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Gregg Jaeger

Quantum Objects

Non-Local Correlation, Causality and Objective Indefiniteness in the Quantum World

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Non-Local Correlation, Causality and Objective Indefiniteness in the Quantum World

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To my parents.

Preface

The conceptual elements of quantum theory that now underlie our picture of the physical world include objective chance, quantum interference, and the objective indefiniteness of dynamical quantities. Quantum interference, which is directly observable, was readily absorbed by the physics community. Objective chance and indefiniteness, being of more philosophical significance, gained acceptance only after much debate and conceptual analysis, when it was recognized that observed phenomena are better understood through these notions than through older ones or hidden variables. Of the results understood via these notions, the failure of quantum systems always to obey the constraint of Bell's inequality and its testable successor, the CHSH inequality, has been the most decisive. These inequalities are now recognized as central results not only of quantum theory but of physics as a whole. They concern the strength of correlations between properties of physical systems that are expected given the finiteness of the speed of light and common experience but are found not to encompass all of those found in the extended range of experience recently attained by science, which now reaches far beyond the familiar. These new observations breached the conceptual core of physics in a way that decades of previous arguments to the effect that Quantum mechanics gives rise to paradox had not, because non-local property correlations were shown to conflict with the deepest classical mechanical notions rather than just explaining particular anomalies. The success of quantum theory strongly urges us to accept the notions mentioned above and others which require the modification of our conception of what a generic physical object *is* and how it can *be*.

Earlier discussions and surprising results of quantum theory did not lead to such a thoroughgoing questioning of cherished notions regarding the content and structure of the world, being typically limited to processes occurring at atomic scales. Those previous results conflicted primarily with common sense or showed only the inadequacy of particular laws or rules rather than of our basic notions regarding physical existence. It is for this reason that the question, for example, of which position was victorious in the Bohr–Einstein debates over the viability of the then newly formulated quantum theory was and is not widely considered as significant, for example, as that between the Ptolemaic and Copernican positions.

However, the transition to the quantum world picture must be seen as at least as significant as the transition to Copernican cosmology as grounded in Newtonian mechanics or to the acceptance of biological evolution at the species level.

In the wake of the Bohr–Einstein debate, a more penetrating conservative response to those physicists who were content with early quantum theory was mounted by Einstein, Podolsky, and Rosen (EPR). The argument was relatively more philosophical than previous ones and made explicit several conditions that had arguably been implicit in the foundation of classical theory; it pointed out a conflict between these conditions and behavior predicted by Quantum mechanics for a pair of particles in a specific quantum state. The argument of EPR later led John Bell more or less directly to his inequality, because it pointed directly to an example of classically inexplicable behavior within an extended bipartite physical system: The Bell inequality quantitatively demarcated a boundary of classically explicable behavior that is trespassed in Quantum mechanics, providing a generalizable distinction between classical mechanical and extreme quantum mechanical correlations. It also proved less obscure to many than both the EPR argument and the reasoning of Bohr by making no use of terms such as *reality* or *phenomenon*.

The EPR paper itself had already prompted Schrödinger to produce his own touchstone articles on the predictions of quantum theory and its relationship to space-time, which also included his famous cat example and for the first time clearly defined and named the characteristic feature associated with distinctly quantum correlations: entanglement. Even before the EPR article, Bohr and Heisenberg had begun emphasizing the limitations inherent in the joint specification of physical quantities in the quantum world that challenged those naive forms of metaphysical realism which could comfortably be held together with classical physics. Through a series of incisive inquiries, beginning with this earlier work and leading to experimental testing of the validity of Bell-type inequalities, it became entirely clear to physicists that the novelties of quantum theory involve more than the discreteness of basic physical quantities and differences in predictions on the merely technical level, for example, of atomic spectral distributions.

Attempts to produce a satisfying corresponding worldview began with their appearance and have continued, with quanta being increasingly central to this picture. The investigation of the underlying characteristics of quantum systems—the pervasiveness of property indefiniteness and extremely strong property correlation at non-local spatiotemporal separations—have also provided a context for the emergence of quantum information science, to which some now look for a new quantum perspective. The associated questions of the implications of quantum mechanics for the conception of physical objects in space-time and of the form of causation appropriate to the quantum world are the primary subjects of this book.

Lexington, USA Gregg Jaeger
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Chapter 1
Quantum Theory and Locality

Abstract The relationship of quantum theory and space-time theory is clarified in historical context. In particular, non-local property correlation is described in relation to various notions of locality. This lays the ground for a search for the ontology most appropriate to quantum theory and the role and nature of causation in quantum theory, where Einstein's metaphysical views, the EPR argument, Gleason's theorem, and the Bell-type inequalities play key roles. The fundamental principles of quantum theory, such as the Superposition principle and the Born rule, and the relevance of the theory of communication to quantum theory are also explained. It is shown that various presumptions about the above relationships are unwarranted, the most significant of these being the presumption that the failure of correlations between properties of quantum systems always to have local explanations, in Bell's sense of locality, requires a rejection of metaphysical realism.

The world view offered by science was repeatedly challenged in the twentieth century by incisive physical and philosophical investigations. Among the first issues to emerge during the resulting physical and philosophical course corrections, which were driven by the discovery of specifically relativistic and quantum phenomena, is the question of the relationship between measurement outcomes and the physical objects to which the associated values are assigned. A significant portion of what is now known as *the interpretation of quantum mechanics* developed from this question; in its narrowest sense, *interpretation* indicates providing meaning to the elements of a theoretical formalism, its predictions, and its relationship to its referents and their properties [154].

Quantum physics was initially distinguished by its allowing values of, and so changes in, a number of basic physical quantities only in discrete amounts, for which it gives theoretical predictions involving the unit of action $\hbar$, for example, for energy. This enabled the solution of a number of puzzles surrounding the behavior of electromagnetic radiation and the stability of atomic matter. These solutions were breathtaking demonstrations of the ability of physics to explain how the objects of everyday life can persist and appear the way they do—from explaining how tables

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and chairs remain solid to explaining why the sky appears blue on a sunny day. Moreover, quantum theory enabled an understanding of the structure of the atomic nucleus and the properties of the many elementary particles, extending the scope of physics to length scales far beneath those it had ever non-speculatively ventured before, doing so with unprecedented precision and destroying old conceptions of the world of the very small. The clarification of the relationship of submicroscopic objects to the larger ones, with which we are more familiar, and the extent to which the former are identifiable parts of the latter, that is, the extent to which an ontological reduction of known objects to elementary particles can be coherently accomplished has emerged as an important task. Aspects of this task are addressed here. In particular, various steps toward and challenges to its accomplishment are explored, generally and then more specifically.

It is assumed in the following that quantum theory is applicable in all physical situations, including those considered here. Nonetheless, as Gerard 't Hooft has commented, "Modern theories of matter and forces at the most fundamental level require that we subject the fabric of space and time…to the rules of quantum mechanics as well. Direct attempts to do this were always bound to fail. The gravitational force is non-renormalizable, and this means that the language we use to describe the structures at the tiniest scales for time and distance must be inappropriate" [304]. Let us therefore not forget that the joint applicability of quantum theory and relativity in their current forms must ultimately be limited. Relativity, which is an approach to space-time theory that correctly describes physical phenomena at speeds and gravitational strengths orders of magnitude above those which had ever before it been non-speculatively treated, was nearly as revolutionary as quantum theory. Like relativity theory, quantum theory brought into question basic metaphysical assumptions about the relationship of light and matter, some of which had gone unchallenged by the Special theory of relativity. It also brought into question assumptions regarding the relationship of physics to human experience and human knowledge. The work of Albert Einstein was central to the emergence of both of these powerful, fundamental theories. Einstein was aware of the philosophical significance of quantum theory and explicitly explored it in less well known but invaluable contributions to the development of physics. His views and concerns continue to play an important role in the clarification of the nature of the quantum world, as they do here.

The most important tool for providing answers to the question of the relationship of quantum theory to the physical world was provided by Max Born, through the rule for which he received a Nobel prize. Born's rule provides probabilities in terms of the complex-valued components c_i of the quantum-physical state-vector $|\psi\rangle$: $p_{|\psi\rangle}(a_i) = |c_i|^2 \equiv c_i^* c_i$ when $|\psi\rangle$ is written in the eigenvector basis $\{|a_i\rangle\}$ of A. Depending on the interpretation of the quantum formalism followed, this vector is taken to serve either as the mathematical representative of the state of being of a quantum object providing the probability distribution $p^A_{|\psi\rangle}$ of the allowed values to be found as outcomes of genuine measurements of the relevant set of physical magnitudes A of the system, or as the representative of an observer's state of belief regarding the system. Here, it is taken to serve as the former. The Born rule is the most secure of the postulates of Quantum mechanics; it is crucial to almost all accepted and proposed alternative understandings of quantum theory.[1] The other fundamental principle provides the state space of the quantum system, such as the complex-valued vector space known as Hilbert space, $\mathcal{H}$.

Superposition principle: Any linear combination of system states is a possible system state.

A fortiori, the sum $\sum_i c_i|\phi_i\rangle$ is an allowed physical state when the states $\{|\phi_i\rangle\}$ are taken from a set $\{|\phi_j\rangle\}$ that constitutes a countable basis for $\mathcal{H}$. For example, the complex linear combination of the eigenstates $|E_i\rangle$ of the energy

$$|\psi\rangle = \sum_i c_i|E_i\rangle \ , \tag{1.1}$$

with the c_i being a set of complex numbers the squared moduli $p_i = |c_i|^2$ of which sum to unity (in accordance with a basic requirement for a well defined probability measure), of a system acting as a quantum simple harmonic oscillator (such as an atom within a diatomic molecule) is a valid state. It is noteworthy that for the simple harmonic oscillator only special sums (coherent states $|\alpha\rangle$), rather than individual discrete energy states $|E_i\rangle$, exhibit oscillations closely resembling those of a classical oscillator (cf., e.g. [251], pp. 96–97). This marks an important way in which Quantum mechanics predicts a broader set of behaviors than does Classical mechanics, which one would like to reduce to the former, at the very least in appropriate limits.

Even though quantum theory has several versions, e.g. the quantum mechanics of discrete systems and quantum field theory,[2] and though the reduction of classical theory to it has not been rigorously demonstrated, none of the predictions of Quantum mechanics have been shown to be clearly false.[3] Instead, or more precisely for that reason, its greatest difficulties appear in relation to the interpretation of its formal structure and its relation to space-time theory, the latter also being a very powerful theory and empirically well supported. In addition, the most important problems related to these, such as how best to understand the nature and process of quantum measurement, are shared by both the non-relativistic and relativistic versions even though the ways quantum objects are represented in these two cases are quite different.[4]

Not surprisingly, Einstein was among the first to consider explicitly the question of how Quantum mechanics is best interpreted. He divided the possible interpreta-

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[1]Nonetheless, it continues to be probed, cf., e.g. [156]. Here, *Quantum mechanics* with capital *Q* designates the standard theory of quantum mechanics, such as in the Dirac–von Neumann formulation summarized in Sect. 1.2.

[2]Here, the former will be called simply Quantum mechanics.

[3]Within the appropriate context in relation to relativity, of course. For a recent attempt to find fault, see [320].

[4]They each also have issues arising from their own peculiarities such as the appearance of infinities in the latter.

tions into two distinct classes: (i) those taking the quantum state $|\psi\rangle$ as complete in the sense of fully describing the physical state of the individual object, and (ii) those taking it as incomplete and therefore, at best, describing only ensembles of objects. In addition to being uncomfortable with the idea that a fundamental physical theory should provide predictions which are essentially probabilistic, Einstein questioned the consistency with modern space-time theory of Quantum mechanics under any individual-system interpretation, because the latter requires objective chance and predicts the existence of strong distant property correlations which he called "spooky."

This central interpretational distinction can be seen as corresponding to that of whether: (a) the quantum probabilities are *reducible*, that is, arise only from a failure to capture precisely somehow fully pre-determined behavior of quantum objects as elements of collections with which they can be associated by common preparation, or (b) their probabilistic behavior is not reducible in such a way. Closely related to this distinction is another one regarding the interpretation of the state $|\psi\rangle$: whether to understand it primarily epistemically or primarily ontologically. The former position views the probability $p_{|\psi\rangle}(a_i)$ in quantum theory as arising at least in part from a lack of knowledge of the present or future property A of the object, whereas the latter position views it as due to the indefiniteness of the property itself. Einstein held that, to be considered complete, fundamental physical theories must be both (1) local (in a specific sense) and (2) objective (realist). He thought the former feature necessary for the existence of distinct physical objects to be describable by physics in the first place; the latter feature is necessary in order to ensure that the lack of descriptive detail is not masked by, in some way, having the theory describe quantities other than those of its objects alone and is an assumption long accepted in physics. After Einstein found that the extraordinarily strong correlations between properties of spatially separated systems sometimes predicted by Quantum mechanics brings these requirements into question, he together with Boris Podolsky and Nathaniel Rosen (as EPR) disputed the completeness of the theory. The EPR trio argued that Quantum mechanics is incomplete on the grounds that the assumption to the contrary, that the theory is complete, brings it into conflict with a set of fundamental ideas about the nature of physical objects with which relativistic classical physics closely accords [92]. These ideas are related to the above two conditions (1) and (2) which Einstein took as essential to physics itself and so as prior to Quantum mechanics.[5]

A fundamental mathematical result later produced by Andrew Gleason demonstrated that Quantum mechanics *is complete* in an important sense: the probability measures compatible with Quantum mechanics are restricted more or less precisely to the $p_{|\psi\rangle}(a_i)$ of Born's rule [121]. It was followed by results of John Bell

[5]More broadly, Einstein believed that the invariants of the scientific approach are: (i) "The truth of theoretical thought is given exclusively by its relation to the sum total of [the experiences of sense perception]", (ii) "All elementary concepts are reducible to space–time concepts," and (iii) "The spatiotemporal laws are complete" [91].

who, by assuming a condition on joint probabilities now known as *local causality* which was itself inspired by the EPR assumptions, provided—in one of the most cited articles of modern physics—an inequality regulating correlations between spacelike-separated events regarding parts of bipartite systems such as EPR had considered [15].[6]

Following Bell's initial theoretical work, John Clauser, Michael Horne, Abner Shimony, and Richard Holt (CHSH) derived a closely related inequality,

$$|S| \leq 2\,, \tag{1.2}$$

where S is a simple combination of expectation values of four correlated outcomes of measurement events involving such systems, that could be and was used in *practicable empirical tests* of local causality and that does not assume perfect correlations between the distant events involved, as Bell's theorem does [60]. Such inequalities, now collectively referred to as *Bell-type inequalities*, precisely specify the extent to which models satisfying Einstein's criteria[7] that might serve as alternative theories of quantum mechanics constrain the set of distant correlations between properties of such pairs of systems. Experimental results of Alain Aspect et al. and later workers, which left only minor loopholes to be fully closed and which have since been further supported by subsequent experiments, confirmed with a high degree of confidence that, in some situations, $|S| > 2$: The CHSH inequality is violated and strong "non-local" correlations do exist in nature. They also always accord with the predictions of Quantum mechanics, contrary to common sense and classical mechanical intuition [3]. These results imply that, if the world it describes is objective—assuming that a radical break from current notions of space-time itself is not made—then the physical world picture of quantum theory is one in which the properties of objects are, in general, objectively indefinite and ruled by objective chance.

Given the lack of empirical support for any particular radical revision of space-time theory, the approach of this book is *to accept* interpretative position (ii) *and to reject* position (i) in the senses they were understood by Einstein, given the results of Gleason and Bell and the above later experimental results. Two of its goals are to describe the world picture that results from such an approach and to relate it to historically important and intuitive physical notions, which now must be weakened to remain adequate. It develops alternatives to the approach taken by EPR that satisfy less strict criteria for a realist physics than the ones chosen by them. One finds that naturalistic metaphysical and epistemological assumptions about the physical world remain acceptable because they remain compatible with Quantum

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[6] Here by *event* is meant an occurrence in a particular location, represented by a point in space-time, as it will here in the sequel; an explicit definition of *local causal*, also know as *Bell locality* is given in Sect. 1.4, below. Note, however, that in General relativity an event is defined as an intersection of world lines.

[7] That is, that assume 'local causality,' which is defined below.

mechanics and experience as manifest in experiment. A noteworthy fact in this regard is that the formalization of Quantum mechanics of John von Neumann paid close attention to causality and the principle of psycho-physical parallelism, namely, that subjective and objective phenomena remain, at a minimum, consistent with each other.[8] The formulation of von Neumann presented in the notation of Dirac, some naturally developed metaphysical insights of Werner Heisenberg relating to state change, and subsequent results in elementary particle physics and field theory are our main tools here.

The metaphysical notion of substance, which had long been used by physics by virtue of its assumed association with matter, is now rarely discussed in relation to the interpretation of quantum theory. However, the notion of particle traditionally but not necessarily associated with it is. As Paul Teller has pointed out, "Interpreters of quantum theories almost never address the idea that particles, as material objects, are thought of as substantial. Many of us, in our prequantum thinking, think of particles as composed of bits of substance, vaguely thought of as 'stuff' in which properties can inhere" ([303], p. 10). Unlike the notion of substance, some notion of the causal particle itself remains relevant because as seen in the sequel an identity can be attributed to the small physical objects of quantum mechanics which, under certain circumstances such as those of Compton scattering, can be seen causally to influence each other; the causal particle notion remains central to our understanding of quantum physics, even though it must be weakened in order to remain generally applicable. As Brigitte Falkenburg has noted,

> Modern physics rests on the assumption that phenomena consist of substances with causal powers. Based on this assumption, the experimental method pursues the mereological and causal analysis of phenomena with the goal of finding these substances and powers. By the way, these substances are called *particles* in the universe of discourse constructed by the founders of modern physics. ([95], p. 37)

However, like that of substance, the traditional notions of part and cause are seen in light of the results of quantum theory and of experimental micro-physics described above to be too strong to be universally applicable to the quantum world. The elementary particles that have been discovered are no longer best thought of as substantial (in the philosophical sense of the word) as "atoms" had before been (in their various guises and at various times throughout the centuries). Rather, they are essentially a co-presence of properties, which in quantum theory are distinguished by their obeying space-time symmetry requirements, and can come into and go out of existence.

In quantum theory there are tensions, not present in the classical context, between different manners of describing physical systems, such as the spatiotemporal, wherein they are associated with trajectories, and the 'causal,' where momenta are primary, as famously argued by Niels Bohr. Therefore, only a considerably weakened particle notion can apply in the quantum context. The elementary particles of modern physics cannot consistently be considered particles in the traditional sense

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[8] See [154], Sect. 2.5.

because, for example, they are not inherently localized as the particles of Classical mechanics are and, in the relativistic regime, cannot be localized for more than an instant—even then, this localization is only so in the measurer's frame of reference.

A degree of localization is, of course, necessary for a space-time trajectory to be well defined but need not be absolute. Understanding this already in 1934, the year before the appearance of the EPR paper, Einstein remarked

> I still believe in the possibility of giving a model of reality, a theory, that is to say, which shall represent events themselves and not merely the probability of their occurrence. On the other hand, it seems to me certain that we have to give up the notion of an absolute localization of the particles in a theoretical model. [89]

As a result, the ability to individuate quantum systems via precise space-time trajectories is generally unavailable.[9] Subsequent results militate against Einstein's hope that physical phenomena might all be local and deterministically described; objective chance has since been found to be required in order that events themselves be described in the realm of quantum theory.

For those who, like Einstein, require a realist understanding of the formalisms of Quantum mechanics and Quantum field theory, objective irreducible probability is thus a necessary element of a world picture consistent with physics. Quantum objects are best understood as both governed by irreducible chance and dependent on the details of their environments, that is, on other systems in a way that is qualitatively different from classical physics. Einstein's interpretational rival Bohr, who was first to use successfully the quantum discreteness of physical quantities to model the structure of the atom, was also the first to propose a radically new interpretation of mechanics. Bohr's view emphasized the tension between spatiotemporal and causal descriptions in quantum theory rather than attempting to avoid them. He also emphasized that relating the quantum formalism to experience, that is, to observed events involves reference to system *preparation and measurement processes*. Bohr stressed what he called "the inability of the classical frame of concepts to comprise the peculiar feature of indivisibility, or 'individuality,' characterizing elementary processes" while also arguing that these concepts are necessary for obtaining knowledge of those processes [31]. By contrast with events, physical processes must occupy *regions* of space-time because, for example, at a minimum they involve more than a single moment of time (Fig. 1.2). Preparation and measurement processes are, one finds, the essential elements of signaling in the theory of communication. Because communication can be viewed as the establishment of correlations between the preparer and measurer of signal-bearing systems, information-theoretical concepts also naturally play a role in the analysis of contemporary physics. Indeed, various attempts to provide a foundation for quantum theory from *within* information theory have been made in recent years.[10]

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[9] Contemporary results bring into question even such modest substitutes for particle trajectories as space-time tubes, as seen in Chap. 4.

[10] See [154], Chap. 3. Arkady Plotnitsky has argued that Bohr's approach to Quantum mechanics had already involved an informational dimension, see ([223], p. 43 and quotes therein).

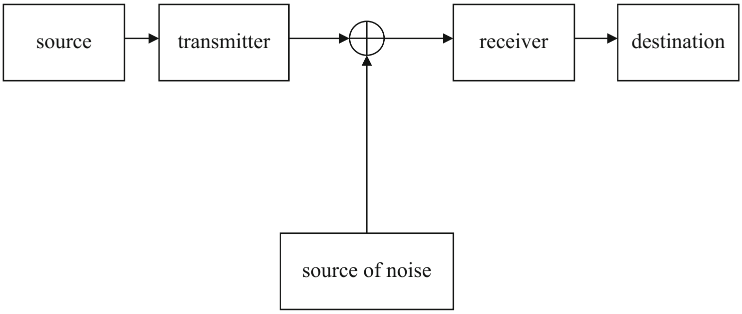

Fig. 1.1 Schema for a standard communications system, which can be implemented in space-time. Signals are prepared and sent from source at location s_{source} to destination at location $s_{\text{destination}}$ at which they are measured, being transmitted by a transmitter at the former to a receiver at the latter over some time interval Δt. These signals also typically encounter environmental noise in the process [274]

Communications signals have typically been electromagnetic. The *Nahwirkungsprinzip*, that is, principle of locality requiring that causal influences propagate continuously from point to point was introduced by Michael Faraday in the context of electromagnetism and helped James Clerk Maxwell in formulating electromagnetic field theory as a set of partial differential equations for field strength, greatly influenced Einstein's thinking, with positive effect in the case of Relativity (cf., e.g., [126], Sect. I.2). The expression "spooky action-at-a-distance" [*spukhafte Fernwirkung*] to which allusion was made above, was used by Einstein to characterize the particular sort of non-local property correlations predicted by Quantum mechanics which he found unacceptable in foundational physics. However, the sort of unlocalized process that is involved in the quantum mechanical processes which Einstein called "spooky" is a very specific one which does not directly conflict with locality in the sense required by Relativity. The probabilistically regulated results of quantum measurements specified by the Born rule in the context of the complex-vectorial nature of the quantum state do not allow for signals, electromagnetic or of any other kind, to be produced that could support superluminal communication, that is, does not allow *controllable influences* violating this sense of locality. Hence, this spook, which is a property of the quantum state rather than of a physical influence, is less frightening than might be first thought, just as are most spooks, after they are fully investigated. Indeed, dropping the unjustified assumption that *all* physical states *whether their implications for phenomena are probabilistic or not* must exhibit correlations which change only locally removes a great deal of discomfort over the quantum correlations predicted between distant systems and observed in the laboratory.

The requirements on signaling, that (a) the process of signaling should allow for the realization of communication tasks consistently with fundamental information-theoretic categories and (b) superluminal signaling be precluded on physical grounds, have been essential to the investigation of the mechanics of systems violating property locality in the sense assumed in Bell's and related derivations. There have recently been attempts to find simpler conditions or different formulations that uniquely distinguish Quantum mechanics from among possible physical theories capable of describing non-local correlations from the information-theoretical perspective, where physics plays only an indirect role. Indeed, contemporary investigations of the foundations of quantum mechanics often look beyond physics proper by examining physical situations also as information-scientific ones, cf. [154].

The appearance of these recent analyses emphasizes the need to reassess the now often-ignored relationship of quantum theory to space-time first made evident in the work of EPR and Bell, which focus on correlations which may occur at non-local separations. Bell emphasized the significance of the relationship between quantum theory and space-time theory as follows.

> For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory. . . ([17], p. 172).

Let us begin by considering the various ways that these two pillars have been connected in foundational investigations of quantum theory and the sources of tension between them in order assess whether the tension which is the source of Bell's concern could pose a genuine threat to realism, as has often been suggested.

1.1 Physical Systems and Space-Time

Einstein was concerned about the implications of the strong correlations predicted by Quantum mechanics between properties of extended composite physical systems when they are in certain states now called *entangled states*.[11] His concern lay not only in relation to the requirements of Relativity but at the metaphysical level: He believed that the adoption of Quantum mechanics would preclude any realist conception of the physical world. In the 1935 EPR article, Einstein and his collaborators argued that Quantum mechanics must be an *incomplete theory* given what they viewed as essential realist preconditions which they presented in the form a set of basic fundamental criteria later taken to define "local realism."

As indicated above, the EPR argument was made in a historical context in which Quantum mechanics was thought to be an adequately formalized, fundamental theory. The need for such an argument was to a great extent due to von Neumann's successful capturing of the mathematical essence of previous formulations of the theory together with the success of Bohr in defended his "Copenhagen" method of applying the theory to various challenging "thought experiments." Bohr had this

[11]The mathematical relationship involved here is specified in Sect. 1.3

success in a range of challenging situations Einstein put to him in discussions during the Solvay conference of 1927, raising the philosophical stakes for this alternative to Einsteinian realism [8]. As Einstein's saw it, Bohr's physical and rhetorical achievement had led to an unhealthy complacency in the face of both the tension between Quantum mechanics and Relativity and the growing prospect of a fundamental, non-epistemic probability in physics.

Relativity, like Quantum mechanics, concerns motion but, unlike it, did not as fundamentally challenge the previously assumed metaphysics of matter. For example, there are no inherent restrictions on the simultaneous definiteness of properties of physical systems or of space and time in Relativity which would be analogous to the Heisenberg "uncertainty" relations. The latter include that for position and momentum: for a collection of quantum systems each prepared in the state $|\psi\rangle$ one finds, for example, in one dimension

$$(\Delta X)_{|\psi\rangle}(\Delta P)_{|\psi\rangle} \geq \hbar/2\ , \tag{1.3}$$

$\Delta(\cdot)$ being the square-root of the dispersion, which is given by $\mathrm{Disp}_{|\psi\rangle}(A) \equiv \langle (A - \langle A\rangle \mathbb{I})^2\rangle_{|\psi\rangle} = \langle A^2\rangle_{|\psi\rangle} - \langle A\rangle^2_{|\psi\rangle}$ where the angle brackets indicate the expectation value, X being the position operator and P being the momentum operator.[12] However, in light of the explanatory power of Relativity, which had eliminated deep conceptual difficulties present in the Newtonian theory of gravitation and in the propagation of electromagnetic waves in Newtonian space-time, all mechanics was expected to be local, that is, the light-speed constraint on propagation velocities was expected to apply to all relations between physical events, including those involving signals. Furthermore, the centuries of success of the apparently strictly deterministic theory of Classical mechanics had relegated probability in physical theory to a secondary role: because previous probabilistically specified states had been shown to be reducible to underlying non-statistical states, the newly introduced such states in quantum theory were assumed to be so as well.

Einstein believed that a fundamental physical theory ought to be captured by a set of basic axioms and should be required by virtue of its fundamental role to provide a full description of independently existing localized objects that obey Relativity. Thus, for him, if Quantum mechanics were taken to be a theory of individual objects then it could not be a fundamental one, because of the extraordinary property correlations it allows between distant objects: In a number of situations, Quantum mechanics predicts that properties shared by two systems will be fully correlated, even while being indeterminate in value, *independently of their space-time separation*.

Einstein, Podolsky, and Rosen argued that Quantum mechanics must provide an incomplete (local) description of the system constituted by two systems, no matter how accurate are its (typically statistical) predictions because, in some states, for

[12] Operators on Hilbert space, a theoretical novelty introduced to physics with the introduction of Quantum mechanics, are discussed in detail in the next section.

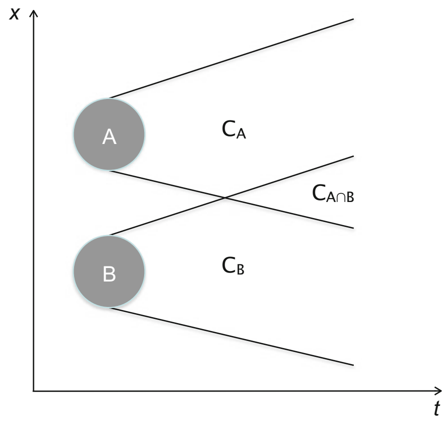

Fig. 1.2 Distinct space-time regions A and B with the regions C_A, C_B, respectively formed by the union of the light cones of the individual event-points within them, and the region of their overlap, $C_{A\cap B}$. The spatial location is indicated by x and the time axis t extending rightward into the past toward earlier values of t. Note, that their intersection includes no point of either A or B ([17], Chap. 7)

both of two incompatible properties once a property of one subsystem becomes determinate (e.g. upon measurement) the same property of the however-distant other subsystem will also instantaneously become determinate, whichever property is measured.[13] The conception of physical properties in space-time which underlies his standard of theory completeness is closely tied to his metaphysical views. His notion of physical property is a straightforward one based on the requirement of the separability of localized independent objects and is applicable, for example, to a particle conception of light and matter such as that of elementary particles capable of free motion, to a field ontology wherein properties are associated with normal modes or space-time points, and to a combination of the two (Fig. 1.2).

Einstein's physical world picture is reflected in the following comment regarding spatially extended composite physical systems, from a letter to Born.

> We all of us have some idea of what the basic axioms of physics will turn out to be. . . whatever we regard as existing (real) should somehow be localized in time and space. That is, the real in part of space A should (in theory) somehow 'exist' independently of what is thought of as real in space B. When a system in physics extends over the parts of space A *and* B, then that which exists in B should somehow exist independently of that which exists in A. That which really exists in B should therefore not depend on what kind of measurement is carried out in part of space A; it should also be independent of whether or not any measurement at all is carried out in space A. If one adheres to this programme, one can hardly consider the quantum-theoretical description as a complete representation of the physically real. ([34], p. 164)

For Einstein, the ontology of field theories, such as that of General relativity, was considered unproblematic in this regard despite the inherent unity of the field in some ways of conceiving it.[14] He viewed the *Principle of contiguity* obeyed by them

[13]Einstein's views on the question of the interpretation of Quantum mechanics to this effect have been carefully argued, for example, by Fine ([107], p. 61).

[14]See Sect. 2 of [143] for a discussion of this as contrasted with the quantum theoretical case. More about the different ways of approaching quantum fields appears below.

as enforcing this independence and believed that it, or something similar, should hold also in the quantum context.

> The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B; this is known as the 'principle of contiguity,' which is used consistently in the field theory. If this axiom were to be completely abolished, the idea of the existence of (quasi-) enclosed systems, and thereby the postulation of laws which can be empirically checked in the accepted sense, would become impossible. ([34], p. 171)

Concerns about the unusual strong correlations between distant events involving subsystems of composite systems were relatively easily dismissed early in the history of quantum theory; at that time, one could still be reassured by the accumulation of centuries of empirical successes using Newtonian gravity, with which Quantum mechanics is more easily combined. Moreover, relativistic quantum theories such as Dirac's theory of the electron, although exceptionally powerful, had seemingly unacceptable consequences, such as an infinite 'negative-energy sea,' and seemed to preclude particle localization.[15] However, as the list of successes of Special relativity in nuclear physics and of General relativity also grew, the issue of the compatibility of quantum theory with space-time theory was less easily ignored. At the very least, it became important to consider correlations among quantum systems in relation to the requirements of Special relativity because the separation of systems is customarily specified in spatial or spatiotemporal terms. Moreover, despite its apparent difficulties, Dirac's relativistic theory in particular continued to provide predictions and explanations which were otherwise unavailable.

The following is a helpful formalization of the requirement of logical consistency of a theory with Relativity.[16]

> *Relativistic constraint.* A theory is compatible with Relativity if it can be formulated without ascribing to space-time any more or different internal structure than the (special or general) relativistic metric. ([189], p. 292)

It is this requirement that will ultimately be looked to here as the standard of compatibility with Relativity, rather than the locality conditions of EPR and Bell discussed below, which address the preclusion of unidentified possible inter-system influences and the relationship between composite system correlations and individual system properties, respectively. This constraint more directly precludes a radical break from our current understanding of space-time than those conditions which, unlike it, portray quantum mechanics as supporting "action at a distance" even though the consistency of the probabilistic predictions of measurement outcomes of joint measurements on extended systems with the requirements of Special relativity is known.

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[15]These difficulties, which remained, are taken up in Chap. 4.

[16]The compatibility of Quantum mechanics and Relativity has been called their "peacefully coexistence."

Although there are indeed *prima facie* strong reasons for concern about the compatibility of quantum theory with relativity, this issue requires careful treatment and has been approached in several ways. It has been doubted that there would be consistency if Lorentz invariance were considered a principle of Special relativity because, for example, some forms of quantum theory are not Lorentz invariant.[17] There is also the more proximate and not easily evaded issue of the full set of consequences of the preclusion of correlations occurring at spacelike separation. This condition is often referred to simply as the *no-signaling* condition and is often also called so here.

The following principles are more standardly held: (A) the Principle of special relativity, that is, the kinematical equivalence of all inertial reference frames, and (B) the Light principle, that is, that the speed of light is the same in all inertial reference frames. These are then understood together to imply the following.

> *First-signal principle.* Nothing (causal) that can travel at speeds less than or equal to that of light can ever travel faster than light.

Thus, it is understood that the no-signaling condition applies for us.[18] However, the following two points must be kept in mind: the relationship between relativity and the constraint on signal propagation has not been made entirely clear, and although the fact that the assumption of a speed of light in vacuum that is invariant is applied to space-time in General relativity suggests that superluminal signaling is impossible, it is not strictly precluded by it and Einstein himself made no explicit argument of the latter kind [170]. On the other hand, by the time of Einstein's relativity, the following notions were not entirely new: the speed of light is finite (essentially shown by Rømer [245, 250] much before) and the speed of light can serve as a universal speed limit for matter (as considered, for example, by Poincaré, cf. [20], pp. 231–232). Einstein did, in any case, explicitly provide a signal-based rule for attributing times to distant events, which does not involve the introduction of absolute time, that coheres with the requirement of Lorentz invariance: To attribute times to distant points, one sends a light signal from a reference clock to a distant clock, from which it can be reflected back, providing the time one should attribute to the event of reflection at the distant clock, namely, the temporal midpoint between the times of the sending and receipt of the signal.[19] He similarly stated that the distance attributed this event be the speed of light times half the time between the sending and receipt of the light signal.

Examining Fig. 1.3, one sees that, according to the First-signal principle, space-time events in the temporal interval (t_0, t_2) at the point s_0 cannot causally interact

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[17] Strictly speaking, the no-superluminal-signaling condition strongly associated with Relativity is not one of its basic principles. In any event, Quantum mechanics and General relativity are incompatible for other reasons.

[18] Cf., e.g., [143]. Note also that Einstein's signal-based rule for time attribution can be shown to provide coordinates equivalent to those given via the Lorentz transformation if some additional assumptions are made, cf. [185], Chap. 4.

[19] Cf., e.g., [185, 307], and Fig. 1.3.

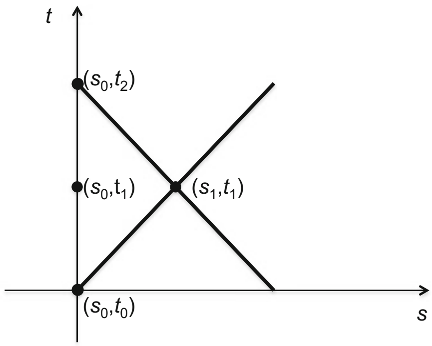

Fig. 1.3 The Einstein clock synchronization procedure in space-time. A light signal is sent from space-time point (s_0, t_0), reaches a mirror at point (s_1, t_1), from which it is reflected back to the source at (s_0, t_2). This provides the value $t_1 = (t_0 + t_2)/2$ at both s_0 and s_1, where the right hand side contains only local values ([185], p. 59)

with the event (s_1, t_1).[20] The sense of *signal* involved here can also be made more specific. It is sometimes said that (1) a signal has been transmitted between two locations—for example, two ends of a linear accelerator—whenever a correlation arises between distant laboratories. However, alternatively and more correctly, it can be held that (2) a signal has been transmitted whenever *information* could have been communicated via these correlations. One way that a difference can be made between (1) and (2) is via the question of whether one can have *control* over the establishment of these correlations that enables the latter situation that could actively be used for communication. This is a very important, indeed crucial, notion in relation to quantum theory, because Quantum mechanics precludes just such control. It is because the term *signal* suggests that the correlation in question is or could be used for communication that the latter is preferable. This distinction is important for our purposes because it may be that there are correlations between two distant events but these cannot be used to communicate between the two locations in which they occur because they arise entirely at random.

Propositions (1) and (2) do not refer to the *propagation of an influence*, which is also important to note in the quantum context because there the notion of propagation is far less straightforward than in the classical case. In the context of classical electromagnetism, one can distinguish a number of related speeds that could be seen to pertain here, for example, the front speed, the information speed, and the signal speed. Imagine that a change that might be used for signaling occurs, in that at a time t_0 the electric field is zero everywhere and after that instant the field begins to quickly increase in magnitude. A field front created in this way will travel with the *front speed*, which is the speed of light in vacuum, c, because the sudden turning on of the field can induce no immediate response in media present, see e.g. [36]. If a signal-defining pulse is produced and the field returns to zero afterward, the peak moves at a speed known as the signal speed, typically equal to the group velocity of the associated wave packet. The speed at which any information

[20]The principle that no (causal) signal may propagate faster than the speed of light is often called *Einstein locality*.

transmitted through the use of the pulse is the *information "velocity,"* v_{info}. Although in practice such information typically is received along with the pulse peak, the arrival of the pulse front in principle allows for the opportunity of information communication, which again will be c. The standard of "peaceful coexistence" between quantum theory and space-time theory in the study of the foundations of quantum theory is taken as the requirement that superluminal signaling be impossible. Under this requirement, Quantum mechanics cannot be interpreted in a way that such signaling could take place. Hence, the upper limit of the allowed quantum information velocity is, in effect, c. More importantly, as Einstein noted, superluminal signaling contradicts our collective experience. Although manifest Lorentz invariance of the theory is not imposed in Special relativity, three sorts of experiment have been performed that empirically support it. These are the Michelson–Morley type experiments [193], which have verified the isotropy of the speed of light, the Kennedy–Thorndike type experiments [168], which have verified the independence of c from the apparatus speed, and the Ives–Stilwell type experiments [150], which have measured time dilation via the Doppler effect [173].

Einstein was not alone in his concern over the broader implications in the context of the space-time structure of Bohr's interpretation of the theory, which includes the idea that space-time and causal descriptions of physical phenomena are mutually exclusive. The relationship between space-time theory and quantum theory was understood by many of the founders of Quantum mechanics as central to understanding it and to the constitution of the modern physical world view. For example, Erwin Schrödinger commented

> It has even been doubted whether what goes on in an atom can be described within a scheme of space and time. From a philosophical standpoint, I should consider a conclusive decision in this sense as equivalent to a complete surrender. For we cannot really avoid our thinking in terms of space and time, and what we cannot comprehend within it, we cannot comprehend at all. [266]

Einstein repeatedly emphasized the importance of having physical theory describe a spatiotemporally local and objectively existing world, as in the remarks shown above, as a precondition of physics itself. More generally, he held that all elementary physical concepts should be reducible to spacetime concepts and that the corresponding spatiotemporal laws are complete. Because of this emphasis on the independence of objects from others not in their immediate vicinities, Einstein's preconditions for well formulated physical theory, as represented in the EPR assumptions they motivated, have come to be known in the physics literature as *local realism*, or more specifically *local-causal objectivism*.[21] Although the EPR preconditions have turned out to be too strong, an alternative realist world view can still be consistently maintained as shown below.

[21] For discussions of the relevant spectrum of metaphysical positions see, for example, [95, 154], and [107]. Arthur Fine has called Einstein's position "motivational realism" because, he argues, realism "is the main motive that lies behind creative scientific work and makes it worth doing" ([107], pp. 109–110).

1.2 Quantum Mechanics

Relativity introduces a dependence of the values of physical quantities on equivalence classes of reference frames, the inertial reference frames. Nonetheless, this sort of relativity does not arise from there being a dependence on an observing subject, such as a human being: reference frames are not defined via conscious subjects. Similarly, a representative of a physical magnitude in quantum theory, namely, a Hermitian operator O acting on the state $|\psi\rangle$ of a quantum system,[22] which in the formalism of standard quantum theory is associated with a Hilbert space $\mathcal{H}$, corresponds to an equivalence class of measurement apparatus. The elements of this class differ from those of any other apparatus capable of precisely measuring quantities of other Hermitian operators with which they are *incompatible* in the sense that they both cannot be simultaneously measured with full precision.

Like the inertial frames of reference of Relativity, the theoretical dependence of measurements in Quantum mechanics on differing apparatus equivalence classes is not a dependence on the minds of observing subjects who may use them. The incompatibility of measurement apparatus in some pairings and dependence on the order of performance—first an A-measurement and then a B-measurement versus first one of B and then of A—of two incompatible measurements is defined via the Hermitian operator algebra, which is reflected in the failure of subsets of the operators to commute: Operators A and B of incompatible measurements are such that the commutator

$$[A, B] \equiv AB - BA \neq \mathbb{O} \, , \tag{1.4}$$

$\mathbb{O}$ being the zero operator.[23] Therefore, one cannot assume that the corresponding physical magnitudes in general take pre-existing exact values that are unaffected by measurements. Dirac accordingly distinguished the quantities given via the Hermitian operators, which he called “q-numbers,” from commuting quantities which he called “c-numbers,” cf. ([271], p. 15).

Examples of the state $|\psi\rangle$ are the two z-spin angular momentum eigenvectors of the Hermitian operator S_z (for the component of the *total spin* S along a chosen direction $\mathbf{z}$) of a quantum system, such as an electron, having always an S value of 1/2, which can only take on the values $+\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$; these are often written in Dirac’s notation as $|\uparrow\rangle$ and $|\downarrow\rangle$ or $|0\rangle$ and $|1\rangle$. Correspondingly, after the passage of an initial beam of electrons through an inhomogeneous z-axis-oriented magnetic field in a Stern–Gerlach apparatus, a given member of the collective beam will appear in one of the two distinct detected final beams corresponding to these states, as illustrated in Fig. 1.5. The space of probability values for the

[22] An operator is *Hermitian* if it is equal to its Hermitian conjugate $O = O^{\dagger}$, that is, its complex conjugate transpose, as considered in the matrix representation.

[23] Note that the inequality conditioning this commutation is exactly the negation of that defining the backward light-cone, cf. Fig. 1.4.

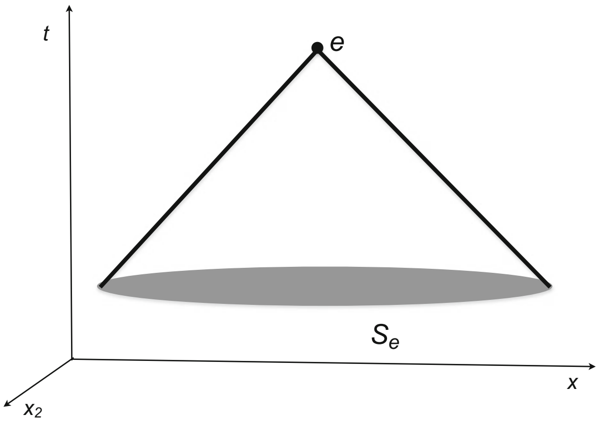

Fig. 1.4 An event e in space-time (time flowing vertically and space extending perpendicularly to it in this schematic) and a cross-section (at *bottom*) of the corresponding backward light-cone of past events timelike related to it. The events within this backward cone lie in the causal past of e, where S_{e} is a cross-section of its backward light-cone: The local influences on e in the sense of Relativity are present only within this cone of space-time points ($\mathbf{x}$,t), satisfying the inequality $c^2(t_{\mathrm{e}}-t)^2 - (\mathbf{x}_{\mathrm{e}} - \mathbf{x})^2 \geq 0$, where $t_{\mathrm{e}}-t \geq 0$, $\mathbf{x} = (x_1, x_2, x_3)$, and c is light speed. Past or present events outside this cone are spacelike related to e

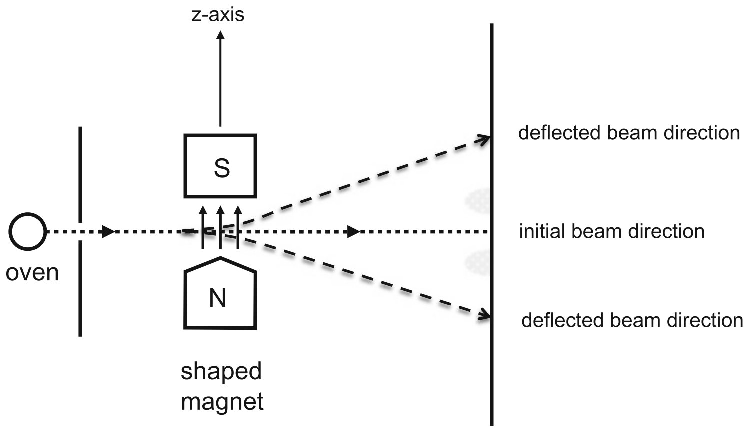

Fig. 1.5 A Stern–Gerlach apparatus acting on a spin-1/2 system [117]

possible spin-1/2 angular momentum components along the three spatial directions and labeled by corresponding quantum states is shown in Fig. 1.6 in the real-valued representation of the expectation values of the three Pauli operators σ_i $(i = 1, 2, 3)$ [153] corresponding to measurements of these three incompatible Hermitian spin-projection operators. The state-vectors $|\psi\rangle$ of the members of the pure statistical ensembles with states $P_{|\psi\rangle}$ have values lying on the ball's periphery, known as

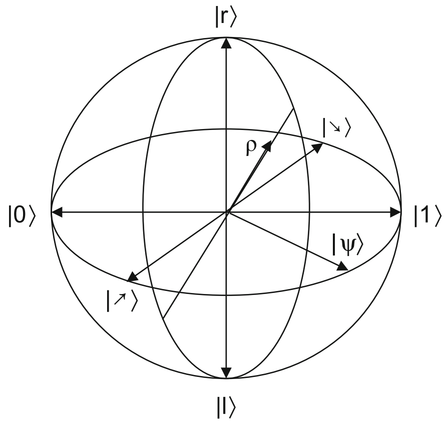

Fig. 1.6 The representation of the probability vectors labeled by the corresponding to the quantum state-vectors and statistical operators of the quantum two-level system, for example, photon polarization or spin angular momentum of a spin-1/2 particle, as a unit-radius ball known as the *Bloch ball*

the *Poincaré–Bloch sphere* [224]; those for the mixed statistical states ρ lie in the interior, with those of the fully mixed state $\frac{1}{2}\mathbb{I}$ at the origin.

In the complex-valued matrix representation of normalized quantum state-vectors, a row vector $(\alpha_1^* \alpha_2^* \cdots)$ represents each Dirac "ket" vector $\langle v|$ and a column vector $(\beta_1 \beta_2 \cdots)^{\mathrm{T}}$ represents each "bra" vector $|w\rangle$. Then the "bracket" $\langle w|v\rangle$ is their inner product. By contrast, the general "ketbra" $|v\rangle\langle w|$ is an operator given by the outer product

$$|v\rangle\langle w| \doteq \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix} (\beta_1^* \ \beta_2^* \ \cdots). \tag{1.5}$$

Note that $P^2_{|v\rangle} = |v\rangle\langle v|v\rangle\langle v| = P_{|v\rangle}$, because $|v\rangle$ has norm 1 and that $P_{|\psi\rangle}$, which is the projector onto the finite Hilbert subspace spanned by the state $|\psi\rangle$, also serves as the statistical operator for a collection of individuals described by the same state $|\psi\rangle$.

Any statistical operator can be written as a linear combination of such (pure) statistical operators $P_{|u_i\rangle}$ with corresponding weights p_i summing to 1: The statistical operator is always decomposable (although not uniquely so) as a convex sum of pure statistical states. For example, a collection of electrons may be described by the statistical state

$$\rho_{\mathrm{e}^-} = \frac{1}{4} P_{|\uparrow\rangle} + \frac{3}{4} P_{|\downarrow\rangle} \ , \tag{1.6}$$

where $P_{|\uparrow\rangle} = |\uparrow\rangle\langle\uparrow|$ and $P_{|\downarrow\rangle} = |\downarrow\rangle\langle\downarrow|$; this total collection can be prepared by combining two sub-collections in which one quarter of the members are prepared in the first state and three quarters in the second. Notably, von Neumann, in his operator-based based formulation of Quantum mechanics, nonetheless never referred to the mixed statistical operators as *states* (*Zustände*) but only as *mixtures* (*Gemische*); only the projectors, which bear a one-to-one relationship to the state-vectors, are *states* in his terminology.

The matrix $[O_{ij}]$ corresponding to an operator O has elements $\langle i|O|j\rangle \in \mathbb{C}$, each corresponding to the probability amplitude for a transition between the two states $|j\rangle$ and $|i\rangle$. The representation of an operator by a set of matrix elements is thus given relative to the choice of eigenbasis. Physical magnitudes correspond to Hermitian operators (observables) and have matrices with real diagonal elements O_{ii} and possibly non-real complex off-diagonal elements such that $O_{ij} = O^*_{ji}$. In the electron spin state above, for example, the only non-zero elements of the matrix ρ_{e^-}, when it is written in the basis $\{|\uparrow\rangle, |\downarrow\rangle\}$, are the diagonal ones: $O_{11} = 1/4$ and $O_{22} = 3/4$, with O_{ij} for $i \neq j = 0$. Such a statistical state is an *incoherent* mixture, in that there are no off-diagonal elements with phases that would support quantum coherence between eigenstates which might lead to the observation of interference phenomena in quantities with operators not commuting with O and to which they could otherwise contribute, as predicted by the Born rule for statistical states, given below.[24] The matrix representation of a statistical state ρ, which is necessary to describe mixtures, is known as a *density matrix* and is designated by the same symbol.[25]

The *spectral theorem* guarantees that every Hermitian operator O can be written

$$O = \sum_i o_i P_{|o_i\rangle} , \tag{1.7}$$

where o_i are the eigenvalues of O, called the *spectral decomposition* (or *eigenvalue expansion*) of the operator O.[26] Thus, for example, for the z-spin S_z, one has $S_z = (+\frac{\hbar}{2})P_{|\uparrow\rangle} + (-\frac{\hbar}{2})P_{|\downarrow\rangle}$. Following the Born rule for quantum probabilities $p_{|\psi\rangle}$, the expectation value of an observable O, which is an Hermitian operator by definition, for a quantum system in the statistical state ρ is the weighted sum of the probabilities of the alternative values $p^O_{|\psi_i\rangle}$, namely,

$$\langle O\rangle_\rho = \mathrm{tr}(\rho O) . \tag{1.8}$$

[24]Interference is discussed below in Sect. 2.5.

[25]These "matrices" are not necessarily susceptible to an explicit matrix description.

[26]However, note that this theorem does *not* hold for operators in *infinite-dimensional* Hilbert spaces, even when there exists a countably infinite set of basis vectors. Such a decomposition does not exist in general in that case, because there may not exist a countably infinite set of *eigenvectors* that form a basis.

A *pure state* is a statistical state describing a collection of identically prepared individuals and can be written as a projector $P_{|\psi_i\rangle} \equiv |\psi_i\rangle\langle\psi_i|$, where $|\psi_i\rangle$ indicates the corresponding state-vector in the Hilbert space $\mathcal{H}$; for pure statistical states, the *purity* $\mathrm{tr}\rho^2 = 1$. Thus, for the impure example ρ_{e^-} given above, one has $p_{|\uparrow\rangle} = 1/4$, $p_{|\downarrow\rangle} = 3/4$ because, for instance, $p_{|\uparrow\rangle} = \mathrm{tr}(P_{|\uparrow\rangle}\rho_m) = \frac{1}{4}P_{|\uparrow\rangle}P_{|\uparrow\rangle} + \frac{3}{4}P_{|\uparrow\rangle}P_{|\downarrow\rangle} = \frac{1}{4}||\uparrow\rangle|^2 + 0 = \frac{1}{4}$, and similarly $p_{|\downarrow\rangle} = \mathrm{tr}(P_{|\downarrow\rangle}\rho_m) = \frac{3}{4}$.

The term *observable* was given to Hermitian operators by Dirac in part to indicate that they generally don't have any numerical value in themselves, the latter being assigned by measurements and their statistics being assigned by the Born rule. Care must be taken when using the term because it has a resonance beyond the theory. In particular, its usage in physics differs from standard philosophical usage. In order to clarify the sense in which it might be considered a legitimate choice, Hans Reichenbach related it to the philosophical terms *phenomena* and *sense data* as follows.

> Using the word 'observable' in the strict epistemological sense, we must say that none of the quantum mechanical occurrences is observable; they are all inferred from the macrocosmic data which constitute the only basis accessible to observation by human sense organs. There is, however, a class of occurrences which are so easily inferable from macrocosmic sense data that they may be considered as observable in a wider sense. We mean all those occurrences which consist in coincidences, such as coincidences between electrons, or electrons and protons, etc. We shall call occurrences of this kind *phenomena*. The phenomena are connected with macrocosmic occurrences by rather short causal chains; we therefore may say that they can be 'directly' verified by such devices as the Geiger counter, a photographic film, a Wilson cloud chamber, etc. ([241], pp. 20–21)[27]

Although Reichenbach's first point is generally correct, it remains the case that the unaided human eye *can* directly detect light in the optical range of wavelengths at near-single-photon levels and distinguish it from darkness in the optical-range by virtue of a process that involves absorption by the rhodopsin molecule in the retina; human sense organs may directly connect with phenomena in such cases. For example, Čerenkov radiation, now familiar as that responsible for the blue glow in nuclear reactor cooling water, was discovered by its namesake through naked-eye observations carried out after hours of adaptation in full darkness of light produced when gamma rays enter uranium-salt solutions as well as ordinary water [59]. Recently and importantly for the current subject, experiments involving photons from an entangled pair have been detected by the human eye in an experimental situation [272].[28] One can see from these examples that reliable and quite direct access to quantum phenomena is possible. These examples also undermine any strict distinction between classes of so-called direct and indirect measurements in quantum mechanics. Furthermore, they tell against the suggestion that what could be considered a classical apparatus external to the human body is *required* for all quantum measurements.

[27] This relates to the "causal particle" concept critiqued by Falkenburg [95] and discussed in Sect. 2.3. Cloud chambers and other particle detectors are discussed later in some detail.

[28] Entanglement is defined and characterized in Sect. 1.4.

According to Reichenbach, the introduction of *observables* together with the above sense of *phenomenon* brings with it the question of the status in the quantum world of occurrences "between" the observation of phenomena associated with quantum observables. This led him to introduce the new term *interphenomenon*.

