Linear Operators and Linear Systems

An Analytical Approach to Control Theory

Jonathan R. Partington

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Linear Operators and Linear Systems
An Analytical Approach to Control Theory

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University of Leeds

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$$|| \; 1$$

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$$\frac{1}{} \quad ^2 \quad \text{og } f( )$$

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L tt g $r \to 1$ th t og(1) h h b th o o
t t to b q t to (1)
o d t o f o t t hk od t ho
o th o o f th f h o o t d b
d th

e e f H^1 B as

$$(\ ) \qquad \frac{\ }{1 \ ^{-}} \qquad | \ |=$$

( ) a z s f a a s y
a s s a H y z s a ( )
s ( ) ≤ 1 a ( ) 1 a s y

: W t o th to o o d g to th t od t
o o ot th t

$$1 \quad ( ) \quad \frac{(1 \quad) \; -}{1 \quad -} \leq (1 \quad) \frac{1}{1}$$

h h o b d th th hk o d t o (1 5) g t o o o
g o th t od t to t o th th q d o

t o to th t o C ( ) ≤ 1 o
th tt g (1) h h o hk od t
h

$$( ) \leq \frac{1}{} \quad ^{2} \left| \quad ( ) \right| \quad \frac{1}{} \quad ^{2} \left| \quad ( ) \right|$$

d tt g → th t

$$\frac{1}{} \quad ^{2} \left| \quad ( ) \right| \quad 1$$

o th t ( ) 1 o t h

o g th hk to t o tho t o d do ot
h g th H^p o th bo d o t o od
o t h W th obt th o o g to t o t

C **ie z** *f* H^p *f* *a* ( ) *ss y*
) B as *s* *z* *s* ( ) *f* *f*( ) ( ) ( )
s H^p $f_{\ p}$ $_{p}$ *s a a y z* *f* H^1
a *as f* S *s a B as* S *s a s* *a*
z s) a *s a* *s a* *za*
s *s a s* *s*

Th xt t x h t o tho t o d
g t o o (1) b o to (1) x t th t th
t g o t k th t to g th t o o t d
o t o L b g th th ( )

e e *a* *z s* *s a*
s *as* μ *s* *a* *s* *s* *as* *a* *a*
s a *s s* *a*

$$( ) \quad \text{x} \left(\quad ^{2} \quad \text{——} \quad \mu( ) \right) \quad (\quad) \quad (1 \quad)$$

e : Choo b h o og h o o th t og ≥ W o th th o o g ot h h t th t o g t h o t o th o o t g o o t μ o d d d t t μ( ) th k t o t $r \to 1$ o q o $\frac{1}{2}$ h ( ) (r) t o o th t ^{h} h

$$( ) \quad ^{2} \text{———} \quad \mu( ) \quad (\quad)$$

d o t t Th t th t μ g o o o th t th t d th th d t o h h (to o t t) q $\underline{\mu}$ o o t h

o t to t o ho d o th d o th ght h Th hk o d t o o th o ( ) o $H^{p}(_{+})$ t o o th o o g

$$\sum \frac{}{1 \quad ^{2}}$$

d th o o d g hk od t h th o

$$( ) \quad \left(\frac{1}{1}\right) \quad _{=1} \frac{1 \quad ^{2}}{1 \quad ^{2}} \frac{}{—}$$

o o t o g t d b o t d o th bo d b t th th h t t ” h h t t xt to Th g o o g t o th

$$( ) \quad \text{x} \left(\quad \text{———} \quad \mu( ) \right)$$

h μ o o o x ^{1} t o o o d g to o t t th o g

t t o $H^{p}(_{+})$ h th o

$$f( ) \quad \text{x} \left(\frac{1}{—} \quad \text{———} \quad ( ) \frac{}{1 \quad ^{2}} \right) \qquad (1 \quad)$$

h 1 th t o d d

$$( ) \frac{}{1 \quad ^{2}} \quad \text{o} \quad \text{g}$$

o h t o f h og $f(\quad)$ ( ) o t h

.4 to - al d a dy a

t o b to d d o t o t k g b t
h b t th t od t o t o h o to ook t
t g t o o th o t d o h th
o h d t d h t t o to th o t
o t o th th o t t t g th t b o k
o od g t th t b o t d o t t

t o th d o o x to
h th o ( ) f_1( ) $_1$ f_2( ) $_2$ f ( ) h f_1 f o d
o x d t o d $_1$ b o W th th t
t d o h o f_1 f t Th d t o
to b d d t o th ho o b
t th t o to d d th t t t
h o th t o d t o th h h o d
th th t d d E d o

e i i 4 s a H^2( ) s s s a a a y s $\rightarrow$
s a y

$$\left(\frac{1}{{}_1} \quad {}^2 \quad f(r \quad)^2 \right)^{1\ 2}$$

s

t th t th b d d o t h o to ^{2}( )
h th d d (f_1 f) th

^{2} f_1 ^{2} f ^{2}

Th oth o t t x t x d o o H th t
d o t t g th d t ( ) th th to
o $\times$ t To do th o t th o to th th
t x ( ) b th o $_{=1}$ h $_1$ d $_1$
th o tho o b o d t Th o to o
o g b

x $_1$

h $_1$ th g o S t o 1 1 th t $^2_1$ ^{2}
th g o th o g t djo t t x So t
t $_1$( ) o to o d o t o o o bo th o o g
d t o

e i i 4 *s a H* (( )) *s s s a a a y*
s → ( ) *y*

$_1$(( ))

g b d *H* (( )) d o t to th
(( ))

S to d d o th ght h b d d
h $H^2(\ _+\ )$ d *H* ($_+$ ( )) W th d to t do
th ogo d t o h q d

e

Th t th o o b t b o d 9 11 11 1 9
1 d oth

Th b d th th o o d o t d d d b
o d o o 39 5 9 1 11 o x

Th o t to t o ho d b o d th t d d o o
t o o g h o x bo d d o to h
a s h ( )$^{1\ 2}$ o t o to
d *a a s y* (o to th t o t o (k) o to
(k) d o o k)

e i e

1 o th t o to th t o t o t th th
b h th t $x \leq 1$ h $x \leq$ d d d th t
bo d d th ≤ 1

Sho th t S d t o $(\mathcal{X})$ th $S \leq S$

3 Th *a a* $^2(\ 1) \to\ ^2(\ 1)$ d d b

(f)(x) $f(\ )$ (f $^2(\ 1)$)

U th C h S h q t to ho th t (f)(x) ≤ √x f 2
D d th t ≤ 1 √

L t R ℓ² → ℓ² d ot th ght h t th t

R(1 2) (1 2)

o th t R 1 o th t (R) ⊆ th t p(R) th t R
h o g Sho ho th t (R)

5 L t ( ) = b x d bo d d q o o x b d d
o to o ℓ²(+) b ((x)) (( )) h x o h
th t bo d d o to d ( ) L t 1 2
o th t h g o d h ( ) d th t
th th o x t d bo d d D d th t
( )

d th djo t o th k o o to d d b x x ⟩
(x) h d x d t o b t

L t ( ) b t o to Sho th t ( ) o tho o
b o h ( )

L t f th o o t o t o o t d
t f b th t t o o to o ²( ) g b (f)( )
f( ) ( ) o ²( ) d t o f h th t f f
Sho th t f o o to Wh t t Wh
t t

9 L t ( ) Sho th t ( ) ( )

1 L t R b Ex o th t R ℓ² → ℓ² g b th t
h t R (1 2) (2) Sho th t p(R) D d th t
(R) ho th t R ot o o to

11 Sho g th C h S h q t th t H ⊆ H² ⊆ H¹ d
g x to ho th t th o t t

1 S o th t f $^2(\ )$ h th o $=$ Sho th t t h o xt o to th d g b $f(r\ )$ $=$ $r^{|\ |}$

13 L t $f(\ )$ og((1) (1)) o f bo d d f bo d d Sho th t f H^2 b t ot H

1 o t $\to 1$ (1 $^{-}$) b th od g k th t $f\ \rangle\ f(\ )$ o f H^2 Wh t $_2$

15 o $_+$ t th L t o o th t o $f(\ )$ $^2(\ )$ d th t f $\mathbb{C}$ $\sqrt{\ }$ f ,

1 th t th t o $\to$ (1) $^2(\ )$ C t t o thog o oj t o o to $H^2(\ _+)$

1 d d t o d t o o t o t o $f(\ )$ $_{=1}(\ )$ $_{=1}(\ )$ o t to b H ( ) d ( ) d (b) o t

1 o th t o ( ) H ($_+$) ho th t $H^2(\ _+)$ o t o t o th t L t o o t o $^2(1\ )$ d th t $H^2(\ _+)$ $H^2(\ _+)$ o o d to $^2(\ 1)$ th

19 th d t o th k t h d oo S t o 1 3

r

r r

Th th o th h t th t d o bo d d o to b o
th g h th t t o (*x* *x*) U bo d d o to o
t o o x th t g o to

( ) ( )

h h od o o t o o h t o t b o
th t o d d t q t o

— ( ) ( )

bo d d o [2]( ) d b t d d b th t h q

o g o o x o bo d d o to o th th o o
g o (o x th h t g o) d t d th g t
d t Th g t t od d h d t d t o
th o t o th o t d o Ch t

. a of an o ato

o d to t t bo d d o to g thod o th th
g b t h q o t od th *a* o o to ot
th t th t b g d th t o *a*
x th th b h o g h th o

e i i $\mathcal{X} \to$ *a a* *s s* *s* g h *s*
s s ( ) $\subseteq \mathcal{X} \times$ *y*

( ) (*x* *x*) *x* $\mathcal{X}$

$\mathcal{X}$ d to d th t t ght o d to
th t ( ) b o th to $\mathcal{X}\times$ (th x)

L t o ook t th h $\mathcal{X}$ d o d d
Th $\mathcal{X}\times$ t o d th th to o og g b b
o o b o o x o $1 \leq p$ d

$$(x \quad)_p \quad (x^p \quad {}^p)^{1\ p} \qquad (\quad 1)$$

(th p b o t h $\mathcal{X}$ d od t
b th $\mathcal{X}\times$ o od t t) oth
o b o g b

$$(x \quad) \qquad \mathrm{x}(\ x \quad)$$

o to th d to b s t g h o d Th q t to
th o d t o th t h $(x\ )$ q $\mathcal{X}$ d x to $\mathcal{X}$
t h th t $x \to x$ d $x \to$ th h x C
o t o (bo d d) o to o d d th o o g o d g h
th o o h g t o th h $\mathcal{X}$ d h
(o t o d) oo b o d book g
11 1

e e C e e e $\mathcal{X}\to$ a a a
Ba a s a s s a ( ) s a s s s a $\mathcal{X}\times$ s

Wh o to ook t bo d d o to th o o t o th t
d to b t k d tho gh t o b to ho th x t o h
d d bo d d o to b t h (th x) th oo
ot o t t d th o to th t o o t t o o
d d o o o t b o h

L t $\mathcal{X}$ d b o d o to d d o b o
$\mathcal{X}$ th th t b th a d d ot t b ( ) o
t t o o to t k bo t o to b t $\mathcal{X}$ d tho gh t
ot d d o th ho o $\mathcal{X}$

t to th t b $\subseteq \mathcal{X}\times$ th g h o o to
(d d o o b ( ) $\subseteq \mathcal{X}$) d o h h (x_1)
d (x_2) h $_1$ $_2$ o q t th o to
( ) g ( )

e a s a a as s ( ) $_{=1}$ a
a a y $_{=1}$ $\rightarrow$ $_{=1}$ s
s a s

( ) $\sum_{=1}$ $\sum_{=1}$ }

s a $_{=1}$ ( ) a y $_{=1}$ ^{2} ^{2} s ( ) s a
s s s a s s a a a s s
)

o tho gh ( ) ot o d Ex 1 3 t t o t th t th
g h o o d Th b g b

( ) $(x$ $x)$ x ( )

d t th d t o th t g To th t th g h o d
o th t $(x$) $_{=1}$ d x ( ) th x $\rightarrow x$ d x $\rightarrow$ o o
to (Th t ot t o o q g b t o t
h) L t x $_{=1}$ d $_{=1}$ Th o h th th oo
d t o x t d to d th th oo d t o x t d to
d th x

( ) $\rightarrow$ o to th o d g h ( ) $\subseteq \mathcal{X} \times$ h
( ) $\subseteq \mathcal{X}$ d $\mathcal{X}$ d h th ( ) t b o h
th th a

x (x ^{2} x ^{2})$^{1\ 2}$

d b t $\mathcal{X}$ d b t Th o o b th
( ) o t o o h to ( ) d th g $x \rightarrow (x$ $x)$
t k p (1) ot th t ( ) h b t o d
b o th h $\mathcal{X} \times$

t o t h th t th g h o o to ot o d b t
th t t o t th g h o o to th th t
sa d d t s to b th o to h th t ( ) ( )
Th o o g t t h o to o b

i i **4** a a a ( ) $\subseteq \mathcal{X}$ a a
s sa a y $(x$) s a s ( ) a a
x $\rightarrow$ a x $\rightarrow$ a

: C th o d t o ( ) th o o
th g h o d ( ) t g h th Co th
o d t o t d th ( ) b th o o
b b d t t th d t
o d t o g bo o b to b th g h o o to th t
t o t o to ( ) th

t bo d d o to o b t o t o L t
x o o o b o to

e *a* *s a* *a as s* ( ) $_{=1}$ *s*
( ) *a* *a* *a* *a y*
s *a* *a* *a s* $_{=1}$ *s*) *s* *a s a a*
→ *y* *a* *a* *y* *a y*
s *sa s y* → *a* *a*
s *sa*

W d d th djo t o bo d d o to o b t
d d (1 1) t o b to th xt t to h h k
h d t o o bo d d o to h h ot b d d o th ho

e i i *a* *s a s a* ( ) → *a*
a *a* *a* ( ) ⊆ ( ) → *s sa* *a*
djo t ( ) ⊆ *a*

⟩ ⟩ o ( ) d ( )

t t d th b djo t d d th d t o t d
d ( ) d ( ) o o o th t ( ) d
d t d o o T k ( ) to b th t o o h h th
t x t th ⟩ ⟩ o ( ) S h
q ( ) d th ⟩ o t d t d o k o
t o ( )

o d g d t d th t th k to
o to djo t to d th t t x th th t oth
o to djo t to t t o o to bdo W o t
o d t *a j* o t ot h d to th t th d t o
od th djo t th th t bo d d d d d o th
ho o

e E ot *a* o d t o b oo t h
d d d djo t o → d → th

$w\rangle$ → $w\rangle$ → $w\rangle$

o *w* ( ) d o

th o th djo t to o d o o g h x d
th xt t ot th t ( ) ⊆ × d ( ) ⊆ × o o
t ( ) o th *s a* o th t

( ) ( ) × ( )

e e *a* *s a s a* ( ) → *a s*
a a *s a* ( ) ⊆ ( ) *s a*
( ) ×

: W ot th t to ($_1$ $_1$) o thogo to th ho o ( )
d o ( ) ($_1$ $_1$)⟩ o ( ) Th g th o d t o

$_1\rangle$ $_1\rangle$ ( )

o $_1$ $_1$ th t h $_1\rangle$ $_1\rangle$ o ( )
d o ( ) ho d th oth h d ( ) do ho d th $_1$ th do
o b d $_1$ $_1$ to t th t o D t o 1
Th $_1$ $_1$

ot g th t o thogo o t th o d b th
d t b t d b x d o t o
o o

C *a* *s a s a* ( ) → *a*
s s y *a* *a* ( ) ⊆ *s a s*
s y *a* *a* ( ) ⊆

: t o Th o 1 th t h o d g h o o
d d d oth th o d x t to th ( )
b t th ( ) ( ) h h ( ) Th o b ( ) th
g h o o to

W th d to (b ook g t g h) th t th d t t ( )
t ho d th

. i o

o t t o o d bo d d o to th th o o
g o o o to h h t h t o g k th th th o o
t o d g d o o th b t h

e i i $\mathcal{X}$ *a Ba a s a* *a* t o g o t o
g o g o) (( )) *s a* *a s* ( )
$\geq$ *sa s y* *s*

( ) *y a* $\mathcal{X}$

( ) ( ) ( ) *y* $\geq$

3 *a* + $\mathcal{X}$ *y* $\rightarrow$ ( )*x s* *s* *y*
x $\mathcal{X}$

o t t d x obt d b d g ( ) h
x d bo d d o to o $\mathcal{X}$ th th g $\rightarrow$ ( )
o t o th o to o og (th x) Th o o g t
ho th t th o to ( ) ot g o o t th x o t

Le (( )) *a s* *s* *s a s*
s a ( ) $\leq$ *a* $\geq$

: Co d t th o to ( ) o $\leq$ ≤ 1 Th o
bo d d o ( ) q 1 th t h o
g t b q ( ) o g g to o t 1 Th (( )*x*)
o g to ( )*x* o h *x* $\mathcal{X}$ th h St h (o bo d d
) th o 9 11 th b q (( )) o bo d d o
d th th t th o t t h th t ( ) $\leq$ o
$\leq$ ≤ 1 t o g b $\geq$ t *r* h
o g t t g d $\leq r$ 1 Th

( ) $\leq$ (1) (*r*) $\leq$ $^{+1} \leq$ th d

W h o th t o g o o t g *a*
( ) ogo o to x t tho gh t ot g b
bo d d

e i i (( )) *a s* *a Ba a s a* $\mathcal{X}$ *s*
t g to *s* *a* *a* ( ) $\rightarrow \mathcal{X}$ *y*

$$x \quad {}_{h\to +}\frac{(\ )x \quad x}{}$$

$a \quad (\ ) \subseteq \mathcal{X} \qquad y$

$$(\ ) \quad x \quad \mathcal{X} \quad {}_{h\to +}\frac{(\ )x \quad x}{} \quad \text{x t}$$

t ot h d to th t ( ) b d th t (th x) Th do o o t d h ( ) th xt t ho

i i 4 $s \quad a \quad a \quad a \quad s \quad (\ (\ ))$ $\mathcal{X} \quad a \quad (\ ) \quad s \quad a \quad (\ ) \quad as$ $(\ ) \quad (\ ) \quad a \quad (\ ) \quad (\ ) \quad a \quad \geq$

: Ch k g th o d t o D t o 3 g o

$$\frac{(\ )\ (\ ) \qquad (\ )}{} \qquad (\ )\frac{(\ )}{}$$

o ( ) th t d to ( ) → Th ( ) ( ) d ( ) ( ) t d

Th o o g x b o t t to t

e $^2(\quad) \quad a \quad (\ (\ ))$ ght h t g o y

$$(\ (\ )f)(\ ) \quad \begin{cases} f(\quad) \quad \geq \end{cases}$$

$s\ s$

$$(\ f)(\ ) \quad {}_{h\to +}\frac{f(\quad) \quad f(\ )}{} \quad f(\ )$$

$s \quad a$

$(\ ) \quad f \quad ^2(\quad) \quad f \quad \text{C} \quad f \quad ^2(\quad)$

$C \quad s \quad a\ s \quad y \quad s \quad s \quad s \quad a \quad a$ $ss \quad as \quad a\ s \quad ^1 \quad s) \quad By \quad a\ s \quad a \quad a \quad a\ s$ $ay \quad ass \quad a \quad y \quad a \quad s \quad (\ (\ )) \quad a \quad y$ $s\ a \quad H^2(\ _+) \quad a \quad a \quad y\ (\ (\ )\ )(\ ) \quad (\ )$ $H^2(\ _+) \quad s \quad s \quad a \quad a \quad s \quad s \quad s \quad a \quad y$ $(\quad)(\ ) \quad (\ )$

E tho gh th t g to d ot b bo d d o to t
b h d th xt t ho

e e s a a a s (( ))
$\mathcal{X}$ s a s a a ( ) s s $\mathcal{X}$

: To ho th t ( ) d t k x th ox t to x th t
h g b

$$x \quad \frac{1}{} \quad ( )x$$

h g th t g o o t o h d t o
$\to \mathcal{X}$ b d d t o

$$( ) \quad \to \quad \frac{1}{}\sum_{=1} (\quad (\quad) \;)$$

t th t $x \to x$ $\to$ o o h

$$\frac{( )x \quad x}{} \qquad \frac{1}{}\left(\begin{matrix} +h \\ h \end{matrix} \quad ( )x \qquad ( )x \right)$$

$$\frac{1}{}\left(\begin{matrix} +h \quad h \end{matrix} \right) \; ( )x \;\; x \to \frac{( )x \quad x}{}$$

$\to$ Th x ( ) th

$$x \quad \frac{( )x \quad x}{} \qquad (3)$$

d o ( ) d

o o th t (x) q ( ) th $x \to x$ d $x \to$
$\to$ W d to ho th t x ( ) d x Th t ob t o
th t o ( ) th d t o th t o $\to$ ( ) x t d q
( ) d d

$$\frac{(\quad) \quad ( )}{} \qquad ( )\frac{( )}{} \to ( ) \qquad \to$$

d o h

$$\frac{(\quad) \quad ( )}{} \qquad \frac{(\quad)(\quad ( ))}{} \to ( ) \qquad \to$$

$$\frac{(\quad)(\quad(\ )\ )}{(\ )} \qquad (\ (\quad)\quad(\ ))\left(\frac{(\ )}{\quad}\right)$$
$$(\ )\left(\frac{(\ )}{\quad}\right)$$

d both t t d to o →

Th o h

$$\frac{(\ )}{\quad}\ \underline{1}^{\ h}\ (\ ) \qquad \text{o} \qquad (\ ) \qquad (\ )$$

tt g x ( ) d tt g → obt

$$\frac{(\ )x \quad x}{\quad}\ \underline{1}^{\ h}\ (\ )$$

t → to o d th t x x t d q

W o t o t b ght h th o o g t ho

i i (( )) a s $a\ Ba\ a$ $s\ a$ $\mathcal{X}$ s a a a s s a ( ) ≤ a s s s a $s\ a$ a ( ) x x a x $\mathcal{X}$ a ( )x x a x ( ) ( )

: Eq t o (3) th ( ) d b ( ) ho th t

$$^{h}\ (\ )x \quad x \quad (\quad) \quad ^{h} \quad (\ )x$$

o x $\mathcal{X}$ d ≥ (ot th t th t g to o th g o (( )) d) L tt g → d g th t th t o d th t

$$x \quad (\quad) \quad (\ )x$$

S o x ( ) obt

$$x \quad (\ )(\quad)x$$

C . C O O O

b ( ) d th ( ) ^{1} x t d g b

( ) ^1x ( )x ( )

h h og o to d L t o

Wh o k b t th th g h
g o b h d t k g djo t Th o o g t h
ot g

e e (( )) a s a s a
s a a (( )) s a s a s a s s a
a s

: W k o b Co o 1 9 th t h o d o to th
d do th h th o t S o th t ( ) $\leq$ o
$\geq$ L t hoo x ( ) d d t k b th
Th

(^{h} ( ))x $\rangle$ x (^{h} ( )) $\rangle$
$\Big\langle$( )x ^{h} ( )

o o t o Th o t k g th o ≤ 1 obt

(^{h} ( ))x $\leq$ ( )x $\rightarrow$ $\rightarrow$

t o o th t ( ) o g o o o

$\Big\langle$ $\underline{(\quad ^h ( ) \quad)x}$ $\Big\langle$( )x $\underline{1}$ ^{h} ( ) $\rightarrow$ ( )x $\rangle$

$\rightarrow$ ho g th t th t g to o th g o
(( )) d h g t (( ))

o to d to b a ≤ 1 d g o (( ))
o o to d to b a s h ( ) o t t o
Th o o g b t d th o o h h k t h th oo g
d t o d t o o (( )) to b o t t o g o
t o th o to

e e **Hi e i** a a a Ba a
s a $\mathcal{X}$ a ( ) s s a a a
a s a y s

s a s a s a

. . *O*

x x $\subseteq \rho(\ )\ a$ ( )^{1} ≤ 1 a

: Th d to d $^{2}(\ )^{1}$ ( )^{1} o
Th bo d d o to d g t g o (( )) d d
b ( ) o $\geq$

t th o b to th t $x \to x$ $\to$ o h x ( ) d
th t ( )x $\to$ ( )x x t d d g o ho t
g to

( ) $\leq$ x ($^{2}(\ )^{1}$) $\leq$ 1

o h d ho g th t (( )) o t t o g o

Co d o th o o g d t q t o o t) (t
o o b to o d t t d)

$$\frac{x(\ )}{x(\ )\quad x}$$

(5)

th t g to o th g o (( ))o $\mathcal{X}$ th th
t o $\to x(\ )$ d to b *s* to (5) x $\mathcal{X}$ d $x(\ )$ ( )x

ot th t ho bo th d t o $\to$ ( )x d d ( ) x
h h th ( )x h x ( ) Th $\to x(\ )$ *as*
s a s to (5) x ( ) d $x(\ )$ ( )x g th
$x(\ )$ x o $(\mathcal{X})$ h t go g o h

W d g o to t od th o t o *a ss s a a*
th tog th th th o t d *ss j* h t t d g t
d o t h To do th o d th o o g d t
q t o

$$\frac{x(\ )}{(\ )\quad x(\ )}\qquad x(\ )\quad x(\ )\quad x$$

( )

