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Nonparametric Statistics for Applied Research

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Preface

I have been working as an applied psychologist for many years, and there are a few things that have consistently stood out, for me at least, in the course of my experiences. Possibly the single, most constant "truth" is that human behavior is messy. It's messy in all sorts of interesting ways, and most of the time, people's messiness also messes with any type of inference you can make about their behavior. So, people may not behave, as a group, in a normally distributed fashion, or as a "single humped camel," as the authors say in this book.

In fact, applied research is messy. For example, take how you get participants. You put out feelers, such as links on various websites; you advertise you need participants for a study on whatever it is you happen to be studying. The individual decides to respond or not—as the researcher, you pretty much have to take who you can get. You also don't always have the opportunity to use measurements that you'd like. So, you may be reduced to asking yes/no questions, simply because you cannot pass an ethics board, people wouldn't answer the questions you really want to ask or both.

And, of course, when you're dealing with messy behavior, there isn't always a nice, tidy way of determining whether you've found anything significant. That's right; I'm talking about parametric statistics. In the real world, the parameters are so often violated that you need to find another way.

To this end, nonparametric statistics offer a delightful smorgasbord of alternatives from which to sample. No matter how sloppy, no matter how imprecise, and no matter how ad hoc the behavioral measurement, nonparametric statistics promise some light at the end of the tunnel, a way to assess whether your findings are potentially pointing to something significant.

While there are a number of textbooks on nonparametric statistics, none of them offers what this book does. This book is unique in a number of ways. For one, the text provides a context for statistical questions: there are applied problems that drive the analyses, and the problems are linked to each other so that the reader gets a real appreciation of how applied science works. The data set used by the book is consistent, too. What this means is that the reader is allowed to become familiar, and confident, with one set of numbers, rather than changing each data set with a new statistical test (the traditional statistics book approach). Also unusual and highly valuable is the decision tree for tests of differences and of association. I am convinced that these trees will facilitate the problem solving process for students of psychology as well as seasoned researchers.

The book also departs from the standard in that it provides the reader with a narrative of real people, doing real things and interacting with each other in real ways. The issues are real, the consequences serious. The reader is introduced to a context in which statistics get applied, and as a consequence, the rationale for using a test is grounded in an understandable example. This is in stark contrast to the standard, abstract, detached examples normally provided in statistics books.

I am most fortunate to have known these three authors for a few years now. I have worked with them all on many projects and have had the good fortune to sit for many hours, discussing all manner of things with them. They have produced a book that will not only educate you but also give you a good read.

Bon Appétit!

Debra Bekerian

Acknowledgments

We would like to take the opportunity to express our gratitude to the many people who have helped make this book possible.

We would like to thank all of our family, friends, and loved ones who patiently supported us as we worked on this book. Their love and support helped us to make this possible and for that we are forever grateful.

We would also like to thank Kristin Rodgers, MLIS, Collects Curator, The Medial Heritage Center of the Health Sciences Library at the Ohio State University, Columbus, Ohio, for assistance with statistical tables and permissions and Dan Bell, Ph.D., Associate Professor of Mathematics, School of Arts and Sciences, Tiffin University, Tiffin, Ohio, for advice and support.

Finally, we would like to thank Marc Strauss, Hannah Bracken, and the editorial staff at Springer Science and Business Media for their guidance and expertise. This book would not have been possible without their belief in our work.

Contents

Chapter 1
Introduction

Abstract In this chapter, a basic conceptualization of parametric and nonparametric statistical usage is presented as well as a basic layout of the text. Two decision trees are introduced which provide a framework from which the rest of the text will flow. The decision trees are considered in great detail with specific attention to the questions presented in the trees. These questions help direct the researcher toward a specific test appropriate for the kind of data that exists in the study. Such topics as significance, ranked data, magnitude, cumulative data, dichotomous data, related and unrelated samples, independent and dependent variables, and covariates are discussed.

The goal of this text is to provide readers with an applied understanding of nonparametric statistical procedures. The authors have taken great care to arrange the book in such a way that is helpful and straightforward when considering the issue of choosing a statistical procedure for research. This is not a typical statistics textbook. Several changes to the structure and format have been made to facilitate the goal of the text.

Chapters in this text are vastly different from chapters in other statistical texts. This book does not assume that the reader is sufficiently familiar with all statistical procedures or that he or she could turn to a specific test and know immediately whether or not to use that test for his or her research. In this book, chapters are laid out based upon a research question. Contrary to traditional texts, the statistical tests are then presented in terms of the research question.

This book presents the reader with a real-world scenario, introduced in Chap. 2 and carried throughout the entire book, where a multidisciplinary team of behavioral, medical, crime analysis, and policy analysis professionals work together to answer specific empirical questions regarding real-world applied problems. The reader is introduced to the team and the data set and follows the team as they progress through the decision-making process of narrowing the data and the

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research questions to answer the applied problem. In this way, abstract statistical concepts are translated into concrete and specific language. Throughout the book, the reader will notice certain terms in boldface type and others italicized. The boldface type identifies the first occurrence of specific statistical terms which can be found in the glossary at the end of the book. Each subsequent occurrence of the glossary terms can be identified by the italicized type.

The chapters reflect three general categories: **Violation**, **Association,** and **Difference**. *Violation* tests are discussed in Chap. 3. *Association* tests are discussed in Chaps. 4–6, while *Difference* tests are discussed in Chaps. 7–12. These three categories form the basis for almost any statistical test that can be used. This book highlights those tests where the data do not conform to the assumptions for common parametric tests.

This text uses one data set from which all examples are taken. This is radically different from other statistics textbooks which provide a varied array of examples and data sets. Using only one data set facilitates teaching and learning by providing multiple research questions that are integrated rather than using disparate examples and completely unrelated research questions and data. Clear and succinct summaries will be presented at the beginning and end of each chapter. A set of conceptual and practical questions will be provided at the end of each chapter which will serve to facilitate teaching and learning and provide additional practice where understanding may be shallow.

Before one can venture through the analyses considered in this text, he or she must first understand what kind of data he or she has. A deeper analysis of this concept will be discussed in Chap. 2, but here the reader must decide whether he or she has recurring themes and patterns over a narrative (qualitative data) or data which uses numbers that denote meaning (quantitative data). If the researcher has qualitative data, this text will not address that kind of analysis. If, however, the researcher has quantitative data, the researcher may continue to examine his or her data to determine whether parametric tests or nonparametric tests are appropriate.

Some types of data lend themselves to certain types of research questions. Some of those research questions help the researcher decide whether parametric tests can be used or nonparametric tests need to be used. The *Violation* tests covered in Chap. 3 consist of nonparametric statistics that allow a researcher to test the **Parameters**, or assumptions, for the usage of the parametric tests which are more widely taught in many locations. Despite these parametric tests being more widely taught, oftentimes at least one of the following assumptions is violated causing an issue when it comes to the usefulness of the test. The following five main *Parameters* for parametric tests are needed and will be considered in greater detail in Chap. 2:

- Randomly sampled data
- Independent sampling
- At least interval data
- Homogeneity of variance
- Normally distributed data

Violation tests are so named because they allow a researcher to test the *Violation* of some of the above assumptions for parametric tests. Once a researcher realizes, by using *Violation* tests, that he or she cannot use a parametric test, the other two categories of tests contain the possible options for statistical analysis.

In addition to assisting the reader with understanding the nature of a test based upon the corresponding research question, two decision trees have also been constructed to provide an "at a glance" determination of the most appropriate nonparametric statistical test. The two decision trees presented here are termed the **Association Decision Tree** and the **Difference Decision Tree** for nonparametric statistics. The tests found in the *Association Decision Tree* will result in an *Association*, and the tests found in the *Difference Decision Tree* will result in a *Difference* between the specified **Variables**. In order to get to the decision trees, the reader must first make a determination about whether he or she is studying an *Association* between *Variables* or a *Difference* between *Variables*. The reader can proceed to the appropriate decision tree once that determination is made.

The decision trees are separated based upon the type of research question that is being asked and subsequently the type of test that will answer that research question. The research questions and tests fall into *Association* and *Difference* tests. The A*ssociation* tests assess similarities between the *Variables* involved in the analysis. Some A*ssociation* tests assess simply whether or not *Variables* are similar or related, while others assess how similar or related those *Variables* are. *Difference* tests assess differences between the *Variables* involved in the analysis. These differences can be small or they can be large. When a difference is large, it is said to be significant. A **Statistical Significance** (or **Probability Level**) is a statistical term for the likelihood that an event will occur. If, based on **Probability**, it is highly unlikely that an event will occur and that event occurs anyway, it is said to be statistically significant. Significance indicates how sure the researcher can be that an association or a difference actually exists.

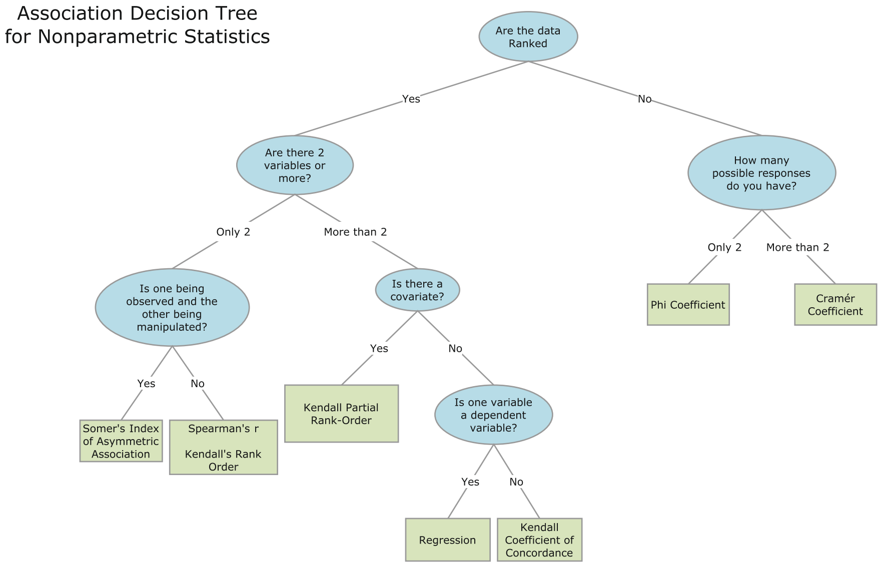
Association Decision Tree for Nonparametric Statistics

In order to use the decision tree for A*ssociation* tests, several concepts must be covered. The first question in the A*ssociation* tree asks about **Ranked Data**. In order to have *Ranked Data*, the original numbers collected must be transformed into the corresponding position when the numbers are sorted from smallest to largest. For example, suppose that ages are collected for 5 people in a class. Those ages are 21, 27, 19, 20, and 23. The corresponding position or ranking would be 3, 5, 1, 2, and 4.

Age	Rank
21	3
27	5
19	1
20	2
23	4

The second question presented in the A*ssociation* tree asks about the number of *Variables* present in the research. When there are only 2 *Variables* in the research, the next question asks about observed and manipulated data. Observed data is that which a researcher has no control over. The researcher is merely observing what takes place. The observed data is called the **Dependent Variable**. The *Dependent Variable* is often thought of as the *Dependent* measure because the researcher can only measure the results and cannot exhibit any control over that result. Manipulated data is that over which a researcher has control. The manipulated data are within the researcher's field of control. The manipulated data are called **Independent Variables** because the *Variables* are independent of the **Experiment,** and therefore, the researcher is able to control for those *Variables*.

When there are more than 2 *Variables* in the research, the first question that must be asked is whether or not a **Covariate** exists. A *Covariate* is a variable that the researcher believes plays a part in the observed effect but wants to hold that variable out of the mathematical equation to test his or her theory. After it is determined that no *Covariate* exists in the *Experiment*, it must be determined if there is a variable that is only being observed in the *Experiment* and not manipulated. The last question deals with the presence of a *Dependent variable* that must be factored into the equation.



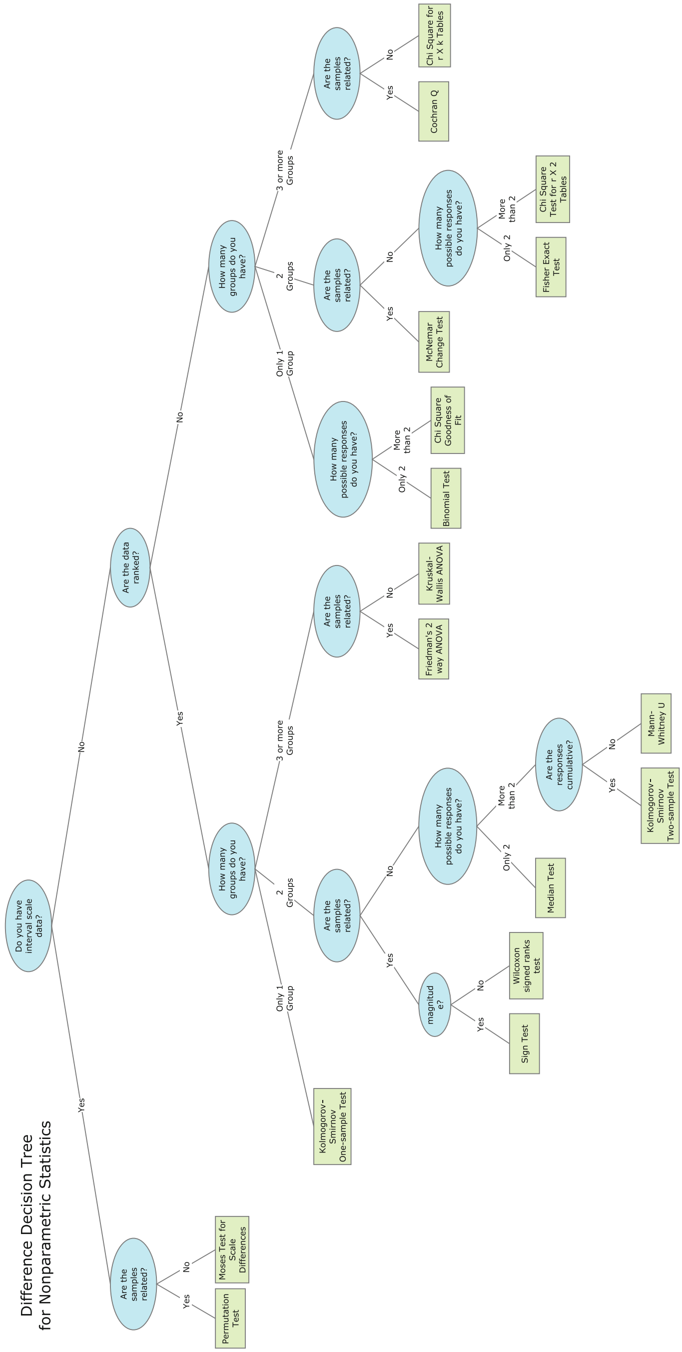
Difference Decision Tree for Nonparametric Statistics

The first question that is considered in the **Difference Decision Tree** for nonparametric statistics is whether or not at least interval scale data is present in the study. At least interval scale means that those data that are either interval, i.e., temperatures in Fahrenheit from freezing to boiling, 32° to 212°, or are ratio, i.e., distance from one object to another, 0–100 miles, are appropriate for some *Difference Tests*. In order words, ratio and interval scale data are appropriate for the "at least interval" requirement.

Ratio
Interval
Ordinal
Nominal

If one has data that is at least interval scale, then the researcher needs to establish whether the **Groups** in the data are **Related** or **Unrelated**. If the samples are *Related*, it means that the numbers in the data set were taken from the same individual; or the numbers were taken from two different individuals who were matched together based upon certain factors. One individual providing the data for an analysis is a *Related Sample* because the participant is obviously related to himself or herself. On the other hand, two individuals matched on certain factors are related because they are related or matched on some dimension. For example, two individuals may be matched based upon their age, sex, ethnicity, and occupation making them more similar than different. This, then, makes them *Related*.

Unrelated Samples are those samples where the information was not collected from the same individual or was collected from individuals who were not matched on any dimensions or factors. This means that *Unrelated Samples* are those where a researcher collects information from one *Group* of people and then visits a completely separate *Group* of people and collects the same information. The two *Groups* of people could be, for example, college students and nursing home residents. They are obviously two completely separate *Groups* of people and are not matched on any factors, thus, making them *Unrelated Samples*.



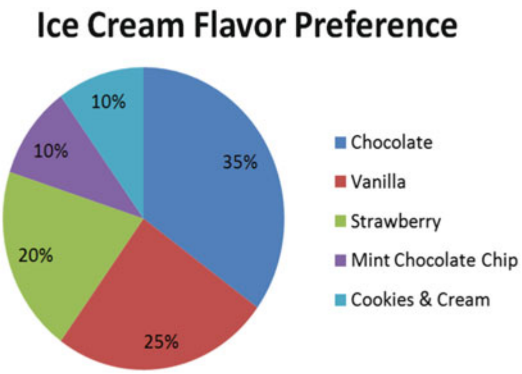
Difference Decision Tree for Nonparametric Statistics
Do you have interval scale data?
Yes
No
Are the samples related?
Yes
No
Permutation Test
Moses Test for Scale Differences
Are the data ranked?
Yes
No
How many groups do you have?
Only 1 Group
2 Groups
3 or more Groups
Kolmogorov-Smirnov One-sample Test
Are the samples related?
Yes
No
magnitude?
Yes
No
Sign Test
Wilcoxon signed ranks test
How many possible responses do you have?
Only 2
More than 2
Median Test
Are the responses cumulative?
Yes
No
Kolmogorov-Smirnov Two-sample Test
Mann-Whitney U
Are the samples related?
Yes
No
Friedman's 2 way ANOVA
Kruskal-Wallis ANOVA
How many groups do you have?
Only 1 Group
2 Groups
3 or more Groups
How many possible responses do you have?
Only 2
More than 2
Binomial Test
Chi Square Goodness of Fit
Are the samples related?
Yes
No
McNemar Change Test
How many possible responses do you have?
Only 2
More than 2
Fisher Exact Test
Chi Square Test for r X 2 Tables
Are the samples related?
Yes
No
Cochran Q
Chi Square for r X k Tables

All other tests in the *Difference Decision Tree* require data that are nominal, i.e., discrete categories, or ordinal, i.e., ranked ages. For these tests in the *Difference Decision Tree*, the next question is whether the data are *Ranked* or not *Ranked*. If the data are ranked, possible tests include the Sign Test and Kruskal–Wallis ANOVA. The third question takes into consideration how many *Groups* are being assessed. In research, one *Group* is often thought of as the **Control Group**. Being designated as the *Control Group* usually means that the researcher does not introduce any manipulation, so that they serve as a baseline for the other *Group(s)* which has some manipulation introduced. For example, suppose a researcher is interested in how effective different treatments are for sexual offenders. The researcher might include one *Group* where the participants receive no treatment (*Control*), another group that receives medication for treatment (treatment *Group*), and a third group that receives both medication and therapy (mixed treatment *Group*). This example has three groups to compare.

In the *Difference* tree, the decision maker is again asked to determine whether the samples are related or unrelated. Since *Related* and *Unrelated Samples* were covered earlier, no additional discussion here is required. The next question encountered inquires about the number of possible responses the participants of the study have provided. If the participants have only two options when answering a question, the two responses are considered to be **Dichotomous**, for example—male/female, yes/no, and compliant/in violation.

When more than two responses are possible, the data are considered to be continuous even though the term continuous is a bit misleading. While **Continuous Data** can be numerical in nature, not all continuous data *are* numerical. Continuous, in this sense, may include a question where the possible responses are ethnicities which are clearly not numerical, but they are also clearly not *Dichotomous*. *Continuous Data* can also refer to **Cumulative Response Data**. *Cumulative Responses* are a variety of responses where the relative frequency as expressed as a percentage adds up to equal 100 %. A pie chart easily illustrates this point: 100 people are asked one question about which ice cream flavor they prefer. The information is presented below for quick reference:

Flavor	Number of people who prefer that flavor
Chocolate	35
Vanilla	25
Strawberry	20
Mint chocolate chip	10
Cookies and cream	10



Most of the tests identified in the *Difference* decision tree are concerned with a significant difference, but there are some that are interested in magnitude. **Magnitude** describes how large that significance actually is. *Magnitude* is a great tool for when a researcher is not content knowing that there is a difference so he or she wants to know how much bigger that difference is.

Sample Sizes in statistics are usually a very sensitive and important thing. However, with the utilization of nonparametric tests, sample size can be as small of 2 participants for some of the tests. Sample sizes can be thought of as small (1–15 participants), medium (16–39 participants), and large (40+ participants), although some tests have specific sample size requirements in order to be considered large or small. One example is the Sign Test where a small sample is considered to be less than 35 and a large sample is more than 35 participants. For nonparametric statistics, a small sample size is alright. In contrast, parametric tests all need large sample sizes of 40 or more **Data Points**. This means that if a researcher has fewer than 40 *Data Points*, nonparametric tests are the most appropriate for the research.

Chapter Summary

- A basic conceptualization of parametric and nonparametric statistical usage was presented.
- A basic layout of the text was presented.
- Two decision trees were introduced to provide a framework from which the rest of the text will flow.
- The decision trees were considered in great detail with specific attention to the questions presented in the trees. These questions help direct the researcher toward a specific test appropriate for the kind of data that exists in the study.
- Such topics as significance, ranked data, magnitude, cumulative data, dichotomous data, related and unrelated samples, independent and dependent variables, and covariates were discussed.

Check Your Understanding

1. Two variables are being assessed for their similarity to one another. Is this a question of difference or association?
2. Four variables are being looked at to determine which predicts a fifth variable the best. Is this a question of difference or association?
3. Three groups are being used to determine how different one is compared to the other two. Is this a question of difference or association?
4. Identify two examples of qualitative data and two examples of quantitative data.

5. Rank the following data:

Participant	Income	Rank
1	$25,500	
2	$16,000	
3	$29,900	
4	$9,900	
5	$59,000	
Participant	**Age**	**Rank**
1	18	
2	26	
3	19	
4	39	
5	21	
Participant	**Weight**	**Rank**
1	157	
2	235	
3	190	
4	143	
5	145	
Participant	**Grade**	**Rank**
1	F	
2	C	
3	B	
4	A	
5	D	

6. A variable that is believed to impact other variables and is, therefore, statistically held constant is called ________________.

   a. Ranked data
   b. A covariate
   c. Qualitative
   d. Quantitative

7. Describe the similarities and differences between related and unrelated samples. Provide an example of each.

Chapter 2
Meeting the Team

Abstract In this chapter, we want to introduce you to the group of individuals you will be following throughout the remainder of this text. The following story will also start introducing statistical terms and concepts that will help you to answer research questions using nonparametric statistical tests. The Data Set which will be utilized throughout the book will be introduced and briefly explained. These concepts are further explained in the glossary at the end of the text.

Governor Nathanial Greenleaf, a successful governor for the State of California over the past 7 years, is looking for a way to further his political career now that his final term as governor is coming to an end. Recently, one of the US Senators for the State of California has announced that he will not be seeking reelection to the Senate. Governor Greenleaf is viewing this as the perfect opportunity to continue on in politics and has begun a campaign to secure the nomination for the election next year. His main opponent in the primary election is Grayson Devins, the former mayor of San Francisco, who has also proven to be very popular among California voters. Given how close these two are in the polls, Governor Greenleaf decided to meet with his campaign committee to discuss some possible election platforms.

One campaign worker suggests that Governor Greenleaf run on a platform of strengthening California's sex offender laws. Over the past year, there had been several high-profile incidents of child molestation by individuals known to be registered as sex offenders; incidents which have garnered an intense amount of media scrutiny. One particular sex offender, known only as the "Midnight Rapist," has targeted several wealthy women who reside throughout the State of California. Governor Greenleaf believes that this is a wonderful political platform and charges his Campaign Manager, Jennifer Parsons, with creating clearly delineated policy solutions that he can then take to the people of California as a major component of his election campaign.

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Jennifer Parsons asked some of the campaign workers to collect data concerning registered sex offenders in the State of California. So, three campaign workers looked up the zip code for the campaign headquarters on the sex offender registry website and selected all of the registered sex offenders within a 20 mile radius of the building. Then, the campaign workers went out and surveyed the sex offenders on such topics as whether or not they were in compliance and whether or not they were taking any medications, their sex, their age, etc. Out of 100 sex offenders surveyed, the results are as follows:

Participant	Age	Sex	Ethnicity/Race	Offense[a] (Count One)	Sentence (In Years)	Living Location	30 Days Post Release Status[b]	90 Days Post Release Status[b]	Current Status[b]	Estimated Yearly Income	Level of Meanness Rater 1	Rater 2	Rater 3	General Aggression Score (GAS) (20 to 80)	Currently Taking Medication	Total Testosterone Level (ng/dl)[c]	Type of Medication	Hormonal Treatment	Treatment[d] Testosterone Level (ng/dl)[c]
1	62	Male	Black	261(4)	7	Incorporated Area	0	1	1	$26,104	4	5	5	65	Yes	710	Mood Stabilizer	No	
2	83	Male	White	288(A)	8	Incorporated Area	0		0	$5,274	4	4	4	59	Yes	687	Antidepressant (SSRI); Antiandrogen	Yes	20
3	33	Female	White	288(A)	3	Incorporated Area	0		1	$19,648	2	3	2	41	Yes	51	Antidepressant	No	
4	77	Male	White	243.4	1	Incorporated Area	0		0	$8,762	1	1	1	43	No	307			
5	19	Male	Black	288(C)	1	Incorporated Area	0		0	$738	2	2	2	53	No	480			
6	37	Male	Hispanic	288(A)	3	Incorporated Area	0		1	$22,933	1	1	1	49	Unknown	579			
7	74	Male	White	288.5	12	Incorporated Area	0		0	$16,756	4	3	4	54	Yes	464	Antidepressant (SSRI); Antiandrogen	Yes	17
8	32	Male	White	288(A)	3	Transient	0		0	$0	2	2	2	58	No	552			
9	72	Male	Hispanic	288(A)	3	Incorporated Area	0		0	$9,870	3	3	3	47	Yes	387	Antidepressant	No	
10	37	Male	Black	288(A)	6	Unincorporated Area	0		0	$8,593	3	3	3	52	Yes	623	Mood Stabilizer	No	
11	39	Male	White	261(2)	5	Incorporated Area	0		1	$17,076	4	4	4	58	No	892			
12	45	Male	Hispanic	288(A)	3	Incorporated Area	0	0	0	$29,512	4	4	3	58	Yes	711	Antidepressant (SSRI); Antiandrogen	Yes	
13	45	Male	Hispanic	288(A)	3	Incorporated Area	0		0	$27,949	2	2	2	45	No	516			
14	20	Female	Black	266	1	Transient	1		1	$0	2	2	2	59	Unknown	37			
15	64	Male	Hispanic	288(C)	1	Transient	1		0	$0	3	3	3	45	No	495			
16	64	Male	Asian	220	4	Transient	0		1	$0	5	5	5	75	No	607			
17	28	Male	Hispanic	288(A)	3	Incorporated Area	0		0	$3,923	1	1	1	53	No	538			
18	52	Male	Hispanic	647.6(a) (1)	1	Incorporated Area	0		0	$20,509	1	2	1	40	No	557			
19	36	Male	Black	647.6(a) (1)	1	Incorporated Area	0		0	$32,706	2	2	2	41	Yes	669	Antidepressant	No	
20	84	Male	Hispanic	243.4	2	Incorporated Area	0		1	$16,871	4	4	4	60	No	650			
21	73	Female	Hispanic	288A(B) (1)	1	Incorporated Area	1	0	0	$29,811	2	2	2	48	No	42			
22	22	Male	White	288(A)	3	Incorporated Area	0		0	$4,037	1	2	1	59	No	695			
23	64	Male	Hispanic	288(B)	8	Transient	0		0	$0	5	5	5	78	Unknown	757			
24	37	Male	Hispanic	288(C)	1	Transient	0		0	$12,462	1	1	2	53	No	862			
25	58	Male	White	261(2)	5	Transient	0		1	$0	5	4	4	64	Yes	491	Mood Stabilizer	No	
26	71	Male	Black	261(2)	5	Incorporated Area	0		0	$8,566	4	4	4	60	No	515			
27	48	Male	Hispanic	288(A)	6	Incorporated Area	1		1	$16,085	2	1	1	50	No	614			
28	73	Male	Hispanic	288(A)	3	Transient	1		0	$2,946	1	1	1	47	Unknown	570			
29	71	Male	Asian	264.1	5	Incorporated Area	0		0	$31,490	5	5	5	73	No	586			
30	54	Female	White	288(A)	3	Incorporated Area	0		0	$16,206	3	3	3	62	No	60			
31	47	Male	Black	261(2)	9	Transient	0	1	1	$0	4	5	4	65	Unknown	439			
32	58	Male	Asian	243.4	2	Incorporated Area	0		0	$22,078	3	2	2	43	No	530			
33	28	Male	Hispanic	288(A)	3	Transient	0	0	0	$0	3	3	3	54	No	467			
34	68	Male	Hispanic	288(A)	3	Transient	1	1	1	$0	1	3	1	52	No	506			
35	66	Female	Hispanic	261(2)/ 264.1	5	Incorporated Area	0		0	$24,013	5	5	5	74	Yes	67	Anxiolytics	No	
36	56	Male	Black	261(2)	5	Incorporated Area	0		1	$14,703	3	3	3	47	Unknown	664			
37	44	Male	Hispanic	288(A)	8	Transient	0		0	$0	4	5	5	68	Yes	583	Mood Stabilizer	No	
38	47	Male	Black	243.4	1	Unincorporated Area	0		0	$19,936	2	2	2	43	No	525			
39	51	Male	White	647.6(a) (1)	1	Transient	0		0	$0	3	1	1	50	Yes	556	Antidepressant (SSRI); Antiandrogen	Yes	23
40	40	Male	Hispanic	261(2)	5	Transient	0		0	$7,823	4	4	4	60	Yes	734	Anxiolytics	No	
41	71	Male	Hispanic	220	2	Incorporated Area	0		1	$5,238	3	3	1	43	Yes	587	Mood Stabilizer	No	
42	57	Female	White	288(A)	3	Incorporated Area	0		1	$17,762	2	2	2	42	No	62			
43	77	Male	Hispanic	288A(C)	3	Transient	1	1	0	$0	2	2	4	44	Yes	547	Mood Stabilizer	No	
44	66	Male	Hispanic	647.6(a) (1)	1	Incorporated Area	1		1	$23,503	1	1	1	44	No	629			
45	48	Male	Black	288(A)	3	Incorporated Area	0	0	0	$26,353	2	2	3	42	No	686			
46	38	Male	Hispanic	243.4	3	Incorporated Area	0		1	$19,492	2	2	2	53	Unknown	874			
47	51	Male	Asian	288A(C)	6	Transient	0		0	$0	4	5	4	65	No	490			
48	46	Male	Hispanic	220	2	Incorporated Area	1		0	$33,426	2	2	2	48	No	622			
49	50	Male	Hispanic	288A(C)	3	Incorporated Area	0		1	$10,886	3	2	2	43	No	492			
50	49	Male	Asian	288(C)	1	Incorporated Area	0		1	$21,934	1	1	1	42	No	799			
51	22	Male	Asian	288(A)	3	Incorporated Area	0		0	$11,232	1	1	1	42	No	539			
52	37	Male	Hispanic	288(A)	3	Transient	1		1	$0	3	3	3	63	Unknown	810			
53	73	Male	White	288(A)	6	Incorporated Area	0		0	$12,184	4	4	4	60	Yes	477	Antidepressant (SSRI); Antiandrogen	Yes	13
54	36	Male	Black	288(A)	3	Incorporated Area	0	0	0	$7,612	2	2	2	53	Yes	864	Antidepressant	No	
55	63	Male	Hispanic	220	6	Incorporated Area	0		1	$24,698	5	5	5	76	Yes	780	Antidepressant (SSRI); Antiandrogen	Yes	25

56	46	Male	Hispanic	288(A)	3	Incorporated Area	0		0	$20,268	2	2	1	40	Yes	524	Antidepressant (SSRI); Antiandrogen	Yes	14
57	29	Male	Hispanic	288(A)	3	Incorporated Area	0		1	$5,764	3	3	3	55	No	457			
58	82	Male	Hispanic	288(A)	3	Incorporated Area	0		0	$2,726	1	2	2	51	Yes	637	Antidepressant	No	
59	56	Male	Hispanic	288(A)	3	Transient	0		1	$0	3	3	3	46	Yes	458	Mood Stabilizer	No	
60	51	Female	White	288(C)	3	Transient	1		1	$3,546	1	1	1	56	No	31			
61	61	Male	Hispanic	288(A)	6	Incorporated Area	0		1	$31,783	2	2	2	43	No	669			
62	34	Male	Black	647.6(a) (1)	1	Transient	0	1	1	$0	3	3	2	43	Yes	747	Mood Stabilizer	No	
63	82	Male	Black	288(C)	1	Incorporated Area	0		0	$29,075	1	1	1	55	No	413			
64	53	Male	Hispanic	288(A)	3	Unincorporated Area	0		0	$21,518	2	2	2	56	Yes	662	Mood Stabilizer	No	
65	56	Male	Asian	288.3	3	Incorporated Area	0		1	$34,678	3	3	3	47	Yes	595	Anxiolytics	No	
66	66	Male	Black	647.6(a) (1)	1	Incorporated Area	0		1	$6,826	4	4	4	59	Yes	667	Antidepressant (SSRI); Antiandrogen	Yes	18
67	47	Male	Hispanic	288(C)	1	Incorporated Area	1		1	$12,427	1	2	2	44	Unknown	642			
68	78	Male	Hispanic	311.3(B)	1	Incorporated Area	0		0	$23,732	1	1	1	42	No	446			
69	41	Male	White	288(C)	2	Incorporated Area	0		0	$10,832	1	1	1	51	Yes	804	Antidepressant (SSRI); Antiandrogen	Yes	
70	67	Male	Hispanic	243.4	1	Incorporated Area	0		0	$10,704	4	4	4	67	Yes	699	Antidepressant (SSRI); Antiandrogen	Yes	
71	32	Male	White	261(2)	7	Incorporated Area	0		1	$10,218	5	5	4	70	No	523			
72	53	Male	Hispanic	286(C)	3	Incorporated Area	0		0	$24,866	3	2	3	45	No	585			
73	79	Male	Hispanic	647.6(a) (1)	1	Transient	0		0	$0	3	3	2	56	No	497			
74	52	Male	Black	243.4	3	Incorporated Area	0		0	$3,380	4	4	4	69	Yes	575	Antidepressant (SSRI); Antiandrogen	Yes	
75	78	Male	Black	288(A)	3	Incorporated Area	0		1	$32,705	3	3	3	58	Unknown	437			
76	48	Female	White	288(A)	3	Unincorporated Area	0		0	$7,746	3	3	3	47	No	40			
77	42	Male	Black	288(A)	3	Transient	0		0	$15,577	2	2	3	40	No	772			
78	66	Male	White	261(2)	5	Incorporated Area	0		1	$12,495	4	4	4	65	No	507			
79	70	Male	Hispanic	288(A)	6	Transient	0		0	$1,767	4	5	4	68	Yes	548	Anxiolytics	No	
80	30	Male	White	243.4	2	Incorporated Area	1		0	$19,077	3	3	3	55	No	773			
81	60	Male	Hispanic	220	2	Incorporated Area	0		0	$33,118	3	4	4	56	Yes	550	Antidepressant	No	
82	64	Male	Hispanic	288(A)	3	Incorporated Area	0		1	$16,551	3	3	3	53	Yes	373	Mood Stabilizer	No	
83	31	Male	Asian	288(A)	3	Incorporated Area	1		0	$25,038	3	3	3	50	No	578			
84	25	Male	Hispanic	288(A)	6	Transient	1		1	$5,208	3	3	2	44	No	544			
85	77	Male	Black	288.5	16	Incorporated Area	0	0	0	$28,575	5	5	5	74	Yes	636	Antidepressant (SSRI); Antiandrogen	Yes	21
86	24	Male	Hispanic	288(C)	1	Transient	0		0	$21,034	4	3	1	56	No	537			
87	73	Female	White	288(A)	3	Incorporated Area	0		0	$27,244	3	3	3	45	No	53			
88	48	Male	Black	288(A)	3	Incorporated Area	0		0	$6,058	2	3	3	44	No	590			
89	59	Male	Hispanic	288A(B) (2)	2	Incorporated Area	0		1	$19,170	1	2	2	40	Unknown	737			
90	51	Male	White	314.1	1	Incorporated Area	0		1	$15,262	1	1	1	45	No	793			
91	38	Male	Asian	647.6(a) (1)	1	Incorporated Area	0		0	$12,419	3	3	3	45	No	662			
92	28	Male	Hispanic	288A(C)	3	Transient	0		0	$17,407	5	5	4	70	No	702			
93	27	Male	Asian	264.1	5	Incorporated Area	0		1	$20,504	5	5	5	78	Unknown	775			
94	46	Male	Black	289	6	Incorporated Area	1	0	0	$28,002	2	2	2	40	No	653			
95	45	Male	Hispanic	288A(C)	3	Incorporated Area	1		1	$12,976	3	3	3	47	No	510			
96	45	Male	Black	288(C)	1	Incorporated Area	0		0	$144	1	1	1	53	Yes	620	Mood Stabilizer	No	
97	28	Male	White	288(A)	3	Incorporated Area	0		1	$11,244	3	3	3	55	No	456			
98	54	Female	Hispanic	288A(B) (1)	1	Unincorporated Area	0		0	$5,448	2	2	1	49	No	46			
99	31	Male	White	288(A)	3	Transient	0		1	$4,471	3	3	3	55	Unknown	673			
100	73	Male	Hispanic	288(A)	3	Incorporated Area	0		0	$23,965	2	2	3	42	No	342			

[a] A list of Penal Code descriptions can be found in Appendix A.
[b] Offenders' post releases statuses indicate whether an offender is complying with parole conditions (0) or is in violation (1).
[c] Measurements in Conventional Units
[d] Testosterone level after 6 months of hormone treatment.

After data collection is complete, the campaign manager hires four consultants to help the governor determine what should be done about sex offender legislation and to analyze the data collected by the campaign workers. The first consultant is Michael O'Brien, a medical doctor who specializes in biomedical research on ways of treating sexual offenders. Another consultant, Theron Barr, is a policy analyst from Washington who has made his career in conducting policy analysis on sex offender laws across the nation. Robin Gogh is a clinical psychologist who works for the California Department of Corrections and Rehabilitation, assisting with the determination of sex offenders eligible for parole. The final consultant, Dakota Cachum, is a crime analyst from Los Angeles who has been compiling and analyzing sex offender data for the Los Angeles County Sheriff's Department since the time of the enactment of Megan's Law in 1996.

Once the consultants agree to work for the campaign, they asked Governor Greenleaf to give them data the State of California has available from probations department about sex offenders in a major metropolitan area of California.

Campaign workers then took their **Data Set** (the proper terminology for a collection of data) of registered sex offenders and disseminated this *Data Set* to the consultants. One evening, all four consultants decided to meet in the conference room of the campaign headquarters to discuss the *Data Set* in person. After all the campaign workers had left for the evening, the four-person team looked over the Excel spreadsheet spread out over the conference room table. Michael angrily yanked out one of the chairs scattered about the conference table and dropped into it.

"What are we supposed to do with this?"

Dakota started riffling through her purse while the other consultants looked down at the floor. Even though they did not want to agree with Michael, they were also at a loss with where to begin. Dakota then snatched up a pen at the bottom of her purse and started circling various items on the *Data Set*.

"Well, we know it's a **Quantitative Data** Set."

Theron began riffling through some of the bins in the conference room and pulled out three different calculators and placed them about the conference table. Dakota's face took on a look of disappointment, concentrating instead on the *Data Set* before her.

"I was hoping they would have done **Qualitative Data Analysis**. It's one of my specialties."

Michael's face turned a dark shade of red as the other three analysts went about their work.

"Since some of us have more important things to do with our lives than to read about statistics all day, would someone mind explaining this to me in a language I can understand."

Dakota looked up from her notes and started pointing to various sections of the *Data Set*.

"*Quantitative Data* is data that can be measured on a numerical scale and analyzed using statistical procedures. *Qualitative Data* looks at the content of what people say, think, or do in terms of patterns which is hard to define or has not yet been defined or is something which is fairly abstract, like 'the ways in which people feel loved'."

Michael angrily pointed to some sections of the *Data Set*.

"Wait a moment. 'Sex' is a category, so how can this be a *Quantitative Data* Set?"

Dakota nodded her head.

"'Sex' is not really an abstract idea and is pretty well defined, something I am sure you have seen firsthand in your medical practice. I think Quantitative would be better suited to help the Governor. What do you all think?"

The other consultants nodded their heads as Michael fumed silently to himself. Theron continued to jot down notes. Dakota began quickly circling several areas of the *Data Set* with some quick strokes of her wrist.

"So, what are our **Variables**?"

Robin looked at her with a snicker.

"I don't really like wearing underwear."

Theron and Michael were appalled at what just slipped out of Robin's mouth. Dakota fought to retain her composure; clearly Robin was going to make this process rather interesting. Still refraining from looking at her (now thoroughly embarrassed) male colleagues, Dakota pointed to the top of the *Data Set*.

"Not 'unmentionables,' *Variables* are just sets of attributes about a construct that someone wants to research."

Michael, still recovering from Robin's rather shocking admission, jumped into the conversation.

"You mean like '*Independent*' or '*Dependent*' *Variables*?"

Dakota was about to answer when Robin chimed in over in her section of the room.

"In your case, I would say more 'Codependent'."

Michael's face grew red as Robin just beamed up at him like an innocent schoolgirl.

Clearly, these two were not willing to play nice with one another. Dakota just continued.

"You are on the right track, Dr. O'Brien. In this study, the people gathering the data wanted to know certain aspects about the registered sex offenders. Some of the *Variables* they chose were income, age, type of offense..."

"And 'Meanness' level."

Dakota looked at Theron, who was pointing at sections further down on the data sheet.

Robin dropped close to the *Data Set*, hoping that the intimacy of distance would transfer to a more thorough knowledge.

"What is a 'Meanness' level?"

Robin lurched away from the *Data Set*. Apparently, the distance did not help her understand the data any better. Dakota flipped through the scant amount of information they were given about the data.

"Apparently, it is the data from a research tool that is being used to help determine whether or not someone should be released on parole for sex offenses. According to this, three raters would assign a number '1' through '5,' and that would help them make a determination as to what their 'meanness' was."

Robin rolled her eyes.

"Well, it didn't seem to be a big determinant in helping to decide who got released. These scores are all over the place."

Theron pointed to the next column over.

"And the General Aggression Score?"

Dakota once again flipped through her notes.

"It also appears to have served the same function as the 'Meanness' scores, except this one seems to be on a scale of '1' to '100.' The one I don't quite understand is this pre- and post- release status *Variable*."

Robin had a thought about what that could be and proceeded to enlighten the rest of the team.

"I bet that is the *Variable* that describes whether or not the individual has registered with the national sex offender registry."

Dakota's voice billowed through the room as she affirmed Robin's suspicions.

"Oh, yes, you are right. My notes indicate that this *Variable* is in fact related to the national sex offender registry."

Theron lowered his head.

"I don't get it."

Dakota adjusted her glasses. She completely agreed with Theron.

"You're right. We need to find out as much as we can about these *Variables* from Jennifer as soon as possible. Until then, I suppose the most logical thing to do is to talk about **Data Scale** or **Level of Measurement**."

"Harumpgh."

Dakota heard Michael grunt in the background but chose to ignore him. Instead, she focused on Theron, who was intensely watching her mark up the *Data Set*.

"Okay Dr. Cachum, but isn't it all just data? Doesn't it all mean something anyway?"

Robin hoisted herself onto the conference table, so she could get a better look at about what the two were commiserating. Robin leaned in and pointed to some of what Dakota had just circled.

"It's all going to tell us something. But what we can do with this data is very much influenced by the scale of the data we have. *Data Scale* refers to the type of data you have to work and in what manner that data exists."

Dakota slid the cap back on her pen and pointed to the *Data Set* labeled "Sex of the Offender."

"See here. This is known as **Nominal Data**. *Nominal Data* are basically categories. For example, if you wanted to examine the eye color in this room, Dr. O'Brien and I have green eyes while you both have blue eyes. Those are the categories for eye color represented in this room."

Theron nodded in understanding, while Michael snorted in contempt.

"My eyes are hazel. This woman clearly has no idea what she is talking about."

Robin subtly rolled her eyes as Dakota smiled apologetically towards her colleague.

"My apologies."

Michael was pacified by this response, as Robin reached over and pointed to another set of data on the spreadsheet.

"If I am not mistaken, isn't this **Ordinal Data**."

Dakota emphatically nodded.

"That's right. *Ordinal Data* gives you a sense of greater than or less than. You see this in those surveys where you are asked to rank whether you 'strongly agree' or 'strongly disagree'."

Theron looked at Robin.

"You mean **Likert-Type Surveys**?"

Dakota continued to circle the other *Ordinal Data* sets on the spreadsheet.

"That's right. Although there are numbers in *Ordinal Data*, they really are just there to help give you the sense of greater than or less than. The numbers themselves have no real meaning. Here, look at the raters for the Level of Meanness values; they range from 1 to 5. Clearly 1 is a lower meanness score than 5, but the number really has no meaning. It is simply a way to show that 5 is more mean than 4 and so on."

Robin then pointed to the "Income" section of the spreadsheet.

"Well, these numbers certainly have meaning."

Dakota could not help but let out a chuckle at that.

"That leads us to the last two types of *Data Scale*. **Interval Data** is data which uses numbers, and the intervals between the numbers have meaning. You could think about it in terms of temperature; 45 degrees is five degrees less than 50. Look at the General Aggression Scores, they range from 20 to 80. There is no zero starting point, and 60 is ten aggression units more than 50. However, 40 is not twice as aggressive as 20, that's our next *Data Scale*."

Theron cocked his head slightly.

"So, what's the last type of *Data Scale*?"

Dakota put down her pen and thought about the best way to explain this.

"The last type of *Data Scale* is **Ratio Data**. *Ratio Data* also uses numerical data and has equal interval points between numbers, but it also has a '0' which denotes nothingness. That *Variable* you mentioned earlier, income, is *Ratio*. You know it is *Ratio* because it has a natural zero starting point and that zero means you are volunteering your time. Think of it like 0%, or . . ."

"Or my patience."

Robin shot Michael a dirty look, yet he just sat in his chair, a wide grin across his face. He certainly was enjoying making this process difficult. Instead of acknowledging her colleague, Dakota merely retrieved her pen and pulled off the cap.

"Okay, the next thing we should do is to figure out the **Measures of Central Tendency**."

Michael slammed his hand down on the conference table.

"What is this, high school?!? Why are we wasting our time on this?!?"

Dakota raised her hand in an effort to pacify her irate colleague.

"I understand why you are so angry Dr. O'Brien. I know this seems like something that we all should know, and I am certain we all have a good idea as to what these concepts are. What we all need to remember is that we have been hired to work on a very contentious political campaign. Our work is going to be made public by the Governor's campaign, and then it will be scrutinized by his opponents. If we make a mistake, it could cost him the election and damage the reputations of everyone in this room. Surely you don't want that, do you?"

Michael's face suddenly blanched when he realized just what was at stake, and how it could reflect back on him. He shifted uneasily in his chair as Robin began to highlight certain numbers in each of the columns of the *Data Set*.

"Okay, the first *Measure of Central Tendency* we should find is the **Mode**."

Theron leaned over Robin's shoulder.

"That's the most frequently occurring score, right?"

Robin smiled.

"You got it."

The whole room was silent for a few minutes as each of the consultants tabulated the *Mode* for each section of data. Theron then scratched his head as he was looking at the data.

"Can you get the *Mode* for categories?"

Dakota nodded her head.

"Oh yes, you just figure out what is the most frequently occurring category. In fact, the *Mode* is the only *Measure of Central Tendency* that you can calculate for *Nominal Data*."

Robin wrote all the numbers down on a legal pad in the center of the room. After all four agreed on the final numbers, Dakota looked back up at the *Data Set*.

"Okay, so the next *Measure of Central Tendency* we should get is the **Median**."

Michael chimed in.

"That's easy. The *Median* is the number in the middle that separates the higher half of a *Data Set* from the lower half of the *Data Set*."

The other consultants nodded in bewilderment at Michael's statement. Michael just leaned back in his chair, grinning from ear to ear.

"Well, I did go to Harva. . ."

"Wait, how do we find the *Median* for the *Nominal Data*?"

