Bill Atweh · Mellony Graven
Walter Secada · Paola Valero
Editors

Mapping Equity and Quality in Mathematics Education

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This book is dedicated to the life and work of Leone Burton (1936–2007), teacher, academic, activist, and a friend

Preface

The discourses of equity and quality in mathematics education have permeated the international debates about mathematics education whether they occur in the contexts of research, policy, curriculum or teaching and learning. Few would doubt that both provide valuable objectives to aim for—yet they provide serious challenges to confront in the planning and implementation of any endeavour in mathematics education. However, rather than being directly articulated, they often remain implicit and assumed. When they are articulated, their understandings are not clearly theorised. Arguably, of greater importance is that the relationship between them is often left unexamined. For some it may seem that equity and quality are distinct aspirations—yet not necessarily mutually exclusive. Others may see a necessary unity between them that one cannot be promoted without the other. Still others place more emphasis on the potential tension between attempts and resources targeted towards their promotion.

In putting together this compilation of chapters, we do not take these terms to have an essentialist meaning. Perhaps many debates in mathematics education can be constructed as debates about the meaning of equity and quality as much as a debate about their relative worth and ways to promote them. In our call for chapters, we identified from our review of the literature, some common associations of the two terms.

Concerns about *quality* mathematics education are often posed in terms of the types of mathematics that are worthwhile and valuable for both the student and society in general, and about how to best support learners so that they can develop this mathematics. Quality mathematics is sometimes measured from within the discipline of mathematics itself and is seen as a reflection of its rigor, formality and generalisability. Alternatively, the value of mathematics is often argued based on perceptions of its utilitarian importance such as individual mathematical literacy, the economic and technological well-being of a society, the participation of an informed citizenry in the challenges of a democratic society, and/or for opening up future opportunities for students in terms of their career goals and access to higher education. Trends gleaned from international comparisons have ignited debates within many countries about the low level of achievement of their students internationally regardless whether mathematics is valued for its academic rigor or utilitarian literacy.

Concerns about *equity,* on the other hand address issues about who is excluded from the opportunity to develop quality mathematics within our current practices and systems, and about how to remove social barriers that systematically disadvantage those students. Equity concerns in mathematics education are no longer seen at the margins of mathematics education policy, research and practice. Issues relating to ability, gender, language, multiculturalism, ethnomathematics, the effects of ethnicity, indigeneity, and the significance of socio-economic and cultural backgrounds of students on their participation and performance in mathematics are widely discussed in the literature. This is not to say, however, that the problem of equity is exclusive of students who are positioned as disadvantaged due to their association with any of the categories above; nor that the growing focus on the issue has in any way been totally resolved across countries and within any society. Rather, insofar as access to quality mathematics is thought to confer benefits on individuals and the larger society, concerns for equity and access revolve around the impacts on an individual's life and social participation and on the larger society's continued well-being when that access and its benefits are systematically restricted from and/or systematically provided to people on the basis of their or their parents' social placements.

In our international invitation for chapters, we identified one overall aim behind this collection as mapping the terrain of mathematics education research and practice—that is on how to understand and advance the quality and equity agendas. The main requirement for chapters was that they consider both agendas and how they relate to each other. We did not have a vision that this collection would provide a comprehensive inventory or a summary of all our individual and collective learnings about them. Rather, we attempted to illustrate the different views and perspectives on the issues in order to move forward the debate on their importance and promotion in the field.

Process Adopted in the Compilation: The initial idea of the book came out of a plenary panel discussion at the International Congress of Mathematics Education in Mexico in 2008 under the topic of Quality Mathematics Education for All[1]. A call for chapter proposals was distributed electronically using several electronic lists of mathematics educators and teachers. Potential authors were encouraged to send printed copies of the call to others who may be interested but may have had limited access to email or international contacts. Similarly, we targeted our own individual contacts from countries that are less technologically developed.

The submitted proposals represented a wide range of academic and professional backgrounds (school teachers, researchers and university academics), levels of expertise in publications and academic writing (first-time authors, recent doctoral graduates, published authors and authors of books) and from a range of

[1] The plenary discussion was lead by Bill Atweh (Australia), Olimpia Figueras (Mexico), Murad Jurdak (Lebanon) and Catherine Vistro-Yu (The Philippines).

methodological investigations (from theoretical to empirical qualitative and quantitative studies) and theoretical perspectives (critical, social justice, postmodern and ethical). We were less successful, unfortunately, in attracting voices from countries less well represented in international dialogue and from countries of non-European language background. This remains a challenge to all international collaborations.

Submitted draft chapters were peer reviewed by the authors[2] in a non-blind review process. Our intention was to create a community of critical friends dedicated to the improvement of the quality of our publication rather than the traditional gatekeeping roles. It is fair to say that the reaction of the authors to this process was mixed. The modified chapters have undergone a second round of review by us as Editors.

The Structure of the Book: The chapters in the book are grouped into four parts; each part contains several contributions and a response chapter by one of the editors. The Part I, The Theoretical Landscape, consists of eight chapters which adopt different theoretical stances on the issues of equity and quality. As Secada observes in his reaction to the chapters:

1. Equity and quality are inherently political terms whose common political bedrock is obscured by being taken for granted.
2. Equity and quality have nuanced meanings in everyday use and philosophically.
3. Scholarly inquiry about the nature of equity and of quality—either alone or linked—has taken a decidedly qualitative turn, focused on textual deconstruction and/or interviews with key informants.

Part II, Mapping Social Constructions and Complexities, consists of 11 chapters which address issues concerning quality and equity as well as their relationships, and highlight particular dimensions of what could be called the social and political constitution of the discourses of equity and quality in mathematics education. As Valero notes in her reaction contribution, the chapters in this section illustrate with empirical material, analysis and discussions, the way in which the discourses of equity and quality move in constant construction and recontextualisation from broad societal trends to the constitution of subjectivities, passing through policy, the media, pedagogy and reaching the learners. Valero concludes her comments on the chapters by raising the question of the social construction of quality and equity and the personal responsibility of an academic or teacher to attempt to promote them.

Part III, Landmarks of Concern, consists of ten chapters dealing with the special needs of different social groups traditionally identified as equity groups. The different authors cover a wide range of areas of disadvantage and exclusion from gender and social class; to race and ethnicity, and to physical and social alternative abilities. As Graven points out in her reaction to the chapters, discussion in this Part points to the need to dispel the myth that ‘same education’ for all results in equity. They illuminate the way in which a one size fits all as an approach, as often reflected in slogans such as ‘education for all’, tends to only provide quality

[2] Special thanks to two additional reviewers Jeanne L Higbee and Irene Duranczyk.

education for dominant groups. Specific groups require that curriculum and programs acknowledge their needs, the resources they bring and, perhaps more of a challenge is that conceptualisations of 'quality' need to be reconsidered from the perspective of marginalised groups.

Part IV, No Highway, No Destination, consists of ten chapters representing different lessons learnt by academic researchers and/or school practitioners from attempts to manage equity and quality within various educational contexts and with a variety of marginalised populations. As Atweh notes in his reaction to the chapters, collectively the chapters in this section point to the fact that action towards the objectives to raise the levels of both equity and quality in mathematics education is not only essential (as the many other chapters in this book argue) but that it is also possible. The message that there are many different paths towards promoting equity and quality and that the pathway may not always be smooth and journey remains always incomplete.

We submit this collection to the international community in mathematics education, not as summary of our collective knowledge in the area, nor as a catalogue of the different perspectives and views; rather as a means for continuing the dialogue on the discourses of equity and quality in mathematics education for the general benefit of the discipline itself and the societies it serves.

The Editors

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Part I
The Theoretical Landscape

Chapter 1
Disrupting ‘Development’ as the Quality/Equity Discourse: Cyborgs and Subalterns in School Technoscience

Anna Chronaki

Anita: I used to like mathematics and I was good at geometry. Yet, there is this oxymoron [...]. And, indeed, everybody believed –towards the end of secondary school- that I will become a scientist.

Anita: All these years [...] I have come to realise that mathematics and technology are fields unfamiliar to women.

Anita: No, look, to be more specific the case is different for young children. If they, already, have family support [..], if they have a computer at home, they can work creatively. The issue is access to computer at home. Then, there will be time and space for girls. I believe that the children of tomorrow will show us that certain taboos can be broken.

Anita, 37 years old, female school teacher, Greek
[Gender, Mathematics, Technology Project Data File]

Giorgos: […] men are more into technology. They like it. Whilst women –those who get involved-because not all of them get involved, they do so, I believe, out of necessity. In other words, men have a passion (for technology), they buy magazines about technology […] whilst women do not care much.

Giorgos: Look, in my school (engineering dept) … men might get involved much more with computers, with technology and the like, but, I think that the girls in my school […] also cover this gap. Because, they handle whatever is required from technology. They do not go beyond it. Only to cover the school demands. Whether they like it or not.

Giorgos, 20 years old, male engineering student, Greek
[Gender, Mathematics, Technology Project Data File]

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Afrodite: I feel very repressed in all possible ways in what concerns school […]. I can manage many challenging things, but I fear [...] something I cannot explain even to myself. I have discovered that I have the abilities to improve my life. And I am not saying this in order to praise myself. At some stage, [...] my family encouraged me [...]. They (people from her family) were saying '*you make the beginning of a new life*'. And this was for me an important point in my life. It made me think of school as the most important thing in the world. It made me think like this until I started coming into contact with completely different things [..]. I then started re-considering how helpful school is since I knew that I would not use it in my future. For I knew that I would repeat my parents' story. Namely, I will get married, I will raise children, and I will be involved in housekeeping and child caring. In this way, my world was demolished. My whole being was demolished. Why? At this moment, I started taking the words of others seriously, that it is a shame for somebody, especially for a girl, to attend elementary school.

Afrodite, 12 years old, female school student, Gypsy Greek [quoted in Dafermos (2005): 257–259]

Women, Mathematics, Technology and other Dangerous Things

Lakoff (1987) used the catch phrase *'women, fire, and dangerous things'* as a title of a book concerned with how human thinking is totally immersed in metaphors and depended on their role to produce meaning in everyday talk. His choice to place the word 'women' next to 'fire' and next to 'dangerous things' intended to show the power of metaphor-use in language-use. It, also, served to produce a certain 'image' of the possible meanings concerning the category 'woman'. First, a woman is a *thing*—not really a person. In addition, a woman, like fire, is a *dangerous thing*. The semantic categories of 'mathematics' and 'technology' along with those of 'women' and 'fire' in Lakoff's choice of words seem to exemplify, when placed together, a similar 'dangerous' liaison, for good reasons, as will be shown in the sections below.

'[M]athematics and technology are unfamiliar fields to women' says Anita, a primary school teacher in her late 30s, whilst Giorgos, a young engineering student, argues that although some female students can cope well with what is required to do with technology during coursework, they lack a passion for it. Coping well with school subjects, including mathematics and technology, creates emotional conflicts for Afrodite, an adolescent Greek Gypsy girl, who senses that she will soon need to abandon school for an early marriage—repeating her parents' story. Education, and specifically mathematics education, provides her with a promise of joining the desired 'modern' ways of imagining, organising and controlling her life. Simultaneously, this very desire soon becomes an unfulfilled promise, creating frustration, pain and feelings of failure. Schooling turns out to be an (almost) impossible path for Afrodite, who, despite being a successful learner, wonders what might be the

real value of school for her. Schooling demands a cultural border crossing, and a constant compromise amongst conflicting 'values' related either to community or school formalities. Afrodite becomes 'voiceless', 'hopeless' or a 'subaltern' in Spivak's (1992b) words as her struggle for recognition proves futile or un-ending.

Anita remembers being good at mathematics (geometry), but contrary to her family's and companions' belief in her capacities, chooses not to study mathematics since she feels that 'science' is not really suitable for her as a woman. Despite her choice not to engage in what was perceived as natural for her, she recognises the fact that the 'new' generation has the potential to reverse such stereotypes if, as she argues, access to both resources and expertise is safeguarded. However, Giorgos, a young male who belongs to this 'new' generation, seems to espouse that women's pursuit of science is not out of pure interest or passion but of mere necessity to acquire the skills required in modern society. Lack of passion and 'pure interest' show that women's relation to technology is weak, subordinated and marginal. As such their pursuit of technology is taken as 'different' and becomes 'other'.

Giorgos, like Anita, invests on hegemonic discourses which naturalise young women as non-passionate, non-dedicated participants in techno-scientific practices arguing that they *'get involved [...] out of necessity'*. Taking into account the fact that the discourse of an intrinsic '*passion for science*' is predominant when scientific creativity and innovations are taken into consideration (Turkle 2008), one easily concludes, as Anita does, that *'women are not really made for the worlds of mathematics and technology'*. In contrast, the case of Afrodite shows that passionate desire alone does not seem to safeguard a continuous participation to education (including mathematics education). Afrodite lives at the borders of two competing discourses; the one depicting school as '*the beginning of a new life*' and the other emphasizing that *'it is a shame for a girl to attend school'*. Schooling represents the risky path towards a 'new', yet 'uncertain', life. In a similar vein, Anita rejects 'uncertainty' and *chooses* a safer area for study and work.

The narratives offered by Anita, Giorgos and Afrodite are inscribed within discourses that carry a 'negative' sense of female experience with technology, mathematics and education. Not only Giorgos, but also Anita and Afrodite seem to be captured within gendered discourses espousing a fixed view of women's relation to technoscience. Their stories are not interpreted in a positive way, but instead perpetuate the projection of stereotypic images. According to Foucault (1972, p. 49) discourses function constitutively towards producing 'truths' which *'systematically form the object about which they speak'*. This approach explains how hegemonic discourses serve to reproduce women as having distinct ways of knowing (Belenky et al. 1986) or that technoscience[1] is mainly a masculine route towards realising the rational, modern 'self' and developing systemic societal change (Ellul 1964). Within this realm, school mathematics and technology are not seen a 'female'

[1] According to Wikipedia *'Technoscience is a concept widely used in the interdisciplinary community of science and technology studies to designate the technological and social context of science. The notion indicates a common recognition that scientific knowledge is not only socially coded and historically situated but sustained and made durable by material (non-human) networks'* (http://en.wikipedia.org/wiki/Technoscience).

choice since women deal with technomathematics in 'different' ways—ways that potentially can work towards 'disrupting' commonly held assumptions and expectations. Women's relation to technoscience can be seen 'disruptive' as they embrace technology and mathematics without revealing a devoted passion. Instead, their engagement seems to be a continuous struggle towards fitting technoscientific materialities in the multiplicities of their everyday working, studying and living. Yet, women's struggling to appropriate technoscientific knowledge is often read as problematic. The issue of 'woman as a problem' has been discussed extensively in relation to technology (Wajcman 2007), but also in relation to mathematics (Fennema and Leder 1990). And, its assumed 'normality' can be oppressive as it does not allow 'other' subjectivities to emerge and does not voice alternative positioning(s).

Following Michel Foucault and Judith Butler, the present chapter attempts to re-read hegemonic discourses of female relation to techno-mathematics. Foucault (1972, p. 151) observes insightfully how discourse *'obeys that which it hides'* and becomes *'the path from one contradiction to another'*. He argues that *'to analyse discourse is to hide and reveal contradictions; it is to show the play that they set up within it; it is to manifest how it can express them, embody them, or give them a temporary appearance'*. Discourse as contradiction comes close to notions of 'disruption' and 'trouble' as promoted by Butler (1990, 1997) arguing for the need to deconstruct what are often seen as 'normal' or 'natural' assumptions on agency, subjectivity and identification. Along these lines, the present chapter aims to move beyond a negative interpretation of women's relation to technology and mathematics as passive, indifferent or marginal. It argues that female partial and at times marginal positionings could problematise technological determinism (Ellul 1964) and bring forward an alternative reading concerning our understanding of technoscientific practices where the complex incompatibility of using technology and mathematics is not concealed but spoken out and negotiated. It is suggested, here, that an alternative reading might be closely related to disrupting assumed normalities of human-technoscience relation(s) by means of disrupting 'development' as the quality/equity discourse in technology-mediated mathematics education.

Such an alternative approach to technoscience then becomes a dangerous gesture to development discourse(s). Danger is encountered at several levels. First, opting out technology and science is 'dangerous' for the 'modern' individual as it blocks the development of the rational subject and perpetuates the 'savage' and emotional self. Second, this very fact becomes dangerous as it holds up the development of modern systemic changes where 'self' and 'society' is interchangeably linked. Development here entails both a quality direction (i.e. towards becoming the rational subject) and an equity dimension (i.e. all subjects need to become rational or else techno-mathematically literate). However, narratives concerning women's relation to school technoscience do not profess this very notion of development. Instead, they exemplify a 'dangerous liaison' as far as women instrumentally make use of their right to opt out, to resist, or to become marginal actors (see also Chronaki 2008). The vision of an equitable future within mathematics education is, also, critically dependent on the potential of reworking what it means to assume a sense of 'I'—an agency that is subjectively negotiated. Specifically, women, instead of committing themselves to the 'risky' path of a 'passionate' experience with formal education and 'new' technologies they turn towards a 'modest' relationship carefully negotiating boundaries. Haraway (1997, p. 130), considers technoscience as the story of globalisation and argues for the significance of a 'modest witness' position as a space for feminist work—a space where technoscientific knowledge is regarded as situated *'deep and wide throughout the tissues of the planet, including the flesh of our personal bodies'*.

This chapter attempts to provide a type of bird-eye view over a very complex area that, at present, pressurises teachers and students towards adopting 'new' media, 'new' roles and 'new' identities. This intensity for change is being discussed under the caveat of development by providing *access for all* (i.e. equity) via technology-mediated mathematics education curricula. However, what do we really mean by development? And how do these relative links among school technoscience appropriation, development, quality and equity affect the daily lives of women and men and especially of women and men who belong to marginalised, oppressed and voiceless groups? Taking into account the above, the following sections attempt to re-read hegemonic discourses on 'development' as 'quality' and 'equity' in mathematics education with an eye to disrupt assumed positioning(s)—or, in other words, to analyse how discourses can 'hide and reveal contradiction' (Foucault 1972, p. 151).

Development as Quality: Intensity for Change

Whilst equity, as Secada (1992, 1995) claims has been marginally explored in the research field of mathematics education, quality has been well emphasised. From the 1980s onwards, mathematics education research has greatly invested on promoting innovative curricula design in order to promote *quality* teaching and learning. Main sources for theorizing *quality* have been certain psychological perspectives based primarily on either constructivist or socio-cultural approaches to learning. Curricula innovations included the cognitively guided instructions for mathematics, contexts for authentic learning, realistic mathematics education etc. (Schoenfeld 1994; Greeno and The Middle School Mathematics Through Applications Project Group 1998; Treffers 1987).

Issues of quality were mainly discussed in relation to the *micro-context* of mathematics classrooms taking into account primarily didactic and pedagogical aspects. Emphasis on how children develop mathematical skills and competences has led towards focusing on cognitive and meta-cognitive strategies as they relate to social interactions (teacher intervention, group work, classroom activity). Despite commonalities, constructivist and socio-cultural perspectives could hardly agree on fundamental principles concerning learner agency and knowledge status. On the one hand, a constructivist perspective[2] directs attention to the learner as an active

[2] The notion of 'a constructivist perspective' is used, here, in an excessive way, but one needs to keep in mind that more than one constructivist perspectives have been formed within the field of (mathematics) education, such as interactive, dialectic, radical, social etc. (Chronaki 1992, 1997).

autonomous subject who potentially reflects on and negotiates ideas by means of experimenting with suitable materials. Learning and knowledge development, thus, depend mainly on explorative activity, reflection and active engagement with task parameters. A mainstream socio-cultural approach[3], on the other hand, emphasises semiotic mediation, tool-use, and collective engagement with purposefully organised activity. The learner is conceived as a motivated subject who needs to actively interact with more knowledgeable others and to purposefully use tools that bridge the gap among past, present and future historical practices, forming zones of proximal development (Wertsch 1991).

Stressing the urgency for quality at the micro-level is not isolated from the macro-level reform agendas in mathematics education at national and international levels (TIMMS 2007). Certain curricula politics (i.e. prescriptions for content, skills and competences, assessment methods) act as 'ideological state apparatus' (Althusser 1971) that regulate behaviour at the micro-level of human interactions. In that way, reform implementation mediates the macro-level societal structuring and creates micro-spaces (e.g. didactic innovations) where self and society develop together. As such, the stress for quality in mathematics education curricula cannot be considered neutral. Mainstream constructivist and socio-cultural perspectives work synergistically towards this end and provide a language for re-producing and legitimizing discourses of 'quality'. Such discourses materialise by means of curriculum reforms and innovations in schools and classrooms (i.e. mathematical content and competences such as active learning, collaborative work, technology-use, etc.) *thus* producing fixed identities of the 'good' learner and teacher. Walshaw (2001, p. 96) highlights that the learner is seen as constructing *'…viable theories of the ways in which the world works'*, the teacher as facilitating and empowering learners to *'… give voice to their subjugated knowledge'* and that learner's personalised and localised knowledge *'…generate not only visibility but also are said to offer agency in terms of identity and position from which they might act for change'*. However, she critiques the view that subject agency can be easily fixed through suitable didactic interventions.

Within this realm, technology-mediated mathematics learning enjoys a prominent position within recent curricula reforms in mathematics education. For example, there is evidence that certain digital tools suitable for dynamic geometry, computer algebra, data handling, statistics, programming and modelling can be instrumentally utilised towards encouraging the development of specific mathematical skills and competences such as visualising, representing and manipulating symbolic entities such as mathematical ideas. At the same time, they foster certain ways of working such as collaboration, reflection, active experimentation, etc. (NCTM 2000; Hershkowitz et al. 2002; Ruthven et al. 2004). Technology-based mathematics education becomes a political arena for teachers, learners and curriculum designers towards producing a particular collective identity change in the name

[3] In a similar vein, one needs to mention diversification across a variety of socio-cultural perspectives ranging the emphasis from psychological to cultural, anthropological and critical approaches to learning and communicating (Kontopodis et al., in press).

of the 'new' math teacher who safeguards 'quality' learning. Specifically, the 'new' maths teacher is required to be a flexible facilitator of knowledge construction, as opposed to knowledge transmitter in the traditional paradigm (Chronaki 2000). Stressing the transformative role of 'new' technology is an old issue that reflects broader socio-economic politics in the so-called 'new' information age (Castells 1996/2000). The sense of 'new' becomes a reference to the most glamorous recent past and implies that 'new' equals 'better' and thus 'new' is associated with quality. The 'new' signifies 'the cutting edge', the avant-garde, the place for forward-thinking people to perform (and become) designers, producers and practitioners. Thus, discourses of 'change' tend to become avenues to 'new' and relate to long-lasting modernist views of social progress and development as smoothly delivered by technology (Lister et al. 2003).

Investment on such discourses emphasises the revolutionary impact of technology towards producing profound transformations of maths teachers' everyday life in terms of evolving techniques, skills, relations, feelings, communicative practices and organisational structures. The transformative impact of 'new' technologies has been mainly discussed as far as it concerns epistemological, pedagogic and didactic potential for change through the analysis of focused teaching experiments (Marrioti 2002). Despite the benefits outlined in such exemplary cases, and the high investment on time and economic resources, widespread technology integration in mathematics classrooms remains a challenge (Ruthven et al. 2004). In addition, a number of studies indicate how female teachers and students do not choose related fields to study and work and rarely report long-lasting transformative experiences (Wajcman 2007). In a similar vein, Anita, Afrodite and the female engineers position themselves in discourses that inscribe them as 'different' when compared to men on the basis of lacking not only passion, but also the flair for active engagement and the competence for deliberative decision making. However, the discourse on 'difference' can easily slip into discourses of 'gender gap' and 'female danger'. Although, Anita, Giorgos and Afrodite are different cases in terms of age, gender, race and school role (teacher, undergraduate student and secondary school student), they all seem to support the view that women's liaison with mathematics and technology is not only uneasy, but it can be a marginalised or a 'dangerous' one. How else, could one explain Anita's choice to withdraw from a successful future in mathematics since she senses that aspects of her everyday life might be in danger, and Afrodite's conflicting experiences that lead her to consider quitting school? But also, how could one predict where young female engineering students might end up in their careers since they, according to Giorgos, lack a passion for technology and science?

In this realm, female teachers and students easily fall into the stereotypic image of resisting technology. An alternative reading is that some teachers and students do not resist technology itself but the stressful requirements for immediate 'change' towards a predefined quality agenda. They realise technology as a risky terrain and they set boundaries on technology use. Illich (1972) has argued that good and evil are not attributes of technology per se, but of technologies-in-use. For example, Anita realises how incompatible is for her to invest on mathematics or technology as a career pursuit. Similarly, female students reject a passionate relation to technology and concentrate, instead, on more pragmatic approach in specific localities. Whilst, males are believed to be passionately attracted by 'new' technology, as Giorgos, the young engineer states, their female counterparts, although competent, do not perform a passionate desire.

Instead of pursuing uncritically a path towards identity 'change', our data of women narratives urge us to consider the human-machine relation as situated in everyday practices. This view agrees with feminist perspectives on technoscience that alert for the importance to move away from a view of 'change' as development towards a full masculine self-realisation. Specifically, Haraway critiques a number of Marxist and psychoanalytic epistemological positions on feminism and turns to explore the complex production of woman/difference/other in relation to technoscience. Striving for a move away from dichotomies, dualisms or binaries situated in discourses that indicate a 'lack' (e.g. lack of passion, interest, competence) and reproduce gendered technological essentialism or technophobia, Donna Haraway introduces the notion of 'cyborg'[4] as a metaphor for a hybrid entity that blurs the boundaries between organic and mechanic. The cyborg refers to the ontology of an enhanced command-control-communication-intelligence system (c3i) where human-machine organisms are integrated into a symbiosis that transforms both (Haraway 2004, 2006/2009).

The cyborg, short for *cyb*ernetic *org*anism, is taken to be the image of an 'augmented human' suitable for extra-terrestrial explorations, scientific experiments and science fiction narratives. But, Donna Haraway uses the term as a prime resource to imagine an alternative kind of material-semiotic world, an alternative perspective of identity politics, and in consequence an alternative optic of feminist technoscience. The notion of cyborg denotes that dichotomies and dualisms such as nature/culture, woman/man, body/mind can no longer be used to figure or create the other. She claims that; *'[...] Instead, the cyborg is resolutely committed to partiality, irony, intimacy, and perversity. It is oppositional, utopian, and completely without innocence. Cyborgs are not reverent; they do not re-member the cosmos. They are wary of holism, but needy of connection'* (cited in Schneider 2005, p. 64).

This particular 'cyborg' point of view allows us to re-consider female relation with technoscience by appreciating its intense partiality. In this sense, Anita's and female engineers' experience as non-passionate, partial, disloyal could be considered as a 'cyborg' position. They can be seen as 'augmented' human creatures as they appropriate, utilise and negotiate varied uses and productions of technology in their

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[4] Cyborg, short for cybernetic organism, is a term coined by the research scientists Manfred Clynes and Nathan Kline in the '60s as they tried to imagine the kind of augmented man that would be necessary for extra-terrestrial exploration or space flight. It refers most particularly to an imagined and actual mix of machine and organism so as to constitute an integrated information circuit. […] The first cyborg, from Clynes and Kline's lab was a white lab-rat with an osmotic pump implanted to allow the researchers to inject chemicals to control and observe aspects of the rat's physiology. […]. Donna Haraway has taken cyborg as a metaphor to draw together an array of critical questions about human-machine relations and varied embodied forms of technoscience as part of socialist feminism. Recently, the cyborg had emerged as a figure in popular culture and especially in science fiction (Clynes and Kline 1960; Haraway 1991).

everyday lives. Through cyborg a more nuanced and complex angle of vision is offered that sees the technoscientific as a field for the contestation of meaning and the possibility of remoulding and redirecting what looks repressive into something more subversive and even democratic. While fully aware of the fact that the image of the cyborg could be as much about global control and domination, or about pre-emptive strikes and imperialism masked as deterrence or defence, Haraway offers an alternative possibility; *'[A] cyborg world might be about lived social and bodily realities in which people are not afraid of their joint kinship with animals and machines, not afraid of permanently partial identities and contradictory standpoints. The political struggle is to see from both perspectives at once because each reveals both dominations and possibilities unimaginable from the other vantage point'* (cited in Schneider 2005, p. 72). As such, the cyborg is not only an image or figure, an entity in reality or imagination, but it is also a standpoint, a way of thinking and seeing.

Calling the late twentieth-century understanding of the relationship between organism and machine a 'border war', Haraway (1997, 2006/2009) recommends instead a pleasure to be found in bringing about the destabilisation of these boundaries and an accompanying heightened '*responsibility in their construction*'. This means that intensity for 'change' via discourses of 'quality' cannot be taken for granted as if it constitutes a 'normal', 'neutral' or 'static' path for development. Discourses of quality as identity change towards developing a 'fixed' list of goals promotes 'fixed' and 'static' identities and denies a 'cyborg' view on women's experience with technology. Specifically, it conceals the fact that 'development' happens in multiple, complex and hybrid ways where boundaries between humans and machines are disintegrated and destabilised. In the following section, development as equity will be discussed as the urgency *for all* to change with/in school maths.

Development as Equity: The Urgency for All to Change

In the field of mathematics education, gender inequity has been, mainly, explored in two interrelated dimensions using, at large, comparative analysis; first in relation to boys' and girls' achievements in specific mathematical curriculum content areas, and second, in relation to male *versus* female participation in areas of study and work that require mathematical knowledge and competences. As far as achievement in particular curricular areas of mathematics (geometry, algebra, problem solving) is concerned, the quantitative data gathered during the last decade inform us that male-female differences have started not only to disappear but even to reverse, since in some countries (e.g. Iceland and Cuba) we, recently, witness some female advantage (Xin Ma 2008 based on a meta-analysis of regional and international studies on student assessment). A number of meta-analytic review studies concerning the relative interdependence of variables such as gender, class, achievement, attitudes, cognitive and meta-cognitive strategies seem to agree that the gender gap has gradually been eliminated (Hanna 2003; Xin Ma 2008). When the dimension of women's career paths is considered, recent research outcomes point out that although there is some considerable increase in the presence of females in areas of study, research and work, their participation in scientific fields still remains unsatisfactory (Jutting et al. 2006). For example, the American Mathematical Society (AMS) reports a slight increase in the representation of women in academic editorial boards. Specifically, they explain that between 1994 and 2003 women representation rate has increased from 9% in 1994 to 16% in 2003.

The situation is far more devastating in countries of the so-called developing world, where women still have limited access to work and education. Dunne and Sayed (2002), for example, explain that, in southern African countries only 5% of all female students enrol in mathematics, computing and engineering. Frantzi (2008) investigating women who enrol in mathematics related higher degrees in Greece observes that whilst before the Second World War women mathematicians were a rare phenomenon and mainly came from the middle or upper classes, during the period 1940–1964, more women enrolled to study mathematics and they came mostly from a lower middle class background. Gender inequity, thus, is not an isolated phenomenon but rather greatly related to class, colonial, racial and cultural constraints experienced by the individual as s/he struggles for access and participation in related practices.

One might observe that although there is noticeable increase in female achievement and participation, the gap between males and females continues to create social inequity. Even though female students are as competent as male and enjoy practising technoscience, they continue not to choose the subject as a main field for study or work. The narrative of Afrodite, a young Gypsy Greek girl, as seen in the introductory vignette, indicates how both gender and race discourses prevent not only her continuous participation in schooling practices thus making her a case at risk, but also constitute her 'voiceless'. Spivak uses the term 'subaltern'[5] to talk about how certain colonial and postcolonial discourses constitute not really the 'voiceless subject', but the subject who realises the impossibility of 'voice'. In exemplary cases of female struggles in imperial India she problematises how the colonial world has always been defined by the West. According to Spivak (1999, 1992a) civilisation, progress and even self-identity itself always eludes the subaltern. In other words, the West is defined by differentiating amongst the 'present', 'past' and 'future' as well as by excluding the other. The colonial world has no such self-identity, at least as the Western viewer perceives it. The cry in Afrodite's diary-writing, perhaps, denotes exactly this awareness of the impossibility to speak and become heard about non-easily fixed, almost un-resolvable, issues.

Based on Spivak we realise how Afrodite becomes doubly the 'other' as a woman and as a gypsy woman and how she realises her fragile and fractured self as

[5] Gramsci has originally coined the term 'subaltern' in order to address the economically dispossessed, and today Ranajit Guha reappropriates Gramsci's term in an effort to locate and re-establish a voice or collective locus of agency in postcolonial India. In her essay "Can the Subaltern Speak?", Spivak acknowledges the importance of understanding the 'subaltern' standpoint but also criticises the efforts of certain subaltern studies emphasis towards creating a 'collective voice' through westernised mediating practices.

she attempts to cross cultural borders amongst home and school. She struggles to live in-between two worlds that require her to continuously cope with conflicting choices and feels emotionally devastated as her diary-writing reveals (Dafermos 2005, p. 257–259). Anzaldua (1987) argues that crossing 'borders' is not a simple but instead a process of learning to accept transformations and learning to tolerate contradictions and ambiguities—a 'mestiza' rhetoric in her words. Mendick (2005) refers to Anelia, a young Turkish Cypriot girl from a UK-based immigrant family, who also experiences the home/school divide. Anelia, as Afrodite, has a passion for (mathematics) education, but she also perceives that making the choice to study could be incompatible for her life because she holds that 'mathematics' is not suitable for a woman. Participation in formal educational practices means for Anelia, as well as for Afrodite, engaging in identity-work that creates multiple contradictions in her life and leads her towards limiting the study of science or considering quitting school (see the case of Afrodite). While both women cope well with formal educational activities—including mathematics and technology—they risk being characterised as 'savage', primitive or other. Although this view is highly criticised by contemporary anthropological thinking for being an imposed 'gaze' at non-Western cultures (Appadurai 1996; Harding 1998, 2008), such differentiation serves to reinforce the epistemic chasm between savage and rational mind by perpetuating knowledge hierarchies. In addition, this chasm shows, as Spivak (1992b) argues, a concern for the processes whereby postcolonial studies rehearse neo-colonial imperatives of political domination, economic exploitation and cultural erasure—an issue referred to by Spivak as 'epistemic violence'. It can be claimed that this is due to the fact that 'development' for Afrodite and Anelia is counted on an imperialist conception of the world and of technoscience. Spivak's post-colonial critique addresses the western, male, privileged, academic, institutionalised discourses which classify the 'other' in the same measures used by colonial dominance that, ironically, seek to dismantle.

Most Gypsy girls do not perform as individual 'entrepreneurs of self', using Paul Du Gay's words, as their decisions in life are depended on extended family and community values, needs or habits (Du Gay 1996). Living between two cultures, Afrodite has to confront conflicting discourses about either 'attachment to community life' or 'pressure to lead a modern life' (Chronaki 2009). For her, it is not an either/or situation but instead a desire to be both and this very fact places her in a painful situation. Afrodite's dilemma whether to quit school or not is connected with pathologising her as incapable of making a sensible choice and as destined to remain subaltern. How could we, then, reconsider 'equity' in view of Afrodite? This means that gender equity cannot be simply viewed in terms of comparative studies of male's and females' skills and attitudes rooted in quantitative analysis or positivistic interpretations. Afrodite's urgency to develop is also linked to her urgency to move towards a certain quality of modern 'life' inscribed through masculine and imperialist agendas of development. Her case, in particular, urges us to consider this 'move' as an unfulfilled promise or as potentially unending.

A major consequence of hegemonic discourses of equity is the constitution of subjects as marginalised, oppressed or voiceless. In an almost pessimistic tone Spivak concludes that the ‘subaltern’ cannot speak in the context of cultural imperialisms and moreover the ‘subaltern’ cannot be given a voice via a mediator. She specifically suggests that any attempt from the outside to grant subalterns a ‘collective voice’ is problematic as first, it assumes cultural solidarity among members of a heterogeneous group of people, and secondly it depends upon western intellectuals to ‘speak for’ their condition. Spivak argues that through such a process the subalterns, in fact, re-inscribe their marginalised and subordinate positions. Afrodite seems to fall into this category. As a gypsy woman she is required to perform a ‘normal’ gendered positioning as constructed by her community. Taking into account that ‘normal’ is a fictional category one can claim that there is no normal way for any gender to act. Gayatri Chakravorty Spivak optimistically argues that although we cannot ‘give’ a voice, we can clear the space for the subaltern to speak. She suggests that instead of urging for a ‘collective voice’ by means of the Western logos, it is preferable to focus on clearing the subalterns’ path so that their voice can be heard. The subaltern, be it a Gypsy adolescent girl or a Western woman who, though competent with computers and maths, chooses not to make them a priority in life, seems to live at the margins of hegemonic discourses of ‘development’. Clearing the path for them to be heard, in the context of this study, is closely related to troubling and disrupting—in Butler’s words—‘development’ within hegemonic discourses by revealing contradictions and taking seriously the contextual processes that constitute marginalised and voiceless positioning(s). Growth, progress, development all seem to safeguard quality. And access to *quality for all* is assumed to be the measure for equity. Quality and equity, thus, become the two opposite sides of the same coin called ‘development’.

Conclusionary Remarks

Technoscientific practices, and school technoscience in particular, are central to both self and society ‘development’. A recent anthropological study shows how ‘namba tok’ (number talk or the use of statistics by colonial officials) is coupled with ‘kaantri’ (country) creation in the consciousness of the Nimakot people of central New Guinea who see their lives changing from nomadic to settled inhabitancy (Wesch 2007). What we have come to call ‘modern’ society has emerged through production and appropriation of a variety of ‘technologies’ including arithmetic, archiving and spacing structural systems utilised to organise and control daily mind-body practices. Dunne (2008), based on Foucault (1977/1980, 1991) discusses the political role of mathematics and technology as core values of modernity and explains how both are utilised to define and fulfil goals of ‘development’. This happens simultaneously at two levels; *first*, they are used to measure the achievement of certain predicted economic, social and educational outcomes, through a broad application of statistics, and *secondly*, by applying pressure, via local and national educational policies, mathematical and technological literacy is promoted. An imperialist (and sometimes a post-colonial) agenda of governmentality means that women's access to and participation in these subject areas are measured and evaluated against that of men and western culture. In other words, the dominant discourse of development serves to legitimise 'women' as 'others' (i.e. women, as primitives, need to develop and progress).

From this point of view, technology-mediated mathematics education is not merely a tool for better understanding mathematical concepts, but can be seen as a tool for introducing learners to certain standards of 'modern' life—and for some (including women) this can be a risky, unsafe and uncertain terrain. Hegemonic discourses, based mainly on constructivist and socio-cultural agendas, tend to overemphasise the 'active', 'rational', 'autonomous' learner who is able to instrumentally utilise any accessible technology and make timely choices and decisions. However, such a view eschews the ideological underpinnings of an over-simplified adherence to modernist and neoliberal ideologies. Walkerdine (1993) and Rose (1999), amongst others, explain that discourses related to an impetus to govern modern life are based on the virtue of self-reliance (autonomy, self-regulation, self-efficacy, etc.) and reflect mainstream and conservative psychology or sociology. Rose (1999), in particular, supports that the burden of 'choice' conceals the broader social context in which jobs for life have disappeared leaving the fiction of life-long learning instead. Simultaneously, inability to choose, to act or to make appropriate decisions signifies inability to perform as an 'autonomous subject' which then results into lack of development and leads to marginalisation. As explained above, self/society development requires both a quality and equity dimension. Within the confines of imperialist, colonial and patriarchal discourses, development is taken to be equivalent to the construction of a fixed 'rationality' as the ultimate goal for quality. Rational development is also taken to be at the heart of technoscientific practices including mathematics and technology-related literacies. Therefore, quality in mathematics education curricula and practices is taken to be a cornerstone for safeguarding quality/equity and minimising exclusion and marginalisation.

However, women often seem to either resist or embrace partially and without passion certain technoscientific practices affecting their daily life or work. Such a standpoint can be stereotypically interpreted through the 'woman as a problem' optic—an interpretation rooted in hegemonic discourses of quality/equity. As previously seen, on the one hand, some mainstream constructivist and mainstream socio-cultural perspectives strive to prescribe quality in mathematics education, and on the other hand, certain mainstream feminist perspectives focus on investigating gender inequity not only at the level of achievement, competences and attitudes but also at the level of access to and participation in mathematics and technology-related fields. While constructivist and socio-cultural theorists emphasise quality curricula, feminists identify gender gaps. In simplified terms, it may seem that one's work serves (to sustain) the work of the other. In other words, when a gender gap becomes identified, a quality curriculum will be there to fill the gap. But life is not that easy.

In the realm of the present chapter, it has been argued that hegemonic discourses of quality/equity as means for self/society development need to be approached through alternative perspectives that enable subjects to move beyond a pressurising emphasis to a singular 'perfectionist' relation to technoscience. Hegemonic discourses tend to read women as 'others' by their being considered, perhaps unintentionally, as the passionless and subordinate users of technoscience. By re-reading these stories we come to realise, that involvement in mathematics and technology in school practices is *neither* simply a matter of access to equitable sharing of resources, knowledge and support *nor* an issue of a particularly passionate interest and positive attitude towards the subject. Women and men seem to live in complex localities that require them to simultaneously appropriate not one but a number of discourses that often become competing forces in both personal and school lives. The notion of 'cyborg' induces a renewed vision of quality, as far as the subject's involvement in technoscience is concerned, emphasising partiality and hybridity. Women as 'cyborgs' can be fragile and fractured amalgams of a human-machine organism and can claim for themselves the right to 'error', to express 'failure', to demand 'connectivity' and to feel confident with 'partiality'. According to Haraway (2006/2009), it is not the 'machine' that women reject but the insecurity that comes as a result of communication breakdown. In other words, it is the fact that they do not seem to have control over the fluid relation which develops between humans and machines that requires a 'holistic' instead of a 'connectivist' relation to technology. The cyborg metaphor, thus, has the potential to become a way of thinking and re-working subjectivity as situated, hybrid and partial.

In addition, the notion of 'subaltern', as argued by Gayatri Chakravorty Spivak, offers an alternative optic on issues of difference or otherness as they affect marginalised, oppressed and voiceless subjectivities. She claims that by having limited access to cultural imperialism and by being constructed as 'different' or 'other', the subaltern can signify the 'proletarian' whose voice can not be heard as it is structurally deleted from the capitalist bourgeois narrative. Furthermore, she objects to the view that since the subaltern cannot speak, an advocate is required to speak for her, arguing: *'Who the hell wants to protect subalternity? Only extremely reactionary, dubious anthropologistic museumizers. No activist wants to keep the subaltern in the space of difference […] You don't give the subaltern voice. You work for the bloody subaltern, you work against subalternity'* (Spivak 1992a, p. 46). The burden created by the organisation of 'collective' or 'mediated' voices for subalterns constitutes, according to Spivak, a rehearsal of a political domination of 'voice' via neo-colonial exploitation that ultimately exacerbates 'epistemic violence'. Instead, she voices the need to seriously consider clearing the way for the subaltern to speak.

As it has already been shown, clearing the way is a process of disrupting the hegemonic discourses of development that either implicitly or explicitly nurture subalternity. While Haraway promotes a notion of the cyborg that opens up subjectivity to embrace situatedness, hybridity and partiality, Spivak enters the complexities of marginalised and voiceless subjectivity by encountering the subaltern's voice. The impossible task of being heard signifies the impossibility of realising self as part of society or else the impossibility of belonging. Spivak does not hesitate to criticise the postcolonial practices that assume a 'voice' can be given via the mediation of an advocate and passionately argues that the subaltern do not need to be given a 'voice' but instead we need to clear the way for them to walk and be heard. This important gesture means that the responsibility of their having a ‘voice’ is simultaneously our responsibility of listening to their voices—a deeply dialogical gesture. As a final word, I would like to argue for the need to consider involvement in school technoscience as a risky gendered territory where subjects negotiate their positions by taking the boundaries and affordances of their localities into account. A situated notion of agency with/in technoscientific practices rejects the utopian and imperialist politics of ‘holism’, ‘advocacy’, ‘perfectionism’ and, instead, pursues ‘connectivism’ and ‘partiality’. For this reason, a turn towards post-structural and postcolonial theorising of female experiences with school technoscience may prove most valuable.

References

Althusser, L. (1971). Ideology and ideological state apparatuses. In L. Althuser (Ed.), *Lenin and philosophy and other essays* (translated by Ben Brewster), (pp. 121–176). London: New Left Books.

Anzaldua, G. E. (1987). *Borderlands la frontera: The new mestiza*. San Francisco: Aunt Lute Books.

Appadurai, A. (1996). *Modernity at large: Cultural dimensions of globalization*. Minneapolis: University of Minnesota Press.

Belenky, M., Clinchy, B., Goldberger, N., & Tarule, J. (1986). *Women's ways of knowing: The development of self, voice, and mind*. New York: Basic Books.

Butler, J. P. (1990). *Gender trouble: Feminism and the subversion of identity*. New York: Routledge.

Butler, J. P. (1997). *Excitable speech: A politics of the performative*. New York: Routledge.

Castells, M. (1996/2000). *The rise of the network society*. Malden: Blackwell.

Chronaki, A. (1992). *The epistemology of constructivism in mathematics education.* Bath: University of Bath.

Chronaki, A. (1997). Constructivism as an energiser for thinking. *Chreods: Journal on Philosophical Issues of Mathematics Education, Special Issue on Mathematics Education and Society: Radical Visions and Socialist Perspectives, 11,* 15–21 (guest editor Peter Gates).

Chronaki, A. (2000). Computers in classrooms: Learners and teachers in new roles. In B. Moon, S. Brown, & M. Ben-Peretz (Eds.), *International companion in education*. London: Routledge.

Chronaki, A. (2009). An entry to dialogicality in the maths classroom: Encouraging hybrid learning identities. In M. Cesar & K. Kumpulainen (Eds.), *Social interactions in multicultural settings* (pp. 117–144). Rotterdam: Sense Publishers.

Clynes, M. E., & Kline, N. S. (1960). Cyborgs and space. *Astraunatics. September 26–27,* 5–76.

Dafermos, M. (2005). *Marginalisation and educational inclusion*. Athens: Atrapos (in Greek).

Du Gay, P. (1996). *Consumption and identity at work*. London: Sage.

Dunne, M. (2008). Gendered docility: The power of technology and technologies of power for low income countries. In A. Chronaki (Ed.), *Mathematics, technology, education: The gender perspective* (pp. 45–57). Volos: University of Thessaly Press.

Dunne, M., & Sayed, Y. (2002). Transformation and equity: Women and higher education in sub-Saharan Africa. *International Studies in Educational Administration, 30*(1), 50–65.

Ellul, J. (1964). *The technological society*. New York: Alfred A. Knopf.

Fennema, E., & Leder, G. (Eds.). (1990). *Mathematics and gender: Influences on teachers and students*. New York: Teachers College Press.

Foucault, M. (1972). *Archaeology of knowledge*. New York: Pantheon.

Foucault, M. (1977/1980). *Discipline and punish: The birth of the prison*. New York: Pantheon.
Foucault, M. (1991). Governmentality. In G. Bruchell, C. Gordon, & P. Miller (Eds.), *The Foucault effect: Studies in governmentality* (pp. 87–104). Hemel Hempstead: Harvester Wheatsheaf.
Frantzi, K. (2008). Greek women mathematicians in the 20th century: An overview. In A. Chronaki (Ed.), *Mathematics, technology, education: The gender perspective* (pp. 111–114). Volos: University of Thessaly Press.
Greeno, J., & The Middle School Mathematics Through Applications Project Group. (1998). The situativity of knowing, learning, and research, *American Psychologist, 53*(1), 5–26.
Hanna, G. (2003). Reaching gender equity in mathematics education. *The Educational Forum, 67,* 204–214.
Haraway, D. (1991). *Simians, Cyborgs and women: The reinvention of nature*. New York: Routledge, and London: Free Association Books.
Haraway, D. (1997). *Modest_Witness@Second_Millennium. FemaleMan©Meets_ OncoMouse™: Feminism and Technoscience*. New York: Routledge.
Haraway, D. (2004). *The companion species manifesto: Dogs, people and significant otherness*. Chicago: Prickly Paradigm Press.
Haraway, D. (2006/2009). *Feminist epistemology and philosophy of science*. Retrieved March 14, 2010 from http://plato.stanford.edu/entries/feminism-epistemology/.
Harding, S. (1998). *Is science multicultural? Postcolonialisms, feminisms, and epistemologies*. Bloomington: Indiana University Press.
Harding, S. (2008). *Sciences from below: Feminisms, postcolonialities and modernities*. London: Duke University Press.
Hershkowitz, R., Dreyfus, T., Schwarz, B., Ben-Zvi, D., Friedlander, A., Hadas, N., Resnick, T., & Tabach, M. (2002). Mathematics curriculum development for computerized environments: A designer-researcher-teacher-learner activity. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 657–694). London: Erlbaum.
Illich, I. (1972). *Tools for conviviality*. New York: Harper & Row.
Jutting, J. P., Morrisson, C., Dayton-Johnson, J., & Drechsler, D. (2006). *Measuring gender (in) equality: Introducing the gender, institutions and development data base (GID)*. Working Paper No. 247, OECD Development Center.
Kontopodis, M., Wulf, C., & Fichtner, B. (2010). *Children, culture and education*, (series: International Perspectives on Early Childhood and Development). New York: Springer (in press).
Lakoff, G. (1987). *Women, fire, and dangerous things: What categories reveal about the mind.* Chicago: University of Chicago Press.
Lister, M., Dovey, J., Giddings, S., Grant, I., & Kelly, K. (Eds.). (2003). *New media: A critical introduction.* London: Routledge.
Marrioti, M. (2002). The influence of technological advances on students' mathematics learning. In L. English, M. B. Bussi, G. A. Jones, R. A. Lesh, & D. Tirosh (Eds.), *Handbook of international research in mathematics education* (pp. 695–723). Mahwah: Erlbaum.
Mendick, H. (2005). *Masculinities in mathematics education*. Maidenhead: Open University Press.
NCTM: National Council of Teachers of Mathematics. (2000). *NCTM principles and standards for school mathematics*. Reston: NCTM.
Rose, N. (1999). *Powers of freedom: Reframing political thought*. Cambridge: Cambridge University Press.
Ruthven, K., Hennessy, S., & Brindley, S. (2004). Teacher perspectives on integrating ICT into subject teaching: Commitment, constraints, caution and change. *Journal of Curriculum Studies, 37*(2), 155–192.
Schneider, J. (2005). *Donna Haraway: Live theory*. New York: Continuum.
Schoenfeld, A. (Ed.). (1994). *Mathematical thinking and problem solving*. Hillsdale: Lawrence Erlbaum Associates.
Secada, W. (1992). Race, ethnicity, social class, language and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623–660). New York: Macmillan.

Secada, W. (1995). Social and critical dimensions for equity in mathematics education. In W. Secada, E. Fenemma, & L. B. Adajan (Eds.), *New directions for equity in mathematics education* (pp. 146–164). New York: Cambridge University Press.

Spivak, G. C. (1992a). Interview with Gayatri Chakravorty Spivak: New Nation Writers Conference in South Africa (Interviewer Leon de Kock). *Ariel: A Review of International English Literature, 23*(3), 29–47.

Spivak, G. C. (1992b). Can the subaltern speak? In P. Williams, & L. Chrisman (Eds.), *Colonial discourse and post-colonial theory* (pp. 66–111). New York: Columbia University Press.

Spivak, G. C. (1999). *A critique of postcolonial reason: Towards a history of the vanishing present.* Cambridge: Harvard University Press.

TIMMS. (2007). *Trends in international mathematics and science study: Results 2007*. http://nces.ed.gov/timss/results07.asp.

Treffers, A. (1987). *Three Dimensions: A Model Goal and Theory Description in Mathematics Instruction—The Wiskobas Project*. Mathematics Education Library. Dordrecht: D. Reitel Publishing Company.

Turkle, S. (2008). *Falling for science: Objects in mind*. Cambridge: The MIT press.

Wajcman, J. (2007). From women and technology to gendered technoscience. *Information, Communication & Society, 10,* 287–298.

Walkerdine, V. (1993). Beyond developmentalism? *Theory & Psychology, 3*(4), 451–469.

Walshaw, M. (2001). A Foucauldian gaze on gender research: What do you do when confronted with the tunnel at the end of the light? *Journal of Research in Mathematics Education, 32*(5), 471–492.

Wertsch, J. V. (1991) *Voices of the mind: A sociocultural approach to mediated action*. Cambridge: Harvard University Press.

Wesch, M. (2007). Creating 'Kantri' in Central New Guinea: Relational Ontology and the Categorical Logic of Statecraft. *Media/Culture, 11*(5). http://journal.media-culture.org.au/index.php/mcjournal/article/viewArticle/67.

Xin Ma. (2008). *Gender differences in mathematics achievement: Evidence from latest regional and international student assessments*. Paper presented at the ICME 11. Mexico. Topic Study Group 32: Gender and Mathematics Education.

Chapter 2
Beyond Gap Gazing: How Can Thinking About Education Comprehensively Help Us (Re)envision Mathematics Education?

Rochelle Gutiérrez and Ezekiel Dixon-Román

One cannot talk about equity these days without being politically correct. In fact, in the United States, "equity" has become an empty signifier manipulated in/through discourse (Dixon-Román, in press). For example, although many use "the achievement gap" as an important call for school accountability around needed resources and additional support for marginalized students, (e.g., Education Trust 2005), such discourse has done little more than replace "the culture of poverty" in the latest of deficit frameworks. That is, while equity issues are becoming more mainstream in the mathematics education community, theoretical framings continue to reflect equality rather than justice, static identities of teachers and students rather than multiple, fluid, or contradictory ones (Gutiérrez 2002, 2007; Martin 2009) and schooling rather than education. Whenever words like "quality," "democracy," and "equity" are used, we must first unpack what these terms mean and then examine who benefits from the definitions employed.

Let us consider the prevailing equity discourse in the United States: "the achievement gap." The excessive focus that U.S. researchers place on the gap between the mathematics achievement of white, middle-class students and that of African American, Latin@[1], American Indian, working-class students, and English learners and the need to close the gap (termed "gap gazing") sheds light on issues of access and achievement from a dominant perspective (maintaining the status quo) with little concern for how students are constructed in the process, what additional

[1] We use the @ sign to indicate both an "a" and "o" ending (Latina and Latino). The presence of both an "a" and "o" ending decenters the patriarchal nature of the Spanish language where is it customary for groups of males (Latinos) and females (Latinas) to be written in the form that denotes only males (Latinos). The term is written Latin@ with the "a" and "o" intertwined, as opposed to Latina/Latino, as a sign of solidarity with individuals who identify as lesbian, gay, bisexual, transgender, questioning and queer (LGBTQ).

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skills are needed to negotiate the discursive spaces of education, and/or how power relations play out in learning. This phenomenon exists beyond the United States as well, in the form of international comparative studies. Although all comparative studies are not problematic, they continue to privilege and normalize certain groups and practices. As such, international studies that seek to compare nations can echo this preoccupation with an achievement gap. While we recognize the essentialist underpinnings of such terms as "white middle-class students" or "American Indian students," we use these terms as they circulate in discourses and are used to position individuals in education.

In this chapter, we begin by destabilizing the equity definition implicit in the achievement gap discourse by first outlining the dangerous effects of maintaining a gap focus and then by explaining, from a Foucauldian perspective, how the "gaze" operates. From there, we turn to research on comprehensive education to highlight the many ways in which people learn or are educated outside of schooling. Finally, we examine what impact shifting our goals from "closing the gap" to "meaning making in social interactions" might have for the endeavor of mathematics education. Throughout this chapter, we argue that both relying upon discourses like the "achievement gap" and continuing to privilege schooling as the primary institution of education will ensure that students of color and other subordinate populations will continue to be left behind. We show how questioning the concepts of "equity" and "education" leads to equally important questions such as "What counts as mathematics?"—an often overlooked issue in debates about equity, access, and democracy. Considering education comprehensively in this way may offer an opportunity to better unite philosophers, sociologists, and cultural anthropologists of mathematics with those who educate broadly.

Destabilizing the Achievement Gap Discourse

The Problem with Gap Gazing

In order to better understand the meanings operating with respect to the term "equity" as well as who benefits from such meanings, we take the perspective of subordinated individuals in society (defined here as African American, Latin@, American Indian, working-class students, and English language learners). Let us consider the various ways in which the current focus on "the achievement gap" within mainstream mathematics education in the United States is problematic. First, although mainly concerned with the well-being of marginalized students, researchers who focus on the achievement gap support practices that often are against the best interests of those students (Gutiérrez 2008). That is, while documenting the inequities that marginalized students experience daily in mathematics education could be seen as the first step toward addressing hegemony, most research stops there. Examining the gap from its many angles and perspectives (what Gutiérrez calls "gap gazing") has done little to change the will or commitment of a nation to engage its citizens in broader forms of mathematical literacy, in part because closing the achievement gap suggests nothing is wrong with the system. Although some would argue that gap gazing seems to relate only to research studies that document the gap and not those that attempt to reduce the gap, we use the terms “gap gazing” and “achievement gap perspective” interchangeably to denote the fact that both connote a discourse that fails to consider equity beyond the narrow definition of “access.” See Gutiérrez (2008) for further elaboration.

More specifically, because gap gazing draws upon one-time cross sections of data, it offers little more than a static picture of inequities with inadequate information about how those inequities were created. In addition, achievement gap studies often fail to question the validity of measurement tools or the choice to focus on measurement. In fact, the best we can achieve under an achievement gap lens is to close the gap, to show that marginalized students can do *as well as* middle-class whites. Most researchers and practitioners fail to question the underlying assimilationist goal and the ways in which framing the problem as an achievement gap supports deficit thinking and negative narratives about marginalized students. That is, such thinking encourages researchers to focus on ways to make subordinate populations more like dominant ones. And yet, other researchers have made cogent arguments for framing the issue of equity around other kinds of gaps, including the opportunity gap (Flores 2007; Hilliard 2003), the education debt (Ladson-Billings 2006); the gap between whites and Asians (Gutiérrez 2008; Martin 2009).

Gap gazing also accepts a static notion of student identity, presuming that students can be reduced to a set of cultural markers, rather than recognizing they are constantly in flux, dependent upon the social structures and social relations in which they are engaged. By always relying upon a comparison group, the achievement gap discourse perpetuates the idea that subordinate populations cannot be studied for their own sake and/or that such populations have nothing to contribute to more general discussions or theories about education. Ignoring the largely overlapping achievement patterns of groups, the dividing practices (Foucault 1980) common in gap gazing research serve to dehumanize students.

Moreover, the achievement gap discourse provides researchers and practitioners with a “safe” proxy for talking about students of color without naming them or having to discuss the institutions of racism, classism, or politics of language that are endemic in today’s society. And, by failing to interrogate these hegemonic institutions, the achievement gap discourse perpetuates the myth that the problem (and therefore solution) is technical in nature. Finally, gap gazing relies upon narrow definitions of learning and equity, assuming both that today’s school mathematics curriculum is the one to which we should aspire and that access to an unfair system is a sufficient goal. Yet, marginalized populations have historically shunned the idea that white student achievement levels signal excellence. Instead, they have tended to hold broader standards for themselves—defining excellence with an eye toward not just doing well in school, but also maintaining cultural values and a critical attitude in their young, often privileging ties to the community over individual success (Hilliard 2003; Kurzweil 2003; Valdés 1996).

The Impact of the Gaze

Currently, the discourse in the United States around the achievement gap is so prominent and normalizing that it is almost unimaginable that students, teachers, and community members can escape its grip. In fact, we argue they do not. Subordinate students and their local communities are assaulted daily with headlines of continued or growing achievement gaps, constructing them as inferior to middle-class whites and Asians. Much of the mathematics education community is complicit in this construction of subordinate populations. In fact, when Gutiérrez first published an article on gap gazing in July 2008, a search in Google Scholar with the words "achievement gap" + "mathematics" produced 8,000 hits; today that same search produces 137,000 hits, signaling that the research community's absorption with this discourse is not waning. Beyond the research community, our society in general seems to have bought into this achievement gap pre-occupation. The same search in regular Google produces 404,000 hits.

The focus on the achievement gap by mainstream mathematics education allows for only certain "truths" to arise (Foucault 1980; Walshaw 2007). These "truths" are not universal or fixed. Rather, they are constructed by our choice of focus. For example, in the achievement gap story, at best (by closing gaps), we can show that students of color are capable of doing as well as middle-class whites; at worst (by failing to close gaps), we reify the notion that perhaps the intellectual capacity of students falls on a "natural" hierarchy that is coded by ethnicity/race. From a post-structuralist perspective, it is the gaze and the repetition of that gaze in discourse that: (1) makes the achievement gap comprehensible, (2) normalizes, and (3) gives authority to a particular discourse about equity. Here, the discourse is one that focuses on standardized test scores and the kinds of students who are capable of doing well in mathematics. As a result, students of color continue to be framed in comparison to whites; this comparison then becomes normalized, as if it is a "natural" way of thinking about achievement, rather than focusing on the excellence of students of color or the many other ways subordinated students may make sense of their experiences with mathematics.

By providing the categories by which teachers and students see themselves, the gaze further serves to regulate bodies in ways that shut down other possible discourses and technologies within school. Even when students are not in school and/or are grown adults, the achievement gap discourse continues to construct our notions of who is good in mathematics and who is not (Martin 2007), as well as influence how those constructions relate to intelligence overall. The residue of comparisons and testing regimes lingers long into one's life.

Teachers who may have thought of their work in much more complex ways may find themselves ignoring other signs of excellence (e.g., improved inter-group relations in students, greater student participation in advanced mathematics courses, positive dispositions toward mathematics, students having improved/broadened visions of their futures, the ability to see that mathematics is socially constructed). This kind of self-regulation occurs because schools shape, monitor, and discipline the knowledge, modes of operating, and positionings of teachers (Walshaw 2007). So, even if an administrator is not explicitly asking teachers to act in particular ways, the mere threat of broader surveillance, of wanting to fit in with what is deemed acceptable or professional, is enough to affect the technologies (practices) seen to be valid. This is especially true, given that the National Council of Teachers of Mathematics, the professional society for mathematics education in the United States, embraces the discourse of gap-closing when conceiving of "equity" and in its annual conferences.

If we decide that doing mathematics equates to scoring well on "achievement tests," then the issue is not just whether the measurement tools we develop have sufficient accuracy or whether the models can become better at predicting or ensuring achievement in school. We now are in the realm of consequences for the kinds of individuals (e.g., students, teachers, parents) who are constructed, which is an artifact of the original definition of mathematics that was employed, something we return to later in this chapter.

Fortunately, individuals are not mere consumers of the discourses that operate in society. That is, just as gap gazing can close down the identities marginalized students are seeking to create/act, it also can serve as the means for opening up new identities. Subordinated students constantly (re)interpret, (re)use, and (re)invent such discourses for their own purposes (Dixon-Román 2009). As such, what may have begun as consumption can turn into a form of production. Even so, this production does not come without costs. Re-signifying/subverting such discourses (Butler 1993) requires that students do additional cognitive and emotional work (McGee 2009; Stinson 2008).

What is hidden in our preoccupation with testing and achievement is the fact that (mathematics) education happens on so many more levels than schooling. Moreover, there is a false dichotomy between the sciences and the humanities (Davis 1994). As educators, we must challenge the wisdom of using the achievement gap discourse as the means for addressing equity/democracy. Varenne (2008) reminds us that,

> school achievement is but a small part of American education and we must convince policy makers (and I include everyone here from senators in Congress, to school teachers) that the main issue for American democracy is not getting everyone to achieve at grade level…it is our duty, as given by those who maintain our positions as experts, to challenge what policy makers actually enforce on each other. (pp. 364–365)

In this sense, the *debate* about what we want for our children/students/society with respect to mathematics is as important as the products that result from education. Thus, it is not just important, it is our "duty" to challenge the centering of mathematics knowledge in schooling and reveal the multiple levels of mathematics education beyond schooling.

The kinds of research questions we ask influence the knowledge that is created as well as what we might be able to do with that knowledge. We see that maintaining a focus on education as it occurs within schooling ends up (re)inscribing the inequitable conditions that produce "failing" students as if they are attributes of the students themselves. Looking to life more generally, we see people learn everyday from many sources and for many purposes.

Thinking About Education Comprehensively

In his address to the John Dewey Society, Cremin (2007/1975) states that,

> The important fact is that family life does educate, religious life does educate, and work does educate; and, what is more, the education of all three realms is as intentional as the education of the school, though in different ways and in different measures. (p. 549)

He further goes on to set forth three assertions:

> First, we have to think comprehensively about education; second, that we have to think relationally about education; and third, that we have to think publicly about education. (p. 1550)

Cremin's insightful observations and analysis were a challenge to the theory of education within educational research, policy, and practice. That is, education had been (and continues to be) understood as a binary opposition between schooling and society, in which schooling was/is privileged as the site of education. Cremin points to this very contention in John Dewey's *Democracy and Education*, but argues that Dewey falls victim to the very dualism that Dewey attempts to reconcile. It is via this understanding of education comprehensively, relationally, and publicly that Cremin attempts to resolve the dualistic understanding of education and society by speaking to how one contaminates the other, how schools are related to other societal apparatuses, and how each of their relational focus is important in the equitable development of high human potential.

When we recognize the limitations of public schooling, other forms of learning in society become important, not just so they may (re)engage and validate a subordinate population, but also because they have the potential to (re)engage education as something broader. Over the years, partly because of racism, sexism, classism, and politics of language, marginalized populations have not been able to rely upon public schooling to teach their young. They often rely upon schools to transmit dominant values and skills, but recognize the need to supplement those values and skills with other things that affirm the child. Through necessity, families and other institutions supporting subordinate populations have had to become more deliberate in their teaching because their young have had to negotiate schooling as an institution in way(s) different from dominant populations. To be clear, we are not suggesting that dominant populations do not need to negotiate schooling or are not educated outside of school. But, we highlight the fact that by focusing our attention elsewhere (to the margins), we find potential solutions for making education overall better, not just for the subaltern.

For these reasons, and building upon Cremin, others (Gordon et al. 2005; Varenne et al. 2009) have begun to theorize, examine, and consider many of the various forms of supplementary and comprehensive education. Supplementary education refers to all of the learning and developmental experiences that occur outside of schooling; whereas, comprehensive education is concerned with the deliberate and relational educative experiences in all institutions of society, not just schooling. They suggest supplementary and comprehensive education might include libraries, museums, childcare centers, health education and clinics, martial arts, hip hop, after school programs, athletics, parenting practice workshops, financial literacy programs, prenatal services, among many others. It is via each of these community institutions, spaces, and practices that we find the various intentional educational processes that Cremin asserts.

For our work, we rely upon Varenne's distinction of thinking about education comprehensively, rather than thinking of some entity we might call comprehensive education that would replace schooling. Building upon the idea that the arrangements of education are somewhat arbitrary (Garfinkel 2002), Varenne (2007, 2008, 2009) puts forth the idea that we are all ignorant and that leads us to seek knowledge. Individuals are constantly trying to figure out what is happening around them, as well as learning to be adaptive to their environments (Lee 2008). When people try to figure out who they are, they rely upon those around them. When people fool around (do not follow the rules), they are instructed in how to behave. This education happens not just with respect to families, but occurs while one is standing in line at the post office, when one is given medicine, or when one is instructed by labels as to which product is best for us.

Considering a more comprehensive view of education allows us to move beyond distributive models of justice (the redistribution of resources in society) toward affirming multiple (and unsanctioned) ways of knowing and challenging the norms of decision-making processes. Both of these are non-material resources of power that are not addressed by distributive models of justice (Dixon-Román, in press; Young 1990). We turn now to what this might mean for mathematics.

Exploring Mathematics Education Comprehensively

The research in mathematics education has not been completely centered on schooling. In fact, programs of research in ethnomathematics, social justice mathematics, and out-of-school mathematics bring us a step in the direction of challenging school-centered mathematics knowledge (Ascher 2002; D'Ambrosio 2006; Frankenstein and Powell 1994; Miranda 2008; Nasir 2000; Nasir et al. 2008; Nuñes et al. 1985; Saxe and Esmonde 2005). However, policy makers and educators are often left scratching their heads about what to make of the fact that students cannot transfer their knowledge of mathematics in out-of-school contexts to an ability to do mathematics in school. These studies do not tend to offer compelling arguments that doing mathematics as an endeavor should be challenged, as they tend to privilege school mathematics as a frame for identifying (i.e., judging) what happens outside of school. Nor do they explicate the desired relationship between schooling and institutions outside of schooling, the relational piece that Cremin put forth.

At times, these studies seem to imply that schools simply need to do a better job of reflecting the real-world problems that people encounter in their lives. However, such a task would require fabricating false problems that are not really those of the particular individuals in a classroom. In fact, out-of-school studies do not suggest that because we know African American males do mathematics while they are engaged in such "cultural practices" as playing basketball or dominoes, we should necessarily include domino playing or basketball as a means for hooking such students into learning school mathematics or highlighting their intelligence. However, other than chronicling that people do mathematics in various effective ways outside of school and that these ways should be valued, the aims of out-of-school mathematics seem poorly articulated with educational policy in general or schooling as an institution.

More than just thinking about the forms of mathematical practices and whether they are valued—be they social justice-oriented, ethnomathematics, or out-of-school—thinking about education comprehensively pushes us to think about the ways in which mathematics formats our worlds. Such thinking moves beyond a sole focus on the practices themselves toward a greater awareness of the role(s) that mathematics play in decision-making. Without paying greater attention to these current structuring roles, we are unlikely as a community to (re)interpret, (re)use, or (re)invent the roles we would like for mathematics to play in our future.

Rethinking (the Roles of) Mathematics

Most researchers writing about equity in mathematics education fail to question what counts as mathematics and/or what should be its role in helping create a more just society. This kind of question is typically reserved for the philosopher, anthropologist, or sociologist. Yet, what counts as mathematics is important to the endeavor of education because the definition of mathematics is complicit in constructing difference (Gutiérrez 2010 in press; Wiliam 2003). In fact, the high status that society confers on mathematics may relate more to the fact it correlates with intelligence tests and is easy to create large differences in performance between individuals than the fact that there is something inherent in mathematics that makes it powerful. The continuation of this falsely earned status in schooling may be due to the fact that males tend to perform better on such tests (Wiliam 2003; Wiliam et al. 2002).

What would it look like in mathematics to consider education comprehensively? To answer this question, we need to grapple with the practices in which people are mathematizing their world and that happens everywhere, not just in schools. We must ask ourselves when are these practices defined as mathematical, when are they considered something else, and when do they blur the boundaries? In fact, regardless of where they occur, social practices can never really be defined in essentialist terms such as mathematics or non-mathematics; however, they can be seen as more or less consistent with previous inscriptions of mathematics as (e.g., measurement) or something that departs from that. We turn, here, to the work of Ole Skovsmose, in particular his notion of "mathematics in action" and the "formatting power of mathematics" to help uncover the roles that mathematics plays in structuring reality.

Skovsmose and colleagues have put forth the argument that in society, being rational is correlated with being mathematical (Christensen et al. 2008; Skovsmose 2004; Skovsmose and Yasukawa 2004). That is, mathematics is seen as the arbiter of truth.

> When debating the quality of a mathematically based decision, only questions concerning rigour of the mathematical description and analysis appear to be open to serious discussions. (Christensen et al. 2008, p. 78)

This view is supported because individuals generally assume that mathematics has the potential to adequately represent the essential characteristics of all things. If we have two different cell phone plans and we need to make a decision between them, we can model with statistics what are the likely outcomes for the average user to arrive at our answer, even knowing that no such "average" user exists. The idea is that mathematics embues a kind of rational order to things that allows one to choose without emotion or bias. Yet, we need only consider how difficult it is to "quantify" or otherwise capture in "mathematical" ways the cultural significance of things (e.g., the value of a deceased loved one's picture, the significance of being able to speak one's mother tongue freely, the impact of art/music on the psyche) to see how this line of thinking goes awry.

Considering a comprehensive view of education, we note that everyday decision-making is never purely mathematical. While some models become useful for making sense of our world, we must also recognize that at some level they are imprecise, fabrications of our surroundings. We often make decisions partly by what the "mathematics" tells us, partly by what "other things" tell us. How do we justify our decisions to ourselves (and others) when they are inconsistent with the "data" we have before us? The fact that people choose to invest in "green" funds or buy "local" or "organic" suggests much more is being considered than maximizing returns or some other straightforward (universal), "rational choice," or cost-benefit analysis.

The valuable point that Skovsmose and colleagues make is not just that individuals and communities are enculturated into this view of mathematics (e.g., as inherently powerful), but that realities become substituted by false situations. This is what they call the "formatting power of mathematics." They argue further that education becomes the process by which reality is falsified in order to dominate.

Following this logic, D'Ambrosio wonders whether we should educate the indigenous/marginalized or whether such individuals might be better off *not* being indoctrinated with such mathematical formatting of the world that is part of the culture of power. But, here is where D'Ambrosio slips into privileging schooling as the main institution of education. In fact, if we recognize that many institutions educate, then we can see how society in general (families, religions, museums, media, community centers), and not just "schooling," contributes to this formatting power of mathematics. And, yet, drawing from the Program ethnomathematics, we can also acknowledge that not all societies value quantity over quality (privilege the products of measurement).

Studies of adults learning to use mathematics (to learn what they did not learn in school, to get better jobs, to take part in political discussions) offer some interesting findings (e.g., FitzSimons 2002; FitzSimons and Godden 2000). These studies highlight flexible learning (e.g., self-teaching) and the de-institutionalization of education (e.g., workplace valued over the academy). Moreover, these studies illustrate how individuals deal with technologies of power that serve to construct them as either "doers" or "knowers" of mathematics. Drawing on studies of adult learners, we might ask, how do individuals negotiate these technologies of power? How do adults become aware of their role in producing and/or using mathematics? What are the implications of market-driven learning that deprofessionalizes teachers and places the responsibility for life-long learning on workers?

By thinking about education comprehensively and therefore rethinking the roles of mathematics in society, it moves us from tinkering with the current arrangements in school (e.g., developing better lists of how to improve achievement for particular populations, creating better models for measuring or predicting achievement, closing the gaps between haves and have nots) and moves us to trying to better understand the reliability of mathematics put into action. When does mathematics capture the salient aspects of one's surroundings and when does it miss? When it misses, what new creative inventions of mathematics are put into play? What is the relationship between doing mathematics and doing other aspects of everyday life? What might it mean to embody ethical actions when using mathematics (Skovsmose 2004)? A broader dialogue is necessary if we are to coordinate the various institutions of education that operate in society.

Although the main purpose of thinking about education comprehensively is to de-center schooling as the primary source for education, it is not to completely dismiss the role of schooling in the broader enterprise of education. As such, we also consider what can schooling learn from a comprehensive look at education? What do people learn about the value and power of mathematics through schooling? Much attention has been placed on the achievement gap. Beginning at that level, instead of asking how we might close the gap, we might ask: through the discourse of the achievement gap, what do individuals learn about themselves and others (e.g., what people are capable of)? What do people learn about competition and/or inequalities? How does that relate to the formation of self and other? Moving beyond the gap gazing discourse, we might ask what are some of the roles that mathematics plays in structuring our realities? When do we "do" mathematics and for what purposes?

Besides becoming more aware of what counts as mathematics, schooling can learn to recognize the structure of the discipline. Traversing the belief of a universal mathematics and recognizing that individuals produce different mathematics in relation to others (over time) allows us to see that academic mathematics is but one form of ethnomathematics (Frankenstein and Powell 1994) that does not always support people to make sense of and function effectively in their worlds. If mathematics has become a tool by the dominant to justify their position, often to look down upon those seen as less rational, how can students learn to be more aware of when they are (re)interpreting, (re)using, or (re)inventing mathematics? How might teaching mathematics for social justice deliberately connect with or contribute to this endeavor?

Schools can learn from people/institutions that operate outside of schooling (both in content and in form). That is, those who educate can be more deliberate about what supplements life (in recognizing how mathematical practices shift over time, deciding whether to speed up these shifts, try to stop them, etc.). In some ways, technology's impact on mathematics is a good example. From time to time, heated debates arise within mathematics education about whether students should have access to technology (primarily calculators) before they have learned the "basic skills" that technology performs. The question of whether to use calculators/computers in math class assumes schooling can somehow control what students "learn" about mathematics, as if they are not already using such technologies outside of mathematics class to make decisions or to educate themselves. Moreover, rather than stressing the importance of introducing students to "real-world problems" in mathematics classrooms as a means to "hook" (i.e., trick) students into doing school mathematics, schools can learn more (e.g., from studies of out-of-school mathematics) about how individuals make judgments concerning when mathematical description is (or is not) adequate as well as what else needs to be considered.

It is not just that thinking about education comprehensively adds to the mathematics education literature by helping us focus on which practices to attend to (e.g., other cultures, other places besides school). It also moves us beyond the mere chronicling of practices to developing a policy agenda. That is, beyond understanding the structuring roles of mathematics in our lives, we also care to influence (push back on) those roles. There are many decisions to be made. We might ask what are some of the ways we would like mathematics to relate to uncertainty, politics, and/or technologies? Perhaps we want school leaders to educate about both the horrors (applications of destruction) and the beauty of mathematics? Rather than perpetuating an internal sense of power to mathematics, we might want citizens to develop the ability to discern for themselves which kinds of questions can be answered using mathematics and which cannot. By thinking critically about the benefits and drawbacks to formatting realities with mathematics, we could be more deliberate in how and when we want to use/create mathematics in our everyday lives.

Conclusion

A brief review of the problems in research using an achievement gap lens helps illustrate that testing and assessment are the remains of "schooling" as a practice. In contrast, thinking about education comprehensively highlights important policy implications. For example, demanding an obsolete disconnected mathematics and testing students to do well in it will not prevent students from rejecting it as a practice.

Achievement gap-only ways of thinking about equity will only continue to privilege schooling as the primary institution of education, imprinting upon students the residue of the hegemonic manners in which schooling has operated over the centuries, not liberate them from oppressed positions in society.

Our purpose in unpacking the gap gazing trend in the United States and in considering education more comprehensively is not to propose a fully developed policy agenda with respect to mathematics education. However, we have offered a number of questions along the way that may guide the development of a policy agenda. For example, what are the structuring roles played by mathematics in a technological and global society? More specifically, in what way(s) does mathematical formatting convince individuals they are making value-free decisions? How do those structuring roles influence the available identities of individuals and the constructions of "truth" about the world in which we live? In what way(s) does surveillance (by others, by self) play into the project of mathematics education? We believe many more questions still need to be asked, and in ways that better engage the broader public in decision-making. In fact, we have refrained from trying to answer these questions outright because we believe there is much more exploration and theory-building that needs to be done and because offering answers now may close down the kind of dialogue we see as important.

We need to be constantly considering the forms of mathematics and what they seek to deal with. As society presents new demands, new technologies, new possibilities, we must ask ourselves whether our current version of mathematics is adequate for dealing with the ignorance that we have.

Discourses like the achievement gap freeze mathematics into a commodity that needs to be "sold" to students while they are in school. Yet, when we look at how individuals and communities make sense of their surroundings with/through/in mathematics, we begin to open up possibilities for rethinking what mathematics does along the way. It is in rethinking "what is education?" and "how might mathematics participate in the creation of a more just world?" that is at the very heart of the democratic project.

References

Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures*. Princeton: Princeton University Press.

Butler, J. (1993). *Bodies that matter: On the discursive limits of "sex"*. New York: Routledge.

Christensen, O. R., Skovsmose, O., & Yasukawa, K. (2008). The mathematical state of the world: Explorations in the characteristics of mathematical descriptions. *Alexandria: Revista de Educacao em Ciencia e Tecnologia, 1*(1), 77–90.

Cremin, L. (2007/1975). Public education and the education of the public. *Teachers College Record, 109*(7), 1545–1558.

D'Ambrosio, U. (2006). *Ethnomathematics: Link between traditions and modernity*. Rotterdam: Sense Publishers.

Davis, P. J. (1994). Mathematics and art: Cold calipers against warm flesh? In P. Ernest (Ed.) *Mathematics education and philosophy: An international perspective* (pp. 164–183). Bristol, PA: Taylor and Francis.
Dixon-Román, E. (2009). *Deviance as pedagogy: A critical perspective on indigenous cultural capital*. A paper presented at the annual meeting of the American Educational Research Association. San Diego.
Dixon-Román, E. (2010). Inheritance and an economy of difference: The importance of supplementary education. In L. Lin, E. W. Gordon & H. Varenne (Eds.), *Educating comprehensively: Varieties of educational experiences: Vol. 3. Perspectives on comprehensive education series*. Lewiston: The Edwin Mellen Press (in press).
FitzSimons, G. E. (2002). *What counts as mathematics? Technologies of power in adult and vocational education*. Boston: Kluwer Academic Publishers.
FitzSimons, G. E., & Godden, G. L. (2000). Review of research on adults learning mathematics. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 13–45). Dordrecht: Kluwer Academic Publishers.
Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap? *The High School Journal, 91*(1), 29–42.
Foucault, M. (1980). *Power/Knowledge: Selected interviews and other writings, 1972–1977*. New York: The Harvester Press.
Frankenstein, M., & Powell, A. B. (1994). Toward liberatory mathematics: Paulo Freire's epistemology and ethnomathematics. In P. McLaren & C. Lankshear (Eds.), *Politics of liberation: Paths from freire* (pp. 74–99). New York: Routledge.
Garfinkel, H. (2002). *Ethnomethodology's program: Working out Durkheim's aphorism*. Lanham: Rowman and Littlefield.
Gordon, E. W., Bridglall, B. L., & Meroe, A. S. (2005). *Supplementary education: The hidden curriculum of high academic achievement.* Lanham: Rowman & Littlefield Publishers.
Gutiérrez, R. (2000). Is the multiculturalization of mathematics doing us more harm than good? In R. Mahalingam & C. McCarthy (Eds.), *Multicultural curriculum: New directions for social theory, practice, and policy* (pp. 199–219). New York: Routledge.
Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Towards a new equity research agenda. *Mathematical Thinking and Learning, 4*(2 & 3), 145–187.
Gutiérrez, R. (2007). (Re)defining equity: The importance of a critical perspective. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 37–50). New York: Teachers College Press.
Gutiérrez, R. (2008). A "gap gazing" fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education, 39*(4), 357–364.
Gutiérrez, R. (2010). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education* (in press).
Hilliard, A. G. (2003). No mystery: Closing the achievement gap between Africans and excellence. In T. Perry, C. Steele & A. G. Hilliard (Eds.), *Young, gifted, and black: Promoting high achievement among African-American students* (pp. 131–166). Boston: Beacon Press.
Kurzweil, J. (2003). Redefining excellence. *SACNAS News, 5*(2), 1–2, 11.
Ladson-Billings, G. (2006). From the achievement gap to the education debt: Understanding achievement in U.S. schools. *Educational Researcher,* 35(7), 3–12.
Lee, C. D. (2008). The centrality of culture to the scientific study of learning and development: How an ecological framework in education research facilitates civic responsibility. *Educational Researcher, 37*(5), 267–279.
Martin, D. (2007). Mathematics learning and participation in African American context: The co-construction of identity in two intersecting realms of experience. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 146–158). New York: Teachers College Press.
Martin, D. (2009). Researching race in mathematics education. *Teachers College Record, 111*(2), 295–338.

McGee, E. (2009). *A model of mathematical resilience: Black college students negotiating success in mathematics and engineering*. A paper presented at the annual meeting of the American Educational Research Association. San Diego.

Miranda, F. O. (2008). *Ethnomathematics and critical mathematics: Exploring the meaning for everyday teachers*. Unpublished master's thesis, Champaign, University of Illinois at Urbana-Champaign.

Nasir, N. S. (2000). Points ain't everything: Emergent goals and average and percent understanding in the play of basketball among African American students. *Anthropology and Education Quarterly, 31*(3), 283–305.

Nasir, N. S., Hand, V., & Taylor, E. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. *Review of Research in Education, 32,* 187–240.

Nuñes, T., Carraher, D. W., & Schliemann, A. D. (1985). Mathematics in the streets and in the school. *British Journal of Developmental Psychology, 3,* 21–29.

Saxe, G. B., & Esmonde, I. (2005). Studying cognition in flux: A historical treatment of fu in the shifting structure of Oksapmin mathematics. *Mind, Culture, and Activity, 12*(3 & 4), 171–225.

Skovsmose, O. (2004). Mathematics in action. *Philosophy of Mathematics Education Journal, 15*. http://people.exeter.ac.uk/PErnest/pome18/contents.htm. Accessed 20 July 2009.

Skovsmose, O., & Yasukawa, K. (2004). Formatting power of 'mathematics in a package:' a challenge for social theorising? *Philosophy of Mathematics Education Journal, 15*. http://people.exeter.ac.uk/PErnest/pome18/contents.htm. Accessed 20 July 2009.

Stinson, D. W. (2008). Negotiating sociocultural discourses: The counter-storytelling of academically (and mathematically) successful African American male students. *American Educational Research Journal, 45*(4), 975–1010.

Valdés, G. (1996). *Con respeto: Bridging the distances between culturally diverse families and schools: An ethnographic portrait*. New York: Teachers College Press.

Varenne, H. (2007). Difficult collective deliberations: Anthropological notes toward a theory of education. *Teachers College Record, 109*(7), 1559–1588.

Varenne, H. (2008). Culture, education, anthropology. *Anthropology and Education Quarterly, 39*(4), 356–368.

Varenne, H. (2009). Educating ourselves about education—comprehensively. In H. Varenne, E. W. Gordon & L. Lin (Eds.), *Theoretical perspectives on comprehensive education: The way forward: Vol. 2. Perspectives on comprehensive education series.* Lewiston: The Edwin Mellen Press.

Varenne, H., Gordon, E. W., & Lin, L. (2009). *Theoretical perspectives on comprehensive education: The way forward: Vol. 2. Perspectives on comprehensive education series.* Lewiston: The Edwin Mellen Press.

Walshaw, M. (2007). *Working with Foucault in education*. Rotterdam: Sense Publishers.

Wiliam, D. (2003). Constructing difference: Assessment in mathematics education. In L. Burton (Ed.), *Which way social justice in mathematics education?* (pp. 189–207). Westport: Praeger.

Wiliam, D., Bartholomew, H., & Reay, D. (2002). Assessment, learning, and identity. In R. Zevenbergen & P. Valero (Eds.), *Researching sociopolitical dimensions of mathematics education: Issues of power in theory and methodology* (pp. 43–62). Dordrecht: Kluwer Academic Publishers.

Young, I. M. (1990). *Justice and the politics of difference.* Princeton: Princeton University Press.

Chapter 3
Beyond Disavowing the Politics of Equity and Quality in Mathematics Education

Alexandre Pais and Paola Valero

Introduction

Who would publicly deny that mathematics education should be concerned with "equity" and "quality"? Definitely, a concern for "quality" has been behind the constitution of mathematics education as a field of scholarly inquiry from its very beginning. In several accounts of the history of mathematics education and its related institutions, considerations about the "quality" of teachers' instruction and of students' learning have been at the core of the justifications for the development of the field. The constant call for the improvement of the "quality" of mathematics education characterizes a field of research that more often than not documents the shortcomings of "quality" in the mathematics being taught, in the teaching practices of teachers, and in the learning of students.

Nowadays the terms "equity" and "quality" appear side by side, and it seems "natural" to have them together. It also seems "natural" to know what is understood by the terms. In fact, notice that we have already used them without a clear signaling of the meaning given to them. In this paper we react to the "naturalization" of the meaning of these terms and their relationship. Thus, it is our purpose to provide a critique of how equity, and its relation with quality, is addressed in current mathematics education research. For doing so, we adopt a socio-political perspective that, on the one hand, views mathematics education practices as a large network of social, economic, political and historical practices and discourses where mathematical rationalities are constituted (Valero 2010). On the other hand, we view research in mathematics education as a series of practices that contribute to the construction of naturalized discourses about what constitutes mathematical rationalities in the social world. Such a perspective allows us to engage in an examination of the types of discourses that mathematics education research has produced when tack-

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ling "equity" and "quality," and the relationship between both. We see such critique as an important activity of mathematics education research. Without it, it becomes impossible to imagine different alternatives to the language that we use and to the world that such language creates.

In particular, we emphasize the fact that discussions of equity and quality are necessarily political since they allow us to address the values and ideologies that make part of educational practices, as well as of the whole set of practices and social organizations that extend beyond mathematics classrooms. However, it is our observation that the great majority of mathematics education research, by being primarily focused on pragmatic approaches to the improvement of classroom practice, lacks a theoretical comprehension of how "equity" and "quality" are related with broader social and political structures. Without such theoretical comprehension, Baldino and Cabral (2006) argue, we, as researchers, risk moving blindly because we do not "take certain distance and develop consistent research theoretical frameworks to appreciate our practices" (p. 31). In this chapter, we contribute to this comprehension by analyzing how social discourse and ideology permeate the way mathematics education research phrases and tackles "equity" and "quality". In our analysis we draw on the work of Gert Biesta, and Slavoj Žižek. Biesta, a philosopher of education, offers us tools to enter into a critique of mathematics education research in relation to the politics of education as a human activity. Žižek offers us tools that allow us to understand how ideology permeates today the field of mathematics education research.

The Framing of the Research Field

In our recent work, we have been paying attention to the historical constitution of mathematics education research as a field of study. Inspired in the work of Michel Foucault, we are interested in the effect of the fields of academic inquiry in the construction of discourses about the social world that they intend to study. Researchers through their practice formulate languages and forms of doing and acting that constitute the world that they themselves are studying. When we adopt a critical strategy to examine the discursive constructions of mathematics education research, we do not intend to dismiss the achievements of the field in relation to how to diagnose and improve the practices of teaching and learning. We are interested in denaturalizing what seems to be taken for granted with the aim of opening a space for other possibilities of phrasing mathematics education. Such a strategy allows us to transgress established ways of seeing and understanding practices in a search for the impossibilities, disturbances and hidden potentials within the established order (see e.g., Biesta 2005; Stentoft and Valero 2010).

Recently, we have been developing a critique on the theories used in mathematics education research (Pais et al. 2010). We argued that there is a strong tendency to reduce the selection of theories to mainly theories of learning. This is due to the way in which the overarching aim of the research field is formulated. There seems to be a consensus on the proposition that the main concern of mathematics education research is to improve students' performance in mathematics (Boero 2008). Niss (2007, p. 1293) is very clear in stating: "We do research on the teaching and learning of mathematics because there are far too many students of mathematics, from kindergarten to university, who get much less out of their mathematical education than would be desirable for them and for society." If this is the main concern of mathematics education research, it is not surprising that the field has been constructed as a space for researching in a systematic, scientific way "the problems of practice" (Silver and Herbst 2007, p. 45), defined as the predicaments of the teaching and learning of mathematics. The work of mathematics education researchers is "to identify important teaching and learning problems, considerer different existing theories and try to understand the potential and limitations of the tools provided by these theories". Cobb (2007) also suggests that mathematics education research should be understood as a "design science," that is, "the collective mission which involves developing, testing, and revising conjectured designs for supporting envisioned learning process" (p. 7). The ultimate goal of this science is to "support the improvement of students' mathematical learning" (p. 8).

The trend to focus on issues of learning—and thereby of teaching—is not exclusive to the field of mathematics education research, but has over the last two decades also proliferated in educational research in general. The language of education has largely been replaced by a technical language of learning. The contradictions on the role of schooling in society and the goals of education that fueled part of the educational debate during the last century seem to have been surpassed. We seem to have reached a consensus on the benefits of schooling. Therefore, a central concern now is to make it more effective. The problems of schooling and school subjects are not anymore political or ideological, but have become primarily technical or didactical. In most cases, solutions to educational problems are reduced to better methods and techniques to teach and learn, to improve the use of technology, to assess students' performance, etc. Education has progressively been reduced to be a controllable, designable, engineerable and operational framework for the individual's cognitive change. Such tendency is what Biesta calls the *learnification* of education (Biesta 2005). Although the dominance of learning theories for researching mathematics education has allowed us to gain deeper understandings on the processes of teaching and learning mathematics, we suggest that it has also left unattended important difficulties and dilemmas faced by the educational communities in their everyday practices. This reduction of education to learning disavows the political magnitude of education. Learning is conceived as a nominal activity, isolated from what Valero (2010) calls the network of socio-political practices that constitute mathematics education—that is, the entire social, political, economical and historical configuration where mathematics education practices are given meaning. We argue that in order to bring the many difficulties and dilemmas of educational communities seriously into the gaze of research, we need broader theoretical frameworks which allow us to understand mathematics *education*, and not only mathematical *learning*.

Education has given up its place in favor of specialized pedagogy and didactics. In the case of mathematics education research, the discursive construction of students as cognitive subjects and "schizomathematicslearners" (Valero 2004a) is a good example of the way mathematics education research reduces full political and historical human beings to "bare learners," whose cognition can be scrutinized with the interest of devising appropriate and effective techniques for learning mathematics. All the complexity of the social and political life of the student is wiped out from the research focus. The student is reduced to a biological entity, likely to be investigated in a clinical way. For example, some researchers find useful to draw an analogy between mathematics education research and medicine. Mathematics education research is formulated as a science of treatment that, by understanding the symptoms that characterize students' learning difficulties in mathematics, aims at designing and applying proper treatments, with the hope of curing what is a defect in students' learning: "The evolving understanding of the logic of errors has helped support the design of better instructional treatments, in much the same way that the evolving understanding of the logic of diseases has helped the design of better medical treatments" (Silver and Herbst 2007, p. 63).

The Framing of "Equity"

In recent literature, the concern for "equity" is addressed in different forms by different authors. It is actually interesting to notice that few authors clarify their understanding of the term. It is written in between the lines that the problem of equity has to do with the differential achievement in mathematics. The systematic underachievement and its consequences for certain groups of students is not acceptable, particularly at a time when the agenda of "mathematics for all" seems to have permeated policy documents all around the world. The understanding about what it means to address and achieve equity also diverge, and some authors prefer to use terms such as social justice (e.g., Gutstein 2003), democratic access (e.g., Skovsmose and Valero 2008), inclusion/exclusion (e.g., Knijnik 1993). It is also common to find the declaration that research on equity requires social and political approaches that situate the problem in a broader context than the classroom or schools (Valero 2004b, 2007). For instance, Nasir and Cobb (2007) state that all the contributors to their book "view equity as situated and relational and as being informed both by local schooling practices and by practices and ideologies that transcend school" (p. 5). However, when reading the contributions in the book, we find that all the research reported is centered on improving the process of teaching and learning mathematics. Although politics is acknowledged as determinant in equity, and some authors explore the connections between mathematics education and politics (e.g., Gutiérrez 2007), the contributions lack a theoretical analysis on how these "ideologies than transcend school" influence what happens in schools, and its contribution—or not—to equity. As mentioned by Gutiérrez (2007), "little has been written in mathematics education that addresses how mathematics might play a role in broader politics" (p. 38).

One of the most extensive reviews on the issue of equity in mathematics education is the article by Bishop and Forgasz (2007). The authors provide an overview of the different research approaches on the issues of access and equity in mathematics education. Right from the beginning they call our attention to the artificiality present in the construction of groups of people as being in disadvantage—girls, ethnic minorities, indigenous minorities, western "ex-colonial" groups, non-Judeo-Christian religious groups, rural learners, learners with physical and mental impairments, and children from lower class—and how such constructions can in themselves convey discriminatory actions. This problem has been recently labeled by Gutiérrez (2008) as the *gap-gazing fetish in mathematics education*. Gutiérrez' provocative formulation generated a debate with Sarah Lubienski. Roughly speaking, they discuss whether research focusing on the achievement-gap benefits or not the purpose of achieving equity. The position of Gutiérrez (2008) is that there are dangers in concentrating on the "achievement gap" because such research helps "offering little more than a static picture of inequities, supporting deficit thinking and negative narratives about students of color and working-class students, perpetuating the myth that the problem (and therefore solution) is a technical one, and promoting a narrow definition of learning and society" (p. 358). She argues that such research, which usually leans on quantitative methods, does no more than providing a description of the problem without presenting understandings that allow a change. She argues that less research focusing on the "gap" should be made, and more research should analyze qualitatively successful experiences among groups of people considered to be in disadvantage. On the other hand, Lubienski (2008) argues that more skilled and nuanced gap analyses are necessary: "analyses of gaps also inform mathematics education research and practice, illuminating which groups and curricular areas are most in need of intervention and additional study" (p. 351). Lubienski is concerned with the question of whether there is a gap, to what follows studies analyzing when the gaps manifest, under what conditions they grow or shrink, and what consequences underserved students ultimately suffer because of the gap. In contrast, Gutiérrez is concerned with the question of how to diminish the gap, to what follow studies oriented toward effective teaching and learning, making research more accessible to practitioners and more intervention by the researcher.

Some authors have been trying to list which practices can be carried out in order to achieve equity in mathematics education. For Schoenfeld (2002, quoted in Langrall et al. 2008, p. 127), achieving equity requires four systematic conditions to be met: a high-quality curriculum, a stable, knowledgeable and professional teaching community, a high-quality assessment aligned with curricular goals, and stability and mechanisms for the evolution of curricula, assessment and professional development. Alternatively, Lubienski (2002) claims that it is necessary to learn more about the complexities of successful implementation of meaningful instructional methods with students who differ in terms of social class, ethnicity and gender. For Goldin (2008), the most important is "to create teaching methods capable of developing mathematical power in the majority of students" (p. 178). Finally, Gates and Zevenbergen (2009) identify a common basis for how to deal with equity, summarizing existing research: "What might we all agree on then as fundamentals of a socially just mathematics education? Perhaps we can list: access to the curriculum; access to resources and good teachers; conditions to learn; and feeling valued." (p. 165).

Although we can discuss the better ways to do research addressing equity and what needs to be done, there is a fundamental question that seems to be unaddressed: Why is there inequity? Why is there a gap at all? That is, why does school (mathematics) systematically exclude/include people in/from the network of social positionings? Why do schools offer low-quality curricula and do not have a stable group of teachers in schools serving underprivileged population? Why does school perform the selective role that inevitably creates inequity? As Bishop and Forgasz (2007) put it, "in every country in the world mathematics now holds a special position, and those who excel at it or its applications also hold a significant positions in their societies" (p. 1149). Why does society need to have an institution that guarantees an accumulation of credit? These questions are rarely posed by research in mathematics education addressing equity. Research only recognizes it as a fact. Posing the questions above dangerously opens the field of mathematics education to politics, and it seems few researchers are ready to take a risk.

In the previous discussions, it is evident that the problem of equity is recognized as an economical and political problem. However, research in mathematics education transforms it to be a problem pertaining to mathematics teaching and learning. This type of displacement reduces the aim of researching the problem of equity to a matter of developing the best "instructional methods" to allow mathematical success to all students. The absence of a political conceptualization of the problem of equity is evident in much of the existing literature. Disavowing politics as part of the conceptualization of equity is one of the best ways of turning research innocuous for social change.

The Framing of "Quality"

A quality mathematics education research is constructed to be the one that allows students to improve their mathematical learning. Why is this important? The literature on mathematics education research is full of statements that justify the necessity of mathematical learning (e.g., Niss 1996). In most of such statements, mathematics and its education are viewed as powerful knowledge and competence for people to become full citizens and competitive workers. Are these formulations enough to justify mathematics education and to define "quality"?

We look for support in the philosophy of education. Biesta (2009) analyzes the functions that education fulfills in society nowadays. The function of qualification has to do with the role of education to providing people with the knowledge, skills and understanding necessary to fulfill a productive function. The function of socialization has to do with the role of education in enculturating people to become members of a particular society, by the insertion of the "newcomers" into existing social and cultural orders. An analysis of the justifications for mathematics teaching and learning leads us to see that mathematics education builds fundamentally on the fulfillment of qualification and socialization functions. Consider the following assertion as an example:

> Mathematics education in schools is thus seen to have a dual function: to prepare students to be mathematically functional as citizens of their society—arguably provided equitably for all—and to prepare some students to be the future professionals in careers in which mathematics is fundamental, with no one precluded dorm or denied access to participation along this path. (Bishop and Forgasz 2007, p. 1152)

On the one hand, mathematics teaching and learning is important because it allows the nurturing of the next generation of mathematicians and of those who will use mathematics in their work, therefore assuring the development of a working force equipped to compete successfully in the global economy of our high-tech society. On the other hand, mathematics teaching and learning secures the insertion of people in a society where mathematics is seen as an indispensable tool to become a citizen. The goal of citizenship concerns a wide range of competences: providing mathematical skills for dealing with situations of everyday life, intellectual enrichment, acknowledging mathematics as equally a part of humankind's cultural and aesthetic heritage, or making accessible powerful tools to analyze critique and act upon the way mathematics is used in society. The way quality is understood both in mathematics education research and in school mathematics seems to be in resonance. Since school mathematics is posited as indispensable to become both a productive and competent worker and an active and participative citizen, the purpose of mathematics education research should be to improve students' mathematical learning.

What is the problem with this view, then? We would like to argue that this way of conceiving quality conceals the ideology informing what it means to be a worker and a citizen in a capitalist society. At first glance, the aims for school mathematics mentioned above are worthy aims for any compulsory schooling system. Becoming a successful worker and an informed and participative citizen seems to fulfill the desire of students, parents, politicians, teachers and others participants in the educational process. So why do we feel uneasy about these aims? On the one hand, the listed aims for school mathematics are formulated on the assumption that subjects are conscious of themselves (Althusser 2000). The assumption disavows the political substrate that informs what it means to be a worker and a citizen in current societies. On the other hand, these aims conflict with the politics of accountability where quality is often defined as having the best ranking positions both in national and international examinations.

Allow us to explore these two aspects in more detail. Althusser (2000) argues that the ideology of capitalism is based on the idea that individuals are self-conscious subjects, responsible for their own acts. They can be persuaded, consciously, to obey rules that otherwise would be imposed by force. It is only under this condition that human beings become *homo economicus*. Marx (1989) showed that, contrary to the assumption that the subject is a coherent rational being, subjects are not conscious about the "nature" of the place they occupy in the structure ruled by the laws of capitalist society. Marx allows us to understand that behind the ideology that asserts the equality of individuals in the free market lays a profound inequality.

One of the ideological modes dissected by Žižek (1994) conceives ideology as "a doctrine, a composite of ideas, beliefs, concepts, and so on, destined to convince us of its 'truth,' yet actually serving some unavowed particular power interests" (p. 10). The strategy to criticize this mode of ideology is to carry a *symptomal reading* (Althusser 1994) that exhibits the discrepancies between the public discourse and the actual intention of it. The Standards of the National Council of Teachers of Mathematics of the United States (NCTM 2000) is a prolific document to engage in such a reading. The basic discrepancy of discourses in this document has to do with how behind the public discourse of forming students to become active and participative citizens in society, there is a concern in maintaining the economic and scientific dominance of the United States. The NCTM Standards can also be read as a case of what Žižek (2006) called *staged democracy*. The document expresses an official discourse with all the virtues and democratic goals that society stands for, but when put in practice the actions deriving and resulting from the formulations will almost secure that the democratic goals continue to fail. In this case, the ideological critique will be concerned not with the understanding of why in practice those desirable aims continue to fail—as if it were a problem of "implementation"—but to understand how the discrepancy is already being created at the level of the official discourse by completely obliterating the real reasons why inequality and lack of democracy continue to exist. In the case of the NCTM standards, the interesting point is not to focus on why their implementation fails in the hands of "incompetent" authorities, administrators and teachers, and of "deficient" children. An ideological critique would see how the document bears in itself the impossibility of achieving its stated goals.

However, in order to become efficient, ideology must go under a process of "self-disguising," so that we can be able to act as if our actions were deprived of all ideological content: "the very logic of legitimizing the relation of domination must remain concealed if it is to be effective" (Žižek 1994, p. 8). We must not perceive ourselves as being questioned by some big Other[1] but as individual subjects who freely choose to believe and act according to utilitarian and/or hedonistic motivations. When the NCTM standards argue for the importance of educating students mathematically to become active participants in society, the document disregards any pathetic ideological phrases in supporting their argument. The argument is a pragmatic one—competent people in mathematics are needed as the future workers of our high-tech society—or a hedonistic one—people gain power through mathematics. However, we cannot miss here that the choices in the document are highly ideological: The formulations involve a series of assumptions about what it means to be an active citizen in an increasingly commoditized society, and how such type of people are necessary for the reproduction of existing social relations. A staged

[1] In this context, the Lacanian notion of big Other stands for all the State, Justice and Law that give symbolic meaning to our social life.

discourse is needed so that school mathematics continues to perform other roles than those present in the official discourse.

Furthermore, how does one assess the quality of students' mathematical learning? It seems to be an unachievable task to assess if students, at the end, become or not desirable workers and citizens. However, society cannot live in this state of uncertainty regarding the mathematical performance of students. Society craves for results, for evidence that shows if people are becoming desirable subjects. Therefore, rigorous instruments should be created so that it is possible to objectively know if students are performing well in school mathematics. Indeed, such instruments exist under the form of national tests and international comparative studies. In our days where accountability reigns, what counts as quality is the performance of students in high-stakes examinations and in international tests. Ultimately, are not the results from these examinations what define quality in mathematics education? We are confronted with the inconsistency of a system that, on the one hand, defines quality as a matter of achieving the desirable "mathematical subject"—the mathematically competent, informed worker, and critical citizen. But on the other hand, what ultimately decides the quality of mathematics education is the results of the exam. Again, the question to be posed is why the type of society we live in needs a staged discourse concealing what everybody recognizes.

The Disavowing of Politics in Mathematics Education

We explore now the disavowing of politics involved when addressing equity and quality in mathematics education. We argue that the political disavowal keeps research at a "technical" level, which contributes to reaching just the opposite of the stated aims. Furthermore, we argue that this is precisely one of the strongest limitations for bringing equity and quality together. A quality mathematics education is not one that attends mainly to the intrinsic characteristics of mathematics as the foundations for educational practices, neither one that proposes pragmatic and hedonistic justifications for why to teach mathematics to all students, but rather one type of education that recognizes the possibilities of the meeting between human beings and the school subject of mathematics within the social, political and historical frame in which such meeting is being constituted. This means that definitions of equity and quality in mathematics education that do not attend to how both notions as well as the practices of mathematics education are shaped in power relationships are partial definitions that can only place hysterical demands to practitioners.

We showed previously how researchers address the societal demand of mathematics for all and make it a research concern. They engage on this demand by assuming that through their studies on the teaching and learning of mathematics, on better curricular and instructional design, on better connections between researchers and practitioners, they are contributing to achieving equity in school mathematics. However, if equity, or rather, inequity, is an economic and political problem that surpasses school, then the demand of mathematics for all is impossible to satisfy (cf. Žižek 1991). Why then do we keep doing research to address equity issues in mathematics education as if the problem could be solved within the realm of mathematics teaching and learning? Could it be that keeping us occupied doing innocuous research inhibits us from looking at other issues?

As we know, dominant social systems demand for perpetual reforms as a means of integrating what could be new and potential emancipatory acts into well-established social structures. In other words, dominant systems such as capitalism today are constantly changing something so that nothing really changes (Žižek 1991). The novelties research produces on how to promote equity by improving the teaching and learning of mathematics are part of these superficial transformations. According to Gutiérrez (2007), "[e]quity is threatened by the underlying belief that not all students can learn" (p. 3). Although in a first reading we agree with Gutiérrez, we see that other beliefs are at stake, namely the not underlined but publicly assumed belief that all students can learn. The interplay between these two discourses makes visible how ideology works today. The view that all students can learn—the official view, present in curricula, political documents and research, attesting that mathematics is for all—conceals the commonly shared but not assumed belief that there will always be some who will fail. Following Žižek (2010), when we read an abstract "ideological" proclamation such as "mathematics for all," we should be aware that people's experiences are different—for teachers and students know and experience that in any mathematics class there will always be some—or many—who fail. The official discourse functions not as some kind of utopian state to be achieved, a desired good to strive for, but rather as a pure mechanism to conceal the fact that mathematics is not for all. The obliteration of the "background noise"—the voices of those who will always fail—is the very core of utopia. The "background noise" conveys "the obscenity of barbarian violence which sustains the public law and order" (Žižek 2010, p. 10). In the case of mathematics education, the obscenity of the barbarian violence that school exercises year after year when it throws to the garbage bin of society thousands of people, under the official discourse of an inclusionary and democratic school (mathematics education).

As far as society remains organized under capitalist tenets, there will always be exclusion because exclusion is not a malfunction of capitalism, but the very same condition that keeps it alive (Žižek 1989). In such organization, having (certain) students failing in mathematics is not an abnormality of mathematics education, it is the necessary condition for its very same existence. So, why do we need an official discourse affirming that mathematics is for all? It is because such discourse masks a crude fact: The capitalist lie that presupposes equity in schools as an extension of the equality in the market. In other words, the official discourse conceals the inconsistency of a system that, on the one hand, demands mathematics for all while, on the other hand, uses school mathematics as a privileged mechanism of selection and credit.

The denial in confronting the core of the problems of equity is the result of an ideological injunction that systematically leads us to repeat the same "abstract" mottos of discourse: School is a place for emancipation; mathematics is powerful knowledge and competence; mathematics is for all; etc. In order to critically analyze such discourses, we should replace the abstract form of the problem with the concrete scenes of its actualization within a life-form: "In order to pass from abstract propositions to people's 'real lives,' one has to add to the abstract propositions the unfathomable density of a life world context—and ideology are not the abstract propositions in themselves, ideology is this very world density which 'schematizes' them, renders them 'livable'" (Žižek 2010, p. 6). In other words, in order to understand the real aims of school mathematics, or the real motives that students have to be in school, we must not repeat ideologically loaded discourses conveyed by the curriculum, the political statements, and the research. Rather, we need to look at schools selecting the future workers of the labor market by means of credit accumulation. That is, what Gutiérrez calls the "underlying belief that not all students can learn" must be posited not as a threat to equity, but as the truth of a system in which equity is forever postponed. Following Žižek's (1989), this implies asking research to pass from the notion of crisis—in this case, the fact that people fail is school mathematics creating exclusion—as an occasional contingent malfunctioning of the system, to the notion of crisis as the symptomatic point at which the truth of the system becomes visible. Some will say that such an awareness of the problem of equity takes us to a deadlock. Indeed, by realizing that exclusion is something inherent to the school system in a capitalist society, we realize that ending exclusion implies finishing schooling as we know it. In the current myriad of world social structure, this does not seem possible. However, what dooms us to constant failure is precisely experiencing the change as impossible. We acknowledge that the problem of equity requires a fundamental societal change, which may be impossible. The question is whether it is impossible or it is ideologically posited as impossible.

Threshold

> The key feature here is that to see the true nature of things, we need the glasses [glasses as a metaphor for critical ideological analysis]: it is not that we should put ideological glasses off to see directly reality as it is: we are 'naturally' in ideology, our natural, immediate, sight is ideological. (Žižek 2010, p. 6)

We would like to argue that mathematics education research needs such glasses. The "natural" way in which we relate to reality is ideological—in our practice we convey discourses that conceal more than what we know. Ultimately, the purpose of this chapter was to attempt an ideological critique in the way we address issues of equity and quality in mathematics education research. Apparently, there is no doubt that definitions of quality and the discourses for equity live side by side and are equally political. However, it is almost inexistent in mathematics education research studies that aim to understand in depth such problems that are identified as political in their nature. If mathematics education research desires to address them, it must open its gates to research that locates the complexity of mathematics education within the network of social and political practices that permeate all educational act. Without that we run the risk of falling in the trap of what we criticize. On the issue of equity, our premise is that exclusion and inequity within mathematics education and education in general is an integrative part of current school education, and cannot be conceptualized without understanding the relation between school education and the social mode of living that characterizes our current world. On the issue of quality, a serious challenge is also to politicize our understanding of what is taken to be the significance of valued forms mathematical thinking within capitalism.

Therefore, our intention with this text is not to give a solution to the problems of equity and quality, neither is it to propose an alternative way of doing research on these topics. Our purpose is much more modest. We wanted to raise the awareness on the fact that there are broader issues involved when discussing equity and quality in mathematics education, than doing research on better ways to teach and learn mathematics, to improve students' mathematical performance so that they could become better workers and citizens. As mathematics education researchers actively engaged in the field, we find the need for developing a deeper understanding of our practices and the discourses we convey. The way we found to do this was to explicitly look at the inconsistency of discourses that make the apology of equity and quality without considering the meaning that these terms have in a society where capitalism has become the ontologized substrate for all social relations.

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References

Althusser, L. (1994). Ideology and ideological state apparatuses (notes towards an investigation). In S. Žižek (Ed.), *Mapping ideology* (1st ed. 1970, pp. 100–140). New York: Verso.

Althusser, L. (2000). *Freud e Lacan/Marx e Freud*. Rio de Janeiro: Edições Graal.

Baldino, R., & Cabral, T. (2006). Inclusion and diversity from Hegel-Lacan point of view: Do we desire our desire for change? *International Journal of Science and Mathematics Education, 4*,19–43.

Biesta, G. (2005). Against learning. Reclaiming a language for education in an age of learning. *Nordisk Pædagogik, 25*(1), 54–55.

Biesta, G. (2009). Good education in an age of measurement: On the need to reconnect with the question of purpose in education. *Educational Assessment, Evaluation and Accountability, 21*(1), 33–46.

Bishop, A. J., & Forgasz, H. J. (2007). Issues in access and equity in mathematics education. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1145–1167). Charlotte: Information Age Publishing.

Boero, P. (2008). Autonomy and identity of mathematics education: Why and how to use external theories. To be published as part of the *Proceedings of the 11th International Congress on Mathematics Education.* http://www.unige.ch/math/EnsMath/Rome2008/WG5/Papers/BOERO.pdf. Accessed 20 Feb 2010.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. Lester (Ed.), *Second handbook of research on mathematics and learning* (pp. 3–38). New York: Information Age.

Gates, P., & Zevengergen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education, 12*, 161–170.

Goldin, G. (2008). Perspectives on representation in mathematical learning and problem solving. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 176–201). New York: Routledge.

Gutiérrez, R. (2007). (Re)Defining equity: The importance of a critical perspective. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 37–50). New York: Teachers College, Columbia University.

Gutiérrez, R. (2008). A "gap-gazing" fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education, 39*(4), 357–364.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 23*(1), 37–73.

Knijnik, G. (1993). An ethnomathematical approach in mathematical education: A matter of political power. *For the Learning of Mathematics, 13*(2), 23–25.

Langrall, C., Mooney, E., Nisbet, S., & Jones, G. (2008). Elementary students' access to powerful mathematical ideas. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 109–135). New York: Routledge.

Lubienski, S. (2002). Research, reform and equity in U. S. Mathematics education. *Mathematical and Learning, 4*(2 & 3), 103–125.

Lubienski, S. (2008). On "gap gazing" in mathematics education: The need for gap analyses. *Journal for Research in Mathematics Education, 39*(4), 350–356.

Marx, K. (1989). *A contribution to the critique of political economy* (1st ed. 1857). http://www.marxists.org/archive/marx/works/1859/critique-pol-economy/appx1.htm. Accessed 13 Jan 2010.

Nasir, N., & Cobb, P. (2007). Introduction. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 1–9). New York: Teachers College, Columbia University.

NCTM. (2000). *Principles and standards for school mathematics*. Reston: NCTM.

Niss, M. (1996). Goals of mathematics teaching. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 1, pp. 11–47). Dordrecht: Kluwer.

Niss, M. (2007). Reflections in the state and trends in research on mathematics teaching and learning: From here to utopia. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1293–1312). Charlotte: Information Age Publishing.

Pais, A., Stentoft, D., & Valero, P. (2010). From questions of how to questions of why in mathematics education research. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (Vol. 2, pp. 369–378). Berlin: Freie Universität Berlin.

Schoenfeld, A. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13–25.

Silver, E., & Herbst, P. (2007). Theory in mathematics education scholarship. In F. Lester (Ed.), *Second handbook of research on mathematics and learning* (pp. 39–56). New York: Information Age.

Skovsmose, O., & Valero, P. (2008). Democratic access to powerful mathematical ideas. In L. D. English (Ed.), *Handbook of international research in mathematics education. Directions for the 21st century* (2nd ed., pp. 415–438). Mahwah: Erlbaum.

Stentoft, D., & Valero, P. (2010). Fragile learning in mathematics classrooms: Exploring mathematics lessons within a pre-service course. In M. Walshaw (Ed.), *Unpacking pedagogies. New perspectives for mathematics* (pp. 87–107). Charlotte, USA: IAP.

Valero, P. (2004a). Postmodernism as an attitude of critique to dominant mathematics education research. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 35–54). Greenwich: Information Age.

Valero, P. (2004b). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 5–24). Boston: Kluwer.

Valero, P. (2007). A socio-political look at equity in the school organization of mathematics education. *Zentralblatt für Didaktik der Mathematik. The International Journal on Mathematics Education, 39*(3), 225–233.

Valero, P. (2010). Mathematics education as a network of social practices. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. LIV–LXXX). Lyon: Institut National de Récherche Pédagogique.

Žižek, S. (1989). *The sublime object of ideology*. London: Verso.

Žižek, S. (1991). *For they know not what they do: Enjoyment as a political factor*. London: Verso.

Žižek, S. (1994). The spectre of ideology. In S. Žižek (Ed.), *Mapping ideology*. London: Verso.

Žižek, S. (2006). *The parallax view.* Cambridge: MIT Press.

Žižek, S. (2010). *Denial: The liberal utopia.* www.lacan.com. Accessed 20 Jan 2010.

Chapter 4
Does Every Child Count? Quality, Equity and Mathematics with/in Neoliberalism

Anna Llewellyn and Heather Mendick

The Oxford English dictionary[1] defines quality as:

> 1 the degree of excellence of something as measured against other similar things. 2 general excellence. 3 a distinctive attribute or characteristic. 4 archaic high social standing.

Given this collection of meanings, it is easy to assume that policies promoting quality education are a good idea. However, like the terms standards and excellence, quality passes as a universal good which it is difficult to oppose. This can conceal how the quality being promoted within current policy regimes is a particular version carrying particular values. We can illustrate this using a motivating question for this volume: How are concerns for equity and quality contradictory and/or synergistic? For this question to make sense, quality education must be defined independently of equitable education, so that the relationship between the two can be examined. What are these contemporary understandings of quality and equity? And, what are their implications for teachers and learners of mathematics? We address these questions in this chapter by looking at how 'quality' and 'equity' are being constructed within current policy and practice in primary mathematics education in England.

Although the context for our exploration is England, quality and equity are part of what Ball (2008) calls 'global policyspeak' (including, for example: parental choice, privatisation, performance indicators, competition), which England has been at the forefront of developing and disseminating.

> There is a discernible process of convergence, or what Levin (1998) calls a 'policy epidemic', in education. An unstable, uneven but apparently unstoppable flood of closely interrelated reform ideas is permeating and reorienting education systems in diverse social and political locations with very different histories. This convergence has given rise to

[1] http://www.askoxford.com/concise_oed/quality?view=uk

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> what can be called a *generic global policy ensemble* that rests on a set of basic and common *policy technologies*. (Ball 2008, p. 39, original emphasis)

These global policy technologies can be broadly understood as neoliberal. Neoliberalism is characterised by 'the progressive enlargement of the territory of the market' (du Gay 1996, p. 56) as private enterprise and technical rationality come to define behaviours and relations in the public sector. Neoliberalism is affecting education in areas as diverse as Europe, the United States, South America and Australia (Grek et al. 2009; Hultqvist and Dahlberg 2001); although predominant in the global North, it is increasingly imposed on the South through conditions of funding.

Neoliberal managerialist discourses of quality arise from the transformation of private sector management techniques into the public sector's New Public Management. This stressed quality, accountability and internal competition, and created a culture of performativity within English education:

> Performativity is a culture or a system of 'terror'. It is a regime of accountability that employs judgements, comparisons and displays as means of control, attrition and change. The performances of individual subjects or organisations serve as measures of productivity or output, or displays of 'quality', or 'moments of promotion' or inspections. These performances stand for, encapsulate or represent the worth, quality or value of an individual or organisation within a field of judgement. Clearly, the issue of who controls the field of judgement and what is judged, what criteria of measurement are used or benchmarks or targets set, is crucial. (Ball 2008, p. 49)

Thus, quality is implicated in rationalist practices of performativity. Within this, teachers' performances are vital not just to the success of their pupils and the schools 'for which they work, *but also to the enterprise of their own lives*' (du Gay 1996, p. 60, original emphasis). In this way, quality/excellence/enterprise culture brings together institutional concerns for productivity (read: results) and contemporary modes of regulation and subjectivity. What does this mean for our understandings of equity? And what does it mean for the practices of mathematics education?

Our Framework

We draw on two sources of data: recent English policy documents and interviews with student-teachers. In this section, we outline our methodological approach. We selected the policy documents partly systematically and partly eclectically, borrowing from cultural studies approaches (see du Gay et al. 1997). We analysed in detail two recent, high-profile documents on mathematics education: the Williams and Ofsted Reports (Ofsted 2008; Williams 2008). We supplemented this with primary mathematics material produced by the National Strategies[2] and two key government White Papers (DCSF 2009a; HM Government 2009). The White Papers enabled us to look at the broader English education policy context and consider the intersection of mathematics with quality and equity. We compared these policy documents

[2] http://www.nationalstrategies.org.uk/Home.aspx

with the student-teacher interviews, exploring convergences and tensions. The in-depth interviews were carried out with four prospective primary teachers in their final year of a three-year undergraduate programme of initial teacher education in Northern England. The qualitative interviews explored the student-teachers' experiences of teaching mathematics and their understandings of quality and equity. They were conducted by Anna in the participants' final teaching practice schools, lasted between 28 and 40 minutes, and were audio-recorded and transcribed. These interviews are part of a longitudinal study following six student-teachers through their 'training'. By bringing together these two different sources of data, we hope to explore *both* policy as discourses, the frameworks of 'commonsense' within which policy texts are set, *and* policy as texts, which constrain but do not control their implementation (Ball 2008).

Our analysis is poststructural in that we see quality, equity and other objects as existing through discourses or 'fictions functioning in truth' (Walkerdine 1990). These fictions are 'true' not because of their power to describe reality but because of their power to produce it; they are structures of knowledge which set limits on our imaginations and actions (Foucault 1972). For example, discourses of quality include ones that: attach it to elite cultural practices, such as classical music (and to elite groups); view it nostalgically as part of a lost golden age (see Smith, this volume) and see it pragmatically as part of management systems ensuring that products and services meet required standards. It is partly the blurring of these discourses that enables quality do the work that it does. Similarly, there are a range of discourses of equity encompassing different dimensions of inequality (gender, ethnicity, disability, sexuality, etc.) and relating inequality, in various configurations, to opportunities, outcomes, individual actions and social structures. In this chapter, we argue that some versions of quality and equity 'fit' better within neoliberal regimes, while other notions are precluded, and that this has particular consequences for mathematics teaching. We begin with our policy analysis and then explore how the dominant policy discourses are navigated by the student-teachers.

Policy Documents

Although policy documents reference and so authorise each other in an intertextual web, there are differences between (and even within) them. Partly these relate to their different conditions of production:

- The *Williams Report* was an 'independent' review commissioned by government and authored by an individual. It set out to make recommendations on educational best practice for the development of mathematical learning within primary schools and early years settings.
- The *Ofsted Report* presented evidence accumulated through school inspections of primary and secondary mathematics and was authored by an 'independent'

quango. Its aim was to analyse this evidence so as to build up a picture of 'effective' mathematics teaching.

- The *National Strategies material* was prepared by a private company to fulfil a government contract to raise standards, that is results, through teacher development. It consists mainly of a series of short publications presenting educational 'problems' and bullet-pointed strategies for addressing them.
- The *White Papers* enunciate official government policy positions. Both were lengthy documents detailing proposed changes in the structuring and organisation of education.

However, while it is important to note distinctions in authorship (and audience), our concern here is with what it is possible to say and not the intentions of those saying it.

From an initial reading of the documents, it is difficult to discern what the oft-used word 'quality' means. It gets 74 mentions across the two mathematics reports and 145 across the two White Papers. However, how quality teaching is to be distinguished, if at all, from excellent or effective teaching is unclear. What is clear is that quality can be measured and comes along with progress (individual and national), understood as forward/upward movement. Neoliberal education policy in England 'operates like a ratchet screwdriver with no reverse movement allowed' (Coffield and Edward 2009, p. 371). The ongoing drive for quality encapsulates this, with the move from good to best and now excellent practice: 'my priority is excellence, excellence, excellence: we have got to upgrade our skills' (Gordon Brown, cited in Gillies 2008, p. 686). Thus, quality or excellence is explicitly linked to economic imperatives. The seemingly equitable phrase 'every child', which appears repeatedly across current UK government policies (for example, in the 'Every Child Matters' agenda, and the 'Every Child a Reader' and 'Every Child Counts' initiatives), is similar to the United States' 'No Child Left Behind'. The links to progress and forward movement are clear. Edelman (2004, p. 11) identifies how we are mobilised to support policies on behalf of the Child who represents our national future:

> In its coercive universalization, however, the image of the Child, not to be confused with the lived experiences of any historical children, serves to regulate political discourse…by compelling such discourse to accede in advance to the reality of a collective future whose figurative status we are never permitted to acknowledge.

The relentlessness of this version of progress is evident when Ofsted say: 'A substantial amount of teaching is no stronger than satisfactory and, in these lessons, pupils do not learn as quickly as they might' (p. 19) and that 'teachers did not show enough urgency' (p. 19). The emphasis on speed indicates that Ofsted expect, even insist upon, measurable forward movement by *every* pupil in *every* lesson.

'Levelling' captures even more clearly the compulsion to progress. Levelling, like quality, blurs several meanings, including: making flat, knocking down, and placing on the same level. To understand it in this context, it is helpful to know something about the organisation of compulsory schooling in England. Since 1988 this has been structured around four Key Stages (KS):

KS1: ages 5–7
KS2: ages 7–11
KS3: ages 11–14
KS4: ages 14–16

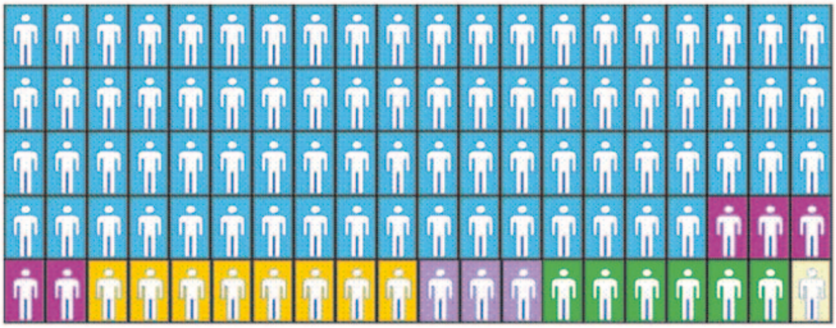
KS1 and KS2 constitute primary schooling, and KS3 and KS4 secondary schooling. KS2 and KS4 end with compulsory national tests in mathematics and English the results of which are widely published in national and local newspapers, websites, etc. National expectations are set for each KS. Pupils are expected to reach level 2 of the National Curriculum at KS1 and level 4 at KS2. Time-demarcated targets are set for the proportion of pupils attaining the expected levels. In addition, expectations are set around the quantity of progress (two levels) required across each KS. As indicated in the Ofsted report, this can be micromanaged to produce expected levels of 'progress' in every lesson.

The levelling discourse can be seen in two recent National Strategies mathematics publications. The title *Securing level 4 in Key Stage 2 mathematics* (DCSF 2009b) clearly ties quality to levelling. While *Making good progress in mathematics* (DCSF 2008) focuses on the ways that children do or do not 'convert' their KS1 results into KS2 results. The meaning of conversion here is imported, like quality, from business. Wikipedia captures it well: 'In marketing, a conversion occurs when a prospective customer takes the marketer's intended action. If the prospect has visited a marketer's web site, the conversion action might be making an online purchase'[3]. Thus, teachers are positioned as marketers and children as customers/consumers within economistic discourses.

This framework of expected levels and conversions, construct a 'normal' developing child, which purports to be, but is not, universal. Any moves away from normal connote danger/risk, and compel intervention to reconstitute the child within the normal. A large part of the Williams Report, and one of only two costed recommendations, is devoted to intervention programmes for learners who 'fall behind' in mathematics. 'Numeracy recovery', the recommended and now implemented intervention, carries, through its use of the word recovery, a sense of returning something to a state of normality (from physical or mental disorders), recovering something lost, going back to an originary state. Thus, the focus on the individual within the mathematics classroom is not about celebrating difference but is tied to normalisation.

We illustrate this by discussing the diagram in Fig. 4.1. This features in the National Strategies' *Making good progress in Key Stage 2 mathematics* and an online spreadsheet where, on the click of a mouse, you can customise the chart for any region in England. As Stronach (2001) observes, a common reaction to such tables is to look up the region that you currently (or previously) live/teach in and compare it to others, producing a performance league table. The spreadsheet format makes this invitingly easy; the culture of comparison and competition makes this pleasurable. The 100 icons, each representing 1% of children in the cohort, are arrayed in

[3] http://en.wikipedia.org/wiki/Conversion_(marketing), emphasis omitted.



77% of pupils achieved national expectations (level 4 or above) in 2007

16% of pupils were at level 3 of which:

- 0% Stuck at L3 or L4
- 5% Falling Behind from L2a or L2b
- 7% Slow Moving at L2c
- 3% Making Good Progress from L1 or W

0% either A, D or no KS1 level recorded

6% at Level 2 or below

1% Absent

National PSA target for 2008 - 85% to achieve L4+

Proportion Below Expected Level Who Are:	%
FSM	28
Statemented SEN	11
Boys	49
BME	24

Source: http://www.teachernet.gov.uk/_doc/12541/KS1-2%20Maths%202007.xls

Note: FSM: Free School Meals

SEN: Special Educational Needs

BME: Black and Minority Ethnic

Fig. 4.1 National Expectations and below Pupil Progression Chart: Key Stage 2 Maths (2007)

neat identical lines. Only the background colourings distinguish them. The normal is defined in comparison to the other as the eye is drawn away from the majority light blue towards the minority, abnormal failures, highlighted in pink, yellow, violet and green: the pink and yellow who failed to convert by making the required two levels of progress, and the violet and green who failed so badly at KS1 that even conversion could not render them within the boundaries of the normal. These icons are pseudo-children with stylised blank bodies; they resemble automata, droids or another science fiction invention. Depicted as such, there is no diversity other than in their mathematics attainment: they are ungendered, unraced, unclassed and without sexuality. What notions of equity are possible within this quality regime?

To address this question, we turn to *New Opportunities* (HM Government 2009), the first education White Paper to be published after the onset of the global economic downturn. This links economic national competitiveness *and* social justice, 'excellence *and* equity' (p. 45, 47, 53, our emphasis). However, the prime minister's foreword marks out a 'modern definition of social justice' based on providing equal access to opportunities for all rather than 'social protection' (p. 1). When presented with these opportunities, it is the responsibility of individuals to aspire to and take them, and so fulfil their potential. Here we see the managerialist convergence noted earlier between productivity, contemporary modes of regulation and subjectivity: self-fulfilment is aligned with effectiveness. Individuals serv(ic)e the state through their lifelong learning which in turn becomes a means to their self-actualisation. However, this model of the self as acquisitive is not equally available to all, and, in particular, is classed (Skeggs 2004). Those who fail are held individually responsible. They 'appear to be unable—or worse, unwilling—to fit themselves into the meritocratic educational system which produces the achievement vital for the economic success of the individual concerned and of the nation' (Francis 2006, p. 193). Thus, the state's failure is transformed into the failure of individual students, teachers and workers (Archer et al. 2010).

Returning to the reports focused on mathematics, there is a general lack of attention to equity. The Williams Report's main reference to social and economic factors is:

> Social factors clearly play a role, and the United Kingdom remains one of the few advanced nations where it is socially acceptable—fashionable, even—to profess an inability to cope with mathematics. Even more seriously, there can be little doubt that economic factors and social deprivation contribute to learning difficulties in all subjects, including mathematics. Given that 15–20 per cent of adults do not have basic functional numeracy skills, many parents will be unable to support their child's learning. (p. 44)

In this extract, social factors are first understood as a general culture of negativity towards mathematics and only second as related to economic inequalities. Further, Williams' final chapter on home/school links is shorter than the others, contains more case studies and is the only one without any recommendations. Within this chapter, while class and ethnicity *are* mentioned (mostly through coded references, such as to 'a deprived housing estate in East London' and 'hard to reach parents', p. 72), Williams seems concerned to downplay the importance of social class. The report's only direct mention is this sentence: 'A 2003 study showed that regardless of class or income, the influence of the parent was the single most significant factor in a child's life' (p. 69). Here parenting is presented as independent of social class, despite research showing how class is reproduced through parenting (Walkerdine and Lucey 1989). Although there are fleeting mentions of 'inclusive teaching' in both reports and, in Ofsted, of the patterns of participation in post-compulsory mathematics by gender, ethnicity and free school meals (a common measure of social class in the UK), the reports' lack of attention to equity appears to be in marked contrast to the broader education policy documents. However, Williams and Ofsted include, respectively, 90 and 40 uses of the word/s opportunity/ies. Thus, they can also be understood to speak the language of the 'modern definition of social justice', of state-facilitated meritocracy.

Crucial to our argument is that the construction of equity as opportunity creates tensions around 'mathematical ability'. Both the Williams and Ofsted Reports discuss the advantages and 'opportunity costs' of practices of grouping by ability. But they make no recommendations and ignore the research evidence that: ability grouping is a means of rationing education in which certain class and 'race' groups are systematically excluded from access to particular knowledges and qualifications (Gillborn and Youdell 2000) and part of broader processes in which some people are given value and others are not (Reay and Wiliam 1999). Both reports attempt to avoid talk of ability as distinct from ability grouping. Ofsted uses the terms 'high-attaining' and 'low-attaining'; Williams also talks about 'low-attainers' (although contrasted with the 'gifted and talented'). The expression 'low-attaining' allows pupils, parents and teachers to disassociate pupils' performances from any underlying, inner ability; they are simply not achieving to expected norms. Thus, we can see a move away from discourses of innate ability, as incompatible with desires for all to progress, and their replacement by levelling, alongside the maintenance of ability grouping in so far as this can be constructed as an efficient, and so quality, pedagogic practice.

To summarise, quality is conflated with measurable progress within neoliberalism where national progress (economic growth and competitiveness) is matched with individual progress (personal growth and self-fulfilment). There are some tensions around 'progress' in the policy texts, such as when Ofsted note 'a surprising finding…that younger pupils, rather than the older and higher attaining, were often more willing to "have a go"' (p. 37). However, the overall drive towards ever-higher performances by pupils and teachers in lessons and tests is clear. Within this, individuals are responsible for their performances and for playing their parts in ensuring national and individual progress. The government role is to ensure that opportunities are available to all. This is exemplified in shifting constructions of ability. In the next section, we look at how these discourses play out in the practices and talk of the student-teachers.

Student-Teacher Interviews

Nicola, Kate, Sophie and Leah (the student-teachers) were in their final year of a three-year undergraduate primary education degree and had worked with Anna as both university students and research participants. Whilst there is not space here for a thorough historical analysis some contextual information is important, particularly that the student-teachers 'training' took place during a time where government policy was prominent and ever-changing. In addition, student-teachers are exposed to many competing discourses and their identities are in flux.

Just as quality is difficult to discern from the policy documents, so it is from the student-teachers' interviews. When asked specifically about quality teaching they each had particular notions. For example, Nicola described it in terms of her main interview theme (teaching for understanding) while Kate also began by discussing her main theme: her current class of pupils. However, she quickly moved onto a more ‘acceptable’ description of quality teaching in ‘an ideal world’:

> Kate: With my class…you have to tell them to do something, get them to put their pencils down, and explain the next bit. …I don’t know what it is with my class. If I was I’d like to be able to, in an ideal world…quick-fire, snappy, teach them something and allow them to put it into action.

Perhaps Kate is positioning herself as a quality teacher by describing quality mathematics teaching as ‘quick-fire, snappy’, drawing on policy discourses of relentless progress. However, associating mathematics with speed also draws on more familiar discourses. In this section, we explore how more ‘traditional’ hierarchical discourses of mathematical ability—as associated with confidence, independence and pace, and as fixed and natural (Mendick 2006)—articulate with the contemporary neoliberal policy discourses discussed in the last section. We can see tensions between these neoliberal and traditional discourses in Sophie’s anxieties:

> Sophie: But there’s such a broad range of abilities, I’ve just found it much more difficult teaching maths this year with them not getting the certain methods that we teach it in. So I don’t know whether that’s because they’ve been taught in different ways each time they’ve come up the school.

Sophie, like the other student-teachers, seems to want to regulate her pupils’ progress in line with the ideas of quality in the policy documents. She becomes frustrated when children fall behind where they *should* be and worries about providing *appropriate* challenges to the most ‘able’. However, her idea that each child has a singular and fixed mathematical ability creates problems. As with Kate, discourses of mathematical ability mean that she has difficulty enacting the prescribed and desired quality teaching.

Despite the move away from individual ability in the policy documents, discussions of ability dominate the student-teacher talk and perhaps explain why none of them spoke of equity in terms of opportunity. In all of their classrooms, children were allocated to sit at either low, middle or high tables according to their levelled or age-ranked attainments. Ability comparisons were carried out throughout the interviews supported by the use of ability tables. Here the freedom offered by working in small groups on tables is in tension with the defining and fixing of each table by ability. Pupils are positioned in the group and in the associated ability. For example:

> Nicola: My lowest group, my Flames, are still on their two and three times tables. My highest group, my Gifted and Talented, can do up to their 15 times tables without a problem. And I’ve got a big sway but then I’ve only got four or five that can do it. The rest of them are stuck on fours and fives. So I’ve got about 80% of the class that really do need targeting for their multiplication.

Comparison is, of course, ‘highly visible as a tool of governing at all levels’ (Grek et al. 2009, p. 123) and is used throughout the quality assurance systems of the wider education marketplace, in league tables, performance indicators, etc. However, the interaction of rational technologies of comparison with hierarchical discourses of mathematical ability creates inequities in access to knowledge and allocation of value.

Inequities in access to knowledge are evident in Sophie's discussion of her use of practical resources:

> Sophie: My high ability, they could probably do it just by sitting looking at that; they can work it out. Their brains have got the steps going logically…but definitely for the less able. They definitely need [practical resources].

They are also evident in Nicola's discussion of the 'using and applying' strand of the mathematics curriculum (that traverses the content strands and relates to thinking and reasoning skills):

> Nicola: It's like using and applying is, I find more important for them [higher attaining pupils]. Because the using and applying is how I can push them without putting them so far ahead that I am causing a problem for later on.

Thus, abstract rather than practical reasoning and the 'higher-level' skills of using and applying are only encouraged and expected from higher attaining pupils. This too is a familiar discourse about mathematical ability, that learners have their limits and need differentiated curricula (Houssart 2001). However, we can also read in Nicola's talk a concern about the policy press for forward movement. Associating progress with acquisition of mathematical content, she has found an 'acceptable' way to push her pupils, but one that is not equitable as 'quality' learning is restricted to those judged able.

As well as restricting access to knowledge, ability was attached to value hierarchies:

> Leah: These can only cope with about Year Three work at a push with a bit of support and they're in Year Four. Then I've got my Year Four table…they're up to Year Five. They're higher level, they're brilliant.

The 'higher', 'brilliant' group are contrasted with those who need 'a push' and 'a bit of support', drawing on the familiar trope of mathematical ability as independence and inability as dependence. All the student-teachers agreed that 'slow' pupils require intervention in the form of extra time and support: 'they need that time so that's why they're doing the intervention' (Leah). Pupils are removed from the 'normal' classroom so that they can, as Nicola says, 'do it at their pace…with children on their level'. This carries ideas that children belong with others of their type but, rather than the policy idea of 'on their level' as their current performance level, here it is also associated with personality. Thus, policy discourses construct level as changing and external to the self and student-teacher discourses construct it as fixed and internal to the self. This is evident in the way that the student-teachers largely understand intervention in terms of building pupils' confidence and giving them attention:

> Sophie: They need a bit of attention and it's just a confidence issue.

> Leah: [The intervention programme's] just like reinforcing their maths skills because with that group and the taking of it is confidence…They're not as good at picking it up as some

> of the others in Year Five and Year Four tables. So they need just that little bit more support. But they are capable with their support, it's just, it needs the support, definitely on a lower level…because it's in a smaller group as well, they're getting more attention then.

Thus, mathematical ability is discursively linked to personal qualities of independence, speed, interest and confidence. Indeed generally, in student-teachers' discussions of pupils' identities, attainment levels and cognitive ability were overshadowed by more affective personality traits. This naturalises particular traits as indicators of ability, ones which are not equally available to all and thus the notion of equitable good quality teaching becomes problematic.

In particular, we would tentatively support other research showing an alignment between middle-class cultural capital, masculinity, whiteness and the 'able' personality traits. For example:

> Sophie: I don't think [ability] really matters. It's all personality, I do think it's personality that has a big, big impact because…Luke will sit there and he'll write sums, write sums, write sums, but because he doesn't enjoy literacy as much…he isn't as creative he just turns off from it and he doesn't work. So I think it's all to do with their personal thoughts about the subject…Because he's very capable but he just has a negative view of his handwriting and his ability. So he doesn't work as hard, but he could easily be in the top two [ability] tables. Whereas in maths…I don't know whether it's because he enjoys it more, he feels more capable.

Sophie attributes Luke's achievement to personality, which she dissects into 'personal thoughts about the subject' and 'a negative view of…his ability'. Luke is described, as someone who 'could easily be on the top two tables', consolidating dominant discourses of ability and suggesting how some, middle-class boys, are read as 'able' despite poor attainment (Walkerdine 1990). In a second example, at Nicola's school pupils chose their own ability table names: Flames (lowest group), Comets (middle) and Spoons (highest). The lower attaining pupils chose the more obviously powerful names and the highest attaining group (mostly boys) chose an ironic name, suggesting they have nothing to prove. Perhaps, like Mac an Ghaill's (1994) 'Real Englishmen' they were engaged in a middle-class masculine performance, mocking, and so indicating their superiority to the system. In stark contrast the Comets were stuck:

> Nicola: I think that my lowers are never going to enjoy maths…they're never going to want to do maths. And I can see that now…by the time they've hit Year Three, Four, it's just gone.

This discourse, that each mathematical ability group is fixed in behaviour and achievement, is very different from the mobile consumers constructed in the policy texts.

From these interviews, we suggest that the rational neoliberal discourses of quality and equity conflict with the traditional hierarchical discourses of mathematics and the caring emotional discourses of primary teaching. This conflict results in levelling and ability groupings leading not to progress for all but to the labelling and normalising of pupils and there being no place (apart from intervention) for those who do not fit. We argue that this is not an inadvertent side-effect of such grouping practices: 'an opportunity cost' that can be eliminated through improved teacher training and development, as Williams would have it. Ability judgements are structurally embedded in our constructions of mathematics and are produced by and reproduce inequitable discourses in the wider society. However, this cannot be spoken within neoliberal discourses of equity as opportunity, with their emphasis on individual responsibility rather than social structures. Thus, rather than levelling simplistically ensuring that every child progresses within mathematics, it supports ability hierarchies, which translate into 'progress', but only for some. Thus, mathematics classrooms, and mathematics itself, fabricate and are fabricated by discourses that cannot be taken on board within neoliberalism. These discourses and the objects they bring into being are self-perpetuating and cyclic, as the assumed hierarchical nature of mathematics and mathematical ability naturalises the processes through which some succeed and others are excluded (Mendick 2008).

Conclusions

'Quality' operates as a 'mobilising metaphor' (Carlile, in press) to muster support for a raft of neoliberal policies that manage teaching and learning as if it were a commercial process. Children are transformed into attainment levels, the inputs and outputs of the educational production process. Teachers' work becomes facilitating the efficient 'conversion' of the input attainment levels to higher output levels; children's work is to ensure that they, like good consumers, take up the educational opportunities available to them. We see the ongoing colonisation of learners, teachers and the learning process by business. Within this, inequity is constructed as a lack of opportunity to progress (naturalised as development) which can be addressed through interventions to recover the deviant child within the bounds of the normal. However, the classed, raced and gendered exclusions constituted through the normalisation of the consuming, progressing self are left unaddressed. When equity is constructed as the opportunity to participate in the conjoined progress of the self and society, then broader structural inequalities cannot be addressed. This is striking in the talk of the student-teachers and their widespread rationing of education and allocation of value through mathematical ability judgements. Normalisation masquerades as intervention as opportunity is lost beneath the daily practices of the levelling and labelling of pupils. Although neoliberal politics claims otherwise, we have argued that you cannot make an economic argument for equity when it is understood in collectivist rather than individualist terms. For quality to fit with social justice as we understand it, it must be defined in something other than economic language.

While we hope this analysis is (at least a little) disturbing, it also, perhaps inevitably raises the question, so what? We feel that the value of analysing the discursive power relations in which we are caught up is that we can begin to open up possibilities for thinking, being and doing otherwise (Butler 1997). For example by intervening, we position pupils as dependent, slow and unconfident in relation to mathematics, partly perpetuating a cycle that keeps them in their place, as ‘unable’. The alternative is not to withdraw support but to question the setting up of independence, speed and confidence as goals and the positioning of support as something needed by those who are lower/lesser. This raises questions about whether we should be teaching mathematics in its current form and what the alternatives might be. This is particularly important given that mathematics is implicated in the neoliberal agenda of economic well-being, prosperity and competitiveness in a high skills global economy, for ‘by the use of mathematics as a language of “the market” so mathematics has become entwined and identified with market economics’ (Woodrow 2003, p. 2). Mathematics is tied to processes of measurement and to their normalising role.

Any attempt to understand policy inevitably has a shelf-life as it is subject to rapid change. It is possible that before this book is published we will have a change of government in the UK. However, the discursive patterns of quality and equity within neoliberalism have a longer history than the current government and will continue (in modified form) after they go. For we live in an era in which education is constructed as central to the knowledge economy and ‘performance data’ have become the measure of quality and the driver of policy globally. These discourses will also live on in the practices of the student-teachers, like those interviewed for this article, who learnt to teach within them.

References

Archer, L., Hollingworth, S., & Mendick, H. (2010). *Urban youth and education: Exploring the identities of educationally ‘at risk’ young people*. Maidenhead: Open University Press.

Ball, S. J. (2008). *The education debate*. Bristol: The Policy Press.

Butler, J. (1997). *The psychic life of power*. Stanford: Stanford University Press.

Carlile, A. (2010). Meeting rooms, documentation, and the policy community: Finding space for social justice in the administration of permanent exclusion from school. *Journal of Power and Education* (in press).

Coffield, F., & Edward, S. (2009). Rolling out ‘good’, ‘best’ and ‘excellent’ practice. What next? Perfect practice? *British Educational Research Journal, 35*(3), 371–390.

DCSF. (2008). *Making good progress in mathematics*. London: DCSF.

DCSF. (2009a). *Your child, your schools, our future: Building a 21st century schools system*. London: The Stationery Office.

DCSF. (2009b). *Securing level 4 in Key Stage 2 mathematics*. London: DCSF.

du Gay, P. (1996). *Consumption and identity at work*. London: Sage.

du Gay, P., Hall, S., Janes, L., Mackay, H., & Negus, K. (1997). *Doing cultural studies: The story of the Sony walkman*. London: Sage.

Edelman, L. (2004). *No future: Queer theory and the death drive*. Durham: Duke University Press.

Foucault, M. (1972). *The archaeology of knowledge*. London: Routledge.

Francis, B. (2006). Heroes or zeroes? The discursive positioning of ‘underachieving boys’ in English neo-liberal education policy. *Journal of Education Policy, 21*(2), 187–200.

Gillborn, D., & Youdell, D. (2000). *Rationing education: Policy, practice, reform, and equity*. Buckingham: Open University Press.

Gillies, D. (2008). Quality and equality: The mask of discursive conflation in education policy texts. *Journal of Education Policy, 23*(6), 685–699.

Grek, S., Lawna, M., Lingard, B., & Varjoc, J. (Eds.). (2009). Special Issue, North by northwest: Quality assurance and evaluation processes in European education. *Journal of Education Policy, 24*(2), 121–133.
HM Government. (2009). *New opportunities: Fair chances for the future*. London: HM Stationary Office.
Houssart, J. (2001). Setting tasks and setting children. *Proceedings of the 24th Annual Conference of the British Congress of Mathematics Education, 21*(2), 136–146.
Hultqvist, K., & Dahlberg, G. (Eds.). (2001). *Governing the child in the new millenium*. London: RoutledgeFalmer.
Mac an Ghaill, M. (1994). *The making of men: Masculinities, sexualities and schooling*. Buckingham: Open University Press.
Mendick, H. (2006). *Masculinities in mathematics*. Maidenhead: Open University Press.
Mendick, H. (2008). Subtracting difference: Troubling transitions from GCSE to AS-level mathematics. *British Educational Research Journal, 34*(6), 711–732.
Ofsted. (2008). *Mathematics: Understanding the score*. London: Ofsted.
Reay, D., & Wiliam, D. (1999). 'I'll be a nothing': Structure, agency and the construction of identity through assessment. *British Educational Research Journal, 25*(3), 343–354.
Skeggs, B. (2004). *Class, self, culture*. London: Routledge.
Stronach, I. (2001). *Quantifying the qualitative as contemporary fool's gold*. Paper presented at British Society for Research in Learning Mathematics, Manchester, March.
Walkerdine, V. (1990). *Schoolgirl fictions*. London: Verso.
Walkerdine, V., & Lucey, H. (1989). *Democracy in the kitchen: Regulating mothers and socialising daughters*. London: Virago.
Williams, P. (2008). *Independent review of mathematics teaching in early years settings and primary schools*. London: DCSF.
Woodrow, D. (2003). *Mathematics, mathematics education and economic conditions*. http://www.esri.mmu.ac.uk/respapers/papers-pdf/Paper%20-%20maths,%20maths%20ed%20&%20economics.pdf.

Chapter 5
Quality and Equity in Mathematics Education as Ethical Issues

Bill Atweh

The discourses of quality and equity have become globalised concerns in the field of mathematics education as reflected in most policy and curriculum documents around the world. While few people would contest their importance to mathematics education theory and practice, their meanings often remain unexamined. A careful reading of their use in various contexts reveals alternative, if not divergent understandings behind them. This chapter[1] is an attempt to contribute to a systematic theorising of the two agendas that taken separately potentially, even though not necessarily, might lead into conflicting actions and outcomes and may lead into lack of achievement of either. In the first part of the chapter, I undertake a critical reconstruction of some of the tensions reflected in the use of the two terms and their interactions. By "reconstruction" I do not mean abandoning or rejecting the understandings of the past. However, an interrogation of the two concepts allows us to examine the assumptions and limitations behind their different uses. As Christie (2005) argues, all concepts are socially constructed and hence are "contingent and contestable" and are to be "rendered permanently contested" (p. 241). In other words we need to be "working *with* and working *against*" (p. 240) the constructs towards alternative understandings that are more likely to deal with contingent problems that any discourse may lead to. In the second part, I present a reconstruction of the two agendas grounded on the discourse of *ethical responsibility* that allows for a viable understanding of both agendas and constructs them as complementary, and is hence more likely to facilitate their achievement.

[1] This chapter is based on my contribution to the Plenary Session 6 (Panel Debate on Equal Access to Mathematics Education) at the International Congress of Mathematics Education in Monterrey, Mexico in 2008.

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Tensions Within the Discourses of Quality and Equity

To start with, here I understand *quality* as a question of values and judgements rather than an objective and decontextualised description of a phenomenon. As Dahlberg et al. (1999) point out, the dominant understanding of "'the discourse of quality' can be seen as part of a wider movement of quantification and objectivity intended to reduce or exclude the role of personal judgement, with its attendant problems of partiality, self interest and inconsistency" (*quote in the original*, p. 87). The authors go on to trace the emergence of this discourse by placing it within the rise of the Enlightenment with its overzealous trust of quantification, comparing the dissimilar by reducing them to the same criteria. They add that in the age of uncertainty, it "offers us confidence and reassurance by holding out the prospect that a certain score or just the very use of the word quality means that something is to be trusted, that it is really good…rather than being a symbol whose meaning can only be arrived by critical reflection and judgement" (pp. 92–93). Thus, the determination of quality involves setting standards of product or service delivery and criteria for the achievement of these standards. Of particular interest here is the argument the authors make that these criteria and standards are often taken to be based on *rational*, *objective* and *universal* grounds.

Although different policy and curriculum documents in mathematics education around the world have been constructed using the discourse of quality, the term is often assumed and not defined. Hence, it remains, and should remain, a contested construct. It seems to me that the discourse of quality in mathematics education is often based on one or both of two considerations:

1. Doing better mathematics and
2. Increasing students' achievement in that mathematics

As Atweh and Brady (2009) argue, in the dominant mathematics education discourse, "better mathematics" often refers to abstraction and the rigour of the discipline of mathematics (e.g. Juter 2006). This includes formalised symbolic language, axiomatic thinking, standard efficient algorithms and proofs. It may also include sophisticated modelling of mathematically based problems—usually from areas such as physical world, engineering and the economy, in which there is a unique or best-fit solution. This is often contrasted with practical mathematics that focuses on social world applications, routine problem solving—on personalised (often called student-invented) algorithms, solutions and non-standard presentations of mathematical arguments. In many Australian curricula, these two types of mathematics are contained in alternative streams that students select (or are assigned to) depending on their previous mathematics performance (often taken as a sign of ability) and post school aspirations. This construction of quality mathematics, in contrast to practical mathematics, is presented as a *common sense* solution for the need to provide a greater choice (a valuable endeavour in neo-liberal politics) for students and to cater for the needs of a larger number of students. Regardless of attempts by education systems and teachers to present the different streams as equally valuable, many students refer to the practical mathematics subjects with the diminutive term "vegi-math[2]".

This binary might be counterproductive by denying the majority of students (that is, those taking the so-called social or practical mathematics), the opportunity and the ability to develop their generalised abstractions of mathematical concepts and procedures and to develop their confidence as users of mathematics. Likewise, it denies the students undertaking the more academic mathematics subjects the opportunity to see the application of mathematics to solve problems in their immediate life. Arguably, in our times, students need both abstract knowledge and practical knowledge. Hence, if quality of school mathematics education is only understood from within the discipline of mathematics, it may lead into alienation of the majority of the student population that fail to appreciate such abstraction, are not capable of achieving it, or fail to see its relevance to their lives.

An alternative understanding quality mathematics education is the focus on students' achievement, in particular based on comparing students' performance with others or with pre-determined standards using frequent national testing. As Apple (2000) argues, neoconservative governments around the world have encouraged privatisation and devolution of decision making in education yet reinforced their control over curriculum and standards through testing regimes. This is the "scientific management of education through legislation" approach to curriculum development and reform as discussed by Neyland (2004). Commenting on the standards movement in the USA and on the attempts to implement the No Child Left Behind policy Mark (2008) raises the question whether such practices are able to achieve equity. He argues that high-stake testing may lead to an image of mathematics as something to be planted in minds of students irrespective of meaning and isolated from their everyday life and experiences. Further, such practices are in danger of reinforcing student alienation and dissatisfaction from their experiences in mathematics school learning.

I will return to the discussion of the different understandings of quality below. However, now I turn to deal with another important challenge to mathematics teaching, namely that of *equity*. Whereas concerns about quality are about what type of mathematics is worthwhile and valuable and about how students can best develop this mathematics, concerns about equity are about who is excluded from the opportunity to participate and achieve in mathematics within our current practices and systems, and about how to alleviate their disadvantage (Burton 2003; Secada 1989). Atweh and Keitel (2007) note that concerns about participation and achievement in mathematics study by different social and cultural groups are no longer seen at the margins of mathematics education policy, research and practice. Issues relating to gender, multiculturalism, ethnomathematics and the effects of ethnicity, indigeneity, socio-economic and cultural backgrounds of students on their participation and performance in mathematics are regularly discussed in the literature.

[2] That is, vegetarian mathematics—in contrast to the academic mathematics which is regarded as meaty (with apologies to the vegetarian readers!).

In a previous article (Atweh 2007), I indicated how the concepts of equity, diversity and social justice are often dealt with in the literature as exchangeable constructs. At the risk of over-generalisation, perhaps there are some regional variations in their use—i.e. concepts of equity and diversity are widely used in the USA, while European literature makes more reference to social justice. In the USA, however, Secada (1989) discusses equity in terms of social justice. Similarly, the three terms are often used to discuss different forms of disadvantage—i.e. equity and social justice are often used—but not exclusively—to look at lack of participation and achievement based on gender, Indigeniety and social class, while diversity is often used—but not exclusively—to look at variation due to ethnicity, language and cultural background, age, sexual orientation and disability.

In spite of the overlap in the aims of both agendas of equity and diversity, there is an important difference between them in that they aspire to potentially contradictory outcomes with regard to group status. Fraser (1997) points out that the diversity discourse might lead to essentialising the differences between the different groups and it may fail to take into consideration the changing constructions of these labels and their contextual understanding in time and place. Similarly, the diversity discourse fails to adequately take into consideration one of the biggest threats to social inequality and exclusion in mathematics education, namely socio-economic background and poverty that are difficult to construct as diversity issues in the same way as, for example, cultural differences. Equity projects aim at reducing group differences, e.g. in achievement and participation, and hence its ultimate aim is to abolish group differences. Diversity discourse, on the other hand aims at enhancing respect for group differences and status. This is the dilemma that Fraser (1997) refers for in discussing the multidimensional model of social justice. There are two further limitations of the equity and diversity agendas. On one hand, remediation equity concerns might lead to a backlash of misrecognition (Fraser 1995) for the target group by constructing them as victims or as needy of special assistance, while diversity construction promotes group status. On the other hand, the diversity agenda might lead to of romanticising difference between groups by treating them as exotic, while the equity agenda highlights their exclusion and disadvantage. As Burton (2003) argues in her introduction to her book "Which Way Social Justice in Mathematics Education", in mathematics education literature there seems to be a "shift from equity to a more inclusive perspective that embraces social justice" (p. xv). She goes on to say "the concept of social justice seems to me to include equity and not to need it as an addition. Apart from taking a highly legalistic stance, how could one consider something as inequitable as socially just?" (p. xvii). Using Fraser's conceptualisation of social justice as having two irreducible dimensions, distributive and recognition, the social justice agenda incorporates both equity and diversity concerns, respectively. Fraser demonstrates that while neither agenda is reducible to the other, the two are not mutually exclusive (Fraser 1997; Fraser and Honneth 2003). In practice, most social justice action contains elements of both.

Relationship Between Equity and Quality

It is perhaps not difficult to point out to both extrinsic and intrinsic values that many industrialised societies might have to explain their attempts to achieve both quality and equity. In terms of quality, excellence is often valued for its own sake. Perhaps the world's fascination with high performance in sports, and the huge amount of resources devoted to it, illustrates the intrinsic values of quality performance. Closer to the topic here however, Stack (2006) discusses the media frenzy around the PISA results in Canada that are undoubtedly mirrored in many participating countries around the world. Regrettably, however, the serious questions about the possible invalidity of these tests to represent real performance of students (Fensham 2008) and the hidden inequity with societies that their results reveal (McGaw 2004) are not seen to be as newsworthy. Likewise, quality in mathematics educations is also extrinsically valued for the significant potential of mathematical knowledge to the society's well-being and economic and technological development. Undoubtedly, it has that potential. However, these assumptions about the value of mathematics education for the student and society should not be accepted uncritically. First, the relationship of mathematics to general economic development is far more complex than is often assumed. For example, Woodrow (2003), citing the example of the development of the Asian economies and the high achievement by their students in international testing, argues that increases in mathematics education standards have occurred after their economic development, and arguably as a result of it, rather than the other way around. Further, Ortiz-Franco and Flores (2001) demonstrate that during the period between 1972 and 1992, the mathematics achievement of Latino students in the USA have increased in comparison with other students, although their socioeconomic status has decreased.

Similarly, concerns about equity in different societies reveal some intrinsic and extrinsic values. Equity in mathematics education can be constructed as a human rights issue for full participation in society by many traditionally excluded groups. Perhaps, the pioneering work of many women in mathematics education represented at different times at International Organisation of Women in Mathematics Education have shown us how addressing exclusion combining research and political action can lead to changes of patterns of participation and achievement. Similarly, concerns about equity and social justice reflect extrinsic values that equitable participation and achievement bring to any society—in particular, values such as social cohesion, and harmony, peace as well as economic benefits. The consistent message from educational economists is that if a society considers that achieving equity is costly, they should realise that the cost of an inequitable world is potentially far greater.

Here I argue that, although not necessarily mutually exclusive, the agendas of quality and equity may lead to undesired contradictory outcomes. As Gough (2006) points out, in many policies "equality (or equity) is understood to be a *necessary condition* of quality" (p. 12). However, in practice, a focus on one without the other is problematic. In the same article, Gough refers to several South African writers who argue that the quality agenda in that country is often used as a means to justify the continual exclusion of black students from further education. In other words, a concern about quality with no concern about equity may lead to "elitism". In the same vein, a concern about equity with no consideration about quality runs the risk of sacrificing it. Luke (1999), referring to the work of Newman and his associates (1996), points out that "the worst enemy of equitable and socially just outcomes is the phenomenon that we could call 'dumbing down'" (p. 11) the curriculum. Hence, the focus on only one demand is not only misguided—by failing to deal with significant determinants of participation and achievement in mathematics—but also counterproductive—in leading to results contrary to what we are aiming to achieve.

This potential conflict between equity and quality is not only hypothetical. In practice, where resources are scares, as often is the case in education, in particular, in many less industrialised countries, this potential can become reality. At the International Conference on Education organised by UNESCO in Geneva (International Bureau of Education, 2005), Mohammad Osman, the Bangladesh minister of education is quoted as saying:

> While access has increased, quality has suffered largely due to systems' inability to provide the requisite number of well qualified and trained teachers and syllabi and curricula that is consistent with the need of a changing world. (p. 51)

In other words, under adverse conditions, the choice may come down to either concentrate on some basic education for a wider range of students, or spend more resources to increase the education of the most likely to reach their high potential.

Is the identification of values as basis for quality and equity agendas sufficient to guide necessary action towards their achievement? There remain a few problems. Firstly, values are socially constructed and can vary from one culture to another and from one time to another. Further, values are open to conflict with each other, and action towards one may lead to a sacrifice of the other. Values alone do not lend themselves to obvious criteria for their own evaluation and critique. Hence their ability to provide normative criteria for action is limited. Lastly, action towards achieving quality and equity in mathematics education based on values is becoming increasingly difficult in our age of uncertainty (Skovsmose 2005). As Foucault (1984) says "people know what they do; frequently they know why they do what they do; but what they don't know is what they do does" (p. 95). Skovsmose goes on to argue that in the age of uncertainty the only option we have to guide our action is a sense of responsibility of one to the other. As Critchley and Bernasconi (2002, p. 26) eloquently put it "the end of certainty can be the beginning of trust". Equally correct, they could have said the beginning of responsibility.

This concept of responsibility brings us to the heart of the discourse of ethics. In the following section, I will articulate a particular understanding of responsibility based on ethics as elaborated by Levinas and argue that this understanding provides alternative constructions of quality and equity and contributes to the normative criterion for action and reflection towards their achievement.

Ethical Responsibility

Atweh and Brady (2009) point out that the demand for responsibility, or more often in its related term accountability, is an ever-increasing concern in educational discourse, policy and practice in many countries around the world. In educational discourse, the term responsibility is used with a variety of meanings. Responsibility is often presented as a requirement or duty that restricts (as in, it is the teachers' responsibility to cover the curriculum), as privilege that enables (as in, the teachers' responsibility to maintain discipline in class), as a placement of blame (as in, who is responsible for the students' lack of achievement?), or in its ethical or moral meaning (as in, it is the teachers' responsibly to tell the truth). In these uses, responsibility is understood as determined by social structures and roles, rules and regulations or codes of behaviour. Such rules and codes assume an individual agent who is independent and with a moral choice of following the rules or not. Further, they are based on a rationality that constructs the "good" as subservient to knowledge of the good and such knowledge is taken to be objective and universal (Cohen 2001).

The argument here is not that rules and codes are not necessary for the well functioning of society and the common good of its members. Rather, the concern is that this construction of responsibility mechanises the relationship between people and, hence, is in danger of eroding the humanity of the human (Cohen 2001). Similarly, it reduces complex decisions to a choice between one rule and another, and hence hides deep ethical concerns. As an illustration of this danger, consider the processes for assuring ethical conduct of research as adopted in many countries. Reducing ethical concerns to filling in forms and ticking of boxes is in the danger of researchers avoiding facing deeper ethical questions as to who benefits from the research, whose concerns are researched and what is the role of the participants in the research process (Groundwater-Smith 2007).

