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Megha Agarwal

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First published 2015 by
PALGRAVE MACMILLAN

Palgrave Macmillan in the UK is an imprint of Macmillan Publishers Limited, registered in England, company number 785998, of Houndmills, Basingstoke, Hampshire RG21 6XS.

Palgrave Macmillan in the US is a division of St Martin's Press LLC, 175 Fifth Avenue, New York, NY 10010.

Palgrave Macmillan is the global academic imprint of the above companies and has companies and representatives throughout the world.

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ISBN 978-1-349-47176-8 ISBN 978-1-137-35992-6 (eBook)
DOI 10.1057/9781137359926

This book is printed on paper suitable for recycling and made from fully managed and sustained forest sources. Logging, pulping and manufacturing processes are expected to conform to the environmental regulations of the country of origin.

A catalogue record for this book is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Agarwal, Megha, 1982–
Developments in mean-variance efficient portfolio selection / Megha Agarwal.
pages cm
ISBN 978-1-349-47176-8 (hardback)
1. Portfolio management – Mathematical models. 2. Finance – Mathematical models. I. Title.

HG4529.5.A36 2014
332.63′2042—dc23 2014025710

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Foreword

Mean Variance efficiency has been the most popular portfolio selection criterion ever since its discovery by Markowitz (1952). Since that time, this criterion has been much appreciated by both practitioners and academicians. *Developments in Mean Variance Efficient Portfolio Selection* tracks the advances made to this renowned criterion. It provides a thorough summary of the results and issues while addressing the concerns of present day investors. Building on the classical quadratic programming portfolio selection model, the book contributes to the area of portfolio modelling by adding multiple constraints to the mean-variance efficiency criterion and supporting the methodology with new empirical investigations.

The book includes a most comprehensive review of existing theories and empirical studies on portfolio selection starting from Markowitz, Sharpe, and Ross, and then building to the Fama and French model and current research. Desires and needs of the investor are also modelled into the general quadratic programming framework. Constraints are included to include accounting variables as well as market-based indicators. In addition to meeting these constraints, the model then ensures that the resultant portfolios are efficient in the Markowitz sense, producing maximum returns for a given level of risk.

The proposed model is tested using data from two prominent global stock exchanges: the London Stock Exchange and India's National Stock Exchange. Sophisticated statistical software such as SPSS, E-views, and the optimisation software Lingo are used in this empirical testing of the model. Moreover, exhaustive and updated databases, such as Thompson Reuters Eikon and the Centre for Monitoring India's Economy's Prowess, have been used for the purpose of data collection. Using these resources, the book provides a detailed comparison of the performance of model with the existing Markowitz efficient frontier in terms of risk, return, and portfolio utility. Forward tests are also conducted on the portfolios to confirm their robustness.

A key strength of the book is its addressing the limitations of current portfolio selection models in accommodating the multiple objectives of the investor while maintaining a mean-variance efficient portfolio. The book elaborates on the practical equity portfolio creation process, thus reducing the gap between theory and practice. The model then can actively assist both individual and institutional investors in making rational and systematic investment decisions.

The book provides a solution to the problems faced by investors by consolidating into the model many constraints faced in creating an optimal portfolio in practice. By placing the analysis in the context of the literature and global events, the book appeals to both current and future interests, especially in light of the global recovery from recession. The creation of balanced portfolios is important not only to the retail investor but also to brokerage houses, market participants, finance students, research scholars, and the market at large. All can benefit from the lessons and analysis described in this comprehensive study of the topic.

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Preface

Over the last few years there have been rapid changes in the securities market, more so in the secondary market. Advanced technology and online-based transactions have modernised the stock exchanges. The number of companies listed and the total market capitalisation in the equities market is relatively large as compared to the stage of economic development. A systematic and rational financial investment decision in a rapidly changing world of equities investment forms the core of this book.

The explanatory power of equity variables such as return, dividend, beta, liquidity, etc. have been studied in isolation thus limiting their application in improvement of the existing portfolio selection models. The mathematical complexity of some existing models hinders its applicability. A balanced portfolio which provides an investor with capital protections and opportunities for superior gains is required. A flexible model capable of accommodating the real-world constraints and objectives of an investor has been formulated using the Quadratic programming approach.

The model was tested on real data drawn from two important stock exchanges of the world, London Stock Exchange's FTSE 100 and Indian National Stock Exchange's Nifty 50. Eight portfolio model formulations, namely diversifier's portfolio, satisficer's portfolio, plunger's portfolio, market trend portfolio, capital gain bias portfolio, dividend gain bias portfolio, equal priority portfolio and the ideal portfolio, were created for investors with different priorities and risk appetite. The objective of risk (variance) minimisation was achieved by optimising variables such as earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price to book value ratio, price-earnings ratio, promoter's shareholding, sales, turnover, beta, unsystematic risk and volume across other important portfolio simultaneously. All the portfolios created were compared with the Markowitz's efficient frontier in the risk-return space. Ideal portfolio was found to be the closest to the Markowitz's portfolio.

Two multiple regression equations have been estimated with returns and excess returns to standard deviation as the dependant variables. Regression models explain the relevance of a new variable, namely impact cost having significant explanatory powers for predicting security return and Sharpe ratio. Granger causality tests we are undertaken to find out the relationship of causation between returns on a security and the variables set as constraints in the programming problem. The null hypothesis that dividend, impact cost, net profit, promoter's holding, sales and volume do not cause returns could not be rejected.

The portfolio utility analysis was undertaken to empirically find the utility derived by an investor from alternate portfolios for changing levels of risk tolerance. A direct relationship between the degree of risk tolerance and the value of portfolio utility was found from the quantitative analysis. The portfolio selection model formulations were plotted in the risk-return space along with the utility curves to find the optimal portfolio choice for different types of investors. The evaluation of the alternate portfolio selection model formulations was attempted by using Sharpe ratio (1966) and Treynor ratio (1965). The Sharpe ratio was the highest for Markowitz portfolio followed by the ideal portfolio. The ideal portfolio performed the best, even better than the Markowitz portfolio, when evaluated according to Treynor's ratio. Tests of equality of mean, variance and portfolio utility for ideal portfolio, Markowitz's portfolio and index portfolio were conducted to further investigate the proximity of these portfolios. Forward tests have been conducted to confirm the robustness of these portfolios.

The mean-variance model formulated and applied here will be of immense use for the investors, both individual and institutional, brokerage houses, mutual fund managers, banks, high net worth individuals, portfolio management service providers, financial advisors, regulators, stock exchanges and research scholars in the area of portfolio selection.

Acknowledgements

It is impossible for me to express adequately my indebtedness to Prof. J. P. Sharma, Head and Dean, Department of Commerce, University of Delhi. His constructive criticism and erudite suggestions have been instrumental in leading this research to its logical conclusion. I shall remain indebted to Prof. Sharma for his keen interest, invaluable guidance and intellectual stimulation, and this would remain a beacon throughout my life.

I am indebted to Prof. Colin Clubb, Department of Management, Kings College London. He was very kind, cooperative and spared his valuable time to review the drafts, discuss and provide suggestions that have helped me improve this work. I have freely encroached upon his time, but for his keen interest this work would not have taken its present shape.

I am thankful to Prof. (Dr.) K. V. Bhanumurthy, Department of Commerce, University of Delhi for his generous guidance and cooperation. I would like to take this opportunity to acknowledge the time and effort he has invested in reviewing my work. I would also like to thank Prof. (Dr) J. D. Agarwal, Chairman, Indian Institute of Finance and Editor-in-Chief Finance India for his invaluable comments and suggestions from time to time.

My sincere gratitude to Mr Mark Wiley, LINDO Systems, Inc, USA, for providing me access to the evaluation copy of LINGO 13.0 used for quadratic programming portfolio selection optimisation with unlimited variables and constraints. I am also thankful to Janis Merton from Thompson Reuters helpdesk for replying to all my queries relating to the database.

I am also thankful to the organisers of various national and international conferences who gave me the opportunity to present my research papers. In particular, I would like to acknowledge the organisers of 2nd IIMA International conference on Advanced Data Analysis, Business Analytics and Intelligence organised by Indian Institute of Management, Ahmedabad; 13th West Lake International Conference on Small and Medium Business (WLICSMB, 2011) organised by Economic Commission, Hangzhou Municipal Government, Zhejiang Provincial Institute of Small and Mid-sized Business and College of

Business Administration, Zhejiang University of Technology, Hangzhou, China and 64th All India Commerce Conference (AICC) organised by Indian Commerce Association, Department of Commerce, School of Management, Pondicherry University, Pondicherry. The comments and suggestions of the participants from these conferences have been duly incorporated.

Am indebted to my Principal, Dr Vijay Laxmi Pandit and my colleagues at Rajdhani College Mr Krishan Kumar, Dr Raj Kumar, Mr Nirmal Kumar, Mr Surinder Kumar Sachar, Dr Renu Gambhir, Dr Rajender Kumar and Dr Priyanka Kaushik for their academic support during the writing of this book.

My thanks are also due to the staff members of the Department of Management, King's College London and Franklin-Wilkins Library for extending their help and cooperation. I am obliged to the authorities and to the library and computer centre staff at the University of Delhi, Ratan Tata Library, Central Reference Library, Rajdhani College and Indian Institute of Finance for extending unconditional support.

I also take this opportunity to express my deep personal gratitude to Dr Manju Agarwal and Prof. Aman Agarwal for their unflinching encouragement and support. It may not be possible for me to adequately thank my husband Prof. Saurabh Agarwal and daughter Vedika who have patiently stood by me and supported me throughout . I dedicate this book to my parents, Mr Ashok Chawla and Mrs Santosh Chawla. I also acknowledge the moral support of my brother Mr Prashant Chawla.

Last but not the least, I am thankful for the immense benefit I have derived from the work of various authors which has enriched my understanding of the subject.

Megha Agarwal

1
Introduction

1.1 Introduction

The sacrifice of current money and other resources for future benefits is referred to as an investment. Investing is done with an aim of earning returns, which involves two key aspects: time and risk. The present outflow of funds is certain, but the future gains are uncertain and involve risk. A deliberate and careful investment decision leads to the creation of a portfolio of assets. Investment decisions are to be taken within the framework provided by the complex of financial institutions and intermediaries comprising the capital market. The capital market also provides the mechanism for channelling current savings into investments. Portfolio analysis starts with information concerning individual securities and ends with conclusions concerning portfolios as a whole.

The importance of an investment decision is emphasised more in this phase of recession and recoveries in the global economies, examples of which include the economic crisis in Cyprus, Greece and Iceland, as well as the aftermath of the UK banking crisis. The rapidly increasing growth of voluminous literature on portfolio selection in the recent years indicates the widespread interest of the academic and business communities in this area. It further emphasises the importance of investment decisions in today's world. This book is written to provide an aid to different types of investors in the selection of securities and the creation of an optimal portfolio of assets.

Mean-variance criterion introduced by famed economist and Nobel Laureate Harry Markowitz (1952) is by far the most widely known efficiency[1] criterion for investment analysis. Efficiency is defined in terms of either lower variance at the same level of mean return or higher mean return at the same level of variance. An optimal portfolio is more than a long list of good stocks and bonds; it is a balanced whole providing an investor with protections and opportunities with respect to a wide range of contingencies (Markowitz, 1959). Markowitz provided the direction to incorporate the multiple objectives of an investor in portfolio construction. The multidimensional nature of this decision is captured through the use of multi-criteria decision making (MCDM) framework, which provides the methodological ground to solve the problem of portfolio construction.

A portfolio optimisation model making use of the potentials of a quadratic programming approach for a real life investor with multiple constraints is developed and tested throughout the course of this book. The preferences of investors and key features of investment opportunities must be clearly understood and linked to each other to ensure optimal investment decisions. Incorporation of fundamental accounting, financial and corporate governance variables in a mean-variance portfolio selection model accounts for the multiple objectives of an investor. The crucial role of portfolio attributes like expected return, dividend returns, variance, the responsiveness of stock's return to index return (Beta), trade volume, price-to-earnings ratio, market capitalisation, operating profit margin, net profit margin, free float, free cash flows and other such factors are identified for creation of efficient portfolios.

The objective function of an investor is to minimise risk that is variability of returns as defined by Markowitz. The real world financial markets, economic conditions/scandals, and attitude and priorities of an investor impose a number of constraints that need to be incorporated for careful selection of securities. The actual scenario faced by an investor, his/her set of constraints and the objective of risk minimisation is simulated in various model formulations. An attempt has been made to identify and suggest the alternate parameters for practical application of the developed model. Empirical testing of the model has been also undertaken for Nifty 50 securities of the National Stock Exchange of India and FTSE 100 securities of the London Stock Exchange, UK. The performance of resulting portfolios created using alternate portfolio selection model formulations as per investor preferences are compared using the Sharpe and Treynor ratio. A comparison and tests of the performance of proposed portfolio selection model, vis-à-vis Markowitz's model for portfolio selection and the index portfolios have also been undertaken.

This chapter introduces the concept of portfolio selection and its relevance in today's world. It discusses the research gaps, significance of portfolio selection decisions, objectives of writing this book, research methodology adopted, research hypothesis, sources of data, chapter scheme and possible limitations.

Background

Introduced more than 60 years ago in the pioneering work by Markowitz, the mean-variance optimisation is one of the most popular approaches to portfolio selection. The basic assumptions of his theory are that a rational investor has either multivariate normally distributed asset returns or, in the case of arbitrary returns, a quadratic utility function. In the validity of these assumptions, Markowitz has shown that the optimal portfolio for the investor rests on the mean-variance efficient frontier. The portfolio of financial assets has been defined as efficient if, and only if, for any given expected return there is no other portfolio with lower variance, and for any given variance there is no other portfolio with higher expected return. The efficient frontier consists of all efficient portfolios. James Tobin (1958), another Nobel Laureate economist, based his theory of investment choice under conditions of uncertainty on the mean and variance of the distribution of returns. The Markowitz-Tobin analysis remains the cornerstone of the work in the field of investment analysis.

The advantage of mean-variance criterion is that the investor can focus on the first two aspects of the distribution of returns: the expected return (E) or mean and the variance (V). The investors tend to diversify risk by building portfolios comprising of a number of common stocks, or stocks and cash, bonds, derivative products, etc. The desire to stabilise the income stream is *sine qua non* for investment diversification. The greater the number of securities included in a portfolio, the lower its variance. However, institutional restrictions and costs limit the actual size of portfolios.

The concept of diversification and efficient frontier provided logical basis for selecting a portfolio based on individual utility curves. An approximation of a utility function by a quadratic in a certain neighbourhood is central to the Markowitz (1959) rationale for mean and variance. Thereafter, single index model (Sharpe, 1964), multi-index models (Fama and French, 1992), utility-based models, stochastic dominance-based models, correlation-based models, and models based on criteria such as safety first, skewness, geometric mean returns and so on have emerged for portfolio selection.

Building an equity portfolio is considered more varied than building debt portfolios because of the multiplicity of objectives. The primary objective of equity portfolios could be to generate absolute returns with low volatility over a long time period, to generate long-term capital growth from a diversified portfolio investing predominantly in equities or to generate capital appreciation and income distribution from an investment which outperforms the specific indices such as Sensex or Nifty. The objective could also be to generate long-term capital growth from an actively managed portfolio comprising equities, equity-related securities and equity derivatives. Another objective could be to generate higher than benchmark returns and long-term capital appreciation. Not only are the objectives multiple but so are the avenues for investment. While creating an equity portfolio, an investor can focus on benefitting from arbitrage opportunities, equity derivative strategies, pure equity investments and some small balance in debt and money market instruments.

New approaches for portfolio selection have emerged in the recent years with developments of new techniques in the field of operations research and management science, advancements in information technology and better accessibility of market information through databases such as Thompson Reuters Eikon. The MCDM approach has been applied to the problem of portfolio selection by researchers in finance. Hurson and Zopounidis (1995) provided the justification for applying the MCDM framework to the composition of an optimal portfolio. They suggest that because risk originates from various sources, its nature is multidimensional. Also, the preferences and objectives of investors are many. By incorporating a number of other criteria in addition to the traditional mean-variance, MCDM builds realistic models. This approach is advantageous as it can create portfolios specific to the preferences of an investor, while incorporating financial market factors at the same time. The rather restrictive norms imposed on the behaviour of investors by the classical approaches of portfolio selection are lifted by the MCDM framework by including the attitude and preferences of a real life investor.

Portfolio management has been defined as an ongoing process involving setting up of investment objectives and constraints, developing investment strategies, composing portfolio, initiation and implementation by managers and traders, performance evaluation, monitoring market conditions and revision/rebalancing (Maginn et al., 2007). It is an integrated set of planning, execution and feedback functions (Xidonas et al., 2009), the planning of which involves formulation of objectives and expectations. Investors' return objectives, risk tolerance, liquidity needs, regulatory and taxation requirements, as stated in the investment policy statement, form the basis for this step. Execution may be carried over by portfolio managers by initiating portfolio decisions based on analysis and implementing them. Monitoring, rebalancing and portfolio evaluation are finally carried out in the feedback stage. The focus of this monograph is on the stage of portfolio creation as per the relevant market conditions and investor preferences.

The various portfolio constraints faced by an investor include illiquidity, short-selling, minimum capital requirements, diversification, dividend, volatility, volume, turnover and many more. These constraints have an impact on the portfolio strategy. The Martingale technique, quadratic programming, the Markovian chain process, the Lagrange multiplier, Riccatti equations, mixed integer and heuristic approaches have all been used by researchers worldwide to study such constraints. Limited empirical research work makes it imperative to develop a portfolio selection model which is best suited to current capital markets conditions and accommodates for multiple objectives of the investor.

Substantial improvements in the availability of large data sets, real time information and software capable of performing complex computations is continuously contributing towards improved research work in portfolio selection. Better understanding of the markets and evolving economic models provide the base to add further to the Modern Portfolio Theory. A distinction needs to be made between the real behaviour of an investor vis-à-vis rational behaviour. Investors' priorities, preferences and the decision rules they follow are instrumental in the selection of securities in the basket of assets.

The traditional approaches of mean-variance portfolio selection, diversification principle, single factor models using beta and multifactor models all have proved to be extremely useful in the past. Although they are necessary for tackling the problem of portfolio selection, they are not sufficient. The incorporation of accounting variables such as operating profit margin, net profit margin, free cash flows; financial market variables such as dividend yield, price-to-earnings ratio, trade volume, etc. and corporate governance variables like promoters' shareholding, free float, the number of independent directors, etc. would help in improving the existing portfolio selection models. These would represent both the fundamentals of securities in question and the personal preferences of an investor. The MCDM approach can facilitate in synthesising together the theoretical as well as practical aspects of portfolio creation. This approach can ease the complexity of the multi-criteria problem and simplify the use of criteria from different context, resulting in portfolios specific to an investor's preferences. Understanding of the emerging portfolio selection theories and prevailing portfolio management practices has paved the way for a new portfolio selection model. The model aims at enhancing the value of the portfolio while optimising across multiple constraints.

1.2 Review of trends in the Indian economy and Indian capital markets

The global financial crises, sovereign debt risk and downgrade by major credit rating agencies across the globe has affected the performance of a large number of stock indices including that of National Stock Exchange's (NSE) Nifty and BSE's Sensex. All possible efforts by the government and regulatory bodies to provide greater depth and liquidity to the financial markets have in fact taken a beating amidst weak global economic prospects. Despite all possible odds, the Indian economy has been able to maintain its position of being one of the fastest growing economies in the Asian Sub-Continent in 2011–2012. Our Gross Domestic Product (GDP) is estimated to grow at the rate of 6.9 per cent in 2011–2012. However, the high amount of current account deficit, 3.6 per cent of GDP (2011–2012), is affecting the exchange rate negatively. Even the fiscal deficit was 5.9 per cent of GDP in 2011–2012. The increasing deterioration of fiscal balance is primarily due to the reduced direct tax revenue and increased subsidies which are also affecting the future economic prospects. The government is aware of these shortcomings and to an extent is trying to limit the damage by effective implementation of the Fiscal Responsibility and Budget Management (FRBM) Act. Tax reform measures related to the Direct Tax Code and Goods and Service Tax (GST) are also expected to contribute to improving the current fiscal situation. The disinvestment policy of the government of raising Rs 30,000 crores (in 2012–2013) and ensuring minimum public shareholding by central public sector enterprises is also expected to boost the capital markets and bring back retail investors to the equities market. The Rajiv Gandhi Equity Saving Scheme[2] is a novel idea of ensuring increased participation by retail investors. Other measures like the simplification of the issue of Initial Public Offers, allowing qualified financial institutions to access the Indian bond market, etc. are also expected to yield positive results for Indian capital markets.

In the primary securities market, measures by Securities and Exchange Board of India (SEBI) like (1) Increase in the monetary limit for retail investors from Rs 1 lakh to Rs 2 lakhs; (2) reduction in the process time lines from 22 days to 12 working day between issue closure and listing; (3) improvement in the refund process of application supported by blocked amount process (ASBA); (4) introduction of pre-announced fixed pay date for payment of dividends and for credit of bonus shares; (5) requirement of minimum public shareholding; (6) voluntary adoption of International Financial Reporting Standards (IFRS) and (7) disclosure of change in shareholding pattern within 10 days of +/–2 per cent change and many others are expected to restore the confidence of investors. Some of the measures in secondary securities market like (1) trading in securities using wireless technology; (2) smart order routing;[3] (3) extension of contract tenure for securities lending and borrowing from 30 days to 120 days; (4) setting up of trading platform for small and medium enterprises (SMEs); (5) mandatory certification for risk managers and (6) enhanced norms for credit rating agencies and a host of changes in derivative markets are expected to improve the internal control and efficiency of these markets. Some of the technology-based measures have yet not been implemented in some of the world's most advanced stock exchanges; most of these measures have been well implemented, but some of them are still facing operational bottlenecks.

Despite a robust economic performance and a host of measures implemented by the regulatory body, the resource mobilisation in primary market sharply declined from Rs 48,654 crores in 2010–2011 to Rs 9,683 crores (up to 31 December 2011). Even the number of Initial Public Offers (IPOs) reduced from 53 (2010–2011) to 30 (up to 31 December 2011). However, mutual funds exhibited a substantial increase in resource mobilisation in 2011–2012. Poor performance was also exhibited by major stock indices like NSE Nifty (20.7 per cent decline) and Nifty Junior (22.6 per cent decline). Even the free float market capitalisation for Nifty (20 per cent) and Nifty Junior (21.8 per cent) decreased. From Table 1.1, it may be observed that Nifty Index had negative returns in 2008–2009 (–36.2 per cent) and in 2011–2012 (–20.7 per cent) with extreme volatility in returns for remaining years. Market capitalisation has exhibited sustainable increase over the years except for 2008–2009. The daily volatility has also remained largely range bound between 1.1 (2010–2011) and 2.6 (2008–2009). Price-to-earnings ratio has been around 20 showing neither over nor under valuation. Nifty Junior also exhibited a similar trend.

Table 1.1 Index returns, volatility, market capitalisation and P/E ratio

Index	**2007–2008**	**2008–2009**	**2009–2010**	**2010–2011**	**2011–2012#**
Nifty					
Return (per cent)	23.9	–36.2	73.8	11.1	–20.7
Market capitalisation (Rs Crore)	1240071	771483	1525162	1755468	1405066
Daily volatility	2.0	2.6	1.9	1.1	1.3
P/E ratio	20.6	14.3	22.3	22.1	16.8
Nifty Junior					
Return (per cent)	16.0	–45.6	148.4	4.7	–26.1
Market capitalisation (Rs Crore)	202809	113523	292316	316529	247531
Daily volatility	2.4	2.8	2.0	1.2	1.1
P/E ratio	16.7	8.7	15.8	17.6	13.5

Notes: Market Capitalisation is calculated on free float basis. P/E ratio is price-earnings ratio.
Figures as of 31 December 2011.
Sources: NSE and Economic Survey 2011–2012.

1.3 Research gaps

Markowitz has explained the concept of diversification for creating efficient portfolios. With expected return on one axis and risk on the other, he drew the first efficient frontier. The riskiness of a portfolio was defined in terms of covariance. The major limitation of Markowitz model is the large amount of inputs required by the model. For a portfolio analysis of N securities it requires:

1. N expected returns,
2. N expected variance,
3. $(N^2 - N)/2$ co-variances.

On a whole, it requires [N (N+3)/2] separate pieces of information. Thus, data input became a limiting factor. This was solved by Sharpe's one factor model. The model linearly related the return on security with return on the index. This reduced the data requirement to only 3N+2.

The arbitrage pricing theory (APT) has been extensively researched as an alternative to mean-variance portfolio selection models and capital asset pricing model. The approach proposed by Ross (1976) provided a consistent and robust method for pricing the risk associated with different assets, on the assumption that each asset's return is a function of factors such as interest rates, yield structures and the return on market portfolio. However, the testability of the APT still remains questionable and arbitrary in terms of the factors. Dybvig and Ross (1985) himself tried to prove the testability of APT refuting Shanken (1982) objection and explained the irrelevance for testing of the approximation error. Testability of arbitrage pricing theory was explained on subsets and was not able to overcome the limitations of the capital asset pricing model in the APT.[4]

1.4 Raison d'être of the book

The limitations of existing portfolio selection models by Markowitz, Sharpe, Ross, Fama and French to accommodate for multiple objectives of an investor in a portfolio optimisation model is the primary reason for writing this book. Most of the existing models have focused on optimality in terms of one or two key variables ignoring minimum performance of the portfolio across other financial variables. The focus on multiple financial variables becomes increasingly important in the light of the contagion effect of financial crisis of United States (US) and European Union (EU) on the performance of stock markets across the globe. Individual investment and portfolio decisions in the capital market are also socially significant as they shape the pattern and growth of real output of the economy.

Two indicators based on distribution of returns are often used to evaluate investments. The profitability of an investment is measured by expected return that is the mean of probability distribution of returns. The other indicator is based on the dispersion of returns. Investors desire high returns but are averse to a high variance, which is an indicator of investment's risk.

The important issue of creation of optimal equity portfolios in the presence of many constraints faced by investors are raised in the course of this monograph. A balanced portfolio which provides an investor with protections and opportunities with respect to a wide range of contingencies needs to be created. The argumentation here leads to development of a model for creation of optimal portfolios, which best suits the needs and objectives of an investor.

A systematic and rational financial investment decision in a rapidly changing world of investment alternatives forms the core of discussion. The contrary thinking abilities, patience, composure, flexibility, openness and decisiveness of a person may make him/her a successful investor. The portfolio optimisation model so created is tested for

Nifty Index of National Stock Exchange (NSE) of India and FTSE-100 index of London Stock Exchange. The discourse is relevant in the current scenario as it reviews the existing modelling framework for portfolio selection which has been developed by Markowitz, Sharpe, Fama and French and Ross. The advantages that individual and institutional investors derive from diversification by building portfolios composed of a number of common stocks selected as per the mean-variance criterion, beta, book-to-market ratio, market capitalisation and so on has been presented.

The work here empirically tests the relevance of portfolio selection models on the stocks listed on the National Stock Exchange. Portfolios are created by employing alternate portfolio selection model formulations for listed stocks part of NSE Nifty. The crucial role of portfolio attributes like expected return, variance, the responsiveness of stock's return to index return, market capitalisation, book-to-equity ratio and other such factors are identified in creation of efficient portfolios. Asset combinations are formed based on the learning's from the existing literature on portfolio modelling. The resulting portfolios created using alternate portfolio selection model formulations have been compared using the Sharpe and Treynor ratio. Quantitative and qualitative comparison of the alternate portfolio selection models enables the researchers to rank them in terms of their effectiveness in the present day Indian securities market. Further, this work has been able to develop a portfolio selection model that can capture valuable statistical information in asset mean returns and variance.

The principle objective of the study is to determine the mean-variance efficient sets and to provide a quadratic programming model to compute them. The objective function of the investor is to minimise the risk that is variability of returns as defined by Markowitz. The real world financial markets impose a number of constraints that have been incorporated in this study. In all, 30 such constraints have been identified and empirical testing of this model has been undertaken. The model is expected to be of immense help to the Financial Institutional Investors (FII's), Qualified Institutional Buyers (QIB's), Mutual Funds and other Indian investors in selecting the optimal portfolio in presence of the plethora of real life constraints.

An attempt has been made to identify and suggest the alternate parameters for practical application of the developed model. Efforts have been made to reduce the gap that generally exists between a theoretical model and its possible industrial application. Investors can create efficient portfolios using the model developed. The risk and return of these portfolios are compared with the risk and return of portfolios created using other portfolio selection models and with Nifty 50. The actual scenario faced by an Indian investor, his/her set of constraints and the objective of risk minimisation is simulated in the model formulations.

The mean-variance analysis undertaken in this research work will be of immense use for Indian investors both individual and institutional, brokerage houses, mutual fund managers, banks, high net worth individuals, portfolio management service providers, financial advisers, regulators, stock exchanges and research scholars in the area of portfolio selection.

1.5 Problem statement

The selection of an optimal set of securities that form a portfolio is indeed a tedious task. In the portfolio creation process, investors may inadequately comprehend return and risk. Investors may vaguely formulate investment policies, make untimely entries and exits, pay high transaction costs, over diversify or under diversify. Investors may not effectively incorporate changes in expectations while extrapolating the past. All these errors in investment management make an investor prone to losses. This most often results in destruction of wealth of a retail investor in an attempt to create it. Review of existing theories and empirical studies on portfolio management outlined the following research problems:

1. Despite a large number of empirical studies in the area of market efficiency and capital asset pricing model, there has been a dearth of research in analysing mean-variance efficient portfolio selection for Indian securities market.
2. Mathematical complexity of some of the recently developed international models limits the applicability of their contribution. In some cases, the non-stationary data makes the empirical testing of the model rather complicated. The testing of models for their robustness and optimality in emerging markets like India has not been investigated in detail.
3. The large numbers of portfolio selection models are a simplified work of correlating risk and return without adding non-market factors like inflation, interest rate, purchasing power or employing techniques such as generalised auto regressive conditional *hetroscedasticity* or auto regressive conditional hetroscedasticity to model the variability

of returns. Even now, most of the models do not incorporate the effect of taxes, transaction costs, short sales, borrowing and lending despite the fact that a large number of scholars have proved the significant explanatory power of each of these factors.

4. Not much empirical evidence exists in India on investigating the impact of corporate governance practices, habits of investors and learning effect on the portfolio selection decisions.
5. The modelling framework needs to be extended to include the futures and options of securities. The existing models could be extended to include other parameter settings to represent large numbers of securities and other forecast models, such as auto regressive conditional hetroskedasticity (ARCH) type models.

1.6 Research objectives

The primary objective of this research work is to create a mean-variance efficient portfolio selection model for an investor in the presence of the plethora of real life constraints he/she faces while selecting a portfolio of assets.

Sub-objectives

1. To review emerging issues in portfolio selection modelling framework including the foundations set by classical works of Markowitz, Sharpe, Fama and French, Ross among others.
2. To provide alternate measures of aspiration values which could serve a guide to create portfolios for different types of investors.
3. To find an algorithm(s) that optimises across multiple constraints, while minimising the variance of the efficient equity portfolio. For developing this algorithm the thesis intends to:
   a) identify the multiple constraints for portfolio selection decision;
   b) set the aspiration values [Quartile three (Q_3), Median, Quartile one (Q_1) or mean] for investors with different risk profiles;
   c) recommend investment portfolios for different types of investors;
   d) to understand and investigate the relationship between portfolio returns, excess return to standard deviation and portfolio selection variables;
   e) to find causation between security returns and portfolio variables;
   f) to identify the relevance of security variables on security returns using the multivariate and causality analysis;
   g) recommend the Ideal portfolio with the most relevant constraints;

4. to graphically compare and plot all the resultant portfolios from alternate portfolio selection model formulations with portfolios on the Markowitz efficient frontier for the same level of upper bounds;
5. to undertake a utility analysis of the resultant portfolios using arithmetical and graphical technique;
6. to measure the performance of the resultant portfolios using the Sharpe ratio and the Treynor ratio;
7. to conduct tests for equality of risk, return and portfolio utility between Markowitz's portfolio, index portfolio and the best performing portfolio.

1.7 Research hypotheses

In the process of creating a model for portfolio selection, the existing Markowitz model providing maximum expected utility to an investor needs to be tested and compared with the performance of the theoretical model recommended in this work. An attempt to test the relevance of Markowitz's model of portfolio selection and the portfolios created on National Stock Exchange has been made. The hypotheses are as follows:

Hypothesis 1 (H_1)

The objective is to compare the mean return of the proposed model with the mean return of Markowitz's model.

H_0: There is no difference in the expected return of the portfolio created by the proposed model and the Markowitz model.

$$\mu_E = \mu_M \tag{1.1}$$

H_1: Expected return of the portfolio created by the proposed model is superior to the return of portfolio created by the Markowitz model.

$$\mu_E > \mu_M \tag{1.2}$$

The expected return on the security has been calculated by the formulae using models given by Markowitz (1952).

Hypothesis 2 (H_2)

The objective is to compare the risk (variance) of the proposed model with that of the Markowitz model.

H_0: There is no difference in the risk (variance) of the portfolio created by the proposed model and the Markowitz model.

$$\sigma_E^2 = \sigma_M^2 \tag{1.3}$$

H_1: Expected risk of the portfolio created by the proposed model is lower than the risk (variance) of portfolio created by the Markowitz model.

$$\sigma_E^2 < \sigma_M^2 \tag{1.4}$$

The variance of a portfolio has been calculated using the formulae given by Markowitz (1952).

Hypothesis 3 (H_3)

The objective is to compare the portfolio utility of the proposed model portfolio with that of the Markowitz portfolio.

H_0: There is no difference between the utility derived by investors from the portfolio created by the proposed model and the Markowitz portfolio selection model.

$$U_E = U_M \tag{1.5}$$

H_1: Portfolio utility in the portfolio created using the proposed model is higher than the utility of the portfolio created using Markowitz portfolio selection model.

$$U_E > U_M \tag{1.6}$$

In hypothesis one, two and three, a comparison has been made between the portfolios created using the proposed theoretical portfolio selection model and the Markowitz model. An attempt has been made further to compare the return, risk and portfolio utility of the proposed model with the return, risk and portfolio utility of the benchmark portfolio (Nifty 50). All these hypotheses have been tested using t test at five per cent level of significance.

1.8 Research methodology

(a) Quadratic Programming Portfolio Optimisation

Alternate portfolio selection model formulations, which attempt to minimise variance in the presence of real life constraints, have been solved using quadratic programming. The conceptual framework for using quadratic programming method for portfolio selection has been discussed in detail. An attempt has been made to provide the rationale for single objective multiple constraints portfolio optimisation and quadratic programming framework to achieve an optimal portfolio. A theoretical model of portfolio selection for an investor faced with multiple constraints has been developed. Empirical analysis of the Markowitz model (1952) and the model developed provide further insights as regards single objective multiple constraints portfolio optimisation. Model improvements are also recommended by suggesting, selected constraints mean-variance efficient ideal portfolio selection model.

(b) Multivariate Regression Analysis

Multiple regression analysis has been undertaken to find the important portfolio variables significantly explaining the cross section of returns. Two multiple regression equations have been estimated with returns and excess returns to standard deviation ratio as the dependant variable respectively. The list of independent variables included earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price to book value ratio, price-earnings ratio, promoter's shareholding, sales, turnover, unsystematic risk and volume. The model forecasts have been presented graphically.

(c) Granger Causality Tests

Granger causality tests are run to examine whether any of the independent variables considered in multiple regression equations cause returns and to find how much of current returns can be explained by past values of returns. Also, whether adding lagged values of independent variable improves the explanation of returns or not.

(d) Portfolio Utility Analysis for various Types of Investors

Utility analysis is undertaken quantitatively and graphically. Portfolio utility has been calculated by subtracting risk penalty from expected return for different levels of risk tolerance of an investor. For the graphical analysis, the portfolio return and variance of all the constructed portfolios are plotted over the indifference curves of various types of investors.

The first type of investor is a "diversifier", a more risk-averse investor whose indifference curves are concave to the y-axis. He/she expects an increasing return for equal increments in risk (along the X-axis). He/she is known as the "normal" investor. The diversifier would be willing to take more risk only for a more than proportionate increase in return to compensate for the additional risk. The second category of investor is a "plunger", a less risk-averse investor whose indifference curves are flatter as smaller risk premiums are expected by them. A plunger would be willing to take additional risk even for less than proportionate incremental returns. The third category of investor is a "risk neutral", an investor who is indifferent to risk. He/she is more concerned with the expected return on an investment rather than the risk. The indifference curve of such an investor is a straight 45 degree line from the origin. At the end of the continuum lies the "risk lover", a risk seeker investor in search for greater volatility and uncertainty in investments for anticipated abnormally higher returns. He/she exhibits irrational behaviour characterised by straight line indifference curves moving from right to left.

By superimposing the risk-return combination of portfolios on the indifference curves, the choice of portfolio by an investor is determined graphically.

(e) Performance Evaluation of Portfolios

To sum up the performance of portfolios and rank them, portfolio evaluation measures as given by Sharpe (1966) and Treynor (1965) have been used. The ratios summarise risk and return of a portfolio in a single measure, categorising the performance of a portfolio on a risk adjusted basis. A larger value denotes better performance of the portfolio. All the modelled portfolios have been ranked accordingly to compare their performance.

(f) Tests of equality

Tests of equality for mean, variance and portfolio utility between the Markowitz portfolio and other modelled portfolios have been conducted. Values and probability figures for F-test, Seigel-Tukey test, Levene test and Bartlett-Forsythe test are reported. The category statistics and Bartlett weighted standard deviation have been also calculated for analysis.

1.9 Sources of data

The study uses secondary data sources for monthly stock returns, beta, free float, promoter's holding, institutional holding, trading volume, turnover and impact cost (a measure of liquidity) for firms at the National Stock Exchange Nifty, 91 days Treasury bill rates and monthly returns on Nifty (NSE's value weighted index) over a period of 12 years starting from April 2000 to March 2012. Also, annual accounting data such as book-to-market equity, market capitalisation, sales, net profit, dividend, earnings per share and price-to-earnings ratio, total assets and other variables from the annual report of the constituent companies have been used.

The data have been collected from the official website of National Stock Exchange Limited (www.nseindia.org), annual reports of companies and Centre for Monitoring Indian Economy (CMIE) database PROWESS. The measure for risk-free rate of interest, 91 days T-bill rate has been taken from the official website of Reserve Bank of India (www.rbi.org.in). All the assets included in the sample are equity shares part of NSE Nifty. Data for London Stock Exchange's FTSE 100 has been collected from the Thomson Reuters database Eikon.

The software used for the research includes Statistical Package for Social Sciences (SPSS 16), E-views 5.1 and Lingo 13.[5] Transpose of matrices of security returns and covariance matrices were attempted using SPSS. The generation of covariance matrix (50x50) of the assets from the series of return, multiple regression estimation, correlation matrix, granger causality tests and tests for equality have all been attempted through E-views. The eight scenarios based on non-linear portfolio selection models have been programmed using the Lingo 13. The local solver and global solver of this software were used to generate solution to these portfolio selection problems.

1.10 Chapter plan

The monograph has been divided in six chapters. Chapter 1 introduces the concept of portfolio selection and its relevance in today's world. It discusses the research gaps, problem statement, objectives of the study, research methodology, research hypothesis, sources of data, plan of the study and possible limitations. It also discusses the significance of the study and main purpose of taking up this research. The discussion on problem statement is based on the following questions. What problems have given rise to research on this topic? What problems still remain to be tackled in existing literature? What are the emerging problems faced by a small retail investors today? Do portfolio managers really understand their investors? What are the objectives and constraints in making an investment in equities? The research objectives are framed in a manner so as to test the relevance of investment management and various theories developed in it and how they can be used for making portfolios in a developing securities market like that of India. An attempt to mention the existing gaps in literature and how they can be filled by this research endeavour has been made.

The advancements in literature in the area of portfolio management have been discussed in Chapter 2. Studies on mean-variance efficient portfolios, asset pricing theories, diversification of portfolios, portfolio optimisation, impact of behavioural factors on portfolio choice and lead-lag relationships between stock and futures markets have been included.

Chapter 3 focuses on the contributions to theory in the area of mean-variance efficient portfolio selection. The important models forming the pillars in the field of portfolio selection are discussed. The utility assumptions, quadratic programming framework, emerging issues and challenges in the Indian capital markets are a few areas discussed in this chapter. This chapter provides the conceptual framework for this book.

Chapter 4 discusses the research design and methodology. A theoretical mean-variance efficient portfolio selection model is developed, taking in view the multiple objectives and limitations of an investor. In all, 30 constraints have been identified in the theoretical model. The methodology for multivariate analysis, causality tests, utility analysis and tests of equality have been discussed.

This is followed by empirical testing of the model for benchmark index NSE's Nifty in Chapter 5. The sources for data collection and software used for analysis have been mentioned. Aspiration values for different types of investors are outlined for creating optimal portfolios. The mean-variance efficient portfolio selection model is programmed in the Lingo 13. The resultant portfolios are presented graphically through bar graphs and pie charts. A comparison of these portfolios with the Markowitz efficient frontier has also been attempted. Multivariate analysis and granger causality tests have been undertaken, and an empirical analysis of the eight portfolios achievements across the multiple constraints has been discussed in detail. A utility analysis for the alternate portfolios made has been attempted using arithmetical and graphical techniques. The portfolio performance evaluation measures as proposed by Sharpe (1966) and Treynor (1965) have been used to evaluate and rank the portfolios. Tests of equality for return, variance and portfolio utility between the Markowitz portfolio, ideal portfolio and index portfolio have also been discussed towards the end.

Testing of the multi-criteria decision making portfolio selection model on London Stock Exchange's FTSE 100 index is carried out in Chapter 6. Portfolios are created for four different types of investors constraining various risk, return, liquidity and governance parameters. Forward tests are attempted to check the robustness of our results. This chapter provides international perspective to this research endeavour.

Chapter 7 includes the summary, conclusions and recommendations for future research. This is followed by endnotes, references and annexure.

1.11 Limitations of the study

1. The study suffers from the assumptions of various statistical methods which are used for hypothesis testing.
2. The model formulations are for a single period and do not incorporate the continuous changes in the environment and its effect on portfolio selection.
3. The quadratic programming portfolio selection analysis is based on ex-post data which may not be truly representative of the future scenario.
4. Certain constraints originally included in the theoretical model were excluded in the empirical analysis due to unavailability of data and programming limitations.
5. Minute by minute real time data incorporating immediate changes may substantially improve the reliability of results. This has not been undertaken in this study.

2
Advances in Theories and Empirical Studies on Portfolio Management

Portfolio selection modelling dates back to the development of mean-variance[1] model of Markowitz. The concept of diversification and an efficient frontier provided the logical basis for selecting a portfolio based on individual utility curves. Roy (1952) provided a specific point on the efficient frontier whereby he attempted to minimise the upper bound of the chance of a dread event. Roy's principle of safety first, further supported the concept of diversification of resources among a wide variety of assets. Utility was defined in terms of minimisation of a chance of a catastrophe. Markowitz used statistical analysis and Roy used econometric analysis for the purpose of their studies. Tobin (1958) provided the basis for two fund separation theorem in the context of portfolio selection whereby an investor allocates his resources among risky and riskless assets. The theory propounded by Tobin was based on the risk avoiding behaviour of investors and was conceptually shown to be superior to the Keynesian theory of liquidity preference.

Sharpe (1964) provided equilibrium for calculating the expected return on an efficient portfolio. It was superior to Markowitz analysis as it did not require finding the covariance in returns of each security with every other security in the portfolio. The Capital Asset Pricing Model nearly related the return on portfolio with the beta of the portfolio. The concept of capital market line and securities market line are currently used for identification of under-/overvalued securities. Samuelson (1969) extended the issue of portfolio selection to lifetime portfolio selection using dynamic stochastic programming. The Samuelson model denied the validity and the concept of business risk. He found that investing and reinvesting on the basis of geometric mean rather than arithmetic mean resulted in greater accumulation of terminal wealth. However, the same may not exhibit the principle of transitivity.

Lintner (1965), using empirical evidence, concluded that securities value varies directly with intercept and the correlation coefficient and is inversely related with the residual variance. The paper provided empirical evidence of diversification gains from combination of negatively correlated securities. Fama et al. (1969), by providing insight on weak form, semi-strong form and the strong form of efficiency in the capital markets, provided the basis for efficient allocation of resources. Fama's model provides the basis for selecting securities of an efficient portfolio, conditional to the application of the fair game model.

Dealing with a more general class of utility function, Black (1972) created zero beta portfolios. Empirical tests of this two factor model suggest that it explains historical returns better than pure CAPM (Black et al., 1972). Black and Scholes (1972) demonstrated that the linear relationship between return on a security and its beta did not always hold true. On the lines of expected utility maxim and limited liability of assets, Merton (1973) developed the Inter-temporal Capital Asset Pricing Model (I-CAPM).

In India extensive empirical work has been undertaken to test the Capital Asset Pricing Model. Bansal (1988), using 200 firm's portfolio returns for the period of 1972–1984 and the Reserve Bank of India Index as market proxy, found CAPM to be applicable to the Indian capital market. Yalwar (1988), using 20 years' of data for 122 firms, found that CAPM is operative in Indian capital markets for active equities. Vipul (1998) found CAPM of limited applicability for earning supernormal returns. However, he found clear evidence supporting applicability of CAPM for Indian capital markets. Sehgal (1996), based on an analysis of 100 securities for a period from April 1984 to March 1993, found that CAPM does not hold for Indian capital markets. Hence, studies by Bansal (1988), Varma (1988), Yalwar (1988), Srinivasan (1988) and Vipul (1998) support the CAPM model. However, studies by Barua and Raghunathan (1990), Obaidullah (1994), Sehgal (1996), Madhusoodanan (1997), Ansari (2000) and Dash and Sumanjeet (2008) question the validity of the CAPM in the Indian markets. Major re-examination of the CAPM model was done after the empirical work of Fama and French (1992).

Fama and French found that three variables: market equity, the ratio of book equity to market equity and leverage variables capture much of the cross section of average stock returns. The three variables were found to have more explaining power compared to market beta. Dash and Sumanjeet (2008) support Fama and French Hypothesis for Indian capital markets on the basis of their study from January 1997 to January 2007 on BSE 500. Market beta was found to have insignificant explanatory power. Pandey (2002), using fixed effects regression model found that beta, book-to-market value (BM) ratio, earnings-price (E/P) ratio and dividend yield have significant powers in explaining the variance of returns on equities. Pay-out and leverage had insignificant explanatory powers in explaining the cross section of returns. The linear relationship between beta and stock returns was also observed. Size rather than BM ratio was found to be the most significant dominant variable. The analysis was based on a data set of 1729 firms per year. In India, the capital market studies mostly focus on market efficiency and CAPM. These findings will provide the theoretical background for synthesising efficient portfolios.

This chapter revisits the existing portfolio selection models for searching undervalued securities and creating efficient portfolios. The purpose is to undertake a comparative analysis of existing portfolio selection models. The review of literature[2] available in the area of portfolio selection has been divided into five parts covering studies relating to (a) mean-variance efficient portfolios, (b) asset pricing theories, (c) diversification of a portfolio, (d) portfolio optimisation and variance-covariance matrix and (e) studies on impact of behavioural and systemic factors on portfolio choice.

2.1 Literature on mean-variance efficient portfolios

The following paragraphs outline recent literature by researchers on attempts to achieve points on the efficient frontier.

Haas (1972) provided a theoretical framework for analysing international capital flows. A survey of portfolio theory focusing on the work of Markowitz, Tobin, Sharpe, Lintner and other models are examined. Assumptions of the model are consistent with contemporary portfolio theory. By using the Tobin separation theorem that a single optimal portfolio consisting of foreign and domestic investments exists, then this optimal portfolio together with the total wealth of the economy determines the desired stock of foreign assets at any point in time. The gross capital outflows are attributed to a change in the optimal ratio of foreign assets to total assets arising from a change in the rates of return or variance-covariance matrix or a change in wealth. Multiple regression equations using Koyck distributed lags were used to test the model with data from United States(US), United Kingdom (UK), Canada, Germany, Switzerland and the Netherlands. Exact and linear approximations of equations from the model were estimated bilaterally. The empirical tests confirm the usefulness of the portfolio approach and the risk variables proved to be significant.

Kazemi (1988) developed a multi-period general equilibrium model of asset prices which is testable without observing the market portfolio or aggregate consumption. Systematic risk was measured through conditional covariance of rate of return with next period risk-free interest rate. When the return on a risky asset was regressed against the short-term risk-free rate, the slope coefficient is negative. A minimum set of assumptions needed to produce an inter-temporal Sharpe Lintner (ISL) model were used. The interest rate based asset pricing model is expressed in real terms, whereas asset returns and short-term riskless rates are expressed in nominal terms. The model is a direct result of the mean-variance efficiency of the market portfolio. Hence, from theoretical point of view, it can be criticised for its rather restrictive assumptions. Also, the conditional covariances of asset returns with the next period riskless rate are likely to be non-stationary; this makes any empirical testing of the model rather complicated. It is an attempt to introduce new insights into the ISL model, and it presents an alternative equilibrium relationship that is in principle testable.

Lee and Chang (1995) test the THP[3] mean-variance instability portfolio selection model on eight Taiwan stocks to demonstrate the effect of instability in preference, on the traditional mean-variance frontier. Instability is defined as frequency of fluctuations in net earnings. They find that Taiwanese investors speculate in high variance stocks as compared to US investors speculating in low variance stocks. Also, short-selling increases the risk of the portfolio of an instability preferred investor. Investors' enthusiasm in "catching the next peak" affects the frequencies of stocks. Investors assume an increasing preference to instability. The model is a single period model and does not incorporate taxes and transaction costs.

Ballestero (1998) provided a new bounding utility theorem to approximate the optimum portfolio of an investor with well-defined preferences for profitability and safety. The lesser the preferences deviate (from the average preference behaviour), the narrower the bounds for the utility optimum on the efficient frontier. Fletcher and Hillier (2001) found empirical evidence of improved Sharpe ratios and abnormal returns by using strategies based on re-sampled portfolio efficiency over traditional mean-variance strategies. An investment universe of US risk-free assets and ten international equity index returns was used to estimate the mean-variance and re-sampled efficiency strategies. The largest equity indexes in terms of market value were selected: Canada, France, Germany, Hong Kong, Italy, Japan, the Netherlands, Switzerland, the UK and the US.

Based on a multi-period mean-variance model proposed by Frauendorfer (1995), refined by Frauendorfer and Siede (2000) and later extended to a complete application model including transaction costs and market restrictions, Steinbach (2001) presents a theoretical understanding of a multi-period mean-variance approach. Complex portfolio optimisation problems are analysed by the derivation of primal and dual solutions. The precise interaction of objective (risk measure) and constraints (wealth distribution) is presented based on scenario trees. Semi-variance problems as well as quadratic programmes with second order approximation of return distribution are developed into structure exploiting numerical algorithms. The multi-period mean-variance problem is found to behave much like their single period counterparts in many respects. It is possible to avoid over performance by allowing the removal of capital. Small target rewards are met exactly at zero risk when all the capital is either invested in cash or removed. Moderate values of rewards over performance cannot be avoided completely, but the semi-variance is minimised. The model presented is not time invariant. The initial objective of the investor is always depicted in optimal decisions even when time has passed. A moving horizon technique instead of a restricted expectation over a sub-tree might provide a more realistic model. Long-term models based on utility of consumption could further the current research.

Zhou and Li (2002) solve mean-variance portfolio selection problems in continuous time with no short-selling. A stochastic optimal linear-quadratic (LQ) control problem is formulated. A continuous function is constructed via two Riccati equations and shown to be a viscosity solution to the Hamilton-Jacobi-Bellman (HJB) equation. The efficient frontier and efficient investment strategies for original mean-variance problem are derived by solving these Riccati equations.

Zhou and Yin (2003) propose a continuous time version of Markowitz's mean-variance portfolio selection model. They consider a market consisting of a bank account and multiple stocks. A finite number of market states modulated by a continuous time Markov chain, with random regime switching decide the market parameters. The techniques of Markov chain modulated diffusion formulation, stochastic linear-quadratic control, Lagrange multiplier techniques and two systems of linear ordinary differential equations are used to derive mean-variance efficient portfolios and efficient frontiers. It is observed that the efficient frontier is not a straight line in the mean-standard deviation diagram. However, if the interest rate is independent of the Markov chain, then the efficient frontier becomes a straight line and the one fund theorem is preserved even if the appreciation and volatility rates of the stocks are random. The wealth process is allowed to take negative values, representing the bankruptcy situation. The model does not introduce transaction costs. In the presence of transaction costs, optimal strategies might not be continuously trading strategies as opposed to the no transaction case. Another limitation is the assumption of Markov chain being independent of the underlying Brownian motion. A corresponding discrete-time model would be more useful in the actual computing.

Leibowitz and Bova (2005) provided the three part efficient frontier. Three segments of frontier are cash to core segment, fixed core segment and the equity extension. They demonstrated how allocation alphas and betas simplify the portfolio allocation process. By dividing the portfolio assets into swing assets and alpha core, volatility is diversified and returns are enhanced. Post and Levy (2005) analyse market portfolio efficiency in relation to benchmark portfolios formed on market capitalisation, book-to-market equity ratio and price momentum. It was found that reverse S shaped utility functions with risk aversion for losses and risk seeking for gains can explain stock returns. Calafiore (2007) proposed an efficient method for determining optimal robust portfolios of risky financial instrument in presence of ambiguity. Polynomial time algorithms are developed for assessing and optimising the worst case risks of a portfolio with known nominal discrete return distribution.

Xiong and Zhou (2007) present a continuous time mean-variance portfolio selection model with multiple stocks and a bond. Efficient strategies based on partial information of stock and bond prices are derived. Optimal filter of the stock appreciation rate processes is employed to develop analytical and numerical approaches to solve the backward stochastic differential equations.

Garlappi et al. (2007) developed a portfolio selection model for an investor having multiple priors and aversion to ambiguity. Multiple priors are characterised by a confidence interval around estimated expected return and ambiguity aversion is modelled through minimisation over the priors. Two additions to the standard mean-variance model (1) additional constraint on expected returns to lie within a confidence interval of estimated value and (2) additional minimisation over a set of possible expected return are imposed. The multi-prior model with parameter uncertainty and uncertainty about the return generating process is applied to a fund manager allocating wealth across eight international equity indices. The percentage of funds allocated to US index from 1980 to 2001 under four different strategies reflect the effect of ambiguity aversion on individual weights in a risky portfolio.

In comparison to classical and Bayesian models, the ambiguity-averse portfolios, as suggested by the study are less unbalanced, fluctuate less over time and deliver higher out-of-sample Sharpe ratios in comparison to standard mean-variance (MV) models and Bayesian models. The multi-prior model which imposes no restriction on short-selling provides strong axiomatic foundations to the literature on portfolio choice. However, the learning effect on portfolio selection is not analysed. The model may be reduced to a simplified mean-variance model and is flexible enough to accommodate cases with expected returns calculated jointly or in subsets and also for real world constraints on the size of trades and position limits.

Lucas and Siegmann (2008) disqualified variance as an appropriate risk measure for hedge funds due to the presence of large downside risk. They examined the effects of shortfall-based risks in portfolio optimisation. Hedge fund data taken from HFR database for 1994 to 2004 showed returns to be fat tailed with positive excess kurtosis. The data are analysed using goal programming technique, robustness tests and simulation experiments. It was proved that negative skewness for optimal mean-shortfall portfolios is larger than mean-variance portfolios and that they reduce the probability of small shortfalls at the cost of increased extreme crash probability. The study warns the investors using expected shortfall as a risk measure in portfolio optimisation. Also quadratic shortfall or semi-variance is suggested as a measure of downside risk under loss averse preferences for the not so attractive hedge funds.

Pellizon and Weber (2008) assess whether actual Italian portfolios are efficient using Italian portfolio data, time series on financial assets and housing stock returns for the period 1989–1998. Household distribute their funds into financial and real assets. The standard tests of portfolio efficiency usually ignore the existence of illiquid wealth, such as housing in a household portfolio. The tests for efficiency must be run conditionally upon housing wealth. The study focuses on issues of efficiency with illiquid wealth and covers cases of correlated returns. Efficiency tests developed by Gourieroux and Jouneau (1999) incorporating a subset of asset holdings to be constrained (that is in housing) are used. Markowitz's expected return, variance-covariance matrix of assets, Bayesian methods of error estimation, multivariate generalised auto regressive conditional heteroscedasticity for second moments, partial correlations and WLS regression are calculated. Due to infrequent housing price adjustments in the stocks, the optimal portfolio is affected by housing price risk. Financing decisions are affected by the need to hedge risks connected with existing illiquid housing wealth when financial assets and housing returns are correlated. The optimal investment in risky assets is still the same as static mean-variance analysis framework with housing as an additional constraint in optimisation. The majority of Italian households are found to have non-diversified and non-efficient portfolios in the mean-variance efficient sense. The study finds that housing wealth plays a key role in determining the efficiency of homeowner portfolios.

Panageas and Westerfield (2009) found that risk neutral hedge fund managers place a constant fraction of funds in a mean-variance efficient portfolio and the rest in riskless assets. They act as constant relative risk aversion (CRRA) investors even in presence of option-like contracts. Risk seeking incentives of option-like contracts are based on combining finite horizons and convex compensation schemes.

2.2 Literature on asset pricing theories

Jobson (1982) developed a multivariate linear regression model to test the arbitrage pricing theory (APT). The central conclusion of the APT is shown to be equivalent to a statement about the intercept term in the multivariate regression function of a vector of (N–k) returns on a vector of k portfolios. Given a linearly independent set of k portfolios with return premium vector r_1 and a set of (N–k) assets with return premium vector r_2, a test of the hypothesis of a k factor APT model can be carried out by testing for a zero intercept term in the multivariate linear regression r_2 on r_1. The k factor APT hypothesis is accepted if the intercept term is zero in the multivariate regression. The simple univariate multiple regression software can be used to carry out the tests. The relationship of these tests with the concept of performance potential and Sharpe's measure of performance are also discussed.

Hall and Tsay (1988) compared the performance of portfolios among value line rank one stocks with random portfolios selected from New York Stock Exchange (NYSE) and American Stock Exchange (ASE). The study provides evidence on investment performance of value one stocks for 1976–1982 from the perspective of survey subscriber. How investment returns get affected by variations in portfolio size and revision periods is also answered. For each of the 13 value line and 54 random portfolios constructed, an asset pricing equation is estimated using 364 weeks of total return data. Three evaluative measures of portfolio performance as proposed by Jensen, Sharpe and Treynor are computed. To determine differences across portfolios, a linear regression technique is used. Active traders investing in value line rank one stocks were found to earn a statistically significant and positive excess return. However, after one per cent one way transaction costs were considered, neither active nor passive traders could achieve any statistically greater return than portfolios of randomly selected stocks. The results were not sensitive to variations in size of the portfolio. The study proves the survey of value one stocks to be misleading in the context of imperfect capital markets in the presence of transaction costs.

