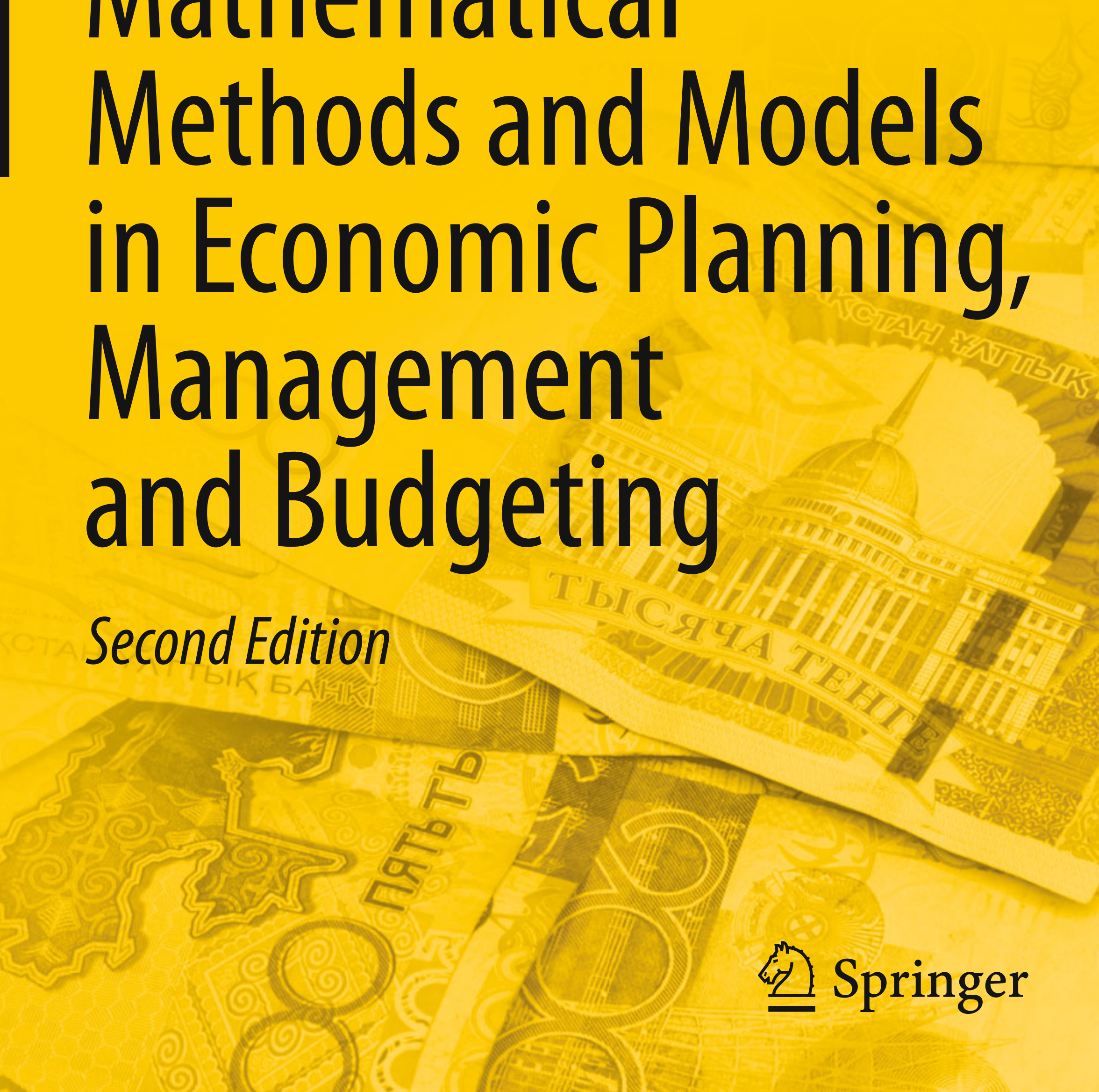
Galimkair Mutanov

Mathematical Methods and Models in Economic Planning, Management and Budgeting

Second Edition



Springer

Mathematical Methods and Models in Economic Planning, Management and Budgeting

Galimkair Mutanov

Mathematical Methods and Models in Economic Planning, Management and Budgeting

Second Edition

Springer

Galimkair Mutanov
Al-Farabi Kazakh National University
Almaty, Kazakhstan

ISBN 978-3-662-45141-0 ISBN 978-3-662-45142-7 (eBook)
DOI 10.1007/978-3-662-45142-7
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014955682

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

A new qualitative stage of the market economy, defined as knowledge economics, differs considerably from the previous stage in terms of nonlinear trends in economic development. Those responsible for reproduction cycle management have to make managerial decisions in conditions of high uncertainty affecting the development of production processes, distribution, exchange, and consumption. It is impossible to estimate the effectiveness of such projects and processes without adequate administrative instruments.

Economic-mathematical modeling is one of the most effective methods for describing complex socio-economic objects and processes in terms of mathematical models in combination with new engineering decisions. Modeling thus becomes part of economics itself. Knowledge economics as a common albeit somewhat abstract category must be expressed in a tangible concrete form. This can be achieved by means of mathematic modeling of its processes as managerial objects.

The present work presents a series of works in the field of scientific and methodological bases of creation of information-analytical systems of management of financial and economic processes and systems in the period from 1995 to 2011 years.

This text reflects the current level of theoretical research and development of the system of mathematical models and methods that can be used to solve real, important economic problems: control of development and operation support at any budget level; quality assessment of economic systems management in terms of energy-entropic approach; and risk management of investment processes.

The proposed mathematical methods and models were tested on the example of Kazakhstan's economy, and developed solutions and models may be used in any level and in any State. In the course of this research were analyzed and developed the ideas of Nobel laureates and leading scientists—V. Leontiev, Prigogine, G. Odum, E. Odum, Harold Kuhn, John Keynes, Christopher Sims, principle of McKinsey matrix and others.

The reader will find presented here a complex of models used to analyze and forecast the flow of budget financial resources. Current calculations and long-term forecasting of budget indicators are the instruments of realization of strategic development plans. Traditional methods of budget program planning are still widely used; they include planning based on rated standards specified by administrative bodies or by the changing dynamics of previous periods. However, the development of information technologies (IT), the requirements of a market economy, and the high pace of development necessitate new, highly intellectual and precise analytical and planning models.

The semantic and frame-based models suggested in this work create the mathematical basis for automated system control. The author presents here his seminal development of a mathematical budget model which with mathematical exactness reflects properties and conditions at any budget level. Methods and mathematical models for program control of budget resources focused on end results such as correct planning are utilized according to the strategic plan of socio-economic development of the nation or region. Benchmarks are thus created against which to estimate the achievability of set goals under certain limitations in budget funds and budget potential as determined in the process of medium-term planning. This work details an effective method for estimating the stability of program movements determining system decisions, based on construction of Lyapunov's function, allowing planners to estimate the efficiency of budget mechanisms for resource distribution. The principles governing the design of an intellectual system modeling program control of budget resources, and permitting correction of the decision by adjustment of the system of indicators, are outlined.

New knowledge often arises at the intersection of different scientific fields when well-known laws of one science are adapted to and interpreted by the other, opening up a phenomenon for approach from another angle, and the results can be exciting. An example is application of the thermodynamic approach to business system management, through mathematical descriptions focused on a decrease in entropy and increase in productive efficiency. The theoretical approach proposed by the author becomes even more valuable as national theory and practice in Kazakhstan have not previously offered developments in the assessment of business system efficiency based on the energy-entropic method. The universality of the proposed method is based on the fact that all systems of the material world—from wildlife and inanimate nature to technology and production—are arenas of ever-present change in amounts of energy and entropy, and studying this dynamic can give new knowledge of the laws governing the functioning and development of such systems. This research undertakes, therefore, the scientifically based application of the energy-entropic method to assessment of the economic efficiency of any production process.

The work further suggests methods and mathematical models which can be used to analyze currency purchase and sale, to make forecasts, to determine profitable cycles, and to structure decision-making in the foreign exchange market. An information system is developed on the foundation of these methods and is applied to a more detailed analysis of the foreign exchange market, leading to practical recommendations for second-rank banks on correction of exchange rates. A description is provided of the software, hardware, and instrumentation used in the proposed system.

In order to protect the safety of financial investments in conditions of information uncertainty, methods and mathematical models are developed to assess innovation projects. This is a strategically oriented approach which enables every project to engage the highest available expertise level with application of advanced information technologies. The role of risk-management and qualitative project assessment becomes even more important if the project in question is innovative—and it must be stressed here that innovativeness of development is one of the main priorities of the economic program of our country. A richly complex method of assessment of innovation projects and a graphic model based on such parameters as innovation, competitiveness, and reduced net cost of a project facilitate in-depth assessment of innovation projects on the basis of absolute positioning. The methods and mathematical models presented here can be used by expert commissions of venture funds, development institutes, and other potential investors to meaningfully assess innovation projects.

Investment in the development of economic processes and systems always entails risk. An investment decision that is not adequately reasoned can cause adverse economic consequences for the investor. Making investment decisions becomes even more complicated with the high degree of uncertainty surrounding economic consequences of a given investment. The series of mathematical methods and models proposed here represent an integrated methodology for making investment decisions that aims to reduce risk by more objectively estimating probability of investment consequences, and thereby to equip the investor with a practical instrument for scientifically-based forecasting. A review of a variety of methodological approaches to studying risk reveals that researchers tend to focus their attention on entrepreneurial risk—that is to say, as the object of analysis they consider individual enterprises, and the subjects of their investigations are statistical variations in stochastic probability distributions of all possible losses and damages. Meanwhile, insufficient attention is given to the investigation of principles of functioning and forms of manifestation of nonstatistical risks, their influence on the entrepreneurial activity and interaction with statistical risks. This research suggests a methodological base for creation of an integral expert system supporting coordinated investment decisions that takes into account assessment and control of project risks.

The methods and models developed by the author have been brought to practical realization in the form of software tools. These are reliable instruments to be used in solving problems of business forecasting, assessment, and management of the development of economic processes and systems.

I would like to acknowledge contributions to this book made by my assistants I.G. Kurmashev, A.U. Shintemirova, Zh.D. Mamykova, E.S. Kutuzova, V.P. Kulikova.

It is my hope that this work will be of both theoretical and practical interest to its readers, to the benefit of all.

Contents

Chapter 1
Mathematical Methods of Budget Modeling

One of the key goals of public administration is distribution of budgetary funds as specified in budget planning.

The budget as an economic form of real, objectively specified distributive relations fulfills a specific public role: It satisfies demands of the society and its governmental/territorial institutions. It can therefore be considered as an independent economic category and, hence, as an independent object of research.

The budget *per se* is not a "popular" object of research in the field of economic-mathematical modeling. Studies devoted to modeling of macroeconomic processes are usually oriented towards the solution of narrow and highly specified problems. For example, the classical macroeconomic approach to budgetary regulation suggests a model of the tax system multiplier as a practical application of Keynes' model; there are also models optimizing applied exchequers and income aspects of the budget.

Most investigations in this field consider the budget globally, not as a specific object of management but as a system disturbing parameters affecting the budget.

Therefore there are not many models based on cash flows in the budget system which would enable analysis of the history of development of budget elements in dynamics and thus could be used to forecast budget behavior in the future.

The platform of such an approach to budget modeling may be summarized as follows: The process of clarification (adjustment) of the draft budget is an *inevitable* and *efficient* stage in control of budget performance *envisaging* usage of formalized criteria in choosing items in addressing allocation of funds. The main results of such investigations are these:

- Specific features of the business system structure enable use of a frame-based representation of budget knowledge, which promotes a more flexible approach to algorithmization and programming.
- Construction and analysis of mathematical models help to identify bottlenecks in the budget system and determine a target in the field of research.
- In light of intensive development of socio-economic activities it becomes feasible to make short-term forecasts of growth of financing and receipts within the budget system.

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_1

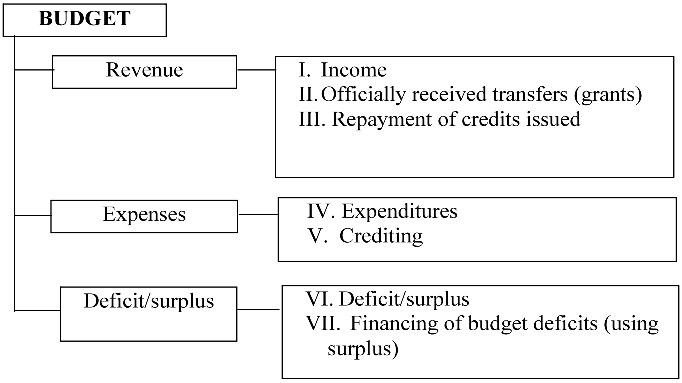


Fig. 1.1 State budget structure

- Information and computing technologies enable development of a sophisticated budget system with *feedback*, such that visualization at any moment in time enables the researcher himself to make corrections.

1.1 Budget as an Object of Modeling and Management

1.1.1 Budget Structure and Contents

The budget system is a set of financial relations between state, legal, and physical entities related to formation and usage of centralized monetary funds, budgets, and methods and techniques of their development and performance, as well as administration bodies. The *state budget* is a centralized monetary fund of the state approved by decision of the representative bodies, designated for support of social and economic development of the country and created at the expense of taxes, other obligatory payments, income from capital transactions, and non-tax and other receipts.

The state budget structure is formed on the basis of budgetary classification and includes seven parts [1] (Fig. 1.1).

The general budget state is one of balance: Sections I, II, and III in the figure refer to balance credit, while sections IV, V, VI, and VII balance debit. Deficit (the excess of expenditures over revenue), and surplus (the excess of revenue over expenditures) have, respectively, “minus” and “plus” signs in the balance structure. The Ministry of Finance provides financing of deficit and usage of surplus, depending on the state of the budget.

Revenue to the budget is classified according to four subdivisions:

• *Category*: tax revenues, income from capital transactions, official transfers, repayment of credits taken from the budget, general financing, movement of other budgetary funds		*For example*: Tax revenues
• *Class*	Every subsequent subdivision gives more detail about income	Tax group
• *Subclass*		Tax type
• *Specificity*		A specific payer or taxable object

Expenses follow the following classification system:

- Functional—provides detailed characteristics of types of activity and participants in budgetary relations, forming a code for functional classification of expenses:

  (1) a functional group;
  (2) a subfunction;
  (3) a state institution—program administrator;
  (4) a program;
  (5) a subprogram.

- Economic—distributes expenses according to allocation and type of expense.
- Departmental—groups budgetary programs according to executors: executive bodies of the Public Health Ministry, Ministry of Finance, Ministry of Education, *akimat* administration, etc.

The budget performance uses a combination of functional and economic classifications in the form of cross-classification including basic subdivisions of all classifications.

Budgetary classification using the same structure—sections, paragraphs, chapters, and articles—corresponds to the principle of unity of the budgetary system, facilitates budgeting, enables unification of budget indicators, and creates a basis for implementation of an automated system of financial calculations in budgeting and budget performance, as well as a wide use of computers [5].

1.1.2 Main Principles of Budget Formation

Assignment, functioning, and interrelation of all budget elements form a single mechanism called a budgetary system. The structure of the budgetary system is determined by the state system of the country; for example, the structure of the budgetary system of Kazakhstan includes the republican budget as well as the local budgets of regions (*oblasts*), cities, and city districts.

The main problem of the operating budget is, in fact, a financial one. Increases in budget revenue and optimization of expenses, budget balancing, distribution of budgetary funds in rapidly changing social and economic conditions—these financial aspects of the budget structure define the level of social and economic development of the country and ultimately the living standards of its people [2–4].

As specified in the Law on Budgetary System of the Republic of Kazakhstan [5, 6, 8], the budgetary system is based on the following principles:

1. The principle of *unity* means the degree of organizational—economic centralization of the budgetary system. Unity is expressed in the existence of a common system of public revenues operating on the territory of the country, and through uniformity of State expenditures. This principle guarantees unity of methodology and organization of budgetary planning, and of its interrelation with social and economic forecasting.
2. The principle of *completeness* means that all financial operations of the government, all collected receipts, and all expenditures are concentrated in the budget; each item in the budget accounts for all state receipts and expenses incurred in the frames of reference of that item.
3. The *reality* principle is necessary for prevention and elimination of falsification of budget inventory. It implies correct representation of state financial operations in the budget, and correspondence of the approved sums to the performance of budget assignments.
4. The *publicity* principle means the requirement to publish data of budget incomes and expenses in the periodical press so that the public may become acquainted with the budget structure, as well as any budget deficit and the methods employed to cover it (cutting back on expenses, increase in incomes, loans, currency issue).
5. The *independence* principle means maintenance of stable specifications of income distribution among budgets of different levels and the right to define the direction of budget expenditures.

The budget is formed at the expense of taxes, levies, transfers, and other obligatory payments specified by the current legislation, and fulfills the following tasks:

– redistribution of internal national product (GNP);
– state regulation and stimulation of the national economy;
– financial support of the budgetary sphere and realization of the social policy of the state;
– control over formation and usage of centralized funds of money resources.

The data of accounting and reporting, the number of taxpayers and objects of financing, and the sum of tax revenues form the initial planning base for revenues and expenses. Budgetary planning relates to concrete items and is calculated for the budgetary period. In forming a projected, consolidated financial balance it is necessary to forecast calculations of the main budgetary indicators.

The consolidated financial balance is a comparison of the total monetary receipts from all potential territorial units with the financing sums allocated for the planned objects of public administration and assistance.

Forecasting of budget development is a complex of probabilistic assessments of possible developmental trends of revenue-producing and expenditure budgetary el-

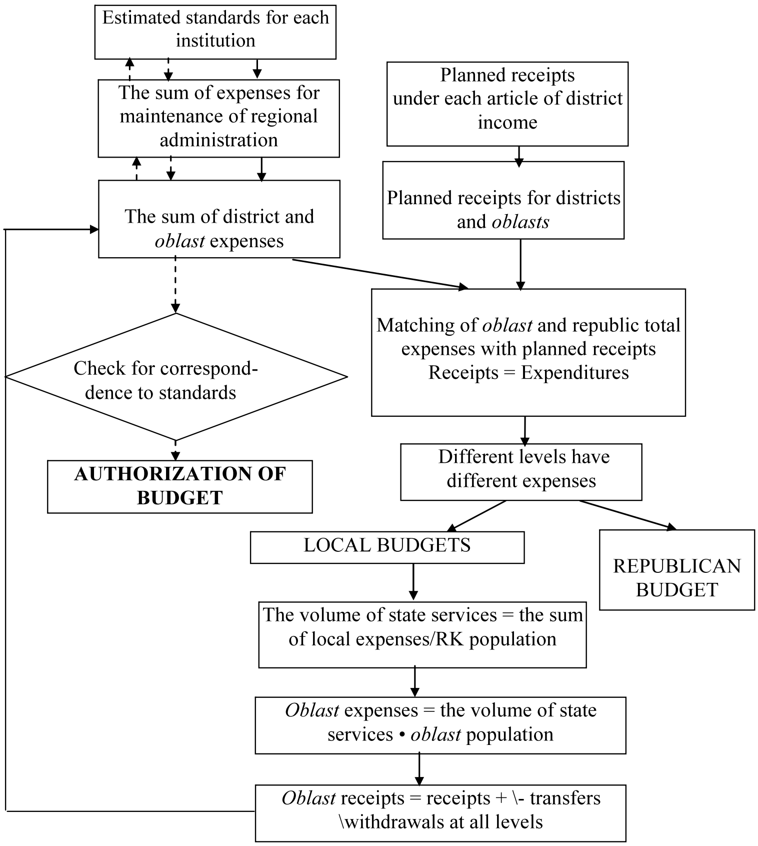


Fig. 1.2 Process of budget formation

ements in order to calculate and justify optimal means of budget development and to propose methods of budget strengthening on the basis of existing tendencies, specific social and economic conditions, and projections.

The process of budget formation at different levels is shown in Fig. 1.2. The disadvantage of this approach is calculation of the sum of budget expenditures and then revenues by regulating receipts, which in some cases leads to approval of local budgets with concealed deficits.

The approved budget can be corrected within the year if new circumstances arise (additional reserves of receipts appear, new governmental programs are introduced, etc.); in this case the estimate of expenditures is recalculated for changed articles. As is noted in [7], “the procedure of budget forming and forecasting at all levels still uses the system inherited from the old centralized regimes, with practically no changes.”

1.2 Budget Models

1.2.1 Models of Knowledge Representation and Budget Functioning

The budget system has a stringent unfolded hierarchical structure. The elements of the system are budget items assigned different values depending on the type of inquiry. In the unfolded form, the budget has a large volume, with the number of all income and expenditure items roughly amounting to 80 and 1300, respectively. Therefore to process such data arrays it is necessary to use information technologies. The usage of computer facilities as means of information processing requires representation of the subject data in a form suitable for digital processing [9–12].

However, there is a class of non-formalized or ill-structured problems which have one or several features typical of the budget system:

– the algorithm of problem solution is unknown or cannot be used because of limited computer resources;
– the problem cannot be defined in numerical form;
– the targets of the problem cannot be expressed in terms of a specific target function.

Attempts to solve non-formalized problems and to eliminate disadvantages of the procedural approach led to formation of a separate direction in artificial intelligence: knowledge engineering.

The idea of this approach used in knowledge engineering is to separate knowledge from the software in the computing system and to turn it into one of the components of its dataware, termed a knowledge base.

Knowledge is represented in a specific unified form which allows its easy identification, modification, and updating. Problems are solved by means of logical conclusions made on the basis of knowledge. For this purpose an independent mechanism for making logical conclusions has been developed as a basic part of the system software.

The systems constructed according to such principles are called *knowledge-based systems*, where “knowledge is a formalized information referred to or used in making logic conclusions.”

All possible knowledge is subdivided into the following types [14]:

– facts (factual knowledge of the type “A is A,” typical of databases and network models);
– rules (knowledge of the type “IF, THEN” for decision-making);
– metaknowledge (knowledge of knowledge, i.e., knowledge concerning modes of its usage, knowledge concerning properties of knowledge; this is needed to administrate knowledge bases, to make logical conclusions, and for identification, training, etc.).

Knowledge is represented in a special form. The form of knowledge representation considerably influences characteristics and properties of the system; therefore knowledge representation is one of the most important problems typical of knowledge-based systems. As logical conclusions and operations with knowledge are made using software, knowledge cannot be presented directly in the form used by man. Therefore formal models of knowledge representation are developed in order to represent knowledge.

Four separate models of knowledge representation are typically defined [15, 17, 18]:

– logic model;
– production model;
– frame-based model;
– semantic network model.

The specificity of budget structure knowledge leads to wider usage of the last two types of knowledge representation models in this context.

The frame-based model of knowledge representation is a psychological model of human memory and mind systematized in the form of a unified theory.

An important point in this theory is understanding of the frame—data structure for representation of a conceptual object. Every frame describes one conceptual object, and specific properties of this object and facts concerning it are described in slots—structural elements of the given frame. In general, the frame can be represented as the following structure:

Frame name:
Name of slot 1 (value of slot 1);
Name of slot 2 (value of slot 2);
...
Name of slot N (value of slot N).

The slot may have the following values: numbers, formulas, texts in a natural language or programs, output rules or references to the other slots of the given frame or other frames. The value of the slot can be presented by a set of slots of the lower level, which realizes the “matreshka” principle in the frame of representations.

All frames are interconnected and form a unified frame system in which declarative and procedural knowledge is integrally united. As the conceptual representation usually has a hierarchy, an image of integral knowledge is constructed as a single frame-based system with a hierarchical structure.

The mechanism of output control can be organized as follows: connections between a specified frame and other frames are given in the special slot, the value of which is an attached procedure—a specific output procedure in the frame.

To provide an output, first one of the attached procedures of the frame is started. Then the return value is estimated, and depending on the result the attached procedures of other frames are successively initiated. During this process slots are generated and destructed, their values change, etc.

It is a process of gradual advancement towards the target value.

The language of frame-based knowledge is especially effective for the structural description of difficult concepts and solution to problems in which, according to the situation, it is desirable to use different output modes.

No standard definition of semantic networks exists, but usually the term implies a system of knowledge comprising a network with nodes corresponding to concepts and objects, and arcs—relations between objects. Hence, every possible network can be considered as a part of a semantic network.

The relations used in semantic networks can be classified as follows:

– linguistic; in particular, case relations including relations of the type “object,” “agent,” “condition,” “place,” “tool,” “purpose,” “time,” and others;
– attributive, such as form, size, color, etc.;
– characteristics of verbs, which include gender, tense, mood, voice, number;
– logic providing performance of operations for calculation of statements (disjunctions, conjunctions, implications, negations);
– quantified, i.e., using universal and existential quantifiers;
– theoretical, including the concepts “an element of the set,” “subsets,” and others.

A specific feature of the semantic network is integrity of the system formed on its basis, which does not allow the separation of the knowledge base from output mechanisms. The semantic network is usually interpreted by means of procedures using it.

These procedures are based on several modes, but the most typical among them is a comparison of parts of the network structure. This mode is based on construction of a subnet corresponding to the problem, and its comparison to the network database.

The main advantage of semantic networks is their visualization and direct connection of concepts through the network, which enables the user to quickly find connections between concepts and, therefore, to control accepted decisions.

In working out a concrete model of knowledge representation one should try to take into account the following requirements [16, 18]:

1. Knowledge representation must be homogeneous (uniform). Homogeneous representation simplifies the mechanism controlling logic output and knowledge management.
2. Knowledge representation must be understandable to experts and system users. Otherwise, the process of knowledge acquisition and its assessment becomes rather difficult.

The development of knowledge representation models enables the user to realize computer variants of data processing tools with usage of control elements able to recognize system elements and their functions [18].

The process of formulation of research and modeling problems is based on systematized representation of the studied object.

The term “system” points out ordering, integrity, and the presence of certain laws. System representations cause interest not only as a convenient generalizing concept but also as a means of formulating problems with a high level of complexity.

Any economic system is a dynamic system of a certain level of complexity. It consists of a large number of parts (subsystems) connected by numerous links expressed as flows of labor force, materials, energy, information, and financial assets [3].

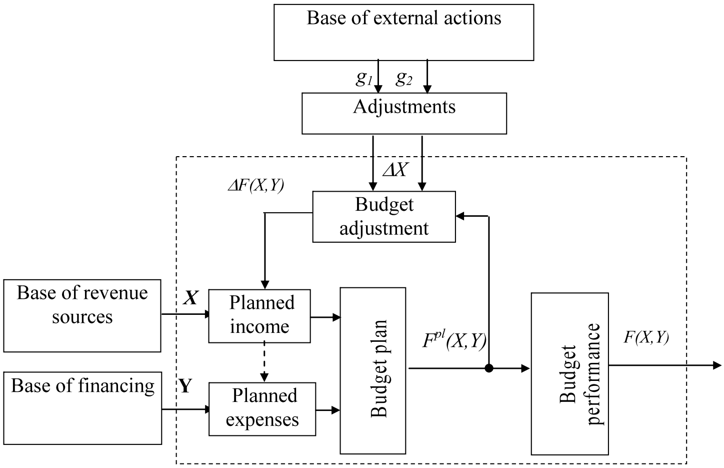


Fig. 1.3 Schema of the static state of the budget system

The budget system is a system providing centralized receipt and purposeful distribution of money resources.

The elements of such a system are budget indicators (items of income and expenses according to budgetary classification).

The flows of money resources between these form a *system of connections*. The operation of the system provides a programmed and well-founded distribution of resources.

The *efficiency indicator* shows the correspondence of the actual execution of budgetary indicators to the planned volumes. Therefore the planning process is the basis of the budget system.

As the expense part of the budget includes financing of the social sphere, public administration, and programs of social and economic development, effective usage (control) of this parameter will be an *object of system control*.

The revenue part of the budget is planned based on the taxable base of the region and calculated indicators of tax proceeds.

Planning of income is a process with a high degree of uncertainty. Its control is a complicated process because it is impossible to plan with a high probability such indicators as State taxes, receipts under penalties and other factors that depend on the situation.

Any system is either in motion (dynamic) or in a static state (constant). It is very important to determine the exact state of the system in time.

The static system is a more definite system as the action of time factors is excluded from the set of external actions.

The static structure of the budgetary system (in a steady or constant period) can be presented as follows (Fig. 1.3): Of all the structures of the budgetary system we consider the subsystem which only includes the elements of income (X), expense (Y), and their balance ratio [8, 19].

Thus, the objects of financing and objects of revenue sources are beyond the frameworks of the studied system. The results of system operation are indicators of budget performance in the reported period $(F(X, Y))$.

The process of budgetary system functioning in such a structure consists of the following stages:

(1) income and expenses for the forthcoming budgetary period (X, Y) are planned on the basis of sources of revenue and financing;
(2) the development of the expense plan is influenced by the fixed plan of incomes, which acts as restrictions in the sources of financing;
(3) income and expense planned targets are brought to the balance ratio $(F^{pl}(X, Y)$, where $\sum x_j = \sum y_i)$;
(4) in the process of budget performance the system is influenced by different disturbing factors caused by political, economic, and natural changes. The result of such influences is reflected in the system in the form of absolute changes in income;
(5) changes in the budget plan are expressed by the function of budget adjustment $(\Delta F(X, Y))$ on the basis of which the income plan (Δx_j) and, accordingly, the expense plan (Δy_i) are adjusted;
(6) as a result of adjustments introduced into incomes $F(X, Y)$ the budget performance differs from the originally planned version;
(7) adjustment of the budget plan is made $(\Delta F(X, Y))$ in order to provide minimal divergence from the actual execution;
(8) the result of system operation is budget performance in the current period $(F(X, Y))$.

Such adjustments of the plan allow control of current budget changes. As the controlled system must have a feedback element, the function of such an operator in the studied system is performed by the function of possible change in expenses caused by income changes and the function of plan adjustment.

The budget system is a component of all economic and financial systems of the state, therefore in each period the system is influenced by various factors: legislative changes, economic development, the political situation, etc.

The model must satisfy the principles of balancing, proportionality, and transparency in the distribution of budgetary funds.

Functioning of the budget system at each stage is provided by the laws of the operating system presented as models of knowledge structuring and mathematical models of element interaction.

1.2.2 Semantic Model of Budget Representation

1.2.2.1 Object Domain Model

The idea of the knowledge model of the *budgetary system* is to create a semantic network of concepts and elements of the object domain, forming budget knowledge.

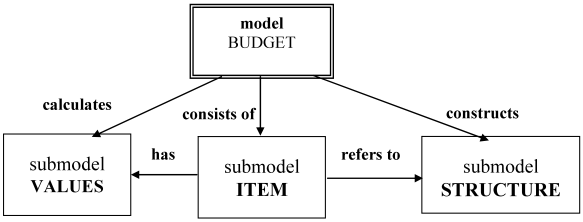


Fig. 1.4 A semantic budget model

This base is used to form the budget databank with observance of all requirements of unified budgetary classification, as well as rules of processing values of system elements and their interrelations. The obtained databank is analyzed in terms of logic models [8].

Taking into account specific features of the budget structure and the need to create a knowledge model, we will represent a knowledge (object) domain as a semantic network. The concepts of the system are placed in the vertices of the graph and connections between vertices correspond to relations between concepts. The result is a model of the data structure that may be visualized as a network.

It is very convenient to use a semantic knowledge graph to form the knowledge base of the budget. In such a graph every element of the budgetary system can have its own level of specification, size, and set of values depending on its level in the budget structure.

A specific feature of the semantic network the is integrity of the system thus created, which does not allow division of the knowledge base from the mechanism by which conclusions are formed.

The semantic network is interpreted by means of procedures using it. These procedures are based on several modes, but the most typical of them is a method of comparison of parts of the network structure. It is based on the construction of a subnetwork corresponding to a certain category of knowledge and its comparison to the database of the unified network.

In this model the concept "budget" is subdivided into three interrelated concepts: item, structure, and item values.

Thus, each concept can be presented in the form of an independent network.

The integral semantic model of the budget is shown in Fig. 1.4.

Every element of the budget system in the semantic model includes two semantic relations: a set of data (values) and a classification category.

According to the general budget structure, two semantic submodels—a submodel of the budget item and a submodel of the budget structure—are considered. The submodel of values is a model of logic rules [8, 13].

Thus, the semantic budget model displays the knowledge base of the item, its level in the budget structure, and possible values.

The elements of the integral model represent three subsystems constructed according to the following relations: The submodel "Item" is a component; the structure is formed in the submodel "Structure"; and the quantitative and qualitative values are calculated in the submodel "Value." The submodel "Item" is a link for all models. Each submodel is further presented as an independent semantic network.

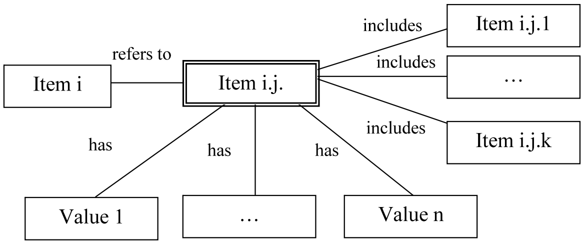


Fig. 1.5 A semantic model of the budget item

The central concept of the considered knowledge domain is the concept "budget item" which contains information about its belonging to a certain program of actions.

According to the hierarchical structure of the budget, any item is an element of the higher-level item and includes items of the lower level.

Thus, any item can be presented as follows:

Item	Name i.j
Includes	List of items {item i.j.1., item i.j.k.}
Refers to	Item i
Value 1	Number/Expression
Value 2	Number/Expression
...	Number/Expression
Value *n*	Number/Expression

The semantic graph of this model is presented in Fig. 1.5, which shows the basic constituting elements and connections between them.

All items have definite values which can be presented as follows:

– quantitative values assigned to the given item;
– values obtained by summation of the values of the lower-level items;
– values obtained by processing the above values;
– logical, textual values of the item.

The arcs display such connections as "refers to," "has," and "includes." Each value of the connection "has" is a result of a logical or computational rule of processing values for the chosen element.

Similarly, the connection "includes" displays the list of all items provided in the program for realization of the given item.

The values of the model items are the summands for obtaining the values of the considered item. The connection "refers to" shows that the given item belongs to the higher-level item; accordingly, its values are summands of the higher item.

Items of intermediate budget levels have similar semantic graphs regardless of their belonging to income or expense budget parts.

If the considered item is the lowest level of the budget hierarchical structure, its semantic graph displays connections with the higher item and assignment of values.

The graph of the lowest-level item of the budget is shown in Fig. 1.6.

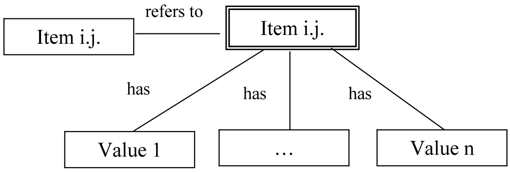


Fig. 1.6 A semantic graph of budget specificity items

If the item forms the first level in the hierarchy of budget items, i.e., the budget level, its graph, according to the budgetary classification, shows connections of value assignments (which can include values of the budget state) and the presence of items of revenue and budget expenses.

This model can be structurally presented as follows:

Item	BUDGET
Income	Revenue
Expenses	Expenditure
State	Deficit/surplus
Value 1	Number/expression
Value 2	Number/expression
…	Number/expression
Value *n*	Number/expression

A corresponding semantic graph is presented in Fig. 1.7.

In the given semantic model the relation "receives" enables determining that the item refers to the budget income, and the relation "is allocated"—a budget expense.

The items "Revenue" and "Expenditure" refer to the items of the intermediate level whose semantic graph was considered earlier.

The connection "state" describes the current state of the budget—deficit/surplus —by comparison of the amount of receipts and expenditures. The connection "has" is a result of the submodel "Value" corresponding to the chosen item level.

Thus, on the basis of the developed semantic model of budget items it is possible to construct an integral model of the budget system.

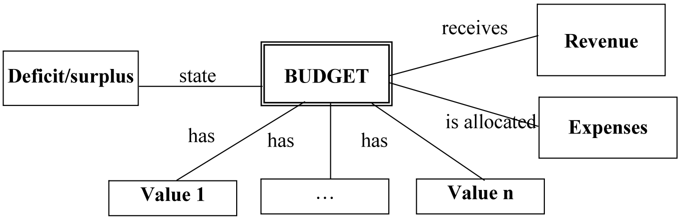


Fig. 1.7 A semantic graph of the BUDGET item

1.2.2.2 A Budget Structure Graph

Figure 1.8 shows a semantic graph of the knowledge base for the regional budget which reflects all interactions of elements of classification groups.

All vertices (nodes of the graph) correspond to budget items. The classifying elements marked by dotted arrows indicate belonging to a certain class (for the arrows directed upwards) or containing elements (for the arrows directed downwards), and represent the connections "refers to" and "includes."

This structure corresponds to the accepted uniform budgetary classification where each vertex has its code corresponding to the code in the state classifier of budget items.

The graph was constructed according to the rules of construction of hierarchical tree-like structures:

– the graph has only one root;
– the graph does not have any isolated (dangling) vertices which are not connected with the higher vertices, except the root;
– the connection is provided in two directions: from top to bottom and from bottom to top;
– a lower vertex can be connected only with one higher vertex;
– items are grouped at one level according to a common criterion (the general program).

The model structure corresponds to the general budget structure [20].

As budgets at all levels are formed according to the same schema, the given model is applicable to any system whose functions coincide with the budget functions.

1.2.2.3 A Graph Representing Budget Values

The key point in the notion "budget" is the budget item, namely the values it takes in different cases.

The item has several values, both fixed in the plan and in the budget performance, and calculated as required to make decisions in the process of budget performance. Every value of the item has a corresponding algorithm of its processing in the general knowledge base.

The aim of the semantic model for the budget item values is to develop a model forming and analyzing numerical and semantic item values.

Therefore the graph for budget value processing corresponds to the item graph and reflects the structure of its formation.

A corresponding semantic graph is represented in Fig. 1.9.

The roles of conditions are played by logical and mathematical rules based on the principles of budget system functioning and processing of the lower-level items included in the given item, which can be written in the following way:

$$\text{Item value i.j.} = \text{condition} \{\text{Value i.j.1}; \ldots; \ \text{Value i.j.k}\}.$$

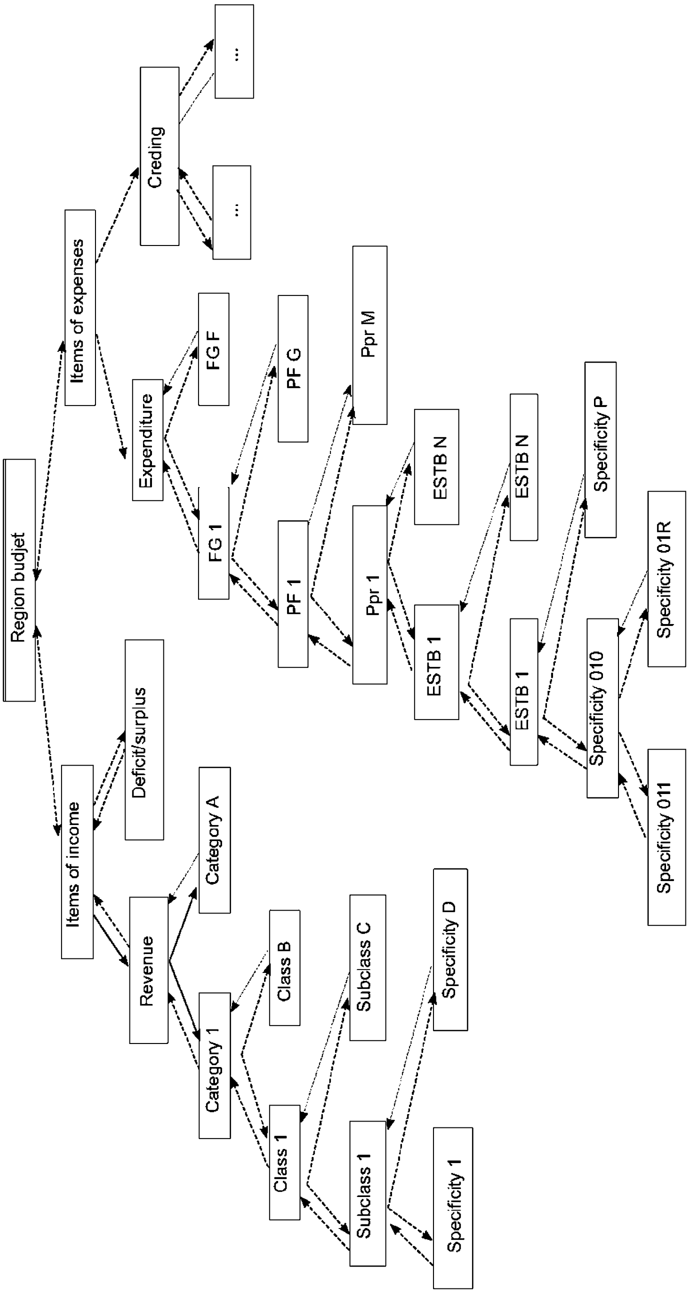


Fig. 1.8 A semantic model of the budget

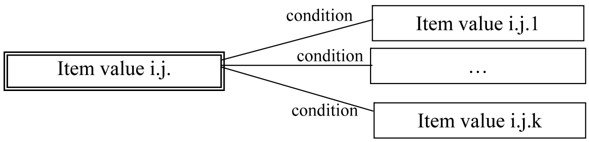


Fig. 1.9 A semantic model of budget item values

Productive knowledge models are widely used to realize formalized systems of data processing and systems based on experts' intuition and experience. As the productive models are realized procedurally, they have a useful property of modularity. That is why rules can be added or deleted without any unexpected side effects, which is very important for the sphere studied in this work.

If the condition has become invalid or if, by contrast, an absolutely new requirement has appeared, these changes can be included in the system.

To represent knowledge related to processing of budget items, module algorithms for data processing are used.

Mathematical budget models can be expressed as algorithms of data processing.

The rules and conditions are algorithmic forms of logical and mathematical budget models including summation of absolute item values and data acquisition on the budget (balance) state. Moreover, the rules of modules can be changed during processing, excluded from or added to the general model.

The elements of the budget model are all types of budget item: Y-expense items, X-income items.

To describe the knowledge domain in terms of the rules of processing of variables and elements, the following symbols are used:

y_r—expense item elements ($r = 1$, expense; $r = 2$, crediting);
y_{ri}—expense item elements of functional groups;
y_{rij}—expense item elements of subfunctions;
y_{rijs}—expense item elements of public institutions;
y_{rijsm}—expense item elements of programs;
y_{rijsmk}—expense item elements of subprograms;
$y_{rijsmkl}$—expense item elements of specificities;
x_d—income item elements of categories;
x_{dq}—income item elements of classes;
x_{dqw}—income item elements of subclasses;
x_{dqwp}—income item elements of specificities.

Indexing of elements enables recognition of the item in the budget structure and preservation of the classification code. The variables are numerical and calculated values assigned to the model elements.

Therefore *the budget analysis can be presented in terms of constructing algorithms in the form of individual modules for data processing*.

The set of modules forms a productive model of knowledge presentation for the functional budget structure. The model enables the introduction of optional formalized knowledge not violating the system structure, as well as the use of any number of variables and methods for processing them.

The semantic budget model expressed as the graphs of submodels of the ratios of system elements enables the developer:

– to exactly define the content and purposes of budget items;
– to present the content in a vivid and demonstrative form;
– to form a system (integral) representation of the content both for developers and users;
– to structure the content according to accepted classification requirements;
– to define functional budget content;
– to switch to digital forms of representation of the content model.

As a whole, the semantic model reflects the existing interrelations between the elements of the budget system.

1.2.2.4 An Example of the Semantic Model

Let us consider local budgets as examples of the semantic model. The main task of a knowledge representation models is recognition of the user's computer information. As an example we will consider the following statement:

Financing of item 5.2.254.39.30.132, Purchasing of medicines and other items used for medical purposes, amounts to 7,312,000 tenghe.

This item referring to specification No. 130 "Purchasing of goods" by a public institution (program administrator No. 254 "Executive office of the Public Health Ministry financed from the local budget"), fulfilling subprogram No. 30 "Sanitary and epidemiologic stations" of the governmental program No. 39 "Provision of sanitary and epidemiologic welfare," which refers to subgroup No. 2 "Public Health Care," functional group No. 5 "Health Care" of the expenditure budget part, was assigned the value of 7,312,000 tenghe.

According to the structural graph, this element is the final vertex providing connection of the type "refers to" to the higher-level items and having the value "has." The item code defines its position in the budget structure. The assigned data are used to get the values of all higher items and are a component of all budgets.

Let us consider another example where the budget element is not the final vertex.

In the framework of the government program "Rendering medical aid to HIV-infected patients," number 5.2.254.31, the sum of 12,351,000 tenghe was allocated.

This items refers to the public institution program administrator No. 254 "Executive Office of Public Health Ministry financed from the local budget," subgroup No. 2 "Public Health Care," functional group No. 5 "Public Health Care," in the expenditure budget part. The value of the item 12,351,000 tenghe is the sum of the values of subprograms included in this program. The expressions "is allocated" and "financing" are equated to the connection "is allocated to," which defines the element as a budget expense.

Income items are recognized in a similar way, for example:

The income tax paid by the legal entities withheld from the payment source has made 4,755 million tenghe.

The above item having classification code 1.1.1.3 refers to the category of tax proceeds of the budget revenues, class "Income tax," subclass "Income tax paid by legal entities." The volume of returns under the given item has made 4,755 million tg.

1.2.3 Frame-Based Model of Budget Knowledge Representation

1.2.3.1 Budget Model

As the knowledge of a specialist about the structure of the subject domain has a hierarchical interrelated character, to represent the above-stated key concepts frame-based models are used. The frame-based model can contain information in a structured and ordered form, and having access to the sections required at the moment makes it possible to input data in a more flexible form [21].

It is suggested to use a frame-based system for the budget, with subsystems for its articles. The active frames with additional information about special levels of budget articles will serve in the database as a transition from the active frame of the previous article to the frame of the lower-level article. In this case the frames of individual budget articles will have the same substructure. This phenomenon is called terminal section and is considered an important part of frames theory.

Unlike the other models, the frame models have rigid structure of information units called a protoframe [13, 14]:

Frame name: (name)
Slot 1: (value of slot 1)
Slot 2: (value of slot 2)
...
Slot N: (value of slot N)

A slot is an unfilled frame substructure; filling it brings it into association with a certain situation, phenomenon, or object. To find the slot value, the set of the slots of the lower level can be used, which enables realization of the "matreshka principle" in the frame model.

When the frame is concretized, it and its slots are assigned names, and slots are filled. Thus, protoframes turn into frames—specimens. A transition from the initial protoframe to the frame—specimen can proceed in several stages because of step-by-step specification of slot values.

Every frame contains the following information: how to use it, what to do if something unexpected happens, and what information is missing in slots. To organize communication between the objects of the subject domain, the network of frames is constructed. The communication can be organized by placing the names of other frames in some slots of the subframe.

The described approach *allows us to model budget changes, to analyze relations between expense and income items, to freely process the data, and to trace the hi-*

erarchy of item formation. The frame-based model is based on the approved budget structure whose basic components are receipts and expenditures.

The structure of the expenditure part, according to the unified budgetary classification, has seven levels:

r.	Expenses/Crediting
r.i.	Functional groups
r.i.j.	Subfunction
r.i.j.s.	Public institution
r.i.j.s.m.	Program
r.i.j.s.m.k.	Subprogram
r.i.j.s.m.k.l.	Specificity

The revenue structure has four levels:

d.	Category
d.q.	Class
d.q.w.	Subclass
d.q.w.p.	Specificity

According to the approved classification of the budget system elements, the superframe "BUDGET" has been created, expressed as:

FRAME NAME: BUDGET
TYPE of SUBFRAME 1: receipts
TYPE of SUBFRAME 2: expenditures
VALUE 1: module 1
...
VALUE F: module F

The subframe "VALUE" in all prototypes of the frame-based model is further presented as a module of the logic knowledge model of the specified frame level. Such structure of the superframe reflects a detailed elaboration of concept "BUDGET." According to the frames theory, the superframe contains as slots individual subframes, which reflect the meaning of content in the element of the concept "BUDGET."

The prototype of subframes at each level is presented as follows:

FRAME NAME: (name)
TYPE of the SUBFRAME 1: (name)
TYPE of the SUBFRAME 2: (name)
...
TYPE of SUBFRAME N: (name)

Here the slot "FRAME NAME" reflects belonging to the higher-class elements, and the slot "SUBFRAME TYPE" contains a classification indicator—the name of the type of budget item.

As the budget consists of two parts, and these have different structures, the subframes are individual submodels of the frame-based budget model.

1.2.3.2 Budget Item Model

The frame-based budget model enables the user to make a query of a single item or a list of budget items by the address (code) from any place in the structure. This follows from *the necessity of analyzing budget items in any section of the budget structure levels*. For this purpose, frame-based item models for the revenue and expenditure parts have been developed [8].

The prototype of the revenue item subframe:

FRAME NAME: RECEIPTS
FRAME TYPE: income item
CATEGORY: code_name d
CLASS: code_name d.q
SUBCLASS: code_name d.q.w
SPECIFICITY: code_name d.q.w.p
VALUE 1: module 1
...
VALUE F: module F

Such a frame structure enables the input of an item address and the output of its value without going through the entire frame-based structure of the budget model.

The subframe of the expense item is presented in the following prototype:

FRAME NAME: EXPENDITURES
FRAME TYPE: expense item
FUNCTIONAL GROUP: code_name i
SUBFUNCTION: code_name i.j
PROGRAM: code_name i.j.m
SUBPROGRAM: code_name i.j.m.k
INSTITUTION: code_name i.j.m.k.s
SPECIFICITY TYPE: code_name i.j.m.k.s.tv0
SPECIFICITY of EXPENSES: code_name i.j.m.k.s.tvn
VALUE 1: module 1
...
VALUE F: module F

A specific feature of the subframe "expense item" is that each slot is the corresponding subframe in the frame-based budget model. After choosing the frame of the first slot, the subsequent slot contains the list of items included in the given frame and their values. If a slot is not fully specified—for example, only the class of items is chosen—then the whole list of items included in the slot is presented. In this case, in "Value" slots the values of the last specified slot are given.

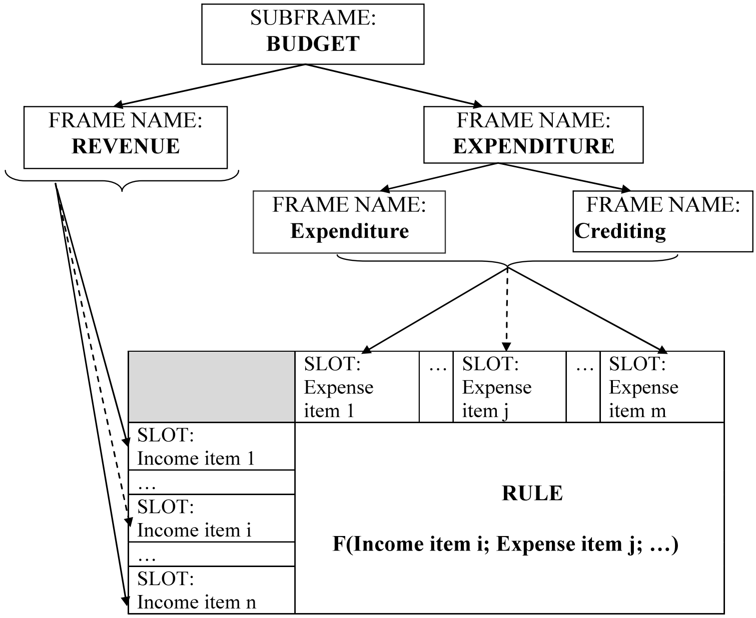


Fig. 1.10 A structural schema of the knowledge base for a regional budget

Figure 1.10 shows the structure of the knowledge base for budget formation. It reflects all interactions of elements of classification groups and their functional roles.

Income and expense frames can be chosen at any level of budget items. The rules are represented by the interaction matrix which gives the principles of distribution of money resources among budget parts. These data can be used for further analysis of budget indicators.

Thus, the expert can input budget data in the frame-based model according to the budget structure. Moreover, it is possible to call up any frame at any level of the budgetary classification which will contain the values and list of all subordinated frames.

1.2.3.3 An Example of the Frame-Based Model

To test the applicability of the above knowledge-representation models, we will consider their ability to recognize items. For example, to describe the following expense item:

5.2.254.39.30.132. "*Purchasing of medicines and other items used for medical purposes" 7.312 million tg*.

The following frame specification can be made:

FRAME NAME: expense item
FRAME TYPE: item

FUNCTIONAL GROUP: 5 Public health services
SUBFUNCTION: 2 Public health protection
PROGRAM: 39 Provision of sanitary and epidemiologic well-being
SUBPROGRAM: 30 Sanitary and epidemiologic stations
INSTITUTION: 254 Executive office of public health services financed from the local budget
SPECIFICITY TYPE: 130 Purchasing of goods
SPECIFICITY of EXPENSES: 2 Purchasing of medicines and other items used for medical purposes
SUM: 7.312 million tg.
PRIORITY: 2
PRIORITY DEGREE: 0.625

Let us consider another example where the budget element is not the final vertex.

In the framework of the government program "Rendering medical aid to HIV-infected patients" number 5.2.254.31, the sum of 12,351,000 tenghe was allocated.

The corresponding frame specification is expressed as:

FRAME NAME: expense item
FRAME TYPE: item
FUNCTIONAL GROUP: 5 Public health services
SUBFUNCTION: 2 Public health protection
PROGRAM: 31 Rendering medical aid to HIV-infected patients
SUBPROGRAM: 31 Rendering medical aid to HIV-infected patients
INSTITUTION: 254 Executive office of public health services financed from the local budget
SPECIFICITY TYPE:
SPECIFICITY OF EXPENSES:
SUM: 12.351 million tenghe.
PRIORITY: 1
PRIORITY DEGREE: 5.25

Let us consider an example of the frame-based model for the income item:

The income tax paid by the legal entities withheld from the payment source has generated 4.755 million tenghe.

For the revenue item the frame specification is made as follows:

FRAME NAME: income item
FRAME TYPE: item
CATEGORY: 1 Tax revenues
CLASS: 1 income tax
SUBCLASS: 1 income tax from legal bodies
SPECIFICITY: 3 income tax from the legal entities withheld from the payment source.
SUM: 4.755 million tg.

Thus, every slot is the subframe and relates to the corresponding frame-specification.

In the database every frame is a "pack of cards" item or a set of homogeneous items.

Thus, the submodels "Structure" and "Item" are presented in the form of the knowledge frame model on the basis of connections presented in the semantic model.

1.3 Mathematical Budget Models

System analysis is a methodology, and modeling is a tool for working out and using means facilitating formation and analysis of aims and functions of control systems of any complexity, including the budget. Among the applied modeling methods an important place is occupied by economic mathematical methods which are applied to the solution to the following practical problems [8]:

1. Improvement of economic information system for budgetary planning: streamlining of registration-accounting information system, detection of drawbacks in the available information, and development of requirements for preparing new information or updating information.
2. Intensification and better accuracy of balance calculations and projections of expenditures and revenues.
3. Deepening of the quantitative analysis of formation processes and use of budgetary funds, quantitative estimation of the consequences of specification and redistribution of estimated expenses within changeable items, etc.
4. Solution of principally new problems of budget planning, formation, and management. The dataware of budget planning and management must be based on up-to-date hardware and software.

Characteristics of the budget model:

- the model operates with data from one fiscal year;
- input data: forecasted values of income and expenses relative to the calculated base of the budgetary system objects;
- output data: the budget project close to the real execution;
- influence of external factors on the system—changes in the receipts and expenditure plans;
- the result of model operation—possibility to adjust the budget project according to the real execution.

Assumptions in the budget model:

- in order to control budgetary funds and their address movement, the amount of change in one income item is transferred to one expense item;
- all revenue items form the general state budget;

– when the budget is distributed for expenses, only the budget volume is considered and sources of receipts are not taken into account;[1]
– every revenue unit is *proportionately* distributed among all expenditure items.

1.3.1 Static Mathematical Budget Model

Let m be the number of income items from which the budget is formed, and n the number of expense items by which the budget must be distributed. Based on the assumption of proportional distribution of each revenue unit among all expenditure items, we will introduce a coefficient reflecting a share of *each* income item in the *given* expense item:[2]

$$\forall i \in \{\overline{1,n}\} \quad a_{ij} \stackrel{\text{def}}{=} \frac{y_i}{x_j}, \quad j = \overline{1,m}, \tag{1.1}$$

where y_i is the absolute value of the i-th expense item; x_j is the absolute value of the j-th income item; and a_{ij} is a coefficient of interaction of income and expense items.

The coefficients of interaction can be considered in two aspects:

- in terms of distribution of an income item;
- in terms of formation of an expenditure item.

In terms of formation of each fixed i-th expenditure item (with participation of all income items), we have:

$$a_{ij} \stackrel{\text{def}}{=} \frac{y_i}{x_j} \Rightarrow y_i = a_{ij}x_j \Rightarrow \sum_{j=1}^{m} y_i = \sum_{j=1}^{m} a_{ij}x_j \Rightarrow y_i \sum_{j=1}^{m} 1 = \sum_{j=1}^{m} a_{ij}x_j$$
$$\Rightarrow y_i m = \sum_{j=1}^{m} a_{ij}x_j \Rightarrow y_i = \frac{1}{m}\sum_{j=1}^{m} a_{ij}x_j. \tag{1.2}$$

So that for each $i = \overline{1,n}$:

$$y_i = \frac{1}{m}\sum_{j=1}^{m} a_{ij}x_j. \tag{1.3}$$

[1] This means that it is impossible to analyze, for example, the influence of expense distribution on future receipts to budget funds, or the necessity of financing of an individual sphere of the state activity in order to increase receipts. This is because the state budget mainly finances those spheres which in real time do not give financial return (public assistance, education, etc.).

[2] The sign "$\stackrel{\text{def}}{=}$" means "equal by definition".

In the matrix form the connection between the revenue and expenditure items can be written as:

$$Y = \frac{1}{m} AX, \tag{1.4}$$

where

$$X = \begin{pmatrix} x_1 \\ \dots \\ x_j \\ \dots \\ x_m \end{pmatrix}, \qquad Y = \begin{pmatrix} y_1 \\ \dots \\ y_i \\ \dots \\ y_n \end{pmatrix}$$

are, respectively, revenue and expenditure vectors of the unified budget classification;

$$A = \begin{pmatrix} a_{11} & \dots & a_{1j} & \dots & a_{1m} \\ \dots & \dots & \dots & \dots & \dots \\ a_{i1} & \dots & a_{ij} & \dots & a_{im} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{nj} & \dots & a_{nm} \end{pmatrix}$$

is an $n \times m$ matrix of interactions between revenue and expenditure items.

The matrix A is a compact data carrier containing data on the internal state of the system and principles of budgetary fund distribution by expense items. One of the states of matrix A can be used as a reference variant of budget planning.

The main principle of the budgetary system is that the budget state must be balanced for any level of budget classification:

$$\sum_{i=1}^{n} y_i^0 \stackrel{\text{def}}{=} \sum_{j=1}^{m} x_j^0. \tag{1.5}$$

According to this principle, the interaction matrix must lead to the balanced state of the budget item and preserve it in all possible current adjustments.

Let vectors X_0 and Y_0 characterize a balanced state of the budget. According to (1.4), we have:

$$Y_0 = \frac{1}{m} A_0 X_0 \tag{1.6}$$

or in the coordinate form

$$y_i^0 = \frac{1}{m} \sum_{j=1}^{m} a_{ij}^0 x_j^0, \tag{1.7}$$

where

$$X_0 = \begin{pmatrix} x_1^0 \\ \dots \\ x_j^0 \\ \dots \\ x_m^0 \end{pmatrix}, \qquad Y_0 = \begin{pmatrix} y_1^0 \\ \dots \\ y_i^0 \\ \dots \\ y_n^0 \end{pmatrix}$$

are, respectively, revenue and expenditure vectors for the balanced state of the budget; m is the number of budget items; and A_0 is the matrix of interactions bringing the budget into a balanced state.

In terms of distribution of each fixed j-th income item among all expenditure items, $i = \overline{1, n}$, we have:

$$a_{ij} \stackrel{\text{def}}{=} \frac{y_i}{x_j} \Rightarrow \sum_{i=1}^{n} a_{ij} = \sum_{i=1}^{n} \frac{y_i}{x_j} \Rightarrow \sum_{i=1}^{n} a_{ij} = \frac{1}{x_j} \sum_{i=1}^{n} y_i. \tag{1.8}$$

The ratio (1.13) is also valid for the balance situation:

$$\sum_{i=1}^{n} a_{ij}^0 = \frac{1}{x_j^0} \sum_{i=1}^{n} y_i^0. \tag{1.9}$$

Relations (1.9) and (1.5) enable us to form a conclusion: *The sum of the j-th column elements of the balance interaction matrix is equal to the ratio of all balanced budget income to the income of the j-th item.*

Actually, for the balanced state of the budget items of the interaction matrix A_0:

$$\left. \begin{aligned} \sum_{i=1}^{n} a_{ij}^0 &= \frac{\sum_{i=1}^{n} y_i^0}{x_j^0} \\ \sum_{i=1}^{n} y_i^0 &= \sum_{j=1}^{m} x_j^0 \end{aligned} \right\} \Rightarrow \sum_{i=1}^{n} a_{ij}^0 = \frac{\sum_{i=1}^{n} y_i^0}{x_j^0} = \frac{\sum_{j=1}^{m} x_j^0}{x_j^0}$$

whence we have (with the same notations):

$$\sum_{i=1}^{n} a_{ij}^0 = \frac{\sum_{j=1}^{m} x_j^0}{x_j^0}. \tag{1.10}$$

An obvious consequence of (1.10) is the statement that every i-th item of the balanced budget is the ratio of the balanced budget itself to the sum of coefficients of interaction for all expense items among which the j-th item was distributed:

$$x_j^0 = \frac{\sum_{j=1}^{m} x_j^0}{\sum_{i=1}^{n} a_{ij}^0}. \tag{1.11}$$

Let the VB budget be formed. In other words, it is necessary to plan revenues for each j-th revenue item and distributions for each i-th item of expenses. Using the interaction matrix of the balanced budget items of the previous years A_0 as the basic estimated indicator and taking into account (1.11), we can *calculate* the elements of the formed vector of incomes by the formula:

$$x_j = \frac{\mathrm{VB}}{\sum_{i=1}^{n} a_{ij}^0}, \quad j = \overline{1, m}. \tag{1.12}$$

According to (1.3) we "*state*" the elements of the expense vectors—distribute planned incomes:

$$y_i = \frac{1}{m} \sum_{j=1}^{m} a_{ij} x_j, \quad i = \overline{1, n}.$$

Analyzing (1.11) and (1.12), we can assign a certain sense to the quantity $1/\sum_{i=1}^{n} a_{ij}$, $j = \overline{1, m}$, i.e., the quantity inverse to the sum of the elements of the j-th column of the matrix of VB budget interaction (characterizes the distribution of the j-th income item) gives the fraction corresponding to the planned j-th income item in the total budget volume.

Consider the sum of shares of the planned budget:

$$\sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}} = \sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} \frac{y_i}{x_j}} = \sum_{j=1}^{m} \frac{x_j}{\sum_{i=1}^{n} y_i} = \frac{\sum_{j=1}^{m} x_j}{\sum_{i=1}^{n} y_i}. \tag{1.13}$$

The following variants of the budget state are known: balance, surplus, and deficit. According to (1.13) we have:

$$\sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}} = \frac{\sum_{j=1}^{m} x_j}{\sum_{i=1}^{n} y_i}$$
$$\Rightarrow \sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}} \begin{cases} 1, & \text{balance } \sum_{i=1}^{n} y_i \equiv \sum_{i=1}^{n} y_i^0 = \sum_{j=1}^{m} x_j^0 \equiv \sum_{j=1}^{m} x_j, \\ <1, & \text{deficit } \sum_{j=1}^{m} x_j < \sum_{i=1}^{n} y_i, \\ >1, & \text{surplus } \sum_{j=1}^{m} x_j > \sum_{i=1}^{n} y_i. \end{cases} \tag{1.14}$$

Hence, (1.14) can be used to analyze the budget state:

- if $\sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}} = 1$ the budget state is balanced;
- if $\sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}} < 1$ the budget state is deficit, and the deficit fraction is:

$$Df = \left(1 - \sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}}\right) \cdot 100\,\%;$$

- if $\sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}} > 1$ the budget state is surplus, and the surplus fraction is:

$$Pf = \left(1 - \sum_{j=1}^{m} \frac{1}{\sum_{i=1}^{n} a_{ij}}\right) \cdot 100\ \%.$$

Therefore *the budget state is determined on the basis of information about the values of elements in the interaction A matrix.*

1.3.2 Mathematical Model of Interaction of Income and Expenditure Items

According to the condition of budget balance, changes in the prognostic revenue values must cause corresponding changes in the expense items.

The increment of the matrix of interaction of budget items caused by the changes in incomes and expenses is equal to:

$$\Delta A = A' - A_0, \tag{1.15}$$

where A_0 is the initial[3] matrix of interaction; A' is the matrix of interaction corresponding to changed (specified) income–expense items.

The elements of ΔA matrix:

$$\Delta a_{ij} = a'_{ij} - a^0_{ij} = \frac{y'_i}{x'_j} - \frac{y^0_i}{x^0_j} = \frac{y^0_i + \Delta y_i}{x^0_j + \Delta x_j} - \frac{y^0_i}{x^0_j}$$
$$= \frac{x^0_j \Delta y_i - y^0_i \Delta x_j}{x^0_j (x^0_j + \Delta x_j)} = \frac{\Delta y_i}{(x^0_j + \Delta x_j)} - \frac{y^0_i \Delta x_j}{x^0_j (x^0_j + \Delta x_j)}, \tag{1.16}$$

where Δs_k is the absolute value of changes in the k-th income/expense item:

$$\Delta s_k = s'_k - s^0_k \begin{cases} >0, & \text{if the } k\text{-th item increased,} \\ <0, & \text{if the } k\text{-th item decreased,} \\ =0, & \text{if the } k\text{-th item did not change.} \end{cases}$$

Taking into account the interaction coefficients (1.1) and (1.16), the matrix of changes in the budget state can be calculated by the following formula:

$$\Delta a_{ij} = \frac{\Delta y_i}{x^0_j + \Delta x_j} - a^0_{ij} \frac{\Delta x_j}{x^0_j + \Delta x_j}. \tag{1.17}$$

---

[3]It is possible to take a matrix with averaged elements of balance matrices for the period of stable state development as the initial (reference) matrix of interaction.

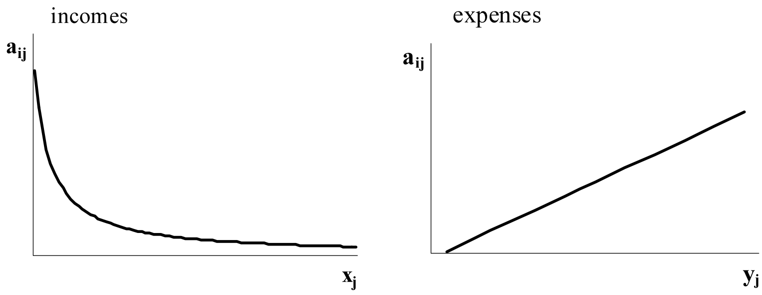


Fig. 1.11 Isoquantum of functions of interaction matrix elements as a function of income and expense items

From (1.17) we can determine the rate of change in the interaction coefficients in case of changes in income items and no changes in expense items $\Delta y_i = 0, i = \overline{1, n}$:

$$\frac{da_{ij}}{dx_j} = \lim_{\Delta x_j \to 0} \frac{\Delta a_{ij}}{\Delta x_j}\bigg|_{\Delta y_i = 0} = - \lim_{\Delta x_j \to 0} a_{ij} \frac{\Delta x_j}{(x_j + \Delta x_j)\Delta x_j} = -a_{ij} \frac{1}{x_j}$$
$$\frac{da_{ij}}{dx_j} = -a_{ij} \frac{1}{x_j} \Rightarrow \int \frac{da_{ij}}{a_{ij}} = - \int \frac{dx_j}{x_j} \Rightarrow \ln |a_{ij}| = \ln \frac{C}{|x_j|}$$

where:

$$a_{ij} = \frac{C}{x_j}, \quad C = \text{const.} \tag{1.18}$$

Hence, if the income item changes and the expense items are constant, the interaction coefficients are inversely proportional to the income items. The isoquantum of the function of interaction coefficients is shown in Fig. 1.11.

From (1.17) we can see that if the income item changes and the expense items are constant $\Delta x_j = 0, j = \overline{1, m}$ the rate of change in the elements of the interaction budget model matrix will be expressed as:

$$\begin{aligned} \frac{da_{ij}}{dy_i} &= \lim_{\Delta y_i \to 0} \frac{\Delta a_{ij}}{\Delta y_i}\bigg|_{\Delta x_j = 0} = \lim_{\Delta y_j \to 0} \frac{\Delta y_i}{x_j \Delta y_i} = \frac{1}{x_j}, \\ \frac{da_{ij}}{dy_i} &= \frac{1}{x_j} \Rightarrow \int da_{ij} = \int \frac{dy_i}{x_j}, \end{aligned} \tag{1.19}$$

where:

$$a_{ij} = \frac{y_i}{x_j} + C, \quad C = \text{const.} \tag{1.20}$$

Hence, if an expense item changes and income items are constant, the elements of the interaction matrix are inversely proportional to the corresponding income item and directly proportional to the expense items.

Thus, provided that the expense items have constant values, the change in the j-th income item will lead to the following changes in the matrix of budget items interaction:

$$\Delta A_{\Delta x_j} = \begin{pmatrix} 0 & \cdots & -a^0_{1j}\frac{\Delta x_j}{x^0_j+\Delta x_j} & \cdots & 0 \\ 0 & \cdots & -a^0_{2j}\frac{\Delta x_j}{x^0_j+\Delta x_j} & \cdots & 0 \\ & \cdots & & \cdots & \\ 0 & \cdots & -a^0_{nj}\frac{\Delta x_j}{x^0_j+\Delta x_j} & \cdots & 0 \end{pmatrix}.$$

Only the j-th column of the initial interaction matrix is changed here.

Similarly, if any changes in the strategy of the state development take place, and an individual program gets additional financing, the j-th line of the budget matrix changes:

$$\Delta A_{\Delta y_i} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \frac{\Delta y_i}{x^0_1} & \frac{\Delta y_i}{x^0_2} & \cdots & \frac{\Delta y_i}{x^0_m} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

For example, let us take a balanced budget $Y_0 = \frac{1}{m}A_0X_0$ and a changed budget $Y' = \frac{1}{m}A'X'$. Let us assume that Δx_j, a change in the j-th income item, caused change only in the i-th expense item (other items did not change):

$$\begin{pmatrix} y'_1 \\ y'_2 \\ \cdots \\ y_i \\ \cdots \\ y'_n \end{pmatrix} = \frac{1}{m}\cdot\begin{pmatrix} a'_{11} & a'_{11} & \cdots & a'_{1j} & \cdots & a'_{1m} \\ a'_{21} & a'_{22} & \cdots & a'_{2j} & \cdots & a'_{2m} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a'_{i1} & a'_{i2} & \cdots & a'_{ii} & \cdots & a'_{im} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a'_{n1} & a'_{n2} & \cdots & a'_{nj} & \cdots & a'_{nm} \end{pmatrix}\cdot\begin{pmatrix} x'_1 \\ x'_2 \\ \cdots \\ x'_j \\ \cdots \\ x'_m \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} y^0_1 \\ y^0_2 \\ \cdots \\ y^0_i + \Delta y_i \\ \cdots \\ y^0_n \end{pmatrix} = \frac{1}{m}\cdot\begin{pmatrix} a'_{11} & a'_{11} & \cdots & a'_{1j} & \cdots & a'_{1m} \\ a'_{21} & a'_{22} & \cdots & a'_{2j} & \cdots & a'_{2m} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a'_{i1} & a'_{i2} & \cdots & a'_{ii} & \cdots & a'_{im} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a'_{n1} & a'_{n2} & \cdots & a'_{nj} & \cdots & a'_{nm} \end{pmatrix}\cdot\begin{pmatrix} x^0_1 \\ x^0_2 \\ \cdots \\ x^0_j + \Delta x_j \\ \cdots \\ x^0_m \end{pmatrix}. \tag{1.21}$$

Taking into account (1.1), i.e., $a'_{ks} \stackrel{\text{def}}{=} \frac{y'_k}{x'_s}$, $s = \overline{1, m}, k = \overline{1, n}$, from (1.21) we get the formulae transforming the elements of the changed interaction matrix:

$$\begin{aligned} a'_{ks} &= \frac{y'_k}{x'_s} = \frac{y^0_k}{x^0_s} \\ &= a^0_{ks}, \quad \text{if } k = 1, \dots, i-1, i+1, \dots, n; s = 1, \dots, j-1, j+1, \dots, m; \\ a'_{is} &= \frac{y'_i}{x'_s} = \frac{y^0_i + \Delta y_i}{x^0_s} = a^0_{is} + \frac{\Delta y_i}{x^0_s}, \quad \text{if } s = 1, \dots, j-1, j+1, \dots, m; \\ a'_{kj} &= \frac{y'_k}{x'_j} = \frac{y^0_k}{x^0_j + \Delta x_j}, \quad \text{if } k = 1, \dots, i-1, i+1, \dots, n; \\ a'_{ij} &= \frac{y'_i}{x'_j} = \frac{y^0_i + \Delta y_i}{x^0_j + \Delta x_j}. \end{aligned} \tag{1.22}$$

Let us determine the response of the vector of expense items to the change in the j-th income item. We will express the change in the abstract vector as follows:

$$\begin{pmatrix} v'_1 \\ \dots \\ v''_i \\ \dots \\ v'_n \end{pmatrix} = \begin{pmatrix} v_1 \\ \dots \\ v_i + \Delta v_i \\ \dots \\ v_n \end{pmatrix} = \begin{pmatrix} v_1 \\ \dots \\ v_i \\ \dots \\ v_n \end{pmatrix} + \begin{pmatrix} 0 \\ \dots \\ \Delta v_i \\ \dots \\ 0 \end{pmatrix}$$

or in the vector form $V' = V + \Delta V$.

Then we have:

$$\left. \begin{aligned} Y' &= \tfrac{1}{m} A' X' \\ Y_0 &= \tfrac{1}{m} A_0 X_0 \\ Y' &= Y_0 + \Delta Y \\ X' &= X_0 + \Delta X \end{aligned} \right\} \Rightarrow \Delta Y = \frac{1}{m}\big(A' - A_0\big) X_0 + A' \Delta X$$

or

$$\Delta Y = \frac{1}{m} \Delta A X_0 + A' \Delta X. \tag{1.23}$$

From the equality (1.23) we see that changes in the budget items linearly depend on the changes in the values of the interaction matrix elements.

The obtained information can be used for analysis and comparison of stable and crisis budget conditions. During a period of stable economic development it is possible to assume that the distribution of budgetary funds among the expense items remains the same during a certain period of time, and receipts from the income sources remain proportional. Such an assumption allows us to leave the elements of interaction at certain levels of the budget matrix unchanged and use them in the next year's budget calculation:

$$Y' = \frac{1}{m} A_0 X', \tag{1.24}$$

where A_0 is the reference balanced interaction matrix; X', Y' are planned income–expense items.

Considering the matrix of interaction of the budget items in the dynamic mode (in the mode of transition from one time period to another), it is possible to assume that in a certain period of time a_{ij} coefficients will be constant and have values obtained according to the strategy of state development.

The above results show that the budget state can be analyzed and forecast on the basis of the matrix of interaction of budget items. The usage of the given model will enable us to trace changes in the expense items caused by change in a certain income item.

1.3.3 Model of Budget Sensitivity

The budget consists of two balanced basic parts: revenues and expenditures of financial funds. Situations in which it is necessary to make a justified choice of an item for current changes in the approved budget are not infrequent. It is logical to assume that, as a rule, it is revenue which deviates from planned expectations. In this case, it is necessary to find ways to make adjustments in expenses [8].

As different budget items have different absolute values, any change (percent of change) in the item in one part of the budget can differently influence changes (percent of change) in the items in the other part.

In studies of different economic processes the concept of *sensitivity* [22, 23] of interrelation of various technical and economic indicators is used. It can be *absolute* (rate of change in the indicator in response to the change in the factor) and *relative* (elasticity showing percentage of change in the indicator in response to a one-percent change in the factor).

One of the basic assumptions of the given model is *proportionality* of distribution of every unit of revenues among all expenses items, which enables us to introduce the concept of the coefficient of revenue and expense interaction (1.1). Hence, $y_i = a_{ij} x_j$. In these limitations[4] the relative sensitivity (elasticity) of the i-th budgetary expense items with respect to the j-th revenue items is equal to:

$$\left.\begin{array}{l} E^i_{x_j} = \dfrac{dy_i}{dx_j}\dfrac{x_j}{y_i} = \dfrac{d(a_{ij}x_j)}{dx_j}\dfrac{x_j}{y_i} = a_{ij}\dfrac{x_j}{y_i} \\ a_{ij} \overset{\text{def}}{=} \dfrac{y_i}{x_j} \Rightarrow y_i = a_{ij}x_j \end{array}\right\} \Rightarrow E^i_{x_j} = 1, \tag{1.25}$$

[4] A mathematical apparatus is adequate for system research to the extent to which the hypothesis and assumptions underlying the mathematical model reflect reality.

where $E^i_{x_j}$ is the elasticity of the i-th expense item with respect to the j-th income item.

Obviously, the reverse statement is also true: The unit elasticity indicates proportional distribution of the budget revenues among the expense items.

Owing to the additivity of differentiation, the elasticity of the whole budget with respect to the j-th income item is also equal to 1:

$$\begin{aligned} E^{\sum_i}_{x_j} &= \frac{d\sum_{i=1}^n y_i}{dx_j} \cdot \frac{x_j}{\sum_{i=1}^n y_i} = \sum_{i=1}^n \frac{d(y_i)}{dx_j} \cdot \frac{x_j}{\sum_{i=1}^n y_i} \\ &= \sum_{i=1}^n a_{ij} \frac{x_j}{\sum_{i=1}^n y_i} = \frac{\sum_{i=1}^n a_{ij}x_j}{\sum_{i=1}^n y_i} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n y_i} = 1, \end{aligned}$$

where $E^{\sum_i}_{x_j}$ is the elasticity of the budget $\sum_{i=1}^n y_i$ (expense items) with respect to the j-th income item.

In budget planning it is also possible to formulate the inverse problem: How sensitive are the income items (or the whole budget) to the changes in the expense items?

Let us make similar calculations of elasticity for the expense item (with the corresponding notations):

$$\left.\begin{aligned} &E^j_{y_i} = \frac{dx_j}{dy_i}\frac{y_i}{x_j} = \frac{d(y_i/a_{ij})}{dy_i}\frac{y_i}{x_j} = \frac{1}{a_{ij}}\frac{y_i}{x_j} \\ &a_{ij} \overset{\text{def}}{=} \frac{y_i}{x_j} \Rightarrow x_j = \frac{y_i}{a_{ij}} \end{aligned}\right\} \Rightarrow E^j_{y_i} = 1;$$

$$\begin{aligned} E^{\sum_j}_{y_i} &= \frac{d\sum_{j=1}^m x_j}{dy_i} \cdot \frac{y_i}{\sum_{j=1}^m x_j} = \sum_{j=1}^m \frac{d(x_j)}{dy_i} \cdot \frac{y_i}{\sum_{j=1}^m x_j} \\ &= \sum_{j=1}^m \frac{1}{a_{ij}} \cdot \frac{y_i}{\sum_{j=1}^m x_j} = \frac{\sum_{j=1}^m \frac{y_i}{a_{ij}}}{\sum_{j=1}^m x_j} = \frac{\sum_{j=1}^m x_j}{\sum_{j=1}^m x_j} = 1. \end{aligned}$$

Therefore, *under the assumptions of the considered model, the use of elasticity for estimating the response of the budget or of its individual items to the change in an income/expense item is not informative.*

Absolute sensitivity of individual budget items and the budget as a whole is expressed as:

$$\left.\begin{aligned} &e^i_{x_j} = \frac{dy_i}{dx_j} \\ &a_{ij} \overset{\text{def}}{=} \frac{y_i}{x_j} \Rightarrow y_i = a_{ij}x_j \end{aligned}\right\} \Rightarrow e^i_{x_j} = a_{ij}, \tag{1.26}$$

where $e^i_{x_j}$ is the rate of change in the i-th expense item in response to the change in the j-th income item;

$$e^{\sum_i}_{x_j} = \frac{d\sum_{i=1}^n y_i}{dx_j} = \sum_{i=1}^n \frac{d(y_i)}{dx_j} = \sum_{i=1}^n a_{ij},$$

where $e^{\sum_i}_{x_j}$ is the rate of change in the $\sum_{i=1}^n y_i$ budget (expense items) in response to the change in the j-th income item;

$$\left.\begin{aligned} e^j_{y_i} &= \frac{dx_j}{dy_i} \\ a_{ij} &\overset{\text{def}}{=} \frac{y_i}{x_j} \Rightarrow x_j = \frac{y_i}{a_{ij}} \end{aligned}\right\} \Rightarrow e^j_{y_i} = \frac{1}{a_{ij}}, \tag{1.27}$$

where $e^j_{y_i}$ is the rate of change in the j-th income item in response to changes in the i-th expense item;

$$e^{\sum_j}_{y_i} = \frac{d\sum_{j=1}^m x_j}{dy_i} = \sum_{j=1}^m \frac{d(x_j)}{dy_i} = \sum_{j=1}^m \frac{1}{a_{ij}},$$

where $e^{\sum_j}_{y_i}$ is rate of change in the $\sum_{j=1}^m x_j$ budget (revenue) if there are changes in the i-th expense item.

Hence, *the assumptions of the considered model enable us to use absolute sensitivity* (*rate of change*) *for estimating the response of the budget or of its individual items to the change in the income/expense item, and information for this purpose is provided by the matrix of interaction coefficients.*

Let us introduce the concept of absolute sensitivity matrices of budget items:

$$\begin{aligned} e_x &= \left(e^i_{x_j}\right)_{n\times m} \middle| e^i_{x_j} = a_{ij}, \\ e_y &= \left(e^j_{y_i}\right)_{n\times m} \middle| e^j_{y_i} = \frac{1}{a_{ij}}, \end{aligned} \tag{1.28}$$

where e_x, e_y are matrices of absolute sensitivity of budget items (expenditures with respect to incomes and income with respect to expenditures, respectively), whose elements reflect the rate of change of each expenditure/income item in response to the change in the income/expenditure item.

Obviously, the higher the values of matrix elements, the faster the corresponding expenditure/income item responds to the changes in the interacting income/expenditure item.

In order to control the distribution of budgetary funds and support governmental development programs, the list of programs admissible for sequestering and the volume of sequestering of budget expense items are established by the government.

According to the Law on the Budget System, no changes in budget items exceeding 10 % are allowed. A bigger change requires reconsideration of the Budget Law for the current year and the list of governmental development programs.

For planning in the framework of the current year the cases of current changes in the budget are considered.

Target financing (such as crediting and target financing of special programs at the expense of additionally allocated funds) is not considered as it is a targeted special-purpose funds transfer. Thus, in the revenue part the changes in the transfer items are not considered, and in the expense part crediting is ignored.

Let:

- $y_{\max}$ be maximal admissible value of y_i;
- $y_{\min}$ be minimal admissible value of y_i;
- $x_{\max}$ be maximal admissible value of x_j;
- $x_{\min}$ be minimal admissible value of x_j.

Let us introduce the concept of *maximal and minimal sensitivity of expense items with respect to the x_j income item*:

$$e_{x_j}^{\max} = \frac{y_{\max}}{x_j} \quad \text{and} \quad e_{x_j}^{\min} = \frac{y_{\min}}{x_j}.$$

Hence, the i-th expense items are admissible to change if they satisfy the condition:

$$0 < e_{x_j}^{\min} \le a_{ij} \le e_{x_j}^{\max}.$$

Having introduced the concept of *maximal and minimal sensitivity of income items with respect to the y_i expense item*

$$e_{y_i}^{\max} = \frac{x_{\max}}{y_i} \quad \text{and} \quad e_{y_i}^{\min} = \frac{x_{\min}}{y_i},$$

one can consider that the j-th income items are admissible to change if they satisfy the following condition:

$$0 < e_{y_i}^{\min} \le \frac{1}{a_{ij}} \le e_{y_i}^{\max}.$$

Therefore the sensitivity matrix gives the quantitative criterion for making decisions on budget correction.

It is also possible to introduce a qualitative criterion of budget control—the degree of priority (for example, the weight determined or calculated by experts) and to construct an integral criterion according to one of the criteria of the decision-making theory.

If the increase in income ($\Delta x_j > 0$) is expected for only one item, it is reasonable to consider a special case for a single expense item (y_i) to get priority for redistribution under the program of social and economic development, i.e., *addressness* of items. This *will result in the reconstruction of the interaction matrix* according to the following formulas (with minimal changes only in the j-th column of the A matrix):

$$\begin{aligned} a_{kj}x_j &= a'_{kj}(x_j + \Delta x_j), \quad \forall k \neq i, \\ m\Delta y_i + a_{ij}x_j &= a'_{ij}(x_j + \Delta x_j), \quad \text{for } k = i, \end{aligned} \tag{1.29}$$

where a_{kj} are elements of the initial matrix of interaction; a'_{kj} are the elements of the interaction matrix reconstructed after the change in the j-th item in order to satisfy the condition of addressness and the assumption about distribution proportionality.

Owing to the singleness of the redistribution item, we get $\Delta y_i = \Delta x_j$. In this case, the elasticity of the expense items (the i-th item is given priority) relative to the changes in the j-th income item are equal to:

$$\begin{aligned} E^k_{x_j} &= \frac{\Delta y_k}{\Delta x_j}\frac{x_j + \Delta x_j}{y_k + \Delta y_k}\bigg|_{k=i} = \frac{\Delta y_i}{\Delta x_j}\frac{x'_j}{y'_i}\bigg|_{\Delta y_i = \Delta x_j} = \frac{1}{a'_{ij}} = \frac{x_j + \Delta x_j}{a_{ij}x_j + m\Delta x_j} \\ E^k_{x_j} &= \frac{\Delta y_k}{\Delta x_j}\frac{x_j + \Delta x_j}{y_k + \Delta y_k}\bigg|_{\substack{k \neq i \\ \Delta y_k = 0}} = 0. \end{aligned} \tag{1.30}$$

As we see, the elasticity depends on the value of the income item (x_j), the increment of the income item (Δx_j), and the interaction coefficient in the initial budget (a_{ij}) corresponding to the addressness of the item.

If the income increment in the item is insignificant, i.e., $\Delta x_j \to 0$, the aim to direct it only to the i-th item of expenses, provided all other elasticities are zero, means an "address" elasticity equal to:

$$E^i_{x_j} = \frac{1}{a'_{ij}} = \frac{x_j + \Delta x_j}{a_{ij}x_j + m\Delta x_j}\bigg|_{\Delta x_j \to 0} \approx \frac{1}{a_{ij}}. \tag{1.31}$$

Therefore, in case of insignificant changes in the income item, in order to estimate elasticity (sensitivity) it is possible to use both initial and transformed interaction matrices.

If a budget income item whose increment considerably exceeds the previous income level for the given item, i.e., $\Delta x_j \gg x_j$, is to be distributed, then, provided other elasticities are zero, we have an "address" elasticity equal to:

$$E^i_{x_j} = \frac{1}{a'_{ij}} = \frac{x_j + \Delta x_j}{a_{ij}x_j + m\Delta x_j}\bigg|_{\Delta x_j \gg x_j} \approx \frac{1}{m}. \tag{1.32}$$

Hence, in the model with the assumption about proportional distribution of income items, the above estimation of elasticity means, for example, attraction of a new income item to allocation (address allocation of funds) to the only expense item with elasticity inversely proportional to the number of income items.

Therefore:

- small changes in income must be allocated to one article with the maximal coefficient $\frac{1}{a_{ij}}$;

- as a rule, it is not rational to allocate new income budget items to a single expense item.

In case of decrease in receipts, the expense item is chosen, by contrast, according to the minimal coefficient $\frac{1}{a_{ij}}$.

All recommendations use initial assumptions of the model and singleness of the changed income item and the expense item corresponding to it. This allows application of the reasoning not more than m times during address budget replanning.

First of all, violation of the assumption of item addressness will entail the necessity to take into account priorities of expense items and nontrivial dependence on the choice of efficiency criterion.

References

1. Melnikov, V.D., Il'yasov, K.K.: Finances. Almaty (2001), p. 512
2. Stanford, J.: Free Trade and the Imaginary Worlds of Economic Modelers (1993)
3. Stiglitz, J.E.: Nobel prize lecture: information and the change in the paradigm in economics (2001)
4. Dugdale, D., Lyne, S.: Budgeting. CIMA Financ. Manag. 32–35 (2006)
5. Il'yasov, K.: State Budget. Rik, Almaty (1994), p. 268
6. Law on the budget system of the Republic of Kazakhstan (1999)
7. Wooster, J.: Current state and recommended directions of further development of the REFORM of the inter-budgetary relations in Kazakhstan. Econ. Stat. **1**, 2–13 (2000)
8. Mutanov, G.M., Shintemirova, A.U.: Mathematical Model of Budget. Astana (2003), 176 p.
9. Cooper, W.D., Morgan, R.G., Redman, A., Smith, M.: Capital budgeting models: theory vs. practice. Bus. Forum **26**(1/2), 15–19 (2001)
10. Derfuss, K.: The relationship of budgetary participation and reliance on accounting performance measures with individual-level consequent variables: a meta-analysis. Eur. Account. Rev. **18**(2), 203–239 (2009)
11. Ostergren, K., Stensaker, I.: Management control without budgets: a field study of "beyond budgeting" in practice. Eur. Account. Rev. **19**(1), 1–33 (2010)
12. Hannan, L., Rankin, F., Towry, K.: Flattening the organization: the effect of organizational reporting structure on budgeting effectiveness. Rev. Acc. Stud. **15**(3), 503–536 (2010)
13. Mutanov, G.M.: Risk Management in the Underground and Mining Enterprises. Gylym, Almaty (1996), p. 291
14. Ostreykovsky, V.A.: Informatics. Vysshaya shkola (2000), p. 511
15. Pospelov, D.A.: Artificial Intelligence. Vol. 2. Models and Methods. Radio i svyaz (1990), p. 304
16. Kusiak, E., Yan, K.T., Chau, U.M., et al. (eds.): Artificial Intelligence—Application in Integrated Production Processes. Machinostroeniye, Moscow (1977), p. 453
17. Tase, A., Gribbon, P., Lui, J. (eds.): Logical Approach to Artificial Intelligence: From Classical Logics to Logical Programming. Mir, Moscow (1990), p. 332
18. Jaeger, M.: Model-theoretic expressivity analysis. In: De Raedt, L., Frasconi, P., Kersting, K., Muggleton, S. (eds.) Probabilistic Inductive Logic Programming. Theory and Applications. Lecture Notes in Artificial Intelligence, vol. 4911. Springer, Berlin (2008)
19. Hussaini, S.W., Kummamuru, S., IEEE: A cybernetic model to understand the context of complex IT transformation programs overcoming challenges in current program management frameworks. In: 7th Annual IEEE International Systems Conference (Syscon), pp. 745–749 (2013)

20. Sullivan, S.D., Mauskopf, J.A., Augustovski, F., Caro, J.J., et al.: Budget impact analysis principles of good practice: report of the ISPOR 2012 budget impact analysis good practice II task force. Value Health **17**(1), 5–14 (2014)
21. Li, M., et al.: Frame layer rate control for H.264/AVC with hierarchical B-frames. Signal Process. Image Commun. **24**, 177–199 (2009). doi:10.1016/j.image.2008.12.009
22. Crognale, C.: Sensitivity and power budget of a homodyne coherent DP-QPSK system with optical amplification and electronic compensation. J. Lightwave Technol. **32**(7), 1295–1306 (2014)
23. Fruth, J., Roustant, O., Kuhnt, S.: Total interaction index: a variance-based sensitivity index for second-order interaction screening. J. Stat. Plan. Inference **147**, 212–223 (2014)

Chapter 2
Methods and Mathematical Models of Budget Management

2.1 Current Trends in Budgeting

Budget is a centralized monetary state fund the designated for financial support of implementation of its tasks and functions. The state put forward the task of creation of a program-based targeted control of budget funds with account for priorities of socio-economic development of the country. Taking into account specific features of the development of the budgeting process in the Republic of Kazakhstan and provisions of the new Law on Budget Planning it is expedient to consider the budgeting process as a management system based on the principles of budget programming.

In the conditions of socio-economic development of the state the problem of allocation of budgetary funds is one of the main tasks of public administration and state regulation of financial and fiscal processes. In the annual program of Government of the Republic of Kazakhstan the first priority task in the budget policy is implementation of long-term planning and construction of trajectories of program actions aimed at budget performance as a system of allocation of budget funds. A proper budget planning must take into account budget targets and expenses needed for their realization and must be focused on solution of key tasks and problems for the planned system as well as planned concentration of funds. It generated a necessity of implementation of a new method called *budget programming* as a means of development of a "clever" economy. This method justifies interrelation between economic forecasts and long-term targets in the development of the country/region; it forms the base for various scenarios of economic development, thus, implementing the program-targeted approach to planning.

The concept of budget programming is in compliance with one of the modern directions of the result-oriented budgeting (ROB), its aim is to interrelate the decisions on expenditures with the expected return of the expenditures, their effectiveness and efficiency. Budget programming is based on the principles of mathematical methods of the control theory, namely, program control which is oriented on the final result and may be considered as an optimization task of a system transfer from the initial state to the required one.

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_2

Budget forecasting is a methodological approach to budget control, in western countries such a methodology was called *budgeting oriented on the result or a concept* (model) *of result-oriented budgeting in the framework of the middle-term financial planning* (ROB). Its essence is the distribution of budget resources among administrators of budget funds and/or budget programs realized by the administrators with account for or in direct dependence on the achievement of concrete results (rendering of services) according to the middle-term priorities of the socio-economic policy and within the budget funds forecasted for the middle-term period.

The pioneers of the result-oriented budgeting are the USA, New Zealand, Australia, the Netherlands, Great Britain and Sweden. France and Germany started to implement the model of result-oriented budget planning later.

The term *result-oriented budgeting* (ROB) means such an approach to the budget process where spending of financial resources is related to expected important social results. Unlike the traditional system of cost-in-no-object approach answering the question, “How much must we spend?” the ROB system enables us to answer the question, “What social result will be achieved at the expense of spent funds?”

The present-day concepts of the result-oriented budgeting are based on the concept of the Program-Targeted Planning developed in the 1960–1970s in the USSR and the Planning-Programming-Budgeting System (PPBS) developed in the USA in the late 1950s–early 1960s.

The key unit in the ROB system is monitoring of the results. Based on its results the officials make decisions on the expediency of financing of a budget program in the sums specified in the previous fiscal year.

The international experience of ROB implementation [1–5] shows that the system of indicators of budget efficiency is used as an instrument of accounting of public authorities to the society.

In [2–4] the author presents a description of the interrelation between priorities at national, regional and municipal levels. The upper horizon of planning is presented by a five-year strategic plan of the territory development. The middle-term level is presented by a two-year plan-schedule of measures taken to realize the strategy (it is updated and specified by the results of each fiscal year) and a three-year financial plan (budget). Targets of the plan-schedule and financial plan concretize the long-term targets presented in the strategic plan. The first financial year is described in more detail than the following years. The next two years are described at a larger scale. Such a system makes it possible to maximally use the potential of the model of the result-oriented regional financial management.

According to [2–4] the experience of Great Britain demonstrates that ROB use makes it possible to develop approaches to solving the following problems:

– Allocation of budget funds not according to the types of expenditures but according to strategic plans;
– Rendering of services really needed by the population;
– Control of allocations of budget services by choosing the most economically efficient method of their rendering;
– Comparison of expenditure programs and choice of the most efficient ones by the results of efficiency of expenditures;

– Comparison of services in terms of their quality with other similar services in other countries, regions and cities;
– Higher transparency and better substantiation of budget expenditures;
– Determination of positive social effect of the service and not only its cost;
– Shift of the accent from the external control of the targeted use of financial allocations to the increase in internal responsibility and internal control of the efficiency of expenditures;
– Account for the consequences of decisions on the volume and structure of the expenditure items of the budget.

The ROB concept is a modern methodological approach to budget control, the principles of which are reflected in the program method of budget control.

2.2 Current State of Budget Control Methods and Mathematical Models

To forecast budget development there are a number of methods that can be used, including mathematical modeling, index, normative projection, expert evaluation, and balance, among others.

The first of these, the mathematical modeling method, is based on the usage of economic and/or mathematical models that allow a great number of interrelated factors influencing budget items to be taken into account. They also allow leeway in determining the technique of budget forecasting, and in choosing from among several budget variants the optimal one corresponding to the accepted strategy of social-economic development of the country and the budgetary policy being pursued. Such methods are considered in [10–27].

The index—or indicative—method is based on various indicators characterizing socio-economic development of the country/region. The indicators connect the decisions on expenditures with the expected returns from such expenditures, their efficiency, and their effectiveness. This concept is also used to assess the quality of budget control in order to improve the effectiveness of financial resource management in the area. Such methods are considered in [19].

The normative method is based on progressive norms and financial budgetary standards required to calculate budget revenue on the basis of established tax rates and a number of macroeconomic factors such as the tax burden, the budget deficit limit (percentage of GDP and budget expenditures), the maximal national debt, etc. This method was used to determine the budget paying capacity in [8, 31].

The method of expert evaluation is used when tendencies in the development of certain economic processes have not been determined, no analogues are available, and it is necessary to use special calculations performed by highly qualified experts. Various methods used to evaluate experts' estimations are outlined in [10, 20, 31, 32].

The balance method based on comparisons (assets with liabilities, the whole and its parts, etc.) enables the expenditures of any budget to be examined in relation to its revenue. It is a useful method for ascertaining proportions in allocating funds amongst different budgets. This method is considered in detail in [29], where it is supposed that each item of budget receipts is proportionately distributed among all expenditure items.

Based on the logic of constructing balance models, the relation between the revenue and expenditure items can be presented as a matrix expression:

$$\vec{G} = \frac{1}{m} A \vec{D}, \tag{2.1}$$

where $A = \{a_{ij}\}$ is a matrix of interaction between revenue and expenditure items, consisting of n lines and m columns; $\vec{D} = (d_1 \ \dots \ d_j \ \dots \ d_m)^T$ is the vector of revenue; $\vec{G} = (g_1 \ \dots \ g_i \ \dots \ g_n)^T$ is the vector of expenditures; $d_1, d_2, \dots, d_m$, $g_1, g_2, \dots, g_n$ are the items of the corresponding budget parts at a certain level of the uniform budget classification.

The elements of matrix A correspond to the values:

$$a_{ij} = \frac{g_i}{d_j}, \tag{2.2}$$

where a_{ij} are the elements of the matrix of interaction of revenue and expenditure, g_i is the absolute value of the i-th item of expenditures $i = \overline{1, n}$, d_j is the absolute value of the j-th item of revenue $j = \overline{1, m}$.

This research is basic for the development of the technique of program budget control, as its structure and characteristics are able to reflect the relation between revenues and expenditures. Such an approach makes it possible not only to estimate the interaction between budget revenues and expenditures but also to show the interaction between current and capital components of the budget expenditure parts, as well as interrelation between indicators of socio-economic development and the current state of budget expenditures.

While a static model characterizes the budget state at a certain moment in time, if budget programming is focused on the program of socio-economic development, the static model will not adequately reflect the state of the budget system [6]. Therefore it is expedient to consider control, distribution, and redistribution of budget funds as a dynamic system.

General formulation of the problem of program control Let we know the initial state $\tilde{g}_i^0$ $(i = \overline{1, n})$ of the expenditure part of the budget. In such a case the rule $u_i^0 = w_0(g_i^0)$ enables us to find an optimal value $\tilde{u}_i^0$ of the controlling action of the i-th budget item at the first step: $\tilde{u}_i^0 = w_0(\tilde{g}_i^0)$. Then the next state of the item i will be unambiguously determined by the equation of the item movement: $\tilde{g}_i^1 = f(\tilde{g}_i^0, \tilde{u}_i^0)$. This property makes it possible to determine the optimal value $\tilde{u}_i^1$ of the control action at the second stage: $u_i^1 = w_1(g_i^1)$. Using the same procedure further we come to the conclusion: the optimal control strategy $u_i^s = w_s(g_i^s), s = \overline{0, N}$ found using the principles of dynamic programming enables us for a given initial state $\tilde{g}_i^0$ of the item i to consequently determine the optimal values $\tilde{u}_i^0, \tilde{u}_i^1, \ldots, \tilde{u}_i^N$ of control action for the whole period of controlling of the item $g_i^{s+1} = f(g_i^s, u_i^s), s = \overline{0, N}$. The set of values of control action can be considered as a program control planned in advance for the item $g_i^{s+1} = f(g_i^s, u_i^s)$ which is in state $\tilde{g}_i^0$ at a given moment [14]. What structure does the control correspond? Let us suppose that there is an object of control—a budget item i and there is a control system—the department of budget planning, which sends impulses $\tilde{u}_i^s$, $s = \overline{0, N}$ to the input of the object calculated in advance for the initial state $\tilde{g}_i^0$ of the item i. It means that the control system must have an a priory information about the state $\tilde{g}_i^0$. The feedback is not needed. At the same time this open control system realizes the same level of control as does the system with the feedback. If we solve the control problem, when we know in advance the behavior of the object and the conditions of its functioning, the feedback principle is not important and closed control can be replaced by an open control i.e. *program control*.

The program control is based on program movements that can transfer the system from the point $x = x_0$ at $t_0 = 0$ to the point $x = x_1$ at $t_T = T$ [15].

This approach considers a relation between socio-economic development of the country/region with budget funds. The presence of the target object as a group of indicators of socio-economic development makes it possible to develop a program control that allows us to transfer the budget system to the desired state defined by a system of indicators and to estimate a possibility of achieving a desired level of development under some limitations of budget resources and budget potential determined for the period of the middle-term planning.

2.3 General Concept of the Programmable Method of Budget Mechanism Control

2.3.1 General Statement of the Problem of Budget Mechanism Control

Problem statement. Consider a budget system as a programmable control system able to respond to change fulfill the movement program and to find the best solution to the given control task [6, 33–36].

Let us consider a macroeconomic overview of the role of the budget in the state structure.

The state as a subject of control acts as a force of unification, cooperation, and integration. The state is a regulator controlling the socio-economic sphere and regulating the economic and political processes that determine the socio-economic development of the country. Therefore a very important role in the macroeconomic process is played by the budget—a centralized monetary fund that acts as an object regulating state financial funds. Its role is to observe and control socio-economic development of the country. To provide high-quality control of the socio-economic sphere the state uses a regulating function allocating budget funds to support or develop industries and regions; it can also purposefully intensify or hamper production rates, accelerate or decelerate growth of capital and private incomes, or change the supply and demand structure [7]. The dynamic nature of economic development makes it necessary to know or at least to have a qualitative picture of the future state of the socio-economic sphere, which depends on state financial resources accumulated in the budget. As a socio-economic sphere is a macroeconomic model, it is quite natural that the results of management are seen after some period of time, and are reflected in the budget [8, 9]. This means that the budget characterizes the level of state development. Therefore, it is possible to manipulate the budget or to develop a program algorithm of budget control, the output information of which will serve as control information for algorithm operation. And further, it is necessary therefore to formulate law regulating the "State–Budget–Socio-Economic Sphere" system, so as to provide optimal management of the object: the budget.