> We then shall consider as unobservable all those occurrences that happen between the coincidences such as the movement of an electron or of a light ray from its source to a collision with matter. We call this class of occurrences the *interphenomena*. …they are constructed in the form of an *interpolation* within the world of phenomena… ([241], p. 21)

The ontological status and conception of interphenomena was the subject of much debate in the early years of quantum mechanics and was deftly dealt with by the Copenhagen school, which originated with Bohr. As Reichenbach put it, "the principle of indeterminacy leads to some ambiguities which find their expression in the duality of waves and corpuscles. … One sort of experiment seemed to require the wave interpretation, another the corpuscle interpretation. … The decisive turn…was made by Bohr in his principle of complementarity…that it is impossible ever to verify the one and falsify the other" ([241], pp. 21–22). Even Einstein opined that "It has no meaning to say, it *is* a wave and it *is* a corpuscle…However, there is no *contradiction*," cf. [293], p. 364.[29] Although Einstein who in practice discovered 'wave–particle duality' (cf. [293], Sect. 4) found the thesis not inconsistent, he expressed continued dissatisfaction with the complementarity approach in relation to the nature of light and matter, at the end of 1952 saying "we are just as far from a really rational theory (of the dual nature [*Doppelnatur*] of light quanta and particles) as fifty years ago!" ([290], pp. 482–483).

Here, I will hold that the differences between phenomena and interphenomena arise from the necessity in measurements of opening the closed system described by $|\psi\rangle$ to the measurement apparatus and the environment, with only the *interphenomena* governed by the unitary state evolution alone. This contrasts with the Copenhagen school position in that the Principle of complementarity plays no role in this picture and interphenomena remain unspeakable. Most particularly for Wolfgang Pauli, although not for Bohr himself, the relationship between the system being measured and the entire environmental configuration around it including any human beings creatively contributes in an essential way to the *existence* of phenomena—at the very least, through the operation of free will in choosing the quantity to be measured and contributing to its taking place and the applicability of Bohr's principle of complementarity.[30]

Complementarity and its role in the interpretation of Quantum mechanics was explicated by Heisenberg as follows.

[29] More on the issue of wave–particle duality can be found in Chap. 3.

[30] For Pauli, there is a "limitation of the applicability of our ways of perception, not only by the possibilities of observation but also by the possibilities of definition (caused by the laws of nature)" ([122, 317], p. 21). Note also that Pauli was less than happy with Reichenbach's attempts to formalize these notions in logical rather than physical terms [317].

> Bohr uses the concept of 'complementarity' at several places in the interpretation of quantum theory. The knowledge of the position of the particle is complementary to the knowledge of its velocity or momentum...; still we must know both for determining the behavior of the system. The space-time description of the atomic events is complementary to their deterministic description... [The change in the course of time of the probability function] is completely determined by the quantum mechanical equation, but it does not allow a description in space and time. The observation, on the other hand, enforces the description in space and time but breaks the determined continuity of the probability function... ([136], pp. 49–50)

Complementarity can thus be seen to relate the novel aspects of Quantum mechanics to issues regarding space-time description of quantum objects, but does so by considering it and causation to be *mutually exclusive*.[31] The basic mathematical relationship between the "wave-like" and "particle-like" aspects of quantum objects is through that of the physical properties most closely associated with them, namely, frequency–wave-number and energy–momentum, respectively, given by the Planck–Einstein and Einstein–de Broglie relations:

$$E = h\nu \,, \tag{1.9}$$

$$\mathbf{p} = \hbar\mathbf{k} \,, \tag{1.10}$$

where E is energy, ν is frequency, $\mathbf{p}$ is momentum, and $\mathbf{k}$ is wavenumber.

A contrasting, less proscriptive position is that offered by Alfred Landé, that all experiments can be explained through *both* interpretations. It has not been possible to construct an experiment which is incompatible with both these 'interpretations.'[32] This perspective is furthered by more recent developments in quantum theory that describe unsharp measurement, which is discussed in later chapters. The two sorts of behavior of quantum objects, wave-like and particle-like, are often taken to be temporarily exhibited under certain conditions and derive from the corresponding classes of classical systems, light and ponderable matter respectively. It has been noted, however, that these two aspects tend to appear asymmetrically in quantum experiments in accordance with the dictum, due to Wolfgang Ketterle, that one generally *prepares waves but detects particles* ([95], pp. 281–284).

In the well known double-slit experiment, which is illustrated in Fig. 1.7, one prepares quantum light in momentum eigenstates, associated with "wave-like" behavior, and detects them locally, and so behaving in a "particle-like" way; similarly so in Aspect's experiments testing the Bell inequality. The latter part of the Ketterle dictum is explicable by the fact that all individual measurements must occur in a local region, or "laboratory." The former half is, by the same token, suspect, as preparations also generally take place in a localized "laboratory."

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[31] However, note that, unlike in Relativity, time in Quantum mechanics serves only as a parameter.

[32] For more on the issue of wave–particle duality, see [154], Sect. 1.2.

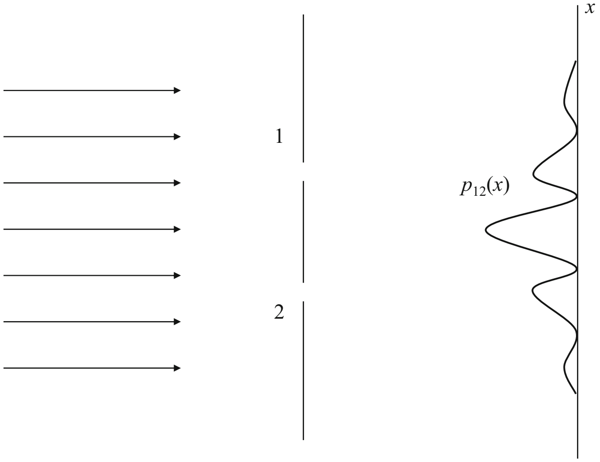

Fig. 1.7 The Young double-slit experiment. One can note of the incoming states, having well-specified momentum, that they exhibit a wave-like character (seemingly passing through both distant slits 1 and 2), whereas the individual detections giving the curve at right are discrete (having highly localized positions x). This example is typically understood as contrasting with the behavior observed in Compton scattering in which particle-like character is exhibited, illustrated in Fig. 2.6

One of the aspects essential to the notion of a particle is such (spatial) localizability. In non-relativistic versions of Quantum mechanics, it is possible to use the momentum eigenstates $\{|\mathbf{k}\rangle\}$ to form an orthonormal set of position states $\{|\mathbf{x}\rangle\}$ parameterized by spatial position $\mathbf{x}$ via the Fourier transform, as follows.

$$|\mathbf{x}\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}}|\mathbf{k}\rangle \ , \tag{1.11}$$

where $\langle\mathbf{x}|\mathbf{x}'\rangle = \delta^3(\mathbf{x}-\mathbf{x}')$, which is an instance of superposition for the case of continuous rather than discrete eigenvalues, as in Eq. 1.1. The position and momentum eigenstates are complementary in this mathematical sense and related by the Fourier transform. However, as shown in Chap. 4, the relationship between position and momentum bases is different in relativistic versions of the theory, where serious difficulties arise for the establishment of a robust intuitive notion of localization appropriate to quantum systems.

One finds Einstein commenting (in 1939) on the question of wave–particle duality as follows.

> I do not believe that the particle-waves have reality in the same sense as the particles themselves. The wave-character of particles and the particle-character of light will—in my opinion—be understood in a more indirect way, not as immediate physical reality. (as quoted in [293], pp. 373–374)

Recall that, nonetheless, Einstein rejected Bohr's more abstract and general notion of complementarity. A useful approach to dealing with this conceptual tension is to understand these two aspects as essentially different from the classical notions of particle and wave traditionally associated with them, contrary to what is done on the complementarity approach. Richard Feynman, who played a central role in precisely describing the interactions of elementary particles through his contribution to Quantum electrodynamics, was a strong advocate of a more modern particle-based approach to quantum objects. Regarding this debate, he noted in relation to behavior appearing analogous to that of classical systems that

> there was a period of time during which you had to be clever: You had to know which experiment you were analyzing in order to tell if light was waves or particles. This state of confusion was called the 'wave–particle duality' of light… ([105], p. 23)

and emphasized that light is best thought of as *particulate* for reasons connected with the *latter portion* of Ketterle's motto.

> [Y]ou were probably told something about light behaving like waves. I'm telling you the way it *does* behave—like particles. …every instrument that has been designed to be sensitive enough to detect weak light has always ended up discovering [that], ([105], p. 15)

regardless of the theoretical difficulties surrounding particle localization.

The elementary particles to which Feynman refers are not particles in the traditional classical sense—the sense of being, among other things continually localized—but are more precisely identified as discrete instantiations of irreducible representations of symmetry groups corresponding to their invariant properties, such as rest mass (for light, zero) and spin (for light, $\hbar$).[33] Nonetheless, for Feynman, they *are* more closely related to the traditional particle concept than to the traditional wave concept, for just the reason noted above, namely, that experiments and observations involving them can and do take place within local regions, the objects being, at a minimum, localizable in this way and found in discrete units involved in processes in which a strict accounting of quantities can ultimately be made. The corresponding group instantiations of theory are the *quanta* of the associated quantum fields.

When identified with field quanta, particles are treated either (i) in terms of their "occupation" of normal modes of the field, or (ii) in terms of their contribution to the energy to the local field [81]. Quantum fields have the desirable property that their equations of motion, as well as their commutation relations, involve only the value of the fields and their derivatives at any point of interest and are local in the fundamental sense that they satisfy a micro-causality condition. Description (i) is one in which the field is considered a many-particle system represented by a direct sum of Hilbert spaces corresponding to all non-negative integer values, that is, corresponding to the number of quanta which could be in the field.[34] The Hamiltonian of the field also has the same mathematical form as that of a collection of independent simple harmonic oscillators each with characteristic frequency $\omega_{\mathbf{k}}$ oscillating along the corresponding direction $\hat{\mathbf{k}}$. One thus has a collection of n

[33] This notion of particle is discussed in more detail in Chap. 4.

[34] This is discussed in detail in Chap. 3.

particles of momentum $\hbar\mathbf{k}$, n being an eigenvalue of an associated number operator $N_{\mathbf{k}}$, and energy $\hbar\omega_{\mathbf{k}}$ associated with each "oscillator." Description (ii) is arrived at instead by field quantization: The field is initially considered directly via the single-particle equation and is then "second-quantized," as follows. The electromagnetic field, for example, can be understood in terms of the local field $\phi(\mathbf{x})$, as for example in the specification of the micro-causality condition.[35] The motion of the field is decomposed via Fourier analysis into a collection of independent elements, each also seen as having the mathematical form of a quantum simple harmonic oscillator. The excitation number providing the energy of each oscillator is the number of corresponding quanta. Each of the (possibly infinite number of) normal modes is associated with a specific frequency, as in the special case of the field in the interior of a cavity; these frequencies are determined by the boundary conditions of this mathematical "cavity," within which a quantum is otherwise entirely delocalized.

It has been argued that in quantum field theory, the non-commutativity of position and momentum underlying the wave–particle duality in the discrete-system context is simply replaced by another one: The occupation-number operator $N_{\mathbf{k}}$ fails to commute with the local field operator $\phi(\mathbf{x})$, (cf., e.g. [240], p. 17). On the other hand, Heisenberg argued that the second quantization approach to field theory, as also suggested by Landé, "demonstrates that in the formalism of the quantum theory the particle and wave pictures appear only as two different manifestations of the same underlying physical reality" [134]. The question is whether concern over this 'duality' is well founded. This duality is no longer in itself of significant concern, given that elementary particles specifically can be viewed as energy-bearing instantiations of the irreducible representations of the group of space-time symmetries, as demonstrated by Eugene Wigner. Consider a single light quantum as an instantiation of the relevant irreducible group representation. The empirical feature perhaps most distinctively characterizing the light quantum is that it is capable of producing *only* a single detection event at a highly sensitive light detector, such as a single-photon number detector [195]. The associated energy is *indivisible*.

Feynman, shortly after the above-quoted remark emphasizing this point, explained better the behavior of the quantum objects he called *particles*.

> In fact, both [electrons and photons] behave somewhat like waves, and somewhat like particles. In order to save ourselves from inventing new words such as 'wavicles' we have chose to call these objects 'particles,' but we all know that they obey these [quantum probability] rules. It appears that *all* the 'particles' in Nature—quarks, gluons, neutrinos, and so forth… behave in this quantum mechanical way. ([105], p. 85).[36]

A strong critic of such language might even argue that the particle concept has been "so thoroughly denuded by quantum field theory that [it] is hard to see how it could possibly underwrite the particulate nature of laboratory experience," including that Feynman described as quoted further above ([61], p. 264). However, there remain

[35] This condition on quantum field operators is defined below in Eq. 1.13

[36] Other names for quantum objects related to the particle concept that have been suggested include "quarticle" [145].

good reasons not to reject the term *particle* including, in particular, that the term continues to be successfully used in practice. The continued use in fundamental physics to objectively existing countable objects, or objects with properties for which a strict accounting can be made, regardless of how counterintuitive their character has shown itself to be, is not simply a matter of historical accident. A particle conception remains extremely valuable for providing physical explanations. Nonetheless, as Schrödinger said, "we cannot escape the conclusion that the new objects are *neither* particles *nor* waves…we must humbly *learn* rather than *prescribe*, what nature is made of" ([56], p. 26); he explained that one must do so based on experimental evidence in a way according with Landé's position.

> A vast amount of experimental evidence clinches the conviction that wave characteristics and particle characteristics are never encountered singly, but always in union; they form different aspects of the same phenomenon, and indeed of all physical phenomena. The union is not a loose or superficial one. … It seems that both concepts, that of waves and that of particles, have to be modified, so as to attain a true amalgamation. [268]

This is, for example, what Feynman's characterization of elementary quantum objects does when given the mathematical underpinning of Wigner's symmetry-based formalization, albeit with a preference for the term *particle*. Moreover, subsequent mathematical developments of the quantum formalism, such as that of positive operator valued measures can be used to formalize this idea.[37]

The entities of elementary particle physics are, as Feynman noted, of a *fundamentally different nature* from classical particles, their essential characteristics being their discreteness and (momentary) localizability; other traditional particle characteristics such as impenetrability, substantiality, unique space-time trajectory, and/or transcendental individuality do not pertain to them, as seen here in later chapters.[38] Despite the utility of Ketterle's generalization in many situations, it is quite possible to prepare a quantum system in a way that appears particle-like, such as in a non-destructive precise measurement of an electron's position or the confinement of an ion to an electromagnetic trap. The Ketterle motto derives from the commonplace need to prepare systems to interact with others or for later detection *elsewhere*, not from any reason of principle. It is possible to measure the defining characteristics of a single electron, that is, its mass, spin, and charge (cf., [288], p. 10), and see one emerging in the decay of a single, localized nucleon, for example.

Feynman described a contemporary solution to the problem of the ultimate ontological character of elementary objects as follows.

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[37]POVMs are discussed in the following chapter.

[38]Falkenburg has identified the following characteristics, which she refers to as "informal predicates," of classical particles: carrying mass and charge, mutually independence, exhibiting point-like behavior during interactions, being subject to conservation laws, having behavior completely determined by mechanical law, following phase-space trajectories, being spatio-temporally individuated, being able to form bound systems ([95], p. 211).

> Quantum electrodynamics 'resolves' this wave–particle duality by saying that light is made of particles (as Newton originally thought), but the price of this great advancement of science is a retreat by physics to the position of being able to calculate only the *probability* that a photon will hit a detector... ([105], p. 37)

This is very similar to Einstein's characterization of the situation with respect to quantum objects, cf. [91].

> It has turned out that by replacing the [electromagnetic] field in the sense of the original field theory by a probability field... The price which had to be paid for the extraordinary success of the theory has been twofold: The requirement of causality, which anyhow cannot be tested in the atomistic domain, had to be given up, and the endeavor to describe the reality of physical objects in space and time had to be abandoned. In its place, an indirect description is used, from which the probability of the result of any conceivable measurement can be computed. [91]

The lesson to be learned from all of this is that one cannot insist that all or even most of the familiar characteristics of commonplace objects continue to apply to newly discovered entities, such as elementary particles, as physics moves forward. Bohr was open to this possibility, in that he believed that

> the present formulation of quantum mechanics in spite of its great fruitfulness would yet seem to be no more than a first step in the necessary generalization of the classical mode of description... we must be prepared for a more comprehensive generalization of the complementary mode of description which will demand a still more radical renunciation of the usual claims of so-called visualization [30]

The deeper question is that of the circumstances under which a quantum mechanical 'system' can be considered an individual object. To the extent that conventionality regarding this term can be reduced in this way, progress will have been made toward a better understanding of quantum objects.[39]

1.3 Einstein Locality

In the famous Solvay conference discussions with Einstein over the apparent limitations of Quantum mechanics, Bohr steadily maintained that the theory was as complete as possible and that there is a fundamental and inescapable tension between the space-time and causal descriptions of phenomena corresponding to complementarity.[40] Despite his being a minimal realist open to the application of probability to individuals, Bohr's view does not allow for a precise objective description of physical objects, but rather only of the phenomena with which they are associated. In the journal *Philosophy of science*, in an article entitled "Causality and complementarity," he wrote

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[39]The investigation of this issue is continued in Chap. 4.

[40]Note that for *Pauli*, as the causal description became less appropriate an *acausal* correspondence between events—a less mystically flavored version of Jungian synchronicity—became more appropriate [175, 317].

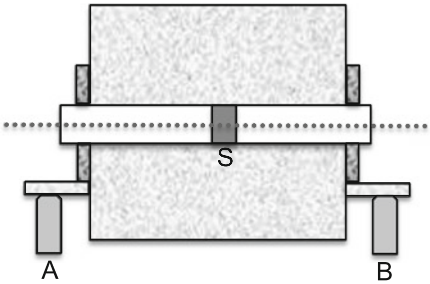

Fig. 1.8 The Wu–Shaknov experiment. Counterpropagating, oppositely polarized photon pairs produced in the annihilation of electron-positron pairs at source S are later detected at separate locations via detectors A and B [332]

> The renunciation of the ideal of causality in atomic physics which has been forced on us is founded logically only on our not being any longer in a position to speak of the autonomous behavior of a physical object, due to the unavoidable interaction between the object and the measuring instruments which in principle cannot be taken into account, if these instruments according to their purpose shall allow the unambiguous use of the concepts necessary for the description of experience. In the last resort an artificial word like "complementarity" which does not belong to our daily concepts serves only briefly to remind us of the epistemological situation here encountered, which at least in physics is of an entirely novel character. ([30], pp. 293–294)

The indication here that complementarity stems from an epistemological novelty is noteworthy. In order to become free of the historical grip of the powerful but imprecise notion of complementarity, a more careful investigation of the character of spatiotemporal description and causation is needed.

Beyond the novelties appearing in the mechanics of lone systems, as mentioned above, Quantum mechanics predicts exceptionally strong correlations between distant events associated with different subsystems that have been confirmed in a range of situations. The first such evidence was discussed after Einstein's time, in an analysis of the observations of electron-positron pairs originating from a common source S made by Chien-Shiung Wu and Irving Shaknov (Fig. 1.8) [332]. Later experiments designed specifically to demonstrate rigorously the non-local character of these correlations were performed on photon pairs by Aspect, after decades of technological progress made them feasible [3]. These and later results have strongly indicated that definite, distant correlated measurement results of the quantum world cannot be straightforwardly accounted for by local causes. Predictions of such 'non-local' correlation in quantum theory can be traced to the tensor-product space structure of composite systems and the quantum Superposition principle, which remains valid for states of joint systems regardless of their separation. The state $|\Psi\rangle$ of a quantum system formed by the composition of two distinguishable quantum systems $\mathcal{H}_A$ and $\mathcal{H}_B$ into a larger system lies within the tensor product space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, with bases composed of elements of the form $|u_i\rangle_A \otimes |v_j\rangle_B$[41]:

[41]Here, the subscripts provide a label attributable by virtue of some property distinguishing to systems, such as spin or rest mass.

Any quantum state $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ can be written in terms of the basis vectors of the subsystem Hilbert spaces, that is, in the form

$$|\Psi\rangle_{AB} = \sum_{ij} a_{ij} |u_i\rangle_A \otimes |v_j\rangle_B \ , \tag{1.12}$$

with $a_{ij} \in \mathbb{C}$, where the sets of vectors $\{|u_i\rangle\}$ and $\{|v_j\rangle\}$ consist of orthogonal unit vectors spanning the space of possible state vectors for the two subsystems, respectively.[42] The experimental evidence regarding highly correlated quantum systems described in Quantum mechanics by states of the form $|\Psi\rangle_{AB}$ is incompatible with the spatiotemporally reductionistic "local realism" of the kind to which Einstein subscribed, on which the EPR conditions were based, and which Bell later more mathematically captured. Nonetheless, as shown in the sequel, one need not abandon less classically inspired forms of realism merely because of the associated failure of so-called "local realism."

Einstein had as early as 1927 provided a thought experiment aimed at demonstrating the inadequacy of Quantum mechanics in describing a quantum system in space-time context in a situation involving possible detection at spacelike separated locations. He considered the wave-function (the configuration space representation of the quantum state) for a single ball restricted to a region consisting of two boxes which can be greatly distanced in space rather than for a composite system as in the later, EPR argument ([161], p. 115). In this early thought experiment, when a measurement is made to check whether the ball is contained in one of the boxes, one is also able to determine whether the ball is present in the other. From this, Einstein argued that the quantum state must provide an incomplete description of system location because otherwise the performance of the measurement on the first would immediately determine the location-state of the ball in the second, arbitrarily far away, something which would before be unspecified. This would, he argued, be at odds with Relativity because a *non-local influence* seems to be present. This early argument failed because the constraint on the speeds of influences is not crucial to it; the wave-function need not be understood as describing the distribution of some substance, particularly in the Newtonian sense as the bearer of properties such as solidity and inertial mass (cf., e.g., [95], p. 27), in which case there could be a conflict.[43] This thought experiment involves only a logical determination but no superluminal causal influence. In particular, no *physical contingency* comes into play. It is simply required for *logical consistency* that one must assume the ball is in one and only one box at any given moment, even when the wave-

[42] The products of unit vectors of Eq. 1.12 are often written in compact form: $|u_i v_j\rangle_{AB} \equiv |u_i\rangle_A \otimes |v_j\rangle_B$.

[43] This may have been overlooked because at the time many, for example Schrödinger, believed some sort of substance might be present. Reichenbach notes that here arises "the question whether the waves have *thing-character* or *behavior-character*, i.e., whether they constitute the ultimate objects of the physical world or only express the statistical behavior of such objects" ([241], p. 22). We return to this question below.

function is not entirely absent from either box [283]. Thus, for example, Bell distinguished quantum correlations from correlations such as "When the Queen dies in London (may it be long delayed) the Prince of Wales, lecturing on modern architecture in Australia becomes *instantaneously* King" ([20], p. 281).[44] The box thought experiment shows that Einstein was aware that the traditional concepts of substance and causation had become problematic for Quantum mechanics, even if the argument itself did not succeed. Similarly, although the EPR thought experiment did not suffer from the above flaw, EPR still failed to address the crucial question of the *controllability* of the distant physical changes involved.

What role does causality play in quantum theory, then? The no-signaling requirement is the enforcer of causal order in contemporary physics and has been taken as an element of the formulation of advanced forms of the theory, more or less explicitly. The *micro-causality* condition is typical in formulations of quantum field theory and is essentially the requirement that operators at spacelike distances commute with each other. In very general terms for operators, such as that of the field, for associated with pairs space-time locations x, y (cf. Fig. 1.4), this is

$$[A(x), B(y)] = \mathbb{O} \quad \text{if} \quad (x_0 - y_0)^2 - (\mathbf{x} - \mathbf{y})^2 < 0 \,. \tag{1.13}$$

Under this condition, the time-evolved version of a quantum mechanical operator representing a physical quantity commutes with the projectors onto the eigenstates for its possible values when two measurements are made at *spacelike separated* space-time locations $x = (x_0, \mathbf{x})$ and $y = (y_0, \mathbf{y})$, where x_0 and y_0 are the time coordinates multiplied by the speed of light, c. The micro-causality requirement allows for the existence of simultaneous eigenstates for $A(x)$ and $B(x)$, which would be precluded were it not to hold.

One motivation for the micro-causality condition is that if simultaneous eigenstates were *not* allowed then superluminal signaling might be achieved by judicious use of measurements at spacelike separations, through the ability to measure or not measure a quantity in that measurement at one location would induce a measurable dispersion at another. Accordingly, one finds in a well-known contemporary textbook on quantum field theory the statement that

> To really discuss causality [rather than particle propagation] we should ask not whether particles can propagate over spacelike intervals, but whether a measurement performed at one point can affect a measurement at another point whose separation from the first is spacelike ([219], p. 28).

emphasing possible effects due to measurement. However, as James T. Cushing pointed out, the micro-causality requirement is not a characterization of causation in quantum theory but rather a condition of *non-acausality* [68]. The focus in quantum theory in relation to possible causes is on certain classes of distant correlations, and

[44] A similar example is considered in more detail in Sect. 2.3.

their relation to measurement, rather than on the properties of system trajectories which are, in general, ill defined.

In the standard quantum theory of measurement, a discontinuous state change is assumed to take place at the end of the process of measurement and relates to micro-causality, for example, as follows.[45] Consider that two observables A and B commute if and only if the expectation value $\langle B\rangle$ is not altered by a specific such state change (namely, a non-selective Lüders operation, defined below) relative to A for any initial state ρ to a new state ρ', that is, if and only if $\langle B\rangle_{\rho'} = \langle B\rangle_{\rho}$. The observables A and B can in Quantum mechanics be observables pertaining to spacelike separated subsystems of a joint system. In this, one can see the motivation for the above-mentioned assumption of local commutativity in relativistic quantum field theory requiring the mutual commutativity of observables, one taken from each of two local algebras associated with two spacelike separated regions of space-time: It is necessary and sufficient for the failure of signaling between these two regions, that is, the impossibility of influencing the outcomes of measurements in one of the two regions through non-selective measurements in the other [48]. Consider the following failure of superluminal signaling via Quantum mechanics. Take a possibly time-dependent normalized state-vector $|\Psi(t)\rangle$ to be measured with maximum precision for some physical quantity with corresponding Hermitian operator O. If $|\Psi(t)\rangle$ before measurement were initially superposed in the O-basis, it will thereby be discontinuously changed to the O-eigenstate corresponding to the measurement outcome and could be a means for sending a communication signal. However, until that outcome comes to be, known the only state attributable the measured system is a statistical mixture of all possible outcomes weighted with the corresponding probabilities for each outcome to occur. That is, there will be no non-zero off-diagonal elements of the corresponding statistical operator $\rho(t')$, that is, the statistical state

$$\rho(t') = \sum_i P_i(t')\rho(t')P_i(t') \tag{1.14}$$

is entirely diagonal, as evidenced by the appearance of corresponding non-zero matrix elements $\rho_{ii}(t')$ only. Now, if the system is later measured at the receiver the expectation value of O at the time t'' of measurement will be

$$\langle O\rangle_{t''} = \mathrm{tr}(\rho(t'')O) = \mathrm{tr}\big(U^{\dagger}(t''-t')\rho(t')U(t''-t')O\big) = \langle O\rangle_{t'}\ . \tag{1.15}$$

Any putative receiver's measurement of the expectation value will yield *no communicated information* in this way. One sees that it is possible to frame the impossibility of signaling quite generally without specific reference to Relativity. Bell commented in relation to conditions like micro-causality that "the only way I know to relate local commutativity to any sort of causality concerns the response of the quantum system

[45]Measurement theory is discussed in greater detail in the next chapter.

to external interventions," such as measurements and the imposition of external fields ([20], p. 222). This was just the sort of approach that Einstein et al. took in the EPR paper.

The EPR argument involves a locality condition that differs only slightly from the following, which Einstein referred to as the *principle of local action*, previously mentioned here in its more general form (cf. [143], p. 234).

> *Einstein locality:* The real, physical state of one system is not immediately influenced by the kinds of measurements directly made on a second system, which is sufficiently spatially separated from the first. ([107], p. 61)

The EPR argument itself involves three explicit, related conditions [92].
The *locality criterion* is "Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system."
The *reality criterion*, which EPR introduced in order to define "physical reality" for the purposes of the scenario, is "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."
The *completeness criterion* of EPR is "Every element of the physical reality must have a counterpart in the physical theory," which is the necessary condition that EPR argued quantum mechanics *fails* to satisfy.
The locality criterion relates to causation and echoes Max Planck, who identified it with determinism, in that, in 1932, he had proposed to define causation as a "lawful connection in the temporal course of events" [221] while "[a]n event is causally determined when it can be predicted with certainty" [222], which is consistent with Laplacian determinism, discussed below.

The EPR reality criterion will also select for situations consistent with state determination under this view. The conclusion of the EPR argument is that in order to be explicable via the elements of reality in each subsystem, if this 'non-locality' is to be disallowed, the predicted correlations must be accounted for by more than what Quantum mechanics, which includes the uncertainty relations, provides.[46] The EPR argument has therefore been construed as an argument that a hidden variables theory, of the sort discussed below, is needed to provide an explanation of the correlations [285]. The original version used an example involving continuous quantities, namely, those of in a two-part system in a one-dimensional setting in the (entangled) quantum state

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp\left[\frac{i}{\hbar}(x_1 - x_2 + x_0)p\right] dp \;, \tag{1.16}$$

[46]The uncertainty principle and relations are discussed below in Sect. 2.3.

where x_1 and x_2 are the subsystem positions, x_0 is a fixed distance and p is momentum [92].

The EPR argument is now instead typically presented in the context of discrete observables, a move which avoids some weaknesses of the above case.[47] Without changing the argument in any essential way, one can apply it to the bivalent observables associated with the spin-singlet state

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\,. \tag{1.17}$$

As a background assumption, EPR invoked the following interpretative rule.[48]

> The *eigenvalue–eigenstate link*. A quantum system magnitude is attributed a definite value *if and only if* the system is in a state that is an eigenvector of the operator corresponding to that magnitude.

This rule is an example of a semantic element of the interpretation of the theory, an account of which quantities can have which values and when they have them, an aspect of theory interpretation sometimes overlooked. EPR use it when they assume, including their first equation, that

> If ψ is an eigenfunction of the operator A, that is, if $\psi' \equiv A\psi = a\psi$, where a is a number, then the physical quantity A has with certainty the value a whenever the particle is in the state given by ψ [92].

Using the Bohm singlet state $|\Psi^-\rangle$, then, the EPR argument can be given in terms of two propositions [276].[49]

(I) If an agent can perform an operation that permits it to predict with certainty the outcome of a measurement without disturbing the measured spin, then the measurement has a definite outcome, whether this operation is *actually* performed or not.

(II) For a pair of spins in the state $|\Psi^-\rangle$, there is an operation that an agent can perform allowing the outcome of a measurement of one subsystem to be determined *without disturbing* the other spin.

By the measurement of the quantity corresponding to $P_{|\uparrow\rangle}$ for one spin, the value corresponding to the projector $P_{|\downarrow\rangle}$ onto the orthogonal state is also fully specified. By II, one can also obtain the values of the same two observables of the second spin without influencing it, because there is perfect anti-correlation of spins in $|\Psi^-\rangle$.

[47] For example, the first of the two states above has unhelpful peculiarities, the most significant of which is that—despite the authors' expectations—the correlations of measurement outcomes at distant locations they take to be characteristic of the state when the subsystems are well separated are not absolutely strict.

[48] This assumption plays an important role in later discussions here. Also, cf. [239].

[49] The result is then called the EPRB argument—the B standing for Bohm [24], who first used the spin singlet state for its study.

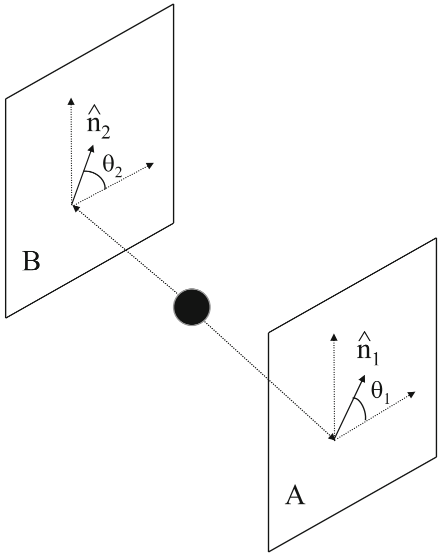

Fig. 1.9 Schematic of a Bell-type inequality test using polarization interferometry shown in space only. Two non-orthogonal states parameterized by angles θ_1 and θ_2, respectively, from a set of specific values, are measured at spacelike separation in planes normal to the axis of system (counter-)propagation in two laboratories A and B

By I, the values of the second spin are definite. However, instead one could have measured the values of the quantities corresponding to another basis, say those corresponding to the spin superposition states $|\nearrow\rangle$ and $|\searrow\rangle$, but those values must then also be definite. Therefore, the values of the states of both systems for all values of ξ must be definite under the assumptions; the description of the joint system by the quantum state $|\Psi^-\rangle$ is seen to be incomplete because it does not supply them all.

The experimental situation involved is shown in Fig. 1.9.[50] Karl Popper argued that the EPR argument in this second form, which is due to David Bohm, actually represents a significant advance from the consideration of the original EPR state because

> [N]obody assumes that if we 'measure' a state of polarization of some system S... with the help of a polarizer, we always measure the state of S as it was immediately before entering the polarizer; on the contrary, we may actually know that the state of polarization of S before entering the polarizer was different from the state 'measured' by this polarizer... For if... the 'measurement' of [subsystem] S_1 consists in S_1 passing through a polarizer, and if *this* 'measurement' of S_1 informs us according to quantum mechanics about the state of S_2, then the kind of action at a distance described by Einstein is not merely part of the *interpretation* [in which S_1 has no definite value of a subsystem variable and that value comes into existence when S_1 is measured]—that is, Bohr's interpretation of quantum mechanics—but part of quantum mechanics itself. ([229], pp. 187–188)

Recall that the conclusion of EPR was that, if non-locality is to be disallowed, the measurement correlations predicted by Quantum mechanics, in order to be

[50]Popper's point was later amplified in the much later GHSZ argument involving three spin-systems [123].

explicable via the elements of reality in each subsystem, require more than Quantum mechanics can offer. It is for that reason that the EPR argument can be construed as an argument that further theoretical elements are needed to provide a complete explanation of the correlations that are predicted by Quantum mechanics—and subsequently to the EPR argument itself, observed in the laboratory.

Hidden-variables theories of quantum phenomena are based on the consideration of a putative *complete state*, λ, which in such theories is taken to render the quantum-mechanically pure state $P_{|\psi\rangle}$ a statistical state in the traditional sense. The "hidden variables," which might provide the parameters that EPR have been reasonably interpreted as arguing must be missing from the description of Quantum mechanics, which complete the state description, can be formally considered by having the quantum observable O taking the value $O_{|\psi\rangle}(\lambda)$ in state $|\psi\rangle$ described by the map $O_{|\psi\rangle} : \Lambda \rightarrow \mathbb{R}$, Λ being the domain of possible values of hidden variables λ.[51] (The conjectured completed states are by definition constrained by statistical principles such as the need to preserve the functional subordination in the space of quantum observables and the preservation of the convex structure of the set of quantum states.)

Assuming, then, a probability measure μ that can be used to characterize the degree of ignorance as to the value of λ, so that $\{\Lambda, \mu\}$ constitutes a standard probability space, one has a probability density function $\sigma_{|\psi\rangle}$ for each $|\psi\rangle$. The probability that the hidden variable lies in the interval $\lambda + d\lambda$ is given by $\sigma_{|\psi\rangle}(\lambda)d\lambda$; the expectation value of O is

$$\langle O\rangle_{|\psi\rangle} = \int_{\lambda} O_{|\psi\rangle}(\lambda)\sigma_{|\psi\rangle}(\lambda)d\lambda \; . \tag{1.18}$$

The values of quantum (statistical) observables are thus treated as random variables over $\{\Lambda, \mu\}$. Bell introduced just such a construction in the proof of his theorem. Various sorts of hidden variables models for quantum mechanics have been conceived, depending on what λ is assumed to provide.[52] When it is taken to provide definite values to all physical magnitudes of a quantum system corresponding to the quantum $|\psi\rangle$, the hidden-variables model is called *non-contextual*. Non-contextual models provide the non-statistical state of the overall system, that is, the system consisting of the measured system together with the measurement apparatus, thereby determining the value of a quantity obtained by measurement, regardless of which other quantities are simultaneously measured together with that quantity. The *contextual* hidden-variables models make use not only of λ but also other relevant parameters related to the conditions of their measurement.[53] Such theories make

[51] The relationship between the potential and actual will be taken up in greater detail in Chaps. 2 and 3.

[52] Importantly, note that the "hidden parameters" are not necessarily truly hidden in the sense of being physically inaccessible.

[53] An example of such a theory was introduced by Bell [16].

use of all these parameters to assign each quantum mechanical projector a definite value.

Bell provided the following example of a (non-contextual) hidden-variables model for the spin-$\frac{1}{2}$ system. In this model, a spinorial representation, ϕ, is used along with a real parameter $l \in [-\frac{1}{2}, \frac{1}{2}]$, which completes the specification of the dispersion-free state λ. System properties are represented by matrices in $H(2)$ of the form $\alpha\sigma_0 + \sum_{i=1}^{3} \beta_i \sigma_i$, with eigenvalues

$$\alpha \pm |\boldsymbol{\beta}| \tag{1.19}$$

and expectation values

$$\left\langle \alpha + \sum_{i=1}^{3} \beta_i \sigma_i \right\rangle = \left(\phi, \left(\alpha\sigma_0 + \sum_{i=1}^{3} \beta_i \sigma_i \right) \phi \right) , \tag{1.20}$$

where $\boldsymbol{\beta}$ is a three-component real vector and the σ_μ ($\mu = 0, 1, 2, 3$) are the Pauli matrices. Bell took $\boldsymbol{\beta}$ to have the component values $\beta_1, \beta_2, \beta_3$ with respect to the z-spin axis. Measurement of the property $\alpha\sigma_0 + \sum_{i=1}^{3} \beta_i \sigma_i$ provides eigenvalues

$$\alpha + |\boldsymbol{\beta}|\text{sign}\Big(\lambda|\boldsymbol{\beta}| + \frac{1}{2}|\beta_3|\Big)\text{sign}\mathcal{X} , \tag{1.21}$$

where $\mathcal{X} = \beta_3$ if $\beta_3 \neq 0$, $\mathcal{X} = \beta_1$ if $\beta_3 = 0$ *and* $\beta_1 \neq 0$, and $\mathcal{X} = \beta_2$ if $\beta_3 = 0$ *and* $\beta_1 = 0$; the sign function is defined by the conditions that sign$F = +1$ if $F \geq 0$, and sign$F = -1$ if $F < 0$. One finds, as desired, that the Quantum-mechanical expectation values are indeed recovered by taking a uniform average over the range of values of the hidden variable l.

The *non-local* hidden-variables theories allow the action on a subsystem of a composite system to have an immediate effect on another spacelike-separated system. The alternative quantum theory which had been outlined by Louis de Broglie in the 1920s [76,77] effectively involving such variables was later developed by Bohm in the early 1950s [25]. Such variables had been considered by Born almost immediately after he introduced the Born rule, when he, like de Broglie, was inclined to view the wave-function as a "guiding field" for particles (in the traditionally sense). Rounding out the common classification of hidden-variables models are the *stochastic hidden-variables theories*, which require the hidden variables and experimental parameters to specify the probabilities of measurement outcomes for quantum pure states.

Bell was concerned that the no-signaling condition in quantum theory, as with situations in quantum mechanics wherein measurements are given special status, might introduce extra-physical considerations into the treatment of apparently perfectly ordinary physical situations, while they should be understandable in terms of more fundamental physical concepts. He rhetorically asked,

> Do we…have to fall back on 'no signaling faster than light' as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have lost the idea that correlations can be explained, or at least this idea awaits reformulation. More importantly, the 'no signaling…' notion rests on concepts that are desperately vague, or vaguely applicable. The assertion that 'we cannot signal faster than light' immediately provokes the question: Who do we think we are? We who make measurements, we who can manipulate 'external fields,' we who can 'signal' at all, even if not faster than light. Do we include chemists, or only physicists, plants, or only animals, pocket calculators, or only mainframe computers? ([17], Chap. 6).

He would rather have seen physics appeal to some new sort of non-local causality than have measurement or communication per se be central to physical theory.[54] Although the non-local correlations of measurement outcomes predicted by Quantum mechanics cannot be used for superluminal communication between distant regions, as we have seen,[55] this does not mean that there is not a potentially serious conflict of the theory with Relativity. It would also be rather odd if Quantum mechanics were to be in tension with Relativity for *no* deep mechanical reason but stopped just short of directly contradicting it.

1.4 Non-local Correlations

Einstein appears to have decided that the search for a specific successor to Quantum mechanics was of higher priority than the probing of classes of possible theories on the basis of general characteristics, despite his clear attention to them in principle. In particular, he was strongly convinced that the future of physics lay in the methods of continuous field theory. Instead, it was Bell, inspired by EPR, who was successfully to explore the general implications of the presence of the sort of states in Quantum mechanics of the sort considered by EPR and Bohm. In the process, Bell found an inequality providing a precise limit for the strength of measurement correlations that can be local-causally explained: He showed that this inequality must be obeyed by theories providing local-causal explanations for property correlations, assuming the relevant property of one subsystem is always perfectly anti-correlated with that of the other subsystem. This assumption was relaxed in later, successor inequalities serving the same purpose, further demonstrating their relevance; the strong assumption of perfect anti-correlation is not needed to achieve the sort of result Bell first discovered. Indeed, the extension known as the CHSH inequality

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[54]Bell's general attitude was summarized by Roman Jackiw and Shimony as follows. "Bell felt Niels Bohr and Werner Heisenberg were profoundly wrong in giving observation a fundamental role in physics, thereby letting mind and subjectivity permeate or even replace the stuff of physics…Bell always maintained that what is there to be known has an objective status and is independent of being observed." ([151], p. 83).

[55]For a more detailed proof, see e.g. [87].

was derived without this assumption, allowing itself to be subsequently tested and shown to be violated by a broad range of quantum-mechanical systems.[56]

The motivation for using a local theory to explain of all correlations between two subsystems forming a compound system, such as that described by Bohm's singlet state $|\Psi^-\rangle$, when the systems are spacelike separated has a deep motivation, namely, to sustain what has over recent centuries become the standard for a well formulated, explanatory mechanical theory. Bell felt the power of this impulse directly. He described his theorem in "plain English" as follows.

> It comes from an analysis of the consequences of the idea that there should be no action at a distance, under certain conditions that Einstein, Podolsky, and Rosen focussed attention on in 1935—conditions which lead to some very strange correlations as predicted by quantum mechanics. ([75], p. 45)

Thus, Bell explicitly took on the EPR assumptions.[57] In particular, he considered two spacelike separated measuring instruments, one in lab A and one in lab B with the capacity to record measurement outcomes of a set of physical quantities. The instruments are assumed to be capable of measuring these quantities of a complete state λ of the pair of systems which fully specifies all "elements of physical reality" posited for the pair and may or may not depend on the details of the how measurements are carried out or the specifics of the measurement arrangement. The description is intended to be complete in the sense that there is nothing in the common past of the two systems that is not captured by λ. He took the spin along a single direction as the pertinent bivalent physical magnitude of each subsystem, as in the EPRB argument.

To carry out his derivation, Bell considered a probability measure, $\mu(\lambda)$, on the entire space Λ of parameters providing complete states λ. The expectation values, $E^{\mu(\Lambda)}$, of the bivalent quantities, as random variables, were therefore taken to be, following the schema of Eq. 1.18 but with a factorable integrand:

$$E^{\mu(\Lambda)}(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) = \int_\Lambda A_\lambda(\hat{\mathbf{n}}_1) B_\lambda(\hat{\mathbf{n}}_2) d\mu(\lambda) \ , \tag{1.22}$$

where $\lambda \in \Lambda$, and $A_\lambda(\hat{\mathbf{n}}_1)$ and $B_\lambda(\hat{\mathbf{n}}_2)$ are measurement outcomes along specific directions $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ on the subsystems in A and B, respectively. The inequality at which he arrived is

$$\left|E^{\mu(\Lambda)}(a,b) - E^{\mu(\Lambda)}(a,c)\right| \leq 1 + E^{\mu(\Lambda)}(b,c) \ , \tag{1.23}$$

[56] Most often, tests involve pairs of photons in the singlet state $|\Psi^-\rangle$ [15] of polarization.

[57] Nonetheless, it has been argued that Bell's sense of locality differs in important ways from that of EPR (cf., e.g. [107], p. 61). At a minimum, it can be said that the EPR conditions emphasize system properties more than measurement outcomes, although Bell also later focused on "local beables," that is, local elements of reality rather than observables (cf. [20], Papers 8 and 17).

where $\{a, b, c\}$ is any set of three angles specifying directions of measurement in planes normal to the line of system propagation [15, 17]. Following this, a description of correlated properties of a bipartite system is said to be *local causal* (or *Bell local*) if a definite probability is assigned to the event of there being a positive measurement outcome for every one of the bivalent physical magnitudes of each subsystem by the complete state of the joint system independently of measurements performed on the other subsystem, including when the subsystems are spacelike separated.

Like Einstein, Bell was specifically interested in the question of whether there might be a conflict between the outcomes of measurements of observables characterized and predicted by Quantum mechanics and the principles of Relativity, but sought to probe them by clarifying the relativistic constraints, couched in terms of causal relations. Bell's result made clear the difference of the behavior of composite physical systems with correlations that might be produced by local hidden variables theories from those with correlations according with the predictions of Quantum mechanics. The theorem is a demonstration that the correlations predicted by local-causal theories must obey an inequality. Those of theories violating "local causality," such as Quantum mechanics, need not. Bell later summarized his local causality as the idea that the "direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light" ([20], p. 224).

Similar inequalities were obtained based on weaker assumptions that also imply the factorability of joint detection probabilities (see below) that still suffice for distinguishing Bell-local from Bell non-local correlations. For example, the *Clauser–Horne* (CH) *inequality* is the relatively straightforward algebraic result that the probabilities constrained by Bell's locality condition obey the relation

$$-1 \leq p_{13} + p_{14} + p_{23} - p_{24} - p_1 - p_3 \leq 0 \tag{1.24}$$

(as well as all inequalities resulting from permutations of the above indices), where p_1 and p_3 are the probabilities that the first system spin is found along the *first* of the four directions $\{a, b, c, d\}$ and the second system state is found along the *third* direction; p_{ij} stands for the joint probability of finding the first system state along the direction i *and* the second system state along direction j, $1 \leq i, j \leq 4$. The states can be considered those of generic quantum two-level systems in a space isomorphic to that of the EPRB scenario.

No special restrictions are placed on the complete-state space Λ or on the probability distribution used in the derivation of the CH result.[58] In the general context, the assumption that the joint measured property value probability factors is written

[58]Indeed, the CH inequality follows from the basic properties of probabilities such as that they lie in the interval [0, 1].

$$p_\lambda(A^{(1)}, A^{(2)}|a^{(1)}, a^{(2)}) = p_\lambda(A^{(1)}|a^{(1)})p_\lambda(A^{(2)}|a^{(2)}) \,, \tag{1.25}$$

where the $\{a^{(i)}\}$ are the measuring instrument settings when measurements are made and the $A^{(i)}$ stand for the property values. This condition is known as *stochastic independence*. When this condition is violated, there is an incentive to provide an explanation as to *why* it is violated, often in the form of some sort of underlying cause or causes. According to Quantum mechanics, correlations between pairs of subsystems in any statistical ρ, including those occurring in the joint measurements discussed above, are describable as a bipartite quantum system with the Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. The pertinent observables of the two subsystems can be notated $A^{(i)}$. $A^{(1)}$ and $A^{(2)}$ are uncorrelated between the two subsystems if one can write the state in the form $\rho = \rho^{(1)} \otimes \rho^{(2)}$, where $\rho^{(i)} \in \mathcal{H}_i$ $(i = 1, 2)$. The expectation value of the product of the $A^{(1)}$ and $A^{(2)}$ on the subsystems can then be factored, that is,

$$\langle A^{(1)} \otimes A^{(2)} \rangle_\rho = \mathrm{tr}\big(\rho(A^{(1)} \otimes \mathbb{I})\big)\mathrm{tr}\big(\rho(\mathbb{I} \otimes A^{(2)})\big) \tag{1.26}$$

In this case, the probability of outcomes of joint measurements of the $A^{(i)}$ is the product of the probabilities of outcomes of the measurements performed separately.

When joint measurements involve correlations, it may be the case that the expectation values of sets of measurements of the physical magnitudes $A^{(i)}$ of Quantum mechanics have values that can be written

$$\langle A^{(1)} \otimes A^{(2)} \rangle_\rho = \sum_{j=1}^{n} p_j \mathrm{tr}\big(\rho_j^{(1)}(A^{(1)} \otimes \mathbb{I})\big)\mathrm{tr}\big(\rho_j^{(2)}(\mathbb{I} \otimes A^{(2)})\big) \,, \tag{1.27}$$

where the statistical states of the subsystems are $\rho_j^{(i)}$ $(j = 1, \ldots, n)$ and p_j are probabilities. Any system with an associated density matrix non-trivially of this form (with $n \geq 2$) is typically classified as 'classically correlated,' whether it is a pure or a mixed state, because such correlations can arise in a pair of classical mechanical systems. In such cases, there is no violation of any Bell-type inequality [322].

The joint-detection interference visibility V_{12}, is a quantity measuring of strength of interference that is often used in experiments testing for Bell inequality violation. It is defined as the difference of maximum and minimum detection-event rates of the pattern, which yield probabilities through the relative frequencies they provide, divided by their sum. The highest coincidence interference visibility obtainable in an experiment using classically uncorrelated states, including results predicted by local hidden-variables theory, is 0.5 [243]. By contrast, entangled states can attain visibilities of two-system interference of up to 1.0. Bell-type inequalities can be violated once the visibility surpasses $1/\sqrt{2} \approx 0.71$.[59]

[59] For more on the relationship between interference visibilities and entanglement, see [160].

A decade after his theorem appeared in print, but before tests had weighed in very strongly against Bell locality, Bell discussed the possibility of *local beables,* a term chosen "to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory" ([17], Chap. 7). Local beables correspond to Bell's notion of elements of physical reality. Under a theory describing local beables, the observables of Quantum mechanics could be " 'describable in classical terms', because they are there. The beables must include the settings of switches and knobs on experimental equipment... The theory local beables should contain, and give precise physical meaning to, the algebra of local observables" (ibid.). Bell argued that

> No one is obliged to consider the question 'What can no go faster than light?'. But if you decide to do so, then... you must identify in your theory 'local beables'. The *beables* of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real.... Local beables are those which are definitively associated with particular space-time regions. The electric and magnetic fields of classical electromagnetism... are again examples.... The total energy in all space... may be a beable, but is certainly not a local one. ([20], p. 219)

The requirement of assignability to a bounded space-time region is a significant restriction of the notion of an element of reality, because it goes beyond simple compatibility with relativity theory; it takes on the situation forwarded by Einstein in which one considers "the real in part of space A" and "the real in part of space B" and considers them as existing independently. This took the form of the separability of physical state which assumes that each of its factors is an exclusively locally described state of affairs, as described above.[60]

Bell understood his locality condition to be not incompatible with indeterminism, that is, by contrast with the determinism of theories such as Maxwell's electromagnetism in which the fields in any region of space at a given time fully bounding the backward light-cone of a given space-time region of later times determines the local values within it. He also emphasized that, unlike similar previously introduced notions, his beables are *not* restricted to apparatus when they are not interacting with their object systems. In particular, he wished to avoid imposing any conceptual division of the world into systems and apparatus and arbitrary limitations on the range and duration of interactions, because he was concerned with "the question of principle and not with that of practical approximation" ([20], p. 43) (Fig. 1.10). Bell required that the assignment of values to some beables Λ implies only a probability distribution for a differently parameterized beable A, localized in the same sort of later space-time region as were the events in a second contemporaneous but distinct space-time region 2, and that a distribution of conditional probabilities $p(\mathrm{A}|\Lambda)$ should not describe causes for events in 1 and vice-versa.[61] In general, the

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[60]If the theory does not assume such localized states, as indeed Quantum mechanics doesn't, then it can be expected to violate Bell-type inequalities.

[61]Note also that conditional probability in itself does not depend in any way on causal or even temporal order.

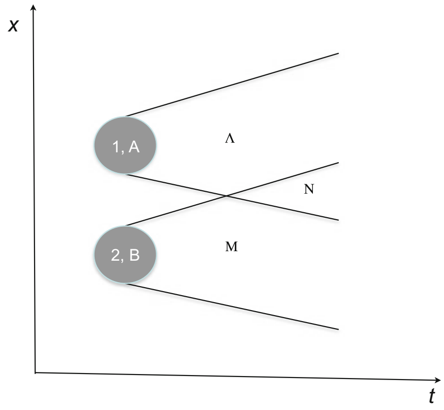

Fig. 1.10 A pair of Bell beables A and B, contained in space-time regions 1 and 2 that are spacelike separated. The regions are assumed to be such that $|A| \leq 1$ and $|B| \leq 1$, cf. Fig. 1.1. The beables Λ, M, and N are specified, N being a complete specification of beables in the space-time region common to the light-cones of 1 and 2 and Λ and M being the regions of their respective light-cones *not* including N ([17], p. 55), cf. Fig. 1.1

conditional probability can be formally defined in terms of joint and single-event probabilities

$$p(A|B) = p(A, B)/p(A)\ . \tag{1.28}$$

Bell defined local causality in this context as follows.

> *Bell locality*. Let N denote the specification of *all* the beables, of some theory, belonging to the overlap of the backward light cones of spacelike separated regions 1 and 2. Let Λ be a specification of some beables from the remainder of the backward light cone of 1, and B of some beables in the region 2. Then in a *locally causal theory* $[p(A|\Lambda, N, B) = p(A|\Lambda, N)]$ whenever both probabilities are given by the theory. ([17], Chap. 7)

Thus, in a local deterministic theory each event is determined by physical law together with the state of affairs in the backward light cone; in the case of probabilistic theory, the probability of the event cannot be changed by conditioning on events at spacelike separation. Events in the two regions may also be correlated due to common causes, an idea brought to the level of principle by Reichenbach, who formalized it as follows.

> *Common-cause principle*. Any two correlated events are either causally connected or arise from a common cause.

When correlations arise that are inexplicable in terms of common causes, that is, fail to satisfy this principle, the laws explaining them are called *cross-sectional laws* ([241], p. 4).[62]

[62]Common causes are discussed in detail in the next chapter.

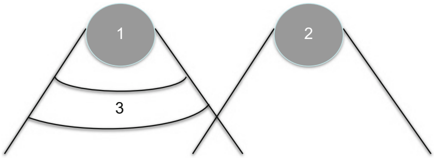

Fig. 1.11 Specification of events in Region 3 renders those in Region 2 irrelevant to predictions regarding Region 3 in the case of locally causal theories as defined by Bell, for example, in the consideration of source-free situations described by Maxwell's equations ([20], p. 225). [Axes (not shown) are rotated 90° relative to the previous figure]

Bell went on to point out that even the *relativistic* version of Quantum mechanics is not 'locally causal' in this sense, because the beables are (Fig. 1.11)

> the settings of switches and knobs and currents needed to prepare the initial [system]. For these are completely summarized, in so far as they are relevant for predictions about [detections]... by the wave function' . ([17], Chap. 7)

He emphasized that

> It is important that region 3 completely shields off from 1 the overlap of the backward light cones of 1 and 2. And it is important that events in 3 be specified completely. Otherwise the traces in region 2 of causes of events in 1 could well supplement whatever else was being used for calculating probabilities about 1. The hypothesis is that any such information about 2 becomes redundant when 3 is specified completely. (ibid.)

This can be written $p(b_1|B_3, b_2) = P(b_1|B_3)$, in relation to an event b_1 in region 1 [206]. Bell also took care to note that "Very often... factorizability is taken as the starting point of the analysis. Here we have preferred to see it not as the formulation of 'local causality,' but as a consequence thereof" ([17], Chap. 22).[63] This emphasizes the connection between his approach and the approach of EPR.