Alternative constructions of responsibility and ethics acknowledge that ethical decisions are often messy and complex. Universal laws are often not helpful in dealing with case by case situations. This of course is not a sanction for an "anything-goes" ethics. On contrary, as Dahlberg and Moss (2005) argue, this ethics is more demanding of the agent than simply following conventions. Ethical decisions are much more of a burden when seen as more than merely following of rules. This of course supposes that the agent is intrinsically an ethical being who acts for good and does not need rules and codes to act responsibly. Are people intrinsically ethical?

Can we turn to philosophy to assure us? Cohen (2005) explains this avoidance of ethical discussion in philosophy as a fear of moralising, preaching and questions of values by philosophical discourses mainly focused on ontology rather than meaning. In Western thinking, there is a movement away from essentialist thinking represented in the universality of ethical principles (Christie 2005) and their foundation on rationality as established by philosophers such as Kant. Going back to the philosophical and ethical discourses of Socrates, who argued for the primacy of the knowledge of the *good* over the knowledge of the *truth*, Cohen raises the question

"Has the philosopher abdicated responsibilities" by only dealing with questions of knowledge rather than values (p. 39). However, this avoidance of ethical discourse is slowly dissolving. As Critchley (2002) indicates, it was only in the 1980s that the word ethics came back to intellectual discourse after the "antihumanism of the 1970s" (p. 2). Further, the post-ontological philosophical writings of Levinas (1969, 1997) have been influential in the re-introduction of ethics within philosophy by establishing ethics as the "first philosophy".

For Levinas, ethics is before any philosophy and is the basis of all philosophical exchanges. It precedes ontology "which is a relation to otherness that is reducible to comprehension or understanding" (Critchley 2002, p. 11). This relation to the other that precedes understanding he calls "original relation". Using a phenomenological approach, Levinas argues that to be human is to be in a relationship to the other, or more accurately, in a relation *for the other*. This relation is even prior to mutual obligation or reciprocity. Roth (2009, p. 31) argues that this original ethical relationship discussed by Levinas consists of an "unlimited, measureless responsibility toward each other that is in continuous excess over any formalization of responsibility in the law and stated ethical principles".

From this perspective, people are neither intrinsically good nor are they intrinsically bad. They are morally ambivalent (Neyland 2004). However, since *being-for-the-other* precedes *being-in-itself*, the self is intrinsically ethical—in the sense that concerns about ethical responsibility towards the other precedes the knowledge about the self. As Neyland argues, it is an "incorrect assumption that the ethical self is caused by—is a product of—social legislation that redeems the pre-ethical self from a prior and unwanted disposition" (p. 56). On the contrary, there is a danger that legislation limits, if not erodes the ethical self. However, he goes on to argue that ethical encounter is not sufficient as a substitute for ethical codes, but needs to be supplemented by "shared ethical ideals, priorities and principles that are open to agonistic negotiation" (p. 57). These should complement rather than override ethical primacy of direct encounter.

So what do the agendas of quality and equity look like within this ethical responsibility?

Constructing Quality and Equity as Ethical Concerns

As discussed above, in mathematics education quality is often understood from within the field of mathematics and articulated in terms of rigour and in the form of standards and means of testing. Very rarely, it is based on a discussion of the aims of mathematics (Jurdak 1999). A discussion that is based on the wider role of mathematics in the lives of the students as well as society would lead an alternative understanding of quality that does not refer to a particular type of mathematics nor achievement in it, but whether or not the practice of mathematics education itself has achieved these aims and what type of mathematics education would promote their achievement.

Undoubtedly, mathematics is useful for economic and technological development of society (Kuku 1995). However, traditional forms of mathematics education based on the development of abstract and objective content is not a guarantee against the misuse of such developments that might lead to inequality, insecurity and environmental degradation—arguably all encompassing threats to our global society. Similarly, mathematics is a useful subject for many jobs and careers. However, often it is used as a badge of eligibility of entry to those careers as much as it is used in those careers themselves—thus leading to exclusion and disadvantage. Mathematics education cannot abdicate its responsibility to deal with arising problems with the content it develops and remain ethical. Further, limiting the aims of mathematics education to social development, constructs the individual as subservient to social structures rather than an active agent in their society. Once again, ethical practice, as discussed above, is based on the responsibility to the *other* before, and as a basis of, responsibility towards the social.

Here, I recognise an encompassing aim of mathematics education as a contribution to the ability of students to meet the demands of their current and future lives—i.e. as their development as responsible citizens. I acknowledge the problematising of the concept of responsible citizenship provided by Popkewitz (2004). In this context, responsible citizenship is not understood as playing a particular social role, obeying laws, following regulations or being pleasing to authority. Rather a responsible citizen is somebody who is both willing and able to take responsibility to expose social problems through mathematics and propose possible solutions for them. Puka (2005) illustrates how the distinction that some feminists make between responsibility and "response-ability" is a significant contribution to ethical thinking. Response-ability highlights the ability to respond to the demands of the other. This is similar to what Roth (2007) points out, that responsibility

> etymologically derives from a conjunction of the particles re-, doing again, spondere, to pledge, and -ble, a suffix meaning 'to be able to'. Responsibility therefore denotes the ability to pledge again, a form of re-engagement with the Other who, in his or her utterances, pledges the production of sense. Each one, on his or her own and together, is responsible for the praxis of sense, which we expose and are exposed to in transacting with others. (p. 5)

In other words, the aim of mathematics education is to develop a response-able citizen. Using Gutstein's terms (2006), a citizen who is able to "read and write the world through mathematics".

Undoubtedly, to achieve this role, care is to be given to develop the power of rigour in mathematical arguments, flexibility in problem solving and generalisation in mathematics. Hence, the contention here is not that the understanding of quality mathematics referred to above is wrong, but that it is limited. The meaning of quality in this case is what kind of mathematics is more likely to promote the response-ability of the student. Quality in mathematics education is measured not as, or not only as, formal abstraction and generalisation, but by its capacity to transform aspects of the life of the students both as current and future citizens. In another context (Atweh 2009), I discussed some curriculum and pedagogical implication of what I and some colleagues have called Socially Response-able Mathematics Education. Perhaps intuitively it is not difficult to understand that the agendas of equity and ethics are associated. In the previous sections I argued that social justice is a wider agenda than equity; hence we will discuss the relationship between social justice and ethics. I will follow with the discussion of ethics as postulated by Levinas and show why ethics needs justice and why justice needs ethics.

As discussed above, Levinas constructs the encounter with the other as the bases of ethical behaviour. He posits the ethical self as prior to consciousness of the self, being and knowledge. The encounter with the other demands nonreciprocal and unlimited commitment to serve the needs of the other. However, the other is not singular. There are many others. How can this unlimited responsibility be shared with two or more others? Hence, by necessity, this primal ethical relationship is restricted by the presence of the Third (Simmons 1999). How can ethics not lead into injustice in treating two or more others the same way? Levinas' answer is that ethics needs justice to regulate it. This should not be taken as a defect in the construction of ethics as an infinite demand. Rather, it is a call for a construction of justice at the service of ethics. If ethical responsibility is to be good for the other without leading to injustice, it needs justice to regulate it in a society that has many others. Although, justice is not reducible to ethics, it is taken to be a subservient to ethics.

What does this construction contribute to the understanding of social justice? Atweh and Brady (2009) posit two reasons why the discourse about ethics supports, and lays the foundation for, concerns about social justice. First, social justice discourse is often constructed as concerns related to the participation of social groups in social activity and their enjoyment of their fair share of social benefits (Fraser 1997). It has less to do with the outcomes achieved by a particular individual—unless the outcomes are due to their belonging to a social group. They are often silent on issues related to the interaction between two people—say of the same social group. Ethics, on the other hand, is concerned with a face-to-face encounter and interaction between people. This understanding of justice as subservient to ethics resolves the problem of dealing with the individual versus a group in social justice concerns. Undoubtedly, dealing with the demands of marginalised groups remains a crucial social justice issue. However, by understanding that justice is justified by ethics, an encounter with a particular member of that group is still subject to unlimited ethical responsibility. In practice, this implies that dealing with individuals in isolation from their social group memberships, thus failing to see the effect of their background on their chances of social participation, is in danger of being unjust. In the same vein, stereotyping an individual only as member of a group, thus focusing on their background and failing to see their possibilities, is in danger of being unethical.

Secondly, as argued above, the foundation of social justice on values that different social groups and countries have is not sufficient. This focus on ethics establishes social justice concerns as a moral obligation, rather than on charity, good will or convenient politics. In other words, adopting a social justice approach places knowledge as a servant of justice, while an ethical approach places justice at the service of the moral (Cohen 2001). Neyland (2004) quotes Cohen (1986) as saying:

> The demands of justice arise out of ethical situations and at the same time pose a danger for that situation. The danger of justice, injustice, is the forgetting of the human face. The human face "regulates", it is the goodness of justice itself. (p. 9).

Conclusions

The demand to re-examine issues of quality and equity in mathematics education arise not only from their increasing role in official and academic discourse and practice in the field. The perceived importance of mathematics and the implications of lack of achievement in it have the potential of increasing pressure and anxiety for students and teachers. Similarly, many countries around the world are investing huge resources for reforms in their mathematics education curricula and teaching. Often these reforms mirror reforms in more industrialised countries. Rather than accepting quality and equity as absolute and non-problematic constructs, this chapter presents alternative possible understandings of them. In particular, I examined the possibility of basing both constructs on the discourse of ethical responsibility as elaborated by Levinas.

By using ethics as a foundation for both constructs, not only is it possible to argue that the two agendas are not contradictory, but also that they are both necessary for an ethical practice in mathematics education. Understanding the educative interaction as ultimately an ethical encounter highlights the responsibility (read response-ability) of the teachers to meet the demands of the responsibility (read response-ability) of the students to meet the demands of their current and future social lives. This understanding necessarily implies the call for "powerful" mathematics (read quality) for every student (read equity). Here, I understand the power of mathematics not as traditional rigour of formal mathematics but as its potential to contribute to active citizenship for reading and writing the world (Gutstein 2006). Furthermore, since ethics is based on questions of what is good to do—and what is good to be, such a discussion should form a normative guidance to practice. By normative role, I do not mean they are sufficient to inform practice in every classroom and with every student around the world. Rather, they establish criteria for decision making in educational planning and practice that allow us to act and reflect on our actions.

References

Apple, M. (2000). Between neoliberalism and neoconservatism: Education and conservatism in a global context. In N. Burbules & C. Torres (Eds.), *Globalisation and education: Critical perspectives* (pp. 57–78). New York: Routledge.

Atweh, B. (2007). What is this thing called social justice and what does it have to do with us in the context of globalisation. *The Philosophy in Mathematics Education, 21*(2), 111–124.

Atweh, B., & Brady, K. (2009). Socially response-able mathematics education: implications of an ethical approach. *EURASIA Journal of Mathematics, Science and Technology Education, 5*(3), 267–276.

Atweh, B., & Keitel, C. (2007). Social justice in international collaborations. In B. Atweh, M. Borba, A. Calabrese Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 95–112). Dordrecht: Springer.

Burton, L. (Ed). (2003). *Which way social justice in mathematics education?* London: Praeger.

Christie, P. (2005). Towards and ethics of engagement in education in global times. *Australian Journal of Education, 49*(3), 238–250.

Cohen, R. (1986). Introduction. In R. Cohen (Ed.), *Face to face with Levinas* (pp. 1–12). New York: Sate University Press.

Cohen, R. (2001). *Ethics, exegesis and philosophy: Interpretations after Levinas.* Cambridge: Cambridge University Press.

Cohen, R. (2005). Levinas, Plato and ethical exegesis. In J. Bloechl & J. Kosky (Eds.), *Levinas Studies: An annual review* (Vol. 1). Pittsburgh: Duquesne University Press.

Critchley, S. (2002). Introduction. In S. Critchley & R. Bernasconi (Eds.), *The Cambridge companion to Levinas*. Cambridge: Cambridge University Press.

Dahlberg, G., & Moss, P. (2005). *Ethics and politics in early childhood education.* London: Routledge-Falmer.

Dahlberg, G., Moss, P., & Pence, A. (1999). *Beyond quality in early childhood education and care: Postmodern perspectives.* London: Falmer Press.

Foucault, M. (1984). *Madness and civilization: A history of insanity in the age of reason.* London: Vintage Publishing.

Fensham, P. (2008). Context or culture: Cam TIMSS and pisa teach us about what determines educational achievement in science. In B. Atweh, M. Borba, A. Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 151–172). Dordrecht: Springer.

Fraser, N. (1995). From redistribution to recognition: Dilemmas of justice in a postsocialist society. *New Left Review*, July–August, 68–93.

Fraser, N. (1997). *Justice interruptus: Critical reflections on "postsocialist" condition.* New York: Routledge.

Fraser, N., & Honneth, A. (2003). *Redistribution or recognition? A political-philosophical exchange.* London: Verso.

Gough, N. (2006). Quality imperialism in higher education: A global empire of the mind? *Critical Perspectives on Communication, Culture and Policy Studies, 25*(2), 1–15.

Groundwater-Smith, S. (2007). Ethics in practitioner research: An issue of quality. *Research Papers in Education, 22*(2), 199–211.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* London: Routledge.

International Bureau of Education. (2005). *Quality education for all young people*. Geneva: UNESCO.

Jurdak, M. (1999). The role of values in mathematics education. *Humanistic Mathematics Network Journal,* (21), 39–45.

Juter, K. (2006). Limits of functions as they developed in time and as students learn them today. *Mathematical Thinking and Learning, 8*(4), 407–431.

Kuku, A. (1995). Mathematics education in Africa in relation to other countries. In R. Hunting, G. Fitzsimons, P. Clarkson, & A. Bishop (Eds.), *Regional collaboration in mathematics education* (pp. 403–423). Melbourne: Monash University.

Levinas, E. (1969). *Totality and infinity: An essay on exteriority* (trans: Lingis, A). Pittsburgh: Duquesne University Press.

Levinas, E. (1997). *Otherwise than being or beyond essence* (trans: Lingis, A.). Pittsburgh: Duquesne University Press.

Luke, A. (1999). *Why equity and social justice still matter but differently*. Paper prepared for the Education Queensland online conference. Brisbane, Department of Education, Training and the arts. education.qld.gov.au/corporate/newbasics/docs/onlineal.doc. Accessed 9 Oct 2008.

McGaw, B. (2004). *Australian mathematics learning in an international context.* Keynote address to the Mathematics Education Research Group of Australasia conference Townsville, Australia. http://www.merga.net.au/documents/keynote32004.pdf. Accessed 5 Oct 2009.

Mark, E. (2008). Leaving no child behind yet allowing none too far ahead: Ensuring (in)equity in mathematics education through the science of measurement and instruction. *Teachers College Record*, *110*(6), 1330–1356.

Newmann, F. & Associates (1996). *Authentic achievement: Restructuring schools for intellectual quality*. San Francisco: Jossey Bass.

Neyland, J. (2004). Towards a postmodern ethics of mathematics education. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 55–73). Charlotte: Information Age Publishing.

Ortiz-Franco, L., & Flores, W. (2001). Sociocultural considerations and Latino mathematics achievement: A critical review. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 233–253). New Jersey: Lawrence Erlbaum.

Popkewitz, T. (2004). The Alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Educational Research Journal, 41*(1), 3–34.

Puka, B. (2005). Teaching ethical excellence: Artful response-ability, creative integrity, character opus. *Liberal Education*, *91*(3), 22–25.

Roth, W. R. (2007). Solidarity and the ethics of collaborative research. In S. Ritchie (Ed.), *Research collaboration: Relationships and Praxis* (pp. 27–42). Rotterdam: Sense Publishers.

Secada, W. (1989). *Equity in education*. Philadelphia: Falmer.

Simmons, W. (1999). The third: Levinas' theoretical move from an-archical ethics to the realm of justice and politics. *Philosophy and Social Criticism, 25*(6), 83–104.

Skovsmose, O. (2005). *Critical mathematics education for the future*. Working Papers on Learning 2. Department of Education and Learning. Denmark: Aalborg University.

Stack, M. (2006). Testing, Read All about It: Canadian Press Coverage of the ***PISA*** Results. *Canadian Journal of Education*, *29*(1), 49–69.

Woodrow, D. (2003). Mathematics, mathematics education and economic conditions. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 9–30). Dordrecht: Kluwer Academic Publishers.

Chapter 6
Ghettoes in the Classroom and the Construction of Possibilities

Ole Skovsmose and Miriam Godoy Penteado

Introduction

Based on teachers' accounts, we are going to discuss equity and quality with reference to mathematics education context. The particular context we are going to consider is set by the basic education system in Brazilian public schools. We interviewed 11 teachers who work in different public schools in Brazilian cities and two prospective teachers, both university students (presented in Table 6.1). All interviews were carried out by e-mail. In the following, we refer to teachers and prospective teachers as teachers, the names are fictitious each chosen by themselves.

We asked the teachers, first, to describe episodes that could give an impression of the diversity of students they had been teaching. Second, we asked what ideas they associated with the notions of 'equity' and 'quality', and how they would combine these two notions with regard to mathematics education.

As shown in the Table 6.1, the majority of teachers were from São Paulo state, one of the richest states in Brazil. Furthermore, several of the teachers have a master's degree in mathematics education; one is a PhD student in mathematics education, while others have been involved in research-based developmental programmes. As a consequence, the group we interviewed does not represent the population of teachers in Brazilian public schools in any statistical sense of the word.

Based on the interviews, we do not try to formulate any statements about teachers' experiences and opinions in general. Instead, our intention is to present some episodes that could give an impression of what is taking place in public schools and at the same time give inspiration for addressing issues about equity and quality.

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Table 6.1 Interviewed teachers

Name	Age	Brazilian State	Years of experience as a teacher
Marina	45	São Paulo	16
Silvana	44	São Paulo	15
Rubens	35	Minas Gerais	15
Rebeca	39	São Paulo	14
Lucas	34	Goiás	8
Luis Manoel	27	Paraíba	6
Alessandra	31	São Paulo	5
Adriana	30	São Paulo	5
Denis	24	São Paulo	2
Gustavo	24	São Paulo	2
Rudá	26	Minas Gerais	1
Tanien	20	São Paulo	Prospective teacher
Daniela Rosa	18	São Paulo	Prospective teacher

Thus, we try to identify some categories 'dense with experiences' by taking departure in the teachers' presentation. We summarise the teachers' descriptions, which have been provided to us in Portuguese. In most cases, we do not provide a word by word translation, however when we do this, we insert the translation in quotation marks.

After a brief overview of the Brazilian school system, Section 'Episodes from the School' presents some episodes experienced by the teachers. Section 'Equity and Quality' presents the teachers' comments on equity and quality with a particular reference to mathematics. Section 'Emerging Categories' summarises emerging categories 'dense with experiences'. Section 'Ghettoes in the Classroom' and Section 'Construction of Possibilities' discuss two issues related to equity and quality drawing on the emerging categories, while Section 'Final Considerations' contains the final considerations.

Episodes from the School

The majority (around 87%) of the Brazilian population from 6 to 18 years old study in public schools which are financed by the federal government, by the state, or by the city council where no fee has to be paid.

In general, a Brazilian public school is organised as is any such school in other countries. One difference could be that the students attend at the school for less time than in some other places. For example, the schools operate with two daily periods: morning from 7:00 to 12:00 and afternoon from 12:30 to 17:30. In some cases, there is also an evening period from 19:00 to 23:00. This means that one group of students goes to the school in the morning, another group in the afternoon and a third group in the evening. In some cases, this also happens to the teachers in the sense that some of them have classes in different periods of the day.

The usual pattern of a mathematics lesson is the traditional one where the teacher makes a presentation of the content of the day's lesson, some examples may also be presented; then, the students try to solve exercises, and the solution will be checked. In other words, the normal pattern of the mathematics classroom in Brazil is similar to the normal pattern throughout the world. The school mathematics tradition has taken an almost universal format.

Diversity

It is early afternoon, and the school is about to start the afternoon session. Luis Manoel describes a scene when a school bus is arriving. The bus is provided by the city council, and it brings students from the rural areas. There is no sign on the bus indicating that it serves as a school bus. It stops in front of the main entrance and the students get out, making a lot of noise. Many of them just bring notebooks, others bring a backpack which has been donated by the local school authority. Some bring their cell phones playing popular music. Some are singing, some are dancing along with the rhythms emanating from their phones. In general, the atmosphere is relaxed and playful, which continues after they have entered the gate and got to the school yard next to their classroom.

Silvana describes a few of her students, all about 12 years old. One girl has huge difficulties; she is not organised and cannot concentrate. She knows how to read the different syllables, but when the syllables come together to form words, she finds it difficult. In mathematics, she does not know how to subtract, and when it comes to addition, she just makes some drawings on the paper because she has not mastered the algorithm. She does not disturb the class, she does not do anything, she remains apathetic.

A boy is completely alienated from what is taking place in the classroom. He smiles and stays absentminded. He has all the equipment he needs for going to school. He arrives spotless and organised, but he does not do anything. Another boy enjoys the confusion and is always disturbing his friends. Once, after he had been fighting with some other students, he was sent to the principal. When he returned he told his friends that he was going to ask his stepfather to take revenge. Another boy does not open his note book, and after Silvana has complained, he tells that he does not have any pencil, nor any pen, and so he cannot do anything. As Rebecca observes, there are also many students who are interested in engaging in the school activities: ‘We also have excellent students who enter the best universities, write poetry, are very good in Sport and get Olympic prizes in mathematics’.

As the students are different, so are their family backgrounds. Gustavo has taught in a poor neighbourhood. Here many students come from broken families. Some may live with their grandparents, some with one or the other of their parents. Gustavo also points out that there are several cases where girls get pregnant when they are only 13 or 14 years old. In order to reduce this problem, they are given information about contraception. But the problem remains far from solved. Several teachers tell about how drug trafficking is part of the business of the neighbourhood. One of Rebeca’s students was an addict and offered drugs to his classmate: ‘He would not be quiet in the classroom and was aggressive to everybody in the school. Once he was found with a knife, and the police were called. He was aggressive toward the police as well'. It also appears that the very school building is suffering in this neighbourhood. Graffiti seems to appear everywhere together with a general damage within the school.

There is a huge diversity among the students. Thus, whatever background a student might come from, some are eager to learn and to pursue further education, while others disturb the class and are seen by the teachers as problematic. This notion of diversity is crucial in formulating the teachers' experiences of extremes, which spreads in all possible directions.

Access to Digital Technology and Prestige

The number of students who have access to computers and the internet is increasing. As Rubens and Adriana emphasize, more and more students, who have no economic conditions to have a computer at home, get access to the internet in cybercafés where they pay per hour. The number of students with cell phones is also increasing, but in the majority of the cases, the cell phone has no access to the internet as this kind of service is too expensive.

Adriana tells that one morning in the middle of a lesson a student cried out in a loud voice: 'Goooooooool'. The word *gol* is the Portuguese word for 'goal', and every time a goal is scored the Brazilian speaker cries 'goooooooool' for as long as his breath allows the word to last. Stretching the word *gol* is common among every enthusiastic football supporter, and Adriana's student was obviously listening to a transmission of a football match during the lesson.

The new technology has clearly entered the classroom of public schools. But it is far from being every pupil who is familiar with the equipment. Once, Tanien was giving a lesson to students about 11 years old. He had brought the students to the computer room, where they would work with some activities using the software Cabri-Géomètre II. In order to get things started, it was necessary for Tanien to install a file from his pen-drive. A student was following what he was doing and asked:

- Teacher, is this a pen-drive?

Tanien answered:

- Well, it is a new form of disk, and its storing capacity is equivalent to about 1,000 disks.

The student was astonished:

- Oh my God, could I use such a thing at my computer (the one he was using at the school)?

Tanien asked the student to click at the icon and open the activities. To Tanien this episode illustrates that one cannot assume that students would be familiar with what other people might take for granted.

In public schools new technology has entered the classroom. However, students are familiar with this technology in very different ways. Some demonstrate much hands-on practice, while many, as Tanien observes, are far from familiar with such things. However, to everybody this familiarity appears to signify prestige.

Poverty and Hope

There are many slums and squatter settlements in Brazil, and students in public schools may also come from such neighbourhoods. Lucas points out that the situation of many of his students is dramatic. He has students who not even have conditions for having a proper bath before going to school. They do not brush their teeth, their clothes are dirty, and they do not have basic things like paper and pencil. The deprived appearance of the students is a direct expression of poverty: families can be missing even the most basic things like hot water, shampoo and soap.

Lucas remembers a student, about 12 years old, who came to ask if Lucas would give a note book, as his parents could not afford to buy one. He was a positive and attentive student with reasonable marks. Lucas could clearly imagine that if the students had better conditions, had possibility to eat properly, and had conditions for concentrating on studying, he might come to do much better. As Lucas emphasises: ‘It is simply not fair that a 12 year old student needs to be preoccupied with how to get a note book’.

Alessandra tells that in her school the students’ union organises a campaign to collect second-hand clothes to donate to poor students’ families. The class which collects most clothes will receive free tickets for the cinema, and popcornas well.

Marina tells about a student of one of her colleagues, who completed some difficult mathematical activities. The teacher was very happy with what the student had done, and she told the student: ‘If you continue doing well like this, I will give you a present’. The student answered: ‘Oh, teacher, if you are going to give me a present, please give me a kilo of beef’. Marina also tells that it happens several times that some students did not feel well, as they had nothing to eat during the day.

Many students are in difficult situations, and they might also be with desperate hopes. In one case, a girl asked Marina if she would become her mother. Some days later, during parents meeting, the girl’s father told Marina that his wife had left home and that he now lived alone with three children. It could well be that the girl had anticipated that her parents’ divorce was on its way and tried to secure some kind of solution. In terms of marks the girl was one of the best students Marina had, and she tells that several of her students have come to her and asked if she would be ready to adopt them.

Stigmatisation

There are many different groups of students in the school, and there are many ways of demonstrating differences. First, with respect to obvious material things, like clothes, shoes and equipment, not least electronic devices like a cell phone. But differences are also marked through differences in languages and use of slang. There are differences in ways of behaviour, for instance with respect to going to parties, music and dances. These choices can also include an expression of priorities with respect to the competing gangs that dominates the neighbourhood.

There seem to be very complex dress codes in operation. The students from rich as well as from poor neighbourhoods do really have to 'dress up' in order to go to school. However, there are students who come to school in clothes which are in a very bad state, and due to their appearance, students may get into difficulties with their peers. Marina tells that, sometimes, when she organises classroom activities where students have to work in groups or in couples, problems could easily emerge. Nobody wants to work together with students in rags, and to solve the problem she worked together with the otherwise excluded pupils. Denis describes how he once was putting up a huge poster in the classroom and asked the students to move up to sit in front of it. A nicely dressed student did not want to sit next to another student. When Denis asked why, the student answered: 'I'm afraid of getting lice from her hair. She is dirty and does not take a bath'. Poverty is reflected in appearance, which in turn becomes a cause for brutal forms of stigmatisation.

None of the teachers refers to racism, but racism exemplifies a most brutal form of stigmatisation, and one can imagine many contexts where this notion would be crucial in order to formulate teachers' experiences and for being able to address issues about equity and quality. Here we restrict ourselves to point out the overall notion of stigmatisation.

Learning Condition

In many cases, students need to repeat a school year. Thus, Gustavo tells that he has experienced situations where 18 years old students are joining the same class as 12–13 year-olds. In many cases, it might seem that students come to school not to learn but to be together with friends. However, one needs to be aware, as Gustavo also remarks, that for the majority of students 'the school environment may be the only possibility they have for interacting with others'. For many students, working with the content is only one possible facet of going to school.

Daniela Rosa tells that many students from secondary school—students from 14 to 18 years—need to work in order to support their family as well as themselves. Thus, she knows students who are studying in the morning while working during the afternoons and evenings. The consequence, naturally, is that it becomes difficult for them to pay enough attention to what is taking place during the lessons.

The teachers experience many differences in students' capacities for learning. Marina tells that several times fast learners get annoyed with students who need too many repeated explanations. However, learning capacity is a complex phenomenon. It can be related directly to the students' situation outside school. Once Adriana asked her class how many had done their homework—only very few hands were (slowly) raised. There can be different reasons for this, and certainly the most direct reason can be that for many students it is not possible to find any space at home for doing their homework. On top of this, a 'culture of studying' can be missing at home. In many homes, any form of reading is a rare preoccupation. At a parents meeting, Adriana asked if any of the parents had seen their children doing homework. The parents got surprised by the question. One stated: 'Uau, in fact I never see my children studying at home'.

Naturally, it is important not to make any generalisations, and the teachers emphasise several times that there are students with any kind of background that do excellent in school, just as slow learners may come from very different backgrounds. Marina tells that she had a student, whose father was in prison, while her mother worked as a prostitute in a nearby city. This girl lived with her aunt. She was excellent in the school and excellent in mathematics in particular.

In the classroom, students may display different learning abilities. However, the term 'ability' is a strikingly misleading concept—most dangerous because it has come to assume an almost universal currency. It designates some phenomena as personal or as individual characteristics of the students, while these phenomena more realistically represent characteristics of the students' learning conditions. 'Ability' is thus a social construct, not a psychological one. In order to address issues of both equity and quality in learning, it is crucial not to read conditional qualities as individual qualities because doing so means that one comes to read differences in performances as individual differences, and not as representing inequalities in the experienced learning conditions.

Equity and Quality

After the teachers had presented episodes that could illustrate the diversity of their experiences, we asked them directly about their interpretation of equity and quality, and here we make a synopsis of their key positions.

Possibility

Adriana emphasises that she does *not* think of equity, as the governor of the São Paulo State has tried to realise it, namely in terms of the same curriculum for everybody. In fact, as a consequence of this state policy there has been a huge effort in implementing a uniform curriculum. But as Adriana emphasises, equity also means respecting differences.

According to Luis Manoel, equity in education means to ensure 'opportunities for obtaining knowledge'. Rudá also emphasises that equity does not simply mean equal opportunities in any direct measurable sense; it also means to ensure opportunities for everybody for getting a dignified life, and this might mean very many different things depending on the situation. Adriana stresses that equity has to do with establishing possibilities for everybody, considering their very different conditions for going to school, for learning and for realising their aspirations. In other words, equity in education has to be searched for in terms of a respect for differences. Also Rubens highlights that equity in education cannot be obtained through unification of the curriculum, but that equity has to do with the opportunities in life that the students are getting as a result of their engaging in schooling.

This brings us to an important observation: One can search for equity in education in terms of the possibilities that becomes established for the students.

Interpreting equity in terms of opportunities can get the most direct form. One can think of opportunities in terms of access to further education, being vocational training, technical schools as well as universities. Naturally, opportunities could mean many more things, but access to further education is an important set of opportunities provided by the educational system. Furthermore, access to further education can be directly addressed: one can simply register how many students from different schools get on to further education courses, and especially who might enter the best universities. Here the statistics reveal huge differences.

This observation brings us directly to political issues. Marina finds that if the government would invest adequately in public schools, they would in fact be able to compete with private schools, which in turn would mean that more quality in education would be ensured. This would also mean that working as a teacher at public schools would become more prestigious. According to Marina, some teachers see the situation in the following way: 'Oh, I'm teaching in a public school, where the students would never come to make a *vestibular* [The exam for entering at the university], so I do not need to do much, just teaching the most basic things'.

Participation

The Brazilian government has taken initiatives in a range of affirmative actions in order to provide more opportunities for black people and students from a poor background. However, a huge challenge is put in front of the teachers, and Rubens stresses that a principal step in order to counteract the strong processes of social exclusion is to have well-educated teachers. This does not simply refer to the teachers' knowledge of the particular disciplines. According to Rubens, to be a well-educated teacher also means to be 'open to dialogue', to be 'open to different experiences', and to be 'open to the students'. In characterising quality in mathematics education Rebeca emphasises the importance of reaching as many students as possible. This means that quality has much to do with students' participation in the learning processes. She sees the involvement of students as crucial in case education should come to make a difference in their life. In a similar way, Denis states that quality in mathematics education means more than just learning mathematics. It is important as well that students come to work with relations around mathematics. This could mean using mathematics for understanding daily life situations, social problems, political discussions, etc.

Several of the teachers highlight that it is necessary to address quality in education, and also in mathematics education, with reference to the cultural diversities of the students. Thus, one should not try to search for equity in terms of 'unification', nor when this unification becomes expressed in terms of a 'unified curriculum'. Qualities have to be searched for in terms of 'sensitivity' to differences; whatever these differences take a cultural, political or economic format. Thus, Adriana emphasises that quality in mathematics education means to ensure conditions for students to understand society, to act in the world, to have access to the produced knowledge and to make connections to different domains of knowledge.

This understanding of quality brings the teacher into a new position, and Rubens states that it becomes important to reconsider the role of the mathematics teacher, and in particular not to maintain the teacher as the centre of the classroom. Instead, it becomes important for the teacher together with the students to work with projects where processes of exploration and collaboration become crucial. It becomes important to establish a new dynamics in the classroom interaction.

Emerging Categories

Some categories, relevant for addressing equity and quality, did emerge from the teachers' comments. A crucial one is *diversity*, which refers to the students, their background, their family situation, their attitude, their behaviour, their learning conditions, etc. There is no homogeneity to be expected in an educational setting. The next two categories have to do with *access to technology* and *prestige*. They are closely connected, but are not identical. Access to computer and internet has assumed a particularly important place in the life of young people. It becomes part of life style and of a youth culture that stretches across other cultural settings. Prestige has to do with many things, the way of dressing for instance, the position among friends, the way one acts, etc. It is not possible to discuss equity and quality in education without addressing *poverty* as well as *hope* which in turn might appear desperate. Brutal processes of social exclusion relate to forms of *stigmatisation*, and racism could make part of such processes. The notion of capacity plays an important role in much discussion of students' achievement in school; however, we would emphasise instead the importance of *learning condition*, which refers to the complexity of the students' contexts. Two more categories that help to address equity and quality are the construction of *possibilities* and the notion of *participation* which are important for the quality in learning as well as for the construction of possibilities.

In short: we want to highlight the following nine notions as being of particular importance: *diversity*, *access to technology*, *prestige*, *poverty*, *hope*, *stigmatisation*, *learning condition*, *possibility* and *participation*. This is our suggestion for a categorical framework for addressing issues about equity and quality in (mathematics) education, which at the same time is sensitive to teachers' experiences.

In the following two sections, we address two challenges regarding equity and quality. The first concerns what we refer to as ghettoes in the classroom, which we see as obstructions for establishing equity as well as quality in education. The second concerns the construction of possibilities, which we see as crucial for trying to establish equity as well as quality.

Ghettoes in the Classroom

Processes are taking place in schools through which ghettoes in society become replicated in the classroom. We choose to talk about ghettoes in the classroom (and not just about ghettoes in the school) in order to emphasise that ghettoising reaches into the micro level of schooling. Based on the categories that emerged from the interviews, we will condense some of the ways in which these ghettoes may emerge. (Certainly, it is relevant to investigate how these ghettoes are experienced by the students and by the students' parents and families, but in this text we draw on the categories that emerged from the teachers' experiences.)

Students can become 'differentiated out' according to appearance. Their shoes might be different, but there are hundreds of ways of noticing social manifestations of economic differences within a school context. Such differences might appear insignificant, but among students certain differences get a gigantic significance and make part of the most brutal form of differentiation. Thus, several of the teachers referred to the importance for the students to 'dress up' for going to school. Missing the 'dress code' means a stigmatisation. It is deeply problematic to appear poor. As a consequence, there are certain things that become prestigious: those which demonstrate distance to poverty, as for instance, access to internet, ability in handling a computer, to speak in a cell phone, and in a clandestine way to follow a football match in the classroom. In the interviews, none of the teachers referred explicitly to racism. However, this form of differentiation cannot be ignored. Racism emerges when perceptible differentiations coagulate as explicit categories.

Some students can be attentive, others apathetic. Differences in learning conditions might appear as differences in students' abilities, but cannot be interpreted in this simple way. Some students might need to spend much of their time working in order to earn money, as pointed out by Daniela Rosa. As stressed by Lucas, it cannot be fair that a 12-year-old student needs to be preoccupied about how to get a note book. Such a preoccupation is certainly a learning obstacle; it has to do with poverty, but it has nothing to do with the student as a person.

The discourse of differentiation often gets an us-them format: 'They' come from a poor community; 'they' have to travel a lot to get to the school; 'they' look poor; 'they' have a different behaviour. The differentiation becomes formed through discursive patterns of a us-them format. The us-them terminology brings about characteristics of 'them'. One could talk about 'them' as being badly behaved, as being difficult, as being not able to learn, etc. It is a labelling that assigns some characteristics to the students. As the labelling becomes formulated in individual terms, referring to some characteristics of the individual student, the labelling can be used as a 'justification' of a differentiation (we have put 'justification' in quotation marks as it might take all kind of artificial and false forms): There is something that separates 'them' from 'us', and this something is a feature of 'them'. It is not 'us' who characterise 'them', it is 'them' who brings about their own characteristics. Furthermore, the characteristics of 'them' are not attractive but refer to limitations, flaws, defects, and weaknesses. A labelling easily turns into stigmatisation, which occurs when a 'justification' is added on top of a differentiation.

Stigmatising is an important element in the formation of ghettoes in schools. A discourse is created which includes a profound labelling, which refers to characteristics of the students themselves. The differentiation becomes engraved on a discourse of essence. Ghettoes in the classroom become created when differentiation turns into an us-them formulation, and labelling turns into a stigmatisation. Many students find themselves located in such ghettoes. Here, there is not much to hope for, not in any realistic way; only desperate hoping is available.

It might be difficult to describe the precise meaning of what equity and quality in education could mean, but they are not empty concepts. We find that ghettoes in the classroom represent a huge challenge for any education aspiring for equity and quality. In fact equity and quality in education are obstructed by all socio-economic process that makes part of the formation of ghettoes in society. Such ghettoes become replicated in the classroom; however, there are very many additional processes that take place in the formation of classroom ghettoes. An education for equity and quality must try to act against all processes, social and educational, which make part of the formation of ghettoes in the classroom.

Construction of Possibilities

Ghettoes in the classroom emerge through complex processes of differentiation, where lack of prestige, poverty and stigmatisation turns into general discourses, which in turn coagulate as ghetto-walls. Breaking down such walls constitutes part of the construction of possibilities for students. However, let us address things step by step.

Getting in contact is an important initial step in opening up a ghetto, and not getting caught by the logic of differentiation. Getting in contact can be seen as an initial educational step of steering away from the us-them differentiation. The importance of getting in contact was emphasised many times during the interviews. Thus, in characterising quality in mathematics education, Rubens stated that it becomes important for teachers and students to explore and collaborate and to establish a new dynamic in the classroom. In particular, he emphasised the importance for teachers to be 'open to dialogue'. In fact 'open to dialogue' is closely related to 'getting in contact', which can be identified as an important feature of a dialogue (see Alrø and Skovsmose 2002). In a similar way, Rebeca emphasised the importance of reaching out to as many students as possible.

There is no easy way of getting in contact; nevertheless, it is fundamental to the students' experience of meaning. Ghettoes in classrooms are surrounded by walls which obstruct exchanges of meaning. What is taught appears meaningless to the students, if the students' priorities are not recognised as significant for the learning processes. Ghetto-walls appear to be meaning-proof. It is not easy to remove a ghetto-wall, not even to make hole in it.

The preoccupation in providing mathematics education with meaning is recognised broadly, both in theory and in practice. We see meaningfulness as referring to relationships between what is taking place in school and the students' aspirations and hopes. Meaningfulness for students has much to do with what they might come to see as their possibilities. For a ghettoised student, however, there is nothing to be hoped for through schooling. Here we could refer to a 'ruined foreground', which only make space for desperate hoping (for an introduction of the notion of foreground see Skovsmose 1994, 2005a, b; Alrø et al. 2009). It is painful to hope for something which is unattainable. Students trapped within a classroom ghetto might experience ruined foregrounds which cause a huge obstruction in their experience and consequential construction of meaning.

One important element of the education for equity is to provide possibilities for the students. Construction of possibilities is important for everybody. In the interviews, several of the teachers referred to the notion of possibility. Luis Manoel pointed out that quality in education means to ensure opportunities for obtaining knowledge, while Adriana stated that equity has to do with ensuring possibilities for everybody. Our point is that the notions of equity and quality need to be discussed in terms of construction of possibilities. However, ghettoising obstructs the construction of possibilities: Students become designated to return to the situation they come from: they need only to be 'taught the basics'.

In the interviews, there was no indication of mathematics playing a particular role in processes of differentiation. It appears that processes of labelling, of stigmatising and eventually of establishing ghettoes in the classroom is of general nature and not related to any particular school subject. However, our point is different. Although mathematics may not play any *particular* role in establishing ghettoes in the classroom, mathematics may still provide possibilities for moving beyond such ghettoes. This has to do with the variety of roles of mathematics in the global networking.

Different forms of qualifications which, one way or another, can be related to mathematics becomes of demand at today's labour market. One can think of those groups of peoples who are developing and implementing new forms of information processing and automatisation. In this case, a variety of qualifications are important: for instance, an expertise with respect to programming, input-output analyses, construction of time-efficient work procedures, etc. One can also think of all these people who are going to operate with already implemented techniques, for instance, as bank assistants, as workers in a factory, as shop assistants, etc. The ability to handle certain technique-based procedures becomes a key demand. As consumer, one also needs to be able to operate with automatic processes, not only when one does internet shopping, but with any processing of one's own money, not to forget budget making. Thus, society rapidly develops a growing demand for qualifications related to a wide range of formal techniques including mathematics. Furthermore, it becomes important to reflect on what is done and what can be done through such techniques, and as for instance underlined by Denis, mathematics education might provide possibilities for such reflections.

Such general observations indicate many particular ways in which mathematics education could take part in constituting possibilities for students, not least for students who tend to be caught up by ghettoes in the classroom (see, for instance, Gutstein 2006; Greer et al. 2009; Penteado and Skovsmose 2009). As an illustration of such possibilities, we can refer to a statement made by Lupes, one of Gutstein's students:

> With every single thing about math that I learned came something else. Sometimes I learned more of other things instead of math. I learned to think of fairness, injustices and so forth everywhere I see numbers distorted in the world. Now my mind is opened to so many new things. I'm more independent and aware. I have learned to be strong in every way you can think of it. (Lupes, quoted from Gutstein 2003, p. 37)

Final Considerations

One could think of social ghettoising as being the cause of educational ghettoising. At least, it seems farfetched to assume that ghettoes in the classroom can be identified as the cause of ghettos in society. However, the situation is much more complex than these two cause-effect formulations indicate. One should not think of some one-way cause-effect relationship. Instead, one could think of educational and social phenomena as being connected and as interacting in extremely complex patterns. So, even though one should not assume to be able to solve social problems through educational initiatives, such initiatives could have impacts and make a difference for some students in some situations.

As a consequence, we find that it makes sense to formulate, as an educational aim, to pursue education for both equity and quality, in particular, with respect to groups of students who tend to be caught by processes of ghettoising. We find that a crucial element in such an education is to provide possibilities for the students.

We find that aiming for both equity and quality makes sense even though we see educational processes as dominated by social processes. However, domination means *domination* and not determination. A dominant system can still be undetermined and include contingencies.

In order to explore possibilities that accompany contingencies, clusters of categories becomes important. If one wants to address equity and quality in (mathematics) education, we find that categories like *diversity*, *access to technology*, *prestige*, *poverty*, *hope*, *stigmatisation*, *learning condition*, *possibility* and *participation* become important. Naturally, one can search for categories in different directions, and most common is to search for categories by exploring theoretical and, sometimes, philosophical frameworks. Here, we have searched for categories in a different direction. We have tried to present categories, dense with experiences, by relating them directly to teachers' expressions of their lived experiences.

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References

Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht: Kluwer.

Alrø, H., Skovsmose, O., & Valero, P. (2009). Inter-viewing foregrounds: Students' motives for learning in a multicultural setting. In M. César & K. Kumpulainen (Eds.), *Social interactions in multicultural settings* (pp. 13–37). Rotterdam: Sense Publishers.

Greer, B., Mukhopadhyay, S., Powell, A. B., & Nelson-Barber, S. (Eds.). (2009). *Culturally responsive mathematics education*. New York: Routledge.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, *34*, 37–73.

Gutstein, E. (2006). *Reading the world with mathematics*. New York: Routledge.

Penteado, M. G., & Skovsmose, O. (2009). How to draw with a worn-out mouse? Searching for social justice through collaboration. *Journal for Mathematics Teacher Education, 12*(3), 217–230.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer.

Skovsmose, O. (2005a). Foregrounds and politics of learning obstacles. *For the Learning of Mathematics, 25*(1), 4–10.

Skovsmose, O. (2005b). *Travelling through education: Uncertainty, mathematics, responsibility.* Rotterdam: Sense Publishers.

Chapter 7
Identity as the Cornerstone of Quality and Equitable Mathematical Experiences

Margaret Walshaw

Introduction

Quality pedagogical practices that are simultaneously socially just, are, by all accounts, at the crossroads in mathematics education (Walshaw 2009). Recent analyses of international test data have revealed patterns of social inequity that provide a sobering counterpoint to claims of an equitable and quality mathematical experience for all students. Such analyses show that for specific groups of students mathematics presents as an impossible challenge (Anthony and Walshaw 2007). The phenomenon is extremely complex and is not easily explained by conventional liberal democratic mechanisms. An equitable and quality mathematical experience will not be achievable for specific groups of students if the mechanisms that contribute to the types of mathematical identities offered them in the mathematics classroom are not explored. Such an exploration is the aim of this chapter.

In democratic societies, *all* students have right of access to knowledge. A key lever to this access is the classroom teacher. Research (e.g., Alton-Lee 2003; Hayes et al. 2005; Normore and Blanco 2008) has confirmed that it is the classroom teacher who has a significant influence over students' learning. This is not to suggest that the teacher makes instructional decisions in isolation from structures and other people: the teacher's classroom practice is always situated within a web of wider influences. In this expanded view, social and political factors that impact on teaching are hugely significant. Within the context of a larger sociopolitical environment, effective teachers enhance students' access to powerful mathematical ideas, irrespective of socioeconomic background, home language, and out-of-school affiliations. Such teaching is able to signal how persistent inequities in students' mathematics education might be addressed. This is crucially important in light of trends of systemic underachievement that provide a sobering counterpoint to claims of equitable learning opportunities.

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Problems and difficulties associated with how to ensure that quality mathematics experiences are also equitable are nested within a much larger phenomenon of a changing student demographic (see Banks 2007). In the contemporary scene, to deal with issues associated with diversity, policy makers are tending to opt for the classic deficit response—blame schools, teachers and students, introduce new initiatives, and intensify teacher surveillance—without also understanding how the inequities themselves are produced (see Nasir and Cobb 2007). Government incursions might heighten social awareness to the problems faced by teachers in schools, yet they cannot shore up the possibility of the production of a quality and equitable practice. Policy initiatives whose explanatory power lies in pathologising identities not immediately identifiable as "middle class and white" simply cannot get to the heart of the problem.

Nor is the quality/equity intersection helped very much by confusing equity with equality—important though equality is—as though unequal approaches, unequal access, and unequal opportunities would fully explain why many students do not succeed with mathematics and why many are disaffected and continually confront obstacles to engage with the subject. At a certain level within the contemporary debate about democratic provision, equality is privileged over any other advocacy, based on the understanding that equal outcomes, approaches and access, summed together, yield a comprehensive picture of equitable practice for students. This kind of approach has been seriously undermined by people like Foucault (e.g., 1972). However helpful the concept of equality might be in enhancing students' engagement with mathematical ideas, in trying to paint a picture of equitable arrangements in mathematics education, issues of structures, as well as interactions between contexts and people, cannot fail to intervene.

Cobb and Hodge (2007) have argued that issues relating to student diversity are among the most complex and challenging issues facing mathematics education today. However, in New Zealand and in most other western countries, diversity is now part of the way of life. These changed and continually changing demographics will require mathematics teachers to cater for increasingly diverse groups of students. How can we advance our understanding of the quality/equity conflation in order to deal with these issues? In addressing that question, I ground my discussion in the understanding that the quality of pedagogy is a social justice issue of momentous importance. In seeking a model of pedagogy that is equitable for the contemporary social context, I raise thorny questions about the generalized discourse of equity within mathematics education. I plan to do this by rethinking equity itself in ways that demand attention to interrelationships and the intersubjective negotiations that ensue.

The book edited by Nasir and Cobb (2007) provides a background for that discussion not only because it adds another dimension to ideas about the subject's fragmentation, but also because it represents a turn to theory that takes relationships as fundamental to the social justice project. There are arguments in the book, and, specifically, the discussion about communities of practice within the chapter written by Cobb and Hodge, that are helpful in advancing our understanding of equity. It is my contention that whilst the arguments put forward in their chapter are useful for understanding the multiple layers of teaching experience in a context of classroom diversity, a quality and equitable experience in mathematics education is, however, not created solely from rational decision making and negotiations of self with social structures. I propose a first-steps approach to equity that resonates in some ways with the approach provided by Cobb and Hodge *and* moves it forward.

Thus, the chapter is an exploration into a range of theoretical issues about how students' diverse sociopolitical realities impact on the types of mathematical identities and the level of mathematical proficiency offered them in the mathematics classroom. No matter how this endeavour differs from that of Cobb and Hodge, there is a consensus on the continuing political promise of the radical democratic project.

Identity at the Core of Quality Mathematical Experiences

In a publication that synthesised the literature on effective pedagogy in mathematics (Anthony and Walshaw 2007), I, together with my co-writer, developed a professional language to define a quality pedagogical practice. The intent was that the model of pedagogy developed from the teacher effectiveness literature would be used by teachers and educators for the purposes of talking about progressive, socially just pedagogic action. We were drawn to a view of pedagogy that magnified more than what teachers know to support mathematical learning. We tended to look beyond narratives of improved test scores, simply because explanations tied to high-stakes assessment told us only one aspect of the story. We broadened the scope of the goals of mathematics education, as advanced by policy makers, and the business and industry communities. For us, enhancing the intellectual capacities required for future employment and citizenship in a technologically oriented, knowledge-driven society, was not sufficiently far-reaching. We also wanted to encompass more than is typically required in day-to-day routines, namely, the skills, understandings, and numerical literacy needed for dealing confidently with everyday life.

If effective pedagogy is about understanding "what students know and need to learn and then challenging and supporting them to learn it well" (National Council of Teachers of Mathematics [NCTM] 2000, p. 16), we believed that a quality pedagogy conceives of the learner as a producer of knowledge, and understands teachers as co-producers. An effective mathematics teacher recognises that any pedagogic action involves the imposition of a cultural arbitrary. It is through the core dimensions of pedagogy that effective pedagogic action is able to attend to power relations that unfold from that cultural basis. These core dimensions include the cognitive demands of teaching, as well as the structural, organisational, management, and domain-specific choices that teachers make. These are all part of the large matrix of practice that involves systemic and policy support for focusing on quality and for working with and valuing diversity. The choices that a teacher makes include the negotiation of mathematics curriculum policy and carry over to decisions about the human, material and technological infrastructural arrangements that allow students to achieve specific outcomes.

A “quality” or an “effective” pedagogical practice becomes intimately focused on enhancing student outcomes and achieves its purpose. That is to say, a pedagogical practice that is effective is linked to a range of student outcomes, and these include achievement outcomes, encompassing conceptual understanding, procedural fluency, strategic competence and adaptive reasoning (National Research Council 2001). These outcomes characterise an apprentice user and maker of mathematics and are appropriated by the student through effective classrooms process. Added to those outcomes, is another set that underwrite a quality mathematical experience which are often tend to be overlooked. These are the social and cultural outcomes relating to affect, behaviour, communication, and participation (Anthony and Walshaw 2007).

Quality mathematical experiences that enhance a range of student outcomes are premised on the understanding that knowledge is necessarily social. If opportunities to learn arise in the community that the teacher develops, then people, relationships and trusting classroom environments are critically important. In the synthesis of the literature, we found evidence that teachers who truly care about their students have high yet realistic expectations about enhancing students’ capacity to think, reason, communicate, and develop mathematical argumentation (Walshaw and Anthony 2008). The tasks, activities, and tools they choose are aligned with these expectations and significantly influence the development of mathematical thinking, allowing students to access important mathematical concepts and relationships, to investigate mathematical structure and to use techniques and notations appropriately. Teachers’ pedagogical language and action associated with fairness and consistency works to counter the effects of social or material disadvantage. In developing inclusive partnerships, effective teachers ensure that the ideas put forward by students are received with respect and become commensurate with mathematical convention and curriculum goals. For them, a quality mathematical experience is more about transformative relationships than about transmitting and consuming knowledge. Such relationships always involve reciprocity and a pedagogical attention that moves students towards independence.

Crucially, transformative relationships make particular identities, and not others, available and realisable for students. Identity is, in a very real sense, at the core. It involves the ways in which students “think about themselves in relation to mathematics and the extent to which they have developed a commitment to, and have come to see value in, mathematics as it is realized in the classroom” (Cobb et al. 2009, p. 40–41). Thus, it is deeply implicated in the development of a quality pedagogical experience. Proposed in this way, effective pedagogy is able to reveal how the development of mathematical proficiency and aptitude over time is characterised by an enhanced, integrated relationship between teachers’ intentions and actions, on the one hand, and learners’ disposition towards mathematics learning and development, on the other. Importantly, both parties bring to the teaching and learning encounter a history that is enmeshed with the experience of the social and political world. But the scope of this history extends far beyond their interpersonal interactions: power and affective dimensions cannot fail to intervene.

Identity at the Core of Equitable Mathematical Experiences

Transformative interactions between contexts and people are at the root of effective and equitable pedagogic action. In turn, it is the concept of identity that underlies transformative interactions, and hence, is crucial for explaining pedagogy. Importantly, the experience of identity is rendered meaningful by particular groups and particular classroom practices (Britzman 1998). In the mathematics classroom, this is precisely because mathematics knowledge is created in the spaces and activities that the classroom community shares within a web of economic, social and cultural differences. Hence, knowledge creation cannot be separated from the axes of social and material advantage or deprivation that operate to define students. Quality pedagogy takes into account the ways of knowing and thinking, language, and discursive registers made available within the physical, social, cultural, historical, and economic community of practice in which the teaching and learning is embedded.

These ideas are most keenly expressed by Cobb and Hodge (2007). For them, an individual learner's socially just relationship with mathematics is essentially situated and relational. Equity, here, is defined not as a property of people, but as a *relation* between settings and the people within those settings. The proposal comes hard on the heels of a renewed respect for "the other" within mathematics education (e.g., Ernest et al. 2009; Gutstein 2006; Sfard and Prusak 2005; Stinson 2006; Valero and Zevenbergen 2004). In questioning the criteria for interpreting equitable experience, Cobb and Hodge have re-evaluated the long-standing idea that student experience in mathematics classrooms is consistent across groups. Their heightened sense of awareness of the limits of past efforts to counter differential performance in mathematics has crystallised into the development of an interpretive framework, providing insight into the meaning of a quality and equitable experience in mathematics.

The thrust of the argument endorsed by Cobb and Hodge (2007) is that equity is not a property of people. Nor is it a static process. Instead, it is a "as an artefact of cultural settings and the relations between them" (Nasir and Cobb 2007, p. 6). In their view, equity amongst mathematics learners depends to some extent "on how students' identities as learners are enabled as they participate in classroom mathematical practices (through social interaction and participation structures) vis-á-vis their identities as 'doers' of other practices" (p. 8). Practices at both the micro level of the classroom and the macro level of the institution of schooling—and the power plays within—all work to inform the development of an equitable practice within mathematics education. Put simply, equity here means protection from and resolution of those processes or structures that serve to undermine a student's sense of self and others as legitimate mathematical learners within the context of the practices of the mathematics classroom and within other communities in which the student participates. The student's identity, then, is able to tell us about the nature of a quality mathematical experience.

Thus, in an effort to understand the local, systemic and typically flexible conditions of identity construction, Cobb and Hodge offer a view of equity that foregrounds engagement within communities of practice. Three constructs are proposed to analyse students' identities in mathematics classrooms consisting of normative identity, core identity, and personal identity. These three constructs operate to deal with the methodological difficulty in accounting for an individual's differential engagement—with respect to the individual's role and position—between and within social groups, cultures and institutions. Normative identity is set within the highly localised context of the classroom. It refers to the identity of a student fully engaged within the practices—both mathematical and social—of the classroom. The activity normalised within the classroom involves not only rights and obligations, but also interaction and reciprocity, all of which are oriented towards enhancing students' capacity to think, reason, communicate, reflect upon and critique what they do and say in class.

Core identity has a more expansive reach than the classroom. It is "concerned with students' more enduring sense of who they are and who they want to become" (Cobb and Hodge 2007, p. 167). Core identity, as a construct, takes on board Gee's realisation that an individual's engagement within the communities with which she is associated tends to take a unique "trajectory," and this is apparent even when life histories appear to be similar. Specifically, the core identity of a student is embodied with a double valence: on the one hand, the student is an agent; on the other hand, the student has a connotation of being subjected to. In other words, core identity has both the status of being acted upon by wider social and political structures, and simultaneously, the status of position of agency within those sociopolitical structures.

Personal identity brings us full circle back to the classroom. Its focus is on "who students are becoming in particular mathematics classrooms" (p. 168). The construction of a personal identity is an ongoing process, ever-changing as the student works at reconciling her core identity with the normative activity as established within her mathematics classroom. The construct of personal identity is formulated in such a way as to reconcile participation and engagement within specific communities outside of the classroom, with the specific mathematics learner a student aspires to be within the classroom. Thus, the identity of a student comes into being *in relation to* the negotiations that she undertakes with other individuals and communities. Crucially, a change in a student's personal identity within a particular classroom context may also result in a change in their core identity—their long-term assessment of sense-of-self. That is to say, a change in personal identity operating within the mathematics classroom has a direct bearing on the kinds of mathematical identities that students might take up and the kinds of proficiencies to which they might aspire.

Understandings like these, developed from Vygotsky's work (see, for example, Gee 1999, 2001; Lave 1988; Lave and Wenger 1991; Rogoff 1990; Roth 2004; Valsiner 1987; Wenger 1998; Wertsch 1991), propose that what students say and do within the discourses made available as a result of the social categories of gender, race, and so on, has the effect of contributing to the development of their identity as apprentice mathematicians at a given time and place. In Gee's (2004) understanding, identities develop from knowledge which "is distributed across people and their tools and technologies, dispersed at various sites, and stored in links among people, their minds and bodies, and specific affinity groups" (p. 33). Similarly, for Cobb and Hodge, identities develop from "locally instantiated practices that are dynamic and improvisational in nature" as people engage in "joint activities that involve the directly negotiated use of artifacts" (p. 164), bearing in mind, also, the power structures that enable and constrain agency within such practices. These practices determine the spectrum of speech acts, and actions, within a classroom, that can be taken seriously at any given historical moment. It is these practices and activities that circumscribe the possibility of thought concerning what an equitable and quality mathematics experience might look like.

Equity and Quality Under Erasure

Leading edge work like Cobb and Hodge's (2007), with its emphasis on socially constituted identities, has allowed us to problematise the tendency to assume that "social structures and the ideologies they give rise to" (Nasir and Cobb 2007, p. 7) play out in the same way for all, irrespective of one's history, interests, social categorical descriptions, affiliations, and circumstances. Their work is illuminating in the sense that a student's mathematical identity is influenced by her membership within shifting social networks and her engagement with and negotiations amongst members of those communities. The suggestion is that mathematical identity is formed from a reconciliation of existence in the "borderlands of various communities" (Cobb and Hodge 2007, p. 162), not the least of which is the community of the classroom. Insightful though this work is, there are, however, no easy reconciliations between borderlands. Findings from my research with others on connections between teaching and learning (e.g., Walshaw et al. 2009) have revealed that mathematical identities are formed in a very slippery space. Affective issues always intervene.

The procedure of Vygotskian-inspired work like that of Cobb and Hodge is to steer a middle course between supporting long-held epistemological and ontological preoccupations surrounding a stable rational identity, and in participating in the wider epistemic shifts for theorising conflict and tension as they play out in the process of reconciliation. The conceptual tools proposed by Cobb and Hodge allow us to deal with the interplay between social practices and the processes of self-formation that are at work in mathematics schooling. However, the three constructs rest on the presumption that the effects of power, privilege, and disadvantage in identity construction have, to all intents and purposes, been countered. The approach, they point out, makes "the notion of identity as it relates to mathematics teaching and learning both tractable and relatively concrete" (Cobb et al. 2009, p. 41). Thus, the stable self, presumed by Cobb and Hodge, leaves intact the characterisations of identity, consciousness and agency put forward by traditional social science. As a consequence, these understandings provide a limited perspective of how mathematical identities are constituted within the realities of the classroom and the wider sociopolitical context.

Theoretical and methodological issues to do with the concept of the stable self have been critiqued on a number of fronts within the discipline (e.g., Brown 2008; Cabral 2004; Hardy 2004; Fleener 2004; Valero and Zevenbergen 2004; Walshaw 2001, 2004). As a counterpoint to the stable self, Stentoft and Valero (2009) have advanced the notion of *fragile identities in action* to draw attention to instability of identification processes embedded within discourse. They use this concept to focus on what is typically considered "noise" or "impossibilities" in classroom interactions. I wondered if it might be possible to think about an equitable and quality experience in mathematics in a way that captures the fragility of identity and, at the same time, highlights the fact that "[m]ore often than not…identities are not a matter of deliberate rational choice" (Sfard and Prusak 2005, p. 18). Is it possible to account for emotions, both positive and negative and often in conflict within the tentative and shifting self, engaging within a range of communities, including the community of the mathematics classroom?

Rational processes take us only so far in the development. In the next section, I make a case for the strategic use of theory from a framework that focuses on the emotive and unconscious aspects of identity construction. Specifically, it offers insights from Lacan's (1977a, b) psychoanalytic horizon. Like the approaches offered by Bibby (2009) and by Appelbaum (2008), the interest is "toward the relationality of the teaching/learning encounter" (Appelbaum 2008, p. 52). Neither wholly focused on the teacher nor the student, but on the relation between both, the approach attempts to grasp the complexity of people and the cultures they create. It does that by drawing on a theory of the subject/identity that can analyse the fluidity and complexity of the self/community relation not merely through rational links. The methodological interest here is to explore identity through unfamiliar ways of thinking more deeply in order to reveal the significance of that theory for a pedagogy that is simultaneously effective and equitable.

The Lacanian Response

Lacan is, by any criteria, a most significant theorist for understanding democratic provision in mathematics classrooms. Broadly speaking, his philosophy seeks to expose and make sense of the potential for fairness and equity in any social setting. Identity formation is at the heart of his thesis. He provides conceptual tools that allow us to deal with the complex interplay between social practices and the processes of self-formation that are at work in schooling. What is of primary importance for him is the transparency of the relation between the person and the social. In particular, he offers a definition of identity to explain how one's sense of self is a product of discursive diffusion, and it is this concept that allows him to explore the dynamic self/social relation. A psychoanalytic approach, like his, can be "one of the most helpful [to education] in its theories of learning, and in its curiosity toward what is not learned" (Britzman 1998, p. 68).

Identity, for Lacan (1977a, b), is not constituted by consciousness. Rather, conscious subjectivity is fraught and precarious. Unconscious processes will always interfere with conscious intentionality and experience (Britzman 1998). Clearly then, the student is not a coherent rational self, her own source of meaning, knowledge, and action. The central issue here is that if the student's consciousness is not the strategic organiser of her intentions or her experience of mathematics pedagogy, then it is not very helpful to differentiate the "cognitive" aspects from the "social" aspects of teaching and learning. The meanings of mathematics that the student produces are often beyond the reach of consciousness. They involve relationships and experiences that are not in any way straightforward, but are rather, "mediated by multiple historical and contemporary factors, including social, schooling and psychodynamic relations" (McLeod and Yates 2006, p. 38). Desires, hopes and anxieties become highly influential.

Claiming that identity is never completely constituted, is to claim that identification will never be reducible to it. Specifically, students' category distinctions do not have the full measure of the identifications laid upon them. Categories such as "working class," "elite," "African American", "Māori," "high achiever," "mathematically challenged," are all fluid. If there are unrealisable aspects inherent in the discursive constitution of identity, it is important to consider those aspects. If the self is not the abstract universal with "human" attributes or rights, nor a fundamentally passive, "marginalised," or "excluded" human in need of advocacy, then, in the Lacanian assessment, ethical action as a theoretical construction is neither resourced by strategic essentialism nor organised around the category of the "other." For Lacan, ethical deliberation is always relative to a particular situation. What needs to be attended to is the set of relationships implicated, and specifically, the complex network of relations ethical action sustains.

Lacan develops a psychoanalytic approach for understanding the gap at the heart of identity and offers a treatment for examining the way in which pedagogical processes are lived out by the individual student (Britzman 1998; Ellsworth 1997). He grounds his development in what he calls the Symbolic and the Imaginary registers. The Symbolic identification (constitutive identification) allows the student to assume the place from where she is being observed, from where she looks at herself as likeable, and worthy of being liked (Žižek 1989). In particular, the student's social and cultural determinants foreground a particular subjective position within the classroom and, as a consequence, the identity that she is led to endorse is often not chosen through rational deliberation. For all their power in guiding actions and thinking, symbolic identifications are merely the product of discursive dissemination—they are simply stories that exist in social spaces.

Lacan (1977a, b) would maintain that it is through the unconscious that the student comes to understand these stories and the network of symbolic social relations that structures what she can and cannot do, say, or think. Through the unconscious, she can explain how she is positioned in a cultural network of the Big Other in which some relations are sanctioned and other relations are prohibited. If the Symbolic represents the *constitutive* identification, the Imaginary represents the *constituted* identification through which the subject identifies herself with the image that represents what she would like to be. Its focus is on images with which she chooses to identify. Arguably, the Imaginary and the Symbolic registers of identity are responsible for processing different sets of "data"—the Symbolic (words, laws, numbers and letters), and the Imaginary (visual-spatial images as well as illusions of self and world)—yet both function interdependently, working together to inform the subject's experience of self-in-mathematics and sense of self-as-learner.

What needs to be emphasised here is that between the identifications the student has of herself, and "others" (specifically the teacher) have of the student, there will always be a divide. There is always a trace of mis-recognition that arises from the difference between how one party perceives itself and how the other party perceives it. Teachers want to provide quality mathematical experiences. Indeed, they are bound by statutory obligation to enhance students' outcomes. Students, for their part, have developed an understanding about what it means to be a learner in the classroom. Both teacher and student, independently, "dream up" characteristics that designate a mathematical identity for the ideal student. That is not to say that the student will necessarily perceive herself as fitting that designation. Indeed, between the understandings and taking into account the meanings presupposed by the teacher, lies a fundamental mismatch. Žižek (1989) puts it this way: The subject "put(s) his identity outside himself, so to speak, into the image of his double" (p. 104).

These kinds of speculations about teaching/learning sit uneasily with psychology's liberal-humanist discourse. Within the discourses of liberal humanism within education, particular identities are ascribed to students on the basis of social categories such as socioeconomic status, gender, and ethnicity. These identities, as Stentoft and Valero (2009) have noted, are used to predict achievement and student disposition to the extent that "it becomes almost impossible to think about these students outside of or beyond these categories" (p. 57).

Psychoanalytic work in mathematics education attempts to get around the problem of categorization. It does this by illuminating "the relationality of the teaching/learning encounter" (Appelbaum 2008, p. 52). It is a relationship in which the boundaries between "inside/outside," "mind/social" become blurred. In that respect, the student's construction of herself as a learner in the classroom is highly dependent on the teacher's image of the student as a learner. In some teachers' eyes, some students can "never be good enough." Teaching and learning become sites of tension between differential positions of knowing: the meanings that a student produces of herself as a mathematics learner and those that the teacher produces of the student. It is a recursive and uneven process. It is also never completed.

In the Lacanian tradition, reconciling identities involves engaging, confronting, making decisions, and resolving conflicts between Symbolic and Imaginary images. That is to say that the meanings that a student produces of herself as, on the one hand, a proficient and positively disposed mathematics learner, or, on the other hand, as in some way deficient, together with the meanings that the teacher produces of the student, are the result of political struggles involving personal, psychic and emotional investments. The key point to be taken from an engagement with Lacan's theorising, is that by minimising those tensions and by tracking the relationship, a productive approach to an open-ended socially just and quality mathematical experience might be articulated.

Minimising those tensions is an ethical obligation on the part of the teacher. It involves helping students to deconstruct the wider social discourses of power and the categorisations that position students. It also, as Britzman (1998) has argued, demands attention to knowing the self. It requires a hard look, with the students in mind, at the multiple exigencies demanded of pedagogical practice: planning, creation and organisation of classroom community, tasks and activities selection and use, management of classroom discourse, the cognitive structure and feedback provided to students, the kinds of questions asked, the depth of on-the-spot reflection and action, and the assessment measures employed. It demands an attention to how one's own "otherness" is characterised by enacting all of these practices and how that otherness predisposes one towards habituated thought and a "blindness" towards others; how it circumscribes the capacity to think about, understand, and be ethically responsive to students in the classroom. A pedagogical project like this, unsettling though it may be, is, however, a form of praxis dedicated to producing long-term change with respect to teaching and learning mathematics. In the larger order of things, the project makes a valuable contribution not only to the political imagination of teachers and students, but also to the advancement of an ethical sociality.

Conclusion

Recently, mathematics educators who have embraced social theories have worked diligently to map out what constitutes an equitable and quality mathematical experience. Cobb and Hodge (2007), for example, have moved beyond the traditional approaches and have done so by sketching out the importance of identity to improving teaching and learning of central mathematical ideas. Their approach does not simply overlay identity with social processes and practice, but directly *connects* it with such practices. Their conceptual tools provide clarity and definition, particularly to "mathematics educators whose primary concern is to contribute to the improvement of classroom processes of learning and teaching" (Cobb et al. 2009, p. 43). The tools offer a means to deal with the interplay between social practices and the processes of self-formation that are at work in the mathematics classroom. However, their approach rests on a presumption of a universal human subject. As a result, all ethical deliberation and action is reduced to questions of human rights and humanitarian actions.

The concept of identity at the heart of a quality and equitable mathematics experience is an extremely complex phenomenon that is not easily explained by conventional democratic mechanisms. The notion of identity as offered by Cobb and Hodge, grounded in the "colloquial meaning of *identifying*, namely, to associate or affiliate oneself closely with a person or group" (Cobb et al. 2009, p. 40) cannot provide a totally compelling perspective of the dynamics of identity within a self/social relation "collectively shaped even if individually told" (Sfard and Prusak 2005, p. 17), and, therefore, subject to change. The concept, as formulated by Cobb and Hodge, sustains the hold of psychological ways of thinking within mathematics education, failing to theorise adequately the complex ways that disadvantage and privilege work in shaping a quality mathematical experience.

Identity is, in Lacan's estimation, generated by the structural discursive rules that govern thought, action and speech. For all the apparently "tractable and relatively concrete" (Cobb et al. 2009, p. 40) appearance of students in mathematics, they are all merely productions of practices through which they are subjected. Self-conscious identifications and self-identity are not simple, given, presumed essences that naturally unfold but, rather, are produced in an ongoing process through a range of influences, practices, experiences and relations, some of which operate beyond consciousness. They are constructed in an often contradictory space. Teachers who provide a quality and equitable experience embrace the contradiction that lies in that space. They recognise that people have different histories and different "presents." Such teachers do more than respectfully understand difference; they preserve the difference of the student by suspending their own intentionality. What emerges is a creative tension that is central to the development of a positive identification with mathematics.

I would argue for a psychoanalytic approach to explaining a quality and equitable experience in mathematics, specifically because it is able to offer a conceptual apparatus for exploring intrapsychic processes that are at work in the constitution of identity, and hence in teaching and learning. Engagement with and resolving conflicts between the contents of the two registers of identification (Imaginary and Symbolic) is precisely what gets to the heart of teaching and learning. Committed to political mobilisation, the approach offers a way in which mathematics education could develop genuinely inclusive democratic provision. In advocating for this approach, the overarching aim has been to stimulate reflection and discussion that will act as a catalyst for not merely changing how teachers view their world but to offer an interpretation that might change the texture of mathematics pedagogy itself. What mathematics pedagogy might become stands as the political, ethical, social, and philosophical problem for mathematics education today.

References

Alton-Lee, A. (2003). *Quality teaching for diverse students in schooling: Best evidence synthesis*. Wellington: Ministry of Education.

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES]*. Wellington: Ministry of Education.

Appelbaum, P. (2008). *Embracing mathematics: On becoming a teacher and changing with mathematics*. New York: Routledge.

Banks, J. (2007). Series foreword. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. vii–x). New York: Teachers College Press.

Bibby, T. (2009). How do pedagogic practices impact on learner identities in mathematics? A psychoanalytically framed response. In L. Black, H. Mendick, & Y. Solomon (Eds.), *Mathematical relationships in education: Identities and participation* (pp. 123–135). New York: Routledge.

Britzman, D. (1998). *Lost subjects, contested objects: Toward a psychoanalytic inquiry of learning*. Albany: State University of New York Press.

Brown, T. (Ed.). (2008). *The psychology of mathematics education: A psychoanalytic displacement*. Rotterdam: Sense Publishers.

Cabral, T. (2004). Affect and cognition in pedagogical transference: A Lacanian perspective. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 141–158). Greenwich: Information Age.

Cobb, P., & Hodge, L. (2007). Culture, identity, and equity in the mathematics classroom. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 159–172). New York: Teachers College Press.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education, 40*(1), 40–68.

Ellsworth, E. (1997). *Teaching positions*. New York: Teachers College Press.

Ernest, P., Greer, B., & Sriraman, B. (2009). *Critical issues in mathematics education*. Greenwich: Information Age.

Fleener, J. (2004). Why mathematics? Insights from poststructural topologies. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 201–218). Greenwich: Information Age.

Foucault, M. (1972). *The archaeology of knowledge and the discourse of language* (trans: Sheridan Smith A.). London: Pantheon.

Gee, J. (1999). New people in new worlds: Networks, the new capitalism and schools. In B. Cope & M. Kalantzis (Eds.), *Multiliteracies: Literacy learning and the design of social futures* (pp. 43–68). London: Routledge.

Gee, J. P. (2001). Identity as an analytic lens for research in education. *Review of Research in Education, 25*, 99–125.

Gee, J. P. (2004). Discourse analysis: What makes it critical? In R. Rogers (Ed.), *An introduction to critical discourse analysis in education* (pp. 19–59). Mahwah: Lawrence Erlbaum.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Towards a pedagogy for social justice*. New York: Routledge.

Hardy, T. (2004). There's no hiding place: Foucault's notion of normalization at work in a mathematics lesson. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 103–120). Greenwich: Information Age.

Hayes, D., Mills, M., Christie, P., & Lingard, B. (2005). *Schools and teachers making a difference? Productive pedagogies and assessment*. Sydney: Allen & Unwin.

Lacan, J. (1977a). *Ecrits: A selection*. London: Tavistock.

Lacan, J. (1977b). *The four fundamental concepts of psycho-analysis*. London: The Hogarth Press.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life*. Cambridge: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

McLeod, J., & Yates, L. (2006). *Making modern lives: Subjectivity, schooling, and social change*. New York: State University of New York Press.

Nasir, N., & Cobb, P. (Eds.). (2007). Introduction. In *Improving access to mathematics: Diversity and equity in the classroom* (pp. 1–9). New York: Teachers College Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: NCTM.

National Research Council. (2001). *Adding it up: Helping children learn mathematics* (Report prepared by the Mathematics Learning Study Committee). Washington: National Academy Press.

Normore, A. H., & Blanco, R. I. (2008). Leadership for social justice and morality: Collaborative partnerships, school-linked services, and the plight of the poor. In A. H. Normore (Ed.),

Leadership for social justice: Promoting equity and excellence through inquiry and reflective practice (pp. 215–240). Charlotte: Information Age.
Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social contexts*. New York: Oxford University Press.
Roth, W.-M. (2004). Identity as dialectic: Re/making self in urban school. *Mind, Culture, and Activity, 11*(1), 48–60.
Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22.
Stentoft, D., & Valero, P. (2009). Identities-in-action: Exploring the fragility of discourse and identity in learning mathematics. *Nordic Studies in Mathematics Education, 14*(3), 55–77.
Stinson, D. (2006). African American male adolescents, schooling (and mathematics): Deficiency, rejection, and achievement. *Review of Educational Research, 76*, 477–506.
Valero, P., & Zevenbergen, R. (Eds.). (2004). *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology.* Dordrecht: Kluwer Academic Publishers.
Valsiner, J. (1987). *Culture and the development of children's action: A cultural-historical theory of developmental psychology.* Chichester: Wiley.
Walshaw, M. (2001). A Foucauldian gaze on gender research: What do you do when confronted with the tunnel at the end of the light? *Journal for Research in Mathematics Education, 32*(5), 471–492.
Walshaw, M. (Ed.). (2004). *Mathematics education within the postmodern*. Greenwich: Information Age.
Walshaw, M., & Anthony, G. (2008). The role of pedagogy in classroom discourse: A review of recent research into mathematics. *Review of Educational Research, 78*(3), 516–551.
Walshaw, M., Ding, L., & Anthony, G. (2009). Enhancing mathematical identities at the expense of mathematical proficiency? Lessons from a New Zealand classroom. *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 313–320). Thessaloniki: PME.
Wenger, E. (1998). *Communities of practice: Learning, meaning and identity.* Cambridge: Cambridge University Press.
Wertsch, J. V. (1991). *Voices of the mind: A sociocultural approach to mediated action*. Cambridge: Cambridge University Press.
Žižek, S. (1989). *The sublime object of ideology.* London: Verso.

Chapter 8
A Framework for Evaluating Quality and Equity in Post-Compulsory Mathematics Education

Gail E. FitzSimons

Introduction

The aim of this chapter is to address the issues of quality and equity in mathematics education across the various sectors of formal education by reviewing the literature on quality in education in the post-compulsory years and to design a framework that will inform decision-making in relation to these. The framework is intended to be flexible and to encourage policy makers, educators, and practitioners at all levels to consider whether their mathematics curriculum and teaching effectively meets the needs of all their learners in their particular contexts.

For the purposes of this chapter, I take the concept of equity as access to and successful (as defined by the learner at least) participation in mathematics education set in the context of the economic, social, cultural, and political conditions of the time and place. This entails the recognition of, respect for, and valuing of the diversity of learners. Or, in D'Ambrosio's (2009) words, respect for human cultural dignity. In terms of mathematics education, this means problematising the curriculum and pedagogy rather than the individual learners or particular identified subgroups of learners.

This chapter will critically review the literature on quality in post-compulsory education in both vocational and higher education sectors with a focus on equity. It will conclude with a proposed framework to enable mathematics educators, individually or collectively, to evaluate their current situations within their larger contexts up to global levels, and then to move forward with strategies for resolving the tensions and contradictions as part of an ongoing process.

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Quality in Post-Compulsory Education

The Vocational Education and Training Sector

The European Centre for the Development of Vocational Training [Cedefop] commissioned a study to develop indicators of quality on the level of VET systems in Europe (Seyfried 2007) with the intention of supporting a European strategy for improving quality in VET. It collected and analysed over 200 existing indicators.

> In operational terms, indicators produce information which helps relevant VET actors to assess the extent to which their pre-defined objectives have been met, identify influential factors and take informed decisions …indicators often contribute to a common understanding of relevant criteria for quality. …Since an indicator is not a value in itself, defining and selecting indicators presupposes clarification of the objectives to be attained in order to improve quality. (p. 9)

The policy priorities set by Member States, the European Commission, and the social partners were as follows:

- Better employability of the labour force
- Better match between training supply and demand
- Better access to vocational training, in particular for vulnerable groups on the labour market (p. 9)

These priorities were broken down into more concrete measurable objectives, which were then related to indicators which had to be related not only to the objectives but also to one another. In this study, vulnerable groups clearly defined at a European level included:

- Early school leavers (drop-outs)
- Young unemployed people (less than 25 years)
- Long-term unemployed people (more than one year)
- Older people (over 55 years of age)
- Handicapped people (according to national definitions) (p. 47)

Other groups included migrants from non-EU countries and ethnic minorities. The question arises: What contribution might a different quality and quantity of mathematics education have made in avoiding or alleviating the situations in which people end up being labelled as “vulnerable”? In the following, I will discuss selected policy objectives for vocational education from a critical mathematics education perspective.

The study listed proposals for indicators for each objective of the three policy priorities. Context indicators applicable to all policy priorities included structural indicators of economic growth, employment, unemployment, and expenditure on VET per capita. These are clearly important factors influencing the education system overall—especially at the post-compulsory level—but ones about which little can be done at the regional or local level, particularly with regard to mathematics education. On the other hand, the policy priority of employability had the objective of preparing learners with the following competences, among others:

- Basic skills in literacy and numeracy meeting the requirements of the recognised demand
- Basic social skills meeting the requirements of the recognised demand
- ICT skills (p. 85)

The most interesting point here is that the conception of "basic skills in numeracy" is almost always taken for granted in documents such as these—as if there is a well understood and shared agreement about what this means in practice—when this is far from the actual case. The kinds of mathematical skills that workers of the future will need to use and to develop on-the-job are a long way from simplistic lists of tasks found in primary or even early secondary schooling (FitzSimons 2008a). Mathematical skills and techniques are clearly of value in using certain work-related technologies, such as spreadsheets and databases or graphics packages (Hoyles et al. 2002). There is also a need for a mathematics curriculum and pedagogy which links social and ICT skills with mathematics, especially but not only via the medium of communication technologies (FitzSimons, 2010) to support not only the learning of mathematics but also participation in the world of work and personal activities beyond formal education.

Considering the stated policy objective of completion of VET/avoidance of dropping out (p. 85), one indicator suggested is the existence of an active policy. The subject of mathematics has been long recognised by adult and vocational teachers as a significant factor contributing to school or post-compulsory students deciding to drop out of a course or in preventing them from achieving completion. I believe that this objective needs to be made more prominent specifically in relation to mathematics education, proactively, at all levels. It is not enough to simply monitor attrition rates after mathematics education is no longer compulsory since this may be years after the point where students ceased to be engaged.

The policy objective concerning transition to employment considers as an indicator, inter alia, the effectiveness of transition between formal and non-formal learning and the labour market (p. 86). Here, the distinction between different types of learning is made prominent, and this is particularly pertinent to mathematics education. It is commonly presumed that mathematics can really only be learned in formal education settings, whereas in fact its ideas and techniques can be developed through social interaction from birth, and at the workplace (paid or unpaid), in particular. In other words, both *what* mathematics is learned and *how* it is learned outside of institutional settings need to be valued. Recognition and valuing of the informal ways that mathematics can be learned and alternative culturally validated mathematical knowledges are essential to prepare current students at all levels for an unknown and unknowable future. They need to feel that what they bring to the learning situation through their own life experience is of value (see also Mellin-Olsen 1987).

The policy objective of quality of employment (stability, income, desired working time) and finding employment in new sectors (p. 86) underlines the fact that mathematics education has a role to play in supporting learners to pursue and, ideally, achieve their career goals. The text also includes a statement related to the existence of "active policies to train unemployed people from the old sector to enter a new sector, e.g. ICT" (p. 86). The nexus between mathematics and technology was briefly discussed above (see also FitzSimons 2007, 2010). Also important is the fact that school leavers and graduates of today will almost certainly have to retrain in as yet undeveloped or unknown fields before they reach retirement age. Learning how to learn, especially in mathematics, is a critical skill which needs to be developed in the compulsory years of schooling together with the disposition to learn mathematics. As noted earlier, there are many adults, young and old, who would never willingly choose to study mathematics again. This generalised experience needs to be addressed in current and future schooling, starting from the early years where, in my experience, learning can often be joyful, and not letting out of sight this engagement, cultural relevance, and ownership of the knowledges and skills until the end of compulsory education at least (FitzSimons 2008b, suggests possible activities of relevance to students' personal cultures). As Ernest (2009) proposes, the mathematical content of curricula for the large majority of students not destined for advanced academic mathematics study may need to be revised.

Under another objective: responsiveness (reaction to knowledge about recognised demand) (p. 87), one of the criteria is "Utilisation of acquired skills at the workplace, from the perspective both of the employer and the employee." Note the reciprocity between employer and employee perspectives. The following objective of "broadening access [to] everyone who can benefit" (p. 88) sums up the necessity of physical, intellectual, social, and cultural inclusion in mathematics education. In other words, the diversity of learners in formal education should not be the problem, but rather a valuable and valued resource. In my own practice of vocational and tertiary teaching, this meant learning about what students bring to the class from their wide range of backgrounds as citizens and workers. My willingness to learn has been richly rewarded by the wealth of examples drawn from actual lived experience, and this has often been the source of seriously engaged discussion and debate among class members, either in support of or in contrast to the underpinning theory of the lesson concerned. The policy objectives conclude with: "opportunities for vulnerable groups," for example, indicators of the "existence of special offers (outreach activities, guidance, orientation, motivation, courses, qualifications, competences)" (p. 87) and "permeability" which pays attention to group-specific dropout rates through active policies to monitor and to reduce them. Although the range of outreach activities, guidance, orientation, and motivational activities are offered to some mathematics students in different places around the globe, which of these are made specifically available to the most vulnerable of students? (See Wedege and Skott 2006, for a school mathematics competition with a focus on inclusiveness, intended to challenge and to motivate all students.)

Quality Assurance (QA) in Higher Education

In this section, I attempt to clarify the often complex terminology associated with quality and then draw upon critiques of traditional context-free models of QA.

Stensaker (2008) offered a brief summary of the four main trends of QA in the Higher Education sector:

1. Power: QA processes support the development of a stronger institutional leadership; it also triggers discussions and debates about institutional identity. Although individual academics may have lost power, QA offers a more legitimate role for students and other stakeholders.
2. Professionalisation: There has been an increase in bureaucracy but also in the possibilities of collegiality and co-operation. Formalisation of QA processes may also remove some of the mysteries surrounding them.
3. Public Relations: The outcomes of QA processes may be used as a marketing and branding tool; they also lead to a greater emphasis on teaching and learning outcomes.
4. Permeability: Ranking and performance indicators increase visibility and offer more informed decision-making processes.

Considering Stensaker's (2008) first point, in terms of mathematics education across the non-university sectors, it seems that students have been largely if not totally excluded as stakeholders, except for others presuming to speak on their behalf. When have students' opinions on the mathematics education they receive been solicited, taken seriously, and acted upon (cf. Mellin-Olsen 1987)? Making quality criteria explicit, even open to debate, together with an expectation of collegiality and co-operation should, in theory at least, help the cause of professionalisation if mathematics teachers feel that they are indeed part of the process and not helpless and unwilling participants (even scapegoats). The emphasis on mathematics teaching and learning outcomes needs careful treatment—to ensure avoidance of increasingly ubiquitous externally motivated and selective or culturally inappropriate state-mandated or international testing regimes (e.g., Civil and Quintos 2009; Jablonka 2003). On the other hand, publicity of those mathematics programs judged by the local community to be socially and culturally valued and successful in fulfilling student and community needs is to be welcomed. In particular, the local mathematics teaching community should be positively encouraged and acknowledged by the education institution and the larger community for collegial sharing and critique of efforts made on behalf of all stakeholders including, at the forefront, students.

Critique of QA in Higher Education According to Stensaker (2008) the dominant perspective of QA is a focus on implementation of pre-defined criteria; involvement supports an external agenda and not the perceived problems of those working in institutions. There is a simplistic view of the problem definition and agreement on aims and objectives of policies to address the problem. There is a strong focus on design issues, with domination of technicalities; measuring outcomes conceived mostly as a methodological problem. This is in the face of growing evidence of shortcomings and misconceptions: Quality issues are multifaceted and there is little agreement on aims and objectives of policies. Change is not only conditioned by control but also by trust and dialogue.

Stensaker (2008) proposes that an alternative perspective is to critically review assumptions. The context must be taken seriously; cause and effect are sometimes difficult to separate; there is a need to move beyond quantification. The benefits of qualitative approaches include a focus on creating meaning, and he recommends a narrative approach, although issues of power must still be addressed. He continues that there is "a need to consider non-traditional ways and means of achieving quality and to show a willingness to abandon standardised schemes for innovative practices…" (p. 9). He gave an example of recognition of high quality paramedic work where 90% of experienced practitioners could discriminate between an expert and novice trainees, while only 50% of students and 30% of their teachers could do so. Stensaker concludes that it is insufficient to evaluate quality by solely focusing on rules, systems, and procedures, thus possibly failing to address issues related to excellence, innovation, and renewal.

Houston (2007), in his critique of the adoption of Total Quality Management (TQM) in higher education, adopts a critical systems perspective which he argues "help to deal with the wholeness, interconnectedness and emergent properties of complex situations" (p. 4). This approach requires critical questioning of taken-for-granted assumptions and prescriptive approaches to problem solving—TQM is often taken at face value as a "good" thing without consideration of its fitness for purpose within the particular problem context of higher education. Although originally designed with a focus on manufactured products and the productive process, even Deming (1993, quoted in Houston 2007, p. 5) notes that the choice of aim or purpose should be a matter for clarification of values, should never be defined in terms of activity or methods, and should relate to improvement in life for all concerned. There have been different kinds of critique of TQM in higher education, including from pragmatic, prescriptive, and philosophical perspectives, and of its use in various political agendas. However, in relation to who decides what quality is and against which criteria, the concept of customer is problematic: student, government, and industry all have different boundaries to their value judgements in this sphere, with complex positionings and ambiguous relations to the concept of customer focus. There are

> unresolved tensions between the dual purposes of control, and of quality improvement and learning, …and between the rhetoric and reality of quality…. Harvey (2002) has noted that audit processes pay little attention to educational processes, educational theory or student learning. (p. 10)

Houston (2007) notes that the National Institute of Standards and Technology (NIST[1]) had also queried the assumptions of applying a competitive business model to education, thus downplaying organisational uniqueness and emergent characteristics and encouraging inappropriate comparisons—especially where fundamental values systems are so different. Externally developed frameworks, with assumed advantages and purposes, "should not be acceptable in universities with their fundamental roles of critical evaluation and higher learning" (p. 11). "The purpose of

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[1] See http://www.baldrige.nist.gov/Education_Criteria.htm

higher education, rather than conformity, should be to promote diversity: to extend each student towards realising their own individual potential" (p. 11). These processes are contested and value-bound. Recognising that recent years have seen the creation of quasi-markets in higher education, Houston maintains that the "primary purpose is to contribute to society in specified ways, including contributing to the economy" (p. 12). A similar argument could be made for mathematics. Clearly, concludes Houston, the language, concepts, and tools of TQM do not really match the substance of higher education. Nor indeed do they match a mathematics education which has equity as its priority.

A Systems Thinking Approach

From an Australian perspective, mathematics educator Galbraith (1999) recommended adopting a systems thinking approach to QA within the university sector, with the goals of nurturing a valued but endangered species and attending to the good of the whole rather than opportunistic individual parts. He argued that university management should move forward from a strict adherence to a corporate framework, with its dynamics that lead to a dissociation of causes from effects, to locate itself within an ecological world view. This is consistent with Buchanan's (2006) approach towards a skill ecosystem, which considers workforce skills and knowledge in an industry or region as a complex whole. Houston (2007) develops Galbraith's idea: "Authentic quality theory is essentially systemic; attending to values, purpose, and optimising performance relative to the aim of the system" (p. 13). He recommends exploring the development of locally appropriate systemic approaches to improving quality in and of higher education. Such a perspective would accommodate "the variability of students, adaptability and flexibility of processes, the interactions of components and expectations about the final outcome: learning" (p. 14).

It is essential to keep in mind that universities, as with all education institutions, are workplaces in themselves—it is much more common to consider them as places where other people (i.e., learners) are prepared for the workplace. Adopting a systems approach, the NIST (2009) offers a framework connecting and integrating seven categories of "Education Criteria for Performance Excellence" (p. 1) in a systems perspective (see p. iv for model). The categories are: (a) leadership; (b) strategic planning; (c) customer focus; (d) measurement, analysis, and knowledge, management; (e) workforce focus; (f) process management; and (g) results. From the perspective of this chapter, their customer focus and workforce focus in particular raise serious issues for consideration. These will be addressed further below.

A systems thinking approach to quality in the workplace has striking parallels with intended outcomes of mathematics education. As noted in FitzSimons (2002), an Australian government report (NBEET/ESC 1996), adopted a human capital development perspective, spanning a lifetime and across multiple career paths. Along with team work, communication, and continuous learning at both the organisational and the individual level, they observed that the competency of information literacy is:

> a literacy that combines information collection and analysis and management skills and systems thinking and meta-cognition skills with the ability to use information technology to express and enhance those skills. In a society of information 'glut' the ability to detect 'signal' from 'noise' will become increasingly valued. (p. 74)

There are clear implications for mathematics education in supporting the development of these kinds of skills. Drawing on the work of Salner (1986, cited in NBEET/ESC 1996), systems thinking was described as context-oriented and context-dependent, involving the following competencies:

- the ability to see parts/wholes in relationship to each other and to work dialectically with the relationship to clarify both similarities and differences. In effect, this means the ability to balance the processes of both analysis and synthesis;
- the ability to abstract complexity so that organising structures (visual, mathematical, conceptual) are revealed rather than imposed;
- the ability to balance flexibility and real world change against the conceptual need for stable system boundaries and parameters;
- command of multiple methods for problem solving as opposed to employing a limited range of algorithms to the widest variety of situations; and
- awareness that the map is not the territory, and the ability to act accordingly in the use of systems models. (pp. 75–76)

Once again, these competencies can be readily identified with the projects of the institution of mathematics. At all education levels, how might appropriate mathematics education be developed to support competencies such as these in all learners, not just the most "advanced," along with a increasing sense of personal agency?

Towards a Quality Framework

From the literature reviewed above, there are complex issues to be addressed. For many, it may require a shift from consideration of the micro and meso levels of the classroom, school, and region to the macro levels of national and global levels, encompassing factors well beyond mathematics education. A business approach to quality which demands compliance with externally set agendas is inappropriate and unlikely to achieve significant improvement educationally. The European Union vocational education model was designed to assess the extent to which predetermined policy objectives agreed upon by member countries were met. Although the EU model raises issues of employment and access by vulnerable groups, both models come under the category of control. In this section I offer two frameworks for planning and/or evaluation. The first focuses on the macro level of setting the context for the education activity in terms of global or national conditions, then regional and, finally, local conditions. The second focuses on the meso and micro levels of the individual institution and the mathematics classroom or other learning setting. (See FitzSimons 2002, Chaps. 4–6, for development of these levels of knowledge production, distribution, and recontextualisation in vocational education following the work of Basil Bernstein.)

Planning and Evaluation at the Macro Level

In order to set the context, Kettunen's (2008) concept of a quality map as a tool to take into account the environmental context, including global, national, and regional dimensions, provides a useful starting point. Information should be collected following Dill and Soo's (2005) five principles of validity, comprehensiveness, relevance, comprehensibility, and functionality—bearing in mind the interests of the range of stakeholders internal and external to the institution. The information collected qualitatively and quantitatively could consider economic, environmental, social, cultural, and historic data, as these impinge upon the education system in general, and mathematics education, in particular. Specifically, following Ritsilä et al. (2008), information on the labour market and employment trends, socio-ecological development (human and environmental well-being), regional development, public social debate, and innovation (local, in the case of schools), combined with EU vocational policy objectives of, inter alia, employability, quality of employment, and access and opportunities for groups identified as vulnerable (Seyfried 2007). Having set the context, Kettunen (2008) proposed linking this to a comprehensive strategic quality plan incorporating feedback and continuous improvement.

A critical systems approach to QA was recommended by Houston (2007) (and NIST 2009, among others) in order to "help to deal with the wholeness, interconnectedness and emergent properties of complex situations" (p. 4). If nothing else, education is certainly a complex system, and mathematics education is a case in point. Houston reminds us that students, government, and industry all have different boundaries to their value judgements in this sphere, with complex positionings. Accordingly, any evaluation framework needs to take into account which stakeholder perspective(s) and whose values predominate. As far as equity in mathematics education is concerned, Houston's goal for universities of promoting diversity and extending each student "towards realising their own individual potential" (p. 11) holds true throughout the entire education system.

Houston (2007) underlines his belief that authentic quality activities should be essentially systemic, attend to values and purpose, and optimise performance "relative to the aim of the system" (p. 13). But what if the system's espoused and enacted purposes are contrary to the values of equity held at the local level? Houston's recommendation to explore the development of locally appropriate systemic approaches to improving quality in and of education is possible, even at the level of the individual teacher. From personal experience as a vocational mathematics teacher, it is always possible to find ways of enacting one's strongly held personal

Table 8.1 A proposed framework for establishing contexts at the macro level for evaluating mathematics education agendas with a focus on equity

	National/global	Regional	Local
Socio-economic issues – Labour market – Access to education	• Global financial crisis • Automation at all skill levels redefining work • Availability of appropriate and affordable post-compulsory education	• Downturns in manufacturing and primary industries • Chronic youth unemployment	• Skills shortages in health and aged care
Socio-ecological issues – Community health and well-being	• AIDS • Mental health • Obesity • Malnourishment • Smoking and disease		
Environmental issues – Global warming – Soil and water conservation	• Extreme climate events • Lack of appropriate soil and water for subsistence and industrial uses • Loss of biodiversity		
Public social debate issues – Drug and alcohol abuse – Gambling	• Adolescent substance abuse • Government-sanctioned gambling • Rise of domestic and public violence		
Innovation issues – Partnerships with local industry – New educational uses for IT	• Creative educational uses for mobile [cell] phones • Evolving uses of internet	• Regional and local partnerships between education and industry, including apprenticeships, internships, professional practicums, taster days for school leavers, etc.	

beliefs and values—ones supported by high quality peer-reviewed research—even under a rigid TQM system and surrounded by colleagues entrenched in the most traditional attitudes and beliefs about teaching and learning (and even transferring these to online courses!).

In terms of setting the context, it may be useful to prepare a broad framework according to the criteria nominated as being of importance from an equity perspective, depending on the "baskets" of criteria selected. The ones indicated in Table 8.1 are merely examples, and other users would ascertain their own priorities.

Planning and Evaluation at the Meso and Micro Levels

This section focuses on the specific education institution and the particular learning setting which may encompass formal classroom instruction, online delivery, and interactive technology-supported participation, workshop or other practical activities, or some combination of these.

Activity theory offers a foundation which stresses the importance of inter- and intra-personal communication in the development and transformation of culture, and social practice. It is discussed in depth by Engeström (1987), and from a technology perspective by Kuutti (1996) who synthesised the work of Leont'ev and Engeström to devise a two-dimensional framework for analysing the use of technology from an activity theoretical perspective. One dimension incorporates Leont'ev's three hierarchical levels of unconscious *operations*, goal-directed *actions*, and collective *activity* with an overarching motive; the second dimension incorporates Engeström's (1987) six components of the mediational triangle: subject, object, and mediating artefacts, all set in the socio-cultural context of rules, community, and division of labour—also elaborated in Engeström (2001). Mellin-Olsen (1987) was among the earliest to have built on an activity theory foundation in relation to mathematics education. (For further information on activity theory at the post-compulsory level see my web page for *Adult Numeracy and New Learning Technologies,* http://www.education.monash.edu.au/research/projects/adult-numeracy/, an Australian Research Council Post-Doctoral Fellowship Research Project [2003–2006].)

Adapting Engeström's (2001) criteria (illustrated in Table 8.2) interrogates: (a) who are learning, (b) what do they learn, (c) why do they learn, and (d) how do they learn. These questions apply not only to students, but also—in relevant contexts—to teachers, managers, parents, employers, the larger community, and policy makers as stakeholders in the education domain. Engeström elaborated five principles to summarise activity theory (the activity system as a unit of analysis, multi-voicedness, historicity, contradictions, and expansive learning) and cross-tabulated these with the four questions above which he described as central to any theory of learning (see FitzSimons 2003, for elaboration from an adult mathematics educator's perspective). In practice, working with this matrix with vocational education teachers and trainers, I have found it easier to use alternative heading descriptors of: (a) who are we really talking about, who is involved in this program? (b) what do learners bring to class, what else do they do in life? (c) what is known of the teaching and learning history of the program and its learners? (d) what are the tensions and contradictions? and (e) what can be done to move forward? (Apologies to Yrjö Engeström!) Critically, in responding to these five prompts, educators at all levels should imagine that the study of mathematics is not compulsory for the students concerned or that the students have the hypothetical freedom to attend an alternative institution for their mathematics education, even to learn via the internet instead.

In Table 8.2, adapted from Engeström (2001), I offer an illustration of how the teaching-learning situation of an individual program for adults returning to study mathematics may be mapped for purposes of description and analysis. This is in order to provide a starting point for evaluation of the current situation and to provide the basis for future planning.

This example, based on one class of learners, can be systematically developed to encompass increasingly complex layers; for example, all relevant mathematics classes: (a) in a school or institution at a given year level or program level, (b) at

Table 8.2 An adaptation of Engeström's (2001) matrix for planning and evaluating post-compulsory mathematics education at the meso and micro levels

	Who is actually involved in this program?	What do learners bring to class?	What is the teaching and learning history of program and learners?	Tensions and contradictions	Expansive learning (Moving forward)
Who are learning?	Adult women returning to study mathematics (aged 16 and over)	Wives, partners, mothers, daughters, carers, etc. Family business or other work experience Social and sporting association work Hobbies (e.g., cooking, dressmaking, cat/dog breeding)	Learners have expectations based upon the often negative experiences of traditional school instruction The program has been evolving based explicitly upon meeting learners' needs (in an era when this was possible and no predetermined curricula)	Neoliberal policies imposing increasing costs on adult learners	Learners develop the confidence in their own mathematical abilities towards leadership roles at home, in the school, and at work
Why do/might they learn?	Prove something to themselves Gain entry to further education (vocational or university) Gain access to new employment Find a new direction in life Help their children with homework	To understand and counter technologies of management used by employers—especially mathematical or statistical data To support their family business and social activities mathematically (e.g., banking, book-keeping)	Traditionally mathematics has been taken as a proxy for intelligence and a requirement for entry to many careers even if used inappropriately	Increasingly adult education viewed as Human Resource Development rather than individual growth or social cohesion focus	Learners are enabled mathematically to continue studies in vocational and higher education or in-house workplace training Learners are enabled to participate fully as citizens in spheres of their own choosing

Table 8.2 (continued)

	Who is actually involved in this program?	What do learners bring to class?	What is the teaching and learning history of program and learners?	Tensions and contradictions	Expansive learning (Moving forward)
What do/might they learn?	School mathematics/statistics up to year 12	To recognise and value their numeracy skills developed outside of school	Traditionally the teacher and text have been sole sources of authority in mathematics	Introduction and imposition of pre-determined competencies; increasingly audited	Learners conducting mathematical research into historical, social, environmental, cultural, etc. topics of interest and disseminating findings via posters etc.
How do/might they learn?	Meeting for 3 hours once a week in a small room of a converted house	To make sense of new knowledge by integrating it with knowledge gained from practical experience	Adult education incorporating innovative theories of teaching; mathematics education evolving to include problem solving and modelling, use of IT; constructivism	Limited material resources available to support adult education Not all teachers specifically qualified in mathematics/education Limited professional development in mathematics education	

a particular educational site or campus, (c) at a regional level, or (d) at a systemic level. Not only should the learners be the focus or unit of analysis, but also the mathematics teachers themselves as a community of practice. Just as the history of the development of the discipline of mathematics reflects its intimate connection with the social and natural world (FitzSimons 2002), so any evaluation could consider formal and informal linkages with other school subjects and the world beyond the institutional walls. It could involve stakeholders, such as the wider school community including non-teaching staff, volunteers, and parents, and the world of business and industry as but one important stakeholder (see, e.g., OECD Global Science Forum 2008). Keeping in mind the objectives of equity and participation, a democratic process would see teachers critically involved in an ongoing workshop process, drawing upon the input of important stakeholders, such as learners and the local community, business and industry.

If all students enrolled in mathematics or mathematics-containing subjects were treated with the same respect and dignity as adults returning to study mathematics in the post-compulsory sector, this may have a positive impact on equity. On the website for my project into adult numeracy (discussed briefly above) there are around 70 pages of research-based prompts questioning various aspects of mathematics education. In addition, from the 2009 to 2010 criteria for educational excellence (NIST 2009), there are questions which enable organisations to examine, among other things, how well they engage students, their workforce, and other stakeholders and to build an appropriate culture for doing so. Adapting this text specifically for mathematics education leads to the following pertinent questions:

1. "How do you engage students and stakeholders to serve their needs and build relationships?" (p. 13).
    (a) How do you identify and innovate mathematics programs and support services to meet the requirements and exceed the expectations of your students and other stakeholders? How do you attract and retain new students, and provide opportunities for expanding relationships with existing students and stakeholders as appropriate?
    (b) How do these vary for different students and stakeholders?
    (c) How do you build an organisational culture that ensures a consistently positive student and stakeholder experience and contributes to greater engagement?
2. "How do you obtain and use information from your students and stakeholders?" (p. 15).
    (a) How do you listen to your students and stakeholders to obtain actionable information and feedback on your mathematics programs and support services? How does this vary with different students and stakeholders at different stages? How do you follow up on these?
    (b) How do you listen to former students, potential students, and relevant stakeholders in relation to your mathematics programs and support services?
    (c) How do you manage student and stakeholder complaints in relation to these?

3. “How do you engage your workforce to achieve organizational and personal success?” (p. 19).
   (a) How do you determine the key factors that affect your mathematics education and relevant support staff? How are different factors affecting satisfaction for different groups determined?
   (b) How do you foster an organisational culture that is characterised by open communication, high quality work, and an engaged staff? How do you ensure that you benefit from the diverse ideas, cultures, and thinking of your total mathematics education workforce (teachers and support staff)?
   (c) How do you recognise, reward, and create incentives for high quality practice and engagement by this staff?
   (d) How do you address the learning and development needs of your mathematics education workforce?

Having mapped the broader context and then the detail at the particular level of interest in mathematics education, the next logical step is to prepare a strategy plan. Often the proposals implicit in Engeström’s (2001) expansive learning, incorporating moves towards resolving identified tensions and contradictions, require serious discussion and dialogue in an atmosphere of mutual respect. In what Engeström describes as relatively long cycles of qualitative transformations, questioning and deviation from established norms can escalate into a deliberate collective change effort or “a collaborative journey through the *zone of proximal development* of the activity” (Engeström, p. 137).

Conclusion

In this chapter, I have drawn extensively on the quality literature in the post-compulsory sector together with my own experience as a vocational mathematics practitioner and adult learning and development lecturer to propose a framework for evaluating mathematics programs and support services across the education sectors from a quality and equity perspective. I have suggested an approach to establish broad-scale contextual information appropriate to the location and situation of the user and an approach to a more detailed analysis of the actual teaching-learning situation. In the case of post-compulsory education where student participation is often voluntary and teachers’ employment is increasingly dependent upon student satisfaction, students’ motives and goals are given high priority. Of course, there are many other stakeholders whose concerns need to be addressed and this requires sensitive value judgements by the relevant decision-makers, keeping in mind quality and equity considerations.

Van Kemenade et al. (2008) raise the question: Is the quality focus on:

1. Providing society with the graduates that have the knowledge and skills society needs?

2. Exceeding the learning results that are asked for by the students and the world of work?
3. Transforming the students into citizens of the world?
4. Preparing the students to be leaders in their future society?

Ultimately, evaluating quality and equity is not about judging others and finding them wanting—as in the 'deficit' model of judging students, especially in the case of mathematics when used inappropriately as a sorting mechanism. It is about humility and respect, and listening to the voices of those most intimately concerned with the best interests of learners, such as committed teachers and parents, and working together to understand what is and how things might be different in ways that empower all learners.

References

Buchanan, J. (2006). *From 'skill shortages' to decent work: The role of better skill ecosystems.* Sydney: NSW Board of Vocational Education and Training.

Civil, M., & Quintos, B. (2009). Latina mothers' perceptions about the teaching and learning of mathematics: Implications for parental participation. In B. Greer, S. Mukhopadhyay, A. B. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 321–343). New York: Routledge.

D'Ambrosio, U. (2009). Foreword. In B. Greer, S. Mukhopadhyay, A. B. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. vii–xii). New York: Routledge.

Dill, D., & Soo, M. (2005). Academic quality, league tables, and public policy: A cross-national analysis of university ranking systems. *Higher Education, 49*(4), 495–533.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research.* Helsinki: Orienta-Konsultit.

Engeström, Y. (2001). Expansive learning at work: Toward an activity-theoretical reconceptualization. *Journal of Education and Work, 14*(1), 133–156.

Ernest, P. (2009). New philosophy of mathematics: Implications for mathematics education. In B. Greer, S. Mukhopadhyay, A. B. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 43–64). New York: Routledge.

FitzSimons, G. E. (2002). *What counts as mathematics? Technologies of power in adult and vocational education*. Dordrecht: Kluwer.

FitzSimons, G. E. (2003). Using Engeström's expansive learning framework to analyse a case study in adult mathematics education. *Literacy & Numeracy Studies: An International Journal of Education and Training of Adults, 12*(2), 47–63.

FitzSimons, G. E. (2007). Globalisation, technology, and the adult learner of mathematics. In B. Atweh, A. C. Barton, M. Borba, N. Gough, C. Keitel, C. Vistro-Yu, et al. (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 343–361). Dordrecht: Springer.

FitzSimons, G. E. (2008a). Mathematics and numeracy: Divergence and convergence in education and work. In C. H. Jørgensen & V. Aakrog (Eds.), *Convergence and divergence in education and work. Studies in vocational and continuing education series* (Vol. 6, pp. 197–217). Zürich: Peter Lang AG.

FitzSimons, G. E. (2008b). Mathematics and the personal cultures of students. In E. Barbin, N. Stehliková, & C. Tzanakis (Eds.), *History and epistemology in mathematics education: Pro-*

ceedings of the 5th European Summer University, ESU 5 (pp. 651–658). Plzeň: Vydavatelský servis.

FitzSimons, G. E. (2010). *Lifelong learning—Theoretical and practical perspectives on adult numeracy and vocational mathematics*. New York: Nova Science Publishers.

Galbraith, P. L. (1999). Systems thinking: A missing component in higher educational planning. *Higher Education Policy, 12*(2), 141–157.

Houston, D. (2007). TQM and higher education: A critical systems perspective on fitness for purpose. *Quality in Higher Education, 13*(1), 3–17.

Hoyles, C., Wolf, A., Molyneux-Hodson, S., & Kent, P. (2002). *Mathematical skills in the workplace. Final report to the Science, Technology and Mathematics Council.* London: Institute of Education, University of London: Science, Technology and Mathematics Council.

Jablonka, E. (2003). Mathematical literacy. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *2nd International handbook of mathematics education* (pp. 75–102). Dordrecht: Kluwer.

Kettunen, J. (2008). A conceptual framework to help evaluate the quality of institutional performance. *Quality Assurance in Education, 16*(4), 322–332.

Kuutti, K. (1996). Activity theory as a potential framework for human-computer interaction research. In B. A. Nardi (Ed.), *Context and consciousness: Activity theory and human-computer interaction* (pp. 17–44). Cambridge: The MIT Press.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: Reidel.

National Board of Employment, Education and Training/Employment and Skills Council (1996). *Education and technology convergence: A survey of technological infrastructure in education and the professional development and support of educators and trainers in information and communications technologies.* Commissioned Report No. 43. Canberra: Australian Government Publishing Service.

National Institute of Standards and Technology (2009). *2009–2010 education criteria for performance excellence*. The Baldrige National Quality Program at the National Institute of Standards and Technology. Gaithersburg: National Institute of Standards and Technology.

Organisation for Economic Co-operation and Development Global Science Forum (2008). *Report on mathematics in industry*. http://www.oecd.org/dataoecd/47/1/41019441.pdf.

Ritsilä, J., Nieminen, M., Sotarauta, M., & Lahtonen, J. (2008). Societal and economic engagement of Universities in Finland: An evaluation model. *Higher Education Management and Policy, 20*(2), 118–137.

Seyfried, E. (2007). *Indicators for quality in VET. To enhance European cooperation.* Cedefop Panorama series #134. Luxembourg: Office for Official Publications of the European Communities. http://www.trainingvillage.gr/etv/Upload/Information_resources/Bookshop/469/5167_en.pdf.

Stensaker, B. (2008). Outcomes of quality assurance: A discussion of knowledge, methodology and validity. *Quality in Higher Education, 14*(1), 3–13.

Van Kemenade, E., Pupius, M., & Hardjono, T. W. (2008). More value to defining quality. *Quality in Higher Education, 14*(2), 175–185.

Wedege, T., & Skott, J. (2006). *Changing views and practices? A study of the KappAbel mathematics competition.* Trondheim: Norwegian Center for Mathematics Education, Norwegian University of Science and Technology [NTNU].

Chapter 9
The Theoretical Landscape

Editor's Reaction to Part I

Walter G. Secada

Through much of the late twentieth century, notions of equity and quality in school mathematics were closely tied to interpreting the common finding of socio-demographic group-based differences in mathematics achievement. The often-virulent nature/nurture debates were grounded in psychological theories of individual difference (Jensen 1972); whereas sociological theories of social and cultural capital sought to explain inequalities in how society distributes its desiderata (Bowles and Gintis 2002). Explanatory theories that invoked constructs, such as the "culture of poverty" (Lewis 1965; Office of Policy, Planning and Research 1965; see Wilson's critique 2009) and even intervention studies, such as *Subtracting Bias, Multiplying Options* (Fennema et al. 1981) were grounded in one or another of these disciplines and thereby constrained by their discursive practices. Not surprisingly, work from the 1960s through 1980s is often criticized for "blaming the victim."

It was within this context—where scholarship involving equity was dominated by psychological and sociological theories that constrained the possibilities for those who worked in the field—that alternative conceptions for equity were developed in the mid-to-late 1980s by first asking the question: "What is equity?" Responses ranged from notions of justice tied to Aristotle's and legal notions of equity; through more critical neo-Marxist, feminist ideas, and multicultural critiques; and evolving socio-cultural perspectives on the nature of learning (for more on this work, read the edited collections by Fennema and Leder 1990; Secada 1989; Secada et al. 1995; Secada and Meyer 1989/1991). All sought to distance themselves from classical views of individual difference, equality/inequality, and socially enlightened self-interest (a foundational cornerstone for what is now referred to as neoliberal thought). The work coming from the 1980s may strike current readers as slightly naïve. My friend and colleague Elizabeth Fennema repeatedly told me: if she were starting today, she would not start with the Fennema and Sherman studies (1977). And I would agree.

As I read the chapters that constitute the first section of this book, I am heartened by the breadth of perspectives that are taken, by the many voices that are speak-

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ing, and by directions in which they seem to be taking as they answer the question: "What is equity?" From these chapters, I take away three theses that—were I starting today—would anchor my work:

1. Equity and quality are inherently political terms whose common political bedrock is obscured by their being taken for granted and by their being seen as normal.
2. Equity and quality have multi-vocal and nuanced meanings as found in everyday use and philosophically.
3. Scholarly inquiry about the nature of equity and of quality—either alone or linked—has taken a decidedly qualitative turn that is focused on textual deconstruction and/or interviews with key informants.

Equity and quality are inherently political terms whose common political bedrock is obscured by being taken for granted. In this section of the book, *Chronaki's* chapter argues that economic development is rife with imperialistic, patriarchal and colonialist discursive functions. She positions herself within feminist and post-colonial discourses to critique views of school mathematics that would reduce its concerns to highly technical concerns for things such as learning, narrowly construed.

Gutierrez and Dixon-Roman adopt what they call a social-political stance to mathematics education. Their goals are to destabilize schooling as the primary vehicle of education; to reject group-based "equality" in achievement outcomes; and question the central importance that mathematics seems to have acquired within Western and industrialized society.

Pais and Valero argue that mathematics education must be understood within a capitalist setting—calls for quality and equity within those settings serve particular interests. According to their critique, current theorizing about equity and quality seems narrowly technical and would be strengthened, were educators to seriously debate the purposes of schooling.

Llewellyn and Mendick also make explicit the neo-liberal uses of the terms in current-day England. They help us understand that much current writing conflates economic with personal growth; or national competitiveness with personal self-fulfillment.

It may seem obvious that equity and quality both share a common economic and political foundation in such a way as to link their fortunes inextricably to one another. Though most critiques in this volume locate current uses of the terms equity and quality within neo-liberal political and capitalistic economic settings, I would hazard the guess that writers living in societies that adhere to alternate political and economic systems would make similar claims.

I would further hazard the guess that none of the authors would deny the empirical finding that—as Lucy Sells so famously noted in 1973—mathematics is a critical filter in later-life economic opportunity (see also Sells 1976). Indeed, one could argue that without Sell's insight and rhetorical flourish, mathematics achievement differences would not have their present-day salience in the social policy arena such that an icon from the U.S. Civil Rights era, Bob Moses (Moses and Cobb 2001) would declare that algebra is the new civil right. Instead, what I take from the above arguments is that the existence of empirical ties between mathematics achievement in school and later-life opportunity should not be used to trump discussion as to the positioning of equity and quality within present-day political and economic settings. Rather, as we build back up from that positioning and ask ourselves why this is so, we are likely to encounter social, political, and economic interests and processes that shape how we frame our scholarship (in much the same way that psychology and sociology shaped the scholarship of the 1960s through 1980s) and that place limits on the empirical interventions that are considered reasonable or even possible. The intellectual foundations for such a program of scholarly inquiry are now firmly in place. The challenge is to embark on it.

Equity and quality have nuanced meanings in everyday use and philosophically. In a wide-reaching review of prior work, *Atweh* draws upon philosophical notions of ethics to deconstruct how quality and equity are portrayed within an Australian and the politically-dominant American settings. He argues that "better maths" are portrayed in terms of abstraction and rigor as dominant; that equity seems to be replaced by diversity; and that ethical responsibility has become absent from such formulations.

In an interesting inclusion of teachers' voices, *Skovmose and Penteado* asked teachers how they think about terms like equity and quality. Their emergent categories include diversity, access to technology, prestige, poverty, hope, stigmatization, learning conditions, possibility, and participation. They challenge educators, especially researchers whose interests involve the promotion of equity agendas to recognize teachers' everyday work lives and to help them create possibilities for themselves and for their students.

Walshaw reminds us that students create identities as learners and as people; and that those identities go beyond cognitive outcomes to include social and cultural components that are related to affect, communication, behavior, and participation. Equitable practices, whatever their formulations, must attend to and develop these identities.

In her analysis of post-compulsory mathematics education, *FitzSimmons* makes the telling point that the needs of the many are sacrificed to imperatives for the few. Whereas Atweh notes that mathematics education follows from a society's economic development, this article echoes Sells' argument that *personal* economic opportunity depends upon competence in math where that competence supports participation in that economy. FitzSimmons further argues that post-compulsory mathematics should be tied to the everyday and work lives of people who are studying it; she further notes that quality and its assessment are not context-free.

All four of these chapters make the point that in the everyday, we must attend to wide variations in people's personal and professional lives, in their identities, and in the ethical provision of educational opportunity. For many students and their teachers, personal social, political, and economic imperatives drive their interests in mathematics education; those imperatives cannot be addressed through abstract and formal mathematics. Across these articles, I see the charge being placed upon equity-educators to provide hope and opportunity to teachers and students across the spectrum; in a word, critique is not enough.

Scholarly inquiry about the nature of equity and of quality—either alone or linked—has taken a decidedly qualitative turn, focused on textual deconstruction and/or interviews with key informants. All of the authors of this section rely heavily on different kinds of qualitative research methods ranging from critical, to post-modern deconstruction and analyses of power relations, and onto analysis of discourse. *Chronaki's* examples illustrate her points. *Skovmose and Penteado* looked across their cases to create their themes. Atweh relies on a broad reading of international studies and philosophical writing to contest and interrogate the terms equity and quality as particular historical artifacts. *Gutierrez and Dixon-Roman*'s social-political analysis and *Pais and Valero's* contribution draws upon methods used by Foucault in his studies of how discourse is used in prisons and other, similar, settings.

Across these chapters, one sees the importance of very careful analysis that stresses deep understandings of the myriad meanings that can be attached to such seemingly simple terms as equity and quality. Such a micro-analytic stance to what one studies represents an important shift away from the over-broad formulations that seem to be part of the large-scale policy studies that so often rely on quantitative research methods. It remains an open question whether a rapprochement between localized, qualitative methods and more-general quantitative methods will be possible. However, these chapters should remind us that the details found in micro-analytic methods should inform the larger studies; they should not be swept over as too often happens within present-day research and policy.

Across the chapters that comprise the first section of this book, three big lessons seem to be that equity and quality derive from the same political and economic systems; have multiply voiced and nuanced meanings; and are studied through careful close readings of texts, prior studies, and local circumstances. Other readers of this book, steeped in their own traditions and social contexts, are likely to find additional lessons as they read. However, were I to be starting my career at this particular historical moment, I would accept the above insights and ask the question with which I leave the reader: Now what?

References

Bowles, S., & Gintis, H. (2002). The inheritance of inequality. *The Journal of Economic Perspectives, 16*(3), 3–30.

Fennema, E., & Leder, G. (Eds.). (1990). *Mathematics and gender.* New York: Teachers College Press.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and affective factors. *American Educational Research Association, 14*, 51–71.

Fennema, E., Wolleat, P. L., Pedro, J. D., & Becker, A. D. (1981). Increasing women's participation in mathematics: An intervention study. *Journal for Research in Mathematics Education, 12*(1), 3–14.
Jensen, A. (1972). *Genetics and education.* New York: Harper & Row.
Lewis, O. (1965). *La vida: A Puerto Rican family in the culture of poverty.* New York: Random House.
Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights.* Boston: Beacon Press.
Office of Policy, Planning and Research. (1965, March). *The Negro family: The case for national action* (Moynihan Report). United States Department of Labor. http://www.dol.gov/oasam/programs/history/webid-meynihan.htm. Accessed 23 May 2010.
Secada, W. G. (Ed.). (1989). *Equity in education.* London: Falmer Press.
Secada, W. G., Fennema, E., & Adajian, L. B. (Eds.). (1995). *New directions for equity in mathematics education.* Cambridge: Cambridge Press.
Secada, W. G., & Meyer, M. R. (Eds). (1989/1991). Needed: An agenda for equity in mathematics education. *Peabody Journal of Education, 66*(Special Issue 2), 22–56.
Sells, L. W. (1973). *High school mathematics as the critical filter in the job market.* Unpublished manuscript, University of California, Berkeley (as cited in Fennema and Sherman, 1977).
Sells, L. W. (1976). *The mathematics filter and the education of women and minorities.* ERIC # ED121663 (Paper presented at Annual Meeting of the American Association for the Advancement of Science, Boston, Massachusetts, February 18–24, 1976).
Wilson, W. J. (2009, January). The Moynihan report and research on the black community. *Annals of the American Academy of Political and Social Science, 621*, 34–46.

Part II
Mapping Social Constructions and Complexities

Chapter 10
Equity in Quality Mathematics Education: A Global Perspective[1]

Murad Jurdak

It is important to examine inequities in mathematics education at the global level because these inequities are likely to filter into the national system and eventually into the classroom. World mathematics education can be thought of as a complex nested hierarchical four-layer system: The classroom system, school system, national system, and the global system (Fig. 10.1). Because each system is nested within the next higher one, the societal relationships of power of a higher system carry over to the lower systems and eventually to the student at classroom level. Using a hypothetical example, I illustrate how the interactions of the factors and their attributes generate inequities in the international system, and how the inequities filter to the classroom. Consider the mathematics education communities in two countries, one being a developing country (low socioeconomic status) and the other a developed country (high socioeconomic status). It is likely that the quality of mathematics education is better in a developed country than in a developing country and eventually this quality differential will result in better teaching and learning of mathematics. Moreover, it is likely that the mathematics education community in the developing country does not have as much access or ownership of internet or knowledge of English as in the developed country. This by itself might generate an inequity between the two countries in terms of ownership of two essential tools for generating and sharing mathematics education knowledge, thus generating a chain reaction which results in an inequitable participation of the two countries in mathematics education at the international level. Even if a mathematics educator in the developing country succeeds in submitting a proposal to an international conference, it may

[1] Based on the author's presentation in ICMI 11 in the plenary panel on equal access to quality mathematics education and on the author's book (Jurdak 2009).

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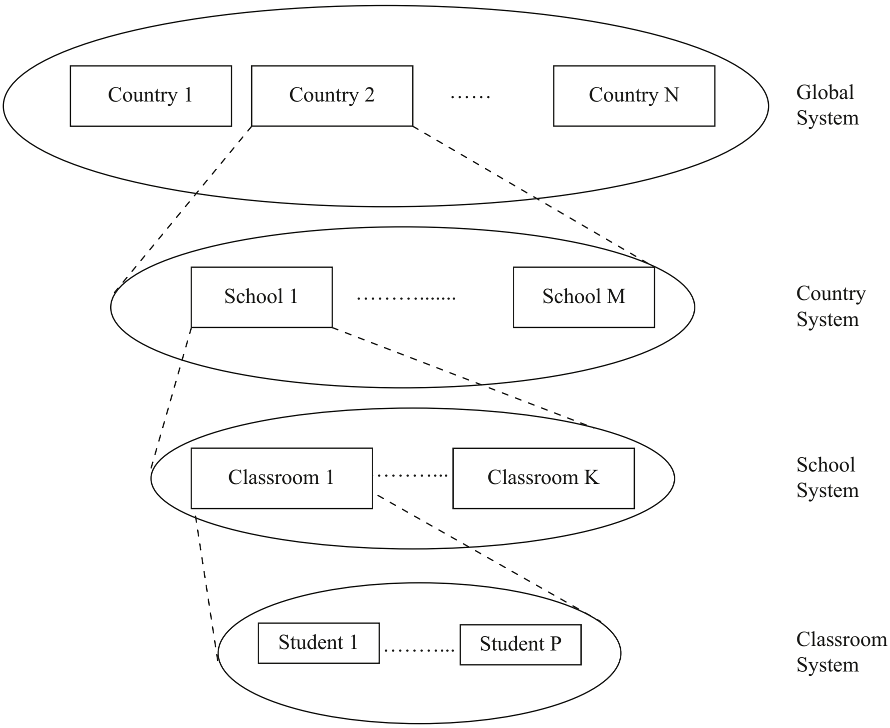

Fig. 10.1 The nested hierarchical world system of mathematics education

not be accepted on the basis of inadequate 'quality' or questionable 'relevance' to the international community. If against all odds, a submission is accepted, its author will not likely have the financial resources to travel in order to participate in the conference. Obviously, the interaction of these factors may eventually lead to the exclusion of the developing country from participating in mathematics education at the international level.

Using TIMSS 2003 database, this chapter addresses the issues of equity in quality mathematics education at the global level. I define mathematics education at the global level to include the teaching and learning mathematics in different countries as well as production of mathematics education knowledge. In this chapter, I use the country's TIMSS 2003 score as an indicator of the quality of mathematics education in that country. Also, an indicator of equity in mathematics education in a certain country is defined in terms of the size of between-school variance in TIMSS 2003 scores in that country. The first section of the chapter attempts to identify the potential inequity factors among countries. The second section explores the correlation between the quality of a country's mathematics education, as measured by the countries TIMSS national score, and its socioeconomic and educational factors. The third section addresses the relationship between equity and quality of mathematics education. The fourth section explores how the relationship between equity and quality is moderated by socioeconomic and educational levels. The fifth section contrasts the optimal and separate modes of development in mathematics education. The chapter ends with concluding remarks.

Potential Inequity Factors in Mathematics Education at the Global Level

One significant attribute of a country, which may be critical for education and for mathematics education, in particular, is the *country's socioeconomic status*, which in turn affects mathematics learning in schools (see for example, Adler 2001; Zevenbergen 2001). Another attribute is the *country's human capital* (defined as people's innate abilities and talents plus their knowledge, skills, and experience that make them economically productive; World Bank, Development Education Program-Glossary) and which depends significantly on the *educational capital* represented by the spread of basic education as measured by the adult literacy rate as well as the level of education in the country as measured by enrollment rates at the secondary and tertiary levels. The nature and structure of a country's education system, being closely related to its political system and its history, is a third attribute which impacts mathematics education. Last, but not least, the *country's culture* in its ideological, social, and technological components impact the quality of mathematics education (see for example, Bishop 1988; Cobb and Hodge 2002). The identity of the country's mathematics education is not only shaped by these factors but also by their interaction. For example, a country in which socioeconomic divisions coincide with cultural divisions in society would be different from the one whose socioeconomic and cultural divisions do not coincide.

Accounting for mathematics achievement differences among countries in terms of cultural differences is complex. Stevenson et al. (1986) pioneered studies which attempted to account for mathematics and reading achievement differences among American, Japanese, and Chinese children not only in terms of educational input but also in terms of cultural differences. The authors concluded that the cognitive abilities of the children in the three countries were similar, but large differences exist in the children's life in school (for example, time spent on academic activities), the attitudes and beliefs of their mothers (the belief regarding the relative importance of the child's ability or effort in success at school), and the involvement of parents and children in school work. The authors implied an association between the lag in mathematics achievement of American children in comparison to Japanese and Chinese children, and the differences in cultural practices and beliefs in the three countries. The much debated 'learning gap' between the United States and other developed countries, as reflected in TIMSS studies, led to the question, much debated in the United States, whether educational policies and practices can overcome cultural effects. Stigler and Hiebert (1999) addressed this question in their book *The Teaching Gap* arguing that while it is impossible to change the culture of the society as a whole, it is possible to change the classroom culture by making use of the best ideas from the world's teachers.

The between-country inequities in mathematics education at the global level result from complex interactions among the triad consisting of country, international mathematics education community, and distribution of roles among countries. Many of the inequities in mathematics education among countries may be accounted for in terms of the interaction between the country's socioeconomic economic status and culture. The country's socioeconomic status and culture are likely to impact the human capital in the country in terms of the spread and level of education in the country. In general, a country with higher socioeconomic level is likely to have more human capital, including mathematics education, than a low socioeconomic country. However, the interaction between a country's socioeconomic status and its culture may result in low mathematics achievement in a relatively rich country. An example of this are some oil-rich countries which, though are relatively high in economic terms, fall low in international mathematics achievement studies probably because of historical and cultural reasons. Similarly, those countries have low participation in and contribution to the international mathematics education community in spite of the fact that in principle they can afford to avail funds for research and travel. As the English language has become the language of international conferences and journals, lack of competency in that language prevents mathematics education researchers from participating in such conferences or publishing in such journals. In some countries, the teaching of English is viewed as a cultural issue (Jurdak 1989).

Language is a cultural factor that may affect mathematics achievement differentially, at least in comparative international studies. The tests used in such studies are translated to the language of the country; however, because language is a cultural carrier even in mathematics education (Jurdak 1989), it is likely that much is lost, and much cultural load is carried in such translations. Another cultural factor which may have a differential impact on between-countries mathematics achievement is the over-inculcation of ideologies, such as religious ideologies. The effect of this ideological factor on mathematics education is two-fold: First, such ideologies are normally taught through rote learning methods which transfer to the teaching of mathematics, and second, the instructional time given to such valued ideologies being commensurate with its value to the society may take away from the instructional time allotted to mathematics in the curriculum (Jurdak 1989).

The policies and practices of the international mathematics education community may exclude some countries from effective participation and contribution to that community (Jurdak 1994). The obstacles that face mathematics educators from developing countries when attempting to participate in international conferences are many and varied. Some of those obstacles relate to the policies and practices of the organizing bodies of these conferences, and some to the countries themselves. Normally, international conferences are organized in developed countries in cities that have the infrastructure and the specialized human resources to support such large conferences. For mathematics educators of developing countries, the cost of attending such conferences can be daunting, as there are neither resources nor traditions in their countries to support their participation (Atweh et al. 2003). However, there is increasing awareness on the part of some international mathematics education organizations of the need to alleviate some of the financial burden on mathematics educators from developing countries to enable more of them to participate in international conferences. There is also an effort to provide assistance in editing the English of their contributions.

Another policy which puts some mathematics educators at a disadvantage with regard to their participation in international conferences is the *de facto* adoption of English as the language of such conferences. Moreover, the call for these conferences is usually done through emailing lists which, in most cases, are based on previous participation. Furthermore, the policies that govern acceptance of contributions do not have enough flexibility to allow a wide range of diverse profiles in content and format, though such contributions may be perceived as meaningful in the contexts of the authors' countries. However, there is increasing awareness on the part of international mathematics education organizations for the need to provide assistance in editing the English of their contributions.

The same can be said about international journals of mathematics education. The publication policies of such journals are almost standardized along Western scientific journals and consequently exclude contributions that address local issues perceived by their authors as meaningful in both the local and the international contexts. Because of the stringent standards in refereed journals, the English language is more of a barrier in international mathematics education journals than it is in conferences.

Country Indicators and Mathematics Achievement

The relationship between the quality of mathematics education in a country and its economic and educational indicators is explored through correlation analysis. The country's mathematics achievement as measured by the country's TIMSS 2003 mathematics achievement score (hence referred to as *quality index*) was correlated with each of a set of economic indicators taken from the World Bank database and with each of a set of educational indicators taken from the UNESCO database.

The educational indicators were drawn from the UNESCO Institute for Statistics mostly for the year 2005 (*UNESCO* Institute for Statistics (UIS), Data Center). The selected educational indicators are:

1. Net primary school enrollment (% of primary school-aged enrollment to the number of primary school-aged children; usually children 6–11)
2. Gross primary school enrollment (% of primary school enrollment to the number of primary school-aged children; usually children 6–11)
3. Net secondary school enrollment (% of secondary school-aged enrollment to the number of secondary school-aged children; usually children 12–17)
4. Gross secondary school enrollment (% of secondary school enrollment to the number of secondary school-aged children; usually children 12–17)
5. Gross tertiary school enrollment (% of tertiary enrollment to the number of young people in the five-year age group following the secondary school leaving age)

6. Children of primary school age who are out of school (%)
7. School life expectancy ISCED 1–6 years
8. Pupil teacher ratio (primary)
9. Expenditure on education as % of GDP
10. Expenditure on education as % of total government expenditure
11. Primary completion rate (% of relevant group)

The economic indicators were taken from among the World Development Indicators on the World Bank web site (World Bank, Key Development Data and Statistics). These are:

1. Ratio of girls to boys in primary and secondary education
2. Adult literacy rate (% of people aged 15 years and above)
3. Gross National Index (GNI) defined as the value of all final goods and services produced in a country in one year (Gross Domestic Product; GDP) plus income that residents have received from abroad, minus income claimed by nonresidents (World Bank, Development Education Program-Glossary)
4. GNI per capita
5. GDP defined as the value of all final goods and services produced in a country in one year (World Bank, Development Education Program-Glossary)
6. GDP per capita
7. Poverty rate (% of population on less than $ 2 per day)
8. GDP growth rate (%)

A file for the 45 countries that had valid data in TIMSS 2003 was created and their respective economic and educational indicators for year 2005 were retrieved from the UNESCO and World Bank home pages. The indicators that had significant correlations with the national mathematics score are listed in Table 10.1.

Three economic indicators had the highest impact on mathematics achievement. The GDP per capita and the GNI per capita correlated significantly and positively with the national mathematics achievement score. Poverty rate had a significant negative correlation with the national mathematics achievement score. The significant negative correlation between expenditure on education as a percent of government

Table 10.1 Significant correlations of TIMSS 2003 national score with the World Bank economic indicators and the UNESCO educational indicators

Economic indicators		Educational indicators	
Positive correlation			
GDP per capita	+0.51	Primary school net enrollment	+0.37
GNI per capita	+0.43	Secondary school net enrollment	+0.55
		Tertiary enrollment	+0.61
		School life expectancy	+0.37
		Adult literacy	+0.63
Negative correlation			
Poverty rate	−0.52	% out of school primary age children	−0.37
Government expenditure on education	−0.37	Primary pupil teacher ratio	−0.45

expenditure and mathematics achievement indicates that the quality of mathematics co-varies with the government expenditure on education in opposite directions. One possible explanation/implication for this phenomenon is that the government expenditure on education normally goes to the improvement of the input of the educational system (schools, teachers, equipment), and these do not necessarily impact the quality of classroom mathematics learning. The between-country variance in mathematics achievement accounted for by each of these three indicators was as follows:

1. GDP per capita: 26%
2. GNI per capita: 19%
3. Poverty rate: 27%

Three educational indicators had the highest impact on mathematics achievement. Adult literacy rate was positively correlated with the national mathematics score and accounted for 40% of the between-country variance; tertiary enrollment rate was positively correlated with the national mathematics score and accounted for 37% of the between-country variance; and, secondary school enrolment positively correlated with the national mathematics score and accounted for 30% of the between-country variance. These correlations indicate that student mathematics learning in a country, as measured by achievement, is significantly related to the educational capital of a country. Adult literacy reflects the spread of education in a country whereas tertiary education enrollment reflects the level of education in a country. Both the spread and level of education in a country are closely related to the level of student parental education which in turn impacts mathematics achievement as TIMSS 2003 data indicate (Jurdak 2009). This is in line with Bourdieu's emphasis on the relationship between the *home habitus*, of which parental education is a factor, and meaningful classroom learning (Bourdieu et al. 1994).

When the economic and educational indicators were entered in a stepwise multiple regression model, using the national mathematics score as a dependent variable, tertiary enrollment rate was the only indicator that entered into the equation, accounting for 56% of the between-country variance in national mathematics score. This indicates that the tertiary enrollment rate was the dominant indicator in the set of economic and educational indicators that were entered in the stepwise regression equation. This means that, in general, a high tertiary enrollment rate in a country is associated with a high TIMSS 2003 national score. However, we should not interpret this to mean that the country's tertiary enrollment rate is the single predictor of the country's mathematics achievement, but rather should be interpreted as a 'composite proxy' indicator for the significant educational and economic indicators previously identified.

Relationship Between Equity and Quality at the Country Level

In this chapter, I define the mathematics education *inequity index* of a country as the percentage of between-school variance in the school mathematics score in the country to the total variance. The between-school variation is theoretically accounted for by variation in the aptitudes and attitudes of students attending different schools, and/or the quality of education provided by the schools. The between-school variance indicates the extent of variation, and hence inequity among schools in mathematics achievement due to schools' educational quality. The larger the between-school variance in mathematics achievement in a country, the more is the inequity in educational provisions among schools in the country. The percentage of between-school variation was calculated for each country by using the variance component model taking TIMSS 2003 student mathematics score as a dependent variable and the school as a random variable.

A representative sample of 18 countries was drawn from the 45 countries which participated in TIMSS 2003 because the inclusion of all countries would have required tedious work with possibly little added value to the conclusions. The sample of 18 countries, stratified by population size and the region to which the country belongs was selected. The 45 countries were assigned to one of the eight geographical regions according to the UNESCO classification. Also each of the 45 countries was classified into one of the three categories according to their population size (high, medium, low). From each geographical region, one country was selected randomly from each of the three population categories. If a region has less than three countries, then all the countries in that region were included. The inequity and quality indices and their ranks for each of the 18 countries in the sample are shown in Table 10.2.

Correlation analysis showed that there was no significant correlation between quality and inequity indices or their ranks. For example, the highest three scoring countries, namely, Singapore, Hungary, and the Netherlands have different inequity index ranks. The Netherlands has the highest inequity index, Singapore an average inequity index, and Hungary the lowest inequity index. On the other hand, the

Table 10.2 Quality and equity indices with their ranks for the sample of 18 countries

Country	Quality index	Quality index rank	Inequity index	Inequity index rank
Singapore	605	1	0.41	10
Netherlands	536	2	0.76	1
Hungary	529	3	0.37	14
Russia	508	4	0.39	12
Australia	505	5	0.52	6
United States	504	6	0.43	9
Italy	484	7	0.31	15
Armenia	478	8	0.20	18
Romania	475	9	0.38	13
Lebanon	433	10	0.54	5
Indonesia	411	11	0.62	3
Iran	411	11	0.40	11
Egypt	406	13	0.45	8
Chile	387	14	0.62	3
Botswana	366	15	0.21	17
Saudi Arabia	332	16	0.23	16
Ghana	276	17	0.46	7
South Africa	264	18	0.71	2

lowest three scoring countries, namely, Ghana, South Africa, and Saudi Arabia also differ in their inequity levels. South Africa had a very high inequity index, Ghana an average inequity index and Saudi Arabia a low inequity index. This does not mean that there is no relationship between the equity and quality indices of a country, but rather it implies that the relationship between equity and quality of mathematics education is too complex to be captured by a simple correlation and that this relationship is moderated by many factors. In the next section, I shall explore the way the socioeconomic status of a country moderates the relationship between equity and quality in mathematics education.

How Does the Country's Economic and Educational Status Moderate the Relationship Between Equity and Quality of Mathematics Education?

In this section, three levels for the country quality index of mathematics education are defined as follows:

1. High quality level: TIMSS 2003 country mathematics score is greater than 525
2. Average quality level: TIMSS 2003 country mathematics score is between 475 and 525
3. Low quality level: TIMSS 2003 country mathematics score is less than 475

Three levels of the country inequity index are defined as follows:

1. High inequity level: Country inequity index is greater than 0.60
2. Average inequity level: Country inequity index is between 0.60 and 0.40
3. Low inequity level: Country inequity index is less than 0.40

The sample of 18 countries is mapped in a matrix whose two dimensions are quality index and inequity index (Table 10.3). Nine countries were classified as low quality

Table 10.3 Quality-inequity matrix

<table>
<tr><th colspan="2"></th><th colspan="3">Quality Index</th></tr>
<tr><td rowspan="4">Inequity Index</td><td></td><td>High</td><td>Average</td><td>Low</td></tr>
<tr><td>Low</td><td>Hungary</td><td>Armenia
Romania
Russian Federation
Italy</td><td>Botswana
Saudi Arabia</td></tr>
<tr><td>Average</td><td>Singapore</td><td>United States
Australia</td><td>Iran
Egypt
Ghana
Lebanon</td></tr>
<tr><td>High</td><td>Netherlands</td><td></td><td>Chile
Indonesia
South Africa</td></tr>
</table>

level in mathematics education, six as average quality level, and three as high quality level. On the other hand, seven countries were classified as low inequity, seven as average inequity, and four as high inequity.

Optimal and Separate Modes of Development in Mathematics Education

In the last half of the past century, the decline of colonization was a major reason for the emergence of the two-tiered system of mathematics education. During the age of colonization, the two-tier system did not exist because colonized countries, mostly developing countries, adopted the mathematics education of their colonial rulers. However, as colonization started to be dismantled, the developing countries had to invest most of its resources in providing public education to its increasing number of students. This was often done at the expense of the quality of education and educational research and development. Hence most of the developing countries did not have the chance to accumulate enough 'credentials' in mathematics education to fully participate in the international mathematics education community.

This situation led to the formation of a two-tiered system of math education at the global level. The upper tier, referred to as the *optimal mode of development*, includes the developed countries that are integrated in the international mathematics education community. The lower tier, referred to as the *separate mode of development*, consists of the marginalized countries which have yet to be integrated in the international activities of mathematics education.

A close examination of the quality-inequity matrix (Table 10.3) reveals that eight of the nine countries having average or high quality index and low or average inequity index generally fit the optimal mode of development in mathematics education model (the shaded area in Table 10.3). According to international comparative studies, these countries have high or average mathematics achievement performance, contribute significantly to international research in mathematics education, and assume leadership roles in international mathematics education organizations and conferences.

On the other hand, the nine countries having low quality index in mathematics education (third column of Table 10.3) fit in the separate mode of development model. According to international comparative studies, these countries have low mathematics performance, have little contribution to international research in mathematics education, and normally have humble participation in international mathematics education conferences, such as the ICME's. In other words, they are marginalized by the international mathematical education community and left to follow their own path in developing their mathematics education. Except for the Netherlands, the three countries that have high inequity index (the third row in Table 10.3) fit the separate development level.

It is to be noted that the quality of the mathematics education of a country is more critical to optimal development than equity of access to the former. Nine of the

18 countries did not fit the optimal mode of development in mathematics education because of low quality of their mathematics education rather than high inequity index. On the other hand, only one of the 18 countries (the Netherlands) did not fit the optimal development mode because of high inequity index rather than low quality index.

Contrasting the Developmental Profiles of Optimal and Separate Modes of Development

How do the developmental profiles of separate and optimal modes of development contrast in terms of developmental indicators? In Table 10.4, the 18 countries in the sample are classified according to their developmental mode in mathematics education (first column in the table), percentile rank of the country in terms of tertiary enrollment rate (column 3) and GNI per capita (column 4), and the region to which it belongs (column 5). A close examination of Table 10.4 supports the following assertions:

1. With the exception of Armenia, all the countries classified as fitting the optimal development mode, belong to three regions considered to be highly developed: North America, Western and Eastern Europe, East Asia and the Pacific. On the

Table 10.4 Percentile rank of tertiary enrollment rate and GNI per capita for each of the sample of countries classified by mode of development and region

Development mode	Country	Tertiary enrollment rate	GNI per capita	Region
Optimal development	Armenia	20	20	Central Asia
	Australia	80	90	East Asia and the Pacific
	Hungary	70	70	Central and Eastern Europe
	Italy	70	90	North America and Western Europe
	Romania	60	40	Central and Eastern Europe
	Russian Federation	80	40	Central and Eastern Europe
	Singapore	–	80	East Asia and the Pacific
	United States	100	100	North America and Western Europe
Separate development	Botswana	10	50	Sub-Saharan Africa
	Chile	60	50	Latin America and the Caribbean
	Egypt	40	10	Arab States
	Ghana	10	10	Sub-Saharan Africa
	Indonesia	20	10	East Asia and the Pacific
	Iran, Islamic Rep. of	20	10	South and West Asia
	Lebanon	60	50	Arab States
	Netherlands	70	100	North America and Western Europe
	Saudi Arabia	20	70	Arab States
	South Africa	10	50	Sub-Saharan Africa

other hand, with the exception of Indonesia and the Netherlands, all the countries classified as following the separate development mode, belong to three regions considered to be developing: Arab states, Latin America, and Sub-Saharan Africa.
2. With regard to GNI per capita, six of the eight optimal development countries are in the upper 30% of the countries in the sample in terms of GNI per capita. For the ten of separate development countries, eight of them are in the lower 50% of the countries.
3. With regard to tertiary enrollment ratio, seven of the eight optimal development countries are in the upper 30% of the countries in the sample, whereas, six of the ten separate development countries are in the lowest 20%.
4. More or less, the classification of countries along the line of mode of development in mathematics education approximates the well-known north-south division in terms of geography, economy, and education.

In summary, a country classified as fitting in the separate mode of development of mathematics education is likely to be relatively poor, low in the spread and level of education among its population, and belongs to a socioeconomically developing region. On the other hand, a country classified as following the optimal mode of development of mathematics education is likely to be relatively rich, high in the spread and level of education among its population, and is part of a developed region.

Concluding Remarks

There seems to be a divide between developing and developed countries in mathematics education, and some of the significant factors that contribute to that divide seem to be out of the reach of mathematics educators and even national governments. Factors such as poverty or wealth of a country or the spread and level of education of its population cannot be changed immediately by national policies.

One can account for the inequity in mathematics at the global level among countries in terms of interaction between the following three factors and their attributes in the mathematics education system at the global level as follows:

1. Socioeconomic status, educational capital, and culture of the country
2. Policies that govern international organizations and conferences
3. English as the international language in mathematics education and access to international mathematics education literature

Obviously, the socioeconomic status of a country, its educational capital, and its culture are factors beyond the sphere of influence of local or international mathematics education communities whereas the other factors are not. The international mathematics education community has a responsibility to find ways and means to encourage and enable mathematics educators to be integrated in the international mathematics education community because such integration is likely to provide exposure to research and development in mathematics education which hopefully may be translated into improved mathematics learning and teaching. The participation in and contribution to international mathematics education conferences and international mathematics education journals are critical for such integration. One measure in this regard would be to make the policies that govern international mathematics education international organizations more favorable to the participation of mathematics educators from developing countries. Another measure is to intensify and broaden efforts to avail resources to promising mathematics educators whose institutions or countries cannot support their travel and accommodation. Writing and presenting in English is a major barrier to the participation of many mathematics educators in international conferences. Mathematics educators who are qualified to engage in international conferences, except for their proficiency in English, would have a better chance of being integrated in the international community if some form of mentoring volunteered by their colleagues who can provide their support in reviewing and editing manuscripts. Providing opportunities for presentations in international conferences in languages other than English by using increasingly more affordable technologies, such as simultaneous translation, would broaden access to such conferences. All these measures would hopefully help enhance the integration of more mathematics educators in the international community and hence make the latter more inclusive.

References

Adler, J. (2001). Resourcing practice and equity: A dual challenge for mathematics education. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 185–200). Mahwah: Lawrence Erlbaum Associates, Publishers.

Atweh, B., Clarkson, P., & Nebres, B. (2003). Mathematics education in international and global contexts. In: A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & L. Leung (Eds.), *The second international handbook of mathematics education* (pp. 185–230). Boston: Springer.

Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics, 19,* 179–191.

Bourdieu, P., Passcron, J. C., & de Saint Martin, M. (Eds.). (1994). *Academic discourse: Linguistic misunderstanding and professional power* (pp. 35–79). Stanford: Stanford University Press.

Cobb, P., & Hodge, L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4,* 249–284.

Jurdak, M. (2009). *Toward equity in quality in mathematics education*. New York: Springer.

Jurdak, M. E. (1989). Religion and language as culture carriers and barriers in mathematics education. In C. Kietel (Ed.), *Science and Technology Education, Document Series No. 35. Mathematics, education and society* (pp. 12–14). Paris: UNESCO.

Jurdak, M. E. (1994). Mathematics education in the global village. In D. Robitaille, D. Wheeler, & C. Kieran (Eds.), *Selected lectures from the 7th International Congress on Mathematical Education* (pp. 199–210). Quebec: Laval University.

Stevenson, H., Lee, S., & Stigler, J. (1986). Mathematics achievement of Chinese, Japanese, and American children. *Science, 231,* 693–699.

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the class room*. New York: Free Press.

UNESCO Institute for Statistics (UIS). Retrieved on October 15, 2007 from *Data Center*. http://stats.uis.unesco.org/unesco/tableviewer/document.aspx?ReportId=143.
World Bank. Retrieved on October 15, 2007 from *Key Development Data & Statistics*. http://web.worldbank.org/wbsite/external/datastatistics/0,,contentMDK:20535285~menuPK:1192694~pagePK:64133150~piPK:64133175~theSitePK:239419,00.html.
World Bank. *Development Education Program-Glossary*. http://www.worldbank.org/depweb/english/beyond/global/glossary.html.
Zevenbergen, R. (2001). Mathematics, social class and linguistic capital: An analysis of mathematics classroom interactions. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 201–215). Mahwah: Lawrence Erlbaum Associates, Publishers.

Chapter 11
Effects of Student-Level and School-Level Characteristics on the Quality and Equity of Mathematics Achievement in the United States: Using Factor Analysis and Hierarchical Linear Models to Inform Education Policy

Mack C. Shelley and Wenyu Su

Introduction and Literature Review

Certain specific circumstances make the U.S. context distinctive from that of other countries. Historically, often dramatic inequities in parental educational attainment, family income and wealth, and funding for public education have been exacerbated by legacies of slavery, racial segregation, and a "separate but equal" legal structure that institutionalized lower resources and consequently lower achievement for many non-majority students, together with lack of full linguistic and cultural integration and the No Child Left Behind (2001) national mandate to move toward equal achievement for all major sociodemographic groups. Mathematics education in the United States also needs to be understood in the context of the lack of a national curriculum and a strong tradition of decentralized curriculum control. The county's public school system contains over 14,500 school districts, with over 100,000 schools and about 50,000,000 students. Many districts and buildings are small and underfunded, although over $ 10,000 is spent per student (http://www.schooldatadirect.org/app/location/q/stid=1036196/llid=162/stllid=676/locid=1036195/site=pes).

Concerns about equity in educational outcomes in the United States have taken on new urgency, amid persisting achievement gaps between white and African American students (Campbell et al. 2000; Lee 2002; Lubienski and Bowen 2000). Family socioeconomic status (SES; Secada 1992; Weis 1988) differences account for much of race-related achievement gaps, with teacher expectations, school structure, student motivation, and student resistance (Ferguson 1998). Race-based differences in students' beliefs and classroom experiences are related to mathematics achievement gaps, but such differences might be due to SES more than race (Strutchens et al.

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B. Atweh et al. (eds.), *Mapping Equity and Quality in Mathematics Education,*
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2004). Although overall student performance varies only a few points by strand, significant and persisting variations exist across strands (Lubienski 2001).

Disparities in Implementation of Curriculum

Shelley and Lubienski (2005) concluded that implementation disparities may explain existing achievement gaps and inform future instruction reform efforts (Newmann et al. 2001) and attempted to identify which reform-oriented instructional practices correlate with achievement for disadvantaged students. Two-level hierarchical linear models (HLM) (Bryk and Raudenbush 1992; Lee and Bryk 1989; Newmann et al. 2001; Raudenbush and Bryk 2002) were used to estimate parameters for models with students (at level 1) nested within schools (level 2), to assess the contribution of reform-based instructional practices to explaining variation in mathematics achievement controlling for race, ethnicity, SES, gender, language spoken at home, disability, and appropriateness of mathematics courses. Correct estimates for "plausible values" were produced, using the model-based multiple imputation strategy for data missing at random (e.g., Schafer 1997).

Social Structures

Shelley and Lubienski (2003) applied HLM to National Assessment of Educational Progress (NAEP) data from 1990, 1996, and 2000 to investigate the role of social structures in the relationship between mathematics instruction and student achievement. Level-1 results showed that higher SES, ethnicity (White and Asian/Pacific Islander), and male gender were associated with higher mean mathematics achievement scores. For level 2, at 4th-grade baseline, mean mathematics achievement was affected significantly negatively by school SES, and by school race both positively through the level-1 intercept and negatively through student gender. For the 4th-grade-enhanced model, mean mathematics achievement was affected significantly negatively by school SES (through the level-1 intercept), and by school race both positively through the level-1 intercept and negatively through student gender. For 8th grade, mean mathematics achievement was affected significantly negatively by school SES, negatively by a greater degree of ruralness, positively by school race, and positively by school race (through student race). For 12th grade, mean mathematics achievement was affected significantly negatively by school SES, negatively by a greater degree of ruralness, and positively by school race and through student SES.

Home and Parental Effects

Even when gaps appear to be rooted in SES differences, parent education, occupation, income, and educational resources in the home are important, as are teacher expectations, school structure, student motivation, and student resistance (Ferguson 1998).

Instructional Practices

Lubienski et al. (2004) showed that teacher-reported collaborative problem solving and teacher knowledge of the NCTM *Standards* were positive predictors of achievement at both grades 4 and 8. Interaction effects suggest that teachers need to monitor whether students—particularly those who are low-SES or African American—are benefiting from calculator and collaborative group work as intended.

Some studies have revealed that low-SES students do not interpret mathematics classroom discourse and open-ended, contextualized problems as middle-class educators intend (e.g., Cooper and Dunne 2000). In addition, Hickey et al. (2001) found that *Standards*-based instruction improved low- and high-SES students' problem-solving skills, but increased the SES-related gap in students' performance on the concepts and estimation portion of the Iowa Test of Basic Skills. Still other studies have suggested that reform-minded practices are particularly beneficial for lower-SES children and minority students (e.g., Schoenfeld 2002).

Although NAEP scores increased between 1990 and 2000 for white, Hispanic, and African American students, and for both low- and high-SES students (Braswell et al. 2001), race-related achievement gaps did not improve (Lubienski and Shelley 2003). Both race- and SES-related differences affect students' beliefs; these correlations with race persisted even after controlling for SES, and suggest that White, middle-class students experience more of the fundamental shifts called for in the NCTM *Standards*.

Race-Related Effects

SES differences involving parent education, occupation, income, and educational resources in the home account for much of the race-related achievement gaps (Rothstein 2004).

Data and Results

Data Source

The data analyzed in this paper came from the 2003 Program for International Student Assessment (PISA), which contains information from more than 250,000 students from 41 countries on student performance in four domains of assessment and responses to the student and school questionnaires. Mathematical literacy is covered in terms of both mastery of the school curriculum and knowledge and skills needed in adult life (www.pisa.oecd.org).

We focus on quality and equity agendas in mathematics education in the United States. Accordingly, only U.S. data are employed in this analysis. The U.S. data included information on 5,465 students nested within 274 schools. In subsequent statistical analysis, variables specific to students will be referred to as level-1 measures and variables specific to schools will be referred to as level-2 measures.

Student-Level and School-Level Measures

The student-level variables and indices include:

PV1MATH—a measure of students' mathematics achievement. Student proficiencies (or measures) are not observed; they are missing data that must be inferred from the observed item responses. PISA uses the imputation methodology usually referred to as plausible values (PVs). PVs are random numbers drawn from the distribution of scores that reasonably could be assigned to each individual—that is, the marginal posterior distribution. PISA provides five plausible values for the combined mathematics scale. The first plausible value is selected in this analysis.

GENDER, coded as 1 = female, 2 = male.

ESCS—a broader socioeconomic measure called the index of Economic, Social, and Cultural Status, which is derived from the highest occupational status of parents, the highest educational level of parents, and an estimate related to household possessions. Positive scores indicate higher levels of SES.

INTMAT—an index of students' interest in and enjoyment of mathematics. Higher scores indicate higher levels of interest in and enjoyment of mathematics.

INSTMOT—an index for instrumental motivation to learn mathematics. Higher scores indicate higher levels of instrumental motivation to learn mathematics.

MATHEFF—an index of mathematics self-efficacy. Higher scores indicate higher levels of self-efficacy.

ANXMAT—an index of mathematics anxiety. Higher scores indicate higher levels of mathematics anxiety.

SCMAT—an index of mathematics self-concept. Higher scores indicate higher levels of self-concept in mathematics.

INTMAT and INSTMOT are indices of students' motivations in mathematics; MATHEFF, ANXMAT, and SCMAT are indices of students' self-related cognitions in mathematics.

WEIGHT—To make valid estimates and inferences, a composite final student survey weight was derived from school base weight, student base weight, and five adjustments.

The school-level variables and indices include:

SMRATIO—an index of student/mathematics teacher ratio, obtained by dividing the number of enrolled students by the total number of mathematics teachers.

MACTIV—an index of mathematics activity at school, computed by simply counting the number of different types of activities occurring at school.

PCGIRLS—proportion of female students enrolled at school, computed by dividing the number of female students by the total of female and male students in each school.

SCMATEDU—an index of quality of schools' educational resources, derived from school principal's perceptions of what potentially hinders instruction. Larger scores indicate more positive evaluations of the quality of educational resources.

The next section explains how factor analysis methods are employed to derive two sets of constructs—motivations and self-related cognitions—related to student mathematics achievement. These constructs and other measures are used as independent variables in regression models to predict student achievement outcomes (PV1MATH).

We begin the analysis with an initial explanation of the meaning and usefulness of both exploratory and confirmatory factor analyses (EFA and CFA). Both methods are relevant to understanding patterns among these variables and are instrumental for the essential task of reducing the complexity of subsequent statistical models and facilitating the interpretation of results.

Factor Analysis Methods

EFA and CFA were employed to uncover and estimate the above factors for students' motivations and self-related cognitions in mathematics. In EFA, correlations between variables are used to generate a factor structure based on those relationships. CFA (Brown 2006) permits testing hypotheses that relationships exist between observed variables and underlying latent variables. Both are based on the common factor model, postulating that each indicator in a set of observed measures is a linear function of one or more common factors and one unique factor. In matrix notation, a fundamental equation of the common factor model is

$$X = LF + \varepsilon, \tag{11.1}$$

where X is a $(q \times 1)$ vector of observed variables, F is a $(s \times 1)$ vector of common factors, L is a $(q \times s)$ matrix of factor loadings relating the observed x's and latent F's, and ε is a $(q \times 1)$ vector of the unique factors. It is assumed that the number of observed variables is greater than the number of factors. Both the obtained and latent variables in Eq. (11.1) are assumed to be measured as deviations from their means. Thus, the expected value of each vector is a vector containing zeros: $E(X) = 0$; $E(F) = 0$; and $E(\varepsilon) = 0$.

Factor Analysis Results for Student Motivations

Eight items (Tables 11.5, 11.6) in the student questionnaire were used to measure students' motivations in mathematics. The item categories were "strongly agree,"

Table 11.1 CFA fit statistics for motivations

Covariance structure analysis: maximum likelihood estimation	
Fit function	0.0544
Goodness of Fit Index (GFI)	0.9868
GFI Adjusted for Degrees of Freedom (AGFI)	0.9736
Root Mean Square Residual (RMR)	0.0145
Parsimonious GFI	0.6344
Chi-square	271.4445
Chi-square DF	18
Pr > Chi-square	<0.0001
RMSEA estimate	0.0531
RMSEA 90% lower confidence limit	0.0476
RMSEA 90% upper confidence limit	0.0588
Bentler's comparative fit index	0.9909
Bentler and Bonett's non-normed index	0.9859
Bentler and Bonett's NFI	0.9903
James et al. parsimonious NFI	0.6366

"agree," "disagree," and "strongly disagree." Four of these items asked about students' interest in mathematics as a subject as well as their enjoyment of learning mathematics, and the other four items asked to what extent they are encouraged to learn by external rewards such as good job prospects. After observing the correlations among the eight items measuring students' motivations, first a simple CFA model was estimated without considering possible double-loadings and correlated errors. Then the model was respecified based on both the modification indices and the rationality of making a modification. Table 11.1 summarizes the fit statistics.

For this CFA model, the χ^2 value (271.445, df = 18, p < 0.0001) indicates that the variances and covariances estimated by the model, $\hat{\Sigma}$, do not sufficiently reproduce the sample variances and covariances, S (i.e., the model does not fit the data well). However, χ^2 is inflated by sample size, and thus large sample solutions (here, n = 4,995) routinely reject the null hypothesis of model fit based on the χ^2 value even when the difference between S and $\hat{\Sigma}$ is negligible. Therefore, it is rarely used as a sole index of model fit. RMSEA indicates the average amount of unexplained variance, or residual. The value of RMSEA obtained (0.0531) suggests adequate model fit (Browne and Cudeck 1993). GFI (0.9868), CFI (0.9909), NNI (0.9859), and NFI (0.9903) values also meet the criteria for acceptable model fit (Meyers et al. 2006).

Parameter estimates for this CFA model are significant (p < 0.01). The first factor indicates students' intrinsic motivation in mathematics, which are internally generated motives such as interest in mathematics. The second factor is the indicator of students' instrumental motivation derived from external rewards for good performance such as praise or future prospects (Fig. 11.1).

Factor Analysis Results for Student Self-Related Cognitions

Eighteen items (Tables 11.7–11.9) in the student questionnaire measured students' self-related cognitions. Eight items asked to what extent students believe in their

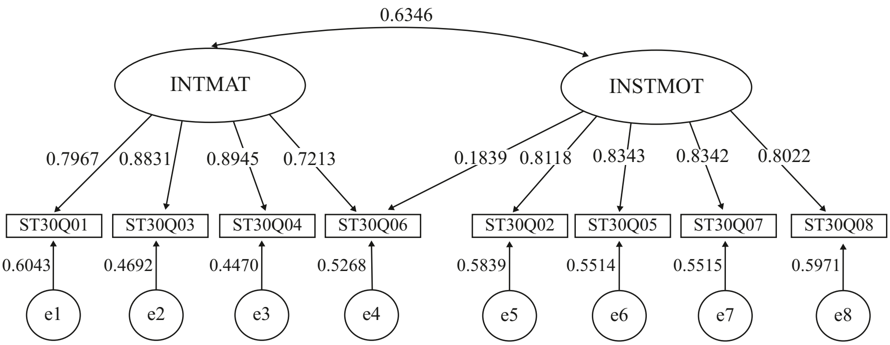

Fig. 11.1 CFA model for motivations

own ability to handle learning situations in mathematics effectively and overcome difficulties. Item categories were "very confident," "confident," "not very confident," and "not at all confident." Six items asked to what extent they feel helpless and under emotional stress when dealing with mathematics; and the other four items asked about students' belief in their own mathematical competence. Items categories were "strongly agree," "agree," "disagree," and "strongly disagree." The CFA model with the standardized coefficients is presented in Fig. 11.2. Table 11.2 summarizes the fit statistics.