2.3.2 Cybernetic Approach to the Description of Budget Mechanism

In providing an overview of budget planning we will refer to the three principles of budget programming: determination of the program strategy for development, evaluation of budget resources, and planning of budget programs.

Definition 2.1 *The budget mechanism* is the process of management of budget resources.

Definition 2.2 *The program method* of budget management mechanism is the method based on the principles of budget programming.

Definition 2.3 *Budget programming* is the methodology of the middle-term planning and is focused on the final product, which must connect budget expenditures with the expected socio-economic results.

In terms of system analysis, the system of the budget management mechanism is determined by system objects, their properties, and their relations. Analysis of the process of budget management mechanism in terms of the system approach is based on the logical model described in the following schema (Fig. 2.1) [22].

The input in this schema is the budget dynamics, the system of indicators of socio-economic development.

The output is the forecast budget state, which can be corrected through the feedback system as the purpose of the suggested management model is sustainable budget development and achievement of a predetermined level of socio-economic development of the country/region.

The main purpose is realization of the strategy of national/regional development in terms of budget mechanism, taking into account resource limitations qualitatively

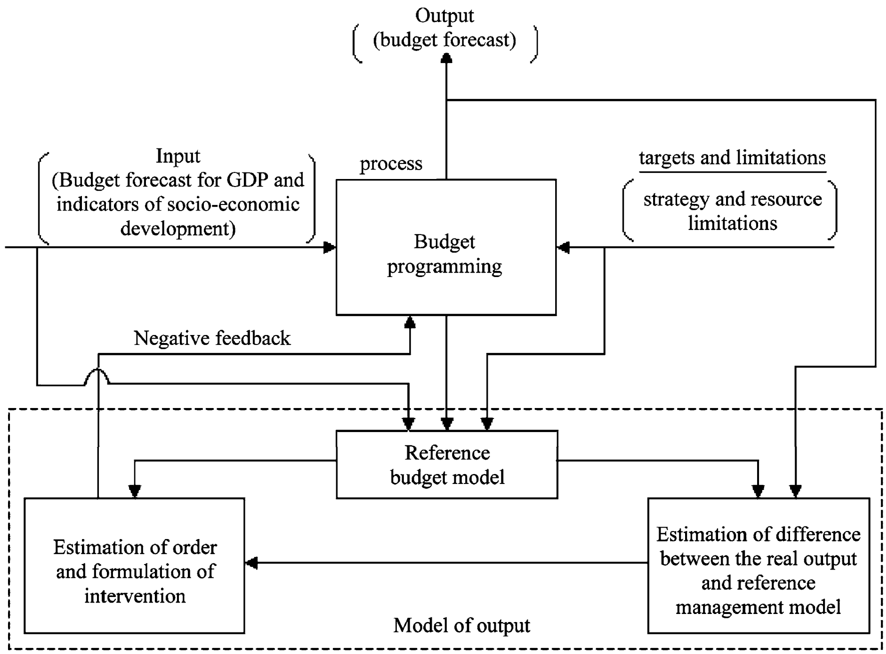


Fig. 2.1 A logical model schema

expressed in the general concept of sustainable budget development as a balance of income and expenditure budget parts.

The system is studied in terms of the process of budget mechanism management and the main principles of system operation, retention, and development.

Limitation is the amount of money allocated for development according to the strategic plan. However, in practice limitations and targets are not always coordinated in the solution of stated problems.

The main process is the process of budget programming focused on the final result through actions aimed at strategic distribution of budget resources for the main sectors of socio-economic development. The method focuses on a program-oriented control of the budget mechanism.

A block output model is made to compare the factual state of budget management with its predetermined value. This is a reference model which enables correcting actions aimed at system improvement.

The schema outlined above is a general functional one, where each system object is an independent model.

The main procedures of the program method of budget control are as follows:

Stage I: Development of a middle-term (3-year) plan of strategic development on the basis of the long-term development strategy: separation, definition, and analysis of targets and macroeconomic indicators of socio-economic development.

Stage II: Analysis of the budget system, namely data preparation as a forecast of the budget revenue using statistical forecast models; forecast of the expenditure base as a percentage of the GDP; and statistical analysis of the forecast dynamics of the main macroeconomic indicators of a basic scenario of economic development.

Stage III: Budget planning according to the targets of development. This stage solves the problem of allocation of budget resources among the expenditure items according to the concept of budget programming.

Stage IV: Evaluation of budget resource management and correction of priority directions of socio-economic development.

Budget management is transformation of the budget system to the desirable state or behavior.

The above stages of the program method may, in their turn, be subjected to further decomposition.

Let us describe the budget mechanism by means of the program method, taking into account decompositions at each stage.

The **first procedure**—development of the middle-term fiscal policy—is decomposed into the following stages:

1.1. Formulation of targets and resource limitations.
1.2. Preliminary forecast of the main macroeconomic factors according to the base scenario of the middle-term economic development formulated by the government; analysis and processing of indicators of the socio-economic and financial state of the country/region.
1.3. Identification and verbal (qualitative) description using scenario of system targets, their analysis and decomposition.
1.4. Development and analysis of qualitative criteria of target achievement.
1.5. Development and analysis of programs providing achievement of identified targets.

The first procedure describes operation of system objects—input, targets, and limitations which form a preparatory stage of the general model of the budget mechanism operation.

The second procedure is the stage of data generation by the mathematical analysis module and data designing for planning.

This **second procedure** of the program method for budget system analysis can be subdivided into the following stages:

2.1. Collection and processing of budget data (revenue and expenditures).
2.2. Analysis of the resource (revenue) budget potential.
2.3. Analysis and identification of the priority budget recipients.
2.4. Forecast of expenditure and revenue parts according to the basic development scenario, i.e., as percentage of GDP.

The **third procedure** describes strategic budget formation—formation of a reference budget by solving the problem of optimal control: distribution of capital investments among budget recipients.

Stages of construction of management mechanisms:

3.1. Formation and analysis of the forecast procedures for control systems—the task of middle-term budget distribution among recipients.
3.2. Formation and analysis of planning procedures inside the system—organization of program movement.
3.3. Formation of systems used to analyze, control, and assess functioning of elements, including research of stability and assessment of the budget system state.

The **fourth procedure** depicts development of scenarios of budget control. This procedure describes the system object—output.

The main functions of this mechanism are:

– Interpretation of obtained results.
– Formation of budget scenarios.
– Determination of budget type.
– Development of recommendations.
– Development of procedures to correct program strategy and budget state.

2.3.3 System Approach to the Mathematical Model of Budget Mechanism

Let us consider the budget system as an integrated single unit, and begin by formulating of the purpose of its functioning.

The purpose of the budget system functioning is planning of budget incomes and execution of the budget with regard to priorities established according to the tasks of socio-economic development [6].

A structural approach to the budget system enables us to determine the system elements and their interrelations. The budget structure may be presented as:

$$\vec{D} = (d_1 \quad \dots \quad d_j \quad \dots \quad d_m)^T$$

is a vector of receipts,

$$\vec{G} = \vec{X} + \vec{Y} = (g_1 \quad \dots \quad g_i \quad \dots \quad g_n)^T = (x_1 \quad \dots \quad x_i \quad \dots \quad x_n)^T + (y_1 \quad \dots \quad y_i \quad \dots \quad y_n)^T$$

is a vector of expenditures, where g_i is the absolute value of the i-th item of expenditures $i = \overline{1,n}$; x_i is the current component of the i-th item of expenditures $i = \overline{1,n}$; y_i is the capital component of the absolute value of the i-th item of expenditures $i = \overline{1,n}$; d_j is the absolute value of the j-th item of revenue $j = \overline{1,m}$.

Capital expenditures are investments allocated to the country's economy and development, i.e., investments in the development of infrastructure, urban development, creation and development of information systems, science, investments in human resources, etc. Current expenditures are expenditures for fulfillment of current needs of the state.

To accomplish the above task it is necessary to construct a mathematical model of the budget system—a model of optimal distribution of budget resources. This model refers to the category of analytical models in which the processes of operation of system elements are expressed as functional relations (algebraic, integral-differential, finite-difference, etc.) or logical conditions. The analytical model can be studied by the following methods: (a) the analytical method, applicable when explicit solutions for the unknown characteristics can be obtained in the general form; (b) the numerical method, for when the equations cannot be solved in the general form, and only numerical solutions for certain initial data can be obtained; and (c) the qualitative method, which is used when, in the absence of an explicit solution *per se*, some of its properties can be determined (for example, to estimate the solution's stability) [12, 28].

The model of the object of modeling, i.e., the budget system S, may be expressed as a set of quantities describing the process of a real system functioning, and in general forming the following subsets [12, 28, 33, 34]:

The set of input actions (revenue and expenditures) on the system

$$d_j \in D, \quad j = \overline{1, m_D}; \qquad g_i \in G, \quad i = \overline{1, n_Z};$$
$$x_i \in X, \quad i = \overline{1, n_X}; \qquad y_i \in Y, \quad i = \overline{1, n_Y}.$$

The set of external actions

$$f_l \in F, \quad l = \overline{1, n_F}.$$

The set of controlling actions of the system

$$u_k \in U, \quad k = \overline{1, n_U}.$$

The set of output (revenue and expenditures) system characteristics

$$d'_j \in D', \quad j = \overline{1, m_{D'}}; \qquad g'_i \in G', \quad i = \overline{1, n_{Z'}};$$
$$x'_i \in X', \quad i = \overline{1, n_{X'}}; \qquad y'_i \in Y', \quad i = \overline{1, n_{Y'}}.$$

In modeling of system S the input actions and external actions on the system E are independent (exogenous) variables which are expressed in vector form as follows:

$$\vec{d}(t) = \big(d_1(t), d_2(t), \ldots, d_{mD}(t)\big);$$
$$\vec{g}(t) = \big(g_1(t), g_2(t), \ldots, g_{nZ}(t)\big);$$
$$\vec{x}(t) = \big(x_1(t), x_2(t), \ldots, x_{nX}(t)\big);$$
$$\vec{y}(t) = \big(y_1(t), y_2(t), \ldots, y_{nY}(t)\big);$$
$$\vec{f}(t) = \big(f_1(t), f_2(t), \ldots, f_{nF}(t)\big),$$

whereas output characteristics and control actions are dependant (endogenous) variables and are expressed in vector form as follows:

$$\vec{d}'(t) = \big(d_1'(t), d_2'(t), \dots, d_{nD'}'(t)\big);$$
$$\vec{g}'(t) = \big(g_1'(t), g_2'(t), \dots, g_{nZ'}'(t)\big);$$
$$\vec{x}'(t) = \big(x_1'(t), x_2'(t), \dots, x_{nX'}'(t)\big);$$
$$\vec{y}'(t) = \big(y_1'(t), y_2'(t), \dots, y_{nY'}'(t)\big);$$
$$\vec{u}'(t) = \big(u_1'(t), u_2'(t), \dots, u_{nU'}'(t)\big).$$

The process of S system functioning is described in time by the operator R_S, which in the general case transforms exogenous variables into endogenous according to the law of system functioning by the rules:

$$\vec{d}'(t) = R_S(\vec{d}, \vec{f}, \vec{u}, t); \qquad \vec{g}'(t) = R_S(\vec{g}, \vec{f}, \vec{u}, t). \tag{2.3}$$

Expression (2.3) is a mathematical description of the behavior of the object (system) of modeling as a function of time t, i.e., it reflects dynamic properties of the system.

For static models the mathematical model (2.3) is an image of the properties of modeled objects D and G, $\{D, F, U\}$ and $\{G, F, U\}$, in two subsets which can be expressed in vector form as follows:

$$\vec{d} = r(\vec{D}, \vec{F}, \vec{U}); \qquad \vec{g} = r(\vec{G}, \vec{F}, \vec{U}). \tag{2.4}$$

The relations (2.3) and (2.4) can be predetermined in various ways: analytically (by means of formulas), as a graph, as a table, and so on. In some cases such relations can be obtained through the properties of the budget system S at certain moments of time called states. The state of the budget system S is characterized by vectors

$$\vec{s}' = \big(s_1', s_2', \dots, s_k'\big) \quad \text{and} \quad \vec{s}'' = \big(s_1'', s_2'', \dots, s_k''\big), \tag{2.5}$$

where $s_1' = s_1(t')$, $s_2' = s_2(t'), \dots, s_k' = s_k(t')$ at the moment $t' \in (t_0, T)$; $s_1'' = s_1(t'')$, $s_2'' = s_2(t''), \dots, s_k'' = s_k(t'')$ at the moment $t'' \in (t_0, T)$, etc., $k = \overline{1, n_S}$.

If the process of functioning of budget system S is considered as a sequential change of states $s_1(t), s_2(t), \dots, s_k(t)$, these states can be interpreted as point coordinates in the k-dimensional phase space. Each realization of the budget process will correspond to some phase trajectory. The set of all possible values of the budget state $\{\vec{s}\}$ is called the space of states of the modeled object—the budget system S, where $s_k \in S$.

In studying this model we did not take into account the external impact on the system, therefore in constructing the planning model the set of external impacts $f_l \in F, l = \overline{1, n_F}$ was not taken into account.

The state of the budget system S at the moment of time $t_0 < t^* \le T$ is fully determined by the initial conditions $\vec{s}^0 = (s_1^0, s_2^0, \dots, s_k^0)$, input conditions, and controlling actions $\vec{u}(t)$, undertaken during the time interval $t^* - t_0$, and is expressed by two vector equations:

$$\vec{s}(t) = \Phi\big(\vec{s}^{\,0}, \vec{d}, \vec{g}, \vec{u}, t\big); \tag{2.6}$$

$$\vec{d}(t) = R(\vec{s}, t), \qquad \vec{g}(t) = R(\vec{s}, t). \tag{2.7}$$

The first equation including the initial state $\vec{s}^{\,0}$ and exogenous variables $\vec{d}, \vec{g}, \vec{u}$ defines the vector-function $\vec{s}(t)$, and the second equation for the obtained values of states $\vec{s}(t)$ gives endogenous variables at the system output. Therefore the chain of equations of the object "input–states–output" enables us to determine characteristics of the system:

$$\vec{d}'(t) = R\big[\Phi\big(\vec{s}^{\,0}, \vec{d}, \vec{u}, t\big)\big], \qquad \vec{g}'(t) = F\big[\Phi\big(\vec{s}^{\,0}, \vec{g}, \vec{u}, t\big)\big]. \tag{2.8}$$

In this general case, the time in the system S model can be considered in the modeling interval $(0, T)$ as both continuous and discrete.

Therefore the mathematical model of the object—the budget system—is a finite subset of variables $\{\vec{d}(t), \vec{g}(t), \vec{u}(t)\}$ with their mathematical interrelations and characteristics $\vec{d}(t)$ and $\vec{g}(t)$.

2.4 Mathematical Models of Budget Expenditure

2.4.1 Construction of Program Movements for Budget Expenditure

Definition 2.4 The *program control* of the budget is budgeting focused on the final product as determined by the development program.

In the schema of middle-term budgeting the regulation time considerably exceeds the planning time, which makes the process of middle-term budgeting interrelated and coordinated in time, thus allowing us to consider the process of budget planning as discrete. Such a schema enables to reconsider (correct) the plan for future planning periods as they move closer, in order to reduce disproportions in development, and points out the underlying nature of the program-targeted planning method: program control [1, 6, 24].

Let us consider how program movements are constructed in the simplest case, in linear management systems [15, 28].

Let the expenditure budget part be described by a system of n differential equations in vector form, which describes the simplest case of the linear system of budget expenditure control

$$\dot{g}(t) = Pg(t) + Qu(t), \tag{2.9}$$

where

$$g^{(1)} = \big(g_1^{(1)} \quad g_2^{(1)} \quad \dots \quad g_n^{(1)}\big)^T,$$

$$g^{(2)} = \big(g_1^{(2)} \quad g_2^{(2)} \quad \dots \quad g_n^{(2)}\big)^T, \quad \dots, \quad g^{(n)} = \big(g_1^{(n)} \quad g_2^{(n)} \quad \dots \quad g_n^{(n)}\big)^T$$

is a vector characterizing the state of budget expenditure items; $P = \{\frac{y_i}{x_j}\}$ is $(n \times n)$ is a matrix describing the current state of the budget expenditure items whose elements reflect interconnection between capital (y_i) and current (x_j) expenditures; Q is a $(n \times r)$ matrix characterizing the program of achievement of the development level formulated in the strategic plan of socio-economic development for the middle-term period [28].

Let two constant vectors g_0 and g_1 define the initial and final state of the system (2.9). It is necessary to find such a vector-function $u = u(t)$ of r dimension, that solution to the system (2.9) starting at $t_0 = 0$ in the point $g = g_0$ goes to point $g = g_1$ at $t_T = T$ and the integral $\int_{t_0}^{t_T} u^T(\tau)u(\tau)d\tau$ is limited. Such controls $u = u(t)$ are called program controls [13].

Along with Eq. (2.9) let us consider a homogeneous equation

$$\dot{g} = Pg. \tag{2.10}$$

Let

$$g^{(1)} = \begin{pmatrix} g_1^{(1)} & g_2^{(1)} & \dots & g_n^{(1)} \end{pmatrix}^T,$$
$$g^{(2)} = \begin{pmatrix} g_1^{(2)} & g_2^{(2)} & \dots & g_n^{(2)} \end{pmatrix}^T, \quad \dots, \quad g^{(n)} = \begin{pmatrix} g_1^{(n)} & g_2^{(n)} & \dots & g_n^{(n)} \end{pmatrix}^T$$

be n linearly independent solutions of the homogeneous equation (2.10). Any such system of solutions is called a fundamental system of solutions.

Let us denote as $\Phi(t)$ a fundamental matrix of solutions to the system of Eqs. (2.10), at initial conditions $\Phi(t_0) = I_n, t_0 = 0$, where I_n is a unitary matrix of the order n [15, 28].

In the system (2.9) we will make a nonsingular linear transformation over vector g:

$$g = \Phi(t)z, \tag{2.11}$$

where vector z is a new vector-function. This vector-function satisfies the condition

$$\dot{z} = B(t)u, \tag{2.12}$$

where $B(t) = \Phi^{-1}(t)Q$ [15, 28].

For simplicity let us assume that $r = 1$. The integration of Eq. (2.12) from $t_0 = 0$ to $t_T = T$ gives

$$z_1 - z_0 = z(t_T) - z(t_0) = \int_{t_0}^{t_T} B(\tau)u(\tau)d\tau. \tag{2.13}$$

The system of linear integral equations (2.13) enables us to find program control $u(t)$.

A solution to Eqs. (2.13) will be sought in the form [15, 28]

$$u(t) = B^T c + v(t), \tag{2.14}$$

where c is a constant vector to be determined, $v(t)$ is a function summed up with its square in the interval $[t_0, t_T]$ such that

$$\int_{t_0}^{t_T} b_s(t)v(t)dt = 0, \quad s = 1, 2, \ldots, n. \tag{2.15}$$

Here $b_s(t)$ are components of vector B, and equality (2.15) expresses the condition of orthogonality of function $v(t)$ to all components of vector B.

Substituting (2.14) into (2.13) we find [15, 28]

$$z_1 - z_0 = A(t_T)c, \tag{2.16}$$

where $A(t_T) = \int_{t_0}^{t_T} B(t)B^T(t)\,dt$.

Equation (2.16) has a solution if $\det A(t_T) \neq 0$ or if matrix $A(t)$ has the same rank as the extended matrix $\vec{A} = \{A(t_T); z_1 - z_0\}$.

In order to understand the existence of program controls correctly, let us formulate the analogue of the classical theorem of program control [28] existence in terms of the budget system.

Theorem 2.1 *In order to obtain a program control which transfers the budget system (2.9) from any initial state g_0 to any other state g_1 within time t_T, it is necessary and sufficient that matrix $A = \int_{t_0}^{t_T} BB^t dt$, where $B = \Phi^{-1}Q$, be nonsingular. In this case the entire subset of program controls is determined by the formula*:

$$u = B^T c + v,$$

where $v(t)$ is a function summed up with its square in the interval $[t_0, t_T]$ and

$$\int_{t_0}^{t_T} Bv dt = 0, \qquad c = A^{-1}(t_T)\big[\Phi^{-1}(t_T)g_1 - g_0\big].$$

2.4.2 A Model of Program Control of the Expenditure Budget Part

After introduction of the general notations of the system and general procedure of budget mechanism control, let us consider the general formulation of the mathematical model of optimal distribution of budget resources.

The optimal distribution of budget resources is realized according to the priority directions and general concept of socio-economic development for the middle-term planning period. The task in allocation of budget resources, i.e., in developing a plan for dividing these resources among expenditure items, is to enable socio-economic development for the main macroeconomic indicators up to the desired level of the basic development scenario [6, 33–36].

The expenditure budget part consists of $i = \overline{1, n}$ expenditure items whose qualitative state is denoted as z_i.

The mathematical formalization [35] of the budget mechanism control is presented as a task of optimal distribution of budget resources ΔC among the expenditure items and budget recipients ($i = \overline{1,n}$) in order to bring budget potential of budget recipients to the planned state $\breve{Z}$ (from the initial state Z^0), where $\breve{Z} = (\breve{z}_1, \dots, \breve{z}_i, \dots, \breve{z}_n)$ is a vector characterizing the state of an expenditure budget item in the future, i.e., at the end of the middle-term planning period. Formulating the problem in this way can be considered a task of long-term planning. Plan $\breve{Z}$ ($\breve{Z} \geq 0$) is a final result of strategic development determining the final target of the budget mechanism functioning which defines the qualitative state of the reference budget model at the end of the middle-term planning period. In the plan $\breve{Z}$ the component $\breve{z}_i$ denotes the planned qualitative state of the expenditure item i ($i = \overline{1,n}$).

Vector $\breve{Z}$ is expressed as a sum of two vectors $\breve{Z} = \breve{X} + \breve{Y}$. Let us consider these parameters in the framework of investment distribution. $\breve{X} = (\breve{x}_1, \dots, \breve{x}_i, \dots, \breve{x}_n)$, $\breve{X} \geq 0$ is a vector of future expenditures designated to provide a guaranteed budget standard, vector $\breve{Y} = (\breve{y}_1, \dots, \breve{y}_i, \dots, \breve{y}_n)$, $\breve{Y} \geq 0$, is a vector of capital expenditures designated to enable formation of a budget characterizing socio-economic development in the framework of the guaranteed standard state defined by the basic development scenario.

The procedure of deriving vector $\breve{Z}$ is an auxiliary optimization task. Let us consider how the perspective state is derived.

Suppose that every year t, $t_0 \leq t \leq T$ the development of the budget recipient (an expenditure item as a functional group of budget classification) is proportional to the invested capital, i.e., if $u_i(t)$ is a capital investment directed to the development of the budget recipient i in a year t then the following relation can be written [6, 28]:

$$\begin{aligned} &z_i(t + \Delta t) = z_i(t) + \lambda_i u_i(t), \\ &\quad t_0 \leq t \leq T,\ i = \overline{1,n},\ \big(z_i(t) = x_i(t) + y_i(t)\big), \end{aligned} \tag{2.17}$$

where λ_i is a weight coefficient, with the formula for its calculation being given below, and Δt is a period of discretization equal, for example, to 1 year, 1 quarter, etc.

Every year t for item i the following balance relations must be fulfilled for item i [6, 28]:

$$\sum_i \big(x_i(t) + y_i(t)\big)\alpha_{ij} = y_j(t), \quad j = \overline{1,n}. \tag{2.18}$$

The element α_{ij} in the relation (2.18) is an element of the balance matrix $A = \{\alpha_{ij}\}$, $\alpha_{ij} \geq 0$; in matrix form the relation (2.18) is written as $(\breve{X} + \breve{Y})A = \breve{Z}A = \breve{Y}$ [29], where

$$\alpha_{ij} = \frac{y_i}{x_j \sum_{i=1}^{n} \frac{z_i}{x_j}}, \quad i = j = \overline{1,n}. \tag{2.19}$$

This coefficient shows which fraction corresponds to the capital component of the i-th item from all volume of the budget expenditure fund.

To determine the perspective plan of budget resource allocation it is necessary to know the planned volume of expenditures by the budget items for each period of the middle-term planning, $\sum_{i=1}^{n} \hat{u}_i(t),\ t_0 \le t \le T$, which is included in the development program as a rough estimation of the budget state for the planning period. Here $\hat{u}_i(t)$ is capital investments planned in the basic development scenario for a year t for the upper-level items in the budget classification functional group. Such a distribution plan is called a desirable plan of distribution of capital investments for the year t. This plan $\{\hat{u}_i(t)\}$ is used to determine the desired state of capital investments

$$\breve{z}_i = z_i(t) + \lambda_i \left(\sum_{t=t_0}^{T} \sum_{i=1}^{n} \hat{u}_i(t) \right). \tag{2.20}$$

Thus, we have found the perspective state of the expenditure part at the end of the middle-term planning.

The coefficient $\lambda_i, i = \overline{1, n}$ is a weight coefficient which characterizes the desired socio-economic development. The basic scenario contains a forecast of the main macroeconomic indicators which will be denoted as $I_q, q = \overline{1, k}$. In the formulation of the task we are interested in the average growth rate ($\overline{TR}_q$) of the above indicators in the planned period [6, 28]:

$$\overline{TR}_q = \sqrt[T-1]{\frac{I_T}{I_1}} 100\ \%, \tag{2.21}$$

where I_1, I_T are the values of the factor (indicator) at the beginning $t = t_0$ and end of the middle-term planning $t = T$.

In order to determine the weight coefficient it is also necessary to estimate the relation between the current state of the budget expenditure part and the current revenue base. This relation is expressed as a matrix of interaction of budget items $\tilde{A} = \{a_{ij}\},\ i = \overline{1, n},\ j = \overline{1, m}$.

The relation between the desired state of the main macroeconomic indicators $I_q, q = \overline{1, k}$ and the structure of the budget expenditure part is expressed as the weight coefficient calculated by the formula [30]:

$$\lambda_i = \frac{1}{\sum_{q=1}^{k} \frac{\overline{TR}_q}{\sum_{j=1}^{m} a_{ij}/m}}, \quad i = \overline{1, n}. \tag{2.22}$$

This coefficient characterizes the fraction of influence of the current state of the i-th budget item on the forecasted average growth rate of the main macroeconomic indicators. Such a scheme requires more precise estimation of macroeconomic indicators of economic development, which will enable us to get a more justified scenario of the budget expenditure plan for the forthcoming middle-term planning period and to coordinate the main economic forecasts with the middle-term targets and tasks of national/regional development.

Having determined the desired normative state of the expenditure item, one must develop an admissible plan of allocation of capital investments—which is called a reference plan—transferring the budget system from state Z^0 to state $\breve{Z}$ simultaneously for all expenditure items. According to the plan the resources allocated for the expenditure item are proportional to the lag of this item from the values specified in the basic scenario. The reference plan is determined by the following formulae:

$$\overline{u}_i^1(t) = \frac{\breve{x}_i - x_i(t)}{\sum_i (\breve{z}_i - z_i(t))} \Delta C(t), \tag{2.23}$$

$$\overline{u}_i^2(t) = \frac{\breve{y}_i - y_i(t)}{\sum_i (\breve{z}_i - z_i(t))} \Delta C(t), \tag{2.24}$$

$$\overline{u}_i(t) = \overline{u}_i^1(t) + \overline{u}_i^2(t), \quad i = \overline{1, n}. \tag{2.25}$$

The reference plan is such that every year $t, t = 0, 1, \ldots, T - 1$ the state of the budget expenditure part satisfies the balance equation (2.18) if the initial task is correctly formulated.

The state of the budget expenditure part for the period $t + \Delta t$ is determined by the formulae:

$$Z(t + \Delta t) = X(t + \Delta t) + Y(t + \Delta t) \tag{2.26}$$

where

$$\begin{aligned} x_i(t + \Delta t) &= x_i(t) + \lambda_i \overline{u}_i^1(t), \\ y_i(t + \Delta t) &= y_i(t) + \lambda_i \overline{u}_i^2(t), \quad i = \overline{1, n}. \end{aligned} \tag{2.27}$$

When the budget is transferred from an initial state $Z^0 = X^0 + Y^0$ to state $\breve{Z}$, the following conditions must be satisfied:

1. $\breve{Z}$ level must be achieved simultaneously for all expenditure items.
2. The balance equation (2.18) must be fulfilled at each stage of development.
3. The plan of budget resource distribution among the expenditure items must be supported by the external resources needed to provide guaranteed normative funds to cover $\breve{X}$ in addition to the development budget. If the plan of budget resource distribution is not supported by resources, then part of the budget funds must be allocated for enhancement of the budget revenue part (i.e., attraction of investments, improvement of fiscal policy, or transfers from higher offices, which last may result in passiveness of authoritative regional bodies in forming their own incomes).

The reference plan guarantees:

(A) Simultaneous achievement of state $\breve{Z} = \breve{X} + \breve{Y}$ by all expenditure items, balance, and fulfillment of the plan of budget resource distribution with accompanying development of the revenue base.
(B) Smoothing of disproportions in the development of expenditure items, as realization of this plan enables lags in expenditure items to be overcome proportionally to their values.

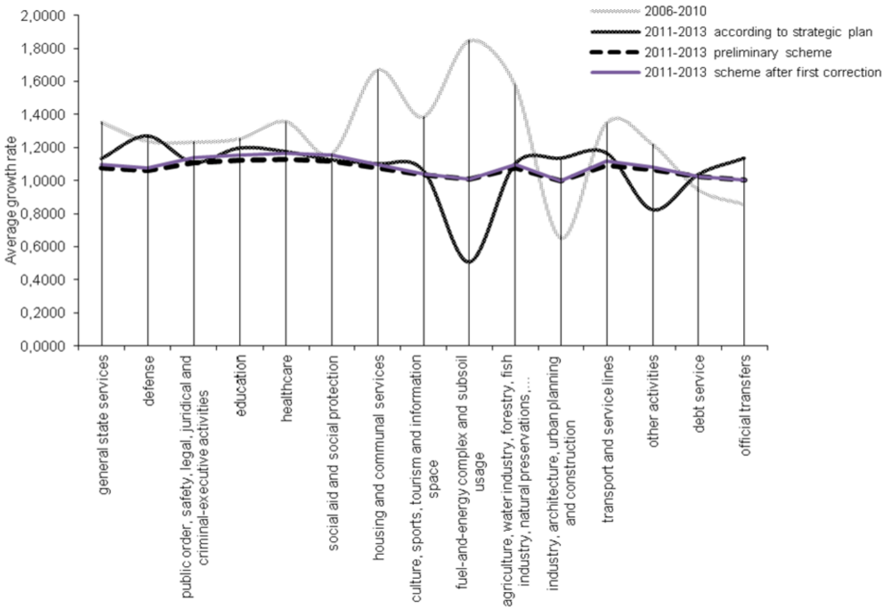


Fig. 2.2 A diagram showing variations in the average growth rates of budget expenditure items

Existence of the reference plan $\overline{u}^1(t)$, $\overline{u}^2(t)$ guarantees the existence of at least one reference plan of distribution of capital investments satisfying conditions 1, 2, and 4, which enables formulation of the problem of optimal development [6, 28].

In the computational experiment we considered the statistical data for performance of the 2008–2013 state budget of the Republic of Kazakhstan for the upper level of the budget system classification (categories of revenues and functional groups of expenditures). Therefore the elements of the revenue vector (D) and the expenditure vector (Z) include the following components: d_1—tax earnings; d_2—non-tax earnings; d_3—earnings from operations with capital; z_1—general state services; z_2—defense; z_3—public order, safety, legal, juridical and criminal-executive activities, z_4—education; z_5—healthcare, z_6—social aid and social protection; z_7—housing and communal services; z_8—culture, sports, tourism and information space; z_9—fuel-and-energy complex and subsoil usage; z_{10}—agriculture, water industry, forestry, fish industry, natural preservations, protection of environment and wildlife, land-use relations; z_{11}—industry, architecture, urban planning and construction; z_{12}—transport and service lines; $z_{13}-$ other activities; z_{14}—debt service; z_{15}—official transfers.

According to the suggested method of budget performance it is necessary to analyze the average rate of growth in expenditure items for the retrospective period planned in the strategic plan and the average rate of increase in the normative states during the planned period (Fig. 2.2).

The results of the analysis show a stepwise trend for the previous period, which is demonstrated by the chaotic statistics. The plan of strategic development also envisages nonuniform development of budget items, with allocation of budget funds on program control smoothing disproportions in the development of budget items, which is clearly seen in the diagram. The program movement guarantees uniform development of the expenditure part of the budget preventing unequal development of budget items.

2.4.3 Model of Management Adjustment

Having solved the problem of budget resource distribution, we get some states of budget expenditure items for the middle-term period. In modeling the process of budget resources control a very important modeling stage is estimation of correctness of the obtained solution.

A correct solution corresponds to such a normative state of budget items at the end of the middle-term period which is lower that the desired state defined by the strategic development plan.

IF the quantitative value of the obtained normative state ($z_i(T)$, $i = \overline{1,n}$) of all expenditure part of the budget system at the end of the middle-term period exceeds the quantitative value of the predetermined volume of monetary funds in the strategic plan ($u_i(T)_i$),

THEN it is necessary to correct the target reference point or weight coefficients (λ_i) in order to reduce disproportions between the strategic planning and planned state of budget performance,

OTHERWISE the solution is not correct.

In case the solution needs correction it is necessary to find the ratio:

$$\delta_i = \frac{z_i(T)}{\breve{z}_i}. \tag{2.28}$$

The weight coefficient of item $i, i = \overline{1,n}$ is corrected according to the following rules:

IF $\delta_i = 1$,
THEN the value of the weight coefficient is not changed $\lambda_i^k = \lambda_i$;
otherwise IF $\delta_i \neq 1$,
THEN the value of the weight coefficient λ_i^k is corrected according to the gradient method $\lambda_i^k = (\delta_i - \lambda_i)\lambda_i(1 - \lambda_i)$.

The value of the weight coefficient is reduced in order to bring the system closer to the desired result. The algorithm acts on all the system because the budget system is an integral system and it is desirable to provide centralized control of the system in order to avoid excessive attention to one of the programs.

Then the algorithm of allocation of budget resources is called where calculations are made with the corrected target.

After getting a correct solution i.e. such a normative state of the expenditure budget items which does not exceed the desired state one can switch to estimation of the state of the revenue budget part.

Let $\Delta Z(t) = \Delta X(t) + \Delta Y(t)$ be the increment of the development of the revenue budget part per year t. To simplify the task let us assume that the development of expenditure budget items is proportional to the budget resources allocated for their development i.e. $\Delta z_i(t) = \lambda_i u_i(t), i = 1, \dots, n$, where $u_i(t)$ are budget resources allocated for the development of budget item i [6, 28].

As the budget is a financial system consisting of revenue and expenditure parts, its revenue part requires replenishment in the form of input of resources, needed for its development, which are further accumulated in expenditures characterizing socio-economic and financial development of the country/region.

Let $\Delta W^1(t)$ be a vector of resources needed for socio-economic development of recipients of the expenditure budget part available at a given planning period (this year), which can be used for the development (funds formed by the income part of the budget). Let $\Delta W^1(t) = \{\Delta W^1(t)\} \geq 0$ and $\Delta W^2(t) = \{\Delta W^2(t)\} \geq 0$ be vectors of resources which require additional budgetary funds needed for full realization of the budget development plan (additional investments to the budget guaranteeing achievement of the stated purposes).

In order to determine vector $\Delta W^2(t)$ it is necessary to estimate the excess of the expenditure base over the revenue base, and to do this one must find the ratio:

$$\eta = \frac{\sum_{t=t_0}^{T} Z(t)}{\sum_{t=t_0}^{T} \Delta W^1(t)} \tag{2.29}$$

where $\sum_{t=t_0}^{T} Z(t)$ is the total consumption volume for the middle-term planning period determined by the program control; $\sum_{t=t_0}^{T} \Delta W^1(t)$ is the total forecasted receipts volume for the middle-term planning period.

For ratio η let us find such $\Delta W^{1*}(t)$ which will be a balance with respect to normative states of the budget expenditure component, i.e. η is a coefficient providing balance between the expenditure and revenue components:

$$\Delta W_j^{1*}(t) = \eta \Delta W_j^1(t), \quad j = \overline{1, m}, \tag{2.30}$$

where m is the number of budget income items.

According to $\Delta W^{1*}(t)$ the revenue budget part needs additional budget resources in amount $\Delta W_j^2(t) = v_j(t)$ where $v_j(t)$ are monetary funds (capital) needed for the development of the j-th revenue item in a year t.

The plan of budget resources allocation $\overline{u}(t) = \{\overline{u}_1(t), \dots, \overline{u}_i(t), \dots, \overline{u}_n(t)\}$ satisfies the following conditions: level $\breve{Z}$ must be reached simultaneously for all expenditure items; balance equation (2.18) must be satisfied at each stage of development. Then, due to linearity, the system of the type $\alpha(t)\overline{u}(t) = \{\alpha(t)\overline{u}_1(t), \dots, \alpha(t)\overline{u}_i(t), \dots, \alpha(t)\overline{u}_n(t)\}$ has the same property, where $\alpha(t) > 0$ is the growth coefficient for the development of the expenditure budget base in the year t.

IF resources $\Delta W^1(t)$ are sufficient for realization of $\alpha\overline{u}(t)$,
THEN $\Delta W^2(t) = 0$,
OTHERWISE

$$\Delta W^2(t) = \sum_{j=1}^{m} \mu_j v_j(t), \tag{2.31}$$

where $v_j(t)$ are monetary funds (capital) needed for the development of the j-th revenue item in the year t; μ_j is the average rate of growth of budget revenue items in the middle-term planning period.

The budget fund $\Delta C(t)$ allocated on the development of the expenditure base consists of two parts:

$$\Delta C(t) = \Delta C_1(t) + \Delta C_2(t). \tag{2.32}$$

Here $\Delta C_1(t) = \alpha \sum_i \overline{u}_i(t)$ are resources directed directly to the development of the expenditure base of budget recipients, $\Delta C_2(t) = \sum_j v_j(t)$ are resources directed to the accompanying attraction of capital resources for the development of income base needed for the development of expenditure base of budget recipients.

The plan $\alpha(t)\overline{u}(t)$ will be provided with resources if

$$\sum_j \mu_j v_j(t) \geq \alpha(t) \sum_i \overline{u}_i(t)\lambda_i$$

$$v_k(t) \geq 0, \ \alpha \geq 0. \tag{2.33}$$

This condition means sufficiency of additional resources providing the predetermined level of expenditure base for the period t.

In case the expenditure base is developed according to the maximal growth scenario in the period of observations, the necessity of financing needed to support the predetermined level of development is transformed into the task of maximization of development by attracting financial resources:

$$\max \alpha \quad \text{for condition (2.33)}. \tag{2.34}$$

Thus, we get a model correcting the forecast of the budget revenue part preserving balance with the planned expenditure part and providing development of the budget expenditure part.

2.4.4 Description of Algorithms of Basic Processes

Program control is a process of planning of probable or logical future and forecasted future states. It is the process which enables to understand how to achieve the desired aims, how to use knowledge to turn the logical future into a desired one, and how to make such actions.

The basic component in the program control is a plan having three general elements—an initial state, a target (or final state) and processes connecting these states. The target of planning is to connect the elements in such a way that it will enable to reach maximal efficiency with minimal expenditures.

The first element of any plan is *the initial state*.

The second element is the *target*. The target must be properly formulated, achievable and restated or changed according to the changed circumstances.

The third element of the plan is *processes*. This element is the plan itself as it contains the description of the method providing transition from the initial state to the target.

The planning process mainly has one direction i.e. it is a time-ordered sequence of events beginning at the present time $t = 0$ and ending at a certain moment in future $t = T$. This sequence that is called a *direct process* considers current factors and suggestions which give a certain logical result. In the second sequence that is called a *reverse process* the sates are considered beginning from the desired result at a certain moment of time T in the reverse time direction, to the initial state, in order to estimate factors and intermediate results needed to reach the desired final result.

The direct planning process gives assessment of the state of probable final result. The reverse planning process gives the means of control and management of the direct process on the way to the desired state.

Let us consider the algorithms of direct and reverse processes [6].

Direct process algorithm

Step 1. Acquisition and processing of input information.

At this stage the current budget state and the strategic plan are analyzed according to the basic scenario of economic development. To do this it is necessary to present the vector of interaction between the two main indicators and the revenue items as weight coefficients, expressed as vector $Q(t)$, the target indicator of performance of the strategic development plan. At this stage the volumes of financing of expenditure items in the strategic development plan for the middle-term planning period are calculated.

Step 2. Direct pass: Determination of the desired states of the budget expenditure part.

At this step the desired state is calculated by formula (2.20), this state is a transformation of the target indicator into a concrete state which is to be reached to provide a required level of development.

Step 3. Distribution of resources by the expenditure items for the middle-term period.

Knowing the desired and current state of the budget expenditure part, we can find the control which can transfer the system into the desired state by formulae (2.23)–(2.25). The obtained control is a program of system transfer from one state to the other according to the formulae (2.26) and (2.27), this procedure enables to determine normative states of expenditure items in the middle-term planning period.

Step 4. Comparative analysis of obtained solutions.

At this step works the control function which finds incorrect solutions by comparing the normative state in the last period of the middle-term planning and the desired state at the end of the planning period. At this step the decision to correct the obtained solution is taken, it includes the following operations:

IF the quantitative value of the obtained normative state of all expenditure part of the budget system at the end of the middle-term period exceeds the quantitative value of the planned state that was predetermined in the middle-term fiscal policy,
THEN it is necessary to correct the target reference point or weight coefficients in order to reduce disproportions between the desired and planned states,
OTHERWISE The algorithm must be stopped.

Step 5. Reverse pass: Correction of weight coefficients.
Weight coefficients are corrected using the gradient method. After correction of weight coefficients it is necessary to repeat steps 2–4.

Reverse process algorithm
Step 1. Estimation of stability margin of obtained solutions.
At this step it is necessary to determine the confidence interval for every item which will characterize the stability margin of the item.
Step 2. Estimation of stability of obtained solution.
At this step to estimate stability of obtained solutions Lyapunov's function is used.
To go to the next step it is necessary to fulfill the following rules:

IF the obtained solution is not stable,
THEN in the direct process algorithm it is necessary to correct values of forecast items,
OTHERWISE give qualitative interpretation of the obtained solution.

Step 3. Correction of the forecast of the budget revenue part.
At this step formulae (2.29) and (2.30) are used to correct the budget revenue part in such a way as to provide balance between revenue and expenditure parts.
Step 4. Determination of the development coefficients for expenditure items for the period t.
At this step according to the development coefficient the volume and financial resources required for the development are determined by formula (2.32), and possibility to achieve planned development level is estimated by formula (2.34).
The above algorithms are used to perform a computational experiment; the algorithms are the base for programming of the program control process.

2.5 Mathematical Models of Budget Revenue Part

2.5.1 Basic Provisions Describing Interactions of Budget Items

In the authors presented a mathematical model of the static budget state (2.1) which enables to answer the following questions:

1. Are there principles according to which budget resources are distributed by the items of state programs of strategic development?
2. Is it possible to assess the budget (deficiency/proficiency) state?
3. How does the budget allocation change when the budget items are changed?
4. What is the reaction of budget indicators on the change of external conditions?
5. Is it possible to use the current budget state for further planning?

Answers to the above questions will help to find optimal variants of the current and future budget development.

The constructed static model reflects the current budget state. A specific feature of the suggested model is that the conditions of operation of the budget model are well coordinated with the program control model, which enables to get correct values of the budget revenue. Let us reformulate the conditions of static model functioning for the planned period $t \in [t_0, T]$ in terms of program control of budget resources:

1. the model operates the data within one planned period $t \in [t_0, T]$;
2. the initial data are the dynamics of revenue and expenditure budget values before the moment $t = t_0$;
3. the output data are the corrected plan of budget receipts—a balance with respect to the expenditure plan;
4. the computational scheme of the static model is based on the principle of matrix budget representation as a matrix of interaction of budget items whose elements are indicators of the internal budget state.

2.5.2 Learning Elements of Budget System

The model of learning elements is based on the principle of matrix representation of the budget system in order to simplify calculations on the nature of object domain [20].

Problem formulation Consider a budget system as a matrix of interaction of revenue and expenditure items a_{ij} formed to determine the budget state. The initial elements of the system are revenue items, denoted as $D = \{d_j;\ j = \overline{1, m}\}$, and expenditure items $G = \{g_i;\ i = \overline{1, n}\}$. These elements form $m \times n$ matrix for interaction of budget items $A = \|a_{ij}\|$. The system operates in the dynamic regime $\tau = \overline{1, t_0}$. The matrix elements are indicators of the internal budget state generating learning elements of the mathematical model of budget forecast ε_{ij}^{+} and ε_{ij}^{-}, which can reflect development and crises moments in time needed to find an indicator of the internal budget state in future.

Definition 2.5 *Learning elements of budget system* are such elements which can correct the future budget; their purpose is to bring the system to the balance as close as possible.

Essence of learning elements These elements are numbers but these numbers initially contain information about processes in national economy i.e. numbers with history. A learning element of any item gives the coefficient of future variation of this item relative to its current state as it was obtained from dynamics analysis.

As the statistics increases such elements will also be corrected i.e. they will be taught. A learning element with plus characterizes the degree of item development in future, whereas a learning element with minus shows the degree of criticality of the item in dynamics and its influence on future.

The forecast system based on such system can give more reliable information on system development and crisis. A dramatic effect on the item future is imposed by its current state; such concept enables to speak about dynamic memory of budget elements.

The above elements play a role of an auxiliary reference mark in the direction of balanced state of the item and budget as a whole achieved by coefficients of items interactions reflecting mathematical relations between budget elements.

The elements of the budget system form a matrix of interactions of revenue and expenditure budget items consisting of n lines and m columns:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{im} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nm} \end{pmatrix}.$$

The elements of A matrix correspond to the values in (2.2).

The relation (2.2) is an internal indicator of the system state or, in other words, it is the coefficient of interaction of input and output system parameters.

The object of modeling will be considered in dynamics, therefore for every time period we will construct its matrix of budget items interaction. The time series is presented as $\tau = \overline{1, t_0}$, the forecasted year is denoted by t.

Every matrix element in the current period can be expressed as the value of the previous year plus the difference between the current and the previous years:

$$a_{ij}^{\tau} = a_{ij}^{\tau-1} - e_{ij}^{\tau} \tag{2.35}$$

where $a_{ij}^{\tau-1}$ is an element of the interaction matrix in the previous time period; e_{ij}^{τ} is the difference between the values of elements in the interaction matrix in the current and previous time periods.

Learning elements of this model contain all past information, thus, they predetermine the future internal indicator of the system state ($a_{ij}^{\tau+1}$) retaining balance budget state. If it is not the case, it means that the budget has deficiency or proficiency.

This model has two types of learning elements: one of them contains information about development ε_{ij}^{+} (a tendency of the budget item to increase which is the result of development of the state economy), the other element contains information about crisis situations in the internal system state ε_{ij}^{+} (when there is a tendency to decline i.e. there are sharp jumps from rise to fall and vice versa), such information is needed to determine the internal state of the system in future.

After careful analysis of the budget dynamics the authors heuristically got expressions for calculation of learning elements.

The learning element ε_{ij}^{+} for each budget matrix element can be expressed as:

$$\varepsilon_{ij}^{+} = \frac{\sum_{\tau=1}^{t_0} a_{ij}^2}{\sum_{\tau=1}^{t_0} \tau \cdot \sum_{\tau=1}^{t_0} \tau a_{ij}}; \quad i = \overline{1,m};\ j = \overline{1,n}, \tag{2.36}$$

where $\tau = \overline{1, t_0}$ is the period of observations; a_{ij} are budget matrix elements.

The learning element ε_{ij}^{+} can be expressed as:

$$\varepsilon_{ij}^{-} = \frac{\sum_{\tau=2}^{t_0} e_{ij}}{\sum_{\tau=1}^{t_0} \tau}; \quad i = \overline{1,m};\ j = \overline{1,n}, \tag{2.37}$$

where $\tau = \overline{1, t_0}$ is the period of observations; ε_{ij} is the difference between the values of budget matrix elements.

Matrix representation of learning elements is expressed as $\|E_{ij}^{+}\|$ and $\|E_{ij}^{-}\|$, respectfully.

The obtained learning elements give the forecast indicator of the internal budget state i.e. the elements of the budget forecast matrix:

$$a_{ij}^{t} = a_{ij}^{t_0} + \varepsilon_{ij}^{+} + \varepsilon_{ij}^{-} \tag{2.38}$$

where $a_{ij}^{t_0}$ is the value of the budget matrix for the current period (t_0); ε_{ij}^{+} and ε_{ij}^{-} are the elements of the learning model.

The expression (2.38) gives a forecast matrix of budget items interactions:

$$A^{\tau+1} = \begin{pmatrix} a_{11}^{\tau+1} & a_{12}^{\tau+1} & \dots & a_{1j}^{\tau+1} & \dots & a_{1m}^{\tau+1} \\ a_{21}^{\tau+1} & a_{22}^{\tau+1} & \dots & a_{2j}^{\tau+1} & \dots & a_{2m}^{\tau+1} \\ \dots & \dots & \dots & \dots & \dots & \\ a_{i1}^{\tau+1} & a_{i2}^{\tau+1} & \dots & a_{ij}^{\tau+1} & \dots & a_{im}^{\tau+1} \\ \dots & \dots & \dots & \dots & \dots & \\ a_{n1}^{\tau+1} & a_{n2}^{\tau+1} & \dots & a_{nj}^{\tau+1} & \dots & a_{nm}^{\tau+1} \end{pmatrix}$$

According to the main principle of the budget system the budget state must be balanced i.e. the condition $\sum_{i=1}^{n} g_i = \sum_{j=1}^{m} d_j$ must be fulfilled for any level of budget classification. From the above-stated it follows that the budget matrix must give the balance state of the budget item and retain it under possible current corrections.

From the condition of balance budget state it follows that the following relation must be fulfilled:

$$\sum_{i=1}^{n} a_{ij}^{t} = \frac{\sum_{i=1}^{n} g_i^t}{d_j^t} = \frac{\sum_{j=1}^{m} d_j^t}{d_j^t}. \tag{2.39}$$

Hence, the expression (2.39) is a sufficient condition of the balance state of the budget matrix: the sum of the j-th column of the budget matrix elements is equal to the fraction of the budget revenue per the j-th revenue item.

Let us check the balance of the forecast matrix:

$$E(A) = \sum_{j=1}^{m} \left(\frac{1}{\sum_{i=1}^{n} a_{ij}^{t}} \right) = 1. \tag{2.40}$$

The expression (2.40) is a sufficient condition of the budget state analysis. If the above condition exceeds 1, then fractions of the expenditure items exceed the budget, which corresponds to the proficiency state. Similarly, if the expression is less than 1, the state of the budget is called deficiency [29].

Fulfillment of the condition of balance enables to make a conclusion about the balanced budget state for the forecasted period. Now we can switch from the elements to finding forecasted values of budget revenue items.

2.5.3 Model of Correction of Budget Revenue Forecast

After solving the problem of budget resource distribution among budget recipients for the middle-term planning period, it is possible to determine the volumes of budget resources (corrected) $\sum_{i=1}^{n} z_i(t)$ for the planned periods $t \in [t_0; T]$ used to calculate the volumes of budget receipts for certain time periods based on the principle of balance of revenue and expenditure expressed in the matrix of interaction of budget items.

Taking the forecast matrix of item interaction and the corrected budget $\sum_{i=1}^{n} z_i(t)$ for the planned periods $t \in [t_0; T]$ as the basic calculated parameter, it is then possible to calculate vector elements by the formula:

$$d_j = \frac{\sum_{i=1}^{n} z_i(t)}{\sum_{i=1}^{n} a_{ij}^{t+1}}. \tag{2.41}$$

Thus, the connection between the static model and the program control model is based on the natural connection between budget components—revenue and expenditure.

2.6 Model of Information System for Program Budget Control

As an instrument controlling budget process we propose to use the information system of budget control based on the principles of budget programming, i.e. a budget process modified in the framework of the program-targeted approach. The information system must enable us to solve the following problems: support of decision making, organizational control, control of the budget system, system diagnostics, complex informational search and other operations. All these tasks have a high degree of information complexity.

The information system of budget control must be able to fulfill the following functions:

– automation of solution of current and standard tasks of the budget performance: collection, processing and storage of information, making reports, compiling statistics, etc.;
– development of budget programs and projects;
– definition of targets and resource limitations in budget formulation;
– program control of allocation of budget funds among budget recipients according to the strategic plan of socio-economic development.

The information system of budget control is a complicated multi-level information system guaranteeing automatic control of all subprocesses of budget system and types of budget activities. The model of budget control information system can be presented as an interaction of 6 subsystems (Fig. 2.3) reflecting the system of criteria used for description of a scheme of multi-level (middle-term) budgeting. The program realization of such a system will become an important instrument in such an approach to budgeting, which makes it possible to solve complicated information problems related to organization and control of result-oriented budgeting.

As it is seen in Fig. 2.3 the structure consists of 6 major subsystems: interface, structure, targets and target formulation, mechanism of program control, automation and data bases. Let us consider each subsystem:

Interface is a dialogue regime of contact with the user which provides flexible control of the system.

The *structure* consists of three modules. Each module is an independent element of the budget process that forms commands for users in the form specific for the given area.

The subsystem "*targets and target formulation*" is an addition to the *structure* as each element of the budget process must have a purpose and a final product in order to take into account state purposes with the volumes of revenues needed for their achievement, and must be oriented on solution of key targets and tasks for the planning system as well as planned allocation of budget funds.

Automation is a subsystem designated for solution of current and standard problems of budget process: collection, processing and storage of information, making reports, compiling statistics and, besides, this subsystem has a module for input of

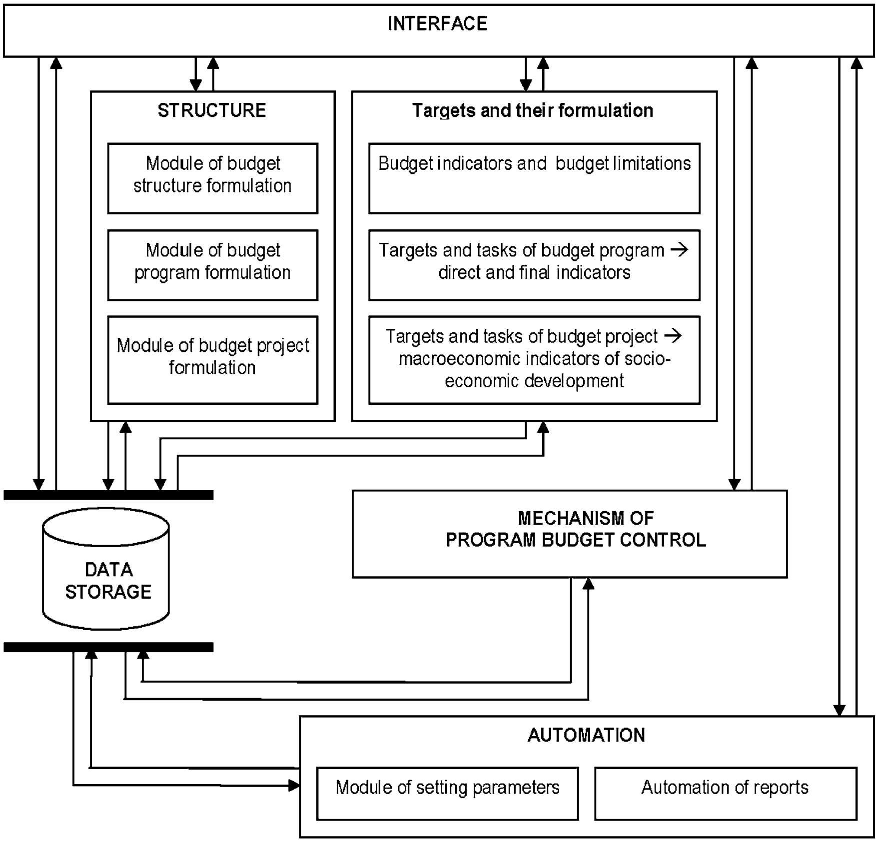


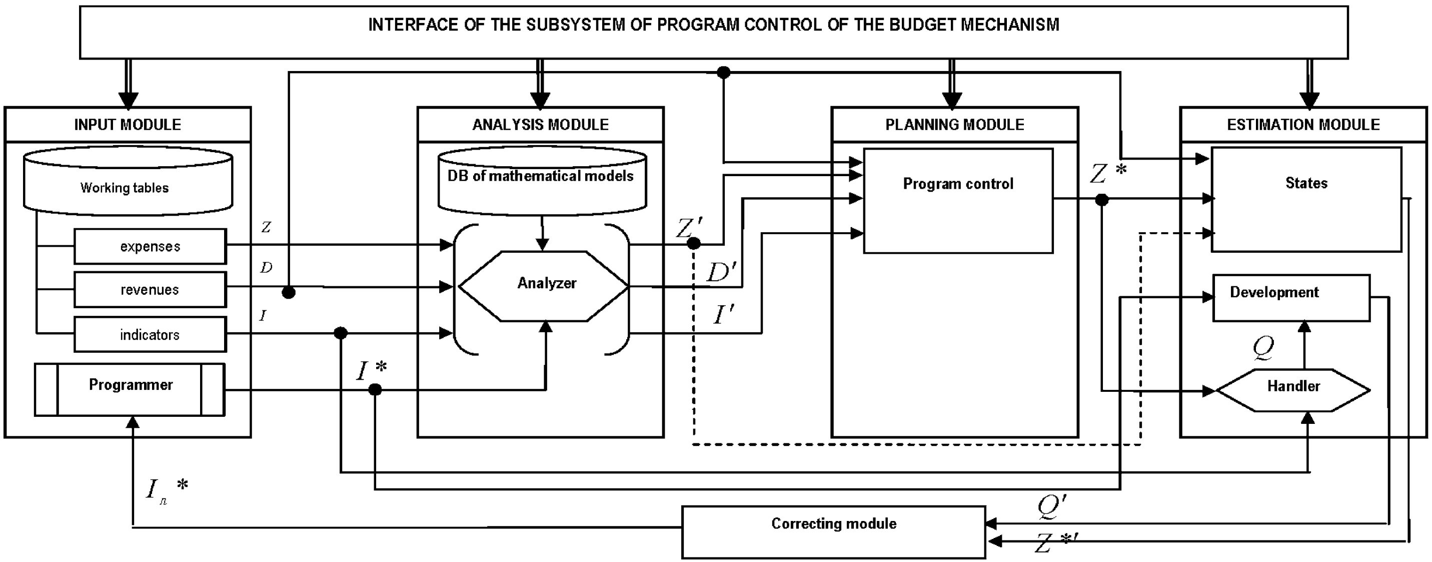
Fig. 2.3 A model of IS budget control of the result-oriented program-targeted budgeting approach

parameters that can set parameters needed for budget modeling and making managerial decisions.

Data storage is a database the organization of which satisfies the requirements of emergency use of the budget system and result-oriented budgeting technology.

The mechanism of program control of the budget is a modeling block of program control of budget funds in the form of an optimal allocation scheme. This block realizes the following criterion from the system of criteria used to describe the middle-term budgeting scheme—procedures and methods of estimation of demands (budget funds) for achieving purposes in the long-term time periods.

In terms of cognitive approach the information model is a set of the following subsystems: a complex of distributed technical means; a complex of mathematical models; a complex analyzing states and making decisions; data bases; a system of information processing and mapping. The information system includes the three main components: database, base of mathematical models and program subsystem.



Notes: Z is the dynamics of expenditure items; *D* is the dynamics of revenue items; *I* is the dynamics of the main indicators of socio-economic development; *I** is the forecast of the indicators for the planned middle-term period; *Z'* is the forecast of the expenditure items for the middle-term period; *D'* the forecast of the revenue items for the middle-term; *I'* is transformation of indicators into dimensionless form; *Z** is a forecasted state of expenditure items obtained as a result of optimal expenditure of budget funds; *Z**' is a comparative estimation of the state of the expenditure base; *Q* is calculation of new values of the main indicators of socio-economic development; Q' is a comparative estimation of the state of socio-economic development; I_k* is a correcting value of the forecasts of indicators for the planned middle-term period; *DB* is the database with the frame data organization; *Analyzer* is a mathematical module transforming information from one type into the other by mathematical models stored in the database; *Programmer* is a module for input of target indicators; *Handler* is a gateway providing calculation of new values of indicators; *State* is a block for analysis of the state of the budget system in the current and planned periods; *Development* is a block for analysis of the sate of socio-economic development of the country/region; *Correcting* module is a block correcting the strategy of development in the form of adjustment of indicators.

Fig. 2.4 Architecture of the subsystem of program control of budget mechanism

The main purpose of development of the IS program control is a complex information support of the processes of budget control with the use of possibilities of information and telecommunication technologies.

In order to achieve the main purposes it is necessary to solve the following problems:

- to form a common data bank reflecting the state of the budget system and providing timely and operative allocation of full, objective, reliable and unambiguous information on the budget process;
- to provide an information environment common for all users, to provide common standards for preparation of information and normative-reference materials;
- to provide efficient two-way communications and feedback channels;
- to develop a common style of paper work, to provide a centralized access to reports and user-friendly navigation in the information system.

Characteristics of the "portrait" of the IS for program budget control:

- methodology of control aimed at achieving strategic purposes of the administration of a state institution expressed in the IS as a system of controlling actions regulating user's activities;
- possibility of access to data for many users connected in a local network of the organization;
- availability of communication means;
- advanced, user-friendly graphic interface of the terminal user;
- regimes of processing of operative information close to the real-time regime;
- means of access authentication and differentiation, which make it possible to dose information according to job responsibilities of users, high level of protection from unauthorized access;
- a common server for databases, possibility of processing of thousands and millions of records for compiling reports;
- use of standard languages and protocols for data presentation and manipulation.

The program control module of the information system (Fig. 2.4) simulates program control of budget funds as an optimal distribution scheme providing a possibility of corrections by means of adjustment of the system of indicators. This module clearly demonstrates the algorithm for budget planning and control formulated on the base of mathematical methods and models.

References

1. Implementation of the middle-term financial planning in the budget process of the Russian Federation. Report of the Fiscal Policy Center. Contract OUT-PER-I-00-99-00003-00 (2002)
2. Serpilin: The Great Britain experience in regional management and result-oriented budgeting: materials of the "International Cooperation" program. http://www.unicom-ms. 10 March 2006
3. Melkers, J., Willoughby, K.: The state of the states: performance-based budgeting requirements in 47 out of 50. Public Adm. Rev. **58**(1), 66–73 (1998). doi:10.2307/976891

4. Lee, J.Y.-J., Wang, X.: Assessing the impact of performance-based budgeting: a comparative analysis across the United States, Taiwan, and China. Public Adm. Rev. **69**, 60–66 (2009)
5. Result-oriented budgeting: international experience and possibilities of its use in Russia. Report of the Fiscal Policy. Contract OUT-PER-I-00-99-00003-00 (2002)
6. Mutanov, G.M., Mamykova, Zh.D.: Methods and Mathematical Models of the Software Budget Management. University, Almaty (2008), p. 144. ISBN 978-601-208-014-8
7. Bruins, R.J.F., Munns, W.R. Jr., et al.: A new process for organizing assessments of social, economic, and environmental outcomes: case study of wildland fire management in the USA. Integr. Environ. Assess. Manag. **6**, 469–483 (2010)
8. Narver, J.C., Slater, S.F.: The effect of a market orientation on business profitability. J. Mark. **54**, 20–35 (1990)
9. Hurley, R.F., Hult, G.T.M.: Innovation, market orientation and organizational learning: an integration and empirical examination. J. Mark. **62**, 42–54 (1998)
10. Mutanov, G.M., Kulikova, V.P.: Mathematical Modeling of Economic Processes. Economics, Almaty (1999), p. 356
11. Intriligator M. Mathematical Methods of Optimization and Economic Theory. Progress (1975), p. 607. Translation from English
12. Sovetov, B.Ya., Yakovlev, S.Ya.: Modeling of Systems. Moscow (1985), p. 239
13. The Theory of Forecasting and Decision Making: A Teaching Aid. Higher School (1977)
14. Neimark, Yu.I., Kogan, N., Savelyev, V.: Dynamic Models of Management Theory. Nauka, Moscow (1985), p. 400. Main editorial of physico-mathematical literature
15. Zubov, V.I.: Lectures on Control Theory. Nauka, Moscow (1975)
16. Eliseyeva, I.I.: Econometrics. Moscow (2003)
17. Dougerti, K.: Introduction to Econometrics. INFRA-M, Moscow (1999), p. 402. Translated from English
18. Chernyak, Y.A.: System Analysis in Economic Control. Economics, Moscow (1975), p. 191
19. Yudin, D.B., Yudin, A.D.: Extreme Models in Economics. Economics, Moscow (1979), p. 288
20. Shelobayev, S.I.: Mathematical Methods and Models. Moscow (2001)
21. Kholod, N.I., Kuznetsov, A.V., Zhikhar, Y.N., et al.: Economic and Mathematical Methods and Models. BGEU (2000), p. 412
22. Sulitsky, V.N.: Methods of Statistical Analysis in Management. Moscow (2002)
23. Lewis, K.D.: Medium-Term Forecasting. Moscow (1993)
24. Teece, D.J., Pisano, G.P., Shuen, A.: Dynamic capabilities and strategic management. Strateg. Manag. J. **18**, 509–533 (1997)
25. Yefimova, M.P., Petrova, E.V., Rumyantsev, V.N.: General Theory of Statistics. Moscow (2004)
26. Zhdanov, S.A.: Economic Models and Methods in Management. Business and Service, Moscow (1988), p. 176
27. Dubrova, T.A.: Statistical Methods in Forecasting. UNITY, Moscow (2003)
28. Mutanov, G.M., Bykova, I.Yu., Mamykova, Zh.D.: Software control of budget funds. Bull. Natl. Acad. Eng. Repub. Kaz. **4**(26), 33–38 (2007)
29. Mutanov, G.M., Shintemirova, A.U.: Mathematical Budget Models. Astana (2003), p. 176
30. Afanasyev, V.N., Kolmanovskii, V.B., Nosov, V.R.: Mathematical Theory of Constructing Control Systems. "High School" (1998), p. 574
31. Odintsov, B.E.: Designing Economic Expert Systems: Teaching Aid for Higher Education Institutions. Computer, UNITY (1996), p. 166
32. Heerdt, J.A., Coutinho, D.F., Mussa, S.A., Heldwein, M.L.: Control strategy for current harmonic programmed AC active electronic power loads. IEEE Trans. Ind. Electron. **61**(8), 3810–3822 (2014)
33. Adi, V.S.K., Chang, C.-T.: A mathematical programming formulation for temporal flexibility analysis. Comput. Chem. Eng. **57**, 151–158 (2013)

34. Grigoroudis, E., Phillis, Y.A.: Modeling healthcare system-of-systems: a mathematical programming approach. IEEE Syst. J. **7**(4), 571–580 (2013)
35. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. Syst. Man Cybern. **15**(1), 116–132 (1985)
36. Rachel, F.M., Cugnasca, P.S.: The multi-agent programming paradigm use for railway applications. In: Allan, J., Arias, E., Brebbia, C.A., Goodman, C., Rumsey, A.F., Sciutto, G., Tom, N. II (eds.) Computers in Railways XI: Computer System Design and Operation in the Railway and Other Transit Systems, vol. 103, pp. 641–650 (2008)

Chapter 3
Energy-Entropic Methods in Assessment and Control of Economic Systems

In spite of differences between the nature of infrastructure and that of technology (power industry, trade, services, etc.), their development is governed by common mechanisms.

The three laws of thermodynamics—the law of energy conservation, the law of energy dissipation (increase in entropy), and the law of maximum energy usage—form the foundation of the energy-based approach to the description and control of production systems.

Well-known laws of the one science adapted and interpreted by the other science help to:

- provide understanding of existing knowledge;
- draw conclusions from available facts;
- make qualitative and quantitative assessment of different influences on the studied phenomena;
- predict consequences of decisions; and, finally,
- control risk and safety of complex economic systems.

The entropy assessment of the state of production-system parameters enables us to use the same relative index *to estimate* changes in the state of these parameters and *to combine* these estimations to create an integrated economic image of the existing production situation.

The process of production management can be characterized by the tendency towards ordering caused by the reduction in the number of states of the production system.

Thereby the efficiency or inefficiency of the production process can be estimated both in terms of energy conservation and of quality of economic-system management.

3.1 Arguments in Favor of Application of the Thermodynamic Approach to Economic Systems

Let an abstract enterprise be a complex system which at some moment of time t is characterized by the state defined by the following *resources* X_i $(i = \overline{1, m})$: equip-

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_3

ment, the number of items in the production phase, available orders, finished products, raw materials inventory, number of workers, etc., and by *managerial actions* f_k $(k = \overline{1,l})$:

$$S_t = S(X_{it}, f_{kt}), \tag{3.1}$$

where S_t is the state of the production system at the point of time t; X_{it} $(i = \overline{1,m})$ are the types of system resources at the point of time t; f_{kt} $(k = \overline{1,l})$ are managerial actions at the point of time t.

Taking into account the most important system properties, all resources can be subdivided into material, energy, and information resources. Therefore the state of the production system is described by the relation

$$S_t = S(M_t, E_t, I_t, f_{kt}), \tag{3.2}$$

where S_t is the state of the production system at the point of time t; M_t are material resources of the system at the point of time t; E_t are energy resources of the system at the point of time t; I_t are information resources of the system at the point of time t; f_{kt} $(k = \overline{1,l})$ are the controlling actions on the system at the point of time t.

The operating production system, i.e., the system in the dynamic mode, can transfer from one condition to the other during a time interval Δt, $S_t \rightarrow S_{t+\Delta t}$.

The specific features of energy resources explain their key role in determining the intensity of system processes. For example, in the absence of energy resources:

– the system may be absolutely static;
– or all processes in the system are fully determined by the external factors.

A traditional set of indexes, which describes the production system in terms of stability and enables formulation of the optimization task with respect to decisions on material and technical development, includes the volume of goods production in cost indexes and calculated indexes of the physical volume of production in industry, agriculture, and construction. In conditions of stable economic development it is reasonable to use methodology based on production statistics. However, in conditions of crisis and poor economic management such a methodology does not work because of, for instance, rapid structural changes and disproportional price increase for various types of goods. As a result, it is difficult or even impossible to observe the requirements for incoming information (completeness, reliability, relevance, etc.) [1, 2].

The other criterion of progressive (that is, successful and effective) development of the production system—the growth rate of fund intensity—must be accompanied by a growth rate in efficiency (the output of goods per consumption of 1 tenge by the production part of the system) that would exceed the growth rate of the coefficient of overhead expenses.

The next natural step of progressive development of the production system is use of entropic assessments instead of (or at the same time as) cost price assessments of production efficiency. The basis of the suggested method is the laws of thermodynamics, which are valid in systems of *any nature* and have *definite properties*.

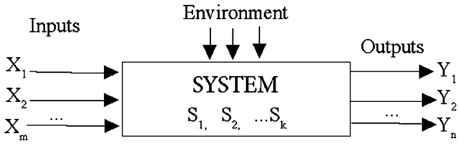


Fig. 3.1 A control system

The system is said to be in *equilibrium* (quasi-static) if all $\{a\}$ parameters of the system vary infinitely slowly with time, i.e., the rate da/dt of change of a specific parameter is *much* lower than the average rate of change of this parameter during the time of relaxation τ:

$$da/dt \ll \Delta a/\tau.$$

Otherwise the system is said to be in *non-equilibrium*.

The transition of the system from state S' to state S'' is termed *reversible* if the system returns from S'' to the initial state without any changes in its environment. If such is not the case, the process is termed *irreversible*.

According to thermodynamic theory [3–6], a system that can exchange matter as well as energy with its surroundings is called an *open* system. A system that can exchange energy but not mass with its surroundings is called a *closed* or *isolated* system (that is, isolation is understood as a thermal and mechanical property) [7, 8].

The second law of thermodynamics states the *existence* of entropy in any *equilibrium* system and its *nondecreasing* in any processes in *isolated* (closed) systems. According to thermodynamics theory:

– *any irreversible process is nonequilibrium*;
– *any nonequilibrium process is irreversible*, if in addition to the second law of thermodynamics *any state is achieved in a nonequilibrium way when it is achievable from the given equilibrium state*.

Thus, the second law of thermodynamics stating that *entropy increases in the closed system in nonequilibrium processes* enables us to characterize entropy *as a measure of irreversibility of processes in the closed system* [8, 9].

The main function of the production system is the use and successive transformation of the material and energy potential of nature into material values suitable for direct human consumption. Hence, the productive system is:

– *irreversible* as its products cannot spontaneously transform to the initial state (though the production process can be called circular as it is always accompanied by the deterioration–restoration of the production system);
– *open* as it represents a set of man-made objects and material existing due to exchange of substance and energy with the environment [10].

Let us represent the process of goods production as operation of the system transforming $\{X_i\}$, $(i = \overline{1,m})$ energy and other resources supplied to the input into finished products $\{Y_j\}$, $(j = \overline{1,n})$ w obtained at the output (Fig. 3.1).

According to the first law of thermodynamics, the system transition from state S' to state S'' can be characterized by the amount of internal energy V depending on:

– the initial S' and final S'' states of the system;
– total interaction with the environment in the form of work, heat, and energy mass transfer.

For reversible processes the *total interaction* does not depend on the means of transition from state S' to S'' and is defined by the expression

$$V'' - V' = A + Q + Z, \tag{3.3}$$

where $V'' - V' = \Delta V$ is the change of internal energy; A is the work done by the system and over the system; Q is heat brought to the system; Z is energy mass transfer.

Thus, separately, the work A, the amount of heat Q, and energy transfer[1] Z depend on the trajectory of system transition from state S' to state S''. That is to say, they are functionals of transition history.

It is obvious that control of influence on the system must be organized as a targeted change in the internal energy of the system.

For infinitely small changes in the state of the system, the first law of thermodynamics is written as

$$dV = \delta A + \delta Q + \delta Z, \tag{3.4}$$

where dV, the change in the internal energy V, is a total differential; δA, δQ and δZ, infinitely small changes in work, heat, and transfer energy, are not total differentials of any function.

According to the first law of thermodynamics, in production of goods nothing disappears completely and neither does it arise without equivalent expenses of substance or energy.

According to the law of energy dissipation, in all processes some part of energy loses its ability to do work and deteriorates in quality, i.e., when the working process is over, most part of energy loses its ability to do work.[2]

So, the intensity of exchange processes in the system is quite adequately characterized by current energy expenses. Therefore any production process consuming energy is accompanied by its dissipation in the environment as the efficiency coefficient is not unlimited. In other words, some part of energy of ordered processes (kinetic energy of moving bodies, energy of electric current, etc.) transforms into energy of disordered processes (finally—to heat), which is lost irrevocably. As a result, the system becomes less efficient, and for its further functioning it must use energy from the outside—the productive system virtually becomes an open system. Efficiency (the ability of a system to survive) is defined not by the amount of consumed energy but by the way the energy is used by the system. From these positions

[1]Energy Z transferred by mass can depend on the means of transition, whereas the amount of mass must remain the same.

[2]Considering the material model of production, we do not analyze many non-material types of service, including so-called "services of financial intermediaries".

the production system (as an economic subsystem) may be *artificially isolated* (or for some period of time we can assume that it is isolated).

Thus, according to the first postulate of thermodynamics, *the isolated macroscopic system eventually reduces to the state of thermodynamic balance and can never leave it spontaneously*. Therefore it is necessary to take into account some limiting factors, for example:

– self-sufficiency of the system with all things needed for its functioning (to a large extent this depends on the nature of the system);
– duration of system operation.

In material energy models such an indicator is characterized by the ratio of the free and total system energy. In economics terminology this means the ratio of free residue of finished products to expenses, on the self-maintenance of the enterprise. It is natural to consider the following industrial systems more progressive (that is, more effective):

– a system creating an "identical" market basket of goods with lower public expense;
– a system whose industrial "motor" at equal volume of consumed "fuel" (goods and funds spent for manufacturing) creates more material production, i.e., is *less power-intensive*.

It should be noted that it is possible to determine the dependence of the efficiency indicator on the internal organization of industrial systems.

From the second law of thermodynamics it follows that:

$$\text{for irreversible processes} \quad \oint \frac{\delta Q}{T} < 0, \tag{3.5}$$

$$\text{for reversible processes} \quad \oint \frac{\delta Q}{T} \geq 0, \tag{3.6}$$

where δQ is an infinitely small variation in heat supplied to the system; T is absolute temperature of the source of heat.

In real working conditions at a given point of time each value of the state of controlled object corresponds to a certain value of entropy. For entropy[3] S we can write:

$$\text{for irreversible processes} \quad \oint \frac{\delta Q}{T} \geq 0, \quad (S_{(2)} - S_{(1)}) > \int_{(1)}^{(2)} \frac{\delta Q}{T} \tag{3.7}$$

$$\text{for reversible processes} \quad dS = \frac{\delta Q}{T}, \quad (S_{(2)} - S_{(1)}) = \int_{(1)}^{(2)} \frac{\delta Q}{T} \tag{3.8}$$

[3] Visually identical designation of the state of system S and entropy S in this context does not cause misinterpretation.

where dS is the change in entropy; $S_{(1)}$, $S_{(2)}$ are entropies of the system in states S', S'', respectively; δQ is an infinitely small variation in heat supplied to the system; T is the thermodynamic temperature of the heat source.

The state of the productive system depends on disturbing X_i and controlling f_k influences (3.1), therefore the production control process will be characterized by the tendency to ordering caused by reduction in the possible variety of states of the productive system [11].

Let us introduce the notions that:

– the thermodynamic phenomenon is *an ideal process* in which an elementary type of energy, work, or heat is consumed or produced;
– generalized thermodynamic X_i forces and corresponding to them generalized Y_i coordinates are used for quantitative description of the measure of work δA and heat δQ [8].

Let us define the concept of *generalized forces* (GF) X_i as equivalent to:

– the concept of initiating impact of the environment on the system;
– or the interaction of processes inside the system.

The reaction of the system to such actions is expressed in the change of the corresponding *generalized coordinates* (GC) Y_i [12]. Depending on the conditions, the role of GC can be played by GF, and vice versa. Therefore changes in the elementary work can be written as:

$$\delta A_i = d(X_i dY_i) \tag{3.9}$$

The work done by the system reduces its internal energy (considered as negative), and the work done by external forces increases the internal energy (considered as positive).

The entropy assessment of the state of the industrial system parameters enables us to use the same relative indicator to estimate changes in these parameters and to synthesize these estimations in a unified economic image of the developed production situation [13].

Therefore, *entropy can be used as a measure of deviation of actual production parameters from the standard parameters*.

The basic characteristic of the management process is the aprioristic entropy of the controlled object taking into account a set of states of the production process. If the process is irreversible, then:

$$\sigma = \frac{d(\rho S)}{dt} = \sum_i \frac{\partial(\rho S)}{\partial a_i} \cdot \frac{da_i}{dt}, \tag{3.10}$$

where ρS is the production of entropy in the system; $\sigma = \frac{d(\rho S)}{dt}$ is the rate of entropy production in the system; a_i are local thermodynamic parameters; $i = \overline{1, n}$ is the number of GC (GF) of the production system.

Let us consider increase in the entropy production with change in local parameters as the reason for the irreversible process:

$$\sigma = \sum_i f_i X_i, \tag{3.11}$$

where σ is the rate of creation of entropy in the system; $X_i \equiv \frac{\partial(\rho S)}{\partial a_i}$ are thermodynamic forces; $f_i \equiv \frac{da_i}{dt}$ are thermodynamic flows determining the rate of change in parameters a_i.

The entropy of all nonequilibrium systems is an additive function of the entropy production in different parts of the system:

$$S = \int_{S'}^{S''} (\rho S) dV, \tag{3.12}$$

where S is entropy of the whole system; ρS is entropy produced in the system; dV is the change in the internal energy of the system as a result of infinitely small changes in work, heat, and energy mass transfer during system transition from state S' to S''.

In the equilibrium state the thermodynamic forces, flows f_i, and rate of entropy production σ are equal to zero. Therefore for small deviations from the balance it is natural to use a linear function:

$$f_i = \sum_{j=\overline{1,n}} k_{ij} X_i, \quad i = \overline{1,n}, \tag{3.13}$$

where X_i are thermodynamic forces (disturbance); f_i are thermodynamic flows (control); k_{ij} are kinetic (or phenomenological) factors.

It should be noted that the linear law (3.13) includes different independent kinetic parameters k_{ij}, the number of which can be reduced taking into account time and spatial symmetry in the interaction: The increase in flow f_j caused by the increase by the unit of force X_i is equal to the increase in the flow f_i caused by the increase by the unit of force X_{ij}.

From (3.11) and (3.13) it follows that for linear irreversible processes the law of entropy production can be written in the quadratic form with respect to thermodynamic forces (the same notations):

$$\sigma = \sum_i f_i X_i = \sum_{i,j} k_{ij} X_i X_j. \tag{3.14}$$

According to the second law of thermodynamics for irreversible processes $\sigma > 0$, i.e., the quadratic form (3.14) is positively defined:

$$\sigma = \sum_i f_i X_i = \sum_{i,j} k_{ij} X_i X_j > 0. \tag{3.15}$$

The basic laws and the equations of thermodynamics for irreversible processes are generated in Onsager's reciprocity relation equivalent to the principle of the least energy dissipation [14]. Thus, (3.14) gives us a local measure of irreversible processes in the system.

According to this principle, in case of irreversible processes the rate of entropy flow through the system boundary in isolated systems is equal to zero, whereas in stationary processes in open systems the total entropy of the system is constant. In other words:

- in open systems, in stationary processes the energy dissipation is minimum;
- in isolated systems stationary processes are impossible, as energy flow is needed to support these processes.

In the matrix form the coordinate expressions (3.13) and (3.14) are written as:

$$F = K \cdot X,$$
$$\sigma = X^T \cdot F = X^T \cdot K \cdot X, \tag{3.16}$$

where

$$F = \begin{pmatrix} f_1 \\ \dots \\ f_n \end{pmatrix}, \qquad X = \begin{pmatrix} X_1 \\ \dots \\ X_n \end{pmatrix}$$

are column vectors; $K = (k_{ij})_{i,j=\overline{1,n}}$ is a matrix of kinetic coefficients; X^T is a series vector, a transposed vector X.

3.2 Energy-Entropy Model for Assessment of Economic System Management

An example of entropy [15] calculation (quantitative influence of entropic processes on production) [16] in the case of two irreversible processes:

(1) according to the linear law (3.13) the flows are expressed as
(2)

$$f_1 = k_{11}X_1 + k_{12}X_2,$$
$$f_2 = k_{21}X_1 + k_{22}X_2, \tag{3.17}$$

where X_1, X_2 are thermodynamic forces (disturbance);
(3) for the entropy source in (3.14) we get the square-law form
(4)

$$\sigma = k_{11}X_1^2 + (k_{11} + k_{22})X_1X_2 + k_{22}X_2^2. \tag{3.18}$$

According to (3.15) the square-law form must be positive for all values of X_1 and X_2 except for $X_1 = X_2 = 0$ when the entropy production is equal to zero (but nothing changes in the system). According to [17] this requirement gives the following inequalities:

$$k_{11} > 0, \qquad k_{22} > 0, \\ (k_{12} + k_{21})^2 > 4k_{11}k_{22}. \tag{3.19}$$

Hence, coefficients k_{11}, k_{22} are positive, whereas reciprocity coefficients k_{12}, k_{21} can be both positive and negative but their values are limited by the condition (3.19).

For the general case we will introduce the following notations:

$$k_{ij} = k_{(ij)} + k_{[ij]}, \\ k_{(ij)} = k_{(ji)}, \\ k_{[ij]} = -k_{[ji]}. \tag{3.20}$$

Therefore $k_{(ij)}$ and $k_{[ij]}$ are symmetric and asymmetric parts of kinetic coefficients. According to [14], the asymmetric part does not make any contribution to the production of entropy (2.14) and, hence:

$$\sigma = \sum_{i,j} k_{(ij)} X_i X_j \geq 0 \tag{3.21}$$

or in matrix representation:

$$\sigma = X^T \cdot K \cdot X \geq 0, \tag{3.22}$$

where

$$X = \begin{pmatrix} X_1 \\ \dots \\ X_n \end{pmatrix}, \qquad X^T = (X_1, \dots, X_n)$$

are the force vector and transposed force vector; $K = (k(_{ij}))_{i,j=\overline{1,n}}$ is a symmetric part of matrix K.

According to the theory of matrices of a special type [17], for any quadratic form there exists such a *linear* transformation which transforms this quadratic form into a quadratic form in new variables with its subsequent transformation into a diagonal form. In particular, for the real symmetric matrix K there is such a real nonsingular matrix T that matrix $\tilde{K} = T^T \cdot K \cdot T$ is diagonal. The transformation to the main axes normalizes the given quadratic form:

$$X^T K X \equiv \tilde{X}^T \tilde{K} \tilde{X} \equiv \sum_i \tilde{k}_{ii} \tilde{X}_i^2 \equiv \sum_i \lambda_i \tilde{X}_i^2, \tag{3.23}$$

where λ_i is the eigenvalue of matrix K found as a solution to the characteristic equation $\det(K - \lambda E) = 0$ for

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix};$$

$\tilde{X} = (\tilde{X}_1, \ldots, \tilde{X}_n)$ are normal coordinates (correspond to the eigenvalue); $X = T\tilde{X}$, $T = (t_{ij})_{i,j=\overline{1,n}}$ or $X_i = \sum_{k=\overline{1,n}} t_{ik}\tilde{X}_k$; $\tilde{K} = T^T \cdot K \cdot T$ or $\tilde{k}_{ik} = \sum_{j,h=\overline{1,n}} k_{(jh)} t_{ji} t_{hk}$.

An additional transformation (normalization) $\tilde{X}_i = Y_i/|\sqrt{\lambda_i}|$, $i = \overline{1,n}$ transforms formula (3.23) to the canonical form:

$$X^T K X \equiv \sum_i \xi_i Y_i^2, \tag{3.24}$$

where coefficients

$$\xi_i = \begin{cases} +1, & \text{if } \lambda_i > 0 \\ -1, & \text{if } \lambda_i < 0 \\ 0, & \text{if } \lambda_i = 0. \end{cases}$$

For practical purposes we recommend:

- an *estimation* relation

$$\left.\begin{array}{l} \mathrm{CF} = \lambda_1 \tilde{X}_1^2 + \lambda_2 \tilde{X}_2^2 + \cdots + \lambda_n \tilde{X}_n^2 \\ \lambda_{\min} = \min_{i=\overline{1,n}}\{\lambda_i\}, \qquad \lambda_{\max} = \max_{i=\overline{1,n}}\{\lambda_i\} \end{array}\right\}$$
$$\Rightarrow \lambda_{\min}\left(\tilde{X}_1^2 + \cdots + \tilde{X}_n^2\right) \le KF \le \lambda_{\max}\left(\tilde{X}_1^2 + \cdots + \tilde{X}_n^2\right)$$

  or

$$\lambda_{\min} \sum_i \tilde{X}_i^2 \le KF \le \lambda_{\max} \sum_i \tilde{X}_i^2.$$

  Here $\sum_i \tilde{X}_i^2$ is the square of the length of the deviation vector $\tilde{X} : \|\tilde{X}\|^2 = \sum_i \tilde{X}_i^2$. As the linear transformation does not distort the sizes, $\|\tilde{X}\|^2 = \|X\|^2$ is valid, i.e., the estimation is correct:

$$\lambda_{\min} \sum_i X_i^2 \le KF \le \lambda_{\max} \sum_i X_i^2; \tag{3.25}$$

- for the *comparative* analysis the following fact [17] is used: For any two real, symmetric forms $X^T M X$ and $X^T K X$ there is a real transformation such as (3.13) or (3.16) simultaneously transforming the given forms to the canonical form.

The possibility to transform two quadratic forms to a canonical form enables comparison of two single-type manufacturing departments at different enterprises or to compare products of different technological levels produced at the same enterprise.

For non-closed systems the quadratic form is not necessarily positively defined. The comparative analysis is based on the following theorem [18]: *If at least one of the two real quadratic forms is positively defined, there is a basis in which both forms will get a canonical form*. The results of the theorem can be applied to the matrices defined numerically. For example, for a closed system one can use the model (the standard) for comparison of results and conclusions of a real non-closed system.

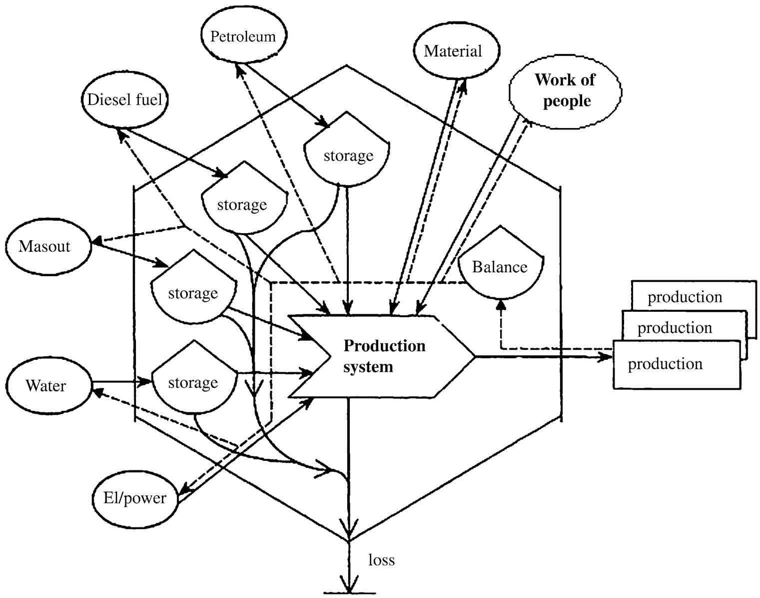


Fig. 3.2 Power consumption in the production process

Comparing open systems with the standard, it is possible to rank them according to the production efficiency and operating quality.

3.3 Energy-Entropy Approach as the Basis of System Estimation of Production Management Quality

3.3.1 United Measuring System of Energy Resources

An abstract power production scheme presented in Fig. 3.2 shows energy movement during the production process and reflects the action of laws of thermodynamics in economic systems. It is known that *all* types of energy can be fully transformed into energy of lower quality, thermal energy, which enables it to be employed as a universal equivalent [15, 19]. For this purpose the power resources consumed in the production process must be transformed to the united measuring system by means of a "nominal" power expense per 1 tenge of output goods defined at the state, sector, and other levels.

The following method can be used to determine *the nominal coefficient of power consumption per unit of output goods*:

1. All types of power resources consumed by the economic system in a certain year [15] and measured in different units are reduced to the unified system of

Table 3.1 Average specific combustion heat of the main fuel types

Coal	kcal/ton	6,450,000
Natural and accompanying gases	kcal/10^3 m^3	8,250,000
Petroleum	kcal/ton	10,870,000
Masout	kcal/ton	9,560,000
Diesel fuel	kcal/ton	10,160,000
Liquefied hydrocarbon gases	kcal/ton	16,260,000
Aviation fuel	kcal/ton	10,250,000
Electric power	kcal/kWh	862.07

measurement. This is done using reference information such as that in Table 3.1, and correlation[4]

$$Q\ (\text{kcal}) = V_i \cdot K_i, \tag{3.26}$$

where V_i is the amount of energy resources of the i-th type (in due units of measurement); K_i is the coefficient of transformation of the i-th power resource into calories.

The results of calculations based on the statistical materials of the Republic of Kazakhstan are presented in Table 3.1.

2. We found coefficients of power consumption by GDP[5] for different years. Part of a calculation based on the statistical data from the Republic of Kazakhstan is shown in Table 3.2.

The indicators vary from year to year because of changes in the output volumes and consumption of power resources.

The *nominal* is the coefficient that *characterizes the ratio of all types of power consumption by the RK industrial complex in a certain year to the industrial production for the same period.*

We will further use the obtained nominal to compare power consumption of different types of products produced by the enterprise.

[4]Actual specific power intensity can differ from the calculated values because of the usage of homogeneous but qualitatively different types of energy resources. In other words, it is possible to obtain different quantities of calories by burning the same type of fuel. To get a more precise estimation of the caloric content of power resources, for example, on the territory of Kazakhstan, it is necessary to correct the obtained coefficients for the discrepancy (in %) between calculated and actual power intensity in Kazakhstan based on the assumption that the entire territory has the same type of consumption and the same quality of power resources.

[5]Gross domestic product (GDP) is used as a key economic indicator with which power indicators are compared. In conditions where the role of the non-productive sphere is a strong part of the increase of the total efficiency of usage of major production factors, the GDP is a more adequate indicator of economic development than the national income, covering as it does only the sphere of goods production.

Table 3.2 Indicators of specific power consumption of the gross domestic product in the Republic of Kazakhstan (partial list)

Indicators	Units of measurement	2003	2006	2007	2008	2013
Consumption of HERE (calculated)	10^{12} kcal	953.520	988.340	922.970	818.920	893.880
Specific power consumption (HERE = $0.7 \cdot 10^7$ kcal)	kcal/Tenge	400.701	663.334	799.087	902.572	915.763
Specific power consumption (calculated)	kcal/Tenge	543.161	980.204	1,239.551	1,316.592	1,426.101
Discrepancy	%	73.8	67.7	64.5	68.6	64.3
Including industry						
Specific power consumption (HERE = $0.7 \cdot 10^7$ kcal)	kcal/Tenge	211.892	322.066	369.782	434.66	440.376

3.3.2 Methods Used to Estimate Power Consumption (Efficiency of Power Resources Usage) at the Enterprise Level

To estimate power consumption by output products one must use the following procedures:

1. Allocate all expenses for power production;
2. Transform units of measurement for different types of power resources to the uniform system of measurement;
3. Calculate specific power consumption of output products;
4. Compare the result obtained at the enterprise with the nominal value;
5. Estimate the obtained result;
6. Make a decision about expediency of enterprise operation on the basis of obtained result.

Let us illustrate the technique of estimation of power consumption of output products on the example of a concrete product.

At the first stage, from all expenses which go towards production we allocate power expenses under the schematic diagram (Fig. 3.3).

If several plants of the enterprise are involved in the production cycle of manufacturing the product in question, to get an objective estimation of power consumption it is necessary to consider consumption of electric and thermal energy in physical units (kWh, Gcal) in all the plants.

The second step is revaluation of power consumption by the product (transformation of power consumption to the unified system of measurements) using the formula (3.26). A partial charting of consumption of the electric thermal energy (in kilocalories) needed to manufacture the studied product is presented in Table 3.3.

In the third step we calculate the power consumption in the production (characterizes the ratio of energy (in calories) to costs (in tenge)) substituting the value of

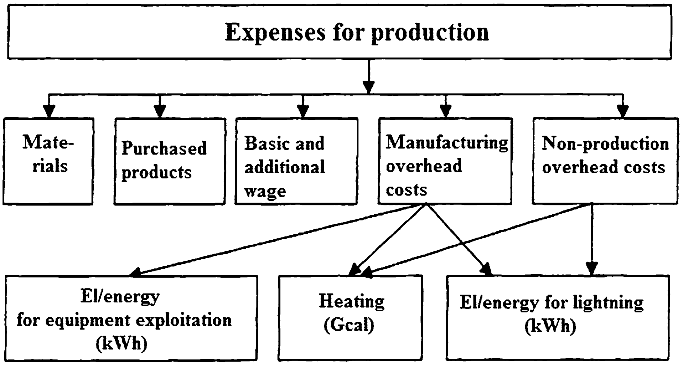


Fig. 3.3 A schematic diagram of distribution of energy consumption

energy resources in kilocalories (3.4) in the formula:

$$C_{\text{sp.pr.}} = E_{\sum}/(P_{\text{whsle}} \cdot N), \tag{3.27}$$

where $E_{\sum}$ is the total energy consumption (total consumption of electric energy, heat in the form of steam and hot water, compressed air); P_{whsle} is the wholesale price; N is the output of products.

The fourth step compares the value of power consumption at the enterprise with the standard; as standard one can choose, for example, the average energy consumption of Kazakhstan enterprises as given in Table 3.3.

Such comparison can be made visually (various computer software is available for plotting, construction of graphs, etc.) or analytically.

This process makes it possible to evaluate the level of stability of specific energy consumption used in production of certain products along with its probable dynamics, as well as to identify "bottlenecks" in different time periods, to plan means of problem solving, etc. Further steps in evaluating energy consumption are taken thorough evaluation of obtained results and decision making.

3.3.3 Entropic Evaluation of Production Efficiency

To estimate production efficiency the following method is used:

1. Decomposition of calculated items according to expense groups.
2. Calculation of divergences of obtained parameters (in tenge) per month (from the model).
3. Transformation of divergences measured in tenge into kilocalories.
4. Calculation of quadratic form determining the entropy level.
5. Comparison of the entropy level of the production being studied, with a similar production or a model.

Table 3.3 Overall consumption of electric and thermal energy for production of the item "deep-well pump ECW-6" during one year

Plant No.	Actual electric energy consumption		Overall consumption of electric energy	Actual thermal energy consumption for maintenance of buildings
	Maintenance of buildings	Maintenance of equipment		
1	2	3	4	5
February				
1	78,312.69	1,706,368.1	1,784,680.7	10,160
2	17,037.08	771,189.75	788,226.83	5,970
8	1,187.81	94,315.62	95,503.43	340
9	1,489.19	46,639.8	48,128.99	840
11	1,394.64	89,824.4	91,219.04	780
14	9,366.56	359,297.78	368,664.16	2,680
18	6,843.2	42,784.78	49,627.98	4,780
Total	115,631.17	3,110,419.9	322,6051	25,550
March				
1	64,579.02	1,316,831.6	1,381,410.6	8,020
2	16,032.47	679,444.76	695,477.23	5,360
8	1,093.26	95,379.33	96,472.59	300
9	1,164.17	82,350.73	83,914.9	420
11	1,087.35	81,101.89	82,189.33	700
14	7,304.14	193,482.93	200,787.07	2,160
18	5,348.1	31,751.74	37,099.84	3,610
Total	96,608.51	2,480,742.8	257,751.3	20,570
April				
1	133,897.45	24,798,443.7	2,932,341.1	6,040
2	26,775.94	1,163,432.8	1,190,208.7	3,250
8	2,198.33	167,410.22	169,608.55	220
9	3,971.18	333,331.25	337,302.43	690
11	2,617.91	162,156.68	164,774.59	510
14	13,461.84	389,105.11	402,566.95	1,340
18	8,220.11	49,391.6	57,611.71	1,970
Total	191,142.76	5,063,271.2	5,254 413.9	14,020
…				

6. Detection of production elements giving maximal increase in entropy.
7. Making decisions on regulation of production processes.

The schematic of interaction of the production system processes (3.2) is presented in Fig. 3.4, which shows the feedback mechanism realized as an information

Table 3.3 (Continued)

Plant No.	Actual electric energy consumption		Overall consumption of electric energy	Actual thermal energy consumption for maintenance of buildings
	Maintenance of buildings	Maintenance of equipment		
1	2	3	4	5
October				
1	23,194.79	354,564.09	377,758.88	...
2	17,563.03	418,977.64	436,540.67	...
8	2,044.69	59,490.94	61,535.63	...
9	3,090.67	82,827.55	85,918.22	...
11	1,110.99	21,232.83	22,343.82	...
14	5,696.76	81,427	87,123.76	...
18	5,336.28	9,100.63	14,436.91	...
Total	58,037.21	1,027,620.6	1,085,657.8	...
Annual	1,100,088.8	25,185,235	26,285,323	60,140

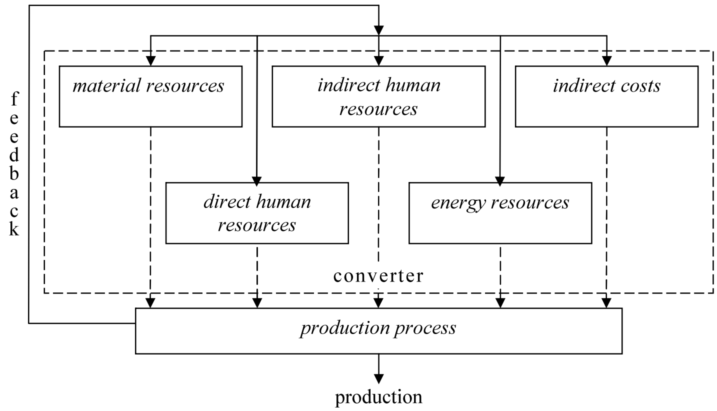


Fig. 3.4 A schematic diagram of interaction of production processes

process: Increase in knowledge about the system gives increase in information and decrease in entropy of the system (decrease in uncertainty).

The requirement of uniformity of measurements of all processes in the system makes it possible to use other terms for the estimation of economic production than economic criteria (cost, payback, profit, etc.) [20], where the universal equivalent is money.

As a measure of evaluation of different processes in the production system we can use consumed and "saved" energy.

The main disadvantage of this approach is that it difficult to take into account all circumstances to "close" the energy subsystem.

Table 3.4 Quantitative characteristics of the elements of production process (tenge)

Elements	February	March	April	May	June	July	August
Production	30,870.5	38,573.72	28,053.36	191,078.83	52,153.99	47,747.36	70,088.29
Raw materials	13,956.0	15,915.9	11,218.49	12,117.58	9,519.99	12,272.28	11,752.13
Energy	5,033.4	7,330.9	2,082.2	12,193.3	3,499.9	3,184	5,008.5
Direct human resources	3,663.75	4,289.99	4,255.35	27,109.7	9,959.51	10,896.18	14,433.25
Indirect human resources	2,673.9	3,985.8	3,074.4	24,509.6	11,987.9	10,052.8	12,849.1
Plant costs	2,049.7	3,552.5	2,632.2	25,780	9,176.8	6,457.9	13,158.5
All-factory costs	2,493.6	3,108.4	19,156.7	14,746.8	6,745.8	5,982.9	7,958.6

The production process shown in Fig. 3.4 has the following elements (Table 3.4):

X_1 is production including production costs;
X_2 is raw materials (material resources) including the cost of raw materials spent on output products and components;
X_3 is the energy in the form of cost of energy spent on the output products;
X_4 is direct human resources (wages with all deductions of workers directly employed in the production process);
X_5 is indirect human resources (wages with all deductions of indirect workers and management personnel);
X_6 is plant costs (all plant expenses spent on output products except elements X_3, X_4, X_5);
X_7 is overall expenses (all factory expenses spent on production except elements X_3, X_5).

A signal of possible entropic process at the enterprise is information about detected *deviations* of the chosen quantitative characteristics of the product [21, 22], either from standards or in time interval[6] (Table 3.5).

The data obtained in the form of vectors-deviations[7] $X = (X_i)_{i=\overline{1,7}}$ are used to calculate the values of the quadratic form for the entropy source by the for-

[6]It is necessary to note differences in the entropy values obtained from deviations of the production process parameters from the planned values and deviations from the statistical data for previous periods of time. The choice of the method of calculation depends on the research purpose and conditions.

[7]The elements of the system and vectors-deviations have the same notations to avoid misinterpretations.