Michael glowered at Theron for interrupting him. Theron smiled sheepishly at his colleague and then turned to Robin and Dakota for help. Dakota pointed at various sections of the *Data Set*.

"That's actually not a bad question. Actually, you need data which can be arranged in a numerical sequence from lowest to highest. Since *Nominal Data* is essentially categories, you won't be able to find the *Median* for this data. You are going to have to at least have *Ordinal Data* to find the *Median*."

The consultants then focused on those sections of the *Data Set* that had numerical data and proceeded to find the *Medians*. Once completed, Dakota leaned back in her chair, stretching out her back.

"Okay, one more *Measure of Central Tendency* to go. We need to find the **Mean**."

Theron smiled.

"That's the arithmetic average for the different categories."

Dakota nodded her head

"You got it. But remember, you should only calculate the *Mean* for data that is at least *Interval Scale*."

Once they figured out the *Means* for those groups of data which were of *Interval Scale*, the consultants looked over all the scribbled writing on the legal pad. Michael cleared his throat, alerting the group that he was read to make a contribution.

"You forgot the **Midrange**."

Robin slammed her pen on the table, exacerbated by her colleague's comment. Theron just mouthed the word '*Midrange* to himself, hoping that the silent repetition would jar some long-forgotten memory of statistics class where this topic could have possibly been discussed. Dakota felt a very subtle smile cross her lips; refusing to be undone by her colleague and his attempt to prove some type of mathematical superiority.

"You're absolutely right, I nearly forgot all about that. The *Midrange*, or Mid-extreme, is the mathematical average of the highest and the lowest scores in a *Data Set*. It isn't commonly calculated, but Dr. O'Brien is right that we should be as thorough as possible."

Dakota outstretched her hand towards Michael.

"Would you care to do the honors of figuring this out?"

Michael shook his head sheepishly, trying to avoid the triumphant look in Dakota's face. With a snort of approval, Robin dropped her pen and pulled her cellular phone out of her jacket pocket.

"I'm starving. Pizza?"

Dakota and Theron both responded in unison.

"Cheese."

Robin began dialing the number in her phone.

"What about you Dr. O'Brien?"

He looked up from the legal pad.

"Sausage."

Robin shook her head as she eased herself out of her chair and began chatting on the phone.

"Hi, I'd like to place an order for one large cheese pizza and one small sausage pizza. Can we get that delivered to..."

Robin eased into the hallway, closing the door behind her. Dakota tore off the sheet of paper with all the information about the *Measures of Central Tendency* and put it off to the side.

"Now that we have that out of the way, it's time to move on the **Measures of Variability**."

Theron arched his eyebrow.

"Would we really need to do that?"

Dakota nodded her head.

"Absolutely, it is crucial that we see how spread apart the scores in the categories are from one another."

"You mean the **Variance**."

Dakota smiled at Michael, who was impatiently doodling on the sheet of paper with the *Measures of Central Tendency* on it; trying desperately to look as if he had no interest in the conversation going on around him.

"That's right. The first thing we need to know is the difference between the highest and the lowest scores, also known as the **Range**."

Theron had a quizzical expression on his face as he began looking at the different columns of data.

"So, I am guessing that we really can only figure out the *Range* for the data that are in *Interval Scale*."

"You got it."

All three consultants popped their heads up to see Robin standing in the doorway, holding a stack of pizzas in one hand and a six-pack of sodas in the other. She gingerly placed the food down at the corner of the conference table and helped Dakota and Theron with their calculations. The three of them patiently figured out all of the different *Ranges* for the *Interval Data* sections, as Michael sat quietly grazing on his pizza. Once all of the data was written down, Dakota plopped her pen on the table and retrieved food for the rest of the group. Robin pulled the *Data Set* over to her, letting out a sigh as she looked it over.

"We still have one more *Measure of Variability* we need to calculate.

Dakota nodded her head.

"Yep, and this one isn't as straight forward as the others. We need to see how different each score is from the *Mean*, and what these scores would look like as a distribution."

"The sthumfard dividdation."

The group looked quizzically at Michael, who was trying to talk past all the food which was puffing out his mouth. Theron popped open his can of soda, trying to stifle the laugh he felt bubbling up inside him.

"You mean the **Standard Deviation**."

Michael just stared at the group, gulping down even more of his food.

"That's what I said."

Robin raised her hand as a wide-eyed look of terror crossed her face.

"Isn't the *Standard Deviation* kind of hard to find?"

Dakota pulled a piece of paper from her legal pad and began to write out an equation.

"Yes and no. With computer programs like Excel or SPSS, the *Standard Deviation* can be found with a few keystrokes. Heck, even some calculators can do it with relative ease. But I think this time we may have to do it the old-fashioned way."

Dakota finished writing and turned the page around so the whole group could see what she had written:

$$S = \sqrt{\frac{\sum (\chi - \overline{x})^2}{n - 1}}$$

Michael took one look at the formula and shook his head.

"That's not right."

Michael lunged towards Dakota, snatching the pen out of her hand. He then scrawled out another formula on the piece of legal paper:

$$S = \sqrt{\frac{n(\sum \chi^2) - (\sum \chi)^2}{n(n - 1)}}$$

Dakota quickly nodded in agreement.

"There are lots of different formulas which one can use to determine the *Standard Deviation*. These two are essentially the same; the first formula doesn't have the repetitious 'n' in the numerator and denominator. Both are correct and will get you to the same answer, but each one arranges the data a bit differently to get to that answer. Regardless of the formula used, the end result will still tell you how different each score is from the *Mean*."

Robin began plugging numbers into her calculator but was stopped when Dakota gently pushed the calculator down to the table.

"I think it might be best to wait until we can double-check our work with something more sophisticated than an old calculator."

Robin breathed a sigh of relief.

"Good. This one doesn't even have a **Square-Root** button on it. Just so we are all on the same page, what are all of these symbols and where do we find that information?"

Michael finally chimed in with a helpful remark.

"Well, I remember that Σ means that we need to add all the numbers together and that x is the set of numbers for any given column of data. As for the n, that is just the number of participants or people we have. You could also say that n is the number of numbers that we have in a given column of data."

Theron smiled as he moved all the completed papers off to the side.

"That is very helpful, Michael, but this isn't going to do much to help the Governor, is it?"

Robin shook her head in defiance to what Theron just said.

"I wouldn't say that. We got a lot of the **Descriptive Statistics** out of the way."

Michael snorted in contempt.

"*Descriptive Statistics*. That tells you nothing."

Dakota shook her head and pointed to all the work they had done.

"*Descriptive Statistics* give you a lot of good frequency information, and it also tells you a lot about the data you have to work with. While it may not be as 'sexy' as running tests like Regressions[1] or parametric statistical analyses, it is absolutely vital to help you determine what tests are appropriate for your data."

Theron jotted down some notes and then asks the team "Okay, I haven't had a statistics class since college, and we have officially surpassed all of my statistical abilities. What exactly are we looking for?"

All of the consultants watched as Robin scribbled out "Rules for Parametric Statistics" at the top of her legal pad.

"Now we need to figure out what we can and cannot do."

Dakota leaned in close to Robin.

"From my understanding, in order to use Parametric Statistics, the first, and most important, thing we need is data which is **Orthogonal**."

Dakota could see the obvious distain on her colleagues faces as Theron timidly blurted out what everyone else was thinking.

"No way! I had braces when I was in high school. There is no way I am going back to that mess."

Dakota, unable to control her laughter, realized that Theron was quite serious about his remark.

[1] Regressions are discussed in greater detail on page 168 in Chap. 7.

"Well, Theron, fortunately for you, **Orthogonality** has nothing to do with orthodontics. *Orthogonality* is achieved when data are independent of one another. One *Variable* has no impact on another *Variable*; one participant's response has no impact on another participant's response, much the same way you and I have no impact on Jennifer. She is just going to do what she wants to do, and there is nothing that any of us can do to impede that."

Dakota's explanation seemed to pacify the group for now.

"The second thing we need is data that are from a **Random Sample** of your target population."

"Well, we already have that" chimed Michael.

Dakota quickly glanced over the *Data Set* and shook her head.

"I am not sure if we do. There is no real explanation as to how the campaign workers collected this information. Did they randomly select possible participants who all had an equal likelihood of being selected for participation in the study, or did they just go to every registered sex offender that lived near the campaign headquarters."

Theron looked over some of the information that Jennifer had given him.

"I think they just went to all the sex offenders living around here, but I can't be certain."

Robin shook her head. "That sounds like a **Convenience Sample**."

Dakota watched as Robin continued to write down bits of information.

"Okay, we are going to have to speak to Jennifer about that one at a later time. For right now, let's keep going through the other assumptions for parametric statistics. If any of these other assumptions are violated, then we know for a fact that we cannot do parametric tests. We also need data which is at least *Interval Scale*.

Michael began scratching his head.

"What does that mean? Does that mean we can use *Ordinal Data*, or do we need *Ratio Data*?"

Theron stood up from the conference table and began to pace the room. Finally, Theron made his way back to his chair and pulled his legal pad close.

"It means that we need data that is *Interval Scale* or higher when the scales are placed in this order."

Ratio
Interval
Ordinal
Nominal

Dakota nodded her head.

"Okay, that will be a little problematic. Only some of these categories have data which are *Interval*. Can we just focus on these particular categories?"

Robin leaned over the *Data Set* and then looked at all of the writing on the legal pad.

"This is getting complicated."

Dakota put her pencil on the conference table, noticing the coffee maker at the far side of the room. She eased herself out of her chair and crossed the room.

"True, but it needs to be done. We can't do anything until we know more about the data we are working with. Coffee anyone?"

All three consultants shook their heads. Dakota poured herself a cup and then returned to her seat.

"The fourth thing we need is **Homogeneity of Variance**."

Michael's eyes grew large.

"What is that? Is that even a real term?"

Dakota laughed.

"Oh it is, and it also is known as **Homoscedasticity**. It just means that if you graphed out the data, there would be a constant *Variance* for all data points. Remember, *Variance* is how spread out or close together the data are."

"Oh come on!!!"

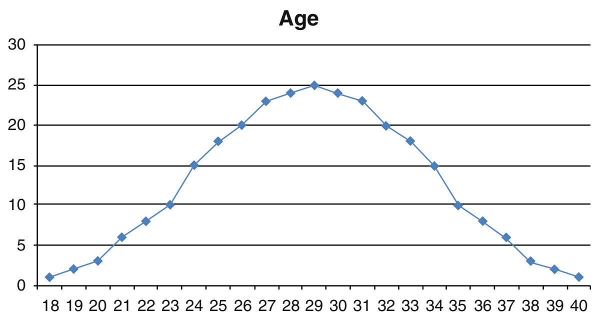
The group all snapped their attention to Michael, who was clearly becoming irritated with everything going on.

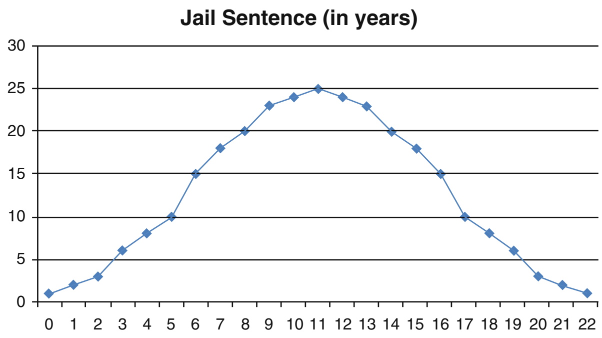
Dakota attempted to calm Michael a bit.

"*Homoscedasticity* just means that if we look at two *Variables* they will have the same shape when they are graphed out."

Moving toward the whiteboard, Dakota describes the term with a picture.

"All we have to do is make sure that whatever shape the first *Variable's* graph takes, that the second *Variables* graph should be the same or very similar. For example, they might both look like this if we were looking at the age of the offender and the length of jail sentence:





Can you see how those two pictures look the same? Does that help to clarify things, Michael?"

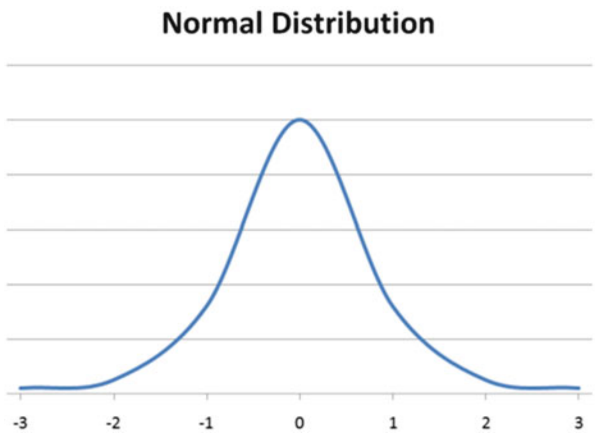
Michael just nodded silently. Dakota continued on with her list.

"Finally, we need data which is **Normally Distributed.**"

Robin looked at the *Data Set*, overwhelmed by the information on the spreadsheet in front of her.

"Do we have that?"

Dakota roughly sketched out something that looked like a bell.



"Data in a normal distribution is supposed to look like a bell-shaped curve when it is graphically presented. It has no **Skew** and a unique type of **Kurtosis**."

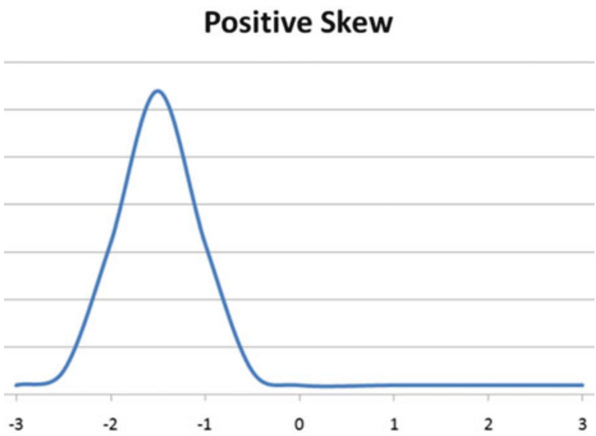
Theron raised his hand.

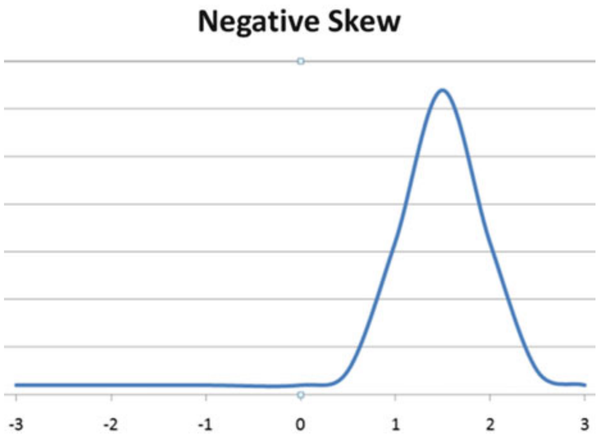
"*Skew*. Isn't that determined by **Outliers**?"

Dakota nodded.

"Yes. *Outliers* are extreme scores within the *Data Set* which have an impact on the *Mean*. If the *Outliers* are pulling the *Mean* higher than the majority of scores in the Data Set, then you have **Positive Skew,** making the *Tail* on the right side of the graph longer than the *Tail* on the left side. If the *Outliers* are pulling the *Mean* lower than bulk of the scores in the Data Set, then you have **Negative Skew**. *Negative Skew* means that the *Tail* on the left of the graph is longer than the *Tail* on the right."

Dakota added the *Positive* and *Negative Skew* graphs to her drawings.





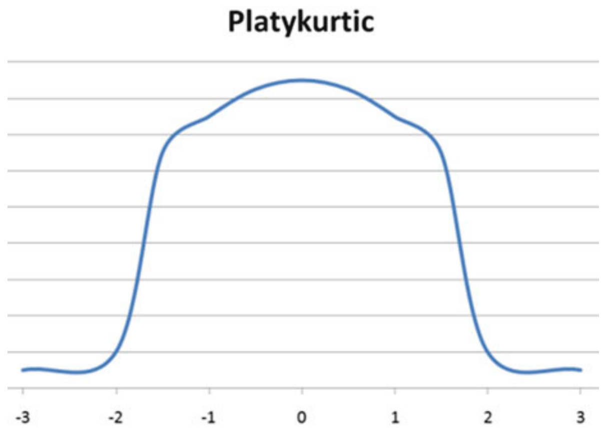
Michael patiently listened until Dakota was finished with her explanation.

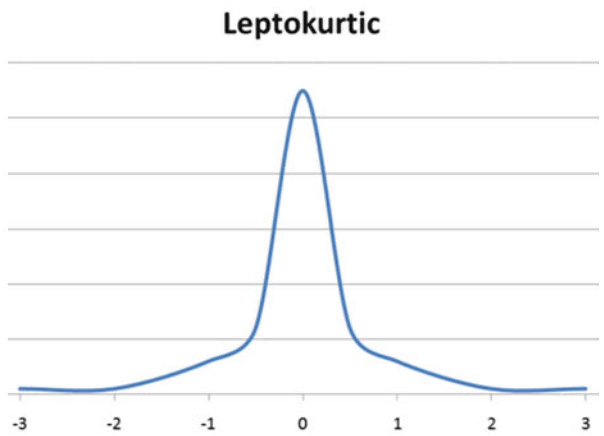
"So, you are telling me that if I have a *Mean* of 24 and the bulk of my scores are between 15 and 19, then my graph will be *Positively Skewed*? And if my *Mean* is 24 and the bulk of my scores are between 27 and 30, then my graph will be *Negatively Skewed*?"

"Yes, Michael, that is exactly what I have been saying."

"So, then, what is *Kurtosis*?"

Dakota roughly sketched out two other pictures next to the one of the bell. One was of a relatively flat line, while the other looked like a misshapen bell with an exaggerated peak in the center.





"Think of *Kurtosis* as a way to visually examine the *Variance* within your *Data Set*. If the scores are very spread out, the *Variance* creates a graph that resembles a plateau. That is known as **Platykurtic**."

Robin leaned over Dakota's shoulder and pointed at the relatively flat line.

"Is that what *Platykurtic* data would look like?"

Dakota nodded and then pointed to the other diagram.

"That's correct. And this is known as **Leptokurtic**. The scores are roughly similar, with the high peak in the middle resulting from the *Measures of Central Tendency* and there is little-to-no *Variance* among the scores."

Michael pointed to the bell-shaped curve.

"And what's this? I suppose the bell gets its own fancy name as well."

Dakota cracked a smile.

"As a matter of fact, it does. The bell-shaped curve is known as **Mesokurtic**."

Theron sighed in discontent, echoing the sentiments of all the consultants in the room.

"So, what do we have?"

Dakota looked over the *Data Set*.

"Well, there are some statistical tests we can run on the *Interval Data* sets to see whether or not we have are working with a normal distribution. For example, we could use the **Test for Distributional Symmetry.**[2] "

Michael leaned back in his chair.

"Look, it's obvious we cannot do anything tonight. We need to be able to examine this data using some type of sophisticated software."

All the consultants nodded in agreement. Dakota then started passing out photocopies of the *Data Set*.

"Okay. How about each of us generates some research questions we want to answer based on the data. Let's meet tomorrow night and bring our research questions with so we can discuss them among the group. I will conduct a couple statistical tests to see if our data is *Normally Distributed* or *Mesokurtic*, and we can move forward from there."

Chapter Summary

- The data set was introduced describing qualitative and quantitative data in terms of four levels of measurement: nominal, ordinal, interval, and ratio.
- Several of the data set variables were briefly discussed to provide more clarity in terms of the data set and how to interpret the numbers therein.
- Descriptive statistics such as measures of central tendency were discussed: mean, median, mode, and midrange.
- Descriptive statistics such as measures of variability were also considered: variance, range, and standard deviation were also discussed.
- The five assumptions that must be met in order for parametric tests to be utilized were explained. The five assumptions are orthogonality, at least interval scale data, random sampling, homoscedasticity, and Normally Distributed data.
- Examples of outliers, skew, and kurtosis were also discussed and considered in terms of usefulness in understanding the need for descriptive statistics.

[2] The Test for Distributional Symmetry is discussed in greater detail on page 53 in Chap. 3.

Check Your Understanding

1. Identify each of the variables from the data set as being qualitative or quantitative.
2. Identify the level of measurement for the following variables from the data set as nominal, ordinal, interval, and ratio:
   a. Age
   b. Total Testosterone
   c. Offense
   d. Sentence
   e. Estimated Yearly Income
   f. Currently Taking Medication
   g. Level of Meanness
3. Calculate the four measures of central tendency for the first 10 participants for each of the variables in the data set—mean, median, mode, and midrange:
   a. Age
   b. Sentence
   c. Estimated Yearly Income
   d. Level of Meanness Rater 1
   e. General Aggression Score
   f. Total Testosterone Level
4. Calculate the three measures of variability for the first 10 participants for each of the variables in the data set—variance, range, and standard deviation:
   a. Age
   b. Sentence
   c. Estimated Yearly Income
   d. Level of Meanness Rater 1
   e. General Aggression Score
   f. Total Testosterone Level
5. The mean of a variable is 100, but the bulk of the data points are between 90 and 200. What kind of distribution is it?
   a. Positively skewed
   b. Mesokurtic
   c. Platykurtic
   d. Negatively skewed
   e. Leptokurtic

Chapter 3
Questions, Assumptions, and Decisions

Abstract In this chapter, the newly developed team will raise multiple research questions that will address issues which may be of concern to the political campaign. The team will also learn that some of the assumptions necessary for parametric tests have been violated in gathering the data. Several tests will be utilized to determine whether or not other assumptions have been violated. Research questions that are addressed in this chapter include the following: (1) Are the data Normally Distributed? (2) Are the data random? (3) Do the data have homoscedasticity? The conclusion that the team reaches is that parametric tests should not be used in this situation because of the type of data that was collected and the way that data was collected.

Dakota sat quietly in a conference room chair, the morning sun flittering through the partly closed blinds and dancing on her eyelashes. The weather had turned unseasonably cold over the past few days, and Dakota's grueling work schedule deprived her of any leisure time outdoors. So, she enjoyed this quiet moment of relaxation, even if it was partially obscured by some fairly unattractive window treatments. She could hear Theron's pencil scratching away at the New York Times crossword puzzle (to be fair, she heard him utilize the eraser far more often than the pencil's graphite point). At the far end of the table, Dakota heard Michael clear his throat for what must have been the fifth time in the past ten minutes. Her eyes gently fluttered open to spy the clock over the door of the conference room; their meeting was to have started almost an hour ago, yet Robin was nowhere to be found. Michael cleared his throat once more, although it was now painfully obvious that Michael was clearing out irritation and not phlegm.

"Where is that blasted girl? Is it that hard to come back to a building you were just at the day before?"

Dakota's eyelids fell open, and her eyes caught sight of Theron as he furiously erased the entire upper right-hand corner of his crossword puzzle. He lazily

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8_3,

continued to read his paper, as if Michael did not even exist. There was something about Theron which always brought a smile to Dakota's face. Maybe it was his unyielding enthusiasm, or maybe it was the fact that even when he wore a very expensive suit he lost none of his sloppy boyish charm. Theron continued to scratch about his crossword puzzle, absent-mindedly lost in his own little world.

"Come on, we all knew that she was going to meet with Jennifer to get a little clarification on the data set."

Michael slouched down into his chair, acting more like a petulant child than a well-respected member of the medical community. From his furrowed brow, it was obvious that the mere mention of Jennifer elicited a reaction which was anything but pleasant. Dakota just stared at her sulking colleague, studying the lines in his face which were no doubt the result of years of bureaucracy taking their toll. Even though Dakota surmised that they were just a few years apart in age, his forays into government circles aged him well beyond his years. Michael just grumbled quietly to himself.

"Ha. I doubt the ice queen is going be helpful to any of us."

Dakota felt herself shudder from the chill which entered her mind as she thought of Jennifer. In truth, there was nothing all that terrifying about her at first blush; she had an unconventional beauty, wore only fashions designed specifically for her by Ivanka Trump (apparently, they grew up together; a point of fact that came up in almost every conversation one had about Jennifer), and was immensely self-possessed. What made Jennifer stand out was that she wielded power, and she could use it to do awesome or terrifying things depending upon which need suited her at that moment. All those in political office were not only aware of her charms but of the damage she could do to a promising career with one phone call. The fact that Jennifer worked for the governor spoke volumes about Nathanial Greenleaf's character. Dakota smoothed out the lines in her skirt and rose towards the dry-erase board.

"Okay. Yesterday, we all agreed that we would look over the data to see if we could formulate some research questions relevant to current policy issues. What do we have?"

Theron shoved his hand into his pocket, pulled out a sheet of paper tightly folded into a triangle, and slid it over to Dakota. She adeptly stopped the paper with her index finger and began the arduous process of unwrapping the tiny package which contained Theron's work. Still, Dakota was glad that he refrained from doodling all over it. Once the paper was opened to something a little less shaped like a football, she began jotting his questions onto the dry-erase board. She read the questions aloud as she wrote them out, trying to see if this recitation would manage to cause a reaction from Michael.

"Okay, so Theron's questions are related to a couple of potential policy areas. Regarding the use of instruments:

– Is sentence length associated with the General Aggression Score?
– Is total testosterone level associated with the General Aggression Score?

- Are there significant differences among raters' Level of Meanness scores when offenders are grouped by income?
- Are the General Aggression Scores different for 'in compliance' and 'in violation'?

To address potential differences in living location and transient status:

- Is ethnicity/race associated with living location?

To address issues of compliance:

- In terms of pre and post-release status, is there a difference between those with an annual income of less than $15,000 a year compared to those with an annual income of more than $15,000 a year?

And some questions that examine the type of participants in our data set:

- Is there a difference between the observed frequency of offense and the expected frequency of offense? This will allow us to see any potential differences between our participants and offenders in the State, as a whole. And
- Is there a difference between the observed frequency of sentence length and the expected frequency of sentence length?"

Once she finished writing out the last question, Dakota took a step back to see what her colleague had given her. She was actually very impressed with the questions and told him so with a subtle nod of her head. Theron picked up on this and smiled back, pleased that he was able to impress her with his work. Dakota then looked to Michael, who was lounging at his end of the conference table. He solemnly rose from his chair and handed her an ivory sheet of paper. Dakota could feel the parchment under her fingers as she looked over Michael's monogrammed stationary, trying not to let her bemusement show on the lines of her face. This piece of paper was much nicer than her Doctorate diploma, something which both depressed and bemused her. She carefully placed Michael's questions on the desk and dutifully began to write his questions under Theron's. Michael's questions were all specifically related to biological considerations and related treatment and assessment issues:

- Is sentence associated with the General Aggression Score?
- Is sentence associated with the General Aggression Score when testosterone level is fixed?
- Is compliance status associated with whether they are currently taking medication?
- Is there a difference between the probability that sex offenders will take medication and the probability that sex offenders will not take medication?
- For those on antiandrogens, are General Aggression Scores significantly different from one another on the basis of race/ethnicity?

Again, Dakota took a step back to admire what was on the board before her. Even though only half of the group's questions were on the board, she was pleasantly surprised with what they had produced thus far. She looked towards Michael and smiled.

"I like your questions Dr. O'Brien."

Michael swiveled in his chair.

"I knew you would."

Dakota just shook her head and began writing her questions on the dry-erase board, tacking them onto the bottom of their fairly well-developed list. When formulating her questions, she considered the consistency of assessment administrators and variables potentially related to compliance, all issues receiving attention in the field of sex offender management.

– Is the size of the difference significant between rater 1 and rater 2 on the Level of Meanness scale?
– Is there a difference in Current Release compliant/in-violation status based upon race/ethnicity?
– Is the variability the same for the General Aggression Score based on offense?
– Is the probability of changing from compliant to in violation and in violation to compliant the same for 30-days post-release to the present time?[1]

Dakota took a step back to see that the dry-erase board was almost completely covered with their questions about the data set. Theron sighed quietly to himself, a look of confusion and fear falling across his eyes as he looked towards the chaotic number of hypotheses laid out before him.

"Okay, so where do we begin?"

Dakota felt a smile cross her lips, a smile so subtle that it would rival only the "Mona Lisa." She picked up the marker and wrote one more question at the top of the dry-erase board.

– Is the data *Normally Distributed*?

Once she was done writing, she circled the question with the marker.

"That's where we begin."

Michael continued to listen with only passing indifference. Theron's brow began to sweat, his mind racing to solve the riddle as to how this is done.

"And why are we starting there?"

Dakota heard his question, yet her eyes remained transfixed on the dry-erase board.

"We need to know as much about this data as possible. The **Tests of Distributional Symmetry** allow us to see whether or not the data are Normally Distributed, and it gives us a chance to understand exactly what it is we are working with."

"Okay, so how do we do that?"

[1] Another research question, "When presented with only two sex offenders at a time, do the raters agree on ranking of the level of meanness for each sex offender?," will be posed later in the narrative to the consultants as events begin to unfold in the media.

Michael sneered towards Theron, his displeasure with his colleague bubbling over his feigned nonchalant indifference.

“It’s obvious boy. We let our captain over here show us how it’s done.”

Dakota placed her hands on her hips and locked eyes with Michael, their battle for dominance once again playing out within the cramped confines of the conference room. Still, Dakota felt a sinking feeling in her stomach; Michael was determined to make her out to be more like Captain Ahab than Horatio Hornblower. She silently let her apprehension pass and began turning the white board around, thus preserving their questions for posterity. After steadying herself for a moment, she began writing some equations on the dry-erase board.

“Typically, the first thing we would like to do with our *Tests of Distribution* is to determine whether or not the *Interval Data* we have are *Normally Distributed*.”

“You mean, the distribution looks like the back of a dromedary?”

Dakota looked incredulously at Michael, who was smirking at his supposedly insightful comment. Theron sat on his end of the table, eyes widened in shock and horror at the useless nomenclature being thrown about the room. Dakota reached over and patted his arm with a serene gesture.

“It’s okay. Typically, we say that the *Normal Distribution* looks like a bell.”

Theron seemed to sigh with relief, still jotting out the word “dromedary” on the top of the page of his notebook. Apparently, this was one vocabulary word that he would learn by the end of the day. Dakota wrote out a phrase at the top of the dry-erase board.

“The first test we can discuss is the **Kolmogorov-Smirnov One-Sample Test**.”

Theron and Michael looked at each other, their faces shocked in fear over the awe-inspiring name of the test. Theron meekly stuck his hand up into the air while simultaneously shrinking into his seat.

“The Kremlin-Vodka Test?”

Michael glowered at him.

“That’s not even close to correct. I swear; it’s like I am surrounded by morons.”

Theron looked as if someone smacked him in the face.

“I am just trying to understand this.”

Dakota felt the hairs on the back of her neck bristle as Michael continued to chastise Theron with his icy expression. She continued writing on the dry-erase board, hoping that the icy bitterness which was frosting over her colleagues would invariably subside.

“The *Kolmogorov-Smirnov One-Sample Test* tells you whether or not your data is the same as a theoretical probability distribution.”

Michael leaned forward in his chair, crossing his arms over his chest. He was desperately trying to appear disinterested, but was failing miserably.

“Why would you care about this?”

Dakota quickly sketched a *Bell-Shaped Curve*, dividing it into two sections.

“Well, think of it this way. In order to be able to run statistics, you have to be certain that your data are Normally Distributed. The *Kolmogorov-Smirnov One-Sample Test* allows you to do that by letting you examine the data you have to see if it comes from the population with your hypothetical *Bell-Shaped Distribution*.”

Dakota saw the confusion in her colleagues, as if her explanation were mudding the intellectual waters instead of making them more translucent. She closed her eyes for a moment, tilting her face up to the sky. After a few deep breaths, her mind was able to formulate something which might help.

"Okay, let's see if this helps. Suppose that I am running a coffee company."

Michael cackled with mocking laughter.

"Why on earth would you want to do that?"

Dakota rolled her eyes, actually feeling more flattered than insulted.

"Please, Dr. O'Brien. I am trying to go somewhere with this."

He graciously nodded his head, giving her permission to continue. She felt as if Michael was baiting her into a trap; this was definitely his way of comparing her to Captain Ahab. Dakota gritted her teeth behind a forced smile and continued on with her metaphor.

"We will call this coffee shop 'Pequods'."

Theron grinned, enjoying the irony of this all.

"You mean like that other coffee chain named after the first mate of the Pequod in 'Moby Dick.' You know, Starbu..."

Dakota raised her hand, stopping Theron's stream of consciousness from continuing on another tangent.

"That is beside the point. Let's say you want to see what the profits for Pequod's were over the past fiscal year, from July to June, and compare it to the projections which were made by your Board of Directors. In this instance, the projection made by the Board of Directors becomes the hypothetical distribution you wish to compare your data to in terms of determining normality."

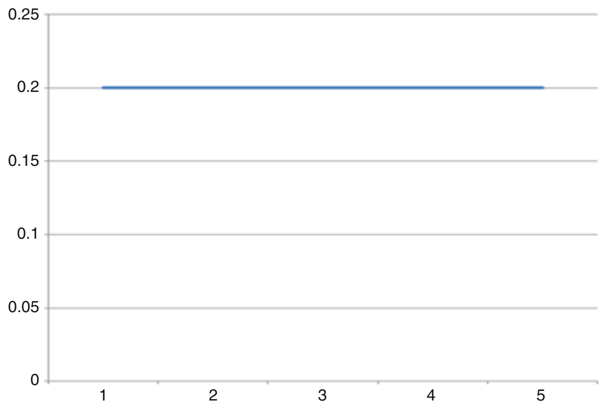
Michael leaned back in his chair, an ironic grin plastered across his face.

"Now my dear lady, I hardly feel that a good businessman would want his yearly profits to resemble the back of the dromedary. In fact, no one wants yearly projections which look anything like a bell."

Dakota smiled in triumph.

"You are very correct, Dr. O'Brien. Fortunately, there is more than one distribution method which could be used."

Dakota quickly sketched out three different distributions on the board.



Theron jumped to his feet, pointing at the diagram on the board.

"It's a pedestal!!!"

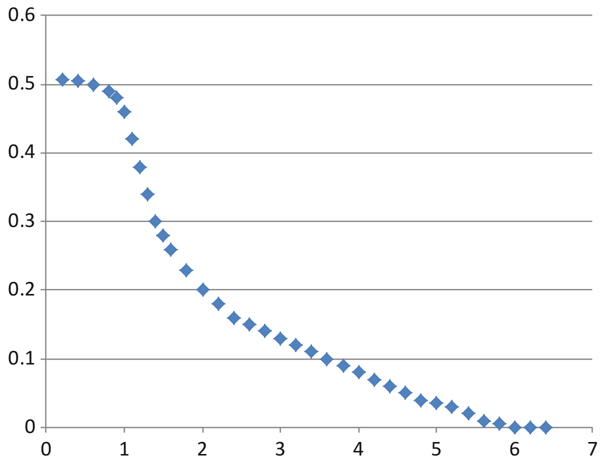
Dakota and Michael were flabbergasted by his outburst. Theron just flashed his boyish grin and meekly sat back down.

"I thought we were guessing what the drawing was supposed to be."

Michael just shook his head, as Dakota once again found herself stifling another fit of laughter.

"This is a **Uniform Distribution**. For this type of distribution, all possible outcomes between the endpoints have an equally probable likelihood of occurring."

Theron was furiously writing down as much as he could from Dakota's lecture, hoping to retain as much information as possible. Dakota spied Michael off in her periphery, his smile fading from his face. She just continued on to the next of her drawings.



"This is known as a **Poisson Distribution**. These are often conducted when you wish to see the likelihood of a rare event occurring, which is why you often see these distributions having a *Right Skew* and then tapering off as the likelihood of the occurring event becomes negligible."

Theron shuddered.

"Wow, if I were running a coffee shop, I definitely wouldn't want it to be poisoned."

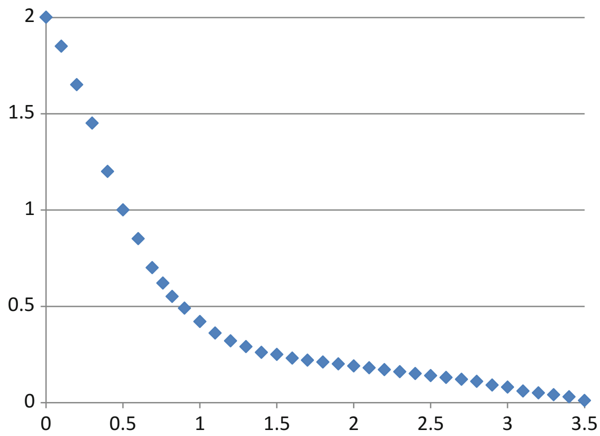
Michael shook his head, choosing to focus on Dakota rather than his incessant ramblings.

"Poisson, not poison."

Dakota let the momentary bickering pass before continuing on.

"Actually, Dr. Barr has a point. I personally would be horrified if I saw my profits decline to the point of a *Poisson Distribution*, and no one wants their profits to remain stagnant over time like in the *Uniform Distribution*. I am pretty sure most businessmen are greedily hoping for this."

Dakota pointed dramatically to the last distribution on the board.



"This is an **Exponential Distribution**."

Robin cocked her head to the right as confusion clouded her eyes.

"I thought exponents were those little numbers that were in superscript next to the big numbers and that they signified multiplication or something."

Theron arched his eyebrows.

"Did you just start understanding the world through font settings on Microsoft Word?"

Robin just shook her head.

"You know what. . ."

Dakota firmly grasped that dry-erase marker and started tapping it violently against the dry-erase board.

"Okay all. The *Exponential Distribution* is used to help you understand the amount of time you have to wait before an event will occur during a particular temporal interval. Basically, all successive points on the distribution continue to decrease along the axis in a **Continuous Probability Distribution.**[2] "

Robin's look of confusion did not lessen. She started to open her mouth to ask a question when Michael's voice bellowed from his corner of the conference table.

"Time period."

Dakota cast a sidelong glance in Michael's direction, chastising him with her eyes. Still, even she had to admit that there was some relief in not having to answer more of Robin's questions. She pointed at the chart on the board.

"Typically, this model can be used to help determine how often something will happen based on a constant average rate of occurrence, typically for a continuous random variable."

Theron stopped taking notes in his legal pad, glancing at the graph on the board.

[2] Continuous Probability Distributions are discussed in greater detail on page 191 in Chap. 8.

"I get it. You would use this distribution to help determine how often a person will come to a bank teller window between when the bank opens and when it closes on Saturdays."

Dakota smiled.

"That's correct. As you can see from the diagram, we have three separate distributions. The distribution you use will be based on your needs for a specific situation.[3] "

Michael felt his dominant tendencies rising up inside him and could not bear to listen to this anymore.

"This is all very interesting young lady, but what do these three drawings of yours have to do with anything we discussed?"

Dakota snapped the cap back onto the marker for the dry-erase board.

"Well, these are important because you can tailor your *Kolmogorov-Smirnov One-Sample Test* to run with any of these possible hypothetical distributions. However, the one most people would use is the Bell-Shaped Distribution, since that is the distribution which will allow you to conduct most *Parametric Tests*."

Michael mulled over her answer for a moment, trying to find a flaw in her logic. Finding none, he allowed her to continue on with her example. Dakota popped off the cap to the dry-erase marker, sketched out all the months of the year (from July to June), and continued writing down numbers.

"Okay, so in this example, the actual amount becomes the **Observed Frequency**, while the projections become the **Expected Frequency**. Then, you just look to see if there is an agreement between the two."

Theron leaned back in his chair.

"So, the goal is to have your projections be in line with the actual profits from the coffee shop?"

Dakota felt her stomach flutter, an obvious sense of pride washing over her.

"That's correct. Or, to compare it to the *Kolmogorov-Smirnov One-Sample Test,* our sample would match our idealized distribution."

Theron smiled.

"And if your data from the Board of Directors did not line up with the actual information from the coffee shop sales, you would say that. . ."

Dakota allowed herself to drop into her chair, finishing his sentence off as she settled herself down.

"You would say that your data set did not line up with that of your theoretical distribution."

Both men seemed content with that explanation. Dakota looked at the data set, pointing to the "Age," "Length of Sentence," "Income," "General Aggression Score," "Total Testosterone Level," and "Treatment Testosterone Level" categories.

[3] The interested reader should consult another text for additional details about exponential distributions.

"Well, all these variables are *Interval* or *Ratio Scale* but let's look at these 'Length of Sentence' scores from the data set. After all, in order to run the *Kolmogorov-Smirnov One-Sample Test*, you need at least an *Interval Data* set which can allow you to see the **Cumulative Frequency** of the *Observed Value* and compare it to the *Predicted Value*."

Theron began to rub his jaw with the palm of his hand.

"So, how do we get those?"

Dakota smiled.

"Well, the *Cumulative Frequency* is just a running total of all the frequencies we are observing or expecting over a given time period. Let's say in July they sold 200 pounds of coffee, and in August they sold 150 pounds of coffee. In July, the *Cumulative Frequency* would be 200, but the *Cumulative Frequency* for August would be 350. And so on."

Month	lbs. of coffee sold	Cumulative frequency
July	200	200
August	150	350

Dakota finished sketching out her drawing and looked back at Michael and Theron. Neither one had a trace of confusion in their faces, so she felt it was safe to continue on with her explanation.

"Next, you need to calculate the **Cumulative Relative Frequency**."

Theron's face once again became a mask of confusion.

"I don't suppose that is how often a relative visits your house."

Michael dropped his pen in disgust, physically turning himself away from Theron. Dakota just smiled at this statement, choosing instead to be patient rather than infuriated.

"The *Cumulative Relative Frequency* is just taking your data and transforming it into decimal values rather than whole numbers."

Dakota quickly wrote another formula on the board:

$$\textit{Cumulative Relative Frequency} = \frac{\textit{Cumulative Frequency}}{\textit{Sample Size}}$$

Dakota just looked at the men in the room.

"Then, you use the exact same logic that you did with the *Cumulative Frequency*, except that this time you will be continuously adding decimal values."

"And these decimal values should add up to 1.0, correct?"

Dakota shook her head.

"Not quite. The final value for the *Cumulative Relative Frequency* will be 1.0, not the sum of them."

Michael was listening so intently that he was not even aware that he asked Dakota a question; thus conceding some sort of mathematical superiority to Dakota. She allowed herself to savor this for only a moment before continuing on with the discussion.

"Now, once you have obtained the *Cumulative Frequency* and the *Cumulative Relative Frequency*, you just run the equation":

$$D = max|F_o(X_i) - S_N(X_i)|$$

Michael snorted with contempt.

"Humph. After that amazingly helpful example, I thought you'd give us something a little more impressive than finding the largest discrepancy between what you expected and what you observed, and then seeing whether this biggest discrepancy is big enough to conclude that there is a violation."

Dakota couldn't help but smile at Michael's verbal barb towards her. There was something almost sweet in the way he addressed her, regardless of how gruff he tried to appear. She placed one hand on her hip and smoothed her hair out of her face with the palm of her other hand.

"Well, Dr. O'Brien, I didn't create the test. So, shall we continue?"

Michael nodded, thus giving them permission to continue on with their work. Theron started scrawling out the equation on his piece of notebook paper, while Dakota worked the list of numbers that would be needed for her to work on her own version of the equation on the dry-erase board.

D = *the largest absolute value of* $F_0(X_i) - S_N(X_i)$; *the maximum deviation*
$F_0(X_i)$ = *theoretical Cumulative Relative Frequency distribution; the theoretical distribution associated with the* ***Null Hypothesis*** *or* H_o. *The Null Hypothesis in this case is that the observed sentences are Normally Distributed.*
$S_N(X_i)$ = *observed Cumulative Relative Frequency distribution of N observations*
$i = 1, 2, \ldots N$
μ = *mean*
σ = *standard deviation*
$z = \frac{X_i - \mu}{\sigma}$ = *score used to determine the theoretical relative frequency (probability)*

Sentences for each of the $N = 100$ offenders are listed below. Each box represents a separate offender:

$\mu = 3.41$
$\sigma = 2.458$

7	5	1	9	2	3	6	7	2	1
8	3	3	2	3	3	1	3	3	3
3	3	8	3	3	6	1	1	3	5
1	1	1	3	1	3	3	3	6	6
1	1	5	5	3	6	3	3	16	3
3	4	5	5	3	3	1	3	1	1
12	3	6	8	6	3	1	3	3	3
3	1	3	1	2	3	1	5	3	1
3	1	5	1	3	3	2	6	2	3
6	2	3	5	1	3	1	2	1	3

Dakota continued with her explanation of the procedure.

"After the frequency for each sentence is determined, the cumulative *Observed Frequency* is calculated.

Next, the observed *Cumulative Relative Frequency* is calculated by dividing the observed *Cumulative Frequency* by the *Sample Size*":

$$S_{100}(X_1) = \frac{24}{100} = .24$$

Sentence (in years)	Frequency	Cumulative frequency: Observed	Cumulative Relative Frequency: Observed $[S_N(X_i)]$	z-score	Predicted $[F_0(X_i)]$	$\lvert F_0(X_i) - S_N(X_i)\rvert$
1	24	24	0.24			
2	8	32	0.32			
3	41	73	0.73			
4	1	74	0.74			
5	9	83	0.83			
6	9	92	0.92			
7	2	94	0.94			
8	3	97	0.97			
9	1	98	0.98			
10	0	98	0.98			
11	0	98	0.98			
12	1	99	0.99			
13	0	99	0.99			
14	0	99	0.99			
15	0	99	0.99			
16	1	100	1.00			

"The theoretical *Cumulative Relative Frequency Distribution* can be any distribution specified by the researcher. In cases where predictive frequencies are known, they can be used. Cumulative relative predicted frequencies are calculated the same as the observed *Cumulative Relative Frequencies*.

In this case, we want to compare the observed sentences to the *Normal Distribution*. To do this, we need to calculate something else, a **z-score**. A *z-score* is a standardized value where the *Mean* is usually 0 and the *Standard Deviation* is usually 1 and is used to determine an exact probability of an event occurring. In order to calculate the normal cumulative distribution for the *Kolmogorov-Smirnov One-Sample Test*, we use the *Mean* and *Standard Deviation* derived from the observed sentence data and calculate the *z-score*":

$$z = \frac{X_1 - \mu}{\sigma} = \frac{1 - 3.41}{2.458} = -0.98$$

"We use this same formula for each of the sentence lengths."

		Cumulative frequency	Cumulative Relative Frequency			
Sentence (in years)	Frequency	Observed	Observed $[S_N(X_i)]$	*z*-scores	Predicted $[F_0(X_i)]$	$\lvert F_0(X_i) - S_N(X_i)\rvert$
1	24	24	0.24	−0.98		
2	8	32	0.32	−0.57		
3	41	73	0.73	−0.17		
4	1	74	0.74	0.24		
5	9	83	0.83	0.65		
6	9	92	0.92	1.05		
7	2	94	0.94	1.46		
8	3	97	0.97	1.87		
9	1	98	0.98	2.27		
10	0	98	0.98	2.68		
11	0	98	0.98	3.09		
12	1	99	0.99	3.49		
13	0	99	0.99	3.90		
14	0	99	0.99	4.31		
15	0	99	0.99	4.72		
16	1	10	1.00	5.12		

"Now, even though we have calculated all of these *z-scores*, we are not going to use them in any equation. They are only used to find the predicted $[F_o(X_1)]$. By consulting the standard normal distribution table,[4] we find that the $z = -0.98$ gives us $p = .1635$ which is our predicted $[F_o(X_1)]$. This calculation is continued for each sentence length. The *Absolute Value* of the difference between $S_N(X)$ and $F_0(X)$ is calculated for each sentence":

$$D = |F_0(X_1) - S_N(X_1)| = |.1635 - .24| = 0.08$$

		Cumulative frequency	Cumulative Relative Frequency			
Sentence (in years)	Frequency	Observed	Observed $[S_N(X_i)]$	*z*-scores	Predicted $[F_0(X_i)]$	$\lvert F_0(X_i) - S_N(X_i)\rvert$
1	24	24	0.24	−0.98	0.1635	0.08
2	8	32	0.32	−0.57	0.2843	0.04
3	41	73	0.73	−0.17	0.4325	0.30
4	1	74	0.74	0.24	0.5948	0.15
5	9	83	0.83	0.65	0.7422	0.09
6	9	92	0.92	1.05	0.8531	0.07
7	2	94	0.94	1.46	0.9279	0.01
8	3	97	0.97	1.87	0.9693	0.00
9	1	98	0.98	2.27	0.9884	0.01

(continued)

[4] The standard normal distribution table can be found in the appendix Table Q.