Murphy (1990) developed a model of asset pricing relaxing the restrictive assumptions of standard theories. Every investor is assumed to maximise the expected value of a Von-Neumann-Morgenstern utility of consumption function. The Lagrangian formula for this function is proved. A risk-free asset having a return variance of zero is said to exist. Formulae for an individual's expectation of return on an asset and the market weighted average expectation of excess return on a security are derived. Also, the probability distributions of underlying random variables embodying all public information are specified. A tractable CAPM framework is reached which can test empirical hypotheses and draw important insights on capital market equilibrium.

Manjunatha et al. (2006) tested for the applicability of Capital Asset Pricing Model in the Indian capital markets using a sample of the 30 companies constituting the Sensex. The data of daily closing price histories for 2000–2003 are extracted from the Centre for Monitoring Indian Economy (CMIE) database and Bombay Stock Exchange's website. Hypothesis testing for both the intercept and slope were conducted. Time series regression, cross section regression, t-tests, ANOVA (F-test) are run on daily prices data. Security returns were found to be dependent on market returns. The results indicated intercept to be significantly different from the risk-free rate of return and slope not equal to the difference between market returns and risk-free rate. Also, an inverse relationship was found to exist between portfolio returns and their betas. Thus, CAPM did not hold in the Indian context. This apparent contradiction may be due to the short time period or the use of Sensex as the market proxy.

Cheng et al. (2007) study the relationship between private transactions of executives with their firms and future returns on firm's equity. The dual motives of insiders are disentangled using a novel disclosure feature of certain trades. Form-5 trades for Standard and Poor's 500 firms for a period 1998–2001 are collected from Thomson Financial database in WRDS, analyst forecasts are collected from COMPUSTSAT and data on stock returns from Center for Research in Security Prices.

Twenty-four per cent of sample firm years contained non-zero insider sales. A total of 445 firms containing 1457 firm years were analysed. Three multivariate regression with and without various controls and fixed effects were modelled. They find that insider sales disclosed via Form-5 of large firms are predictive of negative future returns to the extent of 6–8 per cent and lower future annual earnings in comparison to the forecasts. Managers acquire private information on future profits at the time of sale of the form which investors infer only upon public filing. The Sarbanes-Oxley Act curtailing the use of Form-5 was thus justified in the research work.

Griffin et al. (2007) study the dynamic relationship between market wide trading activity and returns in 46 markets. The weekly data of market returns, traded value and total market capitalisation for ten and a half years from 1993 to 2003 collected from DataStream International is analysed. An EGARCH (1, 1) model was fit to measure volatility. Cross-sectional variation in relation is used to understand the determinants of return-volume relation. Impulse response functions (IRF) were estimated. A trivariate vector auto regression (VAR) of market return, market volatility and turnover are run on the weekly data. Also, information of behaviour of investor types, dynamics from previous times, asymmetries in reaction of turnover to past returns and daily data are further studied for a detailed analysis. Augmented Dickey-Fuller tests for stationarity of turnover series rejected the null hypothesis of presence of unit root. Both past weekly and daily returns exhibit a positive relation with turnover. The magnitude of return-turnover relation is large with one standard deviation (SD) shock in returns leading to 0.46 SD increase in turnover. The relationship is found to be more statistically and economically significant in countries having a high level of corruption, high volatility and short-selling restrictions. The study signifies the importance of the relationship between past returns and liquidity in the markets for an individual investor deciding his/her portfolio and paves the way for the development of a theoretical model on these lines.

Kumar et al. (2008) investigate the role of information on stock returns and cost of capital with the presence of estimation risk and learning from noisy signals of uncertain quality in the investment arena. They develop an information dependent conditional CAPM in the face of continually arriving information. The returns are multivariate normal in the model with the first and second moments of the joint distribution of returns and signals being information dependent and hence stochastic. The theoretical model is empirically tested using daily value weighted market index for the period from 1964 to 2005. Empirical results strongly support the predictions of the model. The innovations in market volatility, oil prices, exchange rates and dispersion of forecasts explain the cross section of stock returns and also influence stock's systematic estimation risk. Dividend and share repurchase lead to downward announcement effects on betas and their standard error as found by the event study. The research provides a different perspective to the existing results besides yielding new facts.

To understand the asset pricing dynamics induced by market clearing mechanism, Cochrane et al. (2008) solved a model with two Lucas trees with each tree's dividend income following a geometric Brownian motion. The investors were assumed to have log utility and consume the sum of two trees dividends. With ever changing values, investors seek to rebalance their portfolio by adjusting prices in the fixed-size tree. Due to this phenomenon, expected returns, excess returns and return volatility vary through time. They found that returns displaying serial correlation were predictable from price-dividend ratio, and volatility in returns was different from volatility in cash flows. Shocks in returns could occur without news about cash flows. Their theoretical work could lead to the construction of an asset quantity model rather than an asset pricing model.

Kabito (2009) tested CAPM for 50 randomly chosen UK listed companies for the period of 1980–2006. Strathclyde University database was accessed for financial and market data. To test for the validity first pass (time series) and second pass (cross-sectional) regressions were used. No relation between actual returns and CAPM returns was found to exist though unsystematic risk was found to effect returns. Tests for higher risk and higher returns and linear relationship of returns and beta contradicted CAPM. Tests for the influence of firm size and growth opportunities on excess returns were consistent with CAPM. Thus, the study results did raise doubts on the validity of the model. The reasons for this contradiction could be the unrealistic assumptions of the original model or the use of market proxies hinting towards further improvements in the model.

Wachter and Yogo (2010) developed a life cycle consumption and portfolio choice model for households having non-homothetic utility for basic and luxury goods. The degree of increasing share of risky assets in the portfolio of US households from 1989 to 2004 as reported by the Survey of Consumer Finances is explained. The survey reveals that the share of wealth invested in stocks rises in wealth. A model explaining cross-sectional variation between wealth and share of wealth in stocks is calibrated and solved using a labour income process (having low correlation with stock returns) and simulated for an ex-ante 10,000 identical households having idiosyncratic income shocks. The households try to maximise the expected discounted sum of future utility flow and accordingly choose between consumption and shares. The model tests for four predictions: (1) expenditure share falls for one good and rises for the other, (2) portfolio share rises in wealth, (3) portfolio share is relatively stable over the life cycle and (4) the portfolio share can fall in response to increase in wealth for a specific household. The model quantitatively explains all four predictions. The tools of consumer demand analysis, multivariate regression model and Engel curves are used. Highly educated households tend to be wealthier and tend to keep a higher proportion of wealth in shares. Basic consumption falls more than proportionally and luxury income rises more than proportionally with permanent income. The two offsetting effects of wealth on portfolio choice are predicted to be operating: (1) through cash-on-hand and (2) through permanent income. The model may have important implications for asset pricing with heterogeneity in risk aversion of the investors.

Taneja (2010) examines CAPM and the Fama and French model to explain the cross-sectional stock market returns in India. A sample of 187 Indian companies for a period of five years from June 2004 to June 2009 is studied. The sample selection is based on the continuous presence in Standard and Poor's (S&P) CNX 500 index for at least ten years. A comparison of the two models to test the validity of each one of them is also undertaken. The study creates six portfolios according to the size and value of the companies and three portfolios according to the price-to-book value (P/V) ratio. The standard multivariate regression technique of Fama and French (1993, 2000) has been used. Beta captures 89.1 per cent of systematic risk and all betas are statistically significant. Hence, CAPM cannot be defeated. Fama and French explain 92 per cent of total systematic risk with statistically significant betas. A positive relation between size and monthly returns is found, which rejects the claims of inverse relationship of size and returns by Fama and French (1995, 1996), Corner and Sehgal (2001), Ajili and Sakkout (2003), Bundoo (2006) and Bahl (2006). An inverse relationship is witnessed between average monthly returns and value of the firm. The Fama and French model was found to be a good estimator in the Indian context. However, the two factors (size and value) have a high degree of correlation among them and either of them can explain excess average monthly returns.

Srivastava (2010) tested the Indian stock markets for the existence of the least weak form of market efficiency in the changing market environment with increasing uncertainty, especially after the recent global financial crisis. The five major equity indices of NSE namely Nifty, Defty, Nifty Junior, CNX Midcap and CNX 500 are analysed for a period of January 1998 to December 2009. The random walk nature of stock markets is tested using run test, autocorrelation function ACF (k) and unit root test. The returns are found not to be generated by a normal distribution as is evident from the descriptive statistics (skewness and kurtosis) and Kolmogrov Smirnov goodness of fit test. The null hypothesis of no serial dependence in the return series of all the five market indices is rejected indicating the markets to be weak form inefficient during the sample period. The results of all the three tests employed which are in broad agreement conclusively reject the presence of random walks in daily returns. The findings are contrary to the available literature on market efficiency in India providing an avenue for future research on improvements in form of efficiency.

Mehta and Chander (2010) empirically tested the Fama and French three-factor model on Indian stock markets. The monthly observations of stock prices and firm specific parameters for 219 listed BSE 500 companies from February 1999 to December 2007 were collected from PROWESS database of Centre for Monitoring Indian Economy and analysed. The non-parametric Kruskal-Wallis H test and parametric t-test and one way ANOVA (F-test) have been applied. Tests for seasonality in size and value were conducted. The January effect and April effect reported no strong evidence of variability in the mean returns. However, November and December effects were prominent. The superiority of the Fama and French model, vis-a-vis its other variants is established in explaining the variability of returns of the six size and value sorted portfolios, suggesting its usage to investors and fund managers. The firm's characteristics, namely size and book-to-market ratios, had significant explanatory powers in addition to the CAPM's beta.

Saleh (2010) investigated the validity of value-glamour strategy for Amman Stock Exchange during a period of 1980–2000 using book-to-market equity and size. It also explores the effects of volatility of the stock on the returns of the portfolio. Monthly stock returns for all non-financial firms, three month T-bill rates, monthly returns in ASE index, book-to-equity, market capitalisation, trading volume and total assets of the firms are studied. The returns of value and glamour stocks are calculated as per the formula[4] used by Fama and French (1993). The paper constructed the high-stock-volatility-minus-low-stock-volatility

(HSVMLSV) factor and added it to the Fama and French three-factor model to create a four-factor model. Evidence in favour of value-glamour strategy could not be established for Amman Stock Exchange. However, its underperformance is found to be related to stock volatility which is consistent with the Fama and French prediction. Small and high book-to-market stocks outperform small and low book-to-market stocks as returns in the former are more volatile. Similarly, large and high book-to-market stocks underperform large and low book-to-market stocks as returns in the first are more volatile. Also, stock volatility explains the difference in returns of value and glamour stocks at least for emerging markets. The results assert that stock volatility should be modelled as a risk factor over the one year period by splitting the sample into different market conditions.

2.3 Literature on diversification of portfolio

Lintner (1965) empirically analysed and concluded that the value of the securities varies directly with intercept and the correlation coefficient and inversely with the residual variance (or "standard error of estimate") of their regression on external index of business conditions. The "income effect" and "risk effect" changes the slope coefficient and affect stock values in opposing direction. The securities, having returns independent of general business conditions or returns which are positively (but less than perfectly) correlated with the general market, should sell at a price low enough to provide an expected return higher than the pure interest rate prevailing in the market. The gains from diversification come from combining negatively correlated stocks and "averaging over" the independent components of the returns and risks of individual stocks. Strikingly, even the best possible diversification merely minimises the risk due to residual uncertainty at any given level of return. It was impossible to construct a portfolio that is efficient as per Markowitz's mean-variance efficient frontier. Hence, through diversification, one could only achieve favourable combination of risk and expected return. Even prudent selection and broad diversification by the investor could not substantially reduce the risks associated with given expected returns.

Diversification may give superior portfolios from the reduction of variance and hence increasing the geometric mean return, but this does not appeal to risk averse investors to justify diversification (Evans and Archer, 1968). Latane and Tuttle (1966) found that a proper amount of borrowing and diversification pays in the long term. However, their concept of pure risk yields and risk, which are not eliminated by diversification, will prove to be useful tools for intraday short sellers. Perhaps, there are doubts concerning the justification of increasing portfolio sizes beyond ten securities or so; this indicates the need for analysts and private investors to include some form of marginal analysis in their portfolio selection models (Latane and Young, 1969).

Diversification as a strategy was found to be less paying in mutual funds then it would in case an individual investor maintains his/her own strategy. The conclusion of Friend and Vickers (1965) shows no evidence of mutual fund performance being any better than that realisable by random or mechanical selection of stock issues. There are limitations on diversification. Even fewer, between six and 11, gave good average return when compared to various types of common stock funds (Gaumnitz, 1967). Even 20 or less than that can give good returns with less risk. Benartzi et al. (1997) found that investors are better off with a lesser number of choices of portfolios to choose from. Also, a large numbers of investors will prefer taking the middle path that is if there are three funds, for example the first with 30 per cent equity, the second with 50 per cent equity and the third with 70 per cent equity. The majority of the investors may choose the option of fund with 50 per cent equity. Hennessy and Lapan (2003) used group and majorisation theory to study portfolio allocation. Preferences over allocations are found to be partially ordered by majorised convex hulls generated by a permutation group. Group transitivity ensures complete portfolio diversification.

Ivkovik et al. (2008) study the phenomenon of excess returns of individuals with concentrated portfolios in comparison of investors with a diversified portfolio. They test for informational advantage being the source of such increased returns by the concentrated investor. All investments made by 78,000 households are analysed through monthly position statements for 71 months from January 1991 to November 1996. Common stocks constitute two-thirds of the total value of investments and hence are focused. Risk-adjusted returns are calculated using the four-factor model.[5] A key regression specification relating excess return to indicator variable, the Herfindahl index, interaction between the two, industry and momentum controls is run. The study finds that purchases made by diversified households underperform the Fama and French (1992) benchmark by 1 to 2 per cent, whereas those made by concentrated households with large portfolios outperformed the benchmark by 2.2 per cent. The excess returns are stronger for local stocks (those which are not included in S&P index) reflecting concentrated investors exploitation of informational asymmetries. Sharpe ratios of concentrated portfolios are lower indicating higher risk but information ratios are higher suggesting better risk return trade-offs and superior returns when combined with rest of portfolio. Robustness tests indicate the concentration results are not driven by industry specialisation, inside information, repeated trades, market timing or regional differences among investors. Indian mutual fund industry also experiences a similar phenomenon with highly diversified units not being able to perform well.

Calvet et al. (2009) investigate the dynamics of an investor's portfolio choice through the data set of wealth of 4.8 million households in Sweden between 1999 and 2002. Decomposition of risky portion of portfolio into passive and active rebalancing, rebalancing regressions by ordinary least squares to capture the dependence of active change on initial risky share and robustness checks are undertaken. An adjustment model with different target risky shares across households is developed and how households characteristics effect the decision to trade individual assets is examined. Although little aggregate rebalancing is witnessed during the period of study, strong household level evidence of active rebalancing could be found. Wealthy, educated investors rebalanced more actively and towards a more risky asset as they become richer. Households exit direct stockholding if stocks perform well. This further justifies the disposition effect of Leal et al. (2010). However, this relationship is weaker for mutual funds. Households rebalance primarily by purchasing risky assets if a risky portfolio performed poorly and adjust fund purchases and sell stocks if a portfolio performed well. This tendency was not very strong for wealthy investors with a diversified portfolio.

Jeyachitra et al. (2010) analyse the portfolio performance and measure impact of diversification on risk of Nifty stocks in the continuously changing environment. Data relating to daily, weekly and monthly closing prices of NSE Nifty listed companies for the period from April 2004 to 2009 were collected from PROWESS database. Eight portfolios of five stocks each with equal weights of each of the 40 actively traded stocks at NSE were constructed and compared with Standard and Poor's CNX Nifty Index. A high positive and linear correlation between portfolio return and risk was found with unsystematic risk declining due to diversification in the Indian Stock Market. The study is a simplified work of correlating risk and return without adding non-market factors such as inflation, interest rate, purchasing power or employing techniques such as generalised auto regressive conditional heteroscedasticity (GARCH) and auto regressive conditional heteroscedasticity (ARCH) to model the variability.

2.4 Literature on portfolio optimisation and variance-covariance matrix

Winkler and Barry (1975) presented a general model for portfolio selection. Bayes theorem is used to revise the distributions of returns and closing prices as new information in the form of observed prices becomes available. A single period model of a myopic investor maximising wealth of only the next time period and a multi-period model of an investor maximising total wealth over a finite time period are considered. Goal programming is used in the former case, while the latter is solved using dynamic programming with backward induction. The effects of transaction costs on purchase and sale of securities are also incorporated. The models are supported by examples of linear utility, more practical quadratic utility and other simplified numerical examples. Information is gathered by investors from hard data and soft data. The learning effect of the process of generation of security prices and of other important variables is studied. Extensions of the model including tax effects, short sales, borrowing and lending, as well as extraneous factors are suggested by the researchers to make it more realistic.

Bawa (1976) tried to find out the admissible set of portfolios for all individuals with utility functions monotonically increasing in wealth, inclusive of all risk averse, risk neutral and risk preferring investors as well as individuals with Friedman-Savage type utility functions and more complex increasing utility functions with several concave and convex segments. The admissible set may be used for portfolio selection by risk preferring investors or by a group of investors who are neither risk lovers nor averters. The admissible set consists of Markowitz-Tobin mean-variance efficient set plus a set of non-diversified portfolios and a set containing portfolios of at most two securities. This admissible set may be obtained for all investors using Bawa's algorithm and the Markowitz-Sharpe critical line algorithm. The first order stochastic dominance rule is used to determine the admissible set of portfolios with no restriction on the probability distribution of returns. Portfolios using Roy's safety first rule are obtained formally using optimising techniques with prescribed critical (disaster) level. The paper helps reduce several portfolio selection problems to simple parametric quadratic programming problems which can be efficiently solved using the algorithm created in conjunction with the Markowitz-Sharpe algorithm. The results can help in developing an equilibrium model of financial markets in which all kinds of investors are considered.

Richardson (1989) solved for a self-financing portfolio trading policy which achieves a specified level of terminal wealth with minimum risk. A continuous trading framework with a choice of a riskless bond and a stock whose price is governed by a geometric Brownian motion is considered. Markets are assumed to be frictionless (with no transaction costs) permitting unlimited borrowing at a risk-free rate. Hilbert space theory, Martingale theory and stochastic calculus are used to find an explicit representation of optimal policy. In discounted form, the investor maintains funds in the stock so that the amount invested is proportional to the difference between a certain constant and the accumulated portfolio gain. The optimal trading policy suggests a highly leveraged investment in stocks in the early stages and an accumulation of bonds in the later stages. The problem will become more complex when borrowing is not permitted and would require a different solution.

Taksar et al. (1988) presented a diffusion model with two assets for optimal portfolio selection. The rate of growth of one asset is deterministic (r) while the other grows according to Brownian motion (μ, σ^2). The shift in money from risky asset to the non-risky one involves brokerage fees. The objective of the research is to introduce a continuous time model to maximise the rate of growth of funds. The ratio of money in two assets is governed by a diffusion process with state dependent coefficients. Using rigorous mathematical procedures, the problem was formulated, optimality equation derived, the control limit policies constructed, the sufficiency of the optimal equation proved and the solution was obtained. The problem was formulated as minimisation of expected average cost of the integral function. This function (X) was in terms of stock to bank ratio process. Further, modifying the control functional X (t) is described as the cost of transferring funds from stock to bank. There exist two constants $A < B$ such that as soon as the stock to bank money ratio drops below A (or exceeds B), funds must be transferred (minimal amount) from bank to stock (or from stock to bank) restoring the ratio back to A (to B). It is deduced that optimal boundaries depend only on the quantity $(r-\mu)/\sigma^2$. It is found that an optimal policy keeps the ratio of funds in risky and non-risky assets within a certain interval with minimal effort. The model does not allow for the borrowing of cash or the short-selling of stocks.

Young (1998) introduced a new principle for choosing portfolios based on historical returns data. His optimal portfolio is the solution to a simple linear programming problem. Minimum return and not variance measures risk. The mini-max principle is used. Portfolios which minimise maximum loss over the past observation periods for a given level of return are chosen. Framing the portfolio selection process as a linear optimisation problem makes it feasible to constrain certain decision variables to be integer, or 0–1, valued. This facilitates the use of more complex decision making models. Under weak conditions, the mini-max principle corresponds approximately to an expected utility maximising principle, with the implied utility function representing an extreme form of risk aversion. If an investor's utility function is more diversified than is implied by a mean-variance analysis, or if returns data are skewed, or if the portfolio optimisation problem involves a large number of decision variables, including integer valued variables, the mini-max rule provides a sensible approach to portfolio selection. The chosen portfolios are well diversified and are nearly equivalent to the ones chosen by mean-variance rule.

Polson and Tew (2000) used a dynamic asset–allocation approach based on re-estimating and then rebalancing the portfolio weights on a prescribed time window. Predictive distribution of expected returns and predictive variance-covariance matrix are the main inputs to optimisation process followed. The optimum portfolio constructed using Hierarchical Modelling outperforms the underlying benchmark of Standard and Poor's index. Campbell et al. (2002) drive unbiased quartile correlation estimates capturing the increasing correlation in extreme market conditions. It provides a pragmatic approach to understanding correlation structure in multivariate return distributions. Empirical evidence of significant increased correlation in international equity returns in bear markets is found. The study highlights the importance of providing a tail-adjusted mean-variance covariance matrix.

Optimising a consumption-investment game with many securities, many time periods, transaction costs and changing probability distribution of asset returns remain the objective of financial analysts all over the world. Markowitz and Dijk (2003) consider the cost of following one heuristic over the other which can be scaled to handle such large games. A simple dynamic investment model is defined with mean-variance (MV) surrogate heuristic. The MV surrogate heuristics may be scaled to larger problems for which optimum solutions cannot be computed. An investment game with two assets (stock and cash), 11 portfolio states (0 per cent stock,10 per cent stock, 20 per cent stock... 100 per cent stock), transaction costs of changing the portfolio state and forecasting stock returns that can be in one of five prediction states: very optimistic, optimistic, neutral, pessimistic and very pessimistic are considered. The system has 55 states. The optimum strategy is an 11x5 action matrix A that specifies choice of next portfolio as a function of the current portfolio and prediction state. The MV heuristic is found to do as well as the optimum strategy in all the games considered. Dynamic programming has been used to select mean and variance and to evaluate the discounted expected utility of the mean-variance surrogate versus other heuristics. The assumption of an investor having detailed simulation model with which he/she can select his/her expected return and variance in the MV surrogate function is questionable. The model could be extended to include other parameter settings to represent investment situations with large numbers of securities and other forecast models, such as auto regressive conditional heteroscedasticity-type models.

Chacko and Viceria (2005) examined the optimal consumption and portfolio choice problem of long horizon investors having access to risk-free asset with constant returns and risky asset (stocks) with constant expected return and time varying precision (the reciprocal of volatility). An exact solution for investors with unit elasticity of inter-temporal substitution of consumption is worked out. A negative inter-temporal hedging component with larger than one coefficient of risk aversion and negative correlation between volatility and returns are estimated.

A simulation based approach for discrete-time portfolio choice problems is presented by Brandt et al. (2005).The situations involving non-standard preferences, a large number of assets with arbitrary returns and large number of state variables with potentially non-stationary dynamics are considered. The issues of intermediate consumption, portfolio constraints, model uncertainty and learning are accommodated in the method. A large number of sample paths of asset returns and state variables are simulated from the known and bootstrapped joint dynamics of returns and state variables. The problem is then solved for optimal portfolio policies recursively in a standard dynamic programming fashion. For each simulated path portfolio weights are computed such that they maximise Taylor series expansion of investors expected utility. The method consists of simulating the asset returns and state variables, computing a set of across path regressions for each period and then evaluating the closed form solution of the approximate portfolio problem. It is found that the investor chooses the portfolio anticipating that future data realisations will contain useful information to learn about true parameter values. The work provides a solution to a reasonably realistic discrete-time portfolio choice problem with learning about all parameters of the return generating process. The empirical results show that parameter uncertainty and effect of learning reduce an investor's allocation to stocks. Also, learning about the parameters of data generating process creates a negative hedging demand for stocks.

Liu (2007) solved for dynamic portfolio choice problems up to the solution of an ordinary differential equation (ODE) with quadratic asset returns and constant risk relative aversion (CRRA) coefficient of the agent. The utility function is defined over intermediate consumption and terminal wealth with complete financial markets. Several properties of dynamic portfolio weights come to light different from static portfolio weights. Three applications of the model (1) bond portfolio selection, (2) stock portfolio selection and (3) bond and stock portfolio selection problem were presented. Bond returns and stock return volatility display stochastic volatility. The dynamic portfolio weight of a risky asset did not always decrease with risk aversion, even when risk premium was positive. The negative portfolio weights existed due to dynamic portfolio choice. The ratio of bond to stock portfolios is found to be increasing with risk aversion. The study provides a new dimension to the problem of portfolio selection by considering the stochastic environments in which the decision is needed to be taken.

Buraschi et al. (2008) developed a framework for multivariate intertemporal portfolio choice model suitable for economies having stochastic degree of correlation across industries, countries or asset classes. The model studies volatility and covariance hedging in various settings. It is found that multivariate nature of second movements has important consequences for optimal asset allocation.

Singh et al. (2008) undertook an empirical analysis of the Markowitz portfolio selection model using equities, commodities and bonds for the Indian Securities Market. Markowitz's model was used for the purpose of analysing the daily returns of 2005. The analysis was performed for equities part of CNX Nifty Index and Nifty Midcap (100); commodities part of Comdex and randomly collected bonds from NSE website. Nine alternate portfolios were created based on existing portfolio creation rules. It was found that by decreasing the number of equities and increasing the number of commodities negatively affected returns and positively affected variance. Even low beta equities, when arranged in descending order of return, perform efficiently according to mean-variance criteria. High-risk return combinations were achieved by an all equity portfolio. An all commodity portfolio has very low risk return and hence is not recommended in the Indian Securities Market. Inclusion of a bond in an equity and commodity portfolio positively affected return without an increase in the risk. The lowest coefficient of variation was found for a portfolio having a combination of equity, commodities and bonds.

Dumas et al. (2009) identify the trading strategies that allow investors to take advantage of excessive stock price volatility and sentiment fluctuations. A difference of opinion general equilibrium model is constructed. Overconfident investors overreact to a public signal and add volatility while rational investors choose a conservative portfolio. Technical virtues of continuous time, rational expectations equilibrium asset pricing models are combined to analyse consequences of a specific deviation. Angerer and Lam (2009) investigate the relationship between portfolio choice and labour income risk. Permanent income risk reduces the share of risky assets in household's portfolio choices, while transitory income risk does not. Thus, a household's portfolio choices respond to labour income risks in a manner consistent with economic theory.

Singh and Agarwal (2009) presented a conceptual framework for effective investment management. From an extensive review of literature, they found that investment choices made by individuals are heterogeneous in nature. The portfolio choices made by individuals are a function of gender, age, number of dependents, marital status, income, habit, wealth, years to retirement and many other such factors. Mean-variance models, capital market models and existing asset pricing models are incomplete if they do not incorporate the effect of individualistic choice in portfolio selection decisions. Researchers in India were found to have focused on empirical validity of international models and their adaptability for the Indian Securities Market particularly Sharpe's CAPM and Fama and French's three-factor model. The results of a large number of Indian research studies were found to be contradictory as most of them were based on sample data over short periods. An attempt has been made in this research work to overcome this shortcoming. The plethora of issues related to an investor's behaviour, mean-variance efficiency, diversification and strategies for creating optimal portfolio provide an interesting insight into optimal portfolio creation.

Kryzanowski and Singh (2010) found that the minimum portfolio size of an investor depends upon the investment opportunity set, metrics used to measure the benefits of diversification and the criteria chosen to determine when the portfolio is well diversified. The portfolio size issue is revisited for Canadian stocks listed on Toronto Stock Exchange for 1975–2003 with recently developed (un)conditional metrics of investment opportunity (IO) sets (un)differentiated by cross listing status and market capitalisation. Five thousand equal weighted portfolios with different sizes are created using Monte Carlo approach for all stocks in the six IO sets. The average correlation coefficient is used to measure the rate and measure of maximum risk reduction and average covariance depicts minimum risk level. Mean derived dispersion (MDD) metrics, mean realised dispersion (MRD) metrics and normalised portfolio variance (NV) are differentiated by portfolio size and IO set to determine the minimum portfolio size required to achieve a sufficiently well diversified portfolio. The required number of stocks is 70 for cross listed firms, 60 for IT firms and over 100 for other IO sets. These numbers reduce as the effect of preferred positive skewness and negative kurtosis by investors is taken into account. Also, the investor's concern about impact of diversification on reward for bearing risk alters the number of stocks in the efficient portfolio ranging between 40 to100. Mutual funds and pension funds however did not find a place in any of the IO sets.

2.5 Literature on the impact of behavioural and systemic factors on an investor's portfolio choice

Schneller (1975) tried to investigate the impact of lengthening revision horizon on the optimal portfolio of a mean-variance investor. The emphasis was on how the long-term investor will construct his portfolio in the mean-variance framework. The returns on risky and riskless assets were assumed to be stationary and inter-temporarily independent. A sequence of definitions and corollaries was developed. It was found that long-run portfolio shifts towards the asset with the highest Ω index. When more than one asset is included in the long-run portfolio, these assets will be represented by a weight proportional to the multiplicity of their Ω index. In case the returns on assets are uncorrelated, the optimal portfolio is constructed from the assets with the highest ratio between their first and second moments around zero in equal weights. As the revision horizon lengthens, the relative sizes of the expected returns and the variances and covariances which represent the risk characteristic undergo a power transformation. The point of tangency of the market line and the efficient set is not invariant to the length of the revision horizon. This non-linearity of the time transformation explains the fact that the optimal portfolio varies with the revision period length.

The effect of quality of corporate governance on investors' portfolio selection decision is studied by Giannetti and Simonov (2006). Comprehensive data on 621,764 stockholders on the Swedish stock market are investigated. Data on corporate return and risk characteristics from SIX Trust, which provides information on the closing prices and dividend yields of the companies listed on the Stockholm Stock Exchange, and accounting variables from Market Manager are analysed for evaluating investors connections with insiders. The proxies for corporate governance include ratio of control to cash flow rights,[6] control premium[7] and dummy variable proxying for the level of control entrenchment.[8] Investor i's choice is modelled by using a binary variable, Y_i, f, which equals 1 if investor i holds shares in firm f, and 0 otherwise. The probability that investor i holds shares in firm f, that is Pr (Y_i, f = 1), is estimated using a probit model. The findings suggest a distinction between investors with only security benefits from those with additional private benefits. The former are reluctant to invest in weak corporate governed companies while those connected with insiders are more likely to invest in such companies. The fear of expropriation may have a greater impact on markets with lower investor protection and less effective law enforcement compared to the relatively strong Swedish markets.

Polkovnichenko's (2007) study of the effect of additive and endogenous habits on portfolio selection by an investor with stochastic uninsurable labour income, his/her wealth accumulation and dynamics with age. The habit wealth feasibility constraints are analytically derived and found to be dependent upon the worst possible path of future income and on habit strength, but not on the worst possible income in future. The model is set in partial equilibrium with an investor receiving income subject to transitory and permanent income shocks during his/her working life and constant for retirement periods. The model suggests conservative portfolios when there is a slim chance of severe income shocks. A portfolio's share of stocks increases with wealth for low and moderate wealthy households. Younger households have more conservative portfolios than their middle-aged counterparts. The findings are robust to income smoothing via borrowing of flexible labour supply. The life cycle model predicts counterfactually high wealth accumulation for high values of habit strength parameter. Habits die hard and have an impact on the portfolio selection by an investor. They must be considered for modelling techniques for choice of portfolios.

Bekaert et al. (2007) studied the impact of local market liquidity on the expected returns in 19 emerging markets. The data on monthly returns, excess returns and dividend yield for global equity market indices is collected from Standard and Poor's Emerging Markets Database (EMDB). Bivariate VAR (1), Wald tests, Asymptotic tests and Monte Carlo simulation are carried out on the data. Liquidity and market portfolios are identified as risk factors with transaction costs proportional to liquidity in a simple asset pricing model considered to interpret the liquidity effects. The study suggests that liberalisation of economies have not been able to fully eliminate the impact of transformation of zero daily firm returns averaged over a month. Liquidity significantly predicts future returns unlike turnover. Elevated political risk, poor law and order conditions served as effective segment indicators and liquidity explained returns more in countries where these indicators were operative. Unexpected liquidity shocks are found to be positively related with contemporaneous return shocks and negatively related to the shocks to dividend yield. A clear and global picture of the impact of liquidity on returns can be formed by including the more developed markets with richer data availability.

Hvidkjaer (2008) studied the relationship between retail investors trading behaviour and the cross section of future stock returns using the volume arising from small trades. Retail investors' behaviour is constructed from transactions of low dollar value in US stocks listed on NYSE and ASE for a period from 1983 to 2005 with ISSM and TAQ data sets. The signed small-trade turnover (SSTT) is calculated and portfolios are formed on its basis. Time series regressions similar to the Fama and French three-factor model are performed to analyse systematic risk and provide alternative risk adjustment of returns. The stocks with sell-initiated small-trade volume outperform stocks with buy-initiated small-trade volume. This return difference continues to be large and statistically significant for up to two years of portfolio formation and thereafter stocks favoured by retail investors experience prolonged underperformance in comparison to the stocks not favoured by them. This may be due to retail investors not acting in accordance with the fundamental values or because of them facilitating mispricing by buying the overvalued stocks sold by informed investors. The study very well links the systematic component of the retail investor's behaviour with future returns.

Ansari and Jana (2009) through an extensive survey proved that the real investment arena comprises of not only rational investors but also noise traders. Although the Efficient Market Hypothesis does not acknowledge the effect of these noise trader's decision on the stock prices, they do affect the sentiment of the Indian stock markets. Their portfolio decision is guided by self-attribution bias, loss aversion bias, confirmation bias and overconfidence bias in addition to the normal mean-variance characteristics. The research is a primary study using questionnaire technique to collect data through convenience sampling procedures and applying simple statistical tools to check the effect of biases on the portfolio decision. It shows that traders use both fundamental analysis and technical analysis for stock selection, which is contrary to the view of finance theorists. The deviation of Indian capital markets from Efficient Market Hypothesis (EMH) may not only be an anomaly but probably could be process generated.

Indian investors to date prefer investments with low risk, high safety and liquidity. Through a survey of 117 respondents, Jasmeen (2009) found that the majority of investors choose low risk investments. It shows the effectiveness of investor awareness programmes and lack of confidence among Indian investors. Questionnaire techniques of data collection and Chi-square tests using the statistical package for social science has been used. Women were found to be more risk seeking than men. Graduates take the highest degree of risk. The trend of low risk, high risk and moderate risk is seen in all categories. Also, the association between the profile (age, gender, qualification, income and profession) of investors and risk capacity is found to be insignificant.

Sahoo and Rajib (2010) evaluated the price performance of initial public offers and examined their usefulness at the time of issue to explain the post issue price performance. Ninety-two Indian IPOs issued during the period 2002–2006 have been studied. Market adjusted abnormal return (MAAR) for the listing day is calculated to depict the degree of under-pricing. The long-run performance of IPOs up to 36 months are measured using wealth relative (WR) and buy-and-hold abnormal rate of return (BHAR) techniques adjusted with market index Nifty. Multivariate regression (OLS) is used for testing influence of explanatory variables. Indian IPOs were found to be 46.55 per cent under-priced on the listing day compared to the market index. This under performance is most pronounced during the initial year of trading followed by over performance. Factors such as the under-pricing rate, offer size, leverage, uncertainty and the timing of issues are statistically significant in influencing underperformance. Investors directly subscribing to IPOs earn a positive market adjusted return throughout the period of study. On the contrary, investors buying IPOs on the listing day earned a negative return until one year. The study provides guidance to IPO investors and provides a new dimension to research on IPO value and its effect on post-listing prices.

Leal et al. (2010) investigate the disposition effect[9] on the Portuguese stock market through the trading records of 1496 individual investors studied for trades, volume and value traded from 1999 to 2002. Only the investors who traded exclusively in the Portuguese market and had no trade in derivatives were studied. Realised gains, realised losses, potential gains, potential losses, proportion of gains realised (PGR) and proportion of losses realised (PLR) are calculated. Hypothesis testing is done to test for statistical significance of differences in proportion of PGR and PLR using t-test. Strong evidences of disposition effect were found every month of the year globally for all individual investors. The PGR is 20 per cent higher than PLR. Even the fiscal effect had no significant impact on investor preference. The data period, which was divided into bull and bear periods, felt more prominent intensity of disposition effect in the bull period. Also, investors in high percentile as per trading frequency, volume of transaction and portfolio value are found to be less prone to disposition effect. The results of this study seem similar to the conclusions of Prospect Theory in behavioural finance whereby investors codify their wealth changes in terms of gains and losses using a reference point.

Dhatt (2010) examined the effect of buyback announcements on the Indian stock prices. The signalling hypothesis motivated by asymmetric information between a firm's manager and market place remains to be the main reason behind buybacks to correct the valuation of shares. The sample consisted of all share buybacks executed between March 2004 and April 2009. The data were obtained from SEBI's Status Report on Buybacks in India, websites of BSE, NSE and the Centre for Monitoring Indian Economy's database PROWESS. Forty buyback offers consisting of 33 open market operations and seven tender offers were studied. Event study methodology with market model is applied to estimate normal return for a security. The abnormal returns and cumulative average abnormal returns (CAR) are computed depicting the difference of ex-post return over the event window and normal return. A positive impact of the announcements was found and incorporated in the share prices on the announcement day itself. Tender offers generated a higher announcement return than open market offers. The announcement day return was found to be 2.549 statistically significant at one per cent level. Also, the pattern of underperformance was greater and significant preceding open market offers. Thus as observed, the company's announcement signalling undervaluation is proved beneficial for it in correcting its value.

Sudhakar and Kumar (2010) captured the perceptions of Indian investors towards mutual funds and particularly the Unit Trust of India (UTI). Investment objectives, age, occupation, income, risk tolerance levels, expected returns, choice pattern and preference pattern of schemes affect the decision of investors to invest in mutual fund schemes. A survey study of 500 UTI investors in Hyderabad was conducted. Strata sampling, questionnaire technique, direct personal interviews, Chi-square tests of goodness of fit and independence of attributes were used for the study. Majority investors choose a portfolio of mutual funds which gives regular income, tax benefits and capital appreciation. Fifty-two per cent of investors were found to be willing to take moderate risk, while 57 per cent accorded preference to growth schemes. UTI is still considered to be a safe and secure investment by the investors although concerns over returns, diversification and out performance of the market were felt.

Agarwalla and Pandey (2010) study the information and liquidity effect of Block Trades at NSE, India. A large data set of 500 companies was studied at one minute time intervals over a time horizon of 108 months from 1999 through 2007. Based on a multiple criteria formula using trade value and trade volume the block trades are identified. Block trades are classified as all-or-none (AON) and not-AON. Transaction time event approach is used to study the speed of market response to information provided by block trades. Mixed signalling effects were observed. Block purchases are found to be more informative with a higher permanent price impact than block sales. The temporary/liquidity impact is more than permanent impact for block purchases. This observation is in contradiction with other developing and developed markets. Price impact was found to be higher for not-AON and arrival of multiple block trades increased market's confidence on information. Prices were observed to increase eight minutes before block purchases. In the case of block sales, prices revert quickly and a very small permanent price impact is felt. Information on block trades not considered so far in any portfolio selection model does have an impact on an investor/professional trader in choosing his/her portfolio.

Chiang et al. (2011) tested the Taiwanese stock markets for the presence of bubbles during a sample period of eight years (2001–2008). The study used secondary data of 28 construction companies listed on the Taiwanese stock exchange. The unit root test is applied on the asset price and fundamental factor under four scenarios. Investor sentiment indicator (BSI) is created to study how investor sentiment correlates with bubble indicator and hypothesis of co-integration tested. The study found that although the business cycle of the housing market is the main determinant of stock performance in construction, there are deviations from the fundamental value time and again. The bubbles are found to be highly related to the behaviour of foreign investors. The herding behaviour of influenced retail investors further pushes up the stock price and creates bubbles. The study suggests picking up stocks that have fewer foreign and retail investors to avoid buying at the peak of bubbles.

Agarwal (2011), from their empirical survey of 326 retail investors, found that investors in India prefer to invest for long term. A large number of investors prefer to invest based on advice received from investment advisers. Brokers' advice resulted in gains for those following them. Return was given more importance than risk by retail investors. Savings was the main source of funding the investment in equities and most of the investors relied on fundamental analysis before making the investment decision. Using conditional probability, the effect marital status, gender, occupation, age on source of investment advice was analysed. The conditional effect of retirement, family responsibility and type of investor type on the possibility of investment in next 12 months was also analysed. With excess of disposable income, an investor prefers to invest in equities and mutual funds. Most of the investors preferred to keep their savings in the form of fixed deposits or self-constructed portfolios. The paper provided deep insights into the psychological factors affecting the dynamics of portfolio selection.

2.6 Literature on the lead-lag relationship between the stock and futures market

For the temporal relationship[10] among the spot and futures (options) market, several studies attempted to examine the lead-lag relationship between the spot and the futures market both in terms of return and/or volatility. Most of the studies have suggested that a leading role of the futures market varies from five to 40 minutes, while the spot market rarely leads the futures market beyond one minute. While explaining the causes behind such a relation, Kawaller et al. (1987) attributed the stronger leading role of the futures market to the infrequent trading of component stocks. Although, Stoll & Whaley (1990) and Chan (1992) proved the existence of such relation even in case of highly traded stocks or after adjusting for infrequent trading of component stocks.

Koch et al. (1987) estimated the lead-lag relation between S&P 500 index futures and S&P 500 index. The lead-lag effects were found using simultaneous equation model estimated by three stage least squares regression. Based on the minute-to-minute changes in both the index and the futures prices, a model was constructed to describe the dynamic intraday price relationship between the index and futures prices.

Harris (1989) observed increased volatility after the introduction of index futures by comparing daily return volatilities during the pre-futures

(1975–1982) and post futures (1982–1987) between S&P 500 and a non-S&P 500 group of stocks controlling for differences in firm attributes (beta, price-level, size and trading frequency). He pointed out other index-related instruments and developments such as growth in index funds and increase in foreign ownership of equity as possible explanations of higher volatility in stock markets.

Stoll and Whaley (1990) used the ARIMA model and ordinary least squares to estimate the lead-lag between S&P 500 index futures, Major Market Index futures and the underlying spot market. The results indicated that S&P 500 and Major Market Index futures lead the cash market by ten minutes and they attribute this to faster dissemination of information into futures market. The findings were consistent with the evidence gathered by Koch et al. (1987) and MacKinlay and Ramasamy (1988).

Hodgson (1991) studied the impact of All Ordinaries Share Index (AOI) futures on the Associated Australian Stock Exchanges over the All Ordinaries Share Index. Standard deviation of daily and weekly returns is estimated to measure the change in volatilities of the underlying index. The results indicated that the introduction of futures and options trading did not affect the long-term volatility, which reinforced the findings of the previous US studies

Chan (1992) estimated the lead-lag relation between Major Market Index and Major Market Index futures under conditions of good and bad news, different trading intensities and under varying market wide movements. ARMA models, as proposed by Stoll (1990), were used. It was found that the futures market leads the spot again attributed to faster information processing by the futures market. However, under bad news it is the cash index that leads over the futures market while there is no effect on the lead-lag relation during different trading intensities.

Kamara et al. (1992), observed the stability of S&P 500 index returns with the introduction of S&P 500 index futures. They assessed the change in the volatility of the S&P 500 index due to the introduction of futures trading for the period 1976 to 1987. The changes in the volatilities were examined using parametric and non-parametric tests. Apart from F-tests, Kolmogorov-Smirnov two-sample test and Wilcoxon Rank sum test are used to find out if the dispersion is significantly high in the post futures period. The results showed that the daily returns volatility was higher in the post futures period while the monthly returns remain unchanged. They concluded that increase in volatility of daily returns in the post futures period is not necessarily related to the inception of futures trading.

Jegadeesh and Subrahmanyam (1993) compared the spread in NYSE before and after the introduction of futures on the S&P 500 index as volatility can also be measured in terms of individual stock bid ask spread. They found that average spread has increased subsequent to the introduction of futures trading. When they repeated their test by controlling for factors like price, return variance and volume of trade, they still found higher spreads during the post futures period. The study suggested that the introduction of index futures did not reduce spreads in the spot market and there is weak evidence that spreads might have increased in the post futures period.

Abhyankar (1995) investigated the lead-lag relationship between hourly returns in the FTSE 100 stock index futures and the underlying cash index using hourly data for the period 1986–1990. The lead-lag relation for periods of differential transactional costs, good and bad news (measured by the size of returns), spot volume and spot volatility were tested. The results revealed that when transaction costs for the underlying asset fell (post "big bang"), the futures lead of the spot index reduced, implying that transaction cost differential is the major driver for the lead-lag relationship. It was found that the futures lead over spot was insensitive to variations in spot transaction volume. An AR (2) – EGARCH (1,1) model was then fitted to spot and futures returns to give a time series of estimated volatilities and it was observed that during periods of high volatility, futures markets led spot market returns. Support was also found for the hypotheses that lower transactions and entry costs in the stock index futures market is one of the reasons why traders with market wide information prefer to use the futures markets. This causes the arbitrageurs to step in quickly to bring the cost-of-carry relationship into alignment.

Darrat and Rahman (1995) examined whether futures trading activity has caused stock price volatility. The study is conducted on S&P 500 index futures for a period of 1982–1991. The study also examined the influence of macro-economic variables such as inflation, term structure rates on the volatility of the S&P 500 stock returns. Granger causality tests are applied to assess the impact on stock price volatility due to futures trading and other relevant macro-economic variables. The results indicated futures trading did not cause any jump volatility (occasional and sudden extreme changes in stock prices). Term structure rates and OTC index have caused the stock price volatility while, inflation and risk premium have not influenced the volatility of stock prices.

Gregory and Michael (1996) examined how volatility of S&P 500 index futures affected the S&P 500 index volatility. The study also found the effect of good and bad news on the spot market volatility. The change in the correlation between the index and futures before and after the October 1987 crash is also examined. Volatility is estimated by the EGARCH model. It was shown that the bad news increased the volatility more than the good news and the degree of asymmetry is much higher for the futures market. The correlation between the S&P 500 index futures and S&P 500 index declined during the October 1987 crash.

Abhyankar (1998) revisited the relationship using five minute returns by regressing spot returns on lagged spot and futures returns, and futures returns on lagged spot and futures returns using EGARCH. It was found that the futures returns led the spot returns by 15 to 20 minutes. Chatrath and Song (1998) examined the intraday behaviour of the spot and futures market following the release of information and also investigate the role of such information in the volatility spill over among the two markets. Their results supported one market leading to greater volatility in the other is being partly driven by information. The leading role played by the futures market was attributed to the new information efficiently reflected in the futures market.

Frino and West (1999) found the cost of trading SIMEX Nikkei 225 futures to be significantly lower than trading a similar nominal exposure using OSE Nikkei 225 futures. The cost differences are attributable to lower margin requirements, minimum tick size and bid ask spreads on SIMEX as well as the existence of negotiated brokerage commission versus the fixed rate regime operating in Japan. While SIMEX innovations are found to strongly cause index innovations, no such relationship is documented for OSE innovations. The difference was attributed to higher cost of trading on the OSE discouraging informed traders.

Frino et al. (2000) examined the temporal relationship among the spot and the futures market around the release of different types of information. They found that the lead of the futures market strengthens significantly around the release of macro-economic information, while the leading role of the futures market weakens around stock specific information release. Therefore, according to them, the disintegration in the relationship between the two markets is mainly driven by noise associated with trading activity around the release of different types of information.

Chris et al. (2001) estimated the lead-lag relation between the FTSE 100 stock index futures and the FTSE 100 index. Based on the results obtained, they developed a trading strategy based on the predictive abilities of the futures market. The study used the co-integration and error correction model, ARMA model and vector auto regressive model. The results indicated that futures lead the spot market attributable to faster flow of information into futures market mainly due to lower transaction costs. It was shown that the error co-integration model predicted the correct direction of the spot returns 68.75 per cent of the time.

Thenmozhi (2002) examined the changes in the volatility of Nifty index due to the introduction of Nifty futures and movements in the futures price providing predictive information regarding subsequent movements in the index prices. The study highlighted the inception of futures trading leading to reduction in the volatility of spot index returns. The information flow was higher in the post futures period resulting in decline in spot index volatility in the post futures period. The lead-lag analysis showed futures had little or no memory effect and infrequent trading was virtually absent in futures market. The futures market transmitted information to the cash market and futures market was faster than spot market in processing information. The futures returns led the spot index returns by one day. The cash index returns did not lead the futures returns.

Raju and Karande (2003) estimated a lead-lag relation between S&P CNX Nifty and its futures. The major findings were that the futures market responds to deviations from equilibrium and price discovery occurs in both futures and cash market. The volatility in the spot market came down after the introduction of stock index futures.

Kenourgios (2005) showed the presence of bi-directional causality between stock index spot and futures markets, indicating that futures market serve as a focal point of information assimilation and contributes to price discovery. Shenbagaraman (2003) explores the impact of the introduction of derivative trading on cash market volatility using data on stock index futures and options contracts traded on the S&P CNX Nifty (India). The results suggested that futures and options trading did not lead to a change in the volatility of the underlying stock index, but the nature of volatility changed after introduction of futures.

Mukherjee and Mishra (2006) investigated the possible lead-lag relationship, both in terms of return and volatility, among the Nifty spot index and index futures market and also explored the possible changes in such relationship around the release of different types of information. They suggested that although there is a strong contemporaneous and bi-directional relationship among the returns in the spot and futures market, the spot market has been found to play a comparatively stronger leading role in disseminating information available to the market and therefore said to be more efficient. The results exhibit that though the leading role of the futures market wouldn't strengthen even for major market wide information releases, the role of the futures market in the matter of price discovery weakened and sometime disappeared after the release of major firm specific announcements.

The literature on the lead-lag relation between the index futures and the spot index indicates that futures market is the main source of market wide information and the futures lead the spot market. There is little evidence of spot index leading the futures market. Most of the studies use simultaneous equation modelling solved by ordinary least squares method to examine the lead-lag relationship between the futures and the spot market. Serial correlation tests and ARIMA models have been used to eliminate effects of infrequent trading and bid ask price effects. It is shown that trading frequency cannot account for the observed lead, rather it is the speed of price adjustment to information in futures markets that makes investors trade in futures first as they receive new market wide information. Support is also found from the earlier studies that lower transactions and entry costs in the stock index futures market is one of the reasons why traders with market wide information prefer to use the futures markets.

2.7 Summary and conclusions

This chapter is an attempt to review the most relevant research studies[11] in the area of portfolio selection. A comprehensive but not exhaustive review of literature has been presented. Researchers have come a long way on the road of portfolio selection starting from Roy's safety principle, Markowitz's efficient frontier, Tobin, Lintner and Sharpe's Capital Asset Pricing Model (CAPM), Fama and French's three-factor model, Ross's Arbitrage Pricing Theory (APT) to the recent day work by Pandey and Chee (2002), Saleh (2010), Ivkovik et al. (2008) and others. The classical works on portfolio selection served as the roadmap for present day researchers. Testing of the existing portfolio selection models for robustness and optimality in the ever changing financial markets all across the world has been a significant contribution in this area.

The area of portfolio selection has witnessed application of all possible mathematical models and quantitative techniques. The methodology, techniques and theories applied in this research includes unit root tests, goal programming techniques, dynamic programming, quadratic programmes, polynomial time algorithms, structure exploiting numerical algorithms, variance-covariance matrices, multiple regression equations, decision tree approach, scenario trees, Lucas trees, Engle curves, utility theory, Markov chain modulated diffusion formulation, stochastic linear-quadratic control, Riccati equations, Hamilton-Jacobi-Bellman equation, backward stochastic differential equations, partial correlations, multivariate GARCH for second moments, EGARCH, impulse response functions, group and majorisation theorem, Hilbert space theory, Martingale theory and stochastic calculus, Bayesian inferential procedures use of fuzzy techniques and stochastic dominance concepts, alternative risk measures, performance evaluation techniques, use of mean derived dispersion metrics, mean realised dispersion metrics and the application of evolutionary algorithms for dealing with non-smooth conditions.

Studies on mean-variance efficient portfolios are adding to the Markowitz efficient frontier by applying utility optimum bounds, alternate variance measures and latest quantitative techniques. However, restrictive assumptions of the model, mathematical complexity and ignorance of existence of multiple constraints faced by the investor raise concern. A flexible model to accommodate for the real world constraints and objectives of an investor inclusive of the effect of derivatives segment on portfolio selection is the need of the hour.

Existing studies on asset pricing theories are over focusing on testing of the Capital Asset Pricing Model (CAPM), Efficient Market Hypothesis (EMH) and the Fama and French three-factor model. The understanding of the existing scenario of an investor, his/her desires and limitations, development and testing of new portfolio selection models maximising investor's utility is required. The studies on diversification of a portfolio indicate the benefits of diversification and problems of non-performance associated with over diversification. Industrial and company diversification of a portfolio must be ensured by the investor while taking the decision of portfolio choice.

The studies on portfolio optimisation and variance-covariance matrix depict the development of equilibrium models, simulations, rules and heuristics for optimal portfolio selection. These models can be extended to accommodate the preferences of different types of investors and their real life constraints. There is always an impact of behaviour and systemic factors on an investor's portfolio choice. The presence of disposition effects and repeating habits is witnessed in an investor's decision. Systematic components of an investor's behaviour has been linked with future returns. Model formulations incorporating behavioural finance aspects needs to be considered. The lead-lag relationship existing between the derivatives and cash market is reviewed. The faster processing and absorption of information and recent news effect has been witnessed in derivative markets. Model formulations must incorporate the role of leading derivative markets on the lagging equity spot markets while creating an optimal portfolio.

The explanatory power of equity variables such as return, beta, liquidity and so on has been studied in isolation limiting its application in improvement of existing portfolio selection models. The issue of assignment of weight to a security in a portfolio needs to be further investigated. There also exists a gap in model development and its applicability. To bridge these gaps, it is imperative to develop a portfolio selection model which is best suited to investors accommodating for their multiple objectives and constraints.

3
Contributions to the Portfolio Theory

An optimal portfolio is more than a long list of good stocks and bonds; it is a balanced whole providing an investor with protections and opportunities with respect to a wide range of contingencies (Markowitz, 1959). The important criterion identified by the investors are high returns which are rather consistent that is, have less variability. Efficient portfolios are the ones yielding the highest returns for a given degree of risk or providing least risk for a given level of return. Mean-variance criterion provides an intuitive explanation for diversification. Investors would most often choose the portfolios which maximises their expected utility, while taking into consideration any other constraints they might be facing.

3.1 The standard mean-variance portfolio selection model

In the standard portfolio selection model,[1] an investor has to choose a fraction $X_1, X_2 \ldots X_n$ invested in n securities subject to constraints

$$\sum_{i=1}^{n} X_i = 1 \tag{3.1}$$

$$X_1 \geq 0 \quad i = 1 \ldots n \tag{3.2}$$

The returns on individual securities are assumed to be jointly distributed random returns. The expected (mean) return on the portfolio is

$$E = \sum_{i=1}^{n} X_i \mu_i \tag{3.3}$$

where μ_i represents the return on a security.

The variance of return V on the portfolio is

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij} \tag{3.4}$$

where, $$\sigma_{ij} = E[(r_i - \mu_i)(r_j - \mu_j)] \tag{3.5}$$

σ_{ij} is the covariance between r_i and r_j.

The portfolio $X_1, X_2 \ldots X_n$ which meets the two constraints (Equation 3.1 and 3.2) is the feasible solution of the standard model. It is regarded as an obtainable or legitimate portfolio. An obtainable mean-variance (EV) combination is inefficient if another obtainable combination has either higher mean and no higher variance or less variance and no less mean return. Efficient EV combinations are the ones which are not inefficient.

3.2 Advances in portfolio selection theories

Roy's (1952) principle of safety first further supported the concept of diversification of resources among a wide variety of assets. Tobin (1958) provided the basis for two fund separation theorem in the context of portfolio selection whereby an investor allocates his resources among risky and risk less assets.[2] Sharpe (1964) provided equilibrium for calculating the expected return on an efficient portfolio. The capital asset pricing model (CAPM) linearly related the return on portfolio with the beta of the portfolio. Samuelson (1969) extended the issue of portfolio selection to lifetime portfolio selection using dynamic stochastic programming.

Fama (1969), by providing insights on weak, semi-strong and strong forms of efficiency in capital markets provided the basis for efficient allocation of resources. Fama and French (1995) found that three variables, market equity, the ratio of book equity to market equity and leverage variables capture much of the cross section of average stock returns. The three variables were found to have more explaining power as compared to market beta. Saleh (2010) constructed high-stock-volatility-minus-low-stock-volatility (HSVMLSV) factor and added it to the Fama and French three-factor model to create a four-factor model.

Ross (1976) developed the arbitrage pricing theory (APT) as an alternative to CAPM. The random returns were expressed by a simple factor model with a mean zero common factor and a mean zero vector which is independent to permit the applicability of law of large numbers. The APT has been used extensively for measuring portfolio performance (Connor and Korajczyk, 1986; Connor and Korajczyk, 1991; Lehmann and Modest, 1987; Rubio, 1992; Chang and Lewellen, 1985; Berry et al., 1988; Frohlich, 1991).

Beaver et al. (1970) concluded that the accounting measures of risk are impounded in the market price based risk measures and advocated their use for selection of stocks in a portfolio. The accounting measures of risk included in their analysis were dividend pay-out, growth, leverage, liquidity, asset size, variability of earnings and covariance of earnings. Dealing with a more general class of utility function Black (1972) created zero beta portfolios. On the lines of expected utility maxim and limited liability of assets Merton (1973) developed inter-temporal capital asset pricing model (I-CAPM). Ballestero (1998) provided a new bounding utility theorem to approximate the optimum portfolio of an investor with well-defined preferences for profitability and safety. The lesser the preferences deviate (from the average preference behaviour), the narrower are bounds for the utility optimum on the efficient frontier.

Zhou and Li (2000) formulated a continuous time mean-variance portfolio selection model as a bi-criteria optimisation problem. The objective was to maximise the expected terminal return and minimise the variance of the terminal wealth. Alexander and Baptista (2002) related value-at-risk (*VaR*) to mean-variance analysis and examined the economic implications arising from mean-VaR framework. Pardalos et al. (1994) and Elton et al. (2007) provide an extensive review of the portfolio selection models based on the Markowitz mean-variance criteria and other techniques developed, their methodologies and computational results. Ogryczak (2000) developed a multiple criteria linear programming model for portfolio selection. Their model is based on the preference axioms for choice under risk. It is shown that the classical mean-risk approaches solved using linear programming models correspond to specific solution techniques applied to this multiple criteria model.

Saaty et al. (1980), Lee and Chester (1980), Evrard and Zisswiller (1983), Nakayama et al. (1983), Martel et al. (1988), Szala (1990), Colson and Zeleny (1979), Hurson and Zopounidis (1993) have used the multi-criteria approach for stock valuation, stock evaluation, use of alternative measures of risk, security selection, modelling of investor preferences, portfolio construction, portfolio ranking and construction of efficient frontier. The methodologies commonly used include ELECTRE 1, ELECTRE II, ELECTRE IS, ELECTRE III, MINORA system and ADELAIS system. Ballestero and Romeo (1996), Ogryczak (2000), Zopounidis and Doumpos (2000), Arenas Parra et al. (2001), Ballestero and Pla-Santamaria (2003), Ehrgott et al. (2004) are a few research studies using the MCDM framework for supporting the decision of portfolio construction.