Table 3.5 Deviations of quantitative characteristics of production process elements per month (tenge)

	Divergence vectors					
	February–March	March–April	April–May	May–June	June–July	July–August
X_1 (output products)	7,703	−10,520	163,025	−138,925	−4,407	22,341
X_2 (raw materials)	1,960	−4,697	899	−2,598	2,752	−520
X_3 (energy)	2,298	−5,249	10,111	−8,693	−316	1,825
X_4 (direct human resources)	626	−35	22,854	−17,150	937	3,537
X_5 (indirect human resources)	1,312	−911	21,435	−12,522	−1,935	2,796
X_6 (plant costs)	1,503	−920	23,148	−16,603	−2,719	6,701
X_7 (all-factory costs)	615	16,048	−4,410	−8,001	−763	1,976
Vector length	*8,556*	*20,481*	*167,980*	*142,033*	*6,300*	*23,914*

mula (3.16):

$$F = K \cdot X \quad \text{or} \quad \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{pmatrix} = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} \\ k_{21} & k_{22} & 0 & 0 & 0 & 0 & 0 \\ k_{31} & 0 & k_{33} & 0 & 0 & 0 & 0 \\ k_{41} & 0 & 0 & k_{44} & 0 & 0 & 0 \\ k_{51} & 0 & 0 & 0 & k_{55} & 0 & 0 \\ k_{61} & 0 & 0 & 0 & 0 & k_{66} & 0 \\ k_{71} & 0 & 0 & 0 & 0 & 0 & k_{77} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix},$$

$$\sigma = X^T \cdot F \quad \text{or} \quad \sigma = (X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad X_6 \quad X_7) \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{pmatrix}.$$

Coefficients k_{ii} $(i = \overline{1, 7})$:

- give the value of the production coefficient inherent to every parameter of production process;
- characterize the level of technological process and are accepted as conditionally equal to identical processes assuming the same level of productive forces in the uniform production system;
- are not equal to zero, because although every value of production elements participating in the evaluation is obligatorily present in the final product, it can be spent on itself.

From the condition of positive values of the quadratic form it follows that the nonzero coefficients must be considerably less than the diagonal ones. Taking into account that the studied production system is metal-consuming (hence, a large proportion of expenses relates to raw materials), coefficients $k_{12} = k_{21}$ must be chosen larger than others; for example, the following relations for undetermined coefficients are possible: k_{11}, $k_{12} = k_{21} = 0.2k_{11}$, $k_{13} = k_{31} = 0.1k_{11}$, $k_{14} = k_{41} = 0.1k_{11}$, $k_{15} = k_{51} = 0.1k_{11}$, $k_{16} = k_{61} = 0.1k_{11}$, $k_{17} = k_{71} = 0.1k_{11}$. In this case the coefficient k_{11} acts as a part of the scale coefficient in evaluating entropy of the system. The experimental calculations show that in the considered example possible increase in the proportionality coefficient α in the relation $k_{12} = k_{21} = \alpha k_{11}$ in the range $\alpha \in [0.2; 0.9]$ does not contradict any conclusions referring to the competing enterprise (see below (3.29), (3.30)).

In other words:

– one can assume admissibility of usage of any value of the proportionality coefficient α, most suitable for the given enterprise;
– suggested "qualitative" method[8] of quantitative assessment of production coefficients is sufficiently stable with respect to various assumptions.

With sufficient statistics and information about technological processes, one can directly evaluate kinetic coefficients k_{ij} and calculate entropy production rate σ. If such information is not available, the approximate estimators are used.

If it is assumed that the system is closed, then it is possible to construct a positively defined canonical quadratic form (3.23) with $\lambda_i > 0$ $(i = \overline{1,7})$ or (3.24) with $\xi_i = +1$ $(i = \overline{1,7})$. To evaluate the quadratic form an inequality can be used. The matrices of technological coefficients and the quadratic form can be considered identical for all single-type productions with approximately identical technology level. Hence, the diagonalized matrices will contain the same eigenvalues λ_i, and thus the same $\lambda_{\min}$, $\lambda_{\max}$.

The value of entropy and, respectively, the value of the quadratic form is calculated *for each time interval* being studied, at fixed moments of time $t = \overline{1,T}$. As a positively defined quadratic form creates an ellipsoid, the inequality (3.25) by its lower and higher limits forms a spherical layer in the same space [23, 24]. *The modulus of the divergences vector* $X = (X_i)_{i=\overline{1,n}}$ *is set to be equal to radius* R of a sphere: $\sqrt{\sum_i X_i^2} = R$. For each $t = \overline{1,T}$ we calculate $\sqrt{\sum_i X_{it}^2} = R_t$ and suppose $R_{\min} = \min_{t=\overline{1,T}}\{R_t\}$, $R_{\max} = \max_{t=\overline{1,T}}\{R_t\}$.

Spheres with radii $\sqrt{\lambda_{\min}} \cdot R$ and $\sqrt{\lambda_{\max}} \cdot R$ apparently lie inside the spherical layer defined in (3.25) for every $t = \overline{1,T}$, and the values of the quadratic form from the vector of divergences lie between the boundaries:

$$\lambda_{\min} R_{\min}^2 \le \lambda_{\min} R^2 \le K\Phi \le \lambda_{\max} R^2 \le \lambda_{\max} R_{\max}^2.$$

[8] Checked by repeated computing experiments.

In this case it is obvious that $R^2_{\min} \le R^2$, $R^2 \le R^2_{\max}$, that is

$$R_{\min} \le R \le R_{\max} \quad \text{or} \quad \min_{t=\overline{1,T}}\left\{\sqrt{\sum_i X^2_{it}}\right\} \le R \le \max_{t=\overline{1,T}}\left\{\sqrt{\sum_i X^2_{it}}\right\}. \tag{3.28}$$

Thus, quadratic forms can be compared by comparing limitations of *radii R—modulus of divergence vectors*.

Estimation of radii in the considered example gives the following values ((3.28), Table 3.5):

$$6.3 \cdot 10^3 < R < 1.7 \cdot 10^5, \tag{3.29}$$

$6.3 \cdot 10^3$—June–July radius; $1.8 \cdot 10^5$—April–May radius.

For comparison we give the estimation of entropy level at the other enterprise, "a competitor":

$$2.9 \cdot 10^3 < R < 5.4 \cdot 10^3. \tag{3.30}$$

As the intervals of radius values at the studied enterprise and its competitor do not overlap, productions are distinguishable, but level of entropy of the studied system is higher, which means that it is less effective than the competitive enterprise.

Different economic systems at *different* times have *different* money–energy ratios [14]. The production data calculated in monetary terms (for example, tenge) can be converted into calories as follows (we stop "feeling" possible change in the type of obtained dependencies):

$$E_{\text{kcal}} = E_{\text{mon.un.}} \cdot C_{\text{sp.pr.}}, \tag{3.31}$$

where E_{kcal} is the expenditure element giving the amount of energy used in production, expressed in kilocalories; $E_{\text{mon.un.}}$ is the expenditure element giving the amount of energy used in production, expressed in monetary units (for example, tenge); $C_{\text{sp.pr}}$ is the value of specific energy consumption in a certain period of time calculated by formula (3.27).

The energy-entropic assessment is made according to the technique described above, using the data expressed in kilocalories.

3.3.4 Usage of Energy-Saving Criterion to Assess Production Control Quality

3.3.4.1 A Thermodynamic Approach to Constructing Systems Controlling Production Processes

The technological process follows the laws of thermodynamics and, consequently, can be considered as the system interacting with the environment by mass and power exchange. The problem of determining coordinates of the technological process model can be solved by the energy-entropic approach due to its universality: The choice of coordinates of various technological process models does not depend on their functional and organizational structure [12].

To this effect, at first the system is decomposed as the finite set of subsystems. The state of every i-*th* subsystem is described by the amount of internal energy, which is defined as the sum of integrated elementary work. The identification of phenomena significantly influencing the internal energy enables us to generate *a vector of model co-ordinates*:

$$R = X \cup Y, \quad X = \{X_i\}_{i=\overline{1,n}}, Y = \{Y_i\}_{i=\overline{1,n}},$$

where X is a subset of generalized forces (GF); Y is a subset of generalized coordinates (GC); $R = X \cup Y$ is a united set of subsets of GF and GC.

In constructing a model of the type "input–output," R is partitioned into the vector of controlling coordinates U, the vector of disturbing coordinates f, and the vector of output coordinates Z based on the following prerequisites. Vector Z includes all set of GC from the number of significant phenomena ($Z \subset Y$). The subset GF is partitioned into two further subsets: X^U including a set of component vectors controlling coordinates $U|U \subset X^U$, and subset X^f including components of the vector of disturbing coordinates. The criteria of identification of subsets X^U and X^f in the set is the sign of direction of the corresponding elementary work δA in (3.9):

$$\begin{aligned} X_i &\in X^U, \quad \forall \delta A_i > 0, \\ X_i &\in X^f, \quad \forall \delta A_i < 0. \end{aligned} \tag{3.32}$$

Such division means that the vector of controlling coordinates is formed from the GF which are responsible for energy supply to the system from the environment (in particular, the controlling environment). The GF corresponding to the phenomena related to energy consumption in the system must be placed into X^f. If for some phenomena satisfying the first condition of the criterion (3.32), the energy is supplied to the system purposefully, then the GF of such phenomena is excluded from vector X^U and included into X^f [25].

In addition to the considered conditions the choice of integrated thermodynamic (TD) coordinates and forces is limited by the Onsager principle using the concept of source [14]. In this case TD forces are a variety of gradients Y_i acting in the system—for example, concentration, temperature, potential gradients, etc. Gradients in the system initiate corresponding fluxes f_i (diffusion flux, heat flux, electric current, etc.) tending to bring the system into the equilibrium state. The processes described by the Onsager principle refer to the category of irreversible processes. The interdependence between forces X, fluxes f, coordinates Y, dissipative function δ, and energy A is shown structurally in Fig. 3.5.

Here $K = (k_{(ij)})_{i,j=\overline{1,n}}$ is the matrix of Onsager coefficients; $G = (g_{ml})_{m,l} = \overline{1,n}$ is the Onsager matrix of backward transformation for vectors GF and GC.

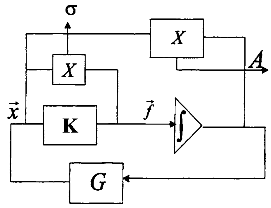


Fig. 3.5 The interdependence between forces X, fluxes f, coordinates Y, dissipative function δ, and energy A

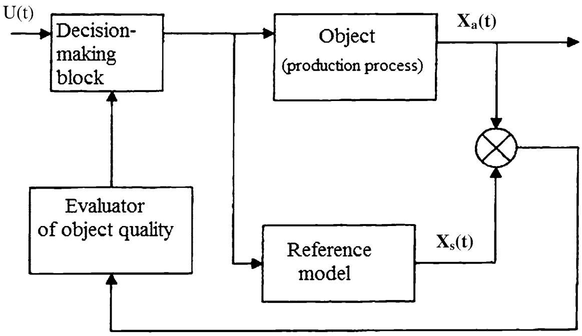


Fig. 3.6 A control system with the reference model

In equilibrium conditions one more requirement is imposed on the choice of thermodynamic forces and coordinates: observance of the involution condition. Application of the energy-entropic approach for formation of model coordinates introduces corrections in the criterion identifying the domain of determination of the modeled technological process. In terms of thermodynamics a technological process can be considered as a finite set of subsystems localizing different types of energy. The subsystems interact with each other and with the environment in the form of energy exchange. As technological processes are nonequilibrium TD systems, their status must be also characterized by degradation conditions [26].

The control of technological processes is formalized as a *temporary* procedure. A model describing the *behavior* (including control subsystem) and *state* of subsystems in the conditions of interaction is constructed. Then the aim of control is reduced to achieving a required status in one of the subsystems (e.g., in the subsystem of technological links, due to interaction of subsystems with each other) [25].

The principle of operation of production control systems according to the energy-entropic criterion is to decrease the degree of uncertainty of the controlled object in order to direct the object's movement along the desired line according to the criterion of minimization of the object's divergence from the set parameters. This requirement is satisfied by the control system with the reference model (Fig. 3.6). In this case *characteristics of the reference model* are defined by the standard, statistical, or expert method.

The stated problem is realized by minimizing discrepancy between the standard and actual object on the basis of corresponding criteria.

A comparison of the values of the object output characteristics $X_a(t)$ with the values $X_s(t)$ built into the reference model gives estimation of the error in the object operation $\varepsilon = X_a(t) - X_s(t)$. The quadratic deviation between, for example, the modeled amount of raw materials and actual amount of raw materials will be the entropic assessment of the management quality.

3.3.4.2 Comparison of Production Processes in Terms of Energy-Entropy

Basic assumptions:

– Estimation of economic efficiency of production management control based on energy savings is determination of the *degree of correspondence* of the energy expenditure comparable with the production volume, to *the nominal value*.
– In the illustrative example the nominal value of the energy expenditure per 1 tenge of GDP is the average amount of energy consumption in the RK branch of industry to which the studied enterprises refer. JSC "I," JSC "II," and JSC "III" are enterprises of Kazakhstan's industrial complex, therefore the specific energy consumption of the industrial complex is taken as "nominal." In the period from 2008 to 2013 it lay between 434.77 kcal/tenge and 440.38 kcal/tenge.
– Enterprises are considered as closed systems—external connections of the production system are not taken into account. The only factors that are taken into account are the production process and energy expenditure as a result of which finished products are manufactured from raw materials and half-finished products. The energy expenditure for the delivery of raw materials and sales of finished products are not taken into account.
– Several plants are involved in the production cycle of manufacturing the studied products, therefore to make an objective estimation of energy expenditure it is necessary to calculate consumption of electrical and heat energy in manufacturing each of the products in all plants where they are processed.
– For the single-type productions JSC "I," JSC "II," and JSC "III" we assume that the matrix of kinetic coefficients $K = (k(_{ij}))_{i,j=\overline{1,n}}$ is the same for all studied production systems.

Energy consumption of products is estimated according to the following procedures:

1. Conversion of energy inputs of all kinds into a unified system of measurement using the formula (3.26). At the moment of calculation the rate of conversion of electric energy to kilocalories corrected for specificity of energy consumption in Kazakhstan was taken equal 590.95 kcal/kWh.
2. Calculation of energy consumption by formula (3.27) on the basis of data for consumption of all energy resources used in industry reduced to a single coefficient. A partial chart of values of calculated specific energy consumption is given in Table 3.6.

Quantitative characteristics of elements of the standard are presented in Table 3.7.

Comparative evaluation of productive efficiency according to the energy-entropic criterion involves *calculations by the quadratic form* (3.16) defining entropy level

Table 3.6 Calculation of specific energy consumption of production

Month	Overall consumption (kcal)	Wholesale price (tenge)	Output (items)	Specific heat capacity (kcal/tenge)
JSC "I"				
January	129,355,938	31,800	636	6.40
February	289,597,128	31,800	637	14.30
March	85,217,268	31,800	160	16.75
…				
JSC "II"				
January	361,968,750	27,500	810	16.25
February	288,640,000	27,500	512	20.50
March	249,920,000	27,500	512	17.75
…				
JSC "III"				
January	6,860,451	29,800	2	115.11
February	415,873,486	29,800	70	199.36
March	614,155,322	29,800	111	185.67

Table 3.7 Quantitative characteristics of standard elements (of model production process (tenge))

Elements	Model
Products	20,430
Raw materials	11,735
Energy	4,210
Direct human resources	715
Indirect human resources	1,745
Shop's expenses	855
All-factory costs	1,170

for all enterprises participating in the comparative analysis. In this case entropy is used as a measure of divergence of the production process parameters:

– from the planned values (in monthly dynamics) in cost (tenge) and energy (kcal);
– from the modeled value[9] (in cost (tenge) and energy (kcal)).

As some part of information is absent, *comparison of the entropy level* in manufacturing of products produced at JSC "I," JSC "II," and JSC "III" is made in the following way: Instead of direct calculation of quadratic form (3.16) we will make its estimation (3.25).

[9] In order to check the difference in deviation values obtained in different ways.

To simplify the calculations we suppose that "approximate" comparison of quadratic forms is sufficient, i.e., in assumption $\lambda_{\min} \sim \lambda_{\max}$ the formula (3.28)[10] can be used.

The radius analysis shows:

- Cost evaluation (in tenge) gave the biggest range of radius variation for JSC "I":
  - $1.15 \cdot 10^3 \leq R \leq 9.59 \cdot 10^3$ are divergences by month;
  - $1.52 \cdot 10^3 \leq R \leq 1.27 \cdot 10^4$ are divergences from the model.

The other two enterprises have smaller ranges, i.e., JSC "I" has the worst situation in terms of uncertainty. We consider that evaluations in tenge are not always reliable because of high mobility of the financial sphere.

- Evaluation in kilocalories detected the biggest range of radius variation:
  - $1.5 \cdot 10^5 \leq R \leq 3.56 \cdot 10^6$ for divergences per month of JSC "III";
  - $1.47 \cdot 10^4 \leq R \leq 7.57 \cdot 10^6$ for divergences from the model "leader" JSC "II."
- Evaluations of quadratic form radii in kilocalories simultaneously at two "nominals" gave the following results:
  - JSC "I" $5.35 \cdot 10^4 \leq R \leq 6.00 \cdot 10^5$
  - JSC "III" $1.50 \cdot 10^5 \leq R \leq 3.56 \cdot 10^6$
  - JSC "II" $1.47 \cdot 10^4 \leq R \leq 7.57 \cdot 10^6$

The data show complete "overlapping" of ranges of the quadratic form radii, where the range of JSC "II" "covers" the range of JSC "III," which, in turn, "covers" the range of JSC "I."

Hence, the enterprise JSC "I" is less subjected to the entropic influence although, in general, all productions do not differ greatly from each other.

Having representative statistical material, one can set a problem of detecting the dependence of the entropy level of a production process on important characteristics of this process, for example, cost price level.

References

1. Mutanov, G.M., Useinov, B.M., Kutuzova, E.S.: New technologies in application of the laws of thermodynamics to studying of economic systems. In: Proceedings of International Scientific Conference Current Scientific Achievements and Fundamental Physical Training. Vestnik KazGU, p. 72 (1999)
2. Mutanov, G.M., Useinov, B.M., Kutuzova, E.S.: Application of the laws of thermodynamics to description and studying of economic systems. In: NKU Vestnik, Penropavlovsk, p. 80 (2000)
3. Ajibola, O.O., Aviara, N.A., Ajetumobi, O.E.: Sorption equilibrium and thermodynamic properties of cowpea (Vigna unguiculata). J. Food Eng. **58**(4), 317–324 (2003)

[10] An example of the estimation procedure is derivation of estimate (3.29) using Table 3.5.

4. Da Silva, G., Kim, C.-H., Bozzelli, J.W.: Thermodynamic properties (enthalpy, bond energy, entropy, and heat capacity) and internal rotor potentials of vinyl alcohol, methyl vinyl ether, and their corresponding radicals. J. Phys. Chem. **110**(25), 7925–7934 (2006)
5. Tavakkoli, M., Masihi, M., Ghazanfari, M.H., Kharrat, R.: An improvement of thermodynamic micellization model for prediction of asphaltene precipitation during gas injection in heavy crude. Fluid Phase Equilib. **308**(1–2), 153–163 (2011)
6. Ferro, R., Cacciamani, G.: Remarks on crystallochemical aspects in thermodynamic modeling. Calphad **26**(3), 439–458 (2002)
7. Chen, W.-H.: Business process management: a thermodynamics perspective. J. Appl. Manag. Stud. **8**, 241–257 (1999)
8. Bazarov, I.P., Thermodynamics. Vysshaya Shkola (1991), p. 376
9. Jaber, M.Y., Nuwayhid, R.Y., Rosen, M.A.: A thermodynamic approach to modelling the economic order quantity. Appl. Math. Model. **30**, 867–883 (2006)
10. Fisk, D.: Thermodynamics on main street: when entropy really counts in economics. Ecol. Econ. **70**, 1931–1936 (2011)
11. Cutrini, E.: Using entropy measures to disentangle regional from national localization patterns. Reg. Sci. Urban Econ. **39**, 243–250 (2009)
12. Yakovlev, V.B., et al.: Construction of Models of Continuous Technological Processes: Teaching Aid. KazPTI, Alma-Ata (1988), p. 31
13. Wehrl, A.: General properties of entropy. Rev. Mod. Phys. **50**, 221 (1978)
14. Glensdorf, P., Progozhin, I.: Thermodynamic Theory of Structure, Stability and Fluctuations. Mir, Moscow (1973), p. 280
15. Markvart, T.: Solar cell as a heat engine: energy-entropy analysis of photovoltaic conversion. Phys. Status Solidi (a) **205**(12), 2752–2756 (2008)
16. Peng, L., Sun, H., Sun, D., Yi, J.: The geometric structures and instability of entropic dynamical models. Adv. Math. **227**, 459–471 (2011)
17. Korn, G., Korn, T.: Reference Book on Mathematics for Scientists and Engineers. Nauka, Moscow (1973), p. 831
18. Kurosk, A.G.: Course of Higher Algebra. Nauka, Moscow (1975), p. 432
19. Odum, G., Odum, E.: Energy Basis of Man and Nature. Progress, Moscow (1978), p. 380
20. Cafaro, C., Giffin, A., Ali, S.A., Kim, D.-H.: Reexamination of an information geometric construction of entropic indicators of complexity. Appl. Math. Comput. **217**(1), 2944–2951 (2010)
21. Georgescu-Roegen, N.: Entropy Law and Economic Process. Cambridge, MA (1971)
22. Kapur, J.N., Kesavan, H.K.: Entropy Optimization Principles with Applications p. 408. Academic Press, San Diego (1992)
23. Caticha, A.: Entropic dynamics. In: Fry, R.L. (ed.) Bayesian Inference and Maximum Entropy Methods in Science and Engineering. AIP Conf. Proc., vol. 617, p. 302 (2002)
24. Jost, L.: Entropy and diversity. Oikos **103**(2), 363–375 (2006)
25. Mutanov, G.M., Kutuzova, E.S.: Electro-Entropic Methods of Assessment and Control of Economic Systems. Galym, Almaty (2002), p. 142
26. de Marchi, N.B., Blaug, M.: Appraising Economic Theories: Studies in the Methodology of Research Programs. Edward Elgar, Brookfield (1991)

Chapter 4
Currency Trading Methods and Mathematical Models

4.1 Currency Market Research and Management

In the conditions of the world crisis, the state rather often has to manipulate with the exchange rate in order to change the conditions of foreign trade using such methods of currency regulation as double currency market, devaluation and revaluation. Exchange rates have a considerable influence on the foreign trade of different countries acting as an instrument connecting cost parameters of national and world markets, influencing price relations between export and import, causing changes in the economic situation in the country and also changing the behavior of companies working for export or competing with import.

Using the exchange rate the entrepreneur compares his production costs with world prices. It makes it possible to estimate the result of foreign economic operations of individual enterprises and the country as a whole. Based on the current exchange rate and specific weight of each country in the world trade, the efficient exchange rate is calculated. The exchange rate influences the ratio between export and import prices, competitiveness of companies and profit of enterprises.

Currency trading is these days quite a common activity—around two trillion dollars a day are being traded in the worldwide FOREX market (FOREX = FOReign EXchange), and not less than 80 % of all transactions are speculative ones aimed at making profits by gambling on exchange rate differences. This game involves many players including both financial organizations and individual investors. The global interbank foreign exchange market (FOREX) is one of the largest segments of the financial market. The modern FOREX market formed in the mid-1970s, after the Bretton Woods system of monetary management stopped functioning in 1971 and countries started gradually switching to “floating exchange rates” driven by trade. Since that time, FOREX has become the most dynamic and liquid market. It is unique in that it operates 24 hours a day and it is not tied to a specific trade platform. Structurally, FOREX is a network of currency dealers working as a single mechanism from all leading financial centers in the world [1–3].

The most tradable currencies in the FOREX market are the US dollar (USD), Euro (EUR), Japanese yen (JPY), Swiss franc (CHF), and pound sterling (GBP).

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_4

These make four currency pairs: EUR/USD, USD/CHF, GBP/USD, and USD/JPY [2].

The past decades witnessed changes in the character of trade on the currency market with removal of accent on the date of trade—more and more bargains are concluded in future. On the one hand, it resulted in the increase in the sensitivity of the currency market to variations of market conditions and to a considerable increase in currency fluctuations, on the other hand, it gave wider possibilities for making highly efficient investments. The exchange courses are so unstable that variations by several percent occur rather often, which enables clients to make several operations every day. If one has a reliable trade technology, it is possible to make a business the efficiency of which is much higher than that of any other business. The largest banks buy expensive electronic equipment and hire staffs of hundreds of traders trading in various sectors of the currency market. Operations with currency, which not long ago were a privilege of large banks-monopolists, are now widely accessible due to the electronic trade system. And large banks themselves often prefer electronic trade system to individual bilateral operations. At present electronic trade systems cover 11 % of the total turnover of the FOREX market [1–3].

As a whole, devaluation of the national currency provides a possibility to the exporters of the country to reduce prices in the foreign currency getting a profit exchanging a more expensive foreign currency to the cheaper national currency and can sell goods at prices lower than the world prices, which gives enrichment at the expense material loss of your country. Exporters increase profits by exporting large batches of goods. However, reduction in the exchange rate of the national currency rises prices on import as to get the same sum in the foreign currency exporters have to increase prices, which stimulates increase in prices in the country, reduction in import of goods and consumption or development of national production of goods instead of import. Reduction in the exchange rate reduces national indebtedness in the national currency but increases foreign debts expressed in foreign currency. It becomes unprofitable to carry abroad profits, interest rates and dividends obtained by foreign investors in the country of residence. Such profits are reinvested and used for purchasing of goods in internal prices and further export.

In the conditions of freely-converted currencies it is very important to determine the balance of exchange rates. The exchange rates will be balanced if there is no possibility of getting speculative profit in the closed successive operations of currency purchasing. In case of imbalance it is necessary to determine a sequence of operations leading to the speculative profit.

Small companies and individuals have recently had much better opportunities for participating in the FOREX market, which has become accessible even to those who do not have a lot of money. Brokers, offering their services in margin trading, ask for a collateral deposit and make it possible for a client to trade in currency using amounts 40 to 50 times bigger than the deposit. The clients bear the risk of loss, while their deposits are collaterals that insure the broker. Nearly everybody can access the FOREX market, and many companies are seeking clients [2, 3].

Attractiveness of FOREX market for individual investors is mainly related to a possibility of getting high profits in a short period of time. Indeed, the graphs of movement of currencies show that a successful bargain is an efficient investment decision.

The other important property of the currency market is its stability. It is well-known that the main property of the financial market is its unexpected falls. Unlike stock market FOREX does not fall. If stocks are depreciated, it is a collapse. If the dollar goes down, it means that the other currency goes up. It makes the market and business connected with it stable: currency is an absolutely liquid commodity and will be always traded [2, 3].

The FOREX market operates 24 hours a day. It is not tied to any certain periods of time when exchanges are open; trade transactions are constantly going on between banks located in various parts of the globe. Exchange rates keep fluctuating by a fraction of a percent or even several percent, giving opportunities to market players to make several transactions a day [2].

Classification of Currencies [2, 4–6]:

1. Major currencies are those that are freely available in all segments of the FOREX market. They are fully convertible in large volumes and can be used in various types of spot and forward (concluded for future periods) transactions. The top five major currencies are the US dollar (USD), Euro (EUR), Swiss franc (CHF), Japanese yen (JRY), and British pound sterling (GBP).
2. Minor currencies are also completely convertible, but they may exhibit insufficient liquidity (difficulties while executing transactions in excess of USD 50 million) and can be subject to some other restrictions (on using financial derivative instruments, such as futures). This group includes currencies of Ireland, Canada, Australia, and Greece.
3. Exotic currencies are those that are quoted at all times, but can be subject to restrictions on the type and volume of transactions including forward market deals, which may be unavailable. National governments may apply restrictions on trade in their currency. This is true for Indonesia, Thailand, Hong Kong, Malaysia, Vietnam, China, and Philippines.
4. Emerging market currencies are the currencies of Eastern European countries, former USSR republics, South American countries, and South Africa.
5. Proxy currencies are those that undergo identical changes in value for some period of time (e.g., Euro and Swiss franc to the US dollar). Those who trade in FOREX usually deal only with the G5 currencies, as they always give unrestricted trade opportunities. However, short-term illiquidity can happen from time to time, when a trader cannot get requested quotations in due time because of an uncertain situation in the market (caused by central bank interventions).

Market participants [2, 7–10]:

1. Exporters and importers. They buy and sell foreign exchange when they need it for their own business, rather than at the moment when such deals can bring high profits.
2. Investors. They invest money in foreign securities, real estate, expansion of their business in other countries, etc. These people just need a certain currency for their business and do not focus on the exchange rate.

3. Central banks. Every country whose currency is traded in the world market wants to keep fluctuations of exchange rate quotations within a certain corridor. When a trip outside the corridor is detected, the central bank must interfere and support its weakening currency by buying it, or by selling if it appreciates. These operations are called interventions. From the political and economic point of view, they are intended to keep up the balance of interest of importers and exporters. A high exchange rate is not good for exporters, as their goods can become restrictively expensive in the importing country and the demand for their product will drop. But this situation is advantageous for importers, who can afford more goods for their money. Therefore, foreign banks are constantly striving for the golden mean.
4. Hedgers. Hedging consists of the purchase or sale of equal quantities of the same or very similar commodities in two different markets at approximately the same time (financial resources and financial instruments).
5. Market makers. These are dealers who regularly quote both bid and ask prices for currency, while there are no actual buyers. Many major banks are market makers. They counteract with other banks or financial institutions to make the buy and sell process uninterrupted. Thus, the currency market is always active, even when other market participants slow down their activities. Banks, which make the market, benefit from spread rather than from price movements.
6. Speculators. They stimulate the operation of the market mechanism, balance the demand and supply sides, and smooth out the trade process. But here we can find some unlucky buyers and sellers, who want to promptly close their loss-making positions.

4.2 Mathematical Models of Equilibrium Exchange Rates

4.2.1 Model Development and Analysis

The proposed mathematical model of equilibrium exchange rates is an example of mathematical modeling in finance. This model can help the banks and other financial institutions to make balanced and informed decisions.

Currently, banks are offered a wide choice of financial, credit, and commercial projects for obtaining funds. Obviously, there is an extensive sweep of factors that can affect bank financial performance, and it is most unlikely that elementary calculations or intuitive reasoning can help. Therefore, it is important for banks to have adequate models as well as mathematical and software tools in order to be able to reject non-viable projects and choose only feasible ones.

Now, when currencies are freely convertible, it is essential to be able to determine the equilibrium exchange rate.

In saying that the exchange rate is in equilibrium we denote the situation when there is no opportunities to earn a speculative profit while engaging in a closed sequence of currency purchase transactions. In the event the exchange rate is in disequilibrium, the task arises of determining a sequence of transactions yielding speculative profits [1].

If exchange rates are anchored to the main currency (anchor currency), then the rates are in equilibrium. Disequilibrium arises when there is no anchor currency.

Here we develop a model for currency purchase and sale transactions. The model takes into consideration restrictions, terms, and conditions pertaining to deals in currency; and the problem of equilibrium is formulated. The test for the model will be to find an equilibrium exchange rate. The model can be used for carrying out simulated calculations and varying the conditions and parameters when various market assumptions are used.

How the Mathematical Model Is Built [1] Let n be the number of types of currency. Each type will be associated with a vertex of a directed graph $G = (N, E)$, $N = \{1, \dots, n\}$. Each edge $(i, j) \in E$ will be associated with a positive number $b(i, j)$, which is the rate of conversion of the currency i into the currency j. Difference between the bid and offered rates is given by the inequality $\alpha(i, j) \cdot b(j, i) < 1$, $i \neq j$ (commission). We shall assume that the graph G has all its edges and loops; that is, $(i, i) \in E$ and $b(i, i) = 1$, $i = 1, \dots, n$.

A closed path (cycle) $K = ((i_1, i_2), (i_2, i_3), \dots, (i_p, i_1))$ determines the sequence of currency purchasing (exchange) transactions. Let $|K| = p$ be the number of edges in the cycle:

$$a(K) = a(i_1, i_2) \cdot \dots \cdot a(i_p, i_1). \tag{4.1}$$

The cycle K will be called profitable, if $b(K) > 1$. A profitable cycle determines sequence of transactions that yield a speculative profit.

The graph G will be balanced, if it does not have any profitable cycles. Any balanced graph corresponds to the equilibrium exchange rate.

Note that if K is a profitable cycle, then $|K| \geq 3$.

The model in question has an important feature, which is a closed path in the oriented graph that helps to simulate feedback. Feedback is the most important element of any economic system. There are paths that enhance or amplify changes moving a system away from its initial state. These paths are called positive feedback cycles. Negative feedback cycles are those that dampen or suppress changes that tend to move the system from its initial state.

A positive feedback cycle contains an even number of edges with a minus mark. Otherwise, it will be a negative feedback cycle.

If there are many positive paths in a system, then the system will be unstable. On the other hand, if there are many paths counteracting a change, it may cause disequilibrium of another type, namely oscillations with increasing amplitude. When oscillations attenuate, and the system falls into a certain state with indicators reaching a certain level, then this system will be stable.

A specific feature of tasks with many components, is that model systems built of directed graphs (digraphs) can include various social, economic, and environmental indicators. The indicators can be of different nature, such as statistical and non-statistical ones, as well as quantitative and qualitative indicators. Many-component tasks can be used to assess system development trends. When refining the model, it will be possible to quantitatively forecast changes in the system variables and to study various scenarios of interactions that influence the system in question.

Models based on weighted digraphs with time lags can give us trends of variable change with time. Thus, even such a simplified model can be used to project the behavior of quite a complicated system.

4.2.2 *Equilibrium Exchange Rate: Statement of the Problem and Ways to Solve It*

The problem of equilibrium exchange rate will be formulated as follows. For a given graph $G = (N, E)$ and given numbers $b(i, j)$, which satisfy the above conditions, it will be required to determine whether the graph G is balanced or not. If not, it will be required to find at least one profitable cycle.

The general problem is to find all profitable cycles (or cycles with $\alpha(K) > 1$) [1, 21].

One of the solution techniques here is to reduce the equilibrium exchange rate problem to the assignment problem.

For this purpose, the rates of conversion have been replaced by their respective logarithms, which are called edge lengths (or weights). Then the problem is reduced to the one of finding paths with positive lengths.

Assume that $v(i, j) = \lg b(i, j)$ and $v(K) = \lg b(K)$. From (4.1), it follows that $v(K) = \beta(i_1, i_2) + \cdots + \beta(i_p, i_1)$. Each edge (i, j) of the graph G will be associated with its number $v(i, j)$, which will be termed the length of the edge (i, j). The path K will be profitable, if and only if the length $v(K)$ of this path is positive.

A simpler problem has been formulated and solved; and an algorithm has been designed to determine whether a profitable path exists or not.

A simple polynomial algorithm is suggested to solve the problem; thus, the assignment problem is a simple polynomial problem. To find paths with positive lengths, let us formulate the following problem.

$$f(x) = \max \sum_{i=1}^{n} \sum_{j=1}^{n} \beta(i, j) \cdot x_{ij} \tag{4.2}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, \ldots, n, \tag{4.3}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, \ldots, n, \tag{4.4}$$

$$x_{ij} \geq 0, \quad i, j = 1, \ldots, n. \tag{4.5}$$

The problem described by (4.2)–(4.5) is the assignment problem. In this problem, the components x^0_{ij} of the optimal angular vector $x^0 = (x^0_{11}, \ldots, x^0_{1n}, \ldots, x^0_{n1},$

$\dots, x_{nn}^0$) are equal to 0 or 1. The vector x^0 defines a certain set $\{K_1, K_2, \dots, K_r\}$ of mutually disjoint (or mutually exclusive) paths in the graph G [1]. The equation $x_{ij}^0 = 1$ means that the edge (i, j) lies in one of these paths. Any vertex of the graph G lies in one of the paths.

To solve the problem, the equilibrium condition theorem has been proven and a method has been proposed for finding profitable paths (the theorem is formulated below) [1, 21].

Theorem 4.1 *Let x^0 be an optimal angular vector in the problem* (4.2)–(4.5) *and* $\{K_1, K_2, \dots, K_r\}$ *be a set of closed paths* (*cycles*) *defined by the vector* x^0. *Further*:

(1) $f(x^0) \geq 0$;
(2) *if* $f(x^0) = 0$, *then the graph G is balanced*;
(3) *if* $f(x^0) > 0$, *at least one of the cycles* $K_1, \dots, K_r$ *will be profitable.*

Proof The vector x^1 having the components $x_{ii}^1 = 1, i = 1, \dots, n$; $x_{ij}^1 = 0, i \neq j$, is an allowable vector in the problem (4.2)–(4.5). As far as $\beta(i, i) = 0$, $i = 1, \dots, n$, then $f(x^0) = 0$. Therefore, $f(x^0) \geq 0$. From the content of variables x_{ij}, it follows that

$$f\left(x^0\right) = \beta(K_1) + \cdots + \beta(K_r) \tag{4.6}$$

Assume that $\beta(K_i) < 0$ for a particular $i \in \{1, \dots, r\}$. Let P be a subset of vertices that form the path K_i. Let us take the set $\{K_1, K_2, \dots, K_r\}$ and replace the path K_i with cycles (loops) $((j, j))$, where $j \in P$. The allowable vector x^2 satisfying the condition $f(x^2) > f(x^0)$ will conform to the derived set of closed paths. But it goes against the statement that x^0 is an optimal vector. Therefore,

$$\beta(K_i) \geq 0, \quad i = 1, \dots, r. \tag{4.7}$$

It follows from (4.6) and (4.7) that if $f(x^0) = 0$, then $\beta(K_i) = 0$, $i = 1, \dots, r$; and it means that the graph G is balanced. If $f(x^0) > 0$, then it follows from (4.6) that there exists such $i \in \{1, \dots, r\}$ that $\beta(K_i) > 0$; and it means that K_i is a profitable cycle. The theorem is proved. □

Pursuant to the theorem and by solving the assignment problem, it can be determined whether a graph is balanced or not. If not, then we proceed to solving the problem (4.2)–(4.5) to try and find at least one profitable cycle. If there are no profitable cycles, then the graph is balanced. But if there are profitable cycles (not all of them), then the graph is not balanced.

The problem of determining a closed sequence of operations that bring a speculative profit has been reduced to the assignment problem, which is a polynomial problem and can be solved on a computer within a reasonable time, even for a large number of variables.

To improve the reliability of currency purchase-and-sale models, it will be necessary to conduct the following additional studies:

- how to incorporate various markets (exchanges) into the model;

- how to differentiate various purchase-and-sale transactions with respect to time;
- how to state the problem for more general initial assumptions; and
- choosing the most appropriate methods for solving generalized problems.

In particular, the proposed model can be used to formulate the problem of currency exchange rate adjustments, so that any opportunities for speculative gains would be excluded. This kind of task can find applications with the central banks that control exchange rates. Our findings are new and can be of practical application when analyzing the freely convertible currency markets. The proposed model can be further developed in order to improve its adequacy when describing real processes.

To continue the studies, a problem has been formulated of defining the cycle edges (arcs) having maximum average weights (maximum density cycles). The need for solving this problem arises when studying the mathematical models of equilibrium exchange rate. The problem solution has been reduced to solving the assignment problem. An estimate has been given of the computational complexity of the proposed method and a case study is presented for computing the maximum density of a digraph.

4.2.3 *Optimal Adjustment of Currency Exchange Rates*

Let $N = \{1, \ldots, n\}$; $G = (N, E)$ be a complete digraph containing loops (i, i); in addition, $l(i, j)$ is the weight (length) of a graph edge $(i, j) \in E$ and K is a simple closed path in the graph G. Let us set $m(K)$ as the number of edges in the cycle K and $l(K) = \sum_{(i,j)\in K} l(i, j)$ as the weight of the cycle K.

Definition 4.1 We shall define $\rho(K) = \frac{l(K)}{m(K)}$ as the density of a simple cycle K.

Pursuant to the above definition, a path density is the average value of edge weights of the path. In particular, the density of a cycle having only one loop is equal to the loop weight.

Let us formulate the problem of determining the maximum density of a simple cycle in a graph G. One of the applications of this problem is to determine the maximum speculative profit as an average per operation in the mathematical model of currency purchase-and-sale transactions [1, 11–22].

Suggested below is a method for solving this problem and an estimate of its computational complexity, such that the complexity is majorized by $T(n)2^n$, where $T(n)$ is the computational complexity of the assignment problem with n^2 variables.

Every edge $(i, j) \in E$ will be associated with a Boolean variable x_{ij} and we shall deal with the following problem of the Boolean linear programming:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} l(i, j)x_{ij} \to \max; \tag{4.8}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, \ldots, n; \tag{4.9}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, \ldots, n; \tag{4.10}$$

$$x_{ii} = \alpha_i, \quad i = 1, \ldots, n; \tag{4.11}$$

where $\alpha_i \in \{0, 1\}$, $\sum_{i=1}^{n} \alpha_i \neq n - 1$.

The problem defined in (4.8)–(4.10) is the assignment problem. The problem complexity of (4.8)–(4.11) for given α_i is not higher than the complexity of (4.8)–(4.10). Consequently, the time for solving the problems (4.8)–(4.11) is majored by

$$\sum_{r=0}^{n} \binom{n}{r} T(n) = T(n)2^n, \tag{4.12}$$

where $T(n)$ is the complexity of the assignment problem.

By implication of the variables, the solution for (4.8)–(4.11) at fixed α_i bijectively determines a set of pairwise disjoint simple cycles. In this connection, if $x_{ii} = 1$, then there is such a cycle in this set, which is a loop (i, i).

Theorem 4.2 *Let K^* be a cycle having the maximum density as compared to all the other available cycles that are derived when solving the* (4.8)–(4.11) *problems. Then K^* is a simple cycle having the maximum density in the graph G.*

Proof Let K be any simple cycle in the graph G and let N_1 be the graph vertices contained in the cycle K, $N_2 = N \backslash N_1$. Assume that $\alpha_i = 1$ if $i \in N_2$ and $\alpha_i = 0$ if $i \in N_1$.

For these values of α_i, the vector x that satisfies (4.9), (4.10), and (4.11) corresponds to the set of cycles K, (i, i), $i \in N_2$. Let $K_1, \ldots, K_s$, (i, i), $i \in N_2$ be the cycles that correspond with the optimal vector of the problem defined in (4.8)–(4.11), with α_i value being the same. As the value of the target function (4.8) is equal to the total weight of the cycles defined by the allowable vector of the (4.8)–(4.11), then

$$\sum_{j=1}^{s} l(K_j) \geq l(K). \tag{4.13}$$

The total number of edges in the cycle $K_1, \ldots, K_s$ is equal to the number of edges in K:

$$\sum_{j=1}^{s} m(K_j) = m(K). \tag{4.14}$$

As $\frac{l(K^*)}{m(K^*)} \geq \frac{l(K_j)}{m(K_j)}$, $j = 1, \ldots, s$, then it follows from (4.13), (4.14) that

$$\sum_{j=1}^{s} l(K_j) m(K_j) = l\big(K^*\big) m(K) \geq \sum_{j=1}^{s} l(K_j) m\big(K^*\big) \geq l(K) m\big(K^*\big).$$

Finally,

$$\frac{l(K^*)}{m(K^*)} \geq \frac{l(K_j)}{m(K_j)}.$$

The above inequality means that K^* is the maximum density cycle in the graph G. The theorem is proven. □

So, to find a maximum density cycle, it will be sufficient to solve the problems stated in (4.8)–(4.11), with their number being not more than 2^n. Before we proceed to solving the next problem, it is required to save the information on the best cycle identified when solving the previous tasks. Task computing time will be calculated based on the above formula (4.12) [1, 22].

Solution method We shall solve the problem (of assignment) for cycles with the number of edges:

(1) 3
(2) 4
(3) 5
...
(n) n.

At the step k (cycles with $k+2$ edges), we solve the assignment problem and get the cycles $K_1, \dots, K_p$. Prior to this step, all the previous problems have been solved. We denote $n_1, \dots, n_p$ as being the numbers of edges/arcs in these cycles; $l_1, \dots, l_p$ are weights (lengths) of these cycles. Then the density of the cycle K_i will be the ratio of the length to the number of edges (or arcs) in this cycle.

4.2.4 Building a Balanced Directed Graph

We start with the definition of a balanced directed graph (digraph) and then go on to build a balanced digraph based on a weakly connected subgraph. The proposed method can be used when solving problems of equilibrium exchange rates and when harmonizing the quality indicators for a managed system based on their relative priority [1].

Definition 4.2 A complete digraph $G = (N, E)$ with the arc lengths $l(i, j)$ is balanced if the length of any cycle in the digraph is zero.

Statement 4.1 A complete digraph with the arc lengths $l(i, j)$ is balanced if and only if there exists such a vector $x = (x_1, \dots, x_n)$ that $x_1 = 0$, $x_j - x_i = l(i, j)$.

Proof

(1) Let G be a balanced digraph. Assume that $x_1 = 0$, $x_j = l(j, 1)$. Then, as per the definition, $x_j - x_i = l(j, 1) - l(i, 1) = l(j, 1) + l(i, 1) = l(i, j)$.

(2) Let $x_1 = 0$, $l(i, j) = x_j - x_i$. Then the length of an arbitrary cycle $K = ((i_1, i_2), (i_2, i_3), \ldots, (i_{m-1}, i_m), (i_m, i_1))$ will be $l(K) = l(i_1, i_2) + \cdots + l(i_m, i_1) = (x_{i_2} - x_{i_1}) + (x_{i_3} - x_{i_2}) + \cdots + (x_{i_1} - x_{i_m}) = 0$.

The statement is proven. □

Let $H = (N, F)$ be a digraph having n vertices, and let $h(i, j)$ be the length of an arc $(i, j) \in F$. We set a problem of building a balanced digraph $G = (N, E)$, $F \subset E$ based on its subgraph H, in such a way that the arc lengths in the digraph H would be as close as possible to the lengths $l(i, j)$ of their corresponding arcs in the digraph G. The optimality criterion will be taken as the sum of squared deviations:

$$\min \sum_{(i,j)\in F} \bigl(l(i, j) - h(i, j)\bigr)^2. \tag{4.15}$$

Pursuant to Statement 4.1, the problem (4.15) reduces to minimization of the function:

$$f(x_2, \ldots, x_n) = \sum_{(i,j)\in F} \bigl(l(i, j) - h(i, j)\bigr)^2, \tag{4.16}$$

where $x_1 = 0$ in the right-hand side of (4.16).

Statement 4.2 The linear equation system

$$\frac{\partial f}{\partial x_j} = 0, \quad j = 2, \ldots, n, \tag{4.17}$$

has an unambiguous solution, which is the minimum point of the function $f(x_2, \ldots, x_n)$.

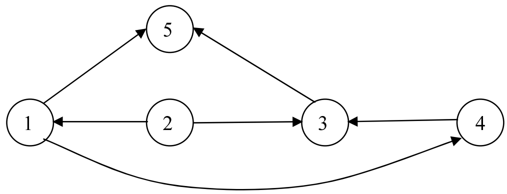
Proof Let us first prove that f is a strictly convex function. Assume that for certain $x^1 = (x_2^1, \ldots, x_n^1)$, $x^2 = (x_2^2, \ldots, x_n^2)$, $0 < \alpha < 1$, the following equality holds:

$$f\bigl(\alpha x^1 + (1-\alpha)x^2\bigr) = \alpha f\bigl(x^1\bigr) + (1-\alpha) f\bigl(x^2\bigr). \tag{4.18}$$

Note that any function of two variables—as presented in (4.16)—will be convex. If we fix one of the variables in this function, then the function will be strictly convex with respect to the other variable. Therefore, it follows from (4.18) that $x_i^1 = x_i^2$, $i \in J_1$, where J_1 is a set of vertices in the digraph H, which are connected with the vertex 1 by either an incoming or an outgoing edge. Since the variables $x_i = \alpha x_i^1 + (1-\alpha)x_i^2 = x_i^1$, $i \in J_1$ in Eq. (4.18), then (4.16) will contain strictly convex functions of the functions x_i, $i \in J_2$, where J_2 is a set of vertices, which are connected with the vertices of the set J_1 by either incoming or outgoing arcs. Therefore, it follows from (4.18) that $x_i^1 = x_i^2$, $i \in J_2$. As the digraph H is weakly connected, then as a result we get $x^1 = x^2$. This means that f is a strictly convex function.

The function f is limited from below ($f(x) \geq 0$) and is continuous. Consequently, the minimum is achieved at $x^0 \in R^{n-1}$. Since f is a strictly convex function, then x^0 is the only minimum point. In addition, f is a differentiable function

Fig. 4.1 Subgraph



over the entire function domain R^{n-1}. Therefore, x^0 is a stationary point (it is a solution of the linear equation system (4.17)). It is common knowledge that in case of a convex differentiable function, the following inequality holds

$$\left(\text{grad}\, f\left(x^1\right), x^2 - x^1\right) \le f\left(x^2\right) - f\left(x^1\right) \tag{4.19}$$

for any $x^1 \in R^{n-1}$ and $x^2 \in R^{n-1}$. If x^1 is a stationary point (grad $f(x^1) = 0$), then (4.19) implies that $f(x^1) \le f(x^2)$, viz. x^1 is the point of the minimum. Therefore, x^0 is the only stationary point and the only solution for the system (4.17). Statement 4.2 is proven. □

Let us consider a specific example of building a balanced digraph based on a subgraph H, shown in Fig. 4.1 [1].

Given the edge/arc lengths in the digraph H:

$$h(2,1) = -1.5; \quad h(1,4) = 2.4; \quad h(1,5) = 1; \quad h(2,3) = -1.6;$$
$$h(4,3) = -2.8; \quad h(3,5) = 1.2,$$

then

$$f(x_2, x_3, x_4, x_5) = (-x_2 + 1.5)^2 + (x_4 - 2.4)^2 + (x_5 - 1)^2 + (x_3 - x_2 + 1.6)^2 + (x_3 - x_4 + 2.8)^2 + (x_5 - x_3 - 1.2)^2.$$

$$\frac{\partial f}{\partial x_2} = 4x_2 - 2x_3 - 6.2;$$
$$\frac{\partial f}{\partial x_3} = -2x_2 + 6x_3 - 2x_4 - 2x_5 + 11.2;$$
$$\frac{\partial f}{\partial x_4} = -2x_3 + 4x_4 - 10.4;$$
$$\frac{\partial f}{\partial x_5} = -2x_3 + 4x_5 - 4.4.$$

By solving the system (4.17) for this example, we get the only solution $x^0 = (x_2^0, x_3^0, x_4^0, x_5^0) = (1.433; -0.233; 2.483; 0.983)$.

Pursuant to Statement 4.1, vector x^0 defines lengths $l(i, j)$ of all arcs in the balanced digraph G, built on the basis of the given subgraph H. For example, $l(2,1) = x_1^0 - x_2^0 = 0 - 1.433 = -1.433$; $l(1,4) = 2.483$; $l(1,5) = 0.983$; $l(2,3) = -1.666$; $l(4,3) = -2.716$; $l(3,5) = 1.216$.

Comparing $l(i, j)$ with $h(i, j)$, it can be seen that the values $h(i, j)$ have undergone only insignificant adjustments [1].

Let us study some applications.

(1) Let n be a number of the types of currencies; $v(i, j)$ is the exchange rate of the currency j with respect to the currency i; $h(i, j) = \ln v(i, j)$; H is the digraph having n vertices and arc lengths $h(i, j)$; $w(i, j) = e^{l(i,j)}$. The values $w(i, j)$ are the adjusted exchange rates, for which there are no cyclic sequences of the currency purchase-and-sale transactions that would yield a speculative profit [1, 2].
(2) Let n be a set of performance indices of a certain management system; $v(i, j)$ is a quantitative measure of a relative priority of the index j with respect to the index i. These values can be predetermined using some kind of expert evaluation. They correlate if $v(i, j) \cdot v(j, k) = v(i, k)$. Otherwise, it will be necessary to build a balanced digraph using the method discussed in paragraph (1) above.

4.2.5 Equilibrium Exchange Rates: Problem-Solving Procedures

In this chapter, we shall solve a number of assignment problems using a simple polynomial algorithm.

4.2.5.1 Statement of the Assignment Problem

Suppose there is a set of positions $P = \{p_i\}_{i=1}^n$ and a set of elements $E = \{e_i\}_{i=1}^n$, where each of the elements can take only one position, and only one element can be placed in each position. Every variant of placing the i-th element in the j-th position can be assigned the cost c_{ij}. The task is to place all the elements in such a way that the integrated cost is minimal. To put it differently, the task is to find such a permutation of elements specified by bijective mapping $\Phi : E \to P$, which gives the minimum

$$\sum_{i=1}^{n} C_{\Phi(i),i} \to \min \tag{4.20}$$

The number of feasible permutations is $n!$

In various tasks, elements and positions can be interpreted as employees and jobs (the assignment problem *per se*); machines and parts machined by them; types of work and work periods or output work queue; customers and services (in a store or in a distributed computer environment); electrical/radio components and their mounting modules (the task of layout design); or electrical/radio components and their mounting areas (the task of component arrangements on a chip card).

If the number of elements in a specific task is not equal to the count of positions, then quite costly phantom elements or phantom positions can be introduced (which would be excluded from the optimal solution).

4.2.5.2 Assignment Problem as a Linear Programming Problem

Let us introduce an assignment matrix X of the size $n \times n$, the elements of which will be calculated according to the following rule

$$x_{ij} = \begin{cases} 1, & \text{if the element } i \text{ is assigned to the position } j \\ 0, & \text{if not} \end{cases}$$

We come to the following task of linear programming: minimize

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

provided the conditions below are met:

$$\sum_{j=1}^{n} x_{ij} = 1; \quad i = 1, 2, \ldots, n$$

(every element has been assigned to only one position);

$$\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1, 2, \ldots, n$$

(for each position, there is only one element assigned to it);

$$x_{ij} \in \{0, 1\}$$

(variables x_{ij} can take on only the values 0 or 1).

Therefore, the assignment problems can be solved using methods of linear integer (discrete) programming.

4.2.5.3 Assignment Problem as a Transportation Problem

The assignment problem is a special case of the transportation problem [25], where the workers to be assigned are the points of departure, and the assigned jobs are the points of destination; here, the demand and supply sizes are equal to 1. Therefore, the assignment problem can be solved as the transportation problem. Since the demand and supply sizes are equal to 1, a more effective method of solution has been developed, namely: the Hungarian Method.

The Hungarian Method [23–26] was suggested by an American mathematician (H. Kuhn, 1955), who used the matching theory taken from publications of the Hungarian scientists D. Kenig (1884–1944) and E. Egervary (1891–1958).

Assume that the cost matrix is $C = \|c_{ij}\|$, $i, j = 1, 2, \ldots, n$.

Step 1. We shall find the smallest element p_i in each i-th row of the matrix and subtract it from every element in the row: $c_{ij} = c_{ij} - p_i$ $(i = 1, \ldots, n)$.

Step 2. In the new matrix, we shall find the smallest element q_j in each j-th column and subtract it from every element in the column: $c_{ij} = c_{ij} - q_j$ $(j = 1, \ldots, n)$.

There will be at least one zero element in each column (i.e., job position) and in each row (i.e., worker) of the resulting matrix; these zeros will indicate candidates to be assigned.

Having applied this algorithm, we may find that the resulting optimal solution is not feasible; that is, none of the workers can be assigned to any job. If this is the case, then we should proceed to the next step of the algorithm.

Step 3. Select rows and columns through which you draw lines in such a way that all the zeros are covered and that no more lines have been drawn than necessary. Thereafter, find the smallest element that is not covered by any of the lines. Then subtract it from each entry that is not covered by the lines and add it to each entry that is covered by a vertical and a horizontal line. This procedure must be iterated until we get a feasible solution.

Modified Hungarian Method [14] Let us slightly modify the original task. Assume that we have m customers, whose demands are equal and who can each be served by only one vendor out of n available vendors. Each i-th vendor can serve more than one customer, but not more than a_i. A matrix $C = \|c_{ij}\|$ is given, which determines the cost of service provided by an i-th vendor to a j-th customer. It is sought to assign customers to vendors in such a way that the total cost of service is minimal.

Let us introduce a matrix $X = \|x_{ij}\|$, where an element x_{ij} equals 1 if an i-th vendor is assigned to a j-th customer; otherwise, the elements equal zero. It follows from the problem statement that the sum of all elements in the i-th row shall not exceed a_i, while the sum of all elements in the j-th column shall equal 1.

Thus, we are solving the minimization problem for the function:

$$z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

under the following restrictions

$$\begin{aligned}
&\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1, \ldots, m; \\
&\sum_{j=1}^{m} x_{ij} \le a_i; \quad i = 1, \ldots, n; \\
&\sum_{i=1}^{n} a_i \ge n; \\
&x_{ij} \in \{0, 1\}.
\end{aligned}$$

The algorithm of the modified Hungarian method includes a preparatory step and consists of not more than $(m - 2)$ iterations. Each successive iteration process consists of three steps. Every step is an equivalent transformation of the cost matrix. The purpose of every successive iteration process is to increase the count of independent zero elements. A single zero element in a column of the matrix c_{ij} will be called an independent zero element (or just independent zero). A row can have more than one independent zero depending on restrictions set for the relevant vendor. Each successive iteration process brings one more independent zero. A problem is considered solved when the count of independent zeros reaches the count of customers. We shall denote independent zeros as 0^*. When applying the algorithm, it will be required to cover and uncover the rows and columns of a cost matrix. Uncovered rows/columns will be marked with $+$ and the covered ones, with *. Unmarked rows/columns will be considered uncovered.

Preparatory Step. The purpose of this step is to carry out equivalent transformations of a cost matrix and to obtain at least one zero in each column of the matrix and put a tag on the independent ones. To this end, we shall find the smallest element in each column and subtract it from every element in the column (as in the conventional Hungarian method). As a result, we shall get a matrix with nonnegative elements, which will have at least one zero in each column.

Thereafter, we shall examine all the columns of the resulting matrix and tag with an asterisk those zero elements that belong to rows for which restrictions are set (independent zeros). Thus, the number of restrictions goes down by one. Having reviewed all the columns, we shall get several independent zeros.

Next iteration. If the matrix has m independent zeros, we finish the calculations. Otherwise, we go to the next iteration. Before starting, we cover all columns containing an independent zero element (it will be remembered that there can be only one independent zero in a column) and uncover all the rows. Then we proceed to the three steps described below.

Step 1. Examine uncovered elements (the ones at the intersection of uncovered columns and rows) and find the smallest element. If the smallest element $c_{kl} > 0$, go to step 3. If it is zero, then check whether the same l-th column contains an independent zero element and whether there are restrictions set for the row k. Four scenarios are possible:

(1) There is an independent zero and restrictions are kept;
(2) There is an independent zero and restrictions are not kept;
(3) There is no independent zero and restrictions are kept;
(4) There is no independent zero and restrictions are not kept.

For cases 1 and 3, mark the identifies zero element c_{kl} with a prime, cover the k-th row and uncover the columns, which give independent zeros at the intersection with the k-th row. Then, go on to step 1. Go to step 2 (scenario A) for case 2 and to step 2 for case 4 (scenario B).

Step 2 (scenario A). Decrease by 1 the number of restrictions in the row containing $0'$. Then, build a chain of elements as follows: initial $0'$; zero tagged with an asterisk and located in the same column; primed zero located in the same line as the previous zero tagged with an asterisk, etc. Thus, the chain is formed by moving from $0'$ to 0^* along a column and from 0^* to $0'$ along a row. The end element of the chain is $0'$.

Zeros tagged with an asterisk will be replaced by zeros (unmarked), while primed zeros will be replaced by zeros with an asterisk. This is the end of a routine round of iteration. The count of independent zeros (and, consequently, of assigned customers)

Table 4.1 Example 4.1

#	1	2	3	4
1	1.00	4.00	0.50	0.30
2	0.20	1.00	0.15	0.10
3	1.50	6.00	1.00	0.60
4	2.80	9.00	1.20	1.00

went up by 1. Now, we need to cover the columns with independent zeros while counting their numbers. If the count of zeros is less than m, we need to uncover all covered rows and go to the first step of the next iteration process. But if it equals m, then the computation is finished.

Step 2 (scenario B). The smallest element will be tagged with an asterisk, and the restriction set for the row containing this element will be decreased by 1. The routine iteration is finished. It will be required to cover the columns with independent zeros. If the count of independent zeros is less than m, it will be necessary to uncover all the covered rows and go to the first step of the next iteration. But if it equals m, then the calculation is over.

Step 3. It will be remembered that the third step is started (after the first step is done) if the smallest element of all the uncovered ones is strictly greater than zero. If this is the case, subtract the smallest element from all the elements of uncovered columns and add it to all the elements of covered rows. Now we have zeros among the uncovered elements; hence, we go to the first step. Upon completion of the first step, we go to the second step for cases 2 and 4 and complete the routine iteration. But for cases 1 and 3, we should go to the third step (since all the zeros in the matrix are covered); we run the third step, get a matrix with uncovered zeros and go back to the first step.

If we need to find the maximum total cost rather than minimum, the algorithm will be same, but it will be required to find the biggest element during the preparatory stage rather than the smallest one.

4.2.6 *Experimental Study of the Model of Equilibrium*

Example 4.1 Let us solve the problem for numbers $\alpha(i, j)$ given in Table 4.1.

When solving the equilibrium problem for this example, we will get the optimum vector $x^0 = (0, 1, 0, 0,\ 0, 0, 0, 1,\ 0, 0, 1, 0,\ 1, 0, 0, 0)$. The vector corresponds to the two cycles: $K_1 = ((1, 2), (2, 4), (4, 1))$, $K_2 = ((3, 3))$. Cycle K_1 is profitable: $\alpha(K_1) = 1.12$.

Example 4.2 Assume that $n = 6$ and the numbers $\alpha(i, j)$ are given in Table 4.2.

The solution yields: $x^0 = (1, 0, 0, 0, 0, 0,\ 0, 1, 0, 0, 0, 0,\ 0, 0, 1, 0, 0, 0,\ 0, 0, 0, 1, 0, 0,\ 0, 0, 0, 0, 1, 0,\ 0, 0, 0, 0, 0, 1)$, $f(x^0) = 0$. There are no profitable cycles in this example; therefore, the graph is balanced, which is easily checked by exhaustive enumeration of possibilities using the equilibrium problem software.

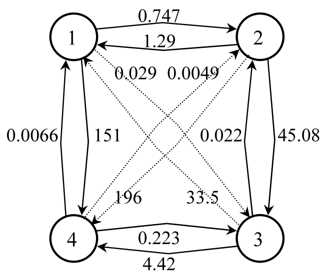
Table 4.2 Example 4.2

#	1	2	3	4	5	6
1	1.00	0.06	0.08	0.01	1.33	0.40
2	14.00	1.00	1.10	0.19	19.11	5.25
3	10.55	0.74	1.00	0.12	14.50	4.50
4	55.00	4.20	6.30	1.00	96.50	32.00
5	0.75	0.03	0.04	0.01	1.00	0.30
6	2.30	0.16	0.21	0.02	2.45	1.00

Table 4.3 Example 4.3

#	1_{USD}	2_{EUR}	3_{RUB}	4_{KZT}
1_{USD}	1	0.747	33.5	151
2_{EUR}	1.29	1	45.08	196
3_{RUB}	0.029	0.022	1	4.42
4_{KZT}	0.0066	0.0049	0.223	1

Fig. 4.2 Example 4.3



Example 4.3 Let us consider a specific example of solving a problem of equilibrium exchange rates (data source: a bank's exchange point). Actual data have been used to build a directed graph G and a table of exchange rates. In this case, the exchange rates are anchored to the US dollar and exchange rates between non-anchor currencies are unambiguously determined by their rates of exchange to the anchor currency (Table 4.3). The task is to determine whether the exchange rates are balanced and whether it is possible to get a speculative profit in the course of closed currency purchase-and-sale transactions (Fig. 4.2). In other words, we need to find whether a profitable cycle exists.

1. We solve this problem by simply enumerating the possibilities.

   (1) We start by considering cycles at 3 vertices:

$$1\text{–}2\text{–}3\text{–}1 \quad K = \big((1,2),(2,3),(3,1)\big)$$
$$\alpha(K) = 0.747 \times 45.08 \times 0.029 = 0.976$$
$$1\text{–}3\text{–}2\text{–}1 \quad K = \big((1,3),(3,2),(2,1)\big)$$
$$\alpha(K) = 33.5 \times 0.022 \times 1.29 = 0.951$$
$$1\text{–}2\text{–}4\text{–}1 \quad K = \big((1,2),(2,4),(4,1)\big)$$

$$\alpha(K) = 0.747 \times 196 \times 0.0066 = 0.966$$

$$1\text{–}4\text{–}2\text{–}1 \quad K = \big((1,4), (4,2), (2,1)\big)$$

$$\alpha(K) = 151 \times 0.0049 \times 1.29 = 0.954$$

$$1\text{–}3\text{–}4\text{–}1 \quad K = \big((1,3), (3,4), (4,1)\big)$$

$$\alpha(K) = 33.5 \times 4.42 \times 0.0066 = 0.977$$

$$1\text{–}4\text{–}3\text{–}1 \quad K = \big((1,4), (4,3), (3,1)\big)$$

$$\alpha(K) = 151 \times 0.223 \times 0.029 = 0.976$$

$$2\text{–}3\text{–}4\text{–}2 \quad K = \big((2,3), (3,4), (4,2)\big)$$

$$\alpha(K) = 45.08 \times 4.42 \times 0.0049 = 0.976$$

$$2\text{–}4\text{–}3\text{–}2 \quad K = \big((2,4), (4,3), (3,2)\big)$$

$$\alpha(K) = 196 \times 0.223 \times 0.022 = 0.962$$

Inference: There are no profitable cycles among the cycles built at 3 vertices of the graph, since $\alpha(K) < 1$.

(2) Let us now consider cycles built at 4 vertices of the graph:

$$1\text{–}2\text{–}3\text{–}4\text{–}1 \quad K = \big((1,2), (2,3), (3,4), (4,1)\big)$$

$$\alpha(K) = 0.747 \times 45.08 \times 4.42 \times 0.0066 = 0.982$$

$$1\text{–}2\text{–}4\text{–}3\text{–}1 \quad K = \big((1,2), (2,4), (4,3), (3,1)\big)$$

$$\alpha(K) = 0.747 \times 196 \times 0.223 \times 0.029 = 0.946$$

$$1\text{–}3\text{–}4\text{–}2\text{–}1 \quad K = \big((1,3), (3,4), (4,2), (2,1)\big)$$

$$\alpha(K) = 33.5 \times 4.42 \times 0.0049 \times 1.29 = 0.936$$

$$1\text{–}3\text{–}2\text{–}4\text{–}1 \quad K = \big((1,3), (3,2), (2,4), (4,1)\big)$$

$$\alpha(K) = 33.5 \times 0.022 \times 196 \times 0.0066 = 0.953$$

$$1\text{–}4\text{–}2\text{–}3\text{–}1 \quad K = \big((1,4), (4,2), (2,3), (3,1)\big)$$

$$\alpha(K) = 151 \times 0.0049 \times 45.08 \times 0.029 = 0.967$$

$$1\text{–}4\text{–}3\text{–}2\text{–}1 \quad K = \big((1,4), (4,3), (3,2), (2,1)\big)$$

$$\alpha(K) = 151 \times 0.223 \times 0.022 \times 1.29 = 0.955.$$

Inference: There are no profitable cycles among the cycles built at 4 vertices of the graph, since $\alpha(K) < 1$. As a result, we find that there are no profitable cycles and the graph is balanced. Therefore, there are no currency purchase-and-sale transactions that would bring a speculative profit.

2. Let us reduce this problem to the assignment problem and solve it:

   We will solve the problem (4.2)–(4.5), given that $v(i, j) = \lg b(i, j)$:

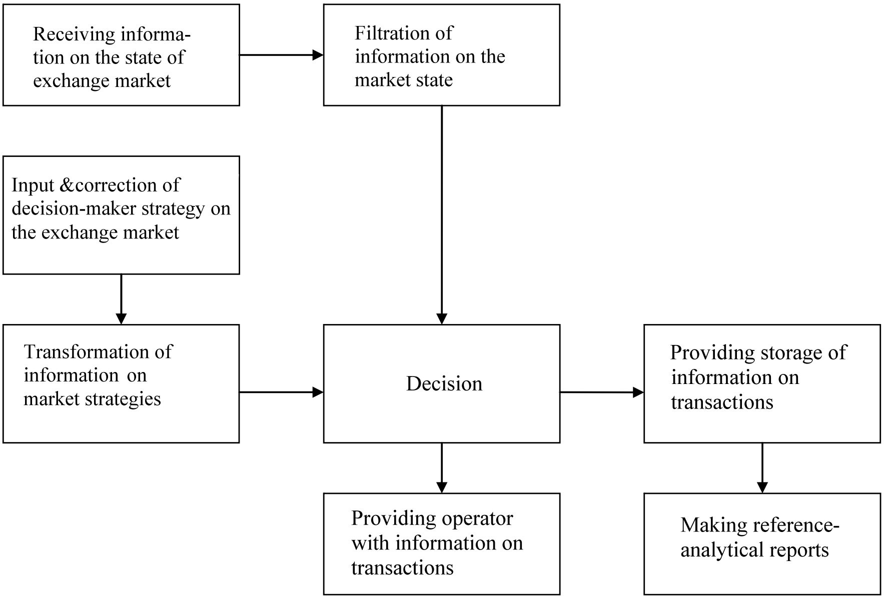


Fig. 4.3 Functional schematic of an information system

$$
\begin{aligned}
f(x) = {} & x_{11}\lg 1 + x_{12}\lg 0.747 + x_{13}\lg 33.5 + x_{14}\lg 151 + x_{21}\lg 1.29 + x_{22}\lg 1 \\
& + x_{23}\lg 45.08 + x_{24}\lg 196 + x_{31}\lg 0.029 + x_{32}\lg 0.022 + x_{33}\lg 1 \\
& + x_{34}\lg 4.42 + x_{41}\lg 0.0066 + x_{42}\lg 0.0049 + x_{43}\lg 0.223 + x_{44}\lg 1 \\
& \to \max
\end{aligned}
$$

$$
\begin{aligned}
& x_{11} + x_{12} + x_{13} + x_{14} = 1 \\
& x_{21} + x_{22} + x_{23} + x_{24} = 1 \\
& x_{31} + x_{32} + x_{33} + x_{34} = 1 \\
& x_{41} + x_{42} + x_{43} + x_{44} = 1 \\
& x_{11} + x_{21} + x_{31} + x_{41} = 1 \\
& x_{12} + x_{22} + x_{32} + x_{42} = 1 \\
& x_{13} + x_{23} + x_{33} + x_{43} = 1 \\
& x_{14} + x_{24} + x_{34} + x_{44} = 1.
\end{aligned}
$$

In this example, the solution of the problem (4.2)–(4.5) gives an optimum vector $x^0 = (1, 0, 0, 0,\ 0, 1, 0, 0,\ 0, 0, 1, 0,\ 0, 0, 0, 1),\ f(x^0) = 0$. The graph is balanced [Fig. 4.3].

For this example, the solution outcome was the same when using PER (Hungarian method) as when using the equilibrium problem solver.

4.3 Mathematical Projection Models for Currency Transactions

4.3.1 Forecast Problem of Risk Minimization

The last decades have seen a change in the trade pattern of the currency market, affecting trade transaction maturity; a significant growth of futures contracts has been observed. These developments have led, on the one hand, to an increase in currency market sensitivity and to significant currency fluctuations, but, on the other hand, they have brought better opportunities to investors [11, 12, 17, 19].

In order to continue investigations of the exchange rate balancing, it was necessary to solve the problem of determination of arcs of the loop with the maximal average value of the arc weight (maximal density loop) in the directed graph with given weights (lengths).

The necessity of solving this problem arises in studying mathematical models of exchange rate balancing. The solution to such a problem is reduced to the solution of several assignment problems.

A generalization of this problem gives the problem of determining conditions when operations can last for several days (study of day-by-day graphs). This brings up the problem of determination of a sequence of operations (loops) on multigraphs. A directed graph with loops and multiple edges (arcs) is called a pseudograph. A directed graph with multiple edges (arcs) but without loops is called a multigraph.

Information about possible operations of currency purchases and sales giving speculative profit may be useful for planning central bank actions. In forecasting speculative operations it is necessary to forecast changes in currency exchange rates for some period of time [27, 28]. The data for the expected exchange rates can be obtained by extrapolation or by other methods, in particular by the method of expert estimates.

Information which is, at least to a certain extent, reliable makes it possible to determine closed sequences of currency exchange operations giving a speculative profit [1, 21–23].

In this chapter the above-formulated problem is formalized as a combinatorial problem on the multigraph, its computational complexity is estimated, and a method for its solution is discussed.

Let n be the number of currencies; t a forecasting interval (equal to the number of days or weeks for which the information about forecast exchange rates is available); $\alpha(i, j, k)$ the exchange rate of the i-th currency to the j-th currency on the k-th day (week), $k = 1, 2, \dots, t$.

Let us consider a complete directed multigraph with n vertices and arc multiplicity equal to t. Denote (i, j, k) as the k-th arc connecting vertex i with vertex $j, k = 1, 2, \dots, t$.

Let us call a loop $(i_1, i_2, k_1), (i_2, i_3, k_2), \dots, (i_m, i_1, k_m)$ in the multigraph an admissible loop of length m, if $m \le n$, $i_p \ne i_q$, for $p \ne q$ and

$$1 \le k_1 \le k_2 \le \cdots \le k_m \le T \tag{4.21}$$

The admissible loop corresponds to the admissible sequence of currency exchange operations with return to the currency from which the operation began. This condition (4.21) means realizability of the sequence of operations in time.

Let $\alpha = \prod_{s=1}^{m-1} \alpha(i_s, i_{s+1}, k_s) \cdot \alpha(i_m, i_1, k_m)$.

Then $(\alpha - 1)$ is a relative profit obtained from the sequence of operations. The admissible loop is called profitable if for this loop $\alpha > 1$.

Let us formulate the problem of determination of all profitable loops in the set of many other admissible loops. This problem has the NP level of complexity even for $T = 1$, i.e., in case of a simple graph.

As it is necessary to determine all profitable loops, one can hardly find a method much more effective that the method of enumeration of all possible loops [1, 23]. It is quite natural to ask for which n, m, T values the problem can be solved by computer. The computational complexity of the problem is estimated from the following theorem.

Theorem 4.3 *The number of admissible loops of length m is equal to*

$$D(n, m, T) = A_n^m \cdot B(m, T) \tag{4.22}$$

where $A_n^m = n(n-1) \cdot \dots \cdot (n-m+1)$ *is the number of combinations of n by m;* $b(m, t)$ *is the number of different pairwise tuples* $(k_1, k_2, \dots, k_m)$ *satisfying the condition* (4.21).

Proof Let $(k_1, \dots, k_m)$ be an arbitrary tuple satisfying condition (4.21). The number of different arcs (i_1, i_2, k_1) is equal to $n(n-1)$. For each arc (i_1, i_2, k_1) there are $(n-2)$ different arcs (i_2, i_3, k_2), etc. This means that A_n^m different loops of length m correspond to one tuple $(k_1, \dots, k_m)$. This yields formula (4.22), etc. □

Numbers $b(m, t)$ in the formula (4.22) can be calculated by recurrence formulae according to the following theorem.

Theorem 4.4 *Let* $\beta_p(m)$ *be the number of tuples of length m satisfying condition* (1) *where* $k_m = p$, $p \in \{1, \dots, t\}$. *Then*

$$\beta_p(m+1) = \sum_{r=1}^{p} \beta_r(m); \tag{4.23}$$

$$\beta(m+1, T) = \sum_{p=1}^{T} (T - p + 1) \cdot \beta_p(m). \tag{4.24}$$

Proof The recurrent formula (4.23) follows directly from condition (4.21). From the same condition it follows that from one tuple of length m, where $k_m = p$, one can get $(t - p + 1)$ tuples of length $m + 1$ adding numbers $k_{m+1} = p, p + 1, \dots, t$. This means that numbers $b(m, t)$, $m = 1, 2, \dots$ can be calculated using formula (4.24). □

As an example, numbers $b(m, 6)$, $m = 1, \dots, 7$ calculated by formulae (4.23) and (4.24) are given in the Table 4.4.

Table 4.4 Example, numbers $b(m, 6)$

M	1	2	3	4	5	6	7
$B(m, 6)$	6	21	56	126	252	462	792

At relatively small values of parameters n, m, t the number $d(n, m, t)$ is very big and quickly increases with the growth of these parameters. For example, $d(20, 5, 4) \approx 10^8$, and hence solution to the problem, is at the limit of operational possibility for computers with a speed of about 10^8 operations per second.

Nevertheless, it is possible to use the enumeration method for solving the problem of forecasting speculative operations in real conditions [1, 21]. It can be justified by the following circumstances.

(1) The number of currencies which are actively used in the currency market is not very high. It is much smaller than the number of national currencies.
(2) The number m of operations in the closed sequence of operations providing speculative profit is, as a rule, not more than 4–5. This is confirmed both by the experimental calculations and by some general reasons along the lines of "the game is not worth the candle."
(3) It is also not reasonable to make number t high, as forecasting for large time intervals is unreliable.
(4) As loops (i_1, i_2, k), (i_2, i_1, k) of length 2 correspond to negative profit, it is possible to state that the number of profitable loops is much smaller than the number of admissible loops.

The list of all profitable loops of length $2, \ldots, m$ (if there are any) for the given value of parameter m is determined after considering all admissible loops. This information may be useful for making decisions at the level of central banks.

4.3.2 A Collocation Model for Forecasting Operations on the Currency Market

4.3.2.1 Background of the Collocation Model

The most important role in forecasting is played by optimization models (extremum models). Optimization models are a system of equations that in addition to limitations (conditions) include a special equation called a functional or an optimality criterion. This criterion is used to find the best solution for a certain parameter. In forecasting speculative operations it is necessary to forecast fluctuations in exchange rates for a certain time period. Information about expected exchange rates can be obtained by extrapolation or other methods, and in particular by expert evaluations.

The need to forecast through specific and applied analysis—aimed at the future and taking into account the uncertainty of this future—arises in various fields of human activity: policy, international relations, economy, finances, education, social processes, medicine, and so on. Forecasting is the scientific investigation of perspective development of a phenomenon, and estimation of parameters characterizing this phenomenon in the near or distant future [19, 20].

Ever since finance theory first appeared as a science in the 1920s to 1930s, the main challenge confronting scientists has been improvement of forecasting methods and development of new methods to be used in forecasting of specific financial parameters. The main task of forecasting of the stock market state is to reduce risk caused by uncertainty of efficiency of any currency transaction [19, 20].

The hypothesis of "random wandering" eventually triggered development of the concept of the functional (efficient) market, all participants of which are equally informed and take optimal decisions on the basis of this information.

The term "collocation" (meaning mutually shared location or arrangement, and used in language study to describe words that often appear together) is widely applied in modern computational mathematics to obtain approximate solutions to differential equations. Collocation in mathematics means determination of function by choosing analytical approximation to a definite number of given linear functionals. Mathematical ("pure") collocation is widely used in technical applications for solving interpolation problems [28].

4.3.2.2 Development of Mathematical Model for Forecasting Exchange Rate

Formulation of the problem. The problem of forecasting a dynamic (time) series of financial/economic information—in particular, financial indices—is one of the main tasks of financial theory. In every forecasting model the exchange rate a_n at a time moment $n = 0, 1, 2, \ldots$ of discrete time is presented as [24]:

$$A_n = A_0 \cdot e^{H_n} \tag{4.25}$$

$$H_n = h_0 + h_1 + \cdots + h_n, \tag{4.26}$$

$$h_i = \begin{cases} 0 & \text{for } i = 0, \\ \ln \frac{A_i}{A_{i-1}} & \text{for } i > 0. \end{cases} \tag{4.27}$$

Giving a "logarithmic profit" at moment $i \geq 0$.

Further, we will consider how the mean square collocation model is used to forecast exchange rates [28]. The sequence $(h_i)_{i\geq 1}$ (4.27) is supposed to be stationary, i.e., its members are random quantities such that their mathematical expectation

$$E(h) = m, \tag{4.28}$$

and covariances

$$Cov(h_i, h_{i+k}) = C_{hh}(\tau) \tag{4.29}$$

Do not depend on $i \geq 0$.

Let a_i be observed exchanged rates till the moment of discrete time $i = n$ [24]:

$$A_0, \quad A_1, \quad A_2, \quad \ldots, \quad A_n. \tag{4.30}$$

Hence, the values of stationary dynamic series h are also known:

$$h = (h_1, h_2, \ldots, h_n)'. \tag{4.31}$$

The task is to construct a forecast of exchange rate A for a certain moment of time in the future $i = n + k$ in the framework of the stationary model (4.28), (4.29).

Let us denote the forecast by $\widehat{A}_{n+k}$. We see that according to (4.25) and (4.26)

$$A_{n+k} = A_0 \cdot e^{H_{n+k}} = A_0 \cdot e^{H_n} \cdot e^{\Delta H}, \tag{4.32}$$

where

$$\Delta H = \sum_{i=n+1}^{n+k} h_i = S(h) \tag{4.33}$$

Is the value of linear functional s for the stationary dynamic series h. The forecast $\widehat{A}_{n+k}$ will be made taking into account (4.32):

$$\widehat{A}_{n+k} = A_n \cdot e^{\Delta \widehat{H}} = A_n \cdot \exp\{\Delta \widehat{H}\}, \tag{4.34}$$

where $\Delta \widehat{H}$ is the forecast (estimation) of the quantity (4.33).

The estimation $\Delta \widehat{H}$ will be determined in the class of linear procedures

$$\Delta \widehat{H} = g' \cdot h = \sum_{i=1}^{n} g_i h_i. \tag{4.35}$$

A zero-trend model Let us first assume that the model trend is expressed as a mathematical expectation of the dynamic series

$$M = e(h) = 0 \tag{4.36}$$

(the main precondition of the pure collocation model). Then for any vector of coefficients $g = (g_1, g_2, \ldots, g_n)'$ in the procedure (4.35) estimation, $\Delta \widehat{H}$ will be unbiased as the equality for g is fulfilled:

$$E(\Delta \widehat{H} - \Delta H) = 0. \tag{4.37}$$

Hence, in order to get an optimal mean-square forecast (4.35) it is sufficient that g satisfy the condition

$$\sigma_\varepsilon^2 = E\left(\varepsilon^2\right) = E\left\{(\Delta \widehat{H} - \Delta H)^2\right\} \to \min. \tag{4.38}$$

Vector g giving an unconditional minimum r^n to function σ_ε^2 can be determined in solution to the linear algebraic equations of the type:

$$C_{hh} \cdot g = C_{h,\Delta H}, \tag{4.39}$$

where

$$C_{hh} = \left(C_{hh}(i - j)\right) \tag{4.40}$$

Is a square nonsingular matrix of size $(n \times n)$ whose elements are the values of the covariance function $C_{hh}(\tau)$ of the stationary dynamic series h for $\tau = ij$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, n$.

Vector $C_{h,\Delta H}$ is a vector of covariance of h_i values, $i = 1, \ldots, n$ of the stationary dynamic series and values ΔH of the linear functional

$$C_{h,\Delta H} = \begin{pmatrix} Cov(h_1, \Delta H) \\ Cov(h_2, \Delta H) \\ \vdots \\ Cov(h_n, \Delta H) \end{pmatrix} \tag{4.41}$$

To calculate the covariance $Cov(h_i, \Delta H)$, $i = 1, 2, \ldots, n$, we will use the rule of covariance transformations.

$$Cov(h_i, \Delta H) = Cov\left(h_i, \sum_{l=n+1}^{n+k} h_l\right) = \sum_{l=n+1}^{n+k} Cov(h_i, h_l) = S\big(C_{hh}(\tau)\big)$$
$$= \sum_{l=n+1}^{n+k} C_{hh}(i-l). \tag{4.42}$$

After solving the system of Eqs. (4.39) and substituting solutions into (4.35), we get the formula of the optimal mean-square forecast for estimation of the linear functional

$$\Delta\widehat{H} = C_{\Delta H,h} \cdot C_{hh}^{-1} \cdot h, \tag{4.43}$$

where $C_{\Delta H,h} = C_{h,\Delta H}$.

The auto-covariance matrix of the forecast errors of the functional ΔH is determined by the formula

$$C_{\varepsilon\varepsilon} = C_{\Delta H,\Delta H} - C_{\Delta H,h} \cdot C_{hh}^{-1} \cdot C_{h,\Delta H}, \tag{4.44}$$

the forecast (4.43) has minimum dispersion

$$\sigma_\varepsilon^2 = \sigma_{\Delta H}^2 - C_{\Delta H,h} \cdot C_{hh}^{-1} \cdot C_{h,\Delta H}. \tag{4.45}$$

If condition (4.36) is satisfied. In the expression (4.45) dispersion $\sigma_{\Delta H}^2$ of the linear functional (4.33) is calculated as

$$\sigma_{\Delta H}^2 = SS\big(C_{hh}(\tau)\big) = \sum_{i=1}^{k}\sum_{j=1}^{k} C_{hh}(i-j). \tag{4.46}$$

Therefore if condition (4.36) is fulfilled (lack of the trend in the dynamic series data $(h_i)_{i\geq 1}$), then to construct an optimal forecast of the linear functional ΔH the following algorithm is used [24]:

(1) The values of "logarithmic profit" $(h_i)_{i\geq 1}$ are calculated using the data of the dynamic series of exchange rate $A_0, A_1, A_2, \ldots, A_n$ by the formula (4.27);
(2) The covariance function $C_{hh}(\tau)$ of the process $(h_i)_{i\geq 1}$ is estimated;
(3) Using the rule of transformation of covariances (4.42), the main covariance function $C_{hh}(\tau)$, cross-covariance functions of the process values $(h_i)_{i\geq 1}$, and linear functional values ΔH are obtained;
(4) The forecast (4.43) of the linear functional ΔH is fulfilled; and

(5) The formula (4.44) is used to estimate the forecast precision.

Substituting the functional estimate $\Delta\widehat{H}$ obtained by the descriptive algorithm into (4.34), we get the forecast of the exchange rate a_i for the moment $i = n + k$. The mean-square error of the forecast is determined as

$$\sigma_{\widehat{A}_{n+k}} \approx A_n \cdot e^{\Delta\widehat{H}} \cdot \sigma_\varepsilon = \widehat{A}_{n+k} \cdot \sigma_\varepsilon. \tag{4.47}$$

Formula (4.45) shows that optimality of the forecast for the functional $\Delta\widehat{H}$ (4.41) provides optimal forecast for exchange rates (4.32) in terms of relative mean-square error. It should be added that in differentials the forecast (4.32)–(4.41) is unbiased.

$$E(\widehat{S}_{n+k} - S_{n+k}) = 0. \tag{4.48}$$

It should be pointed out that during trade sessions and business operations the financial data (in particular, exchange rates) are fixed, unlike macroeconomic parameters for which this is not typical. In the overwhelming majority of applications the successive values of the time series are not independent; for example, the presence of autocorrelation in financial time series is caused by specific features of forming quotations under the action of numerous internal and external factors [1, 24].

4.4 Information Decision Support Systems in Currency Operations

4.4.1 Development of Information Model for Decision Support System in Currency Exchange Operations

The information model is a formalized description and documentation of information processes used to make technical decisions underlying the logical structure of the automated information system controlling currency exchange operations and the conceptual model of the currency exchange database [29].

The information model of decision making in currency exchange operations has four main types of information processes:

– information processes related to acquisition, accumulation, and storage of statistical data, called **information processes of data acquisition**;
– information processes related to analysis and forecasting of statistical data, called **information analytical processes**;
– information processes related to recommendations on currency operations by means of mathematical models for forecasting and correction of currency exchange operations; and
– information processes related to solution of various calculation operations (current financial operations, making reports, etc.), called **calculation information processes**.

The information system is structured to support decision making on the basis of certain predetermined rules. Knowledge bases contain logical rules, recommendations on the basis of which decisions can be made, depending on the situation on the currency exchange market.

The decision-support information system fulfills the following functions:

(1) Receives information about the state of the market in real time from communication systems. This function is fulfilled by interfaces supplied by the firm developing communication systems.
(2) Filters information received from communication systems and stores it in the database. The list of data to be filtered is defined by the types of transactions in the framework of which risk management is provided. Every type of transaction has a table of databases containing information on the state of the market segment at the moments of time when the characteristics of open positions corresponding to the type of transaction are revalued [30].

    The query to the communication system provides:

    - reading of required information by means of the conjugation protocol;
    - syntactic analysis of the message structure;
    - semantic message control (checking of data reliability according to formal rules);
    - conversion from formats of communicative system to DB management system, data storage in DB tables, control of data integrity and consistency;
    - operator message about information from the communication system and error report;
    - maintenance of adjustment of protocol parameters.

(3) Enables the decision-maker to input and adjust exchange rate information, and to choose models to analyze currency exchange rates:

    - model of balanced exchange rates;
    - model forecasting speculative exchange operations;
    - methods of technical analysis;
    - collocation model for forecasting the time series of currency exchange rates.

(4) Provides the decision-maker concluding transactions, with information about characteristics and parameters of transactions to be concluded;
(5) Provides DB storage of information about concluded transactions, and makes reference-analytical reports on concluded transactions.

A functional schematic of the information system is shown in Fig. 4.3.

There are three types of specialists working with the system:

(1) a system administrator—a technical specialist providing correct system operation, adjustment of communication protocols, and binding of the types of transactions with formats of communication systems;
(2) an analyst—a specialist determining parameters of transactions types, characteristics of open positions and their restrictions;

(3) an operator—a specialist directly making transactions offered by the automated system.

Limitations of characteristics are input data for algorithms of DSS problems. The quality of decision depends on how precisely limitations of characteristics reflect the required state of the owner's assets. That is why the procedure of restrictions is very important.

4.4.2 IS Software

As a result of rapid development of the FOREX market, a wide range of software-analytical complexes for traders and analysts has appeared recently. However, creation of such systems in present-day conditions implies not only software implementation of a mathematical model or approach to decision making on the currency exchange market, but also collection of basic data and creation of a reporting system [5, 6, 31].

The information-analytical system includes the following subsystems: information, analytical, and decision support systems (Fig. 4.3). To create the information subsystem one must first develop a module for data storage, which for the FOREX market must contain:

1. Numerical values of exchange rates: open and close prices as well as maximal and minimal prices.
2. Main macroeconomic indicators of the country issuing currency, considered in details in.
3. State of the trading account and operations made with it. These data are needed to analyze the efficiency of earlier decisions.

The analytical subsystem which undertakes major transformations of the data received from the information subsystem consists of software implementation of mathematical models:

1. An analysis and forecasting of exchange rate behavior (neural networks, mathematical methods in economics, technical analysis). To analyze exchange rate history it is recommended to take values in different periods of the day. This must be done because FOREX unites four regional markets, each of which has different demand and supply rates for the same currency and different trading psychology.
2. An analysis of the main macroeconomic indicators.
3. An analysis of the efficiency of previous decisions.

   The decision support subsystem is realized in the form of a visual interface for the trader, showing the main results of calculations made in the information and analytical subsystems.

As an example for this task consider the designed program module with the balancing model. This module enables you to get current exchange rates, to see the history of exchange rates, and to determine a profit loop (Fig. 4.4).

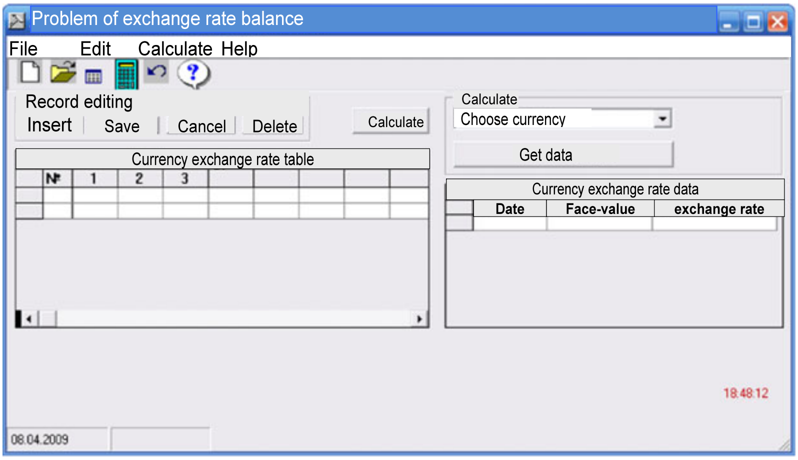


Fig. 4.4 A program module of the balancing problem

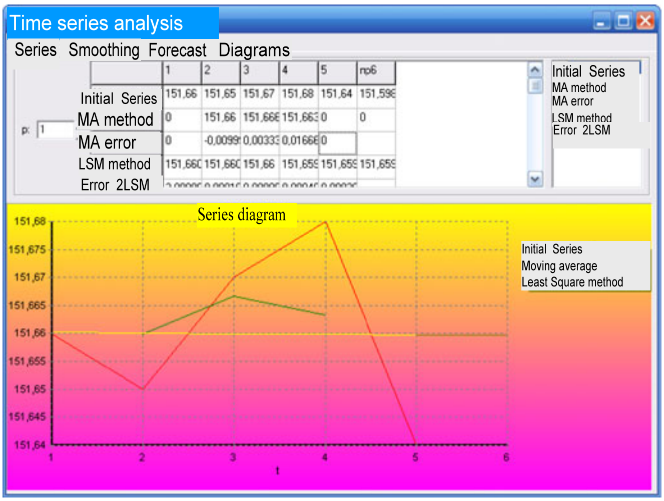


Fig. 4.5 A program module for time series analysis

To work with the “Analysis of time series” program module it is necessary to input the number of time series elements, then input series values or load an already constructed time series (Fig. 4.5).

The information system modeling decision support in currency exchange operations is closely integrated in the MS Office package, in particular MS Excel and MS Access. The information model of decision support system for in currency exchange operations is a tool that provides online access to a variety of data, data analysis, organization of reliable multi-variant calculations, and issuing recommendations to second-level banks for planning currency exchange operations.

References

1. Mutanov, G.M., Kurmashev, I.G.: Information System of Decision Support for Currency Exchange Operations. Ust-Kamenogorsk (2009)
2. Likhvidov, V.N.: Fundamental Analysis of World Currency Markets: Methods of Forecasting and Decision Making. Teaching Aid. Vladivostok (1999), p. 234
3. Berger, D., Chaboud, A., Chernenko, S., Howorka, E., Wright, J.: Order flow and exchange rate dynamics in electronic brokerage system data. J. Int. Econ. **75**, 93–109 (2008)
4. Moore, M.J., Roche, M.J.: Less of a puzzle: a new look at the forward FOREX market. J. Int. Econ. 387–411 (2002)
5. Bunkina, M.K., Semenov, A.M.: Fundamentals of Monetary Relations: Teaching Aid. Yurait, Moscow (2000), p. 192
6. Suren, L.: Exchange Operations: Basic Theory and Practice. Trans. from German. Delo (2001), p. 176
7. Piskulov, Yu.D.: Theory and Practice of Currency Dealing. INFRA-M (1996), p. 224
8. Neyman, E.L.: Small Trader's Encyclopedia. VIRA-R Alpha Capital (1999), p. 236
9. Polyakov, V.P., Moskovkina, L.A.: Structure and Functions of Central Banks. Foreign Experience. INFRA-M (1996)
10. Sapozhnikov, N.V.: Currency Exchange Operations of Commercial Banks. Legal Regulation: Practical Reference Book. YUrist (1999), p. 256
11. Gyunter, M.: Aksioms of a Stockbroker. Piter, St. Petersburg (2008), p. 208. Trans. from English, editor: V.V. Ilyin
12. Mark, N., Changing, C.: Monetary policy rules, learning, and real exchange rate dynamics. J. Money Credit Bank. **4**(6), 1047–1070 (2009)
13. Christofides, N.: Graph Theory: Algorithmic Approach. Mir, Moscow (1978), p. 432
14. Kofman, A.: Introduction into Applied Combinatorial Analysis. Nauka, Moscow (1975)
15. Hendry, D.F.: Dynamic Econometrics. Oxford University Press, Oxford (1995)
16. Hase, D.: Analysis of Causes in Statistical Investigations. Finances and Statistics (1981)
17. Erlikh, A.A.: Technical Analysis of Commodity and Financial Markets. INFRA-M (1996), p. 176
18. Kiyanitsa, A.S.: Fundamental Analysis of Financial Markets. Piter, St. Petersburg (2005), p. 288
19. Sheremet, A.D., Scherbakova, G.N.: Financial Analysis in the Commercial Bank. Finansy i Statistika (2001), p. 256
20. Lukasevich, I.Ya.: Analysis of Financial Operations. Methods, Models and Calculation Techniques. Finansy, YUNITI (1998), p. 400
21. Kurmashev, I.G.: Problem of balanced exchange rates. SKO Vestn. **4**, 5 (1999)
22. Kurmashev, I.G.: Problem of a maximal density loop. SKSU Vestn. **9–10**, 23–26 (2001)
23. Mutanov, G.M., Kurmashev, I.G.: Financial-mathematical model for controlling operations on balanced exchange rates. D. Serikbayev EKSU **3**(41), 137–144 (2008)
24. Kurmashev, I.G.: Use of collocation model for forecasting of operations on currency market. Reg. East. Vestn. **3**(39), 55–61 (2008)
25. Takha, H.A.: Introduction into Investigations of Operations. Trans. from English. Viliams (2001), p. 912
26. Romanovsky, I.V.: Discrete Analysis, 2nd edn. with corrections. Nevskiy Dialekt, St. Petersburg (2000), p. 240. Teaching aid for students specialized in applied mathematics and informatics
27. Morshed, A.K., Mahbub, M., Turnovsky, S.J.: Real exchange rate dynamics: the role of elastic labor supply. J. Int. Money Financ. **30**(7), 1303–1322 (2011)

28. Babeshko, L.O.: Collocation Models for Forecasting in Financial Sphere. Ekzamen (2001), p. 288
29. Larichev, O.I., Moshkovich, E.M.: Qualitative Methods of Decision Making: Verbal Decision Analysis. Nauka, Fizmatlit, Moscow (1996), p. 208
30. Chernorutsky, I.G.: Methods of Decision Making. BHV-Petersburg, St. Petersburg (2005), p. 416
31. Olbright, C.: Modeling with Microsoft Excel and VBA: Development of Decision Support Systems. Dialekyika-Williams (2005), p. 672

Chapter 5
Methods and Mathematical Models of Innovation Project Appraisal

5.1 Current Status of Innovation Project Review and Appraisal

5.1.1 Innovation Project as a Subject of Analysis and Appraisal

Any innovation project is a complex system of actions that are interdependent but are interconnected by resources, time, and performers and are aimed at achieving specific targets in priority areas of development of science and technology.

Innovation projects are, in effect, long-term investment projects characterized by a high degree of uncertainty as to their future outcomes and by the need to commit significant material and financial resources in the course of their implementation.

Uncertainty is inherent to all stages of an innovation project cycle: the initial phase of developing an idea, when selecting a project, and again when implementing it. Moreover, it may well happen that novelties that have successfully passed the testing phase and have found a manufacturing application, are then rejected by the market and their production must be stopped [1].

Even the most successful innovation projects are not foolproof. At any time of their life cycle they are vulnerable to the advent of a more promising novelty offered by a competitor.

It is also quite characteristic of an innovation project—as compared to an investment project—that modified alternative options can be developed during any stage of its life cycle. In case of a long-term investment project, only one option is selected to be implemented, while an innovation project requires that reevaluations and revisions be carried out at every implementation stage using numerous benchmarks and milestones. In fact, any innovation is characterized by its alternative nature, uncertainty, and availability of many options at all phases. Therefore it is quite a challenge to forecast innovation behavior, since this task entails assessing the integral performance index, projected future competitiveness, and adaptation to the market.

Practice shows that while 10 projects have been thoroughly vetted and launched, 4 or 5 of them result in total failure, 3 to 4 result in setting up viable companies that

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_5

do not yield any tangible profit, and only 1 or 2 bring really good outcomes. It is due to the success of such projects that investors—on the average—get a high rate of return (venture funds, etc.).

As compared to investment projects, innovation projects have the following specific features:

– Higher uncertainty as to future costs, period of achieving the intended targets, and future revenues; all these factors affect the accuracy and reliability of preliminary financial and economic assessments and suggest that additional criteria should be used for project appraisal and selection;
– When developing an innovation project, the time factor has to be taken into account to a greater extent;
– Innovation projects have certain advantages as compared to investment projects, in that they can be terminated without significant financial losses;
– Spin-off results of the research involved in innovation projects can be of commercial value, above and beyond the value of the project itself.

Thus, an innovation project must be considered as a complex of interrelated goals and objectives, each with an implementation plan, and it is therefore necessary that a more detailed analysis be carried out of all project stages including operational management, and that strict control be exercised over its implementation [1, 2].

5.1.2 Existing Methods and Tools of Evaluating Innovation Projects

A specific feature of innovation processes that result in innovation programs and projects is that they are characterized by the highest investment risk—risk that furthermore is quite difficult to assess due to the lack of effective assessment methods.

Appraisal of an innovation project is an important and challenging procedure at the research and development stage; it is a continuous process that implies a possible suspension or termination of a project at any point of time when new information is obtained. Thus, ongoing appraisal is one of the procedures for managing the R&D process. Such appraisal requires a clear, formal foundation including the following components [1, 3, 4]:

– Identification of factors relevant to the project;
– Assessment of project proposals as per these factors using quantitative information or expert reviews;
– Acceptance or rejection of project proposals based on the estimates obtained;
– Identification of areas that require additional information, and allocation of resources needed for obtaining new information;
– Comparison of the new data with the data used in the initial appraisal;
– Assessment of how new variables will affect the project;
– Decision on whether the project will be continued or suspended/terminated.

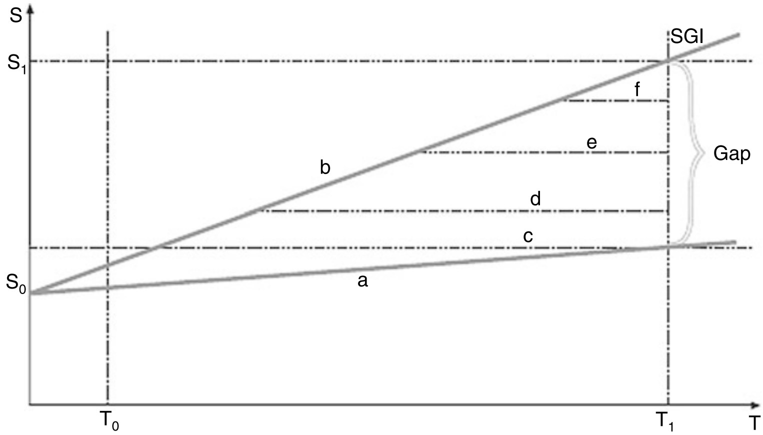


Fig. 5.1 Gap-analysis plot

Main factors to be taken into account in the appraisal procedure include:

– Financial results of the project implementation;
– Impact of the project in question on other projects included in the company's R&D project portfolio;
– Influence of the project on the economy of an enterprise as a whole, if it is a success.

In Kazakhstani and foreign practice there are various methods that have been successfully used to assess innovation projects. Below, four of these assessment methodologies are described in some detail: gap analysis; SWOT analysis; LIFT technology audit; and the TAME evaluation system. In addition, various software products used to aid in assessing innovation projects are also introduced [1].