( ) h t k to b th t g to o g o (( ))
o b t d x ( ) Th o to ( ) $\to$ to
oth b t (th o t t h) d d
s a a od g th W th k o t o d g
ob t o o $x(\ )$ th *s a* o th o g t o th o x

k o th t th o t o ( ) ( )x o d d th t th x o k
t ot o to b bo d d th t to th o o
(t t o d h o t o xt o to th ho o) t
t th t b bo d d th t to th g h o d x
th o t b q g th t th x t o t t 1 2 h th t

$$\leq \quad _1 \quad _2 \quad \text{o} \quad (\quad) \tag{}$$

Th o to d to b) a ss o th o t t
q t g b o th x t o t t
h th t

$$( )x^{\ 2} \leq x^{\ 2} \text{ o } x \quad (\quad) \tag{}$$

o h k g h th g ob t o o to d b ot b
t ght o d o x g d t o to
th ot b b to t do ( ) x t o ( )
t d th t k g th od t o th t o → ( )x th
th t o → o + (th) d g th
C h S h q t $f \rangle \leq f$ obt th s
th t th x t o t t h th t

$$(\quad)^{\ 1} \leq \frac{}{\sqrt{\quad}} \text{ o } + \quad (\quad) \tag{}$$

Th W o j t th t ( ) d ( ) q t t t o t
th t o g o o x g o h h ( ) o
o to o h d o o t t o g o th o j t ho d o
t d o o th o j t do ot ho d o th o
bo d d g o d t d o t o g o t
d o o th t o th o j t th t th h h t
k o to b d o d t o g to o b t d
t th th o o d C o d k o to
W do ot go to th d t h

e th x o g o o d th a
s h h h b t d d o t o o t o
ob b t d o o d th t d h W b g th th
h t q t o o t t d t b t o $x(r\ )$ o)

$$\frac{x}{} \quad \frac{^2x}{r^2} \quad x(r\ ) \quad x\ (r) \quad \text{g} \tag{9}$$

r d ot o t o d t d o th o o th x
th t th t o $r \to x(r\ )$ $^2(\quad)$ o h $\geq$ Co o th

q t o (5) gg t o d t o o g o (( )) th t t
g to t k to b th d t o to ^{2} r^2 th t
og h b o th L g b

$$\sum_{=1} \frac{^2}{r^2}$$

g t to th t t o th L do h o k
o $H^2(\ _+)$ d th g o (( )) d d b (( ))( ) x (^{2}) ( ) th
t g to t g ( )( ) ^{2} ( ) o t o to (9) g
b o o t o th th a th t

$$(\ (\ )x \ )(r) \quad x(r \ ) \qquad \frac{1}{\sqrt{\qquad}} \qquad x \ (\ ) \qquad (\ 1 \ )$$

h h o o d to ( ) b th L t o

Th o o g x t k o

e Th d t o o t d d d b
b od d b th q t o

$$\frac{^2}{^2} \qquad \frac{}{x} \qquad \leq x \leq 1 \qquad \geq \qquad (\ 11)$$

th t o d t o o th o t o d o t

$$(x \ ) \quad {}_1(x) \qquad \text{d} \qquad (x \ ) \quad {}_2(x)$$

g d bo d o d t o

$$(\ ) \quad (1 \ ) \qquad (\ ) \quad (1 \ )$$

d t g th t th b x d t th d o t

W t od th o to $\frac{^2}{x^2}$ th do

$$(\ ) \qquad {}^2(\ 1) \quad \frac{}{x} \text{ b o t } \frac{^2}{x^2} \quad {}^2(\ 1) \ (\ ) \quad (1)$$

W th x (11) th o

$$\frac{}{} \qquad \text{h}$$

$$\left(\frac{}{} \right) \qquad \text{d}$$

$$\left(\quad {}_2 \quad \right)$$

h ( ) b o th b t ( ) ^{2}(1)
q d th th o ($_1$ $_2$) ^{2} ^{2} $_2$ ^{2}

Th o d o b o g o th o d o t od
t h q o t d g o d b d o d o to o b t

. a t i

W t t b d g th ot o o d t b t t o o d b o
b t Th b g g h h b b to d th d t
b t t o o d o to

e i i *a s s s a s a s a*
g *a y* ( ) *s y*

( )

a a j s a
s y

W th th d t o t to th t t (th x) o
o t d th g b d t o d h o ho th t
th t o q t

e e *s s s a s a a s a as*

( ) x ( ) ( )

$a \quad a \quad \leq (\quad) \leq 1 \quad a \quad a$

: W b g th th o o g t x d t t h h d
b g d th t d oj t o

$$\left(\quad \right) (\quad) (\quad) \left(\begin{array}{c} (\quad) \\ (\quad) \end{array} \right) \qquad (1)$$

Th t to o th t h d d o (1) t o t o o h
b t × d d th th d o t o o h b t d
×

$$\left(\begin{array}{c} (\quad) \\ (\quad) \end{array} \right) ,$$

x ( ) ( )

h h q t to th q d t th djo t o ( )
( )

W t ( ) ( ) d t th *a* b t d
S ( )*x* j t th d t o *x* to th t

( ) d t(*x*) d t( ) (13)
, =1 , =1

t t h S o ho d th d t h g d

Th ob h tt t to d th g o b
o g h $\mathcal{X}$ b th o g t th t bo d d
oj t o d x t (d t ot k o th d o
o thogo oj t o h o b t) W o d t d
( ) x ( ) ( ) th ( ) , =1 d t( ) b t
th ot t t d

$_1$( ) x $_1$( ) $_1$( ) h
$_1$( ) d t(S)
, =1

t g S w w 1 o th t h o Th d d
t o th o o o d b o $\mathcal{X}$ th d t b o d

t h tt h d to h t t o th g to b
x t t o o th d o (th x o x o g
th) o ot th t ( ) 1 oo th o t
to o thogo to th ho o th oth o x h o
o b o th oth W h o o t t g x
h o to ook t b d t d b o to

t ho d o b o o o d o ho to d th g
b t t o b t o to ( ) → d ( ) → h
th do ( ) d ( ) b o o o d th
g h ( ) d ( ) o d b o ×

e i i *a as a* g *a*
( ) *s y*

( ) (( ) ( ))

tho gh b g d t o d t d t b t b
d d t b t o to o o o k Th ogo t
o b ho d t th t o d t o th t o o d
o to b t d d d th *a y*

o t h to d ot th o thogo oj t o o to th
g h ( ) o Wh bo d d th b tt do x t
o o

e e 4 → *a a a*
j × → ( ) *s y*

$$\left(\right) \quad \left(\right) (\quad) \, ^{1} (\quad) \left(\right) \quad (\quad)$$

: Th o 1 t

( ) (*x x*) ( )

q h (*x x*) ( ) d ( ) ( ) Th h

x

x

( )*x* d th t o ot th t
djo t o to d *x* 1 th

( )*x* ≥ ( )*x x*⟩ *x* ² *x* ² ≥ 1

o th t th do d d x t
ght g t o o th t b o d Ex 1

C *a y s* ( ) *s y*

: S both to o og g b t t t to h k th t
q o g g o t o o g th oth (d)
( ) q ( ) h th t → o o ( )
th t o Th o 3 th t → o th t th o to
o o g th g h t

Co → th t k g Th o 3
h ( ) ¹ → ( ) ¹ o d h →
o W o h ( ) ¹ → ( ) ¹ o d o t g
o g t q th t → o

e

So t d d t od to t xt th o o to th o 9 11
11 1

C o d o to d d th 5 1 o x

o d d o t o th th o o g o b o d th
o og h 3 5 3 5 1 o th d o d b t d th
W o j t th d to th t 3 5 1 1 1 3

oo o th o o th g t d t d o h h t t
S o 31 Th oth o g oo th t t o
x th t

e i e

1 Sho th t $\mathcal{X} \to$ g b t to th
( ) b o th to $\mathcal{X} \times$

L t d b o x od t d o to $_1$ ($_1$ $_1$)
d $_2$ ($_2$ $_2$) $\times$ d $_{1\ 2}\rangle$ $_{1\ 2}\rangle$ $_{1\ 2}\rangle$ Sho th t
th k $\times$ to od t d th t th d d o
g b (1) th p

3 L t $\mathcal{X}$ b t d o o d d o th t S x
a as s o $\mathcal{X}$ th t x t o to h th t
t b t d d t (th b ho to x t b
g o) Sho th t to o b t o o
to S q d th t o to d
q g $\mathcal{X} \to$ b g x b t o
x S S o o th t o d Sho th t th
bo d d b t h d d g o $\mathcal{X}$ to

L t b th o to g Ex 1 3 h h d d d d
bo d d C t t djo t d ( )

5 th t th t g to o g o o to
d th t t do o d t o b

L t b bo d d o to o h $\mathcal{X}$ Sho th t
d ( ) x ( ) b o o o g t o th t
(( )) g o h th t $\to$ ( ) o o t o d th t

th t g to o th g o (( ))

L t ( ) =1 b q o o x b d d g o (( )) o b t th o tho o b ( ) =1 b

$$( )\sum_{=1} \qquad \sum_{=1}$$

h =1 to Sho th t o d d th q ( ) bo d d bo th o to ( ) bo d d d o g o Wh t t t g to

S o th t (( )) g o th t g to T k Sho th t (( )) o g o d th t t t g to

9 L t b b t th o tho o b ( ) d t (( )) b th g o o th t g to t g o h h ( ) b t q Sho th t (( )) o g o o th t th o t o d t o ( ) q t to d b t S o th t th ob t o t o ( ) → g b h ( ) o q W t do ( ) x t t o ( ) d ( )

1 d t th t (1) t th h t q t o

11 Ch k th t th g t d d t o th o t o o o d b o (ot th t d o Wh)

1 o th t ( ) ( ) o b ⊆

13 o t oo o Th o 3 t t g o th d t t ( ) ( )

1 L t p b x d C t th g b t th o d o b $(x \ )$ x d $(x \ px)$ x o th t o d o b t ^{2} o t tt ℓ_2^2 Th b do

th b t g do th oj t o x t d g th d t o o
th g o t t b t g th t o d t d g

15 Sho th t th h g 1 t t th t o o
Ex 1 g 1 o

1 S o th t → o d d d d o to th g h
( ) ( ) o o o t g bo d d
o to o d x t o o th o thogo oj t o
× → ( )

1 th oo o Co o 3 5 d x t bo d
o t o d

r 3

i i ri i

h o o h to g d t o to th th t o dd
t o o t o h t (o k o t) d t
t t g t o o o t b t h o t t
o th t t S h o to h g h th t h t
t b d o t o th g o t o to th
o t o t t o to d t d th t o th b

L t t t x Th o to o t t o b th t o
$\rightarrow \xrightarrow{h}$ (th d) o th d $H^2(\ _+)$ b t
d t d th th d d t q t o

— ( )

o d *a* *sys* ( ) h h o t o o t xt
d g o x th t t o to g 1 W h
o d t d t t t o d q t o Ch t b t h
h th h t th t o ” t t t
o o d to o to o t t o

. In a iant b a

S o th t bo d d o to o o x b t th
a a s s a o b $\subseteq$ h th t o
t o d q t o o d t th t o t g h th
o o d t b t o th ob o
x d t (1 o t o th q t o) Th
o o d g q t o o h k o to h g t
g o th ℓ^1 3 9 113 11

d t o to th t b o t t to th s R $\ell^2(\ _+) \to \ell^2(\ _+)$ d d b

$$R(\ _1\ )\ (\ _1\ )\ (\ )\ \ \ell^2(\ _+)$$

(W h th tt S o q t d o to to b d d ho t) Th o to h t xt o to $\ell^2(\ )$ h h h o d ot b R d d b

$$R(\ )\ (\ )\ \text{h}\ _1$$

ot th t R t o to o $\ell^2(\ )$ b t th t R ot o o to o $\ell^2(\ _+)$ (Ex 1 o Ch t) o th t o o to o t d g t t o o th o d t b k o h h h o d W th ook t th ogo t o $^2(\ )$ d $^2(\ )$ to d t o o ot t o th to d t d h t t t h h h g d o to th o d h t t g h (th o b t o d t o to b d d t)

o th t o o d b t $\ell^2(\ )$ d $^2(\ )$ d h h $\ell^2(\ _+)$ o o d to th d H^2 $H^2(\ )$ (Ch t 1) o d th ob o o d t g th o d t b o th o to S o t t o b o $^2(\ )$ g o b th o

$$(Sf)(\ )\ \ f(\ )\ \ f\ \ ^2(\ )$$

d t t t o to H^2

t o t h to to o to o $\ell^2(\ )$ d $\ell^2(\ _+)$ t g th a d s Th q t o to o $^2(\ )$ d $H^2(\ )$ th t th y a

Th t o t o o d t b th t o t $S \subseteq$ h th ( ) S o ( ) S th t d a a o s y a a th o d a a o y a a d t th t S ^{1}

L t b g b g th t b o b t o t χ o th h t t t o o o th t

$$\chi(\ ) \begin{cases} 1 \\ \ \text{oth} \end{cases}$$

Th χ $^2(\ )$ f $^2(\ )$ f o Th t o o th o o g t o th o

e e ie e *s s s a* $^2(\ )$ *sa s s S*
a y χ $^2(\ )$ *s as a s s* $\subseteq$

: W t o th o thogo oj t o o th o t t t o 1 o
S 1 d S o h S 1 $\rangle$ o
b t th th t

$$\frac{1}{} \quad ^2 \quad (\ (\ )\quad (\ )^2) \qquad \text{o}$$

d h ^{2} o t h d ( ) o 1 L t b th t
o h h 1 o χ W d to ho th t χ $^2(\ )$ C
d h χ $^2(\ ) \subseteq$
Co d t o f th f χ $^2(\ )$ th th t

$$\frac{1}{} \quad ^2 \quad f(\ )\chi\ (\ ) \qquad \text{o}$$

d o d th t $f\chi$ o S f d 1 χ d o

$$\frac{1}{} \quad ^2 \quad f(\ )(1\ \chi\ (\ )) \qquad \text{o}$$

Th $f(1\ \chi\ )$ d f Th χ $^2(\ )$ t d
ot th t o o t b χ $^2(\ )$ b o t d H^2 b Th
o 1 3 Th g th th ob t o th t H^2 h o t
b o S

Th t o o 1 t b o $^2(\ )$ ght o o
t d d g b th B – s h h o o o th
th t o H^2 th o t t g

e e *s s s a* $^2(\ )$ *sa s s S* S
a y H^2 *s as a sa s y* ( ) 1
a s y a s a a s

: W th d to th t 1 o th H^2
o d b o $^2(\ )$ d th t S S
L t o t b h th q $_1$ d $_2$ t o t o od
o th h th t $_1H^2$ $_2H^2$ th $_1$ f $_2$ d $_2$ $_1$ o t o
t o f H^2 h h t th b od d h
th x $f \leq 1$ d th ho d o d o d
th t f d o t t

o g h 1 t b h S d o
d t o h h t k to h t o h th t S
t S o 1 Th g

$$\frac{1}{} \quad ^{2} \qquad (\quad)^{2} \qquad \text{o} \qquad \geq 1 \qquad (3\,1)$$

b t b o j g t o th o ho d o $\leq$ 1 d h ^{2} o t t
d th o t t 1 b 1
W o x o t Th o 3 1 1 d ot th t th q $(S \quad)$
o tho o d th t t o d th ho o $^{2}(\quad)$ th
o o t b t o th o h h th t o h o th
o d o $(S \quad)_{=}$ h h H^2 o t d o o o
d ≥ 1 h

$S \quad \rangle \quad S \quad \rangle$

S b o t t o Th S W h th t b h d th t
$H^2 \subseteq$ d $(\ H^2) \subseteq$ d o h q t both

Th o o g o o th g th o d t

C e i *a z s s a* H^2 *a s a*
a S H^2 *s s*
a s a s

Th t o g th o t to t o d b d Th o
1 3 3 to d h o d h t t o o o t t o

C 4 H *s a y* H^2 *s s* H^2

: o H th o o H^2 o d b o H^2 d
h o th o H^2 o o t o E d t o o
H o o t t h o o t d o d o
o t t d H^2 d H^2 Co ot o t th $H^2 \subseteq H^2$
h o o t t t o d d g d o H^2 ot d
H^2

t g to o g d o o t b o bo d d o
to t to th t $(\quad)$ d x th th o d o
$(\ x)_{=}$ o d t b o th th ho o th x
d to b y d th t b ob q t to
th q t o o h th o o to W h j t th t
th to o S (H^2) th o t t o

o o g o to th to d t o h t o o
k o th B – a bo t t b o $^2(\ )$
t d th t R ≥ o ght h t

$$(R\ f)(\ ) \quad \begin{cases} f(\ ) & \geq \\ & \text{oth} \end{cases}$$

o th (to o t t) t q $^2(\ ) \to H^2(\ _+)$
d d b th L t o (Ch t 1) h h t $(R\ f)(\ )$
$(\ f)(\ )$ d th ob to d g th h t t b
o $H^2(\ _+)$ th t tho t d th o to S o t t o b
th t o → o ≥
g th k o o to o $^2(\ )$ d $^2(\ )$ t g th
a d th q t o $^2(\ )$ d $H^2(\ _+)$ t g th y
a Th o th s t t o

e e z s s $s\ a$ $\subseteq H^2(\ _+)$ sa s $s\ S$ $\subseteq$
a ≥ a y $H^2(\ _+)$ s H $(\ _+)$

: C h b $H^2(\ _+)$ d d o d d h t
t

L t $H^2(\ ) \to H^2(\ _+)$ d ot th o t o o h d b d
Th o 1 5 d d b th o o bj t o d d b $(\ )$
(1) (1) t t to ho th t th o d b ^{1} o H^2
t d S S $(\ S\ ^1f)(\ )$ $(\ )f(\ )$ o f $H^2(\ _+)$ t o gh
to ho th t t d t t o b th t o

$$\to \frac{1}{1} \qquad 1 \quad \frac{}{1}$$

t th o to S o t t o b 1 (1) b ox t d b o
b t o o th S S

$$\frac{1}{1} \qquad {}^{1+} \qquad \to \qquad {}^{1+}$$

o ox t g th t t g b
th t 1 (1) bo d d o t t o o b t o o t o
Th o b d th L b g bo d d o g th o ho th t
o f h Sf → $S\ f$ h h S t o b t o
o th S Sf d th o t th oo

W o o o o to d o o th to d d o j t
t o o th o h g t o o th W th o o
t b o $^2(\ )$ d th oth g t o o th g

th o o 1 t b o $H^2(\quad)$ To o d o t o o
t h ot d th ogo h t x t b t
th
t d o t o t k g th t 1 o t h o
d to d th t o ho o thogo oj t o o d g
th t → ( ) *as a j a* t
t th o o g

( ) th o thogo oj t o o to o o d b ( ) o
o o t

Th g → ( )x ⟩ b o x

S ( ) b g d d × t x d t o th o d
o t j t th t (( )) W q g d o
j t o o to o $^2(\quad)$ b o t t t o ot th t

$$f \quad ^2(\quad) \quad f(\quad) \quad (\quad)$$

th 1 o thogo oj t o o j t g → p
h p^2 p th t p o 1 t th b to o Th o 3 1 1
o t to o t t h h o g

e e *s s s a* $^2(\quad)$ *sa s s S a y*
$^2(\quad)$ *s as a j a* →
( ) *s s s as*

: t d th t b $^2(\quad)$ h t
t t o o d q $^2(\quad)$ o g to f
o th t h b q o g g o t h d
o d th t $f(\quad)$ ( )

h o d h t t b t $^2(\quad)$ →
d ot th o thogo oj t o o to t L t $_1$ d ot th o
b o d o r d 1 ≤ ≤ t , d ot th to d
t o d d o b → o h t ( ) d ot th
(h h o d) o th t o to

$$(\quad,)(\quad) \quad r \quad 1 \leq \quad \leq$$

Th d d to th t o D ( ) to b th
o thogo oj t o o o to ( ) t th t

$$\subseteq f \quad ^2(\quad) \quad f(\quad) \quad (\quad)$$

th t o q th th o d x t f $^2(\ )$ h th t $f(\ )$ $(\ )$ b t f S , o r d $1 \leq \leq$ Th

$$\frac{1}{\ } \ ^2 \ f(\ ) \ , (\ ))\rangle$$

o h d o $f(\ )$, $(\ ))\rangle$ Th $f(\ )$ b t to $(\ )$ th t o thogo to g t o $(\ )$

d d b d t o th t o o t t q to th $\subseteq \ ^2(\ )$ d o $\rightarrow$ $(\ )$ b

o d to t t th g L x th o o $H^2(\ )$ d th o t o a a Th t o H $(\ (\ ))$ t g th dd t o o d t o th t $(\ )$ o t o

e e a z s s a $H^2(\ )$ a s a a S s a r $\leq r \leq$ a a H $(\ (\ ))$ s a $H^2(\ )$

: th oo o Th o 3 1 ook t th d b t d S t g S S $\subseteq H^2(\ )$ b t g b th t th d o o t o t ot th t th b S S^2 o thogo S o t d o $S\ w\ S\ w\rangle$ $w\ S\ w\rangle$ h j d w w d d th t th o thogo d t

$$S \quad S^2 \qquad (3\ )$$

o x d x S S^2 th

x o x S d x Sx_1 o o x_1

x Sx_1 S o x_1 o x_1 S d x_1 Sx_2 o o x_2

x S^2x_2 S^2 o x_2

Co t g th th t x b tt x S x th x o h S x $H^2(\ )$ t h x

t o w h o t t od w S w o ≥ 1 o th t

$$\frac{1}{\ } \ ^2 \ w(\ ) \ w(\ ))\rangle$$

d b o j g t o th t th o o o o t o $\rightarrow$ $w(\ )\ w(\ ))\rangle$ o (t t t t th o t to o Eq t o (3 1))

S o th t d r ≤ t → b o t o o h
d o d ( ) ( ) b

( ) ( )( ) o

th t t o o t o H²( ) b tt
f( ) = h d = ² W th t f th
o t = S ( ) d

$$f^2 \sum_{=} {}^2 \sum_{=} S(\ )^2 f^2$$

b (3) o d th t H (( ))

e ob b th o t g o o th g L x th o
b o d 115 h h d h t o to o g b t
o to S h th t S o t d S x → o x (Th
d th h t o H²( ) b t ot th h t o ²( ) h h t)
o to d to b S S S d t o t
(h h th t t h k d o t
o) Th t t o t th t th o d S t b o h
th o h S Th d b tt ho th t th
Th o 3 1 d 3 1 t h ook d t h t t o to th
xt t o

. In a iant o ato

th t o t t d h t t o to b h h
o to ho g h o d h t t b ot th t
bo d d o to o ℓ²(₊) h th t R R th (x x) ( )
h

$$R(x \quad x) \quad (Rx \quad Rx) \qquad (\ )$$

d th ( ) o d h t t b o ℓ²(₊ ²) Th o
o ho d o (Rx R x) ( ) o o x ℓ²(₊) th do ot h
R R

Th t k to d o d h t t o to o ℓ²(₊) d
²( ) (d b th ogo o to o d) b o th
g h b t o th t S t o 3 1 h d t to o
d b t o g to th q t h h t H²( )
H²(₊) ²( ) d ²( ) b o h o ook t th to d

o t t o t o o to h h t q t
t th o *a sa y* th o d t o th t ( ) o
th ( )( ) o W th ot
th t" o t t d th b dg o " ot th t bo d d
h t t o to d d o th ho o $\ell^2(\ _+)$ th $(R^{\ 0}\ )$ $R^{\ 0}(\ )$
h h th t o d t o S t ho d o $^2(\ )$
g o th t o t th ght h t R o $\geq$ o t
to th t th t h t $R\ \ \ell^2(\ )\to\ell^2(\ )$ ot d to t do
x o $^2(\ )$ W t d t o d t S t o 3 3

Wh th t *a sys* h g o d
h t t o to d d o o th bo W b g b t d g
bo d d o to o h h th t t b d d d t

e e $H^2(\ \ )\to H^2(\ \ ^p)$ *a a a*
a s s a S s a
$H\ (\ \ (\ \ ^p))$ *s a a*

( )( ) ( ) ( ) o $H^2(\ \ )$

: W b g th th p 1 L t ( ) ( )( ) h
th t o o t t q to 1 C t t o o
t d ot th t o S o 1 th S S
h ( )( ) $(S\ \ )(\ )$ ( ) ( ) ( ) o th o th
( ) d H^2 d o t o o b o t t th t ( )( ) ( ) ( ) o
H^2

o h w t H^2 d ot th od g k $\to(1\ \ \overline{w}\ )^{\ 1}$
o th t $f\ \ \rangle\ \ f(w)$ o f H^2 o

$\rangle\ \ \rangle\ \ (w)\ (w)\ \ \overline{(w)}\ \ \rangle$

o H^2 d h $\overline{(w)}$ o w $(w)\leq$
d o $\leq$ W d h ( )(w) (w) (w) d o
d d th t $\leq$ h h g

W d th to d to th o o ook g
t d d o o t o th to g d $H^2(\ \ )$ $(H^2(\ ))$
d o $H^2(\ \ ^p)$ h h ( )$^p_{=1\ =1}$ g $p\times$
t x o h t t o to d th o o d to $H\ (\ \ ^p\ )$ d
t o Th t h ( )( ) ( ) ( ) h ( ) d ( )( )

to d ( ) t x o h W h ≤ d
t to ho th o q t

o h ^{p} o d th t o

$$\rightarrow (1 \quad \overline{w}\ )^{1}$$

o o f $H^2(\quad)$ h f ( )⟩ $(w)f(w)$ ⟩ d o

$$(\quad) \qquad (w)$$

L tt g w d o th t h o ^{p} d d th t
≥ d o h q t

Th t o th bo th o th o th t o o
o o d *a s*

C *y* *a* $\ell^2(\ _{+})$ *a* *s*
s *R as* *a* *a*

$$(\quad)(\ ) \quad \sum_{=} (j)\ (\quad j)$$

( ) (1) *a* *s* *a* *H* *a s*

: Th d t t t o o Th o 3 1 g th t d d t
o o d b t $\ell^2(\ _{+})$ d H^2 h h o t ( ) th $_{=}$

ogo t ho d o $H^2(\ _{+})$ o o th t o ≥ th h t
o to S d d o (o b to d) d b th o
$(S \quad)(\ )$ ( )

e e $H^2(\ _{+} \quad) \rightarrow H^2(\ _{+}\ ^{p})$ *a* *a* *a*
a *s* *a s* *a* *s S* ≥ *s a*
$H\ (\ _{+}\ (\quad ^{p}))$ *s* *a* *a*