The χ^2 value (1,690.6526, $df = 129$, $p < 0.0001$) again is inflated by large sample size. RMSEA (0.0492) suggests good model fit. GFI (0.9625), NNI (0.9616), CFI (0.9676), and NFI (0.9651) indicate an acceptable model fit. It can be concluded that the CFA model might be acceptable.

Parameter estimates are significant ($p < 0.001$). The first factor is the indicator of self-efficacy in mathematics, which reflects how well students think they can

Table 11.2 CFA fit statistics for self-related cognitions

Covariance structure analysis: maximum likelihood estimation	
Fit function	0.3385
Goodness of Fit Index (GFI)	0.9625
GFI Adjusted for Degrees of Freedom (AGFI)	0.9503
Root Mean Square Residual (RMR)	0.0219
Parsimonious GFI	0.8115
Chi-square	1,690.6526
Chi-square DF	129
Pr > chi-square	<0.0001
RMSEA estimate	0.0492
RMSEA 90% lower confidence limit	0.0472
RMSEA 90% upper confidence limit	0.0513
Bentler's comparative fit index	0.9676
Bentler and Bonett's non-normed index	0.9616
Bentler and Bonett's NFI	0.9651
James et al. parsimonious NFI	0.8137

handle even difficult tasks. The second factor indicates mathematics anxiety, and the third factor reflects students' self-concept, which indicates their belief in their own mathematical competence.

Developing Underlying Dimensions

The χ^2 value is not the sole index of fit, but the value of $p<0.0001$ challenges the conclusions from the CFA models. Hence, EFA was employed to explore further the latent factors for motivations and self-related cognitions in mathematics, and to obtain factor scores for subsequent analysis.

EFA using maximum likelihood (ML) estimation was used to identify the underlying dimensions for motivations and self-related cognitions, respectively. Two indices for motivations in mathematics—INTMAT for intrinsic motivation and INSTMOT for instrumental motivation—and three indices for self-related cognitions in mathematics were constructed: ANXMAT for mathematics anxiety, MATHEFF for mathematics self-efficacy, and SCMAT for self-concept. The factor scores for these five indices were obtained for subsequent analysis; the scale for each factor score is inverted, so that higher scores indicate higher levels on those indices. These indices derived from factor analysis, combined with other student-level and school-level variables, then are used as predictors in hierarchical models to investigate their relationships with mathematics achievement.

Multi-Level Regression Analysis

Level-1 Model The general *level-1* model with Q predictor variables is

$$Y_j = X_j\beta_j + r_j, \quad r_j \sim N(0, \sigma^2 I), \tag{11.2}$$

where Y_j is an n_j by 1 vector of outcomes, X_j is an n_j by (Q+1) matrix of predictor variables, β_j is a (Q+1) by 1 vector of unknown parameters, I is an n_j by n_j identity matrix, and r_j is an n_j by 1 vector of random errors assumed normally distributed with a mean vector 0 and a variance-covariance matrix in which all diagonal elements are equal to σ^2 and all off-diagonal elements are 0.

Level-2 Model The *level-2* model is

$$\beta_j = W_j\gamma + u_j, \quad u_j \sim N(0, T), \tag{11.3}$$

where W_j is a (Q+1) by F matrix of predictors, γ is an F by 1 vector of fixed effects, u_j is a (Q+1) by 1 vector of level-2 errors or random effects, and T is an arbitrary (Q+1) by (Q+1) variance-covariance matrix.

Given that the data are not perfectly balanced, the unique, minimum-variance, unbiased estimator of γ will be the generalized least squares (GLS) estimator

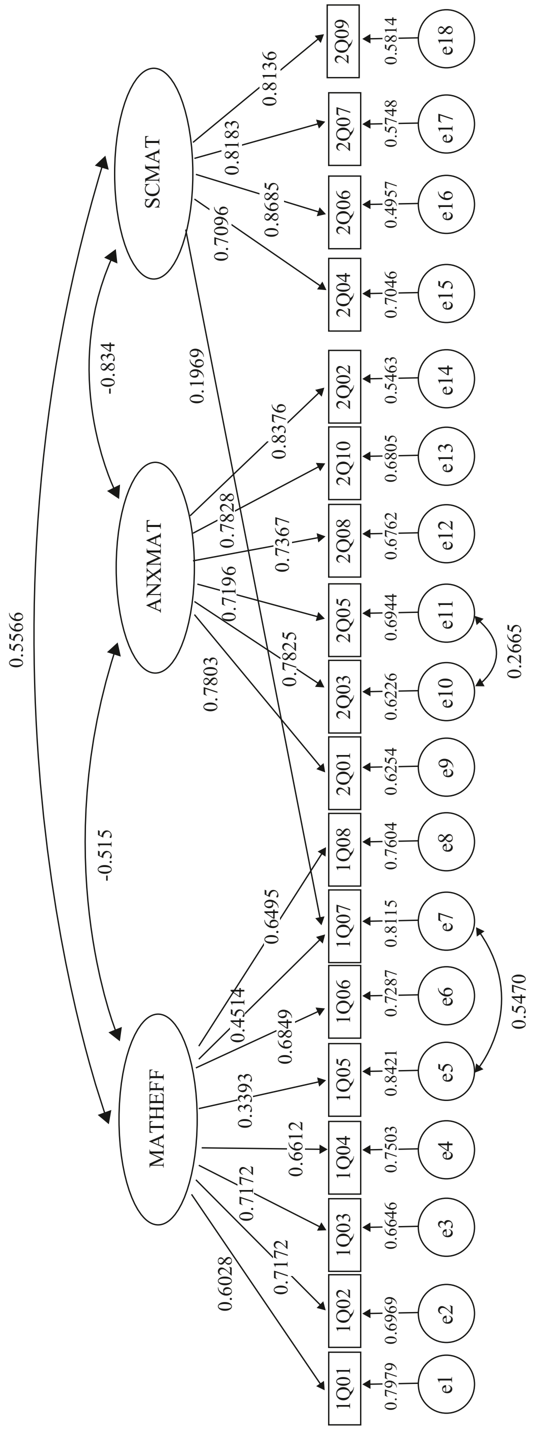

Fig. 11.2 CFA Model for self-related cognitions

$$\hat{\gamma} = \left(\sum W_j^T \Delta_j^{-1} W_j\right)^{-1} \sum W_j^T \Delta_j^{-1} \hat{\beta}_j, \tag{11.4}$$

where

$$\Delta_j = Var(\hat{\beta}_j) = T + V_j = \text{parameter dispersion} + \text{error dispersion}, \tag{11.5}$$

$$V_j = \sigma^2 \left(X_j^T X_j\right)^{-1}. \tag{11.6}$$

Under normality assumptions, (11.4) is also the maximum likelihood estimator for γ. SAS PROC MIXED was used to estimate multilevel models. No structure was imposed on the variance-covariance matrix for the level-2 residuals. This specification is common in school effects analysis.

Model 1: Do Schools Differ in Students' Average Mathematics Achievement?

A multilevel regression analysis reports residual variance at different levels: the between-school variance and the within-school variance that are not explained by the predictors included in the model. Unbiased estimates of the between-school variance and the within-school variances are provided by the following model (Model 1; Table 11.3).

$$Y_{ij} = \alpha_j + \varepsilon_{ij},$$

$$\alpha_j = \gamma_{00} + u_{0j},$$

where Y_{ij} is the performance score for the ith student from school j; α_j represents the school means; the variance of u_{0j} is the school variance; and the variance of ε_{ij} is the within-school variance.

Both variance components are significantly different from zero, which suggests that schools differ in their students' average mathematics achievement and that

Table 11.3 Model 1

Covariance parameter estimates					
Cov parm	Subject	Estimate	Standar Error	Z Value	Pr Z
Intercept	SCHOOLid	2,167.74	226.54	9.57	<0.0001
Residual		6,540.67	134.74	48.54	<0.0001
Fit statistics					
−2 Log Likelihood = 59,055.5					
AIC = 59,061.5					
AICC = 59,061.5					
Solution for fixed effects					
Effect	Estimate	Standard Error	DF	*t*-Value	Pr > |*t*|
Intercept	484.73	3.0865	273	157.05	<0.0001

there is even more variation among students within schools. About 25% of the total variance explained is accounted for by the school, so we cannot ignore the sources of school variation. The estimated intercept of 484.73 tells us the average school-level mathematics achievement score in this sample of schools, which is different from the average student-level achievement score.

Model 2: Results of Full Two-Level Analysis

INTMAT and INSTMOT have positive relationships with mathematics achievement, but are excluded from the modeling because of potentially serious multicollinearity. All other student-level variables are added to the level-1 model. We would like to see if school educational resources influence the effects of students' self-cognitions on mathematics achievement and whether the school-level percentage of female students might predict the relationship between gender and mathematics achievement at the student level. The full model is:

Full Model

$$\begin{aligned} Y_{ij} &= \alpha_j + \beta_{1j}(GENDER)_{ij} + \beta_{2j}(ESCS)_{ij} + \beta_{3j}(ANXMAT)_{ij} \\ &\quad + \beta_{4j}(MATHEFF)_{ij} + \beta_{5j}(SCMAT)_{ij} + r_{ij} \end{aligned}$$

$$\begin{aligned} \alpha_j &= \gamma_{00} + \gamma_{01}(SCMATEDU)_j + \gamma_{02}(MACTIV)_j \\ &\quad + \gamma_{03}(SMRATIO)_j + \gamma_{04}(PCGIRLS)_j + u_{0j} \\ \beta_{1j} &= \gamma_{10} + \gamma_{11}(PCGITRLS)_j + u_{1j} \\ \beta_{2j} &= \gamma_{20} + \gamma_{21}(SCMATEDU)_j + u_{2j} \\ \beta_{3j} &= \gamma_{30} \\ \beta_{4j} &= \gamma_{40} + \gamma_{41}(SCMATEDU)_j + u_{4j} \\ \beta_{5j} &= \gamma_{50} + \gamma_{51}(SCMATEDU)_j + u_{5j} \end{aligned}$$

PCGIRLS, MACTIV, and SMRATIO are not significant in the model. Also, SCMATEDU does not predict the slopes for ESCS, MATHEFF, and SCMAT. A model selection procedure then was conducted to find a reasonable model that can help study the effects of those variables on mathematics achievement. The equation of the chosen model (Model 2) can be written as:

Level-1 Model

$$\begin{aligned} Y_{ij} &= \alpha_j + \beta_1(GENDER)_{ij} + \beta_{2j}(ESCS)_{ij} \\ &\quad + \beta_{3j}(ANXMATT)_{ij} + \beta_{4j}(MATHEFF)_{ij} + \beta_{5j}(SCMAT)_{ij} + r_{ij} \end{aligned}$$

Level-2 Model

$$\begin{aligned} \alpha_j &= \gamma_{00} + \gamma_{01}(SCMATEDU)_j + u_{0j} \\ \beta_1 &= \gamma_{10} \\ \beta_{2j} &= \gamma_{20} + u_{2j} \\ \beta_{3j} &= \gamma_{30} + u_{3j} \\ \beta_{4j} &= \gamma_{40} + u_{4j} \\ \beta_{5j} &= \gamma_{50} + u_{5j} \end{aligned}$$

Table 11.4 Model 2

Covariance parameter estimates

Cov parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	SCHOOLid	943.55	111.73	8.44	<0.0001
UN(2,1)	SCHOOLid	–62.1931	52.4391	–1.19	0.2356
UN(2,2)	SCHOOLid	87.2422	39.5269	2.21	0.0137
UN(3,1)	SCHOOLid	44.0744	42.4458	1.04	0.2991
UN(3,2)	SCHOOLid	15.4362	27.1472	0.57	0.5696
UN(3,3)	SCHOOLid	72.9037	31.2104	2.34	0.0097
UN(4,1)	SCHOOLid	231.68	49.2355	4.71	<0.0001
UN(4,2)	SCHOOLid	–50.2891	29.6954	–1.69	0.0904
UN(4,3)	SCHOOLid	–21.6282	23.7257	–0.91	0.3620
UN(4,4)	SCHOOLid	87.5506	36.0732	2.43	0.0076
UN(5,1)	SCHOOLid	49.8628	43.8162	1.14	0.2551
UN(5,2)	SCHOOLid	3.4400	26.1171	0.13	0.8952
UN(5,3)	SCHOOLid	66.7076	23.7572	2.81	0.0050
UN(5,4)	SCHOOLid	–2.6828	23.0863	–0.12	0.9075
UN(5,5)	SCHOOLid	55.1667	31.8173	1.73	0.0415
Residual		4,191.13	93.7292	44.72	<0.0001

Fit statistics

−2 Log Likelihood = 56,886.1

AIC = 56,932.1

AICC = 56,932.3

BIC = 57,015.2

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	*t*-Value	Pr > |*t*|
Intercept	477.35	3.7452	272	127.45	<0.0001
GENDER	–0.7438	1.9510	4716	–0.38	0.7031
ESCS	20.3275	1.3744	4716	14.79	<0.0001
SCMATEDU	6.7792	1.8373	272	3.69	0.0003
ANXMAT	–22.9517	1.2451	4716	–18.43	<0.0001
MATHEFF	36.9741	1.2870	4716	28.73	<0.0001
SCMAT	10.7759	1.2479	4716	8.64	<0.0001

Accounting for self-related cognitions and school educational resources (Table 11.4), gender no longer has a significant effect on mathematics achievement, while all the other fixed effects are significantly different from zero ($p < 0.05$). Holding other variables constant, higher SES, self-efficacy, and self-concept lead to higher mathematics achievement. Self-efficiency has the strongest effect on mathematics achievement. Also, school-level mathematics achievement is higher with schools having better educational resources.

The between-school residual variance (943.55) tells us about the variability in intercepts, and the within-school residual variance is 4,191.13. First, the intercepts are still different, in other words, schools do differ in average mathematics achievement levels even after controlling for the effects of those level-1 and level-2 predictors, suggesting that there is additional variation in school mean achievement levels that is not explained by these variables. Second, the slopes for ESCS, MATHEFF, ANXMAT, and SCMAT are also different across schools. For example, the association between mathematics achievement and anxiety level varies across schools.

Conclusions and Discussion

An essential aspect of multi-level modeling is to determine whether the second (school) level contributes anything of value that helps to understand what affects student mathematics achievement and thus how policies might be structured to enhance equity. In this case, we have substantial evidence to support the conclusion that schools matter, in addition to individual student characteristics. About 25% of the total variance is accounted for by the school, indicating that we should not ignore school-level variation.

SES segregation exists in the U.S. schools; some are attended by mainly high-SES students, while other schools are attended by mainly students with low SES. Our results provide convincing evidence that SES segregation plays an important role in the lack of equity in students' mathematics achievement. Ending, or at least drastically reducing, the inequity will not be possible until national- and state-level policy initiatives can result in reduced economic disparity. Doing so is a daunting task, particularly for a county in which SES inequity is heavily confounded with race, ethnicity, culture, and a political context in which "progressive" policy initiatives have a very low survival rate.

The gender difference in mathematics achievement is significant before accounting for self-related cognitions and school educational resources; but not after including those variables in the model. The gender difference in mathematics achievement has been attributed to a number of variables, most notably differential course-taking patterns and exposure to mathematics, different learning styles, teacher behavior and learning environment, parental attitudes and expectations, and SES, as well as other background characteristics of students. The findings from this study indicate that the gender difference would become nonsignificant if more support were provided for female students.

At the student level, SES, motivations, and self-related cognitions have positive relationships with students' mathematics achievement, while mathematics anxiety is negatively associated with mathematics achievement. From a policy perspective, these findings suggest some potential avenues for endeavoring to enhance equity in student achievement. The most obvious suggestion is to pursue redistributionist policies targeted to specific families and neighborhoods that transfer income and wealth toward elements of society that currently are underprivileged economically. That certainly is more easily said than done, and by itself probably would be insufficient to rectify generations worth of inequality. A second avenue is to enhance student motivations to learn mathematics, which, for example, could take the form of peer mentoring or supplemental instruction, both of which have shown good results in science, technology, engineering, and mathematics (STEM) disciplines. Enhancing self-related cognitions is not a simple endeavor, but some progress toward the goal of enhancing students' self-competence with mathematics could be achieved by providing positive feedback through emphasizing role models of successful students from similar backgrounds and by offering positive reward structures for improved performance perhaps along the lines of positive behavior supports designed to diminish negative school-related behaviors. Similarly, policies to alleviate mathematics anxiety would need to take the form of innovative curriculum approaches that match individual student learning styles with constructivist teaching approaches designed to facilitate positive teacher-student interaction.

At the school level, better educational resources could improve average school mathematics achievement. Clearly, this would necessitate major policy innovations redirecting resources toward districts and buildings with lower levels of funding, probably combined with the consolidation of smaller and/or underperforming schools/districts. However, our results also demonstrate that the percentage of female students, number of mathematics activities, and student/mathematics teacher ratio do not significantly influence average school mathematics achievement, suggesting that policy initiatives designed to enhance equity would not produce substantial improvements if efforts were directed toward these targets.

The associations between mathematics achievement and ESCS, MATHEFF, and SCMAT also differ across schools. This suggests that policy initiatives designed to enhance equity of student mathematics achievement will need to be targeted differently to schools with varying levels of mean SES, mean student self-efficacy, and mean levels of mathematics self-concept. Thus, a "one size fits all" approach would not be likely to be as effective as one that is targeted to the specific circumstances defining each school's specific combination of needs.

It is essential to adopt policy tools that disrupt the negative synergies that often exist between low-performing students and poorly trained teachers. Results from Mathematics and Science Partnerships projects (http://www.nsf.gov/news/longurl.cfm?id=51) may provide guidance about desirable policy innovations such as summer teaching institutes, learning communities linking K-12 teachers with faculty at institutions of higher education, additional teacher certification or master's degrees, higher education or business "externships," mentoring or cognitive coaching, and training in the use of innovative instructional technology. Policies must be sensitive to the increasing ethnic and cultural diversity of mathematics teachers and of their students, and should be designed to encourage prospective teachers from diverse backgrounds to enter the STEM career pipeline and help them persist to degree completion as well as to facilitate the retention of new mathematics teachers early in their careers, when retention is problematic.

Appendix

Tables for the Items from the PISA Student Questionnaire

Table 11.5 Items for measuring interest in and enjoyment of mathematics (INTMAT)

Thinking about your views on mathematics: To what extent do you agree with the following statements?
ST30Q01 a) I enjoy reading about mathematics
ST30Q03 c) I look forward to my mathematics lessons
ST30Q04 d) I do mathematics because I enjoy it
ST30Q06 f) I am interested in the things I learn in mathematics

Note: Item categories were "strongly agree," "agree," "disagree," and "strongly disagree." Item categories were coded as 1 = strongly agree; 2 = agree; 3 = disagree; 4 = strongly disagree

Table 11.6 Items for measuring instrumental motivation to learn mathematics (INSTMOT)

Thinking about your views on mathematics: To what extent do you agree with the following statements?
ST30Q02 b) Making an effort in mathematics is worth it because it will help me in the work that I want to do later on
ST30Q05 e) Learning mathematics is worthwhile for me because it will improve my career <prospects, chances>
ST30Q07 g) Mathematics is an important subject for me because I need it for what I want to study later on
ST30Q08 h) I will learn many things in mathematics that will help me get a job

Note: Item categories were "strongly agree," "agree," "disagree," and "strongly disagree." Item categories were coded as 1 = strongly agree; 2 = agree; 3 = disagree; 4 = strongly disagree

Table 11.7 Items for measuring mathematics self-efficacy (MATHEFF)

How confident do you feel about having to do the following calculations?
ST31Q01 a) Using a <train timetable>, how long it would take to get from one place to another
ST31Q02 b) Calculating how much cheaper a TV would be after a 30% Discount
ST31Q03 c) Calculating how many square meters of tiles you need to cover a floor
ST31Q04 d) Understanding graphs presented in newspapers
ST31Q05 e) Solving an equation like $3x+5=17$
ST31Q06 f) Finding the actual distance between two places on a map with a 1:10,000 scale
ST31Q07 g) Solving an equation like $2(x+3)=(x+3)(x-3)$
ST31Q08 h) Calculating the petrol consumption rate of a car

Note: Item categories were "very confident," "confident," "not very confident," and "not at all confident." Item categories were coded as 1 = very confident; 2 = confident; 3 = not very confident; 4 = not at all confident

Table 11.8 Items for measuring mathematics anxiety (ANXMAT)

How much do you disagree or agree with the following statements about how you feel when studying mathematics?
ST32Q01 a) I often worry that it will be difficult for me in mathematics classes
ST32Q02 b) I am just not good at mathematics
ST32Q03 c) I get very tense when I have to do mathematics homework
ST32Q05 e) I get very nervous doing mathematics problems
ST32Q08 h) I feel helpless when doing a mathematics problem
ST32Q10 j) I worry that I will get poor <marks> in mathematics

Note: Item categories were "strongly agree," "agree," "disagree," and "strongly disagree." Item categories were coded as 1 = strongly agree; 2 = agree; 3 = disagree; 4 = strongly disagree

Table 11.9 Items for measuring mathematics self-concept (SCMAT)

How much do you disagree or agree with the following statements about how you feel when studying mathematics?
ST32Q04 d) I get good <marks> in mathematics
ST32Q06 f) I learn mathematics quickly
ST32Q07 g) I have always believed that mathematics is one of my best subjects
ST32Q09 i) In my mathematics class, I understand even the most difficult work

Note: Item categories were "strongly agree," "agree," "disagree," and "strongly disagree." Item categories were coded as 1 = strongly agree; 2 = agree; 3 = disagree; 4 = strongly disagree

References

Braswell, J. S., Lutkis, A. D., Grigg, W. S., Santapau, S. L., Tay-Lim, B., & Johnson, M. (2001). *The Nation's report card: Mathematics 2000*. Washington: U.S. Department of Education, National Center for Education Statistics.

Brown, T. A. (2006). *Confirmatory factor analysis for applied research*. New York: Guilford Press.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. In K. A. Bollen & J. S. Long (Eds.), *Testing structural equation models* (pp. 136–162). Newbury Park: Sage.

Bryk, A. S., & Raudenbush, S. W. (1992). *Hierarchical linear models: Applications and data analysis methods*. Newbury Park: Sage.

Campbell, J. R., Hombo, C. M., & Mazzeo, J. (2000). *NAEP 1999 trends in academic progress: Three decades of student performance, NCES 2000*. Washington: U.S. Department of Education, National Center for Education Statistics.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham: Open University Press.

Ferguson, R. F. (1998). Can schools narrow the African American-white test score gap? In C. Jencks & M. Phillips (Eds.), *The African American-white test score gap* (pp. 318–374). Washington: Brookings Institution Press.

Hickey, D. T., Moore, A. L., & Pellegrino, J. W. (2001). The motivational and academic consequences of elementary mathematics environments: Do constructivist innovations and reforms make a difference? *American Educational Research Journal, 38*(3), 611–652.

Lee, J. (2002). Racial and ethnic achievement gap trends: Reversing the progress toward equity? *Educational Researcher, 31*(1), 3–12.

Lee, V. E., & Bryk, A. S. (1989). A multilevel model of the social distribution of high school achievement. *Sociology of Education, 62*, 172–192.

Lubienski, S. T. (2001). *A second look at mathematics achievement gaps: Intersections of race, class and gender in NAEP data*. Paper presented at the American Educational Research Association, Seattle (ERIC, document number ED454246).

Lubienski, S. T., & Bowen, A. (2000). Who's counting? A survey of mathematics education research 1982–1998. *Journal for Research in Mathematics Education, 31*(5), 626–633.

Lubienski, S. T., & Shelley, M. C. (2003). *A closer look at U.S. mathematics instruction and achievement: Examinations of race and SES in a decade of NAEP data.* Paper presented at the 2003 Annual Meeting of the American Educational Research Association, Chicago, April, 21–25.

Lubienski, S. T., Camburn, E., & Shelley, M. C. (2004). *A closer look at mathematics instruction, achievement, and equity: Examinations of race and SES in 2000 main NAEP data*. Ames: Iowa State University.

Meyers, L. S., Gamst, G., & Guarino, A. J. (2006). *Applied multivariate research: Design and interpretation.* Newbury Park: Sage.

Newmann, F. M., Smith, B., Allensworth, E., & Bryk, A. S. (2001). Instructional program coherence: What it is and why it should guide school improvement policy. *Educational Evaluation and Policy Analysis, 23*(4), 297–321.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (2nd ed.). Thousand Oaks: Sage.

Rothstein, R. (2004). *Class and schools: Using social, economic, and educational reform to close the African American-white achievement gap*. Washington: Economic Policy Institute.

Schafer, J. (1997). *Analysis of incomplete multivariate data.* London: Chapman & Hall.

Schoenfeld, A. H (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13–25.

Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623–660). New York: Macmillan.

Shelley, M. C., & Lubienski, S. T. (2003). *Mathematics instruction and student achievement: Multilevel models and social structures in NAEP 2000 data.* Paper presented at the 2003 Annual Meeting of the American Educational Research Association, Chicago, April 21-25.

Shelley, M. C., & Lubienski, S. T. (2005). Do NCTM reform practices matter? Predicting precollege student mathematics achievement using NAEP 2000 data. In *2004 American Statistical Association Proceedings* (pp. 2363–2368). Alexandria: American Statistical Association.

Strutchens, M., Lubienski, S. T., McGraw, R., & Westbrook, S. K. (2004). NAEP findings regarding race/ethnicity: Students' performance, school experiences, attitudes/beliefs and family influences. In P. Kloosterman & F. K. Lester, Jr. (Eds.), *Results and interpretations of the 1990 through 2000 mathematics assessments of the National Assessment of Educational Progress* (pp. 269–304). Reston: NCTM.

Weis, L. (Ed.). (1988). *Class, race, and gender in American education.* Albany: State University of New York Press.

Chapter 12
Equity and Quality Issues in Mathematics Education in Malawi Schools

Mercy Kazima and Chikondano Mussa

Introduction

In this chapter, we use the term "equity in education" to refer to providing equal access and equal opportunities for education to all groups of learners and potential learners. In Malawi the groups of learners that are of concern regarding equity are female versus male learners and learners in rural versus urban areas. Concerns about equity between males and females are mainly about quantities in schools where the proportion of females reduces significantly as they go up the education system, and about performance in mathematics where females perform less than males at all levels. Concerns about equity between rural and urban students are mostly about access to resources where schools in rural areas have access to fewer resources than urban schools, have lower numbers of qualified teachers, and their performance in mathematics examinations is lower than urban schools. We argue that although there are prospects for providing equity and quality mathematics to learners from policies the government has put in place, and some associated initiatives in the form of funded projects, there is a lot that still needs to be done to address the inequities that exist. Furthermore, we highlight that the indicators of quality that the government has developed are all about resources which reflect a simplistic view of quality. We argue further that there is more to provision of quality mathematics than providing resources to schools. We draw upon findings from some studies in Malawi that support this argument.

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Background

School System

Malawi school system has eight years of primary school and four years of secondary school. Transition from one class to the next in primary school is according to merit. Learners have to pass each class of primary school to progress to the next. The tests to determine progression of learners are school based. Transition from primary to secondary school requires passing a national examination called the Primary School Leaving Certificate (PSLC). Admittance to secondary school is granted to learners according to merit on the examination. This exercise is extremely competitive because there are fewer places in secondary school than the student numbers that pass the PSLC examinations. For example in 2007, a total of 161,567 learners sat the PSLC exam, 115,670 passed but only 39,596 were allocated places in government and government-aided secondary schools (Ministry of Education 2008).

Malawi has experienced huge expansion of its education system during the past 15 years. At the primary level, the introduction of free primary education policy in 1994 greatly increased the number of learners enrolled. According to the Ministry of Education Statistics, primary school enrolment rose by almost 50% from approximately 1.9 million in 1994 to 2.9 million in the 1995 academic year (Ministry of Education 2000). This increase was mainly in rural areas where many school age children in the past had failed to attend school because of lack of ability to pay school fees. The increase in enrolment was registered in all classes of primary school which meant that some who had previously dropped out of school had returned. The most recent enrolment was about 3.6 million (Ministry of Education 2008). Table 12.1 shows the exact enrolment numbers for selected years from 1994 to 2008.

As Table 12.1 shows, secondary school sector has also expanded significantly. This has mainly been due to a 1998 Malawi government policy of secondary school expansion (Ministry of Education 2001). The policy provided for “distance education centres” to be converted into secondary schools. “Distance education centres” were widely distributed across the country—especially in rural areas. The centres offered secondary school education to learners that passed the PSLC examinations but did not qualify for a place in conventional secondary schools. The centres offered their education using distance mode with limited face-to-face interactions. The secondary school expansion policy converted each of these centres into what is called “community day secondary school”. These are second-class secondary

Table 12.1 Enrolment in Malawi primary and secondary schools. (Source: Ministry of Education 2008)

	1994	1995	1999	2005	2006	2007	2008
Primary	1,895,423	2,860,819	2,896,280	3,200,646	3,280,714	3,306,926	3,600,771
Secondary	48,360	57,812	75,959	183,854	218,310	210,325	233,573

schools with poorer infrastructure and resources than conventional secondary schools, and the students that are admitted are those that do not make it into the conventional secondary schools. The secondary school expansion policy also encouraged the private sector to contribute to the provision of secondary school education. As a result of this policy, the secondary school enrolment rose from a total of approximately 58,000 in 1995 to 184,000 learners in 2005. The most recent enrolment is approximately 234,000 (Ministry of Education 2008). Although secondary school education is not free, it is heavily subsidised by the government. As is evident from the statistics, enrolment continues to increase at both primary and secondary school levels.

Mathematics Education

Mathematics as a subject of study is taught as a compulsory subject in both primary and secondary schools. Mathematics is also one of the two subjects (other being English) that are compulsory at national examinations. Although a pass in mathematics is not a requirement for candidates to pass the examinations, passing mathematics offers increased opportunities to candidates. For example, entry into primary teacher education colleges requires at least a pass in mathematics, and many job opportunities and training include mathematics as a requirement or added advantage. Similarly, entry into most of the university programmes requires learners to have obtained a good pass in mathematics at Malawi Schools Certificate of Education (O-level equivalent) examinations and also pass university entrance examinations, which include numerical skills. There is therefore huge pressure on learners to demonstrate competence in mathematics; and consequently on teachers to produce mathematically proficient learners. The fact that mathematics is compulsory to all learners in schools makes the demand for qualified mathematics teachers greater than other subjects.

Equity Groups

Our sense of equity in education is that it refers to providing equal opportunities for education to all groups of learners and potential learners. For Malawi, equity concerns can be raised on urban versus rural learners, and male versus female learners.

Male versus Female Learners

Tables 12.2, 12.3 and 12.4 show the enrolment figures distributed by gender for primary, secondary and some higher education institutions in Malawi. It is clear

Table 12.2 Enrolment in Malawi primary schools in 2008 by class and gender. (Source: Ministry of Education 2008)

	Male	Female	Total
Standard 1	435,794	444,623	880,417
Standard 2	329,288	333,669	662,957
Standard 3	295,117	293,981	589,098
Standard 4	216,921	218,375	435,296
Standard 5	176,684	178,792	355,476
Standard 6	138,017	136,667	274,684
Standard 7	110,679	105,190	215,869
Standard 8	103,788	83,186	186,974
Total	1,806,288	1,794,483	3,600,771

Table 12.3 Enrolment in Malawi secondary schools in 2008 by class and gender. (Source: Ministry of Education 2008)

	Male	Female	Total
Form 1	36,279	30,595	66,874
Form 2	38,457	32,712	71,169
Form 3	28,160	19,794	47,954
Form 4	28,878	18,698	47,576
Total	131,774	101,799	233,573

Table 12.4 Enrolment in some higher education institutions in Malawi in 2008 by gender. (Source: Ministry of Education 2008)

	Male	Female	Total
University of Malawi	4,521	2,199	6,720
Mzuzu University	982	466	1,448
Technical colleges	934	434	1,368
Primary teacher education colleges	2,273	1,521	3,794
Total	8,710	4,620	13,330

from the tables that although the proportions of females is a little higher than males as learners enter primary school in Standard 1, the proportion of girls is reducing significantly as they go up the education system.

We note that for primary schools, while the reduction in numbers continuing from one year to another happens for both males and females, the reduction is greater for females than males at the last three years of primary education. This indicates that there are some factors that affect the education of girls much more than the education of boys (see Table 12.5). Some of these factors are external, that is they do not originate from the school system but from the wider social context. Such factors include early marriages, pregnancy and family responsibilities (Chimombo 2005; Ministry of Education 2008). Internal factors that affect the proportion of girls in primary schools include school conditions that are not conducive for girls, for example, lack of toilet facilities, fear of sexual harassment from teachers and fear of unjust punishment in the school (Chimombo 2005; Moleni 2001).

Table 12.5 Dropout rate for Malawi senior primary school in 2008 by class and gender. (Source: Ministry of Education 2008)

	Male	Female
Standard 6	7.38	11.16
Standard 7	4.11	13.21
Standard 8	5.23	20.37
Total	5.57	14.91

For secondary schools, we note from Table 12.3 that there were 5,684 more males than females that were offered places in Form 1 to start secondary education in 2008, and overall there were about 30,000 more males than females in secondary schools. We also note that for both junior and senior secondary the numbers are increasing for males from Form 1 to Form 2 and also from Form 3 to Form 4 while the numbers are decreasing for females. A possible explanation for this phenomenon is that more males than females repeat the years after not succeeding in national examinations at the end of Form 2 and Form 4. Although dropping out of female learners from secondary schools contributes to the large difference in numbers, most of the difference is accounted for by offering more places to males than to females. This is due to an internal factor at policy level which particularly affects proportion of female learners in secondary and higher education. For example, almost all the national boarding secondary schools were designed to have a 1:2 ratio of accommodation places available for girls and boys. Similar designs catering for larger numbers of boys than girls were made for higher education such as the University of Malawi, Mzuzu University, Technical Colleges and Teacher Education Colleges. There have been plans from both government and nongovernmental organisations to address this inequality by design. However, to date the inequality in student gender numbers still exists in almost all higher institutions because enrolment numbers are ultimately restricted by the available accommodation. This is one area where the government could have demonstrated commitment to gender equity beyond policy on paper by constructing additional hostels for females. Alternatively, they could convert some of the males' hostels to females' hostels which would not be as costly as constructing new hostels.

Gender disparity in favour of males in mathematics has persisted over a long time. Several studies have shown that in Malawi, the performance of females in mathematics is lower than that of males at all levels of Education (Condie et al. 2008; Chamdimba 2003; Mbano 2001). One explanation for the disparity is that there is low participation of females in many mathematics classrooms. Chamdimba (2003) studied mathematics teaching at secondary school and concluded that there is low participation of females in mathematics classrooms. She also found that many of the mathematics teachers are male and that they often allow male learners to dominate in class activities. Chamdimba (2003) suggests the use of cooperative learning as a way of encouraging and increasing the participation of females in mathematics classrooms. This dominance by male learners is not surprising especially in boarding schools where there are twice as many males than females. Nevertheless, as Chamdimba (2003) suggests, teachers can find ways of including female learners even when they are underrepresented in the classroom.

Urban Versus Rural School Learners

It is important to note that the majority of the Malawi population is rural; therefore, most of the schools are in rural areas. For example, there are a total of 5,461 primary schools in the country of which 5,088 are in rural and 373 are in urban areas (Ministry of Education 2008). For secondary schools, most of the community day secondary schools are in rural areas while most of the conventional secondary schools are in urban areas. Therefore, it is fair to see community day secondary schools as rural and conventional secondary schools as urban. There are differences in performance at mathematics national examinations between these two types of schools. On average, urban secondary schools consistently perform better than rural schools. For example, Mwakapenda (2002) found that over a period of five years, urban schools learners obtained an average mark of 53% while rural schools learners obtained an average mark of 16% in Malawi Schools Certificate of Education mathematics examinations. This suggests, among other things, that the quality of teaching and learning mathematics in rural schools is poorer than in urban schools.

Many would appreciate the difficulty in achieving equity between urban schools and rural schools because urban areas have access to resources outside school such as national libraries which are not within easy reach of rural schools. However, for resources that are supplied by the education system, an attempt could be made to distribute these fairly equitably. Studies that have studied equity among schools in Malawi (e.g. Chimombo 2005; Moleni 2001; Kuthemba 2000) have revealed that resources are not distributed equitably, with urban schools getting more and better resources than rural schools. Table 12.6 shows distribution of human resource and material resources in primary and secondary schools.

These ratios show the inequity that exists, implying that urban school learners are on average given better opportunities to education than rural school learners. The average teacher-to-learner ratio is the same for the two types of secondary schools but the qualified teacher-to-learner ratio is much higher for the community day secondary schools. However, the average class size is smaller in community day than in conventional secondary school. This is likely due to dropouts which is higher in rural than urban areas.

Table 12.6 Distribution of human and material resources in Malawi schools in 2008. (Source: Ministry of Education 2008)

	Primary schools		Secondary schools	
	Urban	Rural	Conventional	Community day
Teacher:learner ratio	1:49	1:83	1:20	1:20
Qualified teacher:learner ratio	1:51	1:97	1:27	1:68
Textbook:learner ratio	1:3	1:7	NA	NA
Classroom:learner ratio	1:101	1:114	1:52	1:44

NA indicates not available

Indicators of "Quality"

A state president's committee on education quality in Malawi (Kuthemba 2000) described some indicators of quality in Malawi schools. These include: (i) average number of learners per teacher, (ii) average number of learners per textbook, (iii) average number of learners per classroom, (iv) average number of learners per chair, (v) average number of learners per desk and (vi) dropout and repetition rates. According to Kuthemba (2000), the larger the average number of learners per teacher and per textbook the poorer the quality of available education. The assumption is that teachers with large numbers of learners would not have much time, if any, for individual learners. Similarly, the larger the number of learners per textbook the less time individual learners have to access the book. Numbers of classrooms and classroom furniture give an indication of the conditions of the schools the learners are in. Large numbers of learners per classroom show that there is not enough room to accommodate all learners such that some school may resort to the use of open space under trees as classrooms for learning. The large numbers of learners per chair and per desk show that some learners sit on the floor in school when learning. This is very common especially in primary schools. Dropout and repetition rates inform us of how effective the schooling system is to the learners and also how much wastage the system is experiencing. Wastage will be discussed in more detail later in the chapter. Table 12.7 shows the statistics for some of the indicators in primary schools in 1993 (before free primary education), in 1995 (soon after free primary education) and in 2008 (the most recent situation).

Perhaps the most important indicator is the teacher. Quality of teachers in terms of whether or not they are well trained and qualified is important because well-trained teachers are likely to cope better with large classes, poor resources and other situations prevailing in Malawi schools. For this reason, we will pay particular attention to the quality of teachers. Probably the biggest challenge faced by the Malawi Ministry of Education in terms of quality education is the provision of qualified teachers. There is shortage of qualified teachers in both primary and secondary schools. Ministry of Education statistics show that less than 50% of all the teachers teaching in public secondary schools in Malawi are qualified (Ministry of Education 2006). It was not possible to get the distribution of the percentage by subject but it is likely that the percentage of qualified mathematics teachers is lower than this average considering the low enrolment levels of student teachers majoring in mathematics at the various teacher education institutions.

One of the major factors that contribute to the shortage of teachers is low retention of qualified teachers by the Ministry of Education. For example, in 2008, a total of 1,261 teachers (402 for secondary and 859 for primary) left the education system (Ministry of Education 2008). This should be compared with the total number of teachers that are produced every year by teacher education institutions in Malawi. According to Ministry of Education (2008), all public secondary teacher education institutions in the country graduate an average total of 391 teachers per year. This number is less than the total number of 402 teachers that left the Ministry of

Table 12.7 Some indicators of "quality" in Malawi primary schools. (Sources: World Bank Discussion paper No. 350, Ministry of Education 2008)

Indicator	1993	1995	2008
Average number of learners per teacher	68	68	78
Average number of learners per qualified teacher	88	131	90
Average number of learners per textbook	3	7	5
Average number of learners per classroom	102	422	112
Average number of learners per chair	32	56	NA
Average number of learners per desk	18	31	NA
Total number of dropouts	NA	260,086	161,532
Total number of repeaters	329,872	474,806	698,205

NA indicates not available

Education in 2008, which is more or less the same number that leaves every year. It will not be possible for Malawi to overcome this challenge of shortage of teachers, and qualified teachers in particular, if more teachers are leaving than become available every year. This very low teacher retention rate also indicates a very high wastage in terms of teacher education expenditure by government. It was not possible to get teacher attrition numbers by subject, however, it is likely that mathematics teachers suffer the same or higher attrition rate as the average teacher. Since mathematics is taught to all learners, and has more time allocated in school timetable, schools suffer shortage of qualified mathematics teachers a lot more than they do for elective subjects. For the primary sector, all primary school teacher colleges in the country graduate about 1,596 teachers per year (Ministry of Education 2008). Comparing this with the 859 primary school teachers that left the education system in 2008, we see that it is more than half of the teachers produced each year. If this trend continues, it means that for every two years, a whole year's output is lost hence whole year's government's expenditure for a cohort in all primary teacher education colleges is wasted.

Low salaries and poor conditions of service are some of the main reasons that teachers resign. There may not be much that the government of Malawi would afford in the short-distance future, however, the very low retention rate of teachers should trigger plans of improving teachers' conditions of service in the long term. We believe there is another way of reducing attrition especially for newly qualified teachers who have been educated by public funds. Teacher education is free in the government primary teacher training colleges in Malawi, and is heavily subsidised at government secondary teacher education institutions and at public universities. Teachers who get their training through these heavily subsidised or free education should be bonded to teach in the system for a specified number of years. Currently, there is no bond at all such that teachers who benefit from these institutions are free not to teach in the system or not teach at all. This is one of the areas where public funds are wasted. Introducing a bond might also limit interested potential teachers to being those that are indeed willing to serve in the system and so reduce the attrition rate.

The importance of well-trained and qualified teachers cannot be overemphasised. As mentioned before, qualified teachers are likely to be better equipped to teach within the constraints of Malawi schools. The qualified teachers go into teaching fairly well aware of the challenges since they do their teaching practice in the same. Untrained teachers on the other hand, face these challenges while grappling with their own limitations in how to deal with them. While we appreciate the necessity of unqualified teachers where there are no qualified teachers, unqualified teachers should not be a long-term solution to teacher shortage. We would recommend that Malawi invests in teacher education and find feasible ways of training the unqualified teachers. Malawi might not be able to provide well for all its learners in the school system, but should strive at providing qualified teachers. This we believe is a crucial step towards quality education.

We note that while Malawi was trying to achieve quantity in terms of numbers of learners enrolled in schools, quality was overlooked. Free primary education was introduced before any preparations were in place (Chimombo 2005), consequently, the system was overwhelmed by the huge expansion in enrolment. Politically, the large numbers of enrolment might look good but practically the schools are heavily stretched and struggling to cope. It would have been more productive if the free primary education was planned some years ahead of time by increasing teacher education output as well as planning and implementing strategies for teacher retention.

Another area where Malawi could do better is in the utilisation of the limited resources that are available. As observed in Table 12.7, the school system has high numbers of dropouts and repeaters. With free primary education and heavily subsidised secondary education, the costs are wasted when learners drop out or repeat the year. Therefore, there is a lot of wastage of resources (Kuthemba 2000). Looking at the total number of 161,562 learners that dropped out of primary school in 2008, we note that it is equivalent to 2,071 average size classes or 259 single-stream primary schools. We can also look at the number of repeaters in a similar manner. The total number of 698,205 repeaters translates to 8,952 classes and 1,119 single-stream primary schools. As was observed, there are many reasons for dropping out and the school system cannot address them all, however, they can address the issues in the long term. Repetition is one factor that encourages dropout because some learners would rather leave school altogether than repeat the year. We believe that most of the repetition is unnecessary and that with carefully planned support to learners there would be no need for learners to repeat years in school. There is need for some sound policy on repetition; a policy that would significantly reduce repetition rates. Reducing repetition would reduce wastage and also reduce dropout rates and so further reduce wastage.

Prospects of Change

Malawi has stated its commitment to mathematics, science and technology human capacity development. Her policy initiatives in this regard began with the vision 2020. Launched in 1997, the vision is to change Malawi's economic base from reliance on physical labour to a technologically driven economy (Malawi Government 1999) through the development of human capacity in mathematics, science and technology, by the year 2020. This vision is reflected in the National Science and Technology Policy Paper (Malawi Government 2001) which argues for expansion of University education "especially in the scientific, engineering and technological fields" (p. 22) and the expansion of postgraduate programmes. The Malawi Poverty Reduction and Strategy Paper, (Malawi Government 2002) and more recently, the Malawi Growth and Development Strategy Paper (Malawi Government 2006), which represent government's blue print for poverty reduction, also echo the need for mathematics, science and technology human capacity development. These commitments by the government offer prospects for providing quality mathematics to Malawi learners.

There are a number of prospects in the form of in-service teacher education initiative projects. We will give three examples here. The first is the Strengthening of Mathematics and Science in Secondary School Education (SMASSE) project, which started in 2001 and is funded by the Japanese Government through the Japan International Cooperation Agency (JICA). The project aims at improving the teaching and learning of mathematics and science subjects by encouraging mathematics and science teachers to share their professional knowledge and skills, and to assist under-qualified teachers in schools. SMASSE started by mainly targeting under qualified mathematics and science teachers in one of the six education divisions in Malawi. Now the project plans to include all secondary schools in the country. Second example is an initiative funded by African Development Bank, and is called Secondary School Teacher Improvement Programme. The programme emphasises on the improvement of teachers' content knowledge and particularly targets unqualified teachers teaching in community day secondary schools. The programme started in 2004 as in-service without academic qualification. It later developed into a Diploma in Education course through distance mode at Domasi College of Education. The third example is a smaller project called the Hands-on-Activities for secondary mathematics and science. This is funded by USAID and is implemented by the University of Mzuzu in the northern part of Malawi. The purpose of this project is to improve the teaching of mathematics and science by developing and implementing hands-own, interactive teaching methodologies that use readily available and inexpensive materials. The project involves assessing the needs of schools, developing hands-on activities appropriate for the schools and offering in-service training for teachers on the use of such activities in their classes. This project has worked with teachers in some parts of the northern region of Malawi.

Critical Reflection of Prospects

Looking at the prospects for change, government's commitment is in the form of policy and strategy papers, which do not seem to translate into observable actions of implementation. For example, in 2006 the government announced its intentions to establish a new university of science and technology in response to the National

Science and Technology Paper (Malawi Government 2001), but until now—almost five years on, there is no visible evidence to indicate serious plans of having this university. The policy and strategy papers indeed indicate commitment to mathematics and science education; nevertheless, they should have been followed by clearly observable actions which would have demonstrated the commitment. Such actions could include working with mathematics teacher education programmes, mathematics curriculum developers and mathematics teachers in schools to discuss the mathematics to be taught in schools that would suit the objectives of the policy papers, and also possible ways of teaching the mathematics in schools.

To a small extent, some observable actions were done for secondary schools through the in-service teacher education initiative projects presented in the previous section. It is appropriate that all the projects are implemented through teacher education institutions. However, there seem to be no coordination between the projects, each seems to run independently with no acknowledgement of the existence of the others. Coordination among these projects which all have a general aim of improving the teaching and learning of mathematics and science, would have made the projects richer and more likely have stronger products. Looking at the examples of projects which offer prospects for Malawi, one would notice that they are all for secondary school and not primary school mathematics, and that they are all funded by external bodies. This emphasis on secondary education and little attention to primary is problematic because improving teaching and learning of mathematics at secondary can only be effective for learners if their primary mathematics was learned well. The fact that all projects are funded by external bodies is also problematic if continuity is not guaranteed after the project funds expire. It is commendable that teacher education institutions through government manage to secure funding for projects such as these; nevertheless, it is important to have plans for after the project phase. Apart from the SMASSE project where the government contributes to the in-service training of teachers as agreed with the Japanese government, we are not aware of any other commitment for the other projects. This is one area where the government could demonstrate its commitment by supporting the initiatives during and/or after the project funds expire. This support does not have to be in large amounts of funding but in ensuring that some use is made of the products of the project. For example, the teachers trained in the Hands-on-Activities for mathematics project, could be used to train other teachers within their districts at local in-service training.

The prospects, though problematic in many ways, illustrate the efforts and commitment that Malawi as a nation is making towards the teaching of mathematics and towards education in general. However, Malawi still faces some huge challenges in its attempt to provide quality mathematics to all. The drastic increase in school enrolment has put tremendous pressure on the already meagre resources. These problems affect all subjects but are more acute for mathematics because it is attended by all learners.

According to the performance of learners at national examinations, there are some schools that seem to cope well in teaching mathematics despite the challenges Malawi faces. Research on such schools with the aim of learning from them how they cope and manage to teach effectively would be useful in helping other schools. Nampota (2009) studied an exemplary secondary school that has high pass rates for physical science at Malawi Schools Certificate (O-level equivalent) examinations. Nampota found that effectiveness of teaching science at this school was not necessarily due to availability of resources but the culture of the school. At this school, teachers worked closely together, for example, they could discuss lesson plans and they sometimes did team teaching. At the school, learners were encouraged to work in groups both in and outside the classroom. According to the learners, the groups were useful because there were always people to ask questions or discuss with apart from the teacher. Although Nampota did not study mathematics, this school also has high pass rates for mathematics and it is likely that their success in mathematics is also due to the same reasons. Another study, Chamdimba et al. (2008), conducted a base line survey of schools that have consistently performed well at national examinations and those that have not performed well. They found that the differences in the schools were not in resources or number of teachers as they had expected but rather in the administration and organisation of the schools. The main finding was that discipline is a major factor. The well-performing schools had good discipline in both learners and teachers. Teachers arrived at school in time and remained in school throughout the working hours. The teachers prepared their lessons and had records of evaluations and learners' progress. Learners were also disciplined in terms of attending classes and doing their school work. In comparison, the poor-performing schools were not disciplined, teachers could come, teach their classes and go as they pleased during working hours, some could be absent without reason, as such there was very little interaction between the teachers. Learners were also not very disciplined in terms of attendance; there were a lot of absenteeism and skipping classes. Another interesting finding was the teachers' perception of their learners. In the well-performing schools teachers generally perceived their learners as good and hard working while in the poor-performing schools teachers generally perceived their learners as not that good and not hardworking.

These findings are very important because the two sets of schools (well-performing and poor-performing) had similar kinds of resources: teachers, textbooks, classrooms and furniture. The difference was in the administration of the schools. This is intriguing because heavy emphasis is placed on resources such that any lack of effective teaching and learning is blamed on resources. Looking at the indicators of quality that Malawi has developed, we notice that they are all about resources. While resources are indeed an important prerequisite in achieving quality in education, it is not the only factor. Administration of the institutions, work ethics of teachers and attitudes of both teachers and learners towards education are also very important. Schools need good administration, good interaction between teachers, shared vision or understanding of effective and quality education, good attitudes towards education, among other things. Without these no amount of resources can result in effective teaching and learning. Therefore as Malawi continues to strive for quality education, and quality mathematics education, efforts should be made to address these issues. There is a lot that can be done to achieve better quality of mathematics education within the constraints of resources that the Malawi schools face.

Conclusion

In this chapter, we have discussed equity and quality issues in mathematics education in Malawi. Although most of the issues apply to education in general, we have highlighted specific issues in mathematics education. We have shown the inequity that exists in terms of opportunities for mathematics education where male learners and urban school learners have an advantage over female learners and rural school learners, respectively. This implies that females in rural schools are the most disadvantaged. Therefore, there is a need for sound policies and initiatives to address these inequities with special attention to females and rural schools. We have argued that Malawi can achieve better equity among its groups of learners by addressing the causes, most of which originate from earlier policies. We have also argued that the indicators of quality that the government developed assume that the more the resources the better the quality of education. However, this does not necessarily follow as evident from research. Better quality mathematics education can be achieved by taking a more comprehensive view of quality and striving towards it. We have emphasised the importance of providing qualified teachers as a crucial step towards quality mathematics education. While we appreciate the limitations within which the Malawi education system operates, we are of the view that there is more Malawi can do within the limitations to provide more equity among its groups of learners and better quality mathematics education in schools.

References

Chamdimba, P. C. (2003). Gender related differences in working style during cooperative learning in secondary school mathematics: A Malawian case study. In Burton, L. (Ed.), *Which way social justice in mathematics education?* USA: Greenwood.

Chamdimba, P., Kamwanja, L., Condie, R., & Chambers, P. (2008). Baseline survey report for DFID Delphe Project.

Chimombo, J. (2005). Quantity versus quality in education: Case studies in Malawi. *International Review of Education, 51,* 155–172.

Condie, R., Chambers, P., Kamwanja, L., & Chamdimba, P. (2008). *Girls into science and technology in Malawi: Overcoming barriers.* Paper presented at the BAICE International Conference, September 2008, Scotland.

Kuthemba, J. (2000). *The State President's Committee on Education Quality in Malawi.* A report to the State President of Malawi (Unpublished). Lilongwe, Education Planning Division.

Malawi Government. (1999) Vision 2020: Long term development framework. http://www.sdnp.org.mw/~esaias/ettah/vision-2020/chapter-8.htm. Accessed 11 Nov 2004.

Malawi Government. (2001). *National science and technology policy paper.* Lilongwe: National Research Council of Malawi.

Malawi Government. (2002). *Malawi poverty reduction strategy paper.* Lilongwe: Government of Malawi.

Malawi Government. (2006). *Malawi growth and development strategy paper*. Lilongwe: Government of Malawi.

Mbano, N. (2001). Cognitive acceleration through science education in Malawian schools. In. V. Reddy, P. Naidoo, & M. Savage (Eds.), *Gender, science and technology* (pp. 19–35). Durban: University of Durban-Westille.

Ministry of Education. (2000). *Education statistics 2000*. Lilongwe: Education Planning Division.
Ministry of Education. (2001). *Malawi education sector: Policy and Investment Framework (PIF)*. Lilongwe: Government of Malawi.
Ministry of Education. (2006). *Malawi government education statistics 2006*. Lilongwe: Education Planning Division.
Ministry of Education. (2008). *Education statistics. Education Management Information Systems.* Lilongwe: Department of Education Planning.
Moleni, C. (2001). *Factors affecting the quality of education in primary schools.* An overview of needs assessment for CCAP synod of Livingstonia Education Department. Centre for Educational Research and Training (CERT), University of Malawi, Zomba.
Mwakapenda, W. (2002). The status and context of change in mathematics education in Malawi. *Educational Studies in Mathematics, 49*(2), 251–281.
Nampota, D. (2008). *Exploring strategies for effective teaching and learning of physical science in Malawi.* Proceedings of the 16th conference of the Southern Africa Association of Mathematics Science and Technology Education (SAARMSTE). 14–18 January 2008, pages 390–397, National University of Lesotho, Maseru, Lesotho.

Chapter 13
Looking for Equity in Policy Recommendations for Instructional Quality

Enakshi Bose and Janine Remillard

Introduction

In 2008 two national policy reports addressing the improvement of mathematics education in the United States were released. Convened by a presidential executive order, the National Mathematics Advisory Panel (NMAP) released *Foundations for Success* in March 2008. In June 2008, the National Council on Teacher Quality (NCTQ), a nonpartisan advocacy group, published *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by American Education Schools* (National Council on Teacher Quality 2008). Both reports frame their recommendations as responses to a continued crisis of low achievement and under-performance in mathematics of students in the United States. Both cite U.S. students' weak standings in national and international comparisons, and both warn of dire consequences of inaction for the nation's economic competitiveness, innovation leadership, and quality of life. A year later, in June 2009, the Carnegie Corporation of New York and the Institute for Advanced Study (Carnegie-IAS) released *The Opportunity Equation: Transforming Mathematics and Science Education for the Global Economy*. Echoing the urgent call for change, this report positions mathematics and science education as central to all school reform initiatives. Together, these reports consider many components of mathematics education, including curricular content, standards and assessment, and instructional practices; all identify teacher quality as a critical lever for improving the mathematics learning of all students.

In this chapter, we examine these policy reports using an equity lens. It is our view that addressing the system-wide challenges in mathematics education requires understanding the problem of inequitable access to quality mathematics instruction; teaching for quality and teaching for equity are synergistic. We frame equity along

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two dimensions: the kinds of mathematical knowledge to which students have access and the opportunities for students to participate in classroom environments that support significant mathematics learning. In this analysis, we examine the extent to which each report explicitly addresses equity and also what the frameworks and recommendations imply for furthering an equity agenda. We offer our analysis as an examination of a single case of the relationship between the rhetoric of national policy and the challenge of achieving equity and access in an education system. Like every country, the United States possesses a number of particularistic features, including a highly decentralized school system with strong market influences and a political system grounded in democratic ideals. Nevertheless, the struggle to increase educational access and opportunity within its schools is one shared by educators and policy makers in many national and regional school systems. We believe a close examination of recent U.S. policy documents from an equity perspective can be instructive, particularly on account of both the espoused commitment in the United States to equal opportunity and the influential role the United States has in the world.

Policy reports that examine and critique mathematics instruction are not neutral or apolitical; they articulate goals and visions from a particular stance. The reports analyzed here, for example, straddle two presidential administrations and evoke a continued federal/national interest in education. In the United States, this national emphasis sustains a growing shift from a traditionally local and decentralized education system to one in which the federal government assumes roles such as monitor and protector of the nation's mathematical literacy (Lappan and Wanko 2003). How local stakeholders—states, districts, and teacher education programs—take up these recommendations will vary. Nevertheless, national reports do have status within ongoing discussions about mathematics education reform. Spillane (2008) observes:

> National reports are one mechanism through which administrations attempt to lay out, reiterate, and press their agendas. Although not policy, individuals and agencies from Congress to the statehouse use reports, along with other policy texts, to inform and justify particular policy alternatives. …Whether or not the particular recommendations put forth in the national report are implemented, the report will contribute to the ongoing policy discourse by reifying them. (p. 638)

Assessing how these reports speak to equity concerns is critical, as it allows us to understand both the potential and the limitations of these visions for improving mathematics learning.

Equity as an Analytical Lens

In order to examine the stance toward equity implicit in these policy documents, we developed an analytical lens that includes two dimensions: the kinds of mathematical knowledge to which students have access and the opportunities for students to participate in classroom environments that support significant mathematics learning.

Equity as Access to High-Value Mathematical Knowledge

The kinds of mathematical knowledge to which students have access and the purposes ascribed for learning mathematics are integral to any equity agenda. Historically mathematics competence has been applied as a filter for higher education and employment opportunities, creating an inequitable distribution not only of income but of skills and power (Moses and Cobb 2001). A central component of equitable mathematics instruction is access to mathematical knowledge that has social, academic, and economic currency.

For over two decades, educators and researchers have challenged existing conceptions of mathematical knowledge as the ability to memorize and apply rules and procedures, noting both the lack of flexibility and connectedness in this way of knowing and asserting that it has limited value beyond that assigned to it by schools (National Research Council [NRC] 1989; National Council of Teachers of Mathematics [NCTM] 1989, 2000; Skemp 1978). Haberman (1991) used the term "pedagogy of poverty" to describe instructional approaches characterized by low expectations, tight teacher control, and focus on isolated skill development with limited opportunities for critical thinking. This pedagogy, he argued, which is the "coin of the realm" in urban schools, produces graduates who possess basic skills but are "nonthinking, underdeveloped, unemployable youngsters" (p. 294).

Haberman's (1991) call for "good teaching" to replace the pedagogy of poverty bears a family resemblance to the curricular emphasis on conceptual understanding, reasoning, problem solving, and communication advocated by many mathematics educators (Hiebert et al. 1997; Carpenter and Lehrer 1999). This emphasis is central to both sets of curriculum standards published by NCTM (1989, 2000), which outlined mathematics knowledge both in terms of content domains—number and operations, algebra, geometry, measurement, and data analysis and probability—and thinking processes such as problem solving, reasoning and proof, communication, and representation. Carpenter and Lehrer (1999) suggest that such conceptually connected mathematics knowledge is generative: "When students acquire knowledge with understanding, they can apply that knowledge to learn new topics and solve new and unfamiliar problems" (p. 19). Others argue that mathematics knowledge that includes reasoning and problem solving prepares young people to participate in a democratic society and succeed in a technology-based, post-industrial economy (Boaler and Staples 2008).

Some scholars and educators are quick to warn of the potential dangers of placing sole emphasis on conceptual knowledge and process skills. Kilpatrick et al. (2001) apply the term *mathematical proficiency* to describe knowledge that integrates procedural and conceptual understanding. Mathematical proficiency comprises five intertwined strands: conceptual understanding, procedural fluency, strategic competence (ability to represent and solve problems), adaptive reasoning (including reflection, explanation, and justification), and a productive disposition to viewing mathematics as useful. Equity in mathematics education, from this perspective, includes access to the mathematics skills, discourses, and ways of thinking that are valued and rewarded in mainstream culture. Some argue further that mathematical proficiency is necessary for students to engage in critical analysis of social conditions and unjust power relations in their lives—what Gutstein (2006) calls "reading the world"—in order to begin to effect change (see also Gutierrez 2002).

Equity as Opportunities to Learn

In addition to providing students access to powerful and socially valued mathematics, achieving equity includes ensuring that students have the support they need to learn it. The research evidence is clear that the instructional environment and teachers' practices are critical to creating opportunities for students to learn mathematics (Hiebert and Grouws 2007; Franke et al. 2007). The opportunities-to-learn dimension is particularly important in our discussion of equity for two related reasons. First, teaching for mathematical proficiency, as discussed above, is considerably more challenging than teaching a set of rules and procedures (Ball 2001; Schoenfeld 1998). Students must learn to reason about, think with, and apply knowledge of procedures and relationships in a variety of situations. Stein et al. (1996) used the term *high cognitive demand* to refer to instructional tasks that call for and emphasize meaning and thinking. Their research on middle school mathematics classrooms revealed that teachers routinely allowed the cognitive demand of tasks to drop. Teaching for high cognitive demand requires providing adequate support and scaffolding of students that assists them in understanding the task, making connections to their existing understanding, and accomplishing the task without reducing its overall complexity (Henningsen and Stein 1997).

The opportunities-to-learn dimension is also important when considering instructional practices appropriate for students who traditionally have been marginalized in education (and in other social institutions). The tendency for instructional practices in classrooms populated by low-income and racial minority students to focus on low-level skills has been well documented (Anyon 1981; Dowling 1998; Haberman 1991). Moreover, a number of studies have illustrated the ways that discourses valued in school tend to reflect ways of interacting and thinking common in the homes of children from dominant social and economic groups; these discourses are often foreign to children from homes on the margins of society (Delpit 1988; Lerman 2000; Lubienski 2000; Zevenbergen 2000). As a result, teaching for equity in mathematics requires instructional practices that support students in gaining access to and developing the kinds of skills necessary for mathematical proficiency. Research on successful teachers in low-income settings highlights the importance of helping students build on their existing knowledge to develop new understandings and addressing gaps in their knowledge without reducing expectations for conceptually complex thinking and problem solving (Ladson-Billings 1997).

Overview of Reports and Methods

The three reports analyzed here are recent contributions to the ongoing discussion in the United States about improving mathematics instruction. We chose to examine these documents because of their focus and timeliness. Whereas other policy reports have attended to relevant topics such as standards and accountability, or teacher recruitment and retention, these three reports allot considerable attention to the nature and quality of mathematics instruction. Each report speaks to an overlapping audience of educators, policy makers, and local, state, and federal school personnel.

Published within a year and a half of each other, the three reports are contemporaneous, and the more recent reports reference the earlier ones. All three reports frame the problem in similar ways: evidence from countless national and international assessments suggest mediocre mathematics achievement and learning in the United States. The reports share concerns about how such weak and incomplete learning may impact the future of individual citizens and the country as a whole.

Commissioned by an executive order from the Office of the President of the United States and comprised of scholars, researchers, and education officials, the NMAP was charged with examining research on "proven-effective and evidence-based mathematics instruction" and developing recommendations regarding instructional practices and programs, standards and assessments, curricular content, teacher recruitment and professional development, and research agendas (NMAP 2008, p. 71). Released in March 2008, the NMAP report warns that weak student performance in mathematics threatens not only U.S. competitiveness in the global economy but also national security and quality of life. The report attributes this dire state to system-wide failure:

> This Panel, diverse in experience, expertise, and philosophy, agrees broadly that the delivery system in mathematics education—the system that translates mathematical knowledge into value and ability for the next generation—is broken and must be fixed. This is not a conclusion about teachers or school administrators, or textbooks or universities or any other single element of the system. It is about how the many parts do not now work together to achieve a result worthy of this country's values and ambitions. (p. 11)

The NMAP focused on the preparation of students for entry into and success in Algebra, framed in the report as "a demonstrable gateway to later achievement" (p. xiii). The NMAP report heralded a significant and unprecedented federal interest in influencing and shaping school mathematics education.

Three months after publication of the NMAP report, the NCTQ, a nonpartisan advocacy group, released *No Common Denominator*, a report focused on the mathematics preparation prospective teachers experience in education schools in the United States. In this report, a Mathematics Advisory Group of mathematicians and K-12 mathematics teachers analyze the program requirements and mathematics coursework in a sample of teacher preparation programs across the country. The NCTQ echoes concerns over students' poor performance on international and national tests, and attributes this in part to weak content knowledge of elementary mathematics teachers: "…We are now on a treadmill in education. We fail to teach mathematics well, and our weak students become the next generation of adults, some of whom become the teachers who produce the next crop of weak students" (p. 61). It advocates reforms to teacher preparation programs, emphasizing mathematics content coursework.

In June 2009, the Carnegie Corporation of New York and the Institute for Advanced Study (Carnegie-IAS) Commission on Mathematics and Science Education released *The Opportunity Equation.* Comprised of mathematicians and scientists, educators and researchers, and leaders of business and cultural organizations, the commission urges action to bridge the gap between the current system and the demands of the future: "We believe that mathematics and science education as currently provided to most American students falls far short of meeting their future needs or the needs of society" (p. 7). An overarching recommendation is to "'Do school differently' in ways that emphasize the centrality of math and science to educational improvement and innovation" (p. 2). The report also emphasizes repeatedly the dual needs for excellence and equity in math and science education, noting that these are critical for innovation, economic growth, and democratic participation.

Data Analysis

Using document analysis, we first examined how each report addresses equity concerns explicitly. We then considered implications of the report recommendations for the kinds of content students should learn and opportunities to learn. We coded each report in its entirety, but focused on sections describing curricular content, instruction, and teachers and teaching. Finally, we looked across the three reports with respect to the two dimensions of our framework, developing conjectures about the equity stance implicit in each and looking for patterns across the three.

Analysis: Looking for Equity

In the following sections, we present our findings. We begin by describing how the reports attend to equity concerns explicitly. We then discuss the reports' recommendations for content and instruction through an equity lens, noting how each defines the nature of mathematics to be learned and addresses the opportunity-to-learn dimension of instruction.

Explicit Attention to Equity

The Carnegie-IAS report is the only one of the three that explicitly acknowledges equity as a goal of mathematics education reform. The report's first sentence confronts the problem of inequitable access to effective, quality mathematics instruction:

> The United States must mobilize for excellence in mathematics and science education so that all students—not just a select few, or those fortunate enough to attend certain schools—achieve much higher levels of math and science learning. (p. 1)

The report urges a transformation of mathematics and science education to "deliver it equitably and with excellence to all students" (p. 1). Framing mathematics and science knowledge as "fundamental to sound decision making" (p. 1) as well as to a range of fields, from technology and business to health and human services, *The Opportunity Equation* regards deep mathematics and science learning as integral to both individual attainment and national growth and innovation. The report suggests the following goal:

> Knowledge and skills from science, technology, engineering, and mathematics—the so-called STEM fields—are crucial to virtually every endeavor of individual and community life. All young Americans should be educated to be 'STEM-capable,' no matter where they live, what educational path they pursue, or in which field they choose to work. (p. 2)

The Carnegie-IAS attention to equity contrasts starkly with that of the other two reports. The NMAP proposes a qualified call for equity as access to an algebraic course pathway: "All school districts should ensure that all prepared students have access to an authentic algebra course—and should prepare more students than at present to enroll in such a course by Grade 8" (p. 23). The NMAP report primarily refers to inequity using language about achievement and learning gaps between groups of students (by race or socioeconomic status). The NMAP suggests that there are effective techniques to improve early childhood mathematics education, particularly for those students from low-income backgrounds, but cautions that research on efficacy of such interventions is needed. Meanwhile, in its analysis of teacher preparation programs, the NCTQ report does not address issues of equity. Indeed, it appears that expanding access to quality mathematics instruction is not part of the report's agenda. The report states, "The fact that a large and increasing number of teacher candidates applying for admission to teacher preparation programs are transferring from two-year institutions further underscores the need to establish a uniform and higher threshold for admission [to teacher preparation programs]" (p. 53), implying that excluding certain teacher candidates is preferable to developing opportunities to increase their mathematics knowledge and to broaden the pool of prospective teachers.

A Narrow Conceptualization of Mathematics

Notably, all three reports refer to "mathematical proficiency" (Kilpatrick et al. 2001) in their descriptions of the mathematics students need to learn. However, beyond this cursory acknowledgement, the NMAP and NCTQ reports emphasize procedural fluency (with conceptual understanding framed as a facet of fluency) over other aspects of proficiency. The Carnegie-IAS report articulates a more comprehensive vision of mathematical proficiency, but its suggestions of how such a vision might be realized are vague. As such, the reports construe mathematics narrowly, with primary emphasis on procedural knowledge and the accumulation of facts and skills.

Both the NMAP and NCTQ reports suggest a skills-oriented trajectory for learning mathematics. The reports characterize mathematical knowledge as sequential and cumulative, and then assert that this necessitates teaching mathematics through a fixed sequence. The NMAP claims

> …the structure of mathematics itself…requires teaching a sequence of major topics (from whole numbers to fractions, from positive numbers to negative numbers, and from the arithmetic of rational numbers to algebra) and an increasingly complex progression from specific number computations to symbolic computations. The structural reasons for this sequence and its increasing complexity dictate what must be taught and learned before students take course work in Algebra. (p. 17)

Both reports organize lists of these foundational skills as "Essential Components" (NCTQ) or "Critical Foundations" (NMAP). However, while the reports allude to conceptual understanding as important, neither report operationalizes the term. Descriptions of skill development do not mention what deep understanding entails.

Both the NMAP and NCTQ reports privilege instruction of mathematical procedures and suggest that procedural skills must be established in order for students to solve problems. This sequencing is a reversal of the stance advocated by some researchers of mathematics learning that sense-making is part of mathematics learning and problems are authentic places in which learners both apply prior knowledge and construct new understandings. The NMAP report advises:

> By the term *proficiency,* the Panel means that students should understand key concepts, achieve automaticity as appropriate (e.g., with addition and related subtraction facts), develop flexible, accurate, and automatic execution of the standard algorithms, and use these competencies to solve problems. (p. 22)

In its discussion of what teachers need to know about mathematics, the NCTQ's attention to problem solving is limited: "[Teachers] should learn that a large variety of word problems can be solved with either arithmetic or algebra and should understand the relationship between the two approaches" (p. 55).

The Carnegie-IAS report articulates a different view of mathematical thinking:

> What is too often missing today for students at all levels is a focus on acquiring the reasoning and procedural skills of mathematicians and scientists, as well as a clear understanding of math and science as distinct types of human endeavor. Learning math and science from textbooks is not enough: students must also learn by struggling with real-world problems, theorizing possible answers, and testing solutions. (p. 13)

The Carnegie-IAS report contends that mathematical knowledge involves not only procedural prowess but also "habits of mind and methods for discerning meaning that enable students to learn deeply and critically" (p. 7). In this vision of mathematics, grappling with real problems is part of the process of learning mathematics, not an outcome following skill acquisition. However, the Carnegie-IAS report does not develop fully what such a curriculum entails; in contrast with the lists and tables of essential, foundational skills in the NMAP and NCTQ reports, the Carnegie-IAS report provides no comparable suggestions. As a result, how stakeholders take up such recommendations is open to interpretation.

We also examined each report's emphasis on particular domains in mathematics. Both the NMAP and NCTQ share a preoccupation with algebra and its place in the education pipeline, preparing students for a college (calculus) trajectory. In contrast, the Carnegie-IAS report suggests the development of a rigorous high school mathematics course sequence that attends to statistics, data analysis, and discrete mathematics operations. It explicitly notes that such a pathway should be as rigorous as the preexisting calculus pathway, so as not to be confused with an ability-tracking program. This proposal broadens the scope of mathematics emphasized and has the potential to encompass student mathematical sense-making, but at present is not developed in the same depth as the traditional arithmetic/algebra/calculus trajectory.

Limited Attention to Role of Instruction in Opportunities to Learn

The instructional learning environment and teacher practices are critical to supporting opportunities to learn mathematics for understanding. Teaching for mathematical proficiency requires attending to more than the mathematical content to be learned. Dimensions of instruction such as the selection and design of tasks for teaching particular concepts, the use of tools and representations, the management of classroom discourse, and the development of class norms that support reasoning and justification are essential to fostering learning environments in which all students may have access to significant mathematics (Kilpatrick et al. 2001; Boaler 2002; Boaler and Staples 2008; Hiebert et al. 1997; Franke et al. 2007). Across all three reports, the limited attention to instructional practice in the recommendations to improve mathematics achievement is noteworthy.

A prevalent implicit perception is that of mathematics teaching as content delivery. The NCTQ report acknowledges that teacher practices such as using formative assessment to understand student (mis)conceptions are important, but does not suggest how mathematical tasks or classroom discussions might support the development of ideas. The NMAP report constrains its recommendations by focusing on extreme approaches of instruction—teacher-directed or student-centered. While it notes that research does not support the exclusive use of either approach, the NMAP makes few suggestions about what is important to instruction, like student investigation and explanation, or how teachers set up tasks or foster class discussion. The NMAP report recommends "explicit systematic instruction" (p. 48) on a regular basis for students with learning disabilities. Such instruction would include teacher demonstration of specific strategies and opportunities for student practice. How to prevent this instruction from becoming routine and drill-oriented is unclear.

The Carnegie-IAS report diverges from the other reports in its description of the mathematical activity necessary for attaining excellence and equity. It recommends, for example, the use of instructional materials that are "rigorous, rich in content, motivating, and clearly connected with the demands of further education, work, and family and community life" (p. 51). This recommendation implies that instruction is more than content delivery, but is vague on what such practice entails.

Discussion

Our analysis reveals that even as these reports make some useful recommendations with respect to improving the quality of mathematics instruction, they overlook the implications of their proposals for issues of equity. In this section, we draw on our framework to consider the implications for equity of our two key findings—that the reports frame mathematics knowledge narrowly and privilege content knowledge over other aspects of teacher knowledge for practice.

Access to High-Value Mathematical Knowledge

To what extent do these reports emphasize a conception of mathematics knowledge likely to increase access and equity? As indicated earlier, both the NMAP and the NCTQ reports place central emphasis on the procedural aspects of mathematics knowledge and argue for learning sequences that position problem solving and reasoning as advanced skills to be taken up only once basic procedural and factual knowledge is mastered. This segmented conception of mathematical learning ignores research findings that demonstrate positive learning outcomes from instruction in which problem solving and strategy development are integrated with the teaching and learning of number sense and basic operations (Carpenter et al. 1989). Also, in prioritizing automaticity and procedural prowess, these reports overlook the significance of maintaining cognitive complexity throughout instruction (Stein and Lane 1996). Clearly, one intention of these two reports is to guard against instructional practices that disregard procedural fluency as a goal, and we concur that a mathematics education that does not include procedural fluency will not achieve equity.

At the same time, a conception of mathematics that focuses narrowly on procedural skills to be taught in a rigid sequence is equally perilous. In order for all students to gain access to higher mathematics, all students need early and consistent opportunities to develop all strands of knowledge associated with mathematical proficiency (Kilpatrick et al. 2001). Procedural fluency alone is insufficient and, if taught in isolation, will continue to disadvantage those students already at the margins of society by restricting their opportunities to developing conceptual understanding, strategic competence, adaptive reasoning, and productive dispositions toward mathematics (the other four strands). Such an interpretation of mathematics risks leaving the most vulnerable students with limited mathematical skills. Through its attention to mathematics understanding as critical to developing logical and scientific ways of thinking and reasoning, the Carnegie-IAS report acknowledges dimensions of mathematical knowing beyond procedural fluency. Unfortunately, it does not provide any detail on what this kind of mathematics looks like.

Another risk of the narrow conceptions of mathematics promoted by these reports is the way students are positioned as consumers of the knowledge of others and are not encouraged to generate their own knowledge, draw their own conclusions, or apply their knowledge in novel situations. In this sense, the conception of mathematics promoted in the NMAP and NCTQ reports leaves little room for the development of critical mathematics knowledge (Gutierrez 2002; Gutstein 2006), a form of mathematics knowledge that positions students as users of mathematics and encourages them to apply their knowledge to uncover and challenge societal injustices.

Privileging Teacher Content Knowledge Before Other Forms of Knowledge

Designing instruction to teach toward developing mathematical proficiency is demanding, complex work. There is little doubt that deep subject matter knowledge is essential for teachers to be effective in this endeavor. The necessity of mathematics knowledge in teaching is a central theme of all three reports. The NMAP recommends, "Teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior to and beyond the level they are assigned to teach" (p. 37). The NCTQ report concurs, "Future teachers do not need so much to learn more mathematics, as to reshape what they already know" (p. 25). The Carnegie-IAS report advocates that

> We must also aim to build a teaching profession in which all teachers, in every discipline and from the elementary grades on up, are 'STEM-capable,' or sufficiently conversant with math and science content and relevance to infuse their classrooms with rigorous, motivating math and science learning. (p. 35)

We find, however, that the reports privilege teacher content knowledge over other forms of knowledge that are also necessary to teach mathematics effectively and engage different learners. The NCTQ report states, "Simply requiring more mathematics does not necessarily lead to better teaching" (p. 28), but does not elaborate on what other knowledge or skills are necessary. The Carnegie-IAS report outlines in slightly more detail what teachers must know and do:

> Educators need expertise and support in using instructional techniques that address the learning needs of the diversity of American students at all grade levels. Schools must be designed to enable adults to assess students' learning needs and strengths and develop customized approaches to instruction (what activity, at what intensity and over how long, toward what end) to bring all students to high levels. This is fundamentally a new kind of teaching and learning; it challenges teachers to possess and use a larger repertoire of instructional techniques, applied in alignment with the student's needs and the demands of the course work. (pp. 51–52)

In emphasizing teachers' knowledge of subject matter, however, these reports miss an opportunity to extend awareness and understanding of the complex nature of teaching and the multiple knowledge capacities teachers activate in practice.

More than two decades ago, Shulman (1986) conceptualized three categories of content knowledge necessary for teaching—subject matter knowledge, pedagogical content knowledge, and curricular knowledge—and extended the knowledge base to encompass knowledge of students and pedagogy. Teachers cannot rely on content knowledge alone to organize classrooms in which students engage with mathematical tasks at levels of high cognitive demand, or to bridge students' existing knowledge to new concepts, or to orchestrate classroom discussions in which students question, explain, and justify mathematical ideas (Borko and Whitcomb 2008; Boaler and Staples 2008). These reports are either silent or extremely vague about how teachers might use extensive subject matter expertise to develop instructional practices and classroom environments that engage all students in the learning of deep mathematics.

Conclusion

Are these recommendations likely to launch the reforms that are necessary to achieve goals of an equity agenda? Our answer is no. In different ways, these reports have missed an opportunity to embrace a complex view of mathematics and mathematics teaching or to take seriously the relationship between equity and educational quality. In this way, and to the extent that their recommendations are taken up, we can only predict that they will work against the equity agenda.

References

Anyon, J. (1981). Social class and social knowledge. *Curriculum Inquiry, 11*(1), 3–42.

Ball, D. L. (2001). Teaching with respect to mathematics and students. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 11–22). Mahwah: Erlbaum.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239–258.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of railside school. *Teachers College Record, 110*(3), 608–645.

Borko, H., & Whitcomb, J. A. (2008). Teachers, teaching, and teacher education: Comments on the National Mathematics Advisory Panel's report. *Educational Researcher, 37*(9), 565–572.

Carnegie Corporation of New York—Institute for Advanced Studies Commission on Mathematics and Science Education. (2009). *The opportunity equation: Transforming mathematics and science education for citizenship and the global economy*. New York: Carnegie Corporation of New York—Institute for Advanced Studies Commission on Mathematics and Science Education. http://www.opportunityequation.org/TheOpportunityEquation.pdf. Accessed 16 June 2009.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19–32). Mahwah: Lawrence Erlbaum.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499–531.

Delpit, L. (1988). The silenced dialogue: Power and pedagogy in educating other people's children. *Harvard Educational Review, 58*(3), 280–298.
Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths, pedagogical texts*. London: Falmer.
Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Greenwich: Information Age Publishing.
Gutierrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning, 4*(2 & 3), 145–187.
Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy of social justice.* New York: Taylor & Francis.
Haberman, M. (1991). Pedagogy of poverty versus good teaching. *Phi Delta Kappan, 73*(4), 290–294.
Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*, 528–549.
Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Greenwich: Information Age Publishing.
Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning math with understanding.* Portsmouth: Heinemann.
Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics.* Washington: National Academy Press.
Ladson-Billings, G. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education, 28*(6), 697–708.
Lappan, G., & Wanko, J. J. (2003). The changing roles and priorities of the federal government in mathematics education in the United States. In G. M. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 2, pp. 897–930). Athens: National Council of Teachers of Mathematics.
Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport: Ablex.
Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*(4): 454–482.
Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Mathematics literacy and civil rights*. Boston: Beacon.
National Council on Teacher Quality. (2008, June). No common denominator: The preparation of elementary teachers in mathematics in America's education schools. http://www.nctq.org/p/publications/docs/nctq_ttmath_fullreport_20090603062928.pdf. Accessed 1 July 2008.
National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards.* Reston: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics. (2000). *Principles and standards of school mathematics*. Reston: National Council of Teachers of Mathematics.
National Mathematics Advisory Panel. (2008, March). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington: U.S. Department of Education. http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf. Accessed 1 May 2008.
National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington: National Research Council.
Schoenfeld, A. (1998). Toward a theory of teaching-in-context. *Issues in education, 4*(1), 1–95.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.
Skemp, R. R. (1978, November). Relational understanding and instrumental understanding. *Arithmetic Teacher, 3*, 9–15.

Spillane, J. P. (2008). Policy, politics, and the National Mathematics Advisory Panel Report: Topology, functions, and limits. *Educational Researcher, 37*(9), 638–644.
Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50–80.
Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.
Zevenbergen, R. (2000). "Cracking the code" of mathematics classrooms: School success as a function of linguistic, social, and cultural background. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 201–224). Westport: Ablex.

Chapter 14
Whose 'Quality' and 'Equity'? The Case of Reforming 14–16 Mathematics Education in England

Andy Noyes

Everything has been said before, but since nobody listens we have to keep going back and beginning all over again.
Andre Gide (1891)

Introduction

In this chapter, I want to explore how issues of quality and equity are currently being quietly contested in a period of significant change in secondary school mathematics in England. The particular reforms I focus on are part of the Qualification and Curriculum Development Authority's[1] (QCDA) efforts to redesign the mathematics curriculum and assessment for 14–19-year-olds. More specifically, the UK government has remitted QCDA to develop coherent *pathways* for all learners of mathematics in the 14–19 age range. This policy trajectory analysis is primarily about the politics of mathematics education in England but also resonates with a range of international mathematics curriculum reforms. My hope is that this analysis will open up a critical dialogue on current 'reforms' in mathematics education around the world and expose some of the principles and taken for granted assumptions framing those changes.

When I first proposed this chapter, one of the reviewers expressed the view that it had been done before. This raises an important question which I want to explore briefly at the outset. Am I, in the words of André Gide, simply 'going back and beginning all over again'? The challenges of developing more equitable and

[1] QCDA is the national body responsible for developing the curriculum, improving and delivering assessments and reviewing and reforming qualifications.

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better quality mathematics education are as intractable as ever, and are not unique to England. Past critiques of political influences in curriculum design offer useful starting points (e.g. Ernest 1992, 1994) but it is self-evident that in and of itself such scholarly work has not disrupted the generally conservative trend of mathematics education over the last decades; hence the need for this volume. Moreover, the world is changing and in many places education is increasingly framed by neo-liberal market discourses in which mathematics has a heightened importance in global competition. Such social and political changes demand a sustained critique which might require the development of new tools of analysis. Perhaps most challenging of all is the question of how mathematics education scholars can engage with the policy formation process to ensure more equitable/quality outcomes for all learners.

One of the difficulties in redesigning mathematics curricula is that amongst those involved in the policy debates, from civil servants to teachers' representatives (e.g. teacher unions, subject associations, curriculum development agencies), there is little or no attempt to understand how different points and angles of view introduce tensions and contradictions in curricular aims, or of whether and how these tensions should be resolved. In England, references to 'the community' of mathematics educators are often heard without any critique of the multiple perspectives held by various factions of such a community. Those in various positions represent different ideologies, philosophies and social trajectories and use the words equity and quality without questioning whether or not they mean the same thing. For some (mathematics) educators, the idea that 'all students are equal, but some are more equal than others' (to borrow from Orwell) is subtly disguised beneath the surface of the current demand from government for greater 'stretch and challenge' for the 'most able'. And on the notion of *quality*, which is interlocked with varied views on the *qualities* of mathematics teaching and learning (Ernest 2004; Noyes 2007a), there is a notable space between the current 'functional mathematics' discourse in England, which is inspired by economic drivers, and the wonderful 'secret garden of mathematics' that Marcus du Sautoy (2009) wants all school children to discover. (N.B. This metaphor is problematic—the 'garden' would no longer be secret if everyone had discovered it and surely then the loss of mystique would render it far less romantic! See Lakoff and Nunez (2000), for a good critique of the 'romance of mathematics'.)

Bourdieu explained that 'the deepest logic of the social world can be grasped only if one plunges into the particularity of an empirical reality' (1998, p. 2) so this is my *plunging in* to what is a rather complex set of political and educational processes and relationships. I am seeking to draw on Bourdieu's notion of *field* to develop understanding of the positions and power-relations amongst the various individuals and stakeholder groups in the process of curriculum contestation. Such contestation is not as antagonistic as seen in the so-called US 'math wars' (Schoenfeld 2004), as the quiet classificatory work of English schooling tends to render such political contestation invisible. Bourdieu's field is a field of forces, rather akin to a magnetic or gravitational field, with varying directions and strength of force. The problem for any such analysis of the mathematics education field in England is its sheer scale and complexity. Moreover, much of the *force* exerted by individuals and groups is not plain to see but is hidden in private conversations and meetings or is cumulative in effect through historical allegiances and differences. This exploration of current developments in mathematics education for young people in their final two years of compulsory schooling (14–16-year-olds) in England can only really scratch the surface but I try to go into as much detail as is possible in this chapter. Having recently spent time trying to understand the US education system, I am increasingly aware of how little we understand one another's national contexts and so this detail is not intended to deter readers but rather to help readers appreciate the complexity of any national context. This also highlights the difficulty of communicating these issues to an international audience: context is all important and language is problematic.

Much of the 'evidence' for this study comes from my involvement in major studies of 14–19 mathematics education in England. One of these is as co-director of the Evaluating Mathematics Pathways project (www.nottingham.ac.uk/emp). This project brings a large multi-site team into contact with a wide range of stakeholders who are directly involved and/or keenly interested in 14–19 mathematics education. Through this work, I have had privileged access to parts of the mathematics education field.

The Context

The National Curriculum (NC) in England was introduced over 20 years ago through the Education Reform Act of 1988 and has since undergone a number of revisions (1991, 1994, 1999, 2007). Ernest (1992) discussed how various stakeholder groups contributed differentially to the establishment of that original NC. His categories (old humanists, industrial trainers, progressive educators and public educators) are still useful and not simply for England. They are not fixed and, indeed, the discourses of these groups around the form and function of the curriculum shifts as the (education) world changes.

Young people follow the NC until they complete their compulsory schooling at age 16 (Year 11) with the General Certificate of Secondary Education (GCSE) qualifications. Obtaining five or more higher grades (A*-C) allows students access to a wide range of further educational opportunities. The majority of those achieving this level at GCSE proceed to the traditional academic track of Advanced level qualifications (General Certicificate of Education or GCE). The remainder by and large follow a wide range of vocational programmes. Advanced (A) level qualifications are the standard university-entrance qualifications and most students would study three or four subjects over the following two years, up to the age of 18 (Year 13). A level Mathematics is a pre-requisite for most Science, Technology, Engineering and Mathematics (STEM) courses in higher education.

Mathematics education, as part of the broader STEM agenda is increasingly trumpeted as being critical for economic stability and productivity here in the UK (Roberts 2002; Sainsbury 2007), in Europe (Gago 2004) and in the rest of the developed world (e.g. in the United States; National Academies 2007). The Roberts report on the UK's 'supply of people with science, technology, engineering and mathematics skills' led to the announcement by the Chief Secretary to the Treasury, in July 2002 of an inquiry into post-14 mathematics education in the United Kingdom. The resulting Smith Report (2004) *Making Mathematics Count* made a raft of recommendations which have resulted in considerable discussion and policy formation for 14–19 mathematics in England.

Most of the activity resulting from 'Smith' has been structural and a number of changes have occurred at GCSE level (GCSE, taken by 16-year-olds as the exit qualification from compulsory schooling). The recommendations grow out of a broad sense of 'mathematics in crisis', which is perhaps more of a panic about the supply of mathematically well-qualified STEM graduates and technicians. The report states that 'it is clear that the overwhelming majority of respondents to the Inquiry no longer regard current mathematics curricula, assessment and qualifications as fit for purpose' (p. 6), but what purpose did the respondents have in mind? The majority public who do not use any of their school mathematics in any recognisable way and have many negative memories of their school mathematics experience were not contributors to the Inquiry report. So there appears to be a common, taken-for-granted notion of this purpose in large part because certain stakeholder groups (e.g. industrialists and academics), with intersecting concerns about improved quality and quantity in supply, have a disproportionate say. The terms of reference are primarily concerned with creating a better qualified workforce capable of maintaining the UK's position in a changing and increasingly challenging global economic hierarchy.

There has been some criticism of the capitalist agenda for mathematics education (Gutstein 2009; Noyes 2009b) but perhaps not enough is being done to question the conservative trends which shape the day-to-day experiences of learners of mathematics in England. Critical mathematics educators have contributed to this debate but this does not seem to have much of an impact on mainstream mathematics education, in England at least. In Schoenfeld's (2004) analysis of the 'math wars' in the United States, he urges for liberal and conservative antagonists to work towards a middle ground in which multiple purposes for mathematics education can fruitfully coexist. Similarly, Ernest (2004) has more recently rethought his views about the oppositional stance of various stakeholders to argue that the curriculum can accommodate all positions. I remain unconvinced by these arguments, appealing though they are, as such a middle ground seems to be an unstable place; the ridge between entrenched positions. Schoenfeld traces back the divisions in the United States over a century and similar roots to the English GCSE system can be found in the difference between the classical mathematics curricula of the grammar and public schools in England and the emerging practical mathematics for the new industrial classes in the nineteenth century (Rogers 1998). Schoenfeld's plea for a resolution is made on the grounds that casualties result from any war. In contrast, in England there is no war but there are certainly casualties.

Mathematics education in England is now framed by a neo-liberal education culture in which markets, managerialism and performativity (Ball 2003, 2007, 2008) are the tools employed by recent governments in their standards-raising agenda for schools and other public services. With such a taken-for-granted set of discourses influencing educational reform, there is a need for stakeholders to reconsider the purposes for the mathematics curriculum (Gill 2004; Heymann 2003; Noyes 2007a, b). It is worth noting for international readers that standards in the United Kingdom are essentially performative, evidenced through increased test scores at ages 11, 14 and 16. Our improved performance in the latest Trends in International Mathematics and Science Study (nces.ed.gov/timss) was heralded as a sign that standards in science and mathematics were improving. The headlines announced our place in the world's top ten but tucked away in the press reports was the time-bomb that students were enjoying these subjects less and had on-going weaknesses in facility with number and algebra. The implications for quality here are significant and get us back to the question of not only the purpose of the mathematics curriculum but of schooling in general.

This mixture of neo-liberal educational policy and an economic drive for increasing the supply of mathematically well-qualified young people has particular potency for 16-year-olds in England. The 'terrors of performativity' (Ball 2003) are most keenly experienced, for both teachers and learners, at the GCSE (aged 16). For some years so-called 'league tables' of school performance have been used to provide the parent/guardian/carer-customer with a means of comparing schools. The critical measure is the proportion of students in a school obtaining 5 or more A*-C grades at GCSE. Despite the refinement of these comparative tools to take account of progress made between 11 and 16 (Value Added), and then student background (Contextual Value Added), the raw measures continue to have a powerful effect on teacher behaviour. Since 2006, this 'performance measure' has included mathematics, i.e. 5 A*-C grades including English and mathematics. The pressure on teachers to maximise the number of C grades leads to an impoverished curriculum experience through teaching to the test, selective curriculum coverage and so on, as evidenced by the schools' inspectorate's recent report (Ofsted 2008). It is important to note that the factors having the largest impact on quality and equity in mathematics classrooms have originated from outside mathematics education. That said, these educational trends are not easily reversed as they are interlocked with the prevailing neo-liberal economic policy project of recent governments.

The grade C threshold, which is only attained by around half of students in mathematics, is considered an essential pre-condition for many future educational and career opportunities. It continues to be the most effective means of social stratification that school assessments offer. The division at this 'magical threshold' (Bourdieu 1998) favours students from higher socio-economic backgrounds and has mixed impact upon different ethnic groups. Students from the poorest fifth of homes are less than half as likely to achieve this grade C as those from the wealthiest quintile (Noyes 2009a). This situation is inequitable and has complex social causes which go beyond the scope of this chapter. In any discussion about how to increase attainment, the nature of the curriculum and the value of the qualification (beyond its exchange value) often go unquestioned. However, during the last five years, since the Smith Report, a number of strategies have been developed to improve the quality and appropriateness of mathematics qualifications for 16-year-olds, and so I now explore these in a little more depth.

Recent Developments

Following one of the Smith Report recommendations (and also influenced by the parallel Tomlinson Report (DfES 2004) on 14–19 qualification reform), the QCDA has spent the last four years overseeing the Mathematics Pathways project. In response to these reports, the Government's white paper *14–19 Education and Skills* (DfES 2005) indicated the intention that all 16-year-olds should be able to demonstrate their ability to be functional with mathematics—whatever that means. Moreover, attaining a higher GCSE grade in mathematics (A*-C) would be predicated upon passing functional mathematics at level 2.[2] There is no space here to explore what 'functional mathematics' is (Roper et al. 2006) but suffice to say that it has been a challenging process to develop valid assessments of mathematical functionality. The QCDA has published functional skill standards (www.qca.org.uk/qca_15565.aspx) which are helpful but still leave plenty of room for interpretation in practice.

The Smith report recommended that mathematics GCSE be reorganised into two-tiers, one for students who might be expected to get A*-D grades and the other for C-G grade students. This replaced the old three-tier system, within which the lowest attaining students could not attain a grade C which was considered a major disincentive. This does not, of course, alter the fact that around half of English young people do not get a grade C, so this new possibility is, for most, a mere mirage. A third aspect of the Pathways Project was to make GCSE mathematics a 'double award'. Since many people consider mathematics to be more difficult than other GCSE qualifications it was conjectured, and stated as reality, that this would increase the status of the qualification by making it worth two GCSEs. Over the last three years, opinions about what proportion of the cohort should be doing a double-GCSE have varied, revealing some confusion about the purpose of this policy, from about 50% to 'most or all'. This recommendation seems to have become entangled with that of stretching and motivating the top 10%. This point implies that the remaining 90% do not need motivating but it is evident that this is not the case (Nardi and Steward 2003). Our evaluation shows that in many schools only the top 50% of the students were entered for this pilot qualification. This is one example—of which there are many—where qualifications get developed without the necessary time to think through the aims, and possible unintended consequences.

[2] GCSE is a level 1/2 qualification with grades C and above equivalent to level 2.

Two Models for Piloting

In 2005, Phase I of the Mathematics Pathways Project began when QCA contracted two teams to develop pathways models: the University of Leeds and King's College, London working with Edexcel (the latter being one of the three unitary awarding bodies (AB) in England).

The KCL/Edexcel model had a distinctive emphasis on mathematical modelling and the use of ICT, both within the course and in the assessment of the mathematics. Functional mathematics would be assessed by a computer-mediated test with portfolio assessment being proposed for trial at all levels. Their model was designed to engage students and build learner confidence through an emphasis on modelling and a belief that some big mathematical ideas are simply worthy of inclusion in the curriculum, whether of practical use or not.

The Leeds' model aimed to develop 'a curriculum and assessment structure which would encourage more students to study mathematics beyond compulsory schooling, engage and motivate students and provide students with a mathematically challenging experience within their capabilities'. Functional mathematics assessment should consist of a competence and functionality element.

It seems that these two models reflect different visions of mathematics education (thinking back to Ernest's categories) which is not necessarily a problem but needs consideration. The emphasis on mathematical proof and challenge sounds a little different from modelling and the use of relevant ICTs. Whether these distinctions are significant is unclear but suffice to say that the two groups seemed to represent different priorities, albeit with a fair degree of common ground.

Following the development of the Phase I models, QCDA contracted two English awarding bodies to develop various aspects of the Phase I models. The KCL/Edexcel model was not fully implemented, due in part to the quite significant shift in pedagogy that would be required. In addition, the challenges of using ICTs proved problematic. Both piloting contractors designed and ran trial qualifications in 2006–2007. The aims for the second GCSE—*additional* mathematics—were not finalised in the two Phase I models and this lack of clarity has continued into the development of the pilot GCSEs. Initially, the two GCSEs are assessing the same programme of study although the second GCSE aims to have a greater emphasis on mathematical thinking and problem solving. This reflects changes in the new programme of study, within the 2007 NC, in which mathematical process skills (including mathematical thinking and problem solving) have a prominent position.

The pilots of these proposed GCSE qualifications were planned to run from September 2007 and would inform the final form of the qualification when implemented in September 2010. The difficulties of working with such a tight developmental timetable have been further exacerbated by a number of major announcements. I have already referred to the two decisions about functional mathematics. As part of a suite of functional qualifications (English, mathematics and ICT), these seemed to have an increased status when the announcement was made that the qualifications would be 'stand alone' and act as a 'hurdle' to attaining GCSE mathematics. This 'hurdle' role would help to satisfy the critical employers who reported that GCSE mathematics did not equip their new employees with the necessary skills. However, this decision seemed to have a significant flaw; it left the government trapped between a rock and a hard place. If a significant number of students failed to get the functional mathematics qualification at the appropriate level, entries to further education courses would be hit and school performance would be seen to have dropped: all bad press. On the other hand, if the hurdle provided no real obstacle to attaining a grade C GCSE then why bother with it. So, in March 2009, following advice from Ofqual (the Office of the Qualifications and Examinations Regulator), the announcement was made that functional mathematics would not act as a hurdle. Immediately, the future of this qualification was thrown into some doubt as who would now take it. However, the Secretary of State hinted that 'other incentives' would help to ensure take up of the qualification. One of the disappointments in this process was that some (but certainly not all) of the development work on functional mathematics assessments was innovative and had the potential to be a powerful catalyst in influencing changes in mathematics teaching and learning.

The move to a two-tier GCSE has had an unintended impact on learners. For middle attainers trying to get a grade C, schools have experienced some difficulties in deciding the most appropriate tier of entry. The result appears to be a trend of entering a greater proportion of the cohort for the Foundation tier. Although they should still have access to the entire programme of study, there is the distinct possibility that a whole swathe of students will not encounter some of the more challenging aspects of mathematics assessed in the higher-tier GCSE. Combined with this is an increasing tendency, generated by the disciplinary power of league tables, to enter middle attaining students for GCSE early (e.g. at age 15) so that they can have more than one attempt at getting that all-important grade C. For many, GCSE mathematics is simply qualification currency and has little to do with mathematics per se. Few students who have achieved a grade C early will be motivated to retake in order to 'improve their grade'. This means that students effectively underachieve and are far less likely to continue studying mathematics beyond GCSE. Even high attaining students who fail to get the highest grades may find themselves at a disadvantage when they apply to university, where oversubscribed places on popular courses are offered to candidates with A* and A only.

The third aspect of mathematics at GCSE is the double award. Early in the Pathways Project it became apparent that the regulations would not condone two awards arising from one programme of study. The piloted model of two GCSEs was becoming obsolete before one year of piloting had been completed. Meanwhile, the Advisory Committee on Mathematics Education (ACME, then chaired by Professor Adrian Smith), was working with QCDA and the Department for Children Schools and Families (DCSF) to reformulate an alternative vision for a linked pair of GCSEs. In December 2008, an announcement was made that from 2010 a linked pair of GCSEs would be piloted. These would together have to cover the programme of study and each would need to have additional content in order to meet regulatory requirements with regard to content overlap. We need to ask whose interest this proposal serves. ACME intends the linked-pair to be appropriate for all learners but it seems highly unlikely that schools will enter students struggling to attain a grade C for two awards requiring the teaching of further content.

Most of this policy work occurs in and between the DCSF, QCDA, Ofqual and ACME. These are all powerful groups but the awarding bodies must not be ignored in this discussion. Exposed to the force of the examinations market, they dare not make radical changes and so an inherent conservatism acts as an impediment to change in assessments and therefore, most importantly upon pedagogy. The Smith Inquiry terms of reference included recommendations on pedagogy but due to the powerful influence upon teaching that slow-changing, high stakes assessments have, the rate of pedagogic change is systemically limited.

Mapping the Field

In his work on the structure of the scientific field, Bourdieu (2004) describes the 'structural interlockings' between individual scientists, labs, groups of labs, etc., and the same notion is helpful in my context. Here I am primarily concerned with the statutory curriculum but classroom experiences are linked, through a range of *structural interlockings*, to the decisions of policymakers. Any analysis is complicated by the multidimensionality of such interlockings. For example, awarding bodies, regulated by Ofqual but operating under market principles in competition with one another, are 'interlocked' with public-service schools and a managerialist government in quite different ways from the subject associations or ACME. I have referred to these and a number of other important stakeholders above and here want to explore their relationships in the field of mathematics education, particularly policy formation. For mathematics education to become more equitable those social structures and forces which tend, intentionally or not, to reproduce social inequities must be understood and, where possible, challenged. The diagram (Fig. 14.1) is a simple map of the field that gives some indication of the competing positions and purposes for school mathematics and therefore how equity and quality get framed in policy discourse and documentation. The two dimensions of the page are limiting as this *field* is multidimensional and changes over time.

Perhaps it is easier to conceptualise 14–16 mathematics education as a *landscape*—an increasingly common metaphor in education. Elsewhere I used the metaphor of *learning landscape* to explore the impact of policy intervention in 11–14 mathematics education in England (Noyes 2004). That analysis argued that geology, climate, human intervention and time are all important dimensions for mapping the landscape. These are interdependent. Indeed, it might be argued that one of the key differences between geological change (e.g. structure of schooling and society), climate change (e.g. attitudes to learning, pedagogic trends) and human intervention (e.g. policy) is the typical timescale over which they act. Lemke (2000) explores such 'scales of time' in what he calls 'ecosocial systems', a metaphor not wholly dissimilar to that of the landscape. So, for example, in our mathematics education policy context in England, policy decisions can be made in days, implementation of

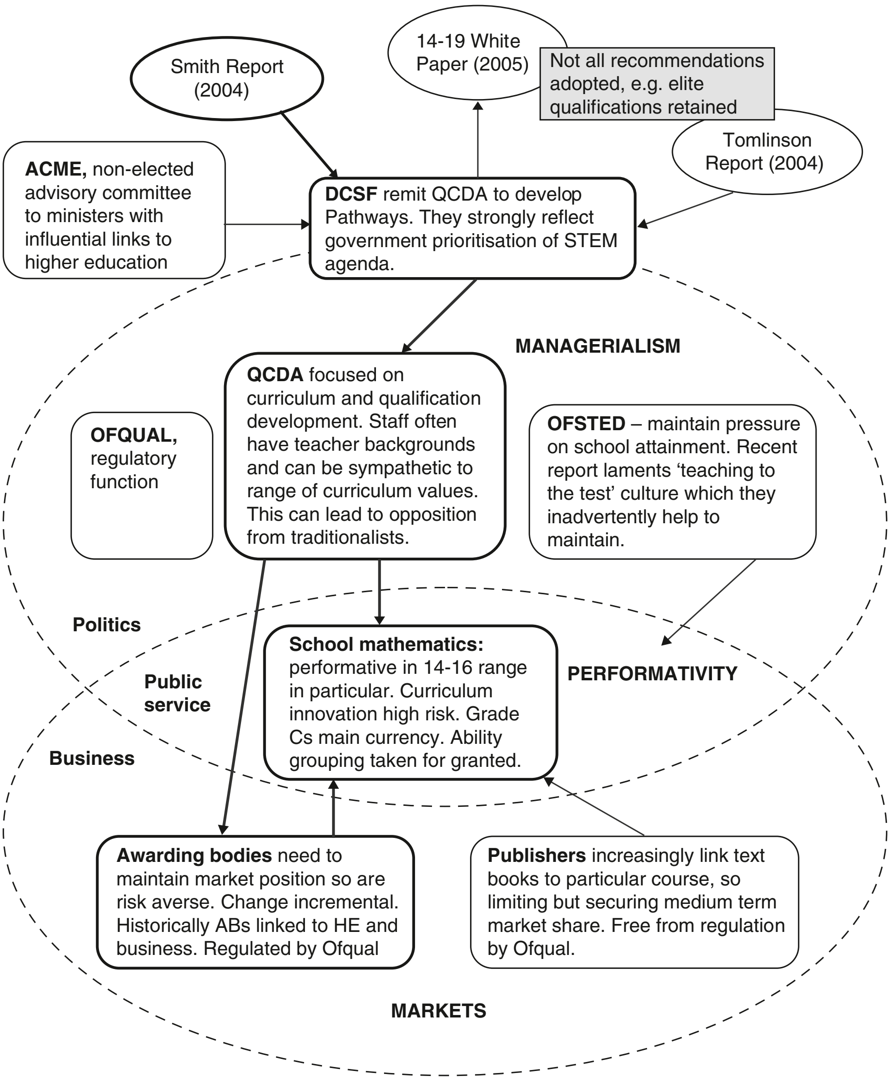

Fig. 14.1 Positioning school mathematics education

project proposals takes weeks, piloting of qualifications takes months, significant changes in the assessment takes years and deep shifts in classroom practice arguably takes tens of years. This is just an example, of course, and we can imagine how these timescales might look different under different circumstances (e.g. the sudden impact of a new government).

Individuals with different values and aims for the curriculum can be found across this space, although there is a *harmonisation of dispositions* (Bourdieu 1984) towards mathematics education in certain places. Where are Ernest's progressive and public educators here? Some can be found in schools but as I have tried to show there is a considerable tension between the performative culture of schools and what many would hold to be the values of public service. The same contrast is perhaps not as great in the managerialism-politics or market-business space. On this diagram it is not possible—indeed, it would be ethically questionable—to name individuals. However, much of the influence is not through these structurally interrelated groups but rather through particular powerful individuals. Anyone immersed in the murky waters of policy making, reforming and lobbying has a sense of who these individuals are and what their networks are but by their very nature the processes by which they exert their influence are non-transparent.

The role of the awarding bodies is unique. They function within a market and are regulated by Ofqual but their influence upon classroom practice is considerable. The adage 'what you test is what you get' to describe the powerful influence of high stakes external testing highlights the ways in which conservative curriculum influences hold sway. Combine this with the obsession with league tables and the UK government's concern to meet its own targets for national examination performance at 16 and changes in assessment becomes high risk. In our system of three awarding bodies, each of which produces a large number of mathematics assessments every year, most assessment writers have been apprenticed into the house style and this too means there is little space for innovation. So the net result of this conservatism is that ineffective and/or inequitable classroom practices are hard to challenge.

Final Comments

Any discussion of equity and quality needs to take account of the difficulties of language. For the different actors in this mathematics education field their unique habitus, historically developed within their particular social milieu, educational and life trajectory means that they interpret these terms in different ways. One of the significant problems is that the vast majority of those involved in policy formation in mathematics have enjoyed success in the subject and the privileges which come as a result. This tends therefore to ensure that the interests of those like this group—the future scientific, business, industrial, educational and political leaders—are protected.

Even the notion of 'mathematics' should be opened up to scrutiny; Popkewitz (2004) has pointed out school mathematics is not the same as academic mathematics. However, academic mathematicians are deeply involved in advising policymakers. In addition, there are the well-rehearsed arguments about the differences between the traditional and new/humanistic mathematics (Ernest 2009). It is evident in the developments over recent years that the traditionalists are still dominant. Interestingly, the more innovative, progressive *functional mathematics* qualification, and the Pathways model emphasising modelling and ICT have been less strongly supported by influential groups/individuals compared with the second GCSE. More mathematics for the highest attainers seems, at the present time, to have a reasonable chance of happening. In contrast, disengaged middle and low attainers (Nardi and Steward 2003), who arguably would gain the most from better pedagogy inspired by innovative assessments (e.g. in functional mathematics), will probably see limited change in the coming years. The influence of two-tier GCSE and the all-important C grade in the performance tables appears to be having a more negative impact upon middle attainers.

Opening up this discussion about what mathematics education should be and what an equitable and quality mathematics education should look like is difficult given the structure of the field. It is, by and large, taken-for-granted that the traditional and economic views of mathematics education are the right ones. There is no 'math war' here in England as the progressive/humanist mathematics educators are barely in the fight, and where they are they experience considerable opposition. So, a curriculum model which is rooted in an elitist education system remains. The question of whether a single GCSE qualification can adequately meet the learning needs of the full range of learners is unresolved. It seems that the answer here depends upon what is considered to be the purposes of the curriculum. The new secondary curriculum has as its aims: successful learners, confident individuals and responsible citizens. Laudable though these aims are it would be easy to construct very different curricula (simply select a few national education systems: Japan, Finland, United States) each of which purported to be striving towards meeting these aims.

If, instead of working backwards from the needs of future employers and higher education courses, we thought about what the generally well-educated 16-year-old should experience in their mathematics learning we might get a very different set of possible solutions to this question. This is not a new idea (Heymann 2003; Noyes 2007b) but one that has little impact in mathematics classrooms in England, despite the valiant efforts of some, because of the structuring of the generally conservative education field described above. So although the programme of study for 14–16-year-olds seeks to define what a rich and worthwhile curriculum entitlement is for all learners, there remains a gap between the rhetoric and the reality. The influence of the educational assessment market is paramount here for any real change in curricular experience is catalysed by significant change in assessment. Unfortunately, the combination of the market, high stakes performativity and managerialism (through inspectorial and league table fabrications) creates strong resistance to assessment change. Evolution is more likely than revolution but any movement from a conservative position arouses suspicion and mobilises the right to resist progressive change. In arguing for a bottom-up curriculum, I am not saying that the needs of future employers are not important but rather that they are all so different it makes complete sense that any standard qualification will not satisfy any of their specific demands, beyond a good general education (Heymann 2003). However, to loosen the grip of a curriculum which essentially acts in a classificatory way to steer learners into particular educational and life trajectories challenges the reproductive function of education and arouses the privileged to defend their position. Any effective attempt to do this would probably lead to our own 'war'—albeit conducted in a peculiarly English way.

Can the question of whose equity and quality be fully answered? Perhaps not, but I have aimed to expose some of the structures and processes which maintain school mathematics as a tool for the utilitarian interests of industry, higher education and economic/political power. Although my analysis has centred on the case of mathematics curriculum reform in England, I am confident that similar field structures are found in other countries, albeit inflected peculiarly by the particular context.

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References

Ball, S. (2003). The teacher's soul and the terrors of performativity. *Journal of Education Policy, 18*(2), 215–228.

Ball, S. (2007). *Education plc*. London: RoutledgeFalmer.

Ball, S. (2008). *The education debate*. Bristol: Policy Press.

Bourdieu, P. (1984). *Distinction: A social critique of the judgement of taste*. Cambridge: Harvard University Press.

Bourdieu, P. (1998). *Practical reason*. Cambridge: Polity Press.

Bourdieu, P. (2004). *Science of science and reflexivity*. Chicago: The University of Chicago Press.

DfES. (2004). *14–19 Curriculum and qualifications reform: Final report of the working group on 14–19 reform*. London: Department for Education and Skills.

DfES. (2005). *14–19 Education and skills*. London: Department for Education and Skills.

du Sautoy, M. (2009, June 23 Tuesday). The secret life of numbers. *The Guardian.* http://www.guardian.co.uk/education/2009/jun/23/maths-marcus-du-sautoy. Accessed 23 June 2009.

Ernest, P. (1992). The national curriculum in mathematics: Political perspectives and implications. In M. Nickson & S. Lerman (Eds.), *The social context of mathematics education: Theory and practice* (pp. 33–61). London: South Bank Press.

Ernest, P. (Ed.). (1994). *Constructing mathematical knowledge: Epistemology and mathematics education* (Vol. 4). London: Falmer Press.

Ernest, P. (2004). Relevance versus utility: Some ideas on what it means to know mathematics. In D. C. B. Clarke, G. Emanuelsson, et al. (Eds.), *Perspectives on learning and teaching mathematics* (pp. 313–327). Goteborg: National Centre for Mathematics Education.

Ernest, P. (2009). New philosophy of mathematics: Implications for mathematics education. In B. Greer, S. Mukhopadhyay, A. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 43–64). Abingdon: Routledge.

Gago, J. M. (2004). *Increasing human resources for science and technology in Europe*. Brussels: European Commission.

Gill, P. (2004). Mathematics. In J. White (Ed.), *Rethinking the school curriculum: Values, aims and purposes* (pp. 104–116). London: RoutlegeFalmer.

Gutstein, E. (2009). The politics of mathematics education in the United States: Dominant and counter agendas. In B. Greer, S. Mukhopadhyay, A. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 137–164). Abingdon: Routledge.

Heymann, H. W. (2003). *Why teach mathematics: A focus on general education*. Dordrecht: Kluwer Academic.

Lakoff, G., & Nunez, R. (2000). *Where mathematics comes from*. New York: Basic Books.

Lemke, J. (2000). Across the scales of time: Artifacts, activities and meanings in ecosocial systems. *Mind, Culture and Activity, 7*(4), 273–290.

Nardi, E., & Steward, S. (2003). Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal, 29*(3), 345–367.
National Academies. (2007). *Rising above the gathering storm: Energizing and employing America for a brighter future*. Washington: National Academies Press.
Noyes, A. (2004). Learning landscapes. *British Educational Research Journal, 30*(1), 27–41.
Noyes, A. (2007a). Mathematics counts…for what? Rethinking the mathematics curriculum in England. *Philosophy of Mathematics Education, 21*(Special Issue on Social Justice, Part 2).
Noyes, A. (2007b). *Rethinking school mathematics*. London: Paul Chapman.
Noyes, A. (2009a). Exploring social patterns of participation in university-entrance level mathematics in England. *Research in Mathematics Education, 11*(2), 167–183.
Noyes, A. (2009b). Participation in mathematics: What is the problem? *Improving Schools, 12*(3), 277–288.
Ofsted (2008). *Mathematics: Understanding the score*. London: Office for Standards in Education.
Popkewitz, T. (2004). School subjects, the politics of knowledge, and the projects of intellectuals in change. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (pp. 251–268). Dordrecht: Kluwer Academic.
Roberts, G. (2002). *SET for success: The supply of people with science, technology, engineering and mathematics skills*. London: Department for Education and Science.
Rogers, L. (1998). *Society, mathematics and the cultural divide: Ideologies of policy and practice 1750–1900.* Paper presented at the Mathematics Education and Society 1, Nottingham.
Roper, T., Threlfall, J., & Monaghan, J. (2006). Functional maths: What is it? In A. Noyes & H. Povey (Eds.), *Research in mathematics education* (Vol. 8, pp. 89–98). London: BSRLM.
Sainsbury, L. (2007). *The race to the top: A review of the government's science and innovation policies.* Norwich: HMSO.
Schoenfeld, A. (2004). The math wars. *Educational Policy, 18*(1), 253–286.
Smith, A. (2004). *Making mathematics count*. London: Stationery Office.

Chapter 15
Equity and Quality of Mathematics Education: Research and Media Portrayals

Helen J. Forgasz and Gilah C. Leder

Introduction

Researchers often report on the dissonance between beliefs and actions with respect to the teaching and learning of mathematics (e.g. Forgasz and Leder 2008). While the means by which beliefs are, or can be, measured are frequently discussed in the literature (e.g. Leder and Forgasz 2002), how beliefs are developed is less often the focus of research papers. In the past, we have claimed that the power of the media to shape public opinion should not be underestimated (Forgasz et al. 2007; Leder and Forgasz 1997). Media reports of specific scholarly articles can often be selective in what is reported and can be misleading. An article by Hyde et al. (2008) published in the highly prestigious and influential journal, *Science*, serves as an instructive example. The authors claimed that "these very current data provide no evidence of a gender difference favoring males emerging in the high school years" (p. 494). This produced a stampede of frenzied and biased reporting in the popular press. In proclaiming that there were no gender differences in male and female mathematics achievement, the media reporters had not taken into consideration Hyde et al.'s cautious reminder that earlier research had indicated that it was particularly on high level, complex items that boys excelled over girls, and that "state assessments designed to meet NCLB[1] requirements fail to test complex problem-solving of the kind needed for success in STEM[2] careers" (p. 295). That students in different schools and locations may not have experienced the same curriculum or quality of teaching was also overlooked.

[1] No Child Left Behind.

[2] Science, Technology, Engineering and Mathematics.

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In this chapter, we examine and compare scholarly research and media coverage of equity and quality issues with respect to mathematics learning. Our focus is on equity issues that the media are likely to cover including mathematics achievements by gender, ethnicity, and socio-economic background, as well as aspects of school practices such as ability grouping and single-sex classes that are associated with potential variations in the quality of mathematics education experienced by some students.

We have restricted the media coverage to a manageable but particularly pertinent period, November to December 2008, and have only examined two Australian newspapers, *The Age* (published in Melbourne, Victoria), and *The Australian* (a national publication). These two newspapers are among the top three broadsheets in Australia (TheNewspaperWorks 2009). We selected this time frame as, together with a range of articles on general educational issues, there were reports on Australian students' performance in the Trends in International Mathematics and Science Study (TIMMS) 2007, Victorian grade 12 high stakes examination results, and the National Program for Literacy and Numeracy (NAPLAN) results for students in grades 3, 5, 7 and 9. Our reliance on articles from the two Australian newspapers is less limiting than might first appear since many articles are reproduced in multiple national and international press outlets because of the widespread syndication of reports and commentaries.

Within mathematics education, the terms *equity* and *quality* have been variously defined and interpreted. Following Bishop and Forgasz (2007), we argue that "without *access* to mathematics education there can be no equity" (p. 1146) and that *equity* can be considered an outcome "for judging or evaluating any educational variable, including access" (p. 1146). Inequities are thus represented by differences in outcomes for identifiable groups within a given context or setting, and may be apparent as access, achievement, enrolment, and/or attitudinal differences. We also accept the duality of Atweh's (2007) definition of *quality in mathematics education* to include the intellectual rigour of the mathematical content encountered, and the capacity of the mathematics to transform aspects of students' lives as citizens now and in the future. Atweh's perspective is consistent with Bishop and Forgasz (2007, p. 1152) who viewed mathematics education as serving two purposes:

> to prepare students to be mathematically functional as citizens of their societies—arguably provided equitably for all—and to prepare some students to be the future professionals in careers in which mathematics is fundamental, with no one precluded from or denied access to participation along this path. The notion of more than one purpose for mathematics education raises issues of what constitutes an equitable mathematics curriculum and raises questions about the equity implications of systemic, school-based, or classroom-based practices in mathematics education such as tracking/streaming or single-sex settings.

In the remainder of the chapter we present, under different headings, summaries of the scholarly research on equity issues followed by popular print media portrayals of these issues.

Scholarly Research on Equity and Quality in Mathematics Learning

As mooted above, the scholarly research examined with respect to equity and quality issues covers:

- mathematics achievement levels,
- ability grouping for mathematics learning, and
- single-sex groupings for mathematics learning.

Achievement (by Equity Variables)

The results of two large-scale mathematics testing regimes are discussed here. Students' achievements were examined to determine if inequities were evident, and comparisons with the results from previous years were also undertaken. Students' achievements in the tests were made public in late 2008 and the findings were also reported in the press. The data sets arise from mathematics testing conducted in Australia: the high stakes results for mathematics subjects in the Victorian Certificate of Education (used for university selection), and the 2008 numeracy results from the NAPLAN testing of students across the nation in grades 3, 5, 7 and 9.

Victorian Certificate of Education (VCE) Mathematics Results

VCE results are made available on the website of the Victorian Curriculum and Assessment Authority (VCAA): http//www.vcaa.vic.edu.au. In 2008, four grade 12 mathematics subjects were offered: Specialist Mathematics (the most challenging), Further Mathematics (the least challenging); and two parallel offered versions of the same subject, Mathematical Methods (graphics calculators mandated) and Mathematical Methods CAS (CAS calculators mandated). With respect to enrolments in the three subjects, males are greatly over-represented in Specialist Mathematics and in Mathematics Methods and the parallel version Mathematical Methods CAS, and slightly over-represented in Further Mathematics (Forgasz 2006).

Each VCE mathematics subject has three assessable components which are reported separately on the VCAA website: a series of school-assessed tasks, and two examinations. For each subject the school-assessed component carries 34% of the final assessment; there is some variation in the weightings carried by each of the two examinations in the three mathematics subjects. The results of the three assessment tasks are provided in the same format for each subject. There are ten possible grades ranging from A+ to E as well as "UG" (ungraded) for each assessment task. The percentages of the cohort obtaining each grade for each assessment task are found on the website for each subject; the percentages within gender cohorts are also available.

The two courses, Mathematical Methods and Mathematical Methods CAS, have run in parallel since 2002. They have provided the opportunity to compare findings on the performances of males and females using two different technologies for the learning of mathematics. In the past, females have been found to be less confident than males about the use of technology for mathematics learning (e.g. Vale and Leder 2004). The enrolment numbers in the CAS version of the subject were very small in the early years, but have since grown considerably. Forgasz and Tan (2009) examined the patterns in performance in each of the three assessment tasks over the seven-year period (2002–2008) by gender. The findings can be summarised as follows:

- For each assessment task, the pattern was more stable for male and female achievements in Mathematical Methods than in Mathematical Methods CAS.
- For both subjects:
  - a higher proportion of males than females received the grade A+ for each of the three assessment tasks, and
  - for each assessment task, the gender gap (i.e. the difference in the percentage of male and female students achieving the grade) was greater for Mathematical Methods CAS than for Mathematical Methods.
- The gender gap favouring males was widest for the A+ grade in Mathematical Methods CAS for Examination 2, the assessment task for which the calculator must be used. Graphs, based on data published on the VCAA website, for the A+ results by gender for Examination 2 in each subject are shown in Fig. 15.1. It should be noted that the same scale was used for both graphs.

Close examination of the data in Fig. 15.1 reveals that from 2002 to 2008, there was an average gender gap of 3.0% for the A+ results on Examination 2 in Mathematical Methods. For Mathematical Methods CAS, the average gender gap was 7.5% over the seven-year period. A similar pattern, with a smaller difference in the gender gaps in the two subjects, was found for the school-assessment task and Examination 1. These data suggest that the mandated use of the sophisticated CAS calculator may be disadvantaging females.

The VCAA website does not publish results by any other student grouping factors. The results are, however, published in a different way in the daily newspapers. For each student in each of their VCE subjects, the VCAA calculates a single score called a "study score". The study score is derived by first combining the results of the three assessment tasks in each subject in some complex, statistical fashion, and then standardising the scores to have a mean of around 30, a standard deviation of about seven, and a maximum possible score of 50. In the newspaper, the names of students and the schools they attend are published for each subject for those achieving scores of 50 down to 40. These data can be analysed to explore achievement patterns by a range of variables including: school type attended (Government/Independent/Catholic); school setting (single-sex/coeducational); school location (metropolitan/non-metropolitan); gender; and ethnicity. School type attended has been used as an indicator of socio-economic status (SES).

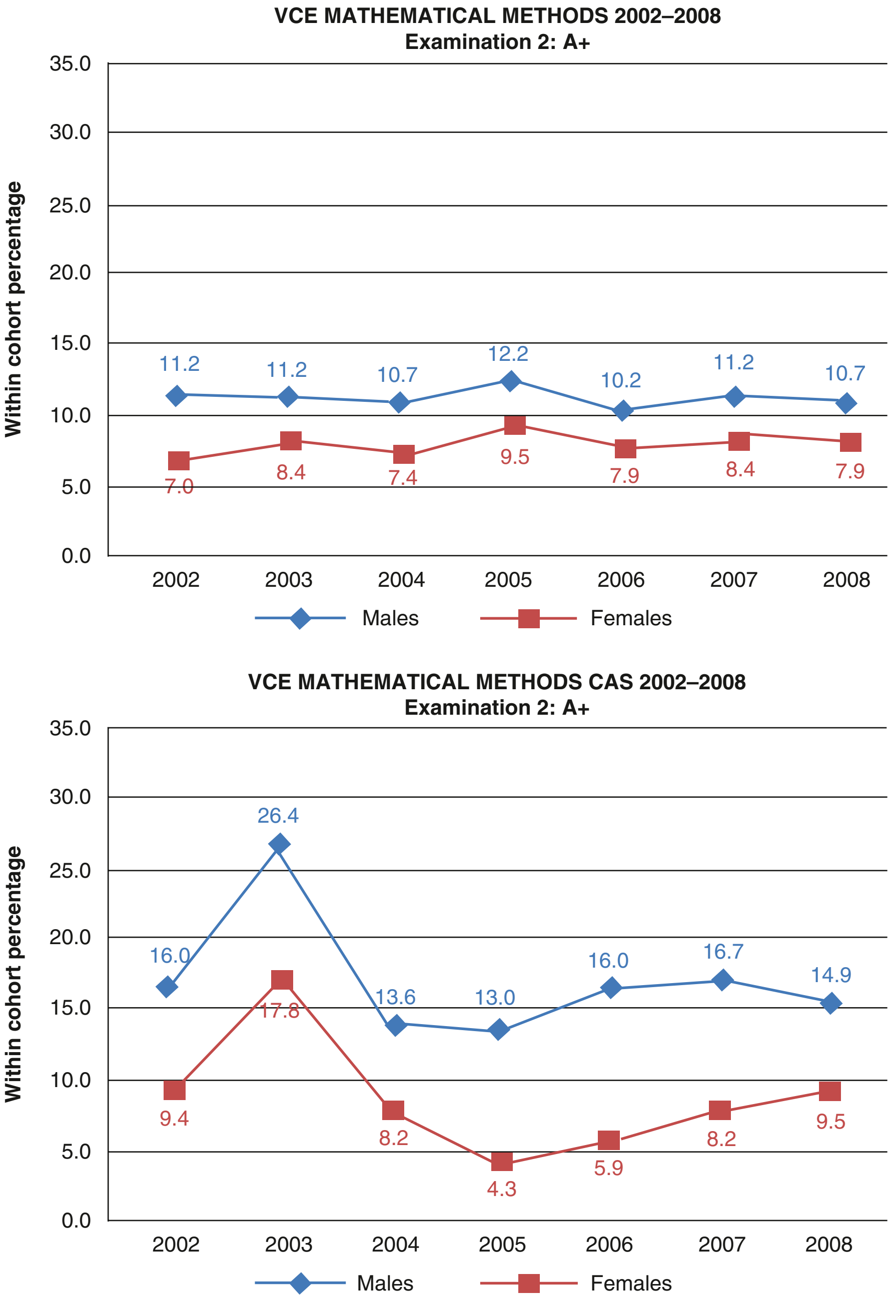

Fig. 15.1 Mathematical methods and mathematical methods CAS 2002–2008: A+ results for examination 2 by gender

Forgasz (2008) used school type in this way to examine the interaction of gender and socio-economic status among the highest performing students—those scoring 46–50 (about 1% of the cohort in each subject) in the 2007 VCE results. "Low" SES was associated with attending a government school (67% of all schools in Victoria) where no fees are paid. Students attending Catholic schools (20% of all schools) where moderate fees are paid were considered to be "medium" SES. Those attending high fee-paying Independent schools (13% of all schools) were designated to be "high" SES. Findings indicated that the proportions of males receiving each of the study scores from 50 to 46 (ranging from 58% of those scoring 46 to 83% of those scoring 49) were higher than their 54% representation in the Mathematical Methods and Mathematical Methods CAS cohorts. With respect to SES, students from Independent schools (high SES) were vastly over-represented among the top scorers (ranging from 30% of those scoring 48 to 59% of those scoring 47), and those from the government sector were grossly under-represented (ranging from 30% of those scoring 46 to 52% scoring 48). Students from the Catholic sector were fairly well represented at around 20% for scores of 46, 48, and 49 and under-represented for scores of 47 and 50. In other words, there were clear inequities in outcomes, with males and those from high SES backgrounds succeeding at the highest levels of achievement well beyond their representation in the cohort profile.

In summary, with respect to enrolments and achievement in the mathematics subjects offered in the VCE, inequities are apparent. Males are over-represented in enrolment numbers and are outperforming females in all the mathematics subjects offered. Females' achievements appear to be further affected by the mandated use of the CAS calculator. Students' SES, as gauged by school type attended, has again emerged as a variable of inequity with respect to the VCE (e.g. Teese et al. 1995).

Media Perspectives

Publication of the VCE results produced a plethora of reports and articles under headings ranging from the neutral: "Wait over for nearly 50,000 VCE students"[3] to the more evocative headlines: "Private school students scoop scholarship pools", "Mac Rob [a selective-entry girls high school] scores top marks seven years running" and "Boys outnumber girls at top VCE level". These headlines succinctly reflect the three distinct yet overlapping themes that permeated the pool of VCE articles: "high-achieving" schools, high-achieving students, and differences in between-group performances—typically comparisons of the performance of males and females, of private and state (public) school students, and less frequently of students at metropolitan and rural schools.

Brief biographical sketches of high-achieving VCE students were included in various articles. Some merely focused on students attending private or single-sex schools. Others included cases deemed particularly noteworthy: a young refugee from Afghanistan who had arrived in Australia only five years before sitting for the

[3] See the reference list for a record of the print media articles on which we drew for this chapter.

VCE examination yet came top of the high school he attended; a student who came top of her school in the north of rural Victoria; and a student from the Victorian College of the Deaf whose VCE score was well above that of other students attending the same institution and of other deaf students who had completed their high school education in mainstream schools.

Students' intended post-school destinations attracted considerable attention. "Almost all of the elite university scholarships offered to the state's highest-ranking VCE students have been scooped by students from private schools." While medicine and law were reported as popular university destinations for the very high achievers, the Victorian Minister of Education advised students who "did not reach the standards they had set for themselves" to consider other university courses or educational opportunities such as Technical and Further Education institutions, apprenticeships, and traineeships. The implication that certain courses and eventual career options were beyond the reach of many was further reinforced by articles centred around interviews with a broader range of students who discussed their course or work choices for the following year.

Considerable emphasis was given to girls' on average higher pass rates than boys' in the VCE examination overall, as well as in mathematics. Mostly, however, results were discussed in general terms rather than by individual subject. The consistent finding that boys outnumbered girls among the highest performers was also stressed in many articles, with the benefits for girls of learning in a single-sex environment espoused in some of these.

Equity issues were invoked subtly rather than directly in these newspaper articles about the VCE. There was, for example, no explicit discussion of the impact on a school's performance of a discriminatory intake through scholarship holders or entry examinations, the SES of the parents, or the parental and peer group educational expectations of students at private or selective intake schools. Issues raised in the letters to the editor section were less equivocal and more direct. "The VCE results this year once again showed a vast disparity between public and private schools. No one would seriously contend that the children of middle- and upper-class families are smarter than their less fortunate counterparts. … The government should look to implement measures of equality throughout the education system."Authorities, some writers argued vehemently, should reject using the VCE results as either a simplistic or an accurate method for ranking schools.

NAPLAN Results

Until 2007, Australian states ran their own literacy and numeracy testing programs. The first nation-wide testing was undertaken in 2008 for students in grades 3, 5, 7 and 9. The official report of the results of the NAPLAN was published late in the same year (see NAPLAN 2008). The NAPLAN results at each grade level for numeracy (and literacy) were reported by a range of variables including: Australian state/territory; gender; Indigenous/non-Indigenous; LBOTE (language background other than English)/non-LBOTE; geographical location (metro/

Table 15.1 NAPLAN 2008 results by grade level. (Source: Data extracted from NAPLAN 2008)

	Grade 3	Grade 5	Grade 7	Grade 9
Overall mean score	396.9	475.9	545.0	582.2
State/territory (6 states; 2 territories)	ACT: 411.5 to NT: 338.4	Vic: 489.7 to NT: 411.4	ACT: 556.2 to NT: 488.1	ACT: 594.9 to NT: 532.6
Male/female	M: 400.6 > F: 393.1	M: 481.6 > F: 469.9	M: 552.3 > F: 537.3	M: 586.5 > F: 577.6
Indigenous/non-indigenous	Non-indig.: 400.5 > indig.: 327.6	Non-indig.: 479.5 > indig.: 408.0	Non-indig.: 548.6 > indig.: 476.2	Non-indig.: 585.7 > indig.: 515.1
LBOTE/non-LBOTE	LBOTE: 401.0 > non-LBOTE: 396.8	LBOTE: 484.9 > non-LBOTE: 474.9	LBOTE: 553.0 > non-LBOTE: 544.4	LBOTE: 594.8 > non-LBOTE: 581.1
Geographical location (4 categories)	Metro: 402.6 to very remote: 306.2	Metro: 482.0 to very remote: 386.3	Metro: 551.4 to very remote: 451.1	Metro: 588.3 to very remote: 493.2
Parental educational background (5 categories + not stated)	Bachelor degree +: 425.1 to below grade 11: 360.8	Bachelor degree +: 508.7 to below grade 11: 440.4	Bachelor degree +: 584.8 to below grade 11: 510.6	Bachelor degree +: 623.3 to below grade 11: 550.9
Parental occupation (5 categories + not stated)	Senior management/professional: 421.4 to unemployed: 360.5	Senior management/professional: 503.0 to unemployed: 440.9	Senior management/professional: 578.0 to unemployed: 508.0	Senior management/professional: 616.0 to unemployed: 549.5

provincial/remote/very remote); parental educational background; and parental occupation. A summary of the 2008 NAPLAN numeracy results at each grade level is found in Table 15.1. These data show a strikingly similar pattern. At all grade levels:

- Males scored higher than females.
- Non-indigenous students scored higher than Indigenous (Aboriginal and Torres Strait Islander) students.
- Students with language backgrounds other than English scored higher than those whose language backgrounds were English.
- There was a direct relationship between high scores and geographical location: students living in state/territory capital cities (metro) scored highest; students living in very remote locations scored lowest.
- There was a direct relationship between high scores and parents' educational levels: students whose parents had the highest educational levels scored higher; students with parents with the lowest educational levels scored lowest.

- There was a direct relationship between high scores and parents' occupations: students whose parents were senior managers or professionals scored highest; students whose parents had been unemployed for the previous 12 weeks scored lowest.

At all grade levels except grade 5, students in the ACT (Australian Capital Territory), the small territory in which the Federal government is the major employer, scored highest; students in the NT (Northern Territory) where the highest proportion of Indigenous and very remotely located students are to be found, scored lowest. At grade 5, students from Victoria, the second most populous state, scored highest.

In summary, NAPLAN numeracy achievements were inequitable at all grade levels tested with respect to gender, indigeneity, language background, geographical location, parental educational levels, and parents' occupations. With the exceptions of gender and language backgrounds, the other variables appear to be directly related to SES, a factor considered to be the most significant contributor to achievement differences (e.g. Teese et al. 1995) and therefore impacting on future life options.

Media Perspectives

The NAPLAN (2008) test data, and their breakdown by state, "sex, location, parental background and indigenous status", were variously reported in the popular print media, with direct between-state comparisons of student performance considered of particular interest. With students throughout Australia sitting for the same test, the uneven performance of students at different schools in Australia could no longer be masked. The "tests gave parents and governments an unprecedented level of information and would enable better targeting of resources to schools and students in specific areas or years". Furthermore, it was pointed out in multiple articles, the high proportion of indigenous students who failed to meet the numeracy (and literacy) benchmarks was now more obvious.

The lower performance of indigenous students, compared with the wider Australian school population, attracted sustained media attention. The discovery that Aboriginal students living in metropolitan areas as a group performed almost as well as their non-indigenous peers received less media attention than the more startling finding that Aboriginal students living in remote communities had an extremely high failure rate of 70–80%, "A combination of low employment and poor social conditions" were explanations offered for the distressingly poor performance results. "Indigenous children are just as smart as other Australian children", one highly regarded educator was quoted as saying, "their different pass rates are the result of different schooling".

The link between parental occupation and student performance in the NAPLAN test, documented in some detail in Table 15.1 above, was also deemed worthy of wider dissemination: "Children whose parents worked in management and business had a less than 10% rate of failing NAPLAN tests compared with an average 22% failure rate for those children whose parents were unemployed."

Gender differences in performance also attracted attention. “The report shows that girls are doing a lot better than boys in reading and writing…. But when it comes to numeracy the boys are ahead—but only marginally and not in the lower grades.”

Thus the media reports faithfully, but in varying levels of detail, indicated which of the factors singled out in the more formal report seemed to affect student performance. Indigenous students, those in remote areas, and those whose parents were unemployed were depicted as the most disadvantaged.

TIMSS 2007 Results

The mathematics performance of Australian grade 4 and grade 8 students, as measured in the 2007 TIMSS, were released at almost the same time as the NAPLAN results. Although space constraints do not allow a detailed description of this third data set, it is worth noting that there were remarkable similarities in the TIMSS and NAPLAN findings. Inequities were again apparent: girls, indigenous students, those from remote areas, those who do not speak English at home, and grade 8 students whose parents have not completed high school were the disadvantaged groups. There was one intriguing difference between the NAPLAN and TIMSS 2007 results. In NAPLAN those who spoke English at home outperformed those who did not; in TIMSS 2007, the opposite was the case. Media reporting of the TIMSS 2007 findings overlapped with the coverage given to the NAPLAN results.

Equity, Quality and Segregated Learning

> Segregation has been a common institutional response to the management of differences in education. The degree of segregation varies historically and cross-culturally for different social groups, and often occurs invisibly through broader patterns of residential segregation, selection procedures and parental choice. (Lynch and Baker 2005, pp. 145–146)

Because mathematics remains a gateway to a number of career options, there is considerable interest in exploring which educational settings optimise equitable access to quality mathematics learning. Scholarly research findings on ability grouping and single-sex settings, with a focus on Australian findings, are discussed in the next sections.

Ability Grouping

The most common finding with respect to ability grouping for mathematics is that the high achievers benefit most, with low- and middle-level achievers losing out.

Ireson et al. (2002) found that students who succeeded at the British Key Stage 2 (end of grade 6) tests "benefit more from setting [ability grouping] than lower attaining pupils" (p. 311), and noted that students incorrectly placed in ability groups were likely to remain in them. Those placed in lower groups were unlikely ever to attain higher examination grades. Boaler et al. (2000) claimed that ability grouping was likely to be "the single most important cause of the low levels of achievement in mathematics in the UK" (p. 646). Linchevski and Kutscher (1998) found that average and less able grade 9 students' achievements in mixed ability settings in Israel were significantly higher than those of their peers in streamed classes. For the highest achievers, performance levels were about the same in both settings. In an Australian study on ability grouping in grades 9 and 10, Zevenbergen (2003) concluded that:

> Most often when students are grouped by ability, the outcomes support the practice—that is, the higher streams perform very well, and the lower streams perform poorly. This can be used as evidence to show that the practice is justified and that the groupings are correct since the outcomes 'prove' the effectiveness of the original groupings. However, questions need to be posed as to whether pedagogy is matching the needs of the students or whether the outcomes are a reflection of the pedagogies being used. (p. 3)

Online surveys designed to gather data on the practices used to group students for mathematics learning in grades 7–10 were recently completed by teachers representing 44 secondary schools in Victoria, Australia (Forgasz and Tan, 2009). The types of grouping practices used and the teachers' views of the practices were also sought. Ability grouping for mathematics was found to be widespread across Victoria, even at grade 7 (the first year of post-primary education). The extent of the practice was greater as grade level increased. Although teachers were largely aware of the various limitations of ability grouping, consistent with those previously reported in the literature, the majority of teachers supported ability grouping, believing that it enabled teachers to cater best for students of different achievement levels.

Media Perspectives

Ability grouping for mathematics learning attracted no explicit media interest during the period monitored. In articles concerned with inequities in the large-scale testing results, the practice of ability grouping was not identified as a potential factor affecting the results of particular sub-groups of students. While no explicit link was made in any of the print media articles surveyed between ability grouping and the fact that a disproportionate number of the highest performing students came from selective-entry high schools and private schools, discerning readers might undoubtedly infer such an association.

Some writers used the TIMSS data as platforms to highlight the plight of those with strong mathematical potential, and argued that it was all too often assumed that unsatisfactory achievement was only of concern for students considered to be failing at school. In the words of one journalist, "Much less worried about, however, is whether there are too many children who are not getting an excellent education." From the beginning of kindergarten, it was argued, there were literacy and numeracy programs to assist students with learning difficulties but "extending the abilities of the mathematics whiz-kids" is rarely considered.

Single-Sex Settings

The extent of single-sex schooling varies around the world. Gender-segregated schooling is a religious or a cultural norm in some countries, while in others it is available as an alternative that parents can choose for their children.

Comparisons of the academic performance and school-related attitudes of students attending single-sex and coeducational schools yield inconsistent findings. Gill (1988) claimed that in Australia (and the United Kingdom), "the distinction between single sex and coeducational schools is interwoven with the division between private and public schooling" (p. 3). This, in turn, is closely associated with differences in SES.

For some, single-sex education is considered anachronistic, reflecting earlier times when males and females had different educational needs that were related to their gendered future roles in society; coeducation was seen as a means to achieve gender equity. This equity argument has been challenged and various single-sex interventions implemented in different settings, for different lengths of time, for different age groups, and with different aims have been trialled to address gender inequities in enrolments and achievements in mathematics learning (see Leder et al. 1996).

Based on an extensive review of the literature, Forgasz et al. (2007) provided a summary of the main findings on the relative benefits of single-sex and coeducational settings that are also pertinent to mathematics learning:

- Coeducational settings appear to be more beneficial to boys than to girls; the benefits of single-sex settings are more equivocal for boys than for girls.
- Benefits for girls in single-sex settings include: greater positive self-concept; less gender-stereotyping of some disciplines; and perceptions of a "comfortable" learning environment.
- Other factors (e.g. organisational support, parental support, curriculum content, teaching approaches, professional development, and socio-economic backgrounds) were more significant than the gender mix of the learning setting and should be considered if the goals of enhancing girls' and boys' learning in single-sex settings are to be achieved.

In Australia, researchers have examined single-sex interventions within regular coeducational secondary school settings (e.g. Forgasz and Leder 1995; Leder and Forgasz 1997; Rennie and Parker 1997), with mixed results. Forgasz and Leder (1995) concluded that although "beliefs were that females would benefit most from single-sex classes, there were signs that males derived equal, if not more, benefit from the program than the females" (p. 44). Teachers reported to Parker and Rennie (1995) that

> single-sex classes appeared to hold the most benefit for specific groups of girls who were experiencing a great deal of harassment from boys in mixed-sex classes, and the least benefit for high-achieving girls and boys and for boys in some classes which were particularly difficult to discipline. (p. 8)

Based on a US study, Becker (2001) concluded that "simplistic solutions, such as single sex classes per se, do not appear to have been successful in themselves in achieving equity and there have been calls for new strategies to be explored… [and that] epistemology, pedagogy and parents' perceptions remain important factors" (p. 323).

In summary, ability grouping and single-sex settings may have some benefits for some students. The research, however, does not provide clear-cut answers whether either can be supported in terms of improving equity of outcomes. It would appear that ability grouping can have more serious longer-term consequences since there are issues related to the quality of the mathematics education offered in the various groups formed. Single-sex settings for mathematics as interventions to dissuade girls from abandoning future mathematical studies have had some small measure of success. Unfortunately, the consequence is that there must be single-sex classes for boys and these, it seems, have many problems associated with them.

Media Perspectives

Articles with a focus on schools with high performing students unavoidably included references to the benefits and disadvantages of schooling at single-sex and co-educational schools. According to one detailed article, supported by the editorial column in the same paper, "Many schools are having it both ways, enrolling boys and girls but separating them in class." This option, that is single-sex classes within a coeducational setting, was strongly lauded and supported with uniformly positive anecdotal evidence from senior staff working at schools which had adopted this option for all or, more commonly, some of their classes.

In an unusually comprehensive newspaper article, Bachelard and Power (2008) provided a list of eight common beliefs about the advantages of single-sex education and three putative research-based conclusions. Both sets are reproduced in full below.

Facts or Fiction?

Commonly held beliefs about the advantages of single-sex education:

- Girls and boys are "wired differently", so need to be taught in different ways.
- Boys get more attention than girls in coeducational schools, and girls learn to defer with boys in class. This reinforces gender stereotypes.

- Boys are prone to bullying and disruptive class behaviour, which makes them a bad influence on girls in coeducational environments.
- Girls in single-sex schools are more likely to study mathematics and science, subjects traditionally viewed as “masculine”.
- Teachers in single-sex schools have a greater ability to tailor classes and activities to suit the specific needs of their students.
- Girls develop more confidence and leadership abilities in single-sex schools.
- Students, particularly girls, develop closer and more meaningful relationships both with each other and their teachers in single-sex schools.
- The world is not a level playing field. Single-sex schools give girls a head start.

What the research shows:

- Boys and girls do learn differently, but single-sex education does not improve academic outcomes.
- The strongest influence on a child's educational outcome is quality teaching.
- Single-sex schools may produce high-achieving students, but this has more to do with the quality of their teachers and learning provisions.

This list offers a generally balanced and fair reflection of consistent research findings relating to single-sex and co-educational schooling, and for mathematics learning in co-educational schools. There is no recognition, however, that socio-economic factors are implicated in the choices available to parents in selecting the learning setting for their children.

Concluding Comments

For each section in this chapter, our overview of the findings from the scholarly literature has taken up more space than the summaries of the “media perspectives” on the same topic. This is consistent with, and an accurate reflection of, the more detailed and nuanced reporting characteristic of the former body of work. Emotive headlines and biased or incomplete reporting of research, coupled with a failure to discriminate between findings from rigorous studies and small, one-off intervention reports, have the potential to distort public perceptions of critical issues in mathematics education or education more generally. Core equity and quality issues are all too commonly overlooked in opportunistic and limited reports.

We contend that even when educational researchers have devised studies incorporating many of the inter-related factors that can influence equity and/or the quality of learning and educational outcomes, rigid space restrictions can lead to media commentators simplifying the problem and minimising or ignoring the complexity of the issues involved. Similarly, media reports on students’ performance in mathematics testing regimes appear to rely heavily on the executive summaries that accompany the full reports of these data. Thus, the more detailed and complex analyses undertaken of entire data sets are often omitted. For example, while the performance of students in academically selective schools is certainly highlighted in the media, there is almost no discussion of the impact of "this creaming off the top" on the students who remain behind. Are such students disadvantaged because there are fewer high achievers to engage in stimulating mathematical discussions in class? Is equity compromised because their access to optimum quality discussions is limited? Or are they instead advantaged because they are less likely to be overshadowed and will be challenged to become active rather than passive participants; to contribute rather than listen? Who benefits, and who loses, when governments increase the number of selective secondary schools, as currently occurs in Victoria, the state in which we, the authors, live? Will such a measure lead to outcomes of "national significance", one of the criteria typically used to rank competitively selected projects? Will increased segregation enhance (and for whom) or impede (and for whom) equity and quality of (mathematics) education? Will it promote the two purposes of high quality and equitable mathematics education to which we referred early in the chapter: "to prepare students to be mathematically functional as citizens…and to prepare some students to be the future professionals in careers in which mathematics is fundamental" (Bishop and Forgasz 2007, p. 1152)? To what extent can, and should, schools and other social entities be expected to counteract inequities in students' home environment? Should the media shape or reflect public opinion on the answers to these questions?

It is not easy to strike a balance when reporting, accurately and in detail, complex data that foreground equity and quality issues for different audiences. This is starkly illustrated when we consider the reporting of results from NAPLAN, the national testing program in numeracy and literacy discussed earlier in the chapter. In the summary reports and in the media, the results are presented at the national level and by state. Within these categories, the performance of various sub-groups is also presented. At the school level, however, individual school performance is set in the context of "like" schools (by socio-economic, enrolment size, etc.) and other measures. Individual student achievement data are also presented to parents in context, that is, against national and school average data, as well as an indication of the range within which 60% of all Australian students at that grade level (MCEECDYA 2009). Parents are more likely to focus on the performance of their offspring. What it means for their children's education in the broader context is less likely to be meaningful. Educators and policy makers, however, are less likely to be interested at the individual level. The questions, then, are for whom, and how, are quality and equity issues identified and interpreted.

Invited "opinion" pieces and "expert" comments on these topics (as well as others) are sometimes sought from those who hold strong views that are not necessarily supported by robust research-based evidence. These writers are typically trusted by the general public as being unbiased. In this way, the media deliberately, or inadvertently, deny their consumers knowledge of the complexities of the interaction between schooling and learning and this can result in re-inforcing some readers' pre-conceived beliefs and stereotypes. But academics must also shoulder some of the blame if their work is not disseminated widely or appropriately:

> What wins the day is not the evidence, but the power of the anecdote, and often that comes through the media. (Academics) tend to write esoteric articles in literary journals that only about four people read and we're not communicating as we should about what the evidence-based research says. (Rowe quoted in Bachelard and Power 2008)

In this chapter, we have demonstrated the effects of the inevitable tension between the measured and detailed academic documentation of research findings and the media reporting of the same findings constrained by space and time pressures. Quality and equity concerns are discussed in both contexts. Often, however, the versions of the story found in the print media fail to convey accurately or completely the issues fuelling or combating equity and quality in mathematics learning and education more broadly. The readership of the print media far outweighs that of academic publications. Mathematics, it is now widely recognised, is not value free. The evidence we have presented here similarly suggests that bias, whether intentional, pragmatic, or opportunistic, is often mirrored in the reporting of the mathematics achievement outcomes. Who reads which accounts, what opinions and views are formed, and the power and influence wielded, may well determine who benefits and who is disadvantaged in their mathematical learning.

References

Atweh, B. (2007). *Pedagogy for socially response-able mathematics education.* Paper presented at the annual conference of the Australian Association for Research in Education, Fremantle. http://www.aare.edu.au/07pap/atw07600.pdf. Accessed 6 May 2009.

Becker, J. R. (2001). Single-gender schooling in the public sector in California: Promise and practice. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education. An international perspective* (pp. 367–378). Mahwah: Lawrence Erlbaum.

Bishop, A. J., & Forgasz, H. J. (2007). Issues in access and equity in mathematics education. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 1145–1167). Reston: NCTM.

Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping—disaffection, polarisation and the construction of failure. *British Educational Research Journal, 26*(5), 631–648.

Forgasz, H. J. (2006). *Australian year 12 mathematics enrolments: Patterns and trends—past and present*. International Centre of Excellence for Education in Mathematics [ICE-EM]. http://www.ice-em.org.au/images/stories/FileDownloads/1.pdf. Accessed 21 Feb 2008.

Forgasz, H. (2008). Gender, socio-economic status, and mathematics: Performance among high achievers. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Mathematical ideas: History, education and cognition.* Proceedings of the Joint Conference PME32—PMENAXXX (Vol. 3, pp. 25–32). Morelia: Cinvestav-UMSNH (Also available on CD-ROM).

Forgasz, H., & Tan, H. (2009). *'Streaming' for mathematics in Victorian secondary schools*. Paper presented at the annual conference of the Mathematics Research Group of Australasia, Wellington, July.

Forgasz, H. J., & Leder, G. C. (1995). Single-sex mathematics classes: Who benefits? *Nordisk Matematik Didaktik (Nordic Studies in Mathematics), 3*(1), 27–46.

Forgasz, H. J., & Leder, G. C. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan & T. Wood (Eds.), *International handbook of mathematics teacher education: Vol. 1.*

Knowledge and beliefs in mathematics teaching and teaching development (pp. 173–192). Rotterdam: Sense.
Forgasz, H. J., Leder, G. C., & Taylor, C. (2007). *Research versus the media: Mixed or single-gender settings?* Paper presented at the annual conference of the Australian Association for Research in Education. http://www.aare.edu.au/07pap/for07148.pdf. Accessed 27 Apr 2008.
Gill, J. (1988). *Which way to school?* Canberra: Curriculum Development Centre.
Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B., & Williams, C. C. (2008). Gender similarities characterize math performance. *Science, 321,* 494–495.
Ireson, J., Hallam, S., Hack, S., Clark, H., & Plewis, I. (2002). Ability grouping in English secondary school: Effects on attainment in English, mathematics, and science. *Educational Research and Evaluation: An International Journal on Theory and Practice, 8*(3), 299–318.
Leder, G. C., & Forgasz, H. J. (1997) Single-sex classes in a co-educational high school: Highlighting parents' perspectives. *Mathematics Education Research Journal, 9*(3), 274–291.
Leder, G. C., & Forgasz, H. J. (2002). Measuring mathematical beliefs and their impact on the learning of mathematics: A new approach. In G. Leder, E. Pehkonen, & G. Toerner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 95–113). Dordrecht: Kluwer.
Leder, G. C., Forgasz, H. J., & Solar, C. (1996). Research and intervention programs in mathematics education: A gendered issue. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (Pt. 2, pp. 945–985). Dordrecht: Kluwer.
Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal of Research in Mathematics Education, 29*(5), 533–554.
Lynch, K., & Baker, J. (2005). Equality in education. An equality of condition perspective. *Theory and Research in Education, 3*(2), 131–164.
MCEECDYA. (2009). *Student report 2009.* http://www.naplan.edu.au/verve/_resources/NAPLAN_yr_3_with_mean_example.pdf. Accessed 2 Dec 2009.
NAPLAN. (2008). National assessment program. Literacy and numeracy. http://www.naplan.edu.au/verve/_resources/2ndStageNationalReport_18Dec_v2.pdf. Accessed 3 Jan 2008.
Parker, L. J., & Rennie, L. H. (1995). *To mix of not to mix? An update on the single-ses/coeducation debate*. Pre-conference draft. Paper presented at the Australian Association for Research in Education annual conference, Hobart, Australia, November.
Rennie, L. R., & Parker, L. H. (1997). Students' and teachers' perceptions of single-sex and mixed-sex mathematics classes. *Mathematics Education Research Journal, 9*(3), 257–273.
Teese, R., Davies, M., Charlton, M., & Polesel, J. (1995). *Who wins at school? Boys and girls in Australian secondary education.* Melbourne: Commonwealth of Australia.
The Newspaper Works. (2009). *Newspaper circulation steady—sales continue to withstand economic crisis*. http://about.theage.com.au/userpdfs/MR_readcirc0905.pdf. Accessed 24 July 2009.
Vale, C. M., & Leder, G. C. (2004). Student views of computer based mathematics in the middle school years: Does gender make a difference. *Educational Studies in Mathematics*, 56, 287–312.
Zevenbergen, R. (2003). Ability grouping in mathematics classrooms: A Bourdieuian analysis. *For the Learning of Mathematics, 23*(3), 5–10.

Media Sources[4]

Arup, T. (2008, December 20). Aboriginal children fail basic schools tests. *The Age*.
Bachelard, M., & Power, L. (2008, November 9). Class divide. *The Age*.

[4] Some of these articles were reproduced in multiple print outlets. All were found in online archives and therefore there are no associated page numbers.

Baker, J. (2008, December 18). Unequal access begins at school. *The Age*.
Buckingham, J. (2008, November 6). Brightest and best miss out. *The Australian*.
Denholm, M. (2008, November 1). Gillard praises Tasmania for lead on school ratings. *The Australian*.
Donnelly, K. (2008, December 10). Wake-up call to education chiefs. *The Australian*.
Drape, J. (2008, December 19). Girls better writers, boys lead in maths. *The Age*.
Dumenden, I. (2008, December 18). More than a number. *The Age*.
Editorial. (2008, November 9). Looking at both sides of the gender question in schools. *The Age*.
Elder, J. (2008, December 14). Best friends face VCE results with a bond that will last. *The Age*.
Ferrari, J. (2008, December 10). Aussies behind in maths, science: Study. *The Australian*.
Ferrari, J. (2008, December 10). Doesn't add up: Borat kids beat Aussies in maths and science. *The Australian*.
Ferrari, J. (2008, December 23). One in three indigenous kids fail test. *The Australian*.
Ferrari, J. (2008, December 24). Call for plan to tackle indigenous youth illiteracy. *The Australian*.
Ferrari, J. (2008, December 27). Indigenous literacy levels higher for city kids. *The Australian*.
Ferrari, J., & Trounson, A. (2008, November 25). Testing literacy and numeracy is vital. *The Australian*.
Perkins, M. (2008, December 15). Wait over for nearly 50,000 students.
Perkins, M. (2008, December 16). VCE results. *The Age*.
Perkins, M. (2008, December 16). Private school students scoop scholarships. *The Age*.
Perkins, M. (2008, December 18). MacRob scores top marks seven years running. *The Age*.
Perkins, M. (2008, December 18). College's VCE result a sign of big success. *The Age*.
Roberts, B. (2008, December 15). Days of the old school yard are over. What's next?. *The Age*.
Scott, E., & Turnbull, J. (2008, December 15). Boys outnumber girls at top VCE level. *The Age*.

Chapter 16
Equity Concerns About Mathematical Modelling

Eva Jablonka and Uwe Gellert

Introduction

Modelling approaches are propagated to enhance the quality of the outcomes of mathematics education by providing students with generic competencies and thereby creating a flexible work-force. At the same time, mathematical modelling is seen as an approach that promotes inclusion of all students, as it allows them to study a problem at the level of mathematics that they are comfortable with. The history of the discourse of mathematical modelling as a solution for many quality and equity problems in mathematics education has yet to be written. A focus on applications and on the technological value of mathematics has been interpreted as a reaction to reforms in the context of "New Math" (e.g. Howson 1989). As is well known, these reforms were underpinned, if not initiated, by arguments that stressed the competitive advantage of a nation with a mathematically skilled work-force. But this did not result in curricula that focus on applications and modelling, but rather in identifying an essence of academic mathematics, which was meant to become accessible to all students. Sfard (1998) observes a move towards the propagation of contextualised mathematics as engendered by the participatory metaphor of learning. The assumption of the contextuality of all knowledge is (mis)interpreted in a way that leads to the contention that mathematical concepts can be meaningfully learned only within a "real-life" context. Jablonka (2009) sees the trend partly linked to a conceptualisation of mathematics as a service-subject for science and engineering. Such a conceptualisation fuels labour-market driven arguments for a focus on applications and modelling, often hand in hand with a bemoaning of falling enrolment in science and engineering programmes and a perceived "threat" of loosing competitive advantage.

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The following quotes can be seen as exemplary for the innovative potential attributed to the inclusion of mathematical modelling activities:

> In a technology-based age of information, educational leaders from different walks of life are emphasizing a number of key understandings and abilities for success beyond school. These include the ability to make sense of complex systems through constructing, describing, explaining, manipulating, and predicting such systems (such as sophisticated buying, leasing, and loan plans); to work on multi-phase and multi-component projects in which planning, monitoring, and communicating are critical for success; and to adapt rapidly to ever-evolving conceptual tools (or complex artifacts) and resources. (English 2006, p. 303)
>
> While technology can remove the computational complexity of mathematical problems, it does not remove the need for students to choose carefully the tools and resources to use and to transform problem data into forms that can be handled effectively by these tools. The results obtained must be interpreted, documented, and communicated in forms that clearly and effectively convey the products of problem solution. One approach to providing students with these competencies is through mathematical modelling (English and Watters 2005; Lesh and Doerr 2003). (English 2006, pp. 303–304)

Mathematical modelling, as reflected in the above argument, is conceptualised as a set of generic competencies, the “key understandings and abilities”, which are trans-disciplinary. One concern about the development of these “key understandings and abilities” in classrooms relates to the fact that the social base for their development is in many cases far from optimal. The ideas of collaborative teamwork, free forms of communication and critical thinking require a sort of an ideal democratic environment where negotiations of meaning are expected to take place. But as Appelbaum (2004) points to, classrooms can hardly be seen as ideal speech communities. Depending on their backgrounds and educational biographies, students will not be equally able to communicate their ideas well and not all will be guaranteed an audience. Another concern about modelling in the classroom is related to the question about the type of knowledge to which different groups of students might or might not gain access in a classroom practice that offers tasks for students without specifying the knowledge domains that form the basis for their solution. A further concern is related to the choice of the problems to be modelled, which can be representative of a range of practices, in which mathematics is applied. Modelling “reality” by means of mathematics implies both a particular construction of this reality as well as an epistemological claim about the mathematical model of that reality. So the introduction of mathematical modelling has the potential for both, socio-mathematical indoctrination on the one hand or demystification of the relation between mathematics and reality on the other.

The chapter addresses these concerns by scrutinising the rhetoric and practice of mathematical modelling in primary and secondary mathematics classrooms from a sociological perspective. By drawing on structurally different examples of modelling activities in mathematics classrooms, the chapter reconsiders issues of access, control and success. The first section of this chapter provides an outline of a view on mathematical modelling that is underpinned by theories of recontextualisation of other discourses by pedagogic discourse. The next section focuses on the practice of mathematical modelling in classrooms. The final section raises issues of curriculum construction that focus on mathematical modelling. By means of examples of how modelling is enacted in classrooms, we show that students who lack the tacit knowledge required about school and classroom structure might not get access to valued forms of school mathematics knowledge through their engagement in modelling activities. Further, we argue that the conceptualisation of modelling as a set of generic competencies that could be provided by mathematics education only seemingly transcends the difficulties arising from cultural differences and economic inequalities. We consider the causality between participating in mathematical modelling activities and the diverse educational potentials attributed to this experience as mythical. We argue, that in order to overcome the problems of discontinuity between everyday knowledge and school mathematics, modelling activities should make the confrontation of the different types of knowledge involved more explicit. This is to enhance the quality of modelling activities as well as to mitigate unequal attainment due to the implicitness of the criteria.

Modelling as Recontextualisation

Mathematical modelling consists of a variety of activities that occur within distinct domains of practice in which mathematics is used. Any universal description of the process of mathematical modelling does not capture the varying methodological standards, criteria for validation and evaluation that are relevant in different practices. Jablonka (1996), for example, found that the examples that were propagated in the emerging discourse of mathematical modelling in school mathematics are extremely varied in terms of methods, models and problems. The methods range from simple algorithms to computer simulations, from ad-hoc constructions to models that are elements of scientific theories (such as physics or economics). The situations to be investigated comprise practical problems from the everyday domain as well as specific problems from a diversity of vocational practices and academic fields as well as from public administration. She also found a number of reports from classrooms, in which teachers and students together had tried to model some issue of relevance to them. Ten years later, a meta-analysis of papers published in the 14th ICMI study volume on "Modelling and Applications in Mathematics Education" (Blum et al. 2007) reveals a similar picture. It is informative to look at the collection of contexts used in the examples from the research studies presented at that conference:

> Examples from everyday life comprise: filling a swimming pool, light intensity needed for reading, cooling rates of coffee (of green tea or of corn soup), planning bus trips for senior citizens or students, distances given on road signs, shaking hands at birthday parties, buying dishes in restaurants, comparing discount percentages, life spans of batteries, taxi prices, dealing with supermarket bills, railway schedules, bank accounts, savings and loans. Some examples deal with a mathematical analysis of cultural artefacts, such as shapes of ice cream, of hats and umbrellas, of churches, modern bridges and airport buildings, of dress designs by Sonia Delaunay and the surface of a Porsche. It is easy to imagine that these contexts reflect or advocate a life-style, which is not that of the majority of the students' families.