1. **Gap analysis of innovation projects** entails measuring the difference between the current trend in development of a scientific institution or an innovation company and its potential for development when implementing an innovation project [5].

A key gap-analysis question posed during expert examination of projects can be formulated as follows: What strategy should a scientific institution or an innovation company select to focus its activities in implementing the project? Four feasible strategies are developed on the basis of the gap analysis, and then the most optimal line of activity is selected to implement an innovation project [1, 5].

The gap analysis implies that a plot is to be built (see Fig. 5.1) using the two most important variables—money and time [1, 5].

The plot is built to extrapolate the current development trend of a research institution or innovation company into the future, and thus determine optimization methods. (In the interests of brevity an innovation company is referred to in the remainder of this example.)

The main variables of this graphic model are the indicators T and S, which are the time period and the expected economic effect, respectively.

Indicator T_0 characterizes the current development period of an innovation company and the line *a* shows the institution's strategic development trend, which has been extrapolated on the basis of the previous-period results of its activities. The line *a* reaches the economic effect S_0 over the strategic period T_1 (T_1 is a five-year period, at a minimum); that is, the period of time when a strategic program of the project implementation or project portfolio is to be completed [1, 5, 6].

Organizations and companies functioning under the conditions of closed internal investments and innovations, are destined to operate in a relatively stable market situation. It would be possible for an innovation company to attain leadership in the market and to grow only by using investment and innovation tools for accelerating its activities (in the above plot, these are marked with *c*, *d*, *e*, and *f*), which link the current development trend with the company's potential growth.

Line *b* shows what will happen if the project is implemented and investments are attracted. There are four main areas of investment activity, or investment strategies, that make it possible to use this opportunity, and that differ in several parameters:

– Expectation period for the benchmark economic effect to be achieved by implementing an innovation project or project portfolio;
– Size of required initial and expected subsequent investments in the project;
– Risk level and likelihood of achieving the strategic goal of an investment (SGI);
– Optimal expected economic effect.

The gap analysis plot illustrates how the four major investment strategies are placed in this scenario. The investment **optimization** strategy (shown as *c* in the plot): The company places an emphasis on attracting additional investments for improving the technologies available for manufacturing innovation products and services [5, 6].

The investment strategy of **innovation** (shown as *d* in the plot above): An innovation company invests its own funds or attracts investments to create new technologies and develop new products or services [5, 6].

The investment strategy of **segmentation** (shown as *e* in the plot): An innovation company attracts investments or invests its own resources with the intent to introduce innovations into new markets [5, 6].

The investment strategy of **diversification** (shown as *f* in the plot): The most expensive and risky way of pursuing the strategy aimed at implementing a project, in this approach significant funds are invested to expand the project portfolio, the activities, and the nomenclature of new products and services [5, 6].

When examining projects, gap analysis shows how the strategic innovation activity of an innovation company is linked to management functions such as marketing, implementation of technology commercialization projects, production, etc. This assessment method highlights the fact that the decision-makers in innovation companies must be highly knowledgeable and experienced, to be equipped to make priority choices among strategies with the aim of maximizing the potential in a situation [5, 6].

2. **SWOT analysis of innovation projects**, and of the companies implementing them, is a consistent study of the internal condition of an organization, its strengths

Table 5.1 Sample of a SWOT analysis matrix

	A leader in the market of Russian manufacturers of innovation technology and products	No own large-scale industrial production and certification as per international standards
Opportunities to expand market outlets to foreign countries	Closed cycle of R&D provides a leading position in the market and gives opportunities to expand market outlets	Expansion to West European and USA markets makes projects cost-effective and attractive for investors, which gives opportunities to attract funds for establishing serial production of innovative products and their certification as per requirements of recipient countries
Strong dependence on partners, including foreign ones	Increase in the number of permanent partners and establishing strong business relations with them	In the event innovative products are to be supplied to foreign markets, contracts stipulate product manufacturing, packaging and marketing in line with the requirements of the market outlets

and weaknesses, as well as opportunities and outside threats. The SWOT analysis is a diagnostic technique on the basis of which a strategy of project appraisal and monitoring is developed that takes into account strengths and opportunities, compensates for shortcomings, minimizes threats, and mitigates risks [5, 6].

Table 5.1 below is the matrix of the SWOT analysis showing strengths and weaknesses, as well as threats and opportunities of a technology commercialization [5, 6].

In the course of the SWOT analysis, a list is compiled of the innovation project strengths, weaknesses, opportunities, and threats. For example, the following features might be listed as strengths of a given innovation project [5–7]:

– Original know-how and technologies used by a project, which are universal and can form the basis for fabricating a new generation of the product;
– Stable growth of the product market in Kazakhstan and elsewhere with good opportunities for further development;
– Company has a team of skilled specialists with experience in doing research and development work and in new technology commercialization, as well as organizational and operational experience;
– Guaranteed sale of the innovation company's products;
– Availability of developed sales outlets where the company's products are marketable;
– Company's products are import-substituting, high-quality products, which are as good as import analogues but more affordable for consumers; and
– Availability of a clear-cut development strategy supported by technologies, knowledge of the market, and realistic assessment of company's capabilities.

The following features, meanwhile, might be listed as strengths of a technology commercialization project:

– An innovation company does not have its own production capacities and equipment to fabricate innovation products or deliver innovation services;
– The necessity to undergo certification of both the production facilities applying innovation technology and the end products as per the standards of the countries to which it is intended to deliver the innovation product.

SWOT analysis results are used to draw up recommendations for developing marketing strategies of innovation projects [5–7].

3. **LIFT methodology and technology audit**. The Linking Innovation, Finance, and Technology methodology was developed within the framework of the 5th European Commission Framework Programme with the involvement of the INBIS Corporation (Great Britain) with the purpose of determining whether an innovation technology can be commercialized or not. The methodology combines technological audit and business planning, and can be used as a practical method of selecting technology commercialization projects for financing. Main elements of the LIFT methodology have been adapted to the Russian requirements by implementing the TACIS BISTRO Project: *Creating Elements of Interregional Innovation System and Approbation of the Siberia Innovation Development Model* [1, 5–8].

The methodology was evaluated and tested by selecting technology commercialization projects in the Tomsk, Novosibirsk, and Krasnoyarsk Regions. The methodology is not qualified to provide comprehensive answers to questions arising in a technology audit process, but it can give a certain profile of a technology commercialization project [5, 6, 9].

The LIFT methodology entails a technology audit conducted by a team of three experts specialized in technology commercialization, intellectual property issues, and technology economics. The technology audit procedure consists of three stages [8]:

– Filling out a questionnaire for a technology commercialization project;
– Experts interviewing design engineers, researchers, and managers of an innovation company; and
– Delivering an expert opinion.

If one organization submits several project proposals, it fills out a separate questionnaire for each of them. Forms are to be filled out within three working days from receipt, and returned via e-mail. Experts then draw up questions for the interviews based on the analysis of questionnaire data. During the interviews, all ambiguities are to be clarified, and factual or quantitative data are to be supported by documentary evidence [5, 6]. The documentation likely to be requested at the interview stage might include, for example:

– Project business plans;
– Potential buyers' opinions of the commercial product (CP) innovation project;
– Documents characterizing the CP market;
– Specifications of the equipment used;
– Curriculum vitae of individuals in charge of project implementation;
– Copies of patents and patent research reports, documentary evidence of CP novelty;

– All available correspondence between the CP designer and manufacturer.

Duration of all the interviews is two hours for each company. Based on the results, experts assign scores to each project indicator. The results of a technology audit are handed over to the applicant on a standardized form [5, 6, 8].

Structure of the LIFT methodology The methodology is developed on the module principle and consists of sections that make it possible to separately evaluate various aspects of an innovation project. These sections include [1, 5, 6, 8]:

(1) **General information.**
(2) **Development stage** of a technology commercialization project.
(3) **Scientific and technical capacity** of a project.
(4) **Legal treatment of intellectual property** and its application strategy.
(5) **Human capacity** of a project team (organization).
(6) **Conformance to international standards.**
(7) **The level of interrelations and interface** of the organization developing a scientific and technical product (STP) and its industrial partner.
(8) **Level of organization (team) management and commercial maturity** of a project.
(9) **Anticipated effect** from implementation of an innovation project.

Assessment of the technology commercialization project is achieved by assigning scores as per the LIFT methodology, using a scoring scale from 1 to 5 for each indicator. Indicators are subdivided into two groups: 10 indicators of project attractiveness and 20 indicators assessing project risks.

The score for each indicator can be determined using the project appraisal card, which shows the criteria for each indicator. It is considered as the expert opinion jointly agreed on by all experts engaged in the audit. After a score for each indicator is determined, total score will be calculated for indicators belonging to the project attractiveness group. The maximum score that a project can be given is 50 (10 attractiveness indicators times 5 potential points) [1, 5, 6].

Project risk indicators play a role in calculation of the final score only when their value equals 2 or 1. When the indicator value is 3 or more, the project can be qualified as non-risky according to that indicator. However, if the indicator value is 2, then the total score of the project attractiveness group will be reduced by 1; and if the indicator value is 1, the said value will be reduced by 3.

Thus, a project with the highest total score assigned for attractiveness (50) can actually receive a negative final score (-10) if all 20 risk indicators are equal to 1. If the grand total of scores is 40 and over (after deducting the total risk score), then the project is ranked as a ‘priority one’ and can be launched right away. In the event the grand total score is in the range from 25 to 40, the project is considered promising and it needs to be improved. If the grand total score is less than 25, then the project has much more weakness than strength [1, 5, 6, 9].

Thus, the task was set to design a technique for assessing innovation projects and tools of information support needed during the assessment process. The task was further specified in a number of separate aspects:

– Development of techniques for assessing project innovativeness and competitiveness;
– Development of a method to assess feasibility and economic effectiveness of innovation projects with due account of their life cycle stages;
– Development of a comprehensive technique for assessing innovation projects based on such indicators as innovativeness, competitiveness, and current net value;
– Design a decision support system using the developed methods and models that support the process of a comprehensive project assessment.

5.2 Development of Methods and Models for Assessing Innovativeness and Competitiveness of Innovative Projects

5.2.1 The Essence of Innovation and Competitiveness

At the present stage of the global economy one of the key elements of national economic safety is to ensure "global" competitiveness, which combines competitiveness of business entities at all levels including enterprises, industries, and the region's economy as a whole [11].

It is interesting to note that at the micro level, competitiveness can be defined at the level of households. Here, it entails competitiveness of household members: able-bodied citizens and their ability to take effective jobs in terms of payment and working conditions [10]. In an implicit form, this concept is used in the social population protection system. Authorized state agencies when deciding on the competitiveness of a particular household, especially for targeted social support, will inevitably take into account revenue generation potential. In so doing they take into account age and sex structure of the household, professional and qualification structure of its able-bodied members, financial assets available, real estate, transport vehicles, etc.

On a larger scale, overall interpretation of the term "competitiveness" refers to the ability of the system to achieve and maintain an advantageous position in a changing environment. This formulation leads to the assumption that to sustain competitiveness in the changing environment, changes in the system—innovations—are required [11].

Thus, the main function of R&D in current conditions is to determine effectiveness and competitiveness of businesses at all levels, while a competitive environment is in turn essential for innovation development. Even the most innovative companies in the world today see not only opportunities to become leaders, constant motivation to maintain and strengthen the advantages they have achieved—to their mind increased innovativeness of the industrial sphere is a constant threat to their own business. The more intense the innovation is and the greater its role in economic growth, the more seriously key enterprises treat innovation as a phenomenon, employing strategies for further productivity improvement and economic growth through the development and use of advanced technology with a policy of innovation [1, 10].

To achieve competitive advantage, firms must find new ways to compete in their niche to enter the market in the only possible way: through innovation [1, 5, 10]. Innovations in the broadest sense include use of new materials, introduction of new technologies, and improving means and methods of production activities. Thus innovation equally includes R&D results of production purposes, and results geared at improving the organizational structure of production. Any innovation activity requires, above all, investments in various production factors: production infrastructure and marketing, training of personnel, and development of their skills and knowledge in technology, including research developments [1, 11].

The relationship between competitiveness and innovativeness originates from definitions of these concepts. Competitiveness can be understood as "the ability of countries or companies to produce goods and services that can compete successfully on the world market." In turn, innovativeness can be understood as "introduction of a new or significantly improved idea, product, service, process, or practice designed to produce a useful result." The quality of innovation is determined by the effect of its commercialization, the level of which can be determined by assessing the competitiveness of products [11].

Thus, there is a certain relationship between competitiveness and innovativeness. In a certain sense, innovative relations are the result of competitiveness, which enables us to consider competitiveness as a function of innovation: $K = f(I)$ [1, 11].

Therefore, innovativeness and competitiveness are the most meaningful indicators for the innovation project appraisal process.

5.2.2 Innovativeness Criteria for Innovative Projects

Currently, and in spite of the wording proposed above, there is no single, clear definition of an innovation.

In the wide context, innovation is related to both development of new techniques and technology transfer. In practice these contribute to the competitiveness of the product and/or the enterprise as a whole.

With regard to the state's economy innovation is considered as a consequence of the traditional market practices, as a consequence of competition in the market for goods and services. In the economic sphere of activity innovation cannot exist without the market, since it is inextricably linked with the community's ability to generate changes.

According to the conclusions of American economists M. Porter and S. Stern, criteria for evaluating the intensity of innovation and research at the state level include [1, 11–13]:

- Number of research staff;
- Volume of investments in R&D;
- Percentage of R&D funded by private industry;

– Share of R&D performed by the higher education sector;
– Expenditure on higher education;
– Level of intellectual property protection;
– Openness of international competition; and
– GDP per capita.

Enterprise innovativeness is understood as the process associated with the formation and use of innovation and the ability for rapid and effective development of innovation, creation and implementation of innovations, and perception of innovation to meet the demand [1].

Organization innovativeness is the capacity for constant renewal. An enterprise's innovativeness as a quality is based on the ability to master both the technical innovations associated with upgrading the technical and technological content of production, as well as on social innovation. Effectiveness of technological innovation depends not only on technical characteristics, but also on a system of arrangements for the staff to change their set of sustainable behavior patterns to meet technical and technological requirements of the innovation [13].

From this perspective, social innovations designed to address corporate culture issues should be focused on innovative type, i.e., rules of conduct to support continuous update processes at an enterprise.

Project innovativeness has to do with the "advanced nature" of technologies and solutions used, including how relevant their application is for the enterprise, region, country, etc. [1].

At the project level, we can say that innovativeness is the extent of demand for innovative products, subject to certain criteria.

To evaluate an innovative project at the R&D stage, the following basic innovativeness criteria are suggested [1, 11]:

(1) Compliance of a project with the priority areas of industrial and innovation strategy;
(2) Relevance of research and product uniqueness (no analogues);
(3) Scientific originality of the solutions proposed within the project;
(4) Technological level of the project (technology transfer, new technology);
(5) Advantages of the project in comparison with analogues existing in the world;
(6) Economic feasibility of the project;
(7) Cost of the project;
(8) Possibility of tax incentives.

5.2.3 Competitiveness Criteria for Innovative Projects

In the modern world, competitiveness has become one of the main strategic goals of economic development of regions and countries as a whole. Economic success mainly depends on determining the degree of competitiveness. As for innovativeness, there is still no strict definition of competitiveness.

A number of different interpretations of the concept of competitiveness can be found in the literature [11].

According to Porter [13], competitiveness includes:

- First, the ability to create an advantage over competitors, allowing achievement of goals;
- Second, effective use of such competitive advantages.

At the same time, the authors differentiate between the notions of *quality* and *value in use* based on the assumption that "*value in use* accumulates all properties of the product which are related to its ability to meet human needs, while *quality* comprises only some part of those properties related to the specified data of a particular product."

Various other works [11, 13–16] note the need for regulation of the terminology used in this area; however, attempts to formulate a definition of competitiveness as an economic category are reduced to the definition of quality. At the same time, analysis of the concept under consideration, the results of which are given in this paper, suggests that this range of issues is beyond the scope of qualimetry. To investigate competitiveness as a generalized economic issue, the following factors should be taken into account:

- Most important is a quantitative assessment of economic entities whose competitiveness is under consideration, otherwise assessments of competitiveness will be entirely subjective;
- There is no universal, commonly accepted concept of competitiveness;
- Main parameters used to determine the level of competitiveness are multilayering, relativism and specificity;
- Competitiveness is determined by comparing either businesses or their products;
- Comparison of economic entities in the framework of the comparative analysis of competitiveness must meet requirements of completeness and correctness.

Competitiveness is a property of an entity characterized by the degree of real or potential satisfaction of specific needs, as compared with the best similar objects represented in the market. Competitiveness determines the ability to withstand competition in comparison to similar objects in this market.

Thus, the overall interpretation of "competitiveness" refers to the ability of the system to achieve and maintain its advantageous position in a changing environment [11]. This formulation leads to the assumption that in order to be competitive in the changing environment it is necessary to introduce changes, i.e., innovations, in the system itself [11].

The leading role of innovation within the structure of competitiveness factors has been acknowledged by M. Porter who made a significant contribution to the scientific understanding of international competition patterns [13]. According to Porter, it is companies rather than countries that compete in the international market. Competitiveness on the national level is only one competitiveness factor for the companies registered in this particular country. Firms achieve competitive advantage by finding new ways to compete in their area of activity and bringing them to the market by producing innovations (use of new materials, new technologies, improving the means and methods of organizing the production activity, etc.).

The study of the conceptual framework of competitiveness has brought us to the following conclusions [1]:

– Depending on the goals and objectives of a study, the concept of competitiveness can be discussed at various levels of hierarchy starting with a particular product and building to the national economy; there is a strong internal and external dependence between all levels of competitiveness;
– The national competitiveness indicator is a synthetic indicator, which combines competitive features of a product, enterprise, or industry and characterizes the situation in the world market. In the most general sense, national competitiveness can be defined as the ability of a country to produce goods and services (in the free competition environment) that meet the requirements of the global market and the sales of which improve wealth of the country and its individual residents;
– Within the hierarchy of competitiveness concepts, the most fundamental notion is the competitiveness of products, applied to various types of products (production and technical end use: consumer products, services, information, etc.);
– Competitiveness is an estimated parameter; therefore, it presupposes the availability of a subject (who estimates) and an object (which is estimated), as well as objectives (criteria) of estimation;
– Competitiveness is the key indicator characterizing significance of an object;
– Innovation is the key factor affecting competitiveness;
– Various parameters (criteria) of competitiveness assessment are used when assessing competitiveness of a specific object, depending on the level of its hierarchy (product, company, region, country).

Let us consider in more detail the notion of competitiveness as an innovation project characteristic $K = f(I)$.

Since the outcome of an innovative project is a specific product of certain type [8] (consumer goods, services, information, etc.) to be assessed by competitiveness indicators, then in order to achieve this goal it will be necessary to define the components of product competitiveness.

Product competitiveness is determined as an aggregate of qualitative and cost characteristics of a commodity, which make this commodity superior to all other competing products. An indicator such as marketing performance of a commodity serves as another competitiveness indicator, which means a state of expanded product characteristics (marketing logistics, service, warranties, advertising, image, packaging, branding, etc.) [1].

A classification scheme reflecting the performance of a product's competitiveness can be represented as a chain: price–quality–service–marketing environment. Table 5.2 shows the key characteristics of these indicators.

Thus, product competitiveness is characterized by three groups of indicators: qualitative, cost, and marketing.

In turn, each group covers a corresponding set of components characterizing each parameter, represented in the diagram in Fig. 5.2.

Table 5.2 Key indicators of product competitiveness

Indicators	Criteria characteristics
Price	Ratio of prices among major competitors. Maturity of price differentiation system depending on supply and demand, as well competitors' policy. Attractiveness for discount system users
Quality	Technical and operational characteristics of a product (functionality, reliability, usability, etc.). Prestige, design, ecological properties of goods
Service	Quality of product delivery. Level of trading service. Availability of spare parts and materials, as well as servicing centers
Marketing indicators	Level of marketing logistics organization. Effectiveness of promotional activities. Level of design and packaging intentionality. Level of development of product branding. Level of warranty service for customers before and after purchase. Access to purchase using multimedia technologies

Quality of products is a collection of properties that determine its suitability to meet certain needs in accordance with its purpose. Product quality is revealed through consumption. While assessing the quality of goods, the consumer links the usefulness of a product to its value in use [1].

Quality of product includes the following components: intended-use parameters, ergonomic parameters, aesthetic parameters, and regulatory parameters.

Intended-use parameters characterize the area of application and functions of a product. They determine the content of beneficial effects, achieved through the use of this product under the particular environment of a consumer.

Intended-use parameters are further classified into:

– Classification parameters that characterize the products attribution to a certain class and are used within assessment only at the stage of selecting the application field for competing products and goods; they serve as a basis for further analysis and are not involved in further calculations (for example: passenger capacity, consumption velocity);
– Technical performance parameters that characterize progressiveness of technical solutions used when designing and manufacturing products (e.g., machine productivity, accuracy and speed of measuring devices, computer storage capacity). They can also be used as classification parameters;
– Design parameters that characterize main engineering design options used at the stage of product design and manufacturing (product composition, structure, size, and weight); some parameters can also serve the purpose of classification.

Ergonomic parameters characterize products in terms of their compliance with the properties of a human body while performing labor operations or consuming (hygienic, anthropometric, physiological features, human body characteristics shown in production and domestic activities).

Aesthetic parameters characterize information significance (expediency of form, integrity of composition, perfection of production performance, and presentation stability of product); they model the external perception of the product and reflect

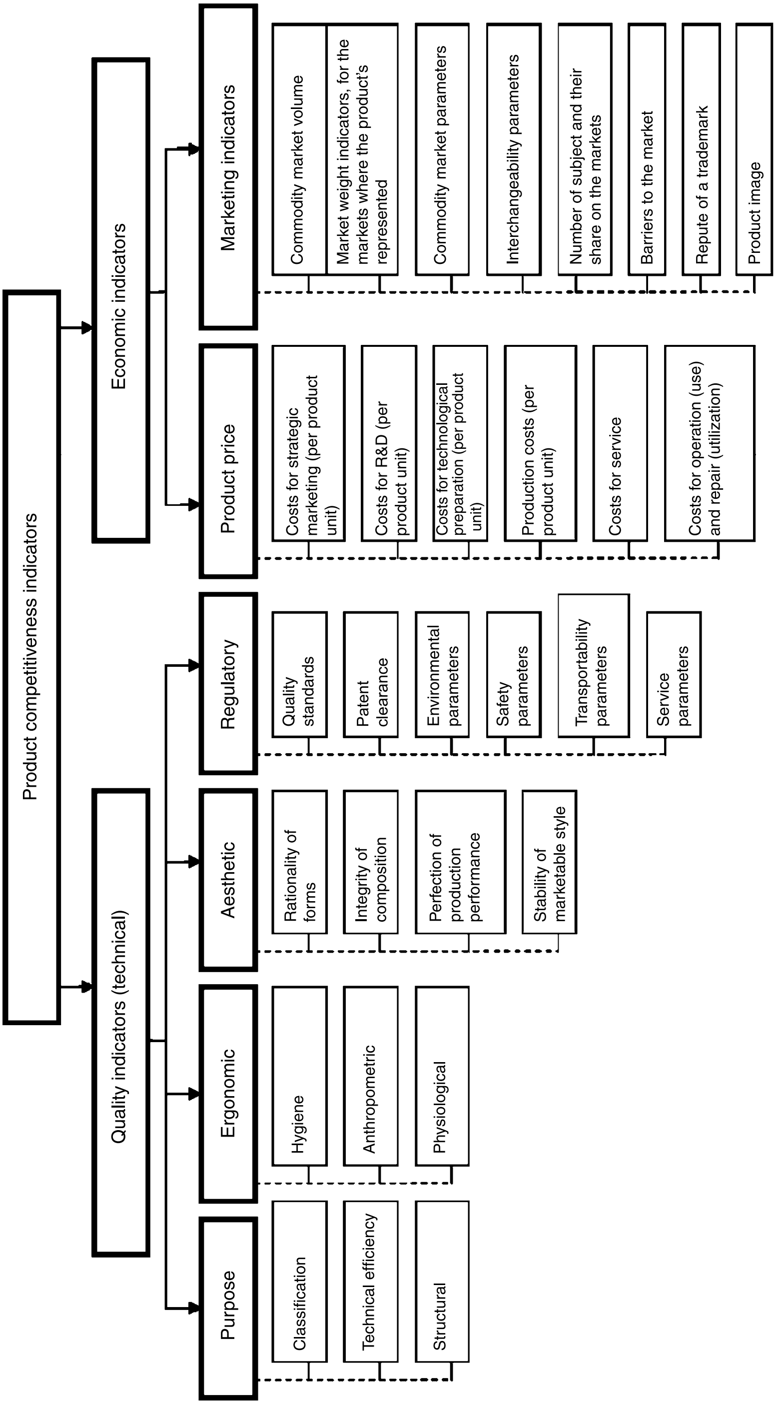


Fig. 5.2 Product competitiveness indicators

exactly those of its external properties that are most important to consumers, ranked in order of importance for a specific type of product [1].

Regulatory parameters characterizing product properties which are regulated by mandatory rules, standards, and legislation in the target market for this product (patent clearance parameters, characterizing incorporation of technical solutions in the product, and products not covered by patents produced in the markets of prospective sale, ecological parameters, safety parameters with the mandatory requirements of applicable international and national standards, technical regulations, standards, legislation set for a particular market).

The group of *economic indicators* includes cost and market indicators.

Cost indicators include full costs incurred by a consumer (consumption cost) to purchase and consume the product, as well as product operational costs. Consumption cost is made up of the market value of a product and costs associated with its operation/use during the entire life cycle.

Marketing indicators characterize the conditions of purchase and use of a product in the market, level of activities and measures undertaken to ensure marketing support for the product (advertising, product image, brand recognition, importance indicators for the markets where the goods are represented, number of subjects and their market share, etc.). To analyze the competitiveness of goods, it is required to determine and compare the structure of marketing indicators for the goods produced and for products of competitors.

Based on the data available on competitiveness components, the competitiveness criteria for innovative projects can be schematically presented as follows by combining them into groups of quality and economic indicators (R&D) (Fig. 5.3).

Thus, such set of criteria makes it possible to conduct an initial assessment of an innovation project.

5.2.4 Method and Graphic Model for Assessing Innovativeness and Competitiveness of Innovative Projects

Here we propose a method of assessment of innovative projects referred to the scientific, technical, and industrial sector, with a system of target indicators.

In developing the method we used a methodological approach based on expert assessment of innovation and competitiveness indicators for innovative projects, accompanied by a graphic model of project innovativeness and competitiveness assessment [11, 16, 17].

Adequacy of the criteria for the complex index is determined by assigning weights to each criterion and using an additive–multiplicative method of calculation [11].

Innovative project assessment, based on the graphic model for assessing project innovativeness and competitiveness, should be carried out in three stages: (a) selecting optimal criteria, (b) determining weight coefficients, and (c) positioning projects in the matrix.

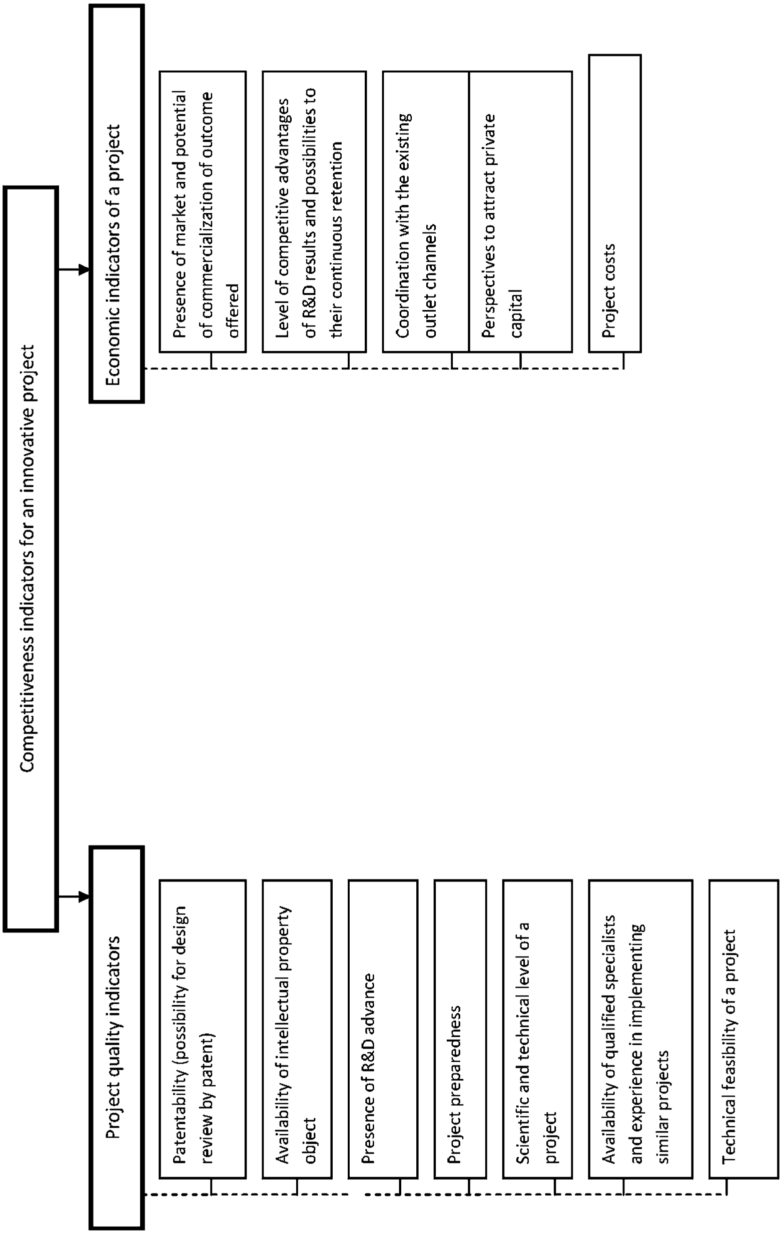


Fig. 5.3 Product competitiveness indicators

Table 5.3 Criteria for innovativeness and competitiveness indicators for innovative project at the R&D stage

Innovativeness criteria
1. Compliance of project with the priority lines of the industrial and innovation strategy
2. Research novelty and project uniqueness (no analogues)
3. Scientific novelty of the solutions proposed by the project
4. Project technological level (new technology)
5. Project advantages as compared with the existing analogues in the world
6. Economic feasibility of the project
7. Opportunities to undertake future R&D on the basis of this project and new technology
8. Possibility of tax incentives
Competitiveness criteria
1. Availability of markets and opportunities to commercialize the proposed project results
2. Level of competitive advantages of R&D results and opportunities to retain them in the long-run
3. Consistency with the existing sale outlets (distribution channels)
4. Patentability (possibility to defend the project by using the patent)
5. Availability of proprietary articles
6. Availability of scientific and technical potential of the project
7. Technical feasibility of the project
8. Project costs
9. Degree of project readiness
10. Availability of a team and experience in project implementation
11. Opportunities to involve private capital (investment attractiveness)
12. Scientific and technical level of project

From the perspective of the market, innovative projects are the objects of two interacting segments: science and business. Therefore, they should be formalized as two-dimensional objects: innovativeness (I) and competitiveness (K), where $K = f(I)$. The main distinctive feature of these indicators is that they are considered as an assessment of project viability and attractiveness to investors and depend on numerous criteria. A set of these criteria is presented in Table 5.3 [11].

To calculate these criteria, we propose the following method. The easiest way to solve this task is related to determination of the average expert values for each innovativeness and competitiveness criterion. Common values of innovation and competitiveness criteria are defined as follows [11, 17]:

$$I_j = \sum_{i=1}^{n} w_i f_{ij}, \quad \sum_{i=1}^{n} w_i = 1 \tag{5.1}$$

$$K_j = \sum_{k=1}^{m} w_k g_{kj}, \quad \sum_{k=1}^{m} w_k = 1 \tag{5.2}$$

$$\begin{aligned} I_{\min} \le I_j \le I_{\max}, \\ K_{\min} \le K_j \le K_{\max}, \end{aligned} \tag{5.3}$$

where f_{ij} is the value of i-th criterion of the j-th object (project) for the innovativeness indicator; w_i—value of weighting coefficient of i-th criterion for the innovativeness indicator; n—number of criteria for the innovativeness indicator; g_{kj}—value of the k-th criterion of j-th object (project) for the competitiveness indicator; w_k—value of weighting coefficient of the k-th factor for the competitiveness indicator; m—number of criteria for the competitiveness indicator; $j = \overline{1, J}$ with J being the number of objects (projects); $I_{\min}$, $I_{\max}$, $K_{\min}$, $K_{\max}$—minimum and maximum values of the innovativeness and competitiveness indicators [11].

In the graphic model for assessing project innovativeness and competitiveness, the range of indicators is split into 9 sectors [11, 17].

In this case, in order to position each project, it is required to define I and K parameters, which are the coordinates of these objects (projects) in the matrix. To determine the coordinates in the model we use weighted average factors (criteria). It is recommended that the values for each factor be assessed using the expert approach (from 1 to 9); in the presence of several experts, the values are averaged [11].

To formalize the criteria rating, expert estimates are the most suitable tool because they contain a complex of logic, mathematical, and statistical procedures, and are based on knowledge of professionals [11].

To determine the weighting coefficients for each criterion and their ranking we used the ranking method.

While ranking, the initial ranks are transformed so that rank 1 turns into n-rank, and so on, while the rank n turns into rank 1. Totals are calculated by these transformed ranks [1, 11, 17]:

$$R_j = \frac{\sum_{k=1}^{M} R_{jk}}{M}, \tag{5.4}$$

where R_j = the sum of the ranks converted across all the experts for j-th factor; R_{jk} = converted rank assigned by k-th expert to j-th factor; and M = number of experts.

Next, the weights of criteria are calculated [1]:

$$W_J = R_J \Big/ \sum_{J=1}^{N} R_J, \tag{5.5}$$

where W_J is the average weight of criterion across all the experts; N is the number of criteria.

An important component of the graphic model of assessment of project innovativeness and competitiveness is the matrix of judgments where element values are based not on accurate measurements, but on subjective judgments (these matrices are produced by experts). Judgment matrix [1]:

$$A = (a_{ij}), \quad i, j = 1, 2, \ldots, J \tag{5.6}$$

where a_{ij} is a number corresponding to the significance of an object (Criteria I and K).

Expert "Quality" while completing the judgment matrix is determined through the conformity relation (OS). OS values $0.5 \leq \text{OS} \leq 1$ are considered acceptable [30]. For the judgment matrix it is required to find the maximum secular $\lambda_{\max}$ value and vector of Z eigenvalues, i.e., it is necessary to solve the equation:

$$A \cdot Z = Z - \lambda_{\max} \tag{5.7}$$

Vector Z components are the weight coefficients.

Based on the above, to calculate criteria values the questioning of 22 experts was undertaken. The questioning process involved scholars from Eastern-Kazakhstan State Technical University and other Kazakh institutions of higher education, as well as the specialists of the regional scientific and technological park "Altai," experts of "TsvetMet" center, and other representatives of development institutions and specialists of industrial enterprises. In qualitative terms, the experts were managers, specialists of scientific research, and innovation managers.

The questionnaire was compiled based on the two sets of criteria outlined above. The total number of questionnaires was 22. The experts were also given the option to expand the list or eliminate unnecessary criteria from the list presented in Table 5.3 as necessary.

Criteria ranking was quite simple: By assigning the highest ranking to the criterion of the lowest value, in their opinion. Importance (rank) of each criterion is determined by the average estimated value and the sum of ranks of expert assessments.

Consistency of expert assessments by criteria was verified by calculating the coefficients of factor variations [18, 19], which are the analogous to dispersion and represented in the following formula:

$$S_i = \frac{m}{m-1} \cdot \frac{(\sum_{j=1}^{n} f_{ij})^2 - \sum_{j=1}^{n}{}^2 f_{ij}}{(\sum_{j=1}^{n} f_{ij})^2}, \tag{5.8}$$

where S_i is the factor variation coefficient; f_{ij} is the average value for the total number of f_{ij}—ranks of i-factor, assigned by the j-th expert; m—number of experts; n—number of criteria.

Since experts come from various entities or groups, there is a need to identify homogeneity of these groups. To solve this problem across various criteria, derived from experts, the consistency of their views is determined by using concordance coefficient. OS concordance coefficient is calculated by using the formula proposed by Kendall:

$$\text{OS} = \frac{12 \cdot S}{m^2 \cdot (n^3 - n)}, \tag{5.9}$$

where S—the sum of squared differences (deviations); m—number of experts; n—number of criteria.

In the case where any expert fails to identify the rating value between several related factors and assigns the same rank, the concordance coefficient is calculated by using the following formula [18–20]:

$$\text{OS} = \frac{S}{\frac{1}{12} m^2 (n^3 - n) - m \sum_{j=1}^{m} T_j}, \tag{5.10}$$

where

$$T_j = \frac{1}{12}\sum_{t_j}(t_{j^3} - t_j),$$

t_j is the number of equal ranks in the series j.

The sum of the squared differences (deviations)—S was calculated by using the following formula:

$$S = \sum_{i=1}^{n}\left\{\sum_{j=1}^{m} x_{ij} - \frac{1}{2}m(n+1)\right\}^2, \tag{5.11}$$

where x_{ij}—the rank of i-factor assigned by j-expert, m—number of experts; n—number of factors (criteria).

The significance of W coefficients was verified for the level of 0.01 (99 %) by x^2 criterion of x^2, minimizing the error of the 2nd type (assuming an incorrect hypothesis), with α significance level—probably to reject a fair hypothesis (error of the 1st type) and the number of freedom degrees—f [18]. Statistics value of x^2 is calculated by the formula:

$$x^2 = m \cdot f \cdot W, \tag{5.12}$$

where m—number of experts; f—number of freedom degrees—$f = k - 1$, W—concordance coefficient.

Provided that the statistical value of x^2 exceeds the critical value of x^2 with a significance level α and the number f of degrees of freedom, i.e.:

$$x^2 = m \cdot f \cdot W > x_{cr}(a; f),$$

and the hypothesis on consistency of expert opinions is not rejected.

Specialist experts excluded two criteria from the groups of innovativeness criteria, namely the ability to perform future research and development on the basis of this project and the new technology, and the potential to use tax incentives. They also excluded three criteria from the group of competitiveness criteria: clarity of setting goals and project objectives, scope and range of practical use of technologies, and provision of project with up-to-date level of innovation management (project management techniques). The criteria selected by experts are presented in Table 5.4 [11].

Table 5.5 shows the values of concordance coefficients across innovativeness and competitiveness criteria.

The values concordance coefficients obtained for each group of the criteria (0.69, 0.54) indicate agreement (0.5–0.7 = significant correlation) between ratings of all experts [1, 17].

When analyzing the results of expert assessments of innovativeness criteria, where $m = 22$, $f = 11$, $\alpha = 0.01$, the value of x^2—statistics is as follows:

$$x^2 = 22 \cdot (12 - 1) \cdot 0.69 = 166.98 > x^2_{cr}(0.01; 11) = 24.725.$$

Table 5.4 Criteria of innovativeness and competitiveness indicators for selection of innovation projects

Innovativeness criteria
1. Compliance of project with the priority lines of the industrial and innovation strategy
2. Research novelty and project uniqueness (no analogues)
3. Scientific novelty of the solutions proposed by the project
4. Project technological level (new technology)
5. Project advantages as compared with the existing analogues in the world
6. Economic feasibility of the project
Competitiveness criteria
1. Availability of markets and opportunities to commercialize the proposed project results
2. Level of competitive advantages of R&D results and opportunities to retain them in the long-run
3. Consistency with the existing sale outlets (distribution channels)
4. Patentability (possibility to defend the project by using the patent)
5. Availability of proprietary articles
6. Availability of scientific and technical potential of the project
7. Technical feasibility of the project
8. Project costs
9. Degree of project readiness
10. Availability of a team and experience in project implementation
11. Opportunities to involve private capital (investment attractiveness)
12. Scientific and technical level of project

Table 5.5 Concordance coefficient values

Indicators	Innovativeness criteria	Competitiveness criteria
Average sum of ranks	−77	−143
Sum of squared difference S	5826	37255
OS-concordance coefficient	0.69	0.54
S_j-variation coefficient	0.84	0.93

Accordingly, the value of x^2 statistics of the expert assessments results of competitiveness criteria is as follows:

$$x^2 = 22 \cdot (12 - 1) \cdot 0.54 = 130.68 > x^2_{cr}(0.01; 11) = 24.725$$

Since the values of x^2 statistics exceed the value of x^2_{cr}, therefore, the hypothesis of expert opinion consistency is not rejected.

The expert evaluation results make it possible to obtain weighting coefficients to determine positioning of innovative projects in the matrix [11].

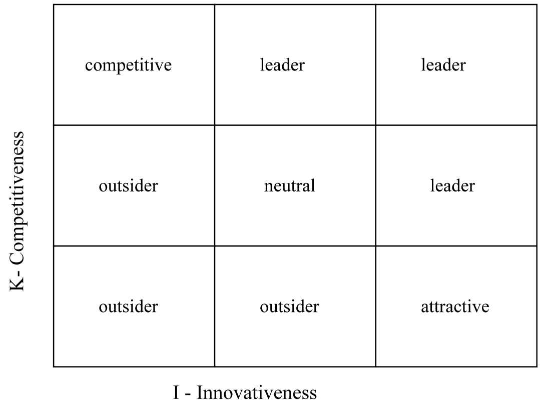


Fig. 5.4 Graphic model for assessing innovativeness and competitiveness of innovative projects

Weights determined at the second stage demonstrate the importance of each criterion. These data indicate that expert evaluations for each group of the criteria differ by their significance.

For innovativeness indicators these coefficients are the following: 0.228, 0.252, 0.102, 0.105, 0.069, and 0.244. Consequently, the most important criterion is the relevance of research and uniqueness of the project (no analogues); the second one is the economic feasibility of a project; the third one, compliance of project with priority lines of the industrial and innovation strategy; the fourth one is the technological level of a project (technology transfer, new technology); the fifth criterion, scientific novelty of solutions proposed in project; and the sixth one describes project advantages as compared with existing analogues in the world [11, 17].

For competitiveness indicators, weight coefficient values are as follows: 0.277, 0.119, 0.033, 0.060, 0.067, 0.067, 0.040, 0.037, 0.041, 0.142, 0.057, and 0.061. Within this group, the criteria are the following: the first one is availability of the market and opportunities to commercialize the proposed project results; the second, availability of a team of qualified specialists having experience in project implementation; the third, level of competitive advantages of R&D results and potential to retain them in the long-run; the fourth, availability of proprietary articles; and the fifth criterion is availability of scientific and technological potential, etc. [11, 17]. Thus, graphic model of innovativeness and competitiveness for innovative projects is as follows (see Fig. 5.4).

The third stage involves positioning of projects within the graphic model of innovativeness and competitiveness of innovative projects. As an example, five projects have been chosen for further assessment by experts by the criteria of innovativeness and competitiveness. Tables 5.6 and 5.7 show the assessments of innovativeness and competitiveness indicators averaged across five experts.

Based on these averaged assessments and weighting coefficients, innovative projects were positioned in the graphical model for innovativeness and competitiveness of innovative projects. The obtained weights and assessment criteria are presented in Tables 5.8 and 5.9 [11].

Table 5.6 Averaged assessments for innovativeness criteria

#	Criteria	Innovative project				
		Project #1	Project #2	Project #3	Project #4	Project #5
1	Project relevance to the priority areas of the industrial and innovation strategy	4.40	3.80	3.60	4.60	8.60
2	Research novelty and project uniqueness	3.80	2.80	2.80	3.60	8.40
3	Scientific novelty of the solutions proposed within the project	3.00	2.40	3.20	3.40	8.80
4	Project technological level (new technology)	3.00	2.00	2.60	3.00	8.40
5	Project advantages as compared to the analogues existing in the world	2.60	1.80	2.60	2.80	8.20
6	Economic feasibility of the project	3.80	3.80	2.40	3.20	8.60

Table 5.7 Averaged assessments for competitiveness criteria

##	Criteria	Innovative projects				
		Project #1	Project #2	Project #3	Project #4	Project #5
1	Availability of market and opportunities to commercialize the proposed project results	3.80	3.20	2.60	3.60	8.00
2	Level of competitive advantages of R&D results and possibility for their continuous preservation	3.60	2.20	2.40	3.40	7.80
3	Consistency with the existing distribution channels	3.80	1.60	2.40	2.80	7.60
4	Patentability (possibility to defend the project by using the patent)	4.20	2.40	3.00	3.00	7.40
5	Availability of the object of intellectual property	3.80	2.00	3.20	2.80	8.20
6	Availability of scientific and technical potential of the project	3.00	2.40	2.60	3.40	7.80
7	Technical feasibility of the project	3.40	2.20	2.60	3.40	7.40
8	Project costs	3.00	3.60	2.20	3.20	8.40
9	Degree of project readiness	3.00	2.60	2.40	3.20	8.00
10	Availability of qualified specialists and experience in project implementation	3.60	2.80	3.40	4.00	8.40
11	Opportunities to involve private capital (investment attractiveness)	2.80	2.00	2.60	3.20	8.40
12	Scientific and technical level of project	3.00	1.60	2.80	3.40	6.80

Figure 5.5 demonstrates graphic model for assessment of project innovativeness and competitiveness. The proposed five projects were positioned according to expert assessments received.

Table 5.8 Utility estimate for projects based on the innovativeness indicators

Criteria	Criteria weights	Normalized estimate of priority vector, Project #1	Normalized estimate of priority vector, Project #2	Normalized estimate of priority vector, Project #3	Normalized estimate of priority vector, Project #4	Normalized estimate of priority vector, Project #5
(1) Compliance of project with the priority directions of industrial and innovation strategy	0.228	1.0014	0.8648	0.8193	1.0469	1.9572
(2) Research novelty and project uniqueness (no analogues)	0.252	0.9576	0.7056	0.7056	0.9072	2.1169
(3) Scientific novelty of the solutions proposed within the project	0.102	0.3061	0.2449	0.3265	0.3469	0.8980
(4) Project technological level (new technology)	0.105	0.3145	0.2096	0.2725	0.3145	0.8805
(5) Project advantages as compared with the existing comparables in the world	0.069	0.1801	0.1247	0.1801	0.1939	0.5680
(6) Economic feasibility of the project	0.244	0.9283	0.9283	0.5863	0.7817	2.1008
Total	**1**	**3.6879**	**3.0779**	**2.8904**	**3.5912**	**8.5213**

The resulting matrix allows positioning each innovative project based on the criteria of innovativeness and competitiveness indicators in a certain sector. Matrix boundaries are the maximum and minimum possible values from 1 to 9, respectively [11, 17].

Thus, three groups are highlighted in this matrix (Fig. 5.5): (1) “leader”; (2) the “outsider”; and (3) the “border.”

Projects that fall into group of “leaders” have the highest values of innovativeness and competitiveness indicators as compared with the other two groups; they are the absolute priority to be implemented at the earliest possible time. Projects that fall into the three sections in the lower left corner of the matrix (“outsiders”) have low values based on many criteria. These projects are problematic as they have more weaknesses than strengths [11, 17].

The three sections located along the main diagonal, going from lower left to upper right edge of the matrix have the classical name of “border”: these include the

Table 5.9 Utility estimate for projects based on competitiveness indicators

Criteria	Criteria weights	Normalized estimate of priority vector, Project #1	Normalized estimate of priority vector, Project #2	Normalized estimate of priority vector, Project #3	Normalized estimate of priority vector, Project #4	Normalized estimate of priority vector, Project #5
(1) Availability of market and opportunities to commercialize the proposed project results	0.277	1.0507	0.8848	0.7189	0.9954	2.2121
(2) Level of competitive advantages of R&D results and possibility for their continuous preservation	0.119	0.4282	0.2617	0.2855	0.4044	0.9277
(3) Consistency with the existing distribution channels	0.033	0.1266	0.0533	0.0800	0.0933	0.2532
(4) Patentability (opportunities to defend the project by using the patent)	0.060	0.2499	0.1428	0.1785	0.1785	0.4403
(5) Availability of the object of intellectual property	0.067	0.2534	0.1334	0.2134	0.1867	0.5469
(6) Availability of scientific and technical potential of the project	0.067	0.1997	0.1598	0.1731	0.2264	0.5193
(7) Technical feasibility of the project	0.040	0.1377	0.0891	0.1053	0.1377	0.2996
(8) Project costs	0.037	0.1118	0.1342	0.0820	0.1193	0.3130
(9) Degree of project readiness	0.041	0.1235	0.1071	0.0988	0.1318	0.3294
(10) Availability of qualified specialists and experience in project implementation	0.142	0.5104	0.3970	0.4821	0.5672	1.1910

Table 5.9 (Continued)

Criteria	Criteria weights	Normalized estimate of priority vector, Project #1	Normalized estimate of priority vector, Project #2	Normalized estimate of priority vector, Project #3	Normalized estimate of priority vector, Project #4	Normalized estimate of priority vector, Project #5
(11) Opportunities to involve private capital (investment attractiveness)	0.057	0.1589	0.1135	0.1475	0.1816	0.4767
(12) Scientific and technical level of project	0.061	0.1829	0.0976	0.1707	0.2073	0.4147
Total	**1**	**3.53**	**2.57**	**2.74**	**3.43**	**7.92**

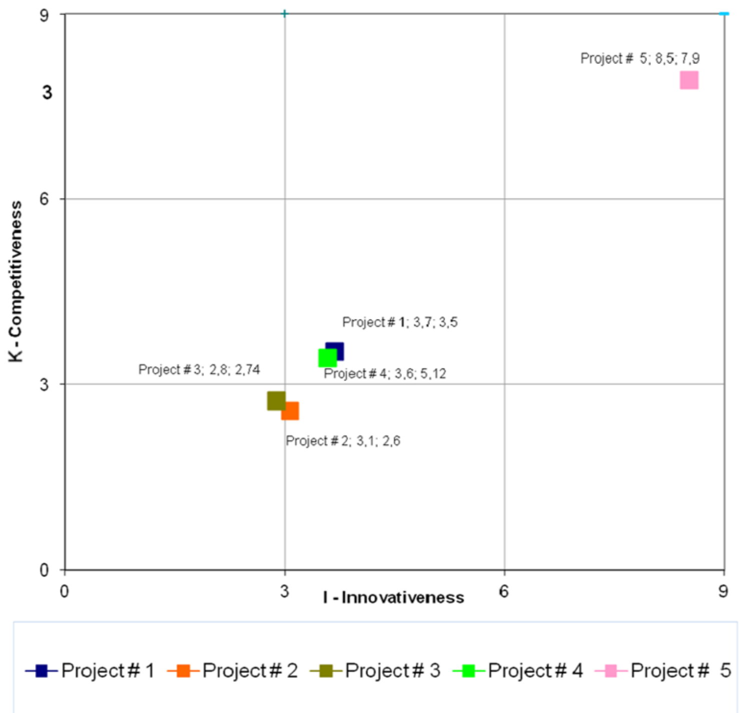


Fig. 5.5 Example of project positioning in graphic model for assessing project innovativeness and competitiveness

competitive sector (at low attractiveness), attractive (at low competitiveness), and neutral. These projects are promising and require finalization work [11, 17].

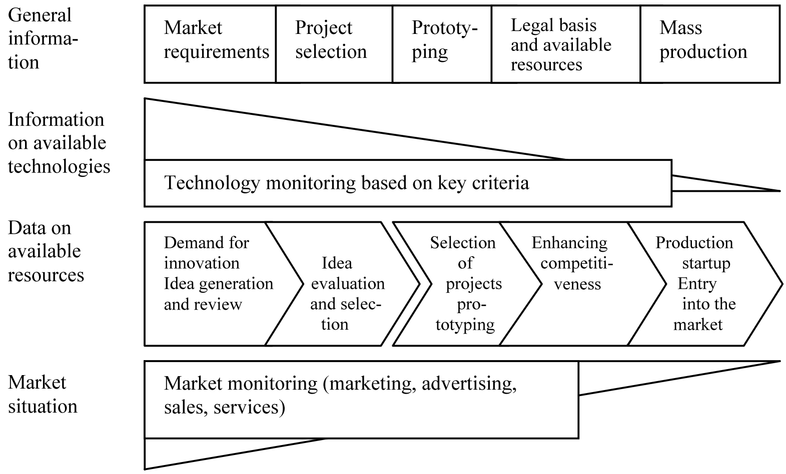


Fig. 5.6 Main steps in creating innovative products

As a solution, let us select the project whose normalized estimates of priority vectors by value occupy the "Leader" section. Such an alternative is Project #5, which according to experts is a priority and ready for implementation.

Project #1 and Project #4 are neutral in the matrix; it is promising, but has some shortcomings that need to be worked on. Project #2 and Project #3 have low values.

In this way, this method allows prioritizing projects on such key indicators as innovativeness and competitiveness.

The second stage of project assessment is to determine the economic viability of the project, the methodology of which is described in the next section.

5.3 Development of Methods and Models for Assessing Feasibility and Cost-Effectiveness of Innovative Projects

5.3.1 Basic Steps in Designing an Innovative Project

The process of introducing a novelty product into the market is one of the key steps of any innovative process, which results in an implemented/realized change or innovation. However, when designing an innovation, it will be necessary to go through certain stages both in the information field and the technological/resource field, all while continuously monitoring market developments [21–23].

The main steps in creating innovative products are as follows (see Fig. 5.6) [1]:

(1) Monitoring the market situation:

   – Undertaking marketing activities in order to assess demand for the new product, its anticipated sales and after-sales service; targeted advertising;

– Understanding market demands for project selection, as well as for innovative products including their prototyping and legal framework.

(2) Monitoring technology:

– Demand for innovation;
– Idea generation, assessment, and selection;
– Project design and selection;
– Identification of competitive features (cost-performance ratios, economic indicators, and environmental criteria);
– Prototyping/manufacturing a pilot product;
– Market entry and developments stemming from it.

An R&D team or an individual produces an idea in the course of their research. This idea is then subject to marketing evaluation. Based on the evaluation results, further development work is conducted and the intellectual property gets patent protection or, alternatively, the underlying idea can be protected by a patent. Such a process requires that a licensing appraiser be involved, either domestic or foreign [1].

Further along in the process, innovation and technology management will require resources provided either by the government or by a private investor. Then a prototype is made and the inventors have to decide whether they reassign the rights or set up a business of their own. Further choices involve whether to sell the technology by setting up a joint stock company and entering the securities market or to start manufacturing and find a niche for innovative commodities in the market.

However, the specific feature of the novelty product market is that intellectual products are oriented towards specific buyers. Therefore, the existing market relations dictate how the innovative products should be handled in the market, with the basic principle being that innovators should not approach the market with their finished goods, they should instead offer their high-quality services accompanied by the ability to satisfy the demands of the prospective customers.

Commercialization of innovations is closely related to the concept of intellectual property, which determines the innovators' rights for protecting results of their intellectual activity in the form of patents, scientific discoveries, works of art, trademarks, etc. [1].

Based on the general market information, it will be required to determine a demand for specific innovations in a given region and assess creativity of the ideas worked out by research teams. Upon development of a business plan and its approval by the scientific and technical council of a technological park (university) and by the economic council, a prototype will be designed and built, for which intellectual property rights can be granted. Further steps include obtaining approvals by the regional expert council and development of project documentation including a Feasibility Study for future capacities. Thereafter, an investor and/or governmental institution should be found who is able to fund the project. At the same time, a management company should be established [1].

A serious barrier to moving a research product into the market is the procedure of registration of title for intellectual property, since the market dictates the rules of the game for intellectual property. Usually, innovators are not rich and cannot afford

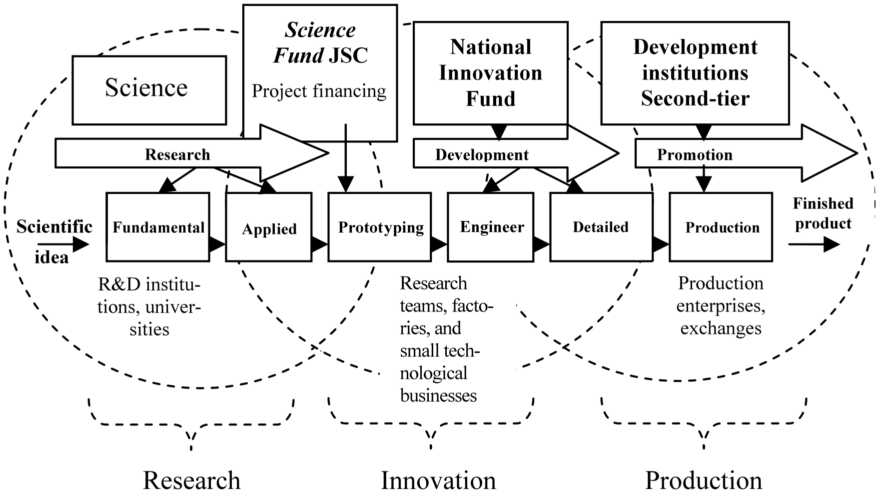


Fig. 5.7 Technological chain of innovations

patenting procedures, and venture funds are not always there to provide the needed seed money.

Commercialization of innovations is now a worldwide phenomenon that sculpts a specific market, which is the platform for trading in intellectual products such as discoveries, inventions, technologies, and technological know-how. Innovations offering great improvements are the basis for technological breakthrough, and their promotion can be supported by the government (state orders and favorable legal regimes). But for the majority of innovations, a market is being formed where the innovative products are tested for their novelty and efficacy. In addition, the market participants have opportunities to discuss their commercial potential.

The Republic of Kazakhstan has its own national model of short-term innovative development, and innovation development patterns have been designed covering such issues as funding and commercialization of innovative projects.

The JSC *National Innovation Fund* (NIF) has suggested a pattern in the form of a consolidated, interconnected chain including all the stages from research right up to production/manufacturing, but it remains problematic to find financing sources for the first stage because the NIF provides funding only for the development engineering stage and for fully designed projects.

The innovation development pattern is shown in Fig. 5.7.

The Science Committee provides base financing for fundamental and applied research conducted by R&D institutions and universities. The JSC *Science Fund* and JSC *National Innovation Fund* finance development engineering projects that have moved to the stage of prototyping. In addition, the NIF specifically states that their objectives are to provide funding only for basic and detailed engineering stages. Other development institutions and second-tier banks are oriented at product manufacturers, such as small innovation enterprises putting out competitive products and flexibly responding to changes in the market.

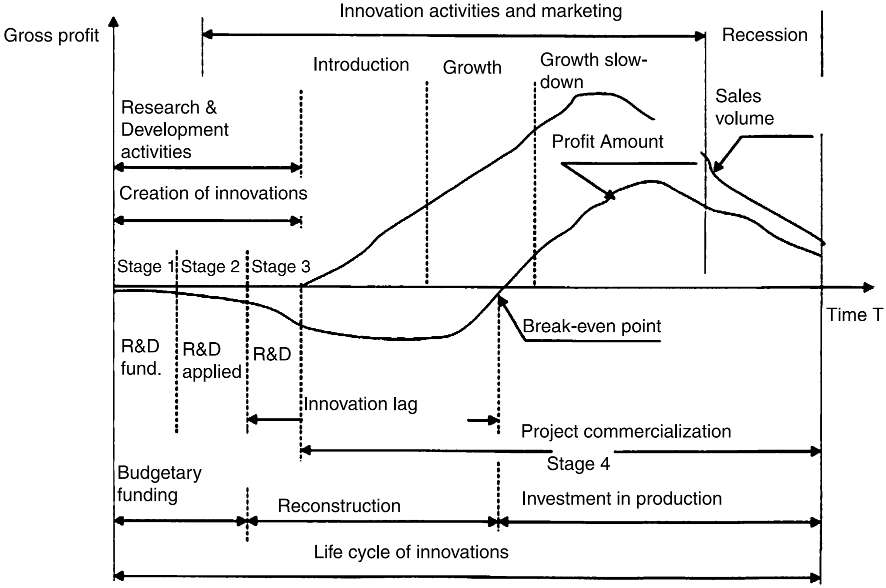


Fig. 5.8 Innovation project life cycle

5.3.2 Basic Life Cycles of Innovation Projects

The life cycle of an innovation project can be defined as the period of time between the conception of an idea, which is followed by its dissemination and assimilation, and up to its practical implementation.

Another way of putting it is that the life cycle of an innovation is a certain period of time during which the innovation is viable and brings income or other tangible benefit to a manufacturer/seller of the innovation product [1, 23].

The life cycle of an innovation project defines specific time-dependent patterns of the development of production, sales, and income generation of a company operating in a certain market; that is, changes in competitiveness of a product in the market. The life cycle of an innovation, in this case, shows every commodity as a product has a limited life span, during which it goes through certain stages, i.e., the development stage followed by introduction, growth, maturity, and decline stages. Figure 5.8 shows the dynamics in sales volumes and income earning by stage of the product life cycle [4, 7, 23].

Let us study the specific features of every stage of the innovation life cycle [1, 4, 7, 23].

Innovation onset This stage is extensive and most important in the entire life cycle of a product. Creation of an innovation is a whole complex of works related to converting the results of scientific research (both fundamental and applied) into new products (commodity items), their assimilation in the market, and involvement in trade transactions [23].

The complexity here includes creation of the new capacities or adaptation of the existing capacities to the new product cycle.

Do not forget that sales of initial/pilot batches of the new products (that, is marketing research) also start at this stage and play a key role for involving the innovation in trading activities.

The initial stage of an innovation project is R&D, during which it is necessary to appraise the likelihood for the project to reach the desired performance [23, 24].

Implementation This stage starts when a product is brought into the market; and it depends on availability of production facilities, sales opportunities, and price-output performance. A producer must make efforts to induce customers to choose a product that is new to them. The marketing strategy at this stage should be aimed at providing attractive and persuasive information to the prospective customers, which will help to increase the number of sales outlets. The requirement for this stage is to project the new commodity prices and production volume [23, 24].

Growth stage When an innovation fits market demand, the sales grow significantly, the expenses are reimbursed, and the new product brings profit. Promotion activities at this stage are quite costly, as the competition grows and the prices remain at the same level or even decline somewhat while the demand grows. The profits go up at this stage because sales promotion expenses account for a bigger sales volume, while costs of production go down. To make this stage as lengthy as possible, the following strategy should be chosen [23, 24]:

– Improve the quality of an innovation,
– Find new segments in the market,
– Use new distribution channels,
– Advertise new purchasing incentives, while
– Bringing down the prices.

Maturity stage This stage is usually the most long-lived one and can be represented by the following phases [23, 24]:

– growth retardation–plateau–decline in demand.

During the maturity stage, the selling race becomes acute and the enterprises choose self-defensive strategies. It will be possible to extend this stage in time by modifying:

(1) **Markets** in the period when a company tries to attract mew customers by
    – Developing new market segments,
    – New promotion efforts, and by
    – Changes in positioning aimed at more attractive segments of the market;
(2) The **product** per se, when a company attempts to attract new customers who earlier have preferred the products of other competitors; this is done by improving the product quality or packaging;

(3) A set of **marketing exercises**, by which a company promotes sales by lowering prices, improving advertising, offering incentives, issuing discount coupons, or by using cheaper distribution channels.

Decline stage At this stage, we see lower sales volumes and lower effectiveness. It is unavoidable that the products become obsolete in the course of time and new products come into the market to replace the old ones; therefore, the demand and sales volumes go down bringing about a poorer economic performance [23, 24].

In the real situation, as we move along through the product life cycle stages we anticipate the ageing and obsolescence of our products and a decline in economic performance. This change is an incentive for us to improve or replace the ageing product.

Given the above sequence, an innovation life cycle is regarded as the *innovation process* [1].

The innovation process can be studied as a multifaceted problem, with the various facets involving different levels of detail.

First, it can be considered as a parallel and serial consummation of research, technical and engineering, innovative, and manufacturing activities including marketing [23, 24].

Second, it can be considered as a time series of an innovation life cycle starting with the conception of an idea, going through it development stage and up to roll-out [23, 24].

Third, an innovation process can be considered as the process of funding and investing in the development of a new product or service. In this case, it will be an innovation project as a particular case of investment projects, which are widely used in the business environment [23, 24].

Generally speaking, an innovation process is the process of registering and commercialization of an invention, new technologies, types of products and services, as well as finding new solutions for production, financial, and administrative issues that can result from intellectual activities [11, 23, 24]. Principal stages and characteristics of the innovation process are given in Fig. 5.8.

It is a common practice to differentiate between scientific research and research-and-engineering work, as well experimental (research and development) work [1, 4, 7].

Scientific work is aimed at getting new knowledge, as well as its dissemination and application, including the following [24, 25]:

- *fundamental research*, which means experimental and scientific work aimed at getting new understanding of the basic laws driving the development of nature and society;
- *applied research* is the scientific work aimed at solving practical problems and getting practical results; and
- *research-and-engineering activity*, the aim of which is to get and use new knowledge in the area of technological, engineering, economic, social, and humanitarian problems in order to ensure that fundamental and applied science, technology, and production function as an integrated system [1, 4, 7].

By *experimental development* we mean a systematic work that is based on knowledge and understanding acquired in the course of scientific research or based on practical experience and aimed at supporting human health and welfare, getting new data, products, and machinery, as well as at introduction of new technologies and further improvement thereof [1, 4, 7, 23].

Fundamental research (Stage I) is conducted by academic institutions, universities, and other types of higher educational institutions, as well as by specialized schools and laboratories. Funding is usually provided by the governmental budget on a non-reimbursable basis [4].

Applied research (Stage II) is conducted by research institutions, with funding provided either by the governmental budget (state research programs or on a competitive basis) or by customers, or both. As it is difficult to foresee whether the result of an applied research project will be negative or positive, the probability of getting a negative result is quite high. Therefore, this is the stage where the investors are likely to lose their money, and such investments are termed 'risk investments.' Only venture funds dare to invest into such businesses [1, 4, 7].

R&D (development engineering) and prototyping work (Stage III) is conducted by specialized laboratories, as well as by design engineering bureaus, pilot-line production facilities, and research-and-production departments of major industrial enterprises. Funding sources are the same as for Stage II, with co-funding provided by the above entities [1, 4, 23–25].

The fourth stage is the commercialization stage, which covers product output and introduction to the market, including further stages of the product life cycle.

At the interface between the third stage and entry to the market, as a rule, there is a need for substantial investments for building new production capacities or modernizing the existing ones, personnel training, advertising, etc. At this period of time, it is not yet clear how the market will respond to the innovative product and the risk of its rejection is quite high; therefore, the investments continue to be risky.

Use of all types of resources is non-uniform during the entire period of R&D project implementation; it depends on the type of work performed. Financial resources used in the course of implementing such programs were shown earlier in Fig. 5.4 here cumulative cash expenditures are plotted for an R&D project. If a need arises to cut the implementation period of an R&D project, then quite tangible additional resources will be required. In the event such resources cannot be made available, the R&D project implementation may be suspended at any stage of the R&D process [1, 4, 7, 24].

Attraction of investments is the final stage of an innovation process. Investments are needed for expanding production capacities, marketing development, and creating a business environment necessary for getting returns on investment.

The stages of an innovation process correlate with the stages of the investment project cycle. An investment cycle can be defined as the period of time between the conception of an investment idea and up to attainment of the set goals [1, 26–28]. An investment cycle can be subdivided into three interrelated phases, namely: pre-investment phase, investment phase, and operation phase (Fig. 5.9). Each phase is linked to corresponding stages of the innovation project (concept formation, planning, implementation).

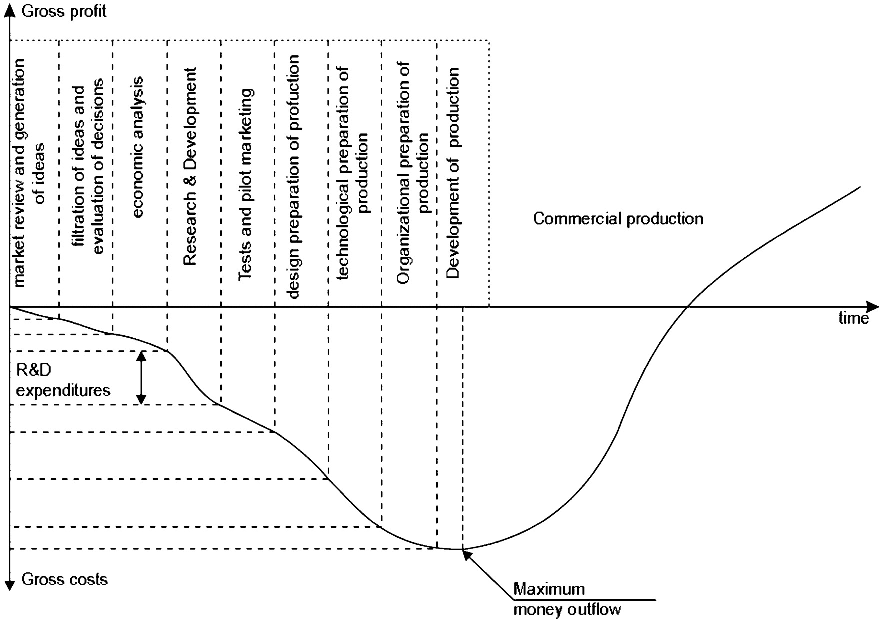


Fig. 5.9 Financial profile of an R&D project

The pre-investment phase includes such steps as setting targets for the investment project, appraisal of investment opportunities, and working out the project rationale. A conceptual project design (idea, concept, or vision) must be formulated prior to setting strategic goals and objectives for the project. A vision of the future depends on the investment policy of the country, region, sector of the economy, and the company. When formulating the idea for a large-scale investment project, it is necessary to correlate the project vision with the strategic goals of the country [1, 26–28].

To draw up the project rationale, it will be necessary to work out the project design options and carry out an environmental analysis and patent examination for the identified options, as well as to run checks for innovativeness and compliance with law and certification requirements. The front-end review of investment opportunities helps to understand whether it is worthwhile to invest in the proposed project. Therefore, it is advisable to study the information available on internal and external factors. The key external factors of an investment project are the ones that affect the investment activities of the country, region, sector of the economy, and the company, as well as diversification opportunities and intersectoral cooperation opportunities. The internal factors include financial standing of the investment project participants; potential investment volumes and anticipated investment efficiency; project implementation feasibility; viable policy decision options; and examination of project alternatives in order to identify their effectiveness, safety level (e.g., environmental safety) and risk level [1, 26–28].

The following issues should be dealt with during the investment phase of a project: project documentation development, project peer review, and project approval. Project documentation includes approved permits for civic construction, expansion, and modernization of a facility; investment options rationale; and design assignment. Project documentation must include the sections listed below:

– An executive summary (covering the design basis for a project);
– Process design (work program, system of production ties, equipment to be used, environmental management plan, ratio of labor to output, mechanization and automation of production processes);
– Economic activity management system (organizational structure, staffing level and personnel qualification requirements, working conditions and safety requirements);
– General layout and engineering infrastructure (construction site description; engineering infrastructure options showing layout of railroads, motor roads, access roads, communication lines, etc.; and surrounding area development plan);
– Construction arrangements;
– Environmental protection;
– Cost estimate documentation (construction cost estimates for two levels of prices: baseline (constant) price level and projected/anticipated price level); and
– Investment effectiveness.

The estimates for production facilities include construction project costs, unit capital investments, fixed assets cost, net profit, level of profitability, internal rate of return, investment pay-back period, and maturity term of loans and other borrowings. When estimating the investment effects, it will be required to compare the collected data and calculations with cost-performance ratios included in the investment rationale section. Depending on the expert examination results, a decision is made as to whether it is worthwhile to invest money in the proposed project. The final step in the second phase (investment phase) of an investment cycle is getting the project approved. The project approval procedure depends on the project funding sources. If an investment project is to be implemented at the regional level, then governmental authorities of the regional and national level will be involved in the approval procedure. If a project is to be funded by private investors (using their own capital or borrowed money), then the project approval procedure will conform to the requirements of their customers (or investors) [1, 23, 26–28].

The third phase of an investment cycle is the operation phase. It includes preproduction activities, production operations, and post-production activities. Preproduction activities are determined by the structure of an investment cycle. Production operations include production and economic performance monitoring. Monitoring results shows whether the targeted performance indicators are achieved or not. Such indicators include production output volume and quality level, revenue on capital employed, increased life cycle duration of a project by upgrading the production capacities, and product improvements. Post-production activities include product promotion systems and finished product sales, as well as after-sale services [16, 21].

Thus, the life cycle concept plays a fundamental role in planning the innovative product manufacturing process, in setting business arrangements for an innovation process, as well as during project design and evaluation stages. This role can be itemized in a series of statements [23]:

– The innovation life cycle concept forces a company CEO to regularly review functioning of the business, from the point of view of both the company's current performance and its future development;
– The innovation life cycle concept justifies the need for planning innovation product output and for acquiring innovations;
– The innovation life cycle concept is the basis for innovation analysis and planning. The innovation analysis makes it possible to understand the current stage of the innovation life cycle, what its prospects are for the short-run, when a sharp decline can be anticipated, and when it will stop existing;
– The innovation life cycle concept determines prospects for further implementation of the project; that is, an innovation project can be viable only if it goes through the complete life cycle.

5.3.3 The Method and Graphic Model for Assessing Feasibility and Economic Effects of Innovation Projects

The life cycle of an innovative project is linked to cash flows resulting from implementation of the project.

Cash flow (or flow of funds) arises as a result of implementation of an innovation project.

Estimation of anticipated cash flows is an extremely significant stage of the innovation project analysis, whereby we estimate and compare the amounts to be invested and paid back [25].

Since the indicators to be compared refer to different time periods, their comparability becomes a key problem. It can be perceived differently depending on the existing objective and subjective criteria such as inflation rate, size of investments and generated income, forecasting horizon, skill of the analyst, etc. [25].

It is helpful to have visual aides when analyzing projects, like the cash flow diagram shown below in Fig. 5.10 [25].

In general, an innovation project P can be described by the following model [29, 30]:

$$P = \{I_i, S_t, n, r\}, \tag{5.13}$$

where I_i stands for investments in the year $i - m$; and $i = 1, 2, \ldots, m$ (most commonly, $m = 1$), S_t is the cash inflow (outflow) per year t; $t = 1, 2, \ldots, n$; n is the project duration, and r is the discount rate.

When analyzing investment projects, a number of things need to be taken into account [25, 29, 30]:

– Any investment project implies a series of cash flows, the components of which are either net outflows or net inflows of cash; sometimes, the analysis uses a series of projected net annual profit values rather than cash flows;

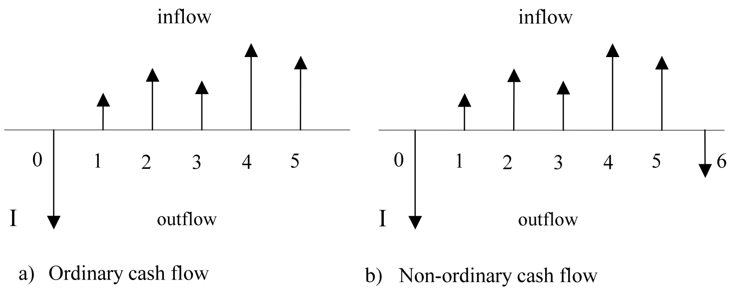


Fig. 5.10 Graphic representation of cash flow

– Most commonly, the analysis is done on a year-on-year basis, though any period can be taken as the base unit (e.g., month or quarter). It is just necessary to connect cash flow components with the interest rate and duration of the period;
– An assumption is made that a lump sum is invested at the end of the year preceding the first income-generating year of the project (investments can be made during a number of subsequent years);
– Cash inflow/outflow will be estimated as of the end of each project year.

Cost methods to estimate effectiveness of innovation projects can be classified into two groups depending on whether the time parameter is accounted for or not [1, 25, 29–31].

I. **Methods based on discounted estimates [25]:**

(1) Net present value (NPV) method of project evaluation;
(2) Profitability index (PI) method to estimate investment profitability;
(3) Internal rate of return (IRR); and
(4) Discounted payback period (DPP).

II. **Methods based on accounting estimates [25]:**

(5) Payback period (PB).
(6) Return on investment (ROI);
(7) Accounting rate of return (ARR) or investment effectiveness ratio.

Let us discuss the above methods in more detail.

I. Methods based on discounted estimates [25]:

(1) The net present value (NPV) method compares the value of initial investment (I_0) with the aggregate sum of discounted net cash inflows generated during the forecasting period [25]:

$$NPV = \sum_{t=1}^{n} \frac{S_t}{(1+r)^t} - I_0, \tag{5.14}$$

where S_t is the net cash flow (the amount of cash, inflow minus outflow) at time t (inflow ("+"), outflow ("−"), unit of money; n is the project implementation period (forecasting horizon), years; r is the discount rate (relative units); $I_0 = S_0$ is the initial investment, $\frac{1}{(1+r)^t}$ is the discount coefficient (discount factor).

NPV should be understood as the project's economic effect normalized to the project launch time. A project may be accepted for implementation if its $NPV > 0$ [25, 29–31].

If it is anticipated that investments will be made over the course of m years, rather than as a lump sum, then the NPV calculation formula will look like this:

$$NPV = \sum_{t=1}^{n} \frac{S_t}{(1+r)^t} - \sum_{t=1}^{n} \frac{I_t}{(1+r)^t}. \tag{5.15}$$

(2) The profitability index (PI) method is applied to estimate net profitability of investments (5.16). The profitability index helps to select a project out of several alternative ones [25].

$$PI = \frac{\sum_{t=1}^{n} S_t (1+r)^{-t}}{I_0}, \tag{5.16}$$

where PI is a relative index that allows us to quantify the amount of value created per unit of investment; that is, to find economic effectiveness of investments.

The best proposed project out of several alternatives is the one with $PI \to \max$. In the above formula (5.16), the numerator is the net present value plus the investment value (initial investment) [25, 29, 31].

(3) The Internal Rate of Return (IRR) method uses the discounted cost concept. It is a tool to find the discount rate level such that the present value of the proposed project earnings will be equal to the present value of required cash investments. In other words, IRR is the discount factor value such that $NPV = 0$ [25].

To determine the internal rate of return R, we solve the following equation for R [25]:

$$\sum_{t=1}^{n} S_t (1+R)^{-t} - I_0 = 0. \tag{5.17}$$

The iteration method is used to solve the equation. A project may be accepted if IRR is equal to or exceeds the existing discount rate (which is used to calculate NPV) [25, 29–31].

IRR can also be calculated by the iteration technique using discount rates. For this purpose, two discount rates $r_1 < r_2$ will be chosen from the tables, such that the function $NPV = f(r)$ in the interval (r_1, r_2) would change its value from '+' to '−' or from '−' to '+'. Then the following formula is used [25]:

$$IRR = r_1 + \frac{f(r_1)}{f(r_1) - f(r_2)} \cdot (r_2 - r_1), \tag{5.18}$$

where r_1 is the tabulated discount rate value such that $f(r_1) > 0$; $(f(r_1) < 0)$; r_2 is the tabulated discount rate value such that $f(r_2) < 0$; $(f(r_2) > 0)$.

Computational accuracy is inversely proportional to the (r_1, r_2) interval length; and, based on the tabulated values, we get the best approximation when the interval length is 1 %; that is, when r_1, r_2 are the nearest neighbors satisfying the preset conditions (may be changing $y = f(r)$ from positive to negative) [25].

When analyzing effectiveness of the proposed investment, we calculate IRR to determine the anticipated profitability of the proposed project and the maximum acceptable relative level of expenditures on the project. The IRR value shows an acceptable ceiling for the interest rate (cost of capital). If this ceiling is exceeded, the proposed project will be creating loss. Average weighted cost of capital can be used as a comparison test provided that a company uses more than one financing source for its investment project [1, 25, 29, 31].

(4) The Payback Period (PP) method is one of the simplest and most commonly used methods of investment estimation [1, 25, 29].

Payback period is the amount of time taken to break even on an investment.

If it is projected that an investment project's cash flow will be constant during several years, then:

$$PB = \frac{I_0}{S_t} \text{ (years)}, \tag{5.19}$$

where I_0 is the initial investment; S_t is the year-on-year cash flow.

If it is anticipated that the earnings flow will change from year to year, then it will be necessary to put together a cash flow balance (cumulative cash flow). The formula below is used to calculate a payback period [25]:

$$PB = \frac{|P_{k-}|}{|P_{k-}| + P_{k+}}, \tag{5.20}$$

where $|P_{k-}|$ is the negative balance of the accumulated cash flow at the step prior to the break-even point; P_{k+} is the positive balance of the accumulated cash flow at the step right after the break-even point.

(5) Discounted break-even period is the period of time for the present value to turn from negative to positive [25]. This method gives us the break-even point of the project. The sum $\sum_{t=1}^{n} S_t \frac{1}{(1+r)^t} \geq I_0$ of discounted cash flows will be calculated [25].

Further calculations will be similar to calculating a normal payback period. If the discounts are made, the payback period will increase, meaning that a project acceptable by the payback period criteria may be not acceptable if we use the discounted payback period.

(6) Return of Investment (ROI) method. In this method, the year-to year profit (before or after tax) is compared with the initial investment [1, 25, 29–31].

$$ROI = \frac{S_t}{I_0}, \tag{5.21}$$

where I_0 is the initial investment;

S_t is the year-to-year cash flow, being the difference between income and expenditures during the period t.

(7) Accounting Rate of Return (ARR) method [25, 29–31].

This method is used by managers as an indicator of the operation's effectiveness. The most general form of it is as follows

$$ARR = \frac{\text{Average book profit}}{\text{Average capital expenditures}}, \tag{5.22}$$

where *average book profit* is the average annual after-tax profit during the entire period of project implementation and *average capital expenditures* are the expenditures assigned to the entire period of project implementation.

It is very important for an investor to understand whether it is worthwhile to invest or not. Therefore, investors should not rely on only one of the above methods. It would be wiser to make a decision having conducted analyses using *all* the above methods and analyzing discrepancies arising from their ranking orders.

It is a universal belief that the NPV method is the most reliable. However, a situation is possible where the NPV method gives results for a number of projects that are very close in value. Then the need arises to use other methods.

The concept of *feasibility of an innovation project* is very important. It implies availability of financial, research, engineering, technological, production, as well as institutional and managerial resources available for the task of handling innovation projects over their entire life cycle.

The entire life cycle of an innovation project depends on the flow of funds that can be positive (inflow) or negative (outflow). Their interdependence is key for determining whether a proposed project (or stages thereof) is viable or not [1].

The project efficiency criterion at the i-th stage is the actual cash flow S_t, which is the difference between the incoming cash Π_t and outgoing cash O_t at every period of project implementation [25, 29]:

$$S_t = [\Pi_{t1} - O_{t1}] + [\Pi_{t2} - O_{t2}] = S_t + S_t'. \tag{5.23}$$

The implementation period of the i-th investment project has been subdivided into arbitrary sub-intervals Δt (day, week, month, quarter, year); and each of them has been assigned its sequential number $t_{0i}, t_{0i}+1, t_{0i}+2, t_{0i}+n$.

We shall represent the results of the proposed project appraisal as $\Pi_t - E_t$, $t \in [t_{0i}, t_{0i}+n]t$, where Π_t is the incoming flow generated in the course of implementing an innovation project (as per the above sub-intervals $t = t_{0i}, t_{0i}+1, \ldots, t_{0i}+n$) and E_t means outlays of the i-th innovation project.

Thus, the aggregate costs $O_t = I_t + E_t$, $t \in [t_{0i}, t_{0i}+n]$ are subdivided into the investment costs I_t and additional current costs/expenditures E_t.

From this point, the i-th project implementation results will be interpreted as the current year results (net cash returns) S_t. Therefore, $S_t = \Pi_t - E_t$.

Figure 5.11 shows the accumulated net cash inflow over the entire period of project implementation, where S_t is the maximum cash outflow of investments over the period t_{0i+n} and S_t' is the maximum cash inflow of the net profit over the periods $t_{0i+n'}$; while PB/DPB is the proposed project break-even point (here, PB means a payback period and DPB, a discounted payback period).

An important tool for making decisions on an investment project is the net present value (NPV) method, which can be applied starting the time t_{0i} and up to the end of a project period.

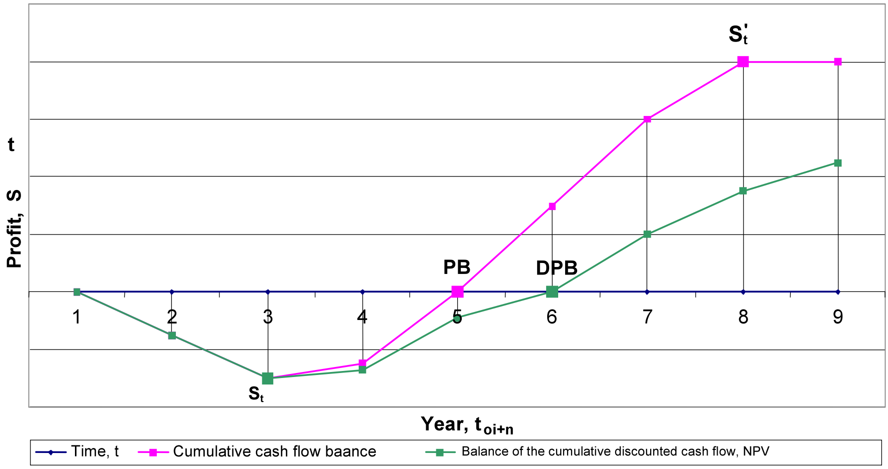


Fig. 5.11 Graphic model of an innovation project's feasibility and economic effect

The formula is as follows:

$$NPV_i(\tau) = -\sum_{t=t_{0i}}^{t_{0i}+n_i} \frac{I_t}{(1+r)^{\tau-t}} + \sum_{t=t_{0i}}^{T} \frac{S_t}{(1+r)^{\tau-t}}. \tag{5.24}$$

This characteristic shows how a modification of the investment program (characterized by the i-th project with the expenditure level I_t within the interval $[t_{0i}, t_{0i}+n]$) affects the net cash inflow (designated as S_t) from the time t_{0i} to the end of the projected period n.

Hence, the above index gives an estimate of both current and future proceeds at any given time $t \in [t_0, n]$, which are earned in the course of the project implementation.

But if we want to know the financial performance that has been achieved by the time τ without considering any further proceeds, then we can use a modified formula:

$$N\tilde{P}V_i(\tau) = -\sum_{t=t_{0i}}^{\tau} \frac{I_t}{(1+r)^{\tau-t_j}} + \sum_{t=t_{0i}}^{\tau} \frac{S_t}{(1+r)^{\tau-t}}. \tag{5.25}$$

This estimate can be helpful for a day-to-day control of the chosen investment option/scenario.

Thus, the $NPV_i(\tau)$ characterizes a discounted effect (with the discount rate being r) of an investment decision made at the time τ.

Let us consider other performance parameters of an innovation project.

The discounted PI can be described by the formula:

$$PI|_{t_{0i}} = \frac{\sum_{t=t_{0i}}^{n} \frac{S_t}{(1+r)^{t-t_{0i}}}}{\sum_{t=t_{0i}}^{t_{0i}+n_j} \frac{T_t}{(1+r)^{t-t_{0i}}}}. \tag{5.26}$$

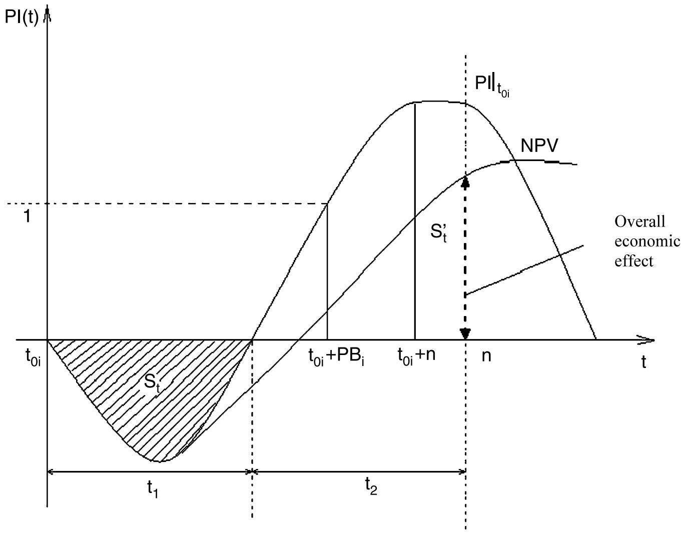


Fig. 5.12 Plotted economic effectiveness of a proposed innovation project

The above indicator (in the discounted form) characterizes how each unit of the invested funds works in the interval $[t_{0i}, t_{0i} + n]$ of a given component of the innovation project and how it affects the future developments ($S_t, t \in [t_{0i}, T]$) as compared to the case where the i-th step is not implemented. If the resulting $PI|_{t_{0i}} > 1$, then the proposed project may be accepted.

The standard project appraisal method allows us to determine the end effect of an investment project only during the final stage of its implementation.

When formulating an investment program, a decision-making process is not limited to selection of efficient projects, but implies a holistic approach that covers the role of the selected projects in achieving overall strategic objectives and requires that appraisal and evaluation techniques be improved.

Therefore, the following profitability index calculation procedure is suggested:

$$PI(t) = \frac{\sum_{\tau=t_{0i}}^{t} \frac{S_\tau}{(1+r)^{1-\tau}}}{\frac{I_\tau}{(1+r)^{1-\tau}}}, \quad t \in [t_{0i}, n]. \tag{5.27}$$

Based on the above estimates, we can build a time profile of payback/return per each unit of funds I_τ invested into the project (Fig. 5.12).

Hence, we can determine the project performance (payback/return per each unit of invested funds) at every stage of its life cycle. The results are shown in Fig. 5.13.

This method characterizes the changes in payback on investments within the interval $[t_{0i}, t_{0i} + PB_i]$, where the discounted outflows are higher than discounted inflows, and within the final interval $[t_{0i} + PB_i, T]$, where the financial performance targets have been attained.

Thus, the following things are to be determined:

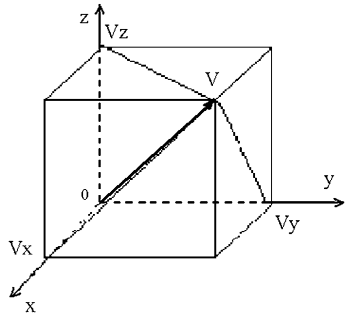


Fig. 5.13 Three-dimensional Cartesian system of coordinates

– Time profile;
– Time values of return on investment;
– Comparison of several investment proposals and choice of the most competitive ones.

Figure 5.12 shows the time $t_{0i} + n_i$ (for a certain n_i) when the financial performance reaches the targeted level of $PI|_{t_{0i}}$ as per the formula (5.15).

The entire system of the project performance indicators is given in terms of the project scale, returns per an investment unit $PI_i(t)$, payback time PB_i, and achievement of the investment goals T_i, as well as investment costs given their distribution in time I_t.

All the above indicators are calculated as time values. If a need arises to estimate undiscounted values, the discount rate r is set equal to zero.

The above formulae can be used to analyze individual projects in more detail, as well as to update investment programs and improve their performance parameters if there are additional managerial (investment) resources.

Further development of the proposed approach makes it possible to modify and refine financial and economic estimates of the future financial and economic performance parameters of a corporation.

To further improve the analysis and make it more comprehensive, it will be worthwhile to include such aspects as effectiveness of investment decisions.

This is the rate of return on investment or internal rate of return (IRR). The internal rate of return is computed by finding the discount rate r that equates the present value of a project's cash outflow with the present value of its cash inflow. A project will be effective if the IRR is equal to or more than the rate of earnings desired by the investor.

IRR can be thought of as the rate of growth a project is expected to generate, or the rate of earnings that an invested capital I_t can bring in terms of inflows S_t, $t \in [t_{0i}, n]$.

However, it can be technically challenging to calculate the IRR and the results obtained can be misinterpreted. Therefore, more accurate recommendations are needed for carrying out computations.

First, we need to have a good estimate for the life cycle (denoted as n in the above) of an i-th innovation project.

It should be noted that if an i-th project continues to perform well beyond the period $[t_{0i} + n]$, it can result in lower estimated IRR_i as compared with the actual one.

On the other hand, an overestimated n will result in the situation where actual growth of the i-th investment option can be zero beyond the period n; therefore, the estimated IRR_i will be low (the requirement $IRR > d$ will not be met) and the project may be rejected.

As is known, IRR_i value is determined as the root of the equation:

$$\sum_{t=t_{0i}}^{t_{0i}+n_i} \frac{I_\tau}{(1+r)^{\tau-t_i}} = \sum_{t=t_{0i}}^{t_{0i}+n_i} \frac{S_t}{(1+r)^{t-t_{0i}}}, \tag{5.28}$$

where $IRR = r$.

Note that IRR_i is the discount level such that the payback time of an investment project equals n; that is, $DPB_i = n_i$.

Thus, the internal rate of return of an investment is the discount rate at which the net present value of costs (negative cash flows) of the investment equals the net present value of the benefits (positive cash flows) of the investment [32].

This value, in a sense, characterizes the investment growth rate because its computation result for a bank deposit equals the interest rate. However, as stated elsewhere, IRR is usually used for ranking proposed projects, rather than forecasting actual returns [1, 32].

The same can be said about effective use of capital borrowed for implementing a proposed investment project. In the event the interest rate is lower than the estimated IRR, then there may be no profit on the invested capital. Such a situation is also described in the policy advice.

Despite the above limitations, IRR is widely used in the analysis of investment opportunities as a coarse estimate of the capital growth rate, which helps to rank various investment project proposals. In addition, it can help to understand the 'structure of involvement' of the available 'disposable capital' and to decide whether it will be worthwhile to borrow.

Figure 5.12 shows the payback period of a project with the time values taken into consideration.

Thus, we have identified four basic indicators for appraising investment projects: Net Present Value (NPV) to characterize the scale of an investment activity; Profitability Index (PI) to characterize the yield per unit of a proposed investment with a possibility to include actual performance adjustments ($PI_i(\tau)$); IRR, which characterizes capital growth rate; and (PB_i), which shows the payback period on a project.

These four indicators can be used for a comprehensive appraisal of an innovation project.

The time value of an investment can be described by the formula:

$$I_{i0} = \sum_{t=t_{0i}}^{t_{0i}+n_i} \frac{I_t}{(1+r)^{t-t_{0i}}}. \tag{5.29}$$

Therefore, we can calculate the corresponding values (based on the above) since:

$$PI_i = \frac{NPV}{I_0} + 1. \tag{5.30}$$

Therefore, the targeted NPV_i, PI_i, and IRR_i will be attained by $t_{0i} + n_i$.

A more comprehensive method should be used at the final project appraisal stage. It is described in the following section.

5.3.4 Method and Graphic Model for Innovation Project Evaluation

Having applied the two above methods, it is considered practical to appraise and compare alternative projects with a more comprehensive technique, which is based on determining vector lengths in the Cartesian system.

The 3D Cartesian coordinate system has three mutually perpendicular axes O_x, O_y, and O_z with the common zero point O and preset scale. Let V be an arbitrary point and V_x, V_y, and V_z be its O_x, O_y, and O_z components (Fig. 5.13). The coordinates of points V_x, V_y, V_z will be denoted as x_V, y_V, z_V, respectively; that is, $V_x(x_V)$, $V_y(y_V)$, and $V_z(z_V)$. Thus, the point V has the following coordinates: $V = (x_V; y_V; z_V)$.

Then the length of vector OV will be calculated by the formula:

$$|V| = \sqrt{x^2 + y^2 + z^2}. \tag{5.31}$$

Thus, the comprehensive method can be used to appraise an innovation project by determining the vector length in a 3D coordinate system (x, y, z), where x is the project innovativeness (I), y is the project competitiveness (K), and z is the net present value (NPV). The values I, K, and NPV are specified both in the innovativeness and competitiveness assessment method and in the economic benefit evaluation method.

The scales for I and K axes have been normalized to conventional standard units with due account for their commensurability and comparability. The scales for I and K axes are given as dimensionless values with a 9-point scoring system making it possible to classify projects into 3 groups, namely: "outsiders," "marginal," and "leaders." The NPV axis has also been normalized to the conventional standard system (with the score from 0 to 9 points). This approach allows us to classify projects into groups based on their profitability and move to the dimensionless unit of measurements.

Monetary units have also been expressed in dimensionless units by assuming that a 10 billion dollar project has its NPV equal to 3 conventional units, the NPV of a 20 billion dollar project will be 6 units, and that of a 30 billion dollar project will equal 9 units. If and when needed, this proportion may be changed as a function of resulting NPV values.

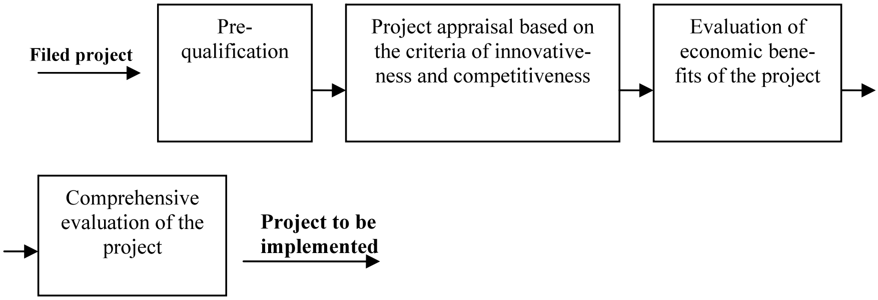


Fig. 5.14 Innovation project appraisal stages

The vector length for Project #5 will be as follows:

$$|V| = \sqrt{I^2 + K^2 + NPV^2} = \sqrt{8.5^2 + 7.9^2 + 3.75^2} = 12.2.$$

The maximum vector length is 15.59, on the basis of the following expression:

$$|V| = \sqrt{I^2 + K^2 + NPV^2} = \sqrt{9^2 + 9^2 + 9^2} = 15.59.$$

Based on the two-stage examination of the project, we can define the project vector length. This method can prove to be effective when ranking alternative projects. The longer the vector, the more preferable the project is.

Thus, to understand whether an innovation project is feasible or not, the project must go through several stages of examination, which are shown in Fig. 5.14.

The above algorithm has scope for improving the existing procedures of project review for both commercial and strategic goals.

5.3.5 Research into the Methods and Models on Innovation Project Evaluation

The methods proposed in this book were developed based on five projects being the product of the East Kazakhstan R&D company *Altai*. Let us consider how the proposed methods can be applied.

(1) The first step in innovation project evaluation is project examination using the innovativeness and competitiveness criteria.

Table 5.10 give the results of expert examination using the innovativeness and competitiveness criteria.

The graphic model of the project innovativeness and competitiveness evaluation is given in Fig. 5.15 and shows how the innovation projects are positioned based on the expert examinations.

It follows from the obtained positioning that, judging by the normalized priority vectors, Project #5 has been classified in the "leader" group, which means that it is a

Table 5.10 Average scores assigned by experts based on the innovativeness and competitiveness criteria

Innovative project	Competitiveness	Innovativeness
Project #1	3.7	3.5
Project #2	3.1	2.6
Project #3	2.9	2.7
Project #4	3.6	3.4
Project #5	8.5	7.9

Fig. 5.15 A sample of project positioning based on the graphic model of evaluating innovativeness and competitiveness of a project

priority project and can be implemented. Project #1 and Project #4 are in the "neutral" group meaning that it should be further engineered and finalized. Project #2 and Project #3 are "outsiders" in this matrix.

Project #5 moves to the next stage of estimation.

(2) The second stage of the project examination procedure is the economic effect analysis of a project using project life cycles.

Financial performance of Project #5 is given in Tables 5.11 and 5.12. To compute the project effectiveness indices, we have used the KZT discount rate of 11.29 %.

Net cash flows during the total 16 periods of project implementation (by year) are given in Fig. 5.16.

As follows from Fig. 5.16, the project NPV for the total life of the project is 12,506,094,589.24. The NPV obtained for each period can be used to compare in-

Table 5.11 Data for Project #5

Performance indicator	Indicator values by projected interval (year)				
	1	2	3	4	5
Net income	−396,800.00	−766,808,399.00	2,497,116,619.78	2,346,068,914.50	2,342,303,918.80
Net income balance	−396,800.00	−767,205,199.00	1,729,911,420.78	4,075,980,335.28	6,418,284,254.08
Discount rate (11.29 %)	0.898553329	0.807398085	0.725490237	0.651891668	0.585759429
Time value of net income	−356,545.96	−619,119,633.15	1,811,633,729.54	1,529,382,778.29	1,372,026,605.23
Time value of net income balance	−356,545.96	−619,476,179.11	1,192,157,550.43	2,721,540,328.72	4,093,566,933.95
Net income	2,344,005,951.46	2,343,889,264.46	2,343,777,314.21	2,343,669,904.32	2,343,566,846.94
Net income balance	8,762,290,205.54	11,106,179,470.00	13,449,956,784.21	2,343,669,904.32	18,137,193,535.47
Discount rate (11.29 %)	0.526336085	0.472941041	0.424962747	0.381851691	0.343114108
Time value of net income	1,233,734,915.01	1,108,521,429.09	996,018,045.84	894,934,316.33	804,110,848.86
Time value of net income balance	5,327,301,848.96	6,435,823,278.05	7,431,841,323.89	8,326,775,640.21	9,130,886,489.07
Net income	2,343,467,962.35	2,343,373,078.59	2,343,282,031.09	2,343,194,662.31	2,343,110,821.40
Net income balance	20480661497.82	22,824,034,576.41	25,167,316,607.50	2,343,194,662.31	29,853,622,091.21
Discount rate (11.29 %)	0.308306324	0.277029674	0.248925936	0.223673228	0.200982324
Time value of net income	722,505,993.49	649183880.16	583303672.66	524109914.91	470,923,858.34
Time value of net income balance	9,853,392,482.56	10,502,576,362.72	11,085,880,035.38	524,109,914.91	12,080,913,808.62

Table 5.12 Experimental data for Project #5

Performance indicator	Indicator values by projected interval (year)
	6
Net income	2,354,354,757.49
Net income balance	32,207,976,848.70
Discount rate (11.29 %)	0.180593336
Time value of net income balance	425,180,780.61
Time value of net income balance	12,506,094,589.24

vestment proposals having different life spans and choose the most effective ones from the point of view of the financial and economic analysis goals.

The project payback period happens to be within the second year. Formula (5.8) will be used to find a more accurate estimate of the payback period.

$$PB = \frac{|P_{k-}|}{|P_{k-}| + P_{k+}} = \frac{-767{,}205{,}199.00}{(-767{,}205{,}199.00 + 1{,}729{,}911{,}420.78)} = 0.80$$

Thus, the more accurate estimate gives the payback period of 1.8 years.

Now we calculate the payback period with the account of discounted cash flows:

$$PB = \frac{|P_{k-}|}{|P_{k-}| + P_{k+}} = \frac{-619{,}476{,}179.11}{(-619{,}476{,}179.11 + 1{,}192{,}157{,}550.43)} = 1.08$$

Then the project discounter payback period will be 2.08 years.

Now, let us take NPV, another project performance indicator. As follows from Fig. 5.17, its value is quite high (KZT12,506,094,589.24).