$(\quad)(\ )$ ( ) ( ) *a* $H^2(\ _{+} \quad)$

: W o d o th d th to
x L t ˆ d ot th $H^2(\ _{+})$ t o ˆ( ) $\frac{1}{+1}$ h h th L
t o o th t o → $^{2}(\quad)$ ot th t th o d
o th t t *R* d $^{2}(\quad)$ $^{2}(\quad)$ d

. . *N* *N* *O* *O*

R ⟩ o ≥ th ( ) o ≥ d o

L t ( ) (ˆ)( ) ˆ( ) o h + C t + d o th t

$$(S \hat{})(\ ) \quad S \quad \hat{} \quad (\ )\hat{}(\ ) \quad (\ )(S \hat{})(\ )$$

o h ≥ o o t o $H^2(\ _+)$ th o d o th S ˆ x t d q → th ( ) ( ) ( ) o h h h ( )( ) ( ) ( ) th t o g o t o

th oo o Th o 3 1 th t th od g k t o g to o

$$\langle\ (\ )\ (\ ) \quad \overline{(\ )}\ \rangle$$

o + d $H^2(\ _+)$ Th ≤ g ( )( ) ( ) ( ) o $_2 \leq$ $_2$ d o h

To t th b k to h t t o o h t t o to o $^2(\ )$ ot t t ght o d Th ob th t t h d to d b th L t o o g $H\ (\ _+)$ t o Th o o g o t d t S o th t $^1(\ )$ th t $\leq\ _1 \leq\ _2 \leq$ d th t $(\ )_1$ th $_{=1}$ Th d o o t o o to $_h$ o t d th th *s s s* d x d b th o ( ) ( ) $_{=1}$ ( ) b

$$(\ _h)(\ ) \underset{=}{} (r)\ (\ \ r)\ r \sum (\ \ ) \tag{3 3}$$

T k g L t o th t $(\ _h)(\ )$ ( )( )( ) o + h

$$(\ ) \quad (\ \ )(\ ) \sum_{=1}$$

d $H\ (\ _+)$ Th d t h d d t Ch t

d t o q o th o t h t t o o th bo d d h t t o to o $\ell^2(\ )$ d $^2(\ )$

C 4 *a a sa s a a a* $\ell^2(\ )$ $^2(\ )$ *s s a ay a s a a a* $^2(\ )$ *s y* $^2(\ )$ *s a H* ( ) *s y H* ($_+$) *s a* ( )( ) ( ) ( ) *a* $^2(\ )$ *s y* $^2(\ )$ *a a a a*

: Th t o $\ell^2(\ )$ b o o S th o to h $\ell^2(\ _+)$ t b d th h $H^2(\ )$ t b o b Th o 3 1 h th o ( )( ) ( ) ( ) o $H^2(\ )$ b t th h t o th o o S $H^2(\ )$ o S S o $\bigcup_{=} S$ $H^2(\ )$ d $^2(\ )$ d o d d th t o th ho o $^2(\ )$

Th t o $^2(\ )$ o o g Th o 3 3

o h to ook t o g t to h h t t o to o o g bo d d d t h th t o o k o g h d b b t

Th o o g th o g h t th o d o to th h t t g h o H^2 W g th to d b t do ot h th d g $H^2(\ )$ o $H^2(\ _+)$ th o d

e e Ge i i $(\ ) \to H^2(\ ^p)$ *a s s a a a* $(\ ) \subseteq H^2(\ )$ *s r* $\leq$ *a s a H (( )) a N H ((^{p})) s a*

$$(\ ) \quad \binom{}{N} H^2(\ ) \quad H^2(\ ) \qquad (3\ )$$

$\binom{}{N}$ *s a s a* $H^2(\ )$

: S th g h ( ) o d h t t b o $H^2(\ ^{+p})$ th to o o th g L x th o bo Th o 3 1 ho th t ( ) h th q d o Th o d t o th t o g (d h $r \leq$) th b d w to th t w o d th t ( ) b g h

W o t o th ogo t o o d h t t t o ^{2}

e e $(\ ) \to {}^2(\ ^p)$ *a s s a a a ( )* $\subseteq {}^2(\ )$ *s s a as a j a* $\to (\ ^{+p})$ *s a*

$$(\ ) \quad {}^2(\ ^{+p}) \quad \binom{}{N} \quad {}^2(\ ^{+p})$$

w s w $^2(\ ^{+p})$ *Nw a s*

Th o o d t o W th o 3 1 th t o d t o b g
th to g t th t ( ) d d g h
th p 1 th b x d o t t o o
ogo x o o th t b h h do ot d g
5

e e ( ) → ^{2}( ) *a s s a a a*
( ) ⊆ ^{2}( ) *a*

( ) ^{2}( )

$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (^{2}) ( ) ^{2} 1 *a a* $_2$( ) 1 *a*

: Th o 3 ( ) ^{2}(^{2}) o o oj t o d
L t ( ) d ot th g o ( ) Th ( ) g h h
( ) ^{2} o th t o o t th t ( ) th th o
b o o d o th h t o t k ( ) $\begin{pmatrix} & \\ & \end{pmatrix}$
oth t t to ( ) $\begin{pmatrix} {}_1(&) \\ {}_2(&) \end{pmatrix}$ th ≤ $_1$( ) ≤ 1
Th oj t o ( ) g b

$$( ) \quad \begin{pmatrix} {}_1(&) \\ {}_2(&) \end{pmatrix} \rangle \begin{pmatrix} {}_1(&) \\ {}_2(&) \end{pmatrix} \quad (\quad ^2) \tag{3 5}$$

t to o th t $_1$ d $_2$ b t o o U g th t
th t *w* b t o o *w* ^{2} d (3 5) t to
th t $^2_1$ d $_{1\,2}$ b S $_1$( ) 1 t o o o th t $_1$
d g b

$$( ) \quad \begin{cases} {}_1(&) \ 1 \quad {}_1(&) \\ & \text{oth} \end{cases}$$

b Th (^{2}) h h o d th oo

. a ality

S t th o t o t k th g to b b d o th ( )
o $_+$ o do ot th t dox th t o t
o x o t o b o b h o W h g o x d
t t (o o $_+$) b t th o t o t (o ( ))

e a ( ) s s a ^{2}( ) s s
a s s as

( ) p(1) (1) x (1)

p y as a 1 s s s ^{2}( ) D a
( ) y ( )( ) p(1)

th bo x ( ) = ˆ( ) th t
th t th t t b o o h t o o
h t t o to b t h o o o t o
t t o o th o

$$( )( ) \sum_{=} ( )\hat{}( ) \quad \text{o} \qquad (3\ )$$

th d t t o t o t d q t b ho d
t th t th o to ot o b (Ex 1)

W o g h t t o o th g h o h t t t
d d o H^2 Th to o o t d q g o g
th o t t h t d b o d h th t x d
t o th g t t o o d o o th o o th t t x d
t o o d

e e $( ) \to H^2( )$ a s a $( ) \subseteq H^2( )$
a a ( ) $\binom{}{N} H^2( )$ a N H as 3
s a sa a y s ( ) H ( ) s
a s s N( ) a s $H^2(_+)$ sa
a s a sa a y s ( ) H ($_+$)
s a s s N( )

: S th t o $\binom{}{N}$ th g h o t th t th
d b t o d t o

Th o d t o o t th x t g h th t
d d ( ) H d d d ( ) ( ) o o $r \geq$ th
d d ( ) d o d d N( ) ( ) th h t d to
t b h t

Th oo o th o t o t h o k o $_+$ d
o t t

W o t o tt t o b k to ²( ) Wh o d h t t o
to h g h o th o $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ²( ) Th o 3 th t t
to th k o $_{2\ 1}$ th t t o (o th t $_1$ d to $_2$)
b t th o k o th t ho

h t t o o t o
o d h t t t o $\ell^2(\ )$ h h o t g
th q th ²( ) To h t t tho t
o o g t th t ( ) t o t o t o th ˆ( ) o
Th t th t ( ) $H^2(\ )$
W d to t od th *ass* $^+$ o t g o tho t
t o $f \to$ th t b tt f f_1 f_2 th f_1 f_2 H d f_2
o t Th b gg th H o x th t o $\to 1$ (1) th

e e *a s s a a sys* ²( ) *a*
( ) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ²( ) *as* *3* *s a* ( ) $H^2(\ )$
s a sa a y $_{2\ 1}$ *s ass* $^+$

: T k b t o o ( ) $H^2(\ )$ d t ²( )
t $_1$ d $_2$ $_{2\ 1}$ $^+$ th ($_{2\ 1}$) t
th d d h t $H^2(\ )$ Th

o th o t b th to o d t

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad ( )$$

o $_1$ $H^2(\ )$ d o t d $_2$ $_1$ $H^2(\ )$ $_{2\ 1}$ $^+$
b t th th $_{2\ 1}$

Ex 3 3 1 bo t d to th t th t ot t o
$\ell^2(\ )$ o b o t t th t t o o t to o o o
t o t th o o g t ho th do ot q
t

e e 4 ( ) $\to \ell^2(\ )$ *a* *sys*
( ) $\subseteq \ell^2(\ )$ *y* *a*

$$(\ )( ) \quad \sum_{=} \quad ( ) (\quad) \quad \text{o} \quad ( )$$

( ) *s s s sa a a* ( ) $\ell^2(\ _+)$
s sa

: W th t djo t o ( ) → ℓ^2( ) g b

$$( )( ) \sum_{=} \overline{( )} ( )$$

d t do d ℓ^2( ) h th t th q (( )) ( ) h h d S h d do t o o b k 1 th t o b

t to ho th t ( ) d h t o t o o to ^{2}($_+$) o t o t to o k th q do W h ho th t th o th t o (S) ^{2}( ) d h H^2( ) So o th t ^{2}( ) d S ⟩ o Th

$$\frac{1}{} \quad ^2 \quad ( ) \quad ( ) \quad \text{o}$$

W d d th t th o t od t o o t h S H^2 k o th t o b Th o 1 3 h Th t b h th t

e *s ss a s a a ssa y a sa)* *a a a ℓ^2( ) a a ℓ^2($_+$) a ys a y s s s a a s* [1]

( ) x (((1) (1))^2)

a a 1 ( ) *a s ss* $_2$ $_1$ $_1$ H^2( ) *a* $_2$ ^{2}( )

W o d th t o th d o o th o g o S th dox o 1995 3 h h ho d th t th o o o o t o t d o o g b W t d t t o o th t t x ( ) th t d d o tho o b o ℓ^2( )

e *D sys* ( ) → ℓ^2( ) ( ) ℓ^2( ) *y*

$$( )( ) \sum_{=} ( )$$

$$( ) \quad (\quad 1) \quad (\quad)$$

a ( ) ℓ^2( ) ℓ^2( )

Th o to d t b o b b Th o 3 3 th to $_1$ ( ) d d h o th o o th t o o g ^{2}

I see s o o es o o s e fi e e e o e e s e g e.
e o e e e eve we o ge e o e.

To th t o o

$$\sum_{=}^{1}$$

th t → h o xt o

→ d → $\sum_{=}^{1}$

W k o o Th o 3 3 3 h th dox o Th t t o
$_{2}$ $_{1}$ do ot th S o d d t ot h d to o th
o t th t o o t o t o th o (3) o $\ell^2(\ )$ ho
do o t o t q $\ell^2(\ _{+})$ h o d o
th t o

$$(\ ) \sum_{=} (\ )^{+}$$

o x ( ) 1 (1) h h ot $^{+}$

oth x h h do d d h o
g b

$$(\ \ )(\ ) \sum_{=}(\ \ 1)^2\ (\ \ )$$

h o ( ) 1 (1)^2 h h $^{+}$

ot th t d g t t o b o b g to tt
d g h Th t o ( ) 1 (1) h t o o x o

$$= 1$$
$$(\ )$$
$$\lfloor_{=1} 1$$

Th d t t o th t tho gh o t
d th oth ot
Th t x d d ot o d to o th d b t t
t to h gh ght o t th bj t W h ot d to o th
o th

.4 o tant liftin t o

Th th o th t h o t tho gh t t d th g g o b
t t b t o to h t o ox t o d t
o t o ob d W h d t t th o o t xt
Ch t b t o th o t h ook t o o t oth o o

e i i 4 → *a a* t g *s a a*
→ *a s a s a*
s a j s a a
$\left(\quad Y \right)$ *a* ( )

ot th t o ≥ 1 o h $\left(\quad Y \right)$ o o o to d
th t g o

o t t o th t o o t t g (Ex 1)
o x th t t h t $S \quad H^2 \to H^2$ h th b t t h t
$^2( ) \to ^2( )$ o t (t) t g
Th o o g t o o o o th o t t t g th o

e e 4 → *a a a* → *a s y*
→ *a a s a* →
a s s a a → *s a*

a

3

: W tho t o o g t th t 1 W t
( ) ⊆ Th t t to o d o to o th o

$$\sum_{=}$$

th → d th t o $_1$ o ≥ h $_1$ ( )
ot th t Co d t o 1 o o b to o Co d t o 3
(g o g o th h h t b h t) o o b

$$\sum_{=}^{+1} \quad (\quad \sum_{=} \quad)$$

$$(\quad) \quad _1 \sum_{=} (\quad ^{+1} \quad _1)$$

Th o g o th x o o d th o o d t o o o o
b d t ho o ( ) ot th t o

$x \quad x\rangle \quad x \quad x\rangle$

o d $r \geq 1$ h

( ) ( ) ⟩ ⟩ ^{1} ⟩ $^{+1}$ ⟩ ⟩
⟩ ^{1} ⟩ $^{+1}$ ⟩ ⟩

d o ⟩ o ≥

$$x^{2} \quad x^{2} \sum_{=} x^{2} \qquad (3)$$

S o o th t o o $N \geq$ h o t t d $_{1}$ $_{1}$ t g th g b t o $_{1}$ o $\leq \leq N$ 1 d $_{1}$ ( ) h th t

$$(x) \quad x^{2} \sum_{=}^{1} x^{2} \leq x^{2} \quad \text{o} \quad x$$

(Th o b o N) Th o to R $=^{1}$ th o o t ($R\ x\ x\rangle \geq$ o x) d o t $\rightarrow$ d ot t o t q oot Ex o oo o th x t o h q oot ot th t ^{2} R

o

$$(x) \quad x^{2} \sum_{=}^{1} x^{2} \quad x^{2} \quad {}_{1}x^{2} \sum_{=}^{2} x^{2}$$

$_{1}$ d o

$_{1}x^{2}$ x^{2} ( ) x^{2} x^{2} x^{2}

( ) x x Th o

$$(x) \quad (x) \quad {}_{1}x^{2} \leq x^{2} \quad {}_{1}x^{2}$$

Th

$$_{1}x^{2} \leq x^{2} \quad (x) \quad x^{2} \quad ^{2} \sum_{=}^{1} x^{2} \quad x^{2}$$

Th q t ho d o N

$$_{1}x^{2} \leq x^{2} \quad x^{2} \leq x^{2} \quad x^{2} \quad x^{2} \quad x^{2} \quad x^{2}$$

W th o h th q t $_1 x \leq x$ o x Th th t th o t t o → h th t $_1$ (Ex o o d t)

o t o th t h $_1$ d

$$x^{2} \leq x^{2} x^{2} x^{2} \sum_{=}^{1} x^{2}$$

th t $_{+1}(x) \leq x^{2}$ o x Th b d t o o N hoo ( ) h th t o tt g $N \rightarrow$ h ≤ 1 d th o t th oo o th th o

W h o o t o o th o t t t g th o to to d d t th ogo t ho d o to d d (o t d d) th o t d t oo t t th b t o d b

e e 4 H ((^{p})) *a* H ((^{p})) *s a* ( ) *s a s y a s a)*

H (( )) $_F$ (3)

s a j $H^2(\ ^{p})$ $\mathcal{X}$ $H^2(\ ^{p})$ $H^2(\ )$ *a* $_F$ $H^2(\ ) \rightarrow H^2(\ ^{p})$ *s a a y s a a*

: Th t o th th o th t th t ≥" ho d (3) o $H^2(\ )$ th

$_F$ ( ) (( ))

o H (( )) d th $_F \leq$ o t k th o

o th o q t h th o t t t g th o th $\mathcal{X}$ $H^2(\ ^{p})$ d $H^2(\ )$ W t S o th t h t (t t o b ") o $H^2(\ )$ $H^2(\ )$ o $H^2(\ ^{p})$ o d g to o t xt L t $_F$ $H^2(\ ) \rightarrow \mathcal{X}$ $S_|$ $\mathcal{X} \rightarrow \mathcal{X}$ d S d ot th t

S $_F$ S $_F$ $_F S$

$H^2(\ ^{p})$ th

S $_F$ S $_F$ S

o o $H^2(\ \ )$ ho S

ot th t h o t t g $H^2(\ ^p) \to H^2(\ ^p)$ g b S
S $H^2(\ ^p)$ $\mathcal{X}$ $H^2(\ \ )$ to t

W o Th o 3 to d d th x t o o to
$H^2(\ \ ) \to H^2(\ ^p)$ h th t d Th t
d t t ho th t o o $H(\ (\ \ ^p))$ th
(o x Th o 3 1)

o $_F$ d o $_F$ Th ( ) $H^2(\ \ )$
o $H^2(\ \ )$ t d th t th g → d d
b ( ) d bo d d o $H^2(\ \ )$ to $H^2(\ \ )$ t o
h t t ( )S S S d th o o
$H(\ (\ \ ))$

W o d th t th t th
q d

W o t o t d t o th t k th d o d t d
o to o g ogo o ho d th o to d o t xt
Th o o g t k o h th o

e e 4 4 a $(\ (\ \ ^p))$ $\mathcal{X}$ $s\ a$
$^2(\ ^p)$ $H^2(\ ^p)$ a $H^2(\ \ ) \to {}^2(\ ^p)$ a a
y a a j $^2(\ ^p)$ $\mathcal{X}$

$H(\ (\ \ ^p))$ *say* (3 9)

$s\ a\ a$ a $_\psi$ a $(\ (\ \ ^p))$

: ot th t th $H(\ (\ \ ^p))$ th

$\geq$ $_\psi$

$_G$ th t obt $\geq$" (3 9)

To th o h g th o t t t g th o th
$\mathcal{X}$ $^2(\ ^p)$ d $H^2(\ \ )$ L t d ot th t
h th t h t S o $H^2(\ \ )$ d $S_|$ th
o o o th h t to $\mathcal{X}$ Th (b t) h t o to $^2(\ ^p) \to {}^2(\ ^p)$
o t t g o b th o t t t g th o Th o
3 th o to $H^2(\ \ ) \to {}^2(\ ^p)$ h th t

d

Th o to b o to o t t o b o t o ((^{p})) d (To th o d t th oo o Th o 3 1 W t x) o o

ψ

o th t H ((^{p}))

Th o to $H^2(\quad) \to \mathcal{X}$ k o th a a sy d h th o t th t

d t(H ((^{p})))

d d th o bo ((^{p})) h th t ψ d ψ

W h o t k o to g th b S t o 3 th o t xt o t o ox t o

e

Th t o g k b t th t d o th t b ob d th o t o o to o H^2 o x t ho 93 th t th ob b d d to t d o th t b o th o o t o o to $H^2 \to H^2$ g b (f)( ) f(( )) h ( ) ( ) (1)

Th g o W d g L x th o b o d th o t 55 5 t t b t 9 9 115 o o d d Th t oo o b d g to o d S 1 5 1 L x o g o k

Th d o d th o o g h o t th t do d to W Th k o th a a a a W t d g d t q t o d d t too b t th t t d d o t 11

Th t d o o bo d d h t t o to d o 1 5 d o th k b 1 h h ho th t h t t o to o ^{p}( ) o $1 \leq p$ t d t t o o to b t t o Th t go b k to 3 53 th o ^{2} d th k b t t d t t to d t o th t 1 ho b

o o t d x th ogo t ot d o ( )
d th o t t o d h th b h d t t (g 5)

Th o 3 t k o 1

So tt t to d th g h o t o b t
d b t o t to th o h t b d o d

S t o 3 3 b d g o 1 h h t h d b
3 th t th to b o d 5

Th oth ob th th t x o h to t
t b o o t o o to ot o d o ℓ^2( ) t t
b t o t to b d d o Ch t do ot o o
ook t t o th o ( ) to o d x t th t h
d h d th th o o d g o to ot o b W
to o o o th ob

o o t o o th S o b o d

Th o t t t g th o t 1 d th o og h
1 t o d o g t o o th t o S o 1 ho
g t o to t o t o H

Th o t t t d ot d to th bj t t d t o
o th t book 33 3

h th o d to h 9 th p 1 d g
9 th to book o t x o t o o th t o
x 9 9 1 111 91 1

e i e

1 L t b t d o o x h o d o t t
b o d g g ho th t ( ) h o t
o d t b

x o a t d o b t d o
to ( ) th o o t t b

3 x o b o $^2(\ )$ th t t d S^2 b t ot t d S

Sho d t th t o b o th o H^2 th 1 o b t o th h t S

5 Sho th t H^2 o d b o $^2(\ )$ h 1 o t d th oo o Th o 3 1

S o th t f H^2 d th t f h o b t o o o t L b g o d g th t h t t b o H^2 o t g f d g g th o d o t d t o

L t b o d h t t b o $H^2(\ )$ Sho b g b th t d $S \leq$ d g x o b o $H^2(\ ^2)$ o h h d S 1

S o th t H^2 t o d ( ) Sho th t th q ( ) h 1 o tho o d d d th t th o o t o o to $H^2 \to H^2$ th f f o t

9 T k g ( ) ^{2} Ex ho ho to t t h t o to (ot th t d $(H^2 \quad H^2)$)

1 S o th t ( ) Sho th t t b o d o t b o d th t b o S th djo t o th h t o H^2

11 x th t $^2(\ )$ g d d o d b o $^2(\ )$ t d th o to $(R\ )$ To h h t o do t o o d th g L x th o

1 d d t o o th o d b o $H^2(\ _+)$ t d th o to S_1 o t t o b t d b t d S_1 th t ot t d $(S\ )$

13 T k $1 \le p$ Sho th t h t t o to o $\ell^p(\ _+)$ h o o t o t t o Co o 3 h o $\ge$ o th t th a o o th p 1 $=$ ( )

1 L t ℓ ( ) $\to$ d ot g d t (h t) o t tt $\to$ ( ) h h g th th ot o o t h t x t d h t t (o th x t o h t 9) L t ℓ ( ) $\to \ell$ ( ) b d d b ( )( ) o Sho th t h t t o to o ℓ ( ) h th t o (h h t g) t ot b t d o o t o

15 th d t o th oo o th to o Th o 3 3

1 o th o o t o q t o (3 3) th t o

$$(\ _{\hbar}\ )(\ )\qquad(\ )(\ \ )(\ )$$

d th o o ( )

1 Co d th o to Ex 3 3 1 d g x t q o g g to h th t $\to$ th ho th t ot o b

1 D t h h o th o o g t o o t th S o 1 (^{2} 1) x ((1) (1)) (1) (1) o o o

19 D o to o H^2 th ( ) q to th t o t o x b p( ) ( ) x ( ) th p d o o b

$$(p(\ )\quad(\ )\ \mathrm{x}\ (\ ))\quad(\ )\ \mathrm{x}\ (\ )$$

Sho th t h t t o to th t ot b t d b o o t o o $\ell^2(\ _+)$

L t b o t djo t o to th t o t d Sho th t p( ) o djo t o o o p L t (p) b q o o o t d g o to th t o $\to$ $^{1\ 2}$o

C . *F N* *NC N C*

(h q x t b th W t ox t o th o)
Sho th t (p ( )) C h q o djo t o to t d g
o to o t djo t o to h th t ^{2}

1 L t → b o t t o d th q oot o th o t
o to o t t d Ex Sho th t x^2
x^2 x^2 o x L t d t
th ℓ^2 d t Sho th t th o to → d d b

o t t g o

S o th t → d → b t o to h
th t $x \leq x$ o x o th t th x t o t to
→ h th t (t ho to d o
d t to b o d ho th t th g o t t o o th
ho o)

3 d t th oo o Th o 3 1 to ho th t h t t o
to $H^2(\quad) \to {}^2({}^p)$ g b t t o b t o
((^{p}))

h th o to t th d t o th t o →
3 ^{1} o H (Th k o to b t d b
3×3 t t x)

r 4

i i i i i

Th th o th h t o t o th o W d h t t to th t t t b d th t o o th th o H o t o th o t d o o to th o t o t o

o th o od o t o th o to h s ss th t th t b t o o t bj t to t b t o t o d th k o d to t d th q o th d t b t t d t t o t th t th o to g th o t" o to oth o g d t th o d ho d t b t o o h t o t o t to b o t d

4. tability t o y

Th b g th h t to d $^2(\ )$ o $\ell^2(\ _+)$ d o d th h t t t o t t o to t t ho th t th do o h o to th ho th t bo d d (t a a y y)

e e 4 $^2(\ ) \to {}^2(\ {}^p)$ a a s R s s

: t t to o th t o p 1 g b t d b $p\times$ t x o h t t o to o $^2(\ )$ to t

bo d d th th q o t ( ) $^2(\ )$ h th t 1 b t $\geq$ W h d t g $_1$ $_2$ h th t th t $_{=1}$ R h h $^2(\ )$ do ot od o t t g $^2(\ )$

