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Elena S. Prassas
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# Engineering Economics and Finance for Transportation Infrastructure 

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# Engineering Economics and Finance for Transportation Infrastructure 

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## Chapter 1 Introduction

This chapter provides an overview of the textbook, and discusses key reasons for the importance and relevance of the subject.

### 1.1 Funding Needs for Transportation Infrastructure

In 2011, the American Society for Civil Engineers (ASCE) released an analysis of the funding needs for the nation's surface transportation systems (highways, railroads, and transit), and the shortfalls in current financing [1]. The report provided a critical wake-up call for this key element of the nation's infrastructure.

The report indicated that a minimum investment of an average of $\$ 220$ billion/year would be needed to keep our surface transportation system operable at a level sufficient to meet the nation's minimal needs. This amount would be needed for at least the 30 -year period between 2010 and 2014, yielding a total 30 -year funding need of $\$ 6.6$ trillion. Over the same 30 -year period, the report indicates a shortfall in anticipated revenues (under the current funding process) of $\$ 2.972$ trillion - a shortfall which is equal to $45 \%$ of the need.

If the absolute shortfall in anticipated revenues were not daunting enough, the report estimates that the impact of a deteriorating surface transportation system on personal income would reach $\$-3.135$ trillion by 2040, and that the impact on the nation's gross domestic product (GDP) would be \$-2.622 trillion by 2040.

The shortfalls for the nation's surface transportation systems do not even reflect the totality of transportation needs. The ASCE Infrastructure Report Card [2] issued in 2009 indicated additional aviation funding needs of $\$ 87$ billion over a five-year period. An update to this report is expected in March of 2013.

### 1.2 Revenues for Transportation Infrastructure

At the federal level, the principal source of funding for surface transportation is the excise tax on gasoline. It produces about $89 \%$ of the federal transportation revenues that support the Highway Trust Fund.

Gasoline tax revenue, however, is on the decline and under attack. As vehicles become more efficient, partially in response to new federal fuel efficiency standards, and partially in response to the new vehicle market, there is an accelerating negative impact on gas tax revenues. This, combined with recent declines in annual vehicle-miles travelled, has caused federal gasoline tax revenues to decline.

In fact, gas tax revenues flowing to the Highway Trust Fund peaked in 1998 at close to $\$ 30$ billion. They had declined to $\$ 27$ billion by 2009, and are continuing to decline. It is estimated that when actual revenue declines and the effects of inflation are considered, that the total purchasing power of federal gas tax revenues has declined by $33 \%$ since 1993 [3]. On the state and local levels, declines in motor fuel tax revenues have been more recent, and more modest. After peaking in 2008 at nearly $\$ 39$ billion, state and local motor fuel tax revenues had fallen to $\$ 37.9$ billion by 2010 [4], with losses continuing to accelerate.

Indeed, federal policy on motor fuel excise taxes has varied greatly over the years in response to both needs, and related and unrelated policy issues. For a good legislative and policy history on federal motor fuel excise tax rates, see Reference [5].

### 1.3 Tools for Analysis

This text addresses two key issues that will help determine the future course of transportation infrastructure in the U.S.

As the nation's needs for transportation improvements increases in a tight fiscal environment, it will be critical for available dollars to be spent in the most efficient and effective manner.

The primary set of analysis tools for ensuring this is engineering economics. Profs. Grant and Ireson once noted that an engineer was someone "who could do for $\$ 1$ what any fool could do for $\$ 2$ " has stood the test of time [6]. Engineers simply must get the most "bang for the buck," whether the "buck" is coming from a private investor or from the public tax base. Money is a limited resource, and must be invested for the best advantage.

Engineering economics is a set of analysis tools that allows engineers and poli-cy-makers to assess the relative economic advantage in making alternative investments. For the purposes of this textbook, the "alternative investments" are competing proposals for transportation infrastructure improvements. As long as there is not enough money to fund every proposed improvement, they must be compared to insure that optimal investments are made. Even if there were enough money to fund every transportation proposal made, engineering economic analysis would still have to be conducted to make sure that the investments "earn" in benefits to transportation users and the general public enough to justify any investment.

As will be seen, engineering economic analysis can be used to assess different proposals for a single project, competing projects on a regional level, selection of multiple projects for implementation by an agency with a limited budget, and even alternative methods for financing a given transportation project.

The second primary challenge is to ensure that there is sufficient funding available to make it feasible to provide, maintain, and operate a transportation infrastructure that support's the nation's social and economic wellbeing. This is a more complicated issue, involving public policy and the inevitable politics that accompanies it.

This textbook provides a history and overview of transportation finance in the U.S., and an exploration of current and new ideas for how to meet the nation's needs for transportation funding over the next 30 years and beyond.

There can be no doubt, however, that the collective failure of the nation and its leaders to find workable solutions to the nation's obvious transportation infrastructure needs will have a dire impact on the U.S. economy, and on the lives of its over 300,000,000 residents.

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# Part I <br> Engineering Economics with Transportation Applications 

## Chapter 2 <br> Banking Formulae

This chapter introduces the theory of basic banking transactions, and their simple application to alternative analysis.

As discussed in the chapter 1, it is useful, as a background to studies of transportation economics, to have a good understanding of the principles of engineering economy, which is extensively treated in the literature [1-4], and of basic banking (compounding) formulae. A study of the methods for handling time differences in the comparison of various project costs (including initial costs, recurring costs, etc) is essential. Every engineer who is called upon to make engineering studies involving cost analysis should be familiar with the following derivations of mathematical formulas used in converting costs that are dissimilar in time to comparable bases. Without these conversions, it would be impossible to compare costs occurring in the future with costs that occur now.

### 2.1 Single Payment Transactions

### 2.1.1 Compound Amount

The simplest form of bank transaction involves a single deposit of \$P that is left in the bank for some amount of time, $n$ (measured in compounding periods), and then withdrawn as a single sum made up of the initial principal amount plus the accumulated interest, at a rate of $i$ per compounding period. For simplification, the number of compounding periods $n$ is referred to throughout as the number of years, that is, a compounding period of one year is assumed. However, all equations hold regardless of whether $n$ represents years, quarters, months, or days. Of course, the correct interest rate $i$ (annual, quarterly, monthly, daily) must be chosen as well. Thus, a deposit left in an account for 10 years, at $8 \%$ per year, compounded quarterly, would use $n=10 \times 4=40$ compounding periods and $i=8 \% / 4$ $=2 \%$ per compounding period, in the equations below.

The derivation of the formula to determine the total amount withdrawn after $n$ periods is based upon the fact that at the end of any given period, you have the initial amount plus the interest for that period; this amount then becomes the input for the next period. In the derivation, $P$ dollars is deposited for $n$ compounding
periods, at an interest rate of $i$ per period. An amount $F$ is then withdrawn in the future, depleting the original deposit plus all interest.

The amount in an account at the end of a given interest period is equal to the amount at the beginning of the interest period plus that amount times the interest rate (expressed as a decimal). Thus if $\$ P$ is the original deposit, then the amount in the account at the end of the first period is $P+P i$ or $P(l+i)$. This then becomes the amount in the account at the beginning of the second period, and the amount at the end of the second period is $(P+P i)+(P+P i) i$. Using numbers to make this even clearer, if the original deposit is $\$ 100$, put into an account paying $5 \%$ compounded annually, then at the end of the first year you would have $\$ 100 *(1+.05)=100 * 1.05=\$ 105$. At the end of the second year, you would have $\$ 105 * 1.05=\$ 110.25$. As can be seen from these examples, compounded interest means that interest is paid on the interest earned, as well as on the principal.

The derivation of the formula then, for $n$ years, is as follows:


Compound Amount
$P(1+i)+P(1+i) i=P(1+i)^{2}$
$P(1+i)^{2}+P(1+i)^{2} i=P(1+i)^{3}$
$P(1+i)^{3}+P(1+i)^{3} i=P(1+i)^{4}$

$$
P(1+i)^{n-1}+P(1+i)^{n-1} i=P(1+i)^{n}
$$

The general expression for a single payment transaction is:

$$
\begin{equation*}
F=P(1+i)^{n} \tag{2.1}
\end{equation*}
$$

Where: $F$ is the future compound amount (\$),
$P$ is the original principal amount deposited (\$),
$n$ is the number of compounding periods, and
$i$ is the interest rate per compounding period (expressed as a decimal).
The term $(1+\mathrm{i})^{\mathrm{n}}$ is called the Compound Amount Factor (CAF), which can also be denoted as $\mathrm{F} / \mathrm{P}_{\mathrm{n}, \mathrm{i}}$. $\mathrm{F} / \mathrm{P}$ is read "Future given Present," and gives the future value when you start with the present value and leave it untouched for $n$ periods at interest rate $i$.

Using the $F / P$ notation, the equation becomes:

$$
\begin{equation*}
F=P^{*} F / P_{n, i} \tag{2.2}
\end{equation*}
$$

This factor can be tabulated in compound interest tables for different interest rates and compounding periods. Table 2.1 shows the compound amount factor for a selection of interest rates and time periods.

Table 2.1 Compound Amount Factors ( $\mathrm{F} / \mathrm{P}_{\mathrm{n} .}$ )

| No. of | Interest Rate (\% per period) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Int. Per. | 1\% | 2\% | 3\% | 4\% | 5\% | 10\% | 15\% | 20\% |
| 1 | 1.010000 | 1.020000 | 1.030000 | 1.040000 | 1.050000 | 1.100000 | 1.150000 | 1.200000 |
| 2 | 1.020100 | 1.040400 | 1.060900 | 1.081600 | 1.102500 | 1.210000 | 1.322500 | 1.440000 |
| 3 | 1.030301 | 1.061208 | 1.092727 | 1.124864 | 1.157625 | 1.331000 | 1.520875 | 1.728000 |
| 4 | 1.040604 | 1.082432 | 1.125509 | 1.169859 | 1.215506 | 1.464100 | 1.749006 | 2.073600 |
| 5 | 1.051010 | 1.104081 | 1.159274 | 1.216653 | 1.276282 | 1.610510 | 2.011357 | 2.488320 |
| 6 | 1.061520 | 1.126162 | 1.194052 | 1.265319 | 1.340096 | 1.771561 | 2.313061 | 2.985984 |
| 7 | 1.072135 | 1.148686 | 1.229874 | 1.315932 | 1.407100 | 1.948717 | 2.660020 | 3.583181 |
| 8 | 1.082857 | 1.171659 | 1.266770 | 1.368569 | 1.477455 | 2.143589 | 3.059023 | 4.299817 |
| 9 | 1.093685 | 1.195093 | 1.304773 | 1.423312 | 1.551328 | 2.357948 | 3.517876 | 5.159780 |
| 10 | 1.104622 | 1.218994 | 1.343916 | 1.480244 | 1.628895 | 2.593742 | 4.045558 | 6.191736 |
| 15 | 1.160969 | 1.345868 | 1.557967 | 1.800944 | 2.078928 | 4.177248 | 8.137062 | 15.407022 |
| 20 | 1.220190 | 1.485947 | 1.806111 | 2.191123 | 2.653298 | 6.727500 | 16.366537 | 38.337600 |
| 25 | 1.282432 | 1.640606 | 2.093778 | 2.665836 | 3.386355 | 10.834706 | 32.918953 | 95.396217 |
| 30 | 1.347849 | 1.811362 | 2.427262 | 3.243398 | 4.321942 | 17.449402 | 66.211772 | 237.376314 |
| 35 | 1.416603 | 1.999890 | 2.813862 | 3.946089 | 5.516015 | 28.102437 | 133.175523 | 590.668229 |
| 40 | 1.488864 | 2.208040 | 3.262038 | 4.801021 | 7.039989 | 45.259256 | 267.863546 | 1469.771568 |
| 45 | 1.564811 | 2.437854 | 3.781596 | 5.841176 | 8.985008 | 72.890484 | 538.769269 | 3657.261988 |
| 50 | 1.644632 | 2.691588 | 4.383906 | 7.106683 | 11.467400 | 117.390853 | 1083.657442 | 9100.438150 |

## Example Problem 1: Future Value of a Bank Deposit

An investor places $\$ 5,000$ in a bank account today that pays $5 \%$ interest per year compounded annually. After exactly eight years, how much is in the account?

Answer: The total future amount in the account, including principal plus interest is:

$$
F=P * F / P_{8,5 \%}
$$

Using the Table 2.1, $\mathrm{F} / \mathrm{P}_{8,5 \%}=1.477455$. Then:

$$
F=5,000 * 1.477455=\$ 7,387.28
$$

Example Problem 2: Problem 1 with Semi-Annual Compounding
An investor puts \$5,000 in a bank account today that pays 5\% interest per year compounded semi-annually. After exactly eight years, how much is in the account?

Answer: In this problem, because interest is compounded semi-annually, the interest used will be $2.5 \%$ and the number of compounding periods will be 16. Further, since $2.5 \%$ is not shown in Table 2.1 (or in the appendix tables at the end of the book), the F/P factor will have to be computed using Equation 2.1. Therefore, the total future amount in the account, including principal plus interest is:

$$
F=P(1+i)^{n}=5,000(1+0.025)^{16}=5,000 * 1.484506=\$ 7,422.53
$$

Compounding semi-annually, as opposed to compounding annually, results in an increase in the compound amount of approximately $\$ 35$. The same amount of $\$ 5,000$ is deposited for the same amount of time, but because it is compounded more frequently, twice per year, the final amount is more.

## Example Problem 3: Problem 1 with Daily Compounding

What if compounding occurs daily throughout the year? In the previous problem, what $n$ and $i$ would be used, and how much would be in the account after 8 years?
Answer: Since compounding is daily, the interest rate per compounding period would be $5 / 365$ or $0.013698 \%$ and the number of compounding periods would be $8 * 365$ or 2,920 compounding periods. Thus the future amount in the account, including principal and interest is:

$$
F=P(1+i)^{n}=5,000(1+0.00013698)^{2,920}=5,000 * 1.491756=\$ 7,458.78
$$

Use of daily compounding adds another $\$ 37$ to the compound amount after 8 years. In general, compounding more frequently increases the compound amount for a fixed interest rate per annum and time period.

Banks, however, usually compound interest on a continuous basis. This means that the interest rate/period approaches " 0 " while the number of compounding periods approaches " $\infty$." The effective interest rate for continuous compounding is found as:

$$
\begin{equation*}
i=\lim _{n \rightarrow \infty}\left[\left(1+\frac{i}{n}\right)^{n}-1\right]=e^{i}-1 \tag{2.3}
\end{equation*}
$$

For the compound amount of a single deposit, the equation for continuous compounding becomes:

$$
\begin{equation*}
F=P\left(1+e^{i}-1\right)^{n}=P e^{i n} \tag{2.4}
\end{equation*}
$$

## Example Problem 4: Problem 1 with Continuous Compounding

Again, the investor deposits $\$ 5,000$ for 8 years at an annual interest rate of $8 \%$. What is the compound amount under continuous compounding?

Answer: Using Equation 2.4, the compound amount is now:

$$
F=P e^{i n}=5,000 e^{0.05 * 8}=5,000 * 1.491825=\$ 7,459.12
$$

Continuous compounding does not add much to the compound amount compared to daily compounding (only 34 cents).

The difference does, however, explain why banks publish both a nominal interest rate and an effective interest rate (sometimes referred to as actual annual
yield). For the situation of Examples 1-4, the nominal interest rate is $8 \%$ per year. The actual annual yield under continuous compounding, however, would be found by Equation 2.3:

$$
i=e^{i}-1=e^{0.08}-1=1.08329-1=0.08329
$$

or $8.329 \%$.

### 2.1.2 Present Value or Present Worth

Equation (2.1) may be used to solve for the principal amount, $P$, that would need to be deposited given a known target value for the compound amount, F, after $n$ compounding periods at interest rate $i$. Solving Equation (2.1) for $P$, results in:

$$
\begin{equation*}
P=F\left[\frac{1}{(1+i)^{n}}\right] \tag{2.5}
\end{equation*}
$$

The value $1 /(1+i)^{n}$ may also be tabulated in compound interest tables for convenience, and is called the Present Worth Factor (PWF), which can also be denoted as $P / F$ (read present given future). It is used to compute the present amount which would have to be invested to provide for a known future amount. In essence, $P$ is the present value, or present worth, of an amount $F$ at some time in the future.

Using the $P / F$ denotation, the equation becomes:

$$
\begin{equation*}
P=F * P / F_{n, i} \tag{2.6}
\end{equation*}
$$

Table 2.2 shows the present worth factor for a selection of interest rates and time periods.

Table 2.2 Present Worth Factors $\left(\mathrm{P} / \mathrm{F}_{\mathrm{n} .}\right)$

| No. of | Interest Rate (\% per period) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Int. Per. | 1\% | 2\% | 3\% | 4\% | 5\% | 10\% | 15\% | 20\% |
| 1 | 0.990099 | 0.980392 | 0.970874 | 0.961538 | 0.952381 | 0.909091 | 0.869565 | 0.833333 |
| 2 | 0.980296 | 0.961169 | 0.942596 | 0.924556 | 0.907029 | 0.826446 | 0.756144 | 0.694444 |
| 3 | 0.970590 | 0.942322 | 0.915142 | 0.888996 | 0.863838 | 0.751315 | 0.657516 | 0.578704 |
| 4 | 0.960980 | 0.923845 | 0.888487 | 0.854804 | 0.822702 | 0.683013 | 0.571753 | 0.482253 |
| 5 | 0.951466 | 0.905731 | 0.862609 | 0.821927 | 0.783526 | 0.620921 | 0.497177 | 0.401878 |
| 6 | 0.942045 | 0.887971 | 0.837484 | 0.790315 | 0.746215 | 0.564474 | 0.432328 | 0.334898 |
| 7 | 0.932718 | 0.870560 | 0.813092 | 0.759918 | 0.710681 | 0.513158 | 0.375937 | 0.279082 |
| 8 | 0.923483 | 0.853490 | 0.789409 | 0.730690 | 0.676839 | 0.466507 | 0.326902 | 0.232568 |
| 9 | 0.914340 | 0.836755 | 0.766417 | 0.702587 | 0.644609 | 0.424098 | 0.284262 | 0.193807 |
| 10 | 0.905287 | 0.820348 | 0.744094 | 0.675564 | 0.613913 | 0.385543 | 0.247185 | 0.161506 |
| 15 | 0.861349 | 0.743015 | 0.641862 | 0.555265 | 0.481017 | 0.239392 | 0.122894 | 0.064905 |
| 20 | 0.819544 | 0.672971 | 0.553676 | 0.456387 | 0.376889 | 0.148644 | 0.061100 | 0.026084 |
| 25 | 0.779768 | 0.609531 | 0.477606 | 0.375117 | 0.295303 | 0.092296 | 0.030378 | 0.010483 |
| 30 | 0.741923 | 0.552071 | 0.411987 | 0.308319 | 0.231377 | 0.057309 | 0.015103 | 0.004213 |
| 35 | 0.705914 | 0.500028 | 0.355383 | 0.253415 | 0.181290 | 0.035584 | 0.007509 | 0.001693 |
| 40 | 0.671653 | 0.452890 | 0.306557 | 0.208289 | 0.142046 | 0.022095 | 0.003733 | 0.000680 |
| 45 | 0.639055 | 0.410197 | 0.264439 | 0.171198 | 0.111297 | 0.013719 | 0.001856 | 0.000273 |
| 50 | 0.608039 | 0.371528 | 0.228107 | 0.140713 | 0.087204 | 0.008519 | 0.000923 | 0.000110 |

## Example Problem 5: Determining an Investment Amount

How much must an investor put into an account today if the investor wants to have $\$ 50,000$ after 10 years? The account pays $4 \%$ interest per year, compounded annually.

Answer: The total future amount in the account must be $\$ 50,000$, including principal plus interest, thus the investor must deposit

$$
F=P * P / F_{10,4 \%}
$$

Using Table 2.2, $\mathrm{P} / \mathrm{F}_{10,4 \%}=0.675564$. Then:

$$
F=50,000 * 0.675564=\$ 33,778.20
$$

Essentially, to accumulate $\$ 50,000$ after 10 years, the investor must deposit \$33,778.20 now.

While the present value of a future amount is useful in considering common investments in bank accounts and other financial instruments, it is even more important in engineering economics as a fundamental concept.

In engineering projects, we often know the future value of things. If a bridge is to be replaced in 20 years, we can obtain an estimate of what it will cost to do so. The overseeing agency would have to develop an investment or savings plan to accumulate the money that will be needed in the future.

Engineers also estimate the benefits of future projects that are accrued both to the users of the future facility, the government or agency, or both. A new bridge may, for example, reduce travel times for users (which has a monetary value) and/or reduce the number and/or severity of accidents. For the government or agency, the new bridge might provide additional toll revenues or increase the income from fuel taxes.

In any event, the costs and benefits of a new bridge are both in the future, but they do not necessarily occur at the same time. The costs to build the new bridge would be accrued over a number of years during the planning, design, and construction phases. Benefits would begin to accrue after the bridge is opened to traffic.

The concept of present worth allows engineers to convert costs and benefits to an equivalent value at a defined point in time. Comparisons between costs and benefits can then conducted without disparities in the time frame.

## Example Problem 6: A Present Worth Application

A small town knows it will have to make major repairs to a historic covered bridge that is the only entrance to town. It is estimated that the repairs will have to be made in 10 years, and that they will cost $\$ 200,000$. If the interest rate in safe investments is $5 \%$ per year, how much would have to be invested now to insure that the funds are available in 10 years?
Answer: What must be invested now is the present worth of $\$ 200,000$ ten years in the future at an interest rate of 5\% per year. From Table 2.2, the value of $\mathrm{P} / \mathrm{F}_{10,5 \%}$ is 0.613913 . Then, using Equation 2.6:

$$
P=F * P / F_{5 \%, 10}=200,000 * 0.613913=\$ 122,782.60
$$

If the town has $\$ 122,782.60$ to invest today at $5 \%$, then it can guarantee that it will have the $\$ 200,000$ needed to repair the bridge in 10 years. Most engineering economic analyses assume annual compounding of interest. This is because both future interest rates and future costs and benefits are only estimates today. The detail of compounding periods is not considered a significant issue against this reality.

On the other hand, the equations for present worth can, of course, reflect any compounding period. If the town of Example 6 were making an actual investment, and the interest rate was known, then it could consider compounding periods. If the interest were compounded quarterly, then $\mathrm{i}=5 / 4=1.25 \%$ and $\mathrm{n}=$ $4 * 10=40$. If daily compounding were used, $\mathrm{i}=5 / 365=0.013698 \%$ and $\mathrm{n}=$ $10 * 365=3,650$. Since these interest rates are not included in Table 2.2, Equation 2.5 would be used to compute the required investment today. Then:

For quarterly compounding:

$$
F=200,000\left[\frac{1}{(1+0.0125)^{40}}\right]=\frac{200,000}{1.6436}=\$ 121,684.11
$$

For daily compounding:

$$
F=200,000\left[\frac{1}{(1+0.00013698)^{3650}}\right]=\frac{200,000}{1.6486}=\$ 121,316.06
$$

If the town can take advantage of quarterly or daily compounding, it would have to invest less to accumulate the $\$ 200,000$ needed in 10 years. Of course, it could even take advantage of continuous compounding if the investment was placed in a typical bank account. To find the present worth under continuous compounding, Equation 2.4 is solved for P :

$$
\begin{equation*}
P=\frac{F}{e^{i n}} \tag{2.7}
\end{equation*}
$$

For Example 6, this would be:

$$
P=\frac{200,000}{e^{0.05^{*} 10}}=\frac{200,000}{1.6487}=\$ 121,307.70
$$

The difference between daily compounding and continuous compounding is not large, but both are more advantageous to the investor than simple annual compounding of interest.

The equations and illustrations of this section all refer to a single payment, that is, a single amount of money is invested or deposited today to accumulate a single amount that will be available as a single future withdrawal. Figure 2.1 illustrates the concept of these single payment transactions. Note in the illustration that the single payment, P , is made at $\mathrm{t}=0$, that is, at the beginning of the first year or period. The compound amount, F , is withdrawn at the end of year " n ."

(a) Time Sequence

$$
\begin{aligned}
& F=P(1+i)^{n} \\
& P=F\left[\frac{1}{(1+i)^{n}}\right]
\end{aligned}
$$

where:

$$
\begin{aligned}
& (1+i)^{n}=C A F_{n, i}=F / P_{n, i} \\
& \frac{1}{(1+i)^{n}}=P W F_{n, i}=P / F_{n, i}
\end{aligned}
$$

(b) Equations

Fig. 2.1 An Illustration for Single Payment Transactions

### 2.2 Uniform Series Transactions

Uniform series transactions involve some uniform amount either being deposited or withdrawn at the end of each interest period for some $n$ compounding periods.

### 2.2.1 Sinking Fund Transactions

A common bank transaction involves the use of an account as a sinking fund. A sinking fund involves a series of uniform deposits of $\$ R$ made into an account at the end of each interest period. At the end of $n$ periods at interest $i$, a single amount $S$ is withdrawn that depletes all deposits plus interest.

To derive the formula for the compound amount, $S$, each of the individual deposits is considered as single payment. The compound amount for each individual deposit is found using Equation 2.1. The amount available after " n " years is the sum of the compound amounts for each payment. The first payment of $\$ R$ is made at the end of the first period and thus would be earning interest for $n-1$ periods.

Thus the first payment would amount to $R^{*}(1+i)^{n-1}$ at the end of $n$ periods at interest rate $i$. It follows that the compound amount for each individual deposit can be expressed as:

| Deposit at end of Period \# | Compound Amount |
| :---: | :---: |
| Deposit 1 | $\mathrm{R}(1+\mathrm{i})^{\mathrm{n}-1}$ |
| 2 | $\mathrm{R}(1+\mathrm{i})^{\mathrm{n}-2}$ |
| 3 | $\mathrm{R}(1+\mathrm{i})^{\mathrm{n}-3}$ |
| 4 | $\mathrm{R}(1+\mathrm{i})^{\mathrm{n}-4}$ |
| - | - |
| - | - |
| N | $\mathrm{R}(1+\mathrm{i})^{\mathrm{n}-\mathrm{N}}$ |
| - | - |
| - | - |
| n | $\mathrm{R}(1+\mathrm{i})^{\mathrm{n}-\mathrm{n}}=\mathrm{R}$ |

Note that the last payment made at the end of $n$ periods earns no interest. It is deposited and immediately withdrawn. The total future amount $S$ is the sum of all of the individual compound amounts shown above, and is itself a compound amount, but of a uniform periodic series of payments, $R$. Thus:

$$
S=R(1+i)^{n-1}+R(1+i)^{n-2}+R(1+i)^{n-3}+\ldots \ldots+R(1+i)+R
$$

This series can be represented in closed form by multiplying the series by (1+i) and subtracting the result from the original series:

$$
\begin{aligned}
S & =\quad R(1+i)^{n-1}+R(1+i)^{n-2}+R(1+i)^{n-3}+\ldots \ldots+R(1+i)+R \\
S(1+i) & =R(1+i)^{n}+R(1+i)^{n-1}+R(1+i)^{n-2}+R(1+i)^{n-3}+\ldots \ldots+R(1+i) \\
\hline S-S(1+i) & =-R(1+i)^{n} \\
-S i & =-R(1+i)^{n}+R \\
-S i & =-R\left[(1+i)^{n}-1\right]
\end{aligned}
$$

Solving the final equation for $S$ yields:

$$
\begin{equation*}
S=R\left[\frac{(1+i)^{n}-1}{i}\right] \tag{2.8}
\end{equation*}
$$

The value $\left.\left[(1+i)^{n}-1\right] / i\right]$ is called the Series Compound Amount Factor (SCA) because it is used to compute the compound amount for a uniform series of periodic deposits. It can also be denoted as $\mathrm{F} / \mathrm{A}_{\mathrm{n}, \mathrm{i}}$, which is read as Future given Annual. Using the factor, the equation becomes:

$$
\begin{equation*}
S=R^{*} F / A_{n, i} \tag{2.9}
\end{equation*}
$$

Of course, Equation 2.8 could be solved for R, given a known value of S. In this context, it is possible to estimate the annual deposit or investment that would be necessary to accumulate a single future value after $n$ periods at interest rate $i$. Then:

$$
\begin{equation*}
R=S\left[\frac{i}{(1+i)^{n}-1}\right] \tag{2.10}
\end{equation*}
$$

The term $i /\left[(1+i)^{n}-1\right]$ is called the sinking fund factor $\left(\mathrm{SFF}_{\mathrm{n} . \mathrm{i}}\right)$. It can also be denoted $A / F_{n, i}$ which is read Annual given Future. The equation then becomes:

$$
\begin{equation*}
R=S * A / F_{n, i} \tag{2.11}
\end{equation*}
$$

While both $n$ and $i$ may be for any uniform time interval, for most engineering applications, they are considered to be annual. Engineers will often be looking to determine how much money must be set aside each year to accumulate the cost of some anticipated future project. Figure 2.2 illustrates the concept of the sinking fund. Table 2.3 gives series compound amount factors (SCA) for a variety of interest rates and time periods; Table 2.4 gives sinking fund factors (SFF).


$$
\begin{aligned}
& S=R\left[\frac{(1+i)^{n}-1}{i}\right] \\
& R=S\left[\frac{i}{(1+i)^{n}-1}\right]
\end{aligned}
$$

where:

$$
\begin{aligned}
& {\left[(1+i)^{n}-1\right] / i=S C A_{n, i}=F / A_{n, i}} \\
& i /\left[(1+i)^{n}-1\right]=S F F_{n, i}=A / F_{n, i}
\end{aligned}
$$

(b) Equations

Fig. 2.2 Illustration of Sinking Fund Transactions

Table 2.3 Series Compound Amount Factors $\left(\mathrm{F} / \mathrm{A}_{n, 1}\right)$

| No. of | Interest Rate (\% per period) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Int. Per. | $\mathbf{1 \%}$ | $\mathbf{2 \%}$ | $\mathbf{3 \%}$ | $\mathbf{4 \%}$ | $\mathbf{5 \%}$ | $\mathbf{1 0 \%}$ | $\mathbf{1 5 \%}$ | $\mathbf{2 0 \%}$ |
| $\mathbf{1}$ | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 |
| $\mathbf{2}$ | 2.010000 | 2.020000 | 2.030000 | 2.040000 | 2.050000 | 2.100000 | 2.150000 | 2.200000 |
| $\mathbf{3}$ | 3.030100 | 3.060400 | 3.090900 | 3.121600 | 3.152500 | 3.310000 | 3.472500 | 3.640000 |
| $\mathbf{4}$ | 4.060401 | 4.121608 | 4.183627 | 4.246464 | 4.310125 | 4.641000 | 4.993375 | 5.368000 |
| $\mathbf{5}$ | 5.101005 | 5.204040 | 5.309136 | 5.416323 | 5.525631 | 6.105100 | 6.742381 | 7.441600 |
| $\mathbf{6}$ | 6.152015 | 6.308121 | 6.468410 | 6.632975 | 6.801913 | 7.715610 | 8.753738 | 9.929920 |
| $\mathbf{7}$ | 7.213535 | 7.434283 | 7.662462 | 7.898294 | 8.142008 | 9.487171 | 11.066799 | 12.915904 |
| $\mathbf{8}$ | 8.285671 | 8.582969 | 8.892336 | 9.214226 | 9.549109 | 11.435888 | 13.726819 | 16.499085 |
| $\mathbf{9}$ | 9.368527 | 9.754628 | 10.159106 | 10.582795 | 11.026564 | 13.579477 | 16.785842 | 20.798902 |
| $\mathbf{1 0}$ | 10.462213 | 10.949721 | 11.463879 | 12.006107 | 12.577893 | 15.937425 | 20.303718 | 25.958682 |
| $\mathbf{1 5}$ | 16.096896 | 17.293417 | 18.598914 | 20.023588 | 21.578564 | 31.772482 | 47.580411 | 72.035108 |
| $\mathbf{2 0}$ | 22.019004 | 24.297370 | 26.870374 | 29.778079 | 33.065954 | 57.274999 | 102.443583 | 186.688000 |
| $\mathbf{2 5}$ | 28.243200 | 32.030300 | 36.459264 | 41.645908 | 47.727099 | 98.347059 | 212.793017 | 471.981083 |
| $\mathbf{3 0}$ | 34.784892 | 40.568079 | 47.575416 | 56.084938 | 66.438848 | 164.494023 | 434.745146 | 1181.881569 |
| $\mathbf{3 5}$ | 41.660276 | 49.994478 | 60.462082 | 73.652225 | 90.320307 | 271.024368 | 881.170156 | 2948.341146 |
| $\mathbf{4 0}$ | 48.886373 | 60.401983 | 75.401260 | 95.025516 | 120.799774 | 442.592556 | 1779.090308 | 7343.857840 |
| $\mathbf{4 5}$ | 56.481075 | 71.892710 | 92.719861 | 121.029392 | 159.700156 | 718.904837 | 3585.128460 | 18281.309940 |
| $\mathbf{5 0}$ | 64.463182 | 84.579401 | 112.796867 | 152.667084 | 209.347996 | 1163.908529 | 7217.716277 | 45497.190750 |

Table 2.4 Sinking Fund Factors $\left(\mathrm{A} / \mathrm{F}_{\mathrm{n}, \mathrm{i}}\right)$

| No. of | Interest Rate $\%$ per period) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| Int. Per. | $\mathbf{1 \%}$ | $\mathbf{2} \%$ | $\mathbf{3} \%$ | $\mathbf{4 \%}$ | $\mathbf{5} \%$ | $\mathbf{1 0 \%}$ | $\mathbf{1 5 \%}$ | $\mathbf{2 0 \%}$ |
| $\mathbf{1}$ | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 |
| $\mathbf{2}$ | 0.497512 | 0.495050 | 0.492611 | 0.490196 | 0.487805 | 0.476190 | 0.465116 | 0.454545 |
| $\mathbf{3}$ | 0.330022 | 0.326755 | 0.323530 | 0.320349 | 0.317209 | 0.302115 | 0.287977 | 0.274725 |
| $\mathbf{4}$ | 0.246281 | 0.242624 | 0.239027 | 0.235490 | 0.232012 | 0.215471 | 0.200265 | 0.186289 |
| $\mathbf{5}$ | 0.196040 | 0.192158 | 0.188355 | 0.184627 | 0.180975 | 0.163797 | 0.148316 | 0.134380 |
| $\mathbf{6}$ | 0.162548 | 0.158526 | 0.154598 | 0.150762 | 0.147017 | 0.129607 | 0.114237 | 0.100706 |
| $\mathbf{7}$ | 0.138628 | 0.134512 | 0.130506 | 0.126610 | 0.122820 | 0.105405 | 0.090360 | 0.077424 |
| $\mathbf{8}$ | 0.120690 | 0.116510 | 0.112456 | 0.108528 | 0.104722 | 0.087444 | 0.072850 | 0.060609 |
| $\mathbf{9}$ | 0.106740 | 0.102515 | 0.098434 | 0.094493 | 0.090690 | 0.073641 | 0.059574 | 0.048079 |
| $\mathbf{1 0}$ | 0.095582 | 0.091327 | 0.087231 | 0.083291 | 0.079505 | 0.062745 | 0.049252 | 0.038523 |
| $\mathbf{1 5}$ | 0.062124 | 0.057825 | 0.053767 | 0.049941 | 0.046342 | 0.031474 | 0.021017 | 0.013882 |
| $\mathbf{2 0}$ | 0.045415 | 0.041157 | 0.037216 | 0.033582 | 0.030243 | 0.017460 | 0.009761 | 0.005357 |
| $\mathbf{2 5}$ | 0.035407 | 0.031220 | 0.027428 | 0.024012 | 0.020952 | 0.010168 | 0.004699 | 0.002119 |
| $\mathbf{3 0}$ | 0.028748 | 0.024650 | 0.021019 | 0.017830 | 0.015051 | 0.006079 | 0.002300 | 0.000846 |
| $\mathbf{3 5}$ | 0.024004 | 0.020002 | 0.016539 | 0.013577 | 0.011072 | 0.003690 | 0.001135 | 0.000339 |
| $\mathbf{4 0}$ | 0.020456 | 0.016556 | 0.013262 | 0.010523 | 0.008278 | 0.002259 | 0.000562 | 0.000136 |
| $\mathbf{4 5}$ | 0.017705 | 0.013910 | 0.010785 | 0.008262 | 0.006262 | 0.001391 | 0.000279 | 0.000055 |
| $\mathbf{5 0}$ | 0.015513 | 0.011823 | 0.008865 | 0.006550 | 0.004777 | 0.000859 | 0.000139 | 0.000022 |

## Example Problem 7: An Investment Sinking Fund

An investor puts $\$ 500$ in a bank account at the end of each year for eight years. The account pays $3 \%$ interest per year compounded annually. At the end of the eight years, how much is in the account?

Answer: The total future amount in the account, including principal plus interest is:

$$
S=500 * F / A_{8,5 \%}
$$

From Table 2.3, the series compound amount for this series of investments is found as $F / A_{8,5 \%}$ is 9.549101 , and thus:

$$
F=500 * 9.549101=\$ 4,774.55
$$

If the investor puts $\$ 500$ into this investment at the end of each year, at the end of the eighth year, he/she will have accumulated $\$ 4,774.55$.

## Example Problem 8: Using a Sinking Fund to Pay for a Future Need

In order to pay for a replacement part that will cost $\$ 15,000$ ten years from now, what amount would have to be invested into an account paying $3 \%$ at the end of each year for the next ten years?
Answer: The annual amount invested into the account for ten years would be:

$$
R=15,000 * A / F_{10,3 \%}
$$

where $A / F_{10,3 \%}$ is found in Table 2.4 as 0.087231 . Then:

$$
R=15,000 * 0.087231=\$ 1,308.47
$$

If $\$ 1,308.47$ in invested at the end of each year at $3 \%$, the total amount in the account would be $\$ 15,000$ at the end of 10 years.

### 2.2.2 Capital Recovery and Annuities

In capital recovery transactions, a single sum of money, $P$, is deposited. Thereafter, a uniform amount, $R$, is withdrawn at the end of each period, such that after $n$ periods at interest $i$, the last withdrawal depletes the deposit, principal plus interest.

The home mortgage is a common example of capital recovery, in which the bank invests a single sum in the home owner. The bank then "recovers" its investment in equal periodic "withdrawals." When the home owner makes the last payment, the full amount of the original loan, principal plus interest, has been repaid. Virtually all bank loans are made on the capital recovery principle, where the bank is the "depositor," and the recipient of the loan is the "account."

The derivation of the formula for capital recovery is again based upon the fact that the amount on account at the end of any given period is equal to the amount at the beginning of the period plus the interest on that amount. In capital recovery, this is complicated by a withdrawal of $\$ R$ made at the end of each period.

As was done with single payment transactions, a series of equations can be constructed describing the amount remaining in the account at the end of any given period:

```
Period No.
    1
    2
    3
    -
    \(\bullet\)
    -
    -
    n
```

```
Amount in Account at End of Period
```

Amount in Account at End of Period
$\mathrm{P}(1+\mathrm{i})-\mathrm{R}$
$\mathrm{P}(1+\mathrm{i})-\mathrm{R}$
$[\mathrm{P}(1+\mathrm{i})-\mathrm{R}](1+\mathrm{i})-\mathrm{R}=\mathrm{P}(1+\mathrm{i})^{2}-\mathrm{R}(1+\mathrm{i})-\mathrm{R}$
$[\mathrm{P}(1+\mathrm{i})-\mathrm{R}](1+\mathrm{i})-\mathrm{R}=\mathrm{P}(1+\mathrm{i})^{2}-\mathrm{R}(1+\mathrm{i})-\mathrm{R}$
$\left[\mathrm{P}(1+\mathrm{i})^{2}-\mathrm{R}(1+\mathrm{i})-\mathrm{R}\right](1+\mathrm{i})-\mathrm{R}=\mathrm{R}(1+\mathrm{i})^{3}-\mathrm{R}(1+\mathrm{i})^{2}-$
$\left[\mathrm{P}(1+\mathrm{i})^{2}-\mathrm{R}(1+\mathrm{i})-\mathrm{R}\right](1+\mathrm{i})-\mathrm{R}=\mathrm{R}(1+\mathrm{i})^{3}-\mathrm{R}(1+\mathrm{i})^{2}-$
$\mathrm{R}(1+\mathrm{i})-\mathrm{R}$
$\mathrm{R}(1+\mathrm{i})-\mathrm{R}$
$P(1+i)^{n}-R(1+i)^{n-1}-R(1+i)^{n-2}-\ldots \ldots \ldots \ldots . . .-R$

```
\(P(1+i)^{n}-R(1+i)^{n-1}-R(1+i)^{n-2}-\ldots \ldots \ldots \ldots . . .-R\)
```

By definition, the account is depleted after the $n t h$ withdrawal, so that the amount remaining after $n$ periods is zero:

$$
P(1+i)^{n}-R(1+i)^{n-1}-R(1+i)^{n-2}-\ldots \ldots \ldots \ldots . .-R(1+i)-R=0
$$

This equation may again be shown in closed form by multiplying it by (1+i) and subtracting the result from the original series:

This equation may be simplified as:

$$
\begin{aligned}
& P(1+i)^{n}[1-(1+i)]+R\left[(1+i)^{n}-1\right]=0 \\
& -P i(1+i)^{n}+R\left[(1+i)^{n}-1\right]=0
\end{aligned}
$$

Solving this equation for $R$ yields:

$$
R=P\left[\frac{i(1+i)^{n}}{(1+i)^{n}-1}\right]
$$

where the term " $\mathrm{i}(1+\mathrm{i})^{\mathrm{n}} /\left[(1+\mathrm{i})^{\mathrm{n}}-1\right]$ " is the Capital Recovery Factor (CRF), so called because it is used to "recover" your capital plus interest in equal periodic payments. The capital recovery factor can also be denoted as $\mathrm{A} / \mathrm{P}$, which is read "annual given present." Using the factor, the equation becomes:

$$
\begin{equation*}
R=P^{*} A / P_{n, i} \tag{2.13}
\end{equation*}
$$

Equation 2.12 can also be solved for P , knowing R. The equation then becomes:

$$
\begin{equation*}
P=R\left[\frac{(1+i)^{n}-1}{i(1+i)^{n}}\right] \tag{2.14}
\end{equation*}
$$

where $\left[\left((1+i)^{\mathrm{n}}-1\right) / \mathrm{i}(1+\mathrm{i})^{\mathrm{n}}\right]$ is the series present worth factor (SPW) because it represents the present value of a series of uniform periodic withdrawals. It can also be denoted P/A, which is read find the present given annual. Using this factor, the equation becomes:

$$
\begin{equation*}
P=R * P / A_{n, i} \tag{2.15}
\end{equation*}
$$

This results in the amount you would need to invest in the present in order to withdraw a known amount, R , at the end of each period.

Figure 2.3 illustrates capital recovery transactions. Table 2.5 contains Capital Recovery Factors (CRF) for various interest rates and time periods; Table 2.6 shows Series Present Worth Factors (SPW).

(a) Time Sequence

$$
\begin{aligned}
& R=P\left[\frac{i(1+i)^{n}}{(1+i)^{n}-1}\right] \\
& P=R\left[\frac{(1+i)^{n}-1}{i(1+i)^{n}}\right]
\end{aligned}
$$

where:

$$
\begin{aligned}
& i(1+i)^{n} /\left[(1+i)^{n}-1\right]=C R F_{n, i}=A / P_{n, i} \\
& {\left[(1+i)^{n}-1\right] / i(1+i)^{n}=S C A_{n, i}=A / P_{n, i}}
\end{aligned}
$$

(b) Equations

Fig. 2.3 Illustration of Capital Recovery Transactions
Table 2.5 Capital Recovery Factors - $\mathrm{A} / \mathrm{P}_{\mathrm{n}, \mathrm{i}}$

| No. of | Interest Rate (\% per period) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Int. Per. | 1\% | 2\% | 3\% | 4\% | 5\% | 10\% | 15\% | 20\% |
| 1 | 1.010000 | 1.020000 | 1.030000 | 1.040000 | 1.050000 | 1.100000 | 1.150000 | 1.200000 |
| 2 | 0.507512 | 0.515050 | 0.522611 | 0.530196 | 0.537805 | 0.576190 | 0.615116 | 0.654545 |
| 3 | 0.340022 | 0.346755 | 0.353530 | 0.360349 | 0.367209 | 0.402115 | 0.437977 | 0.474725 |
| 4 | 0.256281 | 0.262624 | 0.269027 | 0.275490 | 0.282012 | 0.315471 | 0.350265 | 0.386289 |
| 5 | 0.206040 | 0.212158 | 0.218355 | 0.224627 | 0.230975 | 0.263797 | 0.298316 | 0.334380 |
| 6 | 0.172548 | 0.178526 | 0.184598 | 0.190762 | 0.197017 | 0.229607 | 0.264237 | 0.300706 |
| 7 | 0.148628 | 0.154512 | 0.160506 | 0.166610 | 0.172820 | 0.205405 | 0.240360 | 0.277424 |
| 8 | 0.130690 | 0.136510 | 0.142456 | 0.148528 | 0.154722 | 0.187444 | 0.222850 | 0.260609 |
| 9 | 0.116740 | 0.122515 | 0.128434 | 0.134493 | 0.140690 | 0.173641 | 0.209574 | 0.248079 |
| 10 | 0.105582 | 0.111327 | 0.117231 | 0.123291 | 0.129505 | 0.162745 | 0.199252 | 0.238523 |
| 15 | 0.072124 | 0.077825 | 0.083767 | 0.089941 | 0.096342 | 0.131474 | 0.171017 | 0.213882 |
| 20 | 0.055415 | 0.061157 | 0.067216 | 0.073582 | 0.080243 | 0.117460 | 0.159761 | 0.205357 |
| 25 | 0.045407 | 0.051220 | 0.057428 | 0.064012 | 0.070952 | 0.110168 | 0.154699 | 0.202119 |
| 30 | 0.038748 | 0.044650 | 0.051019 | 0.057830 | 0.065051 | 0.106079 | 0.152300 | 0.200846 |
| 35 | 0.034004 | 0.040002 | 0.046539 | 0.053577 | 0.061072 | 0.103690 | 0.151135 | 0.200339 |
| 40 | 0.030456 | 0.036556 | 0.043262 | 0.050523 | 0.058278 | 0.102259 | 0.150562 | 0.200136 |
| 45 | 0.027705 | 0.033910 | 0.040785 | 0.048262 | 0.056262 | 0.101391 | 0.150279 | 0.200055 |
| 50 | 0.025513 | 0.031823 | 0.038865 | 0.046550 | 0.054777 | 0.100859 | 0.150139 | 0.200022 |

Table 2.6 Series Present Worth Factors - $P / A_{n, i}$

| No. of | Interest Rate (\% per period) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Int. Per. | 1\% | 2\% | 3\% | 4\% | 5\% | 10\% | 15\% | 20\% |
| 1 | 0.990099 | 0.980392 | 0.970874 | 0.961538 | 0.952381 | 0.909091 | 0.869565 | 0.833333 |
| 2 | 1.970395 | 1.941561 | 1.913470 | 1.886095 | 1.859410 | 1.735537 | 1.625709 | 1.527778 |
| 3 | 2.940985 | 2.883883 | 2.828611 | 2.775091 | 2.723248 | 2.486852 | 2.283225 | 2.106481 |
| 4 | 3.901966 | 3.807729 | 3.717098 | 3.629895 | 3.545951 | 3.169865 | 2.854978 | 2.588735 |
| 5 | 4.853431 | 4.713460 | 4.579707 | 4.451822 | 4.329477 | 3.790787 | 3.352155 | 2.990612 |
| 6 | 5.795476 | 5.601431 | 5.417191 | 5.242137 | 5.075692 | 4.355261 | 3.784483 | 3.325510 |
| 7 | 6.728195 | 6.471991 | 6.230283 | 6.002055 | 5.786373 | 4.868419 | 4.160420 | 3.604592 |
| 8 | 7.651678 | 7.325481 | 7.019692 | 6.732745 | 6.463213 | 5.334926 | 4.487322 | 3.837160 |
| 9 | 8.566018 | 8.162237 | 7.786109 | 7.435332 | 7.107822 | 5.759024 | 4.771584 | 4.030967 |
| 10 | 9.471305 | 8.982585 | 8.530203 | 8.110896 | 7.721735 | 6.144567 | 5.018769 | 4.192472 |
| 15 | 13.865053 | 12.849264 | 11.937935 | 11.118387 | 10.379658 | 7.606080 | 5.847370 | 4.675473 |
| 20 | 18.045553 | 16.351433 | 14.877475 | 13.590326 | 12.462210 | 8.513564 | 6.259331 | 4.869580 |
| 25 | 22.023156 | 19.523456 | 17.413148 | 15.622080 | 14.093945 | 9.077040 | 6.464149 | 4.947587 |
| 30 | 25.807708 | 22.396456 | 19.600441 | 17.292033 | 15.372451 | 9.426914 | 6.565980 | 4.978936 |
| 35 | 29.408580 | 24.998619 | 21.487220 | 18.664613 | 16.374194 | 9.644159 | 6.616607 | 4.991535 |
| 40 | 32.834686 | 27.355479 | 23.114772 | 19.792774 | 17.159086 | 9.779051 | 6.641778 | 4.996598 |
| 45 | 36.094508 | 29.490160 | 24.518713 | 20.720040 | 17.774070 | 9.862808 | 6.654293 | 4.998633 |
| 50 | 39.196118 | 31.423606 | 25.729764 | 21.482185 | 18.255925 | 9.914814 | 6.660515 | 4.999451 |

## Example Problem 9: A Retirement Annuity

You are about to retire and have saved $\$ 650,000$. It is in an account that pays $3 \%$ annually. If you want to totally deplete the account after 20 years, how much can you take out of the account annually so that you will spend the total principal plus interest?
Answer: The amount that you would be able to withdraw annually would be found using Equation 2.13:

$$
R=650,000 * A / P_{20,3 \%}
$$

where $\mathrm{A} / \mathrm{P}_{20,3 \%}$ is found from Table 2.5 as 0.067216 . Then:

$$
R=650,000 * 0.067216=\$ 43,690.40
$$

Thus you would be able to withdraw $\$ 43,690.40$ at the end of each year for the next 20 years.

## Example Problem 10: Establishing an Annuity

What single amount would you have to put into a bank account today that pays $5 \%$ interest annually if you want to withdraw $\$ 50,000$ at the end of each year for the next 15 years?

Answer: You want to find the present value of a series of future withdrawals. This would be done using Equation 2.15:

$$
P=50,000 * P / A_{15,5 \%}
$$

where $\mathrm{P} / \mathrm{A}_{15,5 \%}$ is found from Table 2.6 as 10.379658 . Then:

$$
P=50,000 * 10.379658=\$ 518,982.90
$$

Thus, to insure an annuity of \$50,000/year for 15 years at $5 \%$, you would have to deposit $\$ 518,982.90$ today. This could also be thought of as the cost to buy such an annuity from a private insurer.

### 2.3 Using Banking Formulae in Engineering Applications

The banking formulae discussed in this chapter relate to investments of various kinds at fixed interest rates and time periods. They deal with a set of variables that can be related to each other:
> $P=A$ single amount invested at time " 0 " for a fixed time period and interest rate,
> $\mathrm{F}=\mathrm{A}$ single future amount from a single investment "P" that accumulated at a fixed interest rate,
> $\mathrm{R}=\mathrm{A}$ periodic deposit or withdrawal made at the end of each compounding period for a fixed time period and interest rate, and
> $\mathrm{S}=\mathrm{A}$ single future amount that accumulates from periodic deposits "R" at the end of each compounding period after a fixed time period and interest rate.

While these formulae govern actual banking transactions, including savings accounts, annuities, and loans, they can be applied as analytic tools to help analyze the economics of a transportation or other project.

The costs and benefits of various transportation and other types of improvement projects occur over a substantial period of time. In general, any economic analysis must include the full service life of the project under consideration. Some costs, like capital construction costs, occur at discrete times in the present and/or future. Some costs are recurring on a regular basis, such as maintenance and operating costs.

To conduct a comparative economic analysis of alternatives, all costs and benefits of all alternatives must be converted to a common point in time and a common analysis period. The banking formulas are applied to make such conversions, so that costs and benefits can be directly compared. Because these applications do not involve actual bank accounts, it is common to assume annual compounding of interest as a base condition.

Single payment formulas can be used to convert costs occurring at different points in time to a common basis. Capital recovery formulas can convert a single cost to an equivalent annual cost over the service life of an improvement, or estimate the present worth of known future annual costs and expenditures. Thus, the banking formulas are used, either alone or in combination, to theoretically alter the time basis of cost and benefit estimates so that direct comparison is possible.

Some simple examples of such applications follow.

## Example Problem 11: Building a New School

A town wishes to build a new school that will cost $\$ 12,000,000$. The school is to be built in 10 years. The town will provide funding for the new school by depositing a uniform amount into an investment fund paying 5\% per year, compounded annually. How much must be set aside in each of the 10 years to provide for the new school?

Answer: The town is, in effect, setting up a sinking fund into which it will make 10 uniform annual payments. At the end of ten years, the fund will have accumulated $\$ 12,000,000$, principal plus interest, which will be withdrawn to pay for construction of the new school. In this case, the compound amount to be withdrawn in the future is known. The annual deposit must be computed. Thus, Equation 2.11 is applied:

$$
R=S * A / F_{10,5 \%}
$$

where:

$$
\begin{aligned}
& \mathrm{S}=\$ 12,000,000 \\
& \mathrm{n}=10 \\
& \mathrm{i}=5 \% \\
& \left.\mathrm{~A} / \mathrm{F}_{10,5 \%}=0.079505 \text { (Table } 2.4\right)
\end{aligned}
$$

Then:

$$
R=12,000,000 * 0.079505=\$ 954,060.00
$$

Note that over the 10 years, the town invests a total of $10 * \$ 954,060.00=$ $\$ 9,540,600$. Interest makes up the remainder of the $\$ 12$ million that is withdrawn after ten years to build the school. The "price" that the town pays for this benefit is that it must wait for 10 years to build the school as it accumulates sufficient funds.

## Example Problem 12: Paying for Future Maintenance

A town has just completed a project that rebuilt all of its local roads. To provide for future maintenance expenses, the town plans to make twenty equal annual payments into an account paying $5 \%$, compounded annually. Thereafter, the town hopes to be able to make thirty equal annual withdrawals of $\$ 55,000$. How much must be invested each of the first twenty years to provide for the future withdrawals?

Answer: This problem is a combination of two types of transactions: The first twenty years, while payments are being made into the account, is a sinking fund. Thereafter, when the town "recovers" the capital in equal annual installments, capital recovery is the form being used. It is important to note that it is the output of the sinking fund (after 20 years) that forms the input for the capital recovery. Figure 2.4 illustrates what is happening.


Fig. 2.4 Illustration for Example Problem 1

For 20 years, the town will have to invest $\$ \mathrm{R} /$ year. After 20 years at 5\%, it will have accumulated an amount $\$$ F. Instead of withdrawing F, however, it will be reinvested at $5 \%$ to allow for capital recovery of $\$ 55,000 / \mathrm{yr}$ for 30 years.

The problem must be solved backwards. The only amount we know is that $\$ 55,000$ will be withdrawn at the end of years 21 through 50. At the beginning of the withdrawals, a single amount must be on deposit sufficient to allow for these 30 withdrawals. The amount can be found as the present worth of $\$ 55,000$ per year for thirty years at $5 \%$ interest, using Equation 2.15:

$$
P=R^{*} P / A_{30,5 \%}
$$

where: $\mathrm{R} \quad=\quad \$ 55,000$

$$
\mathrm{P} / \mathrm{A}_{30,5 \%} \quad=\quad 15.372451
$$

Then:

$$
P=55,000 * 15.372451=\$ 845,484.81
$$

This is the amount that must be available in the account before beginning the annual withdrawals, which start at the end of Year 20. This, then, is also the amount that must be in the account at the end of the first 20 years of deposits; it is now the known amount that is used to find how much needs to be deposited annually during those first 20 years. This is a sinking fund transaction, and the annual amount deposited is found using Equation 2.10:

$$
R=S * A / F_{20,5 \%}
$$

$\begin{array}{llll}\text { where: } & \mathrm{S} & = & \$ 845,484.81 \text { (computed above) } \\ & \mathrm{A} / \mathrm{F}_{20.5 \%} & = & 0.030243 \text { (Table } 2.4)\end{array}$

$$
\mathrm{A} / \mathrm{F}_{20,5 \%}=0.030243(\text { Table 2.4 })
$$

Then:

$$
R=845,484.81 * 0.030243=\$ 25,570.00
$$

Thus, the town must invest $\$ 25,570 /$ year for 20 years to allow it to accumulate an amount that would fund annual withdrawals of $\$ 55,000 /$ year for 30 years thereafter.

### 2.4 Closing Comments

Basic banking formulae provide an important mathematical basis for considering the relative economy of alternative engineering investments. While they describe basic features of actual bank transactions, like savings accounts, mortgages and other loans, and annuities, they can be used in innovative ways to allow engineers to consider the relative economic worth of a project.

Engineering projects may be in the private (such as an office building) or public (such as a highway) domain. Regardless, the engineer is spending the funds from either a single investor or group of investors, or from the tax revenues of the appropriate federal, state, and local governments. In either case, the engineer must get "the best bang for the buck." Money, whether private or public, is a scarce resource, and engineers must insure that it is used with maximum efficiency.

Subsequent chapters will illustrate many complex analyses that are conducted to insure this.

## Problems

## Problem 2.1

Create a generic excel spreadsheet where the interest rate, $i$, can be changed at the top and all tabulated values in the rest of the sheet will change accordingly.

|  | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Yr | Interest = | i\% |  |  |  |  |
| 2 | n | $\begin{aligned} & \text { SINGLE } \\ & \text { PAYMENT } \end{aligned}$ |  | UNIFORM SERIES |  |  |  |
| 3 |  | $\begin{aligned} & \hline \text { CAF } \\ & \text { (F/P) } \end{aligned}$ | $\begin{aligned} & \hline \text { PWF } \\ & \text { P/F } \end{aligned}$ | $\begin{aligned} & \hline \text { SCA } \\ & \text { F/A } \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { SFF } \\ & \text { A/F } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { SPW } \\ & \text { P/A } \end{aligned}$ | $\begin{array}{\|l\|} \hline \text { CRF } \\ \text { A/P } \\ \hline \end{array}$ |
| 4 | n | $=(1+\$ \mathrm{C} \$ 1)^{\wedge} \mathrm{A} 4$ | $\begin{aligned} & =1 \\ & C 4 \end{aligned}$ | $=\frac{(1+\$ \mathrm{C} \$ 1)^{\wedge} \mathrm{A} 4}{\$ \mathrm{C} \$ 1}$ | $\begin{aligned} & =1 \\ & E 4 \end{aligned}$ | $=\frac{(1+\$ \mathrm{C} \$ 1)^{\wedge} \mathrm{A} 4-1}{\$ \mathrm{C} \$ 1^{*}(1+\$ \mathrm{C} \$ 1)^{\wedge} \mathrm{A} 4}$ | $\begin{aligned} & =1 \\ & \mathrm{G} 4 \end{aligned}$ |
| 5 | 1 |  |  |  |  |  |  |
| 6 | 2 |  |  |  |  |  |  |

NOTE: After creating this sheet, all problems can use the spreadsheet to create a table for any interest rate, $i$, or year, $n$, needed, instead of using the formulas.

## Problem 2.2

If $\$ 1000$ is deposited in an account paying $8 \%$, compounded quarterly, what will the compound amount be after 10 years?

## Problem 2.3

Consider an account paying $6 \%$, compounded annually. $\$ 1000$ per year is deposited into the account for 10 years. Thereafter, no further deposits are made. What amount will be in the account after 20 years?

## Problem 2.4

How many years will it take before $\$ 10,000$ invested at $10 \%$, compounded annually, will accumulate to $\$ 50,000$ ?

## Problem 2.5

What deposit must be made now to provide for a series of 10 annual withdrawals of $\$ 1,500$ at the end of years 1 through 10 AND for a series of annual withdrawals of $\$ 2,500$ at the end of years 21 through 30 ? Interest is $8 \%$, and no withdrawals are made during years 11 through 20. Interest is compounded annually.

## Problem 2.6

A city plans to pay off a $\$ 1,000,000$ public debt falling due in 20 years by making an annual deposit out of operating revenues into a sinking fund paying $7 \%$ interest. How much must the city deposit each year to guarantee that the debt may be fully retired at the end of 20 years? If the city must also pay $9 \%$ per year interest on the full outstanding debt during the 20 years before it is repaid, what is the total annual cost of the debt to the city?
If instead of paying the total principal, $\$ 1 \mathrm{M}$, at the end of 20 years, the terms are that part of the principal is paid down each year, how much is the total annual cost of the debt to the city?
Explain why the two are different.

## Problem 2.7

Compute the annual cost of purchasing 70 transit cars at $\$ 1.4$ million each, with $10 \%$ interest and a 40-year useful life.

## Problem 2.8

A sinking fund is to be created, with the same deposit at the end of each of ten years, to yield the following progress payments for a transportation construction project:

| At the end of |  | Yield |
| :--- | :--- | :--- |
| Year 10 |  | $\$ 10$ million |
| Year 11 |  | $\$ 10$ million |
| Year 12 |  | $\$ 10$ million |
| Year 13 |  | $\$ 30$ million |

What is the annual deposit required, with interest $=10 \%$

## Problem 2.9

A taxi service is to be initiated. Revenues of $\$ 4$ million per year are expected for each of the first ten years, and $\$ 7$ million per year for each of the following ten years.
The expenses are estimated at $\$ 3.5$ million annually for each of the twenty years of the analysis period.
What is the present worth of the business? Use interest $=10 \%$

If the business can be sold for $\$ 10$ million at the end of twenty years, what is the present worth of the business?

## Problem 2.10

A sinking fund with a life of 30 years is created as part of a labor agreement, with annual payments into it of $\$ 3000$. Thereafter, there are to be annual withdrawals for fifteen years, depleting the fund. What annual withdrawals are justified?

## Problem 2.11

A person wishes to accumulate $\$ 32,500$ over a period of 15 years so that a cash payment can be made. To have this amount when it is needed, annual payments will be made into a savings account that earns $8 \%$ interest per year. How much must each annual payment be? After 7 years, how much is in the account?

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## Chapter 3

## Use of Banking Formulae in Engineering Economics


#### Abstract

This chapter presents illustrative uses of basic banking formulae for conducting economic comparisons of various transportation alternatives. Examples are kept relatively simple to demonstrate the principles involved in these usages.


The primary types of banking transactions discussed in Chapter 2 form the basis for most engineering economic analyses. These principles are generally applied in two ways:

- To analyze specific plans for financing transportation improvements; or
- To reduce all cost elements in an analysis to a common base.

In the first case, real money transactions are involved. In the second case, the equations are treated as mathematical and philosophical equalities, although no real money transactions are involved.

Capital recovery is a central concept that is used in the actual repayment of public debt (bond issues, see Chapter 7). It is also used to compute equivalent annualized costs for lump sum amounts and/or to compute present values of series of anticipated benefits or costs.

Present worth is another key concept. Whether used in conjunction with a series, as in capital recovery, or in conjunction with single payments, the computation of present worth provides a means to convert all costs and benefits to a single base year.

Often transportation and other engineers need to consider different alternatives for implementing a project, and to make a decision as to which is the most economic alternative. All alternatives must be compared on equal terms. A project that spans ten years cannot be directly compared to a project that spans twenty years; a project that will be implemented in 5 years cannot be directly compared to a project that will not be implemented for 15 years. In essence, applying basic banking formulae allows all cost elements to be reduced to a common point in time, and a common time span.

### 3.1 Simple Engineering Applications

In this section, a number of examples are presented in which basic banking formulae are applied to making alternative decisions.

## Example Problem 1: Surface Treating a Gravel Roadway

A city engineering department finds that by surface treating a gravel road, a maintenance savings of $\$ 25,000$ per year can be achieved. How much can the city justifiably spend to treat the roadway if the service life of the treatment is taken to be 10 years and interest is estimated to be $4 \%$.
Answer: In this problem, the issue is no longer a real money transaction, but the comparison of costs and quantifiable benefits of alternative courses of action. The choice is:

- Leave the road untreated, which incurs a cost of $\$ 25,000 / \mathrm{yr}$.
- Spend some unknown amount of money, P, to surface treat the gravel roadway, which eliminates $\$ 25,000 / \mathrm{yr}$ in annual maintenance costs for a period of 10 years.

Rather than deposits and withdrawals from a bank or investment account, the amounts in this example represent the investment of public monies (tax revenues) in an engineering project, where the benefits are measured in terms of reduced maintenance costs.

The benefits of the investment are known: $\$ 25,000$ per year of reduced maintenance costs for the 10-year service life of the improvement. Logically, then, the maximum amount one should be prepared to pay for such an improvement is the present value of those benefits, including an appropriate interest rate. Using the Series Present Worth equation:

$$
P=R * S P W_{10,4 \%}=R * P / A_{10,4 \%}
$$

where:

$$
\begin{aligned}
& \mathrm{R}=25,000 \\
& \mathrm{P} / \mathrm{A}_{10,4 \%}=8.110896 \text { (Table 2.6) }
\end{aligned}
$$

Then:

$$
P=25,000 * 8.110896=\$ 202,772.40
$$

Note that when dealing with conceptual conversions using the banking formulae, annual compounding is always assumed. The result, $\$ 202,772.40$, is not the price of the improvement, but it is an indication of the maximum price that should be paid for the treatment. If the actual price is less than $\$ 202,772.40$, then surface treating the roadway costs less than maintaining it for 10 years. If the actual price were more than $\$ 202,772.40$, then the roadway would not be treated because the benefits of treating are less than the cost. If the actual price were exactly
$\$ 202,772.40$, it would not matter, in the economic sense, if the roadway is treated or not.

In essence, the result $(\$ 202,772.40)$ is the break-even point at which the two alternatives have equal costs.

## Example Problem 2: A Highway Repaving Project

Construction of a new pavement will result in a savings of 5 cents per vehiclemile in operating costs to each user of the road. If the AADT (Average Annual Daily Traffic) for the segment of highway to be repaved is 2,000 veh/day, and the new pavement has a service life of 25 years, what maximum expenditure per mile of highway can be justified for the construction of the surface? Interest is estimated to be $4 \%$.