Spronk and Hallerbach (1997), discussing the direction of financial modelling, advocate the multi-criteria view of portfolio selection with investor specific decision context and securities specific decision context. In their conceptual framework for portfolio selection, the factors for securities related decisions are further bifurcated into direct return related and indirect return related attributes. They comprise of return, risk, taxability, liquidity, dividends, future sales and earnings, growth expectations and financial stability. Firm size and price ratios such as price-to-earnings ratio, price-to-book ratio are also found to be important as they are indicators of firm value. Interactive programming methods and different ways of using the framework are recommended.

Powell and Premachandra (1997) provide a goal programming model to accommodate the diverse objectives and constraints of institutional investors while selecting a portfolio of securities. Risk-return goals, legal restrictions, cash flow requirements and performance targets are some of criteria they need to consider. Credit rating of the firm, dividend yield, earnings yield, trading volume and social acceptability index are a few factors modelled. Different priorities for various investor groups such as young workers, working mothers and retiring couples are set. This supports the view of attaining multiple objectives rather than the single objective of risk minimisation.

Due to the limitations of both preference axioms based stochastic dominance and expected utility theory models and inability of bi-criteria optimisation to incorporate preferences of decision makers under risk, Ogryczak (2000) developed a multiple criteria linear programming model for portfolio selection. The mean-variance model proved to be a specific aggregation of the multiple criteria model and allows for the use of various criteria's to analyse portfolio selection. The model is based on the preference axioms for choice under risk.

Taking forward the modern portfolio theory from a two-dimensional space of mean and variance to a higher dimensional space, Steuer et al. (2007) focused on investors having more concerns than to just maximise portfolio return and minimise variance. Incorporating liquidity, dividends, the number of securities in a portfolio, diversification through upper bounds, social responsibility, turnover, the amount invested in R&D and excess return over benchmark as additional concerns they developed a multi-criteria portfolio selection model. Heterogeneity of expectations allowed in the model and experiments with incorporating new criteria for suitable investors does not disturb the standard finance theory. The inability of an index-based market portfolio to represent optimal portfolio on the non-dominated frontier further supports the multi-criteria approach. The solvability of model is stressed upon due to the availability of computer software with developments in operational research.

The multiple objectives of risk-averse investors have been incorporated in portfolio selection models by imposing additional constraint to the objective of minimisation of risk by Garlappi et al. (2007). Steuer et al. (2008) advocates multiple criteria formulations for portfolio selection by investors who have multiple argument utility functions. This might happen when the investor has more considerations than just maximising return or when he/she believes that mean, variance and covariances cannot be known with certainty in the beginning of the holding period and hence prefers to base portfolio decision on additional measures such as dividends, growth in sales, investment in R&D and the like. Portfolio optimisation for such investors must be looked upon in the multidimensional space incorporating the two dimensions of mean and variance.

Xidonas et al. (2009) also applied the multi-criteria decision making approach to portfolio selection. The criteria identified for securities appraisal and portfolio optimisation consisted of market dimensions, security fundamentals, and mean-variance efficiency. They also incorporated decision maker preferences. The dividend yield and capital return formed return dimension; market beta and standard deviation of capital return were analysed for risk dimension; marketability and the price-to-earnings ratio determined market acceptance. Various fundamental analysis criteria such as ROA and ROE (profitability characteristics); asset turnover and inventories turnover (management performance) and capital structure of companies were modelled. The minimisation of variance was the objective with constraints on return, beta, dividend yield, marketability and full capital utilisation. The model was tested for different types of decision makers on the Athens Stock Exchange's FTSE-140. The multiple objectives beyond mean-variance help an investor in identifying the optimal portfolio on the extensive efficient frontier as per his/her preferences. Rodríguez et al. (2011) examined interactive methods to solve the multi-objective portfolio selection problem. The method has been tested on data from the Madrid Stock Exchange for two investors where one of them is more aggressive. Return was represented by both capital return as well as dividend yield; variance of a portfolio representing global risk, beta measuring systematic risk and specific risk measured by residuals were modelled as objectives in the problem. The importance of preferences of investor in reaching the optimal portfolio has been emphasised.

Xidonas et al. (2011) take into account the inherent multidimensional nature of the portfolio selection problem and also incorporate preferences of the investor in the decision process. The multi-criteria decision making approach based on multi-objective mathematical programming is implemented using integrated portfolio synthesis and selection information system (IPSSIS) and tested on the Athens Stock Exchange. The objectives include maximising return, dividend yield and minimising beta coefficient. Constraints such as stock preferences, sector preferences, share adjustment, market risk adjustment, capitalisation adjustment and diversification adjustment are modelled in the multi-objective mixed integer linear programming model. It provides for a more realistic and flexible model assisting investors in implementing their investment strategies.

Patel and Subramanyam (1982) created an algorithm for optimal portfolio selection with fixed transaction costs by placing restrictions on the variance-covariance matrix of returns. Inclusion of transaction costs, short-selling and higher moments of returns such as skewness and kurtosis in a portfolio selection model provides to achieve greater flexibility (Yu and Lee, 2011). Incorporation of transaction costs make the model more realistic by incorporating for the market frictions (Choi et al., 2007; Kozhan and Schmid, 2009). Considering skewness and kurtosis of return distribution improves optimal portfolio (Prakash et al., 2003; Li et al., 2010). Yu and Lee (2011) create five portfolio rebalancing models with the additional criteria, solve them using multi-objective programming and compare them. They conclude that the incorporation of the additional criteria produces better portfolios than the TSE50. They also argue for inclusion of more such criteria for dynamic portfolio selection. Zheng et al. (2011) apply constrained ELECTRE TRI model for portfolio construction to select a satisfactory portfolio taking into consideration the preferences of the decision maker at individual as well as portfolio level. The alternatives are evaluated and screened out using a sorting model on the basis of intrinsic performance. The preferences of decision makers are modelled as constraints of the optimisation procedure. Ballestero et al. (2012) incorporate ethical policies of the firms in a bi-criteria model proposed for socially responsible investing. Sensitivity analysis for strong green investors and weak green investors is also undertaken. The resultant portfolio of strong green investors underperforms Markowitz's efficient portfolio. This does not hold true for weak green investors with a low level of aspiration for the ethical goal.

Smimou (2013) incorporates the impact of political instability risk in case of international portfolio diversification using alternative instability risk proxies in a discrete time version of the mean-variance framework. The quadratic programming Markowitz model, with an additional constraint of change of corruption index/governance indicator as a proxy of instability risk, is modelled. As instability risk increases the securities in portfolio shift, from instable to more stable countries, indicating an attainable international Markowitz frontier.

Zopounidis and Doumpos (2013) analyse the importance of multicriteria decision systems for supporting financial decision making. They present an up-to-date review of the use of MCDM approach for portfolio selection and corporate performance evaluation highlighting the improvements made to the existing framework in these areas of finance and operations research. They have emphasised on the multicriteria aspect of return, profitability, wealth, risk and investor preferences. The ever increasing uncertainties in the global financial markets require investors to adapt to new decision factors by comprehensively analysing multiple relevant factors in order to make optimal decisions. The contribution of the MCDM approach in complementing the normative and descriptive portfolio selection models to enhance its application is made.

Ever since the publication of seminal work of Markowitz (1952) on portfolio selection, the use of optimisation models has remained central for portfolio construction. Markowitz (1959) extended the MV approach by introducing other risk measures such as semi-variance, mean absolute deviation and expected loss. To solve complex financial optimisation problems and predict patterns in financial data, the techniques of computational intelligence (Doumpos et al., 2012) have also attracted attention. Their exists immense evidence in literature on the use of MCDM approach for handling the problem of portfolio selection by taking into account multiple conflicting criteria of diverse nature and investor preferences.

Modelling of the stock selection process was first introduced by Hurson and Zopounidis (1995) through the use of outranking and disaggregation techniques on the basis of financial and stock market criteria. The MOO model of portfolio construction used multiple portfolio selection measures such as returns, dividends, systematic risk and marketability among others. Research studies such as Zopounidis et al. (2013), Xidonas et al. (2007), Xidonas et al. (2009) and Xidonas et al. (2011) have used this framework for modelling portfolio selection. Hallerbach et al. (2004), Ballestero et al. (2012) and Tsai et al. (2009) have linked the context of socially responsible investing by adding ethical, social and environmental criteria to the MCDM analysis. Multi-objective linear programming, goal programming, stochastic programming and fuzzy models have also been applied to asset allocation decisions using the MCDM approach (Ogryczak, 2000; Ballestero, 2001). For evaluation and inclusion of assets in a portfolio, discrete multi-criteria models have been attempted (Hurson and Zopounidis, 1997; Xidonas et al., 2009). Zopounidis et al. (1998) and Doumpos et al. (2000) have categorised the studies on portfolio selection using multi-criteria analysis under four/five broad categories. The multidimensional context of security evaluation and apportionment of capital to these selected securities has been studied by MCDM researchers using multi-objective mathematical programming/goal programming, preference disaggregation analysis, multi-attribute utility theory and the like (Xidonas et al., 2009).

In the current capital market situation, with abundant data availability and latest computational techniques, there emerges a need to develop a portfolio selection model analysing efficiency across multiple financial variables that can realistically incorporate the multiple goals and constraints of today's investor. The vast literature available in the field of portfolio optimisation can pave way in understanding the dynamics of investor's behaviour. The increasing uncertainties in financial markets; investors having multi-argument utility functions with more concerns than efficiency regarding mean and variance; and limitations of preference axioms based stochastic dominance and expected utility theory models call for creation of a multi-criteria decision making portfolio selection model to assist investors in implementing their investment strategies. In the present state of emerging stock market activity, the highly complex and multidimensional nature of portfolio selection decision with ever increasing aspirations of investors and their numerous limitations in relation to the intrinsic characteristics of securities can be effectively modelled in a MCDM problem. An effort is made here to model this into a linear-quadratic programming framework. The MCDM framework complements and extends the modern theory of portfolio selection.

3.3 Emerging issues and challenges in Indian equity markets

Over the last few years, there has been a rapid change in the Indian securities market, more so in the secondary market. Advanced technology and online-based transactions have modernised the stock exchanges. In terms of the number of companies listed and total market capitalisation, the Indian equity market is considered large relative to the country's stage of economic development.

The crises of recent years in the midst of the worst financial catastrophe of recent decades marked by massive credit failures, banks and brokerage meltdowns and government bailouts have impacted the capital markets immensely. Since the beginning of 2011, the two adverse developments were witnessed in the advanced economies: (1) a much slower rate of recovery than expected and (2) an increase in fiscal and financial risks. The data reflect a sluggish growth in the advanced economies. The world economy was hit by a number of shocks. Japan was shaken by the devastating earthquake and tsunami. Political turmoil was witnessed in the oil producing nations. The growth in private demand has stalled. Downside risks have increased with fiscal uncertainty, housing market weakness, renewed financial stress and subdued business and consumer sentiment.

These global developments undermined the prospects of self-sustaining recovery in India. The sovereign debt crisis and prolonged slowdown in European Union nations and United States have impacted India's growth prospects. Gross domestic product (GDP) growth, declining index of industrial production (IIP), rising inflation, fiscal management, management of external sector transactions and depreciation of the Indian rupee are some important issues raised in the backdrop of the global slowdown.

The effects are more pronounced on India's financial markets particularly in the securities market rather than on real economy. The resources mobilised through public and rights issue in the primary market, market capitalisation to GDP ratio, average daily market turnover, net foreign institutional investor's investment in debt and equity, all have fallen. The financial markets experienced higher than normal levels of volatility and uncertainty during 2011.

The Securities and Exchange Board of India (SEBI) has concerns over the changes and improvements required in the market structure in the view of the fast pace of technological developments. It has formed Technical Advisory Committees (TAC's) to frame appropriate policies arising out of technological advancements in the areas of wireless trading, co-location, algorithmic trading, smart order routing and application programming interface.

In the light of developments in the secondary market, measures for improving market safety, efficiency, transparency and integrity assume importance. The reduction of transaction costs, simplification and transparency in the systems and procedures of legal framework, quick and efficient handling of investor grievances and a strong regulatory framework especially for intermediaries and mutual funds are a few important areas requiring attention. Some of the emerging issues in the Indian capital market may be enumerated as below.

3.3.1 Risk management

In the present day scenario, the risk management framework for the cash and derivatives segment needs to be reviewed. Changes in the risk management/margin system must be incorporated. The regulatory framework for risk management for the cash and derivative segment requires an in-depth analysis and attempts must be made for reduction of the transmission of risk from other segments. Investor protection measures related to risk management should also be reviewed.

3.3.2 Disclosures and accounting standards

The disclosure requirements in offer documents, application forms, advertisements or any other mode of mass communication are standardised keeping in view the protection of interests of the investor and improving the overall efficiency of the market. The continuous disclosure requirements pertaining to listing of equity or debt of an issuer are framed by SEBI. Also, the disclosure requirements of intermediaries registered with SEBI have been framed.

The continuous disclosure requirements of listed companies and the valuation methods and standard norms of intermediaries operating in the capital market need to be continuously reviewed in the light of changing market scenario. The operational and systemic risks in the primary securities market need to be addressed in these disclosures.

An effort is needed to ensure smooth implementation of accounting standards and statements of the Institute of Chartered Accountants of India (ICAI) pertaining to disclosures in the capital market. Coordination between SEBI and ICAI by constituting study teams for providing inputs to the Accounting Standard Board (ASB) for evolving new accounting standards and reviewing the existing ones would be a positive step in improving the disclosure and accounting standards.

3.3.3 Investor protection and education

Investor protection and education activities directly undertaken by SEBI or through any agency to utilise the SEBI investor protection and education fund must be carefully monitored. Different target groups of investors at varied locations across the length and breadth of the country should be covered. Also, a method to evaluate the effectiveness of the investor education programme needs to be devised.

3.3.4 Wireless trading and co-location

Millions of Wi-Fi internet access points with broadband connectivity have allowed investors to sign on to their brokerage accounts on laptops and mobiles at different locations, which has resulted in lower costs of transactions. With real time data streaming in, it has made the trading of stocks more flexible and convenient from the point of view of an investor. However, increased access requires improved security systems. For robustness of the system, creation of security tokens and passwords, use of smart cards, encryptions and biometric devices, improvements in cell networks and enhancements in the end user devices is important. Systems where users are allowed to shut down user accounts when the devices are stolen must be created.

Extending the internet trading platform in May 2009, SEBI released a proposed framework for using wireless trading subject to security and encryption safeguards. This has provided a boost to equity trading in India and has widened the scope of trading by giving an opportunity to the tech savvy young urban population of mobile subscribers in the equity markets. Measures for user identification, authentication and access control are important and require frequent up gradation in the changing computing environment.

In September 2010, the Bombay Stock Exchange (BSE) was the first to launch mobile trading in India soon followed by National Stock Exchange (NSE). All the brokers providing internet trading and who complied with the security norms issued by SEBI were allowed to provide wireless trading. The method of trading is new to the Indian investor and is still limited to a small population. However, it is gaining pace and the security concerns need to be addressed as and when they arise providing alternate means in case of failure.

3.3.5 Algorithmic trading and high frequency trading

Algorithmic trading is the use of computer programs and software to execute trades based on pre-defined criteria and without any human intervention. High frequency trading is a subset of algorithmic trading which involves buying and selling of thousands of shares in fractions of seconds. Both algorithmic and high frequency trading exist in high volumes in India, although the impact they impose on individual trade is rather very low. However, they are capable of causing large market movements. No real damage has been witnessed so far as a result of these swings, but a word of caution is required for the exchanges.

In March 2012, SEBI released broad guidelines to put a check on algorithmic trading programmes providing measures to check an excessive flood of orders and irregular price quotations. The regulator has advised exchanges to set up systems complying with latest guidelines for reining in algorithmic traders. The brokers would require prior exchange permission before offering algorithmic trading to their clients. Existing algorithmic traders would be vetted for risk management systems. To prevent order flooding and high number of orders as a proportion of actual executed trades, exchanges were asked to give economic disincentives. Algorithms would not be allowed to quote beyond a certain number of securities per order or in violation of price bands. A dummy filter would act as an early warning system to detect sudden surge in prices. Other such moves preserving market integrity have been included in SEBI guidelines which need to be incorporated by the Indian stock exchanges.

3.3.6 Smart order routing

In August 2010, SEBI allowed for the introduction of smart order routing allowing the brokers trading engines to systematically select the execution destination based on price, costs, speed, likelihood of execution and settlement, size, nature or some other consideration relevant to execution of the order. In 2011, smart order routing finally took off in India on the country's two premier bourses after resolution of their long standing dispute over the audit trail of orders.

By using smart order routing technology, investors are able to obtain the best possible price while buying or selling shares, similar to what was being done manually by stock brokers. This technology makes this much faster to execute orders. Smart order routing determines which exchanges offer the best price at any given time. Speed is the key to the success of programme trading. If the price feed is not fast enough, the software will be unable to capitalise on the opportunities that last for a second or less. It helps in better price discovery and induces increased electronic trading volume.

Lack of interoperability between India's two securities clearing houses remains an unresolved issue for smart order routing in the equities market. Clarification was given by the regulator on permission of smart order routing for all types of orders following confusion among the market participants. High clearing charges have negated any improvements from smart order routing. Many front-end trading systems used by buy-side trading desks are unable to split an order that is executed across two venues for confirmation purposes. Competition and interoperability already exists between the Central Depository Services (India) and the National Security Depository.

Some market participants have suggested interoperability agreements such as those by central counterparties in Europe, while others are in favour of a single-central clearing organisation. Clearing the interoperability would allow trading firms to use capital more effectively and move out from the current artificial inefficiency situation. Interoperability arrangements would have to account for risk management implications. A central mechanism would be needed for interoperability to monitor potential margin breaches across both exchanges.

3.3.7 Minimum public shareholding

The Securities and Exchange Board of India (SEBI) has been seriously promoting the idea of minimum public shareholding ever since 2001 when it amended the Clause 40A of the Listing Agreement to provide for mandatory non-promoter shareholding. The same are mandatory under the Securities Contracts (Regulation) Rules, 1957, and Securities Contracts (Regulations) Act, 1956. Minimum public shareholding has been considered beneficial from the perspective of an investor as it ensures increased liquidity, lesser price manipulations, price discovery, low volatility, increased access to capital, enhanced corporate governance and better endorsement of the brand value. The most recent amendment came in 2010, which raised the minimum public shareholding to a uniform of 25 per cent for all companies, listed and seeking to list. For government companies and public sector companies this percentage has been kept as 10 per cent. A lower public shareholding minimum is justified as it would prevent any large scale disinvestment by government companies which may distort the market. From the international perspective, countries such as Singapore, Taiwan, UK, China, Hong Kong and Brazil require a minimum public float of 12 per cent to 25 per cent. The common routes for increasing minimum public shareholding include (1) issuance of shares through an Initial Public Offer or Follow on Public Offer; (2) sale of shares by promoters in secondary market; (3) the Institutional Placement Programme[3] (IPP); and (4) Offer for Sale[4] (OFS) of shares by promoters. Qualified institutional placement, preferential allotment or issue of depositary receipts is not a valid method of increasing public shareholding.

A large number of companies who have not been able to fulfil these requirements are also planning to delist them. However, the delisting norms are quite harsh (expensive and cumbersome) on the company and its promoters and is being considered only as the last option. Despite all the effort by the regulator and amendments year after year (in 1993, 2001, 2005 and 2010), out of 4977 listed companies only 3525 have been able to comply with the regulations for minimum public shareholding. As many as 1259 listed companies have not submitted the shareholding information to the exchanges and the regulator.

Existing norms of not more than 50 per cent of the net offer to the public for Qualified Institutional Buyers[5] (QIBs), not less than 15 per cent of the offer for Non-Institutional Bidders[6] and not less than 35 per cent of the offer size for Retail Individual Bidders[7] ensures greater depth and breadth in the securities market. These limits provide the upper bounds for institutional and non-institutional stock holding affecting portfolio composition. However, it also limits the amount of investment a particular investor group can make.

The two issues which arise from the preceding discussion is that on one hand we have the regulator pushing listed companies to increase their public shareholding while on the other hand a large number of promoters of listed companies not willing to part with their stock holding. The issue becomes extremely important because if these 1259 companies actually comply with the 2013 deadline then it may result in supply of a large number of shares on NSE and BSE. Excess supply may result in fall in share prices of a large number of stocks. Another possible scenario could be that the deadline is extended. This would certainly dampen the spirits of those companies who have willingly complied with the SEBI regulations as the inefficient would be rewarded by this extension. The situation becomes complex in a scenario where a large number of investors are shying away from Indian capital markets in the light of downgrades by credit rating agencies, poor economic performance and policy bottlenecks at the national level. Even the two new measures introduced by SEBI of IPP and OFS have not yielded the desired results. The emerging issues and challenges posed by the requirement of minimum public shareholding require an in-depth research and analysis and would certainly affect the returns and volatility of Indian capital markets in future.

The section on emerging policy issues and challenges related to portfolio selection, raises questions related to risk management, disclosures and accounting standards, wireless trading, co-location, programming interface, investor protection and education, algorithmic trading and high frequency trading and smart order routing. Incorporation of changes in the risk management and margin system of the cash and derivatives segment to reduce the transmission of risk has been emphasised. Coordination between the SEBI and the ICAI, by constituting study teams for providing inputs to the ASB in order to evolve new accounting standards and reviewing the existing ones, has been suggested for improving the disclosure and accounting standards.

The development of a method to assess the impact of investor education initiatives is stressed, and the need for clearing the interoperability issues between the two exchanges is recommended. Incorporation of moves to preserve market integrity in case of algorithmic trading as suggested by the regulator must be done by the exchanges to promote algorithmic trading. In case of mobile trading, measures for user identification, authentication and access control are important and require frequent up gradation in the ever changing computing environment. The issue of mandatory minimum public shareholding of 25 per cent by the deadline in 2013 might lead to excess supply of securities in the secondary market or an extension of time to the 1259 non-complying companies. In both the above mentioned scenarios, the image of Indian capital markets needs to be protected against falling investor confidence and downgrades by the credit rating agencies.

Low levels of gross domestic product (GDP) growth, declining IIP, rising inflation, fiscal management, management of external sector transactions and depreciation of the Indian rupee are important issues affecting Indian capital markets. The effects are more pronounced on India's financial markets, particularly on the securities market rather than on real economy. The resources mobilised through public and rights issue in the primary market, market capitalisation to GDP ratio, average daily market turnover, net foreign institutional investor's investment in debt and equity, all have fallen. The reduction of transaction costs, simplification and transparency in the systems and procedures of legal framework, quick and efficient handling of investor grievances and a strong regulatory framework especially for intermediaries and mutual funds are a few important areas requiring immediate attention.

4
Mean-Variance Efficient Portfolio Selection: Model Development

The limited literature available in India in the area of portfolio selection compared to the efficient markets of the developed economies, such as the United States (US) and the United Kingdom (UK) prompted us to conduct an in-depth study in this field. Although the effect of various financial and accounting factors on security returns has been studied separately, no efforts have been made to integrate these factors for the benefit of an investor. The present quest tries to fills these voids. On the basis of knowledge gained from reviewing the research efforts of the past and the emerging issues in the Indian capital markets, portfolio modelling has been attempted using the quadratic programming approach.

Investors are faced with an array of factors when choosing securities for their portfolio. Making an optimal portfolio selection decision takes serious consideration of investment objectives and limitations. Ambiguity on identifying the important factors from a given list of variables exists. Assimilation of the important factors already listed in literature, identification of new variables impacting portfolio selection decision and formulating a portfolio selection problem with multiple constraints require attention.

The principle objective of this chapter is to develop a model to determine the mean-variance (EV) efficient sets in a constrained environment for a class of portfolio selection problems. The main goal of an investor (minimisation of risk/variance) forms the objective function of the quadratic programming problem. The multiple objectives and limitations of today's investor are depicted through the formulation of linear constraints to this quadratic programming problem. An attempt to derive an efficient frontier[1] for an investor with a single objective and multiple restrictions has been made. Integration and improvement of the existing multi-objective quadratic programming (MCDM) portfolio modelling framework has been attempted.

4.1 Multi-objective quadratic programming

The MCDM approach from the field operations research dealing with decision problems involving multiple criteria has undergone tremendous growth in the last few years. The issue of portfolio construction involves analysis of various aspects by an investor – fundamental accounting, financial as well as governance. Applying MCDM and mathematical programming, a multi-objective quadratic programming model with the objective function of minimising variance (volatility) may be obtained. The multiple decision criteria relating to return (capital and dividend), systematic risk (beta), marketability (trade volume and price-to-earnings ratio), management efficiency (operating profit margin), profitability (net profit margin), governance (free float) and future investment opportunities (free cash flows) could be modelled as constraints in the portfolio construction model. Pareto optimal or efficient portfolios may be generated by applying the model for portfolio selection. Pareto optimal/ efficient portfolios are the solutions which cannot be improvised for one of the criterion without deteriorating the performance in at least one of the other criterion.

The aim of an investor while selecting a portfolio is to minimise the risk for a given return subject to other constraints he/she faces. In this book, an attempt has been made to aggregate a large number of constraints, to provide a solution which will be a trade-off among conflicting constraints without compromising Markowitz's mean-variance efficiency. Mean-variance portfolio selection refers to the problem of finding an allowable investment policy that satisfies all the constraints such that the risk measured by variance is minimised and expected returns are maximised.

The predictive power of variables in explaining a cross section of returns such as beta, dividend, turnover, VaR, promoters holding, net profit, earnings per share, book-to-market ratio, price-to-earnings ratio, portfolio utility, liquidity and conditional volatility have been investigated in isolation. This approach has limited the application of these studies simultaneously in investment management for improving the efficiency of existing portfolio selection models. The resultant solution of standard portfolio selection models may be Pareto optimal theoretically but may not perform well in the real world situations. The accumulation of important explanatory variables in a single portfolio selection model will lead to selection of portfolios which not only satisfy all identified constraints but are efficient in the mean-variance sense.

4.2 Model building and application

4.2.1 The objective function

The standard mean-variance portfolio selection model, standard analysis with upper bounds, the Tobin-Sharpe-Lintner model and Black's model have all tried to find maximum return-minimum risk portfolios. The recent day researchers Steinbach (2001), Zhao and Ziemba (2002), Lai et al. (2002) and Goldfarb and Iyengar (2003) have developed models with minimising variance as the objective function. Mean-variance efficient portfolios result in maximising the expected utility of the investor.

The objective function of our programming problem is to minimise the variance (the most widely used measure of risk) of the portfolio. This measure of risk is by its very nature quadratic.

Objective Z: minimise variance

4.2.2 Calculation of risk/variance of a portfolio

Variance of a portfolio is defined as

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{4.1}$$

where

$$\sigma_{ij} = E\,[(r_i - \mu_i)\,(r_j - \mu_j)] \tag{4.2}$$

is the covariance between r_i and r_j. In particular

$$\sigma_{ii} = E\,(r_i - \mu_i)^2 = V(r_i) \tag{4.3}$$

is the variance of r_i.

4.2.3 Evaluation criteria and constraint set

The quadratic programming technique for optimisation of a quadratic objective function is subject to linear equality and linear inequality constraints. Constraints are conditions that a solution of an optimisation problem must satisfy. The set of solutions that satisfy all the equality and inequality constraints form the feasible set. A model is infeasible if no portfolio can meet its constraints. The present day investor is multitudinous by his/her nature and has diverse objectives. The real world financial markets also impose additional restrictions/limitations on the process of portfolio selection. These objectives as well as limitations are incorporated in the model through the introduction of linear equality and inequality constraints.

The most appropriate criteria to be used for portfolio optimisation have been identified both in theory and in practice. The proposed methodology is based on international literature (Xidonas et al., 2009; Xidonas et al., 2011). These criterion have been listed below:

1. Return aspects
   (a) Capital Return: As used by Markowitz (1952, 1959) the expected mean of a portfolio ensures an aspired level of return to the investor. A positive E represents the desired expected return of the portfolio and is defined as

$$E = \sum_{i=1}^{n} X_i \mu_i \tag{4.4}$$

where

$$\mu_i = E\,(r_i) \tag{4.5}$$

E is the expected value operator and refers to the expected value of return on a portfolio i.e. $E = E\,(R)$.

And the return on the portfolio (R) is

$$R = \sum_{i=1}^{n} X_i r_i \tag{4.6}$$

where r_1, r_2 … r_n are jointly distributed random variables denoting the current period's return on individual securities. The first differenced values of log of securities closing prices are calculated to depict the returns.

Thus,

$$r_t = \ln\,(P_t) - \ln\,(P_{t-1}) \tag{4.7}$$

r_t is the returns on securities/index,
$\ln\,(P_t)$ is the natural log of closing price in time period t and

$\ln(P_{t-1})$ is the natural log of closing prices in the previous time period.
$\hat{E}$ is the required rate of return on the portfolio.

(b) Dividend Yield: To incorporate the current income requirement of an investor the dividend yield constraint has been introduced in the model. Dividends provide investors an incentive to invest in stocks of such companies stable companies even if their growth opportunities are restricted. The dividend constraint will restrict the companies not meeting an investor's desire for a minimum dividend yield from entering the feasible solution.

$D_i = \frac{DPS_t}{P_t}$ where D_i is the dividend yield, DPS is the dividend per share in time period t and P_t is the closing price in time period t.
It is the dividend per share expressed as a percentage of their price.

2. Risk Parameters
   (a) Systematic Risk (Beta Coefficient)

   Sharpe (1964) related return on the security with return on the market portfolio. The measure of systematic risk component 'β' (beta) was calculated. Beta as explained by the capital asset pricing model measures the sensitivity of rate of return of a security to changes in rate of return to the market.

It is the slope of the characteristic line which gives the relationship between market return and security return. β is a measure of sensitivity of a stock return to the return on the market index (Nifty 50 and FTSE 100 in our case). Taneja (2010), Mehta and Chander (2010) found beta to capture systematic risk and all betas to be statistically significant in the Indian stock markets. Beta has been found to have explanatory power to model returns and hence it has been included in the portfolio selection model.

$$\beta = \frac{\sigma_{im}}{\sigma^2} = \frac{\rho_{im} * \sigma_i * \sigma_m}{\sigma_m^2} = \frac{\rho_{im} * \sigma_i}{\sigma_m} \tag{4.8}$$

where

σ_{im} = covariance between security and market returns
σ_m^2 = market variance
ρ_{im} = correlation between security and market returns
σ_i = standard deviation in returns of a security
σ_m = market standard deviation in returns of market portfolio
$\bar{\beta}$ = target beta

The variance of a portfolio is minimum when $\rho = -1$. The benefits of diversification are achieved by combining negatively correlated securities. For equal risk and return combination, the mean return of the portfolio is equal to the mean of each of security in the portfolio, showing no variability over time. Any variance above or below the mean for set of securities in the portfolio is offset by returns of other assets in the portfolio. Risk is zero for such a portfolio. For different risk and return combinations and by combining negatively correlated securities, one can minimise the standard deviation of the portfolio but cannot achieve a perfect zero as the standard deviation of securities are not equal. The variance of portfolio is less when $\rho = 0$ this consists of securities which are not related to each other. The portfolio risk will be highest if ρ among the securities is perfect positive i.e. $\rho = +1$.

(b) Unsystematic Risk

It is desirable to separate the variations in rates of return into two components – one reflecting the portion of asset price movements caused by changes in the market and the other reflecting the portion of assets' price movements caused by factors unique to the company and the industry. Here the unsystematic risk is related to factors such as labour strikes, inventions and research and development which can be reduced through diversification. The statistical model separating total risk into its components is

$$r_t = a + bm_t + e_t \tag{4.9}$$

where

r_t = rate of return of a security in time period t
a = intercept term
b = slope of the regression line
m_t = market rate of return in time period t and
e_t = random error about regression line in time period t.

The statistic representing the random error *e*t about the characteristic line is a measure of that portion of total risk affected by characteristics unique to the company or industry. However in practice, unsystematic risk is calculated as per the following formula:

Unsystematic risk = Total risk – Systematic risk

$$e_t = \text{var}(r_t)t - \beta^2 \text{ var } (m_t) \tag{4.10}$$

where

β	=	measure of sensitivity of asset returns *vis-a-vis* market returns
$var\ (r_t)$	=	variance of the security returns
$var\ (m_t)$	=	variance of market returns.

Kabito (2009) found unsystematic risk to be affecting returns significantly. Jeyachitra et al. (2010) found reduction in unsystematic risk with diversification yielding positive returns for Nifty stocks.

3. Liquidity in the context of stock markets implies a market where large orders can be executed without incurring a high transaction cost. The transaction not in terms of the fixed costs typically incurred like brokerage, transaction charges, depository charges, etc. but refers to the cost attributable to lack of market liquidity. Liquidity comes from the buyers and sellers in the market who are constantly on the look-out for buying and selling opportunities. Lack of liquidity translates into a high cost for buyers and sellers.

The electronic limit order book (ELOB) available on NSE is an ideal provider of market liquidity. This allows all investors in the market to execute orders against the best available counter orders. The market thus possesses liquidity in terms of outstanding orders lying on the buy and sell side of the order book, which represent the intention to buy or sell. When a buyer or seller approaches the market with an intention to buy a particular stock, he/she can execute his/her buy order in the stock against such sell orders, which are already lying in the order book and vice versa.

Market impact cost is one of the most appropriate measures of the liquidity of a stock. It accurately reflects the costs faced when actually trading an index. For inclusion in the index, the security should have traded at an average impact cost of 0.50 per cent or less during the last six months for 90 per cent of the observations, for the basket size of Rs 20 million.

Impact cost is cost of executing a transaction in a security in proportion to the weight age of its market capitalisation as against the index market capitalisation at any point of time. This is the percentage mark-up suffered while buying/selling the desired quantity of a security compared to its ideal price = (best buy + best sell)/2. It is a practical and realistic measure of market liquidity, closer to the true cost of execution faced by a trader in comparison to the bid-ask spread. It is computed separately for buy and sell and varies as per transaction sizes and outstanding orders at any given point of time. When a stock is not sufficiently liquid, a penal impact cost is applied. In mathematical terms, it is the percentage mark-up observed while buying/selling the desired quantity of a stock with reference to its ideal price (best buy + best sell)/2.

As it is for inclusion of a stock in Nifty, the impact cost of a stock should not be more than 0.50 for being a part of the portfolio. This would imply presence of only the most liquid securities in the portfolio.

$$i \leq 0.50 \tag{4.11}$$

where i = Impact cost of the stock.

4. Market acceptance dimension
   a. Volume: It is the total traded volume of a security in a time period.

The liquidity of an investment is a prime concern of an investor. Higher the volume and turnover of a stock, the more liquid it would be regarded. Griffin et al. (2007) found a positive relationship of returns and turnover. This relationship was found to be statistically and economically significant. Hvidkjaer (2008) also studied the impact of small trades on the cross section of stock returns. Tsuchida et al. (2012) analysed the changes in portfolio performance under weight and turnover constraints. Sun (2011) studied the complex relationship between trading volume and securities price. DeMiguel et al. (2011) found that Sharpe ratios of the portfolio and certainty-equivalent return is accompanied by a higher turnover. Many insights into the portfolio theory are gained by analysing the volume and turnover of securities.

   b. Price-to-earnings ratio: P/E ratio is a valuation ratio of a company's current share price compared to its per share earnings.

$$P\,/\,E\ Ratio = \frac{Market\ Value\ per\ share}{EPS} \tag{4.12}$$

$p/e = P_t/EPS_t$ where P_t is the closing stock price in time period t and *EPS*t is the earnings per share in time period t.

A high P/E suggests that investors are expecting higher earnings growth in the future compared to companies with a lower P/E. It is useful for the investors to compare the P/E ratios of one company to other companies in the same industry, to the market in general or against the company's own historical P/E. An important problem that arises with the P/E measure is that the denominator (earnings) is based on an accounting measure of earnings that is susceptible to forms of manipulation, making the quality of the P/E only as good as the quality of the underlying earnings number.

A P/E ratio neither too high nor too low is desirable as a company with high P/E represents overvalued stock vis-à-vis its earning potential and a low P/E ratio represents lack of demand for a script despite good earnings. Due to the presence of this constraint in the portfolio selection problem, the feasible set would be reduced to the stocks of fewer companies i as per investor preferences and distribution of P/E ratios.

c. Market Capitalisation is the total value of a company's issued share capital as determined by its share price in the stock market. This represents the total number of shares outstanding times the price of each share in a time period t. It is calculated as the number of ordinary shares in an issue multiplied by the previous day's closing share price.

$$\text{Market Capitalisation} = (\text{share price} * \text{shares in an issue})/100$$

d. Management Performance dimension: An investors desire to invest in efficiently managed organisations may be captured through inclusion of the operating profit margin in the portfolio selection model. This may be considered as an index of performance by the management of any organisation (Xidonas et al., 2008). It is calculated using published accounting figures.

Operating profit margin: $O_{PM} = OI \ / \ TR$ where *OI* represents operating income and *TR* represents total revenue. This is a measure of management efficiency as reported in company's financial statements.

e. Profitability dimension
   i. Sales Constraint: The inclusion of sales of the company into the portfolio selection model would capture the profitability dimension. This would ensure that only securities of companies making high sales are selected. For financial organisations such as banks total revenue would be included instead of sales. Depending upon the risk appetite of investors the boundary value of sales/total revenue may be set.
   ii. Net profit margin: Ballestero (1998) emphasised on preferences for profitability and safety of an investor. Net profit is a measure of the company's profitability after accounting for all the costs.

Often referred as the bottom line, it is calculated by subtracting company's total costs from the total revenue.

$N_{PM} = NI \,/\, TR$ where *NI* represents the income available to common shareholders excluding extraordinary items and *TR* represents total revenue.

iii. Earnings Per Share

Earnings per share are the portion of a company's profit allocated to each outstanding share of common stock. EPS serves as an indicator of a company's profitability per share.

$$eps = \frac{(net\ income - preference\ dividend)}{average\ number\ of\ shares}$$

f. Book-to-Market Ratio Constraint

Fama and French classified the stocks into three groups of portfolios: one of low book-to-market equity (B/M) ratio, one of medium B/M ratio and the last being of high B/M ratio. Three classes of book equity-to-market equity (B/M) value (low B/M, medium B/M and high B/M) are created. The stocks are divided into three book-to-market groups (high, medium and low) for the top 30 per cent, the middle 40 per cent and the bottom 30 per cent of the book-to-market values.

The book equity value of the stocks is the respective book value of common shareholder's equity plus the balance sheet deferred tax (if any) and minus the book value of preferred stocks and the book-to-market equity ratio is constructed by dividing their book equity value with their market equity value.

Post and Levy (2005), Taneja (2010), Mehta and Chander (2010) and Saleh (2010) emphasised on the superiority of Fama and French's size and book-to-market ratio in explaining the market returns in global as well as Indian context.

g. Governance impact

i. Free Float equities refers to stocks which are not held by the promoters and associated entities (where identifiable) of companies. Promoters' holding, government holding in case of public sector undertaking, shares held by promoters through American Depository Receipts (ADR's) or Global Depository Receipts (GDS's), associate companies, employee welfare trusts, strategic stakes by corporate bodies, investments under Foreign Direct Investment (FDI) category (where identifiable) and public lock-ins are subtracted to arrive at a free float factor.

$FF = n - c_s$ where n represents shares issued by the company and c_s represents any closely held shares. Free float factor (Investible Weight Factor, IWF) for each company in the index is determined based on the public shareholding of the companies as disclosed in the shareholding pattern submitted to the stock exchanges by the companies on a quarterly basis. The free float measures shares outstanding available for trading by the public. A very small free float per cent would be indicative of concentration of share holdings in a few hands. The companies eligible for inclusion of the model should have a minimum free float. Free float factor (Investible Weight Factor, IWF) for each company in the index is determined based on the public shareholding of the companies as disclosed in the shareholding pattern submitted to the stock exchanges by the companies on a quarterly basis.

ii. Promoter's holding

Promoter's holding/government holding in case of public sector undertaking are the shares held by promoters through ADR's or GDS Receipts. Contrary to the above float adjusted market capitalisation constraint, here the shareholdings of the promoters are specified. The share held by promoters should not be extremely low so as to disinterest the promoters in the well-being of the organisation neither should not be extremely high so that the interests of the shareholders are always sub-ordinated.

A constraint for minimum promoter's holding has been included in the portfolio selection model.

$$\xi \geq 0.10 \tag{4.13}$$

where ξ = Securities held by the promoters of a company.

iii. Institutional holding

Similar to the above mentioned promoter's holding constraint, the institutional holding constraint specifies the minimum shareholding with associate companies, employee welfare trusts, strategic stakes by corporate bodies, mutual funds, financial institutions, foreign institutional investments (FII's), investments under Foreign Direct Investment

(FDI) category (where identifiable) and public lock-ins. The amount of securities with the above mentioned institutions should not be less than ten per cent. Including this constraint in the portfolio selection model would assure our investor that various other institutional parties also have a stake in the securities selected.

$$\zeta \geq 0.10 \tag{4.14}$$

$$\zeta = Y_c + Y_{ew} + Y_{cb} + Y_{mf} + Y_{FI} + Y_{FII} + Y_{FDI} + Y_0 \tag{4.15}$$

where Y_c = amount of securities held by associate companies
Y_{ew} = amount of securities with employee welfare trusts
Y_{cb} = strategic stakes by corporate bodies
Y_{mf} = securities with mutual funds
Y_{FI} = securities held by financial institutions
Y_{FII} = investments under foreign institutional investors
Y_{FDI} = investments under foreign direct investments
Y_0 = any other shares held by institutions

h. Future investment opportunities

Free cash flows: $FCF = CF - I - D$ where CF represents cash flows, I represent capital expenditures and D represents the total cash dividends paid for the fiscal year.

The first two criterion namely capital return and dividend yield reflect on the general return/performance of portfolio, beta reflects on the risk dimension, volume and p/e ratio represents general market acceptance, management efficiency, profitability, size, governance and future investment opportunities are reflected by the other criterion. The theoretical presentations of these criteria can be found in Niarchaos (2005), Alexander and Sharpe (1989), Jones (1985), Bodie et al. (2004) and Reilly and Brown (2005). The incremental explanatory power of cash flows to predict returns has been discussed by Clubb (2005). A comparison to the explanatory power of earnings has also been made. Fundamental analysis has been previously used in portfolio selection and allocation by Edirisinghe and Zhang (2007), Greig (1992) and Ou and Penman (1992). Having been regarded as the best fundamental indicator, free cash flows measures the ease with which businesses can grow and pay dividends to its shareholders. It is being increasingly used for valuation purposes. Gutterman (2011) discusses allocation of scare resources using the future cash flows. Wu (2013) considers self-financing mean-variance portfolio selection with stochastic cash flows.

Griffin et al. (2007) signified the importance of relationship between past returns and liquidity in the markets for the individual investor for deciding his portfolio. Bekaert et al. (2007) studied the impact of market liquidity on expected returns in 19 emerging markets. Liquidity of the securities was found to be an important factor in explaining return on a portfolio globally. Agarwalla and Pandey (2010) found that information and liquidity effect of block trades at NSE have a permanent price impact.

In addition to the above mentioned security evaluation criterion serving as constraints to the problem of portfolio selection, the following traditional constraints are also included.

5. Funds Exhaustion

As in the "standard" portfolio selection model, an investor has to determine the percentage of funds that will be invested in various available securities. The selection of these fractions X_1, X_2 ... X_n invested in n securities has to be such that the sum of all the fractions is 1. This constraint implies that 100 per cent of the amount available has to be invested and no money can be kept idle.

$$\sum_{i=1}^{n} X_i = 1 \tag{4.16}$$

6. No Short Sales

The proportion of funds invested in each of the securities must be positive or zero. Negative fractions denoting short selling of securities do not form a part of the feasible set.

$$X_i \geq 0 \quad i = 1 \ldots n$$

7. Upper Bounds

This constraint specifies the maximum investment that is allowed in a security. It would enable investment in a larger number of scripts and lead to company diversification. The quantity of each security i (stock of a particular company) or related equity investments of a single company that is included in the portfolio is limited to ceiling limit. Such ceilings are commonly practised by investors in the stock markets. Regulations related to them are also present for mutual funds in India. According to clause 11 of the seventh schedule of Securities and Exchange Board of India (mutual fund) Regulations, 1996, on restriction of investments, a mutual fund scheme shall not invest more than five per cent of its NAV in the equity shares or equity related investments of a single company in case of an open ended company and ten per cent for close ended schemes. The quantity of each security *i* (stock of a particular company) or related equity investments of a single company that is included in the portfolio is limited within a given interval. A minimum value ε_i and a maximum value U_i for each security *i* is specified such that

$$x_i = 0 \vee \varepsilon_i \leq x_i \leq U_i \tag{4.17}$$

where $i = (1, \ldots, n)$

This constraint enables to consider the aspect of real world finance as securities cannot be purchased in any quantity. Only minimum lot sizes can be bought and the amount of money to be invested in a single equity must be a multiple of a given minimum lot.

If U_i is greater than 1 for all *i* then the obtainable set is unaffected. It may also be unaffected when some U_i are less than 1 provided the corresponding X_i in the solution set is less than U_i at every point on the boundary. As the upper bounds U_i are decreased the set of obtainable EV combinations will eventually shrink.

The boundary of the feasible set even after introducing this constraint will consist of pieces of parabolas and horizontal and vertical line segments. Only a parabolic segment can be contained within the efficient portion of the feasible EV set.[2] In some cases, the efficient portion may consist of just one segment or even a single point.

8. Lower Bounds

This buy-in threshold's constraint specifies a lower bound for securities to form a part of the portfolio. To reduce the transaction costs, this constraint is added to the portfolio selection problem. It prevents assets with small weights from being included in the portfolio. Securities below a lower bound *l*, are not a part of portfolio. Hall and Tsay (1988) and Taksar et al. (1988) found the significance of transaction costs in forming portfolios with higher return.

$$X_i \geq l_i \tag{4.18}$$

9. Industry Diversification

To make the portfolios more stable. industrial diversification becomes important. Hennessy and Lapan (2003), Jeyachitra et al. (2010) and others supported the concept of portfolio diversification. On the contrary, Ivkovik et al. (2008) warned against too much diversification leading to non-performance of the portfolios.

The constituent securities in the portfolio selected as per the model shall not be dominated by any specific industry. Like, the S&P CNX, Nifty is a well diversified 50 stock index comprising for 24 sectors of the economy including information technology, automobiles, the FMCG sector, mining, gas, metals, textiles, banks, construction, etc.

The industry diversification constraint would set the maximum limit on the proportion of funds (weights) that can be invested in the stocks of a particular industry. This limit would break the dominance of any one industry in the chosen portfolio.

$$\sum_{z=a}^{l} I \leq \alpha \tag{4.19}$$

where

$$\sum_{z=a}^{l} I = X_a + \ldots X_l \tag{4.20}$$

$X_a \ldots X_n$ are the proportion of funds invested in securities of a particular industry z. α = A fixed proportion of securities of an industry in the portfolio.

10. Cardinality

An investor may wish to specify the number of assets in his/her portfolio for the purpose of monitoring and control. Jobst et al. (2001) examined the effects of cardinality constraints and transaction round lot restriction on the portfolio selection problem. The constraints of high practical importance make the efficient frontier discontinuous and approaches for computation of this frontier were suggested. Chang et al. (2000) calculated the efficient frontier for cardinality constrained portfolio optimisation problem. Fernandez and Gomez (2007) applied a heuristic method to trace out the efficient frontier for portfolio selection problem including cardinality and bounding constraints. Soleimani et al. (2009) solved using genetic algorithm portfolio selection problem with cardinality constraint. The cardinality constraint is an important variable in determining the portfolio composition of an investor. Hence, it is a part of this model.

The number of securities that can compose the portfolio is bounded, therefore we have two values $k_{\min}$ and $k_{\max}$.

$$1 \le k_{\min} \le k_{\max} \le n\,, \quad \text{and} \tag{4.21}$$

$$k_{\min} \le \sum_{i=1}^{n} z_i \le k_{\max} \tag{4.22}$$

where

k_{min} = minimum number of securities in the portfolio
k_{max} = maximum number of securities in the portfolio
z_i = number of securities in the portfolio

For testing of the model, portfolios are being created with stocks at Nifty, hence k_{max} would be 50, whereas we fix k_{min} to be ten.

4.2.4 Modelling constraints for an investor

After identifying the objectives and constraints of an investor an attempt is made here to form the portfolio selection model. The main goal of the multi-objective programming model developed is to provide Pareto optimal portfolios as per the criterion identified. These criteria are translated into constraints of the model. In addition to this completeness constraint, which assures 100 per cent capital remains invested, upper bounds for a maximum investment of 15 per cent in a security are also incorporated. The decision variables of the model are continuous: X_i represent the weight of *i*th security in the portfolio.

Modelling Optimisation Process

$$\text{Min } \sigma_P^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} x_i x_j \sigma_{ij}$$

subject to:

$$\sum_{i=1}^{m} x_i = 1$$

$$\sum_{i=1}^{m} x_i E(r_i) \ge R_p$$

$$\sum_{i=1}^{m} d_i x_i \ge D_P$$

$$\sum_{i=1}^{m} \beta_i x_i \le \beta_P$$

$$\sum_{i=1}^{m} e_i x_i \le e_P$$

$$\sum_{i=1}^{m} l_i x_i \ge L_P$$

$$\sum_{i=1}^{m} v_i x_i \ge V_P$$

$$\sum_{i=1}^{m} p / e_i x_i \ge P / E_P$$

$$\sum_{i=1}^{m} o_{PM_i} x_i \ge O_{PM_P}$$

$$\sum_{i=1}^{m} S_i x_i \ge S_P$$

$$\sum_{i=1}^{m} n_{PM_i} x_i \ge N_{PM_P}$$

$$\sum_{i=1}^{m} eps_i x_i \ge EPS_P$$

$$\sum_{i=1}^{m} b / m_i x_i \ge B / M_P$$

$$\sum_{i=1}^{m} c_i x_i \ge C_P$$

$$\sum_{i=1}^{m} ff_i x_i \ge FF_P$$

$$\sum_{i=1}^{m} ph_i x_i \ge PH_P$$

$$\sum_{i=1}^{m} ih_i x_i \ge IH_P$$

$$\sum_{i=1}^{m} fcf_i x_i \geq FCF_P$$

$$\sum_{z=a}^{l} I \leq \alpha$$

$$0 \leq x_i \leq X$$

where

x_i	amount invested in security *i*
σ_i	standard deviation (risk) of returns for security *i*
σ_p	standard deviation (risk) of returns for the portfolio
$E(r_i)$	expected return of security *i*
d_i	dividend yield of security *i*
β_i	beta coefficient for security *i*
e_i	unsystematic risk for security *i*
l_i	liquidity measured by impact cost for security *i*
v_i	volume of security *i*
p/e_i	price to earnings ratio of security *i*
o_{pmi}	operating profit margin of security *i*
s_i	sales for security *i*
n_{pmi}	net profit margin of security *i*
c_i	market capitalisation of security *i*
eps_i	earnings per share of security *i*
b/m_i	book-to-market ratio of security *i*
ff_i	free float percent of security *i*
ph_i	promoters holding of security *i*
ih_i	institutional holding of security *i*
fcf_i	free cash flow of security *i*

$$\sum_{z=a}^{l} I = X_a + \ldots X_l$$

$X_a \ldots X_n$ are the proportion of funds invested in securities of a particular industry *z*. α = A fixed proportion of securities of an industry in the portfolio.

R_p	portfolios targeted expected return
D_p	portfolios targeted dividend yield
β_p	portfolios targeted beta

e_p portfolios targeted unsystematic risk
L_p portfolios targeted liquidity
V_p portfolios targeted volume
P/E_p portfolios targeted P/E ratio
O_{pmP} portfolios targeted operating profit margin
N_{pmP} portfolios targeted net profit margin
C_p portfolios targeted market capitalisation
S_p portfolios targeted sales
EPS_p portfolios targeted earnings per share
b/m_p portfolios targeted book-to-market ratio
PH_p portfolios targeted promoters holding
IH_p portfolios targeted institutional holding
FF_p portfolios targeted free float
FCF_p portfolios targeted free cash flow
X the maximum amount of investment in a security/the upper bound

4.3 Multivariate regression: model formulation

It is important to find out the financial variables that explain the cross section of returns on a security. Certain variables have significantly higher explanatory power of returns and must be included in the portfolio selection model. The factors which do not have a predictive power may be considered less important and could be excluded from the portfolio selection model. To find out the variables which can significantly explain returns, this multivariate regression analysis is conducted. The multiple regression analysis undertaken regresses the returns on a security and excess returns to standard deviation ratio on a number of financial factors. Two multiple regression equations were estimated.

4.3.1 Multiple regression model 1

The regression equation with return on the security as the dependant variable and earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-book value ratio, price-earnings ratio, promoters' shareholding, sales, turnover, unsystematic risk and volume as independent variables was formulated.

$$R_i = \alpha + \beta_1 eps + \beta_2 D_i + \beta_3 \lambda + \beta_4 i + \beta_5 \varsigma + \beta_6 \mathbb{C} + \beta_7 \pi + \beta_8 \frac{p}{b} + \beta_9 \frac{p}{e} + \beta_{10} \xi + \beta_{11} S + \beta_{12} t + \beta_{13} \varepsilon + \beta_{14} v \tag{4.23}$$

where

R_i	=	return on *i*th security
α	=	constant term
eps	=	earnings per share
D_i	=	dividend declared
λ	=	free float
i	=	impact cost
ς	=	institutional holding
$\mathbb{C}$	=	market capitalisation
π	=	net profit
p/b	=	price-book value ratio
p/e	=	price-earnings ratio
ξ	=	promoters holding
S	=	sales
t	=	turnover
ε	=	unsystematic risk
v	=	volume
$\beta_1, \beta_2, \ldots, \beta_{14}$	=	Regression coefficients

4.3.2 Multiple regression model 2

The regression equation with excess return to standard deviation on the security as the dependant variable and earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-book value ratio, price-earnings ratio, promoters' share-holding, sales, turnover, unsystematic risk and volume as independent variables was formulated.

$$S_t = \alpha + \beta_1 eps + \beta_2 D_i + \beta_3 \lambda + \beta_4 i + \beta_5 \varsigma + \beta_6 \mathbb{C} + \beta_7 \pi + \beta_8 \frac{p}{b} + \beta_9 \frac{p}{e} + \beta_{10} \xi + \beta_{11} S + \beta_{12} t + \beta_{13} \varepsilon + \beta_{14} v \tag{4.24}$$

where

Excess Return to standard deviation ratio:

$$S_t = \frac{(R_i - R_f)}{\sigma_i} \tag{4.25}$$

R_f = return on the risk free asset

σ_i = standard deviation of the *i*th security

All other terms are the same as in multiple regression equation 1 explained above.

4.4 Granger causality tests

The Granger (1969) approach[3] examines the question whether x causes y to find how much of current y can be explained by past values of y and then find whether adding lagged values of x improves the explanation or not. If x helps in prediction of y then y is said to be Granger caused by x. This would happen when coefficients of lagged values of x are statistically significant. Granger causality measures precedence and information content and does not imply that y is the effect or result of x. Regressions run are of the form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \ldots + \alpha_l y_{t-l} + \beta_1 x_{t-1} + \ldots + \beta_l x_{t-l} + \varepsilon_t \qquad (4.26)$$

where

y_t = returns on security
x_t = the causation factors such as eps, dividend, net profit, promotional and institutional holding, impact cost, unsystematic risk, sales, volume, turnover, etc.
α_t = coefficient of lagged values of returns
β_t = coefficient of lagged values of various causation factors.

F-statistics are reported for each such equation. These are the Wald statistics for the joint hypothesis:

$$\beta_1 = \beta_2 = \ldots = \beta_1 = 0. \qquad (4.27)$$

4.5 A utility approach

The attitude of an individual comprises of a spoken component comprising of a person's belief and an unspoken component reflecting his/her feelings and emotions. Financial risk tolerance captures both of these aspects. Financial risk tolerance may be inferred by the portfolio allocations already made by the individuals (Schooley and Worden, 1996), interpreting the responses to alternate investment choices (Hey, 1999) and subjective questions (Hanna et al., 1998).

An investor is said to be risk averse if he/she prefers less risk to more risk, all else being equal. Such an investor will require compensation for taking additional risk. When the additional expected return for undertaking equal additional risk is increasing, it refers to a "diversifier" who is treated as a rational investor. When the additional return for equal additional risk is expected to fall progressively then such an investor is known as a "plunger". The opposite of risk aversion is risk seeking (sometimes called risk loving). A risk seeking investor prefers more risk for little return, all else being equal. Financial theories generally assume investors are not risk seeking. However, risk seeking behaviour is observable in actual life. People who play lotteries or gamble at casinos accept a negative expected return in exchange for the thrill of financial risk.

Between risk aversion and risk seeking is a state called risk neutrality. An investor is risk neutral if he/she is indifferent to risk. He/she will neither pay to avoid it nor to take it. In a nutshell, risk does not affect his/her decisions. Financial theories generally assume investors are not risk neutral. All individuals keep their investments in the form of a portfolio. An individual would not keep the savings in only one form (cash). An investor tends to allocate the savings in a manner that it gives rise to a portfolio.

A large number of individuals follow the principle of safety first and try to reduce as far as possible the chance of a catastrophe. The possibility of a disaster directly affects utility function of an investor. The utility function if interpreted in terms of minimisations of the chance of disaster, assumes only two values, e.g., one if disaster does not occur and zero if it does. The level of disaster may change depending on the changes in return and loss.

Markowitz extended the work on portfolios to include utility analysis (See Figure 4.1). "The utility function has three inflection points. The middle inflection point is defined to be at customary level of wealth. Except in cases of recent windfall gains and losses,

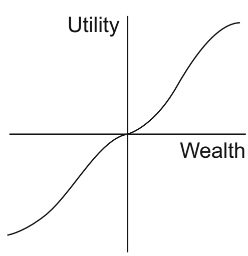


Figure 4.1 Utility and wealth

Source: Markowitz, H. "The Utility of Wealth," *Journal of Political Economy*, Vol. 60, No. 2, April 1952, pp. 151–158.

customary wealth equals present wealth. The first inflection point is below; the third inflection point is above, customary wealth. The distance between the inflection points is a non-decreasing function of wealth. The curve is monotonically increasing but bounded; it is first concave, then convex then concave and finally convex" (Markowitz, 1952).

An investor would try to maximise the utility. Expected value of utility is given by

$$U = U\,(C_1, C_2 \ldots C_t \ldots C_T, W_T) \tag{4.28}$$

where

C_t = real value of consumption in period t
T = time of death
W_T = bequest
U = utility of the investor's lifetime consumption pattern.

