 11^h52^m23.4^s$$

or $$12^h = 11^h52^m23.4^s + \text{ET}$$

or $\text{ET} = 0^h7^m36.6^s$ to be subtracted from the apparent time

or $$\text{G.M.T.} = \text{G.A.T.} - \text{ET}$$

$$= 11^h12^m6.18^s - 47^m53.82^s$$

$$= 10^h4^m29.58^s$$

Now,

$$\text{L.M.T.} = \text{G.M.T.} + \text{longitude}$$

$$= 10^h24^m12.36^s + 4^h53^m4.93^s$$

$$= 15°57'34.51^s$$

Chronometer error $$= 15^h57^m36^s - 15°57'34.51^s$$

$$= \mathbf{1.49^s\ (fast).}$$

Example 7.50 Find the chronometer error from the following data for ex-meridian observations taken on a star east of the meridian.

Latitude of the place	$= 55°0'10.34''$N
Mean observed altitude of the star	$= 37°46'28.65''$
R.A. of the star	$= 16^h39^m44.78^s$
Declination of the star	$= 18°0'31.9^s$
Mean sidereal time recorded by the chronometer	$= 13^h24^m15^s$

Solution:
From Eq. (7.89), we have

$$\cos H = \frac{\sin\alpha - \sin\theta\sin\delta}{\cos\theta\cos\delta}$$

$$= \frac{\sin 37°46'28.65'' - \sin 55°0'10.34''\sin 18°0'31.9''}{\cos 55°0'10.34''\cos 18°0'31.9''}$$

$$= 0.6587299$$

$$H = 48°47'48.9''$$

$$= 3^h15^m11.26''$$

Now,

$$\text{L.S.T.} = \text{R.A.} - H$$

$$= 16^h39^m44.78^s - 3^h15^m11.26^s$$

$$= 13^h24^m33.52^s$$

Thus,

$$\text{Chronometer error} = 13^h24^m15^s - 13^h24^m33.52''$$

$$= -18.52^s$$

$$= \mathbf{18.52^s\ (slow)}.$$

Example 7.51 At a certain place having longitude of 113°33′34.4″ E a star was observed east of meridian at $8^h22^m16^s$ P.M. by a watch keeping L.M.T. The star was again observed at the same altitude to the west of meridian at $10^h21^m19.56^s$. Find the error of the watch if G.S.T. of G.M.N. was $9^h56^m50.54^s$ and R.A. of the star was $19^h18^m45^s$.

Solution:

$$\text{Longitude} = 113°33'34.4''\text{E} = \frac{113.5595556}{15} = 7^h34^m14.29^s$$

$$= 7.5706370^h.$$

$$\text{L.S.T. of L.M.N.} = \text{G.S.T. of G.M.N.} - 9.8567 \times 7.5706370^h$$

$$= 9^h56^m50.54^s - 74.62^s$$

$$= 9^h55^m35.92^s$$

$$\text{L.S.T. of observation} = \text{R.A. of the star}$$

$$= 19^h18^m45^s$$

$$\text{S.I. after L.M.M.} = \text{R.A.} - \text{L.S.T. of L.M.N.}$$

$$= 19^h18^m45^s - 9^h55^m35.92^s$$

$$= 9^h23^m9.08^s$$

$$= 9.3858556^h$$

$$\text{Mean interval after L.M.M.} = \text{S.I. after L.M.M.} - \text{retardation @ } 9.8296^s \text{ per hour of S.I.}$$

$$= 9^h23^m9.08^s - (9.8296 \times 7.3858556)^s$$

$$= 9^h21^m36.82^s$$

$$\text{L.M.T. of transit} = 9^h21^m36.82^s$$

$$\text{L.M.T. when star east of meridian} = 8^h22^m16^s$$

$$\text{L.M.T. when star west of meridian} = 10^h21^m19.56^s$$

$$\text{Mean L.M.T. of observation} = \frac{1}{2}(8^h22^m16^s + 10^h21^m19.56^s)$$

$$= 9^h21^m47.78^s$$

$$\text{Chronometer error} = 9^h21^m47.78^s - 9^h21^m36.82^s$$

$$= \mathbf{10.96^s\ (fast)}.$$

7.15 TIME OF RISING OR SETTING OF A HEAVENLY BODY

The time of rising or setting of a heavenly body can be determined if the latitude of the place and declination of the heavenly body are known.

In Fig. 7.46, M is the position of a star on the horizon when it is rising.

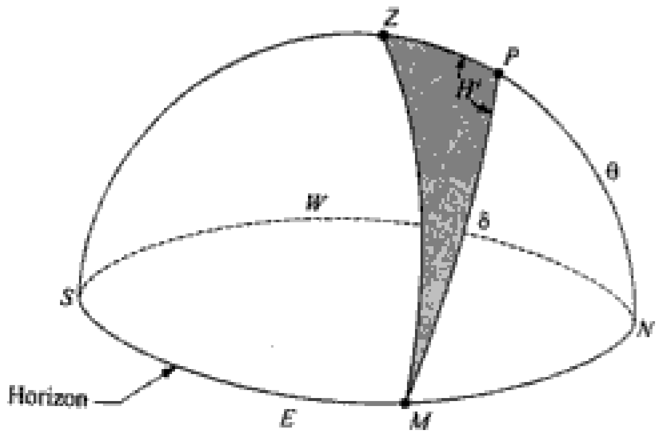


Fig. 7.46 Rising and setting of heavenly body

For the star at horizon from Eq. (7.41), we get

$$\cos H' = -\tan\theta \tan\delta \quad \text{...(7.98)}$$

Knowing the declination of the star and the latitude of the place, the hour angle of the star can be computed using Eq. (7.98).

Thus,

L.S.T. of rising of star = R.A. of star + H of star

The L.M.T. of rising of the star can be determined by converting the L.S.T of rising into L.M.T.

The hour angle of setting of a star is obviously same as that of rising.

7.16 LENGTH OF DAY AND NIGHT

If the declination (δ) of the sun and the latitude (θ) of a place are known, the length of the day or night at that place can be found knowing the hour angle of the sun-rise or sun-set from Eq. (7.98). If the change in the declination δ is ignored,

$$\text{Length of the day} = 2 \times \text{hour angle of the sun}$$

$$= \frac{2H'}{15} \text{ in time unit} \quad \text{...(7.99)}$$

and

$$\text{length of the night} = \frac{2(180° - H')}{15} \text{ in time unit} \quad \text{...(7.100)}$$

or

$$= 24^h - \text{length of day}$$

(1) It can be shown that for all values of δ, the days are equal to the nights for the places at equator. At equator $\theta = 0°$, therefore, from Eq. (7.98), we have

$$\cos H' = -\tan 0° \tan\delta$$

$$= 0$$

or

$$H' = 90°$$

Therefore,

$$\text{Length of days (or nights)} = \frac{2\times 90^\circ}{15} = 12^h$$

(2) For all values of θ at the time of equinox, when the sun is at equator, the declination of the sun is zero, i.e. $\delta = 0^\circ$ and therefore, again

$$\cos H' = -\tan\theta\,\tan 0^\circ$$
$$= 0$$

or

$$H' = 90^\circ$$

Thus,

$$\text{length of the days (or nights)} = \frac{2\times 90^\circ}{15} = 12^h$$

Hence for all values of θ, i.e., at all the places on the earth, the day is equal to night at the time of equinox.

(3) At the places having latitude $\theta = 90^\circ - \delta$, the sun does not set.

$$\theta = 90^\circ - \delta$$
$$\cos H' = -\tan(90^\circ - \delta)\tan\delta$$
$$\cos H' = -\cot\delta\tan\delta = -\cot\delta\tan\delta = -1$$

or

$$H' = 180^\circ$$

Thus,

$$\text{length of day} = \frac{2\times 180^\circ}{15} = 24^h$$

Therefore, the places having day of 24^h, there is no night, i.e., the sun does not set.

(4) At the places having latitude $\theta = 90^\circ + \delta$, the sun does not rise at all.

$$\theta = 90^\circ + \delta$$
$$\cos H' = -\tan(90^\circ + \delta)\tan\delta$$
$$= -(-\cot\delta)\tan\delta$$
$$= 1$$

or

$$H' = 0^\circ$$

Thus,

$$\text{length of day} = \frac{2\times 0^\circ}{15} = 0^h$$

Hence, the sun does not rise at the places where $\delta = 90^\circ + \delta$ and, therefore nights are of 24^h.

7.17 THE DURATION OF TWILIGHT

When the sun sets below the horizon, the darkness does not come instantaneously because the sun rays still illuminate the sky above us due to reflection and scattering of the light in the atmosphere by the particles of vapour and gases present in the atmosphere. The subdued light which separates night from day, is called the *twilight*. The intensity of the diffused light gets gradually diminished with the sinking of the sun. Observations have shown that the diffused light is received so long as the sun does not sink 108° below the horizon.

The duration of twilight at particular place is the time which the sun takes to alter its zenith distance from 90° to 108° in the evening and from 108° to 90° in the morning.

For the spherical triangle *PMZ* shown in Fig. 7.47, from Eq. (7.8), we have

$$\cos z = \cos(90^\circ - \delta)\cos(90^\circ - \theta) + \sin(90^\circ - \delta)\sin(90^\circ - \theta)\cos H$$
$$= \sin\delta\sin\theta + \cos\delta\cos\theta\cos H \qquad ...(7.101)$$

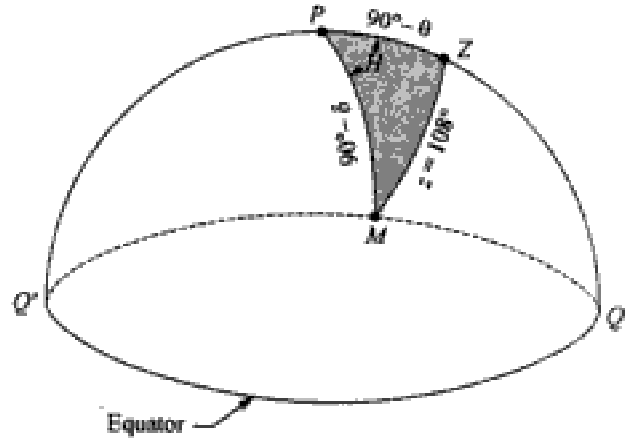


Fig. 7.47

Taking H as H_1 for $z = 108°$ and H_0 for $z = 90°$, respectively, the Eq. (7.101) becomes

$$\cos 108° = \sin\delta\sin\theta + \cos\delta\cos\theta\cos H_1$$

and $$\cos 90° = \sin\delta\sin\theta + \cos\delta\cos\theta\cos H_0$$

or $$\cos H_1 = \frac{\cos 108°}{\cos\delta\,\cos\theta} - \tan\delta\tan\theta \quad ...(7.102)$$

and $$\cos H_0 = -\tan\delta\tan\theta \quad ...(7.103)$$

From Eqs. (7.102) and (7.103), the values of H_1 and H_0 can be calculated for the given values of θ and δ. Hence

$$\text{the duration of twilight } = H_1 - H_0.$$

7.18 THE SUN-DIALS

A sun-dial is generally a plane surface on which a system of lines is drawn in such manner that the coincidence of the shadow of a straight rod, attached to the plane surface at right angles to the surface of the dial, with any one of the lines points out the hour angle of the day in apparent time. The straight rod or edge is called the *stile* or *gnomon* of the dial and the system of lines, the *hour lines.* The intersection line of the plane of the stile with the plane of the dial, is called the *sub-stile.* The apparent time given by the sun-dial can be converted into mean time.

This instrument has lost nearly all its interest and value since the perfection and low cost of watches and clocks have brought these into general use as measure of time. It is useful particularly in places where there are no means available for checking watch or clock times.

A sun-dial may be classified as below:

1. Horizontal dial
2. Vertical dial
3. Oblique dial.

In the horizontal dial, the dial having a system of lines is kept horizontal whereas the vertical and oblique dials have their dials vertical and inclined to the horizontal, respectively. In each case the stile is kept parallel to the earth's axis and therefore, always points north. Since the vertical and oblique dials require calculations which are not so simple, the principle of graduating the horizontal sun-dial shall only be discussed below.

In Fig. 7.48, *NYSX* is the plane of the dial in the horizontal plane and *CP* is the direction of stile which if produced will intersect the celestial sphere in the celestial pole *P*. *NPS* is the plane of the meridian and *M* is the position of the sun at any instant. *CY* is the shadow of the stile on the dial when the sun is at *M*.

Since CP is also the direction of meridian, its shadow will fall on the line CN at apparent noon. The positions 1, 2,3, etc., correspond to the shadows $C1$, $C2$, $C3$, etc., at 1^h, 2^h, 3^h, etc., respectively, after the noon. CY is the shadow of the stile for the position M of the sun which is the intersection of the plane XPY containing the stile and M, with the plane of the dial.

If the small variation in the declination of the sun is neglected, the diurnal path of the sun M will describe a circle uniformly on the celestial sphere about P as the centre. The projections of the equal angular divisions of the diurnal circle of the sun's path will give unequal angular divisions on the dial. The angle MPS is the hour angle of the sun at the instant. The problem is now to locate the points 1, 2, 3, etc., on the dial so as to correspond to the times 1^h, 2^h, 3^h, etc., after the apparent noon.

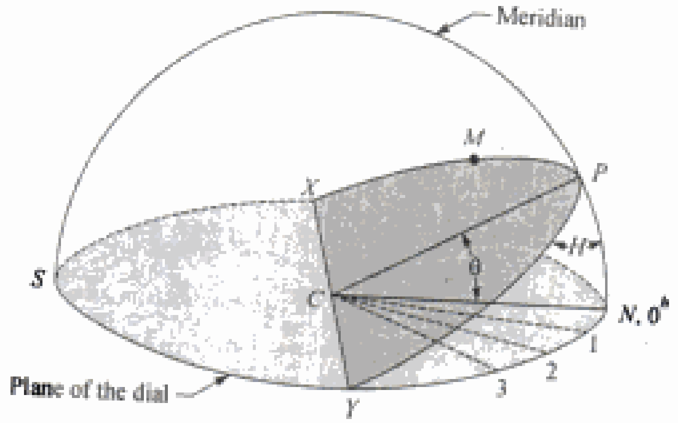


Fig. 7.48 The horizontal sun-dial

In the spherical triangle YPN which is right-angled at N, we have

NP = altitude of the pole = latitude of the place = θ

$\angle NPY$ = hour angle of the sun = H

NY = required angular division along the dial corresponding to the hour angle H.
= η

For the right-angled triangle YPN from Eq. (7.19), we have

$$\sin\theta = \cot H \tan\eta \qquad \text{...(7.104)}$$

The above equation gives the values of η corresponding to the different values of H.

If a horizontal sun-dial is required for Roorkee (latitude = 29°52′), the values for the graduation on the dial are computed for H = 15°, 30°, 45°, etc. corresponding to $1^h, 2^h, 3^h$, etc., respectively, as below:

$$\eta_1 = \tan^{-1}(\sin 29°52' \times \tan 15°) = 7°36'$$

$$\eta_2 = \tan^{-1}(\sin 29°52' \times \tan 30°) = 16°2'$$

$$\eta_3 = \tan^{-1}(\sin 29°52' \times \tan 45°) = 26°28'$$

and so on.

The graduations at 7°36′, 16°2′, 26°28′, etc., correspond to the points 1, 2, 3, etc., and give the local apparent time as $1^h, 2^h, 3^h$, etc. To convert the local apparent time into local mean time approximate value of equation of time must be known.

7.19 DETERMINATION OF AZIMUTH

The term azimuth is somewhat loosely used to denote the direction of a given line with reference to some datum direction. The datum direction is sometimes true north or south, or something close to these or sometimes some arbitrary direction selected to suit the needs of a particular project.

To avoid any confusion due to considering the different types of datum directions, the true (astronomical) north will be considered as the datum direction, and all azimuths shall be expressed in terms of it on the full circle 0° to 360°. A line bearing true north from the observer will have azimuth 0°. In Fig. 7.49 (a and b) the azimuths of a celestial body *M* are shown when it lies in northern and southern hemispheres. In the northern hemisphere, the azimuth *A* of *M* is measured from the elevated north pole if *M* lies to the east of the meridian an if *M* lies to the west of the meridian, its azimuth is (360° – *A*). When *M* is in the southern hemisphere, the azimuth of *M* is (180° – *A*) if *M* lies to the east of meridian and when to the west (180° + *A*).

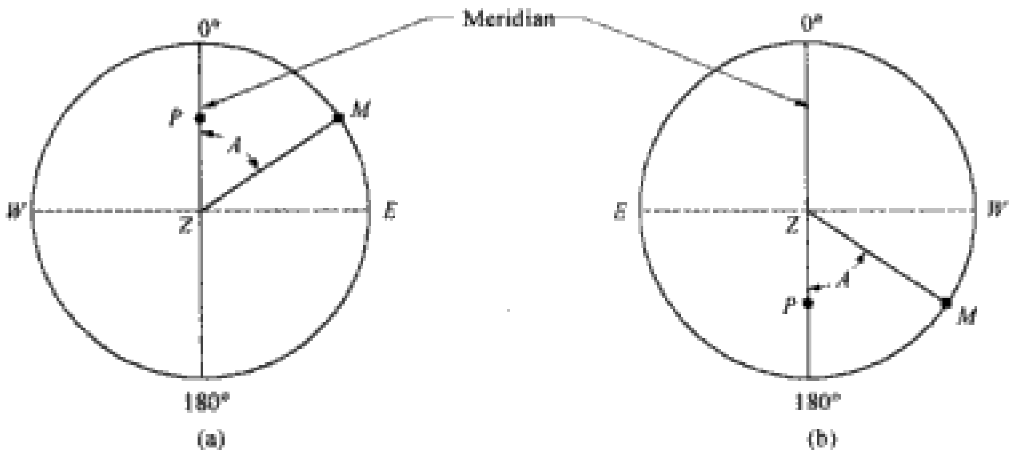


Fig. 7.49 Azimuth of a celestial body in (a) northern hemisphere ; (b) southern hemisphere

The determination of the azimuth of a line is a very important and common operation for the surveyors. In most cases the survey commences from previously established points which help in getting initial azimuth to carry forward the survey. In situations where such points are not available or their reliability is doubtful, the azimuth of the line from the observation point to other station, is required to be determined by observations to the sun or stars. Checking of the azimuth of a line, whose azimuth has been brought forward by some other means, also requires the determination of azimuth by astronomical methods.

7.19.1 Reference mark

When an observation is made by pointing to a celestial body to determine direction, all that is achieved from the subsequent calculation is the azimuth of the line of sight from the instrument to the body (which is moving) at the instant of pointing. In order to be of any practical use, this azimuth must be transferred to a line on the earth's surface between the observing station and some other fixed station, called the *reference mark* (R.M.) or the *reference object* (R.O.). The azimuth is transferred by measuring the horizontal angle between the line to the celestial body and the line to the R.M.

In Fig. 7.50, the azimuth of the celestial body *M* is *A* and the angle between the celestial body and the R.M. at the observing station *O*, is θ. The azimuth of the line joining *O* and *R*. M. is obtained by applying the angle θ in the correct sense to the computed azimuth of the celestial body at the moment of observation. For the case shown in the figure, the azimuth of the line from *O* to R.M. is $[360° - (\theta - A)] = A'$. The azimuth A' of the ground reference line can be used to find the azimuths of other ground lines by triangulation or traversing.

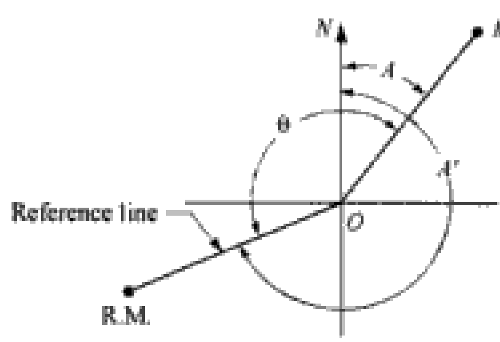


Fig. 7.50 Reference mark

It is desirable that there should be no need of refocussing the telescope after pointing to the celestial body and then directing to the R.M., and this requires the latter should be, where practicable, about a kilometer away. When the stellar observations are taken, the R.M. should be made to imitiate the light of a star as nearly as possible. This may be done by placing a lantern or electric lamp in a box or behind a screen through which a small circular hole is cut to admit the light to the observe. The diameter of the hole should not be more than about 5 mm for a distance of 1 km.

When booking the results of an observation for azimuth, a diagram should always be drawn to show the relative positions of R.M., celestial body and meridian in plan view. This information helps to remove any difficulty in deciding later how to apply the included angle in the correct sense between body and R.M. in finding the azimuth of the latter.

7.19.2 Methods of determining azimuth

The azimuth of a celestial body or the direction of the true meridian may be determined by the following methods:

1. Observations on a star or the sun at equal altitude
2. Observations on a circumpolar star at elongation
3. Observations to two circumpolar stars at elongation
4. Hour angle of a star or the sun
5. Observations on Polaris
6. Ex-meridian observations on a star or the sun.

By observations on a star at equal altitudes

1. Observations on a star or the sun at equal altitude

(a) Stellar observations

The method is based on the fact that if the angle subtended between the R.M. and a star, is measured in two positions of a star at equal altitudes, the angle between the R.M. and the meridian is equal to the half of the algebraic sum of the two angles. In this method, the knowledge of the latitude of the place or the local time is not required, and no calculations are involved. However, the method requires about four to six hours time at night to complete the observations. Also the effects of atmospheric refration may vary considerably during the observation time, affecting the vertical angles to an unknown extent.

In Fig. 7.51, M_1 and M_2 are the two positions of a star at equal altitudes as shown in Fig. 7.45. OP is the meridian of the place which bisects the angle M_1OM_2. The observer's position is O.

To make the observations, the instruments is set at O and levelled accurately. The horizontal circle reading is set to 0°0′00″ for the telescope pointing towards the R.M. The upper clamp is opened and the telescope is turned clockwise and the star is bisected accurately at position M_1. Both the horizontal and as well as the vertical circles are clampled. The horizontal angle θ_1 and the altitudes α of the star are read. Now the star is followed through the telescope by unclamping the upper circle and the star is again bisected when it attains the same altitudes α and the horizontal angle θ_2 is read for this position of the star.

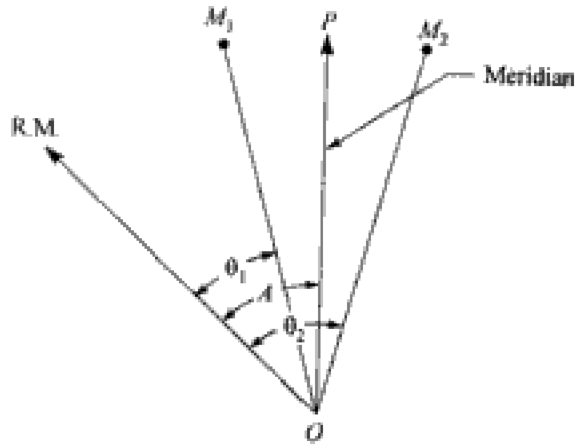


Fig. 7.51 Azimuth by equal altitude

The azimuth A of the line O to R.M. is given by

$$A = \theta_1 + \frac{1}{2}(\theta_2 - \theta_1)$$

$$= \frac{\theta_1 + \theta_2}{2} \qquad ...(7.105)$$

The azimuth measured clockwise from OP is $(360° - A)$.

For any other position of the R.M. with reference to the star's positions, the azimuth may be calculated simply by taking the help of the diagram showing the relative positions of the R.M., star and the meridian.

While taking the observations it should be ensured that the instrument is properly levelled. The altitude bubble should remain central in all positions of the telescope.

The above observation procedure assumes that the instrument is in perfect adjustment. However, if it is not so, still the observations can be made taking at least four observations, two with face left and two with face right, to eliminate the instrumental errors. The procedure of making observations is as follows (Fig. 7.52a).

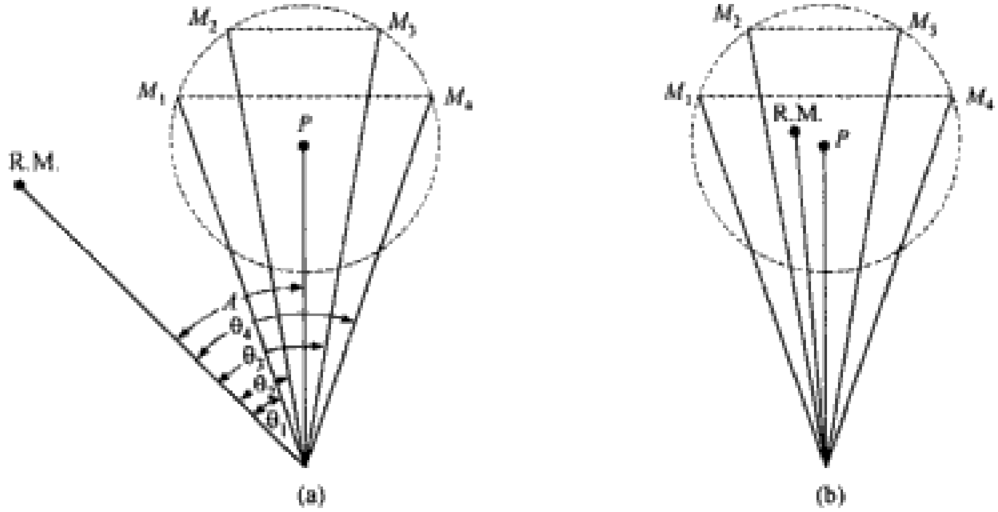


Fig. 7.52 Four observations to eliminate instrumental errors

1. Set up the instrument at O and bisect the R.M. clamp both the circles and take the horizontal circle reading.
2. Unclamp the upper circle and bisect the star at M_1. Note down the horizontal and vertical circle readings and determine θ_1 and α_1.
3. Change the face to the face right and bisect the R.M. again. Set the horizontal circle reading to zero.
4. Unclamp the upper plate and bisect the star at M_2. Clamp both the circles and read θ_2 and α_2.
5. With the vertical circle clamped at α_2, swing the telescope and bisect the star at M_3 when it attains the altitude α_2 again. Read the horizontal angle θ_3.
6. Change the face to the face left and bisect the R.M. Read the horizontal circle reading and set the vertical circle reading to α_1.
7. Unclamp the upper circle swing the telescope and bisect the star at M_4 when it attains the altitude α_1 again. Clamp both the circles.
8. Take the reading of the horizontal angle θ_4 between the R.M. and the star at M_4.
9. Determine the mean of the angle θ_1 and θ_2 to determine the position of the star to one side of the meridian when it is at an average altitude of $\dfrac{\alpha_1 + \alpha_2}{2}$.
10. Similarly determine the mean of the angles θ_3 and θ_4 to obtain the position of the star to the other side of the meridian when the star has an average altitude of $\dfrac{\alpha_1 + \alpha_2}{2}$.
11. Determine the azimuth of the line from O to the R.M. as below:

$$A = \frac{1}{2}\left[\left(\frac{\theta_3 + \theta_4}{2}\right) + \left(\frac{\theta_1 + \theta_2}{2}\right)\right] \qquad \text{...(7.106)}$$

Let δ_1 and δ_2 be the declinations of two stars M_1 and M_2 (Fig. 7.53) which elongate, within a short time, at the place of observation having latitude θ.

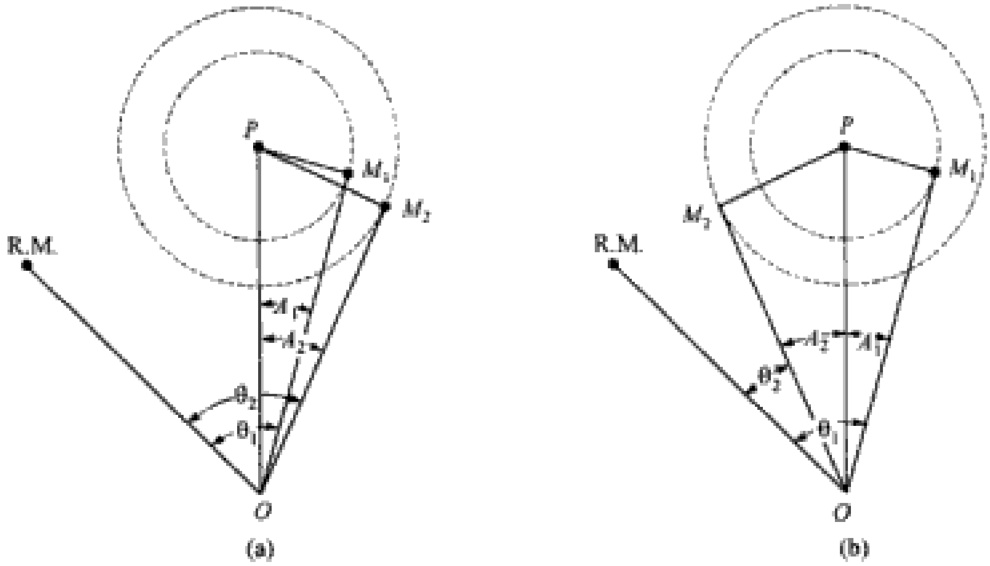


Fig. 7.53 Two circumpolar stars at elongation

Then, $$\sin A_1 = \frac{\cos\delta_1}{\cos\theta}$$

and $$\sin A_2 = \frac{\cos\delta_2}{\cos\theta}$$

or $$\frac{\sin A_1}{\sin A_2} = \frac{\cos\delta_1}{\cos\delta_2} = k, \text{ (a constant).} \quad ...(7.115)$$

Two cases may arise according to the elongation of the stars as shown in Figs. 7.53a and 7.53b.

(a) Elongation on the same side of the pole (Fig. 7.53a)

Let the difference in azimuth $(A_2 - A_1)$ of the stars at eastern elongation be d, then

$$A_1 = A_2 - d$$

or $$\sin A_1 = \sin (A_2 - d)$$
$$= \sin A_2 \cos d - \sin d \cos A_2$$

But from Eq. (7.115), we have

$$\sin A_1 = k \sin A_2$$
$$k \sin A_2 = \sin A_2 \cos d - \sin d \cos A_2$$

or $$k = \cos d - \sin d \cos A_2$$

or $$\cot A_2 = \frac{\cos d - k}{\sin d} \quad ...(7.116)$$

and $$\cot A_1 = \frac{k' - \cos d}{\sin d}$$

where $$k' = \frac{1}{k}.$$

From Eq. (7.116), the azimuth A_2 is determined and knowings A_2, the azimuth of the survey line from O to R.M. can be found.

This method though simple and straight forward, is not very much used due to the following disadvantages:

1. It requires separate observations for determining the chronometer error. However, if the chronometer error is known, the method is better than the ex-meridian observations. If the star observed is near the prime vertical, the errors of time have very little effect on the result.
2. For computation of hour angle from the chronometer time, a linear relationship between the time and the motion of the star in azimuth has been tacitly assumed, which is not correct.

For accurate results, a correction for the curvature of the path of the star must be applied to the mean of the face left and face right observations. The correction to the azimuth is given by

$$C = +\frac{1}{8}\sin A\cos\theta\sec^2\alpha(\cos\alpha\sin\delta - 2\cos A\cos\theta)\times\Delta t^2\times\sin 1'' \text{ (in seconds)} \quad ...(7.120)$$

where Δt = time interval between the face left and face right observations.

The correction is zero at culmination.

5. Observation on Polaris or close circumpolar star

Very precise determination of azimuth may be made by making observations on the polaris or on a close circumpolar star. The polaris at its elongation moves very rapidly in altitude and its azimuth remains constant. Error in altitude if any will not affect the azimuth. Moreover, when the hour angle of the polaris is 90°, error in the assumed value of the latitude of the place will be least. Although any close circumpolar star can be selected, it is preferable to sight the Polaris (or α Ursae Minor). The polaris is the brightest circumpolar star and it is used in preference to other circumpolar stars as far as possible.

The observations on the close circumpolar star or polaris have the following advantages:

1. A large number of observations is possible in a short time since the motion of the star in azimuth is very slow, and thus greater accuracy can be achieved.
2. Observations can be made at any convenient time in the night, without waiting for elongation.
3. It is generally possible to sight Polaris during the twilight when there is sufficient light. Thus there is no necessity of artificial illumination for the R.M. or the instrument.

The chronometer time of the observation of the star is noted to determine the hour angle of the star. The azimuth of the star is then computed by the solution of the astronomical triangle.

Fig. 7.55 shows the position of a close circumpolar star M. Its hour angle H, is computed from the observed chronometer time. The azimuth of the star is given by

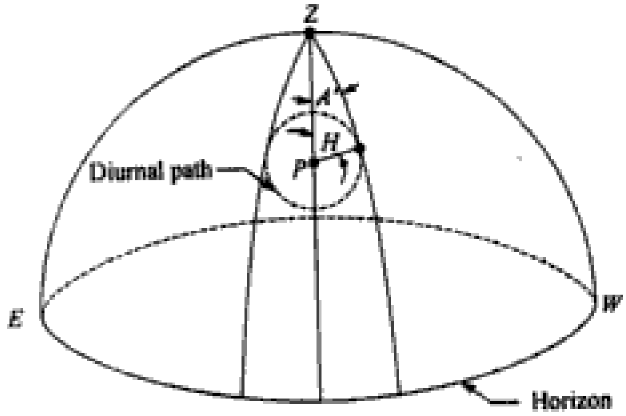


Fig. 7.55 Observation on Polaris

$$\tan A' = \frac{\sin H}{\cos\theta \tan\delta - \sin\theta \cos H}$$

$$\tan A' = \frac{\sin H}{\cos\theta \tan\delta - \sin\theta \cos H} \qquad ...(7.121)$$

where $A' = 360° - A$

θ = latitude of the place

δ = declination of the star

H = hour angle of the star

Eq. (7.121) may also be written as

$$\tan A' = \frac{\sin\theta \cot\delta \sin H}{1 - \tan\theta \cot\delta \cos H} \qquad ...(7.122)$$

or

$$\tan A' = \sec\theta \cot\delta \sin H \frac{1}{(1-a)} \qquad ...(7.123)$$

where $a = \tan\theta \cot\delta \cos H$

In field, the observations are made as follows:

The instruments is set up at the station mark and the R.M. is bisected on face left. The horizontal circle reading is noted. The telescope is rotated in azimuth and the polaris is bisected at the centre of the cross hairs. The horizontal circle and vertical circle readings are taken ensuring that altitude bubble is central. The face is changed quickly to right and the observations are repeated. This constitutes one set. Minimum three sets are taken and finally the R.M. is bisected on the face right to eliminate the collimation error, if any. The chronometer time is noted for each pointing. The hour angle is computed from the chronometer time and then A' is computed using Eq. (7.121). The azimuth of the survey line is computed from the measured horizontal angle between the star and R.M.

