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Gerhard Aust

Vertical Cooperative Advertising in Supply Chain Management

A Game-Theoretic Analysis

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Vertical Cooperative Advertising in Supply Chain Management

A Game-Theoretic Analysis

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To my beloved wife
Katja

Foreword

Principally, the main idea of supply chain management is to collaborate with other firms along the supply chain in order to leverage strategic positioning and to improve operating efficiency. One possible facet of this collaboration—especially along a manufacturer-retailer-supply-chain—is cooperative advertising. While the manufacturer's advertising strategy aims to create a brand image and is long-term oriented, the retailer concentrates on short-term oriented advertising by communicating prices and special offers to customers. The problem may occur that a single retailer is not able to bear local advertising expenditures at a height desired by the manufacturer. Therefore, it can absolutely be in the interest of the manufacturer to support the retailer's advertising financially to increase sales by which the manufacturer profits in the end, too. Such a financial participation is called vertical cooperative advertising, a practice that is very common in a lot of manufacturer–retailer partnership programs.

This outstanding thesis is a cumulative dissertation consisting of four main papers. The first paper represents the first comprehensive literature review on cooperative advertising. This review comprises 110 scientific articles and gives an excellent overview of the state of the art in cooperative advertising. Within the other three papers, Gerhard Aust chooses a game-theoretic approach to analyze manufacturer–retailer supply chains which differ according to demand functions, structure, and distribution of power. He formulates comprehensive mathematical models to determine the advertising participation rate, retail and wholesale prices, local and global advertising expenditures, and profits.

The second paper considers a typical cooperative advertising situation with one manufacturer and one retailer. In contrast to previous articles, Gerhard Aust relaxes the restrictive assumption of identical margins for both players and applies a general demand function leading to differentiated results concerning dominant game structures. The third paper regards a supply chain which consists of one manufacturer and two retailers who sell substitutable products. It is the first time that a cooperative advertising program is applied to such a setting. It turns out that retail competition has harmful effects on each players' profit. However, due to reduced retail prices, customers benefit from this additional competition. In the fourth paper fuzzy set theory is applied for the first time to a single-manufacturer–single-retailer supply chain with cooperative advertising to incorporate uncertainty of demand parameters into analysis.

In summary, this dissertation is distinguished by extremely sophisticated mathematical models, intelligent and clean solutions, and detailed managerial interpretation. Moreover, it contributes significantly to understand and to improve cooperative advertising programs. Therefore, I hope that Gerhard Aust's excellent dissertation finds large distribution.

Prof. Dr. Udo Buscher

Acknowledgments

This cumulative PhD thesis was written during my time at the Chair of Business Management, especially Industrial Management, at the TU Dresden, where I have been working as a scientific assistant since 2011. I would like to use this opportunity to express my sincere gratitude to those who accompanied me during the last years.

First and foremost, I thank my supervisor, Prof. Dr. Udo Buscher. Already during my diploma thesis, he convinced me to take also the next academic step and to continue with my research. Despite many appointments and duties, he always offered me any possible support and feedback, whereby he significantly contributed to the success of my dissertation project. Besides all these professional aspects, I really value him personally, as he is always concerned with creating a friendly and pleasant working environment at our chair.

As every (current or former) PhD student knows, a dissertation can often be an all-consuming and even solitary work, making the colleagues all the more important. Without them, the time wouldn't have been as enjoyable as it was and is! Therefore, I am deeply indebted to Ina Bräuer, Kirsten Hoffmann, Birgit Keller, Janis Neufeld, and Carolin Witek for all those professional and nonprofessional discussions, the coffee and lunch breaks, the numerous cakes and—certainly less numerous—sparkling wines during our meetings, and even for those minor and major troubles with computers and printers, which always are a welcome diversion. Furthermore, my former colleague Dr. Christian Ullerich has to be mentioned in particular, not only for his critical and detailed proofreading of my thesis. Finally, I would like to thank Evelyn Krug, who relieves us from these administrative duties and truly is the heart of our team.

My greatest thanks go to my family. To my parents, Angelika and Gerhard, whose absolute support I can rely on in every circumstance and any decision of my life. I owe everything to you and it can hardly be expressed in words what you have done for me! And, last but not least, to my wife Katja, to whom I would like to dedicate this book. Without her endless patience, tolerance, and willingness to share all the ups and downs and to talk about sometimes really ivory-tower problems even in late hours, this thesis would not have been possible.

Dresden, Germany Gerhard Aust

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List of Symbols

Chapter 2

a_j	Local advertising expenditures of retailer j [$]
$\mathbf{a}$	Local advertising expenditures vector [$]
A_i	Global advertising expenditures of manufacturer i [$]
$\mathbf{A}$	Global advertising expenditures vector [$]
$\mathcal{B}$	Bargaining game
c_{ik}	Variable costs of manufacturer i for product k [$]
c_{jk}	Variable costs of retailer j for product k [$]
C_i	Fixed costs of manufacturer i [$]
C_j	Fixed costs of retailer j [$]
D_{jk}	Customer demand of product k at retailer j [unit]
$\mathcal{G}$	Game
i	Manufacturer index
I	Number of manufacturers
j	Retailer index
J	Number of retailers
k	Product index
K	Number of products
n	Number of players
$\mathcal{N}$	Set of players
p	Player index
p_{jk}	Retail price charged by retailer j for product k [$/unit]
$\underline{\mathbf{p}}$	Retail price matrix [$/unit]
$r_p(\cdot)$	Best response function of player p
s_p	Strategy of player p
s_{-p}	Strategies of player p's counterparts
s_p^*	Strategy on player p's best response function
s	Strategy combination
s^*	Equilibrium of strategies

S_p	Strategy set of player p
$\mathcal{S}$	Strategy space
t_{ij}	Participation rate offered by manufacturer i to retailer j
$u_p(\cdot)$	Utility function of player p
$\mathcal{U}$	Set of utility functions
v_p	Utility value of player p
$v_{\mathcal{N}}$	Total utility value of set of players N
V_p	Achievable utility values of player p
$\mathcal{V}$	Space of utility values
w_{ijk}	Wholesale price charged by manufacturer i to retailer j for product k [$/unit]
x_p	Arbitrary decision variable of player p
$\underline{x}_p$	Lower limit of x_p
$\overline{x}_p$	Upper limit of x_p
y_p	Pay-off of player p [$]
Y	Total pay-off [$]
λ_p	Bargaining power of player p
μ_p	Risk parameter of player p
Π_p	Profit of player p [$]
Π_{1+2}	Total profit of players 1 and 2 [$]
$\Pi_{\mathrm{m}i}$	Profit of manufacturer i [$]
$\Pi_{\mathrm{r}j}$	Profit of retailer j [$]

Chapter 3

a	Local advertising expenditures [$]
a_j	Local advertising expenditures of retailer j [$]
$a(t)$	Time-dependent advertising level
$a_j(t)$	Time-dependent advertising level of retailer j
$a^{\mathrm{L}}(t)$	Retailer's time-dependent long-term advertising level
$a^{\mathrm{S}}(t)$	Retailer's time-dependent short term advertising level
A	Global advertising expenditures [$]
$A^{\mathrm{L}}(t)$	Manufacturer's time-dependent long-term advertising level
$A^{\mathrm{S}}(t)$	Manufacturers' time-dependent short term advertising level
$g(\cdot)$	Price demand function [unit]
$g_j(\cdot)$	Price demand function of retailer j [unit]
$G(t)$	Time-dependent stock of goodwill
$h(\cdot)$	Advertising demand function
$h_j(\cdot)$	Advertising demand function of retailer j
j	Retailer index
J	Number of retailers
k_{m}	Effectiveness of global advertising [$\$^{-1/2}$]
k_{r}	Effectiveness of local advertising [$\$^{-1/2}$]

k_{rj}	Effectiveness of local advertising of retailer j [$\$^{-1/2}$]
p	Player index
p	Retail price [$/unit]
$p_j(t)$	Time dependent retail price of retailer j [$]
t	Time variable [h]
$u(\cdot)$	Utility function
v	Utility value
v_p	Utility value of player p
v_{1+2}	Total utility value of player 1 and 2
$x(t)$	Time-dependent awareness share
α	Demand parameter
α_j	Demand parameter of retailer j
β	Demand parameter
β_j	Demand parameter of retailer j
γ	Demand parameter
$\bar{\gamma}$	Demand parameter
δ	Demand parameter
κ_m^L	Effectiveness of manufacturer's long term advertising
κ_m^S	Effectiveness of manufacturer's short term advertising
κ_r^L	Effectiveness of retailer's long term advertising
κ_r^S	Effectiveness of retailer's short term advertising
κ	Effectiveness of advertising
κ_j	Effectiveness of advertising of retailer j
λ_p	Bargaining power of player p
μ	Risk parameter
ν	Shape parameter
Π	Profit [$]
σ	Goodwill/customer awareness decay rate [1/h]
σ_j	Goodwill/customer awareness decay rate of retailer j [1/h]

Chapter 4

a	Local advertising expenditures [$]
A	Global advertising expenditures [$]
C	Superscript denoting Cooperation
$g(\cdot)$	Price demand function [unit]
$h(\cdot)$	Advertising demand function
$H(\cdot)$	Hessian matrix
$H_i(\cdot)$	ith principal minor of Hessian matrix $H(\cdot)$
k	Advertising effectiveness ratio
k_m	Effectiveness of global advertising [$\$^{-1/2}$]
k_r	Effectiveness of local advertising [$\$^{-1/2}$]

m	Retailer margin [$/unit]
MS	Superscript denoting Retailer Stackelberg equilibrium
N	Superscript denoting Nash equilibrium
p	Retail price [$/unit]
RS	Superscript denoting Retailer Stackelberg equilibrium
t	Participation rate
$u(\cdot)$	Utility function
$u_{\mathrm{m}}(\cdot)$	Utility function of manufacturer
$u_{\mathrm{r}}(\cdot)$	Utility function of retailer
$v_{\mathrm{m+r}}$	Total utility value of manufacturer and retailer
v_{m}	Utility value of manufacturer
v_{r}	Utility value of retailer
w	Wholesale price [$/unit]
α	Initial base demand [unit]
β	Price sensitivity [unit/$]
λ_{m}	Bargaining power of manufacturer
λ_{r}	Bargaining power of retailer
μ	Risk parameter
μ_{m}	Risk parameter of manufacturer
μ_{r}	Risk parameter of retailer
ν	Shape parameter
Π	Profit [$]
Π_{m}	Profit of manufacturer [$]
$\Delta\Pi_{\mathrm{m}}$	Extra profit of manufacturer [$]
$\Pi_{\mathrm{m}}^{\mathrm{max}}$	Maximum profit of manufacturer without Cooperation [$]
Π_{r}	Profit of retailer [$]
$\Delta\Pi_{\mathrm{r}}$	Extra profit of retailer [$]
$\Pi_{\mathrm{r}}^{\mathrm{max}}$	Maximum profit of manufacturer without Cooperation [$]
$\Pi_{\mathrm{m+r}}$	Total profit of manufacturer and retailer [$]
$\Delta\Pi_{\mathrm{m+r}}$	Total extra profit of manufacturer and retailer [$]
$\Delta\Pi_{\mathrm{m+r}}^{\mathrm{R}}$	Relative total extra profit of manufacturer and retailer

Chapter 5

a_j	Local advertising expenditures of retailer j [$]
A	Global advertising expenditures [$]
B	Intensity of saturation effect [unit^{-2}]
D_j	Demand of retailer j [unit]
$f(\cdot)$	Arbitrary function
F	Set of feasible wholesale prices
$g_j(\cdot)$	Price demand function of retailer j [unit]
$h(\cdot)$	Advertising demand function
j	Retailer index

k_{r} Effectiveness of local advertising [$\$^{-1/2}$]
k_{m} Effectiveness of global advertising [$\$^{-1/2}$]
k Advertising effectiveness ratio
l Index
m_j Margin of retailer j [\$/unit]
MS Superscript denoting Manufacturer Stackelberg equilibrium
VN Superscript denoting Vertical Nash equilibrium
p_j Retail price of retailer j [\$/unit]
t Participation rate
$u(\cdot)$ Utility function
w Wholesale price [\$/unit]
$\tilde{w}_l$ Solution candidates for wholesale price
x Arbitrary variable
α_j Price demand parameter (substituted) [unit]
α Price demand parameter (substituted) [unit]
β Price demand parameter (substituted) [unit/\$]
ϵ Price demand parameter (substituted) [unit/\$]
Λ_j Market size of retail channel j [unit^{-1}]
Λ Market size [unit^{-1}]
Π_{m} Profit of manufacturer [\$]
$\Pi_{\mathrm{r}j}$ Profit of retailer j [\$]
$\Pi_{\mathrm{m+2r}}$ Total supply chain profit [\$]
Θ Channel substitutability [unit^{-1}]
$*$ Superscript denoting optimal solution

Chapter 6

a Local advertising expenditures [\$]
D Demand [unit]
$\mathrm{E}[\cdot]$ Expected value
$f(\cdot)$ Arbitrary function
$f'(\cdot)$ First derivative of $f(\cdot)$
$g(\cdot)$ Price demand function [unit]
$h(\cdot)$ Advertising demand function
$\tilde{k}_{\mathrm{r}}$ Fuzzy effectiveness of local advertising expenditures [$\$^{-1/2}$]
L Superscript denoting pessimistic value
m Retailer margin [\$/unit]
p Retail price [\$/unit]
$\mathrm{Pos}\{\cdot\}$ Possibility measure
r Arbitrary value
t Participation rate
U Superscript denoting optimistic value

w	Wholesale price [$/unit]
x	Arbitrary crisp number
y	Arbitrary crisp number
z	Arbitrary crisp number
$\tilde{\alpha}$	Fuzzy initial base demand [unit]
$\tilde{\beta}$	Fuzzy price sensitivity [unit/$]
η	Arbitrary fuzzy variable
φ	Possibility value
Π_{m}	Profit of manufacturer [$]
$\mathrm{E}[\Pi_{\mathrm{m}}]$	Expected profit of manufacturer [$]
Π_{r}	Profit of retailer [$]
$\mathrm{E}[\Pi_{\mathrm{r}}]$	Expected profit of retailer [$]
Ψ	Parameter (substituted)
ζ	Arbitrary fuzzy variable
$\zeta_{\varphi}^{\mathrm{L}}$	φ-pessimistic value of ζ
$\zeta_{\varphi}^{\mathrm{U}}$	φ-optimistic value of ζ
$\tilde{\zeta}$	Arbitrary triangular fuzzy variable

Chapter 1
Introduction

1.1 Motivation

Increasing competition as well as more and more multifaceted customer requirements make great demands to firms nowadays. The resulting concentration on core business however implicates further complexity, because coordination of activities has to be effected not only within one single company, but in fact across the entire supply chain.[1] Hence, it is not surprising that the coordination of firms within a supply chain has gained substantial interest in research in general, but also in operations research.[2] Thereby, subjects considered are manifold: For instance, the coordination of lot sizes at the interface between vendor and buyer is extensively studied under the name of *Joint economic lot sizing*.[3] Another topic is considered in the so-called *channel coordination* literature, which analyzes pricing in multiple downstream firms and proposes instruments to avoid harmful effects like double marginalization.[4]

In this work, advertising is focused on, which constitutes an important part of many firms' marketing strategy. By way of example, €29.7 billion were spent on

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[1]Cf. Zimmermann (2005): *Supply Chain Koordination*, p. 1. A *supply chain* can be defined as "...the network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer" (Christopher (2006): *Logistics and supply chain management*, p. 17). For an overview on other definitions of that term, see also Mentzer et al. (2001): *Defining supply chain management*.

[2]See, e.g., the reviews in Arshinder and Deshmukh (2008): *Supply chain coordination*, Leng and Parlar (2005): *Game theoretic applications*, Maloni and Benton (1997): *Supply chain partnerships*, and Nagarajan and Sošić (2008): *Game-theoretic analysis*.

[3]See, e.g., the review in Sarmah et al. (2006): *Buyer vendor coordination models*.

[4]See, e.g., the book of Ingene and Parry (2004): *Mathematical models*, which contains different approaches to ensure channel coordination.

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advertising in Germany in 2012, compared to even $139.5 billion in the United States.[5] Interestingly, the three biggest advertisers in Germany are Procter & Gamble (with advertising expenditures of €536.6 million in 2012), Ferrero (€409.2 million) and Media-Saturn (€371.9 million), i.e., two manufacturing and one retailing company.[6]

However, characteristics of advertising can differ considerably between manufacturers and retailers, wherefore this also constitutes an important field of coordination within a supply chain. For instance, advertising campaigns placed by manufacturers do mostly have national dimension and are aimed more on the firm's total sales than on single acquisitions at specific points of sale. Thus, manufacturer advertising often concentrates more on the creation of brand image and less on characteristics of single products. In contrast, advertising emanating from retailers is often limited regionally and primarily communicates prices and special offers to the consumers. Hence, the major target is to attract customers to the own retail stores and to induce immediate sales.[7]

These complementary characteristics and goals can lead to a dependency between the two forms of advertising, which can necessitate a coordinated advertising strategy within a supply chain. Besides that, a manufacturer may depend on a minimum amount of retailer advertising to provoke customers' buying decision. In case of small retailers, it could happen that the manufacturer's requirements exceed the retailer's budget, wherefore it can be beneficial to the manufacturer to support the retailer's advertising financially.[8] Such a form of financial cooperation is called *vertical cooperative advertising* and is a very common form of manufacturer-retailer partnership in practice.[9] The most important variable in such programs is the participation rate, which describes the share of retailers' advertising expenditures taken by the manufacturer. Figure 1.1 shows the results of a survey of 2,286 firms in the United States, where the bars denote the number of firms offering each specific participation rate (in logarithmic scale). Obviously, more than 60 % of the firms actually offer cooperative advertising, with 50 % and 100 % being the most common rates chosen by about 95 % of the participating manufacturers. These self-evident values could suggest that their determination bases rather on arbitrariness than on comprehensive analysis. In addition, further importance arises from the fact that such cooperative advertising programs often represent a point of conflict between manufacturers and retailers. A recent example is the controversy between

[5]Cf. Zentralverband der deutschen Werbewirtschaft (2013): *Werbung in Deutschland 2013*, p. 11, and Kantar Media (2013): *U.S. advertising expenditures (Press release, 11.03.2013).*

[6]Cf. Zentralverband der deutschen Werbewirtschaft (2013): *Werbung in Deutschland 2013*, p. 208.

[7]Cf. Jauschowetz (1995): *Marketing im Lebensmitteleinzelhandel*, p. 236, Oehme (2001): *Handels-Marketing*, p. 444, and Pepels (1995): *Handels-Marketing*, p. 260.

[8]Cf. Somers et al. (1990): *Cooperative advertising expenditures*, p. 36.

[9]According to National Register Publishing, $50 billion cooperative advertising funds are available to retailers in the United States (cf. National Register Publishing (2013): *Co-op advertising sourcebook*). A survey of empirical data on cooperative advertising programs is given in Sect. 3.2.

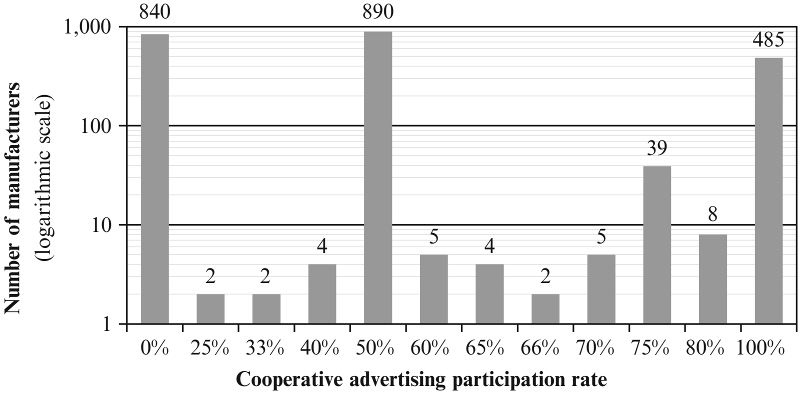

Fig. 1.1 Advertising participation rates commonly offered by manufacturers. Cf. Nagler (2006): *Cooperative advertising participation rates*, p. 96. Please note that all figures in this work are created by the author himself

the German beverage company Krombacher and the German retail chain Kaufland, which even caused a temporary delisting of Krombacher's products twice.[10] This clearly emphasizes the necessity of a well-grounded discussion of the coordination of advertising between the manufacturing and retailing echelon of a supply chain, to which the present work shall contribute.

1.2 Objectives and Research Questions

This work aims at providing decision makers at manufacturing and retailing companies with recommendations on the correct setting of marketing instruments—especially prices and advertising. Particular attention shall be paid to interdependencies, which exist between decisions of firms belonging to the same supply chain. Thus, on the one hand, vertical cooperative advertising programs are considered, representing a widely used example of manufacturer-retailer cooperation. Here, as pointed out in Sect. 1.1, the proper determination of the manufacturer's participation in his[11] retailer's advertising expenditures is focused, which apparently should be as low as possible, but as high as necessary to induce the desired retailer behavior. On the other hand, the underlying distribution of power within the supply chain and its consequences on the best strategies are taken into account.

[10]See Dierig (2012): *Krombacher* and Dierig (2013): *Schweppes*.

[11]Please note that we use the masculine forms "he" and "his" when referring to general notations like manufacturer, retailer, player, competitor, etc. throughout this work for the sake of convenience.

In the domain of operations research, this decision support is derived formal-analytically by means of quantitative models. These mathematical models seek to describe reality in a simplified manner so that it is possible to identify interrelations that can be conferred to real situations in practice.[12] In this work, the consideration of decision making within a supply chain, i.e., of more than one firm, additionally necessitates the application of game theory. Related techniques allow to incorporate not only the aforementioned interdependencies of multiple firms' strategies, but also the aspect of power imbalance which may occur within a supply chain.

Since there already exists a multiplicity of modeling approaches related to this topic, the first step is to review the available studies. This is necessary to identify appropriate mathematical formulations which can be used and further extended to meet the requirements of this work. In addition, a systematic survey allows to reveal both deficits of present models and needs for further research.

As stated above, the main goal is to offer decision support for manufacturers and retailers regarding the best pricing, advertising, and cooperative advertising decisions. Hence, first emphasis shall be placed on the interdependencies and interactions which may occur between the supply chain echelons, i.e., between manufacturing and retailing. Here, it is important that the recommendations derived account for the underlying characteristics of market and customers as accurately as possible. Furthermore, different distributions of power within the supply chain shall be considered, accompanied by advices on the best strategy related to the respective situation.

However, interdependencies do not only exist between different echelons of a supply chain, but also within a single echelon. Therefore, the next goal is to incorporate intra-echelon competition into the consideration, in order to analyze how the firms' decisions are affected. As the topic of this work is not only to find the best strategy regarding pricing and advertising, but also the most advantageous design of a cooperative advertising program, it seems appropriate to concentrate the analysis on retail competition. Thereby, it will not only be possible to analyze the effects of competition on retail prices, which directly impact customer welfare, but also to recommend how the manufacturer should handle this changed situation and how the cooperative advertising program should by modified accordingly.

The importance of the underlying market and customer characteristics on the firms' correct decision on strategies has already been mentioned. However, in practice it is often difficult and complex (if not impossible) for firms to determine them. Thus, this work shall also address this issue and propose an approach which helps to deal with situations where market and customer characteristics are not known in detail. Apparently, this can only be a vague estimation of the real conditions, but this will certainly increase the suitability for decision makers in practice.

These objectives named above can be summarized to four research questions on pricing, advertising, and cooperative advertising in a manufacturer-retailer supply chain:

[12] See, e.g., the introductory section of Jensen and Bard (2003): *Operations research*, pp. 2–12.

Q1: What is the actual state of research on cooperative advertising and what are the directions for further research in that field?

Q2: How should prices, advertising, and cooperative advertising program be set in a manufacturer-retailer supply chain with respect to the underlying distribution of power?

Q3: What are the effects of retail-competition on the firms' decisions and how should the manufacturer adapt his cooperative advertising program?

Q4: How can firms determine their best strategy when data on market and customer characteristics is imprecise or missing?

The structure which is used to work on these research questions is presented in the next section.

1.3 Structure of This Work

As visible in Fig. 1.2, this work consists of seven chapters, which can be roughly grouped into the four blocks *Introduction*, *Theoretical fundamentals*, *Mathematical models*, and *Résumé* as it is illustrated by dashed frames. Furthermore, the figure shows the main contents of each chapter in white boxes, whereby boxes with angular edges denote topics addressed in the correspondent chapter, while rounded edges refer to major properties of the proposed mathematical models.

After pointing out the motivation of the topic and the objectives pursued in this work in **Chap. 1**, some theoretical fundamentals shall be given in **Chap. 2**. This includes introducing general principles of game theory in Sect. 2.1, which shall help to familiarize with the techniques used in the sequel, and a general cooperative advertising model that is successively adapted to the changing requirements during this work (see Sect. 2.2). Subsequently, each of the following four chapters corresponds to one research question stated in Sect. 1.2, as it is indicated in Fig. 1.2.[13]

At first, **Chap. 3** provides a detailed review of existing mathematical models on cooperative advertising. After a short explanation of the searching strategy applied and the different definitions of cooperative advertising found throughout the search (see Sect. 3.1), theoretical foundations of cooperative advertising and empirical data on its occurrence in practice are given in Sect. 3.2. In Sect. 3.3, the review of 58 mathematical models follows, with respect to their general setting (i.e., considered decision variables, underlying supply chain structure, etc.), demand and cost functions they base on, and game-theoretic techniques applied. Finally, we summarize the insights gathered in Sect. 3.4, together with a listing of possible directions for future research on cooperative advertising.

[13]Please note that each of these chapters also represents one separate research article. These four research articles constitute this cumulative dissertation.

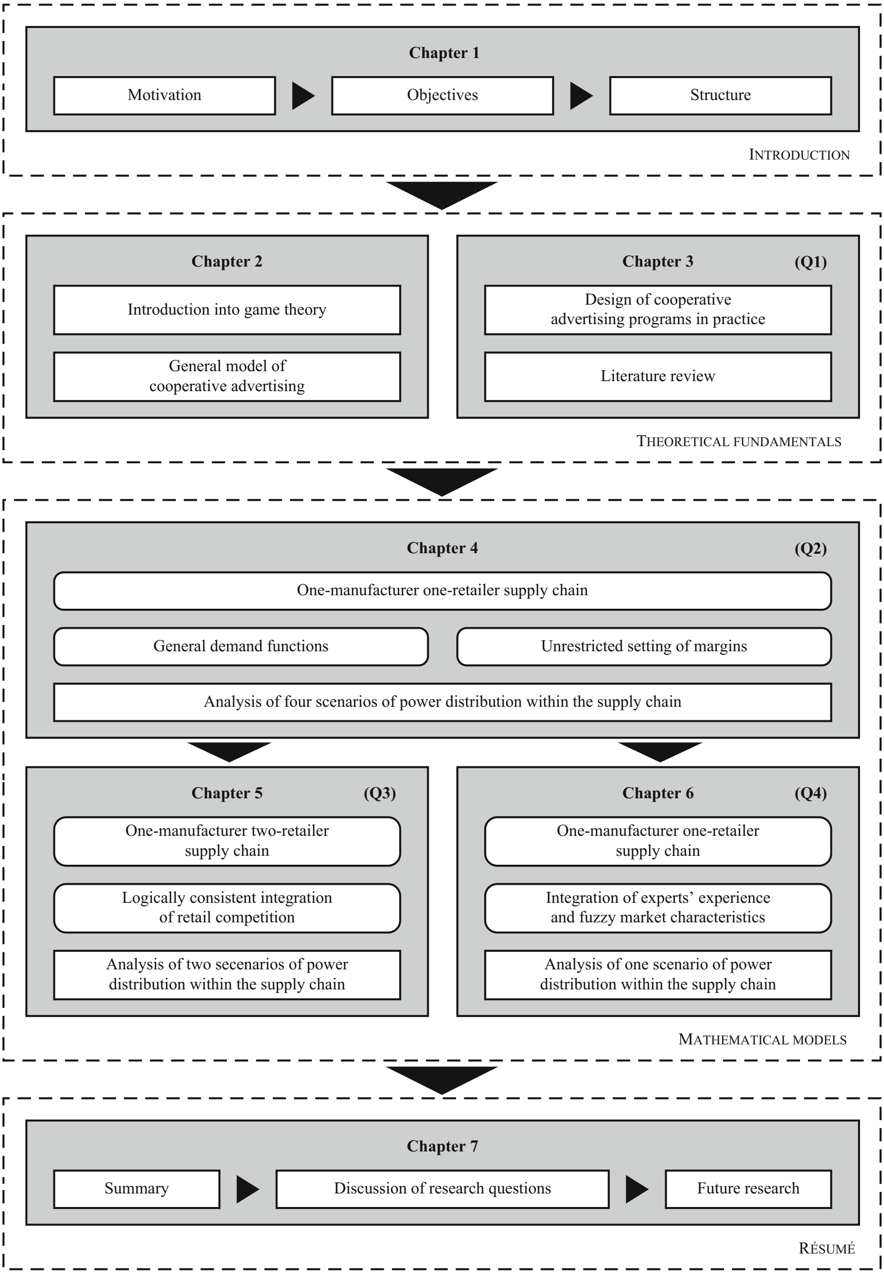

Fig. 1.2 Structure of this work

A model dealing with a one-manufacturer one-retailer supply chain is presented in **Chap. 4**, which shall be used to analyze the interdependencies between the decisions of manufacturers and retailers. The mathematical formulation of this model follows the basic formulation proposed by Xie and Wei (2009) and its extension by SeyedEsfahani et al. (2011).[14] It allows to determine the optimal wholesale and retail prices, the optimal advertising expenditures both of manufacturer and retailer, and the optimal participation rate of the manufacturer in his retailer's advertising cost. Thereby, four different inter-echelon distributions of power are analyzed:

- a symmetric distribution, where no echelon is able to take any competitive advantages,
- two situations with one firm obtaining the channel leadership, respectively, and
- a cooperation of manufacturer and retailer, where both tend to maximize the whole channels profit.

Besides a more general demand function, the major advantage of the model proposed in this work can be seen in a modification, which allows to determine truly unconstrained solutions, while previous studies assume identical margins of manufacturer and retailer in some cases.[15] A detailed description of this issue is given in Sect. 4.1, while the suggested modification is presented at the beginning of Sect. 4.2. After that, the aforementioned scenarios of distribution of power within the supply chain are analyzed and closed-form expressions are given for prices, advertising expenditures, and cooperative advertising participation rate (see Sects. 4.2.2–4.2.5).

Furthermore, we introduce a bargaining model in Sect. 4.2.6, which can be used to divide out the total profit resulting from cooperation between manufacturer and retailer. To the best of our knowledge, this model has not been applied to cooperative advertising models so far. In addition, though it incorporates both risk behavior and bargaining power of the involved parties, it is simpler in use than previous ones. A detailed discussion of the results based on numerical examples follows in Sect. 4.3, with special focus on effects of the different scenarios on the resulting prices, advertising expenditures and firms' profits. A résumé of the most important managerial implications is given in Sect. 4.4.

After the consideration of the inter-echelon interdependencies, the next objective is to analyze the effects of intra-echelon competition on the firms' pricing, advertising, and the cooperative advertising program offered by the manufacturer. For this reason, the model previously explained is extended by a second retailer, which allows to study retail competition (see **Chap. 5**). Though some additional assump-

[14] See Xie and Wei (2009): *Coordinating advertising* and SeyedEsfahani et al. (2011): *Vertical co-op advertising*.

[15] SeyedEsfahani et al. (2011) use this restriction both in the Nash and in Retailer Stackelberg game (cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 266). The same restriction can be found in Xie and Neyret (2009), though this article bases on a different demand function (cf. Xie and Neyret (2009): *Co-op advertising*, p. 1377).

tions are necessary to render possible this extension, the proposed model is still more general than other existing models, as it also allows determining optimal prices, while other studies assume prices to be given exogenously. However, especially the integration of prices as decision variables may—depending on the demand function applied—cause logical inconsistencies when considering a retailer duopoly, which would limit the practical usefulness of the derived recommendations.[16] Hence, the here presented model follows Ingene and Parry (2007) and derives the price demand functions for the two-retailer case from the customers' utility function (see Sect. 5.2 for a detailed explanation).[17] Based on that, two different distributions of power are analyzed in Sect. 5.3:

- a situation where each firm, i.e., the manufacturer and both retailers, have equal power, and
- a situation where the manufacturer obtains the channel leadership, while the two retailers are still equal.

Section 5.4 is dedicated to the interpretation of the obtained results, whereby the focus lies on the comparison of the considered scenarios, inter alia, with respect to the degree of retail competition. The findings are summarized in Sect. 5.5.

Up to now, each model presented here assumes that market and customer characteristics are entirely known to the firms. Obviously, the compliance with this prerequisite is not always given in practice. Reasons for this can be manifold: Maybe, the collection of comprehensive market data is too complex and too expensive or, in case of introducing a new product, there is simply no existing market which can be surveyed. In that case, firms often revert to the experience of experts in order to estimate the missing data. In **Chap. 6**, we demonstrate how these experiences can be included into the mathematical model. Difficulties arise from the fact that expert opinions are usually rather vague and expressed in linguistic terms like *small*, *medium*, or *high* customer demand, which cannot be included by implication. Here, this problem is solved by means of the fuzzy set theory, which was established by Zadeh (1965).[18] This framework allows to convert linguistic terms into fuzzy variables, which still contain the fuzziness of the original statement.

[16]This comment refers to the so-called *Competitive-Substitutability Hypothesis* established by Ingene and Parry (2007): It describes the counterintuitive effect that stronger competition between the retailers leads to rising prices and profits when using the cross-price parameter within a linear price demand curve as a measure for competition (cf. Ingene and Parry (2007): *Bilateral monopoly*, pp. 599 et seq.). For instance, this effect can be observed in Yang and Zhou (2006), who consider a one-manufacturer two-retailer supply chain where only prices are decision variables (cf. Yang and Zhou (2006): *Two-echelon supply chain models*, p. 113). To overcome this issue, Ingene and Parry (2007) propose to derive the price demand function from the customers' utility function.

[17]See Ingene and Parry (2007): *Bilateral monopoly*.

[18]See Zadeh (1965): *Fuzzy sets*.

A short introduction into fuzzy set theory as well as some calculation rules which are necessary in the sequel are given in Sect. 6.2. After that, optimal solutions for a situation where the manufacturer obtains the channel leadership are derived in a simplified version of the model proposed in Chap. 4, where the market and customer characteristics are modeled as fuzzy parameters (see Sect. 6.3). In Sect. 6.4, a numerical example is then used to demonstrate the integration of linguistic expressions into the model. Furthermore, some analyses regarding the effects of parameters' fuzziness on the recommended firms' strategies are conducted. Finally, Sect. 6.5 recapitulates the procedure and addresses limitations of this first approach to apply fuzzy set theory to a cooperative advertising model.

In **Chap. 7**, a summary of this work is given, followed by a discussion of the research questions established in Sect. 1.2. Furthermore, we point out some existing drawbacks, together with possible directions for future research.

Chapter 2
Fundamentals

2.1 The Application of Game Theory

2.1.1 Theoretical Framework

Commonly, the book of von Neumann and Morgenstern (1944)[1] shall be deemed to be the origin of *game theory*, even if there also exist earlier publications related to that field, which indeed rather focus on special problems than on a comprehensive theory.[2] Thereby, the authors intended to establish a mathematical framework which is able to describe the strategic behavior of individuals.[3] In subsequent years, many other researchers contributed to the development of this research discipline, thereunder well-known names like Nash, Selten, and Shapley.[4]

Generally, game theory belongs to mathematics and is often also assigned to operations research. Thereby, we can distinguish two research streams: firstly, rather mathematical works, which are aimed on enhancements regarding theoretical principles and the derivation of new solution methods; secondly, studies that apply game-theoretical concepts and solution techniques to questions emanating from other disciplines. This approach can be observed particularly in economics, but also the application on problems related to business management advances in recent years. This work belongs to the latter group.

[1]See von Neumann and Morgenstern (1944): *Theory of games*.

[2]Cf. Berninghaus et al. (2010): *Strategische Spiele*, p. 1 and Rieck (2010): *Spieltheorie*, p. 21.

[3]Cf. von Neumann and Morgenstern (1953): *Theory of games*, pp. 1–6.

[4]For a summary of the history of game theory, see Berninghaus et al. (2010): *Strategische Spiele*, pp. 3–9, and Peters (2008): *Game theory*, pp. 1 et seq.

G. Aust, *Vertical Cooperative Advertising in Supply Chain Management*,
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But what are the benefits of game theory in comparison to classical optimization methods of operations research? Myerson (1997) proposes the following definition of game theory:

> Game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Game theory provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence one another's welfare.[5]

According to this definition, game theory explicitly considers the interdependencies in decision-making of multiple parties, while classical decision theory only deals with the optimal decision of one single decision maker, which is independent of others' decision.[6] These interdependencies between the parties' decisions can obviously lead to conflicts of interest, when each individual has different objectives.[7] The allowance of such conflicts constitutes another benefit of game theory compared to classical optimization theory.[8] Especially the last point underlines that game theory offers an appropriate framework for the analysis of the interaction between different members of a supply chain intended in this work, because each firm may concentrate rather on its own profit than on favors for other firms.

We start our introduction into game theory with some widely-used nomenclature.[9] When more than one individual has to make a decision and these decisions are interdependent, this situation is called *game*, while the decision-makers are denoted as *players*. Each player p within the set of players $\mathcal{N} = \{1, \ldots, n\}$ has to choose a strategy $s_p \in S_p$, where S_p describes the feasible strategy set available to player $p \in \mathcal{N}$. All possible combinations of the players' (feasible) strategies $s_p \in S_p$ form the strategy space $\mathcal{S} = S_1 \times S_2 \times \ldots \times S_n$, whereas $s \in \mathcal{S}$ describes one single combination of strategies s_p with $p = 1, \ldots, n$. Lastly, the utility function $u_p(s)$ with $u_p(s) \in \mathcal{U} = \{u_1(s), \ldots, u_n(s)\}$ assigns a player-specific utility value to each strategy combination s, which permits to determine the most preferable strategy for player p.[10] This formal nomenclature allows to abstractly characterize a game $\mathcal{G}$ by the triplet $\mathcal{G} = (\mathcal{N}, \mathcal{S}, \mathcal{U})$. In conclusion, for each game the number of players, the space of possible strategies, and the utility functions which evaluate each combination of strategies has to be certain. These three properties are extended

[5]Myerson (1997): *Game theory*, p. 1.

[6]Cf. Riechmann (2008): *Spieltheorie*, p. 18.

[7]Cf. Holler and Illing (2006): *Spieltheorie*, p. 1.

[8]Cf. Borgwardt (2001): *Optimierung*, p. 509.

[9]This explanation is based on Holler and Illing (2006): *Spieltheorie*, pp. 31–42, and Peters (2008): *Game theory*, pp. 73 et seq.

[10]Utility functions were introduced by von Neumann and Morgenstern (1944), who firstly proposed an axiomatic system that allows to quantify utility numerically (see von Neumann and Morgenstern (1953): *Theory of games*, pp. 15–31). Thereby, a utility function is used to assign an individual utility value of a person to a certain event. This also allows to compare different utility values mathematically (cf. Laux (2005): *Entscheidungstheorie*, p. 26).

<table>
<tr><th>Game parameter</th><th colspan="6">Specification</th></tr>
<tr><td>Number of players</td><td colspan="3"><u>2-person</u></td><td colspan="3">n-person</td></tr>
<tr><td>Decision process</td><td colspan="3"><u>simultaneous</u></td><td colspan="3"><u>sequential</u></td></tr>
<tr><td>Time-dependency</td><td colspan="3"><u>static</u></td><td colspan="3">dynamic</td></tr>
<tr><td>Repeat</td><td colspan="3">repeated game</td><td colspan="3"><u>non-repeated game</u></td></tr>
<tr><td>Pay-offs</td><td colspan="2">zero-sum game</td><td colspan="2">constant sum game</td><td colspan="2"><u>non-zero-sum game</u></td></tr>
<tr><td>Side-payments</td><td colspan="3">game with side-payments</td><td colspan="3"><u>game without side-payments</u></td></tr>
<tr><td>Common knowledge</td><td colspan="3"><u>complete information</u></td><td colspan="3">incomplete information</td></tr>
<tr><td>Information on players' actions</td><td colspan="3"><u>perfect information</u></td><td colspan="3"><u>imperfect information</u></td></tr>
<tr><td>Stages of the game</td><td colspan="2"><u>one</u></td><td colspan="2"><u>finite number</u></td><td colspan="2">infinite number</td></tr>
<tr><td>Representation of game</td><td colspan="2">extensive-form game</td><td colspan="2"><u>normal-form game</u></td><td colspan="2">characteristic function</td></tr>
<tr><td>Number of strategies</td><td colspan="2">finite number of strategies</td><td colspan="2">countably number of strategies</td><td colspan="2"><u>infinite number of strategies</u></td></tr>
<tr><td>Strategy space</td><td colspan="3">discrete</td><td colspan="3"><u>continuous</u></td></tr>
<tr><td>Determination of strategy</td><td colspan="3"><u>pure strategy</u></td><td colspan="3">mixed strategy</td></tr>
<tr><td>Type of game</td><td colspan="2"><u>non-cooperative game</u></td><td colspan="2"><u>cooperative game: bargaining</u></td><td colspan="2">cooperative game: coalition game</td></tr>
</table>

Fig. 2.1 Possible properties of a game. Please note that this is only a sample of possible properties of a game, which makes no claim to be complete. This morphological box is prepared on the basis of Borgwardt (2001): *Optimierung*, pp. 512–514, Holler and Illing (2006): *Spieltheorie*, pp. 31–50, and Jost (2001): *Spieltheorie in der Betriebswirtschaftslehre*, pp. 14–29

by further attributes of each specific game, which could be understood as rules of the game. A sample of possible rules which can further describe a game is given in form of a morphologic box in Fig. 2.1. An underline labels a characteristic which applies to the models in Chap. 4 through Chap. 6 of this work.

Besides the **number of players**, where 2-person and *n*-person games are distinguished, the incorporation of time is crucial to the design of a game. On the one hand, this refers to the chronology of **decision making**, which can be either simultaneous or sequential. In the latter case, it depends on the information available to the starter if the possibility to move first is advantageous or not. On the other hand, games differ in the fact whether strategies are **time-dependent** (dynamic games) or not (static games). That means players in a dynamic game have to decide which strategy should be played at which moment of the game and whether to change strategy over time.

According to the possible **pay-offs**, games are distinguished in zero-sum, constant sum, and non-zero-sum games. For example, the first (second) notion indicates that the pay-offs each player achieves at the end of the game compensate one another to zero (a constant). That means each gain of one player leads to a loss at his counterparts.[11] Another distinctive rule is the permission of **side-payments**, which can be used in order to convince other players to choose a certain strategy by compensating eventual deficits resulting from this decision.[12]

A very important criterion is the information available to the players. Thereby, we initially consider the **common knowledge**, i.e., the knowledge available to both players. If only the rules of the respective game are known to each player, this situation is called incomplete information. In contrast, complete information describes a condition where the players are moreover aware of their counterparts' strategy set S_p as well as their utility function $u_p(s)$. With this knowledge, one is even able to anticipate the other players' actions. However, the actual **information on the chosen strategy** is a different matter, which is only given under perfect information.[13]

The most suitable form of **representation of a game** strongly depends on other characteristics that have to be included into consideration. For instance, the representation of a game in an extensive form (i.e., in form of a game tree) allows to display the number of stages of the game, the information available to the players at each stage, etc. Obviously, this form is particularly applicable to dynamic games.[14] On the other hand, static games are often expressed as normal-form games, which simply list every possible strategy of each player, together with the players' utility resulting from each combination. A widely-used display format for this type of representation is a matrix.[15] Lastly, characteristic functions can be applied in case of cooperative games (which are further explained in the sequel) and describe the pay-off a coalition of players can obtain.[16]

The strategies available to the players can also constitute a distinctive feature of a game. For instance, the **strategy set** may be either discrete, i.e., limited to certain values, or continuous, which means that players are able to choose every interim value within a given range. When strategies are discrete and finite, it may occur that no solution of this game can be obtained with these existing pure strategies. In this case, the **determination of strategies** can be effected by means of mixed strategies in order to find a solution of this game. Therefore, it is assumed that players do not

[11]Cf. Rieck (2010): *Spieltheorie*, pp. 102–104.

[12]Cf. Borgwardt (2001): *Optimierung*, p. 514.

[13]Cf. Holler and Illing (2006): *Spieltheorie*, pp. 43–50, and Peters (2008): *Game theory*, p. 59.

[14]Cf. Jost (2001): *Spieltheorie in der Betriebswirtschaftslehre*, pp. 21–25.

[15]Cf. Rieck (2010): *Spieltheorie*, p. 162. Please note that matrices are mostly used for 2-person games, but are also defined for n-player games theoretically.