(continued)

Sentence (in years)	Frequency	Cumulative frequency Observed	Cumulative Relative Frequency Observed $[S_N(X_i)]$	*z*-scores	Predicted $[F_0(X_i)]$	$\lvert F_0(X_i) - S_N(X_i) \rvert$
10	0	98	0.98	2.68	0.9963	0.02
11	0	98	0.98	3.09	0.999	0.02
12	1	99	0.99	3.49	0.9998	0.01
13	0	99	0.99	3.90	0.9999	0.01
14	0	99	0.99	4.31	0.9999	0.01
15	0	99	0.99	4.72	0.9999	0.01
16	1	100	1.00	5.12	0.9999	0.00

"In this case, $D = 0.30$, is the maximum difference between the *Cumulative Relative Frequencies*. Since $N > 35$, a large sample approximation must be used to determine *Statistical Significance*. To help determine *Statistical Significance,* an **Alpha Level** (*a*) must be used. Using the *Alpha Level* of 0.05 and our *Sample Size* (100), we find on the *Kolmogorov-Smirnov One-Sample Test* critical values table[5] that our **Critical Value** needs to be calculated. For small Sample Sizes, those that are fewer than 35 individuals, we don't need to calculate the *Critical Value*, it is merely given to us in the table":

For $\alpha = .05$, $D \geq \frac{1.36}{\sqrt{N}}$ is significant:

$$0.30 \geq \frac{1.36}{\sqrt{100}} = 0.136$$

"Since $D = 0.30$ is greater than the *Critical Value* of 0.136, we reject the *Null Hypothesis* (i.e., the observed sentences are *Normally Distributed*). We conclude that the sentences are not *Normally Distributed*."

Dakota leaned back, admiring all the numbers splayed out before her; but the moment was short-lived as Theron inquired with a pedestrian question.

"So, hold on. What is this business of a *Critical Value*?"

Before the simplicity of the question had sunk in, she responded almost unconsciously.

"A *Critical Value* is simply a 'cut-off' point between the area in a distribution representing the *Null Hypothesis* and the area in the distribution representing the *Alternative Hypothesis*."

Almost unaware of Theron's level of understanding, she looked at the final answer and knew instinctively what it meant.

"Sentencing is not *Normally Distributed*."

Her two colleagues, themselves lost in their own thoughts, only heard a tiniest whisper of what she had said. Theron strained to hear if Dakota would say anything

[5] This table can be found in the appendix Table J.

else; sadly, she just fell silent. Unlike Theron, Michael took a more direct approach in wondering what his colleague had mumbled to herself.

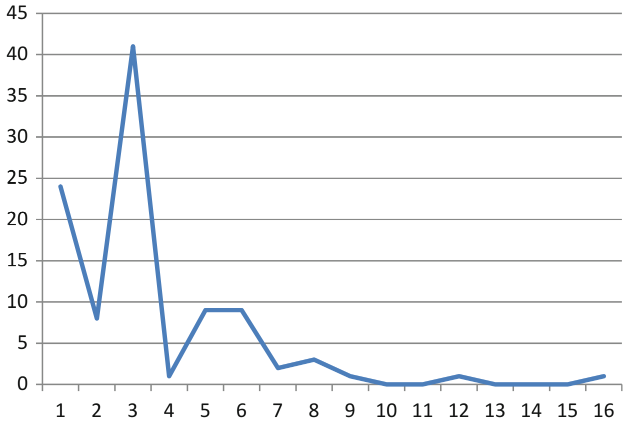
“Beg pardon?”

Dakota drew a large circle over her final answer to the *Kolmogorov-Smirnov One-Sample* equation.

Theron wearily cupped his head in his hands.

“How can we be certain of that?”

Dakota deftly erased all of the equations on the dry-erase board and very methodically began to graph out all the data associated with “Length of Sentence”; with the sentencing scores on the horizontal axis and their frequencies on the vertical axis. It took her only a few moments, but as all the data points began to line up, it was obvious that the data in no way resembled a bell.



The two men just stared in awe at the graph, as Dakota just jotted down a few notes to herself, trying not to look at the graphical elephant in the room. True to form, Michael slammed his palm into the table, breaking the pall which had settled over the conference room.

“Why didn’t we just graph the damn thing in the first place?”

Dakota quietly considered his question for a moment, trying to think of the best way to phrase a response which would not summarily enrage her colleague.

“Dr. O’Brien, graphing all of this data would hardly be prudent. After all, what if I thought the data looked *Normally Distributed*, and you disagreed? With the answer to the *Kolmogorov-Smirnov One-Sample Test* equation, there is little doubt that this set of data is anything but normal in its distribution. Shall we continue on?”

Michael shook his head, causing Dakota’s blood to run cold for a moment. She was mildly afraid of what could possibly come out of his mouth.

“I want to see it.”

Dakota looked incredulously at Michael, yet refused to yield any ground to him.

“Excuse me.”

Michael arched his eyebrow as he continued to stare at Dakota. His face was void of any emotion; he wanted her to justify why she should be the captain of their voyage together and not someone else. Dakota's face was a mask of serenity, all the while she could feel rage bubbling up inside her. She let the clock in the room fall into her periphery, internally noting that it was a little past noon. Dakota swallowed hard, rising to her feet in one elegant motion.

"As you wish."

Theron slowly slid his chair away from the table, as if he was afraid to be physically injured by the fallout from the attack that would be playing itself out between these two behemoths. Dakota kept one eye on Michael, glancing over the data set with another.

"I would be willing to say that out of all of our data, 'income' would most likely be *Normally Distributed*. Theron, would you be so kind as to conduct the *Kolmogorov-Smirnov One-Sample Test* using this data? Also, don't forget to draw out a rough little graph of the data so we can see how 'normal' it is."

Theron pulled out a sheet of paper and methodically began to work the equation with the data associated to "income." All noise in the conference room became absolutely still, with only the occasional scratching from Theron's pencil to alert others that the room was occupied. Dakota and Michael just stared at one another, silently waiting for Theron to prove or disprove Dakota's worth. After what seemed like an eternity, Theron placed his pencil down onto the table.

"I am finished."[6]

Theron slid his work to Dakota. After she had adequately perused his hard work, Dakota slid the paper to Michael, watching as his eyes ran across every number. Michael flicked the paper away, obviously annoyed with the result.

"How in blazes can you call that a *Bell-Shaped Curve*?!? Even if I smoothed out the rough spots, it looks nothing like it is supposed to."

Dakota suddenly felt her stomach unclench. She had locked horns with Michael and had proven her dominance. Hopefully, this would be the last time. Michael glanced at the rough graphical sketch one last time and then turned the full force of his annoyance onto Dakota.

"My dear lady, you seem to be acutely interested in our data falling along with a *Normal Distribution*, correct?"

Dakota nodded somberly.

"*Normal Distributions* are one of the key components which allow us to run many statistics."

Michael's face softened in triumph, as if he had been given information which could damage her credibility. He began to speak in a slow, methodical, and even tone of voice.

"So why did you waste our time on all those other distributions?"

[6] The Kolmogorov-Smirnov worked example for the income data can be found in the appendix on page 371.

Dakota fought the urge to roll her eyes at Michael's question. Although she never liked to admit it, Michael did have his insightful moments when he asked a question. The question teased at her brain, proving one thing to be true; Michael was definitely a worthy adversary. Dakota wearily slid into her chair, her mind wrapping itself around his question.

"Well Dr. O'Brien, the *Kolmogorov-Smirnov One-Sample Test* is flexible enough to be used in all instances. The test can tell you if you have a *Uniform Distribution*, a *Poisson Distribution*, an *Exponential Distribution*, or a *Normal Distribution*; you just need to run the correct version of the *Kolmogorov-Smirnov One-Sample Test* to see if you have the distribution you desire."

Michael leaned back in his chair, studying her with a very peculiar look in his eyes. Dakota felt something inside of her become agitated at this; Michael was not challenging her for dominance, he was testing the limits of her knowledge. She settled down inside herself, bracing for the possible academic onslaught to follow. Secretly, Dakota was hoping that either Michael or Theron would take over in what they were to do next; allowing her to take a momentary reprieve from being their captain. In the corner of her eye, she saw Theron fidgeting with his tie as he started shifting uncomfortably in his seat. For once in her life, the universe had granted her wish. After a great deal of fidgeting, Theron finally opened his mouth.

"Okay, there has to be an easier way to determine normality. I mean no disrespect to the people who invented the Kremlin-Vodka Test. . ."

Dakota patiently interrupted him, hoping to fend off another abusive tirade from Michael.

"Kolmogorov-Smirnov."

Theron nodded a "thank you" to Dakota for politely correcting him and continued on with his thought.

"But there has to be an easier way to calculate this sort of thing."

Dakota thought for a moment and was instantly struck by the muses of inspiration.

"Well, if you wish to know specifically whether or not your data are *Normally Distributed*, you could use **Fisher's Test for Normality of a Distribution.**[7] "

Theron scrambled to write the name down, not realizing that he was speaking as he was writing.

"Fisher, like the peanut?"

Dakota felt foolish for a moment, not entirely sure what he was referring to with the reference. She just answered his question as matter-of-factly as possible.

"All right, we will go with that."

Theron nodded as Dakota went up to the board, wiping away all traces of the *Kolmogorov-Smirnov One-Sample Test*. She wrote down the name of the test and continued on with her explanation.

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[7] Please see the appendix page 374 for a worked example for the Fisher's Test for Normality of a Distribution.

"The *Fisher's Test for Normality of a Distribution* was designed to help researchers determine if there is a significant difference between a **Frequency Distribution** for their sample data, and a **Normal Frequency Distribution** based on the same data."

Michael raised his hand. Dakota was somewhat shocked that he would offer to ask a question of her but decided not to dwell on it. She nodded in Michael's direction, allowing him to speak.

"How do you know that you are comparing two equivalent distributions?"

Dakota pointed to the data set, actually pleased that the question was relevant to the topic at hand.

"Well, this is done by focusing on the *Mean* and the *Variance*. The *Fisher's Test for Normality of a Distribution* takes your data, and compares it to a hypothetically "perfect" *Normal Distribution* using the same *Mean* and *Variance* as the data you are working with."

Michael nodded in agreement, allowing Dakota to turn her attention to Theron. Unlike Michael, Theron looked like a little boy lost in a shopping mall. Dakota reached across the table, holding his hand.

"Okay Theron, let's say I want to make the perfect Mojito."

Theron's ears perked up at the sound of this, and he suddenly became fascinated at what she had to say. Dakota smiled and continued on with her metaphor.

"Now, we know that a Mojito contains Rum, Mint Leaves, Sugar, Lime Juice, and Soda, correct?"

Theron nodded, not entirely sure where she was heading with this example.

"Even though we know what a perfect Mojito tastes like, and what ingredients go into a Mojito, we don't really know what combination of these ingredients go together to make the perfect Mojito. So, what would you do?"

Theron thought for a moment.

"Taste test."

Dakota smiled.

"Exactly. That's what the *Fisher's Test for Normality of a Distribution* does; it's like a taste test for your data. You know what you have in terms of data, and now you want to compare your data to 'perfect' data."

Michael leaned back in his chair, his arms folded over his chest. Even he was surprised at how interested he was in Dakota's lecture. He cleared his throat, signaling to all that he was about to allow the room to feed on his wisdom and his knowledge.

"So, all this test does is allow you to take the information you have from your data set, and compare it to a hypothetical 'perfect' data set which would also have the same information."

Dakota winked at him, the faint etchings of a smile crossing her face.

"That's correct."

Theron stopped writing, his mind racing with a flurry of ideas. He then started flipping furiously through his notebook.

"Wait, so how is this test different from the vodka test?"

Dakota opened her mouth to correct Theron but ultimately decided that it would be a fruitless endeavor. If he can remember the *Kolmogorov-Smirnov One-Sample Test* as "the vodka test," who was she to correct him? Dakota casually crossed her legs and leaned back into her chair.

"The *Fisher's Test for Normality of a Distribution* is specifically designed to determine whether or not you have a *Normal* or *Bell-Shaped Distribution*, while the *Kolmogorov-Smirnov One-Sample Test* can be used for a variety of distributions."

Theron scratched the side of his head.

"So how does this Fisher's thing work?"

Dakota folded her hands in her lap.

"Well, the *Fisher's Test for Normality of a Distribution* requires that you have a pretty decent-sized sample, usually larger than 50 people. Then, the test examines the *Skewness* and *Kurtosis* of your data, and combines them into an overall value. This overall value is then compared to the table value to see if your sample is significant."

Theron nodded vigorously as he continued to write down every word which Dakota uttered. Michael leaned forward in his chair, his visage hardening into a grimace.

"All right my dear, since we just spent the better part of the morning establishing that we have a *Normal Distribution* with some of our data, why would we wish to go through this process again using a test which is going to tell us something we already know?"

Dakota silently listened to Michael's words, feeling that he had an excellent point. After all, there was not a great amount of *Interval Data* contained in their data set, and even she felt as if she was beating a dead horse. Dakota thought for a moment, deciding that it was best to concede to Michael's wishes.

"I agree with you. We already discussed a perfectly wonderful test for distribution, and there really is no point in reinventing the wheel. I just wanted all of us to have this information should it be needed in the future. Should we continue on?"

Theron smiled, allowing Dakota to move on to another topic.

"Okay, now that we have pretty much exhausted our data set in terms of Normality, there is another construct we should probably examine. I think we should look at **Randomness**."

Theron's ears perked up.

"*Randomness*? You mean like my questions or Robin's pop culture references?"

Dakota really could not disagree.

"That's not entirely the same thing. In statistics, *Randomness* is used to determine whether or not your data were collected from random groups of people."

Theron shrugged his shoulders.

"And why would we care about that?"

Dakota pointed to the data set.

"Well, the reason we care is because we want our data to be as realistic as possible. After all, the goal of quite a few statistical tests is to allow us to make inferences about a particular population. Think about it, the Governor needs us to come up with some platforms which could be applicable to all sex offenders living in this State. If our data has *Randomness* to it, then it will better strengthen our recommendations."

Theron nodded in understanding, creating a moment of silence for Dakota to collect her thoughts. Suddenly, a moment of insight struck her like a bolt of lightning. Dakota instinctively rose to her feet and began writing on the dry-erase board.

"You know, there is one test I have used quite a few times. It's not a very difficult test to run, and it can be massively helpful when it seems that nothing else will work. It's known as the **One-Sample Runs Test of Randomness**."

Theron's eyes widened at the mere mention of the test.

"That sounds potentially complicated. Please tell me that this test is not given to us by the good people of the former Soviet Union."

Dakota shook her head, as Michael folded his arms and put his head down on the table. Apparently, Theron and Dakota were no longer stimulating to him, and Michael decided that his time would best be spent sleeping. Rather than be insulted, Dakota hoped that he would sleep through the rest of their meeting. She turned towards the board and began to erase the remnants of the now infamous "Length of Sentence" graph.

"*The One-Sample Runs Test of Randomness* allows someone to make a determination about the overall population, using only the information given in the sample. By doing this, you can see if a data set has *Orthogonal Data*."

Theron nodded his head, surprised that he was able to follow along with what Dakota was saying.

"And if the data are *Orthogonal*, then that increases the likelihood of it being *Normally Distributed*?"

Dakota winked at him.

"That's the idea."

Theron nodded in understanding.

"Okay, so what do we do first?"

Dakota started writing numbers on the dry-erase board.

"Well, first thing we need to do is identify a **Run**."

Theron folded his arms across his chest.

"I am guessing that this test does not involve exercise."

With that, Michael bolted upright in his seat; glowering at Theron with a level of rage that easily raised the temperature of the room by a few degrees. True to form, Theron remained oblivious to Michael, choosing to focus on Dakota and the wisdom she imparted. Dakota just continued to write on the board.

"No Theron, no exercise required. A *Run* is simply a series of repeated symbols which are usually followed by a different set of symbols."

Dakota stepped away from the dry-erase board, allowing everyone to see what she had done. It was nothing all that impressive, just a series of plus and minus signs in a haphazard sequence.

$$+ + - - - + + + + - - + - - - + + + + + - - - + - + + +$$

Michael arched his brow in confusion, while Theron stared directly at the board. After seeing the series of plus and minus signs, he felt more confused than ever.

"What's with all the symbols? Are we reading a Dan Brown novel now? Because I have to tell you, I was definitely not a fan of 'The Da Vinci Code'."

With that comment, Dakota shot Michael a disapproving look that personified the saying "if looks could kill."

"No, Michael! The purpose of this test is simply to test for randomness in the sample data."

Feeling flustered, Dakota quickly glanced over the *Data Set*. Before realizing it, her mouth opened and her stream of consciousness flowed out into the room.

"Okay, that large sample size means we are going to have to use a different equation."

With that, all ears in the room perked up. Suddenly, Dakota realized what slipped out of her mouth. Her mouth became very dry as she tried to correct the mistake.

"Well, for many of these statistical procedures, there are different protocols depending on if you have a small or a large sample size. For the *Runs Test*, if you have a *Sample Size* greater than 20, you need to use some equations to help approximate a dromedary."

Theron rolled his eyes as Michael leaned back into his chair, a grin running from ear to ear. Dakota silently cursed Michael and continued on with her explanation.

"I am sorry, I mean a *Bell-Shaped Curve*."

Theron, still visibly annoyed, slid down into his chair.

"Well, what happens if you have a *Sample Size* that is less than 20?"

Dakota thought for a moment.

"Then you would just count the number of plusses and minuses to see if they were occurring in random order and look to the *Runs Test* Table to obtain the *Critical Value*."

Theron just mouthed the words "*Critical Value*" to himself, clearly unaware of what she was talking about. Dakota steadied herself and continued to explain this in the best possible terms.

"In statistics, the **Calculated Value** is the number we get whenever we calculate an equation. In order to determine whether or not something is *Statistically Significant*, we have to compare it to the *Critical Value*. Oftentimes, the *Critical Value* is obtained by using your **Degrees of Freedom**, and seeing what the value is on some type of table. *Degrees of Freedom* (*df*) indicates how much freedom there is for the numbers to vary."

Dakota paused for a moment, just to see if Theron was following along. Although he was furiously attempting to write everything she was saying, he seemed to understand her explanation. So, after taking a deep breath, she continued on.

"Typically, if your *Calculated Value* is a number higher than your *Critical Value*, we say that your answer is *Statistically Significant* and **Reject the Null Hypothesis**. Got it?"

Theron placed his pencil on the table, his mind lost in thought.

"So, for the *Runs Test*, a *Statistically Significant* result would mean that the plusses and the minuses do not occur in random order?"

Dakota nodded, allowing Theron to breathe a sigh of relief. Michael just spun about in his chair.

"It's nice to know that they will let any moron into graduate school nowadays."

Theron's face stiffened as he turned to his colleague.

"Pot, meet kettle."

Frustrated with her male colleagues, Dakota began working through the steps necessary for the *One-Sample Runs Test of Randomness* for the Ethnicity/Race data.

n_1 = *the number of one type of data*
n_2 = *the number of another kind of data*
$N = n_1 + n_2$ = *the total number of data points*
r = *the number of runs*
μ_r = *the mean for a larger sample than 20 individuals*
σ_r = *the standard deviation for a larger sample than 20 individuals*

"We have a sample of more than 20 individuals which means that we must use these formulas":

$$Mean = \mu_r = \frac{2n_1n_2}{n_1 + n_2} + 1$$

$$Standard\ Deviation = \sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}}$$

$$z = \frac{r - \mu_r}{\sigma_r}$$

Suddenly, Dakota realized that the verbal sparring between Michael and Theron had stopped and that their eyes were now targeted onto her. In an attempt to avoid losing them completely, Dakota continued to explain some of her calculations.

"The data used for the *One-Sample Runs Test* is a modified version of the race/ethnicity data. Here, we were interested in whether or not the collection of Hispanic versus Non-Hispanic offenders was random. First, we denote each Hispanic offender with a plus sign (+). All Non-Hispanic offenders receive a minus sign (−). A "Runs" is a collection of the same type of offender in a row. In other words, it is when the same sign occurs consecutively, so that if we have 3 plus signs then one minus sign, we have two *Runs*."

Offender	Ethnicity		Runs	Offender	Ethnicity		Runs
1	Non-Hispanic	-	1	51	Non-Hispanic	-	
2	Non-Hispanic	-		52	Hispanic	+	30
3	Non-Hispanic	-		53	Non-Hispanic	-	31
4	Non-Hispanic	-		54	Non-Hispanic	-	
5	Non-Hispanic	-		55	Hispanic	+	32
6	Hispanic	+	2	56	Hispanic	+	
7	Non-Hispanic	-	3	57	Hispanic	+	
8	Non-Hispanic	-		58	Hispanic	+	
9	Hispanic	+	4	59	Hispanic	+	
10	Non-Hispanic	-	5	60	Non-Hispanic	-	33
11	Non-Hispanic	-		61	Hispanic	+	34
12	Hispanic	+	6	62	Non-Hispanic	-	35
13	Hispanic	+		63	Non-Hispanic	-	
14	Non-Hispanic	-	7	64	Hispanic	+	36
15	Hispanic	+	8	65	Non-Hispanic	-	37
16	Non-Hispanic	-	9	66	Non-Hispanic	-	
17	Hispanic	+	10	67	Hispanic	+	38
18	Hispanic	+		68	Hispanic	+	
19	Non-Hispanic	-	11	69	Non-Hispanic	-	39
20	Hispanic	+	12	70	Hispanic	+	40
21	Hispanic	+		71	Non-Hispanic	-	41
22	Non-Hispanic	-	13	72	Hispanic	+	42
23	Hispanic	+	14	73	Hispanic	+	
24	Hispanic	+		74	Non-Hispanic	-	43
25	Non-Hispanic	-	15	75	Non-Hispanic	-	
26	Non-Hispanic	-		76	Non-Hispanic	-	
27	Hispanic	+	16	77	Non-Hispanic	-	
28	Hispanic	+		78	Non-Hispanic	-	
29	Non-Hispanic	-	17	79	Hispanic	+	44
30	Non-Hispanic	-		80	Non-Hispanic	-	45
31	Non-Hispanic	-		81	Hispanic	+	46
32	Non-Hispanic	-		82	Hispanic	+	
33	Hispanic	+	18	83	Non-Hispanic	-	47
34	Hispanic	+		84	Hispanic	+	48
35	Hispanic	+		85	Non-Hispanic	-	49
36	Non-Hispanic	-	19	86	Hispanic	+	50
37	Hispanic	+	20	87	Non-Hispanic	-	51
38	Non-Hispanic	-	21	88	Non-Hispanic	-	
39	Non-Hispanic	-		89	Hispanic	+	52
40	Hispanic	+	22	90	Non-Hispanic	-	53
41	Hispanic	+		91	Non-Hispanic	-	
42	Non-Hispanic	-	23	92	Hispanic	+	54
43	Hispanic	+	24	93	Non-Hispanic	-	55
44	Hispanic	+		94	Non-Hispanic	-	
45	Non-Hispanic	-	25	95	Hispanic	+	56
46	Hispanic	+	26	96	Non-Hispanic	-	57
47	Non-Hispanic	-	27	97	Non-Hispanic	-	
48	Hispanic	+	28	98	Hispanic	+	58
49	Hispanic	+		99	Non-Hispanic	-	59
50	Non-Hispanic	-	29	100	Hispanic	+	60

"Second, the number of *Runs* needs to be determined. In this case, $r = 60$.

$$n_1 = 47\ (\textit{the number of Hispanic offenders})$$
$$n_2 = 53\ (\textit{the number of Non} - \textit{Hispanic offenders})$$

In order to determine if $r = 60$ is random, we must now solve $z = \frac{r-\mu_r}{\sigma_r}$:

$$\mu_r = \frac{2n_1n_2}{n_1+n_2} + 1 = \frac{2(47)(53)}{47+53} + 1 = 50.82$$

$$\sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}} = \sqrt{\frac{2(47)(53)[2(47)(53) - 47 - 53]}{(47+53)^2(47+53-1)}}$$

$$= \sqrt{\frac{(4982)(4882)}{(10000)(99)}} = \sqrt{\frac{24322124}{990000}} = \sqrt{24.5678} = 4.9566$$

$$z = \frac{r - \mu_r}{\sigma_r} = \frac{60 - 50.82}{4.9566} = 1.8521$$

With the *One-Sample Runs Test*, a hypothesis can be made about the direction of the deviation from randomness. With the current test, no such hypothesis has been made, so a two-tailed region of rejection will be used. If a direction is hypothesized, only use a one-tailed region of rejection."

Dakota, without turning around to check for understanding, simply continued on with her monologue.

"With $z = 1.8521$, the corresponding *Alpha Level* or p-value[8] (p) for a two-tailed test is 0.0644. Since $p = 0.0644$ is greater than $\alpha = 0.05$, we do not reject the *Null Hypothesis* (i.e., the order of the data collection of Hispanic versus Non-Hispanic offenders was random). We conclude that the order of collection of Hispanic versus Non-Hispanic offenders was random."

Dakota could feel as if her energy was being slowly sapped by the inability of her colleagues to follow her very well-explained train of thought. Her eyes looked towards the clock, while her mind wondered when Robin would arrive. Hopefully, Robin would be able to enlighten them as to how the data were collected. Until then, all of their work was purely speculative. Dakota looked towards Michael and Theron, both of whom were contently sitting in their chairs. Since neither of them seemed all that interested to take over the discussion, Dakota took the helm once more.

"Okay, we should probably determine whether or not our data are **Symmetrical**."

---

[8] This value can be found in the One-Sample Runs Test table in the appendix Table F.

Dakota then looked towards Theron, waiting for him to utter some snide comment or remark. However, he just sat there in absolute stillness. Hearing no one say a word, Dakota began to draw different figures on the dry-erase board.

"In statistics, symmetry tells you if one half of your distribution matches the other half of your distribution."

She paused for questions from her colleagues, still nothing.

"In order to have a normal distribution, you need your data to be symmetrical as well. Therefore, the next test we should discuss is the *Test for Distributional Symmetry*."

Theron looked as if his favorite football team had just been beaten by their archrivals.

"Oh good, more math."

Dakota continued writing on the dry-erase board, feeling Theron's confusion press down upon her. She hoped that her explanation would help to alleviate his confusion or that it would subside before Michael was forced to intervene.

"The *Test for Distributional Symmetry* tells you if your data can fit into the *Bell-Shaped Distribution*, by allowing you to see if both sides of your hypothesized distribution are symmetrical. The *Test for Distributional Symmetry* is a great little test which tells you whether or not your data are *Skewed*."

Michael again leaned forward in his chair attempting to understand the purpose of another test.

"Why would you care about this?"

Dakota quickly sketched a *Bell-Shaped Curve*, dividing it into two sections.

"Well, in order to look like a bell, both sides of the distribution have to look the same."

Dakota saw the confusion in her colleagues; after a few deep breaths, her mind was able to formulate something which might help.

"Okay, perhaps this will help. Suppose that tonight when we are done here you get tacos for dinner. However, instead of getting your tacos the regular way with all the good stuff in the middle of the taco shell, it is all piled up on one side or the other. With all of the taco meat, lettuce, cheese, sour cream, and salsa piled up on one side, you are going to get a few bites of just taco shell and then much of your filling will end up on your plate or in your lap. How many of you would be excited about having 'lopsided tacos'?"

A resounding head shake "NO" was evident in the room and Theron has his 2 cents to expand the metaphor.

"So, essentially, we would have gotten our tacos from that taco joint around the corner?"

Michael, right on cue, shot Theron an odious look, and Dakota pointed a distressed look towards Theron.

"But I like that taco joint!"

Dakota, unable to take the scene anymore, diverted attention back to the matter at hand.

"So, the *Test for Distributional Symmetry* is much the same way. We want the filling, most of the data, distributed evenly throughout the middle of the distribution. We do not want it all piled up on one side or the other."

Michael and Theron looked at one another, each starting to understand the procedure with a little more clarity. Dakota scanned at the data set and pointed to the "Age" category and began writing some of the numbers on the dry-erase board.

	Age
1	33
2	20
3	73
4	54
5	66
6	57
7	51
8	48
9	73
10	54

"Well, let's look at the 'Age' category for the females in our *Data Set*; after all, in order to run the *Test for Distributional Symmetry*, you need to take an *Interval Data* set and break them into smaller groups. We have a total of 10 people. Now, each group is going to contain three numbers, or **Triples**."

Dakota began by writing an equation on the dry-erase board:

$$\frac{N(N-1)(N-2)}{6}$$

"Okay, here is the formula we need to figure out how many *Triples* we need. First, N, again, is the total number of offenders at whom we are looking. So, if we use this formula we find that we need":

$$\frac{10(10-1)(10-2)}{6}$$

$$\frac{10(9)(8)}{6}$$

$$\frac{720}{6}$$

$$120$$

"Well, it appears as though we will have 120 *Triples*."

Theron started writing the numbers onto his legal pad. He was dividing each of the *Triples* into three different categories: Right *Triples*, Left *Triples*, and Neither. Michael and Dakota credulously watched him jot down all of these *Triples* with deft precision. Theron easily placed all of the numbers for the "Age" category into their organized little categories. As Theron began working, Robin interjected.

"Well, this is going to take forever! Anyone for a fresh cup of joe?"

All three consultants looked at Robin as though she had just insulted them. Despite the glare from her colleagues, Robin slipped out of the conference room. Theron continued to work on his legal pad. Theron placed three x's at the top of his list. X_i was the first offender's age, X_j was the second offender's age, and X_k was the third offender's age. All three together make up one *Triple*.

After Robin had made her rounds throughout the entire campaign office chatting with unwitting campaigners about pop culture references, she arrived back at the conference room with a renewed glow on her face.

"Sorry guys, that took way longer than I had anticipated."

Theron shot Robin a typical glance.

"It's alright, I had time to arrange all the data and calculate the *Median* and *Mean* for each of the *Triples*."

Robin's glow began to fade.

"Wait a minute; you didn't say anything about *Medians* or *Means*.[9] I leave to 2 seconds and. . ."

Dakota stopped her dead in her tracks.

"Robin, we needed to calculate the *Median* and *Mean* for each of the *Triples* because they are used to determine the direction which we will need later. When the *Mean* is greater than the *Median*, we classify that direction as 'Right.' When the *Mean* is less than the *Median*, we classify that direction as 'Left.' If they are equal, obviously, it is 'Neither'."

Theron pushed the legal pad towards Robin so she could see what Dakota was talking about.

	Triples = 84	X_i	X_j	X_k	Median	Mean	Direction
1	(X1, X2, X3)	33	20	73	33	42.00	Right
2	(X1, X2, X4)	33	20	54	33	35.67	Right
3	(X1, X2, X5)	33	20	66	33	39.67	Right
4	(X1, X2, X6)	33	20	57	33	36.67	Right
5	(X1, X2, X7)	33	20	51	33	34.67	Right
6	(X1, X2, X8)	33	20	48	33	33.67	Right
7	(X1, X2, X9)	33	20	73	33	42.00	Right
8	(X1, X2, X10)	33	20	54	33	35.67	Right
9	(X1, X3, X4)	33	73	54	54	53.33	Left
10	(X1, X3, X5)	33	73	66	66	57.33	Left
11	(X1, X3, X6)	33	73	57	57	54.33	Left
12	(X1, X3, X7)	33	73	51	51	52.33	Right
13	(X1, X3, X8)	33	73	48	48	51.33	Right
14	(X1, X3, X9)	33	73	73	73	59.67	Left
15	(X1, X3, X10)	33	73	54	54	53.33	Left
16	(X1, X4, X5)	33	54	66	54	51.00	Left
17	(X1, X4, X6)	33	54	57	54	48.00	Left
18	(X1, X4, X7)	33	54	51	51	46.00	Left

(continued)

[9] See Chap. 2 page 18 for detailed descriptions of how to calculate Median and Mean.

(continued)

	Triples = 84	X_i	X_j	X_k	Median	Mean	Direction
19	(X1, X4, X8)	33	54	48	48	45.00	Left
20	(X1, X4, X9)	33	54	73	54	53.33	Left
21	(X1, X4, X10)	33	54	54	54	47.00	Left
22	(X1, X5, X6)	33	66	57	57	52.00	Left
23	(X1, X5, X7)	33	66	51	51	50.00	Left
24	(X1, X5, X8)	33	66	48	48	49.00	Right
25	(X1, X5, X9)	33	66	73	66	57.33	Left
26	(X1, X5, X10)	33	66	54	54	51.00	Left
27	(X1, X6, X7)	33	57	51	51	47.00	Left
28	(X1, X6, X8)	33	57	48	48	46.00	Left
29	(X1, X6, X9)	33	57	73	57	54.33	Left
30	(X1, X6, X10)	33	57	54	54	48.00	Left
31	(X1, X7, X8)	33	51	48	48	44.00	Left
32	(X1, X7, X9)	33	51	73	51	52.33	Right
33	(X1, X7, X10)	33	51	54	51	46.00	Left
34	(X1, X8, X9)	33	48	73	48	51.33	Right
35	(X1, X8, X10)	33	48	54	48	45.00	Left
36	(X1, X9, X10)	33	73	54	54	53.33	Left
37	(X2, X3, X4)	20	73	54	54	49.00	Left
38	(X2, X3, X5)	20	73	66	66	53.00	Left
39	(X2, X3, X6)	20	73	57	57	50.00	Left
40	(X2, X3, X7)	20	73	51	51	48.00	Left
41	(X2, X3, X8)	20	73	48	48	47.00	Left
42	(X2, X3, X9)	20	73	73	73	55.33	Left
43	(X2, X3, X10)	20	73	54	54	49.00	Left
44	(X2, X4, X5)	20	54	66	54	46.67	Left
45	(X2, X4, X6)	20	54	57	54	43.67	Left
46	(X2, X4, X7)	20	54	51	51	41.67	Left
47	(X2, X4, X8)	20	54	48	48	40.67	Left
48	(X2, X4, X9)	20	54	73	54	49.00	Left
49	(X2, X4, X10)	20	54	54	54	42.67	Left
50	(X2, X5, X6)	20	66	57	57	47.67	Left
51	(X2, X5, X7)	20	66	51	51	45.67	Left
52	(X2, X5, X8)	20	66	48	48	44.67	Left
53	(X2, X5, X9)	20	66	73	66	53.00	Left
54	(X2, X5, X10)	20	66	54	54	46.67	Left
55	(X2, X6, X7)	20	57	51	51	42.67	Left
56	(X2, X6, X8)	20	57	48	48	41.67	Left
57	(X2, X6, X9)	20	57	73	57	50.00	Left
58	(X2, X6, X10)	20	57	54	54	43.67	Left
59	(X2, X7, X8)	20	51	48	48	39.67	Left
60	(X2, X7, X9)	20	51	73	51	48.00	Left
61	(X2, X7, X10)	20	51	54	51	41.67	Left
62	(X2, X8, X9)	20	48	73	48	47.00	Left
63	(X2, X8, X10)	20	48	54	48	40.67	Left
64	(X2, X9, X10)	20	73	54	54	49.00	Left
65	(X3, X4, X5)	73	54	66	66	64.33	Left

(continued)

(continued)

	Triples = 84	X_i	X_j	X_k	Median	Mean	Direction
66	(X3, X4, X6)	73	54	57	57	61.33	Right
67	(X3, X4, X7)	73	54	51	54	59.33	Right
68	(X3, X4, X8)	73	54	48	54	58.33	Right
69	(X3, X4, X9)	73	54	73	73	66.67	Left
70	(X3, X4, X10)	73	54	54	54	60.33	Right
71	(X3, X5, X6)	73	66	57	66	65.33	Left
72	(X3, X5, X7)	73	66	51	66	63.33	Left
73	(X3, X5, X8)	73	66	48	66	62.33	Left
74	(X3, X5, X9)	73	66	73	73	70.67	Left
75	(X3, X5, X10)	73	66	54	66	64.33	Left
76	(X3, X6, X7)	73	57	51	57	60.33	Right
77	(X3, X6, X8)	73	57	48	57	59.33	Right
78	(X3, X6, X9)	73	57	73	73	67.67	Left
79	(X3, X6, X10)	73	57	54	57	61.33	Right
80	(X3, X7, X8)	73	51	48	51	57.33	Right
81	(X3, X7, X9)	73	51	73	73	65.67	Left
82	(X3, X7, X10)	73	51	54	54	59.33	Right
83	(X3, X8, X9)	73	48	73	73	64.67	Left
84	(X3, X8, X10)	73	48	54	54	58.33	Right
85	(X3, X9, X10)	73	73	54	73	66.67	Left
86	(X4, X5, X6)	54	66	57	57	59.00	Right
87	(X4, X5, X7)	54	66	51	54	57.00	Right
88	(X4, X5, X8)	54	66	48	54	56.00	Right
89	(X4, X5, X9)	54	66	73	66	64.33	Left
90	(X4, X5, X10)	54	66	54	54	58.00	Right
91	(X4, X6, X7)	54	57	51	54	54.00	Neither
92	(X4, X6, X8)	54	57	48	54	53.00	Left
93	(X4, X6, X9)	54	57	73	57	61.33	Right
94	(X4, X6, X10)	54	57	54	54	55.00	Right
95	(X4, X7, X8)	54	51	48	51	51.00	Neither
96	(X4, X7, X9)	54	51	73	54	59.33	Right
97	(X4, X7, X10)	54	51	54	54	53.00	Left
98	(X4, X8, X9)	54	48	73	54	58.33	Right
99	(X4, X8, X10)	54	48	54	54	52.00	Left
100	(X4, X9, X10)	54	73	54	54	60.33	Right
101	(X5, X6, X7)	66	57	51	57	58.00	Right
102	(X5, X6, X8)	66	57	48	57	57.00	Neither
103	(X5, X6, X9)	66	57	73	66	65.33	Left
104	(X5, X6, X10)	66	57	54	57	59.00	Right
105	(X5, X7, X8)	66	51	48	51	55.00	Right
106	(X5, X7, X9)	66	51	73	66	63.33	Left
107	(X5, X7, X10)	66	51	54	54	57.00	Right
108	(X5, X8, X9)	66	48	73	66	62.33	Left
109	(X5, X8, X10)	66	48	54	54	56.00	Right
110	(X5, X9, X10)	66	73	54	66	64.33	Left
111	(X6, X7, X8)	57	51	48	51	52.00	Right
112	(X6, X7, X9)	57	51	73	57	60.33	Right

(continued)

(continued)

	Triples = 84	X_i	X_j	X_k	Median	Mean	Direction
113	(X6, X7, X10)	57	51	54	54	54.00	Neither
114	(X6, X8, X9)	57	48	73	57	59.33	Right
115	(X6, X8, X10)	57	48	54	54	53.00	Left
116	(X6, X9, X10)	57	73	54	57	61.33	Right
117	(X7, X8, X9)	51	48	73	51	57.33	Right
118	(X7, X8, X10)	51	48	54	51	51.00	Neither
119	(X7, X9, X10)	51	73	54	54	59.33	Right
120	(X8, X9, X10)	48	73	54	54	58.33	Right

Dakota silently admired her work as the wheels in Theron's head began to turn.

"So, how did you figure out which direction the *Triple* went and what do we do with the triplets now?"

Dakota smiled.

"*Triples*, not triplets. We determine right, left, and neither *Triples* based upon how the *Mean* compares to the *Median* using this coding system.

Right triple	Median < Mean
Left triple	Median > Mean
Neither	Median = Mean

And once you are done, you just plug the numbers into this equation":

$$\text{T} = \text{\# of right triples} - \text{\# of left triples}$$

$$\text{B}_\text{i} = \text{\# of right triples involving X}_\text{i} - \text{\# of left triples involving X}_\text{i}$$

$$\text{B}_\text{jk} = \text{\# of right triples involving X}_\text{j}\text{ and X}_\text{k} - \text{\# of left triples involving X}_\text{j}\text{ and X}_\text{k}$$

$$\sigma_T^2 = \frac{(N-3)(N-4)}{(N-1)(N-2)}\sum_{i=1}^{N} B_i^2 + \frac{N-3}{N-4}\sum_{1\le j\le k\le N} B_{jk}^2 + \frac{N(N-1)(N-2)}{6} - \left[1 - \frac{(N-3)(N-4)(N-5)}{N(N-1)(N-2)}\right]T^2$$

"Okay, so we know the N, let's fill that in right away. Then, we can do some of the math to help make this thing a bit more manageable":

$$\sigma_T^2 = \frac{(10-3)(10-4)}{(10-1)(10-2)}\sum_{i=1}^{N} B_i^2 + \frac{10-3}{10-4}\sum_{1\le j<k\le N} B_{jk}^2 + \frac{10(10-1)(10-2)}{6} - \left[1 - \frac{(10-3)(10-4)(10-5)}{10(10-1)(10-2)}\right]T^2$$

$$\sigma_T^2 = \frac{(7)(6)}{(9)(8)}\sum_{i=1}^{N} B_i^2 + \frac{7}{6}\sum_{1\le j<k\le N} B_{jk}^2 + \frac{10(9)(8)}{6} - \left[1 - \frac{(7)(6)(5)}{10(9)(8)}\right]T^2$$

$$\sigma_T^2 = \frac{42}{72}\sum_{i=1}^{N} B_i^2 + \frac{7}{6}\sum_{1\le j<k\le N} B_{jk}^2 + \frac{720}{6} - \left[1 - \frac{210}{720}\right] T^2$$

"We can easily calculate the T at the end of the equation; it is simply the number of right *Triples* minus the number of left *Triples*. We can just count those from the previous work Theron did. We have 44 right *Triples* and 71 left *Triples*. When we subtract those, we get −27. When we **Square** −27, we get 729":

$$\sigma_T^2 = \frac{42}{72}\sum_{i=1}^{N} B_i^2 + \frac{7}{6}\sum_{1\le j<k\le N} B_{jk}^2 + \frac{720}{6} - \left[1 - \frac{210}{720}\right] - 27^2$$

$$\sigma_T^2 = \frac{42}{72}\sum_{i=1}^{N} B_i^2 + \frac{7}{6}\sum_{1\le j<k\le N} B_{jk}^2 + \frac{720}{6} - \left[1 - \frac{210}{720}\right](729)$$

Next, we must determine the sum of B_i *Squared* ($\sum B_i^2$):

$$B_i = \text{Number of Right Triples involving} X_i - \text{Number of Left Triples involving} X_i$$

Dakota attempted to help her colleagues with the next step.

"Now, in order to find B_i, we need to take each offender and add up how many of each *Triple* they are involved in.

	X_i	Right triples	Left triples	Neither triples	Total	B_i	B_i^2
X1	33	13	23	0	36	−10	100
X2	20	8	28	0	36	−20	400
X3	73	13	23	0	36	−10	100
X4	54	14	20	2	36	−6	36
X5	66	11	24	1	36	−13	169
X6	57	14	19	3	36	−5	25
X7	51	16	16	4	36	0	0
X8	48	16	17	3	36	−1	1
X9	73	13	23	0	36	−10	100
X10	54	14	20	2	36	−6	36
							$\sum B_i^2 = 967$

"Then we determine $\sum B_{jk}^2$ by comparing pairs of offenders to see whether the pair is involved in right, left, or neither."

		X_j	X_k	Right	Left	Neither	Total	B_{jk}	B_{jk}^2
X1	X2	33	20	8	0	0	8	8	64
X1	X3	33	73	3	5	0	8	−2	4
X1	X4	33	54	1	7	0	8	−6	36

(continued)

(continued)

		X_j	X_k	Right	Left	Neither	Total	B_{jk}	B_{jk}^2
X1	X5	33	66	2	6	0	8	−4	16
X1	X6	33	57	1	7	0	8	−6	36
X1	X7	33	51	3	5	0	8	−2	4
X1	X8	33	48	4	4	0	8	0	0
X1	X9	33	73	3	5	0	8	−2	4
X1	X10	33	54	1	7	0	8	−6	36
X2	X3	20	73	1	7	0	8	−6	36
X2	X4	20	54	1	7	0	8	−6	36
X2	X5	20	66	1	7	0	8	−6	36
X2	X6	20	57	1	7	0	8	−6	36
X2	X7	20	51	1	7	0	8	−6	36
X2	X8	20	48	1	7	0	8	−6	36
X2	X9	20	73	1	7	0	8	−6	36
X2	X10	20	54	1	7	0	8	−6	36
X3	X4	73	54	4	4	0	8	0	0
X3	X5	73	66	0	8	0	8	−8	64
X3	X6	73	57	4	4	0	8	0	0
X3	X7	73	51	5	3	0	8	2	4
X3	X8	73	48	5	3	0	8	2	4
X3	X9	73	73	0	8	0	8	−8	64
X3	X10	73	54	4	4	0	8	0	0
X4	X5	54	66	4	4	0	8	0	0
X4	X6	54	57	4	3	1	8	1	1
X4	X7	54	51	3	3	2	8	0	0
X4	X8	54	48	3	4	1	8	−1	1
X4	X9	54	73	4	4	0	8	0	0
X4	X10	54	54	4	4	0	8	0	0
X5	X6	66	57	3	4	1	8	−1	1
X5	X7	66	51	4	4	0	8	0	0
X5	X8	66	48	4	3	1	8	1	1
X5	X9	66	73	0	8	0	8	−8	64
X5	X10	66	54	4	4	0	8	0	0
X6	X7	57	51	4	2	2	8	2	4
X6	X8	57	48	3	4	1	8	−1	1
X6	X9	57	73	4	4	0	8	0	0
X6	X10	57	54	4	3	1	8	1	1
X7	X8	51	48	4	2	2	8	2	4
X7	X9	51	73	5	3	0	8	2	4
X7	X10	51	54	3	3	2	8	0	0
X8	X9	48	73	5	3	0	8	2	4
X8	X10	48	54	3	4	1	8	−1	1
X9	X10	73	54	4	4	0	8	0	0
									$\sum B_{jk}^2 = 711$

"We can then solve σ_T^2 by adding in the two numbers we just calculated":

$$\sigma_T^2 = \frac{42}{72}\sum_{i=1}^{N} B_i^2 + \frac{7}{6}\sum_{1\le j<k\le N} B_{jk}^2 + \frac{720}{6} - \left[1 - \frac{210}{720}\right](729)$$

$$\sigma_T^2 = \frac{42}{72}(967) + \frac{7}{6}(711) + \frac{720}{6} - \left[1 - \frac{210}{720}\right](729)$$

$$\sigma_T^2 = 564.08 + 829.5 + 120 - 516.38$$

$$\sigma_T^2 = 997.2$$

Finally, we calculate z using this formula:

$$Z = \frac{T}{\sigma_T}$$

$$Z = \frac{T}{\sigma_T} = \frac{-27}{\sqrt{997.2}} = -0.8550$$

"With $z = -0.8550$, the corresponding p-value for a two-tailed test is 0.39533. Since $p = 0.39533$ is greater than $\alpha = 0.05$, we do not reject the *Null Hypothesis* (i.e., the data are from the same symmetrical distribution). We conclude that the data are generated from a symmetrical distribution."

Without skipping a beat, Michael leaned forward in an attempt to cut short their mathematical adventure.

"Okay young lady, any other statistical tests you wish to berate us with today?"

Dakota's eyes scanned the data set, looking to see if there was anything else that caught her fancy. While she would never admit it, she was also feeling a strong urge to throw up her hands and walk away. Dakota tried to shake off the feeling of hopelessness, when she saw something out of the corner of her eye. She pulled the data set close to her face.

"*Outliers*."

Michael tilted his head in confusion.

"Excuse me?"

Dakota let the data set fall away from her face, allowing her to meet his gaze head on.

"I am wondering about some of these extreme scores we have and if these *Outliers* should be rejected."

Theron folded his arms onto the table and buried his face into his shirtsleeves. Even though his voice was horrifically muffled, Dakota could hear what he was trying to say.

"Please tell me this just involves circling really big numbers."

Dakota wanted to be as soothing as possible and empathized with Theron's frustration. However, in this instance it was better to be cautious in all regards. She leaned forward, whispering in Theron's ear.

"It's all right; this test really isn't that bad."

Michael reached his arms towards the sky and folded his hands behind his head.

"And what test is that."

Dakota lifted her face away from Theron.

"The **Dixon Test for Outliers.**[10] "

Theron's head snapped up from his comfortable hiding place.

"Mason-Dixon? Is the south rising again?"

Michael opened his mouth to scold Theron, yet something stopped him. Apparently the good doctor could appreciate a well-timed pun. Dakota just continued on, as if nothing has happened.

"The *Dixon Test for Outliers* is used to see if there is a significant difference between an extreme value and the rest of the *Data Set*. It's a fairly easy test to calculate, and it is usually used sparingly."

Michael cocked his head.

"Sparingly?"

Dakota nodded.

"That's correct. Typically, the *Dixon Test for Outliers* is only computed once for a *Data Set*."