> Professional practices from which the examples are taken include: designing a 3-question survey, reducing the noise of an aeroplane to a given limit, locating a water reservoir, seismic exploration of oil and gas, data analysis and dynamical systems models in biology, optimising traffic flow, designing a road to limit speeding, selling dishes in restaurants, measuring land, or optimising a relay race.
>
> Two examples presented at the study conference deal with an analysis of fairness (in ranking of Commonwealth games performance and of students' assessment) and one refers to statistics of supposedly rising crimes. These examples differ from the others in that they do not simulate an out-of-school mathematical practice in the classroom as authentic as possible, but aim at evaluating a practice in which mathematics is used by others. (Jablonka 2007, p. 194)

Many of the examples of modelling activities propagated for or reported from classrooms can be seen as recontextualisations of other mathematical practices (e.g. of university mathematicians, of computer scientists, of engineers, biologists, economists, statisticians, of computer users, of skilled manual workers, of consumers), where recontextualisation is understood as the process of subordinating one practice under the (evaluation) principles of another (Dowling 2009). When modelling activities are simulated in the school classroom, the criteria for the performance of mathematical modelling change in accordance with the evaluation criteria transmitted with the pedagogic discourse. By this, modelling in classrooms becomes a more uniform activity than in other practices.

Different Versions of Modelling in the Classroom: Who Has Access to the Principles?

Given the diversity of agendas and examples, the unifying principle of the modelling discourse in mathematics education can be seen in the differences constructed in relation to school mathematics without applications or in the differences to other forms of insertions of non-mathematical practices (such as word problems). By pointing to these differences "mathematical modelling" is constructed as a reform movement. There are some characteristic knowledge claims reflected in what has been termed "the modelling cycle": an ontological realism that acknowledges an independently existing reality that is the object of knowledge and the properties of which provide objective limits to how we can know it. However, these are seen as open to revision: a fallibility principle is acknowledged. This is a difference in comparison to school mathematics with a focus on both procedures and algorithms as well as on mathematical relationships and proof.

Julie (2002) describes a hypothesised activity system for school mathematical modelling: The classroom rules change towards acceptance of different non-equivalent answers, unrestricted time, acceptance of the provisional status of the outcome, and presentation in user-defined format that does not emulate the text in use. The division of labour changes from individualistic to working in collaborative teams. In the work of some teachers, who attended a three-day camp on mathematical modelling, Julie also found indication that the texts, which could have been used, were not seen as objects to be mastered, but as resources to assist the pursuance of the object. In a classroom, such a shift would indicate a shift in the authority relationship between teachers, texts and students. Underpinned by learning theories that stress the agency of the learners, school mathematical modelling activities are also intended to encourage students to communicate their own ideas and to scrutinise the ideas of others, as for example suggested by English (2006):

> The third significant aspect lies in the problems' inherent requirement that children communicate and share their mathematical ideas and understandings. Modeling problems are especially valuable because they provide a rich and varied arena for developing children's mathematical communication skills. (p. 319)

Julie (2002) also draws attention to the fact that the situation chosen as a starting point for modelling might be selected because of mathematical reasons or because of social reasons. In the first case, the context is arbitrary and the mathematical concepts, procedures, etc. are those specified in the curriculum; in the latter, the context is given (or selected by the students) and the mathematics is arbitrary. But any mathematics curriculum ultimately prescribes a certain mathematical knowledge. It also specifies the contexts in which this knowledge has to be applied, but only implicitly (Dowling 1998), if it is not a critical mathematical literacy curriculum that explicitly specifies contexts of political and social relevance.

The changes in the instructional and regulative classroom rules are modifications of *what* knowledge is accessed in classrooms and of *how* this knowledge is made accessible. In an elaboration of Bernstein's sociology of education (Bernstein 1996), the underlying principles can be termed *classification* and *framing*:

> I will now proceed to define two concepts, one for the translation of power, of power relations, and the other for the translation of control relations, which I hope will provide the means of understanding the process of symbolic control regulated by different modalities of pedagogic discourse.…
>
> I shall start first with power. We have said that dominant power relations establish boundaries, that is, relationships between boundaries, relationships between categories. The concept to translate power at the level of the individual must deal with relationships between boundaries and the category representations of these boundaries. I am going to use the concept of *classification* to examine relations between categories, whether these categories are between agencies, between agents, between discourses, between practices. (Bernstein 1996, pp. 19–20; italics not added)

In the context of mathematics education and for the purpose of our analysis, classification refers to categorising areas of knowledge within the mathematics curriculum. Strong internal classification means that clear boundaries between mathematical areas are maintained. Strong external classification indicates that few connections are made to other disciplines or everyday practice.

Framing draws on the nature of the control over the selection of the communication, its sequencing, its pacing, the evaluation criteria, and the hierarchical rules as the social base which makes access to knowledge possible (p. 27):

> I am going to look at the form of control which regulates and legitimizes communication in pedagogic relations: the nature of the talk and the kinds of spaces constructed. I shall use

> the concept of framing to analyse the different forms of legitimate communication realized in any pedagogic practice. (p. 26)

Different versions of mathematical modelling in the classroom imply variations of classification and framing. If the situation chosen to be modelled is selected because of mathematical reasons, the external classification might still be strong whereas the internal classification might become weaker as a mix of different mathematical topics and procedures is legitimate. The framing might weaken in relation to the sequencing of mathematical topics and the pace, but the criteria for what counts as a solution might still be held by the teacher. In some cases, the social base is selected by the students, for instance, when they decide about the use of texts and tools. If, in contrast, the situation chosen for a modelling activity is selected because of social reasons, then the classification might be rather weak. But this does not necessarily imply the weakening of the framing (cf. García and Ruiz-Higueras 2010, report of a modelling activity about silkworm transformation in a kindergarten class).

A version of school mathematical modelling that stresses that the external classification remains strong, is provided by Zbiek and Conner (2006):

> The primary goal of including mathematical modeling activities in students' mathematics experiences within our schools typically is to provide an alternative—and supposedly engaging—setting in which students learn mathematics without the primary goal of becoming proficient modelers. We refer to the mathematics to be learned in these classrooms as 'curricular mathematics' to emphasize that this mathematics is the mathematics valued in these schools and does not include mathematical modeling as an explicit area of study…we recognize that extensive student engagement in classroom modeling activities is essential in mathematics instruction only if modeling provides our students with significant opportunities to develop deeper and stronger understanding of curricular mathematics. (pp. 89–90)

Such a version is reflected in the approach of the *Realistic Mathematics Education*, where models are seen as vehicles to support "progressive mathematization" (Treffers and Goffree 1985), as van den Heuvel-Panhuizen (2003) points to:

> Within RME, models are seen as representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have various manifestations. (p. 13)

A version of school mathematical modelling that stresses that the external classification is weakened considerably constructs modelling as new (but vague) content. This version, which Perry and Dockett (2008) trace back to Lesh and Doerr (2003), is sometimes referred to as *emergent modelling*:

> An alternative perspective on modelling has been developed by Lesh and Doerr (2003). Galbraith et al. (2006, p. 237) have described this perspective in the following way:
>
> This second perspective [RME is the first one] does not view applications and modelling primarily as a means of achieving some other mathematical learning end, although at times this is valuable additional benefit. Rather this view is motivated by the desire to develop skills appropriate to obtaining a mathematically productive outcome for a problem with genuine real-world connections…
>
> While the above approaches differ in the emphasis they afford modelling in terms of its contribution to student learning, they generally agree that modelling involves some total

> process that encompasses formulation, solution, interpretation, and evaluation as essential components. (Perry and Dockett 2008, p. 92)

A description of modelling in the mathematics classroom in terms of classification and framing allows to analyse the potential of access to the forms of school mathematical knowledge that are more or less implicitly valued in the students' performance of the activities. In the following two examples of students' participation are discussed. Both are taken from published reports of such activities.

Equity Concerns About Modelling in the Classroom

Bernstein (1996) is claiming theoretically that variations of classification and framing relate to differential access to institutionalised knowledge. Empirical research in diverse fields, but not in the area of mathematical modelling, has generated evidence that access to the principles of classification and framing is not evenly distributed among social classes and communities (Brown 2000; Chouliaraki 1996; Cooper and Dunne 2000; Gellert and Jablonka 2009; Hasan 2001; Lubienski 2000; Morais and Miranda 1996; Moss 2000). For instance, Cooper and Dunne (2000) studied an extensive database from the Key Stage 2 (ten- to eleven-year-olds) national tests in England. They found that the misreading of the tasks, as being situated in a practice outside school mathematics, is linked to the social background of the students:

> Our key finding is that, compared with service-class children, working- and intermediate-class children perform less well on 'realistic' items in comparison with 'esoteric' items. We have attempted to control for a variety of other possible explanations of this apparent effect of item type in interaction with social class, including children's 'ability', the wordiness of items, the difficulty level of items and, employing N[ational] C[urriculum] attainment targets, the mathematical topic being addressed. The effect of the interaction of social class with item type survives, though it is sometimes reduced. (p. 199)

When, in an experiment, Cooper and Dunne made the classification principle behind the item construction transparent to working-class children, the rate of success increased substantially. However, the impact of the Key Stage 2 results on students' potential careers is rather strong:

> To illustrate this, we have developed a simulation of what would happen to children from different social class backgrounds if a selection process were to occur on the basis of three differently composed tests: one comprising items that behave like our 'esoteric' items, one of items that behave like our 'realistic' items, and one comprising an equal mixture of the two. This process might be realized as a selection examination for secondary school of for 'ability' group placement within the first year of secondary school. …It can be seen that using our results as the basis for predicting outcomes, the proportion of working-class children in this sample who would be selected by an 'esoteric' test is double that which would be selected by a 'realistic' test. (p. 94)

Instead of the 12.1% of working-class children who would have been selected on the basis of the "realistic" test, 24.2% would have passed the "esoteric" test (compared to 33.3% and 30.0% of service-class children).

An Example of Weakly Classified and Weakly Framed Modelling

The different versions of mathematical modelling may impact differentially on the legitimacy of participation in students' collective modelling activities. English's (2006) report of a group of four primary school children's construction of a consumer guide can be considered a type of a modelling activity where the task and the solving process are weakly classified and, in some dimensions, weakly framed. After the teacher had clarified the purpose and working of a consumer guide, each group of students was provided by four different packets of potato chips and instructions what to do: to write a consumer guide that helps people in choosing any snack chip, not just the four provided by the teacher. The students were asked to describe the nature or type of factors that they want to consider and to categorise, and to rate these factors.

The activity is weakly classified as there is no clear indication about priorities of mathematical knowledge and experience with consuming potato chips, respectively. The internal classification is weak as well as no indication is given on the mathematical topics prevalent in categorising and rating factors. The framing is mixed as on the one hand, the teacher clearly initiates the activity, controls the social base and makes criteria for evaluation explicit: "Prepare a short report for your class members explaining why the system you developed for your consumer guide is a good one" (p. 309). By giving this criterion, the activity is clearly framed as pedagogic. On the other hand, the students seem relatively free to decide about the sequencing of their modelling activities, and the teacher is not controlling strictly the pace in which the activity develops.

The description of the students' interactions reveals important differences in students' participation. These differences develop coherently during the activity. Whereas a student named Kelly is repeatedly arguing on a personal and context-bound level, Ahn, another student, seems to reject the limitations of personal experience and looks out for mathematical, that is disembedded, concepts to approach the modelling task:

1. Kelly, after the group has established a list of factors such as flavour, taste and crunchiness, "verified its appropriateness by drawing on her experience in a supermarket: 'That's because they had a chip tasting and they asked all those questions…. They were asking people to taste the chips and they asked questions like, would you be likely to ask your parents to buy this sort of chip?'" (p. 310) Ahn, in contrast, pushed forward the idea to categorise and rank the factors immediately.
2. When arguing on the representativeness of their consumer guide, Kelly commented: "It'll be a bit hard because this is what we like and other people like other flavors." Ahn, in contrast, explained, "that the consumers 'might not like what we like but we're covering all aspects. So they can choose from what we give them so that isn't really a factor.'" (p. 311)
3. Then the discussion "addressed categorizing *cheese taste* according to 'above average cheesiness, below average cheesiness, just right cheesiness,' to which

> Ahn responded, 'But we don't know the average.' Kelly also expressed concern about rating taste: 'I know how you rate but I still don't get it, why we rate, because they are all different factors so it's like harder to rate stuff…so I just don't get how you are supposed to rate them if they're not the same. Like, you can't add centimetres to meters.'" (p. 311)

Weak classification and mixed framing values might amount to everyday insertions by some students while others focus on what they consider the appropriate mathematical concepts and procedures to be applied. This is not to say that knowledge about potato chips is irrelevant for solving the task, quite the contrary. However, it seems to be difficult for some students to decide to what extent and for which purpose this knowledge is relevant: Is the activity accessing knowledge of potato chips, of consumer guides, or of the mathematical concepts and procedures involved? This issue is pertinent in the contexts of assessment. "When children communicate their ideas to their group during problem solution, they engage in formative assessment: They progressively assess and revise their current ways of thinking" (p. 320). However, as the activity is weakly classified it remains unclear towards which ends the formative assessment is directed. If the activity of setting up a consumer guide is embedded in a mathematics curriculum, then students such as Kelly who believe in the value of non-mathematical arguments do not profit from the kind of peer formative assessment that does not take the institutional context sufficiently into account. If the writing of consumer guides occurs within a project curriculum with weaker boundaries between the vertical discourse of mathematics and other disciplines or common sense knowledge, then the project itself and the students' activities are ambivalent: they simulate an out-of-school mathematical practice in the classroom as authentic as possible (the teacher distributes brands of potato chips), but it remains widely implicit in which respect the students' activities can count as mathematical. Students such as Ahn know, at least tacitly, that the mathematics involved needs to be made explicit (e.g. when he argues about average values), but others might entirely miss the mathematical points. In sociological terms, these others are educationally disadvantaged by the weak classification of the activity. The lack of reference to mathematics in the criteria given by the teacher (see above) aggravates this effect.

In weakly classified and framed modelling tasks, the recontextualisation principle is covert. As the openly announced criteria ("prepare a short report", see above) do not refer to any knowledge domain, the students apparently exert control over the recontextualisation principle. They seem to be in a position to decide in which way the practice of selecting potato chips might be subordinated to school practice. However, as school knowledge is institutionalised knowledge, the control over the recontextualisation principle remains in the hands of the institution, that is, the teacher. Mathematical modelling that is weakly classified and framed minimises some students' access to valued forms of school mathematics knowledge, because it masks the differential value of different kinds of arguments for legitimate participation in the mathematics classroom.

An Example of Mixed Classification and Mixed Framing

In order to analyse age 11 to 13 students' operations and arguments in mathematical modelling, Grigoras and Halverscheid (2008) construct a three-step task. It is said that a "salesman has to travel through eight cities of Germany" (p. 107). The students are, first, asked to identify the kind of information necessary to plan the salesman's travel; and, second, to write a one-page letter to the salesman, which provides three suggestions of how to plan the travel. Then the salesman responds by indicating eight German cities he has to visit. He emphasises that he intends to finish the trip as fast as possible. The students are, third, asked by the travelling salesman to "explain me in a detailed manner, how you developed the trip suggestion" (p. 107). The task was handed out to groups of students that have not yet been mathematically introduced to graphs.

Grigoras and Halverscheid (2008) observe that the groups of students struggle with the recontextualisation effected in the salesman tale: "It was often not clear whether the students are in the mathematical world or in the rest of the world" (p. 111). Although this way of characterising the mutual insulation of knowledge domains might be considered as theoretically blank, it nevertheless indicates that the recontextualisation of salesman travel manifest in the modelling task is strongly related to the values of classification and framing underlying the production of legitimate answers to the problem at hand. This point might already be considered a shortcoming of the activity. When looking more closely at the data, we are faced with students' differential understanding of the mathematical purpose of the activity. Although the task formulation is devoid of any mathematical concept or operation, there is a clear indication that some students know about the restricted value of everyday arguments in solving mathematics tasks while others argue within their repertoire of everyday knowledge.

> S.2: Here, when we…look [to S.1] again here. Then we can come back. (pause) That is totally stupid, because one drives around…
> …
> S.2: Firstly, one has to establish a route, so that one saves energy!
> …
> S.2: What does he want in this town?
> S.3: We do not need this! (p. 110)

The student S.2 is repeatedly using everyday arguments while the student S.3 rejects these as irrelevant for solving the task. As in the other example, we do not know about the students' social backgrounds. But again we witness that access to the recontextualisation principle is not equally given. Indeed, from the task and the students' discussion presented in the report, we cannot directly rate the value of classification. As this modelling activity was embedded in an ordinary transmissive geometry course, it is a good strategy for students to expect a strong external classification.

Grigoras and Halverscheid (2008) discuss why the students' mathematical discourse is rather limited:

> If a question leaves the mathematics open, there is a tendency of students to answer the questions rather intuitively than based on classical mathematical reasoning. This is a possible answer why for some students it was not clear what the problem has to do with mathematics. (p. 111)

Apparently, a weak internal classification paralleled by low framing values with respect to the criteria for legitimate problem solutions, might lead to students' intuitive rather than mathematical reasoning. The weak internal classification is due to the absence of explicit mathematical concepts and operations in the modelling task as well as to the solitary character of the modelling task. The combination of strong external and weak internal classification operates in a mode that is particularly susceptible to educational disadvantage. Students that struggle with the recontextualisation principle are faced with the absence of textual indicators that allow identification of any relevant mathematical operation or concept. Whereas those students familiar with the classification of mathematics in school might find an interesting opportunity to spread and develop their mathematical creativity, those who lack the tacit knowledge required about school and classroom structure might be overwhelmed by the indefiniteness of the activity.

Modelling as a Basis for Curriculum Construction: Concerns and Potential

In the section above, two different modes of school mathematical modelling in terms of the relationships between the knowledge domains involved have been described, and it has been shown how these might relate to differential access to mathematical knowledge. If modelling is not subordinated to the principles of school mathematics as specified in a curriculum, then the question arises to the principles of which discourse it relates. As mathematical modelling is not a uniform practice, but a set of interrelated activities in different domains (that is, it might itself be considered as a mediated discourse), there is no set of uniform criteria for performing mathematical modelling. Consequently, the discourse of school mathematical modelling, if it is not subordinated to accessing mathematical knowledge, leaves an open space for promoting different agendas, such as developing human capital by channelling students into an engineering career pipeline, expressing and rethinking cultural identity, educating critical consumers or promoting social change.

The conceptualisation of modelling as a set of generic competencies that could be provided by mathematics education seemingly transcends the difficulties arising from cultural differences and economic inequalities because the activity of constructing mathematical models, through which these competencies are to be developed, is not seen as culture-bound and value-driven. Such a conception masks the fact that the construction of mathematical models depends on the perception of what the problem to be solved with the help of mathematics consists of and what counts as a solution. But depending on the subject position of the "modeller" in a practice, there are different models of the same problematic situation:

> For example, if the problem of a bank employee, who has to advise a client (aided by a software package), is the comparison of financing offers for a mortgage, for the manager of the bank this is a problem of profitability, and for the customer it is one of planning her personal finances. (Jablonka 2007, p. 193)

This is not to suggest that mathematical models should be scrutinised exclusively in terms of the values connected with the underlying interests. But the discourse of mathematical modelling as providing individuals with generic competencies that enable them to become adaptive to the conditions of technological development, to overcome the limitations of specialised knowledge, to gain competitive advantage on the labour market and become critical consumers and democratic citizens, is mythologising mathematical modelling because the causality between participating in mathematical modelling activities and the diverse educational potentials attributed to this experience is mythical. The myth embodies the claim of the ethical neutrality of mathematical modelling practices.

The popularity of modelling can be explained by the fact that it achieves a fictitious marriage between two strands of critique of a strongly classified mathematics curriculum. Such critique is on the one hand an outcome of an attack on a neo-conservative defence of canons of disciplinary specialised knowledge, which (at least historically) comes together with the reproduction of inequality of access to such specialised knowledge. On the other hand, the critique of strongly classified curricular knowledge comes from the side of those called “technical instrumentalists” by Moore and Young (2001) who advertise economic goals. Preparation for the “knowledge-based economy” is a major concern. Moore and Young observe that the scope of instrumentalism has extended from vocational training to general education under the guise of promoting the employability of all students. There is a danger that the myth of the neutrality of generic modelling skills discards the tension between neo-liberal ideology with a focus on human capital preparation and a conception of education for social change.

Instead of focussing on the question of how individuals through acquiring mathematical modelling techniques can best be prepared to adapt to the conditions of technological development and specialised knowledge, the question of how individuals can be prepared to reproduce these conditions as their own could form the starting point for curriculum development. The potential of activities in which modelling takes place, when students and teachers engage in a modelling activity about an issue relevant to them or to their community has to be further illuminated by a sociological analysis of the institutional structures and organisational dynamics, in which mathematics education is embedded.

It has been argued by a range of scholars who concentrate on analysing mathematisation and demathematisation as social processes that reflections on modelling can contribute to deconstructing the myth of the ethical neutrality of mathematical practices (cf. the review by Jablonka and Gellert 2007). The adequate forethought for curriculum construction then consists in the identification of modelling practices that differ in methodological standards and criteria for validation and evaluation.

When mathematical modelling is to mediate access to institutionalised mathematical knowledge, then identifying out-of-school practices, which could profitably be transformed by a mathematical recontextualisation remains a major task. For overcoming the problems of discontinuity between everyday knowledge and school mathematics that have been identified as a main source for unequal attainment, modelling activities should include a more explicit confrontation and transformation of the different types of knowledge involved.

References

Appelbaum, P. M. (2004). Mathematics education. In J. Kincheloe & D. Weil (Eds.), *Critical thinking and learning: An encyclopedia for parents and teachers* (pp. 307–312). Westport: Greenwood.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique*. London: Taylor & Francis.

Blum, W., Galbraith, P., Henn, H.-W., & Niss, M. (Eds.). (2007). *Modelling and applications in mathematics education: The 14th ICMI study*. Berlin: Springer.

Brown, A. (2000). Positioning, pedagogy and parental participation in school mathematics: An exploration of implications for the public understanding of mathematics. *Social Epistemology, 14*(1), 21–31.

Chouliaraki, L. (1996). Regulative practices in a 'progressivist' classroom: 'Good habits' as a disciplinary technology. *Language and Education, 10*(2 & 3), 103–118.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem solving*. Buckingham: Open University Press.

Dowling, P. (1998). *The sociology of mathematics education: Mathematical myths/pedagogical texts*. London: RoutledgeFalmer.

Dowling, P. (2009). *Sociology as method: Departures from the forensics of culture, text and knowledge*. Rotterdam: Sense.

English, L. D. (2006). Mathematical modeling in the primary school: Children's construction of a con-sumer guide. *Educational Studies in Mathematics, 63*(3), 303–323.

English, L. D., & Watters, J. J. (2005). Mathematical modeling in third-grade classrooms. *Mathematics Education Research Journal, 16,* 59–80.

Galbraith, P., Stillman, G., & Brown, J. (2006). Identifying key transition activities for enhanced engagement in mathematical modeling. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces*. Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia (pp. 237–245). Adelaide: MERGA.

García, F. J., & Ruiz-Higueras, L. (2010). Exploring the use of theoretical frameworks for modelling-oriented instructional design. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education: January 28th–February 1st 2009* (pp. 2166–2175). Lyon: Institut National de Recherche Pédagogique.

Gellert, U., & Jablonka, E. (2009). "I am not talking about reality": Word problems and the intricacies of producing legitimate text. In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modelling verbal descriptions of situations* (pp. 39–53). Rotterdam: Sense.

Grigoras, R., & Halverscheid, S. (2008). Modelling the travelling salesman problem: Relations between the world of mathematics and the rest of the world. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the Joint Meeting of PME 32 and PME-NA XXX* (Vol. 3, pp. 105–112). Mexico: Cinvestav-UMSNH.

Hasan, R. (2001). The ontogenesis of decontextualised language: Some achievements of classification and framing. In A. Morais, I. Neves, B. Davies, & H. Daniels (Eds.), *Towards a* *sociology of pedagogy: The contribution of Basil Bernstein to research* (pp. 47–79). New York: Peter Lang.

Howson, A. G. (1989). Curriculum towards the year 2000. In J. Malone, H. Burkhardt, & C. Keitel (Eds.), *The mathematics curriculum: Towards the year 2000* (pp. 13–17). Purth: Curtin University of Technology.

Jablonka, E. (1996). *Meta-Analyse von Zugängen zur mathematischen Modellbildung und Konsequenzen für den Unterricht*. Berlin: transparent verlag.

Jablonka, E. (2007). The relevance of modelling and applications: Relevant to whom and for what purpose? In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 193–200). Berlin: Springer.

Jablonka, E. (2009). Mathematics for all: Why? what? when? In C. Winsløw (Ed.), *Nordic research in mathematics education* (pp. 293–305). Rotterdam: Sense.

Jablonka, E., & Gellert, U. (2007). Mathematisation—demathematisation. In U. Gellert & E. Jablonka (Eds.), *Mathematization and de-mathematization: Social, philosophical and educational ramifications* (pp. 1–18). Rotterdam: Sense.

Julie, C. (2002). The activity system of school-teaching mathematics and mathematical modelling. *For the Learning of Mathematics, 22*(3), 29–37.

Lesh, R. A., & Doerr, H. (2003). Foundations of a model & modeling perspective on mathematics teaching and learning. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modelling* (pp. 3–34). Mahwah: Lawrence Erlbaum.

Lubienski, S. T. (2000). A clash of social class cultures? Students' experiences in a discussion-intensive seventh-grade mathematics classroom. *The Elementary School Journal, 100*(4), 377–403.

Moore, R., & Young, M. (2001). Knowledge and the curriculum in the sociology of education: Towards a reconceptualisation. *British Journal of Sociology of Education, 22*(4), 445–461.

Morais, A., & Miranda, C. (1996). Understanding teachers' evaluative criteria: A condition for success in science classes. *Journal of Research in Science Teaching, 33*(6), 601–624.

Moss, G. (2000). Informal literacies and pedagogic discourse. *Linguistics and Education, 11*(1), 47–64.

Perry, B., & Dockett, S. (2008). Young children's access to powerful mathematical ideas. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 75–108). New York: Routledge.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), 4–13.

Treffers, A., & Goffree, F. (1985). Rational analysis of realistic mathematics education—The Wiskobas program. In L. Streefland (Ed.), *Proceedings of the Ninth International Conference for the Psychology of Mathematics Education* (Vol. 2, pp. 97–121). Utrecht: OW&OC, Utrecht University.

van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*(1), 9–35.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational Studies in Mathematics, 63*(1), 98–112.

Chapter 17
Equity and the Quality of the Language Used in Mathematics Education

Marcus Schütte and Gabriele Kaiser

Introduction

The students of German schools are influenced by multilingualism and various cultural backgrounds due to continual immigration in the recent decades. Currently, almost one-third of the students in German schools have a migration background. The gap between the pupils' high or low socioeconomic background seems to increase exceedingly as well. This circumstance would not be worth considering if each student had equal prospects for a successful school career. However, this is not the case as is confirmed by various official reports (cf. Beauftragte der Bundesregierung für Migration, Flüchtlinge und Integration 2005, p. 30 and 2007, p. 12 ff).

Results of the international comparative OECD study PISA whose first tests were undertaken in 2000 clarify that the German school system does not render high-quality school achievements in its width by an early and intense selection but merely raises the unequal opportunities within the school system to a top international level. This may be surprising since people in Germany generally assume that a strong selection results in a high quality of education. On closer examination, the results of PISA 2000 show a reverse image. Thus, eight out of 11 countries with the highest school equity among the OECD states who participated in the PISA study 2000 achieve a high quality in terms of their students' performance in reading. Merely Spain seems to achieve comparatively high school equity for its pupils with a mean performance in reading literacy, which is statistically lower than the OECD average (cf. OECD 2005, p. 27 ff.).

Therefore, the key to successfully establishing school systems with an internationally compared high quality seems to be the systematic integration of multifarious pupils rather than early selection. A basic formula could therefore be: equal opportunities accomplish quality breadthways. So far, the constructive dealing with

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lingual and cultural plurality of pupils who originate from so-called educationally disadvantaged families with low socioeconomic status only succeeds rudimentary within German schools. Therefore, the success of the entire educational system seems to be endangered (cf. OECD 2006, p. 30).

Great potentiality to change this situation is unanimously located—especially for pupils who live and learn with more than one language—in the German language proficiency. This appears to be plausible. But how do children have to be supported on their way to get there? The results of international comparative studies like PISA 2003 as well as the international primary school study PIRLS (Bos et al. 2003) lead to the result that poor educational opportunities and school achievements—also those in mathematics—of pupils can predominantly be ascribed to causes that lie apart from school such as the socioeconomic background as well as language used in the homes. According to those results, there is a connection between the vernacular used in the parents' homes and the linguistic and mathematic competences of the youths. Youths whose vernacular used in the parents' homes is not coherent with the language used during lessons achieve lower competency scores in all domains of the PISA tests (cf. Deutsches PISA-Konsortium 2004, p. 259 f.).

On the basis of those results during the last few years various attempts have been made especially to diagnose the linguistic abilities of children before entering school and if necessary to improve them. For this reason, obligatory language tests for children have been established in the individual federal states. (cf. e.g. Ministry of Education North-Rhine Westphalia 2009; see also materials by the University of Dortmund[1]).

Criticism concerning those tests is manifold due to the following reasons:

- On the one hand, the diagnostic methods seem to be imperfect and little standardised. Different years are examined by means of highly different tests within the different Federal states of Germany. Also the choice of children seems to be heterogeneous. While in some Federal states of Germany all children are tested, there are some Federal states in which only children with a migration background are tested and others where children are tested who do not attend nurseries.
- On the other hand, it seems unclear how and where children with linguistically too low competences in comparison to their peer group can get the best support. According to the integration plan of the German government, in the future those children ought to be linguistically supported in their homes as well as in the nurseries even before entering school. German politicians seem to perceive the core of the solution to the problem in linguistic education in early childhood.

A connection between the pupils' poor school achievements in German schools and factors from outside school such as language used in the parents' homes and the parents' socioeconomic status is highly plausible. Nevertheless, it cannot be assumed that the structure of school and lessons does not have any influence on these results. To make families and nurseries responsible for success in school as

[1] http://www.delfin4.fb12.uni-dortmund.de/. [Last access: 15.04.2010].

well as educational opportunities in advance of school entry and thereby not taking the school itself in consideration seems disputable against this background. The question arises whether the school—the prevalent place of the education of young people—is able to make a significant contribution to support young people who are not provided with the necessary resources to successfully complete their school career by their homes in such a way that they have comparable prospects as their privileged classmates.

The aim of the present contribution is therefore to spot causes that are responsible for students performing poorly who participate in classes with linguistic and cultural plurality and often originate from educationally disadvantaged families with low socioeconomic status. The reconstruction of phenomena concerning the linguistic shaping of teaching within a school subject such as mathematics, which is still considered to be unaffected by linguistic and cultural influences, might facilitate the transferability of these conclusions to other school subjects. Thus, the results of these analyses of mathematics teaching in primary school classes indicate implications for the studies of students within plural linguistic and cultural classes beyond mathematics and primary school education.

The international discussion in the field of mathematics education uses prominently approaches that allocate language and communicative competence both a special significance for the learning of mathematic contents (cf. Pimm 1987; Steinbring 2006; Zevenbergen 2001). Within the German-speaking countries, Maier (2006, 2004 and 1986) was already concerned 20 years ago with research in the field of language and mathematics. In the following section, we want to reveal on the basis of our own empirical studies how mathematics teachers use language in their teaching to shape their lessons. In the underlying research of this contribution, there were 15 different episodes in total which were analysed. The video recordings on this took place in three classes of the fourth grade in two Hamburg primary schools with an approximate 80% migration contingent amongst the pupils. The enquired classes were instructed by three different teachers. The episodes were systematically compared. Subsequently, an exemplary analysis of a short sequence from one of the 15 episodes will be depicted.[2]

An Episode from the Lesson Sequence "Least Common Multiple"

In the following chapter, a short sequence of an episode of an everyday primary school mathematics lesson during the introduction of a new mathematical concept is described. This scene serves as an illustrative example. According to the approach

[2] This scene under consideration deals primarily with a shortened extract from the original episode, since for reasons of space limitations no analysis of the entire episode was possible. The detailed analysis of this episode can be found in Schütte (2009).

of interactional analysis, first this sequence will be presented as a summarised analysis of the interaction followed by an analysis of the linguistic shaping of the lesson including a theoretical reflection.

Background and Transcript of the Lesson Episode

At the beginning of the scene "least common multiple" Ms. Teichmann along with 25 female and male pupils, 17 of which have a migration background, are situated in the classroom. Ms. Teichmann is formally educated as grammar school teacher. In the preceding mathematics lessons, Ms. Teichmann immersed pupils during the lessons in the basic arithmetic operation of multiplication. In this lesson, the introduction of a new mathematic concept should take place: the LCM—the Least Common Multiple[3].

It is Wednesday morning shortly after the start of instruction. Ms. Teichmann asks initially what the abbreviation LCM stands for. Thereafter, she asks to calculate multiples. Subsequently, the teacher draws two circles on the blackboard. She divides one of the circles into three segments and the other into four segments, with an addition symbol between them and an equals sign. She marks for each circle one of the segments in pink. While one pupil very quietly says, "1/3 plus 1/4", Ms. Teichmann asks the pupils which equation is written on the blackboard. The pupils begin to guess and first give the answer "1 plus 1" or "2", and then somewhat later label the segments with 1/3 and 1/4. The teacher notes this in the drawing on the blackboard, as shown in Fig. 17.1. Several pupils offer many creative solutions for their addition, such as for example "2/7". The teacher adjusts the fractions from 1/3 and 1/4 to 4/12 and 3/12 and then adds the fractions together to get 7/12. In closing, her generalisation of the procedure follows…[4]

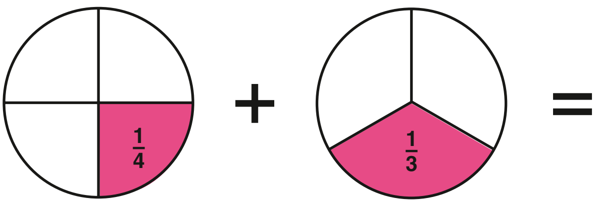

Fig. 17. 1 Illustration for adding fractions created by the teacher

[3] It should be noted that fractions as well as the concept of the LCM are not subject matters of the fourth grade in primary schools in Hamburg and pupils are likely overextended by calculating fractions (cf. Hamburger Rahmenplan Mathematik für die Grundschule 2004).

[4] The transcription conventions are provided in the appendix.

Table 17.1 Classroom discourse while introducing the concept of adding fractions

241	16:30	<L:	Right/you may not- add a large piece of pizza
242		<L:	[Points to the left circle]
242.1		>L:	And a small one and a smaller -. one together
243		>T:	[Points to the right circle]
244		T:	That is not equal right/
244.1		<T:	You must practically…
245			Chop them into such pieces that they are equal\
246		<T:	[Makes a chopping motion with her hand]
247		>T:	..right/these pieces are equal\
248		>T:	[Points to the left circle]
248.1		<T:	[Points to the right circle]
248.2		<T:	These pieces as well\
249			Only here it is less\right/here there are only three-
250		>T:	And here there are four pieces.
250.1		>T:	[Points to the left circle]
251		P:	Ah now I understand it
252	16:57	T:	And for that reason one need this\. if you at all want to (add) fractions-
253			So that you can add together such pieces of cake together\
254			Right/one cannot simply
255			Say three and four is seven and from above
256			We will take two and then I have two sevenths\
257	17:11		Two sevenths is something completely different
258			No that doesn't work\

Summarising Analysis of the Interaction

At the end of the episode, the teacher attempts to show the pupils a generalisation of the addition of fractions. She uses for this purpose the everyday example of the division of a pizza respectively cake and makes the division of them visual through gestures <246>. Hereby both levels of the illustration on the basis of relations to everyday life and the generalisation of the rules of fractional arithmetic melt together. This is shown in the statement by Ms. Teichmann in <252–258>. The reference to "LCM" seems to have been completely lost, respectively left as implicit. Alone the, "…and for that reason one needs this…" in <252> from Ms. Teichmann gives us the idea that there is still a reference to the "LCM", since one needs an "LCM" in order to find the least common denominator for the addition of the two fractions. Ms. Teichmann does not further explicate this connection. Also, the final generalisation by hand of the cake example <252–258> can barely be accounted for as a further clarification of the procedure, since Ms. Teichmann says that one may not simply add three and four together and means thereby apparently the denominators of one third and one fourth.

Through the selected example, however, pupils did indeed have to add three and four in order to ascertain the solution of the task—though, on the level of the numerator. They added 3/12 and 4/12. Moreover, the addition of the numbers three and four are everyday tasks for primary pupils. Why one may no longer carry out this arithmetic remains unexplained. Since one cannot assume, that the pupils are competent to differentiate between numerators and denominators, one can classify the statement of the teacher as contradictory. Consequently, pupils in the end of this episode were merely able to solve an addition task, which they were already capable of solving before and whose correctness would now be put into question.

Methodology and Methodological Approach

The underlying research to this contribution is qualitatively oriented and grounded in interpretative classroom research. More exactly: in the domain of the interactionistic approaches of interpretative classroom research in the field of mathematics education (cf. Krummheuer et al. 1999). Video recordings of everyday[5] mathematics primary school classes serve as empirical basis of this contribution. Through the analysis of the units of interaction in the videotaped instructional episodes, we oriented ourselves to a reconstructive-interpretative methodology and on a central element of the research style of Grounded Theory—the methodical approach of comparative analysis.[6] In order to analyse the lingual shaping of primary mathematics teaching via the teacher, those video episodes were primarily analysed, where new mathematical terms are introduced. The linguistic form is of great importance within these stages because it is a matter of first-time construction of something subjectively new to the pupils. The focus on mathematics teaching is justified since it is a determined factor for the transition to Secondary school in the German school system and constitutes the central selection subject in the fourth grade next to the teaching of the mother tongue German.

The Analysis of the Linguistic Shaping

Here, subsequently follows the analysis of the linguistic shaping of the instruction on the basis of the selected instructional episode. We confine ourselves to two theoretically developed analytical levels that explicitly refer to linguistic approaches [especially Pimm (1987) with regard to the term of register developed by Halliday (1975)]. Alternatively, we primarily refer to sociological pedagogical and linguistic

[5] This means that the researchers did not wittingly undertake an interference during the lessons. The teachers of the research chose the mathematics used in class on their own as well as the materials and their didactic procedures to organise the subject matter.

[6] The extensive transcripts of the episodes as well as the analyses can be found in Schütte (2009).

approaches like the ones developed by Bernstein (1977) and Gogolin (2006). These two perspectives represent the level of the mathematic-linguistic register and the formal language register respectively that are based on one another hierarchically and reflect the amount of the use of *Bildungssprache*, which might be translated as academic language[7] (Gogolin 2006), by the teacher during the introduction of new mathematical concepts.

The Embedding of Mathematical Concepts in a Mathematics Register

The first level of analysis concerning the linguistic shaping of instruction while the teacher introduces new mathematical concepts can be developed by a reference to the approach of Pimm (1987). He compares teachers as a role model of a *native speaker* of mathematics (ibid, p. xiii) and other people, for whom mathematics appears to be incomprehensible, as per a foreign language, to which they are not empowered (ibid, p. 2). In this context, Pimm (1987) is speaking of a *mathematics register* (p. 74) and theoretical concept introduced by Halliday (1975).

Halliday understands a register as an assemblage of meanings that are intended for a particular function of language together with words and structures that are able to express these meanings. Halliday subsequently talks of the mathematics register only when a situation is concerned with meaning, that is related to the language of mathematics, and when the language must express something for a mathematical purpose. Mathematics register in this sense can be understood as not merely consisting of terminology and the development of this register is also not merely a process to which new words can be added (cf. Halliday 1975, p. 65).

Pimm (1987, p. 76) sees the task of pupils, however, as to become proficient in a mathematics register and in this way to be able to act verbally like a native speaker of mathematics. The level of the linguistic shaping of instruction falls into what extent the newly learned mathematic concepts in the researched lesson were integrated into a mathematics register or if they were to be introduced and regarded as isolated units.

The Analysis of the Selected Episode

In the selected episode, the teacher appears to attempt to explain the mathematic concept LCM in connection with the addition of fractions. In the beginning of this episode, the teacher reverted to the concept of multiples in allowing pupils to calculate them. According to the theoretical perspective of Pimm (1987), the attempt by

[7] The translation of *Bildungssprache* as academic language does not cover the full meaning of the expression because the term *Bildung* has a special meaning in European pedagogy.

the teacher to reconstruct the concept of LCM only allows itself to be incorporated, not as an isolated conceptual unit, but through its connection with other mathematic concepts in a mathematics register. According to Pimm, it should be the goal to make pupils competent native speakers of mathematics. In the introduction by the teacher, however, at no point of time in the entire scene the mathematical concepts of denominator, numerator, fractions, fraction strokes, or multiples were verbally and content-wise clarified in the official classroom discourse. They remain implicit and are integrated without reflection in the already familiar calculation routines. Even the teacher herself rarely uses the concepts to be learned actively, such as is shown in the first analysis. She rather reverts back predominantly to the everyday language concepts. The illustration on the blackboard is the only aid for the pupils to develop the meanings of the new concepts. Merely the concept of multiples is offered differently to the pupils. They are given the possibility to actively negotiate the meaning of multiples by calculating them.

It seems questionable that pupils are able to develop the concepts without a verbal contextual explanation of the concepts by the teacher. Pupils must develop the subject with this *implicit procedural method* from their everyday background or from what they already know from their lessons. They will thus be able to take no decisive steps in the direction of becoming a native speaker of mathematics.

The Embedding of the Mathematical Concepts in a Formal Language Register

The second level of analysis of the linguistic shaping of instruction unfolds from the reference of the theoretical explanations of Gogolin (2006) referring to the German context as well as explanations of Bernstein (1977) and Zevenbergen (2001) in an international context. According to Gogolin (2006), pupils in German schools are bound to the normative standard that they are receptively and productively in command of the cultivated linguistic variations in class. This language of school—described by Gogolin as *Bildungssprache* (ibid, p. 82 ff.)—translated by us as academic language—has more in common with the rules of written linguistic communication, regarded on a structural level. It is in large parts inconsistent with the characteristics of the everyday verbal communication of many pupils.

Gogolin relates the concept of *Academic language* to a concept of *Cognitive Academic Language Proficiency* described by Cummins (2000, p. 57 ff.) that Cummins develops in the context of second language acquisition with English being the secondary language. Cummins refers to children quickly acquiring abilities in their secondary language to cope with everyday situations but they require considerably more time to gain competences in the *Academic language* of lessons which are necessary to be educationally successful. A crucial feature of the *Academic language* in class is its conception as a written form. Therefore, it features a high degree of information density as well as a disengagement of the situation. Hereby, the *Academic language* differs significantly from common oral communication of pupils. Empirical research in the German-speaking countries that could characterise *Academic language* in lessons more precisely are not yet available. Gogolin and Roth (2007, p. 42) as well as Kaiser et al. (2009) only name sub-areas, which are relevant for the acquisition of *Academic language*, amongst others the passive, impersonal expressions, the subjunctive, nominalisations, compound words and attributes.

In their discussion concerning the language of instruction, Bernstein (1977) and Zevenbergen (2001) do not intend to differentiate between children with or without migration background but rather to distinguish between children from the working and middle class. According to Bernstein and Zevenbergen, the linguistic abilities of formal language that are required in schools set a line of demarcation in everyday language, that is more in accordance with the abilities of the middle class than with those of the working class. This formal language of instruction stands out through its precise grammatical structure and syntax as well as through its complex sentence structure. Through proficiency in this formal language, pupils—especially those from the middle class—develop a sensibility in regard to the structure of objects as well as the structure of language which helps them to solve problems in life and in school in a relevant and goal-oriented manner. Successfully receptive in *being (a) part (of)* and productive as in *taking part (in)* (Markowitz 1986, p. 9, translated by the authors), a linguistic discourse of instruction is something that is only possible for pupils, according to the above-mentioned theoretical approaches, when they have competence in the formal language or the *Academic language* of instruction. In this way, it is possible for them to understand abstract concepts independent of concrete context and to transfer them into written de-contextualised form.

In this level of the analysis of the linguistic shaping of primary mathematics instruction, there follows the question, to what extent, and how pupils are introduced during instruction to a formal *Academic language* that enables them to take part in the interaction during classes that implies to efficiently cope with language in order to satisfy the criteria of shapeliness and grammaticality of language.

The Analysis of Selected Episode

In her attempt to make a generalisation at the end of the sequence, the teacher says in <241–242> “Right/you may not add a small piece of pizza and a small one and smaller one together”. She also uses the comparative form of the adjective small, for this purpose, but does not go into the “Least Common Multiple” more explicitly. However, it is not self-explanatory that all pupils—most especially those who have grown up multilingual or originate from educationally disadvantaged environments—are familiar with the correct comparative forms of adjectives in the German language. Pupils might not be able to differentiate between Small Common Multiple and Least Common Multiple at once. This interpretation is supported by analysis of previous episodes, in which pupils used the incorrect comparative form when attempting to use the term Least Common Multiple (for details concerning the extensive analyses we refer to Schütte 2009).

Another correlation to this can be seen in the procedure at the beginning of the scene where the teacher allowed the pupils to calculate multiples. At no point in time did the teacher explain the connection between the terms multiple and Least Common Multiple. She did not refer to the meaningful components that form the mathematical concept of least common multiple. In this way, it is made difficult for students to be able to differ between multiples in common and the Least Common Multiple. It is not attempted on the part of the teacher to integrate the new concept into a related text. Hereby the question may be asked if and how the students should be empowered to understand such abstract concepts independent of concrete examples and to be able to transfer them into written form.

Summary of the Analysis of the Linguistic Shaping of Instruction

The comparison of the two hierarchical levels of the linguistic shaping of instruction which are related to each other resulted in the following structural characteristics which were not all apparent in the documented lesson sequence. However, the selected sequence exemplifies central structural characteristics of the linguistic shaping of mathematics classes.

The implicitness of learning processes can be reconstructed as a common structural characteristic that underlies the linguistic shaping while the teacher introduces new mathematic concepts. The implicitness of learning content is reflected in the usage of different mathematical and formal linguistic registers. Concerning the mathematic register, it can be reconstructed that the meaning of concepts as well as content connections between the new mathematic concepts and the already known everyday language concepts are not made clear by the introduction. The meanings or connections are not explicitly taken up in the instructional discourse and find thus no consideration in the classroom discourse.

The formulated goal of Pimm (1987, p. xiii) that students should learn to speak mathematics like a native speaker, will be difficult for students to achieve, as the native speaker of mathematics—the teacher—does not exemplify this active speaking herself or himself.

A similar picture becomes apparent in the way the teachers commit themselves to formal linguistic particularities of formal linguistic register. Also here there is an implicitness that rules the teachings. The teacher only refers back to the grammatical structure implicitly, in which the mathematical concept is embedded, or to that, which characterises the meaning carrying elements. With which linguistic methods the complex and abstract mathematic concept, in the sense of the conceptual writing is expressed to a connected text is left, as regards content or implicitness, in the end of the attempted explanations, unconnected.

Thereby, the few attempted explanations with regard to content or implicitly ending stay unconnected. There is only superficial response to the shapeliness of the pupils' speech in the different episodes to that extent that the articulation and notation of the new mathematical concepts are explicitly taken up by the teacher. Superficially, this takes into account that a high ratio of pupils does not speak German as their native language. A continuous embedding of the mathematic concept in *Academic language* is not noticeable but necessary to describe abstract concepts de-contextualised from a specialised to a general view.

Theoretical Reflection: Implicit Pedagogy as a Barrier for Equity

The implicit proceeding within teaching discourses, containing a formally unshaped mode of speaking may endanger the comprehensive development of new concepts on the side of the students and therefore hinders the understanding of mathematical concepts. What kind of problem can generally arise to pupils of so-called educationally disadvantaged families due to this kind of linguistic shaping of instruction?

The analysed teaching sequence is predominated by a language usage that significantly depends on colloquial everyday language and in spite of introducing new mathematical terms and definitions does not achieve a formal linguistic status. Considering the linguistic shaping of teaching a multifarious body of pupils, it can be supposed that this usage of informal everyday language via the teacher during the introduction of new mathematical concepts benefits especially those pupils who originate from so-called educationally disadvantaged families with a low socioeconomic status and/or migration background. These pupils are according to Bernstein (1977) competent to follow a teaching discourse that is characterised by the usage of everyday language since they are proficient in the German informal everyday language. In that way, the pupils from educationally disadvantaged families in the researched primary school teaching would be "met wherever they were" following a prevalent pedagogical device.

This conclusion contains discrepancies against the background of the new results of Bernstein (1996), Bourne (2003) and Gogolin (cf. 2006) as well as providing only a pretended aid to the pupils. In addition, it can be disproved by the reference to the depicted analytical results of the research at hand. The children who require a linguistic introduction to formal linguistic instructional language within class are not satisfied by this kind of linguistic shaping that resembles everyday discourse by the use of implicitness and informal everyday language. It can be assumed that the school institution is the only possible place for many of these children to learn a formal *Academic language* and their only guide is the teacher. Therefore, the instructors do not offer them adequate guide and the school does not provide an adequate place to learn formal *Academic language* that again is crucial to a successful school career. This kind of linguistic shaping is inconsistent with the normative requirements of the German school system.

Based on the works of Gogolin et al. (2003), it is possible to hypothesise that these competencies of formal speech become relevant during efficiency ratings such as testing and the comprehension as well as development of texts. The present classes do not seem to prepare all the students for the requests of the German school system. Competences in *Academic language*, that is textual competencies that are characterised by verbal linguistic text competences, may enable the students to comprehensively read through thematically and linguistically carefully composed and specialised texts as well as to process this information according to a task and subsequently develop oral and written texts. But these skills are not conveyed to students within this kind of education.

In this respect, the system of primary school fails to disclose an access to a successful school career to its unprivileged participants. The pedagogical approach to meet the pupils—especially those who originate from educationally disadvantaged families—at their profession and to remain at this level may become a stumbling block for the linguistic as well as professional education in German schools. The opportunities to learn about new technical terminology seem to be limited to these pupils in opposition to their classmates who grow up in a monolingual German environment as well as educationally advantaged. Pupils from educationally advantaged families apparently possess the abilities to compensate the deficits that are located in the linguistic accomplishment of instruction which affect the development of meaning of new technical terms and/or the composition and appliance of a formal *Academic language* due to the competences that they acquired at their homes.

However, the reconstructed procedures of the teachers during introducing new mathematical terms could not only be explained by mathematics teaching approaches and for this reason further pedagogical, sociological and linguistic approaches were consulted for the analyses. The use of these approaches allows clarification of the procedure of the teacher during the linguistic shaping of instruction on the basis of a theoretical concept which is called *Implicit Pedagogy* (cf. Schütte 2008, 2009). According to this implicit pedagogy, the teachers are predominately assigned to prepare a learning environment to their students and to observe the development of individual inherent abilities and "talents" of each child. This kind of teaching could be understood as a "pathologic form of open teaching concepts". In these interactions, the teachers do not function as the advanced individuals who encourage the pupils to advance their development even if they partially choose frontal teaching with the whole class or rather "closed forms of interaction" as teaching methods.

The approach of *Implicit Pedagogy* is based on the main idea, that alone on the basis of the abilities the students bring along with into school they can unlock meanings of concepts or underlying contextual and linguistic contexts or it is assumed that the structures are self-explanatory to the students. Neither the lessons, the qualifications of the teachers, nor their efforts will bring in the decisive influence on the possible educational success of students in school, but rather, and above all else, the abilities that the children have already brought with them into school decide on their educational success. Existing social relationships hereafter become reproduced. Besides, distinctions between pupil's performances can be unchangeable classified and legitimised by socioeconomic and social differences.

Prospects

According to international comparative studies, the decisive factor to successfully complete an educational biography in German schools seems to be the pupils' mother tongue or the language used in their everyday life. However, this contradicts the data of the complimentary study of PISA referring to students with migration background (cf. Ramm et al. 2005). In this study, 50% of the youths with a migration background classify themselves to be primarily German-speaking whereas only 13% state that they predominantly use their native language. In regard to these data, it is possible to state that the majority of youths with a migration background attending German schools use an informal everyday language in their social environment. Admittedly, within the complimentary study of PISA 2003 students with a migration background neither reach the average level of competence of the youths without a migration background in mathematics nor in reading abilities despite of their proficiency of the German everyday language.

A special interest of research could accrue from the fact that according to the results of PISA 2003, youths with a migration background who pass their entire educational biography in German schools show worse competency values than those who immigrated at a later time. The thesis suggests itself that pupils who have already been educated in their countries of origin possess formal linguistic abilities that are in a different language to that of the German language and that they benefit from these abilities in the lessons held in German. Although pupils who pass their entire educational biography at German schools seem to possess proper colloquial German language competences but neither acquired abilities of a formal *Academic language* of their countries of origin nor in the German language. In this respect, there is nothing to object to the trials of encouraging children linguistically within their families or in day care centres. But these initiatives begin with colloquial competencies that are undoubtedly necessary conditions for a successful school career in German schools, but do not represent sufficient conditions. This results in the necessity of research and support. Therefore, it is essential to find out how to equate the everyday as well as academic linguistic competencies of students with a migration background or originating from educationally disadvantaged families with low socioeconomic status to their fellow students who grow up monolingual via systematic programs for language acquisition in the subject lessons.

The goal should be to create a concept of language acquisition that enables all pupils to participate actively in the educational discourse of lessons by teaching formal linguistic competences regardless of whether they live and learn in one or more languages.

Appendix

Table 17.2 Transcription Conventions

1	Line numbering
1.1	Additional, belated inserted line
[*Annotation*]	Annotation, commented explanatory remark
[Action]	Action that occurs between two temporally separated sections of the transcript
(Word)	Not an undoubtedly understandable word or sentence
/	Lifting of the voice
-	Voice in abeyance
\	Lowering of the voice
. .. …	Speech pauses given in seconds
T	Teacher
P	Pupils
<T: The house is smaller\ <P: is smaller	Notation of the score: The speakers (in part) act simultaneously. This is indicated by the simultaneous proceedings standing directly among each other
>T:	The shift of the direction of the arrow indicates a new and immediately following block of score

References

Beauftragte der Bundesregierung für Ausländerfragen. (2005). *Bericht der Beauftragten der Bundesregierung für Migration, Flüchtlinge und Integration über die Lage von Ausländerinnen und Ausländern in Deutschland.* Berlin: Bundesregierung.

Beauftragte der Bundesregierung für Migration, Flüchtlinge und Integration. (2007). *Der Nationale Integrationsplan. Neue Wege—Neue Chancen.* Berlin: Bundesregierung.

Bernstein, B. (1977). *Class, codes and control. Towards a theory of educational transmission: Vol. 3.* London: Routledge & Kegan.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity*. London: Taylor & Francis.

Bos, W., Lankes, E.-M., Prenzel, M., Schwippert, K., Walther, G., & Valtin, R. (Eds.). (2003). *Erste Ergebnisse aus IGLU. Schülerleistungen am Ende der vierten Jahrgangsstufe im internationalen Vergleich*. Münster: Waxmann.

Bourne, J. (2003). Vertical discourse: The role of the teacher in the transmission and acquisition of decontextualised language. *European Educational Research Journal, 2*(4), 496–521.

Cummins, J. (2000). *Language, power and pedagogy. Bilingual children in the crosstire.* Clevedon: Multilingual Matters.

Deutsches PISA-Konsortium. (2004). *Der Bildungsstand der Jugendlichen in Deutschland—Ergebnisse des zweiten internationalen Vergleichs.* Münster: Waxmann.

Gogolin, I. (2006). Bilingualität und die Bildungssprache der Schule. In P. Mecheril & T. Quehl (Eds.), *Die Macht der Sprachen. Englische Perspektiven auf die mehrsprachige Schule* (pp. 79–85). Münster: Waxmann.

Gogolin, I., & Roth, H.-J. (2007). Bilinguale Grundschule: Ein Beitrag zur Förderung der Mehrsprachigkeit. In T. Anstatt (Ed.), *Mehrsprachigkeit bei Kindern und Erwachsenen. Erwerb—Formen—Förderung* (pp. 31–45). Tübingen: Attempto.

Gogolin, I., Neumann, U., & Roth, H.-J. (2003). *Förderung von Kindern und Jugendlichen mit Migrationshintergrund.* Expertise für die Bund-Länder-Kommission für Bildungsplanung und Forschungsförderung. BLK-Materialien zur Bildungsplanung und Forschungsförderung, No. 107.

Halliday, M. A. K. (1975). Some aspects of sociolinguistics. In UNESCO (Ed.), *Interactions between linguistics and mathematical education* (pp. 64–73). Copenhagen: UNESCO.

Hamburger Rahmenplan Mathematik für die Grundschule des Bildungsplans Grundschule. (2004). Freie und Hansestadt Hamburg. Behörde für Bildung und Sport. www.hamburger-bildungsserver.de/bildungsplaene/Grundschule/M_Grd.pdf. Accessed 16 May 2008.

Kaiser, G., & Schwarz, I. (2009). Können Migranten wirklich nicht rechnen? Zusammenhänge zwischen mathematischer Sprachkompetenz und allgemeiner sprachlicher Kompetenz. *Friedrich Jahresheft 2009 Migration, 2009,* 68–69.

Krummheuer, G., & Naujok, N. (1999). *Grundlagen und Beispiele Interpretativer Unterrichtsforschung*. Opladen: Leske + Budrich.

Maier, H. (1986). Empirische Arbeiten zum Problemfeld Sprache im Mathematikunterricht. *Zentralblatt für Didaktik der Mathematik, 18*(4), 137–147.

Maier, H. (2004). Zu fachsprachlicher Hyper- und Hypotrophie im Fach Mathematik oder Wie viel Fachsprache brauchen Schüler im Mathematikunterricht? *Journal für Mathematik-Didaktik, 25*(2), 153–166.

Maier, H. (2006). Mathematikunterricht und Sprache. Kann Sprache mathematisches Lernen fördern? *Zeitschrift für die Grundstufe des Schulwesens mit "Mitteilungen des Grundschulverbandes e. V.", 38*(4), 15–17.

Markowitz, J. (1986). *Verhalten im Systemkontext. Zum Begriff des sozialen Epigramms. Diskutiert am Beispiel des Schulunterrichts*. Frankfurt: Suhrkamp.

Ministry of Education North-Rhine Westphalia. (2009). http://www.schulministerium.nrw.de/BP/Schulsystem/Schulformen/Grundschule/Sprachstand/Sprachstandsfeststellung_zwei_Jahre_vor_der_Einschulung/Druckverlage_Fachinformation.pdf/.

OECD. (Ed.). (2005). *School factors related to quality and equity*. Paris: OECD Publishing.

OECD. (Ed.). (2006). *Where immigrant students succeed. A comparative review of performance and engagement in PISA 2003*. Paris: OECD Publishing.

Pimm, D. (1987). *Speaking mathematically*. London: Routledge.

Ramm, G., et al. (2005). Soziokulturelle Herkunft und Migration im Ländervergleich. In PISA-Konsortium Deutschland (Ed.), *PISA 2003. Der zweite Vergleich der Länder in Deutschland: Was wissen und können Jugendliche?* (pp. 269–298). Münster: Waxmann.

Schütte, M. (2008). Die sprachliche Einführung neuer mathematischer Begriffe im Grundschulmathematikunterricht. In E. Vasarhely (Ed.), *Beiträge zum Mathematikunterricht* (pp. 729–733). Franzbecker: Hildesheim.

Schütte, M. (2009). *Sprache und Interaktion im Mathematikunterricht der Grundschule. Zur Problematik einer Impliziten Pädagogik für schulisches Lernen im Kontext sprachlich-kultureller Pluralität.* Münster: Waxmann.

Steinbring, H. (2006). What makes a sign a mathematical sign?—An epistemological perspective on mathematical interaction. *Educational Studies in Mathematics, 61*(1–2), 133–162.

Zevenbergen, R. (2001). Language, social class and underachievement in school mathematics. In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 38–50). New York: Routledge.

Chapter 18
Foundational Mathematics: A Neglected Opportunity

Jennifer S. McCray and Jie-Qi Chen

Early childhood education represents a powerful opportunity to address equity issues in mathematics education in the United States. This is true in two distinct respects. First, there are persistent differences in mathematics achievement between the United States and many other developed nations (National Research Council 1989, 1990; Schoenfeld 1992). Tellingly, these differences appear as early as preschool, where children from other developed and developing countries outperform their American counterparts on such beginning mathematics concepts as number words and early addition (e.g., Ginsburg et al. 1997; Starkey et al. 1999). Second, there is clear evidence of a national mathematics "achievement gap" among U.S. students of different demographic backgrounds, which appears as early as kindergarten (Entwisle and Alexander 1990; Griffin et al. 1994; Jordan et al. 1992). This gap disproportionately favors children from higher socio-economic status backgrounds (Denton and McPhee 2009), indicating that children who lack economic means have an additional disadvantage in terms of readiness to learn primary-level mathematics. Because this gap persists and often widens throughout schooling (National Research Council 2009), early childhood is the best time to eliminate it, creating subsequent equitable access to mathematical learning in elementary school.

Given that approximately 80% of U.S. preschool-age children are in some form of out-of-home care, there is ample opportunity to address mathematics through the early care and education system (Jacobson et al. 2007). While intervention that begins early has effects that extend into later years (Bowman et al. 2001; Clements et al. 2004), early intervention specifically focused on mathematics has positive effects on student learning that generalize beyond it (Fuson et al. 1997). The effectiveness of such intervention points out a dearth of quality teaching that, if remedied, can provide more equitable preparation for elementary school among and for U.S. students.

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Unfortunately, mathematics education for preschool children has been neglected in the United States, and for many years. This is in contrast to early literacy, an educational topic that has received a wealth of attention over the past two decades. So successful have been the efforts of the early literacy community to explain itself that "Today…early childhood literacy is regarded as the single best investment for enabling children to develop skills that will likely benefit them for a lifetime," (Dickinson and Neuman 2006, p. 1). This disparity of attention paid to early literacy versus early mathematics is seen in conference programs, and evidenced in the teaching practices of early childhood teachers. For example, at a recent meeting of the National Association for the Education of Young Children (NAEYC 2008), a conference focused almost entirely on preschool education, there were only 23 presentations on mathematics, compared to 65 on literacy. Further, a recent study of early childhood classrooms in Chicago, Illinois, showed that 90% were likely to conduct literacy-related activities on any given day, while only 21% were observed to conduct mathematics activities (Chicago Program Evaluation Project 2008).

Neglect of early mathematics also occurs among researchers in mathematics education. At the 2008 Research Pre-session of the Annual Meeting of the National Council of Teachers of Mathematics (NCTM), only three of 106 presentations focused specifically on the mathematics education and learning of children before first grade. The Chicago Public Schools, the third largest public school system in the United States, provides math coaches to classroom teachers in grades Kindergarten to 12, but no such provision is made for pre-k teachers and the children they teach. Clearly, early mathematics education for three- to five-year-olds has not yet become part of U.S. mainstream education (Ginsburg et al. 2008), and while quality mathematics teaching is the focus of current reform efforts in elementary and secondary schools (see, e.g., Hill et al. 2005), there is little to no discussion of its relevance to early childhood education.

We argue that a re-definition of early mathematics as *foundational mathematics* may further both quality and equity agendas in the United States. As described in greater detail below, foundational mathematics is that mathematical thinking that can develop prior to primary school. It anticipates arithmetic and does not rely upon the use of secondary symbol systems, such as written numerals, but is clearly mathematical, and susceptible to teaching intervention. Specifically, we propose that the reconceptualization of early mathematics as foundational mathematics: (1) distinguishes it from the mathematics that is more appropriate for elementary school, (2) names this mathematics, legitimizing it as content that can be both understood and taught, and (3) suggests its important role in preparing learners for more advanced thinking.

To advance this case, we begin by describing how in the United States both the early childhood and mathematics education communities have come to neglect early mathematics. We define foundational mathematics at length, endeavoring to make clear its centrality to equitable access to mathematical learning, and delineating how its common acceptance might address those misconceptions that have kept it from being taught. Finally, we report on a proven intervention designed to help early childhood teachers understand foundational mathematics, thereby improving the quality of their teaching.

Early Childhood Education: Why Does It Neglect Mathematics?

There are two related reasons why early education has managed to ignore the importance of improving its teaching of mathematics for so long. First, there is a belief that young children are not ready to engage mathematical thinking before primary school, an idea often (misguidedly) supported through reference to "developmentally appropriate practice," the dominant paradigm for early care and education in the United States. Second, there is a distinct lack of both good mathematical knowledge and mathematical confidence among early childhood teachers, who like all teachers, tend to teach best what they know best. While each of these causes has an unfortunate tendency to feed and fortify the other, each also has roots in different socio-historical events and understandings.

Developmentally Appropriate Practice

In the United States in the early 1980s, there was a convergence between middle-class families who hoped preschool could give their children a "leg up" on academic achievement, and those concerned with promoting educational equity by improving the preparation of children from socio-economically disadvantaged families (Golbeck 2001). As a result of this convergence, more state and local governments included preschool in the public schools (Spodek and Brown 1993), and emphasized early formal academic instruction, based on the idea that it was an enriching experience for young children (Gersten and Carnine 1984; Veras 1975). Direct instruction, drill, and worksheets focused on imparting basic skills tended to be part of this effort to enhance the education of young children, as the typical first grade curriculum was simply "pushed down." This approach to preschool was met with something of a backlash within the decade. Elkind (1987), for example, argued that children's predilection to learn by play and exploration was inappropriately squelched by a curricular focus on early academic achievement. Other child development experts joined the debate and echoed his sentiments (see, e.g., Kagan and Zigler 1987; Sigel 1987), and in 1987, the NAEYC issued its Position Statement on Developmentally Appropriate Practices (DAP) (Bredekamp 1987).

Essentially, the Position Statement on DAP supported a developmentally universalist, child-centered approach to teaching. While it was not tied to any specific curricula, it advocated a set of principles NAEYC felt should govern the education of all young children. Specifically, it recommended that teaching be responsive to the needs and capabilities of individual children, emphasizing teacher observation and support over assessment and the introduction of skills. Though not without controversy, the influence of DAP on early childhood practice in the United States and elsewhere has since been particularly pervasive and far-reaching (Bowman et al. 2001).

In part due to this almost-universal emphasis on child-led teaching at the preschool level, there are many early childhood teachers who firmly believe the study of mathematics before elementary school is developmentally inappropriate. Not unlike many of their elementary counterparts, preschool teachers often think that memorization of facts is the key element of mathematical learning (Sarama and DiBiase 2004). There is also evidence that the math-related knowledge and teaching strategies of early childhood teachers, scant though they may be, are focused on modeling the use of procedures to solve problems and ensuring correct implementation of computational skills (Copley 2004, p. 403). While the larger mathematics education community has made a lot of headway in "redefining mathematics as a dynamic discipline full of opportunity for inquiry and discovery," the early childhood community has not yet gotten the message (Feiler 2004, p. 399). Given their ideas, skill sets, and beliefs about mathematics, it is easy to see how early childhood teachers would think it developmentally inappropriate to teach it to preschoolers.

Lack of Content Knowledge and Confidence

Early childhood teachers' sense that mathematics is not an appropriate topic for preschoolers is further encouraged by their belief that it is not an appropriate topic for themselves (Ginsburg et al. 2006). Most early childhood teachers, unlike their peers at the elementary school level, have received no training in teaching mathematics, even if they have a bachelor's degree in early childhood education (Copple 2004; Ginsburg et al. 2006). This educational lack is both compounded by and compounds a lack of confidence in their mathematical abilities among early childhood teachers, many of whom were counseled into teaching at the younger grades so they could "escape" mathematics (Andews 2009, personal communication). The joint position statement on preschool math by the NAEYC and the NCTM (NAEYC 2005) supports this view, noting that the general lack of knowledge and preparation contributes directly to poor math-related attitudes and a lack of confidence among many early childhood teachers.

While lack of confidence does not, in itself, prevent a teacher from teaching math, it appears to feed an unfortunate tendency to avoid math in the classroom. A recent contribution to preschool math teacher training goes so far as to devote a section to lack of confidence, noting "Math Anxiety—You Can Handle It" (Smith 2001, p. 2), and when surveyed (Carpenter et al. 1988), both pre- and in-service teachers in early childhood classrooms expressed great reluctance to teach mathematics, making comments like "I don't do math." In the United States, we have unwittingly assembled a cadre of early childhood teachers who are less knowledgeable and more fearful of mathematics, on average, than the general population. It is no wonder they are less likely to teach it than might be helpful.

It is also true that this lack of knowledge and confidence on the part of early childhood teachers has the added detriment of diluting the quality of mathematics that is taught. Research shows that when knowledge and confidence are weak, teachers understandably tend to rely on text, and to present content as a collection of unchanging and not highly connected facts (Brophy 1991). Only a solid and nuanced understanding of content allows a teacher to anticipate how thinking about it develops, implement curricula effectively, and most importantly, flexibly capitalize on those moments in classroom life when a key concept can be clarified or reiterated (National Research Council 2009). Describing early childhood educators, Copley notes “to them, mathematics is a difficult subject to teach and one area that they often ignore except for counting and simple arithmetic” (2004, p. 402). There is good reason to assume that the lack of mathematics knowledge among preschool teachers has not only influenced the field to avoid and ignore mathematics as a topic, but has ensured that most of the math teaching that occurs is mediocre, at best.

Mathematics Education: Why Does It Neglect Early Childhood?

Over the last century, the dominant paradigm for analyzing elementary math content and advocating reform in its teaching in the western world has expressed itself as a dichotomy between math *concepts*, on the one hand, and math *procedures* on the other. In this orientation, *conceptual mathematical knowledge* is richly connected mathematical thinking that embodies information about relationships between things, such as the idea that any set of items can be counted, or that adding and subtracting are inverse operations and therefore can “undo” one another. *Procedural mathematical knowledge*, on the other hand, is meant to include the forms, rules, and procedures that make it possible to complete mathematical tasks. Procedures are generally sequential lists of actions that produce desired outcomes, such as “perform operations within parentheses first” or “when multiplying by 10, move the decimal one place to the right.”

While mathematics education reformers spent much of the twentieth century debating whether to emphasize concepts over procedures or vice versa (see, e.g., Bruner 1960; Gagné 1977; McLellan and Dewey 1895; Thorndike 1922; Wheeler 1939), in the early 1980s Resnick and Ford suggested that *relationships between* concepts and procedures might be a better place for educational focus. This suggestion was buoyed by the work of Giyoo Hatano, who, in 1982, opined that the goal of education—whatever the subject matter—should be to foster *adaptive expertise* among students (Hatano 1982). As he defined it, adaptive expertise is “the ability to apply meaningfully learned procedures flexibly and creatively” (Hatano 2003, p. xi). In this construction, teachers should prepare students who can use their existing knowledge to *create* effective procedures that solve problems they have not encountered before. Hatano makes the point that conceptual knowledge gives “meaning to each step of the skill and provide(s) criteria for selection among alternative possibilities for each step within the procedures” (1982, p. 15). Elementary math education reformers latched onto this notion of adaptive expertise as a worthwhile goal, and have advocated teaching methods meant to prevent a child from learning a mathematical procedure with no connection to mathematical meaning, or concepts (see, e.g., Baroody and Coslick 1998; Baroody and Dowker 2003; Clements et al. 2004).

This emphasis on avoiding disconnection rests on the proposition that the mathematics to be learned employs written symbols: namely, numerals and mathematical notation. Disconnection between procedures and concepts is only possible when mathematical procedures are not *implicitly* meaningful, as occurs when written notation is in use. Because of its focus on this problem, the concept-procedure literature includes an unspoken assumption (e.g., Hiebert 1986) that all the important mathematics occurs at the point when the thinker begins to use written symbols to describe and manipulate it. Any mathematical thinking that occurs prior to writing is generally not considered. Hiebert and Lefevre (1986) comment, "connections between conceptual and procedural knowledge still are in place as children enter school," (p. 19), thereby dismissing the preschool period as a time for teaching interventions. Their assumption seems to be that the mathematical thinking that develops before school does so in a manner that is beyond intervention; differences in the educational experiences of the very young are not acknowledged by this argument as significant. This idea about preschool mathematics, combined with the math education literature's characterization of good teaching as imparting new written procedures in ways that are meaningfully connected to concepts, has distracted math educators from a serious consideration of the mathematics teaching that is possible prior to the use of written symbols by its students.

Why Should We Call Early Mathematics Foundational?

While it is undoubtedly true that the use of numerals, notation, and arithmetic propels mathematical thinking into levels of complexity far beyond what humans would be capable of without them, it is also true that mathematical thought can and does occur prior to their use, and that it is highly abstract. In fact, the number system, which many consider the earliest mathematics, rests on the understanding of relational abstractions, and does so prior to the use of any written notation (Dehaene 1997; Wiese 2003).

Take the idea of "three." To illustrate three, we must ask "What is the commonality among three dogs, three houses, and three pencils?" Their *threeness* is a quality of the sets, but not their members. Three, and all number names, are relational abstractions, since they describe similarities *between collections*. To see the threeness of a single collection, one must see past the qualities of the things themselves, and see only quantity. While number is best defined in the relationships between things, it exists mainly in the organized thinking of human beings as an abstract idea (Wiese 2003). This is true of shape as well—"square-ness" is a common characteristic of a cracker and a cocktail napkin, not dependent upon size, and existing only in the imagined two-dimensional world. It is defined by the relationships among its parts—sides and angles. So the two mathematical topics we tend to consider most basic to the mathematical domain—number and shape—are both highly abstract and relatively complex. The concept-procedure literature, by focusing attention on keeping written procedures meaningful, has missed the important mathematics that develops as children see and verbally name quantitative ideas in the world. This thinking—that occurs well before kindergarten—is the raw material that must be elaborated upon in order to construct a meaningful understanding of numerals and notation. It is therefore foundational to the mathematical learning that ought to proceed in elementary school.

While the concept-procedure literature dismisses early childhood as an important time for mathematics teaching, it also provides rich descriptions of the very mathematics we have termed "foundational" in its portrait of the predilections and abilities of toddlers (children between the ages of about 12 and 30 months). For example, Sinclair and Sinclair describe a two-year-old placing a spoon in each teacup as constructing a working concept of one-to-one correspondence (1986). Similarly, when a toddler sorts a set of animal figurines into "families," she is said to be exploring ideas of similarity, difference, and classification. In these depictions, foundational mathematical knowledge is exhibited through action, and has not yet been separated from the world of concrete objects. Sinclair and Sinclair (1986) comment "the young child cannot do without actual experience when logico-mathematical knowledge is in its beginnings" (p. 63).

This literature misses the fact that three- to five-year-olds, unlike their younger counterparts, can be somewhat sophisticated discussants, and are generally interested in actively using language and their burgeoning pictoral representation skills to get their versions of reality "right" (see, e.g., Bodrova and Leong 1996; Lindfors 1991; McLane and McNamee 1990; Paley 1988). Preschoolers are capable of and derive fairly complex benefits from talking about and otherwise elaborating upon their mathematical ideas about and actions taken upon objects. As mathematical actions and ideas are named, language acts to explicitly connect them to a larger world of social convention, while also separating them out as distinctly meaningful—a binding process that helps children sort and organize their thinking (Gentner and Loewenstein 2002). This elaboration of thought through the flexible use of language—one of the very earliest and most powerful symbolic systems to develop—has a profound impact on the development of mathematical thinking that occurs before elementary school, providing a base upon which symbolic numeracy and operations can later be built.

Clements (2004) notes differences between "the intuitive, implicit, conceptual foundation for later mathematics" and the subsequent elaboration that produces something more like conventional math knowledge (p. 11). He calls the process by which the toddler's embedded and foundational ideas become the preschooler's more elaborated and explicit ones "mathematization," noting that without it, children have "fewer chances…to connect their informal experiences to later school experiences in mathematics" (pp. 11–12). Sophian (1999) is specific about mathematization's effects, noting that children's very early conceptual knowledge is inconsistent, in that it is "in evidence at one moment and yet may not be at the next" (p. 17). She goes on to suggest that assisting children to broaden the application of their budding generalizations and make the knowledge they have begun to construct more explicit and transferable is an important preschool math teaching goal. It is the preschooler's growing facility with language and other primary symbol systems that makes such mathematization possible; it is also this key developmental shift and the clearly mathematical thinking it fosters that the mathematics education literature has so far neglected.

Copley (2000) agrees that there is an important shift in mathematical thinking during this period, and adds to this idea, noting "Early childhood educators say that children learn by doing. The statement is true, but it represents only part of the picture. In reality a child learns by doing, talking, reflecting, discussing, observing, investigating, listening, and reasoning" (p. 29). By contending that the early conceptual mathematical constructions of young children might benefit from social interaction with experienced and knowledgeable others, this idea emphasizes the role of teaching. While it is true that children at this stage of development do not need teachers to "re-connect" procedures to the concepts they refer to (as is needed when arithmetic procedures are learned in elementary school math), preschoolers do need assistance establishing and consolidating initial connections between their budding ideas and more generalized concepts. This developmentally informed perspective on mathematical thinking in early childhood makes the relevance of children's experiences clear.

Robust findings about differences in the home language experiences of young children suggest the problem: children from homes with fewer economic resources learn fewer words, have fewer experiences with words in interaction with other persons, and acquire a vocabulary of words more slowly (see Hart and Risley 1995). Given the key role of language in the development of foundational mathematical thinking, it is unsurprising that socio-economic status and math achievements tend to rise and fall together at the preschool level (Denton and McPhee 2009). Children from lower-resourced family environments need enriched early education mathematics experiences if they are to catch up with their peers. Average teaching is clearly not enough to ensure they are equitably prepared for mathematics in elementary school.

Because there is generally no training in mathematics provided to early childhood teachers (Copley 2004), they tend to rely on their own understandings to teach math (Sarama and DiBiase 2004). Unfortunately, foundational mathematics is invisible to most adults. As a means of efficiency, adults have learned not to check that five pennies are still five pennies whether arranged in a circle or in a line. Similarly, early childhood teachers have experienced the fact that a single set of objects can be sorted in more than one way so many times, that have generally forgotten they once did not know it. Young children are just discovering these truths, and learning the formal mathematics of elementary school is dependent upon their becoming explicit and transferable. It is only when these foundational ideas are clear enough that preschoolers can talk about them (or at least listen with understanding) that they can begin to represent them with secondary symbol systems, such as numerals, and learn to act upon them in ever more sophisticated ways. Since early childhood teachers generally miss the existence of foundational mathematical thinking among their students, they lack an explicit awareness of the concepts their teaching ought to help build.

Improving Teachers' Understandings of Foundational Mathematics

To help promote preschool teachers' understanding of the content of foundational mathematics and how it develops in the thinking of young children, we launched the Early Mathematics Education Project in 2007. Designed primarily for preschool teachers, this program includes workshops, on-site coaching, and the use of videotape to promote reflective practice among teachers. Instructors are Erikson faculty, and coaches are experienced preschool teachers, who participate in content training alongside their teachers. Emphasis is placed on addressing both teacher confidence and teacher knowledge, and specially designed adult learning tasks help teachers "see" the foundational mathematics. Finally, teachers are explicitly instructed in the use of math-related language, a powerful tool for helping young children solidify and build upon their mathematical thinking in preparation for the use of written notation to come. Key elements of the program, which has a demonstrated impact on children's mathematical learning, are described below.

Key Program Elements

Addressing confidence. Lack of math-related confidence among preschool teachers was explicitly considered in the design of the professional development. Since most early childhood teachers feel comfortable with stories as a part of their curriculum, children's storybooks are used throughout. Each workshop's activities—both the adult learning experiences and the recommended activities for children—are built out of rich children's literature, and teachers are provided with storybooks to support their teaching. In addition, the professional development is ongoing, with five day-long meetings held once every other month throughout the school year. This design allows the assignment of preschool teaching tasks between sessions and the opportunity to discuss their implementation at the next meeting. As importantly, the extended time allows each cohort of teachers to become familiar with one another, creating a safe environment for the expression of frustration as well as accomplishment. Coaching, too, is meant to encourage real changes in practice. Our coaching process emphasizes teacher strengths, and is designed to help teachers learn to be reflective. By making clear the coach's consultative—rather than supervisory—role, this coaching process helps underconfident teachers use the coach as a non-threatening source of support. Finally, teachers are explicitly engaged as partners in research. Their insights as they implement activities in their classrooms are actively sought by our instructors, and they are encouraged to present their own work to their peers as a means of sharing expertise. In addition to generating new ideas and solidifying understanding, these assumptions of professionalism among the teachers boost their sense of their own competence and promote their active attempts to improve practice.

Big ideas. Recognizing that early childhood teachers lacked knowledge of foundational mathematics and its development among young children, our team developed lists of Big Ideas within each of the content strands identified by the NCTM. Each Big Idea is meant to represent important conceptual material, central to the content area, that elaborates and solidifies the mathematical experiences and thinking of young children between the ages of three and five years. For example, a Big Idea in the content strand of measurement would be that "every object has many different attributes that can be measured, such as length or weight." While three- and four-year-olds can experience this truth on their own, the program operates on the assumption that structured activities and teacher language that make it explicit have a "mathematizing" effect on young children's thinking (see Table 18.1 for sample Big Ideas).

Adult learning tasks. To help teachers recognize and integrate both foundational mathematics and the Big Ideas, activities were constructed that would highlight them while being complex enough to engage adult learners. For example, when studying geometry, teachers are asked to describe a shape without using its name, forcing them to notice and name shape attributes, such as number of sides and size of angles. Similarly, in the study of algebraic thinking, teachers are challenged to name as many different sets of objects as they can find in the story "Goldilocks and the Three Bears." These sets are further analyzed to discover size and sequence patterns among them. Subsequently, teachers are provided with a sample lesson for children in which a large set of concrete objects of three sizes, such as spoons, mittens, and toothbrushes, is sorted in two distinct ways: first by object type, and then by size, so that "each bear can have his or her own things." In this way, adult

Table 18.1 Sample big ideas by mathematics content strand

Content strand	Big idea text
Algebra	The same collection can be sorted in different ways
Number and operations	A collection can be made larger by adding items to it, and made smaller by taking items from it
Measurement	Many different attributes can be measured, even when measuring a single object
Geometry	Two- and three-dimensional shapes can be used to represent and understand the world around us
Data analysis and probability	How data are gathered and organized depends upon the question they address

learning activities were designed to help teachers construct their own understanding of foundational mathematics, and then to suggest ways to emphasize and build upon it in the teaching of young children.

Math-related language. There is interesting evidence that mathematical learning during the preschool year is affected by language input from teachers. In each of three studies (Ehrlich 2007; Klibanoff et al. 2006; McCray 2008), researchers found a significant, positive relationship between the amount of math-related language by teachers and the growth of conventional mathematics knowledge among preschoolers in their classrooms from fall to spring. That is, the more preschool teachers talked about math in their classrooms, the greater the gains in math knowledge made by their students. Incorporating these findings, the workshops and coaching sessions emphasize both awareness and use of math-related verbalization. Teachers learn how to use "mathematizing" language to describe children's daily activities during transition, snack, dramatic play, and outdoor time. Additional emphasis is placed on asking questions that encourage children to describe their thinking using mathematical language. While the findings behind these language-related practices are correlational as opposed to causal, they strongly support the theory that what teachers say and what they invite children to say can help preschoolers solidify their undifferentiated mathematical thinking and link it to conventionally named concepts. For this reason, the professional development urges teachers to talk with children about foundational mathematics, pointing it out when it occurs and urging students to describe how they understand it.

Results of the Intervention

During the 2008–2009 school year, we conducted program evaluation in 28 preschool classrooms in a large urban public school system. The mathematics achievement scores of 236 children from socio-economically disadvantaged backgrounds were gathered once in the fall and once in the spring. The results demonstrated that compared to students whose teachers did not participate in our training program, students with program-participating teachers showed significantly greater growth on both the Child Math Assessment (CMA, Klein and Starkey 2006) and the Applied Problems Subtest (#10) of the Woodcock Johnson III (WJ-III, Woodcock et al. 2001) over the course of the school year. Using Hierarchical Linear Modeling, we controlled for Time 1 scores, since children in our participating classrooms were significantly ahead of their non-participating counterparts in the fall. While average scores increased in both groups, analysis of WJ-III Age Estimate scores attributes 2.74 additional months of growth in mathematics learning to the intervention ($p < 008$). Compared to students whose teachers did not participate in our program, children with program-participating teachers learned almost three months more material in the same amount of classroom time. Teachers, too, report high levels of satisfaction with the training, and credit it with making positive changes in their mathematics teaching.

Conclusion

In sum, recognition and understanding of foundational mathematics, its abstract nature, its role in preschool teaching, its susceptibility to intervention, and its relationship to elementary school mathematics directly addresses the misconceptions of both early childhood educators and researchers in mathematics education. First, it makes clear that there is mathematical thinking and learning that precedes the use of written symbols, that this thinking is based in experience with objects and space, and that it is well placed in a child-centered, developmentally appropriate curriculum. The tendency of some educators to "push-down" the subject matter intended for first grade is thereby pre-empted. Second, describing foundational mathematics helps define the specific kind of mathematical knowledge that preschool teachers need to be more effective supporters of children's mathematical development. It makes clear that their need for more content knowledge cannot be addressed by requiring higher-level mathematics classes, and instead suggests a new type of mathematics class for teachers, focused on numerical and spatial abstraction that occurs prior to the use of written symbols. Finally, by emphasizing the role of a solid understanding of foundational mathematics as a necessary precursor to the development of school-based, conventional mathematics, this argument draws attention to the importance of preschool teaching for lifelong learning in math. Specifically, acknowledgment of foundational mathematics makes clear the role of language as a mechanism for identifying and solidifying its concepts, and highlights the need for adults to help young children find and use the words that will be central in the understanding of written numerals, notation, arithmetic, and algorithms to come.

If mathematics educators become more aware of both the distinct nature and centrality of foundational mathematics as a basis for preparation for elementary school, perhaps they will exhort and encourage early childhood educators to take their charge seriously as it relates to math, sharing their own findings and concerns. If early childhood educators recognize that there is developmentally appropriate mathematics content for their students, and that exposure to this content can make or break the capability of some children to understand the mathematics of elementary school, they may be inspired to learn about it and teach it. The general public must also be educated, so that policymakers and funders will be impelled to support and expand these changes in our understanding. By establishing recognition of foundational mathematics, and helping the public in the United States to understand its relevance, early childhood and mathematics educators will have taken a vital first step in enhancing opportunities for truly equitable mathematics learning.

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References

Baroody, A. J., & Coslick, R. T. (1998). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction*. Mahwah: Erlbaum.

Baroody, A. J., & Dowker, A. (2003). *The development of arithmetic concepts and skills: Constructing adaptive expertise*. Mahwah: Erlbaum.

Bodrova, E., & Leong, D. J. (1996). *Tools of the mind: The Vygotskian approach to early childhood education*. Upper Saddle River: Prentice Hall.

Bowman, B. T., Donovan, M. S., & Burns, M. S. (Eds.). (2001). *Eager to learn: Educating our preschoolers*. Washington: National Academy Press.

Bredekamp, S. (1987). *Developmentally appropriate practice in early childhood programs serving children from birth through age 8*. Washington: National Association for the Education of Young Children.

Brophy, J. E. (1991). Conclusions. In J. E. Brophy (Ed.), *Advances in research on teaching: Teachers' subject matter knowledge and classroom instruction* (Vol. 2, pp. 347–362). Greenwich: JAL.

Bruner, J. S. (1960). *The process of education*. New York: Vintage Books.

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem-solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*(5), 385–401.

Chicago Program Evaluation Project. (2008). *The Chicago program evaluation project: A picture of early childhood programs, teachers, and preschool age children in Chicago.* Final External Report, Mathematica, Princeton, December.

Clements, D. H. (2004). Preface. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. ix–xv). Mahwah: Lawrence Erlbaum.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (Eds.). (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah: Lawrence Erlbaum.

Copley, J. V. (2000). *The young child and mathematics*. Washington: National Association for the Education of Young Children.

Copley, J. V. (2004). The early childhood collaborative: A professional development model to communicate and implement the standards. In D. H. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 401–414). Mahwah: Lawrence Erlbaum.

Copple, C. E. (2004). Mathematics curriculum in the early childhood context. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah: Lawrence Erlbaum.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Denton, F. K., & McPhee, C. (2009). *The children born in 2001 at kindergarten entry: First findings from the kindergarten data collections of the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B) (NCES 2010-005)*. National Center for Education Statistics, Institute of Educational Sciences, U.S. Department of Education, Washington.

Dickinson, D. K., & Neuman, S. B. (2006). Introduction. In S. B. Neuman & D. K. Dickinson (Eds.), *Handbook of early literacy research* (pp. 1–10). New York: Guilford.

Ehrlich, S. B. (2007). *The preschool achievement gap: Are variations in teacher input associated with differences in number knowledge?* Unpublished doctoral dissertation, University of Chicago.

Elkind, D. (1987). *Miseducation*. New York: Alfred Knopf.
Entwisle, D. R., & Alexander, K. L. (1990). Beginning school math competence: Minority and majority comparisons. *Child Development, 61*, 454–471.
Feiler, R. (2004). Early childhood mathematics instruction: Seeing the opportunities among the challenges. In D. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics* (pp. 393–400). Mahwah: Erlbaum.
Fuson, K. C., Smith, S. T., & Lo Cicero, A. (1997). Supporting Latino first graders' tenstructured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*, 738 –760
Gagné, R. M. (1977). *The conditions of learning* (3rd ed.). New York: Holt, Rinehart, & Winston.
Gentner, D., & Loewenstein, J. (2002). Relational language and relational thought. In E. Amsel & J. P. Byrnes (Eds.), *Language, literacy, and cognitive development: The development and consequences of symbolic communication* (pp. 87–120). Mahwah: Erlbaum.
Gersten, R., & Carnine, D. (1984). Direct instruction mathematics: A longitudinal evaluation of low income elementary school students. *The Elementary School Journal, 44*(6), 28–31.
Ginsburg, H. P., Choi, Y. E., Lopez, L. S., Netley, R., & Chi, C.-Y. (1997). Happy birthday to you: The early mathematical thinking of Asian, South American, and U.S. children. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 1–45). East Sussex, England: Lawrence Erlbaum Associates/Taylor & Francis.
Ginsburg, H. P., Kaplan, R. G., Cannon, J., Cordero, M. I., Eisenband, J. G., Galanter, J., & Morgenlander, M. (2006). Helping early childhood educators to teach mathematics. In M. Zaslow & I. Martinez-Beck (Eds.), *Critical issues in early childhood professional development* (pp. 171–202). Baltimore: Paul H. Brookes.
Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22*(1), 3–22.
Golbeck, S. L. (2001). Instructional models for early childhood: In search of a child-regulated/teacher-guided pedagogy. In S. L. Golbeck (Ed.), *Psychological perspectives on early childhood education: Reframing dilemmas in research and practice* (pp. 3–34). Mahwah: Lawrence Erlbaum.
Griffin, S., Case, R., & Siegler, R. S. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning in arithmetic to students at risk for school failure. In K. McGilly (Ed.), Classroom lessons: Integrating cognitive theory and classroom practice (pp. 25–49). Cambridge, MA: MIT Press.
Hart, B., & Risley, T. R. (1995). *Meaningful differences in the everyday experience of young American children*. Baltimore: Brooks.
Hatano, G. (1982). Cognitive consequences of practice in culture specific procedural skills. *The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 4,* 15–18.
Hatano, G. (2003). Foreword. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. xi–xiii). Mahwah: Erlbaum.
Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale: Erlbaum.
Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1–27). Hillsdale: Erlbaum.
Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–401.
Jacobson, C. J., Denton, F. K., McPhee, C., & Park, J. (2007). *Preschool: First findings from the third follow-up of the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B).* U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, Washington.
Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle-and low-income families. *Developmental Psychology, 28,* 644–653.
Kagan, S., & Zigler, E. (1987). *Early schooling: The national debate*. New Haven: Yale University Press.

Klein, A., & Starkey, P. (2004). Fostering preschool children's mathematical knowledge: Findings from the Berkeley Math Readiness Project. In D. H. Clements, J. Sarama, & A.-M. Dibiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah: Lawrence Erlbaum.

Klibanoff, R., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk." *Developmental Psychology, 42*(1), 59–69.

Lindfors, J. W. (1991). *Children's language and learning* (2nd ed.). Needham Heights: Simon & Schuster.

McCray, J. S. (2008). *Pedagogical content knowledge for preschool mathematics: Relationships between teaching practices and child outcomes*. Unpublished doctoral dissertation, Erikson Institute, Loyola University Chicago.

McLane, J. B., & McNamee, G. D. (1990). *Early literacy: The developing child*. Cambridge: Harvard University Press.

McLellan, J. A., & Dewey, J. (1895). *The psychology of number and its application to methods of teaching arithmetic*. New York: D. Appleton.

National Association for the Education of Young Children. (2005). Early childhood mathematics: Promoting good beginnings. *A joint position statement of the National Association for the Education of Young Children* (*NAEYC*) *and the National Council for Teachers of Mathematics* (*NCTM*). http://www.naeyc.org/about/positions.asp. Accessed 5 Aug 2005.

National Association for the Education of Young Children. (2008). *National Association for the Education of Young Children: 2008 Annual Conference Final Program.* Washington, DC: Author.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

National Research Council. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum*. Washington, DC: National Academy Press.

National Research Council. (2009). Mathematics learning in early childhood: Paths toward excellence and equity. Committee on Early Childhood mathematics, C. T. Cross, T. A. Woods, & H. Schweingruber (Eds.), *Center for Education, Division of Behavioral and Social Science and Education.* Washington: National Academies Press.

Paley, V. G. (1988). *Mollie is three: Growing up in school*. Chicago: University of Chicago Press.

Sarama, J., & DiBiase, A.-M. (2004). The professional development challenge in preschool mathematics. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood education*. Mahwah: Lawrence Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning.* New York: Macmillan.

Sigel, I. (1987). Early childhood education: Developmental enhancement or developmental acceleration? In S. Kagan & E. Zigler (Eds.), *Early schooling: The national debate* (pp. 129–150). New Haven: Yale University Press.

Sinclair, H., & Sinclair, A. (1986). Children's mastery of written numerals and the construction of basic number concepts. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 59–74). Hillsdale: Erlbaum.

Smith, S. S. (2001). *Early childhood mathematics* (2nd ed.). Needham Heights: Allyn & Bacon.

Sophian, C. (1999). Children's ways of knowing. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 11–20). Reston: National Council of Teachers of Mathematics.

Spodek, B., & Brown, P. C. (1993). Curriculum alternatives in early childhood education. In B. Spodek (Ed.), *Handbook of research on the education of young children* (pp. 91–104). New York: Macmillan.

Starkey, P., Klein, A., Chang, I., Dong, Q., Pang, L., & Zhou, Y. (1999). *Environmental supports for young children's mathematical development in China and the United States.* Paper presented at the biennial meeting of the Society for Research in Child Development, Albuquerque, NM.

Thorndike, E. L. (1922). *The psychology of arithmetic*. New York: Macmillan.
Veras, R. (1975). *Children of dreams, children of hope*. Chicago: H. Regnery.
Wheeler, L. R. (1939). A comparative study of the difficulty of the 100 addition combinations. *Journal of Genetic Psychology, 54,* 295–312.
Wiese, H. (2003). *Numbers, language, and the human mind*. Cambridge: Cambridge University Press.
Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock-Johnson III*. Rolling Meadows: Riverside Publishing.

Chapter 19
How Many Sides Does a Box Have? The Struggle to Respect Young People's Thinking

Julian Weissglass

Introduction: How a Riddle Helped Me Become More Respectful of Students

My father liked to ask riddles. Two of them had significant consequences for my professional career. In the early 1970s, the University of California started a program that funded graduate students in mathematics to teach in underprivileged schools using a guided discovery approach. This method had someone knowledgeable about mathematics committed to asking youngsters leading questions about mathematical situations in order to help them discover, understand, and become proficient in mathematics. The basic principle is that when asked to explain a wrong answer, students will discover their mistakes and by working together as a group develop their understanding. See Henkin (1995) for a description. My department chair asked me to supervise this program and I agreed upon the condition that I could teach in it. Little did I know that doing so would change my professional life!

I had one day of professional development in the discovery approach before going into a fourth grade classroom. I planned a series of lessons adapted from a booklet by Walter (1970) that would lead the students to investigate spatial visualization, symmetry, transformation geometry, and abstract algebra. I started by asking the students to close their eyes, to picture a box, and then to count the number of sides on their box. I still remember their excitement as they told me their different answers—three, four, five, six, eight, ten, and twelve. I asked the boy who said "ten" to explain his answer. I couldn't understand what he was saying very well—or follow his logic. When I asked him to explain for the fourth time he became frustrated, so I asked some other students to explain their answers. All the time I was wondering, "How did he get ten?" A remark by another student made me think of

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a riddle my father used to ask: “How many sides does an orange have?” People usually said, “None. It is round.” My father would laugh and say, “It has two, an inside an and outside.” So, I turned to the first boy and said, “Did you have a box without a top and you counted the insides and the outsides?” I will never forget the expression on his face—a blend of pleasure at being understood and incredulity that it had taken me so long to understand him. It turned out that all of the students had logical explanations for their answers! The variety of answers was a result of different definitions, different assumptions, or (literally) different perspectives.

I left the classroom that day humbled. How easy it would have been for me to invalidate his thinking, to add one more defeat at learning to the many he had already experienced in school. Even more troubling was that although I was looking for the one (or possibly two) correct answer(s), all of the students thinking had been excellent and mathematically valid. During the next few years, I encountered many young people with brilliant mathematical minds who were labeled as “slow learners.” Most of them were females or students of color. (The term “of color” is often used in the United States to describe people of non-European heritage, (African, Asian, Indigenous, Latin American, Pacific Islands) collectively. In Europe, the word “Black” is used to describe these individuals collectively. In the United States, Black is used to refer to people of African heritage.) I was very troubled at what I found and decided to educate myself about teaching and learning. In addition to teaching mathematics at elementary schools, I read widely, went to workshops, and taught a course on mathematics for elementary teachers at the University.

At that point I decided to fully respect a young person’s thinking—to assume that their thinking is correct and that if their answer to a question or their way of thinking about a situation is different than mine or the society’s, it is most likely because of a difference in definitions or assumptions. There is, of course, the exception when the student, because of their distress or need for attention, decides to deliberately give a wrong answer, or because of fear makes a guess. But overall it has been a good working hypothesis. It leads to more interesting results than assuming that my thinking is the only correct way. To be honest, I still struggle with it in practice.

What Does Respect Mean?

Sometimes people use respect to mean “admire” or “tolerate.” In the context of this chapter, however, I mean more than that. My working definition of *respecting children’s thinking* is: to take seriously, to interact with thoughtfully, to nurture, to engage, to honor [not humiliate, ridicule, or stifle]. My definition and my analysis in this chapter are (obviously) strongly influenced by my experience in the United States. The social and political history of the United States has resulted in a culture in the society and the schools that influences the relationships between students and teachers. You, the reader, have the opportunity to think about the meaning of respect and the struggle to respect young people’s thinking in the context of your educational system.

Respect means more than telling young people that they are smart. It means more than encouragement and praise. It is different than good grades, gold stars, and certificates. Youngsters' ability to think and to express their thinking is nurtured through dynamic and beneficial interaction with adults and other youngsters in a social situation. Young people know that their thinking is respected and valued when adults take time to listen to their thinking and to think with them—and when teachers provide time for them to explore and express their thinking. Most adults give lip service to the importance of thinking, but it takes more than lip service for young people to believe that their thinking is respected.

Adults help and support young people in many ways—providing love, guidance, instruction, protection, being a role model. They also, however, can undermine young people's self-confidence through ridicule and humiliation, by regarding them as less important or inferior to adults, and by not considering their input when making decisions. Young people commonly hear phrases such as "don't be so childish," "go to your room," "don't talk to your parents like that," and "you are not old enough to do that." They are criticized, yelled at, insulted, and intimidated in ways that adults are not. In school, young people are expected to listen to adults, but rarely are young people's concerns or thinking taken seriously. For the most part they are told what to study, when, and for how long. A teacher can yell at a student with impunity, but if a student yells at a teacher, he or she may be punished. In short, their lives are filled with repeated incidents of lack of respect.

Young people without exception always deserve complete respect. They have this right because they are human beings. It is independent of their racial or cultural background, their economic class, their gender, or their abilities. They would deserve complete respect even if there were no societal benefits. There are, however, benefits to society. If people are respected as thinkers when they are young, they will think well as adults. Therefore, as a group, human beings will be more effective in meeting the many challenges we face. Anyone concerned about young people's well-being, their learning, or the future of humanity would do well to reflect on what it means to respect young people's thinking and how we can improve our capacity in this area.

The Relationship of Respect to Equity and Quality in Mathematics Education

Discussing quality and equity in mathematics education is meaningless unless there is a commitment to respect students' thinking. What could quality in mathematics education mean if students do not end up being able to think mathematically? Why would we even seek equity in mathematics education if our goal were not to enable all children to think for themselves in mathematics? Finally, do you think it likely for students to learn how to think mathematically if their thinking is not respected in school?

Nevertheless, the struggle to respect children's thinking—in mathematics and in general often does not get articulated clearly. The reasons for this are complex. Most adults were not encouraged to think mathematically when young. They had to memorize algorithms, rules, and answers. They were told there were one answer and one right way to solve a problem. Their teachers focused on the mathematical content rather than the process of thinking mathematically. Schools valued memorizing information in order to get the correct answer. And finally there was the pressure to do well on standardized tests.

It will help you understand the issues I address in this chapter if you take a few minutes to think about times when your thinking was respected or not respected in school. Even better, pair up with another person and take turns talking and listening to each other about your memories (See Weissglass (1990) for more information about the role of listening and being listened to in improving education.). We all have stories about our thinking being nurtured and not nurtured. My story is that at home when I challenged my parents' views (especially on race and class) I was usually told, "this is just a phase you're going through" or "you will understand when you grow older." In elementary school I do not remember ever being asked to solve a problem that required original thinking. My father provided me some opportunities to think mathematically, because some of these riddles he asked were mathematical in nature. One was a word problem: "A bottle and a cork cost \$ 1.10. The bottle cost a dollar more than the cork. How much did the cork cost?" Most people (including adults) had trouble with this riddle. I heard it often, so I had the opportunity to think about it a lot. As a result, when I got to high school I was good at solving word problems and was able to succeed in algebra. I was good enough in the other parts of school mathematics—memorizing algorithms and later definitions and proofs and then mastering traditional methods for solving problems—that I was able to get a doctorate in mathematics. Looking back on those years I do not remember many attempts to encourage creative thinking. Mostly I was asked to understand the mathematics that others had created and to apply it in fairly routine ways to solve problems that had already been solved. I did enjoy the creative thinking involved in the research for my dissertation and thought that I would spend my life in the proverbial "ivory tower"—teaching mathematics and proving theorems.

My experiences teaching in elementary schools (described above in part), however, led me to attempt to improve mathematics education. I ended up working, with educators in a variety of professional development settings. See Weissglass (1990, 1994a, b, 1996, 1997, 2000), Weissglass and Mumme (1991), and Peterson and Barnes (1996) to learn more about this work (Peterson and Barnes describe the *Improving Mathematics Education in Diverse Classrooms* project anonymously.) I began to understand that the quality of mathematics education, achieving equity in mathematics education, and respect for young people's thinking are all interrelated. This interrelationship is complex because personal and institutional values and practices often work at an unconscious level to the disadvantages of certain groups in this society.

As I mentioned above, for example, I encountered many students of color and females, who were labeled as slow learners but were able to think deeply and creatively about mathematical situations. In addition, I met educators who had low expectations for certain groups of students and made prejudiced remarks—both blatant and subtle—about them or their families. The teachers in my professional development institutes reported that they had rarely been allowed or encouraged to think mathematically, to solve problems, or "to play" with mathematics in school. They did not have a deep understanding of even elementary mathematics. For example, they could use the algorithm for adding fractions, but they could not explain why it worked. As a result, before they attended the professional development institutes they did not have the capability or the will to change their pedagogy in order to encourage their students' mathematical thinking. They taught the way they were taught—through drill and practice. All students are hurt and deterred by this form of instruction, but students who are targets of race, class, and gender prejudice are deterred more than white middle-class students who have developed the ability to function in the culture of school. The teachers I worked with began to change their teaching. But when they attempted to bring about change in their schools and districts, I heard stories about the racial, class, and gender bias that they encountered (See Oakes (1996) for similar stories). I became convinced that racism, classism, and sexism were the key obstacles to reform in mathematics education.

The Struggle in Society

An individual's struggle to gain understanding, to think more clearly, and to act in more human ways often reflects a wider struggle in society. The gains of each individual assist the progress in the society and progress in society assists each individual. For example, the societal movements in the past century for ending racism, sexism, and the oppression of gays and lesbians, have influenced and have been influenced by changes of behavior, attitudes, and understanding at the individual level. Similarly, the struggle to respect young people's thinking is not just an individual struggle. It is also a historical struggle, which in the United States dates back at least to the beginning of public education. I will only mention a few of the highlights.

The foremost advocate, in the nineteenth century, for public education was Horace Mann who was a Congressman, U.S. Senator, and Secretary to the Massachusetts Board of Education. In a report to the Board he wrote:

> the effective labor must be performed by the learner himself [...] Knowledge is not annexed to the mind like a foreign substance but the mind assimilates it by its own vital powers [...] every scholar in a school must think with his own mind. (Winship 1896)

Some of his contemporaries agreed with him. For example, the American essayist, philosopher, poet, Emerson (2009) wrote "The secret of education is respecting the pupil." Many, however, disagreed—among them the Boston Masters [the equivalent of the Boston School Board]. Their position was "[…] implicit obedience to rightful authority must be inculcated and enforced upon children, as the very germ of all good order in future society" (Association of Masters 1971, p. 121).

Early in the twentieth century John Dewey wrote about the relationship between respecting children's thinking and the goals of a democratic society:

> If we train our children to take orders, to do things simply because they are told to, and fail to give them confidence to act and think for themselves we are putting an almost insurmountable obstacle in the way of overcoming the present defects of our [social] system and of establishing the truth of democratic ideas. (Dewey and Dewey 1915, p. 304)

Dewey knew that respecting children's thinking is challenging:

> Perhaps the most difficult thing to get is intellectual sympathy and intellectual insight that will enable one to provide the conditions for another person's thinking and yet allow that other person to do his thinking in his own way and not according to some scheme which we have prepared in advance […] At present we often think that a child has no right to solve a problem or do a sum at all unless he goes through a certain form. (Dewey 1913/1979)

It is no accident that Dewey uses an example from mathematics education where a lack of respect for young people's thinking is so common. In part this is a result of a limited view of the nature of mathematics—namely that it consists of mastering a set of rules, procedures, and facts, rather than being able to analyze, think logically, and use mathematical concepts and tools in thinking about and solving problems that appear in new contexts. The disagreement between those who hold this view and those with a different view led to public and intense controversies in the latter part of the twentieth century. Some people were surprised that mathematics teaching could be the subject of such intense political struggles. It is not surprising, however, once you realize that the reform effort in mathematics education is part of a larger struggle to liberate human minds and any societal struggle to liberate human beings from prejudices, rigidities, and oppression is met with resistance.

Looking back on past struggles, such as the struggle to establish democratic forms of government, to end slavery, to gain equal rights for women, or to end segregation, it is easy to criticize the thinking of the people who resisted progress toward a society in which each human being is respected and given equal rights. For example, although prejudice toward people of African heritage has not disappeared, a much smaller percentage of people believe in their genetic inferiority than did 100 years ago. Most contemporary educators are quite sure that they would not have gone along with the fallacious beliefs of the eugenics movement. And yet eminent scientists held these beliefs. Edward East, a Harvard professor of genetics, for example, wrote: "Gene packets of African origin are not valuable supplements to the gene packets of European origin; it is the white germ plasm that counts." (East 1929, p. 199) and his views were echoed by academics at some of our most prestigious institutions (Tucker 1994).

Present time controversies, however, are more confusing as we can see as we look at the controversy over mathematics education that erupted in the 1990s. It was especially heated in California.

The "Math Wars" in California

In the 1970s and 1980s, increasing numbers of educators, influenced by the principles of constructivism and social constructivism (Ernest 1991) began to think about changing pedagogy in mathematics classroom. In 1975, the California Department of Education adopted a Mathematics Framework for California Public Schools. It stated:

> Mathematics becomes a vibrant, vital subject when points of view are argued, and for this reason interaction among pupils should be encouraged […] A significant feature of mathematics learning environment is the spirit of free and open investigation […] Pupils and their teachers must be able to explore those facets which have particular meaning for them […] Well equipped and organized classrooms allow people to accept the responsibility for their own learning and progress.
>
> The learning climate in the classroom should provide and atmosphere of open communication between the pupil and teacher. The teacher should encourage questions and accept problems from the pupils. The mathematics' instructional materials should be relevant to the pupil's interests and needs and should provide for pupil experimentation.
>
> The establishment of a classroom climate, under the direction of a teacher, should be pupil oriented, self-directed, and not threatening […] The classroom climate should encourage pupils to solve problems in a variety of different ways and accept solutions in many different forms. All pupils should express creative thinking, even when it differs from the pattern anticipated by the teacher or when it produces a different conclusion or result. (California State Department of Education 1982, p. 3)

These principles, which are important for establishing a pedagogy that respects young people, became central tenets of what was called the mathematics reform movement. Gradually, mathematics educators developed an infrastructure to support the reform movement. In 1983 the state established the California Mathematics Project at campuses of the California State University and the University of California to develop teacher leadership to carry out the reforms. Leadership also developed in other states, often supported by federal funding. The 1985 California Mathematics Framework elaborated a vision for a student-centered mathematics education and laid the groundwork for the Standards of the National Council of Teachers of Mathematics (NCTM 1989). California produced replacement curricular units to conform to the 1985 Framework and put publishers on notice that new texts would have to change. The National Science Foundation funded three systemic initiatives (the state, urban and rural initiatives). As the infrastructure developed and the reform movement became more effective, considerable opposition developed, and attracted considerable public attention. For example, late in 1997, headlines on the first page of the Los Angeles Times read, "State Board May Return Math Classes to the Basics" (1997, November 30) and, "State Endorses Back to Basics Math Standards" (1997, December 2). During this period Edward Hirsch, a professor of English at the University of Virginia, clearly stated the basic principle of those opposing the mathematics reform effort in an invited speech to the California Board of Education in April 1997: "[…]varied and repeated practice leading to rapid recall and automaticity in mathematics is a necessary prerequisite to higher-order problem-solving skills in both mathematics and the sciences." (Hirsch 1997)

Comparing this statement with Dewey's gets at the crux of the problem. Some educators believe that drill and practice and memorization are prerequisites for thinking in mathematics and science. Others, including myself, believe that basic skills can be learned in the process of thinking about interesting problems that engage young peoples' minds and that doing so will increase the likelihood that these skills can be used intelligently. More about the "math wars" can be read in Schoenfeld (2004) and Jackson (1997a, b).

This is more than a theoretical debate. As mentioned above, it has long-term consequences for our society. There is a more immediate effect, however, on young people's lives. Students in large numbers are disengaged and alienated from U.S. schools and drop out. *Engaging Schools: Fostering High School Students' Motivation to Learn* documented this problem:

> Some studies have found that forty to sixty percent of high school students are chronically disengaged; they're inattentive, exert little effort, do not complete tasks, and claim to be bored. This figure does not include those who have already dropped out. (National Research Council and Institute of Medicine 2004)

Although student disengagement and alienation is a symptom of larger problems in the society—racism, classism, disrespect of children's native languages, fear of immigrants, all contribute, I conjecture that a major contribution to student disengagement is the lack of respect for their thinking that students endure in school. What policy makers do not understand is that the focus on memorization, practicing standard algorithms, test preparation, and testing is disrespectful of young people and is a major cause of disengagement.

Why Does the Struggle Exist? Why Has There Been Progress?

In a perfect world everyone would agree on what it means to respect children's thinking and it would be natural for everyone to do so. But we don't live in a perfect world. Misconceptions, biased attitudes, and incorrect beliefs about young people exist and may be even harder to correct than those about other groups in our society. After all, every adult has been a young person and there is a tendency for adults to think they are experts about young people. Adults who have attained some prominence or power in this society will be especially likely to not understand or to deny that there is a connection between societal problems (poverty, pollution, global warming, and student alienation, for example) and young people's thinking not being respected.

Although no one is to be blamed for this situation, each educator is responsible for changing it. There are no evil people conspiring to repress young people's thinking. Educators' passivity, however, in challenging the practices, beliefs, and policies within educational institutions, allow disrespectful practices to continue. The causes of this regrettable situation are complex. Among them are:

1. *A widely held (and false) assumption that young people won't learn without pressure, rewards, and punishment.* This contradicts many teachers' experiences as

well as the research. As stated in *Engaging Schools*, "A large body of primarily experimental studies demonstrates that emphasis on rewards and other extrinsic reasons for engaging in an activity can undermine intrinsic interest in the activity." (National Research Council and Institute of Medicine 2004) This is not a new discovery. Aristotle wrote, "All men by nature desire to know."

2. *Adults being fearful of the challenges presented by the thinking of young people.* To quote John Dewey again, "Anyone who has begun to think, places some portion of the world in jeopardy." (www.quoteworld.org/quotes/3614) These fears push adults to belittle or humiliate young people when the latter challenge the thinking of their elders. Growing up I personally heard such condescending remarks as, "when you get older you will understand" or "this is just a phase you're going through" or "you are so innocent. You don't know anything." Adults unconsciously pass on the mistreatment that they experienced when they were young.
3. *Confusion about young people's abilities and intelligence.* Adults mistake lack of information, skills, and capabilities for lack of intelligence. This confusion causes adults to require performance rather than to nurture young people's inherent intelligence.
4. *Pressure from society to preserve the status quo in the society.* Our schools' mission gets narrowed to prepare people for filling roles and jobs in the society.
5. *The working conditions of teachers.* They are often put in almost unworkable situations, with large numbers of students to think about and inadequate resources. They experience heavy pressure to perform on standardized tests. They are often given curriculum that is boring to their students and told to meet developmentally inappropriate standards. In comparison to other professionals, they receive little support (either intellectual or emotional) for their work.

It is hopeful that in spite of the obstacles and challenges there has been progress. Because educators care about young people and love learning, they do many good things for young people. Almost everyone will tell about an educator whose thoughtfulness, caring, passion for a particular subject, and/or individual support, made a big difference in their lives.

We can however do better. We can increase our efforts to make sure that individuals and institutions make a difference in the lives of students. We will have to work especially hard to ensure that students of color and working-class students receive thoughtful and caring experiences and exposure to teacher's passion for mathematics since they receive fewer of these experiences in mathematics and science (Oakes 2004).

Toward a Strategy for Respecting Young People's Thinking

In this chapter, I have attempted to describe the importance of emphasizing respect for young people's thinking as part of any effort to achieve equity in mathematics education. Educators can ensure that their efforts for equity and quality systemically and thoughtfully include a commitment to respect all young peoples' thinking and that we not measure equity or quality by results on standardized tests. This is not the place to describe, nor am I capable of describing, a complete strategy for transforming schools so that they can achieve this goal. I will, however, propose and elaborate briefly on seven principles that could form the foundation of a strategy.

1. **Completely respect young people as emotional, as well as intellectual, human beings.**
   A person's emotional state has a significant impact on her or his ability to think and to learn. Someone who is sad, fearful, or angry will not be able to think or learn as well as when their brain is not occupied with those emotions. One way to think about this is as follows: Each human (with the exception of those having physiological damages to their forebrain) is born with a tremendous amount of intelligence—the ability to process information coming in from the environment and respond in unique creative ways. The distresses humans experience growing up interfere with this intelligence. Certain physiological processes (crying, shaking, perspiring, laughing, tantruming, yawning, talking) either at the time of the hurt or later on, help humans recover from the distress experiences and reduce or remove the effect on our intelligence. The most eloquent expression of the connection between intelligence and emotional release is in the poem written by the Persian Sufi poet Rumi [1207–73],
   The cloud weeps, and then the garden sprouts.
   The baby cries, and the mother's milk flows.
   The nurse of creation has said, let them cry a lot.
   This rain-weeping and sun-burning twine together to make us grow.
   Keep your intelligence white-hot and your grief glistening, so your life will stay fresh.
   Cry easily like a little child.
   I suspect that some readers will have difficulty believing that healing from how you have been hurt by releasing your emotions actually increases your intelligence. That insight, although understood by some people, has been quite absent from the academic profession. So I suggest that you take some time again to pair up with a friend and talk about how emotional release [crying, trembling, "tantruming", for example] was treated when you were growing up? Doing so might help you better understand your attitudes about the idea that emotional release helps you think better.

2. **Promote young people's creative endeavors and incorporate play into learning activities.**
   The importance of play in developing intelligence has long been recognized. Plato, for example, wrote in *The Republic*:

   > There should be no element of slavery in learning. Enforced exercise does no harm to the body, but enforced learning will not stay in the mind. So avoid compulsion and let your children's lesson take the form of play. (Plato 1941)

And the French philosopher Montaigne wrote, "It should be noted that children at play are not playing about; the game should be seen as their most serious minded activity." More recently, the psychologists John Piaget and Valerie Polakow Suransky have emphasized the importance of play. The former wrote "Play is a particularly powerful form of activity that fosters the social life and constructive activity of the child" (Puckett and Diffily 2004, p. 257) and the latter "Play is the mode through which the child realizes herself. It is through play that the child restructures, invents, makes history and transforms her given reality…the child becomes herself through play." (Suransky 1982)

The increasing emphasis in U.S. schools on improving test scores while disrespectful of students thinking in its own right has also had the effect of decreasing the opportunities for students to pursue creative endeavors and play. Fully developing young people's thinking requires that the curriculum include multiple and readily accessible opportunities and encouragement to engage in a variety of artistic endeavors. Emphasizing language and mathematics to the exclusion of other disciplines does a grave disservice to our brains, which are capable of thinking in many different areas.

3. **Encourage communication and cooperation.**
Mathematics is a way of looking at and making sense of the world. It is a beautiful, creative, and useful human endeavor that is both a way of thinking and a way of knowing. The goal of mathematics instruction is to help students develop and deepen their understanding of mathematics as well as their ability to communicate their ideas to others. The process of communication helps students construct as well as express mathematical meanings. Young people communicate their thoughts and understandings in many ways, verbally, physically, through manipulating objects, and using pictures and symbols. Weissglass et al. (1990) identify five reasons for promoting child-directed communication about mathematics: a. It clarifies children's thinking, b. It empowers children as learners. c. It reduces anxiety and alienation. d. It establishes some common understandings. e. It assists the teacher in thinking about the child as learner.

Both communication and thinking are assisted by collaborative learning environments. The Professional Standards for Teaching Mathematics (NCTM 1991, p. 58) states it well:

> Students' learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas. It is a key function of the teacher to develop and nurture students abilities to learn with and from others—to clarify definitions and terms to one another, consider one another's ideas and solutions, and argue together about the validity of alternative approaches and answers….

4. **Engage and support learners in pursuing their own interests (distinguishing between respect and permissiveness) and connect curriculum to students' culture whenever possible.**
Human beings are inherently curious and it is disrespectful of students' thinking to not let them pursue their own interests. There may be a tension between what society deems important for everyone to know and a student's interest, but there

is time for both. At the present time, however, schools for the most part dictate how students spend their time. I am not advocating permissiveness. It is possible to respond to student's interest, to respect their thinking, and still weave in the skills and understanding that are important for students to function in a society.

5. **Strengthen all student's first language, while supporting all students fluency in a second language.**
Strengthening a student's first language is a crucial part of respecting their thinking. When thinking becomes verbal the words are in the language that the young person has been hearing. Not respecting that language is disrespectful of the individual's thinking and interferes with her/his learning. When I first started teaching an undergraduate mathematics course for prospective elementary teachers using a small group laboratory approach, I noticed that the native English speakers were dominating the Latinos/as in the groups. I decided to give the students the choice of being in groups with people whose native language was the same as theirs. One day, I was observing a group as they were discussing a problem that was related to number concepts. At one point there was some confusion about a definition in my handout. The students immediately changed from English to Spanish. After some very animated discussion, they switched back to English. I was surprised and then understood that their early understanding of number concepts was acquired in Spanish and in order to clarify things they naturally switched to their native language.
As early as 1983 (Dawe 1983) there was research indicating that first language competence is an important factor in young people's ability to reason in mathematics when English was not their first language. Many state and school district policies, however, have ignored this research. Similarly, they have mostly ignored the research about achievement in reading, where the recent research is quite clear:

> This consistent finding [positive effects of bilingual education on students' reading achievement] might surprise some readers. But the NLP [National Literacy Panel] was the latest of five meta-analyses that reached the same conclusion: learning to read in the home language promotes reading achievement in the second language. Readers should understand how unusual it is to have five meta-analyses on the same issue conducted by five independent researchers or groups of researchers with diverse perspectives. […]. No other area in educational research with which I am familiar can claim five independent meta-analyses based on experimental studies—much less five that converge on the same basic finding. (Goldenberg 2008, p. 15)

6. **Avoid promoting pretense.**
For many students school becomes an institution to "get through" rather than a rewarding adventure in learning. In addition, young people sometimes feel that success in school is a condition for love, affection, or recognition. As a result, students end up pretending about what they know or don't know. They often try not to avoid or hide their mistakes, rather than accepting mistakes as a part of the learning process. Teachers frequently go along with this pretense. Because criticism, comparison, and ridicule are so common, students also pretend about

what they feel. Making school safe places for students to talk about the reality of their lives, to admit what they do not know, and to make mistakes will assist in developing more powerful thinkers.

7. **Increase understanding of the nature of oppression.**
   I define oppression as *the systematic mistreatment of certain groups of people by societal institutions or by people who have been conditioned by the society to act, consciously or unconsciously, in harmful ways toward the targets of the mistreatment, with the mistreatment generally accepted by the society.*
   Some of the forms of oppression that affect mathematical learning are racism, classism, sexism, and the oppression of people with physical disabilities. Oppression can be subtle or blatant, conscious or unconscious, personalized or institutionalized. It affects people in complex ways, many of which we are unaware of. Oppression is hurtful to everyone, those who are targets, those who perpetrate, and those who are bystanders. When a young person first sees oppression they are confused and often afraid. There is a tendency to internalize the oppression, to believe the negative messages you get about yourself and your group. Understanding how oppression works and its effects will enable educators to have more attention and to think better about people who are members of groups that are underrepresented in mathematics. Increasing our understanding and our effectiveness will take time. It will not happen linearly, by decision, or by argument. As Gross (2006) points out "the nature of prejudice is to make unwarranted totalizing claims, whereas understanding advances through elucidation of careful distinctions." Ending oppression requires constant and repeated efforts that include both telling our stories and listening to the stories of people with different life experiences than our own, healing emotionally from the distress experiences of growing up in an oppressive society, and deciding over and over again to persist and stay hopeful. We need to accept being uncomfortable in this journey and to not look for "neat" or "easy" answers or explanations. Furthermore to achieve equity and respect for young people's thinking, we must allow students to show their struggles.

Conclusion

There is a lot of pressure on educators from elected representatives and policymakers to increase student achievement. The evidence they seek is increased test scores. Most tests, however, especially standardized tests, will not provide reliable evidence of students' ability to think. Educators who value students' thinking have the challenge of educating politicians and policymakers. In the meantime, it is important to remember that schools and society do not require conformity. They only require the appearance of conformity. Teachers get to make choices about what happens in their classroom. I hope you will choose to have your classroom nurture and respect young people's thinking.

References

Association of Masters of the (Boston) Public Schools. (1971). Remarks on the Seventh Annual Report of Horace Mann (Boston, 1844). In M. B. Katz (Ed.), *School reform: Past and present* (pp. 113–124). Boston: Little, Brown and Company.

California State Department of Education. (1982). Mathematics framework and the 1980 addendum for California public schools. Kindergarten through grade Twelve. Sacramento: California State Department of Education.

Dawe, L. (1983). Bilingualism and mathematical reasoning in English as a second language. *Educational Studies in Mathematics*, 14, 325–353.

Dewey, J. (1913/1979). Reasoning in early childhood. In J. A. Boydston (Ed.), *John Dewey: The middle works* (Vol. 7, pp. 369–376). Carbondale: Southern Illinois University Press.

Dewey, J., & Dewey, E. (1915). *Schools of tomorrow*. New York: E. P. Dutton & Company.

East, E. (1929). *Heredity and human affairs*. New York: Charles Scribner's Sons.

Emerson. (2009). http://www.vcu.edu/engweb/transcendentalism/authors/emerson/essays/education.html

Ernest, P. (1991). *The philosophy of mathematics education*. Bristol: The Falmer Press.

Goldenberg, C. (2008). Teaching English language learners: What the research does—and does not—say. *American Educator,* Summer 2008, 8–11, 14–19, 23–23, 42–44.

Gross, J. (2006). *Fear: Anti-semitism in Poland after Auschwitz.* Princeton: Princeton University Press.

Henkin, L. (1995). The roles of action and of thought in mathematics education—One mathematician's passage. In N. Fisher, H. Keynes, & P. Wagriech (Eds.), *Issues in mathematics education,* (Vol. 5, pp. 3–16). Providence: Conference Board of the Mathematical Sciences.

Hirsch, E. D. (1997). *Address to California State Board of Education: Mathematically correct.*" http://www.mathematicallycorrect.com/edh2cal.htm

Jackson, A. (1997a). The math wars: California battles it out over mathematics education (Part I). *Notices of the American Mathematical Society, 44*(6), 695–702.

Jackson, A. (1997b). The math wars: California battles it out over mathematics education (Part II). *Notices of the American Mathematical Society, 44*(7), 817–823.

Los Angeles Times (1997a), November 30, 1997, page 1.

Los Angeles Times (1997b), December 2, 1997, page 1.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *The professional standards for teaching mathematics.* Reston: National Council of Teachers of Mathematics.

National Research Council and Institute of Medicine. (2004). *Engaging schools: Fostering high school students' motivation to learn*. Washington: The National Academies Press. www.nap.edu/books/0309084350/html/.

Oakes, J. (1996). Beyond the technicalities of school reform: Policy lessons from detracking schools. Los Angeles: UCLA Graduate School of Education.

Oakes, J., Joseph, R., & Muir, K. (2004). Access to achievement in mathematics and science: Inequalities that endure and change. In J. A. Banks & C. A. M. Banks (Eds.), *Handbook of research on multicultural education* (2nd ed., pp. 69–90). San Francisco: Jossey-Bass.

Peterson, P., & Barnes, C. (1996). Learning together: The challenge of mathematics, equity, and leadership. *Phi Delta Kappan, 77*(7), 485–491.

Plato, A. (1941). *The Republic of Plato* (trans: Cornford, F. M.) (p. 258). New York: Oxford University Press.

Puckett, M., & Diffily, D. (2004). *Teaching young children: An introduction to the early childhood profession.* Clifton Park: Delmar Learning.

Schoenfeld, A. (2004). The math wars. *Educational Policy, 18*(1), 253–286.

State Board May Return Math Classes to the Basics. (1997, November 30). *Los Angeles Times,* p. 1.

State Endorses Back to Basics Math Standards. (1997, December 2). *Los Angeles Times,* p. 1.
Suransky, V. P. (1982). *The erosion of childhood*. Chicago: University of Chicago Press.
Tucker, W. (1994). *The science and politics of racial research*. Champaign: University of Illinois Press.
Walter, M. (1970). *Boxes, squares and other things: A teacher's guide for a unit in informal geometry*. Reston: National Council of Teachers of Mathematics.
Weissglass, J. (1990). Constructivist listening for empowerment and change. *The Educational Forum, 50*(4), 351–370.
Weissglass, J. (1994a). Changing mathematics teaching means changing ourselves: Implications for professional development. In D. B. Aichele (Ed.), *NCTM Yearbook: Professional development for teachers of mathematics* (pp. 67–78). Reston: National Council of Teachers of Mathematics.
Weissglass, J. (1994b). Reflections on educational change support groups. *People and Education, 2*(2), 225–248.
Weissglass, J. (1996). Transforming schools into caring learning communities: The social and psychological dimensions of educational change. *Journal for a Just and Caring Education, 2*(2), 175–189.
Weissglass, J. (1997). Deepening our dialogue about equity. *Educational Leadership 54*(7), 78–81.
Weissglass, J. (2000). No compromise on equity in mathematics education: Developing an infrastructure. In W. Secada (Ed.), *Changing the faces of mathematics* (pp. 5–24). Reston: National Council of Teachers of Mathematics.
Weissglass, J., & Mumme, J. (1991). Improving mathematics education through school-based change. In N. D. Fisher, H. B. Keynes, & P. D. Wagreich (Eds.), *Mathematicians and education reform 1989–1990* (CBMS Issues in Mathematics Education, Vol. 2). Providence: American Mathematical Society.
Weissglass, J., Mumme, J., & Cronin, B. (1990). Fostering mathematical communication: Helping teachers help students. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 272–281). Hillsdale: Lawrence Erlbaum.
Winship, A. (1896). *Horace Mann: The educator.* Boston: New England Publishing.

Chapter 20
'Sometimes I Think Wow I'm Doing Further Mathematics…': Balancing Tensions Between Aspiring and Belonging

Cathy Smith

Further Mathematics A-level is a small but prestigious secondary school qualification in England and Wales that acts as gateway and 'gold-standard' for advanced mathematics. Despite repeated changes in teaching and assessment practices it remains at the centre of overlapping discourses about rigorous mathematics and quality, widening participation and equity. My research follows Hart (2003) in using a particular context to examine how conceptions of quality and equity in mathematics education have interacted over time, and I link these to Western, liberal understandings of the self as individual project and narrative. The design brings together two approaches: one analysing how further mathematics is constructed through the public documents and practices of mathematics education; the other analysing students' talk about choosing and studying further mathematics. I take 'aspiring' and 'belonging' as processes by which students live out the discursive concepts of quality and equity as practices of the self. This chapter addresses the questions: how are students' accounts of studying further mathematics structured discursively by its sociohistorical positioning, and how is this positioning in turn effected through the accounts of students? What ways of knowing allow/disallow students to identify themselves both as aspiring and belonging?

Why Does Further Mathematics Matter?

A-levels are the traditional academic qualifications in the English and Welsh school system, studied by 40% of 16–18-year-olds as preparation for university. A-level students specialise in only three or four subjects over two years. Math-

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ematics is the only subject area with two qualifications that can be studied alongside each other: 'mathematics' and 'further mathematics' A-levels. Taking both is necessary for entrance to some university courses, and about 1 in 7 mathematics students do this. Further mathematics matters because the ways in which teachers and students talk about studying further mathematics construct our understandings of quality in mathematics education, and we use those constructions to position ourselves within its practices. The same applies to equity. Students do not have equal access to studying further mathematics: they are constrained by 'individual' factors such as prior attainment, and 'social' factors such as school resources. When teachers and policy commentators speak their concerns about these structural inequalities, they create and draw upon particular constructions of equity.

Recent UK policy texts (Matthews and Pepper 2007; Porkess 2006) have linked further mathematics to alarm about declining participation in mathematics. Their primary concern is to encourage the majority of 16-year-olds to continue mathematics, and rightly so. However, this is accompanied by celebration of 'our very brightest young people' studying mathematics and science A-level subjects who 'by doing so are ensuring that Britain has a bright future' (Wright 2009). This hitches concerns for social justice both to advanced mathematics and to a neoliberal economic concern for the mutual benefits and national competitiveness that are assumed when individuals gain a technological edge. By neoliberalism, I mean a way of understanding society and politics that constructs the process of governing as guiding and regulating free individuals in a quest for mutual—although not necessarily equal—economic success (Rose 1999). These policy texts define the equity issues of A-level mathematics according to the dominant 'system of reason' that has underpinned the policy-making of successive UK governments since the 1980s, thereby constructing a problem that seems valid and deserving of attention (Popkewitz 2002). They also construct solutions in those given ways. A national government initiative, the Further Mathematics Network (FMNetwork) was established from 2005 to 2009 to promote further mathematics and provide teaching to students whose schools lacked resources and staff. I have taken this program as the focus of my research because it gives previously excluded students the opportunity/responsibility to make different choices. Their accounts contribute to an understanding of how new teaching practices work alongside traditional representations, producing potentially different conceptions of what it means to them to aspire and belong to further mathematics.

My theoretical base is a poststructuralist perspective. Power circulates within local practices: it is at the levels of schools, teachers and individuals that knowledge is constructed and reconstructed about who can study further mathematics and how (Foucault 1991). Martin (2006) suggests that the best way to understand equity is to ask how students live and explain their day-to-day experiences of mathematics in relation to school, community and sociohistorical contexts, and how this interacts with the senses of the self that are have meaning for them. I find this compatible with a poststructuralist methodology, analysing in detail 'what is given to us as universal, necessary, obligatory' (Foucault 1991, p. 45) about mathematics education, and how this knowledge is legitimated over what is presented as 'singular, contingent, and the product of arbitrary constraints'.

The chapter continues with an analysis of the historical literature on further mathematics, showing how it constructs quality as extremes of measurement and comparison with the past, and inequity as deficits in schools. I then take promotional and regulatory texts of the new FMNetwork and examine how they sustain those old truths alongside constructions of quality as conformity and breadth-plus-depth, and equity as systematised access. In the third section, I draw on interviews with one student to identify how representations of further mathematics intersect with liberal 'practices of the self' to enable and disable student choices.

Historical Constructions of Further Mathematics

Forty years ago some 45,000 students passed mathematics A-level, and a third of these also took the equivalent of further mathematics and so became eligible for mathematically demanding university courses (Hoyles et al. 2001). Schools were free to choose among several syllabuses, but these all had a similar structure with two A-levels called 'pure' and 'applied' mathematics. This division in terms of content represented an implicit educational hierarchy. Pure mathematics was seen as fundamental in its own right and as necessary preparation for science and engineering degrees; applied mathematics was the 'optional extra' giving practice in the pure techniques. This familiar classed abstract/concrete binary (Mendick 2006) configured practical applications as deviations from the higher education route and preferred the abstract, middle-class qualification to assess everyone. In his historical study of further mathematics, Newbould (1981) found that many students achieved relatively low grades in pure/applied mathematics, but that these were invisible casualties with, for example, no records of how many students dropped out of the courses.

Through the 1980s, the United Kingdom saw a gradual evolution of A-level syllabuses under private exam boards. Increasingly configured as businesses, the boards diversified and competed to attract schools and students: the market and choice were entering educational discourse. New A-level syllabuses introduced the current division into mathematics and further mathematics. 'Mathematics' combined the lower levels of the old pure and applied content. 'Further mathematics' contained topics that are relatively isolated from the core mathematics content (e.g. complex numbers), or develop it (e.g. differential equations), or are applied in different contexts (e.g. mechanics/statistics). This format proved popular, in part because students tended to get at least one good grade, and the old pure/applied format disappeared in 1997 when exam boards were regulated by government. During this time national policies had encouraged more 16-year-olds to stay in a broadly academic program, normalising the A-level/university trajectory as an indicator of educational success. Simultaneously the primacy of pure mathematics was cast as unwelcome specialisation, and applied mathematics was re-valued as relevant and necessary to national economic success. So it was not surprising that schools and students increasingly chose to enter students for the single mathematics A-level, whose syllabus covered both pure and applied content and gave a better grade (Kitchen 1999). The whole period saw a steady decline in numbers taking further mathematics, falling to only 5,000 candidates in 1999, a tenth of those taking mathematics A-level.

How does this historical genesis position further mathematics with regard to quality? First, the title and the very existence of further mathematics suggest that the content of A-level is now structured hierarchically. The syllabus split has designated particular mathematics topics—and the experiences of learning them—as 'further', creating a measure by which they are deemed more difficult, less accessible and therefore higher quality than others. Whether measuring content or mathematical thinking, further mathematics is awarded a symbolic role in emphasising difference (Hoyles et al. 2001). It constructs quality as a property of extremes, standing out from the norm in some measure of mathematics. Thus the first meaning for quality constructed as 'given' within further mathematics is that quality in education is measurable and there is a way of ranking mathematical study. It is worth recalling that mathematics and further mathematics A-levels are taught concurrently to the same students so this ranking cannot be solely determined by prior requisite knowledge: 'further' is not simply 'later' but 'better'.

Second, quality is constructed alongside further mathematics as embedded in tradition and the past. Modern society is alert to managing change and positions individual subjects as responsible for negotiating risks; thus stability becomes personally desirable (Bauman 2001). Further mathematics certainly offers an ongoing link with the education of thirty years ago, although feeling familiar reassurance alone is not recognising quality. Quality also requires observation and evaluation. Bauman argues that when the world around us changes, the normative response of modern individuals is to make sense of what is happening to us, to rationalise and compare old and new practices; I take this change-inspired evaluation as legitimating quality. Because the history of further mathematics positions it as relatively stable in a fast-changing educational environment, it evokes narratives of sense-making that heighten its visibility and position it as a context for evaluation. I call this a 'gold-standard' construction of quality: the gold-standard only has meaning because we no longer pay in gold. However, by evoking the rationale of calculating back, it continually reinvents itself. So in further mathematics we have stories of a past golden age in which students were well prepared in science subjects and competed to enter mathematics degrees. These stories have currency today, even as we accept that practices have changed.

However, further mathematics would be of little interest if it were not for the accompanying story of those who resisted the change: several thousand candidates continued to study it, from a minority of schools in England, Wales and abroad, and the elite universities continued to request it. In a culture of choice, why did it matter that some schools and students continued to choose further mathematics? I have suggested above that further mathematics features in neoliberal discourses as a problem that needs addressing both as a search for quality, for 'bright futures', and through the ways it was publically configured as inequitable. I now examine these constructions of equity in more detail.

The ‘rules’ of the A-level curriculum are that subjects should be roughly equal in teaching time and value, for example, they share a common ‘points scale’ for university entrance. This background parity positions A-level grades as a meaningful discriminator of any individual’s ‘reality of mathematics’ achievement’ (Matthews and Pepper 2007, p. 10). But alongside this official knowledge, teachers and the media disclose a hidden, ‘expert’ knowledge that certain subjects and subject combinations have greater exchange value for university entrance, and these include further mathematics even with a lower grade. We know that students from White middle-class backgrounds tend to seek more expert advice about their choices and choose these high-status combinations (Ball et al. 2000) Information about further mathematics is thus differentiated by class and ethnicity. Moreover, student choice is constrained by what their schools can offer. Since the 1980s the smaller, state-funded, non-selective schools have been affected by shortages of qualified mathematics teachers, financial pressures on teaching small classes and measures to compare schools by A-level grades (Smith 2004). Students in state comprehensive schools are three times less likely to study further mathematics as those in independent or selective schools, and 1.5 times less likely than students in 16–18 colleges which tend to be larger and city-based (Vidal Rodeiro 2007). These differences in school provision challenged liberal notions of equity in all three aspects identified by Hart (2003): students did not have equal opportunity, treatment or outcomes in their mathematics education. Further mathematics was a context in which these differences in individual experiences were made visible as structural differences between schools, not explainable as individual choices, and as such it posed a problem to policy makers. For example, the government’s advisory body has distanced itself from its own qualification: until there is ‘universal and equal access to Further Mathematics’, it is not ‘appropriate for higher education tutors to use [it] as a legitimate discriminator’ (Matthews and Pepper 2007, p. 14).

The role of further mathematics in quality and equity is part of a narrative that society tells itself about itself: we understand the decline of class and class distinctions as central to modernity (Atkinson 2007). In this narrative quality and equity are linked, but they function as opposites. Society needs more workers able to use mathematics, so mathematics applications were included in the single A-level and the ‘higher’ pure topics were left out. Students from all schools should have equal access to university mathematics courses so universities had to modify their curriculum. This framing is not simply a zero-sum game but one that is oriented in time. Quality is constructed as the rules of the past; equity as including more students in the future. The opposition seems natural because other factors are taken as unchangeable: the comparability of A-levels, the amount of teaching an undergraduate should have, and mathematics itself. These are not debated but rather crystallised in the practices of teaching and examining that make up school.

In the next section, I turn to the recent FMNetwork and consider the narratives used in its organisation, promotion and evaluation. I do not aim to criticise these choices but to understand more about how they sustain positionings of equity and quality, and how these relate to traditional conceptions.

Changing Further Mathematics

The FMNetwork was commissioned and funded in England from 2005 to 2009. A national hub provided branded materials in the form of a website, promotional texts and teaching resources. Regional teaching centres recruited locally for further mathematics, employed tutors to visit schools and collected performance data. Schools effectively subcontracted further mathematics teaching for a group of their students. The centre agreed to teach on a concentrated schedule, typically only a weekly two-hour session after school. The school and centre negotiated money, timing, duties, access to resources—all means of circulating power at a microlevel. Such deployment of a market model in publicly funded institutions in order to serve particular agendas of quality and equity is typical of applied neoliberalism.

Bringing Quality Up to Date

The constructions of quality discussed above were rooted in the past or in mathematics content that appears timeless, but the FMNetwork supports a new construction that is rooted in present-day technologies. It does so by emphasising that further mathematics is an A-level just like any other, following the rules and practices of the now-regulated examination system. For example, it encourages students to choose further mathematics by stressing the techniques that integrate the two A-levels (such as exchanging modules to get higher final grades). Thus one way that the FMNetwork constructs quality is as a property of rigorously conforming to an improving school system. This quality-as-conformity promises equity in the form of universal access to further mathematics, and the improved life-chances that follow. For example, the FMNetwork tells universities that ‘the new QCA rule changes […] will make it far easier for ordinary schools to offer Further Mathematics’(Stripp 2004, p. 15) positioning ‘ordinary’ schools as the appropriate focus of universities. However, conformity downplays individual and school agency and positions the structure of A-levels as powerful in itself: the main actors here are ‘rule changes’. Stripp adds that schools can ‘increase the supply’ of mathematics students, but ‘it’s up to the universities to ensure this happens by creating the demand’(p. 16). Changing the rules and demands for further mathematics is taken to be enough to change what schools will offer and students choose. This claim suggests the neoliberal framing of modern society as a complex ‘swarm’ of individual trajectories, all choosing according to economic forces but choosing alike (Bauman 2001). The FMNetwork positions itself with universities and policy makers who understand how power works within the swarm and can use that knowledge for change.

I have now traced three constructions of quality. A sociohistorical perspective on further mathematics sees quality as historical continuity and standing-out-by-measurement. Those constructions were reconciled by representing further mathematics as a gold-standard. Because these views of quality were located in the past; the inequities associated with them could be understood as outdated white middle-class privileges. The third, recent, construction was quality as conformity; this time enacted as progress in a presently improving education system and looking to the future for equity. Clearly, these co-existing constructions introduce potential tensions: is quality judged in the past or present; does it concern conforming or standing out; are inequities over or still being ironed out? I have identified one more construction in the FMNetwork texts that functions to resolve these potential conflicts: quality as achieving breadth-plus-depth. The duality in this metaphor manages tensions through flexibility and ambiguity: further mathematics is valuable because it is broad or deep or both as required. This new metaphor was enabled by a specific rule-change that changed the discursive tools available. In 2000 the first half of an A-level course was given its own name—AS-level—allowing separate identities for each year of further mathematics.

How does this breadth-plus-depth construction work? First, the FMNetwork follows many government texts (e.g. Smith 2004) in associating the AS course with breadth. Breadth provides a metaphor for widening access and inclusion, and also becomes a marker for quality when education is seen as aiming to provide universal, flexible skills suitable for an unpredictable working life (Rose 1999). When Porkess describes AS students encountering ‘exciting new ideas, like complex numbers, as the building blocks at the start of Further Mathematics’ (2006, p. 13) he uses ‘building blocks’ to evoke utility, flexibility and progress—all seen as important for future careers. ‘Building blocks’ evokes children and manual work, including them in further mathematics. I find it an unexpectedly practical metaphor for complex numbers. Compare it, for example, with a further mathematics student’s description of them as uncomfortably abstract: *something that doesn’t even exist. Just, it makes me feel sick, the thought of it*. I suggest that the difference illustrates the imperative for the FMNetwork to construct the AS syllabus as accessible.

The second half of the metaphor follows from the historical re-organisation of syllabuses that associated further mathematics with ‘higher-level’ topics. The FMNetwork texts use this association as a given, and rephrase it in terms of depth:

> The new AS will be more a ‘broadening’ than a ‘deepening’ option. This means that AS-Level Further Mathematics is no longer an ‘elite’ qualification, suitable only for A-level Mathematics high-fliers. (Stripp 2004, p. 15)

Here breadth is written up as a modern contender to depth, but there is still ample reassurance that ‘high-fliers’ should be taking further mathematics. Depth is separated from particular mathematical content, and rather defined as being what the ‘elite’ study, and so inherently bound up with exclusion. It is still firmly attached to quality through the continuation of familiar standards: ‘The stretch and challenge for the elite is still provided by going on to the full A-level in Further Mathematics […] which is just as demanding as ever.’ (Stripp 2007, p. 35).

In summary, the FMNetwork justifies itself as an agent for change by arguing for a new construction of quality as broader relevance and participation. However, since breadth departs from the traditional exclusions, the change is only enabled by a successful defence against itself, that is, by simultaneously arguing for depth.

Breadth and depth are thus held together as two forms of quality existing on either side of the AS-level but pulling in opposite directions, one including and one excluding. What holds them together is students' responsibility for choosing: inclusion is systematised by universal access to AS-level, exclusion can thus be left to individuals.

What Is Equity for the FMNetwork?

In my discussion above, I have started to show how constructing quality in certain ways might entail corresponding constructions of equity. I now use two recent evaluatory texts to exemplify how these constructions of quality and equity function together. One reports the independent evaluation (Searle 2008) commissioned by the FMNetwork to justify government funding; the other (Vidal Rodeiro 2007) reports an assessment agency's research into A-level participation. These texts necessarily draw on, and contribute to, the policy discourses of further mathematics.

Examination data show that the FMNetwork program coincided with a revival in further mathematics: from 2004 to 2008 the number of candidates taking the 'one-year' AS-level course more than tripled and the number taking the two-year A-level course nearly doubled. Searle's (2008) evaluation highlights that over three-quarters of this growth was in state schools and concludes that access according to school sector was becoming more equal. It thus prioritises the historical perspective that class-based differences in provision between schools were the primary problem of inequity. This increase strengthens the network's claim to achieving quality-as-conformity alongside equity as systematised access.

Searle then examines equity in more detail by relating school region to socio-economic status. More affluent areas of England accounted for much of the growth in the two-year A-level, but the 'one-year' AS-level grew very significantly in deprived areas. Presenting this data makes a weaker claim for progress in ironing out differences according to class, but it does strengthen the suggestion that the FMNetwork AS-level is broad in its appeal to previously excluded students. Hence the FMNetwork is positioned as partially successful in its aim to achieve quality constructed as breadth-plus-depth. However, I see tensions between this construction and equity as universal opportunity, which are unstated: how can we account for the social differences in who engages with the 'deeper' material and who stops at AS-level. What individual and social factors might be at play? My research includes students who after one year of study choose to stop mathematics—which can be construed as an exercise of individual agency—but also some who are being taught only the AS-level content over 2 years, a structural constraint. A discussion of equity would be further informed by analysis that linked individuals' outcomes to course opportunity. The fact that this type of data is not within the remit of the official data-collection illustrates how neoliberalism averts its gaze from issues of how individual and social factors interact (Atkinson 2007).

As well as socioeconomic status and school type, the other factor reported in detail in Searle's evaluation is gender, perhaps owing to its ease of classification and the longstanding concerns over girls' participation in mathematics. The proportion of further mathematics students who are female, between 30 and 40%, has not changed in the period. This is left without comment: it is not clear whether any change was desired or feared. Other individual background factors are not reported. We know that students who are Black African, Chinese, Indian, Pakistani and from a mixed background choose mathematics/science subjects proportionally more than White students (Vidal Rodeiro 2007), but not how they have engaged with further mathematics over time. Nor can we find out whether students from different socioeconomic backgrounds, but in the same school, choose differently organised lessons and obtain different outcomes. Through the choices made in these texts, no doubt for necessary reasons, equity is constructed as the absence of those differences that relate to institutions, and what affects individual choice is left out of the enquiry.

In summary, the FMNetwork makes use of an educational technology—the decoupling of AS from A-level—to sustain roles for both breadth and depth, and find a compromise where each has a different function but each conforms to institutional requirements. Quality as depth is described in terms of the past and an elite, and thus linked to quality as gold-standard. There is a new understanding of quality as breadth with everyone doing more mathematics, and this links to quality as conformity. Equity is constructed as the opportunity for an individual to start further mathematics no matter what type of school, how teaching is organised, or what was previously learnt. The first year promotes this goal of universality and recruits for the full course, but it also legitimates selection in the second year. This selection is no longer understood as a means by which schools necessarily reproduce privilege because, for the purposes of further mathematics, schools are positioned as operating with an agency that is informed by economic truths. Change is guaranteed by calling on practices aligned with neoliberalism and individuals have the responsibility for choosing further mathematics for themselves. In the next section, I turn to individuals' accounts so as to consider an example of how quality and equity enter one student's account of choosing whether or not to continue with mathematics.

Practices of the Self

From 2006 to 2009 I have conducted research in three sites offering mathematics A-level in school and further mathematics with the FMNetwork. I followed 24 students over 18 months, collecting data from interviews, lesson observations and email questionnaires. My analytic focus was what Foucault (1990) calls 'practices of the self': the knowledges and processes that inscribe what it means to be a successful individual within a particular history or culture. Practices of the self establish the norms and means by which people explain themselves, govern themselves, and engage with others. I have explored the intermingling of discourses of further mathematics and discourses of the self by analysing textual data in the form of observation field-notes, interview transcripts and e-mail exchanges. I chose one student, Mario, to discuss here because he often appeared uncomfortable with seeing himself as a further mathematics student. It is his quote that provides the title of this chapter.

In our conversations Mario positioned himself variously as successful and as struggling, as a natural and as an outsider, and tried out different ways of justifying his decisions to continue. I interpret the ways in which he argues whether doing further mathematics is 'doing any good' as examples of how he participates in constructing quality. Mario also describes what 'actually' threatens his engagement; I see these as examples of ways-of-knowing through which individual agencies contribute to social patterns.

Mario lives in the centre of a relatively deprived English industrial city. The school he went to until age 16 was replaced by a business-sponsored school that offers further mathematics to all its mathematics students. Mario is white and his family show characteristics of both middle-class and working-class cultures (Ball et al. 2000): Mario's father is a graduate engineer but he lives with his mother and receives some government income support. Mario's passion is rock guitar.

In his first interview Mario describes his initial subject choices as based around maths—the 'four core Maths subjects'. He presents evidence he has gathered to support this claim for the centrality of mathematics: all university courses want high grades, and mathematics 'comes into everything'. These claims are based on its power as a widely accepted currency and a knowledge that will be relevant even—and especially—when he attempts a more idiosyncratic career linking science and music.

Choosing further mathematics is also a way of demonstrating success in unexpected ways:

> First of all when I said about Further Maths my mum was… 'This is…. No, you can't do it.' And I was okay at Maths at GCSE but never like that star, that everyone else was like getting full marks all the time. And when I said Further Maths she was quite shocked and didn't think I could do that. And that made me want to do it even more.

Mario ascribes his mother's doubts to his grades in national examinations at age 16 which were good, but not the best. I suggested above that further mathematics invokes quality as a gold-standard. Mario draws on this representation to challenge the defining power of grades: for him, choosing further mathematics is a way of aspiring to stand out as being different but just as good or better. Mario sets the stakes high by making his comparison with 'full marks all the time'. The sociohistorical context that positioned further mathematics as rarified because it was limited to certain schools has been reinterpreted as a practice of individual choice so that further mathematics aligns the chooser with extreme personal qualities of ability and dedication.

So far I have commented on those aspects of Mario's choice that position further maths outside the narrow focus of school, but Mario also uses arguments that further mathematics conforms with schooling. His work-experience mentor has encouraged an academic route to his dream; he could take a physics degree first and then work in acoustics. Mario also cites universities telling him further mathematics is the right preparation for physics, and his further mathematics teacher's view that it gives you a head-start at university. These reasons, and their authoritative institutional sources, emphasise the progressive nature of school mathematics and position further mathematics as 'further' along that path. Together, his justifications suggest that his aspirations match the FMNetwork's breadth-plus-depth construction of quality. He aspires to study further maths because it is broad enough to provide both academic certification and application in a 'real-life' setting, and because it is deep enough that he can get 'ahead', making sure that he is included in the niche he has picked out as desirable.

In these examples Mario uses constructions of quality in further mathematics to maintain overlap between the talented outsider status that comes with his dream and the reassuring possibilities of exam success. When Mario's AS grades were lower than he wanted, he was once again threatened by measurement. Other students in the class excused their low grades and nearly all stopped further mathematics; but Mario and Randall continued. At the end of their second year I asked why they had chosen differently. During the discussion (lines 576–649), Mario deals with conflicting understandings of what studying further mathematics means to them and about them:

> 577 We were a lot more clever than them.
> 594 I think a lot of people say it's the hardest A level.
> 596 And everyone knows it as well. Which makes us feel cool…
> 602 It's just it's…. I didn't mean it makes us feel cool, it makes us look stupid.
> 622 And it doesn't make you…. People makes…think it makes you a genius.
> 624 We should be really clever, but something about it, maybe a bit of common sense, like we just…sometimes we just, like maybe the time of the day, or what mood you are in, but sometimes we feel really stupid.

Mario is conscious of positioning himself as clever, cool and different but, importantly, not alone in 'belonging' to further mathematics. The feelings are described as the result of their common choices and so personal to *both* of them. Mario starts here by comparing himself with others; his belonging is based on their exclusion from 'the hardest A-level'. He ends up worried by how 'it' positions him compared to the 'genius' model he has just helped to build. This illustrates again how exclusion is inherent in these constructions of quality. Mario not only has to defend his sense of belonging against structural threats such as AS grades but also against how he explains his self-practices to himself. If he does not 'feel' clever, how can he belong? In line 624 he calls up an explanation which constructs two oppositions: either his self-doubts are momentary irrational lapses from cleverness, or cleverness is not related to common sense and practical experience. This answers one threat to continuing, but introduces a second threat—how he sees himself as practical and 'hands-on'. This recalls the historical classed constructions of advanced mathematics as removed from work applications. Mario formulates this in terms of his personal qualities when he wonders:

> whether I'm patient enough to actually go through all the Physics and stuff, and be good, really good at it at the end, or go straight into it and build up experience in it.

Again Mario links education with having to be ‘really good’, and contrasts having to ‘go through’ it with actively ‘building’ authentic, direct experience. Staying in post-compulsory education, and studying more abstract disciplines are the types of choices that produce structural class inequalities (Atkinson 2007). Here Mario is constructing them as choices based on truly understanding himself. Randall, too, positions mathematics as inauthentic, and Mario as excluding himself from their dream:

> Randall: I’m going to be there. But Mario’s gonna be like working out all these equations.
> Mario: And I’m gonna be paid ten times more than you.
> Randall: And I’m gonna be the happier one. It’s not all about money Mario.
> Mario: No. I’m gonna be happy.

Mario is on the defensive. He uses mathematics to make a claim for economic success but Randall excludes him not just from practical experience but from happiness. From a neoliberal perspective, the key practices of the self are concerned with gaining the self-knowledge to pursue personal happiness (Rose 1999). Mario’s case is an example of how the constructions of quality and equity in further mathematics can reappear as ways of understanding oneself as included and excluded not simply from mathematics but from one’s self.

Conclusion

In this chapter, I have used the context of further mathematics A-level to analyse how its sociohistorical role provided understandings of equity and quality as privileged access to a ‘gold-standard’ mathematics education; how an influential reform program created new agendas of universal opportunity and had to reconcile them with existing understandings of quality in order to recruit support; and how an individual explains his experiences and choices in ways which translate these issues of quality and equity as saying something about his own self and his choices. Throughout, I have tried to show how the opportunities for choosing built into the education system in England and Wales position schools and government as powerful in guiding rational economic choices but individual students as responsible for making them.

The FMNetwork has tried to preserve quality while addressing inequity by removing barriers to starting further mathematics. However, the ways in which quality is constructed mean that some students must be excluded, or rather must choose to exclude themselves. School factors do exclude students: for example, reduced teaching time affects AS grades so that students opt out. Students also experience questions about belonging that spring from the same discursive representations that made them join. In further mathematics, the associations of quality with measured success and abstract learning is difficult to sustain against the risks associated with competition, and also against a desire for present happiness and authentic experience.

It is easy to focus on ways in which students may find themselves threatened in belonging to further mathematics, but there are also ways in which they can resist. Mario described his final push for success as deciding to change how he thinks about himself and his goals, giving up some of his independence and using FMNetwork connections to find a tutor. In the process of including himself, he strips further mathematics of the quality of separateness that once attracted him: ‘it should just be called different modules’. Quality as depth is a powerful construction, but aspiring to include ourselves is accompanied by consciousness of how to exclude ourselves. Although my policy analysis suggests that the FMNetwork required a dual breadth-and-depth construction in order to defend itself against the past, individual students may show how ‘further mathematics’ can be rethought as an inclusive ‘more mathematics’.

References

Atkinson, W. (2007). Beck, individualization and the death of class: A critique. *British Journal of Sociology, 58*(3), 349–366.

Ball, S., Maguire, M., & Macrae, S. (2000). *Choice, pathways and transitions post-sixteen*. London: RoutledgeFalmer.

Bauman, Z. (2001). *Community: Seeking safety in an insecure world*. Cambridge: Polity.

Foucault, M. (1990). *The history of sexuality: Vol. 3. The care of the self.* London: Penguin.

Foucault, M. (1991). What is enlightenment? In P. Rabinow (Ed.), *The Foucault reader* (pp. 32–50). London: Penguin.

Hart, L. (2003). Some directions for research on equity and justice in mathematics education. In L. Burton (Ed.), *Which way social justice in mathematics education* (pp. 27–49). Westport: Praeger.

Hoyles, C., Newman, K., & Noss, R. (2001). Changing patterns of transition from school to university mathematics. *International Journal of Mathematical Education in Science and Technology, 32*(6), 829–845.

Kitchen, A. (1999). The changing profile of entrants to mathematics at A-level and to mathematical subjects in higher education. *British Educational Research Journal, 25*(1), 57–74.

Martin, D. B. (2006). *Mathematics success and failure among African-American youth*. London: Lawrence Erlbaum.

Matthews, A., & Pepper, D. (2007). *Evaluation of participation in GCE mathematics: Final report*. London: Qualifications and Curriculum Authority.

Mendick, H. (2006). *Masculinities in mathematics*. Buckingham: Open University Press.

Newbould, C. (1981). Grading A-level double subject mathematicians and the implications for selection. *Assessment and Evaluation in Higher Education, 6*(3), 230–239.

Popkewitz, T. (2002). Whose heaven and whose redemption? In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International MES Conference* (pp. 1–26). Copenhagen: Centre for Research in Learning Mathematics.

Porkess, R. (2006, April). *Unwinding the vicious circle.* Paper presented at the IMA international conference on Mathematical Education of Engineers, Loughborough.

Rose, N. (1999). *Powers of freedom: Reframing political thought*. Cambridge: Cambridge University Press.

Searle, J. (2008). *Evaluation of the further mathematics network.* Paper presented at the Improving Educational Outcomes Conference, Durham University.

Smith, A. (2004). *Making mathematics count. The report of Professor Adrian Smith's inquiry into post-14 mathematics education.* London: HMSO.

Stripp, C. (2004). The changes to AS/A level further mathematics for September 2004. *MSOR Connections, 4*(3), 15–16.

Stripp, C. (2007). The further mathematics network. *MSOR Connections, 7*(2), 31–35.

Vidal Rodeiro, C. (2007). *A-level subject choice in England: Patterns of uptake and factors affecting subject preferences*. Cambridge Assessment.

Wright, I. (2009, August 20). New generation of mathematicians and scientists at heart of building Britain's future [DCSF Press release]. http://www.dcsf.gov.uk/pns/DisplayPN.cgi?pn_id=2009_0151.

Chapter 21
Mapping Social Constructions and Complexities

Editor's Reaction to Part II

Paola Valero

This section gathers 11 chapters with contributions from Australia, Germany, Lebanon, Malawi, Sweden, the United Kingdom and the United States of America. The chapters address issues concerning quality and equity as well as their relationships, and highlight particular dimensions of what I would call the social and political constitution of the discourses of equity and quality in mathematics education. As a whole, the chapters offer a mapping of the multiple influences that in a diversity of sites of practice shape the ways of talking, viewing and enacting both quality and equity. The chapters illustrate with empirical material, analysis and discussions, the way in which the discourses of equity and quality move in constant construction and recontextualization from broad societal trends to the constitution of subjectivities, passing through policy, the media, pedagogy and reaching the learners. I will comment on the chapters knitting a network of discursive relationships between the issues raised by the authors. My intention is to articulate my reading of how these chapters map the discursive complexity of equity and quality in mathematics education in society nowadays.

It is clear that the issues of equity and quality are not phenomena that can be placed at a level of individual choice. *Jurdak* argues that it is necessary to conceptualize equity and quality as a global system of nested subsystems of mathematics education. From a macro-global scale to a micro-local scale in classrooms, in(exclusion to the access of diverse populations to mathematics and to the goods of society is being produced and reproduced. Using international statistical data he shows that even if there seem to be some factors that can be strongly related to inequity in mathematics education at a global level—socioeconomic status, educational capital and culture of countries, policies that govern international organizations of mathematics education, and access to English as the language of the international research field—the divide between developed and developing countries in terms of achieving better results in mathematics cannot be accounted for in any way by internal factors of the organization of mathematics teaching and learning. Rather, they

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are related to factors that go beyond the reach of mathematics educators or even national countries. Differences may be connected with particular global economic and political orders that install stratification in society. *Shelley and Su* provide results that resonate with Jurdak's arguments. Using mainly PISA 2003 data, Shelley and Su developed a statistical analysis of the factors that, at a student level and a school level, affect student achievement in the USA. Their analysis provides evidence that schools educational resources matter and that achievement cannot only be explained in terms of individual student characteristics. They point to the fact that the systemic and endemic disparity among school districts in the USA is certainly connected to the inequity in students' achievement. Of course, such a systemic inequity calls for serious interventions in policy to reduce economic disparity.

Policy is an important discursive level when looking at the constitution of ideas and practices of equity and quality in mathematics education. Shelley and Su call for guidance in policy that decidedly has the intention of addressing the achievement gap in the USA. They show the necessity of paying attention to the role that policy makers in national and local governments, as well as interest organizations formulating visions for the future of mathematics education, play in shaping the possibilities of change towards balanced practices in the field. Although researchers have argued the necessity of studying policy and its impact in mathematics education, the amount of studies addressing policy are still scarce when compared to the amount of studies in other subfields of mathematics education. Here, however, there are three chapters that explicitly address policy. *Kazima and Mussa* present the case of the Malawi educational system and the deep divide in children's possibilities to access to quality mathematics education, according to their gender, their age and their rural or urban place of living. Malawi has been involved in a process of expansion in the coverage of its educational system in the last 15 years. The issue of the quality of education when certain degree of coverage is reached becomes a concern. Particular measures directed towards the improvement of mathematics education have been taken. However, serious differences related to gender and the urban/rural gap are still very evident. The authors point to the fact that providing material resources as mechanisms for addressing quality and equity are not sufficient to generate better possibilities for all students. A broadened concept of quality that touches substantially on teachers' qualifications is needed. More often than not, the operational definitions of both quality and equity that policy makers adopt, often connected with the political rationality of accountability that dominates in this time, lead to restricted views of what equity and quality may be, particularly when it comes to the core conceptions of the components of mathematics education.