The problem with portfolios arise when the investor does not wish to consume in one period but desires to carry it over to the next period. This portfolio problem then requires the selection of that combination of investments which will yield him/her maximum utility. In the classical framework, an investor faces infinite possible combinations of risk free asset and the market portfolio but the ultimate allocation depends on the investors' utility function.

Figure 4.2 plots the efficient combination of portfolios and indifference curves of a normal investor (diversifier) in the capital markets. Y axis represents the expected returns and X axis represents the expected risk. The indifference curves A1 to A6 represent the utility to an investor in risk-return space. The utility map has not been restricted only to the positive quadrant. The intercept 'a' of the concave quadratic utility function could be negative depicting the minimum risk investor would need to take with the risk-return combinations of assets available in the capital markets. A positive intercept would depict the risk free rate of return.

RFR represents the risk free rate of return. The efficient frontier proposed by Markowitz consists of all risky assets. All points below this efficient frontier are attainable but undesirable. All points above this efficient frontier are desirable but unattainable with a combination of only risky assets. The capital market line (CML) represents the risk-return combinations of available assets. The point of tangency of the efficient frontier and CML represents the market portfolio M. Market portfolios is that portfolio which consists of all risk assets namely equities, bonds, derivatives, real estate, antiques, art and all other risky assets required for a completely diversified portfolio. The combination of risky assets in market portfolio results in elimination of the unsystematic risk. The ray joining RFR and B is dominant to all points on the Markowitz efficient frontier as they have a combination of risky and riskless assets. With the inclusion of a riskless asset, one can increase the return of the portfolio without increase in the risk of that portfolio. Points to the left of M represents lending portfolios (L). In lending portfolios, investors construct a portfolio having a combination of market portfolio and riskless assets (one can lend money at risk free rate). Points to the right of M represents borrowing portfolios (B). In borrowing portfolios, investors construct a portfolio with partly own funds and partly by borrowing funds at the risk free rate. Points L and B depend upon the indifference curves of an investor and were originally plotted by Sharpe (1964). For a diversifier, the utility curves will shift to the left of M (on ray M-RFR). Risk aversion in investors often results in investors allocating their funds (W) on the ray RFR-M having a combination of riskless asset and market portfolio (M). As the wealth increases, a desire to earn more wealth is often resulting in investors moving from ray RFR-M to M-B thereby creating leverage portfolios.

A diversifier would invariably prefer minimum risk portfolio thereby investing in either market portfolios or lending portfolios thereby minimising the unsystematic risk. A condition of equilibrium would exist when the slope of utility curves becomes equal to the slope of the capital market.

To undertake the utility analysis, risk penalty is calculated using the formula:

$$\text{Risk Penalty} = \text{Risk Squared}/\text{Risk Tolerance} \tag{4.29}$$

Utility is calculated by subtracting the value of risk penalty from the value of expected return.

$$\text{Utility} = \text{Expected Return} - \text{Risk Penalty} \tag{4.30}$$

Risk squared is the variance of return of the portfolio. Risk tolerance is a number from zero to 100. Equations 4.64 and 4.65 are used for quantitative utility analysis. The amount of risk tolerance shows the investor's willingness to bear more risk for more return. Low (high) tolerance

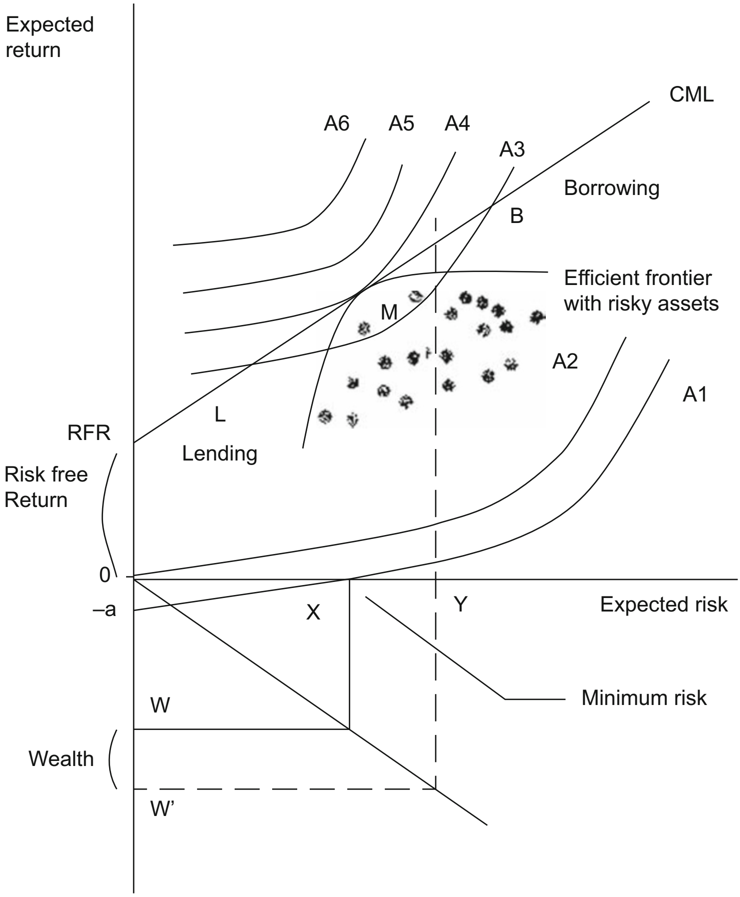


Figure 4.2 Diversifier's utility curves, capital market line and portfolio choice

indicates low (high) willingness. Risk penalty is less as risk tolerance is increased.

Utility analysis is undertaken using both quantitative method and graphical analysis. For the graphical analysis, the portfolio return and variance of all the constructed portfolios are plotted over the indifference curves of various types of investors (diversifier, plunger, risk neutral and risk lover). The indifference curves exhibit the level of utility derived by a particular type of investor from a specific risk-return trade-off. By superimposing the risk-return combination of portfolios on the utility/indifference curves, the choice of portfolio by the investor is determined graphically.

4.6 Performance measures for portfolios

Sharpe (1966) gave a summary measure of portfolio performance. The measure adjusts performance for risk. It measures risk premiums of the portfolio (the excess return required by investors for the assumption of risk) relative to the total amount of risk in the portfolio. His index is given by

$$S_t = \frac{(\bar{r}_i - r^*)}{\sigma_t} \tag{4.31}$$

where

S_t = Sharpe Index
$\bar{r}_i$ = average return on portfolio i
r^* = riskless rate of interest
σ_t = standard deviation of the returns of portfolio t.

The index summarises risk and return of a portfolio in a single measure, categorising the performance of a portfolio on a risk adjusted basis. A larger value denotes better performance of the portfolio. The Sharpe ratios have been computed for the eight portfolios modelled, Markowitz's portfolio and index portfolio Nifty 50 for comparison of performance.

Treynor (1965) provided a measure for portfolio performance based on the concept of characteristic line. This is a linear representation of an otherwise curvilinear relationship between market rate of return and portfolio rate of return. The ideal portfolio lies to the left of the imaginary 45 degree line starting at the origin. The slope of the characteristic line is the beta coefficient. The steeper this line, the more volatility the portfolio possesses. The concept has been incorporated into a single index to measure portfolio performance accurately.

$$T_n = \frac{(\bar{r}_n - r^*)}{\beta_n} \tag{4.32}$$

where

T_n = Treynor Index
$\bar{r}_n$ = average return on portfolio n

r^* = riskless rate of interest
β_n = beta coefficient of portfolio n

The risk and return are summed up in a single number categorising the performance of the portfolio. The Treynor ratio for all the portfolios have also been calculated to rank the portfolios.

4.7 Tests for equality

This tests the null hypothesis that all series in the group have the same mean, median (distribution) or variance.

4.7.1 Mean equality test

This test is based on a single-factor between-subjects analysis of variance (ANOVA). The basic idea is that if the subgroups have the same mean, then the variability between the sample means (between groups) should be the same as the variability within any subgroup (within group).

The between and within sums of squares are defined as:

$$SS_B = \sum_{g=1}^{G} n_g(\overline{x}_g - \overline{x})^2 \tag{4.33}$$

$$SS_B = \sum_{g=1}^{G} \sum_{i=1}^{n} (x_{ig} - \overline{x}_g)^2 \tag{4.34}$$

where
x_{ig} = i^{th} observation in group g
I = $1 \ldots n_g$ for groups g = $1, \ldots, G$
$\bar{x}_g$ = Sample mean within group g
$\bar{x}$ = Overall sample mean

The F-statistic for the equality of means is computed as:

$$F = \frac{SS_B / (G-1)}{SS_W / (N-G)} \tag{4.35}$$

where N = total number of observations.

The F-statistic has an F-distribution with G-1 numerator degrees of freedom and N-G denominator degrees of freedom under the null hypothesis of independent and identical normal distribution, with equal means and variances in each subgroup.

4.7.2 Variance equality tests

Variance equality tests evaluate the null hypothesis that the variances in all subgroups are equal against the alternative that at least one subgroup has a different variance.

F-test

It computes the variance for each subgroup and denotes the subgroup with the larger variance as *L* and the subgroup with the smaller variance as *S*. Then the F-statistic is given by:

$$F = s_L^2 \,/\, s_S^2 \tag{4.36}$$

where S_g^2 = the variance in subgroup g

This F-statistic has an F-distribution with n_L-1 numerator degrees of freedom and n_S-1 denominator degrees of freedom under the null hypothesis of equal variance and independent normal samples.

4.8 To sum up

In this chapter, an attempt has been made to provide the justification for the research design. A mean-variance efficient portfolio selection model is developed theoretically with the objective of risk minimisation and financial, accounting based and corporate governance constraints. The main objective of an investor is minimisation of risk (variance) from the portfolio as this leads to maximisation of his/her utility. In addition to the classical constraints of funds exhaustion, no short sales and upper bounds many new constraints have been added for accommodating the needs and limitations of a present day investor. The investor's desire for capital returns and dividend gains are modelled as constraints. Liquidity measures such as volume, turnover and impact cost (a new variable available from National Stock Exchange's official website) are introduced to tackle investor's desire for a liquid portfolio. Diversification has been incorporated through industrial diversification and company diversification constraints.

To discourage very small fractions of shares in the portfolio leading to high transaction costs, lower bounds have been introduced. Beta factor, which has been found to significantly explain returns, is added as a constraint in the problem. The unsystematic risk of a company and its conditional volatility is also included. Accounting figures and ratios such as net profit, sales, earnings per share, price-earnings ratio and book-to-market ratio are included to accommodate for the earnings from a portfolio. Investors are also interested in securities where other stakeholder's interest is substantive. This desire has been considered through promoter's shareholding and institutional shareholding constraint. The lead-lag relationship between the spot market and derivatives market and its effect on the equity portfolio selection is assimilated through the open interest constraint, volume and turnover of futures as the security constraint.

The Value-at-Risk, cut-off rate, market capitalisation and free float factor are also considered. The number of mutual funds investing in the security depicts the popularity of a script among the technically competent mutual fund managers. This number has been added as a constraint. Risk penalty and portfolio utility measures as given by Markowitz are also set as constraints in the portfolio selection problem. A cardinality constraint to limit the size of the portfolio has also been added.

A multivariate regression technique was discussed to identify the variables which have the explanatory power to estimate returns. Two multiple regression equations are estimated one with returns as the dependent variable and the other with excess return to standard deviation ratio as the dependent variable. The list of independent variables included all the portfolio constraints. The methodology for Granger causality tests to find out relationship of causation between returns on a security and the variables set as constraints in the programming problem has been also discussed.

The portfolio utility concept of Markowitz has been discussed for its application to the research work. The methodology followed for calcu-

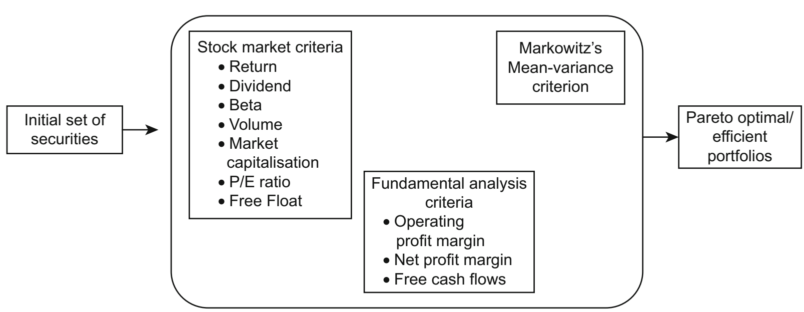


Diagram 4.3 Diagram of the proposed methodology

lating the utility and portfolio evaluation ratios has been elaborated. The performance evaluation measures as proposed by Sharpe (1966) and Treynor (1965) have been explained. The tests for equality of returns, variances and portfolio utilities among the alternate portfolio selection model formulations have been deliberated upon.

5

Mean-Variance Quadratic Programming Portfolio Selection Model: An Empirical Investigation of India's National Stock Exchange

Application of theoretical portfolio selection models to the real life capital markets in order to facilitate the investor in making the optimal decisions requires serious research. The entire purpose of portfolio modelling is defeated if the model created cannot be put to practical use. A portfolio selection model should not be so complex as to discourage the investors from using it. A large number of investors exist in the equity markets at any point of time. All investors in the market may not be identical. They may differ with respect to their risk bearing capacity, preference for quick gains versus regular income or other priorities. Thus, the same model may not be applicable to all of them. The practical application of portfolio selection models assumes significant importance.

An attempt has been made in this chapter to empirically test the model developed so far on India's National Stock Exchange's Nifty 50 index. Eight portfolio selection model formulations depicting different type of investors are constructed from the main model. Alternate portfolio formulations developed for investors present in the equity markets are tested using quadratic programming framework. We have attempted to test the Markowitz portfolio selection model as well. Graphical comparison of the quadratic programming portfolio selection model formulations with Markowitz's efficient frontier shows the performance of the modelled portfolios in the risk-return space. A few constraints have been excluded during the empirical testing due to unavailability of data, computational issues and programming limitations.

Using multivariate regression analysis and Granger causality tests, an attempt has been made to reduce the number of constraints initially mentioned in the model. The actual, fitted and residual values of these estimated equations are presented. The ideal portfolio model formulation is constructed by programming the reduced constraints. Only the variables with high explanatory power (beta coefficients) for returns, excess returns to standard deviation ratio or a proven relationship of causation with security returns are included as constraints here.

Using the portfolio utility concept, the utility derived from ideal portfolio vis-à-vis Markowitz's portfolio and the index portfolio Nifty 50 is examined. The risk-return points of all the eight portfolios constructed are superimposed on the utility curves of various types of investor: the plunger, the diversifier, the risk lover and risk neutral. This exhibits the choice of different types of investors.

A comparison and ranking of portfolios constructed from the eight portfolio selection model formulations is attempted using Sharpe and Trenyor performance evaluation ratios. Using the t-test, F-test, Anova and other statistical measures, we have examined the equality between (1) the returns from the ideal portfolio and returns from the index portfolio Nifty 50; (2) the risk of the ideal portfolio and the risk of Markowitz's portfolio; (3) the risk of the ideal portfolio and the index portfolio Nifty 50; (4) the utility of the ideal portfolio and utility of Markowitz's portfolio; (5) the utility of the ideal portfolio and utility of the index portfolio Nifty 50.

By using the quadratic programming algorithm, an attempt has been made to create portfolios which optimise across various constraints and are also efficient in Markowitz's mean-variance sense. These portfolios, created using the alternate model formulations, provide maximum utility to the investors with varying preferences. The resultant portfolios are a solution suited to the needs of a present day investor.

5.1 Sample size and data collection

The empirical testing of the quadratic programming portfolio selection model has been undertaken using monthly stock returns, beta, free float, promoters holding, institutional holding, trading volume and turnover, impact cost as a measure of liquidity for firms part of Nifty, 91 day Treasury bill rates and monthly returns on Nifty 50 (National Stock Exchange's (NSE) value weighted index) over a period of 12 years starting from April 2000 to March 2012. Also, annual accounting data such as book-to-market equity, market capitalisation, sales, net profit, dividends, earnings-per-share and price-to-earnings ratio total assets from the annual reports of the constituent companies have been used. Industrial diversification has been attempted by dividing the companies that are part of Nifty into 23 different sectors.

The volume of trade and the leadership of this premier stock exchange of India justify the rationale for choosing NSE over other stock exchanges in India. The data have been collected from the official website of National Stock Exchange Limited (www.nseindia.org) and Centre for Monitoring Indian Economy's (CMIE) database PROWESS. The measure for risk-free rate of interest, 91 day T-bill rate, has been taken from the official website of Reserve Bank of India (www.rbi.org.in).

For the quadratic programming portfolio selection model of the companies whose securities are considered for inclusion are domiciled in India, traded on NSE and together compose the S&P CNX Nifty index. All the assets are common shares listed on the NSE (which are of equity and not of a fixed income nature). Convertible stocks, bonds, warrants, rights, futures and options and preferred stock that provide a guaranteed fixed return are not included in the empirical illustration of this model.

5.2 Software used

The software used for the research includes Statistical Package for Social Sciences (SPSS 16), E-views 5.1 and Lingo 13. The generation of covariance matrix (50x50) of the assets from the series of return, multiple regression estimation and Granger causality tests have been attempted through E-views.

The eight scenarios based non-linear portfolio selection models have been programmed using the Lingo 13. Both the local solver and global solver of this software were used to generate solution to the portfolio selection problems.

5.3 Mean-variance portfolio selection model: empirical testing

For testing the mean-variance portfolio selection model in the multiple constraints setting, the 50 securities composing the Standard and Poor's CNX Nifty have been coded from $X_1, X_2 \dots X_{50}$. The security and industry coding is given in Table 5.1.

Table 5.1 Security and industry coding for the basket of assets

Security Code	Securities	Industry Code	Security Classification
X1	A C C Ltd.	I_{13}	Cement and cement products
X2	Ambuja Cements Ltd.	I_{13}	Cement and cement products
X3	Asian Paints Ltd.	I_{23}	Paints
X4	Axis Bank Ltd.	I_1	Banks
X5	Bajaj Auto Ltd.	I_{14}	Automobiles – 2 and 3 wheelers
X6	Bank Of Baroda	I_1	Banks
X7	Bharat Heavy Electricals Ltd.	I_{14}	Electrical equipment
X8	Bharat Petroleum Corp. Ltd.	I_3	Refineries
X9	Bharti Airtel Ltd.	I_{12}	Telecommunication
X10	Cairn India Ltd.	I_{10}	Oil exploration/ production
X11	Cipla Ltd.	I_9	Pharmaceuticals
X12	Coal India Ltd.	I_{16}	Mining
X13	D L F Ltd.	I_{20}	Construction
X14	Dr. Reddy's Laboratories Ltd.	I_9	Pharmaceuticals
X15	G A I L (India) Ltd.	I_{18}	Gas
X16	Grasim Industries Ltd.	I_{13}	Cement and cement products
X17	H C L Technologies Ltd.	I_2	Computers – software
X18	H D F C Bank Ltd.	I_1	Banks
X19	Hero Motocorp Ltd.	I_{14}	Automobiles – 2 and 3 wheelers
X20	Hindalco Industries Ltd.	I_{19}	Aluminium
X21	Hindustan Unilever Ltd.	I_{15}	Diversified
X22	Housing Development Finance Corp. Ltd.	I_5	Finance – housing
X23	I C I C I Bank Ltd.	I_4	Banks
X24	I T C Ltd.	I_1	Cigarettes
X25	Infosys Ltd.	I_2	Computers – software
X26	Infrastructure Development Finance Co. Ltd.	I_{21}	Financial institution
X27	Jaiprakash Associates Ltd.	I_{20}	Construction
X28	Jindal Steel & Power Ltd.	I_{11}	Steel and steel products
X29	Kotak Mahindra Bank Ltd.	I_1	Banks
X30	Larsen & Toubro Ltd.	I_7	Engineering
X31	Mahindra & Mahindra Ltd.	I_6	Automobiles – 4 wheelers
X32	Maruti Suzuki India Ltd.	I_6	Automobiles – 4 wheelers
X33	N T P C Ltd.	I_8	Power
X34	Oil & Natural Gas Corp. Ltd.	I_{10}	Oil exploration

Table 5.1 Continued

Security Code	Securities	Industry Code	Security Classification
X35	Power Grid Corp. Of India Ltd.	I_8	Power
X36	Punjab National Bank	I_1	Banks
X37	Ranbaxy Laboratories Ltd.	I_9	Pharmaceuticals
X38	Reliance Industries Ltd.	I_3	Refineries
X39	Reliance Infrastructure Ltd.	I_8	Power
X40	Sesa Goa Ltd.	I_{16}	Mining
X41	Siemens Ltd.	I_{17}	Electrical equipment
X42	State Bank Of India	I_1	Banks
X43	Steel Authority Of India Ltd.	I_{11}	Steel and steel products
X44	Sterlite Industries (India) Ltd.	I_{22}	Metals
X45	Sun Pharmaceutical Inds. Ltd.	I_9	Pharmaceuticals
X46	Tata Consultancy Services Ltd.	I_2	Computers – software
X47	Tata Motors Ltd.	I_6	Automobiles – 4 wheelers
X48	Tata Power Co. Ltd.	I_8	Power
X49	Tata Steel Ltd.	I_{11}	Steel and steel products 2
X50	Wipro Ltd.	I_2	Computers – software

5.4 Descriptive statistics – returns

The descriptive statistics for monthly returns of the component securities of the Nifty index of the NSE are presented in Table 5.2. Maximum monthly mean returns of 1.16 per cent was offered by Cairn India Ltd. (X10) and minimum monthly mean returns to the extent of –0.09 per cent by Ambuja Cements Ltd. (X2). Cairn India Ltd. did not declare any dividend during the sample period of 12 years but gave the maximum capital gain return to the investors. Median returns exhibiting the mid value was the highest for Jaiprakash Associates (X27) and lowest for Steel Authority of India (X43). Maximum monthly return of 25.50 per cent is offered by Jindal Steel & Power Ltd. (X28). The minimum monthly return of all the equities was negative. This implies all the assets may result in losses in some circumstances or the other.

Investment in equity markets was found to be a risky proposition as the standard deviation (a measure of risk) was found to be higher than the mean (a measure of return) for all of the equities. The maximum volatility in returns measured by standard deviation could be observed for Jindal Steel and Power Ltd. (X28) and Asian Paints Ltd. (X3) were the least volatile. The skewness for any of the 50 securities is not exactly zero. However, it lies close to zero barring a very few securities. Nineteen

Table 5.2 Descriptive statistics: monthly returns for Nifty 50 securities

Securities	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
X1	0.25	0.37	10.51	–9.35	2.77	0.06	5.58
X2	–0.09	0.02	6.80	–7.84	2.45	–0.19	3.74
X3	0.58	0.42	5.50	–3.48	1.72	0.23	3.34
X4	0.54	0.41	8.17	–9.19	2.96	–0.05	3.89
X5	0.97	0.77	8.76	–4.63	2.45	0.51	4.25
X6	0.43	0.43	8.22	–9.98	2.92	–0.12	3.93
X7	0.45	0.39	9.81	–9.59	2.91	–0.35	4.90
X8	0.06	0.37	10.42	–11.08	3.30	–0.66	5.11
X9	0.47	0.43	9.51	–7.05	2.67	0.16	3.88
X10	1.16	1.07	12.22	–5.00	2.61	1.10	6.89
X11	0.39	0.07	9.99	–11.52	2.36	–0.17	8.45
X12	1.12	1.11	11.98	–4.98	3.53	1.45	6.71
X13	0.72	0.65	9.04	–8.14	3.70	0.11	3.30
X14	0.29	0.25	6.21	–10.21	2.14	–0.67	7.15
X15	0.37	0.28	8.05	–5.11	2.24	0.42	3.59
X16	0.08	0.17	8.51	–6.29	2.51	0.04	4.44
X17	0.51	0.48	15.36	–10.08	3.66	0.72	6.14
X18	0.65	0.41	7.53	–5.38	2.06	0.31	3.86
X19	0.28	0.25	11.64	–7.96	2.61	0.50	6.04
X20	0.48	0.21	13.83	–6.98	2.75	0.83	6.57
X21	0.35	0.06	11.42	–5.65	2.58	0.80	5.84
X22	0.50	0.33	17.29	–11.99	2.85	0.68	13.11
X23	0.55	0.47	15.46	–7.91	3.46	0.95	6.63
X24	0.47	0.28	8.13	–6.30	2.01	–0.25	5.33
X25	0.59	0.36	14.18	–7.87	2.96	0.95	7.44
X26	0.84	0.34	14.44	–8.65	3.57	0.85	6.02
X27	1.23	1.19	16.72	–8.12	3.91	0.64	4.81
X28	1.00	0.62	25.50	–7.72	3.94	2.03	13.27
X29	0.89	0.25	20.02	–7.07	3.49	1.51	9.13
X30	0.36	0.43	9.69	–7.38	2.52	0.05	4.54
X31	0.96	0.97	23.99	–12.79	3.36	1.68	18.51
X32	0.91	0.90	8.59	–5.11	2.23	0.06	4.57
X33	0.65	0.52	5.30	–3.79	1.74	0.27	3.26
X34	0.28	0.27	6.67	–6.54	2.33	–0.07	3.15
X35	0.30	0.25	6.40	–6.54	2.15	–0.05	4.56
X36	0.56	0.45	8.60	–7.16	2.84	–0.01	3.40
X37	0.34	0.31	5.03	–6.88	2.22	–0.32	3.27
X38	0.51	0.39	14.45	–5.50	2.63	1.83	11.46
X39	0.28	0.01	7.91	–11.62	2.89	–0.35	4.82
X40	0.54	0.18	13.54	–8.51	3.34	0.50	4.43
X41	0.42	0.33	17.30	–7.76	2.83	1.29	11.48
X42	0.50	0.49	9.94	–9.99	2.61	–0.25	5.10
X43	0.01	0.00	8.48	–9.82	2.96	–0.03	3.74
X44	0.58	0.33	15.90	–6.38	3.55	1.12	6.14
X45	0.04	0.22	5.46	–14.23	2.53	–1.55	9.72
X46	0.58	0.53	7.35	–6.96	2.21	0.01	5.00
X47	0.50	0.37	7.20	–11.05	2.90	–0.32	4.25
X48	0.32	0.34	12.54	–8.87	2.73	0.26	6.57
X49	0.64	0.81	12.26	–10.47	2.97	–0.03	5.01
X50	0.44	0.60	13.69	–15.36	3.17	–0.15	8.51

Note: Std. dev. stands for Standard Deviation.

of the equities are negatively skewed while 31 are found to be positively skewed showing that for most of the securities composing Nifty 50, the return distribution was skewed to the right thereby providing high returns.

For a series to be normally distributed the kurtosis should measure three. In case of the equity return distribution all the securities had a distribution with kurtosis measuring greater than three. For certain securities such as Housing Development Finance Corporation (X22), Jindal Steel and Power Ltd. (X28) and Mahindra & Mahindra Ltd. (X31), the kurtosis measured higher than 13. The monthly return distributions were found to be exhibiting leptokurtic distribution which compared to a normal distribution has a higher and sharper central peak and its tails are longer and flatter. Such distribution is representative of sharper price movements. The Jarque-Bera statistic is more than their critical values for most of the equity return series. This also signals that the monthly returns are not normally distributed. Exhibiting such kind of behaviour is very typical of a financial time series data for longer time periods.

5.5 Data inputs

To formulate the programming problem for minimising the variance of a portfolio, values for monthly returns of securities over 12 years were collected, aggregated and averaged. The quartile values ($Q_1$and Q_3), mean and median of all the securities were also calculated to set the target value of expected return in the eight different scenarios simulated. Similarly, monthly values for beta (measure of systematic risk), earnings-per-share, free float, impact cost (measure of liquidity), market capitalisation, price-to-book value ratio, shares traded (measure of volume), turnover, and price-to-earnings ratio (P/E ratio) were also tabulated.

Annual financial standalone figures for net profit, equity dividend and sales were collected to set the net profit, dividend and sales constraint in the problem. For promoters holding and institutional holding constraint quarterly values available were aggregated, averaged and mean, median, Q_1 (quartile one) and Q_3 (quartile three) were calculated and tabulated. The values for unsystematic risk of these securities were calculated as per the formula enumerated in the theoretical model using returns on the index, total variance and beta. Industrial diversification[1] across the 23 sectors was achieved by limiting the asset weights of a particular sector to 20 per cent.

The no short sales constraint and funds exhaustion constraint were also included in problem formulation. Company diversification/upper bound constraint was set at a level of 15 per cent in all the portfolios. The constraints of mutual funds investment in these securities, portfolio Value-at-Risk (VaR), volume, turnover and open interest of the futures and options segment of these securities, portfolio utility and risk penalty were dropped in the empirical testing of the model due to unavailability of data. The constraints on cut-off rate, conditional volatility and cardinality were also excluded due to computational difficulties and programming limitations.

Input values of all these variables for each of the companies included in the mean-variance efficient portfolio selection model are presented in Table 5.3.

5.6 Model formulations

The investor for the purpose of portfolio modelling is assumed to be a risk averse with indifference curves conclave to the origin and quadratic utility functions. Eight portfolios were created namely diversifier's portfolio, satisficer's portfolio, plunger's portfolio, market trend portfolio, capital gain bias portfolio, dividend gain bias portfolio, equal priority portfolio and the ideal portfolio for investors with different priorities and risk appetite. In all these portfolios the no short sales constraint and funds exhaustion constraint were included. Also, company diversification/upper bounds constraint and industrial diversification constraint are set at the level of 15 per cent and 20 per cent respectively.

1. The diversifier portfolio is for an investor who has smaller risk bearing capacity. Hence, the target values of the variables namely return, beta, dividend, earnings-per-share, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-book value ratio, price-to-earnings ratio promoters holding, sales, turnover, unsystematic risk and volume were set at their Q_1 (quartile one) levels.
2. The satisficer's portfolio is for an investor exhibiting satisficing behaviour i.e. he/she is looking for neither too high nor too low returns. The risk appetite of such an investor is also moderate. The target values of all the programmed variables are set at their median levels.
3. For the plunger portfolio, the Q_3 (quartile three) values of variables are targeted. This portfolio is for a class of investors having extremely high risk bearing capacity. They desire very high levels of return,

dividend, net profit, free float, volume, turnover, price-to-book value ratio, price-to-earnings ratio, beta, other stakeholder's interest and are willing to bear a high degree of risk for this.

4. While creating the market trend portfolio the average values for all the modelled variables have been targeted. Mean rather than mode was chosen to describe the market trend as no single value was repeating itself in most of the series of variables. The targeted mean values for return and beta were marginally higher than their median values in the satisficer's portfolio. Targeted values of all other variables except promoter's holding were also higher than the targets in diversifier's portfolio or the satisficer's portfolio. Of course, all these targets were lower than the Q_3 (quartile three) targets in the plunger's portfolio.
5. The capital gain bias portfolio depicts an investor who aims at very high levels of capital returns (from price movements), beta, earnings-per-share, net profit, price-to-earnings and price-to-book value ratios targeting Q_3 (quartile three) values and is satisfied with a Q_1 (quartile one) level of dividend return. Also, his/her targets for variables such as free float, institutional holding, market capitalisation and sales are set at their median values. Such an investor seems to be more of a speculator aiming at quick returns and not regular income from trading in stock market.
6. On the contrary, the dividend gain bias portfolio is for an investor desiring very high levels of regular income in the form of dividends and is indifferent towards capital gains. The dividend constraint is set at Q_3 (quartile three) level whereas returns, beta, eps, impact cost, market capitalisation, price-to-earnings ratio and price-to-book value ratio constraints are set at Q_1 (quartile one) level. Other constraints namely free float, institutional holding, promoter's holding, sales, turnover and volume are set at their median values.
7. The equal priority portfolio gives same priority to capital returns as well as dividend income aiming at Q_3 (quartile three) levels for both these variables. A high degree of capital gain returns and dividends are desired with lesser emphasis on beta, earnings-per-share, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-earnings ratio, price-to-book value ratio, promoter's holding, sales, turnover and volume positioning these variables at their median values.
8. The ideal portfolio is created for minimising variance of the portfolio by setting the target values of variables in conjunction with the results of the multiple regression and Granger causality tests. Returns,

Table 5.3 Input values for the portfolio selection model

Security	I	Ê	*eps*	D_i	Λ	*i*	ζ	*C*
X1	13	0.25	35.51	2529.66	32026.30	0.09	39.87	104207.91
X2	13	−0.09	10.42	3044.38	34091.61	0.09	42.08	111750.42
X3	23	0.58	35.98	1352.18	0.00	0.07	30.99	91910.67
X4	1	0.54	29.45	2204.55	266004.88	0.08	29.11	176876.63
X5	14	0.97	79.95	5859.68	205799.25	0.09	23.77	285200.44
X6	1	0.43	42.64	3066.95	0.00	0.08	27.92	109983.48
X7	17	0.45	51.24	5282.35	109182.78	0.08	28.15	500067.97
X8	3	0.06	24.05	3621.71	26054.82	0.09	29.36	127723.26
X9	12	0.47	15.40	3797.50	149257.33	0.08	22.58	910057.89
X10	10	1.16	−0.38	1210.07	81729.13	0.08	16.71	474137.98
X11	9	0.39	23.43	14028.88	52767.19	0.07	28.87	140448.36
X12	16	1.12	9.78	1863.55	206903.04	0.07	7.55	2175374.48
X13	20	0.72	8.94	1221.33	71796.15	0.08	12.55	649336.01
X14	9	0.29	43.18	7264.91	47784.87	0.07	38.09	115708.04
X15	18	0.37	22.59	0.00	62756.42	0.08	27.18	257191.28
X16	13	0.08	113.52	1787.70	41161.98	0.09	43.45	139361.13
X17	2	0.51	14.26	3662.93	31039.25	0.08	19.39	160183.60
X18	1	0.65	30.56	2625.62	246820.30	0.09	35.75	387411.64
X19	14	0.28	54.76	6698.65	56269.13	0.07	34.20	163992.38
X20	19	0.48	51.14	1862.74	67849.12	0.09	37.55	161562.72
X21	15	0.35	8.62	13315.09	103961.92	0.07	28.21	489233.58
X22	5	0.50	49.30	5879.32	267176.05	0.07	76.08	413792.46
X23	4	0.55	25.92	7992.40	352687.09	0.08	56.42	530563.01
X24	1	0.47	27.70	13052.76	290082.64	0.07	49.85	608522.69
X25	2	0.59	98.83	11891.48	432810.20	0.07	44.94	821675.73
X26	21	0.84	5.98	1358.30	163625.09	0.09	83.77	159466.90
X27	20	1.23	12.29	927.54	116482.55	0.10	33.87	164477.30
X28	11	1.00	107.16	560.32	237466.77	0.10	19.62	190501.73
X29	1	0.89	9.55	216.15	160343.69	0.10	23.07	131652.73
X30	7	0.36	44.88	4101.39	318049.39	0.09	52.23	440749.26
X31	6	0.96	29.18	2526.59	92659.41	0.08	49.40	149256.71
X32	6	0.91	49.93	1050.20	60367.26	0.08	32.36	248500.35
X33	8	0.65	8.12	20265.57	134565.12	0.09	8.74	1352131.09
X34	10	0.28	63.52	52953.58	139595.54	0.09	9.56	1453407.35
X35	8	0.30	4.85	3302.63	69918.89	0.08	11.14	448629.74
X36	1	0.56	72.45	3200.11	47477.71	0.09	28.40	167667.61
X37	9	0.34	11.83	2113.11	23569.01	0.08	30.80	147836.37
X38	3	0.51	56.08	14125.38	526892.07	0.07	28.93	1684712.99
X39	8	0.28	31.67	1124.37	38787.30	0.09	40.50	130798.32
X40	16	0.54	69.72	1262.51	98828.78	0.09	24.37	87219.54
X41	17	0.42	31.45	938.23	45231.75	0.10	23.46	122541.36
X42	1	0.50	88.88	9768.35	197365.32	0.06	41.62	663615.23
X43	11	0.01	8.48	12037.16	31863.53	0.09	10.33	360904.47
X44	22	0.58	16.07	1525.29	136310.17	0.10	14.51	290213.15
X45	9	0.04	31.66	1459.05	50421.83	0.08	20.86	183276.92
X46	2	0.58	39.42	15120.27	185748.32	0.07	15.47	1191998.79
X47	6	0.50	24.69	5514.61	98200.96	0.07	35.20	222073.93
X48	8	0.32	28.52	1864.05	66293.27	0.10	42.19	136973.91
X49	11	0.64	49.27	6899.10	110987.31	0.07	39.07	262585.82
X50	2	0.44	23.95	5898.28	66923.09	0.08	6.46	618011.45

Note: Security column represents the equities which are part of the portfolio selection model formulations. Column I represents industry classification. A total of 23 industries are represented on the NSE Index Nifty. E represents the monthly mean returns over a period of 12 years (1/4/2000–1/4/2012). However, for some securities complete 12 years data was not available. *eps* represents earnings-per-share. D_i represents the dividend returns. λ represents free float.

Π	*P/B*	*P/E*	ξ	$\hat{S}$	*v*	*t*	β	e_t
7791.27	3.21	15.64	26.85	60184.47	1393118.53	51.38	0.87	4.63
9407.29	2.86	15.99	34.94	53026.53	2372276.24	31.40	0.75	5.91
3656.33	7.93	24.02	46.95	34228.95	56469.23	10.17	0.38	0.00
12667.55	2.66	14.87	49.80	0.00	1073727.91	84.90	1.09	0.00
18908.34	7.39	16.41	49.88	122669.78	423950.09	58.25	0.95	0.00
17572.33	1.00	6.86	59.76	0.00	880960.44	26.12	1.27	0.00
21136.25	4.51	20.17	67.72	185689.66	1021338.37	101.75	1.00	0.00
11900.60	1.58	17.34	62.76	918203.46	840474.90	34.74	0.92	0.00
36765.18	6.15	5292.2	57.93	189518.81	3777365.14	156.16	0.76	0.00
6072.40	1.51	–324.60	63.95	26.48	4151725.83	101.03	0.88	0.00
31923.10	5.67	24.48	39.20	33562.42	901383.21	26.77	0.67	0.00
6128.65	8.78	37.00	90.00	3814.25	5598873.71	192.04	0.74	0.00
5350.58	5.32	55.06	82.47	15853.49	8101956.51	273.20	1.53	0.00
22350.95	3.87	27.91	26.06	29762.07	368360.65	36.66	0.57	0.00
0.00	2.02	10.17	60.01	169422.44	1405719.72	39.22	0.93	0.00
11203.17	2.26	13.13	23.59	76439.91	210235.74	26.71	0.88	5.92
7306.25	5.25	27.82	70.25	33052.48	1142450.24	34.23	1.00	1.10
16632.78	4.30	27.21	23.11	0.00	815581.33	83.28	0.78	0.00
11205.07	6.77	15.07	53.88	105617.52	461521.52	45.26	0.52	0.00
16429.38	1.27	10.48	28.23	133651.75	4169415.16	63.17	1.18	0.00
18812.59	16.71	29.59	51.79	143369.34	2843617.59	66.12	0.60	0.00
18052.38	4.79	22.70	0.00	70.09	972116.51	111.14	0.85	0.00
28729.71	2.04	23.09	5.51	0.00	3496548.44	236.41	1.12	0.00
25199.27	5.61	22.25	0.00	174771.23	4673507.03	105.88	0.54	0.00
35626.29	12.16	35.41	20.69	129707.28	1160480.65	304.36	0.64	0.00
6013.59	2.71	21.43	0.00	80.65	7415496.06	90.22	1.17	0.00
6585.36	3.91	24.35	45.90	47761.88	8737960.83	129.19	1.54	0.00
9065.72	3.88	14.69	55.92	41513.36	656252.13	64.17	1.43	0.00
3005.56	4.83	49.50	55.24	389.70	513269.00	27.71	1.28	0.00
19559.76	4.74	24.90	0.00	201878.57	1519204.60	156.96	1.09	0.00
8973.12	2.96	16.21	24.86	109473.45	1188693.78	65.13	1.02	0.00
13664.25	3.14	18.14	61.03	184134.18	1757539.06	107.05	0.93	0.00
70030.84	2.37	20.02	88.00	310029.55	6019739.66	96.03	0.75	0.00
129269.69	2.19	10.46	76.69	484869.91	1607225.56	117.19	0.98	0.00
12475.12	2.65	22.22	80.71	40595.15	7309472.98	77.18	0.78	0.00
21440.93	1.43	7.15	63.88	0.00	1321059.20	50.53	1.12	0.00
1023.64	5.48	44.90	42.97	39065.68	1286204.78	61.42	0.73	0.00
108973.33	2.61	15.49	46.74	1283211.36	5450848.65	451.46	1.02	0.00
6842.09	1.70	19.65	36.58	55418.76	1311714.50	107.55	1.21	0.00
10348.98	2.75	–1.79	52.27	26582.53	1944660.31	66.17	1.20	0.00
4844.73	7.83	27.89	56.50	60985.47	338960.58	27.13	0.93	0.00
52141.18	1.59	10.93	43.74	0.00	2480308.11	276.62	1.05	0.00
38337.04	2.12	6.75	85.82	331877.75	11394512.51	93.29	1.34	3.27
6720.08	2.67	35.80	62.49	81645.80	2584886.01	85.98	1.33	0.00
6386.01	6.64	26.05	68.22	14719.58	304920.83	24.43	0.42	0.00
52093.79	11.52	60.33	78.51	166127.08	1584793.80	150.42	0.59	0.00
11197.95	2.78	15.50	33.68	243719.53	3195600.83	147.21	1.02	0.00
6796.05	1.73	18.02	32.43	50997.31	828872.52	37.04	1.09	0.00
33200.75	1.73	8.59	29.37	179052.53	5496460.76	217.93	1.28	0.00
25069.58	12.81	39.50	81.36	123499.21	1071244.11	77.02	0.92	2.06

i represents impact cost. ζ represents institutional holding. *C* represents market capitalisation. π represents net profit constraint. *P/B* represents price-to-book value ratio. *P/E* represents price-to-earnings ratio. ξ represents promoters holding. *S* represents sales. *v* represents volume. *t* represents turnover. β represents beta. e_t represents unsystematic risk.

> dividend and impact cost were aimed at high Q_3 (quartile three) level. The two measures of gains of the investor and liquidity measured by impact cost were found to be of utmost importance to the investor. Impact cost had significant degree of predictive power explaining above 30 per cent of variability in returns. Industrial diversification and company diversification were achieved at 20 per cent and 15 per cent respectively. Institutional holding of 31.13 per cent was desired. Funds exhaustion and no short sales constraints were also met. Net profit, promoter's holding, sales, turnover and volume constraint were targeted at median levels. The constraints of beta, earnings-per-share, free float, market capitalisation, price-to-book value ratio, price-to-earnings ratio and unsystematic risk were removed from the problem as they could not significantly explain returns.

In mean-variance portfolio analysis, the investor is assumed to be a risk averse, but the degree of risk averseness may differ for each investor. The targeted values of all the eight portfolio variables created for investors with different priorities and risk bearing capacity are different from each other. Table 5.4 illustrates desired values for all the 20 portfolio variables and eight portfolio selection models considered for empirical testing.

5.7 Mean-variance efficient portfolio selection model formulations: analysis and interpretations

The objective function minimised the variance in the presence of 30 linear, one non-linear portfolio constraints and 50 non-linear variables. A total of 882 non-zero values with 50 non-linear non-zeroes were used to generate the solutions. Eight mean-variance efficient portfolio selection model simulations and one Markowitz formulation were run on Lingo 13x64. To facilitate comparison of alternate portfolio selection modelling frameworks and their comparison with the Markowitz's efficient frontier the analysis has been undertaken.

5.7.1 Diversifier's portfolio

The diversifier's portfolio targeted at achieving the quartile one values of portfolio variables as mentioned in Table 5.5. The portfolio is diversified across 14 companies. A maximum investment of 15 per cent was noticed in Asian Paints Ltd. and Cairn India Ltd. The portfolio was diversified across 12 industrial sectors namely cement, paints, automobiles, refineries, telecommunications, oil exploration,

Table 5.4 Target values for the eight portfolio selection model formulations

	Diversifier Portfolio	Satisficer Portfolio	Plunger Portfolio	Market Trend Portfolio	Capital Gain Bias Portfolio	Dividend Gain Bias Portfolio	Equal Priority Portfolio	Ideal Portfolio
Values used	Q1 values	Median values	Q3 values	Mean values	Q3 and q1 values	Q1 and q3 values	Q1 and q2 values	As per multiple regression
Returns	0.34	0.5	0.63	0.52	0.63	0.34	0.63	0.63
Beta	0.75	0.94	1.12	0.95	1.12	0.75	0.94	Constraint removed
Dividend	1525.29	3200.01	6899.1	6025.07	1525.29	6899.1	6899.1	3200.01
EPS	14.55	30.01	49.78	36.53	49.78	14.55	30.01	Constraint removed
Free float	52180.85	98514.87	188652.57	134458.03	98514.87	98514.87	98514.87	Constraint removed
Funds exhaustion	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1
Impact cost	0.07	0.08	0.09	0.08	0.07	0.07	0.08	0.08
Industry diversification	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Institutional holding	21.29	29.02	39.67	31.13	21.29	29.02	29.02	29.02
Mkt capitalisation	142295.37	235287.14	497359.37	422309.53	235287.14	142295.37	235287.14	Constraint removed
Net profit	6842.09	12667.55	25069.58	21732.38	25069.58	12667.55	12667.55	12667.5
No short sales	X1….X50≥0	X1….X50≥0	X1….X50≥0	X1….X50≥0	X1….X50≥0	X1….X50≥0	X1….X50≥0	X1….X50 ≥0
P/B ratio	2.21	3.18	5.58	4.49	5.58	2.21	3.18	Constraint removed
P/E ratio	14.92	20.1	27.67	120.74	27.67	14.92	20.1	Constraint removed
Promoter's holding	30.14	50.83	63.6	47.96	30.14	50.83	50.83	50.83
Sales	18535.75	57801.62	168598.6	133805.43	57801.62	57801.62	57801.62	57801.62
Turnover	40.73	80.23	115.68	102.11	40.73	80.23	80.23	80.23
Unsystematic risk	0	0	0	0.46	0	0	0	Constraint removed
Upper bounds	X1….X50≤0.15	X1….X50≤0.15	X1….X50≤0.15	X1….X50≤0.15	X1….X50≤0.15	X1….X50≤0.15	X1….X50≤0.15	X1….X50 ≤0.15
Volume	886066.13	1399419.13	3707160.96	2592661.52	886066.13	1399419.13	1399419.13	1399419

Table 5.5 Diversifier's portfolio: targets and achievements

Infeasibilities: 0.000000	**Model Class: NLP**	**Total solver iterations: 166**	
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.1873832	–1.000000
Funds exhaustion	1	0.000000	–0.4314143
Returns	0.34	0.1973907	0.000000
EPS	14.55	13.60833	0.000000
Beta	0.75	0.2372261E-02	0.000000
Dividend	1525.09	3986.761	0.000000
Free float	52180.85	48857.08	0.000000
Impact cost	0.07	0.1142089E-01	0.000000
Institutional holding	21.29	4.419918	0.000000
Market capitalisation	142295.4	278768.6	0.000000
Net profit	6842.09	10372.81	0.000000
P/B ratio	2.21	3.636601	0.000000
P/E ratio	14.92	0.000000	0.7203994E-04
Promoters holding	30.14	24.90651	0.000000
Sales	18535.75	158387.0	0.000000
Turnover	40.73	39.27816	0.000000
Unsystematic risk	0.00	0.3878584	0.000000
Volume	886066.1	1762537	0.000000

diversified, computers, engineering, power, banking and electrical engineering, while not investing more than 20 per cent in any sector. The average weight of each of the securities was found to be 7.14 per cent. Local optimal solution was found after 166 iterations (see Table 5.5).

Dual prices indicate the extent to which the objective value (variance of the portfolio) would be reduced if the constraint is increased by one unit. The dual price for all the variables is zero except funds exhaustion and price-to-earnings ratio. For the price-to-earnings constraint, also the dual price has a negligible value. This indicates that the variance of this portfolio is quite less and can be further reduced only if we make changes in the funds exhaustion constraint, by allowing short-selling of securities in the process of portfolio selection. The selected securities and the proportion of each of them in the portfolio are presented in Figure 5.1 through bar diagrams and pie chart.

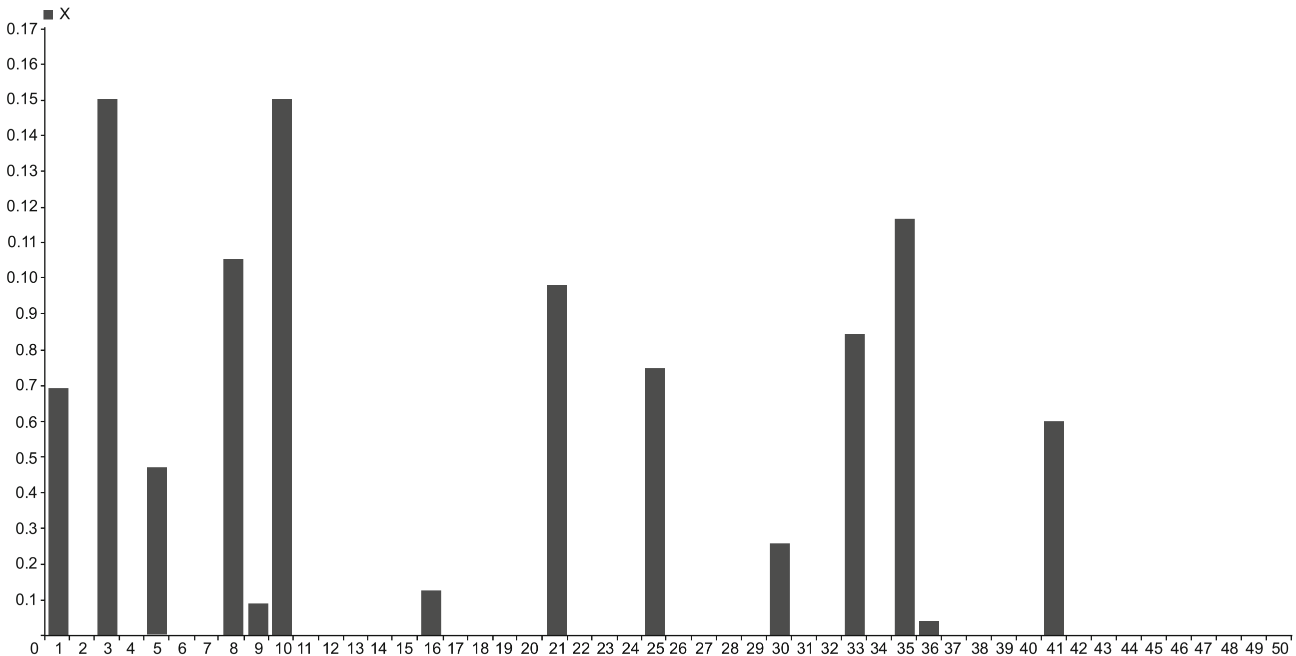


Figure 5.1 Continued

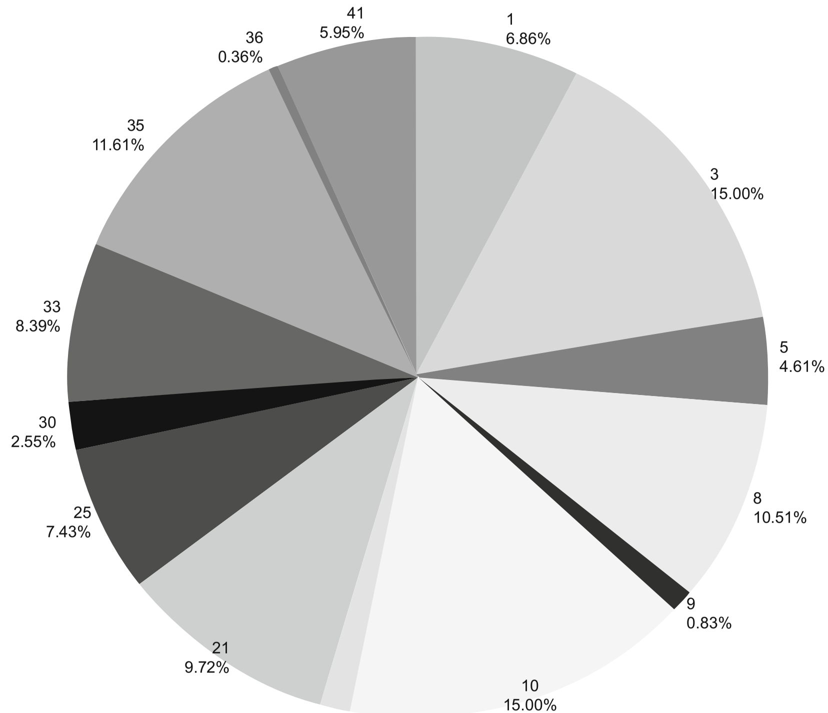


Figure 5.1 Portfolio weights: diversifier's portfolio

5.7.2 Satisficer's portfolio

It is aimed at achieving the median values of the constrained variables. The portfolio is diversified across 15 companies and 12 industries. The maximum weights of 15 per cent were given to Cairn India and Punjab National Bank. Substantive investment was also done in Bajaj Auto, Hindustan Unilever and Larsen & Toubro. The portfolio invested in cement, power, oil exploration, automobiles, mining, banking, electrical equipment and engineering sectors. The telecommunication sector received a very small proportion of investment. The average weight of each security was 6.67 per cent (see Figure 5.2).

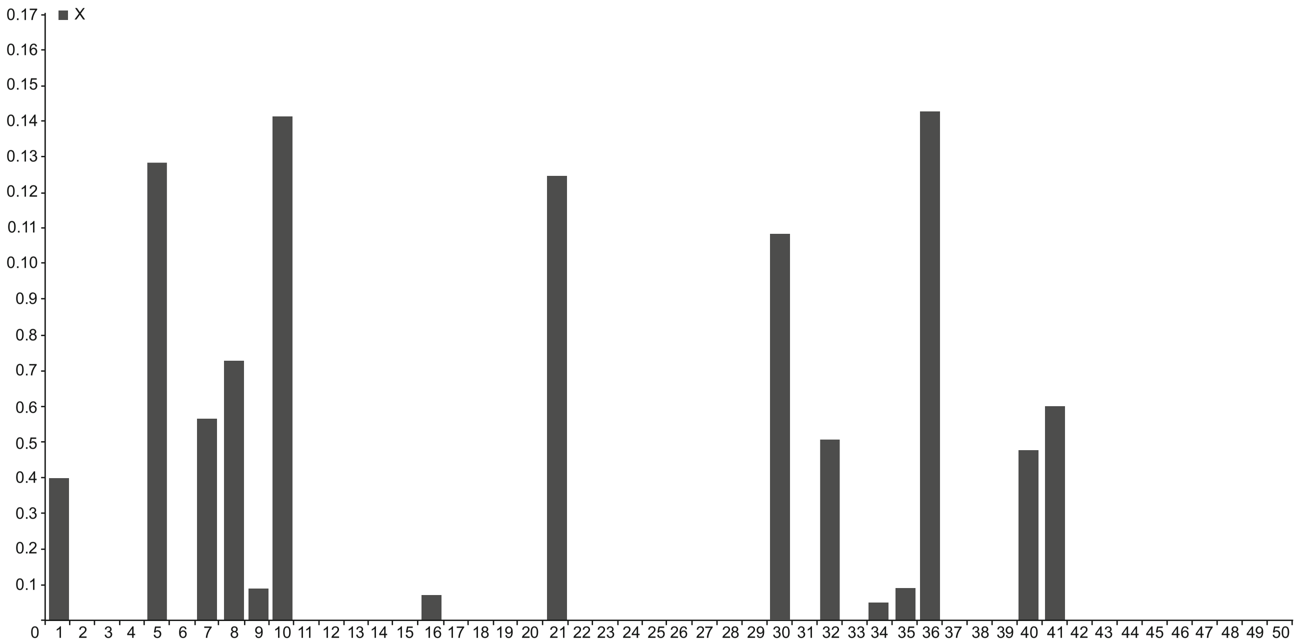
The portfolio achieved a variance of 0.316 with 0.605 levels of returns. All other constraints were achieved at their median levels. Surplus returns of 0.105 over the target of 0.50 per cent were seen. Dividend returns realised by the portfolio were also more than targeted. Systematic risk beta achieved its targeted value of 0.94 without any surplus whereas unsystematic risk of 0.23 was present. Surpluses were also generated for earnings-per-share, free float, impact cost, market capitalisation, net profit, price-to-book value ratio, sales, unsystematic risk and volume, indicating achievement of more than targeted goals for these constraints.

The dual price of 1.145327 for the funds exhaustion constraint indicated further minimisation of variance by alteration/removal of the constraint of neither borrowing nor lending. Portfolio beta also showed a negative dual price, hinting at increase in variance by 1.78 if the beta constraint is increased by one unit. The dual prices of all other variables are extremely small and could be considered negligible (see Table 5.6).

5.7.3 Plunger's portfolio

This portfolio is created for an investor who has the capacity for bearing more risk in anticipation of higher returns. Apart from risk minimisation, he/she prioritises maximising returns, dividend, beta and all other portfolio variables. All risk averters face a diminishing marginal utility of money curve, but for this investor the total utility of money diminishes at a lower than normal rate. The less risk averse investor faces a less concave total utility for money curve, compared to satisficer and diversifier investors. Doubling of money income increases his/her utility of money at diminishing rates.

An increase in the risk of a portfolio requires only a small increase in the expected return, if an investor has to remain equally well off. It implies that the investor is willing to take additional risk for smaller risk premiums (corresponding increase in expected income) in case of a plunger.