6. Ex-meridian observations on a star or the sun

Azimuth is most commonly determined by ex-meridian observation of a star or the sun except for the determination of primary standard. The observations required are the same as those for the determination of time. In fact, the determinations may be combined if the chronometer time of the altitudes are also recorded. From the known latitude of the place and the declination of the star, the azimuth can be determined by the solution of the astronomical triangle.

(a) Stellar observations

Since the refraction correction is very uncertain for stars near the horizon, the star should be selected so that the it has an altitude of at least 30°. The mean refraction for objects at an altitude of 45° is 58" and therefore, it is necessary to correct the observed altitudes for refraction. The star should be sighted when it is changing rapidly in altitude and slowly in azimuth. A favourable position occurs when the star is on the prime vertical when the influence of errors of observed altitude is small. When the star is on the prime vertical the azimuth A in Fig. 7.55, is 90°.

In Fig. 7.56, M is the position of the star when its altitude is α. In the astronomical triangle MZP, we have

$90° - \theta = c$

$90° - \delta = p$

$90° - \alpha = z$

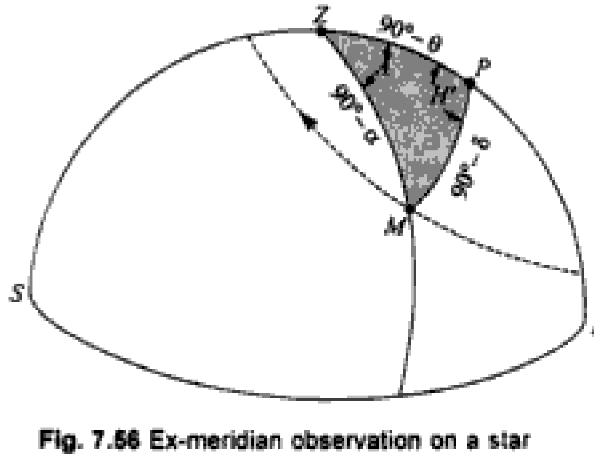


Fig. 7.56 Ex-meridian observation on a star

(c) Effect of error in declination

Differentiating Eq. (7.129), we get

$$-\sin A\, dA = \frac{\cos\delta}{\cos\theta\cos\alpha}\, d\delta$$

or

$$dA = -\frac{\cos\delta}{\sin A\cos\theta\cos\alpha}\, d\delta \qquad \text{...(7.140)}$$

Applying sin rule to the triangle ZMP, we get

$$\frac{\sin A}{\sin(90^\circ - \delta)} = \frac{\sin M}{\sin(90^\circ - \theta)}$$

or

$$\sin A = \frac{\sin M\cos\delta}{\cos\theta}$$

Substituting the value of $\sin A$ is Eq. (7.140), we get

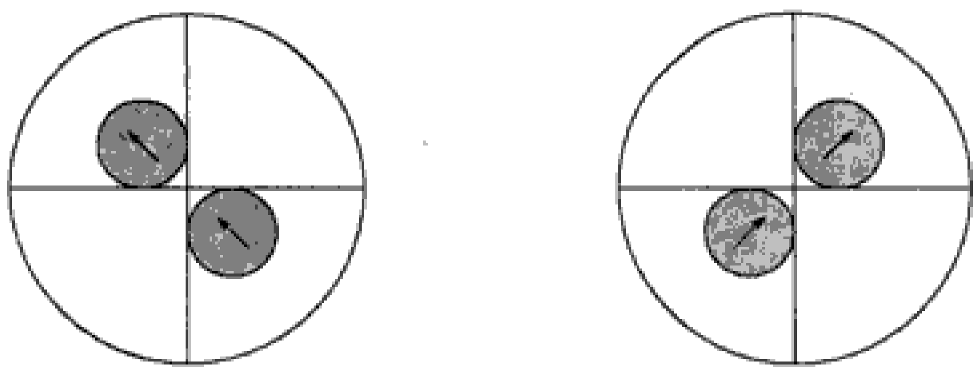
$$dA = -\operatorname{cosec} M \sec\alpha\, d\delta \qquad \text{...(7.141)}$$

Hence, the error in azimuths is a minimum when the parallactic angle is equal to 90°. The error will be less for lower altitudes.

(b) Solar observations

The general procedure of solar observations are the same as for a star. However, since the declination of the sun changes very rapidly, the exact time of observations is required. The corrections for the refraction and parallax should be applied to the observed altitudes to get accurate altitude.

To get the required altitude of the sun and the horizontal angle to its centre, the cross hairs are set tangential to the two limbs of the sun simultaneously, as shown in Fig. 7.58a. The observations for changed face are taken on the opposite limbs as shown in Fig. 7.58b.



(a) (b)

Fig. 7.58 Observations on the sun

The field procedure of making the observations is as follows :

The theodolite filtted with a sun glass and diagonal eyepiece is set up over the station mark. Both the plates are clamped to zero and the R.M. is bisected. The telescope is rotated and the sun is observed by bringing it into the lower left quadrant of the object glass moving upward (Fig. 7.59a). Both the plates are clamped. The vertical wire is kept on the apparent right limb of the sun by moving the tangent screw of the upper plate. When the upper limb touches the horizontal wire, the chronometer time, horizontal angle and the altitude are noted. The face is changed and the sun is observed in the upper right quadrant (Fig. 7.59b). Keeping the vertical wire on the apparent left limb of the sun by moving the tangent screw of the upper plate, when the apparent lower limb touches the horizontal hair, the chronometer time, horizontal angle and the vertical angle are recorded. The telescope is rotated to bisect the R.M. on face right and the horizontal angle is noted.

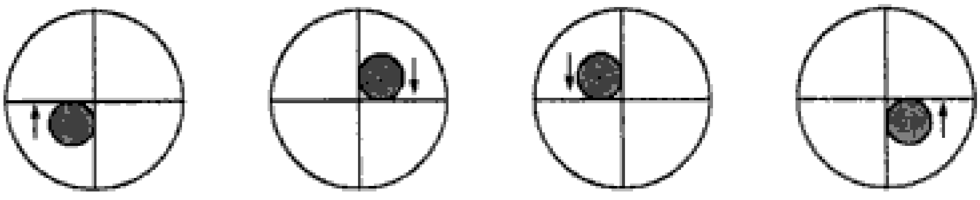


Fig. 7.59 Observations on sun in various quadrants

The above observations constitute one set. Another set of observations is taken in the similar manner for the upper left and the lower right quadrants on face left and right, respectively, as shown in Fig. 7.59 (c and d). The average value of the two azimuths computed from the two sets of the observations is the final value of the azimuth.

For very accurate results, the observed altitude should be corrected for errors due to inclination of the trunion axis, if any

ILLUSTRATIVE EXAMPLES

Example 7.52 A star having declination 72°26′50″N was observed at eastern elongation when the clockwise angle from a reference object *R* was 108°18′30″. Immediately, another star having declination 23°43′08″N was observed at its eastern elongation and the clockwise horizontal angle from *R* was 120°16′05″. Find the azimuth of *R*.

Solution: (Fig. 7.53a):

From Eq. (7.115), we have

$$k = \frac{\cos\delta_1}{\cos\delta_2} = \frac{\cos 72°26'50''}{\cos 23°43'08''} = 0.3294094$$

From the figure, we have

$$A_2 - A_1 = \theta_2 - \theta_1$$

or $$d = 120°10'05'' - 108°18'30'' = 11°57'35''$$

From Eq. (7.116), we have

$$\cot A_2 = \frac{\cos d - k}{\sin d} = \frac{\cos 11°57'35'' - 0.3294094}{\sin 11°57'35''}$$

$$= 3.1313171$$

or $$A_2 = 17°42'40''$$

$$\text{Azimuth of } R = 120°16'05'' - 17°42'40''$$

$$= 102°33'25''$$

True bearing of the survey line to *R* from the station of observation

$$= 360° - 102°33'25''$$

$$= \mathbf{257°26'35''}.$$

Example 7.53 A star was observed at western elongation at a station *A* in latitude 51°30′N and longitude 54°30′W. The declination of the star was 59°55′16″N. The mean observed angle between the reference object *R* and the star was 82°30′24″. Determine (*i*) the hour angle of the star, (*ii*) the altitude of the star at elongation, and (*iii*) the azimuth of the line *AR*.

Solution: (Fig. 7.60)

In the figure, *M* is the position of the star, *P* is the north pole, and *Z* is the zenith of the observer at *A*.

Since the star is at western elongation, the angle *PMZ* is 90° and the triangle *PMZ* is a right-angled triangle.

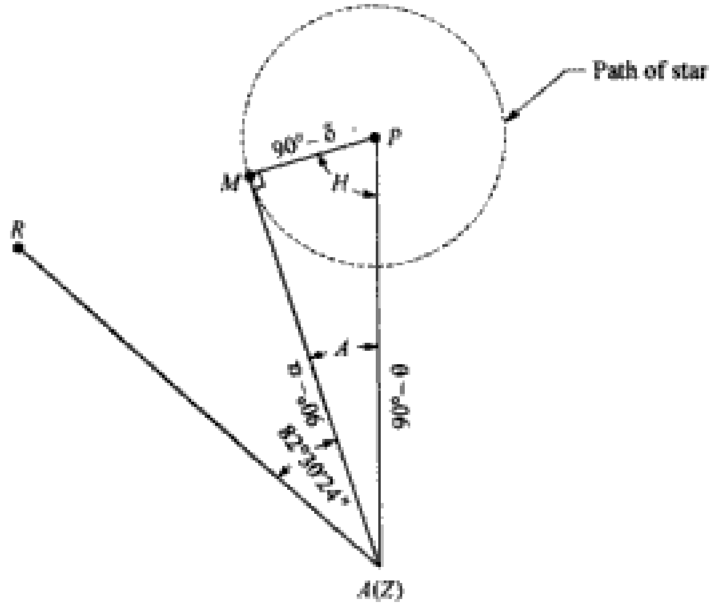


Fig. 7.60

(*i*) From Eq. (7.37), the hour angle is given by

$$\cos H = \frac{\tan\theta}{\tan\delta} = \frac{\tan 51°30'}{\tan 59°55'16''} = 0.7281386$$

$$H = \mathbf{43°16'9.95''}.$$

(*ii*) From Eq. (7.36), the altitude at elongation is given by

$$\sin\alpha = \frac{\sin\theta}{\sin\delta} = \frac{\sin 51°30'}{\sin 59°55'16''} = 0.9043979$$

or $\alpha = \mathbf{64°44'32.3''}$.

(*iii*) From Eq. (7.35), the azimuth of the star at elongation is given by

$$\sin A = \frac{\cos\delta}{\cos\theta} = \frac{\cos 59°55'16''}{\cos 51°30'} = 0.8051087$$

or $A = 53°37'14.71''$.

Therefore,

Azimuth of AR = Azimuth of star + horizontal angle between the line and the star
= 53°37′14.71″ + 82°30′24″
True bearing of AR = **136°07′38.71″**.
= 360° – 136°07′38.71″
= **223°52′21.29″**.

Example 7.54 Find the azimuth and the hour angle of the sun at sunset for a place in latitude 50°, its declination being 18° S

Solution: (Fig. 7.61)

The altitude if the sun at sun set is zero, i.e., $\alpha = 0°$

In the spherical triangle ZPM, we have

$$z = 90° - \alpha = 90° - 0° = 90°$$

$$p = 90° - \delta = 90° - (-18°) = 108°$$

$$c = 90° - \theta = 90° - 50° = 40°$$

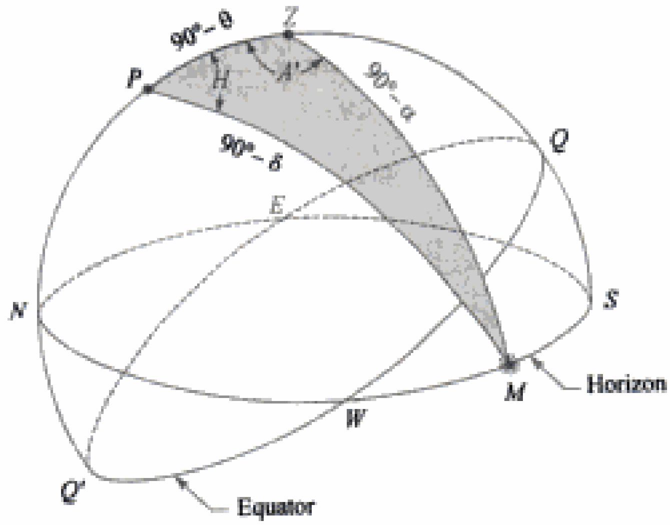


Fig. 7.61

From Eq. (7.9), we have

$$\cos A' = \frac{\cos p - \cos z \cos c}{\sin z \sin c}$$

$$= \frac{\cos 108° - \cos 90° \cos 40°}{\sin 90° \sin 40°} = -0.4807451$$

or $A' = 108°44'2.68''$

Since the sun is in north-west quadrant, its

azimuth = **N 118°44′2.68″ W.**

Again applying the cosine formula Eq. (7.9), we get

$$\cos H = \frac{\cos z - \cos p \cos c}{\sin p \sin c}$$

$$= \frac{\cos 90° - \cos 108° \cos 40°}{\sin 108° \sin 40°} = 0.3872242$$

$H = 67°13.5'19''$

$= \mathbf{4^h 28^m 52.35^s}$.

Example 7.55 A star having declination 60°24′ 11″ N and R.A. $11^h42^m14^s$, was observed at western elongation at a station A in latitude 53°45′N and longitude 54°15′ W. The G.S.T. of G.M.N. was $4^h40^m46^s$. The mean observed horizontal angle between the reference object *R* and the star was 56°36′46ˢ. Find the following:

(*i*) The altitude of the star at elongation.

(*ii*) The azimuth of the line AR.

(*iii*) The local mean time of elongation.

Solution: (Fig. 7.62):

(*i*) When star is at elongation, from Eq. (7.36), we get

$$\sin \alpha = \frac{\sin \theta}{\sin \delta} = \frac{\sin 53°45'}{\sin 60°24'11''} = 0.9274582$$

or $\alpha = \mathbf{68°02'31.18''}$.

(*ii*) The azimuth of a star at elongation is given by Eq. (7.35) as

$$\sin A' = \frac{\cos \delta}{\cos \theta} = \frac{\cos 60°24'11''}{\cos 53°45'} = 0.8352570$$

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Example 7.55 The following observations were made at station *A* to determine the azimuth of the survey line *AB* by the method of ex-meridian observations to the sun.

S.N	Station	Object	Face	Horinzontal circle reading			Vertical circle reading		
				A	B	Mean	C	D	Mean
				° ′ ″	′ ″	° ′ ″	° ′ ″	′ ″	° ′ ″
1.	A	B	L	30 33 19	33 17	30 33 18	—	—	—
			R	210 33 04	33 12	210 33 08	—	—	—
2.		Sun	R	25 52 15	52 10	25 52 13	40 42 12	42 22	40 42 17
			L	205 42 52	43 15	205 43 04	140 17 15	17 12	140 17 14
3.		Sun	L	206 23 38	23 58	206 23 48	140 51 00	50 42	140 50 51
			R	27 41 50	41 30	27 41 40	39 08 00	08 12	39 08 06
4.		B	R	210 33 28	33 32	210 33 30	—	—	—
		—	L	30 33 40	33 40	30 33 40	—	—	—

The latitude and longitude of the station A are 32°46′36″N and 47°49′E, respectively. The declination of the sun at G.M.N. was 1°33′15.6″ N, decreasing at the rate of 60″ per hour. The L.M.T. of the two observations were $3^h12^m10^s$ P.M. and $3^h18^m08^s$ P.M. The chronometer was 3^s slow at noon and gaining 0.9^s per day. The correction for horizontal parallax and refraction are 9.10″cos (apparent altitude) and 50.5″cot (apparent altitude), respectively. The mode of graduations on the vertical circle are given in Fig. 7.63.

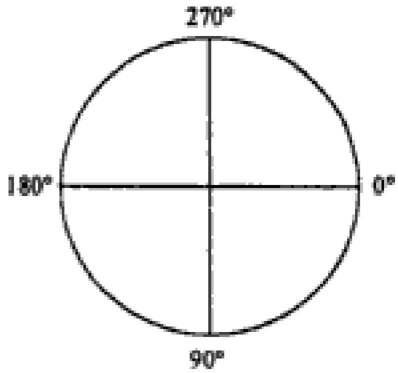

Fig. 7.63

Solution:

Mean of face *L* and *R* readings for S.No. 1

$$= \frac{1}{2} \times (30°33'18'' + 30°33'08'') = 30°33'13''$$

Mean of face *R* and *L* readings for S.No. 4

$$= \frac{1}{2} \times (30°33'30'' + 30°33'40'') = 30°33'35''$$

Mean horizontal reading to *B*

$$= \frac{1}{2} \times (30°33'30'' + 30°33'35'') = 30°33'24''$$

Mean of face *L* and *R* reading for S.No. 2

$$= \frac{1}{2} \times (205°43'04'' + 205°52'13'') = 205°47'39''$$

Mean of face *L* and *R* readings for S.No. 4

$$= \frac{1}{2} \times (206°23'48'' + 207°41'40'') = 207°02'44''$$

Horizontal angle between *B* and the sun at first position

$$= 205°47'39'' - 30°33'24'' = 175°14'15''$$

and when the sun at second position

$$= 207°02'44'' - 30°33'24'' = 176°29'20''$$

Mean horizontal angle between *B* and the sun

$$= \frac{1}{2} \times (175°14'15'' + 176°29'20'') = 175°51'47.5''$$

The vertical angle of the sun at first position

$$= \frac{1}{2} \times [(180° - 140°17'14'' + 40°42'17''] = \mathbf{40°12'32''}.$$

The apparent altitude of the sun in first position = **40°12′32″**.

The vertical angle of the sun at the second position

$$= \frac{1}{2} \times [(180° - 140°50'51'') + 39°08'06''] = 39°08'38''$$

The apparent altitude of the sun in the second position

$$= 39°08'38''$$

Refraction correction for 40°12′32″

$$= 50.5'' \cot 40°12'32'' = 59.74'' \quad (-ve)$$

Refraction correction for 39°08′38″

$$= 50.5'' \cot 39°08'38'' = 62.04'' \quad (-ve)$$

Parallax correction for 40°12′32″

$$= 9.10'' \cos 40°12'32'' = 6.95'' \quad (+ve)$$

Parallax correction for 39°08′38″

$$= 9.10'' \cos 39°08'38'' = 7.06'' \quad (+ve)$$

Corrected altitude of the sun in first position

$$= 40°12'32'' - 59.74'' + 6.95'' = 40°11'39.21''$$

Corrected altitude of the sun in the second position

$$= 39°08'38'' - 62.04'' + 7.06'' = 39°07'43.02''$$

Mean true altitude $= \frac{1}{2} \times (40°11'39.21'' + 39°07'43.02'') = 39°39'41.12''$

The declinations of the sun is computed as below.

Mean time of observation $= \frac{1}{2} \times (3^h\,12^m\,10^s + 3^h\,18^m\,08^s)$

$$= 3^h\,15^m\,09^s$$

$$= 3.2525^h.$$

Correction for L.M.T. @ 0.9^s per day gaining

$$= + \left(3 - \frac{0.9 \times 3.2525}{24}\right) = 2.88^s$$

Correct L.M.T. $= 3^h\,15^m\,09^s + 2.88^s = 3^h\,15^m\,11.88^s$

G.M.T. of observation = L.M.T. – longitude in hours

$$= 3^h\,15^m\,11.88^s - \frac{47°49'}{15}$$

$$= 0^h\,0.3^m 55.88^s = 0.0655222^h$$

The sun's declination at G.M.N. = 1°33′15.6″N

Variation for $0^h\,03^m\,55.88^s$ @ 60″ per hour decreasing

$$= -60'' \times 0.0655222^h$$

$$= -3.93''$$

Declination at the instant of observation

$$= 1°33'15.6'' - 3.93''$$

$$= 1°33'11.67''\text{N}$$

Now, in the astronomical triangle *ZPM*, where *Z* is the zenith, *P* is the north pole, and *M* is the sun, we have

$$ZP = c = 90° - \theta = 90° - 32°46'36'' = 57°13'24''$$

$$ZM = z = 90° - \alpha = 90° - 39°39'41.12'' = 50°20'18.88''$$

$$PM = p = 90° - \delta = 90° - 1°33'11.67" = 88°26'48.33"$$

$$s = \frac{c+z+p}{2} = \frac{1}{2} \times [57°13'24" + 50°20'18.88" + 88°26'48.33"]$$

$$= 98°00'15.6"$$

$$s - c = 40°46'51.6"$$

$$s - z = 47°39'56.72"$$

$$s - p = 9°33'27.27"$$

From Eq. (7.13), the azimuth of the sun is given as

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(s-z)\sin(s-c)}{\sin s \sin(s-p)}}$$

$$= \sqrt{\frac{\sin 47°39'56.72" \times \sin 40°46'51.6"}{\sin 98°00'15.6" \times \sin 9°33'27.27"}}$$

$$= 1.7136563$$

$$A = 119°28'7.71".$$

Since the sun was observed in the afternoon the azimuth of the sun = 119°28′7.71″ W.

Clockwise angle from B to the sun = 175°51′47.5″.

Azimuth of AB from north towards west

$$= 119°28'7.71" + 175°51'47.5"$$

$$= 295°19'55.2".$$

Azimuth of AB from north in clockwise direction

$$= 360° - 295°19'55.2"$$

$$= \mathbf{64°40'4.8"}.$$

Example 7.57 Calculate the approximate direction, east or west of the meridian and the altitude at which the telescope should be pointed to locate the star to make the exact observations on it from the following data:

Latitude of the place	50°20′N
Longitude of the place	20°40′W
Declination of the star to be observed	28°12′N
Right ascension of the star	$16^h08^m25^s$
G.M.T. of observation	17^h18^m
G.S.T. of G.M.M.	$3^h16^m14^s$

Solution: (Fig. 7.64)

To get the L.S.T. of observation of the star the hour angle of the star is required.

$$\text{G.M.T.} = 17^h\,15^m = 17.25^h$$

$$\text{S.I.} = \text{G.M.T.} + \text{acceleration @ } 9.8567^s \text{ per hour of G.M.T.}$$

$$= 17^h\,15^m + (9.8567 \times 17.25)^s$$

$$= 17^h\,17^m\,50.03^s$$

$$\text{G.S.T. of observation} = \text{G.S.T. of G.M.M.} + \text{S.I.}$$

$$= 3^h\,16^m\,14^s + 17^h\,17'50.03^s$$

$$= 20^h\,34^m\,4.03^s.$$

$$\text{L.S.T. of observation} = \text{G.S.T. of observation} + \text{west longitude}$$

$$= 20^h34^m4.03^s + \frac{20°40'}{15}$$

$$= 21^h56^m44.03^s$$

$$\text{H.A. of the star} = \text{L.S.T.} - \text{R.A.}$$

$$= 21^h\,56^m44.03^s - 16^h\,08^m\,25^s$$

$$= 5^h\,48^m\,19.03^s$$

$$= 87°4'45.42"$$

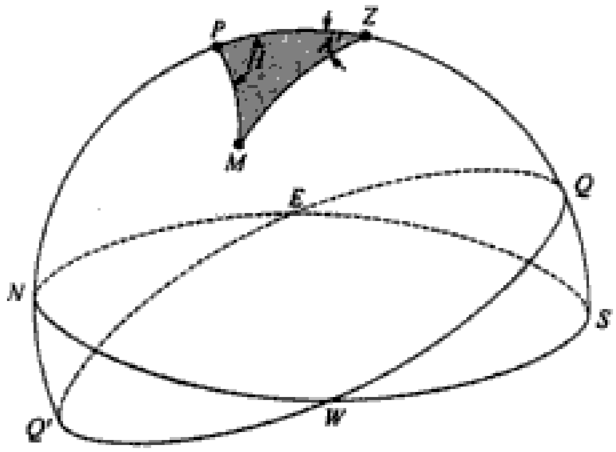

Fig. 7.64

In the astronomical triangle *MPZ*, we have

$$PM = 90° - \delta = p = 90° - 28°12' = 61°48'$$

$$PZ = 90° - \theta = c = 90° - 50°20' = 39°40'$$

$$ZM = 90° - \alpha = z$$

From Eq. (7.9), we get

$$\cos H = \frac{\cos z - \cos c \cos p}{\sin c \sin p}$$

$$\cos z = \cos c \cos p + \sin c \sin p \cos H$$

$$= \cos 39°40' + \cos 61°48' + \sin 39°40' \times \sin 61°48' \times \cos 87°4'45.37''$$

$$= 0.3924203$$

or $$z = 66°53'41.35''$$

$$\alpha = 90° - z = \mathbf{23°6'18.65''}.$$

Now, by sine rule in ΔMPZ, we have

$$\frac{\sin A'}{\sin p} = \frac{\sin H}{\sin z}$$

or $$\sin A' = \frac{\sin p \sin H}{\sin z}$$

$$= \frac{\sin 61°48' \times \sin 87°4'45.42''}{\sin 66°53'41.35''}$$

$$= 0.9569073$$

$$A' = \mathbf{73°7'06.5'' \text{ (West)}}$$

$$A = 360° - 73°7'06.5''$$

$$= 286°52'53.5''. \text{ (in clockwise direction)}.$$

Example 7.58 Calculate the true bearing of the R.M. from a station in latitude 32°30′N and longitude 17°15′E. The angle observed between the R.M. and a star in clockwise direction is 39°12′56″. The R.A. and declination of the star are $12^h 8^m 18^s$ and 17°25′34″N, respectively. The G.M.T. of observation is $18^h 14^m 32^s$ and G.S.T. of G.M.M. is $11^h 14^m 26^s$.

Solution: (Fig. 7.65)

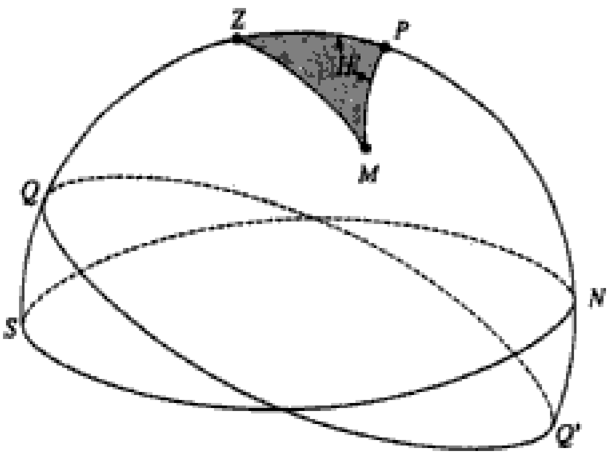

Fig. 7.65

L.S.T. of L.M.M. = G.S.T. of G.M.M. – 9.8567^s per hour of eastern longitude

$$= 11^h14^m26^s - \left(9.8567 \times \frac{17°15'}{15}\right)^s$$

$$= 11^h\,14^m\,14.66^s.$$

L.M.T. of observation = G.M.T. of observation + eastern longitude

$$= 18^h14^m32^s + \frac{17°15'}{15}$$

$$= 19^h\,23^m\,32^s$$

$$= 19.3922222^h.$$

$$\text{S.I.} = \text{L.M.T. of observation + acceleration @ } 9.8567^s \text{ per hour of mean time}$$

$$= 19^h23^m\,32^s + (9.8567 \times 19.3922222)^s$$

$$= 19^h26^m\,43.14^s$$

$$\text{L.S.T. of observation} = \text{L.S.T. of L.M.M.} + \text{S.I.}$$

$$= 11^h14^m14.66^s + 19^h26^m43.14^s$$

$$= 30^h40^m57.81^s$$

$$\text{Hour angle of the star } H' = \text{L.S.T. of observation} - \text{R.A. of the star}$$

$$= 30^h40^m57.81^s - 12^h8^m18^s$$

$$= 18^h32^m39.81^s$$

$$= 278°9'57.12''$$

$$H = 360° - H' = 81°50'2.88'' \quad \text{(smaller H.A. arc)}$$

From Eq. (7.119), the azimuth of the star is given by

$$\tan A = \tan H' \cos\Delta \operatorname{cosec}(\Delta - \theta)$$

where

$$\Delta = \tan^{-1}(\tan\delta \sec H')$$

$$= \tan^{-1}\left[\frac{\tan 17°25'34''}{\cos 81°50'2.88''}\right]$$

$$= 65°39'7.4''$$

$$\tan A = \frac{\tan 81°50'2.88'' \times \cos 65°39'7.34''}{\sin 65°39'7.4'' - 32°30'}$$

$$= 5.2538337$$

$$A = 79°13'24.2''$$

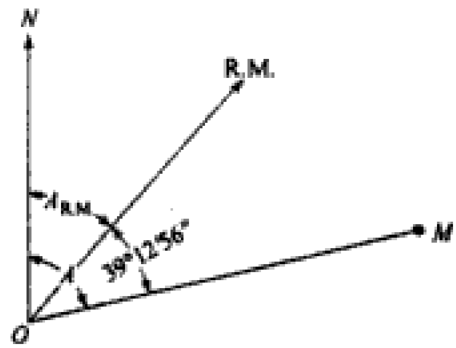

Fig. 7.66

From Fig. 7.66, the azimuth of the R.M. is

$$A_{R.M.} = A - 39°12'56'' = 79°13'24.2'' - 39°12'56''$$
$$= \mathbf{40°0'28.2''}.$$

7.20 DETERMINATION OF LATITUDE

Most of the methods of determining azimuth discussed in Sec 7.19.2, require a knowledge of the latitude of the place. The knowledge of the altitude is also required in determination of local time by means of astronomical observations. Latitude with sufficient accuracy may be scaled off from an accurate map of the area, if available. To get more accurate value, astronomical observations are made using the methods discussed in this section.

7.20.1 General principles of determining latitude

In Sec 7.8.1, it has been proved that the altitude of the celestial pole at any place is equal to the latitude of the place. Since there is no such star which is exactly situated at the poles, the altitude of the star close to the poles (Polaris close to the north pole and σ Octaritis to the south pole) is measured and by applying necessary corrections, the latitude of the place is determined.

The star Almanac gives the accurate angular distances of many stars from the poles (their polar distances which are the compliments of their declinations). If altitude of a star is measured, knowing its angular distance from the pole, the altitude of the pole can be deduced, and hence the latitude.

Most of the commonly used methods of determining latitude depend upon the measurement of altitude and therefore, necessary correction for atmospheric refraction must be applied. To avoid uncertainty in refraction and to get accurate results, pairing of observations at reasonably high latitudes is preferred.

7.20.2 Methods of determining latitude

The following are some of the most commonly employed methods for determining the latitude of a place:

1. Meridian altitude of a star or the sun
2. Zenith pair observations of stars
3. Meridian altitude of a circumpolar star at lower and upper culminations
4. Ex-meridian observation of a star or the sun
5. Transit of a star on the prime vertical
6. Circum-meridian altitude of a star or the sun
7. Altitude of the Polaris.

1. Meridian altitude of a star or the sun method

(a) Stellar observation

The method is based on the fact that the altitude of the pole at a place is equal to the latitude of the place. The altitude of the star is measured when it is at the meridian of the place, i.e., when it is crossing the meridian. For more accuracy in the results, two observations on the same star are made, the face being reversed after the first reading is taken. This is possible with close circumpolar stars when observations are taken with an ordinary 20″ theodolite. The method is not employed when high accuracy is required. The direction of the meridian must be known or must be established before the observations are made.

There can be four cases, as discussed above, arising according to the positions of the star shown in Fig. 7.67. The latitude θ of the place is QZ and the colatitude is ZP.

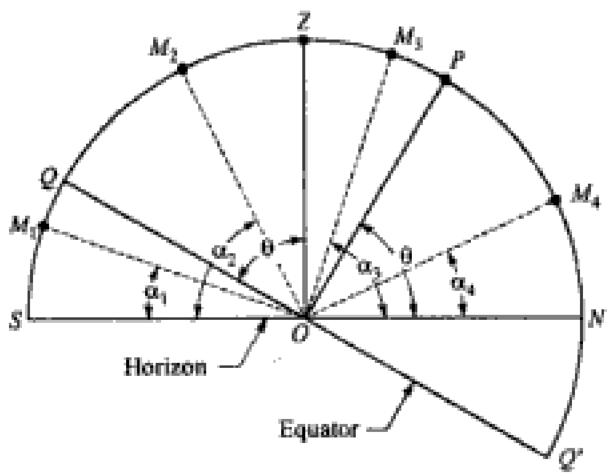

Fig. 7.67 Four positions of star considered in meridian altitude method

Case 1: Star M_1 between the horizon and the equator

In this case,

$$SM_1 = \text{altitude of the star} = \alpha_1$$
$$ZM_1 = \text{zenith distance} = 90^\circ - \alpha_1 = z$$
$$QM_1 = \text{declination of the star (south)} = \delta_1$$

Now QZ = latitude of the place

or $\theta = ZM_1 - QM_1$

$= (90^\circ - \alpha_1) - \delta_1$

or $= z_1 - \delta_1$ ...(7.142)

Hence, latitude = zenith distance − declination.