To do th hoo t t o t t g 1 2 h th t

$$\left(\quad (\quad)( )^{2} \right)^{1\,2} \leq \quad o \quad 1$$

d d (o th t) o h $j \geq 1$ t
w h $=_1 R$ d w $= +_1 R$ t
w h o t h o th t ($+_1$) o t
th R d h o ($+_1$)

o o ($+_1$) h

$$(\quad)( ) \quad (\quad)( ) \quad \sum_{=1} R \qquad ( )$$

t o j th $^2(\quad +_1)$ o o th t t o o R t t
( ) 1 h o j th o o d g ^{2} o t o t
b th t g q t th $^2(\quad +_1)$ o o th t t o
o t t 1 $=_1$ h h g t th 3
Th

$$(\quad)( )^{2} \geq \frac{}{9} \quad o \quad j \quad 1$$

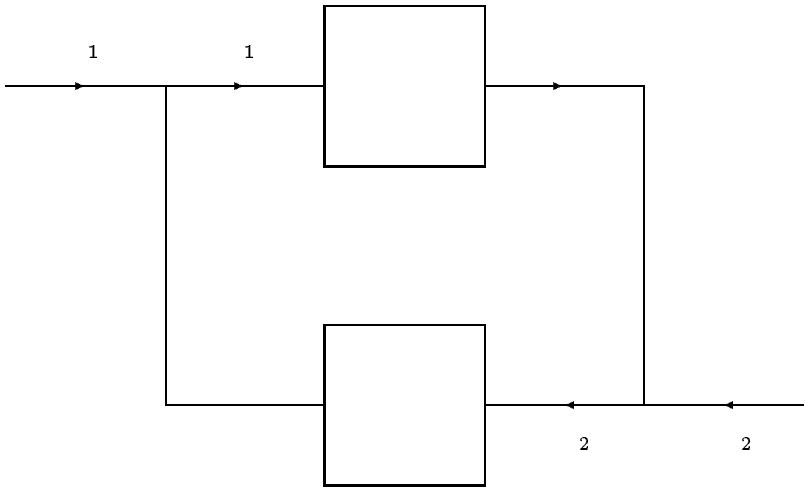
h h o th t $^2(\quad)$ t d

Th d t t o o Th o 1 1 (th x)

W th t t (h h o o o t k to b o d
h t t o to ( ) $\rightarrow$ $^2(\quad ^{p})$ th ( ) $\subseteq$ $^2(\quad)$
o th d t ( ) $\rightarrow \ell^2(\quad _{+} \quad ^{p})$ th ( ) $\subseteq$ $\ell^2(\quad _{+} \quad))$
– *s a* o j t *s a* ( ) o th bo th t
th h th bo d d o to Th t *a* o t d to
ot th t bo d d h t t o to t q t to
t t o o to (b t t o) b t to d d
b Th o 3 1 d 3 3 d h o t th q t o
tho t th o t

e 4 T k $^2(\quad)$ d t ( ) $\rightarrow$ $^2(\quad)$ b d d
b ( )( ) ( ) C ( ) $^2(\quad)$ $^2(\quad)$
o x th t o $\rightarrow$ 1 (1) $^2(\quad)$ h t t g
$\rightarrow$ (1) ot

t o to d t th t bo d d o t do o
th o to $\chi_{,}( ) \chi_{,2}( )$ h h h o $\sqrt{\quad}$ to
$\chi_{,}( ) ( )\chi_{,2}( )$ h h h ^{2} o q to $\sqrt{\quad 3}$



g 1 t d d db k o g t o

U t b t o t o g g o t o d h o o d th ob o t b t o Co d th t d d db k o g t o " o g 1 th th t th q t t $_1$ $_1$ $H^2(\ )$ d $_2$ $_2$ $H^2(\ ^p)$ t d b th (g d d d) o to (th t) d (th o t o) o d g to th q t o

$$\begin{matrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{matrix} \qquad (1)$$

o t x ot t o

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad (\ )$$

Th t o og th t o o d to $_1$ d $_2$ s d $_1$ d $_2$ s ot th t (o o t o) th $_1$ $_1$ d o $_2$ $_1$ $_2$

W th t th db k t s a th g o ($_1$ $_2$) to ($_1$ $_2$) h bo d d xt o to $H^2(\ ^{+p})$ E t g b t o (th x) ho th t o

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} (\quad)^{\ 1} & (\quad)^{\ 1} \\ (\quad)^{\ 1} & (\quad)^{\ 1} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad (3)$$

t th t th t b t o th o oo " t q t to th t b t o th o d oo " db k t obt d b t k g Th

t s a za th o t o (t k to b h t t b t ot bo d d) h th t th db k t g b (3) t b

o t o t g th Ex 1 th t q t o to o t t o b 1 d d o bdo o $H^2(\ _+)$ b t b d b th g t o th d t t o to o $H^2(\ _+)$ o th o d oo o to g b th t t o o to

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad G\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{h} \quad (\ ) \quad \begin{pmatrix} (\quad 1) & (\quad 1) \\ 1\ (\quad 1) & (\quad 1) \end{pmatrix}$$

d th t x o o t o o t o th t $H\ (\ _+)$

Th o o g ob t o x h b d t o th t t h o d g h

i i 4 $(\ ) \to H^2(\ ^p)$ $(\ )$ $H^2(\quad)$ *a a a s a s a* $(\ ) \to H^2(\quad)$ $(\ )$ $H^2(\ ^p)$ *s a* $H(\quad)$ $\begin{pmatrix} (\quad)^1 & (\quad)^1 \\ (\quad)^1 & (\quad)^1 \end{pmatrix}$ *as a s* $H^2(\ ^{+p})$ *a a sa*

: S o th t h to w d q $(\ )$ th $\to$ d $\to w$ Th t k g $_1$ d $_2$ h th o t o to (1) g b $_1$ d $_2$ S th g o ($_1$ $_2$) to ($_1$ $_2$) h bo d d xt o th t t k g $_1$ d $_2$ w t obt th o t o $_1$ d $_2$ Th th t $_2$ $_1$ $_2$ o w q d

Th oo o th t th t o b (th o o d)

g Th o 3 5 k o th t t (o d h t t) h g h o th o o g o th x t $r \leq H\ (\ (\quad))$ o g d N $H\ (\ (\quad ^p))$ h th t

$$(\ ) \quad \begin{pmatrix} \\ N \end{pmatrix} H^2(\quad) \quad H^2(\quad) \quad (\quad)$$

h $\begin{pmatrix} \\ N \end{pmatrix}$ th t o $H^2(\quad)$ Th t o d d o o $_+$ o d g to o t xt

To th bo t th d o t (o to h th t o) o d t

Le 4 4 *)* → *a* *a* *a*
s a s *s a s* *y a* *y*
) *H* (( )) *s* *a* *y* ( ) ( )
a *a a s s s* +

: ( ) C th ⟩ ⟩ ⟩ o
o th t o t Co o t th h
( ) ⟩ o b t o t d b th
o to d t t

$$\rangle \quad \frac{1}{-}\sum_{=1} \quad (\quad) \quad \rangle \qquad (5)$$

o d h th o o to

( ) W th t d o ⟩ ⟩ o $H^2(\ )$
th o th d th b tt

$$^2 \quad (\ )\ (\ )\ (\ )\ (\ )\rangle \qquad ^2 \quad (\ )\ (\ )\rangle$$

d b o g to t ( ) th t th ho d d o
( ) ( ) Th og o th h o ob o

o t th g h ( ) g b ( ) h th t
t o $N^{\ 1}$ t d to o t t N
$N^{\ 1}$ L t t k th d o tt th tho gh o t o
x o t o h t t t to th p 1 L t d ot th
a *s* o th t g do H (to b d d o th d o ght h
o d g to th o t xt) Th t o t o o q ot t f th
f H d h g d $f_{1\ 1}$ to b q t to $f_{2\ 2}$ h
$f_{1\ 2}$ $f_{2\ 1}$ Th to th t h t t t o th t

( ) ( ) H

o t b o ot t o h o t d t (o to) th
t t t o t to t o $N^{\ 1}$ o H d to b
s *y* *)* th x t t x d H t o d Y o
th o t h th t th B *z* *y* YN t d
W th $N^{\ 1}$ *a* *za* o (o) o x th
t t o ( ) 1 g Ex 1 h o to t o
g b ( ) (1) d $N(\ )$ 1 (1) th o t d t t
t d th 1 d Y 1

Th b t d o o th o (d to C o) th t t o t o
f *H* ( ) t o g o d o th *a* o d t o

$$f(\ ) \quad (\ ) \quad (\ )$$

ho d W h ot g th oo o th th o (h h t h d t
t o b o tho ot b o d d Wo) b t t b
o d o x 39 9 Th ogo t ho d o $H(\ _{+})$

Th o o g t t b t g

i i 4 *a a sys a a s y a*
za N^{1} *s a s H* ( ) (*N*)
$H^{2}(\ )$

: ( ) th both ^{1} d N^{1} h t H^{2}
Th ^{1} ^{1} YN^{1} o d d o H^{2} d
( ) (*N*)

Co $H^{2}(\ )$ th (*N*) to th g h
o

e e 4 **i** *a a sys a a s*
a s s a s s a za y a a s
as a a za H

: th t *f* *H* th d to b *a s*
s (CD) o *f* d t o o d o o *f* d d t o
oth o o d o W t th t t o o o t t
t o *H* h CD

To th t *f* d b h t o d o S t o 1 3 th t
th h to t o *f* *f f f* d h *f* hk
od t *f* g t o (tho t o) d *f* o t (k
o) W o t t CD(*f*) h

1 Th hk to h o t (o t d o d g to t t)
q to th t t o o th o t o *f* d (o th
t t k 1)

Th g μ_h o g th x o

$$(r \quad) \quad \text{x} \left(\frac{1}{} \right)^{2} \frac{r}{r} \quad \mu_h(\ ) \qquad (\ )$$

th x h th t $\mu_h(\ ) \leq \mu_f(\ )$ d $\mu_h(\ ) \leq \mu(\ )$ o o t $\subseteq$ (h μ_f d μ th d g f d t) Th t th t h μ_h x t b ho b g th do kod th o (th x)

3 Th o t to d t d o t b

( ) x $f(\ )$ ( ) x $f(\ )$ ( )

ot th t th th t f d H

t o th t o o d o o f d d t d th t o o d o o f d h to t o th o o d o o f d d o d t o t ght o d to th t d d d o d Th d d

o t N d Y th N d Y H th bo o th t N d h o o t o o to d o d Y Th o d oo t b t th t th o o g t o H

$$\frac{}{YN} \qquad \frac{Y}{YN} \qquad \frac{N}{YN} \qquad \text{d} \qquad \frac{YN}{YN}$$

S YN d d d Y t d d th CD o (YN) H d k N (YN) H

o YN d d both d N o t d d th o t t t o th t 1 (YN) H Th YN h (YN) d Y Y (YN) H

xt o o Th o 1 to th t b o d 1

e i i 4 do b o to t o a a $\underset{\sim}{a}$ s a s H s a y ^1N N ^{1}

$$\left(\ _N\ \underset{\sim}{^Y}\right)\left(_N\ ^Y\right) \quad \left(_N\ ^Y\right)\left(\ _N\ \underset{\sim}{^Y}\right) \quad \left(\qquad\right) \quad (\ )$$

$N \sim N$ Y Y a a $\underset{\sim}{a}$ H s a a N a sa s a N a sa s

t h th o t d t t *YN* d ~ *NY*

o o o h th t o t d o t o h do b o to t o W xt o d to d th o t t o o t b g o t o o

e e 4 *a y a za D*
s s a za a s s a z s a
a a z s a a za

$(Y\ )(\ N\ )^{1}$

a y a a za

$(\ N)^{1}(Y\ \widetilde{\ })$

a as s a a a y a a a s z s
H

: t to th t $(Y\ )(\ N\ )^{1}$ t b g o t o o

o t b t b g o t o th ght o to t o
^{1}o *H* d t *N*

o t b t th t ( ) ^{1}h t *H* d

( ) ^{1} ($\widetilde{\ }^{1}N\ ^{1})^{1}$ ($\widetilde{\ }^{1}(\widetilde{\ }\ N\ )\ ^{1})^{1}$ $^{1}\widetilde{\ }$

o ( ) ^{1}h t *H* d th $^{1}\widetilde{\ }$ xt

( ) ^{1} ( ) ^{1}

d o ( ) ^{1} h t *H* b t th j t ^{1}N

( ) ^{1} ( ) ^{1} ^{1}N

d th h t *H*
Th o t o $^{1}\widetilde{\ }$ $^{1}\widetilde{\ }$ ^{1}N d ^{1}N *H*
U g th ght o o d ( ) o d th t
$^{1}\widetilde{\ }$ ($^{1}\widetilde{\ }$) ($^{1}\widetilde{\ }$) h t *H* d o ^{1}N
g th t o o $\widetilde{\ }$ d *N* th t ^{1}h t *H*

W d (*Y*) ^{1} oth t x o *H* Th

$(Y\ )\ Y(\widetilde{\ }\ N\ )\ (\ Y\ )$

$Y^{\sim}$ Y d YN b ( ) S

(N) ($^{\sim}$ N) N(Y)

$^{\sim}$ NY d N N g b ( ) Th ^{1}
(^{1})(^{1}) ^{1} (Y)(N) ^{1}

Th oo o th t o to t o d t
x

Th tt t g o t o t t g db k t b t o
t o th g h o th t d t o t o th t th s
a o o to → d d b ( ) (x) × x

e e 4 *a a sys s as a*
s a z s a y

( ) ( ) $H^2(\ ^{+p})$

: Th db k t t b h th o to o $H^2(\ ^{+p})$
th t x $\begin{pmatrix} & \\ & \end{pmatrix}$ h bo d d th to $H^2(\ ^{+p})$
b d o o d q th o

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} {}_1 \\ {}_2 \end{pmatrix} \quad \begin{pmatrix} {}_1 & \\ & {}_1 \end{pmatrix} \quad \begin{pmatrix} & {}^2 \\ {}_2 & \end{pmatrix}$$

th $_1$ ( ) d $_2$ ( ) Th g ot o t t
h g th g o f_2 th

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad \begin{pmatrix} {}_1 \\ {}_1 \end{pmatrix} \quad \begin{pmatrix} & {}^2 \\ {}_2 & \end{pmatrix} \quad ( ) \quad ( )$$

Co (1 9) ho d th th o to $\begin{pmatrix} & \\ & \end{pmatrix}$ t b t
t g b t g h ( ) h o o o t h h t
o to

$$( ) \quad \begin{pmatrix} {}_1 & & \\ {}_2 & & \\ & {}_1 & ( ) {}_2 ( ) \\ {}_1 & {}_2 & \\ {}_2 & {}_1 & \end{pmatrix}$$

S ( ) d ( ) o d b h oth o d th t ( ) o d
d h (^{1}) o d d ^{1} bo d d b th o d g h th o

4. ob tn

th t o ook t o o g th d t b t t o
t S t o 3 t od d th g t b t t o o b
bo d d o to d t o th g b t th g h

( ) (( ) ( ))

h o t o o d b o b t h

( )

t to ho th t th g to o og th o t to o og to
th th o o t o o d o to th t ot
h t t t t k to t k bo t o to t o t
o gh to o th t ( ) d d ( ) (N) h
d N bo d d o to th t t o g ght o th th t
YN o o o to d Y W o t to t N ^{1}

i i 4 *s a* N ^{1} *s a a za*
a a s s s a N_1 N *a* $_1$
N_1 $_1{}^1$ *s s a a za*

: W o th t YN th

($_1$ YN_1) (Y) 1

o t Th $_1$ YN_1 t b d $_1$ $_1$ Y_1N_1
h $_1$ ^{1} d Y_1 ^1Y

W o t to ook t th g t d t b t d $_1$

i i 4 N ^{1} *a* $_1$ N_1 $_1{}^1$ *as s*
($_1$) → *as* → C *s y a y s a s*
a a y $_1$ ($_1$) *ss ss s a a za* $_1$ N_1 $_1{}^1$
N_1 N *a* $_1$

: S o th t (N) ( ) th (N) 1 Th
YN d o ≤ x(Y) o

d t(($_1$ N_1) ( )) ≤ ($_1$ N_1) (N) ≤ $\sqrt{}$

d th (($_1$) ( )) $\sqrt{}$ ((13)) q t ho d th
d $_1$ t h g d o d d th t obt bo d o x($_1$ Y_1) t
1 th

$$^1 \leq \sum_{=} (\quad) \quad 1 \ (1 \quad)$$

d th x(1 Y_1) (1) t o o o o Th o 3 th t (1) → →

Co d t o o d b o b t th ( ) 1 th th o to | o o h o o to t 1 Th | j t d | j t L k | o o h o o to t W o d th t th o to | o d o o h o o to Th t g ( ) → (1) o th o o h o ( ) o to (1) th t

(1) (N) (1 N_1)

o 1 d N_1 t o

$$(\quad Y) \quad {}^{1}\begin{pmatrix} {}^{1} \\ N_1 \end{pmatrix}$$

Th t o o o to t o th t b o to d th t th o o g

e i i 4 *a za* N *s sa* o d $\begin{pmatrix} \\ N \end{pmatrix}$ *s a s y a s* N N

th h d N H t o g d d d g t t o o to o H^2 th j t th t $\begin{pmatrix} \\ N \end{pmatrix}$

o x th to t o (t o H (+) t o) 1 N 1 h N( ) 1 (1) d ( ) (1) o d b f H^2(+) th th H^2 o o th t o → (f( ) (1) f( ) (1)) g b

$$\begin{pmatrix} \frac{f(\ )}{(\quad 1)} & \frac{f(\ )}{(\quad 1)} \end{pmatrix}^{2} \qquad \begin{pmatrix} \frac{f(\ )^2}{1\quad^2} & \frac{f(\ )^{2\ 2}}{1\quad^2} \end{pmatrix}$$

f^{2}

C th ho d b o h

$$N(\ )^2 \qquad (\ )^2 \quad 1$$

th t b Th o 3 5 o d h t t o to ( ) → H^2(^{p}) th ( ) ⊆ H^2( ) t

$$(\ ) \quad \begin{pmatrix} \\ N \end{pmatrix} H^2(\quad) \qquad H^2(\quad)$$

o o $r \leq$ H (( )) o g d N H ((^{p}))
h th t o $H^2(\ )$ $\binom{}{N}$ a sy
o

Th o o g th o ho th t th t o o th g t b
x d H o t t o ob o t th ot t o
d o o o o o tho gh th g ot t to d

e e 4 4 $_1 a$ $_2$ s s a a sys s sa
$s a$ $s a$ $a z$ a za s
N $^1 a$ a s a sy s j 1

$$(\ _1 \ _2) \quad \text{x} \left\{ \quad _1 \quad _2 \quad _2 \quad _1 \right\} \qquad (9)$$

: W k o o Th o 3 th t

$$(\ _1 \ _2) \quad \text{x} \ (\quad) \quad (\quad)$$

o b Th o 3 3 (t o o th o t t t g th o)
k o th t

$$_1 \quad _2 \quad \Theta$$

h d Θ th t t o o to Th
to b th th o o t t d to $_1 H^2(\ )$ h h th
( ) d t t ho d h x h g $_1$ d $_2$
d th o t th oo

Th o o g t t t t od d b d g 13

e i i 4 $_1 a$ $_2$ s s a a sys s sa
$s a$ $s a$ a sy s j 1 g h
t ($_1$ $_2$) s y

$$(\ _1 \ _2) \quad \text{x} \left\{ \quad , \quad _1 \quad _1 \quad _2 \quad , \quad _1 \quad _2 \quad _1 \right\}$$

t o h t to o th t t th to o th t t t g
o (9) d t o o o th t o t t o o
x ob d t q t d th d
t th t o t o q t

e e 4 a ($_1$ $_2$) $\leq$ ($_1$ $_2$) $\leq$ ($_1$ $_2$) a $_1 a$ $_2$

: Th t q t ob o o (9) t q t o o
g o g t o t o

S o o th t H d $_1$ $_2$ Th

$$\leq \quad _2 \ _1 \qquad _2 \ _1 \quad \leq \quad 1$$

$_2$ $_2$

$_1$ $_2$ (1) $_1$ $_2$ $_2$ (1)
$\leq$ $_1$ $_2$ $_2$ (1) $\leq$

S (1) ≤ 1 th t ($_1$ $_2$) $\leq$ $_1$ $_2$ d th t
o o b t k g th o (d g th o o d g q t
th $_1$ d $_2$ t h g d)

L t o ook b t s t b t o Th d h th t
t t b d b o t o th ho d ho th t o d
o t b t th t o to th g t W h
o o t o th t x d b o ght o to t o
N ^{1} o H th th b t th g t h ght
o to t o o th o $_1$ N_1 $_1$ ^{1} th N_1 N d $_1$

W h k th d o t t o d t d
t b g t o th t t d d to th t xt t d
th ot o d g t o k th g t g o t t (S S)
o o t th o t t b h o th d to Th o 1
h h x th o t xt o th o o g t

e e 4 *a s a a sys a*
a za N ^{1} H *a s a z*
^{1} (Y)(N) ^{1} Y *a* H
a YN 1 *s a z s y a*
N_1 $_1$ ^{1} (N N)( ) ^{1} (N) *a y*

$$\begin{pmatrix} Y \\ N \end{pmatrix} \leq \frac{1}{} \qquad (1)$$

: W h d th t th t b t o $_1$ q t to th
t b t o

$_1$ $_1$ N_1 ( )(N) (N N)(Y)
1 ( )(N) (N)(Y)

C (N) d (1) ho d th

1 ( )(N) (N)(Y)

t b th oth h d o g th t (1) to ho d th
d o t + o h h

$$(Y \quad)(\ )^2 \quad (\quad N \ )(\ )^2 \quad 1 \quad ^2$$

d k $_1(\ )$ b t b ho o (N) th
^{2} N^2 ^{2} th d t b $_1$

Th d t th t th g to o og th o t to o og h h to
o d db k t b t o Th g t o h h $_1$ t b h
(N) d th s ss a d h t t

C 4 s a y s s a a
a a za N ^{1} s a z a
s ss a s y

$$^1 \qquad \begin{pmatrix} Y \\ \end{pmatrix} \quad \begin{pmatrix} \\ N \end{pmatrix} \qquad (1 \quad ^2)^{1\ 2}$$

$H^2(\ _+) \to H^2(\ \ )$ s a a y $\to$ $\mathbb{C}$ $(R \ )$
$H^2(\ _+)$ R Y N ( )

: th t to b ho th d t t o g L t

$$\begin{pmatrix} & N \\ N & \end{pmatrix}$$

o th t

$$\begin{pmatrix} Y & \\ & N \end{pmatrix} \qquad \begin{pmatrix} Y & \\ & N \end{pmatrix} \qquad \begin{pmatrix} R & \\ & \end{pmatrix}$$

Th t o o o o h th o Th o 3 t h
o (th x)

t b ho (1) th t th o t ob t g th g
t o q to th t th t th o o g th o ho d

e e 4 N ^{1} a a a z a za a
a s a z a a (1
s a a

^{1} s s a a $_1$ $(N \quad N)(\quad)^1$ N H
a $\begin{pmatrix} \\ N \end{pmatrix}$

$_1$ s s a a $_1$ ($_1$)

W h o t th oo o Th o 9 d o t th x

e 4 L t ( ) $\frac{1}{1}$ t b t H ($_+$)
Th t o h o to t o N ^{1} o H o x

$$N(\ ) \quad \frac{1}{1} \qquad \text{d} \qquad (\ ) \quad \frac{1}{1}$$

To t th t b g o t o do ot d to o th
to t o d h o k d t th N d d th o t d t t
YN 1 h

$$(\ ) \quad 1 \qquad \text{d} \quad Y(\ ) \qquad \text{o}$$

Th th o t t o o t b g o t o g

$$(\ ) \quad \frac{Y}{N}(\ ) \quad \frac{\frac{1}{+1}\ (\ )}{1 \quad \frac{1}{+1}\ (\ )} \qquad H\ (\ _+)$$

Th t t h do g o k d x
o t t k g g ( ) (o t t g t db k" th
o t o j go) o th t th o d oo o to H( ) g o o t o
1 3 g b

$$H(\quad)(\ ) \quad \begin{pmatrix} (\quad)^{\ 1} & (\quad)^{\ 1} \\ (\quad)^{\ 1} & (\quad)^{\ 1} \end{pmatrix}(\ ) \quad \begin{pmatrix} \frac{1}{+1} & \frac{1}{+1} \\ \frac{1}{+1} & \frac{1}{+1} \end{pmatrix}$$

o ho t H ($_+$) Th ob t g g b

$$^1 \quad \begin{pmatrix} Y & \\ & N \end{pmatrix} \qquad \begin{pmatrix} & \\ 1 & \end{pmatrix} \qquad \sqrt{5}$$

d to ho th o d th t b d t N_1 $_1{}^1$ h

$$(N_1 \quad _1)(\ ) \quad (N \quad)(\ ) \quad \left(\frac{}{5} \ \frac{1}{5}\right)$$

h th t b t o h (N) 1 $\sqrt{5}$ W d th t

$$_1(\ ) \quad \frac{\frac{1}{+1} \quad \frac{2}{}}{\frac{1}{+1} \quad \frac{1}{}}$$

d ($_1$)( ) o th db k t $_1$ t b

To d th o t ob t g t o o th t o
g o d o to t o t th o t o t o b

d b th ob t o th t t o t o th ( ) ( ) o ( ) ( ) o Th o o g o to t o h

$$(N\ N\ )(\ )\ \frac{^{2}}{1\ ^{2}}$$

o h h t ot h d to th t $a\ z$ o to t o g b $N^{\ 1}$ h

$$N(\ )\ \frac{1}{\sqrt{\ }}\qquad \text{d}\qquad (\ )\ \frac{1}{\sqrt{\ }}$$

o o o o k th th ho o N d th o t d t t t d t k ( ) 1 d $Y(\ )$ 1 $\sqrt{\ }$ To t th ob t g t R Y N o

$$R(\ )\ \frac{(\sqrt{\ }\ 1)\ (\ \sqrt{\ })}{\sqrt{\ }}$$

U g th t th t o t t t o H d th t o th d t o 1 ( ) to H ($_{+}$) 1 ( ) (th x) t ot h d to th t d t(R H ($_{+}$)) 1 $\sqrt{\ }$ W o d th t ($\sqrt{\ }$) $^{1\ 2}$