In experimental situations, it is typically impossible to control the putative complete state λ of the composite system of interest. This made practical empirical tests of the early Bell-type inequalities problematic. However, by appropriately modifying the assumptions on the form of measured quantities of Bell, CHSH made testing Bell-type inequalities practical in a broad range of experimental arrangements similar to the arrangement conceived of by Bell, see Fig. 1.9. Their testable form of Bell-type inequality is

$$|S| \leq 2\,, \tag{1.29}$$

where $S \equiv E(\theta_1, \theta_2) + E(\theta_1', \theta_2) + E(\theta_1, \theta_2') - E(\theta_1', \theta_2')$, the Es being expectation values of the products of measurement outcomes given parameter values θ_i and θ_i' of the two different directions $\hat{\mathbf{n}}_i$ for the same laboratory i relative to a reference

[63]The relationship between causes and various conditional probabilities of this sort is treated in detail in the next chapter.

direction as shown [60]; the correlation coefficients contributing to S can be given in terms of normalized system detection rates.

A violation of the left-hand side of the CHSH inequality by a factor of $\sqrt{2}$ beyond its maximum allowed value is possible according to Quantum mechanics. This happens, for example, with a system prepared in the state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ when performing measurements at $\theta_1 = \frac{\pi}{4}$, $\theta_1' = 0$, $\theta_2 = \frac{\pi}{8}$, and $\theta_2' = \frac{3\pi}{8}$, which are steps of $\frac{\pi}{8}$ radians, where the two angles in each lab, that is, on each side of the apparatus differ by $\frac{\pi}{4}$ radians [285]. Here $|\uparrow\rangle$ indicates, for example, photon polarization oriented along one of the orthogonal axes of the plane indicated in Fig. 1.9 and $|\downarrow\rangle$ indicates polarization oriented along the other.[64]

Let us recall Einstein's position on objects well separated in space, for example, a pair of objects A and B, as in the locality criterion of EPR.

> The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B... There seems to be no doubt that those physicists who regard the descriptive methods of quantum mechanics as definitive in principle would react to this line of thought in the following way: they would drop the requirement... for the independent existence of the physical reality present in different parts of space... I still cannot find any fact anywhere which would make it appear likely that (that) requirement will have to be abandoned. I am therefore inclined to believe that the description of quantum mechanics... has to be regarded as an incomplete and indirect description of reality, to be replaced at some later date by a more complete and direct one. ([90], p. 168)

Indeed, as Fine has pointed out, "Einstein is very clear that, in his opinion, the quantum mechanical variables (the 'observables') are the wrong ones. They are not the real physical variables, and that is why it is hopeless to try to complete quantum theory from within" ([107], p. 61). The correlations shown to violate the Bell-type inequalities are the sort of evidence that might have influenced Einstein to reconsider the conclusion of EPR, if any could have.

Although Bell inequality violation was empirically demonstrated without a locality "loophole"—related to the possibility of a dependence of outcomes of detectors on state sources—in the experiments of Aspect et al. [6, 7] as well as later stricter tests (e.g. [320]), other potential experimental loopholes do remain. Nonetheless, their closing is not expected to have an effect on the results obtained; results of CHSH inequality tests show the violation of Bell locality is bounded in accordance with Quantum-mechanical predictions [220]. Unlike in the experimental situation, where challenges are presented by loopholes, the incompatibility of the predictions of Quantum mechanics with Bell-type inequalities can be simply verified: one finds the counterparts in the inequalities and then substitutes quantum

[64]The state $|\Phi^+\rangle$ is also maximally entangled. Since its introduction, the extent of the empirical value of $|S|$ beyond 2 has served experimentalists as a figure of merit for entanglement production. More about entanglement is found in the following section.

probabilities in their place.[65] Bell-type inequalities for pairs of systems of any countable dimension also exist—not only the simple sort discussed above, cf. [65].

The implications for the causal description of microsystems of the above results were described by Bell as follows.

> I cannot say that action at a distance is required in physics. But I can say that you cannot get away with no action at a distance. You cannot separate off what happens in one place and what happens in another. Somehow they have to be described and explained jointly...
>
> [I]f I set up a traditional causal model, in which the cause['s] effects are allowed to be nonlocal, in the sense of propagating instantaneously over large distances, in some frame of reference the cause will come before the effect. So we have to be a bit more subtle than that somehow. I have to find some way out of this situation, which allows something somehow to go from one place to another, very quickly, but without being in conflict with special relativity. And that has not been done. ...The correlations seem to cry out for an explanation, and we don't have one. [19]

The quantum states of composite systems in which pairwise correlations between properties of subsystems can violate a Bell-type inequality are now called *Bell correlated* or *EPR correlated*. When described by state-vectors, these states are entangled. Schrödinger defined state entanglement as vector non-factorability; if a bipartite vector state of a quantum system is entangled, as is the case for example with the Bell (Bohm) singlet state $|\Psi^-\rangle$, then it is Bell correlated with certainty, as was first explicitly pointed out by Sandu Popescu and Daniel Rohrlich [225] and by Nicolas Gisin in the early 1990s [120].[66] However, the class of such entangled states in bipartite systems, such as those known to violate two-party Bell inequalities, although extremely significant, is not the generic class of entangled states, which must also include the statistical states, including the mixed statistical states.

A system in a natural situation, as opposed to an exceptionally well controlled experimental one, often interacts with a number of other systems. Such interactions increase statistical state mixedness; even were a given entangled state of a collection of objects initially pure, it would typically lead to a mixed state, in particular, one obtained from the state of joint system consisting of the system together with its environment by averaging over the degrees of freedom associated with the environment. In light of this, the initial definition of entanglement as non-factorability has been extended to include such mixed statistical states. This was accomplished through the definition of separable states: a bipartite mixed state of a composite system of parts A and B is *separable* if it can be given as convex combination of products of subsystem states:

[65] The bounds on the probabilities and expectation values in Bell-type inequalities are the faces of extreme points in the polytopes of all classically possible correlations.

[66] Not all such states are *Bell states*, that is, elements of the Bell basis as, say, $|\Psi^-\rangle$ and $|\Phi^+\rangle$ are; the Bell basis for the state space $\mathcal{H}_4 = \mathcal{C}^2 \otimes \mathcal{C}^2$ is the set consisting of the following state vectors. $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$.

$$\rho_{\mathrm{AB}} = \sum_i p_i \rho_{\mathrm{A}i} \otimes \rho_{\mathrm{B}i}\,, \tag{1.30}$$

where $p_i \in [0, 1]$ and $\sum_i p_i = 1$, $\rho_{\mathrm{A}i}$ and $\rho_{\mathrm{B}i}$ being states on the respective subsystem Hilbert spaces, and the p_i being classical probabilities.[67] An example of such a state is the following, given by a maximally mixed state, which is proportional to the identity and can be written

$$\rho_{\mathrm{mix}} = \frac{1}{2} P_{|\uparrow\uparrow\rangle} + \frac{1}{2} P_{|\downarrow\downarrow\rangle}\ , \tag{1.31}$$

that is, an evenly weighted ($p_i = 1/2$) mixture of spin-angular-momentum states $P_{|\uparrow\uparrow\rangle} \equiv |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$ and $P_{|\downarrow\downarrow\rangle} \equiv |\downarrow\downarrow\rangle\langle\downarrow\downarrow|$ of an ensemble of pairs of electrons, each statistical state being a product of two identical statistical states. The product states of the form $\rho_{\mathrm{AB}} = \rho_{\mathrm{A}} \otimes \rho_{\mathrm{B}}$, such as $P_{|\uparrow\uparrow\rangle}$ and $P_{|\downarrow\downarrow\rangle}$, correspond to situations in which the states ρ_{A} and ρ_{B} of the two subsystems are *entirely* uncorrelated, that is, are even lacking correlations that do not correspond to violations of the Bell inequality, that is, that are local-causally explicable.[68]

In general for *perfectly correlated* states of composite systems, of which EPR wished to exhibit an example, the outcome of a measurement on one system can be predicted with certainty using the outcomes of appropriate measurements on other subsystems. Remarkably, a decade after Aspect's tests of the CHSH inequality, it was shown by Greenberger, Horne, Shimony, and Zeilinger (GHSZ) that the premisses of the Einstein–Podolsky–Rosen paper become *inconsistent* when applied to systems possessing three or more subsystems, even for the cases involving such perfect correlations [124]. The GHSZ demonstration indicates that the incompatibility of the EPR assumptions with quantum mechanics is stronger than that indicated by the violation of the Bell and CHSH inequalities, in that in the case of a pair of two-level systems there is no internal contradiction at the level of perfect correlations. Indeed, Bell produced an explicit model for the case of a pair of spin-1/2 particles demonstrating the consistency of the EPR conditions with the perfect correlations predicted by Quantum mechanics [16].[69]

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[67]Entangled states of bipartite systems with components labeled A and B are typically denoted using an AB subscript, as in ρ_{AB}, or superscript.

[68]The entangled mixed states ρ are thus precisely the inseparable states. Nonetheless, it is sometimes impossible to tell whether or not a given mixed state is separable. The problem of determining whether a given state of a composite system is entangled is known as the *separability problem*.

[69]Furthermore, the contradiction between quantum mechanical predictions and the Bell and CHSH inequalities are expressions violated only by *statistical* predictions of Quantum mechanics, rather than by individual events.

1.5 Quantum Communication

The conflict between Quantum mechanics and the constraints on the joint probabilities associated with classical behavior discussed above in connection with Bell's theorem created the opportunity to investigate the consequences of quantum theory beyond physics. There are deep ramifications of the violation in laboratory tests of the Bell-type inequalities and the classes of classically inexplicable but correct predictions of Quantum mechanics. Their theoretical significance has now been broadly recognized by physicists and philosophers of science of every stripe, not least for the realist successors to Einstein, Schrödinger, Popper, and Bell.

> A realist might describe the situation this way: Rather than simply providing an opportunity for improving the abstract mathematical laws of quantum theory, the experimental verdict on Bell's inequality offers a new tool for discovering descriptive truths about micro-objects [323].

Bell-type inequalities and the GHSZ result are useful not only for studying microscopic objects: their utility extends to information theory, where entanglement has been shown to play an unexpected role. In particular, there are suggestive implications for the traditional approach to information in the theory of communication, which is formulated in terms of the behavior of probability distributions over sets of possible signal states. This work has involved specific treatments within communication theory itself of *quantum information*, that is, information involving quantum mechanical signal states. These explorations have in turn reflected back on physics, providing novel perspectives [153]. The richness of the study of quantum signals has led to quantum information science being considered a distinct sub-discipline, because it can be extended beyond the requirements imposed by Relativity on the propagation of influences. It now includes the study of the relationship between quantum operations, communication, and entanglement, which have become central to this area of research.

In order to describe quantum mechanically a situation in which signaling might occur, the relationship between the events involved must be precisely specified. In particular, in order to correctly find the pertinent conditional probabilities, one must know which systems are interacting and whether, for example, any pair of events involve a single system or different but possibly quantum-correlated systems. The former involves a single Hilbert space at two different times whereas the latter involves two, possibly at the same time. In classical physics such information may not be needed, even in the case of joint probabilities. In the quantum case, such information is needed for one to specify the Hilbert space(s), states and operators required to find the probabilities for the sets of events involved. Over the decades after Bell's first results, it became clear that for many pure states violating Bell-type inequalities, the further from factorable a state of bipartite system the greater the maximum degree of violation of the CHSH inequality is—the lower limit being that in which the quantum state is simply the tensor product of the component system states [120].[70] A fixed degree of violation is due to the invariance of entanglement under identical local unitary transformations (LUT) of subsystem states, corresponding to the global character of entanglement. An example local unitary transformation is the rotation of the z-spin eigenstate $|\uparrow\rangle$ to the orthogonal state $|\downarrow\rangle$

$$|\uparrow\rangle \rightarrow |\downarrow\rangle \,. \tag{1.32}$$

The effects of other classes of operation on composite quantum systems have been similarly investigated. The most important of these is the general class of local operations (LO), which is that of operations that are carried out on individual subsystems located within the laboratories of their corresponding agents, including unitary operations and measurements occurring with prescribed quantum probabilities. An example non-unitary local operation is the following state transformation of one electron spin of a separable pair. Take the joint state to be initially a tensor product of two states of the form of Eq. 1.6, namely, $\rho_{e^-} \otimes \rho_{e^-}$. Then let one electron initially propagate along the x-direction and the other electron along the x-direction, and let the first electron enter a specific beam of a record-producing Stern–Gerlach device, entering the z-spin state $|\uparrow\rangle$, while the second encounters no such device. The resulting state is[71]

$$\mathcal{O}_{AB}\left(\left(\frac{1}{4}P_{|\uparrow\rangle} + \frac{3}{4}P_{|\downarrow\rangle}\right) \otimes \left(\frac{1}{4}P_{|\uparrow\rangle} + \frac{3}{4}P_{|\downarrow\rangle}\right)\right) = P_{|\uparrow\rangle} \otimes \left(\frac{1}{4}P_{|\uparrow\rangle} + \frac{3}{4}P_{|\downarrow\rangle}\right) . \tag{1.33}$$

The accessibility of states given others also relates to communication. The (subluminal) operations of classical communication (CC) are signal-bearing transmissions between agents in separate laboratories carried out via classical means, and may be in one or two directions, for example, the transmission of a classical signal state encoding the value "0" from the set $\{0, 1\}$ corresponding to one signal bit. The class of "local operations plus classical communication" (LOCC) is that of operations on quantum systems performable by agents acting locally who are also capable of classically communicating. LOCC operations consist of combinations of local unitary operations, local measurement operations, and the addition or disposal of parts of the total system. The distinction between LOCC and LO is significant in that the addition of classical communication between agents to LO, extending it to LOCC, allows the local operations of an agent to be conditioned on outcomes of

[70]For example, Shimony and I noted this in the early 1990s in the course of work on the bipartite-system coincidence interference visibility (entanglement visibility), later allowing for a geometrical means of quantifying entanglement [281].

[71]Note that the operation has been represented here for notational simplicity as a super-operator $\mathcal{O}_{AB}$, which acts on *density operators* in the Liouville space—the space of statistical operators associated with the Hilbert space of state vectors for the joint system. See [153], Sect. B.3 and [106] for a short description of their relationship.

measurements that might be carried out by other agents. Those of its operations that are trace preserving (TP) are referred to as *LOCC protocols*.[72]

The actions of two communicating agents can be correlated in ways describable in Quantum mechanics by global operations which are not necessarily describable as direct products of local operations. However, any quantum operation O_{AB} is implementable by a pair of parties via LOCC when it is separable [314], that is, when it can be written as a convex sum of local operations

$$O_{AB} = \sum_i p_i A_i \otimes B_i \ , \tag{1.34}$$

which guarantees that individual operations are effectively carried out independently in the two laboratories with probabilities p_i. For example, the transformation of the tensor product $D_z(\phi_1) \otimes D_z(\phi_2)$ of two rotations by ϕ_i each of the general form $D_z(\phi) = \exp(-\frac{i}{\hbar} S_z \phi)$ with probability $p_{\frac{\pi}{2},-\frac{\pi}{2}} = 1/2$.[73] The converse is not true, however. Again, the latter is due to the possible influence of communicated classical information on local actions. The class of quantum states that can be prepared from a product state by LOCC is known as the *locally preparable class*.

In quantum information theory, entanglement can be viewed as a resource similar to energy that can take several, interchangeable forms and can be distributed among quantum systems. Questions surrounding quantum resources were among the first considered in quantum information science, which has emerged naturally from the study of physical correlations in the study of quantum systems because it is possible to consider correlations between sender (preparer) and receiver. The separable mixed states, those which can be jointly prepared by N spatially separated observers each preparing one local state $\rho_{A(i)}$ according to a shared set of instructions $\{p_i\}$ [218] 'contain' no entanglement in the resource sense. An agent in one localized region need only sample the probability distribution $\{p_i\}$ and share the his measurement results with an agent in the other in order to create a separable bipartite state. After this, the two agents in separate locations can create their own sets of suitable local states each in classical correlation with the other by appropriate operations on a collection of systems. For fully distributed composite systems, however, because not all entangled states can be converted into each other in this way, the various available transformations give rise to distinct classes of entangled states and different types of entanglement. When there are correlations between properties of subsystems of systems in bipartite separable states, these can

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[72] It is important to note that a LOCC operation is *not* necessarily a TP operation. In the case of operations on a number of copies of a quantum system for any of these classes, the adjective "collective" is added and the above acronyms are given the prefix "C," for example, the CLOCC class is that of collective location operations and classical communication. In cases where transformations are not achievable deterministically, but rather only with some probability, they are considered *stochastic operations* and the adjective "stochastic" is added as well as the prefix "S," as in SLOCC.

[73] Here S_z is defined as above, just below Eq. 1.7.

be fully accounted for locally in the above manner because the separate quantum subsystem states—even when located in spacelike-separated laboratories—provide descriptions enabling such explanations of the joint correlations. The outcomes of local measurements on any system in a separable state can therefore be simulated by a local hidden-variables theory, that is, the behavior of systems described by such states can be accounted for using common-cause explanations, something considered in the following chapter.

In formulating a precise measure of entanglement, it is naturally and conventionally required that the measure be non-negative and normalized in the sense that it be maximum (when normalized, that is, reach 1) for the Bell states, which are those in which the strongest correlations are found. In addition, a fundamental pair of monotonicity conditions has been put forth in contemporary treatments for any candidate measure, below indicated generically as $E_X(\rho)$, to be good a measure of entanglement. This defines the class of entanglement monotones, which are functionals that characterize the strength of genuinely quantum correlations, assuming that no state be convertible by local operations and classical communication (LOCC) to a state having a greater value of the monotone. In particular, a quantity $E_X(\rho)$ is called an *entanglement monotone* if it satisfies the conditions

$$E_X(\rho) \geq \sum_i p_i E_X(\rho_i) \ , \tag{1.35}$$

and

$$E_X\left(\sum_i p_i \rho_i\right) \leq \sum_i p_i E_X(\rho_i) \ , \tag{1.36}$$

for all local operations giving rise to states ρ_i with probabilities p_i, where at the end of the LOCC operation i, classical information is available with probability p_i and the state is ρ_i [315].

Given two sets of entanglement monotones, $E_l^{\Psi} = \sum_{i=1}^{n} |a_i|^2$ and $E_l^{\Phi} = \sum_{i=1}^{n} |b_i|^2$, where $l = 1, \ldots, n$, respectively obtained from the Schmidt decomposition of two bipartite states $|\Psi\rangle, |\Phi\rangle$ and having n components with Schmidt coefficients a_i, b_i, the pure state $|\Psi\rangle$ can be transformed with certainty by local transformations to the pure state $|\Phi\rangle$ if and only if $E_l^{\Psi} \geq E_l^{\Phi}$ for all $l = 1, \ldots, n$ [316]. The Schmidt decomposition is always available for any bipartite pure state $|\Psi\rangle$ in the tensor product space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ of countable dimension [265]: Any such state can be written as a sum of bi-orthogonal terms, namely, in Schmidt form

$$|\Psi\rangle = \sum_i a_i |u_i\rangle \otimes |v_i\rangle \ , \tag{1.37}$$

with Schmidt coefficients $a_i \in \mathbb{C}$, where the sets of vectors $\{|u_i\rangle\}$ and $\{|v_i\rangle\}$ consist of orthogonal unit vectors spanning the space of possible state vectors for the system and the index i runs up to the smaller of the dimensions of the two subsystem Hilbert spaces. For example, the singlet state $|\Psi^-\rangle$ of Eq. 1.17

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\,. \tag{1.38}$$

is of Schmidt form.

The following conditions are those now commonly required of acceptable measures of bipartite entanglement E_X on all states ρ_{AB} of a pair of systems.

(i) $E_X(\rho_{AB}) = 0$ if ρ_{AB} is separable;
(ii) $E_X(\rho_{AB})$ is invariant under *all* local unitary operations $U_A \otimes U_B$, that is, $E_X(\rho_{AB}) = E_X\big((U_A \otimes U_B)\rho_{AB}(U_A \otimes U_B)^\dagger\big)$;
(iii) $E_X(\rho_{AB})$ cannot be increased by *any* LOCC transformation that is, $E_X(\rho_{AB}) \geq E_X\big(\Theta(\rho_{AB})\big)$, where $\Theta(\rho_{AB})$ is a CPTP map.[74]

The entanglement of formation, which takes the form of a von Neumann entropy $S(\rho_X) = -\mathrm{tr}(\rho_X \log_2 \rho_X)$, where ρ_X ($X =$ A, B) is the statistical operator of either one of the two subsystems of the composite system, is the most widely accepted such measure.[75]

In order operationally to find out how much of the resource of bipartite entanglement they share, two parties can concentrate Bell singlet states $|\Psi^-\rangle$ between them. They can "distill," by CLOCC from n copies of an initial bipartite (not necessarily *maximally*) entangled state $|\Phi\rangle_{AB}$, the greatest number $k < n$ of singlet states possible: $|\Phi\rangle_{AB}^{\otimes n} \to |\Psi^-\rangle_{AB}^{\otimes k}$. The resulting state is seen to contain k e-bits of entanglement because each singlet state is assigned one unit of entanglement, that is, one *e-bit*. Distillation can be carried out with an efficiency given by the above von Neumann entropy [227]. This is a reversible process, in the sense that there is an asymptotic scheme in which the inverse conversion

$$|\Psi^-\rangle_{AB}^{\otimes k} \to |\Phi\rangle_{AB}^{\otimes n} \tag{1.39}$$

can be performed, again via CLOCC, with equal efficiency. The monotonicity condition (iii) implies that no entanglement distillation scheme can perform better than such an asymptotic scheme. Entanglement, like heat energy, cannot be increased by local operations on remote subsystems. One thus sees that the shared state contained k e-bits of entanglement, shared between the two parties. The reversible transformations, consisting of only local operations that transform one entangled state into another, produce the analogue of the Carnot cycle. This highly suggestive analogy has stimulated an investigation into the depth of similarity of quantum information theory and thermodynamics.

[74] The class of completely positive trace-preserving (CPTP) linear transformations, $\rho \to \mathcal{E}(\rho)$ often called *operations*, taking statistical operators to statistical operators, each described by a *superoperator*, $\mathcal{E}(\rho)$, satisfying the following conditions. (i) $\mathrm{tr}[\mathcal{E}(\rho)]$ is the *probability* that the transformation $\rho \to \mathcal{E}(\rho)$ takes place; (ii) $\mathcal{E}(\rho)$ is a linear convex map on statistical operators, that is, $\mathcal{E}\big(\sum_i p_i\rho_i\big) = \sum_i p_i\mathcal{E}(\rho_i)$, p_i being probabilities. ($\mathcal{E}(\rho)$ then extends uniquely to a linear map.) (iii) $\mathcal{E}(\rho)$ is a completely positive (CP) map.

[75] For more detail on this, cf., e.g. [153].

The Bell state entanglement resource allows global quantum operations to be performed using quantum state "teleportation."[76] The associated functional, the *entanglement of distillation*, $D(\rho_{AB})$, is defined as the *maximum fraction of singlets* that can be extracted, that is, distilled from n copies of ρ_{AB} by the CLOCC transformation

$$\rho_{AB}^{\otimes n} \to P(|\Psi^-\rangle)^{\otimes k} \tag{1.40}$$

in the asymptotic limit as $n \to \infty$:

$$D(\rho_{AB}) = \lim_{n\to\infty} \sup(k/n), \tag{1.41}$$

where k depends on n. This quantity can be viewed as analogous to thermodynamical free energy and is sometimes called *free entanglement*. It expresses the utility of a state for performing quantum state teleportation.

The consideration of entanglement as a resource for information processing in this way has been shown to bear on issues central to the foundations of quantum theory. Recall from the previous section that the Clauser–Horne–Shimony–Holt Bell-type inequality can be written $|S| \leq 2$, where S is the statistical quantity defined in terms of expectation values for outcomes of collections of joint measurements, and that it has been shown that $|S|$ exceeds 2 both quantum theoretically and in the laboratory. The behavior of quantum property correlations is therefore not in general consistent with 'local causality' but can still be described quantum mechanically, up to and including the upper limiting value of $|S| = 2\sqrt{2}$, a situation that will involve an entangled quantum state, such as the maximally entangled state $|\Psi^-\rangle$ of Eq. 1.17.[77] Indeed, the extent to which $|S|$ surpasses the bound of 2 on the absolute value of the CHSH operator S for local-causal descriptions has been taken as a measure of how 'quantum mechanical' a bipartite system is.

Bell assessed the implications of the violation of the CHSH inequalities by noting the need for an alternative to the world view underlying the EPR program and by noting the connection with limits on signaling for use in communicating information.

> [T]he Einstein program fails, that's too bad for Einstein, but should we worry about that? So what? Now, there are three replies to the question So what? One is that the whole idea of action at a distance is very repugnant to physicists. ...... one is relativity... if we allow the result at one of these experimental set-ups to depend on what an experimenter does at the other, we have a puzzle, because we would not like what he does here to have an effect there, before it is done here. ... [An]other reason is no signals. It is a fact that I cannot use whatever this nonlocal connection is to send signals. When you look at what quantum mechanics predicts, it predicts so long as you look at just one side or other of this experiment, you will simply have no information about what is happening in the other place... we have to invoke some such mysterious power. But it is one which ... I absolutely cannot use to send you a message. [19]

[76] For a discussion of quantum teleportation, see, e.g. [153], Sect. 9.9.

[77] This quantum mechanical constraint is known as the Tsirel'son bound.

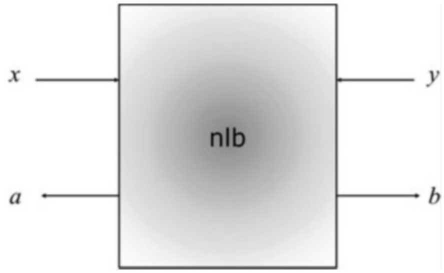

Fig. 1.12 The Popescu–Rohrlich non-local box (nlb). The input bits x, y correspond to settings, resulting in maximally correlated but locally random output bits a, b

Thus, although Bell himself did not directly argue that violation of this inequality distinguishes Quantum mechanics from other conceivable theories besides Classical mechanics, he recognized that the question of whether it does is an important one where communication or its absence matters, even though he considered communication an anthropocentric notion.

For his part, Shimony engaged the question by asking whether "*non-locality* plus *no signaling* plus *something else simple and fundamental*" suffices to uniquely single out Quantum mechanics from the set of conceivable mechanical theories providing correlated local-measurement outcomes [276]. This led Sandu Popescu and Daniel Rohrlich to consider, as a starting point of investigation, whether Quantum mechanics is uniquely distinguished by the conjunction of the properties of '(non-)locality' and causality. In other words, "Is quantum mechanics the *only* causal theory—i.e. theory under which signaling is constrained from above by the speed of light—that violates the Bell inequality?" [226]. Although it is not surprising that the answer to this latter question is "No," this answer adds urgency to the question of what a missing constraint singling out Quantum mechanics might be (Fig. 1.12).

Popescu and Rohrlich then considered a schema they called the *non-local box* (NLB), now often referred to as the *PR box*, to model the *generic* class of 'non-local' theory. This schema is in the realm of thought experiment rather than physics as, it should duly be noted, is the remaining, final portion of this chapter. Like that for tests of the Bell-type inequalities introduced by Bell ([17], p. 151), it accounts for measurement settings, in the sense of conditioning on input bits at distant laboratories, and provides a positive or negative outcome in each laboratory. The PR analysis showed that a theory providing values following this schema could exhibit correlations that *exceed* the quantum mechanical bound. This opened the way for the consideration of the relationship between possible mechanical theories and information in greater generality than before possible. The NLB is defined by its causal provision of maximal correlations. It respects the following conditional probabilities. If the 2-bit outcome string xy is a member of $\{00, 01, 10\}$ then

$$p(m_x^a = 0, m_y^b = 0) = \frac{1}{2} \text{ and } p(m_x^a = 1, m_y^b = 1) = \frac{1}{2};$$

when xy takes the value 11 then

$$p(m_x^a = 0, m_y^b = 1) = \frac{1}{2} \text{ and } p(m_x^a = 1, m_y^b = 0) = \frac{1}{2};$$

all other pertinent probabilities are taken to be zero. The NLB probabilities accord with Einstein locality in that the outcomes on both sides of the measuring apparatus still occur locally at random, as they do in the case of Bell states, but with *stronger* correlations between joint measurement outcomes than those of any quantum state.

It has been shown that a single use of a prior-shared NLB is capable of simulating all outcomes of projective measurements on a Bell state—a maximally entangled pure state—*without* the communication of a bit [58]. Furthermore, there are sets of joint probabilities constrained by the no-signaling conditions that cannot be obtained by measurements on a Bell state. This demonstrates that the non-local box information unit, the *nl-bit,* corresponding to the use of a single NLB is a stronger resource than the e-bit, such as that which can be supported by the state $|\Psi^-\rangle$ of Eq. 1.17 [13, 165]. The existence of quantum protocol known as the *dense-coding protocol* shows that an e-bit allows one to perfectly communicate 2 bits with a single use of a channel that perfectly communicates one qubit, where "perfectly communicates" is shorthand for "communicates the encoding quantum state with perfect fidelity." Furthermore, it has been demonstrated that 1 bit of (superluminal) communication and prior-shared classical randomness together are sufficient to produce all Bell-inequality-violating measurement correlations which are associated with 1 e-bit [58].

Of all the conceivable information-theoretic resources so far explored, 1 bit of supposed super-luminal communication is the strongest resource because, unlike a non-local box, it is not constrained by causality. Thus, in terms of resource strength, there holds a definite ordering relation:

$$1 \;\; \mathrm{e-bit} \;\; \prec 1 \;\; \mathrm{nl-bit} \;\; \prec 1(\mathrm{superluminal})\mathrm{bit}\,, \tag{1.42}$$

here in order of increasing strength, the superluminal bit being unphysical. The communication resource of non-local boxes held by two spacelike separated agents enables them to perform all distributed computations with perfect accuracy given a trivial amount of communication, namely, 1 bit; the single bit can be seen helping "preserve causality" [309].

The natural and physical question then is that of whether what distinguishes quantum mechanics is some physical constraint corresponding to the non-trivial character of the information-theoretic constraint on communication complexity when quantum resources are allowed. This question remains to be answered. In the next chapter, we return to question of the role of a more fundamental factor, namely, causality itself in the quantum world.

Chapter 2
Indefiniteness and Causation

Abstract The basis of the indefiniteness of values of quantum physical magnitudes is explained and related to the theory of probabilistic causation. The motivation for and application of the notion of quantum potentiality and its relationship to the interpretation of quantum probability is explained. Quantum indeterminacy is also discussed and related to quantum property indefiniteness and the notion of wave-particle duality. Various notions of quantum particle are distinguished and those available to modern quantum theory are explicated and critiqued, and in some cases—the mereological and nomological—advocated, in particular, over merely operational notions. The basis for understanding quantum theory as a theory of particles and its consistency with modern notions of light and matter is provided.

The issues surrounding locality explored in the previous chapter are subtly bound up with the assignment of values to physical magnitudes. Before quantum theory, the state of an individual system in fundamental physical theory precisely described all the physical magnitudes pertaining to it; the state of an object readily corresponded to the set of true propositions regarding the values of these quantities, that is, its physical properties. In Quantum mechanics, however, the allowed values are not simply assigned or not assigned, as is the case for previously constructed, deterministic theories such as Newtonian mechanics. Rather, value attributions are probabilistically governed via the Born rule, and not all certainly given. Following Dirac, one says that the functions representing physical magnitudes in Classical mechanics are *c-numbers*, which yield values directly rather than provide probability distributions over allowed values. Logically speaking, the classical theory of individuals fulfills what has been called the *value-definiteness condition*.[1]

[1] In the case of classical statistical situations, states can be given as probability measures $\mu_O : \Delta \to p(O, \mu, \Delta)$ on a phase space specifying the probability that a measurement of the magnitude O will lie in Δ when the system is in the state μ.

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> *Value-definiteness.* "$O \in \Delta$," where O is a quantity describing the physical magnitude and Δ is a Borel subset of the real numbers, regarding a system is assigned a definite truth value, either 0 or 1.

Accordingly, in formal logic, its propositions can be understood—by an approach more common to mathematicians and philosophers than to physicists—as associated with Boolean algebras. Within standard Quantum mechanics, however, the value-definiteness condition has been shown to be impossible to satisfy, in particular, as a consequence of the Kochen–Specker theorem [171]. Although one might have hoped to find some way of accommodating this situation within some classical world picture, a related theorem of Gleason justified the claim, *contra* the conclusion of EPR, that the state description of Quantum mechanics is complete and no classical theory underlies it.

Gleason's theorem identifies the form of all admissible quantum probability measures μ, as functionals of the quantum state $|\psi\rangle$, with that of the standard quantum mechanical measure appearing in the Born rule [121]. Given this complete quantum state description, for example, in the case of position and momentum, neither quantity has even an approximately sharp value in general. They are therefore *not* c-numbers. Furthermore, in general, the more approximately sharp is one of these "incompatible" quantities, the less sharp the other, cf. the Heisenberg relation Eq. 1.3. There is thus an indefiniteness of dynamical variables in Quantum mechanics that varies from situation to situation and is objective, that is, independent of the knowledge of any observer. The EPR article described this objective indefiniteness with respect to a specific spatially represented quantum state of the sort typical to the description of the behavior of a system at the beginning of an experiment, in particular, one assigning a definite value p_0 of momentum to a system moving in one dimension, $\psi = e^{(2\pi/h)p_0 x}$, and in a way that appears stark for realism.

> The definite value of the coordinate for [this] state is thus not predictable, but may be obtained only by a direct measurement. Such a measurement however disturbs the particle and thus alters its state. After the coordinate is determined, the particle will no longer be in [this state]. The usual conclusion from this is that *when the momentum of a particle is known, its coordinate has no physical reality.* [92]

Recall that the EPR criterion for reality is that the value of a physical quantity be *predictable with certainty* without in any way disturbing the system possessing it. The "usual conclusion" mentioned by EPR in the above is, however, more restrictive than necessary, in that it requires that elements of reality be objectively definite, something contrary to what is possible within the probabilistic framework of quantum theory. Furthermore, as in Einstein's earlier ball–box scheme, the state of knowledge—implicitly introduced through the use of predictability in this criterion—and the state of reality are unnecessarily linked together in the EPR approach.[2]

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[2] In this respect, it follows the Laplacian tradition. Unlike the ball–box argument, however, here the main line of argument is unaffected by the linkage.

Given Gleason's result, regardless of the counterintuitive character of the behavior of quantum systems, the following principle must be accepted.

> *Synoptic principle of quantum mechanics*. The quantum state description is the most complete state description possible within its general theoretical framework.

As Reichenbach, who named it, argued, "If the synoptic principle were false, the consequence of the known laws of quantum mechanics without the synoptic principle would be so absurd, so unlikely as compared with the known properties of the physical world, that we can... conclude that the synoptic principle is very likely true" ([242], p. 220).

The possibility of providing a consistent metaphysical realist description of quantum objects remains, despite the violation of Bell-type inequalities. Moreover, physicists do describe the quantum world causally, even though this turns out not to be possible in the traditional and straightforward sense of *deterministic* cause–effect relations. Such a description requires further elements novel to the history of physics not considered, for example, by EPR. Two central metaphysical concepts come into play: (i) indeterminism (less precisely, "acausality"), which regards the relation between physical magnitudes (and thus, states) at different times and which, as seen in the previous chapter, has been long been considered in the history of the foundations of quantum theory, and (ii) indeterminacy (or indefiniteness), which regards magnitudes at just one time.[3] There is a benefit, under-appreciated even by realists, to assenting to the latter: The acceptance of objective indefiniteness, that is, that physical magnitudes can be properties of objects without their all being simultaneously definite, allows for indeterminism without essentially involving any mind in the description of measurement, providing quantum probability with a novel, non-epistemic significance. This notion is in contrast with the characterization of the behavior of physical properties offered by those who argue, as Bohr can be understood as having done early on, that the state of a microscopic system is merely ill-defined *in certain circumstances*, such as during interactions.

Objective indefiniteness is most apparent in the case of the observation of a subsystem of an entangled pair, such as a measuring instrument that has interacted with the object of measurement or one of a pair of photons in the polarization state $|\Psi^-\rangle$, as in the EPRB scenario. Indeed, with the performance of the Aspect experiment in the early 1980s, objective indefiniteness was established as part of physical theory. As Shimony explained not long after, if the Bell singlet $|\Psi^-\rangle$, which is a maximally entangled state for two photons, is a *complete* description of the polarization of each not involving human knowledge of the situation, then "we must accept the *indefiniteness* of the [relevant projections of] polarization of each... as an objective fact, not as a feature of the knowledge of one scientist or of all human beings collectively..." and "must also acknowledge *objective chance* and *objective probability*, since the outcome of the polarization analysis of each

[3] By contrast, *uncertainty* is epistemic and may relate to one time or several times. Although different, uncertainty and indeterminism were both seen by Bohr as subject to complementarity.

photon is a matter of probability" ([280], pp. 177–178). Thus, the rather abstract notion of objective indefiniteness is well motivated not only by mathematics, but also by experience, cf. [154]. Similarly, it will be here argued that the acceptance of *quantum potentiality*, as a mode of physical being intermediate between those of non-existence and actuality with the former leading to the latter as determined by objective chance, is well motivated.

2.1 Probability and Objectivity

The generally non-deterministic character of quantum state transitions makes the theoretical connection between probabilities and the values obtained in measurements of quantum observables, including those in both sharp and unsharp measurements, more direct than in classical (statistical) physics. Probability in physics is typically based on the Kolmogorov axiomatization. The elements involved in that axiomatization are later given an interpretation, that is, probabilities can be connected to quantities and their measurement. Events $A, B, C, \ldots$ and the sample space S of events formed by their union are assumed. The triple (S, F, p), where F is a field of subsets of S, is referred to as a *Kolmogorovian probability space* when the following conditions are satisfied by p, in particular, taking $p(E_i) \in \mathbb{R}$ as the *probability* of the event E_i:

1. For any set of events $\{E_i\}$: $0 \leq p(E_i) \leq 1$;
2. $p(S) = 1$;
3. For any countable sequence of mutually disjoint events $E_1, E_2, \ldots$, $p(E_1 \cup E_2 \cup \cdots) = \sum_i p(E_i)$.

For example, the probability of each of the events $\{E_1, E_2, \ldots E_{38}\}$ of a ball falling into any 1 of the 38 slots of an ideal, standard roulette wheel with 38 pockets is $1/38$ if the ball is dropped at an arbitrary time while the wheel is spinning; the corresponding probability distribution clearly satisfies these conditions.

Interpretations of the quantities governed by such axiomatizations are expected to capture broadly held intuitions, to lend probability predictive significance, to accord with any causal relationships between events and/or processes, and to apply to individual situations [88]. Conceptions of probability, whether Kolmogorovian or of a more general sort, can be broadly distinguished by their being *either subjective or objective*. With regard to probability in physics, it is widely believed that probability should be objective, although recently a subjective Bayesian interpretation has become somewhat more popular in some circles concerned with the foundations of quantum theory, cf. ([154], Sect. 3.7). Probabilities in classical physics are typically needed only when one has imperfect initial-value data, for example, when one lacks knowledge of either the position or the momentum of a point particle or of the specifics of a collection of systems, as in the classical roulette wheel example above. By contrast, probabilities as they arise in quantum theory are necessary even in the case the agent in question, the *subject*, has as much initial value data regarding the system or systems *as is possible* and so, quantum probability is, in general, irreducible.

Accordingly, we take the probabilities of Quantum mechanics to be objective and, in general, irreducible: In the non-statistical-mechanical context, we take them at most to relate indirectly to the state of knowledge of the observer, which is not the primary referent of probability in individual cases. The probabilities arising in Quantum mechanics are broadly recognized as *generalizing* those of Kolmogorovian probability due to their accordance with the Born rule. Recall that Born's rule provides probabilities in terms of the complex-valued components c_i of the state-vector $|\psi\rangle$ when written in the vector eigenbasis $\{|o_i\rangle\}$ of the observable O of interest: $p_{|\psi\rangle}(o_i) = |c_i|^2 \equiv c_i^* c_i$. The probability of a property value o_i being found in a measurement on a quantum system in the *statistical state* ρ is then

$$p_\rho(o_i) = \mathrm{tr}(\rho P_{|o_i\rangle}) , \tag{2.1}$$

where $P_{|o_i\rangle} = |o_i\rangle\langle o_i|$ is the projector corresponding to the eigenvalue o_i. Heisenberg, who first formulated quantum imprecision relations, struggled with the question of whether quantum probability was objective or essentially subjective. In the long run, he viewed only the objective aspect as indispensable, concluding that in Quantum mechanics

> The probability function combines objective and subjective elements. It contains statements about possibilities or better tendencies... [that] are completely objective... and it contains statements about our knowledge of the system, which of course are subjective... the subjective element... may be practically negligible... [which is the] 'pure case'. ([136], p. 53)

Recall that the pure case is that of statistical states of the form $\rho = P_{|\psi\rangle}$, that is, states correctly predicting, via the Born rule, the outcomes of collections of measurements on an individual repeatedly in the same state $|\psi\rangle$ or measurements on collections of individuals each in that state. Heisenberg believed that, as a matter of principle, "quantum theory does not contain genuine subjective features, it does not introduce the mind of the physicist as part of the atomic event" ([136], p. 55) and so takes a realist stance in regard to them.

Although Heisenberg could be interpreted from this as holding that the probabilities assigned to a quantum system merely reflect an ignorance of actual states of affairs and are inferred on the basis of entirely objective data via a statistical syllogism, he instead understood the measurement process as the probabilistic actualization of potential values of the quantity measured. Einstein seems also to have been in some sense open to the idea of something akin to potentiality: Pauli, perhaps the most fervent subjectivist among the investigators of early quantum theory, once said Einstein held that

> Observation cannot *create* an element of reality like position, there must be something contained in the complete description of physical reality which corresponds to the *possibility* of observing a position, already before the observation is actually made. ([175], p. 60)

Pauli understood Einstein as being, like Heisenberg, clearly opposed to any genuinely participatory subjectivism, the notion to which Pauli himself was most drawn [175, 317]. For his part, perhaps surprisingly to some, Bohr also cautioned against attributing creative powers to observation,

> I warned especially against phrases, often found in the physical literature, such as 'disturbing of phenomena by observation,' or 'creating physical attributes of atomic objects by measurements'. ([31], p. 237)

Bohr's one-time post-doctoral understudy John Wheeler was nonetheless among those who did explore the notion of event creation through observation. Wheeler grounded this in the notion of the "participatory universe," one wherein "the elementary quantum phenomenon" is an "act of creation" arising from the choice of the observer to make a measurement.[4]

In contrast to the objectivist understanding of quantum probability even in the setting of a participatory universe, in the subjective interpretations probabilities are not identified with any property of the world but with the *degrees of belief* of rational agents about events and assumes coherence (at a minimum) so that they are personal probabilities, still satisfying the requirements of Kolmogorov. On the subjective interpretations of probability, probabilities are considered to differ categorically from propositions; on these interpretations, quantum probability assignments are not considered propositions within the theory. The subjective conception of probability typically assumes that events are definite and that probabilities arise due to the ignorance of subjects. Specifically, the probability associated with a proposition is a relations between the collection of facts at the subject's disposal and that proposition. A set of alternatives are considered, which are symmetric relative to the agent's ignorance. This then results in probabilities that are uniformly divided over the elements of this set, which is usually assumed to be finite [319]. For example, the increasingly advocated subjectivist approach to probability of Bruno de Finetti takes an agent's degree of belief in an event to be the probability p if and only if *p units of utility* is the price (the so-called 'fair price') the agent would buy or sell a wager that pays one unit of utility if E occurs and 0 otherwise, assuming that there is precisely one such price (an assumption that is often challenged). One first considers a 'Dutch book', that is, a series of bets that guarantees a profit regardless of the outcome in the event about which it is made. It is argued that Dutch books are avoided by an agent if his subjective probabilities obey the Kolmogorov axioms, that is, are coherent, thereby justifying the claim that degrees of belief ought to obey these axioms [167].

An operational definition of subjective probability can readily be given. Upon learning new facts, agents' probabilities are updated in accordance with Bayes' rule and are dependent on their prior probability assignments. One considers an arbitrary sum as being the reward of betting on E and assumes this sum to be

[4]See [324], pp. 189–192. Wheeler stated, however, that only in the case of small numbers of "quantum processes" is the subjectivity significant.

infinitely divisible in principle, in order to guarantee full precision of probability measurement. Utilities are taken to depend linearly on the sums. This approach is more agnostic than other subjective interpretations toward the questions of the existence of objective states of affairs and of mind-independent objects. As applied in the context of quantum theory, it effectively renders quantum physics a theory of beliefs about the world rather than a theory of the world itself,[5] something we reject here.

Several objective interpretations of probability can be clearly distinguished; they include the classical, frequency, and propensity interpretations. The classical conception of probability arose by abstraction from practical situations in which all outcomes are in some sense equally possible.[6] On it, the probability of any one event is the fraction of the total number of events that it represents. An example is the appearance of a sum of the values on the two upward faces of a rolled pair of fair dice. The principle of indifference—that whenever there exists no evidence that favors one possibility over another the two have equal probabilities—is introduced to avoid circularity. The frequency conception of probability is instead based on the direct identification of the probability of events with their relative frequency of occurrence in the total set (reference class) of actual events. A distinguishing element here is the consideration of *actual* outcomes as opposed to possible outcomes. On this conception, probability is defined operationally. This poses a problem in cases where irrational values of probability might be considered necessary because such values clearly cannot exist for finite sets of events, such as *physical* measurements, which clearly cannot ever constitute an infinite class whenever measurement is defined as something which must be carried out by agents. The problem is typically avoided by considering this probability as an *ideal limit* as the number of events becomes infinite, which is counterfactual in character, at some cost to its operational character.

The propensity conception of probability, in which probabilities are understood as dispositions, is closely related to potentiality. It appeared relatively recently, for example in the writings of Charles Sanders Peirce of the early 1930s, reaching a degree of prominence first in the 1950s and being of continuing interest since. Dispositions are ascribed to re-identifiable entities which have properties capable of change, for example, a glass which may break. Under appropriate circumstances the disposition may become manifest. An example is the solubility of a chemical sample: A certain amount of a sample will dissolve if it is mixed with water, whether or not this mixing actually takes place. In contrast to the above alternative objective interpretations of probability, the propensity approach takes probability to be a physical disposition or tendency of a situation in the world to provide each kind of outcome.

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[5] For an extended discussion of the corresponding Radical Bayesian interpretation of Quantum mechanics, see [154], Sect. 3.7.

[6] This is an approach to probability that predates the Kolmogorov axiomatization.

2.2 Possibility and Potentiality

The propensity interpretation of probability is most distinct from other probability interpretations in its relation to singular causal sequences. Indeed, as Wesley Salmon pointed out, the term *propensity* has a causal aspect that is not part of the traditional meaning of the term *probability*; he opined that there are probabilistic causes in the world appropriately called 'propensities' that *produce* relative frequencies and play a role in quantum theory ([255], p. 14). Care must be taken in such a move, however, because dispositions are often taken as most legitimate when they are understood to introduce, at least implicitly, some underlying theory through which a traditional causal account can be given for what they are taken to explain.

Probability can indeed be viewed as deriving its meaning from the role it plays in a given theory, such as quantum theory,[7] but in the process one must be careful not to identify propensity with (i) its structural basis, because one must be able to account for the fact that degrees of propensity can be dependent on relational properties so that inappropriate identifications may arise,[8] (ii) its manifestations, because it would in many cases then be intermittently present or be continually manifest contrary to the idea of un-actualized potentiality, and (iii) disposition in general (cf. [148], pp. 62–63).[9,10] Two prominent approaches to individual systems in Quantum mechanics to which potentialities are related are that of Heisenberg, involving the actualization of properties which are generally already potentially possessed, and that of Popper, for whom they are propensities only to *take on* properties in the future. Our primary interest here is in a potentiality approach to quantum mechanics stemming from that which originated in the thought of Heisenberg, wherein the probabilities given by the Born rule as applied to state-vectors refer to individual situations, rather than in an interpretation of probability itself.

Although Popper's idea of propensity is not restricted to quantum mechanics, and was intended as a notion allowing for application of probability to individual cases in general, it was thought particularly useful in the quantum case [228, 230]. He considered the probability of a measurement outcome of a given type to be the propensity of a repeatable experiment to produce the given outcome with that limiting relative frequency [228]. Henry Margenau made a related important early contribution to the explicit theory of quantum propensity in his "latency" approach, in which he identified a system's having, when in a state $|\psi\rangle$, the "latent" property

[7] One objection to the broad interpretation of probability as propensity is that, although there are inverse probabilities, there are no inverse propensities, as pointed out by Paul Humphreys [148].

[8] Hence, propensities are propensities *for* something to occur, rather than simply propensities *full stop*.

[9] Both Salmon and Humphreys argued that there is a strong reason for rejecting the propensity interpretation of probability itself: there are probabilities that *cannot* be understood as propensities.

[10] Hugh Mellor, has suggested, as an alternative, that one understand propensities as not probabilities but instead as merely related to chances which he views as statistical physical probabilities [192].

of manifesting the eigenvalues $\{o_i\}$ associated with an observable O with the probabilities $\{p_{o_i}\}$ given by the Born rule [188]. It can be argued that dispositional accounts of quantum properties have been implicitly used in quantum physics almost since its formulation because, for example, "transitions between quantum orbitals can be described as stochastic processes that *bring about* certain values of quantum properties with certain probabilities" [300].

By contrast with these conceptions, Heisenberg's notion of potentiality is explicitly metaphysical. According to him, potentiality relates to actuality in that there are transitions between the potential and actual modes of existence by individual systems, specifically during measurements.

> [After the measurement] interaction has taken place, the probability function contains the objective element of tendency...the transition from the 'possible' to the 'actual' takes place during the act of observation...It applies to the physical, not the psychical act of observation. And we may say the transition from the 'possible' to the 'actual' takes place as soon as the interaction of the object with the measuring device, and thereby with the rest of the world, has come into play ([136], p. 54).

Shimony later explicated this as the idea that in quantum world "there is a modality of existence of physical systems which is somehow intermediate between bare logical possibility and full actuality, namely, the mode of potentiality" ([280], p. 177; cf. [284], p. 108). One can understand actualization as a transition of mode of existence *under appropriate physical circumstances*, such as the appropriate application of a reliable measurement instrument.

Deviating from the usual reading of Heisenberg, Kristian Camilleri, has more recently argued that for Heisenberg this transition was not physical but rather due to a necessary change of *description* from a quantum to a classical description. Camilleri argues that one should understand *actual* and *possible* as *two modes of description* in which spatial and temporal language is used "at some level." The transition from potentiality to actuality, according to him, is a transition from one mode of description, the quantum-mechanical mode, to another, the classical space-time mode [55]. However, although it is highly plausible that a need for a change of physical description within quantum theory from a closed system to an open system description can be appropriate during measurement—as Heisenberg put it, "during an act of observation," that "takes place as soon as the interaction of the object with...the rest of the world," that is, on the basis of a physical event—the result of adopting the understanding suggested by Camilleri is for the realist a schizophrenic sort of explanation of what takes place in the measurement process: it suggests a linguistic solution to what is a metaphysical and physical problem. There is a significant threat of circularity in any attempt to explain quantum state transitions during measurement if the linguistic transition involved is explicitly dependent on the observer's consciousness, because as Patrick Heelan argued, "Logically implied in Heisenberg's view of the measurement process is the position that the behavior and pattern of objects in human empirical consciousness are also subject to quantum mechanical laws" ([129], p. 97).

Here, by contrast, a straightforward understanding of quantum mechanics via quantum potentiality that is metaphysical and does not involve an essential dependence on language, or even the presence of a conscious observer is offered. Quantum potentiality is the metaphysical counterpart of objective probability, which in light of the empirical violation of local-causal realism must be accepted by metaphysical realists. It should be noted in particular here that quantum superposition occurs among potentialities, represented by the state-vector components, such as the c_i in Eq. 1.1, and not among the probabilities; according to the Born rule, it is the squares of these components that provide the statistics of actualities occurring upon measurement and are governed by objective chance. Moreover, because of the vectorial nature of the quantum state, the potentialities relating to the various possible actual outcomes in the event of observation can be understood via interference between amplitudes for various processes which are physically possible between the phenomena recorded in preparation and measurement events.[11]

2.3 Quantum Indeterminacy

The precision of simultaneous specifiability of values of a pair of physical magnitudes in Quantum mechanics is limited, in general, as specifically characterized by the Heisenberg–Robertson relations,[12] commonly referred to as Heisenberg *uncertainty relations*. The term *uncertainty* in quantum mechanics need not always, and at the most fundamental level *should not be* understood to be merely epistemic. The Uncertainty principle introduced by Heisenberg was that "canonically conjugate quantities can be determined simultaneously only with a characteristic inaccuracy," which is a statement about imprecision rather than lack of knowledge [133, 135][13]; the Heisenberg–Robertson relations are far from sufficient grounds on which to take Quantum mechanics as merely describing human knowledge of physical objects or situations. Furthermore, one can very well found quantum theory without reference to the Heisenberg relations, the significance of which for the theory has been exaggerated, in the opinions of many. One finds, for example, Feynman downplaying the importance of the Uncertainty principle, as follows.

> I would like to put the uncertainty principle in its historical place: When the revolutionary ideas of quantum physics were first coming out, people still tried to understand them in terms of old-fashioned ideas (such as, light goes in straight lines). But at a certain point

[11]This perspective, discussed below, underlies Feynman's influential approach to quantum mechanics, discussed in Chap. 3.

[12]Given by Eq. 2.3. This is so, arguably, at the level of principle, in that one can adequately formulate quantum theory with the uncertainty taken as one of its principles.

[13]Max Jammer accordingly noted that "The term used by Heisenberg in these considerations was *Ungenauigkeit* (inexactness, imprecision) or *Genauigkeit* (precision, degree of precision). In fact, in his classic paper these terms appear more than 30 times (apart from the adjective *genau*), whereas the term *Unbestimmheit* (indeterminacy) appears only twice and *Unsicherheit* (uncertainty) only three times. Significantly, the last term, with one exception (p. 186), is used only in the Postscript, which was written under the influence of Bohr." ([161], p. 61).

> the old-fashioned ideas would begin to fail. ... If you get rid of [them] and instead use [the addition of amplitudes of indistinguishable processes] there is no need for an uncertainty principle! ([105], pp. 55–56)

Nonetheless, indeterminacy is a basic element of the quantum world view and these relations remain of great value to physics; no matter how their epistemic significance may have been exaggerated by attachments to naive realism, it remains the case that if the Synoptic principle is assented to, as Reichenbach pointed out, "Heisenberg's indeterminacy is inescapable," at a minimum in the form of the Heisenberg–Robertson relations ([242], p. 214).

The most common situation in which the principle is specifically considered is in relation to the trade-off between the precision of specification of position and momentum, as in Eq. 1.3, whether in preparation or measurement. The Heisenberg relations are most often expressed in terms of the dispersions of Hermitian operators for quantum states. The *dispersion* of an operator A, given in a general quantum state ρ, is $\mathrm{Disp}_\rho A \equiv \langle (A - \langle A \rangle \mathbb{I})^2 \rangle_\rho = \langle A^2 \rangle_\rho - \langle A \rangle_\rho^2$. The square root of the dispersion is the 'uncertainty'

$$\Delta A \equiv \sqrt{\mathrm{Disp}_\rho A} \tag{2.2}$$

of A in state ρ. More generally for two non-commuting quantum observables A and B, one deduces the following from the postulates of quantum mechanics,

$$\langle (\Delta A)^2 \rangle_\rho \langle (\Delta B)^2 \rangle_\rho \geq \frac{1}{4} |\langle [A, B] \rangle_\rho|^2 \ , \tag{2.3}$$

namely, the Heisenberg–Robertson relation. Most importantly, operators for canonically conjugate quantities do not commute, so that the right hand side of Eq. 2.3 is non-zero. One can find the range of likely values of individual observables in many situations with great precision, but any observable that does not commute with one that is at any given moment precisely determined will be poorly specified at that time, a fact pertaining, for example, to the EPR argument.