Case 2: Star M_2 between the equator and the zenith

In this case,

$$SM_2 = \text{altitude of the star} = \alpha_2$$
$$ZM_2 = \text{zenith distance} = 90^\circ - \alpha_2 = z_2$$
$$QM_2 = \text{declination of the star (north)} = \delta_2$$

Now, QZ = latitude of the place

or $\theta = ZM_2 + QM_2$

$= (90^\circ - \alpha_2) + \delta_2$

$= z_2 + \delta_2$ ...(7.143)

Hence, latitude = zenith distance + declination

Case 3: Star M_3 between the zenith and the pole

In this case,

$$NM_3 = \text{altitude of the star} = \alpha_3$$
$$ZM_3 = \text{zenith distance} = 90^\circ - \alpha_3 = z_3$$
$$QM_3 = \text{declination of the star} = \delta_3$$

Now QZ = latitude of the place

or $\theta = QM_3 - ZM_3$

$= \delta_3 - (90^\circ - \alpha_3)$

$= \delta_3 - z_3$ ...(7.144)

Hence, latitude = declination − zenith distance

Case 4: Star M_4 between the pole and the horizon

In this case

$$NM_4 = \text{altitude of the star} = \alpha_4$$
$$ZM_4 = \text{zenith distance} = 90° - \alpha_4 = z_4$$
$$Q'M_4 = \text{declination of the star } \delta_4$$
$$PM_4 = \text{codeclination} = 90° - \delta_4$$

Now NP = altitude of the pole = latitude of place

or
$$\theta = PM_4 + NM_4 = (90° - \delta_4) + \alpha_4$$

Since
$$NM_4 = 90° - ZM_4 = 90° - z_4$$

or
$$\theta = (90° - \delta_4) + (90° - z_4) = 180° - (z_4 + \delta_4) \quad \text{...(7.145)}$$

Hence, latitude = 180° – (zenith distance + declination)

The field procedure involves the determination of the meridian from the azimuth, which has already been discussed. On the ground two pegs are fixed at considerable distance apart to define the meridian. The theodolite is set up on the southern peg if the star is in the north otherwise on the northern peg. The distant peg is accurately bisected and both the plates are clampped. The telescope is rotated in the vertical plane till the star is bisected by the horizontal hair. Both the verniers of the vertical circle are read and mean of the readings is taken to obtain the apparent altitude of the star on face left. The apparent altitude is again determined for face right in the similar manner. The altitude of the star is the mean of the two altitudes. It is corrected for refraction to obtain the correct altitude. The declination of the star is obtained from the star Almanac and the latitude is computed from Eq. (7.142) to (7.145) which ever is applicable.

Though the method is quite convenient, it has the following disadvantages:

1. During the interval of changing the face, the star moves out of the meridian and it affects the results.
2. The direction of meridian needs to be determined before making actual observations and it increases the field work.

(b) Solar observation

The altitude of the sun is observed at local apparent noon, i.e., when the sun is at upper transit. The line of sight of the theodolite is placed in the plane of the meridian and the altitude of the sun is observed when the upper or lower limb of the sun touches the horizontal cross hair keeping the sun on the vertical cross hair.

Corrections for instrumental errors, refraction, parallax, and semi-diameter are applied to the observed value to get the correct altitude of the sun. The mean time of observation is also noted. Knowing the altitude and the declination of the sun at the instant of observation, the latitude of the place is computed as below.

In Fig. 7.68, M is the position of the sun.

$$SM = \text{meridian altitude of the sun} = \alpha$$
$$ZM = \text{meridian zenith distance of the sun} = 90° - \alpha = z$$
$$QM = \text{declination of the sun} = \delta$$
$$QZ = \text{latitude of the place} = \theta$$

From Eq. (7.143), we get

$$\theta = z \pm \delta \quad \text{...(7.146)}$$

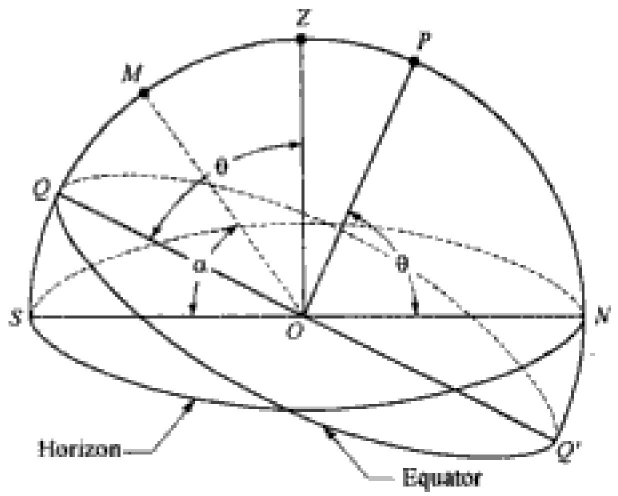

Fig. 7.68 Meridian altitude of the sun

In Eq. (7.146), δ is positive when the sun is to the north of the equator and negative when to the south of the equator.

In order that the observer may be ready for taking the observations at the meridian transit, the standard time or the watch time of the local apparent noon must be known. The standard time of local apparent noon varies throughout the year. The standard time can be determined by applying a correction for the difference in longitude between the local meridian and the standard meridian.

2. Zenith pair observations of stars method

This method is an improvement over the meridian altitude method. The errors of observation, refraction, and instrumental errors can be considerably reduced by making observations upon a pair of stars M_2 and M_3 (Fig. 7.67) which culminate at approximately equal altitude but on opposite sides of the zenith.

The meridian altitude of one of the stars M_2 is observed at its culmination. Then the telescope is rotated in azimuth and the meridian altitude of the other star M_3 is observed. The right ascensions of the two selected stars for observations, should not differ by more then 10 to 30 minutes.

The time of culmination of these two stars will then differ by 10 to 30 minutes giving sufficient time to the observer in observing the second star after taking the observations on the first star.

From Eq. (7.143) for M_2 and Eq. (7.144) for M_3, we have

$$\theta = z_2 + \delta_2 = (90° - \alpha_2) + \delta_2$$

and

$$\theta = \delta_3 - z_3 = \delta_3 - (90° - \alpha_3)$$

$$\text{Average latitude} = \frac{1}{2}[(90° - \alpha_2) + \delta_2 + \delta_3 - (90° - \alpha_3)]$$

$$= \frac{\alpha_3 - \alpha_2}{2} + \frac{\delta_3 + \delta_2}{2} \qquad \text{...(7.147)}$$

From the above expression it is found that the average latitude depends upon the difference in altitudes of the two stars and not on their individual altitudes. Hence any error in the correction for refraction will be common to both the altitudes (which will be approximately equal) and will be eliminated by taking the difference of the two altitudes. If both the stars are at equal altitude, the error in the correction for refraction will be eliminated completely. Similarly, the instrumental errors are also largely eliminated because these will be practically the same for both the observations.

If should be noted that the face of the instrument is not changed when reading the altitude of the second star. To take the reading for the meridian altitude, the telescope is directed along the meridian line established on the ground by two pegs and the altitude is measured when the star intersects the vertical hair as in the case of the method of meridian altitude.

3. Meridian altitude of a circumpolar star at lower and upper culminations

In this method, the altitude of a circumpolar star is observed at its lower as well as at its upper culminations. The method is based on the fact that the north polar distance of a circumpolar star at lower culmination is equal to its north polar distance at upper culmination, i.e., the mean altitude of the circumpolar star at upper and lower culminations is equal to the altitude of the pole and hence equal to the latitude of place.

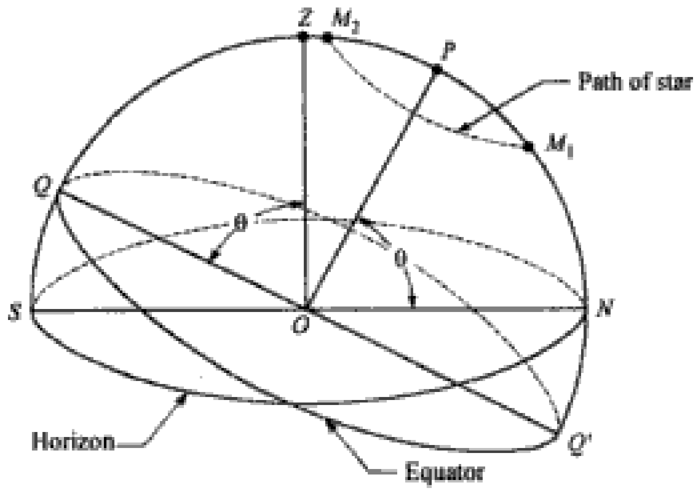

Fig. 7.69 Meridian altitude of a circumpolar star at lower and upper culminations

In Fig. 7.69, M_1 and M_2 are the two positions of a circumpolar star at lower and upper culminations, respectively. The altitude NOP of the pole P is equal to the latitude of the place.

Let NM_1 = altitude of the star at lower culmination = α_1

NM_2 = altitude of the star at upper culmination = α_2

NP = altitude of the pole = latitude of the place = θ

Now $NP = NM_1 + M_1P$

Also $NP = NM_2 - M_2P$

or $2NP = NM_1 + NM_2 + (M_1P - M_2P)$

Since M_1P and M_2P are equal to the codeclination of the star, we have

$$M_1P = M_2P$$

$$NP = \theta = \frac{NM_1 + NM_2}{2}$$

or
$$\theta = \frac{\alpha_1 + \alpha_2}{2} \qquad ...(7.148)$$

Hence, the latitude of the place of observation is equal to half the sum of the altitudes of the circumpolar star at its lower and upper culminations.

The field observations involve determining the meridian of the place and selecting a suitable circumpolar star whose both culminations occur within night. The meridian altitudes of the star at its lower and upper culminations are observed by the method already discussed in the previous methods of determining meridian altitude.

The method is generally not preferred due to the following reasons:

1. The method takes long time because 12 sidereal hours elapse between two observations.
2. If the duration of night is less than 12 hours, one of the culminations of the star will be in day hours.
3. The refraction may change in the time interval between the two observations and, therefore, there may be errors due refraction resulting into inaccurate results.

$$= \frac{R_N - R_S}{2} d \qquad ...(7.158)$$

where R_N = mean reading of north side of bubble

R_S = mean reading of south side of bubble

d = value of one division of the graduations on the bubble tube.

In Fig. (7.157), negative sign is used when Z_1 and P are to the same side of Z and positive sign when Z_1 and P are to the opposite side of Z.

Thus, if the south end of the axis is higher than the north end, Z_1 and P will be to the same side of Z and the level correction ZZ_1 should be subtracted from the calculated value of the latitude to get the true value and vice-versa.

6. Circum-meridian altitude of a star or the sun method

The circum-meridian observations are made when the star or the sun is near the meridian. The method is very accurate. The circum-meridian altitudes of several stars are made for a few minutes before and after transit and reducing them to the mean meridian altitude. The errors due to erroneous value of refraction, personal error, and instrumental errors, are very much reduced by observing an equal number of north and south stars in pairs of similar altitude. Accurate chronometer time and its error are also essential to calculate the hour angles of the individual stars.

The observations of a star is commenced about 10 minutes before the expected time of transit and it is continued for about 10 minutes after the transit. Equal number of the face left and face right observations are taken. However, the observations may be taken only on one face if observations are adequately paired on the north and south stars.

In Fig. 7.73, M is the position of a star on the circum-meridian and M' on the meridian. From the astronomical triangle PZM, using Eq. (7.8), we get

$$\cos z = \cos p \cos c + \sin p \sin c \cos H' \qquad ...(7.159)$$

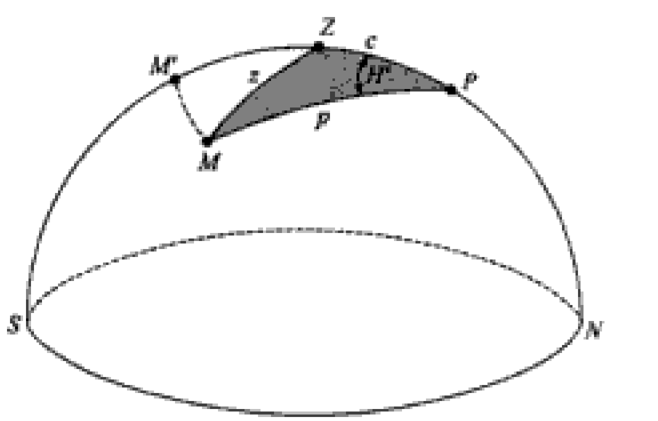

Fig. 7.73 Circum-meridian observation on a star

Let a be the required correction to the observed zenith distance z to obtain the meridian zenith distance when the star is at M'. Therefore,

$$\text{meridian zenith distance} = M'Z = z - a \qquad ...(7.160)$$

When the star is at M',

$$M'P = p$$

$$ZP = c$$

and $$M'Z = M'P - ZP = p - c \qquad ...(7.161)$$

Therefore, $$z - a = p - c$$

Solution: (Fig. 7.67)

For M_3 position of the star, from Eq. (7.144), we have

$$\theta = \delta - z$$

The corrected altitude for refraction from Eq. (7.74) is

$$\alpha = 67°22'44'' - 58'' \cot 67°22'44''$$
$$= 67°22'19.83''$$

Now $$\theta = 49°33'12'' - (90° - 67°22'19.83'')$$
$$= \mathbf{26°55'31.83''\ N}.$$

Example 7.60 The meridian altitude of the sun's lower limb was observed to be 41°12′26″ at place in longitude 72°20′45″ W to determine the latitude of the place. The sun was to the south of the zenith. The declination of the sun at 6 G.A.N. on the day of observation was 19°38′52″N, increasing 7.46″, per hour and its semi-diameter 16′14.24″. Determine the latitude of the place.

Solution: (Fig. 7.68)

The sun's position M is to the south of zenith.

From Eq. (7.146), since the sun is north of the equator, taking positive sign, we get

$$\theta = z + \delta$$

where z and δ are the corrected zenith distance and declination of the sun.

Corrections for the zenith distance or altitude of the sun

(*i*) Correction for refraction $= -58''.\cot(41°12'26'') = -66.24''$

(*ii*) Correction for parallax $= +8.8.\cos(41°12'26'') = +6.62''$

(*iii*) Correction for semi-diameter $= +16'14.24''$

(taken positive since the sun's lower limb was observed)

Total correction $= -66.24'' + 6.62'' + 16'14.24'' = +15'14.62''$

Therefore, corrected altitude $= 41°12'26'' + 15'14.62'' = 41°27'40.62''$

Corrected zenith distance $z = 90° - 41°27'40.62'' = 48°32'19.38''$

Now the L.A.N. is zero when the sun is over the meridian, therefore L.A.T. is $0^h0^m0^s$.

G.A.T. of observation = L.A.T. of observation + longitude (W)

$$= 0^h0^m0^s + \frac{72°20'45''}{15}$$
$$= 4^h\,49^m\,23^s$$

The declination of the sun at G.A.N. = 19°38′52″N

Declination decreasing @ 7.46″ per hour, therefore, the declination of the sun at L.A.N.

$$= 19°38'52'' + 7.46'' \times (4^h49'23'')$$
$$= 19°39'27.98''$$

Thus, latitude $$\theta = 48°32'19.38'' + 19°39'27.98''$$
$$= \mathbf{68°11'47.36''}.$$

Example 7.61 A star is to be observed at its lower and upper culminations at a place in approximate latitude of 75°S. The star to be observed has declination of 48°28′16″ S. Find the approximate apparent altitudes of the star at which it should be sighted in order that accurate observations may be taken upon it.

Solution: (Fig. 7.75)

In the figure the star has been shown in the southern hemisphere where P' is the south pole and Z' is the zenith of the observer. The angles α_1 and α_2 are the apparent altitudes of the star at its upper transit on north side for M_1 position and at lower transit on the south side for M_2 position, respectively.

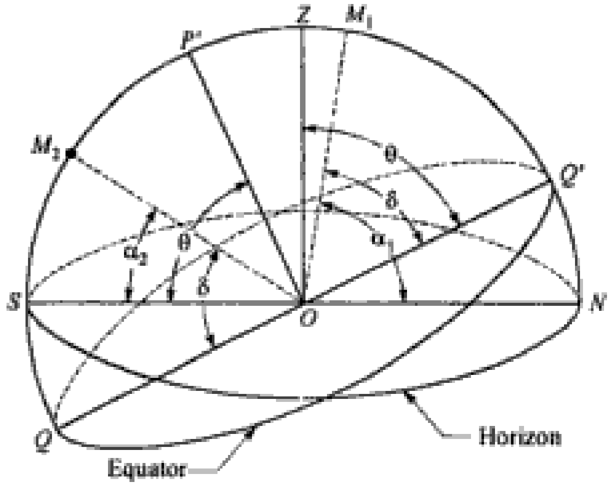

Fig. 7.75

Now $\angle NOM_1 = \alpha_1 = NOZ' - M_1OZ'$

$= 90° - (Q'OZ - Q'OM_1)$

$= 90° - (\theta - \delta)$

$= 90° - \theta + \delta$

$= 90° - 75° + 48°28'16''$

$= \mathbf{63°28'16''\ N.}$

Similarly, $\angle SOM_2 = \alpha_2 = SOP' - M_2OP'$

$= \theta - (QOP' - QOM_2)$

$= \theta - (90° - \delta)$

$= 75° - (90° - 48°28'16'')$

$= \mathbf{33°28'16''\ S.}$

Example 7.62 The altitudes of a star at upper and lower culminations were observed as 61°36′30″ and 20°24′40″, respectively, at a place in north latitude. These values are corrected for refraction. Find the latitude of the place and the declination of the star.

Solution: (Fig. 7.69)

From Eq. (7.148), we have

$$\text{Latitude } \theta = \frac{\alpha_1 + \alpha_2}{2}$$

$$= \frac{61°36'30'' + 20°24'40''}{2}$$

$$= \mathbf{41°0'35''\ N.}$$

$$\text{Declination } \delta = QM_2 = QZ + ZM_2$$

$$= \theta + (90° - \alpha_1)$$

$$= 41°0'35'' + 90° - 61°36'30''$$

$$= \mathbf{69°24'05''}.$$

Also, declination on $\delta = QM_2 = QP - PM_2$

$$= 90° - PM_1$$

$$= 90° - (\theta - \alpha_2)$$

$$= 90° - (41°0'35'' - 20°24'40'')$$

$$= \mathbf{90°24'05''}. \text{ (Check)}$$

Example 7.63 Find the latitude of the place where the following data were collected by making observations on a star:

Corrected altitude of the star $= 38°12'40''$

Declination of the star $= 11°23'30''$N

Hour angle of the star $= 44°34'10''$

Solution: (Fig. 7.70):

The latitude θ is given by Eq. (7.154) as

$$\cos(\theta - n) = \sin\alpha . \sin n . \operatorname{cosec}\delta \quad \text{...(a)}$$

From Eq. (7.152), we have

$$\tan n = \tan\delta . \sec H$$

$$= \tan 11°23'30'' \times \sec 44°34'10''$$

$$= 0.2828241$$

or $n = 15°47'31.86''$

From Eq. (a), we have

$$\cos(\theta - n) = \sin 38°12'40'' \times \sin 15°47'31.86'' \times \operatorname{cosec} 11°23'30''$$

$$= 0.8522941$$

or $\theta - n = 31°32'16.53''$

or $= 31°32'16.53' + n$

$$= 31°32'16.53' + 15°47'31.86''$$

$$= \mathbf{47°19'48.39''\ N}.$$

7.21 DETERMINATION OF LONGITUDE

The latitude and longitude of a place on the earth's surface are the two coordinates of its position. When these coordinates are known, the position of the place relative to other places is known.

The longitude of any place with respect to another place is the difference between the meridians of the two places measured in an arc on the equator. Longitudes are reckoned east or west of the fixed meridian up to 180° when expressed in arcs or up to 12 hours when expressed in time. The fixed reference meridian used by most nations, is Greenwich. The longitude of Greenwich meridian is zero.

Since the difference in longitudes of two places is equal to the difference in their local times at the same instant, the longitude of a place can be determined by determining the local time (mean or sidereal) at the place and subtracting it from the Greenwich time (mean or sidereal) at the same instant. The methods of determining the local time has already been discussed in Sec. 7.14. However, the finding of the Greenwich time at the instant of observation is the main task of the longitude determination. If the local time is ahead of the Greenwich time, the place is to the east of the Greenwich meridian, and if it is behind the Greenwich time, the place is west of the Greenwich meridian.

Some of the commonly used methods for determining the longitude, are as follows:

1. Transportation of chronometer method
2. Electric telegraph method
3. Wireless signals method.

The following methods of determining the longitudes, being of historical interest, are not in use, and will not be discussed:

1. Observations on the moon and the stars which culminate at the same time method
2. Celestial signals method
3. Lunar distances method.

7.21.1 Transportation of chronometer method

In this method, the chronometer time at the instant of making the observations for the local time, is recorded. The chronometer reading is then corrected for its time and rate. To make these corrections, the chronometer should be previously compared with Greenwich time to find its error and rate. Thus, at the instant of observations, the correct Greenwich time is known. The calculated local time is then compared with that of the Greenwich chronometer time to find the longitude of the place of observations.

Chronometer is a delicate instrument and should be handled very carefully. The main difficulty arises from the fact that its rate while being transported is not the same as when stationary. Therefore, the travelling rate of the chronometer when being transported should be ascertained for precise results.

Let us consider two stations A and B situated at different longitudes. The chronometer is regulated to give the time of station A. The rate of the chronometer, i.e., the amount by which it gains or loses in 24 hours, is found at A. The same chronometer is then transported to the station B whose longitude is to be determined and the chronometer error is determined with reference to the meridian of the station B. If the chronometer runs perfectly the two chronometer corrections will differ by just the difference in longitudes of A and B.

This method is rarely used by land surveyors now these days. However, it is still in use for the determination of longitudes at sea.

7.21.2 Electric telegraph method

If the two places are connected by an electric telegraph line, the longitude can be determined very accurately by sending telegraphic signals in opposite directions for the determination of the local chronometer time.

Let A and B be the two stations, A being to the east of B. If t_1 is the local time of A at which the signal is sent from A to B and t_2 is the local time of B at which the signal is received at B, neglecting the transmission time, the difference in longitude considering t_1 being greater than t_2, is given as

$$\Delta\phi = t_1 - t_2. \qquad ...(7.169)$$

If the transmission time s of the signals is considered, $(t_1 + s)$ will be the actual local time of A when the signals are received at B at its local time t_2. The difference of longitudes, in this case, will be

$$\Delta\phi = (t_1 + s) - t_2 = (t_1 - t_2) + s \qquad ...(7.170)$$

Similarly, when the signals are sent from B to A, we have

$$\Delta\phi = t_1' - (t_2' + s) = (t_1' - t_2') - s \qquad ...(7.171)$$

By averaging the two results, we have

$$\Delta\phi = \frac{1}{2}[(t_1 - t_2) + (t_1' - t_2')] \qquad ...(7.172)$$

7.21.3 Wireless signals method

The carrying of the time of the reference meridian has become comparatively easy and more accurate using wireless signals which are sent out regularly from various wireless stations at regular stated intervals. The surveyor can check his chronometer utilizing the wireless signals in almost any part of the world. A list of wireless signals, their time, duration of emission, wavelengths, and type of signals are published annually, in the *Admiralty List of Wireless Signals* and changes or any corrections are notified in the weekly *Notices to Mariners*.

The Greenwich mean time signals are sent and usually continue for a period of 5 minutes. The signals are rhythmic and consist of a series of 61 Morse dots to the minutes. The beginning and end of each minute is denoted by a dash which is counted as zero of the series which follows.

ILLUSTRATIVE EXAMPLE

Example 7.64 At a station *A* on July 14, 1930, the chronometer at 8 P.M. was found to be $2^m 6.5^s$ fast and it was gaining at the rate of 2.58^s in 24 chronometer hours. At another station B on July 15, 1930, the local time was determined by astronomical observations as $9^h 12^m 35^s$ P.M. when the chronometer reading was $9^h 12^m 30.6^s$. Find the difference in longitudes of A and B.

Solution:

Chronometer reading of *A* on July 14, 1930, $= 8^h + 2^m 6.5^s$

$= 8^h 2^m 6.5^s$

Chronometer reading at *B* on July 15, 1930 $= 9^h 12^m 30.6^s$

Time interval $= 24^h + (9^h 12^m 30.6^s - 8^h 2^m 6.5^s)$

$= 25^h 10^m 24.1^s$

Chronometer is gaining at the rate of 2.58^s in 24 hours. Therefore, time gain for $25^h 10^m 24.1^s$

$$= \frac{2.58^s \times (25^h 10^m 24.1^s)}{24} = 2.71^s$$

The chronometer was fast at *A* by $2^m 6.5^s$.

The local time at *A* corresponding to the local time $9^h 12^m 35^s$ at *B* will be

$= 9^h 12^m 30.6^s - (2^m 6.5^s + 2.71^s)$

$= 9^h 10^m 21.39^s$

Therefore, the difference in the longitudes of *A* and *B* is

$\Delta\phi = 9^h 12^m 35^s - 9^h 10^m 21.39^s$.

$= 2^m 13.61^s$

$= \mathbf{0^\circ 33' 24.15''}$.

7.22 IDENTIFICATION OF STARS

Owing to the distortion due to the projection used in the construction of a star chart, the latter cannot always satisfactorily show the relative positions of stars when they are considerable distance apart. A few notes and hints given below with explanatory figures may, therefore, prove useful (Fig. 7.76).

1. Some groups of stars can always be readily identified by well-defined shapes formed by them in the sky.

Some of these shapes are :

(*i*) the belt of Orion
(*ii*) the W of Cassiopeia
(*iii*) the plough of the Great Bear
(*iv*) the sickle of Leo
(*v*) the bright cluster of Pleiades, etc.

2. Another method is to join some of the bright stars by imaginary lines to form simple geometrical figures.

(*i*) The portion of the constellation Ursa Major which is known as 'the Plough', 'consists of seven stars' of which α and β are known as, the pointers, since they point towards α Ursa Minoris is known as Polaris.
(*ii*) A line drawn from the pole star perpendicular to the line from the pointers to the polaris, passes through Cappella Auriga.
(*iii*) If the above line is produced in the opposite direction, it will pass to Vega (α Lyrae).
(*iv*) If the line from the pointers through the pole star is produced, it will strike the centre of the 'Square of Pegasus' formed by β, α and γ Pegari, α Andromeda.

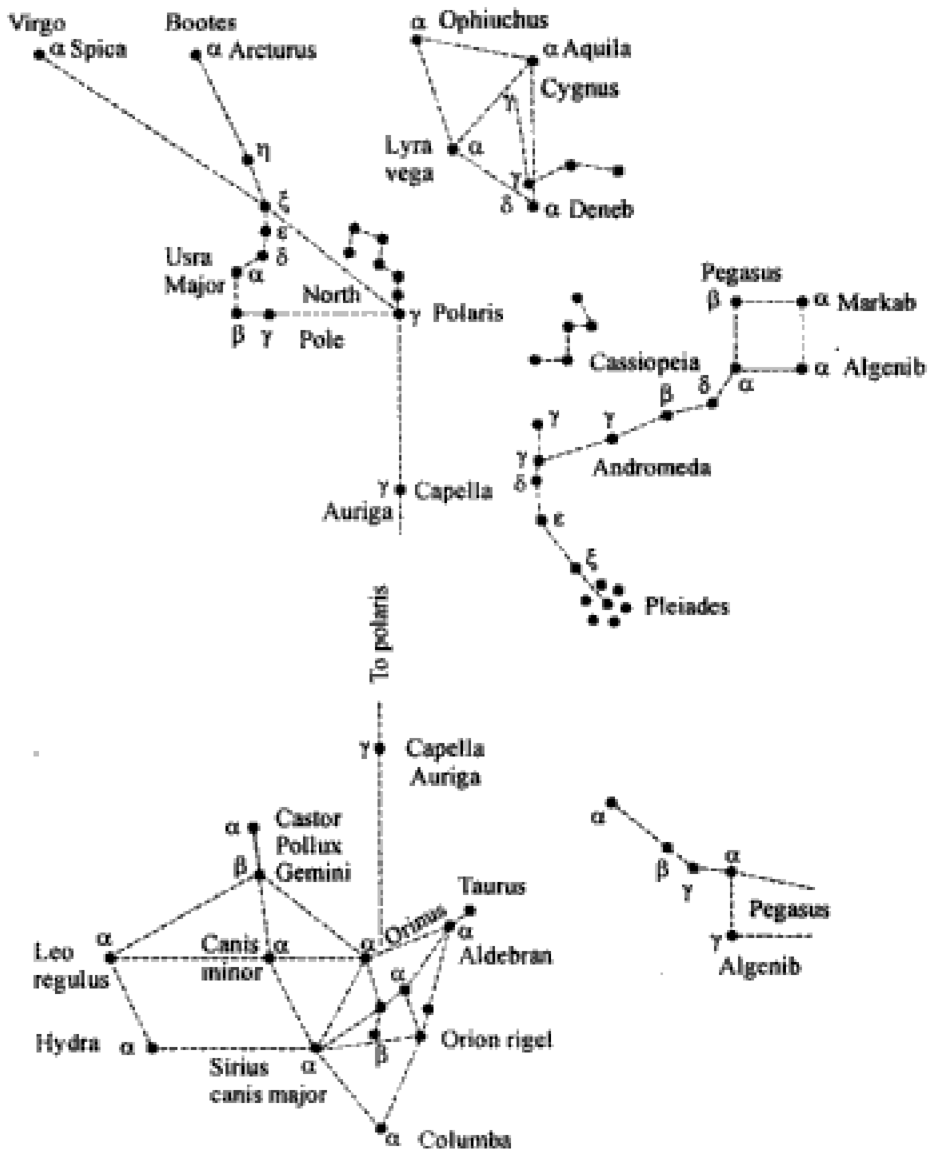

Fig. 7.76 Identification of stars

(*v*) The constellation Cassiopeia whose five brightest stars form the shape of *W*, lie midway between the pole star and the 'Square of Pegasus'.

(*vi*) Between the Square of Pegasus and Vega, and slightly closer to the latter, lies Deneb, the brightest star in the constellation Cygnus which is shaped like a cross with two short and long arms.

(*vii*) The constellation of Perseus lies in a curve reaching to the cluster of the Pleiades.

(*viii*) The constellation of Orion is one of the most conspicuous in the sky. There are three stars in the middle and one star on either side of these three stars. The three stars are known as the 'belt of the Orion'.

(*ix*) The three stars in the belt point upwards to the right to a red star called Aldebaran in the belt of Taurus, the Bull. In the lower direction they point to Sirius (α Canis Majoris), the brightest star in the sky.

(*x*) Batelgeuce (α Orinus) and Sirius (α Canis Majoris) form an equilateral triangle with Procyon (α Canis Minoris).

(*xi*) Procyon and Balegeuce with Pollux (β Gemini) form a right-angled triangle-right angle at Procyon. Castor (α Gemini) lies close to Pollux.

7.42 A star at longitude of 71°15′E transits at $9^h 12^m 25^s$ P.M., recorded with a chronometer keeping Indian standard time. If G.S.T. at G.M.M. on the day of observation was $14^h 36^m 24^s$ and R.A. of the star was $10^h 42^m 17.25^s$, determine the chronometer error.

7.43 A star of declination 82°06′30″N was observed at east elongation when the clockwise horizontal angle from a R.M. was 110°24′50″. Immediately afterwards another star of declination 75°42′20″N was observed at east elongation and the clockwise horizontal angle measured was 125°42′40″. What is the azimuth of the R.M.?

7.44 The Polaris having declination of 89°03′54.9″N was observed west of the meridian when the anticlockwise angle from a R.M. was 28°49′38″. The angle of elevation corrected for refraction was 30°50′19″ and the latitude of the place of observation was 39°19′25″. Calculate the azimuth of the R.O.

7.45 In northern hemisphere in longitude 76°30′E, the observed altitudes of a circumpolar star at lower and upper culminations were 25°37′15″ and 35°24′40″, respectively. Calculate the latitude of the place. (Assume: Refraction in seconds = 58″ × tangent of apparent zenith distance = 58″ × cotangent of apparent altitude).

7.46 An observation was made at a place in longitude 70°20′15″W to find the latitude of the place. The meridian altitude of the sun's lower limb was observed to be 44°12′30″, the sun being to the south of the zenith. Sun's declination at G.A.N. was + 22°18′12.8″, increasing at 6.82″ per hour, and semi-diameter of the sun 15′45.86″. Determine the latitude of the place of observations.

8

MAP PROJECTIONS

8.1 GENERAL

A map projection is a means of representing the lines of latitude and longitude of the globe on a flat sheet of paper. Any such representation is a projection.

In plotting a map of a small and limited area, the curvature of the earth need not be considered. Large scale maps of such areas, are generally plotted by means of a system of rectangular coordinates, considering the level surface of the earth to be a plane.

For maps of larger area, usually on smaller scales, the above simple system of rectangular coordinates, is not satisfactory because the curvature of the earth can no longer be ignored. The geometric shape of the earth is a spheroid with a polar diameter about one-third of 1 per cent shorter than the equatorial diameter. Therefore, a plane passing through the equator would cut it in a circle while a plane through the poles intersects it in an ellipse. However, with the slight difference between polar and equatorial dimensions, the ellipse is very nearby a circle. Consequently the earth is assumed to be a sphere to make easier to visualize the map projections discussed here.