This is one of the points that *Bose and Remillard* raise in their analysis of recent key policy reports for the improvement of mathematics education in the USA. An analysis of the reports reveals the view of the mathematical knowledge that they support, as well as the opportunities for students to engage in significant mathematical learning that they promote. The results of the analysis show that, first of all, policy reports criticizing the state of affairs in education and proposing solutions are not neutral nor apolitical. On the contrary, they take particular political stances not only towards society and the role of mathematical competence in it, but also about what counts as mathematical competence in educational settings. Second, the reports privilege a view of quality and equity in mathematics with a restricted focus on mathematical contents. Such view contradicts a more nuanced and complex view of mathematics connected to research in mathematics education. The authors argue that policy with such a narrow focus has no chances of addressing significantly the already poor practices that lead to poor achievement for many students in the USA. The interesting question here, of course, is which are the kinds of interests that those policy reports and their recommendations are defending. This is precisely the question that *Noyes* addresses for the case of the mathematics reform for the 14–16-year-old students in England. Changes in the curriculum and assessment have been part of political and economic discourses framed by neo-liberal agendas where markers, managerialism and performativity are the tools that governments use to regulate and install educational change. The result of this approach generates a situation where traditional views of quality and equity restricted to "more higher mathematical contents" to as many as possible are uncritically reproduced and strengthened in society. An interesting mechanism for the reproduction seems to be set in operation: while in principle discourses that claim the need of more mathematically competent people as a condition for social development attract the attention of all stake holders due to their apparent appeal to inclusion of all students, curricular change and assessment practices maintain a tradition that effects exclusion. The similarities between the cases of the policy in the USA and in the UK really leave open the question: "whose" quality and "whose" equity is policy promoting?

In the age of information, the role of mass media and communication cannot be ignored as important players in the discursive constitution of equity and quality agendas in mathematics education. One chapter addresses directly and explicitly how media, in particular newspapers, construct partial pictures of key predicaments in equity and quality reported in scholarly literature. *Forgasz and Leder* engage in an examination of how prominent Australian newspapers cover news that attract the public attention regarding practices of significance for equity and quality, namely students' achievement levels, ability groupings and single-sex groupings. The authors evidence the particular representations that the media create, compared to evidence provided in research reports on the same issues. Media portrayals of these topics are often partial and simplified views of the complexity of the problems studied and reported in research. Probably the type of work of journalists—time and space constrained, as well as in need of a common-sense language to report to the general public—shapes the creation of public discourses about the issues and contributes to the reproduction of readers' uninformed conceptions and stereotypes. With such a way of operating, the media may be having a strong impact on how opinions are being formed and, thereby, on who comes to benefit or who becomes disadvantaged in educational practices. Some other chapters in the section address related issues. Particularly the chapter by *Weissglass* exemplifies how the media played an important role in the "Maths Wars" in California and how that helped contributing to the creation of stereotypes of mathematical competence, in particular of disadvantaged youth. *Smith* also refers to how the communication strategies of a program aiming at attracting more students to A-level mathematics in the UK creates particular views not only of the program, but also and specially about what it takes to be mathematically successful at that level. All these chapters show that the meanings for what counts as mathematical competence, as well as what is considered to be equity and quality are also constructed in the public discursive space opened by the media.

The discussion of what counts as mathematics in relation to equity and quality has been present in many of the chapters already mentioned. Particularly Bose and Remillard's paper addressed this issue in relation to policy reports. The chapter by *Jablonka and Gellert* takes a particular stance on the issue and its relation to pedagogy. They examine the particular case of mathematical modeling as both a view of mathematics and an associated pedagogy that has considered to be an alternative for generating quality and equity simultaneously. Taking a Bernsteinian perspective to read the pedagogical discourses taking place in classrooms where modeling is proposed as an alternative form for mathematics education, they evidence defenders of modeling may be creating a myth that can potentially disadvantage many students. The analysis shows clearly that there are fundamental problems with the promises of modeling in teaching and learning. Students lacking tacit knowledge about what is being modeled are at risk, since processes of recontextualization are central to the modeling exercise. The very same lack of acknowledgment on the side of mathematics educators on the contextual nature of modeling competences may lead to an empty type of mathematical activity that does not offer better understanding for students. For these reasons, the authors argue for the need of looking critically at the false promises of pedagogical approaches in mathematics. Pedagogical traditions contribute to disadvantage some types of students. No pedagogy is neutral; it cannot be expected to be beneficial to all students. Pedagogy makes a particular stance, implicitly or implicitly, towards whom reaches quality and inclusion. Jablonka and Gellert showed this for modeling. *Schütte and Kaiser* also exemplify the previous general observation with their study of mathematics education practices in German classrooms for students with an immigrant background. Mastery of the German language is a determinant factor for success in schooling. Immigrant students, often disadvantaged also on the grounds of their ethnicity and socioeconomic status, could keep on being disadvantaged if teaching and learning practices do not develop both their mathematical register as well as their competence in more academic registers in German.

The processes of individualization that have characterized Modern society have impacted understandings of equity and quality in mathematics education. The focus on the learner—who she is, what she lacks, how she can be helped—is yet another dimension of the social and political constitution of the discourses of quality and equity. Some chapters address the issue of how the construction of subjectivities is a complex process that touches deeply individuals, in this case, the mathematics learners. *McCray and Chen* are concerned with small children in preschool age and their mathematical thinking. They argue that attention needs to be paid to early mathematical experiences as a new way to address equity. Focusing on the USA, they examine the potential benefit in addressing imbalances already visible at the beginning of primary school with a more focused attention to early foundational mathematics. The latter refers to a view of early mathematical thinking as a form of knowledge and thinking that all children develop in their participation in the world. Directed attention in preschool to central foundational mathematical ideas—e.g., the same collection of objects can be sorted in different ways—may provide children with a more solid thinking and language for their experiences. A good start may be the beginning of a more positive engagement with mathematics. Taking students seriously is part of McCray and Chen's concern. So it is for *Weissglass*, whose chapter presents an argument for the importance of taking students seriously and to respect them as learners, thinkers and human beings. Respect is at the center of generating pedagogical approaches that can contribute to the construction of strong identities of students as mathematical learners. Weissglass discusses the idea that the lack of respect for the youth is not only a personal matter of some teachers. Rather, it is a social phenomenon where youth is positioned by adults as incompetent, deficient thinkers. Such a positioning has a tremendous impact on the self-confidence of many youngsters, particularly of those who are also positioned as inferior on the grounds of their gender or ethnicity. In the context of the USA, Weissglass proposes seven principles for a mathematics education that builds respect for the learners; the principle reminds us, adult teachers, that most of the time students' incompetence is induced, not genetically determined. Such recognition is important in any consideration of quality and equity in mathematics education.

All learners are building their sense of self in schools, which are, as Jurdak proposed, nested systems of practice where quality and equity are being given meaning. *Smith* shows the discursive constitution of policies, intervention programs and learners' subjectivities. Notions of equity and quality are being formulated and embodied by people and practices in each one of these levels. All levels are discursively connected. Studying the case of A-level mathematics—the same focus of Noyes in his paper—Smith shows brilliantly how school policies argue for the need of opening more spaces for more students to achieve better mathematics. The meaning of "better" and higher-quality mathematics in this case resonates with what has already been discussed in the chapters of Bose and Remillard and Noyes. The particular effort of an intervention program to generate access to more students into A-level mathematics generates a discourse of mathematics with breadth and depth, where inclusion and exclusion live side by side and is experienced differentially by students with different backgrounds. Individual students participating in the intervention become aware of this double construction. They tell stories about themselves, and formulate narratives where they make these contradictions as part of themselves and their own personal choices. In this way, the whole discursive construction builds the mechanism of individualizing what are systemic inequalities. Particular ideas about equity and quality in mathematics are reified in students' subjectivities.

I have tried to show the network of social and political constitution of the discourses of equity and quality in mathematics education. Such network, as illustrated by the chapters in this section, is multilayered and multileveled. The chapters address discursive constructions at a macro-global level, at a level of policy, at the level of the media, at the level of pedagogies and at the level of the constitution of subjectivities. After reading the chapters, one idea keeps on revolving in my thoughts. Even if the discourses of equity and quality are being constructed in this broad network of social practice, are being reified in it in ways that go beyond the control of any individual, its mechanisms posits all its burden on individuals. Why? Whose interest is such functioning serving? Are we as mathematics educators fully aware of the effects of such processes? What can they mean for our well-intentioned actions? I hope that, more than providing definite answers, these chapters help the reader generating sharper questions about the multiple possible meanings of quality and equity in mathematics education.

Part III
Landmarks of Concern

Chapter 22
Students with 'Special Rights' for Mathematics Education

Ann Gervasoni and Lena Lindenskov

Introduction

This chapter explores issues concerning a group of students who we argue have 'special rights' in mathematics education because historically they have not had access to high-quality mathematics programs and instruction. These are students who are visually or hearing impaired, or who suffer from have Down syndrome or other intellectual or physical impairments. Also included are students who underperform in mathematics due to their exclusion from quality mathematics learning and teaching environments necessary for them to thrive mathematically. The arguments presented throughout this chapter are underpinned by a belief that underperformance for the students in focus is too often due to issues associated with both equity and quality. First, many students in this group have been directly excluded from opportunities and educational pathways in learning mathematics because mathematics was deemed an inappropriate field of study for them (e.g., Faragher et al. 2008; Feigenbaum 2000). Second, other students may attend a school where mathematics is taught, but do not receive the quality of instruction or experience that enables them to thrive (Gervasoni and Sullivan 2007; Lindenskov and Weng 2008). These students are indirectly excluded from mathematics education.

Throughout the chapter, we draw upon experience and contributions from the International Congress of Mathematics Education (ICME) 10 (ICME 2004) and ICME 11 (ICME 2008), as well as examples from colleagues around the world. We identify features of programs and approaches that improve quality and equity and generally advocate on behalf of students with special rights.

During ICME 10 it was notable that international dialogue was possible to critically discuss and describe contradictions and synergy between issues of equity or

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'mathematics for all' and issues of quality or 'mathematics for high-level mathematical activity'. Further, at ICME 11 it was noted that many mathematics programs and learning activities for students with 'special needs' attempt to teach mathematics using a conventional approach, but at a slower pace and with a more tunnelled view of a limited range of mathematics. In these cases, instructional innovations were based on deficit models of learners and focused mainly on designing tools to aid communication between the teacher and student that enabled students to access classroom mathematics programs and teaching. Sometimes the mathematics curricula offered in these cases is relevant more for the past, than for the present and the future. For instance, in 2002 it was shown that 55% of special educators in Maryland USA ($n = 35$), had not heard of the NCTM Standards (NCTM 1989, 1991, 1995, 2000), although the first version was published in 1989 (Maccini and Gagnon 2002). We argue that mathematics education research and mathematics learning programs for students with special rights must begin with two equally important foci: (1) placing students at the centre with the stated aims to build on their knowledge, motivation, and communication abilities and (2) a focus on quality mathematics curricula and instruction in response to global challenges. Further, we argue that mathematics programs and innovations need to focus on students' and communities' abilities and strengths in order to build educational capacity so that students with special rights may thrive mathematically. This calls for a new era of cooperation between mathematics teachers, special education teachers, specialists from other support services and community members.

Special Rights in Mathematics Education

A challenge exists for the global community to decide who has special rights in mathematics education. Indeed, the absence of a universally accepted definition is striking. Perhaps at this point in history, drawing attention to this issue is more important than reaching a definitive conclusion. We argue that students with special rights for mathematics education have many and varied characteristics, but that not reaching their mathematical potential is the over-arching characteristic. In this regard, our discussion focuses on two groups. First is the group of students 'who have long-term physical, mental, intellectual or sensory impairments which in interaction with various barriers may hinder their full and effective participation in society on an equal basis with others' (United Nations 2006). For example, these students may be visually or hearing impaired, or have Down syndrome. The less visible second group are students who underperform in mathematics due to their explicit or implicit exclusion from the type of mathematics learning and teaching environment required to maximise their potential and enable them to thrive mathematically (e.g., Gervasoni and Sullivan 2007; Lindenskov and Weng 2008). Too often, both groups experience educational exclusion, and as such have special rights to equitable access to quality mathematics education.

Access to Quality Mathematics Education

When students 'fail to thrive' at school, they often lose confidence in their ability, lose confidence in the school environment as a place to which they belong, develop poor attitudes to learning and to school, and disengage from learning opportunities (Ginsburg 1997). This situation is exactly what school communities seek to avoid, and infringes on a student's right for access to quality mathematics education.

Traditionally, school programs are based on the premise that a class is homogeneous and that each child will gain value from the same type of experience (Ginsburg 1997). This is not the case, and schools and teachers are becoming more aware of the importance of responding to the diversity of individuals in classrooms. This is essential for maximising students' rights to equity and quality. However, Ochiltree and Moore (2001) argue that schools are not equipped to address student differences, which are regarded by some educators as deficiencies. Such views have long-term negative consequences for students.

The phenomenon of learning school mathematics needs to be interpreted in the context of the ecology of the school. Ginsburg (1997) argues that many students in the USA are educationally at risk because they are at the mercy of (a) a culture that devalues mathematics, (b) inhospitable schools, (c) teachers who teach badly, and (d) textbooks that make little sense. We argue that this situation may apply in many countries, and overall suggests that many students lack access to quality mathematics education. Further, students are at greater risk if they have a physical or intellectual disability and this is aggregated significantly if they are from a poor or underprivileged minority. This is a negative view, but highlights the exclusion to quality mathematics education experienced by many students globally.

Possible reasons to explain *failure to thrive* in the school environment include:

- Disparity between students' early environment (including factors such as culture, language, attitudes to education, nurturing, and family harmony)
- The school environment (including socio-cultural norms such as organisational structures, communication, expectations and attitudes of teacher, and school expectations of behaviour)
- Ill health (including physical disabilities)
- Intellectual disabilities and learning disabilities
- Mismatches between a child's approach to learning and the teaching style a child encounters
- Personal experiences such as the death of a grandparent, civil unrest, and poverty

Disparity between formal mathematics curriculum content and students' informal mathematical knowledge may also impede mathematics learning in formal schooling contexts (Doig et al. 2003). For example, students' informal mathematical knowledge is sometimes culturally specific, and may not be obvious to the teacher. Also, some students may not have the chance (or language) to demonstrate their informal knowledge in the context of a formal mathematics program. Thus, a key role of the teacher of mathematics is creating a *bridge* for students as they negotiate the transition from their home environment to the more formal world of the school and the learning of school mathematics. Creating this bridge is essential for the students with special rights to enhance the potential of a quality mathematics education.

Recognising Mathematical Potential

A key issue for the global community and for teachers of mathematics is to recognise all students' potential and ensure that all students access opportunities to achieve this potential. This concern is highlighted in the United Nations Convention on the Rights of Persons with Disabilities (United Nations 2006). We argue that it is important to include mathematical potential when considering students' potential overall. For example, historically, students with Down syndrome and many other intellectual disabilities were expected to have little mathematics learning potential. This has been proven to be untrue. Indeed, Faragher et al. (2008) found that students with Down syndrome had significant mathematical knowledge, but noted that standard assessment procedures often failed to demonstrate this. This reported study used a task-based semi-structured assessment interview that was administered by a teacher experienced with both mathematics and students with Down syndrome, and the study's results were successful in demonstrating that the students had significant mathematical knowledge. The real issue was having a suitable assessment instrument and process for accessing their knowledge. Similarly, students who were blind or deaf were once thought to have little mathematics learning potential. The key to changing these beliefs has been, on one hand, individuals who have learnt mathematics successfully despite expectations to the contrary, and on the other hand, parents and teachers who believed that it was possible for students to learn, when provided with the appropriate opportunities.

There are many theories for explaining disability and how adherents of each theory might view individuals and their participation in education (Evans 2008). Some illustrative examples follow. The first theory is *the medical model* of disability that dates back to the eighteenth century but is still prevalent today. This theory considers disabilities as medical conditions to be treated, often in medical institutions, and students with disabilities as invalids. The focus is on what a person cannot do and people with disabilities are expected to accept and adjust to their conditions, with doctors and other medical professionals responsible for determining how the individual will live his or her life, rather than individuals with disabilities themselves (Evans 2008). Access to education is of little concern in this model. An alternate theory is *the social construction model* that is based on the idea that society creates disability by considering some forms of being and doing as normal and correct and others as dysfunctional and abnormal. In this model, the basis of disability is a biased and excluding environment rather than an impaired individual (Evans 2008). Thus, in an education context, students with disabilities have access to education, and it is the environment that needs to be changed rather than the individual. Proponents of this model work to ensure that school environments are barrier-free and welcoming to all, and this perspective has led to the development of Universal Design principles (United Nations 2006), both in architecture and instruction (Evans 2008). *The social justice theory* takes both the individual and the environment into consideration. This model emphasises the role played by privilege and oppression in determining the experiences of individuals with disabilities, and aims to reinterpret normality so that physical, mental, and sensory differences are viewed as normal, and respect for the human dignity and self-authorship of all students is paramount (Evans 2008). Proponents of this model advocate for all students to have equal access to a quality education at their local school that is based on Universal Design principles, with all students being welcomed, and viewed as responsible for their own decisions, and worthy of respect and consideration.

It is the social justice theory for explaining disability that underpins the perspectives considered in this chapter, and that highlights the importance of access to quality mathematics programs and instruction for all. However, a significant issue in creating educational environments in which all students may thrive is that classroom teachers may require additional knowledge, skills, time, and resources to provide optimal opportunities for students with special rights to learn mathematics and reach their potential. This situation needs to be addressed, and research and development undertaken to provide guidance for teachers and the community about how to provide all students with access to quality mathematics programs and instruction.

Quality Mathematics Education for Students with Special Rights

In many countries such as Australia, Brazil, Canada, Denmark, Japan, Singapore, and the United States, there are examples of school environments that focus on assisting students who have special rights in mathematics education. This section examines the role of assessment in guiding quality instruction and the features of programs that (1) give promising results and (2) develop and maintain students' creativity and optimistic attitudes towards learning mathematics.

Assessment

Doig et al. (2003) argue that once students who underperform in mathematics are identified, there should be an intensive teaching phase during which interpretation and diagnoses are the basis for appropriate action, or else the identification is of little benefit (p. 15). In contrast, a study by Milton (2000) found that although most Australian schools assessed students' mathematics knowledge, a survey of 377 teachers in primary schools indicated that only 14% of them had programs to support students who were underperforming. The challenge remains for communities and nations to respond by providing appropriate experiences and effective instruction to enhance the mathematics learning of all students.

An essential preliminary step to designing quality mathematics programs and instruction is the teacher knowing each student's current knowledge and skills, as well as understanding the likely developmental trajectory of mathematics learning (Bobis and Gould 1999; Clarke et al. 2002; Cobb and McClain 1999). We argue that also important is the teacher knowing students' current interests and motivations. In Australia, task-based one-on-one assessment interviews are widely used by classroom teachers, with responses related to student mathematics growth points along learning trajectories (Clarke et al. 2002; Gervasoni 2004; Bobis et al. 2005), However, in order to effectively assess students, teachers may also need to know how to successfully motivate or communicate with each student, access and use tools to enhance communication, and understand diverse developmental trajectories. For example, an unexplored question until recently was whether students with Down syndrome have the same developmental pathway, but take longer, than most children, or have a different learning trajectory (Faragher et al. 2008). To further inform teaching, there is an obvious need for further mathematics education research into the learning pathways of students with special rights.

Quality Instruction

It is possible to draw from previous studies the promising features of instruction that enable students with special rights to thrive mathematically. A common theme in the literature is the need for instruction and experiences to closely match students' individual learning needs (Ginsburg 1997; Greaves 2000; Wright et al. 2000; Rivera 1997) due to the great diversity of knowledge and backgrounds among and within student subgroups (Shonkoff and Phillips 2000). This highlights the importance of teachers having access to quality assessment instruments and procedures.

Carnine (1997) identified several components of effective instructional design for students with mathematics learning difficulties. These include accommodating differences in prior knowledge, providing scaffolded transition to independent learning, emphasising the 'big', or central mathematical ideas, and using an instructional design that emphasises retention of knowledge.

Another feature of effective instruction for students who do not thrive mathematically is providing rich, challenging programs that promote 'hard thinking' (Means et al. 1991; Resnick et al. 1991; Thornton et al. 1997; Wright et al. 2000). This strategy was one of four themes emerging from mathematics studies involving students with learning difficulties identified by Thornton et al. (1997) and relevant for planning effective high-quality mathematics instruction. Other themes were: (a) providing a broad and balanced mathematics curriculum; (b) accommodating the diverse ways in which students learn; and (c) encouraging students to discuss and justify their problem-solving strategies and solutions. These themes are echoed by others (Clarke 2001; Gervasoni 2004; Griffin and Case 1997; Madden et al. 1999;

Peterson et al. 1991; Resnick et al. 1991; Wright et al. 2000). In summarising research in this area, Gervasoni (2004) found that the following instructional practices are important for enhancing mathematics learning for students who underperform in mathematics education:

- Targeting instruction within each student's zone of proximal development based on current assessment of students' mathematical understandings and the probable course of the child's learning. This assumes that the teacher has the adequate tools and skills to communicate with the student;
- Making adjustments to planned activities on the basis of student responses;
- Within the zone of proximal development and in an area with which the child is learning, presenting rich, challenging problems that promote 'hard thinking'; providing hints to assist the problem solving process, ranging from general metacognitive hints to those specific to the mathematical demands of the task; continually presenting problems to the student of a similar nature, providing as much help as necessary, until the student is able to solve the problems independently;
- Explicitly focusing on 'big', or central mathematical concepts;
- Encouraging students to discuss and justify their problem-solving strategies and solutions;
- Emphasising the retention of knowledge;
- Using peer-mediated instruction with teachers providing guidance, prompts and feedback;
- Encouraging parents to provide encouragement based on knowledge of students' current progress.

Considering these approaches is important when providing quality mathematics programs for students with special rights. These approaches now form a framework for examining the effectiveness of the mathematics programs explored in this section.

Communication Tools

Educational settings for children who are visually or hearing impaired vary around the world. In some countries, these children attend special schools (e.g., Slovakia; Kohanova 2008), and in others they attend local schools. Some countries have specialist services available in all schools to assist with access to quality programs, whilst in other countries only the classroom teacher is available to assist students to learn. In some countries (e.g., Slovakia), the only opportunity secondary students have for learning mathematics is in a local school with no access to specialist support. In such cases, most teachers learn to teach children by trial and error (Kohanova 2008).

Mathematics teaching and learning rely typically on visual and written images presented in textbooks, and on verbal explanations that usually include pointing and gesturing. Such visual cues are not available to students who are visually impaired.

Thus, these students lack access to these image/gesture-reliant explanations. Another issue relates to communication tools that enable students who are blind to record their thinking and problem-solving attempts. Whilst Braille and tactile pictures provide some access to mathematics, this becomes less successful as the complexity of mathematics topics and mathematics notation increases in both primary and secondary school. Further, classroom teachers do not generally understand Braille notation or the principles of Universal Instructional Design. Moreover, many complex mathematical expressions and geometrical constructions are difficult to explain, review and modify orally, and are more easily expressed and modified in notated form. While students may be able to use keyboards and computer software to notate their work, this is usually restricted to a linear form, which is not typical of complex mathematics expressions.

There are a number of computer software programs that assist students who are visually impaired to notate expressions. Most of these are written in English, which highlights another access issue for non-English speakers, and the necessity of developing versions in different languages. For example, a Slovak language version of LAMBDA (Linear Access to Mathematics for Braille Device and Audio-synthesis) has been developed and piloted (Kohanova 2008), but it is not yet established whether this will prove an effective tool for learning and teaching mathematics in schools and universities.

A key issue for students with special rights to mathematics education is that communication tools seem to focus on assisting students who are visually impaired to communicate in ways that others will understand, rather than focusing on the students and the talents they bring to mathematics learning situations. Emphasis seems to be on expecting students to adapt to typical learning environments, rather than quality learning environments being created for students based on their strengths. We propose that the latter is a fruitful area for future research and development.

Building and Using Whole of Community Capacity

Designing quality mathematics programs and instruction for children with special rights is highly complex and benefits from a whole of community approach that enables teachers to draw upon a range of expertise. Dalvang (2008) reports that, in Norway, when difficulties in learning cannot be handled adequately by the school or the parents, then Norway has a support system for special needs education. This system legally enforces the special rights of students for mathematics education. By law, parents have the right to have a report of the situation for their child investigated through psychological and medical examination, and an educational plan developed.

When the results are available the school normally organises an interdisciplinary meeting with the school psychologist, the special teacher, the school nurse, the class teacher, and parents. The student's situation is presented and decisions are made for the future. The participants provide differing information and experiences,

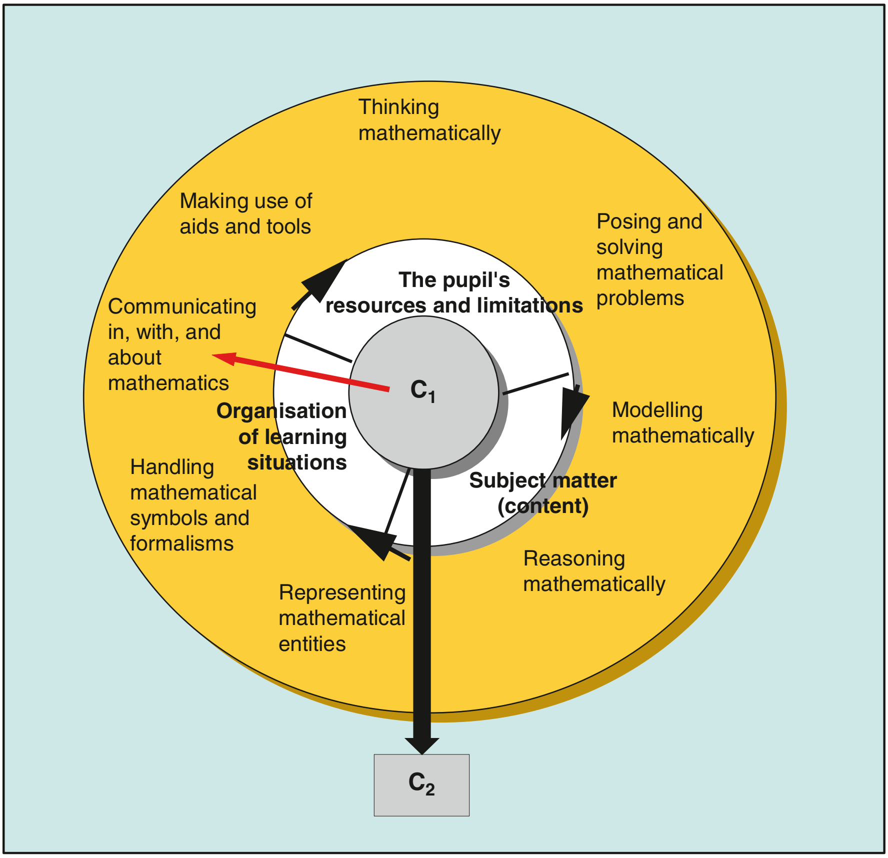

Fig. 22.1 Dalvang's compass model

which they share for the benefit of the student. However, Dalvang (2008) warns that these meetings sometimes focus on a student's deficits, rather than building on their strengths and the resources all can bring to assist the student to learn. To overcome this tendency, Dalvang presents a promising model to provide structure for the conversation and planning of a high-quality program. This Compass Model (Dalvang 2008) is presented in Fig. 22.1. Through moving the dial to different positions, with the student always in the centre, participants are able to use what they know about the child to guide the planning of quality instruction.

Overall, Dalvang concludes that the student's resources and limitations, the subject matter (content) and the organisation of learning environments jointly play an important role in offering understanding of difficulties, and in formulating a program of action. We believe that this is a promising approach for increasing access to high-quality mathematics education for students with special rights. However, many countries and regions are far from having the capacity to so implement this approach at present.

This model may be used for designing programs for students with special rights, including both the first and the second group. The next two paragraphs focus on early in schooling programs that aim to support children with impairments to thrive mathematically and reduce the number of students overall who underperform in mathematics.

Early in Schooling Mathematics Success Programs in Australia

There are several Australian early in schooling programs designed to assist six-year-old children who are not thriving mathematically in the context of the regular classroom. Examples include Extending Mathematical Understanding (Gervasoni 2004) and Maths Recovery (Wright et al. 2000) both, of which, are aligned closely to the features of quality assessment and instruction outlined earlier. An illustrative example follows.

The Extending Mathematical Understanding (EMU) program (Gervasoni 2004) is an early in schooling approach to identifying and assisting students who it appears are not achieving their potential in learning mathematics. The program, successful in increasing students' mathematical knowledge and confidence, is implemented by teachers who undertake a specialist teacher course designed to help them increase student access to quality instruction and curricula. The program comprises daily 30-minute sessions for up to 20 weeks, with specially trained teachers working with three students at a time. The program focuses on children in Grade 1 for early identification and intensive support aimed at accelerating mathematics learning (Gervasoni 2000), and on providing ongoing specialised assistance for underperforming students in Grades 2–6. The EMU program is not remedial in nature, but is built upon constructivist learning principles with students engaged in experiences requiring 'hard thinking'. The students are required to reflect upon their activity and articulate what they had learnt and how they had learnt. In addition, students complete home tasks that promote interaction with parents and caregivers.

The EMU Program focuses on whole number learning and provides varying learning experiences to those possible within the regular classroom setting. In particular, the specialist teachers are skilled in providing intensive instruction and feedback directed to the particular learning needs of each student. They are instructed to constantly focus children's attention on key mathematical ideas, assist children to develop language that facilitates communication about mathematics, and provide manipulatives to support students' mathematical thinking at critical moments in their learning. Overall, students develop the confidence and knowledge necessary to learn mathematics successfully in the regular classroom setting.

Early in Schooling Mathematics Success Programs in Denmark

Traditionally in Denmark, students who do not thrive in mathematics have not had access to early in schooling mathematics success programs. This changed in 2009 with the piloting of a small number of programs. One program is the Early Mathematics Frederiksberg Intervention for individual students in Grade 2 classes at the eight public primary/lower secondary schools at Frederiksberg. Frederiksberg is a municipality with 95,000 inhabitants on 8.7 km^2 in the Copenhagen area. The intervention approach aligns with other intervention programs that recommend early intervention in order to avoid negative self-concepts and attitudes to mathematics evolving (e.g., Gervasoni 2004; Wright et al. 2000; Dowker 2004). The curriculum focuses on whole numbers and calculations, and on geometry and part–whole concepts. The students for the program are identified by their class teacher who observes indications that the students are not achieving their potential in learning mathematics, and are then further ‘diagnosed’ and instructed one by one by one of the special trained intervention teachers, who are instructed to adapt instruction to the individual student’s learning styles and specific motivation.

The intervention approach is based on a new construct proposed in order to stimulate reflection among Danish mathematics teachers, parents, and students. The construct is named ‘mathematics-holes’ because of association with a metaphor of mathematics as a landscape with mountains and valleys and of teaching as guiding students as they build experiences in the mathematics-landscape. When mathematics is seen as a landscape it means that whenever students stop learning and feel stuck it is as if they ‘fall into a hole’. There are several ways for a teacher to cope with a student’s ‘fall’. First, a teacher can invite the student to move to another type of landscape, maybe far away from the hole in which the student was stuck; this means that even when students fail to thrive in one area of mathematics there are still many other mathematics landscapes to experience and learn. Second, teachers can help students ‘fill up’ the hole from beneath with mathematical building stones; or third, teachers can ‘lay out boards over the hole’ in order to let the student experience new and smart mathematical approaches (Lindenskov and Weng 2009). This is an optimistic way to approach programs for students with special rights.

Equitable Access to Quality Curriculum

At the ICME 10 in Copenhagen in 2004, Discussion Group 3 explored two delicate questions: *Mathematics for whom and why?* and *The balance ‘mathematics for all’ and ‘for high-level mathematics’?* (Lindenskov and Villavicencio 2008). This international discussion provided an international perspective on the nature of quality mathematics curricula for students with special rights. There were many similarities in the hopes and aims for mathematical education put forward by more than 100 participants. All agreed that everybody should receive mathematics education, and two arguments supported this view: (1) that learning mathematics develops necessary and relevant thinking tools for work, everyday life, and citizenship; and (2) that engaging in mathematics provides possibilities for enjoyment, creativity, and for personal development. A commitment to equity requires that students with special rights have access to mathematics for both these purposes.

The ICME 10 Discussion Group 3 concluded that a global and common emphasis should be put on three fundamental goals for mathematics instruction for all students:

1. Cultivating mathematical ability and curiosity of students, and not isolated skills and knowledge;
2. Providing students with experiences that put emphasis on mathematical problem solving and thinking abilities, meaning that reasoning and communication deserve priority status;
3. Providing students with experiences that give a broad perspective to the mathematics content structure and the relations among the various disciplines and core ideas, starting from a young age.

These intentionally broad and humanistic goals for mathematics education for all bring to the fore two fundamental problems. One is how to ensure mathematics for all? The other is how to interpret the goals on a local basis, taking into consideration the enormous diversity of communities and their resources across world? The discussion group placed emphasis on students with special needs, i.e. who are visually or intellectually impaired, as citizens who should be given much more attention by all of us (OECD 2004). This emphasis intensifies the question of the balance or dichotomy between 'mathematics for all' and 'for high-level mathematics'. However, it is argued that while everybody needs mathematical literacy, '*mathematics education for all*' should and could ensure, at the same time, the development of capabilities and high levels of performance for some learners by teaching with challenging situations accommodated to different students. This emphasises the importance of providing high-quality mathematics programs for students with special rights. The authors acknowledge that developing such opportunities puts heavy demands on teachers and governments, and raises a renewed global focus on the quality of teacher education.

Still the problem remains of how to locate the fundamental broad and humanistic goals (1–3) and quality instructional practices around the diversified world. One focus must be on all students achieving the mathematical literacy, as, for instance, described by OECD: 'An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen' (http://www.pisa.oecd.org/pisa/math.htm, July 2004).

Considering the world as a global village, where every system and every person depends more and more on other systems and persons, we agree that this OECD definition is valid for a world accelerating towards globalisation; and given that this global village would be unique in its diversity, the mathematical literacy must be the same for all. One argument for this is found in Durgunoglu and Öney (2000) who described the experience of women participating in adult education in Istanbul, Turkey. They noted that these women, like millions of adult literacy program participants all over the world need basic mathematics skills to participate effectively in society.

In actual fact, globally we are very far from this view of mathematics literacy as something that is the same for all. As a first step it might be appropriate for members of the mathematics education community to refer to a more local mathematics literacy that can be national or regional, according to the environment for which a person’s mathematics capabilities permit him/her to respond to the needs of his/her current and future life as a constructive, responsible, and reflective citizen in his/her country or region. Evidently, such necessities vary from one community to another, and from one epoch to another, because, for example, the social-economic and cultural reality of a European city requires quite different mathematical literacy to those who inhabit the Peruvian mountains; and the requirements of today’s corresponding populations are very different to those of fifty years ago and, indeed, in fifty years hence. From this point of view, mathematics literacy is relative; it depends on the demands of the persons’ social, economic, and cultural reality in a given environment and time, with an eye to the future. From a viewpoint of seeing mathematics education as a means to enhance intercultural understanding (Lindenskov 2003), however, mathematical literacy in a broader sense could be realised by providing students from, say, European cities with knowledge of the mathematics culture of, say, Peruvian peers living in rural areas and vice versa.

Advocating for Research and Development for Students with Special Rights

Although there is much willingness across the world for students with special rights to have equitable access to quality mathematics education, there is also an urgent need to provide guidance as to how this might occur. Further, providing the resources for access to quality education for all is an economic challenge for many nations.

Overall, we recommend that the international mathematics education community continue to advocate for all students to have equitable access to quality mathematics education, and for the community to have high though realistic expectations of all students’ potential to learn mathematics, given access to high-quality mathematics education. We also entreat mathematics educators to undertake a program of research and development focussed on providing equitable access to quality mathematics education for students with special rights. This includes the development of advice concerning:

- Whole community approaches that build capacity for providing access and quality, so that all work together for the benefit of students;
- High-quality teacher education that includes a focus on teaching students with special rights;
- Assessment instruments and approaches that enable teachers to identify students’ current mathematical knowledge in relation to learning trajectories, and customise instruction accordingly;

- Quality curricula that include a focus on both mathematics literacy and opportunities for high-level mathematics;
- Quality intervention programs that build confidence and knowledge;
- Quality instructional practices and learning materials;
- Communication tools and approaches that enable students to fully access quality mathematics education. These tools at best will honour and build upon students' communication abilities and strengths, as opposed to compensating for perceived communication deficits.

In summary, there is a great need for further research about what constitutes quality mathematics education for students with special rights. This can be achieved.

Conclusion

This chapter proposed that some students have special rights to mathematics education due to the fact that they have been excluded from accessing quality mathematics programs and learning environments. Whilst acknowledging that some nations struggle to achieve universal primary education for all, let alone universal access to quality mathematics education, we argue that a commitment to equitable access to quality mathematics education for all is an important goal and a special right for those who have been excluded. We therefore entreat the international mathematics community to advocate on behalf of students with special rights to ensure that all have access to quality mathematics education, and to undertake necessary research and development programs to enable the community to better understand the learning potential of students with special rights, and to better understand what constitutes quality mathematics education and how this may be accessed by all. For many reasons, some students 'fail to thrive' when learning school mathematics. The onus is on us all to respond by providing students with special rights with the type of mathematical opportunities they need to learn confidently and successfully…to thrive.

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References

Bobis, J., & Gould, P. (1999). The mathematical achievement of children in the Count Me In Too program. In J. M. Truran & K. M. Truran (Eds.), *Making the difference Proceedings of the 22nd annual conference of the Mathematics Research Group of Australasia* (pp. 84–90). Adelaide: MERGA.

Bobis, J., Clarke, B. A., Clarke, D., Thomas, G., Wright, R., & Young-Loveridge, J., et al. (2005). Supporting teachers in the development of young children’s mathematical thinking: three large scale cases. *Mathematics Education Research Journal, 16*(3), 27–57.

Carnine, D. (1997). Instructional design in mathematics for students with learning disabilities. *Journal of Learning Disabilities*, 30(2), 130–141.

Clarke, D. (2001). Understanding, assessing and developing young children’s mathematical thinking: Research as powerful tool for professional growth. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Proceedings of the 24th annual conference of the mathematics education research group of Australasia: Vol. 1. Numeracy and beyond* (pp. 9–26). Sydney: MERGA.

Clarke, D. M., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, M., Mcdonough, A., Montgomery P., Roche A., Sullivan P., Clarke B. A. & Rowley G. (2002). *Early numeracy research project final report.* Melbourne: Australian Catholic University, Mathematics Teaching and Learning Centre.

Cobb, P., & McClain, K. (1999). *Supporting teachers’ learning in social and institutional contexts.* Paper presented at the International Conference on Mathematics Teacher Education, Taipei, Taiwan.

Dalvang, T. (2008). *The compass model-a possible tool for dialogue, reasoning and understanding the situation in which learners experience difficulties in mathematics education.* http://tsg.icme11.org/tsg/show/8. Accessed 21 July 2009.

Doig, B., McCrae, B., & Rowe, K. (2003). *A good start to numeracy: Effective numeracy strategies from research and practice in early childhood.* Canberra: Commonwealth of Australia.

Dowker, A. (2004). *What works for children with mathematical difficulties.* Oxford: University of Oxford. (Department for education and skills. Research report RR 554).

Durgunoglu, A. Y., & Öney, B. (2000). Numeracy needs of adult literacy participants—Learners’ descriptions of their numeracy needs have a surprisingly strong emotional component. *Focus on Basics, 4*(B), 18–20. (National Center for the Study of Adult Learning and Literacy).

Evans, N. J. (2008). Theoretical foundations of universal instructional design. In J. L. Higbee & E. Goff (Eds.), *Pedagogy and student services for institutional transformation: implementing universal design in higher education* (pp. 11–24). Minneapolis: University of Minnesota. http://www.cehd.umn.edu/passit/docs/PASS-IT-Book.pdf. Accessed: 3 Dec 2009.

Faragher, R., Brady, J., Clarke, B., & Gervasoni, A. (2008). Children with Down syndrome learning mathematics: can they do it? Yes they can! *Australian Primary Mathematics Classroom, 13*(4), 10–15.

Feigenbaum, R. (2000). Algebra for students with learning disabilities. *The Mathematics Teacher, 93*(4), 270–276.

Gervasoni, A. (2000). Using growth-points profiles to identify Grade 1 students who are at risk of not learning school mathematics successfully. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000. Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 275–282). Fremantle: MERGA.

Gervasoni, A. (2004). *Extending mathematical understanding: intervention program specialist teacher manual.* Ballarat: Ballarat Heritage Services.

Gervasoni, A., & Sullivan, P. (2007). Assessing and teaching children who have difficulty learning arithmetic. *Educational & Child Psychology, 24*(*2*), 40–53.

Ginsburg, H. P. (1997). Mathematical learning disabilities: a view from developmental psychology. *Journal of Learning Disabilities, 30*(1), 20–33.

Greaves, D. (2000). Private provider services for students with learning difficulties. In W. Louden, L. K. S. Chan, J. Elkins, D. Greaves, H. House, M. Milton, S. Nichols, J. Rivalland, M. Rohl & C. Van Kraayenoord (Eds.), *Mapping the territory: Primary students with learning difficulties: Literacy and numeracy* (Vol. 1, pp. 135–156). Canberra: Commonwealth of Australia.

Griffin, S., & Case, R. (1997). Re-thinking the primary school math curriculum: an approach based on cognitive science. *Issues in Education, 3*(1), 1–49.

International Congress of Mathematics Education (2004). *Mathematics for whom and why? The balance between mathematics for all and for high level mathematics activity.* http://www.icme10.dk/. Accessed 16 Jan 2010.

International Congress of Mathematics Education (2008). *Activities and programs for students with special needs.* http://www.icme11.org/. Accessed 16 Jan 2010.

Kohanova, I. (2008). *The ways of teaching mathematics to visually impaired students*. http://tsg.icme11.org/document/get/716. Accessed 12 October 2010.

Lindenskov, L. (2003). Intercultural understanding. In M. von Groenestijn (Ed.), *Adults learning mathematics across borders* (pp. 23–30). Hertogenbosch: Cinop, Centrum voor innovatie van opleidingen.

Lindenskov, L. & Villavicencio, M. (2008). Mathematics education for whom and why? The balance between "mathematics education for all" and "for high level mathematical activity". In M. Niss (Ed.), *ICME-10 Proceedings: The 10th International Congress on Mathematical Education.* Roskilde: IMFUFA, Roskilde University.

Lindenskov, L., & Weng, P. (2008). Specialundervisning I matematik – kan det ikke være lige meget? In: S. Tetler & S. Langager (Eds.), *Specialunderisning i skolen* (pp. 211–230). Copenhagen: Gyldendal.

Lindenskov, L., & Weng, P. (2009). "Math-holes"—theory and empirical data—A pilot study in school year 1–3. In K. Linnanmäki (Ed.), *The 4th Nordic Research Conference on Special Needs Education in Mathematics: Different Learners—Different Math?* (pp. 51–70). Vaasa: Faculty of Education, Åbo Akademi University.

Maccini, P., & Gagnon, J. C. (2002). Perceptions and application of NCTM standards by special and general education teachers. *Exceptional Children, 68*(3), 325–344.

Madden, N., Slavin, R., & Simons, K. (1999). *Effects of math wings on student mathematics performance.* Center for Research on the Education of Students Placed at Risk, Johns Hopkins University. http://www.successforall.net/effectsofmath.html. Accessed 4 March 1999.

Means, B., Chelemer, C., & Knapp, M. (1991). *Teaching advanced skills to at-risk students.* San Francisco: Jossey-Bass.

Milton, M. (2000). Numeracy. In W. Louden, L. K. S. Chan, J. Elkins, D. Greaves, H. House, M. Milton, S. Nichols, J. Rivalland, M. Rohl & C. Van Kraayenoord (Eds.), *Mapping the territory—Primary students with learning difficulties: Literacy and numeracy* (Vol. 1, pp. 109–134). Canberra: Commonweath of Australia.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston: Author.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics.* Reston: Author.

National Council of Teachers of Mathematics (1995). *Assessment standards for school mathematics.* Reston: Author.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston: Author.

OECD (2004). *Learning for tomorrow's world—first results from PISA 2003.* Paris: OECD Publishing.

Ochiltree, G., & Moore, T. (2001). *Best start for children: The evidence base underlying investment in the early years (children 0–8 years).* Retrieved 15/8/03, 2003, from http://www.beststart.vic.gov.au.

Peterson, P., Fennema, E., & Carpenter, T. (1991). Using children's mathematical knowledge. In B. Means, C. Chelemer, & M. Knapp (Eds.), *Teaching advanced skills to at risk students* (pp. 68–101). San Francisco: Jossey-Bass.

Resnick, L., Bill, V., Lesgold, S., & Leer, M. (1991). Thinking in arithmetic class. In B. Means, C. Chelemer, & M. Knapp (Eds.), *Teaching advanced skills to at risk students* (pp. 27–53). San Francisco: Jossey-Bass.

Rivera, D. P. (1997). Mathematics education and students with learning disabilities: Introduction to the special series. *Journal of Learning Disabilities, 30*(1), 2–19.

Shonkoff, J., & Phillips, D. (2000). *From neurons to neighborhoods: The science of early childhood development.* Washington: National Academy Press.

Thornton, C., Langrall, C., & Jones, G. (1997). Mathematics instruction for elementary students with learning disabilities. *Journal of Learning Disabilities, 30*(2), 142–150.

United Nations (2006). Convention on the rights of persons with disabilities and optional protocol. http://www.un.org/disabilities/documents/convention/convoptprot-e.pdf. Accessed 3 Jan 2009.

Wright, R., Martland, J., & Stafford, A. (2000). *Early numeracy: Assessment for teaching and intervention.* London: Paul Chapman Publishing.

Chapter 23
Females in Mathematics: Still on the Road to Parity

Lynda R. Wiest

Females have gained ground in mathematics in some countries in recent years. In the United States, for example, high school females compare favorably with males in course grades and amount of mathematics coursework completed (National Center for Education Statistics 2007). However, nonspecific rallying cries of the popular press—and, at times, the mathematics education community—that females are now on par with males in mathematics are troublesome in that they threaten continued, needed support for females in mathematics. Many scholars have found initial or widening gender differences in achievement and, especially, less favorable dispositions in females appearing by at least the middle grades (Andreescu et al. 2008; Blue and Gann 2008; Ginsburg et al. 2005; Halpern et al. 2007a; Lawhead et al. 2005; Ma and Cartwright 2003; Penner and Paret 2008; Stevens et al. 2007). Andreescu et al. (2008) state, "It is during the middle school years, an age when children begin to feel pressure to conform to peer and societal expectations, that they start to lose interest and fall behind in most, but not all countries" (p. 1257).

In this chapter, I provide evidence that many areas of concern and inequity remain for females in mathematics, warranting continued and concerted support from the field of education. Although most data are drawn from U.S. sources, research from a variety of other countries is included to help illuminate this worldwide issue. After sharing brief background information about the current (mis) perceptions of gender equality in mathematics, I compare male and female mathematics achievement, course completion, career paths, and mathematics-related dispositions, discuss the role teachers, parents, and society play in relation to females in mathematics, and suggest strategies for a quality mathematics education for females.

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Perceptions of Gender Equality in Mathematics

Recent research findings in the United States indicating that girls now perform as well as boys on standardized tests in mathematics have garnered much attention. Based on their analysis of state achievement data from ten states, Hyde et al. (2008) conclude: “The general population no longer shows a gender difference in math skills” (p. 495). The study spawned popular press articles that announced such headlines as “In Math, Girls and Boys Are Equal” (Seattle Times News Services 2008). These claims have generated much talk of gender equality in mathematics and even female advantage. In her equity address at the 2008 National Council of Teachers of Mathematics (NCTM) annual meeting, Carol Malloy commented: “Our girls are now doing as well, if not better, than boys in our schools [in mathematics]” (Malloy 2008). In a recent newsletter piece, several members of the U.S. organization Women and Mathematics Education expressed a similar sentiment, one claiming that girls now “do as well or better than boys in middle school math” (Carr 2007, p. 5). *Las Vegas CityLife* reporter Jason Whited said he had “talked to some researchers who say that parity between girls and boys almost exists in math, so it’s time to focus on boys[’] deficiencies [presumably in reading and writing]” (March 3, 2008 email). Likewise, reporter Troy Reinhardt of the Northern Nevada magazine *Family Pulse*, emailed me (March 21, 2008) that one academic informant had told him: “Data prove girls are ahead of boys in almost every department and indicator.” These examples demonstrate perspectives I have experienced in increasing number from individuals both within and outside of the mathematics education community at local through national levels.

It is encouraging that females have made some important strides in mathematics in relation to males in such areas as high school mathematics grades and coursework. Nevertheless, the mantra of “sameness” that has stemmed from these gains poses potential harm to females’ continued progress in mathematics by threatening policy, research, and education efforts to support girls’ continued needs. Concerns persist for females in relation to mathematics performance (e.g., standardized test scores), participation (e.g., career choices), and dispositions (e.g., attitudes and beliefs). A more comprehensive and refined picture is in order to examine group tendencies related to females in mathematics.

Mathematics Achievement

Despite Hyde et al.’s (2008) conclusion of gender equality in standardized test data, females consistently score below males on the mathematics-based sections of important standardized national and international tests, namely, the ACT, SAT (SAT Reasoning Test), GRE (Graduate Record Exam), NAEP (National Assessment of Educational Progress), TIMSS (Trends in International Mathematics and Science Study), and PISA (Programme for International Student Assessment) (Educational Testing

Service 2007; Institute of Education Sciences 2004, 2009; Liu et al. 2008; McGraw et al. 2006; National Center for Education Statistics 2009; Organisation for Economic Co-operation and Development 2007). For example, boys attained 33 points higher than girls on the mathematics portion of the SAT (533 versus 500 of 800 possible points), a gap that has remained fairly stable for some time (Institute of Education Sciences 2009), and they significantly outperformed girls in 35 of 57 countries on the PISA (Organisation for Economic Co-operation and Development 2007). Some of these tests (specifically, the ACT, SAT, and GRE) are considered high-stakes assessments because they can affect decisions about college admissions, including entrance into top-tier universities, as well as scholarships or other awards (Dwyer 2007; Liu et al. 2008; Schmidt 2008). According to Liu et al. (2008), even small differences can have large practical effects, for example, on girls' dispositions (discussed later).

In terms of specific mathematics topics, one prominent achievement area where males outperform females is geometry and visuospatial skills (Halpern et al. 2007a; Liu et al. 2008; McGraw et al. 2006; Newcombe 2007). Halpern et al. (2007a) state:

> Linkage of mathematical and visuospatial skill has important consequences, because high levels of both of these skills are required for careers in fields, such as physics and engineering, in which women are typically underrepresented…. These two variables appear to be more strongly linked in females than males, suggesting that females may be particularly hampered in mathematical domains if they have reduced visuospatial skill. (p. 9)

Thus, individual mathematics topics must be considered in addition to global achievement levels in examining gender differences in mathematics performance.

Gender gaps favoring males are particularly pronounced at the highest achievement levels, including not only standardized test scores but also participation and performance in national/international mathematics competitions and identification among the profoundly mathematically gifted (Andreescu et al. 2008; Halpern et al. 2007b; McGraw et al. 2006; Preckel et al. 2008). For example, only 123 of 1782 participants (6.9%) in 27 top-ranked countries who participated in the precollegiate International Mathematical Olympiad from 1998 to 2008 were female (Andreescu et al. 2008). Figures range from 0% female in Iran, Japan, and Poland to 24% in Serbia/Montenegro, the next highest percent being 15% in Slovakia. Females reach double-digit percents in only 7 of the 27 countries. Andreescu et al. (2008) conclude that there is an "extreme scarcity of females who excel at the highest level in mathematics" (p. 1256). Although not all score differences are statistically significant, they predominantly favor males. Moreover, in some cases this "excellence gap" is growing. The proportion of males who scored at the advanced level of NAEP in both grades 4 and 8, for instance, increased more than that of females from 1996 to 2007, widening the existing gender gap (Plucker et al. 2010).

U.S. females perform well in high school mathematics coursework (National Center for Education Statistics 2007). Both male and female students had higher combined mathematics and science grade point averages (GPAs) in 2005 than in 1990, but the gap that consistently favored females increased from 0.12 to 0.20 points during that time period. (Females' GPAs increased from 2.42 to 2.76 and males' GPAs from 2.30 to 2.56 on a scale of 4.0.) However, an important consideration is the contention that grades can mask true achievement, given that grades may be inflated or diminished by credit given for effort (e.g., homework completion, class attendance) and good behavior. Indeed, research shows a strong positive correlation between effortful/dutiful behavior, such as class attendance, amount of study time, and homework completion, and course grades (e.g., Broucek and Bass 2008; Sarawit 2005). Although these behaviors may yield increased learning, class attendance in and of itself sometimes earns credit that is figured into course grades. Girls have been shown to be more self-disciplined and more motivated to succeed; they thus exert greater and qualitatively different effort by attending class, paying attention, studying, and completing homework more frequently than boys (e.g., Duckworth and Seligman 2006; Institute of Education Sciences 2007; Kenney-Benson et al. 2006; Siebert et al. 2006). Girls' higher grades, then, may to some degree reflect appropriate student behavior rather than pure mathematics achievement.

Course Completion and Career Paths

High school coursework completed appears to associate with future career paths (Ayalon 2003; Ma and Johnson 2008). Overall, U.S. females do well in this arena. In 2005 they earned 0.2 more mathematics and science credits (24 hours of classroom instruction) than males compared with 0.1 fewer credits in 1990, despite the fact that this figure has risen steadily for both sexes (National Center for Education Statistics 2007). Girls and boys take similar course sequences through precalculus (e.g., Bozick and Ingels 2008). However, boys take more calculus, AP (advanced placement) calculus, and computer science coursework and more AP calculus and computer science exams than females (College Board 2008; Halpern et al. 2007a; Institute of Education Sciences 2009). This is a crucial distinction. Ma and Johnson (2008) point out that grade 12 girls who complete high school calculus are *3.16 times more likely* to major in science as girls who do not. (There is no comparable effect for boys.) They explain: "Apparently, completing the most difficult, most advanced, and most rigorous course in school mathematics can promote females to think boldly about prestigious careers…. Calculus is a powerful career filter that critically screens females for prestigious occupations" (p. 75).

Even where mathematics achievement compares favorably by gender, such as high school grades attained and amount of coursework completed, as noted earlier, females' participation in many areas of mathematics, such as mathematics clubs/contests, college majors, and careers, remains low in relation to males (Boaler and Sengupta-Irving 2006; Grevholm 2007; Mendick 2006). Mendick (2006) notes: "In stark contrast to these shifting patterns of results [narrowing gender gaps in mathematics exam scores and grades in England], the choice to study maths once it becomes optional remains highly gendered" (p. 7). Grevholm (2007) describes the situation in Sweden similarly. Likewise, Boaler and Sengupta-Irving (2006) express concern about the lack of continuity between girls' earlier mathematics performance and their later mathematics-oriented choices:

> Girls are opting out of mathematics *despite* their advanced performance in secondary school. The low participation of girls and women at high levels of mathematics and related fields is an important issue, and one that probably begins in school…and becomes more accentuated as levels increase. (p. 210)

Hyde et al.'s (2008) report of gender equality in mathematics stated that 48% of females earn bachelor's degrees in mathematics. This figure, which was several years old at the time of publication, has since declined (see below). Moreover, the authors failed to compare that percent to the proportion of females who earn bachelor's degrees across all majors. They also neglected to discuss trends across advancing degrees. Table 23.1 shows degrees earned by females in the United States in 2006–07. The proportions of women who earned mathematics and statistics degrees fall well below that of females earning those same degrees across all majors. A chi-square test shows that bachelor's degrees earned in mathematics and statistics by gender, for example, differ significantly from expected outcomes based on overall degree figures for all majors, $\chi^2(1, N=14{,}954)=1079.17$, $p<001$. (No other analyses were performed.) The data also tend to reflect even lower proportions of females at higher degree levels for mathematics and statistics.

In 2008, Computer and Mathematical Science occupations ranked third in mean annual wages ($74,500) out of the U.S. Bureau of Labor Statistics' (2009a) 22 major occupational groups. Women comprise only 25% of these workers (U.S. Bureau of Labor Statistics 2009b), indicating that even women who do get mathematics and computer science degrees are translating this preparation into directly related careers in much lower proportions than males. In terms of higher education positions, just 23% of U.S. mathematical scientists with doctorates who are employed in universities and four-year colleges are women, and only 36% of these are tenured, compared with a 62% tenure rate for their male peers (National Science Foundation 2009). Moreover, men outnumber women as both students and faculty at the most competitive, prestigious institutions (Dwyer 2007).

Mathematics-Related Dispositions

A vitally important area to consider for understanding the status of females in mathematics as a foundation for providing them with a quality mathematics education is that of dispositions. Females display more negative affect toward mathematics

Table 23.1 Percent of degrees conferred to females in the United States, 2006–2007. (Source: Institute of Education Sciences (2009))

	Associate's degree (%)	Bachelor's degree (%)	Master's degree (%)	Doctor's degree (%)
Major				
All majors	62.2	57.4	60.6	50.1
Mathematics and statistics	33.7	44.1	41.5	29.8

than males, including poor attitudes in general, anxiety, weak self-concept and self-confidence, lower interest and motivation, less enjoyment and pride, and greater hopelessness and shame (Frenzel et al. 2007; Ginsburg et al. 2005; Halpern et al. 2007a; Ma and Cartwright 2003; McGraw et al. 2006; Preckel et al. 2008). They judge their competence and performance more harshly than males (Chatard et al. 2007; Frenzel et al. 2007; Halpern et al. 2007a; Lloyd et al. 2005), are less likely to attribute high achievement to ability (Dickhäuser and Meyer 2006; Georgiou et al. 2007), and see less value in mathematics (Frenzel et al. 2007). Females tend to perceive mathematics ability as natural rather than developed, a potentially harmful belief (Dweck 2007) that needs to be countered by teaching girls that mathematics ability can be improved (Halpern et al. 2007a).

Dispositions have been found to associate with mathematics performance and participation (Antunes and Fontaine 2007; Crombie et al. 2005; Meelissen and Luyten 2008; Watt et al. 2006). Therefore, some researchers contend that improving girls' attitudes and beliefs will improve their performance and participation in mathematics (Halpern et al. 2007a; Ma and Johnson 2008). Ma and Johnson (2008) say: "Fostering a positive attitude toward mathematics could hold the key to retaining both females and males in advanced mathematics coursework and eventually attract them to the STEM [science, technology, engineering, and mathematics] fields" (p. 77). However, even if girls feel efficacious toward mathematics, they need to believe that mathematics-related careers are both appropriate for and available to them; this and their interest level are important factors in decisions to pursue such occupations (Stevens et al. 2007).

Achievement may in turn influence mathematics dispositions, resulting in a bi-directional relationship (Georgiou et al. 2007; Ma and Johnson 2008). For example, boys' higher mathematics scores may negatively influence girls' dispositions, making girls less likely to enter mathematics careers, whereas greater success might cause girls to engage in mathematics to a greater degree (Liu et al. 2008). However, high test scores and grades are not in themselves enough to benefit women (Antunes and Fontaine 2007; Stromquist 2007). Antunes and Fontaine (2007) explain:

> Good marks [grades] are not enough to sustain girls' maths self concept at the same level as that of boys. Girls have to deal with less favourable stereotypes than boys and need to deal with teachers' practices, which do not support their self-concept. (p. 86)

Thus, relationships among gender, dispositions, and mathematics are important and complex.

Role of Teachers, Parents, and Society

Evidence suggests that females receive less STEM support from salient individuals both in and out of school. In school, both teachers and peers have a strong impact on girls' experiences in the mathematics classroom. Teachers have a heavy responsibility for the way teaching and learning experiences play out. Fredua-Kwarteng

(2005) asserts: “Mathematics teachers are the primary architects of the culture in mathematics classrooms” (p. 8) and thus, teacher leadership is required “to address gender disparity in mathematics learning outcomes” (p. 15). Unfortunately, teachers’ attitudes are one area of concern. In general, teachers hold lower expectations for girls’ performance, give girls less encouragement and sometimes even discourage them, interact more with boys, and ask boys higher-level questions (Asimeng-Boahene 2006; Fredua-Kwarteng 2005; Jones and Dindia 2004). They tend not to accept different ways of learning, which is detrimental to females (Lim 2004). As early as the first and second grades, female teachers’ own mathematics anxiety negatively influences female students’ attitudes and achievement, a serious concern given that most elementary teachers in the United States are female (Beilock et al. 2010). At the doctoral level, women in mathematics have described limited or negative relationships with the predominantly male faculty, feelings of invisibility, a lack of mentoring, advising, and other guidance, feelings of awkwardness and not fitting into this male-dominated field, and a general lack of moral support and encouragement (Herzig 2004). Young adolescent girls, too, have portrayed themselves as invisible or side characters in male-centered classrooms, and they experience harassment from male peers, more passive roles in small-group activities, and use of gender-biased instructional materials (Asimeng-Boahene 2006; Lim 2004).

Parents are significantly less likely to give activity-related STEM materials to daughters and to encourage daughters to participate in out-of-school STEM activities compared with sons (Jacobs and Bleeker 2004; Simpkins et al. 2005). This is important because STEM-related parent expectations, behaviors, and involvement have been shown to influence student performance, participation, and attitudes (Jacobs and Bleeker 2004; Simpkins et al. 2005; Yan and Lin 2005). Mentoring is another prominent area for females in the academic arena. A MentorNet (2008) study of 2,500 higher education STEM students showed that although females are significantly more likely than males to report the importance of mentoring for successful degree completion, they are more likely than males to indicate lack of support in all three factor categories: role modeling, academic/career, and, especially, psychosocial.

Society in general continues to fuel mathematics distaste irrespective of gender, but it targets females more than males. Female-unfriendly STEM messages abound through oral tradition and media transmission and seem to go unnoticed or at least unchallenged by many laypersons and professionals. A quick Web search for the unthinkable message “I’m too pretty to do math” finds many vendors marketing the logo to females on commercial merchandise, such as t-shirts and magnets. In the popular American film *Mean Girls* (Messick et al. 2004), actor Lindsay Lohan’s character is told more than once that it would be “social suicide” for her to join Mathletes, a mathematics competition team. In class, she pretends not to understand mathematics in order to impress a particular boy. (The later undercurrent that this is inappropriate is overshadowed by the fact that this “normal” social scene reflects dominant youth culture.) Steinke’s (2005) research findings on film portrayal of women in science-oriented careers indicate some improvement over past images but that harmful stereotypes persist. Morge’s (2008) recent research showed that college students considered the popular media to influence their beliefs about mathematics and that the media presented successful males more often than females. Thus, female-unfavorable societal images in mathematics-oriented disciplines continue to pervade U.S. culture.

Most experts argue convincingly that gender differences in mathematics are predominantly, if not wholly, culturally driven (e.g., Boaler and Sengupta-Irving 2006). The fact that females show different achievement, as well as course and career choices, within and across cultures and even across community types supports this contention (cf. Andreescu et al. 2008; Guiso et al. 2008; Halpern et al. 2007a; Li 2007; Marks 2008). Based on their study of gender differences in PISA test performance across countries, Guiso et al. (2008) conclude: "Girls' underperformance in math relative to boys is eliminated in more gender-equal cultures" (p. 1165). Another fact supporting culturally driven difference is females' ability to respond favorably to intervention measures rather than being "biologically captive" to perform at a predetermined level (e.g., Wiest 2010). Similarly, Stromquist (2007) points out: "The fact that girls' progress in mathematics has been improving over time…suggests that math ability is not innate but susceptible to social influences and instruction" (p. 37). Further, differences do not appear to exist from birth but rather to manifest themselves at a pivotal time in social development, again supporting sociocultural rather than biological influences. A great deal of recent research also emphasizes gender stereotypes (e.g., "stereotype threat") as another key detrimental social factor working against females in mathematics (e.g., Chatard et al. 2007; Steele et al. 2007). Thus, many contextual factors, including families, peers, cultural norms, teaching environments, and educational policy, craft a culture that provides a different experience for males and females in mathematics, one that has more negative consequences for females (e.g., Boaler and Sengupta-Irving 2006; Geist and King 2008; Halpern et al. 2007b; Marks 2008).

Quality Mathematics Education for Females

Understanding the current status of females in mathematics and related influential factors, as presented in this chapter, is an important backdrop to seeking quality and equity in mathematics education for girls and women. As noted earlier, evidence points to the middle grades as an important crossroad not only for developing appropriate knowledge, skills, and dispositions, but also for considering future course and career paths. It seems reasonable to argue that the seeds of gender differences in mathematics are sown earlier than when they first appear, making the precursor period an important intervention zone. Thus, efforts at supporting and encouraging girls in mathematics would be worthwhile in the elementary grades.

Data provided in this chapter point to a number of recommendations for improving mathematics education for females. I offer the following suggestions as selected strategies for elevating the status of females in mathematics. These approaches apply variably to school personnel (e.g., educators, counselors, and administrators), policy and test makers, parents, researchers, and other education stakeholders.

- *Provide, encourage, and support rigorous, high-quality curricular and supplementary experiences*. In particular, foster girls' geometry and visuospatial skills and encourage girls to take calculus. Develop or suggest additional experiences with clubs, contests, and out-of-school-time programs (e.g., summer, afterschool, weekend, online). Provide and encourage use of mathematics-oriented materials and activities and model their use.
- *Hold high expectations for all students' performance, participation, and dispositions*. Structure equitable learning that requires comparable school experiences and classroom participation for all students.
- *Foster positive dispositions toward mathematics (through discussion, modeling, etc.)*. This includes such areas as interest in the subject matter, awareness of the utilitarian value of mathematics in occupational and everyday life, and the productive role of effort with confidence in personal abilities to improve mathematics knowledge and skills.
- *Improve testing and use of test scores*. Develop quality assessments that emphasize important mathematics knowledge, skills, and reasoning. Teach students test-taking skills, and use tests as only one of varied measures for describing and making decisions about student performance. Consider factoring dutiful/effortful behavior modestly, if at all, into performance measures.
- *Provide networking and mentoring opportunities involving female peers and adult role models*. Posters, online environments, guest speakers, peer tutors, and other such mechanisms can provide positive modeling and support for females in mathematics.
- *Provide information on mathematics-oriented careers and preparation for those careers, as well as encouragement to consider these career options*. Further, promote gender equity in mathematics-related occupations, such as hiring, promotion, and retention practices that are favorable to both sexes.
- *Promote societal change that results in more positive portrayals of mathematics and females in mathematics*. This includes providing feedback to media sources and critically analyzing the media with young people.
- *Use a nuanced approach to researching and analyzing gender differences in mathematics*. Consider, for example, various types of performance, participation, and dispositions, as well as girls' potentially different experiences based on race/ethnicity, social class, and other identities.

Closing Comments

The need for sustaining focused attention to females in mathematics is evident. Policymakers, researchers, educators, parents, and others must continue to address this need. Although some studies indicate no gender differences in mathematics (e.g., Georgiou et al. 2007; Hyde et al. 2008), substantial data reveal disconcerting conditions for females in relation to males in most countries of the world. Even when small, these differences typically favor males, and the cumulative effect of the concerns detailed in this chapter, left unchecked, is potentially disastrous for women's personal lives and society at large. The U.S. House of Representatives formalized this national concern in June 2008 by passing a resolution that calls for recognizing, supporting, and increasing the number of women in the STEM fields (GovTrack.us 2008). At the personal level, mathematics preparation and participation can relate to life quality, including financial security. Factors involved in this relationship are women's low participation in the higher-paying positions afforded by mathematics-related careers and the fact that U.S. women earn 80% of the median weekly earnings of males for full-time work in general (U.S. Bureau of Labor Statistics 2009b), have a 5.2-year greater life expectancy than males (National Center for Health Statistics 2009), and are disproportionately represented among the impoverished (U.S. Department of Health and Human Services 2008).

The conversation about females in mathematics must extend beyond global achievement levels. It must encompass other and subtler areas of performance (e.g., specific mathematics topics), as well as participation (e.g., mathematics courses, degrees, and careers), dispositions, and quality of experience. For example, females who attain mathematics outcomes similar to males but suffer detrimental effects to their dispositions should remain on the radar screen for concerned researchers and educators. Semantic distinctions are important in claims that females "do as well" as males in mathematics. This language must be clarified as to whether it refers to test scores, school grades, dispositions, quality of experience, career choices, or other important indicators of the female condition in mathematics.

This call for continued attention to gender differences is not a look at difference for its own sake or an implication that females have an intellectual shortcoming in mathematics. Rather, it is to encourage a realistic stance that acknowledges the differences described in this chapter and that girls *can* succeed in mathematics on their own merit; however, social and cultural factors mediate females' performance, participation, and dispositions. Females require continued encouragement and support from professionals and the community at large in the area of mathematics. This support includes the types of suggestions made above for providing females with a quality mathematics education. It might also include educating females themselves about gender stereotyping in relation to mathematics (Steele et al. 2007). In addition to finding ways to support females within the existing climate, environmental factors that work against females' mathematics success should come under continual scrutiny—and pressure, where warranted—to function in more positive and productive ways.

To improve the status of females in mathematics, educators, parents, and others should employ the types of strategies suggested in this chapter. Gender issues in mathematics should be actively addressed in pre-service and in-service teacher education, and they must remain on the mathematics education research agenda. Boaler and Sengupta-Irving (2006) note: "It is curious and troubling to note that few researchers study or consider gender and mathematics as an academic field in the twenty-first century" (p. 207). They call for continued research to address lingering inequities. They suggest a focus on contextual factors while acknowledging girls' strengths and cognitive preferences. Attention to affect (e.g., attitudes and beliefs) should also be of concern to researchers and practitioners in efforts to make progress in the area of females and mathematics (cf. Frenzel et al. 2007). Liu et al. (2008) contend: "Gender gaps need to be studied before they can be closed. Especially, so far as there is no solid evidence that gender differences have been eliminated, nor can we justify that the existing difference [in mathematics] is indeed negligible" (p. 20). (See also Lubienski 2008.) Future research should focus on environmental influences (e.g., social, cultural, scholastic) without implication that differences relate to actual gender differences in mathematics ability (e.g., Boaler 2007). Such studies will require sophisticated analyses to "do justice" to examining the complex phenomena involved in gender issues in mathematics (Boaler and Sengupta-Irving 2006; Lubienski 2008), such as examining how other social identities (e.g., race/ethnicity and social class) intersect with gender. One research area made evident by the data presented in this chapter is that of investigating why girls' gains in performance, coursework taken, and early career intentions have not translated into a higher proportion of women in STEM fields. Clearly, the attrition rate needs nuanced explanation. High-quality research is vitally important to provide awareness and information that can help policymakers, education personnel, families, and society at large support girls on a positive trajectory in mathematics that can one day lead to gender parity.

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References

Andreescu, T., Gallian, J. A., Kane, J. M., & Mertz, J. E. (2008). Cross-cultural analysis of students with exceptional talent in mathematical problem solving. *Notices of the AMS, 55*(10), 1248–1260.

Antunes, C., & Fontaine, A. M. (2007). Gender differences in the causal relation between adolescents' maths self-concept and scholastic performance. *Psycologica Belgica, 47*(1–2), 71–94.

Asimeng-Boahene, L. (2006). Gender inequity in science and mathematics in Africa: The causes, consequences, and solutions. *Education, 126*(4), 711–728.

Ayalon, H. (2003). Women and men go to university: Mathematical background and gender differences in choice of field in higher education. *Sex Roles, 48*(5/6), 277–290.

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences of the United States of America* (PNAS), *107*(5), 1860–1863. http://www.pnas.org/content/107/5/1860.

Blue, J., & Gann, D. (2008). When do girls lose interest in math and science? *Science Scope, 32*(2), 44–47.

Boaler, J. (2007). Paying the price for "sugar and spice": Shifting the analytical lends in equity research. In N. S. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 24–36). New York: Teachers College Press.

Boaler, J., & Sengupta-Irving, T. (2006). Nature, neglect and nuance: Changing accounts of sex, gender and mathematics. In C. Skelton, B. Francis, & L. Smulyan (Eds.), *The SAGE handbook of gender and education* (pp. 207–220). Thousand Oaks, CA: SAGE.
Bozick, R., & Ingels, S. (2008). *Mathematics coursetaking and achievement at the end of high school: Evidence from the Education Longitudinal Study of 2002 (ELS:2002)* [NCES 2008-319]. Washington, DC: U.S. Department of Education, National Center for Education Statistics, Institute of Education Sciences. http://nces.ed.gov/pubs2008/2008319.pdf.
Broucek, W. G., & Bass, W. (2008). Attendance feedback in an academic setting: Preliminary results. *College Teaching Methods and Styles Journal, 4*(1), 45–48.
Carr, M. (Ed.). (2007, fall). *Women and mathematics education*. Newsletter.
Chatard, A., Guimond, S., & Selimbegovic, L. (2007). "How good are you in math?" The effect of gender stereotypes on students' recollection of their school marks. *Journal of Experimental Social Psychology, 43*(6), 1017–1024.
College Board. (2008). *The fourth annual report to the nation*. New York: Author. http://professionals.collegeboard.com/profdownload/ap-report-to-the-nation-2008.pdf.
Crombie, G., Sinclair, N., Silverthorn, N., Byrne, B. M., DuBois, D. L., & Trinneer, A. (2005). Predictors of young adolescents' math grades and course enrollment intentions: Gender similarities and differences. *Sex Roles, 52*(5/6), 351–367.
Dickhäuser, O., & Meyer, W.-U. (2006). Gender differences in young children's math ability attributions. *Psychology Science, 48*(1), 3–16.
Duckworth, A. L., & Seligman, M. E. P. (2006). Self-discipline gives girls the edge: Gender in self-discipline, grades, and achievement test scores. *Journal of Educational Psychology, 98*(1), 198–208.
Dweck, C. S. (2007). Is math a gift? Beliefs that put females at risk. In S. J. Ceci & W. M. Williams (Eds.), *Why aren't more women in science? Top researchers debate the evidence* (pp. 47–55). Washington, DC: American Psychological Association.
Dwyer, C. A. (2007). Overview: Gender equity from early childhood through postsecondary education. In S. S. Klein et al. (Eds.), *Handbook for achieving gender equity through education* (2nd ed., pp. 605–607). Mahwah, NJ: Erlbaum.
Educational Testing Service. (2007). *Graduate Record Examinations (GRE): Factors that can influence performance on the GRE General Test: 2005–2006*. Princeton, NJ: Author.
Fredua-Kwarteng, E. (2005). *A perspective on gender disparity in mathematics education*. (ERIC Document Reproduction Service No. ED 493 750). http://eric.gov:80/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/1b/f0/78.pdf. Accessed 26 June 2008.
Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Girls and mathematics—A "hopeless" issue? A control-value approach to gender differences in emotions towards mathematics. *European Journal of Psychology of Education, 22*(4), 497–514.
Geist, E. A., & King, M. (2008). Different, not better: Gender differences in mathematics learning and achievement. *Journal of Instructional Psychology, 35*(1), 43–52.
Georgiou, S. N., Stavrinides, P., & Kalavana, T. (2007). Is Victor better than Victoria at maths? *Educational Psychology in Practice, 23*(4), 329–342.
Ginsburg, A., Cooke, G., Leinwand, S., Noell, J., & Pollock, E. (2005). *Reassessing U.S. international mathematics performance: New findings from the 2003 TIMSS and PISA*. Washington, DC: American Institutes for Research.
GovTrack.us. (2008). *H. Res. 1180, 110th Congress: Resolution recognizing the efforts and contributions of outstanding women scientists,…* http://www.govtrack.us/congress/billtext.xpd?bill=hr110-1180. Accessed 31 Oct 2008.
Grevholm, B. (2007). Critical networking for women and mathematics: An intervention project in Sweden. *Philosophy of Mathematics Education Journal, no. 21*. http://people.exeter.ac.uk/PErnest/pome21/.
Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science, 320*(5880), 1164–1165. http://research.chicagogsb.edu/IGM/docs/ZingalesCulturGenderMath.pdf. Accessed 25 Nov 2008.

Halpern, D. F., Aronson, J., Reimer, N., Simpkins, S., Star, J. R., & Wentzel, K. (2007a). *Encouraging girls in math and science: IES Practice Guide*. Washington, DC: U.S. Department of Education, National Center for Education Research, Institute of Education Sciences.

Halpern, D. F., Benbow, C. P., Geary, D. C., Gur, R. C., Hyde, J. S., & Gernsbacher, M. A. (2007b). The science of sex differences in science and mathematics. *Psychological Science in the Public Interest, 8*(1), 1–51.

Herzig, A. H. (2004). "Slaughtering this beautiful math": Graduate women choosing and leaving mathematics. *Gender and Education, 16*(3), 379–395.

Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B., & Williams, C. C. (2008). Gender similarities characterize math performance. *Science, 321*(5888), 494–495.

Institute of Education Sciences. (2004). *Highlights from the Trends in International Mathematics and Science Study: TIMSS 2003* [Mathematics]. http://nces.ed.gov/pubs2005/timss03/math5.asp. Accessed 31 Oct 2008.

Institute of Education Sciences. (2007). *Student effort and educational progress: Student attitudes and aspirations: Time spent on homework* [Indicator 21]. http://nces.ed.gov/programs/coe/2007/section3/indicator21.asp. Accessed 25 Nov 2008.

Institute of Education Sciences. (2009). *Digest of education statistics 2008*. Washington, DC: U.S. Department of Education, National Center for Education Statistics. http://nces.ed.gov/programs/digest/d08/.

Jacobs, J. E., & Bleeker, M. M. (2004). Girls' and boys' developing interests in math and science: Do parents matter? *New Directions for Child and Adolescent Development, 106,* 5–21.

Jones, S. M., & Dindia, K. (2004). A meta-analytic perspective on sex equity in the classroom. *Review of Educational Research, 74*(4), 443–471.

Kenney-Benson, G. A., Pomerantz, E. M., Ryan, A. M., & Patrick, H. (2006). Sex differences in math performance: The role of children's approach to schoolwork. *Developmental Psychology, 42*(1), 11–26.

Lawhead, P., Loyd, R., Schep, M., Laws, M., & Price, K. (2005). Experiences in math, science and technology summer camps for young females. *ACM-SE 43: Proceedings of the 43rd annual Southeast regional conference* (Vol. 1). New York: Association for Computing Machinery.

Li, Q. (2007). Mathematics, science, and technology in secondary schools: Do gender and region make a difference? *Canadian Journal of Learning and Technology, 33*(1), 41–57.

Lim, J. H. (2004). Girls' experiences in learning school mathematics [online]. *Focus on Learning Problems in Mathematics, 26*(1).http://www.unlv.edu/RCML/win04.html. Accessed 30 Nov 2008.

Liu, O. L., Wilson, M., & Paek, I. (2008). A multidimensional Rasch analysis of gender differences in PISA mathematics. *Journal of Applied Measurement, 9*(1), 18–35.

Lloyd, J. E. V., Walsh, J., & Yailagh, M. S. (2005). Sex differences in performance attributions, self-efficacy, and achievement in mathematics: If I'm so smart, why don't I know it? *Canadian Journal of Education, 28*(3), 384–408.

Lubienski, S. T. (2008). On "gap gazing" in mathematics education: The need for gaps analyses. *Journal for Research in Mathematics Education, 39*(4), 350–356.

Ma, X., & Cartwright, F. (2003). A longitudinal analysis of gender differences in affective outcomes in mathematics during middle and high school. *School Effectiveness and School Improvement, 14*(4), 413–439.

Ma, X., & Johnson, W. (2008). Mathematics as the critical filter: Curricular effects on gendered career choices. In H. M. G. Watt & J. S. Eccles (Eds.), *Gender and occupational outcomes: Longitudinal assessment of individual, social, and cultural influences* (pp. 55–83). Washington, DC: American Psychological Association.

Malloy, C. E. (2008, April). *Iris M. Carl equity address.* Presentation at the National Council of Teachers of Mathematics' Annual Meeting and Exposition, Salt Lake City, UT.

Marks, G. N. (2008). Accounting for the gender gaps in student performance in reading and mathematics: Evidence from 31 countries. *Oxford Review of Education, 34*(1), 89–109.

McGraw, R., Lubienski, S. T., & Strutchens, M. E. (2006). A closer look at gender in NAEP mathematics achievement and affect data: Intersections with achievement, race/ethnicity, and socioeconomic status. *Journal for Research in Mathematics Education, 37*(2), 129–150.

Meelissen, M., & Luyten, H. (2008). The Dutch gender gap in mathematics: Small for achievement, substantial for beliefs and attitudes. *Studies in Educational Evaluation, 34*(2), 82–93.

Mendick, H. (2006). *Masculinities in mathematics*. New York: Open University Press.

MentorNet. (2008). *Students' perceptions of the value and need for mentors as they progress through academic studies in engineering and science*. San Jose, CA: Author. http://mentornet.net/studentperceptions/.

Messick, J., Michaels, L. (Producers), & Waters, M. S. (Director). (2004). *Mean girls* [DVD]. Paramount Home Entertainment.

Morge, S. P. (2008). Media influences on mathematics-related beliefs. *Academic Exchange Quarterly, 12*(4), 232–237.

National Center for Education Statistics (NCES). (2007). *America's high school graduates: Results from the 2005 NAEP high school transcript study* [NCES 2007467]. Washington, DC: U.S. Department of Education, NCES. http://nces.ed.gov/nationsreportcard/pdf/studies/2007467_4.pdf.

National Center for Education Statistics (NCES). (2009). *The nation's report card: Mathematics 2009*. Washington, DC: Institute of Education Sciences, U.S. Department of Education. http://nationsreportcard.gov/math_2009/. Accessed 12 April 2010.

National Center for Health Statistics. (2009). *Health, United States*, 2008. Hyattsville, MD: U.S. Department of Health and Human Services. http://www.cdc.gov/nchs/fastats/lifexpec.htm.

National Science Foundation. (2009). Women, minorities, and persons with disabilities in science and engineering. http://www.nsf.gov/statistics/wmpd/pdf/tabh-27.pdf. Accessed 31 Jan 2010.

Newcombe, N. S. (2007). Taking science seriously: Straight thinking about spatial sex differences. In S. J. Ceci & W. M. Williams (Eds.), *Why aren't more women in science? Top researchers debate the evidence* (pp. 69–77). Washington, DC: American Psychological Association.

Organisation for Economic Co-operation and Development. (2007). *PISA 2006: Science competencies for tomorrow's world* [executive summary]. Paris: Author.

Penner, A. M., & Paret, M. (2008). Gender differences in mathematics achievement: Exploring the early grades and the extremes. *Social Science Research, 37*(1), 239–253.

Plucker, J. A., Burroughs, N., & Song, R. (2010). *Mind the (other) gap! The growing excellence gap in k-12 education*. Bloomington, IN: Indiana University, Center for Evaluation and Education Policy. http://ceep.indiana.edu/mindthegap/. Accessed 12 Apr 2010.

Preckel, F., Goetz, T., Pekrun, R., & Kleine, M. (2008). Gender differences in gifted and average-ability students: Comparing girls' and boys' achievement, self-concept, interest, and motivation in mathematics. *Gifted Child Quarterly, 52*(2), 146–159.

Sarawit, M. (2005). *A study of the relationship between teamwork [sic: termwork] and achievement: A case study of the freshmen class at the Naresuan University International College*. http://www.nuic.nu.ac.th/QA/edoc%5Crsh_005.pdf. Accessed 24 Nov 2008.

Schmidt, P. (2008, March 25). Elite colleges' scramble to enroll high SAT scorers may undermine diversity. *The Chronicle of Higher Education*. http://chronicle.com/news/article/4188/elite-colleges-scramble-to-enroll-high-sat-scorers-may-undermine-diversity.

Seattle Times News Service. (2008, July 25). In math, girls and boys are equal. *The Seattle Times*. http://seattletimes.nwsource.com/html/nationworld/2008071972_math250.html.

Siebert, J., Litzenberg, K., Gallagher, R., Wilson, C., Dooley, F., Wysocki, A. (2006). Factors associated with students' academic motivation in agricultural economics classes. *American Journal of Agricultural Economics, 88*(3), 750–762.

Simpkins, S. D., Davis-Kean, P. E., & Eccles, J. S. (2005). Parents' socializing behavior and children's participation in math, science, and computer out-of-school activities. *Applied Developmental Science, 9*(1), 14–30.

Steele, J. R., Reisz, L., Williams, A., & Kawakami, K. (2007). Women in mathematics: Examining the hidden barriers that gender stereotypes can impose. In R. J. Burke & M. C. Mattis (Eds.), *Women and minorities in science, technology, engineering and mathematics: Upping the numbers* (pp. 159–183). Northampton, MA: Edward Elgar.

Steinke, J. (2005). Cultural representations of gender and science: Portrayals of female scientists and engineers in popular films. *Science Communication, 27*(1), 27–63.

Stevens, T., Wang, K., Olivárez, A., & Hamman, D. (2007). Use of self-perspectives and their sources to predict the mathematics enrollment intentions of girls and boys. *Sex Roles, 56*(5–6), 351–363.

Stromquist, N. P. (2007). Gender equity education globally. In S. S. Klein et al. (Eds.), *Handbook for achieving gender equity through education* (2nd ed., pp. 33–42). Mahwah, NJ: Erlbaum.

U.S. Bureau of Labor Statistics. (2009a). *May 2008 national occupational employment and wage estimates: United States*. http://www.bls.gov/oes/current/oes_nat.htm. Accessed 21 Aug 2009.

U.S. Bureau of Labor Statistics. (2009b). *Highlights of women's earnings in 2008* (Report 1017). Washington, DC: U.S. Department of Labor. http://www.bls.gov/cps/cpswom2008.pdf.

U.S. Department of Health and Human Services, Health Resources and Services Administration, Maternal and Child Health Bureau. (2008). *Women's Health USA 2008*. Rockville, MD: U.S. Department of Health and Human Services.

Watt, H. M. G., Eccles, J. S., & Durik, A. M. (2006). The leaky mathematics pipeline for girls: A motivational analysis of high school enrolments in Australia and the USA. *Equal Opportunities International, 25*(8), 642–659.

Wiest, L. R. (2010). Out-of-school-time (OST) programs as mathematics support for females. In H. J. Forgasz, J. R. Becker, K.-H. Lee, & O. B. Steinthorsdottir (Eds.), *International perspectives on gender and mathematics education* (pp.55–86). Charlotte, NC: Information Age Publishing.

Yan, W., & Lin, Q. (2005). Parent involvement and mathematics achievement: Contrast across racial and ethnic groups. *The Journal of Educational Research, 99*(2), 116–127.

Chapter 24
Quality and Equity in Mathematics Education: A Swedish Perspective

Inger Wistedt and Manya Raman

Introduction

Discussions about quality and equity in schools often focus on the disadvantaged. This focus is understandable, given the depths of inequities in schools in many countries and the tangible price students have to pay if they do not get an adequate education. The case of Sweden offers a somewhat unusual twist on the debate about quality and equity, given that the social welfare system has created a fairly even economic playing field. In a society that prides itself on a certain brand of egalitarianism, the quality and equity question is focused less on whether children are being left behind and more on whether all children are given opportunities to meet their potential.

Note that the teaching of mathematics in Sweden is not completely equitable and unproblematic. There are large refugee and immigrant populations, which raise questions about second-language instruction, and introduce a variety of social and cultural factors that influence how mathematics is taught and learned. In addition, Swedish students, like students everywhere, differ in their abilities and capacities, their interests, and predispositions. Some suffer from learning difficulties, such as dyscalculia, and some will go on to do serious research in mathematics and science-related fields (Blomhøj and Valero 2006; Engström 1999; Magne 2001, 2006; Sjöberg 2006).

There is not a lack of diversity in Sweden, but there are difficulties in giving all students a quality education. The problem, we argue here, lies more with the difficulty of providing quality education than providing some education for all. The state of mathematics teaching in this country reflects a history in which the forces of democracy and egalitarianism have come to be interpreted as a certain kind of leveling rather than a nurturing of individual talents. In this chapter, we trace the

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evolution of this view through the history of Swedish education from the mid-1700s to today, seeing how the tension between providing education for all and developing talents of some create what we see as a sort of false dilemma that, in the case of Sweden, has largely hampered real educational reform.

The Swedish Situation Today

We begin with the present. What does a typical Swedish mathematics classroom look like, and what are the barriers towards a more equitable and quality education for all students?

Mathematics is a compulsory subject in both primary and secondary Swedish schools. Teaching is framed both by school law and by a national curriculum and subject syllabus describing the educational goals. These goals are twofold: to prepare the students for their roles as citizens and to qualify students for further studies. Below we discuss how these goals frame and influence the teaching practice and, hence, the students' learning opportunities.

Whole-Class Instruction and Independent Studies

In many classrooms around the world, a mathematics lesson typically starts with the teacher instructing the class for a shorter or longer period of time followed by individual work (Skolverket 2003, 2008, p. 65). This teaching practice is still common in Sweden (Bentley 2003, p. 10–11). However, in later decades there has been an increasing emphasis on teaching approaches aimed at encouraging the students to take responsibility for their own learning (Bergqvist and Säljö 2008) and today one model of teaching dominates the mathematics classroom in our country:

> The model is characterized by whole-class instruction which sometimes but not always occurs, by independent work in textbooks followed by diagnoses or tests. The teacher walks about in the classroom helping students individually. Planned cooperation among students is relatively rare, collective discussions between teacher and students on mathematics problems and possible solutions to them or laboratory work on mathematics also occur rarely. This is a teaching practice which includes few opportunities for variation in content as well as in ways of learning. (Skolverket 2003, our translation)

According to the TIMSS (Trends in International Mathematics and Science Studies) studies in 2003 and 2007, this model is more dominant in Sweden than in other comparable countries (Skolverket 2003, p. 71; 2005, p. 15; 2008, p. 65). Thus, it is more common than not for Swedish students to work mostly on their own and at their own pace. This means that the students' learning opportunities are restricted to the tasks given in the textbook, which are fairly limited in scope.

Since the model also tends to give the teacher limited time to guide and aid each of the approximately 30 students in the classroom, all textbooks have a "facit," or answer guide for every task in the back of the book. Students often sit and work with one hand in the "facit," checking often to see if they are on the right track. You would rarely see what you would, for instance, in many U.S. classrooms—students called up to the front of the class to work on a problem or students being singled out for being especially talented. It would also be unlikely to find trophies or medals from mathematics competitions, even though Sweden does participate in the Mathematics Olympiad and holds national competitions such as the Kangaroo competition ("Kängurutävlingen") and other national and international initiatives[1].

The Swedish mathematics curriculum is fairly narrow, with arithmetic dominating the elementary and middle school curricula, and with most tasks even at high school and early university level, solvable with rote algorithms rather than real problem solving (Lithner 2000). The simplicity of the tasks has several consequences. One is that they are not really amenable to group work or class discussion. Students might discuss the answer to a particular task, but since no real problem solving is involved, the discussions do not delve deeply into mathematical reasoning. Further, the tasks do not challenge students to structure their mathematical memory, since they are seldom or never asked to draw conclusions from their findings, or to provide proofs or other arguments for their solutions to a task.

It has been pointed out in many studies that the dominant way of teaching in Sweden may be detrimental to students who are less familiar with the norms set up for the Swedish school, such as students with varying ethnical and cultural backgrounds (e.g. Bergqvist and Säljö 2008; Sjöberg 2006). It may also be detrimental to students who are intellectually mature above their age level. Since the textbooks rarely challenge these students, they tend to get bored and some develop behavioral or social problems of various kinds (Mönks et al. 2000, p. 854).

Current Educational Reforms

While considerable resources are spent on students who have difficulties in Swedish schools, there is little attention, to the point where the topic is almost taboo, about the students who are particularly talented. Despite the fact that we have seen major changes in the educational system during the last centuries, it has been virtually impossible to raise any interest in or even understanding of the situation for students who perform above what is expected at their age level (Edfeldt and Wistedt 2009).

However, this situation is just now starting to change. In 2003, the Swedish social democratic government allocated two million crowns to two Swedish universities to develop pedagogy for mathematically and musically gifted students (Prop. 2002/03:1, 15). A year later, and for the first time, the Swedish National Research Council funded a research project on *Gifted Education in Mathematics* (Wistedt 2008).[2] Today the government is allocating money toward developing special programs at the upper secondary school in mathematics and science as well as in

[1] See National Center for Mathematics Education at www.ncm.gu.se.

[2] See www.giftedmath.se.

the humanities and social sciences at ten selected Swedish schools, and more are to come (Regeringskansliet [The Swedish Government] 2008).

These changes are taking place in a climate where nurturing academic talent is seen as elitist. It is yet to be seen if government interventions can change a culture that on the whole (with a few exceptions such as sports and music) does not encourage excellence, and in particular seems to almost discourage certain types of academic excellence (Edfeldt and Wistedt 2009). Below we examine where this climate comes from by providing a historical background for how Swedish mathematics education has developed into what it is today.

A Short History of Swedish Education

As we turn to the past, we will focus on a tension between egalitarianism, which aims at promoting a certain kind of fairness, and excellence, which aims at bringing out the natural talents of all students. We want to contrast excellence with elitism, the latter of which has been viewed in Sweden as the natural enemy of egalitarianism, and the reason we suspect why talent development has been viewed here with scepticism.

Sweden has a long-standing democratic tradition, and the question of nurturing talents has been debated for centuries. Jan Amos Comenius (1592–1670), a Czech scientist and educator still celebrated and cited in European countries (see, e.g., Piaget 1993), held that talent is not socially distributed. In other words, variation in abilities among citizens in a particular society does not necessarily coincide with variation in socioeconomic background (Sjöstrand 1970, p. 247). However, the question was, and still is, if it would be of benefit to society to nurture talent wherever it is found (ibid, 1970, p. 254). Another persistent question is how to find that talent.

In 1737, Gustaf Ruder presented a model for how to identify the most able students regardless of social standing (Kaiserfeld 2008, pp. 4–5). In his book Ruder argued for a successive and continuous choice of the most talented among all students, an issue linked to the political discussion of the day about the necessity for a democratic society to realize a continuous change in social positions. Ruder envisioned an upward stream of intelligence workers and a downward stream of laborers (“ståndscirkulationen”) assuring that people from all social classes would have a chance to be part of the upward stream.

Over the course of the 1800s, as Sweden moved from being an agricultural nation to an industrial one, the question of social mobility was confounded with the difficulty of large-scale education. Multiple models were used for making education more accessible. One was the Bell–Lancaster Method, which used some students as teacher-helpers to help other students, a way to educate more students while keeping costs low (Sjöstrand 1970, p. 84). Another model, with German roots, was the Parallel School System, which was used from 1842 until the 1960s, providing “a set-up which reflected a clearly stratified society: on the one side, an elementary school for the common people, which on the secondary level ran more or less parallel with a highly selective school system, a school recruiting mainly from the upper and middle class" (Husén 1965, p. 181).

However, neither of these models directly addressed the problem of providing quality education to socially and economically underserved populations. This problem persisted, and in the 1950s was conceptualized as a problem for society at large. The term "ability reserve" ("begåvningsreserven") began to be used to refer to children from low-income families who were tested and found to have an untapped capacity for higher educational studies (Husén and Härnqvist 2000).

While the ability reserve has decreased dramatically, there are still some traces of it today. In the academic year 2006/07, 45% of those who were born in the beginning of the 1980s entered higher education (Högskoleverket 2008a). Among those who were born in families with parents holding a doctoral degree, 86% studied at the university, but only 22% of those came from families with no academic traditions (Högskoleverket 2008b). But, on the whole, Swedish education has done fairly well in terms of access to education. In the 1940s only 3–5% of the student population took their secondary-education degree and about 20,000 students in all were enrolled at the university level. Today almost all students study at the secondary level and about half of them, that is, about 85,000 yearly, commence their studies at the university level (Husén 2002; Högskoleverket 2008c).

In the 1960s, the Parallel School System was phased out in place of an integrated school system. Although Sweden had been one of the first countries in Europe to provide detracked education (Tomasson 1965), tracking prevailed in the form of special and general courses in English and Mathematics. Following a broad educational reform in 1994, tracking was removed (Sund 2006), although textbooks with material differentiated for different learning levels are still used in many mathematics classrooms.

Discussion

Throughout the history of Swedish education, there has been a tension between the socializing task, i.e., the purpose of school to bridge differences in knowledge and social standing, and the qualifying task, i.e., to give every student the chance to make the best of his or her abilities (Isling 1974). The solution to the second task, prescribed in current compulsory-school curricula, essentially translates into students working independently in workbooks. Individualization in the integrated Swedish classroom becomes a matter of pace, not quality, within a fairly narrow curriculum.

Seen in the light of history, the current trend in Sweden to allow, or even advocate, the nurturing of academic talent, can be conceived from two different perspectives. The opposition view is that talent development is elitist. This view is consistent with the Parallel School System model that indeed did single out elite members of society for a specialized education. The view we promote here is that developing talent is simply what a democratic society should do to ensure that all of its citizens develop to the full of their potential. This view is in line with Ruder's model of continuous change, with or without the image of the upward and downward streams, and thus has some precedent in the historical development of Swedish education.

The contrast between seeing talent development as elitist versus democratic has particular saliency in Sweden where there is a strong tendency to fit in, a view which has helped shape the culture of Swedish compulsory school (Rubinstein Reich and Tallberg-Broman 2000). There are a number of cultural myths that drive a view that equity means equality, which in practice gets translated into a kind of social leveling.

Back to the Current Situation

In terms of Swedish history and political life, the current trends in education are rather exciting and promising for the idea of helping all students meet their potential. As noted above, the current Swedish government is putting resources into talent development, both in terms of research and opportunities.

However, in the midst of this change, we see traces of history that could threaten to make the changes harder to implement. One example is the language that the popular media uses to describe the educational reforms. The proposed program of providing a challenging curriculum, which is currently being piloted at a few schools around the country, is called "spetsutbildning," which literally means something like "peak performance education" to indicate that the education is supposed to help students reach their full capacity. In the newspapers, the schools piloting such a program are referred to as "elite high schools" ("elitgymnasier"), reflecting a suspicion that the new reforms, like the Parallel School System, would be socially stratified. The main argument against these schools has been that they would only target a small population, an argument that resonates with a deep sense of unfairness that goes against certain egalitarian ideals.

The idea of framing the debate in terms of "all students meeting their potential" might help resolve this conflict. By thinking of mathematics education as a limited resource that only few can benefit from, one is creating a false sort of choice. It seems that education of all students must be sacrificed for the benefit of the few (that is, the talented). However, if one frames the debate in terms of potential, there is no dilemma. All students can be exposed to good mathematics, from early on, and those who show promise and/or interest can be encouraged to pursue it. The details of how to put this view into practice are not trivial, but this view, as such, does not seem in any way to be at odds with either the democratic or egalitarian ideals of Swedish society.

Taking Stock and Looking Ahead

In order to reform Swedish education, one must look beyond the rhetoric of the debate to the underlying assumptions, which are deeply embedded in culture and which, as such, can be hard to identify from within the debate itself. The first assumption we have tried to counter here is that egalitarianism is at odds with excellence. Equity does not mean equality, and even if egalitarianism should be interpreted as some sort of moderation, that by itself is not an obstacle to academic or intellectual ideals. As with Aristotle's idea of the golden mean (Aristotle 350 B.C.), moderation can be a gateway for a higher good.

The second assumption we want to counter is that even in Sweden, which has traditionally been rather homogenous, we are not all alike. The discussion about equity and quality in Swedish mathematics education has benefited a great deal from the large immigrant and refugee populations alluded to earlier who help us see the role culture plays on beliefs about intellectual achievement. For example, many students from fairly poor economic backgrounds, such as refugees from Arab or Asian countries, have very strong social and familial support for studying mathematics and other academic subjects. The solution to the problem of talent selection might not lie in the kind of streams that Ruder suggested, but rather a modern version of the same idea based on bringing out talents of all children regardless of national or cultural origin.

The third assumption we want to question is about where the problem of nurturing talent lies. History tells us that most of the efforts made to optimize learning conditions for talented students in our country, at least thus far, have been organizational. We believe that for real change to take place, one must move past this organization and look closely at what is actually happening or not happening in the classroom (see e.g. Siegel 2004; Lampert 1990). The brief look into a typical Swedish classroom above—dominated by the "facit," routine tasks, and an emphasis of individualized studies—tells us that the activities generally offered to students are not mathematically challenging. To improve the quality of mathematics education for all students, we need to think carefully and openly about how their mathematical experience could be improved.

To this end, we propose a few developmental goals for improving mathematics education on the local level, and providing support for this change on the societal level. We have chosen goals that we think are realistic to achieve, and which could reach across class, culture, and economic barriers:

1. A richer set of educational materials.
2. Teachers trained as much to work with talented students as students with average or less developed abilities.
3. Summer camps or other social outlets for talented students, run by working mathematicians who can give a flavor of what mathematics is like.
4. A culture that supports mathematics as a human discipline, which could include good mathematics problems in national newspapers, public discussions about good teaching practices, economic support for teachers and educational reformers who want to improve the quality of teaching mathematics.

When you walk into a Swedish mathematics classroom in the future, you should see questioning, discovery, rigor, and creativity. You should see teachers with knowledge sharing that knowledge with students who want to learn. What you should not see is any student sitting bored because they finished their workbook early and do not have anything good to do. What you should not hear are stories of students who have become turned off to mathematics without ever seeing what mathematics is really about.

Summing It Up

Swedish classrooms currently lack sufficient stimulation for many students. The discussion around the proposed "spetsutbilding," which attempts to compensate for this lack of stimulation, taps deep worries that quality education is essentially privileged. At the core of both of these problems is a perceived conflict, with a long history, between providing what is fair and providing what is good. This conflict has hampered Sweden in its quest to provide quality education for all, and could very well hamper other countries with similar histories.

Our intent here is not to take sides in a long-standing debate between egalitarian access and talent development, but rather to argue that this choice is a false one. Instead of framing the goal of reform as "mathematics for all" which might unfortunately get implemented as "mediocre mathematics for all," we would like to frame the goal as "helping all children meet their potential." In a country with a fairly open access to education and a strong commitment to not let any child fall behind, we find that many children do fall behind, in another way, by not taking seriously the *quality* aspect of their mathematics education. To us, a child who has not been sufficiently challenged has not been educated, and a country that does not educate its children does not fulfill the promise of a free and democratic society.

References

Aristotle. (350 B.C.). *Nicomachean Ethics* (trans: Ross, W. D.). Constitution Society: Liberty Library of Constitutional Classics, I.3. http://www.constitution.org/ari/ethic_00.htm. Accessed 4 Jan 2010.

Bentley, P.-O. (2003). *Mathematics teachers and their teaching. A survey study.* Göteborg Studies in Educational Sciences 181. Göteborg: Acta Universitatis Gothoburgensis.

Bergqvist, K., & Säljö, R. (2008). Learning to plan. In J. van der Linden & P. Renshaw (Eds.), *Dialogic Learning. Shifting perspectives to learning, instruction and teaching* (pp. 109–124). The Netherlands: Springer.

Blomhøj, M., & Valero, P (2006). Vanskeligheder i/med matematiklæring—behov av øget forskning [Difficulties in/with mathematics—need for enhanced research]. *Nordic Studies in Mathematics Education, 11*(4), 1–6.

Edfeldt, Å. W., & Wistedt, I. (2009). High ability education in Sweden: The Swedish model. In T. Balchin, B. Hymer, & D. Matthews (Eds.), *The Routledge companion to gifted education* (pp. 76–83). London: Routledge.

Engström, A. (1999). *Specialpedagogiska frågeställningar i matematik: en introduktion*. (Special educational issues in mathematics: An introduction). Örebro universitet: Pedagogiska institutionen.

Husén, T. (1965). Educational change in Sweden. *Comparative Education, 1*(3), 181–191.

Husén, T. (2002). Begåvningsreserven då och nu [The Ability Reserve now and then]. *Pedagogisk forskning i Sverige, 7*(3), 164–167.

Husén, T., & Härnqvist, K. (2000). *Begåvningsreserven. En återblick på ett halvsekels forskning och debatt* (The Ability Reserve. A retrospective view of half a century's discussion and debate). Uppsala: Föreningen för svensk undervisningshistoria.

Högskoleverket. (2008a). (Swedish National Agency for Higher Education). *Statistiska meddelanden UF 20 SM 0802: Universitet och högskolor. Högskolenybörjare 2007/08 och doktorandnybörjare 2006/07 efter föräldrarnas utbildningsnivå* (Higher Education. Level of parental education among university entrants 2007/08 and first time students at third circel studies 2006/2007).

Högskoleverket. (2008b). (Swedish National Agency for Higher Education). *Analys nr 2008/11: Fortsatt ökning av antalet nybörjare vid universitet och högskolor* (Continued increase of entrants to higher education). Stockholm: Högskoleverket.

Högskoleverket. (2008c). (Swedish National Agency for Higher Education). *Statistiska meddelanden UF 20 SM 0801: Universitet och högskolor. Studenter och examina i grundutbildningen 2006/08* (Higher education. Students and graduated students in undergraduate education 2006/07).

Isling, Å. (1974). Vägen till en demokratisk skola: Skolpolitik och skolreformer i Sverige från 1880–1970-talet (The path to a democratic school: School policy and school reforms from the 1880's to the 1970's). Stockholm: Prisma.

Kaiserfeld, T. (2008). The persistent differentiation—the education commission's work 1724–1778. CESIS. Electronic Working Paper Series No 113. http://ideas.repec.org/cgi-bin/htsearch?q=Kaiserfeld&cmd=Search%21. Accessed 5 Nov 2010.

Lampert, M. (1990). When the problem is not the questions and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29–63.

Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics, 41,* 165–190.

Magne, O. (2001). *Literature on special educational needs in mathematics: A bibliography with some comments*. Malmö: Lärarhögskolan.

Magne, O. (2006). Historical aspects on special education in mathematics. *Nomad, 11*(4), 7–35.

Mönks, F. J., Heller, K. A., & Passow, A. H. (2000). The study of giftedness: Reflections on where we are and where we are going. In K. A. Heller, F. J. Mönks, R. J. Sternberg, & R. F. Sobotnik (Eds.), *International handbook of giftedness and talent* (2nd ed.). (pp. 833–852). Amsterdam: Elsevier.

Piaget, J. (1993). Jan Amos Comenius (1592–1670). *Prospects, 23*(1–2), 173–196. (UNESCO, International Bureau of Education, 1999).

Prop. 2002/03:1, 15. *Budgetproposition för 2003, 15. Bilaga UO16.* (The Swedish Budget Bill 2003, Appendix UO16). Stockholm: Regeringskansliet.

Regeringskansliet (The Swedish Government). (2008). PM 2008-05-26: Inrättande av försöksverksamhet med riksrekryterande gymnasial spetsutbildning (Establishing experimental work with nation-wide recruiting advanced-placement tracks at the upper secondary level). http://www.regeringen.se/sb/d/10120/a/106291. Accessed 5 Jan 2010.

Rubinstein, R. L., & Tallberg B. I. (2000). *Den svenska skolan i det mångkulturella samhället—Konsekvenser för lärarutbildningen* (The Swedish school in the multicultural society—Consequences for teacher education). Malmö: Malmö högskola.

Siegel, A. (2004). Telling lessons from the TIMSS videotape. http://www.cs.nyu.edu/faculty/siegel/ST11.pdf. Accessed 30 Jan 2010.

Sjöberg, G. (2006). *Om det inte är dyskalkyli—vad är det då?* (If it is not dyscalculia—then what is it?). Umeå University: Department of Mathematics Technology and Science Education.

Sjöstrand, W. (1970). *Pedagogiska grundproblem i historisk belysning* (Educational basic questions in the light of history). Lund: Gleerups.
Skolverket (Swedish Board of Education). (2003). *Lusten att lära—med fokus på matematik. Rapport nr. 221* (The urge to learn—with a focus on mathematics. Report No 221). Stockholm: Skolverket.
Skolverket (Swedish Board of Education). (2005). *En sammanfattning av TIMSS 2003. Särtryck av Rapport nr. 255* (A summary of TIMSS 2003. Off-print of Report No 255). Stockholm: Skolverket.
Skolverket (Swedish Board of Education). (2008). *TIMSS 2007. Rapport nr. 323.* (TIMSS 2007. Report No 323). Stockholm: Skolverket.
Sund, K. (2006). *Detracking Swedish secondary schools: Any losers, any winners?* Swedish Institute for Social Research (SOFI), Working Paper No. 2.
Tomasson, R. (1965). From elitism to egalitarianism in Swedish education. *Sociology of Education, 38,* 203–223.
Wistedt, I. (2008). Pedagogik för elever med förmåga och fallenhet för matematik (Gifted education in mathematics). *Resultatdialog 2008. Forskning inom utbildningsvetenskap* (Result dialogue 2008. Research in Educational Sciences) (pp. 132–136). Vetenskapsrådets rapportserie (Reports from The Swedish Research Foundation) 12:2008. Stockholm: Vetenskapsrådet.

Chapter 25
Equity Issues Concerning Gifted Children in Mathematics: A perspective from Mexico

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Introduction

Our first purpose in writing this chapter is to argue the need for mathematically gifted children to receive particular attention in order to develop their talents as a matter of equity, and second, to provide a perspective that comes from a very particular country that keeps very strong cultural traditions and has a very special political context. We do not intend to enter a discussion about identification procedures, although these procedures have generated a heated debate about their fairness, the under-representation of certain minorities, and the lack of identifying more girls who are gifted in mathematics. We point out, however, that the conception of giftedness has been evolving from a fixed trait to a multi-faceted and changeable characteristic that takes into consideration several aspects of an individual and his or her relationship with the subject matter in question. For example, according to Eriksson (2006, p. 5) excellence has a lot to do with culture, values, and norms:

> As educators of the world's most brilliant children, we have a responsibility to preserve and celebrate the abilities of these endangered cultures. Not only is the loss of this potential one that is individual to these gifted children, but also a loss for the world at large.

Equity issues related to the education of mathematically gifted students have been touched upon in a few studies in the literature so far, viewing the problem from different lenses. Borland (2003, p. 117) expresses the following concerns as to what really goes on in gifted education:

> I think that two things are indisputably true. The first is that professionals in the field of gifted education, no less than any other group of educators, are opposed to racial and other forms of inequity and are committed to fairness in access to education. Indeed, most would argue that educational equity is what brought them to the field in the first place. The second

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> is that, despite the best of intentions, gifted education, as historically and currently practiced, mirrors, and perhaps perpetuates, vicious inequities in our society.

On the other hand, Sriraman and Steinthorsdottir (2007, p. 94) look at the situation posing the following questions:

> 1. Is there a way in which one can resolve talent development, particularly in mathematics education so that that (*sic.*) the curriculum and/or instruction is equitable to all the students in the classroom?
> 2. Can excellence and equity coexist or does attending to one compromise the other, i.e., excellence at the sacrifice of equity; equity at the sacrifice of excellence?

The way these questions are stated implies that it is difficult to consider the education of the gifted and talented as an equity issue inherently; and there are many educators and researchers who feel the same about this issue (see for example Sapon-Shevin 2003). In particular, if we look at the research literature concerning equity in mathematics education, we realize that a great majority of these studies focus on students with disadvantaged backgrounds and learning difficulties. For example, books on this subject hardly contain any studies that focus on gifted children (Secada et al. 1995; Burton 2003).

Sriraman and Steinthorsdottir (2007) study the problem taking into account different traditions in the eastern and western societies, offering an interesting interpretation of the equity issue in the United States. They maintain that the tension between equity and excellence is a symptom of political problems that polarize the society. They suggest that in order to reconcile the elitist Hamiltonian tradition that emphasizes the cognitive traits owned by people and the Jacksonian tradition according to which everybody is equal, a third alternative, namely the Jeffersonian tradition that stresses equality in terms of how people make use of educational opportunities that are available to them, could be considered. The following quote from them shows how they understand 'equality' and offer a solution in terms of this understanding. "The challenge facing society today (in the United States and elsewhere) is to first create this equality in educational opportunity. This is a necessary first step in resolving the tension between equity and excellence" (ibid, p. 99).

Sriraman (2007, p. 1) further argues that "[t]he stifling of creativity in schools is often collectively rationalized under the guise of doing what is supposedly good for the majority of the students." He adds that this is done by invoking the often misused term 'equity', or by "appealing to curricular plans and school achievement goals etc." (p. 1). In the same lines, Benbow and Stanley (1996, p. 252) argue that in the United States as a result of several factors including extreme egalitarianism and "pitting of equity against excellence rather than promoting both equity and excellence in schools," the brightest students are being deprived of opportunities to develop their intellectual potential. Consistent with their belief that all children have the right to develop to their full potential, they maintain that "[e]quity should be viewed as equal access to an *appropriate*[1] education" (p. 257), which means access

[1] Italics in the original text.

to both challenging and obtainable education depending on the academic readiness of each individual.

The term 'equity' indeed is used in many different ways. In the literature, although 'equality of resources' and 'equality of opportunity' are mentioned (Fennema 1990; Winstanley 2006), 'equity' in general is understood or accepted in terms of 'equal outcomes'. For example, when discussing about research on equity in mathematics education, Lerman and Marcou (2008, p. 352) state that "[r]esearching what might lead to more equitable outcomes, across diverse social groups, especially since disadvantaged groups consistently achieve less than children from more advantaged backgrounds in many countries, has become quite common in recent years". Although the concern raised with this statement is of utmost importance, we think that it is equally important to emphasize other aspects of equity, such as development of human potential, especially from the point of view of developing countries. In the first case, concern is on equating the outcomes achieved by different groups, which implies inverting more on disadvantaged groups as they need more attention to be able to improve their attainment of established educational goals. The focus is on reducing the gap, which in turn implies, if we look at the issue from a different angle, less attention for the more able children. In the second case, attention is turned to a different social aspect. In order to provide more well being to all members of the society in which we live, human resources are to be developed to their maximum potential. This requires special attention to gifted children, since their contributions can make the difference.

There is no need for these two objectives to clash, since lack of importance given to either will produce unease in the society and in individuals affected negatively by related policies. However, we point out that the second approach has not been emphasized enough in educational policy and research.

The debate about the attention to gifted students is pointed out by Warwick and Matthews (2009, p. 266) as a conundrum: "gifted programming leads to social inequity, but we incur other social inequities when we do nothing to support gifted development, to say nothing about the loss to society of abilities that are left under-challenged and undeveloped." These authors suggest establishing two types of goals related to gifted education: one in which the aim is to develop the talents and to serve the learning needs of the gifted, and the second where what is learned from the experiences and practices from the former objective is used to foster "high-level ability more broadly across the population" (p. 266). This means raising the percentage and diversity of highly able individuals in the society intentionally, by means of programs specifically geared toward these goals. REAL (Realising Equality and Achievement for Learners) is one such project that takes place in the United Kingdom and concentrates on the needs of Black/Minority/Ethnic and English as a second-language students (Warwick and Matthews 2009). The authors report that through this network schools and teachers are provided with additional resources toward achieving equality and excellence.

Borland (2003) argues for forgetting about identifying and labeling children as gifted, discontinuing with the pull-out programs, hence dispensing with the concept of 'giftedness'. He proposes instead, focusing on curriculum development for differentiated instruction, where all children would benefit from a challenging education at their own level:

> For example, our expectations for students' learning in, say, mathematics would be determined by what they now know and what instruction they demonstrably need in that subject, not on whether their ages mark them for the third-grade curriculum, the fourth-grade curriculum, or whatever. (ibid, p. 119)

We see the problem of equity related to the education of mathematically gifted students at two levels. At an individual level, unidentified talents and lost potential, and at societal level, lost human resources, which becomes especially important for a developing country. In this chapter, we try to address these issues, especially the second one, in the context of Mexico.

Initially, in what follows, we will address the problems some mathematically gifted students face. Second, we will present the Mexican context and discuss some societal concerns from the viewpoint of a developing country. Finally, we will give examples from our work on designing mathematical situations.

Some Concerns About the Education of Mathematically Gifted Students

The foreword to the UNESCO report on Gifted Children in IberoAmerica (Machado 2004, p. 9) starts by stating:

> Every person has a right to receive an education that develops his/her capacities to the maximum and that allows him/her to construct his/her life project. Implementing this right implies guaranteeing the principle of the equality of opportunities, that is providing to each one of them the help and resources that they require, in function of their individual characteristics and needs.

This statement implies that in making curriculum decisions not only the average population would be targeted, but also the groups on both extremes with special needs have to be taken into account.

We pointed out earlier that in general the education of the gifted and talented is not considered as an equity issue. There might be several reasons for this. Perhaps these individuals are thought of as possessing special qualities that give them a head start in the society so they are not considered a priority. Perhaps it is generally thought that gifted children will succeed no matter what kind of education they receive. However, boredom and loss of interest in academic subjects is a major concern, since lack of challenge leads to useless repetition of tasks.

> (Gifted) children have special needs in the educational system; for many their needs are not met; and many suffer underachievement, boredom, frustration, and psychological distress as a result …. The common belief that the gifted do not need special help because they will succeed anyway is contradicted by many studies of underachievement and demotivation among gifted children. (p. xiii) (Collins 2001, cited in Diezmann et al. 2004, p. 8).

Reis (2009) points out another ramification of this lack of challenge: some gifted children never learn to work towards attaining a goal, since they get used to getting good grades with a minimal effort. And there are those cases where a gifted child may end up in serious problems, such as gang involvement or drugs.

Another concern raised by Diezmann et al. (2004, p. 15) is the "negative community attitudes towards the gifted," pointing out that the reasons for this "may be associated with the perception of them as a 'marked' group or 'deviant' population, because the general population finds mathematics difficult and holds negative attitudes towards it (Damarin 2000b)".

Teacher education is another important issue. There are several myths about gifted students and without knowledge about the characteristics of these individuals, teachers can be quite unsupportive of their needs (Diezmann and Watters 2002; Sak 2004). Although underachievement is a subject about which teachers are sensitive, in general, this issue is not associated with gifted students. This might be due to the fact that academic expectations are not geared towards individual potential and interest, rather they are established as common goals to be achieved by all students.

There is also the issue that there might be areas related to mathematics in which individuals might be gifted, but because of the way the mathematics and mathematical activity is defined and valued in the society, they may go unidentified and discriminated. Spatial ability seems to be one such area. When studying this issue, Mann (2005) points out that even though individuals such as Einstein, Edison, and da Vinci probably had difficulties at school, they found avenues to bring about their gifts, otherwise the society would not have experienced the consequences of their talents.

The learning processes of students differ from one another and acknowledging this fact does not mean giving more importance to one group than others: "It is not a matter of giving gifted students more attention or better resources, only of meeting all students' unique learning needs" (Stepanek 1999, p. 2).

Societal Concerns

When discussing the equity and diversity agendas in research, Atweh (2007) notes: "[e]quity projects aim at reducing group differences, e.g., in achievement and hence its ultimate aim is to abolish group differences. Diversity discourse, on the other hand aims at enhancing group differences and status." In our view, it is more productive to view the equity issue in terms of optimal development of human potential instead of concentrating on group differences. As a step toward achieving this goal within the mathematical development of gifted students, our project focuses on designing and applying challenging mathematical situations that would help improving the participants' thinking skills and knowledge.

Attention to gifted children in Mexico has not been systematic and many programs geared toward this population lack a research base and theoretical foundations (Valadez and Betancourt 2004). In 2006, the Mexican Ministry of

Public Education (SEP 2006) prepared an intervention proposal for attention to gifted students at the elementary school level so that this population receives an education according to their specific needs. This proposal promoted an approach that was based on the principle of integration. According to the proposal itself:

> [it] is based on the equity principle that is mentioned in the 2001–2006 National Education Plan, which means not offering the same to all students but offering them what they need, in a differentiated manner and in equilibrium so that each one of them reaches the educational level that corresponds to their learning potential. In this sense, and just like in the case of students who are in situations of different kinds of vulnerability, for the gifted boys and girls equity means giving them something different because they need it. (SEP 2006, p. 22)

Of course, this is a complex issue that calls for approaches that take into account different facets of the problem. This report gives special importance to equity concerns, recognizing the injustice that occurred in this society "where a large population of boys and girls stayed at the margin of educational services" (ibid, p. 37–38). In this same group, both *the population with some disability* and *the gifted population* are mentioned. We make a parenthesis here to mention that one step toward achieving these objectives might be stimulating gifted students through challenging mathematical situations and motivating them to express their ideas and discuss them with their peers and instructors. We will give some examples of our approach of this kind of activity in the next section of this chapter.

Although the above-mentioned proposal was prepared with the participation of several experts and took into account a wide array of research into the area, it was not widely distributed, and it is not very well known among school teachers. Furthermore, the initial enthusiasm that made possible this proposal did not have a follow-up in terms of teacher training or institutional support, which implies for the students to end up not receiving the attention that they need and deserve.

On the other hand, for us it is not surprising that the government show interest in the education of students with special talents. Especially from the point of view of a developing country, these students, especially the ones who are talented in mathematics form an important resource for the future of the country. If not attended, this resource may be lost, or if developed by itself and without special attention, there might be more risk that these individuals migrate elsewhere, such as to more developed countries where they can more easily satisfy their intellectual needs and attain specific goals.

However, it may not be so easy to achieve the desired goals in terms of providing all children with what they need educationally. We give an example from Mexico with the purpose of bringing to the attention how special circumstances (political or otherwise) can affect how the issue is dealt with. In Mexico, a few years ago the government of the Distrito Federal (which we will refer to as the capital government) started a program for talented children (niñ@s[2] talento). In order to understand the context in which it developed, we need to understand the political tensions between the Federal government and the capital government. We interviewed

[2] In some circles in Mexico, recently the character @ has replaced the letters 'o' and 'a' in writing, as in 'niños' (boys) and 'niñas' (girls), to produce gender-neutral words.

Ricardo Cantoral, director of the 'talented children' program, according to whom the capital government had this initiative, because at the federal level not much was being done. Cantoral sees the talented children program as an opportunity "that tries to bring to the population a right":

> **RC**: The possibility of total enjoyment of capacities, interests, science, culture, and sports is also within those rights. In this way the conceptualization of the gifted children program happens in this government, in this city that has the most advanced research centers, that has a policy clearly contrasting with the social policy of the Federal government.

Cantoral goes on to say that the capital government brought together experts from several educational institutions to discuss about what could be done for the children, "as part of a social policy and at the same time to give an incentive to scientific development in the population". However, since the schools depend on the Federal and not on the city government, whatever was going to be done, had to happen outside of schools. Because of this, and although the general intention was to attend all children, there had to be a selection due to restricted resources, using some kind of criterion, which the experts agreed to be *talent*. Even though conceptions of giftedness and talent are not the focus of this chapter, we provide Cantoral's interesting notion regarding talent, since it helps to understand the principles that guide this program:

> **RC**: Here there was a big debate about if it was talent or some other attribute. This issue of talent produced a very interesting debate and the conclusion of the group of specialists was that "talent is something that is distributed democratically," that is, you won't find only some people with talent, but you will find that everybody has it, just that they did not have the opportunity to develop it. […] However, the name 'talented children' was thought of as a distinguishing name that could stimulate the children to be a part of, and not as a conception from inside.

When we asked him if Mexican children talented in mathematics have the necessary means and tools to develop their abilities, we obtained the following response:

> **RC**: I really think not….When you present mathematical activities, you observe quickly more abilities in some than in others. In this case, it is difficult because if somebody is very good in some sports you can help him/her by means of a high performance school and he/she can keep working on the development of his/her talent. Similarly, if it is in music or arts you can send them to Ollin Yoliztli, but if it is in mathematics, actually there is no good place for mathematics, physics or chemistry. You don't find a place where especially this talent can be developed. So there is a lot to be done, and I think that this project is the beginning of something very important.

The effect of this program has to be seen in the long run. It is important to study this kind of initiatives from within and from an outsider's perspective in order to maximize the benefits for the children involved and also in order to avoid the mistakes that have been made elsewhere regarding equity issues concerning minorities and females. Mexico is a land of many languages and cultural traditions, and it requires great sensitivity and knowledge to deal with the equity issues.

We would like to emphasize that we are not necessarily suggesting ability grouping or other means of segregating different groups as a means to address the needs of mathematically gifted students (nor we are condemning such measures, if used appropriately). Some approaches that have been used and that produced several research reports, such as the SMPY (Study of Mathematically Precocious Youth, see Benbow and Stanley 1996), advocate ability grouping and acceleration for the highly gifted, whereas others argue for educational treatments based on integration (Smith 2006). We are aware that in different contexts there might be need for different kinds of interventions for the purpose of attending and serving the gifted and talented in mathematics. As we mentioned before, in Mexico, there has been neither a tradition of research-based attention nor systematic identification of gifted individuals. There is need for research as well as brainstorming in order to come up with the optimal solutions given the limitations and characteristics of each population. This might help from the beginning, as mentioned above, to avoid to a certain extent, mistakes and unwanted situations, such as under-representation of girls and minorities or children with different socio-economic backgrounds.

Our Approach

Zollman (2007) reminds us that the highly gifted students make up about 0.1% of the whole student population, adding: "The mathematically highly and extremely gifted are but a few in number. 'Losing' one is significant" (p. 145). So what can be done in order not to lose them? In our opinion, one part of the answer might be related to special task design and the educational use given to these tasks. This approach is in line with what we understand as quality: "Quality of schooling includes not only time-on-task, but time well spent" (Sirotnik 1983, p. 26, quoted in Benbow and Stanley 1996, p. 258).

Stepanek (1999, p. 9) states that "[b]rain research provides a physical explanation for students' failure to learn. When tasks are not sufficiently challenging, the brain does not release enough of the chemicals needed for learning: dopamine, noradrenalin, serotonin, and other neurochemicals (Schultz et al. 1997, cited in Tomlinson and Kalbfleich 1998)". On the issue of task design for gifted students Watters and Diezmann (2000, p. 14) suggest that "challenging mathematical tasks for gifted students should be authentic tasks that provide opportunities for them to emulate the practices of mathematicians, though at a less sophisticated level". This point of view is also in line with the 'equality of challenge' raised by Winstanley (2006). This notion refers to providing enough challenge to everybody depending on their needs and potential.

In our opinion, one approach that can be used without the involvement of excessive financial resources, and that would benefit the majority of students, is the design of tasks that can be enriched and presented at different levels of mathematical maturation. This way all the students in one class can work on the same task without distraction caused by the boredom of some individuals who finish their job earlier and have nothing else to do, or who find the task easy and uninteresting. This approach requires a collection of mathematical activities with the following characteristics: In the first case, the problem can start by posing questions that most of the class should be able to answer with some effort but without too much difficulty, and the level of difficulty can continue increasing by moving towards generalization and abstraction. In the second case, the same problem can have different versions that can be used considering the mathematical readiness of the students. However, this approach puts considerable demand on the teachers. In our group, we are working on the design of such problems in order to have a resource that consists of activities that contain extensions at different mathematical levels, analysis of these situations in terms of expected conceptual outcomes and possible student difficulties, and suggestions about their application.

The following activity is an example of the first type mentioned above. It allows a general application by means of the central problem posed, and through peripheral activities a challenge to those who need and enjoy it. Before we present the problem, we mention the characteristics that we try to incorporate into the design:

- **The situation is presented via open questions.** The aim is to provide a space for reflection to students so that they can express their knowledge, come up with conjectures, and choose procedures to solve the problem.
- **It is flexible and adaptable.** It can be used at different school levels; the majority of the population for which it is designed can provide an answer (at least partially) to one or several parts of the activity.
- **Motivates the students and it is contextualized.** Apart from being an intellectual challenge, the activity is related to the previous knowledge and backgrounds of the students.
- **It is extendible.** It can be extended in different directions, including generalization, according to the learning objectives.
- **It organizes knowledge.** In order to solve the problem, students would incorporate different kinds of mathematical knowledge that they have constructed or are in the process of constructing and consolidating.
- **Helps generate ideas.** Starting with the presented activity, other problems can be constructed to generalize the mathematical situation and its solution.

The central activity that we present below appears in Rocha et al. (2007) and has been extended through several peripheral activities in order to provide different kinds of challenge.

Internet Cafés

Central Activity

In downtown, there are three places where Internet service with the same quality is offered. The following information shows their rates and promotions:

Place	Hourly price	Promotion
El Chat	$11.00	Does not have a promotion
Intercafé	$12.00	For every 10 hours they give you one hour free
Intermente	$13.00	For every 6 hours they give you one hour free

In order for the promotion to be valid, each place gives a card to the client in which they register the number of hours consumed in different sessions.

- Describe situations in which it would be better to use the service offered by *El Chat;* describe situations in which it would be better to use the service offered by *Intercafé;* and finally, describe situations in which it would be better to use the service offered by *Intermente* (Rocha et al. 2007, p. 216).

To these questions we can add the following:

- If you will use Internet service continuously, which one of the three options would you choose?

This activity is accessible to all students; some may prefer to use random trial and error, others might use the same approach concentrating on the patterns, and still other students may use a functional approach. Further challenge is provided by use of situations derived from the central activity; the aim is to encourage modeling the situations presented verbally, by means of mathematical models.

Peripheral Activity 1: Find a relationship that expresses the total cost in terms of the number of hours spent at "El Chat."

Peripheral Activity 2: For the other two Internet places, find a relationship that expresses the total cost in terms of free hours obtained after having spent n hours using their services.

This activity requires modeling the situation through a function and the limiting processes enter into the discussion as the students try to find out what happens with continuous use of different services. When necessary, students can be provided with help through further questions, such as the following, to help organize their thinking:

For Intercafé fill in the following table, adding as many lines as you need:

Hours spent	Hours free	Total cost
1,024		
1,547		
1,713		
2,220		
n		

Additional questions can be used in order to open up further discussion and to help with the reasoning process: After a large number of hours are spent, which option tends to a less unitary cost? From how many hours on is this unitary cost less than the other two options?

Peripheral Activity 3: Among the consumers a rumor started that the promotion that Intermente offers is not really good and that one ends up paying more than the other two options. For this reason the number of people going to this place went down and the owner decided to establish a new promotion in which the cost would be less than the other two options with a unitary cost not very far from \$11. What hourly rate should this place offer and for how many hours spent should it offer a free hour?

In this problem, students are given the opportunity to construct a new offer based on the information provided in the statement and justify their claims. The functions that they have used previously to model the original situation can be of help in analyzing the new situation and their answers can be used as the starting point of a debate.

Peripheral Activity 4: Draw a graph for the following function and discuss its asymptotic behavior:

$$f(x) = \frac{7x + 3}{7(x + 1) + 3}$$

This question motivates visualization and calls for a discussion in terms of relationships with the previous activity.

This type of activities can be used to cover content objectives (in this case, relations, functions, and proportionality, for example) as well as to provide opportunities to develop the conceptual understanding of students. At the same time, they can be extended to include more advanced topics (such as limits in this case) and deeper mathematical relationships that can be used with those students who need a greater intellectual challenge.

With mathematically gifted students we also make use of mathematical topics that do not form part of their curriculum usually. For example, following Dubinsky's idea of applying situations involving mathematical infinity and paradoxes to gifted children (see Arnon et al. 2008; Dubinsky et al. 2005), we included the tennis ball problem and the problem of Hilbert's hotel in the collection of mathematical activities that we use with this population. We pursue two aims here: on the one hand, we want to understand how young gifted students think about topics that are difficult even for mathematicians, and on the other hand, we hope to provide stimulus for these children to advance their mathematical abilities and talents. The acceptance of actual infinity goes against our natural intuition, and we hope to provide guidance for students to move toward more advanced levels of knowledge construction.

Concluding Comments

Societal problems of a technological and scientific nature can best be understood and resolved by people who know the structure of that society at a profound level. Importing solutions from elsewhere without considerations of the culture and characteristics of the population involved might lead to tensions and failure of efforts. Developing the potential of mathematically gifted students optimally and raising their awareness about their environment and conditions can give rise to a significant national resource which can in the long run promote well being and trust at a societal level, since these individuals can play a crucial role in the society. This way we can assure that the problems of the society can find solutions within the constraints and particularities of the culture involved without exaggerated necessity to import solutions of different sorts that are not sensitive to local needs. We furthermore argue that not attending this population and their needs and not developing their potential talents in mathematics, can give rise to more inequities between international communities. In a way, we see this as a means to attempt to close the knowledge and economic gap between developed and developing countries, and hence the lifestyles of those nations.

However, as already mentioned, there also should be special attention paid to inequities within the national context. Mexico is a land of many languages and cultural practices, and especially the gender issue is a sensitive one, since in some of these practices, girls and women do not receive adequate treatment so as to realize their potentials.

Providing the social and intellectual context to all students so that they can attain their maximum potential can benefit everybody. We have to remember that family, school, and society provide all determining elements for the manifestation and developing (or not) of mathematical talent. If we can aim at obtaining both individual intellectual satisfaction and social responsibility, we think that we can move towards more equitable conditions. Of course, saying that is easy. We have to think deeply about what is involved in preparing such conditions, design appropriate programs, act accordingly, and learn from our mistakes. As Atweh (2007) states:

> Arguably, solving the problems of inequitable achievement and available resources in less-industrialized countries are beyond the capabilities of single academics or even the profession as a whole working in isolation. However, such a call presents a challenge for academics who believe that concerns about social justice do not know any boundaries.

References

Arnon, I., Dubinsky, E., Morics, S., Moses, M., Oktaç, A., Stenger, C., et al. (2008). Thinking processes of GCM: A cognitive inquiry. *Proceedings of the 5th International Conference on Creativity in Mathematics and the Education of Gifted Students*. Haifa, Israel.

Atweh, B. (2007). What is this thing called social justice and what does it have to do with us in the context of globalisation? *Philosophy of Mathematics Education Journal,* 21. Retrieved from http://people.exeter.ac.uk/PErnest/pome21/index.htm.

Benbow, C. P., & Stanley, J. P. (1996). Inequity in equity: How “equity” can lead to inequity for high-potential students. *Psychology, Public Policy, and Law, 2*(2), 249–292.

Borland, J. H. (2003). The death of giftedness: Gifted education without gifted children. In J. H. Borland (Ed.), *Rethinking gifted education* (pp. 105–124). New York: Teachers College Press.

Burton, L. (Ed.). (2003). *Which way social justice in mathematics education?* Westport: Praeger Publishers.
Diezmann, C. M., Faragher, R., Lowrie, T., Bicknell, B., & Putt, E. (2004). Catering for exceptional students in mathematics. Retrieved from http://eprints.qut.edu.au/1855/1/1855.pdf.
Diezmann, C. M., & Watters, J. J. (2002). Summing up the education of mathematically gifted students. *Proceedings 25th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 219–226). Auckland.
Dubinsky, E., Weller, K., McDonald, M., & Brown, A. (2005). Some historical issues and paradoxes regarding the concept of infinity: Part I. *Educational Studies in Mathematics, 58,* 335–359.
Eriksson, G. (2006). Introduction: Applying multicultural and global education principles to the education of diverse gifted and talented children. In B. Wallace & G. Eriksson (Eds.), *Diversity in gifted education: International perspectives on global issues* (pp. 1–8). London: Routledge.
Fennema, E. (1990). Justice, equity, and mathematics education. In E. Fennema & G. C. Leder (Eds.), *Mathematics and gender* (pp. 1–9). New York: Teachers College Press.
Lerman, S., & Marcou, A. (2008). Are all studies on equity in school mathematics equal? *Proceedings of the Fifth International Mathematics Education and Society Conference* (Vol. 2, pp. 352–361). Aalborg University.
Machado, A. L. (2004). Presentación. In M. Benavides, A. Maz, E. Castro, & A. Blanco (Eds.), *La educación de niños con talento en Iberoamérica* (pp. 9–13). Santiago: UNESCO.
Mann, R. L. (2005). *The identification of gifted students with spatial strengths: An exploratory study.* Ph.D. Thesis, University of Connecticut.
Reis, S. M. (2009). Turning points and future directions in gifted education and talent development. In T. Balchin, B. Hymer, & D. J. Matthews (Eds.), *The Routledge international companion to gifted education* (pp. 317–324). London: Routledge.
Rocha, R., González, M. L. G., Rodríguez, M. A., & Flores, E. (2007). *Matemáticas para la vida 1*. Mexico: Pearson Education.
Sak, U. (2004). About creativity, giftedness and teaching the creatively gifted in the classroom. *Roeper Review, 26*(4), 216–222.
Sapon-Shevin, M. (2003). Equity, excellence, and school reform: Why is finding common ground so hard? In J. H. Borland (Ed.), *Rethinking gifted education* (pp. 127–142). New York: Teachers College Press.
Secada, W. G., Fennema, E., & Byrd Adajian, L. (Eds.). (1995). *New directions for equity in mathematics education.* Cambridge: Cambridge University Press.
Secretaría de Educación Pública. (2006). *Propuesta de intervención: Atención educativa a alumnos y alumnas con aptitudes sobresalientes.* Mexico: Secretaría de Educación Pública.
Smith, C. M. M. (Ed.). (2006). *Including the gifted and talented: Making inclusion work for more gifted and able learners.* London: Routledge.
Sriraman, B. (2007). Creativity, innovation and talent development in mathematics education. *Mediterranean Journal for Research in Mathematics Education, 6*(1–2), 1–2.
Sriraman, B., & Steinthorsdottir, O. B. (2007). Excellence and equity in education and talent development: Components of a Hegelian dialectic. *Mediterranean Journal for Research in Mathematics Education, 6*(1–2), 91–102.
Stepanek, J. (1999). *The inclusive classroom. Meeting the needs of gifted students: Differentiating mathematics and science instruction.* Portland: Northwest Regional Educational Laboratory.
Valadez, M. D., & Betancourt, J. (2004). La educación de los niños con talento en México. In M. Benavides, A. Maz, E. Castro, & R. Blanco (Eds.), *La educación de niños con talento en Iberoamérica* (pp. 129–141). Santiago: UNESCO.
Warwick, I., & Matthews, D. J. (2009). Fostering giftedness in urban and diverse communities: Context-sensitive solutions. In T. Balchin, B. Hymer, & D. J. Matthews (Eds.), *The Routledge international companion to gifted education* (pp. 265–272). London: Routledge.
Watters, J. J., & Diezmann, C. M. (2000). Catering for mathematically gifted elementary students: Learning from challenging tasks. *Gifted Child Today, 23*(4), 14–19.

Winstanley, C. (2006). Inequity in equity: Tackling the excellence-equality conundrum. In C. M. M. Smith (Ed.), *Including the gifted and talented: Making inclusion work for more gifted and able learners* (pp. 22–40). London: Routledge.

Zollman, A. (2007). Revisiting the needs of mathematically gifted students: Are students surviving or thriving? *Mediterranean Journal for Research in Mathematics Education, 6*(1–2), 139–147.

Chapter 26
Enhancing Quality and Equity in Mathematics Education for Australian Indigenous Students[1]

Peter Howard, Sharon Cooke, Kevin Lowe and Bob Perry

Introduction

> Education shall be directed to the full development of the human personality and to the strengthening of respect for human rights and fundamental freedoms.
>
> *Universal Declaration of Human Rights 26:2* (United Nations 1948)

Indigenous people have been described as the most educationally disadvantaged group of people within Australia (Council of Australian Governments (COAG) 2009). As a group they do not participate equally with non-Indigenous Australians at all levels of education (Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEETYA 2006)) with their educational outcomes "substantially lower" than non-Indigenous students (MCEETYA 2008). For example,

> While most Indigenous students in metropolitan and regional areas meet the minimum reading standards, the proportion achieving at least the minimum standard of literacy and numeracy skills decreases as the level of remoteness increases.
>
> Australians who do not complete year 12 are less likely to have the same opportunities as those who do. In 2006, year 12 completions for Indigenous Australians were 45.3 %, compared to 86.3 % for non-Indigenous Australians. (COAG 2009, p. 20)

Results from the *Program for International Student Assessment—PISA (2000–2006)* indicated that "[Australian] Indigenous students performed at a substantially and statistically lower level of reading, mathematical and scientific literacy compared

[1] Within Australia, there are two groups of Indigenous people: Aborigines and Torres Strait Islanders. In view of the international character of this publication, the authors of this chapter have chosen to use the term "Indigenous" to represent both groups.

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to their non-Indigenous peers" (De Bortoli and Thomson 2009, p. 51). These results are of significant concern for Indigenous communities and their leaders (e.g., Cape York Partnerships 2009; NSW Aboriginal Education Consultative Group Inc./NSW Department of Education and Training 2004) and mathematics educators (Grootenboer et al. 2009; Perry and Howard 2008; Perso 2005; Warren et al. 2008). They also concern the Australian government.

> Meeting the needs of young Indigenous Australians and promoting high expectations for their educational performance requires strategic investment. Australian schooling needs to engage Indigenous students, their families and communities, in all aspects of schooling, increase Indigenous participation in the education workforce at all levels; and support coordinated community services for students and their families that can increase productive participation in schooling. (MCEETYA 2008, p. 15)

Enhanced educational quality and equity for Indigenous students can only occur through purposeful curriculum change, quality teaching, increased student participation and the engagement of the Indigenous community. Attempts have been made to engage Indigenous students in schooling and to develop strategic ways of co-ordinating school, students, families and community services to enhance the educational outcomes, quality and equity afforded to Indigenous students. However, there is some evidence to suggest that "initiatives to improve the education of Indigenous students through educational policy have had little effect" (De Bortoli and Thomson 2009, p. 52). Quality mathematics curricula that encourage equity, access and engagement with purposeful mathematics learning whilst recognising, maintaining and strengthening the Indigenous students' cultural identity are required (Department of Education, Training and Youth Affairs 2000; Steering Committee for the Review of Government Service Provision (SCRGSP) 2009).

This chapter introduces the voice of an eminent Aboriginal educator to reprise the significant program criteria that are imperative in acknowledging and appreciating the place of social justice in Indigenous students' learning of mathematics. These criteria have been introduced in previous work by the authors (Matthews et al. 2003). Examples of mathematics education programs with which the authors have been involved and that have made substantial differences to the learning of Indigenous students are then measured against these criteria. The chapter argues the importance of the community processes, people relationships and leadership in the enhancement of quality, equity and social capital within communities with high proportions of Indigenous people.

Building Mathematics Curriculum Through Partnership

In 2008, the Australian government introduced a program titled *Closing the Gap on Indigenous Disadvantage* as a focus for the strategic distribution of Federal government funding addressing quality and equity issues for Indigenous families and communities (Department of Families, Housing, Community Services and Indigenous Affairs 2008). Key in this suite of initiatives were the commitments to "halve the gap in reading, writing and numeracy achievements for Indigenous children within a decade" and to "halve the gap for Indigenous students in year 12 attainment or equivalent attainment rates by 2020" (Department of Education, Employment and Workplace Relations 2009). Closing the gap in mathematics quality and equity for Indigenous students can be achieved through building enduring relationships and partnerships amongst Indigenous people, non-Indigenous people and their communities. These relationships and partnerships should be characterised by a sincere and authentic environment of trust and respect (Goos 2004; Howard et al. 2006; Perry and Howard 2008). Such partnerships can result in the school becoming a welcoming place for the students and community members, leading to improved student well-being, learning and achievement (Epstein 2001; Goos 2004). The development of such partnerships is a key strategy for many *Closing the Gap* initiatives including *Make it Count* where the requirements for project schools include the following:

> Specifically, schools will need to commit to developmental work on three fronts:
>
> - Creating a culture and set of practices within the school that genuinely value, actively promote and consistently enable community engagement that supports the learning of mathematics within the school, with an emphasis on the parents of Aboriginal and Torres Strait Islander students and their communities;
> - Curriculum development to ensure that the classroom practices and the activities, assessment and other components of what the students engage with in order to learn mathematics respect and build on the mathematical culture, knowledge and skills of Aboriginal and Torres Strait Islander students and communities; and
> - Professional learning for staff (leaders, teachers and other educators) that builds the necessary cultural competence, respectful and productive strategies for engaging with community and knowledge and skills in mathematics and mathematics pedagogies. (Cooke and Howard 2009, pp. 10–11)

This chapter explores how such objectives might be met in a socially just manner.

Mathematics Education and Indigenous Students—Program Criteria and Reflections from an Aboriginal Educator

The authors of this chapter have previously introduced a set of program criteria against which the success of mathematics education programs for Aboriginal and Torres Strait Islander students in schools might be measured (Matthews et al. 2003). These criteria have been derived from an appreciation of the contexts of Indigenous education (Department of Education, Employment and Workplace Relations 2008; NSW Aboriginal Education Consultative Group Inc./NSW Department of Education and Training 2004; SCRGSP 2009), national mathematics project reports (Department of Education, Science and Training (DEST) 2007), analysis of interviews with Indigenous people (Howard 2001), national professional development initiatives (Australian Principals Associations Professional Development Council 2003) and evidence that meaningful engagement between

Indigenous people and the mathematics education community will enhance the mathematical learning outcomes of Indigenous students (Howard and Perry 2006; Perry and Howard 2008). These criteria form the central organisers for this chapter. For convenience, each is described separately and comment is made on them by an eminent Australian Aboriginal educator. However, it is only through appropriate combinations of these criteria that intervention programs can work towards improvements in Indigenous students' mathematical learning. It should be noted that the order of presentation of these criteria is not linked to any perception of their relative importance.

Sharon Cooke is a Weilwan woman from Brewarrina in north-west New South Wales, Australia. She is an experienced educator in NSW schools and currently holds the position of Aboriginal Education Consultant in a rural Catholic education system. In this section of the chapter, Sharon's thoughts on critical issues in the lives and learning of Indigenous students and how these may impact on their mathematics learning are linked to explanations of the program criteria. Unless otherwise indicated, all quotes in this section are from Sharon.

Enhanced Mathematical Learning

The number of Indigenous students in Australian schools is increasing and the gap between the educational outcomes of Indigenous and non-Indigenous students, as measured by mainstream assessment regimes, is increasing as well (De Bortoli and Thomson 2009; DEST 2007). Enhanced mathematical learning is the key aim of many programs that strive to "close the gap" in Indigenous students' learning. Productive engagement and collaboration amongst governments, Indigenous communities, schools and educational systems can only but enhance the mathematical learning outcomes for Indigenous students. The Australian government has indicated that it will use the *National Assessment Plan—Literacy and Numeracy* (NAPLAN) (MCEEDYA 2009) instrument to measure changes to "the gap." Hence, any mathematics intervention program for Indigenous students needs to show improvements in NAPLAN scores. This may be problematic given that NAPLAN generally reflects the mainstream "school" aspects of mathematics learning most often devoid of any reference to context or culture.

There have been many attempts to develop mathematics programs specifically designed to enhance Indigenous students' mathematics learning (Board of Studies NSW 2007; NSW Department of Education and Training n.d.; Perry and Howard 2003). A national attempt—*Make It Count*—has just begun (Australian Association of Mathematics Teachers 2009). A key aspect of each of these programs has been to investigate curricula and pedagogies that will help Indigenous students see a purpose and meaning in the mathematics they are learning. The need for such approaches is reiterated in the following comments:

> Many of our Aboriginal students either ‘rush’ through their work so they can just get it done and forget about it (or have some time on the computer playing maths games!!) or they dilly dally and go so slow trying to ‘wait out’ the maths lesson or they just refuse to try altogether.
>
> The ‘play’ and ‘fun’ has gone out of it for Aboriginal kids. Gee…It’s a serious subject for them even in Kindergarten. The reality is that even if a page in the book is filled in that doesn’t mean they know what they have done.
>
> The ‘play’ has gone out of teaching and certainly out of maths. It is a scary subject that we all fear by about Year 3 and have given up.
>
> The crowded curriculum is being blamed for teachers not having time to teach other than from stencils and text books because it takes too much time to set up a hands-on classroom for maths. Until maths is taught in other ways and not just from run-off stencils and text books our students will never move forward mathematically.

Social Justice

Social justice is about treating all people with dignity and respect. It is about a community recognising and acknowledging injustices and the development of both appropriate and collaborative actions and processes to address these injustices for individuals or groups so that there is a degree of equality in the overall outcomes. Clearly, Australian Indigenous people have suffered many injustices over many years (Rudd 2008). When considering this construct, it is important to ensure that the very intervention that is designed to enhance mathematical learning outcomes does not introduce other social injustices such as widening the gap that it is hoped to close (Cooke and Howard 2009). For example, many Indigenous families experience significant levels of poverty (SCRGSP 2009). Misunderstandings around the impact of poverty and cultural difference on students’ learning can lead to social justice breaches.

> Many Aboriginal families are dysfunctional because of poverty related issues not because they are black. Many Aboriginal people are poor in soul and poor in spirit because they are poor economically…they live in poor towns…there is a historical background of being poor in Australia.
>
> How is ‘poverty’ addressed in teacher education programs and the development of appropriate curriculum and teaching strategies? How do the teachers cope with being told all the time about the differences between Aboriginal and non-Aboriginal kids?

A human rights approach to education “necessitates a commitment to recognising and respecting the human rights of children whilst they are in school—including respect for their identity, agency and integrity” (UNESCO 2006, p. 2). Valuing the individual Indigenous student’s abilities and history as well as the role language, culture, experiences, expectations and physical factors play in Indigenous student learning will enhance the opportunities for learning mathematics (Matthews et al. 2003).

Empowerment

Empowerment is about Indigenous people gaining the necessary mathematical knowledge and skills to participate in the achievement of authentic educational outcomes. The purposeful engagement between Indigenous people and teachers will strengthen the mathematics learning outcomes for Indigenous students. This is an ongoing process involving mutual respect, critical reflection, caring and group participation where people without an appropriate share of local resources gain a greater control of their own lives (Fridal 1999). In order to achieve well in modern mathematics, students need to develop their problem-solving strategies and their willingness to take some risks in their learning. Many Indigenous children find this quite challenging.

> Aboriginal students need to feel 'safe' to 'have a go'. It needs to be clear to them that it doesn't matter if you 'get it wrong' what matters is having a go.
>
> Many Aboriginal and poor white students come to school with an independence that the school system does not value. They are soon taught not to think independently. They are taught to conform to the school and classroom rules and soon begin waiting for the information to come to them via the teacher rather than to explore, investigate or navigate the world mathematically.

Engagement

Engagement in mathematics learning is about being able to interact purposefully with the learning discourse. It is about Indigenous students being treated with respect and acknowledged as capable learners. Munns and Martin (2005) define student engagement on two levels. For the purposes of this chapter, the second of these levels, "big 'E' engagement," is of particular relevance.

> [B]ig 'E' engagement ('E'ngagement), [is] a wider relationship with school and education. 'E'ngagement is a sense among students that "school is for me". This means that students have a sense that school is a place that works for them and education is a resource that they can successfully deploy in the present and the future. (Munns and Martin 2005, p. 3)

Schools have to be a place of belonging for Indigenous students as much as they are for any other student (Howard 2001). Pedagogies that build strong teacher-pupil relationships, reduce competition, enhance verbal communication, limit direct questioning and emphasise practical experience and group co-operation benefit Indigenous students (Frigo 1999). When such pedagogies are absent, Indigenous students often will distance themselves from learning.

> If an Aboriginal kid is in a boring or mis-managed room they just 'wait it out' and 'dodge' the learning. It is not that they misbehave. They just avoid the learning in a quiet way and it is not until assessment time when the learning decline is noticed.
>
> Many teachers have disenfranchised themselves of teaching our Aboriginal kids by feeling they cannot 'manage the behaviour' so they cannot teach them. I see in so many cases where the Aboriginal kids just leave the classroom or the Principal is called to 'manage' bad

> behaviour so the classroom teacher doesn't 'own' the kids. In the case of kids not engaging because of BORING or UNEXCITING/UNEVENTFUL teaching, particularly in maths, the students is opting to misbehave so they can leave the room either by choice or by invitation. Another way to dodge maths!! Very clever!

Reconciliation

Reconciliation is about "walking in someone else's shoes." It is about taking the time to listen and to care. It is about working together, collaborating to bring about enhanced educational outcomes. It is about sharing and understanding the diversity of culture. It is about appreciating people and their values, language and learning styles. It is about recognising and appreciating difference. For Indigenous students, mathematics learning and teaching is about highlighting strengths in the diversity of their knowledge and their ways of knowing and celebrating what they can do. Enduring, trustful relationships can open up the possibility of genuine reconciliation between the Indigenous students and their teachers.

> Trust!! This is such an important word. If the classroom teacher has not established this with the Aboriginal students then it is a hard road for the teacher.

However, many teachers assigned to rural and remote schools are just beginning their careers and while they might wish to engage in reconciliatory teaching, they do not necessarily have the skills to do this.

> Schools have young, white, inexperienced teachers who have no understanding or concept of what it means to be poor, let alone Aboriginal. These teachers are struggling to find ways to build relationships with our students because they have nothing in common. There seems to be very little common ground between teachers and Aboriginal students. Many primary teachers are often young white females from middle class backgrounds who, in reality, do not have a level of empathy…they don't understand the poorness of the students let alone the culture of the people.

Connectedness

Connectedness is an individual sense of belonging, a feeling of being accepted and knowing that you are valued for who you are. It is about honesty, integrity, being a critical friend in what you bring to any given situation as an important person within the Australian society. Indigenous people relate to their family, place, community and spiritual world. These help determine their beliefs and sense of belonging and, thus, help build aspirations and high expectations of themselves within the educational context.

> Knowing who you are' and having a positive sense of cultural identity is central to Aboriginal and Torres Strait Islander children's social, emotional, intellectual, physical and spiritual wellbeing. (Queensland Department of Education, Training and the Arts 2008, p. 2).

Many Indigenous Australians live away from their traditional country (SCRGSP 2009). This can have ramifications for students' learning.

> My heart and working experiences are telling me that maybe it is more the poorness of our Aboriginal families that is 'blocking' any engagement in the school environment. Yes 'cultural poorness', as well as 'life-soul-prospects poorness'. I am challenging myself in thinking that to drive our teaching of Aboriginal students with a "pure" sense of cultural ways is not the way as many Aboriginal students I work with do not have any real understanding of what it means to be Aboriginal. Yes, they know they are but they don't truly know what that means.

The majority of Australian Indigenous students live and learn in the two worlds of "white" and "black" with external factors affecting their lives and directing their progress. Identity is the basis upon which Indigenous students grow, develop and relate to those about them, including their teachers. Cultural identity is personal and evolves as individuals grow in the knowledge of their cultural backgrounds and as they respond to varying places and circumstances. A significant challenge in educating Indigenous students is for educational systems and teachers to recognise, respect, value and accept the students' *identity* (Howard 2001).

> Our students are struggling with what it means to be "Aboriginal". They know they are but many do not know what that means!!! Many of our students just believe they are 'poor'. They feel poor in knowledge as well as poor in what society has to offer them. They do not see themselves as ever achieving and not necessarily because they are Aboriginal but because they live in isolated areas that they see no way out of or they live in poor large urban areas where they are just as isolated from the 'norms' of society.

Relevance

Relevance is about bringing Indigenous students' environments into the mathematics classroom; providing Indigenous students with the necessary mathematical skills to enable them to look beyond their horizons; and recognising Indigenous students' country in culturally appropriate ways in mathematics curriculum, teaching and learning.

> Engagement of Aboriginal students in the classroom for mathematics is a huge issue because currently there is no real life meaning to them in the way mathematics is being taught. They don't see the way maths fits into their everyday life.

The mathematics knowledge and skills that Indigenous students bring to school, even from a very early age, are often not recognised by teachers. Often, there is a reservoir of numerical knowledge and number relationships in Indigenous communities that is passed on through a myriad of card, number and chance games. The teaching and playing of such games is a latent example of family engagement with numeracy and mathematics.

> Many of our Aboriginal students come to school in Kinder knowledgeable and skilled with numbers. They have sat in and played 'bingo', 'Euchre', 'rummy', 'poker', 'patience', etc for most of their young years with their mums, nans, aunts etc and understand the

> relationship between numbers and the connection to money. They can shop and recognise the change from their purchase as young as 5. Why is it when they commence school in Kinder (many of our Aboriginal students do not attend pre-school) these already learnt skills are not valued and built upon?

When one is striving to develop in Indigenous children the belief that school is for them, it would seem at least counter-productive not to recognise and celebrate the mathematics with which they are already familiar.

Mathematics Programs for Indigenous Students

These program criteria are now used to examine two programs designed to enhance Indigenous students' mathematics learning.

Mathematics in Indigenous Contexts (MIC)

> The aim of MIC was to have school(s) and community work together to develop mathematics curricula that enhanced the knowledge and capacity of the Aboriginal students, community and school(s). MIC was based upon the principle that the mutually beneficial engagement of people and cultures is essential in building a community's capacity for educating Aboriginal students. (Perry and Howard 2008, p. 4)

MIC focuses on the development of culturally and contextually appropriate mathematics teaching units for Indigenous students from Kindergarten to Year 8, to bring about *connectedness* and *relevance* within the curriculum. To achieve the mathematical learning outcomes, targeted schools and teachers were supported by experienced teachers and university mentors. Key features of MIC included:

- Analysis of existing numeracy data to identify where Indigenous students experienced difficulty
- Contextual curriculum design
- Community and parent engagement
- Culturally inclusive curricula and pedagogy in schools to allow Indigenous students to demonstrate their mathematics learning
- Culturally appropriate assessment challenges that are contextual and rich in design and provide opportunities for students to demonstrate real mathematical learning

MIC was implemented in two NSW government primary and two secondary schools (rural and urban) during 2002–2006 (Howard et al. 2006). The project recognised that family *engagement* in Indigenous children's learning in general, and mathematics learning in particular, is of critical importance in that it "provides students with significant positive social capital…heightens parental aspirations for their child's future as well as providing a focus for their expectations on the education system to provide the quality of education necessary to assist Indigenous students achieve these aspirations" (Board of Studies NSW 2002, p. 4). The program emphasised *social justice* and *empowerment* criteria through its active involvement of Indigenous educators, families and community members. The focus was twofold—the professional development of teachers highlighting enhanced mathematical learning and the development and implementation of *connected* and *relevant* contextual mathematics units through the establishment of learning networks.

A central theme of the MIC project was to look at processes for practical and purposeful engagement among schools, teachers and parents of Indigenous students in the development, implementation and evaluation of mathematics curriculum. These improved relationships were seen as having the potential to play a significant role in *reconciliation* by challenging both the negative view about schooling often held by Indigenous students and the views held by schools about Indigenous students' capacity, and willingness to learn and engage in mathematics.

As a result of MIC, Indigenous students found mathematics teaching and learning more relevant and their confidence in mathematics greatly improved. Indigenous students were supported by parents and community members at school and at home and the students themselves were able to contribute to the development of the mathematics learning activities. Indigenous parents and communities developed collaborative partnership with teachers to plan appropriate curriculum. Indigenous parents had opportunities to work in classrooms with students and teachers, increasing their confidence about school mathematics and how children learn. The teachers undertook successful collaborative and cooperative planning with colleagues and the Aboriginal Education Assistants in their schools. They established partnerships with parents and community to develop contextually appropriate mathematics units of work. These teachers increased their understanding of the local community and raised their expectations of the Indigenous students in their class. Indigenous parents and community expressed the view that they had waited a long time to be invited to become real partners in their children's education and such *empowerment* and *engagement* needed to continue (Howard et al. 2006).

Wii Gaay (Clever Child)

From 2002 to 2009, the Catholic Schools Office, Armidale-New South Wales has implemented the *Wii Gaay* mathematics program. The program highlights *social justice* for the gifted Indigenous student and *engagement* across school, family and community. Identified children enter the program at the age of eight or nine years and participate in a four-year program involving biannual residential schools with Indigenous people (Aboriginal Education Assistants) who are positive academic role models for the students. On-line mentorships linking adult Indigenous role models, peer mentors and project co-ordinators feature strongly in the project. The interplay of such groups strengthens the *reconciliation* process between peoples. *Wii Gaay* also focuses on educating teachers about the issues contributing to Indigenous academic underachievement, implementing strategies in a long-term manner that will promote academic achievement.

The residential schools are "theme based" resulting in mathematics *connectedness* and *relevance* to the students and the venues. The teaching/learning activities focus on integrating mathematics teaching strategies with technology such as digital cameras and computers, and the creative arts. All activities are designed to challenge the Indigenous students to bring about *enhanced mathematical learning*. The students continually receive feedback and scaffolding to ensure that they experience ongoing success—increasing their self-efficacy towards learning tasks. A key purpose of the residential schools is to challenge these Indigenous students to work and think mathematically as they use critical reflection in identifying the mathematical strategies they use in moving towards solutions to a number of problem-based questions and activities.

The students' mathematical competencies are assessed orally and with manipulatives. This "testing" is unobtrusive and undertaken in a non-threatening way. Data are presented to the student, classroom teacher and parents. All *Wii Gaay* participants maintain weekly contact through email. There is an online mentoring program during Term 3 each year and school visits are made by the project coordinators and Aboriginal Education Consultant, ensuring regular opportunities to meet and discuss educational progress with the parents, principals, classroom teachers and Aboriginal Education Assistants. It is at these meetings that a DVD of the residential schools is used for teacher professional development and program feedback to parents and communities.

Sharon Cooke summarised *Wii Gaay* in the following way:

> Wii Gaay *empowers* Aboriginal communities by encouraging/inviting parents/carers/guardians of the students involved in the program to attend residential schools (camps) and to be involved in school based development around the program. We constantly seek parent, Aboriginal Educator and community advice/feedback on the program. *Reconciliation* is being addressed by inviting teachers from classrooms who have Wii Gaay students to come and see what happens at our residential schools and how teaching and learning happens without the constraints of a classroom environment. They *walk in the shoes* of our students who are potentially high achievers but need to be 'taught' in others way and with little to no chalk and talk. Self determination will come when these students achieve what they are meant to achieve. They will be the leaders of our future.

Unfortunately, the Indigenous students' success within the residential program is not generally transferred to their mainstream classrooms. There remain many teacher professional development challenges around the modification of teaching practices and teacher attitudes towards raising their expectations for Indigenous students' mathematical success;

Conclusion

Across Australia over the past 30 years, there have been many mathematics intervention programs developed and implemented with the aim of enhancing the mathematical learning outcomes of Indigenous students. While there is some evidence that such enhancement has occurred, the frustrating reality is that it has not occurred at a rate greater than the corresponding rate for non-Indigenous students. If anything, the gap between the achievements of Indigenous and non-Indigenous students on standardised testing in mathematics has widened over these years (De Bortoli and Thomson 2009). With the *Closing the Gap* strategy, the Australian government is determined to halve this gap by 2018.

This chapter has demonstrated the need for mathematics intervention programs to address a number of criteria related to the social, cultural and community contexts of the Indigenous learners and their families, as well as the particular mathematical characteristics of the material to be learned. Such programs need to address the "lack of congruity between the student's home environment or culture and the school's culture" (Boethel 2003, p. 14) and recognise that such differences can place the student at a disadvantage. Sharon Cooke's words provide many examples of how this can happen. While programs would expect the Indigenous students and their families to work towards improving the level of congruity, they should also expect that there will be movement by the school as well. The criteria discussed in this chapter provide important guidance for these endeavours.

Mathematics in Indigenous Contexts and *Wii Gaay* have provided exemplars for the ways in which the criteria can be used to design, implement and evaluate programs of mathematics learning for Indigenous students. These programs, like so much in Australian education, were based in one region or one state. The new national program *Make It Count (2009–2012)* has the opportunity to apply these criteria to develop school-family-community partnerships across Australia that will not only make a difference to Indigenous students' mathematical outcomes but will do so in a way that is sustainable through impact on strengthening people's beliefs in their own worth and their own identity. *Make it Count* is the next phase, a "chronological" attempt to include the criteria discussed in this chapter in a national mathematics project that can impact significantly upon mathematics teaching and learning for Indigenous students. As a nation, Australia is running out of time. We cannot countenance yet another school-centred program. *Make It Count* must assist schools and communities to work together to enhance the mathematical development of the Indigenous students in those communities.

References

Australian Association of Mathematics Teachers. (2009). *Make it count: numeracy, mathematics and indigenous learners*. Adelaide: Author. http://makeitcount.aamt.edu.au/. Accessed 30 Nov 2009.

Australian Principals Association Professional Development Council. (2003). *Dare to lead…Taking it on*. Canberra: Commonwealth of Australia.

Board of Studies NSW. (2002). *Mathematics in indigenous contexts: A project paper*. Sydney: Author.

Board of Studies NSW. (2007). *Mathematics in indigenous contexts K-6*. Sydney: Author. http://ab-ed.boardofstudies.nsw.edu.au/go/mathematics-andamp-numeracy/maths-k-6.

Boethel, M. (2003). *Diversity: School, family and community connections*. Austin, TX: National Center for Family and Community Connections with Schools. http://www.sedl.org/pubs/catalog/items/fam35.html. Accessed 30 Nov 2009.

Cape York Partnerships. (2009). *Education stream*. http://www.capeyorkpartnerships.com/education. Accessed 30 Nov 2009.

Cooke, S., & Howard, P. (2009, August). *"Can WE address the issues surrounding Aboriginal education?" "Yes we can!!!! Together!"* Paper presented to the *Dare to Lead* conference, Canberra. http://makeitcount.aamt.edu.au/Resources/Indigenous-education. Accessed 30 Nov 2009.