Heisenberg provided a thought experiment, namely, the measurement of an electron's position with a gamma-ray microscope [133, 135, 244] to illustrate that an increase of accuracy of position measurement (by shortening the wavelength of the gamma ray) corresponds to an increase of momentum transfer to the observed particle and that "the inaccuracy of the measurement of the position can never be smaller than the wavelength of the light" ([136], pp. 47–48).[14] Using this example, he argued that

> ... in the act of observation at least one light quantum of the γ-ray must have passed the microscope and must first have been deflected by the electron. Therefore, the electron has been pushed by the light quantum, it has changed its momentum and its velocity, and one

[14] Modern technology has made the Heisenberg microscope, which can be viewed as exploiting the Compton effect, more than a thought experiment, cf. [329].

> can show that the uncertainty of this change is just big enough to guarantee the validity of the uncertainty relations. (ibid.)

Although he believed that quantum uncertainty is manifested in the act of observation, Heisenberg explicitly cautioned against considering momentum transfer the *cause of imprecision* in this example.[15]

In debate with Einstein during the Fifth Solvay conference, Bohr provided an analysis along similar lines to Heisenberg's discussions of the microscope but for the double-slit experiment.[16] The Young double-slit experiment is a standard experiment for the consideration of non-commuting observables in quantum mechanics. In the experiment, many identically prepared systems such as electrons are directed precisely normally toward a double-slit diaphragm and, if not absorbed by it, continue on to an opaque screen which acts as an array of area detectors. Einstein argued that the transverse momentum transferred by particles when passing the diaphragm could be measured with arbitrary precision and that the position could also be arbitrarily precisely measured by a sufficient reduction of the width each of the slits. Bohr then pointed out that the transferred momentum was sufficiently uncontrollable in the apparatus that the relation would, in fact, be obeyed.

Three distinct types of Heisenberg relation have now been distinguished. The specific sort pertaining in a given situation is determined by whether values are provided by states, simultaneous measurement accuracies, or measurement sequences, and can be understood as arising in different ways under different interpretations of the quantum formalism.[17] The mathematical relation for quantum statistical states ρ considered above in regard to position and momentum in one dimension, is

$$(\Delta X)_\rho(\Delta P)_\rho \geq \frac{\hbar}{2}\ , \tag{2.4}$$

with the dispersions of the (non-relativistic) position operator X and the momentum operator P (which is the generator of translations) calculated for the same state ρ [244]. The minimum joint uncertainty for this relation is achieved for systems having Gaussian wave-functions; an electron in such a state will be described in one-dimension by a wave-function

[15] This came at the prompting of Bohr, who argued that, "The reciprocal uncertainty which always affects the values of those quantities is... essentially an outcome of the limited accuracy with which changes in energy and momentum can be defined..." ([29], p. 63). Bohr's sense of the term in this regard became so influential that, as Vladimir Fock once said, " 'principle of complementarity' came often to be erroneously understood as a *synonym* for the Heisenberg relations" (here cited in translation by Jammer [161], p. 60).

[16] For a detailed discussion of this encounter, see [161], pp. 127–129.

[17] It also does not simply correspond to 'wave–particle duality.' The first two forms were considered by Heisenberg between 1927 and 1930 [52, 133, 135].

$$\psi(x,t) \approx \left(\frac{2}{\pi}\right)^{1/4} (\Delta k)^{1/2} \exp\left[-\frac{(x - \hbar k_0 t/m)^2}{4(\Delta x)^2}\right] e^{ik_0 x}\ , \tag{2.5}$$

where the $k = p/\hbar$ is the wavenumber, cf. [198].

Heisenberg described the outcome of a joint measurement of position and momentum in terms of the above uncertainties [133]. The Heisenberg relation for simultaneous *measurement accuracies* $\delta x, \delta p$ of the same properties (position and momentum) is

$$(\delta x)(\delta p) \geq \frac{\hbar}{2}\ . \tag{2.6}$$

Another sort of measurement-related Heisenberg relation may also be discerned. It is an accuracy–disturbance trade-off relation for measurement sequences.[18] However, the relationship between these two depends on one's interpretation of Quantum mechanics. The differences of applicability of different sorts of Heisenberg relation can easily lead to confusion in regard to the distinction between indeterminism and uncertainty. This relates to the fact that probabilities associated with statistical states ρ may have both an objective and a subjective aspect, as pointed out by Heisenberg.

There also exist Heisenberg relations between time and energy, even though in Quantum mechanics time is a parameter and is not represented by an operator. The reach of the Heisenberg relations is long. They prove important even for matters such and individuality and identity, which are taken up below. For example, some have argued that, as Schrödinger commented,

> As regards the modification required in the concept of particle, the stress is on Heisenberg's uncertainty relation. We have taken over from previous theory the idea of a particle and all the technical language concerning it. This is inadequate. ... Its imaginative structure exhibits features which are alien to the real particle. ... The uncertainty relation refers to the particle. The particle... is not an identifiable individual. [268]

Schrödinger justifies this final statement, in particular, by the fact that in the quantum field theoretical context, following second quantization, one finds the particle number generally indeterminate in that the expectation value can be non-integral. Nonetheless, an indeterminate number of particles is never observed when the quantity is actually measured.[19] Despite the foregoing caveats and the existence of several versions of them, sometimes confused with each other, uncertainty relations are often fruitfully used to understand fundamental processes as described by the most sophisticated sorts of quantum theory.

[18] This relation was first explicitly discussed by Pauli in 1933: $\delta x Dp \geq \frac{\hbar}{2}$, where Dp indicates the disturbance of what was initially a momentum eigenstate resulting from a position measurement of accuracy δx [114, 213].

[19] The question of the individuality of elementary particles such as the electron and photon and their relation to quantum field theory is taken up in the Chap. 4.

It is often argued that these relations provide bounds on the applicability of wave and particle notions: Relativity associates with a system of mass m a momentum scale $p = mc$ which together with the associated Compton wavelength h/mc provides a bound for the consistent application of the particle notion to the system (cf. [149], p. 46). In the case of the electron, for example, this length is 3.8×10^{-11} cm. To this, it is sometimes added that the attempt to analyze elementary particle systems at the Compton scale or below *requires* the consideration of anti-particles. The latter is significant for the question of the most appropriate notion of particle in quantum theory. In relation to the behavior of forces, Rudolf Peierls used a time–energy uncertainty relation to explain fundamental processes in the following different but interconnected aspects. First, he uses it to explain the stability of electromagnetically bound atoms.

> [A]ccording to quantum mechanics, the electron in a hydrogen atom cannot lose energy. . . If no energy is available, how can photons be produced? The answer is provided by the so-called *uncertainty principle*. . . although energy is strictly conserved, so that it cannot be created or destroyed, but only transferred from one part of a system to another, you cannot verify this conservation in a short time. . . . . . [it is possible] for a system to 'borrow' energy for a short time provided the amount is small enough for the lack of it not to be detectable in time. ([215], pp. 37–38)

Second, Peierls uses it to provide a causal description of the operation of fundamental forces.

> We don't like action at a distance, and prefer to think that. . . any force of interaction between particles is transmitted by some intermediary, which we call a field. . . in quantum theory, with every field goes a quantum, or particle, like the photon. . . think of the electromagnetic interaction as caused by the emission of a virtual photon by one particle and its absorption by another. . . so we can. . . illustrate the [strong nuclear force] interaction. . . of very short range, and this will follow if the particle being exchanged is not massless, like the photon, but has an appreciable mass. . .

In addition, he uses it to explain the concomitant relationship between the range of fundamental forces and the masses of the particles that are understood as their mediators.

> One can estimate the range [of a force] by the following argument. If the amount of energy that has to be borrowed to provide the intermediate particle is E, it must be 'repaid' after a time $\hbar/E$. Since no action can be propagated faster than the speed of light, c, the greatest distance on which it can have an effect is $\hbar c/E$. To create a 'virtual' particle of mass m, the necessary energy is, by Einstein's relation, $E = mc^2$, so the distance is $\hbar/mc$. ([215], p. 42)

In general, in order to probe matter at the scale of elementary particles, high energies are needed, which is one reason why the terms "high energy physics" and "elementary particle physics" are often interchangeably used. It is noteworthy that, as shown by Theodore Duddel Newton and Wigner, for relativistic motions of the electron governed by the Dirac equation, one finds that the quantity just mentioned, the reduced Compton wavelength $\lambda_c = h/mc$ divided by 2π, provides the lower limit for the extent to which an electron can be spatially localized if it is to be understood in terms of positive energy plane-wave states—below this scale antiparticle (negative energy $\equiv$ positron) states are also required to describe the associated phenomena.

2.4 Quantum Properties

Another perspective on the quantities related by the Heisenberg relations can be attained by considering them in the positive-operator-valued measure (POM) formalism. In general, this formalism can be considered in a rather broad mathematical setting where the spectrum of the Hermitian operator O representing a physical magnitude may be continuous or discrete. A measurement of a Hermitian operator can be understood as returning a value within a Borel set $\Delta \in \mathbb{R}$, leaving the state of the system with support (O, Δ) with respect to O. For example, projector $P_O(\Delta)$ from the spectral decomposition of O might describe the quantum mechanically maximally specified state of such a system. For simplicity, however, similarly to considering spin observables in an EPR *gedanken* experiment, instead of X and P let us here instead consider measurements in the simpler case of discrete spectra, as described by Eq. 1.7. Although novel conceptual issues arise from the introduction of POMs, the so-called unsharp measurements described below provide a well-defined set of generalized quantum observables and allow for the extension of previously considered notions, such as Einstein's elements of reality [50, 291].

Given a nonempty set S and a σ-algebra Σ of its subsets X_m, a POM E is a collection of operators $\{E(X_m)\}$ acting on Hilbert space $\mathcal{H}$ that satisfies the following conditions, formally similar to the Kolmogorov axioms.[20]

(i) *Positivity*: $E(X_m) \geq E(\emptyset)$, for all $X_m \in \Sigma$.
(ii) *Additivity*: for all countable sequences of disjoint sets X_m in Σ,

$$E(\cup_m X_m) = \sum_m E(X_m) \; . \tag{2.7}$$

(iii) *Completeness*: $E(S) = \mathbb{I}$.

If the *value space* (S, Σ) of a POM E is a subspace of the real Borel space $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$, then E provides a unique Hermitian operator on $\mathcal{H}$, namely $\int_{\mathbb{R}} \mathrm{Id}\, dE$, where Id is the identity map. The positive operators $E(X_m)$ in the range of a POM are referred to as *effects*, the expectation values of which provide quantum

[20] A *Borel σ-algebra* is the σ-algebra generated by the open intervals (or the closed intervals) on a topological space—for example, in $\mathbb{R}$—which are the Borel sets. The set S is often a standard measurable space, that is, a Borel subset of a complete separable metric space. Because such spaces of each cardinality are isomorphic, they are all measure-theoretically equivalent to Borel subsets of the real line, $\mathbb{R}$. The sequences here are taken to converge in the weak operator topology on $\mathcal{L}(\mathcal{H})$ [50].

probabilities. Each statistical state ρ induces an expectation functional on $\mathcal{L}(\mathcal{H})$, the space of linear operators on the Hilbert space $\mathcal{H}$ providing well defined probabilities because the effects are bounded by $\mathbb{O}$ and $\mathbb{I}$, so that the ranges of the effect spectra are restricted to lie in the closed unit interval, due to the positivity and normalization of the POM; the operator ordering $\leq$ on the effects has the zero operator and identity as its upper and lower bounds.[21] POMs are thus the natural correspondents of standard probability measures in the operator space of Quantum mechanics: The probability of outcome m of a generalized measurement of a system in state $|\psi\rangle$ is given by

$$p(m) = \langle\psi|E(X_m)|\psi\rangle = \mathrm{tr}\big(P_{|\psi\rangle}E(X_m)\big) , \tag{2.8}$$

so that for the statistical state ρ this probability is given by

$$p(m) = \mathrm{tr}\big(\rho E(X_m)\big) . \tag{2.9}$$

The consequence of a POM measurement on the initial state is a post-measurement state ρ'_m, which results with corresponding outcome probability $p(m)$. The post-measurement states of a collection of systems initially described by a statistical operator ρ under a POM $\{E(X_m)\}$ are often taken to be of the form

$$\rho'_m = \frac{M_m\rho M_m^\dagger}{\mathrm{tr}\big(M_m\rho M_m^\dagger\big)} , \tag{2.10}$$

where the $E(X_m)$ can be written $M_m^\dagger M_m$, M_m being called *measurement decomposition operators*, cf. [204]; in the special case that the M_m are projectors, this expression is the *Lüders–von Neumann measurement rule*. The *unsharp measurements* are the class of quantum operations that must be written as (normalized) POMs [73]; when, and only when, the measurement operators M_m are *projectors*—so that the POM is a *projection-valued measure* (PVM)—are they identical to the decomposition operators $E(X_m)$, in which case they are also multiplicative, that is, $E(X_m \cap X_n) = E(X_m)E(X_n)$ for all countable subsets of the corresponding set—equivalently, $E(X_m)^2 = E(X_m)$.

Operationally, when providing positive outcomes, POM elements allow one to eliminate some quantum states from consideration as correct descriptions of the measured system.[22] The effects form a convex subset of $\mathcal{L}(\mathcal{H})$, the extremal

[21] Only if the Hilbert space in question is $\mathbb{C}$ do the effects constitute a lattice; a complementation $^\perp$ defined by $E \equiv 1 - E$ exists that satisfies $(E^\perp)^\perp = E$ and reverses the operator order but is not an orthocomplementation, so that law of the excluded middle does not hold, however. The projection operators *do* form an orthocomplemented lattice with just this order and complement. The lattice of projection operators $\bar{L}(\mathcal{H})$ has the *sharp properties* as its elements.

[22] An example of a POM used in this way is the following [21]. Given the two projectors $P_{\neg|\phi\rangle} \equiv \mathbb{I} - P_{|\phi\rangle}$ and $P_{\neg|\phi'\rangle} \equiv \mathbb{I} - P_{|\phi'\rangle}$, where $\langle\phi|\phi'\rangle = \sin 2\theta$, one can construct a POM $\{E_m\}$ with the

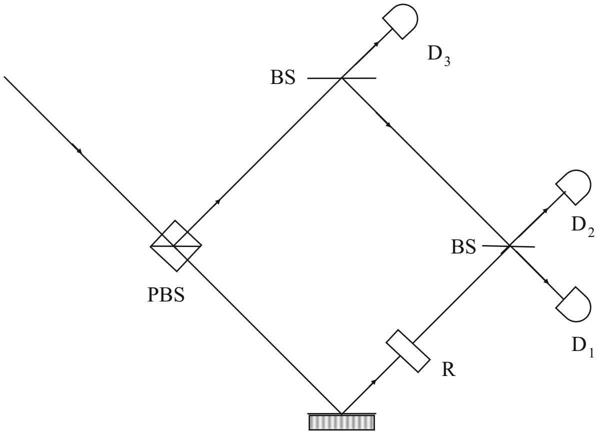

Fig. 2.1 The realization of a projection-operator-valued measurement in linear optics. PBS indicates a polarizing beam-splitter such as a Wollaston prism, BS indicates an ordinary 50–50 beam-splitter, and R indicates a polarization rotator. D_3 provides indefinite-value detections and D_1 and D_2 provide definite-value detections [37]

elements of this subset being the familiar projection operators of quantum theory. A collection of effects is said to be *coexistent* if the union of their ranges is contained within the range of a POM. Any two quantum observables E_1 and E_2 are jointly representable as PVMs on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ exactly when $[E_1, E_2] = \mathbb{O}$, following from results of von Neumann for Hermitian operators. For POMs, however, commutativity remains sufficient but is *not necessary* for coexistence [50]. Thus, the consideration of POMs outside of the subset of PVMs, that is, the measures corresponding to unsharp measurements allows one to circumvent the restriction of commutativity on measurements of non-commuting observables by including *unsharp properties*.

To see this, first consider the so-called *regular effects*, which are those effects with spectrum both above and below $\frac{1}{2}$. One can define *properties* in general by the following set of conditions, given an effect A.

(i) There exists a property $A^{\perp}$;
(ii) There exist states ρ and ρ' such that both $\mathrm{tr}(A\rho) > \frac{1}{2}$ and $\mathrm{tr}(A\rho') > \frac{1}{2}$;
(iii) If A is regular, for any effect B below A and $A^{\perp}$, $2B \leq A + A^{\perp} = \mathbb{I}$ (This renders $^{\perp}$ an orthocomplementation for the regular effects.)

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elements $E_1 = P_{\neg|\phi\rangle}/(1 + |\langle\phi|\phi'\rangle|)$, $E_2 = P_{\neg|\phi'\rangle}/(1 + |\langle\phi|\phi'\rangle|)$, $E_3 = \mathbb{I} - (E_1 + E_2)$, see Fig. 2.1. POM measurements using $\{E_1, E_2, E_3\}$, for example, are more efficient for quantum key distribution and quantum eavesdropping than traditional measurements described by the projectors $\{P_{\neg|\phi\rangle}, P_{\neg|\phi'\rangle}\}$. Similarly, POMs sometimes allow quantum state tomography to be performed with improved efficiency.

The set of properties

$$\mathcal{E}_p(\mathcal{H}) = \{A \in \mathcal{E}(\mathcal{H}) | A \not\leq \frac{1}{2}\mathbb{I}, A \not\geq \frac{1}{2}\mathbb{I}\} \cup \{\mathbb{O}, \mathbb{I}\} \tag{2.11}$$

satisfies these conditions. The set of *unsharp properties* is then $\mathcal{E}_u(\mathcal{H}) \equiv \mathcal{E}(\mathcal{H})_p / L(\mathcal{H})$. A POM is an unsharp observable if there exists an unsharp property in its range [50]. Coexistent observables are those that can be measured simultaneously in a common measurement arrangement; when two observables are coexistent, there exists an observable the statistics of which contain those of both observables, known as the *joint observable*—typically, the two observables are recoverable as marginals of a joint distribution on the product of the corresponding two outcome spaces. This provides additional perspective on the Heisenberg relations by showing, for example, that the quantities appearing together in them can also be simultaneously measured.

Now, take the quantum probability as characterizing the tendency to actualize properties as observed in experimental outcomes in the ontological sense, along the lines of the interpretation of quantum probabilities introduced by Heisenberg and discussed in the previous section. Specifically, let us take the expression

$$p^E_{|\psi\rangle}(X_m) = \langle\psi|E(X_m)\psi\rangle \tag{2.12}$$

for the effect $E(X_m)$ associated with the value set X_m as providing the likelihood of the actualization of the potential property, whether sharp or unsharp, when measured on a system prepared in the state $|\psi\rangle$. Recall that there is no direct dynamical accounting by the Schrödinger time-evolution for this actualization but rather for us it is a theoretical primitive associated with measurement that may be explicable by reference to other notions. Take, as in the standard description of measurement, the measured system to interact with a probe system during an approximate joint measurement of both position and momentum, as justified by the existence of POMs representing the joint observables for momentum and position (see [53]). In particular, allow the couplings between the object system and two probes used in the appropriate measurement to be simultaneously turned on. The resulting measurement scheme allows an approximate measurement of both position, x, and momentum, p. The joint coupling of both probes will result in a change of measurement imprecisions in accordance with the corresponding Heisenberg uncertainty relation. Furthermore, the measurement couplings can be chosen such that the joint measurement is approximately repeatable [48]. As a result, the measured system enters a state in which the position and momentum are both unsharply localized, with corresponding measurement imprecisions $\delta x, \delta p$ appearing in Eq. 2.6. This justifies taking the position and momentum to exist jointly, as in Landé's view of complementarity (see Sect. 1.2).

As suits his needs at a given time, an experimenter can choose either to make a sharp measurement of a single property or to make an optimal joint unsharp measurement of a complementary pair of magnitudes so as to *jointly* specify a pair of corresponding indeterminate properties. Although the sharp physical magnitudes are not actualized during approximate, unsharp measurements, the likelihoods for the value ranges corresponding to the outcomes that are found *can* increase. A clear and consistent meaning is thereby given to pairs of (generalized) quantum observables as approximately specified properties that are always consistently jointly attributable to individual quantum systems. The corresponding elements of reality are discussed in Sect. 3.2.

2.5 Quantum Causation and the Particle

Before the twentieth century, outside the context of gravitation, the deterministic conception of causation was dominant and rarely questioned in physics. In its traditional form, causation is relatively straightforward: It is the necessary relation between cause and effect, in which a cause temporally precedes its effects, as embodied, for example, in time-dependent solutions of the equations of motion of Newtonian mechanics, for example, the motion of a spring after its initial release. In the era of classical physics before quantum mechanics, causation was thus seen as intimately connected with determinism, in particular, in the Laplacian conception of the latter, which has been described as *causal determinism*, according to which the various space-time paths of objects allow for discernible objects to be understood as corresponding to a precisely specifiable causal network wherein effects also serve as unique causes of later effects.

Cause–effect relations holding between events, processes, and physical states are often schematically illustrated as in Fig. 2.2. In the context of relativity, the cause must lie in the backward light-cone of the effect; such a schema may be embedded in a space-time diagram to emphasize this.[23] The pre-quantum mechanical, pre-relativistic picture of matter naturally includes such causal relations and presents them as holding between systems located in an absolute, static spatiotemporal arena and involved in processes during which system states continuously evolve. Such a conception presents little difficulty for the basic ontological prerequisites for well-defined physics, for example, as given by Einstein, with classical continuous field theory being a paradigmatic pre-twentieth century example. However, with quantum mechanics the concept of causation is no longer so easily explicated.

Since the discovery of quantum phenomena, a set of general reasons for skepticism regarding the relevance of causation to physics have been identified in addition to the concerns which have arisen in philosophy, such as Bertrand Russell's famous criticism of the idea of a law of (deterministic) causality. "The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed

[23] See the discussion of light-cones in Sect. 1.1, and Fig. 1.4. Note, however, that in the context of quantum mechanics such arrows should not be confused with space-time trajectories.

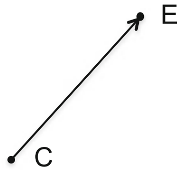

Fig. 2.2 Schematic of a cause–effect relationship, where C is the cause and E is the effect, with time directed upward. Two versions of such a relationship have been considered in physics, one which this connection is a necessary one and one in which it is a probabilistic one

to do no harm" ([248], p. 1). Heisenberg, for example, was quite willing to give up causation in the sense of a necessary connection between cause and effect in order to make further progress in physics. He argued that, "Since all experiments obey the quantum laws and, consequently, the indeterminacy relations, the incorrectness of the law of causality is a definitively established consequence of quantum mechanics itself" ([133], p. 197). However, the failure of deterministic causation has not been a source of great concern to the continued progress of physics. As Feynman put it, "has [physics] been reduced to calculating on the *probability* of an event, and not predicting exactly what will happen? Yes. . . . that's the way it is: Nature permits us to calculate only probabilities. Yet science has not collapsed" ([105], p. 19).

Apart from its technical forms such as local causality and micro-causality discussed in Chap. 1, the term *causality* has been used in a number of different ways by philosophers.[24] For example, one can distinguish between the *principle* of causality and the *philosophical position* that this principle holds universally, that is, that everything can be viewed as an effect having a unique cause, sometimes referred to as *causalism*. Similarly, the term *causation* can refer to instances of relation between a particular cause and a particular effect or to the relationship between cause and effect in general. Causation has since antiquity typically been understood as a necessary connection between two entities, that is, as one in which a given cause C is *always* followed by its effect E. However, David Hume famously regarded causation to be entirely reducible to the *regular temporal succession* or *constant conjunction* of events, C and E, without this regularity arising out of metaphysical necessity.

> We may define a cause to be *an object, followed by another, and where all the objects similar to the first, are followed by objects similar to the second.* [147]

An ongoing tradition in philosophy has arisen from Hume's modern notion, a tradition which holds, contrary to earlier views, that science is *not* founded on an independent metaphysical principle of causality—one independent in the sense that it provides factual content over and above that otherwise present in the laws of

[24] See, for example, [44].

the sciences or that sanctions the substantive application of the adjective *causal* to scientific rules or laws that are based in such a principle.[25]

In the Humean tradition, scientific law is essentially a descriptive catalog of observed regularities. This approach to causation has encountered a broad range of its own difficulties, not least of which being the fact that causes, even outside the quantum realm, are *not always* followed by their effects, primarily due to the existence of background conditions that would render this impossible. One response to this difficulty has been to argue that in order to eliminate the pertinence of background conditions a *ceteris paribus* caveat must be added, that is, the condition "all other things being equal" should be appended to all such statements. Although one might, as J. S. Mill proposed, consider incorporating background conditions together with the cause of interest, the concept would be stretched further than is desirable, particularly in the physical context. Alternatively, one might take the "covering law" approach of Carl Hempel, in which one considers a cause under appropriate background conditions. However, this is also often inappropriate in scientific, as opposed to every day, applications.

More conservatively, one can argue as Russell did that the maintenance of causality as a general 'law' over and above *particular* causal laws is improper; in his view, it is this error that is most problematic in the theory of causation. He supported this claim by producing a Zeno-type paradox using the additional common assumptions (i) that causes and effects are local, that is, that there is no causation over finite spatiotemporal intervals, (ii) that causation requires time-continuity, and (iii) that there is a necessary succession in time between cause and effect. Paul Humphreys has similarly argued that

> Chance by itself has no causal properties or powers, just because there is no such thing as a universal property of chance in the world, any more than there is a universal deterministic tendency in the world. There are particular kinds of physically grounded probabilities, just as there are particular kinds of forces, but in the absence of a successful unifying program, there is no reason to suppose that there is a universal theory of properties or causes.([148], p. 20)

Causation had been extremely successful in physics until the development of quantum theory because it had been almost always identified with a relation between the states of physical systems governed by *law* as characterized as above, following Pierre Simone de Laplace who added that a deterministic mechanics should offer—at a minimum, for the use of a mind sufficiently well informed about the state of physical affairs at any present time—perfect predictability of the future states of affairs, via a "formula" [78]. In physics, due to the nature of the theory of differential equations, the knowledge in question has been that of the system state together with relevant boundary conditions. A failure of determinism in the form of a failure of the current state of affairs to uniquely specify all future states of affairs also occurs in contexts beyond physics. A standard example of this failure in a more ordinary context is that of the relationship between smoking and lung cancer: smoking is

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[25] Cf., e.g. [207, 208].

known to cause lung cancer but not all people who have smoked do later suffer from lung cancer.

It is generally accepted that, in the realm of quantum phenomena, events can be correlated but events that classically would be considered causes are not *always* followed by events that in a classical context would be considered their effects. Given that Quantum mechanical predictions are probabilistic, a probabilistic conception of causation is the most natural in the quantum context, despite the difficulty of grounding causation in statistical relevance relations in full generality, cf. [253]. In the context of quantum theory, physics can been seen as necessitating the use of cluster of concepts relating to causation beyond the traditional pre-theoretic and Laplacian conceptions of causation, such as that of the necessary connection between events involving the propagation and collision in space of particles, something unfounded according to the Humean position.

Elementary particle theory, which can be understood as the realm in which one can seek the greatest detail regarding the interaction and composition of quantum objects, "assumes the validity of three principles that appear to be exactly correct. (1) *Quantum mechanics*... a framework within which we believe any correct theory must fit. (2) *Relativity*... we have no reason to doubt it. (3) *Causality*, the simple principle that causes must precede their effects" [115]. The most widely accepted approach to the relativistic quantum physics of elementary particles is that of *quantum fields*; it is accepted by most physicists familiar with it that any attempt at a relativistic quantum theory of systems other than fields that has the characteristic property of continual localization will be ill-defined[26]; a number of theorems to this effect have been produced, for example, those of Malament [187] and Hegerfeldt [130, 131], the latter having shown that for a generic quantum theory, states with localized particles with an energy bounded from below will have spatial representations which spread at superluminal rates. In field theory, negative energy states are permissible but must then be interpreted as antiparticles.[27]

The particle concept now used in physics can be viewed, following Falkenburg, as having two underlying pre-theoretical meanings, the causal and the mereological (cf. [95], Sect. 3.1). The former is closest to the traditional classical mechanical notion of *particle* but not limited to it. On the former meaning, particles are viewed as the local causes of local events as observed, for example, in detectors. The latter meaning, which has emerged from the discredited metaphysical notion of substance, is that relating to the relationship between parts and the (larger) whole of which they *are* parts. For her, the mereological aspect of the particle notion is provided with its meaning through the analysis of experiments and observations, which she defines very simply: Particles are the constituent parts of matter or light. However, she has argued that the causal particle concept utterly fails, leaving one with only an operational conception, which assumes only that particles are collections of mass,

[26]This is discussed further in Chap. 4

[27]It appears that quantum theory engenders a fundamental conflict between relativistic causality and all but the weakest form of localizability, cf. [61].

energy, spin, and charge, are localizable by particle detectors, and are independent of each other, with states incapable of serving as the causes of physical processes. This is, she argues, because particles are only effects and not causes. This view is intentionally offered in accordance with Ketterle's dictum that one prepares waves and detects particles in practice when dealing with quantum systems. Falkenburg argues that "[T]he causal particle concept is not just *weakened* in the subatomic domain. It simply *fails*. There are particles and there are causes, but the particles are the *effects* and their causes are *not particles* but quantum waves and fields."[28] However, Falkenburg's conclusion is only valid if one assumes that *particle* in any sense stronger than the operational one must also include the requirement that a particle be perpetually localized.

It is true that, as Falkenburg has noted, there is an asymmetry between space-time and momentum parameters in that according to the Schrödinger equation a localized quantum system that is free has a spatial wave-function that spreads out spatially tending in the long-time limit toward a "stable" momentum state associated with a delocalized 'wave' ([95], p. 281). However, this is not sufficient for the rejection of the causal particle concept: Only the notions of causal necessitation and continual localization which have traditionally been associated with the particle concept are problematic. The probabilistic-causal particle notion of elementary particle physics is immunized by the dropping of the requirement of localization *qua* continual localization in favor of *relative localizability*, that is, localizability in a detection frame of reference at a given moment. One must, of course, also remain vigilant against viewing the wave-function as describing some *substance*.

Among physicists the assumption of traditional (deterministic) causality, that is, a relation of necessity between events, has largely been superseded by the assumption of a law-and-rule based approach to state evolution implicitly incorporating *probabilistic causation*. This new approach to causation can be understood as inspired by, but importantly different from Laplace's related conception which includes complete determination of mechanical effects by causes.[29] It is supported by the success of physical laws written as equations, such as the Schrödinger equation for state-vectors or, equivalently, the time-evolution of operators acting on them representing quantum observables, rather than a series of deterministic cause–effect relations between substantial entities. Rather than relying on determinism, physics has increasingly relied on conservation law constraints, symmetry, Einstein locality, and various additional assumptions regarding quantum measurement processes to explain the appearance of definite outcomes in observed individual events; as seen in the sequel, symmetries are fundamental to identifying elementary particles and enforce the conservation laws governing the behavior of quantum systems. Probabilistic causation has been a recognized option since early on in the history of quantum theory. For example, Pauli argued "The simple idea of deterministic causality must... be abandoned and replaced by the idea of sta-

[28]See [95], p. 329. Falkenburg also argues that light quanta are not individuals.

[29]Also cf. [22].

tistical causality" ([70], op. cit., p. 151). Schwinger later similarly commented that "[A]pparent paradoxes in the earlier developments of the theory… are now resolved in terms of this statistical determinate rather than individually determinate theory" ([271], p. 15).

The basic idea of probabilistic causation is relatively easily formalized. For example, Patrick Suppes required that causes be identified by their *raising of the likelihood of their effects*, that is,

$$p(E|C) > p(E) \tag{2.13}$$

(assuming that $p(C)$ lies strictly between 0 and 1), as opposed to the cause *necessitating* its effects. This has the consequence that E is more likely in the presence of the cause than in its absence, that is, that the cause is positively statistically relevant to the effect:

$$p(E|C) > p(E|\bar{C}) , \tag{2.14}$$

where $\bar{C}$ is the negation of C. Equation 2.14 is equivalent to

$$p(E, C) > p(C)p(E) , \tag{2.15}$$

assuming again that $p(C)$ lies strictly between 0 and 1 [301]. Further conditions may then be added to provide a more robust notion. Because of the diverse set of relevant contexts and differing constraints pertaining to its use in science in general, causation is least problematically thought of as grounded in a collection of laws of specific theories, as suggested by Russell and more recently by Humphrey. Further notions can then be introduced to solve conceptual problems such as the accidental generalization problem, that is, the problem of the occurrence of accidental constant conjunctions and the common cause problem, that is, the problem that effects with a common cause will typically be constantly conjoined without one being the effect of the other.[30]

The response to the question of the existence of causation at scales at which the use of quantum theory is necessary for providing explanations has varied considerably between the extremes represented by the positions of Einstein and Heisenberg. In a 1928 article Bohr argued that the quantum postulate entails "a renunciation as regards the causal space-time co-ordination of atomic processes" [27, 28],[31] which relates to the rejection of the causal particle concept. By contrast,

[30] An example in a more ordinary context being when the flash of a cannon is conjoined with and seen constantly before the arrival of the associated boom of the cannon with the probability of the noise being heard being greater than without the flash with the same increase in likelihood as firing the cannon (cf. [74] and Fig. 2.3).

[31] It should be noted, however, that Bohr's use of the term *causal* here is not necessarily the sense of the term as traditionally used in philosophy, in that he believed it to refer specifically to the upholding of conservation laws, cf. [69].

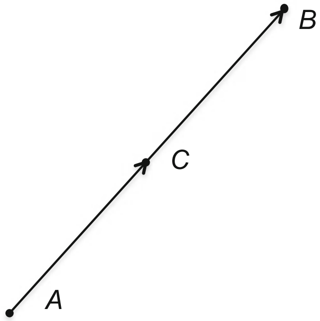

Fig. 2.3 Schematic for a causal chain for three events, in which positive time direction is indicated by *arrows* ([242], p. 189). The chain is transitive if A causes B and intransitive if it doesn't

von Neumann, along with Garrett Birkhoff, argued that Quantum mechanics does involve a form of causation because, although it provides the values of quantities probabilistically, there is

> something else which is causally predictable, namely the so-called wave-function. The evolution of the wave-function can be calculated from one moment to the next, but the effect of the wave-function on observed reality is only probability, [that is,] you do not have complete determination. ([22], p. 486)

It is important to a full appreciation of this view to keep in mind that although probabilities relate directly to what is observed, the phase of the complex wave-functions, that is, of Hilbert-space vectors that provides the wave-like behavior seen in quantum interference, differentiates the behavior of quantum systems from classical probabilistically evolving systems, particularly composite ones.[32] Birkhoff and von Neumann in 1936 considered the evolution of quantum systems in time in the quantum state space to be fully compatible with causation. "The [phase space] point p_0 associated with [a system] S at time t_0, together with a prescribed mathematical 'law of propagation,' fix the point p_t associated with S at any later time t; this assumption evidently embodies the principle of *mathematical causation*," with the qualification that

> [T]he possibility of predicting in general the readings from measurements on a physical system S from a knowledge of its 'state' is denied; only statistical predictions are always possible. This has been interpreted as a renunciation of the doctrine of pre-determination; a thoughtful analysis shows that another and more subtle idea is involved. The central idea is that physical quantities are *related*, but are not all computable from a number of *independent basic* quantities (such as position and velocity). [22]

The view of the scope of causation at microscopic and sub-microscopic scales that was eventually to become most common was articulated by Eugene Wigner as follows: "the acausality of [Quantum Mechanics] manifests itself only at the

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[32]In the following chapters it is shown how this view can be supplemented in order to strengthen the physical conception of causation in quantum physics.

observations undertaken,"[33] which Dirac famously described as a "jump" of the quantum state. For Wigner, the failure of the Schrödinger state evolution to apply continually from the beginning of measurement through to the end of measurement is related to the objective indefiniteness of the quantum world, simply because there is "the possibility of an observation giving various possible results even on a system with a well defined and completely known state" [328]. Bohr's view that quantum mechanics involves an "acausal evolution" at the moment a measurement comes to an end was widely accepted, as was von Neumann's view that so long as measurements were not involved there was an evolution of the quantum state function which is continuous. It is only in relation to *measurement* that these two views clearly diverge, Bohr grounding understanding firmly in the classical world by considering the microscopic to be unspeakable and von Neumann proceeding with an analysis of the interaction entirely *within* Quantum mechanics but requiring a discontinuous state-change rule that brings experience into accord with the description of the situation *after* measurement. Depending on a physicist's purposes, either approach could be used in practice. Less well known and mentioned is the suggestion, made later in Bohr's career, to consider causal relationships in quantum mechanics as obtaining in measurement in that "the experimental arrangement and the irreversibility of the recordings concerning the atomic objects ensure a sequence of cause and effect" [33].[34]

The von Neumann approach to the description of measurement processes has often been criticized despite the fact that it is a well motivated and obvious approach in that, unlike the Copenhagen approach which it is sometimes incorrectly claimed also to represent, it treats measurement from within Quantum mechanics conceived of as a complete and universal theory.[35] Consider, for example, the following critique.

> Although not unmotivated, von Neumann's proposal is completely ad hoc. Measurement processes and measurement devices are not natural kinds, but human beings employ various physical systems as measurement devices if they suit their interests. It is impossible to give a precise physical definition of a measurement process and a measurement apparatus, since there is no physical difference that distinguishes a measurement process from other physical interactions. Measurement devices are an invention of human beings that occurs late in cosmic evolution and that presupposes the existence of macroscopic systems that are not subject to quantum entanglement. [94]

Although von Neumann himself did not lay out in detail the physical conditions under which measurement interactions are certain to take place, there is no question that they *do* take place and, for example, do produce physical records of their results; indeed, this is a primary characteristic of instruments used by people to "suit their interests." It is incorrect to say that there are certainly no physical characteristics

[33] Wigner's comment was made in his Enrico Fermi School lectures of 1970.

[34] For an exception to this, see [321], p. 236.

[35] The later, Everett approach also has this feature, but works with a smaller set of basic principles and encounters greater difficulties as a result [154].

distinguishing measurements from other physical interactions; it appears only that there is no *single, simple one*.[36]

It is largely from the standpoint of objective indefiniteness and a generalization of the von Neumann approach that causation is explored here, without Wigner's belief that consciousness per se plays a role in measurement. Direct observation is viewed here as simply another measurement process in which conscious observers happen *incidentally* to be present and involve the actualization of potentialities and the observation of quantum interference. Similarly, measurement processes are to be viewed as a subclass of physical processes in which robust records of the properties of the object system occur. Although the notion of the robust record is not simple, it is not anthropocentric. It is to be expected that measurement devices will be found to have a specific set of necessary "physical" characteristics beyond the obvious one that they must work reliably. Moreover, measurements need not be natural kinds to play a specific role in physics: A number of noteworthy classes of physical situations, say as described by characteristic Hamiltonians, in which specific processes such as the Zeno effect take place, are not natural kinds.[37] It is sufficient to be aware of the circumstances in which a measurement can be said to have occurred to be able to provide an explanation of the phenomena and behavior observed.

Potentiality relates not only to possible measured property values but also to interference of the possible preceding values; interference can be understood as enabled by the general indefiniteness of values. A basic example is that of interference between two *possible eigenstates* $|\psi_i\rangle$ each corresponding to passage through one of two open slits $i \in \{1, 2\}$. It is sometimes said that this is the interference of probabilities. However, it is the correspondents of complex-valued state amplitudes $c_i(\mathbf{x})$ that interfere: The real-valued squares of the sums of these *amplitudes* provide the probabilities describing interference, not sums of (real-valued) probabilities $p_i = |c_i(\mathbf{x})|^2$. A simple apparatus for observing quantum interference is the Young double-slit interferometer illustrated below.[38] The probability density of finding upon measurement a given quantum system initially prepared in state $|\psi\rangle$ later in state $|\psi_{12}(\mathbf{x})\rangle$, at point $\mathbf{x}$ (and so coordinate x along the detection screen) after the slits is the modulus square of the corresponding sum, that is,

$$p_{12}(\mathbf{x}) = |c_{12}(\mathbf{x})|^2 = \frac{1}{2}|c_1(\mathbf{x}) + c_2(\mathbf{x})|^2 \ . \tag{2.16}$$

[36]The Żurek group, for example, has recently identified physical characteristics of some classes of interactions which could serve as measurements [23].

[37]Indeed, it is quite sensible to consider the appearance of definite measurement outcomes the *measurement effect*.

[38]Detections, as a matter of practice, occur within a finite spatial interval of the detection screen; the $p_i(\mathbf{x})$ are detection-probability *densities*, not probabilities.

In order to be related to the detection frequencies for a collection of measured systems prepared in the corresponding pure statistical state $P_{|\psi\rangle}$, this density is then integrated over the appropriate spatial region Δx of detection.

Salmon has argued that the wave-function can be consistently be viewed as a *wave of propensity* which is capable of interference.

> [T]he quantum mechanical wave is a wave of propensity—propensity to interact in certain ways given appropriate conditions. The results of such interactions are frequencies, and observed frequencies give evidence as to the correctness of the propensity we have attributed in any given case. If we adopt this terminology we can say that *propensities* exhibit interference behavior... In this way we can avoid the awkward necessity of saying that *probabilities* interfere with one another. ([255], p. 15)

Another general view regarding quantum interference is that it occurs among *possibilities*, variously conceived. The most satisfactory way of avoiding talk of interfering probabilities is to say that interference occurs between *potentialities*, which correspond to the complex probability amplitudes of the state-vector, such as the $c_i(\mathbf{x})$ above, these being those of a *single system*. These are the quantities that must be added in the calculation. Because they are complex-valued, the interference between these amplitudes when added is exhibited in relative-phase-dependent cross-terms in the probabilities obtained by squaring the modulus (absolute-value) of the resulting sum.[39] Talk of 'waves of' some quantity or of propagating probability distributions lends an unhelpful connotation of substantiality and of representability in ordinary space, which fails for composite quantum systems. Indeed, it came as a disappointment to Schrödinger when it became evident that the quantum wave-function could not be straightforwardly interpreted in general as a wave in three-dimensional space, because of the rapidly growing size of the configuration space required by the quantum theory of multiple systems.

The double-slit interference experiment was related by Reichenbach to local causation by focusing on the interpretation of theory. In order to help clarify the ambiguities that arise in relation to the appropriate conception of the quantum object,[40] he introduced the idea of a *normal system* of a theory as one that satisfies two conditions: (i) The laws of nature are the same whether or not its objects are observed, (ii) The state of the objects is the same whether or not the objects are observed. He also held that, in the case of a class of descriptions containing a normal system, each description is equivalent to the normal system, which is the one having the greatest descriptive simplicity ([241], pp. 19–20).[41] Reichenbach concluded, however, that the inevitability of state disturbance will rule out (ii) for quantum systems and perhaps even for classical systems (cf. [241], pp. 23–24). Accordingly, he called condition (i) the "*conditio sine qua non* of the normal system." He considered, in particular, the relation of condition (i) to

[39]These are discussed in some detail Sect. 2.6.

[40]For more on the relationship between conception and visualization, see [194].

[41]Reichenbach cited Special relativity as a theory in which such classes are particularly pronounced.

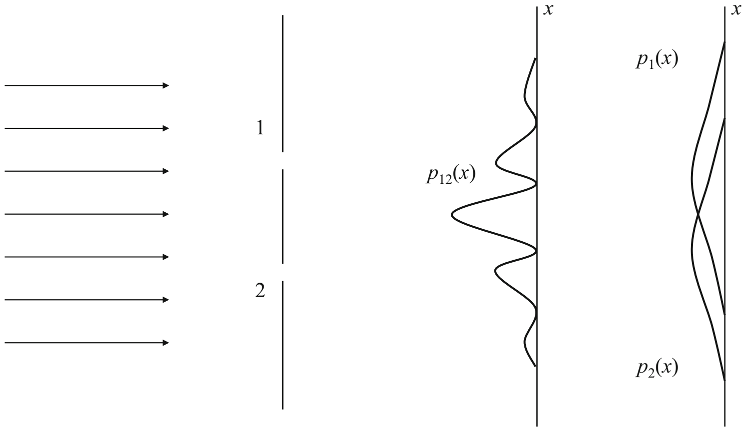

Fig. 2.4 The Young double-slit experiment. Every elementary system, such as an electron, can pass through slit 1 and/or slit 2 and be detected near a point, parameterized by position x along the screen (here only one dimension being relevant), and exhibit interference on an opaque detection screen. In a statistical measurement on a pure collection of many particles coming precisely from the *left*, there will be high values and low values of the detection probability $p_{12}(x)$. If instead only one slit (1 or 2) at a time were available to the particles, two non-periodically modulated distributions ($p_1(x)$ and $p_2(x)$, respectively) would instead result and the detection probability would instead be $p_1(x) + p_2(x)$, the sum of the distributions at far *right*, just as if the systems were prepared in an incoherent state before reaching the two slits, or equivalently were the environment before the slits a decoherence-inducing one. The important difference between this quantum mechanical experiment and the analogous one in which particles are described by *classical* mechanics is that in the quantum case the probability density is not additive, that is, $p_{12}(x) \not\propto p_1(x) + p_2(x)$

experiments involving the various slit configurations possible for an apparatus similar to that of Fig. 2.4, namely, that with only one slit available and that with both slits available. In either case, Reichenbach argued that a causal anomaly will arise in the *laws* governing observable occurrences involving the corresponding entity in one or the other of the two views of quantum systems, as 'waves' or 'corpuscles,' as well as in the pilot-wave interpretation which is a hybrid of them. It is this sort of "causal anomaly" that motivated Einstein's early and failed ball–box thought experiment. The interpretations can be understood as involving the following elements, respectively. In the case of the particle picture, quantum systems only exert influence upon and are only influenced by what occurs in local regions of space. In the case of the wave picture, quantum systems are extended and have "states" at different points in space that are correlated by a phase obeying, as all waves do, a superposition principle.[42]

The situation in the case of the 'wave ontology' is of greater interest to us in this regard because it is one that was identified first by Einstein, as discussed in Sect. 1.1

[42]For an extended discussion of this interpretational distinction, see [99], Sect. 7.

in relation to his ball–box thought experiment, although again similar issues arise for the 'corpuscle' and 'pilot wave' cases as well. Consider the situation in which the quantum system is considered a spatial wave with only one slit is available to it, say, Slit 1. The problem is wave-function collapse:

> so long as the wave has not yet reached the screen it covers an extended surface, namely, a hemisphere [centered at Slit 1]; but when it reaches the screen it will produce a flash at only one point. . . and will then automatically disappear at all other points. ([241], p. 26)

Reichenbach considers this a causal anomaly because it contradicts the laws established for observable occurrences: One does not have a 'normal system' because the laws of the interphenomena are different from the laws of phenomena. Ultimately, he argues that the class of descriptions of quantum interphenomena contains no 'normal system,' something which he calls *the Principle of anomaly*. Regarding anomalies, however, he states that "We must have the courage to face this consequence which is necessarily combined with this interpretation of interphenomena" ([241], p. 27).

2.6 Local Causation

Recall that the experimental demonstration of the violation of Bell-type inequalities has induced many to renounce 'local causality.' The central issue in relation to this empirical evidence is that of the origin of the observed strong correlations correctly predicted by quantum theory in these experiments. One way of testing for the ability of the properties of a composite system to lead to violations of a Bell inequality is to test for a sufficiently large corresponding joint system self-interference visibility.[43] Two general sorts of account of such quantum correlations have been provided: those that turn to interactions between pairs of spacelike separated systems for explanations and those that seek explanations in previous localized physical systems, either by reference to states of the systems before they have become spacelike separated or in the apparatus involved in correlation measurements. Thus, there have been appeals both to non-local interactions and to local interactive forks, which are discussed below. Recall also the setting of a deterministic Einstein-local theoretical description: for every event e in space-time and every cross-section S_e of the backward light-cone of the e, all models must agree with respect to events in the cross section (see Fig. 1.4). Bell pointed out that interactions could propagate from some signal source to the wings lying within such a cross-section at speeds slower than that of light and reach the two wings of the experimental apparatus for measuring two-particle correlations in a way that the

[43]This is grounded in the fact that because, as Gisin first explicitly proved in 1995 [120], any joint quantum system described by an entangled pure state will violate Bell's inequality for at least one set of joint observables, that is, be non-separable, including when they are spacelike distant from each other and that joint-system interference visibility is monotonic in degree of entanglement.

sub-apparatus in the two wings did not behave independently; it must only be the case that they propagate quickly enough to provide the needed effects before each joint measurement is completed ([17], p. 11 and Fig. 1.2). This possibility is known as the *locality loophole.* The experiments of Aspect et al. and others have since largely ruled out such a possibility; they are widely understood to have effectively closed the "locality loophole."[44]

Reichenbach introduced the notion of *screening off* for analyzing correlations in joint measurements, such as those involved in these Bell-type inequality tests, in terms of common causes; it is particularly helpful because it is stated probabilistically.

> *Screening off.* Given events A, B, and C, C screens A and B off from each other if the probability of A given the B and C is independent of B and the probability of B given A and C is independent of A.

Reichenbach first formally introduced the notion of screening off in the context of discussions of causality precisely in order to accommodate the notion in a probabilistic context [242].[45]

The basic idea behind screening off is that if one has a set of events (x, y, z) one can construct a causal network in terms of classes $A, B, \ldots$ of events using a statistically defined relation of *causally between*, assuming both that one is given a triplet (x, y, z) of events between which space-time distances are well defined that reoccurs and that the elements of each triplet can be clearly associated with it. (But note also that, for example, Curt John Ducasse argued that recurrence is irrelevant to causation, in that the cause of a particular event is given "in terms of but a single occurrence of it, and thus in no way involves the supposition that it, or one like it, ever has occurred before or ever will again...; that supposition is relevant only to the meaning of law," something justified on the basis of the similarities between events that could be on that basis grouped together into kinds [85].) This construction appears altogether natural when thought of as formalizing the notion that "when an event of the sort A occurs then an event of sort B will occur with some likelihood."

In a particular instance of a triplet, the relevant events x, y and z belong, say, to classes A, B, and C.[46] If one had found out previously that class C is *causally between* A and B, a relation written $\mathrm{btw}(A, C, B)$, then one could say that z is causally between x and y, as shown below, where reference need only be made to the pertinent classes. Again, if one knows that event x belongs to class A, then one can predict with probability $p(A, C)$ that z will belong to C, and similarly one can predict that an instance of B will occur. Furthermore, once it is known that an instance of C has occurred, it is no longer necessary to know that an instance of

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[44]Gregor Weihs et al. two decades later provided a stronger demonstration with the benefit of yet more modern technology [320].

[45]Salmon later championed the approach of Reichenbach to causality, calling it *causal realism* [254].

[46]Note that here the term *event* is not synonymous with *experimental outcome*.

A has occurred, because an instance of C having occurred itself predicts that an instance of B will occur; this is so because C screens off B from A. Similarly, one can have an instance of C be a common cause for those of A and B.[47]

Now we can see that the Bell locality involves the notion of screening off: its form as a probability factorability statement is tantamount to the independence of the A and B involved. Bas van Fraassen has argued that a correlation between A and B can be understood in terms of a common cause if that putative cause C precedes A and B and the following conditions are also satisfied.

$$p(A \wedge B|C) \;=\; p(A|C)p(B|C) \tag{2.17}$$

$$p(A \wedge B|\bar{C}) \;=\; p(A|\bar{C})p(B|\bar{C}) \tag{2.18}$$

$$p(A|C) > p(A|\bar{C}) \;\text{ and }\; p(B|C) > p(B|\bar{C}) \tag{2.19}$$

These are called *causality, hidden locality,* and *hidden autonomy* [311], respectively.[48] This causality condition is equivalent to $p(A|C) = p(A|B \wedge C)$ conjoined with $p(B|C) = p(B|A \wedge C)$; hidden locality is equivalent to $p(A|\bar{C}) = p(A|B \wedge \bar{C})$ conjoined with $p(B|C) = p(B|A \wedge \bar{C})$, cf. [146], p. 247. The first two conditions are statements of conditional statistical independence, as one might expect to hold if C were a common cause related to a putative hidden variable, λ. Given these, together with Jarrett locality (see below) and strict correlation conditions, van Fraassen was able to derive Bell's inequality; local common cause explanations fail to explain the observed joint correlations.

Much like van Fraassen, Jarrett has argued that Bell's locality condition can be identified with the conjunction of two sub-conditions [163], later descriptively renamed *parameter independence* (PI),[49] which regards the choice of measurement in the distant laboratory, and measurement *outcome independence* (OI).[50] Jarrett showed these two conditions to be logically independent. PI is the condition that the probability of a measurement outcome in one laboratory is independent of the *particular* measurement chosen to be made in the other laboratory, once λ is determined. OI is the condition that the probability of a measurement outcome in one laboratory is independent of the measurement outcome found in the other laboratory, although possibly dependent on the specific choice of measurement

[47] Note, however, that the locality conditions discussed in Chap. 1 are often not written in terms of probabilities that are conditioned on the complete state, but instead indicated the complete state parameter C (i.e. λ) as a subscript or superscript, so that Reichenbach's conditions can be written more carefully as $p_C(A|B) = p_C(A)$ and $p_C(B|A)$. Both notations appear in the literature but the difference in meaning of variant notations is only occasionally pointed out, cf. [80].

[48] The last is more commonly called *positive statistical relevancy*. For cases where common causes have a negative influence on their effects, it must be dropped.

[49] Parameter Independence was called "locality" by Jarrett, which we will also call *Jarrett locality* here, and "surface locality" by van Fraassen [313].

[50] Outcome independence was called *completeness* by Jarrett—we will refer to this here also as *Jarrett completeness*.

made in the other lab and dependent on λ. (A background assumption of Jarrett's analysis is that the probability measure μ is independent of the specific choices of measurements in the two laboratories, cf. ([284], p. 118).[51]) Butterfield has shown that both PI and OI are instances of screening off [54].

Given that the physical world violates the Bell locality condition, the question arises as to *whether just one* of these sub-conditions might be responsible for the failure of "local causality." One can rewrite Bell's locality condition as the *factorization condition* on joint probabilities, namely,

$$p(a = 1, b = 1|\lambda) = p_A(a = 1|\lambda)p_B(b = 1|\lambda); \ldots \tag{2.20}$$

Then, PI is

$$p_A(a = 1|\lambda, b) = p_A(a = 1|\lambda, b') = p_A(a = 1|\lambda); \ldots \tag{2.21}$$

and OI is

$$p_A(a = 1, \lambda, b = 1) = p_A(a = 1, \lambda, b = -1) = p_A(a = 1, \lambda, b); \ldots \tag{2.22}$$

It is OI that is most often considered responsible for the failure of Bell inequalities to hold, on the basis that the no-signaling condition implies OI but not PI; assuming that one could *control* the hidden parameter λ, the marginal probabilities of Eq. 2.9 could in principle be used to signal.[52] More specifically, OI is typically held responsible by the following reasoning. One can argue that if OI were to be violated, then an agent in one wing of the apparatus could send a signal faster than light to one in the other wing.[53] If Bell locality is violated, e.g. $p(a|\lambda, A, B) \neq p(a|\lambda, A, B')$, and an ensemble having the same value for λ were prepared, then an agent in the opposite wing could distinguish measurements of B from those of B' enabling instantaneous signaling. It is then further argued that the violation of PI *does not* enable such signaling.[54] Given that measurements are not reducibly deterministic, their outcomes are beyond the control of measuring agents.

Reichenbach's approach to causation was based on the commonly assumed relationship between *events*. Nonetheless, a given event typically involves a change in object *properties* or *relations*,[55] and may be an element of a *process*. Accordingly,

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[51] The correlations pertaining to the parameter independence and outcome independence conditions are also referred to as act–outcome correlations and outcome-outcome correlations, respectively.

[52] See [162], cf. [54]. Shimony has argued that this would provide *controllable* non-locality, cf. [280], Chap. 11.

[53] This assumes that one can prepare ensembles of pairs that all have the same value for A.