Regardless of whether the earth is considered to be a sphere or spheroid, it is not possible to develop its surface exactly on to a plane. Whatever procedure is used to represent a large area on a map, there will always be some distortion. To minimize the distortion as well as to develop several possibilities, points on the map are represented in terms of parallels of latitude and meridians of longitude. Position on the earth in terms of latitude and longitude, is transformed into scaled linear dimensions on the map. This is accomplished by using the dimensions of the earth and a selected set of criteria for representing the curved earth on the flat map. Such a transformation from latitude and longitude to a map's rectangular coordinates, is the function of map projections.

Although, it is impossible to make a correct map of any part of the earth, it is by no means difficult to maintain certain definite qualities in a projection. These qualities are as follows:

1. Preservation of area
2. Preservation of shape (orthomorphism)
3. Preservation of scale
4. Preservation of bearing
5. Ease of drawing.

On an ideal map without distortion, the following conditions must be satisfied:

1. All distances and areas on the map would have correct relative magnitude.
2. All azimuths and angles would be correctly shown.
3. All great circles on the earth would appear as straight lines.
4. Geodetic latitudes and longitudes of all points would be correctly shown.

It is impossible to satisfy all of these conditions because of the shape of the earth, in the same map. It is possible, however, as stated above, to satisfy one or more of the four conditions.

Different projection systems satisfy different conditions as below:

1. *Conformal projection* results in a map showing the correct angle between any pair of short intersecting lines, thus making small areas appear in correct shape. As the scale varies from point to point, the shapes of larger areas are incorrect.
2. *Equal-area projection* results in a map showing all areas in proper relative size although these areas may be much out of shape and the map may have other defects.
3. *Equidistant projection* represents distances correctly, from one central point to other points on the map.
4. *Azimuthal projection* shows the correct direction or azimuth of any point relative to one central point.

8.2 SCALE FACTOR

If the radius of the globe is R and that of the earth is r, we have:

$$\text{R.F.} = \frac{R}{r}$$

where R.F. = representative fraction

Taking $r = 6370$ km $= 6.37 \times 10^8$ cm and R in centimetres, we have

$$\text{R.F.} = \frac{R}{6.37 \times 10^8}$$

The scale defined above is also called *principal scale.*

The *scale factor* is defined as the ratio of the map distance to the globe distance.

Thus,

$$\text{Scale factor} = \frac{\text{map distance}}{\text{globe distance}}$$

When the surface of the generating globe is projected on a sheet of paper for preparing a map, some dimensions undergo further change. If a certain line becomes double when projected from the globe to the map, the scale factor is said to be 2.0. However, there are always one or more lines which remain unchanged in length, and for such lines the scale factor is 1.0 or unity. These lines are called *standard lines.*

For various points on a map, the scale factor is generally not constant. It varies from point to point. It may also vary in different directions at the same point. Thus the scale of a map is not uniform for the entire map. This is in contrast to a plan which has a uniform scale.

The principal scale is applicable to the generating globe and for the points on the map which have a scale factor of unity. At all other points, there is distortion and the scale is either less than the principal scale or greater than the principal scale.

8.3 GEOMETRY OF THE SPHERE

Since the shape of the earth is assumed to be spherical, the study of the geometry of the sphere will help in understanding the various projection systems. In Fig. 8.1, the sphere shown has its radius R and it has the following properties.

1. Circumference of the sphere $= 2\pi R$ ...(8.1)
   = length of the equator
   = length of the meridian circle
2. Length of the parallel = length of the equator × $\cos\theta$
   $= 2\pi R \cos\theta$ ...(8.2)

where θ is the latitude.

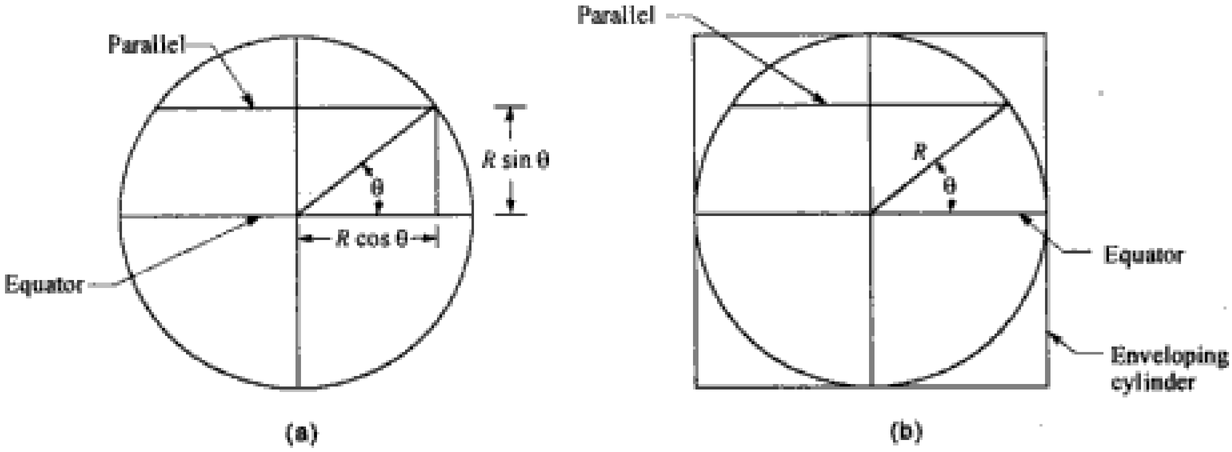

Fig. 8.1 The sphere and an enveloping cylinder

Thus, length of any parallel is less than the length of the equator. It depends upon the latitude of the place.

(*i*) At equator, $\theta = 0°$

Length of the parallel = length of the equator

(*ii*) At poles, $\theta = 90°$

Length of the parallel = 0

3. The partial length of the parallel falling between two meridians also depends upon the latitude. For example, the distance on the earth between meridians through 60°*E* and 90°*E*, is maximum at the equator. It is given by

$$\frac{30°}{360°}\times 2\pi R = \frac{30°}{360°}\times 2\pi R \times 6370 = 3335.32 \text{ km}$$

The partial length of the parallel at a latitude θ is

$$\frac{30°}{360°}\times 2\pi R\cos\theta = 3335.32\cos\theta \text{ km}$$

The partial length of the parallel for latitude 20° is

$$3335.32\cos(20°) = 3134.18 \text{ km}$$

The partial length 3134.18 km on a map of scale having R.F. as 1×10^{-8} will be $3134.18\times 10^5\times 10^{-8}$ = 3.13418 cm. If the scale factor is, say, 1.20, the map distance will be 3.13418 × 1.2 = 3.76102 cm.

4. Surface area of the sphere $= 4\pi R^2$ ...(8.3)

= surface area of an enveloping cylinder having its diameter and height as $2R$

$= (2\pi R)\times 2R$

5. (*i*) Surface area of the zone between the equator and the parallel of latitude θ

= Area of the strip on the enveloping cylinder of the height $R\sin\theta$

$= (2\pi R)(R\sin\theta) = 2\pi R^2\sin\theta$ ...(8.4)

(*ii*) Surface area of the zone between the parallel of latitude θ and the pole

= Surface area from equator to the pole – surface area of the zone between the equator and the parallel of latitude θ

$= 2\pi R^2 - 2\pi R^2\sin\theta$

$= 2\pi R^2(1-\sin\theta)$ ...(8.5)

(*iii*) Surface area between two parallels of latitudes θ_1 and θ_2.

$= 2\pi R^2\sin\theta_1 - 2\pi R^2\sin\theta_2$

$= 2\pi R^2(\sin\theta_1 - \sin\theta_2)$ ...(8.6)

6. Shortest distance between two points A and B whose latitudes are θ_A and θ_B and the difference in longitude is $\Delta\phi$ is

$$\frac{2\pi R}{360^\circ}.D \quad ...(8.7)$$

where D called central angle, is given by

$$\cos D = \sin\,\theta_A \sin\,\theta_B + \cos\,\theta_A \cos\theta_B.\,\Delta\phi \quad ...(8.8)$$

It may be noted that the shortest distance between two points on the sphere is the distance between the points along the great circle.

7. The route of a constant bearing (or azimuth) on a sphere is known as *loxodrome* or *rhumbline*. It crosses every meridian at the same angle. All parallels and meridians are also loxodromes.
8. A small quadrangle on a sphere resembles a trapezoid, but actually it is a curved surface. A 1° quadrangle is the area formed by two parallels and two meridians 1° apart.

8.4 GEOMETRY OF THE CONE

Knowing the geometry of a cone, also helps in map projections. In some map projections a parallel of latitude is drawn as an arc of the circle. The angle of convergence $\Delta\phi$, is 360° for a full circle. On a map of a small area of the earth, this angle is less than 360°.

Fig.8.2 shows a cone with a central angle ω. This angle is related to $\Delta\phi$, but generally it is not equal to it. The central angle in terms of $\Delta\phi$ is given by

$$\omega = k\,\Delta\phi \quad ...(8.9)$$

where k is the constant of the cone. It depends upon the latitude θ of the place, and is expressed as

$$k = \sin\theta \quad ...(8.10)$$

For example, for the complete sixtieth parallel ($\theta = 60°$), shown on a map as a circular arc, the central angle would be

$$\omega = k\,(360°)$$
$$= \sin(60°) \times (360°)$$
$$= 311.769°$$

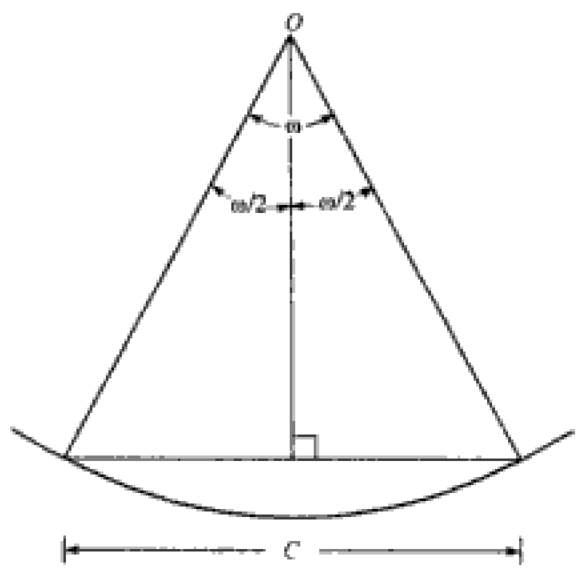

Fig. 8.2 The Cone

The chord length C is given by

$$C = 2l\sin(\omega/2) \quad ...(8.11)$$

The coordinates (x, y) of several points along the circular arc can be calculated as explained below (Fig. 8.3).

Let us assume that a particular parallel has a radius of 100 cm. and the meridians cross the circular arc at an interval of 5 cm, i.e., $OA = AB = BC = CD = 5$ cm.

The central angle for the arc OA is

$$\omega = \frac{OA}{OM} = \frac{5}{100} \text{ radians}$$
$$= \frac{5}{100} \times \frac{360°}{2\pi} \text{ degrees}$$
$$= 2.8648°$$

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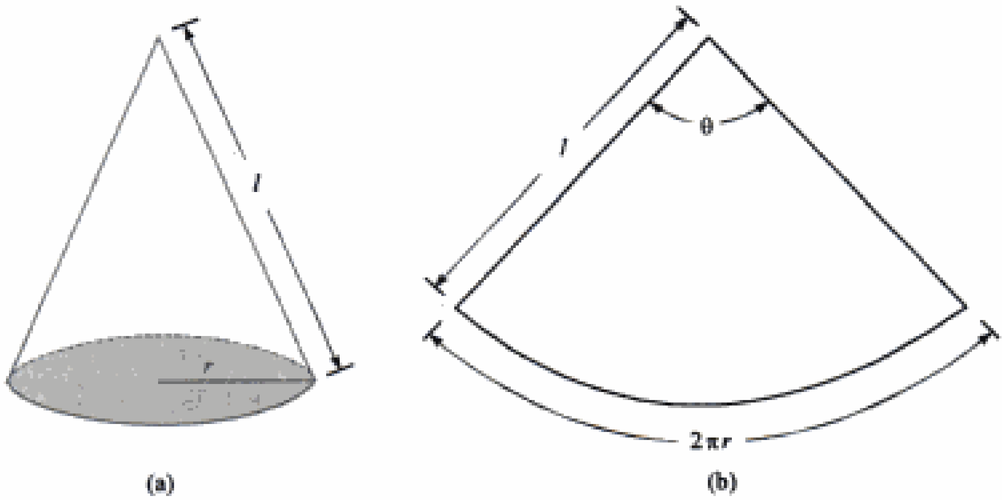


Fig. 8.12 A cone

When the cone is unwrapped or developed, a sector of a circle shown in Fig. 8.12b is formed. The angle θ contained by the bounding radii is given by

$$\theta = 360° \frac{r}{l} \quad ...(8.23)$$

and in terms of θ, the area of the cone

$$= \frac{1}{2} l^2\theta + \pi r^2 \quad ...(8.24)$$

5. Area of a frustum of a cone (Fig. 8.13)

$$= \pi l(r_1 + r_2) \quad ...(8.25)$$

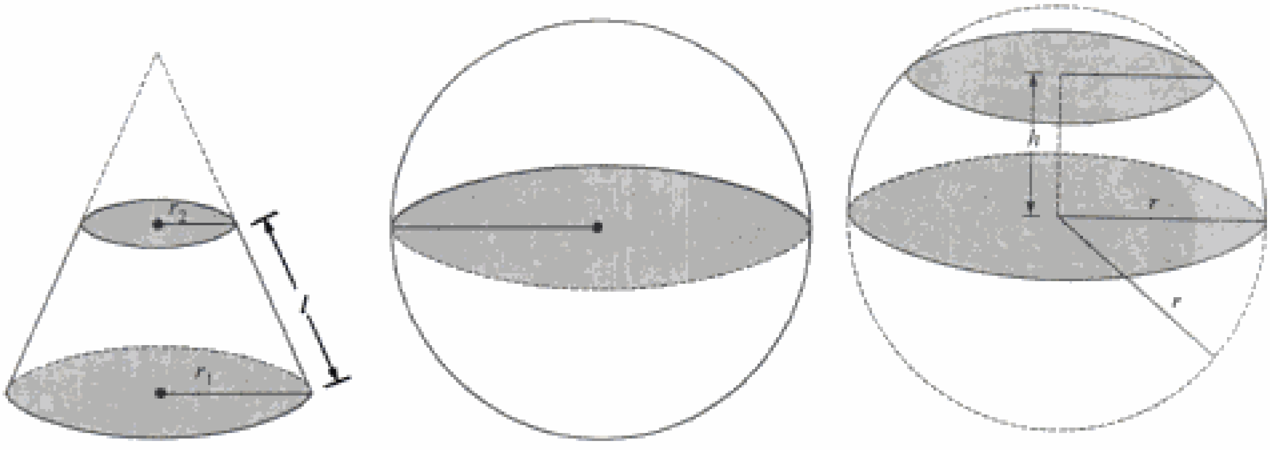


Fig. 8.13 A frustum of a cone Fig. 8.14 A sphere Fig. 8.15 A zone of a sphere

6. Area of a surface of a sphere (8.14) = $4\pi r^2$ ...(8.26)
7. Area of a zone of a sphere (Fig. 8.15) = $2\pi rh$. ...(8.27)

8.6 TYPES OF MAP PROJECTIONS

Although all map projections are carried out by computing the rectangular coordinates (X, Y) for each pair of latitude and longitude (ϕ, λ), there are two methods of projection as below:

1. Geometric projection
2. Mathematical projection

In *Geometric projection* a surface that can be developed into a plane (such as the plane, a cone, or a cylinder) is selected such that it either cuts or is tangent to the earth. A point is then selected as the projection centre from which straight lines are connected to points on the earth and extended until they intersect the selected mapping surface. In *Mathematical projection*, there is no one particular projection point; instead, an equation is used to compute the location (X, Y) of the point on the map from its position (ϕ, λ) on the earth.

In the following sections the various projections have been introduced in groups.

8.6.1 Map Projection to a Plane

The easiest way to visualize geometric map projection is to project on a plane surface which is tangent to the sphere at any point. The point of tangency becomes the central point in the map. There are following three ways of projection depending upon the position of projection centre:

1. Gnomonic projection: In gnomonic projection the projection centre is the centre of the sphere (Fig. 8.16a).
2. Stereographic projection: In stereographic projection the projection centre is the end of the diameter opposite to the point of tangency (Fig. 8.16b).
3. Orthographic projection: In orthographic projection the projection centre is at infinity, in which case the projectors (projection lines) become parallel to each other, and they are perpendicular to the plane of projection (Fig. 8.16c).

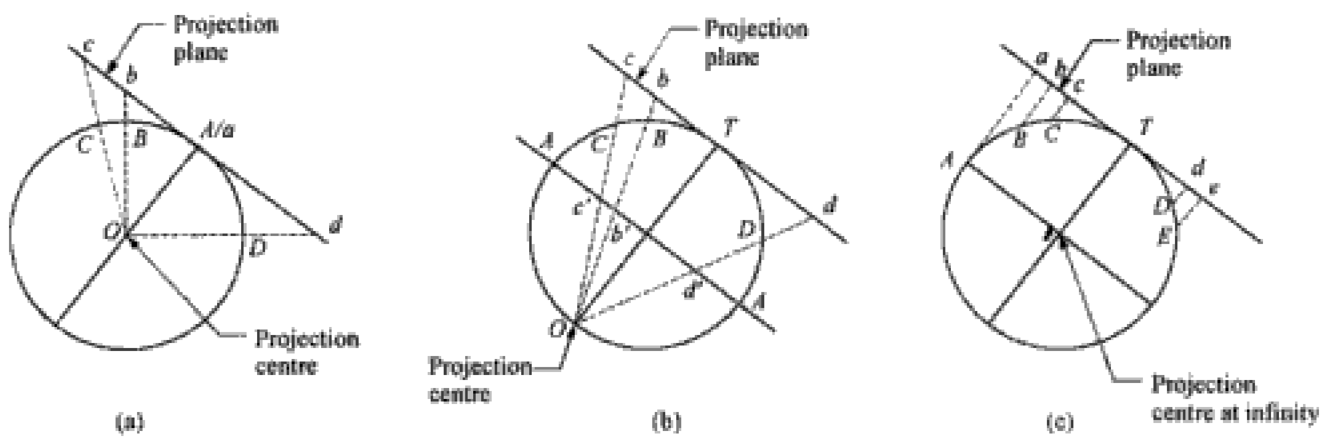


Fig. 8.16 Map projection to a plane: (a) Gnomonic projection, (b) Stereographic projection, and (c) Orthographic projection

Gnomonic projection: This is the oldest true map projection. It is assumed that it was devised by the great abstract geometrist Thales in the sixth century B.C. The projection centre is the centre of the sphere. The projectors through the points B, C, D, etc., passing through the centre O of the earth, are drawn, intersecting the plane of projection at b, c, d, etc., respectively, (Fig. 8.16a). The points b, c, d, etc., are the gnomonic projections of B, C, D, etc., This projection has the following properties:

1. Great circles are projected as straight lines and, therefore, meridians are straight lines.
2. As we go farther from the central tangent point, shapes and sizes undergo extreme distortion.
3. Azimuths of the lines drawn from the tangent point to other points on the map, are correct, because such lines are great circles. Therefore, this is an azimuthal projection.
4. Since the projection centre is at the centre of the sphere, the entire hemisphere cannot be mapped.

This projection system is used mostly for navigational purposes. For other purposes only a limited area around the central point may be used.

Stereographic projection: This projection is credited to the Greek astronomer Hipparchus dating back to the second century B. C. In this projection, the projection centre is taken on the sphere at *O*, diametrically opposite to the point of tangency *T* (Fig. 8.16b). The points *b*, *c*, *d*, etc., are the stereographic projections of the points *B*, *C*, *D*, etc., on the plane of projection tangent to the sphere at *T*. On any other intersecting plane, such as *A–A*, an equivalent projection is possible on reduced scale. This projection is both azimuthal and conformal. It has the following properties:

1. It is the only projection on which circles on the earth still appear as circles on the map.
2. Like the gnomonic projection, in this projection also azimuth lines from the central point are correct because they are great circles.
3. The scale increases as we go away from the tangent point and, therefore, its main defect is that areas are not correctly shown.

This projection is ideally suited for general maps showing hemisphere. It is used for plotting ranges from central objects and for navigational purposes in higher latitudes above 80°, when the plane is tangent at the pole.

Orthographic projection: This is an ancient projection and has been found very useful in space applications. In this projection the projection centre is taken at infinity and consequently the projectors from the points to be projected, are parallel and perpendicular to the plane of projection. In Fig. 8.16c, *a*, *b*, *c*, etc., are the orthographic projections of the points *A*, *B*, *C*, etc., respectively. If the tangent point is one of the poles, each parallel of latitude will be projected correctly to scale as a circle, but the distance between successive parallels becomes rapidly smaller as we move farther from the centre of the map. This means that unlike the gnomonic and stereographic projections, the scale in the orthographic projection decreases away from the central tangent point. In this projection, the azimuths from the tangent point are correct since the lines through it are great circles and, therefore, it is also azimuthal projection just like the gnomonic and stereographic projections. The projection looks like a photograph taken from a deep space. The projection is used for map representations, of moon and planets.

8.6.2 Conical Projection

This group includes a large number of projections, several of which are in common use in atlases. Unlike a sphere, both a cone and cylinder can be developed into a plane without distortion, and, therefore, both are used for map projection. A cone fits over a sphere, touching it in a small circle of latitude, called the *standard parallel*, when the cone axis lies on the polar axis (Fig. 8.17a). As the height of the cone increases, the standard parallel gets closer to the equator (Fig. 8.17b) and, finally, when the standard parallel conicides with the equator, the cone becomes a cylinder (Fig. 8.17c). On the other hand, when the height of the cone decreases, the standard parallel comes closer to the pole, i.e., it moves to higher altitudes (Fig. 8.17d). When the standard parallel lies on the pole, the cone becomes a plane tangent to the sphere at pole (Fig. 8.17e). Thus the projection of a sphere to a plane and a cylinder are actually limiting cases (at least geometrically) to conical projection.

Since a cone has one parallel of latitude common to the sphere, its representation on the map will be at true scale, while scale distortions increase as the distance is extended to the north and the south of parallel. To minimise the distortions in scale, a cone is used that intersects the sphere in two standard parallels. Generally the conical projections are based on mathematical projection though geometric projection on the cone is possible.

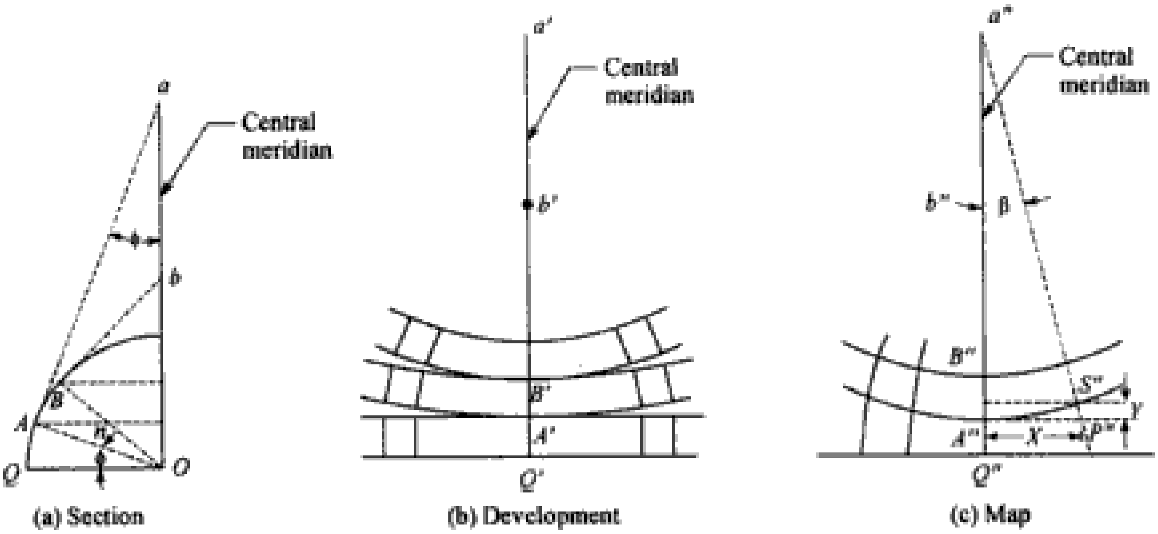


Fig. 8.18 Polyconic projection

When developed, each parallel of latitude appears as the arc of circle. The radius of the arc is equal to the corresponding tangent distance. For example, the parallel through a point A on the earth, has the radius equal to Aa, and that through B, has the radius equal to Bb. The centres a, b, etc., of all the circles lie on the central meridian of the map. If the latitude of the point A is ϕ, the length n of the normal OA is given by:

$$n = \frac{a}{\sqrt{1-e^2 \sin^2 \phi}} \quad ...(8.28)$$

where a = equatorial radius of the earth

e = eccentricity of the ellipse in the meridian section ($e^2 = 0.0067686580$).

The length of the tangent distance aA is

$$= n \cot \phi \quad ...(8.29)$$

The distance $Q'A'$, $A'B'$, etc., along the central meridian on the map (Fig. 2.18b) are true scale representations of the corresponding arc distances QA, AB, etc., in the meridian section (Fig. 8.18a). The parallels drawn on the map may be selected with as small as difference in latitude as may be desired, and each one is drawn with its own particular radius as shown, with the centre of the arc on the central meridian at the proper tangent distance (to scale) above the point where the parallel cuts the central meridian. It should be observed that the method of drawing these areas of the parallels of latitude on the map is such that each parallel is separately developed as the circumference of the base of its own distinct cone, and that the spacing between them increases with increasing differences of longitude from the central meridian, thereby changing the north-south scale of the map from place to place as the longitude differences increase.

The arc distance $A''S''$ (Fig. 8.18c) is the difference in longitudes of the points A'' and S''. The angle $A''a''S''$ shown as β, is given by

$$\beta = \varphi \sin \phi$$

where φ = the arc $A''S''$

ϕ = the latitude

The coordinates (X, Y) of S'' with respect to A'', are given by

$$X = A''P'' = a''S'' \sin \beta = n \cot \phi \ \sin \beta \quad ...(8.30)$$

$$Y = P''S'' = a''S'' (1 - \cos \beta) = n \cot \phi \ \text{vers}\ \beta \quad ...(8.31)$$

A meridian can be plotted by drawing more number of points as S'' and drawing a line passing through them. All meridians are curved lines having concavity towards the central meridian. The polyconic projection has the following properties:

1. Near the central meridian there is little error, but the error increases in proportion to the square of the difference in longitude along any parallel.
2. The map is true to scale along the central meridian and along every parallel. Along the other meridians the scale is somewhat changed.
3. Near the central meridian, the parallels and meridians intersect nearly at right angles.
4. Areas of great extent along north and south may be mapped with a very small distortion.

Although better adopted to mapping an area of great extent in latitude than for an area of great extent east and west, the polyconic projection is sufficiently accurate for maps of considerable areas.

8.6.5 Conformal Projection

In conformal projection the shape of the area is preserved. The angle between any pair of intersection short lines does not change in this projection. In conformal projection, an isometric plane is introduced for correlating the rectangular coordinates (x, y) with the spherical coordinates (β, ϕ).

The isometric plane is defined by an isometric latitude q given by:

$$q = \log_e\left[\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\left\{\frac{1-e\sin\phi}{1+e\sin\phi}\right\}^{e/2}\right] \qquad ...(8.32)$$

where ϕ = latitude.

e = eccentricity of the ellipsoid, given by

$$e^2 = \frac{a^2-b^2}{a^2} \qquad ...(8.33)$$

where a = semi-major axis of the ellipsoid

b = semi-minor axis of the ellipsoid.

For a sphere, the eccentricity is zero.

The coordinates (x, y) are expressed in terms of β and q as

$$x = f_1(\beta, q) \qquad ...(8.34)$$

$$y = f_2(\beta, q) \qquad ...(8.35)$$

For conformal mapping, the following conditions, called Cauchy-Riemann's equations, should be satisfied:

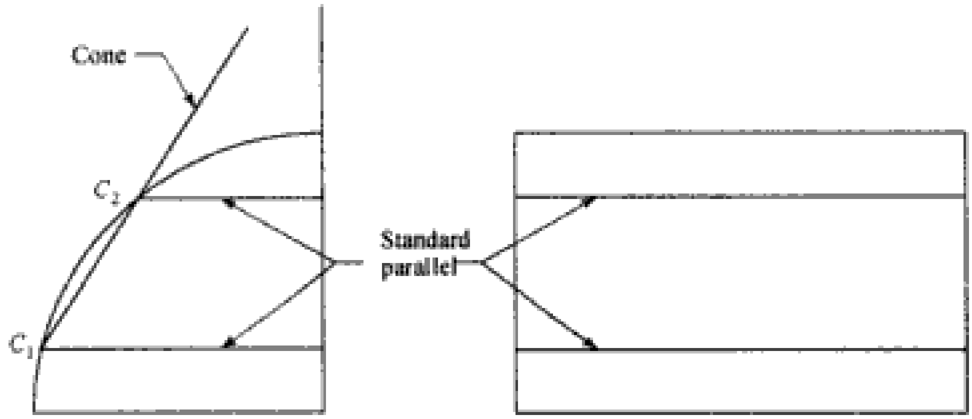
$$\frac{\partial x}{\partial \beta} = \frac{\partial y}{\partial q} \qquad ...(8.36)$$

$$\frac{\partial x}{\partial q} = \frac{\partial y}{\partial \beta} \qquad ...(8.37)$$

8.6.6 Lambert Projection

The projection developed by Johann Heinrich Lambert in 1972, is known as Lambert conformal conic projection and is the one which is the most widely used projections. It is used for the state plane coordinate systems of states of greater east-west than north-south extent.

In this projection the axis of the cone coincides with the axis of the earth. The surface of the cone intersects the earth surface along two standard parallels, passing through C_1 and C_2 shown in Fig. 8.19a.



(a) Section (b) Map

Fig. 8.19 Lambert conformal conic projection

The frustum *ABCD* shown in Fig. 8.20, can be developed and projected on a map. The longitude of the point *A* is equal to that of the central meridian *PEF* minus the angle α at *P*. Similarly, the longitude of *B* is that of the central meridian plus the angle α at *P*. The latitudes of *A* and *B* can be found from their own distances along the meridian knowing the latitudes of the two circles C_1 and C_2. The projections of the points on the earth's surface are made from the earth's centre *O* onto the cone.

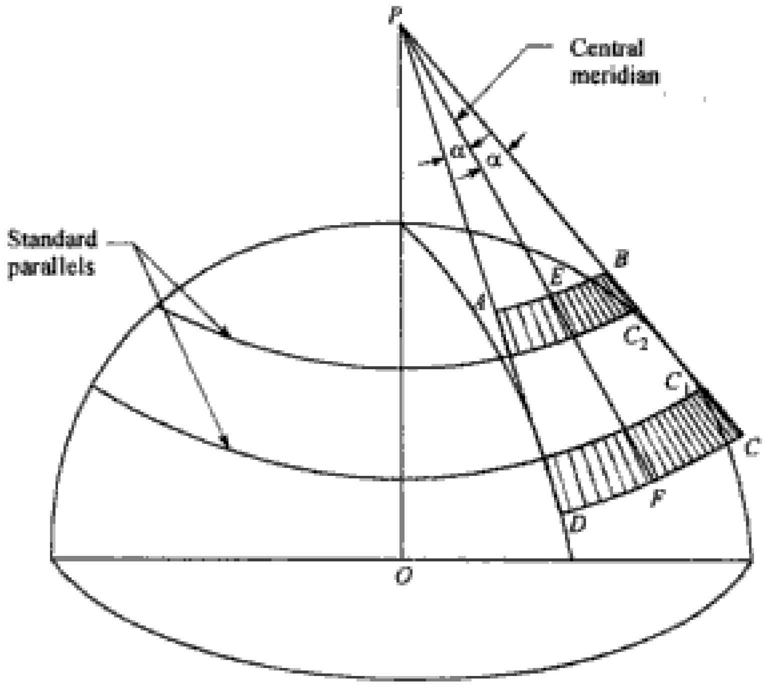


Fig. 8.20

The parametric equations for Lambert projection are:

$$x = Ke^{-lq}\cos(l\beta) \qquad ...(8.38)$$

and

$$y = Ke^{-lq}\sin(l\beta) \qquad ...(8.39)$$

where *K* and *l* are constants whose values depend upon the selected standard parallels C_1 and C_2.

The above equations may be expressed as:

$$x = R\cos(l\beta) \qquad ...(8.40)$$

and

$$y = R\sin(l\beta) \qquad ...(8.41)$$

where

$$R^2 = x^2 + y^2 = K^2e^{-2lq}$$

Since the scale varies rapidly near the poles, the relative sizes of areas in widely different latitudes become misleading. For example, Greenland which is one nineth in area of the South America, appears much larger than South America on the map because Greenland is near the poles. Consequently, such a map is not suited to general use, although because of its many other advantages it is widely used for specific purposes due to the fact that in this projection shape is maintained.

8.6.8 Transverse Mercator Projection

This projection was developed by Lambert in 1772, analytically derived by Gauss 50 years later, and then formulae suitable for calculations were devised by Kruger in 1912. This is perhaps the reason that it is being used most widely world over as conformal map projections.

A transverse Mercator projection is the ordinary Mercator projection turned through a 90° angle so that it is related to a central meridian in the same way that the ordinary Mercator projection is related to the equator (Fig. 8.22). Because the cylinder is tangent to the sphere at a meridian, the scale is true along that meridian, which is called the central meridian. Fig. 8.22a shows a cylinder with one standard line and Fig. 8.22b shows a cylinder with two standard lines.

Although like the ordinary Mercator projection, this projection is conformal, it does not retain the straight-rhumb-line property of the Mercator projection. A portion of the map is shown in Fig. 8.22c in which AA' and BB' are the lines along which the scale is exact.