Answer: As in the previous problem, the solution is essentially the present worth of the future benefits to be derived from the proposed resurfacing. Annual benefits, per mile of roadway, can be computed from the AADT of 2,000 veh/day. Note that the length of the highway segment is not needed, as both the benefits and the justifiable expenditure will be stated on a per-mile basis. Thus:

$$
R=2,000 * 365 * 0.05=\$ 36,500 / \mathrm{mi} / \mathrm{yr}
$$

where R is the annual savings per mile of re-paved roadway over the 25-year service life of the repaving. The maximum justifiable expenditure for the surface is, therefore, the present worth of $\$ 36,500$ per year for 25 years. Thus:

$$
P=R^{*} S P W_{25,4 \%}=R * P / A_{25,4 \%}
$$

$\begin{array}{llll}\text { where: } & \mathrm{R} & = & \$ 36,500 \\ & {\mathrm{P} / \mathrm{A}_{25,4 \%}} & = & 15.622080(\text { Table 2.6 })\end{array}$
Then:

$$
R=36,500 * 15.622080=\$ 570,205.92
$$

This simplified problem introduces a key element in terms of engineering economic comparisons. The savings to road users do not result in any hard-dollar savings for the agency operating the roadway. The analysis, however, recognizes that the proposed repaving will be paid for with tax revenues, many of which come directly from taxes on the road user, such as the gasoline tax.

In this case, the analysis says that it is economically reasonable to spend up to $\$ 570,205.92$ per mile in taxpayer funds to re-pave the roadway, as users will save $\$ 36,500$ per mile collectively each year for the 25 -year expected life of the pavement. Again, $\$ 570,205.92$ is essentially a break-even point. If the repaving costs exactly this much, then the present worth of the road user savings over 25 years are equivalent to the cost. If the actual cost is lower than this amount, the repaving would go forward; if it were higher than this amount, it would not go forward.

Of course, this is a simplified example. In reality, the impacts on ongoing maintenance costs and the economic value of safety improvements would have to be considered in addition to road user costs.

## Example Problem 3: Replacing a Wooden Sign Post

A wooden sign post normally lasts for 15 years and costs $\$ 200$. It is possible to treat these posts in a petroleum-based solution, which increases their service life to 45 years. As the treated posts do not deteriorate completely in place, they must be disposed of at a cost of $\$ 25$ per post after 45 years. If interest is $3 \%$, how much can be justifiably spent to treat the wooden posts?

Answer: As in the previous two problems, the result sought is the maximum allowable amount that could be justifiably spent to treat the wooden posts. Again we are determining the cost at which the two alternatives (treating the post or not) are equally economic - the break-even point.

In this problem, the solution is a bit more difficult in that the benefits involve costs at different times, and over different service lives. The benefit is not easily converted to a monetary value, as it consists of an increase in service life. Since the solution involves a value at which both alternatives are equally economic, a common approach is to express the cost of both on a common base and then to set them equal. In this case, the cost of the treated and untreated posts could be expressed either as equivalent annual costs or as present worths for an equal time period. In this case, converting all costs to equivalent annual costs is more straightforward.

The price of the untreated post is $\$ 200$. This can be converted to an equivalent annual cost by use of the capital recovery factor ( $\mathrm{A} / \mathrm{P}$, that is, looking for annual given we know the present value). Philosophically, we will compute an annual cost equivalent to the amount we would pay each year if we chose to pay for the post on an "installment plan" over its useful life. Thus:

$$
R=P^{*} C R F_{15,3 \%}=P^{*} A / P_{15,3 \%}
$$

$\begin{array}{lllll}\text { where: } & \mathrm{P} & = & \$ 200 \\ & \mathrm{~A} / \mathrm{P}_{15,3 \%} & = & 0.083767 \text { (Table 2.5) }\end{array}$
Then:

$$
R=200 * 0.083767=\$ 16.7534 / y r \Rightarrow \$ 16.75 / y r
$$

The price of the treated post involves several elements. The cost of the wooden post is still $\$ 200$. However, treating the post involves an incremental cost of $\$ T$, making the total cost of the treated post $\$(200+T)$. Further, the treated post involves a disposal cost of $\$ 25$, which occurs at the end of the 45 -year service life of the post. To add this to the total cost of the post (at time " 0 "), its present worth will have to be estimated as:

$$
P=F * P W F_{45,3 \%}=F * P / F_{45,3 \%}
$$

where: $\mathrm{F} \quad=\quad \$ 25.00$

$$
\mathrm{P} / \mathrm{F}_{45,3 \%} \quad=\quad 0.264439 \text { (Table } 2.2 \text { or Appendix) }
$$

Then:

$$
P=25^{*} 0.264439=\$ 6.61
$$

Now, the total present worth cost of the treated post may be estimated as $\$(200+T)+\$ 6.61$, or $\$(206.61+\mathrm{T})$. This cost must now be annualized over the 45 -year service life of the treated post:
where:

$$
R=P * C R F_{45,3 \%}=P * A / P_{45,3 \%}
$$

$$
\begin{array}{lll}
\mathrm{P} & = & 206.61+\mathrm{T} \\
\mathrm{~A} / \mathrm{P}_{45,3 \%} & = & 0.040785(\text { Table 2.5 })
\end{array}
$$

Then:

$$
R=(206.61+T) * 0.040785
$$

To obtain a solution, the two annual costs of the treated and untreated posts are set equal:

$$
\begin{aligned}
& 16.7534=(206.61+T) * 0.040785 \\
& \frac{16.7534}{0.040785}=410.69=206.61+T \\
& T=410.77-206.61=\$ 204.16
\end{aligned}
$$

Thus, if the cost of the treatment to the wooden post is less than $\$ 204.16$, the benefits of increased service life are more than the costs to pay for the treatment, and the treatment will be utilized. If the cost is more than $\$ 204.16$, the untreated posts will continue to be used.

The solution might also have been structured around the present worths for the two alternatives for some uniform period of time, although this approach is slightly more complex. For simplicity, the period of time chosen for analysis should be an even multiple of the service lives of the various alternatives. In this example, with service lives of 15 and 45 years respectively, a 45-year analysis period could be used.

To understand this approach, it is useful to consider a time-line of the costs involved in each alternative. Figure 3.1 shows the time-line for untreated posts; Figure 3.2 shows the time-line for treated posts.


Fig. 3.1 Time-Line for Untreated Sign Posts
To provide for forty-five years of service, the untreated post must be installed at year 0 , and replaced in years 15 and 30 . The last installment is at year 30, as that post will last till the end of year 45 . While a new post would also have to be installed at the end of Year 45, this would cover service beyond the analysis period of 45 years, and is not included. The cost for these renewals must, however, be converted to present worths, as follows:

PW, initial cost $=\$ 200.00$
PW , Year 15 renewal $=200 * \mathrm{P} / \mathrm{F}_{15,3 \%}=200 * 0.641862=\$ 128.37$
PW, Year 30 renewal $=200 * \mathrm{P} / \mathrm{F}_{30,3 \%}=200 * 0.411987=\$ 82.40$
Total Present Worth (untreated posts, 45 yrs ) $=\$ 410.77$
Consider now the following time sequence diagram for the alternative of treating the posts:


Fig. 3.2 Time-Line for Treated Posts
To provide for 45 years of service, the treated post must be installed in year 0 and will last the 45 years. In addition, the disposal cost of $\$ 25$ is incurred at the end of year 45. Converting these to present worths:

| PW, initial cost | $=$ | $\$ 200+\mathrm{T}$ |
| :--- | :--- | :--- |
| PW, Year 45 disposal $=25 * \mathrm{P} / \mathrm{F}_{45,3 \%}=25 * 0.264439$ | $=$ | $\$ 6.61$ |
| Total Present Worth, Treated Posts, 45 years | $=$ | $\$ 206.61+\mathrm{T}$ |

Setting the two values of present worth equal to each other, and solving:

$$
\begin{aligned}
& 410.77=206.61+T \\
& T=410.77-206.61=\$ 204.16
\end{aligned}
$$

The solution and decision are, of course, the same in either method. The results could not be otherwise, since present worths and annual costs are merely different representations of the same costs.

In this case, because the service lives were even multiples of each other, and the untreated post only had to be replaced two times, either method was simple enough. Consider, however, if one alternative had a service life of 3 years and the other had a service life of 11 years. For the present worth method, an analysis period of 33 years would be used. This would mean that the present worth for the 3year alternative would have to be repeated ten times to bring it to year 33. It would be much simpler in that case to compare annual costs.

### 3.2 Applications Involving "Infinite" Service Periods

It is occasionally useful in engineering economic analysis to consider the service lives of various capital investments to be virtually infinite. Any service life of 100 years or greater should be so treated, as the mathematics of banking formulae converge rapidly to infinite compounding periods beyond this point. The problems that follow illustrate some of the special circumstances involving infinite service lives, and the computations that result.

For problems that deal with permanent changes to alignment, for example, the life of the investment is, for all practical purposes, infinite.

Consider the formula for the series present worth factor with $n$ equal to infinity:

$$
S P W_{\infty, i}=P / A_{\infty, i}=\frac{(1+i)^{\infty}-1}{i(1+i)^{\infty}}
$$

Breaking up the faction, this becomes:

$$
S P W_{\infty, i}=\frac{(1+i)^{\infty}}{i(1+i)^{\infty}}-\frac{0}{i(1+i)^{\infty}}=\frac{1}{i}
$$

In general, therefore, the present worth of an Infinite Annual Series is:

$$
\begin{equation*}
P=R * S P W_{\infty, i}=R * P / A_{\infty, i}=R * \frac{1}{i}=\frac{R}{i} \tag{3.1}
\end{equation*}
$$

Example Problem 4: Reducing a Highway Grade
A highway grade is to be reconstructed over a 2-mile length, which will result in a savings to road users of 3 cents per vehicle-mile due to a smoothing of the profile. If the AADT is $8,000 \mathrm{veh} / \mathrm{day}$, how much can be justifiably spent on reconstruction of the grade? Interest is estimated to be $4 \%$.

Answer: As in most problems of this type, the answer is the present worth of the annual savings over the life of the investment. A grade is a sufficiently permanent structure to be considered as having an infinite service life.

Road users will save $\$ 0.03 /$ veh-mile for a virtually infinite period of time. The annual savings to road users will, therefore, be:

$$
R=8,000 * 2 * 365 * 0.03=\$ 175,200 / y r
$$

Since the grade of the highway is essentially a permanent improvement, the present worth of these annual savings is estimated using Equation 3.1:

$$
P=\frac{R}{i}=\frac{175,200}{0.04}=\$ 4,380,000
$$

The highway agency is justified in spending up to $\$ 4,380,000$ re-grading the roadway, which will result in a permanent reduction in annual road user costs of \$175,200.

In Example Problem 4, an annual savings was assumed to be numerically infinite. It is also possible to have a series of infinite periodic costs or savings that do not occur annually. Example Problem 5 illustrates such a case.

## Example Problem 5: Replacing a Railroad Structure with an Embankment_

A railroad trestle is to be replaced by a permanent embankment. How much can be justifiably invested in the embankment if the trestle has a service life of 20 years and costs $\$ 150,000$ each time it is rebuilt? Interest is estimated to be $5 \%$.

Answer: Assuming that at the time the embankment is being considered, the trestle would have to be rebuilt, the solution would be the cost of the trestle plus the present worth of an infinite number of replacements every 20 years. In this problem, the benefits are not in terms of infinite annual savings, but in infinite periodic savings. For all practical purposes, the service life of the embankment itself is infinite.

Consider the present worth of infinite replacements of a facility every $n$ years. If the first cost is not included, then:

$$
P=\sum_{N=1, \infty} S * P W F_{20 N, i}=\sum_{N=1, \infty} S * P / F_{20 N, i}
$$

The replacement occurs every 20 years, so the summation includes values of N from 1 through $\infty$. This series can be expressed as:

$$
P=\frac{S}{(1+i)^{n}}+\frac{S}{(1+i)^{2 n}}+\frac{S}{(1+i)^{3 n}}+\ldots \ldots \ldots \ldots \ldots \ldots \ldots \frac{S}{(1+i)^{\infty n}}
$$

The equation may be represented in closed form by multiplying it by $1 /(1+\mathrm{i})^{\mathrm{n}}$ and subtracting the result from the original series

$$
\begin{aligned}
& P \quad=\frac{S}{(1+i)^{n}}+\frac{S}{(1+i)^{2 n}}+\frac{S}{(1+i)^{3 n}}+\ldots \ldots \ldots \ldots \ldots \ldots \ldots+\frac{S}{(1+i)^{\infty n}} \\
& \frac{P}{(1+i)^{n}}=\frac{S}{(1+i)^{2 n}}+\frac{S}{(1+i)^{3 n}}+\ldots \ldots \ldots \ldots \ldots \ldots \ldots+\frac{S}{(1+i)^{\infty n}}+\frac{S}{(1+i)^{(\infty+1) n}} \\
& P-\frac{P}{(1+i)^{n}}=\frac{S}{(1+i)^{n}} \quad-\frac{S}{(1+i)} 0 \\
& P\left[\frac{(1+i)^{n}-1}{(1+i)^{n}}\right]=\frac{S}{(1+i)^{n}} \\
& P \quad=\frac{S}{(1+i)^{n}-1}
\end{aligned}
$$

Remembering that the sinking fund factor $\left(\mathrm{SFF}_{\mathrm{n}, \mathrm{i}}, \mathrm{A} / \mathrm{F}_{\mathrm{n}, \mathrm{i}}\right)$ is equal to $\mathrm{i} /\left[(1+\mathrm{i})^{\mathrm{n}}-1\right]$, the equation for P may be expressed as:

$$
\begin{equation*}
P=\frac{S}{i} * A / F_{n, i} \tag{3.2}
\end{equation*}
$$

For the problem under consideration, then, the present worth of an infinite series of periodic trestle replacements at a cost of $\$ 150,000$ every 20 years (essentially forever) at $5 \%$ is found using Equation 3.2 where $\mathrm{S}=\$ 150,000, \mathrm{n}=$ 20 years, and $\mathrm{i}=5 \%$. Using Table $2.4, \mathrm{~A} / \mathrm{F}_{20,5 \%}=0.030243$ Then:

$$
P=\frac{150,000}{0.05} * 0.030243=\$ 90,729
$$

However, as Equation 3.2 does not include the cost of a replacement at time $=$ 0 , the first cost must be added to determine the total justifiable expenditure for the permanent embankment. Thus the justifiable expense is the original $\$ 150,000$ plus the cost of infinite replacements or $\$ 90,729+\$ 150,000=$ \$240,729.

This solution can be checked as follows. If $\$ 150,000$ is spent to build the trestle immediately, the remaining $\$ 90,729$ will be invested at $5 \%$ for 20 years to accumulate funds for the next replacement:

$$
F=P * C A F_{20,5 \%}=P * F / P_{20,5 \%}=90,729 * 2.5653298=\$ 240,731
$$

where the CAF $(\mathrm{F} / \mathrm{P})$ is found from Table 2.1.

At the end of the first 20 years, $\$ 150,000$ is utilized to build the trestle, leaving $\$ 90,731$ to be reinvested for another 20 years. The amount remaining for reinvestment will always be sufficient to provide for the next replacement plus another round of reinvestment. Were it not for round-off errors in the use of banking factors to six decimal places, the amount for reinvestment would be exactly $\$ 90,729$ after each replacement.

## Example Problem 6: The Concept of Capitalized Cost

A railroad bridge structure must be replaced every 25 years at a cost of $\$ 250,000$, and it additionally costs $\$ 18,000$ per year to maintain. How much can be spent on a permanent embankment which costs $\$ 3,000$ per year to maintain? Interest is estimated to be $4 \%$

Answer: The result is most conveniently solved by finding the break-even point, as has been illustrated previously. To do this, all costs must be reduced to a common base: either annual cost or present worth for a uniform period of time. To do a present worth analysis requires the use of an infinite analysis period due to the permanent nature of the embankment.
Using an infinite analysis period, the first costs plus the present worths of infinite renewals and infinite maintenance can be compared. By definition, this concept is referred to as Capitalized Cost, that is, the provision, maintenance, and renewal of a capital facility for an infinite period.
The capitalized cost for the bridge involves the present worth of future periodic replacements (every 25 years @ $\$ 250,000$ ), the present worth of all future annual maintenance (every year @ $\$ 18,000$ ), and the first cost of the bridge $(\$ 250,000)$. Equation 3.1 is used to estimate the present worth of the annual maintenance costs, while Equation 3.2 is used to estimate the present worth of all future replacements every 25 years. Then, the total capitalized cost for the bridge is:

$$
\begin{array}{ll}
\text { First Cost }= & \$ 250,000.00 \\
P_{\text {Annual Costs }}=\frac{R}{i}=\frac{18,000}{0.04}= & \$ 450,000.00 \\
P_{\text {Future Periodic }}=\frac{S}{i} * P / A_{25,4 \%}=\frac{250,000}{0.04} * 0.020952= & \$ 130,950.00 \\
\text { Capitalized Cost }= & \$ 830,950.00
\end{array}
$$

The total capitalized cost of the embankment option is the first cost " X " (unknown) plus the present worth of all future annual maintenance costs, or:

$$
\begin{array}{lr}
\text { First Cost }= & X \\
P_{\text {Annual Costs }}=\frac{R}{i}=\frac{3,000}{0.04}= & \$ 75,000.0
\end{array}
$$

The total capitalized cost of the embankment option is, therefore, $X+\$ 75,000$. The capitalized costs of the bridge and the embankment are now set equal to find the break-even point:

$$
\begin{aligned}
& 830,950=X+75,000 \\
& X=830,950-75,000=\$ 755,950
\end{aligned}
$$

As in previous problems, if the cost of the embankment is less than $\$ 755,950$, it will be built. If the cost is more than $\$ 755,950$, the bridge will be replaced. If the actual cost were exactly $\$ 755,950$, both options are equally economic.

## Example Problem 7: Pavement Maintenance

It is estimated that a certain paved road will require $\$ 500$ in maintenance at the end of every 6 months for 5 years, and thereafter $\$ 2,500$ after each 6 months for an indefinite period. Find the present worth of perpetual maintenance if interest is 5\%, compounded semi-annually.

Answer: There are two series of expenses to consider and each must be considered separately. The first consists of ten uniform semi-annual payments of $\$ 500$. The present worth of this series may be straightforwardly computed using the series present worth factor (SPW, P/A) finding the present value given a known periodic payment. The second involves the present worth of future payments every 6 months; this series of expenses, however, only begins in Year 5. The time line in Figure 3.3 illustrates the situation.


Fig. 3.3 Time-Line for Example Problem 7
The present worth of the infinite 6-month payments is easily found using Equation 3.1. The "present worth," that results from Equation 3.1, is indexed to the beginning of Year 5 (or $1 / 2$ Year Period 10). To bring it back to time " 0 ," the true "present" worth is found by multiplying the result by the PWF ( $\mathrm{P} / \mathrm{F}$ ) for 10 periods at $2.5 \%$ per period.
The present worth (at time " 0 ") of the initial 10 semi-annual maintenance costs of $\$ 500$ is computed as:

$$
P=R * S P W_{10,2.5 \%}=R * P / A_{10,2.5 \%}
$$

where $\mathrm{R}=\$ 500$. The interest tables of Chapter 2 do not include a $2.5 \%$ interest rate. Therefore, the SPW (P/F) must be computed using the formula (or using the excel spreadsheet created in Problem 2-1):

$$
\begin{aligned}
& P / A=\frac{(1+i)^{n}-1}{i(1+i)^{n}}=\frac{(1+0.025)^{10}-1}{0.025(1+0.025)^{10}}= \\
& \frac{1.2801-1}{0.025 * 1.2801}=\frac{0.2801}{0.032003}=8.7523
\end{aligned}
$$

Then:

$$
P=500 * 8.7523=\$ 4,376.15
$$

The present worth (at time $=$ " 0 ") of the infinite semi-annual maintenance costs that begin in Year 11 must be treated as a series, which begins in Period 10, and then shifted as a single amount to Period 0 , or:

$$
P=\frac{R}{i} * P W F_{10,2.5 \%}=\frac{R}{i} * P / F_{10,2.5 \%}
$$

where R is $\$ 2,500$ semiannually. Again, as $2.5 \%$ is not included in the interest tables of Chapter 2, it is computed using the equation (or use spreadsheet):

$$
P / F=\frac{1}{(1+i)^{n}}=\frac{1}{(1+0.025)^{10}}=\frac{1}{1.2801}=0.781189
$$

Then:

$$
P=\frac{2,500}{0.025} * 0.781189=\$ 78,118.90
$$

The total present worth of infinite semi-annual maintenance on the facility is the sum of $\$ 4,376.15+\$ 78,118.90=\$ 82,495.05$. This amount, if invested at $5 \%$ interest, compounded semi-annually, would be sufficient to provide for perpetual maintenance on the facility.

As these examples illustrate, the basic banking formulae can be adapted to deal with situations in which the element of time is virtually infinite. While nothing is ever truly "infinite," numerically, the interest factors rapidly approach values for an infinite period whenever the actual time period is 100 years (or periods) or longer.

### 3.3 Methodologies for Engineering Economic Comparisons

The formal procedures for conducting engineering economy studies are mathematically founded on the basic banking principles discussed previously. Full engineering
economic comparisons, and in particular, those involving transportation projects and alternatives, involve other considerations as well. This section explores formal procedures for engineering economy studies and comparisons, with an emphasis on those involving transportation.

### 3.3.1 The Treatment of Transportation Demand

One of the complex components in defining an alternative analysis comparison is the handling of demand volumes. Demand volumes, in turn, are the principal determinants of the scope of the alternatives that must be considered.

The demand volume on any new facility, either highway or public transit, will be subject to growth during the years the facility is in use. The volume on the facility, and its growth over the years, is made up of two major components:

- Current Traffic is defined as the traffic expected on the new facility on the day it opens (or more practically, during its first year of service). It is made up of (1) traffic from the facility it replaces (if one exists) and (2) diverted traffic from parallel or competing facilities.
- Growth Traffic which is itself comprised of several components: (1) normal (or ambient) growth that is created by regional growth patterns in travel; that is, all travel in the region would be expected to grow at some rate reflecting changing per capita trip-making rates and general population trends, (2) development traffic that is created by land development induced by the new facility, which generates additional travel, and (3) induced traffic, which covers trips that were not previously made, or which were made to other destinations due to transportation difficulties in the subject corridor. The existence of improved facilities can 'induce" trips to be made now in the subject corridor.

The problem of demand is complicated by the fact that development growth and induced traffic may vary from alternative to alternative within the same analysis. While this text does not address the process of traffic forecasting, it will address the issues involving which forecasts should be used in comparative analysis, and how to do so.

### 3.3.2 Defining the Scope of Alternatives in an Economic Analysis

The definition of alternatives and their scope is very much related to the consideration of traffic volume forecasts. The following principle provides the key factor in this regard:

[^0]Recall that current traffic represents existing trips being made that are either (1) already on a facility that is being replaced or improved, or (2) diverted to a new or improved facility from parallel facilities in the corridor.

A second rule in defining alternatives for analysis is just as important:

## ALL ANALYSES MUST INCLUDE CONSIDERATION OF THE NULL ALTERNATIVE, THAT IS, THE OPTION OF DOING NOTHING.

It is not enough to answer the question of which alternative improvement is the most economically beneficial. The analysis must also show if any improvement at all is economically better than the status quo. This is accomplished by including the null alternative in all analyses.

Consider the problem of a corridor consisting of three freeways (A, B, and C), each carrying 100,000 vehicles per day with an average vehicle occupancy of 1.5 persons/vehicle. Mass transit is being considered to alleviate congestion in the corridor in one of two configurations, as follows:

1. Construct a busway along one of the freeways, expecting ridership of 20,000 passengers/day, all diverted from the freeway corridor - $1 / 2$ from freeway B , and $1 / 4$ each from freeways A and C.
2. Construct a rail line in the corridor with an expected ridership of 75,000 passengers/day, all of who are diverted from the freeway corridor - $1 / 3$ from each freeway.

The nature of the problem concerns not only the new bus and/or rail facilities, but the totality of 300,000 vehicles per day (vpd) (or 450,000 persons per day ( ppd )) traveling in the corridor. Unless the total corridor is considered in each alternative, it would be impossible to construct scenarios with equal current traffic. The definition of each alternative, therefore, must include all of the existing and proposed facilities in the corridor, as described below in Table 3.1.

Table 3.1 Definition of Alternatives for Corridor

|  | Null Alternative | Alternative 1 | Alternative 2 |
| :--- | :--- | :---: | :---: |
| Freeway A | $100,000 \mathrm{vpd}$ | $96,667 \mathrm{vpd}$ | $83,333 \mathrm{vpd}$ |
|  | $150,000 \mathrm{ppd}$ | $145,000 \mathrm{ppd}$ | $125,000 \mathrm{ppd}$ |
| Freeway B | $100,000 \mathrm{vpd}$ | $93,333 \mathrm{vpd}$ | $83,333 \mathrm{vpd}$ |
|  | $150,000 \mathrm{ppd}$ | $140,000 \mathrm{ppd}$ | $125,000 \mathrm{ppd}$ |
| Freeway C | $100,000 \mathrm{vpd}$ | $96,667 \mathrm{vpd}$ | $83,333 \mathrm{vpd}$ |
|  | $150,000 \mathrm{ppd}$ | $145,000 \mathrm{ppd}$ | $125,000 \mathrm{ppd}$ |
| Busway | ------------ | $20,000 \mathrm{ppd}$ | ----- |
| Rail | ------- | $75,000 \mathrm{ppd}$ |  |

Note that each of the alternatives was defined in terms of equal numbers of people served, that is, a total current traffic of 450,000 persons per day. The economic analysis would then have to consider the user, operating, and capital costs involved in each element of each alternative, and would account for improved conditions on highways as the result of diversion, as well as the costs associated with the proposed bus or transit services. Without this total consideration, it would be virtually impossible to compare a bus alternative serving $20,000 \mathrm{ppd}$ with a rail service serving 75,000 , much less, the current highway system serving 450,000 people/day.

As has been shown, it is possible to define alternatives in such a way that they represent equal values of current traffic. We can then assume that normal growth traffic will also be the same for all alternatives, as this is merely a percentage growth rate applied to current traffic. It is not possible, however, to guarantee that the development and induced growth in traffic will be equal for all alternatives. In the example given in Table 3.1, the rail system will have a far greater impact on development, and therefore development traffic growth, than the bus service. The rail link also has far greater potential for inducing new trips. Thus, while the alternatives are defined for equal volumes of current traffic, the 25 -year traffic forecast for each alternative would most likely be different. This will affect the analysis procedures that can be used, as described later in this chapter.

### 3.3.3 Direct Techniques for Alternative Economic Analysis

The most straightforward analysis techniques deal in the direct conversion of all cost elements to either (1) annual cost or (2) present worth for a specified analysis period. All costs will be modified to a common base in order to be compared directly, so that alternatives with lower total cost are favored over those with higher total costs.

Total costs will include capital costs, operating and maintenance costs, and direct user costs. Each of these costs is identified for the analysis period, either as annual costs, as in the case of user costs, or as single sums occurring at various times throughout the analysis period.

### 3.3.3.1 The Present Worth Method

The present worth method of analysis bases comparisons on the total present value of each alternative for a fixed analysis period (the same for all alternatives). All costs are reduced to present value, that is, indexed to time $=0$ of the analysis period, using the appropriate factors discussed earlier.

If the analysis period chosen is not an even multiple of the service lives of all the components of the analysis, it is often necessary to consider the question of residual value. Residual value represents the value left in the life of a component after the analysis period. It is found using the following formula:

$$
\begin{equation*}
R V=C * A / P_{n, i} * P / A_{N_{r}, i} * P / F_{N_{a}, i} \tag{3.3}
\end{equation*}
$$

where: RV is the present worth of the residual value (at time " 0 "), C is the cost of the component, n is the service life of the component with cost C , $\mathrm{N}_{\mathrm{r}}$ is the years of service remaining at the end of the analysis period. $\mathrm{N}_{\mathrm{a}}$ is the analysis period in years.

This concept is best described by example. Consider the present worth of a bridge structure that costs $\$ 300,000$ and has a service life of 20 years. The analysis period is 50 years, and interest is estimated to be $5 \%$. Figure 3.4 illustrates the problem.


Fig. 3.4 Illustration of Residual Value
The analysis period encompasses the initial cost and the cost of bridge replacements in the 20th and $40^{\text {th }}$ years. The present worth of these costs is given by:

$$
P=300,000+\left(300,000 * P / F_{20,5 \%}\right)+\left(300,000 * P / F_{40,5 \%_{\%}}\right)
$$

where both values of $\mathrm{P} / \mathrm{F}$ can be obtained from Table 2.2 in Chapter 2. Then:

$$
\begin{aligned}
& P=300,000+(300,000 * 0.376889)+(300,000 * 0.142046) \\
& P=300,000.00+113,066.70+42,613.80=\$ 455,680.50
\end{aligned}
$$

The replacement in Year 40, however, provides for service to the 60th year, that is, 10 years beyond the analysis period. Thus the facility has a "residual value" equivalent to 10 years of useful service at the end of 50 years, the cost of which has been included in the initial computation. The present worth of this residual value should, therefore, be subtracted from the initial result of $\$ 455,680.50$.

Since the entire cost of the last replacement occurs in Year 40, it must be annualized in order for one part of it to be separated as the residual value. The equivalent annual cost of the replacement is computed as:

$$
R=300,000 * A / P_{20,5 \%}=300,000 * 0.080243=\$ 24,072.90
$$

where $\mathrm{A} / \mathrm{P}$ is found from Table 2.5 in Chapter 2.

The replacement cost may therefore be treated as a series of annual payments of $\$ 24,072.90$ beginning in Year 41 and ending in Year 60. The last 10 of these annual payments represents the "residual value" of the bridge at the end of the analysis period of 50 years.

The present worth of these last 10 payments must be computed in a two-step process. First, the "present worth" of a series of 10 annual payments of \$24,072.90 is found as:

$$
P=24,072.90 * P / A_{10,5 \%}=24,072.90 * 7.721735=\$ 185,884.55
$$

where $\mathrm{P} / \mathrm{A}$ is found from Table 2.6 in Chapter 2.
This is the single-value equivalent of the 10 annual payments that constitutes the residual value. It is, however, indexed to the 50th year of the analysis period. The true residual value, in Year 0 , is found by finding the present worth of \$185,884.55 in Year 50, or:

$$
R V=185,884.55 * P / F_{50,5 \%}=185,884.55 * 0.087204=\$ 16,208.88
$$

where $\mathrm{P} / \mathrm{F}$ is found from Table 2.2 in Chapter 2.
The true present worth of the cost of building and replacing the bridge every 20 years, over a 50 -year analysis period is the present worth of the initial construction, 20-year replacement, and 40-year replacement minus the residual value computed above, or $\$ 455,680.50-\$ 16,208.88=\$ 439,471.62$.

The concept of residual value is a very valuable one when the present worth method of analysis and comparison is used. It is not always practical to use an analysis period that is an even multiple of all of the component service lives. Comparing two alternatives with respective service lives of 18 years and 15 years would require an analysis period of $18^{*} 15=270$ years, which is not practical. Any reasonable analysis period can, however, be used, as the concept of residual value allows analysts to find the present worth of any analysis period, even if it is not an even multiple of the service life.

Using residual value, it is possible to directly compare any number of alternatives using an arbitrary (but reasonable) analysis period.

### 3.3.3.2 The Annual Cost Method

In the annual cost method, all cost elements are expressed as annual costs. Comparisons are made on the basis of total annual cost of capital costs, operating and maintenance costs, and user costs. Alternatives with lower total annual costs are favored over those with higher total annual cost.

The annual cost method is often very convenient, as many cost elements, such as user costs and operating costs are already given in annual form. Single sums involved in capital investments may be annualized using the $\mathrm{A} / \mathrm{P}$ factor. It must be noted that the annual cost computed must be an annual cost that remains constant throughout the analysis period.

Consider the annual cost for a 30-year analysis period of a highway pavement with the following characteristics:

> Initial Cost $=\$ 1,000,000$ per mile, service life is 20 years.
> Repaving cost $=\$ 300,000$ per mile, service life 10 years.
> Annual maintenance $=\$ 15,000$ per mile per year.
> Interest $=3 \%$.

Maintenance costs are already in annual form and thus do not need conversion. The initial cost and the cost of one repaving in Year 20, however, must be annualized for the thirty-year analysis period.

The problem is that neither the initial paving cost (which provides service for Years 1-20) nor the repaving (which provides service for Years 21-30) are expressed in terms of a 30-year analysis period. If we had a single value, indexed to time $=0$ representing the present worth of the initial paving plus one repavement in Year 20, the resulting present worth could be annualized over a 30 -year analysis period. The present worth of the initial paving is given, and does not have to be converted. The present worth of a repaving in Year 20 does have to be converted. The total present worth of the initial paving plus one repaving in this case is computed as:

$$
\begin{aligned}
& P=1,000,000+300,000 * P / F_{20,3 \%} \\
& P=1,000,000+(300,000 * 0.553676) \\
& P=1,000,000+166,102.80=\$ 1,166,102.80
\end{aligned}
$$

where $\mathrm{P} / \mathrm{F}$ is obtained from Table 2.2 in Chapter 2. This present worth may now be annualized to uniform annual payments over the total 30 -year analysis period as:

$$
R=1,166,102.80 * A / P_{30,3 \%}=1,166,102.80 * 0.05109=\$ 59,576.19
$$

where $\mathrm{A} / \mathrm{P}$ is found from Table 2.5 in Chapter 2 (or use excel spreadsheet).
This is now the annual cost of providing the pavement and one repavement in Year 20. The total annual cost, however, must now add the $\$ 15,000 /$ year maintenance cost, making the total annual cost for the pavement $\$ 59,576.19+\$ 15,000=$ \$74,576.19/mile.

There are, therefore, two direct methods to compare the costs of alternative projects. Either the costs of all alternatives can be converted to a present worth for equal analysis periods, or they can be converted to annual costs. Many times, finding either will require both present worth and annual cost calculations to reduce all costs to a common basis for comparison.

### 3.3.3.3 Examples Using Direct Methods

Direct methods of alternative analysis are the most easily applied, as the final comparison of alternatives is based upon the simple comparison of total annual costs or present worths. Unfortunately, direct methods are limited to cases in
which the value of traffic demand is the same for all alternatives throughout the analysis period. This means that alternative comparisons in which the value of development and induced traffic differ from alternative to alternative, may not be analyzed using direct methods. Indirect methods for cases where the traffic differs between alternatives will be treated later in this chapter.

## Example Analysis 1: Comparison of Pavements

A highway department must decide whether to construct a new portland cement concrete (PCC) pavement of 6 in ., 7 in ., or 8 in . on a given stretch of highway. Table 3.2 shows the cost data for the problem.

Table 3.2 Data for Example Analysis 1

| Pavement <br> Thickness (in) | Initial Cost <br> $(\mathbf{\$ / m i})$ | Service Life <br> $(\mathbf{y r s})$ | Annual Maintenance <br> $\mathbf{( \$ / \mathbf { m i } )}$ |
| :---: | :---: | :---: | :---: |
| 6 | 800,000 | 15 | 15,000 |
| 7 | $1,000,000$ | 18 | 10,000 |
| 8 | $1,150,000$ | 20 | 8,500 |

Assume that at the end of each service life, the pavement is rebuilt at a cost and service life similar to the initial construction. Interest is $3 \%$.

## Solution

This problem is well suited to a direct method of analysis since the comparison is between different structural alternatives for the same highway and traffic demand.

The analysis period chosen is not a major issue for this problem, as each of the pavements is to be replaced at the end of its service life with a similar pavement, and all cost elements are held at uniform levels. Thus, ignoring inflation, the annual cost of each alternative will continue essentially forever. In this circumstance, the annual cost of each alternative is found as:

$$
R=\left(P^{*} A / P_{n, i}\right)+M
$$

where: $\mathrm{P} \quad=\quad$ capital cost to build the alternative;

$$
\begin{array}{ll}
\mathrm{A} / \mathrm{P} & =\quad \text { capital recovery factor (Table 2.5, Chapter } 2) ; \\
\mathrm{M} & =\quad \text { annual maintenance cost of each alternative. }
\end{array}
$$

Then:

$$
\begin{aligned}
& R_{6 \text { in }}=\left(800,000 * A / P_{15,3 \%}\right)+15,000 \\
& R_{6 \text { in }}=(800,000 * 0.083767)+15,000=\$ 82,013.60
\end{aligned}
$$

As the capital recovery factor for 18 years is not included in Table 2.5, it will have to be computed using the equation (or found using spreadsheet):

$$
A / P_{18.3 \%}=\frac{0.03 *(1+0.03)^{18}}{(1+0.03)^{18}-1}=\frac{0.051073}{0.702433}=0.072709
$$

Then:

$$
\begin{aligned}
& R_{7 \text { in }}=\left(1,000,000 * A / P_{18.3 \%}\right)+10,000 \\
& R_{7 \text { in }}=(1,000,000 * 0.072709)+10,000=\$ 82,709.00 \\
& R_{8 \text { in }}=\left(1,150,000 * A / P_{20,3 \%}\right)+8,500 \\
& R_{8 \text { in }}=(1,150,000 * 0.067216)+8,500=\$ 85,798.40
\end{aligned}
$$

Based upon total annual cost, the 6-inch pavement is the most economic alternative. Notice that the annual costs computed do not depend upon the choice of an analysis period. This is true only when all annual costs are uniform for an indefinite period, and all capital investments are replaced at initial cost with the same service life as the initial investment.

## Example Analysis 2: Initiating an Express Bus Route

A major arterial corridor contains a series of local bus routes that carry 40,000 passengers/day. The transit operator is considering the establishment of one or two possible express bus services in the corridor. The first would divert 5,000 passengers/day from local routes, the second would divert 8,000 passengers per day from the local services. Neither of the proposed services would divert passengers from any other source than competing local routes, and neither would induce new development or new trips in the corridor. Relevant cost information is given in Table 3.3. Interest is estimated to be $5 \%$.

Table 3.3 Data for Example Analysis 2

| Cost Element | Alternative |  |  |
| :--- | :---: | :---: | :---: |
|  | NULL | Exp. Route 1 | Exp. Route 2 |
| Capital Investment for <br> Additional Buses (\$) |  | $1,000,000$ | $1,350,000$ |
| Service Life (yrs) |  | 10 | 10 |
| Maintenance and Operating <br> Costs, Local Buses (\$/yr) | $11,000,000$ | $8,000,000$ | $6,000,000$ |
| Maintenance and Operating <br> Costs, Express Buses (\$/yr) |  | 500,000 | 800,000 |
| User Costs, Local Buses <br> (\$/day/user) | 1.50 | 1.40 | 1.35 |
| User Costs, Exp. Buses <br> (\$/day/user) |  | 1.00 | 0.80 |

Solution: Here again a direct approach is valid since neither alternative has development or induced traffic expected. Note that the alternatives are defined to include the entire 40,000 passengers/day using the transit corridor, and to encompass the impact on local bus service, which is obviously affected. A 20year analysis period would be reasonable, and the problem is approached using the present worth method. The service life of an express bus is 10 years. The number of replacement buses for local services is NOT affected by whether or not a new express bus service is offered, so this expense is not included in any of the options.

Present Worth of the Null Alternative:
Annual Cost of Bus Maintenance and Operations (given) $=\$ 11,000,000$
Annual User Costs $=40,000 * 365 * 1.50 \quad=\$ 21,900,000$
Total Annual Costs - Null Alternative $=\mathbf{\$ 3 2 , 9 0 0 , 0 0 0}$
The present worth of 20 years of service at a total of $\$ 32,900,000$ per year is computed as:

$$
P_{\text {Null }}=32,900,000 * P / A_{20,5 \%}=32,900,000 * 12.462210=\$ 410,006,709
$$

Present Worth for Express Route 1:
Annual Cost of Bus M \& O (Local Buses) (given) $=\$ 8,000,000$
Annual Cost of Bus M \& O (Exp Buses) (given) =\$ 500,000
Annual User Costs (Local Buses) $=35,000 * 365 * 1.40=\$ 17,885,000$
Annual User Costs (Exp Buses) $=5,000 * 365 * 1.00=\$ 1,826,000$
Total Annual Costs - Express Route $1=\mathbf{\$ 2 8 , 2 1 1 , 0 0 0}$
Then:

| $P_{\text {Bu S Purchase }}=$ | $\$$ | $1,000,000$ |
| :--- | ---: | ---: |
| $P_{\text {Bus Purchase }, Y r 10}=1,000,000 * P / F_{10,5 \%}=1,000,000 * 0.613913=$ | 619,913 |  |
| $P_{\text {Annual Costs }}=28,211,000 * P / A_{20,5 \%}=28,211,000 * 12.462210=$ | $\$ 351,571,406$ |  |
| Total $=$ | $\$ 353,191,319$ |  |

Present Worth of Express Route 2:
Annual Cost for Bus M \& O (Local Routes) (given) = \$ 6,000,000
Annual Cost for Bus M \& O (Exp Routes ) (given) $=\$ 800,000$
Annual User Costs (Local Buses) $=32,000 * 365 * 1.35=\$ 15,768,000$
Annual User Costs (Express Buses) $=8,000 * 365 * 0.80=\$ 2,336,000$
Total Annual Costs - Express Route $2=\mathbf{\$ 2 4 , 9 0 4 , 0 0 0}$

| $P_{\text {Bus } P_{\text {urchase }}}=$ | $=\$ \quad 1,350,000$ |  |
| :--- | :--- | ---: |
| $P_{\text {BusPur }, 10 \text { Yrs }}=1,350,000 * P / F_{10,5 \%}=1,350,000 * 0.613913$ | $=\$ 8$ | 828,783 |
| $P_{\text {Annual Costs }}=24,904,000 * P / A_{20,5 \%}=24,904,000 * 12.462310$ | $=\$ 299,095,440$ |  |
| Total $=$ | $=\$ 301,274,223$ |  |

Thus, rounding to the nearest million, the Null Alternative would cost $\$ 401$ million, Express Bus Option 1 would cost $\$ 353$ million, and Express Bus Option 2 would cost $\$ 301$ million. Express Bus Option 2 is, by this comparison, the most economic of the alternatives. Note that this is a planning decision made in the context of total economy. The operator of the service would not consider user costs, but would consider revenues. Savings in user travel time, while critical to the planner, are not of great importance to the operator, who is concerned with profitability, not overall economy in the societal sense.

The problem could also have been solved by converting all cost elements to annual cost. This might have been more straightforward, as all elements, except the purchase of express buses, are already expressed in annual terms. Bus purchases for the two express bus options would have then been converted to annual costs over 20 years. This would have been based on the present worth of bus purchases in Year 0 and Year 10. Using the results of the present worth analysis, then the present worth of bus purchases by alternative are:

$$
\begin{array}{llc}
\mathrm{P}_{\text {Null }} & =\$ & 0 \\
\mathrm{P}_{\text {Exp Opt } 1}=1,000,000+619,913 & =\$ & 1,619,913 \\
\mathrm{P}_{\text {Exp Opt } 2}=1,350,000+828,783 & =\$ & 2,178,783
\end{array}
$$

Then:

$$
\begin{array}{ll}
R_{\text {Null }} & =\$ 32,900,000 \\
R_{\text {Exp } O p t 1}=28,211,000+\left(1,619,913 * A / P_{20,5 \%}\right) & = \\
R_{\text {Exp } o p t 1}=28,211,000+(1,619,913 * 0.080243) & =\$ 28,340,986 \\
R_{\text {Exp } O p t 2}=24,904,000+(2,178,785 * 0.080243) & =\$ 25,078,832
\end{array}
$$

As in the comparison of present worths, the most economic alternative is still Express Bus Option 2. The answers do not change; it is only the basis of the comparison numbers that changes.

### 3.3.4 Incremental Methods for Alternative Economic Analysis

Incremental methods of analysis are universally applicable, but are somewhat more complex in that only two alternatives may be compared at a time. Multiple alternatives must be compared in a series of sequential computations, each comparing two alternatives.

Incremental methods are capable of accounting for alternatives with varying demand due to development and induced traffic, and are based upon the identification of:

- Incremental user benefits of one alternative over the other, and
- Incremental system costs of one alternative over the other.


### 3.3.4.1 Incremental User Benefits (IUB)

Incremental user benefits may be expressed as the difference in total user costs between two alternatives, expressed as an annual cost or present worth cost for a specified analysis period. User costs include:

- Travel time costs, and
- Direct costs of tolls and/or public transit fares, and
- Operating and maintenance costs of private vehicles.

Where the alternatives involved have the same value of traffic demand throughout the analysis period, IUB may be simply expressed as total user costs for one alternative minus the total user costs for another. The general formulation for IUB, however, allows for the consideration of alternatives with varying traffic demand:

$$
\begin{equation*}
I U B=\left(U_{1}-U_{2}\right) *\left(\frac{V_{1}+V_{2}}{2}\right) \tag{3.4}
\end{equation*}
$$

where: IUB = Incremental User Benefits (\$)
$\mathrm{U}_{1} \quad=$ Unit user cost per vehicle or person for alternative 1 (\$)
$\mathrm{U}_{2} \quad=$ Unit user cost per vehicle or person for alternative 2 (\$)
$\mathrm{V}_{1} \quad=$ Traffic (vehicles or persons) for alternative 1 (vehs or per/yr)
$\mathrm{V}_{2}=$ Traffic (vehicles or persons) for alternative 2 (vehs or per/yr)
Alternative 1 is always defined as the alternative with the higher unit user costs. Incremental user benefits are generally computed on an annual basis, and may be converted to a present worth, if desired.

Multiplying the difference in unit costs by the average volume of the alternatives being considered is based upon the economic concept of consumer surplus. Consult the literature for a complete discussion of this concept (1).

### 3.3.4.2 Incremental System Costs (ISC)

Incremental system costs refer to the difference in total system costs between two alternatives, expressed as an annual cost or present worth for a given analysis period. NOTE that for a given analysis IUB and ISC must be in the same form (annual or present worth). System costs (SC) for a given alternative would include:

- Capital expenditures, and
- Maintenance and operation of highway and/or public transportation systems involved in the comparison.

The value of ISC is not dependent upon respective traffic demand volumes of the various alternatives, and may be simply expressed as:

$$
\begin{equation*}
I S C=S C_{2}-S C_{1} \tag{3.5}
\end{equation*}
$$

where: ISC = Incremental System Cost (\$)
$\mathrm{SC}_{2}=$ Total System Cost for Alternative 2 (\$)
$\mathrm{SC}_{1}=$ Total System Cost for Alternative 1 (\$)

Alternative 1 is defined as the alternative with the higher unit user cost. It is assumed that the alternative with higher incremental user costs will be the one with lower incremental system costs. If one alternative had both higher incremental user and system costs, it would clearly not be an economic project to pursue.

### 3.3.4.3 Benefit-Cost Ratio Method for Comparing Incremental Costs

A given pair of alternatives can be compared using a benefit-cost ratio, computed as follows:

$$
\begin{equation*}
B C R=\frac{I U B}{I S C}=\frac{\left(U_{1}-U_{2}\right) * \frac{\left(V_{1}+V_{2}\right)}{2}}{S C_{2}-S C_{1}}=\frac{\left(U_{1}-U_{2}\right) *\left(V_{1}+V_{2}\right)}{2 *\left(S C_{2}-S C_{1}\right)} \tag{3.6}
\end{equation*}
$$

where alternative 1 is the alternative with the higher unit user cost and alternative 2 is the alternative with the lower unit user cost. Normally, an alternative with higher user cost would have lower system cost (we are investing money in the system to "buy" user benefits). Occasionally, that will not be true, and one alternative would have both higher user and system costs. The BCR will turn out to be negative in these cases. In such cases, the alternative with the lower costs is obviously the most economically favorable.

The decision concerning the selection of the most economic alternative is based upon the magnitude of the BCR:
(1) If $\mathrm{BCR}>1,0$, then the value of the incremental benefits gained by providing alternative 2 is greater than the incremental cost to provide it. Thus alternative 2 , defined by having the greater system cost, is more economic.
(2) If $\mathrm{BCR}<1.0$, then the value of the incremental benefits gained by providing alternate 2 is less than the incremental cost to provide it. Alternate 1, with the lower system cost, is therefore the more economic alternative.
(3) If $\mathrm{BCR}=1.0$, both alternatives are equally economic.

The results of a benefit-cost comparison of two alternatives are only as accurate as the estimates of user and system costs. There are often uncertainties involved in these estimates. Therefore, it is common practice to establish a range around $\mathrm{BCR}=1.00$ that would be considered to be inconclusive. For example, if the estimates of user and system costs are only accurate to $\pm 10 \%$, then, a BCR in the range of 0.90 and 1.10 might be taken to indicate the two alternatives are essentially equally economic.

## Example Analysis 3: Comparison of Multiple Alternatives

Consider the set of alternatives in Table 3.4 below. All of the incremental system costs and user benefits are given relative to the null alternative, and the benefit cost ratio with respect to the null alternative is computed for each alternative.

All of the alternatives are better than the null alternative, as all of the BCR values are greater than 1.0. From the BCR's of Table 3.4, however, it is not possible to rank alternatives A1 - A7 relative to each other. The fact that alternative A4 has the highest BCR with respect to the null alternative does not mean that it is the best among all the alternatives.

The reason for this is that BCR's are ratios that do not share a common base. In the case of alternative A4, the BCR is 20.0, but the project produces a "profit" of $\$ 190$ million ( $\$ 200$ million - $\$ 10$ million). Alternative A3, for example, has a BCR of only 4.0, but produces a "profit" of $\$ 270$ million ( $\$ 360$ million $\$ 90$ million). To rank the alternatives with respect to each other, BCR's based upon the direct comparison of each alternative to each of the other alternatives would be needed.

Table 3.4 Seven Alternatives with Costs, Benefits, and BCR versus the Null Case

| Alternative | Incremental Sys- <br> tem Cost <br> $\mathbf{( \$ 1 , 0 0 0 , 0 0 0 )}$ | Incremental <br> User Benefit <br> $\mathbf{( \$ 1 , 0 0 0 , 0 0 0 )}$ | BCR Relative <br> to Null Alter- <br> native |
| :---: | :---: | :---: | :---: |
| Null | 0 | 0 | N/A |
| $\mathbf{A 1}$ | 10 | 60 | $60 / 10=6.0$ |
| $\mathbf{A 2}$ | 70 | 300 | $300 / 70=4.3$ |
| $\mathbf{A 3}$ | 90 | 360 | $360 / 90=4.0$ |
| $\mathbf{A 4}$ | 10 | 200 | $200 / 10=20.0$ |
| $\mathbf{A 5}$ | 30 | 180 | $180 / 30=6.0$ |
| $\mathbf{A 6}$ | 70 | 280 | $280 / 70=4.0$ |
| $\mathbf{A 7}$ | 40 | 100 | $100 / 40=2.5$ |

Fortunately, this can be done using the data of Table 3.4. For any alternative " i " that is compared to alternative " j " (where alternative j has the higher incremental user benefits), the incremental benefit-cost ratio may be computed as:

$$
\begin{equation*}
I B C R_{i j}=\frac{I U B_{j}-I U B_{i}}{S C_{j}-S C_{i}} \tag{3.7}
\end{equation*}
$$

Note that since the base data is already in the form of incremental user and system costs, it is expected that alternatives with higher incremental benefits will also have higher incremental costs. The assumption is that if \$X more is spent on alternative $j$, then to be favored, it is expected that alternative $j$ will produce an incremental benefit, $\$ \mathrm{Y}$, that is greater than the incremental cost, i.e., that \$Y > \$X.

When using the IBCR, however, there is a fundamental difference in interpretation from the BCR. When the BCR is negative, both the user and system costs of one alternative are higher than the other. The one with the lower user
costs is therefore the most economic choice. Why would you invest money to increase the costs to users?

When the IBCR is negative, however, the alternative with the higher incremental user benefits has a lower system cost. The economic choice is the alternative with the higher incremental user benefits. The difference is that the IBCR already uses incremental user costs, while the BCR uses the actual user costs of each alternative.

For example, if we wished to compare alternative A6 directly with alternative A3, the result would be:

$$
I B C R_{A 3, A 6}=\frac{360-280}{90-70}=\frac{80}{20}=4.00
$$

Note that since A3 has the higher incremental user benefits, it is designated as alternative "i" in Equation 3.7. Since A3 has the higher incremental user benefits, it is the favored alternative. Table 3.5 contains the results comparing all alternatives to each other using this approach. The null alternative is eliminated, since is already known that all options are economically better than the null case.

Table 3.5 Comparing Alternatives to Each Other for Example Analysis 3

| Atemative with | Altemative with Lover Incremental User Benefits(IUB) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hgher IUB | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
| A1 | NA | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ |
| A2 | 4.00 | NA | ${ }^{*}$ | 1.67 | 240 | + | 6.67 |
| A3 | 3.75 | 3.00 | NA | 200 | 3.00 | 4.00 | 5.20 |
| A4 | + | ${ }^{*}$ | ${ }^{*}$ | NA | ++ | ${ }^{*}$ | ++ |
| A5 | 6.00 | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | NA | ${ }^{*}$ | ++ |
| A6 | 3.67 | ${ }^{*}$ | ${ }^{*}$ | 1.33 | 250 | NA | 6.00 |
| A7 | 1.33 | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | ${ }^{*}$ | NA |

* For these cells, the row alternative has lower IUB than the column alternative.
+ For these cells, the system costs of the two alternatives are equal; the alternative with the higher IUB is favored.
++ For these cells, the IBCR is negative; the alternative with the higher IUB is favored.
Sorting this all out to rank the alternatives in order of economic viability is an exercise in logic.

First, note that there is no BCR entered for the comparison of A4 to A1 or A6 to A2. This is because the incremental system cost for alternatives A4 and A1 are the same, as they are for alternatives A6 and A2. This makes the denominator of Equation 3.7 " 0 ," which does not yield a finite BCR. In these cases, the comparison is actually simplified. Since the two pairs have the same incremental system costs, then the one with the higher incremental user benefits must be the most economic. Thus, A4 is better than A1, and A6 is better than A2. The " + " in Table 3.5 indicates that the alternative of the row is better than the base alternative of the column in both cases. Also, those cells
resulting in negative IBCR values are shown as "++." Again, the alternative of the row is more economic than the alternative of the column.

Now, to rank the alternatives, the results in of Table 3.5 are considered:
(1) From Column A1, all other alternatives are better than alternative A1. This means that A1 is the worst alternative, except for the Null alternative.
(2) From Row A2, alternative A2 is more economic than alternatives A1, A4, A5, A6, and A7.
(3) From Row A3, alternative A3 has higher IUB than all other alternatives, and IBCR values $>1$ vs. all other alternatives. This means that A3 is the best alternative. Taken with item (2), this also means that alternative A2 is the second best alternative.
(4) Things now get more interesting:

- From Row A4, A4 is better than alternatives A1, A5, and A7.
- From Row A5, alternative A5 is better than alternatives A1 and A7.
- From Row A6, alternative A6 is more economic than alternatives A1, A4, A5, and A5.
(5) From this, and the initial analysis vs the null alternative, we can deduce the following sequence:

$$
\mathrm{A} 3>\mathrm{A} 2>\mathrm{A} 6>\mathrm{A} 4>\mathrm{A} 5>\mathrm{A} 7>\mathrm{A} 1>\text { Null }
$$

Figures 3.5, 3.6, and 3.7 illustrate some of the key points in determining the best alternative. Note that these figures plot IUB vs. ISC for each alternative vs. the null alternative. The slope of a line drawn from the origin to any of these points is the BCR for each vs. the null alternative.


Fig. 3.5 Alternatives Plotted vs. Null Alternative for Example Analysis 3

From Figure 3.5, alternative A4 clearly has the highest BCR with respect to the null alternative. It would be logical, then, to next compare all of the alternatives to A4. Fortunately, this can be done graphically with relative ease. All that needs to be done is to "move" the origin in Figure 3.5 to alternative A4 which places it at $(10,200)$. Figure 3.6 shows the results.


Fig. 3.6 Alternatives with Respect to Alternative 4, Example Analysis 3
Figure 3.6 shows that alternatives A5 and A6 have negative IBCR's with respect to alternative A4, and are, therefore, less economically desirable than A4. Alternative A1 would have an infinite IBCR, and as discussed previously, is less favorable than A4, as it has lower incremental user benefits. Alternatives A1, A5, and A7 are, therefore, eliminated from further consideration as the "best" alternative.

Alternatives A2, A6, and A3, however, all have IBCR's > 1.00 relative to A4. Of these, alternative A3 has the highest IBCR (slope). The next step is to move the origin of the graph to A3, as shown in Figure 3.7.

When this is done, it is seen that all the alternatives have both a lower IUB and ISC than alternative A3. Their relationship to the $\mathrm{BCR}=1.00$ line is interesting. The IBCR for each alternative is higher than 1.00, even though the points are "below" the $\mathrm{BCR}=1.00$ line. The slope of a line between the origin and any other alternative, however, will be higher than 1.00 ! They fall "below" the line because both IUB and ISC are negative. However, with IBCR's greater than 1.00 , the alternative with the higher IUB is favored, and that is alternative A3 in all cases. Thus, alternative A3 is the most economically viable alternative among the choices given, including the "null" alternative.


Fig. 3.7 Alternatives with Respect to Alternative 3, Example Analysis 3

## Example Analysis 4: Comparison of Highway Alignments

A State highway department is considering the reconstruction of a 10 -mile segment of two-lane highway along one of three different alignments. The data in Table 3.6 is averaged for a 20-year period.

Table 3.6 Alternatives for Example Analysis 4

| Cost Element | Existing | Alternative 1 | Alternative 2 | Alternative 3 |
| :---: | :---: | :---: | :---: | :---: |
| Length (miles) | 10 | 10 | 9 | 8.5 |
| Construction Cost <br> (\$/mile) | --- | 800,000 | $1,000,000$ | $1,050,000$ |
| Annual Mainten- <br> ance (\$/mi/yr) | 10,000 | 5,000 | 5,000 | 5,000 |
| Demand Volume <br> (vehs/day) | 25,000 | 25,000 | 28,000 | 30,000 |
| Road User Cost <br> $(\$ / v e h-m i)$ | 0.135 | 0.110 | 0.097 | 0.092 |

The service life of new roadways may be taken to be 20 years. Interest is 5\%
Solution: As the various alternatives have different values for traffic volume, an incremental method of analysis is required. As most of the data given is in annual form, all costs will be converted to an annual basis. The benefit-cost ratio method will be used for the comparison. A 20-year analysis period will be used, coinciding with the service life of the potential new alignments of the highway.

System costs (SC) may be computed as the annual cost of the capital investment plus the annual maintenance costs.

$$
S C_{i}=\left(C_{i} * A / P_{20,5 \%}\right)+M_{i}
$$

where: $\quad \mathrm{SC}_{\mathrm{i}}=$ annual system costs for alternative $\mathrm{i},(\$ / \mathrm{yr})$
$\mathrm{C}_{\mathrm{i}}=$ capital cost for alternative $\mathrm{i},(\$)$
$\mathrm{M}_{\mathrm{i}}=$ annual maintenance costs for alternative $\mathrm{i},(\$ / \mathrm{yr})$
and $\mathrm{A} / \mathrm{P}_{20,5 \%}$ is the capital recovery factor for 20 years at $5 \%$. It is found from Table 2.5 in Chapter 2 as 0.080243 . Then:

$$
\begin{array}{ll}
S C_{\text {null }}=0+(10,000 * 10)= & \$ 100,000 / y r \\
S C_{a l t 1}=(800,000 * 10 * 0.080243)+(5,000 * 10)= & \$ 691,944 / y r \\
S C_{\text {alt } 2}=(1,000,000 * 9 * 0.080243)+(5,000 * 9)= & \$ 767,187 / y r \\
S C_{a t 3}=(1,050,000 * 8.5 * 0.080243)+(5,000 * 8.5)= & \$ 758,669 / y r
\end{array}
$$

The unit user costs $(\mathrm{U})$ for each alternative may be computed as:

$$
U_{i}=L_{i} * U C_{i}
$$

and annual volume would be computed as

$$
V_{i}=v_{i} * 365
$$

where: $\quad U_{i}=$ unit user cost for 10 miles for alternative $\mathrm{i},(\$ / \mathrm{veh})$
$\mathrm{v}_{\mathrm{i}}=$ daily demand volume, (vehs/day)
$\mathrm{V}_{\mathrm{i}}=$ annual volume, (vehs/yr)
$L_{i}=$ length of alternative $i,(m i)$
$\mathrm{UC}_{\mathrm{i}}=$ unit user cost for alternative i , (\$/veh-mi)
Then:

$$
\begin{aligned}
& U_{\text {null }}=10 * 0.135=\$ 1.35 ; V_{\text {null }}=9,125,000 \\
& U_{\text {alt } 1}=10 * 0.110=\$ 1.1 ; V_{\text {alt } 1}=9,125,000 \\
& U_{\text {alt } 2}=9 * 0.097=\$ 0.873 ; V_{\text {alt } 2}=11,680,000 \\
& U_{\text {alt } 3}=8.5 * 0.092=\$ 0.782 ; V_{\text {alt } 3}=10,950,000
\end{aligned}
$$

Using the Benefit-Cost Ratio Method:

$$
\begin{aligned}
& B C R_{\text {nulvs } 1}=\frac{(1.35-1.1) * 9,125,000}{691,944-100,000}=\frac{2,281,250}{591,944}=3.85(\text { Alt } 1 \text { favored }) \\
& B C R_{1 v s 2}=\frac{(1.1-0.873) *(9,125,000+10,220,000) / 2}{767,187-691,944}=\frac{2,195,658}{75,243}=29.18(\text { Alt } 2 \text { favored }) \\
& B C R_{2 v s 3}=\frac{(0.873-0.782) *(10,220,000+10,950,000) / 2}{758,699-767,187}=\frac{963,235}{-8,488}=-113.48(\text { Alt } 3 \text { favored })
\end{aligned}
$$

From these results, the economic favorability of these alternatives is ranked, from best to worst: A3 $>$ A2 $>\mathrm{A} 1>$ null

### 3.3.4.4 Rate of Return Method for Comparing Incremental Costs

In all previous comparison methodologies, an interest rate is used directly in computing the annual cost or present worth values to be compared. In the rate of return method, an interest rate is determined that results in:

$$
\begin{equation*}
I U B=I S C \tag{3.8}
\end{equation*}
$$

for a given pair of alternatives. Thus, in order to find the interest rate by manual calculations, values of IUB and ISC for several different interest rates must be computed on a trial basis. The equivalence interest rate may then be estimated by interpolation. Of course, it is much simpler to use an excel spreadsheet and the "what if" command to implement the trial and error analysis.

Consider the case illustrated in Table 3.7, in which two alternatives are compared by computing IUB and ISC for various trial interest rates.

At $13 \%$, incremental costs exceed incremental benefits by $\$ 15,000$, and at $15 \%$ incremental costs are lower than incremental benefits by $\$ 2,000$. The break-even rate of interest is clearly between 13 and $15 \%$.

Table 3.7 An Illustration of the Rate of Return Method

| Interest Rate <br> $(\boldsymbol{\%})$ | IUB <br> $\mathbf{( \$ )}$ | ISC <br> $\mathbf{( \$ )}$ | IUB -ISC <br> $\mathbf{( \$ )}$ |
| :---: | :---: | :---: | :---: |
| 7 | 800,000 | $1,050,000$ | $-150,000$ |
| 9 | 980,000 | $1,080,000$ | $-100,000$ |
| 11 | $1,070,000$ | $1,100,000$ | $-30,000$ |
| 13 | $1,100,000$ | $1,115,000$ | $-15,000$ |
| 15 | $1,120,000$ | $1,118,000$ | $+2,000$ |

If a straight-line interpolation is assumed, the equivalence interest rate may be computed as:

$$
i=13+2\left(\frac{15,000}{15,000+2,000}\right)=14.76 \%
$$

The meaning of interest rate, $i$, so computed is as follows: " $i$ " is the rate of return actually earned on the incremental investment in the alternative with the higher system costs.

Thus, if for the problem given, $\mathrm{SC}_{1}=\$ 1,000,000 /$ year and $\mathrm{SC}_{2}=$ $\$ 1,200,000 /$ year, then the solution indicates that the additional $\$ 200,000 /$ year invested in alternative 2 earns a real rate of return of $14.76 \%$. This "return" is measured in terms of user benefits, not direct dollar amounts.

The decision on which alternative is the most economically attractive is based upon the comparison of the computed interest rate, $i$, with the "minimum desirable rate of return for public investments," $i_{D}$. The latter rate is the interest rate that would have been used in computations with other methods of analysis. Then:

- If $\mathrm{i}>\mathrm{i}_{\mathrm{D}}$, then the rate of return on the incremental investment is more than the minimally desirable rate. Thus the additional investment is justified, and the alternative with the higher system costs is preferred.
- If $i<i_{D}$, then the rate of return on the incremental investment is less than the minimally desirable rate. Thus the additional investment is not justified and the alternative with the lower system costs is more economic.
- If $i=i_{D}$, then both alternatives are equally desirable on an economic basis.

In the sample problem, the computed $14.76 \%$ is clearly higher than any reasonable minimum rate of return, indicating that the alternative with the higher system costs would be the more economically desirable.

As in the case of the Benefit-Cost Ratio Method, the rate of return comparison is valid only for the two alternatives considered in the computation. Analysis of multiple alternatives requires a series of paired comparisons.

## Example Analysis 5: A New Rail Terminal

A new rail terminal is to be built in one of two configurations. Based upon the following information, which of the alternatives is more economic if the service life of the terminal is 50 years.

Table 3.8 Data for Example Analysis 5

| Cost Element | Alternative 1 | Alternative 2 |
| :---: | :---: | :---: |
| Capital Cost $(\$)$ | $39,000,000$ | $25,000,000$ |
| Operating Cost $(\$ /$ year $)$ | $1,000,000$ | $1,250,000$ |
| Maintenance Cost $\$ /$ year $)$ | 150,000 | 120,000 |
| User Cost (\$/passenger) | 0.75 | 0.90 |
| Annual Passengers | $30,000,000$ | $30,000,000$ |

Again, because most cost elements are already in an annual format, all costs will be converted to annual form. The problem is solved using the Rate of Return method.

Solution: Because there are only two alternatives, the comparison is relatively straightforward. The first step is to compute the incremental user benefits (IUB) for the alternatives, using annual costs as a basis. As user costs are directly stated as annual amounts, no conversions are necessary. Then:

$$
\begin{aligned}
& U_{2}=\$ 0.90 \\
& U_{1}=\$ 0.75 \\
& I U B=\left(U_{2}-U_{1}\right) * 30,000,000=(0.90-0.75) * 30,000,000=\$ 4,500,000 / y r
\end{aligned}
$$

Because there are no conversions involved, IUB do not depend upon interest the rate.

The second step is to compute the incremental system costs. Capital costs must be converted to an annual equivalent, using an analysis period of 50 years, which is the service life of the project.