[16]Cf. Holler and Illing (2006): *Spieltheorie*, p. 270, and Borgwardt (2001): *Optimierung*, p. 513.

have to commit themselves to one single strategy from their strategy set, but rather to assign a probability value to each strategy available. Hence, the result of the game is random.[17]

The last characteristic stated in Fig. 2.1—the **type of the game**—is of particular importance, because it describes two streams of game theory with different approaches: While non-cooperative game theory considers the players' possible strategies and actions, which in the end lead to a certain pay-off of the game, cooperative game theory omits these preliminary actions and focuses on the division of the resulting pay-off between the players. In other words, it is assumed that players can agree on their strategies by contract so that everyone can rely on the others' behavior and the only question in dispute is the division of the resulting total pay-off. On the other hand, similar contracts are impossible in non-cooperative games, wherefore players do only decide on a rational basis. Furthermore, it is worth pointing out that the term non-cooperative game does not mean that it is forbidden that players decide to cooperate in order to yield a higher pay-off. It is simply not possible to stipulate this cooperation in form of a contract in advance.[18] The field of cooperative game theory can be further divided into bargaining games[19] and coalition games. The first notion describes a setting where each player acts as an individual, wherefore these games are also called individualistic-cooperative games, while the latter form also allows the formation of coalitions between the players.[20]

2.1.2 Solution Concepts in Non-Cooperative Game Theory

Up to now, we only explained different characteristics of games, without going into concrete solution techniques which can be used to determine optimal strategies of the players. As a start, Table 2.1 shall give a first overview of existing solution methods. Obviously, solution methods can be roughly grouped by the aforementioned types of game, but of course not every method can be applied to each game belonging to this type, e.g., Bayesian equilibria are used in the context of

[17]Cf. Berninghaus et al. (2010): *Strategische Spiele*, pp. 28–34, and Rieck (2010): *Spieltheorie*, p. 80.

[18]Cf. Cachon and Netessine (2004): *Game theory*, p. 36, and Sieg (2010): *Spieltheorie*, p. 91.

[19]Please note that only *cooperative* (or *axiomatic*) bargaining theory is considered in this work, which can be used to determine a fair division of pay-offs as explained above. In addition, there is also a research field called *non-cooperative*, *strategic* or *behavioristic* bargaining theory, which refers to the actual bargaining process. Examples of this area are the Zeuthen-Harsanyi game (see Harsanyi (1977): *Rational behavior*, pp. 162–164, cited in Holler and Illing (2006): *Spieltheorie*, p. 252), or the Rubinstein game (see Rubinstein (1982): *Perfect equilibrium*). For a more detailed elaboration, we refer the reader to Berninghaus et al. (2010): *Strategische Spiele*, pp. 198–229, and Holler and Illing (2006): *Spieltheorie*, pp. 240–266.

[20]Cf. Holler and Illing (2006): *Spieltheorie*, pp. 189 & 267.

Table 2.1 Solution methods of non-cooperative and cooperative games (sample)

	Cooperative games	
Non-cooperative games	Bargaining games	Coalition games
Bayesian equilibrium	Asymmetric Nash solution	Banzhaf index
Bertrand equilibrium	Eliashberg solution	Biform games
Best response	Kalai-Smorodinsky solution	Core of a game
Cooperation	Proportional solution	Deegan-Packel index
Cournot equilibrium	Symmetric Nash solution	Kernel of a game
Nash equilibrium		Nucleolus of a game
Stackelberg equilibrium		Public-Good index
Strategic dominance		Shapley value

Please note that this is only a sample of possible game-theoretical concepts, which makes no claim to be complete. The list is prepared on the basis of Berninghaus et al. (2010): *Strategische Spiele*, Cachon and Netessine (2004): *Game theory*, Holler and Illing (2006): *Spieltheorie*, Nagarajan and Sošić (2008): *Game-theoretic analysis*, Peleg and Sudhölter (2007): *Theory of cooperative games*, and Riechmann (2008): *Spieltheorie*. We refer the reader to these references for further information on particular methods, as a detailed discussion lies outside the scope of this work.

games with incomplete information.[21] As stated in Chap. 1, this work is concerned with the interaction between the manufacturing and retailing echelon of a supply chain as well as with the interdependencies in decision-making between the firms. Thereby, one research subject is the effect of distribution of power between the firms belonging to the supply chain under consideration on the setting of prices, advertising expenditures, and cooperative advertising program.

Hence, non-cooperative game theory seems to offer the appropriate methodology in this regard. In this group of solution methods, Nash and Stackelberg equilibria are the two most important techniques used in supply chain management research, which are, thus, also applied in this work.[22] A Nash equilibrium is used when power is equally distributed between the players, while a Stackelberg equilibrium can represent a situation where one player obtains channel-leadership. Besides these two equilibria, we explain how a Cooperation between the players can be modeled, which is not necessarily a game-theoretic concept, but is often used in this context. In the end, a short introduction into bargaining theory is given, because it can be used to determine a fair split of profits in the case when this is not settled by the equilibrium itself.[23]

[21] See Holler and Illing (2006): *Spieltheorie*, pp. 78–87.

[22] Cf. Leng and Parlar (2005): *Game theoretic applications*, p. 189.

[23] The limitation on bargaining games instead of coalition games results from the fact that only the 2-player game in Chap. 4 additionally requires the application of cooperative game theory. Hence, the analysis of coalitions is dispensable in this context, wherefore we refer interested readers to, e.g., Peleg and Sudhölter (2007): *Theory of cooperative games* for further information.

Nash Equilibrium

The concept of Nash equilibrium was established by Nash (1951, 1950b) and can be seen as a generalization of the duopoly model proposed by Cournot (1838).[24] This solution concept is commonly applied in games when no strategy combination can be identified as the best choice for every participating player a priori.[25] Thereby, the following conditions of a Nash game hold[26]:

- individual and simultaneous decision-making,
- rational behavior and expectations of each player, and
- no agreements between the players.

These items already define some important characteristics of a Nash game. Obviously, Nash games belong to the group of non-cooperative games, because the players act individually and without preliminary agreements. Next, the decision process is simultaneous, i.e., the players do not have knowledge of the strategies chosen by the other players (imperfect information). However, as complete information is assumed, each player knows the characteristics of the game (e.g., the strategy set available to each player and the utility function of his counterparts). Together with the postulated rational expectations, each player is, thus, able to predict which strategy his counterparts will select.

In general, a combination of strategies is called Nash equilibrium if no player can obtain a higher utility value by deviating from his actual strategy. That means, his actual strategy is the best response to the strategies chosen by his counterparts. Formally, the *best response function* $r_p(s_{-p})$ of player p on the strategies of the other players s_{-p} is

$$r_p(s_{-p}) = \left\{ s_p^* \in S_p \,\middle|\, u_p(s_p^*, s_{-p}) \geq u_p(s_p, s_{-p}), \quad \forall s_p \in S_p \right\}. \tag{2.1}$$

Then, the combination of strategies $s^* = (s_p^*, s_{-p}^*)$ with

$$u_p(s_p^*, s_{-p}^*) \geq u_p(s_p, s_{-p}^*), \quad \forall p, \forall s_p \in S_p, \tag{2.2}$$

is called Nash equilibrium, where the strategy s_p^* denotes the optimal strategy of player p, and s_{-p}^* are the optimal strategies of the other players.[27] As game theory generally acts on the assumption of rational behavior, each player only selects

[24]See Nash (1950b): *Equilibrium points in n-person games*, Nash (1951): *Non-cooperative games*, and Cournot (1838): *Recherches*. Please note that the term *Nash equilibrium* characterizes the solution of a so-called *Nash game*.

[25]A strategy fulfilling this condition is called a *dominant* strategy. For more information on the solution technique of strategic dominance see, e.g., Riechmann (2008): *Spieltheorie*, pp. 25–32.

[26]Cf. Sieg (2010): *Spieltheorie*, p. 15.

[27]In the following chapters, we use the superscript N instead of an asterisk to denote a Nash equilibrium in order to distinguish the equilibria introduced in this section.

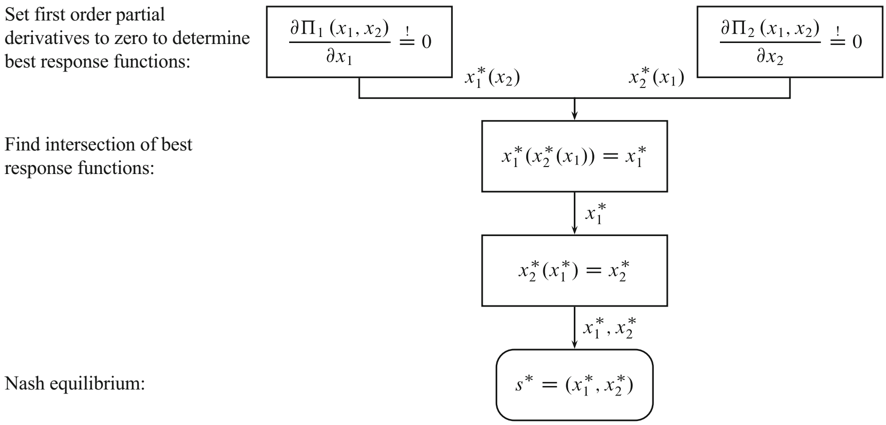

Fig. 2.2 Determination of Nash equilibrium (exemplary solution procedure for a game with $\mathcal{G} = (\mathcal{N} = \{1,2\}, \mathcal{S} = [\underline{x}_1, \overline{x}_1] \times [\underline{x}_2, \overline{x}_2], \mathcal{U} = \{\Pi_1, \Pi_2\}))$

strategies comprised in his best response function. Hence, one can determine a Nash equilibrium of a game by finding the intersection set of the participating players' best response functions.[28]

A brief example shall illustrate this procedure, which is also depicted in Fig. 2.2.[29] We consider two firms ($\mathcal{N} = \{1,2\}$) which both have to set one decision variable x_p which could constitute, e.g., the price or the quantity produced of a product. Hence, each firm's strategy is given by $s_p = x_p$, where the decision variable can be set at will within the strategy set $S_p = [\underline{x}_p, \overline{x}_p]$, which results a continuous strategy space $\mathcal{S}$ with an infinite number of strategies. Each combination of strategies $s = (x_1, x_2)$ leads to certain profits determinable by the two (concave) profit functions $\Pi_1(x_1, x_2)$ and $\Pi_2(x_1, x_2)$, and each firm of course tends to maximize its profit. For the sake of simplicity, it is assumed that profits also constitute the players' utility value, i.e., $u_p(x_p, x_{-p}) = \Pi_p(x_p, x_{-p})$. The resulting decision problem of firm p under the simultaneous and non-cooperative setting of a Nash game is, thus, characterized by

$$\begin{aligned} \text{Max} \quad & \Pi_p(x_p, x_{-p}) \\ \text{s.t.} \quad & x_p \in S_p. \end{aligned} \tag{2.3}$$

[28]Cf. Holler and Illing (2006): *Spieltheorie*, pp. 57–67, Berninghaus et al. (2010): *Strategische Spiele*, pp. 23 et seq., and Riechmann (2008): *Spieltheorie*, pp. 34 et seq.

[29]More examples related to firms competing in prices or quantities can be found in Pfähler and Wiese (2008): *Unternehmensstrategien im Wettbewerb*, pp. 53–102.

As explained above, one has to determine the firms' best response functions $r_p(s_{-p})$ first, which denote the profit-maximizing strategy $s_p^* = x_p$ with respect to the counterpart's strategy $s_{-p} = x_{-p}$. As the profit function $\Pi_p(x_p, x_{-p})$ is concave, we have to set the first order partial derivative $\partial \Pi_p(x_p, x_{-p})/\partial x_p$ to zero in order to identify the value of x_p which maximizes Π_p, given any strategy of the counterpart $s_{-p} \in S_{-p}$. After that, solving the resulting equation for x_p leads to the response function $r_p(s_{-p}) = x_p^*(x_{-p})$. The Nash equilibrium $s^* = (x_1^*, x_2^*)$ can then be determined by solving the system of equations given by $x_1^*(x_2)$ and $x_2^*(x_1)$.

Stackelberg Equilibrium

In contrast to the Nash game, the oligopoly model proposed by von Stackelberg (1934) does not assume a simultaneous, but a sequential decision process with two stages.[30] Thereby, it allows to incorporate a hierarchical structure between the participating players, which is used to model channel leadership in the context of supply chain management. Hence, the player who is able to move first is called *Stackelberg leader* (or simply *leader*), while the second player is denominated *Stackelberg follower* (or *follower*). Again, information plays a decisive role in this game: The assumption of complete information enables the leader to take account of the possible reaction of his follower to his own strategy and to include this knowledge into his decision. On the other hand, a Stackelberg game is played under perfect information, wherefore the follower knows about the strategy chosen by the leader in the first stage and seeks to maximize his utility in the second stage of the game, given the leader's action.[31]

Mathematically, the equilibrium of a Stackelberg game can be identified via backward induction: At first, one has to determine the best response function $r_p(s_{-p})$ of the follower. As this function is known to the leader due to the assumption of complete information, it forms a constraint of the optimization problem of the leader. By solving the resulting optimization problem, the optimal strategy of the leader is found. After that, the follower's best response on this strategy leads to the strategy selected by the follower. Since both strategies lie upon the players' best response functions, no one has an incentive to deviate from his strategy and the Stackelberg equilibrium is found.

Again, the example of two firms playing the game $\mathcal{G} = (\mathcal{N} = \{1, 2\}, \mathcal{S} = [\underline{x}_1, \overline{x}_1] \times [\underline{x}_2, \overline{x}_2], \mathcal{U} = \{\Pi_1, \Pi_2\})$ introduced above shall help to familiarize with this solution concept.[32] The necessary steps are also summarized in Fig. 2.3.

[30]See von Stackelberg (1934): *Marktform und Gleichgewicht*.

[31]Cf. Berninghaus et al. (2010): *Strategische Spiele*, pp. 144–146, Cachon and Netessine (2004): *Game theory*, pp. 27 et seq., Leng and Parlar (2005): *Game theoretic applications*, pp. 191 et seq., and Riechmann (2008): *Spieltheorie*, pp. 132 et seq.

[32]Another example regarding a Stackelberg game with firms competing in quantity can be found in Pfähler and Wiese (2008): *Unternehmensstrategien im Wettbewerb*, pp. 150–157.

Arbitrarily, firm 1 obtains the Stackelberg leadership in this game, while firm 2 acts as follower. As explained above, we start with the optimization problem of the follower, which is given by

$$\begin{aligned} \text{Max} \quad & \Pi_2(x_1, x_2) \\ \text{s.t.} \quad & x_2 \in S_2. \end{aligned} \tag{2.4}$$

Setting the first order partial derivative $\partial \Pi_2(x_1, x_2)/\partial x_2$ to zero leads to the best response function of firm 2, i.e., $r_2(s_1) = x_2^*(x_1)$. This best response function now constitutes a constraint of the leader's optimization problem. Hence, we get

$$\begin{aligned} \text{Max} \quad & \Pi_1(x_1, x_2) \\ \text{s.t.} \quad & x_2 = x_2^*(x_1) \\ & x_1 \in S_1. \end{aligned} \tag{2.5}$$

The optimal value of the leader's decision variable x_1^* is the solution of this problem. By means of the follower's best response function, the missing optimal strategy of firm 2 can be calculated via $x_2^* = x_2^*(x_1^*)$, which completes the determination of the Stackelberg equilibrium $s^* = (x_1^*, x_2^*)$.[33]

Cooperation

The last solution concept for non-cooperative game settings, which shall be introduced in this work, is called *Cooperation*.[34] In case of a Cooperation, it is assumed that players do not try to maximize their own utility, but rather the total utility of all participating players.[35] Hence, the participating players act as a single player, which can lead to the fact that only a collective strategy can be determined. In that case, it is possible that players' individual strategies and utilities are not defined by the Cooperation. That necessitates the application of other methods like bargaining games in order to obtain a fair distribution of utility between the cooperating players.

The aforementioned two-firm game with $\mathcal{G} = (\mathcal{N} = \{1, 2\}, \mathcal{S} = [\underline{x}_1, \overline{x}_1] \times [\underline{x}_2, \overline{x}_2], \mathcal{U} = \{\Pi_1, \Pi_2\})$ as well as the procedure illustrated in Fig. 2.4 shall further explain this. In the first case of individual strategies, the total profit of both players Π_{1+2} can be calculated via

[33] In the following chapters, we use the superscript S to denote a Stackelberg equilibrium.

[34] Please note that this concept is sometimes also named *collusion* or *cartelization* in economics. To avoid any misunderstandings, it shall again be pointed out that the notion *non-cooperative* game only refers to the prohibition of preliminary agreements between the players, which does not forbid to act in cooperation.

[35] Cf. here and in the following Riechmann (2008): *Spieltheorie*, pp. 123–125.

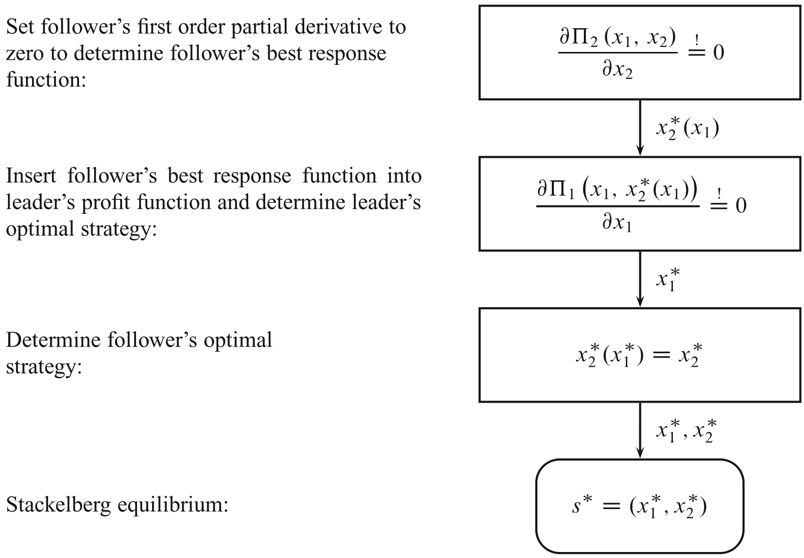

Fig. 2.3 Determination of Stackelberg equilibrium (exemplary solution procedure for a game with $\mathcal{G} = (\mathcal{N} = \{1, 2\}, \mathcal{S} = [\underline{x}_1, \overline{x}_1] \times [\underline{x}_2, \overline{x}_2], \mathcal{U} = \{\Pi_1, \Pi_2\}))$

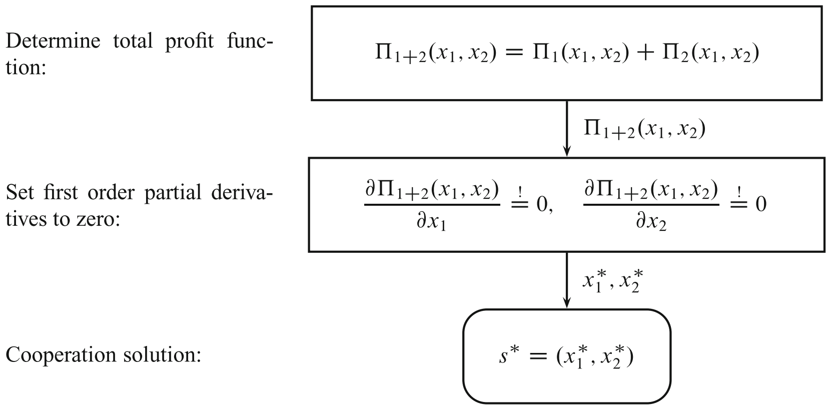

Fig. 2.4 Determination of Cooperation solution (exemplary solution procedure for a game with $\mathcal{G} = (N = \{1, 2\}, S = [\underline{x}_1, \overline{x}_1] \times [\underline{x}_2, \overline{x}_2], U = \{\Pi_1, \Pi_2\}))$

$$\Pi_{1+2}(x_1, x_2) = \Pi_1(x_1, x_2) + \Pi_2(x_1, x_2). \tag{2.6}$$

This leads to the following optimization problem of the cooperating firms:

$$\begin{aligned} \text{Max} \quad & \Pi_{1+2}(x_1, x_2) = \Pi_1(x_1, x_2) + \Pi_2(x_1, x_2) \\ \text{s.t.} \quad & x_1 \in S_1, x_2 \in S_2, \end{aligned} \tag{2.7}$$

The solution of this problem $s^* = (x_1^*, x_2^*)$, which constitutes the optimal strategy in case of a Cooperation, can be obtained by setting the first order partial derivatives $\partial\Pi_{1+2}(x_1, x_2)/\partial x_1$ and $\partial\Pi_{1+2}(x_1, x_2)/\partial x_2$ to zero and solving the resulting system of equations.[36]

Let us now consider a small modification of this problem, namely that the firms' total profit does not depend on the individual decision variables themselves, but on a collective variable, which contains both individual decision variables. For instance, the total profit of both firms Π_{1+2} depends on the sum of both decision variables $x_1 + x_2 = x_{1+2}$, i.e., $\Pi_{1+2} = \Pi_{1+2}(x_{1+2})$. This could be imaginable when both firms produce the same product and x_p denotes the quantity produced. The resulting modified optimization problem is

$$\begin{aligned} \text{Max} \quad & \Pi_{1+2}(x_{1+2}) = \Pi_1(x_{1+2}) + \Pi_2(x_{1+2}) \\ \text{s.t.} \quad & x_{1+2} = x_1 + x_2 \\ & x_1 \in S_1, x_2 \in S_2. \end{aligned} \tag{2.8}$$

Obviously, the solution of this problem only indicates an optimal value of the collective variable $s^* = x_{1+2}^*$ as well the corresponded maximum value of total profit Π_{1+2}, but does not give a concrete recommendation for each firm's strategy or for the division of profits between the two firms. As indicated above, this question can be answered by means of bargaining theory.

2.1.3 Solution Concepts in Bargaining Theory

As aforementioned, bargaining theory can be used to find a fair split of total pay-off in a game where players cannot form coalitions. For instance, players bargain an amount of money which should be divided between both, whereby each player p valuates his share y_p in the total sum Y (with $0 \leq y_p \leq Y$ and $\sum_p y_p = Y$) on the basis of his utility function $u_p(y_p)$, which leads to a certain utility value v_p.[37] Hence, a bargaining game can be characterized by $\mathcal{B} = (\mathcal{N}, \mathcal{V})$, with $\mathcal{N} = \{1, \ldots, n\}$ again denoting the set of participating players, and $\mathcal{V} = \{V_1, \ldots, V_n\}$ describing

[36] In the following chapters, we use the superscript C to denote a Cooperation.

[37] Cf. Berninghaus et al. (2010): *Strategische Spiele*, pp. 158 et seq. Please note that bargaining theory directly refers to utility values v_p, while non-cooperative games are characterized by the players utility functions $u_p(s)$ as explained in the previous section.

the space of the utility values achievable by the players.[38] Thereby, V_p indicates the set of possible utility values v_p player p can receive, which can be calculated via $v_p = u_p(y_p)$ for $0 \leq y_p \leq Y$.

Symmetric Nash Bargaining Solution

As listed in Table 2.1, different bargaining models exist which can be used to determine a solution of a bargaining game. The first one proposed was the so-called Symmetric Nash bargaining solution by Nash (1950a).[39] This approach is based on the following four axioms, which should be fulfilled by a solution of a bargaining game in order to ensure a reasonable division of pay- off according to Nash (1950a)[40]:

1. *independence of equivalent utility transformations*, i.e., the solution shall not depend on the scale applied by the players,
2. *symmetry*, i.e., players with identical characteristics shall obtain identical pay-offs,
3. *independence of irrelevant alternatives*, i.e., new but ineligible pay-off combinations do not affect the best solution, and
4. *Pareto optimality*, i.e., the solution shall be designed in such a way that no player's utility value can be increased without decreasing another player's utility value.

On that basis, Nash (1950a) proposes a model that determines a bargaining solution that maximizes the total utility value $v_{\mathcal{N}}$ of all participating players by

$$\begin{aligned} \text{Max} \qquad & v_{\mathcal{N}} = \prod_p v_p \\ \text{s.t.} \qquad & v_p \in V_p. \end{aligned} \tag{2.9}$$

We again consider an example of two players $\mathcal{N} = \{1, 2\}$ to illustrate this concept. Both players bargain their shares y_1 and y_2 in an amount of money Y. Hence, the

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[38]Please note that this is a modification of the notation $\mathcal{B} = (\mathcal{V}, c)$, which can be found in, e.g., Holler and Illing (2006): *Spieltheorie*, p. 191, and Sieg (2010): *Spieltheorie*, p. 92. Here, c denotes the disagreement point of the bargaining game, i.e., the pay-off value each player receives when no agreement can be settled. However, since this disagreement point is dispensable in this work, it is not considered further. In order to ensure comparability to the notation of non-cooperative games, $\mathcal{G} = (\mathcal{N}, \mathcal{S}, \mathcal{U})$, we also include the set of participating players $\mathcal{N}$.

[39]See Nash (1950a): *Bargaining problem*.

[40]Cf. Berninghaus et al. (2010): *Strategische Spiele*, pp. 162–178, Riechmann (2008): *Spieltheorie*, pp. 171–174, and Holler and Illing (2006): *Spieltheorie*, pp. 195–205.

bargaining model can be reformulated to

$$\begin{aligned} \text{Max} \quad & v_{1+2} = v_1 v_2 = u_1(y_1)u_2(y_2) \\ \text{s.t.} \quad & y_1 + y_2 = Y \\ & 0 \leq y_1 \leq Y, 0 \leq y_2 \leq Y. \end{aligned} \tag{2.10}$$

Obviously, it is necessary to specify the players' utility functions $u_p(y_p)$ to solve this problem. The most simple function would be a linear function in the shape $u_p(y_p) = y_p$. However, according to the symmetry axiom postulated by Nash (1950a), this would result in an equal share of both players, because no specific characteristics of each individual player can be incorporated. Instead of that, a power function

$$u_p(y_p) = y_p^{\mu_p} \tag{2.11}$$

can be used, where the parameter μ_p depends on the risk behavior of player p.[41] Thereby, $\mu_p < 0$ indicates risk aversion, $\mu_p = 1$ risk neutrality, and $\mu_p > 1$ a risk-seeking behavior of player p.[42] With this utility function, the total utility value v_{1+2} given in (2.10) can be rewritten to

$$v_{1+2} = y_1^{\mu_1} y_2^{\mu_2} = y_1^{\mu_1}(Y - y_1)^{\mu_2} \tag{2.12}$$

Setting the first order partial derivatives $\partial v_{1+2}/\partial y_1$ and $\partial v_{1+2}/\partial y_2$ to zero, leads to the following solution of the considered bargaining game,

$$y_p = \frac{\mu_p}{\mu_1 + \mu_2} Y, \tag{2.13}$$

which is called Symmetric Nash bargaining solution. From Eq. (2.13), we can see that the division of the total amount of money depends on the players' risk parameters μ_p, where the more risk-seeking player receives a higher share of Y. However, it is important to note that this unequal distribution of pay-offs is not inconsistent with the symmetry axiom, because this symmetry is only concerned with an equal distribution of utility values between the players. Depending on their risk behavior, players may though valuate their share in the total pay-off differently, which may lead to different individual pay-offs y_1 and y_2. One point

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[41] This form of utility function is applied in Xie and Neyret (2009): *Co-op advertising*, p. 1383, and Xie and Wei (2009): *Co-op advertising*, p. 789.

[42] In general, the risk behavior of an utility function can be determined by means of the Arrow-Pratt measure of absolute risk aversion, which is defined by $\mathbb{R}(y) = -(\partial^2 u(y)/\partial y^2) \cdot (\partial u(y)/\partial y)$ (cf. Pratt (1964): *Risk aversion*, p. 122). $\mathbb{R}(y) > 0$ represents risk aversion, $\mathbb{R}(y) = 0$ risk neutrality, and $\mathbb{R}(y) < 0$ a risk-seeking behavior. For more information on these terms, see Bamberg et al. (2008): *Betriebswirtschaftliche Entscheidungslehre*, pp. 81–84.

of criticism of the Symmetric Nash bargaining solution is that bargaining power or skills of the participating players cannot be included into the bargaining model due to the assumption of symmetry made by Nash (1950a), though this is a very important parameter in real bargaining situations.[43] In order to derive a more general bargaining model, this symmetry assumption will now be abandoned.

Asymmetric Nash Bargaining Solution

A modification of the classic Nash bargaining solution was proposed by Harsanyi and Selten (1972) and Kalai (1977), which is called Asymmetric Nash bargaining solution.[44] In order to integrate bargaining power into the determination of pay-offs, a new parameter λ_p with $\sum_p \lambda_p = 1$ is introduced, which is defined as a measure of each player's bargaining power.[45] Hence, the model given in (2.9) is adapted as follows[46]:

$$\begin{aligned} \text{Max} \qquad & v_{\mathcal{N}} = \prod_p v_p^{\lambda_p} \\ \text{s.t.} \qquad & v_p \in V_p. \end{aligned} \tag{2.14}$$

This general formulation is now applied to the two-player example with utility functions in form of a power function. The optimization problem in this example is

$$\begin{aligned} \text{Max} \qquad & u_{1+2} = v_1^{\lambda_1} v_2^{\lambda_2} = u_1(y_1)^{\lambda_1} u_2(y_2)^{\lambda_2} \\ \text{s.t.} \qquad & y_1 + y_2 = Y \\ & 0 \leq y_1 \leq Y, 0 \leq y_2 \leq Y. \end{aligned} \tag{2.15}$$

By means of the utility function in Eq. (2.11), the total utility value can be rewritten to

$$v_{1+2} = y_1^{\lambda_1 \mu_1} y_2^{\lambda_2 \mu_2} = y_1^{\lambda_1 \mu_1} (Y - y_1)^{\lambda_2 \mu_2}, \tag{2.16}$$

[43] Cf. Holler and Illing (2006): *Spieltheorie*, p. 215. For more information on the term *bargaining power*, we refer the reader to Kunter (2009): *Absatzkanalkoordination*, pp. 25–27.

[44] See Harsanyi and Selten (1972): *Generalized Nash* and Kalai (1977): *Nonsymmetric Nash*.

[45] Cf. Holler and Illing (2006): *Spieltheorie*, pp. 215–217.

[46] An alternative way to integrate bargaining power into a bargaining model can be found in Eliashberg (1986): *Arbitrating a dispute*. Here, the total utility is calculated via $v_{\mathcal{N}} = \sum_p \lambda_p v_p$. This model is used in, e.g., Yue et al. (2006): *Coordination of cooperative advertising*, p. 79, together with an exponential utility function of the shape $u_p(y_p) = 1 - e^{-\mu_p y_p}$. However, as the resulting mathematical expressions a far more complex than those deriving from the Asymmetric Nash bargaining solution, a further discussion of this concept is omitted in this work.

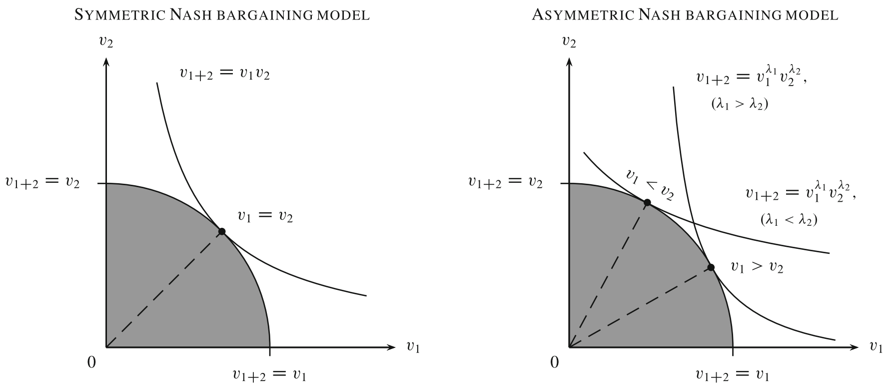

Fig. 2.5 Symmetric and Asymmetric Nash bargaining model (exemplary illustration of a bargaining game with $\mathcal{B} = (\mathcal{N} = \{1, 2\}, \mathcal{V} = \{v_1, v_2\})$). This figure is based on Berninghaus et al. (2010): *Strategische Spiele*, p. 171, and Kunter (2009): *Absatzkanalkoordination*, p. 25

which leads to the following share of player p in Y:

$$y_p = \frac{\lambda_p \mu_p}{\lambda_1 \mu_1 + \lambda_2 \mu_2} Y. \tag{2.17}$$

Obviously, the modification of the classical (Symmetric) Nash bargaining model now allows incorporating not only risk behavior, but also bargaining power of the participating players into the determination of a solution of the bargaining game. Thereby, a higher degree of risk-seeking as well as a higher bargaining power increase the share a player will obtain through bargaining.

Figure 2.5 finally confronts the two bargaining models introduced here. The players' individual utility values v_1 and v_2 are plotted against each other and an identical risk behavior of both players is assumed. The gray area indicates the space of possible utility values $\mathcal{V}$, whereof only the outer border fulfills the Pareto-condition postulated by Nash (1950a), though. This border intersects the axes at $v_{1+2} = v_1$ and $v_{1+2} = v_2$, i.e., where the total utility value v_{1+2} is assigned to one single player entirely. On the left side, we can see that the Symmetric Nash bargaining solution causes an equal distribution of utility, while the distribution resulting from the asymmetric model on the right sight depends on the two bargaining power parameters λ_1 and λ_2.

2.2 A General Model of Cooperative Advertising

In this section, a general (mathematical) model of cooperative advertising is presented, which shall serve as base model in the following. As explained in Sect. 1.1, a cooperative advertising program is a financial agreement between the manufacturing and retailing echelon of a supply chain. In order to offer a framework as universal as possible, the number of firms belonging to each echelon shall not be limited at the moment. Up to now, cooperative advertising research concentrates on these two echelons and does not consider any upstream suppliers or the like, for which reason other supplying echelons are not included.[47] This supply chain under consideration sells one (or more) product(s) to a group of customers.

Each echelon generates a certain margin when selling one unit of a product. Thereby, models either consider each firms' margin directly[48] or indirectly via the price of the correspondent product.[49] In this general model, it is assumed that firms directly set their prices, with w_{ijk} being the wholesale price, which is charged by manufacturer i (with $i = 1, \ldots, I$) to retailer j (with $j = 1, \ldots, J$) for product k (with $k = 1, \ldots, K$). Similarly, p_{jk} is the retail price charged by retailer j for product k.[50]

Furthermore, each firm can invest into advertising in order to increase customer demand, whereby global advertising expenditures A_i of manufacturer i and local advertising expenditures a_j of retailer j are differentiated. Within the offered cooperative advertising program, manufacturer i can also participate in retailer j's advertising expenditures with a participation rate t_{ij}.[51] Further costs like production or transportation costs could either be variable costs c_{ik} (c_{jk}) or fixed costs C_i (C_j) for manufacturer i (retailer j).

Based on the possible decisions of the supply chain echelons, the customer demand D_{jk} of product k resulting for retailer j may generally depend both on retail prices $\underline{\mathbf{p}}$, where $\underline{\mathbf{p}}$ denotes the matrix consisting of the retail prices p_{jk} each retailer j charges for each product k, as well as on the advertising expenditures $\mathbf{A}$ and $\mathbf{a}$, where $\mathbf{A}$ and $\mathbf{a}$ indicate vectors containing the advertising expenditures A_i and a_j of all manufacturers and retailers, respectively. The customer demand function can thus be written as $D_{jk} = D_{jk}(\underline{\mathbf{p}}, \mathbf{A}, \mathbf{a})$. This general formulation of customer demand is necessary to allow for interdependencies between the firms' strategies, like the effects of the prices of substitutable products or the competitors' advertising campaign on each firm's demand.

Figure 2.6 illustrates an example of a supply chain with I manufacturers and J retailers, where the number of products K is limited to one in order to reduce complexity. Solid arrows denote flow of goods between the firms like the quantity demanded of the product, while dashed arrows indicate cash flows, e.g., advertising expenditures. Furthermore, flows between single firms within the network and flows

[47]See the literature review in Chap. 3, especially Table 3.2, for a detailed analysis of existing research.

[48]This approach can be found in, e.g., Huang and Li (2001): *Co-op advertising models*, where margins are, however, exogenously given, or later on in Chap. 4, where margins are decision variables.

[49]This approach can be found in, e.g., SeyedEsfahani et al. (2011): *Vertical co-op advertising* and Xie and Wei (2009): *Coordinating advertising*.

[50]Please note that retailer j's margin can be calculated via $m_{ijk} = p_{jk} - w_{ijk} - c_{jk}$ if necessary.

[51]Cf., e.g., Huang and Li (2001): *Co-op advertising models*, pp. 529 et seq.

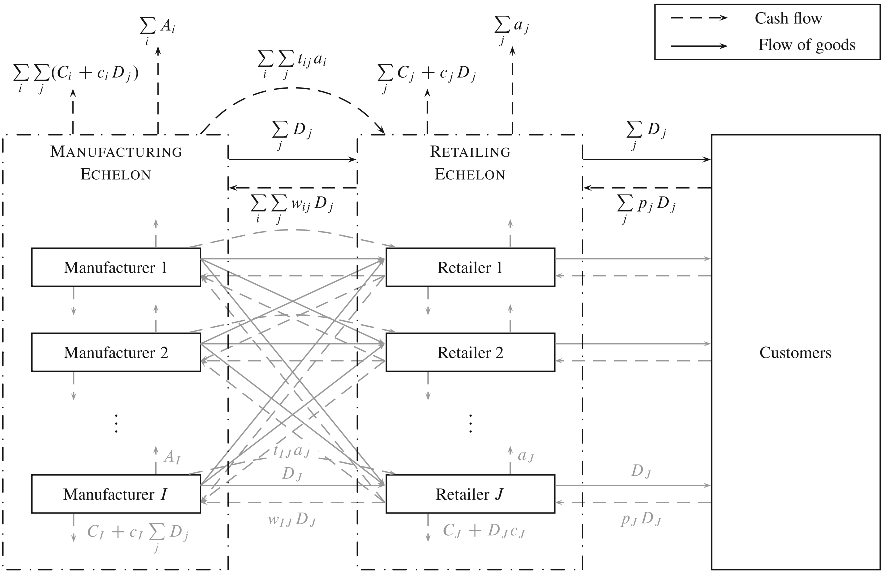

Fig. 2.6 A general one-product ($K = 1$) supply chain with cooperative advertising

which are only related to one single firm, are given in gray color. For the sake of simplicity, only the flows of manufacturer I and retailer J are labeled as an example for the residual flows. On the other hand, the black arrows summarize the flows between the entire manufacturing and retailing echelon.

By means of the financial flows depicted in Fig. 2.6, one is now able to determine the general profit functions Π_{mi} and Π_{rj} of manufacturer i and retailer j, respectively:

$$\Pi_{mi} = \sum_j \sum_k (w_{ijk} - c_{ik}) D_{jk}(\underline{\mathbf{p}}, \mathbf{A}, \mathbf{a}) - A_i - \sum_j t_{ij} a_j - C_i \tag{2.18}$$

$$\Pi_{rj} = \sum_i \sum_k (p_{jk} - w_{ijk} - c_{jk}) D_{jk}(\underline{\mathbf{p}}, \mathbf{A}, \mathbf{a}) - \sum_i (1 - t_{ij}) a_j - C_j . \tag{2.19}$$

For instance, the revenues of manufacturer i are composed of the quantity of each product sold via each retailer $\sum_j \sum_k D_{jk}$, multiplied with the correspondent wholesale price w_{ijk} minus the variable costs of each product in each channel (c_{ik}). From these revenues, the costs of the manufacturer's own advertising A_i, the costs of each cooperative advertising program offered $\sum_j t_{ij} a_j$, as well as the fixed costs C_i have to be subtracted. The profit function of retailer j can be derived analogously.

This framework shall illustrate the general structure of a cooperative advertising model, which can be used to understand and to classify existing models. Obviously, models can differ in the number of firms or products considered, or in the decision variables included. As visible from the brief outlook of models used in this work,

Table 2.2 Outlook of cooperative advertising models used in this work

Model	I	J	K	w_{ijk}	p_{jk}	A_i	a_j	t_{ij}	Demand
Chapter 4	1	1	1	●	●	●	●	●	Deterministic
Chapter 5	1	2	1	●	●	●	●	●	Deterministic
Chapter 6	1	1	1	●	●	–	●	●	Fuzzy

(●) Included as decision variable; (–) not included

which is given in Table 2.2, we contemplate one manufacturer and one product, but one or two retailers. Furthermore, the here presented models always include wholesale and retail price as decision variables, as well as advertising expenditures of both echelons—with the exception of Chap. 6, where only local advertising expenditures are included. Following a common assumption in cooperative advertising literature, both variable and fixed costs are set to zero.[52] Another very important model characteristic is the demand of customers. This refers to the formulation of the underlying demand function on the one hand, and on the data available to the decision makers on the other hand. Here, we consider two models with deterministic customer demand and one model with fuzzy demand, where only imprecise data on market characteristics is available to manufacturer and retailer. However, this differentiation does not comprise every distinction than exists between the various models, but it seems to be appropriate for a first introduction.[53]

[52]See, e.g., SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265, and Xie and Neyret (2009): *Co-op advertising*, p. 1376.

[53]For a more detailed classification, see the literature review in Chap. 3.

Chapter 3
Cooperative Advertising Models in Supply Chain Management: A Review

Abstract This paper reviews articles on cooperative advertising, a topic which has gained substantial interest in recent years. Thereby, we first briefly distinguish five different definitions of cooperative advertising which can be found in operations research literature. After that, we concentrate on vertical cooperative advertising, which is the most common object of investigation. It is understood as a financial agreement where a manufacturer offers to pay a certain share of his retailer's advertising expenditures. In total, we identified 58 scientific papers considering mathematical modeling of vertical cooperative advertising. These articles are then analyzed with regard to their general model setting (e.g., the underlying supply chain structure and design of the cooperative advertising program). After that, we explain the different demand and cost functions that are employed, whereupon we distinguish between static and dynamic models. The last dimension of our review is dedicated to the game-theoretic concepts which are mostly used to reflect different forms of distribution of power within the channel.

3.1 Introduction

A steadily growing stream in operations research literature addresses the interaction of the various members of a supply chain. Thereby, the application of game theory is very common, because it allows to characterize different players' behavior or channel power during decision making.[1] Leng and Parlar (2005) identify four different classes of research[2]: two classes referring to inventory games, a third related to production and pricing competition, and a fourth category named *Games with other attributes*, where one can find game-theoretic analyses of capacity, service, product quality, and advertising decisions. While research in the first categories has been conducted (and reviewed) extensively in the past decades, we want to turn the reader's attention to the latter. The scope of this paper is to give a review of studies which consider the mathematical modeling of cooperative

[1] For a general overview of this field of research and methods in use, we refer the reader to Cachon and Netessine (2004): *Game theory* and Wang and Parlar (1989): *Static game theory models.*

[2] Cf. Leng and Parlar (2005):*Game theoretic applications,* p. 189.

G. Aust, *Vertical Cooperative Advertising in Supply Chain Management*,
Contributions to Management Science, DOI 10.1007/978-3-319-11626-6_3

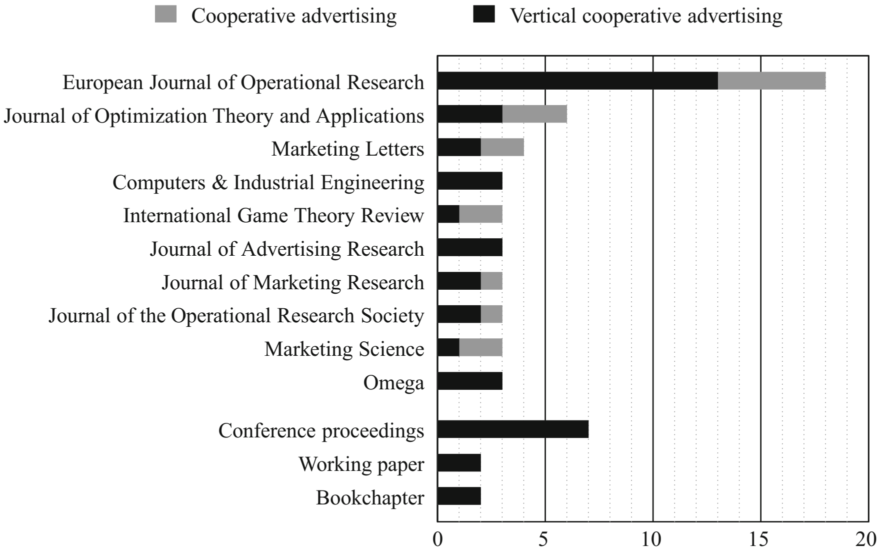

Fig. 3.1 Number of publications on cooperative advertising by journal (as of August 2013; only journals with more than three articles)

advertising, a field which gained substantial interest in recent years' operations research literature. However, existing surveys concentrate on reflecting only singular papers.[3] Therefore, we intend to fill this gap and provide the reader with a broad summary and classification, which also contains recent studies missing in the aforementioned works.

To identify the relevant articles, we performed searches in the databases *ScienceDirect*, *Business Source Complete*, *Web of Knowledge*, and *Google scholar* related to the criteria "cooperative advertising" (and the common abbreviations "co-op advertising" respectively "co-op ad") as well as "advertising coordination". After screening the articles obtained in this way, we furthermore used the bibliographic details given in order to complement our data set. Hence, we are confident that it comprehensively reflects the state of research concerning the mathematical modeling of cooperative advertising.