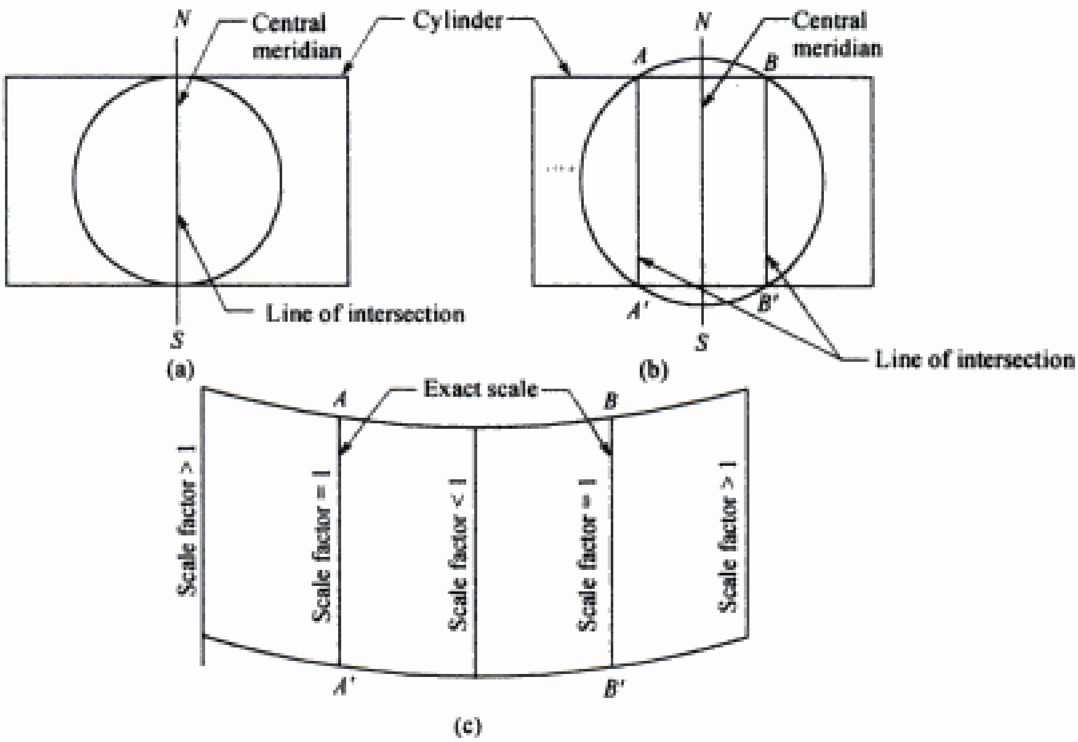


Fig. 8.22 Transverse mercator projection

This projection has the following properties:

1. Both the central meridian and a line perpendicular to it are represented by straight lines.
2. Other meridians are complex curves that are concave toward the central meridian.
3. Parallels are concave curves toward the pole.
4. The scale is true only along the central meridian.

8.6.9 Universal Transverse Mercator (UTM) Projection

This projection is based entirely on the transverse Mercator projection. UTM 1958 has the following specifications.

1. Transverse Mercator projection is in zones that are 6° wide (Fig. 8.23).
2. The reference ellipsoid is Clark 1866 in North America.

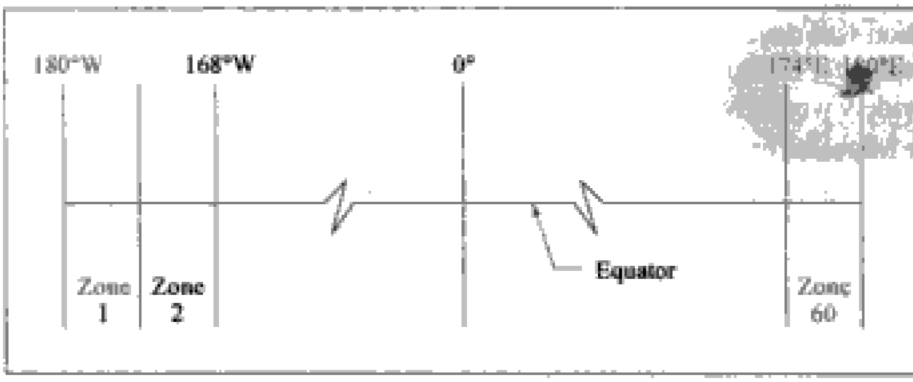


Fig. 8.23 Universal transverse Mercator zones

3. The origin of longitude is at the central meridian (Fig. 8.24).

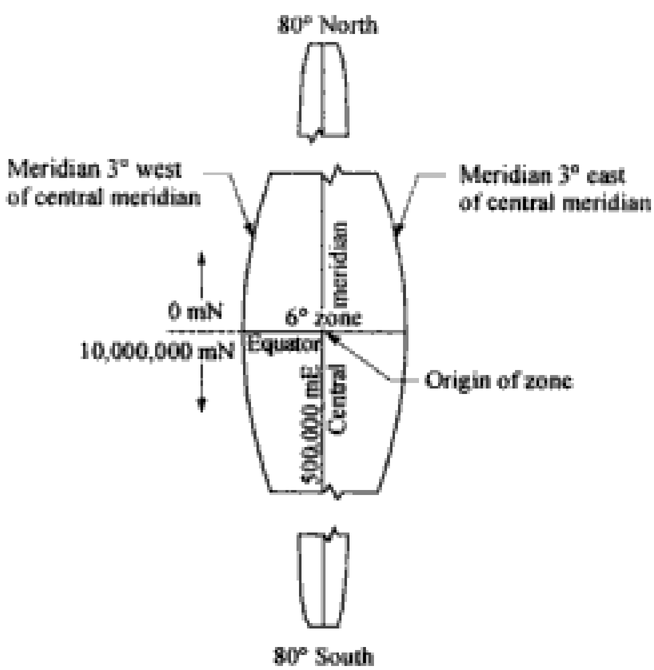


Fig. 8.24 Origin of the coordinates x and y in a UTM grid zone

4. The origin of latitude is at the equator.
5. The unit of measure is the metre.
6. For the southern hemisphere, a false northing of 10,000,000 m is used (to correspond to the equator).
7. A false easting of 500,000 m is used for the central meridian of each zone.
8. The scale factor at the central meridian is 0.9996.

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8.4 Discuss how the following projections are different from each other:
(*i*) Gnomonic projection
(*ii*) Stereographic projection
(*iii*) Orthographic projection.

8.5 Discuss briefly the following:
(*i*) Polyconic projection
(*ii*) Conformal projection.

8.6 Explain the salient features of the following projections:
(*a*) Lambert projection
(*b*) Mercator projection.

8.7 Discuss the salient features of universal transverse Mercator projection.

8.8 What factors are to be considered in making choice for a projection system? Discuss giving suitable examples.

9

PHOTOGRAMMETRY AND PHOTOGRAPHIC INTERPRETATION

9.1 GENERAL

Photogrammetry is the science of making measurements on photographs. *Terrestrial photogrammetry* applies to the measurement of photographs that are taken from a ground station, the position of which usually is known or can be readily determined. *Aerial photogrammetry* applies to the measurement of photographs taken from the air.

Broadly, photogrammetry requires the following:

1. Planning and taking the photography
2. Processing the photograph
3. Measuring the photographs and reducing the measurements to produce the end results, such as point coordinates or maps.

Two broad categories involved in photogrammetry are:

1. Metric or quantitative work
2. Interpretive or qualitative work.

Metric Photogrammetry involves all quantitative work, such as the determination of ground positions, distances, elevations, areas, volumes, and various types of maps. In the second classically called *photointerpretation* (discussed in Sec. 9.27) photographs are analysed quantitatively for purposes of identifying objects and assessing their significance. In recent years records from other imaging systems such as infrared and radar have been used for interpretive purposes, and therefore, the more general name *remote sensing* is used (discussed in Chapter 10.)

When photogrammetry deals with extraterrestrial photography and images where the camera may be fixed on the earth, mounted on board an artificial satellite, or located on a moon or a planet, it is called *space photogrammetry*. Another type of photogrammetry is the *close-range photogrammetry*. It involves applications where the camera is relatively close to the object photographed.

This chapter discusses the elementary principles of aerial photogrammetry as they apply to the work of the survey engineers. Since entire field of photogrammetry cannot be incorporated into a single chapter, the subject matter discussed in this chapter is only an introduction to photogrammetry for the appreciation of the potential applications of the science to surveying.

9.2 LIMITATION OF PHOTOGRAMMETRY

Photogrammetry is particularly suitable for topographical or engineering surveys and also for those projects demanding higher accuracy. Photogrammetry is rather unsuitable for dense forest and flat-sands due to the difficulty of identifying points upon the pair of photographs. It is also unsuitable for flat terrain where contour plans are required, because the interpretation of contours becomes difficult in the absence of spirit levelled heights. Considering these factors, it is evident that photogrammetry may be most suitably employed for mountainous and hilly terrain with little vegetation.

9.3 TYPES OF PHOTOGRAPHS

The photographs used in photogrammetry may be broadly classified in two types as below, depending upon the camera position at the time of photography:

1. Terrestrial photographs
2. Aerial photographs.

9.3.1 Terrestrial Photographs

When the photographs are taken with phototheodolite (Fig. 9.1) having camera station on the ground and the axis of the camera horizontal or nearly horizontal, the photographs are called the *terrestrial photographs*. These photographs present familiar view of the front view or elevation. They are generally used for the survey of structures and architectural or archaeological monuments.

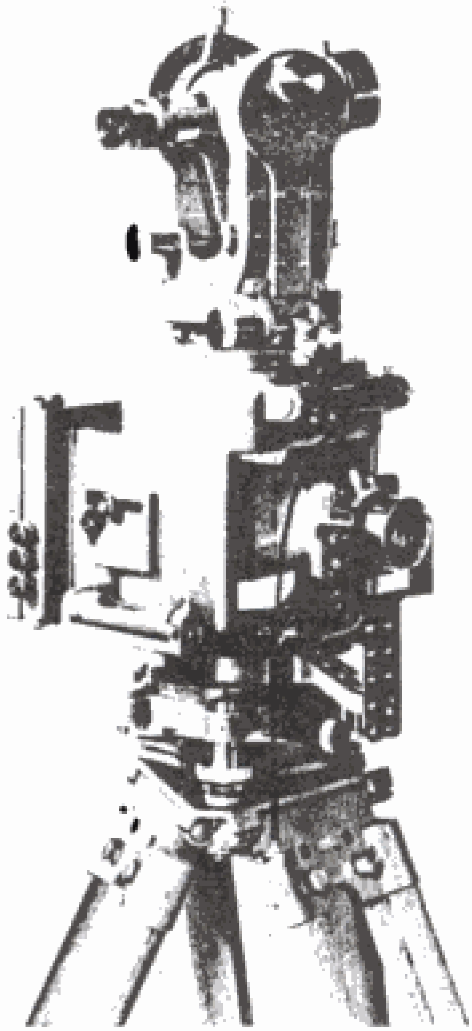


Fig. 9.1 A phototheodolite

9.3.2 Aerial Photographs

The photographs taken with an aerial camera having camera station in air and the axis of the camera vertical or nearly vertical, are called *aerial photographs*.

Depending on the angle between the axis of the camera and the vertical, the aerial photographs may be classified as:

1. Vertical photographs
2. Oblique photographs
3. Convergent photographs
4. Trimetrogen photographs.

Vertical photographs

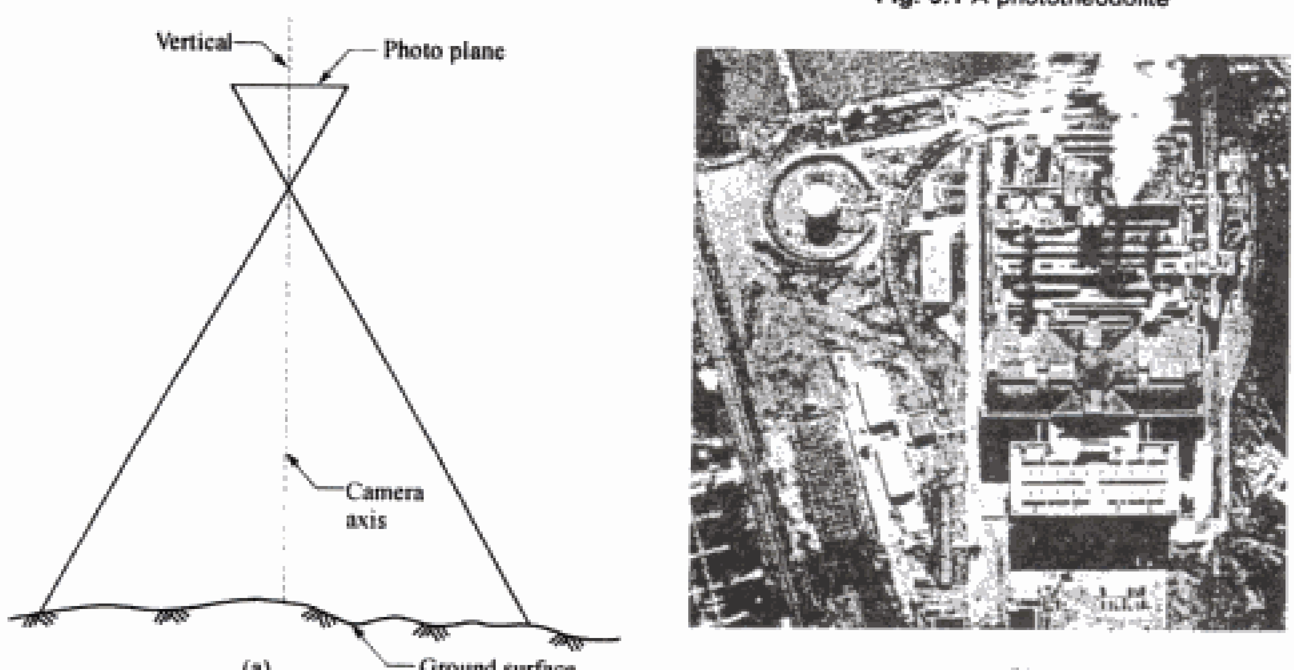
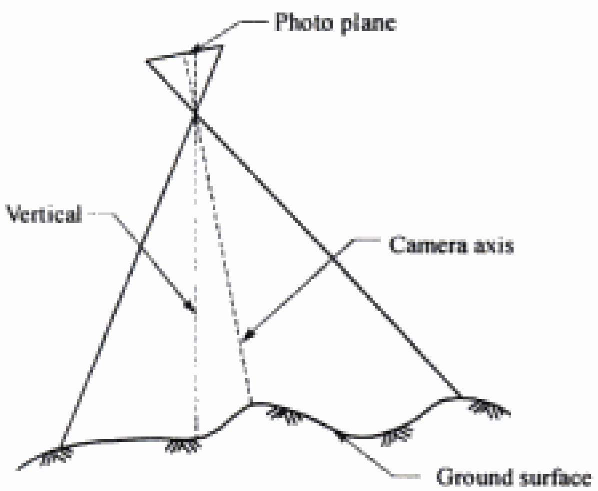


Fig. 9.2 A vertical aerial photograph

Vertical photographs are those aerial photographs which are taken when the optical axis of the camera is vertical or nearly vertical (Fig. 9.2). A truly vertical photograph closely resembles a map. These are utilized for the compilation of topographical and engineering surveys on various scales.

Oblique photographs

Oblique photograph are obtained when the optical axis of the camera is intentionally inclined from the vertical (Fig. 9.3). An oblique photograph covers larger area of the ground. Depending upon the angle of obliquity of the axis of the camera, oblique photographs may be further divided into two categories. An oblique photograph which does not show the horizon, is known as *low-oblique photograph* (Fig. 9.4a).



(a)

Fig. 9.3 An oblique photograph

The aerial photograph which shows the horizon, is called *high-oblique photograph* (Fig. 9.4b).

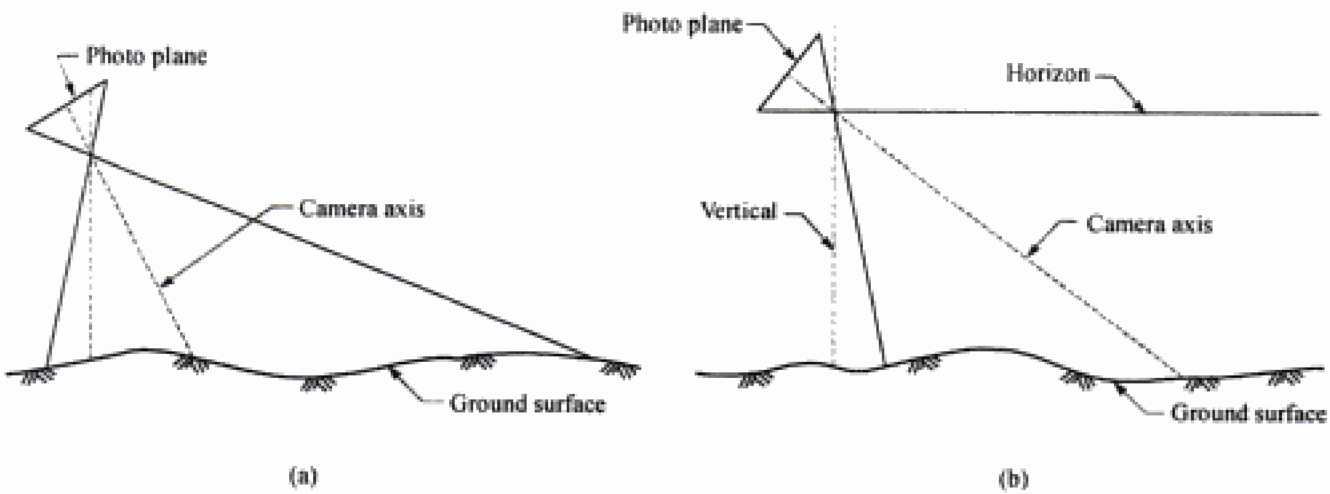


Fig. 9.4 (a) Low-oblique photograph, (b) High-oblique photograph

Low-oblique photographs are generally used to compile reconnaissance maps of inaccessible areas. They are also used for some special purposes such as in estimating water yield from snow-melt. High-oblique are sometimes used for military intelligence.

Convergent photographs

These are low-oblique photographs taken with two cameras exposed, simultaneously at successive exposure stations, with their axes tilted at a fixed inclination from the vertical in the direction of flight, so that the forward exposure of the first station forms a stereopair with the backward exposure of the next station.

Trimetrogen photographs

This type of photograph is a combination of a vertical- and low-oblique photographs exposed simultaneously from the air station from two cameras.

9.4 AERIAL PHOTOGRAMMETRY

Aerial photogrammetry has already been defined in Sec. 9.1. Mapping of large areas using aerial photographs is faster and economical than any other method if the aerial photographs and plotting instruments are already available. Accurate topographic maps can be prepared on various scales ranging from 1 : 500 to 1,000,000 having contours accurate upto contour interval of 50 cm.

The measurement by means of aerial photographs consists of the following stages:

1. Aerial photography
2. Providing ground control
3. Planimetric mapping.

Aerial photography conducted to acquire aerial photographs for large areas involves heavy expenditure and therefore, they are made by the government organizations or large private companies. In India, M/s Air Survey Coy of India has been authorized for making aerial photography and supplying the aerial photographs to the Survey of India.

9.5 PHOTOCOORDINATE SYSTEM

To provide reference lines for the measurements of image coordinates or distances, four marks, called the *fiducial marks*, are located in the camera image plane. These are located either in the middle of the sides of the focal plane (Fig. 9.5a) or in its corners (Fig. 9.5b).

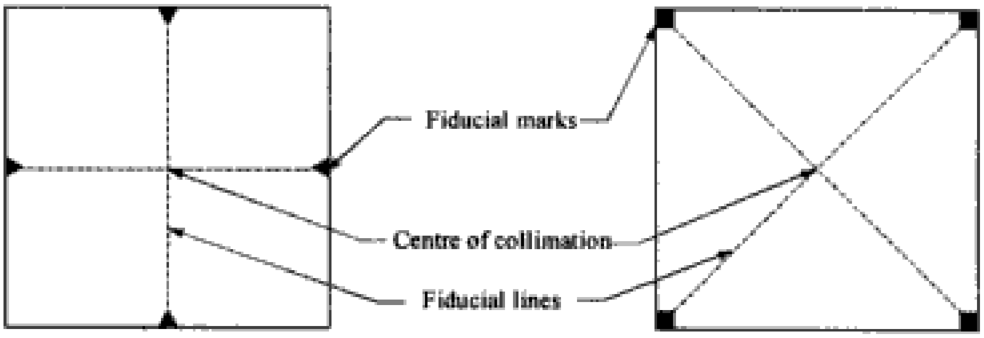


Fig. 9.5 Fiducial marks

The lines joining the opposite fiducial marks are called the *fiducial lines*. The intersection of the fiducial lines is called the *centre of collimation*. In aerial cameras, the centre of collimation is kept at or very close to the principal point which is the reference point. It lies in the focal plane and it is the foot of the perpendicular from the rear nodal point of the lens to the focal plane.

To measure the coordinates of the images appearing on the photograph, a cartesian coordinate system as shown in Fig. 9.6, called the *photocoordinate system*, is used. The x-axis is taken along the direction of flight of the aircraft which is obtained by joining the opposite fiducial markes lying in the direction of flight. The y-axis is taken in a direction perpendicular to the x-axis, joining the remaining two opposite fiducial marks. The intersection O of the x- and y-axes, is the origin of the coordinate system. The coordinates of the image points a and b are (X_a, Y_a) and (X_b, Y_b), respectively. Photographic measurements are usually done on the positive prints on paper, film or glass. Negatives are preserved for records and making additional prints whenever required.

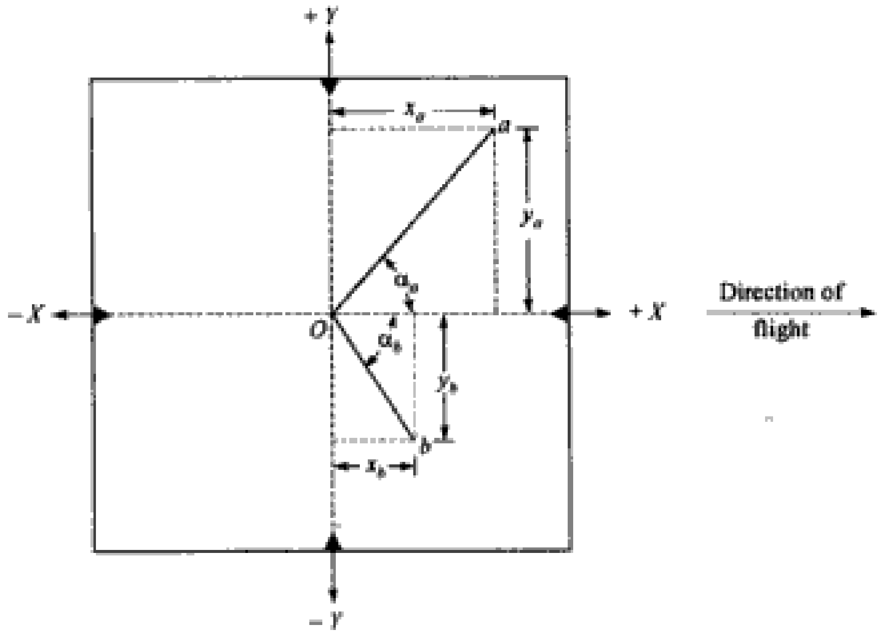


Fig. 9.6 Photocoordinate system

The photo distance ab between a and b, is given by

$$ab = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2} \quad ...(9.1)$$

and the angle aOb is given by

$$\angle aOb = \alpha_a + \alpha_b$$

$$= \tan^{-1}\frac{y_a}{x_a} + \tan^{-1}\frac{y_b}{x_b} \quad ...(9.2)$$

For the measurement of the coordinates, simple scales are commonly used for low order accuracy work. However, when higher accuracy is essential, more accurate scales such as a metal microrule or a glass scale should be used.

In another method, known as *trilaterative method*, the photocoordinates of the points are measured from the fiducial marks as shown in Fig. 9.7. If the distances of a point a from the fiducial marks M_1 and M_2 and d_1 and d_2, respectively, then the coordinates (x_a, y_a) of a are computed from the measured distances using the trigonometric relationships. The trilaterative solution becomes weak when the angle M_1aM_2 is near 0° or 180°. The solution is strongest when this angle is near 90°. The method is more accurate and does not require marking of fiducial lines. A computer programme can be written to generate the coordinates from the distances measured for a number of points.

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Perspective centre (*O*): The real or imaginary point of the origin of bundle of perspective rays, is known is a perspective centre.

Flying height (*H*): Flying height is the elevation of the exposure station *O* above mean sea level.

Line of flight: A line which represents the track of an aircraft on an existing map, is known as the line of flight.

Focal length (*f*): The distance from the front nodal point of the lens to the plane of the photograph, or the distance of the image plane from the rear nodal point, is known as the focal length.

Principal point (*p* and *P*): Principal point is a point where a perpendicular dropped from the front nodal point of the camera lens strikes the photograph. It is also known as photo-principal point (*p*). The principal point is considered to coincide with the intersection of *x*-axis and *y*-axis.

The point where the plate perpendicular *Op* strikes the ground is known as the ground-principal point (*P*).

Nadir point (or Plumb point) (*v* and *V*): The point where a plumb line dropped from the front nodal point, strikes the photograph, is called the nadir point, plumb point or photo-nadir point (*v*).

The point on the ground vertically beneath the exposure station is called the ground-nadir or plumb point (*V*).

Principal plane (*OVP*): The plane passing through *O*, *V*, and *P* is called the principal plane.

Principal line (*vp*): The principal line is the line *vp* of intersection of the principal plane with the plane of photograph. It is thus the line on a photograph obtained by joining the principal point and the photo-nadir point.

Tilt (*t*): Tilt is the angle *vOP* which the optical axis makes with the plumb line. It is also the angle *Ohp* which is the deviation of a plate from the horizontal plane at the time of exposure.

Tilted photograph: At the time of exposure if the camera axis (or optical axis) is tilted unintentionally from the plumb line by a small amount, usually less than 3° (i.e., $t < 3°$) the photograph so obtained is called a tilted photograph.

Isocentre (*i*): Isocentre is the point *i* in which the bisector *Oi* of the angle of tilt meets the photograph. The angle of tilt lies in the principal plane and hence the isocentre lies on the principal line. The distance $vi = ip = f \tan t/2$.

In a truly vertical photograph, the photo-nadir point and the isocentre coincide with the photo-principal point.

Swing (*s*): The horizontal angle measured clockwise in the plane of the photograph from the positive *y*-axis to the plumb point, is known as the swing.

Azimuth of the principal plane (α): The azimuth of the principal plane, also sometimes known as the azimuth of the photograph, is the clockwise horizontal angle α measured about the ground-nadir point from the ground survey north meridian to the principal plane of the photograph.

Horizon point (*h*): The point of intersection on the principal line *vp* produced with the horizontal line *Oh* through the exposure station *O*, is known as the horizon point.

9.8 SCALE OF A VERTICAL PHOTOGRAPH

Map scale is defined as the ration of a map distance to the corresponding distance on the ground. In similar manner, the scale of a photograph is the ratio of a distance on the photo to the corresponding distance on the ground. A map being an orthographic projection, has uniform scale everywhere on the map, but on the other hand, a photograph being a perspective projection, does not have uniform scale. The scale on a photograph varies from point-to-point with change in terrain elevation.

Scales may be expressed either as *unit equivalents*, *dimensionless representative fractions*, *or dimensionless ratios*. If, for example, 1 cm on a map or photo represents 100 m on the ground, the scale may be expressed as:

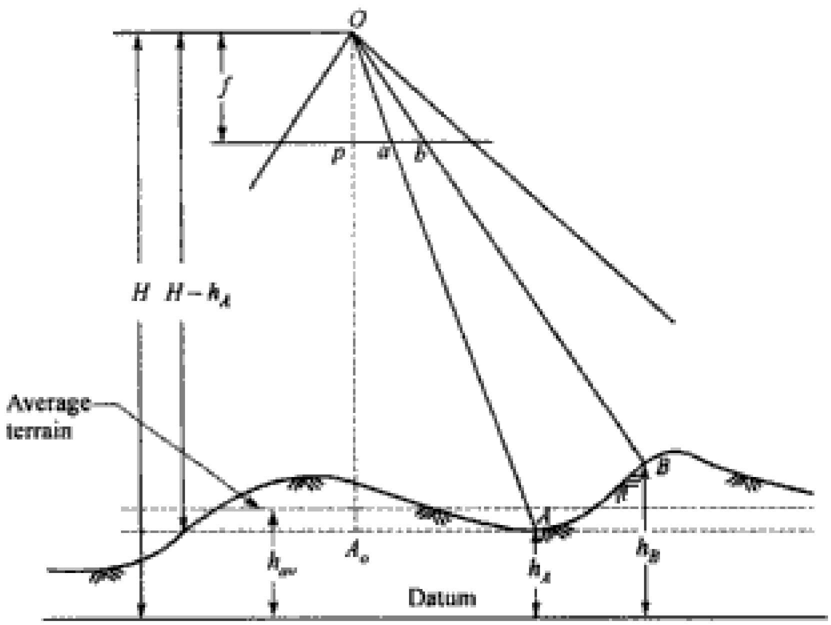

Fig. 9.11 A vertical photograph taken over variable terrain

If the distance of image, *a* of *A* on the photograph from the principal point *p*, is *pa*, the scale at the point *A*, is given by

$$S_A = \frac{pa}{A_0 A}$$

From the similar Δ'*Opa* and OA_0A, we have

$$\frac{pa}{A_0 A} = \frac{f}{H - h_A}$$

or
$$S_A = \frac{f}{H - h_A}$$

Similarly, at the point *B*, we have

$$S_B = \frac{f}{H - h_B}$$

In general, at any point having elevation *h*, the scale is given by

$$S = \frac{f}{H - h} \qquad \text{...(9.6)}$$

where H = the flying height above the datum.

The scale from Eq. (9.6), is called the *point scale*.

9.8.3 Average scale

To define the overall mean scale of a vertical photograph taken over variable terrain, it is often convenient and desirable to use an *average scale* of the photograph. Average scale is the scale at the average elevation of the terrain covered by a particular photograph and is expressed as

$$S_{av} = \frac{f}{H - h_{av}} \qquad \text{...(9.7)}$$

where h_{av} = the average elevation of the terrain covered by a photograph.

Average scale is exact only at those points which lie at average elevation and it is only an approximate scale for all other areas of the photograph.

9.8.4 Other methods of determining scale of vertical photographs

In the previous sections the determination of the scale from focal length, flying height, and elevation has been discussed. Some of other methods which are also used, for determining the scale, are discussed below.

By measuring length on the ground

The scale can be determined by measuring the photo distance between points and the corresponding distance on the ground, and using them in (Eq. 9.3). The scale determined by this method is correct only at the average elevation the two points considered.

By determining the length from map

In this method, the photo distance between any two points, is measured on the photograph. If a map of the photographed area is available, the distance between the two selected points is measured on the map. To get the ground distance, the measured map distance is multiplied by the scale of the map. The ratio of the photo distance to the ground distance is the scale of the photograph.

By determining the length by common knowledge

If a foot-ball field appears in the photograph, the lengths of the sides of the foot-ball field are known. The scale is determined by measuring the length of a side of the field on the photograph and dividing it by the known length of that side of the field.

Determination of average scale of a photograph

(*i*) Few points are selected on the photograph which are uniformly distributed and their location on the map of the area photographed, have been identified. Let these points be *a*, *b*, *c*, *d*, *e*, etc. Various combinations of the points are considered to have different lines such as *ab*, *ac*, *ad*, *bc*, *bd*, *ea*, *eb*, etc. The length of these lines on the photograph are measured as *ab*, *ac*, *ad*, *bc*, *bd*, *ea*, *eb*, etc. The corresponding ground lengths *AB*, *AC*, *AD*, *BC*, *BD*, *EA*, *EB*, etc., are determined from the map. The scales for every line are obtained as below:

$$S_1 = \frac{ab}{AB} = \frac{1}{s_1}$$

$$S_2 = \frac{ac}{AC} = \frac{1}{s_2}$$

$$S_3 = \frac{ad}{AD} = \frac{1}{s_3}, \text{ etc.,}$$

Let

$$s_{av} = \frac{1}{n}(s_1 + s_2 + s_3 + \ldots + s_n)$$

then,

$$S_{av} = \frac{1}{s_{av}} \qquad \ldots(9.8)$$

where *n* is the number of lines considered.

(*ii*) In this method the elevation of the selected points are obtained from the topographic map of the area photographed. Let the elevations of any four points be h_A, h_B, h_C, and h_D, then average elevation of the area is

$$h_{av} = \frac{1}{4}(h_A + h_B + h_C + h_D)$$

If the flying height of the aircraft and the focal length of the aerial camera are known, the average scale is calculated from Eq. (9.7).

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9.9 GROUND COORDINATES FROM A VERTICAL PHOTOGRAPH

The ground coordinates of points whose images appear in a vertical photograph, can be determined with respect to an arbitrary ground coordinate system. The arbitrary, X- and Y- ground axes are in the same vertical planes as the photographic x-and y-axes, respectively. The orgin of the ground coordinates system is considered to lie at the ground principal point.