The system cost is the sum of the annual maintenance and operations costs, plus the capital cost, annualized over 50 years using the $\mathrm{A} / \mathrm{P}_{50, \mathrm{i}}$. Then:

$$
\begin{aligned}
& S C_{1}=39,000,000 * A / P_{50, i}+1,000,000+150,000 \\
& S C_{2}=25,000,000 * A / P_{50, i}+1,250,000+120,000
\end{aligned}
$$

Then:

$$
I S C=S C_{1}-S C_{2}=14,000,000^{*} A / P_{50, i}-220,000
$$

Setting IUB $=$ ISC, we obtain:

$$
\begin{aligned}
& 4,500,000=14,000,000 * A / P_{50, i}-220,000 \\
& 4,720,000=14,000,000 * A / P_{50, i} \\
& A / P_{50, i}=\frac{4,720,000}{14,000,000}=0.33714
\end{aligned}
$$

A capital recovery factor of 0.33714 for 50 years does not appear in Table 2.5 of Chapter 2. The equation for $\mathrm{A} / \mathrm{P}$ can be used to get the rate of return:

$$
\begin{aligned}
& 0.33714=\frac{i(1+i)^{50}}{(1+i)^{50}-1} \\
& i=33.7 \%
\end{aligned}
$$

Because the number of years is high (50), the interest rate is almost equal to the capital recovery factor ( $\mathrm{A} / \mathrm{P}$ ).
What this means is that the additional investment $(\$ 14,000,000)$ in alternative 1 over alternative 2 is effectively earning a return of $33.7 \%$. This is doubtless higher than any reasonable expectation, and dictates that alternative 1 is the most economic of the two alternatives presented.

## Example Analysis 6: Rail Terminal - Another View

The preceding problem was solved in the standard manner from the planner's point of view. User benefits were matched against the cost to purchase those gains. Clearly, in the overall sense, this is correct.

The position and viewpoint of the rail terminal operator, however, is quite different. The operator cannot put "user benefits" in the bank or pay its employees with them. The railroad's concern is for the real money items - capital cost and operating/maintenance costs. If the railroad were to conduct its own economic analysis, user aspects would be ignored. The issue would be simply is the lower operating cost of Alternative 1 worth the additional capital investment? The railroad's analysis would be different.

The incremental user benefits (IUB) would be " 0 ," as the railroad operator would be ignoring these. The incremental system costs (ISC) are the same as in the first analysis. Then, setting IUB $=$ ISC:

$$
\begin{aligned}
& 0=14,000,000 * A / P_{50, i}-220,000 \\
& A / P_{50, i}=\frac{220,000}{14,000,000}=0.015714 \\
& i=-0.9 \%
\end{aligned}
$$

Clearly, the railroad would lose money if it went with Alternative 1, and would choose Alternative 2.

In this oversimplified example, an often-occurring result is clear: while the overall considerations, including user costs, dictate the choice of alternative 1 , the railroad would opt for alternative 2 on the basis of real-world monetary considerations. The true planning issue may then be reduced to one of subsidy - will the government body having jurisdiction to subsidize the more costly alternative to provide users with economically justified benefits?

### 3.4 Defining Inputs to Economic Alternative Analyses

Engineering economy studies are no more than elaborate applications of the banking principles discussed in the Chapter 2, used to determine the optimal choice among alternative projects, in economic terms. The transportation planner and engineer use these techniques to determine the most economic investments for the limited funds available.

The range of problems that may be addressed by these techniques is extremely broad - from single mode comparisons, such as consideration of alternative highway alignments, pavement types, rapid transit vehicle designs, etc., to broad system comparisons, such as the implementation of a rapid transit system vs. highway improvements and increased bus service. The scale and level of detail with which a problem is investigated varies similarly. Smaller problems may be investigated in great detail, while system wide considerations are usually investigated only in the broadest terms.

The principal issue in performing an engineering economy study in transportation is the definition of the problem. Solving it is reasonably straightforward. The sections below discuss and illustrate the various aspects of defining and conducting an alternative economic analysis.

### 3.4.1 Selection of an Analysis Period

A principal ingredient in any alternative analysis is the length of the analysis period selected. In general, the analysis period should encompass the service lives of all components being considered. This, while a good rule in theory, is often
difficult to apply in practice. First of all, the service life of different components of a given alternative may vary considerably: highway ROW has a virtually limitless service life, while pavements last $10-20$ years; a rapid transit facility may have a service life of 100 years, but rolling stock lasts for only 30 years. Further, the service lives of the various alternatives being considered are generally also different: a highway alternate may last 30 years while a rail alternative lasts 100 or more.

Another factor must be considered as well. It would be foolish to analyze a problem for a period beyond our ability to project the facility's utility. A highway may be physically maintained for 100 years or more through periodic reconstruction. However, it is not at all certain what function it would serve in 100 years, given the rapid advancement in transportation technology and changes in urban land use and physical configuration. Consider the problem of a transportation planner in 1900 trying to project the utility of a then-conceived improvement 100 years hence.

Analysis periods, therefore, are chosen in concert with three major considerations:

- The service lives of component elements of the alternatives being considered;
- The period over which the facilities involved can be expected to serve a useful function;
- The period over which demand volumes can be reasonably forecast.

The latter point recognizes the limited range over which most traffic forecasting techniques can be applied with even gross accuracy or reliability.

In general, these considerations lead to the assumption of analysis periods of 25 to 30 years for highways, $10-15$ years for bus systems, and $30-50$ years for rail systems. In the selection of an analysis period, two key principles must be observed:

- The analysis period is selected for a particular problem - and must be applied to all alternatives considered in the problem;
- The use of different analysis periods for different alternatives in the same analysis is NOT permissible.

Where the service lives of various components extend beyond the analysis period, an accounting of their residual value, discussed earlier in this chapter, at the end of the period must be included in the analysis. Note that the analysis period might include the planning period and thus the analysis period could be longer than the project life.

It must be emphasized that the selection of an exact analysis period is subject to judgment. This discussion can focus the considerations that should be taken into account in making a selection, but no firm criteria or standards can or should be formulated.

### 3.4.2 Selecting an Interest Rate

The choice of interest rate used in an analysis can be extremely important. In general:

- Low interest rates favor alternatives with large capital investments, and costs that occur in the more distant future.
- High interest rates favor alternatives with low capital investments and costs occurring in early years of the analysis period.

The cost of capital is also called the hurdle rate, the discount rate, or the minimum attractive rate of return (MARR), that is, it is the minimum rate that is acceptable for an investment. The MARR will be different for different organizations. Private companies normally require a higher MARR, for instance, compared to a public agency.

It is not correct to simply use the "market" rate of interest in analyses. The market rate of interest reflects three factors:

- The real "opportunity cost" of invested capital. Opportunity cost means that you are losing the opportunity to invest this money elsewhere, in the market;
- An allowance for expected inflation; and
- Risk.

If alternative analyses are carried out in terms of "constant dollars", that is, ignoring inflation, then the interest rate used should reflect the real cost of capital and risk (to the investor), but should not provide for expected inflation. The rate that is used for government spending, and which is recommended by FHWA, is usually based on US government treasury bonds with maturities similar to the length of the analysis period used in the economic analysis.

The approach to risk is more complex. "Risk" is generally associated with the investor's probability of being repaid on time and in full, with the agreed-upon interest being paid throughout the life of the investment. In private investments, one does not only choose an investment that will maximize their returns; rather they adjust this decision based on the risk involved. In public investments, the investor is the taxpayer, who is repaid not in cash, but in services received from the facilities built and operated. Risk to the taxpayer, therefore, is uncertainty in the amount and/or value of services received and in the costs to provide them. Two approaches may be taken to account for this uncertainty:

- A range of benefit and cost estimates may be made, with probabilities assigned to the occurrence of the various estimates.
- The interest rate may be varied to reflect the level of risk involved in cost and benefit estimates.

The cost of capital is different for private industry than it is for public agencies, and is itself subject to market fluctuations. For private industry, the cost of risk
would more commonly be included. There are varying theories as to whether risk should be included in the analysis for public investments that can be found in the literature [2-5].

Because alternative economic analyses are sensitive to interest rates, it is often useful to perform such analyses at several interest rates. When comparative analysis is performed, however, the interest must be constant for the comparison. In rare cases, it may be useful to vary the interest rate used for different alternatives in a given analysis to reflect varying risk among the alternatives. If this is done, the difference in such rates should not exceed $0.5 \%$, and a second analysis using a constant rate should be performed for comparison. There is much literature for specific studies dealing with the sensitivity of economic analyses to interest rates $[6,7]$.

As stated earlier, inflation is added to the market rate of interest, but is generally not included in transportation analyses. The primary argument for ignoring inflation is that over a constant analysis period, it would affect the costs and benefits of the various alternatives similarly. Additionally, trying to predict future prices would add unnecessary uncertainty [9].

### 3.5 Closing Comments

This chapter has presented the methodologies for setting up and performing engineering economic analyses, all of which are based upon the basic banking principles of Chapter 2. The methods of analysis have been illustrated with simplified examples to emphasize major points. In more complex situations, however, the goal of the economic analysis should not be lost or less visible: that is, get the most for the money that is being invested. Applications may be extremely complex, but the principles remain simple and straightforward. The goal remains optimal economy, and the computational tools remain no more complex than the banking formulae introduced.

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## Problems

## Problem 3.1

An interim bus service is to have a life of 13 years, with capital investments required: (1) initially - $\$ 1.2$ million, and (2) at the end of year 8-\$1.2 million. The capital costs are for new and replacement buses, which have a service life of 8 years. All other expenses amount to $\$ 450,000$ annually. What is the annualized cost of the service, over its useful life? Interest is $5 \%$.

## Problem 3.2

A temporary bypass road is constructed at a cost of $\$ 0.8$ million to divert traffic around a reconstruction site. The design life of the temporary road is 10 years. The bypass is used for 3 years and destroyed at a cost of $\$ 0.15$ million. What is the annual cost of the bypass at an interest rate of $8 \%$ ?

## Problem 3.3

You have just learned that ABC Corporation has an investment opportunity that costs $\$ 35,000$ and eight years later pays a lump-sum amount of $\$ 100,000$. What interest rate per year would be earned on this investment?

## Problem 3.4

It is estimated that a certain piece of equipment can save $\$ 6,000$ per year in labor and materials costs. The equipment has an expected life of five years and no salvage value. If the company must earn $8 \%$ annual return on such investments, how much could be justified now for the purchase of this piece of equipment?

## Problem 3.5

Maintenance costs for a small bridge with an expected life of 60 years are estimated to be $\$ 1,000$ each year for the first five years, followed by a $\$ 10,000$ expenditure in the $15^{\text {th }}$ year and a $\$ 10,000$ expenditure in year 30 . If interest is $5 \%$ per year, what is the equivalent uniform annual cost over the entire 60 -year period?

## Problem 3.6

A traffic control system for 40 intersections is based upon central computer control. The system is estimated to have 20-year useful life and no scrap value. The cost elements are:
Engineering and field installation:
\$1.2 Million
Computer facilities:
Communication leases:
\$0.3 Million
Maintenance and operation:
\$200,000/year
What is annual cost of system? What percent of annual cost is attributed to the communications expenses? Interest is 5\%.

## Problem 3.7

A road construction project is estimated to have the following expenses:
Initial engineering, right of way acquisition, and construction: \$50,000,000
Major rehabilitation at end of years 15 and 30: $\quad \$ 18,000,000$ ea.
Annual maintenance and operations: $\quad \$ 6,000,000 / \mathrm{yr}$
Lost annual income due to removal of property from tax base: $\quad \$ 3,000,000 / \mathrm{yr}$
What is present worth of total costs, given a 45-year analysis period and interest of $3 \%$ ?

## Problem 3.8

A sinking fund for a bridge replacement is established. The cost is estimated at $\$ 20$ Million, and the present bridge has a thirty year expected life.
a. What is the annual payment to the sinking fund? Assume interest $=8 \%$.
b. At the end of year 13 , what is the balance in the sinking fund?
c. If after year 13 , interest changes to $7 \%$ for the remaining 17 years, how much must your annual payments be in order to still have $\$ 20$ million at the end of thirty years?

## Problem 3.9

A fare collection system has the following annual budget:

Labor - One position ( $24 \mathrm{hrs} /$ day): 5 people @ $\$ 35,000 / \mathrm{yr}$ (including benefits)
Maintenance and Operations: $\$ 15,000 / \mathrm{yr}$

Assume there is an automated system which will require:

Initial capital investment with 20-year service life: \$X
Labor:
\$35,000/yr
M\&O: $\quad \$ 20,000 / \mathrm{yr}$

At $4 \%$ interest, what value of " $X$ " will make the systems equal in annual cost?

## Problem 3.10

How much can a toll bridge authority afford to spend to replace a manned toll booth with an automated device? It costs $\$ 40,000$ per year to man the toll booth, interest is $7 \%$, and the service life of an automated device is expected to be 15 years.

## Problem 3.11

A town is considering two bids for paving. One bid is $\$ 3$ per square yard, with an expected service life of 5 years. The other is $\$ 6$ per square yard, with an expected service life of 10 years. If interest is worth $6 \%$, which bid should the town accept?

## Problem 3.12

A bridge costs $\$ 1,000,000$ to build, and must be fully reconstructed every 30 years. When first built, a single sum of money is invested at $8 \%$ to provide for an infinite number of renewals. How much must be invested?

## Problem 3.13

A viaduct costs $\$ 100,000$ to build and has a service life of 50 years. It also costs $\$ 1,000$ per year to maintain. If interest if $5 \%$, what is the capitalized cost of the viaduct?

## Problem 3.14

An aluminum sign post lasts 15 years compared to 6 years for a wooden post. Wooden posts cost $\$ 50 /$ year to maintain and $\$ 10$ to dispose of when their service expires. Aluminum posts cost $\$ 5$ per year to maintain, and $\$ 25$ to dispose of when replaced. If interest is $3 \%$, and if wooden posts cost $\$ 100$, how much can be justifiably spent to purchase aluminum posts?

## Problem 3.15

Asphaltic concrete pavement costs $\$ 10,000$ per lane-mile and has a service life of 18 years. Portland cement reinforced concrete pavements cost $\$ 25,000$ per lanemile and have an expected service life of 25 years. Maintenance costs for asphaltic concrete pavements are $\$ 4,000$ per lane-mile per year, and for Portland cement concrete pavements $\$ 1,200$ per lane-mile per year. If interest is $5 \%$, which type of pavement is economically more desirable?

## Problem 3.16

A railroad yields a net income of $\$ 400$ million per year. What is the maximum purchase price which can be offered and still attain a rate of return of $10 \%$ ?

## Problem 3.17

In order to meet its needs, a company has $\$ 2.4$ million tied up in inventory. Further, record keeping, control, and operations cost related to the inventory is $\$ 200,000$ annually. What is the annual cost of the total inventory activity? Use interest $=3 \%$.

## Problem 3.18

Two alternatives are being considered for warehousing and distributing canned foods in a sales region. The cans come in cartons of 24 cans per carton. The two alternatives are:

- Alternative A. Have your own distribution system. The administrative costs are estimated at $\$ 43,000$ per year, and other general operating expenses are $\$ 0.009$ per carton. A warehouse will be purchased for $\$ 300,000$.
- Alternative B. Sign an agreement with independent distributor, which asks payment of $\$ 0.10$ per carton distributed.
Assume a study period of 10 years, and that the warehouse can be sold at the end of this period for $\$ 200,000$. Which alternative should be chosen, if they expect the number of cartons to be distributed will be 600,000 per year?


## Problem 3.19

The construction costs and annual maintenance costs of two alternatives for a canal are given below:

|  | Alternative A |  |
| :--- | :--- | :--- |
| Construction Cost | $\$ 25,000,000$ |  |
| Annual Maintenance | $\$ 3,500,000$ | $\$ 50,000,000$ |
|  | $\$ 2,000,000$ |  |

(a) Which alternative would you recommend, using 5\% interest.
(b) What is the capitalized cost just of maintenance for the alternative you choose?

## Problem 3.20

An existing 2-lane highway may be rebuilt as a) an improved 2-lane highway; or (b) a new 4-lane highway. The segment is question is 5 -miles long and reconstruction would take place along the original alignment. The traffic volume for either alternative, as well as for the existing facility is expected to be $\$ 25,000$ vehs/day. The following is known:

| Capital Costs | Alternative |  |  |
| :--- | :---: | :---: | :---: |
|  | Existing | Alt (a) | Alt (b) |
| Grade \& Structures <br> $(\mathrm{n}=40 \mathrm{yrs})$ | --- | $\$ 5 \mathrm{M}$ | $\$ 8 \mathrm{M}$ |
| Pavement $(\mathrm{n}=20 \mathrm{yrs})$ | --- | $\$ 600,000$ | $\$ 1,350,000$ |
| Maintenance/yr | $\$ 65,000$ | $\$ 20,000$ | $\$ 26,000$ |
| User costs $(\$ / \mathrm{veh}-\mathrm{mi})$ | 165 | 145 | 102 |

Using an analysis period of 40 years, and interest of 5\%, rank the alternatives using present worth analysis, annual cost analysis, rate of return analysis, and benefit/cost ratio analysis.

## Problem 3.21

A bus company is considering the purchase of buses from three manufacturers. Due to considerations of bus size and maintenance factors, the number of buses needed to provide similar service varies from manufacturer to manufacturer. To provide for the same service, the bus company must make one of the three following purchases:

| Cost Item | Manufacturer A | Manufacturer B | Manufacturer C |
| :---: | :---: | :---: | :---: |
| No. of Buses Needed | 50 | 45 | 57 |
| Service Life of buses | 12 years | 10 years | 15 years |
| Cost / Bus (\$) | 90,000 | 92,000 | 81,000 |
| Maintenance/bus/yr <br> $(\$ / \mathrm{yr})$ | 1,800 | 2,200 | 1,820 |
| Operating cost/bus/yr <br> $(\$ / \mathrm{yr})$ | 70,000 | 80,000 | 64,000 |

Using an interest rate of 5\%, rank the alternatives in order of decreasing economy.

## Problem 3.22

A project will cost $\$ 5 \mathrm{M}$ to build immediately. Revenues will occur 10 years in the future. Is the $\mathrm{B} / \mathrm{C}$ ratio for this project better when interest is $3 \%$ or $8 \%$ ?

## Problem 3.23

There are three distinct and mutually exclusive alternatives for a transportation corridor project. All benefits and costs shown are relative to the existing or "no build" condition.

The project has a service life of 40 years. At interest rates of $3 \%, 6 \%$ and $9 \%$, which alternative is chosen, and at what is the $\mathrm{B} / \mathrm{C}$ ratio in each case.

| Alternative | Benefits | Costs |
| :---: | :---: | :---: |
| Transit System | \$40 M annually | \$200 Million initial <br> \$25 Million annually |
| Highway Route 1 | \$40 Million annually | \$500 Million initial \$10 Million annually |
| Highway Route 2 | \$30 Million annually | \$300 Million initial \$6 Million annually |

## Problem 3.24

A new transit company needs $\$ 108$ million in initial capital funding, and promises a return to the investor of $\$ 11$ million a year (net, after operating expenses) at the end of each of 20 years, but the first payment is made at the end of Year 2 (and the last at the end of Year 21). What is the anticipated rate of return?
If the payments at the end of years 12 and 13 are not made, but the payment schedule is continued to the end of Year 23 so that the full twenty payments are made, what is the rate of return?

What additional "balloon payment" is needed at the end of Year 24 so that the original rate of return is realized?

## Problem 3.25

(a) An investment costs $\$ 3,400,000$ now. Revenues are $\$ 800,000$ net at the end of each year from the end of year 1 to the end of year 5. Net revenues of $\$ 1,200,000$ per year are made from the end of year 8 to the end of year 15. What is the rate of return on this investment?
(b) If you want to add $3 \%$ interest to that answer, what additional income would you need to recover at the end of year 15 ?

# Chapter 4 <br> The Costs of Transportation for Alternative Economic Analysis 


#### Abstract

This chapter presents an overview of the various costs and benefits usually associated with a transportation project or improvement. It shows how such data should be viewed and utilized in comparative economic analyses.


The words "costs" and "benefits" carry enormous colloquial connotations with them. In general, it is assumed that costs are negative, and to be avoided whenever possible, while benefits are positive events that should be vigorously pursued.

In the context of an engineering economy study of proposed transportation projects, however, both costs and benefits are ultimately the same thing. The benefits of a transportation project are usually quantified as a reduction in the costs of the transportation system in one or more of its myriad elements. Because of this, this text will refer to all elements as "costs."

In any transportation project, costs can be broadly categorized into one of three basic categories:

User Costs
System Costs
Indirect Costs
In general terms, user costs are directly borne by the user of the transportation system. System costs are borne by the private or public entity (or entities) responsible for planning, constructing, maintaining, and operating the transportation facility or system under consideration. Indirect costs refer to impact elements that do not directly affect only users or system owners, but which affect a broader segment of the population. Such costs include, but are not limited to, environmental and social impact costs. The latter may be extremely difficult to quantify in monetary terms, but some progress has been made in addressing these issues in recent years.

The subsections that follow discuss each of these basic elements in more detailed terms.

### 4.1 User Costs

User costs are those elements of cost that are directly borne by the user of the transportation facility. In terms of highway systems, the user bears the following expenses:

```
Purchase or lease of the vehicle
Maintenance of the vehicle
Operating costs of the vehicle (primarily fuel)
Crashes
Travel time
```

Some of these, such as the purchase of the vehicle, or the purchase of fuel, are out-of-pocket costs that are very obvious to facility users. Others, such as the cost of accidents, are felt indirectly through the cost of insurance, vehicle repairs, and medical bills.

In public transportation systems, the user only bears two costs directly:

```
Fare
Travel time
```

The primary difference in the cost structure of auto transportation vs. public transportation is that road vehicles are individually owned and operated by users. In public transportation, vehicles are a part of the transportation system, provided by the owner/operator of the system.

In terms of motor vehicle costs, some elements are fixed (or time-dependent), while others are dependent upon mileage and/or speed. Fixed (or timedependent) motor vehicle costs include:

A portion of depreciation (due to aging of the vehicle)
Interest or loan charges
License and registration fees
Liability and/or collision insurance
Mileage dependent costs vary with extent of vehicle usage, and include:

```
Fuel consumption
Oil consumption
Tire wear
Maintenance
A portion of depreciation (due to vehicle usage)
Crash costs
```

Speed-dependent costs change with the speed of the vehicle, and include the following, many of which are also mileage dependent:

```
Fuel consumption
Oil consumption
Tire wear
Travel time of drivers and passengers
```

In terms of public transportation systems, the fare varies with the number and (sometimes) length of the trip, while travel time is dependent upon speed and schedules. In modern public transportation systems, fares can be quite complex. Fare systems range from a flat fare per use, to fares that vary depending upon the origin and destination (usually based upon trip length), and/or the time of day the
trip is taken. In the latter case, higher rates may be charged during the peak periods to encourage users to travel at other times.

Travel time is the most critical direct user cost element, as the benefits of most transportation improvements include a reduction in this element. Over the years, the perception and use of travel time costs has changed in economic comparisons. In the early days of highway economy studies, a flat rate for the value of travel time was applied. Later, some variation was introduced to reflect the amount of time saved, i.e., saving 5 minutes for each traveler was not worth the same unit value as saving 1 hour for each traveler. Current methods are based primarily on prevailing average wages, and are differentially applied to different categories of users. Adjustments are also made to reflect levels of congestion, resulting in what is essentially a "nuisance surcharge" for being stuck in traffic or other delays.

Crash costs are also an important element. Many projects are developed specifically to provide for increased safety, and the impact of these on crash costs must be assessed to do a proper economic analysis of alternatives. Cost estimates are generally based upon current information involving accident occurrence, insurance payouts, and other measurable costs.

### 4.2 System Costs

System costs are those elements that are borne by the owner (private or public) of the transportation system.

For highways, this is generally a public agency such as a state or local highway or transportation department, or a separately-established toll authority with jurisdiction over one or more toll facilities. State and local transportation agencies get their funding from a variety of state and local taxes, some directed specifically at users, others directed at the general population. While the federal government provides money to state and local transportation agencies, all such funds emanate from the federal Highway Trust Fund, which derives its income from federal user excise taxes, primarily the federal fuel tax.

There is some private ownership involved in highway transportation in the United States (See Chapter 7). Other privately-owned facilities are parking garages or lots.

For highway transportation systems, there are a wide variety of system costs, which may generally be categorized as capital or operating and maintenance costs. Capital costs are those involved in planning, designing, and constructing highway facilities, while operating costs are ongoing expenses incurred to keep the facilities safely operational. Capital costs for highway facilities include:

Acquisition of rights-of-way.
Planning and design expenses for new or rebuilt facilities, including all elements (roadway, roadside, drainage, etc.).
Construction expenses for new or rebuilt facilities.
Purchase and installation/replacement of control devices such as traffic signals, traffic signs, and highway lighting.
Purchase and installation/replacement of detectors as part of a traffic information network and/or control center.

Operating costs for highways include:
Roadway maintenance and periodic repaving.
Traffic markings and periodic renewal.
Roadside maintenance, including mowing, sign maintenance, etc.
Power costs for lighting and signal systems.
Operating costs of traffic information and/or traffic control centers.
Ice and snow treatment and/or removal.
Administrative expenses.
For public transportation systems, both private and public ownership is common. Virtually all heavy and light rail systems are owned and operated by public agencies or public authorities. Bus systems can be either privately owned or publicly owned, but in either case, public subsidies are required to keep them in operation in most cases.

The major difference between highway and public transportation systems is that in public systems, the vehicles are owned and operated by the system, not the individual user. This introduces a major expense to the system side of the equation: operating labor. In public transportation, all personnel involved in operations, including bus drivers/motormen, conductors, fare booth and information kiosk personnel, cleaners, and in some cases, dedicated police units, are all part of the system costs. Further, since the system owns the vehicles, the capital expenses to purchase them, and the maintenance costs of keeping them running, are system costs.

For rail systems, the construction, maintenance, and operation of rights of way and stations are considerable expenses as well. Bus systems, however, operate primarily on existing state and local street systems. If privately owned, bus operators may pay taxes to contribute to state and local highway expenditures. Public operators would not pay such taxes.

Capital expenses borne by public transportation owners/operators include:
Acquisition of rights-of-way for dedicated track/roadbeds.
Construction of dedicated traveled ways (tunnels, structures, trackbeds, exclusive busways, etc.)
Construction of maintenance and/or operational facilities.
Construction of special control systems.
Purchase of vehicles (rail cars, buses, specialized maintenance vehicles, etc).
Public transit operating costs include:
Maintenance of tracks, tunnels, structures, exclusive busways, stations, control and information systems, etc.
Maintenance of vehicles.
Fuel, oil, and/or power.
Maintenance labor costs (salaries plus benefits).
Operating labor costs (salaries plus benefits).
Supervisory labor costs (salaries plus benefits).
Advertising and other public information services.

Insurance costs.
Costs of project financing.
Administrative costs, including labor
The operating costs of public transportation systems are generally grouped into categories based upon the principal unit of measure that controls the expense. The units generally include vehicle-miles of operation, vehicle-hours of operation, the system size (measured as the number of vehicles owned, or the number in peak period operation), and the number of annual revenue-passengers.

Most maintenance costs are directly related to the vehicle-miles traveled within the system. Operating labor, however, is more closely related to the number of vehicle-hours operated. Administrative and supervisory expenses are generally associated with the size of the system, while insurance costs are related to the number of annual revenue-passengers served.

### 4.3 Indirect Costs

Indirect costs of transportation systems are the most difficult to accurately assess, because the magnitude of the impact may be difficult to estimate, and because placing a monetary value on such impacts is extremely difficult, and often subject to dispute. Nevertheless, major transportation system changes are often controlled by these indirect costs. For example, New York City made a number of major transportation decisions that were dominated by indirect costs:

1. In the 1940 's and 1950 's, hundreds of miles of elevated rail rapid transit facilities were dismantled primarily due to their impact on both the aesthetics of such systems on surrounding properties, as well as the impact of noise and air pollution. They were replaced primarily by bus systems with significantly lower capacity than the rail systems that were eliminated.
2. The massive Westway Project, which included complete renovation of the Hudson River waterfront, was eventually derailed by a succession of legal battles lasting more than a decade, mainly involving the environmental impacts on the reproductive cycle of Striped Bass in the Hudson River!
3. The Second Avenue Subway, which has been planned, partially designed, and even partially constructed several times, beginning in the early 1930's, is once again under construction. One major issue has been the environmental impacts of the construction of this facility.

These and other major transportation system decisions have been made on the basis of presumed or quantified indirect impacts of the proposed projects. In the three NYC cases noted, there is little question that tearing down the elevated rail lines, not building Westway, and not building the Second Avenue Subway all had major negative impacts on the traveling public. In each of these cases, however, the indirect effects of these transportation facilities were viewed as negative costs that outweighed the positive benefits to the traveling public.

In 1956, when the authorization and funding of the National System of Interstate and Defense Highways was passed by Congress, highway agencies had little opposition to the construction of this massive system of highways that is still referred to as "the largest public works effort in the history of the world." The nation was in the midst of a post-Korean War robust economy, car ownership and the flight to suburbia were surging, over-the-road truckers were seeking public subsidy in the form of new highways to help them compete with railroads for freight, and the right to eminent domain was unchallenged.

By the mid-1960's, the damage done to many urban areas by the construction of the Interstate System was becoming obvious. Using traditional economic analysis, new interstate highways were invariably built through the poorest sections of many cities. These highways, which inserted significant barriers into the neighborhoods through which they crossed, divided communities and led to serious deterioration of their local economies. In 1968, the Federal Aid Highway Act of that year introduced the requirement for two public hearings on any federally-funded highway project. The second, added for the first time, occurred after the final design of the proposed highway was completed. At this point, the highway project would be a year or less away from construction, and (as the final route was known) everyone knew whose properties and homes were to be dislocated or affected. This second public hearing became a focal point around which community opposition to projects could be organized.

In the 1970 's, environmental concerns were also on the rise, and the introduction of requirements for Environmental Impact Statements on every major project provided yet another forum for opposition to highway and other transportation projects.

The negative indirect costs of transportation projects often include:
Dislocation and relocation of residential and business properties.
Creation of neighborhood barriers with negative effects on the social fabric and economic characteristics of the community.
Air pollution.
Noise pollution.
Aesthetics.
Temporary disruptions to travelers and others during construction.
In recent times, significant effort goes into planning and design of transportation facilities in an effort to minimize or mitigate some of these negative indirect costs. Noise barriers are now common along major highway facilities, and designs attempt to be aesthetically pleasing, properly integrated into its surroundings. The creation of community barriers can be minimized with proper location of the facility. Air pollution is a tricky business, but keeping traffic moving often produces beneficial results.

There are, to be sure, indirect benefits of transportation facilities as well. These are, in general, more difficult to quantify. They include general increases in mobility and accessibility. Improved mobility allows for economic regionalization, with increased competition amongst a broader group of players in any given
market area. Improved accessibility increases the value of land. Residents have a broader set of employment, shopping, recreational, and cultural opportunities due to improved mobility. Such secondary impacts, however, are not as obvious as the indirect negative impacts of transportation facilities.

Economic analysis now tries to quantify some of the more significant indirect costs of transportation projects. Many, however, still lie outside the ability to incorporate them directly into an economic analysis of alternatives, and will continue to affect transportation project outcomes through the political process.

There are a number of references which provide comprehensive discussions of cost classifications for use in engineering economic analysis. AASHTO's User and Non-User Benefit Analysis for Highways (1) categorizes costs as user benefits (treated as a reduction in user costs) and project costs. It also provides the most comprehensive reference for estimating the various elements of cost in economic comparisons (in terms of process).

The Federal Highway Administration (2) classifies costs as agency costs, user costs/benefits from work zones, user costs associated with facility operations, and externalities. The Minnesota Department of Transportation (3) classifies Highway User Benefits as travel time and operating cost reductions, and Costs as capital costs, major rehabilitation costs, and routine annual maintenance costs.

No matter what classification system is used, however, it must be applied consistently across all possible alternatives. The objective is always to include virtually all of the cost elements that will be affected by a particular project or set of project alternatives.

### 4.4 The Costs of Transportation

For any given analysis of alternative transportation plans, all elements of cost must be identified and quantified. The process is best done locally, using facts and figures from relevant local/state agencies. It is impossible to simply provide a central source for transportation cost information. Costs vary widely from region to region, and cost estimates must reflect the specific conditions which exist for any project alternative.

Nevertheless, subsequent sections of this chapter do provide a variety of transportation cost data from various parts of the country and various agencies. They are intended to be illustrative only, and are not presented for actual application in any given comparison of project alternatives.

Because of the nature of published and unpublished cost data, many of the costs cited reflect different base years. Any cited cost can be adjusted either forward or backward in time to mitigate the results of inflation (4):

$$
\begin{equation*}
\$_{\text {base year }}=\$_{\text {data year }} *\left[\frac{\text { Price Index } \text { base year }^{\text {Pr ice Index }} \text { data year }}{}\right] \tag{4.1}
\end{equation*}
$$

where:
$\$_{\text {base year }} \quad=$ Year for which a cost is needed.
$\$_{\text {data year }} \quad=$ Year represented by the data.
Price Index $=$ selected index used to estimate the value of inflation.
There are a wide variety of indexes that can be applied to equation $4-1$, including the Consumer Price Index (CPI), which is often a useful tool for economic analyses.

Table 4-1 shows the average annual urban CPI from 1980 to 2012. Note that an index of 100 reflects costs in 1982 - 1984. It should be noted that CPI values are also available for different sectors of the economy, as well as for different regions, and specifically for 84 defined urban areas.

Table 4.1 Average National Urban CPI, 1980 - 2012

| Year | CPI | Year | CPI |
| :---: | :---: | :---: | :---: |
| 1980 | 82.4 | 1996 | 156.9 |
| 1981 | 90.9 | 1997 | 160.5 |
| 1982 | 96.5 | 1998 | 163.0 |
| 1983 | 99.6 | 1999 | 166.6 |
| 1984 | 103.9 | 2000 | 172.2 |
| 1985 | 107.6 | 2001 | 177.1 |
| 1986 | 109.6 | 2002 | 179.9 |
| 1987 | 113.6 | 2003 | 184.0 |
| 1988 | 118.3 | 2004 | 188.9 |
| 1989 | 124.0 | 2005 | 195.3 |
| 1990 | 130.7 | 2006 | 201.6 |
| 1991 | 136.2 | 2007 | 207.3 |
| 1992 | 140.3 | 2008 | 215.3 |
| 1993 | 144.5 | 2009 | 214.5 |
| 1994 | 148.2 | 2010 | 218.1 |
| 1995 | 152.4 | 2011 | 224.9 |
|  |  | 2012 | $229.4^{*}$ |

*Preliminary estimate based upon 11 months of data.
(Source: U.S. Department of Labor Statistics, Consumer Price Index, All Urban Consumers, Washington D.C., December 2012).

For example, if a database provides information that a particular highway improvement will cost $\$ 6,500 / l a n e$ mile in 2001, the 2012 cost could be estimated as:

$$
6,500 * \frac{229.4}{177.1}=\$ 8,419.54 / \text { lane mile }
$$

It is, of course, preferable to have current cost information from relevant local and regional agencies and contractors to have a more accurate assessment of the cost of a particular improvement.

Once again, it is important to emphasize that the cost data presented herein are intended to be illustrative, and should not be used directly to conduct a thorough engineering economic analysis of a set of transportation alternatives.

### 4.5 The Cost of Travel Time

As noted previously, the single largest benefit of many transportation improvements is a savings in travel time spent by travelers in executing a defined set of
trips. This might be accomplished by providing an improved alignment of an existing highway that makes trips shorter and/or allows them to be made at higher average speeds. It might be accomplished by providing a new public transportation link that provides improved travel times compared to existing highway and transit alternatives. The trick is to turn savings in travel time into a measurable benefit in dollars and cents.

### 4.5.1 Issues Affecting How Travel Time Is Viewed

The value of travel time is a complex issue. While every case is somewhat unique, there has been some consensus over the years on how to approach the issue of travel time in economic analyses.

Travel time is the most noticeable investment a traveler makes in pursuing a particular trip. If it is a long trip involving either public or private transport, out-of-pocket expenses such as fares, tolls, fuel, etc. would also be on the traveler's mind. Mostly, however, travelers are intensely aware of the time they invest, and make many transportation choices based primarily on this factor.

The old adage "time is money" is the basis for the evaluation of transportation system benefits - most of which come in the form of reduced travel time. There are many ways in which the value of travel time can be studied, from simple or complex surveys of travelers to the direct observations of traveler choices for different modes and routes for a well-defined trip. In recent years, most studies of travel time cost conclude that the monetary value of time is most directly related to the traveler's wage, and in the case of commercial vehicle drivers, their total compensation, including fringe benefits.

This is a very simple concept, but is complicated nevertheless. Every individual who travels has a different wage (or total compensation), but systematic economic analysis demands that some representative value be used. The concept of wage-based values of travel time also suggests that those with higher salaries value their travel time more highly - an understandable, but somewhat disturbing, concept. Fortunately, in most economic analyses, the same average compensation levels are assumed for all of the alternates.

Numerous studies, however, have pointed out the many complications that make the monetary valuation of travel time difficult (5-10):

- Some studies have shown that trip purpose affects the value users assign to their travel time; on the way to work, the value of time is higher than on a weekend trip to the beach.
- Some studies show that drivers clearly place a higher cost on travel time in congested or otherwise uncomfortable conditions, and that travel time related to specific delays is also valued at higher than base levels.
- Most economic analyses rely on increments in travel time based upon transportation improvements. The value of the travel time saved may also depend upon the amount of time saved. If each traveler is saved one minute of travel for a one-hour trip, its value is almost nothing. If each driver saved 20 minutes of travel time for a two-hour trip, its value would be much greater. One minute has virtually no "opportunity cost" value - i.e., there is virtually nothing
functional for which the one minute saved would be useful. The 20-minute savings has more value, because there are more "opportunities" for its alternate use by travelers. In effect, the monetary value of travel time may be related to both the absolute amount of travel time saved, and/or to the proportional reduction in travel time.
- Some travel time may have a very low, or even negative value, as travelers enjoy the experience. Examples might include a leisurely drive through a national park, or bicyclists using a bike path on a pleasant, sunny day.
Despite the complications, however, standard approaches have been, and must continue to be developed.


### 4.5.2 Estimating the Value of Travel Time

As previously noted, there is general consensus that the monetary value of travel time is related to prevailing wage rates for various categories of users. AAHSTO (1) relates the value of most car driver, car passenger, and public transit passenger travel time to a percentage of the prevailing wage rate. Travel time values for professional drivers of commercial or transit vehicles are valued as a percentage of total compensation, including benefits.

Todd Litman (11) of the Victoria Transport Policy Institute (VTPI) has done extensive research on the effects of congestion on travel time values. He inserts a multiplier to account for the prevailing level of service, which is a measure of relative congestion levels. The level of service for most facilities can be estimated using the methodologies of the Highway Capacity Manual, published by the Transportation Research Board (12). Level of service (LOS) is a six-letter scale from LOS A to LOS F, with LOS A indicating excellent operating conditions to LOS F, representing a breakdown in continuous flow.

Table 4.2 summarizes many of the recommended rates in various user categories, taken from Reference 11.

Table 4.2 Travel Time Values Related to Percent of Prevailing Wage Rates and Level of Service

| User Category | Level of Service |  |  |  | Wait |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | A-C | D | E | F | Time |
| Private Cars |  |  |  |  |  |
| Driver | $50 \%$ | $67 \%$ | $84 \%$ | $100 \%$ | $100 \%$ |
| Passenger | $35 \%$ | $47 \%$ | $58 \%$ | $70 \%$ | $70 \%$ |
| Transit Vehicle |  |  |  |  |  |
| Driver | $156 \%$ | $156 \%$ | $156 \%$ | $156 \%$ | $156 \%$ |
| Adult Passenger - Seated | $35 \%$ | $47 \%$ | $58 \%$ | $70 \%$ | $50 \%$ |
| Adult Passenger - Standing | $50 \%$ | $67 \%$ | $83 \%$ | $100 \%$ | $70 \%$ |
| Child (under 16) - Seated | $25 \%$ | $33 \%$ | $42 \%$ | $50 \%$ | $50 \%$ |
| Child (under 16) - Standing | $35 \%$ | $46 \%$ | $42 \%$ | $50 \%$ | $70 \%$ |
| Trucks |  |  |  |  |  |
| Driver | $120 \%$ | $137 \%$ | $154 \%$ | $170 \%$ | $170 \%$ |
| Passenger | $120 \%$ | $132 \%$ | $144 \%$ | $155 \%$ | $155 \%$ |
| Pedestrians and Cyclists | $50 \%$ | $67 \%$ | $84 \%$ | $100 \%$ | $100 \%$ |

(Source: Litman, T., Build for Comfort, Not Just Speed, Victoria Transport Policy Institute, Vancouver BC, July 25,2008 ).

Table 4.2 does not break down auto users beyond grouping drivers and passengers, as the study cited was directed towards pubic transportation travel times and their valuation. All values are related to the prevailing wage for the user category. Note that transit drivers, truck drivers, and truck passengers have percentages over $100 \%$. Part of this is to account for total compensation rates, which are higher than wage rates.

Congested conditions (indicated in Table 4.2 by levels of service D, E, and F) involve a higher travel time cost. This represents study results that suggest travelers are more cognizant of travel time when congested conditions are present, and that they place a higher value on it due to the increased inconvenience such conditions cause.

The U.S. Department of Transportation provides more detailed recommendations for the value of travel time (13) that take into account more categories of highway users. The guidelines, however, do not take into account the effects of congestion. Table 4.3 shows the U.S. Department of Transportation guidelines.

Table 4.3 U.S. Department of Transportation Recommendations on the Value of Travel Time

| User Category | Value | Base |
| :--- | :---: | :--- |
| Private Cars |  |  |
| Driver (Commute) | $50 \%$ | of wages |
| Driver (Car Pool Commute) | $60 \%$ | of wages |
| Passenger (Car Pool Commute) | $40 \%$ | of wages |
| Driver/Passenger (Personal - Local) | $50 \%$ | of wages |
| Driver/Passenger (Personal - Intercity) | $70 \%$ | of wages |
| Driver/Passenger (Business) | $100 \%$ | of total compensation |
| Transit Vehicle | $50 \%$ | of wages |
| In-Vehicle (Commute/Personal) | $100 \%$ | of wages |
| Excess (walking, transfer, waiting - non- <br> business) | $100 \%$ | of total compensation |
| In-Vehicle and Excess (Business) |  |  |
| Trucks | $100 \%$ | of total compensation |
| All Travel Time |  |  |

(Source: The Value of Travel Time: Departmental Guidance for Conducting Economic Evaluations, U.S. Department of Transportation, Washington DC, 1997).

It should be noted that the "transit vehicle" values listed in Tables 4.2 and 4.3 were specifically recommended for buses. Their application to light or rail rapid transit vehicles may be problematic.

If the value of travel time, however, is linked to wages and total compensation, it is critical that information on current wages and total compensation be accessed. The best source for such information is the U.S. Bureau of Labor Statistics, which publishes frequent reports showing wages and benefits categorized by industry and job function, private vs. public employers, regional and specific urban areas, etc. Table 4.4 shows a sample of the type of data available from the Bureau, and is meant to be illustrative. In Table 4.4, wages and total compensation are shown by occupational group, segregated by public vs. private employers.

Table 4.4 does not include all categories of workers. It does not include federal employees, for example; nor does it include such state and local workers as police, fire, sanitation and other workers. The Bureau, however, publishes many studies and surveys with varying degrees of specificity related to location, industry, and job function, which may be used to determine appropriate values in any given case.

Table 4.4 National Average Statistics on Wage and Total Compensation Rates by Industry Group (for June 2012)

| Employee Group | Wages <br> $\mathbf{( \$ / h r )}$ | Total <br> Compensation <br> $(\$ / \mathbf{h r})$ |
| :--- | :---: | :---: |
| A/l Private Industry (average) | $\$ 20.27$ | $\$ 28.80$ |
| Al/ State and Local Government (average) | $\$ 26.70$ | $\$ 41.10$ |
| Workers in Private Industry | $\$ 23.17$ | $\$ 33.88$ |
| Construction | $\$ 21.14$ | $\$ 33.07$ |
| Manufacturing | $\$ 13.31$ | $\$ 17.65$ |
| Retail Trade | $\$ 21.96$ | $\$ 31.11$ |
| Wholesale Trade | $\$ 22.17$ | $\$ 34.24$ |
| Transportation and Warehousing | $\$ 35.84$ | $\$ 58.91$ |
| Utilities | $\$ 29.79$ | $\$ 45.08$ |
| Information | $\$ 28.72$ | $\$ 43.11$ |
| Finance and Insurance | $\$ 21.57$ | $\$ 30.25$ |
| Real Estate | $\$ 25.10$ | $\$ 34.80$ |
| Professional and Business Services | $\$ 25.58$ | $\$ 39.11$ |
| Education | $\$ 21.04$ | $\$ 29.65$ |
| Health Care and Social Assistance | $\$ 9.71$ | $\$ 12.20$ |
| Leisure and Hospitality | $\$ 18.70$ | $\$ 25.42$ |
| Other Services | $\$ 30.02$ | $\$ 44.08$ |
| Workers in State and Local Government | $\$ 22.59$ | $\$ 35.88$ |
| Education | $\$ 23.97$ | $\$ 39.64$ |
| Health Care and Social Assistance |  |  |
| Public Administration |  |  |

(Source: Employer Costs for Employee Compensation - June 2012, Bureau of Labor Statistics, U.S. Department of Labor, Washington DC, September 11, 2012.)

The time values indicated in Tables 4.2, 4.3, and 4.4 are per person. Thus, to find a time value per vehicle, the tabulated values have to be multiplied by the average vehicle occupancy for the class of vehicles involved. In general, passenger car occupancy ranges between 1.3 and 1.8 persons per vehicle, depending upon trip purpose and the characteristics of the region in which the travel takes place. Commercial vehicles typically carry only the driver. Bus occupancies are, of course, much higher. A typical intercity bus, fully loaded, would have between 40 and 50 passengers, while a city transit bus could range anywhere between 20 passengers per vehicle in outlying or suburban areas and 70 passengers per vehicle (including standees) in a dense urban area. It is critical that economic analyses reflect the actual occupancies for the alternatives under consideration. This requires, therefore, that appropriate local studies and information be used to determine these values.

As noted previously, a number of studies have determined that the value of time saved is related to the increment. Small increments in travel time have little opportunity value to the user, and therefore have limited economic value. Larger increments in travel time have significantly higher opportunity value to the user, and therefore have higher economic value.

One of the earliest recognitions of this was reported in the $2^{\text {nd }}$ Edition of the Traffic and Transportation Engineering Handbook, published by the Institute of Transportation Engineers (14). It classified the value of travel time saved depending upon whether the increment was 5 minutes or less, between 5 and $15 \mathrm{mi}-$ nutes, or greater than 15 minutes. Using the 5 to 15 minute increment as the base, Table 4.5 shows the multipliers that would result.

It is difficult to actually apply these multipliers, since the studies that determined them are unrelated to the average travel time values discussed in the previous section, and there is no indication of a base time interval that applies to the data in Tables $4.2-4.4$. Nevertheless, the concept is an important one, and the table allows at least a demonstration of the potential impact of this concept on user benefits that are directly related to travel time.

Table 4.5 Value of Travel Time vs. Incremental Time Saved

| Incremental Time Saved | Multiplier |
| :--- | :---: |
| Low time savings ( $0-5$ mins) | 0.12 |
| Medium time savings ( $>5 \leq 15$ mins) | 1.00 |
| High time savings ( $>15$ mins) | 2.17 |

### 4.5.3 An Illustrative Application in the Value of Travel Time

As has been noted, the Highway Capacity Manual (12) provides methodologies that allow the estimation of travel speeds, delays, and other time-related parameters for a variety of traffic facilities, from freeways to signalized intersections.

For this illustration, consider the simple (relatively) case of a potential lane addition to a section of freeway in an urban area. Table 4.6 summarizes known information concerning the case.

Table 4.6 Information for the Illustrative Case

| Item | Data |
| :--- | :--- |
| Type of Facility | 4-Lane Urban Freeway (2 lanes each dir.) |
|  | $65 \mathrm{mi} / \mathrm{h}$ free-flow speed. |
| Length of Subject Section | 8 miles |
| Current Volumes (Each Direction) |  |
| 4 Peak Hours/Day | 4,200 passenger cars/hour |
| 8 Near-Peak Hours/Day | 3,500 passenger cars/hour |
| 12 Off-Peak Hours/Day | 2,800 passenger cars/hour |
| Proposed Improvement | Add 1 lane to each direction. |
| Prevailing Wage Rate for Use | $\$ 25.65 / \mathrm{hr}$ |
| Average Car Occupancy | 1.4 persons/car |

To simplify the example, it is assumed that the addition of a lane will not result in any additional induced traffic volume. Further, the facility is classified as a parkway, and all vehicles are passenger cars. The volumes per hour are shown in three different time periods; in a real case, volumes for each hour of the day would be used.

The fundamental questions are: (1) How much travel time is saved by the addition of a lane, and (2) what is its monetary value?

To investigate the case, the speed-flow curves for freeways, calibrated in the Highway Capacity Manual, must be used to find the impact on speed (and level of service) of adding a lane. The answer will be different for the three demand volume periods defined in Table 4.7. Figure 4.1 shows a simplified version of these curves, showing only the curve for a freeway with a free-flow speed of $65 \mathrm{mi} / \mathrm{h}$.


Fig. 4.1 Speed-Flow Curve for $65-\mathrm{mi} / \mathrm{h}$ Freeways from the 2010 Highway Capacity Manual
(Source: Modified from Highway Capacity Manual 2010, Transportation Research Board, Washington DC, 2010, Illustration 6-11.)

To find the speed, demand volumes have to be reduced to $\mathrm{pc} / \mathrm{mi} / \mathrm{ln}$ for each demand period. Volumes per lane will change depending upon the number of lanes provided: 2 in each direction for the existing case, 3 in each direction with the additional lane. Figure 4.1 is then entered with each of these volumes to find speeds and the prevailing level of service both with and without the additional lanes. Table 4.6 shows the results of these computations. Figure 4.2 then shows how the various speeds are determined from Figure 4.1.

Table 4.7 Demand Volumes Per Lane w/ and w/o an Additional Lane

| Time Period | Total Demand <br> $(\mathbf{p c} / \mathbf{h})$ | Volume/Lane <br> $(\mathbf{2}$-Lanes) | Volume/Lane <br> (3-Lanes) |
| :---: | :---: | :---: | :---: |
| 4 Peak Hours | 4,200 | 2,100 | 1,400 |
| 8 Near-Peak Hours | 3,500 | 1,750 | 1,167 |
| 12 Non-Peak Hours | 2,800 | 1,400 | 933 |


-------- 4 Peak Hrs $\qquad$ 12 Non-Peak Hrs
Fig. 4.2 Speeds Determined for Illustration
From these results, the following can be determined on the effects of lane addition on the various time periods:

- During the 4 peak hours, the addition of a lane increases speed from $58 \mathrm{mi} / \mathrm{h}$ to $65 \mathrm{mi} / \mathrm{h}$, and improves the level of service from E to C .
- During the 8 near peak hours, the addition of a lane increases speed from 63 $\mathrm{mi} / \mathrm{h}$ to $65 \mathrm{mi} / \mathrm{h}$, and improves the level of service from D to B .
- During the 12 non-peak hours, the addition of a lane does not affect speed (it remains $65 \mathrm{mi} / \mathrm{h}$ ) and improves the level of service from C to B .

What remains is to evaluate the monetary value of the travel time consumed with and without the additional lane provided. First, the travel time for each vehicle in each time interval in computed as the length of the section (in miles) divided by the appropriate speed (in mi/h). Then:

$$
\begin{aligned}
& t_{\text {peak }, 2 l a n e s}=8 / 58=0.1379 \mathrm{~h}=8.275 \mathrm{~min} \\
& t_{\text {peak }, 3 l a n e s}=8 / 65=0.1231 \mathrm{~h}=7.385 \mathrm{~min} \\
& t_{\text {near peak, } 2 \text { lanes }}=8 / 63=0.1270 \mathrm{~h}=7.619 \mathrm{~min} \\
& t_{\text {near peak }, 3 l a n e s}=8 / 65=0.1231 \mathrm{~h}=7.385 \mathrm{~min} \\
& t_{\text {non-peak }, 2 l a n e s}=8 / 65=0.1231 \mathrm{~h}=7.385 \mathrm{~min} \\
& t_{\text {non-peak }, 3 l a n e s}=8 / 65=0.1231 \mathrm{~h}=7.385 \mathrm{~min}
\end{aligned}
$$

Two things are now obvious: (1) the addition of a lane does not reduce the travel time during the non-peak hours, and (2) the addition of a lane reduces travel time during the peak and near-peak hours of the day, but by a very small amount (less than one minute).

Since the analysis also provides levels of service, it will continue using the travel time values specified in Table 4.2. The cost of travel time for each vehicle can be computed as:

$$
\begin{equation*}
C_{v e h i j}=\left(V_{d}+0.4 V_{p}\right) * t_{i j} \tag{4.2}
\end{equation*}
$$

Where: $\quad \begin{array}{lll}\mathrm{C}_{\text {vehij }} & = & \text { cost per vehicle in time period } \mathrm{i}, \text { case } \mathrm{j}, \$ / \mathrm{veh} \\ \mathrm{V}_{\mathrm{d}} & = & \text { value of time for drivers, } \$ / \mathrm{h} \text { (from Table 4.2) } \\ \mathrm{V}_{\mathrm{p}} & = & \begin{array}{l}\text { value of time for passengers, } \$ / \mathrm{hr} \text { (from Table } \\ \\ \\ \mathrm{t}_{\mathrm{ij}}\end{array} \\ & & \begin{array}{l}\text { travel time for a vehicle in time period } \mathrm{i}, \text { case } \mathrm{j},\end{array} \\ & & \\ & \end{array}$
For use in economic analysis, this must be converted to a travel time cost per year for all vehicles using the freeway section in question. To simplify the illustration, it is assumed that the same volumes occur each day. In real terms, typical volumes may vary by day of the week and season of the year, and this would have to be taken into account. Note also that the demand volumes used were for one direction; thus there is a multiplier of 2 that must be applied to take into account both directions. The total annual cost of travel time for case $j$ can then be computed as:

$$
\begin{equation*}
R_{j}=\sum_{i}\left(C_{v e h i} * v_{v e h i} * T_{i} * 365 * 2\right) \tag{4.3}
\end{equation*}
$$

Where:

| $\mathrm{R}_{\mathrm{j}}$ | $=$ | annual cost of travel time for case $\mathrm{j}, \$$. |
| :--- | :--- | :--- |
| $\mathrm{C}_{\text {vehi }}$ | $=$ | travel time cost/veh in time period $\mathrm{i}, \$$, as computed in <br> Equation 4.2. |
| $\mathrm{v}_{\mathrm{i}}$ | $=$ | vehicles/hr during time period i. |
| $\mathrm{T}_{\mathrm{i}}$ | $=$ | number of hours in time period i. |
| 365 | $=$ | multiplier for days/yr. |
| 2 | $=$ | multiplier for 2 directions. |

The problem statement indicates that the prevailing wage rate for this area is $\$ 25.65 / \mathrm{hr}$. The speed analysis indicates that the two cases and three time periods involve levels of service ranging from E to B . Table 4.2 will be used to estimate the time values that apply to each time period i and case j . Table 4.8 illustrates the computation of individual vehicle travel time costs, using Equation 4.2.

The value of time per vehicle hour is based upon the given auto occupancy rate of 1.4 persons/vehicle, which consists of 1 driver and an average of 0.4 passengers. For the first case in Table 4.8 (Existing, Peak):

$$
(25.65 * 0.84)+(25.65 * 0.58 * 0.4)=\$ 27.50 / \text { veh }-h
$$

Table 4.8 Travel Time Cost Per Vehicle for Illustrative Case

| Case | Time <br> Period | LOS | Wage <br> $\mathbf{( \$ / h )}$ | Value- <br> Drivers <br> (\% of <br> Wage) | Value- <br> Pass. <br> (\% of <br> Wage) | Value <br> $\mathbf{( \$ / v e h - ~}$ <br> $\mathbf{h})$ | Travel <br> Time <br> (hrs) | Cueh |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Existing <br> (2-Lanes) | Peak | E | $\$ 25.65$ | $84 \%$ | $58 \%$ | $\$ 27.50$ | 0.1379 | $\$ 3.79$ |
|  | Near- <br> Peak | D | $\$ 25.65$ | $67 \%$ | $47 \%$ | $\$ 22.01$ | 0.1270 | $\$ 2.80$ |
|  | Non-Peak | C | $\$ 25.65$ | $50 \%$ | $35 \%$ | $\$ 16.42$ | 0.1231 | $\$ 2.02$ |
| Proposed <br> (3-Lanes) | Peak | C | $\$ 25.65$ | $50 \%$ | $35 \%$ | $\$ 16.42$ | 0.1231 | $\$ 2.02$ |
|  | Near- <br> Peak | B | $\$ 25.65$ | $50 \%$ | $35 \%$ | $\$ 16.42$ | 0.1231 | $\$ 2.02$ |
|  | Non-Peak | B | $\$ 25.64$ | $50 \%$ | $35 \%$ | $\$ 16.42$ | 0.1231 | $\$ 2.02$ |

Table 4.9 illustrates the computations implementing Equation 4-2, which leads to an estimate of the total annual cost of travel time for the existing 4-lane freeway, and the proposed expansion to 6 lanes.

Table 4.9 Annual Cost of Travel Time for Illustrative Case

| Case | Time Pe- <br> riod | Vol/Hr <br> In Period | Hours in <br> Period | Cost/Hour <br> $\mathbf{( \$ )}$ | Multiplier <br> (days/yr) | Multiplier <br> (directions) | Annual <br> Cost (\$) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Existing <br> (2 Lanes) | Peak | 4,200 | 4 | $\$ 3.79$ | 365 | 2 | $\$ 46,480,560$ |  |  |  |  |  |  |
|  | Near-Peak | 3,500 | 8 | $\$ 2.80$ | 365 | 2 | $\$ 57,232,000$ |  |  |  |  |  |  |
|  | Non-Peak | 2,800 | 12 | $\$ 2.02$ | 365 | 2 | $\$ 49,546,560$ |  |  |  |  |  |  |
|  | Total |  |  |  |  |  | $\mathbf{\$ 1 5 3 , 2 5 9 , 1 2 0}$ |  |  |  |  |  |  |
| Proposed <br> (3 Lanes) | Peak | 4,200 | 4 | $\$ 2.02$ | 365 | 2 | $\$ 24,569,664$ |  |  |  |  |  |  |
|  | Near-Peak | 3,500 | 8 | $\$ 2.02$ | 365 | 2 | $\$ 41,288,800$ |  |  |  |  |  |  |
|  | Non-Peak | 2,800 | 12 | $\$ 2.02$ | 365 | 2 | $\$ 49,546,560$ |  |  |  |  |  |  |
|  | Total |  |  |  |  |  |  |  |  |  |  |  | $\mathbf{\$ 1 1 5 , 4 0 5 , 0 2 4}$ |

Adding a lane (in each direction) to the existing 4-lane freeway produces a savings of $\$ 153,259,120-\$ 115,405,024=\$ 37,854,096$ per year. This can be used, in addition to other economic benefits, to estimate how much can be reasonably spent on the 8 -mile lane addition project.

It should be noted, however, that the actual amount of time actually saved by any individual driver is less than one minute. Table 4.5 suggests that in such cases, travel time be discounted to $12 \%$ of the base value, which would change the result considerably. However, as noted previously, it is difficult to apply Table 4.5 directly, as there is no indication that the estimated savings are based on any relationship between the monetary value of time and the amount of time actually saved by any one driver. In this case, part of the savings is also due to the improvement in level of service during the peak and near-peak hours, which would not be affected by the amount of time saved.

### 4.5.4 Closing Comments on Travel Time

Obviously, the monetary valuation of travel time is a significant issue in the engineering economic analysis of alternative projects. Nevertheless, it is not a straightforward issue. This section highlights three fundamental principles used in determining the monetary value of travel time:

- The value of time is tied to local and regional wage rates.
- The value of time is higher under congested conditions.
- The value of time should be related to the actual amount of time each traveler is saved.

The selection of monetary values for travel time is best done in a local and regional setting using appropriate wage rates, always available from the Bureau of Labor Statistics. Local and regional policy will also have to establish the relationships between congestion levels, absolute amount of time saved per person per trip, and the monetary value for travel time.

### 4.6 Vehicle Operating and Maintenance Costs

As was previously discussed, vehicle operating and maintenance costs generally fall into one of two categories: (1) costs related to vehicle usage; and (2) costs related to vehicle ownership. The former are often both time and speed dependent, while the latter are relatively fixed costs that do not rise or fall with actual vehicle operation.

### 4.6.1 Typical Operating and Maintenance Costs

A principal source of general vehicle operating and maintenance cost information is the annual publication, Your Driving Costs, published by the AAA. (It draws its information from a variety of sources). Table 4.10 shows the latest data, for 2011. It is important to note that the data is based upon a vehicle used for personal transportation over five years and 75,000 miles of ownership. As noted in the Victoria Transportation Policy Institute (16), this is somewhat biased towards new vehicles, which, in effect, overestimates items such as depreciation and insurance, while underestimating maintenance and repairs.

Table 4.10 Vehicle Operating and Maintenance Costs

| Cost Category | Small Sedan | Medium Sedan | Large Sedan | Average ${ }^{1}$ | SUV ${ }^{\mathbf{2}}$ | Minivan ${ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Operating } \\ \text { Costs } \end{gathered}$ | \$/mile | \$/mile | \$/mile | \$/mile | \$/mile | \$/mile |
| Gasoline ${ }^{3}$ | \$0.1005 | \$0.1282 | \$0.1416 | \$0.1234 | \$0.1704 | \$0.1508 |
| Maintenance | \$0.0411 | \$0.0429 | \$0.0493 | \$0.0444 | \$0.0480 | \$0.0450 |
| Tires | \$0.0067 | \$0.0111 | \$0.0109 | \$0.0096 | \$0.0116 | \$0.0076 |
| Ownership Costs | \$/Year | \$/Year | \$/Year | \$/Year | \$/Year | \$/Year |
| Insurance | \$951 | \$948 | \$1,006 | \$968 | \$912 | \$853 |
| License/Reg. | \$438 | \$577 | \$769 | \$595 | \$757 | \$618 |
| Depreciation | \$2,560 | \$3534 | \$5,091 | \$3,729 | \$5,052 | \$4,108 |
| Finance | \$584 | \$796 | \$1,089 | \$823 | \$1,071 | \$859 |
| Total/Year ${ }^{4}$ | \$4,533 | \$5,855 | \$7,955 | \$6,114 | \$7,792 | \$6,438 |
| Total/Mile ${ }^{5}$ | \$0.3022 | \$0.3903 | \$0.5303 | \$0.4076 | \$0.5195 | \$0.4292 |
| Total Costs Per Mile ${ }^{6}$ | \$0.4505 | \$0.5725 | \$0.7321 | \$0.5850 | \$0.7495 | \$0.6362 |

1. Average for small, medium, and large sedan.
2. Not included in average.
3. Gasoline cost $=\$ 2.88 /$ gal .
4. Total ownership costs per year (\$).
5. Total ownership costs per mile, assuming 15,000 annual miles travelled.
6. Total of gasoline, maintenance, tire, and ownership costs, \$/vehi-mi.

Such a summary of per-mile operating and maintenance costs is useful where a transportation alternative reduces the number of vehicle-miles travelled in a given study area. It does not, however, treat the issue of the speed dependency of some costs, nor does it include information on any classes of heavy vehicles.

The issue of average vehicle usage per year is easily adjusted using the data of Table 4.10. The average per-mile cost of vehicle ownership is found as:

$$
\begin{equation*}
O_{m}=\frac{A_{o}}{M_{a}} \tag{4.4}
\end{equation*}
$$

Where: $\quad \mathrm{O}_{\mathrm{m}}=\quad$ ownership cost per mile, $\$ / \mathrm{mi}$. $\mathrm{A}_{\mathrm{o}} \quad=\quad$ annual ownership costs, $\$ / \mathrm{yr}$ (Table 4.10)
$\mathrm{M}_{\mathrm{a}}=\quad$ average annual vehicle usage, mi/yr.
In Table 4.9, the ownership cost/yr for the average personal vehicle is given as $\$ 6,114$. If the average vehicle usage is 10,000 miles, then:

$$
O_{m, 10000}=\frac{6114}{10,000}=\$ 0.6114 / m i
$$

This could be added to the per-mile costs of gasoline, maintenance, and tires to come up with a total cost/mile of vehicle operation:

$$
\begin{equation*}
C_{m}=F_{m}+M_{m}+T_{m}+O_{m} \tag{4.5}
\end{equation*}
$$

Where: $\quad \mathrm{C}_{\mathrm{m}}=$ total cost of vehicle operation, $\$ / \mathrm{mi}$. $\mathrm{F}_{\mathrm{m}} \quad=\quad$ fuel cost per mile, $\$ / \mathrm{mi}$. $\mathrm{M}_{\mathrm{m}} \quad=\quad$ maintenance cost per mile, $\$ / \mathrm{mi}$. $\mathrm{O}_{\mathrm{m}} \quad=\quad$ ownership cost per mile, $\$ / \mathrm{mi}$.

For an average vehicle in Table 4.10, with 10,000 miles/yr of usage, the total permile cost of operation and maintenance would be:

$$
C_{m}=0.1234+0.0444+0.0096+0.6114=\$ 0.7888 / \text { mile }
$$

The principal speed-related cost of vehicle operation is fuel, although tire wear and some maintenance items may also be affected by speed. The California Department of Transportation Benefit-Cost Model (17) incorporates fuel consumption rates (based upon 1999 fuel-consumption ratings), which are shown in Table 4.11.

Table 4.11 Fuel Consumption Rates for Autos and Trucks vs. Speed

| Speed (mi/h) | Autos (gallons/mile) | Trucks (gallons/mile) |
| :---: | :---: | :---: |
| 10 | 0.123 | 0.181 |
| 20 | 0.068 | 0.118 |
| 30 | 0.044 | 0.133 |
| 40 | 0.034 | 0.185 |
| 50 | 0.033 | 0.264 |
| 60 | 0.037 | 0.374 |
| 70 | 0.052 | 0.511 |

The same model indicates that the non-fuel costs (including fixed costs of ownership) of trucks are approximately 1.73 times the non-fuel costs of automobiles.

Table 4.11 is based upon vehicle fuel consumption rates in 1999; where more recent data is available, it should be used. Whatever rates are used, however, the costs associated with fuel consumption are based upon the cost of gasoline. Gasoline costs remain very volatile, and depend upon world demand, supply issues, and the availability and cost of alternative fuels.

The latest gasoline and diesel fuel prices can be found at the web-site of the U.S. Energy Information Administration/Gasoline and Diesel Fuel Update, which is revised on a monthly basis. In November of 2012, the national average price of regular gasoline was $\$ 3.437 /$ gallon; diesel fuel was $\$ 4.043 /$ gallon.

### 4.6.2 An Example in Vehicle Operating and Maintenance Costs

As with all of the data in this chapter, it is intended as illustrative. In any given case, local and regional sources of up-to-date information and data should be used. However, it is useful to show how such data could be used in the analysis of highway alternatives.