Through our investigation, we found 110 scientific articles, conference papers, and working papers of scientific institutions written in English dealing with cooperative advertising, with the major part published in academic journals. Though articles were published in altogether 51 journals, Fig. 3.1 indicates that especially the *European Journal of Operational Research* is a popular platform for related publications, followed by *Journal of Optimization Theory and Applications* and

[3]See Taboubi and Zaccour (2005): *Coordination mechanisms in marketing channels*, chap. 3.3 & 4.2, Leng and Parlar (2005): *Game theoretic applications*, chap. 6.4, and Xie and Zhang (2011): *Models in cooperative advertising.*

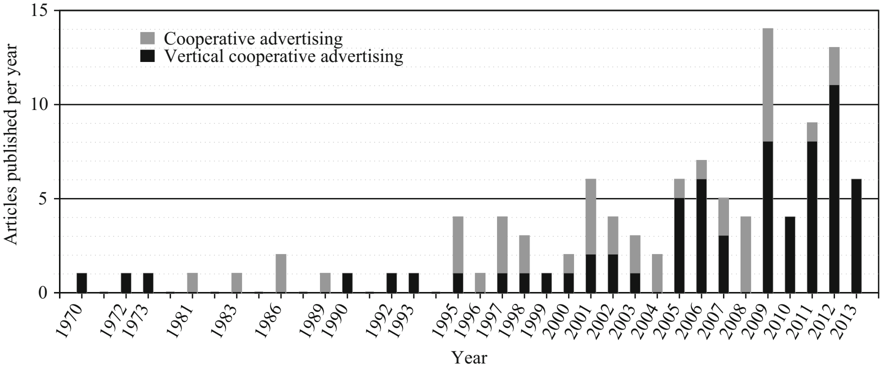

Fig. 3.2 Number of publications on cooperative advertising by year (as of August 2013)

Marketing Letters. The number of publications per year is depicted in Fig. 3.2 and clearly shows the increased interest in this research subject in recent years.
We found five different meanings of the term cooperative advertising, which are briefly described in the following:

Vertical cooperative advertising: This is the most common comprehension of cooperative advertising (used in 68 papers) and describes a financial agreement, where a manufacturer offers to share a certain percentage of his retailer's advertising expenditures.[4] To emphasize the fraction of articles following this understanding, we highlighted related studies by a black bar in Figs. 3.1 and 3.2, while the gray bars refer to all definitions found through the review process.

Cooperative advertising in franchising: A concept similar to the latter is also used in franchisor-franchisee relationships. However, advertising campaigns are mostly implemented by franchisors in order to guarantee uniformity between the different franchisees, who, for their part, participate in the resulting costs by an advertising fee, which is stipulated in the franchise contract.[5]

Horizontal cooperative / generic advertising vs. brand advertising: In contrast to the previous definitions, this group of articles considers collaboration in terms of advertising of firms belonging to the same echelon of the supply chain, which normally act as competitors. Generic advertising is meant as promoting a whole category of products instead of brand-related advertising of single

[4]Cf. Bergen and John (1997): *Cooperative advertising*, p. 357.

[5]Related studies are Bhattacharyya and Lafontaine (1995): *Double-sided moral hazard*, Dant and Berger (1996): *Modelling cooperative advertising*, Desai (1997): *Advertising fee*, Hempelmann (2006): *Optimal franchise contracts*, Michael (2002): *Can a franchise chain coordinate?*, Rao and Srinivasan (2001): *Advertising payments*, Sen (1995) *Advertising fees*, and Sigué and Chintagunta (2009) *Advertising strategies*.

manufacturers.[6] Studies are mostly applied to farming and the agricultural sector.[7]

Cooperative advertising vs. predatory advertising: Following the definition of Church and Ware (2000), cooperative advertising positively influences the own demand as well as the demand faced by the competitors, while predatory advertising detracts consumers from competitors in order to increase own demand.[8]

Joint advertising decisions: The last group we were confronted with simply uses the term cooperative advertising to describe a cooperative (or collusive) decision making concerning the advertising expenditures. It may occur both in intra-echelon as well as inter-echelon competition and focuses on maximizing the joint profit. Some authors propose either contracts or incentive strategies in order to ensure that all players stick to the agreements made.[9]

In the following, we concentrate our review on the first group of articles, which analyses vertical cooperative advertising programs between manufacturer(s) and retailer(s). For the sake of simplicity, we may dispense the prefix vertical and refer solely to cooperative advertising. The remainder of this paper is organized as follows: In Sect. 3.2, we provide a theoretical basis of cooperative advertising, together with empirical data on the diffusion and design of such programs in practice. Subsequently, we review mathematical models dealing with cooperative advertising in Sect. 3.3, with regard to different criteria for categorization, i.e.,

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[6]Cf. Chakravarti and Janiszewski (2004): *The influence of generic advertising*, pp. 488 et seq.

[7]See Alston et al. (2001): *Beggar-thy-neighbor advertising*, Bass et al. (2005): *Generic and brand advertising strategies*, Chakravarti and Janiszewski (2004): *The influence of generic advertising*, Crespi and James (2007): *Bargaining rationale*, Depken et al. (2002): *Generic advertising*, Kinnucan (1997): *Middlemen behaviour and generic advertising*, Krishnamurthy (2000): *Relationship between generic and brand advertising*, Krishnamurthy (2001): *Effect of provision points*, LeVay (1981): *A theory of co-operative advertising*, Lu et al. (2007): *Generic advertising*, Miles et al. (1997): *Advertising budgeting practices*, Simonin and Ruth (1998): *Spillover effects of brand alliances*, Varadarajan (1986): *Horizontal cooperative sales promotion*, and Ward and Dixon (1989): *Fluid milk advertising*.

[8]Cf. Church and Ware (2000): *Industrial organization*, pp. 566 et seq. Studies referring to this definition are Amrouche et al. (2008): *Pricing and advertising*, Depken and Snow (2008): *Strategic nature of advertising*, Erickson (2009): *An oligopoly model*, Friedman (1983): *Advertising*, Karray and Martín-Herrán (2008): *Relationship between advertising and pricing,* Karray and Martín-Herrán (2009): *Advertising and pricing competition*, Ma and Ulph (2012): *Advertising subsidy*, Mariel and Sandonís (2004): *A model of advertising*, Piga (1998): *Review of Industrial Organization*, Slade (1995): *Product rivalry*, Viscolani (2012): *Pure-strategy Nash equilibria*, and Viscolani and Zaccour (2009): *Advertising strategies*.

[9]This approach can be found in Buratto and Zaccour (2009): *Coordination of advertising strategies*, El Ouardighi et al. (2008): *Operations and marketing management*, Forbes (1986): *Market structure and cooperative advertising*, Jørgensen et al. (2001a): *Stackelberg leadership*, Jørgensen and Zaccour (2003a): *Channel coordination*, Jørgensen and Zaccour (2003b): *A differential game*, Karray (2011): *Effectiveness of retail joint promotions*, and Simbanegavi (2009): *Informative advertising*.

the general setting (Sect. 3.3.1), the demand functions (Sect. 3.3.2), and the game-theoretic concepts used (Sect. 3.3.3). In Sect. 3.4, we summarize our findings and give possible directions for future research.

3.2 The Design of Cooperative Advertising Programs

In this section, our scope is to give a brief summary of the theoretical foundations of cooperative advertising as well as on some empirical data on the usage of those programs in practice.[10] Vertical cooperative advertising belongs to promotional support programs which some manufacturers provide to their retailers. More specifically, a manufacturer offers to pay a certain fraction of the advertising cost of his retailer. Thereby, advertising is mostly prepared and organized by the retailer,[11] while the manufacturer solely sets some guidelines like the permitted media etc. After that, the retailer can claim a reimbursement of his expenditures within the predetermined conditions.[12] Crimmins (1970, 1984) explicitly emphasizes that cooperative advertising does not represent an own type of advertising, but rather a financial agreement on the sharing of related cost.[13]

The reasons for such a cooperation between manufacturer and retailer can be manifold. Hutchins (1953) argues that manufacturers adopt cooperative advertising, because it generates immediate sales.[14] To understand this reason, one has to consider the different character and effects of advertising, which depend on the supply chain echelon it emanates from. While manufacturer's global advertising creates a brand image and is more general and nationwide than retailer's local advertising, the latter treats more of promotions and prices. Hence, global advertising makes for publicity and reputation of the product, but does not necessarily lead to real consumer demand.[15]

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[10]For a more elaborate discussion, we refer the interested reader to the books Crimmins (1970): *Cooperative advertising*, Crimmins (1984): *Cooperative advertising*, Hutchins (1953): *Cooperative advertising*, and Young and Greyser (1983): *Managing cooperative advertising*, which also comprise case studies as well as an overview of legal restrictions due to antitrust legislation like, especially, the Robinson-Patman Act (for legal aspects, see also Moran (1973): *Cooperative advertising*).

[11]Cf. Sorenson (1970): *Cooperative advertising*, p. 18.

[12]Cf. Young and Greyser (1983): *Managing cooperative advertising*, p. 4.

[13]Cf. Crimmins (1970): *Cooperative advertising*, p. 21, and Crimmins (1984): *Cooperative advertising*, p. 2.

[14]Cf. Hutchins (1953): *Cooperative advertising*, p. 7.

[15]Cf. Herrington and Dempsey (2005): *Current effects*, p. 62, and Young and Greyser (1983): *Managing cooperative advertising*, pp. 29–37.

Table 3.1 Total amount of cooperative advertising programs in the United States

Year	Amount [bn $]	Source
1957	2	Berger (1972, p. 309)
1970	0.9	Nagler (2006, pp. 91 et seq.)
	3	Huang and Li (2005, p. 174), Young and Greyser (1983, p. 4)
1980	4.8	Young and Greyser (1983, p. 4)
1981	5	Huang and Li (2005, p. 174)
1986	10	Somers et al. (1990, p. 36)
1987	5	Bergen and John (1997, p. 357)
1990	10	Roslow et al. (1993, p. 71)
1993	20	Davis (1994, p. 30)
2000	15	Nagler (2006, pp. 91 et seq.)
2002	60–65	Arnold (2003, p. 4)
2007	25	Chutani and Sethi (2012b, p. 348), He et al. (2011, p. 11)
	50	Kraft and Kamieniecki (2007, cited in Wang et al., 2011, p. 1053)
2008	50	He et al. (2012, p. 74)
2010	50	Yan (2010, p. 510)
2012	50–520	Lieb (2012, p. 3)

Due to these complementary goals and effects, manufacturers are somehow reliant on a certain degree of local advertising. However, it may occur that the retailer's advertising level is not sufficient from the manufacturer's point of view.[16] In this case, a cooperative advertising program can stimulate the retailer's advertising expenditures to a sufficient level. Another reason that can induce a manufacturer to offer a cooperative advertising program is the competition for shelf spaces, which allows retailers to demand promotion support from their manufacturer, or a simple financial consideration: On the one hand, manufacturers mostly do not bear all costs for local advertising, so that the retailers have to take their own share on their part; on the other hand, rates for local advertising may be more economical than rates for global advertising, e.g., in the case of newspapers.[17]

Vertical cooperative advertising programs are widely spread in practice. However, empirical data strongly depends on the source it is taken from and is therefore not completely consistent. Nevertheless, the data collected in Table 3.1 clearly shows an increasing trend of cooperative advertising in practice. Berger et al. (2006) furthermore reports on circa 4,000 existing programs in 52 different product classes,[18] while Dant and Berger (1996) state that 25–40 % of retailers' local advertising is financed by cooperative advertising programs.[19] Two extensive field

[16]Cf. Somers et al. (1990): *Cooperative advertising expenditures*, p. 36.

[17]Cf. Young and Greyser (1983): *Managing cooperative advertising*, p. 36.

[18]Cf. Berger et al. (2006): *Optimal cooperative advertising*, p. 921.

[19]Cf. Dant and Berger (1996): *Modelling cooperative advertising*, p. 1122.

studies with 2,156 respectively 2,286 firms conducted by Dutta et al. (1995) and Nagler (2006) however reveal that most manufacturers offer participation rates of 50 % or 100 %.[20] This may give the impression that most firms determine their participation rate rather arbitrarily than based on detailed analysis and clearly underlines the necessity of a scientific discussion.

3.3 Cooperative Advertising Models

3.3.1 General Setting

To the best of our knowledge, the first mathematical discussion on cooperative advertising is published by Berger (1972), who shows that profits may be increased significantly by quantitative analysis compared to the simplistic fifty-fifty cost sharing which is often used in practice.[21] After this elementary model, where a manufacturer shares his retailer's advertising costs through price discount, several articles picked up this subject and proposed various expansions. In total, 58 of the 68 articles stated in Sect. 3.1 address mathematical modeling of cooperative advertising (see Table 3.2 for a complete listing), while the remaining works are engaged in empirical studies or conceptual theory. In the following, we will give a survey of these models, with particular respect to the general setting of the supply chain, the cooperative advertising program, and the mathematical model, to the formulation of the customer demand, and to the game-theoretic concepts which are used.

We first consider the general setting of the supply chains and models under examination which is used in the articles. From the second column of Table 3.2—which specifies the **topic** of the article, we can see that approximately half of the authors limit their analysis to the determination of players' optimal advertising (indicated by 'A'), while the residual additionally includes further decisions variables. Here, the most prevalent topic is pricing (P), while only few discuss questions concerning product quality (Q), the provision of additional services (S), remanufacturing (R), or inventory and order management (I). As the simultaneous analysis of more than one decision variable provides, e.g., insights into the interdependencies of different marketing instruments on consumer demand, future research will supposedly focus more on multiple decision variables.

The next two criteria refer to **general properties of the mathematical model**, i.e., whether model parameters and variables are time-dependent (dynamic) or not (static) and known before (deterministic) or not (stochastic). Especially the

[20]Cf. Dutta et al. (1995): *Cooperative advertising contracts*, p. 16, and Nagler (2006): *Cooperative advertising participation rates*, p. 96.

[21]See Berger (1972): *Vertical cooperative advertising*.

Table 3.2 General setting

Article	Topic	Model properties		Manufacturer(s)	Retailer(s)	Brand competition	Global advertising	Local advertising	Adv. variable	Coop. ad. scheme	Man. → Ret.	Ret. → Man.
Ahmadi-Javid and Hoseinpour (2011)	A	stat.	det.	M	M		•	•	e	r	•	
Ahmadi-Javid and Hoseinpour (2012)	A	stat.	det.	M	M		•	•	e	r	•	
Aust and Buscher (2012)	A, P	stat.	det.	M	M		•	•	e	r	•	
Bergen and John (1997)	A	stat.	det.	M/D	O/D		•	•	e	r	•	
Berger and Magliozzi (1992)	A	stat.	det./stoch.	M	M			•	o[a)]	r	•	
Berger (1972)	A	stat.	det./stoch.	M	M			•	e	d	•	
Berger (1973)	A	stat.	det.	M	M			•	e	r	•	
Berger et al. (2006)	A, (P)	stat.	det.	M	M		•	•	e	r	•	
Buratto et al. (2007)	A	dyn.	det.	M	M		•	•	l	r	•	
Chen (2010)	A, P	stat.	det.	M	M	•		•	e	r	•	
Chen (2011)	A, I	stat.	stoch.	M	M		•	•	l	(r)[b)]	•	
Chutani and Sethi (2012a)	A	dyn.	det.	M	D			•	l	r	•	
Chutani and Sethi (2012b)	A, (P), P	dyn.	det.	M	D		•	•	l	r	•	
De Giovanni (2011a)	A, P, R	dyn.	det.	M	M		•	•	l	c	•	
De Giovanni (2011b)	A, P, Q	dyn.	det.	M	M			•	l	r	•	
De Giovanni and Roselli (2012)	A, P	dyn.	det.	M	M			•	l	r	•	
Esmaeili and Zeephongsekul (2010)	A, P, I	stat.	det.	M	M			•	e	r	•	
Ghadimi et al. (2013)	A, (P)	stat.	det.	M	D			•	e	r	•	
Guceri-Ucar and Koch (2012)	A	stat.	det.	M	M		•	•	e	r	•	
Haifang et al. (2006)	A	stat.	det.	M	M		•	•	e	r	•	

He et al. (2011)	A	dyn.	stoch.	M	D			•	l	r	•	
He et al. (2012)	A, (P)	dyn.	det.	M	O			•	l	r	•	
He et al. (2009)	A, P	dyn.	det.	M	M			•	l	r	•	
Huang and Li (2005)	A	stat.	det.	M	M		•	•	e	r	•	
Huang and Li (2001)	A	stat.	det.	M	M		•	•	e	r	•	
Huang et al. (2002)	A	stat.	det.	M	M		•	•	e	r	•	
Jørgensen et al. (2000)	A	dyn.	det.	M	M		•[c)]	•[c)]	l	r	•	
Jørgensen et al. (2001b)	A	dyn.	det.	M	M		•	•	l	r	•	
Jørgensen et al. (2003)	A	dyn.	det.	M	M		•	•[d)]	l	r	•	
Jørgensen et al. (2006)	A	dyn.	det.	M	M		•	•	l	a	•	
Kali (1998)	A, P, S	stat.	det.	M	D			•	o[e)]	r	•	
Karray (2013)	A, P	stat.	det.	M	M		•	•	e	r	•	
Karray and Zaccour (2005)	A	dyn.	det.	M	M	•	•	•	l	r	•	
Karray and Zaccour (2006)	A, P	stat.	det.	M	M	•		•	e	r	•	
Karray and Zaccour (2007)	A, (P)	stat.	det.	D	D			•	l	r	•	
Kim and Staelin (1999)	A, P	stat.	det.	D	D			•	e	a	•	
Kunter (2012)	A, P	stat.	det.	M	M		•	•	e	r	•	•
Lei et al. (2009)	A	stat./mult.	det.	M	M		•	•	e	r	•	
Li et al. (2002)	A	stat.	det.	M	M		•	•	e	r	•	
Marchi and Cohen (2009)	A, P, Q	stat.	det.	M	D			•	e	r	•	
SeyedEsfahani et al. (2011)	A, P	stat.	det.	M	M		•	•	e	r	•	
Szmerekovsky and Zhang (2009)	A, P	stat.	det.	M	M		•	•	e	r	•	
Tsao and Sheen (2012)	A, P, I	stat.	stoch.	M	D			•	l	r	•	
Tsou et al. (2009)	A	stat.	det.	M	M		•	•	e	r	•	
Wang et al. (2011)	A, (P)	stat.	det.	M	D		•	•	e	r	•	
Wang (2009)	A, P	stat.	det.	M	M			•	e	r	•	

(continued)

Table 3.2 (continued)

Article	Topic	Model properties		Manufacturer(s)	Retailer(s)	Brand competition	Global advertising	Local advertising	Adv. variable	Coop. ad. scheme	Man. → Ret.	Ret. → Man.
Xiao et al. (2010)	A, P, (P), I	mult.	stoch.	M	M			•	e	r	•	
Xie and Ai (2006)	A	stat.	det.	M	M		•	•	e	r	•	
Xie and Neyret (2009)	A, P	stat.	det.	M	M		•	•	e	r	•	
Xie and Wei (2009)	A, P	stat.	det.	M	M		•	•	e	r	•	
Yan (2010)	A, P	stat.	det.	M	M		•	•	e	r	•	
Yang et al. (2013)	A	stat.	det.	M	M			•	l	r	•	
Yue et al. (2013)	A, (P)[f)], P[f)]	stat.	det.	M	M		•	•	e	r	•	
Yue et al. (2006)	A, P[f)]	stat.	det.	M	M		•	•	e	r	•	
Zhang and Zhong (2011)	A, (P)	stat.	det.	M	D		•	•	l	r	•	
Zhang et al. (2013b)	A, P[g)]	dyn.	det.	M	M		•	•	l	r	•	•
Zhang and Xie (2012)	A, (P)	stat.	det.	M	O		•	•	e	r	•	
Zhang et al. (2013a)	A	stat.	det.	M	M		•	•	e	r	•	•

A . . . Advertising	stat. . . . static	M . . . Monopoly	a . . . Allowances	a) . . . Direct mailing
I . . . Inventory	dyn. . . . dynamic	D . . . Duopoly	c . . . Contract	b) . . . Exogenous parameter
P . . . Pricing	mult. . . . multi-period	O . . . Oligopoly	d . . . Price discount	c) . . . Long-term and short-term advertising
Q . . . Quality			r . . . Participation rate	d) . . . Harmful to manufacturer's image
R . . . Remanufacturing	det. . . . deterministic	e . . . Adv. expenditures		e) . . . Number of advertising messages sent
S . . . Service	stoch. . . . stochastic	l . . . Adv. level		f) . . . Price discount
(·) . . . Exogenous		o . . . Other		g) . . . Advertising influences reference price

distinction between static and dynamic formulations allows to divide the existing articles in two research streams, as we will discuss later on in Sect. 3.3.2. While, for the most part, dynamic articles consider time as a continuous variable, only Lei et al. (2009) and Xiao et al. (2010) propose a multi-period model.[22] Aside from that, it is visible that most studies assume a deterministic environment, though the integration of stochastic influences could improve the accuracy of the results attained. This is clearly a topic for future research, which was recently picked up by Chen (2011), He et al. (2011), Tsao and Sheen (2012), and Xiao et al. (2010).[23]

The predominant **supply chain composition** is a bilateral monopoly consisting of one manufacturer and one retailer (please note that we subsume "manufacturer", "supplier", and "seller" as well as "retailer" and "buyer" for uniformity), while only 14 papers analyze the interaction of more than two players (indicated by 'D' and 'O' for duopoly and oligopoly, respectively). Interestingly, only Karray and Zaccour (2007) and Kim and Staelin (1999) introduced a second manufacturer, whereas one can find more authors that concentrate on retail competition.[24] Nevertheless, competition between two manufacturers could be a promising topic, e.g., when both offer different cooperative advertising programs or, especially, in combination with differences regarding product quality. Seldom, brand competition between a manufacturer's brand and a retailer's store brand is considered.[25] Furthermore, there is no contribution with a three-echelon supply chain consisting of supplier, manufacturer, and retailer as proposed by Chung et al. (2011) for a pricing game.[26] When combined with the analysis of more than two decision variables, e.g., advertising, pricing, and quality, this could also be a promising research area.

Many authors refer to the different effects of **global and local advertising** pointed out in Sect. 3.2. Closely related to that is also the interpretation of the advertising variable, i.e., what the players decide on. Besides two exceptions,[27] advertising variables represent either advertising expenditures/investments (indicated by 'e') or advertising level/efforts (l). Here, advertising expenditures mean a financial amount which is spent on advertising, while an advertising level firstly describes a degree or intensity, which has to be translated into costs afterwards.

[22] See Lei et al. (2009): *Joint advertising channels* and Xiao et al. (2010): *Coordination of a supply chain*.

[23] See Chen (2011): *Coordinating the ordering and advertising policies*, He et al. (2011): *Retail competition*, Tsao and Sheen (2012): *Promotion cost sharing*, and Xiao et al. (2010): *Coordination of a supply chain*.

[24] See Karray and Zaccour (2007): *Effectiveness of coop advertising* and Kim and Staelin (1999): *Manufacturer allowances*.

[25] See Chen (2010): *Manufacturer's co-op advertising*, Karray and Zaccour (2005): *Advertising for national and store brands*, and Karray and Zaccour (2006): *Co-op advertising*.

[26] See Chung et al. (2011): *Price markdown scheme*.

[27] See Berger and Magliozzi (1992): *Optimal co-operative advertising* and Kali (1998): *Minimum advertised price*.

Please note that the classification in Table 3.2 follows this nomenclature, even if this strict differentiation is not always applied throughout the respective articles.

Another distinctive factor is the **cooperative advertising scheme**, i.e., the manner in which the financial participation is modeled, as well as the role allocation within this cooperation. Besides the most commonly used participation rate (indicated by 'r'), where a percentage of the actual advertising costs is shared, we also found a price discount offered by the manufacturer to his retailer (d),[28] or general advertising allowances (a),[29] which do not necessarily have to be spent on advertising completely.[30] Another form is proposed by De Giovanni (2011a), where the manufacturer tries to motivate the retailer to spend more on advertising by means of a revenue sharing contract.[31] Though mentioned by Bergen and John (1997), an accrual rate has not been studied so far.[32] This is an upper limit often used in practice which relates the maximum advertising participation with, e.g., the previous year's sales.[33] Concerning the role allocation, we observed that mostly the manufacturer is assumed to share his retailer's advertising cost, while only three articles allow a bilateral participation, where the retailer may support his manufacturer's advertising program as well.[34]

3.3.2 Demand Functions

Among the most important distinctive features of the aforementioned articles is the underlying demand function, which relates consumer demand with the players' advertising (and pricing). As the demand function strongly depends on the choice between static and dynamic model formulations, it seems to be appropriate to distinguish these in the following discussion.

Demand Functions of Static Games

Table 3.3 lists the static approaches and specifies the shape of the advertising demand and the advertising cost function. Due to the relatively high share of papers also including pricing in their analysis, we also add a classification of the used

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[28]See Berger (1972): *Vertical cooperative advertising*.

[29]See Jørgensen et al. (2006): *Incentives for retailer promotion*.

[30]See Kim and Staelin (1999): *Manufacturer allowances*.

[31]See De Giovanni (2011a): *Environmental collaboration*.

[32]Cf. Bergen and John (1997): *Cooperative advertising*, p. 360.

[33]Cf. Young and Greyser (1983): *Managing cooperative advertising*, p. 33.

[34]See Kunter (2012): *Coordination via cost and revenue sharing*, Zhang et al. (2013b): *Supply chain coordination*, and Zhang et al. (2013a): *Cooperative advertising*.

Table 3.3 Demand and cost functions (static games)

		Advertising demand						Advertising cost			Price demand		
	Article	Linear	Square root	Power	Exponential	Quadratic	Unspecified	Linear	Quadratic	Unspecified	Linear	Power	Other
Advertising expenditures	Ahmadi-Javid and Hoseinpour (2011)			•				•					
	Ahmadi-Javid and Hoseinpour (2012)			•				•					
	Aust and Buscher (2012)		•					•					•[a)]
	Bergen and John (1997)	•						•			•		
	Berger (1972)						•	•					
	Berger (1973)				•			•					
	Berger et al. (2006)				•		•	•					
	Chen (2010)		•					•			•		
	Esmaeili and Zeephongsekul (2010)			•				•				•	
	Ghadimi et al. (2013)			•				•					
	Guceri-Ucar and Koch (2012)			•				•					
	Haifang et al. (2006)			•				•				•	
	Huang and Li (2001)			•				•					
	Huang et al. (2002)			•				•					
	Huang and Li (2005)			•				•					
	Karray (2013)		•					•			•		
	Karray and Zaccour (2006)		•					•			•		
	Kim and Staelin (1999)		•					•			•		
	Kunter (2012)		•					•			•		
	Lei et al. (2009)			•				•					
	Li et al. (2002)			•				•					

(continued)

Table 3.3 (continued)

	Article	Advertising demand						Advertising cost			Price demand		
		Linear	Square root	Power	Exponential	Quadratic	Unspecified	Linear	Quadratic	Unspecified	Linear	Power	Other
Advertising expenditures	Marchi and Cohen (2009)	●								●	●		
	SeyedEsfahani et al. (2011)		●					●					●[a)]
	Szmerekovsky and Zhang (2009)			●				●				●	
	Tsou et al. (2009)			●				●					
	Wang et al. (2011)			●				●					
	Wang (2009)	●						●			●		
	Xiao et al. (2010)			●				●					
	Xie and Ai (2006)			●				●					
	Xie and Neyret (2009)			●				●			●		
	Xie and Wei (2009)		●					●			●		
	Yan (2010)		●					●			●		
	Yue et al. (2013)			●				●				●[b)]	
	Yue et al. (2006)			●				●				●[b)]	
	Zhang and Xie (2012)		●					●					
	Zhang et al. (2013a)		●				●	●		●			

Advertising level	Chen (2011)				●			●					
	Karray and Zaccour (2007)	●							●				
	Tsao and Sheen (2012)	●							●		●		
	Yang et al. (2013)	●							●				
	Zhang and Zhong (2011)	●				●[c)]			●		●		

a) … Shape parameter controls linear, convex, or concave function
b) … Function of price discount
c) … Total demand is multiplied with level of global advertising

price demand function where appropriate. As already adumbrated in Sect. 3.3.1, the functional design is also influenced by the advertising variable itself, i.e., whether it is meant as advertising expenditure or as advertising level. Hence, we separated these two ways in order to make the listing more coherent. Please note that Berger and Magliozzi (1992) and Kali (1998) are not included into the table because they do not consider demand functions comparable to the residual papers.[35] Furthermore, the two articles with multi-period models are also included in this table, because Lei et al. (2009) first propose a static model, which is adapted to n periods afterwards,[36] and Xiao et al. (2010) indeed consider a two-period model, but the demand is only generated in the first period, while the second period is exclusively used for reordering.[37]

One may recognize that most authors either use a square root function or a power function to characterize the form of the consumer demand related to **advertising expenditures**, while only few apply a linear or exponential sales response function. Therefore, we limit ourself to the most common functions.[38]

Let a denote local and A denote global advertising expenditures, Xie and Wei (2009) define a **square root** advertising demand function of a manufacturer-retailer supply chain as

$$h(A, a) = k_{\mathrm{m}}\sqrt{A} + k_{\mathrm{r}}\sqrt{a} \tag{3.1}$$

and justify this with the advertising saturation effect, which indicates a diminishing marginal demand for increasing advertising expenditures.[39] Thereby, the two parameters k_{m} and k_{r} can be interpreted as effectiveness of the corresponding advertising. The same structure was picked up by Aust and Buscher (2012), Karray (2013), Kunter (2012), SeyedEsfahani et al. (2011), Yan (2010), and Zhang et al. (2013a).[40] Zhang and Xie (2012) expand this function to a supply chain with J retailers,

$$h_j(A, a_j) = \alpha_j \left(\beta_j + \sqrt{A} + k_{\mathrm{r}j}\sqrt{a_j}\right), \ j = 1, \ldots, J, \tag{3.2}$$

[35] See Berger and Magliozzi (1992): *Optimal co-operative advertising* and Kali (1998): *Minimum advertised price*.

[36] See Lei et al. (2009): *Joint advertising channels*.

[37] See Xiao et al. (2010): *Coordination of a supply chain*.

[38] We refer the reader to Hanssens et al. (2002): *Market response models*, chap. 3, for an extensive explanation on mathematical modeling of sales response.

[39] Cf. Xie and Wei (2009): *Coordinating advertising*, p. 787.

[40] See Aust and Buscher (2012): *Vertical cooperative advertising*, p. 474, Karray (2013): *Periodicity of pricing and marketing efforts*, p. 637, Kunter (2012): *Coordination via cost and revenue sharing*, p. 479, SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265, Yan (2010): *Cooperative advertising*, p. 512, and Zhang et al. (2013a): *Cooperative advertising*, p. 196.

where the two new parameters α_j and β_j determine base demand and market size, respectively, of retailer j.[41] Obviously, it is assumed that each retailer's local advertising expenditures only affect his own demand and has no effects on his competitors' demand.[42]

Most studies adopt a **power model**, which was first introduced by Huang and Li (2001) to the field of cooperative advertising[43]:

$$h(A, a) = \alpha - \beta a^{-\gamma} A^{-\delta}. \tag{3.3}$$

In contrast to Eq. (3.1), this function depends on four parameters α, β, γ, and δ, which denote the sales saturate asymptote, the impact of advertising on the market demand, and the quasi-elasticity of retailer's and manufacturer's advertising expenditures, respectively.[44] Hence, one is able to control the speed of converging to the saturation level more precisely compared to Eq. (3.1).

An expansion of Eq. (3.3) to a retailer duopoly was proposed by Wang et al. (2011)[45]:

$$h(A, a_j, a_{3-j}) = \alpha_j - \beta a_j^{-\gamma} a_{3-j}^{\bar{\gamma}} (1 + A)^{-\delta}, \; j = 1, 2. \tag{3.4}$$

Thereby, the meaning of parameters α, β, and δ corresponds to Eq. (3.3), while parameters γ and $\bar{\gamma}$ stand for the sensitivity to changes in the own (γ) and in the competitor's advertising expenditures ($\bar{\gamma}$), where own advertising increases and competitor's advertising decreases the resulting consumer demand. Hence, this kind of formulation explicitly refers to rivaling effects of competitors' advertising.

Another formulation of the demand function of a retailer duopoly with similar characteristics can be found in Ghadimi et al. (2013):

$$h(a_j, a_{3-j}) = \alpha_j - \beta_j a_j^{-\gamma} - \beta_{3-j} a_{3-j}^{\bar{\gamma}}, \; j = 1, 2. \tag{3.5}$$

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[41]Besides the two parameters k_m and k_r, which are also used in the following chapters, we omit distinguishing symbols for demand parameters. This section only serves the purpose of giving an overview of existing formulations of demand functions, which are not used further in the sequel. Hence, the meaning of a symbol may change from equation to equation and is indicated in each case when differing from the latter meaning.

[42]Cf. Zhang and Xie (2012): *Cooperative advertising with multiple retailers*, p. 40.

[43]Cf. Huang and Li (2001): *Co-op advertising models*, p. 530.

[44]Cf. Ahmadi-Javid and Hoseinpour (2011): *Coordinating cooperative advertising*, p. 139.

[45]Cf. Wang et al. (2011): *Cooperative advertising models*, p. 1055. The notation of indices j and $3 - j$ permits a general formulation of the demand functions of both retailers, which replaces a separate indication of each individual's function. Thereby, index j denotes the retailer under consideration, while index $3 - j$ stands for his counterpart. This notation is also used in Chap. 5.

Here, demand depends solely on local advertising and the impact of advertising β_j may differ for a_j (while the remaining parameters remain unchanged).[46]

Emanating from an **advertising level** instead of advertising expenditures, we mostly find linear sales response functions. Interestingly, Karray and Zaccour (2007) as well as Zhang and Zhong (2011) assume that also competitor's demand is positively influenced by advertising.[47]

Concerning the mathematical modeling of **advertising costs**, it is obvious that a linear integration is generally used in terms of advertising expenditures, while a quadratically shaped cost function is applied when the advertising variable corresponds to an advertising level. Thus, we can distinguish two cases of handling advertising in (static) mathematical models: Either, advertising expenditures are considered, which go along with a decreasing rise of consumer demand and a linear cost function. In contrast, when talking about advertising level, a linear influence on consumer demand, but a quadratic slope of the advertising cost function is assumed.

Altogether, we can summarize that the assumption of diminishing returns on advertising investment is widely accepted among static models, regardless of which type of advertising variable is used. The two approaches solely differ in the fact whether this is taken into account in the demand or in the cost function. However, there are different approaches to treat effects of advertising between competitors.

Lastly, we briefly review the **price demand function** applied in the articles listed in Table 3.3.[48] One may observe that most authors limit their analysis to linear demand functions of the form $g(p) = \alpha - \beta p$, which are widely used in marketing literature. Here, α indicates the initial base demand, while β can be interpreted as customers' price sensitivity. Recently, SeyedEsfahani et al. (2011) proposed an extension of this function, i.e., $g(p) = (\alpha - \beta p)^{1/\nu}$, where the new parameter ν controls the demand curve's shape in order to obtain a linear, convex, or concave function, which enables a better adjustment to the actual market properties.[49]

Demand Functions of Dynamic Games

One disadvantage of the static models discussed in the previous subsection is the lack of a real distinction between the effects of global and local advertising. As explained in Sect. 3.2, global advertising builds up a long-term brand image, while local advertising tends to generate short-term demand. Obviously, the effectiveness or elasticity parameters used in Eqs. (3.1)–(3.5) cannot reflect these properties entirely due to the missing time dependence. Here, dynamic models have the

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[46]Cf. Ghadimi et al. (2013): *Coordination of advertising*, p. 5.

[47]Cf. Karray and Zaccour (2007): *Effectiveness of coop advertising*, p. 155, and Zhang and Zhong (2011): *Co-op advertising*, p. 1457.

[48]For a comprehensive comparison of price demand functions, we refer the reader to Lau and Lau (2003): *Effects of a demand-curve's shape*.

[49]Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265.

advantage that time is explicitly included into analysis, as we will show in the sequel. As pointed out in a review of dynamic advertising research by Huang et al. (2012), dynamic studies can be classified into six categories according to the **dynamic demand model** they base on[50]: Nerlove-Arrow model, Vidale-Wolfe model, Lanchester model, diffusion models, dynamic advertising competition models with other attributes, and empirical studies of dynamic advertising problems. During our search, we found that cooperative advertising articles only use three of them (see Table 3.4), which are described in the following.

The first (and biggest) group refers to the so-called **Nerlove-Arrow demand model** proposed by Nerlove and Arrow (1962), which explicitly allows for different time-dependent effects of advertising.[51] Besides the impact on present demand (which is similar to the static approach), also effects of advertising on future demand are considered. Therefore, the authors introduce a so-called goodwill stock, which depends on past advertising and by this means allows to separate long-term and short-term effects of firms' advertising on consumer demand. As is visible from Table 3.4, this is the most common dynamic demand model used in cooperative advertising research. We start our discussion with Jørgensen et al. (2000), who were the first to propose a dynamic cooperative advertising model. They assume that both the manufacturer and the retailer can adopt short-term ($A^{\mathrm{S}}(t)$ respectively $a^{\mathrm{S}}(t)$) and long-term advertising level ($A^{\mathrm{L}}(t)$ and $a^{\mathrm{L}}(t)$), where the first type directly affects consumer demand and the latter builds up goodwill. Hence, they define the time-dependent stock of goodwill $G(t)$ by the following dynamic equation:

$$dG(t)/dt = \kappa_{\mathrm{m}}^{\mathrm{L}} A^{\mathrm{L}}(t) + \kappa_{\mathrm{r}}^{\mathrm{L}} a^{\mathrm{L}}(t) - \sigma G(t). \tag{3.6}$$

Thereby, the two parameters $\kappa_{\mathrm{m}}^{\mathrm{L}}$ and $\kappa_{\mathrm{r}}^{\mathrm{L}}$ describe the effectiveness of the players long term advertising, while parameter σ denotes the decay rate of goodwill, i.e., the rate in which existing goodwill fades over time. The demand function is given by

$$h(A^{\mathrm{S}}, a^{\mathrm{S}}, G) = \left(\kappa_{\mathrm{m}}^{\mathrm{S}} A^{\mathrm{S}}(t) + \kappa_{\mathrm{r}}^{\mathrm{S}} a^{\mathrm{S}}(t)\right) \sqrt{G(t)}, \tag{3.7}$$

where $\kappa_{\mathrm{m}}^{\mathrm{S}}$ and $\kappa_{\mathrm{r}}^{\mathrm{S}}$ can be interpreted as effectiveness of manufacturer's and retailer's short-term advertising.[52] Alternatively to this formulation, some articles assume that global advertising only influences goodwill, which goes into the demand function together with local advertising.[53] Another interesting approach can be found in Jørgensen et al. (2003), where retailer's local advertising not only impacts on

[50] Cf. Huang et al. (2012): *Developments in dynamic advertising research*, p. 592.

[51] See Nerlove and Arrow (1962): *Optimal advertising policy*.

[52] Cf. Jørgensen et al. (2000): *Dynamic cooperative advertising*, p. 74.

[53] See Jørgensen et al. (2001b): *Cooperative advertising*.

Table 3.4 Demand and cost functions (dynamic games)

Article	Dynamic demand model			Advertising demand			Price demand	
	Nerlove-Arrow	Vidale-Wolfe	Lanchester	Linear	Square root	Quadratic	Linear	Unspecified
Buratto et al. (2007)	●			●				
Chutani and Sethi (2012a)		●			●		●	
Chutani and Sethi (2012b)		●			●			●
De Giovanni (2011a)	●				●		●	
De Giovanni (2011b)	●			●			●	
De Giovanni and Roselli (2012)	●				●		●	
He et al. (2011)		●	●	●				
He et al. (2012)		●	●	●				
He et al. (2009)		●			●			●
Jørgensen et al. (2000)	●			●	●			
Jørgensen et al. (2001b)	●			●		●		
Jørgensen et al. (2003)	●			●				
Jørgensen et al. (2006)	●			●	●			
Karray and Zaccour (2005)	●			●		●		
Zhang et al. (2013b)	●			●			●[a)]	

All articles refer to advertising level and use a quadratic advertising cost function
a) . . . With respect to the difference between actual price and consumers' reference price

demand, but also negatively influences the goodwill.[54] Furthermore, other variables like pricing[55] or quality[56] can also be included into this Nerlove-Arrow framework.

The second group of articles uses the **Vidal-Wolfe demand model**, which was proposed by Vidale and Wolfe (1957) and expanded by Sethi (1983).[57] The underlying idea is that advertising generates future consumer awareness, which, however, diminishes over time. More precisely, He et al. (2009) define a so-called awareness share $x(t)$ as a fraction of the total market (please note that we neglect the stochastic term for the sake of simplicity)

$$dx(t)/dt = \kappa a(t)\sqrt{1 - x(t)} - \sigma x(t), \tag{3.8}$$

where κ determines the effectiveness of advertising level $a(t)$, and σ is the decay rate of customer awareness.[58] This share is multiplied with the solely price-dependent demand function in order to determine sales. The bilateral monopoly formulation is extended to a retailer duopoly by He et al. (2011) and He et al. (2012), with the following dynamic equation for the market share of retailer 1:

$$dx(t)/dt = \kappa_1 a_1(t)\sqrt{1 - x(t)} - \kappa_2 a_2(t)\sqrt{x(t)} - \sigma_1 x(t) + \sigma_2(1 - x(t)). \tag{3.9}$$

As the market is completely divided between the two retailers, the market share of retailer 2 is given by $1 - x(t)$. Hence, the market share of retailer 1 is increased by his own advertising and decreased by advertising of retailer 2, while the decay of customer awareness can be interpreted contrariwise.[59] Please note that this way of dealing with retail competition may also be categorized into the third group of articles, which base on the Lanchester model. This model was introduced by Kimball (1957) and includes advertising competition in a duopoly into the Vidale-Wolfe model.[60]

Another formulation of retail competition can be found in Chutani and Sethi (2012a), who adopt a model which traces back to extensions of the Vidale-Wolfe model made by Sethi et al. (2008) and Krishnamoorthy et al. (2010)[61]:

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[54] See Jørgensen et al. (2003): *Retail promotions*.

[55] See De Giovanni and Roselli (2012): *Drawbacks of a revenue-sharing contract*.

[56] See De Giovanni (2011b): *Quality improvement vs. advertising support*.

[57] See Vidale and Wolfe (1957): *Sales response to advertising* and Sethi (1983): *Deterministic and stochastic optimization*.

[58] Cf. He et al. (2009): *Cooperative advertising and pricing*, p. 81.

[59] Cf. He et al. (2011): *Retail competition*, p. 12, and He et al. (2012): *Co-op advertising*, p. 77. Please note that He et al. (2011) use $\kappa_1 = \kappa_2 = \kappa$ as well as $\sigma_1 = \sigma_2 = \sigma$.

[60] Cf. Huang et al. (2012): *Developments in dynamic advertising research*, p. 600, and Kimball (1957): *Military operations research methods*.

[61] Cf. Chutani and Sethi (2012a): *Optimal advertising and pricing*, p. 619, as well as Sethi et al. (2008): *Optimal advertising and pricing* and Krishnamoorthy et al. (2010): *Optimal pricing and advertising*.

$$dx_j(t)/dt = \kappa_j a_j(t) g_j(p_j(t)) \sqrt{1 - x_1(t) - x_2(t)}, \ j = 1, 2. \tag{3.10}$$

In contrast to the previous function given in Eq. (3.8), the market share here also depends on price-induced consumer demand $g_j(p_j)$ and the authors do not consider a decay of consumer awareness over time. However, the latter aspect is considered in Chutani and Sethi (2012b), who pick up an extension by Erickson (2009)[62]:

$$dx_j(t)/dt = \kappa_j a_j(t) \sqrt{1 - x_1(t) - x_2(t)} - \sigma_j x_j(t), \ j = 1, 2. \tag{3.11}$$

Taking the above discussion into consideration, we can state that all dynamic models are able to account for time-related effects of advertising. Thereby, the goodwill-oriented models based on Nerlove and Arrow (1962) generally focus on the distinction of short-term and long-term advertising effects (which corresponds well to the commonly made distinction of local and global advertising), while the models referring to Vidale and Wolfe (1957) concentrate on the decay of customer awareness which occurs over time. Interestingly, we can conclude from Table 3.4 that all dynamic models consider advertising levels as decision variable, while the previously discussed static approaches mostly determine advertising expenditures. This focus on advertising levels is combined with a quadratic shape of the advertising cost function. Concerning the price demand, we only found linear demand functions.

3.3.3 Game Theory

Lastly, we consider the solution method which is used in order to determine the channel members optimal decisions regarding advertising (and pricing). As explained in Sect. 3.1, game theory is a popular technique in operations research, because it allows to analyze the channel members' behavior subjected to different relationships and distributions of power within the supply chain. This also holds true for studies on vertical cooperative advertising included in our review, which invariably apply game-theoretic methods—except two cases with similar approaches that are not explicitly denominated game theory, but are calculated in the same manner.[63]

Generally, one can differentiate non-cooperative and cooperative game theory, which mainly differ in the approach which is used to describe a problem: Non-cooperative game theory focuses on the players' actions and strategies which lead to a certain outcome in the end. In contrast, cooperative game theory concentrates on the distribution of possible outcomes between the players and the appendant

[62] Cf. Chutani and Sethi (2012b): *Cooperative advertising*, p. 351, as well as Erickson (2009): *An oligopoly model*.

[63] See Berger (1972): *Vertical cooperative advertising* and Berger (1973): *Cooperative advertising*.

formation of coalitions without considering the actions which are necessary to achieve them.[64]

Our analysis reveals that almost all articles on cooperative advertising apply non-cooperative game theory, i.e., they tend to determine players optimal decisions in order to maximize their profits. The use of cooperative game theory is limited to bargaining models, which are employed to allocate profits in case of a cooperation, with only two exceptions: Marchi and Cohen (2009) propose a biform-game based on the Shapley value,[65] which can be understood as a combination of a non-cooperative and a cooperative game.[66] Furthermore, Ghadimi et al. (2013) construct a three-person cooperative game in addition to their non-cooperative approach, which is also solved by means of the Shapley value.[67]

Hence, we will first consider non-cooperative games which are applied in cooperative advertising literature. Thereby, we differentiate two groups of articles, which are listed in Tables 3.5 and 3.6, respectively: The first group only considers vertical (inter-echelon) interaction, i.e., the interaction between manufacturer(s) and retailer(s), while the second group also takes horizontal (intra-echelon) competition between two or more manufacturers or retailers into consideration. After that, we will discuss articles that use bargaining models in their analysis (see Table 3.7).