X
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.9
0.8
0.7
0.6
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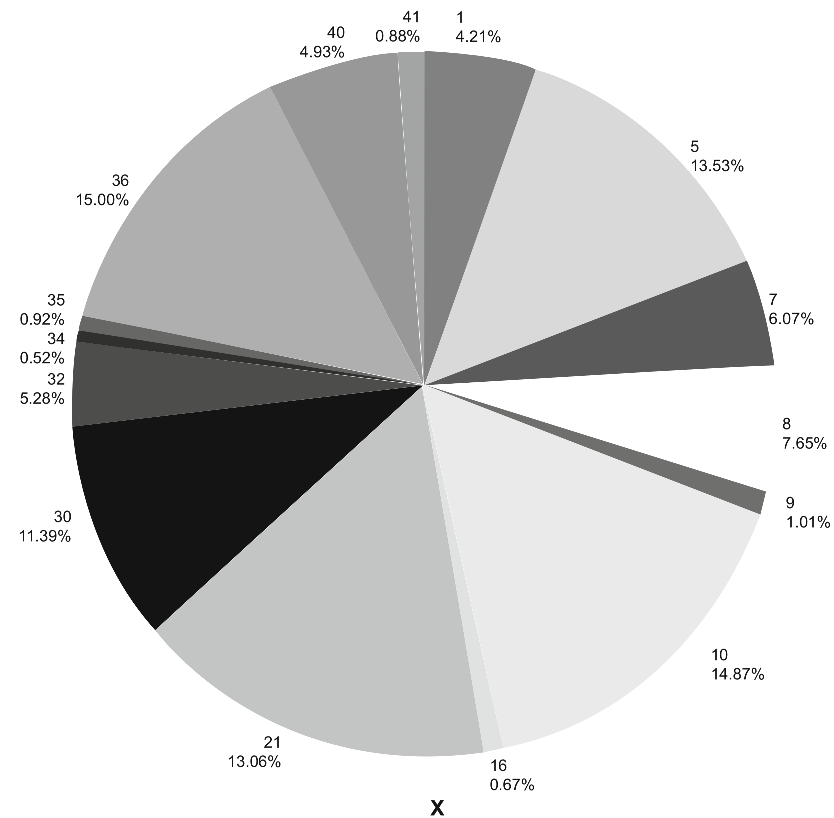


Figure 5.2 Portfolio weights: satisficer's portfolio

Table 5.6 Satisficer's portfolio: targets and achievements

Infeasibilities: 0.000000 Model Class: NLP Total solver iterations: 166			
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.3159617	–1.000000
Funds exhaustion	1	0.000000	1.145327
Returns	0.50	0.1048106	0.000000
EPS	30.01	11.95280	0.000000
Beta	0.94	0.000000	–1.781641
Dividend	3200.01	1464.173	0.000000
Free float	98514.87	19993.59	0.000000
Impact cost	0.08	0.4660980E-02	0.000000
Institutional holding	29.02	0.000000	–0.3366069E-02
Market capitalisation	235287.1	97875.10	0.000000
Net profit	12667.55	3723.638	0.000000
P/B ratio	3.18	1.994125	0.000000
P/E ratio	20.01	0.000000	0.3304722E-04
Promoters holding	50.83	0.000000	–0.8449882E-03
Sales	57801.62	101480.1	0.000000
Turnover	80.23	0.000000	–0.7380637E-04
Unsystematic risk	0.00	0.2350413	0.000000
Volume	1399419	509442.0	0.000000

With the objective of minimising the risk (variance), the other portfolio variables, i.e. returns, dividend, net profit, sales, beta, EPS, other stakeholders share (promoter's holding and institutional holding) were targeted at high Q_3 (quartile three) values. The resultant solution was found to be infeasible. This situation occurs when no single solution can satisfy all the specified constraints. Even after 11,295,317 iterations of the local as well as the extended global solver of the software Lingo 13, no feasible portfolio could be created. It may be inferred that when an investor sets extremely high goals he/she is not able to create a mean-variance efficient portfolio. Hence, realistic and achievable targets must be set (see Table 5.7).

All the funds could not be invested. The institutional holding constraint, promoters holding constraint, no short-selling restriction on 33 securities and upper bounds on 12 of them could not be met making the solution infeasible.

5.7.4 Market trend portfolio

The market trend portfolio targeted the mean values of all the portfolio variables. A monthly return of 0.52 was achieved at 0.364 level of variance. The market trend portfolio aimed at minimising the variance by

Table 5.7 Plunger's portfolio: targets and achievements

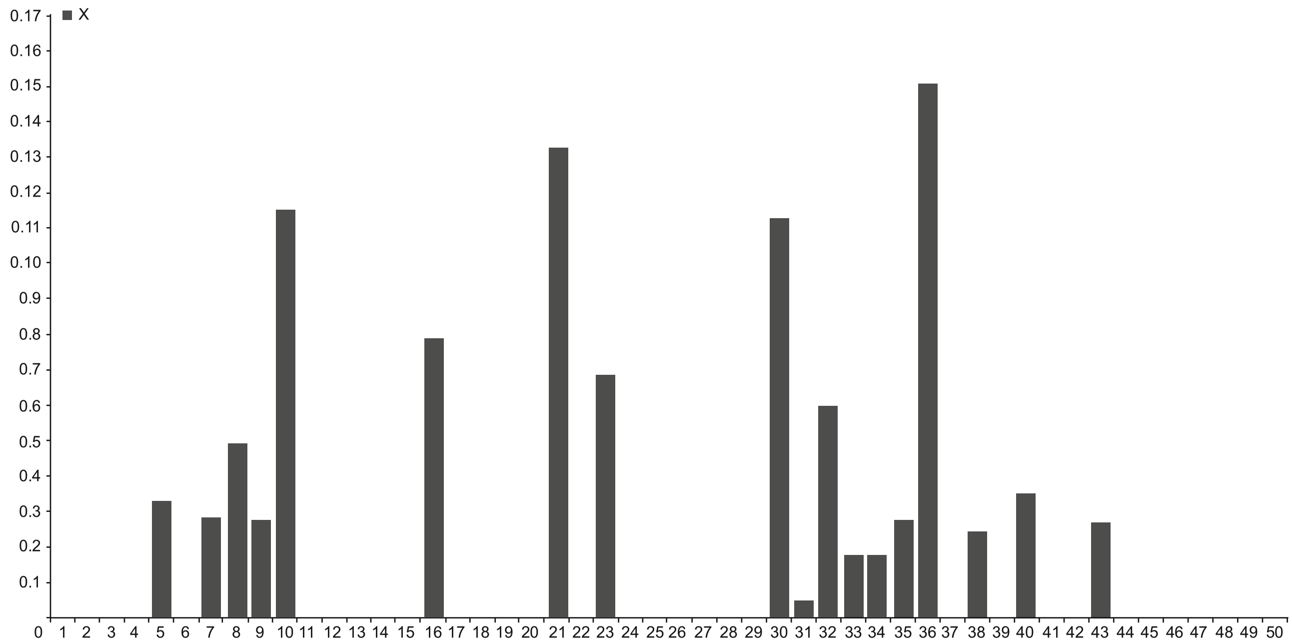
Infeasibilities: 0.000000	**Model Class: NLP**	**Total solver iterations: 11295317**	
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
No feasible solution found			
Variance	Minimise	−0.1000000+308	−1.000000
Funds exhaustion	1	−0.1496409	1.000000
Returns	0.63	0.1594163	0.000000
EPS	49.78	0.000000	−0.1172189E-02
Beta	1.12	0.000000	−0.8432989E-01
Dividend	6899.1	1222.331	0.000000
Free float	188652.6	0.000000	−0.2174883E-06
Impact cost	0.09	0.000000	−0.5774219
Institutional holding	39.67	0.000000	−0.9647344E-02
Market capitalisation	497359.4	343984.7	0.000000
Net profit	25069.58	5792.233	0.000000
P/B ratio	5.58	0.000000	−0.9795396E-02
P/E ratio	27.67	0.000000	0.5730278E-03
Promoters holding	63.60	0.000000	−0.7547559E-02
Sales	168598.6	0.000000	−0.1681082E-07
Turnover	115.68	53.36124	0.000000
Unsystematic risk	0.00	0.9676756E-01	0.000000
Volume	3707161	0.000000	−0.4238346E-08

targeting the mean values of each of the other constrained variables in the problem (see Figure 5.3).

The portfolio diversified across 18 companies and 13 industrial sectors. Maximum investment was witnessed in Punjab National Bank. High investments were made in Cairn India, Hindustan Unilever and Larsen and Toubro. The automobile, electrical equipment, oil exploration, banks, refineries, mining, steel and steel product sectors were the industrial sectors in which investment was made. Power sector received marginal investment. The average weight of each security was 5.55 per cent.

The portfolio generated surplus dividend return, free float, impact cost, net profit, sales, volume and price-to-book value ratio. Returns, beta, institutional holding, market capitalisation, price-to-earnings ratio, promoters holding, turnover and unsystematic risk constraints were almost met. The portfolio is sufficiently liquid with a high impact cost to the tune of 0.0836.

The dual price of 2.133158 for the funds exhaustion constraint implies the extent of reduction in portfolio variance with a unit increase in this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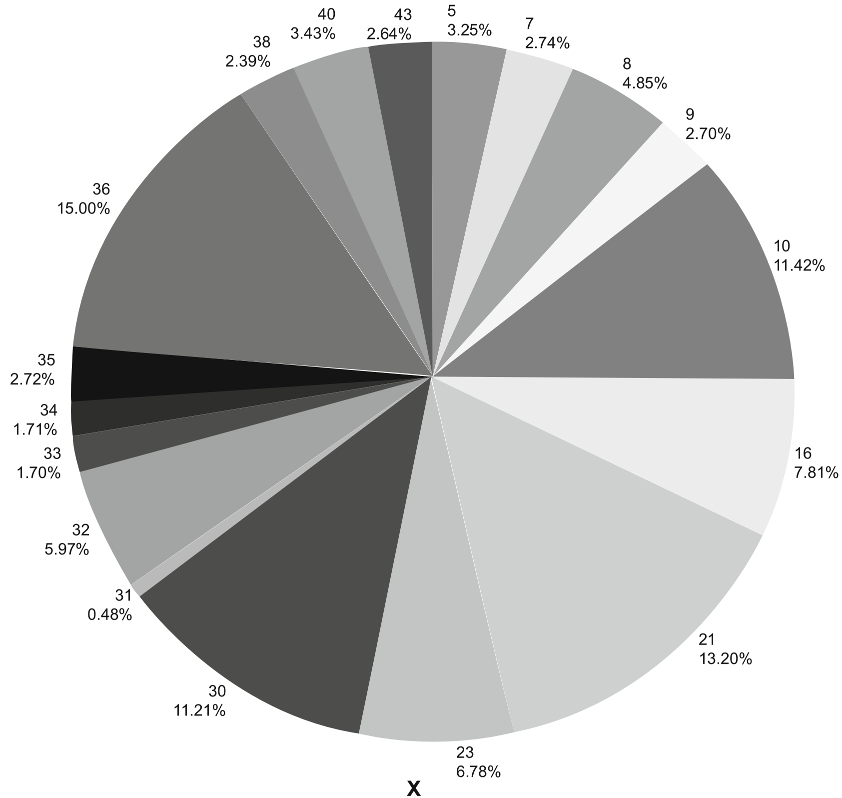


Figure 5.3 Portfolio weights: market trend portfolio

Table 5.8 Market trend portfolio: targets and achievements

Infeasibilities: 0.000000 Model Class: NLP Total solver iterations: 184			
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.3641575	–1.000000
Funds exhaustion	1	0.000000	2.133158
Returns	0.52	0.000000	–0.7730688E-02
EPS	36.53	5.101331	0.000000
Beta	0.95	0.000000	–1.843591
Dividend	6025.070	219.0064	0.000000
Free float	134458.0	946.6538	0.000000
Impact cost	0.08	0.3599698E-02	0.000000
Institutional holding	31.13	0.000000	–0.2052425E-01
Market capitalisation	422309.5	0.000000	–0.1169532E-06
Net profit	21732.38	1011.737	0.000000
P/B ratio	4.59	0.2005439E-01	0.000000
P/E ratio	120.74	0.000000	0.3088719E-05
Promoters holding	47.96	0.000000	–0.7960836E-02
Sales	133805.4	38978.82	0.000000
Turnover	102.1100	0.000000	–0.3333276E-03
Unsystematic risk	0.46	0.000000	0.000000
Volume	2592662	509442.0	–0.1203901E-07

constraint, allowing investment with borrowed funds. The negative dual price of portfolio beta indicates the increase in portfolio variance with decrease in this measure of systematic risk. The negative dual prices for returns, institutional holding, market capitalisation, promoter's holding, turnover and volume also entail the adverse impact on the objective function with increase in these constraint values. A positive but negligible dual price for price-to-earnings ratio indicates the marginal improvement of portfolio variance with unit increase in this constraint (see Table 5.8).

5.7.5 Capital gain bias portfolio

The capital gain bias portfolio aimed at maximising the capital gain returns from the portfolio while keeping the dividend returns at Q_1 (quartile one) level. The portfolio diversified across 12 companies and ten industries. The capital gain portfolio targeted high returns, beta, net profit with less emphasis on other portfolio variables such as sales, free float, dividend, institutional and promoter's holding (see Figure 5.4).

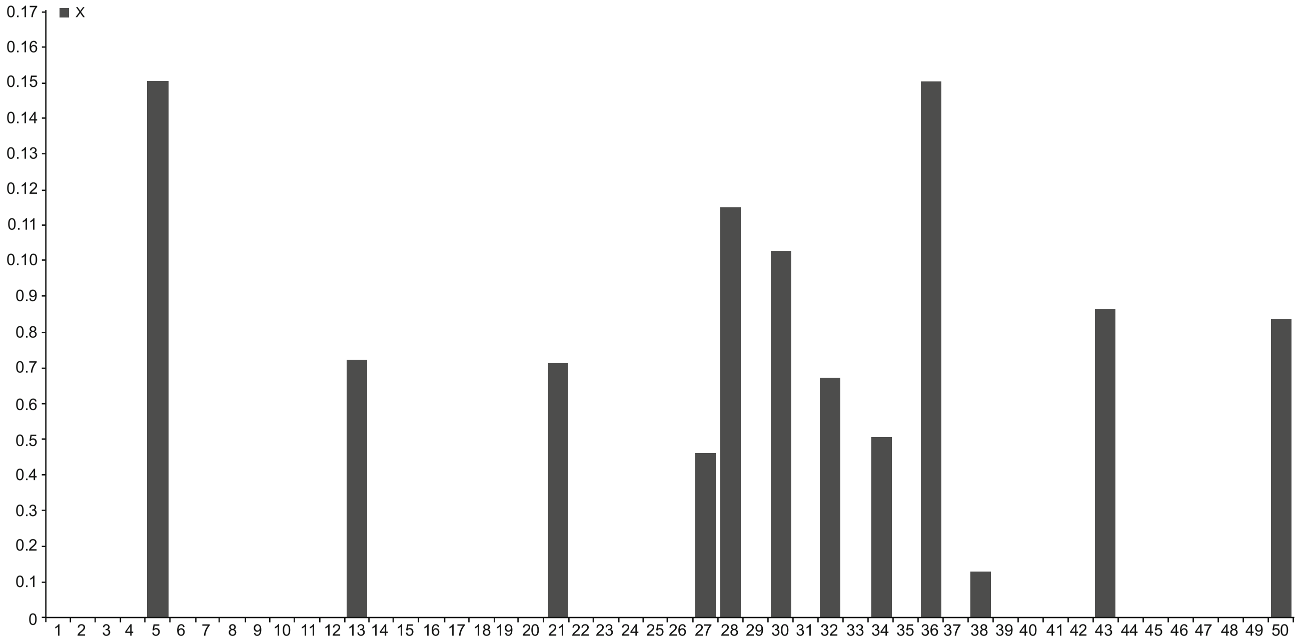


Figure 5.4 Continued

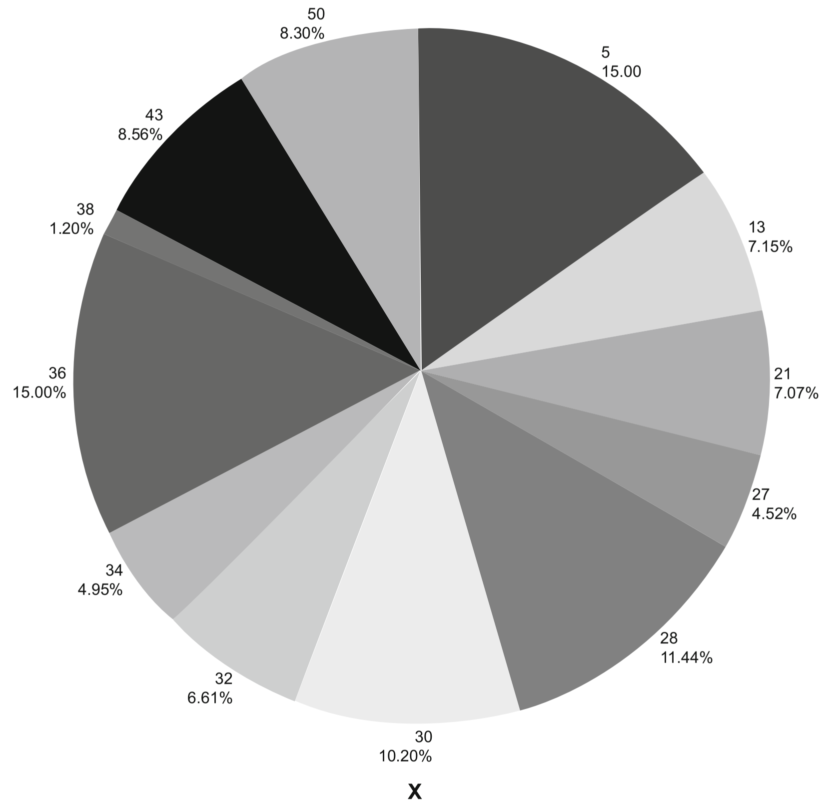


Figure 5.4 Portfolio weights: capital gain bias portfolio

The average weight of each security in the portfolio was 8.33 per cent. Heavy investments were made in Bajaj Automobiles and Punjab National Bank. Steel and construction sector also received substantive investments. Reliance Industries Ltd. in the refineries sector also received marginal share of the portfolio. Cairn India Ltd. the oil exploration company in which each of the earlier portfolios invested heavily did not form a part of this portfolio. Capital gain portfolio had higher variance than all other portfolio model formulations.

High expected return of 0.63 per cent at a high variance of 1.07 was achieved. Surprisingly, high dividend returns of 7292.02 were also realised on this portfolio. Heavy surpluses were also witnessed in free float, market capitalisation, promoter's holding, sales, turnover, unsystematic risk and volume constraints. EPS, impact cost, institutional holding, price-to-earnings ratio constraints were also met with surplus/slack variables (see Table 5.9).

The substantial dual price for funds exhaustion constraint showed the colossal reduction in variance by allowing borrowing/lending of funds for portfolio creation. A negative dual price of returns, beta and

Table 5.9 Capital gain bias portfolio: targets and achievements

Infeasibilities: 0.000000 Model Class: NLP Total solver iterations: 177

Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	1.075562	–1.000000
Funds exhaustion	1	0.000000	6.891842
Returns	0.63	0.000000	–1.163430
EPS	40.78	10.55360	0.000000
Beta	1.12	0.000000	–6.687965
Dividend	1525.29	5766.729	0.000000
Free float	98514.87	42344.85	0.000000
Impact cost	0.07	0.1773638E-01	0.000000
Institutional holding	21.29	2.910939	0.000000
Market capitalisation	235287.1	178627.2	0.000000
Net profit	25069.58	0.000000	–0.1355349E-04
P/B ratio	5.58	0.000000	–0.1140855
P/E ratio	27.67	7.029424	0.000000
Promoters holding	30.14	27.44412	0.000000
Sales	57801.62	89638.85	0.000000
Turnover	40.73	61.66826	0.000000
Unsystematic risk	0.00	0.4508164	0.000000
Volume	886066.1	2106792	0.000000

price-to-book value ratio indicates increase in risk if the value of this constraint is increased. The dual price for net profit constraint is trifling and inconsequential.

5.7.6 Dividend gain bias portfolio

The dividend gain bias portfolio has been constructed for a class of investors desiring a regular dividend income. The portfolio diversified across 16 companies giving each security a weight of 6.25 per cent on an average. The pictorial representation of the securities selected and their respective percentages in the portfolio is presented in Figure 5.5.

The dividend gain biased portfolio targeted high values of dividend gains, promoter's holding, turnover, volume and moderate values of returns, beta, earnings-per-share, P/B ratio, etc. Returns were targeted at a low level Q_1 (quartile one) of 0.34. As expected a lower variance of 0.206 with comparatively high monthly returns of 0.55 were achieved. Although the systematic risk was low (0.75), the unsystematic risk component was at the highest level of 0.67.

Investment was made in over 12e industrial sectors namely cement, paints, automobiles (both two wheelers and four wheelers), refineries, telecommunications, oil exploration, finance, computers, power, banks and diversified. Power Grid Corporation of India and Asian Paints received maximum weights. Quite surprisingly, it was found that Cairn India, a security which did not declare any dividend in the sample period, formed a substantial part of the portfolio aiming high dividend incomes.

Returns, EPS, free float, impact cost, market capitalisation, net profit, price-to-book value ratio, sales, turnover, unsystematic risk and volume constraints achieved more than the target values. Again, the dual price of funds exhaustion constraint hinted at further abatement of variance by allowing borrowing of funds for portfolio creation. A negative dual price for beta and institutional holding indicate the increase in value of variance if these constraints are increased. The dual prices for portfolio dividend, price-to-earnings ratio and promoter's holding are too small to be of any inference (see Table 5.10).

The 16 security portfolio was able to achieve lesser level of variance providing high dividend incomes and good capital gain returns. However, the dividend income achieved by the highly risky capital gain bias portfolio was higher than that of this portfolio.

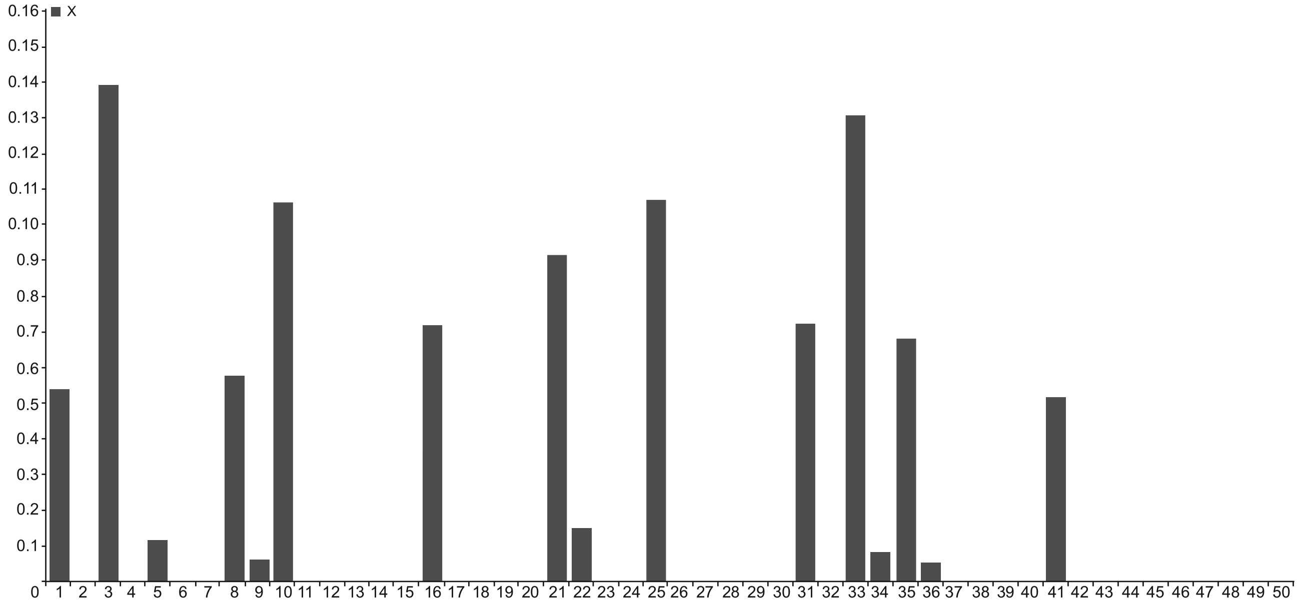


Figure 5.5 Continued

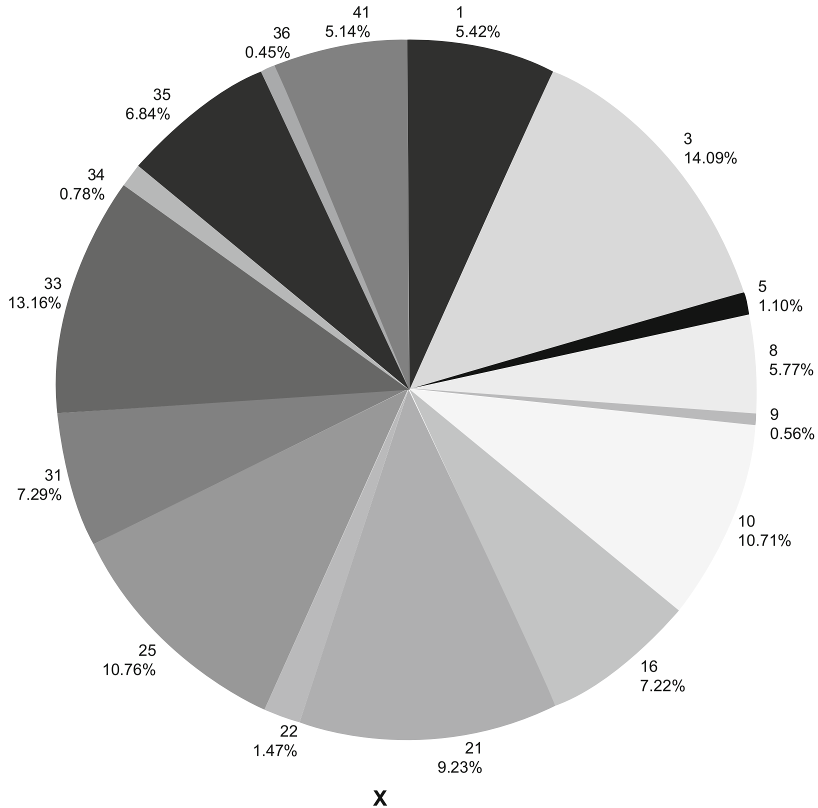


Figure 5.5 Portfolio weights: dividend gain bias portfolio

Table 5.10 Dividend gain bias portfolio: targets and achievements

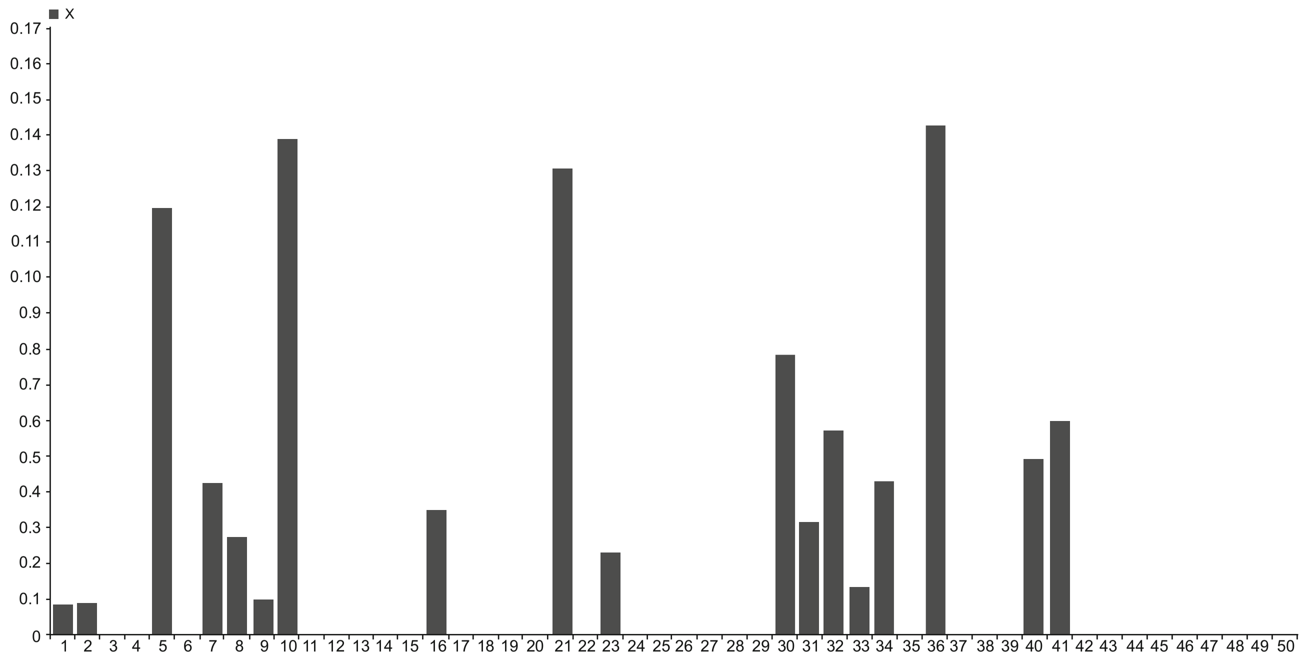
Infeasibilities: 0.000000	**odel Class: NLP**	**Total solver iterations: 166**	
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.2061020	–1.000000
Funds exhaustion	1	0.000000	0.4537292
Returns	0.34	0.2081886	0.000000
EPS	14.55	20.87740	0.000000
Beta	0.75	0.000000	–0.1656815
Dividend	6899.1	0.000000	–0.4557221E-05
Free float	98514.87	12141.14	0.000000
Impact cost	0.07	0.1082289E-01	0.000000
Institutional holding	29.02	0.000000	–0.1432622E-01
Market capitalisation	142295.4	325972.1	0.000000
Net profit	12667.35	8380.579	0.000000
P/B ratio	2.21	3.670022	0.000000
P/E ratio	14.92	0.000000	0.3298965E-04
Promoters holding	50.83	0.000000	–0.5954501E-02
Sales	57801.62	96039.92	0.000000
Turnover	80.23	5.786348	0.000000
Unsystematic risk	0.00	0.6722968	0.000000
Volume	1399419	1041986	0.000000

5.7.7 Equal priority portfolio

The equal priority portfolio gave equal and high preference to both capital as well as dividend returns. The portfolio diversified across 17 companies and 12 industrial sectors (see Figure 5.6).

A high concentration of portfolio weight of around 15 per cent was made in companies like Bajaj Auto, Cairn India, Hindustan Unilever Ltd. and Punjab National Bank. Average weight of each security was 5.88 per cent and that of each sector was 8.33 per cent. The asset mix included investment in cement and cement products, automobiles (two and three wheelers), automobiles(four wheelers), electrical equipment, refineries, telecommunications, oil exploration and production, diversified, banks, engineering and mining sectors. Computers, construction, gas and pharmaceuticals sectors were not found to be suitable for this portfolio.

Q_3 (Quartile three) level of return 0.63 and a dividend of Rs. 6899.1 are achieved without any surplus. All the other variables were targeted at Q_1 (Quartile one) levels. Portfolio variance minimised to the level of 0.32 with funds being entirely invested. Surpluses in EPS, free float, impact cost, market capitalisation, net profit, price-to-book value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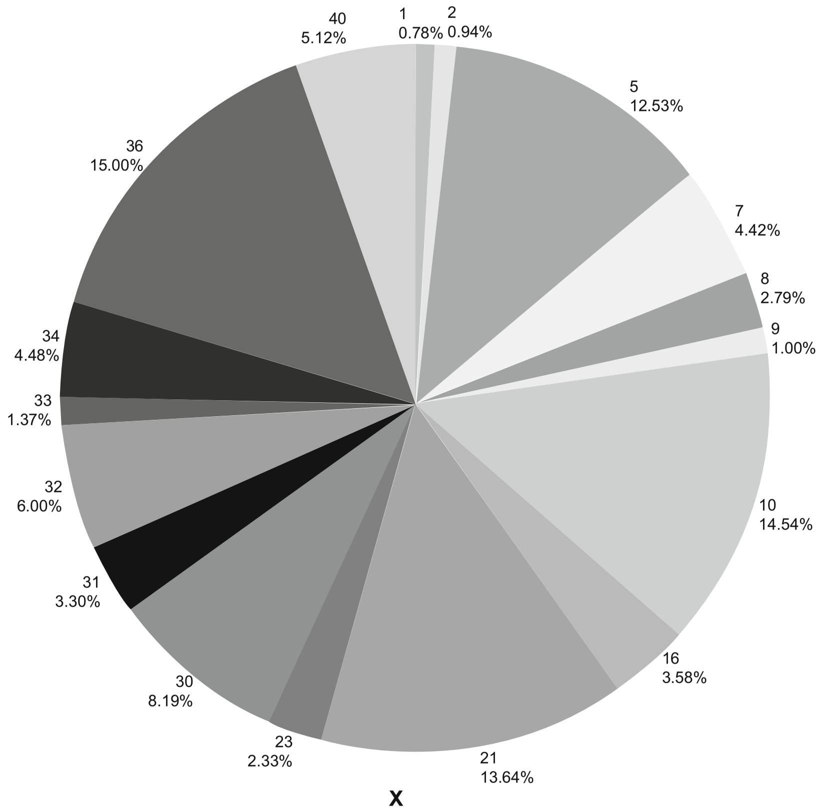


Figure 5.6 Portfolio weights: equal priority portfolio

Table 5.11 Equal priority portfolio: targets and achievements

Infeasibilities: 0.000000 Model Class: NLP Total solver iterations: 184			
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.3223631	–1.000000
Funds exhaustion	1	0.000000	1.304001
Returns	0.63	0.000000	–0.1696794E-01
EPS	30.01	14.28653	0.000000
Beta	0.94	0.000000	–1.752190
Dividend	6899.1	0.000000	–0.5091545E-05
Free float	98514.87	23064.20	0.000000
Impact cost	0.08	0.4108971E-02	0.000000
Institutional holding	29.02	0.000000	–0.6661001E-02
Market capitalisation	235287.1	154105.1	0.000000
Net profit	12667.35	8755.623	0.000000
P/B ratio	3.18	1.922899	0.000000
P/E ratio	20.10	0.000000	0.1923485E-04
Promoters holding	50.83	0.000000	–0.2063938E-02
Sales	57801.62	75521.66	0.000000
Turnover	80.23	3.402580	0.000000
Unsystematic risk	0.00	0.2992494	0.000000
Volume	1399419	575936.5	0.000000

ratio, sales, turnover, unsystematic risk and volume constraint were generated in the solution. The positive dual price for funds exhaustion constraint indicated reduction in variance by allowing borrowing for creating leveraged portfolios. The price-to-earnings ratio showed a positive but small shadow price. The negative dual prices for returns, dividend, institutional holding and promoter's holding show the adverse impact on variance of portfolio with unit increase in each of these constraints (see Table 5.11).

The high return yielding high dividend providing portfolio did quite well in minimising the variance at a level of 0.32, which is lower than the variance of the market trend portfolio and capital gain bias portfolio.

5.7.8 Ideal portfolio

The ideal portfolio is created for an investor who targets to achieve the best outcomes only for a select few portfolio variables. It limited the number of constraints in the quadratic programming portfolio selection model to a select few as per the findings of multiple regression analysis and Granger causality tests. Beta, EPS, free float, market capitalisation, price-to-book value ratio, price-to-earnings ratio and unsystematic risk constraints were excluded in this model formulation. The remaining constraints namely returns, dividends, funds exhaustion, impact cost, industry diversification, institutional holding, net profit, promoter's holdings, sales, turnover, volume were set at their median values (see Figure 5.7).

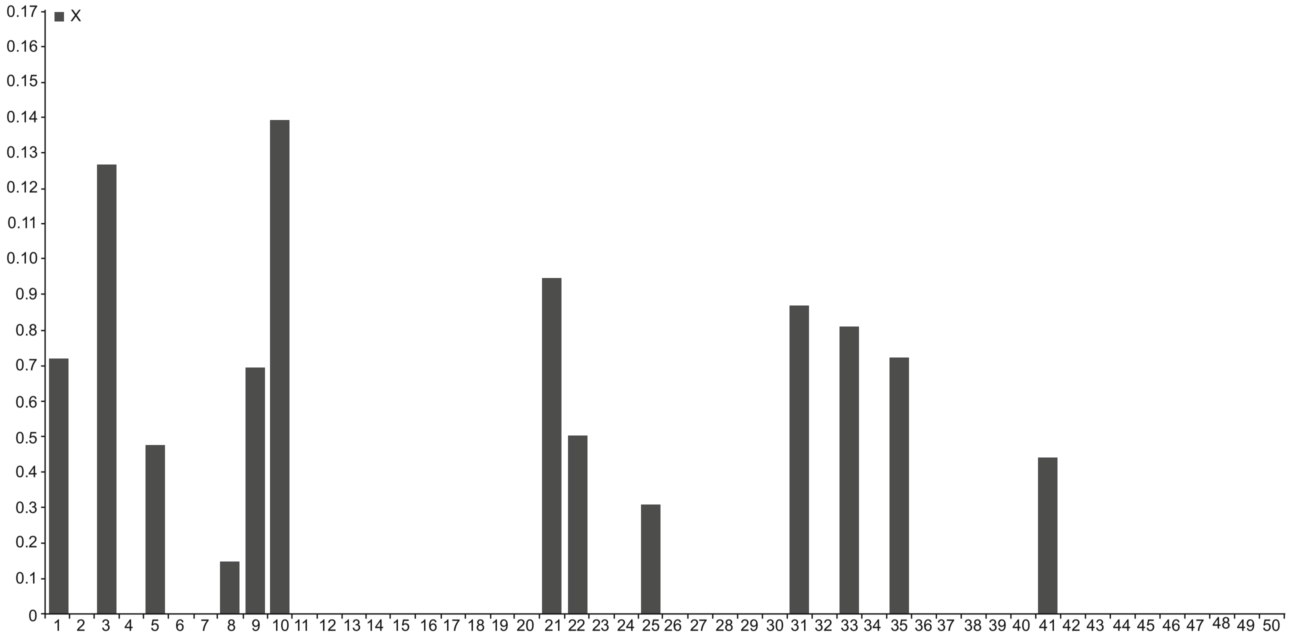
This formulation of the quadratic programming portfolio selection model diversified across 13 companies and 12 industrial sectors. The average weight of each security was 7.69 per cent and that of each sector was 8.33 per cent. Cairn India, which is an oil exploration company, received the maximum weight of 15 per cent. Huge investments were also made in ACC, Asian Paints, Bharti Airtel, Hindustan Unilever, Mahindra & Mahindra, NTPC and Power Grid Corporation of India. Banks, financial institutions, refineries, steel and steel products, construction, pharmaceuticals, aluminium, gas, electrical equipment, cigarettes and metals sectors were not found suitable for this portfolio.

The ideal portfolio yielded the same returns as Markowitz's efficient frontier with marginally higher variance of 0.197. It also fulfils the minimum median requirements for all the other constraints such as funds exhaustion, impact cost, promoter's and institutional holding and turnover, generating surpluses in dividend, free float, net profit, sales and volume. The small and positive dual price of funds exhaustion constraint hinted at reduction in portfolio variance by allowing borrowing of funds. The substantially negative shadow price of impact cost reflects the charge (in terms of effect on variance) for including this constraint (see Table 5.12).

5.7.9 Markowitz's portfolio selection model

The Markowitz portfolio is created using his 1952 portfolio selection model, which focuses on variance, returns and funds exhaustion. This portfolio has been created to facilitate comparison of the portfolios created according to the quadratic programming mean-variance efficient portfolio selection model with multiple constraints developed in the research work and Markowitz's model. The only constraints for Markowitz's model are no short sales, funds exhaustion and Q_3 (quartile three) level of returns (see Figure 5.8).

This is a portfolio created according to the Markowitz portfolio selection model diversified across 12 companies which were same companies as chosen by the ideal portfolio except Housing Development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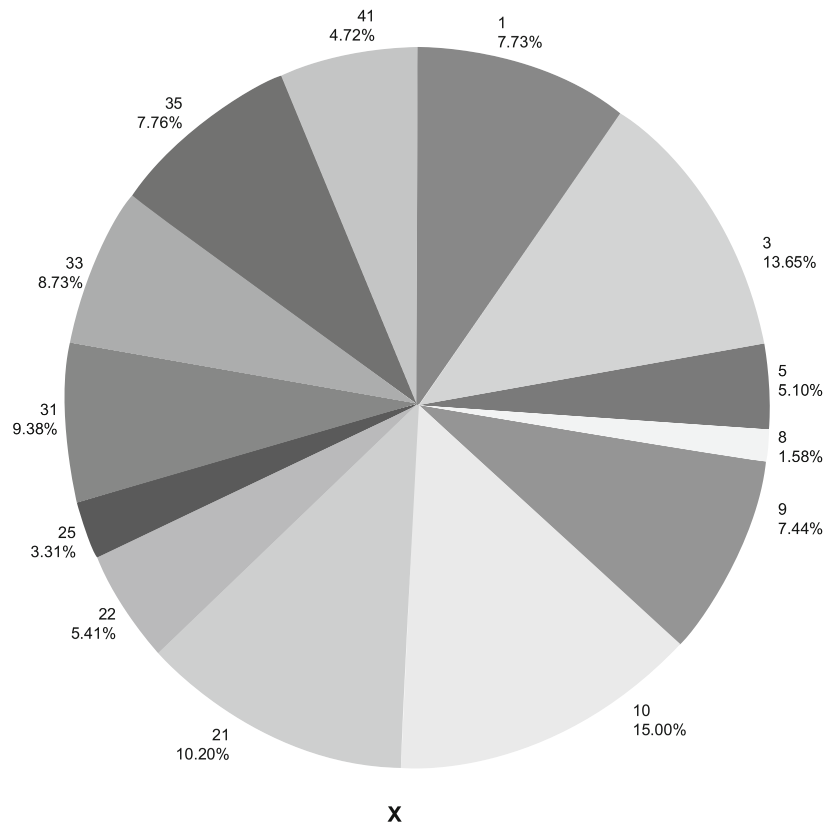


Figure 5.7 Portfolio weights: ideal portfolio

Table 5.12 Ideal portfolio: targets and achievements

Infeasibilities: 0.000000 Model Class: NLP Total solver iterations: 79

Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.1972438	–1.000000
Funds exhaustion	1	0.000000	0.7716190
Returns	0.63	0.000000	–0.6833415E-01
EPS	–	–	–
Beta	–	–	–
Dividend	3200.01	2375.389	0.000000
Free float	–	12141.14	–
Impact cost	0.08	0.000000	–2.458519
Institutional holding	29.02	0.000000	–0.1759535E-01
Market capitalisation	–	–	–
Net profit	12667.35	5455.505	0.000000
P/B ratio	–	–	–
P/E ratio	–	–	–
Promoters holding	50.83	0.000000	–0.8109109E-02
Sales	57801.62	48641.68	0.000000
Turnover	80.23	0.000000	–0.9488568E-04
Unsystematic risk	–	–	–
Volume	1399419	1255697	0.000000

Finance Corporation. The average weight of a security in this portfolio was 8.33 per cent. The portfolio diversified across 12 companies and 11 industrial sectors.

All the constraints were achieved without any surpluses, minimising the variance of the portfolio at 0.176. The negative dual price –0.38 for funds exhaustion constraint shows extent of increase in variance of portfolio with unit increase in the constraint. The returns constraint also displayed a small and negative shadow price (see Table 5.13).

It may be inferred that the ideal mean-variance efficient portfolio was able to decipher the important portfolio variables which were indirectly optimised in addition to mean and variance while creating the efficient portfolio. Accommodating for the multiple objectives of an investor like dividend, liquidity, institutional holding, net profit, promoter's holding, sales, turnover, volume, industrial and company diversification in addition to returns was achieved the ideal mean portfolio.

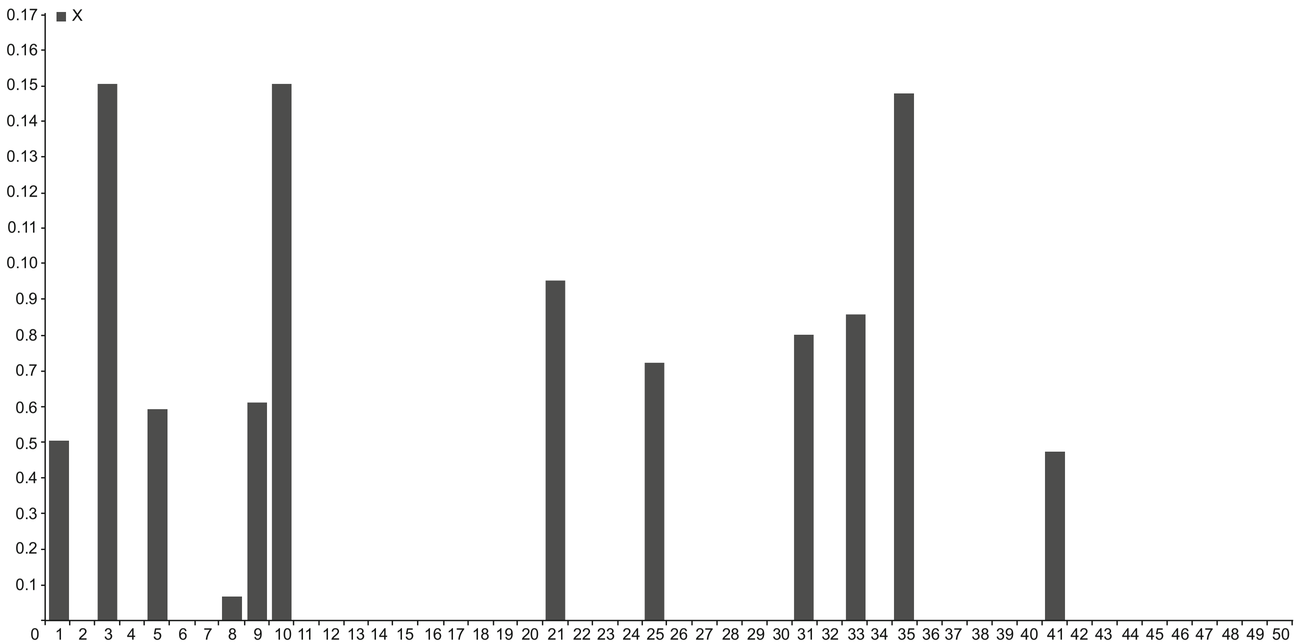


Figure 5.8 Continued

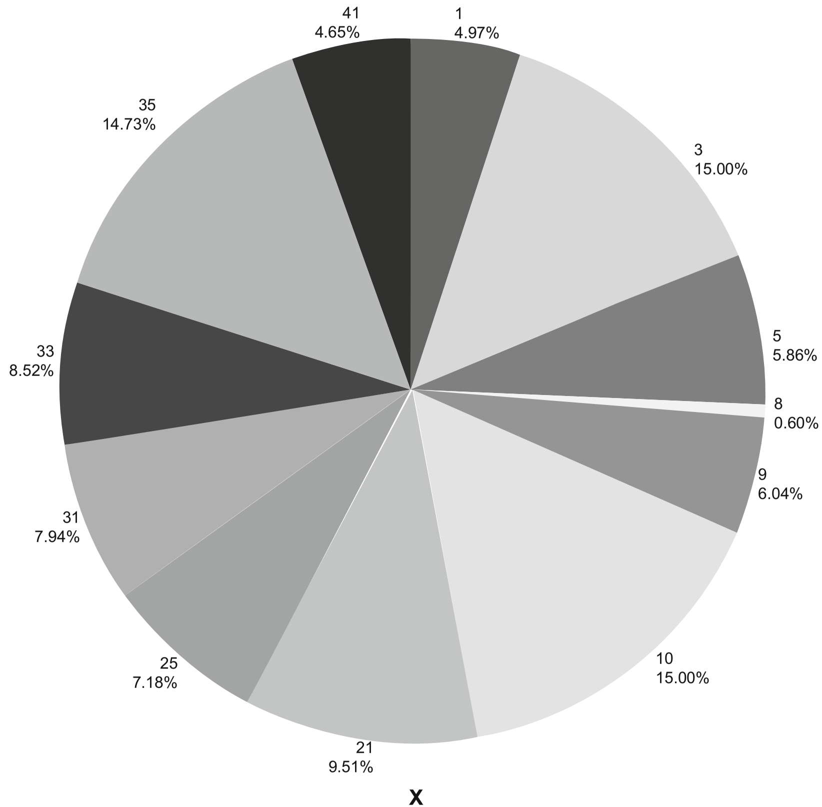


Figure 5.8 Portfolio weights: Markowitz's portfolio

Table 5.13 Markowitz's portfolio: targets and achievements

Infeasibilities: 0.000000	**Model Class: NLP**	**Total solver iterations: 59**	
Variables	**Targets**	**Slack or Surplus**	**Dual Price**
Variance	Minimise	0.1761729	–1.000000
Funds exhaustion	1	0.000000	–0.3801651
Returns	0.63	0.000000	–0.2106992E-01

Table 5.14 Markowitz's efficient frontier

Point	**Mean**	**Variance**	**Point**	**Mean**	**Variance**
1	0.080500	0.175856	11	0.594447	0.175856
2	0.131895	0.175856	12	0.645842	0.176751
3	0.183289	0.175856	13	0.697237	0.185025
4	0.234684	0.175856	14	0.748632	0.198474
5	0.286079	0.175856	15	0.800026	0.224294
6	0.337474	0.175856	16	0.851421	0.320417
7	0.388868	0.175856	17	0.902816	0.516250
8	0.440263	0.175856	18	0.954211	0.831092
9	0.491658	0.175856	19	1.00561	1.26797
10	0.543053	0.175856	20	1.05700	2.03200

Markowitz's efficient frontier

The 20-point Markowitz efficient frontier was derived with the help of programming in the software Lingo 13 (Annex 2). It depicted the minimum variance/maximum expected return portfolios created from 12-year data for the Nifty 50 securities. The 20 mean-variance points obtained are presented in Table 5.14 and graphically depicted in Figure 5.10.

5.8 Comparison of alternate portfolio selection models

Table 5.15 exhibits the weights obtained by Nifty 50 securities in each of the alternate portfolio selection model formulations. All the portfolios except capital gain bias portfolio invested heavily in security X10 (Cairn India). All the portfolios invested in securities X5 (Bajaj Auto) and X21 (Hindustan Unilever). The composition of the ideal portfolio and Markowitz's portfolio was identical with slight differences in secu-

Table 5.15 Weights of securities in the alternate portfolio model formulations

Securities	Diversifier portfolio	Satisficer portfolio	Market trend portfolio	Capital gain bias portfolio	Dividend gain bias portfolio	Equal priority portfolio	Ideal portfolio	Markowitz's portfolio
X1	0.07	0.04	0.00	0.00	0.05	0.01	0.08	0.05
X2	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
X3	0.15	0.00	0.00	0.00	0.14	0.00	0.14	0.15
X4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X5	0.05	0.14	0.03	0.15	0.01	0.13	0.05	0.06
X6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X7	0.00	0.06	0.03	0.00	0.00	0.04	0.00	0.00
X8	0.11	0.08	0.05	0.00	0.06	0.03	0.02	0.01
X9	0.01	0.01	0.03	0.00	0.01	0.01	0.07	0.06
X10	0.15	0.15	0.11	0.00	0.11	0.15	0.15	0.15
X11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X13	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00
X14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X16	0.01	0.01	0.08	0.00	0.07	0.04	0.00	0.00
X17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X21	0.10	0.13	0.13	0.07	0.09	0.14	0.10	0.10
X22	0.00	0.00	0.00	0.00	0.01	0.00	0.05	0.00
X23	0.00	0.00	0.07	0.00	0.00	0.02	0.00	0.00
X24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X25	0.07	0.00	0.00	0.00	0.11	0.00	0.03	0.07
X26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X27	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
X28	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00
X29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X30	0.03	0.11	0.11	0.10	0.00	0.08	0.00	0.00
X31	0.00	0.00	0.00	0.00	0.07	0.03	0.09	0.08
X32	0.00	0.05	0.06	0.07	0.00	0.06	0.00	0.00
X33	0.08	0.00	0.02	0.00	0.13	0.01	0.09	0.09
X34	0.00	0.01	0.02	0.05	0.01	0.04	0.00	0.00
X35	0.12	0.01	0.03	0.00	0.07	0.00	0.08	0.15
X36	0.00	0.15	0.15	0.15	0.00	0.15	0.00	0.00
X37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X38	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00
X39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X40	0.00	0.05	0.03	0.00	0.00	0.05	0.00	0.00
X41	0.06	0.01	0.00	0.00	0.05	0.00	0.05	0.05
X42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X43	0.00	0.00	0.03	0.09	0.00	0.00	0.00	0.00
X44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X50	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00

Note: The security weights have been rounded off to two decimal places.

rity proportions. Certain securities such as X4, X6, X11, X12, X13, X14, X15, X17, X18, X19, X20, X24, X26, X29, X37, X39, X42, X44, X45, X46, X47, X48 and X49 did not form a part of any of the portfolio selection model formulations. Only 27 of the 50 securities were chosen by one or more of the portfolio selection model formulations.

Table 5.16 illustrates values attained for all the portfolio variables in the alternate portfolio selection model formulations. The lowest variance of 0.187 was actualised by the diversifier's portfolio which was slightly more than what was achieved by Markowitz's portfolio selection model (0.176). Ideal portfolio also accomplished a small variance of 0.197. Comparison of the ideal portfolio with Markowitz's portfolio shows that both yield same returns with marginally high variance in ideal portfolio. Even the securities chosen were the same except for HDFC. Thus, the ideal portfolio was able to decipher important portfolio variables optimised by the Markowitz model in addition to mean and variance.

High monthly returns of 0.63 were actualised by capital gain bias portfolio, equal priority portfolio and ideal portfolio, apart from Markowitz's portfolio. Surpluses were generated for portfolio constraints in diversifier's, satisficer's and dividend gain bias portfolios. Highest EPS, beta, dividend, free float, market capitalisation, net profit, price-to-book value ratio, promoter's holding, turnover and unsystematic risk were observed for the capital gain bias portfolio. Beta measuring the degree of responsiveness of security returns to market returns was highest (1.12) for capital gain bias portfolio and lowest (0.75) for dividend gain bias portfolio. The dividend realised was higher in the capital gain bias portfolio than in the dividend gain bias portfolio accompanied by the highest variance of 1.07 among all the modelled portfolios.

The capital gain bias portfolio remained the most liquid portfolio with an impact cost of 0.087 followed by satisficer's portfolio (0.085). All the portfolios were sufficiently liquid with impact cost being equal to or more than 0.08. Highest institutional holding of 31.13 percent was noticed for the market trend portfolio. More than targeted net profits were attained for all the portfolios. Maximum sales were actualised for the diversifier's portfolio which had targeted for the small Q_1 (quartile one) level. Unsystematic risk though targeted at zero level in most of the portfolios was present in all the portfolios ranging from 0.23 to 0.67. The volume of securities traded in each of the portfolios was high.

The plunger's portfolio remained infeasible. The constraints and bounds that caused the infeasibility contained sufficient rows and

Table 5.16 Values achieved by the mean-variance efficient model formulations

Portfolio model	markowitz's model	Diversifier's portfolio	Satisficer's portfolio	Plunger's portfolio	Market trend portfolio	Capital gain bias	Dividend gain bias	Equal priority	Ideal portfolio
σ^2	0.176	0.187	0.316	2.24	0.364	1.07	0.206	0.322	0.197
μ	0.63	0.53	0.605	–	0.52	0.63	0.55	0.63	0.63
EPS	–	28.15	41.91	–	41.63	60.33	35.42	44.33	–
β	–	0.7524	0.94	–	0.95	1.12	0.75	0.94	–
D_i	–	5693.56	4844.12	–	6244.07	7292.02	6899.1	6899.1	5575.4
Λ	–	101037.93	118508.4	–	135404.68	140859.72	110656.01	1008209.07	–
I	–	0.08	0.085	–	0.084	0.087	0.081	0.084	0.08
ζ	–	25.71	29.02	–	31.13	24.2	29.02	29.02	29.02
C	–	421064	333162.20	–	422309.53	413914.34	468267.47	389392.24	–
π	–	17215	16391.19	–	22744.08	25069.58	21047.93	21423.17	18123.05
P/B	–	5.85	5.17	–	4.51	5.58	5.88	5.1	–
P/E	–	14.92	20.1	–	120.74	20.65	14.92	20.1	–
ξ	–	55.04	50.83	–	47.96	57.58	50.83	50.83	50.83
$\hat{S}$	–	176922.75	159281.7	–	172784.25	147440.47	153841.54	133323.28	106443.3
T	–	80.01	80.23	–	102.11	102.4	86.02	83.63	80.23
e_t	–	0.39	0.23	–	0.548	0.45	0.67	0.30	–
V	–	2648603.13	1500899.03	–	2592661.52	2992858.13	2441405.13	1975355.63	2655116.16

Note: σ^2 represents variance of the portfolio. μ represents the capital returns. *eps* represents earnings-per-share. *D* represents the dividend returns. λ represents free float. *i* represents impact cost. ζ represents institutional holding. *C* represents market capitalisation. π represents net profit constraint. *P/B* represents price-to-book value ratio. *P/E* represents price-to-earnings ratio. ξ represents promoters holding. *S* represents sales. *v* represents volume. *t* represents turnover. β represents beta. *e*represents unsystematic risk. The number of constraints in the ideal portfolio formulation has been limited to only the important explanatory variables as per the Granger causality tests and multiple regression estimation.

necessary rows. The *DEBUG* command in Lingo identifies two types of sets: *sufficient* and *necessary*. Removing any sufficient set object from the model is sufficient to fix the entire model. Removing any object from the necessary set fixes the remaining objects within that set. In our model, constraints and bounds that caused the infeasibility contained sufficient rows and necessary rows. Dropping the funds exhaustion constraint, forming sufficient rows would make the model feasible. The constraint could not be met and reported a slack variable of 0.149 in the solution. The necessary rows to make the model feasible were the institutional holding constraint and the promoter's holding constraint. The sufficient variable bounds were no short-selling of securities X11, X14 and X37, which were the three pharmaceutical companies Cipla Ltd., Dr. Reddy's Laboratories Ltd. and Ranbaxy Laboratories Ltd. The necessary variable bounds included no short-selling restrictions on 33 securities and company diversification upper bound on 12 of the securities.

5.9 Markowitz's efficient frontier and mean-variance efficient portfolios

The mean (expected return) and variance (expected risk) of all the modelled portfolios have been plotted along with Markowitz's efficient frontier (see Figure 5.10). Notwithstanding the volume of research conducted on efficient frontiers, in many cases it is still not the easiest thing to compute a mean-variance efficient frontier even when all constraints are linear. This is particularly true of problems having dense covariance matrices (see Figure 5.9). Although the mean-variance efficient frontier of Markowitz's problem is a continuous curve, it is rarely rendered as such. Rather, efficient frontiers are customarily shown in the form of dotted representations. In this way, with so many dots, dotted representations nearly look like the continuous curves they are to represent. It takes an amount of optimisation to generate a representation of Markowitz's efficient frontier.

'M' stands for Markowitz's portfolio with 0.63 expected return and risk at 0.176. The modelled portfolio '1' lies very close to the efficient frontier. It yields the same return with marginally high variance at 0.197^2. This slightly high risk is well compensated by the assurance of a minimum amount of earnings (dividend, sales and net profit), liquidity (impact cost, turnover and volume) and other party's interest (promoter's holding and institutional holding) through multiple constraints in the programming model.