W h o g o d t o b t q t o

1 o o t t o t d t t d

o o t t o d o to t o

Th t o th th d t g W t o t t o o d t S t o b t th o t d o th o t h d th o a a t o th t th d g o t d o to g t th o q to th d g o t to

i i 4 *a a a as a a za N s a a H* ($_{+}$) *s*

: L t p h p d o o th o o o to h th t d g ≥ 1 d d g $p \leq$ Choo o o r o d g h th t r h o o th o d ght h o x $r(\ )$ (1) o to b N p r d r b t to ho th t th o d to t b h o t d t t

o th E d go th t o b to d o o d h th t p 1 d d o tho t o o g t th t

d both h d g t o t 1 ot t w th w o o d d g th p (wp) (w)p 1 d d g(wp) d g d g d g p o th t d wp both h d g t o t 1

o r^2 x o o o x d th d g x d d g o too b tt p o o o th d g d d g o ( )p ( ) d t g th g t o th o g h h h d g $\leq$ ( ) d d g th p (p) d d g p $\leq$ 1 d g p $\leq$ 1

W t th d t t r^2 (x p) p d th

$$\frac{}{r}\ \frac{p}{r}\quad \frac{x \qquad p}{r}\quad \frac{}{r}\quad 1$$

d o t o H ($_+$)

e 4 L t ( ) (1)^2 d t k N( ) (1)^2 d ( ) (1)^2 (1)^2 Th E d go th (o t o) g

$$(\quad) \quad 1(\quad 1)^2 \quad 1$$

d d o g

$$(\quad 1) \quad (\ ^2 \quad 1)(\ ^2 \quad 1) \quad 3 \quad 1$$

o h h d t t

$$(\quad 1) \quad (\ ^2 \quad 3 \quad 1)(\quad 1)^2 \quad 1 \ (\ ^2 \quad 5 \quad)$$

d t g th t YN 1 h

$$( ) \ \frac{^2 \ 3 \ 1}{(\quad 1)^2} \qquad \text{d} \quad Y( ) \ \frac{1 \ (\ ^2 \ 5 \quad)}{(\quad 1)^2}$$

g h t o b t t to o t

Wh t o d t o d th o d ob ho to o t t o d o to t o g h t t d o to th t o Th k th o o g t h h o o th j th o

e e 4 N a a a s H ($_+$) s a

$$N(\ )^2 \quad (\ )^2$$

s a a a H ($_+$) 1 H ($_+$) s a ( )^2 N( )^2 ( )^2 (N) ( ) s a a z a za

: L t $R(\ )\quad N(\ )\overline{N(\ \ ^{-})}\quad (\ )\overline{(\ \ ^{-})}$ Th t o t o o d t o t d bo d d both bo d b o o th g x W tho t o o g t th t R t o t t

o th o d o o R o t bo t th x R d t th th t o t o $R(\ )\quad \overline{R(\ \ ^{-})}$ b th o d o th g x Th w d $R(w)$ th $R(\ \overline{w})$ too (d o o) W th o t $R(\ )\quad (\ )\ (\ )$ h

$$(\ )\quad \overline{\quad\quad p} \quad \text{d} \quad (\ )\quad \overline{\quad\quad \overline{p}}$$

o o th $_1$ d p_1 p t th o d o o th t h (t to th t th b q) Th $R(\ )\quad (\ )\ \overline{(\ \ ^{-})}$ d o th g x

$$N(\ \ )^2 \quad (\ \ )^2 \quad R(\ \ ) \quad (\ \ )^2$$

t o th t N d g o d o to t o

e 4 4 L t t k $N(\ ) \quad (\ \ 1)^2$ d $(\ ) \quad (\ \ 1)^2\ (\ \ 1)^2$ Ex 1 W d o o k th th to d t

$$(\ )(\ \ ) \quad (\ \ 1)^2(\ \ 1)^2 \qquad 3^{\ 2} \quad 1$$

$$(\ ^2 \quad \sqrt{5} \quad 1)(\ ^2 \quad \sqrt{5} \quad 1)$$

d th oot o $^2 \quad \sqrt{5} \quad 1$ both th t h W th o h th o o g o d o to

$$\frac{}{^2 \quad \sqrt{5} \quad 1} \quad \text{d} \quad \frac{(\ \ 1)^2}{^2 \quad \sqrt{5} \quad 1}$$

Th bo o k o *s a a za* o R o g th o t to t o o H t o o o to o t t (o t t g o o) t to t o o b t o t t b t o o ( ) o o

e e 4 R ( ) *s a* R *a a* 1 R ( ) *s a* $H\ (\ _+)$ *y a* $(\ )^2$ $R(\ )$ *a s N a a s* $H\ (\ _+)$

$$N(\ \ )^2 \quad (\ \ )^2$$

s s a $H\ (\ _+) \quad 1 \quad H\ (\ _+)$ *s a*

$$(\ \ )^2 \quad N(\ \ )^2 \quad (\ \ )^2$$

a $(N \quad)\ (\ \ \ )$ *s a a z a za* $H\ (\ _+)$

: t o o o Eq t o (1) th t th t o d d b

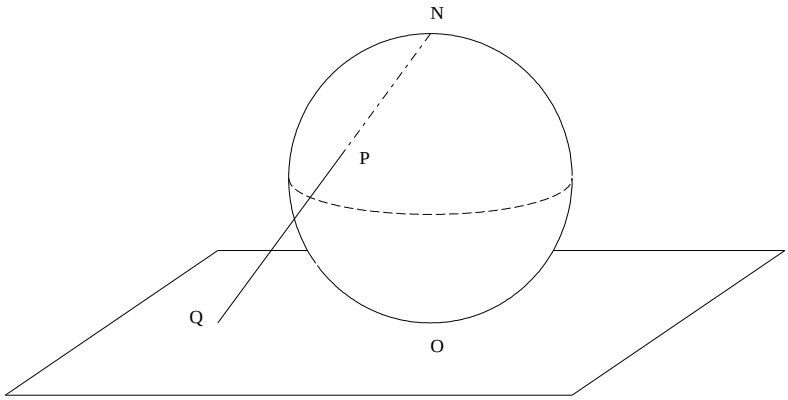
$$( ) \quad x \left(\frac{1}{} \quad \frac{}{} \text{ og } R(\ ) \frac{}{1 \quad {}^{2}} \right)$$

o t t o H ($_{+}$) h th t og ( ) $\frac{1}{2}$ og R( ) o t
h Th g d d t o t ght o d

Th x t th o d th o bo ot o d
x t t o t b k d t th o t th t t to t o
d th o t t o o o to t o g t o t h
d t o x T 131 g x to ho th t t o
th o t o bo d d ot h o t o t to Th
th t th o o t to t o d o t o th o
o d o h to b x d o k g th ox t o
to t o W to 1 d o th d t

4. o dal t i

W o t o tt t o to to th t oot o x
h h o o to t o th o t thod to ob t o t o



g Th h d t og h oj t o

th t th o o t og h oj t o o d o d t
g th xt d d o x ∪ th th h o t d t S

(th a s) g th o t o o d to th o th o o th h o t d t d th o t o o d to t o th o N W th d t o t th th o t o th h h N d o t ght t ot h d to th t th o o d o t o t t b t d S N d d d t d t t o ∪ o o

W d th ho d d t b t t o o t ∪ b g th gth o th ho d b t th o o d g o t o th h

o t o o x b w_1 d w_2 th ho d d t b t th o th h

$$\kappa(w_1\ w_2) \quad \frac{w_1 \quad w_2}{\sqrt{(1 \quad w_1{}^2)(1 \quad w_2{}^2)}}$$

th $\kappa(w \quad) \quad 1\ \sqrt{1 \quad w^2}$

e i i 4 s 1 2 a
a ho d d t κ 1 a 2 s y

$$\kappa(\ _1\ _2) \qquad \kappa(\ _1(\ )\ \ _2(\ ))$$

Th ho d t o t d to th s a d th b

$$\kappa(\ _1\ \ _2) \qquad (\ _1(\ )\ \ _2(\ ))$$

h t t $_1(\ )$ d $_2(\ )$ o to o to (Th d to ob o t b g t o) ot th t th th o t o

$$(\ (\ )) \quad \begin{cases} (\quad (\ )\ ) & (\ ) \quad t \\ (\quad) & (\ ) \end{cases}$$

d th o t t o o th g b t t o o d o b o ^{2} x (d d t Ex 1 o Ch t)

Th o o g t ght o d q t b to

i i 4 w_1 a w_2 s

$$\kappa(w_1\ w_2) \leq \left(w_1 \quad w_2 \left| \frac{1}{w_1} \quad \frac{1}{w_2} \right| \right) \qquad (11)$$

: Th t g th t $\kappa(w_1\ w_2)$ $\kappa(1\ w_1\ 1\ w_2)$ o w_1 d w_2

o th t o t t o to o d th th g h d g t o t o th t t o t o d o t h o t th ( ) ( ) o 1 ( ) 1 ( ) th tt h d o o d t o to h o bo d o κ h h t t o to o o t o 3

Le 4 *a y s* w_1 *a* w_2 *a a y* 1

$$\kappa(w_1\ w_2) \geq \frac{1}{1\quad^2}\qquad \left\{ {}^2 w_1\quad w_2\quad {}^2 \left| \frac{1}{w_1}\quad \frac{1}{w_2} \right| 1\quad {}^2 \right\}$$

: Co d th o o g th h h o t xh t
( ) $w_1 \leq 1$ d $w_2 \leq 1$
(b) $w_1 \geq$ d $w_2 \geq$
( ) th $w_1 \leq$ d $w_2 \geq 1$ o
( ) th o o κ ho th t $\kappa(w_1\ w_2) \geq w_1\ \ w_2\ (1\ \ 1\ \ ^2)$
(b) th th w_1 d w_2 d b 1 w_1 d 1 w_2 d
( ) $\kappa(w_1\ w_2) \geq \kappa(\ \ 1\ \ )\ \ (1\ \ ^2)\ (1\ \ ^2)$

E S kk g o ob t t o th ho d t o h h th o o g t 1 (1) t b d

$$\kappa(\ \ (\ )\ \ (\ ))\quad \frac{1}{3(1\quad 1\quad (\ )\quad ^{1\ 2})^{1\ 2}}$$

th 1 (1) o t b (d th bo d o $\left|\frac{1}{1+}\quad \frac{1}{1+\ '}\right|$) Th o o d to t b g b o t t o t o
U g L 3 3 o o o ob t t th ho d t o d to k th t o t t h o k th S S t d x d o t o H ot th t th t b t q t th o th t (1) H th th t (1) 1 (1) d (1) o H S h t d *s y s a za* b t t th ot o o d th t b b t (13 o th th d t h h b g d d g t o o th o o th o)

e e 4 4 *s a a* $_1$ *a a s s a s a s a s a s a s a s* (1) *s H a*

$$\kappa(\quad _1) \leq (1\ \ 3)\quad 1\quad ^1\quad ^1(1\quad)\ ^1$$

$_1$ *s a s s a z y a s* $_1$ $_1$ (1 $_1$) *H*

: S o th t o t h th t $_1(\ )$ b t $(\ ) \leq$
W t t th ho d d t o $(\ )$ o $_1(\ )$ b g
L 3 3 L t b b th 1 Th
$(\ )$ $(\ )$ $_1(\ )$ $\left|\dfrac{1}{(\ )(1 \quad (\ ) \quad (\ ))}\right| \geq \dfrac{1}{(1 \quad)}$ d
(b) $\left|\dfrac{1}{(\ )} \quad \dfrac{1}{_1(\ )}\right| \quad \left|\dfrac{1}{(\ )}\right| \geq \dfrac{1}{}$
Th

$$\kappa(\quad _1) \geq \frac{1}{1 \quad ^2} \quad \left\{\frac{^2}{(1 \quad)} \quad \frac{^2}{} \quad 1 \quad ^2\right\}$$

t g g t ho th t th t $_1$ bo d d
th th h g o th ght h
Th t o o o o t k g $1\sqrt{\ }$
d t g o b to hoo o d
to obt t ght t t b t do ot do th h
W o o th bo t d ho th t o to $_1$ th
o to $_1$ *H* o o o

e e 4 *U y s s 3*

$$\kappa(\quad _1) \quad (1 \ 3) \quad \left\{1 \quad \frac{}{(1 \quad)(1 \quad (\quad))} \quad \frac{}{(\quad)}\right\}$$

1

: g L 3 3 d th t t

$$(\ ) \quad _1(\ ) \quad \frac{(\ ) \quad _1(\ )}{1 \quad (\ ) \quad (\ ) \ 1 \quad (\ ) \ _1(\ )}$$

d

$$\left|\frac{1}{(\ )} \quad \frac{1}{_1(\ )}\right| \quad \frac{(\ ) \quad _1(\ )}{(\ ) \quad _1(\ )}$$

to bo d κ o b o d th h oth th t $(\ ) \quad _1(\ )$
S t th t o th g to o og o o th ho d t
th o o g t ho

Le 4 *s a* $(\ )$ *a* $_1(\ )$ *a a za s*
H a y $(\ )$ $N(\ )$ $(\ )$ *a* $_1(\ )$ $N_1(\ )$ $_1(\ )$

$$\kappa(\quad _1) \leq \quad \mathrm{x}(\ N \quad N_1 \quad _1\ )$$

s a s a a a a y 1

: S N d o th t o t d t t

$$( ) \quad ( ) \quad Y( )N( ) \quad 1$$

th Y H L t x d Y Th t o h h th t x ( ) N( ) ≥ 1 d h ( ) ≤ 1 x th N( ) ≥ 1 W th t t κ(N N1 1) h ( ) ≥ 1 x

$$\kappa\left(\frac{N}{} \ \frac{N_1}{1}\right) \leq \left|\frac{N}{} \quad \frac{N_1}{1}\right| \leq \frac{N \quad 1 \qquad N \quad N_1}{1}$$

h h b t o t o t t t x(N N1 1) o d d th t 1 1 x t t h ( ) 1 x d N( ) ≥ 1

$$\kappa\left(\frac{N}{} \ \frac{N_1}{1}\right) \leq \left|\frac{}{N} \quad \frac{1}{N_1}\right| \leq \frac{N_1 \quad N \quad N \qquad 1}{N \quad N_1}$$

g o o t o 3 d th bo d d b o t t t

x(N N1 1)

o d d th t N N1 1 x

To o d t h o t o g th xt t t t o o

e e 4 *a s a y s s a a s*

: o o t o d L 3 q ( ) o o t o t t o th t o g to oth o th g t o o g th ho d t

Co → th ho d t th tt g p_1 p b th o o ≥ th o o g o to t o o

$$N( ) \quad ( ) \quad \frac{p}{1} \quad \text{d} \quad ( ) \quad \frac{p}{1}$$

d t o t o to th ho d t h o $p_{,1}$ $p_{,}$ th $p_{,}$ o to p o h j b o h th o $^1 \rightarrow$ ^{1} o o ghbo hood o h p d → o o th o t o th o o th ghbo hood W o h N h

$$N( ) \quad ( ) \quad \frac{p_{,}}{1} \quad \text{d} \quad ( ) \quad \frac{p_{,}}{1}$$

t th t → d t o o th t *N* *N* →
h o g th g t b o o t o

Th t b o o th ho d t th g t d
b d 13 o g dd t o d g b o d t o t o
b to d th t o o th ho d t to o t t o o th
g x th th th t h h h o t t o
d t g

e

Th t t o th to t o t t o h t t o to d
o to o o xt d th d to 1
1 5 1 3

Th o 1 t k o 1 h t q t o o d 13
Th t ho d th t x d too d o obt t o g o
t d ght to t o S o 135 o o o th k b t t b
t o d o to t o

C o o g oo o th o o th o 15

Th o t t o b o d o x 3
135 151 Th o g o 1 1

Th k b t db k t b t d th t t o th g h o th
t d o t o d o d o t t o b tho
S 35 9 153

o o d o o th g t d ob t g h d
o 1 9 1 1 1 13

Th o d d to o g o Th o d to
d g d 13 h Co o o o d

o o th ho d t b o d th book o 5

So d o g o t o th ho d t (o t g t
) 9 99 11 15 Th o 3 d t d t t k
o 99

t h b t d th t th ho d t t o d th th g
t h t t d to t th o t b o o o ( )
tho gh t th ot th 1

e i e

1 o th d t t o o Th o 1 1 th t
h t t o to o $\ell^2(\ _+)$ bo d d

th t th o to o t t o b 1 (d d o bdo
o $H^2(\ _+)$) t q t to th o to Ex 1

3 th t Eq t o ( ) h o t o g b Eq t o (3) h
th o to x t

o th o t o d t t (5)

5 L t μ_f d μ b o t o th oo o Th o 1
g th do kod th o to t μ_f μ d μ μ
h μ μ_f μ ho th t th x o μ_h h
th t $\mu_h(\ ) \leq$ $\mu_f(\ )\ \mu(\ )$ o o t

D th o o th t o o t t o g
Th o 1

o th t th g h t d d t

D d h th o o $H\ (\ _+)$ th d t o th o t t
t g th o o o th ogo o o th d (Th o 3)

9 Sho th t d $f(\ )$ 1 ( ) th d t$(f\ H\ (\ _+))$ $\frac{1}{}$
d th t o t o t th o t t to 1

1 C t o t ob t o t o o Ex 1 b o g th
o t h ob g Ex 9

11 t th t o o Ex 1 o th t b t

$$( ) \quad \frac{1}{-}$$

1 C t o to t o th o o d g o t d t t o
th t $_1( ) \quad \frac{1}{^2 \ 1}$ d $_2( ) \quad \frac{1}{^2}$

13 C t o d o to t o o th t $_1$ d $_2$ g
Ex 1

1 Th *a* o t to th q t t
( ) ^{1} *R* h *R* *H* g o ho o t b
g o t o U th o t t o to o t th
H t o ob t o th t

15 t th t o o Ex 1 o th *s* *s* *y* *za*
h o t q d to ( ) ^{1}

1 Sho th t t o o th o

$$( ) \quad \frac{^{\theta}}{\qquad}$$

th d o o o t o th h
th t t t $\kappa((w_1) \ (w_2)) \ \kappa(w_1 \ w_2)$ o $w_1 \ w_2$ Sho o
th t o t w_1 b d to oth o t w_2 b g th

r

r i i

So h o k d o t t th g o th o $\ell^2(\ _+)$
o $^2(\ \ )$ d th x og h t th o
y s a s h h d o t t th h t
h o k th h t oo b d b d *s a s* o t
o oo *s s s a s*

t t g d o g th o g t o t
h od d o t od g h o g
o g h h th ot o o o ” d d t
b to d o t h th d o h t o g
go o d g o to h t o k t t g g
b t k th t d o t t o t (th t o o
d h) h

. l o t- iodi f n tion

o t od t o d d o th o o t o th t
h b h t d d th 19 th t o to d
th d t o t d to b g o t th t th th th o
o od t o

W b g b g th b o t o od t o o
Th tot t d d t d b o d 9 1 9 155 d
oth → od (( ) ( ) o)
d L b g t g b o t t th t o b g d d
t o $^1(\ )$ b o th o o d

$f(\ \ )\quad (\ )\quad (\ \ )$

t o o d to o x q o o o t g b

$$\hat{f}(\ ) \quad \frac{1}{\ } \quad ^{2} \quad f(\ ) \qquad (\ \ )$$

$f \quad ^{2}(\ )$ th h

$$f(\ ) \quad \sum_{=} \hat{f}(\ ) \qquad (5\ 1)$$

h o g t t d th $^{2}(\ )$ o W o h *a s a s* *y* $f \quad ^{2}(\ )$ th

$$\frac{1}{\ } \quad ^{2} \quad f(\ )^{2} \qquad \sum_{=} \hat{f}(\ )^{2}$$

$f \quad (\ )$ th ot g t o t o g o th o (5 1) t o o o g b t o f b o t j *s s*

$$f(\ ) \quad \sum_{=} \left(1 \quad \frac{\ }{1}\right) \hat{f}(\ ) \qquad (\ \ 1 \ \ ) \qquad (5\ )$$

h h o g o to f h f o t o $\hat{f}(\ )$ th th o d to b *a s* *y* d t o g o to o t o t o g to th th t o t o od t o th o t o q o t go o t o o (So t o th g th th t o d o th th b t th o t d)

L t o th bo t o k g th th t o t o g b

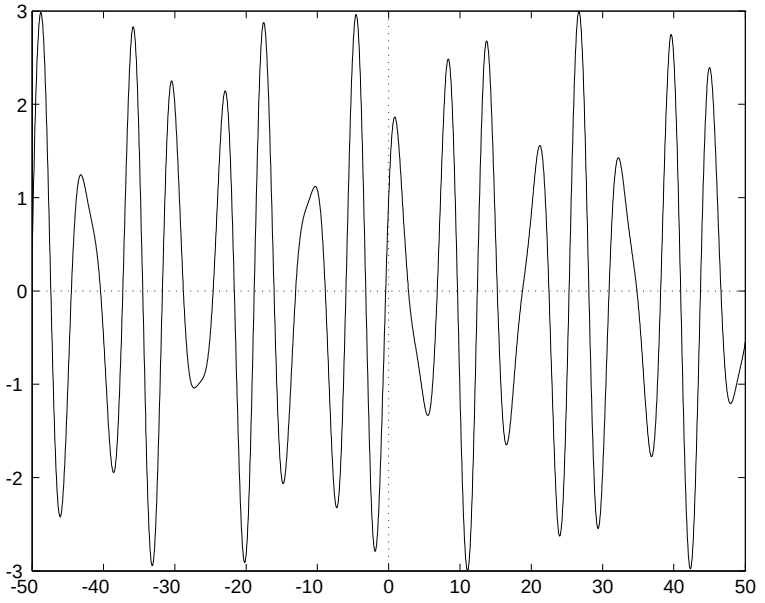
$$(\ ) \qquad (\ \ )$$

h o t t d to g Th d t o g o ot tot t d d (tho gh t h b d h g 19) b t h t t q t to oh o g d t o

e i i *ass* ( ) *y) a s s s* *s a s a* ( ) *s* *s* ( )

S th o t o o t o t o o t o th t ( ) ⊆ ( ) th o o t o bo d d t o o o o t od t o ot g od (th x)

o x g 5 1 ho ot o th t o o √



g 5 1 Th o t od t o o √

th t th ² ( ) o t o L b g t g b t o f → h th t

$f(\ )^2$ o

W t od th o o g od t k ot t o o o t o f ² ( ) d

$$f \quad \frac{1}{\ } \quad f(\ )\overline{(\ )} \tag{5 3}$$

d h th t x t

$$f \quad \rightarrow \quad f \tag{5 }$$

Th h th o o g o thogo t t o

$$\begin{cases} 1 & \mu \\ & \mu \end{cases} \tag{5 5}$$

th d t d to

e t o b to d b t b t k g th o t o o th t o t go o t o o d th o $f\ f^{1\ 2}$ Th d th t o t d ₂( ) h h h o t b o tho o b ( ) o tho gh ( ) b d o t o to th ₂( ) ot o t o d d o W th d to o d h t t t t o b g to th o =1 $\frac{1}{\ }$ x ( ) h h t obj t ₂( ) b t h th d g o

To g t d t o o th ( ) h k o
o o th ght h t R d d o $f \to$ b $(R f)(x)$ $f(x$)

e i i a a s $f \to$
s a a t t o b f

$$R f \quad f \quad \leq$$

a s $f(x$) $f(x) \leq$ a x s $\subseteq$ s sa t
d s s a s a y a a s a

W o t d th o t o t g th o o g o t h h
t o t to b q t to b o g g ( )

e i i **4** say a a $f \to$ s a oh t o f s
s a y s a s a s f s a y
s $f \to$ sa s s o h o d t o s s
a a s a s a s $R f$ s a y a
a y) ( )

o x f od t o o o th
t t o b o d d o f oh t o W
o k g to d ho g th t th t o o d t o bo q t d
th t th o t o t g th o d th th ( ) d d

e e ass B s s sa as ass s
sa s y B s

: W ho t th t th oh t o bo d d o g h
t o f t b h th t t o gth o t 1 t t o
b o f L t x $f( )$ d t x b g
Th th 1 t t o b o f th t o
$f(x) \leq f(x$) $1 \leq$ 1 q d

o o oh t o o o t o o g h
f d t b h th t t o gth o t -
t t o b o f S f o o t o o 1
d th 1 h th t $f(x_1)$ $f(x_2)$ 3 h x_1 x_2 1
d x_1 x_2 $_1$ $_2$ th $_1$ $_2$ d
- t t o b h th t th o t x_1 $_1$ d x_2 $_2$
1 o

$f(_1)$ $f(_2) \leq f(_1)$ $f(_1$) $f(_1$) $f(_2$) $f(_2$) $f(_2)$

t o o th t h $R\ f\ \ R f \rightarrow$ h ( ) q th
$\rightarrow$

o o o b th t f oh t o th t ot o h
t o Th o o h q ( ) h th t $R\ f$
$R\ f$ o j o th b h th t t
o gth o t - t t o b o f W t h
- t t o b d $\leq$ $\leq$ Th $R\ f\ \ R_\delta\ f \leq$ o
h g to b q d b g o th t th
q ( ) o g th t d b o o t t
o th th t $R_\delta\ f\ \ R_\delta f$ o Th j h

$$R\ f\ \ R\ f \leq R\ f\ \ R_\delta\ f \quad R_\delta\ f\ \ R_\delta f$$
$$R_\delta f\ \ R_\delta\ f \quad R_\delta\ f\ \ R\ f$$

h h o t d t o d ho th t oh o h ”

Co f ot oh t o th x t o h h th
t o t t o b o f ot t d T k $_1$ 1 d t
($_2$ $_2$) b t o gth g t th o t g o t o L t
$_2$ ($_2$ $_2$) d t d ( ) t o gth g t th
($_1$ $_2$ $_1$) o t g o t o d t ( )
$1 \leq$ th o

$$R_c\ f\ \ R_c\ f \quad R_c\ _c f\ \ f$$

d o f do ot t th o h o d t o

U g th o h o d t o o to th t th ( ) t o
th o oh t o

C B s a s a s s a ( )
a y ( ) s a B

: f_1 d f_2 oh t o $_1$ d $_2$ o x o t t d
( ) q th b g to b q d b g
o tho t o o g t th t $(R\ f_1)$ d $(R\ f_2)$ o g t
q ( ) t o o o th t $(R\ (\ _1 f_1\ \ _2 f_2))$ o g t
q d th t b h th t $_1 f_1\ \ _2 f_2$ oh t o

o o th o oh t o o d - t t o
b o f d f 3 th

$$R \leq R\ \ R f\ \ R f\ f\ \ f$$

d o t t o b o Th th o o th t o oh t o d t d t o t t o b o b t k g t d t o 3 t t o b o oh t o f th f 3 Th o oh t o