Non-Cooperative Games with Solely Vertical Interaction

In general, we can distinguish three game-theoretic concepts which are used in the articles listed in Table 3.5: First, a **Nash game**, which traces back to Nash (1951, 1950b) and is used to determine a solution of a non-cooperative and simultaneous decision making of two or more equal players.[68] Mathematically, the equilibrium can be calculated by separately setting the first order derivatives of the players' profit function to zero and solving the resulting system of equations.

In contrast, a **Stackelberg game** describes a sequential process, where one player acts as a leader and first sets his decision.[69] Thereby, he is assumed to have perfect information about the second player's reaction, which he can thus incorporate into his decision making. After that, in a second step, the follower tries to find his best decision within the framework set by the leader. This equilibrium is calculated via backward induction: The follower's response function, which is determined by

[64]Cf. Cachon and Netessine (2004): *Game theory*, p. 36, and Nagarajan and Sošić (2008): *Game-theoretic analysis*, p. 720.

[65]See Shapley (1953): *A value for n-person games*.

[66]See Marchi and Cohen (2009): *Cooperative advertising*. For further reading on biform-games, see Brandenburger and Stuart (2007): *Biform games*.

[67]See Ghadimi et al. (2013): *Coordination of advertising*.

[68]See Nash (1950b): *Equilibrium points in n-person games* and Nash (1951): *Non-cooperative games*.

[69]See von Stackelberg (1934): *Marktform und Gleichgewicht*.

Table 3.5 Game-theoretic methods (only vertical interaction)

		Vertical interaction				Dynamic equilibrium	
	Article	Vertical Nash	Man. Stackelberg	Ret. Stackelberg	Cooperation	Open loop strat.	Feedback strat.
Static models	Ahmadi-Javid and Hoseinpour (2011)	•	•				
	Ahmadi-Javid and Hoseinpour (2011)	•	•				
	Ahmadi-Javid and Hoseinpour (2012)	•	•				
	Aust and Buscher (2012)	•	•	•	•		
	Berger (1972)	(•)			(•)		
	Berger (1973)	(•)			(•)		
	Berger and Magliozzi (1992)	•			•		
	Chen (2010)		•				
	Chen (2011)	•			•		
	Esmaeili and Zeephongsekul (2010)	•	•	•			
	Guceri-Ucar and Koch (2012)		•	•			
	Haifang et al. (2006)		•		•		
	Huang and Li (2001)	•	•				
	Huang et al. (2002)		•				
	Huang and Li (2005)		•		•		
	Karray (2013)	•	•	•			
	Karray and Zaccour (2006)		•				
	Kunter (2012)		•				
	Lei et al. (2009)		•				
	Li et al. (2002)		•		•		
	SeyedEsfahani et al. (2011)	•	•	•	•		
	Szmerekovsky and Zhang (2009)		•				
	Tsou et al. (2009)	•	•				

Static models	Xiao et al. (2010)	•					
	Xie and Ai (2006)	•	•				
	Xie and Neyret (2009)	•	•	•	•		
	Xie and Wei (2009)		•		•		
	Yang et al. (2013)		•		•		
	Yan (2010)		•		•		
	Yue et al. (2013)	•	•		•		
	Yue et al. (2006)		•		•		
	Zhang et al. (2013a)		•	•			
Dynamic models	De Giovanni (2011a)	•					•
	De Giovanni (2011b)	•	•				•
	De Giovanni and Roselli (2012)		•		•		•
	He et al. (2009)		•		•		•
	Jørgensen et al. (2000)	•	•				•
	Jørgensen et al. (2001b)	•	•				•
	Jørgensen et al. (2003)	•	•				•
	Jørgensen et al. (2006)		•		•		•
	Karray and Zaccour (2005)	•	•				•
	Zhang et al. (2013b)		•		•	•	

(·) . . . Game theory not explicitly used setting the first order derivative(s) to zero and solving the resulting equations for the follower's decision variables, has to be inserted into the leader's profit function before calculating the first order derivative(s).

The third is not necessarily a game-theoretic concept, though it is often used in this context: **Cooperation** (also denoted as *Vertical Integration* or *Centralized Coordination*) between the players can be understood as a joint profit maximization, where the players act like being coordinated by a superior decision maker. This situation is simply calculated by summing up the individual profit functions before setting the correspondent first order derivatives to zero and solving the resulting system of equations.

As is apparent from Table 3.5, the most prevalent setting in vertical interaction is a Manufacturer Stackelberg game, i.e., a situation where the manufacturer obtains the channel leadership and the retailer acts as follower. This is a common assumption in marketing literature and is often justified by large manufacturers which dominated their retailers in the past. However, this relationship was subjected to changes.[70] An often-cited example is the relationship between the manufacturer Proctor & Gamble and the retail chain Wal-Mart, which evolved into a partnership in the recent years.[71] This change also led to a rethinking in research on cooperative advertising: Huang and Li (2001) were the first referring to this new situation and included a Vertical Nash equilibrium into their consideration, which, at least, can be seen as an equal distribution of power between manufacturer and retailer.[72] This was followed by Li et al. (2002), who analyzed a game where both players cooperate and tend to maximize the total profit of the supply chain.[73] Interestingly, only few articles really take into account a Retailer Stackelberg game, where the retailer dominates the manufacturer.[74]

Another interesting approach is proposed by Karray (2013): In addition to the classical sequential moving which emanates from a Stackelberg game, where each player sets all of his decision variables simultaneously, the model also analyzes sequential decision making, e.g., the players set prices first and advertising expenditures afterwards.[75]

Concerning the influence of the underlying game-theoretic concept on the design of a cooperative advertising program, it is interesting that all studies conclude that a manufacturer is only willing to participate in his retailer's advertising expenditures

[70] Cf. Achenbaum and Mitchell (1987): *Pulling away from push marketing*, p. 38, Buzzell et al. (1990): *The costly bargain of trade promotion*, p. 141, and Kumar (1996): *The power of trust*, pp. 92–94.

[71] See Huang et al. (2002): *Manufacturer-retailer supply chain*, pp. 478 et seq.

[72] See Huang and Li (2001): *Co-op advertising models*.

[73] See Li et al. (2002): *Cooperative advertising*.

[74] See Xie and Neyret (2009): *Co-op advertising* for the first static and Buratto et al. (2007): *Advertising coordination games* for the first dynamic approach.

[75] See Karray (2013): *Periodicity of pricing and marketing efforts*.

Table 3.6 Game-theoretic methods (vertical and horizontal interaction)

	Article	Vertical interaction					Horizontal interaction			
		Vertical Nash	Man. Stackelberg	Ret. Stackelberg	Cooperation	Other	Competition	Horizontal Nash	Stackelberg	Cooperation
Static models	Bergen and John (1997)		●		●		M, R	●		
	Berger et al. (2006)	(●)			(●)	●[a)]	C	(●)		
	Ghadimi et al. (2013)	●			●[b)]		R	●		●[b)]
	Kali (1998)		●		●		R	●		
	Karray and Zaccour (2007)		●				M, R, C	●		
	Kim and Staelin (1999)		●				M, R	●		
	Marchi and Cohen (2009)	●					R	●		
	Tsao and Sheen (2012)	●			●		R	●		
	Wang et al. (2011)	●	●		●		R	●[c)]		●
	Wang (2009)	●					C	●[d)]	●	
	Zhang and Zhong (2011)						R	●		●
	Zhang and Xie (2012)		●		●		R	●		●
Dynamic	Chutani and Sethi (2012a)		●		●[e)]		R	●		●[e)]
	Chutani and Sethi (2012b)		●				R	●		
	He et al. (2011)		●				R	●		
	He et al. (2012)		●				R	●		

All dynamic articles determine feedback equilibria

(·) …Game theory not explicitly used
M …Manufacturer
R …Retailer
C …Channel

a) …Different integration strategies
b) …Also partial cooperation
c) …Cournot game
d) …Bertrand game
e) …Complete integration of all members

Table 3.7 Bargaining games

	Utility function			Bargaining model			Parameter	
Article	Linear	Power	Exponential	Symmetric Nash	Asymmetric Nash	Eliashberg	Risk attitude	Bargaining power
Aust and Buscher (2012)		•			•		•	•
Huang and Li (2001)			•	•			•	
Huang et al. (2002)			•			•	•	•
Huang and Li (2005)	•			•				
Kunter (2012)	•				•			•
Li et al. (2002)			•			•	•	•
SeyedEsfahani et al. (2011)		•		•			•	
Wang et al. (2011)			•	•			•	
Xie and Neyret (2009)		•		•			•	
Xie and Wei (2009)		•		•			•	
Yan (2010)		•		•			•	
Yue et al. (2006)			•			•	•	•

in case of a Manufacturer Stackelberg equilibrium or a Cooperation.[76] Please note that this holds true only for models where the manufacturer sets the participation rate on his own. Buratto et al. (2007) use a different setting and let the retailer decide on the participation rate under a Retailer Stackelberg equilibrium.[77] This approach could be helpful to study cooperative advertising under a dominant retailer in the future. However, it first of all has to be ensured if such an arrangement is possible (and reasonable) in practice.

In the end, we briefly discuss dynamic games, because there exist two types of **dynamic equilibria**, which are given in the last two columns of Table 3.5: One can distinguish open loop equilibria and feedback equilibria, which differ in the underlying information structure. In the first group, the players decisions only depend on the time t and on the initial conditions at time $t = 0$, while the latter assumes that players are able to incorporate the actual condition at time t into their decision-making for time t.[78] Though the determination of feedback equilibria is more challenging, most authors use this information structure when studying cooperative advertising.

Non-Cooperative Games with Vertical and Horizontal Interaction

Now we pass on to articles which involve not only vertical, but also horizontal competition. Please note that this does not only apply for models with more than two players, but also for models which consider competition between two or more retail channels (which may be opened up by one single firm, e.g., a traditional channel and an online channel). The type of competition is thus indicated in Table 3.6. Here, we can see that most studies focus on retail competition (R), as we already noticed in Sect. 3.3.1.

In principle, the same game-theoretic concepts are used in horizontal as in vertical interaction, which have already been illustrated in the previous subsection. Please note that though different notations for the horizontal simultaneous move game with equal players are in use, we only employ the term Horizontal Nash instead of Bertrand or Cournot game. As is visible, this is mostly used for studying horizontal interaction in cooperative advertising. This can be probably explained by the comparatively simple calculation. However, especially an intra-echelon Stackelberg game in combination with an inter-echelon Stackelberg game may lead to very interesting results. This is clearly a task for future research. Furthermore, the study of Ghadimi et al. (2013) has to be pointed out, as it firstly applies the concept

[76] See, e.g., Aust and Buscher (2012): *Vertical cooperative advertising*, p. 478, and SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 267.

[77] Cf. Buratto et al. (2007): *Advertising coordination games*, p. 315.

[78] Cf. He et al. (2007): *Stackelberg differential game models*, p. 389.

of partial cooperation introduced by Cyert and DeGroot (1973) to a cooperative advertising model.[79]

Bargaining Games

As indicated at the beginning of Sect. 3.3.3, some articles which mainly concentrate on non-cooperative games also apply bargaining models, though these belong to the class of cooperative games. In this context, mostly bargaining games are used to determine a fair distribution of the total supply chain profit which results from a Cooperation between manufacturer and retailer.[80] Another possible method for profit allocation was recently introduced by Yue et al. (2013), who assume that each player's profit is linearly related to its bargaining power, without applying a bargaining game explicitly.[81]

A bargaining game generally consists of two components (see Table 3.7), which mainly determine the parameters like players' bargaining power or risk attitude, which can be incorporated into the problem-solving: First, one has to decide which utility function is assumed for the players participating in the bargaining and, attendant to that, which risk attitude they have. During our research, we found three types of **utility functions** which are assumed in cooperative advertising models. The simplest one is a linear function of the shape

$$u(\Pi) = \Pi, \tag{3.12}$$

where the utility function $u(\Pi)$ assigns a certain utility value $v = u(\Pi)$ to a profit Π. Here, the player is assumed to be risk neutral. In contrast, a power utility function

$$u(\Pi) = \Pi^{\mu}, \tag{3.13}$$

allows for risk neutral, risk averse, and risk-seeking players depending on the risk parameter μ. Lastly, an exponential function like

$$u(\Pi) = 1 - e^{-\mu\Pi}, \tag{3.14}$$

can be used for solely risk averse players, where the degree of risk aversion depends also on parameter μ.

The second component of a bargaining game is the underlying **bargaining model**. Here, we found the Symmetric Nash bargaining model by Nash (1950a), the Asymmetric Nash bargaining model by Harsanyi and Selten (1972) and Kalai

[79] See Ghadimi et al. (2013): *Coordination of advertising* as well as Cyert and DeGroot (1973): *Cooperation and learning in a duopoly*.

[80] See, e.g., Huang and Li (2001): *Co-op advertising models*.

[81] Cf. Yue et al. (2013): *Pricing and advertisement*, pp. 498 et seq.

(1977), and the Eliashberg bargaining model by Eliashberg (1986). These models differ in the way the total utility value v_{1+2} of both players is calculated. For instance, the Symmetric Nash model simply assumes a multiplication of the two single utility values v_1 and v_2[82]:

$$v_{1+2} = v_1 v_2. \tag{3.15}$$

Hence, there is no possibility to include other variables like bargaining power during the determination of a distribution of profits which is acceptable for both players. If one also wants to account for bargaining power, either the Asymmetric Nash model[83]

$$v_{1+2} = v_1^{\lambda_1} v_2^{\lambda_2} \tag{3.16}$$

or the Eliashberg model[84]

$$v_{1+2} = \lambda_1 v_1 + \lambda_2 v_2, \tag{3.17}$$

both with $\lambda_1 + \lambda_2 = 1$, can be used.

Whether the obtained solution respects the **parameters** risk attitude and/or bargaining power of the parties involved, thus depends on the combination of utility function and bargaining model. As is obvious from Table 3.7, there are only two pairs that allow considering both: an exponential utility function with the Eliashberg model—as used in Huang et al. (2002), Li et al. (2002), and Yue et al. (2006)—as well as a power utility function with the Asymmetric Nash model—as used in Aust and Buscher (2012).[85]

Particular attention has to be paid to Wang et al. (2011), who, for the first time, apply bargaining to a single-manufacturer two-retailer cooperative advertising model, while the residual works solely consider bilateral monopolies.[86] In this context, coalition forming could be an interesting research topic.[87]

[82]Cf. Nash (1950a): *Bargaining problem*, p. 159.

[83]Cf. Harsanyi and Selten (1972): *Generalized Nash*, p. 96, and Kalai (1977): *Nonsymmetric Nash*, pp. 130 et seq.

[84]Cf. Eliashberg (1986): *Arbitrating a dispute*, p. 966.

[85]Cf. Huang et al. (2002): *Manufacturer-retailer supply chain*, pp. 482–285, Li et al. (2002): *Cooperative advertising*, pp. 353 et seq., Yue et al. (2006): *Coordination of cooperative advertising*, pp. 77–82, and Aust and Buscher (2012): *Vertical cooperative advertising*, p. 477.

[86]Cf. Wang et al. (2011): *Cooperative advertising models*, p. 1065.

[87]We refer the reader to Nagarajan and Sošić (2008): *Game-theoretic analysis* for further reading.

3.4 Conclusion and Further Research Directions

In this paper, we provide a review of mathematical models on cooperative advertising, which enjoy a certain popularity in recent years' operations research literature. Therefore, we conduct a literature research, which reveals that there exist different meanings of the term cooperative advertising. After a short dissociation of terms, we concentrate our analysis on vertical cooperative advertising, which is meant as a financial agreement where the manufacturer offers to bear a certain share of his retailer's advertising cost. By this means, he is able to benefit from the complementary properties of retailer's local advertising compared to his own global advertising.

Our analysis of the general model setting shows that, though many authors consider solely firms' advertising decisions, there is a growing part that includes other demand-relevant variables like price, product quality, or service into their models. We expect that future research will primarily focus on models of that kind, likely even with more than two variables, as these provide more insight into consumer behavior. Another future task is to further concentrate on models consisting of more than two players, i.e., to integrate manufacturer and/or retailer competition. However, first approaches already show a considerably increased complexity of calculus, which may render necessary heuristics or meta-heuristics.[88]

The cooperative advertising program is commonly modeled by a participation rate, which denotes the manufacturer's share in the retailer's advertising. Interestingly, we also found few articles, where the retailer can also participate in manufacturer's advertising, though this clearly represents a minority. However, as Zhang et al. (2013a) show in their analysis, bilateral participation may lead to channel coordination, an objective which apparently has almost been neglected in cooperative advertising literature so far.[89] Future studies could therefore search for other instruments which can improve channel efficiency, like it has been done in literature on pricing to a great extent.

After that, we discuss the demand and cost functions on which the models base on. Due to the diverse approaches, we differentiate between static and dynamic games, whereupon the majority clearly belongs to the first group. Here, the prevalent assumption is that advertising expenditures have diminishing returns, which are commonly expressed by square root or power functions. Comparable assumptions can also be found in dynamic models, which, in addition, have the advantage that time is taken into account. A popular model traces back to Nerlove and Arrow (1962), who propose that advertising causes a certain degree of goodwill, which creates future demand and, however, decays over time. In our opinion, this approach

[88]See, e.g., Sadigh et al. (2012): *Manufacturer-retailer supply chain coordination* and Yu and Huang (2010): *Nash game model* for examples with respect to other topics.

[89]Cf. Zhang et al. (2013a): *Cooperative advertising*, p. 196.

allows representing the different effects of local and global advertising expenditures well.

The last subject of our review is game theory, which is used to analyze the interaction of supply chain members with different underlying power structures. Most studies are limited to vertical interaction between one manufacturer and one retailer, but there also exist articles with horizontal (and mainly retail) competition. Here, the prevalent setting assumes that the manufacturer owns the channel leadership and dominates the retailers, which is a very common assumption in marketing literature. However, more and more authors refer to the growth of large retail chains and the related changes of the distribution of power and consider an equal distribution of power with or without cooperation or even a retailer-dominated channel. This clearly should be continued in future research, presumably with a focus on more complex game structures like Stackelberg games both within an echelon and between echelons.

Though substantial work has been done regarding different power structures within the supply chain, there is no study which analyzes the effects of information asymmetry or information sharing on the decisions and profits of the supply chain members. As some of the game-theoretic studies already revealed, the manufacturer is only disposed to offer a cooperative advertising program when he acts as a leader and, hence, is aware of the local advertising expenditures his retailer is planning. Therefore, the application of principal-agent theory, where the retailer takes on the agent's role, may certainly be of interest.

Lastly, besides these possible extensions of mathematical modeling, further empirical analyses on cooperative advertising programs and related decision processes are of absolute necessity in order to understand the real practical needs and problems which may result from the introduction of a cooperative advertising program.

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http://dx.doi.org/10.1016/j.ejor.2013.08.010.

Chapter 4
Vertical Cooperative Advertising and Pricing Decisions in a Manufacturer-Retailer Supply Chain: A Game-Theoretic Approach

Abstract Manufacturers can increase the advertising expenditures of their retailers by bearing a fraction of the occurring costs within the framework of a vertical cooperative advertising program. We expand the existing research, which deals with advertising and pricing decisions in a manufacturer-retailer supply chain. By means of game theory, four different relationships between the channel members are considered: firstly, three non-cooperative interactions with either symmetric distribution of power or asymmetric distribution with one player being the leader in each case, and one Cooperation where both players tend to maximize the total profit. The latter is complemented by a bargaining model, which proposes a fair split of profit on the basis of the players' risk attitude and bargaining power. Our main findings are as follows: (a) In contrast to previous analyses, we do not limit the ratio between manufacturer's and retailer's margin, which provides more general insights into the effects of the underlying distribution of power within the channel. (b) The highest total profit is gained when both players cooperate. This behavior puts also the customers in a better position, as it produces the lowest retail price as well as the highest advertising expenditures compared to the other configurations.

4.1 Introduction

In recent years, several papers dealt with vertical cooperative advertising in a manufacturer-retailer channel (for the sake of simplicity, we may refer to cooperative advertising in the following). This type of collaboration can be defined as a financial agreement, in which the manufacturer offers to bear either a certain part or the entire advertising expenditures of his retailer.[1] Thereby, he intends to increase the retailer's advertising, which aims at stimulating the immediate demand of the customers. With this financial assistance, the retailer can increase his level of advertising, which leads to higher sales for both, the retailer and the manufacturer.[2] Though being a significant part of many manufacturers' advertising budgets (e.g., a total sum of $15 billion was invested in such programs in the United States in 2000),

[1] Cf. Bergen and John (1997): *Cooperative advertising*, p. 357.

[2] Cf. Somers et al. (1990): *Cooperative advertising expenditures*, p. 36.

G. Aust, *Vertical Cooperative Advertising in Supply Chain Management*,
Contributions to Management Science, DOI 10.1007/978-3-319-11626-6_4

most firms seem to set the participation rate arbitrarily and without detailed analysis to 50 % or 100 %.[3]

The research on cooperative advertising can be roughly divided into two groups. Authors belonging to the first group concentrate their analysis solely on advertising. The first mathematical modeling of cooperative advertising was the work of Berger (1972), who proposed a financing of the retailer's advertising expenditures by a discount on the wholesale price.[4] Dant and Berger (1996) adopt this approach to the context of franchising.[5] Karray and Zaccour (2006) consider a bilateral duopoly and demonstrate that cooperative advertising can also have harmful impacts on the retailers.[6] In contrast to the latter examples which are based on static models, Jørgensen et al. (2000), Jørgensen and Zaccour (2003a), and Jørgensen and Zaccour (2003b) study the effects of cooperative advertising in a dynamic environment by using a goodwill function, on which the retailer's advertising has either positive or negative effects.[7]

One of the first works comparing different types of manufacturer-retailer relationships by game theory was the work of Huang and Li (2001), who use a demand function which depends both on the local advertising expenditures of the retailer and on the global advertising expenditures of the manufacturer. Though emphasizing the changed power structure in favor of the retailers, they consider equal power distribution (Nash equilibrium), manufacturer-leadership (Manufacturer Stackelberg equilibrium) and the case where manufacturer and retailer act in Cooperation and bargain for the division of profits.[8]

Representatives of the second group—to which the present paper belongs to—also include other decision variables like pricing, as can be found in Bergen and John (1997), Kim and Staelin (1999), and Karray and Zaccour (2007).[9] For instance, Yue et al. (2006) extend the model of Huang and Li (2001) by a price-sensitive component within the demand function in order to deliver the optimal advertising expenditures of both channel members as well as the optimal price discount offered to the costumers by the manufacturer. They compare the results of Manufacturer Stackelberg equilibrium and Cooperation.[10] In lieu of a price discount, Szmerekovsky and Zhang (2009) include the resulting retail price in their demand function and calculate a Manufacturer Stackelberg equilibrium. Here—in

[3]Cf. Nagler (2006): *Cooperative advertising participation rates*, p. 96.

[4]See Berger (1972): *Vertical cooperative advertising*.

[5]See Dant and Berger (1996): *Modelling cooperative advertising*.

[6]See Karray and Zaccour (2006): *Co-op advertising*.

[7]See Jørgensen et al. (2000): *Dynamic cooperative advertising*, Jørgensen and Zaccour (2003a): *Channel coordination*, and Jørgensen and Zaccour (2003b): *A differential game*.

[8]See Huang and Li (2001): *Co-op advertising models*. A similar approach with slightly modified demand function can be found in Huang et al. (2002): *Manufacturer-retailer supply chain*.

[9]See Bergen and John (1997): *Cooperative advertising*, Karray and Zaccour (2007): *Effectiveness of coop advertising*, and Kim and Staelin (1999): *Manufacturer allowances*.

[10]See Yue et al. (2006): *Coordination of cooperative advertising*.

Table 4.1 Related cooperative advertising models

	Price demand	Advertising demand	Games			
Huang and Li (2001)	–	$\alpha - \beta a^{-\gamma} A^{-\delta}$	N	MS	–	C
Yue et al. (2006)	$(p/p_0)^{-e}$	$\alpha - \beta a^{-\gamma} A^{-\delta}$	–	MS	–	C
Szmerekovsky and Zhang (2009)	p^{-e}	$\alpha - \beta a^{-\gamma} A^{-\delta}$	–	MS	–	–
Xie and Wei (2009)	$1 - \beta p$	$k_r\sqrt{a} + k_m\sqrt{A}$	–	MS	–	C
Xie and Neyret (2009)	$\alpha - \beta p$	$A - B a^{-\gamma} A^{-\delta}$	N	MS	RS	C
SeyedEsfahani et al. (2011)	$(\alpha - \beta p)^{1/\nu}$	$k_r\sqrt{a} + k_m\sqrt{A}$	N	MS	RS	C

N ... Nash, MS ... Manufacturer Stackelberg, RS ... Retailer Stackelberg, C ... Cooperation

contrast to the latter article—the retailer fully determines the price demand, which previously was influenced only by the manufacturer.[11] The model proposed by Xie and Wei (2009) is based upon a different demand function, which enables the authors to handle (cooperative) advertising and pricing decisions of both channel members contemporaneously. In this context, closed-form solutions of the Manufacturer Stackelberg equilibrium and the Cooperation are derived.[12] Yan (2010) customizes this model in order to adapt it to the e-marketing environment.[13]

The assumption of a dominant manufacturer is indeed very common in marketing literature, but the development of large retailers like Wal-Mart and, according to that, the shift of market power necessitates additional analyses. The first paper which considered not only a leadership of the manufacturer, but also a dominant retailer, was Xie and Neyret (2009). Besides this Retailer Stackelberg equilibrium, also Nash equilibrium, Manufacturer Stackelberg equilibrium and Cooperation are calculated.[14] The work of SeyedEsfahani et al. (2011) applies these four games on the model proposed by Xie and Wei (2009), but relaxes the assumption of a linear price demand function by introducing a new parameter ν, which can cause either a convex ($\nu < 1$), or a linear ($\nu = 1$), or a concave ($\nu > 1$) price demand function.[15] Lastly, a recent paper of Kunter (2012) follows a different approach and concentrates on establishing channel coordination by means of a royalty payment contract.[16]

Table 4.1 summarizes the cooperative advertising models, which are most related to our approach, as well as the corresponding demand functions and games being used. Please note that both Xie and Neyret (2009) and SeyedEsfahani et al. (2011) only initially use the parameters α and β within their price demand function and normalize them to one during further calculus.[17]

[11] See Szmerekovsky and Zhang (2009): *Pricing and two-tier advertising*.

[12] See Xie and Wei (2009): *Coordinating advertising*.

[13] See Yan (2010): *Cooperative advertising*.

[14] See Xie and Neyret (2009): *Co-op advertising*.

[15] Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265.

[16] See Kunter (2012): *Coordination via cost and revenue sharing*.

[17] Cf. Xie and Neyret (2009): *Co-op advertising*, p. 1376, and SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265.

Table 4.2 List of symbols

Variables		Parameters/Functions	
p	Retail price	α	Initial base demand
w	Wholesale price	β	Price sensitivity
m	Retailer margin	ν	Shape parameter
a	Local advertising expenditures	k_r	Effectiveness of local advertising
A	Global advertising expenditures	k_m	Effectiveness of global advertising
t	Advertising participation rate	k	Advertising effectiveness ratio
Π	Profit	$h(\cdot)$	Advertising demand function
		$g(\cdot)$	Price demand function

It is clearly visible that only the latter two really take into account the changed market structure, i.e., the shift of power from manufacturers to retailers, by including Nash and Retailer Stackelberg equilibrium. However, both had to deal with some mathematical difficulties during the calculation of the manufacturer's decision problem for these new games: Following the notation explained in Table 4.2, the profit functions of both articles can be written as

$$\Pi_m = wg(p)h(a, A) - A - ta \tag{4.1}$$

$$\Pi_r = (p - w)g(p)h(a, A) - (1 - t)a, \tag{4.2}$$

where $g(p)$ and $h(a, A)$ denominate the respective price and advertising demand function.[18] As the price demand depends only on the retail price p, the manufacturer will choose the highest possible wholesale price $w < p$ in order to maximize his profit. Given that this behavior would result in $\Pi_r = 0$, Xie and Neyret (2009) assume identical margins (i.e., $w = p/2$) for both players both in Nash and Retailer Stackelberg equilibrium.[19] This ratio between wholesale price and retail price is also adopted by SeyedEsfahani et al. (2011).[20]

In this paper, we intend to relax this restrictive assumption of identical margins to get better insight into the effects of market power on the distribution of channel profits. Through a modification of the profit functions, we are able to extend the existing research by unrestrained Nash and Retailer Stackelberg equilibria. Thereby, we follow the modified price demand function introduced by SeyedEsfahani et al. (2011), as we expect more insight as from the linear function of Xie and Wei (2009). Contrary to the authors, we will not normalize the parameters α and β to one in order to be able to adapt the function to the real price demand.

The remainder is organized as follows: In Sect. 4.2, we explain the model formulation together with our modification of the profit functions (Sect. 4.2.1)

[18]Please note that advertising is commonly assumed to affect demand like a multiplicator of price demand (see Thompson and Teng (1984): *Optimal pricing and advertising policies*, p. 151).

[19]Cf. Xie and Neyret (2009): *Co-op advertising*, p. 1377.

[20]Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 266.

and calculate Nash (Sect. 4.2.2), Manufacturer Stackelberg (Sect. 4.2.3), Retailer Stackelberg equilibrium (Sect. 4.2.4), and Cooperation (Sect. 4.2.5). The latter game has to be complemented by a bargaining model, which is used to determine the profit split between manufacturer and retailer. Therefore, we introduce the Asymmetric Nash bargaining model of Harsanyi and Selten (1972) and Kalai (1977) in Sect. 4.2.6. The results of these four games are compared in Sect. 4.3 via numerical examples. Section 4.4 summarizes our main findings and indicates possible directions of further research.

4.2 Four Forms of Retailer-Manufacturer Relationship

4.2.1 Model Formulation

We consider a supply chain consisting of one manufacturer and one retailer, which is illustrated in Fig. 4.1. According to Choi (1991) and Choi (1996), we introduce the retailer margin m as a new decision variable, with[21]

$$m = p - w. \tag{4.3}$$

Hence, we derive the following modified price and advertising demand functions:

$$g(w, m) = [\alpha - \beta(w + m)]^{1/\nu} \tag{4.4}$$

$$h(a, A) = k_r\sqrt{a} + k_m\sqrt{A}, \tag{4.5}$$

The downward-sloping price demand function $g(w, m)$ in Eq. (4.4) is an advancement of the widely used linear demand function $g(p) = 1 - p$, which is proposed by SeyedEsfahani et al. (2011).[22] The two parameters α and β can be interpreted as initial base demand respectively price sensitivity of the costumers. As described

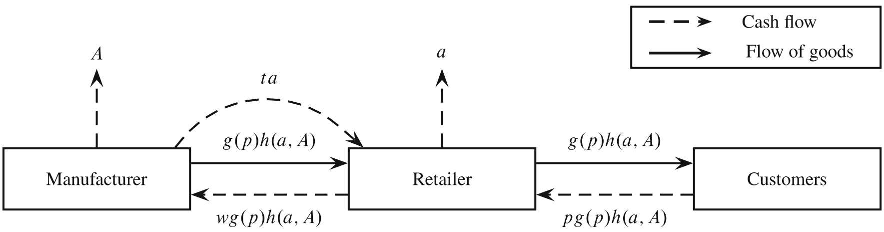

Fig. 4.1 Manufacturer-retailer supply chain

[21]Cf. Choi (1991): *Price competition*, p. 275, and Choi (1996): *Price competition*, pp. 122 et seq.

[22]Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265.

in Sect. 4.1, it can be either concave, or linear, or convex, depending on the shape parameter ν. Therefore, it may be better adaptable to the real reaction of customers on changes in price, which cannot be reduced to one specific shape.

In contrast, there is consent in marketing literature that the advertising response function should be increasing and concave in order to reproduce the diminishing returns to advertising expenditures.[23] Therefore, we use a function based on square roots, which was used similarly by Kim and Staelin (1999), Karray and Zaccour (2006), and Xie and Wei (2009).[24] The parameters k_r and k_m allow us to individually take account of the different effectiveness of local and global advertising expenditures, which strongly depends on the customer perception.

Inserting the functions (4.4) and (4.5) into the general profit functions (4.1) and (4.2), we derive the following profit functions of manufacturer and retailer:

$$\Pi_\mathrm{m} = w\,[\alpha - \beta(w+m)]^{1/\nu}\left(k_\mathrm{r}\sqrt{a} + k_\mathrm{m}\sqrt{A}\right) - A - ta \tag{4.6}$$

$$\Pi_\mathrm{r} = m\,[\alpha - \beta(w+m)]^{1/\nu}\left(k_\mathrm{r}\sqrt{a} + k_\mathrm{m}\sqrt{A}\right) - (1-t)a. \tag{4.7}$$

By splitting the retail price p into wholesale price w and retailer margin m, we generate a profit function of the manufacturer (4.6) where, in contrast to Eq. (4.1), the wholesale price also has an impact on the demand. Hence, we are now able to calculate the first order partial derivative $\partial\Pi_\mathrm{m}/\partial w$ and, therefore, do not need to introduce the assumption $w = p/2$ as described in Sect. 4.1.

Please note that all parameters and variables are positive and that we have to set $w + m < \alpha/\beta$ to ensure a non-negative price demand. The domain of t is set to $0 \le t < 1$ in order to avoid mathematical problems during further calculus.[25]

[23]Cf. Simon and Arndt (1980): *The shape of the advertising response function*, pp. 12–14.

[24]Cf. Kim and Staelin (1999): *Manufacturer allowances*, pp. 65 et seq., Karray and Zaccour (2006): *Co-op advertising*, p. 1010, and Xie and Wei (2009): *Coordinating advertising*, p. 787.

[25]For $t = 1$, the manufacturer would bear the entire local advertising expenditures without any possibility to control the actual amount. Consequently, the retailer would choose a as high as possible, which is an infinite value according to the presented model, because no agreed upper limit of the participation exists. This would violate the condition of positive profits, which shall be assumed for a firm's participation within the supply chain. In practice, where participation rates of 100 % are very common (cf. Nagler (2006): *Cooperative advertising participation rates*, p. 96), such behavior is prohibited by other agreements like an accrual rate, which relates the maximum participation to, e.g., previous year's sales. However, from a modeling perspective, either participation rate or accrual rate can be binding at the same time (cf. Bergen and John (1997): *Cooperative advertising*, p. 360). In combination with $t = 1$, an accrual rate determined by the manufacturer is equivalent to a situation where the manufacturer directly sets a. This would result in a decision in favor of either a or A, depending on the correspondent effectiveness parameter. As this does not represent the situation in practice, the necessary limitation $t < 1$ seems to be an appropriate drawback when considering t as decision variable.

4.2.2 Symmetric and Non-Cooperative Relationship

In this section, we assume a symmetric distribution of power between manufacturer and retailer. This situation can be modeled by means of a Nash game, where both players take their decisions simultaneously and non-cooperatively. Therefore, we can formulate the decision problem of the manufacturer

$$\begin{aligned} \text{Max} \quad & \Pi_\mathrm{m} = w\,[\alpha - \beta(w+m)]^{1/\nu}\left(k_\mathrm{r}\sqrt{a} + k_\mathrm{m}\sqrt{A}\right) - A - ta \\ \text{s.t.} \quad & w < \alpha/\beta - m,\ A > 0 \text{ and } 0 \le t < 1, \end{aligned} \tag{4.8}$$

as well as the decision problem of the retailer

$$\begin{aligned} \text{Max} \quad & \Pi_\mathrm{r} = m\,[\alpha - \beta(w+m)]^{1/\nu}\left(k_\mathrm{r}\sqrt{a} + k_\mathrm{m}\sqrt{A}\right) - (1-t)a \\ \text{s.t.} \quad & m < \alpha/\beta - w \text{ and } a > 0. \end{aligned} \tag{4.9}$$

One can solve these decision problems by setting the first order partial derivatives $\partial\Pi_\mathrm{m}/\partial w$, $\partial\Pi_\mathrm{m}/\partial A$, $\partial\Pi_\mathrm{r}/\partial m$, and $\partial\Pi_\mathrm{r}/\partial a$ to zero. Please note that the manufacturer will choose a cooperative advertising fraction $t = 0$ because of its negative effect on his profit function (4.6). After some algebraic simplification, we derive the following results of the Nash equilibrium.

Proposition 4.1 *If the supply chain is characterized by a symmetric distribution of power and the channel members do not cooperate, this situation can be solved by a Nash equilibrium with:*

(i) $w^\mathrm{N} = m^\mathrm{N} = \dfrac{\alpha\nu}{\beta(1+2\nu)}$ *and* $p^\mathrm{N} = \dfrac{2\alpha\nu}{\beta(1+2\nu)}$.

(ii) $A^\mathrm{N} = \dfrac{\nu^2 k_\mathrm{m}^2}{4\beta^2}\left(\dfrac{\alpha}{1+2\nu}\right)^{2/\nu+2}$ *and* $a^\mathrm{N} = \dfrac{\nu^2 k_\mathrm{r}^2}{4\beta^2}\left(\dfrac{\alpha}{1+2\nu}\right)^{2/\nu+2}$.

(iii) $t^\mathrm{N} = 0$.

Proof of Proposition 4.1 To solve the **manufacturer's problem**, we set the first order partial derivatives $\partial\Pi_\mathrm{m}/\partial w$ and $\partial\Pi_\mathrm{m}/\partial A$ to zero:

$$\frac{\partial\Pi_\mathrm{m}}{\partial w} = \left\{[\alpha - \beta(w+m)]^{1/\nu} - \frac{\beta w}{\nu}[\alpha - \beta(w+m)]^{1/\nu - 1}\right\}\left(k_\mathrm{r}\sqrt{a} + k_\mathrm{m}\sqrt{A}\right) = 0 \tag{4.10}$$

$$\frac{\partial\Pi_\mathrm{m}}{\partial A} = \frac{k_\mathrm{m} w}{2\sqrt{A}}[\alpha - \beta(w+m)]^{1/\nu} - 1 = 0. \tag{4.11}$$

Please note that the manufacturer will set $t = 0$ due to its negative prefix in the profit function. After algebraic simplification, we get

$$w = \nu(\alpha - \beta m)/\beta(\nu + 1) \tag{4.12}$$

$$A = k_{\mathrm{m}}^2 w^2 \left[\alpha - \beta(w + m)\right]^{2/\nu}/4, \tag{4.13}$$

which are the only roots of the derivatives within the domain of definition. To prove the optimality of these solutions, we calculate the Hessian matrix

$$H(\Pi_{\mathrm{m}}) = \begin{pmatrix} \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w^2} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial A} \\ \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial w} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A^2} \end{pmatrix}. \tag{4.14}$$

The second order partial derivatives are as follows:

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w^2} = -\frac{\beta}{\nu^2}\left[\alpha - \beta(w + m)\right]^{1/\nu - 2}(2\alpha\nu - 2\beta\nu m - \beta\nu w - \beta w)\left(k_{\mathrm{r}}\sqrt{a} + k_{\mathrm{m}}\sqrt{A}\right) \tag{4.15}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A^2} = -\frac{k_{\mathrm{m}} w}{4A\sqrt{A}}\left[\alpha - \beta(w + m)\right]^{1/\nu} \tag{4.16}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial A} = \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial w} = \frac{k_{\mathrm{m}}}{2\nu\sqrt{A}}\left[\alpha - \beta(w + m)\right]^{1/\nu - 1}\left\{\nu\left[\alpha - \beta(w + m)\right] - \beta w\right\}. \tag{4.17}$$

The first principal minor of $H(\Pi_{\mathrm{m}})$ at the solution (4.12) and (4.13) is

$$H_1(\Pi_{\mathrm{m}}) = \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w^2} = \frac{-\left(\frac{\alpha - \beta m}{1 + \nu}\right)^{1/\nu - 1}\left[2\beta k_{\mathrm{r}}(1 + \nu)\sqrt{a} + k_{\mathrm{m}}^2 \nu \left(\frac{\alpha - \beta m}{1 + \nu}\right)^{1/\nu}(\alpha - \beta m)\right]}{2\nu}, \tag{4.18}$$

which is always negative. The second principal minor of $H(\Pi_{\mathrm{m}})$ at (4.12) and (4.13) is

$$\begin{aligned} H_2(\Pi_{\mathrm{m}}) &= \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w^2}\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A^2} - \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial A}\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial w} \\ &= \frac{\beta^2\left[2\beta k_{\mathrm{r}}(1 + \nu)\sqrt{a} + k_{\mathrm{m}}^2 \nu \left(\frac{\alpha - \beta m}{1 + \nu}\right)^{1/\nu}(\alpha - \beta m)\right]}{k_{\mathrm{m}}^2 \nu^3 \left(\frac{\alpha - \beta m}{1 + \nu}\right)^{1/\nu + 3}}, \end{aligned} \tag{4.19}$$

which is always positive. Therefore, the principal minors of $H(\Pi_m)$ have alternating algebraic signs $H_1(\Pi_m) < 0$ and $H_2(\Pi_m) > 0$ at the solution (4.12) and (4.13), which means—for the considered parameter framework—that $H(\Pi_m)$ is negative definite at this specific point. Hence, Π_m is concave in w and A at this point, which represents a local optimum of the manufacturer's decision problem (4.8) in this Nash equilibrium.

As Eqs. (4.12) and (4.13) are the only roots of the first order partial derivatives Eqs. (4.10) and (4.11) within the considered domain of definition, there is no other extremum candidate where the function can change its slope from negative to positive. Therefore, a consideration of boundary solutions can be omitted. Hence, we are confident that the local optimum stated above also represents the global optimum of Π_m.

To solve the **retailer's problem**, we set the first order partial derivatives $\partial \Pi_r / \partial m$ and $\partial \Pi_r / \partial a$ to zero:

$$\frac{\partial \Pi_r}{\partial m} = \left\{ [\alpha - \beta(w+m)]^{1/\nu} - \frac{\beta m}{\nu} [\alpha - \beta(w+m)]^{1/\nu - 1} \right\} \left(k_r \sqrt{a} + k_m \sqrt{A} \right) = 0 \tag{4.20}$$

$$\frac{\partial \Pi_r}{\partial a} = \frac{k_r m}{2\sqrt{a}} [\alpha - \beta(w+m)]^{1/\nu} - (1-t) = 0. \tag{4.21}$$

After mathematical simplification, we obtain

$$m = \nu(\alpha - \beta w)/\beta(1+\nu) \tag{4.22}$$

$$a = k_r^2 m^2 [\alpha - \beta(w+m)]^{2/\nu} / 4(1-t)^2, \tag{4.23}$$

which are the only roots of the derivatives within the domain of definition. To prove the optimality of these solutions, we have to calculate the Hessian matrix

$$H(\Pi_r) = \begin{pmatrix} \frac{\partial^2 \Pi_r}{\partial m^2} & \frac{\partial^2 \Pi_r}{\partial m \partial a} \\ \frac{\partial^2 \Pi_r}{\partial a \partial m} & \frac{\partial^2 \Pi_r}{\partial a^2} \end{pmatrix}. \tag{4.24}$$

The second order partial derivatives are as follows:

$$\frac{\partial^2 \Pi_r}{\partial m^2} = -\frac{\beta}{\nu^2} [\alpha - \beta(w+m)]^{1/\nu - 2} (2\alpha\nu - 2\beta\nu w - \beta\nu m - \beta m) \left(k_r \sqrt{a} + k_m \sqrt{A} \right) \tag{4.25}$$

$$\frac{\partial^2 \Pi_r}{\partial a^2} = -\frac{k_r m}{4a\sqrt{a}} [\alpha - \beta(w+m)]^{1/\nu} \tag{4.26}$$

$$\frac{\partial^2 \Pi_r}{\partial m \partial a} = \frac{\partial^2 \Pi_r}{\partial a \partial m} = \frac{k_r}{2\nu\sqrt{a}} [\alpha - \beta(w+m)]^{1/\nu - 1} \{\nu [\alpha - \beta(w+m)] - \beta m\}. \tag{4.27}$$

The first principal minor of $H(\Pi_r)$ at (4.22) and (4.23) is

$$H_1(\Pi_r) = \frac{\partial^2 \Pi_r}{\partial m^2} = \frac{-\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu-1}\left[2\beta k_m(1+\nu)\sqrt{A}+k_r^2\nu\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu}(\alpha-\beta w)\right]}{2\nu}, \tag{4.28}$$

which is always negative. The second principal minor of $H(\Pi_r)$ at (4.22) and (4.23) is

$$\begin{aligned} H_2(\Pi_r) &= \frac{\partial^2 \Pi_r}{\partial m^2}\frac{\partial^2 \Pi_r}{\partial a^2} - \frac{\partial^2 \Pi_r}{\partial m \partial a}\frac{\partial^2 \Pi_r}{\partial a \partial m} \\ &= \frac{\beta^2\left[2\beta k_m(1+\nu)\sqrt{A}+k_r^2\nu\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu}(\alpha-\beta w)\right]}{k_r^2\nu^3\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu+3}}, \end{aligned} \tag{4.29}$$

which is always positive. Therefore, the principal minors of $H(\Pi_r)$ have alternating algebraic signs $H_1(\Pi_r) < 0$ and $H_2(\Pi_r) > 0$ at the solution (4.22) and (4.23), which means—for the considered parameter framework—that $H(\Pi_r)$ is negative definite at this specific point. Hence, Π_r is concave in m and a at this point, which represents a local optimum of the retailer's decision problem (4.9) in this Nash equilibrium.