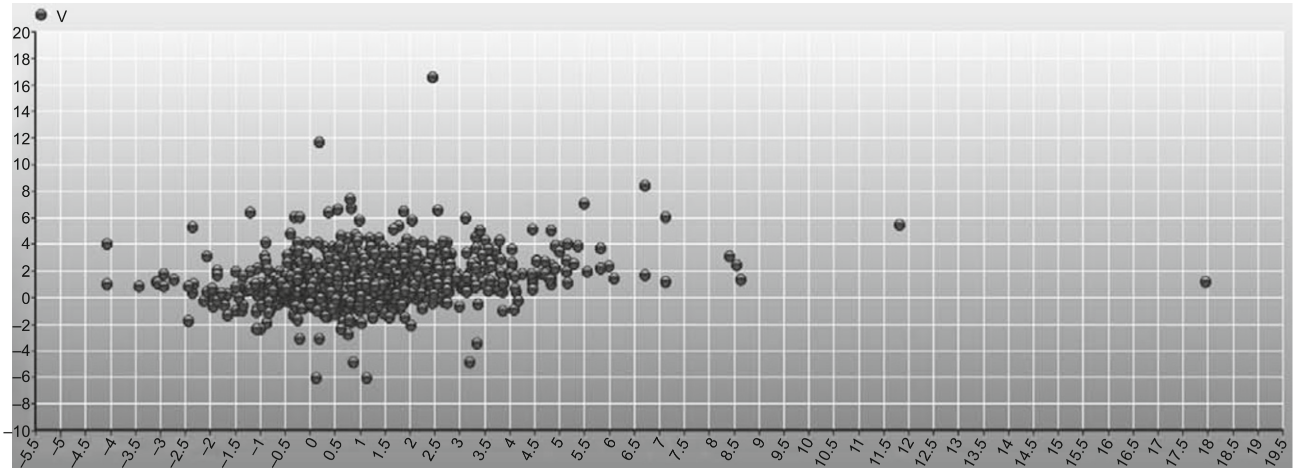


Figure 5.9 Covariance matrix for Nifty 50 securities

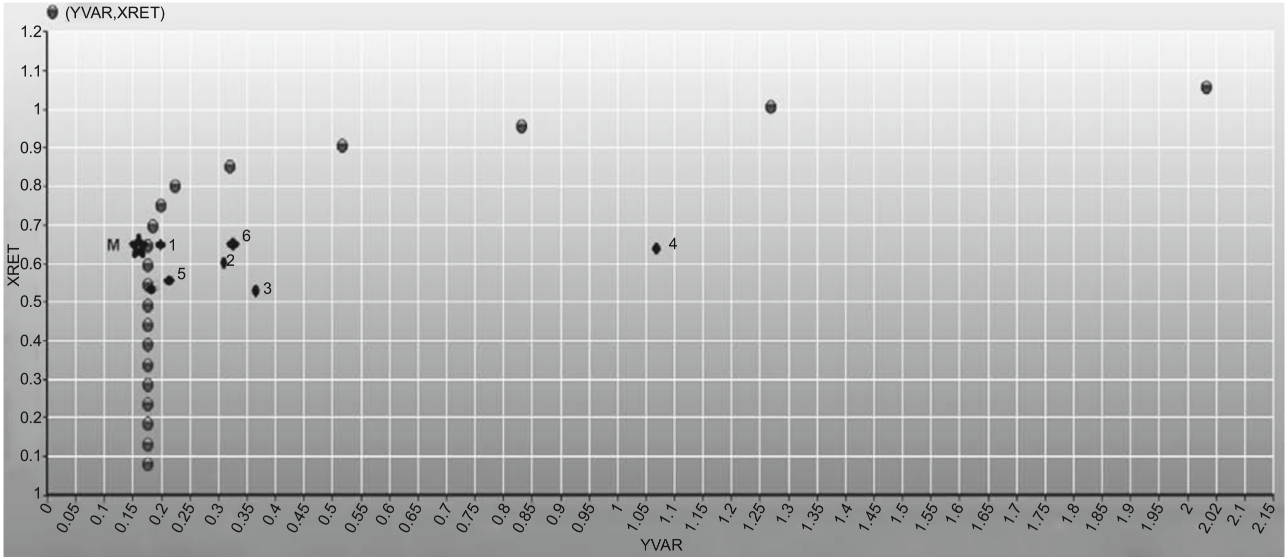


Figure 5.10 Alternate portfolio selection model formulations and Markowitz's efficient frontier

Portfolio '1' for the diversifier's portfolio lies almost on the efficient frontier with 0.187 level of variance. In addition to keeping the variance at a minimum level, this portfolio also achieves more than targeted Q_1 (quartile one) values of many other important portfolio variables such as EPS, returns, beta, dividend, free float, impact cost, sales, net profit, volume, turnover, price-to-earnings ratio, price-to-book value ratio, promoter's and institutional holding. The industrial diversification and company diversification constraints have also been satisfied.

Portfolio '5', the dividend gain bias portfolio, is placed just beside the efficient frontier. This portfolio minimises risk at 0.206, provides returns of 0.55 per cent and achieves high Q_3 (quartile three) level of dividend. High levels of volume, turnover, sales, net profit, free float, institutional holding and unsystematic risk were also attained by the portfolio. Because systematic risk present was at a lower level, the magnitude of unsystematic risk (0.67) increased substantially with the highest value of all the modelled portfolios.

Other modelled portfolios, 2, 3 and 6, namely the satisficer portfolio, market trend portfolio and equal priority portfolio respectively, although below the efficient frontier are in its close proximity. They give little lesser returns for the same level of risk but take care of many other important ambitions of an investor. Portfolio labelled '2' is the satisficer portfolio and realises at least the median levels for all portfolio variables considered in the study. Portfolio labelled '3' is the market trend portfolio, which attains highest institutional holding and second highest sales and turnover among all the modelled portfolios apart from yielding high dividend, net profit, promoter's holding, volume and free float. The equal priority portfolio is able to realise high Q_3 (quartile three) level of capital gains as well as dividends, highest EPS and free float, high impact cost, turnover and volume.

The capital gain bias portfolio depicted as '4' in the figure lies quite below the efficient frontier. Although it achieves very high (at least quartile three) level values for most of the constraints in the problem, the extent of variance realised is too high. It cannot be regarded as efficient in Markowitz's sense. To lie on or near the efficient frontier, the investor should not seek to maximise all the portfolio variables. Accordingly, a select few constraints should be optimised based on investor profile. The plunger's portfolio is also setting high Q_3 (quartile three) values for all constraints which led to an infeasible solution and could not be presented in the figure. Comparison of the modelled portfolios with the Nifty 50 index having a 12-year monthly average return of 1.31 per cent and variance of 0.61 found all the modelled portfolios to yield a better risk-return trade-off than the index itself.

5.10 Multivariate regression analysis: estimating equations

An attempt has been made to examine returns as a function of accounting and financial variables like earnings-per-share, dividend, free float, impact cost, institutional holding, market capitalisation, net profits, price-to-book value ratio, price-to-earnings ratio, promoter's holding, sales, turnover, unsystematic risk and volume with the help of multiple regression equation. The analysis was repeated with excess returns to standard deviation ratio as the dependant variable. The two regression equations with expected returns and excess return to standard deviation as the dependant variables respectively were estimated using the software E-views version 5.1. Generalised Auto Regressive Conditional Heteroscedasticity (GARCH) method of regression was used to deal with the conditional variance and heteroscedasticity issues. The list of regressors included a constant term. Beta was purposefully excluded from the analysis as it has been extensively researched and has been found to have a significant effect on the returns of a security; to highlight the importance of other indicative factors, it was excluded from the analysis.[3]

The results in Table 5.17 indicate the marginal contribution of each of the factors (constraints) set in the portfolio selection model to explain the cross section of returns. The column labelled coefficient depicts the estimated coefficients measuring the marginal contribution of the independent variable to the dependent variable returns, holding all other variables fixed. They show the slope of the relation between the corresponding independent variable and the dependent variable, assuming all other variables do not change. The coefficient of constant term 1.69 is the base level of prediction when all of the independent variables are zero.

The standard error column reports the estimated standard errors of the coefficient estimates measuring its statistical reliability. They are the square roots of the diagonal element of the coefficient covariance matrix. Small standard errors show presence of very little statistical noise in the estimates. The t-statistic computed as the ratio of an estimated coefficient to its standard error tests the hypothesis that a coefficient is equal to zero. The probability of observing the t-statistic shown as p-value is the marginal significance level to accept or reject the hypothesis that the true coefficient is zero against a two-sided alternative that it is different from zero. At 5 per cent level of significance, the hypothesis of zero coefficients for EPS, free float, impact cost and institutional holding can be rejected.

The output Table 5.17 presented here shows impact cost with a coefficient of 44.63 is the most important factor explaining variance in the returns. The p-value of 0.049 showing the marginal significance level rejects the hypothesis of a zero coefficient at 5 per cent level of significance. The other factors explaining the variance in returns are institutional holding, promoter's holding and turnover. The earnings-per-share, dividend and free float inversely explain returns.

R-squared value of 0.31 shows the success of regression model where 31 per cent variance in the dependent variable can be explained through the factors identified. The Durbin-Watson statistic is more than two,

Table 5.17 Results of multiple regression model 1

Dependent Variable: Returns				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	1.694728	2.980290	–0.568645	0.5731*
EPS	–0.014396	0.009208	–1.563426	0.1267*
Dividend	–1.79E-05	6.72E-05	–0.266503	0.7914*
Free float	–5.15E-06	3.34E-06	–1.541429	0.1320*
Impact cost	44.63638	22.97435	1.942879	0.0499*
Institutional holding	0.030387	0.030943	0.982050	0.3326*
Market capitalisation	1.10E-07	8.45E-07	0.130305	0.8971*
Net profit	1.76E-05	2.86E-05	0.613607	0.5433*
P/B ratio	–0.018263	0.075103	–0.243172	0.8093*
P/E ratio	–0.000145	0.000276	–0.525118	0.6027*
Promoters holding	–0.004890	0.022905	–0.213477	0.8322*
Sales	–4.18E-07	1.16E-06	–0.359355	0.7214*
Volume	–3.44E-08	1.05E-07	–0.327481	0.7452*
Turnover	0.008193	0.004768	1.718342	0.0943**
R-squared	0.307978	Mean dependent variable		2.345400
Adjusted R-squared	0.058081	S.D. dependent variable		1.351684
S.E. of regression	1.311843	Akaike info criterion		3.612239
Sum squared residual	61.95358	Schwarz criterion		4.147606
Log likelihood	–76.30599	F-statistic		1.232420
Durbin-Watson stat	2.529222	Prob (F-statistic)		0.029814

Note: *is tested at a 5% level of significance and **is tested at 10% level of significance.

which evidences the absence of positive correlation in the series. The p-value for F-statistic rejects the hypothesis of all the slope coefficients being equal to zero. The standard error of regression 1.312 is a measure of estimated variance of the residuals. The mean and standard deviation of the dependent variable returns has been found to be 2.34 and 1.35 respectively. Akaike Information Criterion (AIC) is used in model selection for non-nested alternatives and smaller values of AIC are preferred.

The F-statistic reported in the table is from a test of hypothesis that all of the slope coefficients in the regression are zero and the p-value is the marginal significance level of F-test. As the p-value is less than a 5 per cent level of significance, the null hypothesis that all slope coefficients are equal to zero can be rejected.

An attempt has been made to improve the results of regression analysis by observing the correlation values of the independent variables and dropping some of them from the list of correlated variables. The correlation matrix has been presented in Table 5.18. The dividend and net profit were found to be highly positively correlated (0.8685). The dividend and market capitalisation were also moderately positively correlated. High degrees of correlation were also found between free float, market capitalisation and turnover. Market capitalisation was found to be moderately positively correlated with dividend, net profit and turnover. Net profit, sales and turnover were also found to be positively correlated with the correlation coefficient ranging between 0.5 and 1.

The high correlation between these independent variables indicates the presence of multi-collinearity in them. Hence, some of the variables namely net profit, market capitalisation and sales were excluded from the regression model. The regression equation with returns as the dependent variable is re-run with EPS, dividend, free float, impact cost, institutional holding, price-to-book ratio, price-to-earnings ratio, promoter's holding, volume and turnover as the independent variables. The results of regression analysis have been presented in Table 5.19.

The negative coefficient of dividend is found to be statistically significant at 5 per cent level of significance. The coefficient of free float is also found to be statistically significant at 10 per cent level of significance. The coefficient of impact cost was the highest even now. A small positive correlation coefficient of free float is also statistically significant. The R-squared value depicted that even with reducing the number of independent variables 23 per cent of variance in returns could still be explained by the set of independent variables.

Table 5.18 Correlation matrix

	Eps	D_i	λ	***I***	ς	***C***	π	***p/b***	***p/e***	ξ	***S***	***V***	***t***
eps	1.0000	0.1377	0.2758	−0.0164	0.1697	−0.0288	0.2382	−0.0423	−0.1041	−0.2179	0.0624	−0.4267	0.1604
Di	0.1377	1.0000	0.2253	−0.2432	−0.1927	0.5367	0.8685	0.0903	−0.0324	0.1537	0.3939	0.0787	0.2365
Λ	0.2758	0.2253	1.0000	−0.2240	0.2665	0.5144	0.4322	0.1388	0.0314	−0.3543	0.3194	0.0974	0.7182
I	−0.0164	−0.2432	−0.2240	1.0000	−0.0932	−0.3390	−0.2486	−0.3592	−0.0800	0.0676	−0.0966	−0.0198	−0.3877
ς	0.1697	−0.1927	0.2665	−0.0932	1.0000	−0.3378	−0.1866	−0.1860	−0.0721	−0.9153	−0.1375	−0.1114	0.0703
C	−0.0288	0.5367	0.5144	−0.3390	−0.3378	1.0000	0.6621	0.2485	0.1592	0.3462	0.4094	0.3162	0.6250
π	0.2382	0.8685	0.4322	−0.2486	−0.1866	0.6621	1.0000	−0.0401	0.0932	0.1588	0.6139	0.1642	0.5196
p/b	−0.0423	0.0903	0.1388	−0.3592	−0.1860	0.2485	−0.0401	1.0000	0.0900	0.1445	−0.1139	−0.1738	0.0357
p/e	−0.1041	−0.0324	0.0314	−0.0800	−0.0721	0.1592	0.0932	0.0900	1.0000	0.0556	0.0382	0.0600	0.0925
ξ	−0.2179	0.1537	−0.3543	0.0676	−0.9153	0.3462	0.1588	0.1445	0.0556	1.0000	0.1358	0.1823	−0.0752
S	0.0624	0.3939	0.3194	−0.0966	−0.1375	0.4094	0.6139	−0.1139	0.0382	0.1358	1.0000	0.1602	0.4235
V	−0.4267	0.0787	0.0974	−0.0198	−0.1114	0.3162	0.1642	−0.1738	0.0600	0.1823	0.1602	1.0000	0.4127
t	0.1604	0.2365	0.7182	−0.3877	0.0703	0.6250	0.5196	0.0357	0.0925	−0.0752	0.4235	0.4127	1.0000

Note: *eps* represents earnings-per-share. D_i represents the dividend returns. λ represents free float. *i* represents impact cost. ς represents institutional holding. *C* represents market capitalisation. π represents net profit constraint. *P/B* represents price-to-book value ratio. *P/E* represents price-to-earnings ratio. ξ represents promoters holding. *S* represents sales. *v* represents volume. *t* represents turnover.

Table 5.19 Results of multiple regression model II

Dependent variable: Returns				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
C	0.070598	0.647934	0.108958	0.9138
EPS	0.000287	0.001908	0.150382	0.8812
Dividend	–1.15E-05	5.54E-06	–2.068977	0.0452**
Free float	1.24E-06	6.51E-07	1.907047	0.0639*
Impact cost	1.419817	5.003968	0.283738	0.7781
Institutional holding	0.000863	0.006759	0.127627	0.8991
Price-to-book ratio	0.005075	0.014772	0.343541	0.7330
Price-to-earnings ratio	–3.12E-05	5.61E-05	–0.556549	0.5810
Promoter's holding	0.003205	0.004607	0.695745	0.4907
Volume	2.22E-08	2.28E-08	0.971605	0.3372
Turnover	–0.000283	0.000933	–0.303083	0.7634
R-squared	0.236328	Mean dependent var		0.516876
Adjusted R-squared	0.040515	S.D. dependent var		0.293789
S.E. of regression	0.287776	Akaike info criterion		0.538269
Sum squared resid	3.229784	Schwarz criterion		0.958915
Log likelihood	–2.456736	F-statistic		1.206905
Durbin-Watson stat	1.737363	Prob (F-statistic)		0.316720

Note: **is tested at 5% level of significance and *is tested at 10% level of significance.

Even after limiting the number of explanatory variables, the R-squared value was low and most of the variables were not significant. Hence, a further investigation to improve the results of regression analysis was undertaken. Returns were regressed only with the two significant variables of the previous model: dividend and free float. A constant term was also included in this model. The results of regression model III are presented in Table 5.20. The coefficients of all the variables and the constant term were statistically significant at 5 per cent and 10 per cent level of significance. The value of adjusted R^2 also improved. By excluding the irrelevant variables the intercept is now capturing the effect of the omitted variables.

Investigations into the improvements of regression model III over regression model II were undertaken. The efficiency index[4] of the models (II and III) was calculated by comparing the actual and fitted values. The mean, standard deviation and coefficient of variation of the two efficiency indexes were then compared. The comparison of the two models in terms of their efficiency is presented in Table 5.21. The mean of efficiency index became closer to hundred (102.88) as the number of variables was reduced. The standard deviation was also

Table 5.20 Results of multiple regression model III

Dependent Variable: Returns				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
Dividend	–9.45E-06	4.90E-06	–1.926049	0.0602**
Free float	8.96E-07	3.58E-07	2.500607	0.0159*
Constant	0.457007	0.062500	7.312076	0.0000
R-squared	0.148695	Mean dependent var		0.516876
Adjusted R-squared	0.112469	S.D. dependent var		0.293789
S.E. of regression	0.276775	Akaike info criterion		0.326901
Sum squared resid	3.600408	Schwarz criterion		0.441623
Log likelihood	–5.172534	F-statistic		4.104683
Durbin-Watson stat	1.594991	Prob (F-statistic)		0.022751

Note: *is tested at 5% level of significance and ** is tested at 10% level of significance.

Table 5.21 Comparing the efficiency of model II and model III

	Model II			**Model III**		
Securities	**Actual**	**Fitted**	**Efficiency Index**	**Actual**	**Fitted**	**Efficiency Index**
1	0.24833	0.36486	68.06172	0.24833	0.4618	53.77436
2	–0.08896	0.41126	–21.6311	–0.08896	0.45879	–19.3901
3	0.57729	0.37991	151.9544	0.57729	0.44423	129.953
4	0.53778	0.68828	78.1339	0.53778	0.67451	79.72899
5	0.96809	0.62011	156.1159	0.96809	0.58604	165.1918
6	0.43076	0.39387	109.366	0.43076	0.42803	100.6378
7	0.45278	0.53495	84.63969	0.45278	0.50493	89.67184
8	0.05667	0.43535	13.01711	0.05667	0.44614	12.70229
9	0.47	0.4341	108.27	0.47	0.55486	84.70605
10	1.16048	0.56882	204.0153	1.16048	0.5188	223.6854
11	0.38556	0.27246	141.5107	0.38556	0.37175	103.7149
12	1.11706	0.81665	136.7856	1.11706	0.62478	178.7925
13	0.72123	0.66828	107.9233	0.72123	0.5098	141.4731
14	0.28868	0.29496	97.8709	0.28868	0.43119	66.9496
15	0.36861	0.51086	72.1548	0.36861	0.51323	71.8216
16	0.08319	0.38285	21.72914	0.08319	0.477	17.44025
17	0.50521	0.46464	108.7315	0.50521	0.45021	112.2165
18	0.65049	0.60759	107.0607	0.65049	0.65334	99.56378
19	0.28292	0.40879	69.20913	0.28292	0.44414	63.70063
20	0.47764	0.47953	99.60586	0.47764	0.5002	95.4898
21	0.35028	0.46042	76.07836	0.35028	0.42436	82.54312
22	0.49924	0.52441	95.20032	0.49924	0.64084	77.904
23	0.55028	0.62473	88.08285	0.55028	0.69749	78.89432
24	0.47028	0.52958	88.80245	0.47028	0.5936	79.22507
25	0.59493	0.70148	84.81069	0.59493	0.73244	81.22577
26	0.84238	0.612	137.6438	0.84238	0.59078	142.5878

Table 5.21 Continued

	Model II			Model III		
Securities	**Actual**	**Fitted**	**Efficiency Index**	**Actual**	**Fitted**	**Efficiency Index**
27	1.23191	0.69566	177.0851	1.23191	0.55261	222.9258
28	1.00319	0.73661	136.1901	1.00319	0.66447	150.976
29	0.89319	0.63551	140.547	0.89319	0.59863	149.2057
30	0.36181	0.60987	59.32576	0.36181	0.70322	51.45047
31	0.95972	0.41635	230.508	0.95972	0.51616	185.9346
32	0.91067	0.51262	177.6501	0.91067	0.50117	181.7088
33	0.65202	0.53579	121.6932	0.65202	0.38612	168.8646
34	0.27632	0.04334	637.5635	0.27632	0.08182	337.7169
35	0.30407	0.65228	46.61648	0.30407	0.48845	62.25202
36	0.56167	0.49276	113.9845	0.56167	0.46931	119.68
37	0.34264	0.38739	88.44833	0.34264	0.45816	74.7861
38	0.50722	0.85935	59.02368	0.50722	0.79563	63.75074
39	0.27847	0.4052	68.72409	0.27847	0.48114	57.87713
40	0.53986	0.55717	96.89323	0.53986	0.53363	101.1675
41	0.41771	0.50708	82.37556	0.41771	0.48867	85.47895
42	0.4991	0.47505	105.0626	0.4991	0.54156	92.15969
43	0.00958	0.61963	1.546084	0.00958	0.37184	2.576377
44	0.57736	0.62371	92.56866	0.57736	0.56472	102.2383
45	0.03521	0.51209	6.875744	0.03521	0.4884	7.209255
46	0.58054	0.55287	105.0048	0.58054	0.48059	120.7974
47	0.50042	0.41701	120.0019	0.50042	0.49289	101.5277
48	0.31917	0.43819	72.83827	0.31917	0.49879	63.98885
49	0.64076	0.43595	146.9802	0.64076	0.49127	130.4293
50	0.44	0.53157	82.77367	0.44	0.46125	95.39295
Mean			109.1085	102.886		
Standard deviation			90.54553	62.45916		
Coefficient of variation			82.98671	60.70716		

lower for model III (62.45) in comparison to that of model II (90.54). Also the coefficient of variation was better for model III (60.70) than model II (82.98).

Figure 5.11 depicts the forecasted returns of model III. Root mean error and mean absolute error provide the relative measure to compare forecasts. Theil inequality coefficient of 0.23 implies a good fit of forecast model. Bias proportion is negligible implying the forecast mean is almost equal to actual mean. Variance proportion and covariance proportion indicate a difference in variance and covariances of the actual values and forecasts.

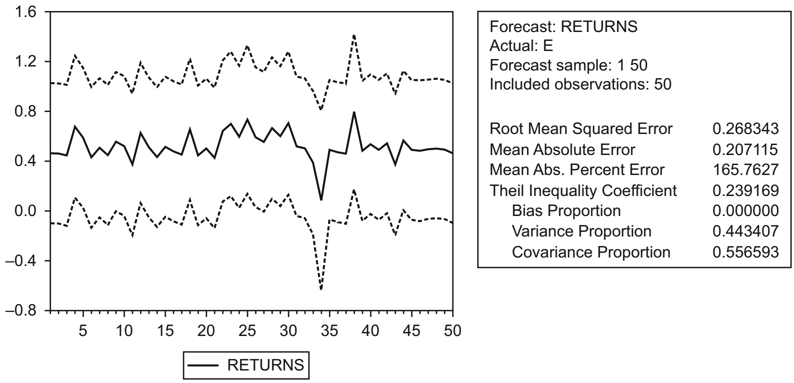


Figure 5.11 Model forecasted returns

The second regression equation was estimated with excess returns to standard deviation ratio as the dependant variable and all other explanatory variables the same. The results of regression analysis are presented in Table 5.22.

The marginal contribution of each of the factors (constraints) set in the portfolio selection model, to the excess return to standard deviation (S_t) is attempted by equation estimation in E-views. The output Table 5.22 shows impact cost with a coefficient of 52.66, which is the most important factor explaining variance in the Sharpe ratio. The p-value of 0.0325 showing the marginal significance level rejects the hypothesis of a zero coefficient at a 5 per cent level of significance. The other factors explaining the excess return to standard deviation are institutional holding and turnover. The earnings-per-share inversely explain S_t.

R-squared value of 0.32 shows the success of regression model where 32 per cent variance in the dependent variable has been explained identified portfolio variables. The Durbin-Watson statistic is more than two, which evidences the absence of positive correlation in the series. The p-value for F-statistic rejects the hypothesis of all slope coefficients being equal to zero.

The small value of R-squared led to further examination of issue of multi-collinearity among the explanatory variables. A high correlation between some of the independent variables indicated the presence of multi-collinearity in them. The variables namely net profit, market capitalisation and sales were excluded from the regression model. The regression equation with excess returns to standard deviation as the

Table 5.22 Results of regression model IV

Dependent variable: excess return to standard deviation ratio

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	–4.065998	3.071934	–1.323596	0.1940*
EPS	–0.013746	0.009491	–1.448347	0.1562*
Dividend	–6.97E-07	6.92E-05	–0.010068	0.9920*
Free float	–4.73E-06	3.45E-06	–1.373027	0.1782*
Impact cost	52.66739	23.68081	2.224054	0.0325**
Institutional holding	0.037824	0.031894	1.185935	0.2434*
Market capitalisation	–4.90E-08	8.71E-07	–0.056268	0.9554*
Net profit	7.86E-06	2.95E-05	0.266474	0.7914*
Price-to-book value ratio	–0.020556	0.077413	–0.265537	0.7921*
Price-to-earnings ratio	–0.000107	0.000285	–0.374803	0.7100*
Promoter's holding	0.002382	0.023610	0.100872	0.9202*
Sales	–3.32E-07	1.20E-06	–0.277143	0.7833*
Volume	–3.63E-08	1.08E-07	–0.335061	0.7395*
Turnover	0.010091	0.004914	2.053303	0.0474**
R-squared	0.325056	Mean dependent var		1.305339
Adjusted R-squared	0.081326	S.D. dependent var		1.410765
S.E. of regression	1.352182	Akaike info criterion		3.672813
Sum squared residual	65.82230	Schwarz criterion		4.208179
Log likelihood	–77.82032	F-statistic		1.333673
Durbin-Watson stat	2.566417	Prob (F-statistic)		0.023623

Note:** is tested at 5% level of significance and *is tested at 10% level of significance.

dependent variable is re-run with EPS, dividend, free float, impact cost, institutional holding, price-to-book ratio, price-to-earnings ratio, promoter's holding, volume and turnover as the independent variables. The results of regression analysis model V have been presented in Table 5.23.

Impact cost and turnover are the two statistically significant explanatory variables at a 5 per cent level of significance. R-squared value shows the explanation of 32 per cent variance in excess returns to standard deviation by the model. The F-statistic is significant at a 10 per cent level of significance rejecting the null hypothesis of all the coefficients being zero.

A further examination into the excess returns to standard deviation is undertaken by estimating a regression equation with only the two significant variables of the previous model and the constant term. The list of independent variables now included only impact cost and turnover (see Table 5.24).

Table 5.23 Results of regression model V

Dependent variable: excess returns to standard deviation ratio				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
EPS	–0.012886	0.008624	–1.494212	0.1432
Dividend	1.38E-05	2.50E-05	0.551612	0.5844
Free float	–4.68E-06	2.94E-06	–1.589292	0.1201
Impact cost	52.51792	22.62135	2.321608	0.0256*
Institutional holding	0.037810	0.030555	1.237456	0.2233
Price-to-book ratio	–0.026230	0.066779	–0.392784	0.6966
Price-to-earnings ratio	–8.32E-05	0.000254	–0.328274	0.7445
Promoter's holding	0.002579	0.020827	0.123832	0.9021
Volume	–3.62E-08	1.03E-07	–0.350902	0.7276
Turnover	0.010298	0.004219	2.441236	0.0193*
Constant	–4.083485	2.929105	–1.394107	0.1712
R-squared	0.323175	Mean dependent var		1.305339
Adjusted R-squared	0.149630	S.D. dependent var		1.410765
S.E. of regression	1.300944	Akaike info criterion		3.555596
Sum squared residuals	66.00574	Schwarz criterion		3.976241
Log likelihood	–77.88989	F-statistic		1.862198
Durbin-Watson stat	2.580881	Prob (F-statistic)		0.081328**

Note: *is tested at 5% level of significance ** is tested at 10% level of significance.

Table 5.24 Results of regression model VI

Dependent variable: excess returns to standard deviation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
Impact cost	43.54693	19.95750	2.181984	0.0341*
Turnover	0.005002	0.002426	2.061521	0.0448*
C	–2.730537	1.737305	–1.571708	0.1227
R-squared	0.121497	Mean dependent var		1.305339
Adjusted R-squared	0.084113	S.D. dependent var		1.410765
S.E. of regression	1.350130	Akaike info criterion		3.496403
Sum squared resid	85.67394	Schwarz criterion		3.611124
Log likelihood	–84.41007	F-statistic		3.250036
Durbin-Watson stat	2.430613	Prob (F-statistic)		0.047640*

Note: *is tested at 5% level of significance.

The coefficient of impact cost and turnover are statistically significant at 5 per cent level of significance. The F-statistic is also significant. Reduction in the number of constraints did not lead to improvement in the value of R-squared. A further investigation into the efficiency of model V and model VI was undertaken by a comparison of their efficiency index (see Table 5.25). The comparison of efficiency index of the two models shows that model VI is a better model with a closer to hundred mean, a smaller standard deviation and a lesser coefficient of variation.

Table 5.25 Comparison of efficiency index of model V and model VI

	Model V			**Model VI**		
Securities	**Actual**	**Fitted**	**Efficiency Index**	**Actual**	**Fitted**	**Efficiency Index**
1	1.79816	1.77789	101.1401	1.79816	1.22793	146.4383
2	2.24736	2.10227	106.9016	2.24736	1.23689	181.6944
3	0.04915	0.33303	14.75843	0.04915	0.36863	13.33315
4	1.20486	0.25486	472.7537	1.20486	0.96018	125.4827
5	0.43563	0.14732	295.7032	0.43563	1.48005	29.43347
6	2.52213	1.03102	244.6247	2.52213	0.88389	285.3443
7	2.42006	1.2809	188.9343	2.42006	1.37107	176.5089
8	0.95292	1.68674	56.49478	0.95292	1.25361	76.01407
9	0.68288	0.88393	77.25499	0.68288	1.3166	51.86693
10	−1.7239	1.30028	−132.579	−1.7239	1.14973	−149.94
11	0.76333	0.5225	146.0919	0.76333	0.45166	169.0054
12	2.54698	0.58352	436.4855	2.54698	1.27832	199.2443
13	0.98408	2.87815	34.19141	0.98408	2.22864	44.15608
14	2.10284	0.81203	258.9609	2.10284	0.61	344.7279
15	1.02095	0.88347	115.5614	1.02095	0.84055	121.4621
16	−0.37263	0.92342	−40.3533	−0.37263	1.32229	−28.1807
17	1.58479	0.79365	199.6837	1.58479	0.81559	194.3121
18	0.55301	1.38611	39.89655	0.55301	1.71414	32.26166
19	−0.12409	0.28744	−43.1707	−0.12409	0.43526	−28.5094
20	2.6731	1.65047	161.9599	2.6731	1.50465	177.6559
21	−0.6625	0.25405	−260.775	−0.6625	0.43076	−153.798
22	−0.0131	1.51569	−0.86429	−0.0131	0.76481	−1.71284
23	3.2204	2.64391	121.8044	3.2204	1.93575	166.3645
24	−0.34398	0.58437	−58.8634	−0.34398	0.73849	−46.5788
25	1.71531	0.84962	201.8914	1.71531	1.73129	99.07699
26	3.90589	3.57412	109.2826	3.90589	1.63996	238.1698
27	3.09526	2.52351	122.6569	3.09526	2.05263	150.7948
28	−1.24048	−0.15861	782.0944	−1.24048	1.72743	−71.8107
29	3.15842	1.44839	218.0642	3.15842	1.76277	179.1737

Table 5.25 Continued

	Model V			Model VI		
Securities	Actual	Fitted	Efficiency Index	Actual	Fitted	Efficiency Index
30	1.11079	1.78024	62.39552	1.11079	1.75608	63.25395
31	2.21803	1.56126	142.0667	2.21803	0.86126	257.5331
32	2.16508	1.67365	129.3628	2.16508	1.39754	154.9208
33	–1.45765	1.19079	–122.41	–1.45765	1.4513	–100.438
34	1.85137	1.28979	143.5404	1.85137	1.55713	118.8963
35	0.92003	0.73111	125.8402	0.92003	1.03043	89.28603
36	0.71412	1.2046	59.28275	0.71412	1.44146	49.54144
37	3.76836	1.33564	282.1389	3.76836	0.8427	447.1769
38	2.51451	2.19611	114.4984	2.51451	2.57598	97.61372
39	2.5133	2.8396	88.50894	2.5133	1.83551	136.9265
40	2.45099	1.02649	238.7739	2.45099	1.62856	150.5004
41	0.89216	1.65644	53.86009	0.89216	1.75987	50.69465
42	0.62925	1.53643	40.95533	0.62925	1.26592	49.70693
43	2.79545	1.5239	183.4405	2.79545	1.54647	180.7633
44	1.23081	1.64175	74.96939	1.23081	1.94538	63.26836
45	–1.43594	0.6543	–219.462	–1.43594	0.98429	–145.886
46	1.15576	0.39643	291.542	1.15576	1.07017	107.9978
47	0.24433	1.63517	14.94218	0.24433	1.05412	23.17858
48	2.77455	2.49991	110.986	2.77455	1.80945	153.3366
49	3.17737	1.95483	162.5395	3.17737	1.29898	244.605
50	–0.12259	0.18445	–66.4625	–0.12259	0.92074	–13.3143
Mean			117.6379			98.03109
Standard deviation			169.9196			121.9463
Coefficient of variation			144.4429			124.3956

Figure 5.12 represents the model forecasts for excess returns to standard deviation ratio. Root mean error and mean absolute error provide the relative measure to compare forecasts. These measures are a little higher than the previous regression model estimating returns. A small Theil inequality coefficient of 0.39 implies a good fit of forecast model. Bias proportion is zero implying the forecast mean is almost equal to actual mean. Variance proportion and covariance proportion indicate a difference between the variance and covariances of the actual values with the forecasts.

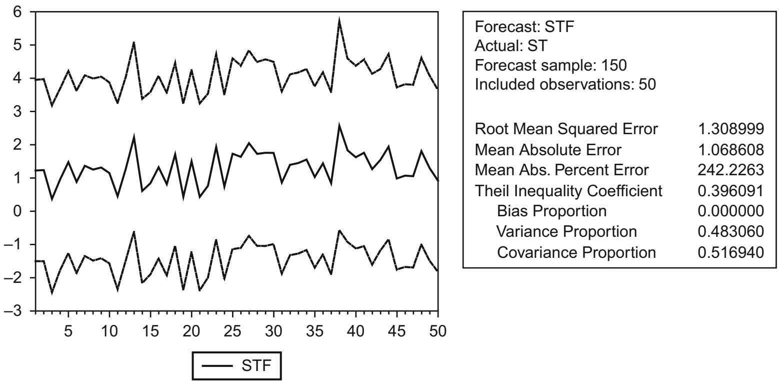


Figure 5.12 Model forecasted excess returns to standard deviation ratio

Table 5.26 Results for pair-wise Granger causality tests

Null Hypothesis	F-Statistic	Probability
AVG_EPS does not Granger cause returns	3.35047	0.04444**
AVG_DIV does not Granger cause RETURNS	0.05303	0.94842*
AVG_FF does not Granger cause RETURNS	4.70811	0.01416**
IMPACT_COST does not Granger cause RETURNS	0.56691	0.57146*
INSTITUTIONAL_HOLDING does not Granger cause RETURNS	5.17766	0.00967**
MKT_CAP does not Granger cause returns	1.71577	0.19191*
AVG_NP does not Granger cause RETURNS	0.18949	0.82807*
PB_RATIO does not Granger cause RETURNS	2.17636	0.12579*
PE_RATIO does not Granger cause RETURNS	3.49237	0.03932**
PROMOTERS_HOLDING does not Granger cause RETURNS	1.48958	0.23687*
SALES does not Granger cause RETURNS	0.26725	0.76674*
VOLUME does not Granger cause RETURNS	0.52373	0.59603*
TURNOVER does not Granger cause RETURNS	3.81329	0.02988**

Note: * is 1% level of significance; ** is 5% level of significance.

5.11 Granger causality analysis

Granger causality tests were conducted to test the causation of different portfolio variables on returns. The results of the Granger causality tests are presented in Table 5.26. High probability values can be seen in the table. One cannot reject the hypothesis that dividend, impact cost, net profit, promoters holding, sales and volume do not cause returns as the probability values are high. Hence, as per the Granger causality tests, these factors assume importance in portfolio selection modelling due to their explanatory power for explaining returns.

5.12 Utility analysis

For the quantitative utility analysis, Equation 4.64 has been used for calculating risk penalty. The risk penalty values are presented in Table 5.27 for risk tolerance level from ten to 100. From the table it can be seen that the value of risk penalty decreases with an increase in the value of risk tolerance. In Table 5.27 the risk penalty values can be observed for all the portfolios at alternate risk tolerance levels. Using Equation 4.65, the utility values are calculated, which are presented in Table 5.28 for risk tolerance level from ten to 100.

It may be observed that as risk tolerance is increasing, the value of utility is also increasing. In Table 5.28, the lowest utility value is witnessed for portfolios at risk tolerance level of ten with Nifty 50 providing the least utility of –4.79 per cent. The highest utility is achieved for the index portfolio Nifty 50 at a risk tolerance level of 100 per cent. Empirically, it has been shown in literature that alternate portfolios will appeal to different type of investors. An attempt has been made to empirically find that at different levels of risk tolerance, the utility derived from alternate portfolios is different. The highest utility is derived for the index portfolio, when the investor has extremely high risk aversion levels of 100 per cent followed by Markowitz's and ideal portfolios.

Table 5.27 Risk penalty for the ideal portfolio, Nifty 50 and Markowitz's portfolio at different risk tolerance levels

Risk Tolerance	**Ideal Portfolio**	**Nifty 50**	**Markowitz's Portfolio**
10%	1.97	6.1	1.76
20%	0.985	3.05	0.88
30%	0.656667	2.033333	0.586667
40%	0.4925	1.525	0.44
50%	0.394	1.22	0.352
60%	0.328333	1.016667	0.293333
70%	0.281429	0.871429	0.251429
80%	0.24625	0.7625	0.22
90%	0.218889	0.677778	0.195556
100%	0.197	0.61	0.176

Table 5.28 Utility values for the ideal portfolio, Nifty 50 and Markowitz's portfolio

Risk Tolerance	Ideal Portfolio	Nifty 50	Markowitz's Portfolio
10%	–1.34	–4.79	–1.13
20%	–0.355	–1.74	–0.25
30%	–0.02667	–0.72333	0.043333
40%	0.1375	–0.215	0.19
50%	0.236	0.09	0.278
60%	0.301667	0.293333	0.336667
70%	0.348571	0.438571	0.378571
80%	0.38375	0.5475	0.41
90%	0.411111	0.632222	0.434444
100%	0.433	0.7	0.454

For extremely low levels of risk tolerance (10 per cent and 20 per cent), all the portfolios yield a negative utility. At 30 per cent risk tolerance level, the utility of Markowitz's portfolio becomes positive, while ideal and index portfolio are still giving a negative portfolio utility. At 40 per cent risk tolerance level, Markowitz's portfolio yields the highest utility, while index's portfolio utility is still negative. Markowitz's portfolio utility is highest at 50 per cent and 60 per cent levels. Similar trends are observed for the ideal portfolio. For the investors with high risk tolerance capacity (70 per cent to 100 per cent), Nifty 50 is yielding the maximum utility. The utilities of these three portfolios are also plotted graphically in a two dimensional space.

The graph clearly shows the utility of an investor from Markowitz's portfolio and the ideal portfolio. The utility plot for the two portfolios move closely for all levels of risk tolerance, with ideal portfolio's utility little lesser than that of Markowitz's portfolio. Nifty 50 gives negative utility at low levels of risk tolerance (i.e.0–40 per cent). If the risk tolerance level of an investor is extremely high (i.e. 70 per cent to 100 per cent, selecting Nifty 50 out of the three portfolios will provide the investor with maximum utility (see Figure 5.13).

For graphical utility analysis, the nine alternate portfolio selection model formulations (Markowitz's portfolio, diversifier's portfolio, satisficer's portfolio, plunger's portfolio, market trend portfolio, capital gain bias portfolio, dividend gain bias portfolio, equal priority portfolio, ideal portfolio and the index portfolio Nifty 50) are plotted in risk-return space (see Figure 5.14).

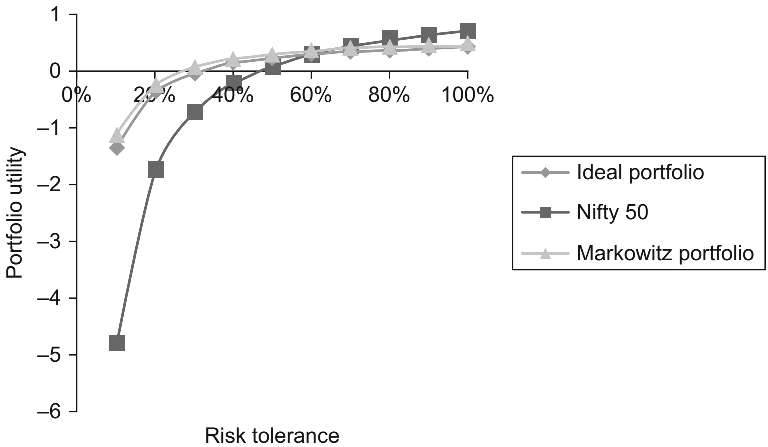


Figure 5.13 Portfolio utility as a function of increasing risk tolerance

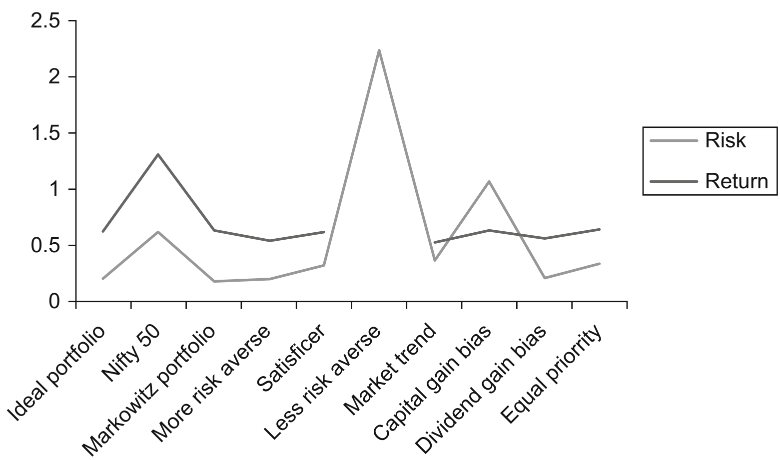


Figure 5.14 Risk and return for alternate portfolio selection model formulations

Plunger's portfolio and the capital gain portfolio are the most risky. Nifty 50 provides maximum return followed by Markowitz's portfolio and ideal portfolio. The break in return line is due to the infeasible plunger's portfolio. The excess return over variance can be graphically depicted through the distance between return line and risk line. This is maximum for the three portfolios namely Nifty 50, Markowitz's portfolio and ideal portfolio. Risk is higher than return for the capital gain bias portfolio signifying the non-commensurate return despite taking additional risk.

Further, the same portfolios are plotted in the risk-return space with risk of a portfolio (variance) on the X axis and returns (mean) on the Y axis. U1, U2 and U3 represent the utility curves or indifference curves representing the preferences of an investor. The utility curves never intersect and move parallel to each other. The higher a curve, the more desirable are situations lying above it. The utility provided by U3 curve is greater than the utility provided by U2 curve, which is more than the utility on U1. The relationship can be exhibited as in equation.

$$U3 > U2 > U1 \tag{5.1}$$

The plunger, diversifier, risk neutral and risk lover investors are considered for utility comparison of these portfolios. The utility curves of plunger are flatter as smaller risk premiums are expected by such investors. The utility curves of diversifiers are steep as they expect higher compensation in terms of return for taking additional risk. The utility curve of a risk neutral is a straight line with constant marginal utility of money/wealth. Risk seekers/lovers utility curves are straight lines from right to left depicting higher levels of utility for high degree of risk and low returns.

It becomes evident from Figure 5.15 that the index portfolio Nifty 50, with the highest return, provides the highest utility to a plunger. Even

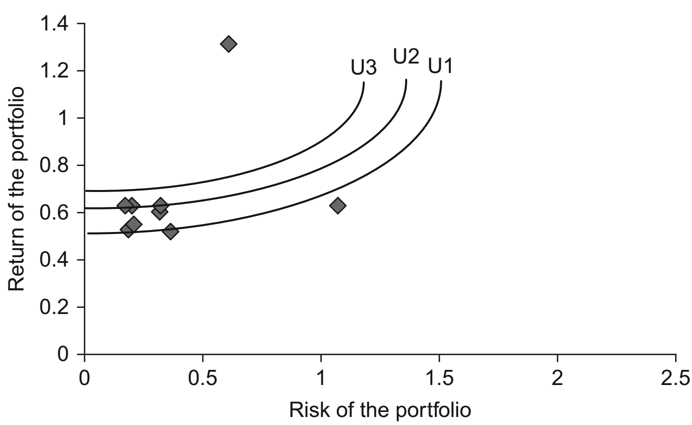


Figure 5.15 Utility curves of a plunger and risk-return on portfolios

for diversifiers, this portfolio lies above the highest utility curve U3 and is most desirable to an investor as shown in Figure 5.18. The market trend portfolio, capital gain bias portfolio and dividend gain bias portfolio are equally preferable by the plunger as they lie on the same utility curve U2. The other four portfolios namely ideal portfolio, Markowitz's portfolio, satisficer's portfolio and equal priority portfolio all lie on the highest utility curve U3 providing the highest level of utility.

For the diversifier desiring high compensation for increasing degrees of risk the steep utility curves U1, U2 and U3 measure utility in ascending order.The capital gain portfolio with highest risk coefficient is the least desirable. The index portfolio Nifty 50 is most desirable. The diversifier's portfolio and dividend gain bias portfolio both lying on utility curve U3 provide equal utility to this investor. The Markowitz's portfolio and ideal portfolio will also provide the same utility, as can be shown by drawing another utility curve U4 (see Figure 5.16).

The utility curves of a risk lover are defined by negatively sloped straight lines from right to left. The closer utility curve to the origin lower its level of utility. The utility increases for a risk lover with increasing risk even when not adequately compensated by increase in returns.

To a risk lover, the Markowitz's portfolio, ideal portfolio, diversifier's portfolio and dividend gain bias portfolio provide the least utility. These four portfolio points lie even below than U1. The market

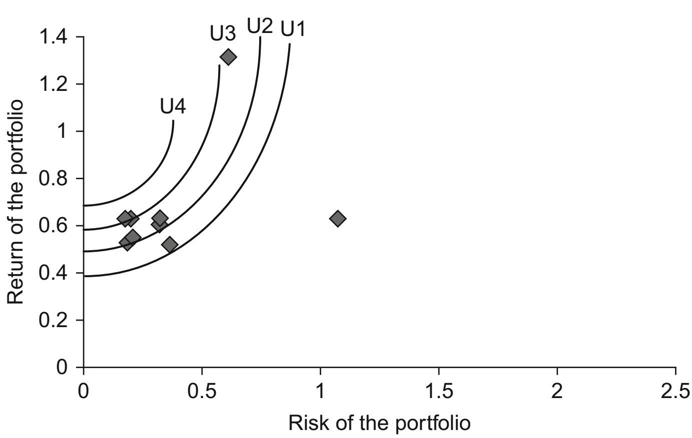


Figure 5.16 Utility curves of a diversifier and risk-return on portfolios

trend portfolio, equal priority portfolio and satisficer portfolio lie on the utility curve U2 and provide the same level of satisfaction to the investor. The capital gain bias portfolio and the index based portfolio Nifty 50 are the most preferred portfolios as they lie on U4 (see Figure 5.17).

The capital gain bias portfolio will be selected by a risk neutral investor because the utility curve of the risk neutral is a straight line from the origin. The capital gain bias portfolio lies on this curve and hence it would be chosen by the investor (see Figure 5.18).

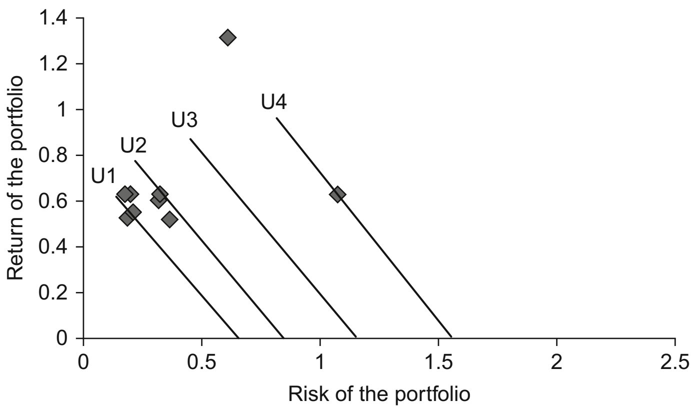


Figure 5.17 Utility curves of a risk lover and risk-return on portfolios

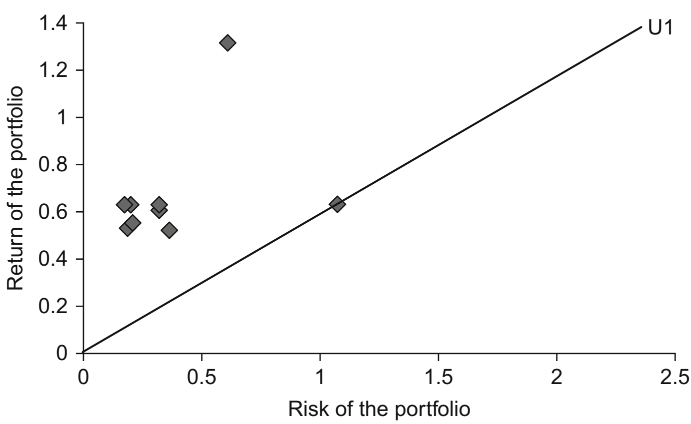


Figure 5.18 Utility curves of a risk neutral and risk- return on portfolios

The graphical analysis of risk-return combinations of portfolios and portfolio utility shows that different class of investors will pick up a distinct portfolio giving him/her the maximum utility as per the slope of his/her utility curves. The plunger would prefer the index portfolio the most. However, the four portfolios namely ideal portfolio, Markowitz's portfolio, satisficer portfolio and equal priority portfolio all lie on the highest utility curve U3 providing the high level of utility. For the diversifier, the index portfolio Nifty 50 is most desirable. The diversifier's portfolio and dividend gain bias portfolio both lying on utility curve U3 provide equal utility to this investor. The Markowitz's portfolio and ideal portfolio will also provide the same and higher utility as was shown by drawing another utility curve U4. Risk lovers will derive high utility from capital gain bias portfolio and index based portfolio Nifty 50. The risk neutral investor picked up the capital gain bias portfolio. Distinct portfolio preferences were noticed for the diverse categories of investors as per the graphical utility analysis.

5.13 Performance evaluation of portfolios: ranking the model formulations

An attempt has been made to rank the alternate portfolio selection model formulations, the Markowitz portfolio and the market portfolio Nifty 50 in order of their performances according to the popular evaluation measures as proposed by the Sharpe (1966) and Treynor (1965) ratios. The Sharpe performance ratios for all the portfolios are calculated using Equation 4.66. The portfolios are then arranged in descending order of their Sharpe ratio and presented in Table 5.29. The Sharpe ratio for Markowitz's portfolio is the highest followed by the ideal portfolio. The market trend portfolio does not do well according to this performance measure. No ratio could be computed for the plunger's portfolio as it had entered the infeasible region of the portfolio selection.

Treynor's performance evaluation ratios are also calculated according to Equation 4.67. The portfolios are then arranged in decreasing order of their Treynor ratio. The ideal portfolio does the best, even better than the Markowitz's portfolio when evaluated according to Treynor's ratio (see Table 5.30).

Equal priority portfolio, capital gain bias portfolio and satisficer portfolio perform well. Dividend gain portfolio and diversifier's portfolio record small ratios hinting at poor performance. Here, the market trend portfolio is the worst performer reporting the least ratio of 0.02.

Table 5.29 Ranking of portfolios as per Sharpe's ratio

Portfolios	**Risk**	**Return**	**Sharpe Ratio**
Markowitz's portfolio	0.176	0.63	0.74
Ideal portfolio	0.197	0.63	0.66
Equal priority portfolio	0.322	0.63	0.40
Satisficer portfolio	0.316	0.605	0.33
Dividend gain bias portfolio	0.206	0.55	0.24
Diversifier's portfolio	0.187	0.53	0.16
Capital gain bias portfolio	1.07	0.63	0.12
Market trend portfolio	0.364	0.52	0.05
Plunger's portfolio	2.24	–	–

Table 5.30 Ranking of portfolios as per Treynor's ratio

Portfolios	**Risk**	**Return**	**Beta**	**Treynor Ratio**
Ideal portfolio	0.197	0.63	0.69	0.19
Markowitz's portfolio	0.176	0.63	0.738	0.18
Equal priority portfolio	0.322	0.63	0.94	0.14
Capital gain bias portfolio	1.07	0.63	1.12	0.12
Satisficer portfolio	0.316	0.605	0.94	0.11
Dividend gain bias portfolio	0.206	0.55	0.75	0.07
Diversifier's portfolio	0.187	0.53	0.7524	0.04
Market trend portfolio	0.364	0.52	0.95	0.02
Plunger's portfolio	2.24	–	–	–

5.14 Hypotheses testing: tests for equality

The ideal portfolio resulted in same returns and marginally high variance in comparison to the Markowitz's portfolio. Hence, detailed yearly analysis of this portfolio is undertaken to examine for the equality of mean, variance and portfolio utility of the ideal portfolio, Markowitz's portfolio and also the benchmark index Nifty 50. The yearly averages of monthly returns and other variables are calculated for the 12-year period starting from April 2000 to March 2012. Twelve portfolios, one for every year of the period under study, are constructed using the mean-variance efficient ideal portfolio selection model.

Yearly averages for the monthly data are calculated for returns, dividends, impact cost, institutional holding, net profit, promoter's holding, sales, turnover and volume. The Q_3 (quartile three) values for each year's returns and median values per year for all other variables are determined to set the target values in the 12 portfolio models. Twelve covariance matrices were composed from the monthly returns for every year for Nifty 50 securities. The other constraints of funds exhaustion, no short-selling and company and industry diversification were applied as in the previous models. These targets and constraints are enumerated in Table 5.31.

Similarly, Markowitz's portfolio for the above targeted level of returns, upper bound constraint of 15 per cent for company diversification, funds exhaustion constraint and no short-selling constraints are formulated for the 12 years. Also, yearly averages for monthly return and variance of the benchmark index Nifty 50 are computed and used. The same level of returns are targeted and achieved in both ideal and Markowitz's portfolio. No surpluses in returns were generated in any of the portfolios in any of the years. The variance of Markowitz's portfolio is lesser than the ideal portfolio. A comparison of the risk (variance) and return (mean) of the mean-variance efficient ideal portfolios, Markowitz's portfolio and composite index NIFTY 50 are presented in Table 5.32.

The tests of equality of mean and variance between series are carried out using the software E-views 5.1.

1. The equality of returns of the index with the returns of the ideal portfolio is tested with the null hypothesis of equal returns.
   H_0: *There is no difference in the expected return of the portfolio created by the proposed model and the Nifty 50 index.*
   H_1: *Expected return of the portfolio created by the proposed model is superior to the return of the Nifty 50 index.*

This test is based on a single-factor between-subjects analysis of variance (Anova). The basic idea here is if the subgroups have the same mean, then the variability between the sample means (between groups) should be the same as variability within any subgroup (within group). The t-statistic is the square root of F-statistic with one numerator degree of freedom. Table 5.33 also presents mean, standard deviation and standard error of mean of these two series of risk of portfolios.

The null hypothesis of equal index and ideal portfolio returns cannot be rejected at a 5 per cent level of significance. It becomes evident that the ideal portfolio is able to yield returns comparable to the benchmark index Nifty 50.

Table 5.31 Target values for the yearly ideal portfolios

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12
Returns	0.20	0.46	0.48	1.221	0.78	0.61	0.29	0.98	1.74	1.18	0.71	0.76
Dividend	615.1	703.4	1000	1335.15	1855.6	2439.7	3409.7	3772.95	3816.05	5588.4	6986	5200.9
Funds Exhaustion	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1	ΣXi= 1
Impact cost	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Industry diversification	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Institutional holding	21.35	21.88	22.78	24.77	26.54	29.81	31.16	30.74	30.14	31.83	31.59	31.10
Net profit	2468.7	2970.4	3898.45	6001.65	8104.7	9901.35	14531.35	17308.2	16906.1	22020.3	26621	23076.6
No short sales	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0	X1….X50 ≥0
Promoters holding	29.55	27.80	30.29	32.35	45.14	44.48	44.85	50.01	50.65	51.07	51.98	52.63
Sales	25638.35	29499.5	36831,2	40582.35	70460.15	90280	114851.2	120617.75	121790.3	123999.2	162538.8	113965.6
Turnover	3.06	2.99	3.43	23.69	30.80	39.55	55.18	82.21	83.37	121.16	114.86	98.86
Upper bounds	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15	X1….X50 ≤0.15
Volume	125366.6	105327.6	156057.8	552084.9	661487.6	682504.7	809422	1200681	2083506	2078340	1504853	2388704

Note: (a) The funds exhaustion constraint would ensure that the entire funds remain invested and no borrowings are made; (b) The no short sales constraint implies that the proportion of funds invested in each of the securities is either positive or zero. Negative fractions denoting short-selling of securities do not form a part of the feasible set; (c) Upper bound constraint ensures company diversification wherein investment in a particular security cannot exceed 15 per cent; (d) The industrial diversification constraint limits investment in a particular industry to 20 per cent; (e) Past data for impact cost were not available. The variable is included in yearly ideal portfolio modelling through data approximation. This was done keeping in view the high explanatory power of this variable in explaining the returns to an investor; (f) Returns are targeted at Q_3 value, all other constraint namely dividend, impact cost, institutional holding, net profit, promoter's holding, sales, turnover and volume are targeted at Q_2 values.