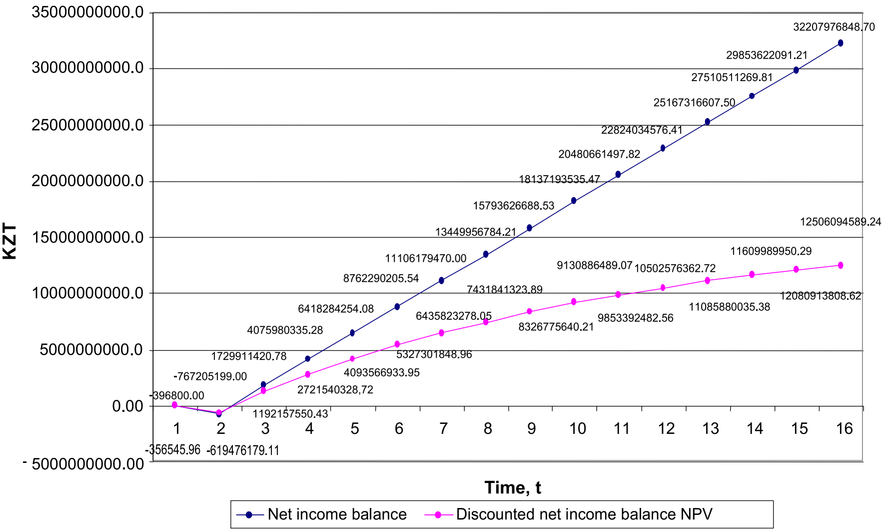


Fig. 5.16 A sample of project implementation as represented by the graphic model of feasibility and cost effectiveness of an innovation project

Fig. 5.17 Graphic model of the comprehensive evaluation of innovation projects

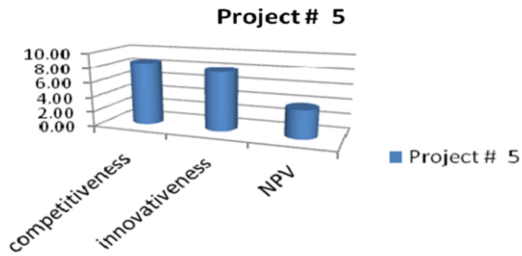


Table 5.13 Main indicators of the project's economic effect

Indicator	KZT
Discount rate, %	11.29
Payback period (PB), months	22
Discounted payback period (DPB), months	25
NPV	12,506,094,589.24
Profitability index (PI)	21.42
IRR (%)	320.25

Formula (5.16) will be used to compute the project profitability index.

$$PI = \frac{\sum_{t=1}^{n} S_t(1+r)^{-t}}{I_0} = \frac{13{,}118{,}630{,}238.99}{-612{,}535{,}649.8} = 21.42$$

Then the NPV for the project will be (see (5.18)):

$$IRR = r_1 + \frac{f(r_1)}{f(r_1) - f(r_2)} \cdot (r_2 - r_1) = 11.29 + \frac{12{,}506{,}094{,}589.24}{12{,}506{,}094{,}589.24 - (-1073.148351)} \cdot (320.25 - 11.29) = 320.25$$

The results of the project's economic effect are summarized in Table 5.13.

The calculations show that the discounted payback period is 25 months, which is quite plausible for such projects. The profitability index (PI) is much higher than 1.0 (it is 21.42). The IRR is also high ($IRR = 320.25$ %). Its modified value is lower (36.25 %).

Therefore, the project has sufficiently high indices of effectiveness and can be accepted for implementation.

(3) The third stage of examining an innovation project is the proposed comprehensive method (Fig. 5.18).

The I, K, and NPV values are 8.5, 7.9, and 12.5, respectively. The NPV value of KZT12.5 billion as expressed in conventional standard units will be 3.75.

Now we go on to defining the vector length for Project #5 (Fig. 5.18):

$$|V| = \sqrt{I^2 + K^2 + NPV^2} = \sqrt{8.5^2 + 7.9^2 + 3.75^2} = 12.2.$$

The vector V length is 12.2.

Therefore, the indices of effectiveness of Project #5 are sufficiently high and it can be accepted for implementation. This method works effectively when ranking alternative projects. The longer the vector, the more chances for the project to be accepted; that is, the absolute positioning approach is used.

It follows from the above that all three methods used for examining the project show that it is acceptable and can be implemented.

In the above example, Project #5 has high estimated performance indicators and can be accepted for implementation. A graphic model of the comprehensive evaluation method is given in Fig. 5.17.

It would be practicable to have a Decision Support System in order to make the comprehensive method of project examination more effective [1, 33].

5.4 Development of an Information System of Innovation Project Examination

5.4.1 Decision Support Systems

The key purpose for designing an innovation project evaluation DSS is to provide an integrated information support for project examination procedures.

The DSS functional role for the purpose of examining innovation projects consists in determining the innovativeness and competitiveness criteria of an innovation project. To do this, the three methods can be used, namely: the feasibility assessment method, the economic (cost-effectiveness) method, and the comprehensive method of project examination. The evaluation results can be given as the graphic model of a project innovativeness and competitiveness evaluation, a graphic model of a project feasibility and cost-effectiveness assessment, and a graphic model of a comprehensive project examination. In addition, a formalized project description document can be issued. The DSS block diagram is given in Fig. 5.18 [34].

User interface supports user interactions with the DSS application. The data input module is a form for origination, input, and output of the data on innovation projects and expert personal data. Information on projects, experts, and their opinions is stored in the database. The rule base is the central part of DSS, and contains knowledge needed for project understanding and appraisal/examination based on the innovativeness and competitiveness criteria. The rule base is the tool for identifying good and bad projects, as well as the ones that need to be refined and finalized.

The project appraisal module is the control module or rule interpreter with built-in capabilities to interpret the information contained in the database (rule base) and formulate opinions; that is, to make decisions based on the assessed feasibility and cost-effectiveness of innovation projects, as well as the comprehensive method of project examination. Graphic models of the proposed innovation projects are created in the display module. The project appraisal knowledge acquisition module is a tool for getting information on the proposed innovation project appraisals, which result from processing expert opinions on the filed projects.

The analysis protocol and selection advice module is a tool that can be used by experts. In addition, it supports and updates the database, and is capable of giving

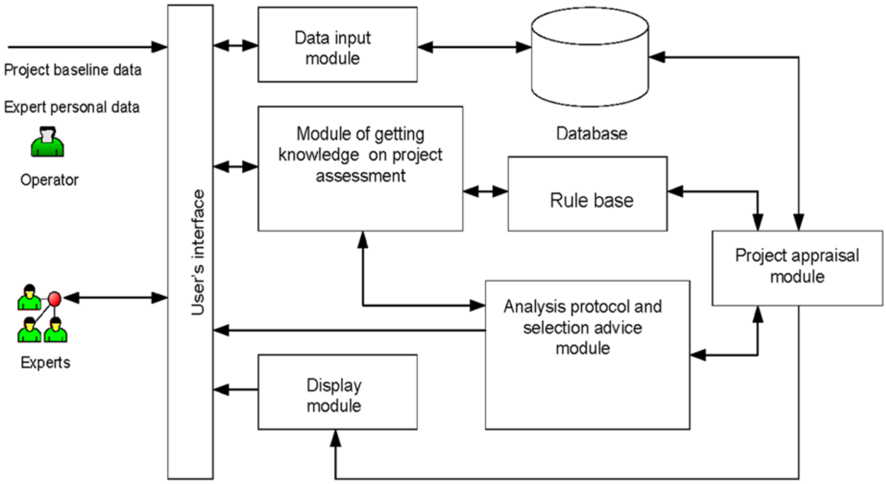


Fig. 5.18 DSS block diagram of project examination

opinions/conclusions as well as comments on the issued opinions accompanied by reasoning.

There are different types of system users: an operator, who is in charge of organizing the entire work of step-by-step project examination and whose main functions are to manage the project evaluation process, manage help systems, and note exception reports in Microsoft Excel. Experts belong to the category of users who study, appraise, and assign scores to the proposed innovation projects.

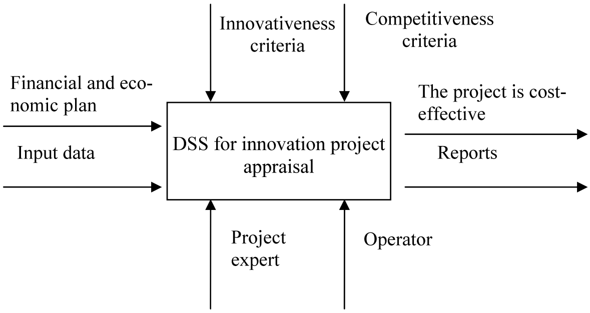
5.4.2 DSS Functional Model Development

Graphic (visual) models are the tools for the system architecture visualization, description, designing, and documentation. Visual simulation has significantly influenced computer software development techniques [34]. A context-sensitive model, shown in Fig. 5.19, describes the purpose of developing DSS for innovation project examination, which is providing a comprehensive support for decision-making.

To support main business processes of innovation project examination, we shall single out functional subsystems; that is, we decompose the system. Project examination management consists of the following processes: receive a project, file/register the project, appraise it using the criteria, appraise the project using economic indicators, run it through expert examination, schedule, reports, and results (Fig. 5.20).

Each process is an individual subsystem, which has its own processes aimed at attaining the common goal of functioning. Let us consider the principle of model functioning. Key objectives of an innovation project examination are to obtain in-

Fig. 5.19 Context-sensitive diagram



formation on whether the new technological and organizational solutions proposed by a project are promising, as well as to understand the project's economic benefits and investment risks. Project examination is an extremely challenging task; and quality of the results strongly depends on qualification of the experts involved and funding available for such examination. At the first stage, project examination is governed by the innovativeness and competitiveness criteria, which describe future behavior of the project.

Input information is as follows: project proposal, applicant questionnaire, project description certificate, and project business plan. The selection criteria are attributes that are used to rank a project.

Interaction of these interfaces in the subsystem *Assessment of the project innovativeness and competitiveness* transforms the data in the output information or *Criterion parameters*, which goes to the input module of other subsystems.

The interfaces in the subsystem *Project feasibility and economic benefit assessment* form economic performance indicators, which are the input data for the subsystem *Comprehensive examination of innovation projects; Conclusions and decision-making*.

Output information of this block is the result of an innovation project examination, which is the basis for decision-making. The output data of all subsystems are accumulated and form a single output information package.

Users of the system are experts and administrators (project developers), who enter the data into the system, while data processing, transfer, and storage are performed by the software of the designed system.

The operations flowchart gives the main rules of the operational sequence, which must be followed. Operations flowcharts can be used independently for visualization, setting specifications, designing, and documenting a set of objects, and they are also good for simulating operations flow. Using the operations flowchart, we shall simulate computational processes in the subsystem named *Decision-making systems for innovation project examination*, which describe the system behavior. Using the operations flowchart, we describe the operations of this subsystem.

The algorithm is as follows:

Step 1—program start;
Step 2—registration of the user and project data;

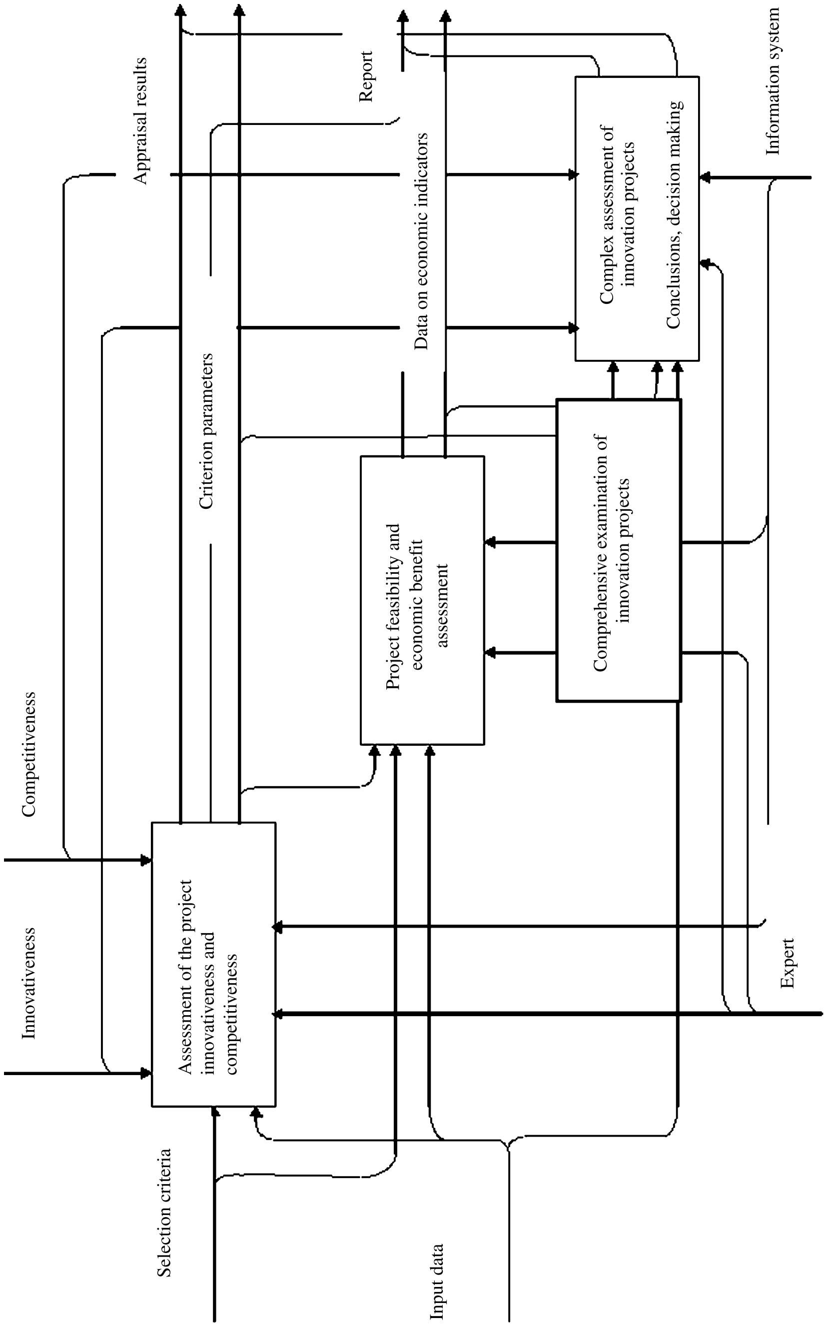


Fig. 5.20 Functional model of DSS software for innovation project examination

Step 3—activation of the innovativeness and competitiveness assessment procedure;
Step 4—activation of the procedure to assess project feasibility and cost-effectiveness;
Step 5—activation of the procedure to conduct the comprehensive examination of the project;
Step 6—output of plots, reports, and recommendations on the project.

The flowchart is given in Fig. 5.21.

This flowchart shows how the entire process operates. First, the program is started, and then a connection with the database is established in order to retrieve the data. Second, registration must be carried out of the user, who reviews the project. Thereafter, a package of required documents is registered.

Project examination procedure is designed to review the proposed project using the innovativeness and competitiveness criteria. Then a graphic model of the project is built to determine project feasibility. If the project is accepted according the two parameters, the feasibility and economic benefit assessment procedure is activated. Thereafter, the parameters are calculated that are needed for building the graphic model of project feasibility and cost-effectiveness assessment. Third, conclusions are made and plots and reports are printed out.

5.4.3 Development of Information Model of Innovation Project Evaluation

Information support (IS) is an aggregate of data, language means of describing data, software for database processing, and also methods of their organization, storage, accumulation, and access thereto, providing the issue of all necessary information in the process of solving functional tasks.

The information model shown in Fig. 5.22 gives a general description of the innovation project assessment process and does not reveal the nature of each information process.

Having analyzed the initial information of an application domain with the purpose of determining the contents and structure of information in order to formalize and develop an information and logical model of data, we defined the attribute content of input and output data. The following infological model has been chosen (Fig. 5.23).

At any time, all data on projects, experts, evaluations, and references from the Criteria Directory are to be available for a user of the subsystem of assessing innovation projects' effectiveness, for use in subsequent calculations.

The user (expert) can carry out the following operations:

– Register a received application for project assessment in the log;

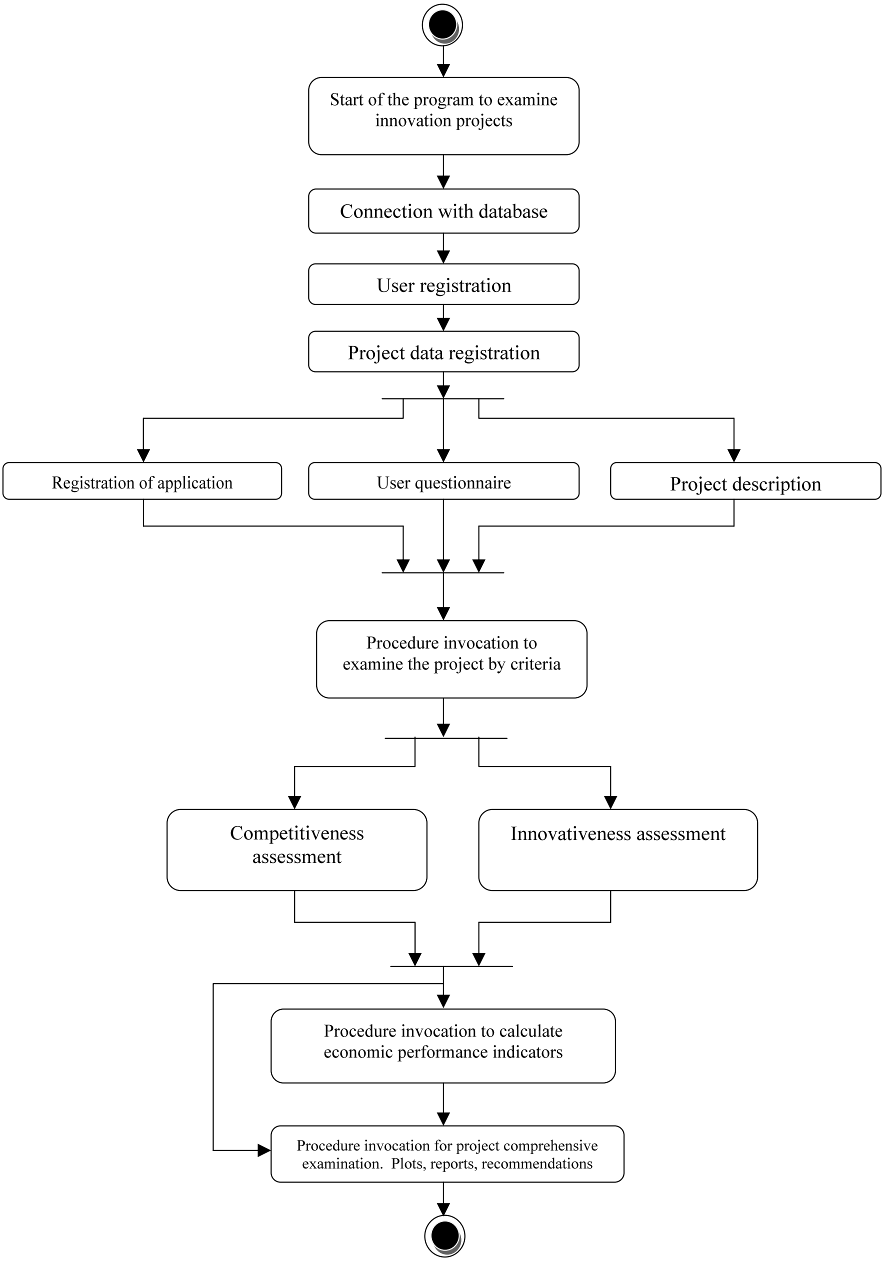


Fig. 5.21 Operation flow chart of a project appraisal subsystem

– Evaluate the project by scoring as per the criteria of the project's innovativeness and competitiveness;

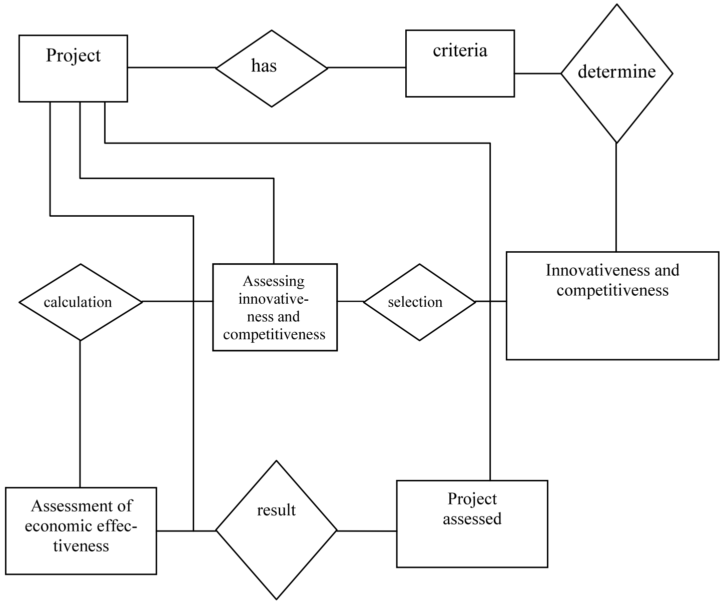


Fig. 5.22 Information model of innovation project evaluation

– Evaluate the project's economic effectiveness;
– Conduct the integrated assessment of the project;
– Edit data;
– Provide a result (recommendation).

A fragment of the database physical model is presented in Fig. 5.24.

PROECT (Project)—this entity is assigned for storing names of projects. Since the name of the project may be quite large (and it will be encountered frequently), then it is expedient to number the projects and refer to these numbers.

KRITERI (Criterion)—is determined by the project's innovativeness and competitiveness.

EXPERTIZA (Expert examination)—is the issue of a result, the number of the result.

RACHET (Calculation)—is related to the project under evaluation.

OCENKI (Evaluations of the project)—the main essence of the project evaluation, including all other attributes of other entities.

The goal is to evaluate the project according to the criteria of the project's innovativeness and competitiveness developed in the rule database, to determine promising projects, projects that need to be improved, and projects that are not worth continuing. The entire process of assessment will be represented in a graphic model of the project's innovativeness and competitiveness (Fig. 5.25).

Let four variables be introduced: I—the innovativeness; K—the competitiveness; S—the result; O—the operation.

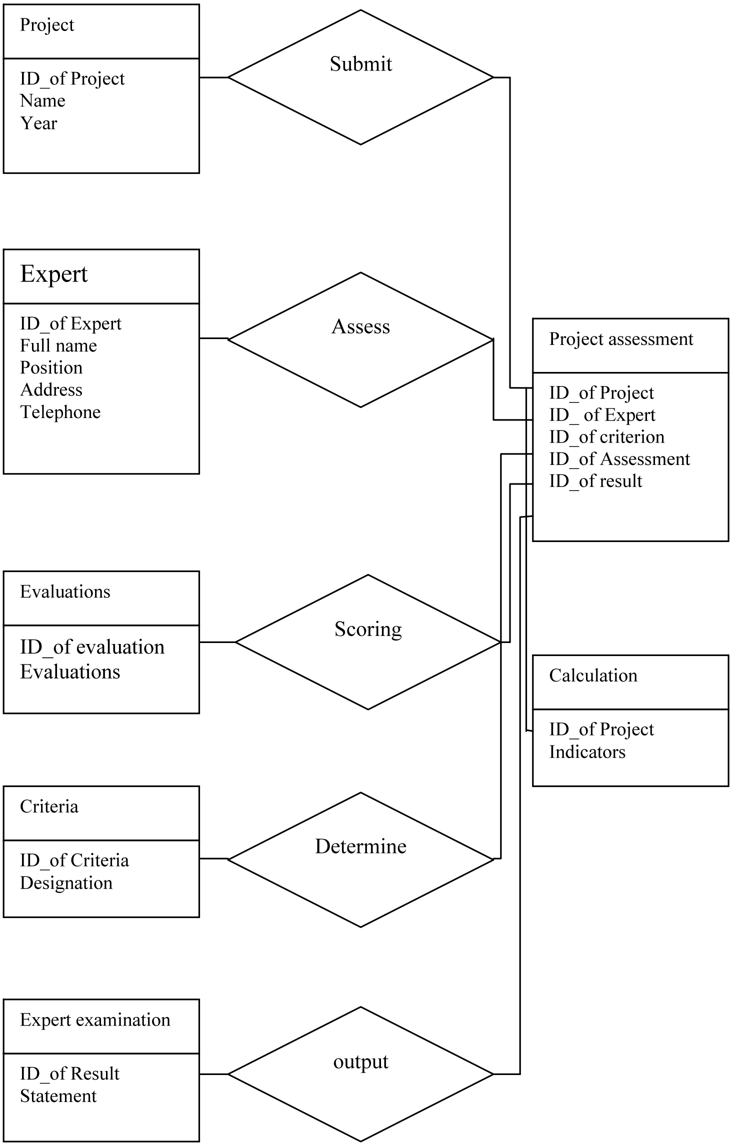


Fig. 5.23 Infological model IS

A number of conditions for innovativeness and competitiveness are also introduced: R—for innovativeness and L—for competitiveness, as provided in Table 5.14.

O—the operation demonstrating the conditions of the logical type I; when the criteria and indicators are met, this condition is fulfilled.

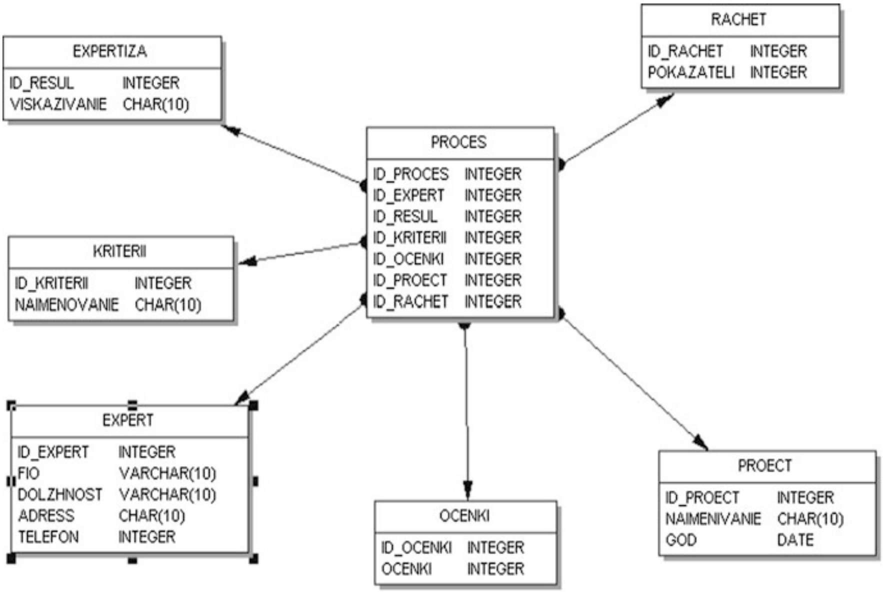


Fig. 5.24 Fragment of database physical model

Fig. 5.25 Graphic model assessing the project's innovativeness and competitiveness

K- Competitiveness

competitive	Leader 2	Leader 1
Outsider 1	neutral	Leader 3
Outsider 3	Outsider 2	attractive

I - Innovativeness

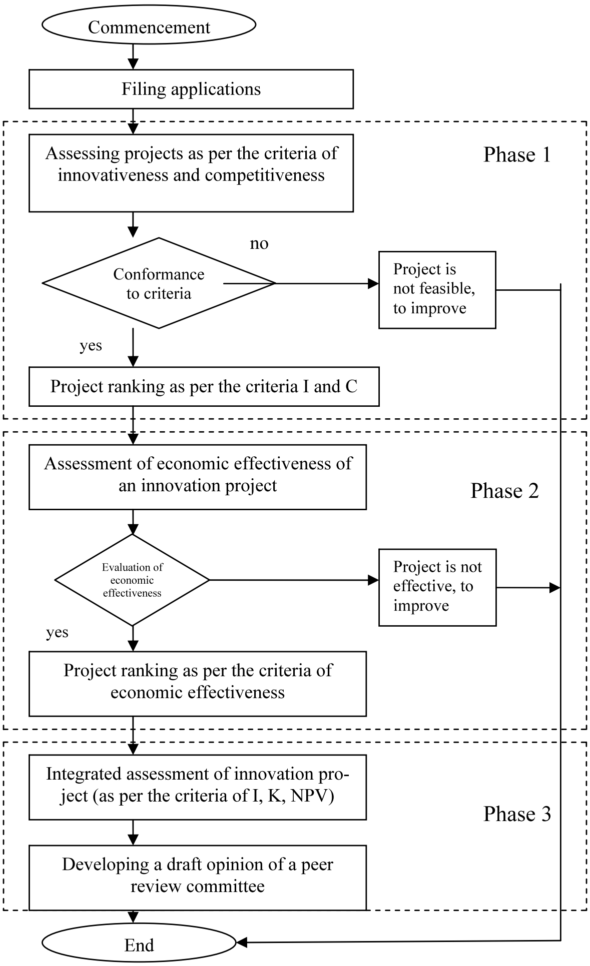
Production models are applied mainly as problem solvers or output mechanisms. Operations of production may consist of active procedures that perform necessary database operations automatically. The scenario of innovation project assessment consists of three phases (Fig. 5.26).

In the first phase, from among applications innovation projects are selected according to criteria of innovativeness and competitiveness. In this phase projects are evaluated by expert examination. Those projects that undergo the initial selection successfully pass on to the second phase of evaluation. In the second phase the economic effectiveness of innovation projects is assessed. Projects conforming to indicators of economic effectiveness pass on to a final phase of assessment. The

Table 5.14 Rule database

No.	*I*	*K*	*O*	*R*	*L*	*S*
1	A	1	1	$0 \leq R < 3$	$0 \leq L < 3$	Outsider 1
2	A	2	1	$0 \leq R < 3$	$3 < L < 6$	Outsider 3
3	A	3	1	$0 \leq R < 3$	$6 < L \leq 9$	Competitive
4	B	1	1	$3 < R < 6$	$0 \leq L < 3$	Outsider 2
5	B	2	1	$3 < R < 6$	$3 < L < 6$	Neutral
6	B	3	1	$3 < R < 6$	$6 < L \leq 9$	Leader 2
7	C	1	1	$6 < R \leq 9$	$0 \leq L < 3$	Attractive
8	C	2	1	$6 < R \leq 9$	$3 < L < 6$	Leader 3
9	C	3	1	$6 < R \leq 9$	$6 < L \leq 9$	Leader 1

Fig. 5.26 Algorithm of assessing innovation projects



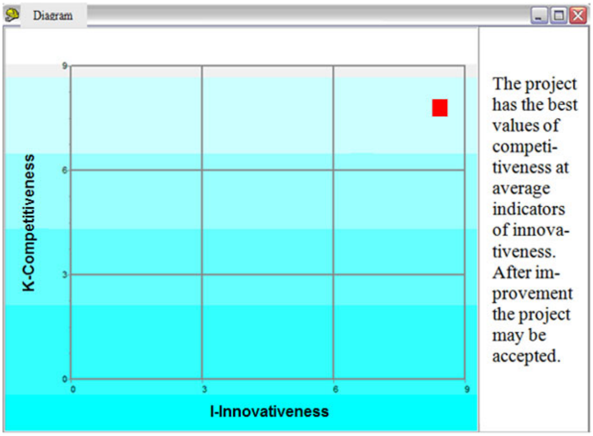


Fig. 5.27 An example of software implementation of a graphic model for assessing innovativeness and competitiveness (Project #5)

third phase of evaluation is an integrated assessment of the project with due account for innovativeness, competitiveness, and economic effectiveness.

Completing the procedure of the assessment by the expert commission results in developing a draft decision.

The decision support system (DSS) of assessing innovation projects is a Windows application with the DBMS Paradox used as a database platform.

Information processing in a dialogue mode is the most effective. The dialogue mode allows referring from a workstation to any information stored in the computer memory. These tools are practically implemented in the DSS applied for assessing effectiveness of innovation projects.

Let's review the available software support using the example of the innovation project.

The first stage of software implementation shows a graphic model for assessing innovativeness and competitiveness. Figure 5.27 shows the results of expert examination for Project #5 as per the first method of assessing the innovativeness and competitiveness.

The second phase is the evaluation of Project #5: *Evaluation of the project's feasibility and economic effectiveness*. This stage involves preparing an estimate and calculating economic indicators. It is quite straightforward, consisting of a number of discrete operations. Figure 5.28 shows the results of the second phase.

Therefore, the project has quite high indicators of effectiveness, and may be accepted for implementation.

The third phase of assessing the project is an integrated assessment of the innovation project. A graphic representation of this method using the example of Project #5 is given in Fig. 5.29.

In this example, Project #5 yields high values for indicators, and can be accepted for implementation. To recap, this method is effective when comparing alternative innovative projects. The lengths of the vectors must be compared for alternative projects, and preference is given to the project with the longest vectors.

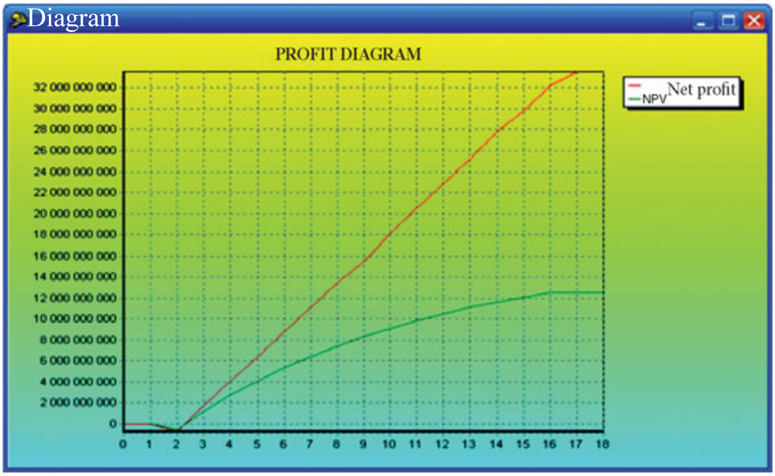


Fig. 5.28 An example of the graphic model software implementation to assess feasibility and economic effectiveness of an innovation project (using Project #5 as an example)

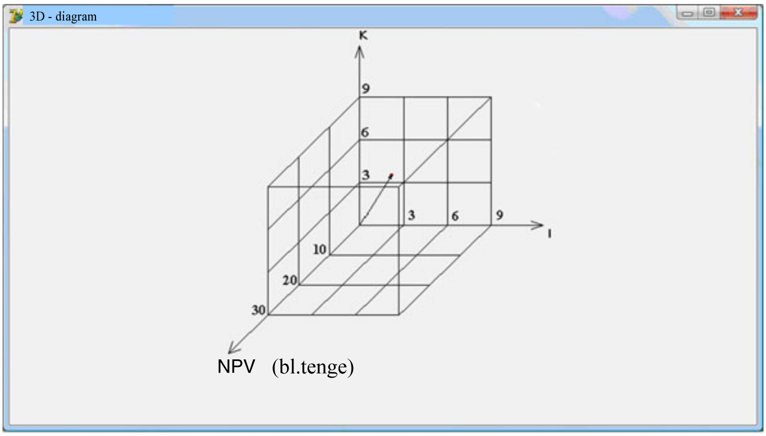


Fig. 5.29 An example of the graphic model software implementation for an integrated assessment of innovation projects (using Project #1 as an example)

Thus, the algorithm given here for assessing innovation projects makes it possible to improve existing procedures of project analysis, both for commercial and strategically specified objectives.

References

1. Mutanov, G., Abdykerova, G.: Information System for Innovation Project Appraisal. VKGTU, Ust-Kamenogorsk (2010), p. 136

2. Kingston, W.: The Political Economy of Innovation. The Hague (1984), p. 245
3. Christensen, C.: The Innovator's Dilemma. Harvard Business School Press, Cambridge (1997), p. 366
4. Evdakimova, L.O., Slesareva, L.S.: Innovation Management, 2nd edn. Adreyev Publishing House (2007), p. 168
5. Kvashnin, A.: As a review of technology commercialization. EuropeAid project "Science and Technology Commercialization" (2006), p. 51
6. Methodological Support Technology Commercialization Centers. Practical Guidelines. Part 1. CSDS RAS (2006), 392. Edited by: O. Lukshi, P. Sushkova, A. Yanovskogo
7. Goldstein, G.: Strategic Innovation Management. TRTU, Taganrog (2004), p. 150
8. Krainev, P.P., Druchok, G.V., Gavrilenko, A.P., Klâvlin, V.V.: Estimation of Innovative Project Based on Prediction of Technical and Economic Indices of Innovation. Finance (2006), p. 504
9. Zagrebelniy, S., Kvashnin, A., Sharonov, I.: Features Innovative Business Planning. SORAN, Novosibirsk (2003), p. 137
10. Porter, M.: Competitive Advantage: How to Achieve High Results and Ensure Its Sustainability. Economics (2005), p. 3058
11. Mutanov, G.M., Esengalieva, Z.S.: Method for assessing innovativeness and competitiveness of innovative projects. Fundam. Issled. **3**(3), 712–717 (2012)
12. Peregozhina, O.N.: Problems of competitiveness of knowledge-intensive industries in modern conditions. High Technol. **1**, 20–27 (2007)
13. Porter, M.: International Competition. Mezhdynarodnie Otnosheniya (1993), p. 604
14. Harina, I.L., Kolenskogo, I.L.: Innovation Management. High School (2003), p. 305
15. Kormnov, Y.: Competitive environment assessment. Econ. Anal., Theory Pract. **2**, 57–63 (2008)
16. Zaviyalov, P., Zaviyalov, P.S.: Competitiveness and marketing. Russ. Econ. J. **12**, 14–18 (1995)
17. Mutanov, G., Yessengaliyeva, Zh.: Development of method and models for assessing innovativeness and competitiveness of scientific-innovative projects. World Appl. Sci. J. **23**(9), 1192–1200 (2013)
18. Beshelev, S.D., Gurvich, F.G.: Mathematical-Statistical Methods of Peer Review. Statistics (1980), p. 263
19. Mutanov, G.: Risk Management in the Underground and Mining Enterprises. Gylym, Almaty (1996), p. 291
20. Kantorovich, L.V.: System Analysis and Some Problems of Scientific and Technical Progress. Dialectic and System Analysis. Nauka, Moscow (1986), p. 215
21. Knudsen, M.P.: The relative importance of interfirm relationships and knowledge transfer for new product development success. J. Prod. Innov. Manag. **24**(2), 117–138 (2007)
22. Abdykerova, G.Zh., Bukayeva, A.D.: An analysis and evaluation of the effectiveness of decision making during the review of scientifically constructed project proposals. Asia-Pac. J. Bus. Ventur. **3**(2), 31–44 (2008)
23. Yessengaliyeva, Zh.S.: Basic implementation life cycles innovative research projects. In: Actual Problems of Modern Science: Intern. Scientific-Practical. Conf., vol. 12, pp. 3–6. Nauka I studia, Przemysl (2012)
24. Bashin, M.L.: Planning Scientific Studies and R&D Work. Economika (1990), p. 250
25. Economic Evaluation of Investment. Comp.: AV Zavodina; NovSU them. Yaroslav Wise. Veliky Novgorod (2000), 105
26. Tychinski, A.V.: Innovation Management Companies: Modern Approaches, Algorithms, Experience. UNIVERSITY, Taganrog (2006), p. 507
27. Carlos, E.C., Amaral, D.C.: Evaluating an agile method for planning and controlling innovative projects. Proj. Manag. J. **41**(2), 73–80 (2010)
28. Abdykerova, G.Zh.: The basic steps to create innovative projects. Bull. East Reg., Ust-Kamenogorsk **2**, 66–73 (2009)
29. Bogatin, Y.V.: Investment Analysis: Textbook. UNITI (1998), p. 423

30. Mutanov, G., Yessengaliyeva, Z.: Development of methods and models for assessing feasibility and cost-effectiveness of innovative projects. In: The 6th International Conference on Soft Computing and Intelligent Systems. The 13th International Symposium on Advanced Intelligent Systems, 20–24 November 2012, pp. 2354–2357 (2012)
31. Yesipova, V.Y. (ed.): Mortgage and Investment Analysis, p. 207
32. Berens, V., Khavranek, P.: Investment Project Appraisal Handbook. Infra (1995), p. 204
33. Mutanov, G., Yessengaliyeva, Zh.: Assessment of scientific and innovative projects: mathematical methods and models for management decisions. In: The 14th International Symposium on Advanced Intelligent Systems (ISIS2013). Advances in Intelligent Systems and Computing, vol. 271, pp. 19–28. Springer, Berlin (2014)
34. Marca, D.A., McGowan, C.: Structured analysis & design technique (SADT). http://www.interface.ru/case/sadt0.htm

Chapter 6
Mathematical Methods for Making Investment Decisions

Structural reorganization of the national economy using various methods including *investment* policy must actively promote a range of kinds of innovation, from technologies and inventions to fresh organization of relations between services, suppliers, and consumers [1]. One of the ways to smooth out the cyclic character of crises in economic processes is to work out long-term complex scenarios and accordingly to develop multiple branches of the national economy including revival of investment activities. Prioritization of investment in enterprises must be based on each enterprise's capability to provide sufficient financial flows, both for "bootstrapping" and for subsequent reaching the trajectory of sustainable development.

In order to estimate and choose among multidimensional alternatives of investment projects to be financed, it is necessary to solve a multi-criterion optimizing problem that seeks to arrive at a compromise between the preferences of potential investors. The ever-present element of uncertainty drives the calculation of reasonable quantitative estimations of investment risk. Risk management of IP-innovations in social and economic spheres is co-directed with the increase in the level of uncertainty: The problem is of course not new but is becoming increasingly significant and hence requires thorough study both in depth and in breadth [1]. The pressing problems of economic science necessitate objective modification and adaptation of previously recognized methods, and development of new modeling tools. The point is not to design the "newest" formal indicators and models just for the sake of newness itself, but to adapt the existing methods to the description of new phenomena, effects, and tendencies, both in response to and to create a deeper understanding of social and economic processes and their evolution.

Present-day financial management is characterized by more active implementation of investment projects, requiring forecasting of not only time structure of payments but also their exact sums, and probabilities of possible deviations from the expected results, that is to say, estimation of the degree of financial operation risk.

Application of *computer* (hardware and software tools for information processing, storage, transfer, and visualization) and *measuring* technologies supporting mathematical modeling expands the possibilities of practical usage of mathematical apparatus. Here are two examples of how this works.

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_6

- The repeated reproduction of a random process by means of computer technology allows replacement of *experimental measuring of values* of real characteristics of random processes, with *studying values* of these characteristics. Technological capacities of the system provide a continuous cyclic process of operation and situational correction of the simulation model. Thus, positive results of simulation modeling are virtually an *expert decision* about the absence of contradictions between the modeled solution and the values of corresponding characteristics obtained in a different way (for example, by retrospective analysis). Hence, they can be used instead of formal *proof* by mathematical methods.
- Application of fuzzy-set theory to the description of economic processes (synthesis of quantitative and expert estimations) is an effective means in the absence of reasonable probabilistic distributions of characteristics.

The problems of planning, estimation, and decision-making [2–5] are all in one way or another connected with the forecasting problem. The formalized statement of the forecasting problem suggested in this research assumes the use of expert estimations. The solution to the problem of expertise in decision-making is to find a compromise between human capabilities (instantaneous decision-making) and capabilities of computer technology (accounting and supervision of a huge number of variables, limitations, and cause–effect relations). Despite the criticism of the objective–subjective ratio in expert estimations, it is reasonable to provide the *possibility* to the decision-makers (experts, specialists) *to make decisions* in the margins of their possibilities and competence. The presence of a good "at-hand" model will expand both possibilities and competence, but will not ultimately prevent the dynamics of dinosaur-like behavior, a reflection of those species that "preferred" to die out rather than to adapt to change [6].

6.1 Basic Concepts of the Risk Theory of an Investment Project

Investment policy is a system of goals and problems—as well as a mechanism of their realization—aimed at control of political, economic, and social development of the corresponding state elements.

As *investments* we will understand all kinds of material and intellectual values invested in the objects of business activities which result in gaining profit or attaining social effect.

An *investment project* (IP) is a basic form of attraction (investment) of capital in order to get an effect in the future.

IP assumes *planning* of three basic cash flows in time:

– flow of investments;
– flow of current (operational) payments;
– flow of receipts.

In terms of mathematics the IP is described by the flows of payments—functions of time whose values are:

– costs (values of functions where payments are negative);
– receipts (values of functions where payments are positive).

Using one of the generally recognized indicators of IP efficiency, NPV, as the IP "model" and *without any limitation in the generality of presentation*, we will understand [1, 8]:

$$NPV = -I_0 + \sum_{k=1}^{K} \frac{A_k}{\prod_{i=1}^{k}(1+r_i)} - \sum_{k=1}^{K} \frac{B_k}{\prod_{i=1}^{k}(1+r_i)}$$
$$= -I_0 + \sum_{k=1}^{K} \frac{A_k - B_k}{\prod_{i=1}^{k}(1+r_i)} \quad (6.1)$$

where I_0 are initial investments; A_k, B_k are incomes and expenses in the k-th period; r_i is the rate of discount in the i-th period; $k = \overline{1, K}$ is the number of periods of IP realization; NPV is the net present IP value.

In theoretical calculations $A_k - B_k$ can be represented as a relatively smooth function of time, which leads to the following model:

$$NPV = -I_0 + \int_o^T f(\tau)e^{-R(0,\tau)}d\tau, \quad (6.2)$$

where $f(\tau)$ is the cash flow density;

$$R(t_1, t_2) = \int_{t_1}^{t_2} r(\tau)d\tau,$$

T is the time of IP realization.

Taking into account the fact that in practice time periods are equal, the rate of discounting is assumed to be constant during all time of IP realization. Taking I_0 to be the initial costs of the project, i.e., $I_0 = CF_0 = A_0|_{A_0=0} - B_0$, the model (6.1) is transformed into the following:

$$NPV = \sum_{k=0}^{K} \frac{CF_k}{(1+r)^k}, \quad (6.3)$$

where CF_k is a prognostic value of the net cash flow in the k-th period; r is the rate of discount.

Assuming that CF_k is a relatively smooth time function and the rate of discount is constant, the theoretical model can be written as:

$$NPV = -I_0 + \int_o^T f(\tau)e^{-\rho\tau}d\tau, \quad (6.4)$$

where $\rho = \ln(1+r)$, as discounting in the infinitesimal time interval gives:

$$\frac{CF_k}{(1+r)^k} \to \frac{f(\tau)}{(1+r)^\tau} \to \lim_{t\to\infty} \frac{f(\tau)}{(1+\rho/t)^{\tau t}} \overset{\substack{\text{remarkable}\\ \text{limit}}}{=} \frac{f(\tau)}{e^{\rho\tau}} = f(\tau)e^{-\rho\tau}$$

Flows of current payments and receipts $f(\tau)$ or CF cannot be planned entirely precisely because of unavoidable uncertainty of information (internal and external[1] instability with respect to IP). Hence, there is an unavoidable risk in investment decisions. That is to say, there is always a possibility that:

– a project recognized as well-grounded will actually turn out to be unprofitable;
– the obtained parameter values will considerably deviate from the planned values;
– some factors will not be considered at all.

We will understand as:

– *threat*, an event which can occur during IP realization, as a result of which the NPV value will deviate to one side or the other;
– *investment losses*, the potential, numerically measurable possibility[2] of less net economic profit or of suffering damages during project realization as a result of threat impact;
– *investment risk*, investment *losses* which *do not satisfy* participants of the project;
– *IP risk level*, the probability that the risk indicator (criterion of IP efficiency) will fall out of the zone of project stability;[3]
– *risk assessment*, the detection and analysis of risk sources, their genesis and determination of risk scale in a concrete situation;
– *risk management*, analysis of the situation, development, and realization recommendations or actions directed at risk reduction to the optimal (acceptable) level.

The system approach to investment policy (Fig. 6.1) also defines the classification of risks.

The interrelation between investments and risks is expressed in the fact that in order to increase return on the capital and/or to minimize possible losses, most resource proprietors have alternative variants of investments (countries, branches, territories).

The methodology of estimating IP risks is based on the assumption that large deviations of economic indicators from possible optimal values are more expensive than are small ones.

Regardless of investment type, volume, and capital structure the procedure of taking an investment decision can be presented as a schematic (Fig. 6.2).

Every stage of the investment decision is connected with a specific set of potential risks and methods for estimating them. Risks are influenced by so many factors

[1] Instability in the IP external environment, as a rule, means fluctuations $r_i(\rho)$.

[2] Remark: Not obligatory "probability."

[3] The boundaries of the stability zone are defined on the basis of the accepted requirements for investment efficiency.

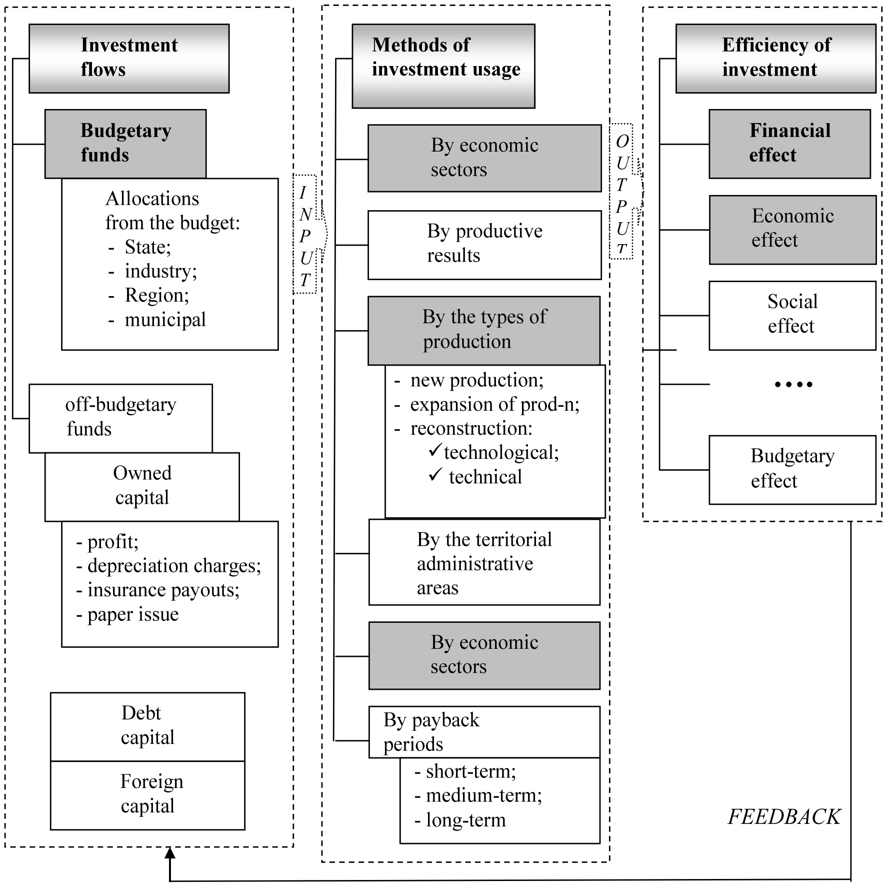


Fig. 6.1 Formation of investment policy

that it is almost impossible to obtain a functional dependence $F(factor) = risk$ or $F(result) = risk$.

Incomplete knowledge of characteristics to be used in modeling, including their values and interrelations, which is typical of the problem of adequate estimation of IP risk, is quite correctly taken into account in the following statement of the problem:

– variables $x_1, \ldots, x_n$ with the corresponding ranges of their definitions $x_1, \ldots, x_n$ are specified;
– limits representing a disordered set of relations or connections between variables $f_i(x_1, \ldots, x_n)$, $i = \overline{1;k}$ expressed in the form of equations, inequalities, logic expressions, etc. are given;
– it is required to find all sets of values $\langle a_1, \ldots, a_n \rangle$, $a_1 \in X_1$ which satisfy all limitations simultaneously.

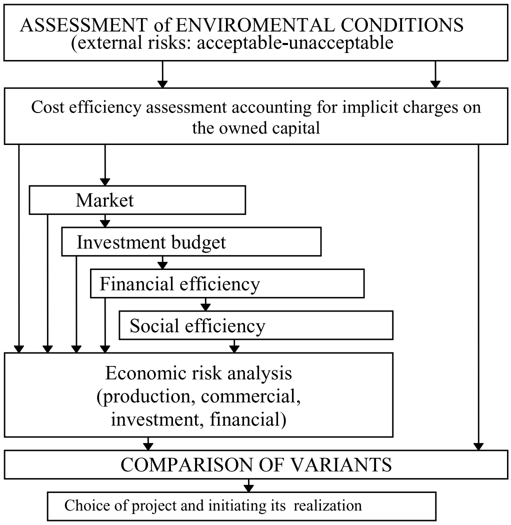


Fig. 6.2 A sequence of steps in making an investment decision

Restrictions on the choice of the best investment variant are determined by the volumes of financial resources and possibilities for using them.

The *direct* investment task is to forecast the effect of investments depending on the economic situation (environment) creating a certain[4] risk level.

The *inverse* task is to determine the risk degree of the investment activity providing the desired effect. Unlike the direct task, the inverse task has no single-valued solution because the same risk level can be created by different causes.

This multi-valuedness of solution leads to the idea of uncertainty, i.e., expansion of the concept "indicator value" from "a single numerical value" to a "set of possible values."

Therefore the area of solutions to investment problems becomes wider due to potential consideration of different types of relations: ill-defined, exactly defined, and over-defined. As applied to IP, the expert/analyst defines [1, 9]:

- which characteristics are set precisely;
- which characteristics are absolutely unknown;
- which are known approximately, i.e., the initial information is given as the description of a set of possible values (indefinite, piecewise, or probabilistic).

It is expedient to consider as risk (threat) factors not all *a priori* risky factors but only "sensitive" factors, that is, factors which actually influence cash flows of the studied IP.

The unknown or imprecisely known data are defined more exactly using the method of satisfaction of restrictions, many of which have available program variants.

[4]The decision can be predicted with satisfactory accuracy on the assumption of sufficient stability, inertia, etc.

The identification of sensitive indicators, analysis of their stability, and forecasting of their possible values are preparatory steps to the quantitative estimation of project risks.

We will call factors sensitive if their change by 1 % gives worsening of risk indicators by more than 1 %. For the function of risk indicator $R(x)$ of factors $x = (x_1, \dots, x_k)$, the absolute and relative sensitivities to the i-th characteristic are calculated as:

$$R'(x) = \partial R(x)/\partial x_i \quad \text{and} \quad R'(x) = \frac{\partial R(x)}{\partial x_i} \cdot \frac{x_i}{R(x)}. \tag{6.5}$$

In practical investment analysis, calculations are often simplified by discretization: the response of $R(x)$ (%) values to the change in the characteristic level x_1 by 1 % is determined:

$$\tilde{x}_i \mid \%\Delta\tilde{x}_i = 1 \Rightarrow \%\Delta R > 1,$$

where $\%\Delta z = \frac{z_2 - z_1}{z_1}$ or $\%\Delta z = \frac{2\cdot(z_2 - z_1)}{z_2 + z_1}$.

The problem of risk management can be solved based on the idea of stability estimation: determination of the limiting levels of the most sensitive characteristics of the project where the basic level of design risks or random losses is preserved within the limits of established "risk-capital":

$$\lfloor \underline{\tilde{x}_i}; \overline{\tilde{x}_i} \rfloor | R(\tilde{x}) \approx R_{basis}. \tag{6.6}$$

Indeed, the determined upper limits of "negative" deviations of projected values solve the problem of possibility of the project "falling" to critical and catastrophic risk zones.

The individual values of risk indicators of concrete projects are defined by predicted levels of their characteristics—financial flows CF or $f(\tau)$ (prices, fixed charges and variable costs, volumes of demand, incomes, etc.), future competitiveness, etc. We formalize the forecasting process as estimation of the state of characteristic x in a certain period of time with preservation of existing tendencies.

Let evolution forecasting x be presented as a sequence of results of observations (6.7), sets of consistent expert estimations in the form of statements of the type (6.8), and let a class of models with linear dependences on parameters (6.9) be chosen:

$$x_t, \quad t = \overline{1; T}, \tag{6.7}$$

$$\alpha^k + \beta^k x_\tau^k \geq \mu^k + \eta^k x_\nu^k, \quad k = \overline{1; K}, \tag{6.8}$$

where τ and ν are the moments of the period of forestalling, α, β, μ, η are estimations given by experts

$$F(t, \theta) = \big(\theta, \varphi(t)\big), \quad t = \overline{1; T}, \tag{6.9}$$

where $\theta = (\theta_1, \dots, \theta_m, \dots, \theta_M)$ is a vector of parameters; $\varphi = (\varphi_1, \dots, \varphi_m, \dots, \varphi_M)$, $\varphi_m(t)$ are known vector time functions; $(\theta, \varphi(t)) = \sum_{m=1}^{M} \theta_m \varphi_m(t)$ is a scalar product.

As the period of observations of investment activities in Kazakhstan may be insufficient for drawing reliable statistical conclusions, and/or the evolution of risk characteristics may deviate from the stationary mode, the task is to construct a forecast sequence most consistent with expert estimations (6.8) and the results of observations (6.9):

$$\widehat{x}_t = \left(\widehat{\theta}, \varphi(t)\right), \quad t = \overline{T+1; T+T_1}, \tag{6.10}$$

where the vector giving estimation of model parameters of the trend $\widehat{\theta} = (\widehat{\theta}_1, \ldots, \widehat{\theta}_m, \ldots, \widehat{\theta}_M)$ is a solution to the problem of minimization

$$\min_{\theta_m} D(\theta) = \sum_{t=1}^{T} d\left(x_t - \left(\widehat{\theta}, \varphi(t)\right)\right) \tag{6.11}$$

with limits (6.8). Here $d(\cdot)$ is a metrics, for example, a module or the square of a real number.

The confidence intervals of the forecast transform the pointed extrapolation forecast (6.10) into the interval forecast:

$$\widehat{x} \pm t_\alpha \cdot s_x, \tag{6.12}$$

where t_α is the value of the Student t-criterion at the significance level α; $s_x = \sqrt{\sum_{t=1}^{T}(x_t - \tilde{x}_t)^2/(T-M)}$ is the mean square deviation of actual observations from the results of calculations; $(T - M)$ is the number of degrees of freedom, M is the number of estimated parameters.

If the expert estimations are given, the forecast problem has a single solution, but often expert estimations depend on the experts themselves—their specialization, experience, etc.

The measurement techniques and the method of forecasting primary characteristics predetermine the method of assessment of the range of negative variances of sensitive characteristics.

In making investment decisions it is difficult to choose the final project variant because the stages of its substantiation differ from each other [1–3]:

- by the types of risk;
- by the reasons causing risk;
- by the estimation procedure.

6.2 Investment Decisions: Project Choice and Risk Management

6.2.1 Methods Supporting Decision-Making

In the traditional approach the system supporting decision-making operates according to the following notions:

- *alternative*—a variant of the problem solution;
- *a set of alternatives*—a set of variants of solutions to the given problem;
- *limits*—a system of fixed values of alternative parameters beyond which the solution becomes invalid;
- *a function of alternative utility* defines transition from the space of alternatives to the other space: brings each variant in correspondence with a set of its estimations, a profit or loss which is a result of choosing this alternative;
- *the criterion of alternative estimation* makes it possible to bring the utility function in correspondence with a certain property of the alternative but not the entire alternative;
- *the criterion of optimal decision* (hereinafter—in the sense of the best correspondence) defines the method the system uses to find the best alternative from the set.

The main task to be solved to make a single decision is:

- either to choose the best alternative for achievement of the given purpose;
- or to range the set of possible alternatives by the degree of their influence on achievement of the purpose.

The multicriterion approach [10] essentially changes the type of the solved problem: The basis for decision making is preferences. The system of preferences is a set of formalizable and non-formalizable methods including the elements of intellectual analysis made by an expert (a decision-maker). To estimate preferences one must define the method of their comparison and identify connections between them. The utility function makes it possible to unify different scales of measurements and different units used to measure preferences. It is difficult[5] to obtain mathematical description of the utility function for complicated, poorly formalizable problems.

6.2.2 Methods Used to Assign the Utility Function Values

One of the main tasks of experts is to prove and anticipate on the basis of information-intuitive assessments events that cannot be calculated statistically and predicted by economic or mathematical models [1–19].

1. *A method of direct estimation.*

Each alternative is assigned a value which, in terms of the specific estimation criterion, shows the degree of correspondence of the given variant of problem solution to the achievement of the IP primary aim. This estimation can be a numerical value on the number scale, a value of the linguistic variable, or the rank of the alternative among other alternatives.

[5]This means not only computing difficulties (modern information technologies smooth over this problem), but also psychological difficulties related to the level of experts' professionalism.

2. *A method of paired comparison.*

The idea of the alternative approach is that experts do not have to specify the weights with which individual quality indicators are included in the total generalized indicator, which enables us to compare objects or projects as a whole, but makes it difficult to determine contribution of individual factors.

Therefore all elements of the set of alternatives are subjected to paired comparison; the preference relational matrix is constructed, with its elements showing how many times the contribution of one alternative to the achievement of the main goal is higher than the contribution of the other alternative. The significance (weight) coefficients are calculated.

– *The operative expert estimation* of alternatives (criteria) can be used by paired comparison with gradation in a three-point scale.

This makes it possible to obtain expertise without worrying about infringement of judgment transitivity using both individual and group opinions, and without preliminary preparation of the group of experts.

To come up with an estimate, every j-th expert ($j = \overline{1; J}$), comparing in pairs a_i ($i = \overline{1; I}$) criteria, forms a preferences matrix where estimations "$\prec$—less important," "$\succ$—more important," and "$\approx$—equivalent, indistinguishable" correspond to the formal points:

$$b_{ik}^{j} = \begin{cases} 0, & \text{when } a_i \prec a_k \\ 1/2, & \text{when } a_i \approx a_k \\ 1, & \text{when } a_i \succ a_k. \end{cases}$$

After summation of all matrices and normalization by the number of experts we get a matrix of normalized estimations $W = (w_{ik}) = (1/J \cdot \sum_{j=\overline{1;J}} b_{ik}^{j})$, where $w_{ik} + w_{ki} = 1$. The zero-order weight approximation is used for reasons of "initial equivalence":

$$g^{(0)} = \left(g_1^{(0)}, \ldots, g_i^{(0)}, \ldots, g_I^{(0)}\right), \quad \text{where } g_k^{(0)} = 1/I.$$

The iterative calculation of the relative importance factors in the s-th iteration continues until the predetermined accuracy is achieved or the predetermined number of iterations is made:

$$g_i^{(s+1)} = \frac{\sum_{k=\overline{1;I}} w_{ik} \cdot g_k^{(s)}}{\sum_{m=\overline{1;I}} \sum_{k=\overline{1;I}} w_{mk} \cdot g_k^{(s)}}, \quad i = \overline{1; I}$$

If $|g_i^{(s+1)} - g_i^{(s)}| < \varepsilon$, where ε is a given accuracy for any $i = \overline{1; I}$, the weight vector (collective preference) is $g = (g_1^{(s+1)}, \ldots, g_i^{(s+1)}, \ldots, g_I^{(s+1)})$.

– *The Saati method of priority determination* (*a nine-grade scale of relations*) is suggested for quantitative estimation of "more inert" alternatives.

The initial data used to determine priorities is the inversely symmetric matrix $B = (b_{ik})$ $(i, k = \overline{1; I})$ formed according to the degree of preference of alternative (criterion) a_i over alternative a_k. The elements b_{ik} are assigned 1 in case of equal importance of criteria, 3 in case of a slight superiority of a_i over a_k, 5 in case of moderate superiority, 7 in case of considerable superiority, and 9 where there is absolute domination of a_i over a_k. The degrees of significance 2, 4, 6, and 8 are interpreted as intermediate placements. In this case:

- natural performance $b_{ii} = 1$;
- the requirement of local coordination follows from the "automatic" condition $b_{ki} = 1/b_{ik}$ $(0.5 \cdot I \cdot (I - 1)$ comparisons are needed).

Absolute coordination can be provided by "automatic" calculation of preference $b_{ki} = b_{is} \cdot b_{sk}, i, k, s = \overline{1; I}$. It should be noted that the above condition means linear dependence of lines and columns, therefore, after making $(I - 1)$ comparisons of paired criteria the others can be defined from already made conclusions. However, in this case the estimation loses its "purity," and the expert loses an opportunity to check sincerity and confidence of his viewpoint. The matrix of preferences reflects human estimations, therefore it is difficult to expect an absolute faultlessness in the presence of a rather wide spectrum of shades of preference. But if the degree of contradictions is inadmissible—that is, if the index of coordination of estimations b_{ik} exceeds the admissible level $CR > 0.1$—it is recommended to reconsider the estimations as the logic of the expert opinion is violated.

The normalized vector of priorities (scales) and the coordination index are determined by the eigenvalue method:

- solving the equation $\det(B - \lambda \cdot E) = 0$, where E is a unity matrix.
- determining the maximal eigenvalue $\lambda_{\max}$ as follows:

$$\lambda_{\max} = \max_{i=\overline{1;I}} \lambda_i \quad \text{where } \forall i \ \lambda_i \mid \det(B - \lambda_i \cdot E) = 0$$

- determining the eigenvector g as the system solution:

$$\begin{cases} (B - \lambda_{\max} \cdot E) \cdot g = 0 \\ \sum_{i=1}^{I} g_i = 1, \end{cases} \tag{6.13}$$

  where the second equation (the normalizing condition) is used instead of any equation of the system $(B(B - \lambda_{\max} \cdot E) \cdot g = 0$.
- the obtained vector g will be the vector of relative weights whose sum is equal to 1.

To solve the above problem it is necessary to solve the i-th order equation (i is the number of alternatives) and the system of i linear equations. To avoid large volumes of arithmetic calculations (especially for large dimensions) the methods of finding approximate values of eigenvalues and eigenvectors are used (i.e., matrix operations are reduced to numerical iterative procedures).

The first scheme used to determine the approximate values of characteristics $\lambda_{\max}$ and g matrices $B = (b_{ik})$:

– the weight vector $g^T = (g_1, \ldots, g_i, \ldots, g_I)$ is actually an eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}$:

$$g_i = \sum_{k=\overline{1;I}} w_{ik}/I, \quad \text{where } w_{ik} = b_{ik} \Big/ \sum_{i=\overline{1;I}} b_{ik}, \quad i = \overline{1;I}. \tag{6.14}$$

– in the explicit form the approximate value of $\lambda_{\max}$ is equal to

$$\lambda_{\max} = \sum_{i=\overline{1;I}} \left(\sum_{k=\overline{1;I}} b_{ik} g_k \right). \tag{6.15}$$

A second[6] scheme is used to determine the approximate values of characteristics $\lambda_{\max}$ and g (note that it is the most precise scheme for the inversely symmetric but uncoordinated matrix of estimations B):

– A vector of approximate relative priorities $g^T = (g_1, \ldots, g_i, \ldots, g_I)$:

$$g_i = \frac{\sqrt[I]{\prod_{k=\overline{1;I}} b_{ik}}}{\sum_{m=\overline{1;I}} \sqrt[I]{\prod_{k=\overline{1;I}} b_{mk}}}, \quad i = \overline{1;I}. \tag{6.16}$$

The maximum eigenvalue of the matrix of pair comparisons $\lambda_{\max}$ is determined as:

$$\lambda_{\max} = \frac{1}{I} \sum_{i=\overline{1;I}} \frac{\sum_k b_{ik} \cdot g_k}{g_i}, \quad i = \overline{1;I}. \tag{6.17}$$

The maximum eigenvalue of an ideally coordinated positive inversely symmetric matrix is equal to the matrix order.[7] The coordination ratio CR of matrix B is calculated as the ratio of the coordination index CI of this matrix to the stochastic random index of coordination RI (an average index of a large sample of coordination indexes of equal inversely proportional matrices B generated in a random way):

$$CR = CI/RI, \quad \text{where } CI = \frac{\lambda_{\max} - I}{I - 1}, \; RI = \frac{1.98 \cdot (I - 2)}{I}. \tag{6.18}$$

If the index value does not exceed 0.1, the mismatch of matrix B is considered admissible.

Forecasting of expert preferences is estimation of priority alternatives as functions of time. If the basic data—expert estimations—contain information about the

[6]One can make a comparative analysis of precision of estimates obtained by the first and second methods.

[7]Proved in the theory of matrix algebra.

change of preference of one alternative with respect to another in some time interval, the estimation of significance (weight) can be set not by a constant but by a function approximating expert's point estimations. Then (6.13) will be rewritten as:

$$\begin{cases} \big(B(t) - \lambda_{\max}(t) \cdot E\big) \cdot g(t) = 0, \\ \sum_{i=1}^{I} g_i(t) = 1, \end{cases}$$

where $B(t) = (b_{ik}(t))$ $(i, k = \overline{1; I})$ is an inversely symmetric matrix of pair comparisons at the moment of time t; $\lambda_{\max}(t)$ is the maximum eigenvalue at the moment of time t; E is a unit matrix; $g(t)$ is a vector of relative weights at the moment of time t.

A technological solution to this problem is interesting in itself. For example, it is possible to approximate not expert estimations, but weight values calculated for them for certain moments of time.

6.2.3 Search for the Best Pareto Point

Let the decision $x \in X$, where $X \subseteq R^n$ is the set of admissible solutions in the space of parameters, be described by the values of criteria $a_i = a_i(x)$, $i = \overline{1; I}$ forming image A of the set X in the criterion space: for $x \in X$, $a = (a_1, \ldots, a_i, \ldots, a_I) = (a_1(x), \ldots, a_i(x), \ldots, a_I(x)) \in A$.

Each of I criteria is transformed so that it must be maximized to improve the quality of solution $x \in X$. Let there be a quality function showing the decision-maker's preferences in terms of efficiency of solution x based on the information about the values of criteria $a_i = a_i(x), i = \overline{1; I}$:

$$F(x) = F(a_1, \ldots, a_I) = F\big(a_1(x), \ldots, a_I(x)\big).$$

Let us assume that the decision-maker for any $x_1, x_2 \in X$ on values $(a_1(x_1), \ldots, a_I(x_1))$ and $(a_1(x_2), \ldots, a_I(x_2))$ can:

- find the best decision based on the comparison of the values of the quality function $F(x_1) > F(x_2)$ or $F(x_2) > F(x_1)$;
- establish that the decisions are equivalent if $F(x_1) = F(x_2)$.

To find the best decision one must determine:

$$x^* = \arg F\big(x^*\big) = \arg \max_{x \in X} F(x). \tag{6.19}$$

The idea of the solution is two-fold:

(1) to reduce the initial set of solutions decisions to the Pareto set;
(2) to reduce the search for the best solution to the search on the specified set.

The solution x or a in the space of parameters is the Pareto-*optimum* if it cannot be improved in one of the criteria without deterioration in at least one other criterion. The Pareto-optimum solutions form a set of Pareto X_p in the space of parameters or A_p in the space of criteria [1, 13].

Let us designate the vector of significance factors as

$$g = (g_1, \ldots, g_I) \in G, \quad G = \left\{ g \,\middle|\, g = (g_1, \ldots, g_I), g_i \geq 0, \sum_{i=1}^{I} g_i = 1 \right\}.$$

Then the Pareto set is described by the following models:

- *For convex sets*

$$\begin{aligned} X_p &= \left\{ x \,\middle|\, \arg\max_{x \in X} \sum_{i=1}^{I} g_i a_i(x),\ g \in G \right\}, \\ A_p &= \left\{ a \,\middle|\, \max_{x \in X} \sum_{i=1}^{I} g_i a_i(x),\ g \in G \right\}; \end{aligned} \tag{6.20}$$

- *For nonconvex sets*

$$\begin{aligned} X_p &= \left\{ x \,\middle|\, \arg\max_{x \in X} \min_{i \in \{1,\ldots,I\}} g_i a_i(x),\ g \in G \right\}, \\ A_p &= \left\{ a \,\middle|\, \max_{x \in X} \min_{i \in \{1,\ldots,I\}} g_i a_i(x),\ g \in G \right\}. \end{aligned} \tag{6.21}$$

If the Pareto-optimum solution x^* is known:

$$a^* = \left(a_1^*, \ldots, a_i^*, \ldots, a_I^*\right) = \left(a_1^*(x), \ldots, a_i^*(x), \ldots, a_I^*(x)\right) \in A_p \subseteq A,$$

then using models (3.20) and (3.21) it can be written as:

- *For convex sets*

$$x^* = \arg\max_{x \in X} \sum_{i=1}^{I} g_i^* a_i(x), \quad g^* \in G; \tag{6.22}$$

- *For nonconvex sets*

$$x^* = \arg\max_{x \in X} \min_{i \in \{1,\ldots,I\}} g_i^* a_i(x), \quad g^* \in G. \tag{6.23}$$

There is a conformity between the best Pareto point x^* and point g^* in the set of all weights G (i.e., there is a possibility to establish conformity between single-criterion and multi-criteria problems).

It is obvious, see (6.21) and (6.20), that change in weight gives different Pareto points. This connection makes it possible to reduce the search for the best solution from the space of criterion A to the space of weights G, which in turn allows us to reduce the dimension of the problem due to normalizability of weight vectors:

$$\sum_{i=1}^{I} g_i = 1 \Rightarrow g_I = 1 - \sum_{i=1}^{I-1} g_i.$$

So, the problem reduces to search for the best points, from the point of view of the decision-maker, such that

$$q_i = g_i, \quad i = \overline{1; I-1},$$

$$0 \le q_i \le 1, \quad i = \overline{1; I-1}, \ \sum_{i=1}^{I-1} q_i \le 1.$$

There are a few rather efficient algorithms to solve this, for example a coordinate-wise descent which assumes descent either to the optimal solution or to the performance of a certain number of steps. This algorithm contains the following steps:

1. Set initial parameters:
   - a step length $h \in (0; 1)$;
   - a constant coefficient of changes in the step length $\eta \in (0; 1)$;
   - the initial point (solution) x_0;
   - the initial weight vector $q_i = \frac{1}{I-1}$, $i = \overline{1; I-1}$.
2. Set $k = 0$.
3. Calculate x_1 by solving the maximization problem:
   - for the convex Pareto set in the criterion space

$$x_1 = \arg\max_{x \in X} \sum_{i=1}^{I} q_i a_i(x);$$

   - for the nonconvex set

$$x_1 = \arg\max_{x \in X} \min_{i \in \{1,\ldots,I\}} q_i a_i(x).$$

4. Check the stop rule (either the optimal solution is found or the limit of iterations is over). If the condition is satisfied, the search must be stopped, and the solution is

$$x^* = x_{k+1};$$

$$\big(a_1(x_{k+1}), \ldots, a_I(x_{k+1})\big);$$

$$g^* = \left(q_1, \ldots, q_{I-1}, 1 - \sum_{i=1}^{I-1} q_i\right).$$

5. Otherwise, $k = k + 1$.
6. Set[8]

$$i_k = k - (I-1)\left[\frac{k}{I-1}\right] + 1, \quad p_k = e_{i_k},$$

where $e_{i_k} = (\underbrace{0, \ldots, 0}_{k-1}, 1, 0, \ldots, 0)$ is the i_k-th coordinate direction.

7. Diminish q_{i_k} by the step value h not changing other components of vector q

$$q^* = q - h \cdot p_k.$$

8. Check other limitations

$$0 \le q_i^* \le 1, \quad i = \overline{1; I-1}, \ \sum_{i=1}^{I-1} q_i^* \le 1. \tag{6.24}$$

If value q^* is inadmissible, pass to item 11, otherwise set $q = q^*$.

9. Determine $\tilde{x}$ after solving the task of maximization with predetermined q value:

– for convex set

$$\tilde{x} = \arg\max_{x \in X} \sum_{i=1}^{I} q_i a_i(x); \tag{6.25}$$

– for nonconvex set

$$\tilde{x} = \arg\max_{x \in X} \min_{i \in \{1,\ldots,I\}} q_i a_i(x).$$

10. The decision-maker compares points (solution) $\tilde{x}$ and x_k. If $\tilde{x}$ is better than x_k take $x_{k+1} = \tilde{x}$ and move on to item 4.
11. Increase q_{i_k} by the step value h, not changing other components of the vector q

$$q^* = q + h \cdot p_k. \tag{6.26}$$

12. Check limitations (6.24). If the value q^* is inadmissible, move on to item 15, otherwise set $q = q^*$.
13. Determine $\tilde{x}$ after solving the task of maximization (6.25) with the predetermined q value.
14. The decision-maker compares points (solution) $\tilde{x}$ and x_k. If $\tilde{x}$ is better than x_k take $x_{k+1} = \tilde{x}$ and go to item 4.
15. $x_{k+1} = x_k$.
16. If $k + 1 > I - 1$ and vector q has not changed during the last $(I-1)$ iterations, change the step length:

$$h = \eta \cdot h.$$

[8] Here $[\cdot]$—the integer part of number $(\cdot)$.

17. Go to item 4.

The algorithm choosing the best Pareto points not only gives the solution x^* (optimal from the point of view of the decision-maker) but also returns the weight vector g^* containing information on the preferences of a certain decision-maker. Indeed, the suggested algorithm can be considered as a method of extrapolation of expert estimations:

– Each object of choice is considered as an I-dimensional vector $a = (a_1, \dots, a_I)$ whose components correspond to the personal (simple) criteria of optimization or some other parameters of the modeled system.
– The expert (decision-maker) can choose from a pair of alternatives the best variant in terms of utility.
– There is a utility function[9] of the following structure:

$$\pi(a) = \sum_{j=1}^{J} g_j f_j(a), \tag{6.27}$$

where $f_j(a)$ are known functions of the vector argument monotonously increasing along each coordinate; g_j are unknown weight parameters.
– To get an unambiguous result let us assume that all criteria a_i are maximized (hence, weights are nonnegative) and introduce a condition of normalization of weights. Then, showing the decision-maker a limited sampling from K pairs of alternatives $(a^k; b^k)$ and assuming that from the point of view of the decision-maker the alternative a^k is better than b^k, we see that the solution of the linear system

$$\begin{aligned} &\sum_{j=1}^{J} g_j \left(f_j\left(a^k\right) - f_j\left(b^k\right)\right) > 0, \quad k = \overline{1; K}; \\ &\sum_{j=1}^{J} g_j = 1, \quad g_j \geq 0 \end{aligned} \tag{6.28}$$

defines the region of admissible values of coefficients of function (6.27). In a concrete situation, setting the direction of optimization by the target function of the system (6.28), one can find a point estimation for the vector of coefficients which ranks $\tilde{g} \approx g = (g_1, \dots, g_J)$ a subset of the best utility alternatives adequate to expert ordering.

6.2.4 Convolutions of Estimation Criteria

There are many functions which allow us to pass from the multicriterion problem to the single-criterion problem. Such transformation, however, gives rise to the prob-

[9] It is supposed that if $\pi(a) > \pi(b) \Leftrightarrow a$ is better than b.

lem of connection of quantities measured in different scales: nominal, ordinal, quantitative, etc. To simplify the investigations the scales of measurement are "unified": As a rule, one of the scales is reduced (in terms of admissible operations) to the level. One of the possibilities of getting a uniform sampling in the space of simple metrics is:

1. Pass from physical units of measurements a_i $(i = \overline{1; I})$ to the relative units f_i by means of translation function $f(a_i)$, such that its values vary in the interval [0; 1], and the working interval of the argument varies from $a_{i\,\min}$ to $a_{i\,\max}$ beyond which the function has constant values.
2. Set weights of criteria g_i showing expert preferences. The degree of confidence in the subjective information is estimated either by Kendalla concordance coefficient, or by the discrepancy function (functions of losses), or by checking coordination of the matrix of pair comparisons, or by estimating the degree of stability of the obtained result: If the distortion g_i does not change the ranking order, the results are stable with respect to g_i and confidence in the obtained estimations is higher.

Let us assume:

- $f(a_1) > f(a_2) \Leftrightarrow f(a_1) \succ f(a_2)$ (preference ratio);
- $\Delta a_1 = \Delta a_2 \Rightarrow \Delta f(a_1) = \Delta f(a_2)$ (utility ratio).

Therefore as a translation function to the relative units we choose a monotonous increasing linear function:

$$f(a_i) = \begin{cases} 0, & \text{if } a_i \le a_{i\,\min}, \\ \frac{a_i - a_{i\,\min}}{a_{i\,\max} - a_{i\,\min}}, & \text{if } a_i \in [a_{i\,\min}; a_{i\,\max}], \\ 1, & \text{if } a_i \ge a_{i\,\max}. \end{cases}$$

Aggregation of criteria in the j-th aspect depends on how the decision-maker understands the degree of compensation of reduced values of some criteria by the increased values of the other criteria.

If compensation of reduction in absolute values of some criteria at the expense of total absolute increase in the others is admissible, generalization is made by means of the additive operator:

$$F_j = F\big(f_1(a), \ldots, f_{I_j}(a)\big) = \sum_{i=1}^{I_j} g_i f_i(a), \quad j = \overline{1, J}, \tag{6.29}$$

where the operator *parameters* are weights of the i-th criteria $g_i \ge 0$, $\sum_{i=1}^{I_j} g_i = 1$; $f_i(a)$ is an estimation of a alternative with respect to the i-th criterion; the operator *values* in the boundary points

$$F_j(1, \ldots, 1) = 1; \qquad Fj(0, \ldots, 0) = 0;$$

$$f_i(a) < 1 \Rightarrow F_j\big(1, \ldots, f_i(a), \ldots 1\big) < 1;$$

$$f_i(a) > 0 \Rightarrow F_j\big(0, \ldots, f_i(a), \ldots 0\big) > 0.$$

The value of weight coefficient g_i is interpreted as the value of the average change in the result F_j when the factor $f_i(a)$ changes by a unit (in case of fractional units used to measure factors we get a percentage change in F_j).

If it is admissible to compensate the total relative decrease in one group of criteria by the total relative increase in the other group of criteria, the multiplicative aggregation function (the same notations) can be used:

$$F_j = F\big(f_i(a), \ldots, f_{I_j}(a)\big) = \prod_{i=1}^{I_j} g_i f_i(a), \quad j = \overline{1, J}. \tag{6.30}$$

The minimax convolution gives the most cautious position:

$$F_j = \min_{i=\overline{1,I_j}} \big\{f_i(a)\big\}, \quad j = \overline{1, J}. \tag{6.31}$$

The minimax convolution does not affect the optimum when new inessential criteria are added, but it has low sensitivity.

6.2.5 *Criteria Used to Choose Optimal Solution*

Knowing the values of utility functions of all experts, it is possible to set a means of choosing optimal solutions. For this purpose different functions can be used. Some of them give solutions that are coordinated to a certain degree, while in the others this is not obligatory. In this case a coordinated solution means that if one of the experts gives an inadmissibly low estimation of the alternative, under no conditions can it be accepted as the optimal alternative. The most widely used criteria are:

– *Criterion of products*. The alternatives with the maximum product values of the utility function of all experts are chosen:

$$Z_p = \max_a \prod_j F_j(a), \tag{6.32}$$

where $F_j(a)$ is the value of the utility function of the j-th expert for the alternative a.

– *Minimax criterion*. The alternatives for which the worst expert estimations have maximal values are chosen:

$$Z_{MM} = \max_a \min_j F_j(a). \tag{6.33}$$

The decision taken according to this criterion cannot be worsened, as it corresponds to the position of maximal caution.

Gurvits's criterion enables us to set a subjective measure of pessimism C.

$$Z_{HW} = \max_a \Big(C \min_j F_j(a) + (1 - C) \max_j F_j(a) \Big). \tag{6.34}$$

It should be noted that the above criterion (as well as the minimax) lets us choose an uncoordinated solution having inadmissibly low expert estimations as an optimal solution. However, if all the alternatives with low estimations are first eliminated, the application of the above criterion can give interesting results [8].

6.2.6 *Choosing a Group Solution on the Basis of Multicriterion Estimation*

The task of making a group solution on the basis of multicriterion estimation can be formally presented as the task of mathematical programming:

$$\begin{aligned} &Z\big(F_1(a), \dots, F_j(a), \dots, F_J(a)\big) \to \max_a; \\ &f_{ij}(a) \geq C_i, \quad \forall j = \overline{1, J},\ i = \overline{1, I_j}, \end{aligned} \tag{6.35}$$

where $F_j = F(f_i(a), \dots, f_{I_j}(a))$ is the convolution of all I_j criteria for the j-th expert; $Z(F(a))$ is a criterion of choice of the optimal solution based on analysis of complex estimations of alternatives by all J experts; C_i is the limitation on the value of alternative estimation by the i-th criterion.

It is also possible to use the other interpretation of the task:

$$\begin{aligned} &F\big(Z_1(a), \dots, Z_i(a), \dots, Z_{I_j}(a)\big) \to \max_a, \\ &f_{ij}(a) \geq C_i \quad \forall j = \overline{1, J}, i = \overline{1, I_j}, \end{aligned} \tag{6.36}$$

where $Z_i(a) = Z(f_{i1}(a), \dots, f_{iJ}(a))$ is a group estimation of the alternative by the i-th criterion.

Method used to choose a single multipurpose group decision Based on the accepted concepts it is possible to suggest the following method for a group of experts to make decisions when it is necessary to achieve several aims "simultaneously":

1. Working out of criteria to be used to estimate variants of solutions on the basis of shared opinion.
2. Estimation of significance of every criterion by calculating its weight by experts' questioning. This allows us to define the utility function for the set of criteria.
3. Working out of variants of problem solution (alternatives).
4. Every expert estimates variants of solutions according to each criterion of estimation. This makes it possible to define a utility function which juxtaposes the valiant of solution and vector of its estimations according to the criteria.

5. Complex estimation of each alternative by each expert taking into account its "vector" utility function is determined. For this purpose "convolutions of criteria" are used. In the other approach at this step a complex estimation of each alternative by each criterion separately—i.e., a group estimation for the criterion—is determined. In this case it is necessary to apply criteria for choosing an optimal solution.
6. Based on the resulting table of complex estimations of alternatives made by all experts (or group estimations by criteria), optimal alternatives are chosen. To this end, criteria for choosing optimal solutions (in case of group estimations—convolutions of criteria) are used.

6.3 Assessment of Investment Project in the Multicriterion Context

Investment decisions depend on a variety of different physical parameters. It is possible to unite these parameters into a uniform mathematical model only on the basis of subjective representations of the decision-maker on the efficiency of alternatives and importance of various criteria. Among the methods of multicriterion optimization the most widely used methods are the T. Saati [6] method of hierarchies analysis and the L. Zade [17] theory of fuzzy sets. It should be noted that the analytical method of calculation of the efficiency factor (hence, risk indicators) is too complicated to be widely used and, as a rule, it is only used with elementary decision rules (for example, Pareto [13]) or for problems with two criteria. In most cases the most acceptable method is the method of statistical tests (Monte-Carlo).

6.3.1 The Hierarchy-Analysis Method as a Synthesis of Quantitatively Measurable Expert Information

The purpose of the hierarchy-analysis method (HAM) is to provide support for decision-making by means of hierarchical decomposition of tasks and estimation of significance of alternative solutions [1, 6].

The first task here is of great importance: that of choosing a set of criteria against which to measure alternatives. The HAM provides decomposition of the main target to such a level of detailing that for the lower layer of the hierarchy it is possible to formulate criteria such that it is possible to obtain a careful breakdown of how nearly the target has been reached by the choice of one of the alternatives (Fig. 6.3).

The sets of values of these criteria are used to describe the outcomes of alternative variants. A utility function is established which reflects the preferences of the decision-maker and is used to match each set of criteria estimates to a number characterizing the efficiency of the alternative. Construction of the utility function on the basis of a formal description of preferences assumes, in particular, the ability of the decision-maker to make substitutions, i.e., in the simplest case to choose the

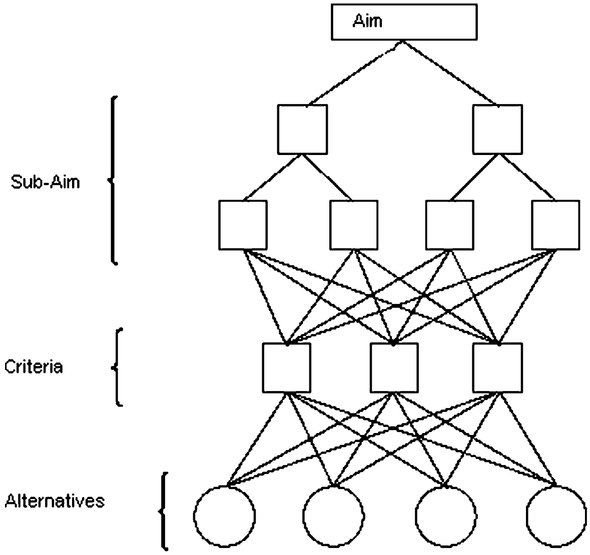


Fig. 6.3 Target decomposition

preferred of two alternatives, when the first is better than the second by the criterion A and worse by criterion B with all other values being equal.

For each "sheaf" of a certain level of the hierarchy tree we construct vectors of priorities of a satisfactory degree of coordination (or the given accuracy) $g^{ij} = (g_1^{ij}, \ldots, g_{I_{ij}}^{ij})$, here $i = \overline{1;n}$ is the number of the hierarchy level, $j = \overline{1;n_i}$ is the number of the i-th level sheaf, $I_{ij} = \overline{1;n_{ij}}$ is the number of criteria of the j-th sheaf of the i-th level (obviously, the number of sheaves is equal to the number of criteria of the previous level). The vector of priorities with respect to the focal point (the main level of hierarchy) is calculated using the well-known principle of choosing branches (links of the tree) according to the logical characters: logical "and" meaning multiplication of weights; logical "or" meaning addition. Therefore, the indicator integrated with respect to each level is the sum of products of the corresponding factors multiplied by the factor "weight." If necessary, the vector $g = (g_1, \ldots, g_I)$ is normalized by the value $(\sum_{i=\overline{1;I}} g_i)^{-1}$.

More often HAM uses the utility function in the form of additive convolution (6.29). The vector g enables us to make a decision in multicriterion conditions, as all its components express the aggregated preferences of the decision-maker concerning alternatives.

The utility function $F = \sum_{i=1}^{I} g_i f_i(a) = g \cdot f(a)$, depending on the informative load of alternatives (criteria) and type of measurement, can be used for traditional purposes:

– *to obtain point and interval estimations of the short-term forecast* F; in this case the time series are analytically aligned with respect to all factors a_i present in the model, and the obtained relations $a_i(t)$ are used to make a point forecast $\widehat{a}_i$ (it would be useful to apply smoothing or adaptive methods), to calculate a standard error and a confidence interval (6.10, 6.12);

– *to optimize allocation of resources* a_i (6.35, 6.36);
– *to estimate management efficiency of the system*: to compare the potential (p) and planned (q) values of the target result F:

$$W = \frac{F^{(p)}}{F^{(q)}} = \frac{\sum_{i=\overline{1;I}} f_i \cdot f_i^{(p)}(a)}{\sum_{i=\overline{1;I}} f_i \cdot f_i^{(q)}(a)}.$$

In particular, in the multicriterion context, the IP (investment project) model (6.3) can be considered as an additive convolution of the criteria "monetary flows" with weights:[10]

$$NPV = \sum_{k=0}^{K} g_k CF_k, \tag{6.37}$$

where CF_k is the forecast value of the pure currency flow in the k-th period; r is the rate of discount.

When the forecast of changes in the currency flows CF with changes in the discount rate r is reliable, the direct calculation (6.37) reduces the multicriterion task of IP estimation to a single-criterion task: comparison of NPV values of all projects taking part in the competition. Otherwise, the calculation of traditional indicators by traditional methods is incorrect. One can either model a new indicator (which is problematic) or suggest a "new" way of estimating the values of the known indicator, for example, to use the HAM methodology: relative estimations of currency flows generated by IP.

Let there be I projects. Let us assume:

– all IPs have the same duration of the life cycle K;
– currency flows are independent, which allows us to compare IPs in terms of concrete CF_k, independent of its dependence on the values of other currency flows.

In this case, the set of currency flows related to a certain IP is determined as

$$CF(IP_i) = \left(CF_0^i, CF_1^i, \ldots, CF_K^i\right), \quad i = \overline{1, I}.$$

In the multicriterion terminology CF,[11] which describe alternative projects, act as their criteria.

In this case the target decomposition will be presented as (Fig. 6.4):

Synthesis, in terms of the integral effect, can be written as (additive convolution (6.29) in the corresponding notations):

$$g(IP_i) = \sum_{k=0}^{K} g^{2k}(IP_i) g^{1k}(CF_k), \quad i = \overline{1, I}, \tag{6.38}$$

[10] Normalization may be required to obtain a unit sum of weights.

[11] It is convenient to use the same notations for the criterion and its numerous values, since it will cause no misunderstandings in the context.

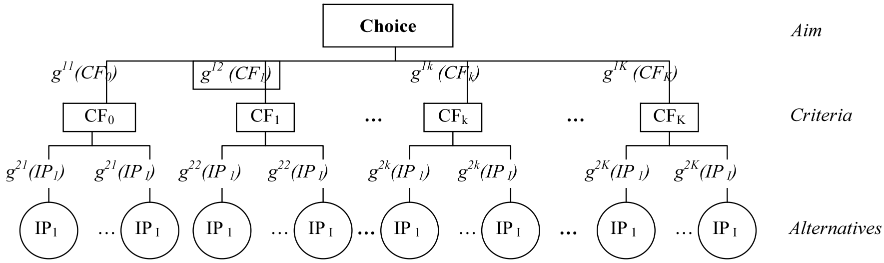


Fig. 6.4 Hierarchy of IP choice targets

where I is the number of estimated IPs; the values of weights, coordination of estimated procedure are compared according to (6.14)– (6.18).

Having constructed an integral vector of priorities $g = (g_1, \ldots, g_I)$, it is not difficult to rate the projects: $g_i < g_j \Leftrightarrow IP_i \prec IP_j$, $g_i = g_j \Leftrightarrow IP_i \approx IP_j$.

For example, as a rule, $CF_0 \prec CF_1 \prec \cdots \prec CF_K$. The degree of priority is established:

- by the expert method (for example, the Saati method or comparison in the three-point scale);
- or by normalizing coefficients $1/(1+r)^k$, $k = \overline{0, K}$ in the IP model (6.3) based on the forecast discount values.

In estimating alternatives the decision-maker uses the paired comparison to determine more precisely what different in significance[12] of currency flows the i-th project generates in each k-th period of calculations.

The first step in choosing an IP is to analyze investment expenses. Let us assume that there is a forecast of initial investments under projects IP_1, IP_2, and IP_3: $CF_0^1 = CF_0^2 > CF_0^3$.

We get the following preferences: $CF_0^1 \approx CF_0^2$, $CF_0^1 \prec CF_0^3$, $CF_0^2 \prec CF_0^3$. However, according to the informal information, the expert or the decision-maker estimates the projects not in terms of the CF_0 value but in terms of opinions (a fragment of the matrix of paired comparisons, Table 6.1).

Using an "unreliable" forecast of the currency flows and taking into account additional information (experience, intuition, situation, etc.), the expert constructs a matrix of paired comparisons for the future periods, defining the superiority and preferability of one IP over the other in terms of criterion CF_k.

It is important to note that HAM is structured such that the conclusions of the decision-maker can be reconsidered in order to achieve an acceptable level of agreement [6, 7].

For IP estimation it is possible to use HAM variations. For example, let A_k values—the incomes of the k-th period in the initial model (1.1)—be known (as would be the case if, for instance, they were specified in the contract) and $B_k = 0$,

[12] Qualitatively, this means "higher."

Table 6.1 A fragment of comparison

CF_0	IP_1	IP_2	IP_3
IP_1	1	1	1/6
IP_2	1	1	1/5
IP_3	6	5	1

i.e., capital investments in the project are concentrated in $CF_0 = I_0$ and are not exactly defined. Then to construct a matrix of pair comparisons of the k-th period ($k = \overline{1, K}$) it is reasonable to use a continuous scale of ratios $b_{ij}^k = CF_k^i / CF_k^j$. The matrix of comparisons for criterion CF_0 is, as before, defined by expert estimations.

In the analysis of investment risk one of the most important points is correct calculation of the degree of influence of a set of factors on IP. HAM and its modifications help solve this problem by means of hierarchical decomposition with subsequent synthesis, which reveals relations through opinions of the decision-maker and their integration according to the global purpose. For example, the following analyses can be used:

– risk assessment (IP models must take into account possible threats and consequences of undertaking the project);
– assessment of priorities in risk management (avoidance or rejection of risks, assumption of risks, prevention of losses, reduction in the amount of losses, insurance, transmission of risks, etc.).

Thus, for example, the system of risk analysis can assume decomposition of a set of elements of the hierarchical structure taking into account complex IP criteria [1, 15].

6.3.2 *Assessment of Investment Project by Complex Criteria*

The choice of criteria to be used to estimate the risk indicator depends on the decision-maker and can be especially individual as it formalizes personal desires and requirements for the quality of investment decision.

At the same time, there is a standard set of simple criteria which the decision-maker can vary according to the specific conditions of the field of application and territorial adjustment of the project.

The *spatial* location of the investment project defines a qualitative set (structure) and power (the number of indicators) of the criterion set which, for convenience of the decision-maker, is subdivided into categories (aspects, directions) that can be used to calculate a quantitative integral indicator.

- *Budgetary* criteria: estimate efficiency of the investment project for the republic budget (region, area, city) and contribution of the project to the economy of Kazakhstan (region).
- *Commercial* criteria: consider commercial economic efficiency of investment activity related to the proposed project, its financial competence, solvency, etc.

- *Financial* criteria: estimate safety of capital investments for commercial structures and the budgetary investor, comparing over time the forecast of revenues with the forecast of necessary expenditures.
- *Marketing* criteria: forecast future competitiveness of production, identify existing and potential competitors, compare the results of IP performance with competitors' analogues and expected consequences of competitors' activities on the market, estimate demand and forecast the sales volume of goods or services, etc.
- *Social* criteria: these have to do with the social aspect of the usefulness of the investment (in particular, potential impacts on the social infrastructure and demographics), attitude of the administration and the population towards the implementation of the project, acceptability of the project to the local culture.
- *Organizational/legal* criteria: these are used to assess impacts of project activity including legal and organizational readiness of the project for implementation and project insurance (including possibility of attracting commercial co-financing).
- *Resource* criteria: refer to characteristics of resources for the technology suggested in the project (including forecast cost, volumes, availability, ecological state of initial resources, etc.).
- *Technological* criteria: take into account requirements and limitations on application of the planned project technology including up-to-datedness, approbation, scale, influence on the environment, etc.
- *Ecological* criteria: include qualitative characteristics and quantitative indicators of ecological and economic efficiency of the investment project.

A set of indicators can be formed according to a range of factors encompassing *economic safety*:

- Economic potential for sustainable growth.
- Support of scientific potential.
- Dependence of the economy on imports for the major production categories.
- Standard of living.
- Quality of life.
- Demographics.
- Ecology.

In terms of financial and socio-economic significance, arriving at an estimate for IP potential assumes a great number of indicators of risk (indices of IP efficiency). The task of estimating significance coefficients becomes more complicated with increase in the dimension of the input data vector: the sum of weights cannot exceed a unit, therefore weights in the multicriterion task are small, which in turn can affect the error of the result.

Let us assume:

- The set of unit criteria is divided into various aspects—subsets of simple characteristics. For example, the aspect {*social structure*} can include simple indices {*fraction of economically active population* (%); *increase in population* (%); *unemployment of the active population* (%); *employment in the scientific and technical sphere* (%); *employment in the industrial sector* (%); *employment in agriculture* (%); *preparation and retraining of personnel under the corporation profile*

(%), *etc.*}. Considering several levels of aggregation, a tree of criteria can be constructed.

- Weights are assigned to simple criteria present in every aspect, independent of other elementary indices. In turn, the significance coefficients of composite characteristics—aspects—are assigned independent of simple criteria.

The system of IP risk analysis includes a multicriterion analysis performed according to the following steps:

1. Formation of a set of criteria.
2. Structuring of the set of criteria—construction of a criteria tree.
3. Unification of individual criteria (to transform to the same scale of measurements).
4. Aggregation of criteria into aspects.
5. Calculation of complex criteria.
6. Determination of preference ratios.

Almost all methods of multicriterion optimization including the Pareto [13] method and the HAM method of preferences can be adapted to investment activities [6].

Investment decisions are of necessity regulated by normative-legislative documents, preferences of all IP participants, interactions at all levels of the investment sphere, etc.

The deterministic character of a real individual IP in a concrete situation defines IP classification, therefore the project (risk) management responds to the particular circumstances surrounding actual decision making.

The task of choosing the best variant for investment is bounded by:

- the volume of financial resources;
- the opportunities for their use.

The above limitations correspond to the two types of investment task:

1. Achievement of one purpose by different methods: choice of a monetary investment variant for achieving the purpose, from among the set of variants differing by organizational, administrative, technological, economic, and ecological solutions.
2. Investment of certain resources in order to achieve different purposes offering possibilities for gaining income, profit, etc.

Methodologically both solutions, in case of not less than two variants, are reduced to the comparison of investment efficiencies and choice of the variant with the best efficiency index in terms of the interests of project participants.

Investment planning means creation of groups of participants forming different private solutions (technical, technical-economic, social, legal, etc.).

Each group has its specific features reflecting specificity of coordination of opinions of individual participants, and each participant has his own interests and individual "weight" in the structure of making investment decisions. Therefore, making investment decisions is a process of coordination of interests on the basis of comparison of "weights" of all decision-making participants and development of aggregated priorities.

For example, the regional investment policy as a set of possibilities and their subsequent results includes coordination of interests of the individual IP with regional aspects. In this context a specific feature of the regional policy is the method of its realization:

- *direct*: allocation of the budgetary funds at the disposal of administration (through privileged taxation, anti-monopoly policy, etc.);
- *indirect*: pushing uncontrollable off-budget resources in a desired direction, by means of special methods and actions (price structuring, customs duties, etc.).

Selection of projects for investment (involving coordination of interests at individual and regional levels) must be oriented towards reaching the best results not only in financial, but also in social and economic spheres. The project must envisage: implementation of higher technologies; development of the raw-material base; development of adjacent regional branches and transport infrastructure; organization (extension) of export-oriented (import-substituting) production.

IP filtering through the set of financial and socio-economic criteria may require multicriterion optimization: determination of a set of effective IP alternatives and their ranking according to certain priorities.

The investment project management (and hence, creation of IP risk) has a hierarchical structure.

The business-mission of a certain level (from individual to republic) is successful if its investment activity develops in coordination with administration, infrastructure of the corresponding region, and its resource potential (scientific-educational, technological, raw resources, etc.).

Let an assumption of agreement of interests be a compromise in the resolution of conflicts: it is important to satisfy interests of all sides of investment activity except in cases of forced decisions. Therefore formulation of the multicriterion task of IP choice contains an objective component (amount of financing, time limitations, ecological requirements, etc.) as well as a subjective component (system of preferences—formalizable and non-formalizable, often intuitive estimates).

Estimation of the preference for some solutions over others is based on definition and comparison of connections between them; that is to say, it is related to the concept of utility (the utility function), which unifies the parameters used to estimate preferences for quantities measured in different scales of measurement or/and different units of measurement.

Let the utility functions of investment results for the sides be denoted $\pi(R)$ and $\delta(R)$, where R are the vectors of risk arguments/indicators with intersecting definition domains. The extreme values of individual and regional utility functions will be denoted as $\pi(R_*^{ind})$ and $\delta(R_*^{reg})$. In the general case points R_*^{ind} and R_*^{reg} do not coincide. For each side it is desirable that the extremum point be the point of making an investment decision, and a point in the nearest vicinity of the extremum point be the end of the investment project. Therefore the functions describing the agreement of interests are considered as a composition of two functions: for ind (individual

Table 6.2 Strategies of individual IPs

IP time interests	*Region resource potential* (*classes*)			
	Low	Medium	High	...
Short-term	(1;1)	(1;2)	(1;3)	
Mid-term	(2;1)	(2;2)	(2;3)	
Long-term	(3;1)	(3;2)	(3;3)	
...	...	...	...	...