[54] Shimony has argued that this would provide only *uncontrollable* non-locality, cf. ([280], Chap. 11). Accordingly, in his analysis of Bell's theorem, Ghirardi has introduced separate notation for controllable and uncontrollable variables [118].

[55] See [287] for a careful discussion of this point.

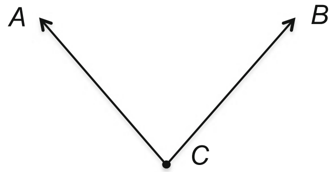

Fig. 2.5 Causation involving three events, in which one, of class C is the common cause of the others, neither of which causes the other ([242], p. 194). This is an example of a causal fork open toward the future, as discussed in the following section

Salmon proposed a more elaborate successor approach to causal analysis wherein causality is treated as primarily a characteristic of continuous processes rather than as a relation between distinct events, where processes exhibit consistency over time. It was designed to be in accordance with the basic assumptions (i) that causation is objective but contingent and (ii) that a good theory of causation should allow for non-necessary causal connections, not be dependent on time, and take into account Humean proscriptions against hidden powers [254]. It is then based on two elements: the *production* and the *propagation* of causal influence. On this theory, causal production is explicable via *causal forks* (see Fig. 2.5) which provide order to causal processes. The "inverse" of a fork is a situation in which a single event has multiple causes, which don't cause each other (an inversion of the situation illustrated in Fig. 2.3) and may or may not act independently of each other.

Salmon introduced the more specific notions of *interactive fork* and *perfect fork* relating to different classes of common-cause. In a *conjunctive fork*, under background conditions, processes can take place in a way potentially not involving laws, so that there are statistical correlations between them due to the common cause screening off their statistical relation. If, for any two events A and B, it is the case that $p(A, B) > p(A)p(B)$ and there an event C such that $p(A, B|C) = p(A|C)p(B|C)$, then A, B, and C comprise a conjunctive fork. In the interactive fork, an intersection of two processes results in their alteration; a correlation between them cannot be screened off by a common cause but rather depends on conservation law. For interactive forks $p(A, B|C) > p(A|C)p(B|C)$ in addition to $p(A, B) > p(A)p(B)$. The perfect fork is the deterministic case of both the conjunctive fork and interactive fork. According to Salmon, when two processes intersect and they undergo a correlated modification that persists after interaction, the intersection constitutes a causal interaction. He provides the setting as follows.

> Let P_1 and P_2 be two processes that intersect with one another at the space-time point S, which belongs to the histories of both. Let Q be a characteristic that process P_1 would exhibit throughout an interval (which includes subintervals on both sides of S in the history of P_1) if the intersection with P_2 did not occur; let R be a characteristic that process P_2 would exhibit throughout an interval (which includes subintervals on both sides of S in the history of P_2) if the intersection with P_1 did not occur.

He then characterizes causal interaction.

> *Causal interaction.* "The intersection of P_1 and P_2 at S constitutes a causal interaction if: (1) P_1 exhibits the characteristic Q before S, but it exhibits a modified characteristic Q throughout an interval immediately following S; and (2) P_2 exhibits R before S but it exhibits a modified characteristic R throughout an interval immediately following S." ([254], p. 171)

This approach to causation has come under criticism for failing adequately to address the hidden powers objection of Hume and for analyzing causation in a fundamentally statistical manner. Salmon's theory was followed by the conserved-quantities approach of Brian Skyrms, [289] then further developed by Phil Dowe. Dowe has argued that causal production can neither be analyzed in terms of statistical relations nor can causal interactions be analyzed in terms of statistical relations, that is, a conjunctive fork is neither necessary nor sufficient for the presence of causation [84].

> The conjunctive fork characterizes two effects arising from a common cause, although it could be generalized to more than two effects...But it has no application where a single effect is concerned, for there is no correlation. ...The development of bacterial poisoning is a causal process whether ten, twenty or only one is afflicted. Clearly there are cases of production of structure of causal processes where no conjunctive fork can be recognized. The statistical characterization is therefore not essential. Second, the existence of a conjunctive fork does not entail the presence of a common cause, and therefore does not entail the presence of a causal process or causal production. [84]

Moreover, argues Dowe, conjunctive forks are not even commonly *associated* with causation.

Having developed a sense of how probabilistic causation can be explicated and potential problems of the statistical approach, let us now consider a specific and well-known experimental example of the application of causation in the quantum realm, namely, Compton scattering [66]. In particular, it has been used to illustrate how common-cause explanations can work in the quantum world. The phenomenon of Compton scattering was initially discovered in the early 1920s and involves an interaction portrayed as analogous to the archetypical example of classical mechanical causation, namely, the elastic collision of billiard balls (see Fig. 2.6); the original observation of the effect was in an experiment in which a metal foil target was struck with an x-ray light beam, giving rise to a deflected (otherwise, a transmitted) x-ray beam. Viewed in this way, it is seen to exhibit a conservation of momentum as the collision of a single photon with a single electron (within the target), as follows. Compton scattering is understood as a process in which such an electron is struck by an x-ray quantum prepared with momentum $p = h/\lambda$, and so energy $E = pc = hc/\lambda$ where c is the speed of light and λ is the x-ray wavelength. The electron can be considered to be free to a very good approximation as well as stationary, because the binding energy of the electron to an atom in the metal corresponds to the ultraviolet range of the light spectrum and so is several orders of magnitude smaller from that of the x-ray involved.[56]

[56]The Compton effect can also be seen when γ-ray light takes the place of the x-ray light.

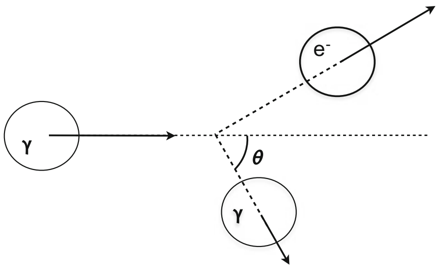

Fig. 2.6 Compton scattering as typically schematically analyzed. In this process, the x-ray (γ) scattered at angle θ transfers some of its energy to the electron, e^-. For this reason, it is sometimes considered an inelastic process. However, in Compton scattering the total kinetic energy of the systems involved, the light and the electron, is conserved, satisfying the definition of an elastic collision; no kinetic energy is converted to other degrees of freedom

In the original experiment, two outgoing beams at different angles of detection with different corresponding central x-ray wavelengths were observed beyond the foil, separated by deflection from a crystal, which acts as a spectrometer, the strongest beam being that in the initial beam direction and being of the initial wavelength and the weaker one corresponding to a different longer, wavelength.[57] The difference $\Delta\lambda = \lambda' - \lambda$ of central wavelength between the two x-ray beams, where the final wavelength is λ' and the initial wavelength is λ, now known as the *Compton shift*, was found to depend on the angle of scattering θ but not on the initial wavelength λ or the specific material of which the target was made; the latter indicated that an *electron* rather than an atom is the system pertaining to the scattering, because the characteristics of the atoms vary with the chemical make-up of the metal used. Thus, this scattering can be explained by assuming that a photon interacts with an electron in a system–system collision. On the basis of that assumption, the difference of energy between the incoming photon energy and the outgoing photon energy in the process is found to be

$$\Delta\lambda = \lambda_c(1 - \cos\theta) , \tag{2.23}$$

where $\lambda_c = h/m_0c$ is the *Compton wavelength* and m_0 is the electron rest mass. The presence of such an electron is seen as a cause of the observed scattering of x-ray light particles in the experiment, illustrated in Fig. 2.6; there is zero x-ray wavelength shift at $\theta = 0$ and a shift of twice the Compton wavelength for direct backscattering, i.e. at $\theta = \pi$.

[57]Those x-rays not Compton scattered can be understood (both quantum mechanically and classical mechanically) as Thomson scattered. Thomson scattering corresponds to the case where in fact the entire atom is involved in the interaction by virtue of the electrons involved being *strongly enough bound not to be ejected,* cf. [93], p. 43, so that (in the classical explanation) they merely oscillate with and radiate the original wavelength.

Compton scattering was also taken as definite evidence of the need to understand freely propagating light, which is typically understood as involving wave-like propagation, as a particle, that is, of the validity of Einstein's *photon hypothesis*—and, unlike in the case of the photoelectric effect, there is an elastic rather than inelastic collision occurring during the underlying process. This was largely because the presence of the scattered beam cannot be explained by reference to classical electromagnetic waves. Classical waves would merely cause the free electron to oscillate with the incoming frequency and would be re-emitted at exactly the same frequency as possessed when incoming, that is, without a shift of wavelength, as is observed in the phenomenon of Thomson scattering. Bothe and Geiger demonstrated experimentally in 1925 that relativistic energy–momentum conservation occurs not only for time-averaged data in Compton scattering but also in *individual* coincidence events; this work was taken as support for the individual-photon explanation of the phenomenon in particular [35].

Salmon argued that Compton scattering can be understood as an example in which an explanation is given via the Common-cause principle.

> Coincidence-counting techniques are a standard part of modern physics. In conducting a Compton scattering experiment, for example, one checks for a correlation between photons scattered at a certain angle and electrons ejected with a particular energy. The observed coincidences provide an entirely satisfactory basis for inferring that *unobserved* collisions between incident photons and (for all practical purposes) stationary electrons have occurred. The appearance of two "tracks" in a cloud chamber emerging from a common point is taken as evidence of an event that constitutes a common cause. It may be, for example, a short-lived neutral K meson (which leaves no track) decaying into a positively charged pion and negatively charged pion. ([254], p. 211)

In fact, cloud chamber experiments to investigate the Compton effect made by Charles Thomson Rees Wilson were interpreted as "experiments in which causality as well as [energy-momentum] conservation were verified in elementary particle processes" ([210], p. 237) because they could be understood as showing the "tracks" of the recoil electrons. In Wilson's experiments, α-particles were observed to produced apparently continuous tracks, whereas for electrons and positrons distinct points of measurement along a putative trajectory were observed. The cloud chamber detector, which detects charged systems, comprised in those experiments a chamber filled with supersaturated steam. The tracks are produced when charged particles ionize the hydrogen atoms in the steam, producing visible droplets of condensation [96].

Furthermore, in coincidence measurements designed by Compton and A. W. Simon,[58] it could be seen that the individual secondary photons and recoiling electrons in Compton scattering appear *at the same moment*. This appears to contradict the general claim of Bohr that there is a necessary incompatibility of the causal and spatiotemporal description of quantum processes. One can see a series of events played out in and traceable through network tracks in investigations of

[58]These were and independently developed also in Germany by W. Bothe and H. Geiger, cf. [205].

Compton scattering in detection chambers. In bubble chamber experiments carried out in early particle accelerator laboratories, ionization due to systems entering the chamber gives rise to gas bubbles and photographed within liquid hydrogen rather than in steam as in the Wilson cloud chamber.

In contemporary electronic particle detector apparatus, such as drift chambers, tracks are typically from an electronically detected and recorded by computer and algorithmically reconstructed. Indeed, the sheer amount of data in some situations is remarkable.

> In fact, experiments at the LHC will witness something like 1 billion collisions per second. Only 100 collisions per second, at 1 megabyte of data per collision, can be recorded for later analysis. It is a major challenge to design and build the high-speed, radiation-hardened custom electronics that provide the pattern recognition necessary to select potentially interesting collisions. . . . During one second of [Compact Muon Solenoid] CMS running, a data volume equivalent to the data in 10,000 Encyclopedia Britannicas will be recorded. The data rate to be handled by the CMS detector (approximately 500 gigabits per second) is equivalent to the amount of data currently exchanged by the world's telecommunication networks. ([201], pp. 59–60)

The succession of localized detection events of a track can be explained via the causal particle notion whereas the divergence of a single track into multiple tracks can be explained via the compatible notion of the mereological particle (see Chap. 4). Despite the clear value of studying scattering phenomena in this way, the tracks produced in particle detectors cannot correspond to *continuous trajectories* of the sort produced by classical point particles. One must be cautious in interpreting such detector data, in particular, in take care how one attributes observed effects to interphenomena. As Schrödinger noted,

> Now we do observe single particles; we see their tracks in the cloud chamber and in photographic emulsions; we register the practically simultaneous discharges caused by a single swift particle in two or three Geiger counters placed at several yards' distance from each other. Yet we must deny the particle the dignity of being an absolutely identifiable individual. [268]

This denial arises from the unusual statistics fundamental particles obey in contexts where more than one of the same type is present, which conflict with the Maxwell–Boltzmann statistics associated with always identifiable, distinct individual particles.

Falkenburg, who has carried out detailed analyses of the various particle concepts that have been in circulation since quantum mechanics was discovered, goes further in her criticism of the causal particle notion.

> [A]ccording to [the causal particle concept] particles are the local causes of local effects in particle detectors, in particular of event sequences or particle tracks. *First*, one event does not [deterministically] cause the next event. . . *Second*, in general the causes do not act locally, as the identification of the causes with particles would require. ([95], p. 329)

Nonetheless, these two points are insufficient for a rejection of the *modern* particle concept, for example, as characterized by Feynman because, for it, first the question of strict determinism is not relevant. In regards to the second point, local action, for example with the detectors mentioned, the data regard a succession of events in which a quantum particle can be understood to be repeatedly localized by detector measurements; (relative) localizability suffices for a causal explanation of the appearance of a track caused by an individual object.

Falkenburg views the causal particle concept as failing for photons because on her definition it necessarily involves both an aspect of individual causation and an aspect of localization, some form of which pertains to the modern particle notion.[59] However, she argues that the strongest argument for rejecting anything but a purely operational notion of particle is that current quantum theories "are at odds with any attempt to explain the experimental results of particle physics in terms of individual causes. (...) In particular, the results of recent experiments of quantum optics *cannot* be due to local causal agents. There are no single (read: agent) photons taking this or that (read: local) part through the branches of a Mach–Zehnder interferometer" ([95], p. 329).[60] This was thought to appear so only because one could approximate a single-photon Fock state within the interferometer by sufficiently attenuating the source of its input. Light interference is independent of light intensity, as was first observed in a related "feeble light" diffraction experiment by Geoffrey Ingram Taylor more than a century ago. These experiments only hinted that single quanta would participate in the production of an interferogram but could not definitely prove it [38, 302].[61] Moreover, Aspect and co-workers showed that the well-known feeble light experiments do not demonstrate definitely the discrete nature of light itself because it could in such cases in principle be ascribed instead to the behavior of its detectors, something in turn due to the fact that the sources used in early arguments did not produce single-photon light pulses but attenuated semi-classical light [5]. However, now nearly perfect single-photon sources are available in quantum optics, showing that the claim that there can never be only a single photon in this interferometer is incorrect.[62]

Even before more contemporary purpose-designed single-photon sources were available, Aspect and Philippe Grangier showed that one *can* have single photons in the interferometer by making use of an atomic radiative cascade source, such as was used their demonstration of the CHSH inequality [5].[63] They showed how an individual atomic radiative cascade, such as used in the earlier demonstration of the failure of the Bell inequalities, if properly excited, does produce such pulses which are readily put into a Mach–Zehnder interferometer [5]. The conclusion above is notably in direct contradiction with Dirac's much earlier characterization the situation in such interferometers.

[59]Note that *local causal* here is not meant in the sense discussed herein Chap. 1, which regards the behavior of distant correlations.

[60]Two other examples are also offered, that of an attenuated laser beam and of a polarizer, said not to create single-photon beams. These other examples, however, are uncontroversial and certainly not designed to produced beams of individual photons.

[61]This independence is also seen in the interference of neutrons, cf. [334, 335].

[62]The Mach–Zehnder interferometer is shown in Fig. 2.7 below.

[63]Indeed, Aspect and Grangier argue strongly that their demonstration is the first to definitively demonstrate the particle-like aspect of photons [5].

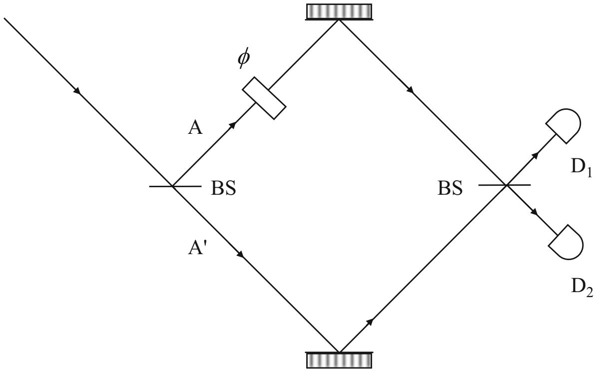

Fig. 2.7 A Mach–Zehnder interferometer realizing a discrete two-beam experiment in which detectors D_1 and D_2 are placed after the two orthogonal light beams (A and A′) have merged. This apparatus has the advantage over the original two-slit apparatus of Fig. 1.2 that no intensity is lost from the original beam during quantum state preparation. "BS" indicates a 50–50 beam-splitter and "ϕ" a variable phase-shifter allowing the sinusoidal interference patterns to be observed at the D_i

> Suppose we have a beam of light consisting of a large number of photons split up into components of equal intensity. . . [and the] two components are made to interfere. . . [Quantum theory], which connects the wave function with probabilities for one photon. . . [makes] each photon go partly into each of the two components. Each photon. . . interferes only with itself. Interference between two photons never occurs ([82], p. 7).

Regarding the other aspect of the above argument—that there is no *single clear path taken* by any one photon—the following can be said. On the one hand, this is uncontroversial in standard quantum mechanics. On the other hand, the light in the interferometer *is* confined within the interferometer with no loss of intensity (as there is the case of light input to the diaphragm of the double-slit interference experiment). So, the photons are localized to within the boundaries of the apparatus (and ultimately to the extent of the detector openings). Furthermore, if no photons are input to it, *no* detections will occur. Thus, relatively localized single photons can be understood as responsible for the detection events observed with this interferometric system, consistently with the modern particle notion.

Falkenburg argues that ultimately only the *operational particle* notion can be justified over and above the detector clicks and other macroscopic phenomena associated with the observation of particles, because "bare quantum 'objects' are just bundles of properties which underlie superselection rules and which exhibit non-local, acausal correlations" ([95], p. 206). These operational particles are: (i) collections of mass m, energy E, spin s, charge q, that (ii) localizable by a particle detector, and (iii) independent of each other. Their properties are observed regularly together, and can so be observed under controlled conditions.[64] Falkenburg

[64]These 'bundles' are sometimes called Lockean 'empirical substances.'

claims that no unambiguous axiomatic counterpart to the operational particles is available and concludes that "without a macroscopic measuring device which is itself local, nothing is localized and no re-identification of the same kind of particle in subsequent measurements is possible" ([95], p. 221). Here, to make a definitive judgment in any particular class of situations one would need, at least, to know the details of the measurement in question. (It would also be helpful to have a resolution of the quantum measurement problem.) Although it appears likely in light of practice that a localized macroscopic measuring device would suffice, it is unclear that it is *necessary* for such re-identification. Thus, the claim that particles in quantum theory can be understood *at best* operationally is premature.

In his treatment of quantum causation, von Neumann distinguished three "degrees" between 'causality' and 'acausality' ([318] pp. 213–214). In the case of first degree, the result of any measurement could always only be statistically predicted, with a second measurement made immediately afterward having a dispersion regardless of the value first obtained. In the second case, the first value would have a dispersion but the second measurement made immediately afterward would not. In the third case, there would be no dispersion for either measurement. The analysis of the Compton–Simon experiment indicates that the second is occurring (cf. [273], p. 140); it has been understood as an example of an interactive causal fork, because the scattering energy of the electron that is struck is found to be strongly correlated with that of the resulting photon without being predetermined, as discussed above. In particular, Salmon considered Compton scattering as an example of why an interactive fork, rather than a conjunctive fork, is sometimes needed in a causal analysis [252].

The schema for Salmon's analysis is the following. There are two correlated events A and B to be explained as arising from a common cause C that provides a full explanation of the relationship between A and B in a conjunctive fork. Recall that, as Salmon puts it, "in the conjunctive fork, the common cause C absorbs the dependency between the effects A and B, for the probability of A and B given C is equal to the product of the probability A given C and the probability of B given C." In the interactive fork, by contrast, "the common cause C does not absorb the dependency between the effects A and B, for the probability of A and B given C is greater than the product of the two separate conditional probabilities." In Compton scattering specifically, at the end of the process,

> there is a certain probability that a photon with a given smaller energy will emerge, and there is a certain probability that the electron will be kicked out with a given kinetic energy. However, because of the law of conservation of energy,... the probability of getting a photon with energy E_1 and an electron with energy E_2, where $E_1 + E_2$ is approximately equal to E (the energy of the incident photon), is much greater than the product of the probabilities of each energy occurring separately. ... The probability that the electron will be ejected with kinetic energy E_2 given an incident photon of energy E is not equal to the probability that the electron will emerge with energy E_2 given an incident photon of energy E and a scattered photon of energy E_1. ... Given a high energy photon impinging on the electron in a given atom, there is no way, even in principle, of predicting with certainty the energies of the photon and electron that result from the interaction.

Hence, the interactive fork is more appropriate in the case of Compton scattering relative to the conjunctive fork. The intersection of the x-ray with the electron, preceded by the processes of their convergence also constitutes an interactive fork.

Finally, note that subatomic processes such as Compton scattering are sometimes cited as grounds for the consideration of "backwards causation," for example, via the idea that the positron can be viewed as a negative energy electron traveling backward in time.[65] This idea appears very naturally when interactions are considered from the point of view of Feynman's schematic space-time diagrams which symbolically represent possible subatomic processes in scattering theory. It was first introduced by John Wheeler, who suggested it to Feynman in 1941 (as mentioned in cf. [102, 103]). Ernst C. G. Stückelberg presented it in publications around the same time [297, 298], with Feynman first using it in a 1948 article and later expanding upon it in a 1949 article.[66]

> [T]he 'negative energy states' appear in a form which may be pictured (as by Stückelberg) in space-time as waves traveling away from the external potential backwards in time [103].

In his 1983 Mautner Lectures, Feynman described the notion of propagation of subatomic particles backward in time, in the general context, as follows.

> This phenomenon is general. Every particle in Nature has an amplitude to move backwards in time, and therefore has an anti-particle. When a particle and its anti-particle collide, they annihilate each other and form other particles. ... Photons look exactly the same in all respects when they travel backwards in time... so they are their own anti-particles. ([105], p. 98)

This view is encouraged by the fact that the Schrödinger evolution of quantum theory is time-reversal invariant.

In fact, Reichenbach later considered these articles of Stuc̆kelberg and Feynman on positrons. Taking him by surprise, he commented about this idea that "the number of material particles... is contingent upon the extension rules of language. However, the interpretations thus admitted for the language of physics differ in one essential point from all others: they require an abandonment of the order of time" ([242], pp. 263–264). For him, the interpretation represented

> the most serious blow the concept of time has ever received in physics. ... Quantum physics, it appears, cannot even speak of a unique time order of the processes, if further investigations confirm Feynman's interpretation, which is at present still under discussion. ([242], p. 268)

[65]Negative as well as positive energies are allowed by the relativistic constraint on the energy-momentum: $E^2 - (pc)^2 = (mc^2)^2$. Note also that, in contrast to the conjunctive fork, the interactive fork does not seem to involve an asymmetry in time.

[66]"This idea that positrons might be electrons with the proper time reversed was suggested to me by Professor J. A. Wheeler in 1941" [101]. (Wheeler was Feynman's Ph.D. advisor.)

The status of time remains to some extent an open question, but one which is beyond the scope of our investigation here. Let us simply note that the Quantum mechanical treatment of measurement, which is involved in the description of any empirically relevant phenomenon, involves elements beyond the Schrödinger evolution and, in particular, involves an aspect of state evolution that is irreversible in time.

Chapter 3
Measurement and the Quantum World

Abstract The quantum theory of measurement and its development are explained. The basis for viewing measurement in quantum mechanics as a genuine physical, rather than psychophysical process or a process that depends on consciousness or the mental, is given and defended against the critiques of Bell and others. The notion of quantum measurement as the actualization of quantum potentiality as grounded in the related versions advocated by Heisenberg and Shimony is explicated in the context of the theory of positive-operator-valued measures. Quantum interference is discussed as a process of interference of quantum potentialities in contradistinction to the interference of some material substance or the interference of probability waves. This provides the basis for a valid realist interpretation of quantum theory.

Quantum theory and measurement have been closely connected since Quantum mechanics was first formalized. The benchmark treatments of the theory by Dirac and von Neumann, the latter more prominently, included prescriptions for the quantum states resulting from measurements and portrayed the corresponding probabilistic state change rule as fundamental. These operations are non-unitary: density matrix diagonalization in the latter case and the quantum "jump" in the former. These early rules were later supplemented by a more precise non-unitary transformation, namely, a projection of the state-vector onto a state of the measurement eigenbasis. The presence of the non-unitary evolutions of state among the elements of standard modern quantum theory indicates the crucial role of probabilistic state-change in quantum theory. It also relates to the issues considered in previous chapters. Its importance for understanding quantum theory can be seen, for example, in the contemporary division of the interpretations of Quantum mechanics into those which accept state-projection in measurement (so-called 'collapse' interpretations) and those which reject it (so-called 'no-collapse' interpretations) which can be viewed as a re-conception of the first classification of interpretations given initially by Einstein and discussed in Chap. 1. The most well known of the latter is the so-called Many worlds interpretation, but several other approaches of this kind have also been offered where non-unitary changes of state are seen as redundant

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formal elements at best, cf. [154]. Although the prominence of measurement in quantum theory has always been greater in comparison with the case of classical theory, the theoretical significance of measurement became more evident with the improvement and generalization of elements of the basic formalism of Quantum mechanics, significantly so with the development of the positive-operator-measure formalism discussed in the previous chapter which relates importantly to unsharp values and uncertainty relations.

The most prominent theoretical tool to emerge in the development of quantum measurement theory is the Lüders rule [169, 186], which corresponds to a projection-valued measure and is generally viewed as an improved alternative to von Neumann's so-called "projection postulate"; the rule is an efficient and accurate means for specifying the quantum states resulting from sharp measurements. POVMs were later developed to describe the unsharp measurements, which have their own transformation rule generalizing von Neumann's original prescription. Lüders' rule is accepted here as a valid but possibly reducible prescription for the state resulting from sharp measurement. Two valuable characteristics of this rule are: (i) it provides a single definite, although not predetermined, object-system state upon completion of every precise measurement when combined with the eigenvalue–eigenstate link, and (ii) it preserves state coherence in the case of non-maximal such measurements. These are of great importance to a realist understanding of Quantum mechanics when the quantum state-vector is assumed to be a complete state description, given that measurements are seen to have definite individual outcomes in practice.

3.1 Measurement

The *quantum measurement* or *quantum objectification problem* has long been a central technical problem of quantum theory—a particularly important one for those who are realist in their attitude to the quantum state. This problem has been evident at least since von Neumann's formalization of the theory, which reflects it in the designation of two distinct state-evolution processes, I and II. It has resisted all straightforward mathematical attempts at its resolution [49]. The problem can be understood most simply as emerging from the fact that the unitary time-evolution of the quantum state, that is, the deterministic Schrödinger evolution of process II, which governs interphenomena, cannot provide the quantum state of a measured system that correctly predicts later behavior after a non-trivial measurement has occurred. The second aspect of quantum state evolution reflected in process I is therefore required. A unification of these two processes would involve a modification of the Schrödinger evolution, that is, of process II so as to incorporate process I and to produce the corresponding measurement results, perhaps as deviations from it under special circumstances. Such special circumstances could involve, for example, the presence of an external macroscopic system. In some treatments, spontaneous state reductions are postulated rather than a standard "projection postulate" operating specifically and only during measurements.

Two alternative approaches to a modified state evolution are: (i) to consider effect of other systems on the evolution of the measured one, for example, by considering all systems within the universe as *open* quantum systems, and (ii) to explicitly restrict the interpretation of the states legitimately considered, for example, to the state of the entire universe alone. The most common alternative approach has been of the first kind, that is, to obtain a post-measurement object-system state from a stochastic model. The most popular responses of the second kind have been to understand the quantum state of a system as referring to something other than an individual quantum system or to understanding it as being of only relational significance. Another alternative has been to turn away from the state as a predictive tool within the theory toward an emphasis on conditional probabilities of measurement outcomes.

Related to the measurement problem is the fundamental role that measurement is given in some popular interpretations of Quantum mechanics, particularly when a projection postulate is introduced in connection specifically with it. Such a postulate is sometimes assumed to come into play in contexts in which it is also the case that *observation* in the sense of the perception of some piece of apparatus by a conscious subject has been introduced, either explicitly or implicitly; this leads to discomfort because observation is understood as a concept external to physics. In some cases, consciousness is attributed an explicit role, for example, Wigner suggested that consistency might be achieved between the description of measurement processes and that of other quantum processes by appealing to observation as an act involving consciousness, even though he understood the problematic nature of dealing with quantum measurement by linking it directly to the conscious observations of experimenters:

> ... it seems dangerous to consider the act of observation, a human act, as the basic one for a theory of inanimate objects. It is, nevertheless, at least in my opinion, an unavoidable conclusion. If it is accepted, we have considered the act of observation, a mental act, as the primitive concept of physics... [328]

At the same time, he commented that by accepting measurement as fundamental to the theory one "explain[s] a riddle by a mystery" [328] Bell was concerned that, in any event, giving measurement a special role produces conceptual imprecision. Instead of the term *measurement*, he suggested the term *experiment* be used. "[I]n fact the [former] word [*measurement*] has had such a damaging effect on the discussion, that I think it should now be banned altogether in quantum mechanics... the latter word [*experiment*] is altogether less misleading" ([18], p. 20). He posed a number of rhetorical questions to express his concern over the notion of measurement as used in quantum theory.

> If the theory is to apply to anything but idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time more or less everywhere? ... Is there then ever then a moment when there is no jumping and the Schrödinger equation applies? The concept of 'measurement' becomes so fuzzy that it is quite surprising to have it appearing in physical theory *at the most fundamental level*... does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? ([17], pp. 117–118)

Although there is undoubtedly a degree of conceptual imprecision in the approach of working physicists to quantum measurement that may be encouraged by the use of such terminology, changing it would make little difference to foundational analyses of experimentation. Most contemporary investigators of quantum measurement agree with these points and seek to understand measurement in terms of more fundamental elements of theory and other physical factors, and are not misled by terminology. Advocates of realism must take Bell's points very seriously. Nonetheless, if the first premiss is accepted, at most one is obliged further to accept that measurement-like processes occur in situations where they do occur; although they may occur all the time and in most places, they do not occur all the time in *every* physical system. They do seem in the experience of human beings constantly to have occurred whenever these beings are attending to their surroundings. What needs to be teased out with regard to measurement in quantum theory is precisely what is involved in measurements that distinguishes them from all other physical processes yet makes it the case that the world of our everyday perception of the large seems to be one in which measurement-*like* processes are regularly occurring while system behavior does not seem otherwise to deviate from that predicted by the Schrödinger evolution.

Clearly, there are many physical processes that *cannot* serve as measurements, for example, those in which no stable record results during the interaction between one object, which might be thought of as a system of interest, and another, which might be thought of as a measurement apparatus. The answers to the second and third of Bell's questions is a clear "yes" and should be kept in mind from here on. Although one can make use of state projection and measurement compatibly with the potentiality approach to Quantum mechanics advocated here, these two notions need not be understood as irreplaceable or irreducible. Moreover, on this approach there *are* notions more fundamental than measurement with which the process can be understood, namely, *potentiality* and *actuality*, which are metaphysical.[1] Let us therefore consider whether and, if so, the extent to which the introduction of potentiality and its actualization in the analysis of the measurement process of affects the significance of the fundamental problem of measurement for quantum theory.

Under the potentiality interpretation of the quantum state, measurements are physical situations in which potentialities are actualized and in which records irreversibly come into being, de facto. This provides a base camp from which one may discover the presumably complex set of physical factors leading to successful measurements. Measurement is standardly described as taking place in the following way. A physical system of interest, the *object* system, is brought into contact with another, the *apparatus* system, with the result that the apparatus indicates the value of a physical quantity of the system under investigation after they have sufficiently interacted. This indication is accomplished through a quantity in the apparatus or a

[1] Bell's own choice for the actual properties of the world was *local beables*, which are described in Sect. 1.4. However, as we have seen, the locality of his beables renders them problematic.

subsystem thereof, the *pointer*, which takes on a definite value corresponding to a value of the system quantity being measured.

A measurement result is given as the registration of such a property of the pointer, the *pointer observable*. The connection between measurement results and physical magnitudes is made explicit through the following condition, taken as a necessary one for a measurement process to measure a given observable.

> The *calibration condition*. If the system is in an eigenstate of the observable to be measured, then the corresponding eigenvalue will with certainty appear as the measurement outcome.

If the calibration condition is taken as a postulate, the appearance of an outcome indicates that the measured system must be in this eigenstate at the moment the measurement ends [196]. The evolution of system plus apparatus conforming to the calibration condition must therefore be such that the apparatus state at the end of the joint evolution will be an eigenstate of the associated pointer (sub)system.[2]

Recall that Quantum mechanics assumes a formal relationship between physical magnitudes and eigenstates, namely, the eigenvalue–eigenstate link.[3] Recall also that quantum theory is distinguished from classical theory by the distinctive theoretical elements of state superposition and the Born rule for the associated probabilities, which imply both the Heisenberg relations, which apply even to single systems, and the occurrence of super-strong correlations of entangled states of composite systems once the Hilbert space of the latter has been assigned using the principle that the Hilbert space of compound systems is formed as the tensor product of the component subsystems. If the state of the system plus apparatus is a superposition of eigenstates of the observable being measured then, according to the minimal interpretation available, the probability of the occurrence of a particular pointer value is given by the squared modulus of the amplitude of the corresponding object eigenstate.

Similarly, there is a qualitative difference between the classical and quantum measurement processes. The discreteness of fundamental quantities in quantum mechanics and the particulate nature of microscopic ontology play a decisive role in distinguishing quantum measurement as, for example, Julian Schwinger clearly explained.

> There is no half of an electron. The electron has a definite mass; it has a definite charge. If the interactions that I am concerned with are electrostatic in nature, I cannot reduce them arbitrarily in strength because there is no half of a unit of charge. This indicates to you immediately, I think, the basic difference between the laws of microscopic measurement and macroscopic measurement. I must take into account the fact that the strength of the interaction - which must be present if I am to talk of measurement at all and, therefore, talk meaningfully of physical phenomena - cannot in general be made arbitrarily small because

[2]Note also that pointer-magnitude values can differ from those of the related measurement memory register, as long as there is a well defined *pointer function* serving to bring the elements of the two sets of values into one-to-one correspondence.

[3]Although some interpretations of quantum mechanics deny this assumption, these interpretations inherit other problems. For more on these see, for example, [80], p. 22.

> the physical objects that interact (the atoms, the electrons) in general have relevant physical properties which come in certain units - quanta, the origin of the name of the subject that we are discussing: quantum mechanics. ([271], 12)

Furthermore, in accordance with Feynman's general view of the probabilistic nature of quantum processes, Schwinger comments here that

> The measurement act involves a strong interaction - I repeat: on the microscopic scale it is necessarily strong because we cannot cut the strengths of the charges in half; we cannot change the properties of these fundamental particles... so the measurement unavoidably produces a large disturbance, which we cannot correct for in each individual instance, for we cannot control what happens in each individual event in any detail. We can only predict or control what happens on the average, never in any individual instance. Therefore, the program of computing what the effect of the disturbance was and correcting for it is, in general, impossible. ... [O]nce we recognize that the act of measurement introduces in the object of measurement changes which are not arbitrarily small, and which cannot be precisely controlled, then we must acknowledge that every time we make a measurement we introduce a new physical situation that is essentially different from the situation before the measurement. (ibid.)

The resulting change, the "new physical situation" arising out of this objectively chancy interaction can be seen as underwriting the non-unitary nature of the result of measurement interaction. Moreover, Peter Mittelstaedt has shown that the quantum mechanical probability emerges as an approximately definite property of a large ensemble of identically prepared systems represented by the same state [196]. In that sense, the quantum mechanical probability postulate is deducible from the eigenvalue–eigenstate link [47].

Unlike the classical state, the quantum state is also typically influenced by its preparation, which may be performed in the same way as a measurement and may determine the system's quantum state, allowing the prediction of its future quantum state. In this way, the probabilities specified by the quantum state can also be seen as having an implicitly conditional character.

When one takes the Schrödinger evolution alone to describe the closed system constituted by the measuring apparatus and the system under measurement (including their environments when appropriate), inconsistencies appear: If the superposition principle is enforced, such a description predicts a number of different but equally valid measurement outcomes if one assumes, in accordance with the calibration condition and the eigenvalue–eigenstate link, that measurement outcomes exist whenever the appropriate one-to-one correlation of measuring apparatus states and object system states occurs.[4] This problem is sometimes also referred to as the problem of 'the reduction of the wave packet,' as if the evolution of the quantum eigenstate must be that of a substantial wave that becomes localized upon the completion of a measurement and must in some way contiguously reduced. A metaphysical transition of mode of being, namely, the actualization of a specific potentiality provides a better explanation of the appearance of a measurement outcome because a mechanistic state reduction does suggest among

[4]This has been seen as both a weakness and a strength in the case of the Collapse-free approach.

other objectionable things that there is a superluminal propagation of portions of a physical ‘wave packet.’

By comparing the predictions as to what should occur during measurements with what *is* observed, one can readily see why the Schrödinger equation fails as a description of the state evolution during the measurement process (cf., e.g., [42, 239]). Consider a measuring apparatus system initially in an eigenstate $|p_0\rangle$ of its pointer variable. First, take the system measured for a magnitude of interest to have corresponding Hermitian operator O with discrete non-degenerate eigenvalues $\{o_j\}$ and to be in an eigenstate $|o_i\rangle$ before the measurement process begins. Then assume that the measured-system state remains unchanged during the measurement process, so that (assuming the eigenvalue–eigenstate link) the value of physical magnitude measured is that before as well as that after measurement has finished. In such situations, measurement should then result in composite-system state transformations

$$|\Psi_j^{(i)}\rangle \equiv |p_0\rangle|o_j\rangle \to |\Psi_j^{(f)}\rangle \equiv |p_j\rangle|o_j\rangle\ , \tag{3.1}$$

for each value of j that is a possible measurement outcome. Much as in the EPR argument, the system being measured must also be capable of being successfully measured for the same quantity corresponding to O were it instead initially *not* in an eigenstate of that operator but instead in any state $\sum_j c_j|o_j\rangle$, which is also a state allowed by the Superposition principle. Assuming, then, that the two systems together form a closed system, the state of the system formed by the apparatus subsystem together with the measured subsystem will be acted on linearly by the temporal evolution operator. The measurement process must therefore be a transformation in which one has

$$|\Psi\rangle \equiv |p_0\rangle \sum_j c_j|o_j\rangle \to |\Psi'\rangle \equiv \sum_j c_j|p_j\rangle|o_j\rangle\ ; \tag{3.2}$$

in the case of a collection of systems this is $\rho(0) \to \rho'(t) = U^\dagger(t)\rho(0)U(t)$, where

$$\rho = P_{|\Psi\rangle}\ , \quad \rho' = P_{(\sum_j c_j|p_j\rangle|o_j\rangle)}\ . \tag{3.3}$$

Note that this is the transformation of a vector to a vector in the first case, and a pure statistical state to a pure statistical state in the second, because unitary transformations preserve the purity of $\rho(t)$. However, what is needed in either case for a description of measurement that accords with a situation in which definite measurement outcomes are produced with the probabilities according with the Born rule is the transformation of the overall system resulting in a final state of the form

$$\rho^{(f)} = \sum_j |c_j|^2 P_{|\Psi_j^{(f)}\rangle}\ , \tag{3.4}$$

that is, a mixed state describing a collection of distinct states with probabilities $|c_j|^2$, where a specific definite outcome is obtained for each measurement. Because, when beginning with $|\Psi^{(i)}\rangle$, the composite system evolves into a coherent superposition involving several distinct measuring system states for the ensemble of measurements rather than just one, the unitary evolution does not provide an adequate measurement description. Indeed, any unitary evolution predicts that the measurement of such a quantity O yields neither a definite outcome nor an appropriate mixture. Moreover, including a pure environmental state in the description makes no difference in this regard.

Von Neumann demonstrated that changing the locus of the boundary between the measuring and observed subsystems has no effect on the accuracy of predictions of Quantum mechanics and that there is no requirement that the quantum state projection, that is, process I now commonly referred to as the *von Neumann projection*, occur at a specific moment in the process of measurement, except to the extent required by psycho-physical parallelism. Wigner argued that a unique prescription for the physical stage at which this process takes place is nonetheless needed for an objective characterization of measurement processes [327]. To show this, he produced a thought experiment to probe the issue of the role of consciousness in measurement. It involves a system S designed to flash when in one state, $|\psi_1\rangle$, and not to flash if in an orthogonal one, $|\psi_2\rangle$. A "friend," F, who sees a flash will be in corresponding states $|\chi_i\rangle$ in the respective cases and so the system S–F will have corresponding joint states $|\psi_i\rangle|\chi_i\rangle$. If S is in a superposition state $\alpha|\psi_1\rangle + \beta|\psi_2\rangle$, the state of S–F must then be

$$\alpha|\psi_1\rangle|\chi_1\rangle + \beta|\psi_2\rangle|\chi_2\rangle\ . \tag{3.5}$$

The probability of the friend seeing the flash will be $|\alpha|^2$, while that of not seeing the flash will be $|\beta|^2$. In order to be able provide a correct answer about what he observes, when measuring S the friend must obtain a measurement result that does not contradict the answer he provides when prompted for one.

In von Neumann's treatment, when the point of observation is reached, the object system is the object of the attention of what he called the *abstraktes Ich*, the essence of the observing subject, occurring in parallel with a state that agrees with what is observed ([318], p. 421). Considering his thought experiment, however, Wigner notes that "So long as I maintain my privileged position as ultimate observer" there will be no logical inconsistency between what 'I' see and this standard theory of measurement [327]. However, Wigner goes on, if after the experiment

> I ask my friend, 'What did you feel about the flash before I asked you?' He will answer, 'I told you already, I did [did not] see a flash,' as the case may be. In other words, the question whether he did or did not see a flash was already decided in his mind, before I asked him. If we accept this, we are driven to the conclusion that the proper wave function immediately after the interaction of the friend and object was already either $\psi_1 \times \chi_1$ or $\psi_2 \times \chi_2$ and not the linear combination $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$. This is a contradiction because the state described by the wave function $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$ describes a state that has properties which neither $\psi_1 \times \chi_1$, nor $\psi_2 \times \chi_2$ has. If we substitute for 'friend' some simple physical apparatus, such as an atom which may or may not be excited by the light-flash,

> this difference has observable effects and *there is no doubt that* $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$ *describes the properties of the joint system correctly,* [whereas] *the assumption that the wave function is either* $\psi_1 \times \chi_1$ *or* $\psi_2 \times \chi_2$ *does not.* If the atom is replaced by a conscious being the wave function, $\alpha(\psi_1 \times \chi_1) + \beta(\psi_2 \times \chi_2)$... appears absurd because it implies that my friend was in a state of suspended animation before he answered my question. ([327], p. 293)

If the friend is to have a specific result before being asked, the joint state cannot have been the superposition state in the basis defined by the definite results. The state of the system of friend plus object must be a mixture of the first and second joint states with probabilities $|\alpha|^2$ and $|\beta|^2$, respectively.

Wigner concluded from this example that the measurement result must have become determinate at the moment the friend made his measurement *as a result of his being conscious* and that it follows (i) "that the being with a consciousness must have a different role in quantum mechanics than the inanimate measuring device" and (ii) "the quantum mechanical equation of motion cannot be linear" ([327], p. 294). Any other interpretation, he argued, must deny the friend true consciousness and commits one to solipsism. However, in attributing the difference to *consciousness* Wigner moves too quickly, even if his points (i) and (ii) are correct. In the absence of empirically evidence to the contrary, a set of conditions necessary but not sufficient for attentive observers to become aware of measurement outcomes is all that is required for consistency with the finding of definite results by attentive observers of measurements.

Other critics of von Neumann's measurement theory, Huzihiro Araki and Mutsuo Yanase, argued that it is incorrect to assume that the simple act of looking at a measurement apparatus is a process describable by quantum mechanics (cf., [318], Chap. 6 and [1, 326]). These authors proved that, for a closed system constituted by the measured system and the measuring apparatus, only physical quantities that commute with all bounded additive conserved quantities of that joint system are accurately measurable and repeatable if von Neumann's measurement state transformation $|p_0\rangle|o_j\rangle \rightarrow |p_j\rangle|o_j\rangle$ is assumed.[5] This might lead one to believe that the measurement problem is not a genuine technical problem because it is based on overly stringent requirements on the description of measurement. However, it has been shown that weakening the assumption that the observables measured be sharp by requiring only that measurements be unsharp cannot eliminate the problem [49].

Yet others have questioned the validity of the arguments that there is a measurement problem by critically examining the assumptions from which it follows; the underlying assumptions may not hold in all circumstances typically identified as measurement processes. For example, Hans Primas has argued that proofs indicating that there is a measurement problem are invalid because finite closed systems *simply do not exist*.

---

[5] Wigner showed this to be true in the case of spin-$\frac{1}{2}$ component measurements; Araki and Yanase generalized the result to measurements of any discrete quantity. See [184] for a generalization to imprecise measurements.

> Since all material systems are inextricably coupled to the electromagnetic and to the gravitational field, even 'reasonably isolated' finite systems do not exist. This does not mean that it is not instructive to study the fiction of closed systems, but one should not confuse tentative investigations and the full-grown theory. [232]

Although what is asserted is the case in *most common situations* encountered in the world—indeed, it was believed for some time by a number of physicists including Schrödinger that entanglement was too delicate to survive in realistic situations—there are good reasons for believing that closed physical systems *do indeed exist* as such for non-trivial periods of time. The difficulty of finding such systems in everyday situations is one reason why tests for extreme quantum behavior, for example, tests of Bell-type inequalities long required a great deal of care and have been performed on a relatively small range of physical systems. However, such conditions have been and now regularly are achieved in the best of such tests, such as those in which measurements carried out on entangled photon pairs; because photons are relatively immune to the effects identified in this argument, they are most often used to demonstrate quantum coherence despite its typically limited strength. Given the success of such tests, the claim that "reasonably isolated systems" do not exist for at least short amounts of time is false. On the other hand, measurement situations *are* likely to be those where the environment plays an important role.

At least four assumptions of a largely scientific realist character can be readily identified as underlying the measurement problem ([280], pp. 57–58).

(i) The quantum state of a physical system is an objective characterization of it, and not merely a compendium of the observer's knowledge of it, nor merely an intellectual instrument for making predictions concerning observational outcomes.
(ii) The objective characterization of a physical system by its quantum state is complete, so that an ensemble of systems described by the same quantum state is homogeneous, without any differentiations stemming from differences in "hidden variables."
(iii) Quantum mechanics is the correct framework theory for all physical systems, macroscopic as well as microscopic, and hence it specifically applies to measuring apparatus.
(iv) At the conclusion of the physical stages of a measurement (and hence, specifically, before the mind of an observer is affected), a definite result occurs from among all those possible outcomes (potentialities) compatible with the initial state of the object.

Condition (i) is essential in any realist interpretation of the quantum formalism; Condition (iii) is a necessary condition for Quantum mechanics to be a universal physical theory; Condition (ii) is recognizable as the Synoptic principle; Condition (iv) is empirically well supported. Because these are either very well supported assumptions or indispensable for a realist interpretation of the theory as a fundamental one, Shimony has argued that the measurement problem must be acknowledged by interpretational realists and that the Schrödinger dynamics must be modified, for example, by some physical process occurring in conjunction with measurements. Nonetheless, in light of the fact of successful quantum measurements, one can regard the details of its solution as details to which future empirical results can be expected to lead. Indeed, Lüders himself viewed the projection rule that he introduced as provisional in expectation of a more detailed description of measurement.[6]

The measurement problem currently lacks a clear physical resolution in the form of a unified dynamics modifying the unitary time-evolution of the standard theory; no specific modified equation of the quantum state evolution under the conditions of measurement has been found that is more explanatory than the simple imposition of the Lüders rule. Under these circumstances, it is advisable to set the measurement problem aside and move forward with providing a characterization of the relationship between the quantum formalism and what is found in experiment that is in agreement with the Born rule. Here, we do so in a way that goes beyond simply providing a catalog of correct measurement statistics, that is, we pursue an interpretation of quantum mechanics that helps provide a world picture in which the most general and long-satisfactory categories such as individual object and cause continue to hold.

On the specific version of the Potentiality interpretation I am advocating here, the complex quantum state amplitudes c_i, which can superpose, are the mathematical representatives of potentialities. The potentialities are to be understood as probabilistic causes for effects arising in the corresponding future quantum states. Recall that the state amplitudes are the components of the state-vector as written in the various bases of the Hilbert space of the corresponding system, as shown in the example state-vector $|\psi\rangle$ of Eq. 1.1. By the Superposition principle, which ensures that any linear combination of state-vectors is itself a state-vector, these amplitudes correspond to an array of other sets of potentialities, which can be seen as interfering in appropriate circumstances, such as when a photon can enter from one of two openings of a diaphragm and strike a screen in the Young double-slit experiment.[7] Recall that, in that representative case, consideration of the relevant complex probability amplitudes allows the correct probability of an object detection event in an interval Δx about a point x corresponding to the resolution of the detector-screen to be obtained by integrating the Born probability density $p_{12}(x)$ over the interval yielding the following probability. Interference is reflected in the presence of an additional phase-modulated "interference term" on the right-hand side of Eq. 2.16 (when fully expanded) that is proportional to

$$\sqrt{p_1(x)p_2(x)}\cos\big(\theta_2(x) - \theta_1(x)\big)\ , \tag{3.6}$$

where each of the $\theta_i(x)$ is a phase of the corresponding amplitudes $c_i(x)$ for passage through one slit—equivalently for striking the screen with a particular vector momentum. It is the presence of the interference term in $p_{12}(x)$ that implies a non-zero visibility, V.[8] Thus, precisely at the center of the diaphragm one has

[6]Cf. the private communication discussed in Ref. [2] of [132].

[7]See Sect. 2.4.

[8]V is defined as the difference of maximum and minimum detection-event rates of the pattern, which yield probabilities through the relative frequencies they provide, divided by their sum.

$p_1(0) = p_2(0)$ and $\theta_1(0) = \theta_2(0)$, representing the maximum probability density, whereas in some locations one finds $\theta_2(x) - \theta_1(x) = \pi/2$ and zero probability density, since then $\cos(\theta_2 - \theta_1) = 0$, as shown in Fig. 1.7. The state amplitudes at any moment provide the propensities for various dynamical properties of the individual object to be possessed, and so to condition its actual properties in the future. If the likelihood of a property to occur in the future, as given by the Born rule, is non-zero, the basic requirement on probabilistic causes that they increase the likelihood of their effects (Eq. 2.13) is satisfied.

The current state $|\psi(t_0)\rangle$ of an object at a given time t_0 is thus a cause of future states of the object, that is, for times $t > t_0$ and deterministically so if the object remains a closed system or probabilistically so when measured for properties that do not commute with the corresponding projector $P_{|\psi(t)\rangle}$, that is, if the observable O is such that

$$[O, P_{|\psi(t)\rangle}] \neq 0 , \tag{3.7}$$

for example, when $|\psi(t)\rangle = |\uparrow\rangle$ and $O = P_{|\nearrow\rangle}$, where $|\nearrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$.

Note that in Quantum mechanics applied as in the double-slit example considered above, where states are associated with paths, *all possible paths* from each event of a system being in one space-time region to being in another play a role in determining the likelihood of the later event occurring there, not only the intuitively most expect path or paths. This is most clearly seen in Feynman's approach to Quantum mechanics in which such a schematization is typical and represented by Feynman diagrams. The same quantum probability of a given final event is found using the rules provided by Feynman for finding the probabilities of state transitions as is found when using the standard formulation. In particular, it can be shown that these rules imply exactly the same results as obtained through the use of Lüders rule for updating the quantum state-vector in the case of discrete quantities (cf. [292], Appendix 3). Indeed, John Stachel has shown that the eigenvector–eigenstate link and the Lüders rule are implied by Feynman's calculus of quantum amplitudes ([292], pp. 314–315) which we interpret here as corresponding to potentialities.

The Feynman probability rules are the following (cf. [102] and [136], pp. 59–62).

(3.1) There is an amplitude for each distinguishable possibility leading from an initial value to a final (registered) value for a physical quantity. The probability for that process is equal to the modulus squared of the corresponding state-vector amplitude, which must be a complex number of modulus $\leq$ 1. A possibility is distinguishable if it provides a measurement outcome capable of indicating that only it has occurred.

(3.2) If several alternative subprocesses, indistinguishable within the given physical arrangement, lead from the initial state to the final (registered) result, then the amplitudes for all the indistinguishable processes must be added to get the total amplitude for their combination (*quantum law of superposition of amplitudes*).

(3.3) If several distinguishable alternative processes lead from the initial preparation to the same final result, then the probabilities for all these processes must be added to get the total probability for the final result (*law of addition of probabilities*).

(3.4) If a final event can be reached through an indistinguishable sequence of processes, the amplitudes for all these processes must be multiplied to get the total amplitude for that process (*law of multiplication of amplitudes*).
(3.5) If a final event can be reached through a distinguishable sequence of distinct processes, the probabilities for all the processes must be multiplied to get the total probability for that process (*law of multiplication of probabilities*).

Feynman viewed the following expression, which combines Rule 3.2 and 3.4, as fundamental to his formulation of Quantum mechanics.

$$a_{ac} = \sum_b a_{ab} a_{bc} \ , \tag{3.8}$$

where a_{ij} designates the amplitude for obtaining outcome j in a measurement of physical magnitude J conditionally upon performing a measurement of physical magnitude I and obtaining outcome i, for a sequence of three measurement events in which magnitudes A, B, C are measured in that order; the corresponding probabilities are $P_{ij} = |a_{ij}|^2$, cf. Rule 3.1. The demonstration that the above rules imply the eigenvector–eigenstate link and the Lüders rule is as follows [292]. If a non-maximal measurement b of a discrete-valued magnitude is followed by a maximal measurement, the probability that the measurement outcome c for the second measurement, with the initial state corresponding to eigenvalue a and the outcome of the first measurement being one of b_i, allows for the identification of a number of different possible processes corresponding to the various values of i. The evolution of the system from a to c can be considered as built up of m indistinguishable such alternative processes each of which can be considered an evolution in two stages, one from a to b_i, having amplitude $\langle a|b_i\rangle$, followed by another from b_i on to c having amplitude $\langle b_i|c\rangle$, so that in each

$$\langle c|a\rangle_{b_i} = \langle c|b_i\rangle\langle b_i|a\rangle \ , \tag{3.9}$$

according to Rule 3.4. Then, by Rule 3.2, the process from a to c has amplitude

$$\begin{aligned}
\langle c|a\rangle_b &= \sum_{i=1}^{m} \langle c|a\rangle_{b_i} && (3.10)\\
&= \sum_{i=1}^{m} \langle c|b_i\rangle\langle b_i|a\rangle && (3.11)\\
&= \sum_{i=1}^{m} \langle c|P_{b_i}|a\rangle && (3.12)\\
&= \langle c|P_b|a\rangle \ , && (3.13)
\end{aligned}$$

P_b being the projector onto the Hilbert subspace corresponding to the outcome b of the non-maximal measurement, cf. Fig. 1.10 (and note that subscript kets are supressed here). Thus, the probability of the transition from a to c is

$$p_{a\to b\to c} = |\langle c|P_b|a\rangle|^2 \,. \tag{3.14}$$

Furthermore, by Rule 3.5,

$$p_{a\to b\to c} = p_{a\to b}\, p_{b\to c} \tag{3.15}$$

which implies that

$$p_{b\to c} = p_{a\to b\to c}/p_{a\to b} \,. \tag{3.16}$$

One can then find the probability of $a \to b$ by considering all the possible values for c

$$p_{a\to b} = \sum_c p_{a\to b\to c} \tag{3.17}$$

$$= \sum_{i=1}^{m} \langle a|P_b P_c P_b|a\rangle \tag{3.18}$$

$$= \sum_{i=1}^{m} \langle a|P_b \Big(\sum_c P_c\Big) P_b|a\rangle \tag{3.19}$$

$$= \sum_{i=1}^{m} \langle a|P_b|a\rangle \tag{3.20}$$

since $\sum_c P_c = 1$ and $P_b P_b = P_b$. Thus, one has probability of $b \to c$ given by

$$p_{b\to c} = \langle a|P_b P_c P_b|a\rangle / \langle a|P_b|a\rangle \,, \tag{3.21}$$

namely, the Lüders' rule.