W h d th t t o oh t o t od ( ) th o d o th o t t o oh t o

e t d dd t o th t th oh t o o o d b g b o ( ) th t to ho th t th o t od t f_1 f_2 o t o oh t o oh t o Th oo o d th t t o Co o 5 1) g th ob t o th t $(R \ f_1)$ d $(R \ f_2)$ o o g t th o $(R \ (f_1 \ f_2))$

W o h to o th d t th t oh t o t o t go o t o o th t t o b t o o th t o To do th d to d t h h q t o t od o t o d h th d t od t o f d f d d (5 3) d (5) q d o o

i i f a B

$$f \ 1 \ \rightarrow \ \frac{1}{} \ f( )$$

s s

$$f \ \rightarrow \ \frac{1}{} \ f( ) \overline{( )}$$

s a B s f a

: S o th t d t f t b h th t t o gth o t t t o b o f W t

$$\frac{1}{} \ f( ) \ \frac{1}{} \sum_{=}^{1} \ ^{+1} \ f( ) \qquad (5 \ )$$

d o th t x Th t t o b o f h

$$^{+1} \ f( ) \qquad f( ) \qquad (f(\ ) \ f( ))$$

$$f( ) \qquad ^{+} \ f( )$$

$$^{+1}$$

d bo d th o d th d d o th t o th ght h d d
g th o t o " t gth o t " b d
t dd g th t (5) obt

$$f\ 1 \quad f\ 1 \quad \leq \quad \frac{}{} \quad \frac{}{} \leq 3$$

t $f\ 1$ bo d d o t g hoo
h th t $\leq$ (1) Th

$$f\ 1 \quad f\ 1 \quad \leq \left| f\ 1 \quad \frac{}{} f\ 1 \right| \left| \frac{}{} \quad 1 \right| f\ 1$$

$$\leq \frac{1}{} \quad \frac{1}{} \ f\ 1$$

$$\leq \frac{}{} \quad \frac{1}{} \ f\ 1$$

t g Th

$$f\ 1 \quad f\ 1$$

t g Th g $f\ 1$ $f\ 1$ h d
t g g th x t o th t $f\ 1$

ot o th t f $f^{-}1$ d d o oh t o f d
th od t f^{-} o oh t o b k 5 1

Th g o to d d b $\rightarrow$ f o t d th B
a s Th b f d th s o f

W h d t to t od x t o th a o
a a o t o oh t o f_1 d f_2 Th d d o x
b

$$_{f\ ,f}(x) \quad R \quad f_1 \ f_2 \quad \underset{\rightarrow}{} \quad \frac{1}{} \quad f_1(x \quad)\overline{f_2(\ )} \qquad (5 \ )$$

W h o o th d S t o 5 3

i i f_1 a f_2 B s s s a a
$_{f\ ,f}$ R f_1 f_2 $\rightarrow$ $_{f\ ,f}(x)$ y x as $\rightarrow$
s R f_1 f_2 f_1 R f_2

: S

$$(x \quad) \quad (x) \quad \underset{\rightarrow}{} \quad \frac{1}{} \quad (f_1(x \quad) \quad f_1(x \quad))\overline{f_2(\ )}$$

th t

$$R \leq R\, f_1 \quad f_1 \quad f_2$$

o h h o d th t o oh t o t t o b o f_1 t t o b o oo f_2 $\leq$

o x d x th o g o R f_1 f_2 to (x) o o o t o 5 1 o g th o h o t o f_1 to d x_1 x h th t o h x th th R f_1 R f_1 Th

$$R \quad f_1 \; f_2 \qquad R \quad f_1 \; f_2 \qquad f_2$$

o o o th b h th t

$$R \quad f_1 \; f_2 \qquad R \quad f_1 \; f_2$$

o $\geq$ o th t o t o 1 o th t g q t th t $\geq$ th

$$R \quad f_1 \; f_2 \qquad R \quad f_1 \; f_2 \qquad (1 \qquad f_2 \quad)$$

o x

ot th t

$$R \quad f_1 \; f_2 \qquad \frac{1}{\qquad} \qquad f_1(x \quad)\overline{f_2(\ )}$$

$$\frac{1}{\qquad}^{+} f_1(\ )\overline{f_2(\qquad x)}$$

$$f_1 \; R \; f_2^{+} \qquad (\ )$$

h $(\ ) \leq x \; f_1 \quad f_2$ d o $R \quad f_1 \; f_2 \quad f_1 \; R \; f_2$

S th o g o R f 1 to f 1 o x th q t t f 1 b t t d b t g t g o t o g t x x o h t th g R f 1 o g

U g th too o d to o o h o o th o oh t o h h d to oo th t t o d th ( )

ot th t th g $(f \quad) \rightarrow f$ t th x o o od t o th o oh t o x t o b th o t d t o d t o (t t t th too b t h d th to t d d t o o) Th b to d o od t

th o o o t od t o th t od t 1 2 o tho o q d x to th tt g

$$\sum_{=1} x \quad \rangle$$

h th t x o thogo to d h t o thogo to t th go th o o g

$$x^{2} \quad x \quad ^{2} \quad ^{2} \geq \quad ^{2} \sum_{=1} x \quad \rangle^{2} \qquad (5 \ )$$

h h *B ss s a y* o o b t o b t o o 1 th $(x \quad) \ (\quad) \ $ d o

$$x \quad ^{2} \ (x \quad) \ (\quad)^{2} \quad x \quad ^{2} \quad ^{2} \geq x \quad ^{2}$$

h h th b t ox t o ” o t o

e e *f a B B ss s a y s*

$$f \ f \geq \sum_{=1} f \quad ^{2}$$

a s 1 *f a s a a s*

: W obt q t d t o (5) t g f o x d o Th th t o $N \geq 1$ h f 1 N o t o t t th tot b o o o o o t f t o t o t b

W o o th t $(f \ ) \rightarrow f$ g od t o th oh t o th t t o t d t

e e *f a B a s a y z* $f \ f$

: f ot d t o th th d h th t $f(\ )$ o t t d h th t $f \geq$ o th t ( )

L t b h th t t o gth o t - t to b o f Th o ≥ 1 h

$$\frac{1}{} \quad f( )^{2} \geq \frac{^{2}}{1}$$

h t (1) o t - t to b o f g th t f ≥ o ( ) (1) th h gth t t (1) f f ≥ $\frac{\delta}{1}$ q d

e S o th t (f) q o oh to o h h f f f → o o oh t o f dd t o o t ho d th t o h t to b o f t to b o th to f th t o o b th g t bo th t (f) t d to f y o (f f)( ) th t obt f f ≥ o o t ( ) h d d to t b t to b o f d h o f o d t o h h f f ≥ d t t t g o o

t t d to o th d t q th o t t g th t th o o t d t th to q th t f o th f d t o To do th d

Le *s a f s a B s a f a* *f* → *as* → o

: W h o th o t d d o t d to So o th t f ≥ o q ( ) d t d g to b th t o

$$f \quad \frac{1}{} \quad f( )$$

$$\frac{1}{} \quad \begin{matrix} + \\ + \end{matrix} \quad f \quad -$$

h h th t

$$f \quad \frac{1}{} \quad f( ) \quad f \quad - \quad ( )$$

h th t t t d to o o → b th o o t t o f d

$$( ) \quad \frac{1}{} \quad \begin{matrix} + \\ + \end{matrix} \Bigg) f \quad - \quad (\quad ^{1} \quad ^{1})$$

t o o th t th g q ( ) t bo d d d h o
g t b q b g o tho t o o g t th t
→ W t h →

o o to 5 1 9 $R f \to R f f R$ o
→ o d b h th t $R f$ o
≥ d o o g t N o o
th d N both d d g o Th o

$$f \quad \frac{1}{N}\sum_{=}^{1} \quad {}^{2+1} \quad \frac{1}{} \quad f(\quad (j \quad 1 \quad N) \;) \quad {}^{\delta}$$

t $^{\delta} \to 1 \to$ o o o th t f
o t g Th o t d to d th t o o

e e 4 i e e e e *f a B s a*
f a f s a y z

: W b g b d g o h x to f th t
q f o th t ( ) d od Th f h o

$$f( ) \quad \sum_{=}$$

d d t t g

$$\frac{1}{} \quad f( )^{2} \quad \sum_{=} \quad {}^{2}$$

Th oo o o d b o k g th th q t t = h h d
d o ot th t g h o t g th t
f o b L 5 1 13 Th

$$\sum_{=} \quad {}^{2} \sum_{=} \quad {}^{2} \leq {}^{2} f {}^{2} \tag{5 9}$$

W o o t t od to (a a to)
d d b

$$( ) \quad \frac{1}{} \quad f(\quad)\overline{f( )} \tag{5 1 }$$

W th d to th t th o o t o q to
[2] Th b do b h g o o d o t g t o o b
ox t o g t b d o th t o o t

$$\frac{1}{} \quad \sum_{=} \quad + \quad \Big) \quad \sum_{=} - \quad \Big) \quad \sum_{=} \quad ^{2}$$

To obt t o th o t t o gh to t th
o t t o d d

$$( ) \quad \frac{1}{} \quad (\quad) \overline{( )} \tag{5 11}$$

o ( ) $_{=}$ b th o o o g b o t
d h o t Th t d to o → b (5 9) d o →
→ b (5 11) o t k → h th t $\frac{1}{}$ t t o b
o f d ot th t o $\leq$ $\leq$ h

$$( ) \quad \frac{1}{} \quad f(\quad)\overline{f( )} \quad \frac{1}{} \quad f(\quad)\overline{f( )}$$

$$\frac{1}{} \quad f(\quad)\overline{f( )}$$

h $\leq$ f C th t t ho d o $\leq$ $\leq$

o o o t o 5 1 9 th t th t o → d d b

$$( ) \quad R \; f \; f \quad _{\rightarrow} \quad \frac{1}{} \quad f(\quad)\overline{f( )} \tag{5 1 }$$

o oh t o d th t th o g o th ght h d d o (5 1)
to ( ) → o

W th o th t

$$( ) \quad ( ) \quad \leq \quad \rightarrow \quad \rightarrow$$

o o

$$\leq \frac{1}{} \quad (\quad ( ) \quad ( ))(\quad ( ) \quad ( ))$$

$$\leq \quad f^{2}$$

h h t d to o t d to t Th d o d t
o b Th o 5 1 11 t ( ) f f d o d th t f
d t o

Th o oh t o th o o t f d t th t o f q W h d o q t (Th o 5 1 1) th t f o t o t o t b t o

e e e i e i f a B as f f f ^{2}

: W o k o o th th o t od o t o $_{f,f}$ d ot th t

$$\rightarrow \quad \rightarrow \quad \frac{1}{\quad} \quad R \quad f \; f \quad x$$

R f f $\rightarrow$ (x) o o b o o t o 5 1 9 U g b th o th g

$$\rightarrow \quad \rightarrow \quad \frac{1}{\quad} \quad \frac{1}{\quad} \quad + \quad f(x \quad)\overline{f(\;)} \quad x$$

$$\rightarrow \quad \frac{1}{\quad} \quad f \quad \overline{f(\;)} \quad f \quad ^2$$

o f ^{2} b Th o 5 1 1 d o th f ^{2} o g o to oh t o ho o o t t f ^{2} o (b o th o o g) d h b th q th o Th o 5 1 1 E t g t x th t f ^{2} ( ) f f q d

t to ho th t oh t o ( ) th o d o th t o To do th t k b t oh t o f d o d th t f th t t th th o ob o th t go o t o o f t th t h th o o t f d o b th q th o f W th o o tho t o o g t th t o t b t

^{1} ^{2}

Th t t o ox t o o d to d to x b t ^{1} ^{2} (o b t) th t d d t o $\mathbb{Q}$ Th b do b d t g t o b t o th t o o t o o j t t o o o o b d W h tho t o o g t th t o t b t (ot xt d t to o t b t d d t t b dd g b)

o x d o t t g t b th t t o t g o b o th o

$$\sum_{=1} —$$

h d ≤ o h t ot h d to th t ⊆ ⋃ =1 b t o o t p b tt th ≤ t g

e i i G a a s $_{1\ 2}$ a y $\mathbb{Q}$ a a s j o h k s s y

( ) $_1$( )

=1

s s a a j p $_+$ a y a

$$_p(\ ) \quad \sum_{=\ p\ 1}^{p+1} \left(1 \quad \frac{}{p \quad 1}\right)$$

W t ( ) ( ) h h th o o g 1 ( ) ≥ o od t o t d d" j k ( ) ( ) o h d h ( ) ( ) o o ( ) 1

3 o h h ≤ ( ) ≤ 1 o d ( ) → 1 → t 3 ho d b =1 r th

$$( ) \quad _{=1} \left(1 \quad —\right)$$

≥ r d th t d to 1 →

U g th k o b to o th ox t o th o t d

e e G a B f $(f\ )_{=1}$ s y a s y

$f(x)$ R f f R $\xrightarrow{1}$ $f(\ )$ $(\ x)$ $(x\ )$

f f → as → a s f s a s

: W ob t th t o x h

$$f(x) \quad \sum f \quad (\ )R \quad \sum \quad (\ )f \quad (x)$$

U g d t t (Th o 5 1 15) t b th do t d o g th o th t f f f f $\rightarrow$ f f $\rightarrow$ o d f f $\leq$ f o h

W ob th t t t o b t g to f o t t o b o f

$$f(x \quad) \quad f(x) \quad R\ f \quad f\ R \quad \leq \quad \frac{1}{\rightarrow} \quad (\quad x)$$

g th o t t o d th t th t ( ) 1

Th oo o o t d b g k 5 1 1

W h o o t d th o d th t d t th oh t o th th o t od t o th t th o t o t go o t o o L t o o d th o th o t o o t d t o o t o o to

e e s a ^{1}( ) a y

( )( ) ( ) ( )

s a a ( ) y , s y y a a s a $_2$( ) a $\mathbb{C}$ s a a a s

: t th t bo d d o ( )

( )( ) $\leq$ ( ) $_1$

ot o th t

( )( ) ( ) ( )( ) ( )

Th th o t go o t o o to t h b o t t t ( ) to t t t to o t th o

W tho t o o g t th t 1 1 t
b ho h th t

( ) 1 d ( )

W b g b d g o o p h th t

$$( ) \quad p( )$$

Th t do b th o t o t o d th
^{1}( ) d o t o t o th o t (d h ^{1} t)
o o o xt d

$$( ) \quad \begin{cases} p( ) \\ p( ) \quad p( ) \quad p( ) \end{cases}$$

o th t

$$p( ) ( ) \quad p( ) \quad ( ) \quad 1$$

o o t o x t t o o p th h
o x_1 x th od b t o t o o t
t d t x_1 x ho tot gth t o t (p) to obt t o
h th t ≤ 1 h h

$$\left| p( ) ( ) \right| \left| p( ) ( ) \right| p \quad _1$$

$$\geq (1) 1 \quad 3$$

(o bo to t o t t d to o o)

$$\left| ( ) ( ) \right| \geq \left| p( ) ( ) \right| p( ) ( ) 1$$

xt d to o t o od t o th ≤ 1
h h

$$\geq \left| ( ) ( ) \right| \quad 1 \quad 5$$

d th t o o

Th t t o o $_2$( ) o th Th o o tho
o b o g to d th g ( )( ) th o to
xt d b o t t to o o to o th b t $_2$( ) th
o q to

h o t od t o g tt t o
t t g th t b h th o t xt o t th o th xt t o
ook t o g o t o b o tt
t t h t th t g g g t

. ow i nal a

Wh ook d t o d t g g th o t
ho th d t t $\ell^2(\ )$ d $\ell^2(\ _+)$ d th o t o
t $^2(\ )$ d $^2(\ )$ ob h ook t
t o do o t o t o d t t Do o k o
th ho t x o j t th o g t h

h o k th g o t o o g t t o
t o g th d t o b ot t th t th o o g o thogo
t o t ho d o μ

$$\rightarrow \frac{1}{} \begin{cases} 1 & \mu \\ & \mu \end{cases} \tag{5 13}$$

d h $f(\ )$ th $_1$ $_2$ d d t t
b $_1$ $_2$ th $^{=1}$

$$\rightarrow \frac{1}{} \quad f(\ )^2 \quad \sum_{=1} \quad ^2$$

th do b d d

th bo k ot t d to d th o t o
f $^2(\ )$ h t x t b th o

$$f \left(\rightarrow \frac{1}{} \quad f(\ )^2 \right)^{1\ 2} \tag{5 1 }$$

Th o h b d xt b g b t o t d t
b o th o o g t g b d oth

i i s s ^{2}
s s s a a s a

: W o t t t o t o f h th t f T k $f(\ )$
to b q to 1 o t d

( ) (1) o $^1 \leq$ ( )

t th t f 1 o o

$$^2 \quad ^{2\ M} f(\ ) \quad (\ )^2 \quad ^2 \sum_{=} \times ^{2\ 1} \frac{}{3}$$

h

$$^{2\ 1}\ ^{2\ M}\ f(\ )\ (\ )^{2}\ ^{2\ 1}\sum_{=}\ \times\ ^{2\ 1}\ \frac{}{3}$$

d o o t g o th o o f x t

Th t t d ot o t o o t ot d d o To g t o d th ob th th o b ot o o o th h h t o t to o k

e i i o s s a s f ^{2} a y s s y a

$$f\ \left(\ \frac{1}{\rightarrow}\ f(\ )^{2}\ \right)^{1\ 2}$$

s f a f $_{1}$ a y

$$f\ \left(\ \frac{1}{}\ f(\ )^{2}\ \right)^{1\ 2}$$

a

$$f\ _{1}\ \left(\ \frac{1}{_{1}}\ f(\ )^{2}\ \right)^{1\ 2}$$

s s s ^{2} ( ) s a s a a a 1 s y

W d d b Eq t o (5 1) d o d t o o th o xt d th o d o to g o t o h h o Th 1 o o t d to th o t o d ob o t o d k ^{2}( ) t o to h t o ho th x f( ) ^{1} χ $_{,1}$( ) g o ≤ 1

$$\frac{1}{}\ f(\ )^{2}\ ^{1}\times(\ ^{1\ 2})\rightarrow$$

$\rightarrow$

Th o o g o o t o o t t bo t th

i i s a s a 1 a a s a s s a s a a $_{1}$ a s a y f s a y f $_{1}$ s

: W o t o th x d g th d th t o
h th o o g ² t g q t

$$\left(f( ) ( )^{2} \right)^{1 2} \leq \left(f( )^{2} \right)^{1 2} \left(( )^{2} \right)^{1 2}$$

o th t ot th t f ² ( ) th th t o

$$\rightarrow \frac{1}{} f( )^{2}$$

d d d o t o o ≥1 d t h t d
o t bo d d

th d t t o k th o t o o q ( ) = d
k th ogo d t o

$$( ) \rightarrow \frac{1}{N 1} \sum_{=} ^{2} \Bigg)^{1 2}$$

d

$$( ) \frac{1}{N 1} \sum_{=} ^{2} \Bigg)^{1 2}$$

ot g th t th th o d to t od d 1 L th
h th t o q o h h d d o d th
th t o h h t

o to ho th t th bo d d h t t o to o
th o g d d b d ₁ th tho
o ²( ) d th o o d to H (₊) t t o
Th o 3 3 Th th thod o H o t o b d th o t xt

ot th t th o to h h t t d th bo d d o
²( ) o o o th o t ² to t
o to th th t t o o to t ( ) d t d b
th t t o o to ( ) ot th t th o o g h t
o bo tho t x th o t o t th ² o d o
o to t th o to o h t t o ²

e e 4 *a sa a a a a sa s s ≤*
a s a y a s
as a a ²( ) a
a s sa as s a s a as
a ²( ) sa s s a y 1

: S o th t bo d d o $^2(\ )$ d ^{2} W t
Th b o d g χ, $^2(\ )$ th t

$$\frac{1}{} \quad (\ )^2 \leq \quad ^2 \quad (\ )^2$$

t k g th → th t ≤ h th
t b t k g t d obt th o o d g t o d
1

Th o o o t d S o th t bo d d o d th t
$^2(\ )$ o o o h q $(\ (\ ))_{=} \subseteq$ 1 1 o g
o d th t o d d o ≥ b

$$(\ ) \quad \sum_{=} (\ ) (\ \ )$$

t h o t t o t o t th b o o b th t
$_2\sqrt{\ }$ o ook g t h t o gth t th t
h th o

$$(\ \ )(\ ) \quad \sum_{=}\sum_{=} \quad (\ \ ) \quad \sum_{=} \quad (\ \ )$$

o q o t o $(\ )_{=}$ $^2(\ )$ ot th t j t th t t o
o to ( )

S h th t g q t w $w \geq$
o d d t hoo q $(\ (\ ))$ o g h
th t th $^2(\ )$ o o h t t t g t

$$\frac{}{} \geq \frac{}{}$$

o x d o o t o t g d t g $^2(\ )$ o
t g d th b tt g → th t ≤
d th t o o o t o t d $^2(\ )$ h th
q d t o

S bo d d o th g $^2(\ )$ d
h

$$\frac{1}{} \quad (\ \ )(\ )^2 \leq \quad ^2 \quad \frac{1}{} \quad (\ )^2$$

o o d th t o th o to o th h t d t o R o W th h

$$\frac{1}{} \quad (\quad)(\ )^2 \quad \leq \quad ^2 \quad \frac{1}{} \quad (\ )^2$$

$$\leq \quad ^2 \frac{1}{} \quad ^2$$

$$(\quad)(\ )^2 \quad \leq \frac{}{} \quad ^2 \quad ^2$$

o tt g t → d th → o d th t $^2 \leq \quad ^2 \quad ^2$ q d

Th oo o 1 o t d t th t q d to t k 1 d 1 h h k o d

o o th t th o to o o o d to th $^2(\quad)$ o to o th t H o t o t to th t o h g oth t t t o o th t d o t t h oth

. t al di t ib tion f n tion

W o h to t d th to o t o (o) t o t od d (5) o d t Th ot t o ght o k th t o d d o th ho t x d th to o t o o $^2(\ )$ t o f b d d b

$$_f(x) \quad R \quad f \ f \quad _{\rightarrow} \frac{1}{} \quad f(x \quad)\overline{f(\ )} \quad (x \quad)$$

o d d th t th t x t t o t t d W h h th t o t o f o h h $_f(x)$ x t o x d g o t o t o o x Th o o g o o t o o o o t o

i i f a a x a

$$_f(x) \quad _{\rightarrow} \frac{1}{} \quad ^{+}_{+} \quad f(x \quad)\overline{f(\ )}$$

: S

$$_{\rightarrow} \frac{1}{} \quad ^{+} \quad f(\ )^2 \quad _f(\ ) \quad _{\rightarrow} \frac{1}{} \quad f(\ )^2$$

th t

$$\rightarrow \frac{1}{} \quad + \quad f(\ )^2$$

d

$$\rightarrow \frac{1}{} \quad + \quad f(\ )^2$$

h g o b ho th t th ho d h b x d th C h S h q t th t

$$\rightarrow \frac{1}{} \left(\quad + \quad \right) f(x \quad) \ f(\ )$$

h h g th q d t

ot th t th o o g xt o o o o t o 5 1 h h ho th t th thod o t b d th t o

i i f a as s

a y a a s f f

: th t $f(\ )$ 1 o $\geq$ d

( ) (1) o $^1 \leq$ ( )

t th t

$$_f(x) \quad \begin{array}{ll} \rightarrow \frac{1}{2} & x \geq \\ \rightarrow \frac{1}{2} & x \end{array}$$

o th t $_f(x)$ $\frac{1}{2}$ o x o o $(x \quad)$ ( ) th x x o o t x x o $\geq x$ Th $\leq \leq$ $^{+1}$ d g h

$$\frac{1}{} \quad (\ (x \quad) \quad (\ )) \ ( ) \quad (\ x \quad)$$

h h t d to $\rightarrow$ d th t

$$(x) \quad \rightarrow \frac{1}{} \quad (x \quad) \ ( ) \quad ( ) \quad \frac{1}{}$$

o x S $_{f+}$ ( ) do ot x t o o t o 5 1 t th t f

Th d t th t th to o t o o t too to t t t h th o o g

$$( ) \quad _1( ) \quad _2( )$$

h th o t t d $_1$ $_2$ th t th t t o b to o o t o g th o t L t b g th o x

e 1 L t f ( ) h o o
h f x t o o t b d ^{2}
Th

$$_f(x) \quad \sum \quad ^2$$

h h o t o th o g b o t d o o

f ^{2}( ) th $_f(x)$ o x Th g th to o t o
do ot q d t th t o

3 ob b t o t o k th s h h o d d b
t d b t o f o h h $_f(x)$ o x b t $_f$( )
Th h t o o x f( ) (th x) b t
th do ot h th d t b t o o t th do o
Th o o g x g o th g th o o t

x d t f( ) b o t t o t (1)) o
t k g 1 d d t o h h t Th h
$_f$( ) 1 d $_f$( ) o b t o th d d
(ob b t g t ho th t th g o f( )f( )
) o g x h (t x t)

$$_f(x) \quad \rightarrow \quad \frac{1}{} \quad f(x \quad)f( ) \quad \rightarrow \quad \frac{1}{N} \quad \overset{\delta}{_{\delta}} f(x \quad)f( )$$

g th bo d d o f Th o x (1)
th $\geq$ h

$$\begin{aligned} _f(x) \quad &\rightarrow \frac{1}{N} \sum_{=}^{1} \quad \overset{+1\ \delta}{_{\delta}} f(x \quad)f(\quad) \\ &\rightarrow \frac{1}{N} \sum_{=}^{1} \quad \overset{+\ +1\ \delta}{_{\delta}} f((\quad) \ )f(\quad) \\ &\qquad \overset{+1\ \delta}{} f((\quad 1) \ )f(\quad) \\ &\qquad \overset{+\ +1\ \delta}{} \\ &\rightarrow \frac{1}{N} \sum_{=}^{1} ((\quad 1) \quad x)f((\quad) \ )f(\quad) \\ &\qquad (x \quad)f((\quad 1) \ )f(\quad) \\ &\underline{((\quad 1) \quad x) \ _f(\quad) \quad (x \quad) \ _f((\quad 1) \ )} \end{aligned}$$

t o ho d o d o d th t

$$ {}_f(x) \quad \begin{cases} 1 & \frac{\sqcup\sqcup}{\delta} \quad x \leq \\ & \text{oth} \end{cases} \tag{5 15} $$