"And what is the point?"

Dakota pointed to the "General Aggression Scores" section of the data.

"Well, if one of these scores is significantly higher than all the others. The *Null Hypothesis* is that the outlier score is in line with other scores in the sample, while *Rejecting the Null Hypothesis* means that the score is an extreme value."

Michael smiled grimly.

"Haven't we already determined that the 'Aggression' scores are not *Normally Distributed*? So, why do we care about *Outliers*?"

Dakota knew he was correct.

"You are right, Dr. O'Brien. We need Robin to tell us how this data was gathered. Until then, I feel as if we are just spinning our wheels."

As soon as these words left Dakota's lips, the doors for the conference room flew open.

"I have some bad news."

All three consultants spun in their chairs to see Robin standing at the door. She was still wearing her usual multicolored shirtwaist dress, but her once carefree attitude was tainted with bureaucracy. She stomped over to a seat next to Theron and dropped herself into it, dropping her purse with an exasperated sigh. Robin tried to collect herself for a moment, oblivious to the fact that the other consultants were anxiously hovering over her like gnats. She casually glanced at Theron's notebook to see what was covered in her absence. Her mind was overwhelmed by all the equations spread out before her, yet her eyes were inexorably drawn to a word scrawled at the top of Theron's notebook. She mouthed the word with a fair degree of disgust.

[10] Please see the appendix page 379 for a worked example for the Dixon Test for Outliers.

"Dromedary?"

Instantly, she turned to Michael.

"Camel. Would it have killed you to say camel?"

Theron smiled as his mind added this bit of knowledge to its storehouse. Michael glowered at Robin, slowly answering her shock with measured degrees of anger.

"Dromedary is the correct term for describing a *Normal Distribution*."

Robin just shook her head, letting her agitation simply fade away. Dakota could feel her insides screaming as she watched her band of merry men fly off on a tangent worthy of a heated game of "Trivial Pursuit." She deftly slammed her palm onto the table, shocking everyone away from discussing camels and turning the attention back to the pressing problem at hand.

"Robin, what did Jennifer say?"

Robin pulled her own legal pad out of her purse and began to flip through it.

"Honestly, she said quite a bit. I swear that woman must be amazing with a riding crop."

Theron shot her a look of surprise.

"I didn't know Jennifer rode horses."

Robin stopped riffling through her papers, thought about Theron's comment for a moment, and then continued looking through her notes once more. This was one of the few times where Robin felt being glib was going to be more trouble than it was worth. Finding the page she was looking for, she turned her notes to her colleagues.

"Apparently, the campaign workers who gathered this data for us drew a circle around campaign headquarters and just surveyed every sex offender within the circle."

Michael looked crestfallen as this bit of information washed over him.

"So, our participants weren't *Randomly Sampled*?"

Robin pulled out the map that was used to determine the area which would be used to poll the sex offenders. A large lopsided circle was fairly obvious to the naked eye, with imperceptible writing filling up the margins.

"I'd say the only thing random here is that some moron actually considers this to be a circle."

Michael banged his fist on the table as Dakota steeled herself against yet another tirade from her coworker.

"Are you kidding me?!? This is completely unacceptable. Now what are we supposed to do? I am as mad as hell. . ."

". . . and you're not going to take it anymore."

Robin smiled broadly at Michael, who was now glowering in her direction. Apparently, this was one glib comment which she was going to use. Theron covered his face with his hand, his body shaking from the laugh he was trying to suppress. Dakota just looked nonchalantly at Robin, scolding her colleague with her eyes. Robin shrunk meekly into her chair, feeling the need to justify her remark.

"What, he is the one having the 'Howard Beale' moment over here."

With that, Theron collapsed into a heap of laughter. Even Dakota felt the subtle edges of a smile form on her lips; it didn't help that "Network" was one of her favorite movies, and Michael did always look like someone who was perilously close to having a nervous breakdown (and yes, he would most likely capitalize on it). Dakota just took a few deep breaths, turning her attention to Robin.

"Do you have your research questions concerning the data?"

Robin handed her a plain slip of notebook paper.

"Right here. Not sure what this is going to help you to discover, unless you plan on taking this sheet of paper and folding it into an incredibly aerodynamic paper airplane."

Dakota once again spun around the dry-erase board and proceeded to add Robin's questions to their list. Her questions addressed issues of compliance, consistency in assessment, and differences in sentencing based upon the severity of the offense:

- For those who consistently report their post-release status, is their status significantly different across all time periods (i.e., current status, 30 days post-release, and 90 days post-release)?
- Is there a difference in the way the raters rated based on ethnicity/race?
- Are minority ethnicities/races receiving higher General Aggression Scores?
- Is there a difference between those charged with Rape and those charged with lewd acts concerning sentence length?

Dakota admired the finished product before them, her mind mapping out a trail as to the best way they could precede with this data. Still, she could not help but hear Michael growling in the background. Dakota just cleared her throat, a noise designed to tell the others in the room that the situation was not lost.

"Okay, given that we have violated a major assumption inherent to conducting *Parametric Tests*, I say we use **Nonparametric Procedures**."

Michael was now looking down at the floor, trying to prevent Dakota from realizing that he had no idea what she was talking about. Robin and Theron looked at one another, not sure how to best approach this issue. Theron raised his hand slightly, allowing Dakota to acknowledge his question.

"Are those as good as the real statistics I learned about in college?"

Dakota took a deep breath.

"Oh yes. *Nonparametrics* allow a researcher to still conduct statistical procedures, but these tests don't require all of the assumptions of *Parametric Tests* to be met."

Robin shrugged her shoulders.

"Well, couldn't we just run the *Parametric Statistics* anyway? I mean, who is going to know?"

Dakota shook her head.

"No. Given how contentious political campaigns can be, I would rather we play it safe than put the Governor in an uncomfortable position later. I am sure Jennifer would agree."

Everyone in the room nodded in agreement. Dakota began ticking off certain questions on the dry-erase board. She put an asterisk next to the following questions:

* Is sentence length associated with the General Aggression Score?
* Is estimated yearly income associated with the General Aggression Score?
* Is ethnicity/race associated with living location?
* Is testosterone level associated with the General Aggression Score?
* Is testosterone level associated with the General Aggression Score when age is fixed?
* When presented with only two sex offenders at a time, do the raters agree on ranking of the level of meanness for each sex offender?
* Do the three raters agree on their meanness rating of each sex offender?

The group looked about the room, confused why Dakota chose those questions. Dakota just put down the marker and began to gather her things.

"Okay, I think we have gone as far as we are going to get today. Tomorrow we start addressing these questions."

Robin just looked at Dakota, speaking aloud what everyone was thinking.

"Wait, why these questions?"

Dakota just continued to gather her things.

"Well, we need to start somewhere, and the *Tests of Association* seem to be the best place to begin. The *Tests of Association* not only provide a practical framework, but they also provide us with a conceptual framework from which we can understand the *Tests of Difference*. The practical framework is the understanding that some *Tests of Difference* require a lack of association in order for the results to be useful. The conceptual framework is the understanding that in **Empirical Research** association is used as a basis for making conclusions about causality."

The group was bewildered, yet they dutifully gathered their possessions, turned off the lights, and followed their captain out of the building.

Chapter Summary

- In this chapter, several analyses were explored that help a researcher to determine whether the traditional parametric statistical analyses can be used with the researchers data.
- Four types of distributions were considered. The four are uniform, Poisson, exponential, and normal distributions.
- The Kolmogorov–Smirnov One-Sample Test was used to determine the distribution of a set of data.
- The One-Sample Runs Test of Randomness was utilized to determine whether or not the data were orthogonal.
- The Test for Distributional Symmetry was employed to ensure that both sides of a distribution were the statistically the same.

Check Your Understanding

1. Match the following descriptions with the correct distribution:

a. A horizontal line with no increasing or decreasing fluctuations	Exponential
b. A rapid increase initially that continues to increase but at a slower and slower rate	Poisson
c. A bell-shaped, symmetrical curve with the bulk of the data piling up in the middle	Normal
d. A rapidly decreasing curve that slowly tapers off as the distribution continues	Uniform

2. List and define the assumptions for parametric tests.
3. Why are sampling procedures important for parameter testing?
4. What is the difference between observed and expected frequencies?
5. What does "rejecting the null hypothesis" mean for the Kolmogorov-Smirnov One-Sample Test, the One-Sample Runs Test of Randomness, the Test for Distributional Symmetry, and the Dixon Test?

Chapter 4
Understanding Similarity (with a Little Help from Big Bird)

Abstract In this chapter, the team begins to experience the dynamics of working with such a diverse group. There are confrontation and disagreement, but ultimately the group is able to work through two research questions: (1) Is ethnicity/race associated with living location? (2) Is compliance status associated with whether the sex offenders are currently taking medication? The first research question is addressed by using the Cramér Coefficient, and the second research question is addressed by using the Phi Coefficient. The concepts of Association and Correlation are also discussed by the team.

"One of these things is just like the other..."

Robin quietly sang to herself as Theron lowered the corner flap of his newspaper, giving her a quizzical expression. Robin just turned away from him and continued to sing to herself.

"... two of these things are really the same."

Theron finally folded the newspaper in half and tossed it aside. Apparently, his new colleague was challenging him to a debate on popular culture. Naturally, he could not refuse such a tempting invitation.

"Okay, why are you bastardizing an old Sesame Street song?"

Robin stopped humming.

"I am not."

Theron rolled his eyes.

"Yes you are. You are messing up the lyrics to 'one of these things is not like the others.' It's an old song they used to sing on Sesame Street."

Robin became very still, her mind a whirl of ideas.

"I thought it was 'one of these things is just like the others.' You know, teaching kids to identify how things are similar."

Theron just shook his head.

"In a word—no."

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
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Robin shrugged.

"Well, we are doing *Tests of Association* today, so my version of the song is more appropriate."

Theron and Robin's witty banter was interrupted by Michael, who was snickering in the corner. Michael just sat very still, running his finger around the rim of a steaming cup of coffee which was only a few inches away from him.

"Given what I know about you young lady, 'Sesame Street' seems to match your maturity level."

Theron whistled under his breath as Robin's eyes narrowed.

"You know, the only doctors who are even kind of interesting are the ones on 'General Hospital.' At least they don't sleep alone every night."

"Is everyone ready to get started?"

All three consultants jumped as Dakota appeared in the doorway; her arms loaded with all manner of office supplies and textbooks. She gracefully dropped the supplies onto the table and marched over to the dry-erase board. As she was searching for a clean space to write, Dakota called out to the group.

"I am guessing you all have heard the news?"

Robin, who was still somewhat startled by Dakota's entrance, looked towards Theron.

"News?"

Without saying a word, Theron picked up his newspaper and deftly flipped it to the front page. In bold letters, the headline screamed out:

"Midnight Rapist" Attacks Elderly Woman.

Robin just shrugged her shoulders.

"And?"

Theron nonchalantly tossed the newspaper into the wastebasket near the door.

"Well, don't you think this bit of information is a little relevant given that we are working on a political platform to address sex offenders?"

Robin huffed as this piece of information filtered through her brain.

"Oh yea, well you get your news from a dying medium."

Dakota stared blankly at the dry-erase board, filtering out the chattering nonsense around her. Even though she honestly believed that the *Tests of Association* were the most logical place to start, it was still daunting to figure out just where to begin. Michael, who had managed to stay quiet during the ridiculousness of Robin and Theron's verbal sparring, finally chimed in with his stereotypical brusqueness.

"You know little lady, some of us have actual jobs we could be doing right now. As much fun as it is listening to these two bicker and to watch you stare vacantly at a glorified chalkboard, I am off to go do actual work."

Dakota's arm shot towards Michael, pointing down towards his seat.

"No. Dr. O'Brien, we all have a job to do. Whether you like it or not, the four of us are in it for the long haul. Besides, you heard Jennifer; we all have a lot to lose if we do not do our jobs and do them well. So I suggest you sit back down, and let me figure out where we should begin."

Michael's face turned bright red.

"Who made you our leader?"

Dakota defiantly brushed the hair out of her eyes.

"Would you like to lead the group?"

Theron and Robin watched in silent awe as the red faded from Michael's face, and he silently eased back in his chair as a defeated look crossed his eyes. While it may have been his natural inclination to lead, he also knew that there was too much at stake for him professionally. Despite his misgivings, he had to let Dakota lead. Dakota just looked at Theron and Robin.

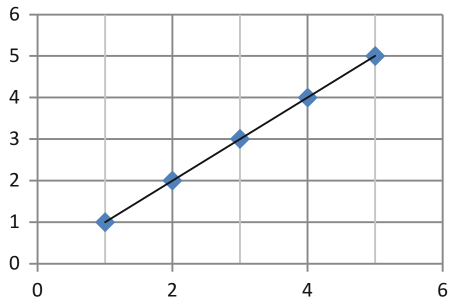
"May I continue?"

All three consultants nodded in unison as Dakota went back to the dry-erase board. Suddenly, a screeching voice broke the silence of the conference room.

"So, what does *Association* tell us?"

Dakota stopped cold. She hated to admit it, but it was an excellent question, one that could lead to a lot of confusion if not handled correctly.

"Well, oftentimes *Tests of Association* are used to show us whether or not there is a *Linear Relationship* between two *Variables*. Usually, we are looking for a positive *Linear Relationship*, which means that as one *Variable* increases, there is a corresponding increase in the other *Variable*. This relationship would look something like this."



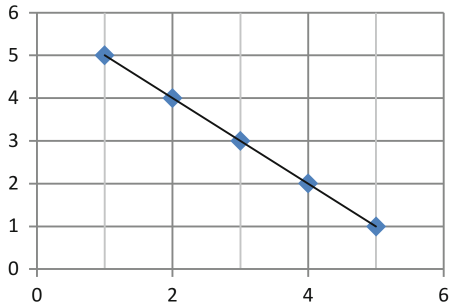
Dakota just continued on with her explanation.

"Now, it is also possible to have a negative *Linear Relationship*. All that means is that as one *Variable* increases, there is a corresponding decrease in the other *Variable*. That means. . ."

Michael gruffly jumped into the conversation.

"It means that as one goes up, the other goes down."

Dakota listened to Michael's brusque explanation. It wasn't very diplomatic, but it was correct. She just sketched out a quick diagram and kept going.



Dakota pointed to the graphs.

"Most of the time, *Correlations* are often gauged between −1.0 and +1.0; a −1.0 implies a perfect *Negative Correlation*, and a +1.0 implies a perfect *Positive Correlation*. Zero is in the middle, and 0 implies that there is no correlation at all."

Robin just swiveled in her chair.

"And if these *Variables* are highly associated, does that mean that one causes the other?"

Michael snorted in contempt.

"No you stupid little girl. There is the old saying that '*Correlation* does not constitute causation.' Have you never taken a stats class before?"

Dakota shot Michael a threatening glare, hoping that it would force him to back down. She steadily tried to repair some of the damage which Michael caused with his demeanor.

"Dr. O'Brien is correct. Just because *Variables* are highly associated, it does not mean that one causes the other."

Dakota's explanation seemed to turn a few light bulbs on, but now Theron had a question.

"So, if the *Correlation* is not one of those 3 numbers, how do we explain what kind of *Correlation* it is?"

Dakota turned to see if someone else was going to handle this one, but Michael and Robin were just staring at the conference table.

"Well, Theron, the closer you get to the middle or zero, the weaker the *Correlation*; and closer you get to −1.0 and +1.0, the stronger the *Correlation*. If you have a *Correlation* of, say, 0.87, you might call that a strong *Positive Correlation*. Everyone got it?"

The group nodded, allowing Dakota to go back to the dry-erase board to figure out where they should begin. Suddenly, her eyes grew wide as she circled a particular question.

"I know. We will start with answering whether or not race/ethnicity is associated with living location."

Theron squinted at the board, trying with see why Dakota chose to start on that particular question.

"Um, okay. Why are we starting there?"

Dakota reached towards the board and drew a large circle over the question.

"Well, the reason I chose to start here is because it is a test to see if two *Categorical Variables* are related."

Theron tilted his head to the side, his mind slowly linking the pieces of information together.

"Wait a moment. Categorical... isn't that the same thing as *Nominal Data*?"

Dakota nodded.

"That's correct. All we are doing is seeing if these two *Nominal Data* sets are associated."

Robin leaned forward.

"And we are doing that how?"

Theron just shrugged his shoulders.

"That's easy. How do you do any *Tests of Association*? We are just going to be doing a whole bunch of **Pearson's Correlations**."

Dakota smiled as she shook her head.

"You're on the right track, but that's the wrong test. *Pearson's Correlation* is what most people think of using when they need to find out if two *Variables* are associated with one another. However, *Pearson's Correlation* is a *Parametric Test*, so it wouldn't be applicable here."

Dakota just turned towards the dry-erase board and started writing.

"Easy. We are going to use the **Cramér Coefficient**."

Robin whistled in mock delight.

"That sounds very French. This could be fun... in a tediously boring kind of way."

Dakota ignored Robin, writing the equation on the dry-erase board. Suddenly, she froze in her tracks, only completing some of the equation. Her arm dropped limply to her side, and she stepped out of the way, revealing the partially finished equation to the group:

$$C = \sqrt{\frac{X}{N(L-1)}}$$

Dakota looked out to her colleagues and felt awash in a sea of confusion. She knew that this equation would have some components which would only confuse them; that would be unavoidable. The problem was that the *Cramér Coefficient* required another statistical procedure in order to run the equation,[1] one that would answer a very different research question than what they were searching for at present. Her mind raced as she tried to figure out how she could alleviate their confusion. Suddenly, Dakota's mind seized on a solution, and her hand instinctively altered the *Cramér Coefficient* equation. It was time to be the leader this group needed:

[1] The other statistical procedure refers to the Chi-Square test and is explained as such on page 196 in Chap. 8.

$$C = \sqrt{\frac{X}{N(L-1)}}$$

N = the total number of cases
L = the total number of rows or columns (whichever is the smallest)

All three consultants looked at this new equation on the dry-erase board, confused as to what the strange symbol represented. Theron meekly raised his hand. Dakota steeled herself as she braced for the question she knew was coming. She just nodded in Theron's direction, allowing him to speak.

"Okay, I understand what all the other symbols mean, but what about that strange 'X'?"

Dakota flashed her best reassuring smile.

"The 'X' represents a statistical procedure which answers a question that is not our concern at present. Since we need to figure out this problem first, the 'X' is just something we have to figure out in order to successfully figure out this equation."

Robin leaned back, chortling with laughter.

"So, basically, 'X' is just marking the spot."

Dakota could only chuckle at this response.

"Trust me; it's not all that important. Not yet, anyway."

Theron just shrugged.

"Okay, what do we do first?"

Dakota snapped the cap back onto the dry-erase marker, her hand shaking against the weight of it. She quickly scanned the conference table with her eyes and yanked out the data table which Jennifer gave to the group.

"Okay, the first thing we need to do is to create a **Contingency Table** with race/ethnicity as one *Variable* and living location as the other."

Robin squinted her eyes in an effort to read the writing on the dry-erase board.

"*Contingency Table*?"

Dakota quickly erased a segment of the dry-erase board, sketching out a rough draft of a *Contingency Table*.

"A *Contingency Table* is just a matrix-type table which displays the frequencies of the *Variables* you are looking at by category; based on the question you are trying to answer. For example, let's say the governor's campaign workers are conducting exit polling to see who is voting for and against Governor Greenleaf. Suppose these are the responses they receive."

	Voter political affiliation			
	Democrat	Republican	Independent	Total
Voting for	12	14	10	36
Voting against	6	9	5	20
Undecided	3	1	0	4
Total	21	24	15	60

Michael folded his hands in his lap, casually swinging in his chair.

"Isn't that just a **Cross Tabulation**?"

Dakota looked towards Michael, unaware of whether or not he was asking a question or still trying to challenge her authority. Either way, she had to answer the question.

"Kind of. *Cross Tabulations* are the process of creating these tables, not the result. I can understand why you could confuse this, given that a lot of statistical software often refers to the process of creating a *Frequency Distribution* as 'Cross Tabs'."

Michael nodded his assent to Dakota's answer, signaling that his question was purely information based and not a threat to her authority. Maybe there was hope for him yet. Dakota noticed that Theron's head was down, quickly sketching something on his legal pad as his face was turned to the data set.

"So, we are just putting one *Variable* on the horizontal axis and the other on the vertical axis and then parsing out the exit poll participants into the tables created by the different categories?"

"That's correct. This is also known as an **R x K Contingency Table**."

Dakota just nodded, allowing Theron to continue. After a few moments, he stopped once more.

"Wait a moment. Does it matter which *Variables* go on the horizontal axis and which go on the vertical?"

Dakota hesitated for a moment and then resolutely shook her head.

"No. It doesn't matter which *Variables* are in which axis.[2] "

Michael casually glanced over towards the data sheet, pretending to have no interest in what the group was doing. However, Robin was less than subtle.

"So, what is each of these little numbers telling us?"

Dakota gingerly pulled the data sheet away from Theron.

"Think of these numbers as parsing out the data set into certain sociodemographic categories. Look at this number in the upper left-hand corner."

Robin craned her neck.

"It's a '12'."

Dakota just nodded her head.

"Exactly, that tells us that out of the 60 people in our example, 12 of them are democrats and are voting for the governor. Make sense?"

Robin nodded as Theron cautiously continued writing. Finally, he triumphantly tore the page from the legal pad and slid it towards Dakota.

"You mean, like this?"

	Race/ethnicity			
Living location	Asian	Black	Hispanic	White
Incorporated area	8	15	29	16
Unincorporated area	0	2	2	1
Transient	2	4	16	5

[2] It does not matter which *Variable* is in each axis because the test is based on the idea that there is a symmetrical relationship between the two *Variables*.

Dakota's eyes widened, impressed with the information Theron handed to her. It was quickly becoming apparent that Theron had the requisite skills to work their examples. She just nodded and slid the paper to the center of the table so that Michael and Robin could see this as well.

"That's excellent. Now we need to determine the *Expected Frequency*."

Robin shook her head.

"I am assuming that this is some dumb math term and has nothing to do with a woman drinking tequila and forgetting to take her birth control pills."

Theron's eyes grew wide as he turned his head towards Robin. Apparently, she really would say whatever popped into her head; whether they wanted her commentary or not was irrelevant. Robin just stared ahead, waiting for someone to address her concerns, and Dakota had no choice but to oblige her.

"Um... the *Expected Frequency* is a theoretical value based on the *Sample Size* and *Probability* for the *Variables* in your study. Based on your data, it's what you would expect the results to be."

Theron folded his arms over his chest.

"I think I get it. It's like polling for a political campaign. Let's see if I get this right. Say we were doing exit polling with the same people that participated in the pre-voting survey for Governor Greenleaf, and we found that the poll was showing he got 60 % of the vote. We would expect that out of the total number of ballots cast, 60 % of them would have voted for the Governor."

Dakota nodded.

"That's right. And it's also a good way to introduce the idea of the *Observed Frequency*, which is just the actual number you observed with your data. Let's say in our mock election that the Governor actually got 65 % of the vote. The *Expected Frequency* was 60 %, but the *Observed Frequency* is 65 %."

Robin shrugged her shoulders.

"And we care about this because..."

Dakota was captivated by the question posed by at Robin.

"Well, we care about this because we need to know whether the *Observed Frequency*, or actual number, of people who voted for the governor, 65 %, is really different from what we expected. That would be important for us to know because it would tell us how good we were at predicting outcomes!"

Robin appeared to be pacified for the moment by such a response but still not willing to give Dakota the last word.

"Ok, enough of the exit polling; that is just hypothetical anyway. What do you say we get back to our different race/ethnicity categories?"

Theron looked at his chart.

"Because we need to know the *Expected Frequency* of our different race/ethnicity categories in order to compare them to the actual data we have."

Dakota held up her hand.

"The correct term is *Observed Frequencies*."

Robin shrugged.

"And how do we figure out these *Expected Frequencies*? Do we just make up numbers?"

A grin etched itself on the corners of her mouth as she sketched an equation on the dry-erase board:

$$E_{ij} = \frac{R_i C_j}{N}$$

Robin and Theron just stared at the board with a dumbfounded look in their eyes. Dakota nonchalantly glanced at Michael, who was intently listening to every word she said. Unable to read him, she returned her attention to Robin and Theron.

"The equation isn't as bad as it looks. The first thing we need to do is just figure out the *Cumulative Totals* for the columns and the rows."

Theron looked somewhat confused.

"Wait, we are just adding?"

Dakota nodded.

"Trust me. Just add up all of the numbers in the columns and in the rows."

	Race/ethnicity				
Living location	Asian	Black	Hispanic	White	Total
Incorporated area	8	15	29	16	68
Unincorporated area	0	2	2	1	5
Transient	2	4	16	5	27
Total	10	21	47	22	100

Robin once again squinted at the dry-erase board.

"Okay. So we added these numbers up. How does that equation help us?"

Dakota pointed to cell in the upper left-hand corner of the *Contingency Table*.

"Well, we can see that we have 8 Asians who reside in unincorporated areas, correct?"

Robin nodded, as Theron jumped forward in his seat.

"So, we have the *Observed Frequency*, and we now need to find the *Expected Frequency*?"

Dakota smiled.

"That's right. And the way we do that is through the formula. All the formula requires us to do is to take the column total and multiply it by the row total. Then, we divide the product by the overall number in the *Sample Size*."

Theron looked at the table he created on the legal pad.

"I get it. We multiply the column total which is 10 and the row total which is 68 and divide it by the total number of offenders, which gives us a total of 6.8!"

$$\frac{R_1 C_1}{N} = \frac{68 * 10}{100} = 6.8$$

Dakota could hardly contain her enthusiasm as she admired Theron's ability to perform simple math.

"Yep, and this process is continued in order to determine the *Expected Frequency* for each cell."

Theron lowered his head for a few moments, stopping only after his calculations were complete. Dakota just glanced at it, thrilled that Theron was able to run these calculations as quickly as he could.

	Race/ethnicity								
	Asian		Black		Hispanic		White		
Living location	Actual	*Expected*	Actual	*Expected*	Actual	*Expected*	Actual	*Expected*	Total
Incorporated area	8	*6.8*	15	*14.28*	29	*31.96*	16	*14.96*	68
Unincorporated area	0	*0.5*	2	*1.05*	2	*2.35*	1	*1.1*	5
Transient	2	*2.7*	4	*5.67*	16	*12.69*	5	*5.94*	27
Total	10		21		47		22		100

Robin just glanced at the chart for a few moments.

"Okay. . . now what?"

Dakota forced a grin across her face in an effort to calm her colleagues. She stuck out her thumb and casually pointed to the dry-erase board behind her, purposely minimizing everything she was saying.

"Now we deal with 'X.' After all, it does mark the spot."

Michael rubbed his palms together. He could see that their fearless leader was purposely trying to hide something, but he had no idea what it was. Michael's voice was very slow and distinct as he queried Dakota.

"And how do we find 'X'?"

Dakota stood straight as a rail and turned her waist so she could write an equation[3] onto the dry-erase board:

$$\boldsymbol{X}=\sum_{i=1}^{r}\sum_{j=1}^{k}\frac{\left(n_{ij}-E_{ij}\right)^2}{E_{ij}}$$

n_{ij} = *the* ***observed****, actual, frequency in a particular cell*
E_{ij} = *the* ***Expected*** *Frequency in a particular cell* = R_iC_j/N
r = *the number of categories for Variable one*
k = *the number of categories for Variable two*

Robin's eyes grew as wide as saucers.

"Oh. Is that all? Are we also going to need to quest for the Holy Grail as well?"

Dakota took a deep breath, steadying her voice as she talked.

"I know it looks scary. But our *Contingency Table* already fills in all of the components of the equation."

Dakota slid the legal pad back to Theron, deliberately trusting him to calculate the equation. It wasn't so much that she was utilizing his keen mathematical

[3] This is the equation for Chi-Square and is covered in greater detail on page 196 in Chap. 8.

abilities; it was more to help reassure him that this equation was nothing all that serious. Theron took a deep breath and just started writing away. After a few silent moments, he slid the legal pad back towards the center of the conference table.

"Is this what you were talking about?"

$$\begin{aligned}
\boldsymbol{X} = \sum\nolimits_{i=1}^{r} \sum\nolimits_{j=1}^{k} \frac{\left(n_{ij} - E_{ij}\right)^2}{E_{ij}} &= \frac{(8-6.8)^2}{6.8} + \frac{(15-14.28)^2}{14.28} + \frac{(29-31.96)^2}{31.96} \\
&+ \frac{(16-14.96)^2}{14.96} + \frac{(0-.5)^2}{.5} + \frac{(2-1.05)^2}{1.05} + \frac{(2-2.35)^2}{2.35} + \frac{(1-1.1)^2}{1.1} \\
&+ \frac{(2-2.7)^2}{2.7} + \frac{(4-5.67)^2}{5.67} + \frac{(16-12.69)^2}{12.69} + \frac{(5-5.94)^2}{5.94} \\
&= 0.21 + 0.04 + 0.27 + 0.07 + 0.5 + 0.86 + 0.05 + 0.01 + 0.18 \\
&\quad + 0.49 + 0.86 + 0.15 = 3.69
\end{aligned}$$

Out of the corner of her eye, Dakota could see that Theron's math was correct. She just smiled and transferred the number onto the dry-erase board.

"Wonderful! Now that we have 'X,' we can figure out the *Cramér Coefficient*. Remember that N is the total number of cases and L is the total number of rows or columns (whichever is smallest)."

Dakota pulled all of their information together and presented it to Theron. For his part, he dutifully picked up his pen and got right to work. A smile crossed his lips as he double-checked his work and slid the paper towards Dakota.

"How's this?"

$$C = \sqrt{\frac{X}{N(L-1)}} = \sqrt{\frac{\mathbf{3.69}}{100(3-1)}} = 0.14$$

Dakota smiled.

"It's perfect."

Robin glanced at the paper.

"0.14. That tells us what?"

Dakota placed her hands on her hips.

"Well, we need to figure out the *Critical Value*."

Robin turned towards Theron, slapping him on the back.

"You're on math-boy."

Dakota just giggled.

"Don't worry. There is little-to-no math involved in this. The *Critical Value* just establishes a threshold to help someone determine whether or not their results are *Statistically Significant*. The *Critical Value* is usually based on things like *Sample Size* of the number of *Variables* you have, thus meaning that it is more tailored to your data.

Typically, if your *Calculated Value* is higher than your *Critical Value*, your result is *Statistically Significant*. If it is not, your answer is not *Statistically Significant*."

Michael leaned back in his chair.

"And in this instance, a *Statistically Significant* result would mean that there is a high association between race and living location."

Dakota just leaned against the wall next to the dry-erase board.

"Okay. Most of these statistical procedures come with *Tables* which list out various *Critical Values* which a researcher might need for their data. Now, if my memory is correct, we need to figure out the *Degrees of Freedom* for *Cramérs Coefficient*."

Dakota quickly jotted something on the dry-erase board while reminding the team about the key parts of the calculation.

"Now, remember from earlier that r is the number of rows we have and k is the number of columns":

$$df = (r - 1)(k - 1)$$

Theron just raised his eyebrow, quickly etching out the solution to the equation. Dakota peered over the edge of his legal pad and put the completed equation onto the dry-erase board:

$$df = (3 - 1)(5 - 1) = 8$$

Robin sighed.

"So, 8 is the *Critical Value*?"

Dakota shook her head.

"You're on the right track. But '8' is the number we look up on the *Critical Values* chart."

Dakota instinctively darted towards the stack of papers she brought in with her, rifling furiously through all of the papers. Seizing upon the one she wanted, she yanked it out and flicked it into the center of the conference table. Robin giggled as she read the title of the table.

"Chai-Square? Awesome. I love chai."

Dakota shook her head.

"Not chai, chi. Like the Greek letter. This is the chart we use for *Cramér Coefficient* to help us determine *Statistical Significance*."[4]

Theron's eyes widened.

"Um, which decimal point level are we using?"

Dakota gritted her teeth. It wasn't so much that she was infuriated with the question; it was more that she felt foolish for not covering this topic earlier.

[4] Please refer to the *Cramér Coefficient* table during this explanation. Looking at the table while reading through the following passage may be useful in understanding how to read the table.

"The decimal points signify different **Alpha Levels**. *Alpha Levels*, or p-levels, allow the research to determine how often a *Statistically Significant* result occurs by chance. The lower the *Alpha Level*, the more likely you are to draw an erroneous conclusion about whether something is actually affecting the behavior. Most people typically use an *Alpha Level* of 0.05, so that is what we will use as well."

Michael grunted.

"And do people actually get a *Statistically Significant* result purely by chance?"

Dakota locked eyes with her colleague.

"Well, is it possible to guess your way to a perfect score on the GREs?"

Michael muttered.

"Well, it's possible, but not probable."

Dakota pointed at her colleague.

"Remember, you as the researcher choose the *Variables* which will be analyzed through statistical analysis. So, it's possible that the combination of *Independent Variables* and *Dependent Variables* you have chosen are what is causing a significant result, even though no Significance exists."

Realizing that she had seized upon an excellent teachable moment, Dakota stopped and glanced at her colleagues.

"Actually, Michael does make a very good point."

Robin just shook her head at this statement.

"No, he really doesn't."

Dakota chose to ignore Robin and focus her efforts on Michael.

"Now, finding a *Statistically Significant* result can happen by mistake on the part of the statistician. These mistakes are known as **Errors**. There are two major *Errors* which a researcher can come across when finding *Statistical Significance*: a **Type I Error** and a **Type II Error**."

Theron just narrowed his eyes as his mind tried to link these concepts to something he could understand.

"You mean like Type I or Type II diabetes?"

Dakota just shook her head, trying to break Theron of his current line of thinking.

"A *Type I Error* is a false positive; it means that you found a *Statistically Significant* result when in fact there is none. A *Type II Error* is a false negative; it means that you did not find a *Statistically Significant* result when in fact there was one. Typically, *Type II Errors* are much more difficult to correct by the researcher."

Dakota paused for a moment and saw that her colleagues were lost in a sea of confusion. So, she tried to address this concept in another way.

"Okay, let us think of *Type I Error* and *Type II Error* like a woman taking a home pregnancy test. So, let's say a woman takes a home pregnancy test and it comes up positive. What is that woman going to do?"

Robin excitedly pointed to Dakota.

"Have a nervous breakdown."

Dakota shook her head.

"No, she would go to the doctor and have a blood test done to confirm whether or not she is pregnant."

Michael grinned.

"I get it; the false pregnancy test is like a Type I Error. If you got a *Statistically Significant* result, you would do further work to verify the result. Just like a Medical Doctor running further blood work to confirm whether or not a woman is pregnant."

Dakota nodded.

"That's correct. Now, let's say a pregnant woman takes a home pregnancy test and it comes out negative, even though she is pregnant. What would she do?"

Theron smiled.

"Probably assume she had the stomach flu and go about her life as she normally would."

Michael once again grinned in delight.

"I get it, if a research does not find a *Statistically Significant* result, they would go on to the next test not realizing that an *Error* was made. This would be just like the young woman who took the pregnancy test and got the false negative result; she wouldn't know something was wrong until much later."

Dakota just smiled back at Michael.

"And that is why a *Type II Error* is much harder to address than a *Type I Error*. Researchers are willing to double-check and verify *Statistically Significant* results, whereby they won't double-check and verify statistics when there is no *Significance*."

Theron was still looking at the table.

"So, I am looking at α of 0.05, and going for *Degrees of Freedom* of 8? Let's hope we don't have an *Error* here."

The whole team grinned at one another as a result of Theron's aspiration. Dakota nodded, and Theron found the spot on the chart for the *a* and the *df*.

"It's 15.51."

Dakota wrote that on the dry-erase board.

"Okay, so our *Critical Value* is 15.51. Since our *Calculated Value* is only 0.14, we know that we cannot reject the *Null Hypothesis*. That means that there is no association between race/ethnicity and living location."

Theron just pulled the piece of paper off of the legal pad and handed it to Dakota. She filed the paper away.

"Shall we move on?"

The group nodded silently, thus allowing Dakota to take Theron's notes and file them into a manila file folder. She then returned her attention to the dry-erase board, where she proceeded to cross out the question of "whether or not race/ethnicity are associated with living location." After she crossed off the question, she silently muttered to herself under her breath.

"One down..."

Michael's ears perked up at the sound of Dakota's voice.

"Beg pardon?"

Dakota methodically shook her head.

"Just talking to myself. I think that the next thing we should discuss is the question of compliance status and whether or not it is associated with the sex offenders currently taking their medications."

Robin narrowed her eyes in an attempt to discern what was written on the dry-erase board.

"Is that even one of our questions?"

Dakota whipped out her marker and deftly circled the question on the dry-erase board. Robin just slumped in her chair.

"A simple 'yes' would have sufficed."

Dakota turned her back to her colleagues so she could face the dry-erase board and the copious amounts of information scrawled upon it. She leaned into the table behind her, searching her mind for the best possible way to answer the question. The room fell into an eerie silence as Dakota was lost in thought; the only sound in the room was the incessant hum of the air conditioner. After a few agonizing minutes, Robin leaned towards Theron.

"I want pie."

Suddenly, Dakota spun around and pointed at Robin with a maniacal smile across her lips.

"That's it!!! Robin, you are a genius."

Robin and Theron sat in a state of total bewilderment, while Michael just shook his head at the inane notion of the annoying girl with the penchant for shirtwaist dresses being considered a genius. Dakota was furiously writing notes on the dry-erase board, her colleagues being nothing more than an afterthought. Once her notes were completed, she once again turned to her colleagues. Theron finally asked the question which was all on their lips.

"Um, how does pie help us?"

Dakota shook her head.

"Not pie, Phi. We can use the **Phi Coefficient** to see is compliance status associated with whether they are currently taking medication."

Robin shook her head.

"Um, what's the *Phi Coefficient*?"

Dakota spoke quickly as she etched a drawing on the dry-erase board.

"The *Phi Coefficient* is a terrific *Test of Association* which you use when you have *Dichotomous* data which also happens to be *Nominal*."

Robin scratched her head.

"*Dichotomous*?"

Dakota just continued drawing, keeping her back to the group.

"Yes, you remember *Dichotomous* data! *Dichotomous* data just means that all participants or all data are going to be assigned to one or the other like 'yes/no.' The example most often given is 'male/female.' Almost everyone is one or the other."

Robin shook her head.

"What about drag queens or transgender people?"

Dakota deftly turned her head to wink at Robin.

"I said 'almost'."

Robin shrugged her shoulders as Dakota continued on with her work. Theron was trying to copy what Dakota was doing but was having limited success seeing the dry-erase board past her frenzied writing. Finally, he turned and looked to the data set in order to see if he could make sense of this on his own. After looking for what seemed like an eternity, something in his mind finally clicked.

"Oh, I get it. In our data set, the sex offenders can either be in compliance with their notification status or not and either are taking psychotropic medication or are not. Both of these *Variables* are *Categorical* and can really have only two possible values, 'yes' or 'no'."

Having erroneously assumed that they had moved past this point in the explanation, Dakota stopped drawing and stepped away from the dry-erase board.

"You got it. The key to the *Phi Coefficient* is that because it only uses two separate *Variables* which have *Dichotomous* scoring, it can be placed in a 2 X 2 *Contingency Table*.[5] Thus, it would look something like this."

	Current compliance status		
Currently taking medication	No (0)	Yes (1)	Total
Yes (1)	*A*	*B*	*A* + *B*
No (0)	*C*	*D*	*C* + *D*
Total	*A* + *C*	*B* + *D*	*N*

Michael pointed to the dry-erase board.

"So, you are going to teach us the alphabet now?"

Dakota slid her legal pad over to Theron.

"Trust me; the letters will make sense once you see the equation for the *Phi Coefficient*. Now, we just need to organize our *Observed Frequencies* in the 2 X 2 *Contingency Table*."

"Grrr."

The group became still as they heard Michael's grumbling in the background. Dakota resisted the urge to roll her eyes and attempted to "play nice" with her colleague.

"Something wrong?"

Michael just nodded.

"You said that this test was a 2 X 2 design, correct?"

Dakota slowly nodded in his direction. She felt an uneasy feeling grip her stomach as she waited for him to finish his thought. Michael just pointed to the data set.

"Well, wouldn't this research question be a 2 X 3 design? I mean, if we are asking about whether or not these sex offenders are maintaining compliance with their medications, aren't we dealing with 'yes,' 'no,' and 'unknown'?"

Dakota became absolutely still; Michael was correct. She had become so focused on the correct statistical procedure to use that she had forgotten to take into account all of the individuals on the data set who did not have any information about their current medications. Robin just snatched the data set, hoping she could

[5] A more detailed explanation of the 2 X 2 Contingency Table can be found on page 72 of this chapter.

prove him wrong. However, it was glaringly obvious that Michael was indeed correct. She angrily shoved the data set away from her.

"I hate it when he is right."

Suddenly, Theron stuck his hand up in the air, calling attention to himself amidst the confusion which was now brewing in the room.

"Um, why can't we just change the research question?"

Michael started muttering to himself as Robin's eyes grew wide with astonishment.

"We can do that?"

Theron just shrugged.

"Well, we don't have any information about the unknown people, so why don't we just run the test on the data we have; those people we know are taking medication."

Dakota seized on the opportunity and started revising the question on the dry-erase board.

"So, we change the question to 'is compliance status associated with whether they are known to be currently taking medication'?"

Dakota then turned her attention back to Michael.

"Does that answer your question?"

Michael leaned back into his chair, his face plastered with a look of smug self-satisfaction. He proved his intelligence to the group and was now basking in his own glory. Still, Dakota couldn't help but notice that he was not flaunting his superiority at her mistake, a thought that suddenly made him oddly endearing to her. Dakota pointed to the legal pad in front of Theron.

"Can you do the math?"

Theron meekly pulled the legal pad towards him, as he was once again given the task of organizing the data. After several minutes of counting, with a little "help" from Robin (which consisted of pointing out sections of the data set that had no bearing on what he was trying to do), Theron was able to slide the finished product back over to Dakota. She quickly looked over his work and filled in the blank 2 X 2 *Contingency Table* which she had set up on the dry-erase board.

	Current compliance status		
Currently taking medication	No (0)	Yes (1)	Total
Yes (1)	23	10	33
No (0)	36	18	54
Total	59	28	87

As the group was digesting the *Observed Frequencies* they had concerning sex offender compliance and whether or not they were on medication, Dakota was quickly scrawling another equation on the dry-erase board. Once completed, she stepped away from the board so that everyone could see what she had written:

$$r_{\varnothing} = \frac{|AD - BC|}{\sqrt{(A+B)(C+D)(A+C)(B+D)}}$$

Theron leaned forward, allowing his mind to process what he was seeing. It only took him a few moments for his brain to register everything.

"Oh, I get it. The equation is using the *Observed Frequencies* in each of the quadrants from the 2 X 2 *Contingency Table*."

Dakota nodded in his direction.

"That's pretty much it. Can you handle it?"

Theron shrugged his shoulders. There really was only way to determine whether or not he was up to the challenge. He silently pulled the legal pad over to him and quickly jotted down the numbers from the dry-erase board. After a few moments, he turned his work over to Dakota:

$$r_{\varnothing} = \frac{|AD - BC|}{\sqrt{(A+B)(C+D)(A+C)(B+D)}} = \frac{|(23)(18) - (10)(36)|}{\sqrt{(33)(54)(59)(28)}} = 0.031$$

Dakota looked it over to see if she agreed with Theron's conclusions. After determining that he was correct, she etched all of the information onto the dry-erase board.

"Okay, of the 100 offenders in the sample, whether or not an offender is currently taking medication is known in 87 cases. In 13 cases, whether or not the offender currently takes medication is unknown. That means that our *Phi Correlation value* is 0.031, which is a pretty weak correlation given that we want a number close to positive or negative 1.0. But we should see this through to the end. So, we now need to figure out our corresponding 'X' value for this information in order to determine if there is a significant association between *Variables*."

She quickly wrote out another equation under the first one:

$$x^2 = \frac{N(|AD - BC| - N/2)^2}{(A+B)(C+D)(A+C)(B+D)}$$

Robin's hand shot up once more.

"Is this one of those 'X marks the spot' things?"

Dakota nodded.

"Bingo. However, 'X' is going to help us to determine *Statistical Significance* with this operation. Once we solve this equation, we will be able to determine whether or not we can *Reject the Null Hypothesis*. Theron, can you do this one too?"

Theron just shrugged his shoulders.

"There are a lot of similarities with the last equation, so it shouldn't be that hard."

Once again, Theron hunched over the legal pad, deftly combining all of the numbers in his head. After a few moments, he slid the final product over to Dakota to get her final approval on his work. She just took the piece of paper and transposed the numbers on the dry-erase board:

$$= \frac{87(|(23)(18) - (10)(36)| - 87/2)^2}{(33)(54)(59)(28)}$$
$$= 0.0033$$

“Okay, our ‘X’ value is 0.0033. Since we only have a 2 X 2 design, we know that our *Degrees of Freedom* is going to be 1 because $df = (r - 1)(k - 1)$. For a 2 X 2 design, the df will always be 1 because $(2 - 1)(2 - 1) = (1)(1) = 1$. Now we just compare the *Critical Value* to the *Calculated Value* and see what we get.”

Out of the corner of her eye, Dakota saw the **Chi-Square Table** and instantly went to the information concerning $df = 1$ and an α of .05. After finding the appropriate number, she just jotted the information onto the dry-erase board.

“Okay, the *Critical Value* is 3.84. Since $x^2 = 0.0033 < 3.84$, we fail to reject the Null Hypothesis (i.e., there is no association between an offender’s current compliance status and whether or not an offender is taking medication). Given that our *Calculated Value* is only 0.0033, we have no choice but to conclude that there is no association between an offender’s current compliance status and whether or not an offender is taking medication. We conclude that offenders’ current compliance statuses are not related to whether or not offenders are known to be currently taking medication.”

Dakota quickly noted the information onto her legal pad and slid the final numbers concerning the *Phi Coefficient* into the manila folder with the rest of their notes from the day. Once that was done, Dakota absent-mindedly brushed the errant strands of hair from her forehead, accidentally smudging her cheek with the dry-erase marker. She let out a long sigh and turned to the group.

“Okay, I think that’s enough for one day. How about we pick up with this tomorrow?”

Robin whistled.

“About time we get out of here.”

Theron already grabbed all of his belongings and hustled out the door, with Robin close at his heels. It was obvious that they were both saturated with math for the day and were afraid that Dakota’s inspiration would keep them there for even more time. Unlike his counterparts, Michael slowly rose to his feet and walked towards Dakota. Aware of what he was doing and his presence near her, Dakota just stood firm and waited to be on the receiving end of some withering insult about her leadership abilities. All Michael did was brush his hand against the smudge on her cheek.

“You got some ink on your face.”

With that, he turned his back to her and waded into the sea of busy campaign workers. He did not notice her face flush red, nor did he notice that her hand was now caressing the spot on her cheek where his hand had been only moments prior.

Chapter Summary

- *Tests of Association* are introduced and the basics of *correlation* are discussed.
- The *Cramér Coefficient* is used to answer the research question: "is ethnicity/race associated with living location?"
- The research question "is compliance status associated with whether they are currently taking medication?" is addressed using the *Phi Coefficient*.

Check Your Understanding

1. Describe the following correlations as strong or weak and positive or negative:

–0.26	0.11
0.98	–0.87
0.58	–0.33

2. What is the differentiating factor that helps a researcher know whether to use the Phi Coefficient or the Cramér Coefficient?
3. *True or False*: A perfect positive correlation means one variable causes another.
4. How does an observed frequency differ from an expected frequency?
5. Cramér Coefficient utilizes what common nonparametric test to arrive at an end result?

   a. Pearson's Correlation
   b. Phi Coefficient
   c. Chi-Square
   d. None of the above

6. Both the Cramér Coefficient and the Phi Coefficient utilize:

   a. Categorical data
   b. Ranked data
   c. Contingency tables
   d. Only A and C

Chapter 5
The Bourgeoisie, the Proletariat, and an Unwelcomed Press Conference

Abstract Despite the turmoil surrounding Governor Greenleaf's competitor, the team rallies together to concentrate on two more research questions. Those research questions are as follows: (1) Is Total Testosterone Level associated with the General Aggression Score? (2) Is sentence length associated with the General Aggression Score? The first research question is discussed using the Somer's Index d, and the second research question is addressed by using Spearman's Rho. Antecedents and consequents are discussed as well as concordant and discordant pairs as they relate to Somer's Index. In this chapter, Pearson's r is also considered. The team has to navigate through the reasons why Pearson's r is not an appropriate choice for their research.