Because the Lüders rule is the *only* rule prescribing the quantum state of a system measurement that provides the correct generalization of conditional probability given the quantum probability measure, the above result is to be expected. In particular, it provides the classical rule for conditional probability when the two operators related to the pertinent events of preparation and measurement commute [41, 146, 294]. The uniqueness of the Lüders rule can be understood by considering a generalized probability function q on the set of subspaces of Hilbert space $\mathcal{H}$. First, because any such function is additive over orthogonal subspaces, it is defined entirely by the assignments of probabilities to the one-dimensional subspaces of $\mathcal{H}$. Second, Gleason's theorem shows that q is provided via a density operator, ρ_q. Finally, if such a q assigns the value 1 for a projector Q, then it assigns the value 0 to projectors onto rays in the *complement* of the subspace onto which Q projects so that $\rho_q|v\rangle = \mathbf{0}$ for all (not necessarily normalized) vectors along such rays. For any such ray $\bar{v}$,

$$q(\bar{v}) = \mathrm{tr}(\rho_q P_{|v\rangle}) = \mathrm{tr}\big(\rho_q\big(Q + (I - Q)\big)P_{|v\rangle}\big) \tag{3.22}$$

$$= \mathrm{tr}(\rho_q Q P_{|v\rangle}) \;, \tag{3.23}$$

for all $|v\rangle \in \bar{v}$, because of the linearity of the trace and because $(I - Q)|v\rangle = \mathbf{0}$ by the definition of $\bar{v}$.

Thus,

$$q(\bar{v}) = |c|^2 q(P_{|u\rangle}) \tag{3.24}$$

for some $c \in \mathbb{C}$, where $|u\rangle$ is the normalized vector along the ray, so that *any* q such that $\rho_q = 1$ is fully specified by the values it assigns to vectors in the ray onto which Q projects. Hence, for any generalized probability function p on the subsets of $\mathcal{H}$, there is a unique q on those subsets such that for all subspaces of the subspace onto which Q projects one has

$$q(P) = p(P)/p(Q) \tag{3.25}$$

and, in turn, q is uniquely represented by ρ_q. Note then that

$$\frac{Q\rho Q}{\mathrm{tr}(Q\rho Q)} \tag{3.26}$$

is a statistical operator, so that one can write

$$\rho_q = \frac{Q\rho Q}{\mathrm{tr}(Q\rho Q)} \;. \tag{3.27}$$

Thus, Lüders' rule provides the unique generalized probability function q such that, for all projectors into subspaces of the space onto which Q projects, $q(P) = p(P)/p(Q)$. This result also supports the approach of standard quantum measurement theory despite the quantum measurement problem.

3.2 Potentiality

It is generally held that the measurement problem as a problem of physics takes a different form in relation to quantum probability under the Potentiality interpretation but is not fundamentally changed.[9] It will be argued here that a solution of the measurement problem, say, through a modified quantum dynamics would provide more physical detail but not affect the relationship between measurement

[9]See, for example, [280], p. 57.

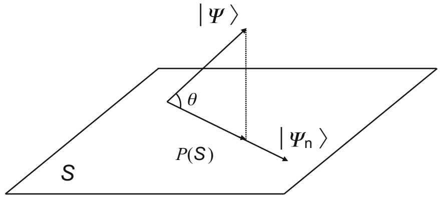

Fig. 3.1 Projection of a quantum state $|\Psi\rangle$ onto a Hilbert subspace S

and the actualization of potentialities. Such a solution would be a significant contribution to theoretical physics, but given the completeness of the quantum state description its result must be empirically equivalent to the use of the Lüders rule in all those many instances in which it has been predictively correct. The Potentiality interpretation of Quantum mechanics would not itself be fundamentally altered, but realist understandings of the theoretical formalism would be lent greater credence by such a solution.

The assumption of a potential mode of being allows one to both assent to the completeness of the Quantum mechanical description of physical systems via the state-vector and to understand causally the corresponding probability distributions for measurement outcomes in a way that remains internal to the theory—for example, without depending on classical theory.[10] Under this approach, for each potentiality there is a corresponding probability, given by the Born rule, that is understood in terms of physical propensity for the various possible values of physical magnitudes to occur. Empirical support for the potentiality approach is found in the increasingly careful and precise confirmation of the quantum mechanical predictions of violations of Bell-type inequalities. As explained in previous chapters, these violations require the objective indefiniteness of the values of physical magnitudes corresponding to the irreducible quantum probability which in turn requires an explanation. The potentiality interpretation enables the explanation of definite measurement outcomes in a way consistent with the objective indefiniteness of physical properties, even though it does not provide the sort of detailed physical explanation that would constitute a full solution of the measurement problem.

Although potentiality is related to propensity, as explicated here it is specific to quantum physics and does *not* imply a general propensity interpretation of probability. In relation to observed phenomena, a given potentiality can be considered to correspond to the (typically limited) capacity of the object system to induce the associated measurement outcome (cf. [47]). In particular, the quantum mechanical probability $p_i = |c_i|^2$ of a specific outcome o_i corresponds to the likelihood for that outcome to arise when an appropriate measurement is made of an observable O whenever the object system is prepared in (or have evolved to) a state $|\psi\rangle$ having a vector component $c_i|o_i\rangle$ in the eigenbasis of O (Fig. 3.1). Thus, the individual system after having been so prepared will with the probability given by the Born rule

[10]Note that this does not require a potentiality interpretation of probability *in general* but only of the probabilities appearing in quantum mechanics.

induce a specific measurement outcome in the measurement apparatus because of the existence of the corresponding potentiality at the moment of initial measurement interaction. More generally, every quantum object has the potential, when in contact with another system capable of serving as a measuring apparatus, to actualize *each* of its properties, that is, of the values of its physical magnitudes (observables) upon an appropriate measurement. Moreover, quantum property indefiniteness can be seen as underlying the *indeterminism* of the outcome of measurement: A property with no definite value before a measurement can only probabilistically induce a single outcome among the set of possible outcomes in a sharp measurement of it [47]. In the exceptional case that a measurement is repeatable, in the sense that the same outcome would be recovered upon immediate repetition, then the physical magnitude is an element of reality according to the EPR reality criterion and will pre-determine the only possible measurement outcome.

Recall that Heisenberg viewed the quantum probabilities provided by the statistical operator ρ, in general, as involving both objective and subjective elements, where the former are "statements about possibilities or better tendencies ('potentia' in Aristotelian philosophy)" [136] and the latter are due solely to the observer's lack of knowledge of the system described by it. He viewed the subjective element to be negligible in the case of a pure case description, that is, one provided by $|\psi\rangle$ via

$$p_\psi(O) = \langle\psi|O|\psi\rangle\ , \tag{3.28}$$

which are the squared magnitudes of the amplitudes corresponding to the potentialities and the sharp properties of the system. Heisenberg emphasized that, in general, a course of events in itself is not determined by necessity but that

> the possibility or rather the 'tendency' towards a course of events possesses itself a kind of reality—a certain intermediate level of reality midway between the massive reality of matter and the mental reality of an idea or picture... [47, 137]

The stage at which the actualization of potentialities occurs was also specified.

> the transition from the 'possible' to the 'actual' takes place as soon as the interaction between the object and the measuring device, and thereby with the rest of the world, has come into play; it is not connected with the act of registration of the result in the mind of the observer. [136]

Shimony later articulated potentiality as a modality of existence of physical systems that confers on properties an intermediate status between that of bare logical possibility and full actuality. It also taken here as so, rather than as resulting from the presence of mind as suggested by Wigner [276].

Approaches to interpreting probabilities as propensities have been seen by some to conflate the possession of a propensity with the manifestation of the (sharp) value of the property. In particular, it has been argued that "to be coherent a propensity view must deny a common proposition behind the [eigenvalue–eigenstate link], namely that it is legitimate to ascribe a property to a system if and only if the system takes a value of the property," because "it would then follow in accordance with the [eigenvalue–eigenstate link] that a system possesses a property if and only if the system's state is an eigenstate of the operator that represents the property. But any coherent propensity… account must ascribe a property without manifestation" [300]. This difficulty, if entertained in relation to the potentiality approach, is mitigated by the extension of properties to correspondents of non-projective POMs. Actualization corresponds one-to-one to the attribution of a sharp property with unit probability: Sharp values are attributable if and only if the system state is an eigenstate of the corresponding Hermitian operator. Unsharp values are attributable even when the system is *not* in a state corresponding to a projector onto a subspace for any value of the eigenvalue.

We take the quantum probability amplitude, which is a complex quantity, rather than the quantum probability as the mathematical representative of potentiality; by clearly differentiated the two, arguments against the propensity interpretation of probability are rendered ineffective [299]. One can nonetheless understand the potentialities compatible with a given measurement outcome as interfering even though, as pointed out in Sect. 2.4, quantum probabilities do not interfere.

3.3 Elements of Reality and Measurement

Our understanding the evolution of the physical state in Quantum mechanics is further advanced by extending and generalizing the standard theory of quantum measurement, keeping in mind the basic requirements on probability. Interpretations of Quantum mechanics can be categorized on the basis of the manner in which properties values are prescribed; this can in turn be understood via the choice of a "preferred basis," which is tantamount to providing an observable which is always definitely valued [42]: Interpretations based on the eigenvalue–eigenstate link such as that described here have as their preferred observable the identity observable $\mathbb{I}$, which always has a definite value by default[11]; other interpretations must provide some extra-mathematical justification for the definiteness of their non-trivial choices of preferred observable.

The traditional notion of an observable as a self-adjoint operator can be subsumed to the broader category of generalized observables, namely, the class of POMs, as the subclass of projection valued measures referred to as the *sharp observables*, the remaining observables being *unsharp*, as discussed in Sect. 2.4. The elements of reality correspond to the subset of observables attributed by the Born rule a degree of reality equal to one.

The state $|\psi\rangle$ of an isolated quantum system is identified with a point p_0 of its phase space that evolves with certainty between measurement interactions along a trajectory given by the Schrödinger time-evolution, which in this way induces a continuous evolution of the potentialities corresponding to the components of $|\psi\rangle$ in the corresponding eigenbases. The potential elements of reality corresponding to

[11] As sketched in [47].

the projective measures can be considered simultaneously real to a degree given by

$$d_E(P_{|\psi\rangle}) = \mathrm{tr}[P_{|\psi\rangle}E] = \langle\psi|E|\psi\rangle \ , \tag{3.29}$$

for the various effects E. The operations described by POMs, besides those involving projectors, are measurements of unsharp values relatable by approximation to the values of non-commuting sharp properties. This allows for a form of joint measurement of properties for which the corresponding sharp observables do not commute: Even though the sharp observables are never jointly measurable, they can be approximated by related unsharp observables (POMs) which *can be* jointly measured. Moreover, not unlike the Lüders measurements, which are strictly repeatable, joint approximate measurements which are approximately repeatable can lead to equal or increased degrees of reality for the measured properties.

Recall from Sect. 2.4 that the quantum formalism can be used to provide the probabilities of the outcomes of measurements on collections of identically but not necessarily precisely prepared collections of systems described by the statistical operator ρ. Consider the generalized observable being measured be a POM E on a σ-algebra Σ of subsets of the value space Ω. One then has an associated probability measure

$$X \mapsto p^E_\rho(X) := \mathrm{tr}[\rho E(X)] \in [0,1]. \tag{3.30}$$

Consider a measurement where the quantum probability $p^E_{|\psi\rangle}(X) = \langle\psi|E(X)\psi\rangle$ is the likelihood of actualization for the effect $E(X)$, whether it is sharp or unsharp, associated with the value set X of the corresponding magnitude, when measured on a system prepared in pure state $|\psi\rangle$.[12] If we assume that the couplings between the object system and appropriate measurement probes are turned on simultaneously, there will be a re-adjustment of the corresponding individual measurement imprecisions in accordance with a quantum uncertainty relation.

In this framework, it is seen that a joint measurement of approximate position and momentum can be made in such a way that the joint measurement is approximately repeatable [48], that is, the position and momentum distributions will have the statistics of mean values q, p and uncertainties $\delta q, \delta p$, respectively.

The assumption of the eigenvalue–eigenstate link can be justified as follows [47]. One can take, as a sufficient condition for a physical quantity to be attributed a definite value, the Einstein–Podolsky–Rosen criterion for a magnitude to be an *element of reality*, that is, that the value or property can be known without changing the state of the system. This can be done because there exists a class of measurements that do so, namely, the operations of the Lüders rule [51], which are the following restrictions of linear, positive, trace-non-increasing maps on the linear space of self-adjoint trace-class operators.

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[12]The existence of such schemes is warranted by the existence of POMs that represent joint observables for momentum and position (cf. [47, 53]).

$$\rho \mapsto \phi_L^P(\rho) := P\rho P, \tag{3.31}$$

where P is a projector. One sees that ρ remains unchanged exactly when it is an eigenstate of P because the projectors are idempotent, that is, $P^2 = P$ for the projectors, and so $P\rho P = PPP = PP = P = \rho$ when $\rho = P$. We see then that if the system's state is an eigenstate of a given observable, the corresponding projector corresponds to an element of reality. Thus, if we assume that whenever the state of a system is an eigenstate of an observable it is actual, then the EPR condition is automatically incorporated.[13] As a necessary condition for actuality, one can appeal to the realist intuition that, for example, elements of reality can influence one another, in particular, those of a measurement apparatus can be accordingly influenced by those of an object system: An observable is actual if an appropriate measurement results in the corresponding outcome with certainty. Note that this is just the calibration condition, the defining requirement for a given measurement scheme to qualify as a measurement of a given observable [51]. When it is satisfied, a property's actuality is in accordance with the system's being in an associated eigenstate. Thus, the eigenvalue–eigenstate link serves both as a necessary and a sufficient condition for the reality of a property of a quantum system [47].

While a measurement is being performed, in general, a stochastic change of state takes place, not only of that of the measured system but also that of the measuring apparatus, independently of whether the measurement made is of a sharp or unsharp observable. The set of all effects then serves to indicate the various possible ways in which the system may behave in measurement-like interactions. In cases where the measurement made is a repeatable one, the properties associated with sharp observables compatible with that observable become actual and definite and those associated with incompatible sharp observables become indefinite and potential. In cases where the measurement is of an unsharp observable, the value of the associated physical magnitude can become less indeterminate. Using the POM formalism, as opposed to the PVMs alone, one is able to provide more explanations of the behavior of quantum systems.

One can use unsharp observables to explain the results of joint measurements of physical properties that are traditionally thought of as completely incompatible and so useless for providing explanations: Better predictions of subsequent measurements can be provided by updating probabilities of future measurements based on the outcomes of unsharp as well as sharp measurements, especially when the corresponding uncertainty product is reduced.

The generalization of the Lüders rule for an effect $E(X)$ is

$$\rho \mapsto \phi_L^E(\rho) = E^{1/2}(X)\rho E^{1/2}(X).$$

[13]The Einstein–Podolsky–Rosen condition of elements of reality was first weakened to a condition regarding elements of approximate reality, represented accordingly by approximately true propositions by Paul Busch, in his discussion of the EPR–Bell experiment [45].

If $\text{tr}[\rho E(X)] \geq 1 - \varepsilon$, then the corresponding state change (as measured by the trace-norm) is small to the order of $\sqrt{\varepsilon}$ [46]. Hence, one sees that approximately real properties can also be found almost certainly with little change to the measured system state. This justifies viewing effects as elements of *approximate reality*, that is, unsharp but real system properties, whether these effects are elements of POMs or are PVMs [47]. Given the necessary condition for a POM to support an element of reality, any effect has an associated likelihood of occurring, quantified by its degree of reality, which depends on the state of the measured system. If the influence on the system is described by the generalized Lüders operation, one can ascertain the actuality of a property of the system: because it can be observed without disturbance, it must already have been real. Discrete observables, such as sharp dichotomic measurements, for example, of the z-spin component of a spin-1/2 system admit repeatable measurements, in which the measured property is fully correlated with the pointer property for every possible measurement outcome, something that allows the measured system property value to be inferred directly from the corresponding pointer property value, once it has become definite.

However, from our discussion of the measurement problem above, one sees that a definite measurement outcome is generally *not* available at the end of the measurement process as described by a unitary evolution of the joint system state of the measured system together with the measurement apparatus; a definite pointer value is not obtained under those conditions, and so a rule such as the Lüders rule is needed. This is sometimes referred to as *the problem of pointer objectification*. Nonetheless, if the actuality of physical magnitudes is taken as generally indeterminate, as it has been here, a process of *unsharp objectification* can be sought which might avoid the apparent need to alter the unitary evolution of Quantum mechanics by considering the pointer observable as unsharp. If both the pointer and system properties are indefinite but nearly definite, they can still be considered approximately real in the sense introduced above. The natural representation of simultaneously unsharp but approximately real magnitudes, for example, position and momentum for large systems such as a macroscopic value-indicating needle can have intrinsic inaccuracies that are very large compared to Planck's constant but are nonetheless small relative to the scale of macroscopic measurement accuracies [48].

For the individual quantum system described by the state $|\psi\rangle$ which, following Birkhoff and von Neumann as above, can be viewed as a point in quantum phase space evolving along a unique trajectory given by the Schrödinger equation, there is a continuous evolution of the potentialities representing degrees of reality $d_E(P) = \text{tr}[P_{|\psi\rangle} E] = \langle\psi|E\psi\rangle$ for all the effects E of the system, which thus are all simultaneously real to just this degree. By virtue of reference to elements of reality via POMs, the simultaneous measurement of approximate values of non-commuting properties can be represented which are jointly measurable. In particular, $p^E_{|\psi\rangle}(X) = \langle\psi|E(X)\psi\rangle$ quantifies the likelihood of the actualization of the potential property corresponding to $E(X)$ when measured on a system prepared in $|\psi\rangle$. Then, one can choose joint measurements that are approximately repeatable [125], so that upon obtaining a joint outcome, say, of x and p, the quantum system will have position and momentum values localized about that point (x, p) with widths of the associated probability distributions equal to the joint measurement imprecision $(\delta x, \delta p)$.

In summary, when a measurement is performed of any observable that is not identical to the projector corresponding to the current system state, there is a change of the states of both the measured system and the measuring system, whether a sharp or an unsharp observable is measured. After a *repeatable* measurement is made of a sharp observable of the measured system, the properties associated with sharp observables compatible with that observable end up actual and definite and those associated with incompatible sharp observables end up indefinite and potential. When a measurement is made of an *unsharp* observable, that observable remains indefinite but can be less indeterminate than before the measurement; properties associated with compatible unsharp observables remain indefinite and but can be less indeterminate. One is able to provide better explanations of the behavior of quantum systems in that one can make use of unsharp observables to explain the results of joint measurements of physical properties which have traditionally been thought of as incompatible and unspeakable, because better predictions of later measurements can be provided using the updated probabilities corresponding to the observed outcomes, so long as the corresponding uncertainty product is reduced.

3.4 Actualization of Potentiality via Measurement

The adoption of the Potentiality interpretation makes it possible consistently to maintain that a quantum-mechanical apparatus records a specific outcome when it makes a measurement, and that when it does so in a repeatable or approximately repeatable manner there is an element of reality or approximate element of reality (actuality), respectively, corresponding to this outcome. It also has the virtue of explaining why measurements produce definite results no matter their degree of precision. Nonetheless, as with most proposed interpretations of Quantum mechanics, the quantum measurement problem raises a set of interpretation-specific issues. Let us consider these and their implications for the potentiality approach, as well as appropriate responses to them.

In a useful measurement, a definite result appears in the form of a property of a system serving as the measurement apparatus. Due to the nature of human cognitive facilities, this “pointer position” is typically a macroscopic apparatus or a portion of the nervous system in cases when a measurement is performed intentionally or recognized as such after the fact. However, as Anthony Leggett has noted, in one sense it is problematic to treat measurement apparatus and measured systems differently.

> [The QM] formalism is itself a seamless whole, extending. . . all the way from the realm of subatomic particles to the macroscopic, everyday world,. . . any interpretation of its meaning which changes radically between the microscopic and macroscopic levels must violate a principle of continuity. [177]

Acknowledging this and being an advocate of the potentiality approach, Shimony has sought to circumscribe a "mechanism" of a possibly but not necessarily psychophysical nature underlying the measurement process, remarking that

> Inevitably other systems become entangled with the object and the apparatus: notably, the physical environment, the space-time metric, the molecules of the observer's sensory and cognitive faculties, and finally the observer's psyche...

$$\Psi = \sum c_i u_i \otimes v_i \otimes w_i \otimes x_i \otimes y_i \otimes z_i \tag{3.32}$$

> In order to have a definite perceptual result, the 'chain of statistical correlations' in the state Ψ must be 'cut' (terminology of London and Bauer, cf. [182], Sect. 11), producing the nonlinear transition or 'reduction'

$$\Psi \to u_i \otimes v_i \otimes w_i \otimes x_i \otimes y_i \otimes z_i \tag{3.33}$$

> If one accepts this way of posing the measurement problem... and does not attempt to solve the problem by a hidden-variables theory, a decoherence theory, or a related strategy, then one must ask where and how the reduction occurs. [282]

Shimony has offered the following conjecture as to the form of a possible answer to this question.

> A possibility...is that the locus of reduction is the macromolecules of the sensory and cognitive faculties. A concrete example will provide some motivation for my conjecture. The photoreceptor protein of the rod cells, rhodopsin, is known to absorb a photon and initiate a biochemical cascade that eventually produces a macroscopic pulse in the optic nerve.
>
> ...The two components in rhodopsin are retinal, which can absorb a photon, and opsin, which acts as an enzyme that effects the binding of about five hundred mediating molecules when it is triggered by the excited retinal.... [W]hat if the unitary dynamics of evolution of the photon and the retinal produces a superposition of the cis and the trans conformations? ...Would not such a superposition produce an indefiniteness of seeing or not seeing a visual flash, unless, of course, a reduction occurred further along the pathway from the optic nerve to the brain to the psyche? My conjecture is that the reduction occurs at the retinal molecule itself: that there is a superselection rule operative which prevents a superposition of molecular conformations as different as cis and trans from occurring in nature. A general superselection rule of this kind would have the desirable consequence that in intracellular processes a molecular 'switch' is never in a superposition of 'off' and 'on,' since these correspond to different conformations.[14] [282]

Such a thing allows for experimental testing, the results of which could be telling. Such a rule would differentiate molecular from atomic physics.

It is important to keep in mind that our intellectual grasp over many everyday phenomena related to those involved in measurements is far less advanced than is typically acknowledged. As Feynman once put it, in general, "Most phenomena we are familiar with involve such *tremendous* numbers of electrons that it's hard for our

[14] A *superselection rule* is a rule precluding certain classes of states from being subject to the Superposition principle.

poor minds to follow that complexity" ([105], p. 8). For this reason alone, skepticism is warranted regarding the common claim that nothing physically distinguishes measurement processes from other processes. Measurements involve large numbers of elementary systems and are complex when considered at small scales, that is, in terms of the natural kinds involved, such as electrons and photons.

Finally, recall that the quantum measurement problem depends on the assumption of the eigenvalue–eigenstate link, that is, that a particular value of a physical magnitudes is possessed if and only if the system state is an eigenstate of the corresponding quantum observable. The rejection this assumption by allowing a system's physical magnitudes to possess determinate values even when its state is a superposition of eigenstates of the corresponding observable has been seen by some as another means of avoiding the measurement problem. Such interpretations are often called *modal* [128]; van Fraassen introduced this term to designate that characteristic of an interpretation that on it a system has a property when the Born rule assigns it the value 1 and can also have a property when the Born rule assigns it a value *less than* 1 [311]. For most such interpretations, the underlying intuition is that (a) the quantum state of a system in interaction with an environment is a mixed state, namely its reduced state, that describes the possible rather than actual properties of the system and (b) that the spectral decomposition of the state selects the preferred variables of the system corresponding to properties typically found to be definite [9]. Modal interpretations also do not typically assume the measurement is accompanied by a non-unitary evolution of state. The cost of the modal approach is the giving up of realism, that is, Shimony's assumption (i) given in Sect. 3.1. Accordingly, the modal approach will not be considered further here.

Chapter 4
Quantum Objects: Parts and Wholes

Abstract The notion of object appropriate to quantum theory is discussed in the context of the analysis of the notion of quantum particle and in light of results in quantum field theory. The whole-part relationship and the question of the applicability of Leibniz's Principle of the identity of indiscernibles in quantum theory are considered together with recent arguments against its applicability in the quantum realm. After the discussion of a range of alternative prescriptions for a quantum principle of individuation, one is advocated that helps clarify the relationship between the individual and ensemble interpretations of quantum mechanics and allows for a well-defined and adequate ontology of individual objects that includes subatomic particles. As a culmination of the foregoing, the prospects for physical reductionism, both explanatory and ontological, are analyzed through the discussion of a wide range of examples, including the reduction of classical physics to quantum theory and the reduction of structural chemistry to quantum theory both when approximations are allowed and not allowed.

The strongest correlations predicted by Quantum mechanics between properties of subsystems greatly surpass the strongest correlations that are classical in the sense of obeying the assumptions of Bell's theorem. This has ramifications for the fundamental characteristics of the objects to which the theory is committed. These predictions have their origin in specific mathematical elements of quantum theory such as the representation of states in Hilbert space. A deeper understanding of the origins of this state representation is generally considered lacking, as reflected in the ongoing history of attempts to provide a simple axiomatic basis for Quantum mechanics and field theory, and for the apparently dualistic nature of quantum dynamics. Moreover, it is clear that a metaphysical realist understanding of composite objects in quantum theory must differ in important respects from that of classical physics. Let us now look more closely at the question of individuality

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in quantum mechanics, its relationship to the structure of physical objects and to space-time.[1]

In light of the property correlations observed between distant quantum systems that exhibit tension with relativity, some have suggested that metaphysical realism should simply be abandoned, either because they are incompatible with Naive realism or because, in practice, the term *local realism* has been used in physics as a synonym for the 'local causality' assumed in proofs of Bell-type inequalities.[2] For most physicists, however, realism or, at a minimum, most of its characteristics are essential preconditions of the physical world view. Indeed, for many of the latter it is the most crucial element of the scientific world view, more so than, say, determinism. Vivid evidence of this is that one finds Einstein writing to one colleague regarding the philosophical controversies surrounding Quantum theory that "The sore point [*Der wunde Punkt*] lies less in the renunciation of causality than in the renunciation of a reality thought of as independent of observation" (quoted in [293], p. 374).

Einstein's primary concern was, as ours is here, what is needed for quantum theory to describe the world in an objective fashion, over and above the construction of a framework merely for reliably predicting what is directly observed. Rather than rejecting realism, one does better at the current stage of the history of physics to continue to try to understand what *is* known about quantum theory that challenges our traditional world view and how an objectively existing reality can be most naturally described by quantum theory, given the nature of the novel correlations it correctly predicts. At first sight, one might wonder how the latter *could be* consistent with metaphysical realism because, for example, the non-contextual local hidden variables theories which are natural under Naive realism are inconsistent with empirical evidence.[3] To understand this, one needs to understand better what the referents—the events, objects, and processes—of Quantum mechanics are and to find a way in which their identities can be established. As emphasized in the foregoing, this can be done with the assistance of the notions of objective indefiniteness, potentiality, and non-local property correlation. Such a realist approach, despite the challenges presented by non-local property correlation and property indefiniteness, has the extremely valuable characteristic that—unlike the Copenhagenist philosophy of physics which Einstein referred to as a "tranquilizing philosophy" that "furnishes to the true believer a soft pillow that he has a hard time leaving"—it does not risk taking physical understanding into what Popper referred to as to the "end of the road,"[4] that is to say, a dead end for scientific discovery. Indeed, it requires a continuation of the quest for a more detailed understanding of difficult questions in quantum theory, such as the physical details of the production

[1]Portions of this chapter closely follow my article [155] addressing individuation and Leibniz's Principle of identity of indiscernibles discussed below.

[2]See [154], Chap. 3.

[3]See [154], Sect. 2.6 for a discussion of Naive realism.

[4]See [230], p. 13.

of outcomes of measurement processes, rather than considering these questions *Scheinprobleme* or denying their relevance to physics.

In this approach, a more detailed description of situations that are currently poorly understood in quantum theory, such as the measurement of a quantum object by a quantum-mechanically described measurement apparatus, is not precluded but is seen as a valued prize. Indeed, the realist expectation is that any significant future progress regarding such questions will involve the discovery of richer physics, such as that of the relationship between quantum mechanics and space-time. Moreover, it is shown below that everything that exists in different regions of space need not always be independent and localized for one to coherently identify and describe in physical theory a world of individual objects and processes, despite realists' own concerns about distant property correlations, for example, the concerns expressed by Einstein in the EPR article and elsewhere.

Willard Van Orman Quine supplied the following definition of the ontological commitment of a theory, which is useful for providing the setting of our investigation of the ontology of quantum theory.

> [A] theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true. [235]

According to this standard, Quantum mechanics is committed under realist interpretations to a set of entities corresponding to the vector spaces in which the states used to make predictions regarding measurement outcomes, expectation values, etc., lie or operate. The predictions of quantum theory within the ontology so circumscribed do not violate of the prohibition of superluminal signaling: As has now been shown in numerous ways, the measurement outcomes predicted by Quantum mechanics satisfy just such a prohibition.

It is helpful here to recall just what metaphysical realism involves. As a philosophical position, metaphysical realism is most concisely described as the assertion that there are entities that exist independent of the mental. In philosophy, as Hilary Putnam has importantly noted, emphasizing the relationship between entities and statements regarding them,

> Whatever else realists say, they typically say that they believe in a 'correspondence theory of truth'... When they argue *for* their position, realists typically argue against a form of idealism—in our time, this would be positivism or operationalism... And the typical realist argument is that ... if these objects [of science] do not really exist at all, then it is a *miracle* that a theory which speaks of curved space-time [, for example,] successfully predicts phenomena. ([234], pp. 140–141)

Although this often-made realist argument is just one of the arguments that can be put forward in support of metaphysical and scientific realism, it is an important one worthy of mention here not only because it is a common feature of a range of 'realisms,' as Putnam points out, but also because it captures the intuition of the overwhelming majority of physicists regarding physical objects and how they would likely explicate the meaning of *truth* if put on the spot, which is via the correspondence of true statements with the state of affairs in the world. To the above common argument for realism more can be added, for example, that realism can help avoid the incommensurability of successive scientific theories by guaranteeing continual reference and that only realism can explain the instrumental reliability of scientific methodology.[5]

Note that accepting metaphysical realism does not preclude one from approaching physics from the point of view of operations, for example, defining 'quantum logical' connectives by filters, etc., but only from notions such as there is *nothing more* to physical quantities than operations in terms of which they can be considered. In philosophical discourse, realism is said to involve several "levels," not only that of truth, but also those of entities and theories. The level of interest in the current context and, most likely, with most physicists is that of *entities,* in particular, physical objects and processes. For the realist, science works and its success is not miraculous because scientific statements make reference to things that exist objectively, that is, independently of our thoughts of them, however much our thoughts may contribute to our personal understandings and experience of them. Thus, if our theories of these things were to change the things in themselves would not; our theories will continue to refer to the same objects through theory change so long as the named objects appear in valid explanations all the while. It is uncontroversial to physicists that they study what the world is made of and how it is put together, it being the same world it has always been while theories improve.

Before moving on to specific answers to the important questions regarding the ontology of quantum theory, let us also briefly sketch the place of truth in relation to physics. The basic idea of the *correspondence theory of truth* is that individual statements, beliefs, and theories are considered true to the extent that they mirror the reality of the situations they can be understood to describe.[6] The relationship between realism and truth is often considered most strongly made through "disquotational theories" of truth and reference. In the disquotational theory of truth, the meaning of 'true' is established by our coming to know a grand collection of facts, each representable by the schema (referred to as that of a *T-sentence*)

(4.1) '*P*' is true if and only if *P*.

Here P is a proposition, e.g., 'The electron has mass m' in ' 'The electron has mass m' is true if and only if the electron has mass m.' On it, this collection of facts establishes the meaning of the term *true* ([233], p. 69ff). Such an approach to truth is a correspondence theory in that, according to it, propositions are true by virtue of their correspondence with such facts. The terms within the propositions P are thought of as referring, for example, to the corresponding objects and properties, as is the case in our example with *the electron* and *mass m*; the latter is what Einstein would say corresponds to an element of reality and is uncontroversial in the classical

[5] Cf., e.g., [178, 279].

[6] Keep in mind, here, that the relationship of truth to beliefs and theories is more complicated than that of a statement regarding a specific state of affairs, to say the least.

mechanical context. The relationship of (4.1) is similar to that given in the EPR completeness criterion: a relationship between reality and its direct representation.

These aspects of the realist interpretation of Quantum mechanics are significant to physics because they clarify issues which might otherwise remain obscured. For example, on some popular approaches, such as the Copenhagen interpretation of Bohr which emerged from his Como lecture, microscopic objects are *not* considered the theory's direct referents; the referents of quantum theory are on it taken instead to be *phenomena*[7] which are assumed to be accessible only via macroscopic apparatus with microscopic parts. The most common realist view of the world has been one of independently existing spatially localized objects of composites thereof, that is, classical bodies. Indeed, it is just this characteristic that serves to secure physical descriptions for Bohr, although, given that measurements of the microphysical are thereby assumed macro-physical, one may then ask about this approach, as Shimony has,

> is it not strange that the macrophysical has to be described in microscopic terms, and specifically quantum mechanically, in order to understand how it works—which would be a peculiar sort of macrophysicalism? [275]

Physicists should take pause regarding this point, given the historical popularity of the cluster of notions identified as the "orthodox" Copenhagen interpretation.[8] It points out the benefit of seeking objectivity within quantum theory itself, allowing one to avoid the nearly circular interdependence of quantum and classical descriptions involved in the Copenhagen theory of measurement.

Returning to the entities to which quantum theory refers, the realist must clearly identify what confers *individuality* upon its objects. In the study of Quantum mechanics, very often and not always helpfully, one finds the words *system*, *object*, and *particle* used more or less interchangeably to describe the theory's referents. However, the traditional notion of a physical object, that is, of a repeatedly identifiable individual that can be always attributed entirely definite properties, at least some of which are unique to it so that it can always be uniquely indicated, is no longer applicable. In a realist conception of the physical world, this naturally brings in the question of the relation between objects in the sense of parts and wholes, that is, individuals that may be or may have been composed from or decomposed into other individuals, as well as their persistence, that is, their identity over time.

For the most part, the systems of quantum physics *are* considered objects that are, have been, or will be in some sense, composed of small parts—molecules, electrons, photons, nucleons, etc. The smallest of those, namely, the elementary particles are involved in *fundamental processes* and can come and go out of existence. However, for such systems one may conclude, as Heisenberg did, that "words such as 'divide' or 'consist of' have to a large extent lost their meaning" or, at least, their

[7]In the highly specific sense he meant the term, see [223].

[8]For an discussion of the various forms of the Copenhagen interpretation see [154], Sect. 3.3.

common-sense meanings [138]. This is something he saw as having been clearly demonstrated empirically in particle 'showers' (cf. [209]).

> At the end of the forties, Powell discovered the pions, which play the major part in these showers [arising in photographic plates due to cosmic rays]. This showed that in collisions of high-energy particles, the transformation of energy into matter is quite generally the decisive process, so that it obviously no longer makes sense to speak of a splitting of the original particle. The concept of 'division' had come, by experiment, to lose its meaning. In the experiments of the fifties and sixties, this new situation was repeatedly confirmed. . . [139]

Furthermore, our access to elementary particles—with the exception of photons which are in any event encountered in a way that is distant from our intellectual conception of them—is highly indirect. As Frank Close has explained,

> Ultimately it is the quantum relation between short distances, the consequent short wavelengths needed to probe them, and the high energies of the beams that creates this apparent paradox of needing ever bigger machines to probe the most minute distances. These were the early aims of those experiments to probe the heart of the atomic nucleus by hitting it with beams of high-energy particles. The energy of the particles in the beam is vast (on the scale of the energy contained within a single nucleus, holding the nucleus together), and as a result the beam tends to smash the atom and its particles apart into pieces, spawning new particles in the process. This is the reason for the old-fashioned name of 'atom smashers'. ([62], Chap. 3)

Thus, the early modern approach to atomic physics involved the indirect probing of objects via the examination of the parts emerging after collisions with α-particles. The current approach often involves precision probing with electrons.

> The key to progress was to ionize atoms, liberating one or more of their electrons, and then accelerate the accumulated electron beam by means of electric fields. . . . The electrons scattered from the protons and neutrons began to reveal evidence of a deeper layer of structure within those nuclear particles. . . . At energies above 10 GeV, electrons can probe distances of 10^{-16} m, some ten times smaller than the proton as a whole. When they encountered the proton, the electrons were found to be scattered violently. This was analogous to what had happened 50 years earlier with the atom; where the violent scattering of relatively low-energy alpha particles had shown that the atom has a hard centre of charge, its nucleus, the unexpected violent scattering of high-energy electron beams showed that a protons charge is concentrated on pointlike objects—the quarks.[9] (ibid.)

Both in regard to the manner in which they combine and in their identity, there is a tension in modern physics between the traditional notion of particle and the way, for example, subatomic particles are and are observed. This tension is also reflected, for example, in the EPR scenario in relation to the reality criterion discussed in Chap. 1.

[9] "(pointlike in the sense that we are not able to discern whether they have any substructure of their own). In the best experiments that we can do today, electrons and quarks appear to be the basic constituents of matter in bulk." ibid.

4.1 Individuation

A standard element of the explications of the notions of individuation and identity is that of a *label* or name. This sometimes seen as corresponding to an identity attributable independently of the properties an object possesses because, in the absence of any explicit assumption or logical constraint to the contrary, the possession of properties and the possession of an identity could be differentiated. The characteristic underlying such an identity is called *haeccity*[10] or *primitive thisness* and can play a role similar to the philosophical notion of substance, in that properties might be thought somehow to be attached to or inhere in whatever is "of the object" over and above its properties. This is sometimes also referred to as a *transcendental individuality*. The difficulties that arise over its applicability in the quantum context are discussed below. Another traditional approach to individuation, which runs even more quickly into difficulties in this context than does labeling, is that based on the relationship between an object and space-time. Such *space-time individuation* takes spatiotemporal continuity as providing identity to an object. This often involves additional physical assumptions, such as the assumption that all objects are impenetrable. Difficulties arise because precisely specifying a unique continuous trajectory is rendered impossible by the first principles of quantum theory, despite the apparent presence of such trajectories based on what is observed, for example, in bubble chamber tracks. A minimal alternative approach is that of considering an individual to be identifiable only through some collection of properties found consistently associated with each other. Below, a natural alternative notion of individuation specifically tailored to quantum theory is developed.

It is commonly thought to be and is often desirable to have objects with structure that is analyzable, or *reducible*, all the way down to the elementary particles. The unchanging properties collectively specifying the kind of elementary particle one has, such as the rest mass, electric charge, and spin-angular momentum, say along the lines that have been laid out by Wigner, can serve along with changeable properties as a basis for the establishment of an individual identity. As shown below, with an appropriate principle of quantum individuation, even elementary particles can be understood as, under appropriate circumstances, possessing an individual identity despite any two of the same kind being commonly called "identical particles" and their obeying non-classical statistics.[11] The strong inter-system property correlations such as those that can arise due to the superposition principle applied to multiple quantum systems also do not pose, in and of themselves, a fundamental threat to the description of a world of objectively existing objects. Nonetheless, one does find that there are limitations on ontological reduction at different scales and levels of complexity.

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[10]This term is typically used in the modal context.

[11]It remains true, however, that in many circumstances the object itself loses its identity by being absorbed into a greater collective object, as seen below.

When discussing quantum ontology, one must be clear as to the sort of quantum theory under consideration, whether it is basic non-relativistic quantum mechanics, multiple-particle quantum mechanics, or quantum field theory (and in the latter case, which approach to quantization is taken). The reference of the term *system* will generally be different in each case; each form of quantum theory has its own, somewhat different ontology and will differently exhibit important characteristics such as entanglement. In single-system quantum mechanics, one considers a system to be associated unambiguously with a Hilbert space $\mathcal{H}$. In the multiple-particle quantum mechanics of n particles, there are different possibilities for mathematical description, for example, considering the tensor product Hilbert space $\mathcal{H} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_n$, and states with appropriate permutation symmetry. In the case of quantum field theories, numerous approaches to conceiving the structure of the field system and to formally grounding its description have been taken, for example, the perturbative and the algebraic approaches. In each case, it is helpful to have a well defined and workable notion of individuation to supplement the formally identified systems in order to provide a structured collection of objects or properties. The conception offered here classifies as objects the well-known entities typically described by quantum theory and, because these objects cannot be used to communicate faster than light, allows one to coherently conceive a physical causal order, although not one of the same form hoped for, for example, by Einstein.

In a much-quoted comment, David Lewis stated that identity is utterly simple and unproblematic, noting that "Everything is identical to itself; nothing is every identical to anything else except itself…two things can never be identical" ([180], pp. 192–193). The Humean metaphysical setting of this comment is one in which quantum objects and space-time are considered solely in terms of their properties and relations. Lewis explains how this might be accomplished, as follows.

> It is the doctrine that all there is to the world is a vast mosaic of local matters of particular fact…We have geometry: a system of external relations of spatio-temporal distance between points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether or fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. There is no difference without difference in the arrangement of qualities. All else supervenes on that. ([181], pp. ix–x)

It is shown below that it is problematic to flesh out even this simple picture in a strongly reductionist way. Our first task is the clarification of the notion of the physical individual in quantum theory, that is, the explication of the manner in which identity can be conferred upon a putative physical object, such as a quantum system, rather than to, say, a number.

Unless care is taken, difficulties surrounding identity can easily arise in quantum theory. For example, as Steven French and Décio Krause have recently argued, "if self-identity is tied to the notion of individuality and quantum entities are understood as objects which are non-individuals"—then "such entities fail to be self-identical" ([111], p. 5). The consideration of physical systems as individuals depends on which sorts of putative entities, for example, events, space-time points or regions, or particles, are assumed variously to occur, obtain, exist, or persist, and on the conception of individuality most appropriate in each case. A number of approaches and formalizations of self-identity have been considered by philosophers and serve as a background for the development of a well defined notion of identity appropriate to quantum mechanics.[12] There are also several ways to approach the question of the persistence of an entity through physical history. For example, specific properties such as system mass or charge can be considered essential to the identity of an entity. If such properties are lost then the entity ceases to exist as such, for example, when a neutron decays 'into' a proton, electron, and electron anti-neutrino. However, particles arising by decomposition (or decay) to previously present particles can be related to them, for example, using such basic quantities and conservation laws.[13]

We begin with the consideration of a set of physical magnitudes, considered as properties, and relations between them (relational properties), in order to see how individual identity can be conferred without the need to refer to any intrinsic thisness or haeccity and in accordance with basic logical principles. An identifiable entity is one that can be attributed a name or label for some instant or period of time, but is not necessarily defined by it. The repeated establishment of a momentary identity can be seen as providing a sort of *history* for an object. In particular, the *genidentity* [*Genidentität*] of an object—which is an identity established when it can be viewed as a succession of momentary entities, such as physical states at various times, that are connected with one another by virtue of their having progressively developed successively from one to the next rather than because of invariant characteristics—is therefore pertinent to physics [14,179]. In classical mechanics, a system's history, either a space-time or phase-space *trajectory* including a spatial trajectory—for example, definable by reference to a precise position and momentum at one time—is typically used to establish identity because in principle there trajectories are unique. In quantum physics, the relation between states at different times is more complex and potentially problematic; what quantum theory formally refers to as a *system* may not have an identity in this sense, even when attributed a label in its mathematical description.

What is a general method by which the class of individuals can be circumscribed? An apparently clear and common answer is simply to use Leibniz's Principle of identity of indiscernibles, originally formulated in the context of logic, to serve this purpose.

> *Principle of identity of indiscernibles* (PII). "If, for every property F, object x has F if and only if object y has F, then x is identical to y."

The PII has long been used as a valuable tool for clarifying physical ontology as well as serving as a basic logical principle, although it is more controversial than its

[12] Cf., e.g. [111].

[13] It is also important to note that there are situations involving composite entities in which it is counterintuitive to say that an entity ceases to exist if parts of it are replaced or even if all of its parts are replaced one at a time. The persistence of objects in time is discussed in detail in the following section.

inverse which provides the implication of an identification for properties, known as Leibniz's law or

> *Principle of indiscernibility of identicals*. "If x is identical to y, then for every property F, object x has F if and only if object y has F."

Leibniz viewed the PII as following from his *Complete notion of the individual* (CNI), under which an entity *amounts to* a collection of properties, often referred to as a *bundle,*[14] along the lines sketched by Lewis as recounted above. The CNI requires that an individual object contain all the logical predicates that the individual will possess; if an entity at any instant of its history *is* just the set of the properties it possesses then there clearly can be only one entity that is constituted by that set. These Lebnizian notions can be brought to bear on physical objects if properly applied consistently with the notion of genidentity. Unique identities may be distinguished by differences between individuals in regard to their unchanging properties, such as internal angular momentum, and/or their dynamical states, such as their location in phase space.

A hierarchy of versions of Leibniz's PII of differing strengths, depending on the sorts of properties that are included, can be given as follows.[15] The weakest form, PII_1, is that including all properties, that is, non-relational properties (such as the magnitude of the angular momentum) as well as relational properties (such as the spin component in a given direction of a particle *relative* to that of another particle). The form PII_2 is that which includes all properties with the exclusion of *spatiotemporal relations*. The strongest form, PII_3, is that which includes only *non-relational* properties. How useful or problematic the PII is in a given situation can turn on the version (1, 2, or 3) assumed. In Classical mechanics, unique identities can be uncontroversially established by differences in unchanging properties or in dynamical states in a way which is consistent with the PII in the form PII_1 and somewhat less obviously in the other two forms. In the quantum case, the application of the PII in any form has been controversial.

4.2 Space-Time and Individuation

In Classical mechanics, the entire set of physical magnitudes describing the individual system is fully specified by a collection of six parameters, the *dynamical variables* of vector position $\mathbf{q}$ and vector momentum $\mathbf{p}$, together determining the state, for example, in accordance with the partial differential equations governing the Hamiltonian function $H(\mathbf{q}, \mathbf{p})$; the domain of the dynamical variables constitutes the classical state space. Such classical states can relatively unproblematically be

[14]This term comes with the possible difficulty of an unintended unhelpful suggestion that there be something besides the properties *binding* the bundle ([249], Chap. 5).

[15]Cf. [111], pp. 10–11 and [237], pp. 24–25.

so considered because the ultimate components of classical systems, bodies or corpuscles, are readily discernible individuals, in turn because, for example, they are standardly assumed to be impenetrable[16]: the impossibility of interpenetration suffices for locations in space, and *a forteriori* in phase space, at any given time to be unique. Thus, classical systems can be individuated and are discernible by reference to the collection of such individual components (parts) to which they can be ontologically reduced because the latter can be identified by their space-time paths, that is, the paths of their centers, which cannot intersect. They can readily be considered physical objects within an ontological hierarchy that allows for the reduction of composite systems to a collection of fundamental components.[17] Elementary quantum particles, by contrast to classical particles, both can be in the same location, in accordance with the inherent joint uncertainty of position and momentum, and can have no finite spatial extent under some circumstances, often precluding a similar straightforward means of discernment and individuation. Statistical requirements on the behavior of collections of similar quantum particles introduce additional complications.

Given the success of the spatio-temporal approach to individuation in classical physics, it was natural to contemplate the explication of the individuation of objects in spatiotemporal terms in the quantum mechanical context, as Einstein did. It has been argued that spatiotemporal continuity is a necessary condition for identity over time there as well, that is, that it is necessary in order for an entity A present at time t_1 to be identified with entity B present at time t_2 that there be a space-time trajectory connecting the A and B [110, 286]. On one approach of this sort, the following three necessary conditions have been placed on such a trajectory [63]. (i) The trajectory must be spatiotemporally continuous, at least in a qualitative sense. (ii) The trajectory must be such that any individual stage along the trajectory must be qualitatively similar to a neighboring individual stage on the trajectory. (iii) The corresponding succession of stages are underlain by the trajectory.

Although elementary classical (Newtonian) bodies obeying the impenetrability assumption always have unique locations, so that a spatiotemporal approach to individuation works for them in cases in which they have continuous space-time trajectories, this approach to individuation clearly fails to hold comprehensively for quantum systems for the reasons just mentioned: Two quantum systems can share an elemental dynamical region, namely, that circumscribed by the position–momentum and time–energy Heisenberg relations, which makes it impossible to associate uniquely initial positions with final positions within regions into which more than one system can enter (see Figs. 4.1 and 4.3, below). Moreover, whereas classical particles are described in accordance with Maxwell–Boltzmann (MB)

[16]For example, Newton wrote in the *Optics* that, "…it seems to me that God in the Beginning formed Matter in solid, massy, hard, impenetrable, moveable Particles…" and Boltzmann assumed in his *Principles of Mechanics* that "two different material points never occupy the same place at the same time or come infinitely close together" ([111], pp. 40–41).

[17]See Sect. 4.6 for a classical mechanical example.

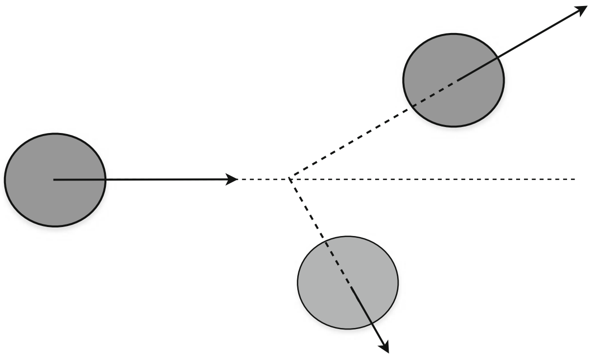

Fig. 4.1 Collision between two classical bodies, billiard balls, with velocities indicated by *arrows* and trajectories by *dotted lines*, with one ball in motion, shown at *left* with initial position and velocity, and another, initially positioned at *center* and at rest but not shown. Both balls are shown at *right* with final positions and velocities shown. Compare this with Compton scattering, schematized in Fig. 2.6

statistics wherein each permutation of particle labels corresponds to a distinct state, broad classes of quantum particles are described by Bose–Einstein (BE) statistics wherein a permutation of particle labels can be made without indicating a different joint-system state, which raises further difficulties for spatiotemporal individuation.

Einstein can be understood in his approach to physics to view physical systems as individuated in a manner typical of classical physics, that is, of Newtonian physics after adaptation to the requirements of Relativity, that is, based on spatial location. It has been argued that the approach to individuation preferred by Einstein is wanting because of failures in the case of entangled systems. In particular, Don Howard has provided a detailed historical argument to the effect that, "For Einstein, any non-null spatio-temporal interval was a sufficient condition for individuating two physical systems," finding this wanting because "even if the world-lines of two systems intersected, from the moment after their intersection Einstein's spatiotemporal individuation principle would count them as, once again, separable systems," something Howard finds problematic because he believes that it implies that they must in every instance become entangled as a result [144]. This is indeed problematic in those cases when the systems do in fact become entangled in the process: In Einstein's treatment, any two systems separated in this way would be described by different objective states and fail to violate the Bell-type inequalities, which are seen to be violated in practice.

Howard argues that a spatiotemporal approach to individuation remains nonetheless possible, in particular, that it can be successfully adapted to the evidence through a shift of emphasis from space-time intervals between systems to the "topology" of their world lines. He has argued that the difficulties encountered by the Einsteinian approach arise ultimately because this view of individuation is based on the assumption of a *point ontology* for the space-time manifold, that is, that space-time is to be identified with the set of points of the space-time manifold, and further that Einstein's conception of "non-null spatio-temporal separation may not be taken as a principle or ground for the individuation of otherwise identical fundamental physical systems" [144]. The first argument is supported by the fact the dynamical character of all space-time robs space-time points of haeccity. In support of the second, Howard points to the empirical violation (for broad classes of spacelike distant pairs of systems) of Bell-type inequalities which model the behavior of independent localized particles and constrain the statistics of joint measurements.

Recognizing that any two quantum systems described by an entangled pure state will violate Bell's inequality for at least one set of observables [120], Howard offers an alternative spatiotemporal principle of individuation, the "QM-GR principle of individuation." The QM–GR principle of individuation focuses on the relationship between world lines, that is, paths of systems. This proposal appears to be plausible given, for example, the above-mentioned use of continuity of space-time trajectory for establishing identity. Howard argues that, leaving aside purely gravitational interactions, "general relativistic space-time already has within it structure that to some extent mimics the pattern of individuation typical of quantum mechanics for all interacting systems" [144]. However, this might also strike one as surprising, given the difficulties of marrying General relativity and Quantum mechanics.[18] In fact, it is claimed that a relativity-based principle may serve as the basis for such a "Pauli program." The QM–GR principle focuses in particular on the "topology" of world lines rather than space-time points to ground a notion of separation of would-be individuals.

> (I) Any two systems, A and B, that were anywhere separated by a non-null spatio-temporal interval in their causal pasts are to be regarded as separable systems at all points in their causal future, unless (1) their world-lines intersect, or (2) their world-lines are both intersected by the world line of a photon. In either of these cases, systems A and B are to be regarded as nonseparable at all points in the causal futures(s) of the intersection(s), unless condition II obtains.
>
> (II) Any two nonseparable systems, A and B, are to be regarded as separable systems at all points in the causal futures of any intersection of either A or B with a third system C, as in I(1) above, or with a photon emitted by a third system C, as in I(2) above.[19]

Howard goes on to say that it would "be desirable perhaps to try to free the formulation of [the QM–GR] principle from all vestiges of notions borrowed from the point structure, such as talk of non-null spatiotemporal interval between two systems during their history prior to some interaction of interest, as in clause I. Though I will not attempt such an explicit reformulation here, there should be no obstacle to doing so..." [144]. Note that, here, "interactions" are mentioned in relation to Clause I, whereas they are *not* mentioned in relation to the second clause. Presumably, it is the intersection of world lines involved in local physical *interactions* that is important here, which are special cases in which world-line

[18] These were pointed out at the very outset of this book.