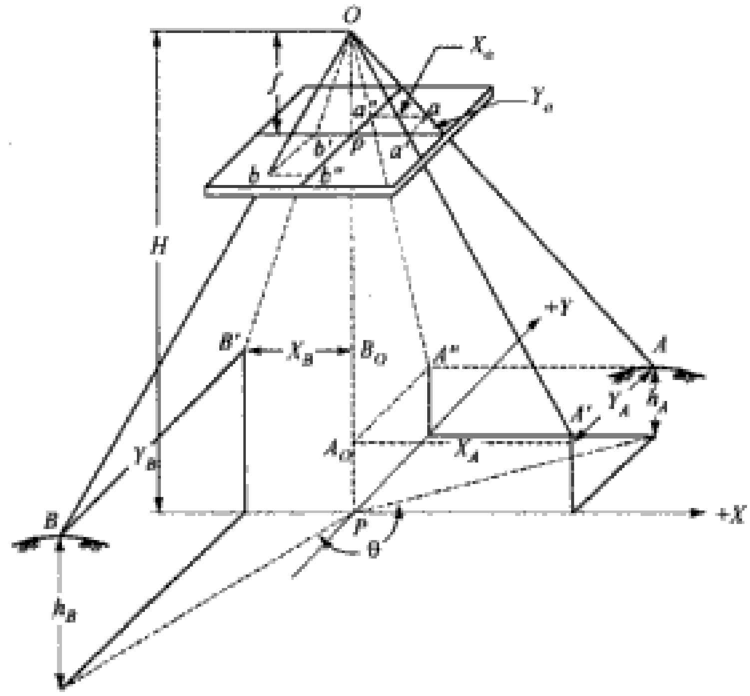

Fig. 9.12 Ground coordinates from a vertical photograph

In Fig. 9.12, let

$(x_a, y_a), (x_b, y_b)$ = the photo coordinates of the points a and b, respectively.
$(X_A, Y_A), (X_B, Y_B)$ = the ground coordinates of the points A and B, respectively.
h_A, h_B = the elevations of A and B, respectively.
H = the flying height of the aircraft.
f = the focal length of the aerial camera lens.

From similar Δ^s $Oa'p$ and $OA'A_0$, we have

$$\frac{Oa'}{O_0A'} = \frac{f}{H-h_A} = \frac{x_a}{X_A}$$

or
$$X_A = \frac{(H-h_A)}{f} x_a \qquad ...(9.9)$$

Similarly,

$$X_B = \frac{(H-h_B)}{f} x_B \qquad ...(9.10)$$

Also, from similar Δ^S $Oa''p$ and $OA''A_0$, we get

$$\frac{a'a}{A'A} = \frac{f}{H-h_A} = \frac{y_a}{Y_A}$$

or
$$Y_A = \frac{(H-h_A)}{f} y_a \qquad ...(9.11)$$

Similarly,

$$Y_B = \frac{(H - h_B)}{f}\, y_b \qquad \text{...(9.12)}$$

From Eq. (9.9) to (9.12), we find that, X and Y ground coordinates of any point are obtained by simple multiplying x and y photo coordinates by the inverse of photo scale at that point.

The ground distance between the two points A and B, can be calculated as

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \qquad \text{...(9.13)}$$

and the horizontal angle θ between the two points at P, may be calculated as

$$\theta = \angle APB = 90° + \tan^{-1}\left(\frac{XA}{YA}\right) + \tan^{-1}\left(\frac{XB}{YB}\right) \qquad \text{...(9.14)}$$

In order to solve the Eqs. (9.9) to (9.12), the data required are f, H, h, x and y. The photo coordinates x and y are measured, f is commonly known, and H is calculated by methods described in Sec. 9.11. Elevation h of the points may be obtained directly by field measurements, or they may be taken from available topographic maps.

9.10 RELIEF DISPLACEMENT ON A VERTICAL PHOTOGRAPH

The shift or displacement in the photographic position of an image caused by the relief of the object, i.e, its elevation above or below a datum, is called *relief displacement*.

Fig. 9.13 shows a vertical photograph taken from flying height H above datum. Focal length of the camera is f, and p is the principal point. The image of ground point A, which has elevation h_A above datum, is located at a on the photograph. An imaginary point A' is located vertically beneath A in the datum plane and its corresponding imaginary image point is at a'. Since the lines AA' and OP are the vertical lines, the plane, $AaOPA'$ is a vertical plane. The plane $A'a'OPA'$ is also a vertical plane which coincides with plane $AaOPA'$. These two planes intersect the photo plane along the lines pa and pa', respectively. Since the lines pa and pa' are coincident, the line aa', which is relief displacement of point A due to its relief h_A, is radial from the principal point.

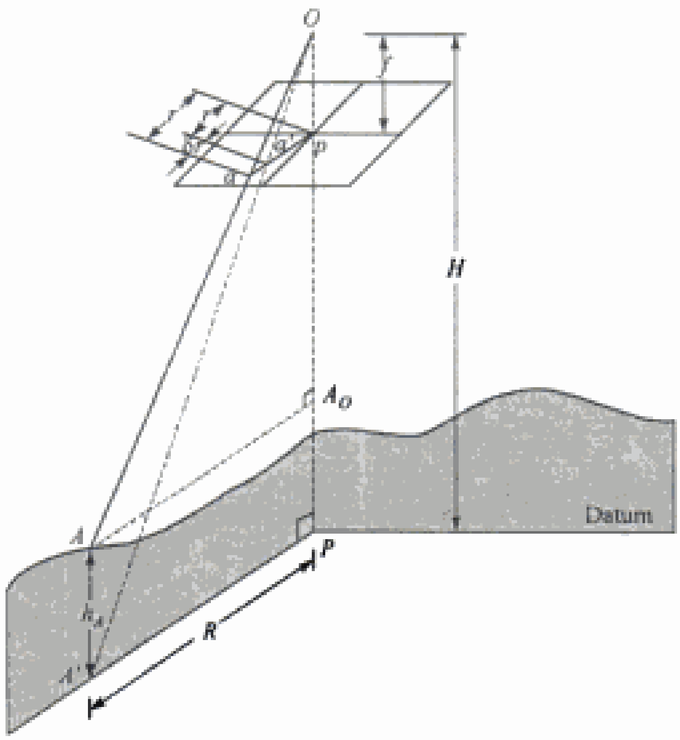

Fig. 9.13 Relief displacement on a vertical photograph

The expression for evaluating relief displacement may be obtained as below:

From similar $\Delta^S Oap$ and $OAAo$, we get

$$\frac{r}{R} = \frac{f}{H - h_A}$$

or $$f.R = r(H - h_A) \qquad \text{...(9.15)}$$

Also, from similar $\Delta^S Oa'p$ and $OA'P$, we get

$$\frac{r'}{R} = \frac{f}{H}$$

or $$f.R = r' H \qquad \text{...(9.16)}$$

Equating the two values of $f.R$ from Eqs. (9.15) and (9.16), we get

$$r(H-h_A) = r'H$$

or $$rH - rh_A = r'H$$

or $$H(r-r') = rh_A$$

Let $(r-r')$ be δr, then

$$H\delta r = r hA$$

or $$\delta r = \frac{rh_A}{H}$$

In general $$\delta r = \frac{rh}{H} \quad ...(9.17)$$

Eq. (9.17) is the equation for relief displacement. On examination of Eq. (9.17), the following conclusions may be drawn:

1. Relief displacement increases with increase in radial distance to the image.
2. Relief displacement increases with increase in elevation of the point above the datum.
3. Relief displacement decreases with increase in flying height above datum.
4. Relief displacement occurs radially from the principal point.

Relief displacement often causes straight roads, fence lines, etc., on rolling ground to appear crooked on a vertical photograph.

In order to determine the height of the objects, Eq. (9.17) is written as

$$h = \frac{H\delta r}{r} \quad ...(9.18)$$

To determine δr, the distances of the top and bottom of the object, are measured from the principal point. The difference of these distances is δr. Eq. (9.18) is of particular interest to the photo interpreters who are often interested in relative heights of objects rather than absolute elevations.

9.11 FLYING HEIGHT OF A VERTICAL PHOTOGRAPH

Flying height of a vertical photograph may be determined by the following methods:

1. By knowing the scale of photograph and focal length of the camera.
2. By knowing the distance between two points and their elevations.
3. By successive approximations.

9.11.1 By scale of photograph and focal length

Flying height above datum may be obtained by using either Eq. (9.4) or (9.6) if a ground line of known length appears on the photograph. This method yields exact flying heights for truly vertical photographs if the end points of the ground line lie at equal elevations. Accurate results can also be obtained by this methods, however, even though the end points of the ground line are at different elevations, if the images of the end points are approximately equidistant from the principal point of the photograph and on a line through the principal point.

By knowing the ground length of the line and measuring its distance on the photograph, the scale S of the photograph is determined. If the focal length of the camera is f, then from Eq. (9.4), flying height

$$H' = \frac{f}{S}$$

If the elevation h of the line is also known, then from Eq. (9.6), flying height is given by

$$H = \frac{f}{S} + h$$

The errors in computed quantities have been discussed as propagation of errors in Chap. 2 of Plane Surveying. For example, flying height above ground may be calculated using Eq. (9.4), as follows:

$$H' = \frac{f}{S}$$

where $$S = \frac{ab}{AB}$$

Thus, $$H' = f \cdot \left(\frac{AB}{ab}\right) \quad ...(9.23)$$

Assuming the values of f, ab, and AB as 150 mm, 127.08 mm, and 1525 m, respectively, the computed value of H' from Eq. (9.23) is

$$H' = 150 \times \left(\frac{1525}{127.08}\right) = 1800.05 \text{ m}$$

To calculate the error dH' in H' caused by errors $d(AB)$ and $d(ab)$ in measured quantities AB and ab, respectively, Eq. (9.23) is differentiated as under:

$$dH'_{AB} = \frac{\partial H'}{\partial AB} . d(AB) = \frac{f}{ab} . d(AB) \quad ...(9.24)$$

and $$dH'ab = \frac{\partial H'}{\partial ab} . d(ab) = -f . \frac{(AB)}{(ab)^2} . d(ab) \quad ...(9.25)$$

Since, due to the compensating nature of error, total error is not generally the sum of the individual contributing errors. The total combined effect may be calculated as the square root of the sum of the squares of the individual errors as:

$$(dH') = \sqrt{(dH'_{AB})^2 + (dh'_{ab})^2} \quad ...(9.26)$$

Suppose that error $d(AB)$ in AB is ±0.3 m and that the error $d(ab)$ in ab is ± 0.25 mm. The corresponding errors in H' from Eqs. (9.24) and (9.25) are

$$dH'_{AB} = \frac{-150 \times (\pm 0.3)}{127.08} = \pm 0.35 \text{ m}$$

and $$dH'_{ab} = \frac{-150 \times 1525}{(127.08)^2} \times (\pm 0.25) = \pm 3.54 \text{ m}$$

The total error is

$$dH' = \sqrt{(0.35)^2 + (3.54)^2}$$
$$= \pm 3.56 \text{ m}$$

The expected error in the computed value 1800.05 of H' is ±3.56 m.

ILLUSTRATIVE EXAMPLES

Example 9.4 The points A and B which appear in a vertical photograph taken from a camera having focal length of 220 mm and from an altitude of 2800 m, have their elevations as 400 m and 600 m, respectively. Their corrected photo coordinates are as under:

Point	Photo coordinates	
	x (mm)	y (mm)
a	+23.8	+16.4
b	−13.6	−29.7

Determine the length of the ground line AB.

Now comparing this approximate length with the correct ground length we get

$$\frac{H - h_{AB}}{H_x - h_{AB}} = \frac{\text{Correct length}}{\text{Computed length}}$$

$$\frac{H - 400}{2532.24 - 400} = \frac{545}{553.45}$$

$$H = \frac{545 \times (2532.24 - 400)}{553.45} + 400$$

$$= 2499.69 \text{ m}$$

Using the new value of H, the ground coordinates of A and B are again computed as below:

$$X_A = \frac{2499.69 - 500}{20} \times 2.65 = 265.0 \text{ m}$$

$$X_B = \frac{2499.69 - 300}{20} \times (-1.92) = -211.2 \text{ m}$$

$$Y_A = \frac{2499.69 - 500}{20} \times 1.36 = 136.0 \text{ m}$$

$$Y_B = \frac{2499.69 - 300}{20} \times 3.65 = 401.4 \text{ m}$$

Now the ground length

$$L = \sqrt{(-211.2 - 265.0)^2 + (401.4 - 136.0)^2}$$

$$= 545.2 \text{ m}$$

$$\simeq 545 \text{ m}$$

Since the computed value of AB agrees with the correct value within reasonable accuracy the flying height is **2499.69 m = 2500 m(say).**

9.13 OVERLAPS

Vertical aerial photography is usually done along flight strips of suitable width, having some common coverage or overlapping in the successive photographs. The common coverage in the photographs in the direction of flight or photo strip, is called the *end lap, longitudinal overlap* or *forward overlap* (Fig. 9.14). The common coverage between the photographs of two adjacent flight strips, is called the *side lap* or *lateral overlap* (Fig. 9.15).

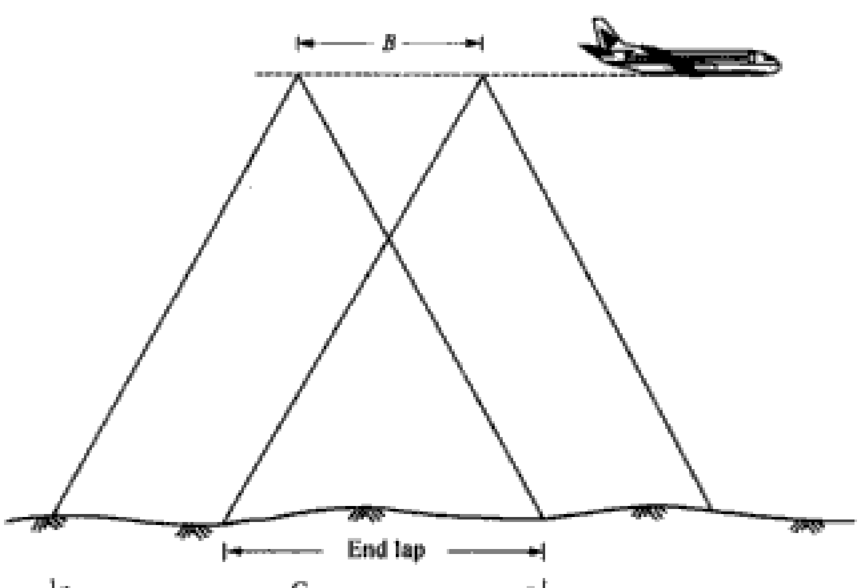

Fig. 9.14 End lap, the overlapping of successive photos along a flight strip

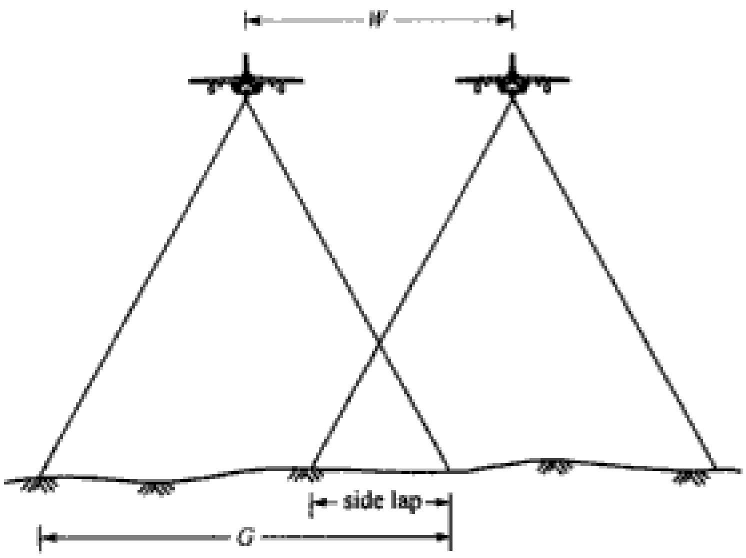

Fig. 9.15 Side lap, the overlapping of adjacent flight strip

If G represent the dimension of the square of ground covered by a single photograph, and B is the distance between two successive exposure stations, called the *air base*, the amount of end lap given in percent is

$$E_{\%} = \left(\frac{G-B}{G}\right)100 \quad \text{...(9.27)}$$

where, $E_{\%}$ = *the percent end lap. If W is the spacing between adjacent flight lines, the amount of side lap in per cent is*

$$S_{\%} = \left(\frac{G-W}{G}\right)100 \quad \text{...(9.28)}$$

where, $S_{\%}$ = the percent side lap.

The end lap is usually kept 55 to 65%, with an average of 60% of the total area covered by a photograph. The side lap generally varies from 25 to 35%, with an average of 30%.

End lap is required for stereoscopic viewing of the photographs whereas side lap is required to prevent gaps occurring between flight strips as a result of drift, crab, tilt, flying height variations and terrain variations. *Drift* is the term applied to a failure of the pilot to fly along planned flight lines (Fig. 9.16). It is often caused by strong winds.

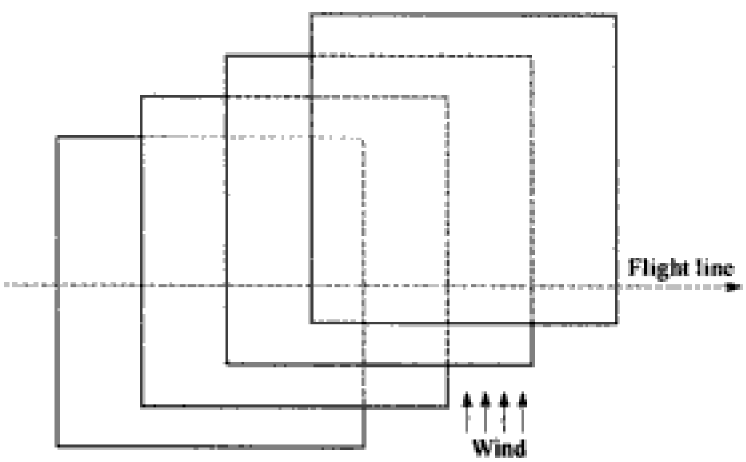

Fig. 9.16 Drift

9.14 COMPUTATIONS OF FLIGHT PLAN

For a flight plan, the following quantities are required to be computed:

1. Number of photographs
2. Number of strips
3. Exposure interval

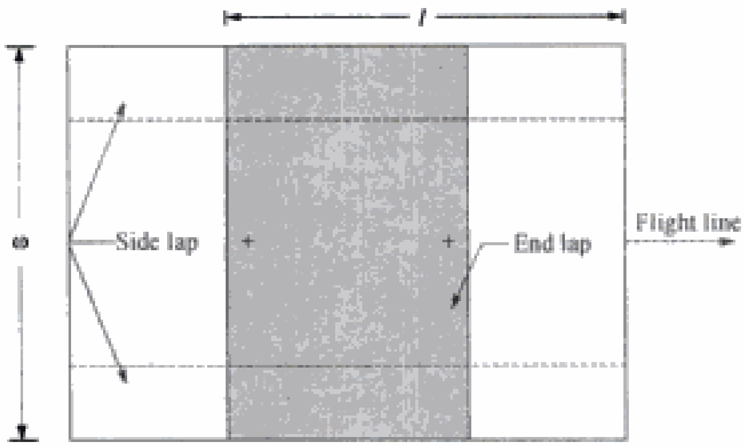

Fig. 9.17 Area covered by overlapping photographs

In Fig. 9.17, let

l = the length of the photograph in the direction of flight,
w = the width of the photograph normal to the direction of flight,
l_e = the end lap between the successive photographs,
l_s = the side lap between the successive flight strips,
L = the net ground distance covered by each photograph in the direction of flight,
W = the net ground distance covered by each photograph in the transverse direction,
A = the total area to be surveyed,
a = the net area covered by each photograph, and
S = the scale of the photograph (f/H).

The scale S of the photograph may be written as

$$S = \frac{l\,(1 - l_e)}{L}$$

or $$L = \frac{l\,(1 - l_e)}{S} \qquad \text{...(9.29)}$$

Also, $$S = \frac{w\,(1 - l_s)}{W}$$

or $$W = \frac{w\,(1 - l_s)}{S} \qquad \text{...(9.30)}$$

The net area covered by each photographs

$$a = L\,W \qquad \text{...(9.31)}$$

Substituting the value of L and W from Eqs. (9.29), and (9.30) in Eq. (9.31). we get

$$= \frac{l\,w\,(1 - l_e)\;(1 - l_w)}{S^2}$$

Therefore, the required number of the photographs is

$$N = \frac{A}{a} = \frac{AS^2}{l\,w(1-l_e)(1-l_s)} \quad ...(9.32)$$

If the length and width of the area to be photographed, are L_0 and W_0, respectively, then the number of photographs (N_1) in each strip, is given by

$$N_1 = \frac{L_0}{L} + 1 \quad ...(9.33)$$

The number of strips (N_2) is given by

$$N_2 = \frac{W_0}{W} + 1 \quad ...(9.34)$$

Actual spacing between the strips

$$d = \frac{W_0}{N_2 - 1} \quad ...(9.35)$$

Total number of photographs for the entire area

$$N' = N_1 \times N_2 \quad ...(9.36)$$

The exposure interval (I) is given by

$$I = \frac{\text{Ground distance between exposures}}{\text{Ground speed of aircraft}} \quad ...(9.37)$$

The ground distance between the exposures is L.

Selection of flying height

One of the main deciding factors for flying height is the contour interval in the photographic mapping. Other factors are scale, relief displacement, and tilt. The accuracy of mapping generally increases with decrease in flying height. The flying height may be expressed as

$$H = C\,.\,\text{contour interval}$$

where C is a factor which varies from 500 to 1500 depending upon the conditions map-compilation process.

ILLUSTRATIVE EXAMPLES

Example 9.10 An area 40 km in the north-south direction and 36 km in the east-west direction, is to be photogrammetrically surveyed. For this, aerial photography is to be made with the following data:

(*i*)	Photographs size	= 20 cm × 20 cm
(*ii*)	Average scale of photographs	= 1 : 15000
(*iii*)	Average elevation of the terrain	= 450 m
(*iv*)	End lap	= 60%
(*v*)	Side lap	= 30%
(*vi*)	Ground speed of aircraft	= 220 km/hr
(*vii*)	Focal length of the camera lens,	= 30 cm

Calculate the following data:

(*a*) Flying height of the aircraft

(*b*) Number of photographs in each flight (i.e., strip)

(*c*) Number of flights (i.e., strips)

(*d*) Total number of photographs
(*e*) Spacing of flight lines
(*f*) Ground distance between exposures
(*g*) Exposure interval

Solution: Fig. (9.17):

Given that

$$S = 1/15000$$
$$f = 30\text{ cm}$$
$$l = 20\text{ cm}$$
$$w = 20\text{ cm}$$
$$L_0 = 40\text{ km}$$
$$W_0 = 36\text{ km}$$
$$h = 450\text{ m}$$
$$l_e = 0.6$$
$$l_w = 0.3$$
$$V = 220 \times \frac{1000}{3600}\text{ m/sec}$$

(*a*) Flying height

From Eq. (9.6), we have

$$S = \frac{f}{H - h}$$

or

$$H = \frac{f}{S} + h$$
$$= \frac{30 \times 15000}{100} + 450$$
$$= \mathbf{4950\ m}.$$

(*b*) Number of photographs in each flight (i.e., strip)

From Eq. (9.33), we have

$$N_1 = \frac{L_0}{L} + 1 \quad \ldots(a)$$

where $L_0 = 40\text{ km}$.

From Eq. (9.29), we have

$$L = \frac{l\,(1 - l_e)}{S}$$
$$= \frac{20 \times (1 - 0.60) \times 15000}{100} = 1200\text{ m}$$

Thus,

$$N_1 = \frac{40 \times 1000}{1200} + 1 = 34.3 = \mathbf{35}.$$

(*c*) Number of flights (i.e., strips)

From Eq. (9.34), we have

$$N_2 = \frac{W_0}{W} + 1 \quad \ldots(b)$$

where $W_0 = 36\text{ km}$.

From Eq. (9.30), we have

$$W = \frac{w(1 - l_w)}{S}$$

$$= \frac{20 \times (1-0.30) \times 15000}{100} = 2100 \text{ m.}$$

Thus, $$N_2 = \frac{36 \times 1000}{2100} + 1 = 18.1 = \mathbf{19}.$$

(*d*) Total number of photographs

From Eq. (9.36), we have

$$N' = N_1 \times N_2$$
$$= 35 \times 19 = \mathbf{665}.$$

(*e*) Spacing of flight lines

From Eq. (9.35), we have

$$d = \frac{W_0}{N_2 - 1}$$
$$= \frac{36000}{19-1} = \mathbf{2000\ m}.$$

(*f*) Ground distance between exposures is

$$L = 1200 \text{m}$$

(*g*) Exposure interval

From Eq. (9.37), we have

$$I = \frac{L}{V}$$
$$= \frac{1200 \times 3600}{220 \times 1000} = 19.6 \text{ secs.}$$

9.15 GROUND CONTROL FOR AERIAL PHOTOGRAMMETRY

The ground points appearing in a photograph are required to be linked to the ground points of known locations. To achieve this, there must be few points appearing in photograph whose coordinates with respect to the ground reference coordinates system, are known. Ground control is also required for the orientation of the photographs in space relative to ground. The number of ground control points depends upon scale of the map, navigational control, and cartographical process of mapping.

Ground control for aerial photogrammetry is classified as:

1. Horizontal control
2. Vertical control.

The planimetric control is provided by normal methods of traversing, triangulation, or trilateration. The elevation of the ground control points may be determined by trigonometric levelling. If better accuracy in heights is aimed at, the control points are connected by network of differential levelling.

The position of ground control points may be obtained on the photographs by *post-pointing* or *pre-pointing*.

In Fig. 9.18, G_1 and G_2 are the existing ground control stations with reference to which a network of basic control points, B_1, B_2, B_3, B_4, and B_5, has been established. These basic control points are then used to generate photo control points P_1 and P_2.

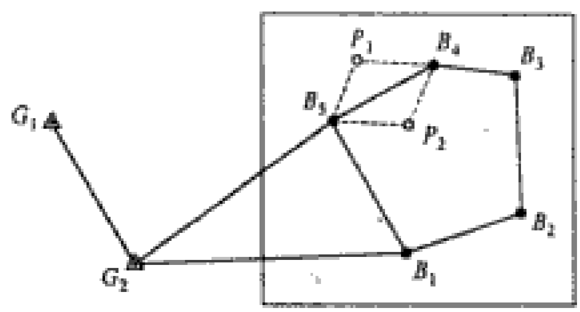

Fig. 9.18 Ground control for aerial photogrammetry

9.16 PLANIMETRIC MAPPING

The methods commonly employed for planimetric mapping are:

1. Direct tracing method
2. Tracing with instruments
3. Radial-line triangulation.

In direct tracing method, a number of photo control points are plotted on a tracing sheet of mylar or any other suitable material, at the scale of the photograph. The tracing of details is done by matching the positions of the plotted photo control points with their corresponding photo images. At a time three photo images forming a triangle are usually selected. The plotted map may be enlarged reduced, if necessary.

For tracing with instruments, projection instruments such as shown in Fig. 9.19, are used. In this method also the positions of the plotted photo control points at selected scale, are matched with the corresponding photo images projected on the drawing sheet having the plotted position of the photo control points.

The radial-line triangulation method is based on the principle that the angles which have vertices at the principal point of a vertical photographs, are truly horizontal angles. The positions of photo exposure stations are located by the method of resection and positions of various points are located by the method of intersection by drawing rays from two or more exposure stations already plotted.

With the advent of digital computers, the radial-line triangulation can be performed by numerical methods.

Planimetric methods are not as accurate as those methods which use stereoplotting instruments. Planimetric methods are generally employed for small areas. The accuracy is considerably increased in numerical methods.

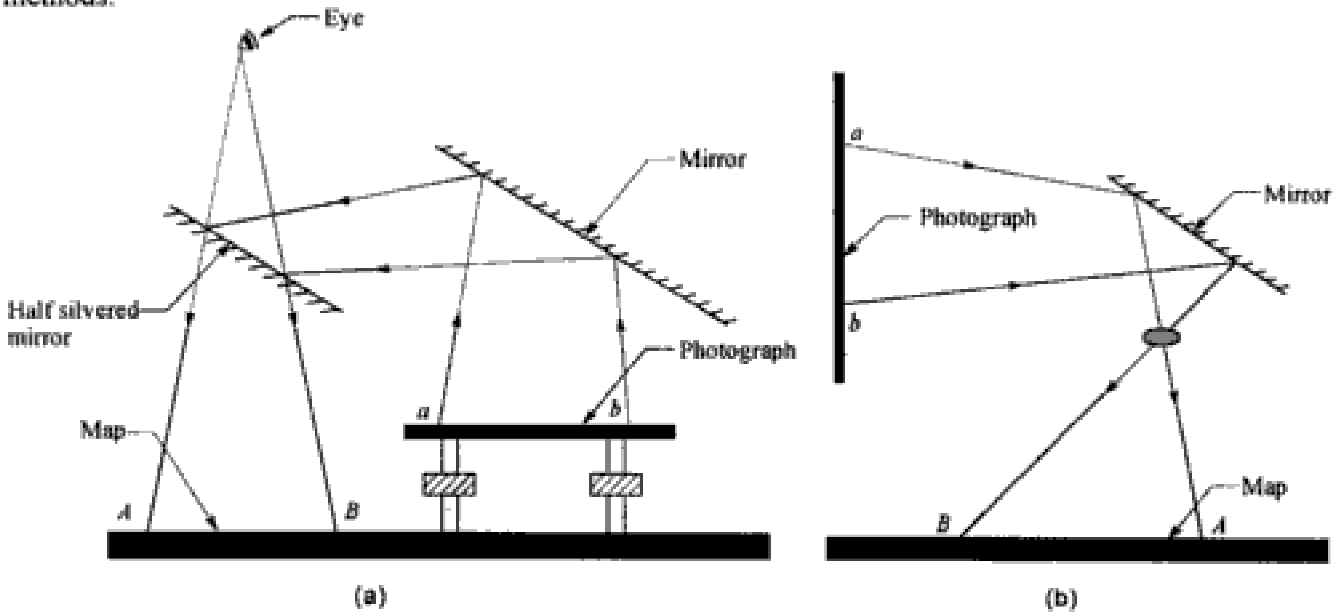

Fig. 9.19 Projection instruments (a) Vertical sketchmaster; (b) Reflecting projector

9.17 PHOTOMAPS AND MOSAICS

Photomaps and mosaics are used as map substitutes. The *photomap* may consist of one photographa, but the photomaps obtained by assembling two or more photographs to form a single picture of an area, is called the *mosaic*. Mosaics are similar to maps in many respect. They have a number of advantages over maps. They show relative planimetric locations of an infinite number of objects. The objects are easily recognized by their pictorial qualities, whereas objects on maps, which are shown with symbols, are limited in number. Mosaics of large areas can be prepared in much less time and at considerably lower cost than map. Mosaics are easily understood and interpreted by people without having knowledge of photogrammetry or engineering.

The disadvantage which the mosaics have, is that they are not true planimetric representations. They suffer from image displacements and scale variations, and cannot be used for quantitative analysis.

9.18 STEREOSCOPIC VISION

Methods of judging depth of the objects from the observer may be either *stereoscopic* or *monoscopic*. Persons with normal vision, i.e., capable of viewing with both eyes simultaneously, are said to have *binocular* vision and perception of depth through binocular vision is called *stereoscopic viewing. Monocular vision* is the term applied to viewing with only one eye, and methods judging distances with one eye are termed *monoscopic*.

9.19 STEREOSCOPIC DEPTH PERCEPTION

With binocular vision, an object is viewed simultaneously by two eyes and the rays of vision converge at an angle called the *parallactic angle* or *angle of parallax*. Nearer the object, the greater the parallactic angle and *vice versa*. In Fig. 9.20, the optical axis of the two eyes L and R are separated by a distance b, called the *eye base*. When the eyes are focussed on point A, the optical axes coverage, forming parallactic angle ϕ_A. Similarly, when sighting point B, the parallactic angle formed, is ϕ_B. The brain automatically and unconsciously associates distances D_A and D_B with corresponding parallactic angles ϕ_A and ϕ_B, respectively. The depth $(D_B - D_A)$ between A and B, is perceived as the difference $\delta\phi$ in the two parallactic angles ϕ_B and ϕ_A. This difference is called the *differential parallax*.

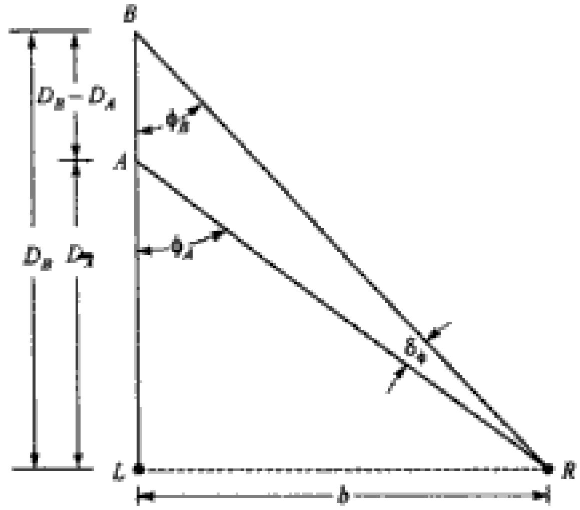

Fig. 9.20 Stereoscopic depth perception and parallactic angle

The shortest distance of clear stereoscopic depth perception for the average adult is about 25 cm and the eye base is between 63 and 69 mm. The maximum parallactic angle formed by the eyes, assuming 25 cm as the minimum focusing distance and average eye base as 66 mm, is therefore approximately:

$$\phi = \tan^{-1}\left(\frac{66}{250}\right) = 15°$$

The maximum distance at which the stereoscopic depth perception is possible for the average adult, is approximately 600 m. Beyond that distance parallactic angles become very small and changes in parallactic angle necessary for depth perception, cannot be discerned. A man with normal vision can discern parallactic angle changes even up to 3″ of arc. Some persons can discern the changes in parallactic angle of even 1″.