Consider the case of a 20-mile segment of suburban freeway carrying an average annual daily traffic (AADT) of 75,000 vehicles/day. Its current alignment has many vertical and horizontal curves, and produces average speeds of $40 \mathrm{mi} / \mathrm{h}$. It is proposed to reconstruct this segment along a new alignment that would (a) reduce the length of the segment to 16 miles, and (b) increase average speeds to $50 \mathrm{mi} / \mathrm{h}$. Fifteen percent of the AADT consists of diesel-fueled trucks, and the remaining $85 \%$ of the vehicles are privately-owned cars. What would the annual savings in operating and maintenance costs to vehicle operators be due to the change in the alignment? It may be assumed that the change in alignment will not induce any new traffic to the facility.

The first need is to divide the AADT into the categories of cars and trucks. Given the input information:

$$
\begin{aligned}
& A A D T_{\text {cars }}=75,000 * 0.85=63,750 \mathrm{veh} / \text { day } \\
& A A D T_{\text {trucks }}=75,000 * 0.15=11,250 \mathrm{veh} / \mathrm{day}
\end{aligned}
$$

At this point, what is needed is the average per-mile total operating and maintenance costs for cars and trucks for use in the comparison. The before situation involves an average speed of $40 \mathrm{mi} / \mathrm{h}$, while the after case involves an average speed of $50 \mathrm{mi} / \mathrm{h}$. This, of course, is a very simplistic view, adopted for illustrative purposes. In reality, the before and after speeds of cars and trucks are likely different, and the use of an average speed would be abandoned for an hour-by-hour look at both hourly volumes and related speeds. Fuel cost will be estimated from the fuel consumption estimates of Table 4.11. Non-fuel costs will be estimated for cars from the data of Table 4.10 (excluding fuel costs). Truck non-fuel costs will use the car value multiplied by 1.73 , as suggested by Ref. 17. Table 4.12 illustrates the information used to estimate vehicle and maintenance costs for the problem as described.

Table 4.12 Data Relevant to Example Problem

| Vehicle | Speed (mi/h) | Fuel Consumption (gals/mi) | Fuel Cost ${ }^{1}$ (\$/mi) | Non-Fuel Cost ${ }^{2}$ (\$/mi) | $\begin{aligned} & \text { Total Cost } \\ & (\$ / \mathrm{mi}) \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cars | 40 | 0.034 | X 3.437=\$0.1169 | \$0.4216 | \$0.5385 |
|  | 50 | 0.033 | X 3.437=\$0.1134 | \$0.4216 | \$0.5350 |
| Trucks | 40 | 0.185 | $X 4.043=\$ 0.7480$ | \$0.7294 | \$1.4774 |
|  | 50 | 0.264 | X 4.034=\$1.0650 | \$0.7294 | \$1.7944 |

1. Fuel price/gallon for Nov. 2012, as noted previously.
2. Cars: Table 4.10 for average car; total of Maintenance, Tire, and Ownership costs; for trucks, 1.73 times the value for cars.

Looking at Table 4.12, the interesting fact is that the higher speeds of the proposed alignment will increase the operating cost per mile for trucks. They would still, however, gain an advantage due to the shorter distance the new alignment entails, and would (although not included in this example) benefit from decreased travel time both due to increased speed and reduced trip length.

Table 4.13 illustrates the computation of annual user operating and maintenance costs for the old and new alignments.

Table 4.13 Annual Operating and Maintenance Costs for Example

| OPTION | VEH TYPE | $\begin{gathered} \text { Cost }^{1} \\ (\$ / \mathrm{mi}) \\ \hline \end{gathered}$ | $\begin{gathered} \text { x AADT } \\ \text { (Veh/Day) } \end{gathered}$ | x Segment <br> Length (mi) | X Days/Year | $\begin{gathered} \text { = Annual } \\ \text { Cost }(\$ / \mathbf{Y r}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Old Alignment | Cars | \$0.5385 | 63,750 | 20 | 365 | \$250,604,438 |
|  | Trucks | \$1.4774 | 11,250 | 20 | 365 | \$121,331,475 |
|  | Total |  |  |  |  | \$371,935,913 |
| New Alignment | Cars | \$0.5340 | 63,750 | 16 | 365 | \$198,808,200 |
|  | Trucks | \$1.7944 | 11,250 | 16 | 365 | \$117,892,080 |
|  | Total |  |  |  |  | \$316,700,280 |

1. From Table 4.12.

From Table 4.13, the annual savings in operating and maintenance costs due to the proposed realignment of the roadway would be $\$ 371,935,913-\$ 316,700,280$ $=\$ 55,235,633 / \mathrm{yr}$. The value of other benefits due to the new alignment would have to be added in a complete analysis. These would include reduced travel time, perhaps increased safety, and reduced highway maintenance costs. A present worth analysis of the value of total savings over the expected service life of the realigned segment would result in an estimate of what amount could justifiably be invested in planning, designing, and constructing the new alignment.

### 4.7 The Cost of Highway Crashes

The impact of highway improvements on the number and severity of crashes that occur is significant. In many cases, highway improvements are specifically aimed at improving safety. Thus, it is important to be able to assess the likely benefits of an improvement on accidents.

There are three factors related to the overall cost of crashes on any given section of highway:

1. The frequency with which crashes occur.
2. The severity of crashes - generally categorized as fatal, injury, or property damage only (PDO).
3. The economic cost of crashes, related to severity.

To estimate the economic cost of accidents on any given section of highway, measures of frequency and severity must be determined for the base case (an existing facility) and for a proposed improvement. The latter requires the ability to relate crash occurrences (or decreases in crash occurrence) due to specific features of a highway or proposed improvement.

The most comprehensive source for such information is the Highway Safety Manual (18), a three-volume document published by the American Association of Highway and Transportation Officials.

The Highway Safety Manual provides a fundamental predictive methodology for estimating the impact of highway improvements on crashes.

$$
\begin{equation*}
N_{\text {pred }}=N_{\text {sppx }} *\left(C M F_{1 x} * C M F_{2 x} * \ldots . \ldots C M F_{i x}\right) * C_{x} \tag{4.6}
\end{equation*}
$$

Where:

$$
\begin{aligned}
\mathrm{N}_{\text {pred }} & =\text { predicted crash frequency for a specific year for site type } \mathrm{x}, \\
\mathrm{~N}_{\text {spfx }} & =\begin{array}{l}
\text { predicted average crash frequency for base conditions for site } \\
\text { type } \mathrm{x},
\end{array} \\
\mathrm{CMF}_{\text {ix }} & =\begin{array}{l}
\text { crash modification factors for site type } \mathrm{x} \text { and specific geome- } \\
\\
\\
\\
\text { tric/control conditions at the site, and }
\end{array} \\
\mathrm{C}_{\mathrm{x}} & =\text { calibration factor to adjust for local conditions, site type } \mathrm{x} .
\end{aligned}
$$

The manual then provides data on the determination of average crash frequencies $\left(\mathrm{N}_{\text {spfx }}\right)$, crash modification factors $\left(\mathrm{CMF}_{\mathrm{ix}}\right)$ based upon specific geometric or control improvements, and the determination of local calibration factors $\left(\mathrm{C}_{\mathrm{x}}\right)$.

For example, the average crash frequency for a rural two-lane, two-way highway is found as:

$$
\begin{equation*}
N_{s p f r s}=\frac{365 * A A D T * L}{1,000,000 * e^{0.312}} \tag{4.7}
\end{equation*}
$$

Thus, a 20 -mile segment of rural two-lane, two-way highway carrying an AADT of $8,000 \mathrm{veh} /$ day would be expected to have an average accident frequency of:

$$
N_{\text {spf rs }}=\frac{365 * 8,000 * 20}{1,000,000 * e^{0.312}}=\frac{58,400,000}{1,366,155}=42.7 \text { crashes } / \mathrm{yr}
$$

In this case, the base conditions for which this equation was calibrated include 12ft lanes, $6-\mathrm{ft}$ paved shoulders, 5 driveways/mile, no vertical or horizontal curvature, and no grade (as well as some other specific features).

The methodology then offers CMFs for 12 different geometric or control conditions that could affect crashes on two-lane, two-way rural highways: lane width, shoulder width and type, horizontal curves, superelevation on horizontal curves, grades, driveway density, centerline rumble strips, roadside design, lighting, and automated speed enforcement. In each case, the CMF only applies to those types of crashes that would be affected by the geometric or control measure.

For example, if the existing roadway had $10-\mathrm{ft}$ travel lanes and only 2 -ft paved shoulders, the $C M F_{l w}$ would be 1.30 (based upon the AADT of 8,000 veh/day) and the $C M F_{s w}$ would be 1.30 (also based upon the AADT of 8,000 veh/day).

In both cases, the lane and shoulder width situations apply to accidents in the following categories: single-vehicle run-off-the-road, multiple vehicle head-on, and multiple-vehicle sideswipe crashes. The methodology also provides information on the proportion of accidents that occur in various categories. The types referenced above make up a total of $57.4 \%$ of all crashes on two-lane, two-way highways.

The manual provides a simple equation for adjusting the CMF to reflect the proportion of total crashes that would be affected by the particular geometric or control condition under consideration:

$$
\begin{equation*}
C M F_{i a d j}=p_{a} *\left(C M A_{i}-1.0\right)+1.0 \tag{4.8}
\end{equation*}
$$

Where: $\mathrm{CMF}_{\mathrm{i} \text { adj }}=\quad$ adjusted crash modification factor for item i , $\mathrm{CMF}_{\mathrm{i}}=\quad$ crash modification factor for item i , and $\mathrm{pa}_{\mathrm{a}}=\quad$ proportion of crashes affected by item i.

For each of the two CMF values in this case, both of which affect $57.4 \%$ of all crashes, the adjusted CMF would be:

$$
C M F_{l w a d j}=C M F_{\text {swadj }}=0.574 *(1.30-1)+1=1.1722
$$

Essentially, the narrow lanes ( 10 ft ) and narrow shoulders ( 2 ft ) of the existing facility each cause the number of accidents to be increased by $17.22 \%$. Using equation 4.6 , the number of crashes per year on the existing facility may now be estimated. Note that the local calibration factor, $\mathrm{C}_{\mathrm{x}}$, is assumed to be 1.0. In reality, if existing crash data suggested that the base number of crashes was under- or overstated, the equation would be adjusted by the ratio of the observed crashes to the predicted base crashes. Now:

$$
N_{\text {pred }}=42.7 * 1.1722 * 1.1722 * 1.0=58.7 \text { crashes } / \text { yr }
$$

Essentially, the existing 20-mile segment of rural two-lane, two-way highway (with $10-\mathrm{ft}$ lanes and $2-\mathrm{ft}$ shoulders) is expected to experience 58.7 crashes year. If the highway were improved to the base case - 12 -ft lanes and 6 - ft shoulders the number of crashes would be expected to decrease to $42.7 / \mathrm{yr}$, a reduction of 16 crashes per year.

This, of course, is hardly the end of the process. A monetary value of the 16 reduced crashes per year must be determined. The most comprehensive crash cost study was published by the Federal Highway Administration in 2005 (19). Table 4.14 summarizes the average cost per highway crash categorized by severity. The table summarizes "human capital costs," which include direct costs of medical care, emergency services, property damage, and lost productivity. The category of "comprehensive costs" includes these, but adds costs associated with long-time losses in quality of life. The costs in Table 4.14 have been updated using the CPI to reflect 2012 dollars.

Table 4.14 Monetary Value of Crashes by Severity (\$2012)

| Crash Severity | Human Capital Costs ${ }^{\mathbf{1}}$ (\$/Crash) | ${\text { Comprehensive } \text { Costs }^{\mathbf{1}} \text { (\$/Crash) }}^{\text {Fatal }}$ |
| :---: | :---: | :---: |
| $\$ 1,613,400$ | $\$ 5,192,700$ |  |
| Disabling Injury | $\$ 144,300$ | $\$ 279,800$ |
| Evident Injury | $\$ 54,300$ | $\$ 102,300$ |
| Possible Injury | $\$ 36,800$ | $\$ 58,200$ |
| Property Damage Only (PDO) | $\$ 8,300$ | $\$ 9,600$ |

1. Rounded to the nearest $\$ 100$.

The Highway Safety Manual gives the proportion of crashes on two-lane, twoway rural highways that fall into each of these categories: $1.3 \%$ Fatal, $5.4 \%$ Disabling Injury, 10.9\% Evident Injury, 14.5\% Possible Injury, and 67.9\% PDO.

Thus, the average cost per crash for two-lane, two-way rural highways would be (using comprehensive cost for the example):

| $0.013 \times 5,192,700$ | $=$ | $\$ 67,505$ |
| :--- | :--- | :--- |
| $0.054 \times 279,800$ | $=$ | $\$ 15,109$ |
| $0.109 \times 102,300$ | $=$ | $\$ 11,151$ |
| $0.145 \times 58,200$ | $=$ | $\$ 8,439$ |
| $0.679 \times 9,600$ | $=$ | $\$ 6,518$ |
| Total | $=$ | $\$ 108,722$ |

Thus, improving the two-way, two-lane rural highway to $12-\mathrm{ft}$ lanes and $6-\mathrm{ft}$ shoulders decreases the number of annual accidents by 16 , and saves:

$$
16 * 108,722=\$ 1,739,552 / y r
$$

This annual amount can be subjected to a present worth analysis for the service life of the improvement to obtain an estimate of how much could be reasonably spent on implementing the wider lanes and shoulders proposed.

Clearly, this is an example of the type of analysis that is made possible in the Highway Safety Manual. The manual contains data and predictive information on a wide variety of facility types, from freeways to intersections, relative to the crash-reductions that could be expected from a wide variety of potential improvements. The manual should be consulted directly for information on other facility types and improvements.

Of course, information on the occurrence and costs of highway crashes are available from a wide variety of sources. For general information on rates, the Fatality Analysis Reporting System (FARS) and General Estimates System (GES), both run by the Federal Highway Administration (20), provide annual updates on the number, types, and severity of highway crashes across the U.S., and by state. While some of the data relates to specific geometric and control features of a site, most of it is more general, focusing on overall rates and general severity characteristics. Table 14.15 compares the number of crashes in 2000 and 2010, as well as the distribution of crash severity over that time.

Table 4.15 General National Crash Data from FARS/GES

| Year | Total Number | Crash Severity (\%) |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | of Crashes | Fatal | Injury | PDO |
| 2000 | $6,394,000$ | $0.60 \%$ | $32.37 \%$ | $67.03 \%$ |
| 2010 | $5,419,000$ | $0.56 \%$ | $28.46 \%$ | $70.99 \%$ |

(Source: Modified from FARS/GES 2010 Data Summary, U.S. Department of Transportation, Washington DC, 2010, Exh. 2)
A number of states also publish annual studies of crash occurrence and/or costs. Many of these are available on-line. New York State, for example, publishes extensive information on crash severity distributions and costs for various types of highway facilities in the NYS highway system. While the information does not isolate the costs related to specific geometric and/or control features, it provides an overview of crash costs related to highway types. An illustrative sample of the data (for 2011) is shown in Table 4.16 (21).

The information in Table 4.16 would have to be paired with information on crash rates at the same facility type. New York State provides these in other data bases. The information in Table 4.16 is only a brief summary of what is available in the full NYS tables. Many more facility types are included, as are separate average costs for fatal, injury, and property damage only accidents, and distributions by crash type for each type of facility shown. The main point is that many states provide detailed crash occurrence and crash cost data on an annual basis that can be used in estimating crash costs for any particular highway segment.

Table 4.16 Average Cost Per Crash for New York State (2011)

| Type of Highway ${ }^{1}$ | Average Cost/Crash ${ }^{2}$ (\$/Crash) | Type of Highway | Average Cost/Crash ${ }^{2}$ (\$/Crash) |
| :---: | :---: | :---: | :---: |
| Full Access, Divided, Rural | \$24,400 | Free Access, Undivided, Rural | \$40,300 |
| Full Access, Undivided, Rural | \$34,600 | Free Access, Divided, Urban | \$43,300 |
| Full Access, Divided, Urban | \$37,200 | Free Access, Undivided Urban | \$36,200 |
| Full Access, Undivided, Urban | \$50,200 | 3-Leg Rural Signalized Int. | \$45.400 |
| Partial Access, Divided, Rural | \$27,000 | 4+ Leg Rural Signalized Int. | \$58,600 |
| Partial Access, Undivided, Rural | \$38,700 | 3-Leg Urban Signalized Int. | \$40,200 |
| Partial Access, Divided, Urban | \$45,100 | 4+Leg Urban Signalized Int. | \$38,400 |
| Partial Access, Undivided, Urban | \$38,700 | 4+Leg Rural Int., No Control | \$58,600 |
| Free Access, Divided, Rural | \$36,400 | 4+Leg Urban Int., No Control | \$38,400 |

1. Full Access = Full Control of Access; Partial Access = Partial Control of Access; Free-Access = No Access Control.
2. Includes intersection accidents in average for segment types that include intersections.

The cost of highway accidents is a significant element in many analyses involving highway improvements. The Highway Safety Manual is the fundamental reference on the impact of various design, control, and operational features on crash occurrence, and on costs. There are, however, other sources of information, primarily those available from state and local highway agencies.

### 4.8 Highway System Costs

Highway system costs are the costs borne directly by the state or local agency having jurisdiction of the highway(s) under consideration. They generally fall into four broad categories:

- Planning and design costs.
- Construction costs.
- Maintenance costs.
- Operating costs.

Planning and design costs involve pre-construction activities that determine what projects are to be undertaken, and the specific characteristics of those projects. Final design always involves a specific improvement, and design costs can easily be associated with particular projects. Many planning tasks, however, are more general, and may not apply to any one given project or alternative. In some cases, these costs would have to be distributed amongst a variety of projects and alternatives involved in planning activities. These costs occur prior to the start of construction, and may be summarized as a present worth for some defined point in time.

Construction costs are those involved in the building of a particular project. They include right-of-way acquisition and all construction activities such as earthwork, structures, drainage systems, control devices, etc. Construction is not a "point" activity. Depending upon the size of the project, construction may take place over days, weeks, months, or years. They occur, however, once for each project.

Maintenance costs are ongoing. They fall into two categories: annual or periodic costs. Such regular activities as the repair of potholes, plowing and de-icing activities, maintenance of control devices (markings, signs, and signals), maintenance of lighting, mowing the grass and other roadside maintenance activities. These are generally expressed as annual costs that continue throughout the life of the project. Periodic costs could include repaving, major structural inspections and repairs and similar items.

Operating costs are similar to maintenance costs, in that they occur on an ongoing basis. They involve items that are technically not being "fixed," but are necessary to "operate" the highway. These would include, for example, the power costs associated with operating signals and lighting. Because they are ongoing, operating costs are most often presented as annual recurring costs.

Such costs are best estimated using local data and taking into account the precise characteristics of the proposed project. Generalizations are hard to draw, and costs very much depend upon the local conditions that apply to the specific project(s) under review.

In 1989, the U.S. House of Representatives Committee on Public Works and Transportation challenged the Department of Transportation to examine the "costs, benefits, and national economic implications associated with a broad array of highway investment options" (22). The result of this challenge was the development of a computer tool to assist in making engineering economic analyses of various highway project alternatives. The tool is the Highway Economic Requirements System (HERS), which is monitored by the Federal Highway Administration (23).

HERS software is available to states. Cambridge Systematics was heavily involved with the FHWA in creating and implementing the system, which can be used to conduct a complete comparison of highway alternatives based upon incremental life-cycle benefit-cost analysis. The software reflects current information on all aspects of economic analysis, including user, system, and external costs. As such, it is a significant source of information for highway construction, maintenance, and operation costs.

Table 4.17 shows costs for a variety of highway improvements. The original costs were based upon 2008 dollars, but have been updated using the CPI to represent 2012.

Table 4.17 Typical Highway Improvement Costs (HERS) - 2012 (\$1,000/Lane-Mile)

|  | Type of Improvement |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Reconstruct | Reconstruct | Resurface | Resurface | Improve | Add | Add | New | New |
| Highway | Existing | \& Widen | Existing | \& Widen | Shoulder | Lane - | Lane - | Alignment - | Alignment - |
| Type | Lane | Existing | Lane | Existing |  | Normal | High | Normal | High |
|  |  | Lane |  | Lane |  | Cost | Cost | Cost | Cost |
| RURAL HIGHWAYS |  |  |  |  |  |  |  |  |  |
| Interstate |  |  |  |  |  |  |  |  |  |
| Level | \$1,204 | \$1,843 | \$427 | \$1,043 | \$80 | \$2,370 | \$3,285 | \$3,285 | \$3,285 |
| Rolling | \$1,235 | \$2,067 | \$455 | \$1,201 | \$131 | \$2,569 | \$4,158 | \$4,158 | \$4,158 |
| Moutainous | \$2,705 | \$3,919 | \$673 | \$1,990 | \$273 | \$7,999 | \$9,364 | 99,364 | 99,364 |
| Other Principal Arterial |  |  |  |  |  |  |  |  |  |
| Level | $\$ 964$ | \$1,439 | \$343 | $\$ 871$ | \$53 | \$1,899 | \$2,717 | \$2,717 | \$2,717 |
| Rolling | $\$ 991$ | \$1,625 | \$383 | $\$ 989$ | \$67 | \$2,033 | \$3,281 | \$3,281 | \$3,281 |
| Mountainous | \$2,231 | \$3,157 | \$540 | \$1,917 | \$117 | \$7,175 | \$8,263 | \$8,263 | \$8,263 |
| Minor Arterial |  |  |  |  |  |  |  |  |  |
| Level | $\$ 847$ | \$1,386 | \$304 | $\$ 811$ | \$50 | \$1,725 | \$2,423 | \$2,423 | \$2,423 |
| Rolling | $\$ 938$ | \$1,517 | \$327 | \$1,009 | $\$ 92$ | \$1,978 | \$3,120 | \$3,120 | \$3,120 |
| Mountainous | \$1,731 | \$2,300 | \$450 | \$1,917 | $\$ 208$ | \$6,057 | \$7,269 | \$7,269 | \$7,269 |
|  | URBAN HIGHWAYS |  |  |  |  |  |  |  |  |
| Freeways/Expressways |  |  |  |  |  |  |  |  |  |
| Small Urban | \$2,082 | \$3,007 | $\$ 505$ | \$2,370 | $\$ 93$ | \$3,772 | \$12,348 | \$5,084 | \$17,354 |
| Small Urbanized | \$2,100 | \$3,338 | \$598 | \$2,451 | \$123 | \$4,149 | \$13,541 | \$6,852 | \$23,393 |
| Large Urbanized | \$3,437 | \$5,155 | \$802 | \$3,796 | \$463 | \$6,898 | \$23,135 | \$10,051 | \$34,312 |
| Major Urbanized | \$6,875 | \$10,310 | \$1,329 | \$7,367 | $\$ 926$ | \$13,796 | \$57,527 | \$20,102 | \$76,900 |
|  |  |  |  |  |  |  |  |  |  |
| Small Urban | \$1,769 | \$2,620 | \$424 | \$2,168 | $\$ 94$ | \$3,206 | \$10,473 | \$4,008 | \$13,679 |
| Small Urbanized | \$1,790 | \$2,803 | \$501 | \$2,266 | \$126 | \$3,474 | \$11,390 | \$4,945 | \$16,878 |
| Large Urbanized | \$2,623 | \$4,005 | \$630 | \$3,317 | \$404 | \$5,084 | \$16,985 | \$6,787 | \$23,168 |
| Major Urbanized | \$5,248 | \$8,009 | \$1,016 | \$6,633 | $\$ 808$ | \$10,167 | \$39,413 | \$13,574 | \$58,762 |
|  |  |  |  |  |  |  |  |  |  |
| Small Urban | \$1,336 | \$1,931 | \$310 | \$1,640 | \$68 | \$2,368 | \$7,669 | \$2,892 | \$9,871 |
| Small Urbanized | \$1,351 | \$2,023 | \$353 | \$1,655 | $\$ 83$ | \$2,495 | \$8,106 | \$3,548 | \$12,112 |
| Large Urbanized | \$1,806 | \$2,723 | \$433 | \$2,263 | \$227 | \$3,459 | \$11,484 | \$4,618 | \$15,761 |
| Major Urbanized | \$3,613 | \$5,447 | \$720 | \$3,423 | \$454 | \$6,917 | \$39,413 | \$9,236 | \$48,740 |

(Source: "2010 Status of the Nation's Highways, Bridges, and Transit: Conditions and Performance," Federal Highway Administration, Washington D.C., 2010, Table A4, modified).

The costs shown in Table 14.17 are, as in previous instances, illustrative. For actual economic analyses, local information and cost estimates on a projectspecific basis would have to be developed for use. For rural highways, HERS separates facilities by function and terrain. In urban areas, functional classifications are used with urban area size categories by population:

Small Urban:
Small Urbanized:
Large Urbanized:
Major Urbanized:

Population 5,000 to 49,999
Population 50,000 to 200,000
Population 200,000 to approx. 1,000,000
Population greater than approx. $1,000,000$

The category of "major urbanized" was added after the collection of cost data, and the precise cut-off point is not clearly defined.

Where cost estimates are shown for both "normal" and "high" cost ranges, "normal" is intended to represent cases in which sufficient right-of-way is available (or can be readily acquired at low cost) to allow additional lanes or realignment to take place. "High" cost situations are those in which adding lanes or realigning the highway would require construction of either temporary or permanent parallel facilities, double-decking, tunneling, or the purchase of new right-of-way at significant cost.

To illustrate how data such as that presented in Table 14.17 would be used, consider the case of a project on a 6-mile stretch of a 6-lane freeway in a large urban area that would: (1) add one lane in each direction, (2) widen the 6 existing lanes from 11 ft to 12 ft , and (3) provide an improved shoulder area on each side of the freeway. What would construction cost for such a project be?

From Table 14.17, the following data can be found:
(1) The cost to add a lane, assuming normal costs, would be $\$ 6,898,000$ per lanemile for 2 lanes;
(2) The cost to widen and repave existing lanes would be $\$ 3,796,000$ per lane-mile for 6 lanes;
(3) The cost to improve shoulders would be $\$ 463,000$ per lane-mile for 2 shoulder lanes.

Therefore, the estimated construction cost for the proposed improvements would be:

$$
C=(6,898,000 * 2)+(3,796,000 * 6)+(463,000 * 2)=\$ 37,498,000
$$

This is, of course, illustrative. Estimates based upon the specific conditions of the proposed project would have to be used to develop a more appropriate estimate of the total cost.

Another source of interesting information on highway costs is the Florida Department of Transportation (24). The FDOT specifications and Estimates Office maintains a set of "generic cost per mile models" that can be used for general reference purposes, although users are cautioned not to use them to predict specific or future estimates. Nevertheless, they do provide another source of illustrative costs. Table 4.18 shows a sampling of the costs indicated for 2012.

Unlike Table 14.17, FDOT presents costs in terms of \$/centerline mile, not per lane-mile. Thus, the costs in Tables 14.17 and 14.18 are not directly comparable.

Table 4.18 Construction and Maintenance Costs - FDOT Generic Cost/Mile Models ( $\$ 1,000 /$ Centerline Mile)

| Facility Type | 2-Lane | 4-Lane | 6-Lane |
| :---: | :---: | :---: | :---: |
| NEW CONSTRUCTION ${ }^{1}$ |  |  |  |
| Rural Freeway | NA | \$5,422 | \$6,307 |
| Rural, Undivided | \$2,388 | \$3,424 | NA |
| Rural, Divided Arterial | NA | \$4,488 | \$5,481 |
| Rural, Additional Lane, Freeway | NA | \$644 | \$644 |
| Rural, Additional Lane, Non-Freeway | \$582 | \$582 | \$582 |
| Suburban, Divided Arterial | NA | \$4,783 | NA |
| Urban Freeway | NA | \$8,875 | \$9,858 |
| Urban Undivided Arterial | \$4,794 | \$7,123 | NA |
| Urban, Additional Lane, Arterial | \$1,664 | \$1,644 | \$1,644 |
| Urban, Additional Lane, Freeway | NA | \$703 | \$703 |
| MILL/RESURFACE |  |  |  |
| Rural Freeway | NA | \$1,180 | \$1,667 |
| Rural, Undivided | \$432 | \$991 | NA |
| Rural, Divided Arterial | NA | \$1,052 | \$1,480 |
| Rural, Additional Lane, Freeway | NA | \$276 | \$276 |
| Rural, Additional Lane, Arterial | \$224 | \$224 | \$224 |
| Urban, Undivided | \$421 | \$830 | NA |
| Urban, Divided Arterial | NA | \$846 | \$1,315 |
| Urban, Additional Lane, Arterial | \$176 | \$176 | \$176 |
| WIDENING (Adding 1 Lane in Each Direction) ${ }^{\mathbf{2}}$ |  |  |  |
| Rural, Freeway (Addl Lanes in Median) | NA | \$3,670 | \$4,003 |
| Rural, Freeway (Addl Lane on Outside) | NA | \$3,394 | NA |
| Rural, Undivided | \$2,029 | NA | NA |
| Rural, Divided | NA | \$2,275 | \$2,537 |
| Urban, Freeway | NA | \$6,008 | \$6,505 |
| Urban, Undivided | \$3,758 | NA | NA |
| Urban, Divided | NA | \$3,526 | \$4,617 |

1. Includes typical shoulders and bike lanes (in some cases).
2. Includes repaving remaining lanes and shoulders.

Again, the costs shown in Table 4.18 are illustrative. FDOT data includes other categories as well, and is updated annually. If a 10 -mile stretch of rural 2-lane highway was being upgraded to an undivided 4-lane highway, the estimated cost from Table 4.18 would be:

$$
C=\$ 2,029,000 * 10=\$ 20,290,000
$$

When dealing with construction, maintenance, and improvement project costs, it is imperative that local site-specific estimates be developed. All of these costs can be greatly affected by local and site specific conditions. Also, none of the costs in this section include acquisition costs for rights-of-way that may be required. Land values are particularly subject to local markets and conditions.

### 4.9 Public Transportation Costs

The costs associated with the provision of public transportation services are varied and complex. Firstly, "public transportation" is not a single mode or system, but a collection of modes and systems. In general, public transportation services can be divided into four major types or classes:

1. Heavy rail rapid transit.
2. Light rail.
3. Bus.
4. Demand-responsive.

Heavy rail systems exist in several of the nation's major cities, and involve separated rights-of-way, often using tunnels (subways) or structures (elevated), and in some cases, at-grade rights-of-way. Some systems have been in place for a very long time, e.g., New York, Chicago, and Boston. Others are newer, e.g., Washington D.C. and San Francisco. Commuter railroads fall into this general category, but differ in that they are classified as railroads (falling under the Federal Railway Administration), as opposed to urban rapid transit. Heavy rail systems generally feature high speeds, and are capable of carrying very large numbers of passengers. Along New York's E-F train corridor, for example, the express tracks often carry up to 50,000 passengers per hour (on just one track). Such high capacity systems, however, are not needed in many areas.

Light rail systems were referred to as "trolleys" in their early history, although modern light rail systems differ in some ways from traditional trolleys. These systems involve railed vehicles that may travel on separated rights-of-way (in some cases in tunnels or on structures), or may travel down arterials and streets, sharing the right of way with other vehicles. Such systems can be molded to meet a variety of demand configurations, but typically carry $10,000-20,000$ passengers per hour on a track.

The vast majority of public transportation systems in the United States use buses. A typical bus can fit from $20-50$ seated passengers, and accommodate $50 \%$ more as standees. Buses are extremely flexible. Local buses use existing streets and highways, mixing with other vehicular traffic. In some cases, exclusive bus lanes are provided to expedite their progress. Separate busways are, in some cases, provided. Buses can also be configured to provide express services. Such services usually have a defined pick-up area (where there the bus operates as a local bus), and a defined drop off area (usually in a Central Business District), operating without stops between the two, often using freeways and expressways. Bus schedules can vary widely, from as little as one bus every half-hour or hour, to buses scheduled minutes apart. Hillside Avenue (in Queens, NYC) has the highest observed volume of buses on a single city street - approximately 180 buses/hr. These buses carry almost 12,000 people on a single street, although it
should be noted that service involves several bus routes that use the same street. Passenger volumes in more typical cases carry $100-2,000$ people/hour on a single route.

Demand-responsive services are not tied to fixed routes, and fill the gap between taxi and local bus services. They generally involve smaller vehicles - from small buses to vans - that operate on an on-call basis. Unlike taxis, however, they serve multiple riders going to and from closely-spaced origins and destinations.

The costs of these systems vary considerably. As has been noted previously, public transportation differs from highway systems in that the vehicle is provided and operated by the system owner. The public transportation user invests a fare and his/her travel time. The primary source of information on public transportation costs is the American Public Transportation Association (APTA), which maintains a database on capital, maintenance, and operating costs of public transportation operators.

### 4.9.1 Construction Costs

It is virtually impossible to provide "typical" numbers for the cost of constructing public transportation services. One of the largest construction projects in recent time is the ongoing building of the Second Avenue Subway in New York City. This project was first proposed as part of a massive subway expansion program in 1919. It has been funded by bond issues three times, with money being diverted to other needs of the system. Now, Phase I of a 3-phase plan to build the subway is underway. The first stage consists of 3 miles of two-track subway and four stations, with a projected cost of $\$ 4.45$ billion -- $\$ 1.48$ billion $/ \mathrm{mile}$. Phase I is expected to be complete, and is scheduled to open in 2016. The total 4 -phase project will produce an 8.5 -mile subway at a projected cost of $\$ 17$ billion.

The project is being built using deep tunneling techniques through bedrock, with more traditional cut-and-cover work being done to build the stations, and the connections between stations and the street. After tunneling is done, and the tunnel structures are built, track will have to be laid, control, power, and communications systems installed, ventilation provided, etc. Any other infrastructure encountered - sewer and water lines, power lines, etc. - would have to be relocated before proceeding. It is a massive and complex project, accompanied by massive costs.

On the other hand, simple busways can be built at costs similar to those indicated for roadways in the previous section. Therefore, no transit capital costs should be considered "typical" or representative.

Table 4.19 summarizes the total 2012 capital expenditures of the public transportation industry by mode and category (25).

Table 4.19 Public Transportation Capital Expenditures for 2010 (\$ Million)

| Category of Expense | Total Industry Capital Expenditures |
| :--- | :---: |
| Rolling Stock Purchases - Total | $\mathbf{\$ 5 , 2 0 1 . 0}$ |
| Bus | $\$ 2,598.3$ |
| Commuter Rail | $\$ 409.0$ |
| Heavy Rail | $\$ 881.3$ |
| Light Rail | $\$ 328.4$ |
| Demand Responsive | $\$ 694.5$ |
| Service Vehicles | $\$ 91.5$ |
| Other | $\$ 197.9$ |
| Guideways | $\mathbf{\$ 6 , 2 8 7 . 1}$ |
| Passenger Stations | $\mathbf{\$ 2 , 8 2 7 . 3}$ |
| Administrative Buildings | $\mathbf{\$ 3 1 8 . 4}$ |
| Maintenance Facilities | $\mathbf{\$ 1 , 0 6 2 . 5}$ |
| Other Capital Expenditures - Total | $\mathbf{\$ 2 , 1 2 8 . 2}$ |
| Fare Collection Equipment | $\$ 190.9$ |
| Communication \& Information Systems | $\$ 1,195.0$ |
| Other | $\$ 742.3$ |
| TOTAL CAPITAL EXPENDITURE | $\mathbf{\$ 1 7 , 8 2 4 . 5}$ |

(Source: Compiled from 2012 Public Transportation Fact Book, American Public Transportation Association, Washington D.C., September 2012, Appendix A, Tables 46. 47. and 48).

### 4.9.2 Rolling Stock

The American Public Transportation Association provides some recent data on the cost to purchase public transportation vehicles in 2010 and 2011. Table 4.20 summarizes some of the information.

Table 4.20 Average Cost of New Transit Vehicles (2010-2011) (\$)

| Type of Vehicle | Cost Per Vehicle (\$) |
| :---: | :---: |
| Commuter Rail Car (Locomotive-Hauled) | \$2,176,350 |
| Heavy Rail Car (1 level, 1 Cab) | \$1,875,793 |
| Light Rail Car (1 level, 2 Cabs) | \$3,600,000 |
| Standard Transit Bus | \$479,585 |
| Demand Responsive (Minibus/Van) | \$65,629 |

### 4.9.3 Operating and Maintenance Costs

In general, the best way to project operating and maintenance costs of a public transportation system (or portion thereof) is using the technique of "unit cost modeling." In this type of modeling, cost components of a given transit agency or operator are categorized as being best related to:

- Vehicle-miles of service.
- Vehicle-hours of service.
- No. of vehicles in peak period service (or vehicles owned).
- No. of "unlinked" passengers served.

The category of "unlinked passengers served" does not consider transfers between modes or lines. Thus, a passenger taking a bus to a rail station, then boarding the train would count as two unlinked passengers.

Depending upon the available data on cost breakdowns, each element of cost is associated with the measure that best explains it. For example, cost items such as fuel (or power) are most related to the vehicle-miles of service, although speed also has an impact. Operating labor is most related to vehicle-hours of service, while maintenance costs, including labor, are best related to vehicle-miles of service. System costs such as insurance, public information systems, administrative costs, etc., tend to be related to measures of the overall size of the system. Such variables as the number of unlinked passengers and the number of vehicles in peak period service would be of great utility for these.

Tables 4.21 and 4.22 display total operating expenditures for the public transportation industry in 2010. Table 4.21 divides costs by mode and functional classification of expenses. Table 4.22 divides costs by mode and type of expenditure.

Table 4.21 Operating Expenditures for the Transit Industry, 2010 (I) (\$ Million)

| Function | Bus | Commuter <br> Rail | Heavy <br> Rail | Light <br> Rail | Demand- <br> Resp. | Trolley- <br> Bus | Other | TOTAL |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vehicle <br> Operations | $\$ 9,949.3$ | $\$ 1,637.3$ | $\$ 2,763.6$ | $\$ 545.9$ | $\$ 1,591.3$ | $\$ 118.0$ | $\$ 403.3$ | $\$ 17,008.7$ |
| Vehicle <br> Maintenance | $\$ 3,463.7$ | $\$ 1,014.1$ | $\$ 1,084.2$ | $\$ 287.2$ | $\$ 337.2$ | $\$ 48.6$ | $\$ 138.9$ | $\$ 6,373.9$ |
| Non-Veh <br> Maintenance | $\$ 739.9$ | $\$ 716.2$ | $\$ 1,574.6$ | $\$ 249.5$ | $\$ 50.7$ | $\$ 18.9$ | $\$ 72.9$ | $\$ 3,422.6$ |
| Administration | $\$ 2,963.9$ | $\$ 693.1$ | $\$ 801.1$ | $\$ 289.9$ | $\$ 653.0$ | $\$ 57.0$ | $\$ 184.3$ | $\$ 5,731.2$ |
| Purchased <br> Transportation | $\$ 1,714.7$ | $\$ 579.0$ | $\$ 57.3$ | $\$ 131.4$ | $\$ 2,554.9$ | $\$ 0.0$ | $\$ 181.1$ | $\$ 5,218.4$ |
| TOTAL | $\mathbf{\$ 1 8 , 8 3 1 . 4}$ | $\mathbf{\$ 4 , 6 3 9 . 7}$ | $\mathbf{\$ 6 , 3 6 9 . 7}$ | $\mathbf{\$ 1 , 5 0 3 . 8}$ | $\mathbf{\$ 5 , 1 8 7 . 2}$ | $\mathbf{\$ 2 2 4 2 . 4}$ | $\mathbf{\$ 9 8 0 . 5}$ | $\mathbf{\$ 3 7 , 7 5 4 . 9}$ |

(Source: 2012 Public Transportation Fact Book, American Public Transportation Association, Washington D.C., September 2012, Table 23.).

Table 4.22 Operating Expenditures for the Transit Industry, 2010 (II) (\$ Million)

| Type | Bus | Commuter <br> Rail | Heavy <br> Rail | Light <br> Rail | Demand- <br> Resp. | Trolley- <br> Bus | Other | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  <br> Wages | $\$ 7479.4$ | $\$ 1,572.8$ | $\$ 3,147.2$ | $\$ 531.9$ | $\$ 1,136.9$ | $\$ 112.4$ | $\$ 304.9$ | $\$ 14,285.5$ |
| Fringe <br> Benefits | $\$ 5,343.6$ | $\$ 1,269.7$ | $\$ 2,556.0$ | $\$ 381.7$ | $\$ 570.5$ | $\$ 84.1$ | $\$ 140.0$ | $\$ 10,341.6$ |
| Services | $\$ 1,118.3$ | $\$ 415.1$ | $\$ 365.3$ | $\$ 222.9$ | $\$ 270.0$ | $\$ 23.8$ | $\$ 90.3$ | $\$ 2,505.7$ |
| M \& S | $\$ 2,432.3$ | $\$ 510.6$ | $\$ 406.8$ | $\$ 108.1$ | $\$ 391.6$ | $\$ 16.0$ | $\$ 175.1$ | $\$ 4,040.5$ |
| Utilities | $\$ 227$ | $\$ 319.9$ | $\$ 556.1$ | $\$ 104.3$ | $\$ 38.2$ | $\$ 5.0$ | $\$ 17.0$ | $\$ \$ 1,267.5$ |
| Insurance | $\$ 511.5$ | $\$ 117.7$ | $\$ 138.3$ | $\$ 28.3$ | $\$ 124.4$ | $\$ 8.9$ | $\$ 41.1$ | $\$ 970.5$ |
| Purchased <br> Trans. | $\$ 1,714.7$ | $\$ 579.0$ | $\$ 57.3$ | $\$ 131.4$ | $\$ 2,554.9$ | $\$ 0.0$ | $\$ 30.7$ | $\$ 5,218.4$ |
| Other | $\$ 4.7$ | $-(\$ 145.1)$ | $-(\$ 853.4)$ | $-(\$ 4.8)$ | $\$ 100.7$ | $-(\$ 7.7)$ | $\$ 30.7$ | $-(\$ 874.9)$ |
| TOTAL | $\mathbf{\$ 1 8 , 8 3 1 . 4}$ | $\mathbf{\$ 4 , 6 3 9 . 7}$ | $\mathbf{\$ 6 , 3 6 9 . 7}$ | $\mathbf{\$ 1 , 5 0 3 . 8}$ | $\mathbf{\$ 5 , 1 8 7 . 2}$ | $\mathbf{\$ 2 4 2 . 4}$ | $\mathbf{\$ 9 8 0 . 5}$ | $\mathbf{\$ 3 7 , 7 5 4 . 9}$ |

(Source: 2012 Public Transportation Fact Book, American Public Transportation Association, Washington D.C., September 2012, Table 24.).

In Tables 4.21 and 4.22, the category of "Trolley-Bus" refers to rubber-tired buses powered by electricity from an overhead centenary wire. The "Other" category contains such systems as the cable cars of San Francisco, ferry systems such as the Staten Island Ferry, and a variety of taxi-like services that do not quite meet the definition of "Demand-Responsive." In Table 4.22, the cost category of
"Purchased Transportation" refers to service subcontracted to agencies or companies other than the agency itself. For example, many demand-responsive systems will subcontract a good deal of their work to local taxi operators. The "other" expense category in Table 4.22 is a catch-all for any expenses that cannot be placed in one of the other categories. In some cases, the number shown is negative, indicating that some expenses are double-counted in more than one category.

Table 4.23 gives national operating statistics for the transit industry in 2010.
Table 4.23 Operating Statistics for the Transit Industry, 2010

| Mode | Revenue <br> Veh-Mi <br> (Millions) | Revenue <br> Veh-Hrs <br> (Millions) | Average <br> Speed <br> $(\mathbf{m i} / \mathbf{h})$ | No. of <br> Rev. Veh. <br> In Peak Service | Unlinked <br> Passengers* <br> (Millions) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bus | $2,090.9$ | 162.3 | 12.9 | 55,580 | $5,003.1$ |
| Commuter Rail | 317.6 | 9.7 | 32.9 | 6,143 | 472.8 |
| Heavy Rail | 647.4 | 32.0 | 20.2 | 9,198 | $3,418.1$ |
| Light Rail | 92.0 | 6.2 | 15.0 | 1,494 | 443.5 |
| Demand-Responsive | $1,447.7$ | 96.8 | 9.7 | 56,677 | 186.8 |
| Trolley-Bus | 11.7 | 1.6 | 7.1 | 421 | 93.5 |
| Others | 222.9 | 8.8 | 25.3 | 14,505 | 205.7 |
| TOTAL | $\mathbf{5 , 4 5 5 . 1}$ | $\mathbf{3 1 7 . 4}$ | $\mathbf{1 7 . 2}$ | $\mathbf{1 4 , 0 1 9}$ | $\mathbf{9 , 8 2 3 . 5}$ |

*Estimated as revenue passenger-miles (APTA Table 6) divided by average trip length (APTA Figure 4).
(Source: Compiled from 2012 Public Transportation Fact Book, American Public Transportation Association, Washington D.C., September 2012, Tables 6, 7, and 8, and Figure 4).

The data from Table 4.21 and 4.23 can be used to develop "unit cost" models for the various modes of public transportation on a national average basis. In actual practice, a unit cost model should be calibrated for the specific public transit service and agency involved in any given project alternative.

Consider the calibration of a national average unit cost model for bus operations in the U.S. Each functional cost element shown in Table 4.20 should be assigned to the operating variable that most effectively explains the expenditure. Thus, for U.S. bus systems:

| Cost Element | Cost | Best-Related Operating Statistic |
| :--- | :--- | :--- |
| Vehicle Expenses | $\$ 9,949,300,000$ | $162,300,000$ Veh-Hrs |
| Vehicle Maintenance | $\$ 3,463,700,000$ | $2,090,900,000$ Veh-Mi |
| Non-Veh Maint. | $\$ 739,900,000$ | $5,003,100,000$ Passengers |
| Administration | $\$ 2,963,900,000$ | $5,003,100,000$ Passengers |
| Purchased Trans. | $\$ 1,714,700,000$ | $5,003,100,000$ Passengers |

Vehicle expenses include all operating labor costs and fuel, both of which are heavily dependent on time. Therefore, these expenses are assigned to the explanatory variable "vehicle-hours." Vehicle maintenance, which also has a large labor component, is, however, mostly dependent upon usage. Therefore, these expenses are assigned to "vehicle-miles" as an explanatory variable. The other categories reflect more on the general size of the operation in question. Both "vehicles in peak hr service" and "unlinked passengers" describe the overall size of the operation, and either could be used. We have chosen to use passengers in this case. The "unit cost" of each expense element can now be computed:

$$
\begin{aligned}
& \text { Vehicle Expenses }=\frac{9,949,300,000}{162,300,000}=\$ 61.30 \text { per veh }-h r \\
& \text { Vehicle Main. }=\frac{3,463,700,000}{2,090,900,000}=\$ 1.66 \text { per veh }-\mathrm{mi} \\
& \text { Non }- \text { Veh Main. }=\frac{739,900,000}{5,003,100,000}=\$ 0.15 \text { per passenger } \\
& \text { Ad } \min \text { istration }=\frac{2,963,900,000}{5,003,100,000}=\$ 0.59 \text { per passenger } \\
& \text { Pur. } \text { Trans. }=\frac{1,714,700,000}{5,003,100,000}=\$ 0.34 \text { per passenger }
\end{aligned}
$$

Based on these computations, a model for the annual cost of a bus transit operation would be:

$$
\begin{align*}
& A C_{i}=61.30 V H_{i}+1.66 V M_{i}+(0.15+0.59+0.34) P_{i} \\
& A C_{i}=61.30 V H_{i}+1.66 V M_{i}+1.08 P_{i} \tag{4.9}
\end{align*}
$$

where: $\mathrm{AC}_{\mathrm{i}}=$ annual cost of bus operations, system $\mathrm{i}(\$)$. $\mathrm{VH}_{\mathrm{i}} \quad=\quad$ annual veh-hrs of bus service, system i (hrs). $\mathrm{VM}_{\mathrm{i}}=\quad$ annual veh-mi of bus service, system i (mi). $\mathrm{P}_{\mathrm{i}}=\quad$ annual unlinked revenue passengers, system i.

Assume that a new set of bus routes is to be added to a system that can be adequately represented by the national average model (based upon 2010 costs) of Equation 4-8. If the additional routes have the effect of adding 100,000 vehicle miles of revenue service at an average speed of $15 \mathrm{mi} / \mathrm{h}$, and 250,000 additional unlinked passengers to the system, what increase in annual costs are expected?

First, the number of vehicle-hours is estimated as the number of vehicle-miles divided by the average speed, or $100,000 / 15=6,667$ vehicle-hours. Then:

$$
\begin{aligned}
& A C=(61.30 * 6,667)+(1.66 * 100,000)+(1.08 * 250,000) \\
& A C=408,687.10+166,000.00+270,000.00=\$ 844,687.10 / y r
\end{aligned}
$$

As previously noted, in practice, a model would be calibrated for the subject transit system (if it exists), or for a similar existing service in the same general area as the proposed project. The data shown here for the nation as a whole is available from APTA for most transit operations in the United States. Models should be calibrated for each mode. Thus, for a city like New York, separate models for commuter railroads, the subway system, the bus system, and any specialized systems would be developed.

### 4.10 Closing Comments

This chapter has presented a wide variety of costs associated with transportation improvement alternatives. Any given analysis, however, is unique, and may involve issues not treated here. Further, the costs shown are intended to be illustrative, and are used to demonstrate how the cost of transportation projects can be determined. In practice, it is imperative that the analyst acquire the most up-to-date cost information available, and that the cost data used are actually applicable to the specific case at hand.

## Problems

## Problem 4.1

The following highway maintenance and operations cost estimates are available for a given jurisdiction. The data, however, come from several different years. Using the Consumer Price Index, adjust these data to reflect 2012 dollars:

| Mowing Operations: | $\$ 1,015 /$ mile (2008) |
| :--- | :--- |
| Pothole Repairs: | $\$ 2,530 / \mathrm{mile}(2010)$ |
| Light and Signal Power: | $\$ \quad 575 / \mathrm{mile}(2011)$ |

## Problem 4.2

An existing 10-mile stretch of urban freeway services the following demand volumes each day, with the average speeds and levels of service shown in the table below. Fifteen percent of the vehicles are commercial trucks and buses, while the remaining $85 \%$ consist of private passenger cars. The average private car

| Average Daily Vol <br> $($ Veh/Day $)$ | Average Speed <br> $(\mathbf{m i} / \mathbf{h})$ | Level of Service |
| :---: | :---: | :---: |
| 7,000 | 65 | B |
| 10,000 | 60 | B |
| 25,000 | 53 | D |
| 30,000 | 51 | D |
| 32,000 | 49 | E |
| $\mathbf{1 0 4 , 0 0 0}$ |  |  |

If prevailing wages in the area of the freeway are $\$ 27,85 / \mathrm{h}$ for individuals and $\$ 46.35 / \mathrm{h}$ for commercial vehicle drivers, what is the estimated cost of travel time expended by all users of the facility in one year?

## Problem 4.3

If the average price for fuel for a private automobile is $\$ 3.75 /$ gallon, how much would fuel cost for a trip of 25 miles, where $1 / 2$ is made at a speed of $30 \mathrm{mi} / \mathrm{h}$ and the other $1 / 2$ at $50 \mathrm{mi} / \mathrm{h}$ ?

## Problem 4.4

A 10 -mile segment of rural freeway carries an average annual daily traffic (AADT) of 40,000 veh/day, where $10 \%$ of the traffic is trucks and the rest, cars. The current alignment, which has many grades, vertical curves, and horizontal curves allows traffic to maintain an average speed of $45 \mathrm{mi} / \mathrm{h}$. A plan has been developed to provide for an improved alignment that will reduce the length of the segment to 8 miles, and increase average speeds to $60 \mathrm{mi} / \mathrm{h}$. What savings in vehicle operating and maintenance costs and travel time costs will the improvement produce? The average wage is $\$ 22.80$ for the area for general users, and $\$ 35.00 / \mathrm{hr}$ for commercial vehicle drivers. Average auto occupancy is 1.3 people per car.

## Problem 4.5

The average crash frequency for a fifteen-mile stretch of 4-lane suburban undivided highway under base conditions ( $12-\mathrm{ft}$ lanes, 6 - ft shoulders, $55-\mathrm{mi} / \mathrm{h}$ freeflow speed) may be estimated as:

$$
N_{\text {sppx }}=\frac{365 * A A D T^{*} L}{950,000 * e^{0.41}}
$$

The existing fifteen-mile segment, however, only has $11-\mathrm{ft}$ lanes, 2 - ft shoulders, and a $45 \mathrm{mi} / \mathrm{h}$ free-flow speed. Local calibrations have determined that Crash Modification Factors (CMFs) for the three non-standard conditions of the existing site are:

$$
\begin{array}{ll}
\mathrm{CMF}_{\text {lane width }} & =1.21 \\
\mathrm{CMF}_{\text {shoulder }} & =1.19 \\
\mathrm{CMF}_{\text {free-flow speed }} & =1.35
\end{array}
$$

It may be assumed that $90 \%$ of all crashes are affected by the lane width, shoulder width, and free-flow speed conditions.

It is proposed to upgrade the fifteen-mile segment to bring it to base conditions. What will be the observed crash frequency per year for the existing and proposed highway?

Assume that in both cases, $1.5 \%$ of all accidents are fatal, $5 \%$ involve disabling injuries, $10 \%$ involve evident injuries, and $2 \%$ involve possible injuries. Remaining accidents fall into the category of Property Damage Only (PDO).

What will be the annual savings in crash costs due to the proposed improvements to the facility?

## Problem 4.6

What is the estimated cost to:
a. Resurface a 10-mile segment of 4-lane freeway in a large urbanized area?
b. $\quad$ Resurface and widen a 2-mile stretch of principal arterial of 4 lanes in a major urbanized area?
c. The cost to add a lane to a 20-mile stretch of 4-lane rural interstate highway in rolling terrain?

## Problem 4.7

How much would it cost to build 30 miles of new rural 6-lane freeway?

## Problem 4.8

Construct a national average unit cost model for light rail operations in the U.S.

## Problem 4.9

Using the model calibrated in Problem 4.8 and other information as needed, what would it cost (per year) to build and operate a new light-rail line given the following information:

| Length of new line: | 20 miles |
| :--- | :--- |
| Cost for right-of-way: | $\$ 12,500 / \mathrm{mile}$ |
| Construction cost: | $\$ 1,250,000 / \mathrm{mile}$ |
| No. of revenue vehicles needed: | 15 |
| Cost/light-rail vehicle: | $\$ 2,950,000 / \mathrm{car}$ |
| Veh-miles of annual operation: | 950,000 veh-miles |
| Average speed of operation: | $18.4 \mathrm{mi} / \mathrm{h}$. |
| No. of unlinked annual passengers: | $21,000,000$ |

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## Chapter 5 <br> Illustrations of Complex Economic Analyses

The word "complex" in the context of a textbook presentation is, at best, relative. It would be impossible to completely illustrate an application that involves all of the complexity that can be found in real cases of alternative economic analysis in the transportation field. This chapter, however, does present more "complex" applications to illustrate many of the techniques discussed previously for addressing complex situations.

The sample problems of this chapter touch on many elements of complexity in comparative engineering economic analysis of transportation alternatives, including, but not limited to:

- Estimating user costs and benefits,
- Estimating system costs,
- Comparing alternatives using estimated costs,
- Dealing with different service lives of alternatives and components,
- Dealing with varying demand levels for different alternatives, and
- Dealing with varying travel time values.


## Sample Problem 5.1: Comparing Pavement Types

Problem Statement: A highway department is considering two types of pavement for a new highway: Portland Cement Concrete (PCC), and Asphalt. Relevant information on the two different pavements is shown in Table 5.1.

Table 5.1 Cost Information for Sample Problem 5.1

| Item | Portland Cement <br> Concrete | Asphaltic <br> Concrete |
| :--- | :---: | :---: |
| Initial Cost (\$/mile) | $\$ 1,400,000 / \mathrm{mi}$ | $\$ 900,000 / \mathrm{mi}$ |
| Service Life (Years) | 20 | 13 |
| Repavement Cost (\$/mi) | $\$ 400,000 / \mathrm{mi}$ | $\$ 400,000 / \mathrm{mi}$ |
| Repave Service Life (Years) | 10 | 10 |
| Maintenance Costs: |  | $\$ 20,000 / \mathrm{mi} / \mathrm{yr}$ |
| Before Repavement | $\$ 10,000 / \mathrm{mi} / \mathrm{yr}$ | $\$ 20,000 / \mathrm{mi} / \mathrm{yr}$ |
| After Repavement | $\$ 20,000 / \mathrm{mi} / \mathrm{yr}$ | $3 \%$ |
| Interest Rate for Analysis | $3 \%$ |  |

Definition of Alternatives: In this case, the alternatives are clearly defined. The question is reduced to consideration of two different types of paving materials for use in the initial construction of the highway. There is no question that the facility in question will be built, so there is no "null" case to consider. The two choices are to:

1. Provide a rigid Portland Cement Concrete pavement at a cost of $\$ 1,400,000 / \mathrm{mile}$, with a service life of 20 years.
2. Provide a flexible Asphaltic Concrete pavement at a cost of $\$ 900,000 / \mathrm{mi}$, with a service life of 13 years.

In either case, subsequent repaving will be done using asphalt, and each repaving will have a service life of 10 years. The other cost differential is that during its initial service life, the Portland Cement Concrete pavement will cost $\$ 10,000 / \mathrm{mi} / \mathrm{yr}$ for maintenance, compared to $\$ 20,000 / \mathrm{mi} / \mathrm{yr}$ for asphalt.

Analysis Period: In this illustration, the selection of an appropriate analysis period is the most complicating factor. The service lives of the two pavements, plus the service lives of subsequent repavings will never result in both options being repaved in the same year. The Portland Cement Concrete pavement will be repaved in years 20, 30, 40, etc. The Asphaltic Concrete pavement will be repaved in years $13,23,33,43$, etc. A reasonable analysis period will have to be chosen, and at least one of the options will involve a residual value. "Reasonable" would involve a period of time over which other elements of the new highway structure would remain relatively stable, i.e., there would be no major reconstructions of the entire roadway anticipated within the analysis period. An analysis period of 30 years would satisfy this requirement.

Methodology: A direct method of comparison may be used, as there are no issues of variable demand involved. In this case, Present Worth values for a 30-year analysis period will be used for the comparison.

Figure 5.1 illustrates the costs involved over thirty years for both options. In Option A (Portland Cement Concrete), an initial construction cost of $\$ 1,400,000 / \mathrm{mi}$ is experienced at time $=0$. One repaving, which occurs in Year 20, costs $\$ 400,000 / \mathrm{mi}$. Maintenance costs for the first 20 years are $\$ 10,000 / \mathrm{mi} / \mathrm{yr}$. After the repaving in Year 20, maintenance costs rise to $\$ 20,000 / \mathrm{mi} / \mathrm{yr}$.

In Option B (Asphaltic Concrete), the initial construction cost is $\$ 900,000 / \mathrm{mi}$ at time $=0$. Two repavings at $\$ 400,000 / \mathrm{mi}$ occur in this option: one in Year 13, the second in Year 23. The second repaving, however, lasts until Year 33, which is 3 years beyond the analysis period. The residual value of the pavement in Year 30 will have to be deducted from the total present worth of this option. Maintenance costs are $\$ 20,000 / \mathrm{mi} / \mathrm{yr}$ throughout the analysis period.

Note that it is assumed that annual maintenance costs will be experienced even in years in which a repavement is completed.


Fig. 5.1 Costs Considered in Sample Problem 5.1

## Solution

Note that as all costs are stated on a per mile basis, the length of the highway segment being built is not relevant. As indicated, the comparison will be based upon the present worth of the total costs for each alternative for a 30 -year analysis period. All conversion factors are taken from the tables in Chapter 2 or the appendix.

Converting Costs to Present Worth for Option A
Initial Pavement Cost:
\$1,400,000.00

Repaving in Year 20:
\$ 221,470.30
$400,000 P / F_{20,3 \%}$

$$
400,000 * 0.553676=221,470.40
$$

$$
\begin{aligned}
& 10,000 P / A_{20,3 \%} \\
& 10,000 * 14.877475=148,774.75
\end{aligned}
$$

Maintenance, Last 10 Years:
\$ 94,459.33

$$
\begin{aligned}
& 20,000 * P / A_{10,3 \%} * P / F_{20,3 \%} \\
& 20,000 * 8.530203 * 0.553676=94,459.37
\end{aligned}
$$

## Total PW of Option A:

\$1,864,704.38
Converting Costs to PW for Option B:
Initial Pavement Cost:
\$ 900,000.00
Repaving in Year 13:
\$ 272,380.54

$$
\begin{aligned}
& 400,000 P / F_{13,3 \%} \\
& 400,000 * 0.680951=272,380.40
\end{aligned}
$$

Repaving in Year 23:
\$ 202,676.70

$$
\begin{aligned}
& 400,000 P / F_{23,3 \%} \\
& 400,000 * 0.506692=202,676.80
\end{aligned}
$$

Residual Value of Repaving in Year 23:

- \$ 54,645.84

$$
\begin{aligned}
& 400,000 * A / P_{10,3 \%} * P / A_{3,3 \%} * P / F_{30,3 \%} \\
& 400,000 * 0.117231 * 2.828611 * 0.411985= \\
& 54,645.84
\end{aligned}
$$

Maintenance for 30 Years:
\$ 392,008.83

$$
\begin{aligned}
& 20,000 P / A_{30,3 \%} \\
& 20,000 * 19.600441=392,008.82
\end{aligned}
$$

## Discussion

Note that in the residual analysis computation, the $\$ 400,000$ cost of repaving in Year 23 is annualized over its 10 -year service life. The "residual value" in Year 30 is the present worth of the last 3 years of this annual cost. The residual is then brought back to Year 0 for inclusion in the analysis.

The results indicate that Option B is favored over Option A. The total cost over 30 years for Option A is $\$ 1,864,704.38$ vs. $\$ 1,712,420.22$ for Option B, a difference of $\$ 152,284.16$.

## Sample Problem 5.2: Reducing the Severity of a Rural Grade

This problem involves reducing the severity of an existing 5,000-ft, $8 \%$ grade on a two-lane, two-way rural highway serving primarily intercity travel. Two alternatives, shown Figure 5.2, are proposed for this:

Option A: The total rise of the existing grade $(5,000 \times 0.08=400 \mathrm{ft})$ is cut in half while maintaining the length of the grade. This results in a $5,000-\mathrm{ft}, 4 \%$ grade, and has a capital cost of $\$ 30,000,000$.

Option B: In this option, the total rise of the grade ( 400 ft ) is maintained, and the severity is reduced by doubling the length of the grade. This produces a $10,000-\mathrm{ft}, 4 \%$ grade, and has a capital cost of $\$ 22,000,000$.

The Average Annual Daily Traffic (AADT) on the facility is 10,000 passenger cars per day, and 2,000 tractor-trailer trucks per day. Auto occupancy is an average of 1.3 persons/car; average truck occupancy is 1.0 persons/truck. The prevailing wage of passenger car drivers and passengers in the region is $\$ 32.00 / \mathrm{hr}$, while the prevailing total compensation of truck drivers is $\$ 56.00 / \mathrm{hr}$. An analysis interest rate of $4 \%$ may be used. The service life of the grade is 40 years.

Table 5.2 gives some additional information concerning average speeds and vehicle operating expenses on the highway, based upon local studies.

Table 5.2 Speed and Operating Cost Data for Sample Problem 5.2

| Grade | Passenger Cars | Trucks |
| :---: | :---: | :---: |
| Speeds $(\mathbf{m i} / \boldsymbol{h})$ |  |  |
| Level (0\%) | $55.0 \mathrm{mi} / \mathrm{h}$ | $46.0 \mathrm{mi} / \mathrm{h}$ |
| $+4 \%$ | $52.5 \mathrm{mi} / \mathrm{h}$ | $40.8 \mathrm{mi} / \mathrm{h}$ |
| $-4 \%$ | $58.3 \mathrm{mi} / \mathrm{h}$ | $50.2 \mathrm{mi} / \mathrm{h}$ |
| $+8 \%$ | $50.1 \mathrm{mi} / \mathrm{h}$ | $38.3 \mathrm{mi} / \mathrm{h}$ |
| $-8 \%$ | $59.0 \mathrm{mi} / \mathrm{h}$ | $43.5 \mathrm{mi} / \mathrm{h}$ |
| Operating Costs $(\$ /$ Veh-Mile $)$ |  |  |
| Level $(0 \%)$ | $\$ 0.58 / \mathrm{veh}-\mathrm{mi}$ | $\$ 1.47 / \mathrm{veh}-\mathrm{mi}$ |
| $+4 \%$ | $\$ 0.62 / \mathrm{veh}-\mathrm{mi}$ | $\$ 1.67 / \mathrm{veh}-\mathrm{mi}$ |
| $-4 \%$ | $\$ 0.55 / \mathrm{veh}-\mathrm{mi}$ | $\$ 1.40 / \mathrm{veh}-\mathrm{mi}$ |
| $+8 \%$ | $\$ 0.69 / \mathrm{veh}-\mathrm{mi}$ | $\$ 1.90 / \mathrm{veh}-\mathrm{mi}$ |
| $-8 \%$ | $\$ 0.57 / \mathrm{veh}-\mathrm{mi}$ | $\$ 1.75 / \mathrm{veh}-\mathrm{mi}$ |

Note that on the $-4 \%$ grade, trucks take advantage of the downgrade in both speed and operating cost. For the $-8 \%$ grade, this is not so, as many trucks will be forced to operate in low gear while descending such a severe grade.

## Definition of Alternatives

As the problem considers replacement of an existing grade, the Null Alternative, i.e., that of simply leaving the existing grade in place, must be considered. Since one of the replacement options involves lengthening the grade to $10,000 \mathrm{ft}$, all alternatives, including the Null Alternative, must consider 10,000 ft of highway. Thus, the three alternatives for consideration are:

- Null Alternative: 5,000 ft of level grade $+5,000 \mathrm{ft}$ of $8 \%$ grade.
- Option A: $5,000 \mathrm{ft}$ of level grade $+5,000 \mathrm{ft}$ of $4 \%$ grade.
- Option B: $10,000 \mathrm{ft}$ of $4 \%$ grade.

The three alternatives are shown in Figure 5.2.

(a) Null Alternative - Existing Grade

(b) Alternative A

(c) Alternative $B$

Fig. 5.2 Grade Reduction Options for Sample Problem 5.2

## Analysis Period

As the service life of the grades is 40 years, this would be the obvious analysis period to select. All of the alternatives, including the Null Alternative, may use this service life.