As Eqs. (4.22) and (4.23) are the only roots of the first order partial derivatives Eqs. (4.20) and (4.21) within the considered domain of definition, there is no other extremum candidate where the function can change its slope from negative to positive. Therefore, a consideration of boundary solutions can be omitted. Hence, we are confident that the local optimum stated above also represents the global optimum of Π_r.

With respect to $t = 0$, we can solve the equation system described by Eqs. (4.12), (4.13), (4.22), and (4.23) to get the expressions stated in Proposition 4.1. This completes the proof of Proposition 4.1. □

Part (i) of Proposition 4.1 reveals that a symmetric distribution of power leads to identical margins for both players. Therefore, we can—at least for the Nash equilibrium—confirm the correctness of the assumption $w = p/2$ made by Xie and Neyret (2009) and adopted by SeyedEsfahani et al. (2011). Furthermore, p and m are increasing functions of the parameters α and ν and decreasing functions of β.

Comparing the optimal expressions for local and global advertising expenditures in Part (ii) of Proposition 4.1, it is obvious that both parties will spend the same amount if the effectiveness of a and A is identical (i.e., $k_r = k_m$), while the retailer absorbs the major part of advertising expenditures for $k_r > k_m$. Hence, it follows that advertising activities are carried out by the echelon with the higher effectiveness. An increase of the other two parameters α and β causes only an increase respectively a decrease of the advertisings expenditures, but does not affect the repartition within the supply chain.

From Part (iii) of Proposition 4.1, we can see that the manufacturer is not willing to share the local advertising expenditures in a Nash equilibrium. This can be traced back to the assumption of this equilibrium that nobody has information about the other player's activities.

4.2.3 Asymmetric Relationship with Manufacturer-Leadership

Now we confer more power to the manufacturer in order to get an asymmetric relationship, where the retailer is dominated by the manufacturer. This corresponds to the common assumption in marketing literature, which has been valid in the retail industry for many years and is still valid in other sectors, e.g., the automotive industry. According to Huang and Li (2001) and Xie and Wei (2009), we use a Stackelberg game to solve this situation.[26] That means, the manufacturer is aware of the retailer's response on his decision and includes it into his determination of pricing and advertising. Formally, we first solve the decision problem of the retailer to identify his best response functions, which then constitute the constraints of the manufacturer's decision problem.

The retailer's decision problem in the Manufacturer Stackelberg game is identical to (4.9) in the previous section, as well as his best response functions:

$$m = \frac{\nu(\alpha - \beta w)}{\beta(1+\nu)} \tag{4.30}$$

$$a = \frac{k_{\mathrm{r}}^2 m^2 \left[\alpha - \beta(w+m)\right]^{2/\nu}}{4(1-t)^2}. \tag{4.31}$$

After substituting Eq. (4.30) into Eq. (4.31), we can formulate the manufacturer's decision problem:

$$\begin{aligned}
\text{Max} \quad & \Pi_{\mathrm{m}} = w\left[\alpha - \beta(w+m)\right]^{1/\nu}\left(k_{\mathrm{r}}\sqrt{a} + k_{\mathrm{m}}\sqrt{A}\right) - A - ta \\
\text{s.t.} \quad & m = \frac{\nu(\alpha - \beta w)}{\beta(1+\nu)} \\
& a = \frac{\nu^2 k_{\mathrm{r}}^2}{4\beta^2(1-t)^2}\left(\frac{\alpha - \beta w}{1+\nu}\right)^{2/\nu+2} \\
& w < \alpha/\beta - m, \;\; A > 0 \text{ and } 0 \le t < 1.
\end{aligned} \tag{4.32}$$

[26]Cf. Huang and Li (2001): *Co-op advertising models*, p. 530, and Xie and Wei (2009): *Coordinating advertising*, p. 787.

The constraints are first inserted into the objective function in order to eliminate the retailer's decision variables. After setting the first order derivatives of the manufacturer's variables to zero, some mathematical simplification leads to the results stated in Proposition 4.2. For the sake of simplicity, we only name the decision variables as functions of the wholesale price.

Proposition 4.2 *If there is an asymmetric distribution of power within a supply chain, where the manufacturer obtains the leadership, this situation can be solved by a Manufacturer Stackelberg equilibrium with:*

(i) $w^{\mathrm{MS}} = \dfrac{2\alpha\nu k^2(\nu+1) + \alpha\nu\sqrt{4k^2(1+\nu)^2(k^2+1)+(\nu+2)^2}}{\beta\left[(\nu+2)^2+4k^2(\nu+1)^2\right]}$

with $k = \dfrac{k_{\mathrm{m}}}{k_{\mathrm{r}}}$, $m^{\mathrm{MS}} = \dfrac{\nu(\alpha-\beta w)}{\beta(1+\nu)}$ *and* $p^{\mathrm{MS}} = \dfrac{\alpha\nu+\beta w}{\beta(1+\nu)}$.

(ii) $A^{\mathrm{MS}} = \dfrac{k_{\mathrm{m}}^2 w^2}{4}\left(\dfrac{\alpha-\beta w}{1+\nu}\right)^{2/\nu}$ *and* $a^{\mathrm{MS}} = \dfrac{k_{\mathrm{r}}^2(\beta\nu w + 2\beta w + \alpha\nu)^2}{16\beta^2(\nu+1)^2} \cdot \left(\dfrac{\alpha-\beta w}{1+\nu}\right)^{2/\nu}$.

(iii) $t^{\mathrm{MS}} = \dfrac{\beta w(2+3\nu)-\alpha\nu}{\beta w(2+\nu)+\alpha\nu}$.

Proof of Proposition 4.2 As the retailer confronts the same decision problem as in the Nash game, Eqs. (4.22) and (4.23) also apply to the Manufacturer Stackelberg game. Therefore, we obtain the manufacturer's objective function by inserting these equations into Eq. (4.6):

$$\begin{aligned}\Pi_{\mathrm{m}} = {} & w\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu}\left[\frac{\nu k_{\mathrm{r}}^2}{2\beta(1-t)}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu+1} + k_{\mathrm{m}}\sqrt{A}\right] \\ & - \frac{\nu^2 k_{\mathrm{r}}^2 t}{4\beta^2(1-t)^2}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu+2} - A.\end{aligned} \tag{4.33}$$

Setting the first order partial derivatives $\partial\Pi_{\mathrm{m}}/\partial w$, $\partial\Pi_{\mathrm{m}}/\partial A$, and $\partial\Pi_{\mathrm{m}}/\partial t$ to zero, yields:

$$\begin{aligned}\frac{\partial\Pi_{\mathrm{m}}}{\partial w} = {} & \frac{\nu k_{\mathrm{r}}^2}{2\beta(1-t)}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu+1} - \frac{k_{\mathrm{r}}^2(2+\nu)}{2(1-t)(1+\nu)}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu} w \\ & + k_{\mathrm{m}}\sqrt{A}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu} - \frac{\beta k_{\mathrm{m}}\sqrt{A}}{\nu(1+\nu)} w\left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu-1} \\ & + \frac{\nu k_{\mathrm{r}}^2 t}{2\beta(1-t)^2}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu+1} = 0\end{aligned} \tag{4.34}$$

$$\frac{\partial \Pi_{\mathrm{m}}}{\partial A} = \frac{k_{\mathrm{m}} w}{2\sqrt{A}} \left(\frac{\alpha - \beta w}{1 + \nu} \right)^{1/\nu} - 1 = 0 \tag{4.35}$$

$$\frac{\partial \Pi_{\mathrm{m}}}{\partial t} = \frac{\nu k_{\mathrm{r}}^2}{2\beta} \left(\frac{\alpha - \beta w}{1 + \nu} \right)^{2/\nu + 1} \left[\frac{w}{(1-t)^2} - \frac{\nu(\alpha - \beta w)(1+t)}{2\beta(1+\nu)(1-t)^3} \right] = 0. \tag{4.36}$$

From (4.35) and (4.36), we derive:

$$A = \frac{k_{\mathrm{m}}^2 w^2}{4} \left(\frac{\alpha - \beta w}{1 + \nu} \right)^{2/\nu} \tag{4.37}$$

$$t = \frac{w(2\beta + 3\beta\nu) - \alpha\nu}{w(2\beta + \beta\nu) + \alpha\nu}. \tag{4.38}$$

One can first insert Eq. (4.37) into Eq. (4.34). Hence, we derive

$$\begin{aligned} &\nu^2 k_{\mathrm{r}}^2 (\alpha - \beta w)^2 + \beta\nu k_{\mathrm{m}}^2 w(1+\nu)(\alpha - \beta w)(1-t)^2 \\ &\quad - \beta\nu k_{\mathrm{r}}^2 w(\alpha - \beta w)(1-t) - \beta^2 k_{\mathrm{m}}^2 w^2 (1+\nu)(1-t)^2 \\ &\quad - \beta\nu k_{\mathrm{r}}^2 w(1+\nu)(\alpha - \beta w)(1-t) = 0. \end{aligned} \tag{4.39}$$

Eq. (4.38) can have both positive and negative values, but according to Sect. 4.1, the domain of definition of the participation rate is $0 \le t < 1$. Hence, we conduct a case-by-case analysis, with Eq. (4.38) being valid for $w \ge \alpha\nu/(2\beta + 3\beta\nu)$, and $t = 0$ otherwise. In the first case, we derive the following expression by inserting Eq. (4.38) into Eq. (4.39):

$$\beta^2 \left[-k_{\mathrm{r}}^2 (\nu + 2)^2 - 4k_{\mathrm{m}}^2 (\nu + 1)^2 \right] w^2 + 4\alpha\beta\nu(1+\nu) k_{\mathrm{m}}^2 w + \alpha^2 \nu^2 k_{\mathrm{r}}^2 = 0. \tag{4.40}$$

The only feasible solution of this equation is

$$w = \frac{2\alpha\nu k_{\mathrm{m}}^2 (1+\nu) + \alpha\nu\sqrt{4k_{\mathrm{m}}^4 (\nu+1)^2 + k_{\mathrm{r}}^4 (\nu+2)^2 + 4k_{\mathrm{m}}^2 k_{\mathrm{r}}^2 (\nu+1)^2}}{\beta \left[k_{\mathrm{r}}^2 (\nu+2)^2 + 4k_{\mathrm{m}}^2 (\nu+1)^2 \right]}. \tag{4.41}$$

In the latter case, we set $t = 0$ in Eq. (4.39). The resulting equation has no feasible solution with $w < \alpha\nu/(2\beta + 3\beta\nu)$. Hence, Eq. (4.41) is the only feasible solution of w in the Manufacturer Stackelberg game and can be rewritten by using the parameter k with $k = k_{\mathrm{m}}/k_{\mathrm{r}}$:

$$w = \frac{2\alpha\nu k^2 (1+\nu) + \alpha\nu\sqrt{4k^2(\nu+1)^2(k^2+1) + (\nu+2)^2}}{\beta \left[(\nu+2)^2 + 4k^2(\nu+1)^2 \right]}. \tag{4.42}$$

To prove the optimality of these solutions, we have to calculate the Hessian matrix

$$H(\Pi_{\mathrm{m}}) = \begin{pmatrix} \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w^2} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial A} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial t} \\ \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial w} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A^2} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial t} \\ \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial t \partial w} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial t \partial A} & \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial t^2} \end{pmatrix}. \tag{4.43}$$

The second order partial derivatives are as follows:

$$\begin{aligned} \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w^2} = {} & \frac{k_{\mathrm{r}}^2 (2+\nu)(-2\alpha\nu + 2\beta\nu w + \alpha\nu t - \beta\nu w t + 2\beta w - 2\beta w t)}{2\nu(1-t)^2(1+\nu)(\alpha-\beta w)} \left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu} \\ & + \frac{\beta k_{\mathrm{m}} \sqrt{A}(-2\alpha\nu + 2\beta\nu w + \beta w - \beta\nu w)}{\nu^2(1+\nu)(\alpha-\beta w)} \left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu-1} \end{aligned} \tag{4.44}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A^2} = -k_{\mathrm{m}} w \left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu} A^{-3/2}/4 \tag{4.45}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial t^2} = \frac{\nu k_{\mathrm{r}}^2 (2\beta w + 4\beta\nu w - 2\beta w t - \beta\nu w t - \alpha\nu t - 2\alpha\nu)}{2\beta^2(1+\nu)(1-t)^4} \left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu+1} \tag{4.46}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial w} = \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial A} = \frac{k_{\mathrm{m}}\,(\alpha\nu - \beta\nu w - \beta w)}{2\nu(\alpha-\beta w)\sqrt{A}} \left(\frac{\alpha-\beta w}{1+\nu}\right)^{1/\nu} \tag{4.47}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial A \partial t} = \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial t \partial A} = 0 \tag{4.48}$$

$$\frac{\partial^2 \Pi_{\mathrm{m}}}{\partial t \partial w} = \frac{\partial^2 \Pi_{\mathrm{m}}}{\partial w \partial t} = \frac{k_{\mathrm{r}}^2(-2\beta w - 3\beta\nu w + 2\alpha\nu + 2\beta w t + \beta\nu w t)}{2\beta(1+\nu)(1-t)^3} \left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu}. \tag{4.49}$$

Due to the complexity of the expressions stated above, we are not able to prove the optimality of our solutions analytically. Instead of that, we computed a numerical study with 3,000,000 randomly generated sets of parameters with $0.1 \leq \alpha, \beta, \nu, k_{\mathrm{r}}, k_{\mathrm{m}} \leq 10$. In each of these cases of our simulation, the principal minors of the Hessian matrix $H(\Pi_{\mathrm{m}})$ had alternating algebraic signs with $H_1(\Pi_{\mathrm{m}}) < 0$, $H_2(\Pi_{\mathrm{m}}) > 0$, and $H_3(\Pi_{\mathrm{m}}) < 0$ at Eqs. (4.37), (4.38), and (4.42), which means—for the considered parameter framework—that $H(\Pi_{\mathrm{m}})$ is negative definite at this specific point and, accordingly, that Π_{m} is concave in w, A, and t at this point. Hence, we are confident that our solutions truly represent a local optimum of this Manufacturer Stackelberg equilibrium.

As Eqs. (4.37), (4.38), and (4.42) are the only roots of the first order partial derivatives Eqs. (4.34)–(4.36) within the considered domain of definition, there is no other extremum candidate where the function can change its slope from negative to positive. Therefore, a consideration of boundary solutions can be omitted. Hence, we are confident that the local optimum stated above also represents the global optimum of Π_m. One can obtain the remaining expressions in Proposition 4.2 by substituting Eqs. (4.22) and (4.38) into Eq. (4.23) and Eq. (4.22) into Eq. (4.3). This completes the proof of Proposition 4.2. □

Part (iii) of Proposition 4.2 shows that the manufacturer will participate in the retailer's investment in local advertising, which is somehow expected as the manufacturer has knowledge about the reaction of his retailer in the Manufacturer Stackelberg equilibrium. Due to the complexness of the expressions, we refer the reader to the numerical examples in Sect. 4.3 for further interpretation.

4.2.4 Asymmetric Relationship with Retailer-Leadership

Now we confer more power to the retailer in order to analyze the situation which is valid mostly in the sector of retailing. According to the previous section, we use a Stackelberg game to solve this situation (i.e., Retailer Stackelberg game). The manufacturer confronts the same problem as in Eq. (4.8), which has the following solutions:

$$w = (\alpha - \beta m)/2\beta, \tag{4.50}$$

$$A = k_m^2 w^2 [\alpha - \beta(w+m)]^2/4, \tag{4.51}$$

$$t = 0. \tag{4.52}$$

After eliminating variable w in Eq. (4.51) by means of Eq. (4.50), we can describe the retailer's decision problem by:

$$\begin{aligned} \text{Max} \quad & \Pi_r = m[\alpha - \beta(w+m)]\left(k_r\sqrt{a} + k_m\sqrt{A}\right) - (1-t)a \\ \text{s.t.} \quad & w = (\alpha - \beta m)/2\beta, \\ & A = k_m^2(\alpha - \beta m)^4/64\beta^2, \\ & t = 0, \\ & m < \alpha/\beta - w \text{ and } a > 0. \end{aligned} \tag{4.53}$$

The solution of the Retailer Stackelberg equilibrium is carried out similarly to the previous sections, i.e., we first insert the constraints for w, A, and t into Π_r and set the corresponding first order partial derivatives to zero afterwards. Solving the resulting system of equations leads us to the optimal expressions shown in Proposition 4.3:

Proposition 4.3 *If there is an asymmetric distribution of power within a supply chain, where the retailer obtains the leadership, this situation can be solved by a Retailer Stackelberg equilibrium with*[27]*:*

(i) $m^{\mathrm{RS}} = \dfrac{-\alpha\nu\left[1+\nu-k^2(2+3\nu)+\sqrt{(\nu+1)^2-2k^2\nu(1+\nu)+k^4(2+\nu)^2}\right]}{2\beta(1+\nu)(2\nu k^2-\nu-1)}$

with $k = \dfrac{k_{\mathrm{m}}}{k_{\mathrm{r}}}$, $w^{\mathrm{RS}} = \dfrac{\nu(\alpha-\beta m)}{\beta(1+\nu)}$, *and* $p^{\mathrm{RS}} = \dfrac{\alpha\nu+\beta m}{\beta(1+\nu)}$.

(ii) $A^{\mathrm{RS}} = \dfrac{\nu^2 k_{\mathrm{m}}^2}{4\beta^2}\left(\dfrac{\alpha-\beta m}{1+\nu}\right)^{2/\nu+2}$ *and* $a^{\mathrm{RS}} = \dfrac{k_{\mathrm{r}}^2 m^2}{4}\left(\dfrac{\alpha-\beta m}{1+\nu}\right)^{2/\nu}$.

(iii) $t^{\mathrm{RS}} = 0$.

Proof of Proposition 4.3 As the manufacturer confronts the same decision problem as in the Nash game, Eqs. (4.12) and (4.13) as well as $t = 0$ also apply to the Retailer Stackelberg game. Therefore, the retailer's objective function in (4.7) can be rewritten as follows:

$$\Pi_{\mathrm{r}} = m\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu}\left[k_{\mathrm{r}}\sqrt{a}+\frac{\nu k_{\mathrm{m}}^2}{2\beta}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu+1}\right]-a. \tag{4.54}$$

Taking the first order partial derivatives $\partial\Pi_{\mathrm{r}}/\partial m$ $\partial\Pi_{\mathrm{r}}/\partial a$ and setting them to zero, we obtain:

$$\frac{\partial\Pi_{\mathrm{r}}}{\partial m} = \left[1-\frac{\beta m}{\nu(\alpha-\beta m)}\right]\left[k_{\mathrm{r}}\sqrt{a}+\frac{\nu k_{\mathrm{m}}^2}{2\beta}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu+1}\right] - \frac{k_{\mathrm{m}}^2 m}{2}\left(\frac{\alpha-\beta m}{\nu+1}\right) = 0 \tag{4.55}$$

$$\frac{\partial\Pi_{\mathrm{r}}}{\partial a} = \frac{k_{\mathrm{r}} m}{2\sqrt{a}}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu} - 1 = 0. \tag{4.56}$$

From (4.56), we derive:

$$a = \frac{k_{\mathrm{r}}^2 m^2}{4}\left(\frac{\alpha-\beta m}{1+\nu}\right)^{2/\nu}. \tag{4.57}$$

Substituting Eq. (4.57) into Eq. (4.55) yields

$$\beta^2\left[-(\nu+1)^2k_{\mathrm{r}}^2+2\nu(\nu+1)k_{\mathrm{m}}^2\right]m^2 + \alpha\beta\nu\left[(\nu+1)k_{\mathrm{r}}^2-(3\nu+2)k_{\mathrm{m}}^2\right]m+\alpha^2\nu^2k_{\mathrm{m}}^2 = 0. \tag{4.58}$$

[27]Please note that m is only defined for $\nu \neq (2k^2-1)^{-1}$.

The only feasible solution of this equation is

$$m = \frac{-\alpha\nu\left[k_{\mathrm{r}}^2(\nu+1) - k_{\mathrm{m}}^2(3\nu+2) + \sqrt{k_{\mathrm{r}}^4(\nu+1)^2 - k_{\mathrm{m}}^2 k_{\mathrm{r}}^2 \nu(\nu+1) + k_{\mathrm{m}}^4(\nu+2)^2}\right]}{2\beta(\nu+1)\left[-k_{\mathrm{r}}^2(\nu+1) + 2\nu k_{\mathrm{m}}^2\right]}, \tag{4.59}$$

which can be simplified by means of parameter $k = k_{\mathrm{m}}/k_{\mathrm{r}}$:

$$m = \frac{-\alpha\nu\left[1+\nu-k^2(3\nu+2) + \sqrt{(\nu+1)^2 - 2k^2\nu(\nu+1) + k^4(\nu+2)^2}\right]}{2\beta(\nu+1)(2\nu k^2 - \nu - 1)}. \tag{4.60}$$

To prove optimality of these solutions, we have to calculate the Hessian matrix

$$H(\Pi_{\mathrm{r}}) = \begin{pmatrix} \frac{\partial^2 \Pi_{\mathrm{r}}}{\partial m^2} & \frac{\partial^2 \Pi_{\mathrm{r}}}{\partial m \partial a} \\ \frac{\partial^2 \Pi_{\mathrm{r}}}{\partial a \partial m} & \frac{\partial^2 \Pi_{\mathrm{r}}}{\partial a^2} \end{pmatrix}. \tag{4.61}$$

The second order partial derivatives are as follows:

$$\begin{aligned} \frac{\partial^2 \Pi_{\mathrm{r}}}{\partial m^2} &= \frac{\beta k_{\mathrm{r}}\sqrt{a}(-2\alpha\nu + \beta\nu m + \beta m)}{\nu^2(\alpha-\beta m)^2}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu} \\ &\quad + \frac{k_{\mathrm{m}}^2(\nu+2)(-\alpha\nu+\beta\nu m+\beta m)}{\nu(\nu+1)(\alpha-\beta m)}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu} \end{aligned} \tag{4.62}$$

$$\frac{\partial^2 \Pi_{\mathrm{r}}}{\partial a^2} = -\frac{k_{\mathrm{r}} m}{4a\sqrt{a}}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu} \tag{4.63}$$

$$\frac{\partial^2 \Pi_{\mathrm{r}}}{\partial m \partial a} = \frac{\partial^2 \Pi_{\mathrm{r}}}{\partial a \partial m} = \frac{k_{\mathrm{r}}(\alpha\nu - \beta\nu m - \beta m)}{2\nu(\alpha-\beta m)\sqrt{a}}\left(\frac{\alpha-\beta m}{\nu+1}\right)^{1/\nu}. \tag{4.64}$$

Due to the complexity of the expressions stated above, we are not able to prove the optimality of our solutions analytically. Instead of that, we computed a numerical study with 3,000,000 randomly generated sets of parameters with $0.1 \le \alpha, \beta, \nu, k_{\mathrm{r}}, k_{\mathrm{m}} \le 10$. In each of these cases of our simulation, the principal minors of the Hessian matrix $H(\Pi_{\mathrm{r}})$ had alternating algebraic signs with $H_1(\Pi_{\mathrm{r}}) < 0$ and $H_2(\Pi_{\mathrm{r}}) > 0$ at Eqs. (4.57) and (4.60), which means—for the considered parameter framework—that $H(\Pi_{\mathrm{r}})$ is negative definite at this specific point and, accordingly, that Π_{r} is concave in m and a at this point. Hence, we are confident that our solutions truly represent a local optimum of this Retailer Stackelberg equilibrium.

As Eqs. (4.57) and (4.60) are the only roots of the first order partial derivatives Eqs. (4.55) and (4.56) within the considered domain of definition, there is no other extremum candidate where the function can change its slope from negative to positive. Therefore, a consideration of boundary solutions can be omitted. Hence, we are confident that the local optimum stated above also represents the global optimum of Π_r. To obtain the remaining expressions in Proposition 4.3, one has to substitute Eq. (4.12) into Eqs. (4.3) and (4.13). This completes the proof of Proposition 4.3. □

One can easily see from Part (i) of Proposition 4.3 that the expressions for w and p conflict with the assumption of identical margins (i.e., $w = p/2$) made by Xie and Neyret (2009) and adopted by SeyedEsfahani et al. (2011). Due to the complex structure of these expressions, we refer the reader to the numerical examples in Sect. 4.3 for further interpretation.

4.2.5 Cooperation

The last game to be analyzed is a Cooperation between the two supply chain members, which is usually calculated via the maximization of the total profit function Π_{m+r}. Hence, we have the following decision problem:

$$\begin{aligned} \text{Max} \quad & \Pi_{m+r} = p\,(\alpha - \beta p)^{1/\nu}\left(k_r\sqrt{a} + k_m\sqrt{A}\right) - A - a \\ \text{s.t.} \quad & p < \alpha/\beta \text{ and } a,\ A > 0. \end{aligned} \tag{4.65}$$

As evident from the objective function, only p, A, and a are decision variables when manufacturer and retailer cooperate. Neither the margins w and m nor the participation rate t have influence on the total profit, but solely on the division of profits between the two channel members. Therefore, we can only determine the characteristics of this Cooperation which are visible outwards, while the internal arrangement remains undetermined at the moment. The solution of this decision problem is determined analogously to the previous sections and is as follows:

Proposition 4.4 *A Cooperation between manufacturer and retailer, which targets a joint profit maximization, can be solved by the following equilibrium:*

(i) $p^C = \dfrac{\alpha\nu}{\beta(1+\nu)}$.

(ii) $A^C = \dfrac{\nu^2 k_m^2}{4\beta^2}\left(\dfrac{\alpha}{1+\nu}\right)^{2/\nu+2}$ *and* $a^C = \dfrac{\nu^2 k_r^2}{4\beta^2}\left(\dfrac{\alpha}{1+\nu}\right)^{2/\nu+2}$.

Proof of Proposition 4.4 The total profit function Π_{m+r} is calculated via addition of the two single profit functions Π_m and Π_r given in Eqs. (4.6) and (4.7), together with the substitution of $w + m = p$. We get

$$\Pi_{m+r} = \Pi_m + \Pi_r = p\,(\alpha - \beta p)^{1/\nu}\left(k_r\sqrt{a} + k_m\sqrt{A}\right) - A - a. \tag{4.66}$$

We then set the first order partial derivatives $\partial\Pi_{\mathrm{m+r}}/\partial w$, $\partial\Pi_{\mathrm{m+r}}/\partial A$, and $\partial\Pi_{\mathrm{m+r}}/\partial a$ to zero:

$$\frac{\partial\Pi_{\mathrm{m+r}}}{\partial p} = \left[(\alpha-\beta p)^{1/\nu} - \frac{\beta p}{\nu}(\alpha-\beta p)^{1/\nu-1}\right]\left(k_{\mathrm{r}}\sqrt{a}+k_{\mathrm{m}}\sqrt{A}\right) = 0 \tag{4.67}$$

$$\frac{\partial\Pi_{\mathrm{m+r}}}{\partial a} = \frac{k_{\mathrm{r}}p}{2\sqrt{a}}(\alpha-\beta p)^{1/\nu} - 1 = 0 \tag{4.68}$$

$$\frac{\partial\Pi_{\mathrm{m+r}}}{\partial A} = \frac{k_{\mathrm{m}}p}{2\sqrt{A}}(\alpha-\beta p)^{1/\nu} - 1 = 0. \tag{4.69}$$

By solving this system of equations, we obtain:

$$p = \frac{\alpha\nu}{\beta(1+\nu)} \tag{4.70}$$

$$a = \frac{k_{\mathrm{r}}^2 p^2}{4}(\alpha-\beta p)^{2/\nu} \tag{4.71}$$

$$A = \frac{k_{\mathrm{m}}^2 p^2}{4}(\alpha-\beta p)^{2/\nu}. \tag{4.72}$$

To prove optimality of our solution, we have to calculate the Hessian matrix

$$H(\Pi_{\mathrm{m+r}}) = \begin{pmatrix} \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial p^2} & \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial p\partial a} & \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial p\partial A} \\ \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial a\partial p} & \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial a^2} & \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial a\partial A} \\ \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial A\partial p} & \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial A\partial a} & \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial A^2} \end{pmatrix}. \tag{4.73}$$

The second order partial derivatives are as follows:

$$\frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial p^2} = -\frac{\beta}{\nu^2}(\alpha-\beta p)^{1/\nu-2}(2\alpha\nu-\beta\nu p-\beta p)\left(k_{\mathrm{r}}\sqrt{a}+k_{\mathrm{m}}\sqrt{A}\right) \tag{4.74}$$

$$\frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial a^2} = -\frac{k_{\mathrm{r}}p(\alpha-\beta p)^{1/\nu}}{4a\sqrt{a}} \tag{4.75}$$

$$\frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial A^2} = -\frac{k_{\mathrm{m}}p(\alpha-\beta p)^{1/\nu}}{4A\sqrt{A}} \tag{4.76}$$

$$\frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial p\partial a} = \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial a\partial p} = \frac{k_{\mathrm{r}}(\alpha-\beta p)^{1/\nu-1}(\alpha\nu-\beta\nu p-\beta p)}{2\nu\sqrt{a}} \tag{4.77}$$

$$\frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial p\partial A} = \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial A\partial p} = \frac{k_{\mathrm{m}}(\alpha-\beta p)^{1/\nu-1}(\alpha\nu-\beta\nu p-\beta p)}{2\nu\sqrt{A}} \tag{4.78}$$

$$\frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial a\partial A} = \frac{\partial^2\Pi_{\mathrm{m+r}}}{\partial A\partial a} = 0. \tag{4.79}$$

The first principal minor of $H(\Pi_{\mathrm{m+r}})$ at the solution (4.70), (4.71) and (4.72) is

$$H_1(\Pi_{\mathrm{m+r}}) = \frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial p^2} = -(1+\nu)\left(k_{\mathrm{r}}^2 + k_{\mathrm{m}}^2\right)\left(\frac{\alpha}{1+\nu}\right)^{2/\nu}/2, \tag{4.80}$$

which is always negative. The second principal minor of $H(\Pi_{\mathrm{m+r}})$ at the solution (4.70), (4.71) and (4.72) is

$$H_2(\Pi_{\mathrm{m+r}}) = \frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial p^2}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial a^2} - \frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial p \partial a}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial a \partial p} = \frac{\beta^2 (1+\nu)^3 \left(k_{\mathrm{r}}^2 + k_{\mathrm{m}}^2\right)}{\alpha^2 k_{\mathrm{r}}^2 \nu^2}, \tag{4.81}$$

which is always positive. The third principal minor of $H(\Pi_{\mathrm{m+r}})$ at the solution (4.70), (4.71) and (4.72) is

$$\begin{aligned} H_3(\Pi_{\mathrm{m+r}}) = {} & \frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial p^2}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial a^2}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial A^2} - \frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial A \partial p}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial a^2}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial p \partial A} \\ & - \frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial A^2}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial a \partial p}\frac{\partial^2 \Pi_{\mathrm{m+r}}}{\partial p \partial a} = -\frac{2\beta^4 (1+\nu)^5 \left(k_{\mathrm{r}}^2 + k_{\mathrm{m}}^2\right)}{\alpha^4 \nu^4 k_{\mathrm{r}}^2 k_{\mathrm{m}}^2 \left(\frac{\alpha}{1+\nu}\right)^{2/\nu}}, \end{aligned} \tag{4.82}$$

which is always negative. Therefore, the principal minors of $H(\Pi_{\mathrm{m+r}})$ have alternating algebraic signs $H_1(\Pi_{\mathrm{m+r}}) < 0$ and $H_2(\Pi_{\mathrm{m+r}}) > 0$ at the solution (4.70), (4.71), and (4.72), which means that $H(\Pi_{\mathrm{m+r}})$ is negative definite at this specific point. Hence, Π_{m+r} is concave in p, A, and a at this point, which represents a local optimum of the decision problem (4.65).

As Eqs. (4.70)–(4.72) are the only roots of the first order partial derivatives Eqs. (4.67)–(4.69) within the considered domain of definition, there is no other extremum candidate where the function can change its slope from negative to positive. Therefore, a consideration of boundary solutions can be omitted. Hence, we are confident that the local optimum stated above also represents the global optimum of $\Pi_{\mathrm{m+r}}$. By inserting Eq. (4.70) in Eqs. (4.71) and (4.72), one can determine the optimal expressions shown in Proposition 4.4. This completes the proof of Proposition 4.4. □

From Part (ii) of Proposition 4.4, we can see that the advertising expenditures of manufacturer and retailer will differ solely if the effectiveness parameters k_{m} and k_{r} take on different values. This was also a property of the Nash equilibrium in Sect. 4.2.2.

We assume that both players will only agree to cooperate if they receive a higher profit than in any other game described above[28]:

$$\Delta\Pi_{\mathrm{m}} = \Pi_{\mathrm{m}}^{\mathrm{C}} - \Pi_{\mathrm{m}}^{\max} \geq 0 \tag{4.83}$$

$$\Delta\Pi_{\mathrm{r}} = \Pi_{\mathrm{r}}^{\mathrm{C}} - \Pi_{\mathrm{r}}^{\max} \geq 0. \tag{4.84}$$

Here, $\Pi_{\mathrm{m}}^{\mathrm{C}}$ respectively $\Pi_{\mathrm{r}}^{\mathrm{C}}$ denote the manufacturer's and retailer's profit in case of Cooperation and $\Pi_{\mathrm{m}}^{\max}$ respectively $\Pi_{\mathrm{r}}^{\max}$ the players' maximal profit in any other game. A Cooperation fulfilling this inequalities is called feasible. Please note that this is a very restrictive assumption, because the players may be satisfied with less profit in reality if there is no chance to obtain the desired market structure. Furthermore, one has to be aware that changes in channel leadership may not be possible in short term. Examples like the relationship between Wal-Mart and Procter & Gamble though show that a manufacturer leadership can also pass into a symmetric leadership or even into a retailer leadership during longer time.[29] Hence, we derive for the total extra profit

$$\Delta\Pi_{\mathrm{m+r}} = \Delta\Pi_{\mathrm{m}} + \Delta\Pi_{\mathrm{r}} = \Pi_{\mathrm{m+r}}^{\mathrm{C}} - \Pi_{\mathrm{m}}^{\max} - \Pi_{\mathrm{r}}^{\max} \geq 0. \tag{4.85}$$

The total profit in case of Cooperation ($\Pi_{\mathrm{m+r}}^{\mathrm{C}}$) can be determined by inserting the optimal expressions listed in Proposition 4.4 into the profit function in Eq. (4.65), while it is necessary to compare the results of the other equilibria from Sect. 4.2.2 to Sect. 4.2.4 in order to find $\Pi_{\mathrm{m}}^{\max}$ and $\Pi_{\mathrm{r}}^{\max}$.[30] Once the shareable extra profit $\Delta\Pi_{\mathrm{m+r}}$ is given, the cooperating players have to agree on its division. A mathematical model to locate a solution, which is satisfying for both, is proposed in the next section.

When $\Delta\Pi_{\mathrm{m}}$ and $\Delta\Pi_{\mathrm{r}}$ have been found, one can calculate the profits of manufacturer and retailer in the Cooperation game via Eqs. (4.83) and (4.84). However, the determination of the remaining variables w, m and t cannot be effected unambiguously, as there is an infinite amount of sets $(w^{\mathrm{C}}, m^{\mathrm{C}}, t^{\mathrm{C}})$ which can yield the particular division of profits $(\Pi_{\mathrm{m}}^{\mathrm{C}}, \Pi_{\mathrm{r}}^{\mathrm{C}})$. If a complete solution is needed, one has to decide on a certain participation rate t^{C} arbitrarily, and then calculate the proper wholesale price w^{C} by inserting the profit function in (4.65) and the solutions stated in Proposition 4.4 into Eq. (4.83). As we expect no benefit for our analysis due to this necessary arbitrariness, we desist from further illustrations of this approach.

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[28]Cf. Xie and Neyret (2009): *Co-op advertising*, p. 1378, and SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 268.

[29]Cf. Achenbaum and Mitchell (1987): *Pulling away from push marketing*, p. 38, Kumar (1996): *The power of trust*, p. 92, and Huang et al. (2002): *Manufacturer-retailer supply chain*, pp. 470 et seq.

[30]We refer the reader to the numerical examples in Sect. 4.3 for further information.

4.2.6 A Bargaining Model

Bargaining models are commonly used in literature to identify a suitable division of pay-offs between two or more players. Results depend both on the underlying utility functions of the players and on the selected bargaining model. For instance, Xie and Wei (2009), Xie and Neyret (2009), and SeyedEsfahani et al. (2011) used power functions of type $u(\Pi) = \Pi^{\mu}$ to determine the players' utility in combination with the bargaining model of Nash (1950a).[31] The exponential function $u(\Pi) = 1 - e^{-\mu\Pi}$ is another possible utility function, on which one can apply either the bargaining model of Nash (1950a) as found in Huang and Li (2001)[32] or the bargaining model of Eliashberg (1986) as found in Yue et al. (2006).[33] These combinations mainly differ in the parameters the results depend on: While a power function in Nash's model incorporates the players' risk attitude, the same function in Eliashberg's model can only be used to represent the bargaining power or skill of each player. With the assumption of an exponential utility function, the latter model can even include both—the risk attitude and the bargaining power.

In this paper, we will introduce another bargaining model, which has—to the best of our knowledge—not been used in this context yet: the Asymmetric Nash bargaining model of Harsanyi and Selten (1972) and Kalai (1977). Compared to the already used models, its advantage is that it allows us to incorporate both the risk attitude and the bargaining power while assuming that the players' utility follow the simple form of a power function, which is considerably easier in calculus and interpretation than the above mentioned exponential function. We can formulate the bargaining model by

$$v_{\mathrm{m+r}} = v_{\mathrm{m}}^{\lambda_{\mathrm{m}}} v_{\mathrm{r}}^{\lambda_{\mathrm{r}}}, \tag{4.86}$$

where λ_{m} and λ_{r} are positive parameters with $\lambda_{\mathrm{m}} + \lambda_{\mathrm{r}} = 1$, which reflect each player's bargaining power, v_{m} and v_{r} denote the utility values of manufacturer and retailer, which can be calculated by means of the utility function, and $v_{\mathrm{m+r}}$ indicates the total utility value of both firms.[34] In combination with Eq. (4.85) as well as the utility functions of manufacturer and retailer, represented by

$$v_{\mathrm{m}} = u_{\mathrm{m}}(\Delta\Pi_{\mathrm{m}}) = (\Delta\Pi_{\mathrm{m}})^{\mu_{\mathrm{m}}} \tag{4.87}$$

$$v_{\mathrm{r}} = u_{\mathrm{r}}(\Delta\Pi_{\mathrm{r}}) = (\Delta\Pi_{\mathrm{r}})^{\mu_{\mathrm{r}}}, \tag{4.88}$$

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[31] Cf. Xie and Wei (2009): *Coordinating advertising*, pp. 788 et seq., Xie and Neyret (2009): *Co-op advertising*, pp. 1383 et seq., and SeyedEsfahani et al. (2011): *Vertical co-op advertising*, pp. 269 et seq. For the Nash bargaining model, see Nash (1950a): *Bargaining problem*.

[32] Cf. Huang and Li (2001): *Co-op advertising models*, pp. 538–540.

[33] Cf. Yue et al. (2006): *Coordination of cooperative advertising*, pp. 77–82. For the Eliashberg bargaining model, see Eliashberg (1986): *Arbitrating a dispute*.

[34] Cf. Harsanyi and Selten (1972): *Generalized Nash*, p. 96, and Kalai (1977): *Nonsymmetric Nash*, pp. 130 et seq.

where μ_m and μ_r are positive parameters reflecting the players' risk attitude, we derive the following optimization problem:

$$\begin{aligned} \text{Max} \quad & v_{m+r} = v_m^{\lambda_m} v_r^{\lambda_r} = (\Delta\Pi_m)^{\lambda_m\mu_m} (\Delta\Pi_r)^{\lambda_r\mu_r} \\ \text{s.t.} \quad & \Delta\Pi_{m+r} = \Delta\Pi_m + \Delta\Pi_r \\ & \Delta\Pi_m, \Delta\Pi_r > 0. \end{aligned} \tag{4.89}$$

The corresponding optimal expressions for $\Delta\Pi_m$ and $\Delta\Pi_r$ are given in Proposition 4.5:

Proposition 4.5 *The asymmetric Nash bargaining model leads to the following division of profits:*

(i) $\Delta\Pi_m = \dfrac{\lambda_m\mu_m}{\lambda_m\mu_m + \lambda_r\mu_r}\Delta\Pi_{m+r}$.

(ii) $\Delta\Pi_r = \dfrac{\lambda_r\mu_r}{\lambda_m\mu_m + \lambda_r\mu_r}\Delta\Pi_{m+r}$.

Proof of Proposition 4.5 We start from the optimization problem stated in (4.89). Inserting the constraint into the objective function yields

$$v_{m+r} = \Delta\Pi_m^{\lambda_m\mu_m} (\Delta\Pi_{m+r} - \Delta\Pi_r)^{\lambda_r\mu_r}. \tag{4.90}$$

The first order partial derivative $\partial v_{m+r}/\partial\Delta\Pi_m$ is set to zero:

$$\begin{aligned} \frac{\partial u_{m+r}}{\partial\Delta\Pi_m} =& \lambda_m\mu_m\Delta\Pi_m^{\lambda_m\mu_m-1} (\Delta\Pi_{m+r} - \Delta\Pi_m)^{\lambda_r\mu_r} \\ & - \lambda_r\mu_r\Delta\Pi_m^{\lambda_m\mu_m} (\Delta\Pi_{m+r} - \Delta\Pi_m)^{\lambda_r\mu_r-1} = 0. \end{aligned} \tag{4.91}$$

Solving this equation leads to

$$\Delta\Pi_m = \frac{\lambda_m\mu_m}{\lambda_m\mu_m + \lambda_r\mu_r}\Delta\Pi_{m+r}. \tag{4.92}$$

The optimal expression of $\Delta\Pi_r$, which is shown in Proposition 4.5, can be calculated in the same way. We refer the reader to Harsanyi and Selten (1972) and Kalai (1977) for further discussion of optimality.[35] This completes the proof of Proposition 4.5. □

Looking at the risk attitude parameters μ_m and μ_r (for $\lambda_m = \lambda_r = const.$), it is clearly visible that an equal risk attitude leads to $\Delta\Pi_m = \Delta\Pi_r$. If we assume that the manufacturer is more risk-seeking than the retailer, i.e., $\mu_m > \mu_r$, the

[35] See Harsanyi and Selten (1972): *Generalized Nash* and Kalai (1977): *Nonsymmetric Nash*.

manufacturer will receive the bigger fraction of the total extra profit. For example, a ratio of $\mu_{\mathrm{m}} = 2\mu_{\mathrm{r}}$ yields a profit split of $2/3\Delta\Pi_{\mathrm{m+r}}$ for the manufacturer and $1/3\Delta\Pi_{\mathrm{m+r}}$ for the retailer.

Now we set $\mu_{\mathrm{m}} = \mu_{\mathrm{r}} = const.$ in order to analyze the effects of the bargaining power parameters λ_{m} and λ_{r}. As expected, an equal bargaining power of both players results in a homogeneous division and otherwise the player with the higher bargaining power will be able to get the bigger share of profit. Similar to the above example, a ratio of $\lambda_{\mathrm{m}} = 2\lambda_{\mathrm{r}}$ leads to $\Delta\Pi_{\mathrm{m}} = 2/3\Delta\Pi_{\mathrm{m+r}}$ and $\Delta\Pi_{\mathrm{r}} = 1/3\Delta\Pi_{\mathrm{m+r}}$.

4.3 Discussion of the Results

4.3.1 Margins and Prices

In the previous section, we identified the optimal solutions of the four game scenarios Nash, Manufacturer Stackelberg, Retailer Stackelberg, and Cooperation, which are summarized in Table 4.3. Due to their mathematical complexness, we

Table 4.3 Optimal expressions in each game scenario

	Nash	Manufacturer Stackelberg
w	$\frac{\alpha\nu}{\beta(1+2\nu)}$	$\frac{2\alpha\nu k^2(1+\nu)+\alpha\nu\sqrt{4k^2(1+\nu)^2(k^2+1)+(v+2)^2}}{\beta\left[(\nu+2)^2+4k^2(\nu+1)^2\right]}$
m	$\frac{\alpha\nu}{\beta(1+2\nu)}$	$\frac{\nu(\alpha-\beta w)}{\beta(1+\nu)}$
p	$\frac{2\alpha\nu}{\beta(1+2\nu)}$	$\frac{\alpha\nu+\beta w}{\beta(1+\nu)}$
A	$\frac{\nu^2 k_{\mathrm{m}}^2}{4\beta^2}\left(\frac{\alpha}{1+2\nu}\right)^{2/\nu+2}$	$\frac{k_{\mathrm{m}}^2 w^2}{4}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu}$
a	$\frac{\nu^2 k_{\mathrm{m}}^2}{4\beta^2}\left(\frac{\alpha}{1+2\nu}\right)^{2/\nu+2}$	$\frac{k_{\mathrm{r}}^2(\beta\nu w+2\beta w+\alpha\nu)^2}{16\beta^2(\nu+1)^2}\left(\frac{\alpha-\beta w}{1+\nu}\right)^{2/\nu}$
t	0	$\frac{\beta w(2+3\nu)-\alpha\nu}{\beta w(2+\nu)+\alpha\nu}$
	Cooperation	Retailer Stackelberg
w	–	$\frac{\nu(\alpha-\beta m)}{\beta(1+\nu)}$
m	–	$\frac{-\alpha\nu\left[1+\nu-k^2(2+3\nu)+\sqrt{(1+\nu)^2-2k^2\nu(1+\nu)+k^4(2+\nu)^2}\right]}{2\beta(1+\nu)(2\nu k^2-\nu-1)}$
p	$\frac{\alpha\nu}{\beta(1+\nu)}$	$\frac{\alpha\nu+\beta m}{\beta(1+\nu)}$
A	$\frac{\nu^2 k_{\mathrm{m}}^2}{4\beta^2}\left(\frac{\alpha}{1+\nu}\right)^{2/\nu+2}$	$\frac{\nu^2 k_{\mathrm{m}}^2}{4\beta^2}\left(\frac{\alpha-\beta m}{1+\nu}\right)^{2/\nu+2}$
a	$\frac{\nu^2 k_{\mathrm{r}}^2}{4\beta^2}\left(\frac{\alpha}{1+\nu}\right)^{2/\nu+2}$	$\frac{k_{\mathrm{r}}^2 m^2}{4}\left(\frac{\alpha-\beta m}{1+\nu}\right)^{2/\nu}$
t	–	0

sustain our discussion on numerical simulations. The expressions depend on the five parameters used in our analysis, i.e., α, β, and ν of the price demand function as well as k_m and k_r of the advertising demand function.