Table 5.32 The risk-return combinations: ideal portfolio, Markowitz's portfolio and Nifty 50 (index)

Year	Returns Markowitz's and Ideal Portfolio	Markowitz's Variance	Ideal Portfolio Variance	Index Returns	Index Variance
2000–01	0.200	0.089	0.372	–1.805	0.650
2001–02	0.460	4.627	6.180	–0.190	0.460
2002–03	0.480	0.081	0.087	–1.021	0.408
2003–04	1.221	0.358	0.372	4.932	0.530
2004–05	0.780	0.042	0.123	1.764	0.508
2005–06	0.610	0.120	0.164	4.731	0.363
2006–07	0.290	0.157	0.890	1.041	0.396
2007–08	0.980	0.198	0.280	2.699	0.842
2008–09	1.740	0.941	1.198	–1.875	1.393
2009–10	1.180	0.355	0.375	5.093	0.977
2010–11	0.710	0.116	0.117	0.633	0.298
2011–12	0.760	0.165	0.251	–0.301	0.411

Table 5.33 Tests of equality of returns on ideal portfolio and Nifty 50 index

Sample: 1 12

Included observations: 12

Method	**df**	**Value**	**Probability**
t-test	22	0.699844	0.4914
Anova F-statistic	(1, 22)	0.489781	0.4914
Analysis of Variance			
Source of Variation	df	Sum of Sq.	Mean Sq.
Between	1	1.648417	1.648417
Within	22	74.04364	3.365620
Total	23	75.69206	3.290959

Category Statistics

Variable	**Count**	**Mean**	**Std. Dev.**	**Std. Error of Mean**
Mean Returns	12	0.784250	0.438900	0.126699
Index Returns	12	1.308403	2.557070	0.738163
All	24	1.046326	1.814100	0.370302

2. Variance equality tests evaluate the null hypothesis that the variance of Markowitz's portfolio is equal to the variance of ideal portfolio.
   H_0: *There is no difference in the risk (variance) of the portfolio created by the proposed model and Markowitz's model.*
   H_1: *Expected risk of the portfolio created by the proposed model is lower than the risk (variance) of portfolio created by Markowitz's model.*

The test statistics decisively accepts the null hypothesis of equal risk (variance) in the series of the portfolios designed by the ideal portfolio modelling and Markowitz's technique as the reported probability values are higher than the level of significance of 5 per cent. The table shows decomposition of total sum of squares (2.205) into the between (0.416) and within (2.28) sum of squares (see Table 5.34).

It may be concluded that the risk-return combinations of the ideal portfolio can be considered equal to those of Markowitz's portfolio. The ideal portfolio, in addition to being mean-variance efficient in the Markowitz sense, tends to guard an investor in the sense of providing a median level of dividend, liquidity (measured through impact cost), institutional holding, promoter's holding, net profit, sales, turnover and volume. This portfolio is also well diversified across companies and industries due to the company and industrial diversification constraints.

Table 5.34 Test of equality of variance of the Markowitz portfolio and the ideal portfolio

Included observations: 12

Method	**Df**	**Value**	**Probability**
F-test	(11, 11)	1.748860	0.3679
Siegel-Tukey		0.317612	0.7508
Bartlett	1	0.811374	0.3677
Levene	(1, 22)	0.187888	0.6689
Brown-Forsythe	(1, 22)	0.104804	0.7492

Category statistics

Variable	**Count**	**Std. Dev.**	**Mean Abs. Mean Diff.**	**Mean Abs. Median Diff.**	**Mean Tukey-Siegel Rank**
Markowitz's portfolio	12	1.289996	0.726639	0.503250	12.00000
Ideal portfolio	12	1.705948	0.944292	0.697083	13.00000
All	24	1.485201	0.835465	0.600167	12.50000

Bartlett weighted standard deviation: 1.512340

3. A comparison of the variances of the ideal portfolio and Nifty 50 index is also undertaken. The null hypothesis of no difference in variance of ideal portfolio and Nifty 50 is tested.
   H_0: *There is no difference in the risk (variance) of the portfolio created by the proposed model and Markowitz's model.*
   H_1: *Expected risk of the portfolio created by the proposed model is lower than the risk (variance) of portfolio created by Markowitz's model.*

The F-test strongly rejects the hypothesis of equal risk (variance) in the ideal portfolio and Nifty 50. The Siegel-Tukey test, Bartlett test and Levene test also reject the hypothesis of equal risk. However, the Brown-Forsythe test replacing the absolute mean difference in Levene test with absolute median difference does not reject the hypothesis (see Table 5.35).

4. Tests for equality of portfolio utility between the ideal portfolio and Markowitz's portfolio are undertaken (see Table 5.36).
   H_0: *There is no difference between the utility derived by investors from the portfolio created by the proposed model and Markowitz's portfolio selection model.*

Table 5.35 Tests for equality of variance between the ideal portfolio and Nifty 50

Included observations: 12

Method	**Df**	**Value**	**Probability**
F-test	(11, 11)	28.40738	0.0000
Siegel-Tukey		1.992292	0.0463
Bartlett	1	21.35438	0.0000
Levene	(1, 22)	2.997065	0.0974
Brown-Forsythe	(1, 22)	1.013007	0.3251

Category statistics

Variable	Count	Std. Dev.	Mean Abs. Mean Di ff.	Mean Abs. Median Diff.	Mean Tukey-Siegel Rank
Ideal portfolio	12	1.705948	0.944292	0.697083	9.583333
Nifty index	12	0.320074	0.241749	0.213689	15.41667
All	24	1.207930	0.593020	0.455386	12.50000

Bartlett weighted standard deviation: 1.227336

Table 5.36 Tests for equality of portfolio utility between the ideal portfolio and the Markowitz's portfolio

Included observations: 10				
Method		**df**	**Value**	**Probability**
t-test		18	0.265558	0.7936
Anova F-statistic		(1, 18)	0.070521	0.7936
Analysis of Variance				
Source of Variation		df	Sum of Sq.	Mean Sq.
Between		1	0.018917	0.018917
Within		18	4.828300	0.268239
Total		19	4.847217	0.255117
Category Statistics				
Variable	Count	Mean	Std. Dev.	Std error of Mean
Ideal Portfolio	10	0.052993	0.546212	0.172727
Markowitz's Portfolio	10	0.114502	0.487986	0.154315
All	20	0.083747	0.505091	0.112942

H_1: Portfolio utility in the portfolio created using the proposed model is higher than the utility of portfolio created using Markowitz's portfolio selection model.

The high probability value figures reported did not reject the null hypothesis of equal portfolio utility provided to the investor by these two portfolios.

5. Tests for equality of portfolio utility between the ideal portfolio constructed and the benchmark index portfolio Nifty is conducted.
   H_0: There is no difference between the utility derived by investors from the portfolio created by the proposed model and the Nifty 50 index.
   H_1: Portfolio utility in the portfolio created using the proposed model is higher than the utility of the Nifty 50 index.

The probability value figures of 35.85 per cent of the t-statistic values and F-statistic values does not result in the rejection of the null hypothesis of equal portfolio utility provided to the investor by these two portfolios at a 5 per cent level of significance (see Table 5.37).

Table 5.37 Tests for equality of portfolio utility between the ideal portfolio and Nifty 50 index

Included observations: 10				
Method		**df**	**Value**	**Probability**
t-test		18	0.942393	0.3585
Anova F-statistic		(1, 18)	0.888105	0.3585
Analysis of Variance				
Source of Variation		df	Sum of Sq.	Mean Sq.
Between		1	1.402716	1.402716
Within		18	28.43006	1.579448
Total		19	29.83278	1.570146
Category Statistics				
Variable	Count	Mean	Std. Dev.	Std error of Mean
Ideal Portfolio	10	0.052993	0.546212	0.172727
Nifty 50	10	−0.476670	1.691316	0.534841
All	20	−0.211839	1.253055	0.280192

5.15 To sum up

This chapter applies the mean-variance efficient portfolio selection model developed in the previous chapter to the component securities of NSE's index portfolio Nifty 50. The rationale, genesis and sample data used for the mean-variance efficient portfolio selection model has been discussed in the chapter. The sample consists of Nifty 50 securities analysed over a period of 12 years. The programming undertaken to ease the empirical testing of the model has been stated towards the end (see Annex 1). Alternate model formulations exhibiting the preferences of various investors in the market are developed and empirically tested by programming the constraints in software Lingo 13. Aspiration values for investors with different risk appetite are set at Q_1 (quartile one), median or Q_3 (quartile three) values of the respective distribution for the 16 constrained variables. Maximum investment in an industry and in a company is limited to 20 per cent and 15 per cent respectively. No short sales of securities are allowed. Utilisation of the entire amount for investment purposes is ensured through the funds exhaustion constraint. Investors considering the securities listed at NSE's Nifty 50 as an investment avenue may possibly use these algorithms (according to their attitude towards risk or preferences) for creating an efficient portfolio meeting all their constraints.

The diversifier investor targeting to achieve the Q_1 (quartile one) values got diversified across 14 companies and 12 industrial sectors. The portfolio in addition to being mean-variance efficient achieved the targeted dividend returns, beta, liquidity, market capitalisation and the other constraints. The satisficer's portfolio designed for an investor with moderate risk appetite also diversified across 15 companies and 12 industries. Funds exhaustion constraint showed a significant dual price indicating a further reduction in variance by allowing for borrowing and lending of funds for creation of portfolios. No feasible solution could be generated for the model formulation of a plunger targeting high Q_3 (quartile three) values for the constrained variables.

The market trend portfolio targeting the mean values for variables was also a well-diversified portfolio with investment in 18 companies across 13 industrial sectors. The average weight of each security was found to be 5.55 per cent. The smallest returns of 0.52 per cent per month were observed for this portfolio. A capital gain bias portfolio was created for investors with desire of high capital returns. High Q_3 (quartile three) values were targeted and achieved for capital returns. Surprisingly, high dividend returns were also realised on this portfolio. This portfolio, which diversified across 12 companies and ten industries was found to have the maximum risk (1.07). The dividend gain bias portfolio constructed to depict the class of investors with a preference for regular dividend income diversified across 16 companies and 12 industrial sectors. The equal priority portfolio giving equally high (Q_3) priority to capital returns and dividend returns diversified across 17 companies and 12 industrial sectors. The average weight of each security was found to be 5.88 per cent.

Finally, an ideal portfolio was formulated with reduced constraints. The more important variables according to their explanatory power for returns, excess returns to standard deviation ratio and causation relationship with returns were constrained. The average weight of each security was found to be 7.69 per cent with the portfolio diversifying across 13 companies and 12 industrial sectors. All the constraints were satisfied (with surpluses) in all model formulations except for the plunger's portfolio where the funds exhaustion constraint, institutional and promoter's holding constraint and no short-selling requirements could not be met rendering the formulation to be infeasible.

The Markowitz portfolio was also created on Nifty 50 securities with upper bound of 15 per cent. The portfolio diversified across 12 companies and 11 industries. The minimum risk of 0.176 was achieved for 0.63 level of returns. A 20-point Markowitz efficient frontier drawn from the same data set eased the comparison of models formulated for efficiency parameters. A graphical comparison of the risk-return trade-off points of the portfolios generated with Markowitz's portfolio found that the ideal portfolio lies closest to Markowitz's portfolio. The ideal portfolio meets the median requirements for other constraints in addition to minimising the risk for return of 0.63 per cent per month. The minimum variance achieved in Markowitz's portfolio (0.176) is lesser than ideal portfolio (0.197). All the modelled portfolios lie below the Markowitz frontier, but all of them except capital gain bias portfolio are quite close to it.

The results of multivariate regression analysis found impact cost to have the highest explanatory power for returns and excess returns to standard deviation ratio with the highest beta coefficient of 0.44 and 0.52 respectively. The other important explanatory variables included institutional holding, promoter's holding and turnover. It became evident that liquidity of an investment and other party's stake in the holding are the most important considerations of an investor. The graphical representation of estimated equations attempted through actual, fitted and residual graphs showed the model to have a good fit as the actual values were quite close to fitted values and residuals were small. Also, the model forecasted returns having small Theil inequality coefficient of 0.21 and 0.34 implying a good fit of the forecast model.

Looking at the observations of high probability values from the Granger causality tests, one cannot reject the hypothesis that dividend, impact cost, net profit, promoter's holding, sales and volume do not cause returns. Hence, as per the Granger causality tests, these factors assume importance in portfolio selection modelling due to their explanatory power for returns.

On the basis of the multiple regression analysis for both the securities returns and excess return to standard deviation ratio as the dependent variable as well as the Granger causality tests, it may be concluded that the explanatory factors for returns to an investor that must be considered while creating a portfolio are impact cost, turnover, volume (depicting liquidity of stocks), dividend, net profit, sales (depicting the earning capacity of the company) and institutional as well as promoter's holding (depicting other stakeholders share). The ideal portfolio is created by taking only these constraints in the model formulation and removing all the other constraints.

As the degree of risk tolerance increased, the value of portfolio utility also increased for Markowitz's portfolio, index portfolio and the ideal portfolio. At the lowest risk tolerance level of 10 per cent, Nifty

50 yielded the least utility. This portfolio had the maximum utility of 0.7 at the highest risk tolerance level of 100 per cent. The portfolio utility analysis attempted to empirically find that the utility derived by the investor from alternate portfolios is different for changing levels of risk tolerance. The graphical representation of portfolio utility highlights the close and overlapping movement of the utilities provided by Markowitz's portfolio and the ideal portfolio. The portfolio utility of index portfolio is lesser at small risk tolerance levels and high for greater risk appetite.

Plotting of the modelled portfolios on the utility curves of plunger, diversifier, risk neutrals and risk lovers in risk-return space, finds out the distinctive portfolio selection as per the category of risk taking capacity of an investor. The plunger is seen to prefer the index portfolio. However, the four portfolios namely ideal portfolio, Markowitz's portfolio, the satisficer portfolio and equal priority portfolio all lie on the highest utility curve U3 providing a high level of utility. For the diversifier, the index portfolio Nifty 50 is most desirable. The diversifier's portfolio and dividend gain bias portfolio both lying on utility curve U3 provide equal utility to an investor. The Markowitz portfolio and the ideal portfolio also provided the same and a higher utility as was shown by drawing another utility curve U4. The risk lovers preferred capital gain bias portfolio and index based portfolio Nifty 50. The risk neutral investor would select the capital gain bias portfolio for maximum utility.

The evaluation of all these portfolios is attempted by using measures as proposed by Sharpe (1966) and Treynor (1965). The Sharpe ratio for Markowitz's portfolio is the highest followed by the ideal portfolio. Portfolios in decreasing order of their utility are equal priority portfolio, satisficer's portfolio, dividend gain bias portfolio, diversifier's portfolio and capital gain bias portfolio. The market trend portfolio performs the worst according to this performance measure.

The ideal portfolio does the best, even better than the Markowitz's portfolio when evaluated according to Treynor's ratio. Equal priority portfolio, capital gain bias portfolio and satisficer portfolio also perform quite well. Dividend gain portfolio and diversifier's portfolio record small ratios hinting at poor performance. Here also, the market trend portfolio is the worst performer reporting the least ratio of 0.02. Further investigations into the proximity of the Markowitz portfolio and the ideal portfolio were attempted through the tests of equality. A detailed analysis from yearly averages was conducted to examine the equality of mean, variance and portfolio utility of the ideal portfolio, the Markowitz portfolio and index portfolio Nifty 50. By analysing the results of t-test and Anova F-statistic values and probability figures, null hypothesis of equality of returns from ideal portfolio and index portfolio could not be rejected. Also, tests statistics could not reject the hypothesis of equality of variance of the ideal portfolio and the Markowitz portfolio. However, the variance of the index portfolio could not be considered equal to that of the ideal portfolio. The tests for equality of portfolio utility found that the utility from the three portfolios could be considered equal to each other.

Hence, it may be concluded that the ideal portfolio created by identifying important portfolio variables not only meets the Markowitz mean-variance efficient criterion but also taps the developments in the stock market. It is able to decipher and identify the important variables, which when targeted, maximise the utility of an investors by meeting their objectives in a multiple constraint setting.

6 Mean-variance Portfolio Analysis using Accounting, Financial and Corporate Governance Variables-Application on London Stock Exchange's FTSE 100

The issue of portfolio construction involving analysis of various aspects by an investor – fundamental accounting, financial as well as governance – has been dealt with here through the application of MCDM approach from the field of operations research. A multi-objective quadratic programming model with the objective function of minimising variance (volatility) and constraints relating to multiple decision criteria such as return (capital and dividend), systematic risk (beta), marketability (trade volume and price-to-earnings ratio), management efficiency (operating profit margin), profitability (net profit margin), governance (free float) and future investment opportunities (free cash flows) has been obtained. The portfolio selection model has been applied to London Stock Exchange's FTSE 100 to generate Pareto optimal portfolios.

6.1 Securities and evaluation criteria

The sample considered in the study consists of securities included in the FTSE 100 index of the London Stock Exchange. FTSE 100 is the most widely used UK stock market indicator representing more than 80 per cent of the total market capitalisation of the London Stock Exchange. FTSE index consists of 100 companies, but there are 101 listings because Royal Dutch Shell has both A and B class shares listed. The study period includes the records of monthly closing prices for five years between 1/1/2001 to 31/12/2005. Data for eight constituent securities of FTSE was not available for this period and hence they were dropped from the analysis. The proposed methodology is based on international literature (Xidonas et al., 2009, 2011). The most appropriate criteria to be used for portfolio optimisation have been identified both in theory and in practice. These have been listed below:

i. Return aspects

(a) Capital Return: $R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ where R_t is return on a share in time period *t*, P_t is the closing share price in time period *t* and P_{t-1} is the share price in previous time period *t–1*.

(b) Dividend Yield: $D_i = \frac{DPS_t}{P_t}$ where D_i is the dividend yield, *DPS* is the dividend per share in time period *t* and P_t is the closing price in time period *t*.

It is the dividend per share expressed as a percentage of their price.

ii. Systematic risk (beta coefficient) $\beta = Cov(R_i, R_m)/VAR(R_m)$ where R_i is the return of the share *i*, R_m is the return of market portfolio. Beta as explained by the capital asset pricing model measures the sensitivity of rate of return of a security to changes in rate of return to the market.

iii. Market acceptance dimension

a. Volume: Total traded volume of a security in a time period
b. Price-to-earnings ratio: $p/e = P_t/EPS_t$ where P_t is the closing stock price in time period *t* and EPS_t is the earnings per share in time period *t*
c. Market capitalisation: This represents the total number of shares outstanding times the price of each share in a time period t
d. Management performance dimension

Operating profit margin: $O_{PM} = OI/TR$ where *OI* represents operating income and *TR* represents total revenue. This is a measure of management efficiency as reported in the company's financial statements.

e. Profitability dimension

Net profit margin: $N_{PM} = NI/TR$ where *NI* represents the income available to common shareholders excluding extraordinary items and *TR* represents total revenue.

f. Governance impact

Free float: $FF = n - c_s$ where n represents shares issued by the company and c_s represents any closely held shares. The free float measures shares outstanding available for trading by the public. A very small free float per cent would be indicative of concentration of share holdings in a few hands.

g. Future investment opportunities

Free cash flows: $FCF = CF - I - D$ where CF represents cash flows, I represent capital expenditures and D represents the total cash dividends paid for the fiscal year.

The first two criteria namely capital return and dividend yield reflect on the general return/performance of portfolio, beta reflects on the risk dimension, volume and P/E ratio represents general market acceptance, management efficiency, profitability, size, governance and future investment opportunities are reflected by the other criterion. The theoretical presentations of these criteria could be found in Niarchaos (2005), Alexander and Sharpe (1989), Jones (1985), Bodie et al. (2004) and Reilly and Brown (2005). The incremental explanatory power of cash flows to predict returns has been discussed by Clubb (2005). A comparison to the explanatory power of earnings has also been made. Fundamental analysis has been previously used in portfolio selection and allocation by Edirisinghe and Zhang (2007), Greig (1992) and Ou and Penman (1992). Having been regarded as the best fundamental indicator, free cash flows measures the ease with which businesses can grow and pay dividends to its shareholders. It is being increasingly used for valuation purposes. Gutterman (2011) discusses allocation of scare resources using the future cash flows. Wu (2013) considers self-financing mean-variance portfolio selection with stochastic cash flows.

6.1.1 Data and software used

Data for 93 of 101 FTSE 100 securities has been extracted from Thomson Reuters Eikon Datastream for Office. Closing price data for eight of the present FTSE constituent securities was missing and hence these eight securities were dropped from the analysis. The monthly returns for period from 2001 to 2005 were calculated from monthly closing prices. Monthly values for criterion such as P/E ratio and traded volume were derived. Annual figures for accounting based operating profit margin and net profit margin were used. The real time values as on 17 July 2013 for beta, market capitalisation, free float and yield were used for our analysis. The latest available figures for free cash flows were used since data for 2001–2005 was missing. For optimised portfolio generation, Markowitz's mean-variance efficiency criterion along with the additional constraints identified are used to construct Pareto optimal portfolios. Table 6.1 summarises the evaluation of 93 securities in these ten criteria. Table 6.1 sum-

Table 6.1 Evaluation of securities in identified criterion

Security	Code	Capital return	Dividends	Beta	Trade volume	PE ratio	Operating profit margin	Net profit margin	Market capitalisation	Free float	Free cash flows
/AAL.L	X1	0.0152	3.8791	1.9428	117130536	0.0000	16.95	13.00	18846917798	91.45	–1015000000
/ABF.L	X2	0.0106	1.5612	0.5562	37340634	16.1543	8.19	6.49	14831807660	42.71	327000000
/ADML.L	X3	0.0315	4.8582	0.6874	27016893	11.8260	0.00	35.51	3753143093	71.65	–9800000
/ADN.L	X4	–0.0034	3.2898	1.2663	46203622	352.3109	–4.21	–8.24	4758594141	76.12	221800000
/AGGK.L	X5	–0.0017	1.3350	1.2673	26065677	357.6556	15.82	9.22	4801503066	88.07	–125300000
/AMEC.L	X6	0.0040	3.4402	1.1282	42729101	35.9643	2.36	0.61	3144504061	98.76	110000000
/ANTO.L	X7	0.0304	1.6874	1.7992	51281919	6.5183	41.98	19.44	8203326905	34.00	1501700000
/ARM.L	X8	–0.0087	0.4956	0.6488	368370441	64.2325	21.32	17.28	12646426532	99.25	79500000
/AV.L	X9	–0.0005	5.1941	1.4132	245607996	289.8045	0.00	2.27	10742145827	98.80	1299000000
/AZN.L	X10	0.0007	5.2865	0.3969	142108570	32.0950	23.29	17.32	41011485781	99.65	–1336000000
/BAB.L	X11	0.0188	2.1935	0.7177	14337748	8.4916	3.75	1.42	4326196030	97.91	74200000
/BAES.L	X12	0.0089	4.5571	0.7937	517504123	18.2033	3.74	–0.29	13786630686	99.40	1151000000
/BARC.L	X13	0.0043	2.0991	2.5058	656219356	11.3978	0.00	20.44	39703016430	98.88	–15747000000
/BATS.L	X14	0.0183	3.8455	0.4494	201883485	20.6106	22.68	13.24	66748534527	94.44	1085000000
/BG.L	X15	0.0147	1.4115	0.8633	329505901	18.1363	36.00	22.42	40062397032	99.68	–2838000000
/BLND.L	X16	0.0149	4.3636	1.0067	81140167	25.5922	99.15	68.16	5983288412	98.95	–685000000
/BLT.L	X17	0.0250	3.7966	1.5307	335295694	0.0000	25.16	15.81	106016508001	99.79	–2550000000
/BNZL.L	X18	0.0076	2.0494	0.5088	49087774	365.9490	6.83	4.18	4561073778	97.80	156600000
/BP.L	X19	0.0022	4.7931	1.1124	1414854286	25.7919	10.69	6.61	87912061757	99.05	–7975000000
/BRBY.L	X20	0.0209	1.9333	1.3277	49509035	511.0278	18.87	12.69	6619348266	98.82	135500000
/BSY.L	X21	–0.0099	3.2536	0.5200	273000914	45.3276	3.82	–5.04	13278790317	59.60	633000000
/BT.L	X22	–0.0097	2.8443	1.0264	1116634334	11.3733	8.89	8.04	26233106527	98.64	2131000000
/CCL.L	X23	0.0242	2.9736	1.0324	33622633	0.0000	0.00	0.00	18605767574	73.20	–112000000
/CNA.L	X24	0.0024	4.2943	0.4285	533868561	15.2169	8.07	4.26	19608113601	98.74	–362000000

/CPG.L	X25	−0.0122	2.5000	0.6769	292951109	39.3590	4.37	1.67	15984054513	99.14	276000000
/CPI.L	X26	0.0014	2.2683	0.5195	109241021	44.6907	10.26	6.18	6790653627	98.82	172300000
/CRDA.L	X27	0.0119	2.3080	0.9599	9879140	281.4653	12.93	7.39	3488302902	97.92	9100000
/CRH.L	X28	0.0084	4.0894	0.9038	29631651	1093.6110	9.19	6.25	9655247342	99.87	0
/DGE.L	X29	0.0052	2.2539	0.5240	358839284	87.4318	18.73	13.53	49945282715	99.40	455000000
/EXPN.L	X30	0.0148	1.8664	0.7955	234745090	13.4413	8.29	5.14	12087079972	99.04	601000000
/EZJ.L	X31	0.0103	1.5926	0.6734	71492388	23.5229	7.02	5.99	5333547938	57.47	−131000000
/GFS.L	X32	0.0051	4.1850	0.4420	113760503	20.7761	2.66	0.03	2993563526	98.67	7000000
/GKN.L	X33	−0.0002	2.1563	2.2174	201194820	323.9546	3.29	0.59	5446651910	99.42	7000000
/GSK.L	X34	−0.0021	4.3681	0.4763	413824055	24.1947	27.72	18.91	84001203996	99.55	−959000000
/HMSO.L	X35	0.0143	2.9036	1.2576	34365050	17.8922	152.69	104.81	3757954625	99.47	−546700000
/HSBA.L	X36	−0.0005	4.3435	1.0319	837617154	18.2213	0.00	21.72	135836027641	99.83	−17407000000
/IHG.L	X37	0.0250	2.1434	1.2393	87071352	141.4713	11.73	8.92	5181364487	97.15	−335000000
/IMI.L	X38	0.0120	2.4038	1.7040	36570442	20.3361	8.21	4.71	4261030459	98.17	66700000
/IMT.L	X39	0.0220	4.9659	0.4076	93526094	18.5331	8.94	4.85	21260551624	99.09	812000000
/ITRK.L	X40	0.0131	1.3438	0.7563	20637664	432.1739	15.45	6.90	4904872258	98.90	60600000
/ITV.L	X41	−0.0019	1.6818	1.7263	628428447	428.7208	5.35	3.12	6057864459	91.76	249000000
/JMAT.L	X42	0.0081	2.0675	1.5184	22784617	19.6313	3.91	2.63	5629970195	96.62	−166500000
/KGF.L	X43	−0.0032	2.4520	0.8171	355529063	20.9568	5.61	1.94	9122588821	98.71	64000000
/LAND.L	X44	0.0114	2.5397	1.2666	56170054	17.0892	64.55	41.32	7376148137	95.45	−539300000
/LGEN.L	X45	−0.0003	4.0713	1.7650	766773936	18.6353	0.00	1.06	11076536451	97.11	2346000000
/LLOY.L	X46	−0.0029	0.0000	2.1357	1469846984	5.6763	0.00	22.16	49758545165	60.82	46000000
/LSE.L	X47	0.0148	1.9932	1.1678	32789241	20.5515	31.34	24.13	3998351640	77.56	218700000
/MGGT.L	X48	0.0150	2.1592	1.5408	39553137	23.4517	15.37	7.89	4299221825	99.46	196300000
/MKS.L	X49	0.0169	3.5895	0.7034	359603691	27.0259	8.43	4.89	7625545131	98.69	39200000
/MRON.L	X50	0.0088	2.8221	1.7455	9088526	0.0000	3.00	−1.44	3399185459	96.54	−77500000
/MRW.L	X51	0.0041	4.1404	0.3072	243092808	19.5378	3.76	2.30	6599185701	81.45	−146000000
/NG.L	X52	−0.0003	5.4106	0.3064	315836595	14.9445	24.44	12.66	27612870960	95.62	−449000000

(Continued)

Table 6.1 Continued

Security	Code	Capital return	Dividends	Beta	Trade volume	PE ratio	Operating profit margin	Net profit margin	Market capitalisation	Free float	Free cash flows
/NXT.L	X53	0.0133	2.1898	0.5204	59967349	15.5108	14.58	10.08	7527656754	92.68	429700000
/OML.L	X54	0.0025	3.5176	1.6409	259634875	13.4043	0.00	2.70	9709925042	96.73	631000000
/PFC.L	X55	0.0828	3.1805	1.0292	79007015	0.0000	7.44	6.71	4391630041	73.48	–1171000000
/PRU.L	X56	–0.0039	2.5741	1.7476	307339765	28.1236	0.00	1.63	28921470756	99.55	–348000000
/PSN.L	X57	0.0302	0.0000	1.1101	37681708	7.5142	19.37	12.19	3736553703	95.76	159200000
/PSON.L	X58	–0.0095	3.6145	0.5879	119234059	61.4954	7.01	1.60	10142578589	97.97	279000000
/RB.L	X59	0.0147	2.8811	0.3150	76736003	29.4751	17.94	13.11	33265996970	88.31	795000000
/RBS.L	X60	0.0039	0.0000	2.9754	91626890	56.2196	0.00	21.28	19615697495	34.13	–46898000000
/RDSa.L	X61	–0.0008	4.1438	0.8371	306699896	11.9183	11.76	6.61	143461493917	98.81	6174000000
/RDSb.L	X62	0.0007	4.8238	0.8301	269278819	43.2436	11.76	6.61	143461493917	98.81	6174000000
/REL.L	X63	–0.0021	2.8660	0.5629	159799556	90.5179	12.75	6.18	18124178517	99.08	–267000000
/REX.L	X64	0.0138	3.2262	0.8288	65803652	78.6424	7.79	2.24	3853730369	98.76	51000000
/RIO.L	X65	0.0170	3.6628	1.7202	163703186	34.4231	21.45	18.03	56023711302	89.99	–11128000000
/RR.L	X66	0.0224	0.0000	0.9413	385734105	25.0561	6.08	2.87	22147304421	99.21	570000000
/RRS.L	X67	0.0508	0.7046	0.3633	797834	6.5003	34.14	34.03	4136998114	98.98	–105304000
/RSA.L	X68	–0.0085	5.7109	0.4334	509234297	80.1311	0.00	–1.65	4663461845	95.56	–166000000
/SAB.L	X69	0.0156	2.0658	0.9405	95978102	0.0000	14.96	7.64	51697893114	58.25	1105000000
/SBRY.L	X70	–0.0003	4.3971	0.6118	265402645	18.3232	2.45	1.16	7189775207	86.75	–420000000
/SDR.L	X71	–0.0006	1.7388	1.3259	19914887	35.4545	10.91	11.89	6687099711	55.22	372300000
/SGE.L	X72	0.0001	3.0102	0.8394	180350460	23.0800	26.44	17.53	3994919232	95.67	108200000
/SHP.L	X73	–0.0038	0.4967	0.7063	84434374	54.8362	–18.12	–24.75	12356451810	98.52	1103500000

/SMIN.L	X74	0.0083	2.8141	0.9465	73685429	23.3121	7.70	3.87	5402710955	99.36	96200000
/SN.L	X75	0.0117	2.1079	0.4877	141034470	31.3055	14.79	11.04	7078227615	98.58	451000000
/SRP.L	X76	−0.0043	1.6251	0.5993	49718346	28.2349	4.42	2.69	3091765011	96.99	158900000
/SSE.L	X77	0.0101	5.2298	0.3879	101619419	13.8716	13.85	9.12	15472111150	99.15	−158500000
/STAN.L	X78	0.0056	3.6855	1.5289	177133273	13.6897	0.00	18.90	36395622891	99.49	16406000000
/SVT.L	X79	0.0087	4.3293	0.2137	47011734	19.9782	20.35	7.88	4170680726	99.36	−108500000
/TATE.L	X80	0.0163	3.0483	0.8110	68264738	12.3169	6.07	3.27	3992828708	98.36	0
/TLW.L	X81	0.0070	1.0889	1.0128	52757733	32.1174	30.01	14.25	9975154777	93.21	−520200000
/TPK.L	X82	0.0143	1.5309	2.4354	11836686	11.7390	10.42	6.39	3988603312	92.88	102200000
/TSCO.L	X83	0.0069	4.0745	0.5835	785978968	18.0641	5.69	3.72	29167061473	99.68	−1353000000
/ULVR.L	X84	0.0035	2.9750	0.5048	140571301	62.8372	11.44	3.91	81654657715	96.74	1757000000
/UU.L	X85	0.0061	4.8338	0.3178	74452563	10.7727	29.71	15.29	4824601147	99.49	−254700000
/VED.L	X86	0.0421	3.3859	2.7002	43010081	0.0000	16.94	5.68	2979450302	34.78	970400000
/VOD.L	X87	−0.0069	5.2990	0.5493	7003466696	120.1438	−21.19	−34.15	92700337145	99.22	−2814000000
/WEIR.L	X88	0.0102	1.7375	1.8898	19036822	14.6925	7.21	5.30	4644746892	99.03	90900000
/WG.L	X89	0.0027	1.2420	1.5478	30103938	11.6329	6.46	2.60	3365513374	92.73	10600000
/WMH.L	X90	0.0194	2.2359	0.7617	139445679	0.0000	7.99	3.86	4009412878	100	156900000
/WOS.L	X91	0.0196	1.9801	1.6844	15629311	58.4251	5.70	3.70	8810393489	99.27	323000000
/WPP.L	X92	−0.0002	2.4409	1.3250	212996166	34.4012	10.51	5.44	15515760289	98.97	271600000
/WTB.L	X93	0.0105	1.6953	0.9459	37980957	15.2250	12.34	5.15	5643840829	96.65	−14400000

marises the criterion identified and proposed methodology for creation of Pareto optimal portfolios.

Bank of England's annual average rate of discount for three month treasury bills, sterling has been extracted for a five year period: 2001–2005. The risk free rate used has been computed from the average of five years annual average rate of discount for three month treasury bills and then calculating the corresponding monthly rate. This riskless rate has been used as an input for the calculation of portfolio performance evaluation ratios. Again, for conducting the out of sample tests, the monthly security returns have been computed from the closing price data for the two year period of 2006–2007.The software used for our analyses includes MS Office Excel, Thomson Reuters Eikon Datastream for Office, E-views 5 and Lingo 14.0x64.

E-views has been used basically for the purpose of generation of the covariance matrix and obtaining a transpose of various matrices used in the study. The quadratic programming has been attempted through Lingo's extended unlimited constraints and variables version. Data extraction from Thomson Reuters Eikon database has been done through the use of Datastream for Office. MS Office Excel has been used for various calculations and performance evaluation comparisons.

6.1.2 Modelling constraints for an investor

The main goal of multi-objective programming model developed is to provide Pareto optimal portfolios as per the criterion identified. These criteria are translated into constraints of the model. In addition to this completeness constraint, which assures 100 per cent capital remains invested, upper bounds for a maximum investment of 15 per cent in a security are also incorporated. The decision variables of model are continuous i.e. X_i represent the weight of i^{th} security in the portfolio.

Objective function

The objective function as identified by Markowitz (1952) is minimisation of variance of the portfolio created. An investor would desire minimum deviation in the returns generated from the portfolio.

Objective Z: Minimise variance

Variance of a portfolio:

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{6.1}$$

where x_i and x_j are the amount invested in security *i* and *j* respectively; σ_i and σ_j are the standard deviation of returns for security *i* and *j* respectively; and σ_{ij} is the covariance between r_i and r_j the returns on security *i* and *j* respectively.

Modelling optimisation process

$$\text{Min } \sigma_P^2 = \sum_{i=1}^{m}\sum_{j=1}^{m} x_i x_j \sigma_{ij}$$

subject to:

$$\sum_{i=1}^{m} x_i = 1$$

$$\sum_{i=1}^{m} x_i E(r_i) \geq R_p$$

$$\sum_{i=1}^{m} d_i x_i \geq D_P$$

$$\sum_{i=1}^{m} \beta_i x_i \geq \beta_P$$

$$\sum_{i=1}^{m} v_i x_i \geq V_P$$

$$\sum_{i=1}^{m} p / e_i x_i \geq P / E_P$$

$$\sum_{i=1}^{m} o_{PM_i} x_i \geq O_{PM_P}$$

$$\sum_{i=1}^{m} n_{PM_i} x_i \geq N_{PM_P}$$

$$\sum_{i=1}^{m} c_i x_i \geq C_P$$

$$\sum_{i=1}^{m} ff_i x_i \geq FF_P$$

$$\sum_{i=1}^{m} fcf_i x_i \geq FCF_P$$

$$0 \leq x_i \leq X$$

where

x_i	amount invested in security i
σ_i	standard deviation (risk) of returns for security i
σ_p	standard deviation (risk) of returns for the portfolio
$E(r_i)$	expected return of security i
d_i	dividend yield of security i
β_i	beta coefficient for security i
v_i	volume of security i
p/e_i	price-to-earnings ratio of security i
o_{pmi}	operating profit margin of security i
n_{pmi}	net profit margin of security i
c_i	market capitalisation of security i
ff_i	free float percent of security i
fcf_i	free cash flow of security i
R_p	portfolios targeted expected return
D_p	portfolios targeted dividend yield
β_p	portfolios targeted beta
V_p	portfolios targeted volume
P/E_p	portfolios targeted P/E ratio
O_{pmP}	portfolios targeted operating profit margin
N_{pmP}	portfolios targeted net profit margin
C_p	portfolios targeted market capitalisation
FF_p	portfolios targeted free float
FCF_p	portfolios targeted free cash flow
X	the maximum amount of investment in a security

6.1.3 Alternative model formulations

Four different model formulations for different types of decision makers were designed by changing the level of investor preferences in the constraints demanded for the portfolio.

1. The risk averse portfolio is for an investor who has smaller risk bearing capacity. He/she would desire a regular income from the way of dividends and hence his/her dividend criterion is set at Q_3 (quartile three) level. Capital returns from the price change becomes a non-binding constraint for such an investor because he/she knows to desire high capital return would require additional risk. Due to his/her risk averse nature, this investor is not willing to take this additional risk and hence capital returns and beta (systematic risk) are set at lower Q_1 (quartile one) levels. Trade volume and P/E ratio are

also set at lower Q_1 (quartile one) levels since a risk averse is hesitant of high trading due to risk attached to it. However, he/she would desire high level of management efficiency, profitability and market capitalisation. This portfolio would demand Q_3 (quartile three) levels of these criteria and a decent level (91.76 per cent) of free float for the portfolio. The free cash flows are set at Q_3(quartile three) level since companies having lower/negative free cash flows may be viewed as risky propositions. The values targeted are summarised in Table 6.2.

2. The risk moderate portfolio is for an investor exhibiting satisficing behaviour: he/she is looking for neither too high nor too low returns. The risk appetite of such an investor is also moderate. The target values of all security evaluation variables are set at their mean levels for constructing a portfolio for the risk moderate.
3. For the risk seeker portfolio, the Q_3 (quartile three) values of variables are targeted for capital return, beta, volume and P/E ratio. This portfolio is for a class of investors having extremely high risk bearing capacity. They desire very high levels of return and are willing to bear adequate risk for achieving it. Evaluation criteria such as dividend, operating profit margin, net profit margin and size do not assume much importance for them and are set at lower Q_1 (quartile one) levels. Preference for free cash flows are set at Q_1 (quartile one) levels as companies with low/negative levels of free cash flows are viewed as the ones having ample investment opportunities and perhaps more return giving.
4. While creating indifference threshold portfolio, the median values for all the modelled variables have been targeted because an indifference threshold investor is willing to take additional risk for an equal compensation in returns.

   It should be noted that Q_3 (quartile three) and Q_1 (quartile one) values instead of maximum and minimum are used since maximising more than one criterion at the same time was resulting in infeasible portfolios. Thus, an attempt of dropping the criterion value from maximum by 5 percentile was made. Solutions for achieving 95, 90, 85 and 80 percentile for more than one criterion was also infeasible. Feasible solutions could be generated for Q_3 (75 percentile) and hence Q_3 (quartile three) and Q_1 (quartile one) were used instead of maximum and minimum values for various stock evaluation criterion. The values targeted for various portfolio formulations are summarised in Table 6.2.

Table 6.2 Constraint values for portfolio selection models

	Return parameters		**Risk**	**Marketability**		**Management efficiency**	**Profitability**	**Size**	**Corporate governance**	**Investment opportunities**
Types of investor	**Capital return**	**Dividend return**	**Beta**	**Volume**	**P/E ratio**	**Operating profit margin**	**Net profit margin**	**Market capitalisation**	**Free float**	**Free cash flows**
Risk averse	Q_1 (–0.00015)	Q_3 (4.0713)	Q_1 (0.5628)	Q_1 (43010080.88)	Q_1 (13.68)	Q_3 (16.93)	Q_3 (13.11)	Q_3 (19615697495)	Q_1 (91.76)	Q_3 (324000000)
Risk moderate	Mean (0.0084)	Mean (2.8625)	Mean (1.0388)	Mean (287303839.8)	Mean (72.12)	Mean (13.48)	Mean (9.43)	Mean (20803818070)	Mean (88.92)	Mean (–696028610)
Risk seeker	Q_3 (0.0147)	Q_1 (1.9801)	Q_3 (1.3276)	Q_3 (292951109.1)	Q_3 (44.69)	Q_1 (3.75)	Q_1 (2.29)	Q_1 (4561073778)	Q_1 (91.76)	Q_1 (–284000000)
Indifference Threshold	Median (0.0061)	Median (2.8221)	Median (0.9038)	Median (101619419.3)	Median (20.77)	Median (8.28)	Median (6.18)	Median (7625545131)	Median (98.52)	Median (62300000)

6.2 Results and discussion

6.2.1 Formation of Pareto optimal portfolios

The multi-objective portfolio selection problem is solved using the global solver LINGO 14 – the programming software of Lindo Systems, US. The educational licence for the extended unlimited constraints-variables version of the software was made available. A total of 93 non-linear variables, 11 linear and one non-linear constraints, 1089 non-zero values and 4371 non-linear non-zero values were used in each of the alternative model formulation to construct Pareto optimal portfolios.

Each of the portfolio is very well diversified. The number of securities in each of the optimal portfolios varies from 15 to 19. An attempt is made to get an idea of the securities which are most often present in the portfolios through calculation for the number of appearances of each security in the portfolios, minimum average and maximum weights of each of them. The portfolios created are summarised in Table 6.3. The maximum weight of any security in the portfolio is 15 per cent as set by the upper bound constraint. This is achieved by securities 3, 34, 50, 55, 77, 82 and 85. The minimum weight of any security to enter the portfolios is two per cent as attained by security 19 in the risk averse as well as risk seeking portfolio. Securities 3, 34 and 50 get an average weight of more than 10 per cent in all the portfolio formulations. Securities 37, 49, 55, 77 and 85 get weights between 5–10 per cent on an average. The other securities selected have average weights less than 5 per cent. The multi-objective programming model aids an investor in expressing his preferences regarding basic characteristics of his desired portfolio and thereby reducing the search space only to the Pareto optimal choices. The selection space is reduced by more than one-third from 93 securities to 27 securities.

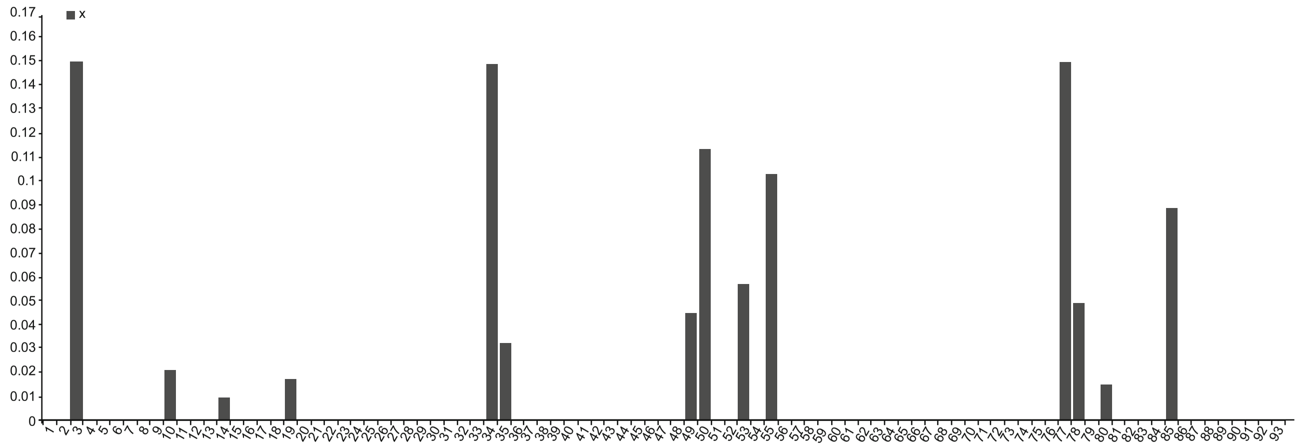
The portfolios constructed are presented graphically in Figure 6.1. The height of a bar shows the weight of security in the portfolio. All the portfolios generated are quite different in composition from each other. This is evidently due to the differences in preferences of investors for whom these portfolios have been constructed.

6.2.2 Portfolio performance evaluation

The performance of portfolios constructed for investors is evaluated in the following ways:

Comparison of output values

The output values achieved by the alternative model formulations for the entire security evaluation criterion are presented in Table 6.4.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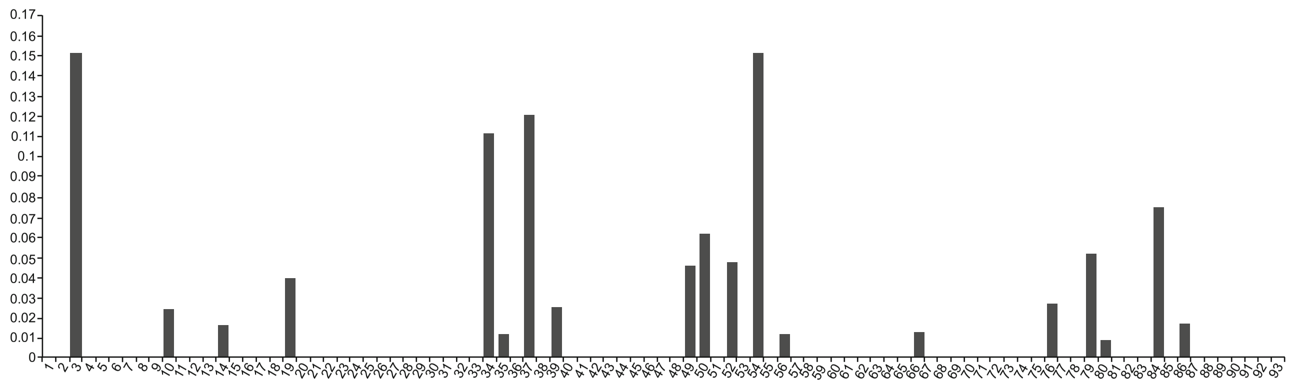

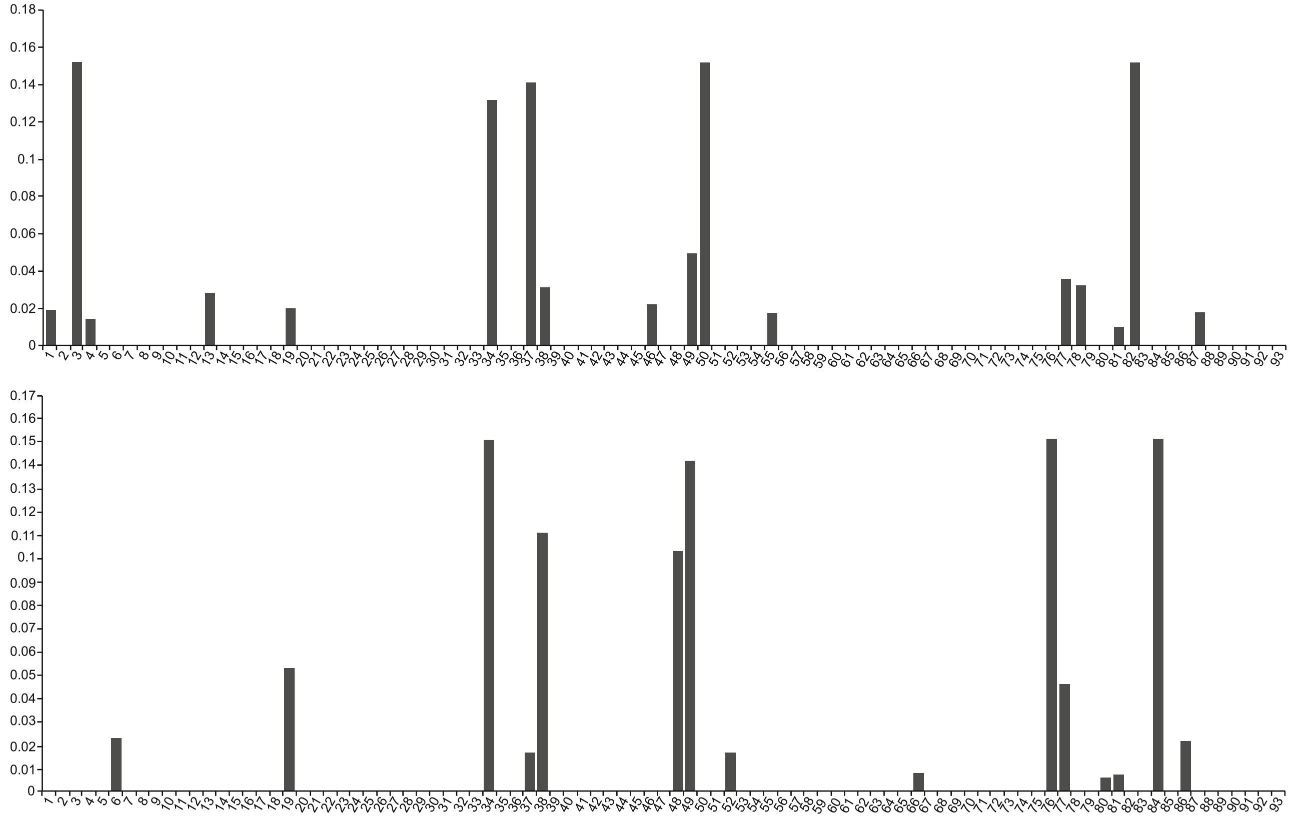

Figure 6.1 Constructed portfolios for alternative model formulations

Table 6.3 Portfolio weights achieved as per evaluation criterion

SECURITIES	Codes	Risk averse	Risk moderate	Risk seeker	Indifference threshold	Max	Min	Average
/AAL.L	1	0.00	0.00	0.02	0.00	0.02	0.00	0.00
/ADML.L	3	0.15	0.15	0.15	0.00	0.15	0.00	0.11
/ADN.L	4	0.00	0.00	0.01	0.00	0.01	0.00	0.00
/AMEC.L	6	0.00	0.00	0.00	0.02	0.02	0.00	0.01
/AZN.L	10	0.02	0.02	0.00	0.00	0.02	0.00	0.01
/BARC.L	13	0.00	0.00	0.03	0.00	0.03	0.00	0.01
/BATS.L	14	0.01	0.02	0.00	0.00	0.02	0.00	0.01
/BP.L	19	0.02	0.04	0.02	0.05	0.05	0.02	0.03
/GSK.L	34	0.15	0.11	0.13	0.15	0.15	0.11	0.13
/HMSO.L	35	0.03	0.01	0.00	0.00	0.03	0.00	0.01
/IHG.L	37	0.00	0.12	0.14	0.02	0.14	0.00	0.07
/IMI.L	38	0.00	0.00	0.03	0.11	0.11	0.00	0.04
/IMT.L	39	0.00	0.02	0.00	0.00	0.02	0.00	0.01
/LLOY.L	46	0.00	0.00	0.02	0.00	0.02	0.00	0.00
/MKS.L	49	0.04	0.05	0.05	0.10	0.10	0.04	0.06
/MRON.L	50	0.11	0.06	0.15	0.14	0.15	0.06	0.12
/NXT.L	53	0.06	0.05	0.00	0.02	0.06	0.00	0.03
/PFC.L	55	0.10	0.15	0.02	0.00	0.15	0.00	0.07
/PSN.L	57	0.00	0.01	0.00	0.00	0.01	0.00	0.00
/RRS.L	67	0.00	0.01	0.00	0.01	0.01	0.00	0.00
/SSE.L	77	0.15	0.03	0.03	0.15	0.15	0.03	0.09
/STAN.L	78	0.05	0.00	0.03	0.05	0.05	0.00	0.03
/TATE.L	80	0.01	0.05	0.00	0.00	0.05	0.00	0.02
/TLW.L	81	0.00	0.01	0.01	0.01	0.01	0.00	0.01
/TPK.L	82	0.00	0.00	0.15	0.01	0.15	0.00	0.04
/UU.L	85	0.09	0.07	0.00	0.15	0.15	0.00	0.08
/VOD.L	87	0.00	0.02	0.02	0.02	0.02	0.00	0.01
	Total	1.00	1.00	1.00	1.00			

Table 6.4 Output of the model formulations

Types of investors	**Risk averse (Q1–Q3)**	**Risk moderate (Mean)**	**Risk seeker (Q3–Q1)**	**Indifference threshold (Median)**
Variance	0.00032	0.00022	0.00051	0.00048
Capital return	0.01875	0.02543	0.0147	0.00791
Dividend	4.0713	3.674	3.1901	3.9031
Beta	0.3387	0.8175	1.3276	0.9038
Volume	155620480	287303839.8	292951109.1	369663019
P/E Ratio	13.68	113.852	52.59	20.77000
Operating profit margin	16.93	13.48	8.829	13.785
Net profit margin	17.046	13.894	11.959	8.492
Market capitalisation	22352169495	20803818070	21209783778	26061285000
Free float	91.76	90.324	91.76	98.52000
Free cash flows	324000000	–696028610	–284000000	0.6230000E+08
Corresponding points on efficient frontier (variance)	0.000235	0.000214	0.000232	0.00031

The minimum variance could be achieved by the risk moderate portfolio, which was extremely close to Markowitz's efficient variance for the same level of return. Risk (variance) of portfolio was 0.00022 for a return of 0.02543. It got diversified across 19 companies. Meeting investor preferences relating to ten other evaluation criteria, this portfolio is able to maintain low risk comparable to a mean-variance efficient portfolio. Not only are the capital returns high but the dividend yield achieved is more than targeted (3.67 per cent). Funds exhaustion constraint showed a significant dual price indicating a further reduction in variance by allowing for borrowing and lending of funds for creation of portfolios.

The risk averse portfolio is also a low risk portfolio close to the corresponding point on Markowitz's efficient frontier. It depicts an investor with smaller risk bearing capacity. The portfolio got diversified across 15 companies. Meeting the high dividend constraint targeted it has a dividend yield of 4.0713 per cent. It invests in securities having low beta thereby reducing the systematic risk of the portfolio considerably (0.3387). High levels of operating profit margin, net profit margin and market capitalisation have been achieved by the risk averse investor.

Meeting all his constraints, the risk seeker ends up with a high degree of risk (variance) with not a very high level of return. A risk seeker portfolio with targets set at Q_3 values of variables such as return, beta, volume, price-to-earnings ratio and free float was formulated for a class of investors having extremely high risk bearing capacity. Diversifying across 17 securities the portfolio achieves 0.00051 level of risk. By investing in high beta stocks the systematic variance of his portfolio is quite high at 1.3276. Dividend yield achieved is more than targeted. The management efficiency criterion and profitability criterion perform much better than demanded. Free cash flow of the portfolio is negative (representing good investment opportunities) but not as low as the risk moderate.

The indifference threshold portfolio resulted in lowest return (0.00791) attached with comparatively a higher degree of risk (0.00048). This portfolio aiming at median values for all the security evaluation criteria represented an investor who is willing to take additional risk for an equal compensation of return. This was also a well-diversified portfolio with investment in 15 securities. Higher than targeted levels for dividend yield, operating profit margin and net profit margin were attained. The free cash flow of portfolio is positive representing idle cash available with such companies and inadequate investment opportunities. The smallest returns (among all model formulations) of 0.00791 per cent per month were observed for this portfolio with risk at a level of 0.0048.

Comparison to Markowitz's efficient frontier

An attempt has been made to display the proximity of modelled portfolios with the Markowitz efficient frontier. Figure 6.2 presents the mean-variance efficient Markowitz portfolio together with the portfolios generated in the risk-return space. The risk moderate and risk averse portfolios perform well in terms of mean-variance efficiency. However, risk seeker portfolios did not perform well, with the indifference threshold being the worst performer.

Performance evaluation ratios

To sum up the performance of portfolios and rank them, portfolio evaluation measures as given by Sharpe (1966) and Treynor (1965) have been used.

$$S_t = \frac{(\bar{r}_i - r^*)}{\sigma_t} \tag{6.2}$$

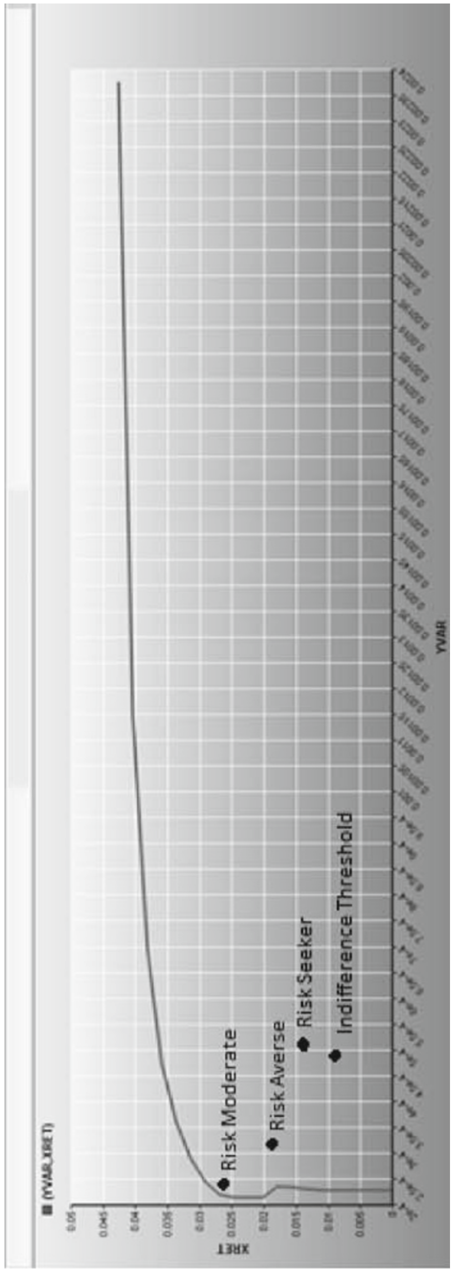

Figure 6.2 Constructed portfolios for alternative model formulations

where

S_t = Sharpe Index
$\bar{r}_i$ = average return on portfolio i
r^* = riskless rate of interest
β_n = standard deviation of the returns of portfolio t.

$$\text{And } T_n = \frac{(\bar{r}_n - r^*)}{\beta_n} \tag{6.3}$$

where

T_n = Treynor Index
$\bar{r}_n$ = average return on portfolio n
r^* = riskless rate of interest
β_n = beta coefficient of portfolio n

The annual average rate of discount of Treasury Bills of the Bank of England for a five year period between 1/1/2001 and 31/12/2005 has been used as the riskless rate of interest for computing the above. Table 6.5 presents the portfolio evaluation Sharpe and Trenyor ratios and ranking for our portfolios.