## Methodology

This problem involves looking at the capital costs of reducing the severity of a grade vs. the savings in travel time and vehicle operating costs that are achieved as a result. It may be assumed that annual maintenance costs for all three options are the same, and may be ignored in the comparison. The comparison may be made using the present worth of 40 years of service, or total annual cost. As most of the costs involved (travel time, operating cost) would be computed as annual costs, the Annual Cost Method is most appropriate for this case. Capital costs will have to be converted to their annual equivalents.

## Solution

Most of the complexity of this sample problem involves keeping track of all of the input information, and converting it to a convenient form for analysis. There are three components of cost for Sample Problem 5.2:

- Annual Cost of Construction
- Annual Cost of Travel Time
- Annual Cost of Vehicle Operation


## Annual Cost of Construction

The annual equivalent cost of construction for each alternative is found as:

$$
A=P * A / P_{40,4 \%}
$$

where $\mathrm{A} / \mathrm{P}_{40,4 \%}=0.050523$. Then:

$$
\begin{aligned}
& A_{\text {NULL }}=0.0 * 0.050523=\$ 0.00 / y r \\
& A_{\text {OPTION } A}=30,000,000 * 0.050523=\$ 1,515,690.00 / y r \\
& A_{\text {OPTION } B}=22,000,000 * 0.050523=\$ 1,222,506.00 / y r
\end{aligned}
$$

## Annual Cost of Travel Time

To determine the total annual cost of travel time, two things must be established:

- The monetary value of travel time ( $\$ / v e h-h r$ ), and
- The number of annual vehicle-hours of travel time involved in each of the alternatives.

The former is based upon prevailing wage rates (for private vehicles) and prevailing total compensation rates (for trucks), both of which are given in the problem statement.

Table 4.3 (Chapter 4) indicates that the value of travel time for intercity drivers and passengers is $70 \%$ of the prevailing wage. The prevailing wage for this
problem was stated as $\$ 32 / \mathrm{hr}$, with a passenger-car occupancy of 1.3 persons/car. Therefore, the value of travel time for passenger cars is:

$$
0.70 * 32 * 1.3=\$ 29.12 / \mathrm{veh}-\mathrm{hr}
$$

For all trucks, Table 4.3 indicates that travel time is valued at $100 \%$ of total compensation, which is given as $\$ 56 / \mathrm{h}$. Truck occupancy was stated as 1.0 person/truck. Thus, for trucks, the value of travel time is:

$$
1.00 * 56 * 1.00=\$ 56.00 / \text { veh }-\mathrm{hr}
$$

The total number of annual vehicle-hours of travel time involved in each of the three alternatives is based upon the AADT and the average speed of travel. The AADT values are stated as a two-direction total, while the average travel speeds, given in Table 5.2 are different for the upgrade direction and the downgrade direction. Over the course of a full year, however, it is reasonable to assume that the split of the AADT is $50 \%$ in each direction. Thus, for each grade, the total average speed in both directions may be taken as the average of the upgrade and downgrade speeds. Average speeds in both directions are computed in Table 5.3 using the data from Table 5.2.

Table 5.3 Average Speeds (Both Directions) for Sample Problem 5.2 (mi/h)

| Grade | Passenger Cars | Trucks |
| :---: | :---: | :---: |
| $0 \%$ | 55.00 | 46.0 |
| $4 \%$ | $(53.2+58.3) / 2=$ | $(40.8+50.2) / 2=$ |
|  | 55.75 | 45.5 |
| $8 \%$ | $(50.1+59.0) / 2=$ | $(43.5+38.3) / 2=$ |
|  | 54.55 | 40.9 |

AADTs for the three alternative grades include 10,000 passenger cars/day and an additional 2,000 tractor-trailer trucks per day. The number of vehicle-hours of travel time for vehicle type " i " over highway segment " j " is computed as:

$$
T_{i j}=\frac{A A D T_{i j} * L_{j} * 365}{S_{i j}}
$$

where:

$$
\begin{array}{rll}
\mathrm{AADT}_{\mathrm{ij}} & = & \begin{array}{l}
\text { average annual daily traffic, both directions, } \\
\text { vehicle type } \mathrm{i} \text {, segment } \mathrm{j} \text { (veh/yr), }
\end{array} \\
\mathrm{L}_{\mathrm{j}} & = & \begin{array}{l}
\text { length of segment } \mathrm{j}(\mathrm{mi}),
\end{array} \\
\mathrm{S}_{\mathrm{ij}} & = & \begin{array}{l}
\text { average speed of vehicle type } \mathrm{i} \text { in segment } \mathrm{j} \\
\text { (mi/h), and }
\end{array} \\
\mathrm{T}_{\mathrm{ij}} & =\quad \begin{array}{l}
\text { annual travel time, vehicle type } \mathrm{i}, \text { traversing } \\
\text { segment } \mathrm{j} \text { (veh-hrs). }
\end{array}
\end{array}
$$

The Null Alternative involves 5,000 ft of level roadway and 5,000 ft of an $8 \%$ grade. Therefore, the annual vehicle-hours of travel time is as follows:

$$
\begin{aligned}
& T_{\text {Null,PC,0\% }}=\frac{10,000 *\left(\frac{5,000}{5,280}\right) * 365}{55.00}=62,844.35 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{\text {Null,PC, } 8 \%}= \\
& T_{\text {Null,PC,Total }}=62,844.35+63,362.78=126,207.13 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{\text {Null,TR,0\% }}= \\
& T_{\text {Null,TR,8\%/ }}=\frac{2,000 *\left(\frac{5,000}{5,280}\right) * 365}{46.00}=15,028.00 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{\text {Null,TR,Total }}=15,00028.00+16,901.90=31,929.90 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr}
\end{aligned}
$$

Option A includes 5,000 ft of level roadway and $5,000 \mathrm{ft}$ of a $4 \%$ grade. Therefore, annual vehicle-hours of travel time are as follows:

$$
\begin{aligned}
& T_{A, P C, 0 \%}=\frac{10,000 *\left(\frac{5,000}{5,280}\right) * 365}{55.00}=62,844.35 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{A, P C, 8 \%}=\frac{10,000 *\left(\frac{5,000}{5,280}\right) * 365}{55.75}=61,998.91 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{A, P C, T o t a l}=62,844.35+61,998.91=124,843.26 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{A, T R, 0 \%}=\frac{2,000 *\left(\frac{5,000}{5,280}\right) * 365}{46.00}=15,028.00 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{A, T R, 8 \%}=\frac{2,000 *\left(\frac{5,000}{5,280}\right) * 365}{45.5}=15,193.14 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
& T_{A, T R, T o t a l}=15,028.00+15,193.14=30,221.14 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr}
\end{aligned}
$$

Option B is a single $10,000-\mathrm{ft}$ grade of $4 \%$. The annual vehicle-hours of travel time for this option are:

$$
\begin{aligned}
T_{B, P C, 4 \%} & =\frac{10,000 *\left(\frac{10,000}{5,280}\right) * 365}{55.75}=123,997.83 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr} \\
T_{B, T R, 4 \%} & =\frac{2,000 *\left(\frac{10,000}{5,280}\right) * 365}{45.5}=30,386.28 \mathrm{veh}-\mathrm{hrs} / \mathrm{yr}
\end{aligned}
$$

The total annual cost for travel time for each of the three alternatives is found by multiplying the number of annual vehicle-hours for cars and trucks by the appropriate value of travel time. Table 5.4 illustrates these computations.

Table 5.4 Annual Cost of Travel Time for Sample Problem 5.2

| Vehicle Class | Annual Travel Time (veh-hrs) | Value of Travel Time (\$/veh-hr) | Total Annual Cost (\$) |
| :---: | :---: | :---: | :---: |
| Null Alternative |  |  |  |
| PC | 126,207.13 | 29.12 | 3,675,151.25 |
| TR | 31,929.90 | 56.00 | 1,788,074.40 |
| Total |  |  | \$5,463,225.65 |
| Option A |  |  |  |
| PC | 124,843.91 | 29.12 | 3,635,454.66 |
| TR | 30,221.14 | 56.00 | 1,692,383.84 |
| Total |  |  | \$5,327,838.50 |
| Option B |  |  |  |
| PC | 123,997.83 | 29.12 | 3,610,816.81 |
| TR | 30,386.28 | 56.00 | 1,701,631.68 |
| Total |  |  | \$5,312,448.49 |

## Annual Cost of Vehicle Operation

The last cost element is vehicle operations over the $10,000-\mathrm{ft}$ section of two-lane highway being considered for grade reduction. The annual cost of vehicle operations is based on:

- The number of annual vehicle-miles of travel for each alternative, and
- The average operating costs of passenger cars and trucks per vehicle-mile.

As the AADT demand volume may be expected to have a directional distribution of $50 / 50$, the average operating costs of vehicles going up and down the $4 \%$ and $8 \%$ grades may be used in the determination of annual costs. These are shown in Table 5.5.

Table 5.5 Average Operating Costs (Both Directions) for Sample Problem 5.2 (\$/veh-mi)

| Grade | Passenger Cars | Trucks |
| :---: | :---: | :---: |
| $0 \%$ | 0.58 | 1.47 |
| $4 \%$ | $(0.62+0.55) / 2=$ | $(1.67+1.40) / 2=$ |
|  | 0.585 | 1.535 |
| $8 \%$ | $(0.69+0.57) / 2=$ | $(1.90+1.75) / 2=$ |
|  | 0.63 | 1.825 |

The annual operating cost for vehicle type "i" over highway segment "j" may now be computed as:

$$
A C_{i j}=A A D T_{i j} * L_{j} * 365 * M_{i j}
$$

where: $\quad \mathrm{AC}_{\mathrm{ij}}=\quad$ annual operating cost, vehicle type i , segment j (\$/yr),

$$
\mathrm{AADT}_{\mathrm{ij}}=\quad \text { average annual daily traffic, vehicle type } \mathrm{i},
$$ segment $\mathrm{j}(\mathrm{mi} / \mathrm{yr})$,

$L_{j} \quad=\quad$ length of segment $j(m i)$, and
$\mathrm{M}_{\mathrm{ij}} \quad=\quad$ unit operating cost, vehicle type i traversing segment j ( $\$ / \mathrm{veh}-\mathrm{mi}$ ).

The Null Alternative includes 5,000 ft of level roadway ( $0 \%$ grade) and 5,000 ft of $8 \%$ grade:

$$
\begin{aligned}
& A C_{\text {Null,PC, } 0 \%}=10,000 *\left(\frac{5,000}{5,280}\right) * 365 * 0.580=\$ 2,004,734.85 / \mathrm{yr} \\
& A C_{\text {Null,PC,8\% }}=10,000 *\left(\frac{5,000}{5,280}\right) * 365 * 0.630=\$ 2,177,556.82 / \mathrm{yr} \\
& A C_{\text {Null,TR, } 0 \%}=2,000 *\left(\frac{5,000}{5,280}\right) * 365 * 1.470=\$ 1,016,193.18 / \mathrm{yr} \\
& A C_{\text {Null,TR, } 8 \%}=2,000 *\left(\frac{5,000}{5,280}\right) * 365 * 1.825=\$ 1,261,600.38 / \mathrm{yr} \\
& A C_{\text {Null,Total }}=2,004,724.85+2,177,556.82+1,016,193.18+1,261,600.38 \\
& A C_{\text {Null,Total }}=\$ 6,460,075.23 / \mathrm{yr}
\end{aligned}
$$

Option A includes 5,000 ft of level roadway ( $0 \%$ grade) and 5,000 ft of $4 \%$ grade:

$$
\begin{aligned}
& A C_{A, P C, 0 \%}=10,000 *\left(\frac{5,000}{5,280}\right) * 365 * 0.580=\$ 2,004,734.85 / y r \\
& A C_{A, P C, 4 \%}=10,000 *\left(\frac{5,000}{5,280}\right) * 365 * 0.585=\$ 2,022,017.04 / \mathrm{yr} \\
& A C_{A, T R, 0 \%}=2,000 *\left(\frac{5,000}{5,280}\right) * 365 * 1.470=\$ 1,016,193.18 / \mathrm{yr} \\
& A C_{A, T R, 4 \%}=2,000 *\left(\frac{5,000}{5,280}\right) * 365 * 1.535=\$ 1,061,126.89 / \mathrm{yr} \\
& A C_{A, \text { Total }}=2,004,724.85+2,022,017.04+1,016,193.18+1,061,126.89 \\
& A C_{A, T o t a l}=\$ 6,104,061.96 / y r
\end{aligned}
$$

Option B includes $10,000 \mathrm{ft}$ of $4 \%$ grade:

$$
\begin{aligned}
& A C_{A, P C, 4 \%}=10,000 *\left(\frac{10,000}{5,280}\right) * 365 * 0.585=\$ 4,044,034.09 / \mathrm{yr} \\
& A C_{A, T R, 4 \%}=2,000 *\left(\frac{10,000}{5,280}\right) * 365 * 1.535=\$ 2,122,253.79 / \mathrm{yr} \\
& A C_{A, \text { Total }}=4,044,034.09+2,122,253.79=\$ 6,166,287.88 / \mathrm{yr}
\end{aligned}
$$

The total annual costs for the three alternatives are compared in Table 5.6.
Table 5.6 Total Annual Costs Compared for Sample Problem 5.2

| Alternative | Capital Cost <br> $(\mathbf{\$ / \mathbf { y r } )}$ | Travel Time <br> Cost $(\mathbf{\$ / \mathbf { ~ } )}$ | Operating <br> $\mathbf{C o s t}(\mathbf{/} / \mathbf{y r})$ | TOTAL COST <br> $(\mathbf{\$ / \mathbf { y r } )}$ |
| :---: | :---: | :---: | :---: | :---: |
| Null | 0 | $5,463,225.65$ | $6,460,075.23$ | $\mathbf{\$ 1 1 , 9 2 3 , 3 0 0 . 8 8}$ |
| Option A | $1,515,690.00$ | $5,327,838.50$ | $6,104,061.96$ | $\mathbf{\$ 1 2 , 9 4 7 , 5 9 0 . 4 6}$ |
| Option B | $1,222,506.00$ | $5,312,448.49$ | $6,166,287.88$ | $\mathbf{\$ 1 2 , 7 0 1 , 2 4 2 . 3 7}$ |

## Discussion

Although the total annual costs of the three alternatives do not vary greatly, it is clear that the cost of the Null Alternative is the most economic. Therefore, no reconstruction to reduce the severity of the $8 \%$ grade would be undertaken, based upon the information given. The cost of accidents, not considered in this example, might be investigated to see if that element tipped the scales in favor of one of the reconstruction alternatives.

## Sample Problem 5.3: Upgrading a Commuter Rail Line for High-Speed Trains

An existing 60 -mile commuter rail line is facing a complete replacement of its rolling stock. Two alternatives are possible:

- Option A: Replace all rail cars with conventional rolling stock with similar characteristics.
- Option B: Replace all rail cars with a new generation of high-speed rolling stock. This option would require making upgrades to the track and signal systems, and would require the elimination of some lightly-used stations on the line.

The line currently serves an average of 150,000 passengers per weekday and an average of 90,000 passengers/day on weekends. At its peak load point on a typical weekday, the peak-hour volume is 25,000 passengers/hr in the primary direction. The volume in the hour after the peak is 18,000 passengers $/ \mathrm{hr}$ in the primary direction. If the high-speed option is chosen, it is anticipated that both the total and peak load point volumes will increase by $20 \%$. The current average fare is $\$ 12.00$ per passenger (one-way). If high-speed is provided, the fare would increase to an average of $\$ 14.00$ per passenger. Key information concerning the two options is summarized in Table 5.7.

Table 5.7 Base Data for Sample Problem 5.3

| Cost Element | Conventional Rail Cars | High-Speed Rail Cars |
| :--- | :---: | :---: |
| Additional Track, Signal Work | $\$ 0.00$ | $\$ 75,000,000$ |
| Cost of Rail Car | $\$ 2,000,000$ | $\$ 3,850,000$ |
| Capacity of Rail Car | 100 passengers | 130 passengers |
| Maximum Speed | $70 \mathrm{mi} / \mathrm{h}$ | $110 \mathrm{mi} / \mathrm{h}$ |
| Average Speed Including Stops | $45 \mathrm{mi} / \mathrm{h}$ | $65 \mathrm{mi} / \mathrm{h}$ |
| Average Station Access Time | $15 \mathrm{~min} /$ passenger | $17 \mathrm{~min} / \mathrm{passenger}$ |
| Service Life | 30 years | 25 years |
| Service Life, Track, Signal Work | 50 years | 50 years |
| Percent of Cars in Service | $10 \%$ | $15 \%$ |
| Maintenance Cost | $\$ 8,000 / \mathrm{yr} / \mathrm{car}$ | $\$ 12,000 / \mathrm{yr} / \mathrm{car}$ |
| Weekday Ridership | 150,000 | 180,000 |
| Weekend Ridership | 90,000 | 108,000 |
| Peak-Load Demand Volume | 25,000 | 30,000 |
| Next Hrs. Peak-Load Demand <br> Volume | 18,000 | 21,600 |
| Average Trip Length | 32.5 mi | 36.0 mi |
| Average Fare | $\$ 12.00$ | $\$ 14.00$ |
| Prevailing Wage of Passengers | $\$ 42.15 / \mathrm{hr}$ | $\$ 42.15 / \mathrm{hr}$ |
| Analysis Interest Rate | $5 \%$ | $5 \%$ |

## Definition of Alternatives

The alternatives are clearly stated, but involve a number of elements. Option B involves higher speed rail cars, fewer stations, more passengers, capital upgrades to trackage and signals, and higher maintenance costs per rail car. User costs for both options include the direct cost of fares and the cost of travel time.

## Analysis Period

If one assumes that all costs are renewable at the end of their service lives, i.e., rail cars would be replaced by similar cars with similar costs, the analysis period is almost irrelevant. In this case, all cost elements will be reduced to equivalent annual costs, so that the analysis period is, in effect, one year.

## Methodology

Because the two options involve different demand levels, an incremental methodology must be used. As there are only two options (the rail line exists and the rolling stock must be replaced), a simple benefit-cost analysis would be appropriate.

A benefit-cost analysis requires a clear separation of user costs and system costs. User costs include the fare and travel time; all other elements of cost are system costs borne by the operator of the rail line.

## Solution

## Annual System Costs

System costs include the purchase of rolling stock, upgrading of tracks and signals (for Option B), and maintenance of rolling stock. Note that operating costs are assumed to be the same for both alternatives.

The cost of rolling stock cannot be computed until the number of cars needed for each option is determined. Four factors affect this: (1) the peak-point passenger load, (2) the capacity of the rail car, (3) the turn-around time for a car in service (how long before a car is used for a second trip), and (4) the percentage of cars undergoing maintenance at any given time.

The time to complete one round trip on the 60-mile line for a rail car in service is found as:

$$
T_{R T}=\frac{120}{S_{a v}}
$$

where: $\quad \mathrm{T}_{\mathrm{TR}}=\quad$ round-trip time (hrs), and
$\mathrm{S}_{\mathrm{av}} \quad=\quad$ average speed in service, including stops (mi/h).

Then:

$$
\begin{aligned}
& T_{\text {TR,OPTION } A}=\frac{120}{45}=2.67 \mathrm{hrs} \\
& T_{\text {TR,OPTION } B}=\frac{120}{65}=1.85 \mathrm{hrs}
\end{aligned}
$$

This means that to provide continuous service, enough cars must be available to cover 2.67 hours of peak and near-peak load conditions for Option A, and 1.85 hours of peak and near-peak load conditions for Option B.

For Option A, peak loading is 25,000 passengers/hour, and near-peak loading in adjacent hours is 18,000 passengers $/ \mathrm{hr}$. The capacity of conventional rail cars is given as 100 passengers/car. The number of cars that must be purchased for this option is:

$$
\left[\frac{25,000+(1.67 * 18,000)}{100}\right]=\frac{55,060}{100}=551 \text { rail cars }
$$

This assumes, however, that all cars are in service. The data suggests that $10 \%$ of conventional rail cars will be "out of service" at any given time, raising the number of cars needed to $551 / 0.90=\mathbf{6 1 2}$ cars.

For Option B, peak loading is 30,000 passengers/hour, and near-peak loading in adjacent hours is 21,600 passengers/hour. The capacity of high-speed rail cars is given as 130 passengers/hour. The number of cars that must be purchased for this option is:

$$
\left[\frac{30,000+(0.85 * 21,600)}{130}\right]=\frac{48,360}{130}=372 \text { rail cars }
$$

Again, this computation expects that all cars would be in service at peak times. As $15 \%$ of the high-speed fleet would be down at any given point, the number of cars that need to be purchased is $372 / 0.85=\mathbf{4 3 8}$ rail cars.

All aspects of system costs for both alternatives must now be converted to annual equivalents based upon the service lives of the components and the analysis interest rate of 5\%.

## OptionA

Annual Cost, Tracks/Signals: \$ 0/yr
Annual Cost, Rolling Stock:
\$ 79,622,424/yr

$$
\begin{aligned}
& 612 * 2,000,000 * A / P_{30,5 \%} \\
& 612 * 2,000,000 * 0.065051=\$ 79,622,424
\end{aligned}
$$

Annual Cost, Maintenance:
\$ 4,896,000/yr

$$
612 * 8,000=4,896,000
$$

Total Annual System Cost, Option A:
\$ 84,518,424/yr
Option B
Annual Cost, Tracks/Signals:
\$ 4,108,275/yr

$$
\begin{aligned}
& 75,000,000 * A / P_{50,5 \%} \\
& 75,000,000 * 0.054777=4,108,275
\end{aligned}
$$

Annual Cost, Rolling Stock:
\$119,642,985/yr

$$
\begin{aligned}
& 438 * 3,850,000 * A / P_{25,5 \%} \\
& 438 * 3,850,000 * 0.070950=119,642,985
\end{aligned}
$$

Annual Cost, Maintenance:
\$ 5,256,000/yr

$$
438 * 12,000=5,256,000
$$

## Total Annual System Cost, Option B:

\$129,007,260/yr

## Annual User Costs

Annual user costs consist of the direct payment of the fare, and the cost of travel time for passengers. To compute either, the total annual passengers for each option must be computed. As separate weekday and weekend daily volumes are given, it must be noted that the typical year includes 260 weekdays and 105 Saturdays and Sundays. Thus, the total annual passengers for each option are:

$$
\begin{aligned}
& A P_{\text {OPTION } A}=(260 * 150,000)+(105 * 90,000) \\
& A P_{\text {OPTION } A}=39,000,000+9,450,000=48,450,000 \\
& A P_{\text {OPTION } B}=(260 * 180,000)+(105 * 108,000) \\
& A P_{\text {OPTION } B}=46,800,000+11,340,000=60,140,000
\end{aligned}
$$

The cost per passenger (user) consists of the fare, which is given for both options, and the cost of travel time. From Table 4.3 (Chapter 4), in-vehicle travel time for transit users (commuting and personal trips) is $50 \%$ of the wage rate, while station access time could be classified as "excess" travel time, valued at $100 \%$ of the prevailing wage rate. Thus, in-vehicle time will be valued at $0.50 * 42.15=$ $\$ 21.075 / \mathrm{hr}$. Station access time will be valued at $1.00 * 42.15=\$ 42.15 / \mathrm{hr}$.

Station access times are given as 15 minutes ( 0.25 hrs ) for Option A and 17 minutes ( 0.283 hrs ) for Option B. In-vehicle travel times are based upon the average trip length, and the average speed of the train, including stops. In-vehicle travel time for Option $\mathrm{A}=32.5 / 45=0.722 \mathrm{hrs}$; for Option B , it is $36.0 / 65=$ 0.554 hrs.

The travel time cost for Option A is $(0.25 * 42.15)+(0.722 * 21.075)=$ $\$ 25.7537 /$ trip; the travel time cost for Option B is $(0.283 * 42.15)+(0.554 * 21.075)$ $=\$ 23.0371 /$ trip.

The total unit user cost is the cost of travel time (per trip) plus the cost of the fare (per trip). The unit user cost for Option A is, therefore:

$$
U_{A}=(12.0000+25.7537)=\$ 37.7537 / \text { passenger }
$$

The unit user cost for Option B is:

$$
U_{B}=(14.000+23.6040)=\$ 37.6040 / \text { passenger }
$$

## The Benefit-Cost Ratio

The benefit-cost ratio is computed as:

$$
B C R=\frac{\left(U_{A}-U_{B}\right) *\left(\frac{V_{A}+V_{B}}{2}\right)}{S C_{B}-S C_{A}}
$$

where $V_{A}$ and $V_{B}$ are the annual passenger demands for Options $A$ and $B$ respectively. Therefore:

$$
\begin{aligned}
& B C R=\frac{(37.7537-37.6040) *\left(\frac{48,450,000+60,140,000}{2}\right)}{129,007,260-84,518,424} \\
& B C R=\frac{8,12796}{44,488,836}=0.1827
\end{aligned}
$$

## Discussion

The BCR of 0.1827 clearly favors the option of conventional rail. User benefits are primarily in the form of travel time savings, as the fare actually increases for the high-speed service. The savings to users are not worth the cost to provide them.

These results can also be viewed entirely from the point of view of the system operator. The system operator will have to spend $\$ 44,488,836$ more (on an annual basis) than simply replacing the rolling stock with conventional rail cars to provide the high-speed service. What they get in return is an increase in passengers and a higher fare. The operator's net increase in revenues is:

$$
\begin{aligned}
& (60,140,000 * 14)-(48,450,000 * 12)= \\
& 721,680,000-581,400,000=\$ 140,280,000 / y r
\end{aligned}
$$

Thus, the operator's revenues rise by over $\$ 140$ million/yr for a cost of only $\$ 44.5$ million/yr. For the operator, this is a "no-brainer," an investment they should enthusiastically embrace.

The ultimate decision in this case is interesting. The operating agency would clearly like to invest in the high-speed rail option, but the "benefits" to users are a fare increase and an insufficient savings in travel time to justify the cost.

Looking at it a third way: Each user would save the average equivalent of $\$ 25.7536-\$ 23.0371=\$ 2.7166$ in travel time with high-speed rail for an additional cost in the fare of $\$ 2.00$ - maybe not a bad deal after all.

Why the discrepancy? The fare increase distorts the comparison. Each user gets a net benefit, but not large enough to justify the expenditure of the additional funds needed for high-speed rail. The operator does well too. The missing link is the fact that there is undoubtedly some public subsidy going into the investment that is not factored into the "user cost."

## Sample Problem 5.4: Considering a By-Pass

An existing 4-lane highway runs through the center of an isolated town, becoming its "Main Street." Because local traffic mixes with a substantial number of through vehicles, traffic conditions have become difficult in the town. Two plans for building a by-pass roadway around the town are under consideration, as illustrated in Figure 5.3. The existing roadway will continue to exist to service local traffic and those through vehicles who wish to go through the town center.

The three alternatives for consideration are:

- Null Alternative: no new construction, existing roadway remains without a bypass,
- Option A: Construct a by-pass along Route B (Figure 5.3), with a connection roadway (D, Figure 5.3), or
- Option B: Construct a by-pass along Route C (Figure 5.3), with no connecting roadway to the town.


Fig. 5.3 By-Pass Options for Sample Problem 5.4
The existence of a by-pass is not expected to cause any additional growth in traffic. The current roadway carries an AADT of 16,000 veh/day. If By-Pass B is built, $50 \%$ of this traffic will be diverted to the new roadway, with $20 \%$ of these also using Connecting Roadway D to make a stop in the town. If By-Pass C is built, $60 \%$ of the traffic will be diverted to it. Table 5.8 shows how the AADT's will be affected by either of the proposed by-pass routes.

Table 5.8 AADT Values for Sample Problem 5.4

| Alternative | Expected AADT <br> (veh/day) | Average Trip Length <br> (mi) |
| :--- | :---: | :---: |
| Null Alternative: | 16,000 | 5.3 |
| Existing Roadway |  |  |
| Option A: | 8,000 | 4.0 |
| Existing Roadway | 8,000 | 5.6 |
| By-Pass B | 1,600 | 0.5 |
| Connecting Roadway D |  |  |
| Option B: | 6,400 | 4.0 |
| Existing Roadway | 9,600 | 5.0 |
| By-Pass C |  |  |

Note that the average trip length on the existing roadway is less than the 6.2mile length of the segment. As a good deal of the traffic on this roadway is local, not all trips travel the full length of the segment. In Options A and B, through traffic is basically diverted from the existing roadway, leaving only local traffic. Thus, the average trip length on the existing roadway declines in Options A and B. Ten percent of the traffic on all segments is commercial trucks.

Table 5.9 summarizes the capital construction and maintenance costs associated with the various options under consideration.

Table 5.9 Capital Construction and Maintenance Costs for Sample Problem 5.4

| Alternative | Construction Cost <br> (\$/mi) | Service Life <br> (yrs) | Annual Mainten- <br> ance Costs <br> $(\$ / \mathbf{m i})$ |
| :--- | :---: | :---: | :---: |
| Null Alternative | 0 | NA | 35,000 |
| Existing Roadway | 0 | NA | 30,000 |
| Option A | 25 | 20,000 |  |
| Existing Roadway | $6,500,000$ | 25 | 15,000 |
| By-Pass B | Connecting Roadway | $4,000,000$ | NA |
| Option B | 0 | 25 | 28,000 |
| Existing Roadway | $7,000,000$ | 22,000 |  |
| By-Pass C |  |  |  |

Average speeds of vehicles on each roadway and each option are shown in Table 5.10. For the purposes of evaluating travel time, the prevailing wage for car drivers and passengers has been determined as $\$ 24.50 / \mathrm{h}$; for commercial vehicle drivers, total compensation is $\$ 50.00 / \mathrm{h}$. Of all car traffic, $40 \%$ of drivers and passengers are commuting to work, and $60 \%$ are making personal and/or shopping trips. Average auto occupancy is 1.4 persons/vehicle; truck occupancy may be taken as 1.0 persons/vehicle.

Table 5.10 Average Speeds of Vehicles for Sample Problem 5.4

| Alternative | Average Speed - <br> Passenger Cars <br> $\mathbf{( m i / h )}$ | Average Speed - <br> Trucks <br> (mi/h) |
| :--- | :---: | :---: |
| Null Alternative | 26 | 18 |
| Existing Roadway | 30 | 25 |
| Alternative A | 50 | 43 |
| Existing Roadway | 35 | 28 |
| By-Pass B |  |  |
| Connecting Roadway | 30 | 25 |
| Alternative B | 62 | 60 |
| Existing Roadway | By-Pass A |  |

Regional studies show that the total operating cost of passenger cars is 4.5 times the cost of fuel, and that the total operating cost of trucks is 3.2 times the cost of fuel. National studies may be used to find fuel costs that reflect the variation with speed. The cost of fuel for this computation may be taken as $\$ 3.50 / \mathrm{gal}$ for gasoline, and $4.25 / \mathrm{gal}$ for diesel. It may be assumed that all trucks in this area use diesel fuel.

## Definition of Alternatives

The problem statement clearly defines the alternatives in this analysis. The Null Alternative will be compared to Options A and B to determine the most economic choice for implementation.

## Analysis Period

As the capital costs of Options A and B provide roadways with a service life of 25 years, this would be an appropriate analysis period. As in many comparisons, it would be assumed that renewals would repeat initial costs, so the choice of analysis period is not critical.

## Methodology

Because all alternatives will carry the same amount of traffic, a direct method of comparison may be used. As most of the costs will be estimated as annual costs, an annual cost comparison would be the most straightforward approach. An analysis interest rate of $3 \%$ may be used.

## Solution

## Annual Costs of Capital Construction Costs and Maintenance

Maintenance costs are already stated on an annual basis. Capital construction costs must be converted to an annual equivalent using the $\mathrm{A} / \mathrm{P}_{25,3 \%}$ ( 0.05748 ). As all of the construction and maintenance costs given in Table 5.9 are stated on a per-mile basis, they must be multiplied by the length of the roadway involved. Table 5.11 illustrates these computations.

## Travel Time Costs

Travel time costs are relatively complex. The depend upon the amount of travel time expended by cars and trucks, based upon the average speeds given in Table 5.9 and the average trip lengths and AADT's given in Table 5.8. The problem statement also notes that $10 \%$ of all vehicles are trucks. The unit value of travel time for cars and trucks is based upon prevailing wage and compensation rates, vehicle occupancy, and trip purposes as stated previously.

Table 5.11 Annual Cost of Construction and Maintenance for Sample Problem 5.4

| Alternative | Construction | Maintenance | Total |
| :--- | :---: | :---: | :---: |
| Null Alternative | $\$ 0$ | $\$ 35,000 * 6.2=\$ 217,000$ | $\mathbf{\$ 2 1 7 , 0 0 0}$ |
| Existing Roadway |  |  |  |
| Option A | $\$ 0$ | $\$ 30,000 * 6.2=\$ 186,000$ | $\$ 186,000$ |
| Existing Roadway | $6,500,000 * 5.6 * 0.05748=\$ 2,092,272$ | $\$ 20,000 * 5.6=\$ 112,000$ | $\$ 2,04,272$ |
| By-Pass B | $4,000,000 * 0.5 * 0.05748=\$ 114,960$ | $\$ 15,000 * 0.5=\$ \$, 500$ | $\$ 122,460$ |
| Connecting Roadway |  |  | $\$ \mathbf{2 , 5 1 2 , 7 3 2}$ |
| Option B | $\$ 0$ | $\$ 28,000 * 6.2=\$ 173,600$ | $\$ 173,650$ <br> Existing Roadway <br> By-Pass C |
|  | $7,000,000 * 5.0 * 0.05748=\$ 2,011,800$ | $\$ 22,000 * 5.0=\$ 110,000$ | $\mathbf{\$ 2 , 1 2 1 , 8 0 0}$ |

Estimating the cost of travel time is a relatively complex multi-step process. First, it would be convenient to determine the value of travel time per vehicle hour. From Table 4.3 (Chapter 4), we find that the value of travel time equals:

- $50 \%$ of wages for commuting drivers,
- $40 \%$ of wages for commuting passengers,
- $50 \%$ wages for drivers and passengers engaged in local personal trips, and
- $100 \%$ of total compensation for truck drivers.

For trucks, which comprise $10 \%$ of the AADT demands in this sample problem, this is straightforward. Given the total compensation rate of $\$ 50.00 / \mathrm{hr}$ for truck drivers, and a truck occupancy of 1.0 persons/truck, the value of travel time is:

$$
1.00 * 50.00 * 1=\$ 50.00 / \text { veh }-\mathrm{hr}
$$

Auto trips are a bit more complicated. The problem states that $40 \%$ of auto-trips consist of commuters, and $60 \%$ consist of personal trips. The auto occupancy for all auto-trips is 1.4 passengers per vehicle. Prevailing wages in the region are given as $\$ 24.50 / \mathrm{hr}$. For commuting trips, the value of travel time per vehicle-hour is $(0.50 * 24.50)+(0.40 * 24.50 * 0.4)=\$ 11.17 /$ veh-hr. For personal trips, the value of travel time per vehicle-hour is $0.50 * 24.50 * 1.4=\$ 17.15 / \mathrm{veh}-\mathrm{hr}$. Given the split of trip purposes, the average value of travel time per vehicle-hour for cars is:

$$
(0.40 * 11.17)+(0.60 * 17.15)=\$ 14.76 / \text { veh }-h r
$$

Now, the number of annual vehicle-hours of travel time consumed by users for each of the three alternatives must be determined. For each type of vehicle and
each component roadway of each alternative, the number of annual vehicle-hours of travel time may be found as:

$$
T_{i j}=\frac{365 * A A D T_{i j} * L_{j}}{S_{i j}}
$$

where: $\quad \mathrm{T}_{\mathrm{ij}} \quad=\quad$ annual veh-hrs of travel time for vehicle type i on roadway component j ,

$$
\begin{array}{ll}
\mathrm{AADT}_{\mathrm{ij}} & =\begin{array}{l}
\text { average annual daily traffic for vehicle type } \mathrm{i} \\
\text { on roadway component } \mathrm{j}(\mathrm{veh} / \mathrm{yr}),
\end{array} \\
\mathrm{L}_{\mathrm{j}} & =\begin{array}{l}
\text { average trip length on roadway component } \mathrm{j} \\
(\text { mi) , and }
\end{array} \\
\mathrm{S}_{\mathrm{ij}} \quad=\quad \begin{array}{l}
\text { average speed of vehicle type i on roadway } \\
\text { component } \mathrm{j}(\mathrm{mi} / \mathrm{h}) .
\end{array}
\end{array}
$$

Table 5.12 shows this computation for the three alternatives under consideration. Note again that trucks make up $10 \%$ of all AADT volumes.

Table 5.12 Annual Vehicle-Hours of Travel Time for Sample Problem 5.4

| Alternative | AADT <br> (veh/day) | Ave Trip <br> Length <br> (mi) | Average <br> Speed <br> (mi/h) | Travel Time <br> (veh-hrs/yr) |
| :--- | :---: | :---: | :---: | :---: |
| Null Alternative |  |  |  |  |
| Existing Rdwy-Cars | $0.90 * 16,000$ | 5.3 | 26 | $365 * 14,400 * 5.3 / 26=1,071,415$ |
| Existing Rdwy-Trucks | $0.10 * 16,000$ | 5.3 | 18 | $365 * 1,600 * 5.3 / 18=171,956$ |
| Option A |  |  |  |  |
| Existing Rdwy-Cars | $0.90 * 8,000$ | 4.0 | 30 | $365 * 7,200 * 4.0 / 30=350,400$ |
| Existing Rdwy-Trucks | $0.10 * 8,000$ | 4.0 | 25 | $365 * 800 * 4.0 / 25=46,720$ |
| By-Pass B-Cars | $0.90 * 8,000$ | 5.6 | 50 | $365 * 7,200 * 5.6 / 50=294,336$ |
| By-Pass B-Trucks | $0.10 * 8,000$ | 5.6 | 43 | $365800 * 5.6 / 43=38,028$ |
| Conn Rdwy-Cars | $0.90 * 1,600$ | 0.5 | 35 | $3651440 * 0.5 / 35=7,509$ |
| Conn Rdwy-Trucks | $0.10 * 1,600$ | 0.5 | 28 | $365 * 160 * 0.5 / 28=1,043$ |
| Option B |  |  |  |  |
| Existing Rdwy-Cars | $0.90 * 6,400$ | 4.0 | 30 | $365 * 5,760 * 4.0 / 30=280,320$ |
| Existing Rdwy-Trucks | $0.10 * 6,400$ | 4.0 | 25 | $365 * 640 * 4.0 / 25=36,376$ |
| By-Pass C-Cars | $0.90 * 9,600$ | 5.0 | 62 | $365 * 8,640 * 5.0 / 62=254,323$ |
| By-Pass C-Trucks | $0.10 * 9,600$ | 5.0 | 60 | $365 * 960 * 5.0 / 60=29,200$ |

To obtain the annual cost of travel time to users, the vehicle-hours of Table 5.12 are multiplied by the unit value of travel time for cars and trucks, as computed previously. Table 5.13 illustrates.

Table 5.13 Annual Travel Time Costs for Sample Problem 5.13

| Alternative | Cars | Trucks | Unit Operating Costs (\$/veh-mi) |
| :---: | :---: | :---: | :---: |
| Null Alternative Existing Roadway | 1,017,415*14.76=\$15,017,054 | 171,956*50.00=\$8,597,800 | \$23,614,854 |
| Option A <br> Existing Roadway <br> By-Pass B <br> Connecting Roadway | $\begin{gathered} 350,400 * 14.76=\$ 5,171,904 \\ 294,336 * 14.76=\$ 4,344,399 \\ 7,509 * 14.76=\$ 110,833 \end{gathered}$ | $\begin{gathered} 46,720 * 50.00=\$ 2,336,000 \\ 38,028 * 50.00=\$ 1,901,400 \\ 1,043 * 50.00=\$ 52,150 \end{gathered}$ | $\begin{gathered} \$ 7,507,904 \\ \$ 6,235,799 \\ \$ 162,983 \\ \mathbf{\$ 1 3 , 9 0 6}, 685 \\ \hline \end{gathered}$ |
| Option B <br> Existing Roadway <br> By-Pass C | $\begin{aligned} & 280,320 * 14.76=\$ 4,137,523 \\ & 254,323 * 14.76=\$ 3,752,807 \end{aligned}$ | $\begin{aligned} & 36,376 * 50.00=\$ 1,818,800 \\ & 29,200 * 50.00=\$ 1,460,000 \end{aligned}$ | $\begin{gathered} \$ 5,956,323 \\ \$ 5,212,807 \\ \mathbf{\$ 1 1 , 1 6 9 , 1 3 0} \end{gathered}$ |

## Annual Operating Costs

The last cost element is vehicle operating expenses experienced by road users. The problem states that total operating costs should be related to fuel costs. Table 4.11 (Chapter 4) may be used to determine the average fuel consumption of cars and trucks at various speeds (in gals/mi). Interpolation may be used in this table to obtain more precise values. The fuel cost per mile is found by multiplying the consumption rate by the price of fuel/gallon. Total operating cost per mile is found by multiplying the fuel cost/mi by 4.5 for cars and by 3.2 for trucks, as given in the problem statement. Table 5.14 shows these computations.

Table 5.14 Operating Cost Per Vehicle-Mile for Sample Problem 5.4

| Alternative | Speed <br> $\mathbf{( m i / h )}$ | Fuel Cons. <br> (gals/mi) | Multiplier <br> for <br> (\$/gal) | Total <br> Operating <br> Cost | Total Operating <br> Cost Per Mile <br> (\$/mile) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Null Alternative |  |  |  |  |  |
| Existing Rdwy-Cars | 26 | 0.0536 | 3.50 | 4.5 | $0.0536 * 3.50 * 4.5=\$ 0.845$ |
| Existing Rdwy-Trucks | 18 | 0.1306 | 4.25 | 3.2 | $0.1306 * 4.25 * 3.2=\$ 1.771$ |
| Option A |  |  |  |  |  |
| Existing Rdwy-Cars | 30 | 0.0440 | 3.50 | 4.5 | $0.0440 * 3.50 * 4.5=\$ 0.693$ |
| Existing Rdwy-Trucks | 25 | 0.1495 | 4.25 | 3.2 | $0.1495 * 4.25 * 3.2=\$ 2.033$ |
| By-Pass B-Cars | 50 | 0.0330 | 3.50 | 4.5 | $0.0330 * 3.50 * 4.5=\$ 0.520$ |
| By-Pass B-Trucks | 43 | 0.2087 | 4.25 | 3.2 | $0.2087 * 4.25 * 3.2=\$ 2.838$ |
| Conn Roadway-Cars | 35 | 0.0390 | 3.50 | 4.5 | $0.0390 * 3.50 * 4.5=\$ 0.614$ |
| Conn Roadway-Trucks | 28 | 0.1300 | 4.25 | 3.2 | $0.1300 * 4.25 * 3.2=\$ 1.768$ |
| Option B |  |  |  |  |  |
| Existing Rdwy-Cars | 30 | 0.0440 | 3.50 | 4.5 | $0.0440 * 3.50 * 4.5=\$ 0.693$ |
| Existing Rdwy-Trucks | 25 | 0.1255 | 4.25 | 3.2 | $0.1255 * 4.25 * 3.2=\$ 1.707$ |
| By-Pass C-Cars | 62 | 0.0400 | 3.50 | 4.5 | $0.0400 * 3.50 * 4.5=\$ 0.630$ |
| By-Pass C-Trucks | 60 | 0.3740 | 4.25 | 3.2 | $0.3740 * 4.25 * 3.2=\$ 5.086$ |

The annual operating cost for any vehicle type on a roadway section is found as:

$$
A_{i j}=365 * A A D T_{i j} * L_{j} * O_{i j}
$$

where: $A_{i j}=$ annual operating cost, vehicle type i on roaway segment j (\$),
$\mathrm{AADT}_{\mathrm{ij}}=\quad$ average annual daily traffic, vehicle type i and roadway segment j (veh/day),
$\mathrm{L}_{\mathrm{j}} \quad=\quad$ average trip length on roadway segment $\mathrm{j}(\mathrm{mi})$, and,
$\mathrm{O}_{\mathrm{ij}} \quad=\quad$ unit operating cost/veh-mi (\$).
These computations are illustrated in Table 5.15.

Table 5.15 Operating Cost Computations for Sample Problem 5.4

| Alternative | AADT (veh/day) | Avg Trip Length, L (mi) | Unit Oper. Cost (\$/veh-mi) | Total Annual Operating Cost (\$) |
| :---: | :---: | :---: | :---: | :---: |
| Null Alternative Existing Rdwy-Cars Existing Rdwy-Trucks | $\begin{aligned} & 0.90 * 16,000 \\ & 0.10 * 16,000 \end{aligned}$ | $\begin{aligned} & 5.3 \\ & 5.3 \end{aligned}$ | $\begin{aligned} & 0.845 \\ & 1.771 \end{aligned}$ | $\begin{aligned} 365 * 14,400 * 5.3 * 0.853 & =\$ 23,761,850 \\ 365 * 1,600 * 5.3 * 1.771 & =\$ 5,481,599 \\ & \$ 29,243,449 \end{aligned}$ |
| Option A <br> Existing Rdway-Cars <br> Existing Rdwy-Trucks <br> By-Pass B-Cars <br> By-Pass B-Trucks <br> Conn Rdwy-Cars <br> Conn Rdwy-Trucks | $\begin{aligned} & 0.9 * 8,000 \\ & 0.1 * 8,000 \\ & 0.9 * 8,000 \\ & 0.1 * 8,000 \\ & 0.9 * 1,600 \\ & 0.1 * 1,600 \end{aligned}$ | $\begin{aligned} & 4.0 \\ & 4.0 \\ & 5.6 \\ & 5.6 \\ & 0.5 \\ & 0.5 \end{aligned}$ | $\begin{aligned} & 0.693 \\ & 2.033 \\ & 0.520 \\ & 2.838 \\ & 0.614 \\ & 1.768 \end{aligned}$ | $\begin{array}{rlr} 365 * 7,200 * 4.0 * 0.693 & =\$ 7,284,816 \\ 365 * 800 * 4.0 * 2.033 & =\$ 2,374,544 \\ 365 * 7,200 * 5.6 * 0.520 & =\$ 7,652,736 \\ 365 * 800 * 5.6 * 2.838 & =\$ 4,640,698 \\ 365 * 1,440 * 0.5 * 0.614 & =\$ & 262,800 \\ 365 * 160 * 0.5 * 1.768 & =\$ & \frac{51,626}{} \\ & \mathbf{\$ 2 2 , 2 6 7 , 2 2 0} \\ \hline \end{array}$ |
| Option B <br> Existing Rdwy-Cars <br> Existing Rdwy-Trucks <br> By-Pass C-Cars <br> By-Pass C-Trucks | $\begin{aligned} & 0.9 * 6,400 \\ & 0.1 * 6,400 \\ & 0.9 * 9,600 \\ & 0.1 * 9,600 \end{aligned}$ | $\begin{aligned} & 4.0 \\ & 4.0 \\ & 5.0 \\ & 5.0 \end{aligned}$ | $\begin{aligned} & 0.693 \\ & 1.707 \\ & 0.630 \\ & 5.086 \end{aligned}$ | $\begin{array}{rlr} 365 * 5,760 * 4.0 * 0.693 & =\$ 5,827,853 \\ 365 * 640 * 4.0 * 1.707 & =\$ & 1,595,021 \\ 365 * 8,640 * 5.0 * 0.630 & =\$ & 9,933,840 \\ 365 * 960 * 5.0 * 5.086 & =\$ & 8,910,672 \\ & \mathbf{\$ 2 6 , 2 6 7 , 3 8 5} \\ \hline \end{array}$ |

At this point, all of the annual costs are known. Total annual costs of each alternative may now be directly considered, as shown in Table 5.16.

Table 5.16 Total Annual Costs for Sample Problem 5.4

| Alternative | Capital \& Maint. <br> $\mathbf{( \$ / \mathbf { y r } )}$ | Travel Time <br> $\mathbf{( \$ / \mathbf { y r } )}$ | Vehicle Operation <br> $\mathbf{( \$ / \mathbf { y r } )}$ | Total Annual <br> $\mathbf{C o s t}(\mathbf{\$ / \mathbf { y r } )}$ |
| :--- | :---: | :---: | :---: | :---: |
| Null Alternative | $\$ 217,000$ | $\$ 23,614,854$ | $\$ 29,243,449$ | $\mathbf{\$ 5 3 , 0 7 5 , 3 0 3}$ |
| Option A | $\$ 2,512,732$ | $\$ 13,906,685$ | $\$ 22,267,220$ | $\mathbf{\$ 3 8 , 6 8 6 , 6 3 7}$ |
| Option B | $\$ 2,295,450$ | $\$ 11,169,130$ | $\$ 26,267,385$ | $\mathbf{\$ 3 9 , 7 3 1 , 9 6 5}$ |

## Discussion

Clearly, both by-pass options are far more attractive economically than the existing route, which would continue to serve both local and through traffic. The numbers slightly favor Option A over Option B, that is, building By-Pass B with a connecting roadway to the town center.

It is also likely that local business owners strongly favor Option A over Option B, as it would allow "through" vehicles to make a stop in town for fuel, food, or shopping.

## Sample Problem 5.5: Considering Regional Public Transportation Options

A medium density suburban county is considering several options for providing a public transportation system.

At the current time, the vast majority of trips are made by car on the county's freeways, arterials, and streets. This generates $3,000,000$ auto trips/day (AADT). These trips have an average length of 25 miles at an average speed of $38 \mathrm{mi} / \mathrm{h}$. Travel time for auto-users (drivers and passengers) is valued at an average of $\$ 20.00 /$ person-hr. The average auto occupancy is 1.3 persons/vehicle. The average cost of vehicle operation is $\$ 0.75 / \mathrm{veh}-\mathrm{mi}$.

There is a small local bus system that carries 50,000 passengers per day at average speeds of $12 \mathrm{mi} / \mathrm{h}$, including stops. The bus system operates 10,000 bus-miles of service per day. The average passenger experiences a total of 32.6 minutes of travel time, at an average value of $\$ 15.00 /$ person-hr. Bus fare is $\$ 1.00$.

Two options are under consideration:

## Option A: Build a Regional Rail Rapid Transit System

Three heavy rail lines totaling 70 route-miles will be built. Of these, 15 miles will be in tunnels, which will cost $\$ 150,000,000 / \mathrm{mi}$ to build. The remaining 55 miles will be at-grade or on structures, and will cost $\$ 50,000,000 / \mathrm{mi}$ to build. There will be 8 underground stations, costing $\$ 15,000,000$ apiece, and 20 above-ground stations, costing $\$ 8,000,000$ apiece. High-speed rail cars will provide service at average speeds of 40 $\mathrm{mi} / \mathrm{h}$, including stops. The system will require 700 rail cars, which cost $\$ 3,500,000$ apiece. The system will operate an average of 90,000 carmiles per day, and is expected to carry 350,000 passengers per day. The average trip length for rail users is expected to be 25.0 minutes. The average fare for the rail system is $\$ 5.40$. The service life of the railroad structures and stations is 50 years; the service life of the rail cars is 25 years.

Under Option A, the bus system will be re-organized to provide feeder service. Ridership will be 25,000 passengers/day, with an average trip time of 8.6 minutes. The down-sized bus system will operate 4,000 busmiles/day at a speed of $10 \mathrm{mi} / \mathrm{h}$, including stops.

The travel time of rail users is valued at $\$ 21.00 / \mathrm{hr}$; bus users's travel time is valued at $\$ 15.00 / \mathrm{hr}$. The fare on the bus system remains $\$ 1.00$.

Option A will increase local highway speeds to an average of $41 \mathrm{mi} / \mathrm{h}$.

## Option B

Option B replaces the entire bus system with an expanded regional light rail system. One hundred miles of light rail lines will be built, all atgrade, at a cost of $\$ 20,000,000 / \mathrm{mi}$. Two hundred small station enclosures will be built at a cost of $\$ 75,000$ apiece. The system will require 1,000 light rail cars at a cost of $\$ 1,800,000$ apiece. The system will serve 200,000 passengers/day. The system will operate 100,000 car-miles/day at average speeds of $35.0 \mathrm{mi} / \mathrm{h}$. The average passenger trip will take 19.5 minutes. The service life of physical structures and station enclosures is 40 years, and the service life of light rail vehicles is 22 years.

The travel time of light-rail users is valued at $\$ 18.00 / \mathrm{hr}$. The fare for the light-rail system is $\$ 3.50$.

Option B will increase the average speed on the highway system to an average of $39 \mathrm{mi} / \mathrm{h}$.

Initially, rail transit trips will consist of trips diverted from the existing bus system (all of them), with the rest being diverted from the highway system. In Option A, riders using the new feeder bus system are part of the rail transit user demand.

Thus, when the proposed system(s) open, there will be no change in total demand. However, over 25 years, the ambient growth on the existing highway system is expected to be $10 \%$, with no growth on the local bus system. For the heavy rail system, rail usage will grow by $20 \%$ over 25 years (as will bus usage), while the usage of the highway system grows by $5 \%$. For the light rail system, passenger growth over 25 years is expected to be $15 \%$, while highway traffic is expected to grow by $9 \%$.

An analysis rate of interest of 5\% may be used for this analysis.
For the sake of simplicity, it may be assumed that the operating conditions described for the highway, bus, and rail systems do not meaningfully change due to demand increases over the analysis period.

## Definition of Alternatives

This is a large, regional set of alternatives. Smaller details of the specific workings of each sub-system are less important when attempting to make a major system decision such as this one.

The components of each of the alternatives are as follows:

- Null Alternative: Includes the current highway system and the local bus system.
- Option A: Includes the new heavy rail system, the down-sized feeder bus system, and the remaining highway system.
- Option B: Includes the new light rail system and the remaining highway system.


## Analysis Period

The service lives of component parts of the system options vary somewhat from 22 years to 50 years. Demand forecasts are for a 25 -year period. If the assumption is made that all cost components renew at the end of the service lives with no changes in cost, the choice of an analysis period is less critical. Because the demand forecasts are for a 25 -year period, this would be the most appropriate analysis period to choose.

## Methodology

As a result of the demand forecasts, each of the options will have different average demands. Thus, direct methods of comparison could not be used. The benefitcost ratio approach would appear to be most applicable here. To apply this, all cost elements have to be reduced to a common basis. For simplicity, it is easier to convert all cost elements to equivalent annual cost.

To use the benefit-cost ratio, all system costs must be reduced to a total annual equivalent. User costs, however, should be reduced to an annual cost per user for use in computing benefit-cost ratios.

## Solution

## Dealing with Demand

A set of traffic and passenger demand levels is specified for the point at which any option is implemented. Twenty-five year forecasts for each alternative, however,
vary. Because the problem states that basic operating conditions are not affected by the growth in demand, each alternative should be based upon the average demand level over the 25-year analysis period.

Starting with the existing demand configuration, initial demands for Options A and B are estimated based upon diversions from the local bus and highway systems. The demand in 25 years is computed from the initial demand and the growth percentages given in the problem statement. Once initial and 25-year demand levels are established, the averages may be computed and used throughout the analysis.

Tables 5.17, 5.18, and 5.19 illustrate the computation of demand levels for each of the three alternatives.

Table 5.17 Analysis Demands for the Null Alternative, Sample Problem 5.5

| Component | Initial Demand <br> (Person-Trips/Day) | 25 - Year Demand <br> (Person-Trips/Day) | Average Demand <br> (Person-Trips/Day) |
| :--- | :---: | :---: | :---: |
| Highway System | $3,000,000 * 1.3=$ | $3,900,000 * 1.10=$ | $4,290,000$ |
| Local Bus System | $3,900,000$ | 50,000 | $4,095,000$ |
| Total Person-Trips | 50,000 |  | 50,000 |

Table 5.18 Analysis Demands for Option A, Sample Problem 5.5

| Component | Initial Demand <br> (Person-Trips/Day) | $\mathbf{2 5}$ - Year Demand <br> (Person-Trips/Day) | Average Demand <br> (Person-Trips/Day) |
| :--- | :---: | :---: | :---: |
| Heavy Rail System | 350,000 | $350,000^{*} 1.20=420,000$ | 385,000 |
| Feeder Bus System | 25,000 | $25,000^{* 1.20}=30,000$ | 27,500 |
| Highway System | $3,900,000-300,000^{*}=$ |  |  |
| Total Person-Trips | $3,600,000$ | $3,600,000^{*} 1.05=3,780,000$ | $3,690,000$ |

*Note: 50,000 of the person-trips on the new rail system are diverted from the local bus system.
**Note: The 27,500 trips on feeder buses are part of the heavy rail system ridership, and are NOT included in the total person-trip.

Table 5.19 Analysis Demands for Option B, Sample Problem 5.5

| Component | Initial Demand <br> (Person-Trips/Day) | 25 - Year Demand <br> (Person-Trips/Day) | Average Demand <br> (Person-Trips/Day) |
| :---: | :---: | :---: | :---: |
| Light Rail System | 200,000 | $200,000 * 1.15=230,000$ | 215,000 |
| Highway System | $3,900,000-200,000=$ | $3,700,000^{*} 1.09=$ |  |
|  | $3,700,000$ | $4,033,000$ | $3,866,500$ |
| Total Person-Trips |  |  | $\mathbf{4 , 0 8 1 , 5 0 0}$ |

## Annual System Costs

System costs include the capital investments necessary to build either of the public transportation options. It also includes the annual operating and maintenance costs for the system. The annual operating and maintenance costs for the bus system must also be included in the Null Alternative and Option A. The bus service is eliminated in Option B.

Highway maintenance is not included in this analysis, as it is assumed to be unchanged, independent of whether or not any rail system is implemented. This is also true of highway capital costs, which are not affected by the option chosen. Capital costs for the three alternatives are illustrated in Table 5.20.

Table 5.20 Capital Costs Included for Sample Problem 5.5

| Alternative | Cost <br> (\$) | $\begin{gathered} \hline \text { Service } \\ \text { Life } \\ \text { (Yrs) } \\ \hline \end{gathered}$ | Annual Cost <br> (\$/Yr) <br> Cost*A/P $\mathrm{P}_{\mathrm{n}, 5 \%}$ |
| :---: | :---: | :---: | :---: |
| Null Alternative Highway System Local Bus System | $\begin{aligned} & 0.00 \\ & 0.00 \end{aligned}$ | $\begin{aligned} & \text { NA } \\ & \text { NA } \end{aligned}$ | $\begin{array}{r} 0.00 \\ \underline{0.00} \\ \mathbf{\$ 0 . 0 0} / \mathbf{y r} \\ \hline \end{array}$ |
| Option A: <br> Rail: ROW, Tracks (in tunnel) <br> Rail: ROW, Tracks (above ground) <br> Rail: Stations (in tunnel) <br> Rail: Stations (above ground) <br> Rail: Rolling Stock <br> Highway System <br> Feeder Bus | $\begin{gathered} 15 \mathrm{mi} @ 150,000,000=\$ 2,250,000,000 \\ 55 \mathrm{mi} @ 50,000,000=\$ 2,750,000,000 \\ 8 @ \$ 15,000,000=\$ 120,000,000 \\ 20 @ 8,000,000=\$ 160,000,000 \\ 700 @ \$ 3,500,000=\$ 2,450,000,000 \\ \$ 0.00 \\ \$ 0.00 \end{gathered}$ | $\begin{aligned} & 50 \\ & 50 \\ & 50 \\ & 50 \\ & 25 \\ & \text { NA } \\ & \text { NA } \end{aligned}$ | $\begin{aligned} & * 0.054777=\$ 123,248,250 \\ & * 0.054777=\$ 150,636,750 \\ & * 0.054777=\$ 6,573,240 \\ & * 0.054777=\$ 8,764,320 \\ & * 0.070952=\$ 173,832,400 \\ & \$ 0.00 \\ & \$ 0.00 \end{aligned}$ |
| Option B: <br> Light Rail: ROW, Tracks <br> Light Rail: Stations <br> Light Rail: Rolling Stock <br> Highway System | $\begin{gathered} 100 @ 20,000,000=\$ 2,000,000,000 \\ 200 @ 75,000=\$ 15,000,000 \\ 1,000 @ 1,800,000=\$ 1,800,000,000 \\ \$ 0.00 \end{gathered}$ | $\begin{aligned} & 40 \\ & 40 \\ & 22 \\ & \text { NA } \end{aligned}$ | $\begin{gathered} * 0.058278=\$ 116,556,000 \\ * 0.058278=\$ 874,170 \\ * 0.075971=\$ 136,747,800 \\ \$ 0.00 \\ \mathbf{\$ 2 5 4 , 1 7 7 , 9 7 0 / \mathbf { y r }} \end{gathered}$ |

The annual costs of operating and maintaining the public transportation systems that are part of the three alternatives must be added to the annual equivalent capital costs of Table 5.18.

In Chapter 4, a national average cost model is calibrated for bus systems. The annual cost of operating and maintaining a bus system may be estimated as:

$$
A C_{b u s}=61.30 V H+1.66 V M+1.06 P
$$

where: $\mathrm{AC}=$ annual cost of maintenance and operation (\$/yr),
$\mathrm{VH}=$ number of annual vehicle-hours operated by the system (veh-hrs/yr),
$\mathrm{VM} \quad=\quad$ number of annual vehicle-miles operated by the system (veh-mi/yr), and
$\mathrm{P} \quad=\quad$ annual revenue passengers carried (persons/yr).
Table 4.23 (Chapter 4) provides similar data for heavy rail and light rail systems which can be used to calibrate unit cost models of the form used for buses. Again, these would be based upon national average statistics reported by the transit industry. Then:
For heavy rail systems:

$$
A C_{\text {Heary Rail }}=86.36 \mathrm{VH}+1.57 \mathrm{VM}+0.46 \mathrm{P}
$$

For light rail systems:

$$
A C_{\text {Light Rail }}=88.05 \mathrm{VH}+3.12 \mathrm{VM}+1.51 P
$$

Note that for rail systems of all types, VH and VM refer to car-hours and carmiles, NOT train-hours and train-miles.

The problem statement provides information on daily vehicle-miles of travel, average speeds, and daily passenger totals. These can be used to determine annual vehicle-hours, vehicle-miles, and passengers. Table 5.21 illustrates these computations.

Table 5.21 Annual Maintenance and Operations Parameters for Sample Problem 5.5

| Alternative | Annual Veh-Mi | Avg Speed (mi/h) | Annual Veh-Hrs | Annual Passengers |
| :---: | :---: | :---: | :---: | :---: |
| Null Alt Highway Local Bus | $\begin{gathered} \text { NA } \\ 10,000 * 365=3,650,000 \end{gathered}$ | $\begin{aligned} & \text { NA } \\ & 12 \\ & \hline \end{aligned}$ | $\begin{gathered} \text { NA } \\ 3,650,000 / 12=304,167 \end{gathered}$ | $\begin{gathered} \text { NA } \\ 50,000 * 365=18,250,000 \\ \hline \end{gathered}$ |
| Option A <br> Heavy Rail <br> Feeder Bus <br> Highway | $\begin{gathered} 90,000 * 365=32,850,000 \\ 4,000 * 365=1,460,000 \\ \text { NA } \end{gathered}$ | $\begin{aligned} & 40 \\ & 10 \\ & \text { NA } \\ & \hline \end{aligned}$ | $\begin{gathered} 32,850,000 / 40=821,251 \\ 1,460,000 / 10=146,000 \\ \text { NA } \end{gathered}$ | $\begin{gathered} 350,000 * 365=127,750,000 \\ 25,000 * 365=9,125,000 \\ \text { NA } \end{gathered}$ |
| Option B <br> Light Rail <br> Highway | $\begin{gathered} 100,000 * 365=36,500,000 \\ \text { NA } \\ \hline \end{gathered}$ | $\begin{array}{r} 35 \\ \mathrm{NA} \\ \hline \end{array}$ | $\begin{gathered} 36,500,000 / 35=1,042,857 \\ \text { NA } \\ \hline \end{gathered}$ | $\begin{gathered} 200,000 * 365=73,000,000 \\ \text { NA } \\ \hline \end{gathered}$ |

Using these parameters, the unit cost models can be used to estimate the annual M\&O costs of each alternative:

$$
\begin{aligned}
& A C_{\text {Null, Bus }}=61.30 \mathrm{VH}+1.66 \mathrm{VM}+1.06 \mathrm{P} \\
& A C_{\text {Null, Bus }}=(61.30 * 304,167)+(1.66 * 3,650,000)+(1.06 * 18,250,000) \\
& A C_{\text {Null, Bus }}=18,645,473+6,059,000+19,345,000=\$ 44,049,473 / \mathrm{yr} \\
& A C_{\text {Opt A,Rail }}=86.36 \mathrm{VH}+1.57 \mathrm{VM}+0.46 P \\
& A C_{\text {Opt A,Rail }}=(86.36 * 821,251)+(1.57 * 32,850,000)+(0.46 * 127,750,000) \\
& A C_{\text {OptA,Rail }}=70,923,236+51,574,500+58,765,000=\$ 181,262,736 / \mathrm{yr} \\
& A C_{\text {Opt A,Bus }}=(61.30 * 146,000)+(1.66 * 1,460,000)+(1.06 * 9,125,000) \\
& A C_{\text {Opt A,Bus }}=8,949,800+2,423,600+9,672,500=\$ 21,045,900 / \mathrm{yr} \\
& A C_{\text {Opt A,Toal }}=181,262,736+21,045,900=\$ 202,308,636 / \mathrm{yr} \\
& A C_{\text {Opt B,Rail }}=88.05 \mathrm{VH}+3.12 \mathrm{VM}+1.51 P \\
& A C_{\text {Opt B,Rail }}=(88.05 * 1,042,857)+(3.12 * 36,500,000)+(1.51 * 73,000,000) \\
& A C_{\text {Opt B,Rail }}=91,823,559+113,880,000+110,230,000=\$ 315,933,559 / \mathrm{yr}
\end{aligned}
$$

The total annual system cost for each alternative is the sum of the equivalent annual capital costs and the annual M\&O costs. These are summarized in Table 5.22.

Table 5.22 Total Annual System Costs for Sample Problem 5.5

| Alternative | Annual Capital Costs <br> $\mathbf{( \$ / \mathbf { y r } )}$ | Annual M \& O Costs <br> $\mathbf{( \$ / \mathbf { y r } )}$ | TOTAL ANNUAL SYSTEM <br> $\mathbf{C O S T S} \mathbf{( \$ / \mathbf { y r } )}$ |
| :--- | :---: | :---: | :---: |
| Null Alternative | 0.00 | $44,049,473$ | $\mathbf{\$ 4 4 , \mathbf { 0 4 9 } , \mathbf { 4 7 3 / \mathbf { y r } }}$ |
| Option A | $463,054,960$ | $202,308,636$ | $\mathbf{\$ 6 6 5 , 3 6 3 , 5 9 6 / \mathbf { y r }}$ |
| Option B | $254,177,970$ | $315,933,559$ | $\mathbf{\$ 5 7 0 , 1 1 1 , 5 2 9 / \mathbf { y r }}$ |

## Annual User Costs

Annual user costs include the cost of the fare and travel time on the public transit portions of the system, plus the cost of travel time and vehicle operation on the highway portion of the system.