IP) this is a composition of $\pi(R)$ and $\alpha(R)$, i.e., $f^{ind}(\pi(R), \alpha(R))$ while for reg (region) it is $\delta(R)$ and $\beta(R)$, i.e., a composition $f^{reg}(\delta(R), \beta(R))$. If the point of reaching agreement is denoted R_c, its sides tend to the following values:

$$\left|\pi\left(R_*^{ind}\right) - f^{ind}\left(\pi\left(R_c^{ind}\right), \alpha\left(R_c^{ind}\right)\right)\right| \to \min;$$
$$\left|\delta\left(R_*^{reg}\right) - f^{reg}\left(\delta\left(R_c^{reg}\right), \beta\left(R_c^{reg}\right)\right)\right| \to \min.$$

It is obvious that in the general case

$$\left|\pi\left(R_*^{ind}\right) - f^{ind}\left(\pi\left(R_c^{ind}\right), \alpha\left(R_c^{ind}\right)\right)\right| \neq \left|\delta\left(R_*^{reg}\right) - f^{reg}\left(\delta\left(R_c^{reg}\right), \beta\left(R_c^{reg}\right)\right)\right|.$$

A compromise tactic for reaching agreement is expressed as:

$$A\left|\pi\left(R_*^{ind}\right) - f^{ind}\left(\pi\left(R_c^{ind}\right), \alpha\left(R_c^{ind}\right)\right)\right| = B\left|\delta\left(R_*^{reg}\right) - f^{reg}\left(\delta\left(R_c^{reg}\right), \beta\left(R_c^{reg}\right)\right)\right|,$$

where A, B are the agreement vectors.

Let us assume that in the two-factor space 'time'–'resource potential' there exist strategies of participants in the individual IP which represent characteristics of targets and tools for implementation of investment strategy (Table 6.2).

Coordinate positioning of the strategy assumes:

(1) calculation of A_{iv}, $i = \overline{1, I}$, $v = \overline{1, V}$—the aggregated indicator of the i-th resource potential of the v-th region according to the IP branch positioning;
(2) clustering of the regional potential (a method of rating on the basis of cluster analysis).

For example, applying Euclid's metrics to calculate the distances between the integral elements of the set of regions $\{1, \dots, V\}$, we get

$$d_{sk} = \sqrt{\sum_{i=1}^{I}(A_{is} - A_{ik})^2}, \quad (s = \overline{1; V}, k = \overline{1; V}).$$

Using iteration clustering methods the set of regions is divided into clusters in semantic meaning, for example the cluster {low; middle; high}. The region belonging to a certain cluster and time interests of investors defines a realizable investment strategy giving the acceptable risk level, and hence the utility function.

For the mid-term and long-term (especially important) investment processes it is possible to monitor the dynamics of resource potential of the region and IP time horizon, making it possible to adjust the strategy of investors' interaction in response to regional aspects and thus to minimize the IP risk and/or to keep it below the acceptable level.

The investment decision often implies assessing the degree of similarity of the two compared strategies (interests) Pr_1 and Pr_2 by their qualitative characteristics presented in the binary matrix

$$\delta_{ij} = \begin{cases} 1, & \text{is the } i\text{-th indicator of the } j\text{-th object} \\ 0, & \text{otherwise.} \end{cases}$$

In this case the degree of coordination Pr_1 and Pr_2 is calculated as

$$d_{1,2} = \frac{2\sum_{i=1}^{I} \delta_{i1}\delta_{i2}}{\sum_{i=1}^{I} \delta_{i1} + \sum_{i=1}^{I} \delta_{i2}}.$$

Compromise strategies with the minimum measure of similarity cannot be considered, as they lead to a mismatch of interests [16].

6.4 Probabilistic Approach to Quantitative Risk Assessment

The idea of quantitative probabilistic assessment of project risk is to determine the IP risk level: the probability that the risk indicator will fall beyond the stability zone [14]. In this case probabilities of "negative" deviations of sensitive characteristics only are considered. (Fig. 6.5).

Let risk indicators R_d, $d = \overline{1, D}$ be defined for each IP and let sensitivity characteristic x_j, $j = \overline{1; J_d}$ be defined for each indicator; numerical boundaries of the stability zone of each indicator's parameters for negative deviations of sensitivity characteristics (6.5, 6.6) and probabilistic distribution of negative deviations of the values of every sensitivity characteristic are determined.

Let $A = \bigcup_{j=\overline{1,J_d}} A_j$ be an ordered union of the set of negative deviations of the j-th index determined by the random number generator $A_j = \{a_1 \le a_2 \le \cdots \le a_{sj}\}$ for each d-indicator; $a_j^{R_1}, a_j^{R_2}, \ldots, a_j^{R_d}, \ldots, a_j^{R_D}$ be maximal (limit) negative values of the j-th index for which the stable level of indicators R_d, $d = \overline{1, D}$ is preserved.

In the above conditions:

– the boundary value of the j-th index stability zone is

$$\bar{a}_j = \max_d\{a_j^{R_d}\} = \max\{a_j^{R_1}, \ldots, a_j^{R_D}\};$$

– the boundary value of the stability zone of indicator R_d is defined as:

$$\bar{a}^{R_d} = \max_j\{a_j^{R_d}\} = \max\{a_1^{R_d}, \ldots, a_{J_d}^{R_d}\};$$

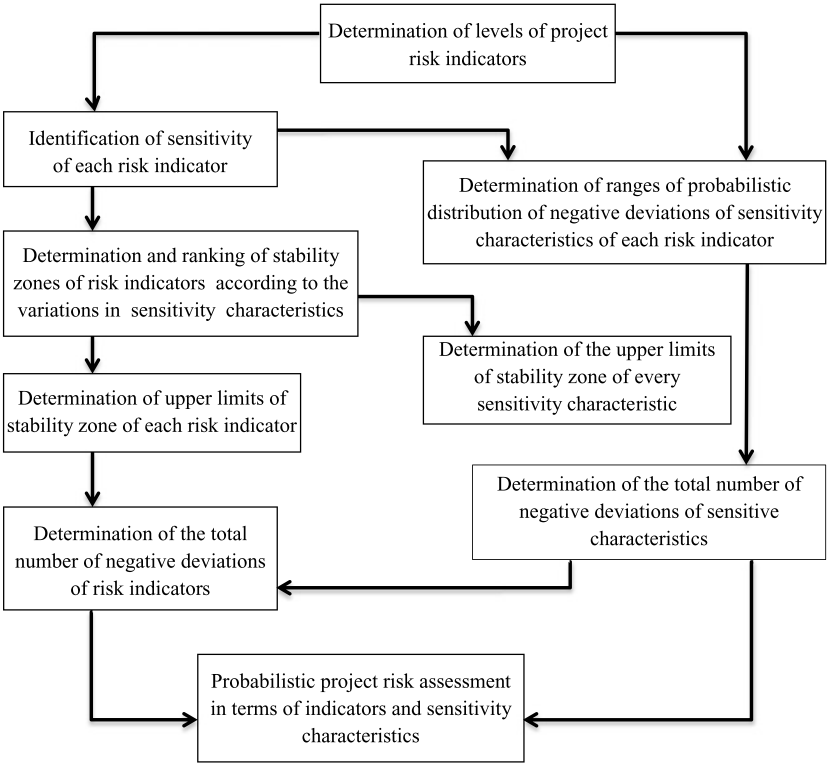


Fig. 6.5 Schematic representation of probabilistic quantitative risk assessment

– the common boundary of the stability zone of all sensitivity characteristics of the project risk indicators can be defined as:

$$\bar{a} = \max_d \{\bar{a}^{R_d}\} = \max\{\bar{a}^{R_1}, \ldots, \bar{a}^{R_D}\} = \max_d \max_j \{a_j^{R_d}\}.$$

The determination of boundary values allows us to subdivide sets A and A_j into subsets of negative deviations of sensitivity characteristics which fall and do not fall in the stability zone:

– for each j-th characteristic:

$$A_j = A_j' \cup A_j'' = \{a_1 \le a_2 \le \cdots \le a_k \le \bar{a}_j\} \cup \{\bar{a}_j < a_{k+1} \le a_{k+2} \le \cdots \le a_{s_j}\};$$

– for each risk indicator:

$$\begin{aligned} A_{R_d} &= A_{R_d}' \cup A_{R_d}'' \\ &= \{a_1^{R_d} \le a_2^{R_d} \le \cdots \le a_h^{R_d} \le \bar{a}^{R_d}\} \cup \{\bar{a}^{R_d} < a_{h+1}^{R_d} \le a_{h+2}^{R_d} \le \cdots \le a_{fd}^{R_d}\}; \end{aligned}$$

– for the project as a whole:

$$A = A' \cup A'' = \{a_1 \leq a_2 \leq \cdots \leq a_m \leq \bar{a}\} \cup \{\bar{a} < a_{m+1} \leq a_{m+2} \leq \cdots \leq a_s\}.$$

Hence, the probability of project risks is defined as:

– for negative deviations of the j-th characteristic:

$$P_j = \frac{s_j - k}{s_j},$$

where s_j is the total number of negative deviations of the j-th characteristic; $s_j - k$ is the number of negative deviations falling beyond the project stability zone for risks;
– for each indicator R_d: $P_{R_d} = \frac{f_d - h}{f_d}$, where $f_d = \sum_{j=1}^{J_d} s_j$ is the total number of negative deviations of all characteristics sensitive to indicator R_d; $f_d - h$ is the number of negative deviations falling beyond the stability zone;
– for the project as a whole:

$$P = \frac{N - m}{N},$$

where $N = \sum_{d=1}^{D} f_d = \sum_{d=1}^{D} \sum_{j=1}^{J_d} s_j$ is the total number of negative deviations of all sensitivity characteristics for all indicators; $N - m$ is the number of negative deviations falling outside the project stability zone.

It is assumed that the probabilities of escaping from the zone of investment attractiveness (stability zone) for every sensitivity characteristic (discount rate, expenses, price, volume of production, demand, etc.) are often directly or indirectly linked. Therefore, it is reasonable to determine general risks of individual projects using the rule of multiplication of probabilities (it is possible to use conditional probabilities).

Risk management assumes iteration of the process of in-depth factor–factor analysis of sensitive characteristics (search for risk sources and solution for their compensation):

$$\begin{aligned} &\textit{sensitivity analysis} \rightarrow \textit{stability analysis} \\ &\qquad \rightarrow \textit{risk assessment} \rightarrow \textit{alteration of sensitive features} \\ &(\textit{improvement of given data}) \rightarrow \textit{new sensitivity analysis} \rightarrow \cdots . \end{aligned}$$

6.4.1 Simulation Modeling of Investment Risks

The essence of simulation modeling is repetitive reproduction of a random process by means of replacement of experimental measurement of the values of real characteristics by random processes by studying the values of these characteristics.

A random process is reproduced by means of a mathematical model; herewith the simulation model is in a constant cyclic operation process and situational correction. Simulation modeling makes it possible to replace a formal mathematical proof by expert decision based on the absence of contradictions between the model solution and the values of the corresponding characteristics determined by other methods, for example by methods of retrospective analysis. If the model conclusions and the conclusions obtained by other means coincide within the tolerance limits, more concrete model results can be considered as base data.

In terms of technology simulation modeling is a set of numerical experiments designated to produce empirical assessments of the degree of influence of various initial characteristics on some result depending on these characteristics (indexes, criteria).

Nowadays, the study of models of any complexity is apt to use software-programmable simulation methods based on numerical iterative procedures.

Simulation experiments on assessment and investment risk management can be classified according to the target criterion:

- Simulations aimed at improving correspondence of the risk model to the displayed investment process [correction methods on the basis of a gradually moving time base used to estimate coefficients of regression equations by the least-square method; insertion of the adaptive identification algorithm into models with variable structural coefficients].
- Simulations aimed at studying complex internal interactions in the social-technical system presented by IP; studying of the influence of changes in the investment project (risk) characteristics on project (risk) indicators [apparatus of production functions including variable parameters; adaptive approach taking dynamics into account].
- Simulations aimed at studying IP risk properties including dynamic properties such as time stability, fluctuating motion, etc. [study of models with constant coefficients related to a certain moment (period) of time; the concept of a model with variable coefficients requires dynamic investigation methods].
- Simulations aimed at constructing a forecast risk model and IP scenarios at all stages of the life-cycle. These provide a way to successfully direct the investment process in order to achieve the maximum effect from its implementation [simulation modeling in combination with adaptive approach].

From the mathematical point of view, the main idea of the Monte Carlo method is to construct an IP model with indefinite values of parameters and to obtain a distribution of project profitability that accounts for probabilistic distributions of project parameters and correlations between changes in parameters.[13]

[13] The existence of correlated variables in the project analysis can lead to incorrect results: computer iterations are a completely computerized part of the project risk analysis, therefore in considering key variables to be independent one can generate unrealistic project scenarios. For example, if there are two negatively correlated variables, say price and sales volume, and if their correlation coefficient is not exactly defined, there can be scenarios randomly generated by the computer where both variables are either high or low, which will negatively affect the result.

The generalized schematic of the method includes the following stages:

(1) to choose the criterion of project efficiency and to determine interrelations between input and output indices in the form of a mathematical model;
(2) to set a frequency distribution (to determine the type of probabilistic distribution, at least, approximately) for every variable influencing formation of the cash flow;
(3) to carry out computer simulation of the key parameters and to use them to calculate the resultant index value (in the framework of the model of probabilistic risk analysis it is necessary to make a large number of iterations, which determine the limits of criterion variation and its distribution if different values of key variables are substituted in the model in accordance with the given distribution);
(4) to use the obtained values of the chosen criterion of project efficiency to construct its probability density function;
(5) to calculate the main numerical characteristics of distributions of input and output indices (mathematical expectation, standard deviation, etc.);
(6) to calculate the coefficient of criterion variation and individual risk of the project;
(7) to interpret the obtained results (to make a statistical and probabilistic analysis) and to make a decision.

The results of the simulation experiment can be used as an informational support for making investment decisions provided that they are given correct statistical treatment [1, 10, 11, 18].

6.5 Quantitative Risk Analysis Based on the Methods of Fuzzy Mathematics

Let A be a universal set. For any object of the set $a \in A$ only two possibilities are considered: either the object belongs to the subset of F objects having a general property or the object does not belong to the subset (the attribute is defined by function μ_a):

$$\mu_a = \begin{cases} 1, & \text{if } a \in F, \\ 0, & \text{if } a \notin F. \end{cases}$$

In fact, the fuzzy boundaries between different gradations of a certain quality of the same notion are fuzzy. Let function μ_a take up any values from the interval [0; 1]. Hence, the object $a \in A$ may not belong to $F(\mu_a = 0)$, may belong to F to a small degree (the μ_a value is close to zero), may belong to F to a considerable degree (value μ_a is close to one), or may be beyond doubt an object of F (value $\mu_a = 1$).

Therefore, to describe fuzzy arguments fuzzy mathematics uses fuzzy sets [19–22] $F|F \in A$; $F = \{(a, \mu_a)\}$: a point (an object) is characterized by the degree of its belonging to the set, i.e., the characteristic functions μ_a of sets F can take any values from the whole interval [0; 1]. According to this definition, ordinary subsets are a subclass of the class of fuzzy sets.

In this context the investment risk assessment is the assessment of a measure of possibility of undesired events during the investment process.

In this case the membership function of the corresponding fuzzy numbers defining such events is known or is determined by special methods.

After analysis of all sensitive input design parameters, according to the principle of generalization, the membership function of the project output parameter is determined $R = R(t_1, \dots, t_m)$:

$$\mu_R(S) = \sup_{t_1..t_m | R(t_1..t_m) = s} \min\big(\mu_{x_1}(t_1), \dots \mu_{x_m}(t_m)\big),$$

where X_i are fuzzy numbers (fuzzy sets with domain in the form of a real axis interval R^1) with carriers $S_{X_i} = (t_{il}; t_{i2}), t_{il} > t_{i2}$; $\mu_{X_i}(t_i)$ is a possibility that a fuzzy value X_i will take up value t_i; $R(t_1, \dots, t_m) : R^1 \times \dots \times R^1 \to R^1$ a functional dependence of the output parameter on the input parameters, known by assumption.

The fuzzy number carrier R can be found according to the rules of interval arithmetic as

$$S_R = \big\{s \mid s = t_1 \otimes \dots \otimes t_m, t_i \in S_{X_i}\big\},$$

where $\otimes$ is an arithmetical operation.

The degree of information density defines a concrete interpretation of uncertainty (in probabilistic, interval, or fuzzy aspect) and leads to a situation biased either to the probabilistic distribution, to the fuzzy-interval assessment of the input/output information, or to the combined operation by principally different mathematical methods.

The reduction of requirements to the degree of information density enables us to broaden the constructive possibilities of the fuzzy subsets theory: fuzzy interval arithmetic is a much simpler construction than probability theory or mathematical statistics.

The following method can be used to calculate the distribution of possibilities π_X of a certain value X on the basis of its probabilistic distribution $p(x)$. If the variable X takes values from the set A, the possibilities distribution π_X on the set A of the variable X values is called the function

$$\pi_X : A \to [0; 1],$$

which assigns $\pi_X(a)$ to each $X \in A$. $\pi_X(a)$—the degree of possibility that a variable X takes a value a.

If a fuzzy set F of possible values of this variable with the membership function μ_F is given, then

$$\pi_X(a) = \mu_F(a), \quad a \in A.$$

In case of a discrete variable X with possible values $\{x_i\}$, $x_i \in \Omega$, and corresponding probabilities $\{p(x_i)\}$, the values $x_i \in \Omega$ must be reordered in such a way that

$$p_{i_1} \geq p_{i_2} \geq \cdots \geq p_{i_j} \geq \cdots \geq p_{i_M}.$$

For the above-mentioned ordering of the set Ω we have:

$$\begin{aligned} &\pi_{i_1} = \sum_{i=1}^{M} p_i = 1, \\ &\pi_{i_j} = j p_{i_j} + \sum_{k=j+1}^{M} p_{i_k}, \quad j = \overline{1; M-1}, \\ &\pi_{i_M} = M p_{i_M}. \end{aligned} \tag{6.39}$$

It is also possible to get a reverse transformation by solving a system of equations;

$$p_{i_j} = \sum_{k=j}^{M} \frac{1}{k}(\pi_{i_k} - \pi_{i_{k+1}}), \quad j = \overline{1; M}.$$

In case of continuous Ω the set of possible values $[x_{\min}; x_{\max}]$ is divided into J intervals $[x_{j\,\min}; x_{j\,\max}]$, where $x_{1\,\min} = x_{\min}$; $x_{j\,\min} = x_{(j-1)\,\max}$ $(j = 2, 3, \ldots, J)$; $x_{J\,\max} = x_{\max}$ and the problem of finding the function of possibility distribution π_X is reduced to solution of Eq. (6.39).

It is possible to change frequency for intervals whose width is defined by the α-significance level according to the following schema: to construct a model of the studied system in the assumption of absence of uncertainties; to replace the deterministic parameters by their exact or fuzzy interval estimations (fuzzy intervals must have the same α-significance level); to calculate fuzzy intervals for output model variables.

Such a transformation preserves quantitative information about the magnitudes and location of possible values and qualitative information about probabilities. This procedure is pictured schematically in Fig. 6.6.

Fuzzy interval A is a combination of α-level sets A_α:

$$A = \bigcup_{\alpha \in (0;1]} \alpha A_\alpha,$$

where $A_\alpha = \{a \mid \mu_A(a) \geq \alpha\}$ is a precise set; $\alpha A_\alpha = \{(\alpha; a) \mid a \in A_\alpha\}$ is a fuzzy set. The result of operations over fuzzy intervals will also be a fuzzy interval whose parameters can be found according to the following rule:

$$\text{if } A = \bigcup_{\alpha} \alpha A_\alpha; \qquad B = \bigcup_{\alpha} \alpha B_\alpha,$$

$$\text{then } A + B = \bigcup_{\alpha} \alpha (A+B)_\alpha = \bigcup_{\alpha} (\alpha A_\alpha + \alpha B_\alpha).$$

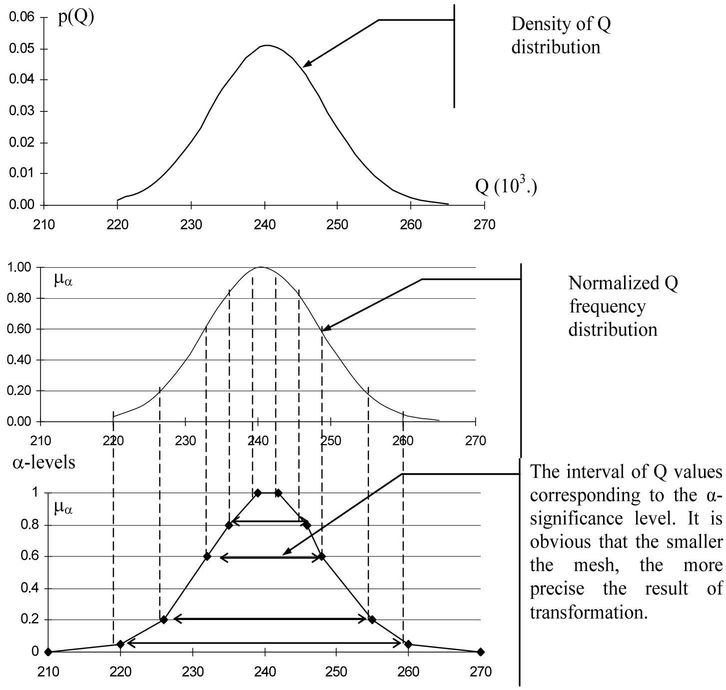


Fig. 6.6 Transformation from frequencies to intervals

The following arithmetic operations can be made with precise intervals $A = [A_1; A_2]$ and $B = [B_1; B_2]$ expressed as operations with real numbers, with the interval boundaries:

$$A + B = [A_1 + B_1; A_2 + B_2],$$

$$A - B = [A_1 - B_2; A_2 - B_1],$$

$$A \cdot B = \left[\min(A_1 \cdot B_1; A_2 \cdot B_2; A_1 \cdot B_2; A_2 \cdot B_1);\right.$$
$$\left.\max(A_1 \cdot B_1; A_2 \cdot B_2; A_1 \cdot B_2; A_2 \cdot B_1)\right],$$

$$A/B = [A_1; A_2] \cdot [1/B_2; 1/B_1],$$

$$[A_1; A_2]^k = \left[A_1^k; A_2^k\right].$$

An example of the results of modeling using the probabilistic and fuzzy procedures is shown in Fig. 6.7. The most probable values obtained by probabilistic and fuzzy methods do not always coincide; to a large extent, the divergences depend on the assumptions made at the stage of constructing the balance deterministic model and simulation presumptions.

For financial managers and appraisers of projects with investments of relatively small volumes, the most acceptable method of integral fuzzy estimation of the de-

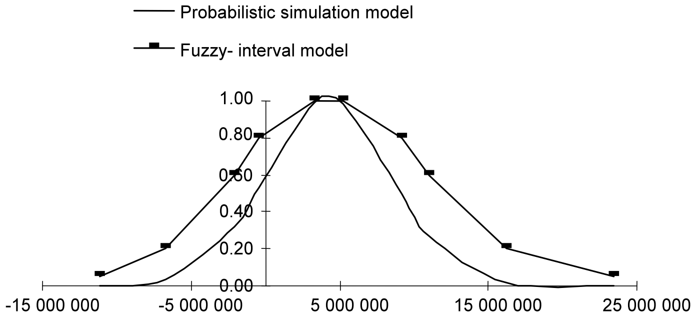


Fig. 6.7 Simulation results

Fig. 6.8 A membership function of the 'triangular' number

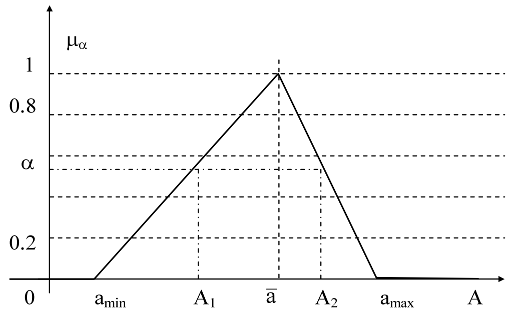
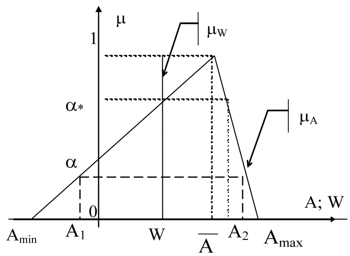


Fig. 6.9 A geometric illustration of making an investment decision



gree of risk index R_α is based on the approximation of the initial data by "triangular" fuzzy numbers. A "triangular" number A with the membership function μ_α (Fig. 6.8) is equivalent to the formalization of the following statement: Parameter A is approximately equal to $\bar{a}$ and is unambiguously located within the range $[a_{\min}; a_{\max}]$: $A = (a_{\min}; \bar{a}; a_{\max})$. In investment analysis such a simplification is rational as it corresponds to the most commonly used "pessimistic, normal and optimistic" scenarios for the initial data. In contrast to the probabilistic approach, the notion of randomness is replaced by the notions of expectedness and possibility.

Let A be the efficiency index of the project with fuzzy boundaries, and let W be a criterion of project efficiency. A geometric interpretation of investment decision-making is shown in Fig. 6.9.

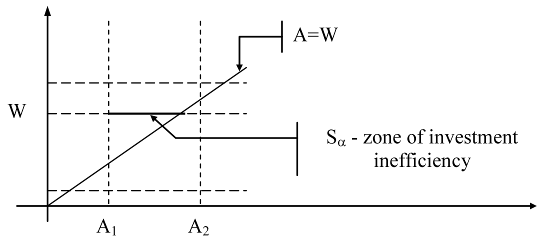


Fig. 6.10 Phase space (A, W)

Let the ordinate of the cross point of the membership functions of the IP efficiency indicator μ_A and the efficiency criterion μ_W be equal to α^* which is the upper boundary of the risk zone. In fact, a predetermined α-level defines the corresponding membership intervals $[A_1; A_2]$ and W (degenerate, precise number). If $\alpha > \alpha^*$, we have $A_1 > W$, which means a one-hundred-percent confidence in the project efficiency; in other words, the degree of risk of the project inefficiency is equal to zero.

If $0 \leq \alpha \leq \alpha^*$, the intervals intersect. This means there is an α-zone of inefficient investments (Fig. 6.10), a region limited by lines $w = W$; $A = A_1$; $A = A_2$ and by the bisecting line of the coordinate angle $A = W$.

The interrelations between parameters W, A_1 and A_2, corresponding to the given level (Fig. 6.10) give the following calculation of the zone of investment inefficiency (length of α-risk zone):

$$S_\alpha = \begin{cases} 0, & W < A_1 \\ W - A_1, & A_1 \leq W \leq A_2 \\ A_2 - A_1, & W > A_2. \end{cases}$$

Under the given membership level α all realizations (A, W) are equally possible, i.e., for $\alpha \in [0; 1]$ it is correct

$$r(\alpha) = \frac{S_\alpha}{A_2 - A_1} = \begin{cases} 0, & W < A_1 \\ \frac{W - A_1}{A_2 - A_1}, & A_1 \leq W \leq A_2 \\ 1, & W > A_2. \end{cases}$$

The integral measure of expectation of negative investment results or the index of the risk degree is

$$R_\alpha = \int_0^{\alpha_*} r(\alpha) d\alpha \qquad (6.40)$$

where α_* is the upper boundary of the risk zone.

It should be noted that:

- the values of the inverse function $\mu_A^{-1}(\alpha_*)$ are W' and W (according to the definition of upper boundary of the risk zone α_*);
- the values of $\mu_A^{-1}(0)$ are $A_{\max}$ and $A_{\min}$;

- $\bar{A}$ is the greatest expected value of the fuzzy number A.

The upper boundary of the risk zone can have the following values:

$$\alpha^* = \begin{cases} 0, & \text{for } W \le A_{\min}; \\ \frac{W - A_{\min}}{\bar{A} - A_{\min}}, & \text{for } A_{\min} < W < \bar{A}; \\ 1, & \text{for } W = \overline{A}; \\ \frac{A_{\max} - W}{A_{\max} - \overline{A}}, & \text{for } \overline{A} < W < A_{\max}; \\ 0, & \text{for } W \ge A_{\max}. \end{cases}$$

Indeed, if, for example, $A_{\min} < W < \bar{A}$, then from the properties of similar triangles (Fig. 6.9) it follows that $\frac{\alpha^*}{1} = \frac{W - A_{\min}}{\bar{A} - A_{\min}}$, etc. The unit value of the level α_* is marked for better illustration of variations of possible values.

From the similarity of triangles we have:

$$\frac{\alpha}{1} = \frac{A_1 - A_{\min}}{\bar{A} - A_{\min}} \Rightarrow A_1 = \alpha(\bar{A} - A_{\min}) + A_{\min}$$

$$\frac{\alpha}{1} = \frac{A_{\max} - A_2}{A_{\max} - \bar{A}} \Rightarrow A_2 = A_{\max} - \alpha(A_{\max} - \bar{A})$$

In this case the degree of risk for the given level α is

$$\begin{aligned} r(\alpha) &= \frac{W - A_1}{A_2 - A_1} = \frac{W - \alpha(\bar{A} - A_{\min}) - A_{\min}}{A_{\max} - \alpha(A_{\max} - \bar{A}) - \alpha(\bar{A} - A_{\min}) - A_{\min}} \\ &= \frac{W - \alpha\bar{A} - A_{\min}(1 - \alpha)}{(1 - \alpha)(A_{\max} - A_{\min})} \\ &= \frac{1}{(A_{\max} - A_{\min})} \cdot \left[W \frac{1}{(1 - \alpha)} - \bar{A} \frac{\alpha}{(1 - \alpha)} - A_{\min} \right], \end{aligned}$$

and the integral risk level (6.40) is:

$$\begin{aligned} R_\alpha &= \int_0^{\alpha^*} r(\alpha) d\alpha \\ &= \frac{1}{A_{\max} - A_{\min}} \left[-W \ln(1 - \alpha) + \bar{A}\big(\ln(1 - \alpha) + \alpha\big) - A_{\min} \cdot \alpha \right] \Big|_0^{\alpha^*} \\ &= \frac{\bar{A} - W}{A_{\max} - A_{\min}} \ln\big(1 - \alpha^*\big) + \frac{\bar{A} - A_{\min}}{A_{\max} - A_{\min}} \alpha^*. \end{aligned}$$

Let us consider the following case $\alpha_* = \frac{W - A_{\min}}{\bar{A} - A_{\min}}$, $A_{\min} < W < \bar{A}$.

A transformed special value of the risk degree will be:

$$R_\alpha = \frac{(\bar{A} - A_{\min}) - (W - A_{\min})}{A_{\max} - A_{\min}} \ln(1 - \alpha_*) + \frac{\bar{A} - A_{\min}}{A_{\max} - A_{\min}} \cdot \frac{W - A_{\min}}{\bar{A} - A_{\min}}$$

$$= \left(\frac{\bar{A} - A_{\min}}{A_{\max} - A_{\min}} \cdot \frac{W - A_{\min}}{W - A_{\min}} - \frac{W - A_{\min}}{A_{\max} - A_{\min}} \right) \ln\bigl(1 - \alpha_*\bigr) + \frac{W - A_{\min}}{A_{\max} - A_{\min}}$$

$$= \frac{W - A_{\min}}{A_{\max} - A_{\min}} \left[\left(\frac{1}{\alpha_*} - 1 \right) \ln(1 - \alpha_*) + 1 \right]$$

$$= \frac{W - A_{\min}}{A_{\max} - A_{\min}} \left(\frac{1 - \alpha_*}{\alpha_*} \ln(1 - \alpha_*) + 1 \right).$$

Denoting $R = \frac{W - A_{\min}}{A_{\max} - A_{\min}}$, we will have

$$R_\alpha = R\left(\frac{1 - \alpha_*}{\alpha_*} \ln(1 - \alpha_*) + 1 \right) \quad \text{for } \alpha_* = \frac{W - A_{\min}}{\bar{A} - A_{\min}}, \; A_{\min} < W < \bar{A}$$

To derive the formula of risk degree in case $\alpha_* = \frac{A_{\max} - W}{A_{\max} - \overline{A}}$ for $\overline{A} < W < A_{\max}$, we must calculate the auxiliary quantities:

$$R = \frac{W - A_{\min}}{A_{\max} - A_{\min}} \Rightarrow \left| \begin{array}{l} W = R(A_{\max} - A_{\min}) + A_{\min}; \\ 1 - R = \frac{A_{\max} - W}{A_{\max} - A_{\min}}; \end{array} \right.$$

$$\begin{aligned} \alpha_* = \frac{A_{\max} - W}{A_{\max} - \overline{A}} \Rightarrow \overline{A} &= -\frac{A_{\max} - W}{\alpha_*} + A_{\max} \\ &= -\frac{A_{\max} - R(A_{\max} - A_{\min}) - A_{\min}}{\alpha_*} + A_{\max} \\ &= A_{\max} - \frac{(1 - R)(A_{\max} - A_{\min})}{\alpha_*}. \end{aligned}$$

Substituting the obtained expressions in the formula of the integral risk level (6.40), we get:

$$\begin{aligned} R_\alpha &= \frac{A_{\max} - \frac{(1-R)(A_{\max} - A_{\min})}{\alpha_*} - R(A_{\max} - A_{\min}) - A_{\min}}{A_{\max} - A_{\min}} \ln(1 - \alpha_*) \\ &\quad + \frac{A_{\max} - \frac{(1-R)(A_{\max} - A_{\min})}{\alpha_*} - A_{\min}}{A_{\max} - A_{\min}} \alpha_* \\ &= \frac{(A_{\max} - A_{\min})(1 - \frac{1-R}{\alpha_*} - R)}{A_{\max} - A_{\min}} \ln(1 - \alpha_*) \\ &\quad + \frac{(A_{\max} - A_{\min})(1 - \frac{1-R}{\alpha_*})}{A_{\max} - A_{\min}} \alpha_* \\ &= (1 - R)\left(1 - \frac{1}{\alpha_*} \right) \ln(1 - \alpha_*) + \left(1 - \frac{1 - R}{\alpha_*} \right) \alpha_* \\ &= \alpha_* - (1 - R)\left(\frac{1 - \alpha_*}{\alpha_*} \ln(1 - \alpha_*) + 1 \right). \end{aligned}$$

A generalized formula of the risk degree taking into account the boundary conditions is written as:

$$R_\alpha = \begin{cases} 0, & \text{if } W \le A_{\min}, \\ R \cdot (\frac{1-\alpha_*}{\alpha_*} \cdot \ln(1-\alpha_*) + 1), & \text{if } A_{\min} < W \le \bar{A}, \\ \alpha_* - (1-R) \cdot (\frac{1-\alpha_*}{\alpha_*} \cdot \ln(1-\alpha_*) + 1), & \text{if } \bar{A} \le W < A_{\max}, \\ 1, & \text{if } W \ge A_{\max}, \end{cases} \tag{6.41}$$

where

$$R = \begin{cases} \frac{W - A_{\min}}{A_{\max} - A_{\min}}, & \text{if } W < A_{\max}, \\ 1, & \text{if } W \ge A_{\max}, \end{cases} \tag{6.42}$$

$$\alpha_* = \begin{cases} 0, & \text{if } W \le A_{\min}, \\ \frac{W - A_{\min}}{\bar{A} - A_{\min}}, & \text{if } A_{\min} < W < \bar{A}, \\ 1, & \text{if } W = \overline{A}, \\ \frac{A_{\max} - W}{A_{\max} - \overline{A}}, & \text{if } \overline{A} < W < A_{\max}, \\ 0, & \text{if } W \ge A_{\max}. \end{cases} \tag{6.43}$$

It is obvious that the greater the uncertainty of the initial data, the higher the risk level. The limiting cases of index R_α (for $W = A_{\min}$; $W = W' = \bar{A}$ and $W = A_{\max}$, i.e., extremely low, average, and extremely high risks) give $R_\alpha \in [0; 1]$. In other words, the constructed risk index satisfies the requirements of the measure function.

The IP model taking into account the unaggregated cost characteristics of the project is expressed as:

$$NPV = \sum_{t=1}^{n} \frac{NCF_t}{(1+r)^t} + \frac{S}{(1+r)_n} - I_0 \tag{6.44}$$

where NCF_t is the amount of the net flow of annuity payments for the period t:

$$NCF = \left[Q(P-V) - F - A\right] \cdot (1-T) + A$$

Q is the production volume;
P is the price (1 item);
V is variable costs;
F is constant costs;
A is amortization;
T is income tax;
r is discount rate;
n is the duration of the project life cycle;
S is the depreciated cost of the project;
I_0 is initial investments.

Let sensitive parameters of the model (6.44) be defined with a certain degree of initial or calculated (from the probabilistic distribution) fuzziness:

$I_0 = (I_{0\min}; \overline{I_0}; I_{0\max})$—the expert cannot exactly estimate the volume of investments at the moment of making decisions;
$r_t = (r_{t_{\min}}; \overline{r_t}; r_{t_{\max}})$—the expert cannot exactly estimate the cost of capital used in the project (the debt-to-equity ratio, the interest rate for long-term credit, etc.);
$NCF_t(NCF_{t_{\min}}; \overline{NCF}; NCF_{t_{\max}})$—the expert forecasts the range of variations of money making as a result of project realization, taking into account possible variations in price for marketable goods, costs of consumed resources, terms of taxation and other factors;
$S = (S_{\min}; \overline{S}; S_{\max})$—the expert does not exactly realize potential conditions of the future sale of the ongoing business or its liquidation.

Setting the α-membership level defines the reliability intervals $[A_1; A_2]$ (Fig. 6.8). Applying interval arithmetic rules to the initial data $[I_{01}; I_{02}]$, $[r_{t1}; r_{t2}]$, $[NCF_{t1}; NCF_{t2}]$ and $[S_1; S_2]$, we get:

$$[NPV_1; NPV_2] = \left[\sum_{t=1}^{n} \frac{NCF_{t1}}{(1+r_{t2})^t} + \frac{S_1}{(1+r_{n2})^n} - I_{02}; \sum_{t=1}^{n} \frac{NCF_{t2}}{(1+r_{t1})^t} + \frac{S_2}{(1+r_{n1})^n} - I_{01}\right].$$

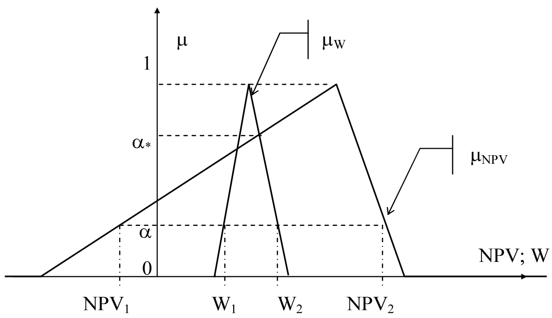
Thus, we can reconstruct the fuzzy number $NPV = (NPV_{\min}; \overline{NPV}; NPV_{\max})$ by two methods:

(1) by fixing the acceptable α-level of discretization and approximating the membership function μNPV by the broken line on interval points;
(2) by converting the fuzzy number NPV to a triangular form using calculations on significant points of fuzzy numbers of the initial data (the condition $\alpha = 0$ defines key parameters in the risk assessment, not approximately but on the basis of analytical relations).

A fuzzy risk assessment (6.41) for the precise criterion W of the indicator NPV is expressed as:

$$R_{NPV}(\alpha) = \begin{cases} 0, & \text{when } W \le NPV_{\min}, \\ R \cdot (\frac{1-\alpha_*}{\alpha_*} \ln(1-\alpha_*) + 1), & \text{when } NPV_{\min} < W \le \overline{NPV}, \\ \alpha_* - (1-R) \cdot (\frac{1-\alpha_*}{\alpha_*} \ln(1-\alpha_*) + 1), & \text{when } \overline{NPV} \le W < NPV_{\max}, \\ 1, & \text{when } W \ge NPV_{\max}, \end{cases}$$

Fig. 6.11 The membership function for *NPV* and *W*



where auxiliary quantities (6.42) and (6.43) are:

$$\alpha_* = \begin{cases} 0, & \text{for } NPV_{\min} > W, \\ \frac{W - NPV_{\min}}{\overline{NPV} - NPV_{\min}}, & \text{for } NPV_{\min} < W < \overline{NPV}, \\ 1, & \text{for } W = \overline{NPV}, \\ \frac{NPV_{\max} - W}{NPV_{\max} - \overline{NPV}}, & \text{for } \overline{NPV} < W < NPV_{\max}, \\ 0, & \text{for } NPV_{\max} \le W, \end{cases}$$

$$R = \begin{cases} \frac{W - NPV_{\min}}{NPV_{\max} - NPV_{\min}}, & \text{for } W < NPV_{\max}, \\ 1, & \text{for } W \ge NPV_{\max}. \end{cases}$$

The calculations of the fuzzy risk can be modified by using more complicated conditions (higher degree of uncertainty). For example, let a fuzzy criterion of the indicator NPV be added to the previous fuzzy conditions:

$W = (W_{\min}; W_{\max})$—there is no precise decision for the criterion according to which the project can be recognized as efficient.[14]

A geometric interpretation of the procedure of making an investment decision is shown in Fig. 6.11. Let the ordinate of the cross point of the membership functions of the IP μNPV efficiency indicator and the efficiency criterion μW be α_* be the upper boundary of the risk zone. In fact, a predetermined α-level defines the corresponding membership intervals $[NPV_1; NPV_2]$ and $[W_1; W_2]$. If $\alpha > \alpha_*$, we get $NPV_1 > W_2$, i.e., the membership intervals do not cross, which means that there is a one-hundred-percent confidence in the project efficiency—or in other words, the degree of risk of the project inefficiency is equal to zero.

If $0 \le \alpha \le \alpha_*$, the intervals intersect; this means that there is an α-zone of inefficient investments (Fig. 6.12), a crosshatched region limited by lines $W = W_1$, $W = W_2$, $NPV = NPV_1$, $NPV = NPV_2$ and by the bisecting line of the coordinate angle $NPV = W$—the phase space $(NPV; W)$.

[14]The classical criterion is $W = 0$. However, based on the strategic plans W can take any value, even negative: if the project diversifies the investor's activities and improves reliability of his business or the investor consciously takes an increased risk for the sake of increase in the weighted average capital return, the marginal project is accepted.

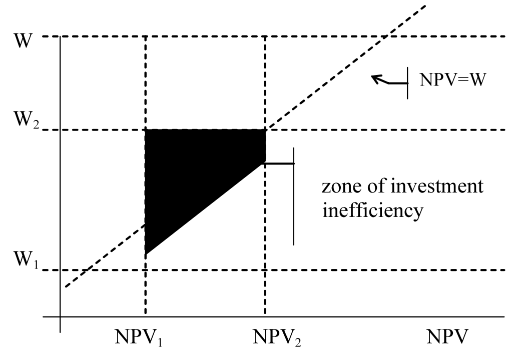


Fig. 6.12 Phase space (NPV, W)

Taking into account interrelations between parameters W_1, W_2, NPV_1, and NPV_2 the area of the α-risk domain is calculated as follows:

$$S_\alpha = \begin{cases} 0, \quad \text{for } NPV_1 \geq W_2, \\ 0.5 \cdot (W_2 - NPV_1)^2, \quad \text{for } W_2 > NPV_1 \geq W_1; NPV_2 \geq W_2, \\ 0.5 \cdot (W_1 - NPV_1 + W_2 - NPV_2) \cdot (W_2 - W_1), \\ \quad \text{for } NPV_1 < W_1; NPV_2 \geq W_2, \\ (W_2 - W_1) \cdot (NPV_2 - NPV_1) - 0.5 \cdot (NPV_2 - W_1)^2, \\ \quad \text{for } NPV_1 < W_1 \leq NPV_2 < W_2, \\ (W_2 - W_1) \cdot (NPV_2 - NPV_1), \quad \text{for } NPV_2 \geq W_1. \end{cases}$$

All realizations of the phase space (NPV; W) corresponding to the α-level are equally feasible. Therefore the degree of risk of an inefficiency $r(\alpha)$ is a geometric probability of the event that point (NPV; W) falls in the zone of investment unattractiveness:

$$r(\alpha) = \frac{S_\alpha}{(W_2 - W_1) \cdot (NPV_2 - NPV_1)}.$$

With an extensive information database a conclusion can be drawn about the expediency of constructing a functional dependence (regression) of the probabilistic and fuzzy risk factors as well as their reliability levels.

It should be noted that the risk assessment can be corrected during the life cycle of project realization by recalculating the interval factor to account for changes in factual data after a passage of time.

As the international system of enterprise efficiency factors envisages several risk indicators, each of which has its own acceptability threshold (probably, a fuzzy efficiency criterion), the resultant risk degree can be estimated as $R_\alpha = \max_i R_{\alpha_i}$, where R_{α_i} is assessment of the i-th criterion.

The degrees of risk are characterized linguistically as, for example, 'risk levels', along with their range {Minimum, Average, Possible, Extreme, Disastrous}. Therefore, having defined five membership functions $\mu(R_\alpha)$, every investor can make a description of the corresponding fuzzy subsets, referring to the particular project [1, 15, 16].

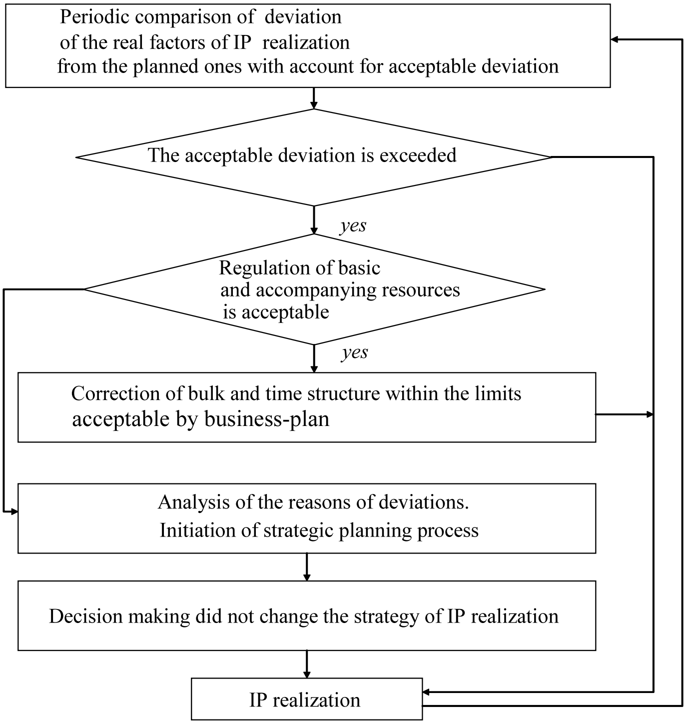


Fig. 6.13 Tasks in strategic control of IP logistics

6.6 Information Support for the Investment Project Analysis

In terms of risk analysis of the IP [23, 24], the most interesting aspect is the strategic control of the IP logistics, the main tasks of which are presented as a block diagram in Fig. 6.13.

Risk is the possibility of adverse events—the potential for destabilizing factors to impact IP elements, which can happen at any moment of time if certain conditions are formed. A rough quantitative description of properties important for studying the real system—a source of information for quantitative risk assessment of the investment project—is obtained by forecasting methods which must give practically the same results:

- choice of the same variant from a variety of possible variants;
- the same assessment of the consequences of the chosen variant.

The forecasting stage is preceded by the iterative procedure of constructing a mathematical model which is presented by:

- mathematical objects reflecting factors of the process-flow and influences on the system;
- relations between mathematical objects.

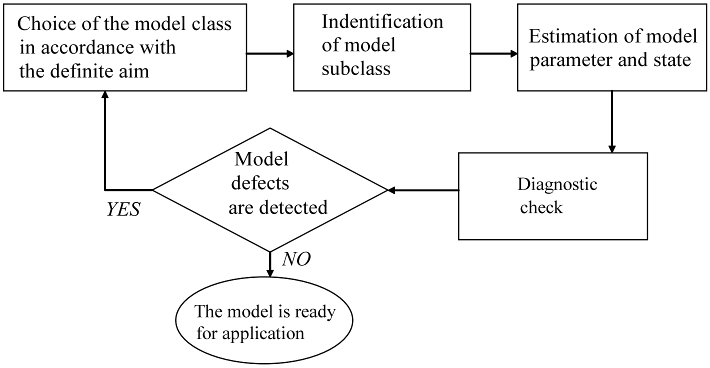


Fig. 6.14 Modeling IP system

The system approach to modeling presupposes studying successively *decreasing* levels of generalization of information about the main factors and their interrelations. In other words to formulate the IP targets one must:

- *detect* factors 'sensitively' influencing the IP cash flows;
- *subdivide* all factors into deterministic factors (with unambiguous estimate of influence), risk factors (with probabilistic estimate of influence), and uncertainty factors (with fuzzy estimate of influence);
- *estimate* the information field of the investment decision-making depending on the dominance of one or another group of factors: future of the IP, calculated alternatives of the future IP, multiplicity of IP outcomes within the intervals of parameter variations; complete uncertainty of the future IP.

The analytical models must take into account the ratio of the sensitive risk factors and uncertainty. Therefore modeling of the dynamic system of investment planning is a parametric iterative process (Fig. 6.14) as it is a real-time control system for the object whose characteristics change in time and *a priori* information is absent or insufficient.

The information for the model can be subdivided into three types:

- information about system structure (in the form of mathematical equalities, schematics, networks, diagrams, coupling matrices, etc.);
- information about parameter values (quantities not depending on input);
- information about state values (dependent variables) at a fixed moment of time (or time function).

The main purpose of the modeled system is to provide information support for the group investment decision on the project after preliminary comparison of different variants accounting for risk. The functional structure of the IP management system is shown in Fig. 6.15.

Risk is characterized by at least two parameters—possibility (probability in some cases) of occurrence of an undesirable event (a threat) P and estimated scope of its consequence X: $R = \{P; X\}$.

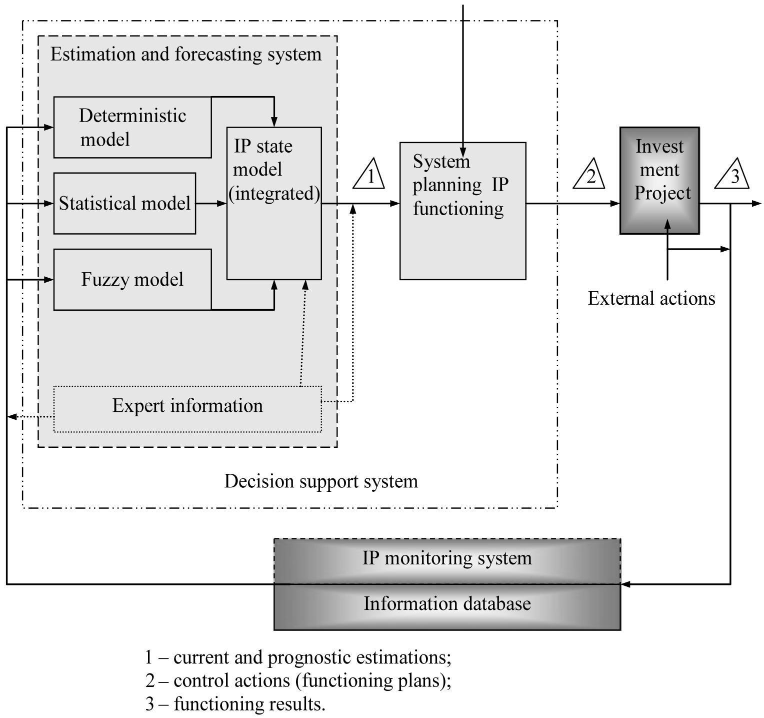


Fig. 6.15 A functional system of IP management

In such an approach the generalized scheme for analyzing the investment project risk has the following structure (Fig. 6.16), where much attention is paid to strategic controlling of the IP logistics (Fig. 6.13).

It is necessary to provide data quality control, as well as control and description of uncertainties and assumptions—in other words, qualitative methodological and informational support (Fig. 6.17).

6.6.1 *Filtration of Investment Projects*

Figure 6.18 is a schematic of investment project filtration.

- The idea can be formalized according to the *acceptability criterion* (due to limited funding allocated to the enterprise) in the following way.

Let every project pr_i, $i = \overline{1; I}$ be characterized by a complex of criteria (quantitative and qualitative) $\{R_1, R_K\}$; let every j-th investor $(j = \overline{1; J})$ be characterized by a complex of preference functions f_j. The limiting volume of investment resources I_0 (not taking reinvestment into account) is determined.

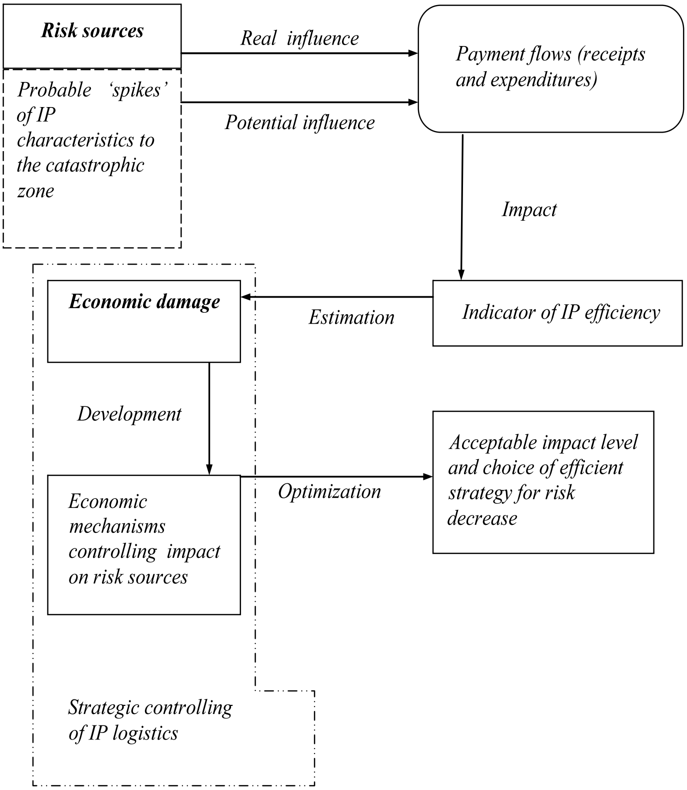


Fig. 6.16 A conceptualization of IP risk analysis

Under these conditions in the finite set of investment projects $Pr = \{pr_1, pr_I\}$ we choose pairwise disjoint subsets: Pr_{D^*}—the best projects for which the loan capital volume required for financing does not exceed I_0; Pr_D—projects with acceptable efficiency; and Pr_D—projects that are financially unacceptable, i.e.

$$Pr \xrightarrow{R,f,I_0} \begin{bmatrix} Pr_{D^*} \\ Pr_D \\ Pr_D \end{bmatrix} \quad \text{such that}$$

$$Pr = Pr_{D^*} \cup Pr_D \cup Pr_D;$$

$$Pr_{D^*} \cap Pr_D = \varnothing; \qquad Pr_{D^*} \cap Pr_D = \varnothing; \qquad Pr_D \cap Pr_D = \varnothing;$$

$$\sum_{pr_i \in Pr_{D^*}} R_{I_0}(pr_i) \leq I_0,$$

where $R_{I_0}(pr_i)$ is the volume of investment (loan) resources for financing the i-th project pr_i by one of the criteria $R_{I_0} \in \{R_1, \ldots, R_K\}$.

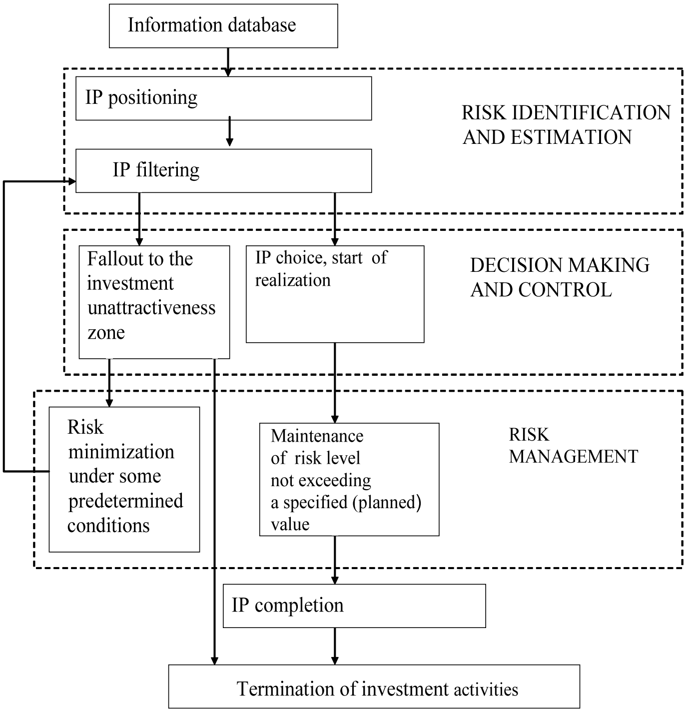


Fig. 6.17 Sequence of stages of risk investment activity

When a single investment project is analyzed its belonging to one of the above subsets is checked.

- Quantitative IP filtration according to the *profitability criterion*.

The essence of the procedure is a preliminary quantitative assessment of the profitability of the set $Pr = \{pr_1, pr_I\}$, thereby reducing the number of projects proposed for financing.

In case of a single project there is a possibility to approximately estimate its expediency (effectiveness, profitability), in other words whether it belongs to the set Pr_R.

The idea of quantitative filtration is to separate a filtered subset of profitable projects $Pr_R \subseteq Pr$ from the set of investment projects $Pr = \{pr_1, pr_I\}$ in accordance with the optimality principle.

Every element (project) $pr_{i_R} \in Pr_R, i_R = \overline{1; I_R}, I_R \leq I$ of this subset either satisfies the boundary W_k $(k = \overline{1; K})$ of economic efficiency (in accordance with the chosen criterion $R_k \in R$) or is in agreement with the acceptability threshold (critical value) established by the decision maker, exceeding this boundary $\Re_k \succ W_k$.

"Free" setting of the acceptability threshold provides flexibility according to changes in R preferences of different investors. For example, the classical criterion

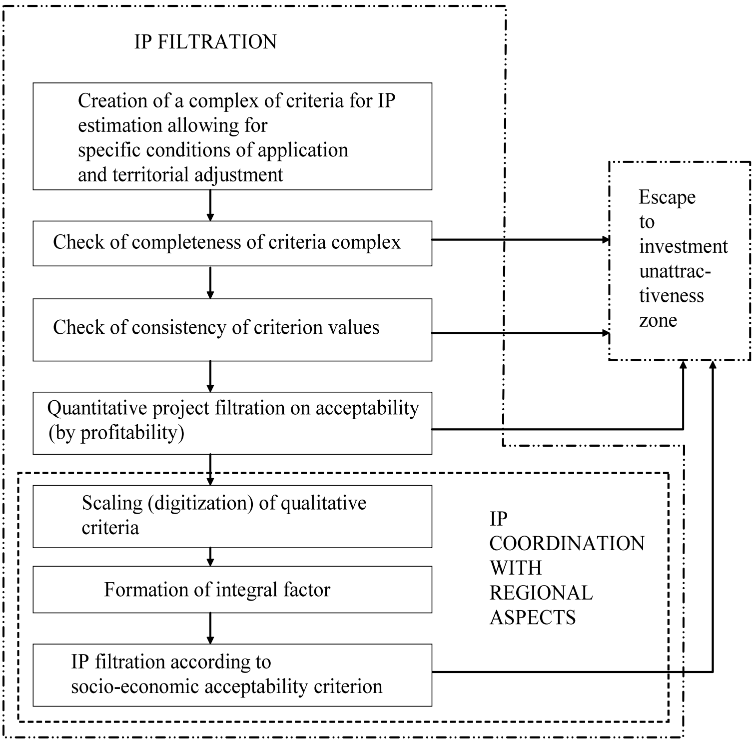


Fig. 6.18 Stages of IP filtration

of investment decision-making on indicator NPV is comparison with the base value $W = 0$. However, according to the agreement it can be practically any W value, even negative, depending on the investment purpose.

The principle of optimality is generally expressed by the multiple choice function:

$$F_R(Pr) = \left\{ pr_i \in Pr_R \subseteq Pr \,\middle|\, R_k(pr_i) > \Re_k, \Re_k \neq \text{const} \right\},$$

where $R_k\ (pr_i)$ is estimation of the i-th variant of the investment project by the k-th quantitative criterion R_k.

The projects that do not pass screening for acceptable profitability fall into the investment unattractiveness zone.

Efficient (allowing for preferences) suggestions are subjected to further analysis, for example on adaptability from the social point of view.

Filtration on profitability is based on project regulation not only according to one criterion NPV:[15] each criterion describes a particular aspect of the financial state

[15]The choice of this indicator for illustration of some situations is explained by its popularity.

of the project, and only taken together do they give a clear picture of investment solvency.

It is necessary here to take into account the possibility of exogenously assigned reinvestment rate and the corresponding models for calculation of the modified IP efficiency index.

The method of ranking projects with substantially different implementation periods can be based on the principle of "repetition recurrence of unlimited number of times":

$$NPV(K,\infty) = \lim_{n\to\infty} NPV(K,n) = NPV(K)\cdot\frac{(1+r)^K}{(1+r)^K-1},$$

where K is the project duration in case of single implementation; $NPV(K)$ is a net reduced effect of the initial project.

- Expansion of IP filtration by the *values of aggregate factors*

Here the idea of filtration is interpreted as shrinkage of the filtered set Pr_R of projects acceptable in terms of profitability: $Pr_A \subseteq Pr_R$ (or expansion of the conditions for screening unacceptable projects).

Using the method of superposition of limitations (upper and/or lower acceptability threshold for each k-th category) we sift out the projects whose aspect magnitudes do not satisfy the acceptability threshold $\Re_k$ established by the user. The choice function takes the form:

$$F_A(Pr_R) = \left\{pr_i \in Pr_A \subseteq Pr | A_k(pr_i) > \Re_k, \Re_k \neq \text{const}\right\},$$

where pr_i is the i-th IP variant from the set of projects acceptable in terms of profitability; $A_k(pr_i)$ is the estimation of the i-th IP by the k-th aspect; $\Re_k$ is the acceptability threshold by the k-th aspect [25].

6.7 Examples of Investment Decision-Making

6.7.1 Assessment of Investment Project Variants

Much printed material is published on the principles of a small-scale business operation. A narrower specialization is the publication of scientific-technical texts, which is typical of practically all large educational institutions.

To take into account all possible risks of investing in the project of organization and operation of a university publishing centre (PC), we constructed a risk tree formed in parallel with construction of the tree of PC targets. This refers to the class of iterative procedures, as the tree of targets is formed by two sorts of expert/logical operations:

- identification of interrelations between the structure of requirements (services and the facilities to provide them), types and forms of services;
- numerical estimation of identified interrelations

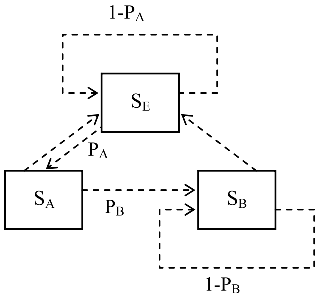


Fig. 6.19 Problem formalization

which are not completely formalized and which are implemented on the basis of an expert-simulation model. The model represents a formal schema 'target → target programs → resources → resource programs → plan' that enables us to simulate the system control procedure for which the main object of efforts is a person (an employee, a student of North Kazakhstan State University) with his needs. The model is constructed on an invariant basis (irrespective of external add-ins). If it is necessary to take into account the environmental influence, a complementary element is included in the model but the model structure, the element order, relations, and interrelations do not change [26–28].

6.7.1.1 Problem formalization

For PC operation (as a complex social-technical system S) it is difficult to formulate the notion of 'failure'—as a rule, physical ageing and/or obsolescence of certain elements leads to some deterioration of its 'life' quality but not to a full stop (catastrophe) of the system.

Taking into account a short life cycle of information technologies (the main PC elements), the following situation is typical—from time to time it is necessary to take one of the alternative decisions on investments. In particular, for the PC under analysis these two main strategies were formulated:

V_1—an extensive strategy increasing the efficiency of system functioning: production accretion (tactics—lowering of expenses; strategy—lowering of incomes and/or the degree of needs satisfaction);

V_2—an intensive strategy increasing the efficiency of system functioning: technological breakthrough (tactics—increased expenses; strategy—increasing incomes and/or the degree of needs satisfaction).

Waiting applications are removed if there is not enough production capacity or there is no appropriate technology.

Figure 6.19 shows formalization of the problem.

The state E reflects 'stationary' object functioning with efficiency w_E, moreover, there is a possibility of investment.

The use of strategy V_1 with probability P_A brings the system to a state A characterized by the response (efficiency) w_A. After some time the system returns to its 'initial' condition—the need to make a decision.

Table 6.3 System state

System state at phase start	System state at the end of the phase		
	E	A	B
E	$P_A \leq R < 1$	$0 \leq R < P_A$	–
A	–	$P_B \leq R < 1$	$0 \leq R < P_B$

The use of strategy V_2 with probability P_B brings the system to a state B characterized by the response (efficiency) w_B. Thus, the stage of 'accretion' is an intermediate state, as after a certain period of time (the obsolescence of technology) the necessity of making a certain decision arises again (the state B can be considered as the state E).

The choice of strategy/strategies and rules is predetermined by the information about the system state, its prehistory and functioning purposes.

In this case it is necessary to take into account a large number of possible states as the specificity of the system does not allow it to be simplified, for example, reduction to mass publishing, reduction in the range of products, etc.

Therefore, in spite of the popularity of analytical study of practical decision-making problems (formalized in the framework of the theory of controlled random processes, Markov processes, non-Markov processes reduced to Markov processes, etc.), in practice the simulation approach is valid.

S modeling implies construction of the sampling of a standard basic (critical) number R and a state table S at the end of the phase of application processing for a certain service (Table 6.3), which is used to imitate the state of the system (service start from state A is possible only under the second strategy).

Here we compare the results of different strategies, all functioning in the same situation: without stops, under the same initial conditions, with the same random numbers used in model testing.

In this connection a statistical problem arises—the use of common random numbers leads to correlation between responses.

It is obvious that to compare two strategies it is possible to examine not absolute values of system responses but their difference, i.e., the dispersion of the difference between the two compared models is:

$$D_{A-B} = D_A + D_B - 2\text{cov}_{AB} = D_A + D_B - 2r_{AB}\sigma_A\sigma_B.$$

The use of strategies V_1 and V_2 corresponds to the two systems with identical direction of response variation (the systems show the same reaction on the values of input variables—both strategies stimulate increase in the efficiency of IP operation), that is, we observe positive correlation between the responses of the systems being compared.

To synchronize events in the simulated system, instead of two flows of random numbers, a single flow where some random numbers are omitted is used (Fig. 6.20).

N is a planned number of served applications (if required it is possible to consider nomenclature), $S_{\max}$ is the required number of simulation periods. Blocks 2–3: The initial values are set, the system is in the state of "necessity of decision-making."

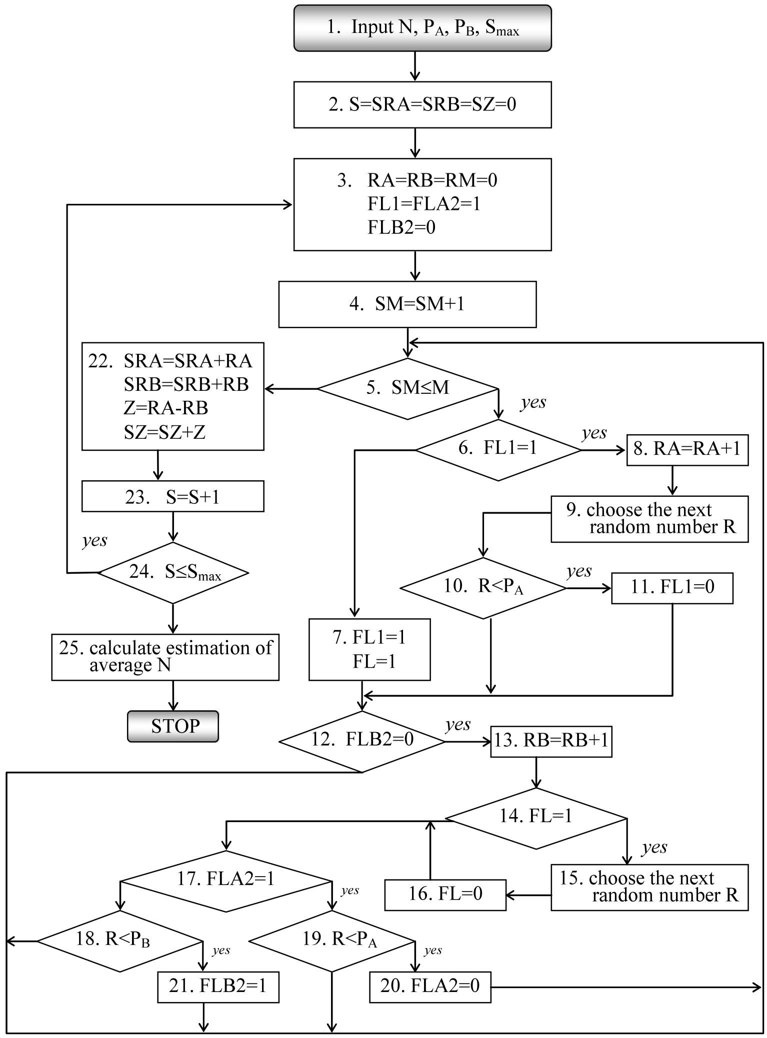


Fig. 6.20 A synchronization algorithm

Block 4 reflects the counter SM of incoming applications for services for one period, with block 5 checking completion of simulation for this period. The next period S is imitated in block 23. The counters of served applications correspond to strategies RA and RB.

Notations of logical variables (flags):

$$FL1 = \begin{cases} 1, & \text{the system is in state } E \text{ for strategy } V_1; \\ 0, & \text{otherwise;} \end{cases}$$

$$FL2 = \begin{cases} 1, & \text{the system is in state } E \text{ for strategy } V_2; \\ 0, & \text{the system is in state } A \text{ for strategy } V_2; \end{cases}$$

$$FLB2 = \begin{cases} 1, & \text{the system is in state } B \text{ for strategy } V_2; \\ 0, & \text{otherwise.} \end{cases}$$

The synchronization itself (parallel functioning of two models) proceeds as follows: *A* random number R_i is generated and is used to imitate operation *S* first for strategy V_1, then for V_2; then the next number R_j is generated, and so on. The state of the system is estimated at the end of the period, and therefore, to avoid usage of the same random number in two successive periods of application handling in the same strategy, a logical variable is introduced:

$$FL = \begin{cases} 1, & \text{the } \textit{system} \text{ is in state } B \text{ for strategy } V_1; \\ 0, & \text{otherwise} \end{cases}$$

which provides generation of a new random number *R* for strategy V_2, which was not used in strategy V_1 (the 'extra' *R* for strategy V_1—state *B* is rejected).

A concrete estimation of strategies was made by comparison of the average number of completed orders for the corresponding transition probabilities and/or calculation of IP operating efficiency (specificity of the given IP is that profit is not always important). The decision sensitivity is analyzed by setting different values of transition probabilities as input data and studying models in all possible ranges of variables [24, 29].

6.7.1.2 Creation of Information Database

The task solution—organization and support of operation of the publishing centre—had a variety of scenarios because of numerous answers to numerous questions, for example: Must we open our own publishing house? Do we have a material base? Where can we get means to organize production and circulating assets for the first editions? What technology should we choose and, accordingly, what is the sequence of equipment purchase? Will the new enterprise survive in its competitive environment? This variety of problems generates numerous sources of risk, hence in addition to financial estimation of the business plan (investment project) it is necessary to make a risk analysis.

As the information base we used expert (professionals in similar enterprises, information technology specialists, specialists with appropriate qualifications) estimations of diagnostics of the university investment environment and specialized literature materials. Expert estimations were taken into account with the following corrections:

– for traditional (popular) and latest technologies;

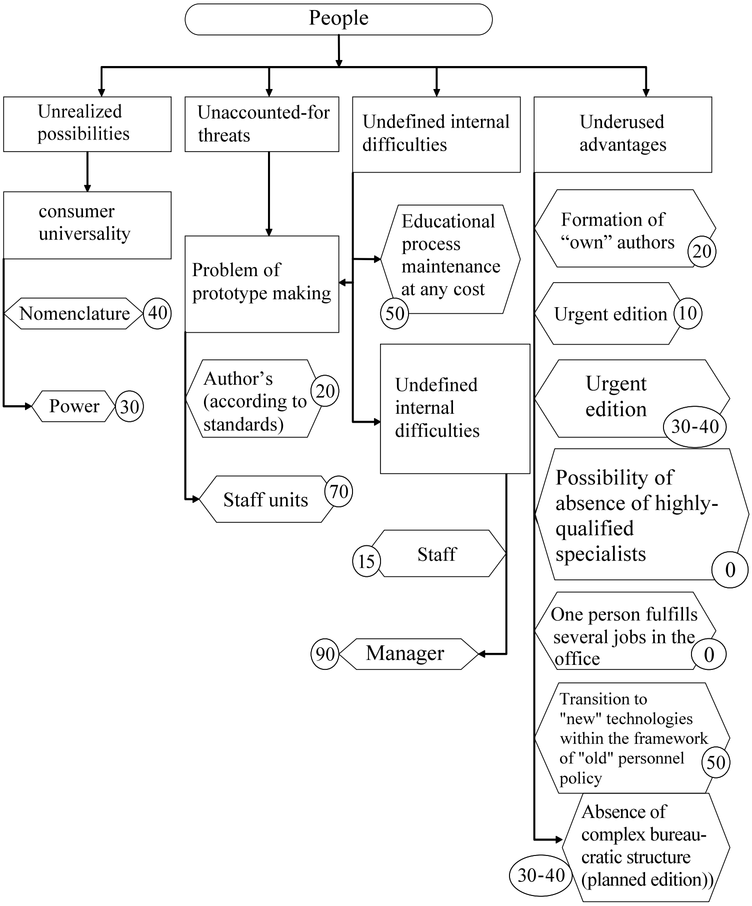


Fig. 6.21 The result of processing information on the main characteristics of the model

– for the main idea of the model of the university publishing house—a supplement of the new parameters (satisfaction of creative needs of publishers and printers at the stage of production and marketing) to the traditional model (characteristics: supply, demand, circulation, profit, production costs, etc.).

The following concept of detecting risk objects and sources is suggested:

– the risk objects are grouped using the 5π principle: people, product, price, place, promotion;
– the sources of risk are: underused internal advantages, unsettled internal difficulties, unrealized external possibilities, unaccounted-for external threats.

The result of processing information on the main characteristics of the model of an abstract publishing house is construction of enlarged risk trees (Figs. 6.21–6.25).

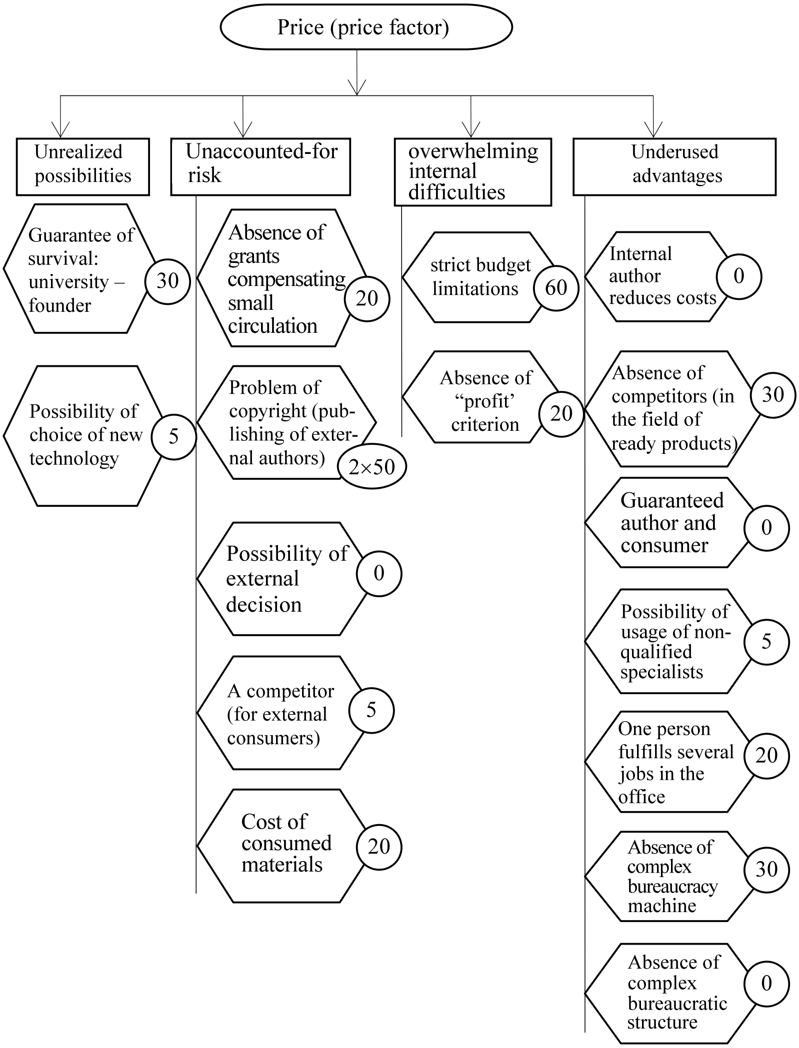


Fig. 6.22 The result of processing information on the main characteristics of the model

6.7.1.3 A Computer Experiment

Quantitative information material is obtained with the help of simulation models detailing the PC functioning under different strategies (the concrete model is realized in the special modeling environment Stratum).

A simple model corresponding to the extensive strategy has two processing channels—a computer (channel 1) and a copier (channel 2). The following variants of the order of processing can be used:

- Only computer typing (type 1)

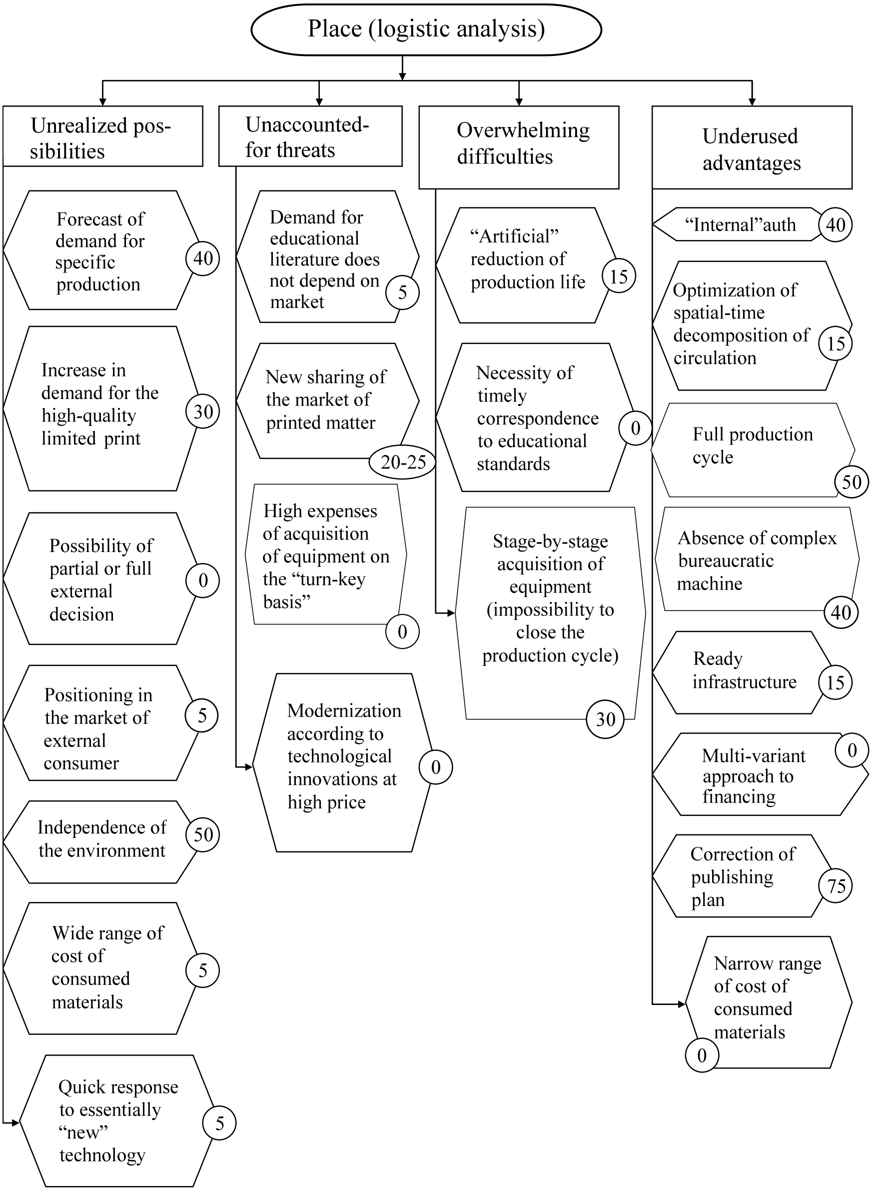


Fig. 6.23 The result of processing information on the main characteristics of the model

- Only copying (type 2)
- First typing and then copying (type 3)

Every channel has its specific feature—the amount of time needed to process one page of the order.

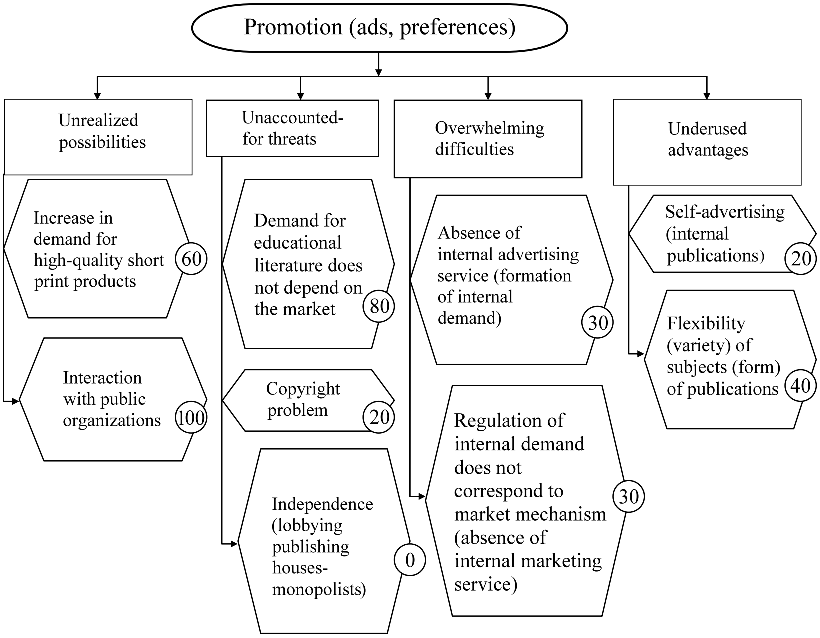


Fig. 6.24 The result of processing information on the main characteristics of the model

The simulation model has five images calculating the data.

Image 1 is the source where the initial data for the event are formed:

- It checks if the event has happened (in this case using the Poisson law); the rate of orders was chosen experimentally and was set equal to 2.5;
- The type of event (a uniform random event);
- The number of pages in the order (a uniform random number, type 1—less than 50, type 2—50 to 150, type 3—more than 150.

Further all calculated information is transmitted by connections to images, which model operation of channel 1 and 2.

Image 2 is responsible for operation of channel 1, computer:

- If an event of type 1 or 3 happens, it is checked if the channel is occupied or not;
- If the channel is occupied, the number of rejections increases by 1 point,
- If the channel is free, the type of the order is checked.
  - If type 1, the number of processed orders and pages increases and the other parameters of the channel are re-calculated (occupation, dead time rate, etc.);
  - If the event is type 3 the data on the volume are transmitted to channel 2;
- If the channel is free the order processing starts, and if the event is type 3 the number of pages in the order is stored in the buffer.

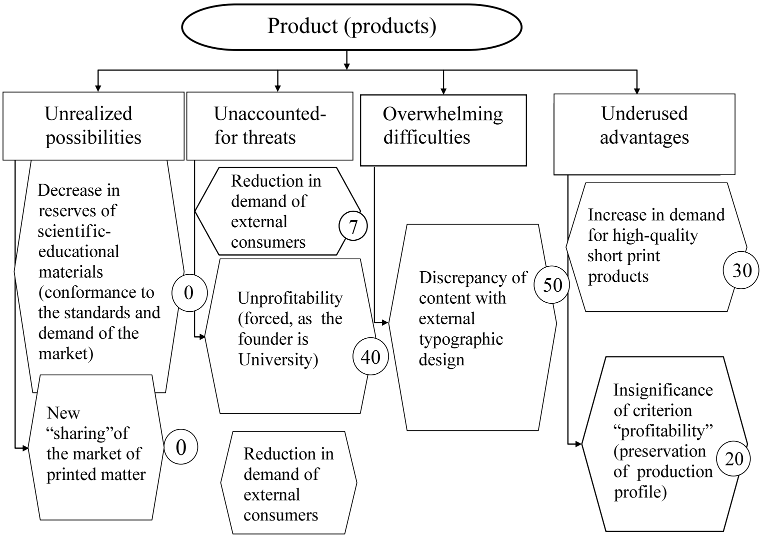


Fig. 6.25 The result of processing information on the main characteristics of the model

Image 3 is responsible for operation of channel 2 and the copier. Its operation is similar to that of channel 1. The only exception is that information is not transmitted to channel 1 (image 2).

Images 4 and 5 calculate performance of the channels (service parameters, dead time rate, average service time, etc.) getting all necessary information through the links with the first three images.

An example of service system functioning during a one-month period: 211 pages were typed (30 rejections), 271 pages were copied (15 rejections).

A more complicated model satisfying an intensive strategy includes six processing channels: 2 computers (channel 1, channel 2—backup), a copier (channel 3), an ink-jet printer (channel 4), a risograph (channel 5), a thermo-binding machine (channel 6). The following variants of order processing are available (+ means "in addition to"):

- Computer typing (type 5),
- + copying (type 6),
- + printing on the ink-jet printer (type 7),
- + printing on the risograph (type 8),
- + printing on the risograph and stitching on the thermo-binding machine (type 9),
- Only copying (type 1),
- Only printing on the ink-jet printer (type 2),
- Only printing on the risograph (type 3),
- Only printing on the risograph and stitching on the thermo-binding machine.

Every channel has its own parameter—time needed to process one sheet of the order.

The simulation model contains 7 basic images for calculating data and 5 service images for determining characteristics of channels.

Image 1 is a source where the initial data of the event are formed. It determines:

- whether the event has happened (using the Poisson law); the rate of orders was chosen experimentally and was set equal to 2.5;
- the type of event (a uniform, random event), but the fractions are not uniform;
- the number of pages in the order; depending on the type of order the number of pages can vary from 50–4,500 pages.

Then, by the connections, all calculated information is transmitted to the images, which model operation of the channels.

Image 2 is operation of channel 1, computer 1:

If the event has type 5–type 9, it is checked whether the channel is occupied:

- If the channel is occupied, the information is transmitted to channel 2 (backup computer);
- If the channel gets free, the type of the order is checked.
  - If the event has type 5—the number of processed channels and pages increases, other channel parameters are recalculated (occupancy, dead time rate, etc.)
  - If the event has type 6–9—the data about the volume is transmitted to the corresponding channels;
- If the channel is free—the order processing starts and if the event has type 6–9, the number of pages in the order and the type of the event are stored in the buffer.

Image 3 is operation of channel 2, the backup computer. The information to this image is transmitted only from channel 1. All other procedures are similar to those of channel 1 except for transition of the order to channel 1 (instead of this the number of rejections increases).

Images 4 and 5 simulate operation of the copier (channel 3) and the ink-jet printer (channel 4), respectively. Their operation is similar to that of the first two channels, but the information is not transmitted further and only characteristics of channels are recalculated.

Image 6 simulates operation of the risograph. In case the order requires processing on the thermo-binding machine, after processing on this channel the information is transmitted to image 7 (thermo-binding machine) [25].

6.7.1.4 Quantitative Risk Assessment

Formation of money flows (\$1 = $T76$ [16]).

The first strategy. Startup costs of the equipment are \$2,000, the salary is \$100 (per month), a ream of paper (500 sheets) is \$4 (50 % for two-side printing), filling

[16] The exchange rate is taken for the moment of work execution.

of the copier for 1,200–2,000 sheets is $10, one copy costs $0.2, typing of one page of the text costs $0.3 (the fraction of typing from the number of sheets ordered for printing is 30 %).

The second strategy. Startup costs of the equipment: risograph $14,000, computer $750, thermo-binding machine $9,500, ink-jet printer for A3 $1,500, salary $200 (per month), a ream of paper (500 sheets) $4, a roll of master-tape $60 (200 frames), a tube of mastic $20 (for 7425, 50 % for two-side printing), a copy costs $0.2 (60 copies per minute, operates 2–3 hours per day), cost of typing of one page is $0.3 (the fraction of typing from the number of sheets ordered for printing is 30 %).

The teaching aids are published in 50–100 copies of an average volume of 50 pages; usually one-page orders are made in 400 copies. The ratio of the orders is 30 % to 70 %. The results of operation of a simulated publishing center and assessments of experts are documented as the interval data and are processed according to (6.44)

The First Strategy

Input parameters:
Production quantity = 400–600
Price per item = 0.2–0.3
Direct expenses = 0.025–0.037
Fixed expenses = 100
Amortization = 20–25
Profit tax (in fractions) = 0.30
Discount rate (in fractions) = 0.2
Depreciated cost = 0
Startup investment = 2,000
Project term = 2 years

Result: Net present value (NPV): [−2033.4; −1913.68]
Conclusion: The project is unprofitable.

The Second Strategy

Input data:
Production quantity = 30,000–70,000
Price per item = 0.25–0.3
Direct expenses = 0.01
Fixed expenses = 200
Amortization = 3,862
Profit tax (in fractions) = 0.3
Discount rate (in fractions) = 0.2
Depreciated cost = 7,000
Startup investment = 25,750
Project term = 5 years

Result: Net present value (*NPV*): [−4,817.93; 22,605.98]
The risk is 3.5 %

The first variant for creation of the publishing center was rejected according to the criterion of profitability (meaning that the PC can operate only on the grant-in-aid auxiliary base).

The second strategy shows a low rate of risk according to the criterion of efficiency. But the assessment using risk trees (Figs. 6.21–6.25) gives a higher risk level. The additive risk indicator, which is used here, estimates nonlinear relationships created by the trees caused by the dead time of service channels: The rate of received orders logically decreases and can be taken into account by the parameters of the simulation model.

This approach, developed on the basis of imitations, can be recommended where there are no alternative business plans or when necessary for administration of long-term projects.

6.7.2 Comparative Assessment of Business Plans in Terms of Risk

Different methods of assessment of risk and characteristics of its sources can be used in the course of development of business plans and formation of integral criteria.

To retain risk within the acceptable boundaries is one of the aims of risk management. Therefore the main element of investment activity is to prevent possible problems—a set of conditions and factors leading to adverse changes during realization of the investment project. Instability of agricultural productivity and, in particular, deviation of the economic process parameters from the hypothetical curves of stable (continuous or alternating but logical) time variation has negative influence on projects connected with the agribusiness industry (for example, raw material processing). The degree of such instability is a quantitative assessment of risk of uncertainty initiated by fluctuations in economic processes [26].

One of the initial parameters threatening food supply safety in the region is instability in crop production which, in its turn, directly or indirectly leads to instability in supply to the market of both raw materials (intermediate product) and foodstuffs. In particular, forecasting of the results of long-term influence of crop-producing instability on projects for meat production (sausage, smoked products) includes several stages:

1. Detection of a random component of time series parameters—crop-production.
2. Detection of a random component of time series parameters—rate of consumption of meat products per capita.
3. Calculation of elasticity of the food market conditions (in the corresponding sector) by the crop-producing fluctuations in the long-term period.

As a solution to the first problem we presented the time series $y(t)$ as an additive three-component model:

$$y(t) = y_t(t) + v(t) + e(t),$$

$y_t(t)$ is a trend (a logical component reflecting the main trend of development); $v(t)$ is a periodic component (a logical component reflecting periodical fluctuations); $e(t)$ is a random component.

It helps to shift the main point of investigation from estimation of instability to estimation of errors in filtration of regular dependencies:

$$e(t) = y(t) - \left[y_t(t) + v(t)\right].$$

The trend is determined by the traditional least squares method; in North Kazakhstan regions the main tendencies in the dynamics of the main parameters (the data for the 15-year period)—production of crops, potatoes, vegetables, forage root crops, oilseeds, and hay—are quite reliably (90 %–95 %) approximated by linear downward trends (a small annual increase is only demonstrated by the linear trend for forage root crops).

Testing the hypothesis of absence of a cyclic component by the variance test displayed the presence of a periodical crop-production component in the trend. It is convenient (and not difficult technically as it is built into the software) to use harmonic analysis to measure periodic fluctuations after exclusion of the trend from the initial series, i.e., in the deviations $E(t) = y(t) - [a_0 + a_1 t]$ of the values of real and model crop-production. The cyclic component is filtered by the expansion of the periodical (assumption) function $v(t + kT) = v(t)$ in a Fourier series:

$$v(t) = \sum_{k=1}^{T} A_k \cos(k\varpi t + \varphi_k),$$

where $\varpi = 2\pi/T$ is the (Nyquist) fundamental frequency; frequencies $k\varpi$ are harmonics of the fundamental frequency, A_k, ϖ_k are parameters (the Fourier constant and phase) determined by the methods of the linear regression analysis if T period is known and by the methods of spectrum analysis otherwise. For the practical use of the harmonic regression model $v(t)$, we leave only harmonics determining the main dependencies of the series under consideration. The variance analysis allows us to use successive insertion (stepwise regression) of harmonics with the highest weights in order to calculate the variation of productivity $Q_v = \sum_t (v(t) - \bar{E})^2$ determined by the model and the determination index (explainable fraction of productivity level dispersion); $R^2 = Q_v/Q = 1 - Q_e/Q$, where $Q = \sum_t (E(t) - \bar{E})^2$, generally corrected for the average level of productivity dispersion, $Q_e = \sum_t (E(t) - v(t))^2$—a random variability inside the chosen model. As a rule, three or four harmonics provide a considerable increase in the explainable variation (inclusion of a greater number of harmonics in the model does not give any visible increase in the explainable variation). The model $v(t)$, whose adequacy and accuracy are checked beforehand, "adjusts" the aligned levels of productivity deviations with respect to the trend (regular fluctuations); the deviations have periodicity—the result of overlapping of waves with 1-to-4-year phases. The variation with respect to the theoretical values reflects instability of the studied risk factor—crop productivity. In this case the degree of tolerance $1 - R^2$ (estimates the degree of incoherence of independent parameters) or the "risk factor" $1 - \sqrt{R^2}$ is taken as the measure of uncertainty (irregular fluctuations in crop productivity). Almost all crops growing in the North Kazakhstan region have high level of "risk" (more than 30 %); the highest risk of production uncertainty refers to growing of crops (0.41 %) and forage crops (0.39 %); the lowest production risk was established for hay (0.27 %).

Instability in productivity leads to instability in production of some types of foodstuffs and/or forces the search for additional suppliers of raw materials. As a result, it causes negative consequences for food supply safety in the North Kazakhstan region: instability in supply of agricultural products and their processing on the food market. In particular, this fact is reflected in consumption of the above products per capita. In the framework of this project (the second problem) we approximated the real dynamics of meat consumption per capita—the effective factor has a clearly pronounced trend changing from rising to falling, which according to the minimum criterion of residual variance is best straightened by the quadratic trend (as a whole, according to the dynamics of basic foodstuff consumption per capita, sharp decrease in consumption in the North Kazakhstan region refers to the beginning of the 1990s). A considerable periodical component of the dynamics of average per capita consumption was not detected; factual deviations of per capita meat and meat-products consumption from the quadratic trend with 0.5 reliability level are also described by the quadratic expression.

The third problem is reduced to the assessment of the degree of statistical dependence between irregular variation in the average per capita meat consumption and irregular variations in crop productivity providing supply of raw materials for the production. Taking into account different lags in responses of efficient factors to random factor fluctuations we calculated paired correlation indices of random deviations for a long-term period in the series of meat consumption and productivity of the main cultivated crops. The values of paired correlations reflect sensitivity of the foodstuff market to the variations in productivity and can be used as an estimation of potential threat to the food supply safety in the region and as a risk parameter of the corresponding investment projects. According to the data from the North Kazakhstan region the consumption of meat products has a clearly seen correlation with zero lag for such factors as hay (over 50 %) and crops (over 35 %); correlation with 1-year lag for hay (more than 0.40) and forage crops (more than 0.50); and 2-year lag correlation for forage crops (more than 0.40). It should be noted that correlations less than 35 % were not taken into account [27].

Risk indicators of the investment project are characterized not only by quantitative but also by qualitative aspects, which are more difficult to "measure" as the degree of uncertainty of qualitative information increases due to low formalizability. This research uses several methods of quantitative assessment of qualitative parameters.

For example, in estimation of a vacuum packing manufacturing line three different technologies were used. As the criterion we took the minimal number of deviations from the standard in check measurements (Table 6.4).

According to the theory of single-factor variance analysis and taking into account that every j-th group gradation of the criterion "technology" has a different number

Table 6.4 Minimal number of deviations from the standard in check measurements

The number of deviations in check measurements (i)	Packaging technology (j)		
	1	***2***	***3***
1	1	2	4
2	3	3	5
3	2	2	3
4	1	1	
5	0	4	
6	2		
7	1		

of observations $i = \overline{1; n_j}$, $j = 1, 2, 3$, we tested the zero hypothesis of the absence of influence of packaging technology on the correspondence to the quality standards at the significance level $\alpha = 0.05$.

Actually observed statistics value is $F = 6.52$. The critical value of Fisher's F-statistics at the significance level of $\alpha = 0.05$ and degrees of freedom $k_1 = 2$ and $k_2 = 12$ is equal to $F\alpha; k_1; k_2 = 3.89$. As $F > F\alpha; k_1; k_2$, the zero-order hypothesis is rejected, i.e., with reliability 0.95 one can state that the choice of technological line for vacuum packing can considerably affect the quality of finished products: $R^2 = 0.52$.

The "center of gravity" of deviations[17] from the standard allows us to conclude that the first technology is better than the other two technologies. As the business-plan project _1 implies usage of the first technology, it can be stated that according to this criterion it does not occupy the worst place.

A method that is rather successful in practice is that of numeralization when the preference function[18] π is constructed on the base scale B and reflects a set of alternatives Pr into the number axis R^1 as: $\pi : B \to R^1$, $B \in R^{verbal}$ is such that $\pi(pr_1) > \pi(pr_2) \Leftrightarrow pr_1$ is better than pr_2, where pr_1, pr_2, are alternatives from Pr; R^{verbal} is the space of basic scales; verbal is the number of qualitative criteria.

In other words, a number-estimation is prescribed to every qualitative alternative, equivalent alternatives are estimated by the same values of the preference function, and the better of the two alternatives is prescribed a higher value. Therefore the process of numeralization of the qualitative criterion is reduced to calculation of the preference function:

$$R = \frac{R_{verb} - R_{\min}}{R_{\max} - R_{\min}} + g,$$

where $R \overset{\text{def}}{=} \pi$ is the preference function (numerical value) of the qualitative criterion; R_{verb} is the value of the qualitative criterion on the basic scale; $R_{\max}$, $R_{\min}$ are the upper and lower boundaries of the interval on the basic scale where the value

[17] In this case the sampling is too small to make a conclusion based on expected values.

[18] A numerical value of the linguistic variable.

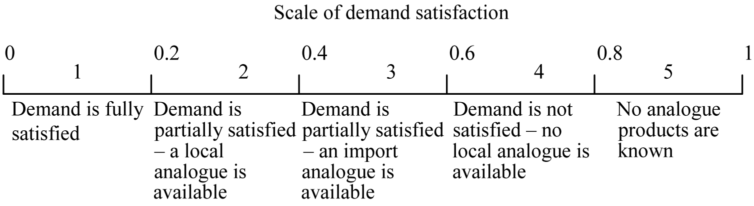


Fig. 6.26 Scale of demand satisfaction

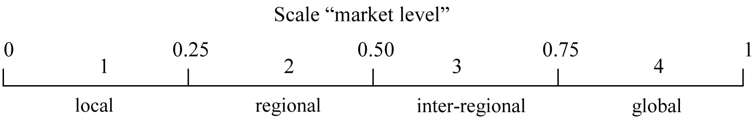


Fig. 6.27 Scale "market level"

of the qualitative criterion is contained; g is a numerical weight assigned to the linguistic variable determined in the considered interval.

For example, the three business plans for organization of sausage-producing outlets have practically the same criteria (future competitiveness, budget, availability of raw materials, levels of processing, organizational/legal, ecological). However, the business-plan_1 with a rather high level of individual financial risk in the aspects (satisfaction of purchase requirements, market level) has a considerable advantage. For this plan, "scenario" expert estimations {minimal, expected, maximal}, probability of losses—41.1 % and fuzzy risk estimation—26.4 % were calculated and basic scales presented in Figs. 6.26 and 6.27 were constructed.

Gallup poll showed that probability of deviation to the "worse" side of the criterion "demand" is about 0.03.

Marketing studies with reliability 0.05 allows us to state that the product will enter the inter-regional market (at the first stage—Astana).

The number of scales and gradations introduced for the qualitative criterion depends on how thoroughly the problem has been studied by experts/specialists. Finally, this method of quantitative estimation of qualitative characteristics can properly describe the situation [28–31].