Council of Australian Governments (COAG). (2009). *National indigenous reform agreement (Closing the gap)*. http://www.coag.gov.au/intergov_agreements/federal_financial_relations/docs/IGA_FFR_ScheduleF_National_Indigenous_Reform_Agreement.pdf. Accessed 30 Nov 2009.

De Bortoli, L., & Thomson, S. (2009). *The achievements of Australian's indigenous students in PISA 2000–2006*. Melbourne: Australian Council for Educational Research.

Department of Education, Employment and Workplace Relations. (2008). *National goals for indigenous education*. http://www.dest.gov.au/sectors/indigenous_education/policy_issues_reviews/national_goals_for_indigenous_education.htm. Accessed 30 Nov 2009.

Department of Education, Employment and Workplace Relations. (2009). *Indigenous schooling*. http://www.deewr.gov.au/Indigenous/Schooling/Pages/default.aspx. Accessed 30 Nov 2009.

Department of Education, Science and Training (DEST). (2007). *What works. The work program: Core issues 4: Numeracy*. http://www.whatworks.edu.au/upload/1252234248223_file_4Numeracy.pdf. Accessed 30 Nov 2009.

Department of Education, Training and Youth Affairs. (2000). *National Indigenous English literacy and numeracy strategy 2000–2004*. Canberra: Author.

Department of Families, Housing, Community Services and Indigenous Affairs. (2008). *Closing the gap on indigenous disadvantage: The challenge for Australia*. http://www.fahcsia.gov.au/sa/indigenous/pubs/general/Documents/closing_the_gap/foreword.htm. Accessed 30 Nov 2009.

Epstein, J. L. (2001). *School, family, and community partnerships: Preparing educators and improving schools*. Boulder: Westview Press.

Fridal, T. L. (1999). The role of indigenous parents in public education: barriers to change in an urban setting. *Canadian Journal of Native Education, 23*(2), 139–225.

Frigo, T. (1999). Resources and teaching strategies to support indigenous children's numeracy learning. Melbourne: Australian Council for Educational Research.

Goos, M. (2004). Home, school and community partnerships to support children's numeracy. *Australian Primary Mathematics Curriculum, 9*(4), 18–20.

Grootenboer, P., Jorgensen, R., Niesche, R., & Sullivan, P. (2009). Reforming mathematical pedagogy in remote indigenous context. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 695–711). Palmerston North: MERGA.

Howard, P. (2001). Beliefs about the nature and learning of mathematics in years 5 and 6: The voices of Aboriginal children, parents, Aboriginal educators and teachers. Unpublished PhD dissertation. Sydney: University of Western Sydney.

Howard, P., & Perry, B. (2006). Issues in teaching mathematics to Aboriginal students. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces* (pp. 293–300). Sydney: Mathematics Education Research Group of Australasia.

Howard, P., Perry, B., & Butcher, J. (2006). *Community capacity research project*. Sydney: Board of Studies NSW.

Matthews, S., Howard, P. & Perry, B. (2003). Working together to enhance Australian Aboriginal students' mathematics learning. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity* (pp. 9–28). Deakin University, Melbourne: Mathematics Education Research Group of Australasia.

Ministerial Council on Education, Early Childhood Development and Youth Affairs (MCEEDYA). (2009). *National assessment program—literacy and numeracy*. http://www.naplan.edu.au/. Accessed 30 Nov 2009.

Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA). (2006). *Australian directions in indigenous education 2005–2008*. Carlton: Curriculum Corporation.

Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA). (2008). *National declaration on educational goals for young Australians*. Carlton: Curriculum Corporation. http://www.mceecdya.edu.au/mceecdya/melbourne_declaration,25979.html. Accessed 30 Nov 2009.

Munns, G., & Martin, A. J. (2005). *It's all about MeE: A motivation and engagement framework.* Paper presented to the 2005 Annual Conference of the *Australian Association for Research in Education,* Sydney. http://www.aare.edu.au/05pap/mun05400.pdf. Accessed 30 Nov 2009.

NSW Aboriginal Education Consultative Group Inc./NSW Department of Education and Training. (2004). The report of the review of Aboriginal education. Yanigurra muya: Ganggurrinmya yaarri guurulaw yirrigin.gurray (Freeing the spirit: Dreaming an equal future). Sydney: Author.

NSW Department of Education and Training. (n.d.). *Count me in too indigenous.* http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/countmeintoo/indigenous/index.htm. Accessed 30 Nov 2009.

Perry, B., & Howard, P. (2003). *Counting On: Evaluation of the impact of the Counting On program.* Sydney: NSW Department of Education and Training. http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/assets/pdf/coeval2002.pdf. Accessed 30 Nov 2009.

Perry, B., & Howard, P. (2008). Mathematics in indigenous contexts. *Australian Primary Mathematics Classroom, 13*(4), 4–9.

Perso, T. (2005). *Improving Aboriginal numeracy.* Adelaide: Australian Association of Mathematics Teachers.

Queensland Department of Education, Training and the Arts. (2008). *Foundations for success: Guidelines for an early learning program in Aboriginal and Torres Strait communities.* Brisbane: Author.

Rudd, K. (2008). *Apology to Australia's indigenous peoples.* http://www.aph.gov.au/house/Rudd_Speech.pdf. Accessed 30 Nov 2009.

Steering Committee for the Review of Government Service Provision (SCRGSP). (2009). *Overcoming indigenous disadvantage: Key indicators 2009.* Canberra: Productivity Commission.

United Nations. (1948). *Universal declaration of human rights: Article 26.* http://www.un.org/en/documents/udhr/index.shtml#a26. Accessed 30 Nov 2009.

UNESCO (United Nations Educational, Scientific and Cultural Organisation). (2006). *A Human-rights approach to education for all.* http://unesdoc.unesco.org/images/0015/001548/154861E.pdf. Accessed 30 Nov 2009.

Warren, E., Cooper, T. J., & Baturo, A. R. (2008). Bridging the educational gap: Indigenous and non-Indigenous beliefs, attitudes and practices in a remote Australian school. *Educational Practice and Theory, 30*(1), 41–56.

Chapter 27
Qualities of Respectful Positioning and Their Connections to Quality Mathematics

David Wagner and Lisa Lunney Borden

In this chapter, we identify qualities of respectful positioning and their connections to quality and equity in mathematics. We consider these qualities in the context of the "Show me Your Math" (SMYM) event, which has run since 2006 and has become increasingly popular amongst Aboriginal communities in Atlantic Canada. Mi'kmaw and Maliseet students are invited to do ethnomathematical investigations to show others the mathematics in the practices of their communities. We draw upon Harré and van Langenhove's (1999) positioning theory to describe the shifting storylines that are contributing to equity and quality within participating mathematics classrooms. Our sense of quality, equitable mathematical experiences focuses on wholeness.

Using examples of students' ethnomathematics and our reflections on the nature of the positioning, we will demonstrate ways of seeing quality mathematics learning and relate these qualities to concerns for equity. We characterize quality learning in terms of wholeness. Thus, our sense of the word "quality" is underpinned by equity. Quality and equity are inseparable.

Wholeness resists fragmentation, thus quality mathematics experiences require *cultural synthesis* bringing together cultures and values from mathematics and the community, *personal holism* including the child's experiential, conceptual, and spiritual development, and *intergenerational interaction*. Our interest in wholeness is another way of talking about equity. We will analyze student work in terms of values developed for mathematics and local community values.

The chapter is structured around stories of student participation to demonstrate students making sense of mathematics in relationship to their places in their community. Though the stories exemplify aspects of wholeness, we identify ways in which we would hope for more wholeness. Before this, we draw on Aboriginal scholarship relating to the context in Canada to give an account of local Aboriginal

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views regarding quality education, which we synthesized to develop a multi-dimensional approach to wholeness. We follow this with an overview of the positioning theory that was instrumental in the development of the SMYM event, and an account of the positioning in the event's development. These sections set up the stories of student participation and our analysis of these stories in terms of the qualities we outline. In our reflection, we consider how the SMYM event might be relevant to other contexts.

Quality Mathematics Education in Context

Our conception of quality education has arisen mostly from our conversations within the communities and is underpinned by concerns for equity. Nevertheless, we outline our sense of what quality means by drawing mostly on publications that reflect this sense. The relevant publications include scholarship relating to Aboriginal education, especially in Canada, and also professional literature relating to mathematics teaching.

The participant Mi'kmaw schools have a stated goal of developing community-appropriate educational standards that include a focus on language and culture yet, by law, they are required to offer comparable curriculum to the public schools. Recent efforts in school improvement initiatives require the participant schools to write provincial assessments in mathematics and literacy. For these and other reasons, no school would openly choose to move away from mainstream mathematics nor would they say that such achievement in mathematics is unimportant. However, the tension between the often-competing goals of community relevance and performance for external standards presents challenges for mathematics education in Mi'kmaw communities. The provincially developed curriculum is underpinned by the National Council of Teachers of Mathematics' (NCTM) *Principles and Standards of School Mathematics* document (NCTM 2000). Although the NCTM (2009) has stated, "a mathematics curriculum should focus on mathematics content and processes that are important and worth the time and attention of students" (p. 1) this assertion appears to be focused on a mainstream view of mathematics. We ask what mathematics content and processes are worth the time and attention of young Mi'kmaw and Maliseet students and, furthermore, we ask who decides. Are the same content and processes most relevant to children in all contexts?

A dominant theme in the literature is that Aboriginal education should seek "to heal and transcend the effects of colonization" (Cajete 2000, p. 181). Hampton (1995) argued that Aboriginal education cannot ignore the reality of colonization but rather must address the issue directly. Aboriginal education needs to move toward decolonization which can be seen as a process of "deconstruction and reconstruction" (Battiste 2004) that "engages with imperialism and colonialism at multiple levels" (Smith 1999, p. 20). This demands the critical examination of the hegemonic structures of mainstream education that continue to perpetuate the values of colonialism (Battiste 2004).

Cappon (2008) argued that Aboriginal education holds the view that learning is holistic, lifelong, experiential, rooted in Aboriginal languages and cultures, and spiritually oriented. He claimed that it is a communal activity with all community members playing a role and that it integrates both western and Aboriginal knowledge. Orr et al. (2002) have shown how this notion of bridging Aboriginal knowledge with mainstream curriculum has been worked at effectively by Mi'kmaw teachers in one of the participant communities. These teachers provided a quality education through the incorporation of cultural practical knowledge along with Indigenous pedagogical, relational, and political consciousness.

Bringing together these articulations of values within Aboriginal communities, we note how they all relate to wholeness in some way. To facilitate analysis, we will use three views of wholeness that we developed through analysis of the data and synthesis of the literature, though we acknowledge that there are not clear boundaries between these three ways of looking. The three views of wholeness relate to cultures, to the person, and to generations.

Firstly, we heard and read a common thread pointing to the importance of connecting mathematical values and community values. When Mi'kmaw and Maliseet children do mathematics they work at the intersection of at least two strong discourses—the mathematics discipline and community tradition. A quality mathematics experience must involve *cultural synthesis*, honoring values and practices from both discourses. Such synthesis addresses the call for decolonization (transcending colonization) not by ignoring or refuting the accomplishments of the colonizing cultures but by adjudicating them in terms of community values.

Secondly, we noted a common interest in the need to integrate all aspects of the child's personhood. It is a human violation to separate them as different and independent aspects, but referring to their distinctness helps recognize their integration. The *personal holism* that is a necessary part of quality mathematics experience is demonstrated well in a variety of North American Aboriginal medicine wheels, which embody the connections of the different aspects of the person. We will focus attention on the experiential, conceptual, and spiritual development of children, all of which are necessary.

Thirdly, a quality mathematics education experience requires connections among the generations. *Intergenerational interaction* connects elders, children, and others. We emphasize the word "inter-action," which emphasizes the necessity that each generation takes action and responsibility in relationships in educational settings.

Our focus on wholeness is in line with current scholarship on equity in mathematics education. Gutiérrez (2007) argued that the conception of equity should include four dimensions: access, achievement, identity, and power. Because research related to equity in mathematics has tended to focus on access and achievement, these aspects are placed on the dominant axis in her model with identity and power comprising the equally important critical axis.

Gutiérrez argued for wholeness with her recognition that many students have been marginalized by mathematics because they are asked to deny their identity in order to participate in the dominant view of mathematics. She pointed to the work of ethnomathematicians and scholars who promote culturally relevant mathematics as a source of identity within mathematics education. These streams of scholarship involve what we call cultural synthesis because they connect mathematics' disciplinary values with contextual values from students' cultures.

The window/mirror metaphor Gutiérrez used for describing identity connects to holistic identity: "students need to have opportunities to see themselves in the curriculum (mirror) as well as have a view onto a broader world (window)" (p. 3). Her take on identity is closely related to power. The importance of power becomes very clear with a focus on positioning. The power dimension involves not only questioning who has power in the classroom in terms of who participates, who talks, and so on, but also as it pertains to using mathematics to question power relationships in the world and seeing mathematics as a human endeavor.

We recognize that our focus on wholeness attends more to the critical axis than the dominant axis of her model, but reiterate that leaders in the communities in which we have been having conversations have been unequivocal about the need for promoting mathematics achievement to bring expertise into the communities while valuing wholeness. The students need access to the mathematics that is recognized and valued outside the communities. Access and achievement are valued, but not above community values. Nevertheless, quality mathematics achievement and participation is a condition of equity.

Positioning Students as Participants in Community Interaction

Positioning theory was instrumental in the development of our research choices. This theoretical perspective on positioning follows from an edited book by Rom Harré and Luk van Langenhove (1999) and from Wagner and Herbel-Eisenmann`s (2009) elaboration of the theory in the context of mathematics education. In general, "positioning" is taken to refer to the way people use action and speech to arrange social structures. Words and associated actions evoke images of known storylines and positions within those stories. When one person invokes a storyline, others in the interaction may be complicit in this positioning or resistant to it.

Davies and Harré (1999) explained how positioning theory draws attention to "immanent" practices, as opposed to "transcendent" discourse structures (e.g. a student's relationship to the transcendent discipline of mathematics), which Wagner and Herbel-Eisenmann (2009) showed to be a common scholarly focus in mathematics education. With positioning theory's attention to immanent relationships, aspects of human interaction become more noticeable because the larger discourses are ignored, even characterized by Davies and Harré as myths. Wagner and Herbel-Eisenmann, however, maintained the promotion of attention to immanent practice without relegating discourses as inconsequential because people take discourses as being real in their own right and powerful in immanent interactions. In this way, so-called mythological disciplines, such as mathematics, are, for example, as real as race, which is said to be a myth (a human construct with no real basis) but which has substantive manifestations in human relationships. Certain people (e.g. teachers) are positioned as mediums or representatives of transcendent influences such as “mathematics.” Wagner and Herbel-Eisenmann’s conclusion is in line with our view of cultural synthesis, as they argue against stripping mathematics of its power (demythologizing it) in favor of inviting new mathematical narratives that recognize mathematics in places that it has often been ignored or marginalized (remythologizing mathematics).

We began our research in Mi’kmaw communities by interviewing Aboriginal elders to identify some of their everyday practices (both traditional and current) that could be deemed mathematical. This typical approach to ethnomathematics research (c.f. Powell and Frankenstein 1997) relies on Bishop’s (1988) articulation of activities that are potentially mathematical (practices that involve counting, measuring, locating, designing, playing, or explaining) and on the assumption that any mathematics is an artifact of a particular culture.

Although we were excited to hear the stories the elders were sharing with us, in reflection on this research we saw ourselves as mediators, interfering with the intended process of connecting students to the mathematics in their communities. We were careful to orient our conversations with participants around mutual respect, yet we still found connections with colonialist storylines as we observed that some participants were trying to be helpful by telling us what we wanted to know. While we appreciated this spirit of cooperation because it seemed generative for our planned research, it was also a little disturbing to have participants ask us if they were telling us what we wanted to hear. On reflection we recognized two concerns. First, we did not want to be seen as the ultimate audience but we often felt positioned in that way (and we were complicit in this positioning as well). Second, we worried about authenticity because participants were subjecting themselves to our agenda and we talked less about their agenda(s).

The interpersonal relationships appeared to be similar to the diagram in Fig. 27.1. The community experts were responding to our request for them to give us information to pass on to teachers who in turn would pass it on to the children. The children would then do something called “work” for the teachers.

Attending to positioning opened up new opportunities. We realized that the conversations would be more authentic if the children themselves talked with elders and others to find mathematics in traditional and modern community practices. We needed to remove ourselves from the position of mediums. Also, reflecting on Morgan’s (1998) research that underscores the importance of audience in students’ mathematical writing, we realized that positioning the children as the ultimate audience in the ethnomathematics conversation affords them no opportunities to address an audience other than their teacher, and certainly no imperative

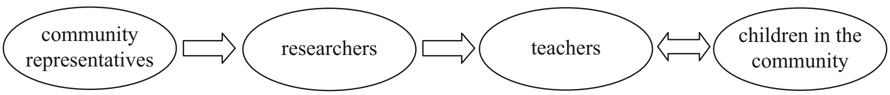

Fig. 27.1 Interpersonal positioning in the initial ethnomathematical work

to engage in real problems/issues faced by their community. New storylines were necessary.

As a result, we initiated a new series of conversations structured to prompt community members to talk and listen to each other about everyday mathematics in traditional and modern community settings. From our dissatisfaction emerged the SMYM event that changed the interpersonal positioning in substantive ways.

We approached some schools with the idea for an SMYM contest in which students would be invited to do projects exploring the mathematics in their everyday lives. We planned to have students submit their work to a web site that would be hosted by the Atlantic Canada First Nation Help Desk, an existing infrastructure that supports communication amongst communities via the internet. Such "contests" are commonly used in this region to promote community-based education and to develop cultural resources for teachers and students. While this plan was well received, the teachers in our initial meeting wanted more than a web site. They suggested schools host local mathematics fairs and send selected students to a regional fair where they could share their work with others.

To substantiate the break from the school tradition of students doing work for teachers as audience, we also suggested that instructions for the contest be given in a video. The teachers in the workshop contributed to the structuring of the video, but we produced it. It featured Aboriginal people, including an elder, a middle-aged teacher, and children, all asking the viewer (the student) to "show their math." We felt that the form of this video helped students see the community as their audience instead of their teachers. In response to this prompt, school children interviewed elders, experts in crafts, and others to explore mathematics done in their communities in historic and modern times. They published their work on the internet site and also presented their work to the region's communities in a math fair.

The interpersonal relationships structured with the SMYM event are more complex than the relationships in our initial work. Figure 27.2 represents an attempt to diagram the relationships. The teachers (and some other community representatives, including elders) were and continue to be in conversation with us. They initiate student investigations that involve students in conversation with experts in the community. The students report back to the community, to their teachers, and to us at mathematics fairs and also to the outside world because their work is put on the web site (http://schools.fnhelp.com/math/showmeyourmath/index.htm).

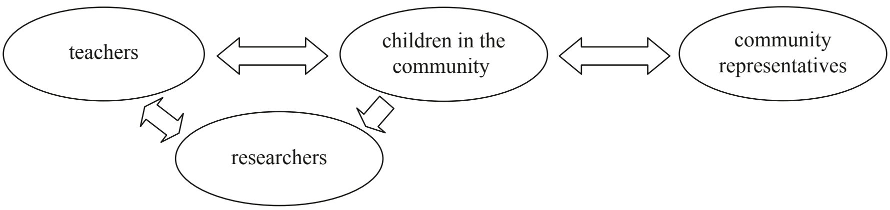

Fig. 27.2 Interpersonal positioning with the "Show me Your Math" event

We elaborate elsewhere (see Wagner and Lunney Borden, in press) on these shifting positioning structures and associated storylines, and on the power of positioning theory for interrogating relational practice. This chapter has a different focus. We draw upon examples of student work in the SMYM event as examples of quality equitable mathematics education. With students positioned as researchers and disseminators of mathematical knowledge, this project has done what Wagner and Herbel-Eisenmann (2009) called for in their theoretical article on positioning: the SMYM project has given students an opportunity to "identif[y] with storylines that are not traditionally a part of mathematics classroom discourse" (p. 13). These new storylines brought about qualitatively different relationships for students with other community members and with mathematics as a field of study.

Examining Student Work

We now turn our attention to the work produced by students for the SMYM event to illustrate the different views of wholeness that we see as central to quality mathematics education. We ask to what extent students demonstrated that their experience involved cultural synthesis, personal holism, and intergenerational interaction. No one example exemplifies our vision for wholeness perfectly, but each example shows aspects of good relationships and helps point the way to an ideal.

Authorities on Efficient Shapes

A unit in *Mathematical Modeling—Book 1* (Barry et al. 2000), the current Grade-10 text in Nova Scotia, prompts engagement in a series of investigations and exercises that would have students examine the geometry of packaging. The unit includes a lesson on "the economy rate"—the ratio of the volume to the surface area of a shape. Through investigations, students discover that a cylinder with its height equal to its diameter is the most economical cylindrical container for a given volume. In 2009, a group of Grade-10 students from one of the participant schools brought this textbook knowledge to the community practice of basket making.

They invited two community elders to come to class and teach them how to make baskets. They were surprised to discover these elders already knew about efficient containers: the elders could identify the baskets that needed the least material for their capacities. A student remarked, "They already knew which [basket] was the most economical. They didn't have to do all the math that we did. We had to do the math to find out which ones it was." We were not present to see "all the math" the students did, but it is clear that they were using formulas to explore a geometry problem and that the presence of the elders prompted them to explain and justify their work more than they would have with only their teacher as an audience. Explaining to a teacher is less natural because the teacher is assumed to already know and understand. Thus, there is evidence of multiple processes promoted by the NCTM (2000): problem solving, communication, connections, and reasoning.

The cultural synthesis is particularly strong in this example, as traditional community knowledge is explicitly set alongside textbook knowledge. This synthesis was especially powerful because students were aware of the juxtaposition and central to arranging it. The textbook was repositioned in that its typical position of dominance in classroom authority structuring was challenged by revelations that there were more local authorities. This challenge of textbook authority could be seen as an attempt to demythologize the powerful discipline of mathematics, but it also could be seen as a remythologizing. Is mathematics diminished when academic authorities are brought alongside local, cultural authorities? No, the sources of authority corroborate each other: both would garner more respect from the students, whose sense of authority also increased as they arranged and interpreted the connection.

The intergenerational aspects of this cultural synthesis are also evident. The complicated mathematics of the students' textbook is remythologized as common sense often used by the elders. No textbook author was needed to tell these elders, or the many generations of basket makers they learned from, which container was the most economical. In the student's observation of this elder knowledge, we see that students see this knowledge as part of their own cultural traditions, a piece of their identity as Mi'kmaw people.

The shift in positioning contributes to the development of personal holism because it relates to the students' identity. Furthermore, the hands-on experience of building the baskets, added to the usual task of observing/measuring capacities, provided students with a more concrete experience of the concept in the textbook, helping to build conceptual understanding: their bodies were involved in the development of their understanding.

We also note that aspects of wholeness were not evident to us in this example. Regarding personal holism, we note the absence of spiritual development. We wonder how this aspect could have been addressed in the classroom experience. Was there a space provided for elders to share stories associated with basket making such as those that provide insight into the proper way to select the tree, gather the wood, make the strips, and so on? We were not present in the classroom during this interaction and we did not explicitly ask questions about spirituality at the math fair so we are unaware of the degree to which this aspect of personal holism was reached. Nevertheless, the intergenerational conversation about basket making opened the possibility for future interaction that may include more spiritual dimensions.

Drum Making

In a project from the 2007 math fair, a Grade-4 class brought community knowledge to the forefront in mathematics class as they made hand drums and prepared a PowerPoint™ presentation to submit for the website documenting their experience. This project differs from the example described above because it started with a community practice rather than from school-based mathematics practices.

The presentation opens with a photograph of three wooden frames sitting on the table and saying, “We started our drums with a 12 sided pine frame. The wood burned turtle you see is a starting point for threading our drums.” On the next slide there is a frame placed on a deer hide with the text: “When making a drum we need to focus on balance and centering. In Mathematics we focus on Symmetry.” The next few slides show pictures of a community member teaching the students how to make the drums. One photograph shows her demonstrating to the children how to measure the sinew using arm lengths, and another shows four children standing at the front of the classroom, each about a meter apart, holding parts of the outstretched sinew. Next, a slide draws attention to mathematical connections with these statements: “We learned the importance of measurement in preparing our materials” and “We measured the diameter and compared our new shape to a circle.” A series of photographs showing the children making their drums explain, “As we thread the sinew through we first go across the diameter of the drum frame starting at the sign of the turtle” and “It is important that we bring everything to the centre to maintain the balance of our drums.” The concluding text, accompanied by an image of the entire class proudly showing their drums, proclaimed, “We learned many things when making our drums. The most important is to maintain balance and centre in our lives.” Throughout the slide show, the Mi’kmaq Honor song plays in the background.

There are significant storylines in this example that differ from typical classroom mathematics storylines. Again, there is intergenerational interaction as the children learned from a community member known for her expertise in drum making as well as her significant traditional spiritual knowledge, which is a shift from the usual storyline of learning from the teacher. We note that there could be a greater connection made between the community knowledge and the school mathematics as it is unclear how explicitly the connections were made. Nevertheless, there is a degree of cultural synthesis with the recognition that the drum-making values align with mathematical obsessions, namely symmetry, and working from a point of origin. The drum making seems to be the dominant aspect of learning with the mathematics added in. We wonder to what extent the students recognized the importance of the mathematical connections. We can imagine them seeing the connection as a glib assertion. This shift to privileging the community knowledge to eclipse mathematics may be completely appropriate in this instance because the typical classroom experience is just the opposite, but leaves open questions about the depth of the cultural synthesis.

Personal holism is evident in the ways that the children were positioned as active agents using their hands (and eyes), discovering both mathematical and cultural knowledge in a way that is rooted in spiritual traditions, consistent with Cappon’s (2008) vision of Aboriginal learning. They were learning from doing, which develops their experiential knowledge, yet we wonder what mathematics they learned. There was ample opportunity for strong connections with mathematics in the students’ curriculum (symmetry) and beyond (the significance of an arbitrary origin) but in our conversation with students, they told us mostly about the physical and spiritual aspects of the drum making. Perhaps this is to be expected of children at this age.

We chose this example because it exemplified the spiritual development of the child, which is an aspect we have seen lacking in many other projects. Relating to Doolittle's (2006) concerns, the students are not being led away from the culture, rather they are being led toward important cultural and spiritual lessons that honor the drum making and this experience "pulls in" mathematics. The more usual direction of cultural synthesis seems to be a focus on some mathematics followed by a look at community practices that might be pulled into the mathematics. Doolittle called this pushing the mathematics into the culture. The spirituality is especially evident in the students' concluding statement that the most important thing they learned was a lesson about living a balanced and centered life. This was reaffirmed at the math fair where one of the students was excited to tell us about what his class had done (the student is "T," the interviewer is Lunney Borden, "LLB"):

T: This one that I made, it has twelve sides so that's a dodecahedron, right. While I was making it we took time because usually people take like twelve days but we took an hour and a half. (*He turns the drum over to show the back*). So we used um, I think it was red deer or red moose, I don't know actually but right here it tells you how to make it. (*He leans forward to point out the sheet of instructions posted in front of their drum display.*) We put a sponge in the drumsticks. We took our time with these. (*He pauses for a bit hitting the drum a couple times with his drum stick, then turns the drum over again revealing the back*). The turtle represents the day. The twelve sides is like twelve in the head so that's like an Indian drum so if you hit at the turtle (*He turns the drum over and hits the drum at the turtle [12:00] position*) that's um, you're going to the spirit world. If you hit over here you're praying for the boys (*He moves the drumstick counterclockwise to the west position*) down here (*south position*) you are praying for the girls and over here (*the east position*) is for the ones that are coming.

LLB: Cool, so did you ever think that was math? Building a drum?

T: Yeah, I think it was because we asked a lot of questions, we learned about it and we ended up learning and um…she's going to come back and teach us a little bit of singing.

For this boy, the cultural and spiritual components eclipsed the mathematics learning.

Reflection

As demonstrated in the projects described above, intergenerational interaction in the form of learning from community members was often present in the SMYM projects and this interaction seemed to contribute to the promotion of cultural synthesis and personal holism. This storyline is consistent with traditional educational practices in pre-contact society and continues in many aspects of informal learning to date.

Joanne, a principal in one of the schools, described to us an experience of working with her six-year-old daughter on her SMYM project. She explained how in helping her daughter think about a project idea, she remembered a game her mother and aunts had played when she was a child. The game, *kunte'jl* (little stones) is quite simple but involves counting and coordination. Play involves flipping one's hand and catching the stones in the palm, on the back of the hand, and continuing back and forth. Joanne explained that her daughter was intrigued by the game and immediately began playing it with her sister. Though Joanne is an educator who values academic knowledge and achievement, the most significant aspect of this experience for her seemed to be the opportunity to tell the story to her daughter. Joanne told us that she had almost forgotten about this game that she had played with her mother. The SMYM project prompted three generations of Mi'kmaw women to share a family story. This kind of sharing is highly valued in the Mi'kmaw communities, especially because, as Joanne noted, if her daughter hadn't asked her about the game, it could well have been forgotten and forever lost to the community. Knowledge remains alive as it is engaged in intergenerational experiences.

We claim that the kind of learning we describe here, in which students connect hands-on experiences, mathematical abstraction, and community cultural traditions through intergenerational interactions, exemplifies quality and equity in mathematics education. In addition to drawing on community ideas of quality, we see these projects as connecting to NCTM standards and principles, in particular, those relating to communication, connections, and equity. Students make connections between mathematics and their own community contexts and in turn communicate this learning using the language of mathematics and the language of their community in some instances. The accounts we have given do not demonstrate the effect of wholeness on student achievement but they do show an engagement that embraces cultural identity. We are confident that such engagement translates into qualities that express themselves in achievement and educational choices that improve measures of participation. With mathematical experiences that demonstrate wholeness, students' mathematical reasoning is not positioned as belonging outside of the community. Instead, it is positioned as a part of (or at least connected to) community knowledge and practice. Thus, we believe that such an approach promotes equity by addressing the critical questions of identity and power while also supporting increased access and achievement. The qualities of achievement and participation are a condition of equity, and are meaningless (perhaps destructive) without wholeness and its implications for identity and power.

Ole Skovsmose commented from the floor in a plenary discussion at the Symposium on the Occasion of the 100th Anniversary of ICMI in Rome in 2008 that he would like to see research from Aboriginal or developing world contexts not advertise these contexts in their titles because identifying these contexts suggests that the children in the majority cultures of developed countries are the only normal children. Research from majority cultures is not typically identified by its context: there is no article title like "American students' conceptions of their identity in mathematics" or "European boys and their mathematical positioning." Our title does not locate the research described here in its context, because we want to argue that quality education demands wholeness in any ethnic or socio-economic context.

In our reflections, we are compelled to ask how our experience in an Aboriginal context might speak to other contexts. Is it reasonable to aim for wholeness elsewhere? And, how might our experiences with the SMYM event guide others interested in aiming for wholeness in other contexts? Disengagement from mathematics is not an issue that is exclusive to Aboriginal communities; on the contrary, we see that it is a pervasive concern for many communities. Recalling the NCTM's (2009) claim that mathematics "must be taught and learned in an equitable manner in a setting that ensures that problem solving, reasoning, connections, communication, and conceptual understanding are all developed simultaneously along with procedural fluency" (p. 2), we note how the SMYM event fulfils many aspects of this demand.

We acknowledge that in any mathematics classroom, multiple and perhaps competing discourses may be at play. We see the ethnomathematical approach used in the SMYM event as providing a path for any student to investigate the intersection of competing discourses from their own cultural view and to examine the role of school-based mathematics through a critical lens. We have demonstrated above that this approach can enable cultural authorities to corroborate mathematical authorities, promoting greater cultural synthesis. The kinds of interactions and the wholeness they embrace can come from Aboriginal students working with baskets, drums, and games, but they can just as well come from Anglo-Canadians working with their grandmas' dishes (as suggested by Doolittle in his 2006 plenary address to the Canadian Mathematics Education Study Group). Cultural synthesis, bringing mathematical and local knowledge together with intergenerational experiences that value the whole person, can be valuable in any context. In such an environment quality mathematics is inseparable from equitable experience.

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References

Barry, M., Small, M., Avard-Spinney, A., & Burnard Wheadon, L. (2000). *Mathematical modeling: Book 1*. Scarborough: Nelson.

Battiste, M. (2004, May 29). *Animating sites of postcolonial education: Indigenous knowledge and the humanities*. Plenary address to CSSE. Manitoba.

Bishop, A. (1988). *Mathematical enculturation: A cultural perspective in mathematics education*. Dordrecht: D. Reidel.

Cajete, G. (2000). Indigenous knowledge: The Pueblo metaphor of indigenous education. In M. Battiste (Ed.), *Reclaiming indigenous voice and vision* (pp. 181–191). Vancouver: UBC Press.

Cappon, P. (2008). Measuring success in first nations, inuit, and Métis learning. *Policy Options, May,* 60–66.

Davies, B. & Harré, R. (1999). Positioning and personhood. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 32–51). Blackwell: Oxford.

Doolittle, E. (2006). Mathematics as medicine. *Proceedings of the Canadian Mathematics Education Study Group Conference* (pp. 17–25). Calgary.

Gutiérrez, R. (2007). Context matters: Equity, success, and the future of mathematics education. In T. Lamberg & L. R. Wiest (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* Stateline (Lake Tahoe): University of Nevada, Reno.

Hampton, E. (1995). Towards a redefinition of Indian education. In M. Battiste & J. Barman (Eds.), *First nations education in Canada: The circle unfolds* (pp. 5–46). Vancouver: UBC Press.

Harré, R.,& van Lagenhove, L. (Eds.). (1999). *Positioning theory: Moral contexts of intentional action*. Blackwell: Oxford.

Morgan, C. (1998). *Writing mathematically: The discourse of investigation.* London: Falmer.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: The National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2009). *Guiding principles for mathematics curriculum and assessment.* http://nctm.org/standards/content.aspx?id=23273. Accessed 20 July 2009.

Orr, J. Paul, J. & Paul S. (2002). Decolonizing Mi'kmaw education through cultural practical knowledge. *McGill Journal of Education, 37*(3), 331–354.

Powell, A., & Frankenstein M. (Eds.). (1997). *Ethnomathematics: Challenging Eurocentrism in mathematics education.* Albany: State University of New York Press.

Smith, L. T. (1999). *Decolonizing methodologies.* London: Zed Books.

Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics*, *72*(1), 1–15.

Wagner, D., & Lunney Borden, L. (in press). Aiming for equity in mathematics education research amongst marginalized communities. In B. Herbel-Eisenmann, D. Wagner, J. Choppin, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies.* Springer: New York, NY.

Chapter 28
The Effects of Poverty and Language on Mathematics Achievement for English Language Learners

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From 1991 to 2001, the enrollment of English language learners (students whose first language is not English, but who are learning English because it is the language of instruction) in U. S. public schools increased by 95%. During the same time period, the general student population increased by only 12% (Padolsky 2002). The rapidly growing number of English language learners (ELLs) in public schools creates a critical mass, that cannot be ignored, of students who are learning English and at the same time they are learning mathematical content. One of the most serious issues related to this increasing population of ELLs is the wide achievement gap in reading and mathematics existing between these students and those whose first language is English (National Assessment of Educational Progress (NAEP) 2005). The achievement gap in mathematics concerns educators since success in mathematics has been linked to science, technology, engineering, or mathematics (STEM) professions that provide increased lifetime earning power, suggesting that mathematics is the gatekeeper to higher paying professions (Lubienski 2007). However, those whose first language is not English are under-represented in these fields. For example, income comparisons among ethnic groups show that 65% of Latinos with limited English proficiency, 50% of English proficient Latinos, 44% of African Americans, and 29% of Caucasians made less than $30,000/year in 2002–2003 (Fry 2003; National Center for Education Statistics (NCES) 2003). In 2003, NAEP also reported that 36% of all fourth graders scored at or above the proficiency level in mathematics while only 11% of ELLs reached at or above proficiency level (NCES 2003).

These statistics indicate that achievement in mathematics is an issue of equity as ELLs are not performing at the same level as those whose first language is English and therefore are ill prepared to enter science, technology, engineering, or mathematics (STEM) careers. By equity, we mean "high expectations and strong support for all students" (National Council of Teachers of Mathematics (NCTM) 2000). NCTM also

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suggests that “All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students” (NCTM 2000). When mathematics teachers work to increase the mathematics achievement of ELLs, this increased achievement becomes “a powerful ladder of mobility” (Lubienski 2007).

A question, then, arises: What can mathematics teachers do differently to increase ELLs’ mathematics achievement? The answer can be quite simple, focus on what students know and can do and build upon this knowledge (Chenoweth 2009). When teachers focus on student learning and build structures that support students, they abandon their traditional approaches of direct instruction and skill-oriented practices, and create a quality mathematics education classroom. If the answer is so simple, then why do ELLs continually lag behind their fully English proficient (FEP) peers? In finding answers to this question, we first need to examine the existing system for teacher beliefs that impede ELLs’ achievement in mathematics and find ways to address those beliefs so that ELLs have opportunities to learn and succeed in mathematics. Apple’s (1995) words below reiterate why we must learn to understand the issues that create unnecessary barriers for many students.

> For without a recognition of the socially situated character of all educational policy and practice, without a recognition of the winners and losers in this society, without more structural understanding of how and why schools participate in creating these winners and losers, I believe we are doomed to reproduce an endless cycle of high hopes, rhetorical reforms, and broken promises. (p. 331–332)

In this chapter, we will argue that English proficiency, which is commonly thought to be the cause of ELLs’ low achievement in mathematics, may not be the primary culprit of such phenomenon. On the contrary, poverty seems to be a stronger predictor for low mathematics achievement and may play a larger role in ELLs’ mathematics performance than their below grade level English proficiency. We suggest that poverty is closely intertwined with discourse, which in turn, affects academic communication between teachers and students of different socioeconomic strata. Before discussing how we can best assist ELLs to achieve at their maximum potential in mathematics, we will address the beliefs held by decision-making stakeholders, mainly classroom teachers that hinder the success of ELLs (Nieto 2010; Weissglass 1997). These beliefs are then challenged by current research regarding effective schools for ELLs and poverty students. Below we delineate beliefs within mathematics education that create inequity for ELLs.

System Factors That Impede ELLs Mathematics Achievement

We suggest that the prevailing dominant view of mathematics education for ELLs espouses a deficit view, where the victim is blamed for inadequate achievement. For example, many teachers insist ELLs would eventually perform better when they are more fluent in English because they could better understand instructions. However, while investigating mathematics achievement differences between ELLs and fully English proficient students (FEPs) on a literacy-based performance assessment (LBPA), Brown (2005) found that the impact of socio-economic status (SES) was larger on FEPs than on ELLs. While high-SES FEPs outperformed high-SES ELLs, there was no significant difference between low-SES FEPs and low-SES ELLs. Most interestingly, however, the study revealed that high-SES ELLs outperformed low-SES FEPs. Non-school factors, such as the presence of a print-rich environment, are associated with higher cognitive academic language proficiency for both ELLs and FEPs. Therefore, high-SES ELLs would not do as well as high-SES FEPs because of less developed academic English, yet, the advantages of their high SES allowed high-SES ELLs to outperform low-SES FEPs, which might suggest that FEPs from low-SES backgrounds lack academic language proficiency. The low-SES FEPs' only "advantage" then was superior conversational English, of little use for performing academic tasks. Thus, poverty may be a stronger factor in determining mathematics achievement. Or at least, poverty and language are closely intertwined in determining students' success in mathematics. While not as profound as Brown's study, the NCES reports in an Issue Brief (2008) that "among students living in poverty, English Language Learners scored 3 points lower, on average, than English Proficient students whose second language was English, but not measurably different than students whose primary home language was English." What is evident is that English proficiency cannot be the major reason why ELLs' mathematics achievement is lagging behind that of FEPs. The National Literacy Panel reports similar findings in reading. Their executive summary states that oral fluency and proficiency in the first language can be used to facilitate literacy development in English, suggesting that tapping into the first language can confer advantages to ELLs.

Contrary to deficit views, it can be said that English proficiency itself does not guarantee academic success nor does exposure to a second language, by itself, help ELLs be academically successful (Brown 2005; Kieffer 2008). In fact, the negative effects of concentrated poverty may be less severe for ELLs than for native speakers of English. One of the possible explanations would be that ELLs are more resilient to the effects of poverty because of their ELL status. Studies have shown that bilingual children are more cognitively flexible and display cognitive advantages over monolingual children (Peal and Lambert 1962). Peal and Lambert report, from a study that explored the relationship between bilingualism and intelligence, that bilinguals scored higher than monolinguals on both verbal and non-verbal intelligence and concluded that bilingualism is more advantageous in that bilinguals are "more adept at concept formation and abstract thinking than the monolinguals" (Peal and Lambert, p. 14).

High-SES students have other advantages associated with their status, such as more books available at home and parents with higher levels of education (Krashen and Brown 2005). As a result, parents, guardians, and family members are more likely involved in assisting with homework, and dinnertime conversations may be more stimulating and educational in terms of cognitive development, whether the conversations take place in English or in another language. The disposable incomes associated with high SES might also suggest that family members take their children to local libraries or museums for educational enrichment that are not necessarily readily available at school. Regardless of culture or the country of origin, it seems that ELLs from high-SES levels consistently outperform low-SES FEPs and "opportunities provided to students at home, such as access to reading materials, were the most common characteristics that discriminated schools whose students achieved at a high level from those scoring at a low level on the TIMSS mathematics and science assessment" (Kitchen et al. 2007, p. 7).

A deficit view is also apparent when teachers believe that ELLs' poor mathematics achievement is directly caused by their ELL status. Low expectations and negative perceptions of their students based upon race, class, and/or language cause teachers to behave differently toward their students (Anyon 1980; Finn 1999; Secada 1992). As a result, ELLs are often taught less mathematics through traditional approaches of direct instruction and skill-oriented practice (Campbell and Langrall 1993; Hunt and Pritchard 1993), further marginalizing these students. Such methods of instruction not only ignore the needs of ELLs or low-SES students, but also deprive them of cognitive stimulation and reinforce negative self-images (Silver et al. 1995). Since the mathematical success of students from high-poverty schools is directly related to the expectations held by their teachers (Kitchen et al. 2007), we must urge teachers to hold high expectations for ELLs (Kitchen et al. 2007). When teachers have high expectations for their students, they are more likely to offer challenging mathematics content along with high-quality mathematics instruction, thus, promoting equity for every student.

We agree with Boaler (1997a, b, c) when she suggests that differences between traditional and reform classrooms are issues of equity, since students in traditional classrooms are rarely expected to explain their thinking and thus, lack an ability to explain what they understand conceptually or in real-world terms. Students also fail to retain the information presented in traditional instructional approaches over a long period of time. These instructional practices alienate talented students as well as ELLs and low-SES students, therefore, depriving them of opportunities to accelerate so that they can continue a pursuit of mathematics (Boaler and Greeno 2000). Furthermore, we know from the TIMMS study, that most mathematics teachers use these traditional practices, especially when working with low-SES and ELL students. It is an unfortunate reality in the United States that second-language learners and poverty are closely associated (DeVillar 2000).

An alternate vision that would empower ELLs and students living in poverty would be to allow the non-traditional ways that ELLs communicate. Moschkovich (2002) calls it "situated learning", where teachers value ELLs' sociocultural background and focus attention on how students actually construct knowledge, negotiate meaning, and participate in mathematical communication in order to maximize their mathematical learning. In a situated mathematics classroom, ELLs can use gestures to communicate. They can draw to show how they solved problems. They can facilitate their communication by using concrete objects that ELLs are more familiar with in their community. They can use their own funds of knowledge based on everyday experiences to explain mathematical concepts. Most importantly, they can use their first language to communicate mathematically (Ovando et al. 2003; Ramirez 1992). This situated perspective broadens the traditional notion that prescribes the ways that mathematical concepts are communicated. The situated perspective acknowledges what ELLs bring to mathematical learning by valuing ELLs' sociocultural backgrounds and knowledge and recognizes them as rich resources that assist them in the learning of mathematics (Ganesh and Middleton 2006). That is, within a situated learning environment, language differences existing for ELLs become an asset, not a handicap.

This situated perspective becomes critical when we look at the recommendations of the NCTM standards documents and publications (1989, 1991, 1995, 2000, and 2007) which suggest a vision of the mathematics classroom where students are engaged in real-world problem-solving activities. These activities expect students to create mathematical models and communicate their thinking orally and in writing, while making and testing hypotheses. This view stands in sharp contrast to traditional mathematics classrooms that posit the teacher as the sole demonstrator of algorithms and presenter of rules and definitions for students to memorize and regurgitate. The reform view suggests that teachers incorporate mathematical examples from bilingual communities when solving problems in order to enhance the learning of mathematics.

However, the classrooms envisioned by NCTM can inadvertently promote inequity in the classroom where a certain group of students is alienated because its vision favors one discourse pattern over others. Here, we draw on the work of Bernstein (1975), who points out the distinctive discourse patterns among people of different socio-economic backgrounds. He discovered that syntactical complexity and lexical diversity varies among people's discourse depending on their occupation. For instance, people with white-collar, managerial positions use more elaborate speech since their work-related community values diverse opinions among workers, thus, it is necessary for them to engage in communication based on negotiation and collaboration. The elaborated speech style is characterized by a variety of vocabulary and complex and longer sentences with modifiers and subordinate clauses. People who employ such an elaborated speech code do not assume the interlocutors have background knowledge on the topic and provide more information to make the context accessible to those with varying backgrounds.

On the contrary, working-class people usually hold jobs in manufacturing where they are required to take orders from their superiors and follow orders without questioning. The communication is unilateral in nature in that negotiation is not allowed and collaboration is not necessary (Finn 1999). Working-class people thus do not need to engage in communication where elaborated discourse style is a necessity. Language expressions used among working-class people are characterized by less variety in vocabulary and simpler syntactic structures. People who employ this language style also tend to assume that their audience would understand without much elaboration on the topic being discussed.

The discourse style dictated by work affects the way family members communicate at home (Bernstein 1975). For example, middle-class parents (white collar) encourage their children to think divergently, ask them to provide different opinions, and demand their children use language that elaborates and clarifies meaning when helping their children with their schoolwork. Middle-class parents also ask their children to express what they do or do not understand when guiding them in solving problems; but, working-class parents focus more on the context of the problem and do not engage in discussion regarding the process involved in problem solving (Ganesh and Middleton 2006). Working-class parents also expect their children to follow their orders and demands without negotiation or discussion, just as they are expected to do at work (Finn 1999).

In explaining disparity in mathematics achievement, discussing the discourse style at work or home is important because examining discourse patterns unique to social class reveals how they actually affect school learning and mathematics achievement in particular. The discourse style valued in academic settings espoused by NCTM standards documents encourages elaborated speech using questioning, hypothesizing, and argumentation. Students who experience this discourse style at home have greater advantages in school. By the same token, students from low-SES groups are placed at a great disadvantage because their dominant mode of communication at home is not compatible with the one valued in the mathematics classroom. Discussion-oriented mathematics, where justification and questioning are required, benefits higher-SES students while frustrating students from the working class (Lubienski 2002, 2007). Anyon's (1980) seminal work clearly illustrates how discourse style shaped by SES backgrounds influences the classroom discourse. She observed that mathematics teachers teaching in exclusively working-class communities taught their students to follow procedures without engaging students in discussion. The differences in communication styles between high-SES and low-SES students were apparent when students described a drawing of squiggly lines and a male's facial expressions. Students from middle-class and low-SES backgrounds describe the drawing with remarkably different language in terms of syntactical complexity and lexical diversity (Heider 1968, cited in Cazden 1968, p. 607).

High-SES student's description: It's a figure which is even on both of its sides and has an opening at the top and it's curved at the bottom or he has his left eyebrow raised more than any of the other faces.

Low-SES student's description: It looks like two snakes are fighting at each other or he looks surprised.

The description of the middle-class student is distinguished by a long compound sentence with subordinate clauses and a variety of vocabulary. More importantly, the student provided extremely elaborative descriptions in a decontextualized manner in which the student did not assume the audience shared the same context. On the other hand, a student from low-SES backgrounds wrote a simpler sentence without elaborating on what he or she meant by "two snakes are fighting at each other." It indicates that this student made assumptions that the audience would understand what he or she is describing in absence of added details. The differences between these two sentences are striking in terms of explicitness of language or lack thereof.

We, thus, contend that poverty, academic discourse, and mathematical achievement are closely intertwined, even more so than we would want to acknowledge, and cannot be easily separated. The point of our discussion is, however, not that teachers in mathematics should change their discourse patterns. In fact, teachers must help students of low-SES backgrounds acquire academic discourse. Unfortunately, in real-life, the opposite seems to be true. Anyon's (1980) working-class schools unequivocally showed that teachers deliberately did not engage their students in rich academic discourse and limited their learning experiences to mechanical, procedural aspects of mathematics believing that their students would not be able to handle academic discourse. Inequity results when teachers do not offer challenging curriculum or engage students from working-class backgrounds in discussions because they believe these students are not cognitively able. Mathematics educators must challenge these teachers' beliefs and provide information regarding supporting low-SES students in acquiring academic discourse, since the current discourse in mathematics classrooms favors students of high SES. Unfortunately, more than half of ELLs are concentrated in city schools that serve high-poverty families (Crawford 2000) and live in low-SES households where the discourse patterns of classrooms may not be the dominant mode of communication (DeVillar 2000).

Additionally, findings from research indicate that examining one's beliefs needs to be the first step to insuring quality and equity in mathematics education (Nieto 2010; Weissglass 1997); that discussions and proficiency in the language spoken at home facilitates conceptual understanding for ELLs (Ovando et al. 2003; Ramirez 1992); and that it is important to discuss the issues of identical education versus equitable education. In 1974, the Lau versus Nichols Supreme Court ruling manifests that the school district must take affirmative steps to remedy the learning situations caused by ELLs' low English proficiency by making instruction comprehensible to them. This ruling clearly specifies that ELLs sitting in the same classroom, taught by the same teacher, using the same textbook, may be identical education, but it is not considered equitable education. Research findings have provided us with information about ELLs that suggest theories about possible solutions to improve ELLs' mathematics achievement. How can these theories play out in practice?

Closing the Gap Between Theory and Practice

> Much like Michelangelo saw the finished sculpture inside the slab of marble and knew his work was to find it, teachers must believe students have mathematical understandings that need to be uncovered and their work as teachers is to find the means by which to do this.

Almost 20 years ago, Secada and Carey (1990) recognized that ELLs benefit from Cognitively Guided Instruction (CGI). We suggest that CGI is one viable approach to teaching mathematics to ELLs and thus CGI can help close the gap between theorists, who propose solutions based on research, and practitioners, who are teachers and administrators responsible for the decisions in the classroom regarding the teaching and learning of mathematics.

The National Research Council in their book *Adding It Up* describes CGI as a successful research-based program that:

> …focuses on helping teachers construct explicit models of the development of children's mathematical thinking in well-defined content domains…teachers develop their own instructional materials and practices from watching and listening to their students solve problems. (National Research Council 2001, p. 400)

The premise underlying CGI is that if teachers learn how their students come to understand mathematical ideas they can provide better instruction for the students in their classes. This premise connects to Chenoworth's (2009) belief that teachers need to determine what students need to know and assess what their students already know and are able to do. The teachers' role is to determine how to move students from where they are to where they need to be, analyze what students have learned, and decide whether further instruction is necessary. Teachers throughout the United States have attended CGI workshops and implemented CGI in their classrooms for topics involving whole numbers, algebra, geometry, and measurement. Even though the focus of CGI has been at the K-5 level, CGI has implications for higher education (See Otto et al. 1999; Lubinski and Otto 2004, b).

Research conducted on CGI teachers has consistently shown that they involve their students in more problem solving; accept a wider variety of problem-solving strategies from their students; expect children to participate actively in problem solving and to share their thinking with one another and the teacher; and base instructional decisions on the understanding of the students in their classes (Fennema et al. 1996). CGI has been implemented in various locations with a large number of minority and low-income students as well as in Hispanic communities (Secada and Carey 1990). Research has also consistently shown that students in CGI classes score better on both problem solving and computation tests than do children in control classes (Carpenter et al. 1989). Since CGI is not a curriculum that can be adapted, it is difficult to quantify the number of classroom teachers that ascribe to its philosophy of assessing and building upon students' mathematical understandings. Of particular importance to this chapter is that CGI professional development sessions prepare teachers to attend to individual children's thinking (Cady et al. 2006) and, thus, to adapt their teaching to the diversity of children in their classes, i.e. to focus on individual differences as envisioned by NCTM's definition of equity.

CGI has all of the components identified by Kitchen et al. (2007) that contribute to optimal mathematics learning in schools that serve the poor: high expectations of students; relevant mathematics, support for teacher-teacher, teacher-student, and student-student relationships. CGI personalizes and individualizes education, two important components in the literature on ELLs. How can these components be realized in the mathematics classroom? The following examples are representative of our work in actual classrooms in which CGI teachers:

1. Routinely encourage students to develop their own sense-making. Pichi is a kindergartener who does not communicate in complete sentences nor does he respond to word problems such as, if you have three marbles and I give you two more, how many marbles will you have? Providing multi-color glass counters

and asking Pichi to pretend they are marbles also does not get a response. Pointing to herself, the teacher said, “I have three blues (pointing to the glass counters in her hand). You (pointing to Pichi) give me two more. How many blues do I have now?” Having Pichi focus on a specific object that he can identify (blue glass counters) and think about a story in which he and the teacher are actors led to a successful outcome. Stories about joining and separating “blues” were part of daily problem solving for Pichi for the following two weeks at which time he could successfully solve problems involving multi-colored glass counters and eventually think of the counters as representing other objects. In upper grades sense-making can be the focus as students with various language strengths are grouped together to discuss a common strategy they’ve successfully used.

2. Provide the necessary mathematics vocabulary that connects to students’ explanations of their own thinking processes to develop students’ mathematical language through negotiation and discussion. This is evident in teachers’ classrooms that incorporate word walls as a reference. Students’ explanations (written in the students’ language by the students) illustrate an understanding of a mathematical term followed by that term (written by the teacher).
3. Introduce symbols after students’ thinking has been communicated, usually verbally, but also with pictures, gestures, and/or symbols. That is, teachers put symbols to students’ thinking, rather than teach the mathematics through symbols. Fifth-grade teacher Ms. Kanya used a word problem to get Jaidee to think about division. She said, “There are 247 pencils. You want to put them in boxes with 12 pencils in a box. Exactly, how many boxes do you need?” Jaidee said he was stuck. Ms. Kanya asked him to try to draw a picture and he did. He began to draw rectangles and he put 12 marks in each rectangle. Ms. Kanya stopped him and noted that she saw him use numerals in his picture last week. Could he do that now? Jaidee put the numeral 12 in the rest of the rectangles he was drawing. As he worked she asked, “What do you know about ten 12’s?” He said, “That would be 120!” She asked, “How could you use that information to solve the problem?” Then he wrote:

$$\left.\begin{array}{l} 12 \times 10 = 120 \\ 12 \times 10 = 120 \end{array}\right. \gg 240$$

Ms. Kanya’s objective was to ask Jaidee about what he knew (12 pencils in 1 box) to help him build on that knowledge to develop his current knowledge (refer to Chenoweth 2009). She asked him what the numerals represented and he knew that the 12 tells the number of pencils in a box and the 10 tells the number of boxes. Ms. Kanya assessed through further questioning that Jaidee understands that $10 \times 12 = 12 \times 10$. He knows that he would have 20 full boxes and 7 left over for the 21st box. So, his answer is 21 boxes are needed. During classroom discussions, Ms. Kanya introduced the terms quotient, dividend, and divisor and encouraged the students to use these terms when explaining their thinking. These discussions connect to Anyon’s idea of rich academic discourse. Representations such as $10 \times 12 + 10 \times 12 + 7 = 20 \times 12 + 7 = 247 \div 12$ emerge from and connect to students' thinking. Ms. Kanya's questioning techniques are communicated to her students' parents who are encouraged to question their children about their school work using such probes as: Tell me what happens in this story. Why did you add these numbers? Why did you start with 10 (or whatever number it is)? (See Jacobs and Ambrose 2008 for additional ideas.)

In several K-6 classrooms we visited, teachers were encouraged to ask their students to "Tell the story" after a word problem had been presented orally. Teachers were asked to not repeat the problem but have students practice "telling the story." Teachers commented that the results of this pedagogical task (many of their students could not do this sufficiently) surprised them and taught them to make sure understanding the problem is the first step toward a problem solution.

Finally, CGI connects to Moschkovich's perspective (2003, 2007) of what is necessary for optimal achievement in a mathematics classroom that serves ELLs. Mathematical communication is an integral part of CGI classrooms. CGI is based on the premise that students construct knowledge; meaning is negotiated among students and teacher; and all students participate in mathematical communication using whatever tools they need. CGI can help close an unnecessary gap between theory and practice.

Conclusions

Mathematics education research provides evidence of characteristics of quality mathematics instruction. Instructional approaches, such as CGI, have shown to be highly effective for language minorities; yet an unnecessary mathematics achievement gap between ELLs and FEPs still remains. As a result, inequity in the mathematics classroom still exists. We believe that dealing with the issues of inequity for ELLs should begin with examining perceptions and beliefs held by the teachers since they are ultimately responsible for classroom instruction. Teachers make instructional decisions based on personal beliefs and values such as the teachers in Anyon's study who believed that a certain group of students who live in low-income housing areas could only handle skill-based mathematics; therefore, skills are what teachers planned for their students to practice. Mathematics educators must work to challenge unsupported beliefs and work with teachers in classroom-imbedded professional development, such as CGI, to provide an alternate vision of mathematics instruction for ELLs and those living in poverty. We think that when teachers believe that ELLs can learn while acquiring English language proficiency, and that ELLs and those living in poverty can handle cognitively challenging curriculum when given the necessary support, they will change their classroom instruction to provide a quality mathematics program where each student is challenged and working at his/her potential. This is necessary for equity to be achieved.

Creating frank and honest dialogue among teachers regarding the influence of their beliefs upon their instruction and their students' mathematical understanding can be highly challenging; yet, it has to happen so that teachers can identify and examine one's subconscious beliefs and pre-conceived notions toward ELLs, and realize how these beliefs influence their instructional decisions. When an atmosphere of trust exists among colleagues, honest reflections about their students' mathematical understanding in relation to their teaching practice can lead to discussions regarding the socially situated character of all educational policy and practice. These discussions will help us answer the question previously posed by Apple (1995) "who is the winner and who is the loser?" For without reflection of how and why schools participate in creating winners and losers, we will be unable to find solutions to the problems identified in this chapter.

We also think that mathematics teachers need to recognize the value of ELLs' language diversity in mathematical learning and allow ELLs to use the language that leads to optimal understanding. Acquiring academic language, the language of mathematics, and English can be burdensome for ELLs. Allowing ELLs to use the language spoken at home can relieve some of the burden and aid ELLs in the learning of mathematics. When mathematics teachers have several students who speak the same language, creating opportunities for these students to use this language as part of classroom dialogue supports the optimal learning of mathematics.

Finally, we firmly think that each student is unique and deserves the support necessary for him/her to succeed. Schools must remedy the situation for ELLs so that they have access to challenging curriculum and effective instructional practices. Equity requires acknowledging the difference of ELLs, enabling languages other than English to be a resource, and above all maintaining high expectations of what these learners are able to achieve.

References

Anyon, J. (1980). Social class and the hidden curriculum of work. *Journal of Education, 162*(1), 67–92.

Apple, M. W. (1995). Taking power seriously: New directions in equity in mathematics education and beyond. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions in equity in mathematics education* (pp. 329–348). New York: Cambridge University Press.

Bernstein, B. (1975). *Class, codes and control: Theoretical studies toward a theory of educational transmissions* (Vol. 3). London: Routledge and Kegan Paul.

Boaler, J. (1997a). Equity, empowerment and different ways of knowing. *Mathematics Education Research Journal, 9*(3), 325–342.

Boaler, J. (1997b). Setting, social class and survival of the quickest. *British Educational Research Journal, 23*(5), 575–595.

Boaler, J. (1997c). When even the winners are losers: evaluating the experiences of "top set" students. *Journal of Curriculum Studies, 29*, 165–182.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171–200). Stamford: Ablex.

Brown, C. L. (2005). Equity of literacy-based math performance assessments for English language learners. *Bilingual Research Journal, 29*(2), 337–363.

Cady, J. A., Meier, S. L., & Lubinski, C. A. (2006). Developing mathematics teachers: Transition from preservice to experienced teacher. *Journal of Educational Research, 99*(5), 295–305.

Campbell, P., & Langrall, C. W. (1993). Research into practice: Making equity a reality in classrooms. *Arithmetic Teacher, 41*(2), 110–113.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499–531.

Cazden, C. B. (1968). Three sociolinguistic views of the language and speech of lower-class children-with special attention to the work of Basil Bernstein. *Developmental Medicine and Child Neurology, 10,* 600–612.

Crawford, J. (2000). *At war with diversity: US language policy in an age of anxiety.* New York: Multilingual Matters.

Chenoweth, K. (2009). It can be done, it's being done, and here's how. *Phi Delta Kappan, 91*(1), 38–43.

Crawford, J. (2000). *At war with diversity: US language policy in an age of anxiety.* New York: Multilingual Matters.

DeVillar, R. A. (2000). Literacy and the role of technology: Toward a framework for equitable schooling. In J. V. Tinajero & R. A. DeVillar (Eds.), *The power of two languages 2000: Effective dual-language use across the curriculum* (pp. 320–336). New York: McGraw-Hill.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. et al. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403–434.

Finn, P. J. (1999). *Literacy with an attitude: Educating working-class children in their own self-interest.* Albany: State University of New York.

Fry, R. (2003). *Hispanic youth dropping out of U.S. schools: Measuring the challenge.* Washington: Pew Hispanic Centero.

Ganesh, T. G., & Middleton, J. A. (2006). Challenges in linguistically and culturally diverse elementary settings with math instruction using learning technologies. *The Urban Review, 38*(2), 101–143.

Heider, E. R. (1968). *Style and effectiveness of children's verbal communications and within between social classes.* Unpublished Doctoral Dissertation, Harvard University.

Hunt, N., & Pritchard, R. (1993). Technology and language minority students: Implications for teacher education. In D. Carey, R. Carey, D. Willis, & J. Willis (Eds.), *Technology and teacher education annual* (pp. 25–27). Charlottesville: Association for the Advancement of Computing in Education.

Jacobs, V. R., & Ambrose, R. C. (2008). Making the most of story problems. *Teaching Children Mathematics, 15*(5), 260–266.

Kieffer, M. J. (2008). Catching up or falling behind? Initial English language proficiency, concentrated poverty, and the reading growth of language minority learners in the United States. *Journal of Educational Psychology, 100*(4), 851–868.

Kitchen, R. S., DePree, J., Celedon-Pattichis, S., & Brinkerhoff, J. (2007). *Mathematics Education at Highly Effective Schools that Serve the Poor: Strategies for Change.* Mahwah: Lawrence Erlbaum.

Krashen, S., & Brown, C. L. (2005). The ameliorating effects of high socioeconomic status: A secondary analysis. *Bilingual Research Journal, 29*(1), 185–196.

Lubienski, S. T. (2002). Research, reform, and equity in US mathematics education. *Mathematical Thinking and Learning, 4*(2–3), 103–125.

Lubienski, S. T. (2007). What we can do about achievement disparities. *Educational Leadership, 65*(3), 54–59.

Lubinski, C. A., & Otto, A. D. (2004a). How learning to make sense of mathematics in a content course can affect the beliefs of preservice teachers. In G. Krauthausen & P. Scherer (Eds.), *Mit Kindern auf dem Weg zur Mathematik: Ein Arbeitsbuch zur Lehrerbildung* (pp. 92–100). Donauwörth: Auer.

Lubinski, C. A., & Otto, A. D. (2004b). Preparing K–8 preservice teachers in a content course for standards-based mathematics pedagogy. *School Science and Mathematics, 104*(7), 336–350.

National Assessment of Educational Progress. (2005). *The nation's report card*. Washington: National Assessment of Educational Progress.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards.* Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics.* Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2007). *Mathematics teaching today: Professional standards for teaching mathematics.* Reston: National Council of Teachers of Mathematics.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* Washington: National Academy Press.

Nieto, S. (2010). *Language, culture, and teaching.* New York: Routledge.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4,* 189–212.

Moschkovich, J. (2003). *What counts as mathematical discourse?* Paper presented at the International Group for the Psychology of Mathematics Education Conference. www.eric.ed.gov. Accessed 21 Apr 2008.

Moschkovich, J. (2007). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*(2), 121–144.

Otto, A. D., Lubinski, C. A., & Benson, C. T. (1999). Creating a general education course: A cognitive approach. In B. Gold, S. Z. Keith, & W. A. Marion (Eds.), *Assessment practices in undergraduate mathematics* (MAA Notes No. 49) (pp. 191–194). Washington: Mathematical Association of America.

Ovando, C. J., Collier, V. P., & Combs, M. C. (2003). *Bilingual and ESL classrooms: Teaching in multicultural contexts* (3rd ed.). New York: McGraw-Hill.

Padolsky, D. (2002). How has the limited English proficient student population changed in recent years? http://www.ncela.gwu.edu/expert/faq/08leps.htm. Accessed 30 May 2005.

Peal, E., & Lambert, W. E. (1962). The relation of bilingualism to intelligence. *Psychological Monographs: General and Applied, 76*(27, Whole No. 546), 1–23.

Ramirez, J. D. (1992). Executive summary of the final report: Longitudinal study of structured English immersion strategy, early-exit and late-exit transitional bilingual education programs for language-minority children. *Bilingual Research Journal, 16*(1–2), 1–62.

Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623–660). New York: Macmillan.

Secada, W. G., & Carey, D. A. (1990). *Teaching mathematics with understanding to limited English proficient students* (Urban Diversity Series, No. 101). New York: ERIC Clearinghouse on Urban Education. (ERIC Document Reproduction Service No. ED322284).

Silver, E. A., Smith, M. S., & Nelson, B. S. (1995). The QUASAR Project: Equity concerns meet mathematics education reform in the middle school. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 9–56). New York: Cambridge University Press.

U.S. Department of Education, National Center for Education Statistics (NCES). (2003). *Condition of education.* Washington: NCES.

U.S. Department of Education, National Center for Education Statistics (NCES). (2008). *National Center for Educational Statistics Issue Brief.* Washington: NCES.

Weissglass, J. (1997). Deepening our dialogue about equity. *Educational Leadership, 54*(7), 78–81.

Chapter 29
Toward a Framework of Principles for Ensuring Effective Mathematics Instruction for Bilingual Learners Through Curricula

Kathleen Pitvorec, Craig Willey and Lena Licón Khisty

Over the past decade or so, discussion about meeting the needs of marginalized cultural groups has increased (Darling-Hammond and Bransford 2005; Gándara and Contreras 2009). Many countries have expressed concern over the poor mathematics performance of students who are less proficient or not proficient at all in the official language of schools. These students often represent a political minority population in their country, and thus, improving their performance involves attention to issues of equity. Associated with this is a concern that educators in these contexts may not be appropriately or adequately prepared to instruct students who are learning in a second language. For example, in the United States, over 40% of the teachers have language minority students (LMS)[1] in their classrooms, but only 12.5% of the teachers have had more than eight hours of training to prepare them to teach LMS (NCTM 2004). Thus, there are critical questions about how to support teachers and what specialized pedagogical content knowledge they may need (Banks et al. 2005). At the same time, countries around the world are concerned about improving the quality of mathematics performance overall. Improving the quality of performance, on the one hand, can be demonstrated in standardized test results, but in addition, "the integrated and balanced development of all five strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) should guide the teaching and learning of school mathematics" (NRC 2001, p. 11).

[1] We have chosen to use interchangeably the terms "language minority student" (LMS), Latinas/os, and "bilingual learner." LMS notes Latinas/os' political status and "bilingual" forefronts their language roots and experiences.

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For one reason or another, these concerns—equity and quality in mathematics teaching and learning—are often dealt with as if they are antithetical. But are they? Do they need to be? Do language minority students require a different or easier mathematics curriculum because they are bilingual learners and may have limited proficiency in the official language of schools, which imply lower content standards? Is it possible for bilingual learners to meet quality standards of mathematics performance through a curriculum that takes into account and builds on their home language while facilitating their development of school-based language?

Our chapter begins to address these questions by examining a curriculum, *Finding Out/Descubrimiento (FO/D)* (De Avila et al. 1987) that was designed in the late 1970s by Bilingual Education scholars for the explicit purpose of meeting the educational needs of Latina/o bilingual learners (De Avila, personal communication December 1, 2008; Navarrete 1983). While *FO/D* was not intended solely to be a mathematics curriculum, students, in fact, gained significantly in mathematics (Cohen et al. 1997)—and thus, it met goals of both equity and mathematical learning. We were drawn to this curriculum because of its success with Latinas/os in mathematics, and wanted to understand what made it effective with this population. Furthermore, research on Latinas/os and other language minority students in mathematics have considered issues of instructional practices that affect students' learning (e.g., Chval and Khisty 2009; Hufferd-Ackles et al. 2004; Khisty 1995; Khisty and Willey 2008) and issues of language use (Adler 2001; Moschkovich 1999; Setati 2005); however, there has been little discussion of curriculum materials and their role in Latinas/os' mathematical development (Chval 2010). Although fidelity to a curriculum cannot always be guaranteed, curriculum, nevertheless, embeds a particular direction and can be a starting point for new thinking about effective instruction (Brown et al. 2009) for Latinas/os and other bilingual learners in mathematics.

Our purpose in this chapter is not to promote *FO/D* but to examine it for what mathematics educators can learn about the organization of instruction that both addresses equity and the improved learning of bilingual learners. By uncovering the characteristics that make *FO/D* effective, we have developed a framework that we believe can be used to guide mathematics curriculum development and implementation to better meet the needs of bilingual learners, and to offer insights into effective practices that can assist teachers. While the focus of our discussion is on Latinas/os in the United States, this framework and these insights might equally assist educators in other contexts.

We begin by providing background to *FO/D*, including its theoretical basis. We follow this by describing our process for identifying what we consider are its key characteristics. We then describe these characteristics and a framework that derives from them. We close our discussion with implications of this framework for curriculum development and implementation and for equitable mathematics instruction.

Background

Finding Out/Descubrimiento as a Curriculum

As noted earlier, *FO/D* was developed in the late 1970s. Lena Licón Khisty, an author of this chapter, became familiar with the curriculum at that time and has followed it over the years since then. *FO/D* was intended to serve students in grades first through fifth as a supplement that could be used once or twice a week or as often as a teacher desired. The materials were originally designed to target the simultaneous development of language and academic content among children of migrant workers in California (De Avila et al. 1987). Unfortunately, it received little attention as a resource for mathematics instruction even for multilingual contexts. Nevertheless, where it was used with bilingual learners, it was found that students made statistically significant gains in mathematics in all subscales (Cohen et al. 1997). Given the popular perception that Latinos/as need repeated practice with basic computation skills, this outcome is especially intriguing given that the mathematics content is embedded in rich, problem-solving, science-based activities, unlike more traditional curriculum materials with explicit presentation of mathematics skills and concepts. Nowhere in *FO/D* will one find computation practice as in traditional mathematics programs, yet students improved in computation (Cohen et al. 1997). Arguably, this is the result of repeated contextualized situations in which students need to utilize computation in order to accomplish the objective at hand. As De Avila (personal communication, December 1, 2008), one of the developers of *FO/D*, put it, "we confuse the means with the ends," referring to educators' tendency to focus on students' ability to perform algorithms correctly as the primary objective.

Because Bilingual Education scholars developed *FO/D*, many of its characteristics are based on and driven by teaching and learning principles known to make a difference in achievement among bilingual learners (Garcia 1991). *FO/D* is organized around thematic science-based concepts such as sound and measurement, concepts that lend themselves to students' curiosity, engagement, and background experiences. Each activity integrates literacy, science, and mathematics such that activities students read and write for meaning and problem solving. Also, *FO/D* comes with substantial suggestions for the teacher on how to establish and facilitate collaborative group work among students. Moreover *FO/D's* primary focus is to advance bilingual learners' critical thinking skills through experiments in science that involve using mathematics and literacy as tools for performing and discussing the experiments (De Avila et al. 1981). In fact, *FO/D* is characterized by a substantial high-level cognitive demand of students, non-routine problem solving, conceptual development, and thinking and reasoning in sustained and thoughtful ways (Cohen et al. 1997; Hernandez 1991)—all goals of current mathematics reform efforts in the United States.

The activities are designed so that Latinas/os will have access to content areas and not be held back because they do not know English (the official school language) or do not have specific content skills. To this end, a unique and key characteristic of *FO/D* is its use of bilingual activity cards (see examples in the next section). Each activity is presented on duplicate cards—one in Spanish and the other in English. Students are free to use the cards as they wish to make meaning of the task: using either the Spanish or English card or both simultaneously. This characteristic is particularly important in that it gives equal status to students' home language and English, and reaffirms the home language as a resource for learning, thus satisfying a dimension of equity (Valencia 2002). Interestingly, Neves (1997) found that *FO/D*'s intrinsic capacity to promote "talk" (which we describe further in a later section) in either language among students working collaboratively translated into gains in English proficiency: students "with the largest gain scores in English had the higher rates of talk in Spanish…[and] the lowest rates of talk in English" (p. 188). This is a somewhat counter-intuitive finding but one supported by current literature on bilingual education (e.g., Cummins 2000; Diaz 1983; Genesee et al. 2006). Lastly, the activity cards are in the form of a colorful cartoon that depicts the series of steps to be taken by collaborating groups of students. We also discuss this characteristic in more detail in a later section.

The Theoretical Basis for FO/D

As we worked to understand *FO/D*, it became clear that it is also rooted in a sociocultural activity perspective of learning (Vygotsky 1978; Engeström 1999), one that would capitalize on students' four levels of resources: the object itself, pictures, language, and peers (De Avila, personal communication, December 1, 2008). Since sociocultural activity theory plays a significant role in understanding second language development (see, for example, Lantolf and Thorne 2006; Razfar et al. in press) and similarly a growing role in understanding mathematics development (see, e.g., Lerman 2001), it is relevant to briefly consider how this theory is reflected in the curriculum that meets the goals for both language and mathematics content development. Some key aspects of sociocultural activity found in *FO/D* include the following: activity, interactional spaces, and mediation through multimodal semiotic tools, including language. First, human development is fundamentally social in nature but also rooted in concrete communicative activity (Vygotsky 1978; Engeström 1999) or participation in practice. In activity are the conditions of social interactions and cooperation. Meaning resides in the activity, its actions, and the language attached to them. "It is through activity that new forms of reality are created, including the transformation of self" (Lantolf and Thorne 2006, p. 215). The activity, tools, and social interactions, then, are the mediational resources for development. De Avila (personal communication, December 1, 2008), noted: "when kids are arranged as they are [in a traditional arrangement of rows of desks], we eliminate these resources." However, when these resources are made available and students are socialized to use them, they are positioned to solve problems collaboratively and make meaning of the content—two crucial tenets of learning and development.[2] Second, development is mediated through interactions with others and thus, interactional spaces are a key component of activity. In *FO/D*, these spaces are carefully constructed to socialize students into norms of rights, roles, and responsibilities to help one another complete a goal-directed task. Students collaborate in a community of practice (Wenger 1998) and become responsible for their own and each other's learning. Third, thinking and development also are mediated by the use of tools (in *FO/D*: concrete objects, pictures, and oral and written communication). Changes in the routine of how tools are used are signs of development (Sfard 2008). This idea is consistent with current thinking in reform mathematics approaches that deemphasize learning procedures without connections and emphasizes problem solving (NCTM 1989, 2000).

Examination of FO/D

In this section, we present what we believe to be the key characteristics of *FO/D*. Again, *FO/D* provides us with the opportunity to consider key characteristics of a curriculum that has evidence of supporting the simultaneous development of mathematical concepts, language for bilingual learners, and academic progress. Our examination of *FO/D* followed a two-step process. First, we compared it to current research in Bilingual Education in order to distill the characteristics that make it relevant to Latina/o students. Here Willey and Khisty have extensive experience with this literature based on teaching university courses in both Bilingual Education and courses that integrate this area with mathematics education; one of the authors has extensive experience in mathematics curriculum development, and altogether, we could compare and contrast curricula and teaching practices from both areas. Second, in order to refine our identification and description of characteristics so that they would be meaningful in a broader context than *FO/D*, we randomly selected one elementary school and one middle school *Standards*-based curriculum[3] whose characteristics we could contrast with those we identified in *FO/D*. However, only identifying key characteristics can be misleading.

For example, the rationale for the visual images in *FO/D* is significantly different than in other curricula. We interpret the visual images in *FO/D* as minimizing the

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[2] As explained in *FO/D* teacher resource materials (De Avila et al. 1987) and teacher trainings (Lotan, personal communication, December 2, 2009), a fundamental design principle of *FO/D* was to challenge traditional classroom organizational structures of sitting and working in isolation. It is worth noting that the traditional pattern still dominates the classrooms of poor and minority students (see Oakes 1990).

[3] We use the term *Standards*-based curriculum to refer to curricula that are aligned with the National Council of Teachers of Mathematics *Standards* from 1989 and 2000 (NCTM 1989, 2000).

influence of any language proficiency factors and as a deliberate means of engaging students in making meaning in the context of the activity. The visual cues of the activity cards open up possibilities for interaction and dialogue as students negotiate and build consensus on both what the card means and what they should do to complete the activity (see Fig. 29.1). On the other hand, we interpret the visual image in a *Standards*-based curriculum as more of a reference for technical definitions of key words in the activity (see Fig. 29.2 as an example). In light of these differences, we focused not only on the characteristics but also their purposes.

The *FO/D* student activity card pictured in Fig. 29.1 illustrates how we believe these cards serve an additional function of supporting student discussion. The images invite discussion because there are sufficient visual cues to allow students to interpret what they are going to do (find and compare the circumference and diameter of objects); to figure out how they are going to do it (the method used in the illustration is to wrap a string around a round object, then to stretch the string out to compare it to a ruler in order to measure the length of the string and, therefore, the circumference); and to interpret or define various aspects of the activity (e.g., the meaning of *circumference*, how to use the tools, the process of recording measurements, and the expectation that more than one object will be measured)—all processes of critical thinking. Student meaning making is, therefore, not reliant on the teacher's facilitation. In a *Standards*-based curriculum, on the other hand, student discussions often revolve around teacher or textual prompts or instructions. This places the burden on bilingual learners' comprehension of oral or written communications, most likely in a second language, which may not be their academic strength (Khisty 1997).

A Framework for Equity and Quality in Mathematics

By making these types of comparisons, we began to develop a list of guiding questions for understanding how engaging students in *FO/D* activities contributed to significant improvements in student achievement scores related to mathematics content and language skills. How do students know what to do, how to approach problem situations, and how to engage in doing mathematics? What roles do students and teachers have in the problem-solving process? What purpose do mathematical activities have and in what way are activities structured to promote engagement in meaningful mathematical activity? How is literacy defined and developed during mathematical problem solving?

Attending to key questions like these allowed us to construct feature descriptions for a framework that would have meaning across curricula. The framework begins by setting forth an overarching ideology that emerges from *FO/D*, one that respects students and the resources they bring to the classroom and to learning. Learning is seen as occurring in interactional spaces. The teacher and the curriculum materials are not positioned as the authorities who hold the mathematical knowledge; instead

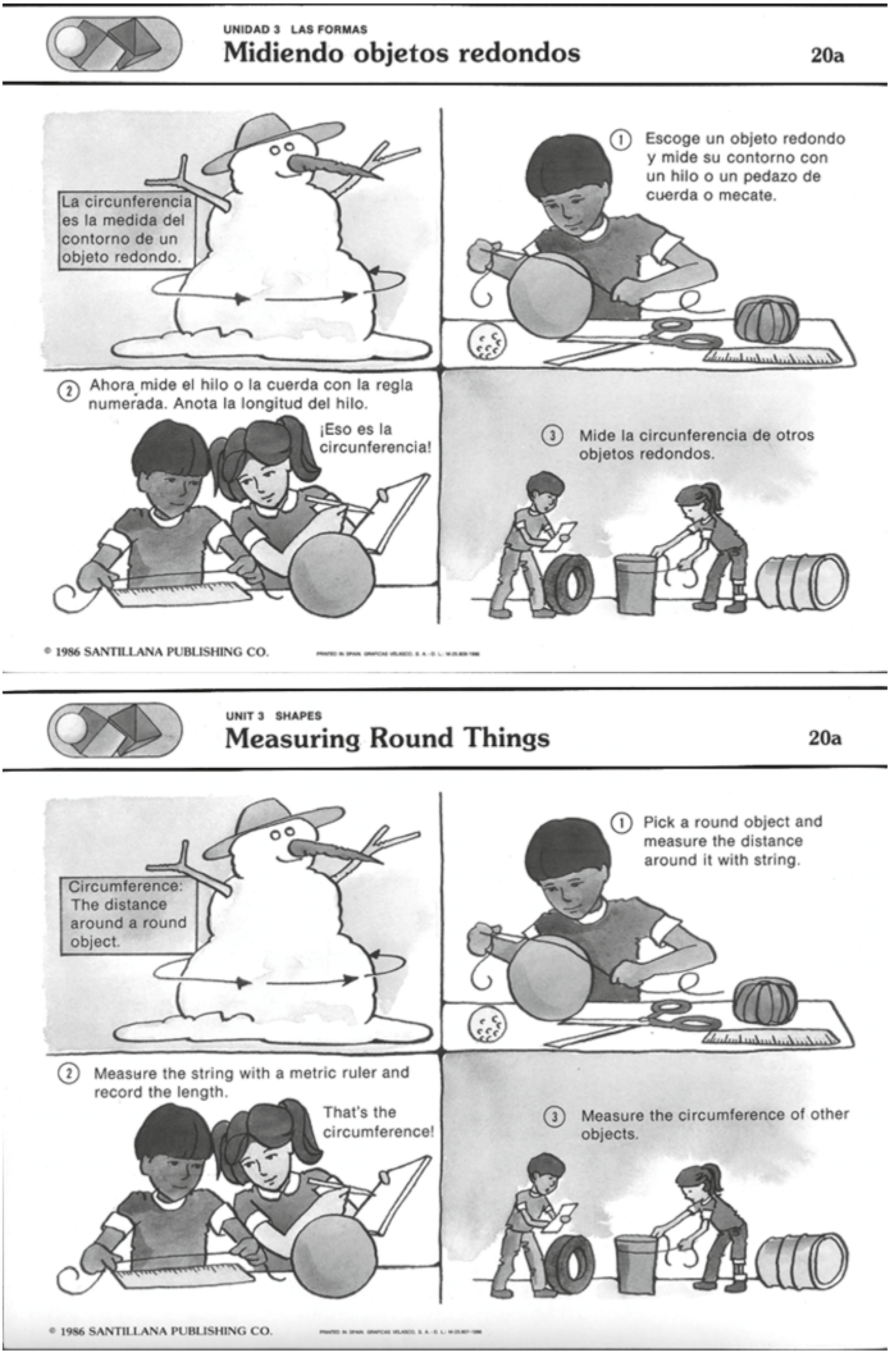
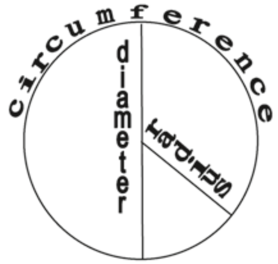


Fig. 29.1 Student activity cards from *FO/D* in Spanish and English

Fig. 29.2 Image and instructions are similar to those found in a *Standards*-based curriculum

Exploring circumference and diameter



1. Use a tape measure.
2. Measure the circumference and diameter of various round objects.
3. Use metric units for all measurements.
4. Use the table below to record your data.
5. Use a fraction to record the ratio of circumference to diameter.

it is the learning community members (teacher and students) working together to make meaning who generate mathematical knowledge. The curriculum materials, the problem context, and the mathematics needed to solve problems are all defined as tools for mediating student learning. This ideology is manifest in three broad categories of features—features related to learning communities, features related to the curriculum materials, and features related to language and communication. We

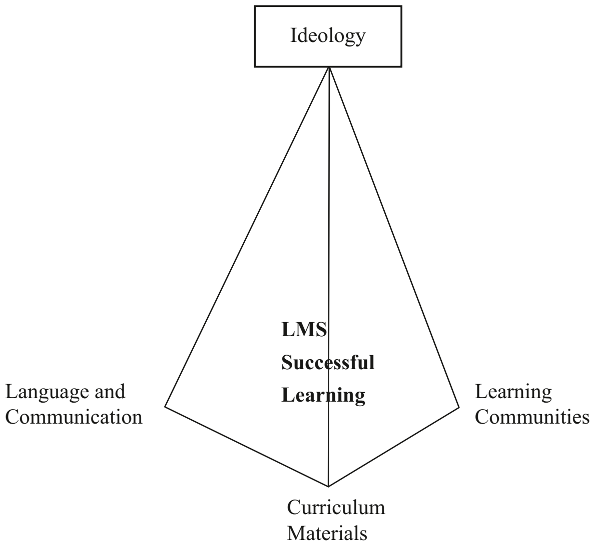


Fig. 29.3 Pyramid of success for Language Minority Students (LMS)

argue that these three categories together with the overarching ideology form the foundation of support for bilingual learners' mathematics learning (see Fig. 29.3). In the following sections, we discuss these three categories.

Learning Communities

As mentioned earlier, the learning community holds the key to the generation of mathematical knowledge. According to the United States National Research Council report (NRC 2001), there are four features of classrooms that support the teaching and learning of mathematics for understanding: ideas and methods are valued; students have autonomy in choosing and sharing their methods of solving problems; an appreciation of the value of mistakes as sites of learning; and the recognition that the authority for whether something is both correct and sensible lies in the logic and structure of the subject rather than the status of the teacher or the person making the argument.

A curriculum that supports the learning of bilingual learners is designed to promote student agency, students' interdependence, and students' independence from reliance on the teacher and the curriculum materials as the only source for instructions, ways to think about a problem, and ways to work. Activities represented pictorially provide students with access to activity instructions via visual descriptions of the activities (see Fig. 29.1). Students rely on each other—the community—and serve as resources for each other in the problem-solving process. In essence, they take responsibility for their learning. The curriculum positions the learning community as central; students are thereby positioned genuinely as knowledge generators rather than knowledge recipients. This approach to learning is no small matter for nondominant or Latina/o students and cannot be taken for granted. It opens the way for students to use all of their resources for learning (Moll 1999). It provides for greater participation of students who too often are marginalized because of one presumed "deficit" or another (Gándara and Contreras 2009). It also supports students in redefining themselves (and teachers in redefining students) as capable of knowing. In essence, the "community" structure is a critical feature of instruction for Latina/o students since too often, this student population is subject to negative and/or deficit beliefs and actions that ultimately subjugate them (Valencia 2002). Ensuring equity and schooling processes that do not fail students, means challenging these factors.

The teacher's role in this learning community includes facilitating the preparation of the mathematical situations and promoting student engagement in the situations. The teacher refrains from doing the intellectual work for students. Although the teacher supports students during the problem-solving process, the teacher does not give students preset solution strategies or approaches, nor does he/she provide answers to the problems. The teacher's participation in these small groups tends to be in the form of supporting student ideas, inserting thoughtful questions that facilitate students' work of meaning making, probing students to make their thinking visible to others in the groups, making connections between their informal strategies and thinking, and more formal ones from the content, and highlighting their strengths and successes so that all students realize the importance of their contributions in completing the mathematical activities (Cohen 1994). Generally, students work together in small groups to define the problems and to establish how they will approach and solve the problems. Together, they make mathematical meaning through their engagement in the situation. The interactional spaces created among students are qualitatively different than students just "talking about their mathematical ideas." The structure of the activity situations requires students to engage in authentic mathematical talk.

Curriculum Materials

A second category of features is related to the design of the curriculum materials (i.e., the tasks and task-related writing students do). Each task or problem situation presents a context in which mathematical meaning making can occur. We use the following example from *FO/D* to illustrate some features from this category (see Fig. 29.4). In the example, students begin to explore ratio and proportion in the context of systematically diluting juice with water. Students record their observations as the concentration of juice is decreased—that is, the ratio of juice to water is changed in each step. Since they change the ratio by adding specified numbers of teaspoons of water, their observations can include everyday and mathematical language focused on ratios and proportion. Mathematics functions as a problem-solving tool in the investigation. Rather than teaching students the strategies for solving ratio problems and then having students practice those strategies, the activity creates a problem situation for which students need to discuss and figure out, first, what it is they are going to do (dilute the juice with water in steps and observe); second, how they are going to do it (measure juice and water into a glass and smell and taste it at each step); and third, the mathematical relationships and connections in what they are doing (the effect of diluting the juice on the smell and taste of the resulting liquid, that, as there is more water, the smell and flavor become weaker). As they discuss the problem situation, they are able to draw on their own experiences and knowledge related to such a situation, and able to explore and use the language of proportion and ratio in an authentic mathematical situation. Students engage in multi-modal learning that involves all their senses and modes of communication.

Language and communication

The third category of features is related to language and communication. One definition of academic literacy is that a person has the ability to read, write, speak,

Fig. 29.4 An *FO/D* science activity in Spanish and English

listen, and think effectively within a particular context, and furthermore, "[b]eing literate enables people to access power through the ability to become informed, to inform others, and to make informed decisions" (Meltzer 2001, p. 1). In the context of a mathematics curriculum, literacy is a functional component of doing the mathematics. Literacy involves reading, writing, listening, and discussion skills. As indicated by findings from research on *FO/D*, English literacy for bilingual learners can develop together with competency in mathematics when the curriculum provides a conducive combination of features (Cohen et al. 1997).

Several key structural features of a curriculum related to language and communication contribute to the simultaneous development of literacy in students' first and second languages (i.e., biliteracy), and mathematical proficiencies. First, both languages should have equal status. As much as possible, materials, explanations, vocabulary, and conversation about the mathematical problem solving are available in both Spanish and English. As students explore the problem situations, they are encouraged to use the language (Spanish, English, and/or a hybrid of both) with which they can best express their meanings and ideas. Dialogue among students is critical for thinking but also for developing proficiency in both languages in a reciprocal relationship (Neves 1997).

Second, in addition to discussions in both languages, students have opportunities to read and write in both languages. In both cases, the level of literacy is more complex and extended. Students interpret the problem situation, using each other and the curriculum materials as resources, and then have opportunities to process their experiences through recording their individual ideas, explanations, questions, observations, and arguments. Instead of limiting students to just filling in blanks on a page, student pages should provide spaces for students to communicate "what I think happened," or "what I observed," or "why I think it works," consequentially tapping into reservoirs of skills that are classified as higher order thinking skills (Vygotsky 1978). When engaging in such literacy activities, students gain extensive experience using their biliteracy skills and have opportunities to refine them. Furthermore, the approach to language development (i.e., active, dialogic, and complex) described in this framework conforms to current work on a functional approach to second language development (Mohan and Slater 2005) and leads to simultaneous development of both language and content knowledge (Cohen et al. 1997; Lotan 2007).

Lastly, a curriculum should support the development of what Hufferd-Ackles et al. (2004) describe as a high-level Math Talk Community—students are the source of mathematical ideas; students take responsibility for their learning; students are questioners; and students explain and articulate their ideas. In addition, as part of students' negotiations around the mathematical meaning embedded in problem-solving situations, they also have to respectfully challenge each other, build arguments for and justify their thinking, and build on each other's ideas. All of these promote authentic talk about mathematical content and thus provide a context in which bilingual learners further develop biliteracy skills.

Implications and Conclusions

We began our discussion questioning whether it was possible to have quality mathematics instruction and outcomes given student populations, such as Latinas/os, who have a history of underachievement in mathematics, and thus, are too often deemed in need of remediation and basic skills, and who are not proficient in or still learning the official language of school. We also questioned if bilingual learners needed a different sort of mathematics curriculum, and if it was possible for these students to meet standard notions of quality mathematics. We set out to answer these questions by examining a curriculum that was specifically designed to reverse patterns of underachievement and to challenge assumptions that Latinas/os cannot do "quality school work." We found that indeed there is evidence that such a curriculum can achieve these goals. At the same time, we found that Latinas/os or bilingual learners do need a different sort of mathematics curriculum, but not one that is easier. They need one that is designed with certain features, features based on creating different kinds of learning spaces, features based on capitalizing on students' resources—in fact, defining Latinas/os' individual traits as learning resources—and features based on a principled approach to curriculum and instruction that includes principles from research that has focused on bilingual learners.

Our description of what we believe to be the most salient features of *FO/D*, leads us to various conclusions about curriculum development, curriculum evaluation, instructional practice, and professional development. First, it is not enough to insure that Latinas/os and other bilingual learners simply will have *access* to quality mathematics programs. Our framework suggests that curriculum development must consider the needs of bilingual learners from the beginning in the design of a curriculum. In the case of the two reform curricula with which we compared *FO/D*, access for bilingual learners, narrowly construed as access to vocabulary and instructions, was added on after the development of the original materials.[4] Second, more attention must be given to being explicit about the ideology in which a curriculum is embedded. The ideology for nondominant students should clearly respect and support the agency of students and see students and their home language as resources for developing mathematical proficiency; this ideology must influence the way mathematical activities are designed, organized, and presented. Third, when learning is seen as occurring in interactional spaces, as described in the framework, activities must be designed to support the existence and use of those spaces. Language ought to not be conceived of as an entity that needs to be taught one technical word at a time, but rather as a natural tool that is continually being refined from students' everyday words to a specialized style of language related to the mathematics discipline (Gee 1996; Lantolf and Thorne 2006). Fourth, where the curriculum materials, the problem context, and the mathematics needed to solve problems are all viewed as tools for mediating student learning, mathematical activities must be constructed

[4] Note that one author of this chapter was part of the author team of a *Standards*-based curriculum and so is familiar with how the curriculum addresses the needs of bilingual students.

to provide students with opportunities to make use of these tools. These opportunities should not be prescriptive but should invite exploration by students with a follow-up that requires an explanation and comparison of student ideas about the problem. While some current mathematics curricula may strive to do this, in their implementation the effort sometimes falls short (Brown et al. 2009) and instead results in eliminating critical resources for language minority students. Clearly from the foregoing, new directions are needed in teacher development—directions that focus on helping teachers skillfully implement student-centered, community-based, dialogic, non-biased, multilingual mathematics instruction.

What *FO/D* has demonstrated is that none of the features of a curriculum that we have described can be an afterthought. They must be part of the fabric and structure of the curriculum and corresponding instruction from the beginning. Instead of only focusing on content first, we advocate a curriculum development process that reflects current sociopolitical and cognitive research on bilingual learners and that demonstrates a shift toward mathematics equity in the form of student-centeredness and community-centeredness as described in *How People Learn* (Bransford et al. 2000). The features that we derived from *FO/D* and identified and described in the framework provide us with a starting point for transforming existing mathematics lessons so that implementation of those lessons provides both quality and equity for Latinas/os and all bilingual learners and their dominant classmates together.

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References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms.* Dordrecht: Kluwer.

Banks, J., Cochran-Smith, M., Moll, L., Richert, A., Zeichner, K., LePage, P., et al. (2005). Teaching diverse learners. In L. Darling- Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world* (pp. 232–274). San Francisco: Josey-Bass.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (2000). *How people learn: Brain, mind, experience, and school.* Washington: National Academy Press.

Brown, S. A., Pitvorec, K., Ditto, C., & Kelso, C. R. (2009). Reconceiving the intended curriculum: An investigation of elementary whole number lessons. *Journal for Research in Mathematics Education, 40*(4), 363–395.

Chval, K. (2010). Mathematics curriculum and Latino English language learners: Moving the field forward. Presentation given at the *CEMELA—CPTM—TODOS Conference,* Mar 4–6, Tucson, AZ.

Chval, K., & Khisty, L. L. (2009). Bilingual Latino students' writing and mathematics: A case study of successful teaching and learning. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms.* Clevedon: Multilingual Matters.

Cohen, E. G. (1994). *Designing groupwork: Strategies for the heterogeneous classroom.* New York: Teachers College.

Cohen, E. G., Bianchini, J. A., Cossey, R., Holthus, N. C., Morphew, C. C., Whitcomb, J. A., et al. (1997). What did students learn?: 1982–1994. In E. G. Cohen & R. A. Lotan (Eds.), *Working for equity in heterogeneous classrooms* (pp. 137–165). New York: Teachers College Press.

Cummins, J. (2000). *Language, power, and pedagogy: Bilingual children in the crossfire*. Clevedon: Multilingual Matters.

Darling-Hammond, L., & Bransford, J. D. (2005). *Preparing teachers for a changing world: What teachers should learn and be able to do*. Indianapolis: Jossey-Bass.

De Avila, E. A., Cohen, E. G., & Intili, J. K. (1981). *Improving cognition: A multicultural approach*. Stanford: Stanford University School of Education.

De Avila, E., Duncan, S. E., & Navarrete, C. (1987). *Finding out/Discubrimiento*. Northvale: Santillana Publishing Co., Norvale, NJ.

Diaz, R. M. (1983). Thought and two languages: The impact of bilingualism on cognitive development. *Review of Research in Education, 10,* 23–54.

Engeström, Y. (1999). Activity theory and individual social transformation. In Y. Engeström, R. Miettinen, & R.-L. Punamaki (Eds.), *Perspectives on activity theory* (pp. 19–38). New York: Cambridge University Press.

Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Cambridge: Harvard University Press.

Garcia, E. E. (1991). *The education of linguistically and culturally diverse students: Effective instructional practices*. Santa Cruz: National Center for Research on Cultural Diversity and Second Language Learning.

Gee, J. P. (1996). *Social linguistics and literacies: Ideology in discourse* (2nd ed.). New York: RoutledgeFalmer.

Genesee, F., Lindholm-Leary, K., Saunders, B., & Christian, D. (2006). *Educating English language learners: A synthesis of research evidence*. Cambridge: Cambridge University Press.

Hernandez, J. S. (1991). Getting to higher ground: The development of thinking skills for Spanish-speaking students. In M. E. McGroarty & C. J. Faltis (Eds.), *Languages in school and society* (pp. 317–334). Berlin: Mouton de Gruyter.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

Khisty, L. L. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 279–297). Cambridge: Cambridge University.

Khisty, L. L. (1997). Making mathematics accessible to Latino students: Rethinking instructional practice. In J. Trentacosta & M. Kenney (Eds.), *Multicultural and gender equity in the mathematics classroom: The gift of diversity, 1997 Yearbook of the National Council of Teachers of Mathematics,* (pp. 92–101). Reston: National Council of Teachers of Mathematics.

Khisty, L. L., & Willey, C. (2008). *The politics of language and schooling in the mathematics education of bilingual Chicana/o students*. Paper presented at the International Congress of Mathematics Education, Monterrey, México.

Lantolf, J. P., & Thorne, S. L. (2006). *Sociocultural theory and the genesis of second language development*. Oxford: Oxford University Press.

Lerman, S. (2001). A review of research perspective on mathematics teacher education. In F.-L. Lin & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 33–52). Dordrecht: Kluwer.

Lotan, R. A. (2007). Developing language and mastering content in heterogeneous classrooms. In R. M. Gillies, A. Ashman, & J. Terwel (Eds.), *The teacher's role in implementing cooperative learning in the classroom* (pp. 189–203). New York: Springer.

Meltzer, J. (2001). Supporting adolescent literacy across the content areas. *Perspectives on policy and practice*. http://www.alliance.brown.edu/pubs/perspectives/adlitcontent.pdf. Accessed July 2009.

Mohan, B., & Slater, T. (2005). A functional perspective on the critical "theory/practice" relation in teaching language and science. *Linguistics and Education, 16,* 151–172.

Moll, L. C. (1999). *Captives of words: Challenging the pedagogy of control.* Invited address, AERA Distinguished Lecture, Montreal.

Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Navarrete, C. (1983). *The effect of various input on the second language acquisition of Mexican-American children in nine elementary school classrooms.* Unpublished doctoral dissertation, Stanford University.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics.* Reston: National Council of Teachers of Mathematics.

NCTM. (2000). *Principles and standards for school mathematics.* Reston: National Council of Teachers of Mathematics.

NCTM. (2004). *NCTM Research Catalyst Conference.* Reston: National Council of Teachers of Mathematics.

Neves, H. A. (1997). The relationship of talk and status to second language acquisition of young children. In E. G. Cohen & R. A. Lotan (Eds.), *Working for equity in heterogeous classrooms: Sociological theory in practice* (pp. 181–192). New York: Teachers College Press.

NRC Mathematics Learning Study Committee (2001). *Adding it up: Helping children learn mathematics.* Washington: National Academy Press.

Oakes, J. (1990). *Multiplying inequalities: The effects of race, social class, and teaching.* Santa Monica: Rand.

Razfar, A., Khisty, L. L., & Chval, K. (in press). Re-mediating second language acquisition: A sociocultural perspective for language development. *Mind, Culture, and Activity*.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*(5), 447–466.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing.* Cambridge: Cambridge University Press.

Valencia, R. (2002). The plight of Chicano students: An overview of schooling conditions and outcomes. In R. Valencia (Ed.), *Chicano school failure and success: Past, present, and future* (pp. 3–51). New York: Routledge.

Vygotsky, L. S., & Cole, M. (1978). *Mind in society: The development of higher psychological processes.* Cambridge: Harvard University Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* New York: Cambridge University Press.

Chapter 30
Reconceptualizing Quality and Equity in the Cultivation of Minority Scholars in Mathematics Education

Roni Ellington and Glenda Prime

The Intersection of Equity and Quality

Concerns about equity in mathematics education continue to be relevant in light of the persistent achievement, attainment and persistence gaps between African Americans and their White counterparts. It is well documented that minority students are underachieving with respect to their majority peers (Anderson 1990; Oakes 1990; Powell 1990), fail to pursue mathematics-related disciplines (Oakes 1990), and are underrepresented in mathematics-related fields. Stinson's (2004) strong documentation of the ways in which mathematics has historically and currently served as a gatekeeper to "economic access, full citizenship, and higher education" (p. 11) provides support for the view that the achievement of equity, often seen as closing the achievement and persistence gaps, is a moral imperative. Moses and Cobb (2002) argue that knowledge of algebra is the next civil rights issue and that the knowledge gap in mathematics could turn students of color into the "designated serfs of the information age" (Moses and Cobb 2002, p. 11). These inequities exist in spite of the fact that there has been a growing interest in issues of equity in mathematics education over the past 25 years (Apple 1992, 1995; Hart 2003; Secada et al. 1995). The concept of equity itself has been the subject of much discussion in the literature. This discussion of how the concept is to be defined is more than an academic one because how we define equity has implications for how we attempt to achieve it and how we could know when we have.

Conceptions of equity in education have traditionally focused on equity of opportunity, equity of treatment, and equity of outcomes. In the context of mathematics education, traditional definitions would suggest that equity of opportunity be achieved through enhanced opportunities for minority students to access mathe-

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matics courses in numbers comparable to White students, and not be tracked into fewer or lower level mathematics courses. Equity of treatment would suggest that all students be exposed to learning experiences and student–teacher interactions that would optimize their learning. Most reform efforts aimed at addressing inequities are based on these views of equity. We would argue that the inadequacy of both of these conceptions of equity might be resulting in the ineffectiveness of the solutions implemented to address inequities in mathematics education. Fennema (1990) suggests that equity conceived of as equity of outcomes "offers the most promise for achieving justice" (p. 5). This conception of equity suggests that at the end of schooling, there would be no racial, ethnic, or gender differences in achievement levels, persistence in mathematics courses, or representation in the math-related pipelines; in short, equity of outcomes would be evidenced by a closing of the gaps.

National calls for access, diversity, and equity in mathematics education and mathematics-related careers have resulted in various initiatives designed to increase the numbers of minorities in mathematics and mathematics education (AAAS 1989, 1993; Anderson 1990; Committee for Economic Development 2003; National Academies of Sciences 2005; Johnson 1984; NCTM 2000; Oakes 1990). Many of these initiatives seem to suggest that providing African American students with increased opportunities to study mathematics and providing them with fair treatment in mathematics learning experiences is sufficient to achieve equity in mathematics education. *Mathematics for All,* an ideal embedded in the 2000 Standards document, and programs such as summer bridge programs and mentorship programs that allocate various financial, institutional, and human resources to minority-serving institutions, are examples of such initiatives. Despite these calls for equity and these initiatives aimed at addressing racial disparities in mathematics, the gaps still exist. African American students are not proportionally represented in courses that position them to gain access to those fields nor are they achieving at the level of Whites, and there is little evidence to suggest that the gaps will close in the near future. What is more, when these programmatic solutions fail to achieve the desired equity, there is a tendency to blame the students, the teachers, the curriculum, or the school districts.

We posit that the failure of these initiatives, even after more than 25 years of equity discussions, is due to the inadequacy of the conceptions of equity that underlie them. All of these programs and policies are based on the first two of the three conceptions of equity described earlier, equity of opportunity and equity of treatment. We concur with Fennema that conceptions of equity, as determined by equity of outcome hold the greatest promise to address the inequalities. We argue that there are two major problems with the notions of equity as equity of access and equity of treatment. Typically attempts to address equity of access focus on increasing the availability of mathematics courses and programs, thus removing the overt barriers to access. A deeper analysis of the reasons why African Americans fail to avail themselves of these provisions would reveal hidden social, cultural, and economic barriers that continue to deny them access even in the face of increased provisions. This idea that if we increase the representation of African Americans in mathematics courses, and if, in the service of equity of treatment, we ensure that they receive comparable allocations of resources, are treated fairly by their teachers, and have equally challenging curricula, we will remove inequities takes a static view of these inequalities. This notion fails to recognize the social structures that continually reproduce the inequities. The second problem with these notions of equity is that such views remove the imperative to address inequity by preparing disadvantaged minority students to disrupt the social structures, and challenge the rhetoric that produces the inequities in the first place.

A promising line of scholarship has expanded the notion of equity and views it from a social justice perspective. Scholars who espouse social justice perspectives suggest that attempts to remove inequities in mathematics education must address the systemic social inequities that are at the heart of inequitable outcomes in mathematics (D'Ambrosio 1990; Martin 2003; Stinson 2004). These conceptions of equity view the goal of equity initiatives as obtaining socially just outcomes in mathematics (Kreinburg 1989). They require that mathematics learning and opportunity be situated inside the social and structural realities of marginalized students. Much of the social justice discussion of equity encourages educators to question traditional conceptions about the nature of mathematics and examine the ways in which mathematics privileges some groups while marginalizing others. In questioning the social constructions of mathematics, scholars who adopt a social justice approach to mathematics education advocate that the teaching and learning of mathematics be used to help marginalized groups improve the conditions of their lives and foster agendas that liberate them from systemic oppression (Martin 2003; Guistien 2002). The works of social justice scholars are situated within many different literatures including ethnomathematics (D'Ambrosio 1997), critical mathematics education (Skovsmose 1994; Skovsmose and Valero 2001), culturally relevant pedagogy (Ladson-Billings 1995, 1997; Tate 1995), feminist pedagogy (Solar 1995), social just pedagogy (Burton 1996; Guistien 2002), and sociocultural frameworks in mathematics education (Moody 2001). Although this social justice scholarship is built from various theoretical foundations, the common focus of this work is to develop pedagogies for creating justice in mathematics education and society (Hart 2003).

Among such scholars, calls by Martin (2003) and Stinson (2004) for *empowerment* as a goal for mathematics education are particularly compelling. Specifically, Martin urges mathematics educators to "situate equity concerns within a broader conceptual framework that extends beyond classrooms and curricula" (Martin 2003, p. 11). This must be done, Martin argues, to empower students to address issues of unequal power relations between dominant and marginalized groups and to position students to address the systemic inequities that are at the root of inequities of access, treatment, and outcomes in mathematics. Stinson (2004) argues that equity cannot be achieved unless marginalized students are taught "to use mathematics knowledge in libratory ways to change and improve the conditions of their lives outside of school" (p. 14).

We suggest that this social justice perspective is ultimately an expansion of the third concept of equity, equity of outcome. Hence, our agreement with Fennema's views that equity, as equity of outcome is the most promising conception of equity. Our position is, however, based on an expanded view of the idea of educational outcomes, a view that takes educational outcomes beyond the walls of classrooms and schools to the wider society. Thus, social justice must itself be seen as an educational outcome. A social justice outcome must encompass at least three things. First, student achievement outcomes in mathematics must be equitable across all groups. Second, education in mathematics must seek to develop sensitivity to the ways in which mathematics has been used to exclude some from social and economic benefits. Third, it should foster a commitment to a reversal of this exclusion.

The social justice perspective not only provides a more encompassing notion of equity but urges on us a reconceptualization of the notion of *quality* in mathematics education. This perspective forces us to move beyond traditional quality indicators such as teacher qualifications, achievement scores, availability of resources, and achievement levels to a view of quality as determined by the extent to which mathematics education results in social justice. The notion of empowerment provides a powerful intersection between equity as equity of outcomes, and quality in mathematics education. It suggests that a quality mathematics education is one which moves us in the direction of social justice. It is an education that uses mathematics to empower all children, but especially marginalized groups, to discern, interrogate, expose, and ameliorate the conditions that result in unequal opportunities and power inequalities. From this empowerment/social justice perspective, a quality mathematics education is one which prepares its recipients to use mathematics and mathematical reasoning to dismantle the walls that exclude some from its benefits. It is this outcome that is the determinant of quality in mathematics education. Achieving this outcome might involve radical shifts in what mathematics is taught, to whom it is taught, how mathematics is viewed, and what outcomes are valued and assessed. Historically Black colleges and universities (HBCUs) have historically been spaces where this view of equity as social justice has been practiced.

So far in this chapter, we have advanced a position that argues for a new conception of quality and equity. Arguing from a social justice perspective, we suggest that the achievement of social justice must itself be the outcome of mathematics education, if equity of access, treatment, and participation are to be achieved. Because the sources of these inequities lie in entrenched social and economic structures and cultural values, mathematics education must intentionally adopt the goal of empowerment of students so that they are equipped to recognize and challenge the structures that debar them from full participation in mathematics. The extent to which education in mathematics does this is the ultimate criterion of quality. This is not to suggest that conceptions of equity as access or equity as treatment are erroneous. Indeed, equity of outcomes cannot be achieved in the absence to equity of access and treatment. The point is that these aspects of equity do not go far enough and that attempts to address these forms of inequity are not likely to succeed in closing the gaps between majority and minority students. Our view is that the way to achieve equity of access and treatment is by a focus on social justice as an outcome of mathematics education.

Our position is both conceptually and empirically based. In what follows, we draw on our experiences as faculty working in an HBCU setting, as well as on the experiences of the students we teach. The experiences of students were captured through empirical work documenting the successes and failures of our program. We use this work to illuminate the ways in which quality and equity are experienced by diverse students in our program.

HBCUs as Spaces for Equity as Social Justice

HBCUs were established during American segregation with the expressed mission of providing collegiate education to African Americans (Sissok and Shiau 2005; Willie et al. 2006). These institutions are "united in a mission to meet the educational and emotional needs of Black students" (Roebuck and Murty 1993). Thus, they have a history of educating socially marginalized students who may not otherwise have access to higher education (Hale 2006). Their mission clearly reflects a commitment to social justice with the goal of empowering African Americans to better their own lives and the lives of others (Hale 2006). In particular, HBCUs have been consistent in their efforts to change the landscape of mathematics education by changing who is learning mathematics and where mathematics is being taught. For example, the 105 HBCUs nationwide represent slightly more than 1.5% of the U.S. colleges and universities, yet they enroll over 11% of African American undergraduates and confer nearly 22% of all Bachelor's degrees granted to African Americans. Of the 6,105 Science, Technology, Engineering, and Mathematics (STEM) degrees awarded by HBCUs in 2001–2002, 87% were awarded to Black students. Of all degrees in STEM awarded to African American students in that year, 33% were awarded by HBCUs (Kim et al. 2008). This is some evidence that HBCUs confer a significant proportion of the mathematics degrees awarded to African Americans.

HBCU graduates are more likely to attend graduate school than their counterparts from White institutions (Baskerville et al. 2008). They are more likely to plan on entering graduate programs in the sciences and engineering (Wenglisky 1997). Of these, a significant number go on to earn doctoral degrees (Soloronzo 1995). Hence, HBCUs have been and continue to be vital to the achievement of equity in mathematics education by producing a cadre of mathematicians and mathematics educators who are both highly competent and sensitive to the needs of their community.

HBCUs are also changing the environment in which mathematics is taught. They are known for providing socially and culturally supportive environments, particularly for students in mathematics-related majors where negative stereotyping often serves to diminish their success (Fleming 1984; Harper et al. 2004; Parcella and Terenzini 2005; Perna 2009). Further, scholars suggest that an understanding of equity and social justice permeates the teaching and learning at HBCUs (Perna 2009; Parcella and Terenzini 2005), resulting in what Stinson refers to as epistemological empowerment, "individuals' growth of confidence in not only using mathematics, but also the personal sense of power over the creation and validation of knowledge" (Stinson 2004, p. 13). The mentoring that students receive at HBCUs is significant and has a positive impact on their success and persistence (Perna 2009). These institutions advance the cause of social justice by increasing the numbers of African Americans who persist in mathematics and by empowering students and communities in mathematics.

HBCU values are evidenced in their mission and goals. Typically, they seek to produce “highly competent Black students who have no ambivalence about who they are and how they should use their skills and talents to maximize their own and their community’s interests” (LeMelle 2002, p. 192). The criteria by which HBCUs judge their worth goes beyond student performance on traditional assessments and graduation rates. HBCUs evaluate the quality and impact of their programs by the extent to which their graduates are “giving back” and “changing the game” in mathematics and science. Although individual HBCUs may have differing emphases, fundamentally embedded in the mission of HBCUs is an inherent concern for equitable outcomes for the students they serve. It is this expanded view of quality that currently shapes the way Morgan State University (MSU) assesses its doctoral program in mathematics education.

The Doctor of Education (EdD) in Mathematics Education at MSU

MSU is the designated urban university for the state of Maryland. The University has a student population of approximately 7,000 students enrolled in 40 undergraduate, 26 masters and 13 doctoral programs. The university has been rapidly transforming itself from being primarily a teaching university to a research-focused institution. In 2005, it was awarded the Carnegie Foundation’s Doctoral Research University classification.

Although HBCUs have a strong history of educating African Americans in mathematics at the undergraduate level, they are relative newcomers in the arena of doctoral education, and even more so in the fields of mathematics and mathematics education. Further, there are only few African Americans who attain degrees in mathematics-related disciplines, including mathematics education. At MSU, our stated goal is to educate African American mathematics education scholars who possess the kind of sensitivity and skills needed to educate minority students and who will have far-reaching positive effects on their students’ persistence in mathematics and on scholarship in mathematics education.

The EdD in mathematics education program at MSU was started in 1997 with an initial cohort of three students. At present there are 22 students pursuing the degree, of whom over 90% are African Americans. In what follows, we attempt to illustrate what quality indicators, from a social justice perspective, look like in the context of Morgan’s EdD program in mathematics education, one of the only two mathematics education doctoral programs offered by an HBCU. Through the voices of students in this program, we attempt to illustrate how this HBCU is achieving quality by advancing a social justice agenda. Interview data (initially collected for a larger study) from eight of the students were analyzed to look for evidence of social justice outcomes.

We suggested earlier that social justice as an outcome means addressing inequities in student achievement in mathematics, sensitizing students so that they are able to recognize inequities in the outcomes of mathematics education, and fostering in them a commitment to reversing the unequal social structures that make mathematics the exclusionary endeavor that it is. Morgan's doctoral program in mathematics education addresses all three of these elements of this social justice agenda.

Enhancing Achievement Outcomes

The literature on social aspects of mathematics education makes it evident that in the United States, mathematics is the most segregated area of the curriculum. A trend of underrepresentation of Blacks begins in the upper levels of the high school and becomes more marked at the undergraduate and graduate levels (Anderson 1990; Oakes 1990). It is clear that if this underrepresentation is the case in mathematics, it is even more the case with mathematics education. The demographic data described above for Morgan State, where over 90% of the mathematics education doctoral students are African American is in stark contrast to this. This is significant not only because it represents a change in the racial and ethnic profile of who is pursuing mathematics-related fields, but because these teachers are drawn from schools that have predominantly African American student populations. Nationally, it is the case that African American high-school teachers teach in predominantly African American schools. Thus, by enhancing the mathematical pedagogical skills of these teachers we are positioning them to enhance mathematics achievement in their own students. This represents a change in the racial and ethnic profile of who is doing mathematics. In addition, the very presence of a doctoral program in mathematics education at an HBCU is for these students a counterpoint to the implicit and explicit messages that they have received about who can and cannot do mathematics or pursue advanced degrees. One African American female student recounts how these negative messages made her reluctant to enter a doctoral program.

> I was very hesitant because it was doctoral work. My educational experiences from the beginning since middle school, elementary, people would always say, administrators and some teachers… "You are not going to amount to anything; you are not going to do anything in life…" (Tonya)

It is plausible that the stereotyping of Blacks with respect to their ability to succeed in mathematics is at least a part of the reason for the inequities in access, treatment, and outcomes in mathematics education and that unless these are refuted they will continue to dampen the aspirations of African American students with respect to mathematics. Another student spoke of the fact that she never envisioned herself doing doctoral work until she learned of the program at Morgan.

In many ways the students who come to do doctoral study at MSU are nontraditional students. The average age of those interviewed was 36 and all were mid-career professionals. Most were pursuing the degree on a part-time basis. The program was structured to facilitate the needs of part-time students. By doing this we are changing who does mathematics education. The accommodation to the needs of such students is more than a matter of scheduling classes for working professionals, it involves deviations in the content of our courses and our research work, from the typical structure of doctoral programs, such that there is a deliberate intent to leverage the experiences, interests, and passion which the students bring to their doctoral work. Our goal is to equip these teachers to enhance the mathematics achievement of their own students.

Increasing Sensitivity to the Exclusionary Nature of Mathematics Education

As faculty in this doctoral program, we are very conscious that our social justice agenda must be as evident in *how* we teach as it is in whom we admit to the program. We are clear that it is our teaching, advising, and mentoring that have the potential to transform mathematics from an instrument for exclusion and selection, to one of inclusion, and to remove the demarcation between those who have access to the power that derives from mathematics and those who do not (Skovsmose and Valero 2001). We are very aware that what is at stake is not just the success and inclusion of the students in our program, but the possibility of extending our social justice agenda to the students and teachers whom they teach, and in the case of those in administrative positions, the policies that they influence. Teaching from this perspective implies a recognition of the nontraditional circumstances of our students. It requires that we respect the interests, experiences, and passions that they bring to the program, and that we provide nurturing and mentoring that is both professional and personal.

Many of the students who seek a doctoral degree in mathematics education at MSU do so because they are already sensitive to the issues of inequity in mathematics education, and they perceive that the doctoral experience at an HBCU would equip them to address the problems of inequity in mathematics education which they confront on a daily basis. When asked why they chose an HBCU, their replies suggested a passion for making a difference in the lives of marginalized students. Tonya's response captured her social justice concern, "*I thought if I can go through this program…I would be in a position to make decisions. I can be the mouth piece for these students and that was the focus of my goal*."

She further expressed the idea that another school "would not have the focus that was relevant to the urban problems I wanted to address." Another student, David, said that when making a decision about where to do his doctoral study, he explored other schools but "did not feel a connection to the passion of the program."

Our program seeks to heighten the students' sensitivity to the issues of inequity in mathematics education and deepen their understanding of the nature and sources of these inequities by engaging them in such issues in all of our courses. Our urban focus is evident in all of the experiences we provide for students, and they are encouraged to question the taken-for-granted assumptions about the causes of the persistent underachievement of African American students in mathematics. Courses in the foundations of education such as Contemporary Issues in Urban Education lend themselves easily to such emphases, but our concern for developing sensitivity in our students means that we seek to embed these ideas even in those courses that address the more technical aspects of mathematics education such as curriculum development and mathematics pedagogy. Thus, issues to do with equity and how it might be achieved run like a unifying thread throughout our students' coursework experiences.

Finally, our approach to students' research agenda affords us another opportunity to enhance their sensitivity to issues of inequity in mathematics education. Their research agendas, grounded in the urban settings in which most of them work, often reflect their concerns for social justice. Marcus spoke about how his experiences in the doctoral program afforded him the opportunity to pursue research with a social justice agenda.

> I already knew too that I wanted to do research at schools that I have attended… I knew that I was going to do research in the communities in which I had lived. I just didn't know how I was going to take the angle that would allow me to address the issue of racism and race.

He went on to explain how his course work introduced him to the theoretical perspectives that he would eventually use to frame his work. This is some evidence that our students begin to serve as social justice agents even before they have completed the program.

Some of the dissertations that our students are currently working on reflect their social justice concerns:

A Critical Ethnography of Black Middle School Students' Mathematics Education and Lived Realities,

A Phenomenological Study of Mathematics Meaning; A Possible Factor Affecting the Mathematics Achievement of African-American Male high School Students.

Urban Middle School Teacher Beliefs about Their African-American Students and the Influence of these on their Instructional Practice.

Working with students on their dissertations provides opportunities for the kind of personal and professional mentoring that we think is critical to a social justice outcome. Marcus spoke about the mentoring he received:

> My mentors encouraged me to keep going and supported me with preparing conference papers. This mentor had an off campus office where me, Jackie and Dr. Williams would work well into the night on conference papers and other work…. I needed a lot of support around this [writing conference papers], my writing was all over the place. Dr. Williams supported me with this while he was working on his own work…close access to this professor helped me get through.

Developing Commitment to Social Justice

We encourage our students to adopt attitudes, dispositions, and instructional practices that will dismantle the barriers that African American students face in mathematics. Hence, we insure that the structure of our doctoral program provides ongoing opportunities for them to change their practice to reflect a social justice mathematics agenda. This is achieved in various ways.

In keeping with the structure of most EdD programs, our program espouses both the theoretical and the practical emphases. In all of our courses, we encourage our students to apply what they have learned in our classes to their own practice, thus establishing a reciprocal relationship between their emerging knowledge and understanding and their practice. Their practice not only drives their research agendas, it provides the grist for their theorizing in their course work. Consequently, the insights gained in class are immediately applicable to their practice. Tonya explains, "*the coursework we are working on, the research that we are doing, I am always able to go to work and apply….*"

Finally, we offer courses that merge the theoretical and the practical. All of our students are required to take a practicum course in which they design, implement, and evaluate an action research-based intervention. In this course, students must reflect on what they have learned about teaching mathematics, particularly with a social justice orientation, and create an intervention that will impact their students' learning outcomes. Although the scope of these projects varies, one of the main goals of the practicum is to demonstrate how theoretical ideas about how to improve African American students', performance, persistence, and attitudes in mathematics translate into practical interventions. Our action research course gives students another opportunity to apply theoretical discussions to practical interventions. Hence, it is our goal that our students' sensitivity to social justice moves beyond the boundaries of the university classrooms and impacts the ways in which mathematics is taught and learned by the African American students they serve.

Conclusion

We have argued in this chapter that an explicit focus on social justice as an outcome of mathematics education is the only way to achieve equity in mathematics education. This represents an expanded view of the "equity as equity of outcomes" notion, from merely closing the outcome gaps in achievement and participation to one which seeks to address and ameliorate the causes of the inequities. We have attempted to broaden the notion of outcomes of mathematics education to one that is based on a social justice perspective. We argue that efforts to reverse inequities in mathematics must address the systemic social inequities that produce inequitable mathematics outcomes. The extent to which a mathematics education program does that is the true indicator of quality. It implies that a quality mathematics education is one that not only results in equitable outcomes with respect to mathematics achievement, but also equips students to discern the ways in which mathematics might be used in the creation of a socially just society, one in which all people have access to the power that knowledge of mathematics affords.

In the chapter, we explored some of the ways in which a doctoral program in mathematics education enacts a social justice agenda through its approach to teaching, advising and mentoring. Students in the program are predominantly African American and in many ways are nontraditional students, and their very presence as the majority component in a mathematics-related field attests to the social justice agenda of the program and the institution. What evidence do we have that the program is successful in achieving its equity-as-social-justice agenda? This is a long-term goal and one that is not easily assessed or objectively measured, but we judge our success by the extent to which our students evidence a commitment to social justice in their personal and professional lives. We have some anecdotal evidence of this in the active social justice research agendas of some of our graduates, and in the placements they seek after graduation. Of particular significance is the fact that many of our graduates choose to maintain their placements in urban schools where they are able to bring the insights and competencies they have gained to bear on the complex issues of unequal outcomes that affect minority students in urban settings.

Our reflection on our work towards the achievement of a social justice agenda through mathematics education, has brought us to the understanding that our success depends at least as much on who we are as agents of social justice ourselves as on what we do. Attempts to describe a social justice curriculum in terms of content and practices and program designs never capture the essence of a social justice agenda, and almost trivialize the process. When faculty and teachers personally commit to a social justice agenda it becomes the ethos of their institutions and classrooms and profoundly influences their ways of being with students. In the doctoral program we have described, it is the faculty's commitment to equity that manifests itself in the nurturing and mentoring and modeling that we do. It might be the case that a commitment to social justice must be caught rather than taught.

References

American Association for the Advancement of Science (AAAS). (1989). *Project 2061: Science for all Americans and benchmarks for science literacy.* New York: Oxford University Press.

American Association for the Advancement of Science (AAAS). (1993). *Benchmarks for science literacy.* New York: Oxford University Press.

Anderson, B. (1990). Minorities and mathematics: The new frontier and the challenge of the nineties. *Journal of Negro Education, 59*(3), 260–272.

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics educatio\n. *Journal for Research in Mathematics Education, 23,* 412–431.

Apple, M. W. (1995). Taking power seriously: New Directions in equity in mathematics education and beyond. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 329–348). Cambridge: Cambridge University Press.

Baskerville, L., Berger, L. N., & Smith, L. (2008). The role of historically black colleges and universities in faculty diversity. *American Academic, 4*(1), 11–31.

Burton, L. (1996). A socially just pedagogy for the teaching of mathematics. In P. Murphy & C. Gipps (Eds.), *Equity in the classroom: Toward Effective pedagogy of girls and boys* (pp. 136–145). London: Famler.

Committee for Economic Development and Research Policy Committee. (2003). *Learning for the future: Changing the culture of mathematics and science education to ensure a competitive workforce.* New York: Committee for Economic Development.

D'Ambrosio, U. (1990). The role of mathematics education in building a democratic and just society. *For the Learning of Mathematics, 10*(3), 20–23.
D'Ambosio, U. (1997). Where does ethnomathematics stand nowadays*? For the Learning of Mathematics, 17*(2), 13–17.
Fennema, E. (1990). Justice, equity, and mathematics. In E. Fennema & G. C. Leder (Eds.), *Mathematics and gender* (pp. 169–187). New York: Teachers College Press.
Fleming, J. (1984). *Blacks in college: A comparative study of student success in Black and White institutions.* San Francisco: Jossey-Bass.
Guistien, E. (2002). *Roads toward equity in mathematics education: Helping students develop a sense of agency.* Paper presented at the meeting of American Educational Research Association, April 2001.
Hale, F. W. (2006). *How Black colleges empower Black students: Lessons for higher education.* Sterling: Stylus.
Harper, S. R., Carini, R., Bridges, B., & Hayek, J. (2004). Gender differences in student engagement among African-American undergraduates at historically Black colleges and universities. *Journal of College Student Development, 45*(3), 271–284.
Hart, L. E. (2003). Some directions for research on equity and justice in mathematics education. In L. Burton (Ed.), *Which way social justice in mathematics education?* (pp. 27–49). Westport: Praeger.
Johnson, M. (1984). Blacks in mathematics: A status report. *Journal for Research in Mathematics Education, 15*(2), 145–153.
Kim, J., & Crasco, L. (2008). *Quality stem education leads to success: Vision of National Science Foundation's historically Black colleges and universities-undergraduate program.* Norwood: Systemic Research.
Kreinburg, N. (1989). The practice of equity. *Peabody Journal of Education, 66*(2), 127–146.
Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal, 32*(3), 465–492.
Ladson-Billings, G. (1997). It doesn't add up: African-American students mathematics achievement. *Journal of Research in Mathematics Education, 28*(6), 697–708.
LeMelle, T. J., (2002). The HBCU: Yesterday, today and tomorrow. *Education, 123*(1), 190–196.
Martin, D. (2003). Hidden assumptions and unaddressed questions in mathematics for all rhetoric. *The Mathematics Educator, 13*(2), 7–21.
Moody, V. (2001). The social constructs of mathematical experiences of African-American students. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 255–276). Mahwah: Lawrence Erlbaum.
Moses, R. P., & Cobb, C. E., Jr. (2002). *Radical equations: Math literacy and civil rights.* Boston: Beacon.
Oakes, J. (1990). Opportunities, achievement and choice: Women and minority students in science and mathematics, In C. B. Cazden (Ed.), *Review and research in education*, *16*(pp. 153–222). Washington: American Educational Research Association.
National Academy of Sciences. (2005). Committee on prospering in the global economy of the 21st century: An agenda for action. In *Rising about the gathering storm: Energizing and employing America for a brighter economic future.* Washington: National Academy of Sciences.
National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston: National Council of Teachers of Mathematics.
Parcarella, E., & Terenzini, P. (2005). *How college affects students: Volume 2, a third decade of research.* San Francisco: Jossey-Bass.
Powell, L. (1990). Factors associated with the underrepresentation of African-Americans in mathematics and science. *Journal of Negro Education, 59*(3), 292–298.
Perna, L. (2009). The contribution of HBCUs to the preparation of African-American women for Stem careers: a case study. *Research in Higher Education, 50*(1), 1–23.
Roebuck, J. B., & Murty, K. S. (1993). *Historically Black colleges and universities*. Westport: Praeger.

Secada, W., Fennema, E., & Adajajian, L. (1995). *New directions for equity in mathematics education.* Cambridge: Cambridge University Press.

Sissok, M., & Shiau, L. R. (2005). Minority enrollment demand for higher education at historically Black colleges and universities from 1976 to 1998: An empirical analysis. *Journal of Higher Education, 76*(2), 181–209.

Skovsmose, O. (1994). *Toward a philosophy of critical mathematics education*. Dordrect: Kluwer Academic Press.

Skovsmose, O., & Valero, P. (2001). The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 37–56). Hillsdale: Erlbaum.

Solar, C. (1995). An inclusive pedagogy in mathematics education. *Educational Studies in Mathematics, 28,* 311–333.

Soloronzo, D. G. (1995). The doctorate production and baccalaureate origins of African-Americans in the sciences and engineering. *Journal of Negro Education, 64*(10), 15–32.

Stinson, D. W. (2004). Mathematics as "gatekeeper"?: Three theoretical perspectives that aim toward empowering all children with the key to the gate. *The Mathematics Educator, 14*(1), 8–18.

Tate, W. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory into Practice, 34,* 166–173.

Wenglisky, H. (1997). *Students at historically Black colleges and universities: Their aspirations & accomplishments. Policy and informational report.* Princeton: Educational Testing Service.

Willie, C. V., Reddick, R. J., & Brown, R. (2006). *The Black college mystique.* New York: Rowman & Littlefield.

Chapter 31
What Does Quality Mean in the Context of White Institutional Space?

Danny Bernard Martin

Framing Quality

My goal in this chapter is to engage in a race-critical analysis of the concept of *quality* as it has been invoked in recent discourse associated with mathematics education reforms in the United States. A race-critical analysis draws from sociology (e.g., Bonilla-Silva 1997, 2001, 2003; Moore 2008; Omi and Winant 1994) and calls not only for an examination of how race and racism structure the very nature of the mathematics education enterprise but also for an examination of how mathematics education research, policy, and reform contribute to the dynamics of race and racism in the larger society. The goal of such an analysis is not to focus on individuals within the domain but rather on the configurations of power and practice, including discursive practices, which necessarily link mathematics education to racialized structures, process, and agendas in the rest of society (Martin 2006, 2009a, 2010).

I focus on quality because of the ways it has been linked to almost every aspect of these reforms—highly qualified teachers, high-quality curriculum, high-quality assessments—and because of the ways that mathematics education has subsequently been linked to maintaining the quality of life for the U.S. citizens and helping to maintain U.S. international standing. *Mathematics for All* and the *Final Report of the National Mathematics Advisory Panel* (U.S. Department of Education 2008) will serve as points of reference throughout. In my view, the underlying focus on quality in both is neither haphazard nor without political motivation and racial intent.

As pointed out by Apple (1992), Popkewitz (1980), and Tate (2004), words like *quality* are often associated with larger slogan systems like *back-to-basics, scientifically based research, standards-based education, global education,* and *multicultural education,* usually for the purpose of evoking a desired set of emotional

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responses and policy-related outcomes. Popkewitz (1980), in particular, noted the following about these key words and slogans:

> The importance of these phrases is that they are emotive…. The slogans potentially establish a mood or a form with which people can feel comfortable and affiliate with particular pedagogical practices…. The potency of a slogan is that it can create the illusion that an institution is responding to its constituency, whereas the needs and interests actually served are other than those publicly expressed. The slogan may suggest reform while actually conserving existing practices. (pp. 304–305).

With respect to "conserving existing practices," I claim that in recent years of U.S. mathematics education reform, *quality* has been infused into the discussions not only for the sake of accountability and improving outcomes such as test scores but also to preserve the interests of the larger white population, including economic and educational advantage, even while making claims about *equity* and *diversity* (Martin 2008, 2010).

My discussion in this chapter is not meant to imply that quality is a signifier for whiteness or that whiteness is a signifier for quality (Staiger 2004). Nor is my discussion meant to imply that other social groups are not interested in quality or that there is no overlap between white interests and the interests of non-dominant groups. However, I do claim, and history bears this out, that quality within the context of dominant white interests does not always align with quality as conceptualized by those who are in marginalized social positions (Bell 1980); nor do positive benefits previously experienced by whites translate into benefits for non-whites.

My race-critical analysis is buttressed by a critical discourse analysis of reform rhetoric and text. These analyses help to expose the subtleties of the *new racism* (Ansell 1997; Bonilla-Silva 2003; Giroux 2006) as it is produced and fostered through powerful channels like research and policy. As noted by van Dijk (2000):

> Especially because of their often subtle and symbolic nature, many forms of the new racism are discursive: they are expressed, enacted and confirmed by text and talk, such as…policies, laws,…political propaganda,…scholarly articles…. Discourse may thus be studied as the crucial interface between the social and cognitive dimensions of racism. (pp. 34–36)

Race-critical and critical discursive analyses raise several important questions: Why is there a heavy emphasis on quality as a dominant theme in the discourse of reform? Why is it that a word search of the *Final Report of the National Mathematics Advisory Panel* produced 98 instances of the word *quality* yet zero instances of the word *equity* in its recommendations for educating, presumably all, children in U.S. schools? What constitutes quality? Who decides? In what kinds of ideological and material spaces do these conceptualizations and interests unfold? How do dominance-serving conceptualizations of quality conflict with notions of quality and equity called for by non-dominant groups? Full responses to these questions are beyond the scope of a single chapter but it is important to raise them in order to make discussions of mathematics education reform more honest and to reveal its deeper political roots, aims, and goals (e.g., Gutstein 2008, 2009; Martin 2008).

In the latter part of this chapter, I give particular attention to Black learners in the United States relative to these quality-focused discussions. I argue that the structural practices, norms, and ideologies, including racism that have long arbitrated quality, standards, and resources in favor of white learners cannot simultaneously serve the diverse needs of Black learners.

Quality in Service to Nationalism—Nationalism in Service to Racism

The racial dynamics referenced in the opening paragraph of this chapter include the racism inherent in nationalist and security-focused political agendas of previous Republican administrations. In forming the *National Mathematics Advisory Panel,* for example, former President George W. Bush was able to extend these agendas into mathematics education. This is a serious claim but one that can be best understood in the context of (a) the administration's prevailing racial politics and (b) the expected role that these reforms will play in helping to shield the United States from perceived threats to U.S. national security and preserve the quality of life for its citizens, on the other. Consider the first few sentences of the Final Report's *Executive Summary:*

> The eminence, *safety,* and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas…. Much of the commentary on mathematics and science in the United States focuses on national economic competitiveness and the economic well-being of citizens and enterprises. There is reason enough for concern about these matters, but it is yet more fundamental to recognize that the *safety* of the nation and the *quality of life*—not just the prosperity of the nation—are at issue. (p. xi; italics added)

To further stress the role that mathematics education should play in this nationalist and security-focused agenda, the *Final Report* goes on to state:

> During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and *quality* of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century. This report is about actions that must be taken to strengthen the American people in this central area of learning. Success matters to the nation at large…. Ignoring *threats* to the nation's ability to advance in the science, technology, engineering, and mathematics (STEM) fields will put our economic viability and our basis for *security* at risk. (pp. 1–2, italics added)

Gutstein (2009), in a searing analysis of former President Bush's American Competitiveness Initiative (ACI) (Domestic Policy Council 2006), to which the formation of the National Mathematics Advisory Panel is linked, noted similar quality-focused and economic-threat language in other documents, such *as Rising Above the Gathering Storm* (National Academies 2006). In that document, it is stated:

> Without high-quality, knowledge-intensive jobs and the innovative enterprises that lead to discovery and new technology, our economy will suffer and *our people* will face a lower standard of living. (p. 1, italics added)

Two key questions can be asked about the excerpts presented above. First, what threats to national security and quality of life in the United States is the report referring? Second, how is the identification of these threats related to "the organizing principles that generate, shape, and sustain white supremacy designed to exclude other human beings by virtue of their race, language, culture, and ethnicity so that they can be exploited" (Macedo and Gounari 2006, p. 3)? It is important to note that while the statements above make mention of economic and security threats, the United States, in reality, has been, and remains, the most dominant economic and military superpower in the world, and history bears witness to the fact that it has gone to great lengths to maintain that standing. So, what is the subtext of these fears? Considering the political origins of the National Math Panel, I believe these quality-focused and security concerns can be linked to ongoing post-9/11 anti-Muslim sentiments, the intensification of a new racism within the U.S. borders that is often more sophisticated and implicit than earlier versions, and the globalization of U.S. racism and white privilege beyond its borders (Abbas 2004; Macedo and Gounari 2006; Mosse 1995; Winant 2004). This new racism can be associated with New Right politics that was a hallmark of the Bush administration. Giroux (2006) noted the following analysis by Ansell (1997):

> The new racism actively disavows racist intent and is cleansed of extremist intolerance…. It is a form of racism that utilizes themes related to culture and nation as a replacement for the now discredited biological referents of the old racism. It is concerned les with notions of racial superiority in the narrow sense than with the alleged "threat" people of color pose—either because of their mere presence or because of their demand for "special privileges"—to economic, socioeconomic, political, and cultural vitality of the dominant (white) society. It is, in short, a new form of racism that operates with the category of "race." It is a new form of exclusionary politics that operates indirectly and in stealth via rhetorical inclusion of people of color and the sanitized nature of its racist appeal. (pp. 21–22)

Macedo and Gounari's (2006) cogent analysis of the racialized nature of the "threat" is particularly helpful in the race-critical analysis presented in this chapter:

> The dichotomy [between "us" and "them"] has been astutely used by the Bush administration to conduct its war on terror and expand its imperial ambitions unimpeded by a domestic opposition. By constructing a terrorist enemy that encompassed all Muslims (a "group" that amounts roughly to 1.2 billion people worldwide and comprises numerous countries, societies, traditions, languages and lived experiences), the Bush administration, aided by a compliant media, exacerbated the racism present in U.S. society so that all Muslims became suspected terrorists. And it legitimized racist treatment of Muslims, as when "Muslim-looking" individuals are deplaned by major airlines because white folks fear of flying in their company. However, the same racial profiling was never applied to white males resembling Timothy McVeigh after the terrorist bombing of the federal building in Oklahoma City, where more than one hundred fifty people died, including women and children. (p. 5)

The us-versus-them sentiment is also present in the economic and quality-focused arguments of mathematics education reform, where it is suggested that threats from abroad (them) are encroaching on U.S. (us, our people, American) international standing and quality of life. The push for quality is therefore a pushback against this threat. Moreover, the push for quality represents a commitment to maintaining and exploiting racial conflict via anti-terror nationalist arguments. In my view, this nationalist fervor has spilled over into efforts calling for *Mathematics for All,* given that such efforts often link the aims and goals of mathematics education to workforce, international competitiveness, and security concerns (e.g., RAND Mathematics Study Panel 2003; U.S. Department of Education 1997).

Does All Really Mean All?

Gutstein (2009) makes a compelling case for questioning the benefits to *all* that are promised in reform initiatives. In his critical analysis, he stated:

> …history shows that when U.S. productivity increases, the wealthiest benefit, not the majority …the impression is clear that this national situation affects all U.S. people. When the documents frame the problems as, for example, losing control of global markets, they imply a negative impact on the people as a whole rather than the potential decline of U.S. corporate profits. (pp. 138–154)

A word search of the *Final Report of the National Mathematics Advisory Panel,* clarifies for whom the quality-of-life referenced in the report applies. The search produced 21 instances of the word *American* (with repetition of some sentences), 11 instances of the word *citizen* (with repetition of some sentences), two non-repeated references to the word *minority,* and only one mention of the word *resident.* Such references, according to van Dijk (2000), contribute to the discursive construction of the *Other* that is needed in nationalist and racist ideologies. This implicit distinction between citizens and non-citizen, American and non-American, despite the rhetoric about "all our people" is more clearly understood in the context of anti-immigrant policies and sentiments flowing from former President Bush's Republican Administration. This includes, as an example, the passing of the Secure Fence Act of 2006 (Pub.L. 109–367), which:

> …allows for over 700 miles (1,100 km) of double-reinforced fence to be built along the border with Mexico, across cities and deserts alike, in the U.S. states of California, Arizona, New Mexico, and Texas in areas that have experienced illegal drug trafficking and illegal immigration. It authorizes the installation of more lighting, vehicle barriers, and border checkpoints, while putting in place more advanced equipment like sensors, cameras, satellites and unmanned aerial vehicles in an attempt to watch and control illegal immigration into the United States. (retrieved on December 1, 2009 from http://en.wikipedia.org/wiki/Secure_Fence_Act_of_2006)

In his official statement to the press following passage of the bill, former President Bush stated the following:

> This bill will help protect the American people. This bill will make our borders more secure…. We must face the reality that millions of illegal immigrants are already here. They should not be given an automatic path to citizenship; that is amnesty. I oppose amnesty. (retrieved on December 1, 2009 from http://georgewbush-whitehouse.archives.gov/news/releases/2006/10/20061026.html)

In my view, the underlying rhetoric of both the *Final Report of the National Mathematics Advisory Panel* and *Mathematics for All* have been constructed to appeal to both white liberal and conservative consciousness. The appeal in the *Final Report* is to nationalism and nativism. In *Mathematics for All,* there is an underlying appeal to white middle- and upper-class liberalism to convince whites that others must now share in the opportunities that they have long enjoyed; that is "their needs—for more and better jobs, access to education and health care…can be linked to those of the minority poor if the 'wedge issue' of race can be blunted" (Winant 2004, p. 60).

However, as noted by Schoenfeld and Pearson (2009), the appeal to white consciousness is sometimes met by resistance, revealing the racial dynamics at play in public and political negotiations of excellence (i.e., quality) and access (equity):

> Simply put, the anti-reform forces in reading and mathematics grew strong at a time of the resurgence of the right wing in California politics. San Diego politician Pete Wilson had ridden "wedge politics" (appeals to the fears of the White middle-class voting majority regarding the rising populations and rights of minorities) to become mayor of San Diego. Wilson was a strong supporter of Proposition 187, a 1994 ballot initiative designed to deny illegal immigrants social services, healthcare, and public education. (The proposition won at the ballot box, with non-Latino Whites being the largest voting block in favor; it was later declared unconstitutional.) In 1996, California voters passed Proposition 209, which abolished affirmative action programs in public institutions (Office of Legislative Analysis, State of California 1996). In 1998, voters passed Proposition 227, which "requires all public school instruction be conducted in English" (California Voter's Guide 1998) and severely curtailed bilingual education. The *Standards* represented a clear tilt toward the "democratic access" view of education. Advocates of reform believed in "mathematics for all"—in particular that it was possible to achieve excellence *and* equity, without sacrificing one for the other. There are many who believe that the goals of equity and excellence [i.e., quality] are in tension, and that making mathematics accessible to many more students necessarily entails "dumbing down" the mathematics. If one believes this, then two consequences of the democratization of mathematics as proposed by reform are (a) a weakening of the mathematical preparation of our best students, and a concomitant weakening of the nation's base of mathematically and scientifically prepared elite and (b) a different demographic mix of those who are considered to be prepared for entry into elite institutions and professions. (p. 573)

So, while such efforts have equity-oriented veneers and are based on an underlying appeal to quality, it would appear that there are other ideologies at play that are not based exclusively on moral and humanistic concern for those who are marginalized in mathematics (Martin 2003). In my view, it is inconceivable that the real goal of *Mathematics for All* is to reconstruct the opportunity structure associated with mathematics education in such a way that we move from an arrangement that has long served white males and the wealthy and defined quality on their terms to an arrangement where Blacks, Latinos, and Native Americans sit atop the hierarchy of material benefits and power. As the statement from Schoenfeld and Pearson (2009) demonstrates, such a rearrangement is likely to be met with resistance (Berry and Bonilla-Silva 2008; Brantlinger 2003; Lipman 2004). Some compromise, for the sake of public appearance, is likely to emerge. Very rarely, if ever, has it materialized that Black, Latino, and Native Americans have collectively enjoyed access to the best learning opportunities, best teachers, best curriculum, most funding, and greatest levels of social and economic reward. In the context of these limitations, *Mathematics for All,* and all other mathematics education reforms, must be analyzed for their implicit racial content, racial signification, and hidden agendas despite their rhetoric about equity and quality (Martin 2003).

White Institutional Space and Mathematics Education Reform

The quality-focused rhetoric of *Mathematics for All* and the *Final Report of the National Mathematics Advisory Panel* can be further understood via the characterizations of mainstream mathematics education reform and policy contexts as instantiations of *white institutional space* (Feagin 1996; Moore 2008). I distinguish *mainstream* mathematics education research and policy as that which has relied on traditional theories and models of teaching and learning (e.g., information processing, constructivism, situated cognition) and research approaches (race-neutral analyses, race-comparative analyses) developed primarily by white researchers and policy makers to normalize the mathematical behavior of white children (Martin 2009a, 2009b, 2009c, 2009d). The term *white institutional space* comes from the work of sociologists Feagin (1996) and Moore, who, in her book *Reproducing Racism: White Space, Elite Law Schools, and Racial Inequality* (2008), examined the white space of law schools and how the ideologies, discourses, and practices in these schools serve to privilege white perspectives, white ideological frames, white power, and white dominance all the while purporting to represent law as neutral and objective.

Based on her analysis, Moore (2008) claimed that the historical development of law schools as white institutional space is characterized by four foundational elements: (1) exclusion of people of color from elite law schools and positions of power in legal institutions which results in the accumulation of white economic and political power, (2) the development of a white frame that organizes the logic of these institutions and normalizes white racial superiority, (3) the historical construction of a curricular model based on the thinking of white elites, and (4) the assertion of law as a neutral and impartial body of doctrine unconnected to power relations.

What is highlighted by the characterization of mainstream mathematics education reform and policy contexts as instantiations of white institutional space is that the enterprise of mathematics education is not immune to the structural and institutional racism that characterizes many other areas of U.S. society. In my view, it is no different than other racialized spaces and should be subjected to the same antiracist scrutiny (Martin 2008, 2009a, 2009d, 2010).

Although they vary in ideological and epistemological orientation, white scholars and policy makers continue to be overwhelmingly represented in elite positions of power in mathematics education, able to determine the tone, direction, and impact of reforms, and able to define standards for quality, even in the contexts of changing population and school demographics. However, with a few notable exceptions (e.g., Silver et al. 1995; Treisman 1985), the gestalt of their ideologies, policies, and practices have produced only marginal gains for non-white children, even as the political context of education has resulted in calls for greater accountability.

Moreover, the relative exclusion of those African American, Latino, and Native American scholars and policy makers who are likely to call into question these power dynamics is readily apparent in contexts ranging from handbook chapters to national panels and discussion groups. As noted in Martin (2008), no African American mathematics education researchers were on the National Mathematics Advisory Panel. To what degree might those in power, and who have the ability to exclude, offer quality-oriented rationalizations as an excuse?

Within white spaces, it is also true that the needs of non-dominant groups are often conceptualized and framed in ways that are contingent on and tied to the well-being and status of the larger white society. Secada (1989) has called this *enlightened self-interest.* Race-critical theorists refer to this as *interest convergence,* which suggests that "gains for blacks [and other minority groups] coincide with white self interest and materialize at times when elite groups need a breakthrough for African Americans [and other minority groups], usually for the sake of world appearances or the imperatives of international competition" (Delgado 2002, p. 371).

In his seminal article, *Brown versus Board of Education and the Interest-Convergence Dilemma* (1980), critical legal scholar Derrick Bell demonstrated how white elites in positions of power manipulated discourse, policy, and outcomes on school desegregation litigation for political, rather than moral, reasons. Delgado (2002) poignantly summarized Bell's argument about why the Supreme Court ruled the way that it did in that landmark court case:

> To explain the rise and fall of black fortunes, according to Bell, one must attend to such matters as the labor market, the need to placate working class whites, wartime needs for solidarity and bodies to serve in industry or on the front, and the exigencies of Cold war competition. Taking as his principle example, Brown v. Board of Education, Bell posited that this remarkable decision came about when it did due to Cold War politics. Bell invited his readers to consider how the NAACP Legal Defense Fund had been litigating school funding and desegregation cases for decades throughout the South, generally losing or winning, at most narrow victories. Then, in 1954, the skies opened—the Court declared, for the first time in a school desegregation case, that separate was no longer equal. Why then? Bell pointed out that the country had just celebrated the end of a bloody world war against Germany and Japan, during which many black men and women had served gallantly. Having risked their lives for the cause of freedom, they were unlikely to return meekly to the former regime of menial jobs and segregated facilities. For the first time in decades, the prospect of serious racial unrest loomed…. The balance of interests shifted; elite whites now saw a powerful reason to advance blacks' cause. For Bell, the Brown decision came about when it did, not because of altruism or advancing notions of social morality. Rather, elite whites on the Supreme Court, in the State Department, and in other circles of power simply perceived that America's self-interest lay in publicly supporting blacks so as to gain an edge in the Cold War with Russia. (p. 372)

Interest convergence helps to explain the duplicitous nature of rhetoric and policy efforts championed by mainstream mathematics education researchers and policy makers, efforts that are promoted as being driven by quality and being beneficial to non-dominant groups while simultaneously functioning to preserve powerful white interests. Consider the statements below from *Everybody Counts* (National

Research Council 1989) and the *Final Report of the National Mathematics Advisory Panel* (U.S. Department of Education 2008), respectively.

> Currently, 8 percent of the labor force consists of scientists or engineers; the overwhelming majority are White males. By the end of the century, only 15 percent of net new entrants to the labor force will be White males. *Changing demographics have raised the stakes for all Americans. Never before have we been forced to provide true equality in opportunity to learn.* (National Research Council 1989, p. 19, italics added)

> "[O]ver the past 40 years, there has been a significant decrease in the proportion of doctorates earned by U.S. citizens and permanent residents in STEM fields. In 1966, they earned 83.5% of all STEM doctorates awarded, but in 2004, they earned just 59.8%" (Babco 2006). This strategy may not work in the future, however, because the supply of immigrant and temporary nonimmigrant STEM professionals may become more uncertain for reasons addressed above. It is therefore in the *national interest* to increase the number of *domestic students* studying and receiving degrees in STEM areas. (U.S. Department of Education 2008, p. 2, italics added)

What is particularly interesting about this reform-oriented discourse is how the needs of African American, Latino, Native American, and poor children have often been subsumed under what is good for *all* in lieu of focusing on what may be appropriate and specifically needed for particular groups. Note that the concern in the first excerpt presented above is with the *decline* in the number of white males, not with the increasing number of minorities. Also, the assumption across these discursive contexts seems to be that mere access to highly qualified teachers, high-quality teaching, high-quality curriculum materials, and high-quality assessments long enjoyed by whites will insure that all children have the opportunity to achieve up to their potential and that inequities in participation, achievement, and persistence will be eliminated. I would claim that a generic focus on *all* or the *nation* contains the following contingencies: (1) if it is good for whites, then it will be good for other groups and (2) before it can be considered good for everyone, it must be considered good for whites.

Quality, Good Intentions, and the Needs of Black Children

> To provide schooling for everyone's children that reflects liberal, middle-class values and aspirations is to ensure the maintenance of the status quo, to ensure that power, the culture of power, remains in the hands of those who already have it.… Several black teachers have said to me recently that as much as they'd like to believe otherwise, they cannot help but conclude that many of the "progressive" educational strategies imposed by liberals upon black and poor children could only be based on a desire to ensure that the liberals' children get sole access to the dwindling pool of American jobs. Some have added that the liberal educators believe themselves to be operating with good intentions, but that these good intentions are only conscious delusions about their unconscious true motives. (Delpit 1995, pp. 28–29)

As an African American scholar in a field numerically dominated by white scholars and as someone who is committed to meaningful mathematics education for Black children in the United States, I remain ever mindful of the quote by Lisa Delpit presented above. Delpit's (1995) critique of 1980s literacy education reform was directed at the proliferation, and misapplication, of process-oriented approaches to writing by liberal white scholars and teachers. Delpit claimed that limited attention to skills development, in favor of the these process approaches, often hindered Black children from gaining access to the linguistic codes of power necessary for negotiating success in the larger society. White middle-class children, she argued, were already familiar with these codes and reform-oriented school practices that were being promoted only served to further empower these children.

Inherent in Delpit's critique is the issue of whether those in power to decide what children should learn, how they should learn, for what purposes they should learn, and how they should be assessed, can truly make their decisions based on what is best for those who are outside the culture of power or whether there are vested interests in preserving existing social relations and hierarchies. Delpit's quote also acknowledges that white scholars, policy makers, and practitioners continue to serve as the principal architects of education for other people's children.

Because of the well-documented oppression, subjugation, and stigmatization faced by Black children, and, to date, the inability of reforms to significantly alter negative outcomes, I can only share Delpit's and other Black scholars' "pragmatic infusion of suspicion" (Alston 1995) about the good intentions of these architects and their reforms. I find Delpit's critique particularly relevant as I consider recent mathematics education reform relative to the needs of Black children. I find it necessary to ask where the needs of Black children *really* fit within these converging interests of quality-focused reform, national security, and workforce preparation? Surely, the needs of Black children as *Black* children must figure prominently in these concerns, given their historical underrepresentation in mathematics and their disproportionate and continuing struggles against difficult social realities. Wouldn't these realities also represent a compelling convergence of interests?

Although much of my argument in this chapter has focused on a critical analysis of *quality,* I have described elsewhere (Martin 2007b) how other key words like *achievement,* have simultaneously been used to socially construct Black children as deficient in service to dominant racial ideologies that render Black culture, families, and communities as pathological. A term like achievement, when used in conjunction with a race-comparative research paradigm in mathematics education facilitates ranking hierarchies which suggest that Black learners should become more like learners who are identified as White and Asian if they are to become high academic achievers (Martin 2007a, 2009a, 2009b, 2009d). This race-comparative approach is one that usually begins with Black inferiority and the normalization of white behavior, implicitly associating the latter with quality (Staiger 2004), and having the outcomes of white children serve as the benchmark of progress for all others. For example, the *Final Report of the National Mathematics Advisory Panel* (U.S. Department of Education 2008) cites the following statement:

> The achievement gap between students of differing ethnic and socioeconomic groups can be significantly reduced or even eliminated if low-income and minority students increase their success in high school mathematics and science courses. (Evan et al. 2006, p. 11).

This statement is merely a variant of the colloquial solution to "closing" the so-called racial achievement gap, which is typically framed as raising Black, Latino, Native American, and poor students to the level of white students (see Martin 2007b, 2009a, 2009b, 2009c, 2009d for more extensive critiques of racial achievement gap rhetoric). Little attention is given to the structural and institutional forces that help to create and maintain such gaps. Moreover, what would happen if low-income and minority students not only matched, but also outperformed, white students on high-status assessments and the so-called racial achievement gap was one that placed white students in a subordinate position. Would the discourse shift to defining quality on terms dictated by the performance of minority students? Would the discourse reflect beliefs about cultural deficits and intellectual inferiority of white students? Evidence from a study of Black students in England suggested that if such a reversal occurred, those in power to set standards and define quality would resort to such drastic strategies as changing the test (Gilborn 2008).

A few years ago, I was a participant on a panel that included a discussion of mathematics achievement data for Black and white students in one region of the United States. One of the panelists pointed out that several years prior, white students had achieved levels that were considered acceptable and that put them on the path toward college and meaningful employment. A few years later, when Black students had achieved those very same levels, and after white students had reaped the benefits of those standards, these levels were no longer deemed acceptable in the environment of accountability. Upward shifts in *both* white and Black student achievement, it appeared, had mostly benefited white students.

Of course, these comments are not meant to suggest that Black learners should not strive for higher levels of academic achievement or full and meaningful participation in the larger opportunity structure. However, alternative framings of the goals for mathematics education are needed that do not require one to accept deficit-oriented characterizations of Black children in relation to the performance of white students or do not reduce Black children's efforts to servicing those in power (Martin 2009a, 2009b, 2009c, 2009d; Martin and McGee, 2009; Perry, Steele, and Hilliard 2003). Martin and McGee (2009) and Martin (2009b, d) have stressed liberatory framings that focus on mathematical, epistemological, and social empowerment (Ernest 2002), but in ways that positively affirms Black children's racial, mathematical, and academic identities.

Conclusion

In recent years of U.S. mathematics education reform, *quality* has been linked to almost every aspect of these reforms—highly qualified teachers, high-quality curriculum, and high-quality assessments. These reforms have, in turn, been linked to maintaining the quality of life for the U.S. citizens and helping to maintain U.S. international standing. I have argued in this chapter that this focus on quality is neither haphazard nor without political motivation and intent.

I claim that quality has been infused into reform not only for the sake of accountability and improving outcomes such as test scores but also to preserve the interests of the larger white population. Definitions of quality, for example, are not race-neutral and the linking of quality-focused mathematics education reforms to liberal and neoconservative political agendas demonstrates a convergence of interests that renders real equity concerns as secondary considerations, at best.

References

Abbas, T. (2004). After 9/11: British South Asian Muslims, Islamophobia, multiculturalism and the state. *American Journal of Islamic Social Sciences, 21*(3), 26–38.

Alston, K. (1996). Race consciousness and the philosophy of education. In A. Nieman (Ed.), *Philosophy of Education 1995* (pp. 149–156). Urbana: Philosophy of Education Society.

Ansell, A. E. (1997). *New right, new racism: Race and reaction in the United States and Britain.* New York: New York University Press.

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education, 23*, 412–431.

Bell, D. A. (1980). Brown v. Board of Education and the interest convergence dilemma. *Harvard Law Review, 93*, 518–533.

Berry, B., & Bonilla-Silva, E. (2008). They should hire the one with the best score: White sensitivity to qualification differences in affirmative action hiring decisions. *Ethnic and Racial Studies, 31*(2), 215–242.

Bonilla-Silva, E. (1997). Rethinking racism: Toward a structural interpretation. *American Sociological Review, 62*, 465–480.

Bonilla-Silva, E. (2001). *White supremacy and racism in the post-civil rights era.* Boulder: Lynne Reinner.

Bonilla-Silva, E. (2003). *Racism without racists: Color-blind racism and the persistence of racial inequality in the United States.* Lanham: Rowman and Littlefield.

Brantlinger, E. (2003). *Dividing classes: How the middle class negotiates and rationalizes school advantage.* London: Falmer Press.

Delgado, R. (2002). Explaining the rise and fall of African American fortunes: Interest convergence and civil rights gains. *Harvard Civil Rights-Civil Liberties Law Review, 37*, 369–387.

Delpit, L. (1995). *Other people's children: Cultural conflict in the classroom.* New York: New Press.

Domestic Policy Council. (2006). *American competitiveness initiative.* Washington: US Government Office of Science and Technology Policy.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Journal*, 15. Retrieved April 18, 2003, from http://www.ex.ac.uk/~PErnest/pome15/contents.htm.

Evan, A., Gray, T., & Olchefske, J. (2006). *The gateway to student success in mathematics and science*. Washington, DC: American Institutes for Research.

Feagin, J. R. (1996). *The agony of education: Black students at white colleges and universities.* New York: Routledge.

Gilborn, D. (2008). *Racism and education: Coincidence or conspiracy?* London: Routledge.

Giroux, H. (2006). Spectacles of race and pedagogies of denial: Antiblack racist pedagogy. In D. Macedo & P. Gounari (Eds.), *The globalization of racism* (pp. 68–93). Boulder: Paradigm Publishers.

Gutstein, E. (2008). The political context of the National Mathematics Advisory Panel. *The Montana Mathematics Enthusiast, 5*, 411–418.

Gutstein, E. (2009). The politics of mathematics education in the US: Dominant and counter agendas. In B. Greer, S. Mukhopadhyay, S. Nelson-Barber, & A. Powell (Eds.), *Culturally responsive mathematics education* (pp. 137–164). Mahwah: Erlbaum Associates.

Lipman, P. (2004). *High-stakes testing*. London: Routledge.
Macedo, D., & Gounari, P. (2006). Globalization and unleashing the new racism: An introduction. In D. Macedo & P. Gounari (Eds.), *The globalization of racism* (pp. 3–35). Boulder: Paradigm Publishers.
Martin, D. (2003). Hidden assumptions and unaddressed questions in mathematics for all rhetoric. *The Mathematics Educator, 13*(2), 7–21.
Martin, D. (2006). Mathematics learning and participation as racialized forms of experience: African American parents speak on the struggle for mathematics literacy. *Mathematical Thinking and Learning, 8*(3), 197–229.
Martin, D. (2007a). Mathematics learning and participation in African American context: The co-construction of identity in two intersecting realms of experience. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 146–158). New York: Teachers College Press.
Martin, D. (2007b). Beyond missionaries or cannibals: Who should teach mathematics to African American children? *The High School Journal, 91*(1), 6–28.
Martin, D. (2008). E(race)ing race from a national conversation on mathematics teaching and learning: The national mathematics advisory panel as white institutional space. *The Montana Mathematics Enthusiast, 5*(2–3), 387–398.
Martin, D. (2009a). Researching race in mathematics education. *Teachers College Record, 111*(2), 295–338.
Martin, D. (2009b). Liberating the production of knowledge about African American children and mathematics. In D. Martin (Ed.), *Mathematics teaching, learning, and liberation in the lives of Black children* (pp. 3–38). London: Routledge.
Martin, D. (2009c). Does race matter? *Teaching Children Mathematics, 16*(3), 134–139.
Martin, D. (2009d). Little Black boys and little Black girls: How do mathematics education research and policy embrace them? *Proceedings of the 33rd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Atlanta, GA.
Martin, D. (2010). Not-so-strange bedfellows: Racial projects and the mathematics education enterprise. In U. Gellert, E. Jablonka, & C. Morgan (Eds.), *Proceedings of the Mathematics Education and Society 6th International Conference* (pp. 57–79). Berlin, Germany.
Martin, D., & McGee, E. (2009). Mathematics literacy for liberation: Reframing mathematics education for African American children. In B. Greer, S. Mukhophadhay, S. Nelson-Barber, & A. Powell (Eds.), *Culturally responsive mathematics education* (pp. 207–238). New York: Routledge.
Moore, W. (2008). *Reproducing racism: White space, elite law schools, and racial inequality.* New York: Rowman & Littlefield Publishers.
Mosse, G. (1995). Racism and nationalism. *Nations and Nationalism, 1*(2), 163–173.
National Academies. (2006). *Rising above the gathering storm: Energizing and employing America for a brighter economic future.* Washington: National Academies Press.
National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington: National Academy Press.
Omi, M., & Winant, H. (1994). *Racial formation in the United States.* New York: Routledge.
Perry, T., Steele, C., & Hilliard, A. (Eds.). (2003). *Young, gifted and black: Promoting high achievement among African-American students.* Boston: Beacon Press.
Popkewitz, T. (1980). Global education as a slogan system. *Curriculum Inquiry, 10*(3), 303–316.
RAND Mathematics Study Panel. (2003). *Mathematics proficiency for all students: Toward a strategic research and development program in mathematics education.* Santa Monica: RAND.
Secada, W. (1989). Agenda setting, enlightened self-interest, and equity in mathematics education. *Peabody Journal of Education, 66*(2), 22–56.
Schoenfeld, A., & Pearson, D. (2009). The reading and math wars. In G. Sykes, B. Schneider, & D. Plank (Eds.), *Handbook of education policy research* (pp. 560–580). New York: Routledge.
Silver, E., Smith, M., & Nelson, B. (1995). The Quasar project: Equity concerns meet mathematics education reform in the middle school. In W. Secada, E. Fennema, & L. Adajian (Eds.), *New*

directions for equity in mathematics education (pp. 9–56). Cambridge: Cambridge University Press.