[19] Namely, those indicated in (his) Fig. 3 similar to that of Fig. 4.2

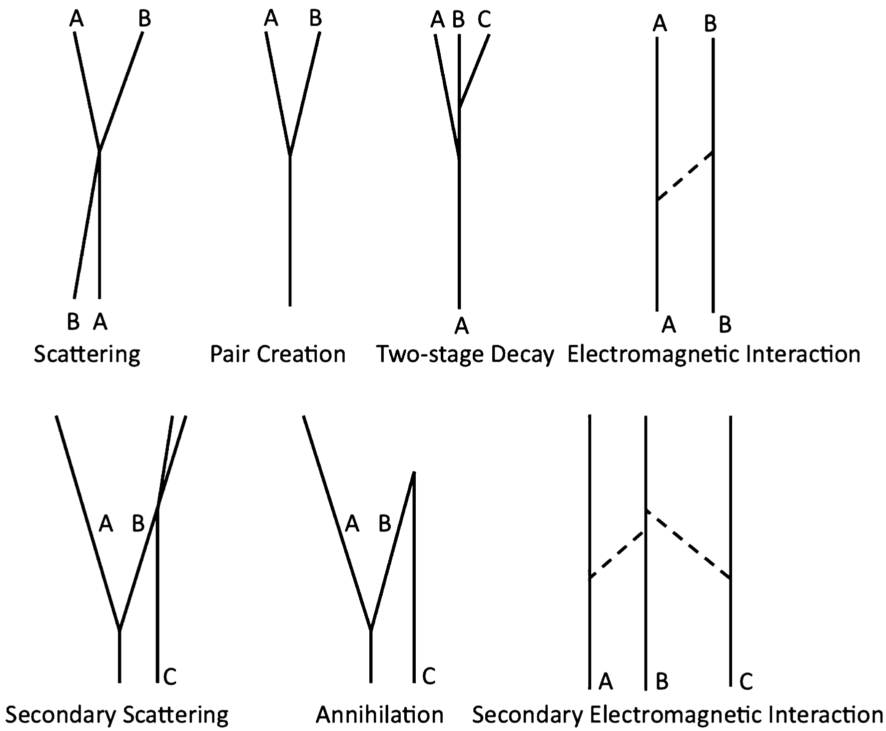

Fig. 4.2 QM–GR individuation illustrations. Spatial position is horizontal, time *vertical*. *Solid lines* indicate massive particles, *dotted* ones photons. One can discern various patterns (x, y, and λ type) considered, for example, by Salmon in relation to causation. See [254], p. 203

intersections occur; those special cases are not explicitly mentioned in either clause.[20]

Although it does avoid pitfalls of the traditional spatiotemporal approach, the QM–GR principle of individuation also fails, and does so with respect to both of these clauses as they are stated, as shown by the following examples. First, consider the simple case of two initially spacelike distant photons each with a wave packet centered about a common central frequency with the same packet width at half-maximum, etc., in a joint separable state, with world lines that meet at some point of space-time. These two initial photons can be considered separable after their having met, in particular, as soon as their wave packets again fail to overlap (again to a corresponding degree of precision[21]), *contra* the QM–GR principle—with regard to Clause I, because Clause II does not pertain.

[20]This remains so even if Howard did have them in mind to some extent.

[21]As shown below the viability of the notion of photon localization is not trivial and only clearly workable if a finite degree of precision is demanded.

The "exception" to this would be when the point of their intersection involves *interaction with matter* because they continue to be located in different field modes by virtue of which they can be distinguished, even though they might have crossed paths in space-time. However, this has neither been assumed in this example nor is it required by QM–GR. It is worth noting here that it is because of the need for the involvement of *matter* that one goes through the non-trivial step of, for example, using optically non-linear crystals,[22] in which spontaneous parametric down-conversion can occur, to produced the non-separable states of light regularly used in experimental demonstrations of Bell-type inequality violation [158].

Second, there are situations in which two initially spacelike-distant particles, A and B, are initially in a non-separable state (that is, are initially non-separable) and *never* meet in space-time, yet are still found to be non-separable in the causal future of *an intersection of one of them with another system* C, in contradiction with QM–GR, in particular, with regard to Clause II.[23] Consider the following laboratory situation involving the doubled use of the so-called "entanglement swapping protocol"[24] on two spacelike-distant *pairs* of non-separable spacelike-distant particles (the pair A-B of interest and other D-E, each pair initially in a Bell singlet state $|\Psi^-\rangle$), where no two among the four world lines of the particles ever intersect. The four particles are initially described by a tensor product state of two singlet states. In the first application of the protocol to them, two of the four particles, namely, *one particle from each singlet* (say, A and D) are subjected to a joint measurement—each particle being in a separate location—of the sort ideally used to show the failure of Bell-type inequalities to hold.[25] Particle A will come into contact with another system (let us call it C) in the process. The resulting state of the initial four particles is such that the remaining two, unmeasured systems are a pair B-E in a different (maximally entangled) Bell state (as does the measured pair) and so are non-separable.[26] In this situation, let us consider in particular the case, among the results which are possible, in which two Bell singlets result in the first application of the protocol. Then, after carrying out the entanglement swapping protocol for a *second time*, involving these resulting pairs of system states, one will receive once again the initial two *non-separable* pairs initially present, namely, A-B and D-E, *contra* the QM–GR principle (noting, again, that system C need not be a photon such as considered in I(2), which II might call on).

In this second example, there is a failure of the QM–GR principle in relation to each of its two clauses, I and II, one relating to engendering non-separability and the

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[22]In particular, typically involving a $\chi^{(2)}$ nonlinearity, cf. [159].

[23]The invocation of Clause I by Clause II will make no difference in some cases, including that considered here.

[24]See [211] for details as to the implementation of this protocol.

[25]Such joint measurements are now commonly referred to as a "Bell measurements."

[26]The four particles thus form *different* non-separable pairs, A-D and B-E; the particular one of the four possible Bell states of each pair that actually results depends on the particular one of the four possible outcomes found by the joint measurement.

other relating to engendering separability.[27] One could imagine a modification of the QM–GR principle such that Clauses I and II would refer only to massive particles and the intersections of their paths with those of photons, and adding two analogous additional clauses with restatements referring instead to massless particles and the "topology" of their paths and those of massive particles. However, upon reflection, one sees that this also would not improve matters in regard to the above counter-examples.

Although previously unrelated systems typically enter non-separable states through mutual interaction, a "composite" quantum system can be *created* entangled after coming into being (and, whenever pure, therefore created non-separable), say if two particles have emerged as a product of particle decay (curiously, the second example offered) in which case the subsystems themselves have not had an intersection of world lines, having not existed before and being required to come into being at a relative distance greater than the Compton wavelength of the particle(s) from which they emerge.[28] Moreover, the use of space-time points or trajectories to solve the problem of individuation in quantum mechanics privileges a particular physical magnitude (spatial position), something that has proven problematic in attempts to resolve foundational problems in quantum mechanics, such as the use of a privileged basis in the description of quantum measurement. Thus, one sees there must be more to the appearance or disappearance of non-separability than the occurrence of intersections of world lines; the attempt to reduce pure state non-separability to space-time considerations alone fails. Indeed, these considerations reveal that the element missing from clauses I and II, whether one is considering engendering or removing (pure state) non-separability, is *interaction*: It is the interaction of a pair of systems rather than the intersections of the world lines per se that engenders non-separability (or removes it), even though entanglement itself is non-dynamical in nature. The QM–GR principle appears plausible at first glance mainly because the example situations provided are relatively simple ones.

For quantum systems described by state-vectors, entanglement may imply Bell inequality violation [120], provided that the joint measurements involved are well-defined, that is, that the systems are susceptible to individual measurement that can be statistically considered marginally relative to compound non-local events. However, unlike Bell non-locality, entanglement itself does *not* depend on spatiotemporal properties; it depends only on a (conventional) choice of the "systems" to which Hilbert spaces are attributed. Even though it is a basic property of entanglement that it cannot be increased (on average) when the systems under

[27] One might imagine that the QM–GR principle was *intended* to apply only to particles of non-zero mass. Indeed, the associated illustrations (see Fig. 4.2) have labeled particles moving at sub-light speeds—the only exception is the single dashed line in one sub-case that represents a virtual photon moving at light speed. However, as Howard himself showed before offering his principle [144], Einstein's worries regarding separability began with his concern over the bosonic nature of light quanta, which are massless.

[28] See the Peierls discussion of the operation of intermediary force particles quoted in Sect. 2.3.

consideration are spatially distanced and the world line topological connectivity described in the formulation of the QM–GR principle may be a necessary condition for the increase of joint system entanglement, such connectivity is not a sufficient condition for it.

4.3 Indiscernibility

Let us continue the investigation of individuation in quantum theory by returning to the consideration of the way in which elementary particles and measurement apparatus,[29] which are the smallest of microscopic systems and the class of macroscopic systems through which we most often come to know them, respectively, have been conceived and related. One example is provided by a detailed discussion of typical tests of Bell-type inequalities. In one such discussion, provided by Linda Wessels, it is assumed that the systems and apparatus involved can be treated as quantum "bodies," where a *body* is defined as "an object that is contained in a relatively well-defined, spatial surface, thus has a well-defined spatial location, and in addition remains distinguishable from other objects even while its physical characteristics (including location and spatial surface) change," adding that, "We commonly conceive of objects as bodies, and this conception of objects also underlies many theories" [323].[30]

Before and at least into the era of early quantum theory, the conception of bodies had indeed been one based on our everyday experience. For example, as Giuliano Toraldo di Francia noted, "when we think of a physical object, we generally have in mind a *solid body*. Accordingly, our code has been developed to suit the situation" ([306], p. 24), despite our everyday experience of different states of matter and their consideration as far back as the pre-Socratic natural philosophers. The difficulties with such a picture are now clear.

Macroscopic measurement apparatus, taken as solitary composite objects, are indeed bodies that can be readily identified by traditional means, despite being quantum mechanically described, because they are unique sets of systems localizable in space at all times during their existence, something which eliminates possible conflicts of the sort described just above. However, the admissible senses of *object* and *body* in quantum theory, which is our best theory in the case of the very small, are far more spare than those just described, amounting to persistent bundles of properties or sets of such properties at points in space-time and very little more.

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[29] In the case of directly observations the apparatus would be the brain, optic nerve, eyes, and so on.

[30] The premisses associated with bodies Wessels offered include "A body has objective properties—properties it has independently of whether it is measured, or observed in any other manner," and "Interactions among bodies satisfy either a contiguous interaction model or the influence model or some combination of the two" (ibid.). This contrasts with views such as Falkenburg's that hold that there are severe restrictions on the extent to which these premisses, particularly the first, are allowed [95], p. 206.

For example, in addition to points already mentioned, in the case of the light particle, the photon, there is zero rest mass, something alien to the traditional body and particle concepts.

In the realm of subatomic physics, it has proven valuable to think of matter and light in terms of the notion of the *nomological object*, that is, in terms of entities with invariant properties determined by physical laws or conservation rules that also have changeable dynamical properties, because the systems in this realm are elementary particles. Falkenburg has described the notion as follows.

> Quantum theory alone does not admit of constructing objects in the sense of individual spatio-temporal systems with position and temporal duration. Abstracting particle properties such as mass, charge, or spin from their experimental or environmental contexts means dispensing with spatio-temporal objects and keeping nothing but *bundles of dynamic properties* of the respective kinds. These bundles are made up of those magnitudes that can be measured dispersion-free at the same time in *any* experiment. Somehow these bundles of dynamic properties propagate through an experimental context, respecting the conservation-laws of mass–energy, charge, and spin (as well as Einstein's causal condition…). ([95], p. 205)

One can then go further and develop a mereology based on the intrinsic properties of a few sorts of these nomological particles. The collection (bundle) of properties is best thought of as associated with PVM or POMs on Hilbert space, which space one can think of as formalizing the common presence of object properties and not necessarily measurable entirely dispersion-free. When being by their very nature precisely the same (identical) in their prescribed intrinsic properties, individual subatomic objects (elementary particles) can only be differentiated via their dynamical properties. Difficulties relating to the PII can therefore then arise, for example, when collections of elementary particles of exact the same type are brought together.[31] The PII has somewhat controversial implications, which ultimately depend on the particular relations considered relevant, that is, whether one adopts PII_1, PII_2, or PII_3.

Elementary particles can be thought of as the objects occurring in nature of central interest to quantum theory and can be considered either as systems in their own right or as quanta of field modes. A number of confusions are possible in relation to individuation in the quantum theory of elementary particles. Any two elementary systems of the same kind, for example, electrons are identical in these properties and may differ only in their remaining, dynamical properties and then often only relative to each other, for example, it may be that two of them may have opposite but indeterminate momenta; individual quantum particles typically have indistinct locations according to the Heisenberg relation and, when interacting, have ill-defined paths, as illustrated in Fig. 4.3, below. As a relatively detailed illustration of the importance of considering how properties are attributed in connection with the application of the Principle of identity of indiscernibles,

[31] Again, with objects described as bundles of properties, which is a notion closely associated with Leibniz's notion of the individual which can be seen as reducing individual to just such bundles. See, for example, the discussion in [111], p. 8.

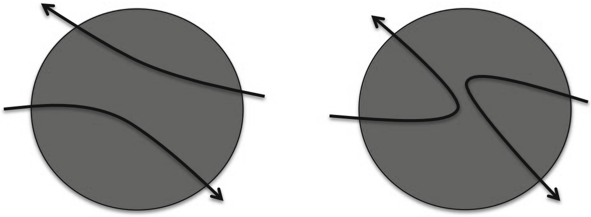

Fig. 4.3 Possible space-time trajectories in quantum theory ([104, 251], p. 358). When one traces to the limits of precision allowed by the Heisenberg uncertainty relations—spatial precision indicated here by the *shaded region*—the trajectories of two interacting systems such as a pair of electrons approaching each other, these are found to be indistinct: No one-to-one correspondence of a single incoming trajectory with a single outgoing trajectory is possible as it would be if these were classical particles, because such a correspondence requires precisions higher than allowed

consider an ostensible pair elementary particle systems arbitrarily labeled "1" and "2," each attributed a *complete set of quantum observables* $\mathbf{q}$ (as opposed to a single quantity, such as momentum). One might consider attributing to the pair the joint quantum state $|\mathbf{q}'\rangle|\mathbf{q}''\rangle \in \mathcal{H}_{12}$ with $\mathbf{q}' \neq \mathbf{q}''$ lying the tensor product Hilbert space $\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2$ of the respective individual-system Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, in that order.[32] Then note that the state $|\mathbf{q}''\rangle|\mathbf{q}'\rangle \in \mathcal{H}_{12}$ is another possible state, one in which the eigenvalues of the first in the above instance are instead those of the second and vice-versa. In either of these cases, if the properties of the system were measured for the particle $\mathbf{q}$-values, one value would be seen to be $\mathbf{q}'$ and the other to be $\mathbf{q}''$. Because all properties that could differentiate the two subsystems, which have been arbitrarily labeled "1" and "2," are included by assumption in the $\mathbf{q}$-values, it would seem that there is no empirical way of differentiating them, making them the *same object* according to PII_3, despite these labels.

Before concluding that we have reached an impasse, let us consider what quantum mechanics allows in the way of state specification based on measurement outcomes. Without assuming that the state should be of product form, the joint state specification for the two-particle system based on the suggested measurements would appear instead to be

$$|\Psi_s\rangle = a_1|\mathbf{q}'\rangle|\mathbf{q}''\rangle + a_2|\mathbf{q}''\rangle|\mathbf{q}'\rangle \,. \tag{4.1}$$

Because such a_i are constrained by the normalization requirement on $|\Psi_s\rangle$ but are not otherwise specified, it might appear from this that a unique state-vector cannot be attributed to the system but only an eigen-subspace of $\mathcal{H}_{12}$. However, the quantum theory of many-particle systems imposes an additional permutation symmetry requirement in the cases of two particles of the same kind: Symmetry

[32] See [251], pp. 361–362 for this textbook example. It is an example of an approach to describing multi-particle states that has been called the "Labeled tensor product Hilbert space formalism" (LTPHSF) [303].

picks out a specific state from this subspace for our pair of 'identical' particles, depending on the class of particle they both are, that is, depending on whether they are bosons or fermions. All permutation-asymmetrical cases, such as $|\mathbf{q}'\rangle|\mathbf{q}''\rangle$ and $|\mathbf{q}''\rangle|\mathbf{q}'\rangle$ and other examples of the form of Eq. 4.1 in which $|a_1| \neq |a_2|$, are precluded—otherwise, 1 and 2 would be (partially) empirically distinguishable and their labels would be justifiable on that basis, in contradiction to our initial assumption that *all* empirically determinable properties *are* given by the values of $\mathbf{q}$.

The only Hilbert-space states of 'identical particles' that are allowed in quantum theory are those that are symmetric or anti-symmetric under change of particle label. Let us define the *permutation operation* P_{ij} the effect of which is to exchange the *labels* of the ith with the jth system. In the above case, we have, for example, $P_{12}|\mathbf{q}'\rangle|\mathbf{q}''\rangle = |\mathbf{q}'\rangle|\mathbf{q}''\rangle$. In general, depending on the spin of the particles involved, given states of N representationally identical (same rest mass, spin, charge, etc.) bosonic (integral spin) and fermions (half-integral spin) systems, respectively, one has the quantum *Symmetrization postulate*, given as the conjunction of the two following equations.

$$P_{ij}\,|\text{N identical bosons}\rangle = +|\text{N identical bosons}\rangle\ , \tag{4.2}$$

$$P_{ij}\,|\text{N identical fermions}\rangle = -|\text{N identical fermions}\rangle\ . \tag{4.3}$$

This postulate has been empirically confirmed in the sense that states violating the symmetry requirements for bosons and fermions haven't been observed in the corresponding sorts of systems, respectively. The rule provides the statistics observed in such systems, the former known as Bose–Einstein statistics and the latter as Fermi–Dirac statistics. When the two systems considered above are fermions, such as electrons, the joint state is required to be antisymmetric under the exchange of the subsystem labels, so that we have

$$|\Psi_s^-\rangle = \frac{1}{\sqrt{2}}|\mathbf{q}'\rangle|\mathbf{q}''\rangle - \frac{1}{\sqrt{2}}|\mathbf{q}''\rangle|\mathbf{q}'\rangle\ , \tag{4.4}$$

a unique state-vector.[33]

The Pauli exclusion principle states that two electrons cannot both simultaneously be attributed all the same atomic quantum numbers; the rule takes the form of the 'principle' that two fermions cannot occupy the same dynamical (pure) state but is also an immediate consequence of the Symmetrization postulate. With this symmetry requirement imposed, both the inherent and dynamical properties of our two particles again appear to be identical (for details, see below), in contradiction with PII_3. Again, however, this overlooks the fact that one cannot properly attribute quantum numbers to the fermions unless they've become distinguished in the first

[33]In quantum field theory, this rule can be deduced and is, therefore, no longer a postulate but a theorem, the *spin-statistics theorem* [86].

place, allowing them to be uniquely labeled. The Hilbert-space description in itself provides no means of distinguishing between, for example, two elementary particles of the same kind; in order non-arbitrarily to label the two Hilbert subspaces "1" and "2," that is, to associate each particle with a corresponding $\mathcal{H}_i$, there must be some distinguishing feature, something which above has been precluded by assumption, and so is not included in the complete set of observables under consideration above. Due to questionable examples like this, the validity of the PII has nonetheless come into question in the quantum mechanical context.[34]

The invariant properties associated with elementary particles considered as nomological objects are most precisely specifiable using a method of Wigner which defines an elementary particle as an irreducible projective representation of the Poincaré group of space-time symmetries [325]. These representations are characterized by their values of mass m and (total) spin s; spin itself corresponds to the spatial rotation group symmetry of SU(2) only if m^2 is non-negative.[35] The allowed spin values are integral or half-integral. A representation of this sort is mathematically given as a Hilbert space $\mathcal{H}$ together with the unitary continuous $U(a, \Lambda)$ action of the Poincaré group on it such that $U(a, \Lambda)U(b, M) = \omega U(a + \Lambda b, \Lambda M)$, where $a, b \in \mathbb{R}^4$, Λ and M are Lorentz matrices, and ω is a scalar.[36] Particles, in this contemporary sense, are standardly called 'identical' when these non-dynamical properties are all the same, even though the particles may differ at any given moment in their variable properties, that is, in some values of the variables describing their states of motion. Wigner argued that his method of identifying elementary quantum systems is justified because "there must be no relativistically invariant distinction between the various states" of such a system [202]. If there were any relativistically invariant subspaces, then the system in question could contain a smaller identifiable subsystem. This provides us with a valuable tool for beginning a quantum mereological analysis, that is, for determining the smallest distinct parts of any quantum system based on property additivity.

Although Wigner's approach has been criticized for failing to answer the question of what is necessary for an entity to be considered an elementary particle and whether a particle interpretation is possible for specific quantum theories (cf., e.g., [43]),[37] together with T. D. Newton he did answer the first as follows.

> [T]wo conditions seem to play the most important role in the concept of an elementary particle. The first one is that its states shall form an elementary system in the sense [of being initial and final states of collision phenomena, and hence their connection with the theory of the collision matrix]. The second is less clear cut: it is that it should not be useful to consider the particle as a union of other particles. . . . Only the first condition is fulfilled for a hydrogen atom in its normal state and we do not consider it to be an elementary particle. [202]

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[34] See, cf., e.g., [110].

[35] Note that in the case of $m = 0$, that of the photon, the SU(2) symmetry does not relate to spin.

[36] For further work related to this approach see, for example, [296].

[37] See Falkenburg [95] for summaries of necessary conditions for various notions of *particle*.

Newton and Wigner also considered and illustrated the notion of an *elementary system*. "It is the second condition which has no counterpart in the definition of an elementary system. As a result of this circumstance the concept of an elementary system is much broader than that of an elementary particle; ... a hydrogen atom in its normal state forms an elementary system", ibid.

If several particles are identical in the limited sense of having the same invariants of mass, spin, electric charge, etc., one can only label subsystems if particles can be distinguished by a dynamical variable or by a property of the quantum field modes they are said to "occupy" as quanta in the quantum field-theoretical context.[38] As seen above, when values of the *dynamical variables* of two elementary particles of exactly the same kind are *also* all the same, problems can arise for various notions of particle beyond issues concerning trajectories. Indeed, problems are said to arise not only for fermions but also for bosons, for which the requirement is that Bose–Einstein statistics rather than Fermi–Dirac or Maxwell–Boltzmann statistics apply [312]. Falkenburg has argued that the mereological particle concept—that particles are the (microscopic) proper parts of physical entities—*fails* for bosons.

> Many bosons may occupy the same quantum state or field mode. Hence, the photon and the field quanta of the other interactions cannot be distinguished in terms of their quantum states... Here, *any* criterion for mereological distinctness fails. ... Therefore, ... one should *not* say that light consists of photons. ([95], pp. 327–328)

This statement, of course, contradicts the clear statement of Feynman, quoted in Sect. 1.2, that "light is made of particles," that is, photons, which are bosons. This disagreement nicely sets the stage for our consideration of a number of arguments against the consistency of jointly holding to Leibniz's PII and the laws of particle physics. In one such instance, Toraldo di Francia argued that there are instead problems for electrons, which are fermions.

> Of course, if we talk about two electrons, we must take into account Pauli's exclusion principle: two or more electrons cannot occupy exactly the same state. For instance, in a nonexcited *He* atom the two electrons, though being both in the 1s state, have their spins pointing in opposite directions; however, one cannot tell which points in which direction. Interchanging the two particles has no effect whatsoever on the state of the atom and nobody can tell whether the interchange has taken place... this contradicts Leibniz's statement that *eadem sunt quorum unum potest substitui alteri salva veritate* (if one thing can be substituted for another, without violating truth, they are one and the same thing. ([306], p. 27)

Although, until the last of the quoted sentences all of the above is correct, the conclusion reached is unjustified: What are described as indiscernible in this situation are the *joint* states of the electrons of the atom before and after label exchange, which are those of the *pair* of electrons (or, of the atomic components), which is identical to itself and one thing, fully compatibly with the PII, rather than the state of the subsystem constituted by a *single* electron.

[38] This context has not been considered above but is considered below.

Falkenburg has argued that the fermionic case, which includes electrons, is relatively unproblematic, that is, that the problems arise in the case of light but not for "matter."[39]

> Fermions have half-integral spin. They obey Fermi statistics and Pauli's principle, according to which each particle of a many-particle system is in another dynamic state, which the states have distinct quantum numbers. Hence, even though the fermion parts of matter are not spatio-temporally individuated like classical particles, they are at least dynamically distinct, i.e., well-distinguished in terms of their quantum numbers. . . . [O]ne may say that matter is made up of electrons, protons, neutrons, and quarks as constituent parts. ([95], pp. 327–328)

However, the state being referred to is that of Eq. 4.4 while the relevant states are, again, those of the *individual electrons*. In that case, the spin state of each of the individual electrons is *indefinite:* the only available states for these are their statistical states, which are each proportional to the identity operator, that is, $\rho_{\text{spin}} = \frac{1}{2}\mathbb{I}$ for each of the corresponding collections of identically prepared such electrons. This points out that it may be an error to view the fermionic case as less problematic: Without the introduction of some restriction on the conditions under which quantum *systems* are to be considered ostensible individuals, there are apparent difficulties for *both* bosons *and* fermions in relation to PII_3. The argument here against the PII is, nonetheless, inapplicable if one accepts the *relative* spin quantum number as the ground on which two electrons are to be differentiated in accordance with the Exclusion principle rather than the expectation value of the individual z-spin for a single electron.[40] Thus, even though PII_3 is seen to encounter a difficulty in this case, PII_1 and PII_2 *do not*, because relative to each other, the z-spin components of the two elections will differ in any measurement direction because the spin singlet state (which can be given in the form Eq. 1.17, that is, that of Eq. 4.4 if the Hilbert space in question is just that of spin) is rotationally symmetric. No matter along which direction the spin is considered, the form of the joint state will remain precisely the same.[41] In that limited sense, the above argument is valid.

PII_3 appears problematic for fermions; at the very least, one should adopt the PII in a weak form. However, there are arguments that are better than those above, against the PII in other forms, also based on the consideration of two elementary particles of the same sort. In particular, van Fraassen has offered the following argument involving bosons. Assume that there are two elementary particles of the same kind—'identical' in the standard quantum mechanical sense of the term, that is, by being of the same species—that can have the same dynamical states, such as when $\mathbf{q}' = \mathbf{q}''$ above. In the case of two 'identical' bosons, there is no restriction

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[39]Presumably, at least in regard to PII_1 and PII_2.

[40]The basis for the consideration of individual electrons *at all* given the unavailability of non-statistical subsystem states here can be found in the discrete values of basis quantities such as mass, spin, etc.

[41]This is not so, for example, for the joint spin state $|\Psi^+\rangle = (1/\sqrt{2})(|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$, even though both are maximally entangled.

on the relative states of the particles as there is for fermions. If two of these do have identical dynamical states, each one of such ‘identical’ particles would ostensibly be described by the same pure state. Considered in this way, they are entirely indistinct in this case, that is, quantum mechanically indiscernible while being assumed to be two entities. In the case of a pair of photons, for example, each can individually be in the same state of motion of the others because the Pauli principle does not apply to them. So, it would appear that the PII in *any* of the three versions above requires the ostensible two photons to be the same *one* individual, and so on. Hence, either there is a contradiction with the PII or the quantum state description of the individuals is incomplete. So, because there are good reasons to believe that the quantum state description *is* complete, as discussed in Chap. 1, it might seem that one should reject all versions of the PII. Again, however, this dilemma depends on a very direct association of Hilbert spaces and individuals via the formal notion of a quantum *system*, which we recognize as conventional. We see next how to resolve the apparent incompatibility of particle physics and the PII, by removing this convention.

4.4 Quantum Individuals

The apparent inconsistency between Quantum mechanics and the PII can be removed by introducing a distinction between the formal notion of a quantum system and that of an individual (quantum object) [155]. The following principle of Quantum mechanics specifies the mathematical correspondent of the quantum system in standard formulations of quantum theory, cf. [278].

> (P1) “Associated with every physical system is a complex linear vector space $\mathcal{V}$, such that each vector of unit length represents a state of the system.”

This specifies the essential mathematical structure of the space of (pure) states of a quantum system S.[42] The mathematical formulation of the theory is structured around these systems and associates to each one a Hilbert space, in accordance with P1 and, depending on the circumstances of that choice, either a ray (or state-vector) within that space or a statistical operator, whether or not the Hilbert space is a tensor product space obtained by applying the rule for composing systems in Quantum mechanics. When individual quantum objects are considered in the formalism of the theory they can be described by state-vectors. That such a description is sufficient for the description of an individual is established, for example, by the

[42] P1 assumes the Superposition principle (SP), which we recall states that any pair of states can (in the absence of superselection rules) be used to arrive at another allowed state of the system by (vector) addition, leading to the possibility of quantum interference; superposition rules are rules precluding the applicability of the superposition principle in specific cases.

evidence supporting the Synoptic principle already considered here.[43] When a system is attributed a state-vector $|a, b, c, \ldots\rangle$, it is certain that if all members of the complete set of observables $A, B, C, \ldots$ (for which $a, b, c, \ldots$ are the corresponding eigenvalues) are measured precisely and in immediate succession, the corresponding outcomes of these measurements on the system will exactly match the values, $a, b, c, \ldots$; these then correspond to Einsteinian elements of reality.

As discussed above, problems may arise in identifying individual objects in this way, given the statistical requirements on complexes of quantum systems, and apparent conflicts arise, given the associated quantum state symmetry requirements, with some or all forms of the PII. For example, in the case that entanglement has originated in the system through the interaction of two initially distinct individuals 1 and 2, such as in the state of Eq. 4.4, "corresponding subsystems" S_i $(i = 1, 2)$ might be formally sought by continuing to consider the product Hilbert space $\mathcal{H}$ of the state of the larger system S emerging from their interaction. However, the S_i are then not each describable by a state-vector but only by a statistical operator on $\mathcal{H}$.[44] Such statistical states allow for the description only of *collections* of individuals. The mixed collections of such individuals are represented as probabilistic combinations of such states, that is, convex combinations of states of collections of identical individual systems which have been identically prepared, each given by the self-outer-product of a state-vector.[45] In addition to appearing when differing collections of identically prepared individuals have been combined together, the mixed states are the only ones which provide a predictively correct (statistical) description of a subsystem of a system involving intra-system interaction. As Schrödinger pointed out,

> …the calculation methods of quantum mechanics allow two separated systems conceptually to be combined together into a single one; for which the methods seem plainly predestined. When two systems interact, their ψ-functions, as we have seen, do not come into interaction but rather they immediately cease to exist and a single one, for the combined system, takes their place. [267]

The description associated with each subsystem is given by a mixed statistical operator ρ_i that is obtained by averaging over the allowed values of the other degrees of freedom of the other subsystem; no state-vector description is available that could be taken to correspond to an individual subsystem given the chosen Hilbert space description. Thus, in the case of the labeled tensor product Hilbert space formalism

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[43] See Chap. 2. For a more complete treatment of this point, including a discussion of how the association of a collection of identically prepared systems with a pure state can be shown to provide the standard quantum statistics, see for example, [79], Sect. 10.2.

[44] Instead, only a statistical state description obtained by partial tracing over a portion of the joint state $|\Psi\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$, leading to $\rho_i = \mathrm{tr}_{j \neq i} P_{|\Psi\rangle}$, is possible. For example, for $|\Psi^-\rangle = (1/\sqrt{2})(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$, one has $\rho = \mathrm{tr}_2(|\Psi^-\rangle\langle\Psi^-|) = \mathrm{tr}_1(|\Psi^-\rangle\langle\Psi^-|) = (1/2)\mathbb{I}$, as mentioned above in regard to the argument of di Francia.

[45] Note that the otherwise noteworthy fact that this decomposition happens not to be unique is not a significant point in the current context.

considered above, there are significant limitations, relating to state entanglement, on the set of situations in which one can identify an individual with each of the system descriptions traditionally available in the quantum formalism. Similarly, Schrödinger argued that

> [T]he elementary particle is not an individual; it cannot be identified, it lacks 'sameness' [that is, genidentity]. ...the particles 'obey' a newfangled statistics, either Bose–Einstein or Fermi–Dirac statistics. The implication, far from obvious, is that the unsuspected epithet 'this' is not quite properly applicable to, say, an electron, except with caution, in a restricted sense, and sometimes not at all. ([268], p. 197)

A natural way to resolve such difficulties in identifying quantum objects, such as elementary particles, is to distinguish them from conventional systems by introducing an explicit *principle of individuation* for quantum systems. However, clearly one must do so differently from the attempts to produce a notion of separate individuals based on space-time properties. By contrast with the aforementioned previous attempts, attention here is focused specifically on quantum theory, and then not on spatial position or any other specific observable but rather on the full states provided by quantum theory itself.

Recall that, under the PII, an ostensible 'pair' of individual systems that differ in no discernible way in the properties considered (under the chosen version of the PII) is to be viewed as a single system identical to itself. This may be the case even if this pair of systems is viewed quantum mechanically as a composite system, that is, when the two are considered jointly also to be a quantum system as per the principles of Quantum mechanics. Thus, according to PII_3, there may be only one entity when permutation invariance requirements on composite-system quantum states are enforced, in that two subsystems under consideration have the same fixed properties and the same fixed dynamical states, while under the system description (without application of the PII) there are two entities (systems) that are being composed, as considered in the previous section, whether this arises due to interaction or not, independent of whether the resulting state is entangled or not. This is in contradiction with the assumption that there is more than one entity, as noted by van Fraassen. More generally, according to PII_3, in cases where two or more putative particles (systems S_1, S_2, ...) are found to have not only identical fixed properties but also identical dynamical states, one finds only *one* entity of the 'common' particle size when the Bose–Einstein permutation invariance requirements for multiple-boson systems such as photons are enforced, as they must be. Let us now see more precisely how such a contradiction can be avoided.

Under the conventional system description, any countable number of systems, with Hilbert spaces $\mathcal{H}_i$, can be composed and formally considered subsystems of a larger compound system associated with the tensor-product Hilbert space $\mathcal{H} = \otimes_{i=1}^{n} \mathcal{H}_i$. This allows for a consistent and practically successful portrayal of a multi-particle system as an individual entity, but there is no suggestion that there is under consideration only a single system of the size of one of the subspaces, as the PII might suggest when taken in form PII_3 and applied in the absence of an explicit quantum principle of individuation. The resulting joint system is an individual identical to itself and defined by the quantum numbers provided by the requirements of quantum theory; it is the set of states of *this* system, that is, of $\mathcal{H}$ and state symmetries which are relevant to the state count according to quantum statistics.

Consider again the case of a pair of electrons, which are fermions, in a spin-singlet state. The single-electron descriptions provided by the reduced state for each as a subsystem, which are the only accurate descriptions in quantum mechanics [174], are identical in such cases. The z-spin value is *entirely indefinite* for each, despite the applicability of the exclusion principle. Under PII_3, in the absence of any further restriction on entities other than being conventional quantum systems, the two would be understood as only one electron, inconsistently with other evidence that two electrons are present, as would be concluded by measuring total system mass, charge, spin, etc. independently of the formal system description. Although this might suggest that the application of the PII in the quantum context is entirely inappropriate, under the versions PII_1 and PII_2 discussed above, inconsistency and contradiction with experiment can be avoided, for example by reference to the exclusion principle dictating their distinct *relative* properties of z-spin. Moreover, as seen below, conflict can also be avoided even under PII_3 if an independent quantum individuation principle is taken on that prevents the PII from bearing on quantum subsystems in the classes of ostensibly problematic situations.

As an example to illustrate what is involved when the required statistical constraints are enforced, let us begin by considering the probability distributions for two individuals and a bivalent property (having values, say, U and D) under the various state statistics, much (in the left half) as Einstein did (in the case of the first three rows and BE and MB columns) in a 1925 letter to Schrödinger (cf. Einstein Archive 22-002 and [144]), cf. [312].

Case	U	D	p_{MB}	p_{FD}	p_{BE}
1	S_1, S_2		$1/4$	0	$1/3$
2	S_1	S_2	$1/4$	$1/2$	(1/
3	S_2	S_1	$1/4$	$1/2$	/3)
4		S_1, S_2	$1/4$	0	$1/3$

This example involves equiprobable weightings for joint states in the context of four *logically possible* states for pairs of physical systems S_1 and S_2, as variously counted according to classical Maxwell–Boltzmann (MB) statistics (which assumes all systems to be, by definition, distinguishable), Fermi–Dirac (FD) statistics, and Bose–Einstein (BE) statistics (which do not). Of course, the appropriate statistics must be enforced in any particular case. According to MB (classical) statistics, the four logical cases *for the compound system* are distinct and are all counted. According to FD statistics (quantum, with the Pauli exclusion principle resulting) only the *second and third* are counted, the other two being physically excluded. According to BE statistics, the first and last cases are each counted, and the second and third logical cases are counted only together as a *single* physical case.[46] Note that for either system S_i *in isolation* the marginal statistics are the same in all cases, the difference between them arising only for the *pair* of systems.

Ideally, one would like to have the PII applicable in all three of the above forms, PII_1, PII_2, and PII_3, consistently with the completeness of the quantum state description for individual systems in as many forms of quantum theory as possible. The range of versions of quantum theory in which this can be done can be broadened by the introduction of an explicit individuality criterion for quantum objects that, for example, blocks van Fraassen's dilemma even in form PII_3.[47] The price of achieving consistency in this situation is that not all of what are formally considered quantum *systems* in the quantum formalism will then be considered individual *objects*. This should not be of great concern to us anyway, because what constitutes a system is largely conventional and does not coincide, for example, with a set of natural kinds, e.g., electrons. An individuality criterion can also serve to reduce the conventionality of the quantum system concept and may assist in the understanding of such phenomena as quasi-particles. By restricting the set of systems that are proper individual objects, problematic PII-based 'identification' of elementary systems with each other after entangling interactions will not be made automatically; in the event that we require the PII to hold in the form PII_3, some elementary particles in some situations will fail to qualify as individual objects and can 'lose their identity' to a larger entity; individuation of elementary particles becomes possible only under appropriate circumstances and the set of physical objects, particularly of individual 'constituents', is restricted.

What sort of principle or criterion would allow for this? Consider the following statistical criterion of individuality, which has recently been offered. "A and B are two individual systems if and only if all probability distributions of values of observables attributable to A and B are statistically independent" [164]. This suggested criterion is problematic because classical systems commonly and unproblematically individuated by the means described previously fail to satisfy it. Furthermore, even two bodies the behaviors of which are correlated in a way that still *satisfies Bell locality* will be excluded as individuals by the requirement of absolute statistical independence. This is clearly too strong a requirement.

A better place to start is, as mentioned above, to focus on quantum theory itself. Let us return to the consideration of the standard Quantum mechanical association of system states with vector spaces. Consider the following promising criterion, formulated more or less directly in terms of the elements of standard quantum postulates, that would apparently remove potential conflict with the PII for the above problematic examples of entangled systems.

> A putative quantum system is an individual object only if its quantum state is pure.

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[46]This differs from the result of enforcing on the subsystems the PII, under which S_1 and S_2 would be replaced by a *single system* S of the same, not larger kind so that neither case would be counted.

[47]This criterion, being external to the quantum formalism, is interpretational in character.

The states of the subsystems will always be impure under this condition when a bipartite system state is entangled, so that they will not be identified with each other under this condition—they will also not be considered individual objects. However, purity is a property of states of collections of systems rather than individuals: The "pure state" is a concept belonging to statistical theories where "pure" refers to the character of a statistical *ensemble*.

Now, recall again that, as Einstein pointed out, the notion of the quantum system is highly conventional. One should not expect that the system which, in the absence of additional properties such as physically meaningful invariances could simply correspond to a subspace of a larger vector space, automatically to represent an object. (In this regard, note, for example, that when distinguishing elementary particles Wigner required the *irreducibility* of the corresponding representation.) Moreover, if we instead take as a necessary condition of individuality that an individual quantum object be described by a *state-vector* within an available, corresponding Hilbert space, the set of quantum objects present in the world may change with time, because given some initial physical situation, the availability of such a non-statistical description may change with time: As Schrödinger pointed out, if the quantum state description of two or more systems is complete, it appears that the set of quantum objects itself can change with time, in light of the manner in which the Hilbert-space description relates to the objects preceding their interaction. This is because there ceases to be a state-vector description of the subsystems of the combined system once the latter is entangled. With this condition, because there does not exist such a state description of the initially individual systems after interaction, the corresponding initial objects cease to exist and conserved properties are passed along to the larger one. This requirement serves as a candidate for an a priori individuality criterion, which we keep in mind.

Consider for a moment von Neumann Quantum mechanics in axiomatic form. It is based on essentially two elements [295]: (A) a one-to-one correspondence between physical quantities and hyper-maximal Hermitian operators in Hilbert space, and (B) linearity of the mean value operator for these quantities. As has been widely noted, for example, by Martin Strauss, there is a considerable conceptual distance between (A) and experimental data, that is, "from a *physical* point of view it can hardly be called satisfactory to base a theory on a postulate whose connection with experimental facts is as little intelligible as it is in the case with postulate (A)" [295]. Given this, an empirical criterion should also be considered. Recall also Dirac's dictum that "Each photon interferes only with itself. Interference between two different photons never occurs" ([83], Sect. 3) and note that it straightforwardly generalizes to "Each quantum system interferes only with itself. Interference between two different quantum systems never occurs" [152]. In light of these, the following principle of quantum individuality, which relates directly to experiment, can also be recognized as a plausible candidate individuality criterion.

> *Interferometric Principle of Individuation* (IPI). An ostensible physical system S is an individual if and only if full interference visibility could be observed in principle in an experiment performable on it.

This allows one to circumscribe the set of objects in the world as governed by Quantum mechanics while retaining the PII.[48] When a quantum system (say a photon pair) is described in a factored Hilbert space $\mathcal{H} = \mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2}$ by a product of state-vectors, one in each of the Hilbert-spaces $\mathcal{H}_{S_i}$, if both systems S_i are appropriately measured maximum interference visibility will be seen in measurements on each subsystem S_i (cf., e.g. [160]); the IPI dictates that there are two distinct, smaller individuals present. By contrast, systems in entangled states yield maximum (joint) interference visibility only of *coincidences*, that is, only for appropriate (joint) measurements on S for at least one observable on the *tensor product* Hilbert space $\mathcal{H}$. In that case, there will be only one, larger individual (the *pair*) present, under the IPI. The inconsistency of QM and the PII (even in form PII_3) in this way dissolves with the use of the IPI.

Another reason to take such a principle seriously is that quantum interference can be understood as resulting from the consideration of all possible system histories, which can be viewed as the generalization to the quantum context of the concept of system trajectory, which itself has been long seen as working consistently with the PII in the classical context, as discussed in Sect. 4.1. In that sense, the IPI offers a similar degree of conception continuity as the QM–GR principle suggested by Howard and, like it, considers system histories, but without a problematic dependency on *space*. Quantum interference arises when there is indistinguishability in principle of quantum 'alternative histories' leading to some single distinct final state, so that it also quite naturally coheres with the PII in the context of 'histories,' which include but are not restricted to spatial histories. The IPI is offered as an empirical principle, that is, a high-level empirical generalization.

Because physicists are in a position of limited knowledge of the history of the world, particularly of *all* its individual objects, it is good to have a principle like the IPI with the pragmatic virtue of helping one identify individuals in an empirical setting. The formulation of the IPI given above makes reference to interference visibility, something requiring an ensemble of measurements, if not systems. Because the PII is fundamentally a principle of logic and is ultimately independent of empirical matters, we should seek a (preferably related) constructive principle of individuality which has empirically significance but without explicitly including *ensembles*. Recall the previous suggested principle of individuality, "a quantum system is an individual if and only if it is described by a pure state." This takes seriously the correspondence between systems and state-vectors in Quantum mechanics and accords with the IPI. However, in addition to the problems pointed out above, this has more of the character of an axiom than a constructive principle, and when the phrases "pure state" or "state-vector" are mathematically explicated in terms of Hilbert space elements one has moved back from the empirical in the

[48] The phrase *in principle* is significant. Under optimally engineered circumstances quite large systems are capable of self-interference, as already seen in the case of C_{60} molecules [2] with no obvious limit to the extent that such phenomena could be observed given appropriate physical resources.

direction of an axiomatic principle such as (A). So, as a further alternative, consider the following form that avoids these two possible shortcomings.

> *Indistinguishability principle of individuation* (IDPI). A system is an individual if and only if for its current state there is at least one physical magnitude with at least two values that are fully indistinguishable in principle by a sharp measurement of it.

A relationship between the IPI and IDPI exists due to the (mathematical) complementarity relationship between the interference visibility and optimum state *distinguishability* (the mathematical complement of the indistinguishability) already established for situations involving states of bipartite states of bivalent quantum observables in [160]. This form could be used as a high-level generalization of the sort advocated by Einstein as helpful as a foundation for progress toward the goal of constructive physical theories. The IDPI is useful for interpreting Quantum mechanics as a theory of individual systems by helping to address at least one of the qualms that Einstein had with the theory, by allowing the individuals to be unambiguously identified consistently with the PII. Note also that, like Feynman's rules for the calculation of quantum probabilities discussed in Sect. 3.6, it can readily be assumed in the context of quantum field theory as well.

In addition to the IPI and the IDPI, which are respectively empirical and operational in character, another individuation principle can be introduced that is neither empirically nor operationally formulated. This third, fundamental specifically realist alternative is explicated in the following section and adopted from there on.

4.5 Parts and Wholes

The cases of potential conflict between the quantum system description of the world and the Principle of identity of indiscernibles arise in the context of the joint consideration of quantum systems. They concern situations where ostensibly larger systems, understood as wholes, are described in terms of descriptors of smaller systems, formally labeled as or understood in some sense as acting like their parts. The formal, logical relationship between wholes and their parts—the *mereology*—in Quantum mechanics has traditionally been centered on the Hilbert-space formalism with no distinction between objects and (the pure formal) systems being made. In order to help construct a fundamental realist individuation principle circumscribing the objects of quantum theory, let us reconsider two of the basic principles of quantum theory. This assists us in resolving the tensions between quantum theory and the PII.

Recall first the principle P1 of standard Quantum mechanics, that which specifies the mathematical correspondent of a quantum system, namely,[49]

[49] Hilbert space is one such complex linear space.

> P1) Associated with every physical system is a complex linear vector space $\mathcal{V}$, such that each vector of unit length represents a state of the system.

This principle specifies the structure of the space of (pure) states of a quantum system. Implicit in it is the Superposition principle, which states that any pair of states can (in the absence of superselection rules) be used to arrive at another allowed state of the system by (vector) addition. This principle is generally supplemented by another structural principle of Quantum mechanics, namely, that the description of the *composite systems* obtained by combining systems is associated with the tensor product of the two Hilbert spaces of those systems as considered alone.[50]

> P2) If 1 and 2 are two physical systems, with which the vector spaces $\mathcal{V}^{(1)}$ and $\mathcal{V}^{(2)}$ are associated, then the composite system 1+2 consisting of 1 and 2 is associated with the tensor product space $\mathcal{V}^{(1)} \otimes \mathcal{V}^{(2)}$.

To assist in relating these theoretically designated systems to the objects of nature, let us introduce a fundamental principle of individuation differing from but related to those considered in the previous section. Noting that, although there is no one-to-one correspondence between the *vectors* of Hilbert space and the quantum states of the associated systems, there *is* one between the *rays* of the Hilbert space (i.e., the equivalence class of complex scalar multiples of the unit vectors for the various directions available in the space), let us adopt the following Hilbert-space based individuation principle.

> *Quantum principle of individuation* (QPI). A system is an individual if and only if its state is entirely specifiable by a ray in the Hilbert space associated with it.

The principles P1 and P2 provide the mathematical ground for and structure of the set of states of putative entities capable of being composed and decomposed, but without clearly distinguishing collective from individual descriptions. Even when of maximum purity, statistical operators can still describe only collections of identically prepared objects providing, via the Born rule, only expectation values. Under the QPI, a ray is necessary and sufficient to specify the state of an individual object. The QPI identifies the proper individual objects from among the quantum systems, for which all rays provide states but not all states are given as rays. It might be thought that the QPI is redundant once P1 has been assumed or the Synaptic principle accepted. However, the QPI is needed to preclude mixtures from being descriptions of individual objects and allows for consistent use of the PII in its strongest form. It also has significant implications for the part-whole relationship and for reductionism in the quantum context.

When a quantum system is described by a ray, there is a corresponding pure statistical operator providing the very same predictions for a collection of such individuals, described by the projector onto that ray. However, the QPI explicitly excludes from the status of an individual object any system (formally so described) the current state of which can be given as a statistical operator ρ on the associated

[50] In relation to the formulations of both P1 and P2, see [278].

Hilbert space but *not* by a ray. An individual system cannot under the QPI be represented by a pure statistical operator; a pure statistical operator description is inappropriate for the description of an individual even though in those cases in which correct predictions of observed behavior can be made using the (pure) statistical operator are the same as those using a ray in Hilbert space. The former is unnecessary and opens the way for a confusion between an individual object description and the description of an ensemble. Moreover, the QPI provides a novel understanding of the description of a system given by statistical operators that are obtained by partial tracing: When impure states are obtained from the state of a greater system, that is, when it is entangled there *is no* smaller individual present under the QPI, because there is no corresponding ray description of such a system.

The set of individual objects identified by the QPI, whether particles, objects built up from particles,[51] or other quantum mechanical entities plays a role in providing the ontological structure of the quantum theory in which they are described. An aspect of quantum theory other than interaction is essential to the exclusion, under certain (often temporary) circumstances, of conventional quantum systems from the class of individual objects or, more precisely, to their being conventionally labeled, as they are only *potential* individuals which *could* appear should physical circumstances change, such as when a measurement-like interaction takes place.[52] In such cases, the operators provide the probabilities obtained *should individuals come into being* by analysis of the larger system, which is non-reductively describable, that is, entangled. An example where this is pertinent is a system S in the well known state $|\Psi^-\rangle = (1/\sqrt{2})(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$, namely, the Bell singlet state. The conventionally indicated subsystems in that case are described by the density operator $\rho = \mathrm{tr}_2(|\Psi^-\rangle\langle\Psi^-|) = \mathrm{tr}_1(|\Psi^-\rangle\langle\Psi^-|) = (1/2)\mathbf{I}$ obtained by partial tracing over the degrees of freedom of the other of the two subsystems S_i and describable by no vector or ray in the associated Hilbert space.[53]

One thus comes to the same conclusion, in a greater context, as Aspect has done on the basis of his observation of the violation of Bell-type inequalities.

> The violation of Bell's inequality, with strict relativistic separation between the chosen measurements, means that it is impossible to maintain the image 'à la Einstein' where correlations are explained by common properties determined at the common source and subsequently carried along by each photon. We must conclude that an entangled EPR photon pair is a non-separable object; that is, it is impossible to assign individual local properties (local physical reality) to each photon. [4]

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[51]See later sections for a preliminary investigation into the limitations of the reduction of the objects to more elementary quantum objects.

[52]Note also that the entanglement and non-separability of systems in the states obtained by application of the Symmetrization postulate to the general form of Eq. 4.1 in accordance with their type–bosonic or fermionic—are essentially non-dynamical in nature, despite the fact that they can be induced dynamically as described in the quotation from Schrödinger above.

[53]The same result is obtained for the cases in which S is in one of the remaining three Bell states.

Such a conclusion, resulting from the QPI, is appropriate for the formally indicated subsystems S_i in the EPRB (singlet) case: the states of the S_i are maximally unspecified, that is, entirely unspecifiable because according to the QPI the subsystems are objects that do not (yet) actually exist, and so cannot have non-statistical states. All that can be said is that there are constraints on the dimensionality of the corresponding Hilbert space dimensions and on conserved quantities *should* the subsystems come into being through later physical division of the larger individual system *into* 'parts', that is, should a product state result; under the QPI, like under the IPI but unlike the classical mechanical situation, only when S is describable by a tensor product state-vector can it be reduced to distinct individual parts. Conflict with the PII in the example offered by van Fraassen is thereby precluded, because it makes no sense to speak of individual parts in it. Furthermore, there is no reference in the QPI (or the IPI) to space-time or preference of any particular quantum observable for the determination of individuality. It is worth noting also here that, under the QPI, a bipartite system in any vector state that violates no Bell inequality has parts that are also individuals, in that they are also describable by rays.[54] Moreover, since such states are precisely the product states, the state of the whole system is *reducible* to those of these parts in that case.

It is an often-noted unique characteristic of quantum states, typically considered paradoxical, particularly in the context of information theory, that in the presence of entanglement a compound-system state may be precisely specified when those of its subsystems cannot be so specified. The Bell states, such as that above, are often provided as examples. The QPI removes the aspect of paradox, because under it the conventionally considered quantum subsystems cannot be considered *individual objects*; the only states attributable to them are mixed statistical states having no single corresponding ray. Tim Maudlin has defined the *ray view* as that in which "a single particle is represented by a ray in the associated Hilbert space," as we have through the adoption of the QPI; the opposing view, *statistical operator view*, is defined by contrast as that holding that quantum states of individual systems are required only to be specified by statistical operators [190]. In regard to reduction, in the case of entangled states, Maudlin distinguishes two possibilities for holders of the ray view such as advocated here: either (i) there *is no state* of the system (as an individual, which is my view and, arguably, von Neumann's and Schrödinger's) or (ii) a relative-state view wherein the subsystems only have states *relative to each other*, such as allows, as discussed above, the PII to be consistently assumed of quantum systems when it is understood in either of the versions PII_1 or PII_2. As Maudlin notes, quantum-state reductionism fails in both cases, and does so immediately in case (i).

[54]Note that this follow from the fact [120] that for the relevant class of systems for every entangled state there exists a Bell inequality it violates.

Maudlin also points out that reductionism fails on the statistical operator view because the singlet state does not supervene on the states of the subsystems, as described above ([190], pp. 54–55), and relates this to Einstein's reductionism.[55]

> Einstein's picture seems to fail not in the assumption that there are 'objects situated in different parts of space,' but rather because the 'being thus' (i.e. the physical states) of those objects are not independently specifiable. One can try to promote Einstein's intuition into an infallible principle by always readjusting one's notion of 'being situated in different parts of space' to fit, but it is hard to see this as a hopeful project. ... If the pair of particles forms an indivisible whole, though, part of the modern account of space-time structure seems to be at risk. ... In the relativistic regime, there is no way of carving up extended spatiotemporal objects into sets of interrelated parts.[56] ([190], p. 56)

Considering composite quantum systems in the relativistic context, one finds that potential difficulties quickly arise. For example, when entangled systems are considered from two distinct inertial references frames, an apparent *inconsistency* arises in the way that a whole can be divided up into parts [190]. Reduction then fails because a number of distinct joint system states, for example, all the Bell states, are compatible with these subsystem states. Moreover, allowing two subsystems to be sufficiently well separated in space, there exists a position P on the world line of one subsystem that in one reference frame occurs before the distant particle is measured and in another reference frame occurs *after* it is measured. The question arises as to *which whole* the state of the distant particle is to be related, that is, whether to the whole describable consistently with the system at P *before* measurement or that *after* measurement. Such difficulties do not arise under the ray view and the QPI.

4.6 Field Theory and Quanta

A natural way of incorporating the exchange-symmetry requirements on states in multi-particle Quantum mechanics is to describe them in Fock's Hilbert space and to perform "second quantization" to achieve a theory of field quanta. One can take as the space associated with the multi-particle system the separable Hilbert space which is the tensor *sum* of a countable number of Hilbert spaces $\mathcal{H}_i$, where the subscript i also corresponds to the number of (non-interacting) particles present, namely, $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3 \oplus \ldots$ where $\oplus$ indicates the direct *sum* (rather than the direct product often considered above and indicated instead by $\otimes$).[57] This space is spanned by the vectors $|n_1, n_2, n_3, \ldots\rangle_A$, where n_j is the number of quanta with

[55]Supervenience is a dependency relation that can exist between sets of properties possessed by similar or by quite different sorts of entity. Supervenience can be defined as follows. A set of properties **A** *supervenes* on a set of properties **B** if and only if two entities a and b that share all properties in **B** necessarily also share all properties in **A**, in which case the properties in **B** are 'base properties' and the properties in **A** are the 'supervenient properties.'

[56]Note that here the phrase "carving up" is not necessarily intended literally.

[57]Cf. [270], Sect. 7a and [303], Chap. 3.

corresponding eigenvalue a_j of the operator A assumed to be a maximal observable on the Hilbert space $\mathcal{H}_j$[58]; $\{|n_1, n_2, n_3, \ldots\rangle_A\}$ is a natural basis for $\mathcal{H}$.