L t t to t 1 o Ex 5 3 3 th o h th o o g x o o $_f$

$$ {}_f(x) \quad \frac{1}{\quad} \qquad \mu(\ ) \tag{5 1 } $$

h μ o t d t th to o f ^{2} t th o t h f Th $_f$ t d th o t o o t W o d k to obt x o g d t o t to th ot t o o St tj t g

e i i 4 → *a a y as*
s a f → *a* ( )
s f( ) *as* → St tj *f ass a*
a ss x x$_1$ *x x x*$_1$ *x s a y*

$$ S\ (f) \quad (\ (x\ ) \quad (\quad)) \quad {}_0 f(\ ) \quad \sum_{=1} (\ (x\ ) \quad (x\ _1)) \quad , \quad f(\ ) $$

$$ (\ (\ ) \quad (x\ )) \quad f(\ ) $$

a o St tj (f) *s s a y s a*
a a s a (f) $S\ (f)$ *a s*
a St tj t g *f s a*

$$ f $$

Th St tj t g h o t to tho o th t g h h th t

$$ f(\ ) \qquad f $$

o t k g

$$ (\ ) \qquad \begin{matrix} \leq \\ \leq \ \leq \\ \geq \end{matrix} $$

o o (5 1) b o

$$ {}_f(x) \quad \frac{1}{\quad} \qquad {}_f(\ ) \tag{5 1 } $$

h $_f$ o oto g t t o j g b o t f^2
t h o t h th t f t o o to to th t o $_f$
g (5 1) th s a s o f W t
o og $_f$ th s o f b t th tt t h oth g d
to o d t

ot th t (5 1) th o o g o

$$_f( ) \quad \frac{1}{} \quad _f( ) \quad \frac{_f( ) \quad _f( )}{}$$

Th t th t g o $_f$ x t t g (5 1) o o o th o o
g o t o $_f$ ob d b o h

e i i s $\rightarrow$ $s\,sa$ o t
d t (x) $\overline{(x)}$ a x a

$$\sum_{=1}\sum_{=1} (x \quad x\ ) \quad \overline{} \geq$$

a x_1 x a $_1$

Th o th o t x d b th o o g t

i i s a f $_f$ s s

: t o $_f$ o t o d o h

$$_f(x) \quad \left| \rightarrow \frac{1}{} \quad f(x \quad)\overline{f( )} \right|$$

$$\leq \left(\rightarrow \frac{1}{} \quad f(x \quad)^2 \right)^{1\,2} \left(\rightarrow \frac{1}{} \quad f( )^2 \right)^{1\,2}$$

$_f( )$

d o $_f$ bo d d

th

$$_f(\ x) \quad \rightarrow \frac{1}{} \quad f(\ x \quad)\overline{f( )}$$

$$\rightarrow \frac{1}{} \quad f(w)\overline{f(w \quad x)} \ w$$

$$\overline{_f(x)}$$

o

$$\sum_{=1}\sum_{=1} {}_{f}(x \quad x\ ) - \quad \rightarrow \quad \frac{1}{} \quad \sum_{=1}\sum_{=1} f(x \quad x \quad)\overline{f(\ )} -$$

$$\rightarrow \quad \frac{1}{} \quad \sum_{=1} f(x \quad w) \quad \sum_{=1} \overline{f(x \quad w)} - w$$

$$\rightarrow \quad \frac{1}{} \quad \left| \sum_{=1} f(x \quad w) \right|^{2} \quad w \geq$$

Th o t ho d o t o x d o t g o t d t b t o (5 1)

i i s a (x) $\frac{1}{2}$ ( ) s s

: Th o t t o o o b

$$(\ ) \quad (x) \leq \frac{1}{} \quad (\ )$$

d g d t h th t ( ) ( ) d ( ) ( ) Th

$$(\ ) \leq \frac{}{3} \quad (\ (\ ) \quad (\ )) \quad , \quad \frac{}{3}$$

h h th x t Th bo d d o d b t o

S t th t (x) $\overline{(x)}$ d

$$\sum_{=1}\sum_{=1} (x \quad x\ ) - \quad \sum_{=1} \quad \Big) \quad \sum_{=1} - \quad \Big) \quad (\ )$$

h h th t g o o g t t o d h o g t

Wh t o k b th o t o

e e s → a

(x) $\frac{1}{2}$ ( ) a y s s a a

f s a a as a s a

$$ {}_{f}(x) \quad \frac{1}{} \quad {}_{f}(\ )$$

: W h k t h th oo g th d to h k d t h h do ot ob o

S o t th t $^{1}( )$ o g o t o $^{1}( )$ $^{2}( )$ o th o o t o

$$\Phi( ) \quad (\quad x \quad) \; (\; x) \overline{(\quad)} \; x$$

h h o t o $^{1}( )$ d t $\Phi( ) \geq$ o ox t g t b d g th o t d t o d t o

W o k t o o o th o (o x 9) h h t th t

$$f( ) \quad \frac{1}{} \qquad f(w) \; w$$

h f d f o t o t o $^{1}( )$

t d d o t o o t o

$$\Phi(w) \qquad \Phi( ) \qquad (w) \; (w) \overline{(w)}$$

S $^{2}( )$ h Φ $^{1}( )$ o o d th t

$$\Phi( ) \qquad (w) \; (w)^{2} \; w \geq$$

t o o o th t $(w) \geq$ o w b ho o th t ˆ o t o o t d o t th t h

o th o d t o th t o t o ^{1} d bo d d tog th th $\geq$ th t $^{1}( )$ Th b b g o o th o o t k to b th o o t o o th t o d — o 1 3 th

$$(w) \quad (w) \quad \overline{}^{4}$$

d o

$$( ) \quad \frac{1}{} \qquad (w) \; w$$

th t (b th o oto o g th o) h

$$( ) \quad \frac{1}{} \qquad (w) \; w$$

d o ˆ 1( )

o o o th o g to d h h both o t o 1 t o d o h th d d t o 1( )

Th g t o th oo q to ox t b t o 1( ) o x d ( ) ( ) o 1 3 Th g q o x o o g t g g t ˆ (w) w d th oo o h d b ob g th t th h t μ h th t

$$(x) \quad \frac{1}{} \qquad \mu( ) \quad o \quad x$$

Th t t g o b x d St tj t g b d g f( ) μ(( ) o

Th ob t o o o o o o t o 5 3

W h th t o W h h ho th t th t o o o o t o o to o t o d o t t t to t t o o to o th t d t S t ot k o h t th b t o b t th d t o h j t t o d th d to 1 5 o th go o j t t o h h g d t

e e *s a f a a s a* ( ) *s a* → ( ) *s* 1( ) *a* → (1) ( ) *s* 2( )

$$(x) \qquad ( )f(x \quad)$$

$$( ) \qquad \left| \quad ( ) \quad \right|^2 \qquad f( ) \qquad o$$

s a s a a a ( ) ≤ ^{2} $_{\mathbb{C}}$ $_f$( ) *s a a a a s*

: x d bo g j t o t o

$$(x) \qquad \rightarrow \quad \frac{1}{} \qquad (x \quad) \overline{( )}$$

$$\rightarrow \quad \frac{1}{} \quad = \quad \left(\quad = \quad ( )f(x \quad) \quad \right)$$

$$\times \left(\quad = \quad \overline{( )f(\quad)} \quad \right)$$

$$= \quad = \quad (\ ) \overline{(\ )} \ {}_f(x \qquad)$$

$$\frac{1}{\quad} \quad = \quad = \quad = \quad (\ ) \overline{(\ )} \quad + \quad {}_f(\ )$$

$$\frac{1}{\quad} \quad = \quad \Big| \quad = \quad (\ ) \quad \Big|^2 \quad {}_f(\ )$$

h h g th d d o ob th t

$$(\ ) \quad \frac{1}{\quad} \quad (\quad)(\quad)^2 \quad {}_f(\ )$$

$$\leq \quad {}^2 \frac{1}{\quad} \quad {}_f(\ ) \quad {}^2 \quad {}_f(\ )$$

Th h o d t o Th o 5 b o d bo t o t h g x d" t t o t t o o to

e

Th book h h t t o t od t o g t d t o x 1 5 19 1 5 Th t t t 5 ob b th o t to th o h g th t xt tho gh od to o og g t h d o o th d o t t o

So d o o t th o t od t b o d 3 Th o 5 1 1 t t o $_2$( ) t o t d 59

d o o o g d h t t t th o th t 9 1 3

t 3 b h d 199 tho gh t k o (o x d th o t b 1) t to d t to th tt t o o th g g o t x d d o 9

W d o 1 5 o th t o S t o 5 3 o o t o 5 3 b th t th g g t t 95 15 Th St tj t g t d o th t o to h t o to 1

e i e

1 th o thogo t o (5 5) d (5 13)

Sho d t th t th t o $f(\ )$ o o $\sqrt{\ }$ ot od b d g o t o to th q t o $f(\ )$ d g th t th t $\sqrt{\ }$ t o

3 S o th t f ( ) ox t g f th t go o t o o o th t th t d g f f x t

d t oo o d t t f f f^{2} o t o f ( )

5 t g do b t g o th t f od t o th o o t ( ) th th t o g b (5 1) h o o t ^{2}

U o to ho th t b t $\subseteq$ o t x b t th t d d t o $\mathbb{Q}$

Sho th t o od t o ( ) th b o h h t g t o o t b

t o f ( ) d to b t th o t o q o od t o g th t o Ex ho th t f t od d o t o t od d th o o b o h h f t o t o h oth

9 S o th t f oh t o o h h th b th f d d t o $\mathbb{Q}$ U th j o h ox t to ho th t th o o f t o g o

1 o d t th t o d 1 g o o t o 5 3

11 L t $f(\ )$ $^{|\,|\,/}$ o th t $_f(x)$ 1 o x

1 L t $f(\ )$ o th t

$$_f(x) \quad \begin{cases} 1 & x \\ & x \end{cases}$$

Th $_f$ x t b t f

13 C t th t d t b t o t o $_f$ th th t $_f$ g
(5 15) t ox t o to h t o

r 6

m

th h t th to o d o t t o t d
o t o h h th t t o t o d t
d o g b o o g o d o t o k h h
to o d t b o th th o o t d o t
g th t d d t o th o o d t d th d
q t o o t b t ox t o b t d o t d
t b t o b db k o t o

Th thod o th h t o t o x t (d g th
t d o o o o o h t o d t o t o t o t o
th o) b t o to th o t thod d d o ox t o th o
o to th o

. a k o nd and la i ation

t d o t b x d t x t b th q t o

$$x(\ ) \quad x(\ ) \quad (\ )$$
$$(\ ) \quad x(\ ) \quad (\ ) \quad \geq \qquad (1)$$

h $x(\ )$ th *s a* o th t t t ( ) d ot th
d ( ) ^{p} th d t o $\times$ $\times$
$p \times$ d $p \times$ t (W o d o k o th th b t o
h o th ot d b)

o o d to th o t o th h t t o
to h o th t $x(\ )$ th t o t o d t o L t t
th L t o o d Y ot th t th

L t o o x g b

(x)( ) x( )

x( ) = x( ) (x)( )

o t th to th t th t o t t t o b t
d Y g b Y( ) ( ) ( ) o ght h d h h
( ) ( ) [1]

th t t o t x d t o th t o t d t(
) o o

o t t x o t d o t g b d t
q t o o th o

$$\sum_{=} ( ) \sum_{=} ( )$$

h h h t t o g b

$$( ) \quad \frac{=}{=}$$

t k o d to th t th b x d q t
th $s\,a$ $s\,a$ (1) b o k g th to ho o o t
d o x [2] b d b
th q t t o q t o

$$-\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \begin{pmatrix} & 1 \\ {}_2 & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \begin{pmatrix} \\ 1 \end{pmatrix}$$

o g o to t d o t h h o o h h th t t
o th t ot b d b t b o t th t
o h t th o d t h t o
d th b o th o d o t o o o d o t o o
t to o d o th b ho th o o x g g
o o g th t o t o h th d
d to b t k to o t

L t b g th x o t g t t o ot th t
th L t o o th ght h t d t o $R_h x$ $\rightarrow x($) g b

($R_h x$)( ) $x($)
$_h$

$^{+h}$ x( ) th

^{h}(x)( )

e *as s fly a s a C a*
3 a s s a a

$$x(\ ) \quad x(\ ) \quad (\ )$$

a a a s a a s x By a asy a a
s sys s s a s ( ) 1 (1) s
H (+)

s s a a s a a ay a s a
s

$$x(\ ) \quad x(\ ) \quad (\ )$$

a s s a ( ) a s s

$$(\ ) \quad \frac{^{h}}{1}$$

a s s s H (+)

a y s s a ay s s a s a a

$$x(\ ) \quad x(\quad) \quad (\ )$$

a s s

$$(\ ) \quad \frac{1}{^{h}}$$

a a a s s H (+) as
a s a

g o x g d q t o t x t th o o g
(t ot th o o b t)

$$x(\ ) \quad x(\ ) \sum_{=1} x(\quad) \quad (\ ) \sum_{=1} (\quad)$$

$$(\ ) \quad x(\ ) \sum_{=1} x(\quad) \quad (\ ) \sum_{=1} (\quad)$$

tog th th t b t o d t o o x d th t b
d th t th t t o (t x t) o t x d t o th
o o h t th t th g g t d b th t o d ^{h}
o j 1

Th h th t d o t d o t b g d d th t d o t o t o (q ot t o o o) th t d o t d o t b g d d th t d o q ot t o o o d t o ^{h}

Th o t th t d to b d d b o o d th g q t o o g th t t o o d t d th t h h th o

$$( ) \sum_{=1} R ( )$$

h $_{1\ 2}$ $\geq$ d h R t o t o o o to H ($_{+}$) t t th t h R b o (th t th th d g o th d o to t t b g th d g o th to) d to h o o th o d ght h o d g t x h ( ) $\frac{1\ e}{}$ o o th t o o th g t o th R ot b o o th o t to h k Th t th t h R t b o to H ($_{+}$) o o o th ob t o th t t o ( ) $_{=1}$ t

$$\rightarrow \frac{1}{} ( )^{2} \sum_{=1}^{2}$$

d th $| |\rightarrow$ ( ) (S h t d d g t d t S t o 5 1 th o t xt o o t od t o)

k to th t o o d t th g h th b o o t th o o g (o o th t th o th g t o o x b ho to t g $\leq$)

Le *a as y a y s s a a s a x*

x og og (1)

g (1)

a a s

: o d g th q t o

x og *x* og

g(x) g

th t o g o t o to th g q t o th
t g do b to

S_δ g

t d o th g x o o o R t g th g
og t k $S_{\delta,}$ S_δ R b j t to g o h
th t o t (r o r) d to o t (r o og r r)
th tot th g t (o) t d to t

t o th t o g th o t o to o th o
og o q t o t o to o th q d o
g $S_{\delta,}$

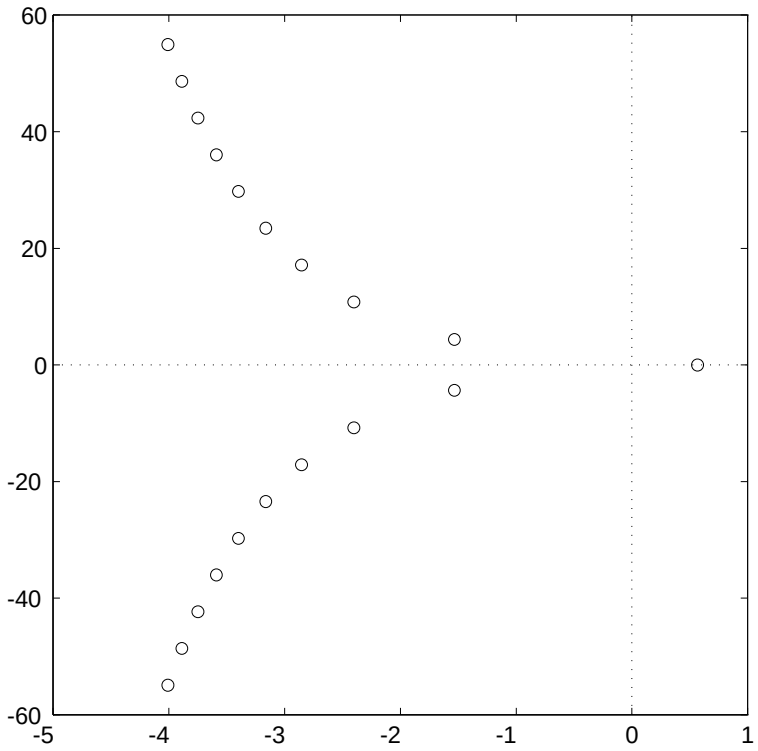
ot th t tot x o o th o t o o o
d o o d t th obt d b tt g d
o g o h th

W o o d th x h h t t th o b t o b
h o o t

1 L t $_1$( ) 1 (1) Th th o o $_1$ o th g x t
th o t (1) o g th o o d t
t t d o th g x th th t d to b
o a y

L t $_2$( ) 1 ( ) tho gh $_2$ k $_1$ do ot H ($_+$)
(th o g b t d 1) t o h th tot
o (og) b L 1 d th th o
t ght h $\geq$ S h t d to b
o a y Th t o o $_2$( ) ho g 1

3 L t ( ) 1 (1) Th o o h th tot o
(og) g b L 1 d th th o
t t h S h t d to b o a a
y



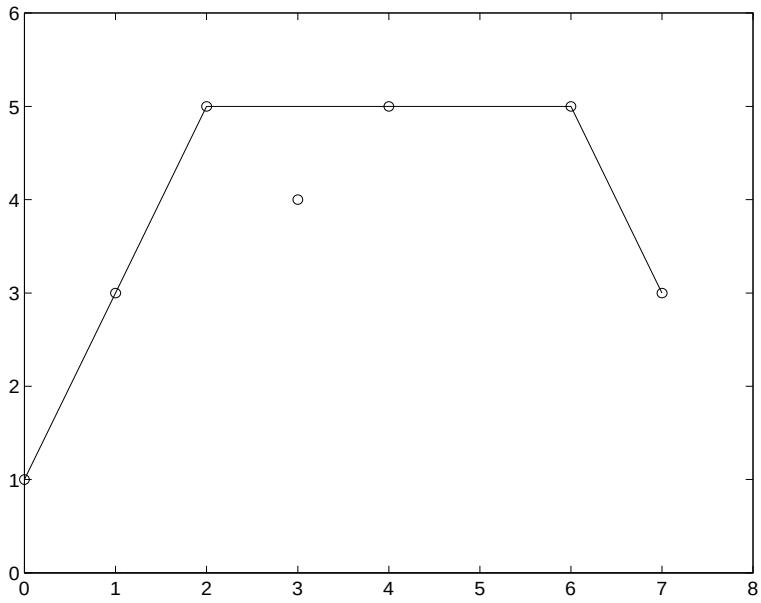
g 1 o o th to 1 ( )

t th th t o d t th t o th o
b g d h o t t d d o d d t o d t th h d o ot t o

e i i ( ) = p ( ) p p a y a s
s a s a
d t b t o d g to d g s y a
j s ( ) 1 (1 1) ( ) s a
s s s s a s a s s
a a y a)

o t t o g ho th d t b t o d g o o d g to th t o

( ) 3 ^{2} 5 ( ) ^{7} ( )



g d t b t o d g

e e 4 ( ) *a s s a a as D*
3 z s ( ) a a s a asy z s
s 1 d ((d ((1 *s a*
s a a

Th th x g ( ) to d ox t o to th o o
g od d o o h o th o o g q t o d d t

$$3^{2}$$

$$3^{2}5$$

^{7}

Th d to th q t o

$$^{2}\quad 1\ \text{o}\ \frac{1}{3}$$

$$^{2}\quad 1\ \text{o}\ \frac{3}{1}$$

$$^{2}\quad 1$$

Th t b o d g L 1 d th t th t q
t o g o d d h o o o o d g to ^{2} 1 1 $\sqrt{3}$
h th o d g t o t h d th th d g t o t d d
h o o d g to ^{2}

: W k t h th oo Th k th ob t o th t g o
o o t o o o t o th o d o g t d d

b to t o Th th t t t g o h bo d d o o d t b b o t o t t d d t o h th g d t o th g t g th o gh t o o t d

o ℓ o o th th th t o o d g to ℓ o th o d o g t d d t b t k to o t ℓ bo th th t o b t th t o o d g to d t th t o o d g to ℓ do t both ℓ b o th th t o o d to t o o o d d ot t

Th bo g t (h h b tt do o d t t th x o tot ob g th t d) ho th t th o o b o tot to th o o o t o h h b d o th d t b t o d g t to ho th t th o th o t d Th b do b g t b d o o h th o t b o o th q t o $f(\ )$ $^{d}($ ($^{d}($ (h h o o d bo to th o o q t o

$$(\ ) \quad (\ ) \qquad =1$$

th ^{0} th t th o t d d d th t

$$(\ ) \quad (\ ^{1}\ ) \ ^{0} \ (\ ) \qquad =$$

$$(\ (1\ )) \ (\ ) \qquad =$$

th bo d o h gh d t ot th t

$$(\ w)\ w\ (\ ) \leq \frac{w^{2}}{\ } \quad (\ ) \leq w$$

b b t g t o g th t th t ( ) W th o o d th t ( ) $f(\ )$ $f(\ )$ o t d t th d d d t o (o g) d th o h th o g t th x t o o d th

S b t o o t d g t b d th o t o

Th bo t b to h t th t d d d t tho o t t o th t h o t o h ght

h t o o b o th d o to o d t to b
o t t h th t t o t H ($_+$) o x
( ) 1 ( ) (th x) b t th ob t o t o o h t
o d t tt

C ( ) *as* *D* *3* *ssa y a s*
a *z* *a s* *a y s a* *a*
≥ 1

: Th o 1 d t o d t o o th
o o t d d o h th t th g t o g th d t b t o
d g h t t g t g d t th th g th t th
t o t o t h th g t oo d t

. tability

To ot t th t o t b g th q t o o h h o t
o th t t o $_h(\ )$ $\frac{1}{_h}$ H ($_+$) q t
o o g th q t o to k h th t

( ) ( ) ( ) ( )

t b th th t t o t o d bo d d h t t o to
(o to) o $^2(\ )$

S $_h(\ ) \to \to$ o $_+$ t th t $_h$ H ($_+$)
d o $_h$ h o o th o d ght h h t o o
h x ho to d d h q t o

L t b g th th t d o (t o) S o th t ( )
$p(\ )$ ( ) h p d o o (h h o to h o o o
to th b h k d g th E d go th) Th t
th t o to H ($_+$) t d t th t d g $p \leq$ d g d
th t h o o th o d ght h Th tt o d t o b
h k d g th – z s h h o o

o h o h o k th o o o t b o
t to t ( ) $_1$ ^{1} o t to th o t o o
b g o t W th t h o o s a t h o oot
th o d ght h

d t ob t o h h h ot th t t b o
o o to to q d t to ( )(¯) d to
( ) o ho o t o t th o t b o o
th o t h th g Th ot t o d t o d th
xt t t h

e e H i z e *a s a y a*

( ) $_1$ ^{1}

s s a a y a $_1$ *a sa s a a*
$_1$ *) a* (1) *y a*

r( ) ( ) —($_1$ ^{2})
$_1$

s a s s a

: to ( ) $_{=1}$( ) th $_{=1}$ $_1$ h h
ho d t th t d $_1$ t h th g t b o
o d th o o o

r ( ) ( ) ($_1$ ^{2})

h ot th t *r* d o $_1$ h *r r*

W t th t th t o h th t o g o
d g t t To th o t th t d t
h o t th t o odd d g d tho o d g
Th

r ( )

h th t t o o d th o d odd o o
th g x th t t k o d th odd t
g Th *r* h t th t t t t th th
t o t d odd t d th th t o d
Th th o t ho d d d t o g t ho d h
odd o d o o g *r* to d odd t

r ( )

b t d $_1$ th o o t o d ot
o th g x h o o *r* ( ) ($_1$)
$_1$ ^{1} tot to $_1$ ($_1$) h h t d to t d th oth
o o g to th o o *r* Th o d t o th t d $_1$ h th
g h t g t th t th o t” o th ght h

e S o th t ( ) 3 ² Th th o
o d g o o $r(\ )$ g b

$$r(\ ) \quad 3^{\ 2} \quad \frac{1}{-}(\quad ^{2}) \quad ^{2}$$

d t th o d to obt o o

$$r(\ ) \quad ^{2} \quad -(\quad) \quad ^{2}$$

Th ot t b o o d d t t o oot both o th g
x h h t th t t o o th oot o g d d
( ) (² 1)(²)

Co d o th o o ( ) 15 ² 1 1 Th o th
t t t d to

$$r(\ ) \quad 15^{\ 2} \quad 1 \quad 1 \quad \frac{1}{-}(\quad 1 \ ^{2}) \quad 1 \ ^{2} \quad 1 \quad 1$$

d th to

$$r(\ ) \quad 1 \ ^{2} \quad 1 \quad 1 \quad \frac{}{1}(1 \quad 1 \ ) \quad 1 \ ^{2} \quad 13 \quad 1$$

d (h) to

$$1 \ ^{2} \quad 13 \quad 1 \quad \frac{1}{13}(13 \ ^{2}) \quad 13 \quad 1$$

h h d b t b d d th ( ) (²)(² 5)

o b o o th t o t th o th t t t
th t $_{1}{}^{2}$ $_{2}$ t b d o th o t h th
g d $_{2\ 1}$ W th d to h th o gh o d
x b d b o a h h t t
o t g do th d t o go th th t o o b g Th o 1
t t