Staiger, A. (2004). Whiteness as giftedness: Racial formation at an urban high school. *Social Problems, 51*(2), 161–181.

Tate, W. F. (2004). What is a standard? In *Proceeding of the NCTM Research Catalyst Conference* (pp. 15–23). Reston: NCTM.

Treisman, P. U. (1985). *A study of the mathematics performance of Black students at the University of California, Berkeley.* Unpublished doctoral dissertation, University of California-Berkeley.

U.S. Department of Education. (1997). *Mathematics equals opportunity.* Washington: U.S. Government Printing Office.

U.S. Department of Education. (2008). *Foundations of success: Final report of the national mathematics advisory panel.* Washington: U.S. Government Printing Office.

Van Dijk, T. A. (2000). New(s) racism: A discourse analytical approach. In: S. Cottle (Ed.), *Ethnic minorities and the media* (pp. 33–50). Buckingham: Open University Press.

Winant, H. (2004). *The new politics of race: Globalism, difference, justice.* Minneapolis: University of Minnesota Press.

Chapter 32
Landmarks of Concern

Editor's Reaction to Part III

Mellony Graven

The chapters in this section have been grouped together because they deal with quality and equity issues and challenges faced by specific social groups. The ten chapters represent a collage of voices from different countries (Mexico, Australia, Denmark, Sweden, Canada, and United States), which highlight concerns about identifiable social groups in relation to access to quality mathematics education. These groups range from children with special rights (visually or hearing impaired, Down syndrome); English language learners (where English is not their home language); learners living in poverty; aboriginal learners (in Canada and Australia); female learners and African American learners; and gifted learners. The stories highlight the ways in which these groups continue to be denied access to and full participation in quality mathematics education. The chapters call teacher educators and researchers to action towards foregrounding issues of quality and equity for these groups in terms of influencing policy, changing one's practice and researching further so as to improve our understanding of the issues and to find ways to move forward. Collectively, the chapters provide lessons on how we may proceed in the form of suggested avenues for further research, ideas for curriculum design, intervention programs and forms of instruction.

Most importantly, the chapters point to a need to examine equity and quality in terms of 'outcomes' for these groups rather than merely focusing on issues of access. Improving mathematical participation in the full range of mathematical offerings (including mathematical careers, post graduate studies, advanced mathematics courses) is emphasised. The chapters also collectively point to the need to dispel the myth that 'same education' for all results in equity. They illuminate the way in which a one size fits all as an approach, as often reflected in slogans such as 'education for all', tends to only provide quality education for dominant groups. Specific groups require that curriculum and programs acknowledge their needs, the resources they bring (e.g. bilingualism) and, perhaps more of a challenge, they require new conceptualisations of 'quality' reconsidered from the perspective of marginalised groups.

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Reading this section, one is encouraged by the creative and innovative forms of action suggested by authors and inspired to research further in order to make inroads into the challenge of achieving greater equitable participation in mathematics education by the marginalised groups.

Exploring the Chapters

The chapter by *Gervasoni and Lindeskov* draws attention to the need for equitable access to quality mathematics education for children with special rights (e.g. visually and hearing impaired, Down syndrome). While traditionally these groups have been identified as those with ‘special needs’, the authors in this chapter talk about them as those with ‘special rights’. They highlight the way in which such learners, in Australia and beyond, are excluded from opportunities and learning pathways in mathematics and call on the international community of mathematics educators to advocate for these rights in their countries and for further research into the learning potential of such students provided with opportunities that cater for their needs. *Wiest* considers the issue on gender and mathematics and cautions us not to simply accept the many gains that have occurred by females in mathematics during the past few decades. She provides evidence in the United States of continued areas of concern including course completion, mathematics-related dispositions, career paths and advanced studies in mathematics. Her chapter concludes with recommendations for addressing these concerns. For example, providing networking and mentoring opportunities involving female peers and role models; holding high expectations of female learners and fostering positive dispositions towards mathematics, and promoting societal changes that result in more positive portrayals of females in mathematics.

Wistedt and Sundström illuminate within the Swedish context, where equity issues are to some extent taken for granted due to the fairly even economic demographics, another form of inequity—namely that bright and exceptionally talented students are not provided quality mathematics education that enables them to reach their full potential. Similarly, *Oktaç, Fuentes and Rodriguez* point to the way in which equity agendas have overlooked gifted children in general and mathematically gifted children in particular. In addition to the equity issue from an individual point of view (dissatisfaction and unmet needs), they address the issue from the point of view of loss of a societal resource of talent in Mexico (and relate this to other developing countries). This, they argue, results in dependence on importing necessary mathematical top-level talent from other countries thus widening the inequities between international communities. Both chapters thus challenge the rhetoric of debate related to gifted children but go on to provide suggestions for providing quality education for these exceptionally talented learners. So, for example, *Wistedt and Sundström* call for a richer set of educational materials; appropriately trained teachers able to work with talented learners; summer camps for such learners and a societal culture that supports mathematics as a human discipline. *Oktaç,* *Fuentes and Rodriguez* call for a specific kind of activity design which involves a central problem posed and several peripheral activities for extending those talented learners. They provide an example of such an activity and stress that activities need to be: presented via open questions; flexible and adaptable; motivating and contextualised; extendable, and should help to both organise learner knowledge and generate further ideas.

The chapter by *Cooke, Howard, Lowe* and *Perry* highlights the importance of relating community connectedness, relevance and belonging to understandings of quality and equity for Australian Aboriginal and Torres Strait Islanders learners. They argue that while there has been some progress in the enhancement of mathematical outcomes for these learners the gap between Indigenous and non-Indigenous students has continued to widen. They call for an increase in intervention programs focused on addressing various criteria related to social, cultural and community contexts. They emphasise that these programs must address the incongruity between student's home and school environments and in order to do this programs must focus on the following criteria: enhanced mathematical learning, social justice, empowerment, engagement, reconciliation, connectedness and relevance. Similarly, *Borden and Wagner* point to the need to characterise quality learning for Canadian Aboriginal learners in terms of local values which they synthesise in terms of three aspects of wholeness, namely, cultural synthesis, personal holism and intergenerational interaction. In so doing, they argue that equity is promoted by addressing issues of identity and power (through deliberate repositioning of learners in more respectful relationships) while supporting increased mathematical access and achievement.

Brown, Cady and Lubinski illuminate the effects of poverty and language on mathematics learning in the U.S. context and examine the way in which discourse diversities are intertwined and cannot be easily separated in relation to mathematical achievement. Their chapter shows that English-speaking students living in poverty might be more severely impacted than English language learners. They suggest Cognitively Guided Instruction (CGI) as an appropriate approach to working with both English Language Learners and learners living in poverty. Similarly, *Pitvorec, Willey and Khisty* explore appropriate frameworks for working with what they term bilingual learners in the United States. They argue that it is possible to address both issues of quality and equity through the use of a particular curriculum framework which considers the needs of bilingual learners from the beginning, supporting the agency of non-dominant students and seeing their home languages as a resource. Both chapters point to the need for broader conceptualisations of language access from its often narrow construction as access to vocabulary and instructions to a recasting of learners' home languages as a resource for learning. Thus language is reconceptualised as a 'natural tool' that is continually being refined by learners rather than an entity which learners need to be taught. In this way increased agency is given to learners.

Ellington and Prime emphasise the need to bring a social justice perspective to the issue of quality and equity in mathematics education. They argue that the continued inequities in mathematical achievement of African American learners and their white counterparts point to inadequate and inappropriate conceptualisations of quality and equity. They emphasise that a social justice perspective on equity must be aligned with the notion of equity of outcomes (and not simply equity of access and treatment). The authors illuminate this through the voices of doctoral students in mathematics education at a Historically Black University. The chapter by *Martin* similarly points to the inadequacy of current notions of quality and equity and argue that mathematics education research, policy and reform can inadvertently contribute to racism when notions of quality and equity are explored within the context of white dominant interests. They examine the way in which such conceptualisations of quality are not aligned with quality for those in marginalised positions. Both these chapters point to a need for awareness that definitions of quality and equity are not race neutral and need deep and thorough interrogation.

The ten chapters in this section collectively point to the need for action by mathematics educators in terms of exploring issues of quality and equity in relation to specific groups. While there are no smooth roads to follow that will guarantee progress on these issues for marginalised groups, the chapters offer a set of landmarks of required attention and action and in addition provide a range of pointers useful in planning our journey ahead. They call us to proactively embrace the journey ahead with our eyes wide open to the various potholes which can be hidden by the smoothness of rhetoric such as ‘quality mathematics education for all’. Without deep examination of the rhetoric surrounding issues of quality and equity in relation to specific groups, our journey is likely to be severely impeded.

Part IV
No Highway and No Destination?

Chapter 33
The CERME Spirit: Issues of Quality and Inclusion in an Innovative Conference Style

Barbara Jaworski, João Pedro da Ponte and Maria Alessandra Mariotti

Introduction

The European Society for Research in Mathematics Education (ERME)

In May 1997, a group of 16 scholars from different European countries met in Osnabrück, Germany, for three days to discuss the formation of a European society in mathematics education. In true European spirit, we decided that we wanted a society which would bring together researchers from across Europe, particularly including colleagues from Eastern Europe, fostering *communication, cooperation* and *collaboration.* We wanted a conference that would explicitly provide such opportunity. We wanted especially to encourage and contribute to the education of young researchers, recognising that they are the future of our discipline. Thus ERME was born and began to take shape.

We decided on a two-yearly conference, or *congress* as it later became known, and the name CERME emerged—*Congress of the European Society for Research*

The authors of this chapter have been deeply involved in ERME development. João Pedro da Ponte was Chair of the Programme Committee for CERME 3 in Italy; M. Alessandra Mariotti was the local organiser of CERME 3 and a member of the ERME Board from 2005 to 2010; Barbara Jaworski was Chair of the Programme Committee for CERME 4 in Spain, member of the ERME Board from 2003 to 2008 and President of ERME 2005–2008. Ponte and Jaworski were members of the group at Osnabrück initiating ERME.

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in Mathematics Education. Considerable time was spent talking about the nature of the conference. How were we going to achieve the communicative, cooperative and collaborative spirit we envisaged? After some discussion, it was agreed that the conference should be more than just a platform for presenting and listening to papers. Many other conferences provided such opportunity. CERME should allow groups in a particular scientific area really to *work* together on their area of research, with sufficient time to get to know each other, to share and discuss their research and to engage in deep scholarly debate.

At the first CERME congresses were held the early meetings of a committee that was to grow in later years into the ERME Board. The committee held open forum to seek views and formulate policy for ERME. Two principles, developed at the Osnabrück meeting, held clear importance, the first to encourage colleagues in Eastern Europe to become part of the society and second to support young researchers (*young* in research terms—not necessarily in age) throughout Europe. The ERME Board has worked hard over succeeding years to further these aims.[1]

During these years, evaluations and other testimonials suggested that we had initiated something exciting, significant and of important consequence for the future. Participants came from these events speaking of *inspirational* experiences. It seemed clear that the events generated something that we came to call *the CERME Spirit*. Based fundamentally on the three Cs, communication, cooperation and collaboration, the CERME Spirit was about the inspiration that derives from a serious scholarly tackling of ideas and concepts in key areas and of mathematics education research with colleagues from multiple nations, facilitated by the group design of the events.

The group design was not without its critics. Some felt constrained by the requirement to spend a conference, largely, in just one group. However, the group work would be seriously disrupted if participants were to hop from group to group, not engaging seriously with the work in any one. Some suggested that perhaps planning could allow participants to take part in two groups, so that engagement in both could be serious. Such ideas have been considered by the ERME Board and Programme Committees but so far we have remained faithful to the initial conception. Many participants have said in evaluation of the events that the opportunity to spend serious time in one group allowed them to really get to know researchers from other countries, and that this contributed significantly to the depth of thinking that was possible.

There are two important issues with which we have been grappling in CERME and YERME (*Young-researchers in ERME*) over the years: the *quality* of scientific work in a group related to papers accepted for the conference and published in the proceedings and the *inclusion* of all people who wish to attend. ERME aims for a high scientific quality of work, reflected in the reviewing and acceptance of papers. Attendance for most delegates requires that they present a paper, but not

[1] It was decided to establish ERME legally with charitable foundation in the UK, and this is now finalised with a formal Constitution and Bye-laws. With a solid legal foundation, ERME is now seeking to develop a strong financial footing.

all papers meet high criteria on quality. To participate, people need to be able to communicate and engage with the scientific discourse; the language of events is English as the only workable common language. However, it is recognised that many participants are disadvantaged by having to work in English. Language is also a factor in writing papers as well as in communicating at the conference. We need to address what exactly we mean by "scientific quality" and what is entailed by "inclusion". We recognise that both terms are deeply embedded sociohistorically in the mathematics education research community with its journals and conferences, and its written and unwritten rules of engagement. ERME, as a still young society, is consciously developing norms and seeking to influence research communication.

At this time in the life of ERME, we have collected data from participants' perspectives on their experience and their associated expectations. In this chapter, we present findings from our analysis of these data and offer a tentative prospective for the ongoing work of ERME.

Locating Concepts and Concerns Within a Wider Frame

In dealing with issues of inclusion and quality within CERME conferences, ERME embarks on an *equity* agenda within a broader frame of *social justice* (Burton 2005). This section will address the question of what such an agenda implies for a European Society and Conference. Atweh and Keitel (2007) suggest that social justice necessitates working for theorising its meanings, working both *with* and *on* the concept. We are working *with* the concept in every conference and through our analysis recognising issues. To work *on* the concept, to start to address what inclusion and quality mean in terms of ERME and CERME, this chapter also begins to construct associated theory. This section introduces the issues, Section 3 presents findings from data analysis and the final section offers a tentative theoretical frame and agenda for the future.

A European Society and Conference

As has been explained above ERME is *European*: although it does not exclude non-Europeans, it seeks primarily to bring together mathematics educators from all the nations in Europe. This implies an agenda of *including* all such mathematics educators, and we address what this means in the European context. We, European mathematics educators, work within the European *Union* and countries closely associated with that union. So, part of the inclusive agenda is about *uniting*, bringing together, sharing our scholarship, developing common understandings, respecting diversity. The three Cs, *communication, cooperation* and *collaboration*, leading to the CERME Spirit, capture elements of the agenda

Communication is about talking to each other in ways that enable sharing and understanding of ideas and traditions that go beyond the superficial. Cooperation implies *working* together, going beyond communication to see how different perspectives and practices can illuminate issues and concerns and open up new possibilities for addressing national agendas. Collaboration means working together to create new agendas with initiatives that cross national boundaries and build cross-national, *European* identities. Atweh, Clarkson, and Nebres (2003, p. 224) quote Hargreaves (1994, p. 45) who writes "one of the emergent and most promising meta-paradigms of the post-modern age is that of *collaboration* as an articulating and integrating principle of action, planning, culture, development, organisation and research" (emphasis in original). Atweh et al. comment as follows:

> The limited resources in some countries imply that they are more likely to copy or import ideas from the more developed regions and countries rather than to critically and empirically reflect on their appropriateness to their local context. (p. 224)

and

> Collaboration should be constructed to empower individual countries to be self-reliant rather than to increase their dependency on ideas from more developed nations. (p. 225)

In mathematics education research and practice particularly, we have seen ideas from certain countries permeating the research agendas of others—for example, with respect to the U.S. reform movement in schools and the associated National Council of Teachers of Mathematics (NCTM) standards (NCTM 1989); and in respect of outcomes of international comparisons such as Trends in International Mathematics and Science Study (TIMSS), where countries around the world have looked to countries of South East Asia to learn how to achieve arithmetic success (Jaworski and Philips 1999). A challenge for ERME/CERME is to provide a forum for collaboration in which "an articulating and integrating principle" (Hargreaves, cited in Atweh et al. 2003, p. 224) can be achieved with open, respectful and non-hegemonic partnership between participants.

There are various issues associated with such an aim. In Western Europe, we have a number of (relatively) rich nations whose mathematics educators have had the privilege of travelling to international meetings and conferences over several decades. Their diverse traditions are well known (if not well understood) by all. A variety of conferences have provided opportunities for participants to hear the theories and approaches of others and to think about implications for their own research and practice. Despite such communication, we see little evidence of collaboration between national traditions on any substantial substantive scale. In Eastern Europe, the picture is different. Only relatively recently have borders been open for communication. Economic resources tend to be much less available for travel and participation beyond national boundaries. Mathematics Education research itself has barely started to exist within some Eastern European countries. It is, of course, inappropriate to generalise.

The relation of mathematics education to mathematics is one factor which varies considerably across Europe. In some countries (for example, the United Kingdom and Portugal) mathematics education research has been largely the province of mathematics teacher educators working at primary and secondary levels, often in university departments of education (not mathematics), although there are notable exceptions. In some countries (for example, Italy and France), many mathematics education researchers are themselves mathematicians, teaching mathematics at university level. Nevertheless, research in mathematics education has a very different character from research in mathematics. Indeed, mathematics education is a different discipline from mathematics, which is not always understood by mathematicians. This has led to conflict between mathematicians and mathematics educators in some countries. In countries where mathematics education as a discipline is not well developed, there can be confusion as to what research in mathematics education means at all. For example, one CERME 6 participant wrote on the evaluation questionnaire: "The conference was very theoretical—I am not used to this in my country (used to Pure Maths)." ERME is currently pursuing links with the European Mathematical Society (EMS) in order to develop a relationship with mathematicians in Europe.

Why Did We Need Another Conference?

It is a legitimate question. We can name several other international conferences which European Mathematics Educators can and do attend and to which *all* Europeans are welcome, such as ICME, PME and CIEAEM. ICME, the *International Congress of Mathematics Education,* is the four yearly congress of ICMI, the *International Commission for Mathematics Instruction* with a membership from 72 countries. ICME attracts thousands of delegates from a variety of constituencies in mathematics education (including teachers, educators and researchers). It is not primarily a research conference. It imposes a *solidarity tax* on delegates to subsidise attendance from less affluent countries (Atweh et al. 2003, p. 192). PME, the annual conference of the *International Group for the Psychology of Mathematics Education* is a research conference. It welcomes research reports from any mathematics education researcher in any country and the main substance of the conference is presentation of research reports. Submitted papers are rigorously reviewed and accepted or rejected. Rejection often means that the authors do not attend the conference since conference funding depends on paper acceptance and publication. CIEAEM, the *International Commission for the Study and Improvement of Mathematics Teaching,* is multilingual; it focuses on teaching mathematics and welcomes teachers and others (Atweh et al. 2003, p. 191). It is not a research conference. Atweh et al. (2003, p. 191) quote the Manifesto 2000 of CIEAEM as suggesting that a challenge for the whole international mathematics education community is "how can communities with different political, cultural and social conditions make ways to learn from each other more productively?" This challenge is overt in ERME.

So, how is CERME different from these other conferences? CERME is first of all a *research* conference in mathematics education, which distinguishes it from

ICME and CIEAEM. It differs from PME in its structure around working groups. The idea is to get away from oral presentations towards work which fosters the three Cs in mathematics education research between group participants. Communication and cooperation have been visible in most CERMEs so far, and we are now starting to see collaborative initiatives across national boundaries (e.g., Prediger et al. 2008).

Equity Agendas in ERME

In considering equity, it seems important to emphasise the difference between equity and equality in educational practice. Zevenbergen (2001) has expressed this as follows:

> Equity refers to the unequal treatment of students (or people more generally) in order to produce more equal outcomes. In contrast equality means the equal treatment of students with the potential of unequal outcomes. (p. 14)

In consideration of social justice in classrooms, Cotton (2001) writes, "the concept of social justice represents a shift in thinking *away from* equality…[since equality]…does not easily accept and value difference" (p. 28). So, for an equitable approach towards organising a conference, we need attention to those factors which do or could disadvantage some (potential) participants, and moreover, a policy towards encouraging certain groups of people for whom participation is problematic.

Atweh (2007) reports from a discussion group at ICME 10 (2004, Denmark) on the topic of *international cooperation.* The topic group organisers identified certain barriers to international contacts which included *financial, language* and *voice.* The first two of these have been part of ERME consideration from the beginning. Recognising the likely financial disparity between the two groups mentioned above and other participants, ERME has, to date, invested most of its funds (gained from members' fees and profit on conferences) into financing summer schools for young researchers[2] and supporting participants to CERME, particularly from Eastern Europe. The available funds have necessarily limited what is possible. Attempts to build up a "support fund" from voluntary contributions have had only very minor success. So, it may be that a "solidarity tax" on ICME lines is called for. Regarding language, while English is the language of the conference—a policy decision agreed at an early stage, which might of course be challenged—group leaders are encouraged to find ways of using other languages in working groups to facilitate full participation. So far practice has been ad hoc with differing reports on success or otherwise. It is an area for further consideration and possibly policy reconsideration.

[2] These are YERME events, known as the YESS 1,2,3,4 & 5 (*YERME summer schools*), taking place in alternate years to the CERME conferences.

The issue of “voice” is related to language but goes beyond language to issues of culture, power and domination. Atweh (2007) reports from the discussion paper in the group at CERME 10.

> Voice: collaboration between educators with varying backgrounds, interests and resources may lead to domination of the voice of the more able and marginalisation of the less powerful. (Atweh et al. 2008, p. 445)

Although language can be a dominating factor, domination extends potentially beyond language per se. Atweh, quoting the leaders of the discussion group, emphasises another factor that relates strongly to the issue of voice. It concerns “missionary attitudes” of some participants in relation to preferred terminologies and their hegemony over ideas, recognising that the result can lead to “a patronising relationship which does not respect and value the diversity of the parties involved”. They suggest, “Instead, an attitude of humility and openness to learn from each other should be the basis of international co-operations” (Atweh, et al. 2008, p. 446). The group structure at CERME is designed to enable participants to go beyond the presentation of papers to discuss ideas and issues and really work cooperatively on the substance of the topic. However, the focus of a group depends on both the papers received and the directions decided by group leaders. It is possible that domination of ideas by certain areas of scholarship in particular parts of Europe could be implicit in group work and remain unchallenged because the dominant voices are those promoting the particular ideas. Gates and Jorgensen (Zevenbergen) refer to Bourdieu’s concept of *habitus* to express this as

> Thus the field…in which the participants engage recognises and conveys power to those whose habitus is represented and privileged in the field. (p. 164)

We look now to the practical realisation of ERME and CERME aims and the issues they raise according to such equity agendas.

Views of Participants Regarding the CERME Activity

Data and Their Analysis

In the registration pack for each participant in CERME 6 in Lyon, France, a short statement explained that members of the ERME Board would be gathering information with regard to *quality* and *inclusion* in CERME conferences. It explained briefly the main aims of ERME and CERME and introduced issues relating to quality and inclusion. Its purpose was both to raise awareness and to promote responses. At the end of the conference, two questionnaires were administered to participants: the first to all participants for evaluation of the congress, with a specific question addressing quality and inclusion as follows:

> Balance of scientific quality and inclusion in your group: please give us here your views on balancing quality and inclusion (see statement in registration pack).

The number of questionnaires returned was 210 out of a participation of about 450, thus just less than half. The second questionnaire was to group leaders, asking for written reflections on their experiences in organising a group at the conference, focusing specifically on issues of quality and inclusion. Out of a total of more than 45 group leaders, 13 responded from 9 out of 15 groups. Their responses addressed the review process, selection of papers, help given to (less experienced) authors to improve papers, language difficulties and inclusion of papers in the conference proceedings. In addition, five interviews were conducted with CERME 6 participants (including two PhD students and three researchers with university positions) on their experiences at CERME and particularly their experience of the review process.

Analysis has involved reading carefully the written comments and listening to the interviews; categorising them in relation to emerging factors. To achieve categorisation, questionnaires were organised according to working group and each written comment was assigned to a category that sought in some way to describe its content. Some touched on several issues and were assigned to more than one category. Particular comments are fed back to group leaders, although here we do not refer to specific groups.

A few participants did not notice the statement in the registration pack and in their questionnaire asked—"WHAT statement?" It could have made a difference to responses whether or not the respondent had read this statement. Some of the responses have a neutral or analytical tone, but most of them are either overtly positive or negatively critical. Many comments were of a telegraphic nature—participants may have offered a quick evaluative comment without deeper thought or analysis. The data sets themselves are limited by those who chose to respond: the findings come from returns from only about half the ERME population at one conference. Within these returns, some participants chose to make no comment on the key question on quality and inclusion. Some responses are ambiguous and their allocation to a category is done on the judgement of the researchers. Those doing the analysis are committed ERME members, active in ERME since its inception. While this allows an insider view of issues and concerns, it might also lead to an overly insider picture of what is offered.

We have organised our presentation of issues thematically, drawing on all the data sources where they offer contributory evidence. Where a quotation is unattributed, this means it is taken from a participant questionnaire; otherwise its origin is stated.

Themes and Issues

Before a conference, the Programme Committee decides what groups to include and invites group leaders. Group leaders initiate a call for papers and organise a review process; they decide on accepted papers and plan a programme of work for their group. Papers accepted for the conference are published on the internet and members of a group asked to read them in advance. Guidelines suggest that oral presentation will occupy only a minimum of group work, perhaps allowing authors of a paper no more than five minutes to present their key ideas/issues. After the conference, selected papers are published in the conference proceedings. Further work on a paper may be required before it can be accepted for publication. Some papers are accepted only for the conference, but not for publication.

From the perspectives of participants, the areas of CERME operation promoting most comments are the groups in which most conference participation takes place, and the review process through which papers are selected for work at the conference and publication in the proceedings. In the following subsections, we take up issues in these areas relating to inclusion for all, scientific quality and the three Cs.

The Review Process and Acceptance of Papers

CERME guidelines suggest open reviewing in which authors and reviewers are known to each other and communication can take place between authors and reviewers. They suggest two levels of acceptance: (1) for presentation at the conference and (2) for inclusion in the proceedings. Further work may be asked for at either or both of these levels. The two-level process has evolved through several conferences in an aim to include as many people as possible at the conference and also to ensure a high quality of published papers after a conference. In theory, this is to achieve a quality–inclusion balance.

There were positive comments about the value of the review process and its contribution in enabling participants to improve their papers and, additionally, in providing experience of reviewing.

> Researchers also want to have their work published—perhaps inclusion can help them to achieve this.

> I think that it is a good idea to do the review process, first of all, because it makes more connections among the members of the working group. So I am obliged to read the work of my colleagues with more thorough, more interest, more accuracy. And it makes, I think it is good experience for someone who has not done a review before. (Interviewee)

In some cases the nature of reviews was criticised as being too short, as containing dubious judgements, or as lacking critically helpful comments and questions.

> The review I got back for my paper was very very short. It only said…. Goal was mentioned, OK. Methodology was mentioned, OK. For the Proceedings [the review was] also very short. So I had to adjust nothing. I don't think it was that good. No difference at all for level 1 and level 2. (Interviewee)

> There were two reviews. One of them had a very helpful suggestion which was about "explain a bit more about the tasks". It was a small but very helpful suggestion. But I don't think it had anything—I wished it would have more questions. I think…. The other was very unethical. One was completely uncritical, the other had some comments about small details. But no big questions—there was nothing in depth to demonstrate any real deep engagement with the paper. Neither one. (Interviewee)

Although not stated, we suspect that these were not named reviews, so no further communication was possible.

In some cases, inclusion was interpreted as meaning that papers were accepted without critical consideration, leading to variable quality, "Too much inclusion—not enough selection." Some authors would have preferred a more critical or "rigorous" review of their paper:

> Inclusion is more than having a paper accepted—need to feel it is valued—needs a more rigorous review process.

> Balance between being inclusive or high standards—I think that the process of achieving that balance is exactly right—the participants reviewing the paper of the other participants. But perhaps there could be a bit more support for the reviews, to be more critical. With some helpful suggestions, I think quality would improve. (Interviewee)

> I think some form of giving people permission to be critical and some sort of encouragement. Some of the papers in our subgroup are of questionable scientific quality. (Interviewee)

So, interpretations of inclusion that lead to uncritical acceptance of papers are inappropriate. The quality of papers is important for all participants. The issue here is how to help authors strengthen weak papers so that they are of sufficient quality and so that authors develop their own critical strength.

There were positive comments about the two level review process, some suggesting that only group leaders should do the final review;

> Review process before conference should be for presentation in conference. For publication Chairs should decide what papers to include.

However, the option of having a paper accepted for presentation but not for the proceedings was seen to prevent some people from attending the conference, therefore running contrary to aims for inclusion. One group leader wrote that papers not accepted for the proceedings were withdrawn since "people cannot get financial support if a paper is not accepted for the proceedings". The importance of having a paper published was emphasised.

> The research community at large does not know what happens between these four walls. But the research community may look at the Proceedings. The maths education world is stressful. [In the proceedings] you know, your paper is permanent. To preserve academic reputation the papers should have a careful publication in the proceedings. (Interviewee)

Some group leaders commented that papers outside the field of the group or of low quality were rejected, or recommended for resubmission as posters. Leaders spoke of trying hard to be inclusive of papers—to include as many as possible (often with no mention of quality). One said that they included papers with "severe weaknesses as long as there was an interesting idea". One leader wrote: "Being all-inclusive and academically qualitative are *a priori* incompatible."

Several leaders spoke of giving help to the less experienced and of being "more severe" to authors from well-represented countries. Some spoke of their organisation of the review process, making decisions as a team. One group leader (A) said the following about her reading of reviews.

> As I am reading through the reviews of papers as part of our "second editing" stage, I want to share an observation about the different styles of reviews…. I notice that some reviewers use the sections on the form to summarize the content of the paper, but make little evaluative comment. This is useful for the group organizers when they see the reviews and are making decisions about the overall structure of the sessions—but it is of little value to the authors. In contrast, other reviews consist of evaluative comments, but may not give any indication of the content of the paper…. This may be of more use to the authors, but is less helpful for the group organizer who wants to know something about the paper. (Group leader)

Another (B) spoke of her experience of using the process as part of the leadership team for the group.

> [The group coordinator] carefully divided all papers in four (the leader and the three co-leaders). Each of us had to review four to five papers together with two other participants of the group. At the end of the review process, each of us made a summary review for each of the papers that we were responsible for, studying first the other two reviews for each paper. In the case that contradictions appeared in the reviews, we invited the other leaders to study the article and express their opinion. If the majority of us agreed to a certain decision, then the group leader adopted that decision. Then we sent only the summary review to the corresponding author. Following the three Cs we tried to include most of the papers in the presentation of the papers. We only rejected a paper which did not meet the scientific guidelines for writing a research paper. (Group leader)

The comments overall suggest aspects of the review process that are not achieving the aims expressed in CERME guidelines. The two quoted above from group leaders (A) and (B) suggest details of the review process that seem to need more attention. For example, group leader (B)'s comment mentions that "summary reviews" were sent to an author, seems to go against the suggested open process allowing communication between author and reviewer. The nature of a review, despite guidance on the review form, does not always satisfy both the needs indicated by group leader (A). How to bring the whole review process closer to CERME aims and the written guidelines seems to need further consideration.

Group Activity and Participation

Although a conference includes keynote presentations and other plenary events, the major part of any delegate's participation is as a member of their selected group. It is likely that they have a paper accepted by the group, or a poster. This will have been published on the internet along with others for the group and participants are asked to read these papers before the conference. They can also read papers for other groups if they wish. Group leaders can plan activity on the assumption that participants have read the group papers.

Group Size and the Number of Papers

CERME 6 had about 450 participants and 15 groups; group size varied from 8 to about 70 participants, with slightly fewer accepted papers. Papers may be up to ten pages in length, so if a group has more than 20 papers the reading task is considerable. One group leader wrote of having 54 papers and 15 posters, another of 55 papers. One solution would be to increase the number of groups, but this places pressure on facilities and resources. A group can be split into smaller groups for at least part of their work but may not have the availability of a separate room for each subgroup. These factors raise a variety of issues for group work and participation. Respondents commented that the size of a group affects what is possible, "participation of all was not easy", that it was hard to read all the papers, and a result was "poor (not in depth) scientific discussion with no clear questions" and "more small group work needed". Despite these practical problems some respondents reported that inclusion was "at a high level".

One of the interviewees indicated that the problem went beyond the possibility to read all the papers to the diversity of content and depth of focus.

> I think there are some working groups that are too big. They have too many articles. Not because it is a problem reading them. But to keep track of so many articles that are not always so homogeneous. It is interesting to have a broad variety of topics but if it is too much it may be difficult to keep things together. So, I would prefer something smaller and discuss more thoroughly. (Interviewee)

Group work is constructed around the papers received and objectives for inclusion suggest that most of these papers will be accepted for presentation. Group leaders therefore have a considerable task in constructing a unifying programme of work. There seems to be a need to give time to each paper, and with 35 papers, even five minutes per paper is very significant. One comment pointed out that "5-min presentations need a quick change of focus between them", indicating issues of transition when many papers are included. Transitions between papers need to make clear links to themes within the work of the group. If participants are unable to keep track of ideas, this might suggest that the programme is not achieving its aims. Charting a scientific path through such diversity is a problem for group leaders and not everyone will agree with choices made.

Organisational Factors

Each group had three or four designated leaders, each one from a different country. One leader was designated as Group Coordinator with the main responsibility for the group. Group coordinators were invited by the PC, and other leaders were decided by the PC in discussion with the coordinator. Comments from participants were overwhelmingly positive about the work and organisation of group leaders, for example, "First class organization from which authors could learn." Most recognised the importance of the work of group leaders and its demanding nature.

However, some comments criticised organisation as "erratic", suggested that group leaders "need to control people who dominate discussion (such as English speakers)", showed "unfair handling of time and papers" and put "too much focus on individual papers rather than big ideas". Some comments suggested that advance communication of the methodology of the group would have been helpful, especially for participants who had not sent a paper, and who had therefore not received prior details of group work. Such comments all suggested feelings of exclusion at some level, although there is an element of not being able to please everyone.

Group Work—Views on Inclusion and Scientific Quality

In the evaluation questionnaires, comments revealed differing perceptions of the terms and concepts of *quality* and *inclusion.* In some cases, the words were used with little further qualification (e.g., "everyone included", "inclusion good", "very inclusive", "inclusion not sufficiently addressed", "over-inclusive"), as if the concept is well understood and the associated judgement unproblematic. Further remarks provided insight into what was understood. For example, "discussion friendly and inclusive", "encouraging and critical". Comments included "good help with English" and "too inclusive—poor English accepted" (the last two comments from the same working group). We therefore recognise a difficulty of interpretation in our analyses. Although seemingly positive comments on inclusion greatly outnumbered the seemingly negative ones, without further illumination on the nature of judgement, it is hard to generalise.

The question had asked about "balance" so many comments made a comparison. Although the majority of comments suggested a good balance, "everyone included—quality high", "excellent in both [Q & I]—newer researchers felt confident—supported by small group discussion", some (also) suggested that inclusion led to a reduction in quality:

> Inclusion good, but therefore scientific quality was very variable.
>
> Inclusion implies a generally poorer quality of paper.
>
> Scientific standards should not be reduced to expand possibility of access.

Such comments reveal not only perceptions of an inverse relationship between quality and inclusion but also the differing values of participants. Some comments qualified the nature of a good balance.

> Discussion on each paper enabled inclusivity *and* movement of papers towards higher quality through richness of critique.

Certain comments referred to inclusion of participants in group activity and dialogue and also to the ways in which accepted papers were addressed in a group. One interpretation of many of these is that inclusion relates to participant interaction in the social setting of the group and quality relates to the nature of papers, the rigour applied to paper acceptance and the ways in which papers were addressed in the group.

> Some high quality papers, some very poor papers—better to raise quality even at the expense of inclusion.
>
> Too strong on quality. One young researcher had paper rejected—it would have helped him to have it discussed.

The quality of discussion within a group was the focus of many comments, some suggesting a high quality ("lively" and "sophisticated"), with "experienced researchers moving talk into deeper reflections", and "Supportive and friendly, with penetrating remarks in response to papers". Others suggested that scientific discussion was poor with "not enough depth" and "Not all key ideas of papers discussed". In one group, the level of discussion was judged to be high, so that "newcomers could not keep up with the standard of the group".

Some comments referred to how oral presentations of papers were conducted in a group. A significant number suggested that, despite recommendations, there was a substantial degree of oral presentation. For example, "too much paper presentation—more time should be given to small group discussion" and "work was almost entirely presentations", with "too much repetition of what is already known", and "not enough time for discussion". This contrasted with other comments: "no paper presentations", "active taking part", and "at least 50% discussion maintained". We note that the comment "not enough time for discussion" could have meant that paper presentation did not allow time for discussion, or it could have referred to the number of papers that were included in that group (or both).

We are aware that language difficulties are more easily overcome in a prepared presentation and that some participants prefer to take that opportunity to feel free and confident to talk. This observation highlights the importance of managing discussion with genuine opportunity for those who have difficulty with English. Surprisingly, there were not many written comments about language. The few comments expressed, not included elsewhere, were as follows:

> Language difficulties and differences in theoretical approach made it difficult to take part in discussion.
>
> Despite being a supportive group, those struggling with English don't have good participation.
>
> Non-English speakers had difficulty to join in and voice ideas.
>
> English speakers talked too much.

A difficulty may be that those experiencing difficulty with English are also not able to express their views on an evaluation form.

Perhaps the strongest message coming across in this section is the diversity in perceptions of group work. Even within a single group, in some cases comments seemed largely in agreement while in others there were (widely) differing views. It therefore seems important to see group activity through these alternative visions when preparing the programme of work.

Issues Raised By Group Leaders

It seems appropriate in this last subsection to give the final voice to group leaders. One very positive comment reported as follows:

> We had four productive days in a friendly atmosphere. Sure some discussions may have become a bit heated, but that is only natural and they were constructive still. As for language, sure there were some participants who had English difficulties, but then other participants would help to translate and it all worked out fine. (Group Leader)

Another commented specifically on the two-stage review process and the decision as to whether a paper would be published in the proceedings:

> According to CERME guidelines, the accept/reject decision should have been communicated before the conference, both with regard to discussion at the conference and with regard to the post-conference proceedings. In fact, following our WG call for papers, we did not communicate the decisions about the publication in the proceedings before the conference. We preferred to discuss the accepted papers in our WG and later to fix the decision with another review process, taking into account all the remarks and comments. This provided opportunity to improve the papers (in particular papers needing help) following the path of "quality and inclusion". I received no complaint about this line from the participants. (Group Leader)

And finally:

> The work in many of the WGs has been a series of paper presentations. The Board needs to be clear that this is not an acceptable format. In addition, it might be necessary to provide a list of acceptable formats that the group coordinators could indicate the one they use or present their alternative format for organizing the sessions. In our group we had 3 different formats that are *not* traditional paper presentations. I would be happy to share these if needed. A proper plan for organizing the sessions might be a requirement for a Chair to be selected. (Group Leader)

The last two comments reflect the thoughtful hard work of group leaders. They also point to a possible tension between leaving group leaders with freedom to construct their work according to their own expertise and professional judgement and requiring that they conform to some pre-given format designed to promote ERME aims and values.

Discussion of Emergent Issues and a Tentative Framework for Further Consideration

Synthesis of Key Issues and Concerns Arising from or Supported By the Data

ERME starts from a position of seeking *equity,* particularly with regard to inclusion of young researchers and delegates from less affluent countries or countries with different traditions in mathematics education. Evidence shows that seeking for equity is both resource and policy based: shown in financial support and organisational structures. The issues that arise relate largely to the *interpretation,* or *operationalisation,* of the organisational structures. The following list highlights the key points:

- Inclusion in recognised as overt in the group activity at CERME—largely this seems to be an *affective* perception of inclusion. Some participants see an inverse relationship between inclusion and quality of accepted papers and scientific debate.
- Quality, that is *scientific* quality, relates to the quality of accepted papers and the quality of scientific work within a group.
- Language is a key factor relating to both inclusion and quality, and perceptions differ with regard to ways of interacting in practice.
- The review process is key to issues of quality in accepted papers. It is interpreted differently from group to group so that outcomes lack consistency.
- The mode of group operation is also variable. Despite recommendations, considerable group time is taken up by oral presentation of papers.
- Group leaders are widely praised and their work recognised and valued. It is difficult for them to rationalise ERME's aims for inclusion and quality.
- Key issues in conference organisation are the number of people in a group and the number of accepted papers to be read and considered. High numbers of both lead to a significant burden on participants in preparation for group work, a significant factor in allocation of group time and a serious challenge to a high quality of discussion and debate.

It is perhaps unsurprising that equity objectives are hard to realise, that participants will perceive their realisation in differing ways and that outcomes will raise issues for operationalisation. While raising issues, criticising outcomes and offering a critical perspective on experiences of inclusion and quality, there is overwhelming praise for the group leaders and their efforts to achieve effective group work. CERME is dependent on its group leaders and a critical review needs to take serious account of the contradictory forces they experience in doing their work. The comments received from the group leaders who responded reflect a deep awareness of issues and a sincere concern to address equity.

It is clear too, regarding policy in ERME, that the policy makers (ERME Board and CERME PCs) have established both an overall vision and important working practices. These are seen both in terms of funding to provide support and guidelines for operational practice. Of course, a policy is indispensable although not a guarantee. The two practical concerns that stand out as being of significant influence on inclusion and quality are the review process and the size of groups. The review process is set out in the guidelines which have been modified and refined over the years. However, it is the review process in practice that matters, and this needs attention at a policy level. The number and topics of groups is decided by each CERME PC. The number of 15 groups has emerged in consideration of a range of topics to fit with interests of participants and also the practical consideration of availability of conference rooms. The PC has no control over the number of papers submitted to a group, but they can learn to some extent from experience at previous conferences—some groups regularly receive a large number of papers. The possibilities for group leaders are severely constrained if they have many participants, too few rooms and inflexible accommodation. It is clear that such factors are of broad general importance.

Getting Beyond the Organisational Factors

The essence of the tensions between inclusion and quality goes deeper than just the organisational issues, although these issues are significant in practice. Differences in experience and culture, mediated through language, have to be recognised and addressed. The expert and the novice, the western participant and the eastern participant have to be able to work together in *non-reductionist* ways. Atweh et al. (2003), writing about the outcomes of international comparative studies, suggest "Outcomes of such studies are also perceived as *necessarily reductionist,* as results cannot do justice to the very complex factors involved" (p. 12, our emphasis). To be non-reductionist, the balance between inclusion and quality within a group needs to take on a scientific nature that goes beyond (perceptions of) the scientific quality of the substance of the topic of the group. A theoretical perspective on this balance needs to take account of "the complex factors involved", and these go beyond organisational constraints.

Experienced CERME participants are aware of the expectation of inclusion within a group, and newcomers are drawn quickly into an inclusive way of being in an *affective* mode. There are almost no comments that suggest that group work was not friendly and welcoming, that participants were not (overtly) encouraged to take part and join in the discussions. Such welcoming encouragement might be seen as a first step towards being drawn in to a scientific depth of ideas. Participating scientifically can be related to what one knows and one's confidence in that knowledge. However, we recall here some of our discussion in section "Equity Agendas in ERME" above: that "the field…in which the participants engage recognizes and conveys power to those whose habitus is represented and privileged in the field". (Gates and Jorgensen 2009 p. 164); that "collaboration between educators with varying backgrounds, interests and resources may lead to domination of the voice of the more able and marginalization of the less powerful" (Atweh, et al. 2008, p. 445); and that an attitude of humility and openness to learn from each other should be the basis of international co-operations (Atweh et al. 2008, p. 446).

It seems to us that those experienced in CERME, in their interpretation of inclusion, are aware of possibilities of privilege, domination and marginalisation and are seeking alternatives. We see, demonstrated in group work and the comments of participants, manifestations of an "attitude of humility and openness". Group leaders and participants try hard to engage everyone and to avoid domination. The difficult challenges are those of interpretation and balance. When participants speak of inclusion being at the expense of scientific quality, they suggest that the balance does not achieve a sufficient depth of ideas or allow deeper scientific considerations to be debated. However, there is no reason why a young researcher cannot enter into the deeper ideas and issues, or why researchers from widely different standpoints in mathematics education cannot seek the roots of their difference and debate them. The great challenge is how to achieve this.

Probably, most recognise that the first step involves communication, and here issues of language dominate. While it is hard to search for the key ideas and to express them, it is even harder if you are trying to do it in an unfamiliar language, or if you are trying to slow yourself down in recognition of your hearers' language difficulties. A consequence is that the "key ideas" get diluted in the language exchange or that those with power over language run away with words and leave others with little sense of what they are talking about. Either way, the ideas remain superficial at the cooperative level. So individuals may have deep ideas but these ideas do not get expressed in the "cooperative frame". We seek here to capture what it means to *cooperate,* and suggest that it is about breaking through the complex barriers not only of differing perspectives but also of the cultural and language differences which underpin them.

It is here perhaps that some consideration of the contribution of the *papers* is relevant. The papers, presented in written form in advance, offer key ideas according to their authors. The review process has both made a selection of papers that are relevant to the group and judged them to be of an acceptable scientific quality. Readers, in advance of group work, can take their time to understand the papers and gain access to the key ideas. They can expand their own visions and formulate questions and alternative perspectives in preparation for the group work. This reading is demanding, not only in tackling an overwhelming number of papers, but in getting to and distilling in some way the key ideas of the papers. The burden falls on the group leaders to identify and synthesise these ideas in order to construct a programme of work. It is here that oral presentations can be counterproductive. A natural tendency is for the authors to try to tell the whole story of their research, rather than to get to the roots of what are the important ideas for the group. Indeed, it is very difficult for each individual author to perceive how their own key ideas, related as they are to many factors of culture, methodology and scientific frame, can fit with the wider interests and concerns of the group.

So, we come to the demands on group leaders. While appreciating the demanding task of reviewing and selecting papers, and composing a programme, it is nevertheless relatively easy to construct a programme in which each paper is addressed one at a time with some level of presentation and some discussion. It may be that links are made in transition between one paper and the next, or that papers are grouped according to some commonalities in their substance, theoretical, methodological or context related. Much harder is to formulate a set of "key ideas" and organise the group around these key ideas as themes for discussion and debate. Members of one CERME group spoke of being asked to prepare one overhead transparency on each of the set of themes in their group. Thus, the themes had been prepared by the group leaders in advance and communicated to participants, and participants had been asked to prepare inputs according to the themes. Work in the group centred on the themes, with all members making an input, but with no oral presentation of the actual papers. Without any judgement on the quality of the themes, it seems that this model offers a sincere possibility for cooperative engagement. There may of course be many other models that seek to reach the key ideas and provide opportunity to engage with them.

ERME recognises and tries to get away from the traditional form of research conference which involves a succession of oral presentations of accepted papers, and tries to progress the field in terms of scientific cooperation, moving towards collaborative possibilities. To collaborate we need to break down the barriers and get to the essence of our substance before we can move forwards. One group at CERME has started to achieve this, evidenced by a cross-nationally authored paper, based on group work, published in a scientific journal (Prediger et al. 2008). ERME needs to learn from such experience and use it to promote models of group work and debate their nature and success.

A Tentative Theoretical Synthesis

So, finally, we seek a balance between inclusion and quality as expressed throughout this chapter. A distinction may be drawn between more affective and more scientific characteristics of inclusion, although these are intricately linked in their influence on the outcomes of discussion and debate in a group. Without inclusion of an affective character, work towards scientific inclusion cannot begin. Scientific quality can be seen in terms of the scientific contribution of accepted papers and the scientific nature of discussion and debate. The essence of scientific quality is about reaching for the key ideas of substance in the scientific area of the group and having the possibility of deep engagement with these ideas.

The following conceptualisation is offered as a tentative beginning to characterise inclusion and quality and to relate the characterisation to the specific aims of ERME in terms of communication, cooperation and collaboration.

Quality **Inclusion**	**Having key ideas in the scientific area**	**(Deep) engagement with key ideas in the scientific area**
Affective inclusion	A Starting to communicate	B Developing cooperation in engaging with debate
Scientific inclusion	C Developing cooperation in recognising ideas	D Enabling collaboration in the development of key ideas and new shared constructs

The two axes represent inclusion and quality. Inclusion is characterised in affective and scientific terms. The distinction is somewhat simplistic, but this is a starting point. Quality is characterised through "key ideas" and their development. The key ideas need to be there for scientific quality to exist at all; they need to be engaged with for scientific quality to start to be overt in the group. Thus, we might see there being progress right to left and up to down in the figure (again, perhaps somewhat simplistic), and hence from top left to bottom right in the figure.

The meanings of boxes A, B, C and D are thus, briefly, as follows:

A: *Starting to communicate:* participants have read the papers; they are together with friendliness and sincere desire to work inclusively together. There are key ideas as recognised through the review process in the accepted papers. Activity and discussion begin to encourage communication related to the ideas where the objective is to know each other's ideas and relate them to each other.

B: *Developing cooperation in engaging with debate:* Group organisation enables a focus on the key ideas. Friendly and considerate interaction, with attention to language enables participants to start to engage with the ideas. The emphasis is on including everyone, possibly at the expense of really probing scientific work.

C: *Developing cooperation in recognising ideas:* Group leaders create activity to encourage a focus on getting participants engaged with the key ideas which are recognised. The emphasis is on reaching a quality of interaction relating to scientific ideas rather than on enabling critical inquiry into the essences of the ideas.

D: *Enabling collaboration:* Here we see deep engagement of a scientific quality with deep probing of ideas and corresponding critical debate. From here, collaboration can begin.

It seems clear that for D to be possible, both B and C have to be achieved. This means dealing with all the organisational challenges recognised above, which is a far from trivial matter.

However, it could be that a theoretical perspective of this sort, of what is involved in achieving inclusion *and* quality in group work in CERME, can act as a basis for thinking about dealing with the challenges and conceptualising in practical terms what we are aiming for in CERME.

References

Atweh, B. (2007). International interactions in mathematics education. In U. Gellert & E. Jablonka (Eds.), *Mathematisation and demathematisation: Social, philosophical and educational ramifications* (pp. 171–186). Rotterdam: Sense Publications.

Atweh, B., Clarkson, P., & Nebres, B. (2003). Mathematics education in international and global context. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpartick, & F. Leung (Eds.), *The second international handbook of mathematics education* (pp. 185–229). Dordrecht: Kluwer.

Atweh, B., Keitel, C. (2007). Social justice in international collaborations. In B. Atweh, M. Borba, A. Calabrese Barton, N. Gough, C. Keitel, C. Vistro-Yu, & R. Vithal (Eds.), *Internationalisation and globalisation in mathematics and science education* (pp. 95–112). Dordrecht, The Netherlands: Springer.

Atweh, B., Boero, P., Jurdak, M., Nebres, B., & Valero, P. (2008). International cooperation in mathematics education: Promises and challenges. In M. Niss & E. Emborg (Eds.), *Proceedings of the 10th International Congress on Mathematical Education (ICME 10), July 4–11, 2004, Copenhagen, Denmark.* (pp. 443–447). Roskilde: Roskilde University.

Burton, L. (Ed.). (2005). *Which way social justice in mathematics education.* Westport: Praeger Publishers.

Cotton, T. (2001). Mathematics teaching in the real world. In P. Gates (Ed.). *Issues in mathematics teaching* (23–37). London: Routledge/Falmer.

Gates, P., & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education, 12*, 161–170.

Hargreaves, A. (1994). *Changing teachers, changing times: Teachers' work and culture in the post-modern age.* London: Cassell.

Jaworski, B., & Phillips, D. (1999). Looking abroad: International comparisons and the teaching of mathematics in Britain. In B. Jaworski & D. Phillips (Eds.), *Comparing standards internationally: Research and practice in mathematics and beyond* (pp. 7–22). Oxford: Symposium Books.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics.* Reston: NCTM.

Prediger, S., Arzarello, F., Bosch, M, & Lenfant, A. (2008). Comparing, combining, coordinating—networking strategies for connecting theoretical approaches. *ZDM, 40*(2), 163–340.

Zevenbergen, R. (2001). Changing contexts in tertiary mathematics: Implications for diversity and equity. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 13–26). The Netherlands: Kluwer.

Chapter 34
Productive Pedagogies in the Mathematics Classroom: Case Studies of Quality and Equity

Martin Mills and Merrilyn Goos

Introduction

Within the field of mathematics education, much has been written about the teaching of mathematics and social justice (e.g., Burton 2003; Gutstein 2006; Gutstein and Peterson 2005; Skovsmose and Valero 2002). Themes within this research have included gender, for example, concerns about the ways in which mathematics education has privileged boys (Brandell and Staberg 2008; Mendick 2005); class, for example, how teachers in schools located in low socio-economic areas have deficit views of students (Zevenbergen 2003); and race/ethnicity, for example, how Indigenous students are marginalised by a Eurocentric mathematics curriculum (Howard and Perry 2007). There is also a body of literature that foregrounds the ways in which social justice can be taught through mathematics (Gutstein 2006). The focus in this chapter is on pedagogy, and in particular "productive pedagogies" (Hayes et al. 2006; Lingard et al. 2001), and mathematics education. We argue that the quality of the pedagogy experienced by students in mathematics classrooms is a social justice issue.

We contend in this chapter that high-quality pedagogies that maximise student achievements must be distributed in socially just ways; that is, those students who are traditionally failed by the schooling process need to experience challenges and equally as, if not, more so, intellectually demanding classrooms as students who have traditionally been good at navigating the expectations of schooling. However, we would suggest that socially just mathematics classrooms also need to engage with a politics of difference that take into account the ways in which some students have been marginalised and excluded from the benefits of schooling. To this end, we draw upon research we have conducted in classrooms using the produc-

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tive pedagogies framework (Mills et al. 2009). This framework works to provide a high-quality education for *all* students, and especially students from disadvantaged backgrounds (Hayes et al. 2006; Lingard et al. 2001). The framework consists of four dimensions—intellectual quality, connectedness, supportiveness and valuing and working with difference. As with other research, we contend that for students to demonstrate high-level intellectual outcomes they must be provided with a learning environment that stimulates intellectual activity (Boaler 2002; Darling-Hammond 1997; Goos 2004; Hayes et al. 2006; Lingard et al. 2003; Newmann and Associates 1996; Sizer 1996). We further recognise that such learning is encouraged, especially for students who have disengaged or are in danger of disengaging from school, when the material covered connects with the students' various worlds (Ashman and Conway 1997; Newmann and Associates 1996; Darling-Hammond 1997). There is also ample evidence to suggest that the supportiveness of a classroom is critical for the achievement of high-level outcomes for students, and again especially for those who have traditionally been failed by the education system (Bourdieu and Passeron 1977; Cope and Kalantzis 1995; Darling-Hammond 1997). In a time and world that is characterised by diversity, complexity, rapid change and conflict we also suggest that achieving positive social outcomes and values requires that students learn to work with and value difference (Keddie and Mills 2007). We would also claim that those students who often feel disconnected from schooling due to a failure to have their "differences" valued within the classroom will benefit academically from such a valuing.

The research on which this chapter is based, the Queensland Longitudinal Study of Teaching and Learning (QLSTL), involved observations of approximately 400 lessons in 18 schools over a period of 18 months. These included primary and secondary schools in rural, regional and urban locations. Classes observed were in the subject areas of mathematics, English, science and social science in Years 4, 6, 8 and 9 (ages 9 to 14 years old). Interviews lasting approximately 45 minutes were conducted with teachers and students from each classroom. Samples of assessment tasks and student work from these classes were also collected. The QLSTL research indicated that in both primary and secondary schools, mathematics is the least intellectually demanding subject in the way it is taught. Likewise, mathematics classrooms in our study also demonstrated the least connections to the world beyond the classroom and students' backgrounds and identities, and the lowest commitment to valuing and working with difference. This chapter focuses on the teaching of mathematics in two of the QLSTL schools: a small low socio-economic inner-city primary school, Magnolia State School and a remote Indigenous community school, Azalea P-10[1]. These schools have been chosen as we are concerned here with the quality of pedagogy provided to students who do not traditionally do well in mathematics, for example, students from low socio-economic and/or Indigenous backgrounds. We regard the provision of high-quality pedagogies to students from such backgrounds as a matter of social justice.

[1] P-10 schools serve students from Preschool to year 10.

Magnolia State School[2] was a case study school in both the QLSTL and an earlier study, the Queensland School Reform Longitudinal Study (QSRLS) (Lingard et al. 2001) conducted between 1998 and 2000. Data from both studies are used in this chapter. The school is a small primary school that in the late nineties was in danger of being closed due to falling numbers. However, the school introduced a number of innovative programs based around philosophy, multi-age classrooms, environmental education and problem-based mathematics that have all worked to make this one of the Queensland state department of education's success stories. The school has grown from approximately 50 students in the mid 1990s to approximately 200 students. Our concern here is with the mathematics reforms, however, other reforms at the school, in particular philosophy, supported the mathematics education work being undertaken there.

Azalea P-10 school is an Indigenous community school located approximately 60 kilometres from a large regional centre. It is a recognised centre of excellence in Indigenous education. The community developed out of a forced settlement of Aboriginal people from different language groups in the surrounding areas in the late 1800s. Most of the children speak a version of Aboriginal English with many words and expressions being very different from standard English (e.g., Bama—person, Bina—ear, Bina gari—deaf, Budda—brother, Popeye—grandfather, Tidda—sister). The school has approximately 450 students who all identify as Indigenous. In all areas of literacy and numeracy the majority of students on Years 3, 5, 7 and 9 tests are below the national minimum standards. The school focus is on improving the literacy and numeracy results of students while retaining a strong focus on the Indigenous culture of the local area.

The mathematics reforms that occurred in the two schools demonstrate the importance of intellectually challenging and engaging classrooms for students who have traditionally not performed well in the subject. Both of these schools have rejected deficit models of students and have set high expectations for students' achievement. However, these high expectations have been accompanied by pedagogical practices designed to support students in their learning. We suggest that this approach of stretching students intellectually within a framework that supports and scaffolds learning is central to a socially just mathematics pedagogy.

Magnolia State School

The mathematics reforms at Magnolia were largely driven by the principal. When she arrived at the school in the mid 1990s she met with staff to find out what they were passionate about and then used these passions to introduce reforms. In describing that time she indicated that she had nothing to lose in the sense that the school, if it maintained its traditions, would be closed. Her focus on what the teachers were passionate about also meant that it was not just she who had an

[2] Pseudonyms are used for schools and participants.

investment in the success of the reforms. Her passions were philosophy and mathematics. She stated:

> So there was no sense of me coming in as any sort of expert in any way, except perhaps maths because I'd just finished a Masters in maths. So obviously I had some expertise there that they may or may not have had, they may have had it, that's fine. But I felt confident in that area and they had competence in other areas. So it was very much a, it was an exciting time 'cos it was I just felt like, as I said, we couldn't make things any worse, and I thought if we do something that's a bit outrageous like philosophy, what's the worst that will happen? They'll give us the sack.

Philosophy has become a key feature of the school and attracts many visitors, local, national and international, to observe its philosophy program in action. However, mathematics has also been central to this school's development. In late 2007, Lorna was asked if the school still had a focus on "open ended problem solving", to which she replied:

> We still do that type of maths. I've still got maths problems outside of my office there. And we still do open ended problem solving. I expect every teacher to do that at least once a week. We can't do it instead of sport any more, which is what we used to (laughs). We do sport now…But the maths, yes we still do do that. We still have open ended problem solving where the teacher may or may not know the answer, where the kids talk things through. So that's part of how we operate here.

One of the teachers, Lisa, we observed teaching mathematics with a Year 3/4 class described her week's mathematics program and how it related to the school's problem-solving approach:

> My program for the week will have one session, we will have problem solving or investigative maths, another session where we might do explicit teaching, where I'm at the blackboard or near the easel, there's, maths rotations where children work in groups and we rotate. So there'll be an area, there'll be a maths rotation where I'm doing work in with a small group, so as opposed to working with the whole class…What I was trying to achieve today was we have been working on 2D and 3D shapes, all of last week, and this fits in with the literacy that we'll be doing around this year, grocery packaging. So it was to give them to, last week the children have been looking at 2D and 3D shapes and analysing the properties of 2D and 3D shapes, things like—how many edges, how many corners, how many right angles, which 2D shapes make up a 3D shape, stuff like that. And that fits in also with the grocery packaging because the end result of the whole unit will be the children designing or making or drawing a diagram of their own cereal box and making the box and then designing the box. So the whole thing I think just fits in together, it's all integrated.

Lisa described the open-ended problem-solving approach as "investigative maths". She indicated that: "I enjoy doing investigative maths." However, she did go on to say:

> I think that for me, also explicit teaching plays a very important part, because I just need to be sure that I'm getting, that the children are getting a solid foundation and I'm able to observe the result.

This explicitness can be critical for students who have not traditionally done well at school. She also saw that importance of making the work connected to students' worlds. For instance, she stressed the importance of the school's efforts to ensure that they had a connected curriculum operating across all classes:

> I think that's my biggest challenge here at Magnolia and this is such a real, you know, a real life or connected to real life school, you know, whatever we do, we try and make sure everything is relevant, it is real like as much as we can make it, and I think that's the biggest issue, my biggest concern would be that these students become disengaged because they can't see the relevance of these studies, and that as educators, I think we need to make sure that in this changing world, you know, we need to really flexible where we need to have positive attitudes about themselves they not going to be, we do not need to prepare them for one career for life, we need to be flexible, so for all those things to happen I think we need to make sure that whatever we teach, we need to make it to real life.

Within Lisa's description of her program, there were many elements of the productive pedagogies model evident. The investigative maths was grounded in intellectual inquiry where students were actively involved in knowledge production. Lisa also saw it as important to make the curriculum connected and to integrate the mathematics across the curriculum to make it more relevant. She also sought to ensure that the classroom supported students' learning by at times employing very explicit teaching, especially with those who were struggling.

The open-ended problem-solving approach used in mathematics is consistent with other central aspects of the school. For instance, Lorna was able to indicate how the mathematics approach tied in with the school's approach to philosophy:

> James, when he was in Grade 5 said philosophy is a good example of how you should behave in the playground with your friends. I mean, the rules are we listen to each other, we think about what each other, what the other person says, we build on each other's ideas, and the most empowering thing of all is no single right answer…I had a child in Grade 7 last year who was very, very learning disabled. He's gone to a special unit and we do lots of oral maths, and he would never get involved in those oral maths sessions, and I remember the day he put his hand up and he said "I just want to build on Chris's answer". And I thought, now this (philosophy) has given this child a way into this discussion. Chris has already said it so it can't be too stupid, you know, and it's just a very empowering tool. So I've had a lot of—I get a real buzz out of that kind of thing. It's the inclusiveness of the process and they don't laugh at each other's comments.

In another interview she stated:

> You know, I like it when in maths they say, "Well I'd like to disagree with that answer", because, you know, and I'm thinking this is philosophy.

The focus in the school is, however, not solely upon academic work. Broader concerns about the well-being of the students were very much in evidence at the school. This was indicated by Lorna when she spoke of the need to work with them on their social skills:

> So making the children understand that we also care about what happens outside, and I keep saying to them you come to school to learn to read and write and do your maths, but you also come to school to learn to get along with each other, and you know, and I think that we have a responsibility to teach them those skills too. I don't think that it's okay to think that they'll automatically get those in time and that they'll learn them by osmosis. So we do actually teach those skills.

This concern was also demonstrated in relation to other skills and qualities. For instance, Lorna commented:

> So it's all those skills like independent learning and knowing yourself well and know what you're good at and not good at, and you know, being sure that you're an okay person, you know, there's so many things in there, but you do see it spilling over.

Within this school there is a very tight community that demonstrates many of the characteristics of what has been referred to as a teacher professional learning community (Louis et al. 1996). Within such a community there are levels of trust and reflection which facilitates teachers supporting each other in their practice. For instance, the principal stated:

> A really nice thing happened last week which in the big picture is a huge step and a really lovely step. Ruth came to me and said our maths is, we feel like we're not having any excitement. Have you got anything that you could give me that we would all love? And I said just the thing, I gave her something and she went away and came back and said it was fantastic.

A key aspect of this school was to also provide support to students by rejecting competition. In the early days of the school's reforms they had even banned competitive sport (although that had reappeared in recent years). She suggested that this non-competitive environment worked well for students who had been scarred by the competitiveness of other school environments. She provided the following anecdote as an example of this:

> There was that little guy last year in grade 7. The year bef…, two years before that, Grade 5, he came at the beginning of Grade 5 from a big, competitive school and he had had a really rough time up to then, and he came here and he settled in well here and—it must have been, it must have been Year 5 because he did his Year 5 test and he did the University of New South Wales maths test, and they all came back around about September/October on the same day, and I remember sitting next to him on the floor and handing out all this and things that they'd done. He'd got a high distinction on maths from the University of New South Wales and he got right up the very end of the graph for his Year 5 test. And I watched him as I just gave them out and he just went pink because he just went, it was Adrian…and I said "Are you OK?" And he said "What?" And I said "Didn't you know you were that good at maths?" He said, "No, I had no idea". And so, because all that other stuff had gone and he was able to just relax into his work, he started to achieve at levels that he'd never achieved before, you know, because of all that other stuff that I call bullshit had gone.

The enthusiasm that the principal and many of the teachers at the school had for mathematics was also reflected in comments made by students. During the site visit to this school numerous focus group interviews were held with students. In contrast to many of the other schools that we had visited all the students we interviewed talked about how they loved mathematics. Indeed, when asked what advice they would give to people wanting to become teachers, one Year 4 student suggested: "Do maths with them (the class)". Another Year 4 student when asked what she liked about school replied: "I like maths because it's fun and I enjoy learning about it". Such responses were not uncommon at this school. Interestingly, in conversations with the Year 6s they indicate that "fun" doesn't equate with something being easy. For many of the students at this school being presented with an intellectual challenge was enjoyable. As one Year 6 student indicated:

> S3: Just before we were doing maths, and it was a bit of a challenge, and I really enjoyed it because it was very challenging and yeah.
> I: What about it was challenging?
> S3: There's this sum, it was really hard and we got to use our brains. It was really difficult.

The Magnolia case study raises some important issues for the teaching of mathematics and social justice. There is a clear commitment to providing intellectually challenging mathematics classrooms for students at this school through the open-ended maths problems or investigative maths. This commitment is to *all* students. However, there is a recognition that the ability to do well at mathematics is not innate or that certain students are predisposed to mathematics. As such there is a very real commitment to make mathematics fun and enjoyable, and to make students see connections between mathematics and the "real world". However, absent from mathematics programs within this school, as in many other schools, is an obvious commitment to valuing difference, although such a valuing was present in other curriculum areas within the school. A similar absence was evident at Azalea P-10.

Azalea P-10

Azalea is a P-10 school that is spread across three different campuses within a relatively small Indigenous community centre. The school is run by one principal whose office is located at the primary school campus. Interviews with and observations of teachers in the high school were conducted, however the focus in this chapter is on the primary school mathematics curriculum. Azalea faces many challenges. Indigenous students in Australia have traditionally not done well at school (Lokan et al. 2001). This is the case for many students at Azalea. Within this school there have been attempts to remedy this situation with selective streaming, sometimes referred to as tracking. They refer to their ability group settings as "journeys". The principal saw the move to ability groupings as a significant reform. She stated:

> Our ability group classes has made a really big difference when we moved to staging which was now four years ago, our home group classes are a whole mix of ability groups, attendance, behaviour. We looked at a whole range and tried to balance those as equally as we can so we haven't got classes loaded with high non-attenders or major behaviour issues or things like that, but that acknowledging the literacy and numeracy needs of kids they do what we do call Journey. They do fifty minutes each day in their ability group. So rather than using our learning support teacher in an intervention way the learning support teacher becomes, in our case, a fifth teacher in that team and takes the lowest of the Journey groups. So we reduce the class sizes and also have them then focused on a narrower ability range for, and we've focused on reading and the number strand of the maths syllabus for the numeracy because we see that as foundational to the rest of the maths learning that they do.

Whilst we recognise that it is sometimes difficult to address all of the needs of students in one classroom, especially in large classes with many students who are disengaged from schooling, we were concerned that those in the "lowest" journey group would not be intellectually challenged. This is a major social justice issue if students who have been marginalised from the curriculum, especially Indigenous students, are not receiving the same intellectual challenge as students elsewhere. Azalea, like Magnolia, also, according to the principal, foregrounded an inquiry approach to mathematics. She stated for instance:

> What the teachers do, we try and embed within our units an investigative approach so the units of work about what are inquiry questions and what are they trying to sort of, we're looking, solving some sort of module or problem. Assessments, as I said, we try and have an investigative approach to the units of work that they do. Same with the maths, we very much, the maths policy is under review it's in draft currently and the staff who've been involved in that have done some professional development and worked around, again the inquiry approach to mathematics.

As with Magnolia, there was also an attempt to integrate mathematics with other areas of the curriculum. For instance, the principal stated:

> We were also part of a Language of Maths Project, a DEST[3] funded project, a few years ago where the focus was on literacies within the curriculum. We chose to look at the literacies particularly within maths and the teaching in English with their maths given our backgrounds so they're in a cluster project with (other Indigenous schools). The focus was a middle years focus but we actually looked at the whole school spectrum and looked at the significance of language within maths teaching as well, and a big part of that was looking at assessment planning.

In order to improve the quality of mathematics teaching across the school, one teacher has been designated as a mathematics specialist who works with both the secondary and primary school. This teacher, Luke, as in many such schools, is a new teacher and at the time of the interview had only been teaching for a year. He indicates that some of the problems he faces stem from students' lack of basic knowledge. For instance, he states that, despite having the "two top maths classes" there are students in the class who "don't know their times tables". However, rather than engaging in rote learning, Luke is still committed to the investigative mathematics approach and claims that these skills can be taught through making the curriculum relevant to students. As he commented:

> I'm aiming to just do things that are more relevant to the kids you know the whole new world thing and trying to engage technologies that are more in their future rather than the chalk and talk and "do this". Do the more active stuff.

One of the Year 4/5 teachers, Clark, described the investigative process in his class.

> Yeah I really like tasks and investigations. I quite enjoy doing those sorts of things; they're a little bit open-ended with the kids. For maths in our home group we usually have investigation maths which is proving a little hard at the moment because we haven't got to the stage we can do considerable investigation on time, which is a bit unfortunate. Last

[3] DEST was the Australian government's former Department of Education, Science and Training.

> term we did an investigation about planning a trip around Australia which was, as long as they started in Azalea, they could choose their own map around Australia and dictate which way they, and that's one of the samples you'll get is their sample of work. So "I travelled south-west of Brisbane" and so forth so they could map around. Investigations like that we sort of, you know it takes on a semi-realistic version of what they're doing and things like creating things like travel brochures. One of the things that I've sort of learnt is the more mainstream we run our classrooms in Azalea and I would argue in like from my experience within (other) Indigenous communities the better results you're gonna have, as long as you respect their culture and the language differences and those sorts of things. Some other things I try and do is make what we do in here as realistic to what you would do in another school and just provide, like provide opportunities to do normal things. School has a set of normal activities, a set of normal units that you would teach and things you would do.

Clark raises some interesting points here that are currently being debated in the literature on Indigenous education. For instance, he suggests that unless Indigenous students experience the same sorts of classrooms as mainstream students, their results will not improve. This principle has been advocated by significant elders in Indigenous communities who have claimed that Indigenous students need to receive the same cultural capital as non-Indigenous students and should not be provided with a watered down curriculum that assumes that they cannot achieve. However, Clark also raises the importance of respecting Indigenous cultures. Whilst this is clearly positive, this respect has to go beyond a token recognition to one where Indigenous cultures are explicitly valued (see McCarthy 2002, for an example of a school mathematics project that was based on the needs and interests of Indigenous students, required the contribution of local Indigenous elders, demonstrated respect for Indigenous knowledge and extended students' mathematical knowledge).

Luke, the mathematics specialist is, however, critical of some of the ways in which this approach has been adopted in the school. Luke is concerned with some of the ways the other mathematics teachers work with their students, especially in relation to supporting them to achieve. When discussing issues of assessment, he made the following observation:

> There's been some things said in staff rooms where you look at people and go "that's probably not what I actually do" and I'm supposedly the head of maths here so yes, sort of, like for an example, one of the other maths teachers gave their students this idea of travelling from here to Townsville or Mt Isa I can't remember and they had to do a whole lot of things along the way and then there was this, there was no construction, there was no supporting of that or no processes to how to do that so and that was assessment so I get wondering well, "what do you hope to achieve by this if you say, how can you possibly get a model answer if you haven't modelled the answer without some sort of constructiveness or on some approach to building".

Luke's comments about the supportiveness of the classrooms are important here. As the productive pedagogies approach has demonstrated, ensuring that students who are struggling with academic expectations, what constitutes high-quality work and how to achieve it have to be made explicit to students. This supportiveness has to also entail working to build students' confidence to take risks with their learning. As with some of the teachers at Magnolia, Luke recognises that there is an issue of confidence in mathematics for many of the students. He stated for instance:

> So I'm trying to build that trust and get them to understand that they are capable of achieving or trusting in the numbers that they come up with and the processes they come up.

On a similar theme, he also notes how they often freeze within exam conditions and do not trust in their own abilities. For instance, he stated:

> Last term I taught kids, the, it was maths percentages, maths and financial literacy, and all the rest of it, we got up to, I taught the kids here the compound interest formula. They were able to do it. I didn't collect enough evidence during the term, I know those kids do it, I saw them do it a hundred times, I gave them an exam and none of them did it. After the exam I said, "what was interest formula, I want you to tell me?" So I've just gone, "you knew it, you knew it now you're talking about it you just said it" but it comes back to that trust issue and the fact that I've put this hell of a blue-grey cover on it and you know with rubric marking criteria and that whole thing and it just made it look stressful to them. And that I think, and the fact you know it was exam conditions and that, now that's not I guess an entirely bad way of drawing knowledge but it's not the best, given that particular experience.

In order to support these students and to help them build confidence in their own abilities, knowledges and skills he works hard to make the curriculum interesting for the students. In discussing some of the challenges he faces in the classroom, Luke made the following comment:

> …probably the hardest thing to create, in my opinion anyway, is trying to create that, the value of education in the students 'cause they just don't seem to really, well the students don't by and large don't put a great value on what they can really get from here. You almost have to trick them into learning, you know play a bit of a game…They're a particularly hard class if I tried, I've tried doing several other things but if I'm trying to get them to do anything that's too big or conceptual, bang they're gone. And it's like you're walking on a razor edge, the whole class is teetering on exploding at any moment, so it has its challenges.

Whilst there is some evidence of deficit thinking about students in this excerpt, Luke clearly does face difficulties in making the students recognise the value of what is happening in the classroom. However, he does also seem to be engaging the students. In a discussion with a group of Year 6 students about what interested them at school, he was singled out as an excellent teacher. For example, it was noted that:

> S1: Tuesdays are fun 'cause we got Mr Luke from the high school.
> S2: Mr Luke.
> I: And what did he teach you?
> S1: He teach us maths and like faster harder stuff than we usually do.

Furthermore, he is clearly a very committed teacher and when students are not achieving he does not attribute this all to the students, their natural abilities, or their backgrounds. When asked how accountable he felt for the success of different groups of students in his classroom, he replied:

> I think I'm entirely accountable…I know we all have bad lessons, I've had my shockers and I've walked away sometimes, as long as I've had a go, I'm okay on myself, but when

> there's times I'll just chuck my arms in the air and say 'word searches or game of knowledge' or something like. You do feel you've let your students down, but then on the other hand sometimes they deserve a break too, so it's, you've got to, I do feel accountable but there's times when "OK guys just do this and chill out. You've done well", or, "it's just going to be too much of a fight today we're not going to get there so let's just pull back for a bit and let's just have a bit of timeout and we'll attack this again tomorrow."

The work on mathematics education in Azalea points to some important considerations for socially just pedagogies in Indigenous communities in Australia. The school is committed to ensuring that students are challenged intellectually in the classrooms through an investigative mathematics approach. Within this approach, as with the approach at Magnolia, students are presented with problems that do not have only one solution. They are thus required to engage in processes of knowledge construction. Such intellectual quality work can only occur, however, if students can see meaning in it, if it is engaging ("fun" as the students suggest) or connects to the real world. Teachers at the school appeared to be committed to make their mathematics classes relevant to the students. While the issue of supportiveness was raised this was not evident in all classes in relation to making criteria explicit or encouraging risk taking. However, Luke, the mathematics specialist, was seeking to make this a key component of the mathematics curriculum at Azalea. Throughout the classes there appeared to be little of the mathematics lessons that engaged with local knowledges or foregrounded Indigenous cultures. This was perhaps a feature of the mathematics specialist's attempt to broaden students' outlook on the world. For instance, Luke sees education as providing an opportunity for students to see beyond the local community:

> …probably the biggest challenge facing this school is trying to instil that importance and to show them what's over the range and beyond and across the ocean. You know the rest of the world is shrinking with, this community just sort of standing on its own and saying, "Oh no we don't need anyone else because we've got it all here".

An issue that confronts many remote Indigenous communities is the extent to which there is a need or desire to engage with the broader Australian society and the role that schools have in encouraging that engagement whilst valuing the knowledge and practices of the local communities.

Conclusion

In this chapter, we have foregrounded the work occurring in two primary schools around mathematics education. These are two very different schools. One is located in an inner city area of an Australian State capital. The other is located in a remote Indigenous community. However, they share a number of similarities. Both schools have a student population that has not always readily engaged with the academic curriculum. The demographics of Magnolia are changing and making this a little less the case, although the principal indicates that they still have a large number of students who come from very challenging backgrounds. Both schools have a stated commitment to providing an intellectually challenging mathematics curriculum to their students. Magnolia does this through its open-ended problem-solving approach, sometime referred to at the school as investigative maths; and Azalea through its own investigative mathematics approach. Teachers at both schools emphasised the importance of making the mathematics curriculum relevant and of being explicit in the teaching of particular concepts for students who were struggling. However, at both schools there was little evidence of a concern with difference in the mathematics classrooms. This need not be the case (McCarthy 2002; Skovsmose and Valero 2002).

We would note, however, that educational research has been very good at determining when social justice is *not* present in the classroom. In presenting these case studies, we have sought to address the much more difficult task of demonstrating its presence in the classroom, albeit in limited ways. We have taken two sites which have gone some way to integrating a concern with social justice into their mathematics classrooms. However, what is apparent from these sites is that it is much easier to tackle issues of distribution, ensuring that students from low SES and Indigenous backgrounds receive as good an education as they would in more privileged locations, than it is recognition, valuing and recognising diversity in the classroom. This is perhaps not unsurprising given the intense theoretical debates that have occurred on the topic of social justice, distribution and recognition (e.g., see Fraser and Honneth 2003). Thus, we would suggest, that whilst there are schools and classrooms that are seeking to challenge the reproductive effects of schooling, there is, even in schools concerned with social justice, still a major need to consider issues of difference in the classroom.

In presenting these two case studies of mathematics teaching and reform we are not seeking to idealise or criticise either or both of these schools. We wanted to present the challenges of real mathematics reforms in real and difficult locations. In so doing, we wanted to emphasise the importance of pedagogies for delivering a socially just mathematics curriculum. Magnolia has gone a long way towards achieving this. Students from a range of backgrounds are demonstrating high academic outcomes on a variety of measures and tests. Azalea's achievements are less obvious. However, the entrenched and dominant effects of colonisation and the failures of education in the past to address the needs of Indigenous communities will not be solved overnight. What Azalea is doing is important in that there are very real efforts to engage the students in mathematics without limiting their opportunities to acquire sophisticated mathematics concepts. We would suggest that one of the major lessons to come from these case studies is that the delivery of socially just pedagogies in mathematics classrooms will always be an unfinished project and that schools, such as Magnolia and Azalea, will need to be continually reassessing the extent to which they are providing their students with socially just pedagogies in all curriculum areas.

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References

Ashman A., & Conway, R. (1997). *An introduction to cognitive education: Theory and applications*. London: Routledge.

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning.* Mahwah: Lawrence Erlbaum.

Bourdieu, P., & Passeron, J.-C. (1977). *Reproduction in education, society and culture.* London: Sage.

Brandell, G., & Staberg, E. (2008). Mathematics: A female, male or gender-neutral domain? A study of attitudes among students at secondary level. *Gender and Education, 20*(5), 495–510.

Burton, L. (Ed.). (2003). *Which way social justice in mathematics education?* Westport: Praeger.

Cope, B., & Kalantzis, M. (Eds.). (1995). *The power of literacy.* London: Falmer.

Darling-Hammond, L. (1997). *The right to learn: A blueprint for creating schools that work.* San Francisco: Jossey Bass.

Fraser, N., & Honneth, A. (2003). *Redistribution or recognition? A political-philosophical exchange.* London: Verso.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35,* 258–291.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* New York: Routledge.

Gutstein, E., & Peterson, B. (2005). *Rethinking mathematics: Teaching social justice by the numbers*. Milwaukee: Rethinking Schools.

Hayes, D., Mills, M., Christie, P., & Lingard, B. (2006). *Teachers and schooling making a difference: Productive pedagogies, assessment and performance.* Sydney: Allen & Unwin.

Howard, P., & Perry, B. (2007). A school-community model for enhancing Aboriginal students' mathematical learning. In J. Watson & K. Beswick (Eds.), *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia* (pp. 402–411). Sydney: MERGA.

Keddie, A., & Mills, M. (2007). *Teaching boys: Classroom practices that work.* Sydney: Allen & Unwin.

Lingard, B., Hayes, D., Mills, M., & Christie, P. (2003). *Leading learning.* Maidenhead: Open University Press.

Lingard, B., Ladwig J., Mills, M., Bahr, M., Chant, D., Christie, P., Gore, J., Hayes, D., & Luke, A. (2001). *The Queensland school reform longitudinal study.* Brisbane: Education Queensland.

Lokan, J., Greenwood, L., & Cresswell, J. (2001). *15-Up and counting, reading, writing, reasoning…How literate are Australia's students? Programme for International Student Assessment (PISA)*. Melbourne: ACER.

Louis, K., Kruse, S., & Marks, H. (1996). Teachers professional community in restructuring schools. *American Educational Research Journal, 33*(4), 757–798.

McCarthy, J. (2002). Connecting numeracy across the curriculum in the outback. *The Australian Mathematics Teacher, 58*(4), 22–27.

Mendick, H. (2005). Mathematical stories: Why do more boys than girls choose to study mathematics at AS level in England? *British Journal of Sociology of Education, 26*(2), 235–251.

Mills, M., Goos, M., Keddie, A., Honan, E., Pendergast, D., Gilbert, R., Nichols, K., Renshaw, P., & Wright, A. (2009). Productive pedagogies: A redefined methodology for analysing quality teacher practice. *Australian Educational Researcher, 36*(3), 67–87.

Newmann, F., & Associates (1996). *Authentic achievement: Restructuring schools for intellectual quality.* San Francisco: Jossey Bass.

Sizer, T. (1996). *Horace's hope.* Boston: Houghton Mifflin.

Skovsmose, O., & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 383–407). Mahwah: Erlbaum.

Zevenbergen, R. (2003). Teachers' beliefs about teaching mathematics to students from socially disadvantaged backgrounds: Implications for social justice. In L. Burton (Ed.), *Which way social justice in mathematics education?* (pp. 133–152). Westport: Praeger.

Chapter 35
Mathematical Literacy in South Africa: Increasing Access and Quality in Learners' Mathematical Participation Both in and Beyond the Classroom

Mellony Graven and Esme Buytenhuys

Introduction

This chapter tells the story of how Mathematical Literacy (ML), as a new subject introduced in South African schools in 2006, opened access to mathematical learning and enabled the mathematical "metamorphosis" of learners in one school. The aim of the chapter is to share the way in which this curriculum intervention has the potential for enabling increased access and quality mathematics education particularly for learners with weak mathematical histories.

The chapter is based on data gathered from two case study classrooms of the first cohort of ML learners in one independent Johannesburg school. The chapter is jointly authored by Esme Buytenhuys, a teacher of one of these classrooms, and Mellony Graven, who at the time of the research[1] was the co-coordinator of the Mathematical Literacy Research and Development thrust of the Marang Centre, Wits University. Esme writes the story of the metamorphosis of the learners in her school based on her experience of working with these learners, reading their journal entries and most importantly reading their "mathematical stories" written on the last day of their 12 years of schooling. Mellony provides the contextual background to the story and some reflective analysis of the story which draws on Sfard and Pru-

[1] Esme and Mellony met at a Mathematical Literacy workshop organized as part of Mellony's work in the Mathematical Literacy Research and Development thrust. At a follow-up ML support group meeting Mellony was drawn to a range of ideas that Esme brought and requested that she and a colleague, Hamsa Venkatakrishnan, visit with her in her classroom to learn about the teaching of the subject. This was the beginning of their relationship which included interviews and visits over the three-year period.

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sak's (2005) narrative definition of identity. Thereafter, we engage with the relationship between the story and the curricula features that supported the more positive evolution of learner mathematical identities.

Our story begins from the point of departure that learners can constantly reauthor their mathematical stories and their lives. It focuses on ways in which learner choices to take ML instead of Mathematics in grades 10–12 freed them from ongoing stories of mathematical failure and enabled increased (and new forms of) mathematical participation, sense making, confidence and enjoyment.

Mathematical Literacy in South Africa

Mathematical Literacy (also commonly referred to as Maths Lit and abbreviated ML) was introduced in schools in the Further Education and Training (FET) post compulsory phase (grades 10–12, learners mainly aged 15–18) in South Africa in January 2006. The subject is structured as an alternative option to Mathematics, and all learners entering the FET phase since January 2006 are required to take one or other of these two options. ML is defined as a subject driven by life-related applications of mathematics that must develop learners' ability and confidence to think numerically and spatially in order to interpret and critically analyze everyday situations and to solve problems (DoE 2003). The rhetoric also foregrounds issues of quality in relation to enabling a learner to become "a self managing person, a contributing worker and a participating citizen in a developing democracy" (p 10). The emphasis in curriculum documents on developing mathematical competence and confidence, and ways of being and acting in the world, highlights the aim of developing positive mathematical learner identities. Evidence from schools suggests that in practice, learners with weak mathematical histories, competence and confidence are mostly guided towards taking this new subject.

The rhetoric of the rationale for ML foregrounds issues of access. The introduction of the subject addresses the concern that in the past approximately 50% of all Grade 10–12 learners did not take Mathematics and there was widespread concern for the high levels of innumeracy and poor performance on international studies. Mathematical participation was furthermore skewed along racial lines (see also Reddy 2006). Thus political will (rather than an initiative led by teachers or educators) led to the introduction of ML with the intention that all learners in the FET band would study mathematics in some form.

Initial design of the curriculum was by a group of department officials appointed by the Department of Education. The names of the members of this group are not publicly known but there was no consultation with the various mathematics education structures that exist in the country. The initial instruction was that it should be an easy mathematics without clear ideas of what this meant (According to A. Brombacher (personal communication, January 2010)). Shortly after the curriculum was designed, a ministerial committee was elected to review both the Mathematics and ML curricula and consultant Aarnout Brombacher (an ex Mathematics teacher and ex president of the national Mathematics Teachers Association AMESA) was brought in to head this review. Following this review, Brombacher developed the Subject Assessment guidelines (DoE 2008) and a Teacher Guide (DoE 2006) for ML. It is in these documents that the curriculum rhetoric begins to veer off from the possibility of being interpreted as a watered down mathematics curriculum. “Mathematical literacy is a different kind of mathematics, not a different lower level of mathematics” (Brombacher 2005).

Thus curriculum rhetoric which emphasized ML as a way of being and acting in the world became foregrounded in subsequent documents. The subject definition “driven by life-related applications of mathematics” (DoE 2003, p. 9) was thus taken to mean learning must be anchored in the real world and mathematics and context must be brought together in a dialectical relationship. Thus the Teachers’ Guide notes:

> the challenge for you as the teacher is to use situations or contexts to reveal the underlying mathematics while simultaneously using the mathematics to make sense of the situations or contexts, and in so doing develop in your students the habits or attributes of a mathematically literate person. (DoE 2006, p. 4)

The purpose of ML is stated in terms of what learners are to become and to be and within this rhetoric an underlying socio-cultural framework is evident. For example, “Mathematical Literacy should enable the learner *to become* a self managing person, a contributing worker and a participating citizen in a developing democracy…” (DoE 2003, p. 10) and “to handle with confidence…enable them to deal effectively with mathematically related requirements in disciplines such as the social and life sciences” (DoE 2003, p. 11).

In contrast, while there is some mention of relatedness to the real world in the Mathematics curriculum, this curriculum states “Mathematics is a discipline in its own right and pursues the establishment of knowledge without necessarily requiring applications to real life” (p. 9). The Mathematics curriculum is a more knowledge-driven curriculum with clear disciplinary boundaries and a focus on vertical mathematical progression necessary for further studies. Thus the curriculum states: “If a learner does not perceive Mathematics to be necessary for the career path or study direction chosen, the learner will be required to take Mathematical Literacy” (p. 11). Key differences between two curricula are summarized in Table 35.1:

Table 35.1 Key differences between ML and Mathematics

Anchored in the real world	Anchored in the discipline of Mathematics
Focus of rhetoric: ways of being and acting confidently in the world	Focus of rhetoric: knowing and understanding
Learner histories: weak competence and low participation	Learner histories: some strength, competence and participation within the disciplinary boundaries
Trajectory—into the world (citizenship) and social and life sciences studies	Trajectory—into further mathematically oriented studies

Thus ML tends to be driven by real life scenarios—and teachers are encouraged to keep these current and relevant to the interests and needs (including future needs) of their learners. For example, investigating electricity consumption and the effects of leaving appliances in standby mode have been suggested by Brombacher as a useful ML scenario. Electricity consumption is currently a "big issue" in South Africa as there is not enough of it, and households and businesses across the country are experiencing power outages and intermittent "load shedding." Other examples of scenarios given in the teacher guide (DoE 2006) include: calculating telephone costs; comparing between cell phone and land line costs depending on the needs of individuals; investigating pyramid schemes, loyalty programs, banking charges, etc. Teachers are encouraged to source data for such activities from current telephone directories, cell phone brochures, newspaper adverts and articles.

Despite such activities one of the greatest issues for learners in choosing to take ML relates to the perceived low status of the subject. Thus while the curriculum document states that ML should be taken "if a learner does not perceive Mathematics to be necessary for the career path or study direction chosen" (DoE 2003, p. 11), the more commonly told story is that it is for those who cannot do mathematics. Such stories (or "stereotypes" as referred to by learners) are problematic and get in the way of the subject achieving its full potential. However, as our story will show, positive learner experiences in relation to this subject allows learners to challenge these stories and create new stories about its value and their mathematical competence.

There are a range of concerns relating to the introduction and implementation of the ML curriculum. These relate to, for example, contradictory messages within curriculum documents (Christiansen 2007), the status of the subject (Sidiropoulos 2008), the validity of its assessment and the value of its currency (Jansen 2009a, b) and teacher shortages (Reddy 2006). These concerns are real and large-scale national research is required in order to reflect on the extent to which ML has met its stated aims and purposes across the country.

Our contribution in this chapter does not aim to address the above issues but rather to highlight, from the case study of one cohort of ML learners in one school (followed from Grade 10 to 12), the *potential* of this subject to transform learners, who defined themselves as mathematical failures and nonparticipators, into mathematical negotiators, participators and sense makers both in and beyond the classroom. In presenting this case the chapter focuses on the way in which ML enabled access to forms of mathematical participation and sense making not previously experienced by these learners in their schooling. The chapter also addresses the issue of "quality" of mathematical learning in ML from the perspective of the learners. In particular, learners' anecdotes of their mathematical learning crossing the boundary of the classroom into their everyday lives challenges the validity of perceptions of the lower quality and status of the subject. While we sometimes use the words of the learners to illuminate our story like Sfard and Prusak (2005: 20) we "urge the reader to remember that what follows is a story about stories."

Before telling Esme's story, we briefly introduce to you Sfard and Prusak's operational definition of identity and its connection to stories.

Defining Identity

The term "identity" is not fully useful in relation to Esme's story unless it is given a clear operational definition. Sfard and Prusak (2005) point out that while "identity" is a term that is widely used in educational literature it is seldom clearly defined. To provide "identity" with an operational definition Sfard and Prusak (2005, p. 16) define identities as "collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable, and significant."

Reification comes with verbs such as "have" (e.g., "I have strong mathematical ability") and I would add with declarations of one's being such as "I am" (e.g., "I am mathematically stupid"). Stories are considered endorsable if the identity builder can answer to them being a faithful reflection of a state of affairs (e.g., "I say I'm mathematically stupid because I constantly fail my tests"). While stories are *significant* if a change in the story is likely to affect the storyteller's feelings about the identified person—e.g., a change in the story that "Math Lit learners are mathematical failures" to "Math Lit learners have a preference for learning life-related mathematics" is likely to lead to a change in feeling by the storyteller about learners.

Thus, within their definition identities are human made, collectively shaped by authors and recipients. They explicitly highlight that their definition presents identities as the discursive counterparts of lived experiences whereas others such as Wenger (1998, p. 151) see such words as only a part of "the full, lived experience of engagement in practice". Sfard and Prusak thus stress "No, no mistake here: We did not say that identities were finding their expression in stories—we said they were stories" (p. 14).

We will return to this notion of identity when reflecting on Esme's story.

Esme's Story of the Mathematical Transformation of Learners in Her School Reading the journals and stories of my learners and reflecting on our three-year journey together made me realize just how enriched I have been by this exercise of committing my findings to paper. Teaching ML is indeed a very rewarding experience and yet at the same time an incredibly difficult one. My learners and I started out not knowing exactly where we were going and how we were going to get there—but with time we trusted and invested in each other and embarked on the journey together.

For me, looking back, I now see this journey as a complete metamorphosis.

The Caterpillar Stage I clearly remember those first few months with my six Grade 10 Maths Lit learners. They started out slinking into my classroom looking for a place to hide—to go unnoticed for 45 minutes. There was a tangible, invisible barrier between the learners and me—created by them. Reflecting on this, in discussion with Mellony, Hamsa and others, helped me to make sense of this. I began to understand the nature of the learner who appears in the Maths Lit class at the outset of Grade 10. These are precious young people who have been mathematically abused and for most as early as in Primary School.

In all the student stories about their mathematical experiences there were repeated emotive words and phrases such as: "failure," "struggle," "stress," "nervous," "hated maths," "worry," "extremely difficult," "no confidence," and "hopeless."

For example, learners wrote:

> During high school I hated Mathematics because it never made sense to me.

> From since I can remember I have struggled with Maths. I would always try my best but never see results

Many learners wrote of their feelings of hopelessness and how they eventually gave up.

> It's no fun knowing that there is no hope in the world that you can pass the tests.

> In Grade 8 and 9 I was told to go to extra Maths before school, but by this time I had lost interest and was tired of trying my best and *never* seeing results.

Many learners also connected their negative mathematical experiences to their broader self image. For example one learner wrote: "I used to hate anything and everything that had to do with Maths. My struggle with Maths also negatively impacted my self-confidence, and left me feeling like I was stupid and useless."

These learners were too scared to partake in discussions. Getting an answer or opinion from anyone was like drawing teeth. One of my vivid memories of one girl's perceived hopelessness is when she put up her hand to answer a question and then quickly put it down again. When I encouraged her she replied "no don't worry, my answer is probably wrong anyway."

The first part of the journey was to get the learners to start changing their perception about themselves. Only by reading their journals I realized just how difficult it must have been for them. Over the three-year period, my class grew from six to fifteen learners. Most of my class stated that they felt like failures because of their mathematical experiences. Not only did they see themselves as failures, they also had the snide comments from the Mathematics learners to deal with. Quite a few of them said that they were embarrassed doing Maths Lit because of the negative opinions and comments of other learners. My initial group, as well as those who changed in drips and drabs, didn't really have a choice in doing Maths Lit in the sense that it was clear that if they continued with Mathematics they would fail.

Interestingly, learners who only joined ML in Grade 11 seemed to show a greater intensity of emotion in relation to their struggles with Mathematics. Many of the girls shared how much crying and stress went into trying to cope with Mathematics. "Maths for me was a daily struggle I got stressed and cried a lot because of my inability to grasp the concepts." One of the boys even stated "before Maths Lit my life was a mess.... It's like there was a large gaping hole that I just couldn't fill." The intensity of the emotions in their stories possibly relates to their extended experience of learning mathematics in the FET band.

Another aspect of emotion that the Maths Lit learners had to deal with was the teasing they received in terms of the lower status of the subject as well as their own feelings that they were taking the subject because they were "stupid." In some cases students who changed much later to Maths Lit were the teasers of the initial group of learners who took Maths Lit from the start of Grade 10. For example one learner wrote: "I always mocked the children that decided to drop to Maths Lit, but that's only because I didn't really understand what it was all about."

It took about six months to get my initial group of learners to accept that I was on their side and that as a team we could achieve a new and positive maths experience. I positioned myself as a colearner—as indeed I was. This curriculum and many of the scenarios we explored were new to me as well. I insisted that nothing they said was stupid and all avenues of thinking would be explored. There was space in the curriculum for increased discussion and allowing for diversions in these discussions. I insisted that they should not look to me for answers—I did not have them. The only way to learn was going to be through engagement. At last they began to gain confidence and were willing to risk participation in discussions. My initial group was quite pleased with themselves when they saw that they were achieving better results than their peers who changed to Maths Lit at a later stage during Grade 10. In their own minds they had the poorest mathematical abilities. Then they began to succeed—*for the first time* the amount of effort expended was proportional to the results they achieved. Their successes were noted enviably by the Mathematics learners.

The Pupa Stage This is the stage when the learners begin to savor the good experiences and build on them. Knowing that success can be repeated the fear of failure diminishes. Classroom discipline becomes so much easier because they feel good about themselves and are not hiding behind a behavioral problem to cover up for their inadequacies. The learners tend to become actively involved in the task at hand and they thrive on the manner in which they engage with the subject. One learner had this to say about Maths Lit, "unlike Maths where you stress and cry over a sum, Maths Lit allows you to go out and see things in action being made; we are put in situations where we must work together in a fun and new way so that we may discover for ourselves the solutions to everyday problems."

Success leads to a greater desire to be challenged and the learners begin to believe that they are able to tackle anything. Their self-esteem in relation to

Maths starts changing—they begin to redefine themselves as learners who are willing to give it a try and as learners who can figure it out.

Getting weaker learners to do their Maths homework has always been an uphill struggle for me. I was under the impression that learners who didn't do homework didn't care or were lazy. I realize now how wrong my perception had always been. A few learners "journalled" about homework and one of them helped me gain new insight regarding this matter: "Since I was young I refused to do Maths homework, not because I didn't want to but because I simply did not understand the work that needed to be done."

In addition, learners linked this ability to make sense and "figure it out" to increased independence. A learner explained: "Its (ML) in English…Its easier to catch up because you can go home and you can read it…Whereas Maths, you need someone to actually like intensely explain it." Homework is not much of an issue in the Maths Lit classroom anymore. The learners actually feel proud of being on top of the situation.

The Butterfly Stage This is the stage when I look at the learners and observe them with pure delight. They are beautiful and whole; and ready to spread their wings. What do I observe?

I see individuals reflecting on answers and the calculations. They check that the answers make sense. They reassess and rework the problem until the answer is sensible and realistic. The Maths Lit learners become inter-dependent: they discuss answers that don't make sense; debate issues mathematically in order to establish meaning for themselves then collectively decide on the most appropriate answer. They are able to make sense of numbers—a skill they thought they didn't have prior to Maths Lit. They have reached a stage where they are able to confidently enter into debate with me. There have been times when their methods have been better than mine. These learners have evolved into mathematical negotiators who no longer shy away from "maths." In addition, the learners' positive experiences spilled over into the exam situation (and their marks were gradually improving) as one learner expressed herself: "Maths Lit has boosted my confidence and now I know I can do well in my exams without the stress of not understanding." Others echoed similar sentiments, "I no longer dread the Maths period, I do really well in exams and I'm always excited to write them."

While many learners at the start were concerned that taking Maths Lit would limit their access to further studies, now some realized that by getting a good symbol for Maths Lit (as opposed to a very low symbol for Mathematics) increased their points quota required for accessing universities. One learner draws this conclusion, "My marks have improved greatly. The average on my report, for Maths Lit, has changed my final symbol which has helped me in applying to university." Another learner writes, "I plan to study Business Management next year and Mathematics is not a requirement, having heard this, my choice to take Maths Lit was easy as it takes the stress off me."

In the final National Senior Certificate Mathematics exit exam, all of the 34 Grade 12 learners passed and 13 of them passed at the highest level (achieving "Level 7—outstanding," i.e., 80–100%) while 16 achieved at the second highest level "Level 6—meritorious," i.e., 70–80%). The metamorphosis is captured by a learner who shares her experience:

> At first, I have to admit, I felt like an idiot; people see it (Maths Lit) as a really easy and pathetic subject, when in reality it is not. It is actually a very interesting and useful subject that teaches you to apply mathematical concepts in your everyday life. We learn maths that you will actually use one day. My decision to change was one of the best decisions I have ever made and I don't regret it one bit…It has made me a happier person.

Similarly, Greg's story of transformation is one that will always live with me. I journeyed with him during his difficult and sometimes painful experiences during the years he was in my class. He came to me in Grade 8 with a mathematical history that spoke of failure. No amount of extra maths or revision helped improve the matter. He joined me again in Grade 10 even more despondent. In his journal he reflected:

> Maths was the most terrible part of my school career. I always used to dread coming to my Maths classes because I never used to know what I was doing.… I always used to get the worst marks in the class. I didn't want to choose Maths Lit because I thought it would be embarrassing but throughout the years I have realized that choosing Maths Lit was definitely the best thing I have ever done in my school career. I loved going to Maths Lit because I know that I'll be using the maths that we learn in and out of my life.

By the end of Grade 10 he achieved 51%. Greg was beginning to remold his relationship with Mathematics. As time passed, Greg became more confident and self-assured often adding value to the class discussions. He was proud of his achievements and was even able to refute the taunts from "Core" Maths learners. He wrote: "Everyone is going to have to buy a house; and calculate electricity and telephone bills—I know I can. I can calculate how to build a house right to the last brick! Can normal Maths kids do that? How's about nooooooo (no)." While Greg's reference to "normal" Maths is somewhat problematic in how it positions Math Lit, his increasing confidence remains clear. Greg attained 61% in his Matric finals and still basks in this success.

I am extremely privileged to have embarked on this pioneering journey—I too have been transformed. ML most certainly has more than just the *potential* to transform learners: it has healed many dysfunctional young adults. It has set them free and given them wings to fly. I think this is a sentiment that is growing. On a Matric Graffiti Wall at another private school, I noticed a learner had written: "Math Literacy 4 future world leaders."

Reflecting on the Story

The story highlights the way in which the teaching of ML as a subject in these classrooms, provided learners with the opportunity for developing new identities in relation to mathematics.

Our chosen definition of identity gives increased agency to the learner and the teacher as it opens the space for the reauthoring of learner identities. It is this agency and space for reauthoring that is particularly appealing for reflecting on Esme's story. It highlights the important role that significant narrators, (e.g., teachers such as Esme), can play in deliberately challenging existing negative stories of learners and the importance of reflecting on their own intentional or unintentional authoring of learner identities.

Thus within this definition of identity, as discursive counterparts of one's lived experiences rather than some intangible (and stable) entity, reauthoring of identities is not only possible but we argue is necessary for enabling and giving momentum to learning. We believe this is especially important in cases where identities have been negatively constructed as stories which are stumbling blocks to learning. Thus Esme notes that the first part of the journey was to get the learners to start changing their perception about themselves.

Freedman and Combs (1996) argue that the metaphor of stories helps one to see how stories circulate in society and how these realities are socially constructed, constituted through language and organized and maintained through narrative:

> When life narratives carry hurtful meanings or seem to offer only unpleasant choices, they can be changed by highlighting different previously un-storied events, thereby constructing new narratives. Or when dominant cultures carry stories that are oppressive, people can resist their dictates and find support in subcultures that are living different stories. (p. 32–33).

The final sentence highlights the opportunity for groups of people in supportive communities or "communities of practice" (Wenger 1998) to enable "living different stories." Supportive communities, such as those formed in the ML classes in this school should open up these alternatives especially when existing stories "carry hurtful meanings," undermine mathematical identities or impede learning. As we see in Esme's story these alternatives were opened up and members of these communities became the new "significant narrators" that told stories of mathematical competence and rejected the stories of other students that they were mathematically stupid. Learners began to argue back to Mathematics students that ML was different mathematics rather than inferior mathematics and began to challenge the appropriateness of its lower status. For example Greg wrote: "I can calculate how to build a house to the last brick! Can normal Maths kids do that? How's about Noooo (no)." Thus, we see that through the development of the ML classroom as a supportive community, learners such as Greg are able to reject the significance of oppressive, negative stories and give more significance to the stories emerging within the community and told by their teachers about their mathematical learning.

Sfard and Prusak (2005) continue to identify two sub categories of stories: *current*[2] identities (told in the present tense and formulated as actual assertions) and *designated* identities (narratives expected to be the case—now or in the future). Learning is then conceptualized as closing the gap between *current* and *designated* identities. In Esme's story we see that the gap between current identities at the beginning of Grade 10 and the designated identity that learners should become mathematically competent problem solvers is large and learners choose not to participate. Designated identities of becoming "competent mathematical problem solvers" are skeptically considered by learners as euphemisms for "mathematical dummies" needing an extra three years to learn basic mathematics.

With a concrete focus on developing learner confidence, constantly encouraging participation and telling new stories about learners' mathematical thinking (e.g., "That is not a stupid idea—in fact it is helpful in solving this problem, tell us more."), current identities begin to shift. Learner talk changes from "I can't" to "I can or at least I'll try," and the gap between *current* identities and *designated* identities begins to sit in productive tension and stimulate mathematical learning.

Esme's story tells of her deliberate and explicit rejection of negative stories and her focus on encouraging participation in the caterpillar stage. This provides the momentum and space for her learners to reauthor their mathematical identities. Indeed, the supportive classroom community created by Esme and her focus on developing mathematical confidence in learners was important. There were, however, several features of the subject ML *per se* which enabled this story to unfold in a way that was different from Esme's experience of teaching "Mathematics" in earlier years. These curricula features are discussed below.

What Curricula Features Support "Living Different (Mathematical) Stories?"

The learning process resulting from the implementation of a curriculum is clearly complex with a multitude of factors impacting on the nature of learning. However, there are various features of the ML curriculum that Esme and learners in this school highlighted as opening the space for prioritizing participation, negotiation, sense making and "preparing learners for life." Table 35.2 identifies several features of the subject that support the emergence of new mathematical teaching and learning stories. While each feature is tabulated separately, they are, of course, complexly interconnected.

[2] Sfard and Prusak do not use the term *current* in their 2005 paper but instead refer to *actual* identity. This term can be misleading. In a personal e-mail correspondence with Sfard in November 2008 she wrote "I decided to replace the term 'actual identity' with 'current identity'. It is just that for some readers, the term 'actual identity' sounded as a declaration." In order to avoid this interpretation for readers of this chapter I too have avoided using this term and have therefore replaced it with the preferred term suggested by Sfard.

Table 35.2 Curricula features that open up new spaces

Curricula feature	Space for new teaching and learning stories
Progression is located in the complexity of contexts with less mathematical "content" covered. From one grade to the next mathematical contents are often repeated with the recommendation that they are explored "in more complex contexts."	Less vertical progression enables slower pacing and increased discussion. Learner centeredness is noted and contrasted to mathematical teaching where the pace is set according to stronger learners in order to "get through the curriculum." Esme: "I am the facilitator and not the teacher, emphasis on understanding concepts rather than being driven by completing the syllabus…More relaxed slower pace…Structure is more informal, cooperative learning."
A focus on contextualization and the use of scenarios.	Exploring various real life contexts and scenarios necessitates discussion and participation in order to make sense of situations, and furthermore brings a personalization of learning, as multiple perspectives are part of the sense making process. Collaborative ways of working are productive of learning (contrasted by learners to group work in Mathematics requiring copying a student "in-the-know"). Contextualization (as well as the "newness" of the subject) also positions the teacher as a colearner and facilitator of discussion rather than the authoritative source of knowledge—opening the space for a more distributed locus of authority in the classroom community. Esme: "We're on a journey together."
A focus on ways of being and acting in the world (supported by an underlying socio-cultural learning theory).	This encourages teachers to focus more holistically on learners and their learning—participation, forms of participation and personalization of learning are of primary importance. Esme: "(Interaction with learners) is stunning—they are real live people and not just a vessel to fill with maths."
Explicit stipulation that ML is not for learners who intend to pursue "mathematically related" studies leads to commonality in learners' trajectories into life and/or non-mathematically related studies.	The explicit departure from preparing learners for further mathematically related studies opens the space for teachers to focus on mathematical engagement necessary for preparing learners for life. Esme: "You know there are a whole lot of things: blood alcohol levels, that is where they are at, teenage pregnancies…it's so important to their lives."
ML is a new subject defined as different to Mathematics. It is explicitly stated that it is not a watered down version of Mathematics.	The "newness" of ML distances teachers from their own apprenticeship experiences of mathematics teaching in their own schooling. This frees them to explore new ways of being in the classroom. The initial absence of external assessment precedents removed the tendency to "teach toward the exam" and supports the focus on learning rather than assessment. Esme: "(The curriculum is) wonderful, allows creativity and freedom to explore, invent, discover for yourself what works, how it works."

As seen in the Table 35.2, various curriculum features work in tandem with teachers' interpretations of the curriculum to enable the development of a supportive community where learners can live and tell a different (mathematical) story and find support in the "subculture" of ML learners. Our story has highlighted learners' opportunities for developing new identities in relation to mathematics and the development of some level of competence and success with mathematical participation both inside and outside the mathematical classroom. But what of the quality of the mathematical learning?

Throughout learner interviews a personalization of learning was evident. Learners' noted that they could both bring their "life" experiences and their opinions to the learning process and extend their classroom experiences into their life. Thus, the boundary between the ML classroom and the world outside was increasingly experienced as permeable with increasing coherence between one's ways of participating and negotiating, being and acting in the world and in the classroom.

Several learners gave examples of how they used the mathematics learnt in class outside of the classroom. For example, one learner spoke of how for the first time he discussed with his father (an architect) the plans of a project he was working on, another explained how she helped her mother (an interior decorator) draw up the plans for redecorating her brother's bedroom, and so on. This, in addition to the strong performance of the majority of learners in this school (85% achieved "outstanding" or "meritorious" results), points to the quality of mathematical learning.

So why did learners experience this in ML and not previously in Mathematics? Overwhelmingly, learners' reasons centre around the nature of participation and engagement afforded in these ML classrooms. Learner comments primarily linked the reasons for this to their changing participation in the classroom in relation to two factors: "real" collaboration and "real" problem solving.

The "realness" of the collaboration and problem solving related to the similarity to real life—opinions and multiple methods are both valued and productive of both mathematical and contextual learning and the nature of the scenarios are messy. ML requires active participation, engagement and negotiation. The freedom to engage with the "messiness" of scenarios and negotiate the way forward without searching for "the right way" opened a learning space that was previously closed in mathematics classrooms. Learners also noted that the nature of engagement with teachers was different and that there was greater independence from the teacher as a result of having their opinions count and influence the direction of the lesson: "I think I understand it more because you like discussing with her… You are not just sitting and just listening to the teacher babble on you actually taking part."

As Esme said, in the butterfly stage, learners became mathematical negotiators. Wenger (1998, p. 210) writes that negotiability can be described with phrases such as: "opening access to information, listening to other perspectives, explaining the reason why,…inviting contributions,…opening decision processes, argumentation, sharing responsibilities…" Indeed, visits to Esme's classroom revealed that these were strong features of her classroom.

Discussion and Conclusions

The story we have told illuminates a mathematical metamorphosis with respect to identity. From the writings of all of the 2008 Grade 12 ML learners in this school, it is clear that for each her/his mathematical story changed substantially as a result of participation in ML. Thus following our use of a narrative definition of identity we have illustrated the subject's potential to support the development of more positive mathematical identities. Learners in this story changed from mathematical outsiders, strugglers and nonperformers to active mathematical participators in their ML classrooms and in the world outside the classroom. We have also highlighted aspects of South Africa's ML curriculum which opened the space for such metamorphoses to occur.

This said the introduction of ML is not without some serious problems and challenges. While there are many other teachers and learners who tell similar stories, including in inner city state schools (see Venkat and Graven (2008) and Graven (2009)), there are also those who find it difficult to teach (and learn). Furthermore, now that the first national Grade 12 exit examinations were written in November 2008, the validity of the assessment of the subject (and with it the quality of the subject) is being called into question (see Jansen 2009a). Differences in perceived validity result in some universities accepting a good result in ML for business- and commerce-related studies while others do not. Entrance criteria are constantly reviewed in relation to stories that circulate about the quality and validity of ML and to debates about access to Mathematics. Racial inequalities in terms of access to scientifically related studies are perpetuated when many state schools only offer ML and not Mathematics to Grade 10–12 learners.

In a recent doctoral study, Sidiropoulos (2008) found that ML teachers she surveyed identified several problematic themes in relation to the implementation of the subject:

1. A threat to the status and identity of mathematics teachers required to teach ML. (Teachers view teaching ML as "inferior" to Mathematics and a demotion from mathematics teaching.)
2. A lack of leadership in ML, as Heads of Department, mostly, do not teach the subject.
3. Thin and disconnected levels of understanding the teaching of "mathematics in context."
4. Many teachers believe the curriculum is too difficult for the learners doing the subject.

The fourth theme emerged from her survey with public school educators. Since ML is a compulsory alternative to Mathematics, some learners enter Grade 10 ML having not managed the mathematical knowledge and skills required in the earliest grades of schooling. This problem is exacerbated in poorer public schools where teacher-learner ratios are higher and access to resources is limited. In contrast, Sidiropoulos' study found private school teachers did not hold this view and instead these teachers made reference to the value that ML added in terms of its benefit for learners' everyday lives.

This difference in Sidiropoulos' (2008) finding between public and private school teachers is particularly worrying in the context of a post apartheid South Africa where the introduction of ML aims to increase mathematical access for *all* learners. As Reddy (2006) so aptly points out in her paper on the state of mathematics and science education—schools are not equal—and the success of new curriculum innovations will therefore differ across schools.

Problems with the implementation of ML must be engaged with and large-scale national research is necessary to contextualize and make sense of these difficulties and to find solutions to them. Our fear is that success stories of the *potential* of ML to meet its aims might be overshadowed by stories of difficulties relating to implementation. Such difficulties are clearly problematic but one should not simply reject the value of this curriculum as a result of these difficulties. Instead, these difficulties should be solved so that the stories of these learners can become the dominant stories of learners across the diversity of schools and contexts in South Africa.

As Reddy (2006) points out, creative interventions often lack the detailed implementation plan and can then be abandoned when they do not produce the expected results. She warns: "We should not move from one intervention to the next and become 'serial innovators'" (p. 412). We hope that our story contributes to raising awareness of the *potential* of this subject to increase mathematical access to quality learning for those with previously negative mathematical histories.

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References

Brombacher, A. (2005, March 1). Maths literacy vital for life today. *The Cape Times*.

Christiansen, I. M. (2007). Mathematical literacy as a school subject: Mathematical gaze or livelihood gaze? *African Journal of Research in Science Mathematics and Technology Education, 11*(1), 91–105.

Department of Education. (2003). *National curriculum statement grades 10–12 (General): Mathematical literacy.* Pretoria: Department of Education.

Department of Education. (2006). *National curriculum statement grades 10–12, teacher guide, mathematical literacy.* Pretoria: Department of Education.

Department of Education. (2008). *Subject assessment guidelines—mathematical literacy assessment guidelines*. Pretoria: Department of Education.

Freedman, J. & Combs, G. (1996). *Narrative Therapy: The social construction of preferred realities*. W.W. Norton & Company, New York.

Graven, M. (2009). Wenger's (1998) perspective on learning is "ready for prime time" in some Mathematical Literacy classrooms. *Proceedings of the 17th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education,* Rhodes University, 1, 71–81.

Jansen, J. (2009a, January 4). Old school: New system produces the same results. *Sunday Tribune*.

Jansen, J. (2009b, January 29). Vital questions for Education. *The Times*.

Reddy, V. (2006). The state of mathematics and science education: Schools are not equal. In S. Buhlungu, J. Daniel, R. Southall, & J. Lutchman (Eds.), *State of the nation: South Africa 2005–2006* (pp. 392–416). Pretoria: HSRC Press.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22.

Sidiropoulos, H. (2008). *The implementation of a mandatory mathematics curriculum in South Africa: The case of mathematical literacy*. A thesis submitted in partial fulfilment of the requirement for the PhD (Education), University of Pretoria, Pretoria, South Africa.

Venkat, H., & Graven, M. (2008). Opening up spaces for learning: Learners' perceptions of mathematical literacy in grade 10. *Education as Change, 12,* 29–44.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* New York: Cambridge University Press.

Chapter 36
Together-and-APart for Quality and Equity in Mathematics Education

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The Conflict Between Quality and Equity

"Quality and Equity for all" in Mathematics education: Why do quality and equity seem to be in conflict with each other? Are they really contradicting goals? Is it impossible to reconcile them? In this chapter we try to examine from where this conflict might emanate. We also suggest a way to moderate this conflict.

In our view this conflict stems from two almost diametrically opposed perceptions of how "quality mathematics" should be cultivated in school. On the one hand, there are those who believe that the quality of mathematics that can be learned depends mainly on the students' mathematical ability. This necessarily leads to same-ability learning groups—a tracking environment where both didactics and mathematics "suit" the students' abilities. In the eyes of these educators, quality is cultivated most promisingly in tracked settings. Their solution is that more mathematically competent students learn advanced mathematics in a faster paced higher track, and less mathematically competent students learn at a slower pace in the lower track. These educators argue that struggling students are better supported in lower tracks where they can get individualized instruction rather than in heterogeneous classes (Loveless 1998). They also justify tracking by the "nature" of the subject. Mathematics is perceived as "graded," "linear," "structured," "serial" and "cumulative"—making it difficult to work with groups of students with different levels of knowledge and ability (Ruthven 1987). On the other hand, there are those who believe that quality mathematics, to a great extent, depends on the group of students with whom the student learns. They argue that the nature and quality of students-teacher and students-students interactions are fundamentally different in low-track and high-track settings (Gamoran 1993). The role and quality of discussions is strongly emphasized in theoretical approaches that describe learning as an individual process nourished by interpersonal interaction (Bransford et al. 2000; Carver and Scheier 1982; Voigt 1994; Wood and Yackel 1990). For these writers,

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the study group is not a mere administrative division, but a crucial component of the learning environment. Moreover, they argue that low-ability settings lead to low-quality teaching. Low-quality teaching is characterized by: teachers' low expectations; a low-status, non-academic curriculum; valuable class time spent on managing students' behavior; and most of the class time devoted to paper work, drill, and practice. They suggest that two hypothetically identical students may end up with different mathematical knowledge if they are allocated to two study groups with significantly different participants and styles of interaction. Following their line of thought, tracking would be an obvious case of creating unequal learning groups within the same school, while learning mathematics in a heterogeneous class would be the preferred method for cultivating equity. In their eyes, equity is cultivated most promisingly in heterogeneous classes. Thus, the conflict between quality and equity finds its expression in the controversy: tracking or heterogeneous classes?

Consequently, the mathematics educating community is split and the controversy is intense; one side pushing for tracking, the other for heterogeneity.

Analysis of past research reveals a similar split—there is conflicting evidence of the effects of tracking versus heterogeneous settings on students' achievements.

The most prevalent finding of tracking studies is that ability grouping does have an effect on achievement. It has been shown that ability-grouping results in an increase in the gap between high- and low-ability students beyond that expected on the basis of initial differences between them, thus contributing to the regeneration of an inequitable society (Ball 1984; Boaler 1997a, b, c; Linchevski and Kutscher 1998; Alexander et al. 1978; Gamoran and Berends 1987; Gamoran and Mare 1989; Kerckhoff 1986; Oakes 1982; Slavin 1990; Sorenson and Hallinan 1986; White et al. 1996). However, studies on performance of high-achieving students studying in heterogeneous groups have shown conflicting results. Some studies have shown that high achievers might be harmed by heterogeneous grouping and that their performance is enhanced by tracking (Brewer et al. 1995; Epstein and MacIver 1992; Kulik 1992). Other studies have revealed no significant differences in the performance of high achievers when they are grouped in heterogeneous classes (Figlio and Page 2002; Mosteller et al. 1996; Slavin 1990).

These inconsistent findings led the research community and mathematics educators to interpretations based mainly on their concealed educational beliefs and accumulated practice. They tend to claim that the conflicting results may be due to the fact that it is usually impossible to disentangle the effects of tracking itself from the effects of differentiated curricula and other factors associated with tracking (Kerckhoff 1986; Lucas 1999; Slavin and Braddock 1993). Those among them who support tracking argue that it is the quality of teaching in the low tracks that is at fault. They maintain that the reason for students' poor achievements is the poor quality of teaching in the lower tracks. Their claim is that it is not the idea of tracking that has failed but rather its application; that with an improved level of instruction specially adapted for students in the lower tracks, these students would profit from learning specifically in these tracks, while the stronger students would be able to learn without having to compromise their level of learning due to the fact that there were less mathematically competent students in their class (Gamoran and

Weinstein 1998; Hallinan 1994; Loveless 1998). According to this interpretation of tracking studies, the negative effects of tracking result from unsuitable teaching approaches in the low-track classes, not from grouping itself.

Those who support learning in heterogeneous groups maintain that it is no coincidence that there are almost no high-quality low-tracked classes. They argue that the way the learning materials are designed and presented is only one small factor in an individual's learning process. The composition of the group, the interpersonal interactions, the level of teacher expectations, the level of discussions determined by the student peers and the class culture have as much of an impact on the level of learning and student development as the learning materials. Thus, allocation in itself, of weak students in a weak group results in an impoverished learning environment where students are exposed to low teacher expectations, low levels of discussions and poor ideas—all this resulting inevitably in low achievements. Moreover, they point out that there are hardly any documented examples of schools in which students in low-track classes received high-quality instruction (Alexander et al. 1978; Ansalone 2005; Gamoran and Berends 1987; Gamoran and Mare 1989; Huang 2009; Heubert and Hauser 1999; Kerckhoff 1986; Lamb and Fullarton 2002; Oakes 1982; Slavin 1990; Sorenson and Hallinan 1986; White et al. 1996).

The discouraging results of tracking studies on the one hand, and evidence of the promising potential of cooperative learning on the other have prompted attempts of the tracking system opponents to cope with student diversity within the heterogeneous classroom (Boaler 2008; Crain and Mahard 1983; Crain et al. 1982; Davidson and Kroll 1991; Goldring and Eddi 1989; Linchevski and Kutscher 1998; Wortman and Bryant 1985; Willie 1990). There seem to be three main strategies used for dealing with heterogeneity within the mathematics class.

The first type of intervention is where the mixed-ability class is divided within the class into homogeneous groups. Thus, although the class is formally heterogeneous, the students' learning environment remains homogeneous (Mills and Durden 1992; Slavin 1990). The rationale for this type of class organization draws from the school of thought that believes that students of different ability-levels should be taught differently (Feldhusen and Moon 1992). However, the fact that students of different abilities are in the same classroom, and not in separate rooms, allows for some mobility between the groups and solves problems related to socio-psychological ones such as students' self-esteem and stigma. Many researchers consider this form of class organization, "always homogeneous settings," as traditional tracking—no different from the situation where students of different levels learn in separate rooms. This learning environment, they claim, does not encourage interaction between students assigned to the different ability levels; consequently students do not benefit much from the class heterogeneity (Mills and Durden 1992; Slavin 1990).

The second type of intervention is where the students always learn in heterogeneous groups. This school of thought believes that all students are capable of working cooperatively all of the time in heterogeneous settings by being provided with activities that are accessible to all—either because of the nature of the activity, or because of the nature of the group interaction (Boaler 2002; Silver et al. 1995). This type of setting, “always heterogeneous settings,” does not take into account that situations might arise where working together on “accessible to all” activities would not be as beneficial to the students and might even in the case of mathematically competent students, lead to mediocrity, and in the case of weaker students to lack of motivation or to a situation where their special needs have not been met (Linchevski and Kutscher 1998).

The third type of intervention is where the students always learn in heterogeneous groups, however, the low-achieving students get extra support by being placed in “out-of-class” mathematics workshops according to teacher recommendations (Burris et al. 2006).

The TAP Rationale

Integrating results from research led us to the following, compelling question:

Is it possible, on the basis of the above reported, ostensibly contradicting results, and on the basis of researchers and educators analyses of these results, to design a teaching model in a heterogeneous class that could contend on the one hand, with the weaknesses portrayed when learning in the tracking system, and on the other with the weaknesses of learning in a heterogeneous class?

Our basic premise was that the learning group is a central component in the student’s development. Therefore, this has to be a central element when organizing learning. Our assumption was that allocation, in itself, of a student to a lower track violates equity, and that the learning group is an essential cultural and educational resource, no less than high-quality learning materials and appropriate teacher training. Consequently, we ruled out a teaching practice where students would learn in ability groups—whether in the same classroom or in separate locations—throughout the whole year. At the same time, in light of research, and according to what every teacher and educator who works in the field knows, there are undoubtedly differences both in mathematical abilities and in learning pace among different students. There are certainly students who want and are capable of learning mathematics at a higher level than the others. Disregarding the abilities and needs of this mathematically competent group of students would also be violating equity. Consequently, an additional basic premise was that the differential mathematical needs of the students with different abilities would have to be met. These two basic premises led us to the conclusion that learning has to take place in a heterogeneous class, but at the same time, the teaching practice would have to contend with two contradictory goals: “acknowledging diversity”—acknowledging the fact that there are differences between students, and, concurrently, “disregarding diversity”—or designing a learning environment that perceives all students as being able to be members of the same learning community. The implication of this statement will soon be clarified.

Our research-based TAP (Together-and-APart) approach was developed in order to accomplish these goals. We first describe the major assumptions and guidelines of TAP. We describe how this approach genuinely supports equity not only through appropriate learning environments, but also by providing learning interventions that enable students to remain a fruitful, mathematics learning community.

TAP's Main Guidelines

As stated above, in our view the above-introduced requirements can be realized only if the learning environment is designed to concurrently "acknowledge diversity" and to "disregard diversity." In TAP, by acknowledging diversity, we mean that we recognize diversity in students' "entry" points and allow and encourage all students to fulfill their mathematical needs, abilities and preferences (APart). Thus, acknowledging diversity should lead to the construction of a learning environment that accommodates differences in the ways learners think about, construct and display mathematical knowledge and understanding.

However, the requirements introduced above also imply that at certain carefully defined points in the learning process, TAP "disregards" diversity. In these cases although TAP acknowledges differences in entry points, it will do everything in its power to ensure acquisition of certain essential mathematical knowledge—in effect reducing diversity in students' exit points. Disregarding diversity means that there is a body of Indispensable Mathematical Knowledge (IMK) that should be owned by all students notwithstanding the acceptance of diversity in other parts of their mathematical knowledge. IMK is that part of genuine school-mathematics that enables the heterogeneous mathematical community to be engaged in fruitful interaction to the satisfaction of all its members, culminating in open doors to higher education. Thus, disregarding diversity should lead to the design of a learning environment that guarantees students' acquisition of IMK.

Acknowledging diversity while disregarding it, two ostensibly contradictory goals, in our perception of equity, are achieved in our teaching model by alternating between two basic types of learning groups: heterogeneous groups and homogeneous groups. Hence, in the TAP project, although the students learn together throughout the year, each student is simultaneously a member of two types of groups: (1) a heterogeneous group (both small heterogeneous groups and the whole heterogeneous class) and (2) a homogeneous group. The composition of these groups is changed from time to time according to specific needs of the students in the different learning situations. In each learning situation, the teachers choose the appropriate class organization according to the different types of interactions they wish to foster. These changes in the student's learning environment lead to situations where a student who is a "leader" in one setting might be the "follower" in another. It improves the student's ability to become an "active listener" and naturally encourages opportunities for direct, indirect and multivocal interactions (Cobb 1994). The various heterogeneous groups are generally engaged in the same activities (Together), while the homogeneous groups are generally engaged in different activities (APart). The heterogeneous groups provide a shared rich learning environment. The learning plan is designed to optimize opportunities for heterogeneous interaction. The heterogeneous groups, whether of the whole class or smaller groups, are provided with meaningful instructional activities suitable for cooperative learning in heterogeneous settings (Together). The small homogeneous groups are provided with differential instructional activities according to the students' prior achievements and inclinations (APart). The various heterogeneous groups are generally engaged in the same activities (Together), while the homogeneous groups are generally engaged in different activities (APart). In the homogeneous groups the students' differential needs are addressed: Some students are provided, for example, with additional opportunities for revisiting IMK, for orientation (preparation) to a subsequent topic, for enrichment, for experiencing mathematical interaction and more (for more details see Linchevski and Kutscher 1998). Homogeneous groups are set up *only* if the differential needs of the students cannot satisfactorily be addressed in the heterogeneous setting. Evaluation is designed and its results analyzed to provide the teacher with the necessary information for structuring the various settings described above, and for planning appropriate interventions for these settings. Evaluation is designed to accommodate, evaluate and reward equally the diverse thinking processes and mathematical knowledge that different students display, as well as to enable the follow-up of IMK.

The TAP project first and foremost makes use of the learning plan recommended by the syllabus, but introduces a number of changes. For instance, one of the changes might be in the order of the topics, where the teaching of a certain topic may be postponed in order to allow the class to study in a heterogeneous setting at this time. For example, in one school Ms. Krispin did not teach the topic "modeling verbal expressions algebraically" at the point where it was supposed to be taught. She felt that in her class this topic required differential learning in homogeneous groups and she preferred to keep the class studying in a heterogeneous setting for the time being. Therefore, she chose at this time to teach a topic more suitable for heterogeneous groups and postponed the above-mentioned topic to a later date.

Sometimes the change in the order of the topics is made by postponing a certain topic in order to allow the class to study in homogeneous settings. For example, in another school, Mr. Rosilio had planned to introduce the topic "collecting like terms" in a heterogeneous setting. He decided, however, to postpone this and first to provide orientation for a certain group of weaker students to this topic so that they would benefit more from the activities presented later in the heterogeneous whole-class setting. Thus, at this point, Mr. Rosilio organized the class into homogeneous groups: He worked with this weaker group on orientation activities. Meanwhile, the other students were involved in differential topics: some groups studied a new, advanced topic independently while others were involved in challenging activities based on the previous topic. Thus, this core material, "collecting like terms," at this time was in favor of the differential topic.

Alternating core and differential topics in this way makes it possible to provide enrichment and orientation and to close knowledge gaps in response to the students' needs. The proportion between core materials and differential topics is about 70% to 30%. Core materials are studied mainly in heterogeneous settings and differential topics in homogeneous settings.

Whatever choices are made, one element stays unchanged: the teaching plan is designed to allow the establishment of a classroom culture where heterogeneity is the norm. This implies that it is necessary to use didactics and activities that allow all students to learn in a heterogeneous setting. Learning continues in this setting for as long as the teacher sees that all students are benefiting. As soon as TAP teachers see that some students are starting to be left behind they consider the possibility of differential learning for a while in homogeneous groups. This is done in order to address the difficulties so that heterogeneous groups are able to be successfully and meaningfully implemented again.

Achieving TAP's Goals

In previous research (Linchevski 1995; Linchevski and Kutscher 1998; 2002a, 2002b) we reported on three studies that investigated the effects of teaching mathematics in mixed-ability settings both on students' achievements and on teachers' attitudes, within the framework of the TAP project. The reported results clearly showed that quality and equity were achieved in the TAP classrooms: all levels of students were able to grow mathematically as attested to by their achievements—the most mathematically competent students' performance was as high as would have been had they been tracked, and all the less competent students' performance was better than would have been, had they been tracked.

In the first two studies, we examined the effect of teaching in mixed-ability mathematics settings on students' achievements. In Study 1, we investigated whether the increasing gap in mathematics performance found in grouping students by ability can be eliminated in mixed-ability classes. In Study 2, we compared the effects of mixed-ability and same-ability grouping on mathematics performance of students classified as high ability, intermediate ability, and low ability. In Study 3, we examined the effects of teaching in mixed-ability classes on teachers' attitudes. The methodologies used in the studies were chosen in accordance with the research question and study (Cahan and Linchevski 1996; Cook and Campbell 1979; Linchevski and Kutscher 1998).

In Study 4, we examined the progress of a traditional teacher whose lessons were teacher-centered in a tracked environment where whole class teaching was the norm. This teacher taught in a typical school in a disadvantaged area in South Africa. Initially, he had no need nor wish to participate in a project propagating learning in mixed-ability classes, this despite low achievement for the majority of the students.

This study used a case study design to follow this teacher for two and a half years (Linchevski et al. 1999, 2000). Results were contradictory: On the one hand, this teacher's practice changed dramatically. He applied TAP's principles and his classes showed similar achievements to those reported above. For example, in an interview the teacher reported: "…looking at the results of last year versus the results that they obtained thus far…out of a class of 48 only five people have not improved on their mark of last year." On the other hand, his beliefs lagged behind—it was clear to him that mixed-ability group-work benefited the "strong" learners: "I find that when they [the "strong"] communicate in the [mixed-ability] group they also learn some other skill—of speaking mathematics, which is of great help for them". And at the same time "I need proof that the strong learners would benefit from working in mixed-ability groups." If previously he was concerned that he "might be neglecting the strong pupils" when they learned independently in the homogeneous groups, he now believed that "within the group there is over enough intelligence to actually run through the activities." But he still had "a difficulty of the letting of one group go ahead." In this case, we see a teacher, notwithstanding his successful record of implementation and evidence of success, still clinging to some of his old beliefs. This teacher's behavior seems to be following a process where practice begins to affect belief (Guskey 1986) which, in turn, affects practice. The process of effect of practice on his beliefs appears to be a cyclical process (Rogers 2007), where changes in his belief are a slow, gradual process that are "incremental rather than monumental" (Ambrose 2004, p. 91).

The main motivation for developing the TAP project was the difficulty we had in accepting the current situation where widespread practice implemented in mathematics classes promotes inequity. Research shows that tracking is one of the main perpetrators of inequity, whereas learning in heterogeneous classes shows promise in cultivating quality and equity in mathematics education. What do we mean by equity in mathematics education? Equity, for us, means that *all* students have the right to be afforded a high-quality mathematics education, curriculum-wise, in an environment that cultivates rich mathematical discussions to the satisfaction of all. This necessarily means learning in a heterogeneous classroom.

But we were faced with a dilemma: Could we disregard research that shows there are topics in mathematics with which low-achievers struggle, and that at times they need different didactical methods and more time in order to successfully cope with these topics? Is it right, even ethical, to create a curriculum that sometimes caters to less mathematically competent students at the expense of more mathematically competent students, or the converse? If our answer to these questions is "no," then we need to implement tracking.

Hence, it would seem that to promote real equity, we would need to create a learning environment where heterogeneity and tracking co-exist. Using principles derived from theoretical approaches and research, we designed the TAP teaching model to include teaching materials and class organization that respond to the different needs of the population while keeping the class as one learning unit. This is done by alternating between two types of groups, heterogeneous groups and homogeneous groups, thus enabling the co-existence of heterogeneity and tracking.

There are other models that have implemented mathematics learning in heterogeneous classes. Some of these approaches also offer extra support for the weaker students in homogeneous settings. This support, however, is given out of the heterogeneous classroom at set times.

As opposed to these models, TAP provides a unique, dynamic solution for responding to all students' differential needs within the classroom setting. The variety of settings and the dynamicity of combinations of these settings, at precisely the right time and for the necessary duration, allow us to provide a genuine response not only to the weaker students' needs but also to the stronger students', so that they can all develop mathematically at a pace they feel comfortable with, be offered opportunities to engage with quality mathematics, and thus be fruitful members of a rich, diverse, challenging mathematical community in which quality and equity is achieved.

References

Alexander, K. L., Cook, M., & McDill, E. L. (1978). Curriculum tracking and educational stratification. *American Sociological Review, 41*(1), 963–980.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7,* 91–119.

Ansalone, G. (2005). Getting our schools on track: Is detracking really the answer? *Radical Pedagogy, 6*(2). http://radicalpedagogy.icaap.org/content/issue6_2/ansalone.html. Accessed 27 Feb 2010.

Ball, S. (1984). Beachside reconsidered: Reflections on a methodological apprenticeship. In R. Burgess (Ed.), *The research process in educational settings: Ten case studies.* London: Falmer Press.

Boaler, J. (1997a). *Experiencing school mathematics: Teaching styles, sex and setting.* Buckingham: Open University Press.

Boaler, J. (1997b). Setting, social class and survival of the quickest. *British Educational Research Journal, 23*(5), 575–595.

Boaler, J. (1997c). When even the winners are losers: Evaluating the experiences of "top set" students. *Journal of Curriculum Studies, 29*(2), 165–182.

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning.* Mahwah: Lawrence Erlbaum.

Boaler, J. (2008). Promoting "relational equity" and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*(2), 167–194.

Bransford, J. D., Zech, L., Schwartz, D., Barron, B., Vye, N. J., & The Cognition and Technology Group at Vanderbilt. (2000). Designs for environments that invite and sustain mathematical thinking. In P. Cobb (Ed.), *Symbolizing, communicating, and mathematizing: Perspectives on tools and instructional design* (pp. 275–324). Hillsdale: Lawrence Erlbaum.

Brewer, D. J., Rees, D. L, & Argys, L. M. (1995). Detracking America's schools: The reform without cost? *Phi Delta Kappan, 77,* 210–215.

Burris, C. C., Heubert, J. P., & Levin, H. M. (2006). Accelerating mathematics achievement using heterogeneous grouping. *American Educational Research Journal, 43*(1), 105–136.

Cahan, S., & Linchevski, L. (1996). The cumulative effect of ability grouping on mathematical achievement: A longitudinal perspective. *Studies in Educational Evaluation, 22*(1), 29–40.

Carver, S. C., & Scheier, M. F. (1982). Control theory: A useful conceptual framework for personality, social, clinical, and health psychology. *Psychological Bulletin, 92,* 11–135.

Cobb, P. (1994). A summary of four case studies of mathematical learning and small-group interaction. In J. P. de Ponte & F. Matos (Eds.), *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education* (Vol. 2, pp. 201–208). University of Lisbon, Portugal.

Cook, T. D., & Campbell, D. T. (1979). *Quasi-experimental design analysis issues for field setting.* Boston: Rand McNally.

Crain, R. L., & Mahard, R. E. (1983). The effect of research methodology on desegregation—achievement studies: A meta-analysis. *American Journal of Sociology, 88*(5), 839–854.

Crain, R. L., Mahard, R. E., & Narot, R. E. (1982). *Making desegregation work: How schools create social climates.* Cambridge: Ballinger.

Davidson, N., & Kroll, L. D. (1991). An overview of research on cooperative learning related to mathematics. *Journal for Research in Mathematics Education, 22*(5), 362–365.

Epstein, J. L., & MacIver, D. J. (1992). *Opportunities to learn: Effects on eighth graders of curriculum offerings and instructional approaches.* (Report No. 34). Baltimore: Center for Research on Elementary and Middle Schools, Johns Hopkins University.

Feldhusen, J. F., & Moon, S. M. (1992). Grouping gifted students: Issues and concerns. *Gifted Child Quarterly, 36*(2), 63–67.

Figlio, D. N., & Page, M. E. (2002). School choice and the distributional effects of ability tracking: Does separation increase inequality? *Journal of Urban Economics, 51,* 497–514.

Gamoran, A. (1993). Alternative uses of ability grouping in secondary schools: Can we bring high-quality instruction to low-ability classes? *American Journal of Education, 102,* 2–22.

Gamoran, A., & Berends, M. (1987). The effects of stratification in secondary schools: Synthesis of survey and ethnographic research. *Review of Educational Research, 57,* 415–435.

Gamoran, A., & Mare, R. (1989). Secondary school tracking and educational inequality: Comparison, reinforcement, or neutrality? *American Journal of Sociology, 94,* 1146–1183.

Gamoran, A., & Weinstein, M. (1998). Differentiation and opportunity in restructured schools. *American Journal of Education, 106,* 385–415.

Goldring, E. B., & Eddi, A. (1989). Using meta-analysis to study policy issues: The ethnic composition of the classroom and academic achievement in Israel. *Studies in Educational Evaluation, 15*(2), 231–246.

Guskey, T. R. (1986). Staff development and the process of teacher change. *Educational Researcher, 15*(5), 5–12.

Hallinan, M. T. (1994). Tracking from theory to practice. *Sociology of Education, 67,* 79–91.

Heubert, J. P., & Hauser, R. M. (Eds.) (1999). *High stakes: Testing for tracking, promotion, and graduation.* Washington: National Research Council.

Huang, M. (2009). Classroom homogeneity and the distribution of student math performance: A country-level fixed-effects analysis. *Social Science Research*, *38*(4), 781–791.

Kerckhoff, A. C. (1986). Effects of ability grouping in British secondary schools. *American Sociological Review, 51,* 842–858.

Kulik, J. A. (1992). *An analysis of the research on ability grouping: Historical and contemporary perspectives* (Research-based Decision Making Series No. 9204). Storrs: University of Connecticut, The National Research Center on the Gifted and Talented.

Lamb, S., & Fullarton, S. (2002). Classroom and school factors affecting mathematics achievement: A comparative study of Australia and the United States using TIMSS. *Australian Journal of Education, 46*(2), 154–171.

Linchevski, L. (1995). Tell me who your classmates are and I'll tell you what you learn: Conflict principles underlying the structuring of the math class. In L. Meira & D. Carraher (Eds.), *Proceedings of the Nineteenth International Conference for the Psychology of Mathematics Education* (Vol. 3, pp. 240–247). Recife, Brazil.

Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus ability-grouping in mathematics. *Journal for Research in Mathematics Education, 29*(5), 533–554.

Linchevski, L., & Kutscher, B. (2002a). Mixed-ability versus same-ability grouping in mathematics. In J. Sowder & B. Schappelle (Eds.), *Lessons learned from research* (pp. 63–70). Reston: National Council of Teachers of Mathematics.

Linchevski, L., & Kutscher, B. (2002b). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability in mathematics. In J. Sowder & B. Schappelle (Eds.), *Lessons learned from research* (pp. 47–61). Reston: National Council of Teachers of Mathematics.

Linchevski, L., Kutscher, B., & Olivier, A. (1999). Assessment in support of equity. In *Proceedings of the Fifth Annual Congress of the Association for Mathematics Education of South Africa* (Vol. 1, pp. 155–167). Port Elizabeth: Association for Mathematics Education of South Africa.
Linchevski, L., Kutscher, B., Olivier, A., & Bennie K. (2000). Diversity acknowledged and ignored: Achieving equity in school mathematics. In J. F. Matos & M. Santos (Eds.), *Proceedings of the Second Mathematics Education and Society Conference* (pp. 199–213). Lisbon: University of Lisbon.
Loveless, T. (1998). The tracking and ability grouping debate. *Fordham Report, 2*(8), 1–27.
Lucas, S. R. (1999). *Tracking inequality: Stratification and mobility in American high schools.* New York: Teachers College Press.
Mills, C. J., & Durden. W. J. (1992). Cooperative learning and ability grouping: An issue of choice. *Gifted Child Quarterly, 36*(1), 11–16.
Mosteller, F., Light, R. J., & Sachs, J. A. (1996). Sustained inquiry in education: Lessons from skill grouping and class size. *Harvard Educational Review, 66,* 797–843.
Oakes, J. (1982). The reproduction of inequity: The content of secondary school tracking. *The Urban Review, 14*(2), 107–120.
Rogers, P. (2007). Teacher professional learning in mathematics: An example of a change process. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (pp. 631–640). Tasmania: MERGA.
Ruthven, H. (1987). Ability stereotyping in mathematics. *Educational Studies in Mathematics, 18,* 243–253.
Silver, E. A., Smith, B. S., & Nelson, B. S. (1995). The QUASAR project: Equity concerns meet mathematics education reform in the middle school. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 9–57). Cambridge: Cambridge University Press.
Slavin, R. E. (1990). Achievement effects of ability grouping in secondary schools: A best-evidence synthesis. *Review of Educational Research*, *60*(3), 471–499.
Slavin, R. E., & Braddock, J. H. III. (1993). Ability grouping: On the wrong track. *College Board Review, 168,* 10–19.
Sorensen, A. B., & Hallinan, M. T. (1986). Effects of ability grouping on growth in academic achievement. *American Educational Research Journal, 23(4*), 519–542.
Voigt, J. (1994). Negotiation of mathematical meaning and learning. In P. Cobb (Ed.), *Learning mathematics: Constructivist and interactionist theories of mathematical development* (pp. 271–298). Boston: Kluwer.
White, P., Gamoran, A., Porter, A. C., & Smithson, J. (1996). Upgrading the high school mathematics curriculum: Mathematics course-taking patterns in seven high schools in California and New York. *Educational Evaluation and Policy Analysis, 18,* 285–307.
Willie, C. V. (1990). Diversity, school improvement and choice: Research agenda items for the 1990s. *Education and Urban Society, 23*(1), 73–79.
Wood, T., & Yackel, E. (1990). The development of collaborative dialogue within small group interaction. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 244–253). Hillsdale: Lawrence Erlbaum.
Wortman, P. M., & Bryant, F. B. (1985). School desegregation and black achievement. *Sociological Methods & Research, 13,* 289–324.

Chapter 37
Research from Practice: Using Technology to Provide Advanced Mathematics and Equity to High School Students in the United States

Benjamin Hedrick and Douglas Edwards

When discussing equity in education, there is often a focus on providing students with equal access so that the average student in one community has the same resources and affordances as the average student in another community. Rarely does the discussion extend to the needs of the high-achieving student. In the United States, the National Council for Teachers of Mathematics (NCTM) writes that equity, "demands that reasonable and appropriate accommodations be made and appropriately challenging content be included to promote access and attainment for *all* students" (NCTM 2000). The emphasis on all students, italicized in the original document, highlights the belief that equity is not simply an issue for students who are performing below standards, but rather an issue for all students at all levels of performance. As such, an important part of achieving equity concerns students who have the potential to succeed in higher level mathematics classes (whom we will term "high-achieving students") but lack access to these classes or access to a highly qualified teacher.

The aim of this chapter is to document a two-year project designed to address the equity disparity faced by a group of high-achieving, underrepresented high school students in the southern United States. Utilizing distance learning technology, two high school teachers at two different schools were able to collaborate and bring Advanced Placement (AP) Calculus BC to high-achieving students who previously had no access to such a course. The authors of this chapter—the two teachers who designed and implemented the program—present this case study to the mathematics education community as an example of how practitioners in the field are working toward achieving equity at the level of advanced mathematics. Though some background information is necessary for a complete understanding of the context and implementation of this program, the bulk of the chapter will be devoted to creation of quality mathematics instruction, outcomes, and a critique of the benefits and limitations of the collaboration.

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The Advanced Placement (AP) Program and Equity Issues

For high-achieving high school students in the United States, access to advanced mathematics content is often synonymous with courses such as those provided through the College Board's AP program. Originally, piloted in 1955 in 27 schools, the AP program was designed to allow motivated students to maximize their learning potential and to avoid content repetition between high school and university courses (The College Board 2009). As of 2007, nearly 1.5 million students at 17,000 schools were involved in the AP program in courses ranging from calculus to art (The College Board 2009). Currently, there are two AP calculus courses, designated AP Calculus AB and AP Calculus BC, which are year-long high school courses. The AP Calculus AB curriculum includes single-variable differentiation and integration and is roughly equivalent to a half-year university course in single-variable calculus (Gollub et al. 2003). The AP Calculus BC curriculum consists of all topics in AP Calculus AB plus additional topics such as parametric integration and differentiation and Taylor Series, and it is roughly equivalent to a one-year university course or two half-year university courses in single-variable calculus (Gollub et al. 2003).

AP examinations are administered by the College Board yearly during a two-week period in early May, which is close to the end of the academic year in the United States. Annually over 200,000 high school students take the AP Calculus AB examination, and over 60,000 take the AP Calculus BC examination (The College Board 2007, 2008). Examination scores range from 1 to 5, where scores of 1 and 2 are considered as failing and scores of 3 to 5 are considered as passing. Students who take the AP Calculus BC examination are given an AB subscore, also graded from 1 to 5, reflecting the elements of the examination that are also AB Calculus topics. In this way, students who choose to take the more challenging BC course are not penalized, and though students may fail the BC examination, they might still receive a passing score consistent with taking the AB examination.

Understanding the AP system and the scoring of examinations is relevant to this chapter for several reasons. First and most important is understanding the level and rigor of AP Calculus BC, which is the course taught in this case study. AP Calculus BC roughly follows the curriculum and pacing of a freshman level collegiate course. For a school to offer any AP course, the teacher must have completed special training, and the course must be certified as AP-worthy through a syllabus audit conducted by the College Board. It is also important to note the expansive and formalized nature of the AP system. High schools across the United States are often judged by how many AP courses they offer and what percentage of students passes the examinations. In addition, colleges and universities increasingly take into consideration the number of AP courses and examinations taken, as well as the scores on these examinations, for admission decisions (Gollub et al. 2003). Students who have not taken these courses are at a severe disadvantage when compared with peers of equal academic quality who have taken AP courses. In some ways one might liken the AP program to the International Baccalaureate program, though the AP program is US-specific.

In addition, relevant is the nature of racial diversity in enrollment in AP mathematics courses, which remains a matter of concern despite a marked increase in participation by underrepresented groups in such courses. According to data compiled internally by the College Board, of the 62,614 students across the United States who took the AP Calculus BC examination in 2007, only 1,404 (or approximately 2.2%) self-identified as African-American (The College Board 2007). Adjusting for unequal population sizes and using enrollment data from the State Department of Education, during the 2006–2007 school year in Georgia (the state in which the following case study took place), for every African-American student taking the AP Calculus BC examination, 13 Caucasian students took the examination (Georgia Department of Education 2007). In addition, the national pass rate for African-American students was 54.99% with an average score of 2.71, which is below the passing mark of 3. The overall national pass rate for the examination was 79.84% with a 3.70 average score, and the pass rate for Caucasian students was 81.11% with a 3.74 average score. The difference in scores and pass rates for African-American students from the national average is significant at the 0.01 level.

The AP Problem at Jefferson High School

Jefferson High School (pseudonym) is a public high school of approximately 2,100 students located in a metropolitan center in the southern United States. The student population is between 98% and 99% African-American, and 42% of the students receive free or reduced-cost lunch (a measure often used in the United States to assess the socio-economic status of the student population). Jefferson High School is also home to one of the district's two math/science magnet schools. Students from across the district who are particularly interested in science or mathematics can apply and take a special placement examination, and successful applicants are grouped together in a "school within a school" for classes that are considered to be superior in quality to the ones offered at their home schools. In order to attend the magnet program, many of these students commute over an hour each way every class day. In the 2006–2007 school year, which was the first year of this two-year program, over 200 of the students at Jefferson High School were enrolled in the magnet program.

Despite the fact that Jefferson is a mathematics magnet school, AP Calculus BC had not been offered to students in seven years and, in the years when it was offered, no students had ever passed the examination. In the year before this case study takes place, 26 students were enrolled in AP Calculus AB. When these 26 students took the AP Calculus AB examination in the spring of 2006, only six (23%) passed. The average student score was 1.77.

Jefferson High School was not able to offer AP Calculus BC for several reasons. First, no teachers at Jefferson had the College Board credentials necessary to teach the course—only 70% of the mathematics faculty had a high school mathematics teaching license. Even had a qualified and certified teacher been available, school policy mandated that at least 11 students enroll in a class for it to be offered, and the number of interested students was consistently too low. In addition, students felt that their previous mathematics classes did not adequately prepare them for such a rigorous course.

A Solution to Jefferson High School's AP Problem

Doug Edwards, the coordinator of Jefferson's math/science magnet program, was a former engineer who left his lucrative job in order to help traditionally underrepresented students receive the education necessary to pursue careers in science and technology. Part of his strategy for success was to form a partnership with a local university; and students from the university would often visit Jefferson as tutors and as role models for the high school students. Through this partnership Edwards learned that the university was to launch a special mathematics course in conjunction with several high schools across the district. Many students in the district were taking AP Calculus AB or BC during their penultimate year in high school, and there were no mathematics classes available for their final year. Using cameras, televisions, and microphones, the university planned to broadcast one section of the appropriate follow-up class to these high schools. The university and the school district shared the costs, and Jefferson High School, as a math/science magnet school, was selected as one of the locations. Ironically, no students at Jefferson were eligible to take the university class, so the equipment was installed in a storage room and covered with boxes.

Perhaps the most interesting feature of this particular program was that it was interactive. With cameras, televisions, and microphones installed both at the university and at the high schools, students could ask the college professor questions in real time, and the professor could also answer the questions immediately. Distance was not a factor.

Though the equipment was designed to be used with the university as the hub, Edwards reasoned that it would be possible to effect the same sort of program between high schools as well. Just as the university was providing high-level mathematics content to schools that lacked it, it would also be possible for one high school to provide AP Calculus BC to another school that lacked it. Edwards contacted the director of mathematics education for the district with his idea, which was quickly accepted. All that remained was to find a school that was willing to share its AP Calculus BC teacher in a collaborative partnership with Jefferson.

Forming a Connection with Washington High School

Washington High School (pseudonym) is a public high school of approximately 2,000 students located in the same school district as Jefferson High School. In 2007, the student population at Washington was 61% Caucasian, 16% African-American, and 13% Asian. Only 3% of the students receive free or reduced-cost lunch. Though Washington is not a math/science magnet school, both AP Calculus AB and BC courses are consistently available and are taken predominantly by students in their penultimate year of high school. All such students who took AP Calculus BC enrolled in the university-sponsored distance learning mathematics class the following year. In the year before the case study takes place, 100% of both the AP Calculus AB and BC students passed the AP examinations. The AP Calculus BC course was taught by Ben Hedrick, and the average student score for the 18 students who took the examination that year was 4.67. Hedrick was suggested as a possible teacher by the Washington High School administration, and after a brief meeting with the Jefferson High School students, parents, and teachers, Hedrick agreed to design and teach the distance learning class, which would consist of 20 Washington students and three Jefferson students.

Since both teachers lacked familiarity with the distance learning equipment, the AP Calculus BC class was structured similarly to the university class taught using the same equipment. Through this system, Hedrick, while in his own classroom with the Washington students in attendance, would speak into the microphone while writing notes on an opaque projector. These notes were displayed on television monitors at both Washington and Jefferson high schools. With an in-class teacher and notes resembling those written on a blackboard, the Washington High School students thus had more or less a typical classroom experience. Students at Jefferson High School, however, saw the teacher through a second television monitor, and explanations and directions were carried through the distance learning equipment and transmitted through speakers. Similarly, a second television monitor at Washington High School enabled Hedrick to see the Jefferson students and interact with them in real-time. The high school students at both schools were also provided with touch-microphones that, when depressed, would allow student voices to be transmitted to the other school. Using these microphones the Jefferson students could interrupt the teacher to ask a question, just as they would in a typical classroom if they so chose.

Creating Quality and Equity in the Classroom: One Class in Two Locations

Parents, students, teachers, and even administrators initially had concerns regarding how this distance learning program would affect students. Washington parents and students were perhaps the most concerned; they had an established and effective course, and it was unclear at first how this collaboration would benefit them. The Washington parents also worried that sharing the class with the Jefferson students would divert time and attention away from their own children. Jefferson students and parents faced their own valid concerns. Washington students clearly had the advantage of an in-class teacher who could be approached before school, during lunch, and after school for help, whereas Jefferson students lacked this access. Furthermore, there was a concern that the physical distance would result in emotional distance, placing the Jefferson students at a lower value level.

Both teachers realized that the success of the program depended on student collaboration. Emphasized from the first day of class was the idea that this program involved only one class, though in two locations. Washington and Jefferson students had equal right to ask and answer questions, and Jefferson students had equal right to ask for help during class or after school. To make this possible, special after-school help sessions were scheduled, and Hedrick would reconnect the equipment in order to answer questions and work through problems with Jefferson students. Washington students would also attend these sessions, and after some practice, students at both schools were able to work together using the distance learning equipment to solve challenging problems.

Physical presence was still a barrier, and Hedrick requested and was granted administrative permission to visit Jefferson once per semester. From there Hedrick was able to teach in exactly the same manner as at Washington, the difference being that Washington students were now viewing class through a television monitor. This simple change had profound effects. The most pronounced was that Washington students were able to understand what class was like daily for the students at Jefferson, which had the unexpected result of increasing respect for and empathy with the Jefferson students. Both groups were highly talented and motivated, and seeing how hard the Jefferson students had to work to gain access to this class made the Washington students realize that they were working with peers. This experience humanized the program and helped the students see each other as fellow learners in the same class. The in-person visits to Jefferson also allowed Hedrick to offer additional assistance to the Jefferson students, who through administrative permission were allowed to miss other classes in order to take full advantage of the visits. These visits also reaffirmed that the Jefferson students were valued and important members of the class. In addition, the Jefferson students were able to visit Washington twice during the school year, putting the entire class together in one room, though it was impossible to arrange for the Washington students to visit Jefferson.

Maintaining quality of the course over the distance proved to be another challenge, though Edwards and Hedrick took steps to make sure that standards remained high. Frequent formative assessments were necessary to insure that Jefferson students in particular were learning and understanding the rapidly presented materials. Brief 5–10-minute quizzes were created for each and every section of material covered, which resulted in quizzes being administered on average more than twice per week. Quizzes had to be created several days in advance and sent as email attachments to Edwards, who would have them printed and ready for distribution through the help of an administrative assistant. Grading quizzes was a more challenging issue. Once taken, the administrative assistant would fax the quizzes to Washington, where they would be graded and faxed back, often the same day. In this manner students at both schools had a strong and immediate sense for what they did and did not understand. The process for giving and grading tests was identical.

Results of the Collaboration

In May of 2007, the first group of distance learning students took the AP Calculus BC examination. The Jefferson High School students traveled to the Washington High School area the night before the examination and stayed with Washington

Table 37.1 Testing results for 2007 and 2008 for Washington and Jefferson students

	Testing results for Washington and Jefferson High School students for the 2007 and 2008 Advanced Placement (AP) Calculus BC examinations								
	2007				2008				
	5	4	3	Avg	5	4	3	2	Avg
Washington	18	2	0	4.90	12	1	0	0	4.92
Jefferson	1	1	1	4.00	1	1	0	1	3.67

students and their families. The following day, all 23 took the AP examination together as one unified class. The 20 Washington students passed the AP Calculus BC examination with an average score of 4.90, and the three Jefferson students passed the examination with an average score of 4.00. As a result, both schools heralded the program as a success. The scores for Washington were higher than the previous year (average score of 4.67), immediately negating the concern amongst Washington parents that the program might have lowered the instructional quality for their children. Jefferson had its first students pass the examination in school history, and as a result, two students were able to participate in the university-sponsored distance learning mathematics course the following year. The third Jefferson student graduated and later attended the same university.

In the second year of the collaboration, 13 Washington students took and passed the AP Calculus BC examination with an average score of 4.92. Two of the three Jefferson students passed the examination with an average score for all three students of 3.67. The one student who failed the examination, however, had an AB subscore of 3, meaning that the student still passed an AP Calculus examination.

These results are summarized in Table 37.1.

All six of the Jefferson students from both years of the distance learning collaboration subsequently chose to pursue a college education following graduation. Two accepted placements at Harvard University and Cornell University, and one turned down an acceptance at Harvard to attend a historically Black college. The remaining three all accepted placements at the university that founded the distance learning program, and they received substantial scholarships. The Washington students also all chose to pursue a college education and are currently in colleges and universities across the United States.

Analysis of the Collaboration: Benefits and Affordances

The primary intent of the distance learning collaboration was to provide access to quality mathematics (specifically AP Calculus BC) to students at both Jefferson and Washington High Schools. Based on the quantitative outcomes, the program was judged to be highly successful. Jefferson students passed the rigorous AP Calculus BC examination at a high rate, and their AP examination scores were strong enough to give them university credit. In addition, students who were not in their final year of high school were subsequently able to enroll in the university-sponsored distance mathematics course that originally inspired the collaboration. Washington students showed a continuous improvement in scores, and the pass rate remained consistent at 100%.

Both students and teachers cited high expectations as one of the primary factors in the success of this program. When interviewed at the end of the school year, the Jefferson students agreed that this distance learning class was the most challenging class they had ever taken at high school due to the intellectual demands placed upon them both by the nature of the material and the expectations of the teachers. The Jefferson students also agreed that the challenge of the program made them feel special and that the constant support and encouragement provided by the teachers and their classmates at both locations enhanced the motivation necessary for success.

It is important to note that these student feelings and concerns were shared by the students at Washington as well. In the end-of-year informal student evaluation of the course and distance learning collaboration, most Washington students also cited AP Calculus BC as the most challenging course they had ever taken at the high school level, and many later took college courses, both mathematical and non-mathematical, that they felt were easier.[1] Expectations for students on both sides of the collaboration were high, and Washington and Jefferson students were treated and graded in precisely the same manner.

Within the classrooms, collaboration took place with strong participation from students at both schools. Though in general students collaborated with their physically present partners, discussion and problem solving utilizing input from students on both sides was not uncommon. For example, students were often invited to come to the front of the room and use the opaque projector to solve problems or show solutions. Students who did so also appeared on a television monitor at the other school, which put the focus on distance peers and their mathematical thoughts and work. Just as in a traditional classroom with a student at the blackboard, other students could chime in and offer thoughts and advice and thus work together to solve problems, even from two distant locations. The teachers actively encouraged such collaboration, emphasizing that both groups should be working together. Jefferson students cited this sense of equality as a major factor in their motivation to succeed.

Students at Washington were also aware of the challenges of the course and were encouraged to form study groups in order to master challenging concepts. This practice, which was in place in the AP Calculus BC class at Washington before the distance learning collaboration began, led to high levels of engagement with the course material and with fellow students. This practice may have been at least in part responsible for the collaboration formed between students not only at their own school but between schools as well. Within the first month, students at both schools had formed localized study groups to do homework and to prepare for tests. One of the Jefferson students, realizing the value of such groups and the potential to learn from the distance classmates as well, requested the email addresses of the

[1] This information comes from many students who remain in contact with the teacher after graduation from Washington.

Washington students to form an online study group where questions could be posted and ideas shared. Only two of the 20 Washington students chose not to participate. Soon ideas were flowing from both sides, and though the teachers had access to the online forum, solutions and ideas were posted by students only.

Dialogue between the students was not limited to calculus work, though such topics constituted the majority of discussion. Soon students were also sharing typical non-school-related interests. In this manner, even though the students predominantly saw each other only through television monitors, friendships formed through both mathematical and non-mathematical shared interests. Several Washington students in particular were impressed with the Jefferson students' work ethic and willingness to challenge themselves, which in turn inspired them (the Washington students) to work harder and to actively support their distance classmates. Though the Jefferson students were initially less participatory, by the second month students on both sides engaged freely in discussion.

The social and academic effects of the program continue even after the class has ended. The Jefferson students remain friends with the Washington students, and many stay in contact even after going away to college. For example, a Jefferson student and a Washington student chose to attend the same university and to be lab partners in a freshmen chemistry class. Similarly, two students continue as friends at Harvard University.

Analysis of the Collaboration: Problems and Concerns

Although there were many positive outcomes as a result of this collaboration, many unresolved problems and equity issues still existed by the end of the second year of the program. These problems ranged from basic teaching and logistical issues to broader and deeper questions regarding the nature of mathematical equity.

Perhaps one of the most glaring issues was ironically that of access. Although this program provided access to some students who otherwise would not have had such an opportunity, many talented students were (and are) still denied access due to issues resulting from prior mathematics courses. Students who participated in the distance learning collaboration cited lack of mathematical preparedness as a major concern. These fears proved to be well founded, and remediation was necessary to provide precursor knowledge in order to access basic calculus concepts. During the second year of the program, interest from Jefferson students led to an initial enrollment of six students; three students, however, were eventually forced to withdraw from the class due to knowledge gaps. Lack of access to quality mathematics earlier in their mathematical careers prevented students from taking advantage of opportunities even when they were offered. As a result, the effectiveness of the distance learning collaboration was limited to a handful of highly talented students who were able to seek outside help or devote time to learning on their own.