As the benefit-cost ratio will be used to compare the alternatives, the approach will be to compute the total annual user costs, then divide them by the number of annual users. Travel time values, speeds and trip times, and vehicle operating costs have been given in the problem statement. Tables 5.16, 5.17, and 5.18 give the number of person-trips per day for each part of the system, based upon the average over the 25 -year analysis period.

Table 5.23 shows the computation of travel time costs for each alternative. Table 5.24 shows the computation of vehicle operating costs and fares for each alternative. It is critical to note that it is only highway vehicle costs that are assigned to users. For public transit components, vehicle operation and maintenance is the responsibility of the system operator.

At this point, the annual user costs must be converted to an annual cost per user or per person-trip. Tables $5.17-5.19$ show the annual person-trips, while Tables 5.23 and 5.24 show all components of the total annual user cost. Annual costs per person-trip are now computed as:

$$
U=\frac{T+O}{V}
$$

where:

| U | $=$ | annual user cost per person-trip $(\$ /$ trip $)$, |
| :--- | :--- | :--- |
| T | $=$ | total annual travel time cost $(\$ / \mathrm{yr})$, |
| O | $=$ | total annual vehicle operating and fare cost <br>  <br> V$\quad$(\$yr), and |
|  | $=\quad$ annual person-trips $($ trips $/ \mathrm{yr})$ |  |

Table 5.23 Travel Time Costs for Sample Problem 5.5

| Alternative | Daily <br> Person- <br> Trips | Average Trip <br> Time <br> $\mathbf{( h r s )}$ | Travel <br> Time <br> Value <br> $\mathbf{( \$ / h r )}$ | Total Annual Cost <br> of Travel Time |
| :--- | :---: | :---: | :---: | :---: |
| $\mathbf{( \$ / \mathbf { y r } )}$ |  |  |  |  |

Table 5.24 Vehicle Operating and Fare Costs for Sample Problem 5.5

| Alternative | Daily PersonTrips | Daily VehicleTrips | Fare (\$) | Veh <br> Oper <br> Cost <br> (\$/veh <br> -mi) | Total Annual Cost of Fare \& Vehicle Operation (\$/yr) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Null Alternative Highway Local Bus | $\begin{gathered} 4,095,000 \\ 50,000 \end{gathered}$ | $\begin{gathered} 4,095,000 / 1.3=3,150,000 \\ \text { NA } \end{gathered}$ | $\begin{gathered} \text { NA } \\ 1.00 \end{gathered}$ | $\begin{gathered} 0.75 \\ \text { NA } \end{gathered}$ | $\begin{gathered} 3,150,000 * 365 * 0.75=\$ 862,312,500 \\ 50,000 * 365 * 1.00=\$ 18,250,000 \\ \mathbf{\$ 8 8 0 , 5 6 2 , 5 0 0} \\ \hline \end{gathered}$ |
| Option A <br> Heavy Rail <br> Feeder Bus <br> Highway | $\begin{gathered} 385,000 \\ 27,500 \\ 3,690,000 \end{gathered}$ | $\begin{gathered} \text { NA } \\ \text { NA } \\ 3,690,000 / 1.3=2,838,462 \end{gathered}$ | $\begin{gathered} 5.40 \\ 1.00 \\ \text { NA } \end{gathered}$ | $\begin{gathered} \text { NA } \\ \text { NA } \\ 0.75 \end{gathered}$ | $\begin{gathered} 385,000 * 365 * 5.40=\$ 758,835,000 \\ 27,500 * 365 * 1.00=\$ 10,037,500 \\ 2,838,462 * 365 * 0.75=\$ 777,028,973 \\ \mathbf{\$ 1 , 5 4 5 , 9 0 1 , 4 7 2} \end{gathered}$ |
| Option B <br> Light Rail <br> Highway | $\begin{gathered} 215,000 \\ 3,866,500 \end{gathered}$ | $\begin{gathered} \text { NA } \\ 3,866,500 / 1.3=2,974,231 \end{gathered}$ | $\begin{gathered} 3.50 \\ \text { NA } \end{gathered}$ | $\begin{gathered} \text { NA } \\ 0.75 \end{gathered}$ | $\begin{gathered} 215,000 * 365 * 3.50=\$ 274,662,500 \\ 2,974,231 * 365 * 0.75=\$ 814,195,736 \\ \mathbf{\$ 1 , 0 8 8 , 8 5 8 , 2 3 6} \end{gathered}$ |

Using the equation, the following annual costs per trip or per user are found as:

$$
\begin{aligned}
& U_{\text {Null }}=\frac{19,815,662,025+880,562,500}{4,150,000 * 365}=\$ 13.663 / \text { trip } \\
& U_{\text {Option } A}=\frac{17,677,450,323+1,545,901,472}{4,075,000 * 365}=\$ 12.924 / \mathrm{trip} \\
& U_{\text {Option } B}=\frac{18,551,592,400+1,088,858,236}{4,081,500 * 365}=\$ 13.184 / \mathrm{trip}
\end{aligned}
$$

## Comparisons

The benefit-cost ratio is computed as:

$$
B C R=\frac{\left(U_{2}-U_{1}\right) *\left(\frac{V_{1}+V_{2}}{2}\right)}{S C_{1}-S C_{2}}
$$

where:

| BCR | $=$ | benefit-cost ratio, |
| :---: | :---: | :---: |
| $\mathrm{U}_{1}$ | $=$ | unit user cost, alternative with the lower value (\$/trip), |
| $\mathrm{U}_{2}$ | = | unit user cost, alternative with the higher value (\$/trip), |
| $\mathrm{V}_{1}$ | $=$ | annual trips for alternative 1, |
| $\mathrm{V}_{2}$ | = | annual trips for alternative 2, |
| $\mathrm{SC}_{1}$ | = | annual system cost for alternative $1(\$ / y r)$, and |
| $\mathrm{SC}_{2}$ | = | annual system cost for alternative 2 (\$/yr) |

Comparing the Null Alternative with Option A:

$$
\begin{aligned}
& B C R_{\text {Null vs } A}=\frac{(13.663-12.924) *\left(\frac{4,150,000 * 365+4,075,000 * 365}{2}\right)}{665,363,596-44,049,473} \\
& B C R_{\text {Null vs } A}=\frac{0.739 * 1,501,062,500}{621,314,123}=1.785
\end{aligned}
$$

This means that the benefits of implementing Option A (in terms of travel time and cost to users) will be 1.785 times the amount of increased investment in construction and operation of the public transit system proposed. The comparison means that implementing the proposed heavy rail system and the revised feeder bus system would be economically viable.

Comparing the Null Alternative with Option B:

$$
\begin{aligned}
& B C R_{\text {Null vs } B}=\frac{(13.663-13.184) *\left(\frac{4,150,000 * 365+4,081,500 * 365}{2}\right)}{570,111,529-44,049,473} \\
& B C R_{\text {Null vs } B}=\frac{0.479 * 1,502,248,750}{526,062,056}=1.368
\end{aligned}
$$

This comparison also shows that the proposed Option B, with its light rail system, would return to users (in the form of travel time and vehicle operating cost savings) 1.368 times the investment in building and operating the new system.

While both of the proposed options are economically favorable when compared to the null case, it is not possible to say that Option A is economically superior to Option B because it produces a higher benefit-cost ratio. This can only be determined by directly comparing Options A and B. Note that for this comparison, Option B has the higher user cost and the lower system cost. Then:
$B C R_{\text {Avs } B}=\frac{(13.184-12.924) *\left(\frac{4,075,000 * 365+4,081,500 * 365}{2}\right)}{665,363,596-570,111,529}$
$B C R_{A v s B}=\frac{0.260 * 1,488,561,250}{95,252,067}=4.063$

This comparison shows that the additional investment in Option A (the heavy rail system) will be returned 4.063 times to users in the form of reduced average trip costs.

Thus, the three alternatives, in terms of economic viability, are ranked as follows: Option A, Option B, Null Alternative.

## Discussion

While the economic analysis clearly favors moving ahead with a public transit system (Option A preferred), the BCR's for both options vs. the null case are not overwhelming. Impacts not included within the economic analysis might be important in this case, given the relatively modest economic advantage of the proposed system. Such things as environmental impacts, neighborhood impacts, the impact of the public transportation system on the value of property (and, therefore, the tax base) and others might weigh on the final decision.

What the analysis does show, however, is that the plans for a regional public transit system are eminently reasonable, and that the direct economic impacts would be positive. Planning of the system should obviously move forward. As specific designs become available, results of environmental impact statements become known, and more detailed analyses of impacts on the general economy of the region are completed, the economic analysis can be expanded and re-done with more detail.

## Sample Problem 5.6: Priority Selection

In previous sample problems, alternatives for action on one particular situation have been considered. An equally important problem faced by most highway agencies is the setting of priorities among economically viable projects, when their implementation is limited by the agency's budget. In this case, a list of candidate projects, each of which already represents a best case among the various options for the project, already exists. The agency's budget, however, does not permit implementation of all approved projects, and a selection of a sub-set for investment must be chosen.

Consider the following case in which a county highway agency has an annual budget of $\$ 10,000,000$. It has already analyzed the following projects, and each is ready for implementation during the budget year. The projects, along with the capital cost and the projected annual "savings" in user and/or system costs are shown in Table 5.25.

Table 5.25 Candidate Projects for Prioritization, Sample Problem 5.6

| Project Number | Capital Cost (\$) | Estimated Annual <br> Savings (\$) |
| :---: | :---: | :---: |
| 1 | $1,500,000$ | 160,000 |
| 2 | $2,000,000$ | 320,000 |
| 3 | $2,500,000$ | 630,000 |
| 4 | $3,000,000$ | $1,025,000$ |
| 5 | $2,200,000$ | 520,000 |
| 6 | $4,000,000$ | $1,240,000$ |
| 7 | $8,000,000$ | $1,480,000$ |

## Definition of Alternatives

The alternatives, in this case, are all possible combinations of project investments that fall within the agency's $\$ 10,000,000$ budget. To simplify the process, the following rules will be applied:

- Each project is assumed to produce a rate of return on investment greater than the market value. Therefore, there is no incentive to not implement any given project to invest the money to earn a higher rate of return.
- A partial investment in a project is not possible. In some cases, particularly where large-scale projects are involved, this is not true, as construction can be spread over several budget years. In this case, the projects are of relatively small scope, and all can be completed in 2-3 months at most.
- The service life of all projects is 10 years.
- The market rate of interest is $5 \%$. Only money that cannot be invested in a full project implementation can be invested to earn this return.

To systematically determine all of the possible combinations for implementation during the budget year, it is easiest to start with one base project, enumerating all of the possible combinations for investment involving that project, then moving to the next project as a base and repeating the process. No combination of projects totally more than $\$ 10,000,000$ of investment is permitted. Table 5.26 enumerates the possible combination of project investments.

Table 5.26 Potential Investment Options for Sample Problem 5.6

| Alt. | Base <br> Project | With <br> Project(s) | Total <br> Investment <br> $\mathbf{( \$ )}$ | Funds <br> Left Over <br> $(\$)$ | Total Annual <br> Savings <br> $\mathbf{( \$ )}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 7 | 1 | $\$ 9,500,000$ | $\$ 500,000$ | $\$ 1,640,000$ |
| 2 | 7 | 2 | $\$ 10,000,000$ | $\$ 0$ | $\$ 1,800,000$ |
| 3 | 6 | $1,2,3$ | $\$ 10,000,000$ | $\$ 0$ | $\$ 2,350,000$ |
| 4 | 6 | $1,2,5$ | $\$ 9,700,000$ | $\$ 300,000$ | $\$ 2,240,000$ |
| 5 | 6 | 3,5 | $\$ 8,700,000$ | $\$ 1,300,000$ | $\$ 1,151,240$ |
| 6 | 6 | 2,4 | $\$ 9,000,000$ | $\$ 1,000,000$ | $\$ 1,346,480$ |
| 7 | 6 | 4,5 | $\$ 9,200,000$ | $\$ 800,000$ | $\$ 2,785,000$ |
| 8 | 6 | 4,3 | $\$ 9,500,000$ | $\$ 500,000$ | $\$ 2,895,000$ |
| 9 | 5 | $1,2,3$ | $\$ 8,200,000$ | $\$ 1,800,000$ | $\$ 1,110,000$ |
| 10 | 5 | $2,3,4$ | $\$ 9,700,000$ | $\$ 300,000$ | $\$ 2,495,000$ |
| 11 | 5 | $1,3,4$ | $\$ 9,200,000$ | $\$ 800,000$ | $\$ 2,335,000$ |
| 12 | 4 | $1,2,3$ | $\$ 9,000,000$ | $\$ 1,000,000$ | $\$ 2,135,000$ |

## Methodology

Sorting out the best alternative is not difficult, once all the choices are enumerated. In each case, the agency is investing $\$ 10,000,000$. The "best" choice is the one which yields the highest annual savings. In this case, the annual savings are the benefits cited in terms of users and system cost, plus what can be earned by investing unused or left-over funds at the market rate of interest, $5 \%$ in this case.

## Solution

Table 5.27 summarizes the total annual savings (or benefits) flowing from the potential investments of the agency's $\$ 10,000,000$ budget.

Table 5.27 Total Annual Benefits for Investment Options in Sample Problem 5.6

| Alternative | Benefits from Projects <br> Funded (\$) | Interest on Left-Over <br> Funds (\$) | Total Annual <br> Earnings (\$) |
| :---: | :---: | :---: | :---: |
| 1 | $\$ 1,640,000$ | $500,000 * 0.05=\$ 25,000$ | $\$ 1,665,000$ |
| 2 | $\$ 1,800,000$ | $\$ 0.00$ | $\$ 1,800,000$ |
| 3 | $\$ 2,350,000$ | $\$ 0.00$ | $\$ 2,350,000$ |
| 4 | $\$ 2,240,000$ | $300,000 * 0.05=\$ 15,000$ | $\$ 2,265,000$ |
| 5 | $\$ 1,151,240$ | $\$ 1,300,000 * 0.05=\$ 65,000$ | $\$ 1,216,240$ |
| 6 | $\$ 1,346,480$ | $1,000,000 * 0.05=\$ 50,000$ | $\$ 1,396,480$ |
| 7 | $\$ 2,785,000$ | $800,000 * 0.05=\$ 40,000$ | $\$ 2,825,000$ |
| 8 | $\$ 2,895,000$ | $500,000 * 0.05=\$ 25,000$ | $\mathbf{\$ 2 , 9 2 0 , 0 0 0}$ |
| 9 | $\$ 1,110,000$ | $1,800,000 * 0.05=\$ 90,000$ | $\$ 1,200,000$ |
| 10 | $\$ 2,495,000$ | $300,000 * 0.05=\$ 15,000$ | $\$ 2,535,000$ |
| 11 | $\$ 2,335,000$ | $800,000 * 0.05=\$ 40,000$ | $\$ 2,375,000$ |
| 12 | $\$ 2,135,000$ | $1,000,000 * 0.05=\$ 50,000$ | $\$ 2,185,000$ |

From Table 5-26, Alternative 8, which implements projects 3, 4, and 6, would be selected as the most economic choice. In this alternative, $\$ 500,000$ is invested at $5 \%$ for the year.

## Discussion

In essence, the analysis period for this example is the one budget year. The analysis would be conducted each year based upon the budget for the year, and the cost of projects, some of which would be carried over from the previous year, and some of which would be new. In the next year, the agency would have access to the amount of its annual budget, plus the $\$ 25,000$ earned on the unused $\$ 500,000$ from the analysis year, plus the $\$ 500,000$ itself that was not spent in the analysis year.

If the exact list of candidate projects were known for a period of upcoming years, a multi-year analysis period could have been implemented.

## Closing Comments

The sample problems of this chapter are, like many elements of this text, intended to be illustrative. Each of the problems illustrates one or more complicating features that must be addressed, and presents ways in which that can be done.

Actual engineering economic analyses of transportation projects are often more complicated than what can be adequately shown in a sample problem. For example, rather than using average demand volumes over an analysis period, an actual analysis would often be done on a year-by-year basis, varying both demand volume and cost values over time. The estimation of the actual costs of various components of each alternative can be quite complicated, and would rely on a wide variety of sources, both national and local, for information. Analyses might also add specific estimates of the costs associated with indirect effects of the proposed projects.

Despite this, the fundamental approaches illustrated here and in other chapters remain the same. In its most straightforward form, engineering economic analysis studies as many costs as can be reasonably estimated, and reduces each to a common basis, so that alternatives can be compared, and best options selected for implementation.

## Problems

## Problem 5.1

A transit system must prepare to replace its entire fleet of buses over period of 5 years. Twenty percent of the fleet will be replaced in each of the five years. The transit system has two options for replacement of the fleet:

| Item | Option A: <br> Conventional Buses | Option B: <br> Articulated Buses |
| :--- | :---: | :---: |
| Capacity of Bus (Seated) | 50 Passengers | 90 Passengers |
| Cost of Bus | $\$ 450,000$ | $\$ 700,000$ |
| Service Life | 13 Years | 10 Years |
| Maintenance \& Operating Cost ${ }^{1}$ | $\$ 1.50 /$ bus-mile | $1.65 /$ bus-mile |
| Operating Labor Cost $^{2}$ | $\$ 63.00 /$ bus-hr | $\$ 63 / \mathrm{bus}-\mathrm{hr}$ |

1. Includes fuel and tire costs, and all maintenance expenditures, including labor.
2. Includes bus drivers, dispatchers, and other labor associated with daily operations.

The system currently serves 75,000 persons/day. The average trip consists of 4.0 minutes of waiting time and 22.6 minutes of travel time on the bus. In its peak hour, the system serves 28,000 persons/day. Passenger demand is not expected to change based upon how the system is re-equipped. The average prevailing wage for passengers $\$ 27.50 / \mathrm{hr}$.

Because of the route system and other factors, buses cannot make more than one trip during the peak hour, but sufficient buses are available to provide services before and after the peak hour. The system currently owns 570 buses. Buses currently operate 45,000 miles $/ \mathrm{yr}$ at average speeds of $13.5 \mathrm{mi} / \mathrm{h}$. At any given time, $25 \%$ of the current fleet is out of service due to maintenance. The existing buses cost $\$ 1.71 /$ bus-mile to maintain and operate, with operating labor costs of \$63.00/bus-hr.

If the buses are replaced by new conventional buses, they will continue to operate an average of 45,000 miles $/$ year at average speeds of $13.5 \mathrm{mi} / \mathrm{h}$. The travel times of passengers will be unchanged. Because the buses will be new, however, only $10 \%$ of the fleet will be in maintenance at any given time.

If buses are replaced by articulated buses, they will operate an average of 50,000 miles $/ \mathrm{yr}$ at average speeds of $15 \mathrm{mi} / \mathrm{h}$. The travel times of passengers will be reduced to 20.7 minutes on the bus. However, fewer buses will be operating at any given time, the average wait time will increase to 4.2 minutes. Because of the more complex mechanics of articulated buses, $13 \%$ of the fleet will be in maintenance at any given time.

An analysis interest rate of 3\% per year should be used for this problem.
a) Should there be a null alternative in this case? Why or why not? If yes, describe it.
b) What would be an appropriate analysis period for this case? Why?
c) What methodology would you apply? Why?
d) Conduct the engineering economic analysis and indicate the most economically viable option. State any assumptions you make in conducting the analysis.

## Problem 5.2

A highway department wishes to reconstruct a 30 -mile section of rural two-lane highway running through a region of mountainous terrain with a 4-lane freeway that will significantly straighten the alignment. Relevant information on the existing and proposed segments is given below:

| Item | Existing 2-Lane H'way | Proposed Freeway |
| :--- | :---: | :---: |
| Current AADT | $9,000 \mathrm{veh} / \mathrm{day}$ | $9,000 \mathrm{veh} / \mathrm{day}$ |
| 20-Year AADT Forecast | $10,000 \mathrm{veh} / \mathrm{day}$ | $24,000 \mathrm{veh} / \mathrm{day}$ |
| Percent Trucks | $12 \%$ | $12 \%$ |
| Operating Cost - Cars | $\$ 0.81 / \mathrm{veh}-\mathrm{mile}$ | $\$ 0.77 / \mathrm{veh}-\mathrm{mile}$ |
| Operating Cost - Trucks | $\$ 2.75 / \mathrm{veh}-\mathrm{mile}$ | $\$ 2.35 / \mathrm{veh}-\mathrm{mile}$ |
| Average Speed | $45 \mathrm{mi} / \mathrm{h}$ | $63 \mathrm{mi} / \mathrm{h}$ |
| Construction Cost | NA | $\$ 19,500,000 / \mathrm{mile}$ |
| Maintenance Cost | $\$ 15,000 / \mathrm{mi} / \mathrm{yr}$ | $\$ 6,000 / \mathrm{mi} / \mathrm{yr}$ |
| Length of Segment | 30 miles | 23 miles |
| Average Wage - Auto User | $\$ 32.00 / \mathrm{hr}$ | $32.00 / \mathrm{hr}$ |
| Average Compensation - <br> Truck Driver | $\$ 57.00 / \mathrm{hr}$ | $\$ 57.00 / \mathrm{hr}$ |
| Auto Occupancy - Cars | 1.4 | 1.4 |
| Vehicle Occupancy - Trucks | 1.0 | 1.0 |
| Analysis Interest Rate | $4 \%$ | $4 \%$ |

a) What would be an appropriate analysis period for this problem? Why?
b) What methodology would you choose to address this situation? Why?
c) Compare the two alternatives and make a recommendation on which should be implemented. Why?

Part II
Financing of Transportation
Infrastructure

## Chapter 6 <br> History of Transportation Finance in the U.S.

This chapter reviews the history of transportation finance in the United States, and traces their development over time.

From the earliest days of the nation, the issue of how to provide for an effective transportation network was a vexing one. Many of the nation's founders recognized the need for an effective transportation system to help knit the disparate colonies into a cohesive nation, and to promote commerce.

George Washington was particularly concerned about western parts of the new nation remaining isolated, and perhaps developing stronger ties to Spanish colonies to the west than to the new nation. The need for improved transportation to the west was heightened by the addition of Ohio as a state in 1803.

Despite these concerns, the new nation faced many other problems, and both state and local governments were hard-pressed to fund all of their public needs. Because of this, the first intercity and interstate roadways were privately-financed toll-roads or turnpikes.

### 6.1 The Early Toll Roads [1-3]

As the nation faced its early needs for transportation, its options were severely limited. The steamship had not yet been invented, and the building of canals was costly. The steam engine and the age of railroads were also in the future. The nation was limited to water transportation using existing rivers, harbors, and oceans, and overland transportation on roads that were merely cleared paths through the natural terrain.

Before the 1790 's, virtually all roads were built by local governments to serve the needs for transport within the town or city. Local governments, however, struggled to pay for the needed construction and maintenance, often resorting to measures requiring able-bodied men to contribute hours of labor to these endeavors. Later, these evolved into fees that could be paid in lieu of such labor.

Recognizing the need for better roads to connect towns and cities, and to facilitate agriculture in more rural areas, private companies jumped into the breach by constructing and operating toll roads or turnpikes. The term "turnpike" actually referred to a British system for collecting tolls on private roads, which consisted of a long "pike" that blocked the road at periodic intervals. When the toll was paid,
the pike was manually "turned" on a swivel device located at the roadside to allow passage. Most of the early turnpikes were simply cleared paths through the wilderness, with roadway surfaces consisting of compacted earth.

The first private turnpike in the U.S. was built in Pennsylvania. It was chartered by the state government (the road was built over publicly-held land) in 1792, and took two years to build. The road traversed 62 miles between Philadelphia and Lancaster, and quickly attracted the attention of merchants in other states who recognized the potential of the new road to divert commerce to the areas it served.

By 1845 , over 1,500 private toll roads across the country had been chartered and built. Many of these produced only modest returns to stockholders, but the indirect benefits to those with homes and businesses nearby the routes were substantial. However, by the late 1840 's, the advent of the steam engine and railroads, as well as state expenditures on canals severely damaged the viability of private toll roads, and most fell into poor condition, and many were simply abandoned to state or local control.

From the mid-1840's to the mid-1850's, approximately 10,000 miles of private toll roads were built as "plank" roadways. Plank roadways were the initial attempt to stabilize the physical condition of roadways through plank construction. Planks were placed over wooden beams placed at each roadside (on compacted earth), usually providing a roadway of between 18 and 22 ft . Because many of these roadways used the rounded side of timber planks, they provided a rough surface that was often referred to as "corduroy," a term now used to describe clothing fabric with a similar pattern. Because of the additional cost of plank roadways, higher tolls were charged at more frequent intervals. Also, while previous toll roads had exempted many local users from payment of the toll, such exemptions were severely limited on plank roadways.

The exact number of private toll roads built throughout the U.S. is not fully documented. Most historians believe that between 30,000 and 50,000 miles of such roadways were built.

Nevertheless, by 1800, public sentiment against privately-operated toll roads, and toll roads in general, spurred a gradual transition to governmental construction and operation. By 1820, there were very few private toll roads in operation, and most road construction and maintenance had been transferred to state, county, and local governments. At the same time, the federal government made its first entry into the provision of a roadway network. Over the next century (1820-1920), the appropriate and legal role of the federal government was strongly debated. The debate led to the current system of highway finance, which was first codified in the Federal-Aid Highway Act of 1916.

### 6.2 The National Road [4-6]

With the development of Ohio and its achievement of statehood in 1803, the need for a road linking the east coast with the west became increasingly evident. The Potomac and Ohio Rivers were major conduits for commerce, but they were separated by the Allegheny Mountains. The idea for a national road connecting the two was developed over a period of years. In 1803, Congress proposed to allocate
a portion of the funds raised through land sales in Ohio to the construction of a "national road" that would travel from Cumberland, MD, to Wheeling, Virginia (now in West Virginia). After much discussion and controversy, President Jefferson authorized construction of the roadway on May 29, 1806, providing \$30,000 in federal funds to do so. Construction began in 1811, and the road was completed to Wheeling in 1819. The road was the first in the U.S. to provide a surface of crushed stone, known as a "McAdam" surface, which provided for greater durability than previous roadways. Congress authorized western extensions to the road in 1820 and 1825.

In 1830, President Andrew Jackson vetoed the "Maysville Road Bill." The Maysville Road was to be part of the National Road, but was located entirely within Kentucky. Jackson indicated that the federal government could not fund any public projects that did not benefit "the entire nation," and particularly not a roadway that was located entirely within one state. If such funding were desired, he declared that a constitutional amendment would be required. This veto had a lasting effect, and has effectively kept the federal government out of the direct construction, maintenance, and administration of roadways in the U.S. After this veto, it was generally understood that road construction and maintenance were to be a function of the various states.

This view has prevailed, even though a subsequent Supreme Court decision (Wilson vs. Shaw, 1907) declared that the federal government could build and administer interstate highways under the provisions of the Commerce Clause [7].


Fig. 6.1 Map of the National Road in 1839 (Source: Wikimedia Commons)
Jackson did not oppose all funding of the National Road, and approved additional funds for interstate segments of the route. Construction of the National Road continued with federal support through 1838, when it reached its western terminus in Vandalia, Illinois. From the mid-1830's, various segments of the National Road were turned over to the states for maintenance and administration.

Over the course of its history, the National Road was often referred to as the Cumberland Road or the National Pike. As the road fell into poor repair and disuse in the late 1800's, its various parts were transferred to state and local jurisdictions. The majority of the original route is now part of U.S. Route 40.

### 6.3 The Good Roads Movement [8, 9]

The "Good Roads Movement" was a grass-roots organization of citizens promoting the idea of a cohesive national roadway system. Initially, the movement was dominated by bicyclists, led by the League of American Wheelmen, which had formally organized in May 1880. The league decried the generally poor condition of rural and inter-community roadways, and began to publish a magazine - "Good Roads" - that highlighted and documented the problem. Primarily in response to the movement, the U.S. Department of Agriculture initiated a national study of roadways in 1893.

As the 1920's approached, the movement had been taken over by the growing number of auto enthusiasts who were interested in promoting a better roadway network for the emerging automobile technology. When Henry Ford introduced his assembly line in 1914, the automobile became accessible to a broad audience, and the automobile began its journey to becoming the nation's primary mode of transportation.

### 6.4 The Lincoln Highway [10]

The Lincoln Highway was the first named transcontinental auto trail in the U.S. It was the first among many similar highways to follow, and was promoted by entrepreneur Carl Fisher. The highway consisted of mapping a continuous route using existing roads. It originally ran through 12 states, linking Times Square in New York to Lincoln Park in San Francisco. The route was promoted by the Lincoln Highway Association, and was a boon to local businesses that were adjacent to it. Various states and local governments began to improve the various portions of the roadway. The route was altered somewhat in 1915, and again in 1928. In its final form, the route crossed 14 states, 128 counties and 700 local jurisdictions.

The success of the Lincoln Highway spurred other state and local governments and business associations to promote additional roadways that became part of the "national trails" movement. Among the many national trails were the Yellowstone Trail, which traversed a northern route linking Massachusetts to Seattle, Washington, and the Dixie Highway, which traveled north-south, linking Chicago, Illinois to Miami, Florida. It was during the creation and upgrading of various national trails that the need for a more organized system of financing these highways, one that involved both states and the federal government, became clear.

### 6.5 The Federal-Aid Highway Program, 1916 to 1955

As the need for a national highway system grew, it became clear that there needed to be a role for the federal government in two critical ways:
(1) If there was to be a system of national highways, the federal government would need to have a role in its planning and coordination, and
(2) Given the enormity of the task and the expense to complete it, most states would need financial assistance from the federal government.

The result was a system in which the federal government provided financial assistance to states to help build roadway networks. Because the federal government was providing at least partial funding, it began to regulate how the funds could be spent.

### 6.5.1 Federal-Aid Highway Act of 1916 [11]

The first formal federal-aid highway act occurred in the midst of turbulent times. World War I had begun in Europe, and the role of the U.S. was a hotly-debated political issue. The temperance movement was pushing the U.S. towards prohibition, which would begin in 1920. Use of the federal income tax was also hotly debated, although the $16^{\text {th }}$ Amendment, adopted in 1913 clearly made it constitutional. In some ways, the expansion of the income tax was made necessary by prohibition, as a significant portion of federal revenues prior to prohibition came from excise taxes on alcoholic beverages.

By 1915, it was estimated that there were approximately 2.5 million miles of roadway in the U.S. Of this total, only $10.5 \%$ were surfaced, and of those, only $12.5 \%$ were paved with bituminous material, brick, or concrete. Intercity travel was not for the faint of heart. Roadway conditions were, in general, horrible, and the condition of many roadways after rain made them virtually impassable. In addition to all their provisions, intrepid adventurers had to carry tire and wheel repair kits, tools to help dig cars out of mud or other hazards, and a variety of spare parts. The need for a significant upgrade and expansion of American highways was evident.

Legislation for the intervention of the federal government into the highway program was introduced by Rep. Dorsey Shackleford of Missouri and Sen. John Bankhead of Alabama, and was passed on July 11, 1916. Interestingly, Bankhead had been an avid supporter of the Good Roads movement, and even had a national trail named after him. The Federal-Aid Highway Act of 1916 contained a number of important provisions:
(1) It provided $\$ 75$ million of federal funding for a five-year period.
(2) Federal funds were allocated to the states in proportion to the state population and the mileage of rural postal routes within the states.
(3) Federal funds could cover up to $50 \%$ of the cost of any individual project, limited, however, to no more than $\$ 10,000 /$ mile of roadway built.
(4) Funds could not be applied to urban roadways, defined as those within a community (based upon census data) with a population of 2,500 or more.
(5) The program was to be administered by the Department of Agriculture, and all project plans had to be approved by the Secretary of Agriculture before federal funds were applied.
(6) All federal-aid highway funds were dispensed from the federal general fund.
(7) Within a prescribed period of time, each state had to form and identify an agency charged with overseeing the state's roadway system.

While the act was a major first step in rationalizing a system of highway funding for the nation, it was deemed flawed by many. It did not provide any funding for highways within cities, where some of the greatest needs for road improvement existed. It also did little to promote the development of a cohesive national network of highways.

Within a year of the passage of the Federal-Aid Highway Act of 1916, virtually every state had developed an administrative unit to oversee and implement highway construction. On a federal level, the Bureau of Public Roads (BPR) was formed, with membership from every state highway agency.

### 6.5.2 Federal-Aid Highway Act of 1921

In 1921, the successor to the first federal-aid highway act was passed. While it did not address the need for funding of urban roadways, it took a number of steps towards identifying and creating a truly national highway network. Key provisions included:
(1) An initial appropriation of $\$ 75$ million was allocated for the first year of the program. Subsequent allocations were handled on an annual basis. This greatly increased the total amount dedicated to the program.
(2) While the $50-50$ split on federal vs. state/local funding was retained, the maximum federal allocation was increased to $\$ 20,000$ per mile.
(3) States were limited to having $7 \%$ of their roadways included in the federal-aid program, and $3 \%$ of that mileage had to be part of the "Primary or Interstate" highway network. Up to $60 \%$ of the total federal-aid received could be allocated to the "Primary or Interstate" system.
(4) The act provided that up to $2.5 \%$ of federal-aid funds could be spent for administration, research, and investigational studies related to federal-aid highways.

To implement the requirements related to the "Primary or Interstate" highway network, such a network had to be identified. In 1922, the Bureau of Public Roads authorized General John. P. Pershing to construct a national highway map that would meet the nation's defense needs in time of war. The resulting Pershing Map became the starting point for defining the "Primary or Interstate" highway system.

The Federal-Aid Highway Act of 1921 required that the Secretary of Agriculture prepare a map of primary or interstate highways by 1923, and issue updates on an annual basis.

### 6.5.3 Federal-Aid Highway Acts from 1921 to 1955

For the 34 years following the second federal-aid highway act in 1921, renewals of the legislation occurred every 2-5 years with only minor changes to their provisions. Funding rose gradually to reflect growing national needs and the increased costs of building better and more modern roadways.

Along the way, both states and the federal government recognized that to sustain the ever-increasing funding needs for highways, additional revenues would be required. Throughout the period, however, all disbursements of federal-aid highway monies were from the federal general fund.

In February of 1919, the first state excise tax on gasoline was inaugurated in Oregon. The tax was set at $\$ 0.01 /$ gallon, and was soon adopted in many other states. The Revenue Act of 1932 added a $\$ 0.01 /$ gallon excise tax on gasoline at the federal level as well. The revenues from this tax were placed in the federal general fund. Over time, both state and the federal gasoline tax were increased, and excise taxes on diesel fuel were added.

The Federal-Aid Highway Act of 1934 contained some significant provisions. It authorized that up to $1.5 \%$ of federal-aid highway funds could be used to conduct surveys, planning, and engineering and economic studies of future highways. It also created a cooperative program between the states and the Bureau of Public Roads to conduct statewide highway planning surveys. These would result in a complete mapping of the states' highway systems along with condition assessments. The program also included regular traffic volume counts and studies aimed at establishing a fair distribution of the costs of highways to various user groups.

Beginning with the Federal-Aid Highway Act of 1938, the idea and concept for what we now call the "Interstate System" was developed. The 1938 act required the Bureau of Public Roads to study the feasibility of a toll-financed highway system. It was to consist of three east-west and three north-south "superhighways." When the BPR issued its report, it indicated that a toll-financed system would not be self-supporting. It instead recommended a 26,700-mile network of "interregional highways."

In 1941, President Franklin D. Roosevelt appointed a commission to evaluate the need for a "national expressway system." The report of the commission, issued in January of 1944, recommended a system of 33,900 miles, with 5,000 miles of additional urban routes.

The commission's report was codified in the Federal-Aid Highway Act of 1944. The act required designation of a national system consisting of up to 40,000 miles that would connect principal metropolitan areas to each other, serve the national defense, and recognize the continental importance of connections to Canada and Mexico. On August 2, 1947, the first 37,700 miles was identified. States had submitted highways to be included in the system, and these were reviewed by the Department of Defense.

While the Federal-Aid Highway Act of 1944 began to define the Interstate System, the act did not appropriate any additional funds to pay for new network. The Federal-Aid Highway Act of 1952 authorized an expenditure of $\$ 50$ million over the fiscal years 1954 and 1955, and a subsequent act in 1954 authorized an additional $\$ 350$ million for 1956 and 1957. The allocations were, however, mere tokens compared to the total cost of building the system, and little was done towards implementing the system.

### 6.6 Federal-Aid Highway Act of 1956 [12, 13]

The Federal-Aid Highway Act of 1956 finally provided funding for construction of the interstate highway system. President Dwight Eisenhower led the effort to find a means to pay for the system that was essentially self-sustaining.

It was indeed fitting that the system began to be realized under Eisenhower's presidency. As a young Lt. Colonel, Eisenhower had been part of the legendary 1919 Motor Transport Corps Convoy. The convoy was one of several authorized to test the nation's highway systems for readiness in times of war. With WWI already underway, the ability to move troops and equipment to various parts of the country had become a critical issue. The convoy started in Washington D.C. on July 7, 1919, and was to travel to Oakland, California, ferrying across the bay to its final destination in San Francisco. The convoy mostly traversed the route of the Lincoln Highway for its journey, which took 62 days to complete, 7 days longer than anticipated.

Along the way the convoy suffered 230 "road incidents," which resulted in 21 vehicles being "retired." Over 80 wooden bridges were substantially damaged and had to be repaired before proceeding. The convoy covered the 3,250 -mile treck at an average speed of $5.7 \mathrm{mi} / \mathrm{h}$. The failure of the highways to adequately serve the convoy became a deep concern of Eisenhower throughout his career. By contrast, Eisenhower's WWII experiences left him quite impressed by the German autobahn system, which he thought would be an appropriate model for a U.S. system.

The Federal-Aid Highway Act of 1956 included the Highway Revenue Act of 1956, which provided the mechanism for funding interstate highways. The combined act included the following provisions:
(1) It changed the name of the system to the National System of Interstate and Defense Highways.
(2) Defined a system of 41,000 miles to be designed and constructed to new standards.
(3) Established a funding proportion of $90 \%$ federal and $10 \%$ state/local monies. This was the first change in the 50-50 funding split established in 1916.
(4) Established the Highway Trust Fund into which a number of road user taxes would be placed, and from which all federal-aid highway funds would be disbursed.
(5) Defined a new method of distributing the funds among states.
(6) Provided an initial investment of $\$ 25$ billion per year to build the system.

The creation of the Highway Trust Fund was critical in allowing the system to move forward. The federal fuel tax was increased from $\$ 0.02$ to $\$ 0.03$ per gallon, with all funds deposited in the Highway Trust Fund. New federal excise taxes were levied on motor vehicle purchases, oil, replacement tires, and other replacement parts. Most of these, with the exception of the fuel tax, were dropped by President Richard Nixon in the 1970s. The trust fund enabled the expenditure, per year, of a total that was higher than the sum of all federal-aid highway disbursements from 1916 through 1956.

The inclusion of a specific defense objective for the system gave Congress the cover it needed to increase the federal share of the costs of the interstate system from $50 \%$ to $90 \%$, which was necessary to avoid placing a crushing expense on the individual states.

Among the physical standards for highways included in the system were:
(1) All highways must have full control of access.
(2) All highways must have at least two lanes for the exclusive use of traffic in each direction.
(3) All highways must have a design speed of 50 to $70 \mathrm{mi} / \mathrm{h}$, based upon the type of terrain.
(4) Lanes must be at least 12 feet wide; right-side shoulders must be at least 10 ft , and must be paved; left-side shoulders must be at least 4 feet wide, and must be paved.
(5) All designs must be adequate to meet traffic volumes expected in 1975.

Further, all interstate-designated highways in the system must begin and end in a junction with another interstate-designated highway. In some cases, a border junction with a major highway in Canada or Mexico would also be permitted.

The allocation of federal-aid highway funds to the various states was revised to reflect the relative cost of completing the interstate system within each state.

With the Federal-Aid Highway Act of 1956, the implementation of the National System of Interstate and Defense Highways would dominate the national highway program for over a decade.


Fig. 8.2 Map of the Completed National System of Interstate and Defense Highways (Source: Federal Highway Administration, Washington D.C.)

### 6.7 Federal-Aid Highway Acts from 1957-1991

With the creation of the Highway Trust Fund and initiation of the interstate system in 1956, federal-aid highway acts entered another period in which renewals every 2 years occurred without major changes in basic funding formulas. Inter-state-included projects received $90 \%$ funding from the federal government, while non-interstate-included projects remained on the traditional system of $50 \%$ federal funding. Nevertheless, there were some significant developments and new programs introduced in some years.

The Federal-Aid Highway Act of 1962 added a requirement that all urban areas with populations in excess of 50,000 develop a comprehensive transportation plan, which would include consideration of all transportation modes. The plans were to be developed by states in cooperation with local governments, and had to be filed by July 1, 1965. This requirement began a trend towards development of metropolitan planning organizations (MPO's) focusing on multimodal regional transportation plans.

The Federal-Aid Highway Act of 1968 included several very important provisions which changed the course of the highway program:
(1) The total mileage of the interstate system was increased from 41,000 miles to 42,500 miles to account for "missing links" in the system.
(2) The public hearing process was changed to require that the public receive information on the social effects of highway location, its impact on the environment, and its consistency with the goals and objectives of urban plans promulgated by the community. These requirements were added to the traditional public hearing process that was limited to the economic impacts of highway location.
(3) A new program, called Traffic Operations Program to Increase Capacity and Safety (TOPICS), was initiated. It provided $\$ 200$ million/year for two years to fund traffic engineering improvements to principal urban streets. Such improvements included channelization and additional lanes at intersections, pedestrian overpasses, traffic signal systems, bus lanes, and elimination of spot safety hazards.

Item (2) greatly expanded the information that had to be presented at public hearings, most of which could only be given after the final routing of the proposed highway was established. These expanded hearings provided opportunities for local opposition to be organized, and led to many lawsuits challenging proposed highway projects. The provision greatly slowed the process of highway construction in urban areas.

Item (3) was significant in that it was the first authorization of federal-aid funds for improvements within existing rights-of-way, many of which did not involve new construction.

The Federal-Aid Highway Act of 1970 (also known as the Highway Safety Act of 1970) contained a number of significant measures:
(1) The funding formula for non-interstate highway projects was changed from the historic 50-50 split to $70 \%$ federal funding beginning in 1974.
(2) Created a new classification of federal-aid urban highways within urban areas of population 50,000 or greater.
(3) Authorized expenditures (from the federal general fund) for a highway beautification program, and created a commission to study such a program.
(4) Created the National Highway Traffic Safety Administration (NHTSA).
(5) Required each state to have an agency charged with development and operation of its highway safety program, and that plans for such an agency be filed with Secretary of Transportation by the end of 1971.
(6) Provided that expenditures for development and research activities related to traffic safety be financed $2 / 3$ from the Highway Trust Fund and $1 / 3$ from the federal general fund.
(7) Delayed the reduction date for certain federal excise taxes from October 1, 1972 to October 1, 1977.

The safety-related provisions of this act incorporated the requirements of two previous acts passed in 1970. The requirements for each state to have an active ongoing highway safety program was critical, as the number of deaths due to highway accidents had grown to over 50,000 per year by 1970 . The new safety requirements included that each state establish a regular system for collecting and analyzing traffic accident data, as well as emergency evacuation programs for serious traffic accidents.

The Federal-Aid Highway Act of 1983 introduced an important program known as the "Interstate Trade-In Provision." By 1983, it was clear that some of the few remaining segments of the interstate system would not be built for a variety of reasons. This provision allowed a state to "trade in" the federal funds they would have received for the highway and receive an equal amount of funds to improve public transportation services. While technically, the public transport funding came from another source, this marked the first instance in which federal-aid highway funds could be used for public transportation improvements. The first two major trade-in requests were the "inner loop" highway in Boston and the Westway project in New York City.

### 6.8 Federal Funding for Railroads

While the federal government developed a program to support highway development and construction, it also provided support for other modes of transportation. Some of this support pre-dates the highway program, specifically the federal support for construction of transcontinental railroads.

### 6.8.1 The Pacific Railway Acts, 1862-1866 [14]

Prior to the Civil War, railroad construction was privately financed, although charters had to be issued by the various states to authorize construction, and much of the right-of-way acquired was owned by the states.

By 1860, there was already an extensive network of railroads throughout the eastern portions of the nation. With the start of the Civil War, railroads became strategically important for the movement of soldiers and war materials. The North held a huge advantage in that its railroads were better coordinated and provided excellent regional coverage. Nevertheless, both the U.S. and the Confederacy invested funds to improve and expand their railroad networks as part of the war effort.

With the addition of the Oregon Territory and California to the U.S. in 1846 and 1848 respectively, interest in creating a transcontinental railroad began to grow. The "way west" could only be traveled by stagecoach or wagon train, and the hardship and danger presented by such travel retarded the settlement of the west, which was considered critical for the cohesive development of the nation that would now proceed on two shores located over 3,000 miles apart.

The first Pacific Railway Act was passed in 1862. It passed after the secession of the Confederate states, and was amended and augmented in 1863, 1864, 1865, and 1866. The first act established the route of the first transcontinental railroad, and assigned the charter to build it to two railroads: the Central Pacific, which would start building from the west coast eastward, and the Union Pacific, which would start building from the mid-west, heading westward. The starting point for the Central Pacific was established as Oakland, California; the Union Pacific would begin at Council Bluffs, Iowa. The eastern terminus was already connected to a network of railroads serving the eastern portion of the U.S. Due to a number of considerations, the western terminus was eventually moved to Sacramento, California.

Both the Central Pacific and Union Pacific were required to build 50 miles of railroad per year. They received government assistance of $\$ 16,000 / \mathrm{mile}$ in level terrain areas, $\$ 32,000 /$ mile on easy grades, and $\$ 48,000 /$ mile in mountainous terrain. Assistance was in the form of federal government bonds, which could be resold. The assistance was, however, really a loan, as the railroads had to repay the principal plus interest on the bonds.


Fig. 6.3 Route of the First Transcontinental Railroad (Souce: Wikimedia Commons, used under Creative Commons Share-Alike 3.0 unported license)

The most direct subsidy to the railroads was in the form of land grants. Each railroad was given 200 ft of land on either side of the track they built. In addition, for each mile of track built, the railroad was given 10 square miles of adjacent land. The right-of-way and land grants were primarily owned by the federal government. To avoid having the railroads exercise a monopoly over development of adjacent lands, the lands (under the $10 \mathrm{sq} \mathrm{mi} / \mathrm{mi}$ of RR provision) granted were arranged in a checkerboard pattern, with the government retaining ownership of alternating blocks. The land grants were used in a number of ways to help finance construction. Bonds were sold based upon the value of the land, but principal plus interest had to be repaid. Lands were also sold outright to prospective developers. While under construction, bonding was the prevalent form of financing. Once completed, however, the land became extremely valuable (in many, but not all areas) and could be sold for excellent prices.

While the federal government played a major role in funding the transcontinental railroad, economists argue over whether the financing really represents a subsidy. As far as the use of bonds is concerned, it is clear that the railroads repaid the debt incurred, plus interest. The land grants, however, were a different story, and railroads benefited greatly from direct sales of adjacent land, and from being a monopoly carrier of freight for farmers and businesses located adjacent to and near the railroad rights-of-way. Over the years, this monopoly created a great deal of tension between settlers and the railroads.

### 6.8.2 AMTRAK [15]

Railroads are the generally considered to be the "forgotten mode" where federal subsidies and direct support are involved. Indeed, railroad freight operations are overwhelmingly private, with Conrail, a government corporation operating without subsidies, serving as a major eastern carrier that consolidated a number of independent railroads.

In the early days of the transcontinental railroads, passenger service was provided (usually on a monopoly basis). Costs were high, but transcontinental travel by wagon train or stagecoach was too daunting for most, and these services quickly disappeared. The federal government, however, did provide a tremendous indirect subsidy to the early railroads. The Post Office placed the vast majority of its intercity mail on passenger trains, including all of the transcontinental mail.

The railroads, however, found that passenger and freight service were often incompatible. Passenger trains had to be reasonably comfortable (emphasis on reasonably), and they had to run on time. Passenger trains required a higher level of track maintenance to maintain a comfortable ride. Freight could be carried over bumpy tracks, and exact schedule adherence was not critical. By the early 1900's, rail passenger service had begun to deteriorate. World War I caused a recovery in passenger rail service, as the government used railroads to transport both soldiers and war materials. After WWI, the decline of passenger rail services accelerated. World War II provided another short-term spike in passenger rail travel due to both the war and the rationing of gasoline, but when the war ended, it was clear that passenger rail service was in dire straits. As the railroads allowed passenger
service to deteriorate in favor of freight, it also lost the indirect subsidy of the mail. The uncertainty of passenger rail schedules, combined with the growth of air transportation, resulted (over time) in most intercity mail service being shifted to the airlines.

In the 1960 's, public calls for the federal government to act to "save rail passenger transportation" began to build. The eventual result was the Railroad Passenger Service Act of 1970. The act created the National Railroad Passenger Corporation to take over and operate most of the nation's remaining intercity passenger rail services. Originally called "Railpax," the system was eventually dubbed "AMTRAK," an amalgam of "American" and "track."

The concept was to operate AMTRAK as a public corporation that would eventually be self-sustaining. Initial subsidies for both operations and capital projects were intended as temporary measures that would eventually be phased out. The concept has never been realized.

In FY 2011, Congress allocated $\$ 563$ million is direct operating subsidies and $\$ 922$ million in direct support of capital projects. In the same year, AMTRAK operated 300 trains/day serving 500 destinations in 46 states. It carried 30.2 million passengers in 2011.

The monies allocated to AMTRAK are drawn from the federal general fund, and are always politically controversial. Congress frequently threatens to cut off AMTRAK's subsidies and turn the system over to completely private operations. To date, it has not done so, but the threat lingers as long as AMTRAK cannot be shown to be self-sufficient.

AMTRAK continues to deal with many age-old problems. It runs most of its services over tracks leased from freight railroads, and frequently shares the tracks with freight trains. The use of passenger rail is highly concentrated in a few dense corridors, leaving the rest of the system with, at best, sparse service. The mostused service is in the Northeast Corridor, from Washington D.C. to Boston, MA, which runs through New York, NY. This corridor has been constantly upgraded, including major station renovations, installation of welded rail, and use of higherspeed trains (up to $150 \mathrm{mi} / \mathrm{h}$ ). The service is quite popular, with peak trains frequently running at capacity. It is still difficult, however, to make the service self-sustaining, given the costs of maintaining high-quality service.

### 6.8.3 High-Speed Rail Program [16]

The issue of high-speed rail in the U.S. has also been controversial. The development for high-speed rail transportation technology has occurred primarily in Asia and Europe. Numerous high-speed systems have been built throughout the world using both conventional rail technology and magnetically levitated vehicles.

The Japanese opened their first bullet-train line in 1964, using conventional rail technology, reaching cruising speeds of $130 \mathrm{mi} / \mathrm{h}$. Subsequently, the bullet-train system was expanded into a national network, with speeds gradually increased to a current (in 2011) maximum of $186 \mathrm{mi} / \mathrm{h}$. While new technology was used to lay fully-welded rails on a separated right-of-way, and develop rail cars with unique
tilting suspensions, the basic system was a conventional steel-wheeled railroad on a steel track.

In 1981, the French opened their first TGV (Train a Grande Vitesse) service between Paris and Lyon. Like the Japanese system, the TGV system used a steelwheeled, steel track system. The system has been very popular, and the latest planned extensions will support speeds of $199 \mathrm{mi} / \mathrm{h}$.

In 2004, China opened the world's first high-speed rail system based upon "maglev" or magnetic levitation technology. There are no wheels, and the train is elevated by magnetic force and operates on a specially-designed track system.

With the elimination of the friction of steel wheels on steel rails, the system operates at a top speed of $268 \mathrm{mi} / \mathrm{h}$. China also has an extensive network of highspeed bullet-trains that operate at top speeds of $217 \mathrm{mi} / \mathrm{h}$. A recent accident causing 40 deaths on one of these has created some concern.

All of the Asian and European systems were developed and built with extensive direct support of the respective national governments in cooperative with private corporations.

The U.S. effort on high-speed ground transportation (HSGT) began with the High-Speed Ground Transportation Act of 1965. It provided an initial investment of $\$ 90$ million to "develop and demonstrate, where possible," HSGT systems. Its principal result was the development of the Metroliner service from New York to Washington D.C., which used conventional rail vehicles with a maximum speed of $160 \mathrm{mi} / \mathrm{h}$, and the Turbotrain service from New York to Boston. Both were developed as part of a public-private cooperative effort with the Pennsylvania Railroad. Both services became part of AMTRAK in 1970.

The Metroliner service experienced a number of operating difficulties. Using independently-powered cars, each with a catenary connection, it was found that at high speeds, oscillations in the catenary cables could lose contact with one or more cars, leading to motor burn-outs. Speeds had to be reduced. The trains were eventually replaced with the Acela Express, which uses an electric engine pulling un-motored passenger cars.

Federal involvement in promoting high-speed passenger rail service in the U.S. has been limited and inconsistent at best. The Passenger Railroad Rebuilding Act of 1980 provided funds for studies of potential high-speed corridors. Subsequent legislation endorsed five high-speed corridors in 1992, but provided virtually no funding to pursue implementation.

In February of 2009, as part of the stimulus effort (American Recovery and Reinvestment Act - ARRA), Congress allocated $\$ 8$ billion for states to pursue intercity rail projects, with an emphasis on those defined as "high speed." As required by the legislation, the Federal Railway Administration (FRA) identified ten potential high-speed corridors (in addition to the Northeast Corridor) for investment of these funds. As a result, 34 states submitted requests for funding that exceeded $\$ 57$ billion. In January 2010, FRA announced a list of 13 corridors, covering 31 states, which would receive funding.

The high-speed rail projects have remained politically volatile. Republicans have argued that such federal spending would be wasteful, and that projects should proceed only if private financing can be achieved. Three states, Wisconsin, Ohio,
and Florida have rejected their allocations, citing the burdens on their states in providing the required state and local monies for the high-speed projects. No funding for these or any other intercity passenger rail projects were included in the FY 2011 or 2012 budgets.

### 6.9 Federal Funding for Aviation

The federal government provides significant support for the nation's air networks. The Federal Aviation Administration provides direct oversight and support for the nation's air traffic control (ATC) system. It provides subsidies for new and upgraded ATC facilities and equipment, construction and upgrading of airports, and for the maintenance of critical air services to remote and/or low-population areas.

### 6.10 Federal Funding for Urban Mass Transportation

The first offering of federal-aid funds for urban mass transportation was defined in the Urban Mass Transportation Act of 1964 [17]. The act allocated $\$ 375$ million (over 6 years) to provide matching funds to state and local governments for rail transit projects. The federal share could be up to $2 / 3$ 'ds of the costs to construct, reconstruct, or acquire rail transit facilities or equipment. The federal share was restricted to $50 \%$ for projects that were not part of a comprehensive transportation plan. The act also created the Urban Mass Transportation Administration to oversee and administer these programs.

The legislation was renewed with the Urban Mass Transportation Act of 1970 [18]. While not changing the funding formulas of the 1964 act, the level of funding was greatly increased to $\$ 10$ billion over 12 years. Two percent of the capital funds and $1.5 \%$ of the research funds provided were to be applied to services for the elderly and handicapped.

The National Mass Transportation Assistance Act of 1974 [19] accomplished several critical changes:
(1) Federal funds could be used to cover operating costs as well as capital costs.
(2) $\$ 11.2$ billion was provided over 6 years, again greatly increasing the amount of funding available.
(3) $\$ 4$ billion of the monies were distributed on the basis of population and population density.
(4) $\$ 7.3$ billion of the monies were for capital projects disbursed at the discretion of the Secretary of Transportation.
(5) $\$ 0.5$ billion of the monies was reserved for rural public transportation demonstrations and projects.
(6) The funding formula for all capital projects was increased to $80 \%$ federal and $20 \%$ state/local; funds used for operations were limited to a $50 \%$ federal match.

The National Mass Transportation Assistance Act set the basic pattern for federal aid to urban public transportation through 1991. Subsequent acts and amendments increased funding levels, and minor changes to allocation models and the applicability of funds.

As noted previously, the 1983 Federal-Aid Highway Act introduced the "interstate trade-in" provision that allowed states and local governments to de-map unfinished portions of the interstate system and receive equal federal subsidies for public transportation projects. The utility of this was limited, however, as the funds for transit would be allocated from public transportation monies disbursed from the federal general fund, and while the interstate highway funds required a $10 \%$ state/local match, the public transit funds that replaced them required a $20 \%$ state/local match.

### 6.11 The Era of Multimodal Funding

Until 1991, Congress dealt with the various modes of transportation in a disjointed manner, authorizing separate legislation and programs for support of highways, railways (essentially ignored), aviation, and urban transit. From 1991 on, the vast majority of federal transportation assistance was bundled into single pieces of legislation that addressed the multimodal nature of the nation's transportation systems.

### 6.11.1 The Intermodal Surface Transportation Efficiency Act of 1991 (ISTEA) [20]

The 1991 act implemented a major change in the focus of federal-aid transportation funds in several ways:

- It treated the transportation system as a single multimodal system, rather than separate modal entities. In doing this, the act also focused increased attention on modal interfaces.
- It treated a major portion of federal transportation monies as block grants to the states, and gave state and local governments significant freedom to make modal allocation decisions.
- It tied federal monies to the development and maintenance of an ongoing State Transportation Improvement Plan (STIP).
- It provided new funding for major maintenance projects on the nation's highways, as well as for environmentally-oriented projects focusing on improvements for bicyclists and pedestrians.
- It created a funding formula of $80 \%$ federal/ $20 \%$ state/local for all noninterstate highway projects.

The act also created a $155,000-$ mile National Highway System (NHS), and reduced the previous four categories of federal aid highways (Interstate, Primary, Secondary, and Urban) to two (Interstate, National Highway System). The NHS was a critical component that answered the growing question of what to do now
that the Interstate System was virtually complete. The NHS will include the Interstate System as a component, and will comprise both existing and new highways as part of a broader, more comprehensive national roadway network. It will include all types of highways, including freeways and a variety of surface rural, suburban, and urban roadways. The network would consist of "primary arterials," and would have to be identified by the Secretary of Transportation by 2005. ISTEA funded the NHS with $\$ 21$ billion over 6 years. An additional $\$ 7.2$ billion was allocated to new and major reconstruction projects on the Interstate System. An additional $\$ 17$ billion was allocated to "rehabilitate, restore, and resurface" the Interstate System. Major reconstruction projects that did not increase capacity were also eligible for these funds.

All other funding for highways and transit capital projects was combined into the Surface Transportation Program (STP). Funding for STP was set at $\$ 23.6$ billion over 6 years. While there were some general guidelines on how these funds were to be used, states had considerable flexibility in assigning them to eligible highway and transit projects.

The Act incorporated ongoing spending for airports and air traffic control systems and special allocations for congestion/pollution migration projects, highspeed rail studies (minor), bridge repair, and other activities.

### 6.11.2 The Transportation Equity Act for the $21^{\text {st }}$ Century (1998, TEA-21) [21]

The Transportation Equity Act followed ISTEA, and, in general, maintained the same programs and approaches, but with higher funding levels. The Act also contained more "special allocations" than its predecessor. The Act allocated:

- $\$ 28.6$ billion for the National Highway System, which was expanded to 163,000 miles;
- $\$ 23.8$ billion for Interstate System maintenance and reconstruction projects;
- $\$ 33.3$ billion for the Surface Transportation Program;
- $\$ 41$ billion of funding targeting transit programs, with $\$ 11.7$ billion coming from the federal general fund.

All allocations were for a 6-year period. Minor funding for high-speed rail projects was included, with $\$ 60$ million allocated to MAGLEV development efforts for FYs 1999-2001.

### 6.11.3 The Safe, Accountable, Flexible and Efficient Transportation Equity Act - A Legacy for Users (SAFETEA-LU, 2005) [22]

This act was perhaps one of the most controversial pieces of legislation in the history of federal transportation assistance programs. When signed in August of 2005, the legislation was more than two years late. During the two-year legislation lapse, Congress authorized continuance of the TEA-21 provisions annually.

The legislation provided $\$ 248$ billion in new authorizations for 21 different programs over a 5 -year period. On the plus side, it continued all of the major programs of ISTEA and TEA-21 with increased funding. On the minus side, the legislation contained what many believed to huge amounts of political "pork" catering to the individual interests of Representatives and Senators. Table 6.1 lists the programs funded and the amounts of funding provided (to the nearest $\$ 0.1$ billion) in order of decreasing allocations.

Table 6.1 Programs and Allocations of SAFETEA-LU

| Program Title | Funding (\$ billion) |
| :--- | :---: |
| Surface Transportation Program (STP) | 32.5 |
| National Highway System (NHS) | 30.5 |
| Interstate Maintenance Program | 25.1 |
| High-Priority Routes Program | 14.8 |
| Congestion Mitigation and Air Quality Improvement Programs | 8.6 |
| Federal Lands Highway Program | 5.8 |
| Highway Safety Improvement Projects Program | 3.8 |
| Recreational Trails Program | 3.7 |
| Appalachian Development Highway System Program | 2.4 |
| National Corridor Infrastructure Improvement Program | 1.9 |
| Projects of National and Regional Significance Program | 1.8 |
| Highways for Life Program | 0.8 |
| Coordinated Border Infrastructure Program | 0.8 |
| Puerto Rico Highway Projects Program | 0.7 |
| Safe Routes to School Program | 0.6 |
| Deployment of Magnetic Levitation Transportation Projects | 0.6 |
| Construction of Ferry Boats and Ferry Terminals Program | 0.3 |
| National Scenic Byways Program | 0.2 |
| National Corridor Planning and Development \& Coordinated Border | 0.1 |
| Infrastructure Programs | 0.1 |
| Highway Use Tax Evasion Projects |  |

### 6.11.4 The Moving Ahead for Progress in the $21^{\text {st }}$ Century Act (MAP-21) [23]

SAFETEA-LU officially terminated in 2010. Once again, with Congress unable to produce successor legislation, annual re-authorizations of the SAFETEA-LU provisions took place.

In 2012, Congress finally passed the Moving Ahead for Progress in the $21^{\text {st }}$ Century Act (MAP-21). Unlike its immediate predecessors, MAP-21 was passed against the backdrop of the "great recession" of 2008, and the painfully slow recovery from its impacts. Funding was "frozen" at FY 2012 levels, with minor allowances for inflation, and the act only covered two fiscal years, 2013 and 2014. The Act included two significant revisions to SAFETEA-LU:

1. The 21 programs of SAFETEA-LU were combined into 5 , and gave states additional flexibility on the allocation of funds. The 5 new or redefined programs are:

- National Highway Performance Program (NHPP)
- Surface Transportation Program (STP)
- Highway Safety Improvement Program (HSIP)
- Congestion Management and Air Quality Improvement Projects (CMAQ)
- Metropolitan Planning Programs (MPP)

Funding for these core programs was set at $\$ 38$ billion/yr for FYs 2013 and 2014.
2. The Act includes several provisions intended to reduce the implementation time for projects funded under the legislation. This includes making it easier for states to acquire right-of-way in advance of project reviews, and providing greater flexibility for states to apply "categorical exclusions" to simplify and waive some environmental impact requirements.

In addition to the five core programs, the Act also authorizes funding for several minor programs. It sets disbursements from the Highway Trust Fund at $\$ 40.4$ billion in FY 2012 and $\$ 41$ billion in FY 2013.

### 6.12 Closing Comments

As we move forward into the $21^{\text {st }}$ century, the nation faces some new challenges. The nation's infrastructure has deteriorated, and there is no cohesive plan identifying how it can be restored and enhanced. Transportation infrastructure is front and foremost in many discussions, as the age of the Interstate System advances, the deteriorating status of the nation's bridges and tunnels accelerate, and the needs for upgrades in both intra-urban and intercity public transportation continue to grow. Paying for this will remain a daunting task as the nation considers its myriad needs and the limitations of public funding. The balance of public vs. private financing of infrastructure investments will continue to be discussed.

It is not very likely that the traditional balance of user taxes and general tax fund subsidies will continue to suffice. A new day is dawning that will require new and innovative approaches, while not stifling economic growth. Solutions will be found; the alternative of further ignoring the nation's infrastructure needs is no longer an option.

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## Chapter 7 Bond Financing

This chapter explores the options for financing transportation improvements through the issuance of bonds.

Due to the extremely high capital costs involved in the provision of transportation facilities, it is often necessary for government agencies to resort to public borrowing in the form of a bond issue in order to pay for these large transportation projects. Bonds are a type of debt financing, and therefore governments need to justify their reasons for borrowing money. There are several reasons why bond financing is used. The most obvious is that there are not enough current revenues to finance a given improvement. The borrowing agency, therefore, has to show that the benefits that are derived from moving ahead with the project sooner are greater than the increased cost incurred by borrowing money. The other option for financing a project, without debt financing, is to "pay-as-you-go" by doing a piece-by-piece implementation, that is, pay for the project with available cash flow, which may significantly delay the implementation of a needed facility.

However, even if there is revenue to "pay-as-you-go," there are other reasons to use debt financing for large transportation projects. With debt financing, the cost of the project is spread over the project's useful life, which puts less of a tax burden on current revenues. Additionally, the tax burden is spread to include future users who will also benefit from the project. Thus bonds allow states to optimize their cash flow and also to go ahead with needed projects sooner than would be possible using only current tax revenues.

### 7.1 What Is a Bond?

A bond is a legal document that states that the owner of the bond(s) has agreed to loan money to the bond issuer. So a bond is simply a loan, but in the form of a security. The issuer is equivalent to the borrower, the bond holder is equivalent to the lender, and the coupon rate is equivalent to the annual interest rate. The term coupon comes from the way some bonds were issued in the past, with physical coupons attached that were detached and turned in to get the interest payments. The term coupon is not used as frequently as in the past, but it is still a term that is occasionally used when discussing bonds.

### 7.2 Types of Bonds

Bonds may be classified in two ways:

1. by the security that underlies the debt, i.e., where is the money coming from that will be used to repay the debt and how secure (or how risky) is that source, or
2. by the manner in which the bonds are retired, i.e., by how the debt is repaid.