Our calculations yield that the parameters α and β, which determine intercept and slope of the price demand, affect solely the level of prices, advertising expenditures, and profits. While an increase of α leads to higher values, an increase of β causes a reduction of all variables and profits. These two parameters do not have an effect on the ratio between manufacturer and retailer variables and profits nor on the relation between the different game scenarios. Therefore we concentrate our further discussion on the remaining parameters ν, k_m, and k_r.

As introduced in Sect. 4.1, the parameter ν determines the shape of the price demand function, with $\nu < 1$ producing a convex, $\nu = 1$ a linear, and $\nu > 1$ a concave curve. The parameters k_m and k_r are part of the advertising demand function and can be interpreted as effectiveness of global and local advertising expenditures. Following Xie and Wei (2009) and SeyedEsfahani et al. (2011), we use a ratio parameter k to describe the relation of the latter parameters with $k = k_m/k_r$. The bigger the value of k, the higher the effectiveness of global advertising compared to the effectiveness of local advertising. As ν and the ratio k are part of many expressions in Table 4.3, it seems appropriate to conduct the sensitivity analysis on these two parameters, while α, β, and k_r are set to constant values (and k_m is calculated via $k_m = k_r k$).

We start our analysis with the **wholesale price** w, which constitutes the manufacturer's margin as we do not consider any manufacturing costs. Please note that the Cooperation game is not included in this figure as it does not determine an optimal value of w. Figure 4.2 reveals six regions we found throughout our computational comparison of the wholesale prices in each game. Each region stands for a different order: For example, in the area in the top right (i.e., for a high value of ν and k) the highest wholesale price can be found in the Manufacturer Stackelberg equilibrium, while the Nash equilibrium yields the lowest price. In contrast, the Nash equilibrium results in the highest wholesale price in the lower left corner, i.e., for small values of ν and k. Low (high) values of k thereby indicate that the

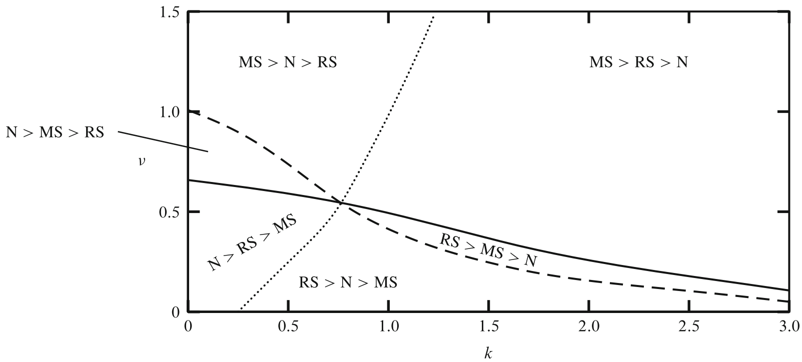

Fig. 4.2 Wholesale price w

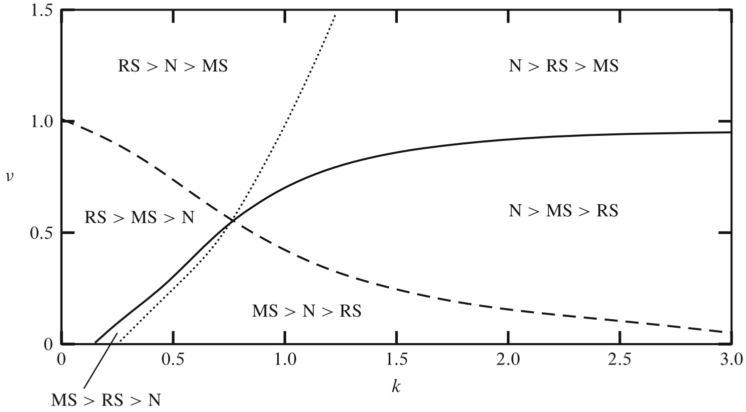

Fig. 4.3 Retailer margin *m*

effectiveness of manufacturer's advertising is substantially lower (higher) than the effectiveness of retailer's advertising.

Moreover, one can notice that each line marks the division line of the considered area—with $0 \leq \nu \leq 1.5$ and $0 \leq k \leq 3.0$—into two regions. Thereby, each line indicates the combinations of ν and k for which two games yield the same wholesale prices w, while each region identifies combinations of ν and k where one equilibrium leads to higher wholesale prices than the equilibrium it is in comparison with. We can identify the following relations in that figure:

- The **dotted line** marks combinations of k and ν where $w^{\mathrm{N}} = w^{\mathrm{RS}}$ is valid, with $w^{\mathrm{N}} > w^{\mathrm{RS}}$ on its left and $w^{\mathrm{RS}} > w^{\mathrm{N}}$ on its right side.
- The **solid line** indicates where $w^{\mathrm{MS}} = w^{\mathrm{RS}}$ is valid, with $w^{\mathrm{RS}} > w^{\mathrm{MS}}$ below and $w^{\mathrm{MS}} > w^{\mathrm{RS}}$ above.
- The **dashed line** denotes parameter combinations resulting in $w^{\mathrm{MS}} = w^{\mathrm{N}}$, with $w^{\mathrm{N}} > w^{\mathrm{MS}}$ below and $w^{\mathrm{MS}} > w^{\mathrm{N}}$ above.

The same computation was done for the **retailer margin** m (see Fig. 4.3 for an overview). Interestingly, one can recognize that the dashed and the dotted line follow the same course as in Fig. 4.2. But compared to the previous illustration, the order is inverse: Considering the retailer margin m, the Nash equilibrium leads to higher values than the Retailer Stackelberg equilibrium in the area left to the dotted line. Like before, Cooperation cannot be included into this analysis.

Totally independent of the combination of k and ν, the lowest **retail price** p results from Cooperation, which is a matter of particular interest, because the retail price can be seen as decisive factor for customer welfare. For simplicity of illustration, we do not indicate this in our overview in Fig. 4.4. One can also find the dashed and dotted lines identically to Figs. 4.2 and 4.3.

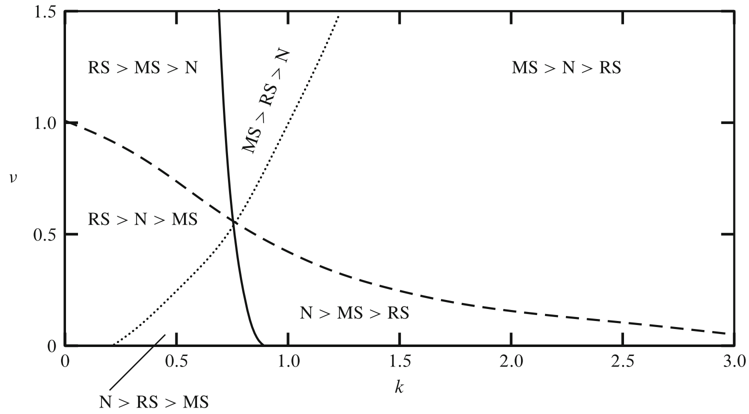

Fig. 4.4 Retail price p

Beyond that, a comparison of our illustrations to the analysis of SeyedEsfahani et al. (2011) shows considerable differences, which can be attributed to the abolition of the assumption of identical margins.[36] With $w = p/2$, the wholesale price of the Retailer Stackelberg equilibrium w^{RS} is always lower than w^{N} or w^{MS}. In contrast, our analysis shows that the wholesale price of the unrestricted Retailer Stackelberg equilibrium can even be the highest of all equilibria. That leads to six areas of price ranking, while previous research identified only two different areas—namely $w^{MS} > w^{N}$ and $w^{N} > w^{MS}$—which are separated by a line very similar to the dashed line identified above. Similar observations also hold for the retail price p.

Lastly, Fig. 4.5 depicts the ratio between wholesale price and retail price within the Retailer Stackelberg equilibrium in order to analyze whether the assumption $w = p/2$ made by Xie and Neyret (2009) and SeyedEsfahani et al. (2011) was correct.[37] It is obvious that this proportion holds only for specific parameters, while the retailer can receive a considerably higher margin than the manufacturer for small values of v and k. This again emphasizes the contribution of the introduction of the new variable m, which allows studying an unrestricted Retailer Stackelberg game.

4.3.2 Advertising Expenditures and Participation Rate

Considering advertising expenditures, both the manufacturer and the retailer will spend the most on advertising when cooperating. This coincides with previous research and was expected, because each player knows exactly how much his counterpart will invest, so that he has a higher willingness to spend more on his part as in any other game structure.

[36] Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, pp. 266–269.

[37] Cf. Xie and Neyret (2009): *Co-op advertising*, p. 1377, and SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 266.

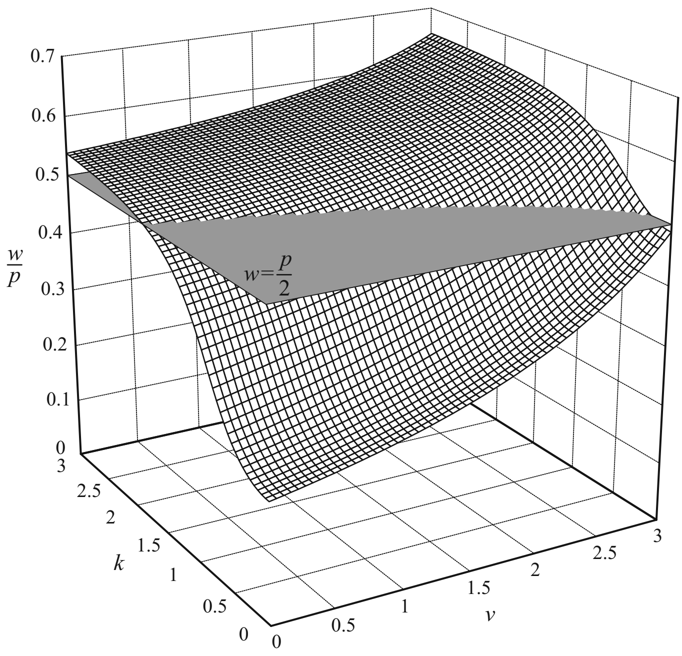

Fig. 4.5 Ratio of wholesale price w and price p in Retailer Stackelberg equilibrium

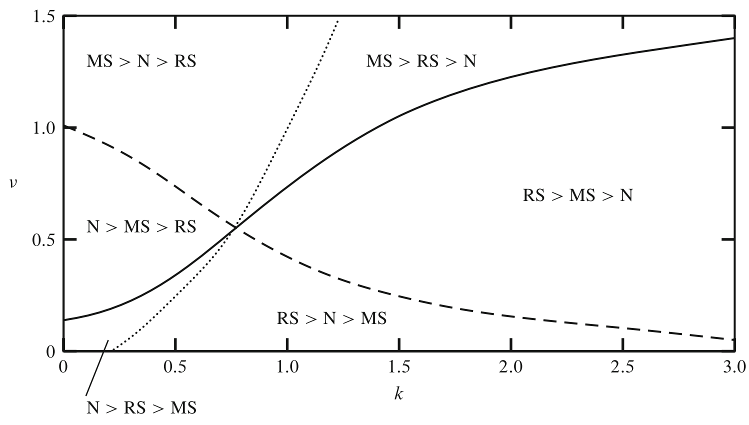

Fig. 4.6 Manufacturer's global advertising expenditures A

From our computation, we derive six different areas for the **global advertising expenditures** A, too (see Fig. 4.6). Furthermore, one can observe the same dashed and dotted line already seen in the previous figures. The distribution of A is somewhat reminiscent of the distribution of w in Fig. 4.2, which is probably due to the fact that the advertising budget is increased by a high manufacturer's margin.

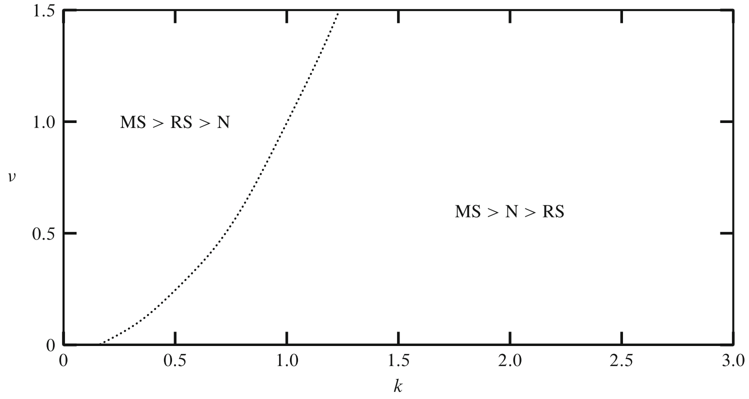

Fig. 4.7 Retailer's local advertising expenditures a

Figure 4.7 shows our results for the retailer's **local advertising expenditures** a. In contrast to the latter illustrations, we derive only two different areas. As stated above, the highest advertising expenditures are received when the players cooperate. The next highest local advertising expenditures are found in the Manufacturer Stackelberg equilibrium, while the order between Retailer Stackelberg and Nash equilibrium depends on the parameters k and ν. The high values of a when the manufacturer is the leader may surprise firstly, but we refer the reader to the overview in Table 4.3, where one can see that this is the only game—besides the Cooperation—where the participation rate t is greater than zero. In this case, the retailer has to pay only a certain fraction of a, while the remainder is funded by the manufacturer.

A comparison to the findings of SeyedEsfahani et al. (2011) reveals that also the results concerning the advertising expenditures of our unrestricted model are more differentiated. When assuming $w = p/2$, the global advertising expenditures of the Nash equilibrium A^{N} do not exceed A^{RS} for any set of parameters.[38] As visible from Fig. 4.6, this can in fact happen for small values of k. Similar to this, we derive that the lowest local advertising expenditures can take place either in Nash or Retailer Stackelberg equilibrium, while $a^{MS} > a^{RS} > a^{N}$ is valid for every set of parameters when $w = p/2$ holds.

Lastly, we discuss the manufacturer's **participation rate**. As aforementioned, the manufacturer will participate in the local advertising expenditures solely when he is either the leader or cooperates with the retailer. In the latter case, participation rate and margins have to be determined via the proposed bargaining model, so that only the Manufacturer Stackelberg game seems appropriate for further analysis. The illustration of the function $t(k, \nu)$ is shown in Fig. 4.8. One can easily see that the highest participation rate will be reached for maximum parameters, i.e.,

[38] Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 267.

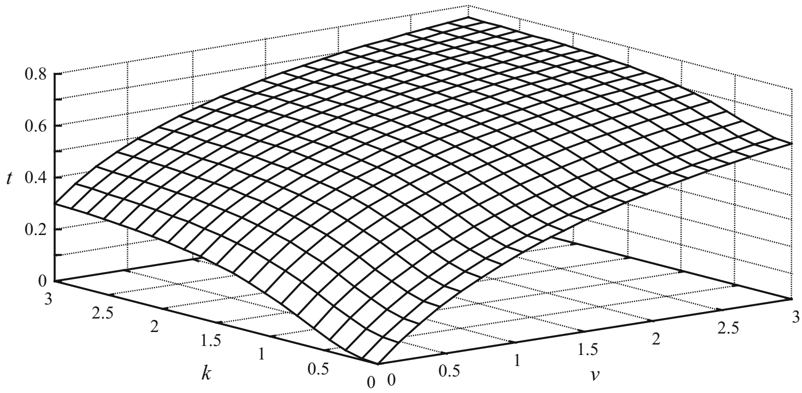

Fig. 4.8 Manufacturer's advertising participation rate t

for a concave price demand function and for an advertising effectiveness of the manufacturer, which is considerably higher than the retailer's. Conversely, there will be no participation if k and ν equal zero. The gradient of the edges furthermore suggests that the shape parameter ν has more effects on t than the advertising ratio k.

4.3.3 Profits

In this section, we analyze the resulting profit for each channel member as well as the corresponding total profit in each game scenario. More than any other variable, the profit can give an indication of the possible behavior and decisions of the players.

Figure 4.9 starts with our findings concerning the **manufacturer's profit** Π_m. It is visible that the manufacturer will receive the highest profit as a leader, if the situation comes within the area to the left of the solid line, while he will obtain the best results as a follower on its right side. Overall, one can state that the manufacturer will always prefer acting as Stackelberg leader to playing a Nash game.

The computation of the **retailer's profit** Π_r, which is summarized in Fig. 4.10, also produces three different regions. The solid line again marks the division line between the areas where either the leadership or the followership is more profitable for the retailer, while the dashed line distinguishes solely Manufacturer Stackelberg and Nash equilibrium. Just as the manufacturer, the retailer will always receive higher profits when he is the leader compared to the Nash game.

For every combination of k and ν, the highest **total profit** results when manufacturer and retailer cooperate. This is a common insight in literature and is conform with intuition, as we modeled the Cooperation by means of a joint profit maximization. Considering the other game structures under examination, no further general conclusions can be derived, because every game may lead to the second highest total profit for a certain set of parameters (see Fig. 4.11).

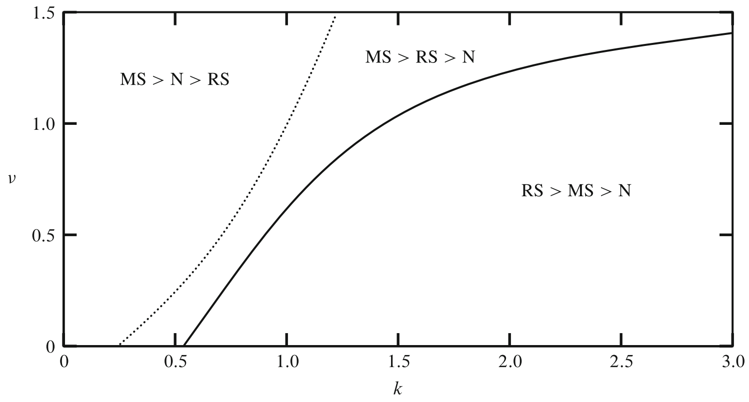

Fig. 4.9 Manufacturer's profit Π_m

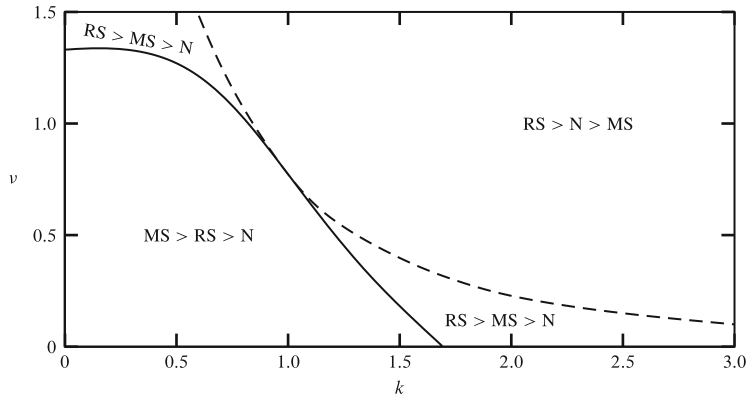

Fig. 4.10 Retailer's profit Π_r

Regarding the analysis of SeyedEsfahani et al. (2011), we can state considerable differences in the regions to distinguish between. Assuming $w = p/2$, for instance, it is always more profitable for the manufacturer to act as Stackelberg follower for small values of ν,[39] while the unrestricted model necessitates a more specific decision: Like shown in Fig. 4.9, the Stackelberg leadership will produce the highest manufacturer's profits even for small values of ν, when parameter k is sufficiently small. The total profit Π_{m+r} in Fig. 4.11 can provide another interesting insight: Even though the highest total profit always results from the Cooperation, we identified a small region with medium values of ν and k, where the second

[39]Cf. SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 268.

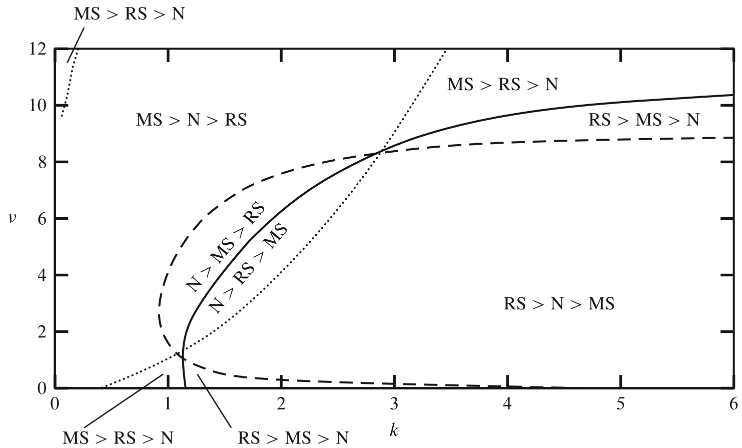

Fig. 4.11 Total profit $\Pi_{\mathrm{m+r}}$

highest profit arises from an uncoordinated behavior (i.e., Nash game). This may be of importance when general frame conditions prohibit a Cooperation and one player has to decide whether to act as a Stackelberg leader or not.

4.3.4 Feasibility of Cooperation Game

The previous section revealed that Cooperation results in the highest total profit $\Pi_{\mathrm{m+r}}$, independently of the parameters. However, this is not a sufficient condition for a cooperative behavior of manufacturer and retailer. As explained in Sect. 4.2.5, we assume that both players will only agree on Cooperation if they individually receive at least the same profit as in any other game structure (see Eqs. (4.83) and (4.84)). Therefore, one has to analyze the validness of Eq. (4.85) in order to prove the feasibility of a Cooperation for certain sets of parameters k and ν.

Again, we use numerical computations to deal with this issue (see Fig. 4.12 for the summarized results). One may notice that a Cooperation is not always feasible, which conflicts with the findings of SeyedEsfahani et al. (2011) firstly. The infeasibility however occurs only for large values of shape parameter ν.

As the detailed example in Table 4.4 with $\alpha = 10, \beta = 1, \nu = 8, k_{\mathrm{m}} = 2, k_{\mathrm{r}} = 1$ (and therefore $k = 2$) shows, both players will realize the highest profits (labeled with bold figures) if they can obtain the Stackelberg leadership. According to our assumption, both will at least claim these profits $(\Pi_{\mathrm{m}}^{\mathrm{MS}} = 59.96$ and $\Pi_{\mathrm{r}}^{\mathrm{RS}} = 43.77)$

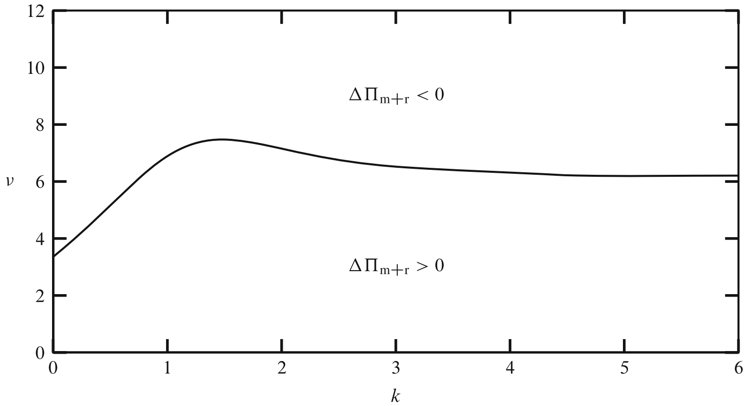

Fig. 4.12 Feasibility of Cooperation

Table 4.4 Numerical example

Game	w	m	p	A	a	t	Π_m	Π_r	Π_{m+r}
N	4.71	4.71	9.41	19.39	4.85	0.00	29.09	43.64	72.73
MS	8.77	1.09	9.86	46.77	13.19	0.88	**59.96**	13.15	73.11
RS	4.45	4.99	9.44	17.13	5.38	0.00	26.72	**43.77**	70.49
C	–	–	8.89	81.12	20.28	–	–	–	**101.40**

during bargaining, because otherwise they would have an incentive to deviate from Cooperation. The sum of these minimum claims is:

$$\Pi_m^{MS} + \Pi_r^{RS} = 59.96 + 43.77 = 103.73 > \Pi_{m+r}^{C} = 101.40.$$

One can see that this sum of minimum claims exceeds the total profit of a Cooperation, which can be divided between the two players. Therefore, there will be no Cooperation, even if the resulting total profit is higher than in any other game.

In order to get more insights into the advantageousness of a Cooperation, we finally consider the relative total extra profit $\Delta\Pi_{m+r}$ within a smaller area, in which the Cooperation is feasible. The relative total extra profit is calculated via

$$\Delta\Pi_{m+r}^{R} = \frac{\Pi_{m+r}^{C} - \Pi_m^{max} - \Pi_r^{max}}{\Pi_m^{max} + \Pi_r^{max}}. \tag{4.93}$$

As visible from Fig. 4.13, a Cooperation can result in a considerable increase of profit for both players in the considered region of k and ν. The exact rate however depends strongly on the underlying set of parameters, which determines the game in which the players would otherwise realize Π_m^{max} respectively Π_r^{max}.

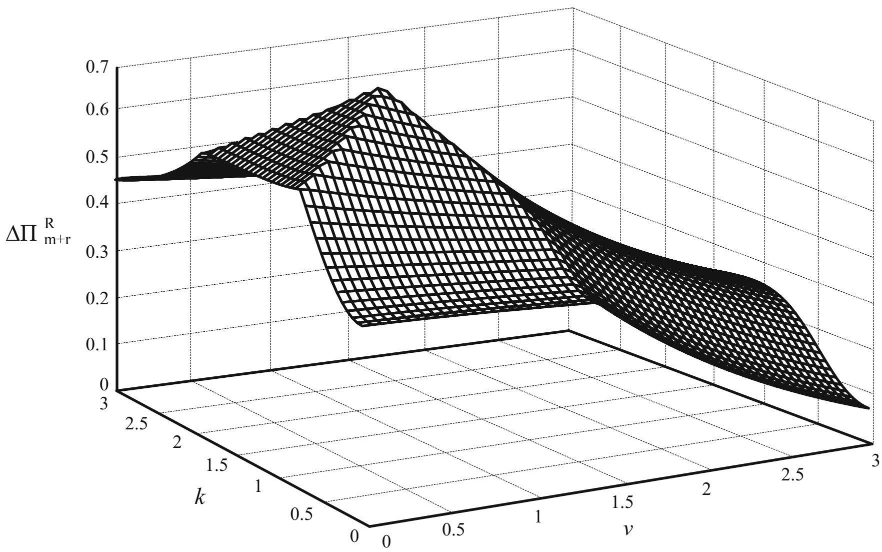

Fig. 4.13 Feasibility of Cooperation with $0 < k \leq 3$ and $0 < \nu \leq 3$

4.4 Managerial Implications and Conclusions

This paper addresses optimal pricing and advertising decisions in a manufacturer-retailer supply chain with consumer demand that depends both on the retail price and on the channel members' advertising expenditures. Additionally, a cooperative advertising program is considered, where the manufacturer can bear a certain fraction of the retailer's advertising costs. By means of game theory, we analyze four different relationships within the supply chain: A non-cooperative behavior with equal distribution of power, two situations in which one player dominates his counterpart, and a Cooperation between manufacturer and retailer.

We adopted a model recently published by SeyedEsfahani et al. (2011) and introduced the retailer margin m as a new decision variable. [40] This modification of the original model enabled us to abandon the restrictiveassumption of identical margins previously used both in Nash and in Retailer Stackelberg games. Furthermore, our model also extends the work of Xie and Wei (2009), which is a special case with linear price demand (i.e., $\nu = 1$).[41]

[40] See SeyedEsfahani et al. (2011): *Vertical co-op advertising*.

[41] See Xie and Wei (2009): *Coordinating advertising*.

The main contributions of our research are as follows: Without the assumption of identical margins, the profit split between manufacturer and retailer can be determined unrestrictedly and solely depending on the underlying set of parameters and game structure. Anyhow, we observed that a Nash equilibrium leads to identical margins on its own, so that the assumption was justified in that case, but not in the Retailer Stackelberg equilibrium. The numerical computations in Sect. 4.3 though showed that our generalized model yields more differentiated results concerning the dominant game structure, which decision makers can use as recommendation for practical problems.

Then, a generalization of the price demand function used in SeyedEsfahani et al. (2011) by introducing the parameters α and β. These parameters do not affect the structure of the results (i.e., the ratio between manufacturer's and retailer's profit), but exclusively the level of prices, advertising expenditures, and profits. Therefore, these parameters can be used to adapt the proposed model more precisely to practical contexts.

The Cooperation, which is characterized by the lowest retail price and the highest advertising expenditures, produces the highest total profit of all considered games. We showed the feasibility of a Cooperation for moderate sets of parameters k and ν and exemplified the Asymmetric Nash bargaining model of Harsanyi and Selten (1972) and Kalai (1977), which allows to consider risk attitude and bargaining power contemporaneously.[42]

Future research could apply our approach of using the retailer margin as decision variable also to the model of Xie and Neyret (2009), which suffers from the assumption of identical margins in the Nash and Retailer Stackelberg equilibrium, too.[43] Moreover, the introduction of additional supply chain members would render possible to analyze not only the interaction between the two echelons, but also the competition between two manufacturers or retailers. The increased complexity could though necessitate the application of heuristics or meta-heuristics.[44] In a multiple player framework, the forming of coalitions during bargaining seems to be another interesting field of research.

Acknowledgements A slightly modified version of this work is also published in Aust and Buscher (2012): Vertical cooperative advertising and pricing decisions in a manufacturer-retailer supply chain: A game-theoretic approach. *European Journal of Operational Research, 223*(2), 473–482. http://dx.doi.org/10.1016/j.ejor.2012.06.042.

[42] See Harsanyi and Selten (1972): *Generalized Nash* and Kalai (1977): *Nonsymmetric Nash*.

[43] See Xie and Neyret (2009): *Co-op advertising*.

[44] See, e.g., Yu and Huang (2010): *Nash game model* for a recent example.

Chapter 5
Game Theoretic Analysis of Pricing and Vertical Cooperative Advertising of a Retailer-Duopoly with a Common Manufacturer

Abstract This paper considers competition of duopolistic retailers, who sell substitutable products supplied by a single manufacturer offering a vertical cooperative advertising program. The price-dependent component of the demand function is derived from the customers' utility function in order to avoid logically inconsistent results. Additionally, each supply chain member can increase the costumers' demand by advertising. By means of game theory, we get the following results: (a) Retail competition harms all players, but is beneficial to the customers. (b) Stronger competition is followed by less advertising. (c) Channel-leadership is not always advantageous to the manufacturer, and—likewise—retailers can also be better off when accepting followership.

5.1 Introduction

Vertical cooperative advertising (we may simply refer to cooperative advertising in the following), which is commonly meant as a financial agreement between manufacturer and retailer about a certain participation in advertising expenditures,[1] has gained substantial attention in operations research in recent years. While the manufacturer's advertising—also referred to as *global advertising*—is mostly aimed at brand image or reputation of a firm, the advertising effected by retailers—also referred to as *local advertising*—works directly on the customers' buying decisions by special offers, promotion activities, etc. As the latter can be seen as a catalyst for the immediate purchase of a product, the manufacturer to some extent depends on his retailers. If we now assume small retailers with a relatively small advertising budget, it will apparently happen that the local advertising (and accordingly the sales) are too low from the manufacturer's point of view. Hence, it can be beneficial to him to share a certain fraction of the retailers' advertising cost in order to increase the local advertising efforts, when the additional revenues exceed the higher costs.[2]

[1] Cf. Bergen and John (1997): *Cooperative advertising*, p. 357, and Crimmins (1984): *Cooperative advertising*, p. 2.

[2] Cf. Somers et al. (1990): *Cooperative advertising expenditures*, p. 36.

G. Aust, *Vertical Cooperative Advertising in Supply Chain Management*,
Contributions to Management Science, DOI 10.1007/978-3-319-11626-6_5

Cooperative advertising programs for retailers are very common in the United States with a volume on the increase from $15 billion in 2000 to $50 billion in 2008.[3] However, despite these big amounts, manufacturers mostly set their cooperative advertising participation rate to 50 % or 100 %,[4] which might be traced back more to arbitrariness than to detailed analysis of profitability. This underlines the necessity of a theoretical examination.

Cooperative advertising literature is primarily limited to supply chains consisting of bilateral monopolies, i.e., the interaction of one manufacturer with one retailer. Berger (1972) was the first to propose a mathematical formulation, where the retailer's advertising is supported by a price discount on the wholesale price.[5] Huang and Li (2001) applied game theory and considered two different types of inter-echelon interaction: an asymmetric distribution of power where the manufacturer holds the channel leadership (Manufacturer Stackelberg) and a Cooperation.[6] In order to gain more comprehensive results, the following works additionally included pricing into their analyses.[7] Besides the above mentioned forms of interaction, some of these works also studied retailer-leadership (Retailer Stackelberg) or symmetric distribution of power between the echelons (Vertical Nash game).

Papers which involve more than two players are primarily limited to a single decision variable like pricing or advertising. In the field of pricing, Choi (1991, 1996) considers a manufacturer-duopoly which sells its product through one or two retailers.[8] While the intra-echelon interaction is always characterized by a symmetric distribution of power, Stackelberg games with manufacturer- and retailer-leadership are applied to the inter-echelon relationship. In contrast to that, Yang and Zhou (2006) compare different intra-echelon interactions (Horizontal Nash, Stackelberg, and Cooperation) in a one-manufacturer two-retailer setting under manufacturer-leadership.[9] Wu et al. (2012) extend this work by a symmetric inter-echelon distribution of power to a total of six different game scenarios.[10] A situation with two manufacturers and a single exclusive retailer under three different inter-echelon power structures is analyzed in Zhang et al. (2012).[11] Recently, Zhao et al. (2012b) use a fuzzy environment in a similar setting to deal with uncertain manufacturing costs and customer demand.[12]

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[3]Cf. He et al. (2012): *Co-op advertising*, p. 74, and Nagler (2006): *Cooperative advertising participation rates*, pp. 91 et seq.

[4]Cf. Dutta et al. (1995): *Cooperative advertising contracts*, p. 16, and Nagler (2006): *Cooperative advertising participation rates*, p. 96.

[5]See Berger (1972): *Vertical cooperative advertising*.

[6]See Huang and Li (2001): *Co-op advertising models*.

[7]See Yue et al. (2006): *Coordination of cooperative advertising*, Xie and Neyret (2009): *Co-op advertising*, and Xie and Wei (2009): *Coordinating advertising*.

[8]See Choi (1991): *Price competition* and Choi (1996): *Price competition*.

[9]See Yang and Zhou (2006): *Two-echelon supply chain models*.

[10]See Wu et al. (2012): *Competitive pricing decisions*.

[11]See Zhang et al. (2012): *Pricing decisions*.

[12]See Zhao et al. (2012b): *Pricing decisions for substitutable products*.

Cooperative advertising within a one-manufacturer two-retailer supply chain in a dynamic environment is considered in He et al. (2011), while Karray and Zaccour (2007) assume a static game.[13] While these authors do not include pricing decisions in their models, Chutani and Sethi (2012b) and He et al. (2012) (dynamic models) and Ghadimi et al. (2013), Wang et al. (2011), and Zhang and Xie (2012) (static models) integrate the retail price or the firms' individual margins as exogenously determined parameters.[14]

To the best of our knowledge, our paper is the first approach to simultaneously analyze pricing and (cooperative) advertising in a one-manufacturer two-retailer supply chain in a static and deterministic environment. Therefore, the remainder of this paper is organized as follows. The mathematical formulation of the model is given in Sect. 5.2. This includes the derivation of the price demand function from the customers' utility function as well as the profit functions of the involved players. In Sect. 5.3, we apply two different game scenarios of manufacturer-retailer interaction on the proposed model and determine Vertical Nash–Horizontal Nash (Sect. 5.3.1) and Manufacturer Stackelberg–Horizontal Nash equilibria (Sect. 5.3.2). This is followed by a comparison of the results in Sect. 5.4, which uses analytical and numerical computations. Here, we will analyze the effects of competition on the players' decisions and profits as well as the advantages of both games for given parameter frameworks. Section 5.5 summarizes the main findings of our research.

5.2 Model Formulation

The supply chain under consideration is composed of one manufacturer and two identical and competing retailers (see Fig. 5.1), which sell substitutable products to a group of customers. Each retailer faces an individual positive demand quantity D_j, while the manufacturer is able to supply the whole quantity of both products $D_1 + D_2$ without any capacity constraints. Each unit of the products is sold at a channel-specific retail price p_j, whereas the manufacturer does not apply price discrimination and invoices the same wholesale price w to both retailers. It is obvious that $0 < w < p_j$ holds.

In order to stimulate customer demand, each channel member has the ability to invest in advertising. Thereby, we distinguish global advertising expenditures A of the manufacturer and local advertising expenditures a_j of retailer j. Via the participation rate t (with $0 \leq t < 1$), the manufacturer can support the retailers' advertising within the framework of a cooperative advertising program. Please note

[13]See Karray and Zaccour (2007): *Effectiveness of coop advertising* and He et al. (2011): *Retail competition*.

[14]See Chutani and Sethi (2012b): *Cooperative advertising*, Ghadimi et al. (2013): *Coordination of advertising*, He et al. (2012): *Co-op advertising*, Wang et al. (2011): *Cooperative advertising models*, and Zhang and Xie (2012): *Cooperative advertising with multiple retailers*.

Table 5.1 List of symbols

Variables		Parameters/Functions	
p_j	Retail price	Λ_j	Market size
w	Wholesale price	B	Intensity of saturation effect
m_j	Retailer margin	Θ	Channel substitutability
a_j	Local advertising expenditures	k_{r}	Effectiveness of local advertising
A	Global advertising expenditures	k_{m}	Effectiveness of global advertising
t	Advertising participation rate	k	Advertising effectiveness ratio
Π	Profit	$\alpha_j, \beta, \epsilon$	Demand parameters (substituted)
D_j	Demand quantity		
		$h(\cdot)$	Advertising demand function
		$g(\cdot)$	Price demand function

that this rate is assumed to be uniform to both retailers, as it is often required by legislative means like the Robinson-Patman Act (1936) in the United States of America.[15]

As we do not consider any manufacturing or transportation costs, the wholesale price w is also the unit contribution margin of the manufacturer, while the margin of retailer j is

$$m_j = p_j - w. \tag{5.1}$$

Hence, we can formulate the channel members' profit functions according to the cash flows in Fig. 5.1, with Π_{m} denoting the manufacturers profit and $\Pi_{\text{r}j}$ the profit of retailer j (see Table 5.1 for a listing of symbols used in this article):

$$\Pi_{\text{m}} = w \sum_{j=1}^{2} D_j - t \sum_{j=1}^{2} a_j - A \tag{5.2}$$

$$\Pi_{\text{r}j} = m_j D_j - (1-t) a_j. \tag{5.3}$$

The demand quantity D_j is assumed to be a function of both, the retail price and the advertising of all channel members. Thereby, we assume that the price of the product directly takes effect on the customers' utility, because the customers weigh additional utility arising from the consumption of the new product against the reduction of available capital. In contrast, advertising does not really generate additional utility, but has more multiplying effects which reinforce the price-induced demand.

Hence, the demand function can be derived as follows: First, in order to determine the price-dependent demand function g_j, we follow the approach of

[15]Cf. Wang et al. (2011): *Cooperative advertising models*, p. 1055.

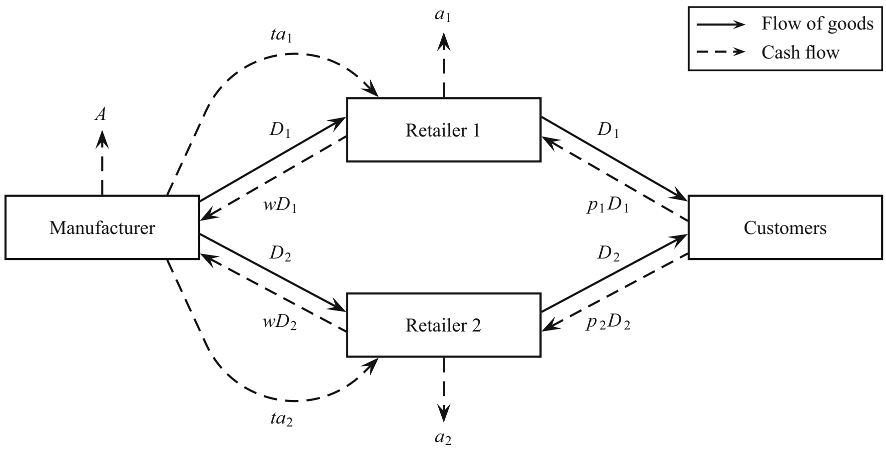

Fig. 5.1 One-manufacturer two-retailer supply chain

Ingene and Parry (2004), Ingene and Parry (2007), and Zhang et al. (2012).[16] They propose the following utility function of the customers:

$$u(g_1, g_2) = \sum_{j=1}^{2} \left(\Lambda_j g_j - \frac{Bg_j^2}{2} \right) - \Theta g_1 g_2 - \sum_{j=1}^{2} p_j g_j, \tag{5.4}$$

Here, the parameter Λ_j (with $\Lambda_j > 0$) can be interpreted as the market size of retail channel j, parameter B (with $B \geq 0$) as the intensity of the saturation effect, which comes along with the already effected acquisitions g_j, and parameter Θ (with $0 \leq \Theta \leq 1$) as channel substitutability. More precisely, $\Theta = 0$ characterizes a market without channel competition, whereas $\Theta = 1$ stands for perfect substitutability and, therefore, intense retail competition. As we assume that the customers are less influenced by channel competition than by saturation effect, we set $B > \Theta$.

By setting the first order partial derivatives $\partial u(g_1, g_2)/\partial g_1$ and $\partial u(g_1, g_2)/\partial g_2$ to zero, we can identify the utility-maximizing demand quantity:

$$g_j(p_j, p_{3-j}) = [\Lambda_j(B - \Theta) + \Theta p_{3-j} - Bp_j]/(B^2 - \Theta^2). \tag{5.5}$$

This expression can be rearranged to the well-known linear demand function

$$g_j(p_j, p_{3-j}) = \alpha_j - \beta p_j + \epsilon p_{3-j}, \tag{5.6}$$

[16]See Ingene and Parry (2004): *Mathematical models*, chapter 11 (especially pp. 493–495), Ingene and Parry (2007): *Bilateral monopoly*, pp. 599 et seq. and its Technical Appendix, and Zhang et al. (2012): *Pricing decisions*, p. 524.

with $\alpha_j = [\Lambda_j(B-\Theta)]/(B^2-\Theta^2)$, $\beta = B/(B^2-\Theta^2)$ and $\epsilon = \Theta/(B^2-\Theta^2)$. This utility-based approach to the price demand function enables us to use parameter Θ as a measure of retail-competition instead of parameter ϵ, where an increase of competition would result in higher prices and profits of all channel members. This counter-intuitive effect, which is also described as *Competitive-Substitutability Hypothesis*, does not occur when using Θ.[17] To ensure a positive price demand, we set $0 < p_j < (\alpha_j + \epsilon p_{3-j})/\beta$.

Second, as explained above, this price-dependent demand can further be increased by advertising. Thereby, we assume that customers react to increasing advertising expenditures with a diminishing marginal demand, which describes the saturation effect visible when analyzing advertising efforts. This effect can be modeled by means of a function based on square roots.[18] Furthermore, we follow Karray and Zaccour (2007) and assume that local advertising of one retailer has the same effects on the demand of the second retailer.[19] This may hold for advertisement, which is not retailer-, but product-oriented, like poster campaigns during the introduction of new cars. It is quite possible that the poster will attract also the attention of customers of retailer 2, even if the campaign is organized by retailer 1. The resulting advertising-induced demand function is

$$h(a_1, a_2, A) = k_{\mathrm{r}} \sum_{j=1}^{2} \sqrt{a_j} + k_{\mathrm{m}} \sqrt{A}, \tag{5.7}$$

with the two positive parameters k_{r} and k_{m} denoting the effectiveness of local and global advertising. Via multiplication of the two elements of demand, one can now determine the total demand function of channel j[20]:

$$D_j(p_j, p_{3-j}, a_1, a_2, A) = g_j h = (\alpha_j - \beta p_j + \epsilon p_{3-j}) \left(k_{\mathrm{r}} \sum_{j=1}^{2} \sqrt{a_j} + k_{\mathrm{m}} \sqrt{A} \right), \tag{5.8}$$

as well as the extended profit functions of all channel members:

$$\Pi_{\mathrm{m}} = w \sum_{j=1}^{2} \left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j}) \right] \left(k_{\mathrm{r}} \sum_{j=1}^{2} \sqrt{a_j} + k_{\mathrm{m}} \sqrt{A} \right) - t \sum_{j=1}^{2} a_j - A \tag{5.9}$$

[17] Cf. Ingene and Parry (2007): *Bilateral monopoly*, p. 599.