The nature of the program itself did nothing to improve mathematics education at earlier levels—the goals were course-specific. Ideally, the distance learning collaboration would have been only a temporary fix as Jefferson strove to create its own strong and self-sustaining program. After two years, however, no such changes had been made despite the efforts of Edwards to do so. Though the distance learning collaboration addressed the needs of a small group of students, it did not address the larger needs of the school or the student body as a whole. In addition, while the distance learning collaboration did address the needs of the students in the short-term, a long-term strategy is necessary to either make such a program sustainable or to bring the school to a level such that higher level mathematics classes can be offered internally. In order to enact sustained systemic change, remediation or improved quality at earlier access points would have been essential.

Outside of purely mathematical issues, Hedrick also raised concerns for the implications of a predominantly Caucasian school with a Caucasian teacher establishing a program to aid a predominantly African-American school, though in this case no direct problems arose. Still, in future collaborations it would be wise for those involved to consider the power implications of such an arrangement (Delpit 1995).

Though technology made such a program possible, technology also hindered the program for a variety of reasons. For example, several times during the course of the year, the district would experience internet blackouts, which prevented the broadcasting of the class to Jefferson students. Washington students, however, were still able to participate in the class as usual and lost no learning time. Perhaps the most prevalent problem, however, was scheduling. High school schedules often changed for special events or for testing, and a change in schedule at either school made simultaneous classes difficult and sometimes impossible. Since the class was taught from Washington, Jefferson students would lose out when such rescheduling could not accommodate their schedule.

Though all of these challenges and problems are of importance, the most decisive obstacle to the effectiveness of this program was sustainability. The program was created through the personal initiative of Edwards and Hedrick, with basic administrative support from both schools. Both teachers volunteered hours of their time every week to make the program work, and trips between schools were funded by the teachers themselves. No financial compensation was received by either teacher during the two years of the program. When Hedrick left Washington High School at the end of 2008 to pursue graduate studies, the program ended. No other Washington teacher was willing to volunteer the time and effort necessary to continue to program, and no teachers at other district schools were interested. Administrative changes at both schools at this time also made the possibility of continuation doubtful, and Edwards left Jefferson High School in October of 2009 in search of the chance to enact his visionary ideas.

Conclusion

Equity and achievement gaps in mathematics education such as those between Washington and Jefferson High Schools are well documented, though there is still debate as to whether more analysis of this gap is necessary and what role it plays in current research. Some researchers, such as Sarah Lubienski, feel that analyses of such disparities are an essential part of efforts to promote equity (2008). Rochelle Gutiérrez, on the other hand, writes, "I suggest a research agenda that focuses on advancement, on excellence, and on gains within marginalized communities. By *excellence*, I mean high performance on standardized tests and broader notions of mathematical literacy" (Gutiérrez 2008, p. 362). Researchers have demonstrated that minority students are less likely to participate in advanced mathematics courses even when they are offered by their schools (Atanda 1999; Horn et al. 2000; Ma and Willms 1999). AP testing data also confirms what researchers have consistently found: minority students who are enrolled in higher level mathematics courses are less likely to succeed on national examinations than their White or Asian counterparts (Gollub et al. 2003). Participation and success in advanced mathematics courses are also predicated on three conditions: (1) the school offers such classes; (2) the school provides access to these courses; and (3) students are prepared for such courses (Gollub et al. 2003, p. 48).

The distance learning collaboration between Washington and Jefferson High Schools is one example of a program that directly and successfully addressed equity issues in American mathematics classes and focuses on Gutiérrez's agenda of excellence. Given that the collaboration was implemented with the primary purpose of granting access to students rather than producing informed research, rigorous examination of this program from a research perspective is challenging. The previous framework and discussion of benefits and problems have been provided so that researchers can study the potential of such a technological collaboration in more depth and produce effective and meaningful research into its effects on mathematical equity. Given the success of the project during its first two years in providing challenging content and access, as recommended by the National Council for Teachers of Mathematics (NCTM 2000), the authors believe that additional study by trained researchers is warranted. Although only one specific group of those in need of equity—high-achieving students—was targeted by this project, much remains to be learned regarding how such a program could potentially affect a broader student audience.

This chapter is the result of teacher research. Having implemented and completed two years of this effective distance learning collaboration, the teachers have stepped back to conduct intentional, systemic inquiry regarding not only what conditions and affordances of the program were either successful or problematic, but also to determine which are necessary, replicable, or fixable (Cochran-Smith and Lytle 1993). In such a manner we hope to "evoke images of the possible…not only documenting that it can be done, but also laying out at least one detailed example of how it was organized, developed, and pursued" (Shulman 1983, p. 495). With the rapid advances in technology, the ability to create such a collaboration becomes continually easier and less expensive. Access to high-level mathematics and to highly qualified teachers through technology directly addresses one aspect of the equity disparity in the United States and also potentially leads to similar benefits to teachers and students across the globe.

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References

Atanda, R. (1999). *Do gatekeeper courses expand education options?* Washington: U.S. (Department of Education, National Center for Education Statistics).

Cochran-Smith, M., & Lytle, S. (1993). *Inside/outside: Teachers, research, and knowledge*. NY: Teachers College Press.

Delpit, L. (1995). The silenced dialogue. In *Other people's children* (pp. 21–47). New York: New Press.

Georgia Department of Education. (2007). *Enrollment by race/ethnicity, gender and grade level (PK-12)* [data file]. http://app3.doe.k12.ga.us/ows-bin/owa/fte_pack_ethnicsex.entry_form. Accessed 13 November 2008.

Gollub, J. P., Berthanthal, M., Labov, J., & Curtis, C. (Eds.). (2003). *Learning and understanding: Improving advanced study of mathematics and science in U.S. high schools.* Washington: National Academies Press.

Gutiérrez, R. (2008). A "gap-gazing" fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education, 39*(4), 357–364.

Horn, L., Nunez, A., & Bobbitt, L. (2000). *Mapping the road to college: First generation students' math track, planning strategies, and context of support*. (NCES 2000-153). Washington: U.S. (Department of Education, National Center for Education Statistics).

Lubienski, S. T. (2008). On "gap gazing" in mathematics education: The need for gaps analyses. *Journal for Research in Mathematics Education, 39*(4), 365–371.

Ma, L., & Willms, J. D. (1999). Dropping out of advanced mathematics: How much do students and schools contribute to the problem. *Educational Evaluation and Policy Analysis*, 21(4), 365–383.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston: National Council of Teachers of Mathematics.

Shulman, L. S. (1983). Autonomy and obligation: The remote control of teaching. In L. S. Shulman, & G. Sykes (Eds.), *Handbook of teaching and policy* (pp. 484–504). New York: Longman.

The College Board. (2007). *State and National Examination Statistics* [data file]. http://www.collegeboard.com/student/testing/ap/exgrd_sum/2007.html. Accessed 13 November 2008.

The College Board. (2008). *State and National Examination Statistics* [data file]. http://www.collegeboard.com/student/testing/ap/exgrd_sum/2008.html. Accessed 13 November 2008.

The College Board. (2009). *The history of the AP program*. http://apcentral.collegeboard.com/apc/public/courses/21502.html. Accessed 13 November 2008.

Chapter 38
Parents and Teachers Collaborate to Achieve Equity and Quality in Mathematics: A Case Study

Virginia R. Nelson

Background

School teachers can often identify families that provide appropriate support at home for the school's mathematics programs. More families would participate effectively if they knew what to do. José Franco (2009) calls the disparity between the academic performance of White English-speaking students and that of immigrant minorities the "opportunity gap." This chapter discusses how a high-quality mathematics program can be achieved by including practices that involve parents. With the combined effects of appropriate rigor in the classroom and strong family support at home, a multi-age group of English learners moved to the privileged side of the achievement gap. The "Project Classroom" in which this took place provides a case study illustrating how quality and equity in mathematics can be achieved in a disadvantaged context.

Improving student achievement must be a whole school aim. However, attempts to achieve this in the secondary schools often come too late. As Han (2008) asserts, the early years are the most important in determining a person's academic outcomes. Quality and equity in the mathematics education of primary grades depend on efforts both at school and at home. Schools whose expectations for immigrant students are both high and appropriate can expect to see enhanced academic achievement. A high-quality program places faith in the conviction that an early successful experience in mathematics leads to continued confidence, interest, and achievement, a "productive disposition," as Kilpatrick et al. (2001, p. 195) refer to it.

In the United States, Canada, and England, immigrant students tend to lag behind English speakers in mathematics (Huang 2000). English learners frequently become high school dropouts (Echevarria and Short 2006). Guo and Mohan (2008) report that English learners who immigrate to Canada in high school are unlikely to

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graduate because of the time it takes to learn enough language to complete the academic program. This is made worse by the often low teacher expectations that accompany marginalized immigrant and minority children (Han 2008). It is commonly believed that Latino parents do not appropriately help their children with their schooling (Ortiz and Ordóñez 2005). Moll notes that schooling for Latino children typically means remediation (Jiménez et al. 1999). With respect to the achievement of children from Spanish-speaking homes in the United States, "there is little disagreement that a crisis exists" (Goldenberg 1996, p. 353). Spanish-speaking student achievement is typically much lower than that of Asian and White English-speaking students, and scarcity of opportunity creates that gap. Low expectations prevent teachers from presenting cognitively demanding lessons to minority students (Ladky and Peterson 2008). Constantino (1994) reports that both English-only and ESL teachers have low expectations for English learners. Some researchers question whether the education system is really interested in the immigrant children's academic needs (Huang 2000). These notions match Holloway's (2004) finding that in the United States, minority students have a less demanding curriculum than White English-speaking students. He correctly asserts that teachers need to expect the same level of performance from all students.

The role of parents should take the central position in efforts to level the playing field. Successful work by parents provides more than academic achievement; it keeps families together, allowing parents to retain the control they took for granted in their homelands. To achieve the goal of equity the education community will have to emphasize a central role for parents. Even parents with little formal education have much to offer in the early years. Using their talents at the beginning of children's schooling will set up their children for subsequent academic success.

To provide a quality mathematics program and stimulate equity by encouraging daily family participation in a classroom in which the parents feel completely at ease, beginning and intermediate-level English learners can learn together. Parents will more willingly participate academically at home and in the school's classrooms. Because of their poverty, poor parents have been viewed as weak partners in the education of children (Guerra and Valverde 2007). Some educators trace student failure to a home life that does not provide adequate support for education (Robinson-Zanartu and Majel-Dixon 1996). Moll notes that parents tend to trust the schools to do their best for students (Jiménez 1999). For several reasons, parents with little or no formal education rely on schools to do the teaching (Peterson and Heywood 2007). One of these reasons is that parents who speak little or no English tend to feel inadequate to participate in school life (Sherris 2008). Just as middle-class parents use their power and social status to advocate for the best possible placement and programs for their children (Gordon and Nocon 2008), the parents of immigrant English learners want the best possible outcomes for their children. They know their own participation would help. According to Moll and Rodríguez-Brown (Jiménez 1999), comfort is a key feature of effective participation. The parents who do frequent the classroom are parents who feel comfortable in the school building (Finders and Lewis 1994). Sadly, many teachers are not prepared well enough to involve parents in the schools (Ortiz and Ordóñez-Jasis 2005).

The quality of the math lesson depends on the quality of the teacher who delivers it. Teachers of English learners must know how to make math accessible to these students. Teacher quality becomes a key issue in providing equity and improvement in the education of immigrant children. Many teachers lack information about second language acquisition (Jiménez et al. 1999), and about how to teach immigrant children (Huang 2000). All teachers need better preparation to effectively teach English learners (Constantino 1994).

Non-communication between schools and immigrant parents can occur because staff might consider it demeaning to give immigrant parents information that the mainstream society takes for granted. For example, a teacher might consider it condescending to tell parents that their children must attend school. Everyone knows that children must attend school. However, a parent who never attended school might not have this basic knowledge. Teachers might scoff at telling parents something so basic. Current practices overlook soliciting, even demanding, partnership with parents at an early age.

Teachers in diverse settings recognize that the practice of equality—giving the same book to each student and teaching the same lesson in the same way—fails to achieve equitable outcomes. Similarly, approaching parents of English learners in the identical ways in which schools approach mainstream families has failed the marginalized parents. Large meetings and document distribution do not provide the information these families need. Equity in the classroom means giving students what they need to become proficient in the academic program. Equity in working with parents means recognizing that there may be more diversity among a classroom's parent population than there is among the students. The students have shared experiences during six hours of every weekday; parents come from a great variety of backgrounds with few common experiences with mainstream society. To increase the achievement of English learners, schools must reach out to their parents in ways that communicate the important skills that they need to develop to be able to contribute to their children's education.

High-Quality Mathematics Instruction for English Learners

In the name of inclusion, some schools abandon and isolate underachieving minority students on the wrong side of the gap. They separate them from each other and place them in cooperative groups with proficient English-speakers. There is a lack of evidence supporting the placement of language-minority students in the mainstream classroom (de Jong 2004). Nevertheless, the separation of young children from classmates with whom they can communicate continues as teachers spread English learners throughout the classroom and throughout the school. They believe that these children are going to learn more English if they cannot speak their primary language in the classroom. Thus silenced, these students cannot negotiate meaning in the content areas. While the mainstream students are learning, the minority English learners are marginalized and isolated from classroom interaction.

Entry age is a major factor in school success. Those who enter early enough can achieve equal footing with the monolingual speakers of English. Even in the primary grades, English learners lack the English language skills of their English-only classmates (Echevarria and Short 2006). When students lack fluency in the language of instruction, extreme measures *might* help them overcome deficiencies (Kishiyama et al. 2009). Schools tend to assume that economically unstable immigrant families cannot provide the advantages that privileged children enjoy. However, these families can provide a rich home life that leads to academic success at school. The schools should provide information to assist families in providing that kind of a home support.

Responsive schools typically attempt to reach out to families by drawing them to the schools. They make friendly gestures and schedule celebratory, cordial meetings which have the trappings of typical family meetings. Such a meeting might include a greeting and interpretation into the families' languages. Or a school system might provide these meetings in the home languages of the students. Just as chalk-and-talk is an inefficient approach for teaching language learners, the typical parent night meeting is only a gesture (Robinson-Zanartu and Majel-Dixon 1996). There will be lectures and pamphlets on how to help students at home. This is what professionals expect when we attend such meetings. It emanates from our thinking that what is good for us is good for everyone. As a procedure, it wastes parents' time and intimidates the most fragile families in our communities. Sending documents in translation can, in fact, diminish communication between the home and school (Waterman and Harry 2008).

When parents participate in students' homework assignments, there are several benefits. Hoover-Dempsey (2001) documents some significant benefits, which include self discipline and responsibility, persistence, and improved behavior at school. Homework should review what the student knows how to do and can do (Waterman and Harry 2008). Schools tend to send homework that might not be reviewed, accompanied by little or no explanation of how to do it (Finders and Lewis 1994). Without a system of homework to reinforce the English learner's concepts, a school cannot provide equity for the learner. Immigrant parents place a high value on homework and expect a lot of it (Waterman and Harry 2008). Completing homework is, in fact, one way to enhance the connection between the home and the school (Ladky and Peterson 2008). At a family meeting prior to school opening for the year, parents learn about homework. Students need mathematics homework to practice number facts, to learn responsibility, and to maintain family cohesiveness. Avoiding early emphasis on procedural fluency in mathematics could lead to students being unable to capably manipulate numbers in sophisticated mathematics problems. Kumon Math encourages self-discipline and self-confidence with number fact worksheets. Singapore Math prepares students for sophisticated problem solving by developing number sense through repetitive practice on unstimulating workbook pages. When preparing mathematics homework for English learners, teachers must remember that any page with a word on it can become a word problem at home. Teachers may send home mathematics fact worksheets. Even opponents of this type of homework will not view the assignment as harmful when they take into account the fact that families can sit together and complete the pages. Parents can maintain their stature in the family.

Some specific practices improve the quality of mathematics instruction for English learners. Secada (1998) includes these ideas: use problem solving, and include discussions about solutions; encourage mental math; and keep expectations high. Oveido (2005) recommends minimizing wordiness. Students can sometimes do the calculations if the wordiness does not get in the way. Perform a clear comprehension check to separate the language from the mathematics. Develop mathematics problems with meaning for the student or the class, something that is more relevant than what is found in textbooks, something that connects to students' lives. Both Secada and Oveido caution against using "key words." Teaching E-learners that *in all* means to add leads to confusion. This question provides an example:

Mary has 5 marbles in all. Mary gives 2 marbles to Tom.
How many marbles does Mary have now?

If beginning English learners have been taught that *in all* always means add, they might calculate an answer of 7.

The use of cognates facilitates comprehension if words in the target language approximate the students' home languages in appearance, pronunciation, and/or meaning. In the ideal mathematics classroom, there is direct teaching of how to reach mathematical solutions. Some components include not only illustrated, one-step situations, but also complex, multi-step problems that take more than one class to solve, and problems on sample tests available from high-stakes assessment websites. Each lesson needs a language as well as a content objective (Echevarria and Short 2006). Linking content and language instruction has proven to be one of the most effective ways to accelerate the academic achievement of English learners. In mathematics, this includes heavy use of manipulatives to present, explain, and visualize concepts. Working in groups to solve problems also promotes math competence (Chamot 1995). Sherris (2008) recommends starting with a review of the standard, a focus on the concept and skills to be employed, and assessments to decide where to begin each math lesson.

Eric. J. Cooper offers twelve possible reasons for the achievement gap, based on reviews of the literature (Cooper 2004). Two of these refer to families. The first suggests that child-rearing practices might differ. The second cites lack of parental involvement. Researchers still have to look at outcomes when a school makes the effort to provide the missing information to undereducated parents so that their parenting may compete with that of privileged families.

Results from the "Classroom Project"

Commencing in September of 2003, the Project Classroom was the only all-English learner classroom in one elementary school district in Oregon, USA. Spanish-speaking students were placed together in a multi-age classroom serving between

26 and 32 students between the ages of seven and ten. All children in the school whose home language was Spanish were invited to participate, and every family wanted to participate. Fortunately, there was space for the children of all interested families. The teacher was a Spanish speaker with certifications in ESOL (English for Speakers of Other Languages) and classroom teaching. In compliance with U.S. Civil Rights legislation, a Specially Designed Alternative Instruction plan described an alternate approach to classroom structure. The structure differed from other classrooms in the school in two ways. First, all students were Spanish-speaking English learners. Second, parents were expected and welcomed to enter the classroom at any time during the school day, and without prior notice.

At after-school meetings and during classroom visits, families shared information about themselves. The teacher recorded data about the families after meetings, when students volunteered information, and upon commentary from members of the immigrant community. Such data showed that the average time the parents had spent in their own formal education was less than two years. All families were classified as poor according to federal guidelines. All adult members of the families were hourly wage earners, mainly in the service sector. More than half of the older siblings had dropped out of school, joined gangs, and/or started families of their own. All had found menial jobs like those of their parents.

Immigrant parents often make assumptions about what they themselves can do to assist their children in school work and what their children could accomplish. In this project, they described it as refreshing and elevating to realize that they had so much information to share with their children. They enjoyed playing mathematics games, and noted that the fights over which TV program to watch disappeared. Their children were asking for mathematics games at home that families had learned together at family meetings. The parents felt valuable.

Meetings that do not provide time for parents to talk with teachers about their own children are not useful (Guo and Mohan 2008). It is possible to establish quality mathematics experiences for students both at school and at home. Family meetings focused on what the parents could do with their young children. Each time the Project Classroom families gathered for family meetings, math activities were presented and practiced.

For example, because combinations to ten form a basis for beginning mathematics mastery (Tang 2001), families were encouraged to work together on these every day. Activities included gluing pebbles or beans to popsicle sticks or tongue depressors in units adding to ten. Creating conversations about mathematics can begin as early as families begin to play math together. Merely as an example, families outlined children's hands and counted fingers. They identified how many fingers are on each hand. They combine the sum of the fingers on one hand with the sum of the fingers on the other. On road trips, they hunted for combinations to ten on passing license plates. Many more ideas are available in publications such as *Family Math* (Stenmark et al. 1986). Families practiced two or three activities at each family meeting.

The effective family meeting did not offer childcare. The entire family was expected to attend (Ortiz and Ordóñez 2005). The lead teacher demonstrated a skill.

Then families practiced that skill. Because the most fragile families might not be literate, reading was not demanded. Families talked about books by looking at the pictures. They practiced making up mathematics problems based on the illustrations. The teacher leading the meeting would say:

- Look at the cover. Students, tell your parents what you see on the cover. What can you count on the cover? Parents, tell your students they are making important observations.
- How many pages does this book have? Tell your parents how you know.
- Students, tell your parents what you see on the first page. What is there one of? Two?

The parents would then provide rich discussions with their children. With students in the early years, this is easy to provide, but parents must learn to provide it. With practice and support, these activities will prepare children to appreciate mathematics. Parents lacked information about how to develop and guide such activities. They, together with their children, benefited from practice. The school must provide such practice. This is a partial list of family activities that schools could assist families to offer at home:

- Look at picture books and invent math stories to match the pictures. With practice parents learn to make math problems out of almost any illustration.
- Count objects and people where more than one appear on a page.
- Recount incidents and memories from parents' early childhood emphasizing how much they liked to learn about math (even if that was not the case).
- Explain shopping decisions and provide an introduction to economics. "We're buying this lettuce because it costs less." "We're buying red lettuce because it has more vitamins."

In addition, some guidelines for providing young people with an environment conducive to home study may include:

- Insist on an appropriate bedtime.
- Locate the best spot in the home for homework completion.
- Specify a daily time for homework.
- Place the TV away from the child's sleeping area.
- Arrange for children to get exercise outdoors.
- Assign age appropriate chores.

Frequent meetings brought the parents' attention to current classroom topics and how the home environment can support them. There is evidence from the literature that regular meetings would support parents' efforts (Ladky and Peterson 2008). In order to encourage parent attendance at meetings, families were contacted personally or by phone. This has proven to be the most effective way to approach the parents of English learners (Chamot 1995).

The Project was designed in the belief that families should keep the heritage language alive in the home. Some parents thought that mastery of the majority language signified intelligence. Children who achieve apparent mastery quickly earn accolades from the monolingual teaching community. Young children who acquire social language quickly and pronounce it perfectly acquire the esteem of their teachers and the community. Immigrant families acknowledge this. Everything works to favor rapid acquisition of the mainstream language, at any cost (Wong Fillmore 1991; Jiménez et al. 1999; Peterson and Heywood 2007).

To seek the best academic outcomes for their children, some families attempt to speak the majority language at home. As students acquire the local language by using it at home they abandon their primary language. Teachers praise the velocity with which students master the new language. People assume that speaking English is one piece of evidence that a child is intelligent (Peterson and Heywood 2007; Jiménez et al. 1999). This praise encourages parents to continue neglecting the language of the home. Soon children can no longer communicate effectively with their grandparents, nor have they mastered the majority language.

Immigrant parents want language mastery for their children. Students who are learning English are sometimes talked about as students with a language problem. Math is a good area to introduce language learning, but it is to be expected that a child will take several years to master the intricacies of the language and be able to participate as well as a native speaker. The school must describe the trajectory of the quick but shallow language learner. Lily Wong Fillmore (1991) and colleagues researched the danger that families face when their children learn the language of the mainstream while losing the home language.

> Parents need to be warned of the consequences of not insisting that their children speak to them in the language of the home. Teachers should be aware of the harm they can do when they tell parents that they should encourage their children to speak English at home, and that they themselves should try to use English when they talk to their children. (pp. 345–346)

At each family meeting, parents heard about the importance of maintaining the home language throughout the children's schooling. No one spoke the home language as well as the parents did, and they were the best source of Spanish their children had. Parents mentioned relatives whose children could no longer communicate with family members who did not speak English. They grew accustomed to the struggle they faced to keep the children speaking and developing their Spanish.

High-stakes testing is typically thought to work against English Learners. According to Boaler (2008, p. 89), "the tests used in America are particularly harmful for children of low income and for English language learners (ELL students)." The Project Class, however, was in a district that uses annual high-stakes assessment to measure achievement. Project Class English learners excelled on the math assessment.

Prior to establishing the Project Class, math scores for Hispanic students at the school lagged more than 20% points behind the scores of White English-speaking students and Hispanic students throughout the State. After just one year as a Project Class, during the 2003–2004 academic year, results from the annual mathematics State assessment for students identified as "Hispanic" show that grade three students at the Project Classroom school outperformed Hispanic students at other District and State schools by 30%; grade five, 20%. The 2004–2005 results were, respectively, 5% and 13%. In 2005–2006, the State added grade four to the published results. "Hispanic" students registered higher percentages in all three grades, ranging between 5% and 18% higher, and paralleling the performance of their English-only classmates classified as "White" (Oregon Department of Education 2008).

This story of success, however, had an unexpected and unfortunate ending. Near the conclusion of the 2005–2006 academic year, a couple of school staff members filed a complaint against the school Principal for approving the establishment of the Project Classroom, claiming discrimination. The District had three options: negotiate with the complainants, invite an investigation, or close down the classroom. The District chose the third option. English learners were distributed among all classrooms. Formulae dictated that each classroom would now have approximately the same number of Spanish-speaking students. Parents no longer entered their children's classrooms. Their students' new teachers scheduled parent helpers for whom they left written instructions in English. They also scheduled parent visits; parents were no longer able to walk in unannounced.

During the next three school years "Hispanic" math achievement at the Project Classroom school lagged behind both the District and the State by between 2% and 20%.

Parents, now disenfranchised, were no longer able to close the gap.

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References

Boaler, J. (2008). *What's math got to do with it: Helping children learn to love their least favorite subject—and why it's important for America.* New York: Viking.

Chamot, A. U. (1995). Implementing the cognitive academic language learning approach *CALLA* in Arlington, Virginia. *The Bilingual Research Journal, 19*(3–4), 379–394.

Constantino, R. (1994). A study concerning instruction of ESL students comparing all-English classroom teacher knowledge and English as a second language teacher knowledge. *The Journal of Educational Issues of Language Minority Students, 13,* 37–57.

Cooper, E. J. (2004). The pursuit of equity and excellence in educational opportunity. In D. Lapp, C. C. Block, E. J. Cooper, J. Flood, N. Roser, & J. V. Tinajero (Eds.), *Teaching all the children: Strategies for developing literacy in an Urban setting* (p. 14). New York: Guilford Press.

deJong, E. J. (2004). After exit: Academic achievement patterns of former English language learners. *Education Policy Analysis Archives, 12*(50), 1–18.

Echevarria, J., & Short, D. (2006). School reform and standards-based education: A model for English-language learners. *The Journal of Educational Research, 99*(4), 195–210.

Finders, M., & Lewis, C. (1994). Why some parents don't come to school. *Educational Leadership, 51*(8), 50–54.

Franco, J. (2009, November). "Making connections with Spanish language communities", 50 Years of Mathematics community: A learning legacy and a look to the future, California Math Council South, Palm Springs Convention Center, Palm Springs.

Goldenberg, C. (1996). The education of language-minority students: Where are we, and where do we need to go? *The Elementary School Journal, 96*(3), 353–361.

Gordon, V., & Nocon, H. (2008). Reproducing segregation: Parent involvement, diversity, and school governance. *Journal of Latinos and Education, 7*(4), 320–339.
Guerra, P. L., & Valverde, L. A. (2007). Latino communities and schools: Tapping assets for student success. *Principal Leadership, 8*(2), 40–44.
Guo, Y., & Mohan, B. (2008). ESL parents and teachers: Towards dialogue? *Language and Education, 22*(1), 17–33.
Han, W. J. (2008). The academic trajectories of children of immigrants and their school environments. *Developmental Psychology, 44*(6), 1572–1590.
Holloway, J. H. (2004). Closing the minority achievement gap in math. *Educational Leadership, 61*(5), 84–86.
Hoover-Dempsey, K. V., Battiato, A. C., Walker, J. M. T., Reed, R. P., DeJong, J. M., & Jones, K. P. (2001). Parental involvement in homework. *Educational Psychologist, 36*(3), 195–209.
Huang, G. G. (2000). Mathematics achievement by immigrant children: A comparison of five English-speaking countries. *Education Policy Analysis Archives, 8*(25), 1–16.
Jiménez, R. T., Moll, L, Rodríguez-Brown, F. V., & Barrera, R. B. (1999). Conversations: Latina and Latino researchers interact on issues related to literacy learning. *Reading Research Quarterly, 34*(2), 217–230.
Kishiyama, M. M., Boyce, W. T., Jimenez, A. M., Perry, L. M., & Knight, R. T. (2009). Socioeconomic disparities affect prefrontal function in children. *Journal of Cognitive Neuroscience, 21*(6), 1106–1115.
Ladky, M., & Peterson, S. S. (2008). Successful practices for immigrant parent involvement: An Ontario perspective. *Multicultural Perspectives, 10*(2), 82–89.
National Research Council. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford, & B. Findell (Eds.), *Mathematics learning study committee, center for education, division of behavioral and social sciences and education.* Washington: National Academy Press.
Oregon Department of Education. (2008). http://www.ode.state.or.us/data/schoolanddistrict/testresults/reporting/PAGRSurpressed.aspx. Accessed 2 Oct 2008, 10 June 2009.
Ortiz, R. W., & Ordóñez-Jasis, R. (2005). Leyendo juntos (reading together): New directions for Latino parents' early literacy involvement. *The Reading Teacher, 59*(2), 110–121.
Peterson, S. S., & Heywood, D. (2007). Contributions of families' linguistic, social, and cultural capital to minority-language children's literacy: parents', teachers', and principals' perspectives. *The Canadian Modern Language Review/La Revue canadienne des langues vivantes, 63*(4), 517–538.
Robinson-Zanartu, C., & Majel-Dixon, J. (1996). Parent voices: American Indian relationships with schools. *Journal of American Indian Education, 36*(1), 33–54.
Secada, W. G. (1998). School mathematics for language enriched pupils. In S. H. Fradd & O. Lee (Eds.), *Creating Florida's multilingual global work force: Educational policies and practices for students learning English as a new language.* Tallahassee: Florida Department of Education.
Sherris, A. (September, 2008). Integrated content and language instruction. *CAL Digest*.
Stenmark, J., Thompson, V., & Cossey, R. (1986). *Family math.* Berkeley: University of California.
Tang, G. (2001). *Math for all seasons.* New York: Barnes & Noble.
Waterman, R., & Harry, B. (2008). Building collaboration between schools and parents of English language learners: Transcending barriers, creating opportunities (pp. 4–19). NCCRES, Tuscon: Arizona State University.
Wong Fillmore, L. (1991). When learning a second language means losing the first. *Early Childhood Research Quarterly, 6*(3), 323–346.

Chapter 39
Critical Mathematics in a Secondary Setting: Promise and Problems

Andrew Brantlinger

Introduction

Critical mathematics (CM) has attracted considerable interest among mathematics educators (Atweh 2004; Gutiérrez 2002). For example, the CM text, *Rethinking Mathematics* (Gutstein and Peterson 2005) has sold 11,000 copies to date (Trokan 2009). The approaches "teaching mathematics for social justice" and "radical mathematics" are similar enough that I include them as "CM." In the U. S. context, progressive CM educators use Freire's (1971) critical literacy ideas to design their mathematics teaching (Frankenstein 1983; Gutstein 2003, 2006). European CM educators similarly incorporate critical and progressive approaches, although they draw more from Frankfurt school theorists than from Freire (Skovsmose 1994). In this chapter, I discuss the U.S. context of Freirean-inspired CM movement.

Similar to mainstream reformers (e.g., the National Council of Teachers of Mathematics 2000), CM reformers critique traditional mathematics instruction found in most American schools (Frankenstein 1983; Gutiérrez 2002). CM educators maintain that, as part of mass schooling under capitalism, mathematics education aligns with the values and interests of the powerful. CM advocate Gutstein (2006) asserts that traditional mathematics instruction and, to a lesser extent, standards-based reform instruction results in "domestication of consciousness" (p. 6) and "mathematics 'mis-education' of low-income students and students of color" (p. 12–13). To counter these adverse effects, he advocates CM which would have such students "reading and writing the world with mathematics" (book title) and would move them from school-learned conformity and passivity toward informed resistance and political agency. Such CM advocates as Gutstein (2003) and Gutiérrez (2002) see CM as a path to mathematics excellence in culturally diverse classrooms, to greater

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equality in mathematics learning and, ultimately, in fewer achievement-related inequitable outcomes in American society. Gutstein (2003, 2006) reports that CM fosters minority student engagement in school mathematics, though his evidence comes from work in a selective setting with students who were institutionally empowered and academically engaged.

CM scholarship remains largely theoretical to date. The small empirical base largely consists of self-reports by teacher-researchers who are committed to developing and promoting CM (Bohl 2000; Frankenstein 1995; Gutstein 2003, 2006; Kitchen and Lear 2000). Further, because CM research has focused on elementary and middle grades mathematics or arithmetic at the college level, there is a scarcity of research on the nature and effectiveness of CM reform at more advanced levels. In this chapter, I address that lack by featuring results from my self-study of a CM geometry course that I taught at an inner-city high school. Through examining student responses to CM, I focus on the gap between its ideals and realities. As an initial enthusiast for CM ideals, I found that trying to implement CM in a secondary classroom to pose deeper problems than those documented the extant literature. In addressing my findings, I respond to the question that is central to this book, namely: what can we learn from specific research studies or programs relevant to quality and equity agendas in mathematics education?

Study Context

This study took place in a remedial night school program at a "high needs" high school in Chicago. The semester-long night course ran for two hours an evening, four evenings a week, for nine weeks. The 28 students enrolled in the course needed geometry credit to make up for past failure and to graduate from high school. Students were between 17–19 years of age and were of Puerto-Rican, Mexican-, and African-American heritage. Most were from low-income families, several lived on their own, and a few worked full-time day jobs. While night school was a convenient alternative for some, about a dozen were banned from attending day school as a result of various offences, including gang involvement, teen pregnancy, drug dealing, and disrespect for school authority. Several of these students were upset about not being able to attend day school.

Students reported that the night school program lacked an academic focus. In a pre-interview, one claimed, "we don't really have to do anything in night school, but just be here, and *don't* talk, and just keep quiet." School security was tight and ever present. In interviews, most students indicated that in the past they only had experienced traditional mathematics instruction. In sum, night school students fit the profile drawn by CM theorists of having been mis-educated in mainstream schooling, including in their mathematics education.

Consistent with conducting practitioner research, I was the teacher, curriculum designer, and researcher of my CM class. Although I had taught for five years in different high-needs Chicago high schools and had taken a graduate course on critical pedagogy, I had never taught CM before conducting this study. I had regularly used standards-based reform techniques and these formed the basis of the night course curriculum.

Because of the dearth of CM instructional materials available at the secondary level, a key component of my study involved curriculum development. While I expected some difficulty in designing CM lessons, they proved to be far more elusive and labor intensive than I had anticipated. Despite spending well over 200 hours working on CM materials before and during the night course, at best, the developed products covered only 11 hours of class time out of the semester-long course. I struggled to create politically relevant tasks that included the sophisticated use of required secondary geometry content (e.g., angle sums in polygons, Pythagorean Theorem). In addition to my CM activities, I included two projects developed by Gutstein (see Gutstein and Peterson 2005). In the end, CM activities comprised about 15% of the course curriculum and reform-oriented activities, and assessments made up the other 85%.

My night course study took place during December and January when Chicago is particularly cold, dark, and bleak. A two-week holiday break separated the nine-week course into two halves. While there were continuities, there also were differences between these two time periods (see Table 39.1).

Table 39.1 Overview of CM activities in night course

Weeks 1–5	My primary instructional goal was to acclimatize students to the reform mathematics foundation upon which CM would build. There was considerable initial resistance by students to reform techniques (lack of teacher telling, non-routine tasks, collaborative work). With time, most students were willing to engage with important aspects of reform instruction while some continued to complain publicly about them. In the first four weeks, I included one CM activity of an hour or less duration per eight-hour week. In Week 1, students were to write responses to a cartoon critical of traditional schooling. In Weeks 2–4, students were to analyze data related to correlations between student race and opportunities for recess; family income and student test scores; and rates of handgun violence in the U.S. Only the race and recess activity ignited a "student-driven" whole class discussion, while the other activities dissipated discursive engagement. While generally writing short but thoughtful individual responses to these activities, several complained that CM was "goofy" and "not what we're here for."
Weeks 6–9	Although never ideal, students engaged in reform activities with less resistance. My relations with them improved considerably with many students coming before or after class to talk with me about concerns in their lives. This period featured longer CM projects, with each taking a few hours to complete. In Week 6, students engaged in the Mercator Map Project (Gutstein & Peterson, 2005). While they did the mathematics, many avoided taking a stand in either writing or whole class discussions on whether the bias (i.e., Eurocentric projection) in the Mercator Map was purposeful. They were evasive despite my requiring the articulation of a political position. In Week 7, students did the three-hour CM project that is discussed in this chapter. In Week 8, students completed a two-hour CM project called "South Central" that dealt with the distribution of community resources (see Brantlinger, 2005). While each lesson resulted in stretches of sustained student engagement, they also featured periods of substantial resistance to CM activities and class discussions of critical themes.

Discourse Analysis of CM Activities

The total study included a discourse analysis, a curriculum analysis, and a self-analysis of my thinking as a CM instructor. The results reported in this chapter come from the discourse analysis of data transcribed from videotapes recorded during the first four hours of the second and seventh weeks of the nine-week night course. These two instructional periods were comparable in that each included one CM activity and three reform activities. As part of the discourse analysis, the eight-lesson transcripts were divided into topically related sets (TRS) or discourse segments that cohere around a topic or theme (see Mehan 1979). To examine discourse structures and who initiated new TRSs (i.e., themes), individual turns within TRSs were recorded as Initiations, Responses, and Evaluations (IREs) (see Mehan 1979, Pruyn 1999). In addition to highlighting structural features of discourse, separating discourse into TRS's and IRE's allowed for an examination of student *subjectification* and *objectification* (Pruyn 1999). Subjectification includes following student-centered goals, teachers encouraging students to think for themselves, and students engaging in student rather than teacher-initiated discussions. Objectification happens when discourse is teacher-centered (lecturing, ignoring student ideas, conveying that students are not capable of reasoning on their own). Student utterances and communicative behavior (putting their heads down, initiating a topic) were coded as exhibiting *engagement, resistance,* or *conformity*. Students' mathematical contributions were coded as indicating *elaborate student engagement* when they measured more than two lines of text on a transcript page. This thematic breakdown was useful in capturing the evolution of instructional patterns and student participation over the semester-long course. In addition, I used Gee's (2005) constructs of *Discourse models* (cultural models) and *social languages* to track students' articulated beliefs about society and mathematics and their use of code-switching from school to home language.

While the discourse analysis included data from both critical and reform activities, in this report I illustrate students' discursive engagement in CM by using an excerpt from one CM lesson. It should be noted that, while unique in many regards, this particular excerpt captures many of the same discursive features and teaching dilemmas of all CM activities.

Inequalities and Area

In the seventh week of the course, I introduced Inequalities and Area (I&A), a three-hour CM project inspired by Williams and Joseph (1993). The mathematical goal of I&A was for students to apply their understanding of area to compute the Gini coefficient of an income distribution graph (see Fig. 39.1). (The greater area of the region enclosed by the two curves, the greater measured income inequality.) A critical goal was to deepen students' understanding of economic inequality. I launched the project by having students respond in writing to two prompts:

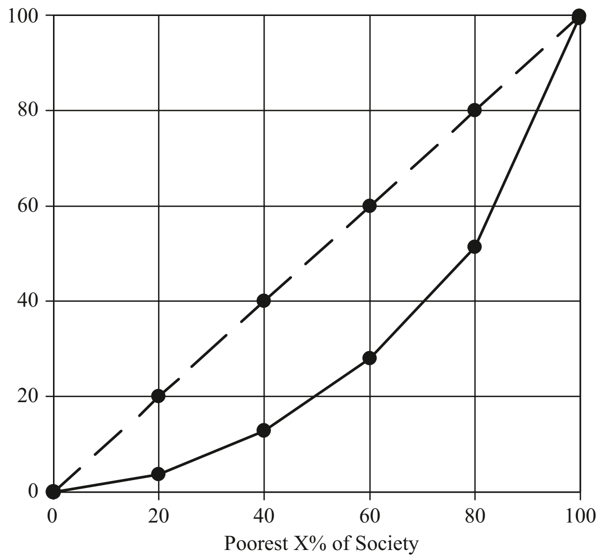

Fig. 39.1 Distribution of income graph

1. Do you think the U. S. economic system is fair to its citizens? Why or why not?
2. Has the distribution of income in the United States become fairer since the Civil Rights Movement? What do you think? What would your family members who work say?

While it could be better phrased, the first writing prompt ignited several impromptu small group discussions. Apparently, because they did not want to be incorrect, many students hesitated to write responses to the second prompt. Following a short whole-class discussion of these prompts, I divided the 25 students present that day into five income bracket quintiles: the first quintile represented the poorest 20% of U.S. wage earners in 2001, the next represented the second lowest 20% of wage earners, and so on to the wealthiest quintile. I gave each group the portion of small candy bars that corresponded to their quintile group's yearly earnings. The poorest group got 4 bars compared to the wealthiest group's 49. We ended the first hour with a whole-class discussion about issues of fairness raised by this distribution. To be clear, while the second and third hours of I&A had a comparatively strong mathematical component, activity in the first hour (featured in Fig. 39.2) had critical rather than mathematical focus.

Although both critical and reform activities were transcribed and analyzed, due to limited space, the discussion that follows focuses solely on CM data (for more detail, see Brantlinger 2007). Using an excerpt from the I&A activity (see Fig. 39.2), I illustrate students' reactions to CM.

Based on transcript analysis of transcribed CM lessons, including this excerpt, I conclude that: (1) participation in CM was less stable and predictable than in reform mathematics, (2) CM activities appeared to trigger certain kinds of student-centered discussions that were not apparent in reform activities, (3) students used vernacular

Key: The double slash (//) represents a break in speech of a half-second or more. Bold speech was louder and "!" indicated further emphasis. Underlined speech indicated a raise in tonal pitch. The "[" and "[[" marks represent overlapping speech.

Osvaldo (sits up in his chair, grins): I think we all should just get paid the same thing //
Kampton: yeah! //
Jayla: but [then we //
Osvaldo: [you know why? // not //
Eddie mutters something about "communists" and smiles.
Osvaldo: not paid the same // but the in[[come // you know?
Lucee: [[okay / Mister B // my turn // I'll go //
Me: okay //
Lucee (to me): are you ready? //
Me: yeah //
Lucee: I think that its not fair because // that if we // they give us // okay // cause some people // don't have higher educations than other people // because most people they simply just don't have enough money // and they have either a large family or a real small family that they gotta help out // either you got someone at home that can't work or whatever // or there's issues (inaudible) whatever // and you gotta work // so you [can't go to school //
Me: [yeah //
Lucee: so all that stuff // but if someone gives someone else the opportunity // and they develop the skills // they can just be as smart as anybody else who went to school // they just didn't have all the time to do all that stuff that they did //
Me: yeah //
Lucee (scrunches body and face up and looks at desk): yeah //
Me: so if you have less money // then you gotta spend more time working // and then you can't go to school and stuff like that //
Lucee: yeah //
Stephie raises hand in back of room.
Me: go ahead Stephie //
Stephie: okay I don't think that // (points to Lucee) // like she was saying that // if you don't go to school you can't work because // I'm going to use myself as an example // I work // and I go to school // and even though I don't have my high school diploma // I moved up little by little // and I became manager // so // it's not that // you just gotta push yourself forward // and you'll do it //
Me: I think both you guys agree with that idea that you have to work harder // maybe than pe[[ople //
Stephie: [[it's not that hard // I don't think it's that much harder //
Kampton: but // for a person with strong willpower // [that person…
Efrain: **[I believe that everything you get //** everything you want you need to work hard for it! //
Robi (has hand raised): yeah! //
Kampton: or // kiss a lot of ass //
Efrain: yep //

Fig. 39.2 Critical whole-class discussion during I&A project

Me: next // huh // I mean I'll be honest // well // I'll take that up another time // (to Efrain) // remember you said that //
Lupe: what [[comes easy // goes easy //
Robi: [[Mister B // I think in a way it is fair // that like rich people get more // because if you went that extra step // to be [like a higher education or something //
Princess: **[but / some people get it / from their parents!** //
Shannon: **right!** //
Robi: you deserve more than // somebody who didn't take that extra step or use it //
Princess: okay but // [[some people // inherit //
Shannon: **[[if you're born rich you**
Me: yeah // did you take I mean so // if you're born to be rich // did you [take an extra step?
Amalia: [you are going to stay rich //
Robi: but if you're already born to then what's the point // of working hard any more? //
A number of students talk over each other at this point.
Lucee: **if any // well you know what he's talking about // if you took that extra step // there's people who wanna take that extra step //**
Shannon: **right! //**
Lucee: **but they don't really/ have the opportunity // they can't // [it's not really their decision // [[they have to go**
Efrain: [too bad for them // [[it's really their way of life //
Lucee (turns towards Efrain): **you just // fucked up** // saying that //
Lucee then covers her mouth and smiles. Many students laugh.
Me: nah // I mean I definitely hear what you're saying // I mean // besides the // f-word there // the rest of what you're saying // and // you don't agree // Malik? //
Malik (who had just raised his hand): no //
Me: that's okay // you [don't have to
Lucee: **[cause // I could go // I would've taken those steps but they're // there's // you know // there's things that are // holding me back right now //**

Fig. 39.2 (continued)

street language to a much greater extent in critical than in reform activities, (4) student responses indicated that CM activities challenged but did not fully counter dominant narratives about social inequality, and (5) a tension between critical and mathematical content surfaced in CM activities with sociopolitical themes presenting a distraction to underlying mathematical goals.

The finding that student participation in CM activities was less stable and predictable than in reform activities was illustrated over the seven CM activities and projects that comprised 11 class periods. Three night class CM activities and projects ignited student engagement in whole-class conversations, whereas four actually decreased or dissipated participation. (In the remainder of this chapter, I refer to the former type as "engaging" CM activities.) In engaging CM activities certain students clamored to be heard in ways they had not in reform discussions. In the case of the four that interfered with sustained participation, planned whole-class discussions of CM collapsed. In these cases, I struggled as a teacher in ways I never had during my years of mathematics teaching. To be clear, I&A was an engaging CM activity and Fig. 39.2 provides an example of student-driven discourse that occurred during the I&A project.

The second finding was that engaging CM activities were structurally and effectively different from whole-class discussions that took place during reform mathematics activities. Such CM activities as I&A were more student-driven and entailed more elaborate student contributions than reform activities. While the discourse at the beginning of the excerpt was directed through me (the teacher), it is apparent that Lucee and Stephie *drove* the conversation. Students mostly were responding to each other's ideas. Analyses indicated that, in general, class discussions about critical themes (e.g., social inequality, racism) in engaging CM sessions were structurally more open and flexible than discussions of mathematics in reform activities (see Brantlinger 2007). Despite my daily attempts to provide accessible reformist mathematics tasks, my students generally did not engage in the types of whole-class discussions about mathematics that reformers imagine (see NCTM 2000, p. 3). Furthermore, while it is not apparent in the Fig. 39.2 transcript, a generally different group of students were discursively engaged in CM activities than were engaged in reform activities. It might be said that CM activities woke up some students (Lucee, Kampton) who usually did the minimum required to pass the class.

Third, while this excerpt only illustrates a limited effect of code switching, students fluctuated between vernacular (street) and scholastic social languages (Gee 2005) to a much greater extent while engaged in CM than in reform activities. During the initial opinion-eliciting and candy distribution activities, students used such vernacular phrases as "fittin to" and taunted each other in Spanish (e.g., "Cállate la boca, Cabrón"). The excerpt above occurred toward the end of the I&A lesson at a point when Lucee and Stephie were directing their talk through me. It was then that they switched back to a more formal scholastic register. That said, a few vernacular outbursts continued (e.g., lines 41 and 68). Student use of vernacular language revealed that they perceived CM discussions as being different and less formal than reform mathematics. It also signaled their ownership of some critical discussions.

Fourth, while CM activities I taught with challenged students' beliefs, they did not fully counter hegemonic narratives about social inequality and economic opportunity (see also Frankenstein 1995). Despite taking part in a CM activity that illustrated substantial economic disparities, a number of students continued to articulate meritocratic beliefs about hard work, the benefits of educational attainment, and a definition of fairness that referred to people getting what they deserve. Countering the few students who claimed that some people face considerable life obstacles and that wealthy people inherit advantage, the majority posited that "you just gotta push yourself forward and you'll do it" or that poor people's "way of life" keeps them down. In other words, they displayed the dominant deficit view of economic disparities. The I&A conversation was not unique in this regard; in a different CM writing assignment, some students described their classmates or classmates' families as personally responsible for the social inequalities they experienced.

Fifth, a tension between the critical sociopolitical and mathematical content surfaced in implementing CM activities. As Fig. 39.2 illustrates, engaging discussions provoked by CM projects often lacked a mathematical focus. In CM lessons taught early in the night course, the sociopolitical contexts and goals presented a distraction to underlying mathematical ideas and goals (see Powell and Brantlinger 2008). In the CM projects featured in I&A, while the critical and the mathematical did not distract each other, they were tangentially related and did not build on each other in particularly meaningful ways. The critical discussion of social inequality had little to do with the mathematical computation of income inequality. In contrast, the political message of I&A essentially was lost in the second and third hours when students computed the Gini coefficient.

Discussion

The results presented in this chapter point to the promise and problems of CM at the secondary level. While some CM lessons engaged students in a manner that reform mathematics never did, other CM lessons fell apart in terms of engaging students in a meaningful manner. On the positive side, engaging CM lessons seemed to strengthen relationships among students and between students and me. Additionally, given the importance of student participation, CM provided the opportunity for some students who rarely spoke up about mathematics in reform activities to contribute to whole-class discussion. However, there was little carryover of this critical engagement to reform lessons, or even from one CM lesson to the next.

I also found that, while some CM lessons gave students an opportunity to discuss issues of interest to them, social rather than mathematics ideas dominated classroom discourse (see Fig. 39.2). Indeed, an issue that is important for CM educators to better understand is how sociopolitical themes relate to scientific or technical learning. Certainly, social contexts have to be taken seriously if critical pedagogy is going to be effective. However, unless social contexts have a mathematical basis or naturally lend themselves to mathematical analysis, they will not facilitate mathematics learning. A related issue that CM educators need to better understand is how the problem solving techniques of secondary mathematics, techniques that were not invented for social application, can be meaningfully applied to actual political problems.

The disconnect between the social and the mathematical was recurring problem that I faced. As an inexperienced critical pedagogue, I certainly share responsibility for failing to better synthesize the critical and the mathematical. However, Gutstein (2006) also cites a "tension" between the critical and the mathematical in his CM teaching and discusses needing to "leave mathematics to the side" to pursue critical goals (pp. 108–109). At the same time, he claims that there is a "dialectical relationship" between the critical and the mathematical in CM and that the two can "facilitate one another, under certain conditions" (pp. 108–109). He is optimistic enough about this that he advocates reconceptualizing school mathematics as a "critical literacy" (p. 6).

While I initially embraced it, I have become skeptical of Gutstein's critical literacy agenda. I am particularly concerned that, similar to vocational mathematics, CM primarily will be used with minority and impoverished students and limit their access to a highly valued academic discipline and its credentials. To be clear, while my students did receive credit for the geometry course, it is likely that they would have learned more college preparatory material had I included fewer CM activities. My students correctly voiced the view that my CM instruction lacked an academic focus. Just as the younger students in Skovsmose's (1994) CM study, many of my secondary students opined that CM was not really school mathematics. A few outspoken students were insistent that, instead of teaching CM, I should focus on the mathematics they needed to know for college.

As the I&A excerpt (Fig. 39.2) makes clear, many of my students expressed hegemonic beliefs about social inequality being due to personal distinctions rather than structural bias in social institutions. Critical scholars (e.g., Gramsci 1971) would predict this to be the case. It might be that students' meritocratic images of opportunity and social mobility can be attributed to youthful hopefulness and their being steeped in the American dream, which they chose to acknowledge over the reality of their daily observations that revealed few economic promises for them. My study illustrated that students' hegemonic thinking was deep and resistant to change. Admittedly, the critical social emphasis in my CM lessons was limited to less than 20% of the curriculum of a nine-week course. Such a short-term intervention could not be expected to counteract the long-term dominance of status quo messages in school, in the media, and in society. For a critical curriculum to effectively counter status quo ideas, it needs to be introduced more prominently and persistently.

To effectively counter dominant messages, CM instructors likely will have to adopt more teacher-centered approaches than current CM theory admits. CM advocates imply that they can avoid teacher telling in part because "doing the mathematics" leads to critical insight (see Frankenstein 1989, p. 3; Gutstein 2003, 44–45). I shared this belief and followed Gutstein (2006, p. 106) who positions himself as a facilitator rather than a teacher who "tells." However, my facilitation stance and student-centered activities were not directive enough to get students to adopt and fully appreciate critical perspectives. In the absence of an authoritative critical presence, my students advanced dominant perspectives more often than critical ones. On the other hand, I realize that had I lectured to students about critical perspectives, my goal of enabling student-driven conversations may not have been realized (see Pruyn 1999). Further, while I attempted to design CM activities that required meaningful mathematical applications, such applications seemed tangential to the social arguments at stake.

It is important to recognize the degree of student resistance that I faced in teaching CM. At times, student resistance—both active and passive—overwhelmed planned CM activities. In such situations, I felt I could react in one of two unsatisfactory ways by: (1) calling on students for a short-answer responses to critical prompts and creating a teacher-centered discourse, or (2) abandoning the whole-class critical discussions I had planned and instead requiring students write individual responses to critical prompts. Either case would require me to explicitly enact my institutional teacher authority. The irony of this observation is that critical pedagogy is theorized as a means to disrupt rather than reaffirm traditional power structures and institutional authority. Shor (1996) describes experiencing similar problems with resistance and authority in his attempt to implement critical pedagogy.

Student internalization of dominant perspectives and resistance to CM forced me to question assumptions CM educators make, and I had made, about urban youth of color, their desires, and their academic needs. Current CM scholarship implies that CM is a natural and "culturally relevant" fit for urban youth of color (Gutstein 2003; Gutiérrez 2002; Tate 1995; Turner 2003). When I began my study, I was drawn to theories that the distinctive needs of lower income urban youth of color meant they would benefit from a substantially different curriculum than that traditionally provided privileged white youth. Consistent with the larger educational reform literature, CM theory paints a portrait of disengaged youth in urban schools and contrasts them with an unnamed "normal" group (i.e., middle-class white suburban youth). Theorized as the anthropological other, urban youth are then assumed to need a markedly distinctive—even oppositional—curriculum. CM advocates (Gutiérrez 2002; Gutstein 2006) and others (e.g., Secada and Berman 1999) claim that the decontextualized or apolitically contextualized problems of mainstream mathematics favor privileged white youth. Based on this, CM advocates argue for appropriately contextualized mathematics curriculum that will foster mathematical engagement and social understanding among urban youth of color.

I initially assumed that, as impoverished youth of color in a remedial secondary program, my students would naturally be drawn to the CM approach. However, I had to work hard to get students to engage with CM. I often had to invoke my institutional authority to do so by, for example, making it clear to students that participation in CM activities would factor in their grades. However, CM often failed despite such invocations of authority and my attempts to provide "realistic" CM tasks. Rather than being a straightforward cultural match, some of my students described CM activities as "goofy," "not what we're here for" and a better fit for social studies than mathematics.

Rather than a uniform reaction, my study revealed considerable variability in minority student reactions to CM. This likely was due to several factors. In interviews, students reported that CM's explicit political focus was unique and absent in other courses. These older students' response to CM revealed complex and conflicting expectations for secondary mathematics. Some students planned to go to college while others did not. A number of students told me in private that they did not want to discuss politics in front of classmates while others seemed to relish the opportunity. Again, it is important to admit that the range of student reaction to my CM instruction may have been due to my inexperience with CM curriculum design and teaching. It is also possible that the compressed timeline of the night school course made instruction and relationship building with students somewhat more difficult than it would have been in day school.

In sum, while my initial goal was to take critical contexts, themes, and goals seriously, once in the classroom, I felt constrained by the required geometry curriculum and student expectation that I prepare them for college. Given the substantial external and internal pressures for teachers to teach the required mathematics curriculum, I suspect that many mathematics teachers, who might otherwise support critical pedagogy, will be uncomfortable "[leaving] the mathematics to the side" (Gutstein 2006, 108–109) as I became in my study.

References

Atweh, B. (2004). Understanding for changing and changing for understanding: Praxis between practice and theory through action research in mathematics education. In R. Zevenbergen & P. Valero (Eds.), *Researching the social dimensions of mathematics education: Theoretical, methodological and practical issues* (pp. 187–206). Dordrecht: Kluwer.

Bohl, J. (2000). On implementing progressive ideals: The wailing of a beginning mathematics teacher. In W. G. Secada (Ed.), *Changing the faces of mathematics: Perspectives on multiculturalism and gender equity* (pp. 105–114). Reston: National Council of Teachers of Mathematics.

Brantlinger, A. (2005). The geometry of inequality. *Rethinking Schools, 19*(3), 53–55.

Brantlinger, A. (2007). *Geometries of inequality: Teaching and researching critical mathematics in a low-income urban high school*. Unpublished doctoral dissertation, Northwestern University, Evanston.

Frankenstein, M. (1983). Critical mathematics education: An application of Paulo Freire's epistemology. *Boston University Journal of Education, 165*(4), 315–339.

Frankenstein, M. (1989). *Relearning mathematics: A different third R—radical maths.* London: Free Association Books.

Frankenstein, M. (1995). Equity in mathematics education: Class in the world outside the class. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 165–190). Cambridge: Cambridge University Press.

Freire, P. (1971). *Pedagogy of the oppressed.* (M. B. Ramos, Trans.) New York: Seabury Press.

Gee, J. P. (2005). *An introduction to discourse analysis: Theory and method.* London: Routledge.

Gramsci, A. (1971). *Selections from the prison notebooks.* New York: International Publishers.

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: towards a new equity research agenda. *Mathematical Thinking and Learning, 4* (2–3), 145–187.

Gutstein, E., (2003). Teaching and learning mathematic for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37–73.

Gutstein, E. (2006). *Reading and uniting the world with mathematics: Toward a pedagogy for social justice.* New York: Routledge.

Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers*. Milwaukee: Rethinking Schools.

Kitchen, R. S., & Lear, J. M. (2000). Mathematizing Barbie: Using measurement as a means for girls to analyze their sense of body image. In W. G. Secada (Ed.), *Changing the faces of mathematics: Perspectives on multiculturalism and gender equity* (pp. 67–73). Reston: National Council of Teachers of Mathematics.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom.* Cambridge: Harvard University Press.

NCTM. (2000). *Principles and standards for school mathematics*. Reston: NCTM.

Powell, A. B., & Brantlinger, A. (2008). A pluralistic view of critical mathematics. *Proceedings of the Fifth International Conference on Mathematics Education and Society.* Albufeira, Portugal.

Pruyn, M. (1999). *Discourse wars in Gotham-west: A Latino immigrant urban tale of resistance and agency.* Colorado: Westview.

Secada, W. G., & Berman, P. W. (1999). Equity as a value-added dimension in teaching for understanding in school mathematics. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 33–42). Mahwah: Erlbaum.

Shor, I. (1996). *When students have power: Negotiating authority in a critical pedagogy.* Chicago: The University of Chicago.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematical education.* Dordrecht: Kluwer.

Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory into Practice, 34,* 166–173.

Trokan, M. (2009). Rethinking schools director of sales. Personal Communication.

Turner, E. (2003). Critical mathematical agency: Urban middle school students engage in mathematics to investigate, critique, and act upon their world. Unpublished doctoral dissertation, University of Texas, Austin.

Williams, J., & Joseph, G. G. (1993). Statistics and inequality: A global perspective. In D. Newlson, G. G. Joseph, & J. Williams (Eds.), *Multicultural Mathematics: Teaching Mathematics from a Global Perspective* (pp. 175–204). New York: Oxford University Press.

Chapter 40
Mathematics Education: What is the Point?[1]

Laura J. Jacobsen and Jean Mistele

Education is the mechanism many countries around the world use to forge a national identity, to generate a viable workforce, and to create an informed citizenry. The question, "What is the purpose of schooling?," almost invariably leads to responses involving support for democratic ideals, or civic and global responsibility, or fairness and justice, or building moral character, or developing the whole person, or gaining knowledge useful for real life and for economic opportunity. Schools ideally help us to develop multiple forms of literacy—for personal growth, community livelihood, the workforce, and responsible, critical citizenship. Mathematics has an important role to play in helping students to understand and even change the world they live in, yet all too often, schools treat mathematics as a discipline to be learned in isolation from other subjects and from students' lives and interests outside of school. Students exit mathematics classrooms often wondering, "What is the point?" Parents communicate to their children the cultural acceptability of struggling with and not understanding mathematics, pointing out without concern that they never "got it" either. Students routinely complete their school mathematical careers never realizing the significance of mathematics in understanding important social, political, and economic issues facing our communities and our world. This is a form of societal negligence that many educators, and others, recognize must change.

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All mathematics educators will likely agree with the importance of mathematics classrooms providing quality mathematical learning opportunities that challenge students to develop deep mathematical understandings and effective problem solving abilities. Most will further agree with the need to support equitable learning opportunities that set high standards and offer strong support to all students, not just the elite few. But as common as these notions of quality and equity are in mathematics education, both descriptions tend to compartmentalize and overlook certain aspects of what is important in education on the whole. To pursue agendas of quality and equity in mathematics education will require rethinking both traditional *and* reform-based disciplinary representations of "common sense." Specifically, mathematics education must revisit education's roots, working to balance emphases on mathematics content, pedagogy, and purpose. This chapter describes current research efforts and challenges in our Mathematics Education in the Public Interest (MEPI) project, initiated at Radford University in Virginia and focused in mathematics teacher education. We contextualize our efforts in broader research-based frameworks for improving mathematics education in pursuit of quality and equity agendas, while also communicating our own understandings of quality and equity.

Alternatives to Traditional and Reform Emphases in Mathematics Education

For many years, researchers have suggested various forms of classroom or knowledge management and instruction to be stratified across social classes (e.g., Anyon 1980; Bowles and Gintis 1976; Knapp and Woolverton 2003; Moses and Cobb 2001; Oakes et al. 2004; Secada 1992; Tate 1997). Content and pedagogies weak in cultural relevance for students or stemming from Eurocentric perspectives may contribute to race and class divisions in access to knowledge (e.g., Atweh et al. 2001; Ladson-Billings 1995; Lubienski 2002; Rodriguez and Kitchen 2005; Tate 1995). Mathematics, "often regarded as the most abstract subject removed from responsibilities of cultural or social awareness" (Boaler and Staples 2005, p. 32), has been associated with such stratification.

Regarding student learning, research has been largely supportive of reform practices of the kind supported in the United States by the NCTM (1989, 2000). In a comparative study of two schools in England, Boaler (1998) suggested that students who receive project-based instruction learn more and different mathematics than students receiving traditional skills-based instruction. In the United States, relatively consistent evidence also exists that students using reform curricula perform equally well on tests of mathematical skills and procedures in comparison with students using traditional curricula and perform better on tests involving mathematical concepts and problem solving (Schoenfeld 2002; Senk and Thompson 2003).

Equity and social justice agendas in mathematics education have been becoming increasingly central in recent years. Recommendations for how to achieve equity goals almost always include requirements for setting high expectations and providing strong support for all students (e.g., Moses and Cobb 2001; National Council of Teachers of Mathematics [NCTM] 2000). But despite many strengths, reform documents such as the NCTM *Standards* (1989, 2000) still do not go far enough. Lubienski's (2002) criticism of *Standards*-based reforms focuses largely on multicultural considerations of discourse and the NCTM's general oversight of such considerations. Others have focused more on the absence in the *Standards* of a critique of societal inequities (e.g., Apple 1992; Gutstein 2003, 2006).

A small but growing number of researchers and projects have provided alternatives to traditional and reform approaches and emphases in mathematics education. Numerous progressive, multicultural, and social justice educators have suggested educators must adjust practices and curriculum to increase mathematical participation and success of diversifying student populations. Consortiums such as the Center for the Mathematics Education of Latinos/as (CEMELA) and MetroMath provide theoretical and practical guidance to achieve these results, as do texts such as *Rethinking Mathematics* (Gutstein and Peterson 2005), *Relearning Mathematics* (Frankenstein 1989), and *Maththatmatters* (Stocker 2006). Further, projects such as Dartmouth's *Mathematics Across the Curriculum* project and Indiana University's *Mathematics Throughout the Curriculum* project have suggested a need for greater interdisciplinarity and a strengthened mathematical infrastructure in the undergraduate curriculum. Quantitative literacy projects such as *Quantitative Reasoning in the Contemporary World* at the University of Arkansas have strong potential to help students make connections between quantitative information and their lives and interests outside of school. These projects help students to understand the relevance and interconnectedness of mathematics with other subjects and with the real world.

Mathematics Teacher Education and the MEPI Project

According to the 2005 National Center for Education Statistics (NCES) report, *The Condition of Education*, as total school enrollment in the United States has increased, the percentage of public school students considered part of a racial or ethnic minority group has also increased, while the percentage of White public school students has decreased. Further, although school enrollments have become increasingly diverse, incoming teachers remain predominantly non-Hispanic White, middle-class, monolingual females having limited experience with students of backgrounds different from their own (Green and Weaver 1992; Hollins and Guzman 2005; Zumwalt and Craig 2005). The increasing diversity of schools and widening cultural gap between teachers and students, paired with the historical and continuing reproduction of educational inequities by socio-economic group, race/ethnicity, gender, and language proficiency, raises serious and immediate concerns.

Disparities and unequal access to mathematics course taking, achievement, and career fields remain a serious problem for American schools and society (Oakes et al. 2004; Secada 1992), and one that must be remedied. Secada summarized:

> Along a broad range of indicators, from initial achievement in mathematics and course taking to postsecondary degrees and later careers in mathematics-related fields, disparities can be found between Whites and Asian Americans on the one hand and African Americans, Hispanics, and American Indians on the other; between males and females; among groups based on their English language proficiency; and among groups based on social class. (p. 623)

The problem we face is not one to be addressed using deficit models involving fixing people.

To close gaps in general—whether they are associated with gender, race/ethnicity, or social class—will require not only educating students with "mathematics" knowledge, for example, but also rewriting learning objectives to necessarily include feminist perspectives, culturally relevant content, and social justice emphases that help students understand and challenge dominant power relations. Mathematics teacher education has a critical role to play in preparing teachers to put at center stage goals to support equity in mathematics teacher education and to diversify student interest and participation in mathematics. These goals must also resonate with broader public interest goals to improve educational and social conditions both in the United States and abroad.

While internationally there is considerable interest among mathematics educators in social justice, the literature on *mathematics teacher education* for social justice is nearly non-existent (Gates and Jorgensen 2009). Among the limited existing literature is research such as DeFreitas and Zolkower's (2009), which described how social semiotics tasks may enhance teachers' preparation to teach for diversity as well as their disposition toward mathematics and beliefs about the relationship between mathematics and social justice. Boylan (2009) emphasized the connection between emotionality and mathematics teaching for social justice, suggesting the need to create space for dialogue about emotional aspects of mathematics teaching and about sometimes oppressive and alienating mathematics classroom practices. However, practically speaking, almost no attention has been given thus far to preparing preservice teachers to teach mathematics for social justice. This is problematic.

In May 2008, the NSF-funded "Connecting Mathematical Funds of Knowledge Conference" held in Tucson, Arizona, helped teacher educators consider what it means to support preservice teachers to connect children's mathematical thinking with children's and community' funds of knowledge in the context of elementary mathematics methods courses. Such emphases, while still very uncommon, are beginning to take root in a small number of mathematics methods courses across the country. However, mathematics *content* courses engaging preservice teachers in learning mathematics in support of equity and social justice emphases, with an eye toward the relevance of mathematics in local and global communities, have been nearly non-existent.

Drawing from theoretical and practical ideas for social justice in mathematics education, our MEPI project strives to improve the quality and relevance of mathematics education for all learners. MEPI has goals to support equity and social justice, to diversify student interest and participation in mathematics, and to broaden and enrich the ways mathematics is viewed as a discipline. Gutstein (2006) proposed an exploratory orientation toward building mathematics curriculum with integrated components of community knowledge, critical knowledge, and classical knowledge. The twelve characteristics of the Connected, Equitable Mathematics Classroom proposed by Goodell and Parker (2001) also support similar emphases in the rethinking of mathematics. MEPI's foundation rests on an assertion that mathematics curriculum and instruction can be improved by maintaining overlapping objectives that: (1) incorporate NCTM *Standards*-based (2000) reform practices, (2) are more culturally relevant and responsive (e.g., Ladson-Billings 1995), (3) make use of individuals' and groups' funds of knowledge (e.g., Civil 2007; Moll and Gonzales 2004), (4) engage learners more fully, more meaningfully, and more responsibly with their communities (e.g., Hart et al. 2007), and (5) explicitly aim to achieve social justice locally and globally (e.g., Frankenstein 1989; Gutstein 2006).

Math for Social Analysis

Current MEPI research investigates preservice elementary and middle school teachers' experiences and learning in a new, required junior-level mathematics course designed for teachers and centered on equity and social justice principles, *Math for Social Analysis*. *Math for Social Analysis* helps students deepen and increase the flexibility of their understanding of mathematics content. The mathematical learning environment is similar to the recommendations proposed by the NCTM (2000), for example, emphasizing classroom discourse and having students make and defend mathematical arguments. However, in *Math for Social Analysis*, mathematical units are placed in interdisciplinary and social contexts encouraging critical analysis and connections to students' lives outside of school. Students study and mathematize issues such as mountaintop removal in Appalachia, gender bias in magazines, the distribution of wealth, and endangered species.

Math for Social Analysis focuses foremost on mathematics, but dualistically aims: (1) to have the learning of interdisciplinary applications or social issues to strengthen and reinforce mathematical understandings and (2) to have the mathematical activities and projects to reinforce and strengthen understandings of the interdisciplinary applications or social issues. Interdisciplinary content and choice of social issues varies depending on current events, preservice teacher interest, and text selection. Interdisciplinary content always includes many diverse relationships to science, social studies, and language arts. Course content includes discussions of political, social, and economic challenges and implications associated with understanding and even changing the world, using mathematics. Table 40.1 describes the outcomes in *Math for Social Analysis*.

In *Math for Social Analysis*, preservice teachers choose between a service learning option and a group MEPI research/teaching project option.

Table 40.1 Outcomes in *Math for Social Analysis*

Outcomes for All Preservice Teachers in *Math for Social Analysis*
• Develop deep and flexible mathematics understandings, largely through interdisciplinary problem solving applications and critical examinations of social issues • Learn to use manipulatives and technology in mathematics, using inquiry-based approaches – Manipulatives include: base 10 blocks, fraction circles, pattern blocks, geoboards, algebra blocks – Technology includes: virtual manipulatives, calculators, geometer's sketch pad • Learn about and critique the NCTM and state standards • Learn to connect mathematics to children's literature • Complete a semester project (described below)

MEPI Research/Teaching Project Option

For the group MEPI research/teaching project, groups of two to four preservice teachers each choose a social issue on a local, national, or global level. The group studies the issue, writing a research paper to answer the research question of their interest. Next, groups create a mathematics unit plan of two or three related mathematics lesson plans, including the relevant NCTM Standards and state standards of learning. They then teach their classmates the issue, including the pertinent mathematics. Teaching the social issue across disciplines is encouraged, and the use of multiple solution methods to enhance the understanding of the mathematics is expected. We further expect preservice teachers to use inquiry-based and hands-on lessons, focusing on helping learners develop a deep understanding of elementary and middle school concepts.

For example, one group of preservice teachers created and presented to their peers in *Math for Social Analysis* a mathematics unit plan written for fourth-graders and focused on water conservation and access to clean water. Their unit plan involved thinking critically about the amount of clean water available in countries across the world and emphasized water conservation. The unit plan began with a presentation of international facts, including number and percent of people who lack access to clean water, times and distances some people have to spend in walking to collect water, and number of deaths associated with unclean water and poor sanitation. The group showed a publicly available online video addressing clean water access issues in Nicaragua.

The mathematics unit plan, aligned with state and NCTM standards, included an activity to estimate individuals' own personal water consumption, given information such as how much water is used for each minute of showering, washing clothes, brushing teeth, and flushing the toilet. A table of data was presented, displaying the water use (gallons/person/day) in each of 13 different countries worldwide. Preservice teachers then computed the average weekly and annual water usage and compared and graphed average per capita water consumption in numerous countries. They discussed difficulties people face in countries having less access to clean water and calculated ways to reduce water usage. To supplement the mathematics activities in the unit plan, the group provided information about illnesses and diseases associated with access to clean water and sanitation.

Service Learning Project Option

For the service learning project option, preservice teachers complete mathematics-related service learning with a local non-profit organization supporting low-to-moderate-income families through economic and educational programs. Preservice teachers attend a service learning orientation and several training sessions provided by the organization, addressing discipline and conflict resolution, and cultural and economic diversity. Preservice teachers work collaboratively with the organization's staff three hours a day, one day a week, in afterschool programs for elementary and middle school children. The preservice teachers must prepare and present five mathematical activities, designed specifically to meet the interests and needs of the children assigned to them. One activity must include literature, two must address an age appropriate social issue, and the remaining two are open. After each activity, the preservice teacher writes a reflection on their experience planning and implementing their activity. In addition, they have responsibilities in mentoring, tutoring across disciplines, mediation, and discipline. They assist during recess, snack time and they ride the bus as a chaperone, where they learn about the children's communities. At the end of the semester, each preservice teacher gives a presentation of their service learning mathematics activities and experiences to peers in *Math for Social Analysis*. Panel discussions with the class offer an open forum for questions and answers from their classmates.

MEPI Research Results

> My attitude about mathematics for a long time has been dread and confusion…I feel that by teaching math in this new method, people may better understand math because they will be able to learn by relating it to real life…This may also better people's attitudes toward math by showing them its importance and relevance to their future. By using this method of teaching math, we have the opportunity to greatly change the way the world sees math for the better. (Mary 12/11/08)

The teaching of mathematics with a social justice perspective is challenging. Many researchers and educators have tried to merge mathematics and social justice in an effort to connect mathematics to students' lives outside of the classroom only to be confronted with the tension that develops from the shifting focus between the social issues and the mathematics. For example, Civil (2007) communicated the tensions arising from trying to incorporate a funds-of-knowledge approach in the mathematics classroom while still maintaining mathematical rigor. She described a building project designed for a second grade class where the children would be asked, "How do you build a house?" (p. 108), and explained the difficulties of focusing on children's funds of knowledge while ensuring the mathematics content did not become superficial.

Gutstein, when teaching seventh graders, found that he focused more on students learning to read and write the world and less on learning the mathematics. He further described "a dialectical relationship between developing mathematical power and teaching students to use mathematics to study, and potentially change, structural inequality," (p. 108) and indicated how the two processes can both facilitate and produce tensions for learning. Gutstein highlighted the challenge that "To learn rich mathematics students, at some point, have to leave the situation in which the mathematics is embedded and focus on the mathematical ideas themselves" (p. 108). He contended this issue will not be resolved until a social justice curriculum that is connected, cohesive, and comprehensive is developed.

Facing similar tensions as Civil and Gutstein, as we create our own curriculum for the *Math for Social Analysis* course, we routinely struggle to strike an appropriate balance of mathematical sophistication and complexity of the social issues. We aim to address social issues of local or global relevance while still ensuring the mathematics does not become weak or artificial to the task. Specifically, we aim to generate, and to have our preservice teachers generate, questions for which the answer is truly valuable and not simply an abstract mathematical exercise. Typically, we create and pose several sample questions about the social issue and involving mathematics, and we ask preservice teachers to do their own research to generate additional mathematics-related questions for the class to consider. We discuss and also collaborate to answer a selected group of these questions. Further, we regularly look at the multiple different problem-solving approaches the preservice teachers use in responding to those questions, and we consider what additional ways may be possible such as using manipulatives, drawings, or other tools. Again, though, maintaining mathematical rigor with social issues always poses challenges, and our interpretations of our own levels of success with fostering this balance as instructors varies substantially from activity to activity and course section to course section. This is a particularly serious concern because we must ensure preservice teachers complete our course with the mathematics knowledge needed to *teach* mathematics. The notion of rigorous mathematics as typically experienced in mathematics courses does not include the necessity of understanding the mathematical concepts so that they can be conveyed concretely, pictorially, and abstractly in more than one way.

We have documented some of the issues and struggles preservice teachers face as they learn MEPI ideas in *Math for Social Analysis*. For example, when preservice teachers created MEPI lesson plans for their projects, they struggled to balance emphases on mathematics, reform-based pedagogy, and social issues (Spielman and Mistele 2010). Some preservice teachers' projects *applied* various mathematical concepts but failed to *teach* any of the mathematics being used; others had interesting projects but used primarily traditional and/or non-challenging mathematics. Some preservice teachers focused on the mathematics but gave only cursory attention to the meaningful aspects of the social issues; others created lessons with disconnect, or artificial connections, between social issues and the mathematics.

Despite these many challenges, on the whole, we have been very pleased with preservice teachers' feedback from *Math for Social Analysis*, especially feedback from those who described previous struggles with or dislike for mathematics. Preservice teachers universally label this course as their first extended experience with learning mathematics in connection with multiple meaningful real-world applications and social issues. Many of our preservice teachers enter *Math for Social Analysis* describing high levels of mathematics anxiety, and many have suggested that integrating social issues into the mathematics classroom has reduced mathematics anxiety (Mistele and Spielman 2009), which generated positive attitudes towards mathematics and teaching mathematics. In Mistele and Spielman (2009), we communicated how *Math for Social Analysis* proved beneficial in reducing mathematics anxiety among our preservice teachers by increasing the utility of mathematics, redirecting attention away from anxiety, and building confidence to teach. Our research raised new challenges to deficit models based on our observations that mathematics anxiety is situated within the dynamics of the classroom, rather than being located within the individual. Some pre service teachers found their attention was redirected away from their mathematics anxiety and negative attitudes as they became immersed in social issues. In focusing on the social issues, they were at times surprised when they realized they were learning mathematics since they were feeling no anxiety.

One strength of the course thus far has been that preservice teachers develop greater understanding of how to integrate social issues and mathematics, and most describe their interest in incorporating social issues into their mathematics classrooms in the future. For example, one preservice teacher, Allie explained:

> When I become a teacher I will be able to look back on this course and remember how easy it is to teach about a social issue and also about a math lesson. Teaching a social issue will engage the student to be interested in the math lesson…. My students will not only be excellent math whizzes but they will also know what is going on with the world. I will encourage them to use the newspaper, book, Internet, and current issues in their learning experience. I hope to also show other teachers at my future elementary school the different ways they can help and teach their students. I now hope that all teachers will use current issues to teach the children.

Rebecca similarly indicated:

> When I signed up for this class, I had no idea what to expect. I thought it was going to be another pointless class, but instead it is one of the very few classes that I have taken that has really made a difference in me as a future teacher. I am walking away with a whole new approach to teaching mathematics…. By teaching mathematics using social issues, you aren't only informing students on important problems, but you are getting them interested and curious to know about that issue and the math behind it. When involving social issues in mathematics, there are number of different manipulatives that you can use to really help students understand. My [own] knowledge of manipulatives has grown a tremendous amount…. Not only has my ability and understanding to teach mathematics changed drastically, but my own overall attitudes have also changed…I am now excited about math.

Based on MEPI research, we have also previously reported on survey results that provided evidence that preservice teachers' views about mathematics and about mathematics teaching changed over the semester (Spielman 2009). They came to see mathematics as increasingly useful for understanding and engaging with important issues and increasingly connected to home and community experiences. Further, based on qualitative data analysis, we concluded that interwoven mechanisms supporting preservice teachers' engagement with and reframing of mathematics included: (1) Learning the relevance of mathematics to something they care about; (2) Developing interest in mathematical applications and in supporting their future students' interest and learning in mathematics; and (3) Shifting their perspectives on mathematics by changing prior assumptions and instructional goals. As preservice teachers increasingly saw mathematics as relevant and important in social issues, they developed new teaching goals to help students integrate math with other subjects and the world outside of school. Finally, they developed a new sense of agency to create mathematical learning opportunities that students will find interesting and relevant.

Discussion

The research of the MEPI project and other related projects yield complex results with both positive and negative outcomes as well as numerous inherent struggles. However, on the whole, the theoretical arguments and practical results achieved thus far by educators working to support equity and social justice agendas are quite remarkable and lend support to the notion that traditional and reform-based platforms are inadequate.

> When we write a thesis or a paper, we learn that the first thing to do is to latch it on to the discipline at some point. This may be by showing how it is a problem within an existing theoretical and conceptual framework. The boundaries of inquiry are thus set within the framework of what is already established. (Smith 1974)

In 1974, sociologist Dorothy Smith questioned the taken-for-granted assumptions of traditional sociological thought—its methods, conceptual schemes, and theories. Smith began a longstanding effort to develop a sociology for women/people that takes issue with the disjunction that at times exists between women's lived experiences in the world and the theoretical schemes available to think about it. Smith argued that supplementing traditional male notions of sociology with components relevant to women's worlds, such as by addressing omitted and overlooked conversations, only serves to produce women's sociology as an addendum while still maintaining existing sociological thought and procedures and also extending their authority.

The common notion of appending real-world problems to the end of mathematics textbook chapters fails to feature those problems as prominent and guiding. Likewise, an insertion of social, economic, and political issues as add-ons to the mathematics curriculum—or as end of chapter exercises—will not be adequate for transforming projections and interpretations of mathematics as a discipline. The "statistics and figures" content approach that takes mathematics as usual and appends social justice concepts will not be enough (Nolan 2009, p. 207). The MEPI project makes an attempt at featuring social, economic, and political issues that need confronting, and generating appropriate mathematics for addressing those issues, so that the mathematics and the issues ideally emerge hand-in-hand.

Spielman (2008) proposed that similar to Smith's development of a feminist sociology, we can argue that appending components of social justice (or other) theoretical or pedagogical viewpoints to certain accepted mathematics disciplinary constructions and assumptions might further sanction and privilege mainstream thought. To secure a transformative and sustainable impact on mathematics equity, rather than appending "radical" concepts to the mainstream, we need to rethink our very understanding of the discipline. We can rethink common sense in mathematics education and in mathematics teacher education, when we revisit education's purpose and put principles such as "freedom, justice and measures of happiness for all" (Goodlad 2004) at the forefront. The goal should not be simply to make mathematics more applied or interdisciplinary in nature, or to emphasize its relevance in the real world. This is important, but oversimplifies the kind of change that is needed. One goal should be to offer students the opportunity to experience this fundamental purpose of education as they encounter and struggle to make sense of the social issues they study. This is a goal we try to support this through our teaching in *Math for Social Analysis*, where the mathematics that preservice teachers use helps to define and clarify aspects of the issues and of their own thinking.

Neither traditional mathematics classrooms nor reform-based recommendations give serious, explicit attention to the fundamental purposes of schooling. We recognize that the twentieth first century is replete in information that is displayed in charts, graphs, and numbers. The skills needed today to address this new form of information are a direct result of the technology that has burst into our society over the last few decades. People require a new set of skills to understand, reason, and make sense of their world outside of school, work, their personal lives, and their lives as citizens. The *NCTM Standards* (2000) promotes this notion, with its emphasis on reasoning and sense-making as students engage in mathematical communication. However, the *Standards* fail to incorporate these notions to address and critique important societal issues that enrich the democratic process. Orrill (2001) believes, "if individuals lack the ability to think numerically they cannot participate fully in civic life, thereby bringing into question the very basis of government of, by, and for the people" (p. xvi). When we reconstruct mathematics by starting from education's broader objectives and purposes to produce an educated citizenship, the need for a more balanced focus on the mathematics, pedagogy, and purpose becomes increasingly clear. Too many generations of students have completed mathematical studies wondering, "What is the point?" This should end.

> Democratic access to powerful mathematical ideas for social justice requires that students have comprehension of global conditions that are driving the global society and how mathematical and technical knowledge can be tools used to develop a more just world. (Malloy 2008, p. 29)

Usiskin (2001) contends that the math curriculum has not caught up to the realities of modern life due to the recent technology advances, and we agree. As we teach and conduct research in *Math for Social Analysis*, we continue to stand out as uncommon or even radical in our field, yet we believe these objectives do inherently represent "common sense" practices in mathematics education, while simultaneously offering a challenge to balance the rigor of the mathematics with the important social issues.

With regard to a "quality" agenda, the choice educators often make between mathematical rigor and utilitarianism is a false choice. What good is one without the other? A quality agenda should imply students know and understand the mathematics and can also meaningfully interpret and apply the mathematics to relevant issues affecting their lives. With regard to an "equity" agenda, in order to bring a more diverse body of students into mathematics and to produce a more critical, engaged citizenry, we need to raise our expectations for mathematics educators. We have an obligation to help students learn mathematics while keeping broader social and civic agendas in mind. We are hopeful that the work of the MEPI project, including the *Math for Social Analysis* course, can facilitate the joining of mathematics and social issues in ways that address the broader objectives for education and also support preservice teachers to interweave these emphases in their own future teaching.

References

Anyon, J. (1980). Social class and the hidden curriculum of work. *Journal of Education, 162*(1), 67–92.

Apple, M. W. (1992). Do the *standards* go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education, 23*(5), 412–431.

Atweh, B., Forgasz, H., & Nebres, B. (Eds.). (2001). *Sociocultural research on mathematics education: An international perspective.* Mahwah: Erlbaum.

Boaler, J. (1998). *Experiencing school mathematics.* Philadelphia: Open University Press.

Boaler, J., & Staples, M. (2005). Transforming students' lives through an equitable mathematics approach: The case of Railside School. http://math.csudh.edu/MathProject/biblio.html. Accessed 5 Jan 2009.

Bowles, S., & Gintis, H. (1976). *Schooling in capitalist America.* New York: Basic Books.

Boylan, M. (2009). Engaging with issues of emotionality in mathematics teacher education for social justice. *Journal of Mathematics Teacher Education, 12*(6), 427–443.

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nassir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105–117). New York: Teachers College Press.

DeFreitas, E., & Zolkower, B. (2009). Using social semiotics to prepare mathematics teachers to teach for social justice. *Journal of Mathematics Teacher Education, 12*(3), 187–203.

Frankenstein, M. (1989). *Relearning mathematics: A different third R—radical maths.* London: Free Association Books.

Gates, P., & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education, 12*(3), 161–170.

Goodell, J. E., & Parker, L. H. (2001). Creating a connected, equitable mathematics classroom: Facilitating gender equity. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 411–431). Mahwah: Erlbaum.

Goodlad, J. (2004). Fulfilling the public purpose of schooling. *School Administrator, 61*(5), 14–17.
Green, J. E., & Weaver, R. A. (1992). Who aspires to teach? A descriptive study of preservice teachers. *Contemporary Education, 63*(3), 234–238.
Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*(1), 37–73.
Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* New York: Routledge.
Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers.* Milwaukee: Rethinking Schools.
Hart, D., Donnelly, T. M., Youniss, J., & Atkins, R. (2007). High school community service as a predictor of adult voting and volunteering. *American Educational Research Journal, 44*(1), 197–219.
Hollins, E. R., & Guzman, M. T. (2005). Research on preparing teachers for diverse populations. In M. Cochran-Smith & K. M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 477–548). Mahwah: Erlbaum.
Knapp, M. S., & Woolverton, S. (2003). Social class and schooling. In J. A. Banks & C. A. M. Banks (Eds.), *Handbook of research on multicultural education* (2nd ed., pp. 656–681). San Francisco: Jossey Bass.
Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal, 32*(3), 465–491.
Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning, 4*(2–3), 103–125.
Malloy, C. E. (2008). Looking throughout the world for democratic access to mathematics. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 20–31). New York: Routledge.
Mistele, J., & Spielman, L. J. (2009). The impact of "Math for Social Analysis" on math anxiety in elementary preservice teachers. *Academic Exchange Quarterly, 12*(4), 93–97.
Moll, L. C., & Gonzalez, N. (2004). A funds-of-knowledge approach to multicultural education. In J. A. Banks & C. A. Banks (Eds.), *Handbook of research on multicultural education* (2nd ed., pp. 699–715). San Francisco: Jossey Bass.
Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Civil rights from Mississippi to the algebra project.* Boston: Beacon Press.
National Center for Education Statistics. (2005). *The condition of education.* http://nces.ed.gov/programs/coe/. Accessed 8 Oct 2005.
National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston: National Council of Teachers of Mathematics.
Nolan, K. (2009). Mathematics in and through social justice: another misunderstood marriage. *Journal of Mathematics Teacher Education, 12*(3), 205–216.
Oakes, J., Joseph, R., & Muir, K. (2004). Access and achievement in mathematics and science. In J. A. Banks & C. A. M. Banks (Eds.), *Handbook of research on multicultural education* (2nd ed., pp. 69–90). New York: Jossey-Bass.
Orrill, R. (2001). Preface: Mathematics, numeracy, and democracy. In L. A. Steen (Ed.), *Mathematics and democracy: The case for quantitative literacy* (pp. xiii–xx). Princeton: Woodrow Wilson National Fellowship Foundation.
Rodriguez, A. J., & Kitchen, R. S. (2005). *Preparing mathematics and science teachers for diverse classrooms: Promising strategies for transformative pedagogy.* Mahwah: Erlbaum.
Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13–25.
Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623–660). New York: Macmillan.

Senk, S. L., & Thompson, D. R. (Eds.). (2003). *Standards-based school mathematics curricula: What are they? What do students learn?* Mahwah: Erlbaum.

Smith, D. E. (1974). Women's perspective as a radical critique of sociology. *Sociological Inquiry, 44*(1), 7–13.

Spielman, L. J. (2008). Equity in mathematics education: Unions and intersections of feminist and social justice literature. *ZDM—The International Journal on Mathematics Education, 40*(5), 647–657.

Spielman, L. J. (2009, September). Mathematics education in the public interest: Preservice teachers' engagement with and reframing of mathematics. *Proceedings of the 31st annual meeting of the North American chapter for the international group for the psychology of mathematics education (PME-NA)*, Atlanta.

Spielman, L. J., & Mistele, J. (2010, April). *Do not do "Connect the Dots": Mathematics lessons with social issues.* Paper accepted for presentation at the annual meeting of the American Educational Research Association (AERA), Denver.

Stocker, D. (2006). *Maththatmatters.* Toronto: CCPA Education Project.

Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory Into Practice, 34*(3), 166–173.

Tate, W. F. (1997). Race-ethnicity, SES, gender, and language proficiency trends in mathematics achievement: An update. *Journal for Research in Mathematics Education, 28*(6), 652–679.

Usiskin, Z. (2001). Quantitative literacy for the next generation. In A. Steen (Ed.), *Why numbers count: Quantitative literacy for tomorrow's America* (pp. 79–86). New York: College Entrance Examination Board.

Zumwalt, K., & Craig, E. (2005). Teachers' characteristics: Research on the demographic profile. In M. Cochran-Smith & K. M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 111–156). Mahwah: Erlbaum.

Chapter 41
Equity and Quality in a Mathematics Program for Under-Represented Students at an Elite Public University

Vilma Mesa and Robert Megginson

Equity and quality are contested terms that permeate and influence what the educational enterprise is about. As one could expect, the terms acquire different dimensions and expressions depending on the context in which they are used. In the particular setting that concerns us, post-secondary mathematics education at an elite university in a Midwestern state in the United States, we associate equity with access, with making sure that students from non-traditional backgrounds (e.g., rural, urban, first-generation, non-white) have the same opportunity as students from more traditional backgrounds to progress forward in their academic degrees; in particular, we are concerned with equalizing their opportunity to access to science, technology, engineering, and mathematics (STEM) majors. Quality for us corresponds to giving students, in particular those of non-traditional backgrounds, the opportunity to experience the process of generating mathematical knowledge instead of absorbing material that has little or no connection to how that knowledge was generated or no insight about why it is important. High-quality mathematics experiences give students the opportunity to live through the same hurdles and challenges that practicing mathematicians experience when they are solving problems.

The program that we describe in the chapter, the Douglass Houghton Scholars Program (DHSP), demonstrates these two dimensions of equity and quality, access to STEM majors from non-traditional students, and a mathematical experience that is akin to the work of mathematics professionals.

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Background

In November 2006, the voters of the State of Michigan chose to ban the use of affirmative action policies, specifically, the use of gender or race preferences, in university admissions. The Michigan vote meant that public funds could not be used to support programs that were targeted to women or specific racial groups. The passing of the proposal raised concerns that programs that have been designed to create an environment that is welcoming to a racially and culturally diverse community of students, faculty, and staff would cease. Framed under the diversity agenda spearheaded by the University of Michigan's (U-M) president (Coleman 2006) and responding to calls for a more diverse national work force that is trained to do STEM work (Committee on Science Engineering and Public Policy 2007; National Science Foundation Advisory Committee to the Directorate for Education and Human Resources 1996) the DHSP, started in August 2005, with a year-long study of historical data regarding students' enrollment patterns in mathematics courses in their first year at the university.

This initial study revealed that students who enrolled in two mathematics courses in their first year were more likely to continue in a STEM degree than students who did not and that when compared to white students who had similar academic characteristics upon entrance to the university, ethnic minority students had lower enrollments in a second mathematics course in their first year. Moreover, among the students whose grade point average (GPA) was one standard deviation below the mean, the proportion of ethnic minority students was larger than that of students whose GPA was one standard deviation above the mean. In this same group, minority students were enrolling in STEM degrees at dismal numbers: From 2003 to 2005, of the 67 students who took Calculus I and Calculus II in the first year and were majoring in a STEM program, only four were ethnic minorities. During Fall 2005 active recruitment for the program started, and in Fall 2006 the program was launched with 29 students.

Modeled after the Emerging Scholars Program (Asera 2001; Treisman 1992), the DHSP encourages students to excel in calculus by offering them opportunities to work on challenging mathematics problems in a year-long workshop that is taken concurrently with Calculus I and Calculus II. As part of the program, and besides attending the workshop, students go to STEM lectures, listen to STEM scientists explain the type of mathematics that is required for their work or talk about career options, and participate in social events (dinners, theater, or movie outings). The content is difficult and the demands (both cognitive and social) on the students and the instructor are quite high. Students receive only course credit for participating; there is no targeted scholarship support for being in the program or other incentives. Since 2006, we have been collecting data on students' performance in the workshop and information on program implementation that document its successes and challenges. We have now a consistent pattern of results that support claims about the success of the program. However, determining whether the program can be sustained in the long term is a question that depends on at least three inter-related factors: maintaining the quality of instruction delivered in the workshops, providing consistent administrative support, and securing funding.

The remainder of the chapter is organized into three sections. We begin by briefly describing U-M and how the program was implemented here. Then, we present results of the on-going evaluation study using current data available from three cohorts of students to substantiate claims about successes and difficulties in implementing the program. We conclude with a discussion of the need to attend to the connection between instructional, administrative, and economic support to ensure the continuation of programs like this one at highly selective, elite universities.

The University of Michigan, the Introductory Calculus Program, and DHSP

The University of Michigan is the state's oldest university, founded in 1817, 20 years before the Territory of Michigan became a state. It is a public research university with 'very high' research activity according to the Carnegie classification (see http://www.carnegiefoundation.org/classifications/) and one of the eight public "Ivies," that is, a public university that provides an educational experience comparable to that of private Ivy League schools. The university, with over 25,000 undergraduate and 15,000 graduate students in 600 academic programs, has been consistently ranked among the 20 best universities worldwide (U.S. News and World Reports 2008). Each year about 6,000 new undergraduate students are enrolled out of almost 30,000 applicants, of which almost 40% are admitted. Among the admitted students about half have a high school GPA above 3.6 (out of 4.0) and American College Testing (ACT) scores above 28 points (out of a maximum of 36). For a public university, the fees are high, but according to Pamela Fowler, the Executive Director of the Office of Financial Aid, the university is "committed to meeting the demonstrated financial need of undergraduate students who are Michigan residents" (Fowler 2009). This statement means that the University makes every effort to ensure that all its Michigan admitted students could afford their studies. By all these measures, the university is considered a highly selective, elite university.

The Introductory Mathematics Program at U-M seeks to develop students' understanding of basic notions of calculus, and reinforces the need to coordinate verbal, symbolic, graphical, and numerical representations in solving real-world application problems. It consists of three courses, Math 105, Math 115, and Math 116, corresponding to precalculus (functions, graphs, limits), Calculus I (derivatives and basic integration) and Calculus II (advanced integration and vector calculus). Depending on an initial placement test, students can place into or out of the sequence. It is taught in small sections of about 32 students who meet three times a week for about 70 minutes. It uses the textbooks developed by the Harvard Consortium and an instructional approach that fosters in-class group work and the development of mathematical communication. Over 3,000 students enroll in the program every year and since its implementation in the early 1990s, the Introductory Mathematics Program has maintained high passing rates (over 80%).

The analyses of two years of data prior to the implementation of the DHSP determined the profile for students who would most benefit from the DHSP once they had earned admission to the university, and for whom the program would be structured (although invitations would actually be issued in strict compliance with Michigan's recently passed anti-affirmative action law): students underrepresented in STEM fields with an ACT math sub-score of at least 27 points and who had placed into Calculus I. Underrepresented students were defined as students coming from rural and urban schools in Michigan, with a family income in the lower 25th percentile of the entering class, or being from an ethnic minority (in Michigan this corresponds to African American, Latino/a, and Native American students). The program targeted students who also manifested some interest in majoring in a STEM field. The workshop portion of the program was offered as a 4-credit, year-long course to be taken concurrently with each calculus course in the first two semesters that met twice a week for two hours each day. The plan was to recruit 36–40 students distributed in two sections of about 20 students each.

During each workshop class session, students receive a worksheet with 5 to 10 problems that might be related to the content that is being used in the calculus

A

1. (a) Find dy/dx when $y = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x}}}}$.

   (b) Where is this function increasing?

   (c) Where is this function concave down??? How might you tell without differentiation dy/dx?

2. Let

$$f_1(x) = \sqrt{x}$$
$$f_2(x) = \sqrt{x + \sqrt{x}}$$
$$f_3(x) = \sqrt{x + \sqrt{x + \sqrt{x}}},$$

   and so on.

   (a) Where does the function in Problem 1 appear in this sequence of functions?

   (b) Graph f_1, f_2, f_3, f_4, and f_5 on your calculator.

   (c) Let $y = f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}$. Find a "closed form" for this function; that is, find an expression for $f(x)$ that does not involve an infinite number of square roots. Here's a step-by-step plan: square both sides, then write a quadratic equation in y with coefficients that may be in terms of x. Solve this quadratic equation for y.

   (d) Graph $f(x)$ along with f_1, f_2, f_3, f_4, and f_5. Stay tuned: in math 116/146, we discuss convergence of sequences of functions.

   (e) Find where $f(x)$ is increasing/decreasing and concave up/down.

   (f) Describe $f(x)$ as the result of transforming $\sqrt{x}$ by a sequence of horizontal/vertical shifts/expansions/contractions.

Fig. 41.1 Problems used in the workshop. **A** Fall 2006. **B** Fall 2007

B

1. The three cities in the picture below are at the corners of an isosceles right triangle whose legs are 10 miles long. The three city managers, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.

The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the city managers (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

(a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

(b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines."

(c) Can you find a value of x which will produce a shorter network than the simple proposal?

(d) Now suppose the angle at B is sharper than 90°; say it is 70°. Can you still improve the network?

(e) What if the angle at B is very obtuse, like say 150°?

(f) Hmmm. Suppose AB = BC = a and the angle at B is θ. Calculate the length of the second network in terms of a and θ.

(g) Can you say for which values of a and θ the network can be improved? Hint: What will happen if you start with $x = 0$ and then increase x a little bit?

(h) Suppose there are four cities at the corners of a square. Can you guess what the length-minimizing network connecting them is?

(i) Now suppose there are lots of cities. If a network of roads between them is the shortest possible, what do you know about it? Hint: Consider a part of it that looks like the three-city picture.

Fig. 41.1 (continued)

class. The instructor assigns students randomly to groups of three or four students; students work on their own with the goal of completing as many problems as they can in the given period. While students work, the instructor observes and asks questions; the instructor's goal is to make sure students can figure out the problems on their own. Students can share with other groups or write solutions on the board for others to see. During the planning year, the bulk of problems for DHSP were developed and refined for our students, because banks of problems for similar programs around the country did not fit the expectations of the Introductory Mathematics Program. In Fig. 41.1, we present examples of problems used in two different semesters.

A key aspect of the workshop is the quality of the teacher-student interaction. One analysis demonstrated that the instructor asked the students open-ended questions only and refrained from giving definite answers; instead he asked students to ask other students or check with their notes. The questions did contain important hints, but there were usually two or three alternatives that students should consider when they sought help from him (Mesa et al. 2007). A second analysis used appraisal theory (Martin and White 2005) to demonstrate the skillful way in which the instructor used linguistic resources to open up the dialog and the opportunities for students to engage in mathematical conversations (Mesa and Chang 2010).

Three main goals were established that would be used to determine the success of the DHSP: (1) to increase the number of students taking two mathematics courses in their first year; (2) to ensure that students earn A or B in the calculus courses they take; and (3) to increase the number of students who choose a science major. Secondary goals of success included increased learning of calculus concepts and improved attitudes toward mathematics. Findings from the three-year long evaluation study are discussed next.

Findings from the Evaluation Study

An ongoing evaluation study collects data on students (characteristics, math achievement, attitudes, and calculus learning), class implementation (observations, interviews with the instructors and with students), and program operation (interviews with key personnel). Similar data from comparable control samples has been collected to serve as contrast to establish the impact of the program. We start providing data that demonstrate the impact of the program, then we describe the nature of the classroom interaction during the workshop with its impact on the overall workshop's quality, and then we present administrative aspects that play a role in the operation and overall success of the initiative.

Documenting Impact on Students

Across three years, 87 students have started the DHSP; a parallel sample of 87 students some of whom were eligible to participate in the program was selected as control sample. Tables 41.1 and 41.2 summarize characteristics of the students who have participated in the study. The groups are comparable except for their Math ACT score, therefore we used this variable as a covariate in the statistical analyses.

Table 41.1 Frequency of students who are female, minority, and express an initial interest in STEM major by group

	DHSP (n = 87)	Control (n = 87)
Female	48	44
Minority	32	29
Initial interest in STEM major	29	19

Table 41.2 Mean and standard deviation of pre-college characteristics by group

	DHSP (n = 87)	Control (n = 87)
Math ACT	28.1 (2.04)	27.4 (1.97)
Math placement score	18.7 (3.23)	18.23 (3.47)

Table 41.3 Frequency and percentage of students taking two math courses, earning high grades, and choosing a STEM major by group

	DHSP		Control		Significance test
	N[a]	# (%)	N	# (%)	
Took two math courses in first year	87	43 (49%)	87	19 (22%)	$\chi^2(1) = 14.2$, $p < 0.001$
Earned A or B in fall course	86	61 (71%)	81	43 (53%)	$\chi^2(1) = 5.65$, $p < 0.05$
Earned A or B in winter course	62	38 (61%)	26	8 (31%)	$\chi^2(1) = 6.84$, $p < 0.01$
Chose a STEM major	67	39 (58%)	65	24 (18%)	$\chi^2(2) = 16.4$, $p < 0.001$

[a] N represents the number of students with available information. The percentages are calculated using this number.

Table 41.3 summarizes findings for the three main goals set for the program, namely, number of math courses taken in the first year besides the DHSP core courses, percent of students who earned A or B in their math courses in the first year, and number of students who choose a STEM major.

Thus, students in the DHSP take more math courses, earn higher grades, and choose a STEM major more frequently than a comparable sample of students who are not in the DHSP. In addition, these figures are higher than historical data. In the academic year of 2004–2005, 39% of students earned A or B in the fall course, and 28% earned A or B in the winter course. In 2003–2004, the figures were 51% and 45%, respectively. The results are more compelling with minority students: 21 out of 86 students in the DHSP earned A or B in the fall course (24%) compared to 15 out of 81 students in the control group (19%) and 13 out of 62 students in the DHSP earned A or B in the winter course (21%) compared to 2 out of 26 in the control group (8%). Moreover, in spite of manifesting similar intentions of majoring in a STEM field, more minority students in the DHSP manifest interest in a STEM major than minority students in the control sample at the end of the year: 16 out of 67 DHSP students (24%) versus 5 out of 65 control students (8%). All these differences are statistically significant as well.

In addition, DHSP students' scores on a course-independent test of calculus knowledge—the Calculus Concept Inventory (Epstein 2005)—administered three times in a year (September, December, and April) show a significant gain from September to December for both groups, but a decline for the Control group from December to April, suggesting that students in the DHSP retained knowledge better than students in the control sample (see Table 41.4). These results were consistent

Table 41.4 Mean and standard deviation of calculus knowledge and attitudes by test and group

	September test		December test		April test	
	N	Mean (SD)	N	Mean (SD)	N	Mean (SD)
DHSP	87	8.29 (3.39)	77	12.60 (3.49)	54	12.94 (4.08)
Control	57	8.00 (3.55)	33	12.06 (3.91)	29	8.83 (2.59)

even for control students who took two math courses in their first year (Mesa et al. 2007; Mesa and Megginson 2006, 2007, 2008).

In summary, the program accomplishes the goals for which it was designed; students are taking two math courses in their first year, earning good grades, and choosing STEM majors in larger proportions than historical data and than comparable samples of students at the university.

These results, we believe, are related to the type of instruction that occurs during the workshop, which emphasizes students' mathematical problem solving with very challenging problems. Before describing in detail how instruction is conducted in the workshop, it is worth mentioning that the Introductory Calculus Program instructors are trained to maintain a problem-solving environment in their classes as well. However, two main differences exist between the two settings: First, instructors teaching calculus need to cover all the material outlined in the course syllabus because the exams are the same for all students. Second, they need to ensure students' competence with that material and assess them throughout the semester. The DHSP has no specified content to cover, no timeline, and no grades are assigned. Thus, compared to calculus instructors, the DHSP instructors have greater freedom in designing the activities and no time constraints for solving any of them. As grades are not assigned either, another source of anxiety is eliminated; in this way, the workshop becomes an opportunity for students to experience authentic mathematics problem-solving sessions, in which failed solutions are more informative than correct ones and where understanding how mathematics works is central. This is the core of the high-quality mathematical experience that we strive for all DHSP students to have.

Instruction Matters

The observation of workshop classes revealed consistency in the way in which the students and instructors interacted, with the exception of one semester. In any given session, students work on the worksheet, as they feel comfortable, selecting which problems to work on, and noting when their group finished a problem. The instructor's main role is to listen and ask questions and to invite students to write answers on the board. Students can voluntarily write their solutions and are free to share ideas with and ask for suggestions from other groups. As a rule, the instructor never gives answers or solutions to the problems and refrains from stating that something is wrong or right. Instructor's questions are geared toward obtaining more information from the students about their thinking on the problems, helping them to state conjectures, or pursuing alternative ways to look at the problem. Consider the following illustrative excerpt from one session in the first year, near the end of October, in which students were using "u substitution" when solving the first problem in Fig. 41.1a:

[Instructor sits by the group and listens for about a minute]
I: What is u substitution and why?
S: I don't know.
I: Well describe the situation where you've used u substitution before.
S: Well apparently normally uses it for intervals (inaudible) Chain rule effect. So is that to record?
S: Well it's just a different form. It's not really…
S: We haven't ever used…
S: "Chain rule 3.4," I know but I used it for the chain rule because we had like $x + 4$ for example to the eighth power (inaudible) then you bring down the 8 so the 8u7 and u's equal to the stuff on the inside so then the derivative of the inside is 1 so then the answer would be $8x + 57$. So if you just simplify it a little bit more without any of the (inaudible).
I: So in this case u substitution is like a record keeping, you know, a record keeping device?
S: Yeah.
I: It simplifies so you can see what the outside function is?
S: Yeah.
I: Right. So then the question is, is it useful here?
S: I don't know 'cause you're…. The book doesn't even mention it at all, the difficult part, that's calc two.
S: Our book doesn't have any.
S: No, this is all stuff from the AP calc last year.
S: Oh.
S: Because if you had a problem that's like super fast or complicated, u substitution helps out a lot.
S: You have to do it on the board.
I: Would you like to give a command performance?
S: Yeah.
S: Demonstrate…
S: I didn't do b and c, I did 1a.
S: Who cares? Just get on up there!
S: I think we all did b and c once we do those equations.
I: I'll hold you to that. [I moves to another group]

Notice that the instructor starts by getting the students to describe to him their understanding of u substitution and helps them "name" the rationale for why they are using it ("a record keeping device"). The instructor asks them whether the choice is "useful" and even though the students avoid answering, the instructor does not press them to give him a response. Instead, he invites them to write their work on the board. This style of instruction was consistent throughout the first year and praised by students as "empowering," as it allowed them to "find answers on [their] own."

The importance of this instructional style became evident during the first semester of the second year, in which there was a radical departure, a consequence of a change in instructors and of a lack of adequate understanding of the meaning and implementation of this way of teaching. During Fall of the second year, the instructor's interaction with the students both in small groups and in the discussions with the whole group, was characterized by rapid exchanges in which he asked questions that required one or two words from the students, overall using a funneling strategy (Steinbring 1989) that sought to align students' solutions with his own. Consider an excerpt from a session toward the end of October in which students were working on minimizing the length of a route connecting three non-collinear cities, A, B, and C (see Fig. 41.1b), which were connected with lines a, b, and c; the students and the instructor used the lower case letters to refer to the corresponding angles in the triangle connecting the cities:

I: Ok, what was c in the picture?
S: The red [line] one from A to B.
I: c. Ok. So what did you do?
S: We had to multiply what we got for that by 2.
I: So what'd you get for c?
S: Like 2 √ 10x…
I: Tell me how you did it?
S: Ok. Law of cosine (inaudible).
I: Ok, what's *a*? (pause 5 seconds) Tell me which segment I should label *a*?
S: (inaudible)
I: A to B? Ok. And what's *b*?
S: x. (pause-4 seconds)
I: Ok. So c^2? (pause-3 seconds)
S: I don't know, I don't know the law of cosines, our group (inaudible).
I: So $a^2 + b^2$…
S: + ab. No, +2
S: +4
S: −2ab.
I: Cosine of? This angle here right, cosine of angle b. I'm sorry, is that right?
S: No.
I: No. What should it be? (pause-4 seconds) Angle? (pause-4 seconds) What should it be Helena [a pseudonym as all other names in the paper]?
S: What?
I: What angle should it be? Cosine of what?
S: Cosine of d.
I: No. Anne? What angle should I put there? Cosine of?
S: b/2
I: It's this angle right here, right? Which happens to be?
S: 45.
I: 45°. Ok, Anne?
S: Yeah.

As seen in this exchange, which was typical of other lessons in this term, the instructor's questions were geared toward getting information from students about what they did, and his answers were about whether their actions were correct or not. There were no instances in which the students were praised for exploring different avenues and very few instances in which the students gave long sentences when they spoke. Instructor and students' interviews revealed a high level of frustration with the workshop sessions, which translated in the highest attrition level recorded (50%) in the three years. The instructor's need for control was running counter the creation of an environment in which DHSP students could experience authentic mathematical problem solving; thus, we engaged in a series of conversations with the instructor geared to illustrate ways in which students can explore solutions on their own. This was an important element addressing both equity (as most of these students had not had access to this type of instruction) and quality (as we wanted students to live through the experience of creating mathematical knowledge, rather than remember something or providing answers to the instructor).

Using transcripts from the previous year and from his own teaching, the instructor received specific guidance about shifting his focus from precision and elegance of the solution to the process of discovery, emphasizing that it was students' responsibility to find an answer, even though the answer was not the most mathematically elegant. A series of three conversations with the instructor, in which the instructor read transcripts of two of his lessons, read transcripts of other DHSP lessons, and worked out an action plan, helped him realize that he, systematically, did not allow students to finish their responses and that his questions sought a precision that was probably not necessary so early in the process. An outcome of these sessions was an immediate change in the atmosphere of the workshop. Students felt at ease and happy that they stayed, with one student saying: "Last semester I dreaded to come to this class. This semester, it is the only class I look forward to come." In spite of the low retention from the first to the second semester, the end results were in line with the findings from the first year. In the third year, the retention from the first to the second semester was almost 90%. Here is an excerpt in the workshop led by the same instructor in February of the third year, in which students were working on a problem that had appeared on an exam a few semesters earlier (find the volume of a table leg that is manufactured using the rotation of the function $f(x) = 4 + \sin(x)$ between $x = 4$ and $x = 16\pi$):

I: All right, Alex, what have you got?
S: I'm reading this first problem, I don't know, I don't understand where does this rotate about.
I: Good question, that's a good question because it doesn't actually say it, does it? What do you think?
S: Well I mean…
I: And that's the way it was on the exam, by the way. So? (pause 5 seconds)
S: (inaudible) Spinning of itself, so it's (inaudible).
I: So let's ask Alice. Hey, Alice, Alex wants to know how to decide where the thing is rotating about. What's the axis of rotation?
S: Ok, well we had it as the x-axis and then we put it as itself and now we have it back to the x-axis.
S: (inaudible)
S: Well I mean yeah, that's reading part b.
S: That only makes sense.
I: Well so somewhere here it's the original of that picture which shows it a little better. And so how did you guys decide it wasn't 4?
S: Well from part b?
I: From part b?
S: Yeah, because it says its average rate is 1 where the average rate, to be 4 it would have to be rotated about the x-axis (inaudible).
I: Well the other thing I heard you guys saying was, I heard Mary saying well if we rotate it about 4 then sometimes it would be infinitely thin, right?
S: Right.

I: And it would just fall apart, which… that's a good point, right. And I think if the picture were a little better you could tell that it wasn't actually infinitely thin. (inaudible)
S: (many at once)
I: Ok.

The instructor refrains from giving a direct answer to a student question, instead seeking another student from a different group to explain; he also suggests making a better picture to illustrate what is going on. The classroom environment was also more relaxed and, it was clear both from students' behaviors and from end-of-semester focus groups, that students enjoyed being in class and they gained confidence in their abilities through it:

> Male Student: Yeah particularly after the last semester, I mean [my confidence is] still been going up, (…) because, (…) it teaches you how to solve problems in general and I don't know about anybody else, but I was always like sort of intimidated by word problems. But now, between taking physics and DHSP, it's pretty much like I'm very used to that. I'm surprised that all I have to do is solve the equation, like it's in the norm to just solve a really complicated word problem, so I'm more confident, which means I'm also more successful.
>
> Female Student: [I think] another thing that adds to your confidence is that this is such a more comfortable environment, so you feel more confidence to ask questions that you don't necessarily feel comfortable in your other math 116 classes. So that adds to your confidence because you can ask the questions because you're in a more comfortable environment, but then after you ask the question, then you learn how to do the problem on your own.

Thus, the quality of instruction was fundamental for ensuring that the program could fulfill the expected goals. Working in difficult problems and a student-centered instructional style ensures that students will have ownership of the solutions.

Keeping the Program Running

Having positive results is important, especially when they are as dramatic as the ones reported here. At the same time, ensuring that there are students who enroll and providing for the needs of the program are key aspects that cannot be overlooked.

Recruitment for the program has been a very intensive and time-consuming process. Once students are admitted, a committee including the director of recruitment at the college level, the instructor, and the coordinator of the Introductory Mathematics Program, selects all students who qualify to participate in the DHSP. These students receive a brochure in the mail describing the program and encouraging them to apply. The committee reviews the applications in order to determine eligibility: students must place in Calculus I and have an interest in majoring in a STEM discipline. When these students visit the campus during orientation, they are encouraged to enroll in the workshop. Students indicate that instructor involvement in this process is fundamental: they learn firsthand what the workshop is about and make their first—and in some cases only—connection with a faculty member. It is necessary to contact about 200 students in order to create a cohort of 60 students. Scheduling conflicts and not placing into Calculus I (either because their placement score is too low or too high) are the main reasons why students do not enroll in the workshop. Recruiting is conducted from October (when descriptions of the program are mailed to target high-schools) until the first week of classes in September. During the summer months (May to August), recruitment requires about two hours every day from the instructor and advisors to monitor which students have been admitted and explain to them the benefits of participating in the program.

Compared to other undergraduate classes that are taught in the department, the program is onerous. Although the size of the sections is in line with departmental standards, it is a faculty member, not a graduate student, who is in charge of the teaching; there are special sessions throughout the year in which students eat together or attend special performances related to STEM (e.g., Complicite's "Disappearing Number"), and there are administrative costs involved in assigning a faculty member for doing the advising over the summer. The program has been funded by an external grant from a private foundation and has received support from the College of Literature, Science, and the Arts, but it is expected that the Math Department will take control and ownership of the program. It is unclear that the department would be able to support the program, given its cost and the relatively "small" number of students who are being targeted. To ensure support, the department faces dilemmas in supporting the DHSP (and other small programs, e.g., Honors): it serves a specialized and small population, maintenance is onerous, it needs external funding, and collective agreement about mission alignment needs to be established.

Articulating Elements for a Successful Program

Three elements of the program—high-quality and challenging instruction, recruitment, and economic sustainability—are fundamental to its success. These elements are part of the overall structure of an Emergent Scholars Program (Asera 2001), and they are depicted in Fig. 41.2. As noted before, during the semester, in which the class implementation departed from the expected practice, an unproductive class environment resulted in high student attrition. This would correspond to a break in the triangle in the "Workshop Setting" box in Fig. 41.2.

The administrative commitment for recruiting and supporting the day-to-day operation of the program has ensured a smooth implementation during the first three years. This corresponds to the connections within the "Administrative Support" box in Fig. 41.2. The program has a strong advocate, who has secured funds and mobilized resources to start up the program. This corresponds to the connections within the "Campus Negotiations" box. As the figure suggests, it would be impossible to run a successful program when one of the connections is missing: without targeting the right students or without students, there would be no workshop; without high-quality instruction the attrition would be very high; and without funding, it would be impossible to support the instructor. Operational costs are high and, because in three years a relatively small number of students have been reached, doubts emerge about the "real impact" of the program. The onerous recruiting process leads to the

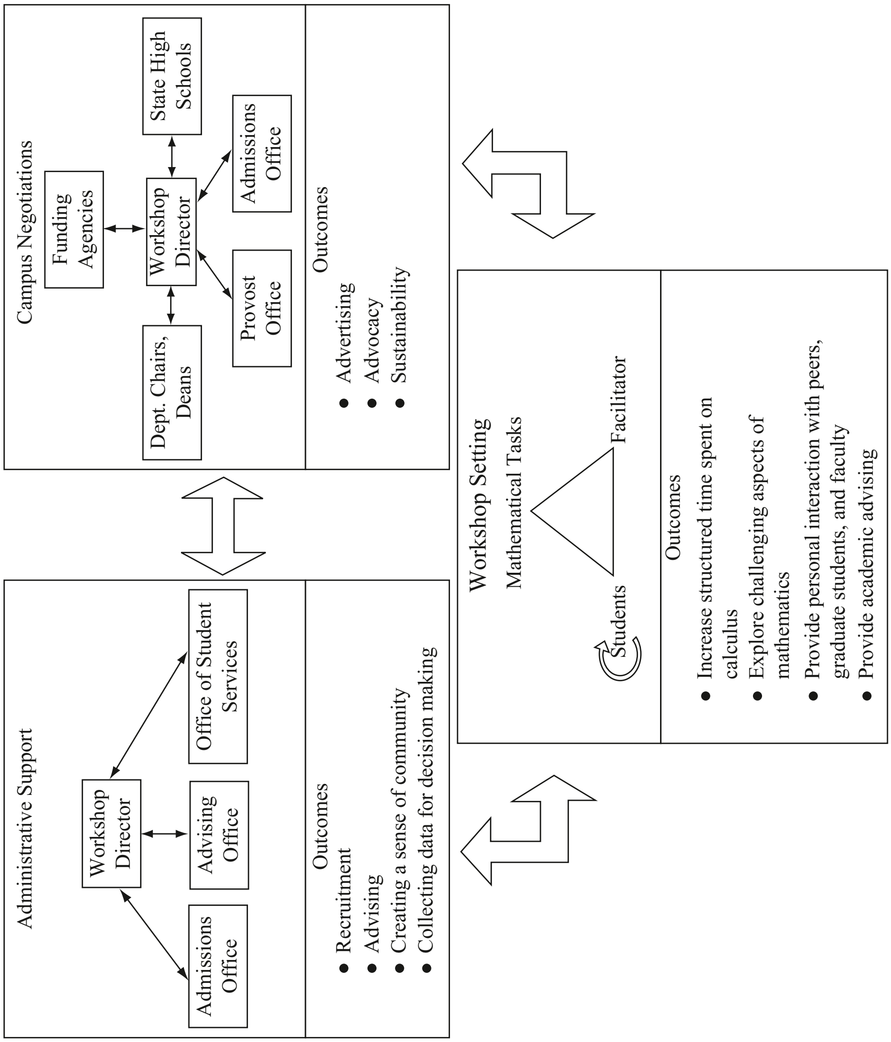

Fig. 41.2 Elements of an ESP program. (Adapted from Asera 2001)

belief that at this elite university there might not be a population for the program, which is contrary to the evidence given by the data.

For us, the question of "what data are convincing enough" is fundamental. When resources are scarce, the decisions that a large research institution makes to support programs are based on impact on large numbers of students—the impact on equitable access needs to be visible. Testimonials from individual students whose lives have been transformed are abundant but not easily quantifiable for making a case for the positive balance between the benefits of the program against its costs: the workshop classes are more diverse than any of their other classes, and beyond gaining confidence in their ability to do mathematics, students make friendships that have lasted beyond that single year together. Maintaining high quality hinges on providing adequate support to sustain the program at all these levels.

Programs that address inequity in access to STEM majors by providing high-quality mathematics instruction are at risk in large elite institutions under a view in which what matters is the 'measurable' economic, short-term return of the 'investment' over the academic and personal gains. Students' lives are transformed in crucial ways when they experience challenging mathematics, or when they choose a STEM major; instructors' understanding of what it means to maintain challenging work with students who are not necessarily considered "good for math" is also transforming and can potentially impact other courses they teach. Quantifying the impact that a steady but continuous increment in the number of students that graduate from STEM majors or documenting the way in which instructors transform their teaching is difficult and takes time. It requires commitment, methodological tools, and a shift in perspective from valuing the short-term over the long-term impact of these initiatives.We believe that the Douglass Houghton Scholars Program has demonstrated convincingly that it has the potential for generating change in the access and retention of under-represented students in STEM fields and look forward to continue documenting its success so it can serve as a model for other post-secondary institutions.

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References

Asera, R. (2001). *Calculus and community: A history of the emerging scholars program*. Washington, D. C.: The College Board. http://professionals.collegeboard.com/data-reports-research/cb/calculus-and-community. Accessed 26 Aug 2005.

Coleman, M. S. (2006). *University accomplishments and challenges: Senate assembly address*. Ann Arbor: University of Michigan. http://www.umich.edu/pres/speeches/050926senate.html. Accessed 19 Jan 2009.

Committee on Science Engineering and Public Policy. (2007). *Rising above the gathering storm: Energizing and employing America.* Washington: National Academy Press.
Epstein, J. (2005). *Development and validation of a Calculus Concept Inventory* (National Science Foundation Grant #0404818). New York: Polytechnic Institute, Brooklyn.
Fowler, P. W. (2009). *Note from the director*. Office of Financial Aid, University of Michigan, Ann Arbor. http://www.finaid.umich.edu/Financial_Aid_Basics/note.asp. Accessed 3 Dec 2009.
Martin, J. R., & White, P. R. R. (2005). *The language of evaluation: Appraisal in English*. New York: Palgrave-Macmillan.
Mesa, V., Boyle-Heimann, K., Mosher, B., Rhea, K., & Megginson, R. (2007). *The Douglass Houghton Scholars Program: Lessons learned in the first year.* Paper presented at the *educating a STEM workforce: New strategies for U-M and the State of Michigan*, University of Michigan, Ann Arbor.
Mesa, V., & Chang, P. (2010). The language of engagement in two highly interactive undergraduate mathematics classrooms. *Linguistics and Education, 21*(2), 83–100. doi:10.1016/j.linged.2010.01.002.
Mesa, V., & Megginson, R. (2006). *Evaluation of the Douglass Houghton scholars program* (Project Description). Ann Arbor: University of Michigan.
Mesa, V., & Megginson, R. (2007). *First year of the Douglass Houghton scholars program: Executive report* (Progress report). Ann Arbor: University of Michigan.
Mesa, V., & Megginson, R. (2008). *Second year of the Douglass Houghton scholars program: Executive report* (Progress report). Ann Arbor: University of Michigan.
National Science Foundation Advisory Committee to the Directorate for Education and Human Resources. (1996). *Shaping the future: New expectations for undergraduate education in science, mathematics, engineering, and technology*. Arlington: National Science Foundation.
Steinbring, H. (1989). Routine and meaning in the mathematics classroom. *For the Learning of Mathematics, 9*(1), 24–33.
Treisman, P. U. (1992). Studying students studying calculus: A look at the lives of minority mathematics students in college. *College Mathematics Journal, 23,* 362–372.
U.S. News and World Reports. (2008). *World's best colleges and universities.* http://www.usnews.com/articles/education/worlds-best-colleges/2008/11/20/worlds-best-colleges-and-universities-top-200.html. Accessed 20 Feb 2009.

Chapter 42
Children's Numerical Thinking in the Early School Grades and How to Foster and Understand Its Development

Álvaro Buenrostro and Olimpia Figueras

In Mexico, different exclusion mechanisms implemented in primary schools are obstacles to providing equity and quality education for the majority of children and turn differences into disadvantages, use a standardized pace for learning and attributing academic failure to children. These three factors adversely impact on children's school performance, particularly among those from families with low income. In order to deal with this issue, the Program to Address Low Academic Performance (PABRE, its acronym in Spanish) was created. A description of the program is the topic of this chapter.

PABRE was set up in 1990 on one of the campuses of the National Autonomous University of Mexico (UNAM) located in a marginalized area of Mexico City. It aimed to investigate:

1. The understanding of the arithmetical knowledge of children attending one of the first three grades of primary school, particularly those deemed by their teachers to be low achievers.
2. The building up of arithmetical knowledge and skills in order that those children can overcome an inequitable situation for learning in their classroom.

Three interwoven components structure the program: Teaching, Service for the Community, and a Research Agenda. These components constitute the means to provide an environment for a learning community in which university students of psychology learn to help children from the early school years to surmount hindrances they face with arithmetical situations and to develop their numerical thinking, guided by the head of PABRE. Since 2000, activities for parents have been incorporated in PABRE in order to help them with different issues regarding psychological assistance and to further support their children at home.

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The Social Setting of Primary School in Mexico

In Latin American societies, and Mexico is no exception, there is an unequal distribution of goods and services that dramatically affects large sectors of the population. Inhabitants of rural communities and marginalized areas of cities live in poverty due to the precarious nature of their financial income. These people have less access to health, housing and educational services which, when available at all, are usually deficient in terms of facilities, equipment and quality. An economic policy that favors a concentration of wealth in very few hands together with a policy to cut public expenditure in social programs accomplishes nothing more than deepening the inequities that already exist (Ortega 2003; Sen and Kliksberg 2008).

Espíndola and León (2002) state that in the educational systems of Latin American countries phenomena such as repeating school years, holding children back a year and low levels of basic knowledge are common, and such phenomena have a greater impact on the poorer sectors of the population. As such, the functioning of Mexico's public schools is impacted by the very asymmetries of society. Access to education, permanence in different educational levels and completion of studies is provided differentially among the various strata of society (Blanco 2009).

While making an analysis of different educational indicators in Mexico, the existence of phenomena that represent barriers to achieving equity came to light. The likelihood of attending school is lower among people from low-income families and communities. A child's socio-economic origin is a determining factor in learning achievement (INEE 2008).

Although it is true that universal access to primary school has been a goal recently achieved in Mexico, it is no less true that many students drop out of school because of their families' economical difficulties. In rural areas, children join their parents in agricultural activities, even when such parents are forced to migrate to other states in the country to get employment in those regions. In the cities one sees that children go out to the streets to sell products, becoming a vulnerable population. Or there are situations in which entire families are compelled to move in order to find employment in another location in the hope of improving their financial circumstances.

This is the situation in which many children fail to complete their studies in the period determined for compulsory education (from 4 to 15 years of age). Some are able to complete those studies later in their lives, while others simply drop out of school never to return, and this leads to functional illiteracy.

Primary School and Low Academic Performance

Basic education in Mexico includes preschool, primary and secondary school. Primary school consists of six grades. It is designed for children aged 6 to 12, and it is organized around school subjects.

Low grades, failing to achieve the target for a school year, repeating school grades and dropping out of school altogether are events in school life that are not unusual among the children classified as students with low academic performance. In Mexican public primary schools, it is quite common to find children who obtain unsatisfactory grades throughout the school year. Those grades, along with other actions of exclusion, lead the child to fail the year and the subsequent need to repeat it or, what is even worse, to drop out of school altogether.

Low academic performance in primary school has multiple facets. It is important to stress that in spite of the improvement in the terminal efficiency attained over the past few years, in Mexico (Martínez 2008, pp. 14–15), many children attending the first three grades of school face situations that lead them to be "low performance students". These children are not exposed to failing and repeating the school year due to an extra-official provision that forbids the school administration failing students in those grades.

If the main objectives of those grades consist of attaining adequate reading, writing and elementary mathematics skills, then one must ask about the teaching of these subjects when a school year is failed. It is true that the low academic performance of some children may be due to situations existing outside of school or to individual conditions, but there is also a consensus in stating that the problem, to a great extent, lies in the quality of the education provided.

School as an institution is characterized by three features that influence student performance in a relevant way: uniformity of the pace for learning, the recognition given to the differences with which children enter school and the processes by which school failures are attributed.

From its earliest existence, the form of a school's organization was taken from the manufacture and production sector of society in which the role of uniformity is fundamental. In education, uniformity can be seen in school curricula, the design of classrooms and in the way textbooks are written. Teachers expect the pace of learning of their students to be uniform, but this is seldom the case. Yet school organization, focusing on the period of time stipulated for awarding grades, discriminates against children whose pace of learning does not meet school requirements.

Uniformity is an aspect that clearly highlights the manner in which schools treat differences. Ignoring them, more often than not leads to the failure of students who, for various reasons, are not performing as the school requires. One could also state that not only are the differences with which students arrive at school ignored, but that those differences are turned into disadvantages. So students who behave differently and who do not meet the school's expectations are labeled in different ways, such as disadvantaged, slow learners, students with learning difficulties and the like.

What is even more worrying is the fact that through a process of attribution of blame, those very students are paradoxically made responsible for the condition with which they are labeled. Many teachers attribute their students' low performance to personal traits before accepting that the situation can be provoked by aspects related to the teacher's behavior and ways of teaching or to conditions prevailing in the classroom. Underlying this type of attribution of blame is a line of logic in which it is the child and child alone who is responsible for his/her academic performance.

Various authors have referred to the role played by these traits in school failure. Briceño (2000) points out, as mentioned before, that the origin of the organization of modern school systems was based on the production systems used in manufacturing and factory production lines. This is associated with a uniform pace for learning, which turns schools into factories of inequalities and of school failure given that they exclude students who learn at a slower pace.

Escudero et al. (2009) relate educational exclusion to the school order and to the discourses that make it up: “It operates by submitting students to certain labeling and classification operations, as well as to a system of attributions and responsibilities, according to which failure is basically due to subjects being <unable> to be successful” (p. 52). They emphasize the importance of looking at the processes by which students’ differences are referred to, valued and treated, because they often become unfair inequalities. Perrenoud (1990) also states that school transforms cultural differences into school inequalities that, through academic evaluations, generate explicit hierarchies of excellence, which determine academic failure or success.

Terigi (2009) classifies academic failure as a psycho-educational problem. She issues a severe criticism of the individual pathological model of school failure and points out that: “in the field of educational practices, the individual pathological model continues to be in effect as the usual interpretation of vast professional sectors faced with academic difficulties” (p. 35). She warns against the use of psycho-educational knowledge as an instrument employed to blame students for failure.

The Psychologists’ Role in Low Mathematics Achievement

How can an educational psychologist contribute to the reduction of the problem of low mathematics performance in primary school? In order to respond to the question, several issues related to the relationship between psychology and mathematics teaching, the roles played by psychologists in education and the processes by which future educational psychologists are trained must be considered.

Kilpatrick (1992) states that psychology has influenced the research carried out in mathematics education. Studies dealing with thought processes and the manner in which mathematics topics are taught and learned are some of the interests in common on both research agendas.

One of the reasons why psychologists participate in the teaching of mathematics is based on a demand originated within the field of educational psychology to draw closer ties between the latter and various academic disciplines. Pintrich (1994) refers to that stated by Shulman (1990) on the subject; he...

> suggested that all too often in the past, educational psychology has not paid sufficient attention to the content and structure of the disciplines that we usually study in school environments and that we have imposed psychological learning models on the contents of those

> areas…the disciplines must be the basis for comprehending the thought of a domain, and the psychological models must adapt to the subject of study and not to the contrary (p. 144).

Similar terms are used by Mayer (2001) when pointing to the development of subject psychologies as an accomplishment of educational psychology; the very intention of subject psychologies is "the learning and the instruction within specific academic subjects such as reading, writing, mathematics, science and history" (p. 84).

The proposals made by Resnick and Ford (1990) also fall within that point of view: "The extent to which psychologists are able to successfully describe what people do when they do mathematics tasks, and how they learn to think mathematically, is the extent to which the psychology of mathematics will be useful for learning" (p. 18).

In recent years, a tendency has been observed that shows greater participation of educational psychologists in school actions and decision making; this is the case in Mexican primary public schools. Right through the levels of the school system, psychologists are able to develop prevention, diagnosis, planning, intervention and research activities. One of the roles of the educational psychologist consists in designing programs for children at risk of school failure.

Despite the important relationship between psychology and the teaching of mathematics, and in spite of the increasingly important role given to educational psychologists regarding low academic performance, in psychology training programs, there is insufficient information to support actions and a lack of ability to intervene in various aspects of low performance in arithmetic.

If indeed it is deemed advisable for educational psychologists to distance themselves from a pathological vision of low academic performance, from a widespread practice that consists of applying psychological tests that frequently lead to a diagnosis of the children in which they are labeled as slow learners, mentally retarded or psychologically immature, then their study programs must change. They need to include information regarding the different processes for building mathematics knowledge and effective ways of furthering it. It is important to create learning environments in which psychology students have the opportunity to identify and evaluate the performance of children considered low achievers in mathematics so as to propose alternatives that enable them to improve their performance and be successful at school (see Buenrostro and Figueras 1999).

The Program to Address Low Academic Performance

PABRE is carried out as part of the educational area curricular activities of the Faculty of Higher Studies Zaragoza (FEZ Zaragoza, its acronym in Spanish) of UNAM. It is a component of a course in the psychology study program.

During the fourth and fifth semesters of the study program, the students review different topics of educational psychology. One of the activities assigned to them, known as Service for the Community, consists of a set of actions similar to those that they would undertake in their professional practices. Students face real-life situations that must be analyzed and solved with the advice of their university teachers. The problem of interest, described in this chapter is low mathematics performance in the first three grades of primary school.

PABRE is organized along three components that comprise basic lines of action to attain the following objectives:

- Teaching:
  Provide students with theoretical and practical knowledge that enables them to give psycho-educational attention to children who have been reported by their teachers as having difficulties articulating language, mastering reading and writing and arithmetic.
- Service for the Community:
  Offer quality psycho-educational support to children from the first three grades of primary school identified as students of low academic performance whose families have low income. The main challenge is to offer these children opportunities to achieve an academic performance that distances them from school year failure, repetition or dropping out.
- Research Agenda:
  Contribute with theoretical and practical proposals that lead to further knowledge of the phenomenon of low academic performance and to search for alternatives to solve this educational problem.

According to PABRE's guidelines, the psychology students see the children twice weekly in working sessions after school. In order to give children the support needed, the students:

- Develop psycho-educational evaluation instruments and apply them to the children who have difficulties in acquiring language, both written language and mathematics.
- Design and apply intervention strategies aimed at solving the difficulties.
- Write psycho-educational evaluation and intervention reports.

Arithmetic in the Early Grades of Primary School

Within the framework of PABRE's teaching and service activities, several qualitative researches and intervention studies have been undertaken (see for example Buenrosto 2003) to progressively build up the project known as "*Arithmetic in the early school grades oriented to understand children's numerical thinking and to foster its development*."

When primary teachers made an application for the admission of a child into the PABRE, several actions aimed at strengthening children's arithmetical knowledge and at improving their academic performance began to be explored. These actions involved the psychology students, the teacher-researcher, the children and their mothers, all working together within an environment that can be characterized as a learning community. The actions carried out are organized into and along three lines of action.

Line of Action 1: Strengthening Children's Knowledge of Arithmetic

In this line of action, emphasis is placed on two core aspects: promoting the children's arithmetical knowledge and contributing to the improvement of their academic performance. Although these aspects are related, they also have different nuances that need to be kept in mind. With respect to the first, the activities carried out with the children are taken from the mathematics education research literature that deals with instruction of arithmetic and the ways of fostering arithmetical knowledge. Regarding the second, the program focuses on the mathematics content that the children must learn in order to achieve a favorable academic performance. At times both aspects go hand in hand, while at others the school practices emphasize content that are not necessarily recommended in the literature.

The task of promoting the building up of arithmetical knowledge is structured around four topics: numeration systems, addition and subtraction, multiplication and sharing, and the numerical processes of quantification (subitizing, counting and estimating), comparison and the part-part whole relationship.

With respect to numeration systems, it is important that the children master both oral and written systems. In terms of the oral system, the program fosters an adequate enunciation of the oral numerical sequence, both from the beginning and from a particular segment. Skip counting is also stressed, especially, 2s, 3s, 5s, 10s, 100s and 1,000s. Regarding the written numeration system, the program focuses on the comprehension of place value together with different forms of written representation of numbers, and on reading and writing of numbers with up to four digits.

In Addition and Subtraction, the children are faced with different types of problems: Change, Combination and Comparison. The program uses a variety of solution strategies to solve the problems, as well as conventional and non-conventional algorithms.

Likewise, for Multiplication and Sharing, the program considers different types of problems and solution strategies together with the corresponding algorithms. Special attention is placed upon mastery of multiplication facts.

The usage of quantification mechanisms is encouraged, particularly "counting on," considered an important element in the generation of more elaborate arithmetical knowledge. The program stresses "counting forward" and "counting of groups", as well as establishing comparisons among numbers and the part-part-whole relationships that exist among them.

Within the work with children, these guiding principles are followed:

- Children's strategies used to solve arithmetical situations, be they considered correct or incorrect, constitute a clue to how the child conceives both the situation and the way to solve it. Hence children's actions represent an opportunity to appreciate their arithmetical knowledge and skills.
- In order to plan activities, use is made of children's informal knowledge as a starting point to favor strategies that are both more economical and of greater conceptual complexity.

- A variety of didactical activities in view of the fact that the children have different learning styles are included in the program. One child may adapt to and feel more comfortable with certain activities than with others; thus flexibility to work with hands-on materials, work sheets and/or educational software is offered to the child.
- The instruction principles expressed by Merrill (2002), when he states that learning is encouraged when students observe a demonstration, apply the new knowledge, activate their previous knowledge, integrate the newly acquired knowledge into their daily experiences and are involved in a task-focused activity are taken into account.
- The use of talk and several actions that foster the building up of children's arithmetical knowledge are used in the program. Through questions, indications and various actions, the teacher and psychology students encourage the children to reflect upon the situations put to them and to seek out more complex and varied responses.

The working sessions with the children are organized in three phases: planning, execution and analysis, considered as a cycle that is repeated. The information obtained from the analysis of the children's actions during the session serves as a basis for the planning and execution phases of the next session, and so on.

One of the objectives of the program is to contribute to improving children's mathematics performance. This can only be accomplished indirectly since no direct intervention in the classroom setting is undertaken; however, a connection is established with the school through the official mathematics textbooks and the children's notebooks.

There are two widely used practices in Mexican primary school classrooms. One is the work done with the textbook; this is a requirement of the education authorities and, for many teachers, it is very important that the exercises contained in the book be solved. The other is the work done with the notebooks, either to reinforce the contents of the textbooks or to include a new content area that the teacher considers important. Analyzing samples of such documents makes it possible to gain an appreciation of the academic demands that must be met by the child in school in order for him/her to be considered of an acceptable academic performance level by his/her teacher. In addition, it provides elements needed to design similar activities to those included in the children's textbooks and notebooks, and to help them to meet the school requirements.

A sample of a specific theme taught to the children is included to illustrate the work done along this line of action.

Addition and Subtraction are a major concern of teachers of the first grades of primary school. Teaching them has traditionally been characterized by emphasizing calculation procedures, on the assumption that once a child has learned to do basic addition and subtraction operations, she/he will be able to solve problems where those operations are needed. For some decades now, researchers have shown that prior to beginning school many children are able to solve some problems using informal strategies, objects and/or fingers.

In recent years, the importance of providing different types of addition and subtraction problems from the very beginning has been highlighted. This change of viewpoint in teaching has made it possible for children to learn on the basis of their previous knowledge, to make sense of the actions they carry out and to master their solution strategies to solve different types of problems. Proposing to teach Addition and Subtraction by problem solving implies taking two important aspects into consideration: the type of problem and the strategies used by the children to solve them.

Story problems (Van de Walle 2004) or whole number situations (Fuson 1992) of four categories: Change Add To, Change Take From, Compare and Combine are posed to the children. The following actions are developed in the problem proposal:

Going from simple to complex situations The semantic structure of the problems differ, consequently, some problems are easier to understand than others. We begin with "Change with Unknown Result" and "Combine All Unknown" problems and go all the way through to "Comparison" problems.

Using transparent wording The manner in which the problems are worded can either facilitate the solution or make it difficult. As a result problems have been worded in independent sentences and in a temporal sequence (for the Change problems), which favors children's comprehension of the problem.

Using diagrams in the problem representation The use of diagrams that represent data relationships is a tool that favors problem understanding and solution. When a problem is posed, a diagram is also presented.

Helping to read the problem Frequently, the children accepted in PABRE are not yet able to read or have a hard time reading the problem. In these cases, either the entire problem or parts of the problem are read to the children. For each sentence read, questions are posed so as to ensure understanding of the word problem.

Depicting problems in different ways Different means are used to pose the problems, for example, using cards that contain both the word problem and a diagram, and situations that occur in everyday life such as purchasing things in a market, movie rental store the bookstore, etc. Work with problems included in the children's school textbooks is also included.

Line of Action 2: Understanding of the Children's Arithmetical Knowledge

The activities undertaken in this line of action are designed to enable the psychology students to:

- Understand the processes by which children build up arithmetical knowledge.
- Identify the changes that are produced in the building up processes.
- Design, choose and carry out didactical activities to promote children's competent use of the arithmetical knowledge.

There are three activities: attendance at seminars, intervention with the children and writing work reports.

Attendance at Seminars

By attending seminars, the university students get relevant information related to arithmetic teaching and learning. Moreover, the students are given tools to identify different strategies and behaviors deployed by the children when different arithmetical situations are put to them.

The information contained in the seminars is divided into three themes. (1) a review of the knowledge features of the four arithmetic topics mentioned previously; (2) informal and formal strategies to solve arithmetical problems, as well as difficulties children face; and (3) the ways of identifying and promoting arithmetical knowledge.

Within the second theme, the students are given general guidelines to conduct their evaluation, intervention and writing a report of the activities carried out. In order to accomplish this, the students review an evaluation instrument that is applied to the children so as to find out what arithmetical knowledge they possess. The students also review several "teacher guides" to orientate their interventions with the children and to report on the changes in the strategies employed by the children.

The third theme refers to the analysis of the arithmetic content of the official textbooks of the first three grades of primary school and the identification of the teaching tendencies underlying the children's notebooks.

Intervention with the Children

For the university students, working with the children is perfect for understanding how they build up their knowledge and how to foster that process. For achieving this, individual working sessions with two students and one child are structured. One of the students conducts the session while the second student records the child's actions. At the next session the roles are reversed. Although there are different ways of recording the data, video recording has been used lately. The videotapes produced become the property of PABRE and can be used subsequently by the university students. In addition to the individual sessions, the program also includes group sessions, where the university students are able to see the advantages of the cooperative learning that takes place when several children undertake a common task.

Writing Work Reports

The students write work reports from the very beginning of their work with the children to the end of those interventions. The reports are documents designed to achieve the following purposes:

- To provide relevant written evidence concerning different aspects of the intervention processes and particularly of the changes in the children's arithmetical performance.
- To serve as the basis for choosing the didactical objectives and activities that are to be undertaken in subsequent working sessions.
- To get information of how students perceive the children's performance, and the way in which they conducted the intervention.

The reports also constitute documents to support the training of students of the next generation of the study program.

In some of the previous didactical experiences, students found it difficult to describe, using words only, the strategies employed by the children. In the majority of the cases, the written information lacked clarity. Consequently, two alternative forms of describing strategies were adopted.

The work of Mayer et al. (1996) was taken into consideration. They used an instruction technique known as "multimedia summary" consisting of a sequence of illustrations accompanied by a brief text to promote student comprehension of scientific explanations. Labinowicz's (1985) ways to describe the actions of children when faced with different mathematics situations has also been used. Children's actions are shown in a series of drawings, each of which is accompanied by a brief description of the action. In overall terms, the following elements are included:

- Description—in written language—of the didactical situation put to the child.
- Graphic and textual description of the child's action.
- Remarks concerning the action.

The second alternative uses short video clips in order to compare the actions of the children before and after the interventions. The video clips are accompanied by an assessment carried out by the student that refers to the type of action and the student's placement of that action within an appropriate context.

The university students record the entire intervention process, and they put it together in an electronic portfolio. That electronic portfolio includes the topics worked with the child. For each child, the portfolio includes the objectives, the materials used, the didactical activities undertaken and, most particularly, the changes in the arithmetical knowledge as a result of the intervention, attempting to make a clear contrast between any given child's initial actions and his/her post-intervention achievements.

An example of the manner in which the theme of Addition and Subtraction is dealt with by the psychology students is included to illustrate the work done along this line of action.

It is useful to recall that one of the concerns of the teacher-researcher, responsible for this program is to distance future educational psychologists from a pathological conception of mathematics difficulties, as well as from the generally accepted idea that children are themselves responsible for low performance in school. The best way to change that conception is by placing psychology students in contact with the knowledge derived from research that deals with arithmetic teaching and learning.

This aim has led to an exploration with the students of teaching situations that encourage a different conception of children's arithmetical performance, as well as the design of educational interventions that promote children's arithmetical thinking. An effort made in this direction is the incorporation of "Communication and Information Technologies" together with an orientation known as "learning by making multimedia products" (Simkin et al. 2002).

The manner in which the aforementioned components are introduced into the study of Addition and Subtraction is described in the following paragraphs.

It is important to know the structure of the additive word problems and the strategies used by the children to solve those problems. These topics are reviewed during the seminars, as are the following activities:

Presenting problems on cards Pairs of students are asked to prepare three examples for each of the eleven types of additive problems, along with their corresponding diagrams. The problems are then reviewed during the seminars, verifying that they are of the type specified and whether the wording is comprehensible for the children.

Preparation of electronic presentations A presentation that includes at least one slide for each of the 11 problems is prepared by the university students. Together with a text, they are required to include an illustration that facilitates understanding of the problem. In addition to reviewing the aspects mentioned in the previous point, an analysis is made of the image's relevance in order to determine whether it is essential to solving the problem or is simply decorative in nature.

Application of educational software Software was designed to display the different types of problems and the most common strategies used by the children to solve the problems. The strategies are shown using video clips of children executing those strategies; the video clips contain data gathered in PABRE itself.

Analysis of textbooks Since mathematics textbooks play an important role in teaching, a review is made of the problems included in those texts. Types of problems, form of presentation and the sequence in which they appear are analyzed.

Once the students have worked with the children, they are asked to make a videotape of the strategies used by the children to solve the problems. The videos are included in an electronic portfolio with which the students can present the changes in the children's strategies and the type of problems that the children are able to solve as a result of the intervention. The videos and the electronic portfolios are collectively analyzed and are used to train subsequent groups of psychology students.

Line of Action 3: Connecting with the Children's Families

The children come to the FES Zaragoza facilities on the UNAM campus accompanied by their mothers and, at times, by their brothers and sisters. This group, together with the university students and the teacher-researcher, forms a learning community. This is a group of people who have common objectives in mind and whose actions will benefit the members of the group.

The support provided to the mothers is structured on the basis of the following five components:

Consultations Two types of consulting are provided. The first deals with a specific subject matter, booklets with information and working materials used by the mothers at home are prepared. One or more talks are given to the mothers to inform them of the importance of the subject matter. The materials are placed at their disposal and activities are practiced so as to further the support for developing children's knowledge. Follow-up is given to the actions undertaken by the mother at home, providing assistance regarding difficulties she may have faced. The second type of consulting is based on the specific needs of each child. As a result of the working sessions, the university students make recommendations to the mothers or suggest that certain activities be done at home.

Consultations of mathematics content In the Mexican primary school system, the textbooks and notebooks are relevant. Consequently, they are reviewed and the way in which mothers can help their children solve the arithmetical situations contained in those materials is explained to them.

Inclusion of the siblings in the activities The children's siblings also benefit from the program. While the children are in the working sessions, their siblings can use board games and books that are available for their use. They are allowed to join the working sessions if they want to. This can be useful since, when the siblings are older, they can help their younger brother or sister at home.

Talks on subjects of family interest In the school setting and in the children's houses, certain problems arise, and it is important to have information concerning those problems. At the beginning of a year, children's mothers are asked on which topics they would like to have more information in order to improve their relationships with their children and families. Based on that information, talks are prepared on the topics chosen. Talks have been given regarding bullying at school, sexual abuse, and family relations, to mention just a few.

Personal marital or family consulting services When the difficulties require individual psychological treatment, the mothers are offered marital or family therapy. Some of the problems that are regularly dealt with include a parent leaving home, alcoholism, sexual abuse, and domestic violence.

The fact that the children and their families attend PABRE generates opportunities for the university students to delve into the problems of their discipline *per se* and to face situations in which they must put their professional skills to the test.

This conjunction of interests and needs characterizes the learning community which, with respect to low academic performance, creates emotional and knowledge-based ties among its participants.

Final Remarks

The program described herein is constantly being adapted since its creation in 1990. Those modifications respond to assessments sustained on reflection regarding different aspects of two components of PABRE—Teaching and the Service for the Community—and the results of the studies undertaken each year (see for example, Buenrostro 2001, 2003).

The factors that adversely impact on children's performance in school, for example, turning difference into disadvantage, uniformity in the pace for learning and attribution of failure to children, are recognized and used for the design of the teaching of the educational component of the psychology study program at FES Zaragoza of UNAM.

Since 1990, around 300 university students have participated in PABRE's activities. The head of this program expects that those who graduated from the study program and now work in education institutions will make efforts to achieve equity in the classroom settings in primary schools. It is also expected that the psychologists would work together with teachers of the first three grades to improve children's mathematical education.

Due to the complexity of the evolution of children's arithmetical knowledge, together with the factors involved in assigning a grade in school, a quantitative measure has not been adopted as a criterion for evaluating PABRE. Emphasis has been placed on the changes in the arithmetical knowledge of each of the children—around 400 in 20 years—changes that are well documented in the reports written by the students and in Buenrostro (2001, 2003).

PABRE has a social recognition in the surroundings of FES Zaragoza's; however, more links between PABRE and the Mexican educational system must be established, in particular, connections with teacher training institutions should be made in order to search for equitable conditions for children to learn mathematics in public primary schools.

Acknowledgments Every child who participated in PABRE's activities had the desire to learn and made efforts for improving her/his academic performance. The children helped the rest of the members of the learning community to understand children's numerical thinking, its development and how to help other children in similar circumstances. We will like to express our acknowledgment to all of them.

During the 20 years of PABRE, different types of support were given to the people responsible for the development of the program. Grants have been received from UNAM (PAPIME—number of reference: PE301606) and the National Council of Science and Technology (Conacyt—number of reference: G37301-S; 2002–2005).

References

Blanco, E. (2009). La desigualdad de resultados educativos. Aportes a la teoría desde la investigación sobre eficacia escolar. *Revista Mexicana de Investigación Educativa, 14*(43), 1019–1049.

Briceño, C. (2000). Escolaridad, calidad y equidad: Convivencia frustrada. Encuentro de Directivos y Altos Funcionarios de los Ministerios de Educación de los países Iberoamericanos. Antigua, Guatemala, 14 al 17 de marzo de 2000.

Buenrostro, A. (2001). Un modelo de enseñanza dirigido a la formación de psicólogos educativos a través del apoyo a niños con bajo rendimiento escolar en aritmética. M. Phil. Thesis. México: Universidad de las Américas.

Buenrostro, A. (2003). Aritmética y bajo rendimiento escolar. Diseño e implementación de dos modelos de enseñanza. Ph.D. Thesis. México: Cinvestav.

Buenrostro, A., & Figueras, O. (1999). The formation of educational psychologists through a program for helping children with a low school performance in arithmetic. In F. Hitt & M. Santos (Eds.), *Proceedings of the twenty first annual meeting of the North American chapter of the international group for the psychology of mathematics education* (Vol. 2, pp. 692–697). Mexico: Cinvestav.

Escudero, J., González, M., & Martínez, B. (2009). El fracaso escolar como exclusión educativa: Comprensión, políticas y prácticas. *Revista Iberoamericana de Educación, 50,* 41–64.

Espíndola, E., & León, A. (2002). La deserción escolar en América Latina: un tema prioritario para la agenda regional. *Revista Iberoamericana de Educación, 30,* 39–62.

Fuson, K. C. (1992). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. Putnam, & R. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 53–187). New Jersey: Lawrence Erlbaum.

INEE. (2008). Panorama educativo de México 2008. Indicadores del Sistema Educativo Nacional. Distrito Federal, México: Instituto Nacional para la Evaluación de la Educación.

Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3–38). New York: Macmillan.

Labinowicz, E. (1985). *Learning from children. New beginnings for teaching numerical thinking*. Menlo Park: Addison-Wesley.

Martínez, F. (2008). *¿Avanza o retrocede la calidad educativa? Tendencias y perspectivas de la educación básica en México*. Informe 2008. INEE.

Mayer, R. E. (2001). What good is educational psychology? The case of cognition and instruction. *Educational Psychologist, 36,* 83–88.

Mayer, R. E., Bove, W., Bryman, A., Mars, R., & Tapangco, L. (1996). When less is more: meaningful learning from visual and verbal summaries of science textbook lessons. *Journal of Educational Psychology, 88*(1), 64–73.

Merrill, D. (2002). First principles of instruction. *Educational Technology Research and Development, 50*(3), 43–59.

Ortega, F. (2003). La equidad en educación básica. *Revista Latinoamericana de Estudios Educativos, XXXIII*(2), 119–134.

Perrenoud, Ph. (1990). *La construcción del éxito y del fracaso escolar*. Madrid: Morata.

Pintrich, P. R. (1994). Continuities and discontinuities: Future directions for research in educational psychology. *Educational Psychologist, 29*(3), 137–148.

Resnick, L. B., & Ford, W. W. (1990). *La enseñanza de las matemáticas y sus fundamentos psicológicos*. Barcelona: Paidós.

Sen, A., & Kliksberg, B. (2008). *Primero la gente: Una mirada desde la ética del desarrollo.* Barcelona: Deusto.

Shulman, L. (1990). Reconnecting foundations to the substance of teacher education. *Teacher's College Record, 91,* 300–310.

Simkins, M., Cole, K., Tavalin, F., & Means, B. (2002). *Increasing student learning through multimedia projects.* Virginia: ASCD.

Terigi, F. (2009). El fracaso escolar desde la perspectiva psicoeducativa: Hacia una reconceptualización situacional. *Revista Iberoamericana de Educación, 50,* 23–39.

Van de Walle, J. (2004) *Elementary and middle school mathematics. Teaching developmentally.* Boston, MA: Allyn and Bacon.

Chapter 43
No Highway and No Destination?

Editor's Reaction to Part IV

Bill Atweh

The chapters in this section deal with lessons learnt by academic researchers and/or school practitioners from attempts to manage equity and quality within various educational contexts (early childhood, primary, secondary, university and the mathematics education profession itself) and with a variety of marginalised populations (African-American, Latino students, low socioeconomic backgrounds, Indigenous students and researchers from non-English-speaking countries). They represent stories from different regions and countries (Australia, Europe, Israel, Mexico, South Africa and the United States). Arguably, they represent different criteria for evaluating quality and equity. Undoubtedly, such a variety makes generalisations from their learnings somehow problematic. There are, however, some overall themes that are worth identifying and some general challenges which require further action and reflection in our practice.

Many Ways to Promote Equity and Quality

Collectively, the chapters in this section point to the fact that action towards the objectives to raise the levels of both equity and quality in mathematics education is not only essential (as the many other chapters in this book argue) but that it is also possible. Experiences in the mathematics education literature about attempts to achieve either equity or quality in mathematics education are rich and varied. What these particular stories reported here have in common is that they acknowledge the need for attempts to achieve both objectives together in the different contexts in which we work. Perhaps there are a few observations that I can make about conditions for productive action in the area. Every program of work reported here has stemmed

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from individuals or groups who have identified a segment of the population who might be excluded from full participation and achievement in mathematics education. Facing this challenge, these authors here shared a (non-naïve) belief and hope that action towards improving participation and achievement in the school subject and the discipline itself is possible, and they shared a determination to be involved in the process of change.

Reading this collection, one cannot but be inspired by the variety of possible types of action and creative solutions that are possible in dealing with the challenge to increase equity and quality in mathematics education. For example, the chapter by *Nelson* discusses the importance of linking with parents of excluded non-English learners in the primary school in looking at the problem of exclusion in mathematics education in the context of a more holistic social problem of exclusion in general society. As that chapter reminds us, parents are not the source of the problem of disadvantage, an opinion which is often implied by reports that blame the family background as a reason for lack of performance, but rather they are an effective source of power to deal with educational problems. The chapter by *Hendrick and Edwards* reports on the collaboration between two teachers from distant schools who offer one advance mathematics subject through the Internet—thus allowing a school with limited resources and low numbers of students who want to pursue such studies the ability to offer their students such a choice. Unfortunately, this creative use of technology is often neglected in the literature in mathematics education. The chapter by *Jacobson and Mistele* develops mathematics activities designed for the increasing students' awareness of social issues thus allowing the discussion of social justice issues in mathematics teaching and at the same time making mathematics more meaningful for the students.

The chapter by *Buenrostro and Figueras* reports on a project that involved university psychology students who undertook a project with low socioeconomic school children at the lower grades to study and improve their development of arithmetical skills. This benefited both the university students themselves and the school children with whom they worked. *Brantlinger* used critical mathematics activities with his secondary Latino and African American school students in order to make mathematics more meaningful and accessible. *Graven and Buytenhuys* report on how a subject on mathematics literacy can transform students from passive non-participants into active negotiators and sense makers of mathematics. The chapter by *Linchevski, Kutscher and Olivier* describes a program of teaching that oscillates between students working together on some common mathematical tasks at times during the lesson, and working separately on more advanced tasks at other times, thus attempting to avoid the problems noted in the literature about streaming of students too early. *Mesa and Megginson* tackle the problem of access to an elite university by students from disadvantaged backgrounds at a time when the State was undergoing a backlash against affirmative action programs. *Mills and Goos* reported on two schools with a large number of students from Indigenous and low socioeconomic backgrounds. They illustrate how disadvantaged schools are often studied for their difficulties and deficit. This chapter points out to the very positive ways in which the schools have attempted to promote equality and quality with their students.

The chapter by *Jaworski, Pone and Mariotti* deals with an important, yet very infrequently researched, problem of exclusion at one of the key international activities in mathematics education, namely, international conferences. For many mathematics educators, participating in international conferences is an essential component of their own professional development as well being a venue for the establishment of collaborations and joint research projects. More importantly, exchanges at conferences are highly influential with regard to learning with and from each other. However, participation in international conferences raises important questions as to who is participating and whose views are given prominence. The chapter discuses how at least one conference attempted to be self critical about its own attempts to promote quality of research exchanges without neglecting its equity commitments.

No Highways

Action to promote equity and quality in mathematics education is not only necessary and possible, but it is not without its difficulties. Most authors in this section were very candid in documenting both the gains achieved and the problems encountered along the way in their endeavours. Questions of equity and quality education do not depend on what happens in the educational settings alone—social conditions and history play crucial roles. Basil Bernstein (1971) was correct in his observation that schools do not compensate for society. However, there is some good news. Research evidence points out that of all the school factors that effected students' achievement, the teacher was the most important. Hence good teaching "can make a difference, but not *all* the difference" (Hayes et al. 2006 p. 178). Collectively, the chapters point out to serious challenges in schools' attempts to reach quality and equity in mathematics education.

Martin and Goos illustrate inspiring stories about principals and teachers who are dedicated to improving the status of their students. However, they point out that in some contexts, in particular, Indigenous education, the historical conditions of neglect and oppression cannot be overcome overnight. Such contexts require special concentrated attention and long-term dedication that require significant resources. They conclude that the agenda of equity will always be an unfinished business of schools and education communities. A similar theme is discussed by *Hendrick and Edwards* who illustrate how successful equity and quality action based on initiatives from a handful of teachers may need significant resources to achieve its aim. Short-term programs can not compensate for long-term disadvantage.

Action to achieve equity and quality often takes the form of special programs that are at times isolated from the general day-to-day running of the teaching of mathematics. This creates some difficulties in achieving higher equity and quality. *Bratlinger* discusses how even the use of critical mathematics posed problems for the teacher—in this case not a very experienced one—in integrating such activities in the teaching of the whole subject. *Jacobson and Mistele* point to the need to maintain a balance between a discussion of social issues and the highlighting of teaching mathematics in classes that use such approaches. Further, *Bratlinger* mentioned some resistance by some students who have perceived these activities as a possible distraction from the main curriculum. His experience points to the urgent need for professional development of teachers as a crucial component of such program implementation. Although not articulated directly in their chapter, the model proposed by *Linchevski, Kutscher and Olivier* demands significant professional development of teachers to achieve its aims. Similar concern was expressed by *Graven and Buytenhuys* who expressed concern that programs that may have great design still leave their implementation open to possibilities of failure because of a lack of teacher expertise.

Nelson points to a great political challenge for equity and quality programs. The chapter reminds us that the rhetoric of equity is not uniformly understood across the profession. More importantly, it is often interpreted in ways that lead to contradictory decisions. Even though the program reported in the chapter was seen to be highly successful by the teacher and parents of targeted Latino students, it was stopped in the school in the name of discrimination towards other students from different backgrounds. Similarly, the chapter by *Jaworski, Pone and Mariotti* points to the ongoing debate in mathematics education at international conferences about the role of the paper presentation review process to maintain and promote the quality agenda without losing sight of the equity implications. Both chapters point to the necessary dialogue on the meaning of both the aims of equity and quality and for looking for creative solutions towards their promotion.

No Destination

Finally, I note that in the above reflection on the chapters, I advisably avoided the use of the term “achieving equity and quality”. From engaging with the stories reported in this section, I became more aware that equity and quality are not states or types of mathematics education to be aspired to and attained. In other words, there is no nirvana where mathematics education is said to be equitable and of the highest quality. Perhaps it is more useful to think of them as challenges to aspire to rather than be accomplished once and for all. Of course, there is a danger that this might lead into stances that argue “no matter what we do, we will not achieve total equity hence there is no need to be too worried about it.” This observation is not a call for defeatism and compliancy—but rather it is a challenge for continual vigilance and dedication to improve the status of the discipline in society and in promoting its power to improve society and the lives of all its members.

References

Bernstein, B. (1971). Class, codes and control. London: Routledge & Paul.

Hayes, D., Mills, M., Christie, P., & Lingard, B. (2006). Teachers and schooling making a difference: Productive pedagogies, assessment and performance. Crows Nest: Allen & Unwin.

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