With regard to the Sharpe ratio, the risk moderate portfolio using mean values of the constrained variables performed the best. Risk averse portfolios ranked second with risk seeker portfolios following them. The indifference threshold portfolio using median target values was the worst performer. Perhaps, to construct Pareto optimal portfolios optimising across a variety of security evaluation criterion without comrpomising much on the mean-variance efficiency parameters, the mean values (as in risk moderate portfolio) of these criteria may be targeted to achieve efficiency.

Table 6.5 Performance evaluation of portfolios

Portfolios	**Return**	**Risk**	**Standard deviation**	**Beta**	**Sharpe ratio**	**Rank**	**Trenyor's index**	**Rank**
Risk free rate	0.003528							
Risk averse	0.01875	0.00032	0.0179	0.3387	0.850917	2	0.044941	1
Moderate	0.02543	0.00022	0.0148	0.8175	1.47661	1	0.026791	2
Risk seeker	0.0147	0.00051	0.0226	1.3276	0.49469	3	0.008415	3
Indifference threshold	0.00791	0.00048	0.0219	0.9038	0.199995	4	0.004848	4

As per the Treynor ratio, the risk averse portfolio was ranked first. The risk moderate and risk seeker portfolio followed in ranks respectively. The indifference threshold portfolio was again the worst performer and resulted in very small Treynor ratios. Sharpe ratio use total risk, whereas Treynor ratio use only the systematic portion of risk to compare portfolios. The comparatively low level of systematic risk (beta) present in risk averse portfolio could be attributed for the better performance of this portfolio according to Treynor ratio.

6.3 Out of the sample tests

To test for the robustness of our model, forward tests have been conducted for the period of two year pre-crisis period beginning from 1/1/2006 to 31/12/2007. The resultant weights from our alternative model formulations have been multiplied by the returns for this forward period to derive the actual returns of the portfolios formulated. The relative performance of these portfolios in out of sample tests has been calculated again through calculating Sharpe and Trenyor ratios. The results of out of sample tests and performance of portfolios for the year 2006 have been summarised in Table 6.6.

As predicted, the risk moderate portfolio performs the best followed by the risk averse portfolio and the risk seeker portfolio respectively; the indifference threshold portfolio performs the worst according to Sharpe's measure of performance evaluation. Also, considering the systematic risk based measure Treynor ratio, the risk averse portfolio outperforms the risk moderate portfolio with the risk seeker portfolio and the indifference threshold portfolio following them respectively. The performace ratios for the indifference threshold portfolio are not negative but are extremely small. The forward tests for 2006 confirms our results from the previous section as the rankings for portfolio match with our prediction.

Table 6.6 Performance evaluation of portfolios for 2006

	Portfolio return	**Sharpe ratio**	**Rank**	**Treynor ratio**	**Rank**
Risk averse	0.025762	1.235	2	0.066	1
Risk moderate	0.026892	1.558	1	0.029	2
Risk seeker	0.028419	1.082	3	0.019	3
Indifference threshold	0.014786	0.512	4	0.012	4

Further, similar forward tests are conducted for the the year 2007. The majority of the securities generated negative or very small returns during this period. It may be noted that this is the time immediately before the crisis and hence negative/small security returns could be expected. All the modelled portfolios except the risk averse portfolio generated negative returns. Looking at the portfolio returns only the risk averse portfolio performs the best and risk seeker portfolio performs the worst.

Even after considering the attached risk measure (standard deviation), the risk averse portfolio is the best performer with indifference threshold portfolio being quite close. The risk seeker portfolio performs the worst. This could be predicted because of the downfall in returns during this period and the high degree of risk such investors are willing to accept. The systematic risk (beta) modelled into the optimisation model for such investors was at Q_3 level, i.e. the investment here happens in high beta securities.

Taking into consideration the systematic risk based measure of portfolio performance, the Treynor ratio, again the risk moderate portfolio performs the best. However, this time risk moderate portfolio and the indifference threshold portfolio have almost equal (negative) performance ratio. Perhaps this is due to the high degree of systematic risk present in the markets at this immediate pre-crisis time period. The risk averse portfolio is ranked third and the risk seeker portfolio performs the worst again.The results for the portfolio performance evaluation measures for 2007 have been presented in Table 6.7.

Analysing the performance of portfolios constructed within the sample as well as out of the sample it may be concluded that optimising across various security criterion without compromising much on the mean-variance efficiency parameters of the risk moderate portfolio does fairly well. The risk averse portfolio gives tough competition to it and even outperforms the risk moderate portfolio considering the system-

Table 6.7 Performance evaluation of portfolios for 2007

Portfolios	**Portfolio return**	**Sharpe ratio**	**Rank**	**Treynor ratio**	**Rank**
Risk averse	0.000362	−0.176	1	−0.00935	3
Risk moderate	−0.0012	−0.315	3	−0.00579	1
Risk seeker	−0.01147	−0.652	4	−0.0113	4
Indifference threshold	−0.00244	−0.271	2	−0.00661	2

atic risk attached as per the Treynor ratio. These two portfolios perform well fairly consistently. The risk seeker portfolio and the indifference threshold portfolio do not perform well in terms of mean-variance efficiency. The indifference threshold portfolio is usually ranked the last of all.

However, the scenario changes for year 2007 when the risk seeker portfolio performs most poorly. Perhaps, the falling security prices and declining/negative returns during this time led to poor performance for this high risk taking portfolio formulation. This is usually the case for risk takers. Under such a scenario, positive returns are achieved only by the risk averse portfolio and it performs the best as per Sharpe's measure of performance evaluation. Also, considering beta based measure of performance evaluation, the risk moderate portfolio does the best. For optimal allocation of assets, investor preferences must adapt to the changes in market conditions.

The multi-criteria portfolio selection model created in this research work incorporates the preferences of an investor. The model is flexible and may be modified according to the changes in the securities market and economy in general. It results in portfolios which are efficient not only in the mean-variance sense but also in relation to a host of other security evaluation criterion. This quadratic programming algorithm may assist an investor in implementing his/her investment strategies.

7
Summary, Conclusions and Suggestions for Future Research

This chapter showcases a reflection of the research work presented. The use of complex programming techniques, the latest research software and the integration of multiple factors describing investor's preferences are able to tap the requirements of an investor from a portfolio beyond just mean and variance. While attempting to minimise risk, an investor is faced with a variety of constraints including earning good returns, dividends, marketability of securities and promising future opportunities. Investors also desire liquidity, management efficiency, profitability, adequate market capitalisation, free float per cent and free cash flows from their portfolio. Substantial interest of promoters and other institutions in the script, free float factor, industrial and company diversification are a few other aspects that an investor seeks from his/her portfolio. An attempt has been made to incorporate all these considerations of an investor for developing and testing of a multi-criteria optimisation model for portfolio selection.

Improved access to databases for financial time series data and availability of software capable of performing complex computations in lesser time contributes in enhancing the quality of research in the area of portfolio selection. The mathematical sophistication in understanding and applying these techniques is imperative for creation of an algorithm to select multi-criteria Pareto optimal portfolios. The need for a balanced portfolio giving the investor protections and opportunities for a wide range of contingencies provided the initial motivation for this research work. An attempt to model the real behaviour of an investor while making a portfolio selection decision has been made. Possible improvements in the existing portfolio selection modelling framework is attempted by optimising across multiple constraints.

In Indian capital markets, there has been the introduction and promotion of a large number of financial and technological innovations introduced by the Securities and Exchange Board of India (SEBI) as well as the stock exchanges. The emerging issues of algorithmic trading, wireless trading, co-location, smart-order routing, high frequency trading and programming interface with the ever changing technology are gaining pace. The minimum public shareholding requirement of the regulator leading to either excess supply of securities or non-compliance by companies may hamper the image of our capital markets. Incorporation of changes in risk management of the cash and derivatives segment, coordination between the regulator and Institute of Chartered Accountants of India (ICAI) for improving the disclosures and accounting standards, clearing the interoperability issues between the two stock exchanges, assessment of impact of investor education initiatives and measures for safety of investor in mobile trading arena are an impending task for all stakeholders of the Indian capital markets.

Increasingly, portfolio creation software is being used by large institutional investors such as mutual funds and pension funds across the globe to assist them in asset allocation across diverse asset classes. Database management modules, input coefficient calculation modules and optimisation modules are required for such software to work. Considerable experimentation is also needed to achieve an efficient asset allocation through understanding the needs and desires of an investor.

A widespread interest of the business and academic community in portfolio selection is witnessed from the growing amount of available literature in this area. This highlights the importance of an investor's investment decision especially after the financial crisis. The mean-variance efficient frontier introduced over 60 years ago by Roy (1952) and Markowitz (1952) and extended by researchers worldwide to include different measures of risk, multi-period analysis, additional variables and constraints serves as the foundation for this research study. Despite a large number of empirical studies in the area of market efficiency and the capital asset pricing model (CAPM), there has been a dearth of research in analysing mean-variance efficient portfolio selection. This research work tries to integrate the existing studies on mean-variance efficient portfolio selection models and extends them using a mathematical programming technique.

Portfolio selection has witnessed use of complex mathematical and quantitative techniques over the years such as polynomial time algorithms, numerical algorithms, structure exploiting algorithms, goal programming, quadratic programming, dynamic programming, a decision tree approach, regression equations, multivariate GARCH, impulse response functions, Hilbert space theory, Martingale theory, Bayesian inferential procedures, fuzzy techniques and many more. The studies on efficient frontier are progressing by including utility optimum bounds, alternate variance measures and the latest quantitative techniques. However, restrictive assumptions of many of these models, their mathematical complexity and ignorance of simultaneous inclusion of multiple constraints are hindrances in their testing and applicability.

The studies on diversification indicated the benefits of diversification and evidence of poor performance related with over diversification. Portfolio optimisation and variance-covariance based studies hinted at model extensions to include preferences of different types of investors and their real life constraints. Model formulations incorporating behavioural finance aspects such as disposition effects and repeating habits were perhaps missing. The fast processing and absorption of information effects in the derivatives market have not been included in equity portfolio selection models for spot markets as yet.

In spite of the recent modelling innovations, the mean-variance model introduced by Markowitz in 1952 and further investigated in his research papers and books is still the best known literature on portfolio selection. The mean-variance approximations provide almost maximum expected utility. It is the starting point for many other models. Despite extensive research, it is still not easy to compute a mean–variance efficient frontier even when all constraints are linear. This is particularly true of problems having dense covariance matrices. Although, the mean-variance efficient frontier of a Markowitz problem is a continuous curve, it is rarely rendered as such. Rather, efficient frontiers are customarily shown in the form of dotted representations (ten point, 20 point, etc.).

With time, the dimensions of portfolio optimality have increased much beyond just mean and variance. Portfolio selection depends upon the preferences of investors regarding various security evaluation criteria. Due to the limitations of both preference axioms based stochastic dominance and expected utility theory models and inability of bi-criteria optimisation to incorporate preferences of decision makers under risk a multiple criteria linear programming model for portfolio selection needs to be developed. The highly complex and multi-dimensional nature of portfolio selection decisions, the intrinsic characteristics of securities and preferences of investors can be effectively modelled in a MCDM problem. Accounting, financial market and corporate governance information remains central as analysts, as well investors attach considerable weight to published accounting numbers, market fundamentals and governance policies while setting their preferences.

An attempt has been made to ease the portfolio selection decision of investors by understanding their desires and limitations. The multiple goals and constraints provide the direction for development and testing of a mean-variance efficient portfolio selection model which optimises across multiple constraints simultaneously. The explanatory power of various equity variables which have been studied in isolation so far has been integrated in this research work. Industrial and company diversification has also been achieved to the extent of 20 and 15 per cent respectively. The preferences of different types of investors have been accommodated by empirical investigation of the model for four different portfolio selection model formulations. An attempt to reduce the gap that exists between theory and application has been made by testing the model on India's National Stock Exchange's Nifty 50 and London Stock Exchange's FTSE 100 securities.

With securities data now widely available at the Centre for Monitoring of Indian Economy (CMIE) database PROWESS as well as Thomson Reuters database Eikon and investors open to the consideration of increased number of opportunities for creating optimal portfolio, portfolio programming assumes significant importance. Selection of a portfolio through the use of appropriate security evaluation criterion as optimiser may often result in saving time and money. This research work is an attempt to fill this gap and provide a frame of reference for those who find themselves confronting with portfolio selection problems.

7.1 Model development

An attempt to aid the decision making of an investor progressing towards achieving the objectives set according to his/her aspiration levels and minimising the risk of his/her portfolio has been made. A multi-criteria mean-variance efficient portfolio selection model in which the distributions are characterised by a series of statistical variables like the expected value, the variance, beta, dividend, volume, price-to-earnings ratio, operating profit margin, net profit margin, impact cost, earnings per share, institutional holding, promoter's holding, unsystematic risk, price-to-book value ratio, turnover, sales, industrial and company diversification market capitalisation, free float and free cash flows has been proposed.

The multi-objective portfolio selection problem has been transformed into a single objective function in which the variance of a portfolio is minimised while constraints are imposed on the other variables. A tractable problem from the computational point of view has been framed. Alternate portfolio selection model formulations are exhibited to simulate various scenarios with realisation of discrete random variables. These model formulations are solved using the quadratic programming approach. The efficient solutions generated are Pareto optimal such that one statistic cannot be improved without worsening of the other. The particular mean-variance efficient solutions of the proposed model are not dominated with respect to minimum variance.

7.1.1 A general model

A plethora of financial, accounting and corporate governance based portfolio selection constraints have been incorporated in the theoretical model. The classical constraints of funds exhaustion, no short sales and upper bounds are also included. Many new constraints are added accommodating for the needs and limitations of a present day investor. Desire for capital returns and dividend gains by the investor are modelled as constraints. Minimum volume, turnover and impact cost (a new variable available from National Stock Exchange's official website) and price-to-earnings ratio are added to tackle an investor's desire for liquid and marketable portfolios. Industrial diversification and company diversification constraints are added to achieve a diversified portfolio. The objective of management efficiency and profitability are incorporated by including constraints on operating profit margin and net profit margin respectively. The objective of high earnings from a portfolio are incorporated by including constraints on accounting figures and ratios such as sales, earnings per share, price-to-earnings ratio and book to market ratio. A market capitalisation constraint has been included for size considerations. Investors are also interested in securities where other stakeholder's interest is substantive, free float, promoter's shareholding and institutional shareholding constraint are added to take care of this desire. The future investment opportunities as captured through free cash flows are also modelled as a constraint. The model aids an investor in expressing his/her preferences regarding characteristics of his/her desired portfolio and thereby reducing the search space only to the Pareto optimal choices. The selection space is reduced by more than one-third from 93 securities to 27 securities for FTSE securities.

7.1.2 Alternate portfolio selection model formulations

Testing on India's National Stock Exchange

The model is tested on real data drawn for the Nifty 50 securities for a period of 12 financial years starting from 2000 to 2012. An analysis of the return distribution of the chosen securities and the Nifty index is presented. The security returns were checked for normality. Very few securities had kurtosis equal to or nearing three, thereby representing non-normal series of returns. This is a common phenomenon for financial time series data.

Eight portfolio formulations namely the diversifier's portfolio, the satisficer's portfolio, the plunger's portfolio, the market trend portfolio, the capital gain bias portfolio, the dividend gain bias portfolio, the equal priority portfolio and the ideal portfolio were created for investors with different priorities and risk appetite. The no short sales constraint, funds exhaustion constraint, company diversification/upper bounds constraint and industrial diversification constraint of 15 per cent and 20 per cent respectively, were common for all the formulations.

The model seeks to achieve the most important objective of an investor, i.e. risk minimisation for the different categories of investors. Not only this, variance is minimised simultaneously achieving more than or equal to targeted levels of other important portfolio variables such as earnings per share, dividend, promoter's holding, institutional holding, impact cost, turnover and volume. This was witnessed with the presence of surplus values for the constraints in most of the model solutions.

The diversifier's portfolio depicts an investor with smaller risk bearing capacity. The target values of the variables namely return, beta, dividend, earnings per share, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-book value ratio, price-to-earnings ratio promoters holding, sales, turnover, unsystematic risk and volume were set at their Q_1 (quartile one) levels. The portfolio was diversified across 14 companies and 12 industrial sectors. The portfolio in addition to being mean-variance efficient achieved the target dividend returns, beta, liquidity, market capitalisation and the other constraints. A small portfolio variance of 0.187 was observed.

The satisficer's portfolio is for an investor with moderate risk appetite. The target values of all the programmed variables were set at their median levels. It got diversified across 15 companies and 12 industries. Funds exhaustion constraint showed a significant dual price indicating a further reduction in variance by allowing for borrowing and lending of funds for creation of portfolios. Risk (variance) of portfolio was 0.316 for a return of 0.605.

A plunger's portfolio with targets set at Q_3 (quartile three) values of variables was formulated for a class of investors having extremely high risk bearing capacity. They desire very high levels of return, dividend, net profit, free float, volume, turnover, price-to-book value ratio, price-to-earnings ratio, beta, other stakeholder's interest and are willing to bear a high degree of risk for this. No feasible solution could be generated for this model formulation. Even after more than 11 lakh iterations of the extended global solver of programming software Lingo 13, the problem could not be solved. The model debugger was run to find the constraints leading to this infeasible solution. The institutional holding constraint, promoter's holding constraint, no short-selling restriction on 33 securities and upper bounds on 12 of them were found to be the necessary and sufficient rows causing the infeasibility.

A market trend portfolio which aimed mean values for all the modelled variables was created. Mean rather than mode was chosen to describe the market trend as no single value was repeating itself in most of the series of variables. It was also a well-diversified portfolio with investment in 18 companies across 13 industrial sectors. The average weight of each security was found to be 5.55 per cent. The smallest returns (among all model formulations) of 0.52 per cent per month were observed for this portfolio with risk at a level of 0.364.

The capital gain bias portfolio depicts an investor who aims at a very high level of capital returns, beta, earnings per share, net profit, price-to-earnings and price-to-book value ratios targeting Q_3 (quartile three) values and is satisfied with a Q_1 (quartile one) level of dividend return. Also, the targets for variables such as free float, institutional holding, market capitalisation and sales are set at their median values. Such an investor seems to be more of a speculator aiming at quick returns and not regular income from trading in stock market. This portfolio, which was diversified across 12 companies and ten industries was found to have the maximum risk (1.07) among all the portfolio selection model formulations. Surprisingly, high dividend returns were also realised on this portfolio.

The dividend gain bias portfolio has been created for an investor desiring very high levels of regular income in the form of dividends. The dividend constraint was set at Q_3 (quartile three) level whereas returns, beta, earnings per share, impact cost, market capitalisation, price-to-earnings ratio and price-to-book value ratio constraints were set at Q_1 (quartile one) level. Other constraints namely free float, institutional holding, promoter's holding, sales, turnover and volume were set at their median values. The portfolio diversified across 16 companies and 12 industrial sectors. A risk of 0.206 for 0.55 levels of returns was achieved for this portfolio.

The equal priority portfolio gave same priority to capital returns as well as dividend income aiming at Q_3 (quartile three) levels for both of these variables. A high degree of capital gain returns and dividends are desired with lesser emphasis on beta, earnings per share, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-earnings ratio, price-to-book value ratio, promoter's holding, sales, turnover and volume positioning these variables at their median values. This portfolio diversified across 17 companies and 12 industrial sectors. The average weight of each security was found to be 5.88 per cent. A risk-return combination of 0.322 (variance) and 0.63 (return) was achieved.

The ideal portfolio was created with lesser number of constraints. The important constraints were identified from the results of the multiple regression and Granger causality tests. Returns, dividend and impact cost found to be of utmost importance to an investor were aimed at high Q_3 (quartile three) level. Institutional holding of 31.13 per cent was desired. Net profit, promoter's holding, sales, turnover and volume constraint were targeted at median levels. The constraints of beta, earnings per share, free float, market capitalisation, price-to-book value ratio, price-to-earnings ratio and unsystematic risk were removed from the problem as they were not found to significantly explain returns. The average weight of each security was found to be 7.69 per cent with the portfolio diversifying across 13 companies and 12 industrial sectors.

A graphical comparison of the risk-return combinations of all portfolios along the 20 point Markowitz efficient frontier showed that the risk-return combinations of the ideal portfolio were lying closest to the Markowitz portfolio. All the modelled portfolios lie below the Markowitz frontier, but all of them except one (the capital gain bias portfolio) were quite close to it. The returns generated by ideal portfolio were extremely near to those of the Markowitz portfolio with their risks being the same. This closeness of the two portfolios has been further investigated through tests of equality on yearly data.

Testing on UK's London Stock Exchange

The model is tested on real data drawn for FTSE 100 securities for a period of five years starting from January 2001 to December 2005. Four portfolios formulations namely the risk averse portfolio, the risk moderate portfolio, the risk seeker portfolio and the indifference threshold portfolio were created for investors with different priorities and risk appetite. Variance is minimised simultaneously achieving more than or equal to targeted levels of other important portfolio criterion. For any portfolio to be feasible, it is not possible to achieve the minimum or maximum values for more than one security evaluation criterion and hence Q_1 (quartile one) and Q_3 (quartile three) of variable's series have been used instead.

A graphical comparison of the risk-return combinations of all portfolios along the 20 point Markowitz efficient frontier showed that the risk-return combinations of the risk moderate portfolio were closest to the Markowitz portfolio. All the modelled portfolios lie below the Markowitz frontier, but all of them except one (the indifference threshold portfolio) were quite close to it. The risk moderate portfolio was extremely near to the Markowitz portfolio with slightly higher risk for the same level of expected return.

7.1.3 Multiple regression analysis

Multivariate regression analysis has been undertaken to identify the variables which have the significantly high explanatory power to estimate expected returns. Two multiple regression equations were estimated, one with returns as the dependent variable and the other with excess return to standard deviation ratio as the dependent variable. The list of regressors included earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-book value ratio, price-to-earnings ratio, promoter's shareholding, sales, turnover, unsystematic risk and volume. Beta has been intentionally excluded from the regression analysis due to its known significant power to explain the cross section of returns.

A new explanatory variable, impact cost emerged in regression analysis as possessing significantly high explanatory powers for predicting security returns and Sharpe ratio. Liquidity of a portfolio is of prime importance to the investor. The other important explanatory variables included institutional holding, promoter's holding and turnover. The stake of other prominent parties in the shareholding also forms an important investment factor. The graphical representation of estimated equations attempted through actual, fitted and residual graphs showed the model to have a good fit as the actual values were quite close to fitted values and residuals were small. Also, the model forecasted returns showing small Theil inequality coefficient of 0.21 and 0.34 implied a good fit of forecast model.

7.1.4 Granger causality interpretations

Causation between returns on a security and the variables set as constraints in the programming problem were tested using Granger causality tests. High probability values were observed for null hypotheses of no causation between security returns and dividend, impact cost, net profit, promoter's holding, sales and volume. Thus, one cannot reject the hypothesis that dividend, impact cost, net profit, promoter's holding, sales and volume do not cause returns. These factors assume importance in portfolio selection modelling due to their explanatory power for returns. The tests rejected null hypothesis of no causation among the other paired variables.

7.1.5 Portfolio utility analysis

The concept of portfolio utility introduced by Markowitz and has been applied in this monograph. The portfolio utility analysis attempted to empirically find that the utility derived by the investor from alternate portfolios is different for changing levels of risk tolerance. The utility of Markowitz's portfolio, the ideal portfolio and the index portfolio have been calculated for the risk tolerance levels of 10 per cent to 90 per cent. A direct relationship between the degree of risk tolerance and the value of portfolio utility was found.

The index portfolio Nifty 50 yielded the maximum utility of 0.7 at the highest risk tolerance level of hundred per cent and least utility (–4.79) at lowest risk tolerance level of ten per cent. At moderate levels of risk tolerance (30 per cent to 60 per cent), the utility of Markowitz's portfolio is the highest, closely followed by the ideal portfolio. From a risk tolerance level of 70 per cent onwards, the index portfolio gives a higher utility than the Markowitz portfolio and the ideal portfolio. However, the volatility in utility of the index portfolio is higher than that of Markowitz's portfolio or the ideal portfolio. It may be concluded that the portfolio utility of the index portfolio is least at small risk tolerance levels and highest for higher risk appetite investors than the Markowitz portfolio.

The intersection of utility curves and portfolios in the risk-return space determined the portfolio choice of a particular category of investor (plunger, diversifier, risk neutrals and risk lovers). The plunger was found to prefer the index portfolio. Four portfolios namely the ideal portfolio, the Markowitz portfolio, the satisficer's portfolio and the equal priority portfolio all lie on the highest utility curve (U3) with a higher level of utility. The diversifier will choose the index portfolio Nifty 50. The diversifier's portfolio and the dividend gain bias portfolio lying on utility curve U3 provided equal utility to this investor. The Markowitz portfolio and the ideal portfolio should also provide the same and a higher utility as was shown by drawing another utility curve U4. The capital gain bias portfolio and the index portfolio Nifty was preferred by risk lovers. The capital gain bias portfolio also yielded the highest utility for the risk neutral investor aa shown in the graphical analysis.

7.1.6 Performance evaluation of portfolios

The performance evaluation of all these portfolios is attempted by using the Sharpe ratio (1966) and the Treynor ratio (1965). Portfolios are then ranked and arranged in descending order of their performance. The performance of portfolios constructed on NSE's Nifty was as follows. The Sharpe ratio was found to be the highest for Markowitz's portfolio followed by the ideal portfolio. The other portfolios in decreasing order of their performance were the equal priority portfolio, the satisficer's portfolio, the dividend gain bias portfolio, the diversifier's portfolio and the capital gain bias portfolio. The market trend portfolio performed the worst according to this performance measure.

When evaluated according to Treynor's ratio, the ideal portfolio performed the best, even better than the Markowitz portfolio. The equal priority portfolio, the capital gain bias portfolio and the satisficer portfolio also performed quite well. The dividend gain portfolio and the diversifier's portfolio showed small ratios hinting at this portfolio's poor performance. As per Treynor's ratio also, the market trend portfolio was found to be the worst performer reporting the least ratio of only 0.02.

The performance of portfolios constructed on FTSE 100 exhibited the following results. The Sharpe ratio was found to be the highest for the risk moderate portfolio followed by the risk averse portfolio and the risk seeker portfolio respectively. The indifference threshold portfolio performed the worst according to this performance measure. When evaluated according to Treynor's ratio, the risk averse portfolio performed the best. The risk moderate portfolio also performed quite well. The risk seeker portfolio and the indifference threshold portfolio showed small ratios hinting at the portfolios' poor performance. As per Treynor's ratio also, the indifference threshold portfolio was found to be the worst performer reporting the least ratio of 0.00048.

7.1.7 Tests for equality: main findings

To further investigate the equivalence of returns, variance and utility of the ideal portfolio with the Markowitz portfolio, a yearly analysis has been conducted. Portfolios for 12 years with yearly revision are constructed as per Markowitz's model and the mean-variance efficient ideal portfolio model. The risk-return trade-offs of these portfolios are identified. No surpluses in returns (than the values targeted) were generated in any of the portfolios in any of the years. The variance achieved in Markowitz's portfolio was lesser than the ideal portfolio for all these years.

Further investigations into the proximity of risk-return trade-off and portfolio utility in the Markowitz portfolio, the ideal portfolio and the index portfolio were attempted through the tests of equality. The tests for equality of returns, variances and portfolio utilities of these portfolios were conducted. Five null hypotheses of equality of (1) returns of ideal portfolio and index portfolio; (2) variance of ideal portfolio and Markowitz's portfolio; (3) variance of ideal portfolio and index portfolio; (4) utility of ideal portfolio and Markowitz's portfolio and (5) utility of ideal portfolio and index portfolio were tested. The same level of returns were targeted and achieved for Markowitz's portfolio and the ideal portfolio, hence no testing was done for this.

By analysing the t-test and Anova F-statistic values and probability figures, null hypothesis of equality of returns from ideal portfolio and index portfolio could not be rejected. Also, tests statistics could not reject the hypothesis of equality of variance of the ideal portfolio and Markowitz's portfolio. The variance of the index portfolio could not be considered equal to that of the ideal portfolio. The tests for equality of portfolio utility found that the utility of the ideal portfolio and the Markowitz portfolio on one hand and the ideal portfolio and the index portfolio on the other could be considered equal. The ideal portfolio created by identifying important portfolio variables not only meets the Markowitz mean-variance efficient criterion but also taps the dynamic changes in the stock market represented by multiple constraints. It tries to maximise the utility of an investor by meeting his/her objectives in a multiple constraint setting.

7.1.8 Out of the sample tests

To test for the robustness of optimisation model, forward tests have been conducted for the period of two year pre-crisis period beginning from 1/1/2006 to 31/12/2007. The forward tests for 2006 confirms our analysis as the rankings for portfolios match with predictions. Risk moderate and risk averse portfolios perform well. The performace ratios for the indifference threshold portfolio are extremely small. The majority of the securities generated negative or very small returns during 2007. The risk averse portfolio is the best performer with the indifference threshold portfolio being quite close. The risk seeker portfolio performs the worst. According to Treynor ratio, the risk moderate portfolio performs the best. Due to the high degree of systematic risk present in the market, the risk moderate portfolio and the indifference threshold portfolio have almost equal (negative) performance ratio. Perhaps with changing market conditions, investors must adapt their preferences to obtain optimal results.

7.2 Conclusions

The proposed multi-criteria portfolio selection model does not dismiss mean-variance or any other portfolio selection models rather it embeds them. The model formulations developed herein complement the existing literature by adding to it. They are not a substitute to any of the existing portfolio selection model. The solutions of proposed models are efficient not only in the mean-variance sense but also for the list of constrained variables identified. The formulated portfolios achieve the minimum risk for a targeted level of returns both capital as well as dividend yield, beta, volume, price-to-earnings ratio, operating profit margin, net profit margin, free float, market capitalisation, free cash flows and so on. The Pareto optimal model formed here makes a positive discrimination between mean-variance efficient solutions and mean-variance efficient solutions with efficiency for other constrained variables. The personal preferences of an investor relating to a variety of fundamental and market characteristics play a crucial role in the selection of securities and portfolio allocation decisions.

The proposed model is tested on real data drawn for the Nifty securities for a period of 12 financial years starting from 2000 to 2012. Eight portfolios formulations namely the diversifier's portfolio, the satisficer's portfolio, the plunger's portfolio, the market trend portfolio, the capital gain bias portfolio, the dividend gain bias portfolio, the equal priority portfolio and the ideal portfolio were created for investors with different priorities and risk appetites. The objective of risk (variance) minimisation is achieved by optimising across other important portfolio variables such as earnings per share, dividend, free float, impact cost, institutional holding, market capitalisation, net profit, price-to-book value ratio, price-to-earnings ratio, promoter's shareholding, sales, turnover, beta, unsystematic risk and volume simultaneously. All of the portfolios created were compared with the Markowitz efficient frontier in the risk-return space. The ideal portfolio was found to be the closest to the Markowitz portfolio.

Two main multiple regression equations have been estimated with returns and excess returns to standard deviation as the dependant variables. Regression models explained the relevance of a new variable namely impact cost having significant explanatory powers for predicting security return and the Sharpe ratio. The Granger causality test was undertaken to discover the relationship of causation between returns on a security and the variables set as constraints in the programming problem. The null hypothesis that dividend, impact cost, net profit, promoter's holding, sales and volume do not cause returns could not be rejected.

The portfolio utility analysis was undertaken to empirically find the utility derived by an investor from alternate portfolios for changing levels of risk tolerance. A direct relationship between the degree of risk tolerance and the value of portfolio utility was found from the quantitative analysis. The portfolio selection model formulations were plotted in the risk-return space along with the utility curves to find the optimal portfolio choice for different types of investors. The evaluation of the alternate portfolio selection model formulations is attempted by using the Sharpe ratio (1966) and the Treynor ratio (1965). The Sharpe ratio is the highest for the Markowitz portfolio followed by the ideal portfolio. The ideal portfolio performed the best, even better than the Markowitz portfolio when evaluated according to Treynor's ratio. Tests of equality of mean, variance and portfolio utility for the ideal portfolio, the Markowitz portfolio and the index portfolio were conducted to investigate the proximity of these portfolios.

After an analysis of the performance evaluation ratios of the portfolios constructed on FTSE 100, it may be concluded that the risk moderate portfolio targeting the mean levels of ten identified security evaluation criteria does fairly well. It is a Pareto optimal portfolio attaining the targeted levels of aspirations without compromising much on the mean-variance efficiency parameters. This portfolio stands extremely close to the Markowitz portfolio in the risk-return space. The risk averse portfolio also performs quite well and even better than the risk moderate portfolio according to market beta based Treynor ratio. It performs the best even during the risky period of 2007 immediately before the crisis. Thus, by specifying preferences through an algorithm, investors may end up selecting a portfolio which is mean-variance efficient and also meets their aspirations. Risk moderate and risk averse portfolios would be most recommended under normal market conditions.

The mean-variance model formulated and applied in this research work will be of immense use for the investors both individual and institutional, brokerage houses, mutual fund managers, banks, high net worth individuals, portfolio management service providers, financial advisers, regulators, stock exchanges and research scholars in the area of portfolio selection.

7.3 Suggestions for future research

Research is a never ending, ongoing and continuous process. This research work may be carried forward in a variety of ways. The multi-criteria portfolio selection model formulated in this study can be extended to include all the companies listed on the London Stock Exchange, National Stock Exchange or other stock exchanges of the world. The same formulations can be tested using daily/minute-by-minute real time data. Also, the inclusion of additional variables such as transaction costs, return on assets, risk penalty, portfolio utility and conditional volatility can be attempted in future. A strong lead-lag relationship of the derivatives market and the stock markets has been observed by researchers in Indian as well as international capital markets. Inclusion of the lead-lag effect of the derivative markets on the cash segment would certainly improve the predictability of equity portfolio selection models. Hence, attempts to include variables from the derivative segment such as open interest, price of futures, price of call/put options, volume and turnover of derivatives of securities may be attempted in future.

Annex 1
Programming for the Multi-Criteria Portfolio Selection Model

MODEL:

SETS:

ASSET/1..93/:Return, Dividend, Beta, Volume, PE Ratio, Operating Profit Margin, Net Profit Margin, Market Capitalisation, Free Float, Free Cash Flows, UB, X;
COVMAT (ASSET, ASSET): V;

ENDSETS

DATA:

```
Return =  0.015216419   0.010639282   0.031536588  −0.003437048  −0.001733693   0.003981138
          0.030445072  −0.00866611   −0.00046       0.000665213   0.01877957    0.008937002
          0.004342333   0.018346047   0.014740185   0.014929555   0.025026666   0.007563315
          0.002185928   0.020851218  −0.009874822  −0.009706045   0.024195942   0.002433834
         −0.012168724   0.001409393   0.011882142   0.00836325    0.005243532   0.014815201
          0.010329913   0.005142648  −0.00017454   −0.002098769   0.014312836  −0.000523803
          0.024977917   0.011968234   0.022020905   0.013134011  −0.001896748   0.008099441
         −0.003184213   0.011418864  −0.000274691  −0.002917929   0.014764617   0.014997215
          0.016948413   0.008833961   0.004144223  −0.000287523   0.01331089    0.002487519
          0.082843515  −0.003879133   0.030197856  −0.009461172   0.014701127   0.003881748
         −0.00081681    0.000700575  −0.00207138    0.013804414   0.016998583   0.022371896
          0.050772286  −0.008506453   0.015605263  −0.000325631  −0.000584795   9.53092E-05
         −0.003810666   0.008291271   0.011694545  −0.004255897   0.010139881   0.005623879
          0.008702021   0.016272383   0.006998249   0.014276156   0.00688453    0.003507798
          0.006132866   0.04207469   −0.006893067   0.0101896     0.002720067   0.019412488
          0.019625175  −0.000154222   0.01052751 ;

Dividend = 3.8791  1.5612  4.8582  3.2898  1.335   3.4402  1.6874  0.4956  5.1941
           5.2865  2.1935  4.5571  2.0991  3.8455  1.4115  4.3636  3.7966  2.0494
           4.7931  1.9333  3.2536  2.8443  2.9736  4.2943  2.5     2.2683  2.308
           4.0894  2.2539  1.8664  1.5926  4.185   2.1563  4.3681  2.9036  4.3435
           2.1434  2.4038  4.9659  1.3438  1.6818  2.0675  2.452   2.5397  4.0713
           0       1.9932  2.1592  3.5895  2.8221  4.1404  5.4106  2.1898  3.5176
           3.1805  2.5741  0       3.6145  2.8811  0       4.1438  4.8238  2.866
           3.2262  3.6628  0       0.7046  5.7109  2.0658  4.3971  1.7388  3.0102
           0.4967  2.8141  2.1079  1.6251  5.2298  3.6855  4.3293  3.0483  1.0889
           1.5309  4.0745  2.975   4.8338  3.3859  5.299   1.7375  1.242   2.2359
           1.9801  2.4409  1.6953;
```

Beta =	1.942838293	0.556217382	0.687407722	1.266331952	1.267267563
	1.128184514	1.799228496	0.648754154	1.413203721	0.396904966
	0.717671965	0.793732387	2.50582032	0.449393225	0.863254487
	1.006707687	1.530684292	0.508818134	1.112438625	1.327698769
	0.519957466	1.02636984	1.032394044	0.428519245	0.676856291
	0.519457723	0.959929363	0.90384379	0.524042435	0.795473624
	0.673446372	0.441952741	2.217442214	0.47628603	1.257593338
	1.03192135	1.239349356	1.703972553	0.40760023	0.756335293
	1.72631691	1.518357629	0.817088858	1.26662286	1.764974284
	2.135661572	1.167759195	1.540824002	0.703351235	1.745549413
	0.307228436	0.306361583	0.520440627	1.640881491	1.029224237
	1.747622182	1.110090465	0.587944161	0.315042108	2.975442892
	0.837094249	0.830122327	0.562873357	0.828752909	1.720207405
	0.94134459	0.363300923	0.433417073	0.940456094	0.611808139
	1.32587768	0.839439193	0.706255138	0.946542788	0.487721387
	0.599335659	0.387900412	1.528938237	0.21368831	0.810967778
	1.012838609	2.435369846	0.58352826	0.504756148	0.317764984
	2.70016396	0.549269718	1.889842811	1.547773065	0.761722503
	1.68440054	1.324969613	0.945854534;		

Volume =	117130535.7	37340634	27016892.63	46203622.17	26065677.49
	42729101.13	51281918.83	368370441.4	245607995.9	142108570.1
	14337748.12	517504123.1	656219355.7	201883485.1	329505901.1
	81140167.05	335295694	49087773.78	1414854286	49509034.83
	273000913.9	1116634334	33622632.96	533868560.7	292951109.1
	109241020.9	9879139.85	29631651.17	358839283.6	234745089.8
	71492388.31	113760503.2	201194820.1	413824055.4	34365049.81
	837617154.3	87071351.83	36570441.92	93526094.17	20637664.02
	628428446.5	22784617.17	355529062.9	56170053.92	766773936.2
	1469846984	32789241.26	39553137.32	359603691.1	9088526.053
	43092808.2	315836594.8	59967348.73	259634875.4	79007015
	307339764.7	37681707.5	119234058.9	76736003.12	91626890.23
	306699895.7	269278818.8	159799555.7	65803651.78	163703186
	385734105.3	797833.5355	509234297.4	95978102.42	265402644.8
	19914887.08	180350459.9	84434373.72	73685429.24	141034469.5
	49718345.6	101619419.3	177133272.6	47011734.07	68264737.9
	52757732.92	11836686.45	785978967.8	140571300.8	74452562.83
	43010080.88	7003466696	19036821.97	30103938.06	139445678.6
	15629311.23	212996166.3	37980956.57;		

PE Ratio = 0	16.15428571	11.826	352.3109088	357.6556134	35.96428571	
	6.518266667	64.23244898	289.8045161	32.095	8.491632653	
	18.20333333	11.39782729	20.61061224	18.13632653	25.59219084	0
	365.9489656	25.79190476	511.0278125	45.32758621	11.37326143	0
	15.21685057	39.35897959	44.69073727	281.4653061	1093.611054	
	87.43183673	13.44125139	23.52285048	20.77607143	323.9546212	
	24.19469388	17.89221492	18.2212911	141.4712935	20.33612245	
	18.53312924	432.1738636	428.7207982	19.63131104	20.95679502	
	17.08919878	18.63532533	5.676267796	20.55150434	23.45165529	
	27.0259399	0 19.5377551	14.94451686	15.51081633	13.40430804	0
	28.12358504	7.514181041	61.49540541	29.47510204	56.21958461	
	11.91833333	43.24355967	90.51792082	78.64239908	34.42307937	
	25.05609355	6.500322581	80.13112367	0	18.32315386	
	35.4544898	23.08	54.83615385	23.31211149	31.3055102	
	28.23489796	13.87163265	13.68971342	19.97815565	12.31693878	

32.11734694	11.7389521	18.06408163	62.83718204	10.77267231	0
120.1437506	14.69244898	11.63292012	0	58.42510555	
34.40122449	15.22501524;				

Operating Profit Margin =

16.948322	8.192830	−4.2137255	15.818112	2.359058	41.97727
21.32033	0	23.291348	3.7532305	3.73519	0
22.683024	35.996632	99.1487305	25.164963	6.831882	10.690616
18.8726875	3.819237	8.8874035	0	8.072692	4.3669025
10.263252	12.934166	9.190238	18.73059	8.287018	7.017451
2.661552	3.290892	27.721796	152.689282	0	11.72763333
8.209886	8.9407125	15.44969	5.353878	3.912491	5.610999833
64.553324	0	0	31.341528	15.373642	8.4327125
3.00111	3.760649	24.4413775	14.58297333	0	7.436118
0	19.366198	7.009372	17.94159	0	11.764652
11.764652	12.74539	7.793398	21.453308	6.08081	34.138654
0	14.95674	2.452804	10.91174	26.4395995	−18.123034
7.704331833	14.793392	4.41509	13.849072	0	20.3490565
6.066614167	30.005668	10.42481	5.686412	11.437178	29.714445
16.937643	−21.1885235	7.209966	6.45964	7.986764	5.701888667
10.505416	12.34053133	;			

Net Profit Margin =

13.003416	6.493721	35.505562	−8.2353215	9.219644	0.612364
19.43954	17.276994	2.268622	17.318516	1.418673	−0.292902
20.443746	13.244654	22.418384	68.161048	15.814292	4.182172
6.609778	12.6867065	−5.042639	8.042066	0	4.262444
1.6662935	6.180758	7.389284	6.253894	13.531221	5.139921
5.98859	0.026932	0.59415	18.913624	104.806598	21.716796
8.9156	4.705148	4.8502125	6.898398	3.116898	2.625127
1.9393705	41.3214345	1.056458	22.157758	24.1315775	7.891104
4.893692	−1.44498	2.2955805	12.66115083	10.08002917	2.696244
6.713058	1.630522	12.187098	1.60021	13.110124	21.27778
6.610398	6.610398	6.18366	2.240532	18.0279	2.873744
34.02962	−1.651152	7.642842	1.157742	11.888878	17.525637
−24.754418	3.874042	11.041644	2.68725	9.1236945	18.900236
7.8847965	3.273313333	14.251398	6.386034	3.722728	3.90912
15.286996	5.678509	−34.1516425	5.296214	2.59903	3.864628
3.7006555	5.438398	5.146918667	;		

Market Capitalisation =

18846917798	14831807660	3753143093	4758594141	4801503066
3144504061	8203326905	12646426532	10742145827	41011485781
4326196030	13786630686	39703016430	66748534527	40062397032
5983288412	1.06017E+11	4561073778	87912061757	6619348266
13278790317	26233106527	18605767574	19608113601	15984054513
6790653627	3488302902	9655247342	49945282715	12087079972
5333547938	2993563526	5446651910	84001203996	3757954625
1.35836E+11	5181364487	4261030459	21260551624	4904872258
6057864459	5629970195	9122588821	7376148137	11076536451
49758545165	3998351640	4299221825	7625545131	3399185459
6599185701	27612870960	7527656754	9709925042	4391630041
28921470756	3736553703	10142578589	33265996970	19615697495

	1.43461E+11	1.43461E+11	18124178517	3853730369	56023711302
	22147304421	4136998114	4663461845	51697893114	7189775207
	6687099711	3994919232	12356451810	5402710955	7078227615
	3091765011	15472111150	36395622891	4170680726	3992828708
	9975154777	3988603312	29167061473	81654657715	4824601147
	2979450302	92700337145	4644746892	3365513374	4009412878
	8810393489	15515760289	5643840829;		

Free Float =	91.45466597	42.70980869	71.64939223	76.11637278	88.07159752
	98.75598003	33.99705309	99.24627371	98.79697117	99.65331713
	97.90802504	99.3954404	98.87937889	94.43818998	99.67668516
	98.94959032	99.79154825	97.79564883	99.05054503	98.82479084
	59.59605017	98.64459143	73.20305439	98.74386656	99.13540249
	98.81905355	97.92394691	99.87293098	99.39976064	99.0366168
	57.4722627	98.66622123	99.42064594	99.54866534	99.472777
	99.83209617	97.15241591	98.16989481	99.08717944	98.89804786
	91.76384842	96.61757362	98.71287293	95.45223258	97.10523779
	60.82191547	77.55510107	99.45619213	98.69273252	96.53601709
	81.44690412	95.62456939	92.6836039	96.7268241	73.48319719
	99.55397345	95.76137693	97.97265833	88.31450939	34.13426771
	98.80704664	98.80704664	99.07773332	98.76073463	89.98513675
	99.20800777	98.97942645	95.55973592	58.25096989	86.74661331
	55.21736008	95.67269958	98.52446109	99.35849342	98.58406107
	96.98620015	99.14858258	99.48751997	99.35571822	98.36201766
	93.20633469	92.87900047	99.67777853	96.74415009	99.48748374
	34.77843789	99.21904099	99.02994872	92.73242444	100.0664188
	99.27054085	98.97435137	96.65238913;		

Free Cash Flows =	–1015000000	327000000	–9800000	221800000	–125300000
	110000000	1501700000	79500000	1299000000	–1336000000
	74200000	1151000000	–15747000000	1085000000	–2838000000
	–685000000	–2550000000	156600000	–7975000000	135500000
	633000000	2131000000	–112000000	–362000000	276000000
	172300000	9100000	0	455000000	601000000
	–131000000	7000000	7000000	–959000000	–546700000
	–17407000000	–335000000	66700000	812000000	60600000
	249000000	–166500000	64000000	–539300000	2346000000
	46000000	218700000	196300000	39200000	–77500000
	–146000000	–449000000	429700000	631000000	–1171000000
	–348000000	159200000	279000000	795000000	–46898000000
	6174000000	6174000000	–267000000	51000000	–11128000000
	570000000	–105304000	–166000000	1105000000	–420000000
	372300000	108200000	1103500000	96200000	451000000
	158900000	–158500000	16406000000	–108500000	0
	–520200000	102200000	–1353000000	1757000000	–254700000
	970400000	–2814000000	90900000	10600000	156900000
	323000000	271600000	–14400000;		

! Upper bound on investment in each;

```
UB= 0.15  0.150 .15   0.15  0.15  0.15  0.15  0.15  0.15  0.15   0.15  0.15  0.15   0.15  0.15
    0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15   0.15  0.15  0.15   0.15  0.15
    0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15   0.15  0.15  0.15   0.15  0.15
    0.15  0.15  0.15  0.15  0.15  0.15  0.150 .15   0.15  0.15   0.15  0.15  0.15   0.15  0.15
    0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15   0.15  0.15  0.15   0.15  0.15
    0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15  0.15   0.15  0.15  0.15   0.15  0.15
    0.15  0.15  0.15;

! Covariance matrix;
V = A 93*93 matrix derived using E-views 5. This cannot be presented here due to space limitations.

! Desired growth rates of portfolio;
Portolio_Return = –0.00015;
Portfolio_Dividend = 4.0713;
Portfoio_Beta = 0.5628;
Portfolio_Volume = 43010080.88;
Portfoio_PERatio = 13.68;
Portfolio_OperatingProfitMargin =16.93;
Portfolio_NetProfitMargin = 13.11;
Portfoio_MarketCapitalisation = 19615697495;
Portfolio_FreeFloat = 91.76;
Portfoio_FreeCashFlows = 324000000;

ENDDATA
! The model;
! Min the variance;
[VAR] MIN = @SUM( COVMAT( I, J):
V( I, J) * X( I) * X( J));
! Must be fully invested;
[FULL] @SUM( ASSET: X) = 1;
! Upper bounds on each;
@FOR( ASSET: @BND( 0, X, UB));
! Desired value or return after 1 period;
[RET] @SUM( ASSET: Return * X) >= Portolio_Return;
[DIV] @SUM (ASSET: Dividend*X) >= Portfolio_Dividend;
[PORTBETA] @SUM (ASSET: Beta*X) >= Portfoio_Beta;
[PORTVOLUME] @SUM (ASSET: Volume*X) >= Portfolio_Volume;
[PORTPERATIO] @SUM (ASSET: PERatio*X) <= Portfoio_PERatio;
[PORT_OPM] @SUM (ASSET: OperatingProfitMargin *X) >= Portfolio_OperatingProfitMargin;
[PORT_NPM] @SUM (ASSET:NetProfitMargin *X) >= Portfolio_NetProfitMargin;
[PORT_Mcap] @SUM (ASSET: MarketCapitalisation*X) >= Portfoio_MarketCapitalisation;
[PORT_FF] @SUM (ASSET: FreeFloat *X) >= Portfolio_FreeFloat;
[PORT_FCF] @SUM (ASSET:FreeCashFlows*X ) >=Portfoio_FreeCashFlows;

END
```

Annex 2
Programming for Markowitz's Portfolio Selection Model

```
MODEL:
SETS:
ASSET: RATE, X;
COVMAT(ASSET, ASSET): V;
ENDSETS
DATA:
! The stocks;
ASSET = AAL   ABF   ADML  ADN   AGGK  AMEC  ANTO  ARM   AV    AZN   BAB
        BAES  BARC  BATS  BG    BLND  BLT   BNZL  BP    BRBY  BSY   BT
        CCL   CNA   CPG   CPI   CRDA  CRH   DGE   EXPN  EZJ   GFS   GKN
        GSK   HMSO  HSBA  IHG   IMI   IMT   ITRK  ITV   JMAT  KGF   LAND
        LGEN  LLOY  LSE   MGGT  MKS   MRON  MRW   NG    NXT   OML   PFC
        PRU   PSN   PSON  RB    RBS   RDSa  RDSb  REL   REX   RIO   RR
        RRS   RSA   SAB   SBRY  SDR   SGE   SHP   SMIN  SN    SRP   SSE
        STAN  SVT   TATE  TLW   TPK   TSCO  ULVR  UU    VED   VOD   WEIR
        WG    WMH   WOS   WPP   WTB;

Rate =  0.015216419  0.010639282  0.031536588 −0.003437048 −0.001733693  0.003981138
        0.030445072 −0.00866611  −0.00046      0.000665213  0.01877957   0.008937002
        0.004342333  0.018346047  0.014740185  0.014929555  0.025026666  0.007563315
        0.002185928  0.020851218 −0.009874822 −0.009706045  0.024195942  0.002433834
       −0.012168724  0.001409393  0.011882142  0.00836325   0.005243532  0.014815201
        0.010329913  0.005142648 −0.00017454   0.002098769  0.014312836 −0.000523803
        0.024977917  0.011968234  0.022020905  0.013134011 −.001896748   0.008099441
       −0.003184213  0.011418864 −0.000274691 −0.002917929  0.014764617  0.014997215
        0.016948413  0.008833961  0.004144223 −0.000287523  0.01331089   0.002487519
        0.082843515 −0.003879133  0.030197856 −0.009461172  0.014701127  0.003881748
       −0.00081681   0.000700575  0.00207138   0.013804414  0.016998583  0.022371896
        0.050772286 −0.008506453  0.015605263 −0.000325631 −0.000584795  9.53092E−05
       −0.003810666  0.008291271  0.011694545 −0.004255897  0.010139881  0.005623879
        0.008702021  0.016272383  0.006998249  0.014276156  0.00688453   0.003507798
        0.006132866  0.04207469   0.006893067  0.0101896    0.002720067  0.019412488
        0.019625175 −0.000154222  0.01052751;

! Upper bound on investment in each;
UB = 0.15;

! Covariance matrix;
V = A 93*93 matrix derived using E-views 5
! Desired growth rate of portfolio;
RETURN = 0.0143;
```

```
ENDDATA
MIN = @SUM( COVMAT( I, J): V( I, J) * X( I) * X( J));
RETURN = @SUM( ASSET: RATE * X);
! Must be fully invested;
@SUM( ASSET: X) = 1;
! Upper bounds on each;
@FOR( ASSET: @BND( 0, X, UB));
! Desired value or return after 1 period;
[RET] @SUM( ASSET: RATE * X) >= RETURN
END
```

Notes

1 Introduction

1. Efficiency defined in terms of either lower variance at the same level of mean return or higher mean return at the same level of variance.
2. Rajiv Gandhi Equity Saving Scheme to allow for income tax deduction of 50 per cent to new retail investors who invest up to Rs 50,000 directly in equities and whose annual income is below Rs 10 lakh to be introduced. The scheme will have a lock-in period of 3 years.
3. Smart Order Routing (SOR) is used by stock brokers/clients to determine the best venue to execute an order, i.e., where to execute the order based on client specified criteria such as best price, liquidity, etc. SEBI has permitted SOR in the Indian securities market for all classes of investors. SOR has been envisaged as a mechanism to provide "best execution" to the investors. It allows the brokers trading engines to systematically choose the execution destination based on factors viz. price, costs, speed, likelihood of execution and settlement, size, nature, etc.
4. Roll, Richard and Stephen A. Ross, "An Empirical Investigation or the Arbitrage Pricing Theory", *The Journal of Finance*, Vol. 35, No. 5, pp. 1073–1103.
5. Mr. Mark Wiley (LINDO Systems, Inc, USA) provided research licence for access to the evaluation copy of LINGO 13.0 used for quadratic programming portfolio selection optimisation in Chapter 5 of this book.

2 Advances in Theories and Empirical Studies on Portfolio Management

1. Return formula $E = \sum_{i=1}^{n} X_i \mu_i$ and Risk Formula $V = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j$
2. The research paper titled, "Recent Advancements in Theories and Empirical Studies on Portfolio Selection" was presented at the 13th West Lake International Conference on Small and Medium Business (WLICSMB 2011) organised by Economic Commission, Hangzhou Municipal Government, Zhejiang Provincial Institute of Small and Mid-sized Business and College of Business Administration, Zhejiang University of Technology, 15–17 October 2011, Hangzhou, China. The comments and suggestions of the participants have been incorporated.
3. Talpaz et al.'s (1983).
4. $R_p - R_f = \alpha_i + \beta\,(R_m - R_f) + s_i\,SMB + h_i\,HML + e_{i,t}$
5. Fama and French (1993) factors and momentum factor (Carhart, 1997).
6. La Porta et al. (1998), and Bebchuk et al. (2000).
7. Defined as the difference between the price per share paid for a control block and the price quoted in the market after the sale announcement (Dyck and Zingales, 2004).

8. Gompers et al. (2003).
9. The tendency of investors to hold losing investments too long and selling winning ones too soon (Shefrin and Statsman, 1985).
10. Research paper titled "Impact of Index Futures on the Index Spot Market: The Indian Evidence" was presented at the 2nd IIMA International conference on Advanced Data Analysis, Business Analytics and Intelligence held on 8–9 January 2011 organised by the Indian Institute of Management, Ahmedabad. The comments and suggestions by anonymous referees and participants have been incorporated.
11. Research paper titled "Advances in Theories and Empirical Studies on Portfolio Management" was presented at the 64th All India Commerce Conference organised by the Indian Commerce Association, 13–15 December 2011, Department of Commerce, School of Management, Pondicherry University, Pondicherry, India. The comments and suggestions of the participants have been incorporated.

3 Contributions to the Portfolio Theory

1. Markowitz (1952).
2. The analysis was deliberately limited to choice among monetary assets.
3. Under the Institutional Placement Programme (IPP), a company can either buy fresh issue of shares or by dilution of promoter's shareholding increase their public shareholding. Bidding through applications supported by blocked amount (ASBA) is mandatory and offers can be made only to Qualified Institutional Buyers (QIBs) with 25 per cent reserved for mutual funds. Also, a minimum of 10 allottees should be there with no allottee getting more than 25 per cent of the offer size.
4. An Offer for Sale (OFS) is a special window of stock exchanges made available to promoters and promoter groups for selling their stockholding. OFS facility is only available to either the non-compliant companies or the top 100 companies as per market capitalisation. A separate trading window is provided under OFS. The bids are supposed to be supported by 100 per cent cash margin in case of non-institutional investors and 25 per cent in the case of the institutional investors. The appointment of one or more BSE/NSE registered brokers is mandatory to carry out the OFS. However, the appointment of a merchant banker is optional. The only limitation of OFS is that the issue size should be Rs 25 crores or less only if undertaken to meet the minimum public shareholding requirement.
5. Public financial institutions as specified in section 4A of the Companies Act, scheduled commercial banks, mutual funds, Venture Capital Funds (VCFs), Foreign Venture Capital Investors (FVCIs) and Authorised Investment Funds (AIFs) registered with SEBI, Foreign Institutional Investor (FIIs) and sub-account registered with SEBI (other than a sub-account which is a foreign corporate or foreign individual), multilateral and bilateral development financial institutions, state industrial development corporations, insurance companies registered with Insurance Regulatory Development Authority (IRDA), provident funds with minimum corpus of Rs 25 crores, pension funds with minimum corpus of Rs 25 crores and a national investment fund

set up by the Government of India, insurance funds set up and managed by the army, navy or air force of the Union of India and insurance funds set up and managed by the Department of Posts, India.
6. Resident Indian individuals, eligible non-resident Indians (NRIs), Hindu Undivided Family (HUFs in the name of Karta), companies, corporate bodies, scientific institutions societies and trusts, sub-accounts of Foreign Institutional Investors (FIIs) registered with SEBI which are foreign corporate or foreign individuals and Eligible Qualified Investors (QFIs).
7. Resident Indian individuals, eligible non-resident Indians (NRIs) and Hindu Undivided Family (HUFs in the name of Karta).

4 Mean-Variance Efficient Portfolio Selection: Model Development

1. The efficient frontier is a smooth continuous curve although made up of several parabolic pieces.
2. See Markowitz (1987), Mean-Variance Analysis in Portfolio Choice and Capital Markets, Basil Blackwell.
3. The Granger causality test is a bivariate test of causality, whereas our estimation is based on multiple regression.

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1. Only 13 industrial diversification constraints exist in the programming whereas there are 23 industries in the problem because ten industries have only one company representation in Nifty 50. The company diversification constraint of 15 per cent makes the industrial diversification constraint of 20 per cent redundant for these ten industries. Hence, it has been excluded.
2. Further tests for equality of the risk and return of the Markowitz portfolio and the ideal portfolio constructed on a yearly basis has been undertaken.
3. A similar regression model was estimated including beta as an independent variable. Beta explained 46 per cent of the variance in returns and coefficient for all other variables were extremely small. To find new and relevant explanatory variables apart from beta, it was excluded from the regression analysis.
4. Efficiency Index is calculated by dividing the actual values with the fitted values and multiplying it by 100. $E.I = \frac{Actual\ Values}{Fitted\ Values} * 100$

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