[18] Cf. Kim and Staelin (1999): *Manufacturer allowances*, pp. 65 et seq., Karray and Zaccour (2006): *Co-op advertising*, p. 1010, and Xie and Wei (2009): *Coordinating advertising*, p. 787.

[19] Cf. Karray and Zaccour (2007): *Effectiveness of coop advertising*, p. 155.

[20] Cf. Yue et al. (2006): *Coordination of cooperative advertising*, p. 68, Xie and Neyret (2009): *Co-op advertising*, p. 787, and SeyedEsfahani et al. (2011): *Vertical co-op advertising*, p. 265.

$$\Pi_{rj} = m_j \left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j})\right] \left(k_r \sum_{j=1}^{2} \sqrt{a_j} + k_m \sqrt{A}\right) - (1-t)a_j. \tag{5.10}$$

5.3 Two Forms of Manufacturer-Retailer Relationship

5.3.1 Symmetric Relationship

We first consider a symmetric relationship between the two echelons, where neither the manufacturer nor the retailer-duopoly can exercise any sort of channel power. As in previous studies dealing with bilateral monopolies, this situation can be modeled by means of a Vertical Nash game, where both echelons make their decisions contemporaneously and without knowledge of the other's activities. Furthermore, we assume a Horizontal Nash competition within the retailer-duopoly, which is—similar to the Vertical Nash game—a non-cooperative and simultaneous decision process.

In order to find the equilibrium of this Vertical Nash–Horizontal Nash game, one has to identify the individual profit maximizing prices and advertising expenditures of each channel member. Starting with the manufacturer, we obtain the following decision problem

$$\begin{aligned} \text{Max} \quad & \Pi_m = w \sum_{j=1}^{2} \left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j})\right] \left(k_r \sum_{j=1}^{2} \sqrt{a_j} + k_m \sqrt{A}\right) \\ & \qquad - t \sum_{j=1}^{2} a_j - A \\ \text{s.t.} \quad & 0 < w < (\alpha_j - \beta m_j + \epsilon m_{3-j})/(\beta - \epsilon), \ A > 0, \text{ and } 0 \le t < 1, \end{aligned} \tag{5.11}$$

while the general decision problem of retailer j is:

$$\begin{aligned} \text{Max} \quad & \Pi_{rj} = m_j \left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j})\right] \\ & \qquad \cdot \left(k_r \sum_{j=1}^{2} \sqrt{a_j} + k_m \sqrt{A}\right) - (1-t)a_j \\ \text{s.t.} \quad & 0 < m_j < [\alpha_j - (\beta - \epsilon)w + \epsilon m_{3-j}]/\beta, \ a_j > 0. \end{aligned} \tag{5.12}$$

By computing the first order partial derivatives $\partial\Pi_{\mathrm{m}}/\partial w$, $\partial\Pi_{\mathrm{m}}/\partial A$, $\partial\Pi_{\mathrm{r}j}/\partial m_j$, and $\partial\Pi_{\mathrm{r}j}/\partial a_j$ and setting them to zero, some rearrangement leads to the following solution of the Vertical Nash–Horizontal Nash game:

Proposition 5.1 *If there is a symmetric and non-cooperative inter-echelon and intra-echelon relationship between the channel members, this situation can be solved by a Vertical Nash–Horizontal Nash equilibrium with:*

(i) $w = \dfrac{(\alpha_1 + \alpha_2)\beta}{2(3\beta - \epsilon)(\beta - \epsilon)}$ *and* $m_j = \dfrac{5\alpha_j\beta - \alpha_{3-j}\beta + 2\alpha_{3-j}\epsilon}{2(3\beta - \epsilon)(2\beta + \epsilon)}$.

(ii) $A = \dfrac{\beta^4 k_{\mathrm{m}}^2(\alpha_1 + \alpha_2)^4}{16(3\beta - \epsilon)^4(\beta - \epsilon)^2}$ *and* $a_j = \dfrac{\beta^2 k_{\mathrm{r}}^2(5\alpha_j\beta + \alpha_{3-j}\beta + 2\alpha_{3-j}\epsilon)^4}{64(3\beta - \epsilon)^4(2\beta + \epsilon)^4}$.

(iii) $t = 0$.

Proof of Proposition 5.1 We start with the **manufacturer's decision problem** stated in Eq. (5.11) and set the first order partial derivatives $\partial\Pi_{\mathrm{m}}/\partial w$ and $\partial\Pi_{\mathrm{m}}/\partial A$ to zero:

$$\frac{\partial\Pi_{\mathrm{m}}}{\partial w} = \left(k_{\mathrm{r}}\sum_{j=1}^{2}\sqrt{a_j} + k_{\mathrm{m}}\sqrt{A}\right)\left[\sum_{j=1}^{2}\alpha_j - \beta(2w + m_j) + \epsilon(2w + m_{3-j})\right] = 0 \tag{5.13}$$

$$\frac{\partial\Pi_{\mathrm{m}}}{\partial A} = \frac{k_{\mathrm{m}}w}{2\sqrt{A}}\left[\sum_{j=1}^{2}\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j})\right] - 1 = 0. \tag{5.14}$$

Please note that participation rate t is set to zero due to its solely negative influence on the manufacturer's profit function. From Eqs. (5.13) and (5.14), we derive:

$$w = \frac{\alpha_1 + \alpha_2 - (\beta - \epsilon)(m_1 + m_2)}{4(\beta - \epsilon)} \tag{5.15}$$

$$A = \frac{1}{4}k_{\mathrm{m}}^2 w^2\left[\alpha_1 + \alpha_2 - (\beta - \epsilon)(m_1 + m_2) - 2w(\beta - \epsilon)\right]. \tag{5.16}$$

Likewise, we calculate the first order partial derivatives of the **retailers' decision problems** (see Eq. (5.12)) $\partial\Pi_{\mathrm{r}j}/\partial m_i$ and $\partial\Pi_{\mathrm{r}j}/\partial a_i$ and set them to zero:

$$\frac{\partial\Pi_{\mathrm{r}j}}{\partial m_j} = \left(k_{\mathrm{r}}\sum_{j=1}^{2}\sqrt{a_j} + k_{\mathrm{m}}\sqrt{A}\right)\left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j}) - \beta m_j\right] = 0 \tag{5.17}$$

$$\frac{\partial\Pi_{\mathrm{r}j}}{\partial a_j} = \frac{k_{\mathrm{r}}m_j}{2\sqrt{a_j}}\left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j})\right] - (1 - t) = 0. \tag{5.18}$$

From these equations, we derive:

$$m_j = \frac{\alpha_j - (\beta - \epsilon)w + \epsilon m_j}{2\beta} \tag{5.19}$$

$$a_j = \frac{k_r^2 m_j^2 \left[\alpha_j - (\beta - \epsilon)w - \beta m_j + \epsilon m_{3-j}\right]}{4(1-t)^2}. \tag{5.20}$$

With $t = 0$, we can now solve the system of equations described by Eqs. (5.15), (5.16), (5.19), and (5.20), which leads us to the expressions stated in Proposition 5.1. This completes the proof of Proposition 5.1. □

Part (iii) of Proposition 5.1 shows that the manufacturer will choose a cooperative advertising participation rate $t = 0$. This is due to its solely negative effect on his profit function given in Eq. (5.9), as the manufacturer is not able to foresee the advertising expenditures of his retailers. As the demand parameters α_j, β, and ϵ are only substitutes of the parameters Λ, B, and Θ, which originate from the customers' utility function, we refer the reader to Sect. 5.4 for further analysis.

5.3.2 Manufacturer-Leadership

Now we assume that an asymmetric distribution of power is given where the manufacturer obtains the channel-leadership. This is a common assumption in marketing literature and is valid, e.g., in the automotive industry. Translated into a Stackelberg game, the manufacturer acts as Stackelberg leader and first sets his optimal wholesale price w, global advertising expenditures A, and cooperative advertising participation rate t, already being aware of the reactions of his retailers. In a second step, the retailers for their part try to find the optimal retail margins m_j and local advertising expenditures a_j within the framework previously determined by the manufacturer. Like in Sect. 5.3.1, the retailer duopoly is assumed to act in Horizontal Nash competition.

Hence, we first have to calculate the retailers' response functions, which can be derived from decision problem (5.12) of the previous section:

$$m_j = \frac{2\alpha_j\beta + \alpha_{3-j}\epsilon + (-2\beta^2 + \beta\epsilon + \epsilon^2)w}{(2\beta - \epsilon)(2\beta + \epsilon)} \tag{5.21}$$

$$a_j = \frac{\beta^2 k_r^2 \left[2\alpha_j\beta + \alpha_{3-j}\epsilon + (-2\beta^2 + \beta\epsilon + \epsilon^2)w\right]^4}{4(2\beta - \epsilon)^4(2\beta + \epsilon)^4(1-t)^2}. \tag{5.22}$$

Eqs. (5.21) and (5.22) are now used as constraints in the manufacturer's decision problem:

$$\begin{aligned}
\text{Max} \quad \Pi_{\mathrm{m}} &= w \sum_{j=1}^{2} \left[\alpha_j - \beta(w + m_j) + \epsilon(w + m_{3-j})\right] \left(k_{\mathrm{r}} \sum_{j=1}^{2} \sqrt{a_j} + k_{\mathrm{m}} \sqrt{A} \right) \\
&\quad - t \sum_{j=1}^{2} a_j - A \\
\text{s.t.} \quad m_j &= \frac{2\alpha_j \beta + \alpha_{3-j}\epsilon + (-2\beta^2 + \beta\epsilon + \epsilon^2) w}{(2\beta - \epsilon)(2\beta + \epsilon)} \\
a_j &= \frac{\beta^2 k_{\mathrm{r}}^2 \left[2\alpha_j \beta + \alpha_{3-j}\epsilon + (-2\beta^2 + \beta\epsilon + \epsilon^2) w\right]^4}{4(2\beta - \epsilon)^4 (2\beta + \epsilon)^4 (1 - t)^2} \\
& w, A > 0, \text{ and } 0 \leq t < 1. \qquad (5.23)
\end{aligned}$$

However, this problem can only be solved analytically for $\alpha_1 = \alpha_2 = \alpha$, i.e., when both retail-channels have the same market size Λ_j. With this simplification, setting the first order partial derivatives $\partial \Pi_{\mathrm{m}} / \partial w$, $\partial \Pi_{\mathrm{m}} / \partial A$, and $\partial \Pi_{\mathrm{m}} / \partial t$ to zero leads to the following solution of the Manufacturer Stackelberg–Horizontal Nash game:

Proposition 5.2 *If there is an asymmetric inter-echelon relationship, where the manufacturer holds the channel-leadership, and a symmetric and non-cooperative intra-echelon relationship between the channel members, this situation can be solved by a Manufacturer Stackelberg–Horizontal Nash equilibrium with*

(i) $m_1 = m_2 = \dfrac{\alpha - (\beta - \epsilon) w}{2\beta - \epsilon}$,

(ii) $A = \dfrac{\beta^2 k_{\mathrm{m}}^2 w^2 \left[\alpha - (\beta - \epsilon) w\right]^2}{(2\beta - \epsilon)^2}$, *and* $a_1 = a_2 = \dfrac{\beta^2 k_{\mathrm{r}}^2 \left[\alpha - (\beta - \epsilon) w\right]^4}{4(2\beta - \epsilon)^4 (1 - t)^2}$,

while the optimal wholesale price w and cooperative advertising participation rate t have to be determined by the following solution procedure:

Step 1: Calculate the possible solution candidates $\tilde{w}_l$ $(l = 1, \ldots, 4)$:

$$\left.\begin{aligned} \tilde{w}_1 &= \tfrac{-y + \sqrt{y^2 - 4xz}}{2x} \\ \tilde{w}_2 &= \tfrac{-y - \sqrt{y^2 - 4xz}}{2x} \end{aligned}\right\} \text{with} \left\{\begin{aligned} x &= k_{\mathrm{r}}^2(-49\beta^3 + 91\beta^2\epsilon - 51\beta\epsilon^2 + 9\epsilon^3) \\ &\qquad + 8k_{\mathrm{m}}^2(-4\beta^3 + 8\beta^2\epsilon - 5\beta\epsilon^2 + \epsilon^3) \\ y &= 2\alpha\beta k_{\mathrm{r}}^2(7\beta - 3\epsilon) + 4\alpha k_{\mathrm{m}}^2(2\beta - \epsilon)^2 \\ z &= \alpha^2 k_{\mathrm{r}}^2(3\beta - \epsilon) \end{aligned}\right.$$

$$\left.\begin{aligned} \tilde{w}_3 &= \tfrac{-y' + \sqrt{y'^2 - 4x'z'}}{2x'} \\ \tilde{w}_4 &= \tfrac{-y' - \sqrt{y'^2 - 4x'z'}}{2x'} \end{aligned}\right\} \text{with} \left\{\begin{aligned} x' &= 4k_{\mathrm{r}}^2(\beta - \epsilon)^2 + 2k_{\mathrm{m}}^2(-2\beta + \epsilon)(\beta - \epsilon) \\ y' &= 5\alpha k_{\mathrm{r}}^2(-\beta + \epsilon) + \alpha k_{\mathrm{m}}^2(2\beta - \epsilon) \\ z' &= \alpha^2 k_{\mathrm{r}}^2. \end{aligned}\right.$$

Step 2: Determine the set of feasible wholesale prices F with
$F = \{\tilde{w}_l | \tilde{w}_l \geq \alpha/(9\beta - 5\epsilon) \text{ for } l = 1, 2; 0 < \tilde{w}_l < \alpha/(9\beta - 5\epsilon) \text{ for } l = 3, 4\}$.

Step 3: Assign the associated cooperative advertising participation rate to all wholesale prices $\tilde{w}_l \in F$ with $t_l = (9\beta w - 5\epsilon w - \alpha)/(7\beta w - 3\epsilon w + \alpha)$ for $l = 1, 2$ and $t_l = 0$ for $l = 3, 4$.

Step 4: Find optimal solution $(w^*, t^*) = \{(\tilde{w}_l \in F, t_l) | (\tilde{w}_l, t_l) = \arg\max \Pi_\text{m}\}$.

Proof of Proposition 5.2 The **retailers' decision problems** in a Manufacturer Stackelberg–Horizontal Nash game are identical to (5.12) and have the solutions stated in Eqs. (5.19) and (5.20). These expressions can be rearranged to

$$m_j = \frac{2\alpha_j \beta + \alpha_{3-j}\epsilon + (-2\beta^2 + \beta\epsilon + \epsilon^2)w}{(2\beta - \epsilon)(2\beta + \epsilon)} \tag{5.24}$$

$$a_j = \frac{\beta^2 k_\text{r}^2 \left[2\alpha_j \beta + \alpha_{3-j}\epsilon + (-2\beta^2 + \beta\epsilon + \epsilon^2)w\right]^4}{4(2\beta - \epsilon)^4(2\beta + \epsilon)^4(1 - t)^2}. \tag{5.25}$$

Constituting the constraints of the **manufacturer's decision problem** (see (5.23)), these response functions have to be inserted into the manufacturer's profit function. In order to reduce the complexity of this problem, we set $\Lambda_1 = \Lambda_2 = \Lambda$, which leads to $\alpha_1 = \alpha_2 = \alpha$. Hence, we can rewrite Eqs. (5.24) and (5.25) as follows:

$$m_1 = m_2 = \frac{\alpha - (\beta - \epsilon)w}{2\beta - \epsilon} \tag{5.26}$$

$$a_1 = a_2 = \frac{\beta^2 k_\text{r}^2 \left[\alpha - (\beta - \epsilon)w\right]^4}{4(2\beta - \epsilon)^4(1 - t)^2}. \tag{5.27}$$

Inserting these equations into the profit function stated in (5.23), we get:

$$\begin{aligned}\Pi_\text{m} =& \frac{2\beta w \left[\alpha - (\beta - \epsilon)w\right]}{2\beta - \epsilon} \left\{ \frac{\beta k_\text{r}^2 \left[\alpha - (\beta - \epsilon)w\right]^2}{(2\beta - \epsilon)^2(1 - t)} + k_\text{m}\sqrt{A} \right\} \\ &- \frac{\beta^2 k_\text{r}^2 \left[\alpha - (\beta - \epsilon)w\right]^4 t}{2(2\beta - \epsilon)^4(1 - t)^2} - A.\end{aligned} \tag{5.28}$$

By setting the first order partial derivative $\partial\Pi_\text{m}/\partial A$ to zero,

$$\frac{\partial \Pi_\text{m}}{\partial A} = \frac{\beta k_\text{m} w \left[\alpha - (\beta - \epsilon)w\right]}{(2\beta - \epsilon)\sqrt{A}} - 1 = 0, \tag{5.29}$$

we can determine the optimal global advertising expenditures as a function of w:

$$A = \frac{\beta^2 k_\text{m}^2 w^2 \left[\alpha - (\beta - \epsilon)w\right]^2}{(2\beta - \epsilon)^2}. \tag{5.30}$$

Setting the first order partial derivative $\partial\Pi_{\mathrm{m}}/\partial t$ to zero,

$$\frac{\partial\Pi_{\mathrm{m}}}{\partial t}=\frac{2\beta^2k_{\mathrm{r}}^2w\left[\alpha-(\beta-\epsilon)w\right]^3}{(2\beta-\epsilon)^3(1-t)^2}-\frac{\beta^2k_{\mathrm{r}}^2\left[\alpha-(\beta-\epsilon)w\right]^4(1+t)}{2(2\beta-\epsilon)^4(1-t)^3}=0, \tag{5.31}$$

leads us to

$$t=\frac{9\beta w-5\epsilon w-\alpha}{7\beta w-3\epsilon w+\alpha}. \tag{5.32}$$

As described in Sect. 5.2, the participation rate is only defined within $0\leq t<1$. However, Eq. (5.32) can take negative values for $w<\alpha/(9\beta-5\epsilon)$. In this case, we have to set $t=0$ to avoid mathematical inconsistencies. The first order partial derivative $\partial\Pi_{\mathrm{m}}/\partial w$ is

$$\begin{aligned}\frac{\partial\Pi_{\mathrm{m}}}{\partial w}=&\frac{2\alpha\beta-4\beta(\beta-\epsilon)w}{2\beta-\epsilon}\left\{\frac{\beta k_{\mathrm{r}}^2\left[\alpha-(\beta-\epsilon)w\right]^2}{(2\beta-\epsilon)^2(1-t)}+k_{\mathrm{m}}\sqrt{A}\right\}\\&-\frac{4\beta^2k_{\mathrm{r}}^2(\beta-\epsilon)w\left[\alpha-(\beta-\epsilon)w\right]^2}{(2\beta-\epsilon)^3(1-t)}+\frac{2\beta^2k_{\mathrm{r}}^2(\beta-\epsilon)\left[\alpha-(\beta-\epsilon)w\right]^3t}{(2\beta-\epsilon)^4(1-t)^2}\end{aligned} \tag{5.33}$$

and is also set to zero. This equation can be simplified by inserting Eq. (5.30):

$$\begin{aligned}&\{(2\beta-\epsilon)\left[\alpha-(\beta-\epsilon)w\right]\left[\alpha-2(\beta-\epsilon)w\right]\\&\quad-2(\beta-\epsilon)(2\beta-\epsilon)w\left[\alpha-(\beta-\epsilon)w\right]\}k_{\mathrm{r}}^2(1-t)\\&\quad+k_{\mathrm{r}}^2(\beta-\epsilon)\left[\alpha-(\beta-\epsilon)w\right]^2t-k_{\mathrm{m}}^2(2\beta-\epsilon)^2w\left[\alpha-2(\beta-\epsilon)w\right](1-t)^2=0.\end{aligned} \tag{5.34}$$

Due to the non-negativity restriction of t, we now have to conduct a case-by-case analysis. For $w\geq\alpha/(9\beta-5\epsilon)$, we insert Eq. (5.32) into Eq. (5.34):

$$\begin{aligned}&w^2\left[k_{\mathrm{r}}^2\left(-49\beta^3+91\beta^2\epsilon-51\beta\epsilon^2+9\epsilon^3\right)+8k_{\mathrm{m}}^2\left(-4\beta^3+8\beta^2\epsilon-5\beta\epsilon^2+\epsilon^3\right)\right]\\&+w\left[2\alpha\beta k_{\mathrm{r}}^2(7\beta-3\epsilon)+4\alpha k_{\mathrm{m}}^2(2\beta-\epsilon)^2\right]+\alpha^2k_{\mathrm{r}}^2(3\beta-\epsilon)=0.\end{aligned} \tag{5.35}$$

The solutions of this expression are given as $\tilde{w}_1$ and $\tilde{w}_2$ in Step 1 of the solution procedure stated in Proposition 5.2. For $w<\alpha/(9\beta-5\epsilon)$, we insert $t=0$ into Eq. (5.34):

$$\begin{aligned}&w^2\left[4k_{\mathrm{r}}^2(\beta-\epsilon)^2+2k_{\mathrm{m}}^2(-2\beta+\epsilon)(\beta-\epsilon)\right]\\&+w\left[5\alpha k_{\mathrm{r}}^2(-\beta+\epsilon)+\alpha k_{\mathrm{m}}^2(2\beta-\epsilon)\right]+\alpha^2k_{\mathrm{r}}^2=0.\end{aligned} \tag{5.36}$$

The solutions of this expression are given as $\tilde{w}_3$ and $\tilde{w}_4$ in Step 1 of the solution procedure stated in Proposition 5.2. This completes the proof of Proposition 5.2.

□

5.4 Interpretation

5.4.1 General Case with Specific Market Size Parameters

We determined the solutions of two games—the Vertical Nash–Horizontal Nash game as well as the Manufacturer Stackelberg–Horizontal Nash game—which correspond to a symmetric respectively asymmetric inter-echelon power distribution. The analytical solutions depend on five parameters, which describe customer behavior: The three parameters α_j, β, and ϵ can be traced back to Λ_j, B, and Θ, which belong to the customers' utility function defined in Eq. (5.4). In addition, the two parameters k_{m} and k_{r} describe the effectiveness of global and local advertising expenditures in generating sales. Previous studies revealed that the ratio between these two effectiveness parameters in particular has an influence on the players' decisions.[21] Hence, we introduce a ratio parameter k with $k = k_{\mathrm{m}}/k_{\mathrm{r}}$ for the sake of simplicity in further discussion.

We first consider the general case $\Lambda_1 \neq \Lambda_2$, for which we identified the channel members' pricing and advertising decisions in the Vertical Nash–Horizontal Nash equilibrium. Parameter Λ_j denotes the market size of retail channel j and goes into Parameter α_j with $\partial\alpha_j/\partial\Lambda_j > 0$ and $\partial\alpha_j/\partial\Lambda_{3-j} = 0$. Hence, we derive the following Proposition:

Proposition 5.3 *If $B > 2\Theta$ holds, an asymmetric market size parameter Λ_i has the following effects on the retailers' decisions in a Vertical Nash–Horizontal Nash equilibrium:*

(i) $\dfrac{\partial m_j}{\partial \Lambda_j} > 0$ *and* $\dfrac{\partial m_j}{\partial \Lambda_{3-j}} < 0.$

(ii) $\dfrac{\partial a_j}{\partial \Lambda_j} > 0$ *and* $\dfrac{\partial a_j}{\partial \Lambda_{3-j}} > 0.$

(iii) $\dfrac{\partial \Pi_{\mathrm{r}j}}{\partial \Lambda_j} > 0$ *and* $\dfrac{\partial \Pi_{\mathrm{r}j}}{\partial \Lambda_{3-j}} > 0.$

Proof of Proposition 5.3 As defined in Sect. 5.2, α_j is a function of Λ_j with

$$\frac{\partial \alpha_j}{\partial \Lambda_j} = \frac{1}{B+\Theta} > 0 \quad \text{and} \quad \frac{\partial \alpha_j}{\partial \Lambda_{3-j}} = 0. \tag{5.37}$$

[21] See SeyedEsfahani et al. (2011): *Vertical co-op advertising*, pp. 266–269, and Aust and Buscher (2012): *Vertical cooperative advertising*, p. 477.

Due to the positive first order partial derivative $\partial\alpha_j/\partial\Lambda_j$ in combination with the chain rule $\mathrm{d}f_1(f_2(x))/\mathrm{d}x = \mathrm{d}f_1(f_2(x))/\mathrm{d}f_2(x)\cdot\mathrm{d}f_2(x)/\mathrm{d}x$, the first order partial derivative with respect to α_j has the same prefix as the first order partial derivative with respect to Λ_j. Hence, one can easily make the conclusions given in Part (i) and Part (ii) of Proposition 5.3 with the following first order partial derivatives:

$$\frac{\partial m_j}{\partial \alpha_j} = \frac{5\beta}{2(3\beta-\epsilon)(2\beta+\epsilon)} > 0 \tag{5.38}$$

$$\frac{\partial m_j}{\partial \alpha_{3-j}} = \frac{-\beta+2\epsilon}{2(3\beta-\epsilon)(2\beta+\epsilon)} < 0, \quad \text{for } \beta > 2\epsilon \;\Leftrightarrow\; B > 2\Theta \tag{5.39}$$

$$\frac{\partial a_j}{\partial \alpha_j} = \frac{5\beta^3 k_{\mathrm{r}}^2(5\alpha_j\beta+\alpha_{3-j}\beta+2\alpha_{3-j}\epsilon)^3}{16(3\beta-\epsilon)^4(2\beta+\epsilon)^4} > 0 \tag{5.40}$$

$$\frac{\partial a_j}{\partial \alpha_{3-j}} = \frac{\beta^2 k_{\mathrm{r}}^2(5\alpha_j\beta+\alpha_{3-j}\beta+2\alpha_{3-j}\epsilon)^3(\beta+2\epsilon)}{16(3\beta-\epsilon)^4(2\beta+\epsilon)^4} > 0. \tag{5.41}$$

Due to the complexity of the resulting first order derivatives $\partial\Pi_{\mathrm{r}j}/\partial\alpha_j$ and $\partial\Pi_{\mathrm{r}j}/\partial\alpha_{3-j}$, we are not able to prove Part (iii) of Proposition 5.3 analytically. Instead of that, we computed a numerical study with 3,000,000 randomly generated sets of parameters within the range $10 \le \alpha_j \le 30$, $0.1 \le \beta, \epsilon, k_{\mathrm{m}}, k_{\mathrm{r}} \le 10$ and could thereby show numerically that $\partial\Pi_{\mathrm{r}j}/\partial\alpha_j > 0$ and $\partial\Pi_{\mathrm{r}j}/\partial\alpha_{3-j} > 0$ holds for each considered combination of parameters—except 18 cases with $\beta \approx 0.1$ and $\epsilon > 5$, which violate the condition $\beta > \epsilon$ resulting from $B > \Theta$ given in Sect. 5.2, though. Hence, we are confident that the given inequalities hold for feasible parameter combinations. This completes the proof of Proposition 5.3. □

Part (i) of Proposition 5.3 reveals that a retailer will achieve a higher margin if the market size of his channel grows, while his margins will decrease if the competing channel's market size is higher. In contrast to that, we can see from Part (ii) and (iii) of Proposition 5.3 that the increase of any market size parameter is followed by higher advertising expenditures and profits of both retailers. As the manufacturer supplies the demand of both retail channels, his pricing and advertising decisions do not depend on specific market size parameters, but rather on the total demand of both channels ($\alpha_1 + \alpha_2$).

5.4.2 Margins and Prices

After this general case, we now consider the special case $\Lambda_1 = \Lambda_2 = \Lambda$, for which we proposed a solution procedure to determine a Manufacturer Stackelberg–Horizontal Nash equilibrium, where the manufacturer holds the channel-leadership. Table 5.2 gives the framework on which the following analyses of the two equilibria

Table 5.2 Framework of numerical analysis

Parameter	Value	Parameter	Value
Market size Λ	10	Effectiveness of global advertising k_m	1
Saturation effect B	2	Advertising effectiveness ratio k	0.5–2
Channel substitutability Θ	0.1–0.9		

Table 5.3 Sensitivity analysis of parameter Θ with $k = 1$

Θ	Game	w	m_j	p_j	A	a_j	t	Π_m	Π_{rj}	Π_{m+2r}
0.1	Vertical Nash	3.39	3.22	6.61	29.94	33.10	0.00	155.87	55.16	266.19
	Stackelberg	4.54	2.66	7.20	36.63	48.17	0.74	132.97	58.46	249.89
0.5	Vertical Nash	3.64	2.73	6.36	27.98	17.13	0.00	115.53	36.69	188.91
	Stackelberg	4.60	2.31	6.91	32.24	40.86	0.78	113.95	43.62	201.18
0.9	Vertical Nash	3.92	2.16	6.08	28.12	8.36	0.00	89.46	23.97	137.41
	Stackelberg	4.68	1.89	6.56	30.67	37.18	0.82	105.04	32.87	170.78

Table 5.4 Sensitivity analysis of parameter k with $\Theta = 0.5$

k	Game	w	m_j	p_j	A	a_j	t	Π_m	Π_{rj}	Π_{m+2r}
0.5	Vertical Nash	3.64	2.73	6.36	27.98	68.50	0.00	378.20	83.81	545.82
	Stackelberg	4.47	2.37	6.84	31.91	163.79	0.77	359.50	132.13	623.77
1.0	Vertical Nash	3.64	2.73	6.36	27.98	17.13	0.00	115.53	36.69	188.91
	Stackelberg	4.60	2.31	6.91	32.24	40.86	0.78	113.95	43.62	201.18
2.0	Vertical Nash	3.64	2.73	6.36	27.98	4.28	0.00	49.87	24.91	99.68
	Stackelberg	4.80	2.23	7.03	32.55	10.14	0.79	52.82	21.43	95.69

are based on. Numerical examples for the effects of parameters Θ and k are listed in Tables 5.3 and 5.4.

First, we consider the manufacturer's wholesale price w, which is in each considered case higher when the manufacturer obtains the Stackelberg leadership. As the retailers likewise achieve lower margins m_j when they act as followers, we can observe that the manufacturer can use his channel power to effect a shift of margin from his retailers to himself. In total, we can state—with respect to these examples—that customers are always better off in a Vertical Nash equilibrium, as the retail prices are always lower. This is consistent with previous research.[22]

Furthermore, Table 5.3 shows that an increase in retail competition (parameter Θ) causes a higher wholesale price but lower retailer margins. As the reduction of the retailers' margins, which is induced by competition, is greater than the increase of the manufacturer's wholesale price, the retail prices p_1 and p_2 will drop, too. Hence, customers may benefit from retail competition through lower prices, which

[22]See Zhang et al. (2012): *Pricing decisions*, p. 528.

shows that the use of parameter Θ instead of ϵ as a measure of competition produces logically consistent results.[23]

Table 5.4 reveals that margins and prices are set independently of ratio parameter k when we assume a Vertical Nash game between the echelons, while an increment in k (i.e., an increase of effectiveness of global advertising k_m with respect to effectiveness of local advertising k_r) slightly raises w and p_j and lowers m_j in a Manufacturer Stackelberg game.

5.4.3 Advertising Expenditures and Participation Rate

As introduced in Sect. 5.2, we distinguish global **advertising expenditures** A of the manufacturer and local advertising expenditures a_j of retailer j. Table 5.3 shows that an increase of retail competition is followed by less advertising of both echelons in a Manufacturer Stackelberg game, while this effect is only visible at the retailers' advertising in a Vertical Nash game. Furthermore, advertising expenditures in a Manufacturer Stackelberg game are always higher than in a Vertical Nash game.

Figure 5.2 analyzes which echelon has higher advertising investments and, hence, confronts manufacturer's advertising A with the advertising of both retailers $a_1 + a_2$. One will observe that—in both considered games—the manufacturer advertises more than the retailer duopoly for large values of k, i.e., for $k_m > k_r$. The exact border also depends on the competition parameter Θ, whereupon low competition instead leads to stronger advertising activity for the manufacturer.

Concerning the **participation rate**, we explained in Sect. 5.3.1 that the manufacturer is not willing to participate in the retailers' advertising expenditures in a Vertical Nash game, i.e., when he has no precognition of the retailers' decisions. In contrast to that, he takes a share of about three quarters of the local advertising

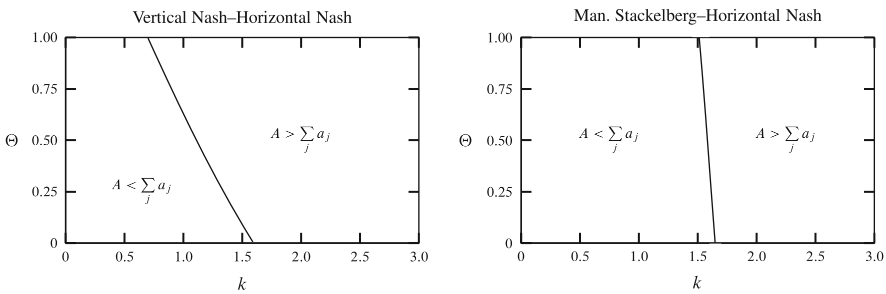

Fig. 5.2 Comparison of the echelons' advertising expenditures

[23]See Yang and Zhou (2006): *Two-echelon supply chain models*, p. 113, for a similar analysis at the basis of parameter ϵ.

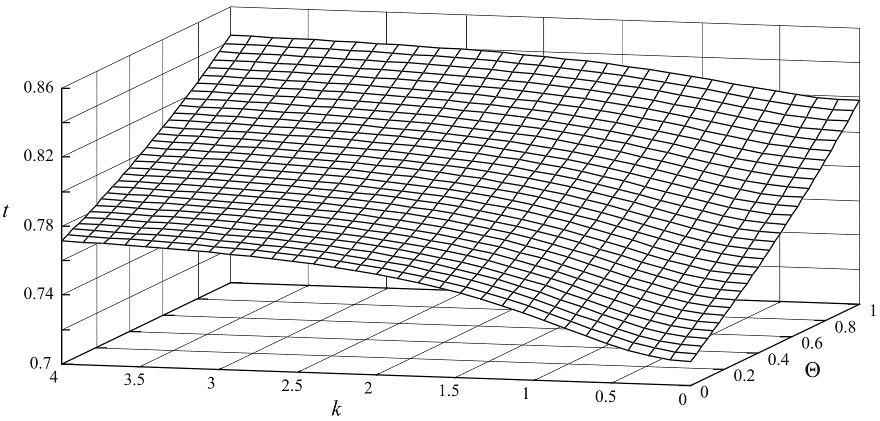

Fig. 5.3 Participation rate t

expenditures as a channel-leader (see Tables 5.3 and 5.4). The effects of retail competition on participation rate t are illustrated in Fig. 5.3. We can see that an increase of competition parameter Θ is accompanied by a higher participation rate, which will nearly reach 85 % if the ratio parameter k takes sufficiently big values. This strong involvement of the manufacturer may be the cause for the higher local advertising expenditures in Manufacturer Stackelberg game, which we stated in Sect. 5.4.3.

5.4.4 Profits

We now consider the resulting profits of the manufacturer, of each retailer, and of the entire supply chain. The examples in Table 5.3 indicate that higher retail competition also harms the manufacturer, as it is followed by less profit. This effect is stronger under symmetric inter-echelon power distribution than under channel-leadership by the manufacturer. Figure 5.4 compares the resulting profit in Vertical Nash (VN) as well as in Manufacturer Stackelberg (MS) game. One can see that it is not always beneficial to the manufacturer to be the Stackelberg-leader, as the Vertical Nash equilibrium yields higher profits in the lower left of the considered area, i.e., for small values of Θ and k. That means, if retail competition is low and the effectiveness of global advertising is not too big compared to the effectiveness of local advertising, the manufacturer will be better off when he does not exert the channel-leadership.

As we concentrate on the special case of identical market size parameters of both channels ($\Lambda_1 = \Lambda_2$), both retailers will receive the same profit, i.e., $\Pi_{r1} = \Pi_{r2}$. Similar to the manufacturer's profit, our analysis shows that competition has harmful effects on the retailers' profits, which clearly coincides with intuition. The results

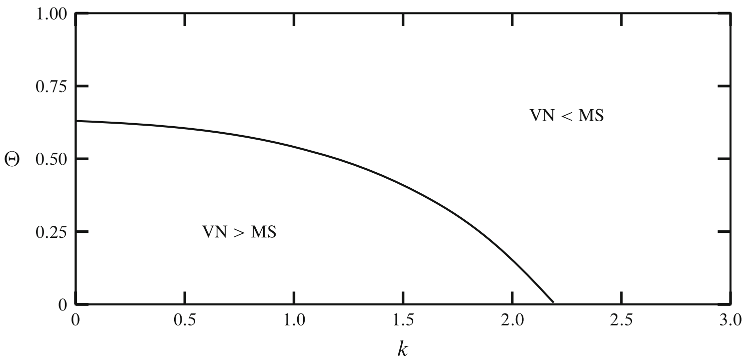

Fig. 5.4 Manufacturer's profit Π_m

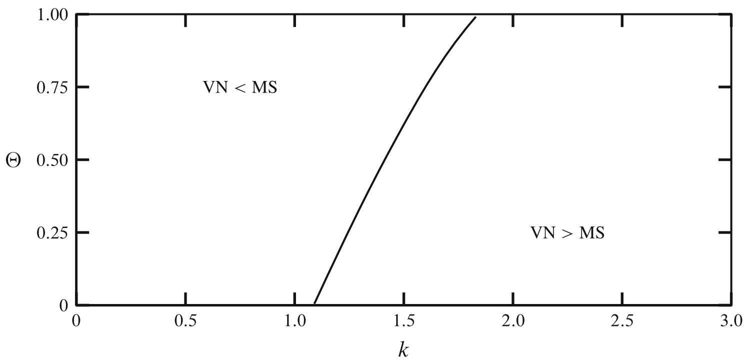

Fig. 5.5 Retailers' profits Π_{r1} and Π_{r2}

of our computational comparison of the two considered games in Fig. 5.5 also reveal that the retailers have to be aware of the underlying market characteristics before deciding whether or not to accept followership. We can observe that the retailers should accept the manufacturer-leadership on the left side of the area under consideration, i.e., when k_m is not too high in comparison to k_r. Interestingly, the intensity of retail competition has only marginal impact on this distribution.

As a consequence of the harmful effects of competition on each players' profit, one can also observe a diminishing total profit for the entire supply chain Π_{m+2r} (see Table 5.3). The comparison of Vertical Nash and Manufacturer Stackelberg equilibrium in Fig. 5.6 furthermore shows that—from the point of view of the total profit Π_{m+2r}—a Vertical Nash game is basically more advantageous than a Manufacturer Stackelberg game when high competition between the retailers can be observed. In the case of low competition, this holds true only for small values of k. This altogether proves the need for an intensive examination of the underlying market characteristics before deciding to either exert or to accept a leadership. As we showed, channel leadership does not always yield the highest profits for the leader and, correspondingly, can also benefit the follower.

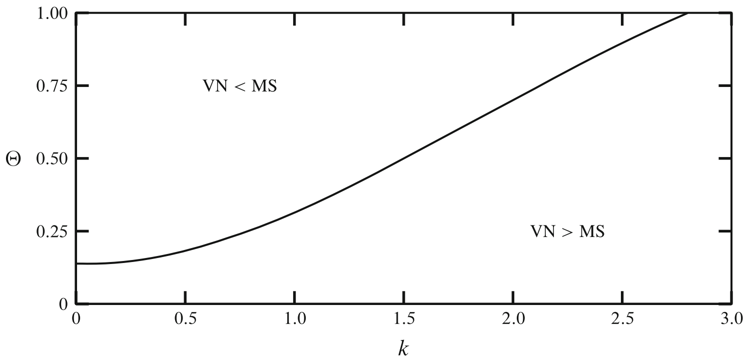

Fig. 5.6 Total supply chain profit Π_{m+2r}

5.5 Managerial Implications and Conclusions

In this paper we consider a supply chain consisting of one manufacturer and two retailers, which offer substitutable products to customers and therefore act under competition. Each player can decide on his margin and advertising expenditures, while customer demand is influenced by retail price as well as local and global advertising. As the retailers' advertising budget may be too low to generate adequate sales, the manufacturer has the possibility to participate in the retailers' advertising within the framework of a cooperative advertising program. We applied two different game scenarios to this supply chain: first, a Vertical Nash–Horizontal Nash game, where all players act simultaneously and under an equal distribution of power within the supply chain; second, a Manufacturer Stackelberg–Horizontal Nash game, where the manufacturer obtains the channel leadership, while the intra-echelon competition of the retailers remains unmodified.

The main contributions of our research are as follows: To the best of our knowledge, this is the first static model which simultaneously analyzes cooperative advertising and pricing of a one-manufacturer two-retailer supply chain, while previous literature either focused on bilateral monopolies and, hence, did not consider the effects of retail competition, or assumed margins to be given exogenously.

We derive the price-dependent component of the demand function from the customers' utility function. This approach allows us to obtain a competition parameter which produces logically consistent results instead of the commonly used cross-price sensitivity parameter of a linear demand function. Thereby, we show that retail competition has harmful effects on each players' profit, but also reduces retail prices and is therefore beneficial for customers. As a consequence of the lower profits, in most cases the players will reduce their advertising expenditures, too. Nevertheless, the manufacturer will set a higher advertising participation rate when the intensity of retail competition grows.

Further, our study reveals that it is not always advantageous to the manufacturer to act as a channel leader, because there exist parameter combinations where he can receive higher profits in a Vertical Nash game. Likewise, the retailers can be better off if they accept followership in some situations. This underlines the need of an intense examination of the underlying market characteristics.

However, this is only a first approach to modeling retail competition in a cooperative advertising framework, which certainly underlies some limitations. Future research could therefore extend our approach by additional game settings, e.g., a Retailer Stackelberg game, where the retailer-duopoly obtains the channel-leadership and the manufacturer acts as a follower. Furthermore, an intra-echelon Stackelberg game between the two retailers would render possible the analysis of the effects of market power on the determination of prices and advertising. Lastly, the introduction of a second manufacturer may reveal some new interesting aspects. However, even in the actual state of the model, we had to assume identical demand parameters for both retailers in order to solve the Retailer Stackelberg equilibrium analytically. The greater complexity of the model might necessitate the application of non-analytical solution methods like meta-heuristics.

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Chapter 6
A Manufacturer-Retailer Supply Chain with Fuzzy Customer Demand: A Vertical Cooperative Advertising and Pricing Model

Abstract In this paper, we apply fuzzy set theory to a single-manufacturer single-retailer supply chain, where both players try to determine their optimal pricing and advertising decisions. The interaction between manufacturer and retailer is analyzed by means of a Stackelberg game. Moreover, a vertical cooperative advertising program is considered, which represents a financial agreement where the manufacturer offers to share a certain fraction of his retailer's advertising expenditures. Even though this topic gained substantial interest in recent years' operations research literature and studies reveal that results strongly depend on demand parameters, most analyses are limited to deterministic model formulations. Here, fuzzy set theory has the advantage that it is not only able to incorporate the uncertainty of demand parameters into analysis. Furthermore, it enables us to take into consideration the experience of decision makers, which is often not expressed numerically, but rather in vague linguistic terms.

6.1 Introduction

Vertical cooperative advertising programs are financial agreements between manufacturers and their retailers on the sharing of advertising expenditures.[1] In most cases, this financial assistance is offered by manufacturers, who thereby intend to increase the retailers' advertising in order to generate sales.[2] Reasons for this form of cooperation can be manifold: Besides cheaper access to local media or better knowledge of local markets, mainly the different effects of manufacturers' and retailers' advertising are mentioned. That means, manufacturers use their advertising campaigns primarily to build up brand image, while retailers' advertising aims on generating immediate sales.[3]

Therefore, vertical cooperative advertising programs are very common in practice. Empirical data clearly shows an increasing trend, e.g., from $15 billion which

[1]Cf. Crimmins (1984): *Cooperative advertising*, p. 2

[2]Cf. Somers et al. (1990): *Cooperative advertising expenditures*, p. 36.

[3]Cf. Hutchins (1953): *Cooperative advertising*, pp. 7 et seq., and Young and Greyser (1983): *Managing cooperative advertising*, pp. 29–37.

G. Aust, *Vertical Cooperative Advertising in Supply Chain Management*,
Contributions to Management Science, DOI 10.1007/978-3-319-11626-6_6

were spent for such programs in the United States of America in 2000 up to \$50 billion in 2008.[4] However, the study of Nagler (2006) reveals that manufacturers mostly set their participation rates to 50 % or 100 % instead of conducting an appropriate analysis on the optimum percentage.[5]

This gap between importance and theoretical background in approaching cooperative advertising has motivated many researchers to study related questions, especially the determination of advertising expenditures and prices of the different echelons of a supply chain. Thereby, the findings of the different analyses reveal that results as optimal participation rate, prices, spending on advertising, or the profit split within the supply chain strongly depend on the underlying demand function as well as on the assumed parameters.[6]

However, as a result of uncertain customer behavior, demand and advertising effectiveness parameters are often unknown in practice. Stochastic models based on probability distributions may be of avail in some cases, but they require extensive historical data, which is often not available to decision makers. At this point, the fuzzy set theory proposed by Zadeh (1965) may be a promising instrument,[7] as it is able to incorporate the experience of decision makers, which is usually expressed in linguistic terms like *low*, *medium*, or *high* price sensitivity. Hence, our scope is to propose how fuzzy set theory can be applied to vertical cooperative advertising models.