Dakota folded her arms over her chest, her eyes glued to the small television screen that one of the campaign staffers rolled into the conference room several minutes ago. Her small cadre of consultants had just walked into the room only a few moments prior, when their cell phones lit up like fireworks. It was Jennifer, demanding that the group get to a television. Dakota knew within moments that the news was going to be grim when she asked what station they should turn to, and Jennifer's only reply was "any of them." Now, she was watching a young morning anchorwoman with a stern face repeating the same sentence for what was likely the fiftieth time.

"To repeat, the serial rapist whom the media has dubbed the 'Midnight Rapist' struck again last night. This time, the victim was 75 year-old Susan Eberling, the mother of current US Senate candidate Christina Eberling..."

Theron wiped his hand across his face, hoping that he could close his eyes and make this problem disappear. Even Robin sat in stunned stillness as she tried to absorb the barrage of information being presented by the news bulletin. All Robin could do was turn to Theron and ask the obvious question that was on everyone's mind.

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8_5,

"Am I insane, or were we just told that the mother of Governor Greenleaf's competition for this election was just attacked by the rapist who has been dominating news headlines for the past few weeks?"

Theron somberly nodded his head. Michael just raised his hand, silencing both of them with a flick of his wrist.

"Shush. Something is happening."

All four leaned into the screen, trying to discern what the anchor was saying.

"We have some breaking news to report. We have just learned that Susan Eberling has died. Eberling apparently suffered severe injuries during her attack, and doctors were unable to revive her. Though Eberling is the seventh victim of the 'Midnight Rapist,' she is the first victim to die from these attacks. Once again, Susan..."

Dakota's arm shot out and hit the power button on the front of the television. There was nothing more they needed to know. Dakota jostled away from the television and walked towards the dry-erase board.

"Okay Theron, you are the closest thing we have to a political consultant, what does Greenleaf's campaign do now?"

Robin and Michael cocked their heads in Theron's direction, waiting to hear what he had to say. Theron just cleared his throat and spoke in a low voice.

"Well, Greenleaf will most likely issue a statement sending his condolences to Eberling's family, and Jennifer will pull any and all negative political ads against Eberling. Other than that, the campaign is going to be pretty much on hold until the police start making headway into their investigation."

Michael folded his hands in his lap.

"What about us?"

Theron locked eyes with Michael.

"I'd say we have just became the lynchpin to this campaign. The people are going to start demanding that Governor Greenleaf cough up his platform concerning sex offenders."

Dakota lowered her head.

"Which means we need to get a report to the Governor ASAP."

Theron somberly nodded his head.

"The campaign... Jennifer... is going to be coming to us."

Dakota jostled the folders around on the table, pulling out her notes from yesterday.

"Well, if people are going to be coming to us for answers, we better have answers to give."

The three other consultants broke away from the television and once again resumed their posts around the conference table. Dakota's eyes glazed over as she stared at the dry-erase board, trying to force her mind past the tragedy which was now playing itself across the network news feeds. Thankfully, she knew what was on the agenda for today; there is something to be said for methodical planning in the face of a tragedy. Dakota just pulled out the dry-erase marker and started circling their research questions on the dry-erase board.

"Okay, yesterday we covered *Nonparametric Tests of Association* for *Categorical Data*, so today we should probably continue on that track."

Michael just leaned forward in his chair as Theron rifled to a clear page of his legal pad. Robin just slumped down in her chair.

"Oh good. We get to talk about this... again."

Dakota stopped circling items, pointing the marker at one question in particular.

"Here... I say we start with the question of whether or not testosterone is associated with the General Aggression Scores."

Robin perked up slightly, staring at Michael on the other end of the table.

"Good... it's time we stick it to the man. Girl Power!!!"

Michael glared back at her.

"Please start lecturing now."

Dakota couldn't help but smile.

"Okay. I say the best test for us to use would be the **Somer's Index d Statistic**."

Robin grinned.

"Wow, I had no idea she was that smart. I mean, first, the 'Thighmaster' and now a statistical procedure. Good for her, especially after she had to play that stupid character on 'Three's Company.' Girl Power!!!"

Theron started to open his mouth but swiftly snapped it shut. Sometimes it really is better to just let some things slide. For her part, Dakota just jotted things down on the dry-erase board.

"The *Somers' d Statistic* is a great *Test of Association* which we can use for two *Variables*, especially when one *Variable* has something distinct or special about it."

At this point, Robin was beaming with glee.

"You mean... *Variables* can be a member of the bourgeoisie or the proletariat?"

Theron instantly patted Robin's hand in an effort to derail the insanity whirling about in her brain.

"Okay... no more quips or discussions about sitcom actors or the tenets of communism for the rest of the day."

Robin glumly leaned back in her chair.

"Spoil my fun."

Dakota rolled her eyes and continued on with her explanation.

"Thank you Theron. With the *Somers' d*, 'distinct' means that there is a particular importance about the *Variable*. More often than not, it means that one of the *Variables* could be *Independent*, while the other *Variable* is *Dependent*."

Theron squinted his eyes, trying to wrap his mind around what Dakota was saying. Suddenly, a faint flicker of inspiration flittered across his face. He nervously cleared his throat in an effort to get Dakota to notice he was trying to volunteer information. Dakota nodded in his direction, giving him the silence to proceed with his question.

"So, would this test be ideal to help determine if some type of event is related to an ensuing behavior?"

Dakota thought quietly for a moment, analyzing how best to respond.

"In a manner of speaking... that is correct. *Somers' d* is used to help us understand whether or not there is an **Asymmetrical Relationship** between

Ordered Variables, and it is used to demonstrate a significant predictive association between those two *Ordered Variables*."

The room grew silent as both Robin and Theron were lost in a sea of statistical jargon. Only Michael seemed to have some vague sense of what she was talking about, nodding his assent to her as she finished speaking. Clearly frustrated, Robin just turned to Theron.

"Okay paper boy, let's hear you dumb down that statement."

Theron just shook his head.

"Nope. Even I know when I am licked."

Robin folded her hands in her lap.

"Quitter."

Dakota quietly thought for a moment, trying to see if there was any way to help clarify what was just said. Sometimes it was just not easy to be a statistics professor. Theron just furrowed his brow.

"Well, haven't we already talked about *Asymmetry* already? You know, back when we talked about *Skew*?"[1]

Dakota grew very still, cautiously collecting her thoughts. It was important for her to be very methodical in how she addressed Theron's questions. After all, he and Robin could go off on some very interesting tangents, and she wasn't entirely keen on fueling that fire. Dakota spoke in a very low, soothing voice.

"You are correct; that is a type of *Symmetry*. But in this instance, *Symmetry* means that it doesn't matter which *Variable* you use as the *Independent Variable* or the *Dependent Variable* when you compute the statistics; you will get the same result either way."

Michael leaned back in his chair.

"I get it. In this instance, *Asymmetrical* means that one of the *Variables* must be identified as the *Dependent Variable* and the other as the *Independent Variable*. You can't interchange them, since that would throw off the statistical calculations."

Dakota nodded in agreement.

"You got it. *Somers' d* allows us to examine whether or not there is an *Asymmetrical* relationship with *Variables* which are *Ordered Data*."

Robin nudged Theron.

"You heard her, put the data in order."

Dakota shook her head.

"Not quite. *Ordered Data* is just a fancy term for *Ordinal Data*. *Ordinal Data* is just data which gives us a sense of greater-than or less-than. Where *Nominal Data* just gave us different categories, *Ordinal Data* gives us a chance to organize information within a particular category. There are numbers associated with *Ordinal Data*, but the numbers just give you a sense of how one number within the data set relates to other numbers in the data set. Think of *Ordinal Data* as being like the Olympic Medals. If you didn't have any other information, all you would know was

[1] Asymmetry is discussed on page 24 in Chap. 2 in the context of skewness.

that the silver medalist did better than the bronze medalist in a particular event, but they didn't do as well as the gold medalist."

Theron nodded.

"I get it."

Dakota quickly checked for the eraser for the dry-erase board. Not seeing one, she tugged her sleeve over her palm and quickly started wiping away some of the markings on the dry-erase board. Robin spun lazily in her chair, smiling at the question which was percolating within her brain.

"Okay, so when we are looking at whether or not there is an *Association* between testosterone level and the General Aggression Scores, which one is the *Dependent Variable*?"

Dakota thought quietly for a moment.

"The General Aggression Scores would be the better choice for a *Dependent Variable*. This is partly because Total Testosterone Levels make a better *Independent Variable*."

Robin, in her usual way, shot a quizzical look at Dakota in an attempt to acquire more information about her perceived random assignment of *Dependent* and *Independent Variables*.

"And, why, pray tell, would General Aggression Scores (GAS) be a better *Dependent Variable*?"

With that question, Robin had thwarted Dakota's attempt to sidestep what was obvious in her mind.

"There are strong theoretical reasons for suggesting that higher levels of testosterone will be antecedent risk factors for more extreme anti-social behaviors—which would lead to higher GAS scores. If we could provide evidence that testosterone was predictively *Associated* with antisocial behavior, this could completely change the way we assess risk of sex offenders. We could use testosterone testing as part of a standard assessment. We could use it to help determine sentencing, amount of supervision, and placement in treatment programs. Also, higher testosterone in adolescence might be predictive of future anti-social behaviors."

Dakota pulled herself away from the table and jotted a new symbol on the dry-erase board:

$$d_{BA}$$

Theron folded his hands behind his head as Robin vacantly twirled her hair between her fingers. All Dakota could do was brace herself for the inane comment which was about to come from Robin's lips. Fortunately, she didn't have to wait long.

"So, is that supposed to mean something? Because it looks like one of the symbols they use for the New York Stock Exchange."

Dakota's lips twisted into a smile. Oh well, at least the comment wasn't as ridiculous as it could have been.

"I can see why you would think that. But in this instance, this particular symbol for *Somer's d* actually tells us which variable is the *Dependent Variable* and which is the *Independent Variable*. As we can also use this test to help determine if there is an Association between an **Antecedent** and the resulting **Consequent**, the equation is just going to help us keep those two *Variables* separate. Trust me; this is going to be crucial when we start to work the equation."

Sensing that he would be tasked to calculate something, Theron pulled out a fresh sheet of paper from his legal pad. As he pulled out his pencil, he looked eagerly at Dakota.

"So what do we do first?"

Dakota popped the cap off her dry-erase marker.

"Well, we're trying to predict that when you compare two participants on their testosterone scores, GAS scores will exhibit the same trend. For example, if John has higher testosterone than Frank, you would also expect John to also have a higher GAS score than Frank as well. In order to make this comparison, you have to have participants who have different testosterone and GAS scores. If the participants are tied, you cannot demonstrate a direction. For example, if John and Frank have the same testosterone level, you are unable to rank them as higher or lower; and therefore, you cannot make a comparison of their testosterone scores. This is also true for the GAS scores: if John and Frank had different testosterone levels, but the same GAS scores, you would not be able to rank them as higher or lower on the GAS *Variable*.

The team sat silent as Dakota continued on with her explanation.

"To make these comparisons, we need to *Rank* everyone from lowest to highest on both *Variables*. This is achieved by first creating an *r X k Contingency Table*."[2]

Robin dropped her head towards the floor.

"Oh not this again."

Dakota shook her head.

"Not quite. This time, we need to set up the *Contingency Table* so that the lowest scores from both testosterone and GAS will appear in the first row and first column. Both *Variables* will then increase as you move across and down the table."

Theron furrowed his brow.

Dakota jotted a quick diagram on the dry-erase board.

"This might help. For example, A has the lowest testosterone and GAS score. B has a higher testosterone level than A but has the same GAS score. D has the same testosterone level as A but has a higher GAS score."

	Total testosterone		
GAS	Low	Medium	High
Low	A	B	C
Medium	D	E	F
High	G	H	I

[2] Contingency tables are discussed in detail on page 72 in Chap. 4.

Michael shifted himself forward, allowing himself to have a clearer view of the dry-erase board.

"I get it. The *Variable* across the top has to be the *Independent Variable*, while the *Variable* down the side is the *Dependent Variable*. However, since we are looking at *Antecedent* and *Consequent*, the *Consequent* would be the *Dependent Variable*, and the *Antecedent* would be the *Independent Variable*."

Theron just shrugged at this statement.

"This makes a certain degree of sense. The *Dependent Variable* is just how you are testing the manipulation of the *Independent Variable*; and the *Consequent* is how you are 'testing' the reaction to the *Antecedent*."

"You got it."

"Once all of our participants and their scores are in the table, we can begin to compare the participants. We start with participant A—the participant with the lowest testosterone and GAS score. We compare participant A with every other participant on the table.

	Total Testosterone		
GAS	Low	Medium	High
Low	A	B	C
Medium	D	E	F
High	G	H	I

"Remember, we want to know in how many cases participants show the same positive trend for both variables, namely—'if testosterone is greater than, then GAS is also greater than.' So, we cannot compare participants who have exactly the same testosterone level (participants in the same column; the dark shaded one), nor can we compare participants who have the same GAS score (participants in the same row; the dark shaded one)."

Again, the team sat mesmerized by the profound explanation that was flowing from Dakota's mouth.

"The reason we cannot compare these participants is because participants who share scores on either *Variable* cannot, by definition, show us direction of both of the *Variables*. We then add up all of the **Concordant** comparisons for participant A—pairs that show a positive relationship. In this case, E, F, H, and I demonstrate a positive trend."

Robin, struggling to stay with Dakota, interjected with a question.

"Great. That gets us one number. How many times do we have to do this?"

Dakota simply continued calmly as she showed her usual patient demeanor.

"Once we are finished with participant A, we continue the same procedure with participant B, who has the next lowest testosterone level but the same GAS score. We compare participant B to everyone who has a higher testosterone level and a different GAS score until we've done all of the possible pairs of participants. Again, we want to see how many participants, who have higher testosterone levels than participant B, also have a higher GAS score. After we have done this for all possible pairs, we add up the number of pairs of participants who follow the positive trend or are *Concordant*, F and I."

	Total Testosterone		
GAS	Low	Medium	High
Low	A	B	C
Medium	D	E	F
High	G	H	I

Dakota stopped to survey the team before she continued on with her discourse.

"However, there are some pairs that actually show the reverse. So, a participant may have a lower testosterone level than another person but have a higher GAS score. This would reflect a negative relationship, or a **Discordant** comparison. For example, H has a lower testosterone level than C, but it has a higher GAS score. Again, participants who share the same testosterone level (column) or share the same GAS score (row) would not be able to provide you with a direction, so you do not examine these comparisons. We compare people who have a lower testosterone level, and see how many of them show higher GAS scores."

Robin couldn't help herself. She realized she was mumbling under her breath after it was too late.

"Yeah, discordant, like my first marriage. Talk about irreconcilable differences."

The team all stopped dead in their thoughts and stared at Robin for a few seconds until Theron pushed through the awkward silence.

"For *Discordant Pairs*, we cannot start with participant A, who has the lowest testosterone level and GAS score, as there is no one who has a lower testosterone level. Instead, we start with the next lowest testosterone level who just happens to be participant B. Right?"

Dakota was beaming from ear to ear.

"Yes, that is right. We then look to see if there is anyone who has a lower testosterone level and a higher GAS score. For example, when comparing participant B, both participants D and G have lower testosterone levels and higher GAS scores. These are *Discordant* pairs.

	Total Testosterone		
GAS	Low	Medium	High
Low	A	B	C
Medium	D	E	F
High	G	H	I

Michael, still partially confused, contributed his two cents.

"This is quite helpful, but how long with this need to go on?"

Dakota sneered at Michael.

"We continue this for all possible pairings. Then, we add up how many pairs of participants follow the negative trend.

Theron's curiosity had finally reached its peak.

"But what do we do with those pairs where participants share the same testosterone level or GAS scores?"

"Well, we add them up, too, and they help us determine the total number of comparisons: the total is the sum of all the positive comparisons, all the negative comparisons, and all of the 'ties'. You then find your answer by subtracting the number of positive pairs from the number of negative pairs, and dividing that number by the total."

Dakota shifted her head towards Theron.

"Can you do it?"

Theron shrugged his shoulders and quickly jotted down a *Contingency Table* using the General Aggression Scores and the Total Testosterone Level from their data set.

	Total Testosterone Level					
GAS	<400	400	500	600	700	800
40 to 49						
50 to 59						
60 to 69						
70 to 79						

Dakota glanced over Theron's work, instinctively knowing that his work was correct. The smile on her face was all the reward he needed.

"That's excellent. Now we just need to find the *Frequencies* for each cell."

Robin grunted.

"So, we need to find out the number of perverts in our data set that have a high testosterone level and have a GAS score of 40–49?"

Theron shook his head as Michael grumbled.

"Okay, how do you not know how to find *Frequencies*? And furthermore, they are sex offenders, not perverts."

Robin parsed her lips to engage her opponent with a barrage of verbal witticism, but Theron clapped his hand over her mouth before she could start.

"I am sure the point of that statement wasn't to debate how to find *Frequencies*, but rather it gave her a chance to utter the word 'pervert' in a campaign meeting."

Suddenly, Theron yelped and wrenched his hand from Robin's mouth, shaking his hand in pain.

"You bit me!!!"

Michael sneered.

"You better hope she had her rabies shots."

Robin glared at Michael as Theron examined his hand. Dakota just shook her head slammed her palms onto the conference table. The loud thud was just what the group needed to shock them back into the task at hand.

"Theron, can you fill in the table?"

Although his hand was still throbbing in pain, Theron gripped his pencil and started scanning the data set. Robin turned her attention away from her nemesis that was seated at the opposite end of the table and started helping Theron in his tabulations. Suddenly, Dakota lurched forward and slammed her hand over Theron's work.

"Wait just a moment."

Robin jumped in her chair, as Theron's eyes grew wide with shock. First, he got bit by one coworker, and now another physically stopped him from doing his job. Whoever said that consulting didn't come with risks? Dakota pointed to the data set.

"I have an idea that may help make our lives a little easier."

In an instant, Robin went from shaking off the shock of Dakota's outburst to feeling a warm sense of joy over having her workload lightened.

"I like the sound of that."

Michael just shook his head in disgust and turned his attention towards Dakota.

"What do you mean?"

Dakota lifted herself up from the conference table and walked over to the silent television set.

"Okay, we just heard that another elderly woman was attacked in her own home, right? That means that a lot of other people are going to be worried about adults attacking other adults for the foreseeable future."

All three consultants just nodded their head, waiting to follow Dakota's logic. Dakota pointed to the data set.

"Now, we have a data set that looks at a wide variety of sex offenders, both child sex offenders and others. Correct?"

Once again, her colleagues just nodded. She continued on.

"Okay, so if adults and senior citizens are now worried about being attacked in their home, wouldn't it make more sense for us to focus only on those sex offenders who attacked adults?"

Theron violently nodded his head.

"Makes sense. The campaign is going to want some empirically sound information on adults who have been assaulted by other adults. That will help them to speak to some of the inevitable concerns which are going to be raised by voters."

Robin shook her head.

"Inevitable concerns?"

Theron just cocked his head in Robin's direction.

"Say what you will about the elderly, but they do make an impressively effective voting bloc."

Robin suddenly grew very quiet. After all, there is no sense in arguing a point when she knows he is right. Dakota pulled the data set off the table so she could quickly glance over it.

"Okay, how many sex offenders on this list only assaulted other adults?"

Theron reached towards Dakota to take back his data set and began counting down the list.

"That leaves us with 27, looking at the nature of the offenses in the data set.[3]"

[3] This information can easily be found in the data set provided at the beginning of Chap. 2. The reader can match the participant number here with the participant number in the full data set to see each individual charge and description.

Dakota pulled over the data set, making certain to focus on the 27 individuals that committed offenses against adults. She carefully jotted the information onto the dry-erase board.

Participant	General aggression score (GAS) (20–80)	Total Testosterone Level (ng/dl)[c]
1	65	710
4	43	307
11	58	892
16	75	607
20	60	650
25	64	491
26	60	515
29	73	586
31	65	439
32	43	530
35	74	67
36	47	664
38	43	525
40	60	734
41	43	587
46	53	874
48	48	622
55	76	780
70	67	699
71	70	523
74	69	575
78	65	507
80	55	773
81	56	550
90	45	793
93	78	775
94	40	653

"So, now we are going to look at these 27 individuals to determine whether or not there is a relationship between Total Testosterone Level and their General Aggression Scores. Everyone okay with that?"

All three consultants nodded their heads, but Dakota was now thoroughly absorbed into staring at the dry-erase board. For a brief moment, she managed to break away from the information on the board and turned her head towards Theron.

"Can you put this into the *Contingency Table*?"

Theron once again pulled his legal pad towards him, and after he was certain that no one was going to stop him from computing the math, he gingerly went back to work. After several minutes, Theron finished filling in the *Contingency Table* and slid it over to Dakota. She snatched up the piece of paper and immediately began to replicate Theron's work on the dry-erase board.

	Total Testosterone Level					
GAS	<400	400	500	600	700	800
40 to 49	1	0	3	3	1	0
50 to 59	0	0	1	0	1	2
60 to 69	0	2	3	2	2	0
70 to 79	1	0	2	1	2	0

Dakota stepped back to admire her work, while Robin seemed slightly less impressed.

"Okay... now what?"

Dakota pointed to certain sections of the *Contingency Table*.

"Now we need to total each row; we'll designate the row total as R_i. We also need to total each column; we'll designate the column total as C_i."

Theron leaned forward, reclaiming his work from Dakota. As he was doing the math on his sheet of paper, Dakota was completing the calculations on the dry-erase board.

	Total Testosterone Level						
GAS	<400	400	500	600	700	800	R_i
40 to 49	1	0	3	3	1	0	8
50 to 59	0	0	1	0	1	2	4
60 to 69	0	2	3	2	2	0	9
70 to 79	1	0	2	1	2	0	6
C_i	2	2	9	6	6	2	27

Still admiring her work, Dakota spoke in a low voice, as if she were speaking only to herself.

"Now that we have done that, we need to find the number of *Concordant* and *Discordant* pairs."

$$Number\ of\ concordant\ pairs = \sum_{i,j} n_{ij} N_{ij}^{+}$$

$i = the\ row$
$j = the\ column$
n_{ij} = *the frequency of people who were in a particular i and j combination*
N_{ij}^{+} = *the sum of frequencies* ***below*** *and to the* ***right*** *of ij; the first subscript (i) refers to a row (i.e., 40–49), while the second subscript (j) refers to a column (i.e., <400)*

"Remember for the *Somers' d* statistic, we are looking for the number of *Concordant* and *Discordant* pairs within the *Rankings*. To find the number of *Concordant* pairs, we first start with row one, column one, which is the lowest testosterone and the lowest GAS. We then find all of the pairs that demonstrate a positive relationship and add up the *Frequencies*."

	Total Testosterone Level					
GAS	<400	400	500	600	700	800
40 to 49	1	0	3	3	1	0
50 to 59	0	0	1	0	1	2
60 to 69	0	2	3	2	2	0
70 to 79	1	0	2	1	2	0

"Look at our example N_{11}; we have already determined that the first row and the first column are not included in the calculation so we simply add up all the frequencies that are below and to the right, and we get 18. Theron, what do you get for N_{12}?"

Theron did the quick math.

"The number of *Concordant* pairs for N_{12} is 16."

Dakota nodded her approval and moved to Robin.

"That is right, Theron. Robin, do you care to calculate the *Concordant* pairs for N_{13}?"

Dakota could see the obvious apprehension on Robin's face. Robin became a bit more relaxed as Theron leaned over to assist in the calculation.

"Sure, I guess… is the answer you are looking for 10?"

Dakota's expression showed her excitement that the group was beginning to grasp the concept.

"Yes, not only is that the answer I am looking for, but it is the correct answer!"

Happy with Dakota's approval, Robin and Theron finished up the calculation for the *Concordant* pairs after a few minutes.

"Here Dakota, have a look at these to make sure we did them right."

$N^+_{13} = 10$	$N^+_{21} = 14$	$N^+_{24} = 4$	$N^+_{32} = 5$
$N^+_{14} = 7$	$N^+_{22} = 12$	$N^+_{25} = 0$	$N^+_{33} = 3$
$N^+_{15} = 8$	$N^+_{23} = 7$	$N^+_{31} = 5$	$N^+_{34} = 2$
			$N^+_{35} = 0$

Excited that Robin was becoming involved in the process, Dakota eagerly continued on with her explanation.

"They look good. Now that we have the *Concordant* pairs, it's time to go to the other side of the coin."

Dakota cautiously went to the dry-erase board, making certain to highlight the differences between the two equations:

$$\textit{Number of discordant pairs} = \sum_{i,j} n_{ij} N^-_{ij}$$

i = the row
j = the column
n_{ij} = the frequency of people who were in a particular i and j combination

N_{ij}^{-} = *the sum of frequencies* ***below*** *and to the* ***left*** *of ij; the first subscript (i) refers to a row (i.e., 40–49), while the second subscript (j) refers to a column (i.e., 400)*

"For *Discordant* pairs, we determine all of the pairs that demonstrate a negative relationship. These are instances when a participant has a lower testosterone level than another person but has a higher GAS score. So, the *Discordant* pairs for N_{11} is 0 because there are no values to left of the 'A' column. If, however, we look at the *Discordant* pairs for N_{12}, we get a value of 1 because that is the number of individuals in the table that falls to the left of the second column. When we look at the *Discordant* pairs for N_{13}, we get a value of 3, and when we look at N_{14}, we get a value of 8. Here, it makes a bit more sense when we look at the table again. Does this make sense now?"

Total Testosterone Level

GAS	<400	400	500	600	700	800
40 to 49	1	0	3	3	1	0
50 to 59	0	0	1	0	1	2
60 to 69	0	2	3	2	2	0
70 to 79	1	0	2	1	2	0

Theron jotted down a few notes and handed the paper back to Dakota. Dakota cast a sidelong glance over his notes, nodding as she double-checked his work.

$N_{14}^{-} = 9$	$N_{22}^{-} = 1$	$N_{25}^{-} = 11$	$N_{33}^{-} = 1$
$N_{15}^{-} = 12$	$N_{23}^{-} = 3$	$N_{26}^{-} = 15$	$N_{34}^{-} = 3$
$N_{16}^{-} = 17$	$N_{24}^{-} = 8$	$N_{32}^{-} = 1$	$N_{35}^{-} = 4$
			$N_{36}^{-} = 6$

"That's fantastic. Now we go back to this symbol."

Dakota popped off the cap to her dry-erase marker and pointed to the first symbol she jotted down on the board, hoping that it could start to tie things together:

$$d_{BA}$$

Dakota pointed at the equation.

"Since we are interested in the *Association* from *Variable* A (Total Testosterone Level) to *Variable* B (GAS), we'll calculate d_{BA}. Or another way, we are treating *Variable* A as an *Independent Variable* and *Variable* B as a *Dependent Variable*."

Michael was now leaning forward, completely absorbed into what Dakota was saying. He gruffly cleared his throat, making certain that Dakota was aware of him. Dakota straightened her spine as she glanced in his direction.

"So, if *Variable* A was a *Dependent Variable* and *Variable* B was an *Independent Variable*, we would find d_{AB}?"

Dakota nodded.

"That's right. And now that we have all of the pieces of the puzzle, it's time we put it together."

To help lead the group, Dakota added a few tweaks to the original symbol she put on the dry-erase board.

"If we want to test for *Significance* where *Variable* A is the *Independent Variable* and *Variable* B is the *Dependent Variable*, we first find the *Variance* of d_{BA}. If we are testing the *Significance* where *Variable* B is the *Independent Variable* and *Variable* A is the *Dependent Variable*, we would find the *Variance* of d_{AB}. There are two equations for each that can be used to calculate this. The first one is rather long and requires that we find $M_i{}^+{}_j$ and $M_i{}^-{}_j$[4] to solve it":

$$var(d_{BA}) = \frac{4\sum_{i=1}^{r}\sum_{j=1}^{k} n_{ij}\left(N_{ij}^{+} + M_{ij}^{+} - N_{ij}^{-} - M_{ij}^{-}\right)^2}{\left[N^2 - \sum_{j=1}^{k} C_j^2\right]^2} \quad or \quad var(d_{BA}) = \frac{4(r^2-1)(k+1)}{9Nr^2(k-1)}$$

$$var(d_{AB}) = \frac{4\sum_{i=1}^{r}\sum_{j=1}^{k} n_{ij}\left(N_{ij}^{+} + M_{ij}^{+} - N_{ij}^{-} - M_{ij}^{-}\right)^2}{\left[N^2 - \sum_{j=1}^{k} R_j^2\right]^2} \quad or \quad var(d_{AB}) = \frac{4(k^2-1)(r+1)}{9Nk^2(r-1)}$$

Robin quickly stepped in to find out more information.

"Wait just a minute. Why do we need to find the *Variance* first?"

This was exactly the type of comment that Dakota was hoping to avoid.

"Well, the reason is twofold. First, we have found that there are differences which exist by chance alone; so, we need to compare everything against this chance position, and finding the *Variance* helps us do that. Second, finding the *Variance* helps us understand the specific equation here."

Before Robin could interject with a snarky remark, Dakota quickly jotted more information onto the dry-erase board.

"The second equation can be used to find an approximation of the *Variance* that is often very close to the first equation; however, it requires an assumption that the sample is **Multinomial**."

Robin just shook her head.

"Sounds like the United Nations."

Theron reached out and patted Robin's hand.

"*Multinomial*... not multinational. It's just another word for **Polynomial**[5] which is really just a string of numbers and letters separated by addition and subtraction signs."

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[4] M_{ij}^{+} is the sum of the *Frequencies* for the terms above and to the left in a contingency table, while M_{ij}^{-} is the sum of the *Frequencies* for the terms above and to the right. Both terms are found using the same procedures for finding $N_{ij}^{+}$and $N_{ij}^{-}$where the row and column that include the specific cell are not included in the count.

[5] A polynomial is an expression or equation of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ where n is a nonnegative integer. This means the powers of x are only whole numbers. A multinomial is a polynomial with at least two terms.

Dakota just continued on with the explanation.

"What we need to do is figure out the *Variance* for the d_{BA}. Let's use the simplified version of the equation. Generally, differences, when they do exist between the equations, are very small.[6] "

Theron shook his head and went to work:

$$var(d_{BA}) = \frac{4(r^2-1)(k+1)}{9Nr^2(k-1)} = \frac{4\left(4^2-1\right)(6+1)}{9(27)4^2(6-1)} = \frac{420}{19440} = 0.0216$$

Once Theron finished the calculations, he shoved the legal pad over towards Dakota. She instantly started transcribing the numbers onto the dry-erase board. As she was writing numbers onto the board, Robin started fidgeting in her chair.

"Now what?"

Dakota became very quiet, allowing her memory to unfold the answer to Robin's question. She quickly cleared off a section of the dry-erase board with a few swipes of her wrist and started to jot down symbols and numbers onto the board.

"Well Robin, now we need to determine *Statistical Significance*. For this test, we need to calculate a *z-score* based on the *Somers' d* $\mathrm{d_{BA}}$":

$$Z = \frac{d_{BA}}{\sqrt{var(d_{BA})}}$$

Theron just shrugged his shoulders.

"This should be easy enough."

Theron quickly pulled the appropriate numbers off the dry-erase board and calculated the final total. Once complete, he slid the paper back towards Dakota, who then transposed his work onto the board:

$$= \frac{-0.0355}{\sqrt{0.0216}} = -0.2415$$

Robin rolled her eyes as Dakota began rifling through the stack of papers on the conference table. Realizing that Dakota was oblivious to her obvious dismay, Robin had no choice but to pipe up.

"I hate to bother you as you sort through your recycling, but aren't you supposed to be telling us what happens now?"

Seizing upon one paper in particular, Dakota yanked it free from the others and laid it out onto the table.

"Now, we need to take our *Calculated Value* and compare it with the *Critical Value*. Since we are using the *z-score* to help determine *Statistical Significance* for

[6] The simplified equation provides an equitable estimate for the longer equation. Using the simplified equation requires the assumption that the frequencies in the cells of the contingency table are fairly equal.

the *Somers' d*, we need to look up the corresponding value on the **z-table** for the score we just calculated.[7]"

Having nothing cute or sarcastic to say, Robin just sat still as Dakota strained to decipher the miniscule writing on the *z-table*. Finally, Dakota glanced back at the dry-erase board in order to remind herself of what number she was looking for. As she returned her attention to the *z-table*, Dakota began muttering quietly to herself.

"Let's see, we have an $\alpha > .05$,[8] and our *Calculated z-value* is -0.2415. That is much less than the *Statistical Significance* cutoff score of -1.96...[9]"

Michael cupped his face in his right hand, inexplicably enchanted by Dakota speaking softly to herself. However, Theron was impatiently itching to write down their final conclusions. Finally, he could no longer contain his eagerness and blurted out his thoughts.

"So... what did we find?"

Dakota jotted some notes onto the paper with all their information scribbled onto it.

"It means that we fail to *Reject the Null Hypothesis*; there is no relationship from one *Variable* to another."

Robin shrugged.

"And that tells us what about testosterone and GAS scores?"

Michael grumbled towards Robin.

"It means that there is no relationship between Total Testosterone Level and General Aggression Scores."

Dakota nodded as she filed away their work using the *Somers' d* test.

"Michael is correct. Since we have no relationship and there is nothing more here to be done, I suggest we move on."

Theron nodded his assent, while Robin whistled her relief. She just glanced at Theron as she started swinging in her chair.

"Good. Listening to the explanation for that test was painful."

Dakota shook her head as a grin crossed her face. It may have been "painful," but it was unfortunately a necessity. She quickly grabbed the eraser and cleared off all the statistical gibberish which had clouded the board since they started working. Once the space was clean, Dakota turned her attention back to the group. Robin, still swaying back and forth within her chair, decided to move the conversation forward with her usual tact.

"All right, what's next?"

Michael grumbled as Theron wiped his hand across his face. Dakota glanced over to the dry-erase board, using her keen intuition in order to identify which research question they should attempt to tackle next. She gently stepped towards the section on the dry-erase board with all of their research questions scribbled onto it.

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[7] A specific discussion regarding how to read these kinds of tables is discussed on page 77 in Chap. 4 in the context of critical values. The process for reading this table is the same for all procedures.

[8] Alpha levels are discussed on page 79 in Chap. 4.

[9] Page 243 in Chap. 10 contains a detailed discussion of statistical *Significance* for z-scores.

Once again, Dakota's voice reverberated through the room as she spoke to her colleagues (and herself).

"Well, we are looking at Association with Ordinal Data... so why don't we just knock out the most obvious one."

Robin stopped swinging in her chair, her eyes fixated on Dakota.

"Did you say obvious or obtuse? Because I agree... these are very obtuse."

Theron shook his head.

"She didn't say obtuse."

Robin just smiled.

"I like obtuse."

Ignoring Robin and Theron, Dakota circled one question in particular.

"Okay, how about we examine whether or not sentence length is associated with the General Aggression Scores for the convicted adults to see if there those being sentenced to longer terms have a higher General Aggression Score or not. Maybe this will help us clarify the direction we should take from here."

Robin's eyes once again glazed over. Michael crossed his arms over his chest, reluctantly allowing himself to engage with Dakota in spite of the others in the room. He leaned forward and snatched the data set from the center of the table.

"Well, from the looks of this, both sentence length and General Aggression Scores are *Interval Data Scale*. Couldn't we just run a *Pearson Correlation*[10]?"

Dakota hesitated at his comment. It was obvious Michael was really trying to be more engaged with the group, no sense in cutting him off when he was actually putting forth an effort. Dakota leaned over in Michael's direction, giving her a better vantage point of the data set. She chose her next words carefully.

"Well, we could run a *Pearson's R*, but you have to remember that we don't have a *Random Sample* of a **Population**. Thus, a **Parametric Test**, such as *Pearson's R*, would be inappropriate."

Michael shrugged.

"I am sure that little fact hasn't stopped people from using *Pearson's R* in the past."

Once again, Dakota had a difficult time responding to Michael. After all, he was correct.

"Well, we could do it. And yes, I am certain that many researchers have just used *Pearson's R* without considering whether or not it was appropriate to use the test based on the data they have. But given what we just saw on the news, we really should play it safe. After all, if we run *Pearson's R*, our results could be called into question, and the campaign could be hurt, simply because it was an inappropriate test to run."

Michael just nodded, quietly absorbing what Dakota had to say. Strange, but Dakota liked this new side of him, actually engaging with the group rather than constantly arguing with them. Theron started riffling through his legal pad, looking for a clean sheet of paper.

[10] A discussion about Pearson's R can be found on page 71 in Chap. 4.

"So, what test do we run instead?"

Dakota just shrugged her shoulders as a smile crossed her lips.

"**Spearman's Rho**."

As soon as Dakota finished speaking, Robin was giggling uncontrollably. Theron just poked her in the ribs with his elbow in an effort to snap her out of it.

"What's up with you?"

Robin finished giggling, pointing at Dakota as she fought to abate the laughter behind her eyes.

"Oh come on. That sounds like a cross between a flavor of chewing gum and a character on 'Star Trek: The Next Generation.' No one is going to take it seriously."

Michael just shot Robin an angry stare.

"What are you talking about?"

Theron just shook his head.

"I think she is confusing spearmint with Ensign Ro Laren."

Robin pouted in her chair.

"No fair. It's not a good pop culture reference if you have to explain it."

Theron smirked.

"Well, that's because it wasn't a good pop culture reference."

Dakota stepped away from the inane conversation and wrote the name of the test on the dry-erase board. Seeing their leader imparting information onto the board seemed to calm the group down, causing all of them to start paying attention to Dakota's explanation. Once she wrote the name of the test on the board, Dakota snapped the cap back onto the marker and returned her attention to the group.

"*Spearman's Rho* is actually a very well-known statistical procedure. In fact, it's probably one of the most well-known *Nonparametric Procedures* out there."

Still sulking, Robin's lips snarled in snarky anger.

"So, why haven't I heard of it?"

Michael suddenly became animated as he sat in his corner of the conference room; Robin discussing her intelligence was something which was too good for him to pass up. He was about to volley a caustic response in Robin's direction, but Michael caught sight of Dakota within his peripheral vision. The exasperated look in her face was enough to stifle the witty comment rattling around in his brain. He quickly slid back into his chair, meekly looking at the ground. Dakota rolled her eyes and just continued on with her explanation.

"Now, *Spearman's Rho* is essentially the *Nonparametric* equivalent to *Pearson's R*. It can tell us whether or not there is a **Correlation** between two *Variables*."

Michael leaned back into his chair, still focusing all of his attention onto Dakota. Theron was also trying to pay attention but was uneasy about what could possibly come from Dakota's explanation of this test.

"I've heard a little about this test. Is this like *Somers' d*, where there has to be something 'special' about one of the *Variables*?"

Dakota just shook her head.

"Nope. Actually, *Spearman's Rho* is extremely versatile. The test can be used when both *Variables* are *Ordinal* or *Ranked*. Now, we can use data where one

Variable is *Ranked* and the other is *Interval Scale* or even when both *Variables* are of *Interval Scale*. However, if we do have data which are *Interval Scale*, that data must be converted to *Ranked Data* in order to use it in the *Spearman's Rho* calculation."

Michael suddenly chimed in on Dakota's explanation.

"Which is what we have with our two *Variables*."

Theron hurriedly jotted down everything that was being said. Suddenly, a question exploded in his mind, and he blurted it out loud.

"Does this mean that if you calculate a *Spearman's Rho* and get an answer of 1.0, it denotes a perfect **Positive Correlation**[11]; that when one *Variable* increases, there is a corresponding increase in the other *Variable*?"

A questionable look crossed Dakota's face as she tried to think of the best way to address Theron's inquiry.

"Essentially, but not quite. Remember, *Spearman's Rho* is using *Ordinal/Ranked Data*. *Interval Data* tends to give us more information; the numerical *Interval* points tend to tell us a lot about the data, and this information is missing in *Ordinal* scores. So, if we were to run *Pearson's R* and *Spearman's Rho* on the data we have for General Aggression Scores and sentence length, there would be some discrepancies in the final calculated values. Remember, we have *Interval Scale* data here, but we do not meet the assumption about *Randomly Sampled* data so *Spearman's* is more appropriate here. You'll see why when we start going through the test."

Michael placed his palms onto the table, leaning his weight onto them as he started to engage with the group once more.

"Okay, what do we do first?"

Dakota marched over to the data set, gripped it firmly, and handed it to Theron.

"First, I want us to determine the sentence length and GAS for each offender who committed acts against other adults. So, isolate and write down all of the General Aggression Scores and Sentence Lengths for the 27 individuals in our data set."

Theron sighed deeply as he took the data set in hand and methodically started pulling the numbers from the data set. Once complete, he slid his legal pad towards Dakota, who started transposing the numbers onto the dry-erase board.

Participant	Sentence (in years)	GAS
1	7	65
4	1	43
11	5	58
16	4	75
20	2	60
25	5	64
26	5	60

(continued)

[11] A discussion about correlations can be found on page 70 in Chap. 4.

Participant	Sentence (in years)	GAS
29	5	73
31	9	65
32	2	43
35	5	74
36	5	47
38	1	43
40	5	60
41	2	43
46	3	53
48	2	48
55	6	76
70	1	67
71	7	70
74	3	69
78	5	65
80	2	55
81	2	56
90	1	45
93	5	78
94	6	40

Once Dakota completed writing the numbers onto the board, she stepped away from the board and looked to her colleagues. Realizing that this next part could be rather confusing, she chose her next words very carefully.

"Now that we have completed that, we need to *Rank* each sex offender's scores on each *Variable*; starting at 1 and going through to however many cases you happen to have in your data."

Robin silently shook her head as Theron meekly asked a question of his leader.

"*Rank* each *Variable*?"

Dakota nodded.

"It's pretty much what you would expect. You start with the lowest number and label it as '1,' the second lowest number would be labeled as '2,' and so on for the first *Variable*. One all the rankings are completed for the scores within the first *Variable*, you then repeat the process with the second *Variable*."

Theron spoke up once more, his voice tinged with uncertainty.

"So, we aren't *Ranking* both *Variables* together?"

Dakota shook her head.

"Not with this test. We *Rank* each *Variable* individually. You got it?"

Theron smiled, pleased that he was able to grasp what Dakota was saying.

"So, in our case, we would go to 27 because we have information on the smaller sample of 27 sex offenders."

Michael sighed, signaling Dakota to pay attention to him and not Theron. She turned away from Theron nodded in his direction, allowing him to speak.

"Do you always have to *Rank* the scores from lowest-to-highest, or can you *Rank* the scores from highest-to-lowest as well?"

Dakota just shook her head at the question.

"Honestly, it's based on your preference. It shouldn't matter either way. However, it will slightly alter your final answer. You'll see what I mean soon enough."

Theron just pulled his arm forward and pulled his legal pad back to him.

"Well, I see that someone was given a sentence of 1 year, so I guess that is the best place to start."

Theron started scanning the numbers but suddenly froze in his tracks.

"Um, what do you do if you have a lot of ties?"

Robin sat in amazement.

"Ties? How many sex offenders only got sentences of 1 year?"

Theron quickly tallied up the numbers from the data set.

"Actually, there are 4 of them."

Dakota held her hand up, as if to stave their confusion at bay.

"It's not as complicated as you might think. What we do is we get the **Average** of the **Assigned Rankings** for those 4 numbers."

Unfortunately, that response elicited nothing but confusion among her colleagues. Once again, she snatched her marker and went to the dry-erase board.

"Okay, you have to remember that we are ranking from lowest-to-highest, so the lowest numerical *Ranking* will be 1, and the highest numerical *Ranking* will be 27. Correct?"

Slowly, all three of them nodded their assent at what she was saying. Dakota just continued on.

"If you have a lot of numbers vying for the same *Ranking*, you need to *Average* the *Assigned Rankings*. In this instance, we have 4 numbers competing for the lowest score, as well as the second lowest score, the third lowest score, and the fourth lowest score."

Robin had an incredulous look in her eyes as Dakota imparted her information. Dakota closed her mouth, giving Robin the opportunity to speak out.

"Wait a sec... you're telling me that we are just trying to figure out an *Average*?"

Dakota started pacing about the conference room, trying to think of a way to best demonstrate this concept.

"Now, suppose we have four medical students who take an exam. Now, three students earn a top score of 1000, and the fourth student earns a score of 750. How would you rate this?"

Student 1	1000
Student 2	1000
Student 3	1000
Student 4	750

Michael leaned back in his chair, his mind trying to wrap itself around this concept.

"Well, the *Rankings* will be from 1 through 4 as well, correct?"

Dakota nodded. Michael just continued on with his hesitant explanation.

"So, we know that the lowest *Ranking* is 4, so we obviously need to get an Average for the first 3 scores."

Dakota went to the board, adding information to her diagram.

"Correct. Now we just need to add up the *Assigned Rankings* and divide the number of terms."

Student 1	1000	2
Student 2	1000	2
Student 3	1000	2
Student 4	750	4

Theron grinned.

"All right, I think I get it. But how this work for our data?"

Dakota just shrugged.

"Essentially, we just have more *Assigned Rankings* to work with. For ties, we compute the average of the assigned ranks. For example, the first 4 offenders all received a sentence of 1 year. The assigned rank will be 2.5."

Dakota pulled out her marker and started working on the *Rankings* for the data set:

$$\frac{1+2+3+4}{4} = 2.5$$

Participant	Sentence (in years) rank	GAS rank
4	2.5	3.5
38	2.5	3.5
70	2.5	20
90	2.5	6

The group looked over the information Dakota placed on the dry-erase board. Suddenly, all three of them looked content; this was sinking in for the group. Dakota just leaned herself next to the dry-erase board, looking at Theron.

"So, the first 4 *Rankings* share an *Assigned Rank* of 2.5. The next lowest score would have an *Assigned Rank* of 5 unless there are more ties. You got it?"

Theron vigorously nodded his head and started working on the rest of the scores in the data set. It took him a few moments, but he was finally able to finish off his calculations. Once completed, he handed his work to Dakota, who in turn put all this information onto the dry-erase board.

Participant	Sentence (in years) rank	GAS rank
1	25.5	18
4	2.5	3.5

(continued)

Participant	Sentence (in years) rank	GAS rank
11	18	12
16	13	25
20	7.5	14
25	18	16
26	18	14
29	18	23
31	27	18
32	7.5	3.5
35	18	24
36	18	7
38	2.5	3.5
40	18	14
41	7.5	3.5
46	11.5	9
48	7.5	8
55	23.5	26
70	2.5	20
71	25.5	22
74	11.5	21
78	18	18
80	7.5	10
81	7.5	11
90	2.5	6
93	18	27
94	23.5	1

Dakota stepped away from the board, allowing the group to see what she was doing. Robin then twirled a lock of her hair between her fingers.

"And... now what?"

Dakota pointed to the two sets of *Rankings*.

"Well, now we have to find the *Difference* between the two *Rankings*."

Robin's mouth dropped open.

"Wait, all we are doing is subtracting?"

Dakota nodded.

"That's it. We are just subtracting the *Rankings* for the GAS scores from the *Rankings* for sentence length and we call that resulting number d_i."

Theron quickly started his calculations but stopped after a few moments.

"Um, is it okay if we get a negative number?"

Dakota nodded in his direction, and Theron just finished up his calculations. Once again, he tore off his sheet of paper from the legal pad and handed it to Dakota. She quickly added these numbers to the growing chart on the dry-erase board.

Participant	Sentence (in years) rank	GAS rank	d_i
1	25.5	18	7.5
4	2.5	3.5	−1
11	18	12	6
16	13	25	−12
20	7.5	14	−6.5
25	18	16	2
26	18	14	4
29	18	23	−5
31	27	18	9
32	7.5	3.5	4
35	18	24	−6
36	18	7	11
38	2.5	3.5	−1
40	18	14	4
41	7.5	3.5	4
46	11.5	9	2.5
48	7.5	8	−0.5
55	23.5	26	−2.5
70	2.5	20	−17.5
71	25.5	22	3.5
74	11.5	21	−9.5
78	18	18	0
80	7.5	10	−2.5
81	7.5	11	−3.5
90	2.5	6	−3.5
93	18	27	−9
94	23.5	1	22.5

Michael looked over the numbers and folded his hands behind his head.

"So what's the next step?"

Dakota pointed at the *Difference* scores.

"Now we *Square* all the *Difference* scores. This is what will get rid of all the negative terms, since all terms that are *Squared* are positive."

Theron cocked his head to the right and quickly *Squared* all of the *Difference* scores. Once more, Dakota took this information from Theron and put all the information on the dry-erase board.