9.20 VIEWING PHOTOGRAPHS STEREOSCOPICALLY

Stereoscopic fusion is a process in which the right images seen by right eye and the left images seen by left eye of the same objects, are fused in the brain to provide the observer with a three-dimensional model called the *stereoscopic model* or *stereomodel*. Figs. 9.21(a) and 9.21(b) show pairs of right and left images. By holding a card between a pair of images, it becomes possible to view closely, the right image by right eye and left image by left eye and seeing the images without blinking the eyes for some time, one can get spatial impression of the objects indicating that the one dot is closer to the observer than the other one. In Fig. 9.21 (b), the view will show a solid pyramid with the apex standing above the base.

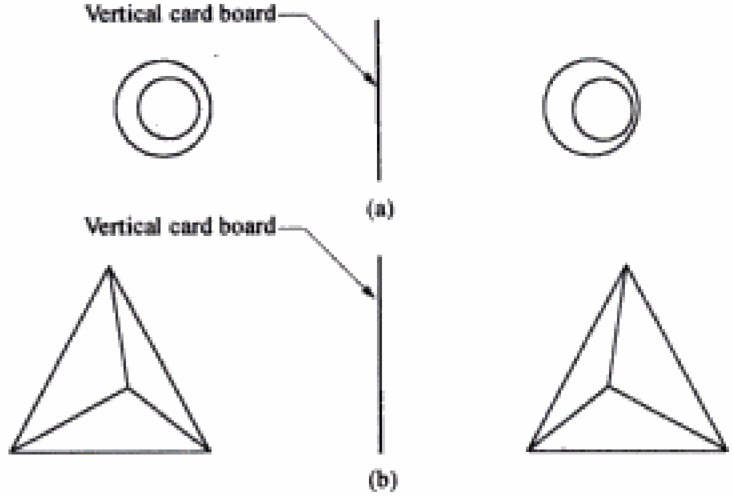

Fig. 9.21 Stereoscopic fusion.

9.21 STEREOSCOPE

A pair of adjacent photographs with end lap, is known as a *stereopair*. The instrument used to view the left images by left eye and right images by right eye, in a stereopair, is called the *stereoscope*. They are of two types:

(*i*) Lens stereoscope or pocket stereoscope

(*ii*) Mirror stereoscope.

A lens stereoscope, shown in Fig. 9.22, consists of two identical convex lenses mounted on a frame with inclined legs. Pocket stereoscopes are cheap and best suited for small photographs.

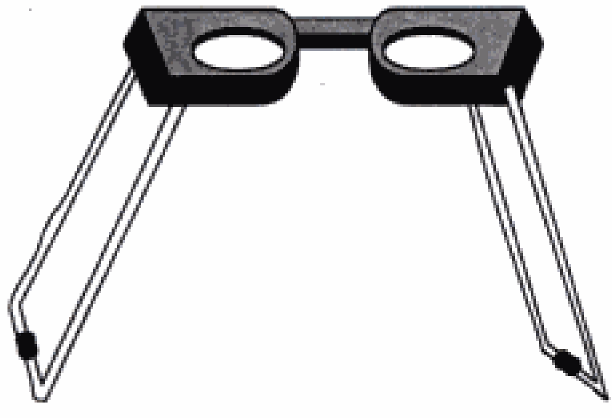

Fig. 9.22 Lens stereoscope

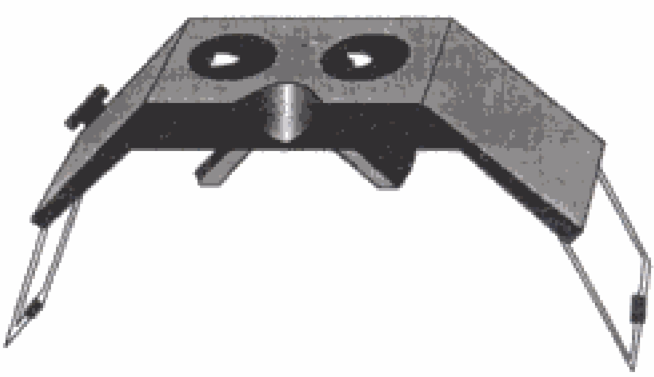

Fig. 9.23 Mirror stereoscope

Fig. 9.23 shows a mirror stereoscope. It consists of mainly two pairs of mirrors inclined at 45° to the plane of photographs with their reflecting surfaces facing each other and two convex lenses. The operating principle of a mirror stereoscope is shown in Fig. 9.24.

The mirror stereoscope permits the two photos to be completely separated when viewing stereoscopically, eliminating the problem of one photo obscuring part of the overlap of the other. It also enables the entire stereomodel to be viewed simultaneously.

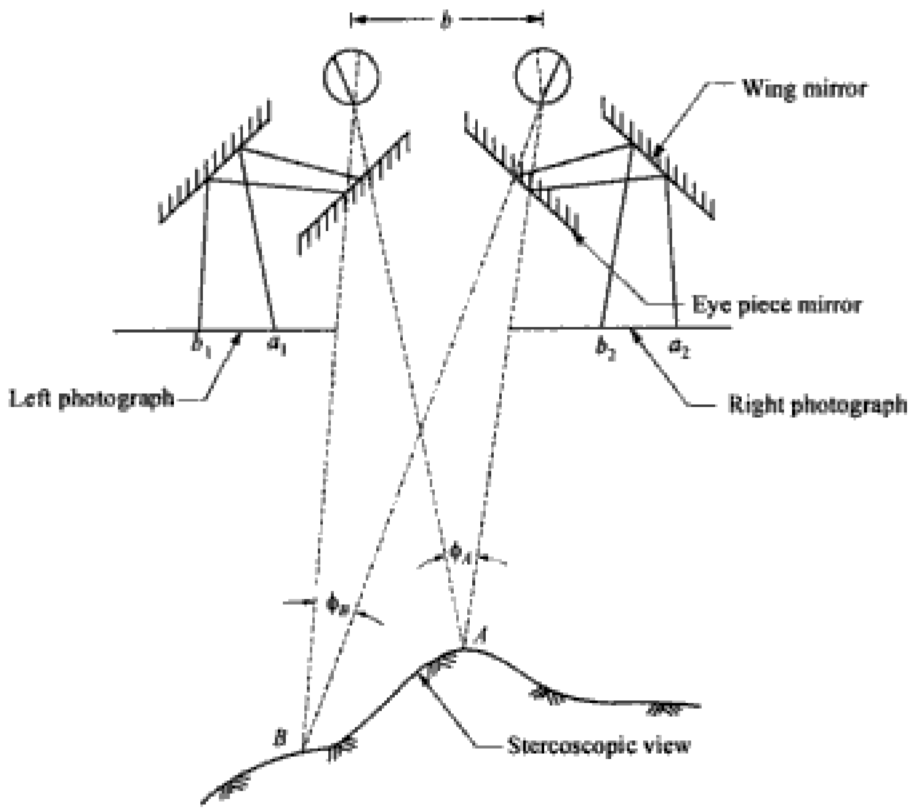

Fig. 9.24 Operating principle of the mirror stereoscope

9.22 PARALLAX IN AERIAL STEREOSCOPIC VIEWS

Fig. 9.25 shows two points A and B which have been photographed as a and b, respectively, in left photograph and as a' and b', respectively, in right photograph, from two aerial camera positions O and O'. p_1 and p_1' are the transferred principal points of p and p'. Oa'' and Ob'' are the lines drawn parallel to the lines $O'a'$ and $O'b'$, respectively.

Since the aircraft taking the photographs is moving at certain velocity, after taking left photograph from O, it has moved to the position O' from where the right photograph has been taken. During this movement the images of A has moved a distance aa'' and that of B, a distance bb''. This displacement of the image on two successive exposures, is called the *parallax*. Since this displacement is along the x-axis, which is assumed to be in the direction of flight, it is known as *x-parallax*. In this case there is no displacement of the images along the y-axis, and hence the *y-parallax* is zero. If y-parallax exists in photographs, it causes eyestrain and prevents comfortable stereoscopic viewing.

It may be observed that the parallax of the higher point B is more than the parallax of the lower point A. Thus, each image in a varying terrain elevation has a slightly different parallax from that of a neighbouring image. This point-to-point difference in parallax exhibited between points on a stereopairs makes possible the viewing of the photographs stereoscopically to gain an impression of a continuous three-dimensional model of the terrain.

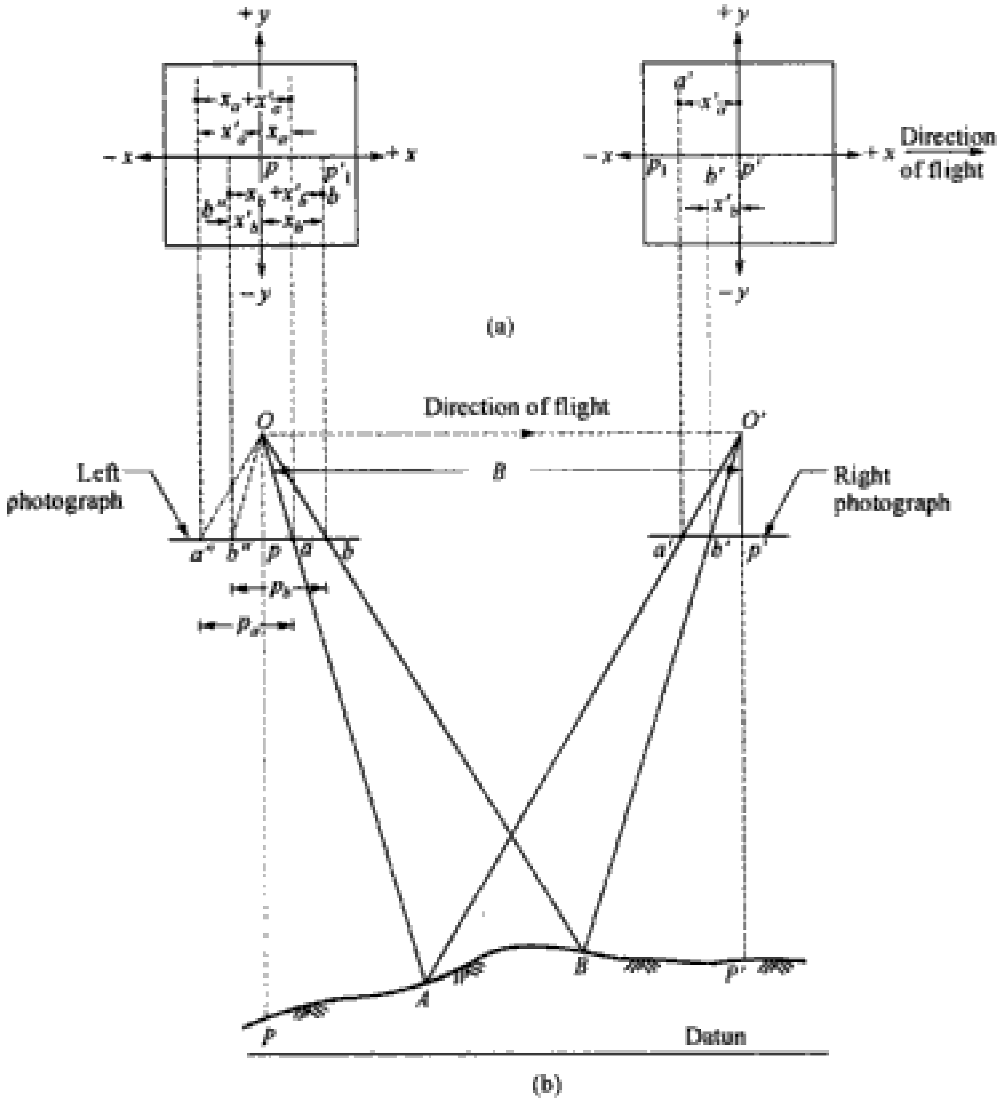

Fig. 9.25 Parallax in aerial stereoscopic views

Algebraic definition of parallax

The x-coordinates of a on the left photograph and that of a' on the right photograph are x_a and $-x'_a$, respectively. Since a'' has been located by drawing the line Oa'' parallel to the line $O'a'$, the x-coordinate of a'' is equal to $x'_{a'}$. The total displacement of the image of A is aa'' which is the x-parallax p_a of A. From the figure we find that,

$$p_a = pa + pa'' = x_a + (-x'_a)$$

$$= x_a + x'_a$$

= Algebraic sum of x-coordinates of left image a and right image a' of A.

Similarly, $$p_b = x_b + x'_b$$

= Algebraic sum of x-coordinates of left image b and right image b' of B.

In general $$p = x + (-x') \quad \text{...(9.38)}$$

Thus, on a pair of overlapping photographs, the parallax of a point is equal to the algebraic sum of the x-coordinates of that point on the left and right photographs.

9.23 SPACE-COORDINATE EQUATIONS

Space-coordinate equations or *parallax equations* are the equations which give ground coordinates of points in terms of stereoscopic parallaxes of the points. Fig. 9.26 shows the ground coordinates x and y with origin at the datum principal point of the left hand photograph and x-axis being in the same vertical plane which contains the line of flight. Let the elevation of A be h and the air base be B.

From similar triangles Opm and OM_0M

$$\frac{f}{H-h} = \frac{x}{X}$$

or $$X = \frac{(H-h)}{f}\,x \qquad ...(9.39)$$

From similar triangles Opn and OM_0N, we have

$$\frac{f}{H-h} = \frac{y}{Y}$$

or $$Y = \frac{(H-h)}{f}\,y \qquad ...(9.40)$$

From similar triangle $O'p'm'$ and $O'M_0'M$, we have

$$\frac{f}{H-h} = \frac{-x'}{B-X}$$

or $$X = B + \frac{(H-h)x'}{f} \qquad ...(9.41)$$

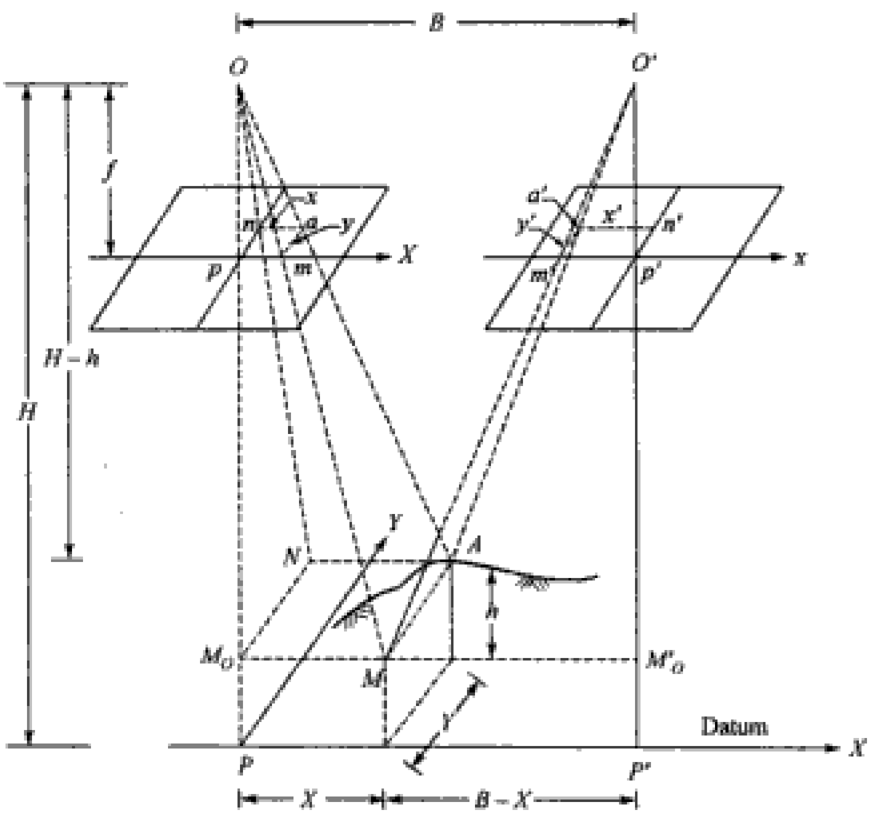

Fig. 9.26 Ground coordinates by parallax

By equating Eqs. (9.39) and (9.41), we get

$$\frac{(H-h)x}{f} = B + \frac{(H-h)x'}{f}$$

or
$$B = \frac{(H-h)(x-x')}{f} \quad ...(9.42)$$

From Eq. (9.38), we have

$$x - x' = p$$

Thus, Eq. (9.42) becomes

$$B = \frac{(H-h)\,p}{f}$$

or
$$h = H - \frac{B\,f}{p} \quad ...(9.43)$$

Substituting the value of h from Eq. (9.43) in Eqs. (9.39) and (9.40), we get:

$$X = B\,\frac{x}{p} \quad ...(9.44)$$

$$Y = B\,\frac{y}{p} \quad ...(9.45)$$

Eqs. (9.44) and (9.45) are extremely useful to photogramist for topographic survey using stereopairs. The ground coordinates obtained from these equations refer to the unique arbitrary coordinate system of the stereopair and they can be transformed to the absolute ground coordinate system.

9.24 DIFFERENCE IN ELEVATION BY STEREOSCOPIC PARALLAXES

If two points A and B have elevations h_A and h_B, and their parallaxes are p_a and p_b, respectively, then the difference in elevations of A and B is given as

$$\Delta h = h_B - h_A$$

and the difference in parallaxes is

$$\Delta p = p_b - p_a$$

From Eq. (9.43), we have

$$h_A = H - \frac{B\,f}{p_a} \quad ...(9.46)$$

$$h_B = H - \frac{B\,f}{p_b}$$

Therefore,
$$\Delta h = B\,f\,\frac{(p_b - p_a)}{p_a\,p_b}$$

or
$$\Delta h = \frac{B\,f\,\Delta p}{p_a\,(p_b + \Delta p)} \quad ...(9.47)$$

In Fig. 9.27, O_1O_2 is the air base B, p_1p_2' and $p_1'p_2$ are the principal bases of left and right photographs, respectively. Since the ground principal points P_1 and P_2 are at same elevations, p_1p_2' and $p_1'p_2$ will be equal to the parallax of principal points or principal base b.

From similar triangles $O_1p_1p_2'$ and $O_1P_1P_2$, we get

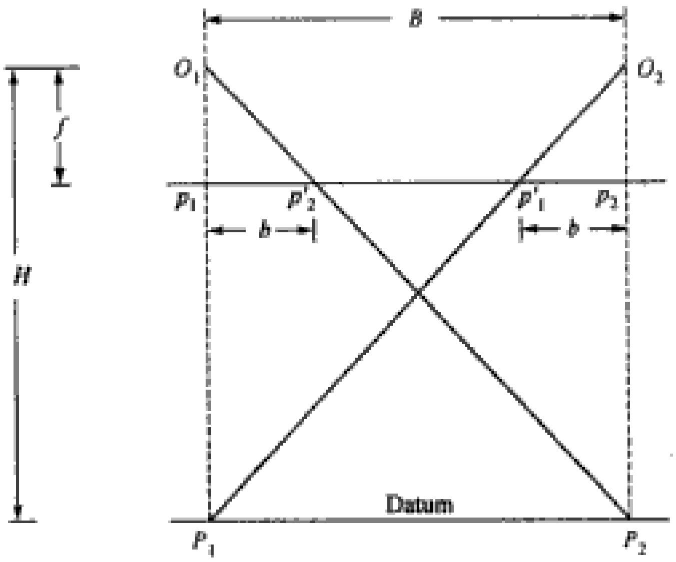

Fig. 9.27 Relation between air base and principal base

If P_1 and P_2 are at different elevations, p_1p_2' and $p_1'p_2$ will not be equal and then

$$p_1p_2' = b_1$$

$$p_1'p_2 = b_2$$

Therefore, if *mean principal base* is b_m, then

$$b_m = \frac{b_1 + b_2}{2}$$

Taking $b_m = b$, Eq. (9.48) becomes

$$\frac{f}{H} = \frac{b_m}{B}$$

or

$$Bf = H\,b_m \qquad ...(9.49)$$

Substituting the value of Bf from Eq. (9.49) in Eq. (9.47), we get

$$\Delta h = \frac{H\,b_m\,\Delta p}{p_a\,(p_a + \Delta p)} \qquad ...(9.50)$$

By taking $b_m = p_a$, Eq. (9.50) may be written as

$$\Delta h = \frac{H\,\Delta p}{b_m + \Delta p} \qquad ...(9.51)$$

with the following assumptions:

(*a*) the ground point (*A*) and two principal point P_1 and P_2 are at same elevation, and

(*b*) the flying height (*H*) is measured above the elevation of the point *A*, Eq. (9.51) may be written as

$$\Delta h = \frac{H\,\Delta p}{b_m + \Delta p} = \frac{H\,\Delta p}{p_a + \Delta p} \qquad ...(9.52)$$

From Eq. (9.46), we have

$$Bf = (H - h_a)\,p_a$$

Hence Eq. (9.52) becomes

$$\Delta h = \frac{(H - h_A)\Delta p}{(p_a + \Delta p)} \qquad ...(9.53)$$

Taking $p_a = b_m$, Eq. (9.53) becomes

$$\Delta h = \frac{(H - h_A)\Delta p}{(b_m + \Delta p)} \qquad ...(9.54)$$

ILLUSTRATIVE EXAMPLES

Example 9.11 In a pair of overlapping vertical photographs the mean distance between two principal points lying on the datum is 6.385 cm. The flying height of the aircraft at the time of photography, was 580 m above the datum. Determine the difference of parallax for top and bottom of a tower of height 115 m having base in the datum surface. The focal length of the camera is 150 mm.

Solution: From Eq. (9.48), we have

$$B = \frac{b}{f} H$$

$$= \frac{6.385}{15.0} \times 580 = 246.89 \text{ m}.$$

From Eq. (9.43), parallax is given by

$$p = \frac{B f}{H - h}$$

For the bottom of the tower, $h = 0$. Hence

$$P_B = \frac{246.89 \times 150}{580} = 63.85 \text{ mm}$$

For the top of the tower, $h = 115$ m. Hence

$$P_T = \frac{246.89 \times 150}{(580 - 115)} = 79.64 \text{ mm}$$

The difference of parallax is given by

$$\Delta p = P_T - P_B = 79.64 - 63.85 = \mathbf{15.79\ mm}.$$

The result can be checked from Eq. (9.51) as below:

$$\Delta h = h_T - h_B = \frac{H \Delta p}{b_m + \Delta p}$$

$$= \frac{580 \times 15.79}{(63.85 + 15.79)}$$

$$= 115 \text{ m (the given value).}$$

Example 9.12 The mean principal base of a pair of photographs taken from an altitude of 5500 m above mean sea level is 92 mm. The difference of parallax between two points is 1.50 mm. If the elevation of the lower point is 480 m above the mean sea level, what is the elevation of higher point?

Solution:

It is given that

$$b_m = 92 \text{ mm}$$

$$H = 5500 \text{ m}$$

$$\Delta p = p_T - p_B = 1.50 \text{ mm}$$

$$h_B = 480 \text{ m}$$

9.28.2 Quantitative Characteristics

The areas, distances, slopes, heights, etc., are the properties which can be measured by normal methods and therefore they are said to be quantitative characteristics.

9.29 CHARACTERISTICS OF PHOTOGRAPHIC IMAGES

Small scale aerial photographs present unfamiliar aspects of the object images. The photointerpreter should learn to pay special attention to the following characteristics of the photographic images. These characteristics help the interpreter in identifying and recognising the objects whose images appear in the aerial photographs.

Shape: This refers to the general form, configuration, or outline of individual objects. In the case of stereographic photographs, the height of the object also defines its shape. Therefore, to facilitate easy recognition objects in profile, the interpreter must use stereographic photographs under stereoscope. The top view of the objects in vertical photography is quite unfamiliar and difficult to interpret, and one requires special training to acquire expertise in studying top views.

Size: It is one of the most important and useful clues to identify the objects. By measuring an unknown objects, interpreter can arrive at more accurate identification. The size of objects on photographs must be considered in the context of the scale. For example, an irrigation ditch and antitank ditch images look alike but differ in size.

Shadows: These are important for two opposing reasons: (1) the shape or outline of a shadow affords the profile view of the objects which aids the interpretation; and (2) objects within shadows reflect little light and difficult to discern on photographs. This hinders interpretation. As an example for the first, shadows cast by towers, bridges, and certain species of trees can aid interpretation.

Pattern: This relates to the spatial arrangement of objects. The repetition of certain general forms is characteristics of many natural and man-made objects. This is what constitutes a pattern that aids recognition. For example, the ordered spatial arrangement of trees in an orchard is distinctly different from that of groups of trees in a forest.

Tone (or hue): This refers to the relative brightness or colour of objects on photographs. In aerial photographs which are generally in black and white, the objects are observed in tones of grey. The tones of photographic images are influenced by many factors. It may be clearly understood by the interpreter that a body of water may appear on a photograph in tone ranging from white to black. Lighter-toned areas are topographically higher and drier while darker toned areas are lower and wetter.

Texture: The texture in aerial photograph is created by tonal repetitions in group of objects too small to be discerned. As the scale of the photograph is reduced, the texture of the object or area become progressively finer. An example would be the smooth texture of green grass as contrasted with the rough texture of green tree crowns on medium scale aerial photographs.

Site: This refers to topographic or geographic locations and is particularly valuable in the identification of types of vegetation.

Association: This refers to the occurrence of certain features in relation to others. For example, a Ferris wheel, which might be difficult to identify if standing in a field near a barn, would be easily identified if it is in an amusement park.

9.30 TECHNIQUE OF PHOTOINTERPRETATION

Most photointerpretation work can be done using:

1. a stereoscope for 3-D viewing,
2. simple plastic or boxwood rule as measuring device, and
3. a coloured pencil for making on the photographs.

PROBLEMS

9.1 What is photogrammetry? Discuss its limitations.

9.2 Differentiate between:

(*i*) Meteric photogrammetry and interpretive photogrammetry.

(*ii*) Terrestrial photogrammetry and aerial photogrammetry.

9.3 What are different types of aerial photographs? Discuss each briefly giving their uses.

9.4 What are the methods of measuring coordinates of photo images?

9.5 Define and explain the following terms giving a neat sketch:

(*i*) Exposure station.

(*ii*) Flying height

(*iii*) Principal point

(*iv*) Principal line

(*v*) Isocentre

(*vi*) Tilt

(*vii*) Swing

(*viii*) Azimuth.

9.6 Derive expressions for various types of scales of a vertical photograph.

9.7 Differentiate clearly between the following:

(*i*) Point scale.

(*ii*) Datum scale

(*iii*) Average scale.

9.8 How would you determine the scale of a vertical photograph?

9.9 Discuss other methods of determining scale of vertical photographs.

9.10 Discuss method of determining ground coordinates from the photo coordinates.

9.11 What do you understand by relief displacement on a vertical photograph? Derive an expression for its determination.

9.12 What are various methods of determining flying height? Discuss the method which gives most accurate value of flying height.

9.13 What are different types of overlaps? What is the purpose of providing them in aerial photographs?

9.14 Write a short note on ground control for aerial photogrammetry.

9.15 Discuss various methods of planimetric mapping from vertical aerial photographs.

9.16 Write a short note on photomaps and mosaics.

9.17 Explain the principle of stereoscopic depth perception.

9.18 Discuss briefly the following:

(*i*) Mirror stereoscope.

(*ii*) Lens stereoscope.

9.19 What do you mean by parallax in aerial stereoscopic views?

9.20 Derive an expression to determine the difference in elevation from parallax.

9.21 Explain the working and use of parallax bar.

9.22 What is floating mark? Explain its principle.

9.23 What do you mean by aerial photographic interpretation?

9.24 Write a short note on characteristics of photographic images.

9.25 Write a short note on technique of photointerpretation.

9.26 Write a detailed note on the application of aerial photographic interpretation.

9.27 Determine the following for a vertical photograph of an area where terrain elevations vary linearly from 200 m to 500 m. The flying height above mean-sea level is 4500 m. The focal length of the camera is 150 mm.

(*a*) Maximum scale.

(*b*) Minimum scale

(*c*) Average scale

(*d*) Datum scale.

9.28 The photo coordinates of the images *a* and *b* of two points *A* and *B* measured on of vertical photograph taken at flying height of 1600 m from a camera having lens of focal length 150 mm, are as follows:

Points	Coordinates	
	x (mm)	*y* (mm)
a	– 46.35	– 48.20
b	+ 38.48	+ 41.62

The elevations of *A* and *B* above the mean-sea level are 140 m and 220 m, respectively. Determine the distance *AB*.

9.29 The vertical photograph of a chimney was taken from an elevation of 700 m above mean-sea level. The elevation of the base of the chimney is 300 m. If the relief displacement of the chimney is 45.1 mm and the radial distance of the image of the top of the chimney from the principal point of the photograph is 120 mm, determine the height of the chimney.

9.30 One kilometer straight length of *AB* of a road has the change of elevation from 200 to 250 above M.S.L. on a vertical photograph taken with a camera having focal length of 200 mm. The photo coordinates of *A* and *B* are:

x_a = + 25.0 mm $\qquad$ y_a = + 12.5 mm

x_b = – 22.5 mm $\qquad$ y_b = + 40.0 mm

If the portion of the road measured directly from the photograph is 80 mm, calculate the flying height of the aircraft.

9.31 An area 30 km long in north-south direction and 24 km in the east-west direction is to be photographed with a camera of focal length 300 mm. The photograph size is 20 cm × 20 cm. The average scale is to be 1 : 12,000 effective at an elevation of 400 m above datum. End lap is to be at least 60% and the side lap is 30%. The ground speed of the aircraft is 200 km per hour. The flight lines are to be laid in a north-south direction on an existing map at scale 1 : 60,000. The two outer flight lines are to coincide with the east and west boundaries of the area. Calculate the data for flight plan.

9.32 A pair of photograph was taken with an aerial camera from an altitude of 4000 m above M.S.L. The mean principal base measured is 88 mm. The difference in parallax between two points is 1.50 mm. Determine the difference in height between the two points if the elevation of the lower point is 450 m. above M.S.L. Calculate the difference in elevation corresponding to the parallax difference of 1.90 mm.

10

REMOTE SENSING

10.1 GENERAL

Remote sensing is a process of obtaining information about an object, area, or phenomenon through the analysis of data by a device without being in contact with the object, area, or phenomenon being studied. It is a methodology employed to study from a distance the physical and chemical characteristics of objects. Human sight, smell, and hearing are examples of rudimentary forms of remote sensing. Photographic interpretation is considered a form of remote sensing, however, it is generally limited to a study of images recorded on photographic emulsions sensitive to energy in or near the visible portion of the electromagnetic spectrum. Remote sensing discussed in this chapter treats sensor systems which record energy in more quantifiable formats over a much broader range of the electromagnetic spectrum. Most of the remote sensing methods make use of the reflected infrared band, thermal infrared band, and microwave portions of the electromagnetic spectrum.

10.2 NECESSITY AND IMPORTANCE

With growing population and rising standard of living, pressure on natural resources has been increasing day by day. It, therefore, becomes necessary to manage the available resources effectively and economically. It requires periodic preparation of accurate inventories of natural resources both renewable and non-renewable. This can be achieved through remote sensing very efficiently since it provides multispectral-multitemporal data useful for resources inventory, monitoring and their management.

10.3 APPLICATIONS AND SCOPE

Remote sensing is being used to collect the information about agriculture, forestry, geography, archeology, weather and climate, marine environment, hydrology, water resources management and assessment, engineering. etc. It has vast applications in exploration of natural resources, analysis of land use and land cover, information about environments, natural hazard studies such as earthquake, land slide, land subsidence, flood, etc.

10.4 ELECTROMAGNETIC ENERGY AND ELECTROMAGNETIC SPECTRUM

Electromagnetic energy is a form of energy which moves with velocity of light in a harmonic pattern consisting of sinusoidal waves of varying wavelengths. Remote sensing makes use of electromagnetic radiation which is not visible to human eye.

Electromagnetic energy is detected only when it interacts with matter. For example, light is seen in dark only when electromagnetic radiation interacts with dust and other particles present in air.

The changes in electromagnetic energy take place when it interacts with the earth's surface and environment. Remote sensing detects these changes and the data obtained is used for determination of the characteristics of the earth's surface.

Electromagnetic waves can be described in terms of three basic parameters:

1. Velocity (c)
2. Wavelengths (λ)
3. Frequency (f).

The following relationship exists between the above three parameters:

$$\lambda f = c = 3.8 \times 10^8 \text{ m/sec}$$

where λ is in metres and f is in hertz.

Electromagnetic spectrum ranges from most energetic γ rays at wavelength less than 10^{-13} m to very long radio waves at wavelength longer than 100 km. Various components of an electromagnetic spectrum with their wavelengths are given below:

1. γ and X rays	Up to 10^{-8} m wavelength region.
2. Ultraviolet	From 10^{-9} to 10^{-7} m.
3. Visible region	(0.4 – 0.7) μm is only one of many forms of electromagnetic energy and mostly used to acquire remotely sensed data for natural resources mapping. This wavelength interval is generally referred to a 'light'.
4. Near –, Middle – Thermal, Far – Infrared	From 0.7 to 20 μm : Near Infrared – (0.7 – 1.3) μm, Middle Infrared – (1.3 – 3.0) μm, Thermal Infrared – (3.0 – 14.0) μm, and Far Infrared – (7.0 – 15.0) μm.
5. Microwave region	Down to a wavelength of 1 m.
6. Radio waves	Wavelengths longer than 1 m.