Let us now assume for simplicity that the number of quanta in the world is finite. The Hilbert subspace $\mathcal{H}_1$ is spanned by those states with exactly one of the n_j being 1 and all others being 0, $\{|1, 0, 0, \ldots\rangle, \ldots, |0, 0, \ldots, 1\rangle\}$, $\mathcal{H}_2$ is spanned by the states with exactly one of the n_i being 2 and all others being 0, $\{|2, 0, 0, \ldots\rangle, \ldots, |0, 0, \ldots, 2\rangle\}$, and so on. The subspaces $\mathcal{H}_i$ correspond respectively to those situations with exactly i quanta present, each with associated Hermitian *number* operator $\hat{N}_i$ such that

$$\hat{N}_i |n_1, n_2, n_3, \ldots\rangle = n_i |n_1, n_2, n_3, \ldots\rangle \ , \tag{4.5}$$

for example, $\hat{N}_2|0, 0, 2, 0, \ldots\rangle = 2|0, 0, 2, 0, \ldots\rangle$. However, superpositions of these cases *are* allowed within the greater Hilbert space $\mathcal{H}$, as is the 'vacuum state' $|0\rangle$ identified as that in which *all* occupation numbers n_i take the value 0, so that the expectation value of the total number operator in a given situation need not be a counting number. The fermionic and bosonic cases are distinguished by corresponding constraints on the eigenvalues of the $\hat{N}_i$: In particular, in the bosonic case there are no restrictions on the values of the n_j, whereas in the fermionic case the values of the n_j are restricted to the set $\{0, 1\}$.

For bosons, there is a number-lowering operator $\hat{a}_i$, defined by the property that $\hat{a}_i|\ldots, n_i, \ldots\rangle = c_i(n_i)|\ldots, n_i - 1, \ldots\rangle$ for non-zero values of n_i and $\hat{a}_i|\ldots, n_i, \ldots\rangle = 0|\ldots, n_i, \ldots\rangle$ for $n_i = 0$, and a raising operator which is its Hermitian conjugate $\hat{a}_i^\dagger$ such that $\hat{a}_i^\dagger|\ldots, n_i, \ldots\rangle = c_i^*(n_i + 1)|\ldots, n_i + 1, \ldots\rangle$, where $\hat{N}_i \equiv \hat{a}_i^\dagger \hat{a}_i$ and $c(n_i) = \sqrt{n_i}$. The result of applying $\hat{N}_i$ is $\hat{N}_i|\ldots, n_i, \ldots\rangle = n_i|\ldots, n_i, \ldots\rangle$. The raising and lowering operators then obey the operator algebra

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij} \tag{4.6}$$

$$[\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0 \tag{4.7}$$

$$[\hat{a}_i, \hat{a}_j] = 0 \ . \tag{4.8}$$

For fermions, one has a similar construction which applies only to the allowed states, that is, states with occupation numbers associated with the raising and lowering operators are 0 or 1. $\hat{a}_i|\ldots, 1, \ldots\rangle = c(1)|\ldots, 0, \ldots\rangle$ and $\hat{a}_i|\ldots, 0, \ldots\rangle = 0|\ldots, 0, \ldots\rangle$, $\hat{a}_i^\dagger|\ldots, 0, \ldots\rangle = c^*(1)|\ldots, 1, \ldots\rangle$, where $|c(1)| = 1$ and $N_i \equiv \hat{a}_i^\dagger \hat{a}_i$; the phase of $c(1)$ must be appropriate chosen depending on the context of the calculation. The raising and lowering operators then obey the operator algebra

$$[\hat{a}_i, \hat{a}_j^\dagger]_+ = \delta_{ij} \tag{4.9}$$

[58]This eigenvalue not to be confused with the referent of the same symbol, which is used to describe generic state amplitude, nor the raising or lowering operators introduced below.

$$[\hat{a}_i^\dagger, \hat{a}_j^\dagger]_+ = 0 \tag{4.10}$$

$$[\hat{a}_i, \hat{a}_j]_+ = 0\ , \tag{4.11}$$

where the subscripts "+" indicate anti-commutation ($[V, W]_+ \equiv VW + WV$ for operators V, W) rather than commutation.

Although the quanta above are defined relative to a given maximal operator A, they could have been made to correspond to a different maximal operator B which does not commute with A. They are typically associated with quantum fields. In the case that one chooses to use such an alternative operator B instead, one has corresponding analogous raising and lowering operators $\hat{b}_i, \hat{b}_i^\dagger$, which are definable in terms of $\hat{b}_i, \hat{b}_i^\dagger$ via the relation $\hat{b}_i^\dagger = \sum_j c_{ji}\hat{a}_i^\dagger$, with the same vacuum state $|0\rangle$. Although operators with discrete spectra are a special case, they can be used systematically as an idealized approximation to situations involving continuous quantities, which aren't fundamentally different from the existential point of view. In the case of the single-quantum state, then, one has $\hat{b}_i^\dagger = \sum_j \langle a_j|b_i\rangle \hat{a}_j^\dagger$, equivalently, $|b_i\rangle = \sum_j \langle a_j|b_i\rangle|a_j\rangle$ and similarly for states with continuous indices, for example, position and momentum $\mathbf{x}$ and $\mathbf{k}$ (cf. [303], p. 55). The micro-causality condition can be shown to impose the commutation relations above for bosons and fermions in their respective cases, in that a failure to satisfy them would imply a failure of micro-causality.

The above formalism can also be understood via the "field quantization" approach rather than the second quantization approach. The field quantization approach is to first consider the set of distinct modes of a classical field which is quantized through the use of boundary conditions. Ray Streater has succinctly summarized this treatment in historical context as follows.

> Quantum field theory was really invented by Dirac in 1927. He considered classical electromagnetic fields in a cubical box with periodic boundary conditions. This theory is described by a collection of independent harmonic oscillators, which Dirac quantized as in the non-relativistic theory. Physical quantities are then ultimately obtained by taking the limit, as the size of the box goes to infinity, of a corresponding quantity in the boxed theory. … These days, the relativistic free fields are rather easily quantized in a rigorous way without first quantizing the box; but in constructing the fields with interaction, we must still use a box and other, much more elaborate approximations.[59] [296]

The superposition principle allows a general state to be given as a superposition of normal spatial modes obtainable by Fourier analysis of the corresponding classical field, that is, the normal modes form a complete set of wave solutions and the energy of a number of excitations is the sum of the energies of the modes weighted by the respective numbers of quanta in them. Each mode can be mathematically treated similarly to a harmonic oscillator: The behavior of the field in the general context is similar and indexed by a continuously indexed mode parameter $\mathbf{k}$ and frequency

[59] The significance of approximation is taken up here in the following, final section.

$\omega_{\mathbf{k}}$, relating to the energy via $\hbar\omega_{\mathbf{k}} = h\nu_{\mathbf{k}}$, with the total energy in the mode being a counting number times this basic energy value, that is, some value $n\hbar\omega_{\mathbf{k}}$.

A quantum field theory is distinguished from a many-particle-system theory by its describing a quantum system by a state specifiable as the value of the relevant physical properties at each of a continuum of space-time points, where the system state is considered an operator rather than a function. As a result, as in the case of classical fields, it involves an infinite number of degrees of freedom. One can construct a localized set of states by making the observables dependent on space-time location (cf., e.g. [305], p. 11); by attributing values of the dynamical quantities to the points of space-time, one obtains the field configuration. Such configurations are the focus of *local canonical quantum field theory*. In non-relativistic field theory, as above, such localized observables suffice to provide us with a robust concept of localizable particles: For each spatial region Δ, there is a number operator N_{Δ} the eigenvalues of which are the number of particles located within the region Δ. Thus, one can determine the presence or absence of a particle in a given region of space without the need for a spatial position operator.

The relativistic case is somewhat problematic, because the particles are only *instantaneously* localizable in an equivalence class of inertial frames of reference. In the case of N free particles, where the field state can be given as $|n(\mathbf{k}_1), n(\mathbf{k}_2), \ldots, n(\mathbf{k}_i), \ldots\rangle$ which is the occupation-number representation of the field in the general case of a countable number of modes, where a given mode i is occupied by $n(\mathbf{k}_i)$ quanta and so possesses energy $n(\mathbf{k}_i)\hbar\omega_{\mathbf{k}_i}$, and $N = n(\mathbf{k}_1) + n(\mathbf{k}_2) + \ldots + n(\mathbf{k}_i) + \ldots$. In experiments involving photons in free-space optical systems, the discrete modes considered are typically those of "traveling waves" associated with the environment, most importantly the region of space, between source and detector. Thus, for example, in the case of the Young double-slit experiment illustrated in Fig. 1.7 or its analogue the Mach-Zehnder interferometer, associated with each spatial mode are: input light waves, waves *filling both paths within the interferometer*, and output waves, all determined by the geometry of the apparatus (cf. [183], pp. 1–2).

One thus has in hand two compatible pictures—that of particles and that of field modes—that are inter-relatable. However, in some situations, enforcing the QPI has the result that only one of the two mathematical descriptions can be considered to be describing *individual* objects. In addition to creation and annihilation operators $a_i^\dagger$ and a_i obeying the commutations relations (4.6)–(4.8), the specification of occupied spatial modes of an optical field involve their description by the space-dependent functions $\mathbf{F}_i(\mathbf{r})$, which are independent if

$$\int d\mathbf{r}\; \mathbf{F}_i^*(\mathbf{r})\mathbf{F}_j(\mathbf{r}) = \delta_{ij}\ . \tag{4.12}$$

Accordingly, for example, a diagonally polarized light quantum can be considered as with equal probability likely to occupy one or the other of the two corresponding orthogonal linear polarization modes. In that example, the system described as two modes (non-trivially being their combination) will have a joint state with specifications labeled as usual by subscripts, say H for the horizontal polarization mode and V for the vertical polarization mode, and will be describable by the entangled state

$$|\Psi^+\rangle = \left(\sqrt{2}\right)^{-1}\left(|0\rangle_{\mathrm{H}}|1\rangle_{\mathrm{V}} + |1\rangle_{\mathrm{H}}|0\rangle_{\mathrm{V}}\right) , \tag{4.13}$$

where mode occupation numbers are indicated inside the kets themselves; both modes will be *individually* described by the same reduced state, a fully mixed statistical state proportional to the unit operator.

If, instead, this is considered as a single particle system, the state description will be as a vector superposition state corresponding to having one or the other state polarization H or V, that is,

$$|\psi\rangle = \left(\sqrt{2}\right)^{-1}\left(|H\rangle + |V\rangle\right) . \tag{4.14}$$

Moreover, considering instead those with the state given by (4.13), the alternative (relatively speaking) orthogonal polarization modes $\nearrow$ and $\searrow$ one has an equally good description using the state

$$|\psi\rangle = |1\rangle_{\nearrow}|0\rangle_{\searrow} , \tag{4.15}$$

when $\hat{a}^\dagger_{\nearrow} = (\sqrt{2})^{-1}(\hat{a}^\dagger_{\mathrm{H}}+\hat{a}^\dagger_{\mathrm{V}})$ and $\hat{a}^\dagger_{\searrow} = (\sqrt{2})^{-1}(\hat{a}^\dagger_{\mathrm{H}}-\hat{a}^\dagger_{\mathrm{V}})$, indicating that factoring also depends on the *modes* under consideration, marking another difference from the ordinary Quantum mechanical description. This points out that a given situation, when differently considered, may correspond to different amounts of entanglement when consider via different pictures.[60] A prior consideration should be whether or not the QPI suffices in a given instance to uniquely determine a set of individual systems, that is, whether even with the QPI in force there might remain an underdetermination of the physical picture. The availability of individual descriptions in both the quantum mechanical and the quantum field theoretical contexts, for example, in both Eqs. 4.14 and 4.15, shows that the picture is, in general, *not* uniquely determined by enforcing the QPI in standard quantum theory, even though it can allow for consistent use of the Principle of identity of indiscernibles.

A central question for the determination of an ontological picture in the quantum field theoretical context is that of localizability, which relates to the situations such as the above where spatial rather than polarization modes are under consideration and which is an essential characteristic of particles. One application of the Fock state construction is the identification of the operators A and B with position and momentum, respectively. Writing, then, the momentum basis as $\{|\mathbf{k}\rangle\}$ and the position basis as $\{|\mathbf{x}\rangle\}$, one can make use of the transformation between these bases to find the field raising operator. In the non-relativistic version, the resulting raising

[60] For further examples, see [310].

and lowering operators can be taken to correspond to modes of the full quantum field and can be written

$$\hat{\Psi}^{\dagger}(\mathbf{x}) = \int d^3\mathbf{k}\big((2\pi)^3\big)^{-1/2}\langle\mathbf{k}|\mathbf{x}\rangle\hat{a}^{\dagger}(\mathbf{k}) \; , \tag{4.16}$$

for the field at the point $\mathbf{x}$, where $\hat{a}^{\dagger}(\mathbf{k})$ is the raising operator corresponding to a field quantum of momentum $|\mathbf{k}\rangle$, where $\langle\mathbf{k}|\mathbf{x}\rangle = \exp[-i\mathbf{k}\cdot\mathbf{x}]$. The operator $\hat{\Psi}^{\dagger}(\mathbf{x})$ corresponds to the value of the field in space *at one moment* and serves as a raising operator for a quantum located precisely at $\mathbf{x}$, in the sense that $|\mathbf{x}\rangle = \hat{\Psi}^{\dagger}(\mathbf{x})|0\rangle$.

In a relativistic (Klein–Gordon) theory, one can proceed similarly, but the resulting field operator must then be written instead as

$$\hat{\Psi}^{\dagger}(\mathbf{x}) = \int d^3\mathbf{k}\big((2\pi)^3\omega(\mathbf{k})\big)^{-1/2}\langle\mathbf{k}|\mathbf{x}\rangle\hat{a}^{\dagger}(\mathbf{k}) \; , \tag{4.17}$$

for the field at the point $\mathbf{x}$, where again $\hat{a}^{\dagger}(\mathbf{k})$ is the raising operator corresponding to a field quantum of momentum $|\mathbf{k}\rangle$, where $\langle\mathbf{k}|\mathbf{x}\rangle = \exp[-i\mathbf{k}\cdot\mathbf{x}]$. In that case, $\hat{\Psi}^{\dagger}(\mathbf{x})$ corresponds to the value of the field in space at one moment and serves as a raising operator for a quantum, again in the sense that $|\mathbf{x}\rangle = \hat{\Psi}^{\dagger}(\mathbf{x})|0\rangle$.

In the relativistic case, however, the state $|\mathbf{x}\rangle$ corresponds to a wave packet lying within one Compton wavelength of the position-value $\mathbf{x}$, but there exists no Hermitian operator serving to indicate position by having eigenvectors $|\mathbf{x}\rangle$ with corresponding eigenvalues $\mathbf{x}$ ranging over the spatial domain. The frequency-dependent term in the covariant case corresponds to a failure of the orthonormality of the states $\{|\mathbf{x}\rangle\}$, so that a quantum described by $|\mathbf{x}\rangle$ is no longer localized in that sense. T. D. Newton and Wigner suggested that one should instead identify the states

$$|\mathbf{x}\rangle = \int d^3\mathbf{k}(2\pi)^{-3/2}\exp[-i\mathbf{k}\cdot\mathbf{x}]|\mathbf{k}\rangle \tag{4.18}$$

as the mutually orthogonal states best serving as precisely spatially localized states, even though they themselves are not Lorentz invariant [202]. In fact, an observer in a different inertial reference frame from one in which the state is well localized about $\mathbf{x}$ at a give $t = 0$ will see this state still concentrated about $\mathbf{x}$ but also having a non-zero probability of detection arbitrarily far away. Similarly, in the same inertial reference frame for non-zero *times* after initial localization, there is a non-zero probability of detection arbitrarily far away.

To what extent, then, can field quanta be thought of as objectively existing particles? The following characteristics, a number of which have been discussed above, have been associated with field quanta that support their being viewed as such (cf. [95], p. 227): they have essential non-dynamical properties of mass, energy, spin, and charge, they are discrete, independent (of each other, in that they may be uncoupled and have uncorrelated initial conditions), are point-like (in their interactions), (at least instantaneously) localizable (in a chosen frame of reference), countable, obey conservation laws, and are capable of binding. The first of these characteristics is that associated with the Wigner classification of identical particles; a correspondence can be set up, namely, between the spin value of the elementary particle and a quantum field—the Klein–Gordon field for spin 0, the Dirac field for spin 1/2, the electromagnetic field for spin 1, etc. From this and the other particulate characteristics, a coherent mereology emerges which supports, in particular, the atomic *Aufbau* process and structural chemistry, providing our best candidates for the specific quantum objects supporting a world picture with an intuitive ontological structure largely consistent with current scientific education and practice.[61]

4.7 Reduction

Quantum mereology naturally involves the question of reduction in the sense of the reduction of wholes to parts, that is, as *micro-reduction*. However, the issues addressed above have implications for reduction in several senses of them term. The standard approach to reduction in science began with the work of Ernst Nagel [199] and Joseph Woodger [331] and was further developed by Kenneth Schaffner [262–264]. This early work has given rise to a broad range of models of how scientific reduction could be accomplished. For example, Sahotra Sarkar [257] has helpfully classified models of reduction into three categories in relation to theories, explanation, and/or supervenience using a set of distinctions introduced by Ernst Mayr [191]: (i) *theory reductionism* which consists of those models that view reduction as a relation between theories; (ii) *explanatory reductionism* which consists of those models that view it as explanation but not as a relation between theories; and (iii) *constitutive reductionism* which consists of those models such as the various types of supervenience which eschew both theories *and* explanation.[62] Each of these categories is relevant here, because all models of reduction share the ontological claim that what happens at the level of the reduced entities (theories or not) is not novel in that it is consistent with what happens at the level of the reducing entities [157].

On the theory-reductionist approach, reduction models are concerned mainly with formal issues and focus on theories within the logical empiricist tradition. On this sort of model, the reduction of one theory to another is considered accomplished when the reducing theory can be used to provide a deductive-nomological explanation of the reduced theory [142].[63] Within this approach Schaffner, unlike Nagel, required only that a *corrected* version of the reduced theory be explained by the

[61] It is, of course, important to recognize the limitations on the *Aufbau* process, as pointed out by Eric Scerri [260, 261].

[62] For models of supervenience see, e.g., [72, 246].

[63] The models of Nickles [203] and Balzer et al. shared this focus on theories. Notably, Quine's [236] notion of "ontological reduction" is also of this sort.

reducing theory. He also required that the terms of the reduced theory be connected to those of the reducing theory by synthetic identities between entities, or sets of entities, which entail ontological commitment. For this reason, Schaffner's approach is more relevant here.

The explanatory-reductionist approach has been pursued mainly by Sarkar [39, 256, 257, 259] along lines first developed by Stuart Kauffman [166] and William Wimsatt [330]. It is perceptibly hostile to formalism, though it remains neutral regarding theories. Sarkar's analysis requires that purported reductions be explanatory but does not incorporate any particular model of explanation. The explanation of the behavior of entities of the reduced theory must be made using only the "strictly separable" parts of that entity having lower-level warrants. It is therefore relevant here, even though it does not provide any specific scheme for ontological reduction.

The constitutive-reductionist approach arose from the minimal model of Donald Davidson [72] and has been advocated by Alexander Rosenberg based almost entirely on the notion of supervenience [246, 247]. Supervenience requires merely that if two things are identical with respect to their specification at a lower level, they cannot differ at a higher level, with alterations at a higher level accompanied by alterations at the lower level, without any explanatory claim regarding the possibility of accounting for changes at the higher level in terms of those at the lower level (see Footnote 55). Davidson's model, for example, provides a scheme for micro-reduction, viewing ontological claims as central to reduction while no explanatory claim is involved, see [257]. Robert Causey distinguished reduction's ontological and nomological aspects [57].

The two general approaches to reduction involving those outlined above that can be most productively considered in relation to quantum theory are explanatory reduction ("epistemological reduction") and ontological reduction. The explanatory approach is primarily concerned with the explanation of one set of theories, laws and/or empirical generalizations by another; by contrast, the ontological approach is primarily concerned with attaining progressively fundamental representations of the world. In the past, the distinction between these two manners of reduction has been muted. This has been the case in models that maintain a logical empiricist perspective [200], and to a lesser extent in the theory-reductionist model of Schaffner [262, 263] and in the fundamentally explanatory-reductionist models of Wimsatt [330] and Kauffman [166].[64] Various aspects of reduction relating to ontological issues have been distinguished by Carl Hempel, who made a strong distinction between the formal and ontological aspects of reduction [141]. Shimony has contrasted its epistemological and ontological aspects in the reduction of wholes

[64]In the last case, ontological aspect of reduction seems to be considered beyond need of explication. These models assume scientific realism. In constitutive-reductionist models [72, 109, 246, 247], an explanatory/ontological distinction is not even suggested because the possibility of explanation is explicitly denied.

to parts in quantum theory specifically [277].[65] Let us assume here that we have an explanation at hand, that is, something that satisfies whatever strictures that one chooses to put on the notion of explanation.

A significant lacuna in the study of reduction has arisen from the relative inattention paid to the role of approximation; approximation has a subtle character and implications for reduction have been inadequately appreciated.[66] Indeed, as shown below, through examples, approximation is particularly important in enabling demonstrations of the coherence of, and the possibility of accomplishing reductions within the realist worldview. Consider now the following set of criteria for reduction, which when applied to various examples allow one to assess attempted reductions and illuminate the significance of various elements involved, such as the significance of approximation in generally accepted instances of reduction. Let us consider several specific criteria for explanation that have been put forward as criteria for characterizing the *strengths* of various reductions.[67] These support a meaningful notion of strength of reduction in terms of the their satisfaction or lack of it. Of the following four criteria, the first is central to explanatory reduction and the second to ontological reduction[68]; the last two characterize both modes but are more obviously important to ontological reduction. These criteria are the following.

(i) *Explanatory fundamentalism*: The reducing factors invoked in the explanation of the reduced feature are present at a level (or levels) different from its own, and the reduced features result only from the rules operating on another level (or other levels).
(ii) *Ontological fundamentalism*: All entities and properties at the reduced level are entirely composed of, and can be entirely replaced by, entities and properties at the reducing level.
(iii) *Hierarchical organization*: The entities being reduced are represented as having an explicitly hierarchical structure in which the reducing entities are present only at levels (of the hierarchy) lower than that of the reduced entities.[69]
(iv) *Spatial instantiation*: The hierarchical structure of any reduced entity is present in physical space.

[65]The significance of there being various aspects of reduction is also evident in more recent discussions of the subject. See, for example [238, 257, 264].

[66]This is so despite numerous discussions of the subject in the context of explanation, for example, [11, 12, 214, 258, 269].

[67]For a detailed account see [257]. Jaeger and Sarkar [157] also discussed some of these criteria, but in a context where the concern was purely explanatory.

[68]These derive largely from Sarkar's criterion (cf. [259], Chap. 3, §6).

[69]This hierarchy, of course, must be constructed based on some criterion independent of the putative reduction. The simplest such hierarchical structure is a (graph-theoretical) directed tree. For an extended discussion, see [257].

The more of these criteria that are satisfied, the stronger the model.[70] Criterion (i) must be satisfied by any successful explanatory reduction, and (ii) must be satisfied for any successful ontological reduction. It is important also to note that (iv) can only be satisfied provided that (iii) is, and neither (iii) nor (iv) can be satisfied if neither (i) nor (ii) is. These criteria can be naturally used to distinguish modes and strengths of reduction. The weakest reductions satisfy (i) or (ii) alone; the next strongest reductions satisfy (i) or (ii), and (iii); yet stronger are reductions that satisfy either (i), (iii) and (iv) or (ii)–(iv); the next strongest reductions satisfy (i)–(iii); the strongest possible reductions satisfy all of (i)–(iv). Several purported reductions, in which the reducing theory is Quantum mechanics, are discussed below to determine their strength or failure. The results demonstrate the value of the above criteria as well as the distinction between explanatory and ontological modes of reduction in the quantum context.

The reduction of scientific theories, laws, and empirical generalizations to others at broader or more fundamental levels of inquiry serves to confirm their validity. For realists, the corresponding ontological reductions are particularly significant. Skepticism about the possibility of reductionist explanation in physics has often been based on the belief that the components of composite systems exhibit collective behavior that cannot be accounted for by an examination of the properties of these parts taken in isolation.[71] Four specific examples, beginning with one from classical physics and then moving on to three from quantum mechanics assist in a preliminary assessment of this claim. Of the four, the last three examples of reduction involve Quantum mechanics: the reduction of Classical mechanics to Quantum mechanics, the reduction of the behavior of composite systems to the quantum mechanics of single systems, and the reduction of the structural chemistry to quantum theory. Of those three, the first two examples demonstrate how interpretive issues in quantum mechanics can be related to those of reduction. The first and last examples demonstrate how approximations can expose the subtleties of reduction and determine which of the explanatory and ontological criteria are met in these in reductions, indicating their strength. The reduction is relatively straightforward in the example involving only classical mechanics.

1. *Forced coupled harmonic oscillators*: Consider a pair of identical classical-mechanical oscillators, each with spring constant k and mass m, and therefore natural frequency $\omega_c = (k/m)^{1/2}$, coupled by a spring and subject to a periodic force. If the damping of their motion is nonzero but negligible then, after a sufficiently large number of cycles, transient effects disappear and the motion of each oscillator will be one of constant amplitude at the frequency ω of the driving force. The equations of motion for this steady state are

$$x_1 = A\, cos\, \omega t\ ,\ x_2 = B\, cos\, \omega t\ , \tag{4.19}$$

[70]The sense of strength that will be used in the sequel is essentially that of Sarkar [257].

[71]The broad reasons for concern are addressed in the previous sections of the chapter.

where x_1 and x_2 are the displacements of the oscillators from their equilibrium positions and A and B are their (constant) amplitudes. The motion of this pair of oscillators is coherent motion in that there are correlations in the oscillators' positions, because both oscillators vibrate with the same, driving frequency; given t, the knowledge of ω allows x_2 to be inferred from x_1 and vice versa. Thus, in this case the behavior of a whole is understood fully through that of its parts and all four criteria (i)–(iv) are satisfied.

After this example, we consider attempts to reduce composite-system non-statistical quantum behavior to individual behavior using Quantum mechanics. In quantum mechanics, as shown below, components of composite systems can exhibit collective behavior that cannot be accounted for by straightforward examination of the properties of these parts taken in isolation as in Case 1 above, precisely when the states in question are entangled. In the quantum description, entangled states are those states of a composite system which cannot be uniquely expressed as a tensor product of states of the individual subsystems but only as a superposition of them, as described in Chap. 1. More particularly, for a composite system involving just two subsystems with Hilbert spaces of countable dimensionality: Each possible state of the composite system is represented by a vector in a Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$, where $\mathcal{H}_1$ and $\mathcal{H}_2$ are the Hilbert spaces of each of the two subsystems when considered in isolation. Recall that entangled states of such a system are those represented by vectors $|\psi\rangle_{1+2} \in \mathcal{H}_1 \otimes \mathcal{H}_2$ such that $|\psi\rangle_{1+2} \neq |\eta\rangle_1|\theta\rangle_2$, for any pair of states $|\eta\rangle_1 \in \mathcal{H}_1$ and $|\theta\rangle_2 \in \mathcal{H}_2$. These states occur when systems interact with each other.[72]

2. *A Rabi oscillator:* Consider an electron capable of being in one of two coupled energy levels, E_+ and E_-, of either an atom, ion, or molecule. The probability of the electron being in either of the two available levels oscillates between them with a frequency that depends on the strength of the coupling and the energy difference between the two levels. The state of the electron is a superposition of the two corresponding states $|\psi_+\rangle$ and $|\psi_-\rangle$, namely,

$$|\psi(t)\rangle = e^{i(\phi/2)}\Big[cos(\theta/2)e^{-iE_+t/\hbar}|\psi_+\rangle + sin(\theta/2)e^{-iE_-t/\hbar}|\psi_-\rangle\Big], \quad (4.20)$$

where θ measures the coupling strength relative to the energy difference, and ϕ is a parameter associated with the coupling connecting energy levels. In this case, a clear quantum mechanical *explanation of its oscillation* within the larger system of which it is a part is obtained without need of explicit reference to the remainder of the system.

3. *Fröhlich systems:* Herbert Fröhlich [113] claimed that states of a macroscopic system for which there exist "macro wave functions" Φ (i.e., states exhibiting off-diagonal long-range order, ODLRO) are necessary to explain a wide range of

[72]The components of the state vector in $\mathcal{H}_1 \otimes \mathcal{H}_2$ of any composite system of two objects can be given for a basis of the form $\{|\alpha_i\rangle|\beta_i\rangle\}$, with $i = \{1, \ldots, n\}$.

ordered biological phenomena.[73] Two important reduced density matrices in this context are

$$\langle j|\rho_1|i\rangle = Tr(a_j \rho a_i^\dagger) \tag{4.21}$$

and

$$\langle kl|\rho_2|ij\rangle = Tr(a_k a_l \rho a_i^\dagger a_j^\dagger)\ , \tag{4.22}$$

where the a's are the annihilation operators of the single-particle states traced out. Other reduced density matrices ρ_n for which n particle states are traced out can be similarly defined.

The subsystems which can exhibit ODLRO consist of a large number of bosons (either simple bosons or fermions forming bosonic quasiparticles). Such a state is approximately represented by the reduced density matrix

$$\Omega_{\text{red}}(r', r'') = N\alpha\Phi(r')\Phi^*(r'') + \chi(r', r'')\ , \tag{4.23}$$

where Φ is the macro wave-function attributed to the relevant subsystem (Φ^* being its complex conjugate), χ is a positive operator, N is the number of particles in the subsystem, α obeys $0 \leq \alpha \leq 1$, and r' and r'' represent two positions in the subsystem. The first term in the above expression represents the infinite-range correlations present in the system, while the second represents local correlations, $\chi(r', r'')$ being small compared to $N\alpha$. As a result of the presence of the first term, $\Omega_{\text{red}}(r', r'') \nrightarrow 0$ even as $|r' - r''| \to \infty$. Therefore, correlation will persist over large spatial distances: ODLRO is present. An approximation is necessary to allow for the consideration of the collective system as a whole described by Φ alone and so to allow Criteria (i)–(iii) to be satisfied.

4. *Two systems in a Bell singlet state*: An example of such a state is the composite system formed by a pair of two spin-$1/2$ particles, such as an electron (e^-) and a positron (e^+), in a singlet state.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\big(|+\rangle_{e^-}|-\rangle_{e^+} - |-\rangle_{e^-}|+\rangle_{e^+}\big)\ , \tag{4.24}$$

where in each term the subscripts refer to the Hilbert space of the corresponding particle.[74] Such a system is an unfactorable "coherent superposition" of composite-system states, one in which the electron has the relevant component

[73] Here, Yang's definition of ODLRO will be used: a subsystem characterized by a reduced density matrix $\rho(r', r'')$ exhibits ODLRO when $\rho(r', r'') \nrightarrow 0$ in the limit $|r' - r''| \to \infty$. See [119, 216, 217, 333].

[74] This state persists only when the e^+e^--system is spatially separated. Otherwise, these two antiparticles annihilate each other and immediately cease to exist.

of its spin in the "up" state while the positron has its spin "down" and one in which the electron has spin "down" while the positron has spin "up." Since both elements of the superposition are possible, the spin state of each particle is indefinite, as described in Sects. 1.3 and 4.4.

Only a statistical description is possible for a collection of such systems, or an ensemble of identical situations involving one system over time, preventing the satisfaction of Criteria (ii)–(iv) but allowing the satisfaction of Criterion (i). As noted above, composite systems in entangled states have subsystems which cannot be individuated and locutions such as "subsystem A" and "subsystem B" do not refer to any precise entity within the entire system [267].

In the descriptions of the structure and explanations of the behavior of quantum systems of Cases (2)–(4), our first criterion of reduction (fundamentalism) is easily satisfied. Thus, none of our examples fail to satisfy the condition for the first sense of reduction. However, there is no hierarchical relation between the subsystem states and that of the related composite systems in these quantum systems in the absence of approximations. Consider, for example, the simplest of atoms, the hydrogen atom consisting of an electron and a proton interacting with each other. In the most complete quantum description even the hydrogen atom can be attributed an entangled state, and so it cannot be represented as a fixed hierarchical structure with identifiable individual states for the proton and electron. The situation is no better in the case of larger atoms. One sees that the usual strong hierarchical picture of the composition of matter is weakened due to the nature of the typical allowed quantum states when systems have interacted. Entanglement eliminates the possibility of strong reduction because an entangled system cannot be described as being clearly hierarchically organized; this does not, of course, imply that this is the only way in which strong reduction can fail in physics.

The prominence and power of approximation to the success of reduction, which becomes yet more so as larger systems are considered, as seen in Case 3, is also under-appreciated. Approximation can influence the strength of any accomplished reduction; as Wimsatt [330], Shimony [277] and Sarkar [256] have previously observed, approximations are central to many actual scientific reductions. Domains of scientific inquiry, such as chemistry and physics, often share entities and properties, treating them in different and relatable manners with the relation in question involving approximations on which the strength and the success or failure of reductions depend, as is shown below. (Some models of reduction, for example that of Schaffner [262–264], blur this distinction between the deduction of theories, laws, or empirical facts and the *derivation* of them from the reducing theory. The lack of this sort of distinction has pained physicists familiar with the philosophy of science.[75]) The sequel will demonstrate through detailed, important examples

[75] See, for example, [176, 231]. Leggett seems to have been the first to note the importance of this distinction. One particularly noteworthy discussion of the intricacies involved in the use of approximations is that of Kurt Friedrichs [112].

the central role of approximation and its relation to the strength of the reduction achieved across larger ranges of theory and ontology to those considered just above.

Briefly consider now the reduction of Classical mechanics to Quantum mechanics. At least two manners of accomplishing this reduction have been considered: (a) recovering Newton's equation of motion from the Schrödinger equation in the limit as the quantum unit of action, $\hbar$ tends to zero[76] and (b) recovering Newton's equation of motion from Ehrenfest's theorem (in instances where the net force acting on the system is approximately a linear function of position). Quantum mechanics is a fundamentally probabilistic theory, whereas (non-statistical) Classical mechanics specifies with certainty the behavior of individual systems. Thus, for any reduction of Classical mechanics to Quantum mechanics to be effected, the relevant quantum mechanical probability distributions must become arbitrarily narrow; reduction requires there to be, when two theories specify the same property, a one-to-one correspondence between instances of that property; any width of the quantum mechanical probability distribution equivocates regarding the value of the corresponding physical quantity, precluding such a one-to-one correspondence.

Inspection of the Heisenberg–Robertson uncertainty relation makes a definite quantum specification of physical quantities appear feasible in the case of particle motions, because in the limit $\hbar \to 0$ it seems to allow both position and momentum to be arbitrarily well specified. However, whether a particular distribution *is* arbitrarily well specified for a given quantum system depends on the full state of the system. For example, if one system has recently interacted with another system, then entanglement is likely to arise, so that the quantum pure states of the individual systems will not be uniquely specified, as seen above. Entanglement aside, the $\hbar \to 0$ limit also doesn't eliminate all the difficulties for this reduction.

The spatial wave-function of a generic quantum system at a position $\mathbf{r}$ at the time t, $\Psi(\mathbf{r}, t)$, can be written as

$$\Psi(\mathbf{r}, t) = A(\mathbf{r}) \exp\left[\frac{iS(\mathbf{r}, t)}{\hbar}\right], \tag{4.25}$$

where $A(\mathbf{r})$ and $S(\mathbf{r}, t)$ are the real and imaginary parts of the wave-function, respectively. The Schrödinger equation gives rise to a continuity equation for conservation of probability density ($\equiv |\Psi|^2 = |A|^2$) and to an expression *formally* equivalent to the classical mechanical Hamilton–Jacobi equation:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + V_q = 0 \tag{4.26}$$

[76]This limit may appear strange, as $\hbar$ is a constant of nature. However, this is a perfectly well-defined mathematical approximation which is justified physically by $\hbar$'s smallness relative to the values of quantities of interest. For reductions using approximations which involve the taking of limits, it is also important that physical quantities remain mathematically well defined throughout the limiting process.

where V is the ordinary potential, $V_q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 A}{A}$ (sometimes called the "quantum potential"), S plays the role of Hamilton's principal function (the generator of trajectories in classical mechanics), m is the system mass, and t is time. Taking the gradient of (4), after writing the probability flux as $\mathbf{J} = P\nabla S/m$ and the corresponding velocity field as $\mathbf{v} = \mathbf{J}/P$, yields

$$m\frac{d\mathbf{v}}{dt} = -\nabla(V + V_q)\ . \tag{4.27}$$

If the quantum potential, V_q, vanishes in the limit $\hbar \to 0$, then Newton's equation of motion is recovered. Though V_q is in fact proportional to $\hbar^2$, it is not always the case that V_q vanishes in the $\hbar \to 0$ limit; that too depends on the exact state of the system—on the behavior of $A(\mathbf{r})$, in particular.

Consider the following important instance in which V_q *can* be seen to vanish in the $\hbar \to 0$ limit: (a) *A free particle in one-dimension.*[77] In this case, the broader the position probability density (and hence the narrower the momentum probability density), the more rapid the vanishing of V_q becomes. Here, because of the width of this position distribution, Criterion (ii) cannot be met because the full state (including both position and momentum) at the ostensibly reducing level (Quantum mechanics) can be mapped, at best, many-to-one to the level to be reduced (Classical mechanics), rendering Criteria (iii) and (iv) irrelevant: ontological reduction fails completely. However, Criterion (i) is still satisfied, so that a weak explanatory reduction is achieved. As will be seen next, the above condition on the position probability distribution is just the *opposite* of what is required of it for achieve a reduction via Ehrenfest's theorem.

Consider a Hamiltonian of the form $H = (\mathbf{P}^2/2m) + V(\mathbf{Q})$, where $\mathbf{P}$ is the Quantum mechanical momentum operator, $\mathbf{Q}$ is the position operator, V the potential and m the mass. The time-evolutions of the average values (or expectation values) of the position and momentum operators bear a relation (in the Heisenberg picture) formally similar to the Hamilton-Jacobi equations, which in this case reduce to Newton's equation of motion: $\frac{d\mathbf{p}}{dt} = \frac{d^2\mathbf{r}}{dt^2} - \nabla V(\mathbf{r})$. The quantum mechanical expression for these

$$\frac{d\langle\mathbf{P}\rangle}{dt} = -\nabla V(\mathbf{Q})\ , \tag{4.28}$$

can be approximated by

$$\frac{d\langle\mathbf{P}\rangle}{dt} = \big[-\nabla V(\mathbf{Q})\big]_{\mathbf{Q}=\langle\mathbf{Q}\rangle}\ , \tag{4.29}$$

[77] See [10], §15.2 for a detailed discussion of this example.

where the position **Q** occurring on the right hand side is evaluated at its expectation value, giving justification for the claim that the quantum mechanical *averages* obey the same equations of motion as those of Classical mechanics.

The approximation involved here holds exactly only when the force, $-\nabla V$, is a linear function of **Q**, as it is in the case of the simple harmonic oscillator.[78] This linear approximation is frequently used, but it is generally a very rough one. There is a significant class of such instances, namely, when: (b) *the width of the position probability distribution is small compared with the typical length scale over which the potential, V, varies*.[79] However, again Criterion (ii) cannot be met—because of a one-many mapping of the momentum and, as in the first case, Criteria (iii) and (iv) are irrelevant. Strong ontological reduction fails, and only a weak explanatory reduction obtains, because again only Criterion (i) is satisfied.

Note that the requirement on the width of the position probability density in (b) is just the opposite of that required for the approximation in (a). Only in such extremes cases as (a) and (b) could one hope to have a successful ontological reduction; when approximations *are* accurate one does not recover the simultaneous position/momentum (phase space) description of classical mechanics. These "reductions" are even less plausible when the required approximations are rendered inaccurate, which is typically so. It is the Hilbert-space description inherent in Quantum mechanics, from which the uncertainty relations can be derived, that renders strict ontological reduction impossible rather than the approximations themselves, since it gives rise to the many-to-one correspondence of system properties. The failure to achieve a one-to-one correspondence between full states is evident in these approximations.[80]

More generally, quantum theory presents serious difficulties for reductionism across large scale ranges, for example, that from sciences describing entities which are localizable only to kilometers in extent to sciences describing entities which are localizable to the nanometer scale, such as geology to physics. Many reductions over such scale ranges must be spelled out as reductions to Classical mechanics which in turn must be reduced to Quantum mechanics. Only fairly trivial explanatory reductions in a limited range of scale and circumstances have been successfully accomplished. The reduction of structural chemistry is an important but exceptional case, because it can be made *directly* to Quantum mechanics.

Recall that composite systems are generally described by entangled states so that definite states cannot be attributed to their individual subsystems (see [277]). Consequently, no ontological reduction is possible (Criterion (ii) cannot be met) and no hierarchical relation can be given between the states of the subsystems

[78] Cf., e.g., [64], Chap. III.

[79] See ibid., Sect. D, 1.γ.

[80] The question of which of these properties of a quantum system are actually possessed at any given time is answered differently by various interpretations of quantum mechanics. As a result, the choice of interpretation can influence the possibility and nature of the reduction.

(Criterion (iii) is unfulfilled) and those of the composite system, let alone be instantiated in physical space (Criterion (iv) cannot be met). In general, even the weakest of explanatory reductions is not accomplished, because the specification of the state requires the use of statistical rules (requiring symmetrization of anti-symmetrization of the state as described in detail in previous sections) not needed to describe individual states. However, in those special circumstances where these rules make no difference to the behavior characterized this weakest reduction (fulfilling only Criterion (i) while Criteria (iii) and (iv) are relevant but unfulfilled) can take place. Again, what has been reduced is neither a theory nor an entity, but simply a limited range of phenomena. For Bell-singlet states, for example, the approximation corresponding to ignoring these rules is a highly inaccurate one. The attempted reduction of structural chemistry to Quantum mechanics, although direct in theoretical terms, requires the introduction of not one, but a *series* of approximations. However, when "decomposed" one can identify entities corresponding to subsystem "components" that were not identifiable beforehand.

In structural chemistry, there are at least two fundamental phenomena to be accounted for: (i) the structure of atoms, and (ii) the structure of molecules. In the reduction of atomic structure in quantum theory, Case (i), the approximations made are those used in the *Theory of atomic orbitals* and the *Method of the self-consistent field*. Two different sorts of approximation can be chosen in the latter method; as will be seen, the choice greatly affects the character of the resulting reduction. In the reduction of molecular structure to quantum mechanical properties, Case (ii), the two standard methods of approximation used are: the *Molecular-orbital method* and the *Valence-bond method*. Here again, the method chosen determines the sort of reduction achievable. When an electron orbital is associated with a molecule, it can be associated with more than one nucleus, these states being called "molecular orbital states" or simply "molecular orbitals." The reduction of atomic structure and the reduction of molecular structure have approximations in common, in particular, that involved in the "method of the self-consistent field."

The work of Bohr [26], George Uhlenbeck and Samuel Goudsmit [308], and Pauli [212] made a proper quantum *Aufbau* of atomic structure from nuclei and electrons possible where a method for finding the wave-functions is available. The introduction of approximations is generally required for any viable method of finding orbital wave-functions. Only in the case of individual hydrogen atoms, which have only one electron (discussed above), is it possible to provide *exact* solutions of the Schrödinger equation. In all other cases, electronic interaction yields an intractable many-body problem, precluding any detailed specification of the quantum state of an atomic system.[81] Let us consider these two cases in succession.

[81] Even in the case of hydrogen, the approximation of ignoring the internal structure of the proton must be made. It will be assumed that this preliminary approximation has been made in all that follows.

Case (i) Atoms.

In relation to atoms, the two methods which are frequently used in attempts to explain atomic structure by Quantum mechanics, namely (a) the Hartree method and (b) the Hartree-Fock method, involve a self-consistent field approximation.

(a) The method of the self-consistent field: the Hartree approximation.

The first approximation made in this treatment is to simply ignore interactions between electrons as such and to characterize them instead by a single "effective potential" acting on each electron. Schrödinger wave-function solutions are guessed for each electron in the presence of the nucleus and an effective background potential (found from the other electron wave-functions); from this, a new estimate is made of the effective background potential (arising from the remaining electrons). A solution to the Schrödinger equation for each electron is then sought. The procedure is repeated, using new estimated wave-functions, until successive iterations make no appreciable difference to the wave-function (and so to the orbital) obtained for each electron. If successful, the resulting field corresponding to the effective potential will be in this sense "self consistent" and the wave-functions obtained accurate approximations. If this process fails to yield a stable, self-consistent result, then different initial wave-functions must be guessed and the entire process begun again.

When this procedure can be carried out successfully, the corresponding many-electron wave-function is obtained from the above individual-electron wave-functions with the assumption that the many-electron wave-function is given by the *product* of the individual-electron wave-functions. The result of the use of this second approximation, first introduced by Douglas Hartree [127], is a (numerical) solution by the *"method of the self-consistent field."* This approach allows successful, strong explanatory and ontological reductions of atomic structure to that of subatomic constituents described by quantum mechanics to be made: *all four criteria* (i)–(iv) for a successful reduction are met using only the rules of Quantum mechanics.

With this approximation, the atom is ontologically reducible to a nucleus surrounded by individuated electrons each in a specified region of space, all at the subatomic level, and much of the behavior of the system is accounted for. However, the reduction appears strong only because this approximation ignores the quantum-statistical requirement that the electrons obey Fermi–Dirac statistics—the requirement that the many-electron wave-function be totally antisymmetric under the interchange of electron wave-function labels; when enforced, this requirement, which is fundamental to many-particle quantum mechanics, automatically renders the product-form wave-function impossible. Unlike in the case of the Bell singlet state, however, here a product-form approximation is often a good one.[82]

[82] See [67].

(b) The Hartree–Fock method.

Let us consider now another version of the self-consistent field method introduced by Fock, called the *Hartree–Fock method*. It is an extension of the Hartree method of calculation and involves a better approximation. It differs by enforcing the additional, fundamental quantum statistical requirement that the many-electron wave-function be totally antisymmetric, thus avoiding the final approximation of assuming that the multi-electron wave-function be of product form [108]. Notably, the *numerical* results of this approximation generally do not vary significantly from those obtained by the Hartree method. Nonetheless, the use of such this so-called *Hartree–Fock* approximate method prevents a strong ontological reduction, as the resulting many-electron wave-function will be *entangled* because the overall wave-function is anti-symmetrized. The second, third and fourth of the stated criteria a reduction fail to be met, and only the weakest form of explanatory reduction obtains.

Although entanglement is involved, Criterion (i) is still met, unlike the case of reducing the electron-positron pair behavior to single-particle behavior, because the reduction here involves the quantum mechanics of *composite* systems and the explanations for atomic behavior is often excellent. This example shows how a *single approximation*, even among several, can affect the character of a reduction, allowing successful strong explanatory and ontological reductions to be accomplished where they would otherwise be considered impossible.

Case (ii) Molecules.

The problem of finding solutions of the Schrödinger equation for molecules is far more difficult than the problem in the case of atoms; more drastic methods of approximation are required. The two most common approximations are: the *Molecular-orbital* approximation and the *Valence-bond* approximation. The Molecular-orbital approximation is an attempt to treat molecules in the same way as atoms were treated above: molecules are understood *in terms of atomic structures* whose distinct character is largely preserved during binding. By contrast, the Valence-bond approximation treats the *molecule* as the fundamental unit; the molecule is understood in terms of molecular structures, one "ionic" and the other "covalent," and the internal structure of its atoms is not preserved.

(a) The Molecular-orbital approximation.

In the Molecular-orbital approximation, the method of the self-consistent field is applied to molecules. In particular, electron wave-functions within molecules, that is, molecular orbitals are sought. By contrast with atomic orbitals, these orbitals are poly-centric, that is, they have loci about more than one nucleus, due to the presence of several nuclei in close proximity. The structure of the molecule is understood in terms of molecular orbitals in the same manner that atomic orbitals are used in the atomic *Aufbau*. In accordance with the poly-centric nature of these molecular orbitals, however, a given electron is no longer typically associated with a *single atom*. The molecule can, therefore, not be understood strictly as a simple concatenation of atoms, but must instead be viewed, at best, as a *rearrangement* of atomic components into a different structure. Near a given nucleus, however, the electron can be expected to have a probability density similar to that of the corresponding atomic orbital when the influence of the potential due to other nuclei is negligible by comparison with that of the nucleus in question; it is in this approximate sense that the atomic parts, which would be seen were the molecular components to be taken apart, are preserved.

In the simplest case, the diatomic case, this leads naturally to the use of the additional, *linear combination of atomic orbitals* (LCAO) approximation. In this approximation, it is assumed that the energies of the atomic orbitals corresponding to the wave-functions ψ_A and ψ_B, where A and B are labels corresponding to the two nuclei, (i) are comparable in magnitude, (ii) have a significant spatial overlap, and (iii) have the same symmetry properties relative to the axis connecting the pair of nuclei in question. When these three conditions are fulfilled, the corresponding molecular-orbital wave-function can be accurately written $\psi = \psi_A + \lambda\psi_B$, where λ is a real-valued constant characterizing the orbital's polarity. One then applies the *variational method* in which the energy function

$$\mathcal{E} = \frac{\int \psi^* H \psi d\tau}{\int |\psi|^2 d\tau} \tag{4.30}$$

is minimized ($d\tau$ being the differential incorporating all relevant coordinates). The conditions for the energy function $\mathcal{E}$ to be stationary allow λ to be eliminated, leaving an equation for the maximum and minimum energy associated with the molecular orbital: E_+, the maximum energy, lies above the larger of the energies E_A and E_B associated with ψ_A and ψ_B, respectively, and E_-, the minimum energy, lies below them. The molecular bonds associated with these molecular orbitals are known as *resonances*. This name is used because it is claimed that if an electron were to begin in one of the states ψ_A or ψ_B, its state would be seen to evolve back and forth (resonate) between these two states.[83]

The corresponding molecular wave-function is constructed from individual-electron wave-functions on the assumption that the many-electron wave-function is given by a *product* of the individual-electron wave-functions, ignoring the requirements of quantum statistics. If this Hartree method of approximation is used, successful strong explanatory and ontological reductions of molecular structure to the quantum mechanics of subatomic entities are accomplished; as in the atomic case discussed above, all four of the criteria for a successful reduction are met.[84] However, when one also introduces the quantum statistical requirement that the many-electron wave-function be totally antisymmetric, that is, one instead makes

[83]Because the Pauli exclusion principle allows two electrons to have the same molecular orbital so long as their spins differ, two electrons can possess the same molecular orbital in a double bond. In this way the Lewis shared-electron (or covalent) bond is introduced.

[84]However, notably, the hierarchical structure within the *atoms* from which the molecules are composed differs from that instantiated when the atoms are alone.

the Hartree-Fock approximation, then the resulting molecular wave-function is again entangled. In that case neither of the criteria (ii), (iii) nor (iv) for a successful reduction is met, and only a *weak* explanatory reduction obtains, as in the case of the reduction of atomic structure to quantum mechanics using the same method.

(b) The Valence-bond approximation.

This primarily, although as seen below, not exclusively quantum-mechanical treatment of the molecule proceeds in five-stages and involves "some aspect of chemical intuition and experience."[85] For clarity, the method will be discussed here as applied to the least complex case, the binding of two hydrogen atoms.

(I) The first step of approximation is to assume that the two-electron wave-function can be factored into a tensor product of two wave-functions, $\psi = \psi_A(1)\psi_B(2)$, one for each electron to one nucleus; each term is of the form $\psi_X = e^{-r_X}$, r_X being the radius of the electron from its assigned nucleus. The variational method is then applied to obtain the electron energies. This yields a binding energy which is smaller than the observed value. The motivation for this step is clearly physical.

(II) The next step, introduced by Walter Heitler and Fritz London [140], is to consider a linear combination of two terms, each of product form as above, with nuclear labels interchanged, to capture the "exchange energy," neglected in (I). The resulting wave-function will, in general, be entangled. This step has a firm physical basis, namely, it is demanded by quantum statistics.

(III) The third step treats the "partial screening" that results when two atoms are near one another. Partial screening is incorporated in the valence-bond approximation by assuming that the resulting spatial distortion of the atomic orbitals of the component atoms can be captured by the exponent of the wave-function. Numerical solutions are then sought and the so-called overlap, Coulomb, and exchange integrals, respectively

$$S = \int \psi_A(1)^* \psi_B(2) d\tau \ , \tag{4.31}$$

$$Q = \int\int \psi_A(1)\psi_B(2)(H - 2E_h)\psi_A(1)\psi_B(2) d\tau_1 d\tau_2 \ , \tag{4.32}$$

and

$$J = \int\int \psi_A(1)\psi_B(2)(H - 2E_h)\psi_B(1)\psi_A(2) d\tau_1 d\tau_2 \ , \tag{4.33}$$

[85] See [67], p. 113.

where E_h is the energy of a hydrogen atom separately, are calculated. From these, the molecular energies

$$E_{\pm} = 2E_h \pm \frac{Q \pm J}{1 \pm S^2} \,, \tag{4.34}$$

are found. This motivation for step, too, is purely physical.

(IV) The next step is explicitly to introduce the polarization induced between atoms into the atomic orbital wave-functions. One replaces the above spherically symmetrical wave-functions by those incorporating polarization effects.

(V) The final step, by contrast, is chemically motivated. One now incorporates the small but non-negligible chance that both electrons may be found at the same nucleus by introducing the *ionic wave-function*

$$\psi_{\text{ion}} = \phi_A(1)\phi_A(2) + \phi_B(1)\phi_B(2) \,, \tag{4.35}$$

where each ϕ_X is taken to have the general form $\phi = e^{-c' r_X}$, c' being a real constant. The previously considered wave-function is called "ψ_{cov}" (for *covalent*) and the linear combination $\psi = \psi_{\text{cov}} + \eta\psi_{\text{ion}}$ is studied to find the corresponding minimum energy using the variational method. The interpretation given to this approximate solution is that the molecule is in a resonant state between two molecular structures, one "purely covalent" and the other "purely ionic." The resulting structure is referred to as a *covalent-ionic resonance*.

In the Valence-bond approach, the entities considered are not subatomic as they were in the molecular-orbital approach but rather are *molecular*; fundamentally chemical intuitions are brought into the method of approximation (Step V). Because this method involves the introduction of entangled states (Step II), which equivocate regarding the quantum states of component electrons, it fails to satisfy Criteria (ii)–(iv) for reduction: ontological reduction fails completely and only a weak explanatory reduction obtains. Furthermore, the assumption of Step (V) renders the weak reduction obtained a reduction to quantum *chemistry:* a reduction to multiple-system quantum mechanics in conjunction with a chemical hypothesis.

There is much more to be said, of course, about the attempted reduction of structural chemistry to quantum theory: many more approximations and concepts, such as hybridization, are needed to capture important phenomena such as directional valence in quantum-mechanical or post-quantum-mechanical terms. What can be clearly said here is that, though approximations can allow for a strong reduction in the case of reduction of *atomic* structure to Quantum mechanics, only a very weak reduction can be accomplished in the case of the reduction of *molecular* behavior to quantum theory.

We see that quantum physics presents significant difficulties for reduction in three important instances—Classical mechanics, multi-system quantum states and chemical structure—usually regarded as quite straightforwardly reduced to Quantum mechanics. In particular, we see that none of these reductions can be made without the introduction of inelegant approximations. The prospects for strong reductionism, with quantum theory serving as the basis of our natural science, are seen through these examples to be dim. In those instances where the systems of interest are entangled—that is, in most realistic cases—it is often that only the weakest type of explanatory reduction is possible. Because the components of entities at the reducing level cannot be precisely specified, ontological reductions are precluded in these cases.

In the case of the reduction of Classical mechanics, the failure results from the radically different characters of the Classical mechanical and the Quantum mechanical specification of states. In the case of structural chemistry, there are important instances wherein entangled states may be accurately approximated by product states: When the Hartree approximation is a good one, and the Molecular-orbital method of studying a chemical system can be followed. However, when the Valence-bond method is needed entanglement is involved so that again only the weakest sense of reduction obtains. Furthermore, these reductions are only accomplished by the introduction of a series of rather inelegant approximations of limited validity. In order to accommodate the required approximation, the reduction of classical behavior to quantum behavior is limited to the statistical situations only.

One is left with the need to approach large objects with relatively weak conditions on the part–whole relationship, similarly to the case of the conditions required for one to consider there to be elementary particles at the lowest levels of our ontology. Nonetheless, this supports key components of current scientific education and practice, which are realist in orientation.

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