Participant	Sentence (in years) rank	GAS rank	d_i	d_i^2
1	25.5	18	7.5	56.25
4	2.5	3.5	−1	1
11	18	12	6	36
16	13	25	−12	144
20	7.5	14	−6.5	42.25
25	18	16	2	4

(continued)

Participant	Sentence (in years) rank	GAS rank	d_i	d_i^2
26	18	14	4	16
29	18	23	−5	25
31	27	18	9	81
32	7.5	3.5	4	16
35	18	24	−6	36
36	18	7	11	121
38	2.5	3.5	−1	1
40	18	14	4	16
41	7.5	3.5	4	16
46	11.5	9	2.5	6.25
48	7.5	8	−0.5	0.25
55	23.5	26	−2.5	6.25
70	2.5	20	−17.5	306.25
71	25.5	22	3.5	12.25
74	11.5	21	−9.5	90.25
78	18	18	0	0
80	7.5	10	−2.5	6.25
81	7.5	11	−3.5	12.25
90	2.5	6	−3.5	12.25
93	18	27	−9	81
94	23.5	1	22.5	506.25
			$\sum d_i^2 =$	1,651

Theron finished off *Squaring* the *Difference* scores and slid the paper to Dakota. She eyed the paper for a few moments, allowing her mind to work through the numbers. She suddenly started adding numbers on her fingers. Robin arched her eyebrows in confusion.

“Why do I feel like I am watching ‘Rain Man’?”

Dakota stopped counting for a few moments, silently giggling at Robin’s comment.

“Sorry. The next step is that we add up all of the *Squared Difference* scores.”

Theron pulled out a fresh page of notebook paper and started calculating out the numbers. Robin just groaned as Theron worked out the numbers.

“Why couldn’t she just tell us that?”

Theron shook his head as he finished calculating the numbers.

“The total is 1651.”

Dakota snapped open the dry-erase marker and wrote the final number on the board. Suddenly, Dakota froze in place as she noticed something within the data from the dry-erase board. Dakota stepped away from the dry-erase board, allowing all the information to wash over her.

“**Ties**.”

Robin and Theron could hear Dakota muttering to herself. Robin leaned into Theron’s direction, trying to keep their conversation subdued.

“Is she talking about neckties?”

Dakota, trying to keep her focus on the board, pointed to several different parts of the dry-erase board.

"We have a slight problem. Our data has a lot of *Ties* within *Squares* of the *Difference* scores. That's going to change things a little bit."

Michael cleared his throat in his side of the room.

"How so?"

Dakota quickly wiped off a section of the dry-erase board and jotted down a new equation onto the board:

$$r_s = 1 - \frac{6\sum_{i=1}^{N} d_i^2}{N^3 - N}$$

N = *total number of participants*
d_i = *the difference in rankings for each participant*

Dakota pointed at the equation.

"Since there are so many *Ties*, we must make a correction to the equation. To do this, we must first determine the total number of *Tied Observations* in the 'sentence' *Variable*. In this sample, we have 6 sets of ties in the sentence variable..."

Dakota quickly highlighted these ties on the dry-erase board.

4 sentences of 1	2 sentences of 3	2 sentences of 6
6 sentences of 2	9 sentences of 5	2 sentences of 7

"Now, in order to understand this equation, we need to know that t_i is the size of the ties ranks for each set. So, the size of the ties rank of 1 is 4, and the size of the ties rank of 2 is 6":

$$\begin{aligned} T_{x(sentence)} &= \sum_{i=1}^{g} \left(t_i^3 - t_i\right) \\ &= \left(4^3 - 4\right) + \left(6^3 - 6\right) + \left(2^3 - 2\right) + \left(9^3 - 9\right) + \left(2^3 - 2\right) + \left(2^3 - 2\right) \\ &= 1008 \end{aligned}$$

"... and 3 sets of *Ties* in the General Aggression Scores *Variable*."

4 scores of 43	3 scores of 60	3 scores of 65

$$\begin{aligned} T_{y(GAS)} &= \sum_{i=1}^{g} \left(t_i^3 - t_i\right) \\ &= \left(4^3 - 4\right) + \left(3^3 - 3\right) + \left(3^3 - 3\right) \\ &= 108 \end{aligned}$$

Michael leaned onto his elbows, trying to take everything in.

"So, how do we correct for the *Ties*?"

Dakota pointed to the equation for *Spearman's Rho*.

"Well, we can't use this equation, but we can use a modification of it. We solve for r_s using the formula that makes the correction for ties":

$$r_s = \frac{(N^3 - N) - 6\sum d^2 - (T_x + T_y)/2}{\sqrt{(N^3 - N)^2 - (T_x + T_y)(N^3 - N) + T_x T_y}}$$

$$T_{x\,or\,y} = \sum_{i=1}^{g} (t_i^3 - t_i)$$

$g = the\ number\ of\ sets\ of\ ties$
$t_i = the\ size\ of\ the\ ties\ ranks\ in\ a\ set$

Theron cocked his head from side to side, trying to figure out the best way to tackle the formula. After turning this equation around in his mind, he just cleared his throat and started plugging away at the math:

$$r_s = \frac{(N^3 - N) - 6\sum d^2 - \frac{T_x + T_y}{2}}{\sqrt{(N^3 - N)^2 - (T_x + T_y)(N^3 - N) + T_x T_y}}$$

$$= \frac{(27^3 - 27) - 6(1651) - \frac{1008 + 108}{2}}{\sqrt{(27^3 - 27)^2 - (1008 + 108)(27^3 - 27) + (1008)(108)}}$$

$$= \frac{9192}{19092.69766} = 0.4814$$

Dakota eyed Theron as he finished conducting the math and snatched up the paper as soon as he was done. She beamed with pride as she transposed the equation onto the board, triumphantly circling the final answer with a bright red circle. Robin just pointed at the circle, firing her caustic wit at the numbers.

"As fun as this is... now what?"

Dakota shrugged her shoulders.

"Same thing we always do. We take our *Calculated Value* and compare it to our *Critical Value* to determine *Statistical Significance*. To determine *Significance*, we have to assume that the participants are a *Random Sample* from the *Population*. If they are, we calculate **t** with $df = N - 2$ since we are using a large *Sample Size*; $N > 10$."

Theron stared off incredulously at Dakota.

"*t*. What is *t*?"[12]

Robin rolled her eyes.

"It's a letter. Did you miss that discussion in kindergarten?"

[12] Further discussion on the t-test can be found on page 208 in Chap. 9 with regard to the Permutation Test for Two Independent Samples.

Dakota gripped the back of the chair in front of her, slightly bowing her head forward. She felt as if she had just finished building a desk from IKEA and forgot to include a piece in the frame, thus requiring her to take the entire thing apart and start all over. Dakota inhaled softly and murmured quietly to herself.

"Well, now I understand why most statistics textbooks start off by teaching the *Difference Tests* before the *Association* ones."

Michael leaned towards Dakota, trying desperately to make out what she was saying.

"Beg pardon?"

Dakota just shook her head.

"It's nothing. We don't need to worry about that right now; all we need to know is that you need to use a **t Statistic**—later we'll probably have to go through and explain things, but this is sufficient for now."

Theron shrugged his shoulders and leaned across the conference table, desperately grasping towards the manila folder which Dakota used to house the Critical Values Table. After a few feeble attempts to grab it, he lunged his arms forward and hooked the folder with his pen. With a loud grunt, he dragged the folder back towards his end of the table. After a few seconds of trying to catch his breath, he started rifling through the papers. One paper in particular caught his eye, and he pulled it free from the others and placed it on the conference table. Dakota slightly lifted her head to glance at the paper.

"That's it. For this problem, we just need to insert our *Spearman's Rho* number into the equation."

Michael arched his eyebrow.

"Equation?"

Dakota wiped her hands onto her skirt as she looked for an empty spot on the dry-erase board. Seeing none, she haphazardly tugged the sleeve of her shirt over her palm and wiped a section of the board clean.

"Sorry. What we need to do is take our Spearman's Value and convert it into a *t-value*. It's a fairly simple equation, as we have calculated everything already, or already have all the information that we need":

$$t = r_s\sqrt{\frac{N-2}{1-r_s}}$$

Dakota glanced towards Theron, patiently expecting him to do math as a trained dolphin is expected to do tricks. Given how quickly he snatched up his pencil, he was more than happy to oblige:

$$t = 0.4814\sqrt{\frac{27-2}{1-0.4814}} = 3.3424$$

Dakota glanced at the calculations and jotted a replica of Theron's work on the board.

"Okay, so our *Calculated Value* is 3.3424. Robin, what is the *Critical Value* for t with a $df = 25$ and an α of .05?"

Robin carefully followed the chart, slowly finding the correct place on the *Critical Values* Table.

"The answer is 2.060."

Dakota suddenly perked up as she heard Robin's response.

"Wait a moment. Since our *Calculated Value* of 3.3424 is greater than the *Critical Value* of 2.060, we *Reject the Null Hypothesis,* which is that there is no association between sentence length and General Aggression Scores. We conclude that there is an Association between an offender's sentence and GAS."

Theron looked bewildered.

"So... is that good?"

Dakota just smiled.

"It means that we have a *Statistically Significant* result. So yes, that is good."

Dakota made a few notes for herself and stuck the papers into her manila folder. Michael leaned back in his chair.

"So, what does one do if they have a smaller *Sample Size*?"

Dakota paused in her movements, smiling in his direction.

"If you have a smaller *Sample Size*, there is a separate *Critical Values Table* which is not based off of a **t Distribution**. You would just use that table instead.[13] "

Michael just accepted this information at face value and let the matter drop. Dakota breathed a sigh of relief that the matter at hand was concluded and mentally readied herself to continue on with their work. Suddenly, Dakota heard the faint whisper of a television in the campaign offices on the other side of the door. Though the volume was muffled, Dakota could discern bits and pieces of a reporter filing a story.

"Governor Greenleaf will be suspending active campaigning... is scheduled to give a press conference later this evening... most likely will touch on the recent homicide..."

Dakota glanced at the faces of her colleagues, all of whom had heard roughly the same thing she did. She knew there was no point in continuing for today, not when so much was left uncertain. Dakota lowered her head as her hands grasped the back of the conference room chair for support.

"Look, I see no point in pretending that we should continue on today. We honestly have no idea what Governor Greenleaf is going to say, and it most likely will have an immense impact on us and his platform about sex offenders. I say we reconvene tomorrow."

Fortunately, all of Dakota's colleagues had also reached the same conclusion, each of them gathering their belongings as she was still speaking. Robin and Theron bolted out the door as quickly as possible, making a beeline for the television. Michael just stood with Dakota, listening to the now-audible report as it echoed through the conference room. He quietly spoke under his breath.

[13] Both the Critical Values Table for Spearman's Rho and the t Distribution are included in the appendices.

"Well, I guess we are going to see just how good Nathanial Greenleaf is at being a politician."

Dakota only stood in silence, feeling oddly disturbed by what Michael had just said. Still, he was correct in that this press conference would tell her a lot about her new boss, even if it was something she wasn't entirely sure she wanted to know.

Chapter Summary

- The Somer's Index d Statistic is employed when attempting to answer the research question: "Is Total Testosterone Level associated with the General Aggression Score?"
- Independent and dependant variables are considered with regard to Somer's Index d Statistic needing a "distinct" variable. In addition, antecedent and consequent are discussed as a means of understanding the relationship between independent and dependant variables.
- A careful consideration of concordant and discordant pairs as they relate to Somer's Index d Statistic is presented.
- Spearman's rho is employed in the attempt to answer the research question: "is sentence length associated with the General Aggression Score?"
- The team continues their discussion regarding the inappropriate nature of a Pearson's correlation with their data.
- The appropriate handling of ties when ranking data is presented.

Check Your Understanding

1. For Somer's d, what does asymmetrical mean?

   a. Ordinal data must be used.
   b. Ordered data must be used.
   c. One variable must be identified as the dependent variable.
   d. Skewed data must be used.

2. How are antecedents and consequents related to independent and dependent variables?
3. Explain concordant and discordant pairs. How is each of them calculated?
4. The act of rearranging data so that its smallest value has a reassigned value of 1 can be referred to as ___________________.
5. Match the correlation value with the term:

a. Positive Correlation	i. −1.00
b. Negative Correlation	ii. 0.00
c. No relationship	iii. +1.00

6. Rank the following data:

Participant	Income	Rank
1	$25,500	
2	$16,000	
3	$29,900	
4	$9,900	
5	$59,000	
6	$25,500	
7	$21,300	
8	$48,050	
9	$25,500	
10	$9,900	

Participant	Age	Rank
1	45	
2	18	
3	53	
4	36	
5	36	
6	27	
7	43	
8	41	
9	38	
10	36	

Chapter 6
Agreeing to Disagree

Abstract As the saga continues, the team now more than ever must be able to navigate through the new developments with regard to Greenleaf's opponent. The team is tasked with addressing two more research questions that are both answered by using one of two Kendall rank-order tests. The two research questions that are addressed in this chapter are as follows: (1) Is sentence associated with the General Aggression Score? (2) Is sentence associated with the General Aggression Score when testosterone level is fixed? Kendall's Rank-Order Correlation Coefficient addresses the former, and Kendall's Partial Rank-Order Correlation Coefficient addresses the latter.

"Well, that was an unmitigated disaster."

Robin's sarcastic comment had a tinge of fear in it as all four consultants sat in stunned silence around the conference table. Rather than go home, they all decided to wait and hear Governor Greenleaf's press conference about the inexplicable horror surrounding the death of his opponent's mother, a murder which was committed at the hands of a recently released sex offender. The group silently hoped that the press conference would give some direction to their task of creating a platform concerning what to do about sex offenders. Unfortunately, it didn't turn out quite as expected. Theron reached up to his collar and pulled off his tie.

"All in all, he did the best he could."

Robin's eyes grew wide with horror.

"Are you kidding me? He stood up there for twenty minutes and didn't actually say anything. It was like watching the Hindenburg disaster, only with a really bad haircut."

Theron shook his head.

"That's not the point; Greenleaf had to walk a fine line on this one. It was a tragedy that directly impacted his campaign, so he had to say something.

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8_6,

However, if he came on too strong, it would have looked as if he were exploiting a tragedy for his own political gain. He had to walk a really fine line on this one."

Robin shook her head.

"You say 'potato', I say 'Hindenburg'."

Dakota was not listening to anything her colleagues were saying but was staring intently at the dry-erase board. She knew in her gut that things were about to go from bad to worse once the political pundits of the world started addressing Governor Greenleaf's speech, which would result in the campaign marshaling their troops to best address this problem. Unfortunately, given that this whole disaster was the result of a recently released sex offender, it pretty much meant that the consultants were now the frontline infantry.

Turning away from the dry-erase board, Dakota refocused her attention onto the statistical information that they had thus far. While the information the team had amassed was interesting from a research perspective, the statistics could hardly be considered as something that would reignite a political campaign. The governor (or his unflinching and unsympathetic right-hand woman) would soon be knocking at their door to help move the campaign past this hurdle, and the team had to be able to present him with something concrete. Sensing that the group was not exactly tired, Dakota eased herself out of her chair and took her place next to the dry-erase board.

"All right, what shall we tackle next?"

Effortlessly, the other three consultants all eased into their respective parts of the conference table. Apparently, Dakota was not the only one who realized that they were the ones who would be on the firing line. Michael leaned forward in his seat, squinting his eyes as he tried to decipher the list of research questions on the dry-erase board. He pointed to an undecipherable line of text at the bottom right-hand corner of the dry-erase board.

"Well, what if we consider whether or not there is an *Association* between General Aggression Scores and sentence length in the entire sample?"

Theron yanked the data set towards the center of the table, allowing all four consultants to analyze the information in parallel with one another. Theron pointed to both sets of data, highlighting the information with his index fingers.

"So, both the General Aggression Scores and the sentence length data are of *Interval Scale*. Since we can't do *Pearson's R*[1] because of the *Sampling Method* that was used to gather the data, wouldn't we just want to run another *Spearman's Rho*?"

Michael and Robin both nodded in agreement as Theron finished speaking. After all, it made sense; they need to find whether or not there is an *Association* between two sets of *Interval Data*, and *Spearman's Rho* was the most logical option to use. However, Dakota hesitated at conducting Spearman's yet again; it was as if something was gnawing away at her mind. There was clearly something else

[1] The reasons why Pearson's Correlation Coefficient is inappropriate are discussed on page 89 in Chap. 5 under the discussion regarding Somer's Index.

going on with the data, and she could not put her finger on it. Dakota intently stared at the data set, feeling their eyes upon her. She was their leader, and they were obviously waiting to take their cues from her.

"All right, I say we..."

Suddenly, another row of data jumped out at Dakota. It was as if the paper was covered in neon; she must have seen this dozens of times, but her mind never assembled the pieces until now. Dakota's voice just continued on in a low whisper as her eyes examined the Total Testosterone Level.

"... we may have a **Partial Correlation**."

Theron and Robin locked eyes with one another, as Michael watched Dakota bolt towards the dry-erase board. The cap for the dry-erase marker skidded across the surface of the table as Dakota tried to explain what she was thinking to the group of bewildered colleagues who were left in her wake. Dakota perched herself at the dry-erase board and started covering it with lines.

"*Partial Correlation*; it basically means that when two *Variables* have a *Correlation* with one another, it is possible that the *Statistically Significant Result* is actually the result of a third *Variable*. The first two *Variables* actually have no Correlation with one another, but they both have high *Correlations* with this third *Variable*."

Robin leaned into Theron and whispered in his direction.

"I think Dakota is turning into Agent Fox Mulder. Next thing you know, she will be telling us that 'the truth is out there' and she will start humming the theme from 'The X-Files'."

Theron furrowed his brow.

"So, does that mean Michael is now Agent Dana Scully?"

Robin's jaw suddenly dropped open as she was instantly appalled at the notion of what just came out of Theron's mouth. She just shook her head in horror, while Dakota quickly finished up her drawing on the board. Michael (who was clearly unimpressed with the Dana Scully reference) spoke only to Dakota.

"So, a *Partial Correlation* will allow us to pull out this third *Variable*, thus allowing us to see what the relationship for these two *Variables* is without its intrusion?"

Dakota nodded.

"Think of it another way. Let's say you found that there was a high *Correlation* between whether or not a woman uses organic shampoo and how well she responds to chemotherapy for breast cancer. If there is a high *Correlation*, the researcher could argue that a woman using a certain organic shampoo will likely have an easier time with treatment. However, suppose many of the women in the breast cancer study were also going through natural alternative treatments while they were also going through chemotherapy."

Michael nodded.

"I get it. The natural treatments were actually the *Variable* that alleviated the chemotherapy treatments, and women who were using natural remedies would be more likely to use organic shampoo than women who were not. But, the researcher conducting this study would only see the linkage between the organic shampoo and chemo, not even considering how that other *Variable* impacted them."

Dakota nodded.

"You are correct. And I think that the Total Testosterone Level might impact a *Correlation* between sentence length and General Aggression Scores."

Theron folded his arms over his chest as he continued to stare at the data set.

"That makes some degree of sense. If General Aggression Scores are *Correlated* with sentence length, it could be because testosterone level is driving up the General Aggression scores. It also makes sense that those with more testosterone were more aggressive with their victims, so judges could have been harsher with their jail sentences."

Robin shrugged her shoulders.

"I have a feeling that this means we won't be doing another *Spearman's Rho*."

Dakota stopped writing on the dry-erase board, only now realizing that the cap to her marker sputtered away from her some time ago. She instinctively dropped to her knees in order to retrieve the top to her ever-faithful teaching companion. All the other consultants heard was Dakota's muffled voice as it reverberated around the conference table.

"You're right; we need to run a **Kendall Rank-Order Correlation Coefficient**."

The consultants just stared down at their captain, who was attempting to raise herself off the floor in her skirt and heels. She braced her arms against the table, jerked herself onto her feet, and snapped the cap of the marker back into place. Dakota tossed her head back and marched towards the board once more.

"As I said, we need to calculate a *Kendall Rank-Order Correlation Coefficient*. It has a similar degree of specificity and **Power** as *Spearman's Rho*. However, the main difference is that the results of the *Kendall Rank-Order Correlation Coefficient* can be used to calculate *Partial Correlations*, whereas *Spearman's Rho* cannot."

Robin grinned, flexing her biceps as if she were a bodybuilder.

"Oh yeah, I got your *Power* right here."

Theron tried not to giggle at the ridiculousness playing out before him, while Michael just shook his head in grim disapproval. Still managing to maintain some degree of composure, Dakota just shook her head and gave Robin an explanation.

"Sorry Robin, *Statistical Power* just refers to how well a specific test can determine *Statistical Significance*. When a test has sufficient *Power*, it means that the test is good enough to detect when there is a systematic effect of one of your *Variables*. So, tests with more *Power* are more likely to demonstrate significant effects, if there are any, than tests with less *Power*. Typically, those tests which use *Interval* or *Ratio* data are considered to have more *Power*, simply because they give you more information with which to work. In our current problem, both *Spearman's Rho* and *Kendall's Rank-Order Correlation Coefficient* have similar levels of *Statistical Power*, meaning that they both have roughly the same ability to determine *Statistical Significance*."

No longer finding her flexing biceps humorous, Robin just slouched back into her chair. Realizing that his colleague was pacified (for now), Michael gestured to the dry-erase board.

"Wait a moment, Kendall's Rank-Order Correlation Coefficient; is that the same thing as **Kendall's tau (τ)**?"

Dakota nodded.

"Yes, both of these are the same statistical procedure that answers the same question. Since we are looking to find a *Partial Correlation* in the near future, I am just going to refer to this as *Kendall's Rank-Order Correlation Coefficient*."

"So, how do we calculate this test?"

Dakota pointed to some of the information they had when they computed the *Spearman's Rho*.

"Well, the first thing we need to do is to determine the x-variable."

Robin snapped her fingers in Dakota's direction.

"I knew it. She is going all Fox Mulder on us."

Dakota shook her head.

"No. We need to identify the x-variable first, and *Rank* both *Variables* from '1' to 'N'."

Robin puckered her lips.

"Great, we now have to *Rank* so many things that we have exhausted all number and have to resort to letters."

Theron shook his head.

"No. She means that you *Rank* from '1'to however many numbers you have within your data set."

Robin's eyes narrowed in anger towards Theron, as he suddenly had a moment of insight. He bashfully turned his head away from Robin.

"You were being sarcastic, weren't you?"

Robin nodded. Theron meekly took his seat and quietly started *Ranking* the data set. As he was doing the work on a piece of paper from his legal pad, an impatient Dakota was conducting her own *Ranking* on the dry-erase board as well. Once both were completed, Theron glanced over his work and was content that both he and Dakota generated the same results.

Participant	GAS	GAS (rank)	Sentence	Sentence (rank)
1	65	85.5	7	93.5
2	59	74.5	8	96
3	41	6.5	3	53
4	43	17	1	12.5
5	53	53	1	12.5
6	49	41.5	3	53
7	54	57.5	12	99
8	58	70.5	3	53
9	47	35.5	3	53
10	52	48.5	6	88
11	58	70.5	5	79

(continued)

(continued)

Participant	GAS	GAS (rank)	Sentence	Sentence (rank)
12	58	70.5	3	53
13	45	28.5	3	53
14	59	74.5	1	12.5
15	45	28.5	1	12.5
16	75	97	4	74
17	53	53	3	53
18	40	3	1	12.5
19	41	6.5	1	12.5
20	60	78.5	2	28.5
21	48	39.5	1	12.5
22	59	74.5	3	53
23	78	99.5	8	96
24	53	53	1	12.5
25	64	83	5	79
26	60	78.5	5	79
27	50	44	6	88
28	47	35.5	3	53
29	73	94	5	79
30	62	81	3	53
31	65	85.5	9	98
32	43	17	2	28.5
33	54	57.5	3	53
34	52	48.5	3	53
35	74	95.5	5	79
36	47	35.5	5	79
37	68	89.5	8	96
38	43	17	1	12.5
39	50	44	1	12.5
40	60	78.5	5	79
41	43	17	2	28.5
42	42	10.5	3	53
43	44	23	3	53
44	44	23	1	12.5
45	42	10.5	3	53
46	53	53	3	53
47	65	85.5	6	88
48	48	39.5	2	28.5
49	43	17	3	53
50	42	10.5	1	12.5
51	42	10.5	3	53
52	63	82	3	53
53	60	78.5	6	88
54	53	53	3	53
55	76	98	6	88
56	40	3	3	53
57	55	61	3	53
58	51	46.5	3	53

(continued)

(continued)

Participant	GAS	GAS (rank)	Sentence	Sentence (rank)
59	46	32	3	53
60	56	66	3	53
61	43	17	6	88
62	43	17	1	12.5
63	55	61	1	12.5
64	56	66	3	53
65	47	35.5	3	53
66	59	74.5	1	12.5
67	44	23	1	12.5
68	42	10.5	1	12.5
69	51	46.5	2	28.5
70	67	88	1	12.5
71	70	92.5	7	93.5
72	45	28.5	3	53
73	56	66	1	12.5
74	69	91	3	53
75	58	70.5	3	53
76	47	35.5	3	53
77	40	3	3	53
78	65	85.5	5	79
79	68	89.5	6	88
80	55	61	2	28.5
81	56	66	2	28.5
82	53	53	3	53
83	50	44	3	53
84	44	23	6	88
85	74	95.5	16	100
86	56	66	1	12.5
87	45	28.5	3	53
88	44	23	3	53
89	40	3	2	28.5
90	45	28.5	1	12.5
91	45	28.5	1	12.5
92	70	92.5	3	53
93	78	99.5	5	79
94	40	3	6	88
95	47	35.5	3	53
96	53	53	1	12.5
97	55	61	3	53
98	49	41.5	1	12.5
99	55	61	3	53
100	42	10.5	3	53

Dakota pointed to the column for the General Aggression Scores.

"All right, we now need to organize the *Rankings* into numerical order based on the *Rankings* for the x-variable."

Michael eased himself back into his chair.

"Organize?"

Dakota nodded.

"Correct. We need to place all the rankings into numerical order. Since the General Aggression Scores is our x-variable, we need to organize these *Rankings* from lowest-to-highest."

Theron poked his head up from his legal pad for a moment.

"Are we organizing both sets of data, or are we just organizing the General Aggression Scores?"

Dakota shook her head.

"No. You need to remember that the scores exist in pairs. Participant one has a General Aggression Score of 65, and was sentenced to 7 years in prison. Even though we are reorganizing the *Ranks* for the General Aggression Scores based on the numerical *Rankings*, the corresponding *Ranking* that person had for sentence length has to remain paired up with that participant."

Theron rolled up his shirtsleeves and slowly rearranged all the *Rankings* for the General Aggression Scores into numerical order.

Participant	GAS (rank)	Sentence (rank)
18	3	12.5
56	3	53
77	3	53
89	3	28.5
94	3	88
3	6.5	53
19	6.5	12.5
42	10.5	53
45	10.5	53
50	10.5	12.5
51	10.5	53
68	10.5	12.5
100	10.5	53
4	17	12.5
32	17	28.5
38	17	12.5
41	17	28.5
49	17	53
61	17	88
62	17	12.5
43	23	53
44	23	12.5
67	23	12.5
84	23	88
88	23	53
13	28.5	53
15	28.5	12.5
72	28.5	53
87	28.5	53

(continued)

(continued)

Participant	GAS (rank)	Sentence (rank)
90	28.5	12.5
91	28.5	12.5
59	32	53
9	35.5	53
28	35.5	53
36	35.5	79
65	35.5	53
76	35.5	53
95	35.5	53
21	39.5	12.5
48	39.5	28.5
6	41.5	53
98	41.5	12.5
27	44	88
39	44	12.5
83	44	53
58	46.5	53
69	46.5	28.5
10	48.5	88
34	48.5	53
5	53	12.5
17	53	53
24	53	12.5
46	53	53
54	53	53
82	53	53
96	53	12.5
7	57.5	99
33	57.5	53
57	61	53
63	61	12.5
80	61	28.5
97	61	53
99	61	53
60	66	53
64	66	53
73	66	12.5
81	66	28.5
86	66	12.5
8	70.5	53
11	70.5	79
12	70.5	53
75	70.5	53
2	74.5	96
14	74.5	12.5
22	74.5	53
66	74.5	12.5

(continued)

(continued)

Participant	GAS (rank)	Sentence (rank)
20	78.5	28.5
26	78.5	79
40	78.5	79
53	78.5	88
30	81	53
52	82	53
25	83	79
1	85.5	93.5
31	85.5	98
47	85.5	88
78	85.5	79
70	88	12.5
37	89.5	96
79	89.5	88
74	91	53
71	92.5	93.5
92	92.5	53
29	94	79
35	95.5	79
85	95.5	100
16	97	74
55	98	88
23	99.5	96
93	99.5	79

Theron compiled the last of the rankings and showed his work to Dakota. Dakota craned her neck slightly and transposed all of the numbers onto the dry-erase board. Robin just slouched in her corner of the conference table, lazily spinning her pen on the tabletop.

"Well, this isn't so bad. It's mind-numbingly boring, but not bad."

Dakota just let Robin's comment wash over her and continued looking over Theron's work.

"Now that all of this is *Ranked* in numerical order, we have to determine the number of **Agreements** and **Disagreements** for each of the ranks of the y-variable."

Robin just shook her head in disgust.

"Names… don't the *Variables* have names?"

Taking a cue from Dakota's playbook, Theron also ignored Robin and focused on the task at hand.

"So, I am guessing that the y-variable is the other *Variable* you are looking at in the *Association* after the x-variable is assigned."

Dakota nodded.

"Correct."

Theron just furrowed his brow.

"So, does it matter which *Variable* is assigned to be the x-variable and which *Variable* is assigned the y-variable?"

Dakota thought quietly for a moment and just shook her head.

"No. Given that we only have two *Variables* for this equation, it should make no difference which you use as 'x' and which you use as 'y.' In fact, they are almost interchangeable."

Theron quietly accepted this explanation, while Michael nodded towards the dry-erase board.

"What do you mean by *Agreements* or *Disagreements*? I know we have discussed this before.[2]"

Theron lurched forward, smiling with pride as if he had some unknown knowledge which the others did not possess.

"I bet I know. *Agreements* would be those times where the *Rankings* would be the same with the others, while *Disagreements* would be those instances where the *Rankings* are incongruous with one another."

Michael and Robin both nodded in silent consensus with Theron's explanation. However, Dakota just shook her head at hearing this, clearly indicating that his theory was incorrect. Upon seeing Dakota's reaction, Theron looked crestfallen. Realizing that she needed to rebuild his self-confidence, Dakota smiled at Theron and tried to explain the situation to the others in the room.

"That's not a bad guess Theron, and it is one that certainly makes sense. However, for *Kendall's Rank-Order Correlation Coefficient*, a *Ranking* is considered to be in *Agreement* or *Disagreement* depending upon whether it falls above or below the *Ranking* you are comparing it to. So the number might not actually be the same, but still be in *Agreement* in terms of the test."

Theron's crestfallen expression melted away to one of personal pride at a moment of understanding. He smiled toward Dakota. Unfortunately, Robin was less than impressed with this explanation.

"Wait, didn't we just have this *Agreement / Disagreement* talk under a different name about six hours ago with that asinine *Somer's d* thing?"

Dakota felt her jaw tighten; she was afraid someone was going to ask this question and was even more afraid that the person who was going to ask the question was going to be Robin.

"Robin, I understand the confusion. You're correct, we have talked about *Concordant* or *Agreements* and *Discordant* or *Disagreements* with both *Somer's d* and with *Kendall's Rank-Order Correlation Coefficient*. However, with *Somer's d*, we were looking for the *Probability* that there was any type of *Concordance* or *Discordance* across all *Variables* when the data is *Ranked*; finding a *Concordant/Discordant* meant adding up the *Frequencies*. Location is still important – whether or not the frequencies were above or below the cell in question – but the actual number we used was the sum of all of the *Frequencies* within the block. For *Kendall's Rank-Order Correlation Coefficient*, you are using your x-variable as a benchmark to determine whether or not the y-variable falls above or below this threshold."

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[2] Agreements and disagreements are discussed in greater detail on page 92 in Chap. 5 when dealing with Somer's Index d. However, in Chap. 5, they are referred to as antecedent and consequent.

Robin shook her head.

"Does not compute."

"Think of it this way, in *Kendall's Rank-Order Correlation Coefficient* we are first comparing individual *Ranks*. *Agreements/Disagreements* are still determined depending on where a ranking falls – is it above or below a specific benchmark? – we then add all of those numbers. So for both, *Agreement/Disagreement* really depends on a number's location. For *Somer's d*, we add frequencies – how many offenders appeared in certain categories. In *Kendall's Rank-Order Correlation Coefficient*, we first compare individual rankings to see if it falls above or below the benchmark, we then get two numbers – all of the *Agreements* and all of the *Disagreement* – then we will need to subtract."

Michael smiled.

"So for both statistical procedures, *Agreement/Disagreement* really depends on a number's location. When we place the x-variable in natural order, we're then comparing the y-variable to itself."

Theron bounced up and down in his seat.

"Oh, I get it. You take one *Ranking* to use as kind of a threshold. If the other *Ranking* is higher than your threshold *Ranking*, then it is in *Agreement*. However, if the other *Ranking* is lower than the one you are using as a benchmark, then you have a *Disagreement*."

Dakota nodded.

"Yes. So I am sure we are all on the same page, when we say a ranking is higher, it's a bigger number. 56 is higher than 2, correct? We are *not* saying 2 is higher than 56 because to be ranked 2nd is better than 56th, right?"

Dakota felt a little silly asking the question, but it was crucial that everyone be on the same page. Still feeling as if the group needed a little more understanding about the task at hand, Dakota cleared off a section of the dry-erase board.

"Maybe it would be easier to understand what we are looking for in terms of *Agreements* and *Disagreement* if we considered one comparison at a time."

Dakota began to sketch out a table on newly cleared space on the dry-erase board. It wasn't too long before she had a bit of work to show the team.

Participant	GAS (Rank)	Sentence (Rank)		Agreement (+), Disagreement (-), or the same (X)
18	3	12.5		
56	3	53	12.5	+
77	3	53	12.5	+
89	3	28.5	12.5	+
94	3	88	12.5	+
3	6.5	53	12.5	+
…	…	…	…	…
23	99.5	96	12.5	+
93	99.5	79	12.5	+

"See here, I've highlighted the first sentence rank so you can see which one we are starting with. Now, we take that first *Ranking*, 12.5, call it our threshold ranking and compare it to every other sentence length. *Rankings* higher than our threshold ranking are an *Agreement*. *Rankings* lower than the threshold ranking are *Disagreements*."

Dakota stepped away from the dry-erase board so that her compatriots could see all the details of her new diagram. She could see a glimmer of understanding flitter across all of their faces. With that, she positioned herself next to a clean space on the dry-erase board and jotted down a new equation:

$$T = \frac{2S}{\sqrt{N(N-1) - T_x}\sqrt{N(N-1) - T_y}}$$

$$T_{x\ or\ y} = \sum t(t-1)$$

N = *the total number of participants*
S = *number of agreements in a comparison of ranks minus the number of disagreements in a comparison of ranks*
t = *the size of the tied ranks in a set*

Theron obediently copied this new equation onto his legal pad, while Robin just narrowed her eyes in disgust.

"Oh good, more math."

Dakota smiled.

"This is equation for *Kendall's Rank-Order Correlation Coefficient*. As you see, we subtract the total number of *Disagreements* from the total number of *Agreements*. Then, we divide that by the total number of pairs."

Theron cocked his head to the right.

"Wait- it's 100. We have 100 pairs within the participants, correct?"

Dakota shook her head.

"In this instance, I can see why you would reach that conclusion. However, the total number of pairs is based off of this":

$$\binom{N}{2}$$

Robin rolled her eyes.

"It's some parentheses with a letter and a number. That's not going to help us with anything."

Michael shook his head.

"Nope, it's a **Binomial Coefficient**."

Robin suddenly shot Michael a very angry look.

"Okay Mensa-reject, what is a *Binomial Coefficient*? And try to explain it to me without sounding like a pompous..."

Dakota held her hand up and slammed her hand into the table. The shock of the noise derailed Robin's tirade and brought the entire room back to her attention.

"That's enough. Michael, you are correct, this is a *Binomial Coefficient*. Mathematically, the *Binomial Coefficient* just expands the powers of a positive integer using algebra.[3] What is important for us at this moment is that the denominator for the *Kendall's Rank-Order Correlation Coefficient* can be understood as *N (N – 1)*. *N (N – 2)* is just going to tell you what the maximum number of what the possible total could be."

Dakota quickly drew another equation next to the T equation which was already prominent on the dry-erase board:

$$T = \frac{2S}{N(N-1)}$$

N = the total number of participants
S = number of agreements in a comparison of ranks minus the number of disagreements in a comparison of ranks

Predictably, Theron altered the equation he already had based on Dakota's explanation, allowing more space for any other wisdom which Dakota may impart. Michael pointed to the equation, engrossed in what Dakota was saying.

"Okay, I understand what the denominator is, but what about the numerator? I've never seen that before."

Dakota pointed to the numerator, highlighting the space with her finger.

"You mean *S*? *S* just stands for the observed sum of the *Agreements* and the *Disagreements*. In order to find the observed sum, you need to take the difference between the *Agreements* and *Disagreements* for each pair, then add the differences for each pair to find the total, which is *S*."

Sensing that her colleagues were comfortable with the explanation thus far, Dakota continued on with her explanation.

"All right, to find *S*, we start with the first rank of the y-variable: 12.5. Now, we determine all of the Sentence *Ranks* below 12.5 which are larger. Larger *Ranks* are considered *Agreements* and are denoted with a '+'. *Ranks* which are smaller are considered to be *Disagreements*, and are denoted with a '-'. *Rankings* that are the same, which often occurs in the case of ties, receive an 'x'."

Michael was very still, his eyes smiling in glee over his ability to grasp the concept before him. However, Theron and Robin did not share Michael's insight into this equation. Theron placed his pencil down in frustration, while Robin just folded her arms and delicately slammed her head onto the table. Realizing that some of her comrades would need a little extra help, Dakota went back to the diagram of the individual comparisons.

[3] The powers of a positive integer are expressed by factorial design. For example, 4! or 4^4.

Participant	GAS (Rank)	Sentence (Rank)		Agreement (+), Disagreement (-), or the same (x)
18	3	12.5		
56	3	53	12.5	+
77	3	53	12.5	+
89	3	28.5	12.5	+
94	3	88	12.5	+
3	6.5	53	12.5	+
…	…	…	…	…
23	99.5	96	12.5	+
93	99.5	79	12.5	+

"Okay, look at the highlighted sections. As you can see, For example, in that first line:

– 53 is larger than 12.5, so it is an *Agreement* and receives a '+'.
– 53 is larger than 12.5, so it is an *Agreement* and receives a '+'.
– 28.5 is also an *Agreement* and receives a '+'.

Now, this process would continue to the final Sentence ranking, 79, which is larger than 12.5, so it is an *Agreement* and therefore receives a '+'."

Dakota halted her explanation, allowing all of this information to be digested by her peers. Theron was busy trying to put all of this information into context by quickly writing all of these explanations onto his legal pad, while Robin fluctuated between staring at what Dakota had written on the dry-erase board and what Theron was drawing out on his pad (whether or not Robin understood any of this information is fairly debatable). Feeling comfortable with how the group was responding to the explanation, Dakota hesitantly continued onward.

Participant	GAS (Rank)	Sentence (Rank)		Agreement (+), Disagreement (-), or the same (X)
18	3	12.5		
56	3	53		
77	3	53	53	X
89	3	28.5	53	-
94	3	88	53	+
3	6.5	53	53	X
…	…	…	…	…
23	99.5	96	53	+
93	99.5	79	53	+

"After all of the comparisons of the first *Ranking* are complete, you start with the second *Ranking* on the Sentence list, 53, and continue as you did in the first row. This process is then continued for every *Ranking* on the list; it is a comparison of every possible pair of rankings for the y-variable."

Theron continued to write some numbers down but suddenly froze in place as a thought raced through his brain and shattered his concentration.

"Wait a moment, how many pairs are we doing this for?"

Dakota pulled out her dry-erase marker.

"Well, our possible numbers are pairs is $N\ (N - 1)/2$."

Theron spoke softly as he mentally inserted numbers into the equation.

"So, that would be $\frac{100(100-1)}{2}$. Overall, that gives us..."

Dakota's voice cut Theron off mid-sentence.

"4950 pairs."

Robin's eyes grew wide with horror.

"Are you kidding me? We have to do this almost five thousand more times!!!"

Dakota vigorously shook her head, trying to ease her panic.

"No, no, no. With such a large number of possible pairs, determining the number of *Agreements* and *Disagreements* would be extremely tedious. By consulting a list of the y-variable *Ranks* that are in natural order, we can simply count the number of *Rankings* that are the same, larger, and smaller than the *Rank* we are comparing. Natural order just means that the smallest rank is first and the largest rank is last."

Theron chimed in with a question to assist with the explanation.

"So, what you are saying is that we are essentially just counting?"

Dakota's face showed her obvious exasperation at the simplicity of Theron's explanation.

"Yes, you could say that we are simply counting."

Robin seemed instantly pacified by this explanation. Theron grinned and slowly drew a breath before continuing on with his work. Just as he was about to start writing once more, Dakota's low voice gently cautioned him.

"Hold on a moment. We must remember to remove the 'Y' *Ranking* we compared from the list before moving onto the next *Ranking*. This will help to make sure that we are only making unique comparisons."

While Dakota's voice was meant to serve as a cautious reminder to Theron, it actually caused him to be overwhelmed with the sheer enormity of the task at hand. Patiently, Dakota cleared off large sections of the dry-erase board and tried to make this seem less daunting, not an easy thing given the size of the data set.

"Theron, I understand why you would feel a little overwhelmed by all of this. However, there is a way to find *Agreements* and *Disagreements* when the data set is large. First, take a look at the list of *Rankings* when they are in numerical order by the x-variable, which in our case is GAS. Next, we want to look at the corresponding y-variable rank. So for participant 18, when the GAS rank is 3, the sentence rank is 12.5."

Participant	GAS (Rank)	Sentence (Rank)	Rankings that are the same (Denoted as X)	Rankings in agreement (Denoted as +)	Rankings in disagreement (Denoted as -)	(# Agreements- # Disagreements)
18	3	12.5	23	76	0	76
56	3	53	40	27	31	-4
77	3	53	39	27	31	-4

To find the number of *Agreements* and *Disagreements* for this pair, we first take a list of the y-variable *Ranks* in natural order. Next, we identify the ranking we are evaluating, which in this case is 12.5. You'll notice when the y-variable *Rankings* are in natural order, you can see that there are 23 other rankings of 12.5. So for our '*Rankings* that are the same' column, we will enter 23."

Dakota paused for a moment to canvass how well her colleagues were grasping all of these concepts. All she saw were three faces staring at her with rapt attention. Feeling cautiously optimistic, Dakota continued on with her explanation.

"Now, *Rankings* in *Disagreements* are all rankings below (larger rankings) than 12.5. If we count all of the *Rankings* below 12.5, we find 76 *Rankings* in *Agreement*. Since there are no *Rankings* above 12.5, this means there are no *Disagreements*. So for *Rankings* in *Disagreement*, denoted as '-', there are 0."

Once again, Dakota noticed that all of her colleagues were utterly engrossed in what she had to say.

"Once you've compared a participant's score against all the others, you do not include that participant again. So, we've ranked 12.5; the next one is 53, so we start with that score. This means for each comparison, our total number of sentence *Rankings* will be reduced by one."

Michael gestured at the dry-erase board with his hand as a question hurriedly flew past his lips.

"Do we want to remove numbers from the list?"

Dakota simply shook her head back at him.

"Not really. We aren't removing the data so much as we are just ignoring it temporarily or like moving down the line demonstrated in our individual comparison table. Each time, the number of comparisons you make is reduced by one because we are no longer interested in the rank we have just looked at. There is no sense in beating a dead horse or a dead rank."

Gently pacified, Theron once again snatched up his pencil and went to work. Michael leaned forward in his chair.

"What happens when we get a handle on the number of *Rankings*?"

Dakota smiled nonchalantly.

"Once we know the number of *Agreements* and *Disagreements* for each *Rank*, we can find *S*."

Theron gripped his pencil tightly in his hand, mentally preparing himself for the arduous task at hand. Robin just scooted herself towards Theron, muttering as she dragged her chair along the carpet.

"You men, always needing a woman to help you out."

Theron leaned over to give Robin some room, and the two spent several minutes in uninterrupted silence as they worked on figuring out the *Agreements*, the *Disagreements*, and the difference between the two. Once the interminably long silence was over, they slid their finished product to Dakota, who nodded her approval at their hard work.

Participant	GAS (rank)	Sentence (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
18	3	12.5	23	76	0	76
56	3	53	40	27	31	−4
77	3	53	39	27	31	−4
89	3	28.5	7	66	23	43
94	3	88	8	8	79	−71
3	6.5	53	38	26	30	−4
19	6.5	12.5	22	71	0	71
42	10.5	53	37	26	29	−3
45	10.5	53	36	26	29	−3
50	10.5	12.5	21	69	0	69
51	10.5	53	35	26	28	−2
68	10.5	12.5	20	68	0	68
100	10.5	53	34	26	27	−1
4	17	12.5	19	67	0	67
32	17	28.5	6	60	19	41
38	17	12.5	18	66	0	66
41	17	28.5	5	60	18	42
49	17	53	33	26	23	3
61	17	88	7	8	66	−58
62	17	12.5	17	63	0	63
43	23	53	32	25	22	3
44	23	12.5	16	62	0	62
67	23	12.5	15	62	0	62
84	23	88	6	8	62	−54
88	23	53	31	24	20	4
13	28.5	53	30	24	20	4
15	28.5	12.5	14	59	0	59
72	28.5	53	29	24	19	5
87	28.5	53	28	24	19	5
90	28.5	12.5	13	57	0	57
91	28.5	12.5	12	57	0	57
59	32	53	27	24	17	7
9	35.5	53	26	24	17	7
28	35.5	53	25	24	17	7
36	35.5	79	8	14	43	−29
65	35.5	53	24	23	17	6
76	35.5	53	23	23	17	6
95	35.5	53	22	23	17	6

(continued)

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Participant	GAS (rank)	Sentence (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
21	39.5	12.5	11	50	0	50
48	39.5	28.5	4	45	11	34
6	41.5	53	21	23	15	8
98	41.5	12.5	10	48	0	48
27	44	88	5	8	44	−36
39	44	12.5	9	47	0	47
83	44	53	20	22	13	9
58	46.5	53	19	22	13	9
69	46.5	28.5	3	41	9	32
10	48.5	88	4	8	40	−32
34	48.5	53	18	21	12	9
5	53	12.5	8	42	0	42
17	53	53	17	21	11	10
24	53	12.5	7	41	0	41
46	53	53	16	21	10	11
54	53	53	15	21	10	11
82	53	53	14	21	10	11
96	53	12.5	6	38	0	38
7	57.5	99	0	1	42	−41
33	57.5	53	13	20	9	11
57	61	53	12	20	9	11
63	61	12.5	5	35	0	35
80	61	28.5	2	32	5	27
97	61	53	11	20	7	13
99	61	53	10	20	7	13
60	66	53	9	20	7	13
64	66	53	8	20	7	13
73	66	12.5	4	30	0	30
81	66	28.5	1	28	4	24
86	66	12.5	3	29	0	29
8	70.5	53	7	20	4	16
11	70.5	79	7	11	12	−1
12	70.5	53	6	19	4	15
75	70.5	53	5	19	4	15
2	74.5	96	2	2	23	−21
14	74.5	12.5	2	24	0	24
22	74.5	53	4	18	3	15
66	74.5	12.5	1	23	0	23
20	78.5	28.5	0	22	1	21
26	78.5	79	6	10	6	4
40	78.5	79	5	10	6	4
53	78.5	88	3	9	11	−2
30	81	53	3	15	1	14
52	82	53	2	15	1	14

(continued)

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Participant	GAS (rank)	Sentence (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
25	83	79	4	9	4	5
1	85.5	93.5	1	4	11	−7
31	85.5	98	0	1	14	−13
47	85.5	88	2	4	8	−4
78	85.5	79	3	6	4	2
70	88	12.5	0	12	0	12
37	89.5	96	1	1	9	−8
79	89.5	88	1	3	6	−3
74	91	53	1	8	0	8
71	92.5	93.5	0	2	6	−4
92	92.5	53	0	7	0	7
29	94	79	2	3	1	2
35	95.5	79	1	3	1	2
85	95.5	100	0	0	4	−4
16	97	74	0	3	0	3
55	98	88	0	1	1	0
23	99.5	96	0	0	1	−1
93	99.5	79	0	0	0	0
					$S = 1{,}391$	

Theron leaned back in his chair, a look of smug satisfaction crossing his face.