References

1. Mutanov, G., Kulikov, V.P., Kulikova, V.P.: Information Support of Investment Decision in Risk Conditions. Astana (2005), p. 456
2. Charnes, A., Cooper, W.W., Rhodes, E.: Measuring efficiency of decision-making units. Eur. J. Oper. Res. **2**(6), 429–444 (1978)
3. Edwards, W.: The theory of decision making. Psychol. Bull. **51**(4), 380–417 (1954)
4. Li, Y., Zhao, L., Li, R.: Novel model on assessment of construction project investment environment. Pak. J. Stat. **29**(6), 989–998 (2013)

5. Das, I., Bhattacharya, K., Canizares, C., Muneer, W.: Sensitivity-indices-based risk assessment of large-scale solar PV investment projects. IEEE Trans. Sustain. Energy **4**(2), 370–378 (2013)
6. Saati, T.: Decision Making. Method of Hierarchy Analysis. Radio i Svyaz (1993)
7. Blome, C., Schoenherr, T.: Supply chain risk management in financial crises—a multiple case-study approach. Int. J. Prod. Econ. **134**(1), 43–57 (2011)
8. Kovalev, V.V.: Financial Analysis (1997), p. 512
9. Mak, B., Schmitt, B.H., Lyytinen, K.: User participation in knowledge update of expert systems. Inf. Manag. **32**(2), 55–63 (1997)
10. Korkhin, A.S.: Regression parameter-estimation as a multicriterion problem. Autom. Remote Control **53**(9), 1403–1409 (1992)
11. Lin, W.T., Shao, B.B.M.: The relationship between user participation and system success: a simultaneous contingency approach. Inf. Manag. **37**(6), 283–295 (2000)
12. Wiesemann, W., Kuhn, D., Rustem, B.: Maximizing the net present value of a project under uncertainty. Eur. J. Oper. Res. **202**(2), 356–367 (2010)
13. Erfani, T., Utyuzhnikov, S.V.: Directed search domain: a method for even generation of the Pareto frontier in multiobjective optimization. Eng. Optim. **43**(5), 467–484 (2011)
14. Raju, K.S., Pillai, C.R.S.: Multicriterion decision making in performance evaluation of an irrigation system. Eur. J. Oper. Res. **112**(3), 479–488 (1999)
15. Lukasevich, I.Ya.: Analysis of Financial Operations. Finansy (1998)
16. Moiseev, N.N.: Mathematical Problems of System Analysis. Nauka, Moscow (1981), p. 488
17. Zadeh, L.: The concept of a linguistic variable and its application to approximate reasoning—I. Inf. Sci. **8**, 199–249 (1975)
18. Omelchenko, I.N., Kolobov, A.A., Ermakov, A.Yu., Kireev, A.V.: Industrial Logistics. MSTU (1997), p. 204
19. Li, Z., Wen, G., Han, Y.: Decision making based on intuitionistic fuzzy soft sets and its algorithm. J. Comput. Anal. Appl. **17**(4), 620–631 (2014)
20. Liang, M.T., Chen, S.Y., Lee, C.W., Yeh, C.J.: Application of fuzzy optimum system hierarchy analysis selection method to determining repair order for existing reinforced concrete bridges. J. Chin. Inst. Eng. **28**(2), 189–204 (2005)
21. Mamdani, E.H., Assilian, S.: Experiment in linguistic synthesis with a fuzzy logic controller. Int. J. Man-Mach. Stud. **7**(1), 1–13 (1975)
22. Matbouli, Y.T., Hipel, K.W., Kilgour, D.M., Karray, F.: A fuzzy logic approach to assess, manage, and communicate carcinogenic risk. Hum. Ecol. Risk Assess. **20**(6), 1687–1707 (2014)
23. Guo, P., Li, Y.: Approaches to multistage one-shot decision making. Eur. J. Oper. Res. **236**(2), 612–623 (2014)
24. Hu, Y.: Energy conservation assessment of fixed-asset investment projects: an attempt to improve energy efficiency in China. Energy Policy **43**, 327–334 (2012)
25. Holcombe, R.: Economic Models and Methodology. Greenwood Press, New York (1989)
26. Orrell, D.: Apollo's Arrow: The Science of Predication and the Future of Everything, Harper Collins Canada, Toronto (2007)
27. Titov, V.I.: Enterprise Economics: Textbook. EKSMO (2008), p. 416
28. Denina, I.G., Saltykov, B.G.: Mechanisms Stimulating Commercialization of Research and Developments. IEPP (2004), p. 604
29. Ghadiali, A.M., Ray, P.S., Lindly, J.K., Batson, R.G.: An expert system for risk assessment of automobile crashes. Int. J. Risk Assess. Manag. **6**, 491–507 (2006)
30. Sedbrook, T.A.: A collaborative fuzzy expert system for the web. Data Base Adv. Inf. Syst. **29**(3), 19–30 (1998)
31. Brinckmann, J., Grichnik, D., Kapsa, D.: Should entrepreneurs plan or just storm the castle? A meta-analysis on contextual factors impacting the business planning-performance relationship in small firms. J. Bus. Venturing **25**(1), 24–40 (2010)

Chapter 7
Multi-Objective Stochastic Models for Making Decisions on Resource Allocation

The development of methods and models for making decisions on resource allocation is an important trend in modern science. However, in spite of a large number of publications on the subject, many problems related to resource allocation still need more thorough investigation. The least studied problems are those that deal with the methods and models of making decisions under conditions close to actual ones, i.e., when the parameters affecting the system are not completely defined or when the functioning system (and/or its objects) has several purposes and/or parameters used to assess the quality of its functioning. The necessity of taking into account a great number of probabilistic factors makes it necessary to develop stochastic models, and the desire of the decision-maker to minimize damage caused by the deteriorating environment makes it necessary to develop methods enabling him to revise the initial plan, which finally results in the development of multi-stage models and methods of their optimization.

The problems of optimization with a large number of criteria were considered in numerous publications of such well-known authors as R. Keeny, Ch. Rife, B. Rois, T.L. Saati, and P.S. Fishburn. The problems of multi-criteria optimization were also studied by O.I. Larichev, V.V. Podinovsky, V.D. Noghin and others. The basics of stochastic programming were formulated in D.B. Yudin's papers. The problems of incomplete information related to model optimization were considered in the works of D.A. Pospelov, F.F. Yurlov, A.N. Kovalyov and others. In the Saint-Petersburg State University there is a group of scientists headed by V.V. Kolbin that studies the problems of decision-making under conditions of incomplete information. The development of the market economy in Kazakhstan makes stochastic programming an area of high potential for scientific research, having important practical application.

Based on the combined target functionals constructed in accordance with classical choice principles, such as egalitarianism and utilitarianism, three new models of resource allocation have been developed. The use of the same selection policy over a long period proves to be inefficient; in practice, a combination of various resource allocation policies is actually more effective. This feature of resource allocation problems makes this method different from other similar methods and models

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_7

for making decisions on resource allocation under conditions of incomplete information.

7.1 Applicability of Multiple Criteria Optimization Methods

From the viewpoint of decision-makers, all known methods of multi-criteria optimization are more or less voluntary combinations of standard heuristic techniques not having any clearly outlined limits of applicability. In cases where applicability areas are indicated, the decision-maker's requirements for the structure of preferences turn out to be unrealistic. In actuality, there are practically no methods that enable us to establish the extent of adequacy of a certain model to the real structure of preferences with regard to a specific task.

As is, any information on the structure of preferences, even if it is used by decision-makers, is arbitrarily interpreted by them as the values of certain parameters within the framework of a selected model.

Here is a typical example of an arbitrary interpretation of information on the relative importance of criteria by decision-makers. As a rule, the decision-maker is unaware of the actual meaning of received information. For example, the decision-maker assumes that in any message "the first criterion is more important than the second one," which may affect the model parameters in various ways and, consequently, affect the final decision.

As is known from practical experience, as a rule decision-makers are inclined to choose a certain model of the structure of preferences for making complicated engineering decisions more or less randomly, based on assumptions that are of secondary importance in terms of model adequacy. This inevitably results in making decisions that are not optimal in terms of the real structure of preferences.

The above-listed models of multi-criterion optimization are devoid not only of a common theoretical basis but also of a well defined terminology. On the other hand, the concept of the choice function reflecting the idea of optimality in the most general sense has been developed and is successfully studied in the general theory of the best option choice. However, due to the high level of generalization of the theory, the properties of the choice function are mostly considered from theoretical combinatorial positions. The structure of the criteria space and other characteristics of multi-criteria tasks of making complicated engineering decisions are taken into account to a much smaller extent. This seems to be the reason why the results and concepts of the decision theory are seldom used in papers considering engineering problems.

The structure of decision-maker preferences, even if it is considered in articles, is treated only as a utility function. In most cases, a model is chosen from a narrow parametric family, which has only some plausible characteristics.

At the same time, it is clear that modeling of the structure of decision-maker preferences must be oriented at the structure itself, expressed as a choice function in the criteria space. As it is impossible to determine the choice function with a high degree of precision, it is necessary to develop a model that would approximate it as the choice function in the criteria space.

7.2 The Decision-Making Problem of Resource Allocation in Terms of Utility Theory

7.2.1 Classical Principles of Choosing Alternative Solutions

Planning economic activities of enterprises is one of the most important factors of free market relations. In market conditions, the effective prices of all products and resources are established by competing manufacturers and consumers themselves, and that is why any enterprise or company determines the range and amount of its output independently. Production plans must be promptly adjusted in response to any changes in the market situation. The production plan must be flexible and easily adaptable to the market environment. To formulate economic goals, to ensure rational business management, and balanced and resource-based planning, it is necessary to develop and study extremal models of choosing decisions on allocation of various and usually limited resources.

In the general case, is not always possible to make a final decision as it depends on two major aspects: firstly, the choice of decision-making policy and purpose; secondly, availability of resources that determine the area of admissible decisions.

Whereas the contextual aspects of the strategies of resource allocation can be coordinated, the resources are limited, and that is why the resources can be distributed only under the condition of ordering relations. To introduce order relations, it is possible to use such choice principles as egalitarianism, utilitarianism, multiple-choice compromises, guaranteed minimum, stability in a certain specified sense, fastest achievement of the required level, etc.

In the theory of cooperative decisions, egalitarianism and utilitarianism are regarded as classical choice principles [1].

Definition 7.1 Egalitarianism is a desire to equalize individual utilities of the coalition agents.

Definition 7.2 Utilitarianism is a desire to maximize the sum of individual utilities.

Based on the above, the problem of limited resource allocation can be presented as a problem of making a collective decision by matching an admissible solution to the levels of the vector of individual utilities $u = (u_1, u_2, \ldots, u_n)$, where u_i is the utility of the i-th agent. In this case the agent means a certain type of resource or an area in which the resource will be used. In the specified range of admissible vectors, the collective decision is presented as a result of the mathematical rule which identifies one vector as the community's choice.

Egalitarianism originates from the ancient principle of justice: equal agents must be equally treated. The application of this choice principle leads to equalization of individual utilities of the coalition agents [2].

The egalitarian utility function $F_{eg} = \max_{i=\overline{1,N}} u_i$ coincides with the value of utility of the least successful agent.

Utilitarianism regards individual utilities only as a way of improving public welfare and may sacrifice interests of an individual agent for the sake of increasing collective utility [3].

The utilitarian decision-making program consists in maximization of the function $F_{ut}(u) = \sum_{i=1}^{N} u_i$ on the set of admissible utility vectors. When such an economic policy is implemented, preference is given to the objects and areas which give maximal increase per unit of investment, whereas other low-profit areas and objects are not considered.

In terms of response to public needs, the principle of a guaranteed minimum of social development and needs satisfaction proves to be the most efficient. The amount of the guaranteed minimum is determined by comparing the level of social development in the country with the international experience. The utility function of the guaranteed minimum principle can be presented as $u_i > P$ for $i = \overline{1, N}$, where P is a fixed constant.

Another important requirement is stability of the choice when it is necessary to foresee sustainability of social and economic development in face of insufficient natural and financial resources, worsening environmental situation, social tension and other factors that may have a negative impact on the implementation of public programs.

7.2.2 *Aggregation of Preferences in the Course of Decision-Making*

If experts, specialists, heads of divisions and other decision-makers act as agents in the process of making decisions on allocation of limited resources, then to construct a utility function, a target functional of the optimization problem, it is necessary to aggregate individual utility functions and preferences of each of the agents. Let us consider the possibility of aggregation of preferences from the viewpoint of an individual making decisions (for example, a CEO at the enterprise) [4].

To choose the best option for resource allocations in the conditions of absolute certainty, it is sufficient to construct a utility function of the decision-maker denoted as v with x consequences. If we denote N criteria as $V_1, V_2, \ldots, V_N$, the evaluations of which will be expressed as utility functions of experts $v_1, v_2, \ldots, v_N$, the aggregation problem will be reduced to the following function:

$$v(x) = v_D\big[v_1(x), v_2(x), \ldots, v_N(x)\big], \tag{7.1}$$

where v and v_D are utility functions of the individual making decisions under conditions of certainty. This representation is based on a number of assumptions.

Firstly, in case of properly selected scales, preferences of the decision-maker for consequences x are characterized by functions v_i. As each v_i function is determined with an accuracy up to the positive monotonous transformation, before comparing individual utilities, it is necessary to normalize the scales used to measure them.

Secondly, for all i the structure of preference of every individual is fully determined by the function v_i.

Thirdly, $v(x)$ contains the assumption that the decision-maker knows v_i functions, otherwise the problem would be uncertain.

Let us consider the possibility of aggregating individual preferences under uncertain conditions. In general terms, the problem formulated by K.G. Errow [5] can be stated as: determine the group ranking of alternatives if the ranking of the set of alternatives made by each member of the expert team is known.

To choose the best alternative under conditions of uncertainty, we will have to use the decision-maker's utility function, which we will denote u for consequences x. Let us use symbols $U_1, U_2, \dots, U_N$ to denote criteria of the utility functions of individuals $u_1, u_2, \dots, u_N$. In case of a "pure" model of decision-making under uncertain conditions, it is necessary to find a suitable type of function u_D, such that

$$u(x) = u_D\big[u_1(x), u_2(x), \dots, u_N(x)\big], \tag{7.2}$$

where u, u_D are the decision-maker's utility functions.

The above model is based on a number of assumptions. The most important one is that the decision-maker's interest can be characterized by the function u_i. It is also assumed that the structure of the individual's preferences can be represented by the function u_i for all i. In contrast to model (7.1), in model (7.2) it is not necessary that the decision-maker must know all u_i with high degree of certainty. In the expression (7.2) $u_1(x)$ is the utility function of Individual 1 for a specific consequence x. At this stage, his preferences with respect to x may include both good and bad feelings towards other individuals.

Let u be a utility function of the decision-maker, and u_i be utility functions of individuals or groups of individuals whose opinion is important for the decision-maker, and that he finds it necessary to take into account while making his decision. The decision-maker is fully responsible for the choice of the alternative, and it is he who must identify possible substitutions between different values of individuals' utility functions $i = \overline{1, N}$. The individuals themselves do not take part in the decision-making process. Only the decision-maker will weigh the advantages that certain individuals might gain by choosing certain alternatives.

J.C. Harsanyi [6] formulated a set of necessary and sufficient conditions under which the group utility function can be written as a weighted sum of individuals' utility functions: $u(x) = \sum_{i=1}^{N} \lambda_i u_i(x)$.

All these tools enable us to consider aggregation of individual preferences as a task of fuzzy mathematical programming, which will allow us to construct a group utility function and group preferences by aggregating individual preferences of each participant of the decision-making process.

7.2.3 Optimality of Making Decisions on Resource Allocation

Due to the limited resources invested in the development of the object in a certain period of time, it is impossible to achieve an ideal state in all respects. That is why it is necessary to determine a means of allocating resources whereby each area can be supported at a certain fixed level that enables the object to remain in an operational condition during a certain period of time. This level is called the *guaranteed minimum*.

The Pareto optimality principle is a cornerstone of most theories on group decisions.

Definition 7.3 According to Pareto, an alternative is optimal if any other alternative that is more preferable for some members of the group is less preferable for the other members of the group.

The Pareto optimality principle states that an alternative must never be selected unless it is optimal in accordance with Pareto's definition. Under this approach it would be possible to enhance the extent of satisfaction of at least some individuals, without infringing the others' interests. The only rational approach would be a Pareto-optimal decision [6, 7].

Definition 7.4 From the viewpoint of egalitarianism, resource allocation is said to be at its best when it provides achievement of a certain reference state of the area (object) development as a result of maximally fast elimination of disproportions in the development of areas (objects).

Definition 7.5 From the viewpoint of utilitarianism, resource allocation is regarded as best when it provides maximal overall development of areas (objects).

Definition 7.6 The principle of choice of the decision-making policy on resource (means) allocation is internally stable if among alternative Pareto optimal decisions no one is more preferable.

Definition 7.7 The principle of choice of the decision-making policy on resource (means) allocation is externally stable if it satisfies the unanimity principle (or if it is Pareto optimal).

The utilitarian policy of resource allocation provides investment in quickly developing areas and objects, which gives maximal profit from invested resources. When such economic policy is implemented, preference is given to objects and areas which yield maximal increase per unit of investment, whereas other low-profit areas and objects are not considered.

Egalitarianism originates from the popular ancient principle of justice: equal agents must be equally treated. An application of this choice principle leads to equalization of individual utilities of the coalition agents. When all agents equally benefit from cooperation, there remains no room for envy and disappointment. However, this principle of choice, seemingly simple, may contradict another postulate of collective decision-making—that of unanimity and efficiency. According to the latter, in case each agent involved prefers decision a to decision v, decision v cannot be taken. In other words, the selected individual utility vector must be Pareto optimal. The unanimity principle may come into contradiction with simple matching of individual utilities. To avoid this, let us formulate the efficiency principle as follows: egalitarianism means pursuit of equality through "augmenting" utilities for the "poor" and not through "destruction" of welfare of the "rich."

In other words, egalitarianism provides for internal stability whereas external stability is ignored. Equal agents gain equal benefits that may be so small that they will bring their cooperative gain down to a negligibly small value.

Utilitarianism provides for external stability and ignores internal stability. The cooperative gain is chosen to be the highest but it is not equally divided among the agents. Consequently, from the viewpoint of "suffering" agents, the benefit is divided unfairly.

If the same classical principle is applied over a long period, social and economic crises become inevitable. Therefore, these principles must be used in combination. It is reasonable to consider the possibility of multi-criteria compromise choice, the components of which are the two classical allocation methods (egalitarianism and utilitarianism). The simplest combination is a linear combination with weighing coefficients the sum of which equals 1.

There are also other feasible combinations of classical methods.

7.2.4 Principles of Choosing Decisions on Resource Allocation Combining Classical Choice Principles

Definition 7.8 The resource allocation is called proportional when financial flows for each area are proportional to the differences between their reference states and the current level.

In this approach it is possible to bring all areas to certain reference states simultaneously. The more the current state of area i lags behind the reference state, the more funds are to be allocated for the development of the area and, hence, for the elimination of disproportions in its development.

The model of making decisions on resource allocation based on the principle of uniform development is a particular case of the linear combination and the principle of proportional development.

Definition 7.9 The resource allocation policy will be called proportional if it is aimed at gradual smoothing of disproportions between areas.

In view of the limited amount of resources being invested in the development of an object within a certain period of time, it is impossible to achieve a reference state in all respects. That is why it is necessary to determine a way of allocating resources whereby each area is supported at a certain fixed level that enables the object to remain in an operational condition during the indicated period of time. It would be logical to call this level "the guaranteed minimum."

Definition 7.10 The resource allocation policy aimed at maintaining the object in an operational condition is called the *resource allocation principle with a guaranteed minimum*.

Financial investments in the development of various areas are aimed at minimization of gross differences between the levels of their development that characterize the current state of the object against its reference state in the economic system.

Definition 7.11 The gross difference between the current level of the area development and its reference state is called the value of disproportion in the development of an economic entity.

Definition 7.12 The principle of resource allocation that enables one to achieve a minimal disproportion in the development of an economic entity is called the policy of distance minimization.

Definition 7.13 The resource allocation must be regarded as best when the disproportion in the object development has the lowest value.

Definition 7.14 The resource allocation must be called optimal if it provides achievement of the reference state of the object.

In practice, the reference state is hardly achievable.

Therefore, usually not an optimal decision but the best of possible alternative decisions on resource allocation is sought.

In the process of management of an economic entity, the resources are often allocated to weak sectors at the expense of better developed areas.

Definition 7.15 The resource allocation is termed a policy of "patching" when priority is given to the needs of areas which, in terms of their development, are the farthest from the reference state.

Let us consider a problem of making a decision on allocation of certain limited resources. Let us assume that there is a fixed amount of physical, financial and other resources to be allocated (or reallocated) to ensure efficient operation and support of the current state of an economic entity. The problem is to allocate the resources in such a way as to ensure a maximal return on the investment.

Let us consider the following model for decision-making on resource allocation. Let us assume that:

$t \in [1, T]$ are the periods of decision-making on resource allocation;

$c(t) = \varphi(t) + \psi(t)$ is a certain amount of resources where:
$\varphi(t)$ are the resources to be allocated at a given point of time t,
$\psi(t)$ are the resources to be retained in the storage or reserve fund of the enterprise. Monetary resources of the reserve fund can be used at a certain point of time $\tau \subset [1, T]$;
$i = 1, 2, \ldots, n$ are areas/sectors to which the resources must be allocated. It should be kept in mind that the development of one area does not depend on the development of the other areas;
n is the number of possible areas for investment and use of the resources available;
$u_i(t)$ is the amount of the resources to be invested in area i within time period t;
$y_i(t)$ is the level of development of area i at a given point of time t;
$\bar{y}_i$ is the reference state of area i at a specified point of time T;
$S_i(t)$ is the efficiency of investments in area i at a certain point of time t, i.e., gain per unit of invested resources.

Let us assume that the value of disproportion in the object development decreases proportionally to the resources invested:

$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t),$$

where $d_i(t)$ is an external factor, it usually has a negative value.

The problem of decision-making on allocation of physical and financial resources can be formulated as follows:

$$F\big(y_1(t), y_2(t), \ldots, y_n(t)\big) \to extr;$$

$$\sum_{i=1}^{n} u_i(t) \le c(t) = \varphi(t) + \psi(t);$$

$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t);$$

$$y_i(1) > 0,\ \ y_i(t) \ge y_i(t-1),\ \ S_i(t) \ge 0,\ \ c(t) > 0,\ \ u_i(t) \ge 0;$$

$$i = 1, 2, \ldots, n,\ \ t \in [1, T],$$

where $F(y_1(t), y_2(t), \ldots, y_n(t))$ is the optimality criterion.

In order to compare the efficiency of the system functioning in different variants of resource allocation, let us move from considering the absolute indicators $y_i(t)$, $i = 1, 2, \ldots, n$ (values with names) to the relative indicators of type $|\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}|$ (dimensionless values). Since $\bar{y}_i \ge y_i(t)$, $i = 1, 2, \ldots, n$, the sign of the module can be omitted in the relative indicators, i.e., $\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}$, $i = 1, 2, \ldots, n$.

For the purposes of resource allocation, we can use as targets functionals that meet both the classical principles of the choice of decisions (egalitarianism and utilitarianism) and the principles of multi-variant compromise choice.

7.3 Formulation and Convolution of Criteria in Monocriterial Decision-Making Models

In forecasting, strategic planning and decision-making on innovations, it is often necessary to solve multidimensional decision-making problems. To solve a multi-dimensional problem on economic planning and scientific and technological progress for a long time interval, it is necessary to analyze decision-making with a number of assumptions to be taken into account [8].

1. The state of an economic system at different points of time is not known with a high degree of certainty. Therefore deterministic models will be too rough and, under the circumstances, it is necessary to use stochastic models.
2. In economic systems, decisions are made periodically, i.e., at approximately regular intervals, and the decisions made affect the objects during finite time periods.
3. Economic quantities, as a rule, are measured at discrete moments in time. Therefore a discrete form of describing decision-making problems in the innovation process better corresponds to reality than an indiscrete form.
4. In search for solutions, it is often necessary to take into account several targets that complement or compete with each other. The problem of decision-making is an optimization problem with several target functions, also known as the problem of vector optimization.

Any economic decision-making problem at any aggregation level must be formulated as a stochastic, discrete, multi-criteria (or multi-objective) optimization problem [9].

Investigation of the multi-objective optimization problem requires the development of methods of specification and control of such key elements as normalization, convolution, priority, studying of specific choice principles and analysis of their properties, such as improvability, sensitivity and sustainability.

In constructing decision-making models, much attention is paid to the choice of targets as they play an important role in the economic interpretation of the problem. In order to satisfy different conditions, it is necessary to use a compromise, which is one of the types of classical policies for making an economic decision. The simplest way to find a feasible compromise decision is to construct a new function for goal f on a number of sub goals f_i $(i = 1, 2, \ldots, n)$ in accordance with a specific rule defining the "optimum" concept. The rule defining the concept of optimum is called "the principle of choosing a solution to a problem of multi-objective optimization." The choice principles include classical choice principles (egalitarianism, utilitarianism) [10]) and other principles that combine classical [11] choice principles with Pareto, Slater, equality, Nash [12], Hurwitz criteria, etc.

Mathematically, classical rules for choosing an optimal decision with a finite number of subgoals n can be formulated as:

$$f_{eg} = \max_{1 \le i \le n} f_i \tag{7.3}$$

$$f_{ut} = \sum_{i=1}^{n} f_i. \tag{7.4}$$

Definition 7.16 Normalization is a single-valued transformation $F \to F$ (where F is the function space in which all components of the multi-objective indicator f are determined) that converts the objective functional $f \in F$ into another element of space F.

Let us describe some basic methods of normalization of egalitarian and utilitarian functionals.

Natural normalization of the egalitarian functional gives the following functional:

$$\frac{\max_{1\le i\le n} f_i - \min_{f_i\in F}\max_{1\le i\le n} f_i}{\max_{f_i\in F}\max_{1\le i\le n} f_i - \min_{f_i\in F}\max_{1\le i\le n} f_i}.$$

Natural normalization of the utilitarian functional gives the following functional:

$$\frac{\sum_{i=1}^{n} f_i - \min_{f_i\in F}\sum_{i=1}^{n} f_i}{\max_{f_i\in F}\sum_{i=1}^{n} f_i - \min_{f_i\in F}\sum_{i=1}^{n} f_i}.$$

Reducing the egalitarian and utilitarian functionals to the dimensionless form, we obtain:

$$\frac{\max_{1\le i\le n} f_i}{\nu[\max_{1\le i\le n} f_i]}$$

and

$$\frac{\sum_{i=1}^{n} f_i}{\nu[\sum_{i=1}^{n} f_i]}$$

Normalization of comparison of the egalitarian and utilitarian functionals gives the following expressions:

$$\frac{\max_{1\le i\le n} f_i}{\max_{f_i\in F}\max_{1\le i\le n} f_i}$$

and

$$\frac{\sum_{i=1}^{n} f_i}{\max_{f_i\in F}\sum_{i=1}^{n} f_i}$$

Savage normalization of the egalitarian and utilitarian functionals gives:

$$\max_{f_i\in F}\max_{1\le i\le n} f_i - \max_{1\le i\le n} f_i$$

and

$$\max_{f_i\in F}\sum_{i=1}^{n} f_i - \sum_{i=1}^{n} f_i.$$

Normalization of averaged egalitarian and utilitarian functionals gives the following functionals:

$$\frac{\max_{1\le i\le n} Mf_i}{\sum_{f_i\in F}\max_{1\le i\le n} Mf_i}$$

and

$$\frac{M\sum_{i=1}^{n} f_i}{\sum_{f_i\in F} M\sum_{i=1}^{n} f_i}.$$

The relations of priority, preferability and importance make it possible to concretize the optimization problem, to analyze the socioeconomic model, and to make economic assessments, for example assessments of profitability and efficiency [13].

Methods and deterministic models of resource allocation were developed based on combinations of classical policies: classical utilitarism + policy of a guaranteed minimum; classical egalitarianism + policy of proportional distribution; policy of proportional distribution + classical utilitarianism + policy of a guaranteed minimum.

Some other combinations are also possible. The coefficients related to particular policies can vary, but the sum of their weighing coefficients must be equal to unity.

As the application of the same classical resource allocation policy for a long period of time is considered undesirable, combined allocation policies have been developed [4, 15, 18, 21]:

1. *Classical utilitarianism + policy of a guaranteed minimum.*

All areas must be brought to the level of a guaranteed minimum, and the remaining resources must be allocated to the most profitable areas. The mathematical model of the problem can be presented as follows:

$$\begin{aligned}
&\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right) \to \min;\\
&\sum_{i=1}^{n} u_i(t) \le c(t) = \varphi(t) + \psi(t);\\
&y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t);\\
&\quad y_i(1) > 0,\ y_i(t) \ge y_i(t-1),\ S_i(t) \ge 0,\ c(t) > 0,\ u_i(t) \ge 0;\\
&\quad 0 < P_i(t) = y_i(t+1) \le \overline{y}_i;\ P_i(t) \ge P_i(t-1);\\
&\quad i = 1, 2, \ldots, n,\ t \in [1, T].
\end{aligned}$$

2. *Classical egalitarianism + proportional allocation.*

In this case a certain part of resources is equally divided among all the areas and the other part of resources is allocated among the areas proportionately to their gain per unit of investment. The mathematical model of the above allocation is written as:

$$\alpha\left(\max_{i=\overline{1,n}}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right) + (1-\alpha)\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right) \to \min, \quad 0 \le \alpha \le 1;$$

$$\sum_{i=1}^{n} u_i(t) \le c(t) = \varphi(t) + \psi(t);$$
$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t);$$
$$\bar{y}_i \ge y_i(t);$$
$$y_i(1) > 0,\ y_i(t) \ge y_i(t-1),\ S_i(t) \ge 0,\ c(t) > 0,\ u_i(t) \ge 0;$$
$$i = 1, 2, \ldots, n,\ t \in [1, T].$$

3. *Proportional distribution + classical utilitarianism + policy of a guaranteed minimum.*

All the areas get resources classical utilitarianism to their profit per unit of investment and the remaining resources are channeled to the development of more promising areas provided all other areas have minimal operational conditions. The mathematical model of such a combination of resource allocations can be presented as follows:

$$\alpha\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right) + (1-\alpha)\left(\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right) \to \min; \quad 0 \le \alpha \le 1;$$
$$\sum_{i=1}^{n} u_i(t) \le c(t) = \varphi(t) + \psi(t);$$
$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t); \quad y_i(t) \ge y_i(t+1),\ y_i(1) > 0;$$
$$\bar{y}_i \ge y_i(t);$$
$$S_i(t) \ge 0,\ c(t) > 0,\ u_i(t) \ge 0;$$
$$0 < P_i(t) = y_i(t+1) \le \overline{y}_i;\ P_i(t) \ge P_i(t-1);$$
$$i = 1, 2, \ldots, n,\ t \in [1, T].$$

Some other combinations could be also considered.

A combined policy of classical egalitarianism and proportional allocation of resources has proven to be efficient for allocation of public investments in the construction of educational institutions. In these cases the available resources are allocated in such a way as to put into service a maximal number of facilities, even at the expense of suspending work in some other sites.

In the conditions of incomplete information, the optimization problems are solved by application of pure and combined strategies. In pure strategies the solution is represented by a vector—an optimal solution to the problem. In mixed strategies the solution is a probabilistic distribution of the optimum plan components. While solving problems through application of both pure and mixed strategies, the results may depend or may not depend on observed realization of random values that appear in the statement of the problem. If the decision precedes the experimental observations, an optimal plan of the problem on decision-making in conditions of incomplete information is found through well-known sampling of probabilistic parameter values (or stochastic characteristics). In this case the problem solution does not depend on current realizations of probabilistic values. If the problem on decision-making is solved after experimental observations, the observation results are taken into account in the optimum plan.

Definition 7.17 Solution of the decision-making problems in pure strategies will be called decision rules.

Definition 7.18 Solution of the decision-making problems in mixed strategies will be called decision distributions.

Definition 7.19 If the solution of the decision-making problem in the conditions of incomplete information is accepted before observation of probabilistic parameters, we say that the decision is determined by the zero-order decision rule.

Definition 7.20 If the solution to the decision-making problem in the conditions of incomplete information is accepted after observation of probabilistic parameters, we say that the decision is defined in deterministic vectors (or the first-order decision rules).

Definition 7.21 If the solution of the decision-making problem in the conditions of incomplete information is accepted before observation of probabilistic parameters of the conditions, we will call the obtained decision rules or decision distributions *a priori* decision rules or *a priori* decision distributions.

Definition 7.22 If the solution of the decision-making problem in the conditions of incomplete information is accepted before observation of probabilistic parameters of the conditions, we will call the decision rules or decision distributions *a posteriori* decision rules or decision distributions.

The decision-making models used in the conditions of incomplete information can be single-stage and multi-stage. In the single-stage models the decision is made once and is not corrected afterwards. In the multi-stage models the decision can be revised many times.

As there are many important reasons for making corrections in the decisions, it is necessary to use multi-stage models of stochastic programming:

1. In the management process, it is not always possible to simultaneously monitor real values of all probabilistic parameters that are included in the target function and limitations of the problem.
2. From the information viewpoint, final corrections of the decision must be made as late as possible when the values of all real parameters are already known.
3. Sometimes, to provide for satisfactory functioning of the process (or object) being modeled, it is necessary to take some actions with strictly determined time intervals.
4. Delayed corrections tend to impair the efficiency of the implemented decision.

5. Shorter time spent on corrections may result in higher risks or additional costs.

However, the necessity of corrections is not a result of deficiencies in the planning system. When planning is implemented in the conditions of uncertainty, corrections to the plan are inevitable.

7.4 Single-Stage Stochastic Models for Limited Resource Allocation with Probabilistic Constraints

To provide for setting of economic tasks, rational management, and balanced and resource-based planning, it is necessary to develop and study extremal decision-making models on resource allocation. However, a final decision cannot be made randomly. Ultimately, the decision depends on two major factors: firstly, the decision-maker's final objective and policy, and secondly, the ratio of available and required resources.

Planning and management processes are associated with uncertainty and risks, and that is why decision-making problems on resource allocation in conditions of insufficient information on the problem parameters can be solved using the mathematical apparatus of stochastic programming.

Statement of the decision-making problem of resource allocation under the conditions of insufficient information largely depends on goals and information structure of the problem. Single-stage decision-making models of resource allocation under the conditions of insufficient information are static control problems when the decision, once made, cannot be corrected. Such problems often contain the requirement that the probability of the decision occurrence within the admissible area must exceed a certain predetermined number $\alpha > 0$.

In cases when possible discrepancies between some assumptions cause various losses, it is reasonable to use differentiated approaches for different conditions. To balance any losses caused by discrepancies between the conditions of the problem, it is natural to limit the probability of their realization from below by various numbers $\alpha_p > 0$. Usually $\alpha_p > 0.5$. Low values of this indicator are indicative of the low competence of the decision-maker. For example, allocating funds for purchasing and maintenance of equipment, in order to enhance the efficiency of the decision to be made, it is often necessary to carry out additional research.

Such formulations of probabilistic decision-making problems are called decision-making models with probabilistic constraints. They occur in two classes of situation. The first class includes planning and management problems that require strict statement of the problem. However, under this approach most problem plans turn out to be empty. Under such conditions the problem will make sense only if we assume that in a certain set of states the constraints are violated. In the second class, expenditures targeting the elimination of discrepancies between conditions with relatively rarely occurring states would not be compensated for by the achieved effect of optimization of the target function. There are the three reasons for statement of a decision-making problem with probabilistic constraints [4, 20]:

(1) Type of decision;
(2) Choice of the indicator of decision quality;
(3) Methods of partitioning the problem constraints.

Depending on the content of the decision-making problem under conditions of incomplete information, plans and solutions to the problem can be calculated using pure or mixed strategies. The solution in pure strategies is a vector and an optimum plan of the problem. Solutions in mixed strategies are probabilistic distributions of the optimum plan components.

The solution obtained using both pure and mixed strategies either depends on or does not depend on the observed realizations of random parameters of the problem. If the decision precedes observation, the optimum plan of the decision-making problem in the situation of uncertainty is determined by stochastic characteristics or the known sample of possible values of the problem parameters. In this case the solution to the problem does not depend on current realizations of the stated problem parameters. If the solution to the decision-making problem follows after the observation, the dependence of the solution on the realized and observed values of the problem parameters must be taken into account in the optimum plan.

In the decision-making process, the constraint elements of the optimization problem are often stochastic, i.e., the elements of the matrix of constraints A and the elements of the vector of constraints c are probabilistic. When solving problems of allocation of limited resources we assume that the elements of matrix A and vector c are independent of each other and are normally distributed values, i.e., $a_{ij} \in N(\bar{a}_{ij}, \sigma_{ij}^2)$, $c_i \in N(\bar{c}_i, \upsilon_i^2)$.

Some single-stage stochastic models of allocation of limited resources with probabilistic constraints have been developed on the basis of combined classical policies: utilitarianism + policy of a guaranteed minimum; egalitarianism + policy of proportional allocation [20].

Therefore a single-stage stochastic problem on limited resource allocation based on the classical utilitarian choice principle and the guaranteed minimum policy with probabilistic constraints will be formulated as:

$$M\left\{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right\} \to \min; \tag{7.5}$$

$$P_1\left\{\sum_{j=1}^{n} a_{ij}(\omega)u_j(t) \le c_i(t,\omega)\right\} \ge \alpha_1, \quad 0.5 < \alpha_1 < 1; \tag{7.6}$$

$$P_2\big\{b_i(t) = h_i(t,\omega)\big\} \ge \alpha_2, \quad 0.5 < \alpha_2 < 1; \tag{7.7}$$

$$\begin{aligned} &y_i(1) > 0,\ y_i(t) \ge y_i(t-1),\ S_i(t,\omega) \ge 0, \\ &c_i(t,\omega) > 0,\ u_i(t) \ge 0;\ \bar{y}_i \ge y_i(t); \end{aligned} \tag{7.8}$$

$$0 < P_i(t) = y_i(t+1) \le \overline{y}_i;\ P_i(t) \ge P_i(t-1); \tag{7.9}$$

$$i = \overline{1,n},\ \omega \in \Omega,\ t \in [1,T]. \tag{7.10}$$

Theorem 7.1 *The linear stochastic problem of limited resource allocation based on the classical utilitarian choice principle with observation of the guaranteed minimum policy* (7.5)–(7.10), *in which the elements of the matrix of constraints A and the elements of the vector of constraints c are independent normally distributed random variables, the solution to which is determined by the zero-order decision rules, is reduced to a deterministic problem of convex programming with a linear target function and quadratic constraints.*

The proof of the theorem includes the method of construction of the deterministic equivalent of the stochastic problem (7.5)–(7.10).

Similar theorems have been formulated and proved for the decision-making model with probabilistic constraints and a stochastic constraint matrix using the *combination of classical egalitarianism and proportional allocation*:

$$M\left\{\alpha\left(\max_{i=\overline{1,n}}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right) + (1-\alpha)\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right)\right\} \to \min,$$

$$0 \le \alpha \le 1;$$

$$P_1\left\{\sum_{j=1}^{n} a_{ij}(\omega)u_j(t) \le c_i(t,\omega)\right\} \ge \alpha_1, \quad 0.5 < \alpha_1 < 1;$$

$$P_2\left\{y_i(t+1) = y_i(t) + S_i(t,\omega)u_i(t) + d_i(t,\omega)\right\} \ge \alpha_2, \quad 0.5 < \alpha_2 < 1;$$

$$y_i(1) > 0,\ y_i(t) \ge y_i(t-1),\ S_i(t,\omega) \ge 0,\ c_i(t,\omega) > 0,\ u_i(t) \ge 0;\ \bar{y}_i \ge y_i(t);$$

$$i = \overline{1,n},\ \omega \in \Omega,\ t \in [1,T],$$

and using *the combined policy of classical utilitarianism + proportional allocation + the policy of the guaranteed minimum*:

$$M\left\{\alpha\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right) + (1-\alpha)\left(\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right)\right\} \to \min;$$

$$0 \le \alpha \le 1;$$

$$P_1\left\{\sum_{j=1}^{n} a_{ij}(\omega)u_j(t) \le c_i(t,\omega)\right\} \ge \alpha_1, \quad 0.5 < \alpha_1 < 1;$$

$$P_2\left\{y_i(t+1) = y_i(t) + S_i(t,\omega)u_i(t) + d_i(t,\omega)\right\} \ge \alpha_2, \quad 0.5 < \alpha_2 < 1;$$

$$y_i(1) > 0,\ y_i(t) \ge y_i(t-1),\ S_i(t,\omega) \ge 0,\ c_i(t,\omega) > 0,\ u_i(t) \ge 0;\ \bar{y}_i \ge y_i(t);$$

$$i = \overline{1,n},\ \omega \in \Omega,\ t \in [1,T],$$

$$0 < P_i(t) = y_i(t+1) \le \overline{y}_i;\ P_i(t) \ge P_i(t-1).$$

The decision-making models of resource allocation with a probabilistic functional have been studied for each of the three combined allocation policies [22]. A method used to reduce stochastic models to the corresponding deterministic equivalents has been presented.

7.5 Multi-Stage Stochastic Models of Limited Resource Allocation with Probabilistic Constraints

Under the conditions of real-life economic processes, the deterministic decision-making problems of resource allocation, in spite of their simplicity and wide application, prove to be incorrect. The deterministic models used in calculations related to decision-making on resource allocation use probabilistic parameters. The necessity of ensuring reliability and maximal proximity to reality makes it necessary to take into account the probabilistic nature of the economic model parameters. On the other hand, production activities of an enterprise are determined by the enterprise management plan and often by certain other regulating plans (for example, municipal education plan, regional development plan, sectoral development plan, etc.). A great number of production situations, external impacts and other parameters of the stochastic model make it necessary to take into account the laws of probabilistic phenomena and processes.

The development of decision-making procedures combining the requirements of fast response and substantiation of plan corrections makes it is necessary to use multi-stage problems of stochastic programming. Correction of the plan in multi-stage problems of stochastic planning does not mean change of parameters of the previously adopted plan, it means the development of measures that will enable one to obtain the predetermined parameters.

Let us list the main reasons for plan correction, which make it necessary to use multi-stage models of stochastic programming:

1. From the information viewpoint, final corrections of the decision must be made as late as possible when the values of all real parameters are already known.
2. In the management process, it is not always possible to simultaneously monitor real values of all probabilistic parameters that are included in the target function and limitations of the problem.
3. Delayed corrections tend to impair the efficiency of the implemented decision.
4. Shorter time spent on corrections may result in higher risks or additional costs.
5. Sometimes, to provide for satisfactory functioning of the process (or object) being modeled, it is necessary to take some actions with strictly determined time intervals.

The necessity to make corrections in the plan is not explained by deficiencies in the planning system. It is necessary to introduce corrections in the plan if planning is made under conditions of uncertainty.

In general terms, the solution to the stochastic decision-making problem is a decision rule or a decision distribution that depends on two groups of factors. The factors of the first group are not related to the observation of current values of parameters stated in the problem. They are determined by *a priori* information, such as some characteristics of the distribution or a sample of possible values of parameters stated in the problem. The factors of the first group can be used in advance to develop or to gradually improve the decision rule or the decision distribution. The factors of the second group are determined by *a posteriori* information that appears as a result of observation of concrete realization of parameters of the decision-making problem.

When the solution precedes the observation, i.e., the chain “solution”–“observation”–“solution”–“observation”–... is used, the decision rules and the decision distributions depend only on the deterministic parameters and statistical characteristics of random parameters of the problem conditions.

When the solution follows the observation, i.e., the chain “observation”–“solution”–“observation”–“solution” ... is used, the decision rules and statistical characteristics of decision distributions are functions, tables or instructions establishing the dependence of the solution on *a priori* information and on the realized values of the random parameters of the problem.

Some multi-stage stochastic models for allocation of limited resources with probabilistic limitations have been developed on the basis of combined classical policies: utilitarianism + the policy of the guaranteed minimum; egalitarianism + the policy of proportional allocation [19, 21].

The method of modeling of *a multi-stage decision-making problem of resource allocation with* a priori *decision* rules is shown below using the example of a combined policy including *classical utilitarianism and the policy of a guaranteed minimum*:

$$
\begin{aligned}
&M_{\omega^n}\psi_0\big(\omega^n, y_i^n(t)\big) = M_{\omega^n}\left\{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t,\omega^n)}{\bar{y}_i}\right)\right\} \to \inf;\\
&M_{\omega_k}\big\{\psi_k\big(\omega^k, u_i^k(t)\big)\big|\omega^{k-1}\big\} = M_{\omega_k}\left\{\sum_{i=1}^{n}\sum_{j=1}^{k}\big\{u_{ij}\big(t,\omega^k\big)\big|\omega^{k-1}\big\}\right\} \le c_k\big(t,\omega^{k-1}\big);\\
&y_i\big(t+1,\omega^{k-1}\big) - y_i\big(t,\omega^{k-1}\big) = S_i\big(t,\omega^{k-1}\big)u_i\big(t,\omega^{k-1}\big) + d_i\big(t,\omega^{k-1}\big);\\
&\quad y_i\big(t+1,\omega^{k-1}\big) \ge y_i\big(t,\omega^{k-1}\big),\ \ y_i\big(1,\omega^{k-1}\big) > 0;\\
&\quad S_i\big(t,\omega^{k-1}\big) \ge 0,\ \ u_i\big(t,\omega^{k-1}\big) \ge 0,\ \ c_k\big(t,\omega^{k-1}\big) > 0,\\
&\quad 0 < P_i(t) = y_i\big(t+1,\omega^{k-1}\big) \le \overline{y}_i;\ \ P_i(t) \ge P_i(t-1);\\
&\quad i = \overline{1,n},\ \ \omega^k \in \Omega^k = \mathop{\times}_{i=1}^{k}\Omega_i,\ \ t \in [1,T].
\end{aligned}
$$

Let us introduce the following notations:

$$\Delta_i\big(t,\omega^{k-1}\big) = y_i\big(t+1,\omega^{k-1}\big) - y_i\big(t,\omega^{k-1}\big)$$

and

$$f_i\big(t,\omega^{k-1}\big) = S_i\big(t,\omega^{k-1}\big)u_i\big(t,\omega^{k-1}\big) + d_i\big(t,\omega^{k-1}\big).$$

Now we will calculate *a posteriori* decision rules, i.e., find the solution among the random quantities:

$$u_i^n\big(t,\omega^n\big) = \big(u_{i1}\big(t,\omega^1\big), u_{i2}\big(t,\omega^2\big), \dots, u_{in}\big(t,\omega^n\big)\big).$$

Let us use p^i to denote the probabilistic measure in the Ω^i set of elementary events W determined as follows:

$$p^i(W) = \begin{cases} p(W \times \Omega_{i+1} \times \Omega_{i+2} \times \cdots \times \Omega_n), & \text{if } W \subset \Omega^i \\ 0, & \text{otherwise} \end{cases}$$

Let us use p_i to denote the probabilistic measure determined as follows:

$$p_i\left(A|\omega^{i-1} \in B\right) = \frac{p^i(A \times B)}{p^i(\Omega_i \times B)}, \quad \text{for all } A \subset \Omega_i,\ B \subset \Omega^{i-1}$$

The measure p^n is continuous.

Let $\Sigma - \sigma$ be algebra of stochastic events in Ω.

Hence, we have determined the probabilistic space (Ω, Σ, P).

The theorem demonstrating the method of construction of recurrent decision rules has been formulated and proved.

The management process provides an opportunity to observe the realization of stochastic parameters of the decision-making model of resource allocation. Knowing how the stochastic parameters are realized in practice, it is possible to make corrections in the accepted decision.

The second possibility of model correction is to take into account *a priori* characteristics at each stage. The sequence of received information and the order of choice and correction of decisions depend on the information structure of the problem—a set of the initial data accumulated at the preceding stages, which may influence the decision at the current stage. If the decision precedes the observation of parameters of random quantities, the solution of the stochastic problem is determined by the stochastic characteristics or the known sample of possible realizations of random parameters of the stochastic model of resource allocation. In the latter case we deal with decision-making based on *a priori* information.

If the decision-maker uses a combined policy including classical egalitarianism and proportional allocation of resources, *the multi-stage decision-making problem of resource allocation in the conditions of incomplete information with* a priori *decision rules will be written as*:

$$M_{\omega^n}\left\{\alpha\left(\max_{i=\overline{1,n}}\left(\frac{\bar{y}_i - y_i(t,\omega^n)}{\bar{y}_i}\right)\right) + (1-\alpha)\left(u_i\left(t,\omega^n\right) - \frac{\frac{\bar{y}_i - y_i(t,\omega^n)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t,\omega^n)}{S_i(t,\omega^n)}\right)}\right)\right\}$$

$$\to \min, \quad 0 \le \alpha \le 1;$$

$$M_{\omega_k}\left\{\psi_k\left(\omega^k, u_i^k(t)\right)\middle|\omega^{k-1}\right\} \le c_k\left(t,\omega^{k-1}\right);$$

$$y_i\left(t+1,\omega^{k-1}\right) - y_i\left(t,\omega^{k-1}\right) = S_i\left(t,\omega^{k-1}\right)u_i\left(t,\omega^{k-1}\right) + d_i\left(t,\omega^{k-1}\right);$$

$$y_i\left(t+1,\omega^{k-1}\right) \ge y_i\left(t,\omega^{k-1}\right),\ y_i\left(1,\omega^{k-1}\right) > 0;$$

$$S_i\left(t,\omega^{k-1}\right) \ge 0,\ u_i\left(t,\omega^{k-1}\right) \ge 0,\ c_k\left(t,\omega^{k-1}\right) > 0,$$

$$u_{ik} \in G_k^0;\ \omega^k \in \Omega^k = \mathop{\times}_{i=1}^{k} \Omega_i;\ i,k = \overline{1,n};\ t \in [1,T].$$

Let us denote the lower boundary of the quality indicator for the solution of the given decision-making problem of resource allocation as $\breve{S}(c^n(\omega^{n-1}))$, i.e.,

$$\breve{S}\big(c^n\big(\omega^{n-1}\big)\big) = \sup_k c\big(t, \omega^{k-1}\big).$$

The decision on resource allocation at the i-th stage is made after realization of random parameters of the problem at the stage preceding the $i-1$-th stage. Hence, the decision rules will be written as:

$$u_i(t) = u_i\big(t, \omega^{i-1}\big), \quad i = \overline{1, n}.$$

The following characteristics of the models were analyzed: stability with respect to functional, stability of stochastic distribution, stability with respect to probabilistic parameter α, stability with respect to i-th limitation, ε-stability with respect to average values.

Combined deterministic and stochastic (single-stage and multi-stage) models can be used in enterprises where management is oriented towards the best final result of decisions on resource allocation [14].

7.6 Use of the Combined Policy Model for Making Decisions on Resource Allocation

The combined policy including classical utilitarianism and the policy of a guaranteed minimum proves to be quite efficient for allocation of enterprise resources if the enterprise ensures uninterrupted operation of all its facilities but does not have enough resources to bring all its objects to an optimal state.

Vivid examples of using such a policy can be seen in repairs carried out by public utilities.

While repairing water pipelines, heat mains, motor roads, schools and other objects, entities in charge bear certain expenses—but if they refuse to repair and maintain these objects, the objects lose their capacity to provide basic services.

The combination of classical utilitarianism and the policy of a guaranteed minimum proves to be efficient for enterprises whose activities include both profitable business lines and loss-making operations as part of their social mandate. Among such enterprises can be found bakeries (losses incurred while baking ordinary bread are compensated for by the income made on buns, rolls, French loaves and other products) and motor transport companies (they cover their losses on inter-city routes by their earnings from intra-city routes) [16, 17].

7.6.1 Allocation of Maintenance Resources by Teplocentral Public Enterprise

Let us consider some problems of resource allocation using the example of the Teplocentral Public Enterprise which provides services to the population. The enterprise supplies hot water for central heating and other needs of the population and organizations of Zyryanovsk City and the District of the same name. The enterprise owns all supply lines and facilities except buildings. The pipes and radiators located inside residential buildings belong to their owners (or organizations). Most of the supply lines and facilities were put in operation simultaneously with the residential blocks many years ago and must now be repaired or replaced.

The Teplocentral Public Enterprise does not have enough resources to replace all worn-out supply lines and facilities in the near future. But leakages of hot water and falling water temperatures caused by disrepair of pipelines and facilities cause losses to the Enterprise. So, the Enterprise has to allocate its available resources (money, pipes, labor) in an optimal way.

The best possible decision must meet the following rules:

1. All the existing service lines must remain in operation. Despite the lack of resources Teplocentral Public Enterprise suffers from, each customer who pays in due time must be provided with full services. To meet the requirement, the Enterprise must maintain its service lines at least to the extent ensuring the state of the guaranteed minimum.
2. If any resources allocated for maintenance during the planned period remain unused, the remaining part of the resources must be spent on major repairs of the most worn-out sections of the heat transport system.

It would be logical to suppose that improvements in the state of each section are proportional to the amount of investment in these sections, and their deteriorating states are due to depreciation and are of random nature.

In order to evaluate the states of different sections of the heat transport system the following criteria are used:

– The number of requests for current repairs (during the preceding period);
– The leakage-caused hot water losses (as a ratio of water released into the section and the water volume that reached customers equipped with water meters);
– Heat losses due to fall in water temperature;
– Temperature of the supplied hot water;
– Temperature in serviced residential buildings;
– Any other criteria used by the individual expert.

The assessment of section i of the service line by the j-th expert y_{ij} is determined by one of the following formulas:

$$y_{ij} = \begin{cases} \prod_{k=1}^{k_j} \frac{y_{ijk}}{\overline{y}_{ijk}}, & \text{if } y_{ijk} \geq \underline{y}_{ijk} \ (k = \overline{1, k_j}) \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} \prod_{k=1}^{k_j} \lambda_{jk} \cdot \frac{y_{ijk}}{\underline{y}_{ijk}}, & \text{if } y_{ijk} \geq \underline{y}_{ijk} \ (k = \overline{1, k_j}) \\ \min_{k=\overline{1,k_j}} \frac{y_{ijk}}{\underline{y}_{ijk}}, & \text{otherwise} \end{cases}$$

where k_j is the number of criteria considered important by the j-th expert; y_{ijk} is the assessment of the k-th partial criterion for the section i of the service line by the

j-th expert; $\underline{y}_{ijk}$ is the minimum permissible value of the k-th partial criterion of the section i of the hot water network by the j-th expert; λ_{jk} is the weighing coefficient of the k-th partial criterion for the j-th expert.

Based on the assessments made by the experts, the condition of the section i of the service line y_i is presented as:

$$y_i = \sum_{j=1}^{k} \chi_j \cdot y_{ij},$$

where k is the number of the experts involved in the analysis of the hot water service line; χ_j is the weighing coefficient of the assessment made by the j-th expert.

In order to achieve better serviceability, the Teplocentral Public Enterprise divided the city into service segments. The Enterprise's mid-term planning covers the upcoming three years.

As it is seen from the above description, in order to allocate the resources the decision-maker uses a model described here as a combined policy model including the models of classical utilitarianism and the guaranteed minimum.

In the deterministic case the model will be written as:

$$\sum_{i=1}^{n} \left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i} \right) \to \min;$$

$$\sum_{i=1}^{n} u_i(t) \le c(t) = \varphi(t) + \psi(t);$$

$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t);$$

$$y_i(1) > 0,\ y_i(t) \ge y_i(t-1),\ S_i(t) \ge 0,\ c(t) > 0,\ u_i(t) \ge 0;$$

$$0 < P_i(t) = y_i(t+1) \le \overline{y}_i;\ P_i(t) \ge P_i(t-1);$$

$$i = 1, 2, \ldots, n,\ t \in [1, T],$$

where $t \in [1, T]$ are periods for making decision on resource allocation; $c(t) = \varphi(t) + \psi(t)$ is a certain amount of resources, where $\varphi(t)$ are resources to be allocated at a given point of time t, $\psi(t)$ are resources to be channeled to the accumulation or reserve fund of the Enterprise. Resources of the reserve fund can be used at a certain point of time denoted as $\tau \subset [1, T]$; n is the number of possible investment areas using available resources, i.e., the number of water pipeline sections; $i = 1, 2, \ldots, n$ are identification numbers of the water pipeline sections where the condition of each section does not depend on the condition of other sections; $u_i(t)$ is the amount of resources being invested in section i at the point of time t; $y_i(t)$ is the condition of section i at the point of time t; $\bar{y}_i$ is the reference state of section i at the point of time T; $P_i(t)$ is the condition of the guaranteed minimum for section i at the point of time t; $S_i(t)$ is the efficiency of investments in section i at the point of time t, i.e., the increase in assets per unit of investment.

We suppose that any reduction in the disproportion of the object development is proportional to the invested resources:

$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t),$$

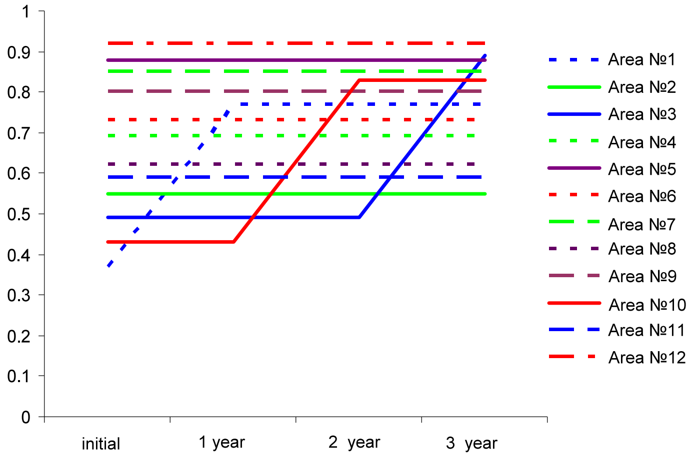


Fig. 7.1 Changes in the state of sections calculated using a combined policy for resource allocation

where $d_i(t)$ is an external factor that is often negative, and that is caused by amortization and other man-caused and natural factors (earthquakes, underground explosions at mining entities, rehabilitation of roadbeds, etc.).

Thanks to long-term observation, the enterprise knows most of the model parameters. The economists of the enterprise have sufficiently precise data on the payments to be made by consumers and resources to be allocated for maintenance and repairs. Market costs of the main materials (pipes, electricity, oxygen for welding, insulation materials) and wages can be forecast with a high level of accuracy.

Based on the results of service line monitoring, it is possible to identify the sections that need major repairs. In the other sections only routine maintenance will be carried out at leakage points. The total cost of routine maintenance will be determined based on the data of preceding years. It is difficult to take into account the random factor $d_i(t)$, which is why it is assumed that complete wearing out of the pipeline from the state denoted as 1 (putting the pipeline into operation) to the state 0 (the state requiring major repairs to provide the guaranteed minimum) is a process that lasts for about 50 years.

Let us consider the changes in the state of utilities when the proposed method is used for resource allocation. It should be kept in mind that the resource allocation for the whole planning period is not final. Funding flows can be regulated on the basis of *a posteriori* information [21].

The charts below reflect the changes in the state of utilities in each maintenance segment using (the same input data):

– Combined allocation policy (Fig. 7.1) including classical utilitarianism and policy of the guaranteed minimum;
– classical utilitarian policy of resource allocation (Fig. 7.2);
– classical egalitarian policy of resource allocation (Fig. 7.3)

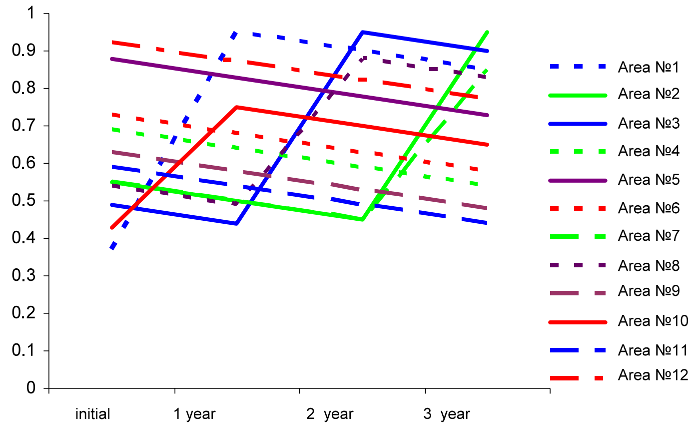


Fig. 7.2 Changes in the state of sections calculated using classical utilitarian policy for resource allocation

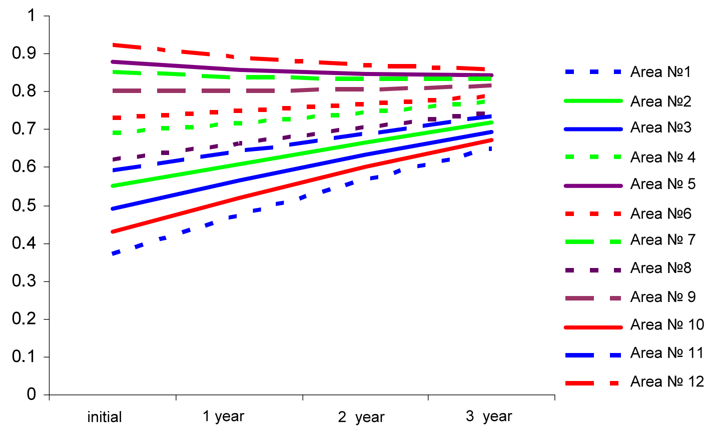


Fig. 7.3 Changes in the state of sections calculated using classical egalitarian policy for resource allocation

7.6.2 Combination of Policies of Resource Allocation in the Investment Management

An analysis of investment projects is based on the comparison of economic benefits provided to the investor after implementation of the project. We will consider the case when the implementation of the investment project does not give any economic benefits. Such projects are usually realized in the social-economic sphere—education, healthcare, ecology, etc.

Many objects are built at the expense of budget. As it is not allowed to transfer money from one budget item to the other, we will consider the objects belonging to

Table 7.1 The example list of investments projects to be implemented at the expense of the oblast's budget development fund

Investments projects	Total cost (10^3 tenge)
Project #1	92,510
Project #2	108,100
Project #3	129,730
Project #4	167,300
Project #5	125,010
Project #6	32,140
Project #7	39,900
Project #8	24,800
Project #9	1,881,870
Total	2,601,360

the same sphere, education. Table 7.1 presents the example of investments projects of education, the construction of which is financed from the budget.

It is reasonable to distribute resources among the projects based on the combined policy including classical egalitarianism and the policy of proportional distribution.

The mathematical model of the problem is formulated as:

Find the distribution of resources

$$u_i(t), \quad i = 1, 2, \ldots, n, \ t = 1, 2, \ldots, T.$$

Satisfying the conditions

$$\sum_{i=1}^{n} u_i(t) \le c(t) = \varphi(t) + \psi(t); \tag{7.11}$$

$$y_i(t) = y_i(t-1) + S_i(t)u_i(t) + d_i(t); \tag{7.12}$$

$$0 \le y_i(t) \le \overline{y}_i; \tag{7.13}$$

$$y_i(t) \ge y_i(t-1); \tag{7.14}$$

$$S_i(t) \ge 0; \tag{7.15}$$

$$c(t) \ge 0; \tag{7.16}$$

$$u_i(t) \ge 0; \tag{7.17}$$

and minimizing the target functional

$$\alpha\left(\max_{i=\overline{1,n}}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right) + (1-\alpha)\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right) \to \min, \quad 0 \le \alpha \le 1 \tag{7.18}$$

The condition (7.11) can be interpreted as: the total funds spent on all n directions cannot exceed the available resources (consisting of expenditures and savings). The condition (7.12) means that the disproportion in the object development decreases proportionately to the invested funds taking into account the external factor. According to the condition (7.13) the current state of any object is not negative but it does not exceed the reference state. The new state of any object is not worse than its previous state follows from the condition (7.14). According to the condition (7.15) the efficiency of investments in any of the directions is not negative. From the condition (7.16) it follows that funds for the distribution cannot be negative. The target function (7.18) means that the total disproportion of all the directions must be minimal. The DM chooses the value of α based on his own experience, in particular, for this problem good α values lie in the interval from $1/3$ to $-2/3$.

As partial criteria of the quality of investment projects in education financed by the oblast's budget one can use:

- Annual saving of funds spent on the pupils' delivery to school;
- Daily saving of pupils' time;
- The number of created jobs;
- Possibility to choose the learning language;
- Perspectives of the development of villages and small towns, and other factors.

Resources are often distributed in the conditions of insufficient information. The decision maker has practically no information about many external factors influencing the components of the system. These factors include natural and technogenic catastrophes. The decision maker has too little information about the future state of the market. Some factors (the efficiency of investments, resource consumption) are only expressed in the form of more or less reliable forecasts. Therefore decision making must be based on stochastic models. Such factors as the efficiency of investments of resources into investment projects $(S_i(t,\omega))$ as well as difficulties related to project implementation $(d_i(t,\omega))$ are not known beforehand, whereas the volumes of delivered resources may change.

Then the problem of distribution of resources among the project is reduced to the problem of making decisions on the distribution of resources with proportional and egalitarian choice principle:

$$M\left\{\alpha\left(\max_{i=\overline{1,n}}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right) + (1-\alpha)\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right)\right\} \to \min,$$

$$0 \le \alpha \le 1$$

$$P_1\left\{\sum_{j=1}^{m} a_{ij}u_i(t) \le c_i(t,\omega)\right\} \ge \alpha_1, \quad 0.5 < \alpha_1 < 1;$$

$$P_2\left\{y_i(t+1) = y_i(t) + S_i(t,\omega)u_i(t) + d_i(t,\omega)\right\} \ge \alpha_2, \quad 0.5 < \alpha_2 < 1;$$

$$0 \le y_i(t) \le \overline{y}_i;\ y_i(t) \ge y_i(t-1);\ S_i(t) \ge 0;\ c(t) \ge 0;\ u_i(t) \ge 0;$$

$$i = \overline{1,n},\ \omega \in \Omega,\ t \in [1,T].$$

Here a_{ij} are the elements of matrix A, the matrix of the conditions of the problem, m is the number of flows of resource investments. If the resources are invested once during each planning period, $m = 1$. The quantities S_i, d_i, c_i $(i = 1, 2, \ldots, n)$

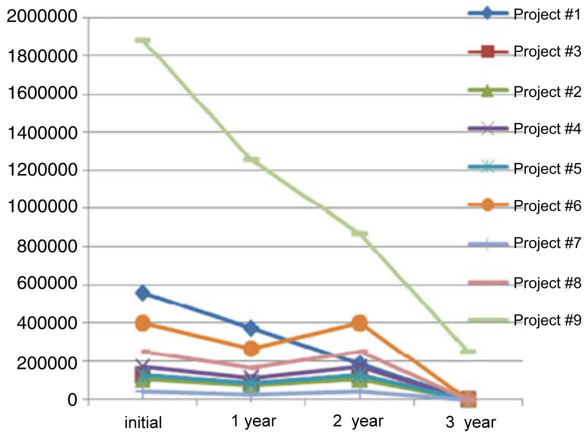


Fig. 7.4 Reduction in the volumes of funds to be allocated

are random quantities in the set Ω of realization of a random parameter ω. It is valid that

$$a_{ij} \geq 0 \quad (i = 1, \ldots, n;\ j = 1, \ldots, m) \quad \text{and} \quad \sum_{j=1}^{m} a_{ij} = 1 \quad (i = 1, \ldots, n).$$

To solve the above-stated problem it is necessary to find its deterministic equivalent.

Let us introduce the following notations:

$$y_i(t+1) - y_i(t) = b_i(t),$$
$$S_i(t,\omega)u_i(t) + d_i(t,\omega) = h_i(t,\omega).$$

Then the initial problem can be written as:

$$M\left\{\alpha\left(\max_{i=\overline{1,n}}\left(\frac{\bar{y}_i - y_i(t)}{\bar{y}_i}\right)\right) + (1-\alpha)\left(u_i(t) - \frac{\frac{\bar{y}_i - y_i(t)}{S_i(t)}}{\sum_{i=1}^{n}\left(\frac{\bar{y}_i - y_i(t)}{S_i(t)}\right)}\right)\right\} \to \min,$$

$$0 \leq \alpha \leq 1$$

$$P_1\left\{\sum_{j=1}^{m} a_{ij}u_i(t) \leq c_i(t,\omega)\right\} \geq \alpha_1, \quad 0.5 < \alpha_1 < 1;$$

$$P_2\{y_i(t+1) = y_i(t) + S_i(t,\omega)u_i(t) + d_i(t,\omega)\} \geq \alpha_2, \quad 0.5 < \alpha_2 < 1;$$

$$0 \leq y_i(t) \leq \overline{y}_i;\ y_i(t) \geq y_i(t-1);$$

$$S_i(t) \geq 0;\ c(t) \geq 0;\ u_i(t) \geq 0;$$

$$i = \overline{1,n},\ \omega \in \Omega,\ t \in [1,T].$$

As a result of usage of the distribution policy the objects are implemented in the order shown in Fig. 7.4.

The figure shows that in case of insufficient financing only one (the most expensive) project will be put into commission. The choice of this object is determined by

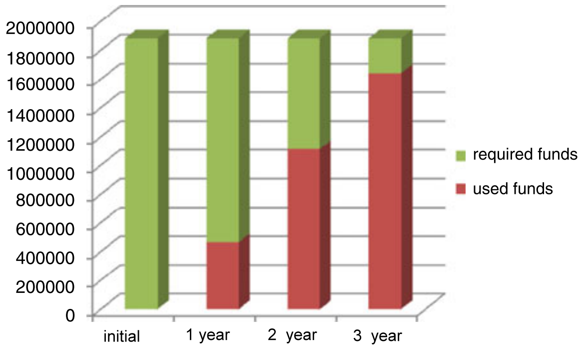


Fig. 7.5 Dynamics of implementation of funds (utility system)

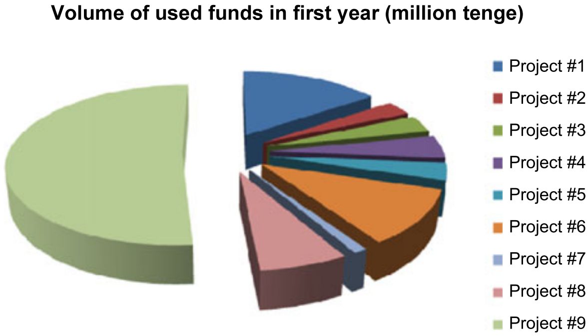


Fig. 7.6 Volume of used funds in first year (utility system)

its high cost (60 % of total cost) at not very high value of the object, practically not higher than the average value of the other objects.

For example Fig. 7.5 show the dynamics of implementation of funds for the project #9.

Diagram in Fig. 7.6 demonstrate the allocation of funds among the projects.

The models enable DMs to make efficient managerial decisions in the conditions of uncertainty and to correct plans when additional reliable data are obtained, which gives better economic effect of taken decisions and contributes to the higher standard of living for the citizens of the republic.

References

1. Mulen, E.: Cooperative Decision Making: Axioms and Models. Mir, Moscow (1991), p. 463
2. Gowdy, J.: Limited Wants, Unlimited Means: A Reader on Hunter-Gatherer Economics and the Environment. Island Press, St Louis (1998), p. 342
3. Mill, J.S., Crisp, R. (eds.): Utilitarianism, pp. 56–67. Oxford University Press, Oxford (1998)
4. Koroteyev, S.V., Bykova, I.Yu.: Investigation of the problem of aggregation of individual preferences in the decision-making process. VKGTU Vestn. **1**, 67–71 (2004)
5. Errow, K.G.: A Collective Choice and Individual Preferences. Mir, Moscow (1973)
6. Harsanyi, J.C., Zelten, R.: General Theory of Choosing Equilibrium in Games. Mir, Moscow (1982)

7. Erfani, T., Utyuzhnikov, S.V.: Directed search domain: a method for even generation of the Pareto frontier in multiobjective optimization. Eng. Optim. **43**(5), 467–484 (2011)
8. Makarov, I.M., Vinogradskaya, T.M., Rubchinsky, A.A., Sokolov, V.B.: The Theory of Choice and Decision-Making. Nauka, Moscow (1982), p. 328
9. Lezina, Z.M.: Procedures of collective choice. Avtom. Telemeh. **8**, 3–35 (1987)
10. Volsky, V.I.: Tournament functions in the problems of collective and multiple criteria choice. In: Theory of Structures, pp. 67–85. Nauka, Moscow (1987)
11. Harsanyi, J.C.: Cardinal welfare, individualistic ethics and interpersonal comparisons of utility. J. Polit. Econ. **63**, 309–321 (1955)
12. Nash, J.F.: Equilibrium points in N-person games. Proceedings of the National Academy of Sciences **36**(36), 48–49 (1950)
13. Petrosyan, L.A., Zenkevich, N.A., Semina, E.A.: Games Theory. Vyshaya Shkola, Moscow (1998)
14. Andreatta, G., Dell'Olmo, P., Lulli, G.: An aggregate stochastic programming model for air traffic flow management. Eur. J. Oper. Res. **215**(3), 697–704 (2011)
15. Koroteyev, S.V.: Multiple criteria utility function in the conditions of certainty. In: Role of Technical Universities in the Development of Innovation Economy: RNTK Materials, pp. 6–9. VKGTU, Ust-Kamenogorsk (2007)
16. Koroteyev, S.V.: Use of the stochastic programming apparatus for the analysis of expediency of equipment purchasing. In: Role of Technical Universities in the Development of Innovation Economy: RNTK Materials, pp. 3–5. VKGTU, Ust-Kamenogorsk (2007)
17. Mutanov, G.M., Koroteyev, S.V., Bykova, I.Yu.: Implementation of preferences in the multiple criteria decision-making problems. In: MNTK "Automation & Control: Perspectives, Problems, Decisions": Collection of Papers, pp. 82–86. KazNTU, Almaty (2007)
18. Mutanov, G.M., Koroteyev, S.V., Bykova, I.Yu.: Problems of multiple criteria optimization. In: MNTK "Automation & Control: Perspectives, Problems, Decisions": Collection of Papers, pp. 79–82. KazNTU, Almaty (2007)
19. Koroteyev, S.V.: Modelling of resource allocation processes with a combined target functional. In: Innovation and Communication Technologies as the Main Factor of the Innovation Society Development: Collection of Papers, MNPK, pp. 225–228. VKGTU, Ust-Kamenogorsk (2007), p. II
20. Koroteyev, S.V.: One-stage stochastic models of resource allocation with a combined target functional. In: Innovation and Communication Technologies as the Main Factor of the Innovation Society Development: Collection of Papers, MNPK, pp. 229–235. VKGTU, Ust-Kamenogorsk (2007), p. II
21. Koroteyev, S.V.: A combined policy of distribution of resources of an enterprise. Sci. Innov., Mater. IV MNPK **11**, 9–12 (2008). Prshemysl (Poland)
22. Yudin, D.B.: Mathematical Methods of Control in Conditions of Incomplete Information: Objectives and Methods of Stochastic Programming. Mathematical Methods of Control with Incomplete Information: Problems and Methods of Stochastic Programming, 2nd edn. (2010), p. 400 (In Russian)

Chapter 8
Mathematical Methods and Models for Monitoring of Government Programs

Government programs at the national and regional level are an essential part of the system of State management of the national economy, making it possible to shape the development of priority sectors and individual regions and attract extra investments and resources. Government programs (GP) are a flexible instrument of the implementation of a long-term economic State policy and an active influence on industrial and economic processes. Kazakhstan is committed to building a strong national infrastructure for the rapid introduction of new information technologies in all spheres of economy and management.

The problems of software support of target programs have been studied by R.A. Kochkarov, V.P. Bocharnikov, S.V. Sveshnikov, Y.V. Yatsyshin, V.V. Kulba, A. Kuzmin, V.F. Kulikov and others [1–9]. In [1], the target program was formalized as a hierarchical structure; the task of organic "covering" of the target program by projects was reduced to a multi-criteria assignment problem on prefractal graphs. In [2], the authors used Fuzzy techniques to solve expert-analytical tasks of formation and support of target programs under conditions of uncertainty. In [3–7], the authors developed procedures for formulation and automated generation of scenarios for the development of complex systems using weighted sign graphs, the methodology of a more efficient monitoring of program stages and targeted use of target program resources. The efficiency of the programs was evaluated by American researchers Michael Patton and Michael Scryven. These publications most fully cover the first stages of the GP life cycle, where the problems of formulation of targeted programs and scenarios of GP development as a complex system are solved [6–9].

The problems of evaluation of the efficiency of government programs in the process of their implementation and budget execution have not been thoroughly studied. These tasks include: evaluation of program performance, assessment of efficiency and effectiveness of expenditures, development of recommendations for improving the effectiveness of programs and further funding, as well as ensuring the transparency of actions of public program administrators in the management of budgetary funds.

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_8

8.1 Government Program as a Targeted System with Program Management

8.1.1 Classification and Stages of Implementation of Government Programs

A government program is a complex of research, production, socioeconomic and other events coordinated in terms of resources, contractors and realization [9].

Government programs combine important characteristics of result-focused budgeting and program-task methods:

- Integrated approach to solving problems;
- Medium-term planning;
- Determination of funding depending on the tasks and objectives; and
- Use of a system of indicators to determine effectiveness.

The government program must determine:

- The main perspective (strategic) objectives and sub-objectives in their hierarchy (the "tree of objectives");
- Steps to achieve the objectives;
- A complex of coordinated measures to achieve the objectives (organizational, socio-economic, scientific, technological, etc.);
- The entities involved in the implementation of the program; and
- The mechanism for implementation, including sources of funding, incentives, liability, etc.

The government program consists of standard sections including:

- Characteristics of the problem to be solved by the program and assessment of current situation;
- The main objectives of the program, evaluation of social, economic and environmental results of program implementation;
- Tasks to be solved in order to achieve the objectives of the program, including the terms and steps of its implementation;
- Activities and indicators characterizing step-by-step solution of problems;
- A list of program steps to be realized to solve each problem;
- Justification of sources and amounts of GP funding;
- Assessment of external factors that may influence the achievement of objectives; and
- A mechanism of program management and a procedure of interaction between GP participants.

The process of GP management can be subdivided into six major groups having different control functions:

- Initiation (making a decision to start development and implementation of the program);

– Planning (determination of criteria of successful implementation and development of network diagrams);
– Performance (coordination of organizations and other resources for program implementation);
– Monitoring and analysis (determination of conformity of program implementation to the objectives and success criteria, and decision-making on the need for corrective actions);
– Management (determination of necessary corrective actions, their harmonization, adoption and application); and
– Completion (execution of the program and analysis of its results).

The GP has the following life-cycle stages [9]:

– Implementation;
– Continuous monitoring;
– Evaluation of effectiveness of implementation; and
– Decision to complete (continue) the GP.

Evaluation of the efficiency of implementation involves the analysis of:

– Implementation of the GP and the action plan;
– Efficient use of materials, labor and financial resources;
– The extent to which the planned objectives and indicators are achieved; and
– Impact of GP implementation on the social and economic development of the country.

The efficiency of GP implementation is evaluated by a State structure responsible for the development of the GP (internal evaluation) as well as by an authorized body (external evaluation).

The authorized body examines the progress reports on the implementation of the government program presented by a responsible public authority, and finally, if necessary, submits a report on the feasibility of further implementation of the program to the Government of the Republic of Kazakhstan.

At all stages of the life cycle, it is necessary to have information on the GP status, and therefore, it is necessary to carry out monitoring of all the activities. Monitoring can be defined as an integration of measurements, research, experiments, information support, analysis, forecasting, organizational administration and management.

8.1.2 Aims and Tasks of Monitoring of Government Programs

The process of monitoring a government program consists of the following procedures [10]:

– Ongoing monitoring of implementation of the GP by the Government agency responsible for its development;

– Timely submission of reports by organizations/contractors on the execution of the sections of the GP and the related parts of the action plan, to the State body responsible for the development of the program; and
– Periodic submission by the State body responsible for the development of the program, of reports on its implementation, to the administration of the President of the Republic of Kazakhstan, the Government of the Republic of Kazakhstan and the authorized body and, if necessary, submission of proposals to amend the program [10].

The primary purposes of monitoring GP implementation are, on the one hand, an objective and timely evaluation of GP implementation and, on the other hand, development of recommendations for GP improvement at the level of adjustment of its aims and objectives, deadlines and milestones, mechanisms of resource provision [11], and a system of management and monitoring of its implementation.

Monitoring of the program must include:

– Use of resources (compliance of spent resources to the plan);
– The process of program execution (conformity of the content and timing of activities to the schedule of works; compliance of technologies, methods and procedures);
– Results (to what extent the objectives can be achieved); and
– Effect (how the program affects the situation, and what this effect is).

The monitoring system consists of a set of elements; the interaction between these elements provides the operation of the system. The elements that make up the structure of the monitoring system are:

– Objects of monitoring;
– A set of monitoring activities (indicators);
– Tools of monitoring activities; and
– Monitoring activities.

The information collected and processed as a result of GP monitoring is, primarily, needed to support management decisions; whereas generalized indicators of GP implementation must be available for free access.

The results of monitoring are structured, processed according to a specially developed methodology [12] and placed for long-term storage in the integrated data warehouses. The data provided by the monitoring system must be adequate, timely and reliable. The system of GP monitoring uses the following types of information:

– Current information, the information on GP status at a certain moment of time or for a certain period of time preceding it: a month, a quarter, six months, a year. This information is based on statistical reports of governing bodies, as well as on the current departmental information;
– Analytical information, the information and estimates obtained after certain procedures of analysis, comparison, identification of trends and patterns in the dynamics of GP implementation relative to some basic periods and phases; and

– Calculated information, the information and estimates based on current, analytical and regulatory information representing the indicators of efficiency of GP implementation.

In general, the information system used for monitoring the government program must be primarily aimed at higher efficiency of managerial decisions.

The main objectives of GP monitoring are:

– Provide the required and timely information to the executive and legislative branches at the governmental and regional levels in order to work out efficient strategic, tactical and operational management decisions when implementing the GP activities;
– Provide objective, factual information on the aims and achievements of GP implementation to mass media (data support); and
– Collect information to carry out research works in order to develop long-term, medium-term and short-term forecasts of GP implementation.

The main functions of the GP monitoring system are:

– Selecting objects of observation at the national and regional levels; formation of conceptual and information models of the objects of observation;
– Collection and processing of reliable and comparable information on the status of implementation of key and intermediate GP objectives;
– Development of systems of coding, processing and submission of information based on modern information technologies;
– Development of efficient systems of data transmission through computer networks and communication facilities;
– Multi-level aggregation of information and its presentation in a form suitable for making management decisions;
– Integrated and thematic processing of information obtained during GP implementation;
– Expert assessment and issuing recommendations on improvement of the GP activities, mechanism of their implementation, better management and resource support; and
– Evaluation of the efficiency of decisions made at the level of legislative and executive authorities, as well as at the regional level.

Thus, the main task of GP monitoring is estimation of the current values of its status indicators, forecast of their changes and analysis of deviations from the accepted values.

8.2 Government Programs in Terms of Systems Theory and General Management Theory

In terms of systems theory and general management theory, the Public Housing Program (PHP) is a target-oriented complex system with software-based management, the structural model of which is shown in Fig. 8.1.

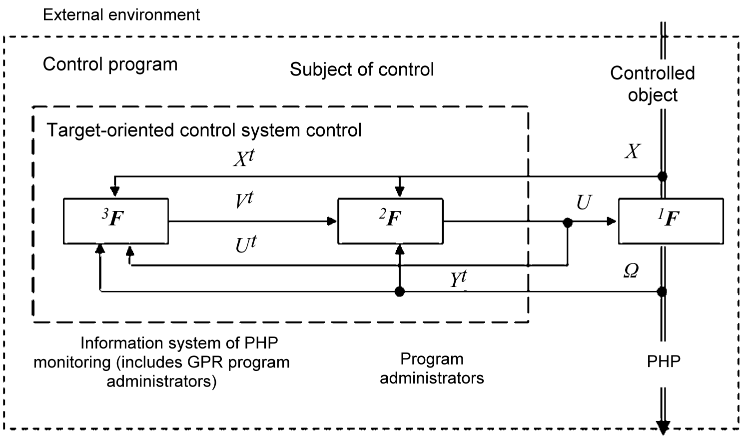


Fig. 8.1 PHP structural model in terms of systems theory and the general management theory

Notations: $^1\boldsymbol{F}$ is a controlled object representing a set of PHP actions; $^2\boldsymbol{F}$ is a subject of control (PHP administrator); $^3\boldsymbol{F}$ is a controlling subsystem assigning the control program $\omega(t)$ with target variables V^t to the $^2\boldsymbol{F}$ component, which carries out control according to a special program. The input variables of the system X determine financial and manpower PHP resources and environmental factors affecting the efficiency of its implementation. U variables are controlling actions on the subject of control aimed at the maximum degree of program implementation. The output variables of the system Ω evaluate the PHP state at each stage of its implementation. $Y^t \subset \Omega$ variables provide feedback and provide the subject of control with information on the state of the subsystem $X^t \subset X$ and $U^t \subset U$ at a certain moment of time t.

PHP is a complex system with a hierarchical structure. Each of the components $^1\boldsymbol{F}$, $^2\boldsymbol{F}$ and $^3\boldsymbol{F}$ of the structural model is a subsystem that can be decomposed. Quantitative and qualitative variables Y^t are directly connected to the PHP indicators, which define the function of its efficiency and effectiveness of implementation. If we denote the function of actual PHP implementation for achieving the objective z as $\varphi(z) : \{U \times X\} \times \Omega$ and the function of planned implementation of the program as φ^{*e}, we can state that the objective of PHP administration is to minimize the proximity function of actual and desired values of indicator $\Delta(\varphi(z), \varphi^*(z))$ by choosing the solutions justified by the subsystem $^3\boldsymbol{F}$: the increase in the efficiency of control of the time of completion of certain stages; optimization of reallocation of resources at *accelerated funding*; organizational management minimizing the impact of external factors; assessment of the degree of achievement of the goals; and analysis of PHP results identifying negative trends.

The scheme of realization of component $^3\boldsymbol{F}$ has required the development of original information technology as a system of scientific and engineering knowledge as well as the methods and models that provide creation, collection, transfer, storage and processing of PHP monitoring data in order to increase the validity of choice of control actions by administrators at various levels by using timely and reliable information resources.

The structural PHP model can be represented as a tree.

8.3 Information and Model Representation of Government Programs and Methods of Monitoring Their Implementation

8.3.1 Formalization of Representation of the Government Program as a Hierarchical Tree

According to the methodology proposed in [4], GP as a program-controlled complex dynamic system can be formally presented as a tuple:

$$Y = \langle P, M, C, \Pi_q, \Omega, R_f, I, \varphi(z), t, U \rangle,$$

where: P is a set of tasks the solution to which must provide the achievement of objectives; M is a set of program activities needed to solve the tasks; C is a set of projects the execution of which will lead to the implementation of activities; Π_q is a set of indicators (activities) of the degree of achievement of objectives $q = 1 \dots Q$, where Q is the total number of indicators of the degree of achievement of program objectives; Ω is a set of GP states described by a linguistic variable and evaluated by fuzzy numbers; R_f is a vector of financial resources, f are the sources of funding, $f = 1 \dots F$; I is a set of GP executors; $\varphi(z)$ is a degree of achievement; t is a time variable; and U is a controlling action of the system.

The results of some GP tasks cannot be expressed in the form of exact quantitative relations. The criteria, evaluations and system of preferences specified on the basis of such sets cannot be exactly defined.

A considerable portion of the information needed for mathematical description of GP status exists in the form of representations and experts' recommendations. The efficiency of GP performance is also influenced by various factors, not only economic but also social, political, environmental and other factors. Thus, in order to realize the technology of solution of analytical tasks supporting GP it is necessary to take into account the uncertainty in the process of information formalization and processing.

The structural model of GP execution can be represented as a tree, the nodes and arcs of which correspond to:

- The main objective—the top level of the tree;
- Tasks;
- Subtasks;
- Activities;
- Projects; and
- Program objects, the lowest level of the tree.

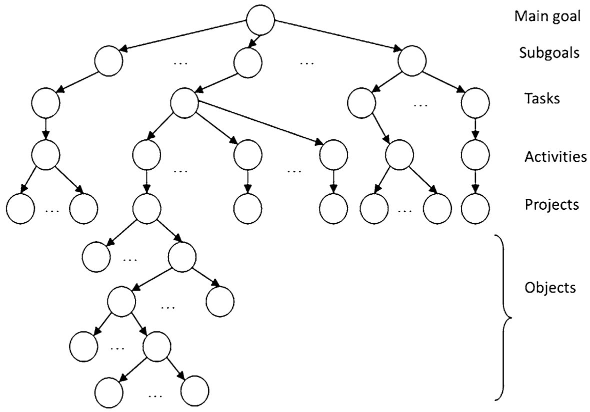


Fig. 8.2 A tree of GP execution

The tree arcs connect the tops according to the logical scheme: “main goal/subgoal, a task the solution to which results in achieving the objective—subtasks that provide the execution of tasks—activities that provide the solution of subtasks—projects that lead to the execution of activities—objects” (Fig. 8.2).

In the model, “projects” and “objects” are the elements of the lowest level; their state can be estimated from the outside, but the higher the level of hierarchy on the GP tree, the more hidden and unclear is the state of execution, the completion status and the efficiency of the program. Therefore, it is necessary to develop a model describing the top of the GP tree and the method of evaluation of its state and the GP status as a whole at a certain point of in time.

8.3.2 A Model Evaluating the State of the Top of the GP Tree

In accordance with the stages of GP execution in all tree tops, some physical processes take place in time (e.g., design, construction, installation, reporting on the steps of implementation, etc.); evaluation of the completion status of these processes is the main purpose of monitoring the status of GP objects.

All processes in the tops or nodes of the GP tree are finite and monotonous.

The problems of identification and recognition enable us to optimize hierarchical structures for a monotonous cost functional but do not enable us to estimate the state of processes in the tops of the hierarchic structure.

Let $t \geq 0$ be a certain moment of time in the tops of the GP tree, then every top of the tree v with the descendants of the same level $\{v_1, v_2, \ldots, v_n\}$ is characterized by its state, which can be described by a set of variables, $v^t(\alpha, \tau, \varphi)$ (Fig. 8.3).

The variable α is the amount of work performed or the percentage of program completion.

The variable τ is the time remaining before the end of the process in the top. The variable φ is the degree of achievement of objective.

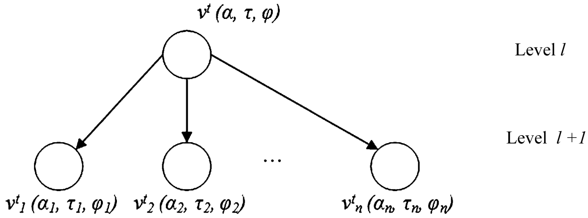


Fig. 8.3 Description of the top v^t of the tree at a moment of time, t

The percentage of program completion, α is defined quantitatively as the percentage of utilized funds from the estimated value. The percentage of completion of the m-th top of the current l-th level, α_m^l is calculated as the sum of elements of the preceding (lower located) level $a_m^l = \sum_{i \in \Gamma_m^1} k_i^{(l+1)} \alpha_i^{(l+1)}$, with $\sum_{i \in \Gamma_m^1} k_i^{(l+1)} = 1$. The percentage of achievement of the goal, φ, in each element of the tree is a function that depends on the percentage of utilized funds, α and the time of evaluation, t.

The execution of the process in each top, v_j provides some level of execution of the process in the higher top, but to a variable degree; that is, the function $\varphi_j(v) : V \to [0\ 1]$ expresses the degree to which the process execution in the top $v_j \in V$ provides the process execution in the top v.

For example, to achieve the GP objectives, the task list $P = \{p_j\}$, $j = \overline{1, N_j}$ is formed.

Fulfillment of each task, p_j provides some level of reaching the goal. The function $\varphi_i(p) : P \to [0\ 1]$ shows how solution of the problem $p_j \in P$ helps to achieve the goal.

For the execution of each of the tasks it is necessary to perform a set of activities (from the approved plan), $M = \{m_k\}$, $k = \overline{1, N_j}$. Each activity m_k, when it is performed, provides some level of solution of P problems, but to a variable degree. The function $\varphi_j(m) : M \to [0\ 1]$ shows how fulfillment of the task $m_k \in M$ contributes to the solution of the problem $p_j \in P$.

The execution of each activity refers to specific works, $C = \{c_q\}$, $q = \overline{1, N_j}$. Then, the function $\varphi_j(c) : C \to [0\ 1]$ expresses the degree to which the execution of project works makes it possible to perform the activity.

Thus, all elements of the GP tree are connected by indicators. Executing agencies give the information (according to the established forms of documentation) according to the bottom-up principle and the administrators assess the degree of program completion and achievement of the objective.

To determine the GP efficiency, it is necessary to evaluate the achievement of objectives throughout the course of the program's implementation. This means that for each reporting period it is necessary to evaluate the extent to which the program is completed, i.e., its status, in order to control and adjust the program.

It is difficult to formalize the assessment of GP implementation in a certain moment of time at each level of the tree as program administrators have to operate with fuzzy, inaccurate data, which cannot be interpreted as true or false [13–15].

To solve the problem one can use the theory of fuzzy sets, in which a linguistic variable is introduced to assess the GP status. The value of the linguistic variable

Table 8.1 The scale of evaluation of root nodes of the GP tree

Fully implemented	$\mu(x)=\frac{1}{1+(x-100)^2}$
The basic indicators fulfilled	$\mu(x)=\frac{1}{1+(x-90)^2}$
Fulfilled in general	$\mu(x)=\frac{1}{1+(x-80)^2}$
Fulfilled in the minimum volume	$\mu(x)=\frac{1}{1+(x-70)^2}$
Fulfillment of program is in critical state	$\mu(x)=\frac{1}{1+(x-50)^2}$
Fulfillment of program is disrupted	$\mu(x)=\frac{1}{1+(x-20)^2}$

"assessment of GP status" is determined by the term-set: "fully completed", "the basic indicators fulfilled", "fulfilled in general", "fulfilled in minimum volume", "fulfillment of program is in critical state", and "fulfillment of program is disrupted".

The linguistic variable "evaluation of GP status" is related to a positive fuzzy number "about A per cent", which is a normal convex fuzzy set with the piecewise continuous membership function $\mu(x)=\frac{1}{1+(x-A)^2}$ defined on the basic numerical scale with dimensions from 0 % to 100 %.

The scale enables us to qualitatively subdivide various states of processes at the top of the tree. Each tree level may have its own scale, but the boundaries of transition from one state to another are described by fuzzy sets.

The scale of evaluation of root nodes of the GP tree used in the study is shown in Table 8.1.

The degree of achieving the goal at the top of the GP tree is determined by the values of this indicator in daughter tops and is calculated according to the Zadeh extension principle as a result of successive multiplication of fuzzy numbers v_1 and v_2 with the functions of membership $\mu_A(v_1)$, $\mu_B(v_2)$ of the sets A and B:

$$\mu_{A\times B}(v)=\sup_{v_1\times v_2}\min\bigl(\mu_A(v_1),\mu_B(v_2)\bigr).$$

The administrator evaluates the GP status in the reported period. If the program does not meet the plan, the administrator must take the required control actions.

8.3.3 Task of Evaluation of Process Completion Time at the Top of the PHP Tree

In the set of variables $v^t(\tau,\alpha,\varphi)$ describing the state of the top of the GP tree at the moment of time t, the most critical variable is the time τ remaining before the completion of the process at the top. The methods of recognition can be used to estimate the time remaining before the end of the process. The necessary condition for implementation of the method is the availability of the data on previously completed objects with the same structure as at the top v. Formally, the task of estimation of the completion time at the top is reduced to a problem of recognition theory.

Let $(\tau^{(i)}, \alpha^{(i)})$, $i = 1, 2, \ldots, m$ be a state of the top similar to v in the i-th previously completed object, the monitoring data for which are stored in the database and can serve as a learning object.

The time of the process completion τ at the top v can be determined as the highest of the process completion times $(\tau_1, \tau_2, \ldots, \tau_n)$ at the tops preceding v and the time $\tilde{\tau}$ forecast for the object $n+1$ based on the state of m learning objects:

$$\tau = \max(\tau_1, \tau_2, \ldots, \tau_n, \tilde{\tau}). \tag{8.1}$$

In this study, the value of $\tilde{\tau}$ is calculated as an approximation of the function $\tilde{\tau} = F_{\alpha_1,\alpha_2,\ldots,\alpha_n}(x^*)$ providing $\inf_{x\in\{x_1,x_2,\ldots,x_n,x_{n+1}\}} \sum_{i=1}^{m} (F_{\alpha_1^{(i)},\alpha_2^{(i)},\ldots,\alpha_n^{(i)}}(x) - \tau^{(i)})^2$ subject to $\alpha = \sum_{j=1}^{n} \alpha_j p_j / \sum_{j=1}^{n} p_j$, where x^* is the vector of artificially introduced variables; p_j is the volume of work of tops-descendants.

In the final form, the problem is reduced to the problem of quadratic programming with n variables and non-trivial constraints m, for the solution of which standard methods of quadratic programming are used:

$$\tilde{\tau} = \min_{x\in\{x_1,x_2\ldots,x_n\}} \left(\sum_{i=1}^{m} \left(\sum_{j=1}^{n} \overline{\alpha}_j^{(i)} x_j - \tau^{(i)} \right)^2 \right), \quad \overline{\alpha}_j^{(i)} = \alpha_j^{(i)} - 1; \tag{8.2}$$

with constraints

$$\sum_{i=1}^{m} \overline{\alpha}_j^{(i)} x_j \geq \max \tau_j^{(i)}, \quad i = 1, 2, \ldots, m;\ x_j \leq 0,\ j = 1, 2, \ldots, n. \tag{8.3}$$

Thus, the time of process completion τ at the top v for volumes $\alpha_1, \alpha_2, \ldots, \alpha_n$ at the tops $v_1, v_2, \ldots, v_n$ can be calculated using the formula (8.1).

The assessment of the time of GP completion obtained from the solution of the problem (8.1) may be different from the target values as each participant of the program is under the influence of external and internal risk factors. Therefore, to effectively manage the GP it is necessary to provide for and assess the risk of the program not being realized in due time.

8.3.4 A Production Model of Assessment of GP Status and Degree of Objective Achievement

A fuzzy production model is used to assess the GP status and the extent to which the objectives are achieved.

In accordance with the approach proposed in [10], the model is a set of fuzzy production rules: (i): Q; P; $A \Rightarrow B : S$; N, where (i) is the notation of a fuzzy production rule; Q is the area of application of fuzzy production that characterizes the subject domain of fuzzy models; P is the condition of use (revitalization) of the kernel of fuzzy production; $A \Rightarrow B$ is the fuzzy production kernel; A is the kernel (antecedent) condition; B is the kernel (consequent) inclusion; $\Rightarrow$ is the sign of logical sequent (succession); S is the method of determination of the quantitative value of the degree of validity of kernel inclusion that determines the algorithm of fuzzy inference in the fuzzy production model; and N is the post-condition of the production rule that defines the actions or procedures to be performed in case of realization of production kernel.

The kernel of fuzzy production $A \Rightarrow B$ is written in the following form:

IF x is A, **THEN** y is B, where x is an input variable, $x \in X$; X is the range of definition of the fuzzy rule antecedent; A is a fuzzy set defined on X; $\mu(x) \in [0, 1]$ is the membership function of the fuzzy set A; y is an output variable $y \in Y$; Y is the range of definition of the fuzzy rule consequent; B is a fuzzy set defined on Y; and $\mu_B(y) \in [0, 1]$is the membership function of the fuzzy set B.

The input variables t, τ, α and the output variables ω and φ are used to form the rules of assessment of the GP status.

The variable τ is the time remaining before the end of the process, which has the following term-set:

$$T1 = \{\text{“Ahead of schedule”, “As scheduled”, “Behind schedule”}\}.$$

“Ahead of schedule” is the situation where the process is fulfilled earlier than planned, “As scheduled” corresponds to the situation where the process is fulfilled according to the plan, and “Behind schedule” corresponds to the situation where the process is fulfilled later than planned.

The variable t is the stage of the process with the term-set $T2 = \{$“Start”, “In process”, “Completion”$\}$.

The term “Start” corresponds to the situation where the assessment is made at the initial stage of the process implementation. The term “In process” corresponds to the situation where the assessment is made during the process. The term “Completion” corresponds to the situation where the process moves into its final stage.

The variable α is the proportion of utilized funds of the plan that has the term-set: $T3 = \{$“completely”, “more than half”, “less than half”, “not utilized”$\}$.

The term “completely” corresponds to the situation where all the funds allocated by the financing plan are utilized. The term “more than half” corresponds to the situation where in the course of the process execution more than half of the funds earmarked by the financing plan are utilized. The term “less than half” corresponds to the situation where in the course of execution less than half of funds earmarked by the financing plan are utilized. The term “not utilized” corresponds to the situation where the funds have not been utilized.

The output variable ω is a linguistic variable “Process status”, which has the term-set $T4 = \{$“completed”, “close to completion”, “acceptable”, “problematic”, “critical”$\}$.

The output variable φ is a linguistic variable “Level of achievement of the process objective”, which has the term-set $T5 = \{$“the objective is achieved”, “high degree of objective achievement", "average degree of objective achievement", "low degree of objective achievement" and "the objective is not achievable"}.

Membership functions for the input and output variables are represented by fuzzy numbers of $(L\text{-}R)$-type. Fuzzy numbers $(L\text{-}R)$ are special fuzzy numbers introduced in order to reduce the amount of computation. Membership functions of $(L\text{-}R)$ fuzzy numbers are defined by non-increasing functions on a set of nonnegative real numbers of real variables $L(x)$ and $R(x)$ satisfying the following properties:

(a) $L(-x) = L(x), R(-x) = R(x)$;
(b) $L(0) = R(0)$;
(c) $L(\infty) = R(\infty) = 0$.

The fuzzy number is generally defined as:

$$\mu(x) = \begin{cases} 1 - (\frac{a-x}{\gamma}), & \text{if } a - \gamma \le x < a; \\ 1, & \text{if } a \le x \le b; \\ 1 - (\frac{x-b}{\delta}), & \text{if } b < x \le b + \delta; \\ 0, & \text{otherwise,} \end{cases}$$

and is denoted by (a, b, γ, δ).

The membership functions for the variable t reflecting the stage of process execution are: $\mu^{start}(x) = (0, 0.2, 0, 0.1)$, $\mu^{in\ process}(x) = (0.4, 0.7, 0.2, 0.2)$, $\mu^{completion}(x) = (0.8, 1, 0.1, 0)$.

The membership functions for the variable τ have the following form:

$$\mu^{lag}(x) = (0, 0, 0, 0.4), \qquad \mu^{as\ scheduled}(x) = (0.5, 0.5, 0.2, 0.2),$$
$$\mu^{ahead\ of\ schedule}(x) = (1, 1, 0.4, 0).$$

The membership function for the variable that reflects the utilized percentage of funds is:

$$\mu^{not\ utilized}(x) = (0, 0, 0, 0.15), \qquad \mu^{less\ than\ half}(x) = (0.3, 0.3, 0.2, 0.25),$$
$$\mu^{more\ than\ half}(x) = (0.7, 0.7, 0.25, 0.25), \qquad \mu^{utilized}(x) = (1, 1, 0.15, 0).$$

The membership function for the output variable reflecting the status of the variable ω at the top of the GP tree, has the following form:

$$\mu^{critical}(x) = (0, 0, 0, 0.2), \qquad \mu^{problem}(x) = (0.3, 0.3, 0.2, 0.2),$$
$$\mu^{acceptable}(x) = (0.6, 0.6, 0.2, 0.2),$$
$$\mu^{close\ to\ completion}(x) = (0.85, 0.85, 0.1, 0.1), \qquad \mu^{completed}(x) = (1, 1, 0.1, 0).$$

The membership function for the output variable φ reflecting the degree of achievement of the objective of the process is written as:

$$\mu^{not\ achievable}(x) = (0, 0, 0, 0.2), \qquad \mu^{low}(x) = (0.3, 0.3, 0.2, 0.2),$$

Table 8.2 Fuzzy production rules

Number	Status/Result	Antecedent/Consequent
Unit 1		
R1	IF	Evaluation is made at the initial stage of the process, AND the process is performed ahead of the schedule, AND less than 50 % of the funds are used
	THEN	The process status is acceptable
...	...	...
R23	IF	Evaluation is made at the stage of process completion, AND the process runs behind schedule, AND financial resources have not been used
	THEN	The process status is critical
Unit 2		
R18	IF	Evaluation is made at the initial stage of the process, AND the process is performed ahead of schedule, AND less than 50 % of the funds are used
	THEN	There is high probability of achieving the process objectives
...	...	...
R47	IF	Evaluation is made at the stage of process completion, AND the process is behind schedule, AND financial resources have not been used
	THEN	The objective of the process is not achievable

$$\mu^{medium}(x) = (0.5, 0.5, 0.2, 0.25), \qquad \mu^{high}(x) = (0.8, 0.8, 0.1, 0.15),$$
$$\mu^{achieved}(x) = (1, 1, 0.1, 0).$$

The fuzzy knowledge base (FKB) is a set of fuzzy rules that reflect the experience of the expert and his understanding of the cause-and-effect relation in the task. The FKB connecting the inputs $X = (x_1, x_2, \ldots, x_n)$ with the output y is expressed as:

$$(x_1 = \tilde{a}_{1j} \Theta_j x_2 = \tilde{a}_{2j} \Theta_j \cdots \Theta_j x_n = \tilde{a}_{nj}) \Rightarrow y = d_j, \quad j = \overline{1, m},$$

where $\tilde{a}_{ij}$ is a fuzzy term that evaluates the variable x_i in the j-th rule; d is the inference of the j-th rule; m is the number of FKB rules; Θ is a logical operation connecting the fragments of the j-th rule antecedent (it can be a logical operation AND or OR); and $\Rightarrow$ is a fuzzy implication.