### 7.2.1 Classification by Type of Security

It is important to recognize the degree of risk to the investor involved in the bond investment. The risk to the investor involves the likelihood that the issuing agency will be able to pay both the interest and the principal on the bond. As the risk to the investor rises, the interest rate that must be paid to attract investors also rises. Types of bonds, classified by type of security are as follows:

1. General Obligation (GO) Bonds. With GO bonds, both the principal and the interest are backed by the "full faith and credit" of the issuer. This means that all of the issuer's revenues (whether private from earnings or public from taxes) are pledged to repay the debt. No revenues for a given year may be committed to other purposes until repayment of the debt for that year is satisfied. This is the most secure type bond, particularly when issued by a public entity such as a state, municipal, or federal government or agency. Because it is the most secure (the risk to the bond holder is low), the interest rate paid to the bond holders is low compared to other types of bonds.
2. Limited Obligation Bonds. These bonds are generally secured by a specific tax created in order to finance the repayment of the bond. For example, a gasoline tax (or an increase in the gasoline tax) may only be used for repayment of a highway bond. The benefit of this type of obligation is that the public can understand the connection between the tax and where the money is being spent. This type of security represents relatively low risk to the investor, but it is not as secure as a GO bond. Consequently, the interest paid is somewhat higher than a comparable GO bond.
3. Revenue Bonds. Repayment of a revenue bond is accomplished by dedicating expected future revenues from the facility being constructed. Revenue bonds are rarely issued directly by governmental units. They are most often issued by public corporations or authorities. For example, a toll authority will issue bonds for a new or improved facility and pledge the future revenues from tolls to pay back the bond holders. This is a less secure bond because of the unpredictability of future traffic volumes (and therefore revenue), and thus will pay a higher interest rate compared to similar GO or limited obligation bonds.
4. Limited Revenue Bonds. These are bonds that pledge future revenues and also pledge one other source, thereby reducing the overall risk. Another name for limited revenue bonds is "double-barreled bonds."

Obviously, the general obligation bond involves the smallest degree of risk for the investor, particularly where public governments or agencies are involved. The chances of an entire government unit going bankrupt or defaulting on repayment is slight, as all tax revenues are pledged versus repayment. At worst, other expenses may cause payment of the debt to be postponed, but this is usually accomplished through refinancing, and has no effect on investors. On the other hand, revenue bonds involve a relatively high degree of investor risk since revenues are difficult to predict and operating expenses must also be covered from the same source(s) of revenue. Consequently, interest rates on revenue bonds are highest, followed by limited-revenue bonds, limited obligation bonds, and then general obligation bonds with the lowest interest rates.

Credit ratings are a major factor in determining the interest that must be paid on a bond in order to attract investors. General obligation bonds issued by the various states, for instance, are not all equally secure or risky because states have different credit worthiness. There are a number of independent private agencies that determine the credit worthiness of the various bond issuers, including the federal government, state governments, and local governments. These agencies prepare annual reports that analyze the financial condition of these entities and then give them a credit rating. Rating scales are different (but similar) for the various agencies, and in general, the lower the issuing agency's credit rating, the higher the interest that must be paid on the bond in order for it to sell. Well known rating agencies include Moody's, Standard \& Poor's (S\&P) and Fitch, but there are others as well.

Table 4.1 [1] illustrates the ratings used by the three credit agencies noted above. Where letter scales are used, triple-A bonds represent the best bonds, shown as Aaa by Moody's and AAA by S\&P and Fitch. Bonds that are rated lower than BBB ( Baa ) are considered to be "junk bonds" (very high risk bonds).

The failure of any type of bond issue is a serious matter. No agency can ever again hope to raise significant amounts of capital at reasonable interest rates after faulting on a debt. For this reason, governmental units would rather "bail out" even limited obligation and revenue bonds that appear to be failing to avoid the deterioration of the ability to raise capital.

Table 7.1 Bond Rating Scales of Three Organizations [2]

| Moody's | S\&P/Fitch | Grade | Risk |
| :---: | :---: | :---: | :---: |
| Aaa | AAA | Investment | Highest Quality |
| Aa | AA | Investment | High Quality |
| A | A | Investment | Strong |
| Baa | BBB | Investment | Medium Grade |
| $\mathrm{Ba}, \mathrm{B}$ | $\mathrm{BB}, \mathrm{B}$ | Junk | Speculative |
| $\mathrm{Caa} / \mathrm{Ca} / \mathrm{C}$ | $\mathrm{CCC} / \mathrm{CC} / \mathrm{C}$ | Junk | Highly Speculative |
| C | D | Junk | In Default |

### 7.2.2 Classification by Method of Repayment

Another way in which bonds may differ is the manner in which they are retired, that is, the manner in which the bond holder is paid back for their loan to the
issuing agency. Some bonds pay interest and part of the principal each year, other bonds pay interest only each year and the total principal at maturity. The following are the most common bond types, classified by their method of repayment.

Serial Bonds In a serial bond issue, a constant amount is paid each year that is part principal and part interest. Bond holders are thus paid off in installments over the period of the debt. Bonds having low serial numbers are paid off first - higher numbered serial bonds are paid off later. In each year, interest on the outstanding debt is paid, plus a portion of the principal is retired. Since each year the total amount paid is the same, as the debt is retired, interest in successive years decreases, allowing a larger portion of the principal to be retired each year. At the end of the bonding period, all bonds have been retired and all interest has been paid. The constant total annual amount paid to repay both the interest and principal on the debt is found using the capital recovery factor, $\mathrm{A} / \mathrm{P}_{\mathrm{n}, \mathrm{i}}$.

## Example 1: Serial Bond Issue

A $\$ 10,000$ serial bond issue in the form of $\$ 100$ bonds, bears $4 \%$ interest payable annually. Construct a schedule for the amortization of this issue in ten equal (or nearly equal) payments. Note: an issue with serial bonds is that it is not possible to redeem a partial bond, so all principal reductions must be in units of $\$ 100$.

Solution. The annual payment at the end of each year for capital recovery is found using the capital recovery factor $(\mathrm{A} / \mathrm{P})$, found in Table 2.5 of Chapter 2:

$$
R=10,000 * A / P_{10,4 \%}=10,000 * 0.123291=\$ 1,232.91 / y r \Rightarrow \$ 1,233 / y r
$$

Since bonds retired in any one year must be an even multiple of $\$ 100$, it is not possible to construct a retirement plan spending exactly $\$ 1,233 / y r$. Table 7.2 illustrates the debt retirement plan. Note that where, in a given year, odd amounts of less than $\$ 100$ are not spent, the remainder carries over to the next year.

Table 7.2 Retirement Schedule for a Serial Bond Issue

|  | (1) | (2) | (3) | (4) | (5) | 6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Amount Owed at Beginning of Year $n$ (1)-(2) ${ }_{n-1}$ (\$) | Interest $=$ <br> (1) * 0.04 <br> (\$) | Available Funds for Principal (5)-(1) <br> (\$) | Bonds Redeemed (\$) | Actual Annual Payment $\mathbf{1 , 2 3 3 + ( 6 )} \mathbf{n - 1}$ | Cumulative Balance $(3)-(4)$ <br> (\$) |
| 1 | 10,000 | 400 | 833 | 800 | 1,233 | 33 |
| 2 | 9,200 | 368 | 898 | 900 | 1,266 | -2 |
| 3 | 8,300 | 332 | 899 | 900 | 1,231 | -1 |
| 4 | 7,400 | 296 | 936 | 900 | 1,232 | 36 |
| 5 | 6,500 | 260 | 1,009 | 1,000 | 1,269 | 9 |
| 6 | 5,500 | 220 | 1,022 | 1,000 | 1,242 | 22 |
| 7 | 4,500 | 180 | 1,075 | 1,000 | 1,255 | 75 |
| 8 | 3,500 | 140 | 1,168 | 1,100 | 1,308 | 68 |
| 9 | 2,400 | 96 | 1,205 | 1,200 | 1,301 | 5 |
| 10 | 1,200 | 48 | 1,190 | 1,200 | 1,238 | -10 |
|  |  | 2,340 |  | 10,000 | 12,575 | 235 |

Note that the actual annual payment includes the amounts carried over from the previous year (Col. 6). Because we cannot retire bonds except in $\$ 100$ units, the actual annual payments turn out to be not exactly uniform. They are, however, in practical terms, relatively uniform. According to the capital recovery factor estimation, over the 10 years, it was expected that $10^{*} 1,233=\$ 12,330$ would be spent. What was actually spent was $\$ 12,575$ minus the sum of the annual balances (which were not spent). This ( $12,575-235$ ) totals $\$ 12,340$, $\$ 10$ more than expected, which coincides with the final cumulative balance of minus $\$ 10$. Even this is somewhat offset by the interest earned on the annual cumulative balances, which is ignored in this analysis.

Term Bonds. In a term bond issue, all bonds are retired at the same time, at the end of the last year of the issue. Thus, the issuer pays interest each year on the entire face value of the bonds, and only pays back the principal at the end of the bond life on maturity date. At the same time, the bond issuer is required to make uniform annual payments into a "sinking fund," in order to guarantee that the money is available when due at the bond's maturity. Consider the bond issue of Example 1, this time issued as a term bond, with the entire $\$ 10,000$ principal paid at the end of 10 years.

## Example 2: Example 1 as a Term Bond Issue

The issuing agency must make two payments each year. It must pay the $4 \%$ annual interest on the entire $\$ 10,000$ principal directly to bond-holders each year. This amounts to:

$$
I=10,000 * 0.04=\$ 400.00 / y r
$$

The second annual payment is into a sinking fund that will accumulate (at 4\%) to $\$ 10,000$ at the end of 10 years. Using the sinking fund factor:

$$
R=10,000 * A / F_{10,4 \%}=10,000 * 0.083291=\$ 832.91 / y r
$$

where $\mathrm{A} / \mathrm{F}$ is found in Table 2.4 of Chapter 2. The total annual cost of the bond issue is $\$ 400.00+\$ 832.91=\$ 1,232.91$. This is essentially the same annual cost as the serial issue.

The cost of the bond issue will be the same for the term and serial bond when the interest paid on the bond is equal to the interest earned in the sinking fund. The term bond is preferred when the interest earned in the sinking fund is higher than that paid on the bond, and the serial bond is preferred when the reverse condition exists. In the previous problem, if the sinking fund investment paid $10 \%$, then the annual interest would remain $\$ 400.00 / \mathrm{yr}$, but the payment to the sinking fund would be:

$$
R=10,000 * A / F_{10,10 \%}=10,000 * 0.062745=\$ 627.45 / y r
$$

and the total annual cost of the bond issue would have been $\$ 400.00+\$ 627.45=$ $\$ 1,027.45 / \mathrm{yr}$ - less than the cost of the serial bond. In simple terms, if borrowed
money can be invested at a rate higher than you are paying on the loan, then it pays to keep the loaned money as long as you can.

## Example 3: Another Comparison of Serial and Term Bonds

Consider a 20 -year $\$ 100,000$ bond issue bearing $5 \%$ interest. Compare the annual cost of a serial and term issue if interest is $3 \%, 5 \%$, or $7 \%$ on sinking fund investments.

In all cases, the cost of the serial issue is:

$$
R_{\text {serial }}=100,000 * A / P_{20,5 \%}=100,000 * 0.080243=\$ 8,024.30 / \mathrm{yr}
$$

The cost of the term issue includes payment of 5\% interest yearly on the full principal and a yearly amount set aside into a sinking fund that will be used to pay the full principal, or $\$ 100,000$, at the bonds maturity in 20 years.

Annual interest will cost:

$$
I=100,000 * 0.05=\$ 5,000.00 / y r
$$

The amount that would have to be invested in a sinking fund to accumulate $\$ 100,000$ in 20 years depends upon the interest rate and the sinking fund factor:

$$
R_{S F}=100,000 * A / F_{20, i \%}
$$

The sinking fund factor, $\mathrm{A} / \mathrm{F}$, can be obtained from Table 2.4 of Chapter 2 for $3 \%$ and $5 \%: \quad \mathrm{A} / \mathrm{F}_{20,3 \%}=0.037216 ; \mathrm{A} / \mathrm{F}_{20,5 \%}=0.030243$. Sinking fund factors for 7\%, however, are not included in the table, and must be computed using the equation for the factor (or the spreadsheet created in Problem 2-1):

$$
A / F_{20,7 \%_{o}}=\frac{i}{(1+i)^{n}-1}=\frac{0.07}{1.07^{20}-1}=\frac{0.07}{2.86968}=0.024393
$$

Then:

$$
\begin{aligned}
& R_{20, i \%}=5,000+\left(100,000 * A / F_{20, i \%}\right) \\
& R_{20,3 \%}=5,000+(100,000 * 0.037216)=\$ 8,721.60 / y r \\
& R_{20,5 \%}=5,000+(100,000 * 0.030243)=\$ 8,024.30 / y r \\
& R_{20,7 \%}=5,000+(100,000 * 0.024393)=\$ 7,439.30 / y r
\end{aligned}
$$

When the sinking fund earns less interest than is paid on the bonds ( $3 \%<5 \%$ ), the total annual cost of the term bond issue is higher than that of the serial issue. When the sinking fund pays the same interest rate as the bonds $(5 \%=5 \%)$, than the total annual costs of the serial and term bond issues are the same. When the sinking fund earns a higher rate of interest than is paid on the bonds ( $7 \%>5 \%$ ), the total annual cost of the term bond is less than that of the serial bond. Table 7.3 summarizes these results, rounded to the nearest dollar.

Table 7.3 Results for Example Problem 3 Summarized

| Interest Rate on: |  | Total Annual Cost of: |  |
| :---: | :---: | :---: | :---: |
| Bond | Sinking Fund | Serial Bond | Term Bond |
| $5 \%$ | $3 \%$ | $\$ 8,024$ | $\$ 8,722$ |
| $5 \%$ | $5 \%$ | $\$ 8,024$ | $\$ 8,024$ |
| $5 \%$ | $7 \%$ | $\$ 8,024$ | $\$ 7,439$ |

Zero-Coupon Bonds: A zero-coupon bond is one in which the principal and interest are paid when the bond is retired (at maturity). To accomplish this, the bond is sold at less than its face value. For example, a 20 -year zero-coupon bond with a face value of $\$ 100,000$ with $5 \%$ interest, would sell for a price of:

$$
P=100,000 * P / F_{20,5 \%}=100,000 * 0.376889=\$ 37,688.90
$$

The investor buys the bond for $\$ 37,688.90$. When the bond matures in 20 years, the investor receives $\$ 100,000$, which includes his principal, plus $5 \%$ annual interest on the original investment of $\$ 37,688.90$.

### 7.3 Establishing a Selling Price for Bonds

Every bond bears an amount (often referred to as the "face value"), an interest rate, and a maturity date on the front of the bond. A term bond showing a face value of $\$ 1,000$ and an interest rate of $7 \%$ means that the holder of the bond will receive $\$ 1,000$ when the bond matures, and will receive a check for $\$ 70$ each year that the bond is held. The bond may or may not, however, be initially sold for $\$ 1,000$.

When bonds are sold initially, or when a bond is sold by one investor to another before it reaches maturity, a selling price must be established. In general, a bond sells:

- At a discount, if the rate of return to be earned by the purchaser is higher than rate shown on the bond.
- At par, if the rate of return to be earned by the purchaser is equal to the interest rate shown on the bond.
- At a premium, if the rate of return to be earned by the purchaser is lower than the rate shown on the bond.

The interest rate shown on the bond is an estimate of what the market value of the bond will be, made by the issuing agency at the time the bond issue is planned. By the time bonds are printed and ready for sale, the market rate of interest may well have changed from the original estimate.

The selling price of a bond depends on:
i $=$ the interest or rate of return that is expected by the purchaser,
$\mathrm{r}=$ the interest rate named on the bond (coupon rate),
c $=$ the face value of bond,

$$
\begin{aligned}
\mathrm{R}= & \text { the redemption value of bond, which is generally the same as the } \\
& \text { face value, unless the issuing agency defaults on the debt, and } \\
\mathrm{n} & =\text { the number of interest periods to maturity. }
\end{aligned}
$$

The selling price of the bond is the present worth of its redemption value plus the present worth of all future interest payments, based upon the desired rate of return of the purchaser. Then:

$$
\begin{equation*}
P=\left(R * P / F_{n, i}\right)+\left(c * r * P / A_{n, i}\right) \tag{7.1}
\end{equation*}
$$

## Example 4: Establishing the Selling Price of a Bond

A 20-year term bond with a face value of $\$ 5,000$ pays interest of $8 \%$ per year. How much should be paid on initial purchase (at time $=0$ ) for this bond in order to receive a rate of return of $10 \%$ on the investment?

Solution: The solution uses equation 7.1 directly, where $\mathrm{i}=10 \%$ and $\mathrm{r}=8 \%$. Then:

$$
\begin{aligned}
& P=\left(5,000 * P / F_{20,10 \%}\right)+\left(0.08 * 5,000 * P / A_{20,10 \%}\right) \\
& P=(5,000 * 0.148644)+(400 * 8.513564)=743.22+3,405.43=\$ 4,148.65
\end{aligned}
$$

Because the investor needed to earn a higher rate of return on his investment than the coupon rate of the bond, the bond sold at a discount.

Often, it is necessary to use this procedure in reverse, that is, to find the actual rate of return knowing the selling price.

## Example 5: Finding the Rate of Return

If the bond of Example 4 is purchased at time " 0 " for $\$ 4,600$, what would be the buyer's rate of return on this investment, assuming no default in the payment of interest or principal?
Solution: The solution involves trials using varying interest rates, and interpolation once trials find interest rates that result in prices both above and below the $\$ 4,600$ target.
Example 4 provides one trial at $10 \%$ which results in a price of $\$ 4,148.65$, which is less than the $\$ 4,600$ price under consideration. This means that the actual rate of return is less than the $10 \%$ of Example 4. A second trial will be made at $8 \%$. No computation is necessary, as $8 \%$ is equal to the coupon rate of the bond. When the expected rate of return is the same as the coupon rate, the bond will sell at par or at its face value of $\$ 5,000$, which is more than the $\$ 4,600$ price under consideration. By interpolation:

$$
\begin{aligned}
& i=0.08+0.02\left(\frac{5,000-4,600}{5,000-4,148.65}\right) \\
& i=0.08+0.02\left(\frac{400}{851.35}\right)=0.08+(0.02 * 0.4698)=0.894 \Rightarrow 8.94 \%
\end{aligned}
$$

## Example 6: Selling a Bond Before Maturity

A US treasury bond that matures in 8 years, has a face value of $\$ 10,000$. The bond stipulates a fixed interest rate of $8 \%$ per year, but interest payments are made to the bondholder every three months.

A prospective buyer would like to earn $10 \%$ on the investment, since interest rates have risen since the bond was issued. How much would this buyer be willing to pay for the bond?

Solution: The price determination remains the present worth of the redemption value at maturity plus the present worth of all interest payments until maturity. In this case, interest is paid four times per year. Thus, $r=8 / 4=2 \%, i=10 \% / 4$ $=2.5 \%$, and $\mathrm{n}=8 * 4=32$ periods until maturity.

$$
P=\left(10,000 * P / F_{32,2.5 \%}\right)+\left(10,000 * 0.02 * P / A_{32,2.5 \%}\right)
$$

As neither the present worth factor nor the series present worth factor needed are included in the tables, they are computed from their equations:

$$
\begin{aligned}
& P / F_{32,2.5 \%}=\frac{1}{(1+i)^{n}}=\frac{1}{(1+0.025)^{32}}=0.453770 \\
& P / A_{32,2.5}=\frac{(1+i)^{n}-1}{i(1+i)^{n}}=\frac{1.025^{32}-1}{0.025 * 1.025^{32}}=21.849203
\end{aligned}
$$

Then:

$$
\begin{aligned}
& P=(10,000 * 0.453770)+(10,000 * 0.02 * 21.849103) \\
& P=4,537.70+4,369.82=\$ 8,907.52
\end{aligned}
$$

Once again, because the investor wishes to make a higher rate of return on his investment than the coupon rate of the bond, the purchase price will be lower than the face value of the bond, i.e., the bond will be purchased at a discount.

If the bond is purchased between interest periods, the problem is somewhat more complicated. The holder of the bond will receive the interest check at the end of the interest period. However, the seller of the bond has equity in that interest, as the seller held the bond for part of the interest period. The fraction of the interest payment must be added to the price of the bond to account for the seller's share. In general, the price of the bond on the interest date prior to the sale is computed as above. The fraction of the interest payment belonging to the seller is then added to the price, in terms of the buyer's expected rate of return:

$$
\begin{equation*}
P=P^{\prime}+\left(f * i * P^{\prime}\right) \tag{7.2}
\end{equation*}
$$

where: $\mathrm{P}=$ price of bond,
$\mathrm{P}^{\prime}=$ price of bond on interest date prior to sale of bond,
$\mathrm{f}=$ fraction of interest payment belonging to seller,
$\mathrm{i}=$ rate of return expected by buyer.

Using the bond in Example 6 above, where the buyer wanted to make a $10 \%$ rate of return and thus purchased the bond for $\$ 8,907.52$, at what price would the bond have sold if it were sold 1.5 months later (halfway into the quarterly compounding period)?

$$
\begin{aligned}
& P=8,907.52+(1.5 / 3.0) * 0.10 * 8,907.52 \\
& P=8,907.52+44.53=\$ 8,952.05
\end{aligned}
$$

### 7.4 Call Privileges

A bond that is callable means that the issuer may retire the bond before the maturity date. This feature is usually added when interest rates are high. This gives the issuing agency a way to pay off the bond early and possibly borrow at a lower interest rate. For the issuing agency, it reduces debt levels if revenues are higher than predicted when the bonds are sold [1]. There are specific dates in the terms of the bond for when it can be called. It is important to consider call dates when buying bonds because it can change the rate of return that you earn.

## Example 7: Calling a Bond

A 15 -year $\$ 10,000$ term bond bearing $6 \%$ interest is purchased at a price that would earn the buyer $4 \%$. What price was paid for the bond?

Solution: Using Equation 7.1:

$$
\begin{aligned}
& P=\left(10,000 * P / F_{15,4 \%}\right)+\left(10,000 * 0.06 * P / A_{15,4 \%}\right) \\
& P=(10,000 * 0.555265)+(600 * 11.118378)=\$ 12,223.68
\end{aligned}
$$

Because the investor only needed to make $4 \%$ buying bonds bearing $6 \%$ interest, the purchase price was above the face value of the bond, i.e., the bond was purchased at a premium.

If after purchasing the bond for this amount, it is then called after five years, what was the actual rate of return earned? Then, using Equation 7.1 again:

$$
12,223.68=\left(10,000 * P / F_{5, i}\right)+\left(600 * P / A_{5, i}\right)
$$

The result is found by using different interest rates, and interpolating when two are found that indicate the range within which the solution is found. The price of the bond will be computed for interest rates of $1 \%, 2 \%, 3 \% \ldots$. , until we find values that surround $\$ 12,223.68$. Factors $\mathrm{P} / \mathrm{F}$ and $\mathrm{P} / \mathrm{A}$ are found from the interest tables of Chapter 2. Then:

$$
\begin{aligned}
& P_{1 \%}=(10,000 * 0.951466)+(600 * 4.853431)=\$ 12,426.72>\$ 12,223.68 \\
& P_{2 \%}=(10,000 * 0.905731)+(600 * 4.713460)=\$ 11,885.39<\$ 12,233.68
\end{aligned}
$$

Then, by interpolation:

$$
i=1.0+1.0\left(\frac{12,426.72-12,233.68}{12,426.72-11,885.39}\right)=1.36 \%
$$

Because the bond was redeemed (or "called") before its expected maturity, the investor earned a far smaller rate of return than originally anticipated.

### 7.5 The Process of Bond Issuance

The issuing of a bond is also called floating a bond. There are generally three parties to every bond issue: the issuing agency, acting in role of management; the public, who, through taxes, must pay for the issuance and will hopefully benefit from the improvement being financed; and the investor, who is looking for a sound investment.

In order to "float" a bond issue:

1. The issuing agency must estimate its capital need and determine the total amount of the issue. It must also estimate the interest rate necessary to attract investors and sell the bond.
2. Approval for issuance must be obtained by the appropriate procedurereferendum, legislative approval, etc. In the US, most states (their political subdivisions or public authorities) use debt financing [3].
3. Bonds are printed and sold, frequently by competitive bid. Bonds may sell at more or less than face value depending upon whether or not the interest rate on the bond is adequate to attract investors. No matter at what price the bond sells, the face value must be repaid, and the stated interest rate on the face value must be paid annually. Bonds are generally sold to banks, investment companies, and similar agencies - rarely are bonds made available to the general public.
4. The issue, principal plus interest, must be repaid according to the predetermined schedule, either as a serial or term issue.

There are some legal limitations on incurring public debt that vary by state. Prohibitions on public borrowing, however, generally refer only to general obligation bonds, and these may be circumvented by legislative action creating a special tax to finance a limited obligation bond, or a public authority empowered to issue revenue bonds. Public authorities have the advantage of not needing legislative action for each of its decisions. They do require periodic action on enabling legislation for the authority itself, as such bills rarely create an agency for periods longer than 5 to 10 years.

Regulations regarding issuance of bonds by local governments vary widely. In many cases, executive action is all that is needed. Most often, approval of a local legislative group or governing panel is required. Local governments seldom require public approval of bond issues, but often work under strict debt ceilings set by state legislatures.

### 7.6 Comparing Bond Financing to Pay-As-You-Go Financing

The transportation planner must be prepared to make alternative analyses on various financing schemes as well as for various physical alternative designs. In comparing credit financing to pay-as-you-go financing, two main items of consideration come into play. The first of these is interest rate - the rate paid on borrowed money may be higher than the minimum desirable rate generally used in economic analysis, particularly if revenue bonds are being used. Secondly, the time frame of achieving benefits will be substantially altered depending upon the effect of financial alternatives on scheduling.

In general, a present worth comparison for an analysis period covering all implementation schedules will work well here. Annual costs for capital items may be computed using actual bonded interest rates, however, all present worth values would be computed based upon the minimum desirable rate of return normally used. In this way, the higher cost of borrowing may be reflected.

## Example 8: Comparing Financial Alternatives

A 10-mile stretch of rural highway is to be reconstructed at a cost of $\$ 30$ million. The road user costs on the existing facility are $\$ 2.8$ million per year and maintenance is $\$ 90,000$ per year. The new facility will reduce these to $\$ 900,000 /$ year and $\$ 10,000 /$ year, respectively.

The highway may be built now if a bond issue bearing 5\% interest is issued for 25 years.

If the project is financed out of current taxes, construction will proceed at 2 miles/year taking 5 years to complete. In each year, $1 / 5$ of the construction cost will be paid, and $1 / 5$ the maintenance and RUC reduction is experienced.

In either instance, service life of the facility is considered to be 30 years. Minimum attractive rate of return is $4 \%$

Solution: Consider a 30-year analysis period. Then:
Bond Financing: The total present worth of this financing alternative will include (1) the present worth of capital recovery costs to retire the bonded debt, (2) the present worth of 30 years of road user costs and maintenance costs. Note that a serial bond is used because the amount that could be earned on a sinking fund is $4 \%$, less than what is being paid to bondholders ( $5 \%$ ).

The annual cost to retire the bond issue (for 25 years) is:

$$
R_{\text {bond }}=30,000,000 * A / P_{25,5 \%}=30,000,000 * 0.070952=\$ 2,128,560 / \mathrm{yr}
$$

The present worth of these costs must be evaluated, but with an interest rate of $4 \%$, the minimum acceptable. If the minimum acceptable rate and the coupon rate of the bonds were equal, the present worth would have been $\$ 30,000,000$, the face value, and no computation would have been necessary. However, the interest rates are not the same, so:

$$
P_{\text {bond }}=2,128,560 * P / A_{25,4 \sigma_{e}}=2,128,560 * 15.622080=\$ 33,252,346
$$

For each year of operation, annual road user costs (RUC) and maintenance costs will be $\$ 900,000+10,000=\$ 910,000 /$ year. The present worth of this is:

$$
P_{\text {RUC } / M \& O}=910,000 * P / A_{30,4 \%}=910,000 * 17.292033=\$ 15,735,750
$$

The total present worth of the bond-financing alternative is, therefore:

$$
P=33,252,346+15,735,750=\$ 48,988,096
$$

Pay-As-You-Go Financing: This option is a bit more challenging. (1) The capital costs of the roadway will be paid in equal installments over the first five years, i.e., at a rate of $\$ 30,000,000 / 5=\$ 6,000,000 / \mathrm{yr}$ for five years. The present worth of these expenditures will have to be evaluated at $4 \%$ annual interest. (2) For each of the first five years, M \& O and RUC costs will decline, with one fifth of the reduction experienced each year. Thereafter, the full reduction in $\mathrm{M} \& \mathrm{O}$ and RUC will take effect, applying for years 6 through 30 of the analysis period. The present worth of these costs will have to be estimated.

The present worth of the capital investments is:

$$
P_{\text {road }}=6,000,000 * P / A_{5,4 \%}=6,000,000 * 4.451822=\$ 26,710,932
$$

Road user costs for the existing roadway are $\$ 2,800,000$ per year, which will be reduced to $\$ 900,000$ per year when the road is fully built. The total reduction in RUC is $\$ 1,900,000 / \mathrm{yr}$. Maintenance and operations cost for the existing roadway are $\$ 90,000$ per year, which will be reduced to $\$ 10,000$ per year when the road is fully built. The total reduction in $\mathrm{M} \& \mathrm{O}$ is $\$ 80,000 / \mathrm{yr}$.

Because we are only building one fifth of the roadway per year, however, these reductions will only be achieved after five years. The total reduction in RUC and $\mathrm{M} \& \mathrm{O}$ of $\$ 1,980,000 / \mathrm{yr}$ will be achieved in five increments of $\$ 1,980,000 / 5=\$ 396,000 / \mathrm{yr}$. We will assume that the first reduction occurs in Year 2, as Year 1 will be consumed with the initial construction. Total RUC and $\mathrm{M} \& \mathrm{O}$ costs by year are:

| End of Year |
| :---: |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| $6-30$ |

$$
\begin{array}{r}
\text { RUC and M \& O Cost for Year } \\
=\$ 2,890,000 \\
2,890,000-396,000=\$ 2,494,000 \\
2,494,000-396,000=\$ 2,098,000 \\
2,098,000-396,000=\$ 1,702,000 \\
1,702,000-396,000=\$ 1,306,000 \\
1,306,000-396,000=\$ 910,000
\end{array}
$$

The present worth of these costs over 30 years may now be computed as:

$$
\begin{aligned}
P_{\text {RUCIM\&O }} & =\left(2,890,000 * P / F_{1,4 \%}\right)+\left(2,494,000 * P / F_{2,4 \%}\right)+\left(2,098,000 * P / F_{3,4 \%}\right) \\
& +\left(1,702,000 * P / F_{4,4 \% \%}\right)+\left(1,306,000 * P / F_{5,4 \% /}\right)+\left(910,000 * P / A_{25,4 \%} * P / F_{5,4 \% \%}\right) \\
P_{\text {RUCIM\&O }} & =(2,890,000 * 0.961538)+(2,494,000 * 0.924556)+(2,098,000 * 0.888996) \\
& +(1,702,000 * 0.854804)+(1,306,000 * 0.821927)+(910,000 * 15.622080 * 0.790315) \\
P_{\text {RUCIM\&O}} & =2,796,184.82+2,305,841.67+1,865,113.08+1,454876.08+1,073,436.66 \\
& +11,235,191.38=\$ 20,730,643.69
\end{aligned}
$$

The total present worth of the pay-as-you-go option is:

$$
P=26,710,932.00+20,730,643.69=\$ 47,441,575.69
$$

Compare this course to the $\$ 48,988,096$ for bond financing, and in this case the increased benefits gained by sooner completion of the project DOES NOT justify the additional expenditure of bond financing, although the comparison is very close.

If the current tax base could only cover the cost of 1 mile per year, or $\$ 3,000,000$ per year, then the total present worth of pay-as-you-go financing would be computed as $\$ 49,419,504$ (computations not shown) and the increased cost of the borrowing money to complete construction sooner would be justified by the sooner completion of the project.

### 7.7 Closing Comments

In this chapter, basic banking formulae and principles are applied to the analysis and description of bond issues, and to the comparison of financial alternatives for funding a given project. Once again, the application of these basic principles covers a very broad range of options, and allows for an orderly review of the details of financing a project, and the alternatives for doing so.

## References

1. Financing Urban Transportation Improvements, National Highway Institute (NHI) Course 15247, Federal Highway Administration, US Department of Transportation, Washington DC
2. Bond Basics: Characteristics,
http://www.investopedia.com/university/bonds/ bonds2.asp\#axzz21TfKFGK6
3. AASHTO Center for excellence in Project Finance, http://www.transportation-finance.org

## Problems

## Problem 7.1

A term bond is constructed with a $\$ 100,000$ face value and an annual yield of $8 \%$ of the face, with a 20 -year life. The bond is not callable. If the opportunity cost for money is $8 \%$, how much is it logical for you to pay for the bond? If the opportunity cost of money is $12 \%$, what would you logically pay for the bond?

## Problem 7.2

A term bond has a face value of \$20,000 and an annual yield of $8 \%$ of the face. The bond is issued with a maturity date in 20 years. Investor A buys the bond at a price that yields a 5\% return. (a) What did Investor A pay for the bond?

Investor B buys the bond from Investor A after ten years for a price that yields a $10 \%$ return on the investment. (b) What did Investor B pay? (c) What rate of return was actually earned by Investor A?

## Problem 7.3

A serial bond with a face value of $\$ 100,000$ is scheduled to make annual payments over 20 years and be self-liquidating. If yield is $8 \%$, what is the annual payment expected?

## Problem 7.4

What are some advantages and disadvantages of a typical AAA-rated municipal bond yielding $6 \%$ compared to a FIDC-insured savings account yielding $6 \%$ ?

## Problem 7.5

A bond will yield the following payments: (1) $\$ 5000$ at the end of each year, years $1-20$ and (2) $\$ 50,000$ at the end of year 25.

1. If you believe the opportunity cost of money to be $10 \%$, what are you willing to pay for this bond?
2. If others believe the opportunity cost of money to be $5 \%$, is the bond worth more or less to them and why? How much is the bond worth to them?
3. If interest climbs, does the value of the bond go up or down? why?

## Problem 7.6

Consider buying a $\$ 1000$ corporate bond at the market price of $\$ 996.25$. Interest will be paid semiannually at the rate of $4.8125 \%$ per payment period over 10 years. Find the rate of return on your investment, assuming that no default occurs?

## Problem 7.7

If a zero-coupon bond will yield $\$ 100,000$ in a single payment 10 years from now, and is rated C by Standard and Poors, what is the most likely present worth of the bond: (a) $\$ 2000$, (b) $\$ 15,000$, (c) $\$ 40,000$ (d) $\$ 100,000$ Justify your answer.

## Chapter 8 <br> Financing Transportation Projects


#### Abstract

The Executive Committee of the Transportation Research Board of the National Academies published a circular on "Critical Issues in Transportation" in 1976. It has been updated every two to four years, with the last update in 2009. Every issue has included some treatment of transportation finance as a critical issue [1, 2]. The 2009 issue addressed the critical issue of inadequate revenues, but also included equity, which considers the impacts on lower income users of newer financing methods. This chapter covers some of the most pressing problems and potential solutions for the financing of future transportation projects.


Funding for transportation projects has historically come from governments, including federal, state, and local levels. This remains the current approach as well. More than $\$ 200$ billion per year is invested in transportation projects [3] across the U.S. The largest source of revenue is the tax on gasoline, both at the federal and state levels.

### 8.1 The Gas Tax and the Highway Trust Fund

More than half of the revenue in the highway trust fund (HTF) comes from the gas tax. After the gas tax, the two next largest sources of revenue in the HTF come from the tax on diesel fuel and the tax on trucks and trailers [4]. Currently the federal gasoline tax is 18.4 cents per gallon; the tax on gasoline began in 1932 at 1 cent per gallon. It has increased very slowly, and was only 4 cents per gallon from 1960 to 1983, when it was increased to 9 cents per gallon, with 1 cent of that earmarked for mass transit financing. This was the first use of HTF monies for public transportation purposes. In 1994 the gas tax was raised to 18.4 cents, where it remains to date [5]. 15.44 cents goes into the highway account and 2.86 cents goes to the mass transit account [4]. The federal gas tax pays for $45-50 \%$ of transportation's capital spending [6]. The gas tax, however, has not risen in almost twenty years at the time of this writing, and it is not indexed to inflation. To put this in perspective: "the gasoline tax has lost one-third of its buying power since it was last raised in 1993" [6]. If a gas tax increase would be considered, it is estimated that every one-cent increase would raise approximately $\$ 1.8$ billion dollars per year [6]. However, with the current political climate and resistance to any tax increases, this seems very unlikely.

Figure 8.1 [7] shows the revenues placed in the HTF and the amounts disbursed for transportation projects. The HTF is so underfunded that in 2008, 2009, and 2010, money had to be transferred from the general fund to the HTF (not included in graph receipts), which at least guaranteed that the HTF would maintain a positive balance.


Fig. 8.1 Annual Receipts and Outlays in the Highway Trust Fund, 2004-2021
(Source: Lee, Joung "Presentation on Potential Program and Budgetary Implications of the Ongoing Use of General Fund Allocations," http://sotfp.transportation.org/Documents/2012-02-29\ Joung\ Lee\ -\ SOTFP\ General\ Fund.pdf, February 2012.)

The Congressional Budget Office (CBO) projects that both the highway account and the transit account of the HTF will be exhausted in Fiscal Year 2015 [8].

State and local governments have their own gas taxes, which vary considerably by state. The average state and local gas tax is 31.1 cents per gallon of gasoline (in 2012) [5]. States put an excise tax on gasoline, which are taxes on specific items such as gasoline, alcohol, tobacco, etc. Some states, however, also apply a general sales tax and sometimes other fees as well, such as environmental fees. When all these taxes and fees are included, New York has the highest tax rate of 49 cents per gallon and Alaska has the lowest at 8 cents per gallon [9]. Often reports only compare the state excise tax on gasoline. For example, it is claimed that Georgia has the lowest gas tax at 7.5 cents per gallon [10]. However, Georgia also adds a $4 \%$ sales tax [11], which makes the overall tax rate higher than Alaska.

### 8.2 The Problem of Revenue Shortfalls

Due to the large costs of transportation projects, using the taxes on gasoline alone is not nearly enough to fund the work that is needed. Former Secretary of Transportation James Burnley has said, "The highway user fee trust fund idea has broken down completely. In the last few years $\$ 34.5$ billion have been transferred from general revenues because we can't pay for the existing highway programs and the transit programs that are funded, in part, from the highway trust fund." [12]

Table 8.1 shows expenditures on highways, by federal, state, and local governments and the percentage of each category that is covered. It can be seen that state governments contribute the largest amounts.

Table 8.1 Direct Expenditures for Highways by Expending Agencies and by Type For 2006

| Type of Expenditure | Billions of Dollars |  |  |  | Percent of Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Federal | State | Local | Total |  |
| Capital Outlay | \$0.50 | \$59.00 | \$19.20 | \$78.70 | 48.9\% |
| Funded by Fed'l Gov't | \$0.50 | \$32.00 | \$1.40 | \$34.60 | 21.5\% |
| Funded by State \& Lcl Gov't | \$0.00 | \$26.2 | \$17.90 | \$44.1 | 27.4\% |
| Non-Capital Exp. | \$1.70 | \$36.50 | \$36.6 | \$74.7 | 46.4\% |
| Maintenance | \$0.20 | \$12.60 | \$18.60 | \$31.30 | 19.4\% |
| Highway \& Traffic Services | \$0.00 | \$4.70 | \$4.40 | \$9.10 | 5.7\% |
| Administration | \$1.50 | \$7.10 | \$4.60 | \$13.20 | 8.2\% |
| Highway Patrol \& Safety | \$0.00 | \$7.70 | \$6.80 | \$14.50 | 9.0\% |
| Interest on Debt | \$0.00 | \$4.40 | \$2.20 | \$6.60 | 4.1\% |
| Bond Retirement | \$0.00 | \$4.60 | \$3.00 | \$7.60 | 4.7\% |
| Total Exp. | \$2.20 | \$100.10 | \$58.80 | \$161.10 | 100.0\% |
| Funded by Fed'l Gov't | \$2.20 | \$32.80 | \$1.40 | \$36.30 | 22.6\% |
| Funded by State Gov'ts | \$0.00 | \$65.10 | \$15.80 | \$80.90 | 50.2\% |
| Funded by Local Gov'ts | \$0.00 | \$2.20 | \$41.60 | \$43.80 | 27.2\% |
| Total Exp. | \$2.20 | \$100.10 | \$58.80 | \$161.10 | 100.0\% |

(Source: Reformatted from http://www.fhwa.dot.gov/policy/2008cpr/chap6.htm\#1).

### 8.3 Additional Funding Sources for Transportation Projects

There are a number of emerging mechanisms that have come into use to help finance transportation projects. Innovative thinking will be needed to help meet the growing investment needs in the renewal and improvement of the nation's transportation infrastructure.

### 8.3.1 Joint Development Funds

Joint Development Charges are charges to developers and/or business owners that try to capture the value added to their land and/or businesses by a transportation improvement. Such approaches are generally called value capture strategies, and
include special assessment districts and tax increment financing districts. In either case, special taxes, or assessments, can be levied with the revenues applied to transportation improvement.

Special Benefit Assessment Districts. A well-defined area that is shown to directly benefit from a public improvement is defined as a special benefit assessment district. The assessment is a charge on the properties within this area. The charge can be a one-time lump sum or an annual charge. Often it is a fee on a per unit basis, such as square footage. Using this tool puts the infrastructure cost on the specific group that benefits from the project instead of spreading the charge over the entire locale. Because of this, however, special benefit assessment districts cannot be used on larger scale projects, such as new roads that cover large areas of the city [13]. States must have legislation that allows the creation of these districts. In order for this type of financing to work, often the assessed properties must agree to participate, otherwise the implementation can be held up in court as developers fight the imposition of this fee. Example special benefit assessment districts include:

- The Tampa Historic Streetcar that covers 2.3 miles and has an average of over one thousand riders per day. In 2006, the special assessment raised $\$ 360,000$, which covered $14 \%$ of the costs of the system. [14].
- Miami used a special benefit assessment district to help finance its Metromover downtown circulator. The assessment was on net leasable square footage, and affected approximately 700 properties [14].

Tax Increment Financing. Improvements to land values caused by transportation improvements can be captured by an increase in property taxes, which are then used to maintain the improvement. This is done by creating a tax-incrementfinancing (TIF) district, which is the well-defined area benefitting from the project. The base year of property values is established before the project begins. Then, as the property value goes up, the increase in property taxes is dedicated to the operation and maintenance of the transportation improvements. A downside to TIF financing is that is takes dollars away from other services needed that are usually financed with property taxes, such as schools and hospitals, water, police, etc [15]. Example TIFs include:

- Denver's 16th Street Transit Mall in downtown Denver is maintained by tax increment financing [15].
- Prince George County in Maryland uses Tax Increment Finance districts to fund various improvements. TIFs have been used to finance infrastructure as well as convention centers and other developments [14].

Negotiated Investments. The developer and the transportation agency may agree to some lump sum to capture the value of the benefit to the property, or in exchange for something that the developer needs, for instance, a zoning change or building permit. This also can include the leasing of land or air rights to developers. Examples include:

- In New York City, the Hudson Yards/7 Subway extension is being partially paid for by charging property owners for zoning changes that allow additional density in the area. The same project is also getting payments for the air rights. [14]

Transit/Traffic Impact Requirements (also known as Development Fees). In this arrangement, developers are charged to mitigate negative impacts caused by the development on the surrounding street network or transit system. This can include new intersections being built or expanded, traffic calming measures installed [14], or to build new facilities to accommodate the new development. California is one of the leaders in using traffic impact fees to help finance transportation improvements [16]. Examples include:

- In California, Placer County has required traffic impact fees from all developers in the County [17].
- The City of Tampa uses traffic impact fees on all land development in the city. The fee is based on a trip generation measure used in planning the development [18].

Leasing of Concessions. Leasing of concession stands to vendors, selling advertising space, etc., are additional sources of funds that can be used to maintain a transportation facility.

### 8.3.2 Less Traditional Financing Approaches

Over the last two decades, many new methods of financing transportation projects have evolved. In 1991, ISTEA and later TEA-21 allowed trial financing methods to test new funding techniques. SAFETEA-LU continued this by funding publicprivate partnerships and eliminating obstacles and limits on such partnerships. Other tools that innovatively finance transportation were also included. MAP-21, the most current transportation bill at the time of this writing, which provides funds through FY 2014, continues this promotion of new financing methods for transforming the framework for how investments are made in the country's transportation infrastructure.

Debt financing for transportation projects has traditionally used tax-exempt bonds, which allows an agency or government to pay a lower interest rate while still attracting buyers. Since 2005, the municipal bond market has averaged \$410B per year, with transportation issues accounting for 12 percent of this total. This includes highway bonds, toll facility bonds, transit bonds, airport bonds, and seaport bonds [19].

Grant Anticipation Revenue Vehicles (GARVEE) Bonds. In 1995, the National Highway System Designation Act expanded the types of transportation costs that could use federal funds. With the 1998 Transportation Equity Act for the 21st century (TEA-21), the amount given to states allowed the use of some of those funds for debt financing qualified projects, through the use of GARVEEs [20].

The GARVEE bond program is a method of financing qualified transportation projects that allows states to use tax-exempt debt financing that is backed by annual federal money appropriated for transportation projects. That is, GARVEEs are a debt-financing tool backed by money that is anticipated from expected future federal-aid grants. Thus the name "grant anticipation vehicle."

In order to be a qualified project, the project must meet certain federal requirements for this type of debt financing. The usual GARVEE projects are large projects with the following characteristics [21]:

1. Large projects where the benefits of moving ahead with the project sooner is greater than the cost of the delay if pay-as-you-go financing is used.
2. The project does not have tolls or other revenues that can guarantee repayment of the loan.
3. The State must be willing to set aside a portion of their future year Federal-aid highway funds to satisfy the debt requirements.

Transportation Infrastructure Finance And Innovation Act (TIFIA). The TIFIA program provides federal loans and/or credit for transportation projects that are deemed to be of national importance or regionally important. The program offers three types of credit, Direct Federal Loans (secured), Loan Guarantees, and Lines of Credit that can be used to assist the financial needs of a project throughout its life cycle. The purpose of the TIFIA program is to use federal funds to encourage private co-investment as well as other non-federal co-investors in these crucial projects. Eligible co-investors include private firms, state governments, local governments, special authorities, and Transportation Improvement Districts. Projects can include highways, bridges, Intelligent Transportation Systems, transit facilities and vehicles, and freight transfer facilities [22, 23].

State Infrastructure Banks (SIBs). SIBs are state run lending "banks" that offer loans, sometimes at below market rates, to attract private and other non-federal investors. The "banks" are capitalized with federal monies as well as State funds.

Private Activity Bonds (PABs). PABs allow private investors the benefits of taxexempt financing, enabling them to borrow at lower interest rates. The PAB bonds are issued by State or Local governments, which use the proceeds to finance qualified projects developed by private companies. The private company then is responsible for paying back the bond issue. Other tax-exempt bonds do not allow for financing of private projects. The purpose of PABs is to encourage more private sector involvement and investment in our transportation infrastructure.

### 8.3.3 Road Pricing

Road Pricing is another source for funding transportation facilities. It refers to charging users directly for access to a specific road or area. This can be a simple toll road or can be a more comprehensive type of charge on all roads based on the vehicle-miles travelled. Congestion pricing also falls under this category, where users are charged more to use a road or enter an area during heavy traffic times, often the morning peak period.

### 8.3.4 Public-Private Partnerships

Public private partnerships, also known as PPP's or P3s, are agreements between private companies and public agencies to be involved in the design and building and/or running and maintaining a transportation facility. This arrangement has the benefit of sharing the costs, which are becoming harder to sustain with only public funds, spreading the risk, and also increasing the benefits to the users. There are many types of P3 arrangements, which run the gamut from small involvement and small financial risk for the private entity to large involvements and large financial risks [24]. Table 8.2, taken from Ref [25] shows this continuum of involvement from Case 1, where all aspects of the project are public, to Case 7, where all aspects of the project are privately controlled.

Table 8.2 Continuum of Projects Involving Private-Public Partnerships

| Case | Development | Delivery* | Operations | Maintenance | Finance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Public | D-B-B | Public | Public | Public |
| 2 | Public | D-B-B | Private | Public | Public |
| 3 | Public | D-B-B | Private | Private | Public |
| 4 | Public | D-B | Private | Private | Public |
| 5 | Public | D-B | Private | Private | Public |
| 6 | Public-Private | D-B | Private | Private | Public-Private |
| 7 | Private | D-B | Private | Private | Private |

(Source: Bailey-Campbell, P. "Challenges Mount for Traditional Transportation Funding," Transportation Research News, No. 274, pg4, Transportation Research Board, Washington DC, May-June 2011).

These types of partnerships allow public agencies access to additional monies for projects. Private management can mean more efficiency in running an operation, as well as the introduction of new ideas. Many public-private partnership projects use the various financing tools described above. Examples of P3 projects are described below.

Chicago Skyway. The first privatization of a toll road in the US was the Chicago Skyway. The Chicago Skyway toll bridge was constructed, operated and maintained by the City of Chicago for 50 years. Then in 2005 it was leased to the Skyway Concession Company, LLC with a 99 -year lease agreement. SCC gave the City of Chicago $\$ 1.83$ billion for the lease and took over responsibility for all operations and maintenance. SCC collects all tolls and concession revenues. [25, 26].

Capital Beltway/I-495 High Occupancy Toll Lanes in Virginia. This project in Virginia was the first to use PABs as part of its financing. The project constructed two HOT lanes per direction parallel to the existing lanes. The HOT lanes use electronic toll technology and dynamic pricing that continuously adjusts toll rates to keep traffic moving freely. The project was a public-private partnership of the Virginia DOT and Capital Beltway Express, LLC. Financing included $\$ 589$ million in PABs, $\$ 589$ million from a TIFIA direct loan, and $\$ 348$ million in private equity as part of the over $\$ 2$ billion dollar construction. An 80 -year operating concession was given to Capital Beltway Express to run and maintain the facility. Buses and vehicles with three or more passengers ride free and other vehicles pay
toll prices that vary based on traffic volume to keep them congestion free [27, 28]. This project was a design, build, operate, and maintain P3 project.

IH 635 Managed Lanes in Texas. Texas DOT is partnering with LBJ Infrastructure Group to operate and maintain this facility for 52 years. The project reconstructs the current lanes and adds four managed lanes underground and six elevated managed lanes. The managed lanes will be equipped with automatic vehicle identification (AVI) technology to read vehicle transponders. The lanes will be dynamically priced. Funding Sources included PABs worth $\$ 615$ million, TIFIA loan of $\$ 850$ million, and other public funds and toll revenues. [29]

Port of Miami Tunnel. Florida has a very active program in private-public partnerships [25]. The Port of Miami Tunnel is a partnership between MAT Concessionaire, LLC and Florida Department of Transportation (FDOT). The responsibility for design-build-finance-operate-maintain is transferred to MAT. During construction, FDOT will make payments to MAT as they complete contractual milestones. In 2044, the tunnel will be returned to FDOT responsibility [30].

### 8.3.5 Conclusion

The examples of public-private partnerships herein are just a few of the myriad of possibilities for using P3s. P3 projects have a great potential for helping to improve our transportation systems and infrastructure. They cannot, however, be the only solution. They are one of a group of tools that will support and improve transportation in the United States and internationally. In many types of P3 projects, funding still must come from the public sector, and we cannot nor would want all of our roads to be privately owned. The goals of private entities are often very different than the goals of the public. Some negatives to private involvement include [31]:

1. Loss of public control. Private entities will want more return on their investment and will most likely increase the cost of a toll road, for instance, more than if it was publicly owned and run. There are ways in the agreement to limit how much a toll can be increased, but it is still likely that the toll will increase more than it would otherwise.
2. With the excessively long leases, is the public receiving full value? Even with the very large sums that the private entity is paying for control of a facility for a period of time, the present value of the future income may be much more than the upfront payment received. The large upfront sum can be attractive to the public sector particularly when there are big budget gaps to fill, but may not be best in the long term.
3. The financing is risky. Depending on the terms of the contract, if the private entity cannot repay the debt or goes bankrupt, who will be responsible. In California, the South Bay Expressway private partner went bankrupt and the highway project went through reorganization. All parties, including TIFIA program, lost money[32].
4. Less transparency. There is concern that these roads that are privately run and maintained are not "public" enough. For instance, is it clear what maintenance is occurring, or how it is operated.
5. Not enough oversight protecting the public interest. When a private entity operates a facility, there is no accountability to the public. There is not the same oversight as on public facilities. For example, are the same safety standards being met?

### 8.4 The Vehicle-Mile Tax and Other Tax Options for the Future

A method that has been proposed to raise money for funding transportation is the VMT tax, or vehicle-miles traveled tax. This tax would either replace or supplement the current gasoline tax. The logic behind the VMT tax is that it more closely relates the amount of usage of roadways to the charge paid. The more you travel, the more you are charged. There is a wide range of gas mileage per gallon amongst the different vehicles on roads. This raises the issue of fairness or equity in the gasoline tax. Newer and more expensive vehicles are getting more miles to the gallon, which is accelerating the depletion of funds available for transportation, as well as creating a more inequitable tax. Lower income individuals often have older automobiles that are less gasoline efficient. Gas efficient vehicles are paying less for driving the same number of miles, not to mention electric and hybrid cars. It is clear that the costs imposed by vehicles on our roadways are more proportional to the vehicle-miles traveled rather than the amount of gasoline consumption.

With the VMT tax, the number of miles traveled by each vehicle must be tracked. Then a fee is charged for each mile driven. The charge could change by time period, that is, be higher during congested periods. It could also be higher for large trucks that put more wear and tear on the roads.

Cities all across the country are considering a VMT tax. Currently there are eighteen states studying this option [33]. Starting in 2006, the VMT tax was tested in twelve cities as part of a federally-funded research program done through the University of Iowa [34]. Oregon was a participating state, and has continued to be one of the leaders in studying the feasibility of a VMT tax. The first pilot studies used GPS devices to track drivers' mileage [34]. There are some privacy issues with using GPS, and Oregon has been testing alternative methods of tracking
mileage data. This current pilot in Oregon gives users four choices as to the method that their mileage will be tracked: (1) GPS, (2) a more simple mileage measuring device (however they are then choosing to be charged for out-of-state driving and driving on private roads), (3) a smart-phone app that uses GPS, or (4) paying a flat fee instead of using any measurement device [35]. Also being studied is whether drivers prefer data to be collected by a government agency or the private sector [35].

Many believe that the VMT tax is inevitable, and the only equitable means of collecting road-use taxes. An interesting result of the Iowa study was the improvement in people's views about the system after having used it. Before the study, only $41 \%$ viewed a VMT tax positively, but after the study that number increased to $70 \%$ [36]. However, there are many issues still to be solved and many years before it could be implemented on a large-scale federal basis.

### 8.5 Closing Comments

All one has to do is look at the AASHTO Daily Transportation Update [37] to see how funding is in the forefront of critical issues facing the transportation community. Everyone agrees that something needs to be done soon to increase the funds available for transportation infrastructure. The issue is how to best do this. The public in general is very resistant to change or to any gas tax increases. Because of this, politicians are reluctant to recommend changes. Many innovative finance methods are currently being used and new methods are being studied across the country. As revenues lag behind the needs of our transportation system, this issue will remain on the critical list for years to come.

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## Appendix <br> Interest Tables

| Interest RATE = |  |  | 1\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.010000 | 0.990099 | 1.000000 | 1.000000 | 0.990099 | 1.010000 |
| 2 | 1.020100 | 0.980296 | 2.010000 | 0.497512 | 1.970395 | 0.507512 |
| 3 | 1.030301 | 0.970590 | 3.030100 | 0.330022 | 2.940985 | 0.340022 |
| 4 | 1.040604 | 0.960980 | 4.060401 | 0.246281 | 3.901966 | 0.256281 |
| 5 | 1.051010 | 0.951466 | 5.101005 | 0.196040 | 4.853431 | 0.206040 |
| 6 | 1.061520 | 0.942045 | 6.152015 | 0.162548 | 5.795476 | 0.172548 |
| 7 | 1.072135 | 0.932718 | 7.213535 | 0.138628 | 6.728195 | 0.148628 |
| 8 | 1.082857 | 0.923483 | 8.285671 | 0.120690 | 7.651678 | 0.130690 |
| 9 | 1.093685 | 0.914340 | 9.368527 | 0.106740 | 8.566018 | 0.116740 |
| 10 | 1.104622 | 0.905287 | 10.462213 | 0.095582 | 9.471305 | 0.105582 |
| 11 | 1.115668 | 0.896324 | 11.566835 | 0.086454 | 10.367628 | 0.096454 |
| 12 | 1.126825 | 0.887449 | 12.682503 | 0.078849 | 11.255077 | 0.088849 |
| 13 | 1.138093 | 0.878663 | 13.809328 | 0.072415 | 12.133740 | 0.082415 |
| 14 | 1.149474 | 0.869963 | 14.947421 | 0.066901 | 13.003703 | 0.076901 |
| 15 | 1.160969 | 0.861349 | 16.096896 | 0.062124 | 13.865053 | 0.072124 |
| 16 | 1.172579 | 0.852821 | 17.257864 | 0.057945 | 14.717874 | 0.067945 |
| 17 | 1.184304 | 0.844377 | 18.430443 | 0.054258 | 15.562251 | 0.064258 |
| 18 | 1.196147 | 0.836017 | 19.614748 | 0.050982 | 16.398269 | 0.060982 |
| 19 | 1.208109 | 0.827740 | 20.810895 | 0.048052 | 17.226008 | 0.058052 |
| 20 | 1.220190 | 0.819544 | 22.019004 | 0.045415 | 18.045553 | 0.055415 |
| 21 | 1.232392 | 0.811430 | 23.239194 | 0.043031 | 18.856983 | 0.053031 |
| 22 | 1.244716 | 0.803396 | 24.471586 | 0.040864 | 19.660379 | 0.050864 |
| 23 | 1.257163 | 0.795442 | 25.716302 | 0.038886 | 20.455821 | 0.048886 |
| 24 | 1.269735 | 0.787566 | 26.973465 | 0.037073 | 21.243387 | 0.047073 |
| 25 | 1.282432 | 0.779768 | 28.243200 | 0.035407 | 22.023156 | 0.045407 |
| 26 | 1.295256 | 0.772048 | 29.525631 | 0.033869 | 22.795204 | 0.043869 |
| 27 | 1.308209 | 0.764404 | 30.820888 | 0.032446 | 23.559608 | 0.042446 |
| 28 | 1.321291 | 0.756836 | 32.129097 | 0.031124 | 24.316443 | 0.041124 |
| 29 | 1.334504 | 0.749342 | 33.450388 | 0.029895 | 25.065785 | 0.039895 |
| 30 | 1.347849 | 0.741923 | 34.784892 | 0.028748 | 25.807708 | 0.038748 |
| 31 | 1.361327 | 0.734577 | 36.132740 | 0.027676 | 26.542285 | 0.037676 |
| 32 | 1.374941 | 0.727304 | 37.494068 | 0.026671 | 27.269589 | 0.036671 |
| 33 | 1.388690 | 0.720103 | 38.869009 | 0.025727 | 27.989693 | 0.035727 |
| 34 | 1.402577 | 0.712973 | 40.257699 | 0.024840 | 28.702666 | 0.034840 |
| 35 | 1.416603 | 0.705914 | 41.660276 | 0.024004 | 29.408580 | 0.034004 |
| 36 | 1.430769 | 0.698925 | 43.076878 | 0.023214 | 30.107505 | 0.033214 |
| 37 | 1.445076 | 0.692005 | 44.507647 | 0.022468 | 30.799510 | 0.032468 |
| 38 | 1.459527 | 0.685153 | 45.952724 | 0.021761 | 31.484663 | 0.031761 |
| 39 | 1.474123 | 0.678370 | 47.412251 | 0.021092 | 32.163033 | 0.031092 |
| 40 | 1.488864 | 0.671653 | 48.886373 | 0.020456 | 32.834686 | 0.030456 |
| 41 | 1.503752 | 0.665003 | 50.375237 | 0.019851 | 33.499689 | 0.029851 |
| 42 | 1.518790 | 0.658419 | 51.878989 | 0.019276 | 34.158108 | 0.029276 |
| 43 | 1.533978 | 0.651900 | 53.397779 | 0.018727 | 34.810008 | 0.028727 |
| 44 | 1.549318 | 0.645445 | 54.931757 | 0.018204 | 35.455454 | 0.028204 |
| 45 | 1.564811 | 0.639055 | 56.481075 | 0.017705 | 36.094508 | 0.027705 |
| 50 | 1.644632 | 0.608039 | 64.463182 | 0.015513 | 39.196118 | 0.025513 |
| 55 | 1.728525 | 0.578528 | 72.852457 | 0.013726 | 42.147192 | 0.023726 |
| 60 | 1.816697 | 0.550450 | 81.669670 | 0.012244 | 44.955038 | 0.022244 |


| Interest RATE = |  |  | 2\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.020000 | 0.980392 | 1.000000 | 1.000000 | 0.980392 | 1.020000 |
| 2 | 1.040400 | 0.961169 | 2.020000 | 0.495050 | 1.941561 | 0.515050 |
| 3 | 1.061208 | 0.942322 | 3.060400 | 0.326755 | 2.883883 | 0.346755 |
| 4 | 1.082432 | 0.923845 | 4.121608 | 0.242624 | 3.807729 | 0.262624 |
| 5 | 1.104081 | 0.905731 | 5.204040 | 0.192158 | 4.713460 | 0.212158 |
| 6 | 1.126162 | 0.887971 | 6.308121 | 0.158526 | 5.601431 | 0.178526 |
| 7 | 1.148686 | 0.870560 | 7.434283 | 0.134512 | 6.471991 | 0.154512 |
| 8 | 1.171659 | 0.853490 | 8.582969 | 0.116510 | 7.325481 | 0.136510 |
| 9 | 1.195093 | 0.836755 | 9.754628 | 0.102515 | 8.162237 | 0.122515 |
| 10 | 1.218994 | 0.820348 | 10.949721 | 0.091327 | 8.982585 | 0.111327 |
| 11 | 1.243374 | 0.804263 | 12.168715 | 0.082178 | 9.786848 | 0.102178 |
| 12 | 1.268242 | 0.788493 | 13.412090 | 0.074560 | 10.575341 | 0.094560 |
| 13 | 1.293607 | 0.773033 | 14.680332 | 0.068118 | 11.348374 | 0.088118 |
| 14 | 1.319479 | 0.757875 | 15.973938 | 0.062602 | 12.106249 | 0.082602 |
| 15 | 1.345868 | 0.743015 | 17.293417 | 0.057825 | 12.849264 | 0.077825 |
| 16 | 1.372786 | 0.728446 | 18.639285 | 0.053650 | 13.577709 | 0.073650 |
| 17 | 1.400241 | 0.714163 | 20.012071 | 0.049970 | 14.291872 | 0.069970 |
| 18 | 1.428246 | 0.700159 | 21.412312 | 0.046702 | 14.992031 | 0.066702 |
| 19 | 1.456811 | 0.686431 | 22.840559 | 0.043782 | 15.678462 | 0.063782 |
| 20 | 1.485947 | 0.672971 | 24.297370 | 0.041157 | 16.351433 | 0.061157 |
| 21 | 1.515666 | 0.659776 | 25.783317 | 0.038785 | 17.011209 | 0.058785 |
| 22 | 1.545980 | 0.646839 | 27.298984 | 0.036631 | 17.658048 | 0.056631 |
| 23 | 1.576899 | 0.634156 | 28.844963 | 0.034668 | 18.292204 | 0.054668 |
| 24 | 1.608437 | 0.621721 | 30.421862 | 0.032871 | 18.913926 | 0.052871 |
| 25 | 1.640606 | 0.609531 | 32.030300 | 0.031220 | 19.523456 | 0.051220 |
| 26 | 1.673418 | 0.597579 | 33.670906 | 0.029699 | 20.121036 | 0.049699 |
| 27 | 1.706886 | 0.585862 | 35.344324 | 0.028293 | 20.706898 | 0.048293 |
| 28 | 1.741024 | 0.574375 | 37.051210 | 0.026990 | 21.281272 | 0.046990 |
| 29 | 1.775845 | 0.563112 | 38.792235 | 0.025778 | 21.844385 | 0.045778 |
| 30 | 1.811362 | 0.552071 | 40.568079 | 0.024650 | 22.396456 | 0.044650 |
| 31 | 1.847589 | 0.541246 | 42.379441 | 0.023596 | 22.937702 | 0.043596 |
| 32 | 1.884541 | 0.530633 | 44.227030 | 0.022611 | 23.468335 | 0.042611 |
| 33 | 1.922231 | 0.520229 | 46.111570 | 0.021687 | 23.988564 | 0.041687 |
| 34 | 1.960676 | 0.510028 | 48.033802 | 0.020819 | 24.498592 | 0.040819 |
| 35 | 1.999890 | 0.500028 | 49.994478 | 0.020002 | 24.998619 | 0.040002 |
| 36 | 2.039887 | 0.490223 | 51.994367 | 0.019233 | 25.488842 | 0.039233 |
| 37 | 2.080685 | 0.480611 | 54.034255 | 0.018507 | 25.969453 | 0.038507 |
| 38 | 2.122299 | 0.471187 | 56.114940 | 0.017821 | 26.440641 | 0.037821 |
| 39 | 2.164745 | 0.461948 | 58.237238 | 0.017171 | 26.902589 | 0.037171 |
| 40 | 2.208040 | 0.452890 | 60.401983 | 0.016556 | 27.355479 | 0.036556 |
| 41 | 2.252200 | 0.444010 | 62.610023 | 0.015972 | 27.799489 | 0.035972 |
| 42 | 2.297244 | 0.435304 | 64.862223 | 0.015417 | 28.234794 | 0.035417 |
| 43 | 2.343189 | 0.426769 | 67.159468 | 0.014890 | 28.661562 | 0.034890 |
| 44 | 2.390053 | 0.418401 | 69.502657 | 0.014388 | 29.079963 | 0.034388 |
| 45 | 2.437854 | 0.410197 | 71.892710 | 0.013910 | 29.490160 | 0.033910 |
| 50 | 2.691588 | 0.371528 | 84.579401 | 0.011823 | 31.423606 | 0.031823 |
| 55 | 2.971731 | 0.336504 | 98.586534 | 0.010143 | 33.174788 | 0.030143 |
| 60 | 3.281031 | 0.304782 | 114.051539 | 0.008768 | 34.760887 | 0.028768 |