"That's not something I want to have to do again."

Michael swiveled his chair in Theron's direction, obviously interested in what he had written down.

"We seem to have a lot of the same *Rankings* here."

Dakota just nodded her head mechanically. She knew instinctively that many of these scores would be equal; there was no avoiding that.

"So, the formula we are working with is the formula that accounts for *Ties*. Remember, that is this formula":

$$T = \frac{2S}{\sqrt{N(N-1) - T_x}\sqrt{N(N-1) - T_y}}$$

"Since there are *Ties*, we must make a correction to the equation, in the event that one of the variables did not have ties, $T_{x\ or\ y}$ would equal 0. If we didn't have ties on the x-variable, for example, but we did on the y-variable, we would insert a 0 for T_x and then whatever T_y equaled into our equation to find T. To do this, we must first determine the total number of tied observations in the x-variable (GAS). In this sample, we have 24 sets of ties in the GAS *Variable*."

5 scores of 40	**6** scores of 47	**7** scores of 53	**4** scores of 60
2 scores of 41	**2** scores of 48	**2** scores of 54	**4** scores of 65
6 scores of 42	**2** scores of 49	**5** scores of 55	**2** scores of 68

(continued)

(continued)

7 scores of 43	**3** scores of 50	**5** scores of 56	**2** scores of 70
5 scores of 44	**2** scores of 51	**4** scores of 58	**2** scores of 74
6 scores of 45	**2** scores of 52	**4** scores of 59	**2** scores of 78

Michael leaned back in his chair.

"So, what is the difference between the typical *Kendall's Rank-Order Correlation* and the *Kendall's Rank-Order Correlation* which corrects for *Ties*?"

Dakota quickly jotted down both equations next to each other on the dry-erase board putting the equation accounting for ties to the right of the equation not accounting for *Ties*:

$$T = \frac{2S}{N(N-1)} \qquad T = \frac{2S}{\sqrt{N(N-1)-T_x}\sqrt{N(N-1)-T_y}}$$

Dakota pointed to both equations.

"As you can see, the *Kendall's Rank-Order Correlation* which addresses *Ties* does so in the denominator, while the *Kendall's Rank-Order Correlation* equation which does not address ties only looks at the possible number of pairs based upon your Sample Size."

Michael nodded his head in agreement and leaned back into his chair. Theron, who was staring blankly at both equations, just spit out the first string of thoughts which flashed through his mind.

"So, what do we do now?"

Dakota pointed to the equation.

"Well, we now need to figure out the T_x value for both the x-variable and the y-variable. In order to do that, we need to find this..."

Dakota erased the other equations from the dry-erase board with the palm of her hand:

$$T_x = \sum t(t-1)$$

Theron smiled.

"I get it. We take each set of *Ties*, then we multiply it by itself minus one, and we do this for every set of *Ties* we have. It's similar to what we did in order to find the possible number of pairs in the denominator for the *Kendall's Rank-Order Correlation Coefficient* equation."

Robin wrinkled her forehead in an attempt to actually understand the test.

"Okay, so, *t* is our bolded number, and we take the bolded numbers in the table, 5, 2, 6, 7, 5, 6 and so on, and multiply each one by that same number minus 1?"

Dakota smiled.

"That's correct. You multiply the set of ties by the number for the set of *Ties* minus 1."

Theron just rolled up his sleeves and set to work.

$$
\begin{aligned}
T_{x(GAS)} &= \sum t(t-1) \\
&= 5(5-1) + 2(2-1) + 6(6-1) + 7(7-1) \\
&\quad + 5(5-1) + 6(6-1) + 6(6-1) + 2(2-1) \\
&\quad + 2(2-1) + 3(3-1) + 2(2-1) + 2(2-1) \\
&\quad + 7(7-1) + 2(2-1) + 5(5-1) + 5(5-1) \\
&\quad + 4(4-1) + 4(4-1) + 4(4-1) + 4(4-1) \\
&\quad + 2(2-1) + 2(2-1) + 2(2-1) + 2(2-1) \\
&= 328
\end{aligned}
$$

After a few moments of work, Theron slid the paper to Dakota, who transcribed the information onto the dry-erase board. As soon as she received the piece of paper, Theron ripped off a new piece of paper and started to work on the y-variable.

"So, I am guessing that the process is the same with the y-variable as well?"

Dakota's eyes were still glued onto the piece of paper which she just received, but she was able to nod her assent to Theron.

"Correct. While the number of *Ties* within the x-variable and the y-variable will be very different, the process to calculate both is exactly the same. We take the bolded t value for each tie and subtract it from 1, then multiply it by that same t value."

Theron never acknowledged Dakota, but he continued on with his mathematical equations as planned:

24 sentences of 1 year	**9** sentences of 6 years
8 sentences of 2 years	**2** sentences of 7 years
41 sentences of 3 years	**3** sentences of 8 years
9 sentences of 5 years	

$$
\begin{aligned}
T_{y(Sentence)} &= \sum t(t-1) \\
&= 24(24-1) + 8(8-1) + 41(41-1) \\
&\quad + 9(9-1) + 9(9-1) + 2(2-1) + 3(3-1) \\
&= 2400
\end{aligned}
$$

Once again, Theron quickly ran all the computations in his head and handed his work to Dakota. She jotted down the information next to everything else, speaking in a clear voice as she worked.

"Perfect. Now, all we need to do is solve the equation. Fortunately, all of the pieces are here."

Theron squinted his eyes, scouring the dry-erase board for all the pieces to the equation which he was now tasked to calculate. After a few seconds of searching, he wrote down all the components of the equation and went to work once more:

$$T = \frac{2S}{\sqrt{N(N-1) - T_x}\sqrt{N(N-1) - T_y}}$$
$$= \frac{2(1391)}{\sqrt{100(100-1) - (328)}\sqrt{100(100-1) - (2400)}}$$
$$= \frac{2782}{\sqrt{9572}\sqrt{7500}} = 0.3283$$

Once his work was completed, Theron once again handed Dakota all of his calculations. Michael watched Dakota hand him the paper and started to slide about uncomfortably in his chair. Dakota only had a few moments to look over Theron's work before Michael would pipe up with more questions.

"So, now what do we do?"

Dakota silently started gathering all of the paperwork concerning *Kendall's Rank-Order Correlation Coefficient* together and placing all of their hard work with the rest of the information which they compiled thus far. Once all of *Kendall's Rank-Order Correlation Coefficient* paperwork was organized, Dakota took her place at the dry-erase board.

"First, we need to be aware that this number is the value for *T* but not just any *T*. We have just calculated T_{xy} which will be important later on when we look at Kendall's next test. So, we have calculated that $\boldsymbol{T_{xy}} = \mathbf{0.3283}$. Now, we determine whether or not we have *Statistical Significance*. To determine *Statistical Significance*, we calculate *z* since we are using a large *Sample Size*, with our $N > 30$.[4]"

Robin just lazily laid her head down on the conference table.

"And what would we do if we had an $N < 30$?[5] I am hoping the answer is that we do nothing."

Dakota smiled at her response. Maybe it was the exhaustion, but she was genuinely tickled by Robin at the moment.

"Since we have a *Sample Size* larger than 30, we need to use this..."

Straining for room on the dry-erase board, Dakota nevertheless managed to squeeze yet one more equation onto the dry-erase board.

"As you can see, we have a specialized equation in order to compute *z* for this sample. Once we know that, we can determine *Statistical Significance*":

---

[4] Page 243 in Chap. 10 contains a detailed discussion of statistical *significance* for z-scores.

[5] When dealing with sample size for Kendall's Rank Order, two other options exist besides the $N > 30$ option. Those two options are $N \leq 10$ and $N > 10$. When $N \leq 10$, the exact probability, or p-value, can be found utilizing an Upper-tail probabilities for *T* table. When $N > 10$, *T* can be assumed to be normally distributed a Mean $= \mu_T = 0$ and Variance $= \sigma_T^2 = 2(2N + 5)/9N(N-1)$. z is then found using the same formula as for $N>30$ as discussed here; however, the value for *T* can be found in a Critical Values table for *T*.

$$z = \frac{3T\sqrt{N(N-1)}}{\sqrt{2(2N+5)}}$$

Theron strained his eyes, managing to gather all the components of the equation from the dry-erase board sketched into his legal pad. Once he was convinced that he had jotted down everything correctly, Theron started in with the calculations:

$$\begin{aligned} z &= \frac{3T\sqrt{N(N-1)}}{\sqrt{2(2N+5)}} \\ &= \frac{3(0.3283)\sqrt{100(100-1)}}{\sqrt{2(2[100]+5)}} \\ &= 4.8397 \end{aligned}$$

Once again, Theron pulled off the completed sheet of paper from his legal pad and handed it to Dakota. She calmly read off the final answer and explained its meaning to the group.

"By consulting a z table, we are able to determine that for $z = 4.8397$, $\alpha < 0.05$. Therefore, we *Reject the Null Hypothesis*; there is no relationship between the *Variables*, and the *Variables* are *Independent*. We conclude that GAS and sentence length are *Associated*; they are not *Independent*."

As soon as she finished speaking, Dakota pulled out all of the work which Theron just completed and laid it out on the conference table in sequential order. Once all of the notes were neatly lined up next to one another, she did something which was rare for her since the group started working on Greenleaf's political campaign; Dakota sat down with the others at the conference table. She placed her elbows onto the table and cupped her chin in her hands as she stared at the information before her.

"There is a *Partial Correlation* here; I just know it."

Michael leaned towards Dakota.

"You said that earlier. Do you really think that there is a third *Variable* which is intruding upon the *Association* between sentence length and the General Aggression Scores?"

Dakota nodded somberly.

"It's certainly plausible. If there is a third *Variable* that is artificially inflating the *Correlation* between the General Aggression Scores and Sentence length, then we need to identify it."

Michael raised his eyebrow.

"That's the *Partial Correlation*, right?"

Dakota nodded in his general direction, telling Michael everything he needed to know. Theron readied his legal pad for a new set of information to be rapidly fired at him.

"So, how do we find this?"

Dakota leaned back in her chair.

"We need to find **Kendall's Partial Rank-Order Correlation Coefficient**.[6]"

Robin just shrugged off what Dakota said with a flippant toss of her neck.

"That sounds like the test we just did."

Dakota leaned her head back, staring up at the ceiling.

"The *Kendall's Partial Rank-Order Correlation Coefficient* allows us to look at the *Association* between two *Variables* when a third *Variable* remains **Fixed**."

Robin swiveled about lazily in her chair.

"*Fixed*... I just did that to my cat."

Without missing a beat, Dakota just responded to Robin's inane chatter.

"No... *Fixed* means that the *Variable* is held constant. With this test, we are interested in whether or not there is an *Association* between GAS and sentence when total testosterone is constant."

Realizing that she needed to return to her post, Dakota stood up and once again went back to the dry-erase board. With a few flicks of her wrist, Dakota placed yet another symbol onto the dry-erase board. However, she was extremely careful not to disrupt any of the other symbols or equations which were written onto the dry-erase board:

$$T_{xy.z}$$

Once she was finished writing, Dakota just went about her explanation of the symbol in a matter-of-fact manner.

"This symbol implies 'the *Correlation* between *Variables* x and y when *Variable* z is held constant.' Now, it's important to remember that the *Variable* letter after the decimal is the *Variable* that is being held constant."

Dakota paused for a moment to make sure that her compatriots were still following her explanation. Once she was suitably reassured that everyone was comfortable with what she was saying, Dakota continued on with the explanation.

"Fortunately for us, this test uses the same procedures as the *Kendall Rank-Order Correlation Coefficient*. Only in this case, we must determine T for each combination of *Variables*: T_{xy} (GAS and sentence), T_{xz} (GAS and total testosterone), and T_{yz} (sentence and total testosterone)."

Theron stopped in his tracks and pointed at the manila folder which Dakota used to house all of their previous work.

"Wait a moment. Does this mean that we can use the *Correlation* result from when we found the *Association* between General Aggression Scores and sentence length?"

Dakota just nodded.

"We can. It's why I preserved all of the information from the last test on the dry erase board. Since we previously used total GAS and sentence for the *Kendall*

[6] *Kendall's Partial Rank-Order Correlation Coefficient* and *Kendall's Rank-Order Correlation Coefficient* share many of the same steps and tasks. Refer to the discussion regarding the *Rank-Order Correlation Coefficient* on page 122 of this chapter to assist in understanding some of the steps for the *Partial Rank-Order Correlation Coefficient*.

Rank-Order Correlation Coefficient, we already know $\boldsymbol{T_{xy}} = \mathbf{0.3283}$. Now, we must find T_{xz}, which concerns the GAS and total testosterone."

Michael jumped out of his seat and gestured to an equation on the dry-erase board. Dakota followed Michael's hand to see which equation he was highlighting:

$$T_x = \sum t(t-1)$$

"Wait a moment; do we use this equation for all three *Variables*?"

Dakota leaned into Michael and patted his hand with her own.

"That's correct. We will use this equation to find the number of ties for all of the *Variables*."

Michael was pacified and went back to his seat. Theron pulled out a clean sheet of paper, calling out a question as he began.

"Are we still looking for the *Rankings* in *Agreement*, the *Rankings* in *Disagreement*, and the difference between the two?"

Dakota smiled in Theron's direction, allowing him to go to work.

Participant	GAS (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
56	3	37	0	63	36	27
18	3	49	0	51	47	4
94	3	70	0	30	67	−37
89	3	86	0	14	82	−68
77	3	89	0	11	84	−73
3	6.5	6	0	89	5	84
19	6.5	75.5	1	22	70	−48
42	10.5	9	0	85	7	78
100	10.5	12	0	82	9	73
68	10.5	18	0	76	14	62
51	10.5	42	0	53	36	17
45	10.5	78	0	20	68	−48
50	10.5	94	0	6	81	−75
4	17	11	0	78	8	70
49	17	28	0	63	22	41
38	17	38	0	54	30	24
32	17	39	0	53	30	23
41	17	57	0	37	45	−8
61	17	75.5	0	20	61	−41
62	17	87	0	11	69	−58
84	23	43	0	47	32	15
43	23	44	0	46	32	14
88	23	58	0	34	43	−9
44	23	65	0	27	49	−22
67	23	68	0	24	51	−27
87	28.5	7	0	69	5	64

(continued)

(continued)

Participant	GAS (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
15	28.5	29	0	52	21	31
13	28.5	35	0	46	26	20
72	28.5	55	0	33	38	−5
91	28.5	71.5	1	21	48	−27
90	28.5	93	0	6	63	−57
59	32	21	0	54	14	40
76	35.5	3	0	65	2	63
9	35.5	14	0	59	7	52
95	35.5	33	0	44	21	23
28	35.5	50	0	35	29	6
65	35.5	59	0	29	34	−5
36	35.5	73	0	19	43	−24
21	39.5	4	0	59	2	57
48	39.5	63	0	24	36	−12
98	41.5	5	0	57	2	55
6	41.5	53	0	29	29	0
39	44	48	0	31	26	5
83	44	52	0	29	27	2
27	44	61	0	25	30	−5
58	46.5	67	0	21	33	−12
69	46.5	95	0	5	48	−43
34	48.5	31	0	35	17	18
10	48.5	64	0	21	30	−9
82	53	13	0	46	4	42
5	53	25	0	37	12	25
17	53	41	0	29	19	10
96	53	62	0	21	26	−5
24	53	97	0	3	43	−40
54	53	98	0	2	43	−41
46	53	99	0	1	43	−42
7	57.5	22	0	34	9	25
33	57.5	23	0	33	9	24
63	61	15	0	37	4	33
97	61	19	0	34	6	28
57	61	20	0	33	6	27
99	61	77	0	13	25	−12
80	61	90	0	4	33	−29
60	66	1	0	36	0	36
73	66	30	0	27	8	19
86	66	40	0	23	11	12
81	66	46	0	21	12	9
64	66	71.5	0	13	19	−6
75	70.5	16	0	28	3	25
8	70.5	47	0	19	11	8

(continued)

(continued)

Participant	GAS (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
12	70.5	84	0	6	23	−17
11	70.5	100	0	0	28	−28
14	74.5	2	0	27	0	27
66	74.5	74	0	10	16	−6
2	74.5	79	0	9	16	−7
22	74.5	80	0	8	16	−8
53	78.5	24	0	20	3	17
26	78.5	34	0	16	6	10
20	78.5	69	0	8	13	−5
40	78.5	85	0	4	16	−12
30	81	8	0	19	0	19
52	82	96	0	0	18	−18
25	83	27	0	14	3	11
31	85.5	17	0	15	1	14
47	85.5	26	0	14	1	13
78	85.5	32	0	13	1	12
1	85.5	83	0	3	10	−7
70	88	81	0	4	8	−4
79	89.5	45	0	9	2	7
37	89.5	54	0	7	3	4
74	91	51	0	7	2	5
71	92.5	36	0	7	1	6
92	92.5	82	0	3	4	−1
29	94	56	0	5	1	4
35	95.5	10	0	5	0	5
85	95.5	66	0	3	1	2
16	97	60	0	3	0	3
55	98	92	0	0	2	−2
23	99.5	88	0	1	0	1
93	99.5	91	0	0	0	0
					$S = 448$	

Theron's calculations lasted for quite some time, which left the rest of the consultants sitting in silent stillness. Dakota could hear the last of the campaign workers trailing out the door to the offices over the constant scratching of Theron's pencil. Once Theron was finished with the math, the rustling of his papers signaled to Dakota that it was her turn to place all of this information onto the dry-erase board. Dakota glanced over all of Theron's calculations, instinctively believing that what he had done was correct. As Dakota was writing everything on the dry-erase board, she could hear Michael clearing his throat.

"Well, are we going to have to deal with *Ties* this time around?"

Dakota continued writing information onto the dry-erase board as she answered Michael's question.

"Since there are *Ties*, we must use the equation for T that makes the correction for ties. We already know $T_{x(GAS)}$ equals 328."

Robin started spinning her pen on the conference table, obviously not paying any attention to what was going on around her. However, Theron was paying attention and was able to ask a question which was pertinent to the overall conversation.

"Wait a moment, how many *Ties* do we have?"

Suddenly, the majority of the consultants were now examining the dry-erase board in an effort to answer the question. After mere seconds, it was Dakota's voice which answered it.

"We have two *Ties* in the total testosterone Variable."

2 levels of 662	2 levels of 669

Theron quickly jotted down this information, as Robin stopped spinning her pen and shook her head.

"I have a feeling that we are going to have to solve an equation here."

Dakota smiled.

"I'd say that's a pretty solid feeling."

Robin rolled her eyes and slid down in her chair, while Theron leaned forward and pointed at the dry-erase board.

"So, which equation are we using?"

Dakota pointed at the air behind her.

"This one":

$$T_{z(testosterone)} = \sum t(t-1)$$

Theron smiled.

"I think I can handle it. After all, it seems like something I did a few minutes ago."

As expected, Theron was able to churn out the equation within a few moments:

$$T_{z(testosterone)} = \sum t(t-1) = 2(2-1) + 2(2-1) = 4$$

Dakota eyeballed his work and jotted it down on the dry-erase board. Rather than wait for Theron to run the computations, Dakota just plowed ahead with the work.

"Fantastic, now we just need to solve for T_{xz}.[7]"

Dakota glanced at the equation which they used earlier to solve for S and used it as the template for the next set of formulas:

[7] The formula to be used when there are no ties can be found on page 139.

$$T_{xz} = \frac{2S}{\sqrt{N(N-1) - T_x}\sqrt{N(N-1) - T_z}}$$
$$= \frac{2(448)}{\sqrt{100(100-1) - 328}\sqrt{100(100-1) - 4}}$$
$$= 0.0921$$

As soon as she was done, Dakota slid some of the papers back to Theron.

"Now, all we need to find T_{yz}; sentence and total testosterone."

Theron raised his eyebrow.

"Same exact process?"

Dakota nodded.

"Start with the *Agreements/ Disagreements* and work onward."

Theron cracked his knuckles and began what seemed like hours of computation. Even with Robin's inability to sit still, Theron continued on with his work and was able to do everything which was asked of him, stopping every so often to pull out clean sheets of paper. Only once did he stop and ask a question.

"We have *Ties*... am I using that equation[8]?"

Dakota nodded, and he continued on. Once done, Theron handed his work to Dakota:

$$T_{xz} = \frac{2S}{\sqrt{N(N-1) - T_x}\sqrt{N(N-1) - T_z}}$$
$$= \frac{2(448)}{\sqrt{100(100-1) - 328}\sqrt{100(100-1) - 4}}$$
$$= 0.0921$$

"So, $\boldsymbol{T_{xz}} = \mathbf{0.0921}$. I am guessing we will need that later?"

Participant	Sentence (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as –)	(# Agreements – # Disagreements)
4	12.5	11	0	89	10	79
5	12.5	25	0	75	23	52
14	12.5	2	0	96	1	95
15	12.5	29	0	71	25	46
18	12.5	49	0	51	44	7
19	12.5	75.5	1	24	69	–45
21	12.5	4	0	91	2	89

(continued)

[8] The calculations here are identical to the calculations discussed previously in this chapter in dealing with Kendall's Rank-Order Correlation Coefficient and Kendall's Partial Rank-Order Correlation Coefficient.

(continued)

Participant	Sentence (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
24	12.5	97	0	3	89	−86
38	12.5	38	0	59	32	27
39	12.5	48	0	49	41	8
44	12.5	65	0	33	56	−23
50	12.5	94	0	5	83	−78
62	12.5	87	0	11	76	−65
63	12.5	15	0	75	11	64
66	12.5	74	0	22	63	−41
67	12.5	68	0	27	57	−30
68	12.5	18	0	70	13	57
70	12.5	81	0	16	66	−50
73	12.5	30	0	59	22	37
86	12.5	40	0	50	30	20
90	12.5	93	0	5	74	−69
91	12.5	71.5	1	21	56	−35
96	12.5	62	0	28	49	−21
98	12.5	5	0	74	2	72
20	28.5	69	0	23	52	−29
32	28.5	39	0	46	28	18
41	28.5	57	0	31	42	−11
48	28.5	63	0	26	46	−20
69	28.5	95	0	4	67	−63
80	28.5	90	0	6	64	−58
81	28.5	46	0	36	33	3
89	28.5	86	0	8	60	−52
3	53	6	0	65	2	63
6	53	53	0	30	36	−6
8	53	47	0	33	32	1
9	53	14	0	56	8	48
12	53	84	0	9	54	−45
13	53	35	0	39	23	16
17	53	41	0	36	25	11
22	53	80	0	11	49	−38
28	53	50	0	30	29	1
30	53	8	0	55	3	52
33	53	23	0	44	13	31
34	53	31	0	39	17	22
42	53	9	0	52	3	49
43	53	44	0	31	23	8
45	53	78	0	12	41	−29
46	53	99	0	1	51	−50
49	53	28	0	36	15	21

(continued)

(continued)

Participant	Sentence (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements − # Disagreements)
51	53	42	0	30	20	10
52	53	96	0	2	47	−45
54	53	98	0	1	47	−46
56	53	37	0	28	19	9
57	53	20	0	37	9	28
58	53	67	0	14	31	−17
59	53	21	0	35	9	26
60	53	1	0	43	0	43
64	53	71.5	0	12	30	−18
65	53	59	0	17	24	−7
72	53	55	0	19	21	−2
74	53	51	0	21	18	3
75	53	16	0	33	5	28
76	53	3	0	37	0	37
77	53	89	0	3	33	−30
82	53	13	0	32	3	29
83	53	52	0	19	15	4
87	53	7	0	33	0	33
88	53	58	0	16	16	0
92	53	82	0	6	25	−19
95	53	33	0	21	9	12
97	53	19	0	26	3	23
99	53	77	0	7	21	−14
100	53	12	0	26	1	25
16	74	60	0	13	13	0
11	79	100	0	0	25	−25
25	79	27	0	19	5	14
26	79	34	0	17	6	11
29	79	56	0	12	10	2
35	79	10	0	21	0	21
36	79	73	0	7	13	−6
40	79	85	0	3	16	−13
78	79	32	0	14	4	10
93	79	91	0	1	16	−15
10	88	64	0	7	9	−2
27	88	61	0	7	8	−1
47	88	26	0	11	3	8
53	88	24	0	11	2	9
55	88	92	0	0	12	−12
61	88	75.5	0	3	8	−5
79	88	45	0	6	4	2
84	88	43	0	6	3	3

(continued)

(continued)

Participant	Sentence (rank)	Total testosterone (rank)	Rankings that are the same (denoted as X)	Rankings in agreement (denoted as +)	Rankings in disagreement (denoted as −)	(# Agreements – # Disagreements)
94	88	70	0	3	5	−2
1	93.5	83	0	1	6	−5
71	93.5	36	0	4	2	2
2	96	79	0	1	4	−3
23	96	88	0	0	4	−4
37	96	54	0	1	2	−1
31	98	17	0	2	0	2
7	99	22	0	1	0	1
85	100	66	0	0	0	0
				$S =$	156	

Again, we make the correction for ties:
$T_{y(sentence)} = 2400$
$T_{z(Testosterone)} = 4$

$$T_{yz} = \frac{2(156)}{\sqrt{100(100-1)-2400}\sqrt{100(100-1)-4}} = 0.0362$$

"Again, we will need this number for our next equation: $\boldsymbol{T_{yz}} = \mathbf{0.0362}$."

Dakota glanced at the dry-erase board and then back to the paperwork which Theron just handed her. Keeping everything organized, Dakota cleared off a significant portion of the dry-erase board and added one final equation to the board:

$$T_{xy.z} = \frac{T_{xy} - T_{xz}T_{yz}}{\sqrt{\left(1-T_{xz}^2\right)\left(1-T_{yz}^2\right)}}$$

Michael glared at the equation, trying to understand all of its working components.

"All of those symbols... they represent all of the varying *Correlations* between the *Variables*... right?"

Dakota smiled.

"You got it. It's the *Correlations* with the General Aggression Scores and sentence length, General Aggression Scores and total testosterone level, and testosterone level and sentence length. The equation just helps to hold testosterone level constant. We have all the numbers already; $T_{xy} = 0.3283$, $T_{xz} = 0.0921$, and $T_{yz} = 0.0362$."

Theron shrugged his shoulders.

"Well, if I have all the numbers..."

After a few moments, Theron was able to swiftly insert all of the *Correlations* into the correct spots and calculated the equation:

$$T_{xy.z} = \frac{T_{xy} - T_{xz}T_{yz}}{\sqrt{\left(1 - T_{xz}^2\right)\left(1 - T_{yz}^2\right)}}$$
$$= \frac{0.3283 - (0.0921)(0.0362)}{\sqrt{\left(1 - [0.0921]^2\right)\left(1 - [0.0362]^2\right)}}$$
$$= 0.3266$$

Stepping back from her work, Dakota began to search her mind for how one determines *Statistical Significance* with *Kendall's Partial Rank Order*. As she was standing in contemplative silence, Dakota could hear Robin thrashing about in her chair.

"It's midnight... do something... work your magic."

Theron, who himself was visibly tired, just leaned towards his childish confidant.

"I hate to break it to you, but you are not Major Nelson or Darrin Stephens."

Robin instantly settled down in her seat. Michael, stunned by her instantaneous change in behavior, was mystified.

"Theron, how did you get her to pay attention? This is fantastic!!"

Theron just shook his head.

"I didn't. I can almost guarantee that right now, Robin is mentally trying to figure out if 'Bewitched' is a better show than 'I Dream of Jeannie'."

Michael saw the absurdity of what Theron had done, but even he had to admit that it was effective. Suddenly, both men grew quiet as they heard Dakota muttering to herself.

"We have a rather large *Sample Size*, so I think we have to solve for z."

Upon hearing her words, Theron started digging through the work that he had done on the *Kendall's Rank-Order Correlation Coefficient*.

"Wait, isn't that what we used on the last problem?"

Dakota nodded.

"Correct. Just as with the *Kendall's Rank-Order Correlation Coefficient*, we are going to determine *Statistical Significance* based on the *z-distribution*."

Even though she continued to stare at the dry-erase board, Dakota instinctively knew that Theron was digging through his notes to find the equation which could compute the *z-score*. After a few more seconds of hearing papers rustling about, Dakota pulled off the tip of a marker and jotted the equation he was looking for on the board:

$$z = \frac{3T_{xy.z}\sqrt{N(N-1)}}{\sqrt{2(2N+5)}}$$

Theron halted his asinine quest for information he did not have and dutifully picked up his pencil to calculate information he could. It did not take him long to reach a conclusion:

$$
\begin{aligned}
z &= \frac{3T_{xy.z}\sqrt{N(N-1)}}{\sqrt{2(2N+5)}} \\
&= \frac{3(0.3266)\sqrt{100(100-1)}}{\sqrt{2(2[100]+5)}} \\
&= 4.8146
\end{aligned}
$$

Michael whistled as he saw the final answer on the paper.

"Wow, given that *z-scores* have significance if they are greater than or less than 1.96 at an *Alpha Level* of .05, I'd say this is massively significant."

Dakota nodded.

That's a pretty fair guess. If we consulted a z table, we are able to determine that for $z = 4.8146$, $\alpha < 0.05$; therefore, we *Reject the Null Hypothesis*, which posits that there is no relationship between the variables when a third variable is fixed. We conclude that GAS and sentence are associated when total testosterone is fixed."

Theron dropped his pencil and noticed that Robin still was gazing off into the distance. He gently nudged her with his elbow, which caused her to start blabbering.

"It's official, 'Bewitched' was better."

Theron just bent over and handed Robin her purse before leading her out through the large doors and towards her car. Michael stopped for a moment, nodded his salutations towards Dakota, and also slipped off into the night. Dakota looked at all the loose paper which was littered around the conference room, as well as all of the incoherent information splayed across the dry-erase board. Dakota just sighed to herself, realizing that they would be back here in a few hours to continue their work. She started to gather her belongings when she noticed that her cell phone had slipped out of her purse and was lying open on the floor. As she bent over to retrieve it, she saw a familiar message which is known to cell phone users everywhere.

"One missed call?"

Dakota switched on the phone and punched in the passcode for her voicemail. The message was short and decidedly to the point.

"This is Jennifer Parsons. Governor Greenleaf has need of your services. You'll hear from me within a few hours."

Dakota deleted the message and curiously checked the time that Greenleaf's right-hand girl tried calling; it was only a few minutes ago. Dakota sighed as she slid the phone back into her purse. Still, one thing was for certain: whatever was coming within the next few hours was not going to be good.

Chapter Summary

- The concept of a partial correlation is raised when discussing which test of association should be used to examine the relationship between sentence and the General Aggression Score.

- Power is a consideration while looking at the reasons why Kendall's Rank Order should be used with these research questions instead of Spearman's Rho.
- Kendall's Rank-Order Correlation Coefficient is employed using the equation that accounts for ties in the data set.
- The expansion of the powers of a positive integer using algebra is explained when the team must grapple with a Binomial Coefficient.
- The constancy of a fixed variable must be acknowledged when the team deals with Kendall's Partial Rank-Order Correlation Coefficient.

Check Your Understanding

1. Under which conditions would you use Kendall's Rank-Order test instead of Spearman's?
2. Under which conditions would Spearman's be a more appropriate choice than Kendall's Rank Order?
3. In the notation $T_{(age)(sentence).(income)}$, which variable is fixed?
4. Explain the concepts of Agreement and Disagreement and identify another test that utilizes these concepts.
5. True or False? Sample size must be taken into account when dealing with Kendall's Rank Order.
6. How is Kendall's Rank-Order Correlation Coefficient equation that accounts for ties mathematically and theoretically different from the one that does not?

Chapter 7
Guesstimating the Fluffy-Maker

Abstract In this chapter, two types of research questions will be addressed. The two general research questions are as follows: (1) whether or not a set of raters agree, which is analyzed by using the Kendall's Coefficient of Concordance, and (2) whether or not a set of raters agree when given only two choices at a time, which is analyzed by using the Kendall's Coefficient of Agreement. These two general questions are used by the team to answer (1) do the three raters give the same "meanness" ratings to each offender and (2) do the raters agree on which of two offenders is "meaner." The team will venture their way through the process necessary to answer these two research questions.

There was a sense of unease in the conference room as Dakota slid behind the door and nervously surveyed her surroundings. She knew that this meeting would not be a good one, especially given that Jennifer called all of the consultants at 5 o'clock in the morning with the urgent message to meet in the conference room in exactly three hours. She left the door open slightly, hoping that the incessant noise of the campaign workers in the offices beyond the door could drown out the ominous sense of dread she now felt in the pit of her stomach.

"Good morning Dakota, coffee?"

Dakota gave Robin an uneasy smile as she gingerly faded into one of the chairs next to the conference table. Robin poured the scalding hot black liquid into a cup and handed the steaming ceramic mug to her colleague. Dakota nodded in Robin's direction as she put the cup to her face and took a tentative swallow. This would make her third cup of coffee in the past hour.

"Thank you."

A sudden rustling noise quickly caught both of their attention as Theron turned over another page of the newspaper. She could see a smile crack on Robin's face as she tried to stifle the nervous laugh which was brewing inside her. They may all be geniuses in their respected field, but they were lousy at pretending to be nonchalant. Theron deftly folded up the newspaper and set it aside on the table, so much for pretending that everything was normal.

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8_7,

"Okay, does anyone know why we were summoned here?"

"You are here because I called you."

Robin physically lurched in fright as all three heads snapped to the door of the conference room. Jennifer stood in the doorway, her arms heavy with a stack of manila files. She calmly glided into the room, dropping the files onto various portions of the conference table. Dakota reached over and put a reassuring hand on Robin's shoulder, feeling her colleague's muscles harden with fright.

"It's okay."

Robin put her hand to her chest, silently feeling her heart thud through her blazer. She nodded at Dakota, trying to regain some composure as Jennifer moved soundlessly through the room, closing all the window blinds. A gloomy pall fell over the conference table as the cheery sun was obliterated with a few quick motions of her wrist. Jennifer then looked about the room, the irritation now cracking her normally hardened expression.

"Where is Dr. O'Brien?"

The doors for the conference room were angrily flung open as Michael stormed through them, his lab coat fluttering in his wake. Robin's hands clenched the arms of the chair as she let out a scream. Dakota now felt as if her heart was pounding through her chest, her hand slightly trembling as she clenched the coffee cup in front of her. These dramatic entrances really must stop. She watched as her colleague march to Jennifer, angrily wagging his finger in her face.

"Lady, who do you think you are calling me out of my shift at the hospital?!?"

Theron's eyes widened as he watched these two figures at the head of the table. Michael looked awe inspiring as his lab coat fluttered around his faded scrubs, while Jennifer had an icy demeanor in a solid black business suit. It was an impressive sight to behold. Jennifer just pointed to a chair; she was obviously in no mood for Michael's medical theatrics. Michael's face was bright red as he fought to control his labored breathing; he would not give this woman the satisfaction of besting him in front of his colleagues. Jennifer's body held firm, her steely gaze fixed upon him.

"Tell me Dr. O'Brien, how are you enjoying your new medical facility inside the prison?"

Michael's eyes shifted nervously as he fought to regain his composer.

"Why do you ask?"

Jennifer's smirk was palpable, the kind of look which silently expressed triumph and dominance over her prey. She stealthily slid up to Michael, her calculated hands smoothing out the lapel of his lab coat.

"You know what is great about a faltering economy? Government officials can ruthlessly eliminate any program they choose, all under the guise of being 'fiscally conservative'."

Robin looked as if the air had been snatched from her lungs as Jennifer's threat slid past her lips and echoed through the room like thunder. Dakota felt icy fingers slide down her spine as she tried to make out the thermometer in the far corner of the room. Over the rims of her glasses, she thought she saw the needle of the thermostat wrenched all the way to the right, yet she swore she could feel frost forming on the lip of her coffee mug. Michael's face blanched white; the fear was radiating off of him like smoke.

"Is that a threat?"

The fear in Michael's voice betrayed his pathetic attempt at bravado. Michael looked like a scolded schoolboy as he plopped himself into the chair. Jennifer quickly moved to the conference room doors and slammed them closed. With a flick of her wrist, all five of them were now isolated from the busy campaign workers.

"I must thank you all for meeting me at such short notice. I have something very important I need you to do."

Dakota shifted uneasily at the artificial pleasantries being sent her way. This was bad. Jennifer opened up some of her file folders and started scattering papers about the table, her voice a monotone rhythm of efficiency.

"Three years ago, Governor Greenleaf was told about a revolutionary new survey which was designed to help assess the 'Level of Meanness' of sex offenders who were up for parole by the State."

Robin's eyes flashed as a moment of insight struck her like lightning.

"Wait, those are the 'Meanness' scores on the data set."

Jennifer nodded somberly as this new piece of the puzzle was shoved into place for the consultants.

"Correct; although the scores you have in front of you are obviously not from the original data collection. The tool was designed by three of the most preeminent clinicians on the East Coast, all of whom have vast experience in working with this population."

Jennifer leaned into the table, her voice dropping to a low hum.

"I am sure all four of you are aware of the budget problems the prisons have been facing. The three clinicians were desperate to find some way of discerning who could be released with minimal risk of reoffending. So, the Governor spent a small fortune employing all three of them, buying the copyright to the test, and setting them to work administering this instrument as part of the sex offender's parole board hearings."

Theron raised his hand, a question burning on his lips. Jennifer nodded in his direction.

"Wait a moment, Jennifer. How much money are we talking here? Are we talking enough money to buy each of us a new Lamborghini?"

Jennifer shook her head.

"Try enough money to buy each of you your very own 747."

Michael whistled as the enormity of this expense was allowed to gestate for a moment. Jennifer folded her arms across her chest.

"Unfortunately, there are now some concerns as to whether or not these three clinicians all are in agreement on the level of 'meanness' scores given between the three clinicians on the instrument they designed. The concerns are related to the subjective assessment of each rater: whether they are all rating the offenders using the same definition of meanness."

Jennifer grabbed Theron's newspaper, opened it to the second page, and tossed it back on the table. The banner headline screamed out "Midnight Rapist

Apprehended." All four consultants looked at the news article with a sense of bewilderment as Jennifer continued on with her story.

"The gentleman to which they are referring in this story was one of the original people granted parole because the offender's meanness rating was so low he was considered to be no danger to the community at large. In fact, the Department of Corrections and Rehabilitation was so impressed with his 'Level of Meanness' scores, that they felt he was no danger to the community at large. He was our 'poster child'."

Dakota's eyes widened in shock at this revelation, even she could see the disastrous effect that this could have on the campaign. Jennifer just continued on, hoping to impress upon the consultants the magnitude of this situation.

"Given that the Governor and Mayor Eberling are now 'neck-and-neck' in the polls, this bit of news could literally ruin the Governor's chances of winning the Senate nomination."

Theron arched his eyebrow. His years of experience in politics were churning away in his brain, questions forming left and right.

"Wait a moment. Why come to us with this? I agree it's bad for the campaign, but there isn't much the four of us can do at this point."

Jennifer pursed her lips as Theron's face blanched white, cursing his experience for giving him clarity around this issue. He looked into the faces of the other consultants, all of whom were now thoroughly confused. Theron shook his head silently, his voice barely a whisper in the room.

"No one knows about the 'Meanness' scale yet, do they?"

Jennifer shook her head.

"No. But given how contentious this campaign is, it's only a matter of time until Mayor Eberling or the press gets a hold of this."

Dakota felt the shock of this melt away, as she added her own question to the conversation.

"Okay, so what do you need from us?"

Jennifer pointed to the copies of the data set now strewn across the table.

"I need you to tell me if there was any agreement among these three clinicians between their 'Level of Meanness' scores. There should be more than enough data there for you to pull something out of it. There are the numbers you have seen before but there is also a sample from that data where the three raters decided which of two sex offenders was meaner than the other sex offender. The raters did this as a little experiment for themselves without knowing that the information may be used later. They only did that for 10 of the participants because it was not sanctioned by the Governor. Do whatever you need to do to get me an answer!"

Robin shot Jennifer a quizzical look.

"Tell you?"

Jennifer nodded.

"Yes. The Governor is not to know any of this. What you find on this matter goes directly to me, and no one else."

Theron nodded in understanding.

"Let me guess, this is so you can formulate an official position for the governor if we do find that the State spent a small fortune on using a test where there is no uniformity in the scores between the people who created it."

Jennifer's face was a blank canvas, all but for a snarl which formed on the edges of her lips.

"You four are the finest consultants money can buy, helping out in one of the most contentious political races this state has ever seen. Surely, I don't need to tell you the *Probability* of the Governor winning this race should this story come to light and we are caught with our pants down. Right now, I am gambling that you four are as good as I think you are."

Dakota laughed at the subtle dig to their statistical prowess. Even she could see the likelihood of this not ending very well. Michael sighed as Jennifer finished her diatribe.

"You sound more like you are running a campaign for the Presidency, not just a race for the U.S. Senate."

Jennifer tilted her head in his direction.

"Dr. O'Brien, our country has a fairly well-established history of moving people from the Governor's Mansion to the White House. Now, can you four help me or not?"

All of the consultants (even Michael) looked anxiously at Dakota, hoping that she had the answer stored somewhere in her brain. It was very obvious that the campaign was not the only thing riding on whether or not they could figure this out. Dakota looked over the data set, slowly nodding her head.

"I think I can help you. There is a test, **Kendall's Coefficient of Concordance W**, which looks at whether or not there is agreement among a group of raters. That test should work."

Jennifer smiled at Dakota, making her very uneasy. There was something very threatening about this woman. Jennifer sauntered over to Dakota, aggressively invading her personal space.

"Good. I expect to see results in one hour."

Jennifer snatched up her purse and slipped out of the conference room. Only after she was gone did the team members breathe a sigh of relief. Theron sighed aloud as Dakota pulled out her laptop and began the always difficult task of finding an outlet in the expansive conference room. Michael fidgeted in his chair, unable to get comfortable.

"So, Kendall's what?"

Dakota felt a wave of empathy for her colleague on the other side of the conference table. Today showed all of them the kind of power that Jennifer wielded, and it was truly awesome. Jennifer had the devastating ability to make their funding go away at the drop of a hat, leaving them with nothing. Robin and Theron were now riffling through the data, trying to make sense of something which seemed to be known only to them. Dakota craned her neck over the table in an effort to see what her colleagues were doing.

"What's up, you two?"

Robin looked over the data and then looked back at her.

"We have no idea what you are talking about."

Dakota could hear the concern on the edges of her voice as she continued in vain to find an outlet for her laptop. Seeing one from the corner of her eye, she triumphantly placed the power cord for her laptop into the outlet.

"The *Kendall Coefficient of Concordance W*. It's basically a test that helps you determine whether or not there is *Agreement* among raters. Given our situation, I'd say it's probably our best bet at the moment."

Theron and Michael gazed intently at the data sheet, trying to absorb all of the rater *Agreements* on the "Level of Meanness scale." Dakota flipped through the book and propped it open to the section on the *Kendall Coefficient of Concordance W*.

"Yes, this is excellent but the first thing we need to do is to *Rank* these scores provided by the three raters. Theron and Michael, do you suppose you could *Rank* those scores for us? The *Rank* will be determined by the scores for the 100 individuals provided by each rater. Each rater's score will determine the 'Level of Meanness' rank from smallest to largest."

Theron and Michael both acknowledged Dakota's request with a reluctant head nod.

"Good, while you are working on that, Robin and I will begin looking at the equation."

Theron stuck his head up from the data sheet.

"Hold it. Is this one of those tests that does two different things depending upon *Sample Size*?"

Dakota nodded.

"It is. If your *Sample Size* is less than or equal to 7, you just need to look at all the possible **Permutations** for whatever your *Sample Size* happens to be. If your sample size is larger than 7, you need to approximate a **Chi-Square Value**."

Robin's face turned towards her.

"*Permutation*? Isn't that what causes some creatures to have a third eye?"

Michael's body dropped forward, and he began fantasizing about slamming his head on the table in frustration. Dakota's face took on a quizzical expression, as Theron leaned in towards his colleague to try and help her out.

"Uh, you are thinking about mutation."

Robin shrugged.

"Is there a difference?"

Dakota just shook her head, trying to tune out the melodic sound of Michael's chair squeaking as he continued to contemplate slamming his head into the top of the conference table.

"*Permutations* are just different ways you can organize a series of numbers. Trust me; it will become more important later.[1] "

[1] Permutations are explained in greater detail on page 208 in Chap. 9 when the Permutation Test for Paired Replicates is considered.

Theron quietly looked over his notes as Michael finally stopped swaying in his chair. Suddenly, something Dakota had mentioned stood out to him.

"Wait a second, isn't *Chi-Square* what we used for the *Phi Coefficient*?"

Dakota winked at her.

"Good memory. When your *Sample Size* gets too large, you cannot get your *Critical Value* from the *Kendall's Coefficient of Concordance* table, so you have to use a *Distribution* that can accommodate large samples; in this case, the *Chi-Square Distribution* works the best."

Theron and Robin both nodded in unison, allowing Dakota to continue on with her explanation. Spying her trusty dry-erase marker on the stand next to the board, Dakota snatched it into her hand, pried off the lid with her index finger, and wrote out the following equation for all to see:

$$W = \frac{12\sum R_i^2 - 3k^2N(N+1)^2}{k^2N\left(N^2-1\right)}$$

Dakota double-checked her work for a few moments. Once she was satisfied, she snapped the cap back onto the marker and let her hand fall away from the book. Michael continued to look through the data set, as Theron hesitantly asked a question.

"Wait a moment. If we want to see how these different scores 'hang together', why don't we just do a **Cluster Analysis**?"

Dakota thought about this question for a moment.

"Well, a *Cluster Analysis* is useful in determining how data are classified into smaller subgroups based upon some *Independent Variable*. The strength of the *Cluster Analysis* lies in maximizing the similarities within each subgroup while maximizing the differences between each subgroup. The *Kendall Coefficient of Concordance W* does somewhat approximate a *Cluster Analysis* in that you are able to see if these scores 'cluster together' in some way. However, we are looking at three different raters who are all assessing the same *Variable*, and this test can give you a pretty immediate 'yes' or 'no' on whether these raters agree. The Cluster Analysis would require some additional examination to determine the same thing."

Theron seemed pacified by this answer, as Dakota's laptop whirred to life. Dakota looked to Robin, silently asking her to bring over a copy of the data set. Robin wordlessly replied, sliding it in her direction. She had seen this column of data at least a dozen times by now, never really understanding why the Governor wished to know more about it. Now that she had her answer, she actually wished she never asked the question. All four members of the team worked in silence, the tension in the room almost becoming too much to bear. Finally, Theron spoke up from his side of the conference table.

"Alright, Michael and I think we have the ranks you asked for. Here they are."

Theron passed around a scribbled on piece of paper with the participant, rater, and rater rank listed on it.