To create a base of rules, various combinations of input variables with all the terms were formed; then, the combinations of antecedents that had no sense were removed, and consequents were defined on the basis of expert knowledge. Table 8.2 shows a fragment of fuzzy production rules of the fuzzy knowledge base formed to determine the GP status (unit 1) and the level of achievement of GP objective (unit 2).

The algorithms of the fuzzy logical inference are based on the Zadeh compositional rule: “If we know the fuzzy relation $\tilde{R}$ between x and y, then for the fuzzy value of the input variable $x = \tilde{A}$ the fuzzy value of the output variable y is defined as: $y = \tilde{A} \circ \tilde{R}$, where the sign "$\circ$" is the "maxmin composition".

We can also use the Mamdani algorithm. The fuzzy Mamdani inference is based on the knowledge base:

$$(x_1 = \tilde{a}_{1j} \Theta_j x_2 = \tilde{a}_{2j} \Theta_j \cdots \Theta_j x_n = \tilde{a}_{nj} \text{ with weight } w_j) \Rightarrow y = \tilde{d}_j, \quad j = \overline{1,m}.$$

The degree of fulfillment of the j-th rule hypothesis for the current input vector $X^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is calculated as:

$$\mu_j(X^*) = w_j\left(\mu_j(x_1^*)\chi_j \cdots \chi_j \mu_j(x_n^*)\right), \quad j = \overline{1,m},$$

where χ_j is a t-norm if in the j-th FKB rule AND is used, and corresponds to s-norm if OR is used.

The fuzzy result is written as:

$$\tilde{y}^* = \left(\frac{\mu_1(X^*)}{\tilde{d}_1}, \frac{\mu_2(X^*)}{\tilde{d}_2}, \ldots, \frac{\mu_m(X^*)}{\tilde{d}_m}\right).$$

To transform to a fuzzy set on the support $|\underline{y}, \overline{y}|$, implication and aggregation operations are used. As a result of logical inference using the FKB j-th rule, the fuzzy value of the output variable y is obtained:

$$\tilde{d}_j^* = imp\left(\tilde{d}_j, \mu_j(X^*)\right), \quad j = \overline{1,m}, \tag{8.4}$$

where *imp* is the implication realized by the minimum operation.

The result of logical inference for the whole FKB is found by aggregation of fuzzy sets (8.4):

$$\tilde{y}^* = agg\left(\tilde{d}_1^*, \tilde{d}_2^*, \ldots, \tilde{d}_m^*\right),$$

where *agg* is the aggregation of fuzzy sets, which is implemented by maximum operation.

MATLAB is used for the implementation of the model. The constructed model enables us to determine the output value of the variable ω from the values of input variables t, τ and α.

In this model, 3 input linguistic variables describing the top of the GP tree, 2 output linguistic variables describing the status and the degree of achievement of the process objective and 2 blocks of 23 production rules are defined.

8.3.5 The Task of Rapid Reallocation of Funds

The government program is a complex system that is subjected to the action of a great variety of external factors. In some cases, the factors may have a negative impact. In this case, the administrator making managerial decisions must optimally define cash redeployment among the GP objects. In this work, this task is solved in two stages. The first step is multi-criteria choice of construction objects for the expedited financing by the method of maxmin convolution [13, 14].

The purpose of this task is to order the construction objects for express financing to achieve the indicators of GP implementation as close to the plan as possible.

There are many object alternatives $A = \{a_1, a_2, \ldots, a_m\}$ and a variety of evaluation criteria $C = \{C_1, C_2, \ldots, C_n\}$.

Let $\mu_{C_i}(a_j)$ be a number in the range [0, 1] which defines the level of evaluation of version $a_j \in A$ by the criterion $C_i \in C$: the greater the value $\mu_{C_i}(a_j)$, the higher the evaluation of version a_j by the criterion C_i, $i = \overline{1, n}$, $j = \overline{1, m}$.

The evaluation of objects by the i-th criterion can be presented as a fuzzy set $\tilde{C}_i$:

$$\tilde{C}_i = \left\{\mu_{C_i}(a_1)/a_1, \mu_{C_i}(a_2)/a_2, \ldots, \mu_{C_i}(a_m)/a_m\right\},$$

where $\mu_{C_i}(a_j)$ is the degree of membership of the element a_j in the set $\tilde{C}_i$.

To determine the degree of membership of fuzzy sets it is necessary to construct membership functions based on paired comparisons.

Fuzzy sets $\tilde{C}_i$ are evaluated on the basis of the formula:

$$\mu(a_i) = \frac{1}{a_{1i} + a_{2i} + \cdots + a_{mi}}. \tag{8.5}$$

The criteria may have different significance when determining the best variant. In this case, the matrix of paired comparisons of criteria is constructed. The weighting factor of criterion β_i is defined on the basis of the calculated values of the right eigenvector of matrix of paired comparisons α_i with the subsequent multiplication by the number of criteria n: $\beta_i = \alpha_i n$, with the weighting factors satisfying the following conditions: $\beta_i \geq 0$; $i = \overline{1, n}$; $(1/n)\sum_{i=1}^{n} \beta_i = 1$, or using formula (8.5).

To choose the best alternative it is necessary to find the intersection of fuzzy sets of corresponding criteria:

$$D = C_1^{\beta_1} \cap C_2^{\beta_2} \cap \cdots \cap C_n^{\beta_n}.$$

The intersection operation corresponds to taking the minimum:

$$\mu_D(a_j) = \min_{i=1,\ldots,n} \left(\mu_{C_i}(a_j)\right)^{\beta_i}, \quad j = 1, \ldots, m,$$

$$\tilde{D} = \left\{ \frac{\min_{i=\overline{1,n}}(\mu_{C_i}(a_1))^{\beta_i}}{a_1}, \ldots, \frac{\min_{i=\overline{1,n}}(\mu_{C_i}(a_m))^{\beta_i}}{a_m} \right\}.$$

The best alternative a^* is determined as the greatest value of the membership function $\mu_D(a^*) = \max_{j=\overline{1,m}} \mu_D(a_j)$.

Thus, we obtain the ordered list of objects for express financing, i.e., the priority of allocation of funds $\mu_D(a_j)$, $j = \overline{1, m}$ for each j-th object.

At the second stage, the administrator must make a decision on the allocation of funds for some calendar period. Let R be the amount of monetary resources available for allocation. The demand of objects $j = \overline{1, m}$ in financing is r_j, $j = \overline{1, m}$. The alternative variables that characterize the choice in the allocation of funds are:

$$x_j = \begin{cases} 1, & \text{if } j\text{-th object is provided with funds} \\ 0, & \text{otherwise} \end{cases};$$

$$j = \overline{1, m}.$$

Taking into account the priorities $\mu_D(a_j)$, $j = \overline{1, m}$, we can conclude that the optimization criterion is maximization of the level of objective achievement for *possible allocation of funds* among the objects to be financed:

$$\sum_{j=1}^{m} \mu_D(a_j) x_j \to \max . \tag{8.6}$$

The constraints are the amounts of money available for allocation:

$$\sum_{j=1}^{m} r_j x_j \leq R. \tag{8.7}$$

By combining the optimization criteria (8.6) and the constraints (8.7) we obtain the following multi-alternative optimization model:

$$\sum_{j=1}^{m} \mu_D(a_j) x_j \to \max,$$

$$\sum_{j=1}^{m} r_j x_j \leq R,$$

$$x_j = \begin{cases} 1 \\ 0 \end{cases} \quad j = \overline{1, m}.$$

To determine the integer optimum-possible plan of funds allocation the clipping method proposed by Gomori is used.

8.3.6 The Task of Optimization of Network Management Model for Construction Works in Fuzzy Environment Based on the "Time-Cost" Criterion

After operative reallocation of funds by the administrator among the GP objects, the executors have to revise network diagrams in order to reduce the time of their fulfillment due to attraction of additional resources, that is, to solve the task of optimization of the network diagram according to the "time-cost" criterion.

It is reasonable to carry out optimization based on the "time-cost" criterion only if the duration of works may be reduced at the expense of higher costs of work performance [15].

To estimate the additional costs caused by higher rates of work performance, either standards or the data on the implementation of similar projects in the past are used.

The initial data for the optimization are:

- $T_{start}(i, j)$ is the ordinary work schedule;
- $T_{expedited}(i, j)$ is the accelerated work schedule;
- $C_{start}(i, j)$ is the cost of work performance according to the ordinary schedule; and
- $C_{expedited}(i, j)$ is the cost of work performance according to the accelerated schedule.

Every work has a certain maximum amount of time by which its duration may be reduced, $Z_{max}(i, j) = T_{start}(i, j) - T_{expedited}(i, j)$. In the analysis of the network model in this type of optimization, the cost increase factor (acceleration factor) is used:

$$k(i, j) = \frac{C_{expedited}(i, j) - C_{start}(i, j)}{T_{start}(i, j) - T_{expedited}(i, j)},$$

which quantitatively assesses monetary funds or costs needed to reduce the duration of work (i, j) by one day.

If we denote works along the critical path as $\{(i, j)_{cr}\}$, its duration L is determined as $L = \sum_{l \in \{(i,j)_{cr}\}} t_l$. In order to reduce L at the expense of additional forces and funds, first of all it is necessary to speed up works along the critical path.

In case of exact values of network model parameters the task of optimization by the "time-cost" criterion is formulated as follows: what additional means $x_1, x_2, \ldots, x_n$ and in what critical works $\{(i, j)'_{cr}\}$ we have to invest in order to reduce the length L of the critical (cr) path.

Let us assume that the investment of additional funds x_l in the work a_1 *reduces the time of work to* $t'_l = f_l(x_l) < t_l$. It is required to determine the values of variables $x_1, x_2, \ldots, x_l, \ldots, x_n, x_l > 0, \forall l = \overline{1, |\{(i, j)_{cr}\}|}$ (additional investments), which would satisfy the following condition:

$$L' = \sum_{l \in \{(i,j)_{New\ cr}\}} f_l(x_l) \le L, \tag{8.8}$$

where $\{(i, j)_{New\ cr}\}$ is the set of works of the new critical (*New cr*) path (after allocation of funds), and the total amount of additional funds would be minimal, i.e.,

$$L = \sum_{l \in \{(i,j)_{New\ cr}\}} x_l \to \min \tag{8.9}$$

In general terms, constraints (8.8) are nonlinear since the investment of any funds in the work a_l does not necessarily cause linear reduction of the time required for the work. Therefore, the task (8.8)–(8.9) generally refers to the class of nonlinear programming problems. However, with small changes in the plan, when the constraints (8.8) are linear, the task of optimizing the critical path is solved by the method of linear programming.

The algorithm of multiple works on the critical path, which may reduce the length by attracting additional funds, consists of the following steps [21, 22].

Step 1. Choice of critical work (i, j) which has a reserve time for reduction and a minimum value of the coefficient of cost increase $k(i, j)$.

Step 2. Estimation of time $\Delta t_{i,j}$ by which it is necessary to reduce the duration of work (i, j), guided by the following considerations:

- Maximum possible reserve time for the reduction of work (i, j) at the current moment $Z(i, j)$ is limited to the value $T_{expedited}(i, j)$, i.e., $Z(i, j) = t_{c.t.}(i, j) - T_{expedited}(i, j)$, where $t_{c.t.}(i, j)$ is the current time of execution (c.t.) of work $(t_{c.t.}(i, j) = T_{start}(i, j))$ only for works that have not yet been reduced;
- In addition to the critical path of length L, the network has a subcritical path of length L_{start}, and the critical path length cannot be reduced by more than $\Delta L = L - L_{start}$ since in this case the critical path will not be critical, and the subcritical path, on the contrary, will become critical;
- The time of reduction of the length of chosen work (i, j) is equal to $\Delta t = t_{c.t.}(i, j) - \min[Z(i, j), \Delta L]$, that is, if the difference between the duration of critical and subcritical paths ΔL is less than the current reserve time for the reduction of work $Z(i, j)$, it makes sense to reduce the work only by ΔL days, otherwise, we can reduce the work fully by the value $Z(i, j)$.

Step 3. We calculate the total cost $C = k(i, j) \times \Delta t$, as a result of rise in the price of accelerated work a new schedule is created, possibly, with new critical and subcritical paths.

Step 4. If the necessary length of critical path is not achieved, pass to step 1.

In the methods of network model analysis described, it was assumed that the time of work execution is precisely known; however, in practice, in the real process of project construction the turnaround time is usually quite uncertain.

In network planning, tasks with vaguely specified duration of work T_{ij} are not precisely known and are presented by fuzzy intervals of $(L\text{-}R)$ type. The estimates of the critical path may appear the early and late dates of completion of the whole complex of works.

Let us assume that we know the earliest date of the start of construction work at the site $T_{start}(1, 1, 1, 1)_{LR}$, the latest date for the termination of construction work $T_0(L_{latest}, L_{start}, 1, 0)_{LR}$ and the duration of work T_{ji}.

The earliest starting (e.s.) date of the i-th work $t_{start}(i)$ is calculated by the formula $\max\{t_{e.s}(j)\} + T_{ji}$, $j \in P_i$, if P_i is nonempty and by $T_{start}(1, 1, 1, 1)_{LR}$ if otherwise, where P_i is the set of works preceding the i-th work. The earliest date of termination $(e.d.t.)$ of the whole complex of works is $L_{latest} = \max\{t_{e.d.t.}(i)\}$, $i \in P$.

The latest start date $(l.s.d.)$ of the i-th work is $t_{e.s}(i)$ and is calculated by the formula $\min\{t_{l.s.d.}(j)\} - T_{ji}$, $j \in S_i$, if S_i is nonempty and by $T_0(L_{L_{latest}}, L_{start}, 1, 0)_{LR}$ if otherwise, where S_i is the set of works following the i-th work. The latest date of termination of the whole complex of works is $L_{start} = \max\{t_{e.d.t.}(i)\}$, $i \in P$.

Arithmetic approximation operations are used to carry out max and min operations. Determination of minimum and maximum of two fuzzy numbers $M = (m_1, m_2, \alpha, \beta)_{LR}$ and $N = (n_1, n_2, \gamma, \delta)_{LR}$ is performed using the formulas:

$$\max(M, N) \cong \big(\max(m_1, n_1), \max(m_2, n_2), \max(m_1, n_1) - \max(m_1 - \alpha, n_1 - \gamma),$$
$$\max(m_2 + \beta, n_2 + \delta) - \max(m_2, n_2)\big)_{LR};$$
$$\min(M, N) \cong \big(\min(m_1, n_1), \min(m_2, n_2), \min(m_1, n_1) - \min(m_1 - \alpha, n_1 - \gamma),$$
$$\min(m_2 + \beta, n_2 + \delta) - \min(m_2, n_2)\big)_{LR}.$$

This option of approximation is used successively for the pairs of fuzzy numbers and results of their comparison. The algorithm for solving the task of optimization by the "time-cost" criterion of network model of work administration in a fuzzy environment is implemented in those environments that support fuzzy technology [21, 22].

8.4 Methods and Models for Evaluation of GP Implementation

8.4.1 Approaches to the Evaluation of Implementation of Government Programs

The final phase of the GP life cycle model is associated with the evaluation of the effectiveness of program implementation.

The analysis conducted during the theoretical review of approaches to the evaluation of GP has made it possible to conclude that the GP evaluation system is based on social impact and budgetary efficiency indicators. Budgetary efficiency means the economic benefits for the State, which is expressed as a ratio of the resulting social impact and costs incurred. The ratio, depending on the method used, can be measured in absolute monetary terms or can define the cost of obtaining a unit of social effect (in relative terms).

The basic general-theoretic methods for evaluating the effectiveness of programs include: cost-benefit analysis (CBA), cost and utility analysis (CUA), cost and effectiveness analysis (CEA) and numerous modifications thereof. Despite the diversity of evaluation techniques, each has several notable limitations in practical application: difficulty in assessing the social impact in terms of money, failing to sufficiently consider the marginal cost of creating the social impact, etc.

Of the GP evaluation methodologies that have been developed to date, the majority are formal in nature and do not represent real experience. The analysis of methods of evaluation of targeted government programs used in Russia has identified the following key disadvantages:

- Focus on the evaluation of quality of programming, rather than on the evaluation of program effectiveness;

– Not one particular GP but many GPs are evaluated with a view to their ranking for financing;
– Complexity of interpreting indicators and groundless choice of weights in evaluating the aggregate;
– High degree of subjectivity in assigning the points for each of the indicators in the analysis of programs; and
– Lack of information on the evaluation criteria prior to its evaluation, etc.

In the Republic of Kazakhstan, the rules for assessing the effectiveness of budgetary programs are approved by decision of the Government [17]. In accordance with the established rules, the administrator assesses:

– Economic efficiency of GP implementation;
– Efficiency of GP management;
– Quality of GP implementation; and
– Performance of GP administrator.

The evaluation of economic efficiency of GP is carried out with the use of the criterion of effectiveness. The economic efficiency of GP is ensured by using the optimum amount of budgetary resources needed to meet GP objectives. The cost efficiency from the implementation of GP is determined by:

– Comparison of the planned volume of cost per unit of services provided by the government (executed works) with the volume of actual costs per unit of services provided by the government (executed works) during implementation of GP in the period under review, or with similar costs under other programs; and
– Analysis of dynamics of GP implementation costs over the past three years.

The management efficiency of GP is defined using the criterion of timeliness by:

– Justification of implementation in full of the activities proposed for financing;
– Analysis of the activities actually carried out during the reporting period, indicating the quantitative indicators and qualitative characteristics in comparison with the planned activities under the GP passport;
– Identifying the reasons for the deviation of events implemented within the GP from the scheduled events;
– Monitoring cash disbursement on GP that reflects the analysis of timeliness of budget utilization in accordance with the plan of GP funding regarding the payments and factors which have entailed budget non-utilization; and
– Analysis of the reasons for the existence of creditor or debit indebtedness for GP.

The final evaluation of GP quality during its implementation is carried out with application of the criterion of quality through:

– The planned and actual levels of satisfaction of requirements of recipients of government services, their comparison and dynamics; and
– Analysis of existing low quality of public services.

The assessment of efficiency is based on the criterion of efficiency through:

– Analysis of how and to what extent the planned activities under GP contribute to the fulfillment of GP objective; and
– Defining the direct and final result of GP administrator's activity.

The direct result of GP administrator's activity is a quantitative indicator, which characterizes the volume of services (executed works) provided by the administrator during the reporting period. The final result of GP administrator's activity is a quality indicator, which characterizes the achievement of public policy objectives by the GP administrator in the process of implementation of the budget. However, in the Republic of Kazakhstan methods for determining this quality indicator are currently not developed [18].

One of the most important indicators of effectiveness of GP implementation is the timely development of the budget because otherwise there will be a reduction in the budget for the next period. Therefore, the administrator, based on the monitoring data, must analyze the factors that act as sources of risk and that result in disruption of GP, and how these factors affect the targets.

In implementing a GP, the government carries out investment activity. Investment activity is notably characterized by exposure to the greatest number of risks that are very difficult to anticipate and assess.

In the existing methods of analysis and risk management, the consequences of risk situations are usually evaluated in the scalar form (e.g., "loss-profit"). To assess the risk of untimely performance of GP, where the funds are allocated from the budget and the State generally does not get profit, the use of these methods is not possible. This determines the need for qualitative analysis and risk management techniques using cognitive modeling as well.

8.4.2 Fuzzy Cognitive Model of Risk Assessment of GP Implementation

Cognitive technologies are intended for modeling and analysis of systems with:

– Multidimensionality of processes occurring in them and their interconnectedness, due to which the identification and detailed study of individual events is not possible: all events occurring in them must be considered in aggregate [19];
– Absence of sufficient quantitative information about the dynamics of processes, that necessitates qualitative analysis; and
– Variability of processes in time, etc.

A cognitive map is a type of mathematical model for formalization of the description of a complex object, a problem or the functioning of a system, and for identifying cause-and-effect links between its elements as a result of impact on these elements or changes in the nature of links [20].

A cognitive map represents a simulated system as a set of concepts that display its objects or attributes and are interconnected by impact relationships or causal relationships. Cognitive maps can be used for the qualitative assessment of the impact of individual concepts on each other and on the stability of the system as a whole, to simulate and evaluate the use of various strategies when making decisions and predicting decisions.

Traditional cognitive maps are plotted as an oriented graph and represent the simulated system as a set of concepts that display its objects or attributes interconnected by impact or causal relationships. These numerical relationships may be positive, negative or neutral, characterizing the impact of concepts on each other.

In a fuzzy cognitive map (FCM) each arc defines not only the direction and character but also the degree of influence of linked concepts. Depending on the approach, instead of arc characters linking concepts, concepts are linked by relationships with values in the $[-1, 1]$ range, or by fuzzy or linguistic variables.

The process of forming and using cognitive maps for training, support of management decision-making and risk assessment of GP can be represented as a sequence of the following procedures:

- Identification of the list of concepts (the agreed list of concepts in the case of a survey by a team of experts);
- Identification of causal relationships between each pair of concepts (the agreed causal relations);
- Plotting cognitive maps;
- Formation of alternate decisions and evaluation of their impacts; and
- Interpretation of cognitive maps and adoption of alternate decisions.

When designing the information and program support for risk assessment, a decision algorithm can be used according to the above sequence of procedures presented in Fig. 8.4 [23, 24].

The effectiveness of initial stages of risk assessment depends on the successful formation of an expert group and obtaining from them the quality of knowledge that forms the basis of any expert system. To select GP action concepts and define relationships between the concepts, one of the methods of expert assessment is used: the direct evaluation method.

Group review of concepts with direct assessment. There are many approaches to solving this problem.

Let m experts assess objects n using indicators l. The results of assessment are represented by values x_{ij}^h, where i is the concept number, j is the expert number, and h is the indicator number. Values x_{ij}^h obtained by direct assessment methods are the numbers within the numeric axis or points.

As a group evaluation for each object, we can take the weighted average of its evaluation

$$x_i = \sum_{h=1}^{l} \sum_{j=1}^{m} q_h x_{ij}^h k_j \quad (i = 1, 2 \ldots n),$$

where q_h are the weighting coefficients of comparison of concepts, and k_j are the coefficients of competence of experts.

Values q_h and k_j are normalized, i.e., $\sum_{h=1}^{l} q_h = 1$, $\sum_{j=1}^{m} k_j = 1$.

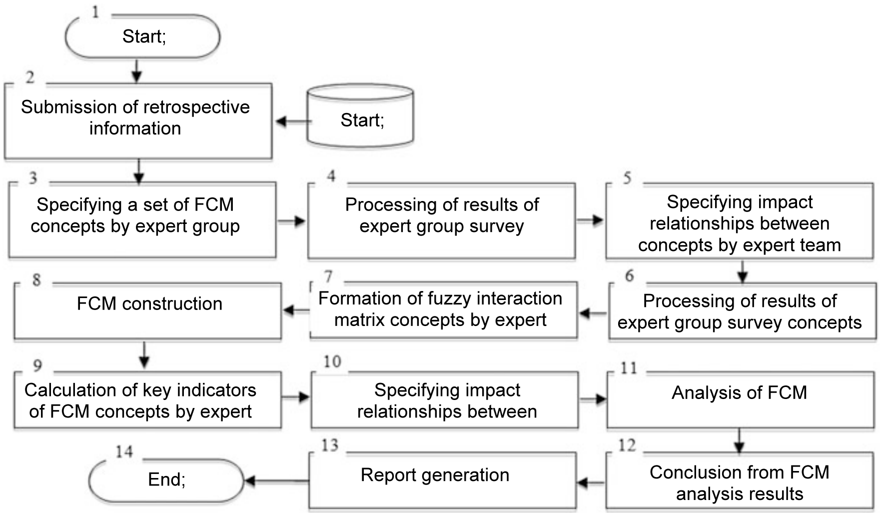


Fig. 8.4 The sequence of procedures to assess the risk of GP

Coefficients q_h can be defined by expertise as an average weighted ratio of h-th indicator for all the experts, i.e., $q_h = \sum_{j=1}^{m} q_{hj} k_j$.

A possibility of obtaining a group expert evaluation by aggregating individual assessments with the competence and importance weights is based on:

- The axioms of von Neumann-Morgenstern utility theory for individual and group assessments; and
- Conditions of indistinguishability of objects in the group if they are indistinguishable in all individual assessments (partial Pareto principle).

The competence rates of experts can be calculated from the aposterior data, i.e., the results of evaluation of objects. The main idea of this calculation is the assumption that the competence of an expert should be estimated by the coherence of his assessment with the group assessment of objects.

Thus, the expert group defines a list of concepts and impact relationships between them. These data are recorded in a database and used for constructing a fuzzy cognitive map and for its analysis [23, 24].

When generating a list of concepts for a cognitive map, to help the experts participating in the formulation of concepts and relationships between concepts, it is necessary to provide additional information about possible causes of GP disruption. To do this, it is necessary to review information from the data warehouse of the GP administrator using the procedure of extraction of knowledge from database tables.

Knowledge acquisition from database tables is a collection of models, methods and algorithms for data analysis in order to obtain the necessary knowledge.

In this case, the process of knowledge acquisition is conducted using a set of production rules making it possible to select data and record them in the required way for later processing. Here, the general rule of acquisition is used: “IF at least/equal/at least M of N conditions $C_1, C_2, \ldots, C_N$ are fulfilled, THEN A, OTHERWISE B", where $M = (\overline{1, N})$, and where the part of the rule after IF is the premise and the part after THEN is the conclusion or action; the conditions $C_1, C_2, \ldots, C_N$ are the facts that describe the current state of the subject area.

The following rules have been generated to obtain information about possible causes of GPHC disruption:

IF the tender date > the user established period, THEN $k = k + 1$ (Late holding of tender procedures);

IF the same object has more than one general contractor, THEN $k = k+1$ (Unfair contractor);

IF the same object has more than one agreement, THEN $k = k + 1$ (Break of contractual relations);

IF the status of object corresponds to "in work: delayed", THEN $k = k+1$ (Commissioning of object);

IF the volume of financing < the volume of spent financial means, THEN $k = k + 1$ (Amount of financing);

IF the amount of financing < the amount of spent financial means, THEN $k = k + 1$ (Defects in the project).

The information obtained as a result of acquisition from the database tables is used as a mechanism to select a list of concepts, as well as to optimize this list, and is given to the expert group during interview.

A fuzzy cognitive map (FCM) is a causal network $G = (E, W)$, where E is a set of concepts $e_i \in E$; and W is a set of relationships between concepts $w(e_i, e_j) \in W$, $w : E \times E[-1, 1]$.

For modeling of fuzzy-target dynamic systems based on fuzzy cognitive maps, in [16] the fuzzy matrix regular algebra defined by the quadruple, is introduced:

$M_n(R) = \langle FM, \vee, \circ, * \rangle$, where FM is the set of fuzzy matrices, $\vee$ is the operation max, $\circ$ is the maxi-triangular composition, and $*$ is the unary closure operator.

Fuzzy matrix regular algebra is used to describe and evaluate the causal impacts of concepts on each other.

The objective of identifying the cross-impact of concepts is to determine the cumulative causal effect (or maximum "weight" of path) from concept e_i to concept $e_j (e_i \to e_{k1} \to \cdots \to e_{kn} \to e_j)$ on the graph of corresponding cognitive map defined by the fuzzy matrix.

Figure 8.5 shows the FCM diagram composed of many of the concepts that belong to one of the following areas: concept areas—sources of risk, basic concepts, target concepts and a set of relationships between concepts.

The causal path is defined as follows:

$$e_i \to e_j\colon \quad \big(i, k_1, \ldots, k_1', j\big) = P_l, \quad l = 1 \ldots m.$$

Then the cross impact of concepts $w(e_i, e_j)$ is defined as follows:

$$w(e_i, e_j) = \bigcup_{l=1,\, p \in P}^{m} T w(e_p, e_{p+1}),$$

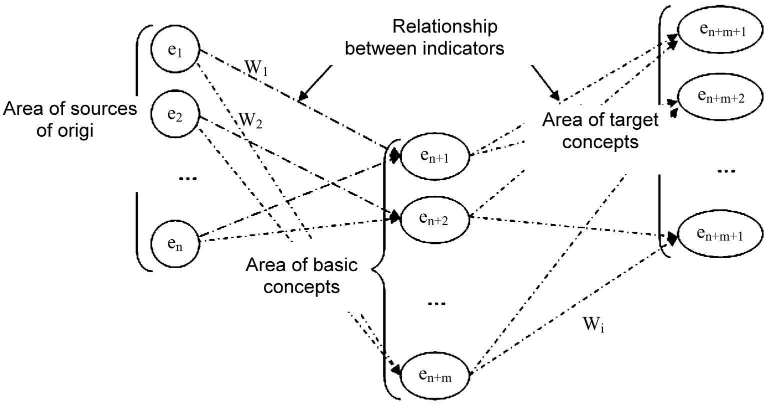


Fig. 8.5 FCM diagram

where T is T-*norm*, which is considered as the operation of taking a minimum or a product.

The cognitive cross-impact interaction matrix and fuzzy cognitive map are formed according to the following procedure.

To determine the cross impact of concepts, a fuzzy matrix R of $2n \times 2n$ size is formed. Its elements are determined from the matrix $W = [w(e_i, e_j)]_{n \times n}$ by the following substitution:

$$\text{if } w(e_i, e_j) > 0, \quad \text{then } r_{2i-1,2j-1} = w(e_i, e_j), r_{2i,2j} = w(e_i, e_j);$$
$$\text{if } w(e_i, e_j) < 0, \quad \text{then } r_{2i-1,2j-1} = -w(e_i, e_j), r_{2i,2j} = -w(e_i, e_j);$$

the remaining elements take on zero values.

The procedure of transitive closure of R allows us to make consistent the cross-impact relationships of concepts: $\hat{R} = R \cup R^2 \cup R^3 \cup \cdots$.

The result consists of positive-negative pairs of elements $(v_{ij}, \overline{v}_{ij})$ formed according to the rule: $v_{ij} = \max(r_{2i-1,2j-1}, r_{2i2j})$, $\overline{v}_{ij} = -\max(r_{2i-1,2j}, r_{2i,2j-1})$.

The elements of matrix $V = [\langle v_{ij}, \overline{v}_{ij} \rangle]$ can be used as indicators characterizing the dynamics to achieve one or more objectives. Figure 8.6 shows the block diagram of FCM analysis. For FCM analysis it is necessary to calculate the basic system parameters of fuzzy cognitive maps [23, 24]:

– Consonance of influence of concept e_i on concept e_j:

$$c_{ij} = \frac{|v_{ij} + \overline{v}_{ij}|}{|v_{ij}| + |\overline{v}_{ij}|};$$

– Dissonance of influence of concept e_i on concept e_j:

$$d_{ij} = 1 - c_{ij};$$

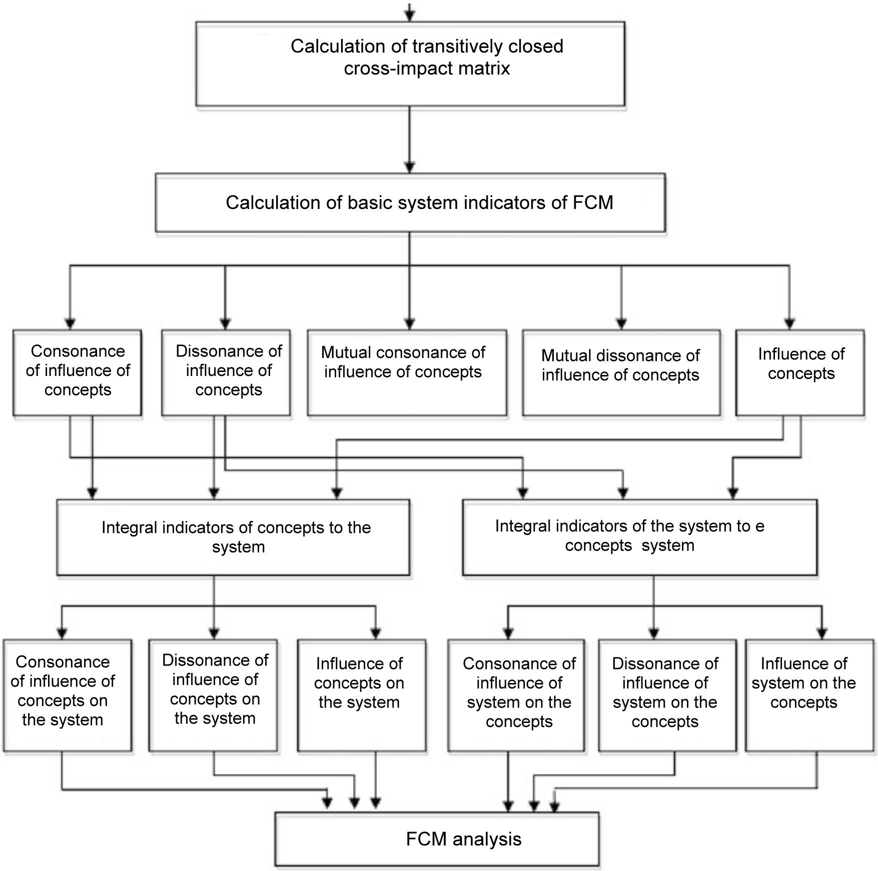


Fig. 8.6 Block diagram of FCM analysis

– Mutual consonance of influence of concepts e_i and e_j:

$$\overset{\leftrightarrow}{c} = \frac{|(v_{ij} + v_{ji}) + (\overline{v}_{ij} + \overline{v}_{ji})|}{|v_{ij} + v_{ji}| + |\overline{v}_{ij} + \overline{v}_{ji}|};$$

– Mutual dissonance of influence of concepts e_i and e_j:

$$\overset{\leftrightarrow}{d} = 1 - \overset{\leftrightarrow}{c}_{ij};$$

– Impact (influence) of concept e_i on concept e_j:

$$p_{ij} = sign(v_{ij} + \overline{v}_{ij}) \max\left(|v_{ij}|, |\overline{v}_{ij}|\right) \quad \text{for } v_{ij} \neq -\overline{v}_{ij}; \quad \text{and}$$

– Mutual positive influence of concepts e_i and e_j:

$$\overset{\leftrightarrow}{p}_{ij} = \overset{\leftrightarrow}{p}_{ji} = (v_{ij} S v_{ji}), \quad \text{where } S \text{ is the corresponding } S\text{-norm.}$$

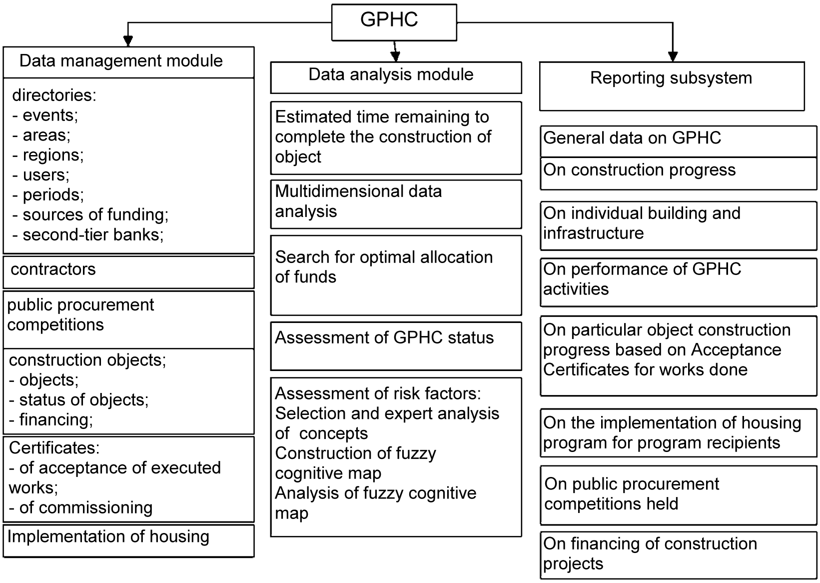


Fig. 8.7 Functional structure of ISMGP

For a more complete FCM analysis, it is necessary to define the following integral indicators of the influence of concepts on the system and of the system on concepts:

– Consonance of influence of the i-th concept on the system: $\vec{C}_i = \frac{1}{n}\sum_{j=1}^{n} c_{ij}$;
– Dissonance of influence of the i-th concept on the system: $\vec{D}_i = \frac{1}{n}\sum_{j=1}^{n} d_{ij}$;
– Consonance of influence of the system on the j-th concept: $\overleftarrow{C}_j = \frac{1}{n}\sum_{i=1}^{n} c_{ij}$;
– Dissonance of influence of the system on the j-th concept: $\overleftarrow{D}_j = \frac{1}{n}\sum_{i=1}^{n} d_{ij}$;
– Impact (influence) of the i-th concept on the system: $\vec{P}_i = \frac{1}{n}\sum_{j=1}^{n} p_{ij}$;
– Impact (influence) of the system on the j-th concept: $\overleftarrow{P}_j = \frac{1}{n}\sum_{i=1}^{n} p_{ij}$;
– Mutual consonance of the i-th concept and the system: $I_i^{SC} = \vec{C}_i S \overleftarrow{C}_i$; and
– Mutual dissonance of the i-th concept and the system: $J_i^{SC} = \vec{D}_i S \overleftarrow{D}_i$.

Indicators $\vec{C}_i$, $\vec{D}_i$, $\overleftarrow{C}_j$, $\overleftarrow{D}_j$, $\vec{P}_i$, $\overleftarrow{P}_j$ characterize impact of each concept on the system.

Selecting the appropriate type of relationship and specifying the level of values, one can get a binary matrix and, therefore, identify the classes of related concepts with this level with respect to a corresponding property (mutual consonance, dissonance, positive and negative impacts). This algorithm can be applied to assess the risk of GP implementation.

The introduction of an information system for monitoring GPHC implementation describes the system engineering solution proposed in the previous sections regarding the description methods, models and algorithms as a tool to support the

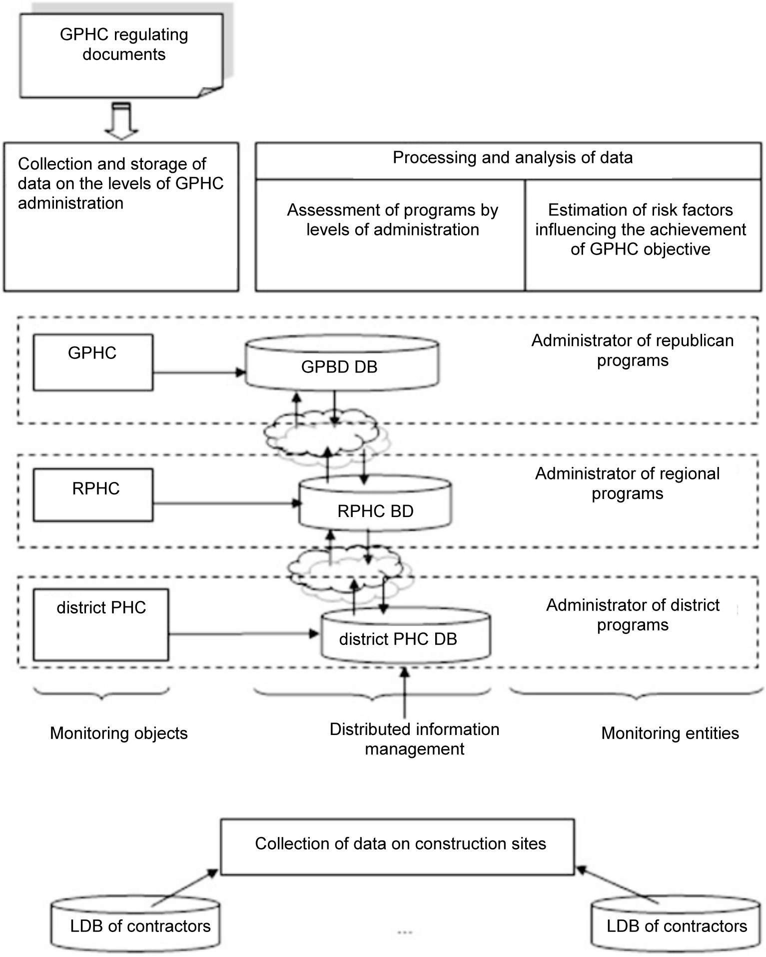


Fig. 8.8 Formation of distributed information resource

technology, and distributed information system for monitoring GPHC implementation (ISMGP) [18, 19].

The functional *structure of ISMGP* includes a subsystem for support of decisions made by administrators (SDMA), a subsystem for monitoring construction projects and a subsystem for evaluation of GPHC implementation (Fig. 8.7).

A core of information *support for ISMGP* is the distributed relational database, the formation of which is shown in Fig. 8.8. For each of the ISMGP subsystems the user roles and data access privileges are defined.

The *ISMGP software* has a client-server architecture. Major subsystems of information system are implemented as web applications, where the client is a browser and the server is a web server on which the application logic is focused.

The results of introduction of information and software components of ISMGP complex are aimed at reducing the complexity and increasing the efficiency and soundness of management decisions by program administrators at the regional level.

The introduction of information technology to monitor the implementation of GPHC will directly affect the final result of the activity of the administrator, which is measured by the quality indicator characterizing the volume of services rendered or work performed by the administrator during the reporting period, by improving the speed and quality of decisions. The use of expert-analytical capacity of ISMGP will improve the evaluation of the impact of costs in relation of the achievement of public policy objectives by the administrator during the implementation of GP [20–26].

References

1. Kochkarov, R.A.: Targeted Programs: Tool Support. Ekonomika, Moscow (2007), p. 233
2. Bocharnikov, V.P., Sveshnikov, S.V., Yatsyshin, Y.V.: Fuzzy Technology: Mathematical and Software Support of Target Programs in Strategic Management. NIKA Center (2005), p. 264
3. Kulba, V.V., Kowalewski, S.S., et al.: Management and Monitoring of Implementation of Social and Economic Programs. LIBROKOM, Moscow (2009), p. 400
4. Kuzmin, A.: Monitoring and Evaluation of Social Programmes. Process Consulting, Moscow (2008)
5. Greenberg, D.H., Robins, P.K.: Incorporating nonmarket time into benefit-cost analyses of social programs: an application to the self-sufficiency project. J. Public Econ. **92**(3–4), 766–794 (2008)
6. Tsygankov, D.B., Smirnova, M.V., Settles, A.: Analysis and Evaluation of National Programs and Sectoral Policies: Reader to Discipline. GU VSE, Moscow (2006), p. 252
7. Bolgarov, A.D., Druzhinin, E.A., Zyanchurina, I.N.: System methodology of assessment of feasibility of the national program of informatization of Ukraine. Collect. Sci. Pap. Kozhedub Kharkov Univ. Air Forces **3**(18), 154–160 (2008)
8. Shakina, E.A.: Multi-level evaluation of target programs as a method of cost management of regional and municipal. The Thesis of Candidate of Economic Sciences: 08.00.10 Place of defense: State Univ., High Sch. Of Ec., 2008, 157 p.: il. RSL OD, 61 08-8/1354
9. Decree of the President of the Republic of Kazakhstan dated June 2, 2003, N 1099. Rules for the formulation and implementation of state programs in the Republic of Kazakhstan
10. Ham, J.C., Swenson, C., Imrohorglu, A., Song, H.: Government programs can improve local labor markets: evidence from state enterprise zones, federal empowerment zones and federal enterprise community. J. Public Econ. **95**(7–8), 779–797 (2011)
11. Bai, X., Liu, Y., Wang, L., Zhong, P.: Model-based monitoring and policy enforcement of services. Simul. Model. Pract. Theory **17**, 1399–1412 (2009)
12. Wu, L., Lu, Z., Sun, G., Li, J.: Study on intelligent monitoring methodology based on the mathematical model of heat transfer for blast furnace stave. Appl. Math. Model. **34**, 2129–2135 (2010)
13. Černorucki, I.G.: Methods of Decision-Making. BHV Petersburg, St. Petersburgh (2005), 416 p.
14. Podinovski, V.V.: Introduction to the Theory of the Importance of Criteria in Multicriteria Decision Making Tasks. Physmathlit, Moscow (2007), 64 p.
15. Shashkin, A.I., Shiryaev, M.M.: Scheduling of project works based on fuzzy input data. Vest. SamGU, Nat. Sci. Ser. **3**(62) (2008)
16. Dyusembaev, A.E.: On the assessment of the status of objects with a hierarchical structure with monotonous processes in the tops. In: Collection of Abstracts of All-Union Conference on Decomposition and Coordination in Complex Systems (1986)

17. Resolution of the Government of the Republic of Kazakhstan dated July 21, 2004. N 779. On approval of rules of evaluation of effectiveness of budgetary programs
18. Urkumbaeva, A.M.: Information system of monitoring of government program of housing construction. D. Serikbayev VKGTU Vestn. **3**, 180–186 (2008)
19. Fukuda, K.: Monitoring unit root and multiple structural changes: an information criterion approach. Math. Comput. Simul. **71**, 121–130 (2006)
20. Maksimov, V.I., Kornoushenko, E.K., Kachaev, S.V.: Cognitive technologies to support managerial decision-making. Inform. Obsh. **2**, 50–54 (1999)
21. Urkumbaeva, A.M., Balova, T.G.: Mathematical and information support for monitoring the implementation of government programs. Shakarim Bull. SSU **4**, 155–159 (2009)
22. Urkumbaeva, A.M.: Network planning and management for the monitoring of public housing programs. Shakarim Bull. SSU **3**, 41–46 (2009)
23. Urkumbaeva, A.M.: Using factor models for risk assessment and forecasting in the information system of monitoring government programs. Shakarim Bull. SSU **2**, 104–107 (2009)
24. Urkumbaeva, A.M.: Information and risk assessment software support government programs. D. Serikbayev Bull. EKSTU **4**, 179–186 (2009)
25. Keller, M.R., Block, F.: Explaining the transformation in the US innovation system: the impact of a small government program. Socio-Econ. Rev. **11**(4), 629–656 (2013)
26. Sternberg, R.: Success factors of university-spin-offs: regional government support programs versus regional environment. Technovation **34**(3), 137–148 (2014)

Chapter 9
Methodology for Identification of Competitive Industrial Clusters

Under current conditions, the development of local areas acts as a driving force for higher competitiveness of individual regions and the country as a whole. World experience provides many examples of how implementation of a cluster strategy of economic development, characterized by close interaction between governmental structures and enterprises, has augmented the competitiveness of the country and its regions. Globalization makes production factors more mobile and increases competition among countries. In this climate, not only innovations and education but also interrelations between enterprises become important factors in gaining and retaining advantage over competitors. This situation has led to the creation of network structures—clusters.

The term "cluster," which has fairly recently appeared in economics, has a wide range of interpretations. There are a great variety of approaches to the definition of the term, with the integral elements being geographical closeness, inter-company communications and inter-company or institutional networks. Taken together, these factors differentiate a cluster from any other socio-economic phenomenon [1].

Scientific investigations in our country are based on the foreign experience of economy clusterization, and mainly on the approach developed by M. Porter [2]; therefore it is necessary to give more precise definitions of cluster-based categories and methods of economy management used in Kazakhstan.

Multi-aspect interpretation of the term "cluster" enables us to classify clusters according to different criteria. Systematization of scientific approaches reveals three conceptually different approaches to cluster identification:

(1) Approaches based on A. Marshall's theoretical principles of economy localization described by S. Rosenfeld; H. Schmitz and K. Nadvi; P. Swann and M. Prevezer;
(2) Approaches centering on inter-branch relations and based on input-output tables. This approach is used in the papers of the following authors: D. Czamanski and L. de Ablas; B. O'hUallachain, J. Redman, H. Roepke et al.; E. Bergman and E. Feser;
(3) Approaches including a wide range of parameters developed in M. Porter's works: economy of localization and urbanization, internal scale effect, value

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7_9

added chain, technological innovations and other parameters explaining grouping of institutions in the geographical space.

An analysis of the existing foreign experience of cluster strategy of economic development shows that there are three "centers" of cluster development: North America, West Europe and Asia.

The development of Kazakhstani oblasts is a priority task at the national level, determining the strategy of territorial development of the Republic of Kazakhstan until 2015. The strategy of regional development is different for different oblasts. This is because different oblasts have different natural resources, different economic structures, different levels of development of certain spheres, different conditions of entering into market economy, different rates of transformation of forms of property and different competitive advantages determined by climatic, demographic, production and geographic factors.

As regional clusters we will consider groups of regions with similar socio-economic situations—which determine competitiveness [2, 3]—in order to involve them in global and regional systems of goods, financial, labor, technology and information markets.

As the current development of Kazakhstan has a clearly pronounced regional context, in order to develop an efficient territorial policy taking into account the differences between the groups of regions (regional clusters) and directed at the development of individual regions, regional clusters and the territory of the republic as a whole, it is necessary to carry out clusterization of the Republic's regions based on similarities in their development.

However, we still do not have any unified mechanisms for creating and stimulating clusters. Therefore in order to realize the vector of economic modernization based on the cluster approach it is necessary to develop a justified system controlling the clusterization process, including a detailed mechanism of cluster formation and state support. In this context it seems reasonable and timely to carry out investigations using cluster methodology and aimed at the development of a theoretical and methodological basis of economic management taking into account the country's regional and sectoral differences.

An important problem of cluster-based economy management is identification of competitive industrial clusters.

An industrial cluster is a set of enterprises or institutions belonging to one or several sectors of economy concentrated in a certain geographic region, which also includes companies interrelated in the value-added process, suppliers of equipment, spare parts and services, research institutes, universities and other organizations complementing and enhancing competitive advantages, as compared to the same sectors in other regions [6].

9.1 Cluster Analysis of Kazakhstani Regions

According to the theory of stable economic growth, the level of economic development of regions can be estimated based on such parameters as innovation activity, private capital, public capital, regional accessibility and regional concentration.

To analyze the economic development of Kazakhstani regions the following factors were chosen:

(1) innovation activity:

- gross expenditures on R&D (P1), which can be considered as resources used for generation of new knowledge and technologies;
- expenditures on technological innovations of enterprises (P2). These are factual expenditures in monetary terms related to the carrying out of different types of innovation activities fulfilled at the enterprise (total expenditures take into account current and capital expenditures).

Depending on the type of innovation activities the following expenditures are specified [4, 5]:

- expenditures on R&D related to implementation of new products and technological processes;
- expenditures on purchasing of intangible technologies—licenses on the use of inventions, production prototypes, useful models, patent rights, nonpatent licenses and technological services;
- expenditures on production design;
- expenditures on instrumental preparation, organization and launching of production, training and retraining due to implementation of new products and technological processes;
- expenditures on purchasing of machinery and equipment related to the implementation of new or modernized products and technological processes;
- expenditures on marketing research on introduction of new products to the market.

(2) Human resources:

- percentage of employed population having higher education (P3);
- percentage of employed population having specialized secondary education (P4).

These indicators show the potential of the region to generate knowledge, to adapt knowledge from other regions and to modernize instruments produced in the region.

(3) Private capital:

- industrial investments into fixed assets per capita (P5). Such investment shows the rate of support and growth of capital assets in the region, and also the level of investment activity and attractiveness of the region for private investors.

(4) Public capital:

- budget investments in fixed assets per capita (P6). These can be interpreted as the regional factor characterizing capital investments [9]. The higher the factor, the higher the investment attractiveness of the region, including attractiveness for private investors.

(5) Regional accessibility (P7). This parameter characterizes the access of the region to the market and the national transport infrastructure (cities of Almaty and Astana). This parameter is calculated as:

$$\frac{\text{The distance from the oblast's center to Almaty}}{\text{The distance from Almaty to Astana}} + \frac{\text{The distance from the oblast's center to Astana}}{\text{The distance from Almaty to Astana}} \qquad (9.1)$$

(6) Regional business concentration:

– the number of active entrepreneurs per 1000 sq. km (P8). This factor characterizes business density and can be interpreted as the level of potential urbanization of regional economy;
– the percentage of employment in industry and construction (P9). This indicator characterizes the degree of potential localization of the national economy and shows the degree of its industrialization.

These data were obtained based on the materials of the RK Statistics Agency and its regional bodies [5–11].

Prior to carrying out cluster analysis, we checked the correlation between the parameters to be used in the analysis as strongly correlated variables tend to dominate and therefore falsify analysis results. If the coefficient of pair correlation between two variables is greater than 0.85, it is recommended to exclude one of them [11]. The coefficients of pair correlation given in Table 9.1 show that there is no close correlation between the chosen variables.

As the parameters taken for the analysis are heterogeneous, the values were subjected to the procedure of z-transformation, i.e., all values were reduced to the interval -3 to $+3$, according to the formula:

$$z = \frac{x - m}{\sigma}, \qquad (9.2)$$

where z is a standardized variable; x is a non-standardized variable; m is a mean value; σ is a standard deviation.

In order to combine regions into larger groups based on their similarity, it is necessary to carry out cluster analysis.

The task of cluster analysis is to subdivide the set of objects I, based on the data contained in the set X, into m clusters (subsets) $\pi_1, \pi_2, \dots, \pi_m$ in such a way that each object I_i belongs to the same subset and the objects belonging to the same cluster are similar, whereas the objects belonging to different clusters are different [12–14].

An important advantage of cluster analysis is the possibility to group objects not by one parameter but by a set of parameters. Moreover, cluster analysis, unlike many other mathematical statistical methods, does not impose any restrictions on the type of studied objects and enables us to consider an arbitrary set of initial data. Cluster

Table 9.1 Correlation matrix

Correlation	P1	P2	P3	P4	P5	P6	P7	P8	P9
P1	1	0.3023	0.4150	0.1710	0.2824	0.0430	−0.1786	−0.1709	0.4374
P2	0.3023	1	0.2813	0.3366	−0.0325	−0.2721	0.2519	−0.2023	0.6247
P3	0.4150	0.2813	1	0.2945	0.6156	0.4017	−0.4485	−0.5074	0.7594
P4	0.1710	0.3366	0.2945	1	0.3249	0.4049	−0.2166	−0.2687	0.6232
P5	0.2824	−0.0325	0.6156	0.3249	1	0.7008	−0.6053	−0.2407	0.6130
P6	0.0430	−0.2721	0.4017	0.4049	0.7008	1	−0.5640	−0.3572	0.2842
P7	−0.1786	0.2519	−0.4485	−0.2166	−0.6053	−0.5640	1	0.2709	−0.2934
P8	−0.1709	−0.2023	−0.5074	−0.2687	−0.2407	−0.3572	0.2709	1	−0.3878
P9	0.4374	0.6247	0.7594	0.6232	0.6130	0.2842	−0.2934	−0.3878	1

analysis also makes it possible to consider quite a large volume of information and to sharply reduce or compress large arrays of socioeconomic information, to make them more compact and convenient.

An important problem is the choice of method for clusterization. The advantage of Ward's cluster analysis is that the set of studied objects is partitioned into the most statistically homogeneous groups [16]. This method enables us to optimize minimal dispersion inside clusters. This target function is known as a within-group sum of squares or the sum of squares of variances (SSV). The formula for the sum of squares of variances is written as:

$$CKO = x_j^2 - 1/n\left(\sum x_j\right)^2, \tag{9.3}$$

where x_j is the value of the indicator of the j-th object.

In Ward's method the groups or objects with minimal SSV increment are united.

A very important problem in cluster analysis is the choice of the optimum number of clusters. This is still among the unsolved problems of cluster analysis because of the absence of an adequate zero hypothesis and complicated nature of multi-dimensional sample distributions [14]. The variation of the corresponding function is very often used as a criterion of grouping (the number of clusters) [15].

For example, in our case it is the square of the Euclidean distance determined through standardized values:

$$dist = \sum_{i=1}^{n} (x_i - y_i)^2. \tag{9.4}$$

The grouping process must provide progressive minimal increase in the value of the criterion. A sharp jump can be interpreted as a characteristic of the number of clusters objectively existing in the studied set, i.e., in the step where the value of the coefficient increases stepwise, it is necessary to stop the process of uniting into new clusters, otherwise clusters located at a rather large distance from each other will be united.

The results of the cluster analysis are presented by:

(1) the proximity (similarity) matrix;
(2) the table of the order of agglomeration;
(3) the table of elements belonging to the cluster;
(4) the tree diagram (dendrogram).

The proximity matrix, obtained after processing of the initial data in SPSS 13, is presented in Table 9.2. The matrix gives information about the similarity and difference in the socio-economic development of regions. The lower the value, the higher the degree of similarity of two oblasts or combinations in the cluster. By contrast, the higher the corresponding value of the proximity matrix, the greater the differences between the two oblasts [10, 15].

Each line describes a step in the factual formation of clusters. The merging process presented in Table 9.3 can be described as follows: the data of the two columns with the common name "merging into clusters" show that in the first step regions 3

Table 9.2 Table of proximity (similarity)

	Oblasts	Square of Euclidean distance													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Akmola	**0.000**	12.496	10.289	31.369	12.579	12.905	25.869	6.731	11.605	33.102	20.039	9.177	9.874	23.280
2	Aktobe	12.496	**0.000**	12.336	15.109	3.726	11.722	17.409	6.130	5.272	16.108	25.480	11.894	14.664	17.949
3	Almaty	10.289	12.336	**0.000**	36.030	8.300	1.076	22.840	4.283	4.787	41.107	9.329	9.549	3.613	16.978
4	Atyrau	31.369	15.109	36.030	**0.000**	20.451	35.211	40.331	26.671	23.409	13.611	45.211	32.475	39.997	31.107
5	West Kazakhstan	12.579	3.726	8.300	20.451	**0.000**	8.958	28.749	4.330	4.008	25.030	19.362	14.793	10.185	19.891
6	Zhambyl	12.905	11.722	1.076	35.211	8.958	**0.000**	19.864	6.156	3.192	38.796	10.177	11.410	7.515	16.781
7	Karaganda	25.869	17.409	22.840	40.331	28.749	19.864	**0.000**	19.403	21.291	24.103	34.753	10.063	26.751	16.798
8	Kostanay	6.731	6.130	4.283	26.671	4.330	6.156	19.403	**0.000**	4.504	24.163	13.894	5.259	3.293	15.809
9	Kyzylorda	11.605	5.272	4.787	23.409	4.008	3.192	21.291	4.504	**0.000**	25.620	17.642	13.638	11.205	18.163
10	Mangistau	33.102	16.108	41.107	13.611	25.030	38.796	24.103	24.163	25.620	**0.000**	52.597	25.851	41.885	25.274
11	South Kazakhstan	20.039	25.480	9.329	45.211	19.362	10.177	34.753	13.894	17.642	52.597	**0.000**	15.315	8.491	25.863
12	Pavlodar	9.177	11.894	9.549	32.475	14.793	11.410	10.063	5.259	13.638	25.851	15.315	**0.000**	6.882	16.716
13	North Kazakhstan	9.874	14.664	3.613	39.997	10.185	7.515	26.751	3.293	11.205	41.885	8.491	6.882	**0.000**	20.097
14	East Kazakhstan	23.280	17.949	16.978	31.107	19.891	16.781	16.798	15.809	18.163	25.274	25.863	16.716	20.097	**0.000**

Table 9.3 Table of agglomeration order (the Ward method)

Steps	Merging into cluster		Coefficients	The step when the cluster first appears		Next step
	Cluster 1	Cluster 2		Cluster 1	Cluster 2	
1	3	6	0.538	0	0	7
2	8	13	2.184	0	0	5
3	2	5	4.047	0	0	4
4	2	9	6.519	3	0	11
5	8	12	10.018	2	0	6
6	1	8	15.177	0	5	10
7	3	11	21.500	1	0	10
8	4	10	28.306	0	0	13
9	7	14	36.705	0	0	12
10	1	3	46.005	6	7	11
11	1	2	59.011	10	4	12
12	1	7	79.588	11	9	13
13	1	4	117.000	12	8	0

and 6 are united (i.e., Almaty and Zhambyl oblasts). These regions have the highest degree of similarity, and are both included in the third cluster as there is no cluster 6 in the table. In the next stages regions 8 and 13 (Kostanay and North Kazakhstan oblasts), 2 and 5 (Aktyubinsk and West Kazakhstan oblasts), etc. are united [10].

To determine the optimal number of clusters it is necessary to determine the step where the coefficient has a sharp jump. In our case there are three sharp jumps in the coefficient: jump No. 1—at the tenth step, from 36.705 to 46.005; jump No. 2—at the eleventh step, from 46.005 to 59.011 and jump No. 3—at the twelfth step, from 59.011 to 79.588.

The optimal number of clusters is the number equal to the difference between the number of observations (14, in our case) and the number of steps after which the coefficient increases stepwise. It means that after formation of two, three or four clusters it is not necessary to continue merging, and the results with such a number of clusters are optimal.

The choice of the optimal cluster from the three possibilities is an intuitive task. In terms of economics it would be more reasonable to choose the result with four clusters.

It is also necessary to explain the last three columns in Table 9.3 showing the order of merging. For example, let us consider the line corresponding to the tenth step. At this step clusters 1 and 3 merge. Before this step cluster 1 took part in merging at the sixth step. Cluster 3 took part in merging at the seventh step. The new cluster 1 will take part in merging at the eleventh step (column: next step).

When the optimal number of clusters is determined, it is necessary to determine which cluster each region refers to (Table 9.4).

Table 9.4 Formation of clusters

#	Oblasts	Cluster number	#	Oblasts	Cluster number
1	Akmola	1	8	Kostanay	1
2	Aktobe	2	9	Kyzylorda	2
3	Almaty	1	10	Mangistau	3
4	Atyrau	3	11	South Kazakhstan (SKO)	1
5	West Kazakhstan (WKO)	2	12	Pavlodar	1
6	Zhambyl	1	13	North Kazakhstan (NKO)	1
7	Karaganda	4	14	East Kazakhstan (EKO)	4

The most common method used for the representation of the proximity matrix is based on the idea of the dendrogram, which is a graphic representation of the process of successive cluster formation (Fig. 9.1).

The cluster analysis enabled us to make a conclusion that even regions located nearby on the map of Kazakhstan differ so considerably from each other by the level of economic development and growth potential that they cannot be referred to the same regional cluster [6–11].

Table 9.4 and the dendrogram show that cluster 1 includes South Kazakhstan, Zhanbyl, Almaty, North Kazakhstan, Pavlodar, Kostanay and Akmola oblasts. Cluster 2 includes Aktobe, West Kazakhstan and Kyzylorda oblasts. Cluster 3 consists of Atyrau and Mangistau oblasts. Cluster 4 is formed by Karaganda and East Kazakhstan oblasts.

Table 9.5 presents the information about cluster profiles. Cluster 1 containing seven oblasts has the lowest level of such factors of growth as the percentage of employed population with higher education (16.1 % as compared with 18.7 % on

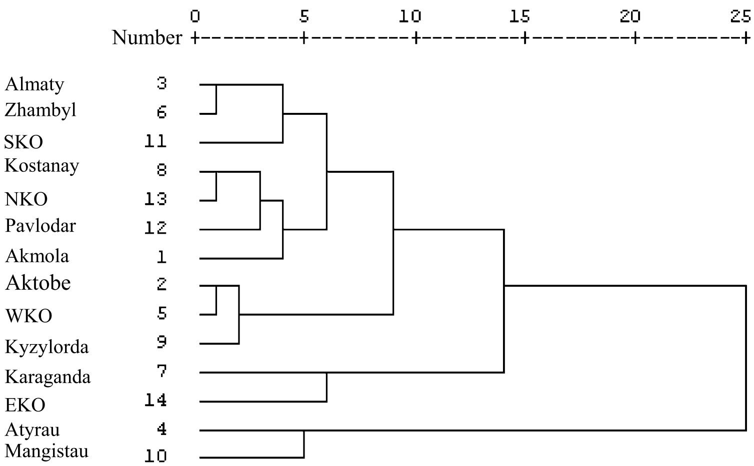


Fig. 9.1 A dendrogram constructed using Ward's method

Table 9.5 Cluster profiles

Average values	Gross expenditures on R&D (millions of tenge, average values)	Expenditures on technological innovations at enterprises (millions of tenge, average value)	Percentage of employed population having higher education (%, average value)	Percentage of employed population having specialized secondary education (%, average value)	Industrial investments into fixed assets per capita (thousands of tenge, average value)	Budget investments into fixed assets per capita (thousands of tenge, average value)	Relative distance	Number of active entrepreneurs per 1000 sq. km (average value)	Percentage of employment in industry and construction (%, average value)
Cluster 1	180.70	1756.02	16.10	29.70	23.90	7.30	0.70	34.11	12.50
Cluster 2	154.10	700.42	20.90	25.80	67.40	17.20	0.40	13.50	14.90
Cluster 3	1285.75	3371.14	25.85	37.70	387.30	20.05	0.28	20.86	32.80
Cluster 4	1992.45	9677.74	21.35	29.25	40.45	6.90	0.71	20.29	25.25
Total	183.35	1936.00	18.70	29.65	34.45	8.75	0.57	24.92	14.95

average in Kazakhstan), industrial investments per capita, and the percentage of employed population working in industry and construction (12.5 % as compared with 14.95 % on average in Kazakhstan) [10].

The average volume of industrial investments per capita for the oblasts of cluster 1 amounts to 23,900 tenge, which is 1.44 times lower than the average value for the country. Such factors of growth as gross expenditures on R&D, expenditures of enterprises on technological innovations, and budget investments into fixed assets per capita are lower than the corresponding average values for Kazakhstan. However, the cluster has the highest level of regional business concentration estimated by the number of active entrepreneurs per 1000 sq. km (34.11 % against 24.92 % on average in the country).

The second cluster has a high level of investment activity. The average volume of industrial investments per capita is 67,400 tenge, which is almost 2 times higher than the average national value. The average value of budget investments in fixed assets per capita is 17,200 tenge, which is also almost 2 times higher than the average value for the Republic. The oblasts included in the cluster have the lowest indicators of innovation activity. Gross expenditures on R&D equal to 154.1 million tenge as compared to the average value for the Republic—183.35 million tenge; expenditures on technological innovations of enterprises amount to 700.42 million tenge, which is 2.8 times as low as the average value for the country. The average value of the indicator showing regional business concentration is 2.5 times as low as the same indicator in the first cluster, and has the lowest value as compared with the other clusters. As the regions included in the cluster are located far from the center of the country and have underdeveloped infrastructure, the average value of the factor reflecting regional accessibility is rather low: the coefficient of the relative distance is 0.4. The other parameters do not differ from the average values for the Republic.

The third cluster consists of the two oblasts rich in oil—Atyrau and Mangistau oblasts. It has the highest average level of such indicators as industrial investments in fixed assets per capita (16.2 times as high as in the first sector and 11.2 times as high as the average value). High values of employment of population having higher and specialized secondary education, and the fact that a third of the employed population work in industry and construction provided the highest growth rates among the oblasts. This cluster also has high average levels of indicators of innovation activities and regional business concentration. The only factor hampering the development of the oblasts is their distance from the markets of Almaty and Astana (the average value of the relative distance is 0.279 as compared with the average value for all the oblasts—0.575).

A distinguishing feature of the fourth cluster including the East Kazakhstan and Karaganda oblasts is the highest indicators of the innovation activity. Gross expenditures on R&D amount to 1992.45 million tenge, which is 10.9 times as high as the average national indicators, while expenditures on technological innovations cost 9667.74 million tenge, which is 5 times as high as the average values for all oblasts of the Republic. The oblasts included in the cluster are characterized by a high level of education of human resources. On average, 21.35 % of the workforce has higher education, whereas the average indicator for all the oblasts taken together is 18.7 %.

The cluster has a high level of industrial investments per capita (1.2 times as high as the average value for the country), high percentage of the population working in industry and construction (1.7 times as high as the average value for the country), and high level of regional accessibility (0.7105 as compared with the average of 0.5745 for all the oblasts). However, budget investments per capita are 1.3 times lower than the average value of this indicator for Kazakhstan. Such indicators as the percentage of employed population having higher and specialized secondary education as well as regional business concentration are close to the average national values. Both oblasts included in the fourth cluster are industrially developed oblasts characterized by high production potential, domination of large enterprises and highly developed mining complex. High values of such factors of economic growth as innovation activity and human capital provided modest rate of economic growth in the past 10 years.

According to the 2015 Strategy of Territorial Development of the Republic of Kazakhstan, the main factors that will provide higher economic potential and competitiveness of the specified regional clusters are:

- Marketing research aimed at determining the directions of positioning (specialization) of regions and reference cities in the national, regional and world economic systems;
- Orientation of regions not only on the efficient use of traditional production factors but also on the development of specialized factors such as innovation capital, qualified workforce, modern infrastructure and institutional environment;
- Combination of the efforts of small and middle-sized companies and other concerned organizations aimed at finding niches in which the regional cluster has potential competitive advantages;
- In the regions dominated by large, vertically integrated companies (mainly in gas-and-oil production and mining industries), the development of higher-level production (based on deeper processing of raw materials), creation of outsourcing mechanisms and strengthening of the local component in large projects with creation of auxiliary, service and processing blocks of small and middle-sized enterprises.

9.2 Methods of Identification of Competitive Industrial Clusters

As mentioned above, cluster initiatives have become an important part of territorial policy all over the world. The cluster concept is a promising instrument to be used in the analysis of areas, forming a new attitude to the role of local authorities, enterprises and organizations making attempts to increase their competitiveness. The application of the cluster approach is especially useful at the level of regions or oblasts as its essential component is close contact among cluster members. An important problem of local authorities is identification of the most competitive industrial clusters in order to speed up their development.

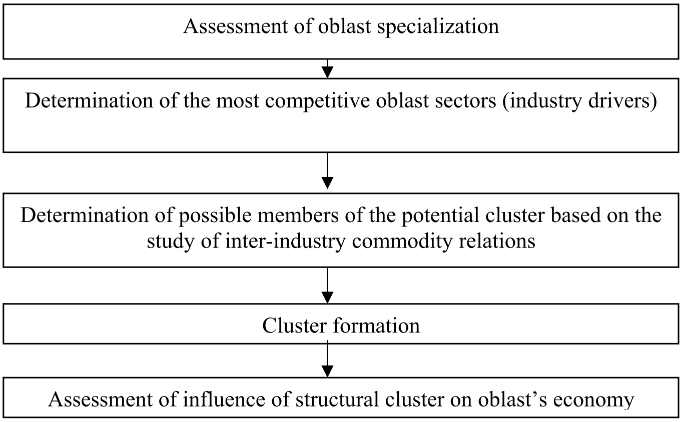


Fig. 9.2 A flowchart of the algorithm of formation of competitive industrial clusters

In order to identify competitive industrial clusters it was proposed to use a combined method that can be schematically presented as Fig. 9.2.

At the first stage of formation of industrial clusters it is necessary to evaluate specialization of the region, to study limitations of production factors and resources in order to determine the sectors where the area has the highest competitive advantages.

To identify a set of branches forming the economic base of the oblast we used the technique suggested by E.W. Hill and J.F. Brennan in [16]. According to this technique, the branches in which the region has the main competitive advantages are defined as industry drivers as they form the basis for the development of regional economy. These industries must form the core of industrial clusters [2, 17, 24, 25]. In order to determine industry drivers a combination of cluster and discriminant analyses is used. Cluster analysis was used to identify different groups of such sectors. The aim of discriminant analysis was to determine differences between homogeneous groups of industries.

The choice of the set of variables for identification of the most competitive specializations in the area is based on the "economic base" theory and the theory of competitive advantages.

The founder of the "economic base" theory was the Danish mercantilist Peter de la Kurt (1659). In his research "'t Welvaren der Stadt Leiden" (On the prosperity of Leiden city) he revealed two basic sectors of the city's economy: export-oriented production and the university. All other elements of the economic structure turned out to depend on the basic elements. The author concluded that in order to increase the population of the city its authorities had to make efforts to stimulate export and import-substitution.

The modern version of the "economic base" theory was formulated by the German economist Varner Sombart. The name of the theory reflects the specificity of the economic model of regional development specifying the concept of the basic sector.

A specific feature of the basic sector is orientation of its activities toward satisfaction of the external demand. The non-basic sector is, by contrast, oriented toward internal regional demands. According to the theory, the equivalent of regional economic development is economic growth measured by the indicators of the dynamics of production volumes, income per capita or employment.

The main driving force of economic development is the reaction of the basic sector on economic demand, increase of which causes increase in regional exports and growth in the basic sector. Changes in the volume of production, income and employment in the basic sector are multiplicatively translated into the other sectors of the regional economy.

In the theory of competitive advantages developed by M. Porter [2], the industry driver must consist of competitive companies or institutions, as the most important advantages of clusterization are created by the competition in product innovations, in quality improvement, in the adaptation of innovations and stimulation of entrepreneurship.

M. Porter states in [2] that competitive advantages of object location are formed by the quality of the environment, which this location provides for achieving high and constantly growing productivity in a certain sphere.

Based on the "economic base" theory and the theory of competitive advantages E. Hill and J.F. Brennan [16] showed that in order to identify industrial clusters it is necessary to take into account the following factors: the level of competitiveness, indicators of the oblast's export orientation, the level of concentration of regional economy and specialization of employment.

The set of indicators for the analysis is limited by the availability and accessibility of required statistical data.

In order to estimate competitiveness the following variables are used:

– the coefficient of localization calculated based on production factors (LQP), which is determined from the formula:

$$\text{LQP}_i = \frac{\text{Specific weight of the regional volume of the industry product in GRP}}{\text{Specific weight of the regional volume of the industry product in GDP}} \tag{9.5}$$

This indicator characterizes the extent of agglomeration of industries and the level of specialization of the territory according to the production factors, and enables us to determine the industries with high and low contribution to the total cost of finished products and services produced by economic units/residents of the region over a certain period of time with respect to the average level in the regional economy. The leading branches of the regional economy will be the branches with the highest level in the region. This indicator is used to estimate specialization of the regional economy and to determine clusters [2, 18, 19];

– absolute change in LQP from 2003 to 2008. (ΔLQP).

This indicator enables us to understand whether the sector gained or lost its competitiveness over the considered period. The obtained data can be interpreted as follows: if LQP is high (>1) and grows with time, it means that such a branch has high competitiveness; if LQP is low (≤ 1) but increasing, it is a sign of increasing competitiveness; if LQP is high but decreasing, it means that this branch is losing its competitiveness; if LQP is low and decreasing, it means that there is no competitiveness;

– absolute change in the place of the regional industry in national employment from 2003 to 2008 (ΔN_i) is calculated using the formula:

$$\Delta N_i = \left(\frac{e_{iR}}{e_{iN}}\right)_{2007} - \left(\frac{e_{iR}}{e_{iN}}\right)_{2003}, \tag{9.6}$$

where e_{iR} is the number of people working in the i-th industry in the region; e_{iN} is the number of people working in the i-th industry in the country.

This variable was used in [16]. The negative value of the indicator shows a decrease in the employment in the region, hence a decrease in the competitiveness of regional enterprises. By contrast, increase in the indicator can be considered as an increase in the competitiveness of the industry, and its enterprises will become leaders in the region.

The following are used as indicators of export orientation of the region [16]:

– the percentage of regional exports in the volume of industry production in the region;
– the percentage of regional exports in an industry, in the total volume of regional exports.

To identify export-oriented industries one can also use the following indicator:

– the percentage of the regional exports of the industry in the total volume of national exports.

High values of the above variable are seen to identify the presence of leading enterprises in the industry.

The level of concentration of the regional economy can be characterized using such indicators as:

– absolute change in the place of the industry in regional employment from 2003 to 2008 (ΔL_i), which is determined by the formula:

$$\Delta L_i = \left(\frac{e_{iR}}{e_{tR}}\right)_{2007} - \left(\frac{e_{iR}}{e_{tR}}\right)_{2003}, \tag{9.7}$$

where e_{tR} is the number of working people in the region.

This variable is used in [16] and its dynamics are interpreted as follows: if the percentage of the people employed by the industry decreases, it occupies weaker positions on the labor market and its regional competitiveness decreases;

– the ratio of the number of registered legal persons in the EKO and the RK per 100,000 residents in the region and in the Republic (Sh). This indicator provides information about spatial concentration of the industry and the level of concentration of a certain industry in the region.

According to [2, 16, 18, 19], the specialization of employment can be estimated using:

– the localization coefficient calculated on the base of employment (LQE), characterizing the degree of agglomeration of industries and the level of specialization of the territory according to the level of employment. To calculate this indicator the following formula is used:

$$\mathrm{LQE}_i = \left(\frac{e_{iR}}{e_{tR}}\right) \Big/ \left(\frac{e_{iN}}{e_{tN}}\right), \tag{9.8}$$

where e_{tN} is the number of employed people in the Republic of Kazakhstan;
– the absolute change in LQE (ΔLQE).

Disproportionately high concentration of employment in a branch of industry is an indicator of cluster economy, especially if LQE is accompanied by growing specific weight of national employment in this branch.

Therefore the leading industries of the territory must have the following characteristics:

– high value of the localization coefficient calculated on the basis of the factors of production and positive dynamics of this indicator;
– high percentage of export of their products;
– disproportionately high specific weight in the export of the region;
– increase in the specific weight of workers employed in the industry at the region level;
– high value of the localization coefficient calculated on the basis of employment and positive dynamics of this indicator.

An analysis of existing methods of cluster identification showed that the most precise and complete information about existing interrelations between industries can be obtained using factor analysis and the method of the main data components presented in the "input–output" tables.

To identify industrial regional clusters we used the method suggested in [20].

In order to identify industries that will form the basis of the industrial regional clusters, it is important to identify the most active relations between industries independent of their location. As the basis of model construction, it is proposed to use national "input–output" tables.

The data of the "input–output" tables enable us to obtain characteristics of interrelations between the sectors producing goods and services and consuming sectors (excluding transport, dealer services and pure taxes on products) and to identify the dependence of domestic production and consumption on the foreign market. A symmetrical "input–output" table establishes relations "product–product," i.e., the same classification is used in the lines and columns of the symmetrical table. The columns of the table present the data on consumption of an industry from other industries and the lines present information about the demand of other industries for the products produced by this industry.

The construction of the "input–output" table is based on Leontyev's model of inter-industrial balance. The main idea of the model can be explained as follows.

Let us suppose that the production sector of the national economy is subdivided into n branches (power-generating industry, machine-building industry, agriculture, etc.).

Let us consider branch $i = 1, 2, \dots, n$. It produces the volume x_i of some products within a given period of time (for example, a year), which is called gross output. Some part of the volume x_i, volume x_{ii} produced by the i-th branch, is used for its own production, some part goes to the other branches $j = 1, 2, \dots, n$ for consumption in their production (this part is denoted x_{ij}), and some other part of the volume y_i goes for consumption in the nonproduction sphere, the so-called volume of final consumption. The above-listed spheres of distribution of the gross product of the i-th branch give the following balance relation:

$$x_i = x_{i_1} + x_{i_2} + \cdots + x_{i_n} + y_i = \sum_{j=1}^{n} x_{ij} + y_i. \tag{9.9}$$

Then the coefficients of direct expenditures a_{ij} are introduced, which show how many units of the i-th branch products are spent on the production of one unit of products in the j-th branch. Now we can write that the volume of production produced in the i-th branch in the volume x_{ij} and transferred for production needs to the j-th branch is equal to:

$$x_{ij} = a_{ij} \times x_j. \tag{9.10}$$

It is assumed that the production technology does not change in all branches (within the time period under consideration), which means that the coefficients of direct expenditures a_{ij} are constant. This gives the following balance relation called Leontyev's model:

$$x_i = \sum_{j=1}^{n} a_{ij} \times x_j + y_i. \tag{9.11}$$

Substituting the vector of gross output X, the matrix of direct expenditures A and the vector of final consumption Y we obtain:

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \qquad A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}, \qquad Y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}. \tag{9.12}$$

Leontyev's model can be also written in the matrix form:

$$X = AX + Y. \tag{9.13}$$

The assumption of stability of the branch production technology used in Leontyev's model and in the compilation of the national "input–output" tables enables us to use the results of these tables at the regional level.

Having the values of general intermediate purchases (p) and sales (s) in each branch, we can write a functional dependence between any two branches i and j for each industry by means of four coefficients:

$$x_{ij} = \frac{a_{ij}}{p_j}, \qquad x_{ji} = \frac{a_{ji}}{p_i}, \qquad y_{ij} = \frac{a_{ij}}{s_i}, \qquad y_{ji} = \frac{a_{ji}}{s_j}. \tag{9.14}$$

Each of the coefficients (9.14) is an indicator of the dependence of branch i on branch j in terms of closely related purchases and sales:

x_{ij} and x_{ji} are specific weights of intermediate purchases $j(i)$ from $i(j)$ in the total volume of intermediate purchases of goods and services. High values of the coefficient mean that the j-th industry depends on the i-th industry as the main source of its intermediate expenditures;

y_{ij} and y_{ji} are specific weights of intermediate sales $i(j)$ for $j(i)$ in the total volume of intermediate purchases of goods and services. High values of this coefficient mean that the i-th industry depends on the j-th industry as the main market.

The coefficients x_{ij}, x_{ji} are included in matrix X, and the coefficients y_{ij}, y_{ji} in matrix Y.

Each column (x) of matrix X is an example of intermediate purchases of the industry located in the columns. Each column (y) of matrix Y is an example of intermediate output sales of the industry located in the columns.

To evaluate the interrelations between each pair of industries based on inputs and outputs of a large number of sectors, one must use correlation analysis.

In order to determine the similarity between the structures of the "input–output" tables of industries l and m, Fezer and Bergman [20] suggested calculating the four correlation coefficients based on the data of matrices X and Y:

$r(x_l, x_m)$ measures the degree of similarity of samples of input purchases of industries l and m;

$r(y_l, y_m)$ measures the degree of similarity of output products of industries l and m; i.e., the degree of similarity of sales of goods to mixed intermediate output buyers;

$r(x_l, y_m)$ measures the degree of similarity of samples of input purchases of industry l with samples of outputs sales to industry m, i.e., the percentage of purchases of industry l from the sectors for which industry m is a seller;

$r(y_l, x_m)$ measures the degree of similarity of the samples of input purchases of industry m with the samples of input purchases of industry l, i.e., the percentage of purchases of industry m from the sectors for which industry l acts as a seller.

The equation for the correlation coefficient is written as [21]:

$$\sigma_{x,y} = \frac{Cov(X, Y)}{\sigma_x \times \sigma_y}, \tag{9.15}$$

where $-1 \le \sigma_{x,y} \le 1$; $Cov(X, Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x) \times (y_i - \mu_y)$ is the covariation coefficient; $\sigma_x = \sqrt{\sum_{i=1}^{n}(x_i - \mu_x)^2 \times p_i}$ and $\sigma_y = \sqrt{\sum_{i=1}^{n}(y_i - \mu_y)^2 \times p_i}$ are the average quadratic variances of x and y; $\mu_x = \sum_{i=1}^{n} x_i \times p_i$ and $\mu_y = \sum_{i=1}^{n} y_i \times p_i$ are mathematical expectations of x and y; x_i and y_i are i-th possible values of x and y; p_i is the probability of appearance of the i-th value of x or y.

In calculations of correlation coefficients only production sectors are taken into account in order to exclude the influence of nonproduction sectors (as technologically different) on the results of the analysis. The industries of the service sector are connected with almost all production industries.

The results of calculations of the maximum coefficient of the four correlation coefficients are used to construct a symmetrical matrix Lv for each pair of industries.

Each column of matrix Lv is a sample of interconnection between the industry in the column and all other production industries. The measures of direct and indirect inter-industrial connection calculated for each sector of national economy and presented in matrix Lv are considered as variables in factor analysis. The purpose of factor analysis is to find such complex factors that most fully explain the observed connections between the available variables.

In the general case the model of the described connection is a set of linear equations. The coefficients of such equations are so-called factor loadings that show the "weight" of each factor for any given indicator. In the matrix form the system of equations can be written as:

$$X = S \times F + E, \tag{9.16}$$

where X is the matrix of indicators (or variables), S is the matrix of loadings, F is the matrix of new "latent" variables, E is the matrix of residuals.

This equation describes the transition from primary variables (indicators) to new variables (factors). Such a transformation enables us:

(1) to identify variables defining the studied set of indicators, to analyze their number and type;
(2) to compress the data—instead of a great number of variables the system is described by a few factors.

At the first stage of factor analysis the given values of variables are standardized (z-transformation); then the standardized values are used to calculate Pierson correlation coefficients (r) between the considered variables:

$$r = \frac{n \times (\sum X \times Y) - (\sum X) \times (\sum Y)}{\sqrt{[n \times \sum X^2 - (\sum X)^2] \times [n \times \sum Y^2 - (\sum Y)^2]}}. \tag{9.17}$$

The initial element for further calculations is the correlation matrix. To construct a correlation matrix it is necessary to determine so-called eigenvalues and their eigenvectors, which are determined through the estimates of the matrix diagonal elements (so-called relative dispersions of simple factors). The dispersion eigenvalues are sorted in descending order; for this purpose all factors whose eigenvalues exceed 1 are taken. The eigenvalues corresponding to these eigenvalues form factors (F); the elements of eigenvectors are called factor loadings (S). They are correlation coefficients between the corresponding variables (X) and factors (F).

To solve the problem of factor determination, many different methods have been developed, the most widely used method being determination of the main components suggested by Pierson in 1901 and developed in detail by Hotteling.

The steps of calculation described above do not give an unambiguous solution to the problem of factor determination. Based on the geometric representation of the problem under consideration, the solution to the unambiguous problem can be found by the rotation of factors. The factors can be replaced by their linear combinations that are not mutually correlated and have dispersions equal to 1. This gives an infinite number of sets of factors satisfying the model.

The procedure of construction of a new set of factors is called orthogonal rotation of factors [26]. After rotation the factor model can be written as:

$$X_i = \sum_{j=1}^{m} c_{ij} F_j^{(R)} + e_i, \quad i = 1, \dots, p, \tag{9.18}$$

where $c_{ij} = \sum_{k=1}^{m} l_{ik} q_{kj}$, $i = 1, \dots, p$, $j = 1, \dots, m$, c_{ij} are loadings of new factors; l_{ik} are estimations of new loadings; q_{kj} are constants.

In order to obtain simple structures it is necessary to minimize the target function depending on the loadings of new factors [26]:

$$G = \sum_{k=1}^{m} \sum_{\substack{j=1 \\ j \neq k}}^{m} \left[\sum_{i=1}^{p} c_{ij}^2 c_{ik}^2 - \frac{\gamma}{p} \left(\sum_{i=1}^{p} c_{ij}^2 \right) \left(\sum_{i=1}^{p} c_{ik}^2 \right) \right] \to \min, \tag{9.19}$$

for $0 \le \gamma \le 1$.

To determine the structure of factors we used the "Varimax" method of rotation maximizing the dispersion of squared loadings for each factor, which increases large values and decreases small values of factor loadings: for $\gamma = 1$

$$\frac{1}{p} \sum_{j=1}^{m} \sum_{i=1}^{p} \left(c_{ij}^2 - \overline{c_{\cdot j}^2} \right)^2 \to \max, \tag{9.20}$$

where $\overline{c_{\cdot j}^2} = \frac{1}{p} \sum_{i=1}^{p} c_{ij}^2$, $j = 1, \dots, m$.

Based on the values of loadings, one can try to interpret the meaning of every factor.

As it is stated in [20], every cluster obtained as a result of analysis of the "input–output" tables consists of a set of primary and secondary industries. Primary industries in this group are the sectors that have maximal factor loading in this factor, the value of which is not less than 0.6. Secondary industries in this group are the sectors that have factor loading in the factor more than 0.35 and less than 0.6.

An important problem is the assessment of structural influence of the industrial cluster on the regional economy. For this purpose economists usually use regional models of general economic development, Regional CGE Models (Regional Computable General Equilibrium Models), a combination of matrix and econometric models [22]. As the basis of such a model, a matrix of financial flows (or a matrix of accounts for the analysis of social processes) is usually used. Under certain conditions regional CGE models enable economists to make prognosis estimations, which is especially important for both short-term and long-term planning. As a rule, to obtain assessments of strategic planning it is sufficient to have only matrix models such as the inter-industry balance ("input–output" tables) and a matrix of accounts for the analysis of social processes.

As mentioned above, the estimation of economic influence using "input–output" tables is based on Leontiyev's matrix equation, which enables us to estimate possible changes in the gross output and the influence of primary factors on a certain change in the final demand.

If ΔY is a vector of changes in the final demand, formula (9.13) can be rewritten as:

$$\Delta X = (1 - A) - \Delta Y. \tag{9.21}$$

Equation (9.21) gives the direct and indirect effects of the changes in the final demand on the gross output (at the expense of additional income, imports, etc.).

Extensive application of the "input–output" method abroad has led to formation of a number of standard operations.

As a rule, 3 types of multipliers are calculated for each sector of the economy [22, 23]:

(1) the multiplier of gross output shows the increase in the gross output in all spheres of the regional economy per unit of increase in the total volume of sales in the i-th industry;
(2) the added value multiplier shows the increase in the gross added value in all sectors of regional economy per unit of increase in the total volume of sales in the i-th industry;
(3) the multiplier of household incomes shows the increase in household incomes as a result of a 1 % increase in the total volume of sales in the i-th industry.

The calculation of multipliers for each product (a pure industry) of the regional economy is based on the determination of direct and indirect effects.

The direct effect is calculated as a ratio of the increment of the corresponding indicator in the industry (ϕ_i^j) to the change in the final demand in the industry (Y_i):

$$K_i^j = \frac{\Delta \phi_i^j}{\Delta Y_i}, \tag{9.22}$$

where i is the number of the industry; j is the number of the multiplier.

The indirect effect is determined as the change of the summarized value of the indicator in the other industries in the region ($\Delta \Phi^j - \Delta \phi_i^j$) divided by the change in the final demand in the industry:

$$H_i^j = \frac{\Delta \Phi^j - \Delta \phi_i^j}{\Delta Y_i}. \tag{9.23}$$

The numerical value of the total multiplier can be evaluated by summing the values of multipliers of direct and indirect effects [25].

The application of factor analysis to the "input–output" structure means reduction in the number of industries to a smaller number of industrial clusters [27–29], which most fully explain the observed connections between the variables of the "input–output" table.

Recommendations/requirements to the leading industries To have high LQP value and positive dynamics of this indicator; to export the major part of its products; to have a disproportionately high weight in the export of the region; to demonstrate growth of the weight of workers employed in the industry at the regional level. The industry drivers must make the core of industrial clusters of the region as they form the basis for the development of the regional economy.

The method of identification of industrial clusters was approbated on the example of the East Kazakhstan Oblast.

The results of estimation of EKO sectoral specialization enable us to conclude that 21 industries in the EKO oblast can be grouped into 7 clusters depending on the level of their competitiveness. Today the industry driver in the oblast is “production of non-ferrous metals,” and the developing driver is “production of machinery and equipment.” These industries are to form the core of the EKO potential industrial cluster as a cluster promoting acceleration of economic development of the oblast.

Based on the results of estimation of EKO sectoral specialization, we identified the mining cluster as the most promising for the development of the oblast economy. The members of the cluster were identified as a result of studying inter-industrial connections based on the “input–output” tables and factor analysis with the main components. These industries are “mining of metallic ores,” “metallurgy and metal working,” “manufacturing and repair of machinery, equipment and spare parts,” and “mining of coal and lignite, development of peat deposits,” with each of the industries forming a link in the price chain, adding its price to the end product.

References

1. Bester, C.A., Conley, T.G., Hansen, C.B.: Inference with dependent data using cluster covariance estimators. J. Econom. **165**(2), 137–151 (2011)
2. Porter, M.: Competition. Trans. From English/A Teaching Aid. Publishing House “Williams” (2001), p. 495
3. Kutyin, V.M.: Territorial economic clusterization (classification) of the regions in Russia: socio-economic aspect. Eurasian Secur. **1**, 525 (2003)
4. The 2003–2015 strategy of industrial innovation development of the Republic of Kazakhstan. Decree of the President of the Republic of Kazakhstan of 17.05.2003, No. 1096
5. Omarova Zh.I. (ed.): Science and Innovation Activities of Kazakhstan in 2008. Collection of Statistical Publications. Astana (2008), p. 79
6. Regions of Kazakhstan 2003. Collection of Statistical Publications. RK Agency on Statistics, Almaty (2004), p. 468
7. Abdiyeva, K.S. (ed.) Yearbook of Statistics of Kazakhstan. Collection of Statistical Publications. RK Agency on Statistics, Astana (2005), p. 502
8. Meshimbayeva, A.E. (ed.): Yearbook of Statistics of Kazakhstan. Collection of Statistical Publications. RK Agency on Statistics, Astana (2007), p. 516
9. Meshimbayeva, A.E. (ed.) Economic Activity of the Population of Kazakhstan. Collection of Statistical Publications. RK Agency on Statistics, Astana (2008), p. 256
10. Official site of the Statistics Agency of the Republic Kazakhstan. http://www.stat.kz
11. Savitskaya, G.V.: Theory Analysis of Economic Activity. INFRA-M (2007), p. 288
12. Duran, B., Odell, P.: Cluster Analysis. Trans. from English. Edited by Boyarsky A.Ya. Statistika (1997), p. 128
13. Ferrary, M.: Specialized organizations and ambidextrous clusters in the open innovation paradigm. Eur. Manag. J. **29**(3), 181–192 (2011)
14. Kim, J.O., Myuller, H.U., Klekka, U.R., et al.: Factor, Discriminative and Cluster Analysis. Trans. from English. Edited by Enyulova I.S. Finances & Statistics (1989), p. 215

15. Baizakov, S., Kalabayeva, A.: On the development of economic school of cluster analysis in Kazakhstan. Econ. Stat. **3**, 11–18 (2004)
16. Hill, E.W., Brennan, J.F.: A methodology for identifying the drivers of industrial clusters: the foundation of regional competitive advantage. Econ. Dev. Q. **14**, 67–96 (2000)
17. Turganbayev, E.M., Kozlova, M.V.: Assessment of sectoral specialization and identification of industries-drivers in the East Kazakhstan Oblast. Econ. Stat. **3**, 107–123 (2009)
18. Guideline on Cluster Development. Almaty: "Center of Marketing & Analytical Research", JSC (2006), p. 56
19. Rocha, H.O.: Entrepreneurship and regional development: the role of clusters. Thesis submitted in partial fulfillment of the requirements for a Doctor of Philosophy degree at University of London, London, 28 December 2004
20. Feser, E.J., Bergman, E.M.: National industry cluster templates: a framework for applied regional cluster analysis. Reg. Stud. **34**(1), 1–19 (2000)
21. Kharchenko, L.P., Dolzhenkova, V.G., Ionin, V.G., et al.: Statistics: Lectures. NGAEU, Novosibirsk (2000), p. 310, edited by cand. Economics Ionin V.G.
22. Tatarinov, A.: Subnational Accounts: Problems of Development and Use in the Regional Analysis. IEPP (2005), p. 163
23. Armstrong, H., Taylor, J.: Regional Economics and Policy. Blackwell, Oxford (2003), p. 437
24. Kozlova, M.V.: Formation and development of clusters as a means of ensuring the competitiveness of the economy of the region (by the example of the East Kazakhstan region). In: The Role of Universities in Building an Innovative Economy: the Materials of the International Scientifically-Practical Conference, Ust-Kamenogorsk, vol. 2, pp. 161–166 (2008)
25. Kozlova, M.V.: Methods of identification of industrial clusters. Vestn. Shakarim Semey State Univ. **3**, 64–70 (2009)
26. Afifi, A., Eisen, S.: Statistical Analysis: Computer-Based Approach. Mir, Moscow (1982), p. 488. Trans. from English
27. Bell, M., Albu, M.: Knowledge systems and technological dynamism in industrial clusters in developing countries. World Dev. **27**(9), 1715–1734 (1999)
28. Guo, J., Guo, B.: How do innovation intermediaries facilitate knowledge spillovers within industrial clusters? A knowledge-processing perspective. Asian J. Technol. Innov. **21**, 31–49 (2013)
29. Iammarino, S., Mccann, P.: The structure and evolution of industrial clusters: Transactions, technology and knowledge spillovers. Res. Policy **35**(7), 1018–1036 (2006)

Conclusion

One of the main issues—and opportunities—in economic development is higher management standards at every level. However, it is impossible to achieve high management standards and to make strategic and tactical decisions without twenty-first-century instruments based on solid mathematical models and methods.

The set of new economic-mathematical methods and models suggested in this book are structured as information support systems for making decisions in problems of budget control, economic efficiency of production systems, currency exchange, and assessment of investment projects and investment decisions.

An analysis of existing practices of budget process modeling reveals a lack of efficient mathematical models for making calculations and forecasts and controlling financial flows. In order to structure and understand functions and organizational budget systems, a unified budget classification system has therefore been established and models of budget knowledge representation have been constructed. A mathematical budget model based on the matrix of interaction of income and expenditure items is presented in the first two chapters of this book. The matrix elements are equal to the ratio of the expense vector elements to the income vector elements. The differential of the budget equation shows budget sensitivity to changeable (controllable) parameters. Statistical processing of the experimental data confirms that the model adequately describes factual budget performance. The model can be used in an arbitrary section of budget classification and in any discretization period. The method of program control is detailed here, along with models for correcting program control and assessing program decisions, the dynamic model controlling flows of budgetary funds, and the information system controlling financial budget flows.

Entropic estimation of the state of production system parameters makes it possible to estimate changes in parameters by a single relative indicator, and to synthesize such estimations into a unified economic image of the current production situation. In the real production conditions at a certain moment of time every value of the state of a controlled object corresponds to a certain value of entropy. As entropy is defined by a quadratic form, by knowing its value for a certain article one can determine the efficiency of its production with respect to other articles. Based on the mathematical statement on reduction of quadratic forms to the canonic form, the

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7

method developed in the third chapter can be used for comparison of several single-type productions. More exact values of quadratic forms mean that this approach is justified mathematically and is applicable for assessment of production systems.

Methods and mathematical models of processes on the currency exchange market are analyzed in the fourth chapter of this work. A mathematical model of balanced exchange rates is presented and analyzed, and balancing problems are formulated and solved. Collocation models presented in this chapter are universal and can be used to solve tasks of exchange market forecasting. Information about expected exchange rates can be obtained by extrapolation. A high degree of information justification makes it possible to identify closed sequences of currency purchase–sale operations giving speculative profit. To facilitate the adjustment of exchange rates by second-level banks, an information system for supporting decision-making is also presented.

To improve the quality of assessment of innovation projects, existing methods of project assessment are analyzed in chapter five. Criteria and methods for assessing innovativeness and competitiveness are developed, along with a graphic model allowing visualization of project assessment in the coordinate scale of the matrix model. Innovation projects being objects of two interacting segments—science and business—are formalized as two-dimensional objects with the dependence $K = f(I)$, where K is competitiveness and I is innovativeness. The chapter establishes a method for estimating realizability and economic efficiency of innovation projects and presents a graphical model based on indicators of pure reduced cost, internal profitability rate, pure profitability index, and payback periods. This model facilitates complex project assessment on the basis of absolute positioning. The corresponding decision support system provides a program-targeted approach leveraging complex expertise in project assessment by such parameters as innovativeness, competitiveness, and economic efficiency. This decision support system is designed to be utilized by expert commissions responsible for venture funds, development institutes, and other potential investors needing to select appropriate innovation projects.

Chapter six explores the methods and mathematical models used to make investment decisions, which form a complex methodology for assessment and choice of multi-dimensional investment project alternatives. Today's financial management is characterized by active implementation of investment projects where it is necessary to forecast not only time structure of payments and their concrete sums but also probabilities of possible deviations from the expected results—that is, to estimate the degree of risk. Computer and measuring support of mathematical modeling expands the possibilities of practical application of the methods and models suggested here.

Chapter seven studies the multi-objective stochastic decision-making models on resource allocation. Constructing methods and models for the distribution of resources is a rather important direction of modern science. Under the condition of incomplete information, combined target functions, built on the classical principles of choice, are used for the analysis and simulation of the distribution of productive and investment resources for regional and industry development.

Mathematical methods and models for monitoring government programs are considered in chapter eight. Such methods and models are aimed at improving the efficiency of the implementation of governmental programs and the transparency of their execution, and also increasing the efficiency and validity of management decisions by the program administrators. The implementation of management technologies to monitor the realization of the governmental housing program will let us improve the evaluation of performance costs, that characterize the achievement of governmental policy aims by the administrator during the process.

The final chapter studies methodology for the identification of competitive industrial clusters. The modern development of Kazakhstan has a distinctly regional context. There is a need to cluster regions of the country according to similarity in economic development, in order to create a dual regional policy, that takes into account differences between groups of regions (regional clusters) and aims at developing not only separate regions and regional clusters, but also the whole territory of the country. The combined technique of Kazakhstani regional cluster analysis consists of assessing the economic level of development and identifying industry-drivers in the region, developing on the level of competitiveness. This constitutes the kernel of a potential industrial regional cluster, and accelerates the pace of economic growth of the area.

Thus, the mathematical models and methods proposed in the book are effective mechanisms of forecasting, synthesis and analysis, and support management in the appropriate spheres of application of economic industry.

About the Author

Galym Mutanov graduated from the Kazakh Polytechnical University where he specialized in "Automatics and Telemechanics" and received two years of scientific training at Moscow Institute of Steel and Alloys. Mutanov took post-graduate and doctorate courses at Moscow State Mining Institute. His Candidate and Doctoral dissertations were devoted to the theory and practice of automated control of technological processes with use of elements of artificial intelligence. Mutanov is the author of over 400 scientific publications, monographs, textbooks, manuals, and inventions in modeling and control of social and economic processes. His works have been published in the USA, Czech Republic, Sweden, Greece, and Switzerland. Mutanov has trained more than 40 doctors and candidates of sciences.

Galym Mutanov has a State Award—order Parasat. He is an Honored Worker in science and technology of the Republic of Kazakhstan, Honored Worker in education, Honored Engineer of the Republic of Kazakhstan, and a full member of international academies. His awards include the grant from the Kazakh Academy of Sciences for outstanding scientists, a silver medal from the International Bibliographical Center (Cambridge), the gold medal "The United Europe" (Oxford), "Name in Science" International medal (Oxford), Medal "For outstanding achievements in science and innovation" of Lisbon Higher Institute of Engineering (Portugal), and Chevalier L'Ordre des Palmes académiques (Ministry of Education, France). Mutanov is a member of the Monitoring Committee of the International Society on Engineering Pedagogics (IGI, Freeburg), President of the Central-Asian IGIP Branch, a member of the UNESCO Steering Council on Information Technologies in Education (Paris), a member of International Association of University Presidents (IAUP), a member of Econometric Society and a member of Royal Economic Society, Pres-

G. Mutanov, *Mathematical Methods and Models in Economic Planning, Management and Budgeting*, DOI 10.1007/978-3-662-45142-7

ident of Kazakhstan Society for Engineering Education (KazSEE) of International Federation for Engineering Education (IFEES).

In 2010 Mutanov was appointed rector of Al-Farabi Kazakh National University.