The remainder of this article is organized as follows: In Sect. 6.2, we first introduce some basic concepts of fuzzy set theory. In the next section, we develop a mathematical model of a single-manufacturer single-retailer supply chain with fuzzy demand and advertising effectiveness parameters (Sect. 6.3.1) and apply a Manufacturer Stackelberg game to that model (Sect. 6.3.2). As a result, we derive closed-form solutions for the players' prices, advertising expenditures, and profits, which are further analyzed in Sect. 6.4. The paper is concluded with a short summary of the main findings and some open topics for future research.

6.2 Fuzzy Set Theory

In this work, we will only give a brief introduction into fuzzy set theory and calculation rules for fuzzy variables which are necessary for the following analysis.[8]

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[4] Cf. Nagler (2006): *Cooperative advertising participation rates*, p. 92, and He et al. (2012): *Co-op advertising*, p. 74.

[5] Cf. Nagler (2006): *Cooperative advertising participation rates*, p. 96.

[6] See Aust and Buscher (2011): *Werbungsbezogene Zusammenarbeit*, pp. 16–19.

[7] See Zadeh (1965): *Fuzzy sets*.

[8] For a more formal introduction and the relevant definitions and axioms, we refer the reader to Zadeh (1965): *Fuzzy sets* and Nahmias (1978): *Fuzzy variables* or to the comprehensive books Liu (2009): *Uncertain programming* and Liu (2013): *Uncertainty Theory*. A more summarized but still formal discussion can be found in Zhou et al. (2008): *Two-echelon supply chain games*, pp. 391–394.

Table 6.1 Calculation rules for fuzzy variables

Operation	φ-Pessimistic value	φ-Optimistic value
Scalar multiplication ($x > 0$)[a]	$(x\zeta)^{\mathrm{L}}_{\varphi} = x\zeta^{\mathrm{L}}_{\varphi}$	$(x\zeta)^{\mathrm{U}}_{\varphi} = x\zeta^{\mathrm{U}}_{\varphi}$
Scalar multiplication ($x < 0$)[a]	$(x\zeta)^{\mathrm{L}}_{\varphi} = x\zeta^{\mathrm{U}}_{\varphi}$	$(x\zeta)^{\mathrm{U}}_{\varphi} = x\zeta^{\mathrm{L}}_{\varphi}$
Addition[a]	$(\zeta + \eta)^{\mathrm{L}}_{\varphi} = \zeta^{\mathrm{L}}_{\varphi} + \eta^{\mathrm{L}}_{\varphi}$	$(\zeta + \eta)^{\mathrm{U}}_{\varphi} = \zeta^{\mathrm{U}}_{\varphi} + \eta^{\mathrm{U}}_{\varphi}$
Multiplication[b]	$(\zeta \cdot \eta)^{\mathrm{L}}_{\varphi} = \zeta^{\mathrm{L}}_{\varphi} \cdot \eta^{\mathrm{L}}_{\varphi}$	$(\zeta \cdot \eta)^{\mathrm{U}}_{\varphi} = \zeta^{\mathrm{U}}_{\varphi} \cdot \eta^{\mathrm{U}}_{\varphi}$
$f(\cdot)$ with $f'(\cdot) > 0$[c]	$(f(\zeta))^{\mathrm{L}}_{\varphi} = f(\zeta^{\mathrm{L}}_{\varphi})$	$(f(\zeta))^{\mathrm{U}}_{\varphi} = f(\zeta^{\mathrm{U}}_{\varphi})$
$f(\cdot)$ with $f'(\cdot) < 0$[c]	$(f(\zeta))^{\mathrm{L}}_{\varphi} = f(\zeta^{\mathrm{U}}_{\varphi})$	$(f(\zeta))^{\mathrm{U}}_{\varphi} = f(\zeta^{\mathrm{L}}_{\varphi})$
Expected value[d]	$\mathrm{E}[x\zeta + y\eta] = x\mathrm{E}[\zeta] + y\mathrm{E}[\eta]$	

[a] Cf. Liu and Liu (2003): *Expected value operator*, p. 201.
[b] Cf. Zhao et al. (2006): *Random fuzzy renewal process*, (cited in Zhou et al. (2008): *Two-echelon supply chain games*, p. 393).
[c] Cf. Zhou et al. (2008): *Two-echelon supply chain games*, p. 393.
[d] Cf. Liu and Liu (2003): *Expected value operator*, p. 204.

Let ζ and η be two independent and nonnegative fuzzy variables, $f(\cdot)$ a function, $\mathrm{Pos}\{\cdot\}$ a possibility measure of a certain event, and φ a possibility value with $0 < \varphi \leq 1$. According to Liu (2009), we can define the φ-pessimistic value $\zeta^{\mathrm{L}}_{\varphi}$ and the φ-optimistic value $\zeta^{\mathrm{U}}_{\varphi}$ of fuzzy variable ζ as follows[9]:

$$\zeta^{\mathrm{L}}_{\varphi} = \inf\{r \,|\, \mathrm{Pos}\{\zeta \leq r\} \geq \varphi\} \quad \text{and} \quad \zeta^{\mathrm{U}}_{\varphi} = \sup\{r \,|\, \mathrm{Pos}\{\zeta \geq r\} \geq \varphi\}. \tag{6.1}$$

Hence, the φ-pessimistic value $\zeta^{\mathrm{L}}_{\varphi}$ is the greatest lower bound that fuzzy variable ζ will reach with a possibility of φ, while the φ-optimistic value $\zeta^{\mathrm{U}}_{\varphi}$ is the least upper bound that ζ will reach with a possibility of φ. $\zeta^{\mathrm{L}}_{\varphi}$ and $\zeta^{\mathrm{U}}_{\varphi}$ can now be used to calculate the expected value $\mathrm{E}[\zeta]$ of fuzzy variable ζ[10]:

$$\mathrm{E}[\zeta] = \frac{1}{2}\int_0^1 \left(\zeta^{\mathrm{L}}_{\varphi} + \zeta^{\mathrm{U}}_{\varphi}\right) \mathrm{d}\varphi. \tag{6.2}$$

Table 6.1 gives an overview of calculation rules for φ-optimistic and φ-pessimistic values as well as for related expected values, which will be used later on during calculus. Thereby, x and y denote normal real-valued numbers, which are also called *crisp numbers* within the context of fuzzy set theory.[11]

After this consideration of general fuzzy variables, we turn our attention to triangular fuzzy variables, which are solely used in the following. Triangular fuzzy variables are of the shape $\tilde{\zeta} = (x, y, z)$ and consist of three crisp numbers

[9] Cf. Liu (2009): *Uncertain programming*, p. 33.

[10] Cf. Liu and Liu (2003): *Expected value operator*, p. 201.

[11] Cf. Liu (2013): *Uncertainty Theory*, p. 23.

$x < y < z$. According to Eq. (6.1), the φ-pessimistic and φ-optimistic values of a triangular fuzzy variable are[12]:

$$\zeta_{\varphi}^{L} = y\varphi + x(1-\varphi) \quad \text{and} \quad \zeta_{\varphi}^{U} = y\varphi + z(1-\varphi). \tag{6.3}$$

By means of Eq. (6.2), we can derive the following expression for the expected value of a triangular fuzzy variable:

$$\mathrm{E}[\tilde{\zeta}] = \frac{x + 2y + z}{4}. \tag{6.4}$$

6.3 A Manufacturer-Retailer Supply Chain Model with Fuzzy Customer Demand

6.3.1 Model Formulation

The first mathematical model on cooperative advertising in a manufacturer-retailer supply chain was proposed by Berger (1972).[13] In the following, many different models and extensions have been published, prevalently with game-theoretic analyses.[14] Although one can realize an increased interest in this field in the recent years, there are only few stochastic approaches,[15] while most authors consider deterministic models. However, to the best of our knowledge, no application of fuzzy set theory to a vertical cooperative advertising model exists yet. Therefore, we take on a deterministic model formulation recently published by Aust and Buscher (2012), which is simplified in order to ensure mathematical tractability, and transform the parameters of the demand function as well as the advertising effectiveness into fuzzy parameters.[16]

Similar approaches of applying fuzzy set theory to supply chain models, which are not related to cooperative advertising, can be found in, e.g., Zhou et al. (2008), who consider fuzzy demand and manufacturing cost in a two-echelon pricing game.[17] This model is further expanded to a manufacturer-duopoly[18] or a retailer-duopoly.[19]

[12]Cf. Zhao et al. (2012b): *Pricing decisions for substitutable products*, p. 410.

[13]See Berger (1972): *Vertical cooperative advertising*.

[14]We refer the reader to a recent review by Aust and Buscher (2014a): *Cooperative advertising models*.

[15]See Chen (2011): *Coordinating the ordering and advertising policies*, He et al. (2011): *Retail competition* and Tsao and Sheen (2012): *Promotion cost sharing*.

[16]See Aust and Buscher (2012): *Vertical cooperative advertising*.

[17]See Zhou et al. (2008): *Two-echelon supply chain games*.

[18]See Zhao et al. (2012b): *Pricing decisions for substitutable products*.

[19]See Zhao et al. (2012a): *Retail competition in a fuzzy environment.*

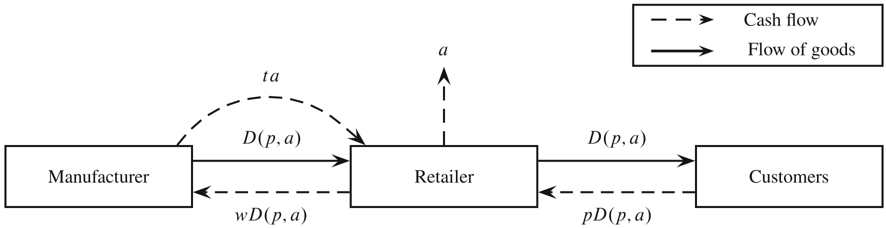

Fig. 6.1 Manufacturer-retailer supply chain

We consider a single-manufacturer single-retailer supply chain, which is illustrated in Fig. 6.1. This supply chain sells one product to the customer market, which demands a quantity D of the product. For each unit, customers pay a retail price p to the retailer, who, for his part, pays a wholesale price w to the manufacturer. The customer demand $D(p, a)$ depends both on the retail price p and on the retailer's local advertising expenditures a. Please note that we do not consider manufacturer's advertising expenditures in order to simplify our analysis.[20] However, the manufacturer has the possibility to participate in his retailer's advertising expenditures by means of a cooperative advertising program. Here, we assume that the manufacturer decides on a participation rate t, with $0 \leq t < 1$ (see Table 6.2 for a listing of symbols used in this article).

With respect to the cash flows in Fig. 6.1, we can set up the profit functions of the manufacturer (Π_{m}) and the retailer (Π_{r}):

$$\Pi_{\mathrm{m}} = wD(p, a) - ta \tag{6.5}$$

$$\Pi_{\mathrm{r}} = mD(p, a) - (1 - t)a. \tag{6.6}$$

Here, m denotes the retailer's margin, which can be calculated via

$$m = p - w. \tag{6.7}$$

As stated above, customer demand depends both on retail price p and advertising expenditures a. Thereby, one can distinguish a price-induced demand component $g(p)$ and an advertising-induced demand component $h(a)$. Following Kunter (2012) and Yan (2010), we assume a linear price demand function[21]

$$g(p) = \tilde{\alpha} - \tilde{\beta} p, \tag{6.8}$$

[20] See Karray and Zaccour (2006): *Co-op advertising* and Yang et al. (2013): *Cooperative advertising*. Though, the distinction between manufacturer and retailer advertising is a common assumption, which can be found in SeyedEsfahani et al. (2011): *Vertical co-op advertising* and Xie and Wei (2009): *Coordinating advertising*.

[21] Cf. Kunter (2012): *Coordination via cost and revenue sharing*, p. 479, and Yan (2010): *Cooperative advertising*, p. 512.

Table 6.2 List of symbols

Variables		Parameters/Functions	
m	Manufacturer's margin	$\tilde{\alpha}$	Base demand
w	Retailer's margin	$\tilde{\beta}$	Price sensitivity
a	Retailer's advertising expenditure	$\tilde{k}_{\mathrm{r}}$	Effectiveness of local advertising
t	Advertising participation rate		
Π	Profit	$h(\cdot)$	Price demand function
D	Demand quantity	$g(\cdot)$	Advertising demand function

where $\tilde{\alpha}$ and $\tilde{\beta}$ are fuzzy parameters. In detail, $\tilde{\alpha}$ describes the initial base demand, i.e., the customer demand which occurs for $p = 0$, while $\tilde{\beta}$ can be interpreted as customers' price sensitivity. In order to ensure a non-negative demand quantity, we set $\mathrm{Pos}(\{\tilde{\alpha} - \tilde{\beta} p < 0\}) = 0$.

Concerning advertising demand, we apply a square root function, which corresponds to the widely spread advertising saturation effect[22]:

$$h(a) = \tilde{k}_{\mathrm{r}} \sqrt{a}. \tag{6.9}$$

The fuzzy parameter $\tilde{k}_{\mathrm{r}}$ determines the effectiveness of advertising expenditures. We assume that advertising affects demand like a multiplicator.[23] With this multiplicative relationship between price and advertising demand, we can now formulate the extensive total demand function as well as the profit functions of both players:

$$D(p,a) = g(p)h(a) = (\tilde{\alpha} - \tilde{\beta} p)\tilde{k}_{\mathrm{r}} \sqrt{a} \tag{6.10}$$

$$\Pi_{\mathrm{m}}(w,t) = w\left[\tilde{\alpha} - \tilde{\beta}(w+m)\right]\tilde{k}_{\mathrm{r}} \sqrt{a} - ta \tag{6.11}$$

$$\Pi_{\mathrm{r}}(m,a) = m\left[\tilde{\alpha} - \tilde{\beta}(w+m)\right]\tilde{k}_{\mathrm{r}} \sqrt{a} - (1-t)a. \tag{6.12}$$

6.3.2 A Manufacturer Stackelberg Equilibrium

For our analysis of supply chain interaction, we use a Stackelberg game, where the manufacturer obtains the channel leadership, while the retailer acts as a follower. That means, the manufacturer has perfect knowledge of the retailer's reaction on his own decision and is therefore able to take this reaction into consideration when determining wholesale price and cooperative advertising participation rate.

[22] Cf. Kim and Staelin (1999): *Manufacturer allowances*, pp. 65 et seq., and Zhang and Xie (2012): *Cooperative advertising with multiple retailers*, p. 40.

[23] Cf. Thompson and Teng (1984): *Optimal pricing and advertising policies*, p. 151.

Mathematically, we first have to calculate the retailer's best response functions by solving the following **decision problem of the retailer**:

$$\begin{aligned} \text{Max} \quad & \mathrm{E}[\Pi_{\mathrm{r}}(m,a)] = E\left[m[\tilde{\alpha} - \tilde{\beta}(w+m)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\right] \\ \text{s.t.} \quad & \mathrm{Pos}(\{\tilde{\alpha} - \tilde{\beta}(w+m) < 0\}) = 0 \\ & m, a > 0. \end{aligned} \tag{6.13}$$

Please note that we assume that both players try to maximize their expected profits $\mathrm{E}[\Pi_{\mathrm{r}}]$ and $\mathrm{E}[\Pi_{\mathrm{m}}]$. Another possible objective could also be the φ-optimistic values $\Pi^{\mathrm{U}}_{\mathrm{r}\varphi}$ and $\Pi^{\mathrm{U}}_{\mathrm{m}\varphi}$, respectively, which can be seen as the maximum profits the players could realize with at least possibility φ. In contrast, the φ-pessimistic values $\Pi^{\mathrm{L}}_{\mathrm{r}\varphi}$ and $\Pi^{\mathrm{L}}_{\mathrm{m}\varphi}$, respectively, stand for the minimum profits the players could achieve with at least possibility φ.[24] Therefore, we first have to determine the expected profit function of the retailer:

$$\begin{aligned} E\,[\Pi_{\mathrm{r}}] &= E\left[m[\tilde{\alpha} - \tilde{\beta}(m+w)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\right] \\ &= \frac{1}{2}\int_0^1 \left[\left(m[\tilde{\alpha} - \tilde{\beta}(m+w)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\right)^{\mathrm{U}}_{\varphi}\right. \\ &\quad \left. + \left(m[\tilde{\alpha} - \tilde{\beta}(m+w)]\tilde{k}_{\mathrm{r}}\sqrt{a} - (1-t)a\right)^{\mathrm{L}}_{\varphi}\right] \mathrm{d}\varphi \\ &= \frac{1}{2}\int_0^1 \left[m\left(\tilde{\alpha} - \tilde{\beta}(m+w)\right)^{\mathrm{U}}_{\varphi}\left(\tilde{k}_{\mathrm{r}}\sqrt{a}\right)^{\mathrm{U}}_{\varphi} - (1-t)a\right. \\ &\quad \left. + m\left(\tilde{\alpha} - \tilde{\beta}(m+w)\right)^{\mathrm{L}}_{\varphi}\left(\tilde{k}_{\mathrm{r}}\sqrt{a}\right)^{\mathrm{L}}_{\varphi} - (1-t)a\right] \mathrm{d}\varphi \\ &= \frac{m\sqrt{a}}{2}\int_0^1 \left[[\tilde{\alpha}^{\mathrm{U}}_{\varphi} - \tilde{\beta}^{\mathrm{L}}_{\varphi}(w+m)]\tilde{k}^{\mathrm{U}}_{\mathrm{r}\varphi}\right. \\ &\quad \left. + [\tilde{\alpha}^{\mathrm{L}}_{\varphi} - \tilde{\beta}^{\mathrm{U}}_{\varphi}(w+m)]\tilde{k}^{\mathrm{L}}_{\mathrm{r}\varphi}\right] \mathrm{d}\varphi - (1-t)a \\ &= \frac{m\sqrt{a}}{2}\int_0^1 \left[\tilde{\alpha}^{\mathrm{U}}_{\varphi}\tilde{k}^{\mathrm{U}}_{\mathrm{r}\varphi} - \tilde{\beta}^{\mathrm{L}}_{\varphi}\tilde{k}^{\mathrm{U}}_{\mathrm{r}\varphi}(w+m) + \tilde{\alpha}^{\mathrm{L}}_{\varphi}\tilde{k}^{\mathrm{L}}_{\mathrm{r}\varphi}\right. \\ &\quad \left. - \tilde{\beta}^{\mathrm{U}}_{\varphi}\tilde{k}^{\mathrm{L}}_{\mathrm{r}\varphi}(w+m)\right] \mathrm{d}\varphi - (1-t)a \\ &= m\sqrt{a}\left[\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \frac{w+m}{2}\int_0^1 \left(\tilde{\beta}^{\mathrm{L}}_{\varphi}\tilde{k}^{\mathrm{U}}_{\mathrm{r}\varphi} + \tilde{\beta}^{\mathrm{U}}_{\varphi}\tilde{k}^{\mathrm{L}}_{\mathrm{r}\varphi}\right)\mathrm{d}\varphi\right] - (1-t)a \\ &= m\sqrt{a}\left[\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\right] - (1-t)a, \end{aligned} \tag{6.14}$$

[24]See Zhou et al. (2008): *Two-echelon supply chain games*, pp. 395–398.

with Ψ being defined as follows:

$$\Psi = \frac{1}{2}\int_0^1 \left(\tilde{\beta}_{\varphi}^{\mathrm{L}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}} + \tilde{\beta}_{\varphi}^{\mathrm{U}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}}\right)\mathrm{d}\varphi. \tag{6.15}$$

In order to determine the retailer's response functions, we have to calculate the first order partial derivatives with respect to m and a:

$$\frac{\partial \mathrm{E}[\Pi_{\mathrm{r}}]}{\partial m} = \sqrt{a}\left[\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\right] - \Psi m\sqrt{a} \tag{6.16}$$

$$\frac{\partial \mathrm{E}[\Pi_{\mathrm{r}}]}{\partial a} = \frac{m}{2\sqrt{a}}\left[\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\right] - (1-t). \tag{6.17}$$

Setting Eqs. (6.16) and (6.17) to zero and eliminating m from $a(m,w,t)$ leads to:

$$m(w) = \frac{\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w}{2\Psi} \tag{6.18}$$

$$a(w,t) = \frac{\left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^4}{64\Psi^2(1-t)^2}. \tag{6.19}$$

Thereafter, we now consider the **manufacturer's decision problem** given by:

$$\begin{aligned}
\text{Max}\quad & \mathrm{E}[\Pi_{\mathrm{m}}(w,t)] = E\left[w[\tilde{\alpha} - \tilde{\beta}(w+m)]\tilde{k}_{\mathrm{r}}\sqrt{a} - ta\right] \\
\text{s.t.}\quad & m = (\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w)/2\Psi \\
& a = \left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^4/64\Psi^2(1-t)^2 \\
& \mathrm{Pos}(\{\tilde{\alpha} - \tilde{\beta}(w+m) < 0\}) = 0 \\
& w > 0, 0 \leq t < 1.
\end{aligned} \tag{6.20}$$

The manufacturer's expected profit function $\mathrm{E}[\Pi_{\mathrm{m}}(w,t)]$ can be determined analogously to the retailer's expected profit given in Eq. (6.14). Hence, we derive:

$$\mathrm{E}[\Pi_{\mathrm{m}}] = w\sqrt{a}\left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi(w+m)\right) - ta, \tag{6.21}$$

with Ψ being defined identically to Eq. (6.15). Inserting Eqs. (6.18) and (6.19) into $\mathrm{E}[\Pi_{\mathrm{m}}]$ yields:

$$\mathrm{E}[\Pi_{\mathrm{m}}] = \frac{\left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^3\left(4\Psi w - 3\Psi wt - \mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]t\right)}{64\Psi^2(1-t)^2}. \tag{6.22}$$

Similar to the retailer's problem, one has to set the first order partial derivatives to zero. If we first consider the manufacturer's participation rate t, we get:

$$\frac{\partial \mathrm{E}[\Pi_{\mathrm{m}}]}{\partial t} = \frac{\left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^3 \left[\left(-3\Psi w - \mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]\right)(1-t) + 8\Psi w - 6\Psi w t - 2\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]t\right]}{64\Psi^2(1-t)^3}. \tag{6.23}$$

From $\partial \mathrm{E}[\Pi_{\mathrm{m}}]/\partial t = 0$, one can derive:

$$t^*(w) = \frac{-\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] + 5\Psi w}{\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] + 3\Psi w}. \tag{6.24}$$

Please note that this expression can take negative values for $w < \mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]/5\Psi$, which would violate the domain of definition given in Sect. 6.3.1. Therefore, it is necessary to check if the obtained solution for w complies with the condition $w > \mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]/5\Psi$. Otherwise, we have to set $t = 0$. Setting the first order partial derivative with respect to w,

$$\frac{\partial \mathrm{E}[\Pi_{\mathrm{m}}]}{\partial w} = \frac{\left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)^2}{64\Psi^2(1-t)^2} \cdot \left[-3\Psi\left(4\Psi w - 3\Psi w t - \mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]t\right) + \left(\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}] - \Psi w\right)(4\Psi - 3\Psi t)\right], \tag{6.25}$$

to zero yields an expression for w which solely depends on participation rate t:

$$w(t) = \frac{\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]}{\Psi(4-3t)}. \tag{6.26}$$

We can now solve the system of equations given by Eqs. (6.18), (6.19), (6.24), and (6.26) in order to obtain closed-form solutions of the Manufacturer Stackelberg equilibrium. The results as well as the corresponding profits are given in Table 6.3.

Table 6.3 Manufacturer Stackelberg equilibrium

	Margins	Advertising	Profits
Retailer	$m = \frac{\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]}{3\Psi}$	$a = \frac{E^4[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]}{144\Psi^2}$	$\Pi_{\mathrm{r}} = \frac{E^4[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]}{216\Psi^2}$
Manufacturer	$w = \frac{\mathrm{E}[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]}{3\Psi}$	$t = \frac{1}{3}$	$\Pi_{\mathrm{m}} = \frac{E^4[\tilde{\alpha}\tilde{k}_{\mathrm{r}}]}{144\Psi^2}$
With $\Psi = \frac{1}{2}\int_0^1 \left(\tilde{\beta}_{\varphi}^{\mathrm{L}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{U}} + \tilde{\beta}_{\varphi}^{\mathrm{U}}\tilde{k}_{\mathrm{r}\varphi}^{\mathrm{L}}\right) \mathrm{d}\varphi$			

It is easy to see that the calculated wholesale price $w = \mathrm{E}[\tilde{\alpha}\tilde{k}_\mathrm{r}]/3\Psi$ always complies with the condition which follows from Eq. (6.24). Hence, the participation rate $t = 1/3$—which results independent of model parameters—is feasible.

6.4 Numerical Studies

This section provides numerical examples of the previously obtained results (see Table 6.3). As described above, one advantage of fuzzy set theory is the ability to include the experience of decision makers, which is mostly verbalized by linguistic expressions like 'customers are *very sensitive*, *sensitive*, or *less sensitive* to changes in prices', which are rather vague than clearly assignable to a single (crisp) value. Therefore, we use triangular fuzzy variables of the form $\zeta = (x, y, z)$, which do not only describe one single number, but rather a range of possible values.

Hence, the first step is to determine appropriate triangular fuzzy variables, which correctly represent the decision makers' experience and estimation. One possible way can be found in Cheng (2004), who proposes a group opinion aggregation model based on a grading process.[25] However, for the sake of simplicity, we arbitrarily choose triangular fuzzy variables for the parameters $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{k}_\mathrm{r}$ in this paper, which can be found in Table 6.4.

Let us now assume a medium base demand $\tilde{\alpha}$ (about 20), a sensitive price sensitivity $\tilde{\beta}$ (about 1.25), and a low advertising effectiveness $\tilde{k}_\mathrm{r}$ (about 2). By means of Eq. (6.3), we can calculate the φ-pessimistic and φ-optimistic values:

$$\tilde{\alpha}^\mathrm{L}_\varphi = 15 + 5\varphi \qquad \tilde{\beta}^\mathrm{L}_\varphi = 1 + 0.25\varphi \qquad \tilde{k}^\mathrm{L}_{\mathrm{r}\varphi} = 0.1 + 0.1\varphi$$

$$\tilde{\alpha}^\mathrm{U}_\varphi = 25 - 5\varphi \qquad \tilde{\beta}^\mathrm{U}_\varphi = 1.5 - 0.25\varphi \qquad \tilde{k}^\mathrm{U}_{\mathrm{r}\varphi} = 0.3 - 0.1\varphi.$$

Table 6.4 Allocation of linguistic expressions to triangular fuzzy variables

	Linguistic expression	Triangular fuzzy variable
Base demand $\tilde{\alpha}$	Low (about 10)	(5, 10, 15)
	Medium (about 20)	(15, 20, 25)
	High (about 30)	(25, 30, 35)
Price sensitivity $\tilde{\beta}$	Very sensitive (about 1.75)	(1.5, 1.75, 2)
	Sensitive (about 1.25)	(1, 1.25, 1.5)
	Less sensitive (about 0.75)	(0.5, 0.75, 1)
Advertising effectiveness $\tilde{k}_\mathrm{r}$	Low (about 0.2)	(0.1, 0.2, 0.3)
	Medium (about 0.3)	(0.2, 0.3, 0.4)
	High (about 0.4)	(0.3, 0.4, 0.5)

[25] See Cheng (2004): *Group opinion aggregation.*

Table 6.5 Numerical example with medium $\tilde{\alpha}$, sensitive $\tilde{\beta}$, and low $\tilde{k}_r$ (see Table 6.4)

	m^*	w^*	a^*	t^*	$E[\Pi_r]$	$E[\Pi_m]$
Fuzzy parameters	5.75	5.75	35.84	0.33	23.89	35.84
Crisp parameters	5.33	5.33	28.44	0.33	18.96	28.44

These values are inserted into Eq. (6.2) in order to determine $E[\tilde{\alpha}\tilde{k}_r]$,

$$\begin{aligned} E[\tilde{\alpha}\tilde{k}_r] &= \frac{1}{2}\int_0^1 \left(\tilde{\alpha}_\varphi^L \tilde{k}_{r\varphi}^L + \tilde{\alpha}_\varphi^U \tilde{k}_{r\varphi}^U\right) d\varphi \\ &= \frac{1}{2}\int_0^1 [(15+5\varphi)(1+\varphi) + (25-5\varphi)(3-\varphi)]\, d\varphi \\ &= 5\int_0^1 \left(\varphi^2 - 2\varphi + 9\right) d\varphi = 41.67, \end{aligned}$$

and, analogously, Ψ:

$$\begin{aligned} \Psi &= \frac{1}{2}\int_0^1 \left(\tilde{\beta}_\varphi^L \tilde{k}_{r\varphi}^U + \tilde{\beta}_\varphi^U \tilde{k}_{r\varphi}^L\right) d\varphi \\ &= \frac{1}{2}\int_0^1 [(1+0.25\varphi)(0.3-0.1\varphi) + (1.5-0.25\varphi)(0.1+0.1\varphi)]\, d\varphi \\ &= \frac{1}{40}\int_0^1 \left(-\varphi^2 + 2\varphi + 9\right) d\varphi = 0.24. \end{aligned}$$

The resulting prices, advertising expenditures, and expected profits of manufacturer and retailer in a Manufacturer Stackelberg equilibrium, which derive from inserting $E[\tilde{\alpha}\tilde{k}_r]$ and Ψ into the expressions given in Table 6.3, are listed in Table 6.5, together with the results of the crisp case. Here, we can see that both players set the same margins m and w. However, the manufacturer can realize a higher profit than his retailer, which can be explained by the participation rate $t = 1/3$: The whole supply chain invests $a = 35.83$ into advertising (fuzzy case), whereof the manufacturer bears one-third, while two-thirds remain in the retailers responsibility. Even if this is only one certain set of parameters, these findings can be generalized to some extent, as it is visible from Table 6.3: Retailer's and manufacturer's margin are always identical ($m = w$) according to this model, and also the inequality $E[\Pi_m] > E[\Pi_r]$ holds for any parameters. Furthermore, the participation rate $t = 1/3$ is constant, as it does not depend on any parameter of the model.

Therefore, we turn our attention to the comparison of fuzzy and crisp case. Here, we can see that, besides the constant participation rate t, each variable assumes

Table 6.6 Variation of fuzziness of $\tilde{\alpha}$, with sensitive $\tilde{\beta}$ and low $\tilde{k}_r$ (see Table 6.4)

$\tilde{\alpha}$	m^*	w^*	a^*	t^*	$E[\Pi_r]$	$E[\Pi_m]$
(10, 20, 30)	5.98	5.98	41.93	0.33	27.96	41.93
(12.5, 20, 27.5)	5.86	5.86	38.79	0.33	25.86	38.79
(15, 20, 25)	5.75	5.75	35.84	0.33	23.89	35.84
(17.5, 20, 22.5)	5.63	5.63	33.06	0.33	22.04	33.06
(20, 20, 20)	5.33	5.33	28.44	0.33	18.96	28.44

higher values, and that both players can expect higher profits under a fuzzy customer demand. The variation of the fuzziness of the market base $\tilde{\alpha}$ in Table 6.6 shows similar results. The higher the fuzziness of the base demand, the higher the players set margins and advertising expenditures, which leads to higher expected profits. This is consistent with previous research on pricing models without advertising.[26]

6.5 Managerial Implications and Conclusions

In this article, we analyze a single-manufacturer single-retailer supply chain with fuzzy customer demand, which is sensitive to prices and advertising. In order to increase the retailer's advertising expenditures, the manufacturer has the ability to participate in his retailer's advertising costs by means of a vertical cooperative advertising program. To the best of our knowledge, this is the first application of fuzzy set theory to a cooperative advertising model. In contrast to deterministic approaches, which require detailed information about customer behavior and market characteristics, we are able to include experience of decision makers into our model, as fuzzy set theory allows us to transform linguistic expressions (e.g., *high* or *low* base demand) into triangular fuzzy variables. Through our numerical examples, we furthermore derive that a higher fuzziness of parameters, i.e., a bigger range of values the parameters may take, leads to higher expected profits, while the participation rate should be constantly set to one-third, independent of the market demand parameters.

However, this is only a first approach of applying fuzzy set theory to cooperative advertising in a supply chain and, therefore, underlies certain limitations: First, in order to reduce mathematical complexity, we had not only to restrict the price demand function to a linear shape instead of the more general form previously

[26]In Zhao et al. (2012b), the profit of the two manufacturers as well as of the total system increases with higher fuzziness both in Manufacturer Stackelberg and Stackelberg Retailer game, while the retailer's profit decreases (cf. Zhao et al. (2012b): *Pricing decisions for substitutable products*, pp. 417 et seq.).

published. Furthermore, we were only able to consider advertising of the retailer, while it is common in research to integrate also the manufacturer's decision on advertising into analysis. Besides this, future research should also consider different membership functions of fuzzy variables instead of the triangular one.

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Chapter 7
Résumé

This work considers various aspects of the interaction between the manufacturing and retailing echelon of a supply chain, whereby special attention is paid to marketing instruments used to influence customer demand. This includes the determination of each individual firms' prices and advertising expenditures on the one hand, but also a special form of cooperation that is called *vertical cooperative advertising* on the other hand. The latter refers to a financial support program which manufacturers offer to their retailers in order to increase their advertising expenditures. Commonly, the support consists of a certain percentage of the advertising cost called *participation rate*. Obviously, setting this rate correctly is a very difficult task, which includes not only an estimation of the retailers' and customers' behavior, but also of the trade-off between related costs and additionally generated profit. Further complexity arises from the interdependencies between the different firms belonging to a supply chain and their decisions made regarding pricing and advertising.

To cope with this complexity, analyses done in this work base on mathematical models to which game-theoretic solutions concepts are applied. In contrast to other maximization methods from the field of operations research, game theory allows incorporating interdependencies between the decisions of participating players into consideration. Thereby, it does not only consider individual optima, but rather determines equilibria between the objectives of all involved parties. Furthermore, an unequal distribution of power between the players may be incorporated, which moreover increases the suitability for practical problems. By this means, the present work's task is to provide decision support for managers in manufacturing and retailing firms, which shall help to derive well-grounded decisions on prices, advertising, and cooperative advertising programs.

Previous to the explicit mathematical analyses, a literature review on existing studies in that research area was conducted. After a confrontation of the different meanings of the term *cooperative advertising* in literature, 68 works were found that correspond to the aforementioned definition. Thereof, 58 studies treating of

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mathematical models are included into the detailed review, which is therefore more extensive than the few existing literature reviews on cooperative advertising. After a general classification of the models according to, e.g., the supply chain structure under consideration, the included decision variables (prices, advertising, quality, etc.), and the design of the cooperative advertising program, follows a detailed examination of the demand functions in use. Due to the diverse approaches, it proved beneficial to distinguish static and dynamic formulations. The last topic this review is concerned with is the application of game theory within these models. Here, an overview of the different models of non-cooperative game theory and bargaining theory is given, whereat a distinction is made between intra-echelon and inter-echelon interaction.

Based on that survey, the model formulation of SeyedEsfahani et al. (2011) was considered as a starting point appropriate to the scope of this work. It allows determining optimal prices, advertising expenditures, and cooperative advertising participation rate in a one-manufacturer one-retailer supply chain. Thereby, a very general demand function consisting of two components is assumed: First, a price demand function depending on the retail price, which can be adjusted via the three parameters initial base demand, customers' price sensitivity, and a shape parameter controlling whether the function is linear, convex, or concave; second, an advertising demand function depending on the manufacturer's (global) and the retailer's (local) advertising expenditures. Here, two parameters allow including the effectiveness of global and local advertising, respectively. Given that framework, the authors analyze four different game structures: a Nash game standing for an equal distribution of power, two Stackelberg games where either the manufacturer or the retailer obtains the channel leadership, and a Cooperation between both. This is more comprehensive than most previous studies, which are limited to manufacturer leadership and Cooperation.

However, despite these advantages, the significance of the obtained recommendations on the setting of prices, advertising, and participation rate is limited, because SeyedEsfahani et al. (2011) suppose that manufacturer and retailer set the same margin in Nash and Retailer Stackelberg game, which is obviously not consistent with reality.[1] To meet the objective of this work, i.e., to offer recommendations to decision makers that correspond to the given market characteristics as accurately as possible, the model is modified to omit this restrictive presumption of firms' behavior. Building on that, each aforementioned equilibrium is recalculated without any simplification of parameters, wherefore it enables a more detailed analysis of the players' best decision and the resulting profits.

The inter-echelon interaction between manufacturer and retailer within a supply chain is only one aspect contemplated in this work. Another task was to study the effects of intra-echelon competition, especially the competition between retailers, on the entire supply chain. Consequently, the model formulation is extended by

[1]As explained in Sect. 4.1, this assumption was first made by Xie and Neyret (2009): *Co-op advertising* and taken up by SeyedEsfahani et al. (2011): *Vertical co-op advertising*.

a second retailer selling a substitutable product that is also fabricated by the same manufacturer. In doing so, the focus still was on a decision support as comprehensive as possible, which is why we continue to incorporate all previous decision variables (prices/margins, advertising expenditures, and participation rate) into our analyses. But especially the interdependencies of competing firms' prices turned out to be critical, because the simple introduction of a parameter denoting the customers' cross price sensitivity can lead to questionable insights, e.g., increasing prices and profits due to a higher degree of competition.

In order to overcome this issue, we follow the suggestion of Ingene and Parry (2007) and deduce the price demand function from the customers' utility function so that logically consistent results are ensured. Another problem was the apparent increase of complexity, which caused a necessary limitation to a linear price demand function. But nevertheless, two games with different distribution of power are considered under this framework: The first game assumes a completely equal distribution of power within the supply chain, i.e., a balance of power between the two retailers and between manufacturing and retailing echelon. This assumption was resigned in the second game, in favor of a manufacturer leadership. By this means, the present analysis again offers insight into the correct setting of prices, advertising expenditures, and cooperative advertising participation rates for at least two scenarios with retail competition.

The last aspect treated in this work is incomplete or even missing data on market and customer characteristics. In practice, it may be too time-consuming and/or too expensive to conduct extensive market surveys and empirical studies on customer behavior. The absence of appropriate data sets, e.g., in case of the introduction of a new product, can represent another reason that firms are not able to give a sufficiently precise estimation of the parameters necessary to the aforementioned mathematical models. To nevertheless offer assistance to decision-makers, we propose an approach that allows to feed the models with the experience of few experts. Here, fuzzy set theory is used to transform these expert opinions—which are expressed rather in linguistic terms, like *medium* or *high demand*, than in form of concrete numbers—into parameters usable in this context. Since this is only a first approach of applying fuzzy set theory to a cooperative advertising model, some simplifications were necessary, e.g., the limitation to a one-manufacturer one-retailer supply chain, a linear price demand function, as well as the omission of manufacturer's determination of global advertising expenditures. Given the imprecise data base, this seems to be acceptable, though, because it can at least be used as a rough estimation of the direction that should be taken.

After this summary of main contents of this work, we shall now discuss the research questions established in Sect. 1.2. The first **research question Q1** is: *What is the actual state of research on cooperative advertising and what are the directions for further research in that field?* This question can be answered by means of the literature review in Chap. 3. It reveals that most actual works consider not only advertising, but also another decision variable like pricing (similar to this work), or even product quality, because this offers more comprehensive insight into the interdependencies in a supply chain. In contrast to that, only few works consider more than two players up to now. Concerning the design of vertical cooperative advertising programs, it turned out that almost all models consider only the manufacturer's participation, whereas the accrual rate[2] has not been analyzed so far. Furthermore, cooperative advertising is generally seen as an unidirectional support offered by manufacturers to their retailers, with the exception of some recent studies that also provide the possibility of a support offered by retailers.

Besides that, existing research can be divided into two groups: Static models without time-dependence and dynamic models where players' decisions can change over time. The selection of an appropriate model depends on the underlying problem to be analyzed. Certainly, dynamic models' advantage is that inter-temporal interdependencies between decisions can be considered, what especially allows an appropriate differentiation of the effects of global and local advertising. On the other hand, time-dependency obviously increases handling complexity, which means that not every problem can be contemplated under a dynamic framework. The last point of the review was the application of game theory. Here, we found out that mostly non-cooperative game theory is used, especially the two solution concepts Nash and Stackelberg equilibrium. This can be explained by the fact that these concepts allow to study the effects of unequal distribution of power within the supply chain, which represents a common research subject in that field. However, in the majority of cases, the manufacturer is assumed to be the channel leader (Manufacturer Stackelberg game), while only few works also compare this structure with retailer leadership (Retailer Stackelberg game) or equal distribution of power (Nash game).

Hence, open topics for future research particularly point towards an integration of more parties into the supply chain framework, an extension that can be related to more firms belonging to the same echelon (like done in this work), or to the adding of other echelons like a supplying echelon, which supplies preliminary products to the manufacturers. In the latter case, other demand-relevant decision variables like product quality could be included. Besides more general formulations of consumer demand, it is thereby important to consider different scenarios of distribution of power, because the commonly made assumption of manufacturer leadership does no longer hold in every market nowadays. However, all these extensions contribute to an increasing complexity of calculus, which might render necessary innovative solution methods like heuristics or meta-heuristics.

Research question Q2 then asks: *How should prices, advertising, and cooperative advertising program be set in a manufacturer-retailer supply chain with respect to the underlying distribution of power?* For this purpose, the one-manufacturer one-retailer model in Chap. 4 was introduced, which represents a more general and unrestricted approach than previous formulations. Here, closed-form expressions of the optimal players' prices, advertising expenditures, and participation rate are

[2] As explained in Sect. 3.3.1, the accrual rate denotes an upper limit of total financial support, which is often related to the last year's sales. In practice, this is a common element of cooperative advertising programs.

derived, together with the profits resulting from this combination of strategies. Concerning the distribution of power, each possible scenario is factored in, i.e., equal distribution of power, manufacturer leadership, retailer leadership, and Cooperation.

On that basis, we conducted extensive numerical studies offering insight into the effects of the different scenarios on the players' strategies and profits. Here, particular attention is paid to the shape parameter, which controls whether the price demand function is linear, concave, or convex, and to the ratio of the echelons' advertising effectiveness. For instance, we derive that cooperating firms should set a low retail price (compared to the other scenarios), but high advertising expenditures in order to increase their profits. Furthermore, the manufacturer will only offer a cooperative advertising program when he obtains channel leadership, whereby the actual participation rate strongly depends on the underlying customer demand parameters and may vary from 0 % to nearly 80 %, just to name a few.

After this consideration of inter-echelon interaction, **research question Q3** shifts the attention to intra-echelon interaction: *What are the effects of retail-competition on the firms' decisions and how should the manufacturer adapt his cooperative advertising program?* Hence, the aforementioned model was extended by a second retailer in Chap. 5. Again, a comprehensive numerical study was conducted for two different game scenarios and with respect to the degree of competition within the retailer-duopoly, which reveals the following insights: First, since both retailers should lower their prices under strong retail competition, we can state a positive effect to the customers. However, these low prices are accompanied by lower advertising expenditures, which together result in decreasing profits for all involved firms. Here, it is important to say that the manufacturer should increase his participation rate in order to absorb the reduction of local advertising—at least partially. And, as a last example of the insights gained from this analysis, it can be observed that channel leadership does not always lead to the highest profit for the manufacturer—it can also be beneficial to abstain from exerting this power.

The fourth **research question Q4** is concerned with the following aspect: *How can firms determine their best strategy when data on market and customer characteristics is imprecise or missing?* One possible answer is given in Chap. 6, where an approach based on fuzzy set theory is proposed. On that basis, firms can use the experience of experts, which can be included into mathematical modeling after a transformation into fuzzy variables. Thereby, each estimation in form of a linguistic expression is transformed into a triangular fuzzy variable that contains the fuzziness commonly immanent in such estimations. Furthermore, necessary modifications of mathematical formulation are conducted which allow to include these fuzzy variables in calculation.

Although each of the four research questions can be answered by this work, it is still subjected to some limitations and drawbacks which should be addressed in future research. The first issue can be traced back to the model formulation itself, wherefore it concerns every model considered here: One of the insights of Chap. 4 is that the manufacturer is only willing to support the retailer through a cooperative advertising program when he obtains the channel leadership or in case of a Cooperation. The reason for this can be found in the commonly used manufacturer's profit function, where the participation rate only arises as a cost factor. Only the knowledge on the relationship between retailer's local advertising and his participation rate, which he achieves as Stackelberg leader, leads to the offer of a cooperative advertising program. Future research should discuss if this effect resulting from the mathematical formulation also holds in practice, or if cooperative advertising is actually also offered when the retailer has greater channel power or, at least, the power is equally distributed. In that case, the commonly-used formulation has to be modified in order to meet the condition given in practice.

Another apparent limitation concerns the one-manufacturer two-retailer model in Chap. 5. Here, only two different game structures are considered, whereof the Manufacturer Stackelberg–Horizontal Nash equilibrium can only be determined under the assumption of identical initial base demand parameters for both retailers. The reason for this is the objective to derive closed-form expressions for each equilibrium, which cannot be met in each possible combination of distribution of power within the supply chain. Since other combinations promise further interesting insights—especially double Stackelberg structures like an inter-echelon Retailer Stackelberg game where one retailer additionally obtains the intra-echelon leadership—other solution approaches like heuristics or meta-heuristics have to be used in future.

Lastly, the assumption of complete (and partly even perfect) information can hardly be seen compatible with real conditions within a supply chain, especially when firms are competing with each other. The omission of this elementary assumption would though necessitate other solution techniques for games with information asymmetry between the players. In this context, problems related to moral hazard and principal-agent theory in general could be of interest, especially with respect to the announcement of planned advertising expenditures by the retailer and the following setting of the participation rate by the manufacturer.

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