| Interest RATE = |  |  | 3\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.030000 | 0.970874 | 1.000000 | 1.000000 | 0.970874 | 1.030000 |
| 2 | 1.060900 | 0.942596 | 2.030000 | 0.492611 | 1.913470 | 0.522611 |
| 3 | 1.092727 | 0.915142 | 3.090900 | 0.323530 | 2.828611 | 0.353530 |
| 4 | 1.125509 | 0.888487 | 4.183627 | 0.239027 | 3.717098 | 0.269027 |
| 5 | 1.159274 | 0.862609 | 5.309136 | 0.188355 | 4.579707 | 0.218355 |
| 6 | 1.194052 | 0.837484 | 6.468410 | 0.154598 | 5.417191 | 0.184598 |
| 7 | 1.229874 | 0.813092 | 7.662462 | 0.130506 | 6.230283 | 0.160506 |
| 8 | 1.266770 | 0.789409 | 8.892336 | 0.112456 | 7.019692 | 0.142456 |
| 9 | 1.304773 | 0.766417 | 10.159106 | 0.098434 | 7.786109 | 0.128434 |
| 10 | 1.343916 | 0.744094 | 11.463879 | 0.087231 | 8.530203 | 0.117231 |
| 11 | 1.384234 | 0.722421 | 12.807796 | 0.078077 | 9.252624 | 0.108077 |
| 12 | 1.425761 | 0.701380 | 14.192030 | 0.070462 | 9.954004 | 0.100462 |
| 13 | 1.468534 | 0.680951 | 15.617790 | 0.064030 | 10.634955 | 0.094030 |
| 14 | 1.512590 | 0.661118 | 17.086324 | 0.058526 | 11.296073 | 0.088526 |
| 15 | 1.557967 | 0.641862 | 18.598914 | 0.053767 | 11.937935 | 0.083767 |
| 16 | 1.604706 | 0.623167 | 20.156881 | 0.049611 | 12.561102 | 0.079611 |
| 17 | 1.652848 | 0.605016 | 21.761588 | 0.045953 | 13.166118 | 0.075953 |
| 18 | 1.702433 | 0.587395 | 23.414435 | 0.042709 | 13.753513 | 0.072709 |
| 19 | 1.753506 | 0.570286 | 25.116868 | 0.039814 | 14.323799 | 0.069814 |
| 20 | 1.806111 | 0.553676 | 26.870374 | 0.037216 | 14.877475 | 0.067216 |
| 21 | 1.860295 | 0.537549 | 28.676486 | 0.034872 | 15.415024 | 0.064872 |
| 22 | 1.916103 | 0.521893 | 30.536780 | 0.032747 | 15.936917 | 0.062747 |
| 23 | 1.973587 | 0.506692 | 32.452884 | 0.030814 | 16.443608 | 0.060814 |
| 24 | 2.032794 | 0.491934 | 34.426470 | 0.029047 | 16.935542 | 0.059047 |
| 25 | 2.093778 | 0.477606 | 36.459264 | 0.027428 | 17.413148 | 0.057428 |
| 26 | 2.156591 | 0.463695 | 38.553042 | 0.025938 | 17.876842 | 0.055938 |
| 27 | 2.221289 | 0.450189 | 40.709634 | 0.024564 | 18.327031 | 0.054564 |
| 28 | 2.287928 | 0.437077 | 42.930923 | 0.023293 | 18.764108 | 0.053293 |
| 29 | 2.356566 | 0.424346 | 45.218850 | 0.022115 | 19.188455 | 0.052115 |
| 30 | 2.427262 | 0.411987 | 47.575416 | 0.021019 | 19.600441 | 0.051019 |
| 31 | 2.500080 | 0.399987 | 50.002678 | 0.019999 | 20.000428 | 0.049999 |
| 32 | 2.575083 | 0.388337 | 52.502759 | 0.019047 | 20.388766 | 0.049047 |
| 33 | 2.652335 | 0.377026 | 55.077841 | 0.018156 | 20.765792 | 0.048156 |
| 34 | 2.731905 | 0.366045 | 57.730177 | 0.017322 | 21.131837 | 0.047322 |
| 35 | 2.813862 | 0.355383 | 60.462082 | 0.016539 | 21.487220 | 0.046539 |
| 36 | 2.898278 | 0.345032 | 63.275944 | 0.015804 | 21.832252 | 0.045804 |
| 37 | 2.985227 | 0.334983 | 66.174223 | 0.015112 | 22.167235 | 0.045112 |
| 38 | 3.074783 | 0.325226 | 69.159449 | 0.014459 | 22.492462 | 0.044459 |
| 39 | 3.167027 | 0.315754 | 72.234233 | 0.013844 | 22.808215 | 0.043844 |
| 40 | 3.262038 | 0.306557 | 75.401260 | 0.013262 | 23.114772 | 0.043262 |
| 41 | 3.359899 | 0.297628 | 78.663298 | 0.012712 | 23.412400 | 0.042712 |
| 42 | 3.460696 | 0.288959 | 82.023196 | 0.012192 | 23.701359 | 0.042192 |
| 43 | 3.564517 | 0.280543 | 85.483892 | 0.011698 | 23.981902 | 0.041698 |
| 44 | 3.671452 | 0.272372 | 89.048409 | 0.011230 | 24.254274 | 0.041230 |
| 45 | 3.781596 | 0.264439 | 92.719861 | 0.010785 | 24.518713 | 0.040785 |
| 50 | 4.383906 | 0.228107 | 112.796867 | 0.008865 | 25.729764 | 0.038865 |
| 55 | 5.082149 | 0.196767 | 136.071620 | 0.007349 | 26.774428 | 0.037349 |
| 60 | 5.891603 | 0.169733 | 163.053437 | 0.006133 | 27.675564 | 0.036133 |


| Interest RATE = |  |  | 4\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.040000 | 0.961538 | 1.000000 | 1.000000 | 0.961538 | 1.040000 |
| 2 | 1.081600 | 0.924556 | 2.040000 | 0.490196 | 1.886095 | 0.530196 |
| 3 | 1.124864 | 0.888996 | 3.121600 | 0.320349 | 2.775091 | 0.360349 |
| 4 | 1.169859 | 0.854804 | 4.246464 | 0.235490 | 3.629895 | 0.275490 |
| 5 | 1.216653 | 0.821927 | 5.416323 | 0.184627 | 4.451822 | 0.224627 |
| 6 | 1.265319 | 0.790315 | 6.632975 | 0.150762 | 5.242137 | 0.190762 |
| 7 | 1.315932 | 0.759918 | 7.898294 | 0.126610 | 6.002055 | 0.166610 |
| 8 | 1.368569 | 0.730690 | 9.214226 | 0.108528 | 6.732745 | 0.148528 |
| 9 | 1.423312 | 0.702587 | 10.582795 | 0.094493 | 7.435332 | 0.134493 |
| 10 | 1.480244 | 0.675564 | 12.006107 | 0.083291 | 8.110896 | 0.123291 |
| 11 | 1.539454 | 0.649581 | 13.486351 | 0.074149 | 8.760477 | 0.114149 |
| 12 | 1.601032 | 0.624597 | 15.025805 | 0.066552 | 9.385074 | 0.106552 |
| 13 | 1.665074 | 0.600574 | 16.626838 | 0.060144 | 9.985648 | 0.100144 |
| 14 | 1.731676 | 0.577475 | 18.291911 | 0.054669 | 10.563123 | 0.094669 |
| 15 | 1.800944 | 0.555265 | 20.023588 | 0.049941 | 11.118387 | 0.089941 |
| 16 | 1.872981 | 0.533908 | 21.824531 | 0.045820 | 11.652296 | 0.085820 |
| 17 | 1.947900 | 0.513373 | 23.697512 | 0.042199 | 12.165669 | 0.082199 |
| 18 | 2.025817 | 0.493628 | 25.645413 | 0.038993 | 12.659297 | 0.078993 |
| 19 | 2.106849 | 0.474642 | 27.671229 | 0.036139 | 13.133939 | 0.076139 |
| 20 | 2.191123 | 0.456387 | 29.778079 | 0.033582 | 13.590326 | 0.073582 |
| 21 | 2.278768 | 0.438834 | 31.969202 | 0.031280 | 14.029160 | 0.071280 |
| 22 | 2.369919 | 0.421955 | 34.247970 | 0.029199 | 14.451115 | 0.069199 |
| 23 | 2.464716 | 0.405726 | 36.617889 | 0.027309 | 14.856842 | 0.067309 |
| 24 | 2.563304 | 0.390121 | 39.082604 | 0.025587 | 15.246963 | 0.065587 |
| 25 | 2.665836 | 0.375117 | 41.645908 | 0.024012 | 15.622080 | 0.064012 |
| 26 | 2.772470 | 0.360689 | 44.311745 | 0.022567 | 15.982769 | 0.062567 |
| 27 | 2.883369 | 0.346817 | 47.084214 | 0.021239 | 16.329586 | 0.061239 |
| 28 | 2.998703 | 0.333477 | 49.967583 | 0.020013 | 16.663063 | 0.060013 |
| 29 | 3.118651 | 0.320651 | 52.966286 | 0.018880 | 16.983715 | 0.058880 |
| 30 | 3.243398 | 0.308319 | 56.084938 | 0.017830 | 17.292033 | 0.057830 |
| 31 | 3.373133 | 0.296460 | 59.328335 | 0.016855 | 17.588494 | 0.056855 |
| 32 | 3.508059 | 0.285058 | 62.701469 | 0.015949 | 17.873551 | 0.055949 |
| 33 | 3.648381 | 0.274094 | 66.209527 | 0.015104 | 18.147646 | 0.055104 |
| 34 | 3.794316 | 0.263552 | 69.857909 | 0.014315 | 18.411198 | 0.054315 |
| 35 | 3.946089 | 0.253415 | 73.652225 | 0.013577 | 18.664613 | 0.053577 |
| 36 | 4.103933 | 0.243669 | 77.598314 | 0.012887 | 18.908282 | 0.052887 |
| 37 | 4.268090 | 0.234297 | 81.702246 | 0.012240 | 19.142579 | 0.052240 |
| 38 | 4.438813 | 0.225285 | 85.970336 | 0.011632 | 19.367864 | 0.051632 |
| 39 | 4.616366 | 0.216621 | 90.409150 | 0.011061 | 19.584485 | 0.051061 |
| 40 | 4.801021 | 0.208289 | 95.025516 | 0.010523 | 19.792774 | 0.050523 |
| 41 | 4.993061 | 0.200278 | 99.826536 | 0.010017 | 19.993052 | 0.050017 |
| 42 | 5.192784 | 0.192575 | 104.819598 | 0.009540 | 20.185627 | 0.049540 |
| 43 | 5.400495 | 0.185168 | 110.012382 | 0.009090 | 20.370795 | 0.049090 |
| 44 | 5.616515 | 0.178046 | 115.412877 | 0.008665 | 20.548841 | 0.048665 |
| 45 | 5.841176 | 0.171198 | 121.029392 | 0.008262 | 20.720040 | 0.048262 |
| 50 | 7.106683 | 0.140713 | 152.667084 | 0.006550 | 21.482185 | 0.046550 |
| 55 | 8.646367 | 0.115656 | 191.159173 | 0.005231 | 22.108612 | 0.045231 |
| 60 | 10.519627 | 0.095060 | 237.990685 | 0.004202 | 22.623490 | 0.044202 |

Interest RATE =
5\%

| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.050000 | 0.952381 | 1.000000 | 1.000000 | 0.952381 | 1.050000 |
| 2 | 1.102500 | 0.907029 | 2.050000 | 0.487805 | 1.859410 | 0.537805 |
| 3 | 1.157625 | 0.863838 | 3.152500 | 0.317209 | 2.723248 | 0.367209 |
| 4 | 1.215506 | 0.822702 | 4.310125 | 0.232012 | 3.545951 | 0.282012 |
| 5 | 1.276282 | 0.783526 | 5.525631 | 0.180975 | 4.329477 | 0.230975 |
| 6 | 1.340096 | 0.746215 | 6.801913 | 0.147017 | 5.075692 | 0.197017 |
| 7 | 1.407100 | 0.710681 | 8.142008 | 0.122820 | 5.786373 | 0.172820 |
| 8 | 1.477455 | 0.676839 | 9.549109 | 0.104722 | 6.463213 | 0.154722 |
| 9 | 1.551328 | 0.644609 | 11.026564 | 0.090690 | 7.107822 | 0.140690 |
| 10 | 1.628895 | 0.613913 | 12.577893 | 0.079505 | 7.721735 | 0.129505 |
| 11 | 1.710339 | 0.584679 | 14.206787 | 0.070389 | 8.306414 | 0.120389 |
| 12 | 1.795856 | 0.556837 | 15.917127 | 0.062825 | 8.863252 | 0.112825 |
| 13 | 1.885649 | 0.530321 | 17.712983 | 0.056456 | 9.393573 | 0.106456 |
| 14 | 1.979932 | 0.505068 | 19.598632 | 0.051024 | 9.898641 | 0.101024 |
| 15 | 2.078928 | 0.481017 | 21.578564 | 0.046342 | 10.379658 | 0.096342 |
| 16 | 2.182875 | 0.458112 | 23.657492 | 0.042270 | 10.837770 | 0.092270 |
| 17 | 2.292018 | 0.436297 | 25.840366 | 0.038699 | 11.274066 | 0.088699 |
| 18 | 2.406619 | 0.415521 | 28.132385 | 0.035546 | 11.689587 | 0.085546 |
| 19 | 2.526950 | 0.395734 | 30.539004 | 0.032745 | 12.085321 | 0.082745 |
| 20 | 2.653298 | 0.376889 | 33.065954 | 0.030243 | 12.462210 | 0.080243 |
| 21 | 2.785963 | 0.358942 | 35.719252 | 0.027996 | 12.821153 | 0.077996 |
| 22 | 2.925261 | 0.341850 | 38.505214 | 0.025971 | 13.163003 | 0.075971 |
| 23 | 3.071524 | 0.325571 | 41.430475 | 0.024137 | 13.488574 | 0.074137 |
| 24 | 3.225100 | 0.310068 | 44.501999 | 0.022471 | 13.798642 | 0.072471 |
| 25 | 3.386355 | 0.295303 | 47.727099 | 0.020952 | 14.093945 | 0.070952 |
| 26 | 3.555673 | 0.281241 | 51.113454 | 0.019564 | 14.375185 | 0.069564 |
| 27 | 3.733456 | 0.267848 | 54.669126 | 0.018292 | 14.643034 | 0.068292 |
| 28 | 3.920129 | 0.255094 | 58.402583 | 0.017123 | 14.898127 | 0.067123 |
| 29 | 4.116136 | 0.242946 | 62.322712 | 0.016046 | 15.141074 | 0.066046 |
| 30 | 4.321942 | 0.231377 | 66.438848 | 0.015051 | 15.372451 | 0.065051 |
| 31 | 4.538039 | 0.220359 | 70.760790 | 0.014132 | 15.592811 | 0.064132 |
| 32 | 4.764941 | 0.209866 | 75.298829 | 0.013280 | 15.802677 | 0.063280 |
| 33 | 5.003189 | 0.199873 | 80.063771 | 0.012490 | 16.002549 | 0.062490 |
| 34 | 5.253348 | 0.190355 | 85.066959 | 0.011755 | 16.192904 | 0.061755 |
| 35 | 5.516015 | 0.181290 | 90.320307 | 0.011072 | 16.374194 | 0.061072 |
| 36 | 5.791816 | 0.172657 | 95.836323 | 0.010434 | 16.546852 | 0.060434 |
| 37 | 6.081407 | 0.164436 | 101.628139 | 0.009840 | 16.711287 | 0.059840 |
| 38 | 6.385477 | 0.156605 | 107.709546 | 0.009284 | 16.867893 | 0.059284 |
| 39 | 6.704751 | 0.149148 | 114.095023 | 0.008765 | 17.017041 | 0.058765 |
| 40 | 7.039989 | 0.142046 | 120.799774 | 0.008278 | 17.159086 | 0.058278 |
| 41 | 7.391988 | 0.135282 | 127.839763 | 0.007822 | 17.294368 | 0.057822 |
| 42 | 7.761588 | 0.128840 | 135.231751 | 0.007395 | 17.423208 | 0.057395 |
| 43 | 8.149667 | 0.122704 | 142.993339 | 0.006993 | 17.545912 | 0.056993 |
| 44 | 8.557150 | 0.116861 | 151.143006 | 0.006616 | 17.662773 | 0.056616 |
| 45 | 8.985008 | 0.111297 | 159.700156 | 0.006262 | 17.774070 | 0.056262 |
| 50 | 11.467400 | 0.087204 | 209.347996 | 0.004777 | 18.255925 | 0.054777 |
| 55 | 14.635631 | 0.068326 | 272.712618 | 0.003667 | 18.633472 | 0.053667 |
| 60 | 18.679186 | 0.053536 | 353.583718 | 0.002828 | 18.929290 | 0.052828 |

Interest RATE = 6\%

| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.060000 | 0.943396 | 1.000000 | 1.000000 | 0.943396 | 1.060000 |
| 2 | 1.123600 | 0.889996 | 2.060000 | 0.485437 | 1.833393 | 0.545437 |
| 3 | 1.191016 | 0.839619 | 3.183600 | 0.314110 | 2.673012 | 0.374110 |
| 4 | 1.262477 | 0.792094 | 4.374616 | 0.228591 | 3.465106 | 0.288591 |
| 5 | 1.338226 | 0.747258 | 5.637093 | 0.177396 | 4.212364 | 0.237396 |
| 6 | 1.418519 | 0.704961 | 6.975319 | 0.143363 | 4.917324 | 0.203363 |
| 7 | 1.503630 | 0.665057 | 8.393838 | 0.119135 | 5.582381 | 0.179135 |
| 8 | 1.593848 | 0.627412 | 9.897468 | 0.101036 | 6.209794 | 0.161036 |
| 9 | 1.689479 | 0.591898 | 11.491316 | 0.087022 | 6.801692 | 0.147022 |
| 10 | 1.790848 | 0.558395 | 13.180795 | 0.075868 | 7.360087 | 0.135868 |
| 11 | 1.898299 | 0.526788 | 14.971643 | 0.066793 | 7.886875 | 0.126793 |
| 12 | 2.012196 | 0.496969 | 16.869941 | 0.059277 | 8.383844 | 0.119277 |
| 13 | 2.132928 | 0.468839 | 18.882138 | 0.052960 | 8.852683 | 0.112960 |
| 14 | 2.260904 | 0.442301 | 21.015066 | 0.047585 | 9.294984 | 0.107585 |
| 15 | 2.396558 | 0.417265 | 23.275970 | 0.042963 | 9.712249 | 0.102963 |
| 16 | 2.540352 | 0.393646 | 25.672528 | 0.038952 | 10.105895 | 0.098952 |
| 17 | 2.692773 | 0.371364 | 28.212880 | 0.035445 | 10.477260 | 0.095445 |
| 18 | 2.854339 | 0.350344 | 30.905653 | 0.032357 | 10.827603 | 0.092357 |
| 19 | 3.025600 | 0.330513 | 33.759992 | 0.029621 | 11.158116 | 0.089621 |
| 20 | 3.207135 | 0.311805 | 36.785591 | 0.027185 | 11.469921 | 0.087185 |
| 21 | 3.399564 | 0.294155 | 39.992727 | 0.025005 | 11.764077 | 0.085005 |
| 22 | 3.603537 | 0.277505 | 43.392290 | 0.023046 | 12.041582 | 0.083046 |
| 23 | 3.819750 | 0.261797 | 46.995828 | 0.021278 | 12.303379 | 0.081278 |
| 24 | 4.048935 | 0.246979 | 50.815577 | 0.019679 | 12.550358 | 0.079679 |
| 25 | 4.291871 | 0.232999 | 54.864512 | 0.018227 | 12.783356 | 0.078227 |
| 26 | 4.549383 | 0.219810 | 59.156383 | 0.016904 | 13.003166 | 0.076904 |
| 27 | 4.822346 | 0.207368 | 63.705766 | 0.015697 | 13.210534 | 0.075697 |
| 28 | 5.111687 | 0.195630 | 68.528112 | 0.014593 | 13.406164 | 0.074593 |
| 29 | 5.418388 | 0.184557 | 73.639798 | 0.013580 | 13.590721 | 0.073580 |
| 30 | 5.743491 | 0.174110 | 79.058186 | 0.012649 | 13.764831 | 0.072649 |
| 31 | 6.088101 | 0.164255 | 84.801677 | 0.011792 | 13.929086 | 0.071792 |
| 32 | 6.453387 | 0.154957 | 90.889778 | 0.011002 | 14.084043 | 0.071002 |
| 33 | 6.840590 | 0.146186 | 97.343165 | 0.010273 | 14.230230 | 0.070273 |
| 34 | 7.251025 | 0.137912 | 104.183755 | 0.009598 | 14.368141 | 0.069598 |
| 35 | 7.686087 | 0.130105 | 111.434780 | 0.008974 | 14.498246 | 0.068974 |
| 36 | 8.147252 | 0.122741 | 119.120867 | 0.008395 | 14.620987 | 0.068395 |
| 37 | 8.636087 | 0.115793 | 127.268119 | 0.007857 | 14.736780 | 0.067857 |
| 38 | 9.154252 | 0.109239 | 135.904206 | 0.007358 | 14.846019 | 0.067358 |
| 39 | 9.703507 | 0.103056 | 145.058458 | 0.006894 | 14.949075 | 0.066894 |
| 40 | 10.285718 | 0.097222 | 154.761966 | 0.006462 | 15.046297 | 0.066462 |
| 41 | 10.902861 | 0.091719 | 165.047684 | 0.006059 | 15.138016 | 0.066059 |
| 42 | 11.557033 | 0.086527 | 175.950545 | 0.005683 | 15.224543 | 0.065683 |
| 43 | 12.250455 | 0.081630 | 187.507577 | 0.005333 | 15.306173 | 0.065333 |
| 44 | 12.985482 | 0.077009 | 199.758032 | 0.005006 | 15.383182 | 0.065006 |
| 45 | 13.764611 | 0.072650 | 212.743514 | 0.004700 | 15.455832 | 0.064700 |
| 50 | 18.420154 | 0.054288 | 290.335905 | 0.003444 | 15.761861 | 0.063444 |
| 55 | 24.650322 | 0.040567 | 394.172027 | 0.002537 | 15.990543 | 0.062537 |
| 60 | 32.987691 | 0.030314 | 533.128181 | 0.001876 | 16.161428 | 0.061876 |

Interest RATE = 7\%

| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.070000 | 0.934579 | 1.000000 | 1.000000 | 0.934579 | 1.070000 |
| 2 | 1.144900 | 0.873439 | 2.070000 | 0.483092 | 1.808018 | 0.553092 |
| 3 | 1.225043 | 0.816298 | 3.214900 | 0.311052 | 2.624316 | 0.381052 |
| 4 | 1.310796 | 0.762895 | 4.439943 | 0.225228 | 3.387211 | 0.295228 |
| 5 | 1.402552 | 0.712986 | 5.750739 | 0.173891 | 4.100197 | 0.243891 |
| 6 | 1.500730 | 0.666342 | 7.153291 | 0.139796 | 4.766540 | 0.209796 |
| 7 | 1.605781 | 0.622750 | 8.654021 | 0.115553 | 5.389289 | 0.185553 |
| 8 | 1.718186 | 0.582009 | 10.259803 | 0.097468 | 5.971299 | 0.167468 |
| 9 | 1.838459 | 0.543934 | 11.977989 | 0.083486 | 6.515232 | 0.153486 |
| 10 | 1.967151 | 0.508349 | 13.816448 | 0.072378 | 7.023582 | 0.142378 |
| 11 | 2.104852 | 0.475093 | 15.783599 | 0.063357 | 7.498674 | 0.133357 |
| 12 | 2.252192 | 0.444012 | 17.888451 | 0.055902 | 7.942686 | 0.125902 |
| 13 | 2.409845 | 0.414964 | 20.140643 | 0.049651 | 8.357651 | 0.119651 |
| 14 | 2.578534 | 0.387817 | 22.550488 | 0.044345 | 8.745468 | 0.114345 |
| 15 | 2.759032 | 0.362446 | 25.129022 | 0.039795 | 9.107914 | 0.109795 |
| 16 | 2.952164 | 0.338735 | 27.888054 | 0.035858 | 9.446649 | 0.105858 |
| 17 | 3.158815 | 0.316574 | 30.840217 | 0.032425 | 9.763223 | 0.102425 |
| 18 | 3.379932 | 0.295864 | 33.999033 | 0.029413 | 10.059087 | 0.099413 |
| 19 | 3.616528 | 0.276508 | 37.378965 | 0.026753 | 10.335595 | 0.096753 |
| 20 | 3.869684 | 0.258419 | 40.995492 | 0.024393 | 10.594014 | 0.094393 |
| 21 | 4.140562 | 0.241513 | 44.865177 | 0.022289 | 10.835527 | 0.092289 |
| 22 | 4.430402 | 0.225713 | 49.005739 | 0.020406 | 11.061240 | 0.090406 |
| 23 | 4.740530 | 0.210947 | 53.436141 | 0.018714 | 11.272187 | 0.088714 |
| 24 | 5.072367 | 0.197147 | 58.176671 | 0.017189 | 11.469334 | 0.087189 |
| 25 | 5.427433 | 0.184249 | 63.249038 | 0.015811 | 11.653583 | 0.085811 |
| 26 | 5.807353 | 0.172195 | 68.676470 | 0.014561 | 11.825779 | 0.084561 |
| 27 | 6.213868 | 0.160930 | 74.483823 | 0.013426 | 11.986709 | 0.083426 |
| 28 | 6.648838 | 0.150402 | 80.697691 | 0.012392 | 12.137111 | 0.082392 |
| 29 | 7.114257 | 0.140563 | 87.346529 | 0.011449 | 12.277674 | 0.081449 |
| 30 | 7.612255 | 0.131367 | 94.460786 | 0.010586 | 12.409041 | 0.080586 |
| 31 | 8.145113 | 0.122773 | 102.073041 | 0.009797 | 12.531814 | 0.079797 |
| 32 | 8.715271 | 0.114741 | 110.218154 | 0.009073 | 12.646555 | 0.079073 |
| 33 | 9.325340 | 0.107235 | 118.933425 | 0.008408 | 12.753790 | 0.078408 |
| 34 | 9.978114 | 0.100219 | 128.258765 | 0.007797 | 12.854009 | 0.077797 |
| 35 | 10.676581 | 0.093663 | 138.236878 | 0.007234 | 12.947672 | 0.077234 |
| 36 | 11.423942 | 0.087535 | 148.913460 | 0.006715 | 13.035208 | 0.076715 |
| 37 | 12.223618 | 0.081809 | 160.337402 | 0.006237 | 13.117017 | 0.076237 |
| 38 | 13.079271 | 0.076457 | 172.561020 | 0.005795 | 13.193473 | 0.075795 |
| 39 | 13.994820 | 0.071455 | 185.640292 | 0.005387 | 13.264928 | 0.075387 |
| 40 | 14.974458 | 0.066780 | 199.635112 | 0.005009 | 13.331709 | 0.075009 |
| 41 | 16.022670 | 0.062412 | 214.609570 | 0.004660 | 13.394120 | 0.074660 |
| 42 | 17.144257 | 0.058329 | 230.632240 | 0.004336 | 13.452449 | 0.074336 |
| 43 | 18.344355 | 0.054513 | 247.776496 | 0.004036 | 13.506962 | 0.074036 |
| 44 | 19.628460 | 0.050946 | 266.120851 | 0.003758 | 13.557908 | 0.073758 |
| 45 | 21.002452 | 0.047613 | 285.749311 | 0.003500 | 13.605522 | 0.073500 |
| 50 | 29.457025 | 0.033948 | 406.528929 | 0.002460 | 13.800746 | 0.072460 |
| 55 | 41.315001 | 0.024204 | 575.928593 | 0.001736 | 13.939939 | 0.071736 |
| 60 | 57.946427 | 0.017257 | 813.520383 | 0.001229 | 14.039181 | 0.071229 |


| Interest RATE = |  |  | 8\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.080000 | 0.925926 | 1.000000 | 1.000000 | 0.925926 | 1.080000 |
| 2 | 1.166400 | 0.857339 | 2.080000 | 0.480769 | 1.783265 | 0.560769 |
| 3 | 1.259712 | 0.793832 | 3.246400 | 0.308034 | 2.577097 | 0.388034 |
| 4 | 1.360489 | 0.735030 | 4.506112 | 0.221921 | 3.312127 | 0.301921 |
| 5 | 1.469328 | 0.680583 | 5.866601 | 0.170456 | 3.992710 | 0.250456 |
| 6 | 1.586874 | 0.630170 | 7.335929 | 0.136315 | 4.622880 | 0.216315 |
| 7 | 1.713824 | 0.583490 | 8.922803 | 0.112072 | 5.206370 | 0.192072 |
| 8 | 1.850930 | 0.540269 | 10.636628 | 0.094015 | 5.746639 | 0.174015 |
| 9 | 1.999005 | 0.500249 | 12.487558 | 0.080080 | 6.246888 | 0.160080 |
| 10 | 2.158925 | 0.463193 | 14.486562 | 0.069029 | 6.710081 | 0.149029 |
| 11 | 2.331639 | 0.428883 | 16.645487 | 0.060076 | 7.138964 | 0.140076 |
| 12 | 2.518170 | 0.397114 | 18.977126 | 0.052695 | 7.536078 | 0.132695 |
| 13 | 2.719624 | 0.367698 | 21.495297 | 0.046522 | 7.903776 | 0.126522 |
| 14 | 2.937194 | 0.340461 | 24.214920 | 0.041297 | 8.244237 | 0.121297 |
| 15 | 3.172169 | 0.315242 | 27.152114 | 0.036830 | 8.559479 | 0.116830 |
| 16 | 3.425943 | 0.291890 | 30.324283 | 0.032977 | 8.851369 | 0.112977 |
| 17 | 3.700018 | 0.270269 | 33.750226 | 0.029629 | 9.121638 | 0.109629 |
| 18 | 3.996019 | 0.250249 | 37.450244 | 0.026702 | 9.371887 | 0.106702 |
| 19 | 4.315701 | 0.231712 | 41.446263 | 0.024128 | 9.603599 | 0.104128 |
| 20 | 4.660957 | 0.214548 | 45.761964 | 0.021852 | 9.818147 | 0.101852 |
| 21 | 5.033834 | 0.198656 | 50.422921 | 0.019832 | 10.016803 | 0.099832 |
| 22 | 5.436540 | 0.183941 | 55.456755 | 0.018032 | 10.200744 | 0.098032 |
| 23 | 5.871464 | 0.170315 | 60.893296 | 0.016422 | 10.371059 | 0.096422 |
| 24 | 6.341181 | 0.157699 | 66.764759 | 0.014978 | 10.528758 | 0.094978 |
| 25 | 6.848475 | 0.146018 | 73.105940 | 0.013679 | 10.674776 | 0.093679 |
| 26 | 7.396353 | 0.135202 | 79.954415 | 0.012507 | 10.809978 | 0.092507 |
| 27 | 7.988061 | 0.125187 | 87.350768 | 0.011448 | 10.935165 | 0.091448 |
| 28 | 8.627106 | 0.115914 | 95.338830 | 0.010489 | 11.051078 | 0.090489 |
| 29 | 9.317275 | 0.107328 | 103.965936 | 0.009619 | 11.158406 | 0.089619 |
| 30 | 10.062657 | 0.099377 | 113.283211 | 0.008827 | 11.257783 | 0.088827 |
| 31 | 10.867669 | 0.092016 | 123.345868 | 0.008107 | 11.349799 | 0.088107 |
| 32 | 11.737083 | 0.085200 | 134.213537 | 0.007451 | 11.434999 | 0.087451 |
| 33 | 12.676050 | 0.078889 | 145.950620 | 0.006852 | 11.513888 | 0.086852 |
| 34 | 13.690134 | 0.073045 | 158.626670 | 0.006304 | 11.586934 | 0.086304 |
| 35 | 14.785344 | 0.067635 | 172.316804 | 0.005803 | 11.654568 | 0.085803 |
| 36 | 15.968172 | 0.062625 | 187.102148 | 0.005345 | 11.717193 | 0.085345 |
| 37 | 17.245626 | 0.057986 | 203.070320 | 0.004924 | 11.775179 | 0.084924 |
| 38 | 18.625276 | 0.053690 | 220.315945 | 0.004539 | 11.828869 | 0.084539 |
| 39 | 20.115298 | 0.049713 | 238.941221 | 0.004185 | 11.878582 | 0.084185 |
| 40 | 21.724521 | 0.046031 | 259.056519 | 0.003860 | 11.924613 | 0.083860 |
| 41 | 23.462483 | 0.042621 | 280.781040 | 0.003561 | 11.967235 | 0.083561 |
| 42 | 25.339482 | 0.039464 | 304.243523 | 0.003287 | 12.006699 | 0.083287 |
| 43 | 27.366640 | 0.036541 | 329.583005 | 0.003034 | 12.043240 | 0.083034 |
| 44 | 29.555972 | 0.033834 | 356.949646 | 0.002802 | 12.077074 | 0.082802 |
| 45 | 31.920449 | 0.031328 | 386.505617 | 0.002587 | 12.108402 | 0.082587 |
| 50 | 46.901613 | 0.021321 | 573.770156 | 0.001743 | 12.233485 | 0.081743 |
| 55 | 68.913856 | 0.014511 | 848.923201 | 0.001178 | 12.318614 | 0.081178 |
| 60 | 101.257064 | 0.009876 | 1253.213296 | 0.000798 | 12.376552 | 0.080798 |


| Interest RATE = |  |  | 9\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.090000 | 0.917431 | 1.000000 | 1.000000 | 0.917431 | 1.090000 |
| 2 | 1.188100 | 0.841680 | 2.090000 | 0.478469 | 1.759111 | 0.568469 |
| 3 | 1.295029 | 0.772183 | 3.278100 | 0.305055 | 2.531295 | 0.395055 |
| 4 | 1.411582 | 0.708425 | 4.573129 | 0.218669 | 3.239720 | 0.308669 |
| 5 | 1.538624 | 0.649931 | 5.984711 | 0.167092 | 3.889651 | 0.257092 |
| 6 | 1.677100 | 0.596267 | 7.523335 | 0.132920 | 4.485919 | 0.222920 |
| 7 | 1.828039 | 0.547034 | 9.200435 | 0.108691 | 5.032953 | 0.198691 |
| 8 | 1.992563 | 0.501866 | 11.028474 | 0.090674 | 5.534819 | 0.180674 |
| 9 | 2.171893 | 0.460428 | 13.021036 | 0.076799 | 5.995247 | 0.166799 |
| 10 | 2.367364 | 0.422411 | 15.192930 | 0.065820 | 6.417658 | 0.155820 |
| 11 | 2.580426 | 0.387533 | 17.560293 | 0.056947 | 6.805191 | 0.146947 |
| 12 | 2.812665 | 0.355535 | 20.140720 | 0.049651 | 7.160725 | 0.139651 |
| 13 | 3.065805 | 0.326179 | 22.953385 | 0.043567 | 7.486904 | 0.133567 |
| 14 | 3.341727 | 0.299246 | 26.019189 | 0.038433 | 7.786150 | 0.128433 |
| 15 | 3.642482 | 0.274538 | 29.360916 | 0.034059 | 8.060688 | 0.124059 |
| 16 | 3.970306 | 0.251870 | 33.003399 | 0.030300 | 8.312558 | 0.120300 |
| 17 | 4.327633 | 0.231073 | 36.973705 | 0.027046 | 8.543631 | 0.117046 |
| 18 | 4.717120 | 0.211994 | 41.301338 | 0.024212 | 8.755625 | 0.114212 |
| 19 | 5.141661 | 0.194490 | 46.018458 | 0.021730 | 8.950115 | 0.111730 |
| 20 | 5.604411 | 0.178431 | 51.160120 | 0.019546 | 9.128546 | 0.109546 |
| 21 | 6.108808 | 0.163698 | 56.764530 | 0.017617 | 9.292244 | 0.107617 |
| 22 | 6.658600 | 0.150182 | 62.873338 | 0.015905 | 9.442425 | 0.105905 |
| 23 | 7.257874 | 0.137781 | 69.531939 | 0.014382 | 9.580207 | 0.104382 |
| 24 | 7.911083 | 0.126405 | 76.789813 | 0.013023 | 9.706612 | 0.103023 |
| 25 | 8.623081 | 0.115968 | 84.700896 | 0.011806 | 9.822580 | 0.101806 |
| 26 | 9.399158 | 0.106393 | 93.323977 | 0.010715 | 9.928972 | 0.100715 |
| 27 | 10.245082 | 0.097608 | 102.723135 | 0.009735 | 10.026580 | 0.099735 |
| 28 | 11.167140 | 0.089548 | 112.968217 | 0.008852 | 10.116128 | 0.098852 |
| 29 | 12.172182 | 0.082155 | 124.135356 | 0.008056 | 10.198283 | 0.098056 |
| 30 | 13.267678 | 0.075371 | 136.307539 | 0.007336 | 10.273654 | 0.097336 |
| 31 | 14.461770 | 0.069148 | 149.575217 | 0.006686 | 10.342802 | 0.096686 |
| 32 | 15.763329 | 0.063438 | 164.036987 | 0.006096 | 10.406240 | 0.096096 |
| 33 | 17.182028 | 0.058200 | 179.800315 | 0.005562 | 10.464441 | 0.095562 |
| 34 | 18.728411 | 0.053395 | 196.982344 | 0.005077 | 10.517835 | 0.095077 |
| 35 | 20.413968 | 0.048986 | 215.710755 | 0.004636 | 10.566821 | 0.094636 |
| 36 | 22.251225 | 0.044941 | 236.124723 | 0.004235 | 10.611763 | 0.094235 |
| 37 | 24.253835 | 0.041231 | 258.375948 | 0.003870 | 10.652993 | 0.093870 |
| 38 | 26.436680 | 0.037826 | 282.629783 | 0.003538 | 10.690820 | 0.093538 |
| 39 | 28.815982 | 0.034703 | 309.066463 | 0.003236 | 10.725523 | 0.093236 |
| 40 | 31.409420 | 0.031838 | 337.882445 | 0.002960 | 10.757360 | 0.092960 |
| 41 | 34.236268 | 0.029209 | 369.291865 | 0.002708 | 10.786569 | 0.092708 |
| 42 | 37.317532 | 0.026797 | 403.528133 | 0.002478 | 10.813366 | 0.092478 |
| 43 | 40.676110 | 0.024584 | 440.845665 | 0.002268 | 10.837950 | 0.092268 |
| 44 | 44.336960 | 0.022555 | 481.521775 | 0.002077 | 10.860505 | 0.092077 |
| 45 | 48.327286 | 0.020692 | 525.858734 | 0.001902 | 10.881197 | 0.091902 |
| 50 | 74.357520 | 0.013449 | 815.083556 | 0.001227 | 10.961683 | 0.091227 |
| 55 | 114.408262 | 0.008741 | 1260.091796 | 0.000794 | 11.013993 | 0.090794 |
| 60 | 176.031292 | 0.005681 | 1944.792133 | 0.000514 | 11.047991 | 0.090514 |


| Interest RATE = |  |  | 10\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.100000 | 0.909091 | 1.000000 | 1.000000 | 0.909091 | 1.100000 |
| 2 | 1.210000 | 0.826446 | 2.100000 | 0.476190 | 1.735537 | 0.576190 |
| 3 | 1.331000 | 0.751315 | 3.310000 | 0.302115 | 2.486852 | 0.402115 |
| 4 | 1.464100 | 0.683013 | 4.641000 | 0.215471 | 3.169865 | 0.315471 |
| 5 | 1.610510 | 0.620921 | 6.105100 | 0.163797 | 3.790787 | 0.263797 |
| 6 | 1.771561 | 0.564474 | 7.715610 | 0.129607 | 4.355261 | 0.229607 |
| 7 | 1.948717 | 0.513158 | 9.487171 | 0.105405 | 4.868419 | 0.205405 |
| 8 | 2.143589 | 0.466507 | 11.435888 | 0.087444 | 5.334926 | 0.187444 |
| 9 | 2.357948 | 0.424098 | 13.579477 | 0.073641 | 5.759024 | 0.173641 |
| 10 | 2.593742 | 0.385543 | 15.937425 | 0.062745 | 6.144567 | 0.162745 |
| 11 | 2.853117 | 0.350494 | 18.531167 | 0.053963 | 6.495061 | 0.153963 |
| 12 | 3.138428 | 0.318631 | 21.384284 | 0.046763 | 6.813692 | 0.146763 |
| 13 | 3.452271 | 0.289664 | 24.522712 | 0.040779 | 7.103356 | 0.140779 |
| 14 | 3.797498 | 0.263331 | 27.974983 | 0.035746 | 7.366687 | 0.135746 |
| 15 | 4.177248 | 0.239392 | 31.772482 | 0.031474 | 7.606080 | 0.131474 |
| 16 | 4.594973 | 0.217629 | 35.949730 | 0.027817 | 7.823709 | 0.127817 |
| 17 | 5.054470 | 0.197845 | 40.544703 | 0.024664 | 8.021553 | 0.124664 |
| 18 | 5.559917 | 0.179859 | 45.599173 | 0.021930 | 8.201412 | 0.121930 |
| 19 | 6.115909 | 0.163508 | 51.159090 | 0.019547 | 8.364920 | 0.119547 |
| 20 | 6.727500 | 0.148644 | 57.274999 | 0.017460 | 8.513564 | 0.117460 |
| 21 | 7.400250 | 0.135131 | 64.002499 | 0.015624 | 8.648694 | 0.115624 |
| 22 | 8.140275 | 0.122846 | 71.402749 | 0.014005 | 8.771540 | 0.114005 |
| 23 | 8.954302 | 0.111678 | 79.543024 | 0.012572 | 8.883218 | 0.112572 |
| 24 | 9.849733 | 0.101526 | 88.497327 | 0.011300 | 8.984744 | 0.111300 |
| 25 | 10.834706 | 0.092296 | 98.347059 | 0.010168 | 9.077040 | 0.110168 |
| 26 | 11.918177 | 0.083905 | 109.181765 | 0.009159 | 9.160945 | 0.109159 |
| 27 | 13.109994 | 0.076278 | 121.099942 | 0.008258 | 9.237223 | 0.108258 |
| 28 | 14.420994 | 0.069343 | 134.209936 | 0.007451 | 9.306567 | 0.107451 |
| 29 | 15.863093 | 0.063039 | 148.630930 | 0.006728 | 9.369606 | 0.106728 |
| 30 | 17.449402 | 0.057309 | 164.494023 | 0.006079 | 9.426914 | 0.106079 |
| 31 | 19.194342 | 0.052099 | 181.943425 | 0.005496 | 9.479013 | 0.105496 |
| 32 | 21.113777 | 0.047362 | 201.137767 | 0.004972 | 9.526376 | 0.104972 |
| 33 | 23.225154 | 0.043057 | 222.251544 | 0.004499 | 9.569432 | 0.104499 |
| 34 | 25.547670 | 0.039143 | 245.476699 | 0.004074 | 9.608575 | 0.104074 |
| 35 | 28.102437 | 0.035584 | 271.024368 | 0.003690 | 9.644159 | 0.103690 |
| 36 | 30.912681 | 0.032349 | 299.126805 | 0.003343 | 9.676508 | 0.103343 |
| 37 | 34.003949 | 0.029408 | 330.039486 | 0.003030 | 9.705917 | 0.103030 |
| 38 | 37.404343 | 0.026735 | 364.043434 | 0.002747 | 9.732651 | 0.102747 |
| 39 | 41.144778 | 0.024304 | 401.447778 | 0.002491 | 9.756956 | 0.102491 |
| 40 | 45.259256 | 0.022095 | 442.592556 | 0.002259 | 9.779051 | 0.102259 |
| 41 | 49.785181 | 0.020086 | 487.851811 | 0.002050 | 9.799137 | 0.102050 |
| 42 | 54.763699 | 0.018260 | 537.636992 | 0.001860 | 9.817397 | 0.101860 |
| 43 | 60.240069 | 0.016600 | 592.400692 | 0.001688 | 9.833998 | 0.101688 |
| 44 | 66.264076 | 0.015091 | 652.640761 | 0.001532 | 9.849089 | 0.101532 |
| 45 | 72.890484 | 0.013719 | 718.904837 | 0.001391 | 9.862808 | 0.101391 |
| 50 | 117.390853 | 0.008519 | 1163.908529 | 0.000859 | 9.914814 | 0.100859 |
| 55 | 189.059142 | 0.005289 | 1880.591425 | 0.000532 | 9.947106 | 0.100532 |
| 60 | 304.481640 | 0.003284 | 3034.816395 | 0.000330 | 9.967157 | 0.100330 |


| Interest RATE = |  |  | 12\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.120000 | 0.892857 | 1.000000 | 1.000000 | 0.892857 | 1.120000 |
| 2 | 1.254400 | 0.797194 | 2.120000 | 0.471698 | 1.690051 | 0.591698 |
| 3 | 1.404928 | 0.711780 | 3.374400 | 0.296349 | 2.401831 | 0.416349 |
| 4 | 1.573519 | 0.635518 | 4.779328 | 0.209234 | 3.037349 | 0.329234 |
| 5 | 1.762342 | 0.567427 | 6.352847 | 0.157410 | 3.604776 | 0.277410 |
| 6 | 1.973823 | 0.506631 | 8.115189 | 0.123226 | 4.111407 | 0.243226 |
| 7 | 2.210681 | 0.452349 | 10.089012 | 0.099118 | 4.563757 | 0.219118 |
| 8 | 2.475963 | 0.403883 | 12.299693 | 0.081303 | 4.967640 | 0.201303 |
| 9 | 2.773079 | 0.360610 | 14.775656 | 0.067679 | 5.328250 | 0.187679 |
| 10 | 3.105848 | 0.321973 | 17.548735 | 0.056984 | 5.650223 | 0.176984 |
| 11 | 3.478550 | 0.287476 | 20.654583 | 0.048415 | 5.937699 | 0.168415 |
| 12 | 3.895976 | 0.256675 | 24.133133 | 0.041437 | 6.194374 | 0.161437 |
| 13 | 4.363493 | 0.229174 | 28.029109 | 0.035677 | 6.423548 | 0.155677 |
| 14 | 4.887112 | 0.204620 | 32.392602 | 0.030871 | 6.628168 | 0.150871 |
| 15 | 5.473566 | 0.182696 | 37.279715 | 0.026824 | 6.810864 | 0.146824 |
| 16 | 6.130394 | 0.163122 | 42.753280 | 0.023390 | 6.973986 | 0.143390 |
| 17 | 6.866041 | 0.145644 | 48.883674 | 0.020457 | 7.119630 | 0.140457 |
| 18 | 7.689966 | 0.130040 | 55.749715 | 0.017937 | 7.249670 | 0.137937 |
| 19 | 8.612762 | 0.116107 | 63.439681 | 0.015763 | 7.365777 | 0.135763 |
| 20 | 9.646293 | 0.103667 | 72.052442 | 0.013879 | 7.469444 | 0.133879 |
| 21 | 10.803848 | 0.092560 | 81.698736 | 0.012240 | 7.562003 | 0.132240 |
| 22 | 12.100310 | 0.082643 | 92.502584 | 0.010811 | 7.644646 | 0.130811 |
| 23 | 13.552347 | 0.073788 | 104.602894 | 0.009560 | 7.718434 | 0.129560 |
| 24 | 15.178629 | 0.065882 | 118.155241 | 0.008463 | 7.784316 | 0.128463 |
| 25 | 17.000064 | 0.058823 | 133.333870 | 0.007500 | 7.843139 | 0.127500 |
| 26 | 19.040072 | 0.052521 | 150.333934 | 0.006652 | 7.895660 | 0.126652 |
| 27 | 21.324881 | 0.046894 | 169.374007 | 0.005904 | 7.942554 | 0.125904 |
| 28 | 23.883866 | 0.041869 | 190.698887 | 0.005244 | 7.984423 | 0.125244 |
| 29 | 26.749930 | 0.037383 | 214.582754 | 0.004660 | 8.021806 | 0.124660 |
| 30 | 29.959922 | 0.033378 | 241.332684 | 0.004144 | 8.055184 | 0.124144 |
| 31 | 33.555113 | 0.029802 | 271.292606 | 0.003686 | 8.084986 | 0.123686 |
| 32 | 37.581726 | 0.026609 | 304.847719 | 0.003280 | 8.111594 | 0.123280 |
| 33 | 42.091533 | 0.023758 | 342.429446 | 0.002920 | 8.135352 | 0.122920 |
| 34 | 47.142517 | 0.021212 | 384.520979 | 0.002601 | 8.156564 | 0.122601 |
| 35 | 52.799620 | 0.018940 | 431.663496 | 0.002317 | 8.175504 | 0.122317 |
| 36 | 59.135574 | 0.016910 | 484.463116 | 0.002064 | 8.192414 | 0.122064 |
| 37 | 66.231843 | 0.015098 | 543.598690 | 0.001840 | 8.207513 | 0.121840 |
| 38 | 74.179664 | 0.013481 | 609.830533 | 0.001640 | 8.220993 | 0.121640 |
| 39 | 83.081224 | 0.012036 | 684.010197 | 0.001462 | 8.233030 | 0.121462 |
| 40 | 93.050970 | 0.010747 | 767.091420 | 0.001304 | 8.243777 | 0.121304 |
| 41 | 104.217087 | 0.009595 | 860.142391 | 0.001163 | 8.253372 | 0.121163 |
| 42 | 116.723137 | 0.008567 | 964.359478 | 0.001037 | 8.261939 | 0.121037 |
| 43 | 130.729914 | 0.007649 | 1081.082615 | 0.000925 | 8.269589 | 0.120925 |
| 44 | 146.417503 | 0.006830 | 1211.812529 | 0.000825 | 8.276418 | 0.120825 |
| 45 | 163.987604 | 0.006098 | 1358.230032 | 0.000736 | 8.282516 | 0.120736 |
| 50 | 289.002190 | 0.003460 | 2400.018249 | 0.000417 | 8.304498 | 0.120417 |
| 55 | 509.320606 | 0.001963 | 4236.005047 | 0.000236 | 8.316972 | 0.120236 |
| 60 | 897.596933 | 0.001114 | 7471.641112 | 0.000134 | 8.324049 | 0.120134 |


| Interest RATE = |  |  | 15\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMENT |  | UNIFORM SERIES |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.150000 | 0.869565 | 1.000000 | 1.000000 | 0.869565 | 1.150000 |
| 2 | 1.322500 | 0.756144 | 2.150000 | 0.465116 | 1.625709 | 0.615116 |
| 3 | 1.520875 | 0.657516 | 3.472500 | 0.287977 | 2.283225 | 0.437977 |
| 4 | 1.749006 | 0.571753 | 4.993375 | 0.200265 | 2.854978 | 0.350265 |
| 5 | 2.011357 | 0.497177 | 6.742381 | 0.148316 | 3.352155 | 0.298316 |
| 6 | 2.313061 | 0.432328 | 8.753738 | 0.114237 | 3.784483 | 0.264237 |
| 7 | 2.660020 | 0.375937 | 11.066799 | 0.090360 | 4.160420 | 0.240360 |
| 8 | 3.059023 | 0.326902 | 13.726819 | 0.072850 | 4.487322 | 0.222850 |
| 9 | 3.517876 | 0.284262 | 16.785842 | 0.059574 | 4.771584 | 0.209574 |
| 10 | 4.045558 | 0.247185 | 20.303718 | 0.049252 | 5.018769 | 0.199252 |
| 11 | 4.652391 | 0.214943 | 24.349276 | 0.041069 | 5.233712 | 0.191069 |
| 12 | 5.350250 | 0.186907 | 29.001667 | 0.034481 | 5.420619 | 0.184481 |
| 13 | 6.152788 | 0.162528 | 34.351917 | 0.029110 | 5.583147 | 0.179110 |
| 14 | 7.075706 | 0.141329 | 40.504705 | 0.024688 | 5.724476 | 0.174688 |
| 15 | 8.137062 | 0.122894 | 47.580411 | 0.021017 | 5.847370 | 0.171017 |
| 16 | 9.357621 | 0.106865 | 55.717472 | 0.017948 | 5.954235 | 0.167948 |
| 17 | 10.761264 | 0.092926 | 65.075093 | 0.015367 | 6.047161 | 0.165367 |
| 18 | 12.375454 | 0.080805 | 75.836357 | 0.013186 | 6.127966 | 0.163186 |
| 19 | 14.231772 | 0.070265 | 88.211811 | 0.011336 | 6.198231 | 0.161336 |
| 20 | 16.366537 | 0.061100 | 102.443583 | 0.009761 | 6.259331 | 0.159761 |
| 21 | 18.821518 | 0.053131 | 118.810120 | 0.008417 | 6.312462 | 0.158417 |
| 22 | 21.644746 | 0.046201 | 137.631638 | 0.007266 | 6.358663 | 0.157266 |
| 23 | 24.891458 | 0.040174 | 159.276384 | 0.006278 | 6.398837 | 0.156278 |
| 24 | 28.625176 | 0.034934 | 184.167841 | 0.005430 | 6.433771 | 0.155430 |
| 25 | 32.918953 | 0.030378 | 212.793017 | 0.004699 | 6.464149 | 0.154699 |
| 26 | 37.856796 | 0.026415 | 245.711970 | 0.004070 | 6.490564 | 0.154070 |
| 27 | 43.535315 | 0.022970 | 283.568766 | 0.003526 | 6.513534 | 0.153526 |
| 28 | 50.065612 | 0.019974 | 327.104080 | 0.003057 | 6.533508 | 0.153057 |
| 29 | 57.575454 | 0.017369 | 377.169693 | 0.002651 | 6.550877 | 0.152651 |
| 30 | 66.211772 | 0.015103 | 434.745146 | 0.002300 | 6.565980 | 0.152300 |
| 31 | 76.143538 | 0.013133 | 500.956918 | 0.001996 | 6.579113 | 0.151996 |
| 32 | 87.565068 | 0.011420 | 577.100456 | 0.001733 | 6.590533 | 0.151733 |
| 33 | 100.699829 | 0.009931 | 664.665524 | 0.001505 | 6.600463 | 0.151505 |
| 34 | 115.804803 | 0.008635 | 765.365353 | 0.001307 | 6.609099 | 0.151307 |
| 35 | 133.175523 | 0.007509 | 881.170156 | 0.001135 | 6.616607 | 0.151135 |
| 36 | 153.151852 | 0.006529 | 1014.345680 | 0.000986 | 6.623137 | 0.150986 |
| 37 | 176.124630 | 0.005678 | 1167.497532 | 0.000857 | 6.628815 | 0.150857 |
| 38 | 202.543324 | 0.004937 | 1343.622161 | 0.000744 | 6.633752 | 0.150744 |
| 39 | 232.924823 | 0.004293 | 1546.165485 | 0.000647 | 6.638045 | 0.150647 |
| 40 | 267.863546 | 0.003733 | 1779.090308 | 0.000562 | 6.641778 | 0.150562 |
| 41 | 308.043078 | 0.003246 | 2046.953854 | 0.000489 | 6.645025 | 0.150489 |
| 42 | 354.249540 | 0.002823 | 2354.996933 | 0.000425 | 6.647848 | 0.150425 |
| 43 | 407.386971 | 0.002455 | 2709.246473 | 0.000369 | 6.650302 | 0.150369 |
| 44 | 468.495017 | 0.002134 | 3116.633443 | 0.000321 | 6.652437 | 0.150321 |
| 45 | 538.769269 | 0.001856 | 3585.128460 | 0.000279 | 6.654293 | 0.150279 |
| 50 | 1083.657442 | 0.000923 | 7217.716277 | 0.000139 | 6.660515 | 0.150139 |
| 55 | 2179.622184 | 0.000459 | 14524.147893 | 0.000069 | 6.663608 | 0.150069 |
| 60 | 4383.998746 | 0.000228 | 29219.991638 | 0.000034 | 6.665146 | 0.150034 |


| Interest RATE = |  |  | 20\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYME | NT | UNIFORM SERIE |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.200000 | 0.833333 | 1.000000 | 1.000000 | 0.833333 | 1.200000 |
| 2 | 1.440000 | 0.694444 | 2.200000 | 0.454545 | 1.527778 | 0.654545 |
| 3 | 1.728000 | 0.578704 | 3.640000 | 0.274725 | 2.106481 | 0.474725 |
| 4 | 2.073600 | 0.482253 | 5.368000 | 0.186289 | 2.588735 | 0.386289 |
| 5 | 2.488320 | 0.401878 | 7.441600 | 0.134380 | 2.990612 | 0.334380 |
| 6 | 2.985984 | 0.334898 | 9.929920 | 0.100706 | 3.325510 | 0.300706 |
| 7 | 3.583181 | 0.279082 | 12.915904 | 0.077424 | 3.604592 | 0.277424 |
| 8 | 4.299817 | 0.232568 | 16.499085 | 0.060609 | 3.837160 | 0.260609 |
| 9 | 5.159780 | 0.193807 | 20.798902 | 0.048079 | 4.030967 | 0.248079 |
| 10 | 6.191736 | 0.161506 | 25.958682 | 0.038523 | 4.192472 | 0.238523 |
| 11 | 7.430084 | 0.134588 | 32.150419 | 0.031104 | 4.327060 | 0.231104 |
| 12 | 8.916100 | 0.112157 | 39.580502 | 0.025265 | 4.439217 | 0.225265 |
| 13 | 10.699321 | 0.093464 | 48.496603 | 0.020620 | 4.532681 | 0.220620 |
| 14 | 12.839185 | 0.077887 | 59.195923 | 0.016893 | 4.610567 | 0.216893 |
| 15 | 15.407022 | 0.064905 | 72.035108 | 0.013882 | 4.675473 | 0.213882 |
| 16 | 18.488426 | 0.054088 | 87.442129 | 0.011436 | 4.729561 | 0.211436 |
| 17 | 22.186111 | 0.045073 | 105.930555 | 0.009440 | 4.774634 | 0.209440 |
| 18 | 26.623333 | 0.037561 | 128.116666 | 0.007805 | 4.812195 | 0.207805 |
| 19 | 31.948000 | 0.031301 | 154.740000 | 0.006462 | 4.843496 | 0.206462 |
| 20 | 38.337600 | 0.026084 | 186.688000 | 0.005357 | 4.869580 | 0.205357 |
| 21 | 46.005120 | 0.021737 | 225.025600 | 0.004444 | 4.891316 | 0.204444 |
| 22 | 55.206144 | 0.018114 | 271.030719 | 0.003690 | 4.909430 | 0.203690 |
| 23 | 66.247373 | 0.015095 | 326.236863 | 0.003065 | 4.924525 | 0.203065 |
| 24 | 79.496847 | 0.012579 | 392.484236 | 0.002548 | 4.937104 | 0.202548 |
| 25 | 95.396217 | 0.010483 | 471.981083 | 0.002119 | 4.947587 | 0.202119 |
| 26 | 114.475460 | 0.008735 | 567.377300 | 0.001762 | 4.956323 | 0.201762 |
| 27 | 137.370552 | 0.007280 | 681.852760 | 0.001467 | 4.963602 | 0.201467 |
| 28 | 164.844662 | 0.006066 | 819.223312 | 0.001221 | 4.969668 | 0.201221 |
| 29 | 197.813595 | 0.005055 | 984.067974 | 0.001016 | 4.974724 | 0.201016 |
| 30 | 237.376314 | 0.004213 | 1181.881569 | 0.000846 | 4.978936 | 0.200846 |
| 31 | 284.851577 | 0.003511 | 1419.257883 | 0.000705 | 4.982447 | 0.200705 |
| 32 | 341.821892 | 0.002926 | 1704.109459 | 0.000587 | 4.985372 | 0.200587 |
| 33 | 410.186270 | 0.002438 | 2045.931351 | 0.000489 | 4.987810 | 0.200489 |
| 34 | 492.223524 | 0.002032 | 2456.117621 | 0.000407 | 4.989842 | 0.200407 |
| 35 | 590.668229 | 0.001693 | 2948.341146 | 0.000339 | 4.991535 | 0.200339 |
| 36 | 708.801875 | 0.001411 | 3539.009375 | 0.000283 | 4.992946 | 0.200283 |
| 37 | 850.562250 | 0.001176 | 4247.811250 | 0.000235 | 4.994122 | 0.200235 |
| 38 | 1020.674700 | 0.000980 | 5098.373500 | 0.000196 | 4.995101 | 0.200196 |
| 39 | 1224.809640 | 0.000816 | 6119.048200 | 0.000163 | 4.995918 | 0.200163 |
| 40 | 1469.771568 | 0.000680 | 7343.857840 | 0.000136 | 4.996598 | 0.200136 |
| 41 | 1763.725882 | 0.000567 | 8813.629408 | 0.000113 | 4.997165 | 0.200113 |
| 42 | 2116.471058 | 0.000472 | 10577.355289 | 0.000095 | 4.997638 | 0.200095 |
| 43 | 2539.765269 | 0.000394 | 12693.826347 | 0.000079 | 4.998031 | 0.200079 |
| 44 | 3047.718323 | 0.000328 | 15233.591617 | 0.000066 | 4.998359 | 0.200066 |
| 45 | 3657.261988 | 0.000273 | 18281.309940 | 0.000055 | 4.998633 | 0.200055 |
| 50 | 9100.438150 | 0.000110 | 45497.190750 | 0.000022 | 4.999451 | 0.200022 |
| 55 | 22644.802257 | 0.000044 | 113219.011287 | 0.000009 | 4.999779 | 0.200009 |
| 60 | 56347.514353 | 0.000018 | 281732.571766 | 0.000004 | 4.999911 | 0.200004 |


| Interest RATE = |  |  | 25\% |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Years, n | SINGLE PAYMEN |  | UNIFORM SERIE |  |  |  |
|  | CAF | PWF | SCAF | SFF | SPWF | CRF |
|  | F/P | P/F | F/A | A/F | P/A | A/P |
| 1 | 1.250000 | 0.800000 | 1.000000 | 1.000000 | 0.800000 | 1.250000 |
| 2 | 1.562500 | 0.640000 | 2.250000 | 0.444444 | 1.440000 | 0.694444 |
| 3 | 1.953125 | 0.512000 | 3.812500 | 0.262295 | 1.952000 | 0.512295 |
| 4 | 2.441406 | 0.409600 | 5.765625 | 0.173442 | 2.361600 | 0.423442 |
| 5 | 3.051758 | 0.327680 | 8.207031 | 0.121847 | 2.689280 | 0.371847 |
| 6 | 3.814697 | 0.262144 | 11.258789 | 0.088819 | 2.951424 | 0.338819 |
| 7 | 4.768372 | 0.209715 | 15.073486 | 0.066342 | 3.161139 | 0.316342 |
| 8 | 5.960464 | 0.167772 | 19.841858 | 0.050399 | 3.328911 | 0.300399 |
| 9 | 7.450581 | 0.134218 | 25.802322 | 0.038756 | 3.463129 | 0.288756 |
| 10 | 9.313226 | 0.107374 | 33.252903 | 0.030073 | 3.570503 | 0.280073 |
| 11 | 11.641532 | 0.085899 | 42.566129 | 0.023493 | 3.656403 | 0.273493 |
| 12 | 14.551915 | 0.068719 | 54.207661 | 0.018448 | 3.725122 | 0.268448 |
| 13 | 18.189894 | 0.054976 | 68.759576 | 0.014543 | 3.780098 | 0.264543 |
| 14 | 22.737368 | 0.043980 | 86.949470 | 0.011501 | 3.824078 | 0.261501 |
| 15 | 28.421709 | 0.035184 | 109.686838 | 0.009117 | 3.859263 | 0.259117 |
| 16 | 35.527137 | 0.028147 | 138.108547 | 0.007241 | 3.887410 | 0.257241 |
| 17 | 44.408921 | 0.022518 | 173.635684 | 0.005759 | 3.909928 | 0.255759 |
| 18 | 55.511151 | 0.018014 | 218.044605 | 0.004586 | 3.927942 | 0.254586 |
| 19 | 69.388939 | 0.014412 | 273.555756 | 0.003656 | 3.942354 | 0.253656 |
| 20 | 86.736174 | 0.011529 | 342.944695 | 0.002916 | 3.953883 | 0.252916 |
| 21 | 108.420217 | 0.009223 | 429.680869 | 0.002327 | 3.963107 | 0.252327 |
| 22 | 135.525272 | 0.007379 | 538.101086 | 0.001858 | 3.970485 | 0.251858 |
| 23 | 169.406589 | 0.005903 | 673.626358 | 0.001485 | 3.976388 | 0.251485 |
| 24 | 211.758237 | 0.004722 | 843.032947 | 0.001186 | 3.981111 | 0.251186 |
| 25 | 264.697796 | 0.003778 | 1054.791184 | 0.000948 | 3.984888 | 0.250948 |
| 26 | 330.872245 | 0.003022 | 1319.488980 | 0.000758 | 3.987911 | 0.250758 |
| 27 | 413.590306 | 0.002418 | 1650.361225 | 0.000606 | 3.990329 | 0.250606 |
| 28 | 516.987883 | 0.001934 | 2063.951531 | 0.000485 | 3.992263 | 0.250485 |
| 29 | 646.234854 | 0.001547 | 2580.939414 | 0.000387 | 3.993810 | 0.250387 |
| 30 | 807.793567 | 0.001238 | 3227.174268 | 0.000310 | 3.995048 | 0.250310 |
| 31 | 1009.741959 | 0.000990 | 4034.967835 | 0.000248 | 3.996039 | 0.250248 |
| 32 | 1262.177448 | 0.000792 | 5044.709793 | 0.000198 | 3.996831 | 0.250198 |
| 33 | 1577.721810 | 0.000634 | 6306.887242 | 0.000159 | 3.997465 | 0.250159 |
| 34 | 1972.152263 | 0.000507 | 7884.609052 | 0.000127 | 3.997972 | 0.250127 |
| 35 | 2465.190329 | 0.000406 | 9856.761315 | 0.000101 | 3.998377 | 0.250101 |
| 36 | 3081.487911 | 0.000325 | 12321.951644 | 0.000081 | 3.998702 | 0.250081 |
| 37 | 3851.859889 | 0.000260 | 15403.439555 | 0.000065 | 3.998962 | 0.250065 |
| 38 | 4814.824861 | 0.000208 | 19255.299444 | 0.000052 | 3.999169 | 0.250052 |
| 39 | 6018.531076 | 0.000166 | 24070.124305 | 0.000042 | 3.999335 | 0.250042 |
| 40 | 7523.163845 | 0.000133 | 30088.655381 | 0.000033 | 3.999468 | 0.250033 |
| 41 | 9403.954807 | 0.000106 | 37611.819226 | 0.000027 | 3.999575 | 0.250027 |
| 42 | 11754.943508 | 0.000085 | 47015.774033 | 0.000021 | 3.999660 | 0.250021 |
| 43 | 14693.679385 | 0.000068 | 58770.717541 | 0.000017 | 3.999728 | 0.250017 |
| 44 | 18367.099232 | 0.000054 | 73464.396926 | 0.000014 | 3.999782 | 0.250014 |
| 45 | 22958.874039 | 0.000044 | 91831.496158 | 0.000011 | 3.999826 | 0.250011 |
| 50 | 70064.923216 | 0.000014 | 280255.692865 | 0.000004 | 3.999943 | 0.250004 |
| 55 | 213821.176807 | 0.000005 | 855280.707230 | 0.000001 | 3.999981 | 0.250001 |
| 60 | 652530.446800 | 0.000002 | 2610117.787199 | 0.000000 | 3.999994 | 0.250000 |

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[^0]:    ALTERNATIVES SHOULD BE DEFINED, INSOFAR AS POSSIBLE, SUCH THAT THE CURRENT TRAFFIC FOR EACH ALTERNATIVE IS THE SAME.