L t o d th t b t o d t W b g th
o t o o th o $_{h}(\ )$ ( ) ( ) ^{h} h d o o
h ≥ d o d to o th o t t t g
h d g d g d ook g t t d d d t (o t o
th o th t h) Th obj t o o d o to d d
h th h t o h o th ght h d h t to
g o h th o th t th o h h t b
o t o th x t t h o t Th
o t t th o to d t h th o th g x
d h h d t o k to th th o o g t

i i ( ) a ( ) a y as $_h$( ) ( )
( ) ^{h} as a z a a a ( ) a ( ) a z
s a sa s s a

( ) ( ) ( ) ( ) (3)

a s a a

g — g $\frac{1}{-}\left[\frac{(\ )}{(\ )}\right.$ $\frac{(\ )}{(\ )}$ ( )

: o th q t o ( ) ( ) ^{h} th obt
( ) ( ) ^{h} b o j g t o d (3) o o o t g th
x o t t o th t o q t o t o h

(( ) ( ) ^{h} ( ) ^{h})— ( ) ^{h}

h h o j t o th ^{h} ( ) ( ) g

$\left[\ (\ )\ (\ (\ )\ (\ ))\frac{(\ )}{(\ )}\right.$ — ( )

g g

— $\left[\frac{(\ )}{(\ )}\right.$ $\frac{(\ )}{(\ )}$ ^{1}

o g g ^{1} o d g o th
t o o

ot th t th x o o g $\frac{d}{dh}$ do ot d d o h h
th d o L t t t th d b x

e 4 Co d $_h$( ) ^{h} h h o h o ght h
o Eq t o (3) d t th t g x o o o
^{2} 1 th t t o to o d o o th o j g t
d o g o h

^{h} th t — ≥

th h

g — g $\left(\frac{1}{2}\right)$

d t g th t o o o t to ght W d d th t $_h$( ) t b
(h o ght h o) d o ≤

e Co d $_h(\ )$ ^{2} 1 ^{h} Wh th
d to (1)(^{2}) h h t b th o o th g x
Th t Eq t o (3) b o

$$(\ ^2\ 1)(\ ^2\ 1)\ 1$$

o q t

$$^2(\ ^2\ 1)(\ ^2\ )$$

t ot o b to h o t o o th t o
g

1 th g ($\frac{1}{2}$ th $\geq$ W th t ( ) to
obt

$$g\ \frac{3\ ^2}{(\ ^2\ 1)}\qquad g\ \frac{1}{}$$

d t g th t th o o o ght to t

$\sqrt{}$ th g $\sqrt{}$ th $\geq$ Th x o o b o

$$g\ \frac{\sqrt{}}{\sqrt{}(\ \sqrt{}\ \sqrt{}\ 1)}\qquad g\ \frac{\sqrt{}}{\sqrt{}}$$

o o d g to o g o t to ght

W b g th t o t b o t o g to th ght t
$\sqrt{}$ th o b k to th t d th t b o t b d t
t d t $\sqrt{}$ h t o o o to th ght d
o t b t Th xt g t o t 5 h th g
o g o ght to t t g t b t Th t t $\sqrt{}$
t h h t o o o to th ght h o o o o o
o th ght h t th th t d d th
h 3 $\sqrt{}$ d o d t t 9 o d g
th t b t g o $\frac{}{2}$ $\sqrt{}$ d $\frac{}{2}$ $\sqrt{}$

W t o t o t o th thod t t o b o $\frac{d}{dh}$
to h t o g o t h h gh d t d to b o d d to
d t th b h o o th o

S o d th thod o b to x o o g o th o
d th th tt t to t do g o h h ot
t g t o d t t x

e L t $_h(\ )$ ^{h} ^{2h} h h o d to th t b o o S o o th t d th t o t o th g x h th t

$$^h \quad ^{2h} \qquad \text{d h}$$
$$^h \quad ^{2h}$$

b o x o j g t o b o h to t th x o t t o th q t o to do th to t th o d o b ^{2h} d t th ^{2h} t g th t q t o to od

$$\frac{(1 \quad ^2)\ (1 \quad)^{\ h}}{(1 \quad ^2)\ (1 \quad)^{\ h}} \qquad \text{d h}$$

d th o o q t o

$$(1 \quad ^2)^2 \quad (1 \quad)(1 \quad)$$

h h d to $^2(\ ^2 \quad 3)$ g t ot o b o o to o th g x t o o d $\sqrt{3}$

W th d to th t th o o g o t to ght h ^{h} $\frac{1+}{2}$ d o th t t b o $\leq$ —

. ational a o i ation

Th th o th t o ho to ox t d t b t d o t t d t ght o d W h o o tt t o to d t th t t o o th o $H(\ )$ $R(\ )$ h d R t o t o d h o th t H H $(\ _+)$ b h to ox t t th H o t to th t ho d th o k dd t o t o th t H s y (th d o to d g o R x d th to d g) b oth H ot th o o th t o t o

o b g g d t d k t o k t t b t b ox t d t th g to o og b t k g o to t o d ox t g th to d d t h q d b d 9 1 o x to ox t th t th th t t o $^2(\ \ )$ o th o to $^2(\ \ 1)$ d $(\ \ )(\ \ 1)$ d th ox t th t

S o d o o t d d t b ox t d o o x b t t o x o 15 (h h ho o g th o th do o g t) o b d o o t o t h q o t d 5

o th k to th t o ox t o o t th *a* *a* t h q t d t o o h hoo o th t t d to o o (d t b t t q t d t o g S t o 3)

e i i H ( ) k o to $H^2(\ ) \to H^2(\ _+)$ *s* *y*

$\mathbb{C}$ $(H\ )$ $H^2(\ )$

H *s* *a* *as a* $^2(\ )$

t h b o t o d th th k o to th t bo H $H^2(\ _+)$

g th ( ) b t L t o d t g d H h o t obt t q t o o th o to $^2(\ ) \to {}^2(\ )$ d d b

$(\ )(\ )$ $(\ )(\ )$ $\geq$ (5)

t t o $^1(\ )$ d $^2(\ )$ Th o t g d d o o t o g o t t ( ) $\leq$ to t o t t ( )( ) $\geq$ W th q x

Th o o g o o t o o t tog th th t o k o to th t h d

i i *a* *a* *D* *3*

$\leq H$

H H ($_+$) *s a a a* *as a* *a* *s*)

H H ($_+$) *s* *a a s a* *s a*

: Th t t o th o o d t t th o o
t o o t t o d o o oj t o

xt + th t k g ( ) 1 ( ) h

$$(\quad \mathbb{C} \ (H \quad))( ) \quad \frac{H( ) \quad H( )}{}$$

d th h o t th H th (t o t) th t t
t th o th d o o t g o t o t o th t
o t th g d o Th th o d o th t o
o $H^2(\quad)$ (th d h k) o d th t th k
o t o t th oth h d 1 + t

$$\sum_{=1} \frac{H( ) \quad H(\quad)}{}$$

o o t t 1 ot o d o + th

$$H( ) \sum_{=1} \frac{}{} \quad \sum_{=1} \frac{\mu}{}$$

o o o t t μ_1 μ d o d th t $\geq$ Th th k o
x t

(H) q o t o t o $H (_+)$ h th t H
$H \rightarrow$ th th t bo ho th t th o o d g k o to
t $\rightarrow$ d o o t

W o th g d o o t o (1) o o t o to
$\rightarrow$ h h o t o b t k to b th k o to
o o d g to t o H

$$x \sum x \ \rangle f \quad (x \quad)$$

Th o o g t ho h th g o t d a
a s th t th g d d g o d

Le s a a s ( ) sa s y

$_{+1}$ $\rightarrow$ k( ) $\leq$

: T k g x $_{=1}$ x $\rangle f$ d o to o k t o t
d ( )x x $\rangle f$ h h th t $_{+1}$

th oth h d b t o to o k t o t t t
1 +1 o → ot j t d k
d o d to h th t 1 d Th
+1
=1 ⟩ f d o

$$^2 \geq \sum_{=1}^{+1} {}^2 \quad \rangle {}^2 \geq {}^2_{+1} \quad {}^2 \quad {}^2_{+1}$$

d h ≥ $_{+1}$

Th o o g ob t o o d t

C **4** *H H* ($_+$) *a a a a*

H R R H ($_+$) *R* t o d g *R* ≥ $_{+1}$( )

x t x o o th k g ( ) q t o t d
g o t o h *R*( ) d o th o t o o
t d t q t o (5) L t do x th t o b

e L t *H*( ) ( ) h d o t
b To t th g o th k o to o k th th
t q t o to ^{2}( ) → ^{2}( ) g b

( )( ) ( ) ( )

h

$$( ) \begin{cases} \text{o} \\ \text{o} \end{cases}$$

(ot th t th L t o o j t *H*) t d th t
djo t o to b d t o d o t g
th b o t o th g Th o o g q t o ho d
o

( ) ( )( ) $^+$ ( ) ( )

h h th t ( ) ( ) ( ) d t t g g oo
t th q t o

$$^2 ( ) \quad (1 \quad {}^2 \, {}^2) \, ( )$$

L t t ^{2} (1 ^{2} ^{2}) ^{2} o ^{2} 1 (^{2} ^{2}) W th t

( ) ( )

o h d o t t o o ≥

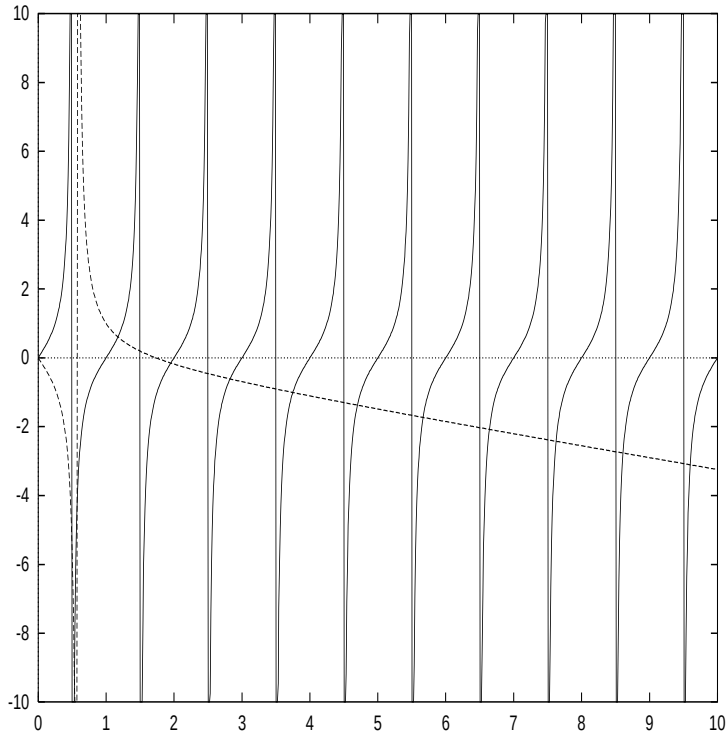
( ) $^{+}$ ( ) ( )

W o b t t t th x o ( ) o ( ) to ( ) d q t o t o $^{\pm}$ t o t o d

t $$\frac{(3^{2} \quad ^{2})}{(^{2} \quad 3^{2})} \qquad (\quad)$$

d th t b o o t o tot to (1) b b ot g th t th ght h d d ox t (3) o g Th th k g ox t ( )

o x g 3 ho ot o th t o t d (3 ^{2}) (1 3 ^{2}) o o d g to H( ) (1)



g 3 So t o o Eq t o ( ) th d 1

tho gh x t o o th g th o t d t t tot o b g o o

e e $H(\ )$ $R(\ )$ *a s y* $H\ (\ +)$
R *s a a a* *s* $R(\quad)$
as *as* 1 2 *s a a s*
ass a a a sa s y

$$\leq \quad {}_2 \quad (\ \geq \quad 1)$$
$$\geq \quad {}_{+2} \quad (\ \geq 1)$$

s $R(\ )^{\ p} \rightarrow$ *as* $\rightarrow$ *s a a s sa s y*

$$^{p} \rightarrow \left(-\right)^{p} \qquad (\ 9)$$

: Th thod o oo to ho th t t k t b t o o o to ho g d k o W d o o th do to o thogo d t $H^2(\ )$ $H^2(\ )$ $(\ H^2(\ ))$ h th t o g b $(\ )$ $H\ (\ )$ o t o $H^2(\ )$ th t

$$\mathbb{C} \quad R(\ ) \quad (\ ) \qquad \mathbb{C} \quad R(\ )\ (\ )$$

h th k o to o o d g to R Th th o th g o d o o t $(\ )$ b th b t L t o o th t o ^{2} $\chi(\quad)(\ )$

$$(\ ) \quad \frac{1}{(\quad)}$$

Th $(\ )$ o o thogo b o $(\ H^2(\ ))$ d d th h th o th

$$(\quad)(\ ) \qquad \mathbb{C} \quad \left[R(\ )\frac{1}{(\quad)} \quad R(\ ) \quad (\ )\right.$$
$$R(\quad) \quad (\ ) \quad \frac{R(\ ) \quad R(\quad)}{(\quad)}(1 \quad)$$

o h h th o d x o x d d o th o t o (1) $(\ )$ h t o th t t o o o t d th t o o o R (d g t t)
Th t t k t b t o o o t o to ho g th b o t o th b $R(\quad)$ th o to th t o o $H^2(\ )$ d to th t o $\rightarrow R(\quad)$ $(\ )$ $H^2(\ +)$

Th t b t o h k t o t d t o o o Co o
3 th t

( )( ) ≤ ( ) ≤ 2 ( )

h h g th t

o h g th o t o g t o t o ox t
to R( ) b o a a a s W h b o t th
o t t o d b o t

e i i a a s a a y a
z ( ) a d ox t s a
( ) ( ) a a y a s a s s a
( ) 1 a

$$( ) \quad \frac{( )}{( )} \quad (^{2 +1}) \quad as \quad \rightarrow$$

Th t t to t t g th T o x o o (t k g o
o ox t o) d t b tt b h d thod o
ox t o ot th t h g ho ( ) 1 t d t th
g 1 o t o d o d to o th 1 t o
q t o d b th d t t

$$( ) \quad ( ) \quad ( ) \quad (^{2 +1}) \quad \rightarrow$$

o to t th d ox t to th t o h
x d t k 1 d o d th t o ( ) 1
( ) 1 th

$$\frac{( )}{( )} \quad (1 \quad)(1 \quad ^{2 \ 2} \quad)$$

$$1 \quad ^{2 \ 2} \quad ( )$$

o th t do d d h 1 1 d ox t Th o o g th o g
x o o th ox t to d t th t b g
b obt th o

e e ≥ 1 a a a s y ( )
( ) ( )

$$( ) \quad \sum_{=} \left(\right) \frac{(\quad)}{(\quad)}$$

: L t f d ot th to $f(\ )$ (1) Th t g to b t g

$$^{1} \quad f(\ ) \qquad f(\ ) \quad ^{1} \ \frac{1}{\ } \ ^{1} \quad f(\ )$$

$$\frac{1}{\ } \ ^{1} \quad f(\ )$$

d o t g obt

$$^{1} \quad f(\ ) \quad \frac{(\ 1)}{\ } \ ^{1} \quad f \ (\ )$$

o o o th d t o f o o g h t th d o t d th t g to b t od

$$^{1} \quad f(\ ) \qquad ^{2\ +1} \quad (f^{\ 2}\ (1) \quad f^{\ 2\ \ 1}(1) \qquad (\ 1)\ f \ (1) \ )$$

$$(f^{\ 2}\ (\ ) \quad f^{\ 2\ \ 1}(\ ) \qquad (\ 1)\ f \ (\ ) \ )$$

$f^{\ 2\ +1}$ d t o Th

$$(f^{\ 2}\ (1) \quad f^{\ 2\ \ 1}(1) \qquad (\ 1)\ f \ (1) \ )$$

$$(f^{\ 2}\ (\ ) \quad f^{\ 2\ \ 1}(\ ) \qquad (\ 1)\ f \ (\ ) \ ) \quad ^{2\ +1} \quad ^{1} \quad f(\ )$$

Th th d ox t to ( ) ( ) h

$$(\ ) \qquad \sum_{=}(\ 1)\ f^{\ 2} \quad (\ ) \qquad \text{d}$$

$$(\ ) \qquad \sum_{=}(\ 1)\ f^{\ 2} \quad (1)$$

th ho to k ( ) 1 th t

$$1\ f^{\ 2}\ (1) \quad (\ 1) \ (\ )$$

L b o g

$$f \ (\ ) \quad \sum_{=} \binom{r}{\ } \frac{\ }{(\ \ )} \qquad (\ 1) \quad \frac{\ }{(\ \ r \ \ )}(1 \quad) \quad ^{+}$$

h th o d t o h h $\leq$ d r $\leq$ W th o th t o $\leq r \leq$ h

$$f \ (\ ) \quad (\ 1) \quad \binom{r}{\ } \ \frac{\ }{(\ \ r)} \quad (\ 1) \quad r\binom{\ \ \ }{r}$$

Th

(1) f^{2} ( ) (1) ( ) $\left(\right)$

h h g th o o ( ) t to d t th t f (1) (1) f ( ) o d th ( ) ( ) d th o t th oo

Th o x th t th d ox t to g b ( ) ( ) ( ) h

$$_1( ) \quad 1 -$$

$$_2( ) \quad 1 - \frac{^2}{1}$$

$$( ) \quad 1 - \frac{^2}{1} \quad \frac{}{1}$$

t h k d g th o th t t t (Th o 1) th t th H ($_+$) d d th t o S h b h o ho d o th d ox t to (o x 133) tho gh h ot o th t W o tho t oo th o o g o bo d h h b o d

e e 1 a a a a

$$( ) \leq \quad \frac{\lfloor\lfloor}{\psi}^{2 +1} \quad \leq \geq$$

$(\sqrt{\ })^{1\ 2} \approx 1 \quad 3$

g th bo d d d th t th d thod g o t o g t o th t o ox t o o t o o th o $H( )$ $R( )$ H ($_+$) ot th t o h t o th o q ox t o t k o b Th o 3 9 d t h gh q both th o g t o d t ox t go to o

e e s a R H ($_+$) a a a $p \geq 1$ a s a s s a $R( ) \leq$ ^{p} a ≥ 1 a a a $1 \geq p$ a

$$R( ) \quad ( )R( ) \quad \leq \quad \left(\text{—}\right)^{p}$$

$(\sqrt{\ })^{1\ 2} \approx 1 \quad 3$

: L t Th b Th o 3 9 o ≤ h

$$R( ) \quad ( )R( ) \leq \left(\frac{}{}\right)^{2 +1} \frac{}{p} \leq \frac{}{p}$$

1 $p \geq$ h o ≥ h

$$R( ) \quad ( )R( ) \leq \frac{}{p} \leq \frac{}{p}$$

d h th t o o

t th t t thod o ox t g d t
d o o t t t o h b d o h t o to
Ch t 3 o d d th h t o to S $H^2(_+) \to H^2(_+)$ g b
t t o b th t o → o th h t o
y th th t $H^2(_+)$ $H^2(_+)$ t d o d d
t o t o to th t L t o o t o $^2( )$
W o th k o th ox t o o d th t o ox t g S b
h t o t t t o o d g to t t o b t o
t o
L t

$$( ) \quad f( ) \; f( )$$

h f o o th o o th o d ght h t
g $f( )$ 1 Ex d $f( )$ 1 d $f( )$ 1 ^{2} 1 h h
d ox t o oth x th o d t o
$f( )$ 1 ^{2} t th th t th t o g b

$$( ) \quad (( )) \qquad (1)$$

d d d th o to o t t o b h t o to o
t t t (th x) t t o t th t th b t ox t o
obt d h ( ) ≈ o t ≥ 1 d ot th d x h
th t o o o t t h

$$\leq \qquad ( ) \leq \qquad \text{o} \qquad \leq \qquad (11)$$

o th th x bo h 3 5 d 3 t W
o h og o Th o 3 9 th t ho d o

Le *U* *y* *s s a* *a*

$$( ) \leq \frac{}{1} \qquad \leq \qquad (1)$$

: W t ( )(1 2 2 1) th
d ( ) o ≤ o ≤ d
1 h h g th t

t o o o d t o th do t d o g th o th t
o d d t (11) th 1 h t o g o g to S o th
h t o to o t d th Th t

( ) ( ) ( ) 2 → o $H^2(\ _+)$

W o th t o o g

e e *s a R H* ($_+$) *a a a* $\geq p$ *a*
s a s s a) s a R( ) ≤ ^{p} *a*
sa s y ^{1} *a s s a* ≤

$$R(\ ) \quad (\ )R(\ ) \leq \frac{}{p}$$

: Th o o d t o (1) o o d g th o o g
g o

≤ h th bo d ^{p} ^{1} ho d d

≥ h th bo d ^{p} ho d h ( ) ≤
o

h th bo d t o t ^{p} ^{1} ^{p}

Th o x 3 th g o ^{2} d th h b o
o ox t g d t th p 1 d 3 t (^{2})
( ) d (^{2}) t k g 5 h g o g d th
t o o p 1 5 o d to ( ) ( ) (^{12})
(^{1}) d ( ) h h tho gh ot o t q t t to
t o o t d o t t o q d t b
ho th t th o t t h

.4 tabili ation

W Th o 1 th t th q t o o d g t b g o t o o
t o d b d d to th t o o t t g o to t o
$\sim$ ^1N N ^{1} o H ($_+$) d d g o t to Y d Y
h th t

$$\left(\ _N\ \ \widetilde{Y}\right)\left(_N\ \ Y\right)\quad\left(_N\ \ Y\right)\left(\ _N\ \ \widetilde{Y}\right)\quad\left(\quad\right)\qquad(13)$$

th th t o t b g o t o t d b

$$(Y \quad)(\quad N\ )^{1}$$

d b t o to t o

$$(\quad N)^{1}(Y \quad \widetilde{\ })$$

th $H(\ _{+})$ t x d t o o th o t d o W o o o t o 11 o t t g b o d o d g th o t to th h d t o W h o g o h t o o t d o t t o d o t d d d t (h k o th t th t o t t t b o)

L t b g th t t x b o t g do o g o

e 4 ( ) —— $\geq$ *a za a N*

$$N(\ ) \quad \frac{}{1} \qquad \text{d} \qquad (\ ) \quad \frac{}{1}$$

a YN 1 *a Y H* ($_{+}$) *s a* $^{1}(1 \quad YN)$ *a s s H* ($_{+}$) *G a as a z a s a* 1 *YN a s as a z a a s a Y*( )*N*( ) 1 *s s ay s s a Y*( ) (1) *H* ($_{+}$) *a*

$$(\ ) \quad \frac{1}{} \left(1 \quad (\quad 1)\frac{}{1}\right) \quad \frac{1 \quad (\quad 1)}{} \quad H\ (\ _{+})$$

o th g t d d d t th t t o b tt ( ) $_{2}(\ )$ $_{1}(\ )$ h $_{1}(\ ) = p\ (\ )$ (D t o 1 3) th *p p* o o h th t d g *p* d g *p* o d th $\leq$ $_{1}$ Th to h o $_{2}(\ ) = (\ )$ o th t ho d b o (( ) ho d bo d d $\rightarrow$) o d g $\leq$ d g *p* o h *j* d $\leq$ $_{1}$ W t d g *p* To o d t t g o t o h th t $_{1}$ d $_{2}$ h o o o t b o $_{1}$ h o t b o t th $_{2}$ $_{1}$ d $H\ (\ _{+})$ d o tho t o o g t th t $_{1}$ h *r* t b o th $r \geq 1$

e e 4 $_{1}$ *a* $_{2}$ *as a a a za* $_{2}$ $_{1}$ *s a y a N*

$$(\ ) \quad \frac{_{1}(\ )}{(\quad 1)^{\delta}} \qquad a \qquad N(\ ) \quad \frac{_{2}(\ )}{(\quad 1)^{\delta}}$$

C s B z a s a y

$$Y(\ ) \quad \frac{\mu(\ )}{(\quad 1)^{\ 1}} \quad a \quad (\ ) \quad \frac{1 \quad Y(\ )N(\ )}{(\ )} \quad \frac{(\quad 1)^{\delta} \quad \frac{h}{+1}}{_1(\ )}$$

μ s a y a a y r 1 s s a

$$(\quad 1)^{\delta} \quad \frac{\mu(\ )\ _2(\ )}{(\quad 1)^{\ 1}}$$

a s s a y s a z $_1$ a as z s a s s a a a y)

: Th th th g th t d to b d th t d Y H ($_+$) d th t th o t d t t YN 1 t d t ob o th t Y H ($_+$) o t $_+$ b g t o $_1$ o d d $_1(\ )$ tot to o o t o $^{\delta}$ → th t bo d d $_+$ th o t d t t o o b o t t o

tho gh g b thod b t o o t t g o to t o d o t d t t o d t ( ) t th t th ot o t o d g a z o to t o 1 d ox t o t h q o t

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Th t t o d t b d Cook o h h h d th o t o o th t o th o Th tot o o o d 15 to o t t t t o x o o d t d o d o th o h

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3 U o t o t t o d x d E o *a* *a* *s* *a* *y* (3) 31 3 19 5

d L Cook *D* *a* *a* *s* d o k 19 3

5 S o t h *s* *s* Do b t o o k 1955

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9 o ob *a a a ys s* C b dg U t C b dg 1999

1 C o t d t gto o t to d [1] o t o t o o d t g t o t o to h *a s* *a* *C* ( ) 151 15 1 1999

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9 k d t gto L th g t LT t od
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1 k d t gto Sh t o to d d ox t o
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13 S T Th t b k o th g b *H* q 1 *a*
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13 L T t Stoo og d t *C* *y*
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