There are certain regions of electromagnetic spectrum which can penetrate through the atmosphere without any significant loss of radiations. Such regions are called *atmospheric windows.*

Electromagnetic radiations are affected by atmospheric effects known as *scattering* and *absorption.*

10.5 INTERACTION OF ELECTROMAGNETIC ENERGY WITH MATTER

Electromagnetic energy does not interact with itself. It is perceived or sensed through its interaction with matter. When radiated electromagnetic energy strikes any object, it can interact with the object in any or all of the following five ways (Fig. 10.1):

1. Reflection
2. Transmission
3. Absorption
4. Emission
5. Scattering.

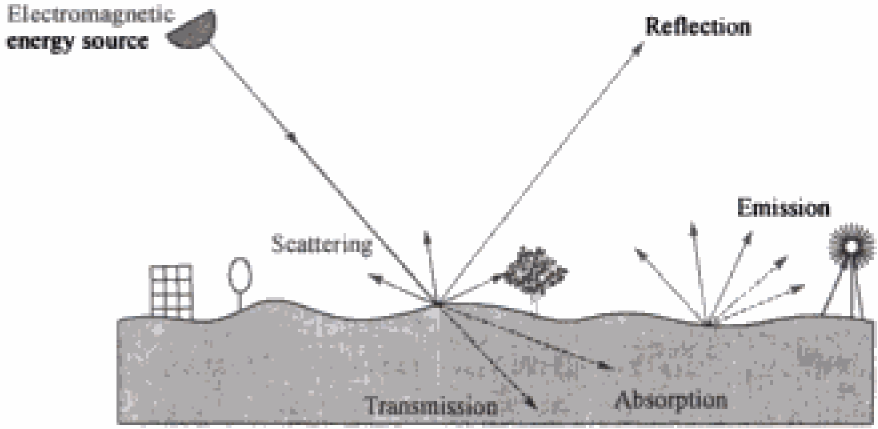

Fig. 10.1 Interaction of electromagnetic energy with matter

The extent and manner of interaction of electromagnetic energy with a given object is dependent upon the physical-chemical nature of the object. Different energy interactions that result from different types of matter, can be discerned by using various types of remote sensors, and this enables different objects to be identified and in some cases quantified.

Interaction of electromagnetic energy with any object is selective by wavelength, i.e., different interactions occur for different wavelengths of energy. Because of the dependency of energy interaction on wavelength, remote sensors have been designed which respond to different wavelengths, a technique which aids in discriminating between different objects.

Ability to define a feature or a surface according to the spectral response is called *spectral signature* of the feature (analogous to the human fingerprints). When the features are identified by a distinctive spectral responses, it does not imply that this identification is necessarily unique.

10.6 AN IDEAL REMOTE SENSING SYSTEM

An ideal remote sensing system shown in Fig. 10.2, consists of the following basic stages:

1. Electromagnetic energy source
2. Energy propagation
3. Energy interaction
4. Return signal
5. Recording
6. Supply of information in the desired form.

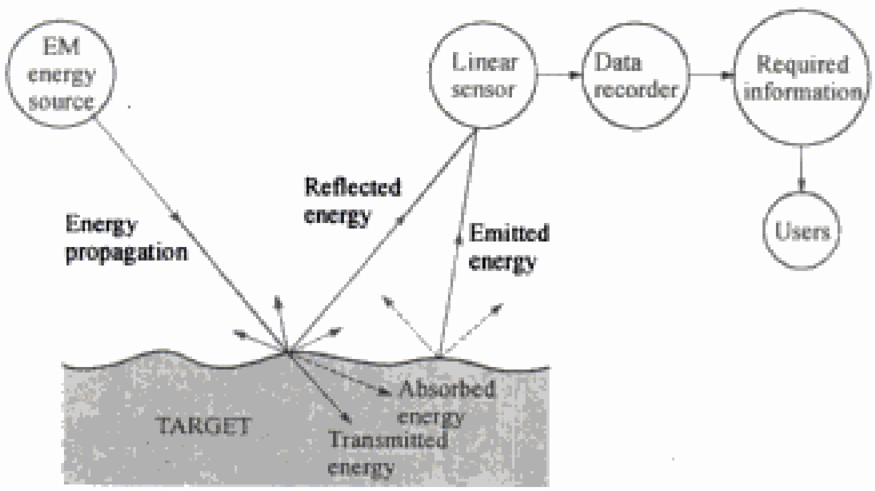

Fig. 10.2 An ideal remote sensing system

The source of energy produces electromagnetic energy and it propagates from the source to a homogeneous target. In an ideal case, produced electromagnetic energy contains all wavelengths and there is no loss of energy during propagation. When the energy interacts with the target, depending upon the characteristics of the target, the energy is transmitted, absorbed, scattered, emitted, or reflected from the target to the sensor. The energy from the target to the sensor is in the form of return signal, reaching a linear sensor which responds linearly to electromagnetic energy of all wavelengths and intensity. The return signal is recorded and processed in real time by the data recorder. The data is then processed into a format which is useful for interpretation. The information about the target collected is made available to the users in the desired form.

10.7 SENSORS USED IN REMOTE SENSING

Sensors used in remote sensing are mechanical devices which collect information, usually in storable form, about objects or scenes while at some distance from them. These sensors simply record in selected wavelength bands, the variations in the amount of energy being reflected or emitted by various objects on the surface of the earth. An ordinary camera which utilizes visible light energy, is probably the most familiar type of remote sensor.

10.7.1 Spectral bands for sensors

The spectral bands for visible and infrared spectrum are:

(*i*) (0.4 – 1.1) μm (window)

(*ii*) (2.0 – 2.5) μm (window)

(*iii*) (3.0 – 5.0) μm (window)

(*iv*) (5.7 – 7.2) μm (H_2O absorption band)

(*v*) (8.0 – 14.0) μm (window)

Similarly, the important microwave frequencies are given below. Band widths at these central frequencies are selected to get enough signal to noise ratio.

(*i*) 19 GHz (window)

(*ii*) 22 GHz (H_2O absorption line)

(*iii*) 35 GHz (window)

The large spectral bands mentioned above are further subdivided into small discrete intervals and the absorption is made in these small parts, called *spectral widths*. The same area of a scene can be observed simultaneously in different spectral regions to get more information about the characteristics of the objects.

10.7.2 Resolvings power and spatial resolution

Resolving power is the ability of a particular sensor to render a sharply defined image. It is inherently poorer as we move to longer wavelength. This difficulty to some extent is overcome by increasing the diameter of the optics, either lenses or antennas that are used to collect the energy. However, another factor that helps to compensate for loss of resolution is decreased atmospheric scattering at longer wavelengths.

The spatial resolution is defined as the ability to distinguish between two closely spaced objects on an image. More specifically, it is the minimum distance, between two objects at which the images of the objects appear distinct and separate.

The term resolving power applies to an imaging system, whereas spatial resolution applies to the image produced by the system. Spatial resolution is different for objects of different shape, size, arrangement and contrast ratio.

10.7.3 Classification of sensors

Sensors may be classified as *active* or *passive*, image or non-image forming, commercial or military. An active sensor provides its own source of energy, directing it at the object in order to measure the returned energy. A passive sensor records the energy that naturally radiates or reflects from an object. Flash photography is an example of active remote sensing in contrast to available-light photography which is passive. The main advantage of passive systems is that they are relatively simple, both mechanically and electrically, and they do not have high power requirements. Their disadvantages are that, particularly in wavebands where natural emittance or reflectance levels are low, high detector sensitivities and wide radiation collection aperatures are necessary to obtain a reasonable signal level. Another disadvantage of passive systems is dependency on good weather conditions.

10.10 MULTISPECTRAL SCANNER (MSS)

A scanner which detects and records simultaneously both reflected and emitted radiations in several spectral bands, is known as *multispectral scanner* (MSS). It operates on the principle of scanning successive lines at right angles to the flight path by means of a rotating or oscillating optical system.

10.11 RETURN BEAM VIDICON (RBV) CAMERA

The direct beam vidicon consists of an evacuated glass envelope containing an electron gun which faces a thin photo-conductive insulating layer (the target), deposited on the end window of the tube. An electron gun scans across the target surface and removes any positive charge so stabilizing the target surface at zero voltage. An exposure is made in conventional manner by means of the shutter and lens assembly. An image is then formed on the target. The illuminated regions induce a positive charge in proportion to the brightness of the reflected signal. This charged pattern is retained on the target surface until it is scanned a second time, in this case the magnitude of the pattern being measured and recorded.

The return beam vidicon differs slightly from the direct beam vidicon in the sense that it relies on the strength of the returning electron beam after it has neutralized the charge on the target, as means of measuring the charge pattern. The advantages are that the camera can operate in poor illumination condition and the resolution is generally higher than that associated with direct beam vidicon.

The development of the vidicon camera has provided satellites with a method of obtaining high resolution pictures without the need for conventional photographic film.

10.12 THEMATIC MAPPER (TM)

The design of thematic mapper was based on the same principle as that of MSS, but with a more complex design to provide finer spatial resolution, improved geometric fidelity, greater radiometric detail, and more detailed spectral information in more precisely defined spectral regions. Whereas the MSS has four broadly defined, spectral regions, the TM records data in seven spectral bands given in Table 10.1.

Table 10.1 Spectral bands recorded by the TM

Band	Resolution	Spectral defination	Application
1.	30 m	Blue-green (0.45 – 0.52) μm	Water penetration
2.	30 m	Green (0.52 – 0.60) μm	Measurement of visible green reflectance
3.	30 m	Red (0.63 – 0.69) μm	Vegetation discrimination
4.	30 m	Near infrared (0.76 – 0.90) μm	Delineation of water bodies
5.	30 m	Mid infrared (1.55 – 1.75) μm	Differentiation of snow and cloud
6.	120 m	Far infrared (10.40 – 12.5) μm	Soil moisture discrimination and thermal mapping
7.	30 m	Mid infrared (2.08 – 2.35) μm	Geological mapping

The spatial resolution of the TM is far better than that of MSS. In TM a pixel size of 30 m is used in all bands except band 6 which has a pixel size of 120 m. This contrasts with the 79 m pixel size, associated with MSS.

10.13 REMOTE SENSING PLATFORM

The base, stationary or moving, on which remote sensors are mounted, is called as *remote sensing platform*. The remote sensing platforms range from a camera on a tripod on the ground to balloon, helicopters, aircrafts, rockets, satellites and space vehicles.

The choice of a particular platform is mainly guided by the choice of sensors which in turn is guided by the objectives of the mission. However, other key factors in the choice of a platform, are the payload, operating height, operating ranges, time, and cost.

10.13.1 Satellites

Satellite platforms enable large area coverage at frequent intervals in uniform solar illumination conditions. They are highly suited for applications based on synoptic measurements over large areas and where periodic observations over the same area are required.

There are following two types of satellites:

1. Geostationary satellites
2. Sun-synchronous satellites.

If a satellite is positioned in the equatorial plane at altitude of 36,000 km, it is known as *geostationary satellite* and the orbit in which it moves is called the *geostationary orbit*. These satellites have same period as the earth and become stationary with respect to the earth's surface. Geostationary orbits are ideal for meteorological or communications satellites designed to maintain a constant position with respect to a specific portion of the earth's surface.

The *sun-synchronous orbit* is an orbit selected so that the satellite placed in this orbit, passes over the same ground track at the same local time each day. The satellites placed in sun-synchronous orbits, are called the *sun-synchronous satellites*. The condition of passing over the same ground tract at the same local time every day, is satisfied by a family of near polar retrograde orbits with altitudes between 300 and 1000 km. Such satellites provide the observation of scenes of the same area under conditions of uniform illumination. This helps in observing and analysing the changes in the appearances of the features within each scene under the same conditions of observation. In reality, satellite images differ greatly because differences in latitude, time of day, and season lead to variations in the nature and intensity of light illuminating each scene.

10.14 DATA PRODUCTS

The data products used in remote sensing are:

1. Photographs
2. Mosaics
3. Orthophoto
4. Satellite imagery or digital image.

10.14.1 Aerial photographs

Aerial photographs may be available as prints made on paper, film, glass or other transparent materials. Prints made on film, glass or other transparent materials may be viewed with transmitted light and they normally show finer detail and sharper definition. Positive transparencies are also known as *diapositives*.

10.14.2 Photographic emulsions

The spectral range of a photographic film largely depends upon the type of emulsion used. Different emulsions have different spectral sensitivity. Some of the most common emulsions used for remote sensing, are discussed below.

Black and white panchromatic

The spectral sensitivity of this emulsion covers the spectral range from 0.4 to 0.7 μm. These emulsions provide good definition and contrast, wide exposure latitude, low cost, and identification of subtle textural variations. The limitation is that sometimes it is difficult to manually interpret ground features due to the enability of the eye to distinguish between subtle differences in grey tones.

Black and white infrared

The spectral sensitivity of this type of emulsion is from 0.4 to 0.9 μm which includes some part of reflected infrared. Since infrared radiation tends to be strongly absorbed by water, the boundaries between water and land are very clearly identified. It is also possible to differentiate between different species of vegetation, such as between coniferous and deciduous trees.

Colour

The human eye can detect about 20 to 30 shades of grey on a black and white aerial photograph, while over 2000 different colours can be discriminated on a colour aerial photograph. Thus a colour photograph provides much more information than the corresponding black and white photograph.

Perception of colour depends largely on the relative amounts of the three primary colours, blue, green, and red which are reflected by a particular object. Combining or adding the three primary colours in different proportions therefore, enables all possible colours to be produced.

An alternative approach to produce a particular colour is that of a subtractive colour mixing. In this case, rather than adding the three primary colours, the three subtractive primaries, yellow, magenta, and cyan, are combined. Subtractive primary colours are so named because each is obtained by subtracting one of the primary colours from white light. For example, yellow is obtained by subtracting blue light and the combination of the remaining two primaries, i.e. red and green.

False colour infrared

This emulsion has spectral sensitivity from 0.4 to 0.9 μm. Unlike normal colour emulsions, colour infrared emulsions are designed in order to record green, red, and infrared energy. In false colour image, a blue image results from objects reflecting primarily green energy, green images result from objects reflecting primarily red energy, and red images result from objects reflecting energy primarily in the near-infrared portion of the spectrum. The advantages are that it can penetrate through haze and it provides accurate identifiable data on vegetation, rocks, soils, water bodies, and moisture distribution.

The disadvantages are that it has lower resolution than colour film and it is expensive than other films.

10.14.3 Mosaics

Photomaps and mosaics have been discussed in Sec 9.17 of chapter 9.

10.14.4 Orthophoto

A pictorial depiction of the terrain derived from aerial photography in such a way that there are no relief or tilt displacements, is an orthophoto. It is equivalent to a planimetric map except that instead of lines and symbols, image tonal variations convey the information about the features on the surface of the earth. Being an accurate map and rendering abundance of detail, makes an orthophoto useful for more applications than either a planimetric map or a normal aerial photograph.

10.14.5 Satellite imagery or digital image

In satellite remote sensing the data pertaining to an area of earth's surface, are collected in digital form using sensors and scanners, and stored on computer compatible tapes (CCT) or some other storage device. This data in the digital form is usually termed as the *imagery. The photographic image as paper prints is called a hard copy*. The smallest ground area in digital form which can be discerned, is called *pixel*. Each pixel is assigned an unique digital number (DN) which is the measure of the reflected energy from the objects. Thus the image is a vast matrix of numbers ranging from 0 to 255.

A satellite imagery covers a large area and provides multi-temporal and multispectral analysis. Collection of field data is minimised. The main advantage of the digital data is that it makes possible digital image processing. The disadvantage of the imagery is that it is expensive. Moreover, the digital analysis requires expensive software and trained personnel.

10.15 REFERENCE DATA OR GROUND TRUTH

Reference data are often referred to by the term ground truth. This term is not meant literally, since many forms of reference data are not collected on the ground, e.g., ground truth may be collected in the air, in the form of aerial photographs to be used as reference data when analysing satellite imagery. Reference data serves the following purposes:

1. To verify information extracted from remote sensing data
2. To calibrate the sensor
3. To aid in interpretation and analysis of remotely sensed data.

10.16 ANALYSIS AND INTERPRETATION TECHNIQUE

Remotely sensed data can be analyzed and interpreted either by visual method or digital method. The visual method of interpretation of the data utilizes the principles of photographic interpretation, discussed in Sec. 9.27 of Chapter 9 and hence, will not be discussed here. This section gives a brief overview of analysing and interpreting the digital data which is called the *digital image processing*.

10.16.1 Image processing

Image processing is a vital part of most remote sensing operations. All digital imagery must be processed in some way in order to be of use in the majority of applications. The image has to be displayed on a screen or as photographic hard copy, for visual interpretation. In many cases, more than just the production of a simple monochrome or colour image is required, and more complex processing is needed in order to produce final product for the uses.

The digital image processing is the processing and analyzing the data stored in CCT or PC based/main frame computer system using image processing algorithm. If relies solely upon multispectral response, tone and colour. The technique has the following broad operations:

1. Image rectification and restoration
2. Image enhancement
3. Image classification
4. Data merging.

Image rectification and restoration

Image rectification and restoration are required to correct the distorted or degraded image data. It is the initial processing of raw image data to correct for geometric distortions, to calibrate the data radiometrically and to eliminate noise present in the data.

Image enhancement

Image enhancement is used for more effectively displaying and recording the data for subsequent visual interpretation. It is a technique for increasing visual distinction between features in a scene.

Image classification

It is to replace visual analysis with quantitative techniques for automating the identification of features in a scene. This normally involves the analysis of multispectral image data and the application of statistically based decision rules for determining the land cover identity of each pixel in an image when the decisions are based solely on the spectral radiance observed in the data. Such classification process is termed as *spectral pattern recognition*. When the classification is based on the geometrical shapes, size, and patterns present in the data image, it falls into the domain of *spatial pattern recognition*. In either case, the intent of the classification process is to categorize all pixels in a digital image into one of several *land cover classes* or *themes*. These categorized data may then be used to produce *thematic maps* of the land cover present in an image and/or to produce summary statistics on the area covered by each land cover type.

Data merging

Data merging techniques are applied to combine data for a given geographic area with other geographically referenced data set for the same area. This technique is frequently used to combine remotelysensed data with other source of information in the context of a geographic information system (GIS).

10.17 SPECIFIC APPLICATIONS IN ENGINEERING

10.17.1 Terrain analysis

Almost every major construction project is controlled to a very high degree by the natural terrain conditions. Since the cost of moving the natural material is usually largest single cost in engineering construction projects, a knowledge of the location of the material is important for proper planning, location, construction and maintenance of engineering facilities. Remote sensing-aided terrain analysis plays a vital role in acquiring knowledge of terrain features.

The same approach is employed in locating construction materials and also in defining the regional physiographic setting and delineating the various land forms present.

10.17.2 Construction material inventories

The availability of suitable construction materials within economic hauling distance is crucial to the planning and execution of engineering projects. Additionally, there is a need to conserve the high-quality material which are becoming scarce.

Remote sensing imagery, particularly aerial photography has proved to be of a great value in performing inventories for construction materials.

10.17.3 Site investigations

Remote sensing techniques have been used successfully and extensively in site investigations for dams, reservoir, bridge, pipeline, crossing of rivers, airstrips, staging and docking facility, etc. Two very common kinds of studies often associated with such site investigations and greatly facilitated by remote sensing, are the location of ground water supplies for towns and industries, and the location of construction materials, chiefly sand and gravel.

Regional data base maps for engineering site planning and route corridor-selection, are now prepared routinely using remote sensing methods.

10.17.4 Erosion assessment

Information regarding soil erosional status is required for soil conservation planning. Remotely sensed data have potential utility for mapping and assessing soil erosion conditions. The study of soil, land cover and drainage characteristics reveals the information about erosion.

10.17.5 Watershed management

Of the varied attributes of watershed management, landform, relief, drainage network, soils, land use, and soil erosion status are amenable to mapping and assessment by remote sensing technique.

10.17.6 Environmental impact assessment

Environmental impact assessment is becoming increasingly important in most countries of the world. With the capability of remote sensing technique for cost-effective mapping of vegetation and other land covers, and the possibilities for digital analysis of the resulting maps, it can be of great assistance in environmental impact assessment, especially where the areas to be studied are large.

PROBLEMS

10.1 What do you understand by remote sensing? How is it different from photogrammetry?

10.2 Write a short note on the importance of remote sensing and its uses.

10.3 Write a brief note on electromagnetic energy. What are the basic parameters which describe the electromagnetic energy.

10.4 Draw a neat diagram of electromagnetic spectrum showing its various regions of particular interest in remote sensing.

10.5 Write a brief note on the interaction of electromagnetic energy with matter.

10.6 Discuss briefly an ideal remote sensing system.

10.7 Differentiate between resolving power and spatial resolution.

10.8 What is a sensor used in remote sensing? Discuss its working and classification.

10.9 Discuss briefly the following:

(*a*) Sensors for infrared region

(*b*) Sensors for microwave region

(*c*) Multispectral scanners

10.10 Write a short note on remote sensing platform. Also discuss the following:

(a) Geostationary satellites

(*b*) Sun-synchronous satellites

10.11 Briefly discuss the data products used in remote sensing.

10.12 What is image processing? Discuss digital image processing and its advantages.

10.13 Discuss the application of remote sensing in:

(*i*) Terrain analysis

(*ii*) Construction material inventories

(*iii*) Site investigations

(*iv*) Erosion assessment

(*v*) Watershed management

(*vi*) Environmental impact assessment.

11

MODERN SYSTEMS IN SURVEYING AND MAPPING

11.1 GENERAL

Significant developments in the last three decades resulting in a variety of sophisticated instruments, have revolutionised the techniques of conventional surveying and mapping. Such innovations are still continuing, leaving behind the conventional methods and making them outdated. As the technological development in instruments and processing techniques is so phenomenal that it is difficult to discuss these comprehensively in one chapter and, therefore, only brief discription of some of the modern instruments and terminology associated with measurement of quantities such as length, angle, area, volume, and coordinates of the points, has been presented in this chapter.

11.2 ELECTRONIC DISTANCE MEASUREMENT INSTRUMENT (EDM)

Recent scientific advances have led to the development of electro-optical and electromagnetic instruments which are of great value to the surveyor for accurate measurements of distances. The electronic distance measuring (EDM) equipment (Fig. 11.1) which can be used for traverse, triangulation, and trilateration as well as for construction layout, is rapidly supplanting taping for modern surveying operations except for short distances and certain types of construction layout.

Electronic distance measurement instruments utilize either infrared (light waves) or microwaves (radio waves). The microwave systems require a transmitter/receiver at both ends of the line to be measured, whereas the infrared systems require a transmitter at one end and a reflecting prism or mirror at the other end. Microwave systems have a usual upper measuring range limit of 50 km. Although the microwave systems can be used in poorer weather conditions (fog, rain, etc.) than infrared systems, the uncertainties caused by varying humidity conditions over the length of the measured line, result in lower accuracy expectations.

Infrared systems come in long range (10-20 km), medium range (3-10 km), and short range (0.5-3 km).

EDM can be mounted on the standards (Fig. 11.2) or on the telescope of most theodolites, additionally they can be directly mounted in a tribrach. EDMs measure slope distance between two points if at different elevations. The slope distance is reduced to its horizontal and vertical equivalents using the slope angle between the two points. EDMs when used with a theodolite, they can provide both horizontal and vertical position of one point relative to another.

EDMs have built-in or add-on calculator/microprocessors which provide horizontal and vertical distances.

11.3 ELECTRONIC THEODOLITE

The most modern theodolites contain circular encoders which sense the rotations of the spindles and the telescope, convert these rotations into horizontal and vertical (or zenith) angles electronically, and display the values of the angles on liquid crystal displays (LCD) or light emitting diode displays (LED). These readouts can be recorded in a conventional field book or can be stored in a data collector for future printout or computations. The circles can be set to zero readings by simple press of button on the instrument. The horizontal circle readings of some of the electronic theodolites can be preset to any desired angle before a back sight is taken.

11.4 ELECTRONIC TACHEOMETER OR TOTAL STATION

Electronic Tacheometers or total stations are instruments which combine electronic theodolite and EDM instrument (Figs. 11.3 and 11.4). They are equiped with a microprocessor for reduction of observed data. Electronic tacheometers can display horizontal angle, vertical angle, slope distance, horizontal distance, difference in elevation, and coordinates. The data can be stored in some form of storage device for further computations using an electronic computer.

11.5 LASER THEODOLITE

Laser theodolite (Fig. 11.5) have all the functions of electronic theodolite with facility of more accurately bisecting the target. By means of cross hairs the laser adaptor enables the observer to centre the laser beam very accurately on the target. The cross hairs can also be rotated as desired.

Laser theodolite has a wide range of applications including tunnelling and mining, piling and fencing, base-line surveys, marking and locating in space, shipbuilding and night surveying, etc.

11.6 AUTOMATIC TOTAL STATION

The total station described in Sec. 11.4 is a manual one in the sense that it requires two persons, i.e., the surveyor and a rod man holding the reflecting prism or mirror. A surveyor always requires some communication system to communicate with the rod man. Another disadvantage of these total stations is that sighting the prism is not only time taking job but also less accurate. The automatic total stations (Fig. 11.6) overcome these problems. Automatic total stations have motorized EDM and theodolite. It can align itself in any specific direction very accurately (up to 0.5"). By combining automatic target recognition (ATR) with motorized system, survey is reduced to a single men's job. The instrument once aligned to prism or reflector, automatically tracks the movement of rod man and is controlled by rod man. It eliminates the need of communication system as the operator of the instrument is rod man himself.

Automatic total stations are particularly very useful in areas of less vegetation and setting out jobs.

11.7 ELECTRONIC FIELD BOOK

Electronic field books (Fig. 11.7 and 11.8) are the data collector units used with the electronic theodolites or electronic tacheometers. The data is stored in prespecified format which can be processed using the software provided by the manufacturer of the instrument. An electronic field book should not be considered a substitute for the standard field book. Conventional field notes still serve to provide a record of descriptive information.

11.8 LASER ALIGNMENT INSTRUMENT AND ELECTRONIC LEVEL

Laser (light amplification by stimulated emission of radiation) alignment instruments (Fig. 11.9 to 11.11) produce a laser beam which is a visible straight line, used for alignment and setting out works in roads and railways. They are also used in the construction of large buildings and alignment of river dams. Laser beam can be used as plumb line. It can generate a reference plane for measurements of inclination and settlements, area levelling or profiles. Laser beam can also be used for placing pipe lines and monitoring machines in direction and elevation.

Laser attachments are also available which can be mounted onto the telescope of theodolites transforming them to laser instruments.

Laser beam instruments can be used in darkened areas indoors without a beam detector.

11.9 DIGITAL LEVEL

The digital levels (Fig. 11.12) use bar-code shafts. After targeting the staff, the reading is taken automatically and displayed. The measurements can be recorded in the field and down loaded into a computer for processing using the software provided with the instrument. Digital levels permit measurements to be made even when light conditions are inadequate. The staff section to be illuminated can be selected by moving the illuminator up or down.

11.10 INSTRUMENT FOR MEASURING TUNNEL PROFILES

The instruments which measure profile are used primarily for tunnelling (Fig.11.13). It basically consists of an electronic tacheometer which does not use a reflector, electronic storage device for the measured data and software to perform the following calculations:

1. Determination of the position of the instrument.
2. Calculation of the relative or absolute coordinates of measured points.
3. Graphic comparison of actual and required measurement, and project data.
4. Calculation of areas and volumes in relation to project area.
5. Calculation of distances between measurement points or project area.
6. Comparisons between periodic measurements.
7. Import and export of data.

11.11 INERTIAL POSITIONING SYSTEMS

An intertial system is a product of very highly sophisticated advanced aerospace technology for the determination of special positions. Its basic components are three *gyroscopes* which keep its three mutually orthogonal axes properly oriented in space (Fig. 11.14). The platform containing the three gyroscopes is kept oriented in the east-west, north-south, and up-down directions. The three axes also contain sensitive accelerometers to measure the acceleration components along the respective axes. The measured acceleration, which is the second derivative of distance travelled with respect to time, is used to compute the distance travelled along each direction, using an on-line computer. The correction for the earth curvature is applied during the computations.

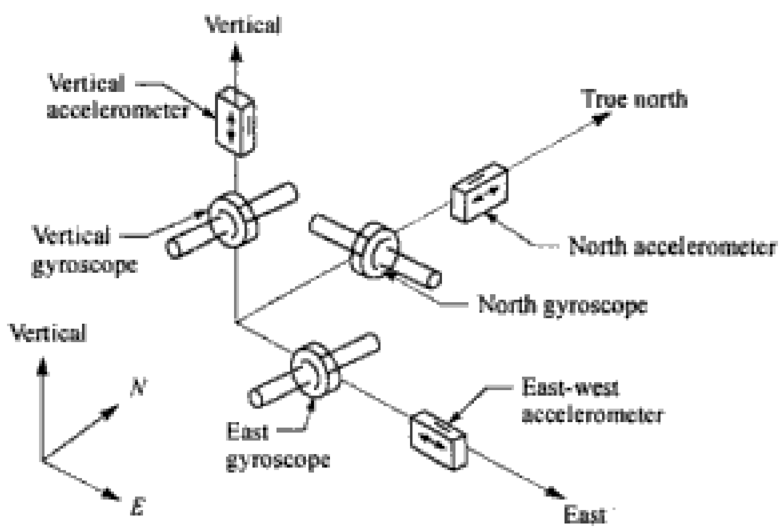

Fig. 11.14 Inertial positioning system

The inertial system may be mounted in either a motor vehicle or a helicopter.

Fig. 11.1 EDM instrument Wild DI 1600, (*Courtesy* : Leica AG, Switzerland)

Fig. 11.2 EDM instrument Distomat DI 20 mounted on a theodolite (*Courtesy* : Leica AG, Switzerland)

Fig. 11.3 Electronic total station D-50 (*Courtesy* : Nikon Corporation, Japan)

Fig. 11.4 Total station (*Courtesy* : Sokkisha Co. Ltd., Japan)

Fig. 11.5 Laser theodolite SLT20 (*Courtesy* : Sokkisha Co. Ltd., Japan)

Fig. 11.6 Automatic total station TPS system 1000 (*Courtesy* : Leica AG, Switzerland)

Fig. 11.7 Data terminal GRE 3 (*Courtesy* : Wild Heerbrugg Ltd., Switzerland)

Fig. 11.8 Electronic field book SDR 2 (*Courtesy* : Sokkisha Co. Ltd., Japan)

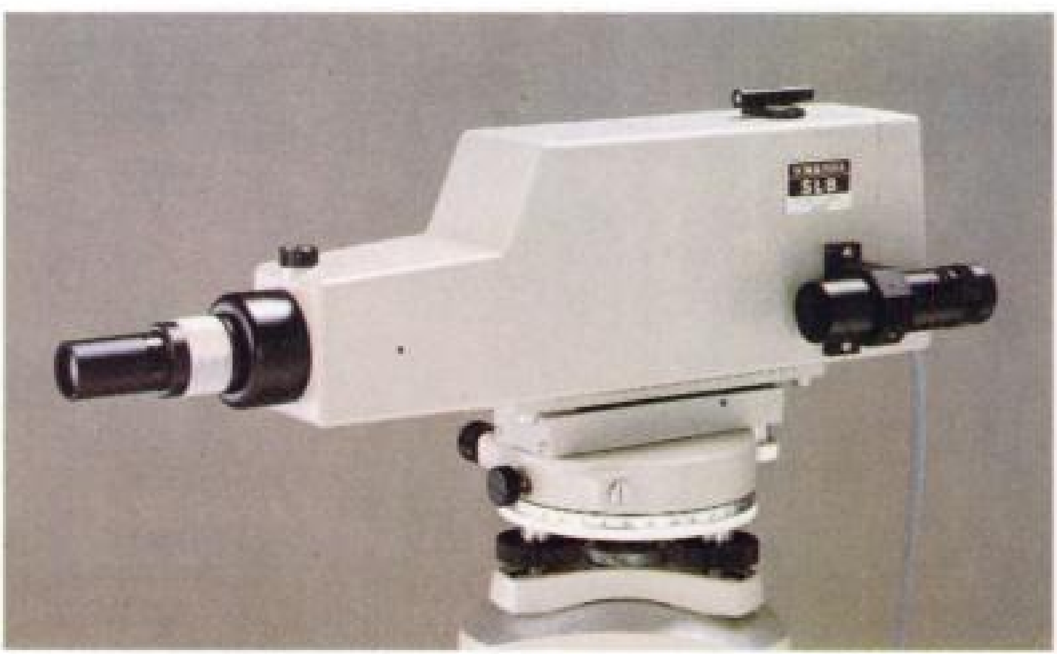

Fig. 11.9 Laser aligner SLB (*Courtesy* : Sokkisha Co. Ltd., Japan)

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Higher Surveying

This book presents a systematic and contemporary treatment of the theory and applications involved in higher surveying. It also highlights some of the modern developments in geomatics.

After explaining the basic survey operations, triangulation and trilateration, the book describes the various adjustment methods applied to survey measurement in detail which is followed by topographic, hydrographic, construction, and route surveying. As engineers and surveyors need knowledge of determining absolute coordinates of points and directions of lines on the earth's surface, a detailed discussion on field astronomy is presented in this book. A chapter on map projection is also included in the book.

Recent advances in land surveying are then highlighted including photogrammetry and photographic interpretation. Remote sensing technique utilizing data acquired through satellites is also explained.

Recent instrumentation techniques and methodologies being used in geomatics are emphasised. These cover a range of modern instruments including EDM, Total station, laser based instruments, electronic field book, GPS, automated photogrammetric systems, and Geographic Information System.

A Large number of worked out examples, illustrations, and photographs are included for an easy grasp of the concepts.

The book would serve as an excellent text for Civil Engineering students. AMIE candidates, and surveyours. Practicing engineers would also find it extremely useful in their profession.

Dr A M Chandra is presently working as Professor of Civil Engineering at the Indian Institute of Technology Roorkee, Roorkee, (formerly University of Roorkee). He graduated in Civil Engineering in 1969, completed postgraduation in Advanced Surveying and Photogrammetry in 1971 and obtained Doctoral degree in 1984 from University of Roorkee.

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