Participant	Rater 1	Rater 1 rank	Rater 2	Rater 2 rank	Rater 3	Rater 3 rank
1	4	81.5	5	93.5	5	96
2	4	81.5	4	80	4	82
3	2	32	3	59	2	34.5
4	1	10.5	1	8.5	1	11.5
5	2	32	2	30.5	2	34.5
6	1	10.5	1	8.5	1	11.5
7	4	81.5	3	59	4	82
8	2	32	2	30.5	2	34.5
9	3	58	3	59	3	59.5
10	3	58	3	59	3	59.5
11	4	81.5	4	80	4	82
12	4	81.5	4	80	3	59.5
13	2	32	2	30.5	2	34.5
14	2	32	2	30.5	2	34.5
15	3	58	3	59	3	59.5
16	5	95.5	5	93.5	5	96
17	1	10.5	1	8.5	1	11.5
18	1	10.5	2	30.5	1	11.5
19	2	32	2	30.5	2	34.5
20	4	81.5	4	80	4	82
21	2	32	2	30.5	2	34.5
22	1	10.5	2	30.5	1	11.5
23	5	95.5	5	93.5	5	96
24	1	10.5	1	8.5	2	34.5
25	5	95.5	4	80	4	82
26	4	81.5	4	80	4	82
27	2	32	1	8.5	1	11.5
28	1	10.5	1	8.5	1	11.5
29	5	95.5	5	93.5	5	96
30	3	58	3	59	3	59.5
31	4	81.5	5	93.5	4	82
32	3	58	2	30.5	2	34.5
33	3	58	3	59	3	59.5
34	1	10.5	3	59	1	11.5
35	5	95.5	5	93.5	5	96
36	3	58	3	59	3	59.5
37	4	81.5	5	93.5	5	96
38	2	32	2	30.5	2	34.5
39	3	58	1	8.5	1	11.5
40	4	81.5	4	80	4	82
41	3	58	3	59	1	11.5
42	2	32	2	30.5	2	34.5
43	2	32	2	30.5	4	82
44	1	10.5	1	8.5	1	11.5
45	2	32	2	30.5	3	59.5
46	2	32	2	30.5	2	34.5
47	4	81.5	5	93.5	4	82

(continued)

(continued)

Participant	Rater 1	Rater 1 rank	Rater 2	Rater 2 rank	Rater 3	Rater 3 rank
48	2	32	2	30.5	2	34.5
49	3	58	2	30.5	2	34.5
50	1	10.5	1	8.5	1	11.5
51	1	10.5	1	8.5	1	11.5
52	3	58	3	59	3	59.5
53	4	81.5	4	80	4	82
54	2	32	2	30.5	2	34.5
55	5	95.5	5	93.5	5	96
56	2	32	2	30.5	1	11.5
57	3	58	3	59	3	59.5
58	1	10.5	2	30.5	2	34.5
59	3	58	3	59	3	59.5
60	1	10.5	1	8.5	1	11.5
61	2	32	2	30.5	2	34.5
62	3	58	3	59	2	34.5
63	1	10.5	1	8.5	1	11.5
64	2	32	2	30.5	2	34.5
65	3	58	3	59	3	59.5
66	4	81.5	4	80	4	82
67	1	10.5	2	30.5	2	34.5
68	1	10.5	1	8.5	1	11.5
69	1	10.5	1	8.5	1	11.5
70	4	81.5	4	80	4	82
71	5	95.5	5	93.5	4	82
72	3	58	2	30.5	3	59.5
73	3	58	3	59	2	34.5
74	4	81.5	4	80	4	82
75	3	58	3	59	3	59.5
76	3	58	3	59	3	59.5
77	2	32	2	30.5	3	59.5
78	4	81.5	4	80	4	82
79	4	81.5	5	93.5	4	82
80	3	58	3	59	3	59.5
81	3	58	4	80	4	82
82	3	58	3	59	3	59.5
83	3	58	3	59	3	59.5
84	3	58	3	59	2	34.5
85	5	95.5	5	93.5	5	96
86	4	81.5	3	59	1	11.5
87	3	58	3	59	3	59.5
88	2	32	3	59	3	59.5
89	1	10.5	2	30.5	2	34.5
90	1	10.5	1	8.5	1	11.5
91	3	58	3	59	3	59.5
92	5	95.5	5	93.5	4	82
93	5	95.5	5	93.5	5	96
94	2	32	2	30.5	2	34.5

(continued)

(continued)

Participant	Rater 1	Rater 1 rank	Rater 2	Rater 2 rank	Rater 3	Rater 3 rank
95	3	58	3	59	3	59.5
96	1	10.5	1	8.5	1	11.5
97	3	58	3	59	3	59.5
98	2	32	2	30.5	1	11.5
99	3	58	3	59	3	59.5
100	2	32	2	30.5	3	59.5

Robin jumped out of her seat, taking her position at the dry-erase board. Under Dakota's equation for the *Kendall Coefficient of Concordance W*, she began to plug actual numbers into the equation.

"Hold on, this isn't right. This formula is only if we have data where each participant has his or her own *Ranking*. Unfortunately, we do not have that luxury. That means that we need to change the formula a bit. It should look like this and we need to find ΣT_j. It should be noted that when there are no *Ties* in the *Rankings*, the ΣT_j is equal to zero. Since we have *Ties* in our *Rankings*, we need to calculate the ΣT_j":

$$W = \frac{12\sum R_i^2 - 3k^2N(N+1)^2}{k^2N\left(N^2-1\right) - k\sum T_j}$$

R_i = *the sum of each individual's rankings*
N = *number of individuals ranked*
k = *number of sets of rankings*
ΣT_j = *sum of the values of T for k rankings*

	Participant							
	1	2	3	4	5	...	99	100
Rater 1	81.5	81.5	32	10.5	32	...	58	32
Rater 2	93.5	80	59	8.5	30.5	...	59	30.5
Rater 3	96	82	34.5	11.5	34.5	...	59.5	59.5
	271	243.5	125.5	30.5	97	...	176.5	122
	73441	59292.25	15750.25	930.25	9409	...	31152.25	14884

$$\sum R_i^2 = 73441 + 59292.25 + 15750.25 + 930.25 + 9409 + \cdots + 31152.25 + 14884 = 2949671$$

$N = 100$
$k = 3$

When *Rankings* are *Tied*, this can have an effect on W. In order to account for the *Ties*, is calculated:

$$T_j = \sum_{i=1}^{g_j} \left(t_i^3 - t_j\right)$$

j = 1,2,3,. . .k where k is the number of sets of rankings
g_i *= the number of groups of ties in row j*

For each group of ties in row, t_i is the number of observations.
So, if rater 1 has 5 sets of ties:

20 offenders are tied at 10.5.
23 offenders are tied at 32.
29 offenders are tied at 58.
18 offenders are tied at 81.5.
10 offenders are tied at 95.5.

$g_1 = 5$. In the first group of ties, there are 20 observations, so $t_i = 20$.
$T_1 = (20^3 – 20) + (23^3 – 23) + (29^3 – 29) + (18^3 – 18) + (10^3 – 10) = 51288$

Rater 2 has 5 sets of ties:

16 offenders are tied at 8.5.
28 offenders are tied at 30.5.
29 offenders are tied at 59.
13 offenders are tied at 80.
14 offenders are tied at 93.5.

$T_2 = (16^3 – 16) + (28^3 – 28) + (29^3 – 29) + (13^3 – 13) + (14^3 – 14) = 55278$

Rater 3 has 5 sets of ties:

22 offenders are tied at 11.5.
24 offenders are tied at 34.5.
26 offenders are tied at 59.5.
19 offenders are tied at 82.
9 offenders are tied at 96.

$T_3 = (22^3 – 22) + (24^3 – 24) + (26^3 – 26) + (19^3 – 19) + (9^3 – 9) = 49536$

Therefore, $\Sigma T_j = 51288 + 55278 + 49536 = 156102$.
These calculations are then plugged into the original equation:

$$W = \frac{12 * 2949671 - 3 * 3^2 * 100(100+1)^2}{3^2 * 100\left(100^2 - 1\right) - 3 * 156102} = 0.920588635$$

To determine whether or not this finding is *Statistically Significant*, χ^2, with $df = N - 1$, is calculated because of the large sample size, $N > 7$.

$$\chi^2 = k(N - 1)W$$
$$\chi^2 = 3(100 - 1)0.920588635 = 273.4148246$$

"If $\chi^2 \geq$ the critical value for *Chi-Square*, the *Null Hypothesis* (i.e. the rankings are unrelated) can be rejected. The χ^2 critical value for 99 *Degrees of Freedom* at the .05 level of significance can be found on the chi square critical value table and is 123.23. Since $\chi^2 > 123.23$, we *Reject the Null Hypothesis* that the *Rankings* are unrelated. We conclude that the raters in this problem do agree when it comes to rating sex offenders on 'Level of Meanness'."

Once everything had been entered, Robin took a step back to see all of these numbers in context. She looked towards Dakota, who was silently double-checking her work.

"Is this correct?"

Dakota nodded her head, the answers on her computer screen mirroring what was scrawled across the dry-erase board.

"Looks okay to me. Anyone else get something different?"

Theron held up a finger in the air, signaling patience for a few moments as he and Michael finished their set of calculations.

"And, we have the same thing as you."

Dakota's mind was ablaze as she saw the writing and numbers before her. Robin sat down in the chair, trying to wrap her head around all of this.

"So, these are *Statistically Significant*."

Dakota, lost in her own thought, could only respond in a very monotone fashion.

"Correct."

Robin squinted, trying in vain to figure out what that meant for them and for the campaign.

"So, *Statistically Significant* is good, right?"

Dakota slowly exhaled, the deep cleansing breath helping to resolve herself for what she must do.

"In this instance, *Rejecting the Null Hypothesis* means that there were no differences among the raters in their opinions."

Michael dropped his head into his hand, a sigh of relief trickling past his lips. Dakota passively stared at Michael, identifying with his emotional outburst. All Dakota could do was sigh as she glanced at the clock. Jennifer would be expecting something from them any minute now. She pulled out her cellular phone and dialed Jennifer's number as the ominous sense of dread gave way to a strange feeling of calmness. In the span of one hour, all four of the consultants had proven their worth, regardless of the results of this test. Michael's jaw clenched as a wave of panic swept over him.

"Is the ice queen going to be okay with these results?"

Theron nodded somberly as Dakota's hand froze while dialing the phone. Even she now regretted that she had considered calling Jennifer. Theron's reassuring smile was of some comfort, but not of much.

"I've known Jennifer for years. This will thrill her."

Dakota tried to find comfort in his words, yet it was hard to do just that. She finished dialing the number. Within two rings, Jennifer's voice slid through the cellular phone.

"Hello Jennifer. We found that there was *Agreement* among the raters on the test. I am emailing the results to you now... you're welcome."

Dakota hung up the phone, noticing the anxious faces among her colleagues in the room. Michael could not let the anxiety continue any longer.

"Well, was she happy or not?"

Dakota just shrugged her shoulders.

"I can never tell with that woman."

Michael nodded somberly. It was not the answer he was hoping for, but it was an answer which would have to be sufficient for now. Dakota turned off her phone and tossed it back into the depths of her purse.

"Well, as long as we are all here, was there anything else we had on our agenda for this project today?"

Michael vigorously nodded his head. Robin sighed as Dakota singled him out among the others in the room.

"Okay Michael, let's hear what you have to say."

Dakota slid the dry-erase marker across the table, where Michael was able to stop it with the flat side of his palm. Theron threw his hands into the air as Michael greedily snatched up the marker and made his entrance at the dry-erase board.

"I say we stop looking at what the data have in common, and start telling the Governor what the data mean for the future."

Robin smacked her hand against her head.

"Okay Marcus Welby, how do you propose we look into the future? Do you have a crystal ball somewhere in your lab coat?"

Dakota's eyes expanded as she realized what her colleague was talking about.

"You mean you want to use the data to make predictions?"

Michael nodded, grinning from ear to ear.

"That's right. I say we start running *Regressions*."

Theron's face became a mask of confusion, his eyebrows furrowing wildly as he attempted to wrap his head around this topic.

"That sounds bad."

Michael looked over at Dakota, his jaw clenching with rage towards Theron.

"Maybe you would like to hold his hand through the process."

Dakota's eyes flashed with anger as Michael began writing furiously across the board. It was one thing to allow Michael a chance to play with the data; it was another to treat his colleagues with contempt. Dakota took a cleansing breath, steeling herself for what was likely to be a long day in the campaign offices.

"Basically, a *Regression* is a procedure which takes information gleaned from *Correlations*, and you use that information to make predictions about the *Variables*."

Robin lazily put her head in her hand, trying to decipher the scrawl that Michael was etching onto the dry-erase board.

"So, it's 'guesstimating'?"

Michael snapped the cap back onto the dry-erase marker and thumped his hands onto his hips.

"Now see here young lady, a *Regression* is taking your *Dependent Variable* and seeing how it changes based on your *Independent Variable*. With that information, you can predict what other points would be like if you graphed them out."

Robin nodded in his direction.

"So, it's 'guesstimating'?"

Michael's face seethed with anger at Robin's impudence. Dakota quietly rolled her eyes, deciding it was better to enter the discussion than to watch these two intellectually fight with one another.

"I can understand your confusion. The procedure itself is far from perfect, and it is one which has to be interpreted with a fair degree of caution. However, it is widely used and can be a good way to determine what the most likely outcome of two *Variables* would be if given this specific situation."

Theron now looked thoroughly confused, raising his hand as if he were once again in a classroom. Dakota pointed at him, allowing him to speak.

"Wait a minute. So, this test can 'predict' the future in only those instances where everything has to work out perfectly?"

Dakota could hear Michael's labored breathing behind her head. Clearly, he was not amused with the confusion of those who were not as enlightened as he was at the art of *Regression*. Dakota could see the image of his red face with his flaring nostrils; it was an image that almost caused her to burst out laughing. She maintained her composure as she steadily answered his question.

"You bring up an excellent point, Theron. There are those, myself included, who believe that *Regression* has very limited practical application. However, many programs and research *Hypotheses* require someone to make such educated predictions about what could possibly happen. I do believe that the Governor may need to make some predictions in order to create an adequate campaign platform, so we should do our best to provide her with as much information as possible."

Michael's breathing quieted as he loudly cleared his throat.

"That's what I said. Now, if you will see here, this is the equation for a **Linear Regression**. As you can see, all we need to do is plot this on a graph and you can follow the line to see what will happen in the future based on the **Criterion Variable** and the **Predictor Variable**":

$$y = mx + b$$

$$m = \frac{(N\Sigma XY - (\Sigma X)(\Sigma Y))}{\left(N\Sigma X^2 - (\Sigma X)^2\right)}$$

$$b = \frac{(\Sigma Y - m(\Sigma X))}{N}$$

Theron dropped his pencil in a wanton act of defiance against the horribly muddled confusion which was now splayed before him.

"A what and a what?"

Michael continued writing, oblivious to the questions forming around him. Dakota just rolled her eyes, taking it upon herself to once again clean up this mess.

"The *Criterion Variable* is basically just the variable you are trying to predict, while the *Predictor Variable* is the one being manipulated so you can predict the *Criterion Variable*."

Theron looked pacified by this answer as Michael began pointing to various parts of his equation.

"Now, this 'b' term is the **Slope**. *Slope* is what is going to tell you how steep your line is going to be when this is plotted onto a line graph. Over here, the 'm' term, is the **Intercept**. The *Intercept* is going to tell you where the line is going to cross the Y-axis on the graph. When "b" and "m" are placed together in this equation, it will help you to position the **Line of Best Fit**."

Dakota felt some of her tension ease as she heard this explanation. To her amazement, Michael had actually done a fairly decent job of conceptually explaining *Slope* and *Intercept*. Now, if he could keep up this level of clarity, the group should be able to grasp this concept quite easily. Robin raised her hand, asking what Dakota knew to be the inevitable follow-up question.

"The *Line of Best Fit*?"

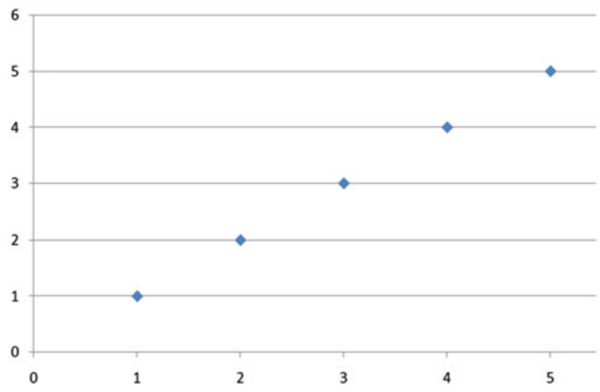
Michael nodded.

"Yes. The *Line of Best Fit* is going to tell you how much uncertainty there is in your line as you try to account for all of the data and random errors like *Outliers*."

Michael circled the equation and snapped the cap back onto the dry-erase marker, completely oblivious to the sea of confusion which was all about him. Dakota's eyes narrowed as she looked at the hieroglyphic writing on the dry-erase board, hoping there was some way she could explain this to the others. She gingerly picked up the marker and began writing in a small corner of the dry-erase board.

"Thank you Michael. That was an excellent explanation of *Linear Regression*. Is everyone on the same page with how this works?"

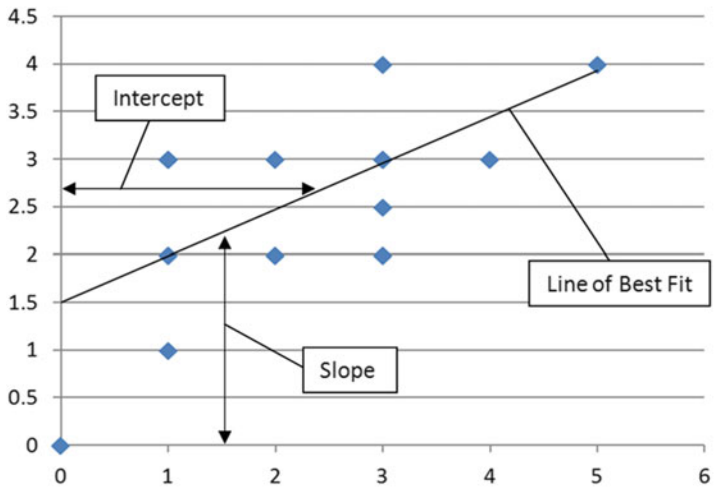
Michael plopped down in his chair, his face beaming with an intellectual superiority that was almost painful to observe. Dakota could see the absolute confusion which was painfully obvious in Robin and Theron's faces. This one was not going to be easy to explain. Dakota deftly drew a small graph on the dry-erase board and then created a small series of ascending dots.



“Okay. The main goal of *Simple Linear Regression* is to take the data you have, and based upon your correlations, you try to create a **Linear Model** based on your data set.”

Dakota could feel the anger welling behind Michael’s eyes as she began drawing all over his work. She tried to let his brewing rage roll off of her, but he was not making the task very easy. To him, she was defiling the sacred knowledge of a god. Dakota just kept pointing to her drawing.

“See here, this would be the *Slope* of the line, while this is the *Intercept*. These are what we use to best position the *Line of Best Fit*.”



Theron meekly raised his hand, his eyes pleading for clarity. Dakota acknowledged him, silently hoping that his mind was able to make something out of this confusion.

“So, it’s like plotting all of the data onto a line graph?”

Dakota mulled this question over in her mind for a moment. Clearly, Michael's explanation of this procedure had caused a great deal of confusion, and it was now up to her to prevent her colleagues from running out of the room.

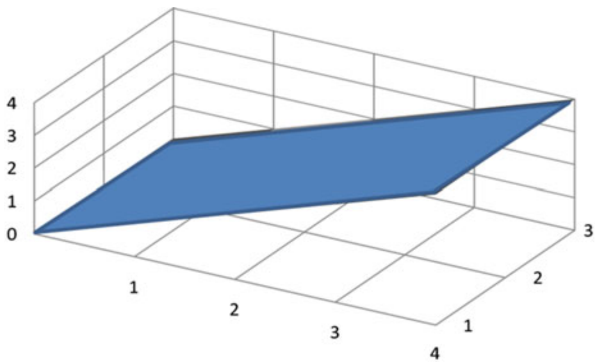
"Since we are using one variable to predict another variable, we can use this *Line of Best Fit* to best predict a 'y' value when we know the 'x' value or the 'x' value when we know the 'y' value. This is where the prediction comes in. If we use this graph here and we want to know what 'y' will be predicted by an 'x' of 6, all we need to do is follow the 'x' of 6 up to the *Line of Best Fit* and at that point follow a horizontal path back to the corresponding 'y'. In this case, it gives us a 'y' of 6."

The light bulbs seemed to be coming on all of the conference room as Michael belted out with obvious exasperation at his colleagues.

"I say we run a **Multiple Regression**."

Dakota felt the muscles in her stomach clench as the wave of confusion once again crashed over her colleagues' faces. In seven little words, Michael all but obliterated their comfortable moment of understanding. Dakota could feel the questions burning in their eyes and snapped up the dry-erase marker to once again make clarity out of confusion. She took a cleansing breath and once again began to write on the dry-erase board.

"A *Multiple Regression* is just like a *Linear Regression*, except you are now looking at multiple *Predictor Variables* to one *Criterion Variable*. The hope is that by adding these extra *Predictors*, you will increase the prediction outcome. Instead of a straight line, you would receive something that looks like this. It is known as a **Plane of Best Fit** because we are now working in three dimensions."



Theron could handle the diagram no more and with disdain in his voice blurted out: "Isn't there a simpler way of predicting data? This seems really complicated for guesstimation."

She thought about this for a few moments and then had a moment of insight.

"I agree that it seems complicated based on the graph but the graph is more complex than the analysis is. Maybe this will help. Think about *Regression* like baking a muffin."

Michael jumped to his feet, no longer able to contain the rage inside of him.

"How dare you!!!! I spend my precious time patiently explaining all of the finer points of this amazing, beautiful statistical procedure, and you reduce it to something any idiot housewife with any old cookbook can whip up. How dare you call yourself a statistician?"

Robin snorted in contempt.

"You had your chance and you blew it. Let's at least hear out the metaphor."

Michael spun on his heels and began to vent his rage directly at Robin.

"Now see here little…"

Instead of feeling intimidated, Robin just looked bemused.

"This know-it-all-Dr.-House routine is a big hit with the ladies, isn't it?"

Michael's reddened face instantly shifted from rage to shame. Robin continued to gloat as she continued on.

"Dakota, please continue."

Dakota just smiled.

"We have all made muffins before and we know that to bake a muffin we need eggs, flour, butter, baking soda, and sugar. However, all of the muffins we have made in the past were just regular muffins. What if we want to make a fluffy muffin? All of these ingredients are pretty standard in baking a muffin, but which ingredient, or combination thereof, makes a muffin fluffy? In fact, I have never seen a recipe which told us which ingredient we should use to make this fluffy muffin. What we are trying to figure out is which ingredients make this muffin fluffier than any muffin we have made in the past. So, what the *Regression* tells us, since we know the ingredients and what we want in the end, is how much of each ingredient is needed to make this muffin a fluffy muffin."

Dakota could see the confusion slowly drain from their faces. Robin smiled and smacked her hand on the desk in triumph.

"So, *Multiple Regression* is simply 'guesstimating the Fluffy-Maker'?"

Dakota couldn't help but laugh at the sound of this comment.

"That is about right."

Michael mulled over the explanation, his face a mixture of joy and shame. He loved this analogy, and was embarrassed to admit it. Theron looked over everything scrawled on the dry-erase board. Something about this explanation did not give him the clarity that he was hoping to have. He pointed to the board with his right hand while picking up the papers for the data set in his left.

"So, how does the 'Fluffy-Maker' help us with our data?"

Dakota squinted at the data set as a squeaking noise sliced through the air like a knife. Robin shuddered as the squeaking grated along her nerves; Michael was swiveling in his chair, demanding to make his presence known.

"Given that this was my idea, I say we run a *Regression* on the testosterone level of the Sex Offenders and the . . ."

"We cannot do that."

Dakota's objection stopped Michael, causing him to freeze in his chair. The two combatants now locked eyes from across the table, Dakota feeling the stinging hatred that was emanating from her colleague. It was as if her simple objection had been a declaration of war. Michael cautiously leaned forward in his chair, all of his muscles preparing to pounce on his prey.

"Excuse me."

Dakota pulled off her glasses, nonchalantly wiping them on the end of her blazer. She was hoping that this did not appear threatening to her colleague, whose jaw was now clenched in rage. Dakota slid her glasses onto the edge of her nose, preparing to do battle once again.

"Michael, I am sure you are aware that *Regression* as you have explained it is one which would require data from a *Random Sample*. As we all know, this is something we simply do not have."

Michael slammed his fist onto the table, slowly rising to his feet.

"So, you are telling me that the one statistical procedure which could actually help the Governor is something we cannot do."

Dakota smiled, shaking her head at Michael.

"Not at all. We just cannot do it this way."

Robin was genuinely intrigued by this.

"Okay, so what's the nonparametric version of the 'fluffy-maker'?"

Dakota just looked at her matter-of-factly.

"**Nonparametric Regression**."

Theron looked a little disappointed. He was expecting something a little less obvious. Still, the procedure could not be as easy as he thought.

"And how is that different from the original?"

Dakota looked at the dry-erase board, hoping that there was something she could salvage from Michael's hard work. Turning back to her laptop, she had an idea.

"Well, let me think. *Nonparametric Regression* is based more on estimations. Now, do you remember the *Predictor Variable*?"

All three of the consultants nodded in unison. Dakota mentally savored this moment, since she knew that there would be fighting soon enough.

"Well, *Nonparametric Regression* operates a little differently than that. In *Nonparametric Regression*, the data you have determines what the *Predictor Variable* is going to be."

Theron furrowed his brow.

"And how is that different from other forms of *Regression*?"

Dakota pointed to Michael's earlier *Regression* equation.

"Well, in parametric *Regression*, you predetermine what the *Predictor Variable* is going to be prior to conducting the analysis."

Theron sat like a sponge, trying to absorb as much of this new information as possible. In truth, he wasn't having much success with this one.

"I don't suppose you have another baking metaphor to help me understand this one?"

Robin giggled as Dakota shook her head.

"Let's see. You guys ever see 'Iron Chef'?"

Michael snorted as he leered at her.

"Maybe you should go back into the kitchen instead of playing statistician."

Robin put her hand on her neck, rubbing out the tension Michael was causing.

"Maybe she should. After all, she is cleaning up after the mess you just made."

Dakota just continued on, being as professional as she possibly could in spite of all the sniping.

"Anyway, you can think of 'Iron Chef' as a good way of understanding how *Nonparametric Regression* works. Suppose you are challenged to make a vegan dessert. You can only use the ingredients you are provided, and you know what is needed for traditional forms of baked goods. Unfortunately, you are having a hard time adapting these traditional recipes, since vegans cannot eat anything even associated with animal products."

Robin nodded her head, trying to wrap her brain around this.

"You mean like using soy milk in place of regular milk, or using carob instead of chocolate?"

Dakota nodded; She was thrilled that someone was able to follow along with her thus far. Theron just rubbed his temples.

"Aren't vegans those aliens on 'Star Trek'?"

Robin and Dakota laughed, oblivious to the anger which now seemed permanently etched into Michael's face.

"Okay. You know something of what you are looking for in the final product: dessert. However, you won't really know what you have until you finish baking and pull your creation out of the oven."

Theron was nodding enthusiastically as his own moment of insight occurred.

"I get it. You may have some idea as to what type of dessert you are making, but you won't know what you have until you pull it out of the oven."

Dakota snapped the cap back onto the dry-erase marker.

"Or, in the case of *Nonparametric Regression*, you won't know what you were modeling or using as predictors until you conduct the *Regression*. Only then will you know what your *Variables* are and what combination of those *Variables* have impacted your model."

Theron leaned back in his chair, stretching his arms over his head.

"Ok, all this talk of food is making me crave pie."

Robin leaned over as the laughter poured out of her and into the room. She could finally appreciate the absurdity about the day the consultants were having. Once she was able to calm herself down, she stood up from the table and headed towards the doors.

"I will see what I can do about getting us some food. Carry on; I am sure that Dr. House will catch me up when I return."

Robin slipped out of the room as Theron raised his hand. Dakota and Michael both nodded towards him to ask his question.

"Okay, so it sounds like you would need some 'Jabba the Hutt' size data set in order to run a *Nonparametric Regression* as the test has to supply so much information for the model. I am wondering, given that our data set is puny, can we actually run this test?"

Dakota and Michael locked eyes with one another, a questioning look upon both of their faces. It was an innocent enough question, but one which these four needed to wrestle with prior to conducting an analysis as complicated as *Nonparametric*

Regression. Theron saw the confusion in their faces and repeated his question, a more pleading tone in his voice.

"So, can we do this test?"

Dakota quietly mulled over the question. For a few moments, it seemed as if these four were onto something rather interesting that they could do with this data. She swallowed hard, feeling a gnawing sensation in her stomach. All she could do was shake her head, her words coming out in a cold stream of negativity.

"No. In order to run a *Nonparametric Regression*, you do need a data set which will not only make predictions, but one which can support the entire model. I honestly do not think that we have data which will allow us to do this. Furthermore, several *Regression* models, this one included, require that you use data which was acquired randomly; something we do not have."

Dakota could see from across the conference room that Michael was beginning to fume.

"Wait a minute. You said that data acquired randomly was an assumption of parametric *Regression*. What makes *Nonparametric Regression* so different?"

Dakota answered Michael without even glancing in his direction.

"You are right, *Randomly Sampled* data are an assumption for both *Parametric* and *Nonparametric Regressions*. However, the difference is that parametric *Regressions* also assume a *Normal Distribution*, *Independent Samples* and at least *Interval Scale Data* while *Nonparametric Regressions* do not."

Michael slammed his hand on the table, his face blustering red with anger.

"I knew it. You are just looking for a way to make me look foolish any chance you get. Would it kill any of you to at least fake an interest in what I have to say?"

"I bet you say that to a lot of women."

Michael's gaze darted towards the door in anger, where Robin stood holding two paper bags full of sandwiches. Dakota rolled her eyes and physically moved herself between Michael and Robin.

"Michael, I am not saying that *Regression* is in and of itself a bad thing. In fact, I personally know a lot of social scientists who have done wonderful things with predictions. What is important is that they abided by the assumptions inherent to *Regression*; thus allowing their work to be beyond reproach. Whatever we produce in this room will be heavily scrutinized, and I don't want our reputations damaged because we acted foolishly."

Dakota reached out and placed her hand on his jacket, her heart softening towards her colleague. She could see at the edges of Michael's anger was something far more damaging than an explosive outburst, utter disappointment. Michael's anger faded as Dakota stood near him.

"So, everything we have done to see how our data are associated has all been for nothing?"

Dakota shook her head.

"I don't think so. We know that we use the *Pearson's R Correlation* as a foundation for parametric *Regression*, but that doesn't mean that the *Nonparametric* correlations tests are any less useful in what we can do with this information.

Spearman's Rho tells you if there is some type of linear relationship between two variables of *Ordinal Scale*..."

"While the *Phi Coefficient* tells you if there is some type of linear relationship between categories."

Dakota smiled as Michael seemed to understand that the *Association Tests* had given them a wealth of good information and that this information was useful to them in its own right. Michael's disappointment faded away as the sun began to set on the other side of the vertical blinds. Michael cracked a smile.

"So, moving on?"

Dakota smiled up at him.

"Yep."

Theron and Robin watched as Dakota single-handedly rebuilt Michael's self-confidence. Both of them silently took their seats, as a weary Michael asked the milieu one exasperated question.

"Anything else?"

Robin looked over the data set, and her face suddenly exploded in a blaze of intellectual fire. She started fidgeting in her chair, something that Dakota instantly noticed.

"What's up?"

Robin then isolated the "Meanness" section of the data set and flipped the pages over for Dakota to see. Michael glanced over, still licking his wounds from this morning's thrashing. Robin pointed to two of the raters.

"So, we have this 'experiment' by the clinicians looking at whether they agreed when they directly compared sex offenders together. I am wondering if there is more we can do with this 'Meanness' test besides what Jennifer asked us to look at."

Michael's eyes widened in horror and his body stiffened in fright at the thought of this.

"Have you taken leave of your senses?!?!? Why do you want to keep playing with this asinine test?!?!?! We found that they agree; what more do you want?"

Robin felt her sympathy for Michael wane slightly.

"I am doing my job. Taxpayers spent their hard earned money on this thing, and the good people of this state believe that this assessment tool can help protect their safety. When I first heard of this assessment tool, I thought it had promise because if it actually works, it would mean that we would have some sort of quota for releasing sex offenders instead of releasing them willy-nilly. Maybe we can help fix this."

Michael just let the comment die in the room. Dakota could feel Robin's passion radiate off of her like heat. Up until this point, Robin has been nothing but helpful; she had earned the right to see where this line of thinking would take her. Besides, the worst of the day was already behind them. Dakota leaned into the data set.

"Okay Robin, let's hear what you got."

Robin beamed at the sound of Dakota's support.

"Okay, we know that these three all agree on how to apply this test. But, let's say that two of these offenders were in a room together, and the clinicians had an option of 'this guy is dangerous' or 'this guy is harmless'; do you think the clinicians might actually agree with one another?"

Dakota felt her glasses slide to the bridge of her nose.

"You mean like a forced-choice?"

Robin nodded excitedly.

"Something like that. Maybe there is some way to analyze this assessment based on how the test is administered."

Dakota crossed her arms over her chest, letting Robin's words construct a puzzle in her mind.

"That might work."

Dakota's face blushed red as she realized that she said that last point out loud. Fortunately, everyone else was oblivious to her faux pas, which allowed it to pass unnoticed. Dakota went to the dry-erase board, deftly erasing the stains of their task this morning.

"Okay, what you are talking about is the **Kendall Coefficient of Agreement u**. If I remember this correctly, the test is used for paired comparisons."

Theron moved around the conference table to Dakota's laptop and opened an Internet page searching for anything on the *Kendall Coefficient of Agreement*. After glancing over the words for a moment, he found his answer.

"That's correct. It says here that the 'objects must be ranked **Pairwise**'. Anybody know what that means?"

Dakota nodded and slid over towards the laptop. As she peered over Theron's shoulder, she spent a few moments greedily absorbing the information. She then began writing on the white board.

"It means that you can only look at this a pair at a time. Let's say our raters were going to assess three different offenders for 'meanness': John, Bill, and Mike."

Michael's ears perked up.

"Now see here…"

Dakota smiled and shook her head.

"I mean no disrespect, but I do know other Michael's."

Michael let the imagined slight pass as Dakota continued to write.

"Let's say we discover that our raters give Mike a higher meanness score than Bill, but give Bill a higher meanness score than John."

Theron quizzically tilted his head.

"So, Mike has the highest 'Meanness' score?"

Dakota emphatically shook her head.

"You can't make those kinds of generalizations with the *Kendall Coefficient of Agreement* because of the nature of the test. It may be logical to come to the conclusion that Mike has the highest meanness score but the generalization cannot be made because the raters are only looking at pairs. All you can do is examine whether or not all of the raters have reached the same conclusions when presented with these forced-choice options. I wear glasses and when I go to the optometrist, he gives me a series of two lenses asking me to choose which lens in each pair gives me a clearer view of the world. This is the very same thing."

Robin flipped to that section of the tattered statistics text she always carried with her.

"Okay, since I do not want to be caught off guard again, what would a *Statistically Significant* result mean for this particular test?"

Dakota gingerly put her hand on the text and moved the pages about with her fingers. This book had been like an old friend to her, and she knew where all of its information was stored.

"For the *Kendall's Coefficient of Agreement*, Rejecting the *Null Hypothesis* means that there is similarity among the ratings."

Robin shivered slightly; she had just heard something similar to this a moment ago.

"So, if we fail to *Reject the Null Hypothesis*, then there is no agreement?"

"Correct and this would be bad if we *Reject the Null Hypothesis*, yes?"

Robin's eyes were now pleading with Dakota.

"Could that work for this 'Meanness' assessment?"

Dakota jotted a little note down on her legal pad.

"I believe it can. If we use this formula to calculate u using the forced-choice *Rankings*, we should get what we need":

$$u = \frac{8\left(\sum a_{ij}^2 - k\sum a_{ij}\right)}{k(k-1)N(N-1)} + 1$$

a_{ij} = the number of times that the object in row i is preferred to the object in row j
N = number of objects
k = number of raters/judges

"For the *Coefficient of Agreement*, you can use either rankings or paired comparisons. For our purposes, we are using *Rankings* provided for only the 10 'experimental' offenders. Remember, these are the 10 offenders the clinicians used to see if they were all in agreement."

	A	B	C	D	E	F	G	H	I	J
Rater 1	7	5	6	2	4	3	10	8	9	1
Rater 2	8	9	5	3	2	10	7	4	1	6
Rater 3	8	6	7	3	4	3	1	2	10	9

"To calculate u, the number of times an offender is ranked above, or, in the case of our data, is ranked as meaner than, another offender is counted. This is done by looking at one pair of offenders at a time. The number of pairs can be determined by using $N(N - 1)/2$. Since $N = 10$, 45 pairs of offenders exist for each rater."

Dakota continued with her explanation and subtly looked in Michael's direction.

"With each pair, one offender is ranked above the other or tied in *Rankings*. Don't get confused when you see the capital letter 'I', it isn't a number 1":

		Rater 1	Rater 2	Rater 3
1	(A, B)	B	A	B
2	(A, C)	C	C	C
3	(A, D)	D	D	D
4	(A, E)	E	E	E
5	(A, F)	F	A	F
6	(A, G)	A	G	G
7	(A, H)	A	H	H
8	(A, I)	A	I	A
9	(A, J)	J	J	A
10	(B, C)	B	C	B
11	(B, D)	D	D	D
12	(B, E)	E	E	E
13	(B, F)	F	B	F
14	(B, G)	B	G	G
15	(B, H)	B	H	H
16	(B, I)	B	I	B
17	(B, J)	J	J	B
18	(C, D)	D	D	D
19	(C, E)	E	E	E
20	(C, F)	F	C	F
21	(C, G)	C	C	G
22	(C, H)	C	H	H
23	(C, I)	C	I	C
24	(C, J)	J	C	C
25	(D, E)	D	E	D
26	(D, F)	D	D	TIED
27	(D, G)	D	D	G
28	(D, H)	D	D	H
29	(D, I)	D	I	D
30	(D, J)	J	D	D
31	(E, F)	F	E	F
32	(E, G)	E	E	G
33	(E, H)	E	E	H
34	(E, I)	E	I	E
35	(E, J)	J	E	E
36	(F, G)	F	G	G
37	(F, H)	F	H	H
38	(F, I)	F	I	F
39	(F, J)	J	J	F
40	(G, H)	H	H	G
41	(G, I)	I	I	G
42	(G, J)	J	J	G
43	(H, I)	H	I	H
44	(H, J)	J	H	H
45	(I, J)	J	I	J

Dakota continued as her subtle comment had gone unnoticed by Michael fortunately resulting in the absence of an outburst.

"The information is summarized in a matrix which illustrates the number of times the offender in each row is determined to be meaner than the offender in the column. When the data includes ties, a count of $\frac{1}{2}$ is included in each of the cells of that pair in the matrix. In the case of multiple raters/judges, as in this example, the total determinations of meanness are summarized in one table":

	A	B	C	D	E	F	G	H	I	J
A	-	1	0	0	0	1	1	1	2	1
B	2	-	2	0	0	1	1	1	2	1
C	3	1	-	0	0	1	2	1	2	2
D	3	3	3	-	2	2.5	2	2	2	2
E	3	3	3	1	-	1	2	2	2	2
F	2	2	2	0.5	2	-	1	1	2	1
G	2	2	1	1	1	2	-	1	1	1
H	2	2	2	1	1	2	2	-	2	2
I	1	1	1	1	1	1	2	1	-	1
J	2	2	1	1	1	2	2	1	2	-

$\sum a_{ij}$ = the sum of a_{ij} above or below the diagonal in the matrix

In this case, we'll use the numbers below the diagonal:

$$\sum a_{ij} = 2+3+3+3+2+2+2+1+2+1+3+3+2+2+2+1+2 \\ +3+3+2+1+2+1+1+1+.5+1+1+1+1+2+1+1+1 \\ +1+2+2+1+2+2+2+2+1+1+2 = 77.5$$

$$\sum a_{ij}^2 = 2^2+3^2+3^2+3^2+2^2+2^2+2^2+1^2+2^2+1^2+3^2+3^2+2^2+2^2 \\ +2^2+1^2+2^2+3^2+3^2+2^2+1^2+2^2+1^2+1^2+1^2+.5^2+1^2 \\ +1^2+1^2+1^2+2^2+1^2+1^2+1^2+1^2+2^2+2^2+1^2+2^2+2^2 \\ +2^2+2^2+1^2+1^2+2^2 = 157.25$$

$$u = \frac{8\left(\sum a_{ij}^z - k\sum a_{ij}\right)}{k(k-1)N(N-1)} + 1$$

$$u = \frac{8[157.25-(3)(77.5)]}{3(3-1)10(10-1)} + 1$$

$$u = \frac{-602}{540} + 1$$
$$u = -0.1148$$

"To determine whether or not this finding is *Statistically Significant*, *Degrees of Freedom* needs to be calculated first. The *Degrees of Freedom* for this example are the number of pairs of comparisons that can vary. In this case, the formula for *Degrees of Freedom* is symbolized by *f*":

$$f = \frac{(2N+5)^3 N(N-1)k(k-1)}{2(k-2)^2\left(2N^2+6N+7\right)^2}$$

$$f = \frac{[(2)(10)+5]^3(10)(10-1)(3)(3-1)}{2(3-2)^2\left[2\left(10^2\right)+(6)(10)+7\right]^2}$$

$$f = 59.1781$$

Next, χ^2 is calculated:

$$\chi^2 = \frac{3(2N+5)N(N-1)k(k-1)}{2(k-2)\left(2N^2+6N+7\right)}|u| + f$$

$$\chi^2 = \frac{3[(2)(10)+5]10(10-1)3(3-1)}{2(3-2)\left[2\left(10^2\right)+(6)(10)+7\right]}|-0.1148| + 59.1781$$

$$\chi^2 = 67.8848$$

If $\chi^2 \geq$ the critical value for *Chi-Square*, then the *Null Hypothesis* can be rejected, meaning that there is no agreement among the raters.

The χ^2 *Critical Value* for 59.1781 *Degrees of Freedom* at the .05 level of significance is 77.9305. Since $\chi^2 < 77.9305$, we fail to *Reject the Null Hypothesis*. We conclude that there is no *Agreement* among the raters when forced to choose between only two participants at a time.

Dakota stood there in silence, enjoying the wave of accomplishment that rolled through her. Even though the day had not started out in the best possible way, it looked like it could end on something of a high note. Methodically, she started to collect her things, all the while keeping Michael in her peripheral gaze.

"Okay everybody, I say we stop for today. Besides, it is good to end on something of a high note."

Theron and Robin silently collected their things, taking Michael's question as their official dismissal for the day. The two of them walked out of the conference room, instantly becoming swept away in the sea of campaign workers muddling about in the offices. Robin quickly turned back to see if Dakota was following her out of the building. All she saw was a brief glimmer of Dakota, walking towards Michael as the heavy conference room door obliterated them from her view.

Chapter Summary

- Two types of general research questions were considered in this chapter. The first was whether or not a set of raters agree. This research question was addressed using the Kendall's coefficient of concordance W.
- The team was tasked with providing an analysis of the three raters of the "Level of Meanness" variable. In doing so, they found that the Kendall's coefficient of concordance W was the best test to use.
- Several forms of regression were considered by the team. After discussion about the relationship of linear regression and multiple regression to muffins, they were deemed inappropriate for the data because the variables do not meet the assumptions for parametric tests.
- Nonparametric regression was briefly discussed. It was concluded that in order to run a nonparametric regression, a much larger data set would be required. In addition, the nonparametric regression does not allow a researcher to make predictions based on the data.
- In an effort to get more from the data, the team considered the use of Kendall's coefficient of agreement u in order to look at a forced-choice option for the "Meanness" variable.

Check Your Understanding

1. How is Kendall's coefficient of concordance (W) different from Kendall's coefficient of agreement (u)?
2. Describe the purpose of a line of best fit.
3. Define and explain slope and intercept in terms of linear regression.
4. The final result of multiple regression is a:

   a. Plane of best fit
   b. Linear model
   c. Fluffier muffin
   d. Line of best fit
   e. Convolved data set

5. For each of the following regression requirements, identify the type of regression, parametric or nonparametric, for each requirement.

   a. Requires a large data set
   b. Requires randomly sampled data
   c. Requires independent sampling
   d. Requires at least interval scale data

e. Requires normally distributed data
f. Requires a predictor variable that is determined before the study is conducted
g. Requires a predictor variable that comes from the data set itself
h. Requires enough data to supply the model for the regression

6. The variable that is being predicted in a regression model is called the:

   a. Predictor variable
   b. Independent variable
   c. Criterion variable
   d. Ranked variable

Chapter 8
X Marks the Spot Revisited

Abstract The team is transitioning from Tests of Association to Tests of Difference in this chapter. The team is wrestling with two tests that are used with only one sample. The two research questions that are addressed are as follows: (1) Is there a difference between the *Probability* that sex offenders will take medication and the *Probability* that sex offenders will not take medication? (2) Is there a difference between the Observed Frequency of sex offender registration and the Expected Frequency of offense for three registration categories: home address, conditional, and undisclosed? The first of these research questions is addressed by using the Binomial Test, and the second is addressed by using the Chi-Square Goodness of Fit Test. Despite some disturbing and politically hazardous news, the team must rally together to plow through these two research questions.

"Optimistic . . . How can I be optimistic? This is like watching my own funeral."

Robin uncomfortably shifted in her chair as she stared at Theron with a look of astonished horror in her face.

"Hey, aren't I supposed to be the morbid one around here?"

Theron had been in a terrible mood all morning. He was the first to arrive at campaign headquarters, and upon arriving the office had been buzzing with news that a major media firestorm was about to break open against Governor Nathanial Greenleaf. With his curiosity piqued, Theron called in some favors with his friends who worked with the Associated Press to see if he could glean any information from them as to the content of the story. The news was not good; someone had leaked information to the press about Governor Greenleaf's involvement with the parole of the sex offender who murdered Mayor Eberling's mother. Now, all four consultants were just sitting at a table, waiting to see how the campaign would respond. Seated in his usual corner of the room, Michael just sat very still with his hands folded in his lap.

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8_8,

“Well, how bad can this get?”

Theron shook his head as a look of horror blanched his face.

“I don’t know. I’ve never worked for a candidate who was responsible for killing the other candidate’s mother before.”

Robin rolled her eyes.

“Isn’t that a little bit melodramatic?”

A smile cracked on Theron’s lips.

“No, that’s politics.”

Robin just gathered her purse into her lap.

“So, are we ready to jump ship and call this whole ‘political advisor’ thing a Failure?”

Theron and Robin turned silently to Dakota and Michael, who were sitting in their usual spaces. Even with no contact between the two of them, it was evident that the sparks which flew between them had smoldered into something much brighter. However, Robin and Theron were more interested in figuring out a way past a political scandal than an office romance. Dakota just sat with a quiet stillness in her chair; she had a difficult time seeing how Nathanial Greenleaf could slip out of this scandal and win his way back into the hearts of the voters. Still, they had a job to do, and Dakota resolved herself to finish it.

“Well, we haven’t lost yet.”

Theron snickered.

“That’s not what the poll numbers are going to reveal.”

Robin shrugged.

“I am with Dakota, I say we plug forward.”

Michael huffed at Robin.

“Ha. You’ve been nothing but a pain since we started this, and whenever we talk about a statistical procedure you look as if you literally die of boredom.”

Robin shook her head.

“I am not disagreeing with you. But this is politics in the United States of America, and I believe that you can kill someone and still have a thriving political career.”

Michael shook his head.

“What are you basing that on?”

Robin shrugged her shoulders.

“Well, Oprah seems to like him . . .”

Dakota couldn’t help but smile at that comment. Still, Robin’s enthusiasm did help her to feel as if they were not rats aboard a sinking ship. Dakota pulled the manila folder out, glancing over the thoroughness of all their previous work. Until they heard a concession speech from Nathanial Greenleaf, she had to believe that he was expecting results from them. Dakota gently closed the folder and started tapping it against the conference table.

“I say we move on. Unless Greenleaf or his fearless protector says otherwise, we move on. Agreed?”

All three of the consultants vigorously nodded their heads, taking Dakota by surprise. She then pulled the dry-erase board around and glanced over all of the research questions which they had crossed off once their questions had been answered. It was time for them to move forward. Dakota pushed the dry-erase board off to the side and pulled out a marker.

"All right everyone, I say it's time we move on. So far, we have examined whether or not there was an *Association* between our *Variables*. . ."

Theron shot up his hand to interrupt Dakota. Dakota stopped mid-sentence and allowed Theron to ask his question.

"Wait, what about the *Regression*? Isn't that making a *Prediction*, not determining an *Association*?"

Dakota smiled.

"Very astute. But remember that *Regressions* are often based on the strength of the *Association*."

Theron was pacified by her answer, thus allowing Dakota to move on.

"As I was saying, it's time for us to change direction. The rest of our research questions are not examining whether or not *Variables* are *Associated* with one another, but how these *Variables* are different from one another. So, from now on we will be using statistical procedures which will focus on the research questions pertaining to *Difference*."

Robin shook her head.

"Not getting it over here."

Dakota silently mulled over the situation.

"Every previous procedure we have run looked at *Associations*; *Statistical Significance* with these procedures often means that the *Variables* are *Associated* with one another; so, we looked to see if testosterone went up, how general aggression scores would act. Now, we want to see if the *Independent Variables* are distinct from one another; *Statistical Significance* now means that the groups are significantly different."

Theron nodded.

"This makes sense I suppose. So where do we start?"

Dakota glanced over all of their questions, settling on one in particular.

"Okay, how about we look at whether or not there is a *Difference* between the *Probability* that offenders are currently taking medication or not currently taking medication. It could be important in light of our *Somer's Index d* results where we found that testosterone levels were associated with aggression which in turn may be related to the *Probability* of taking medication they are taking. In this case, we can look at all 100 offenders to determine the likelihood of them taking medication or not."

Robin shrugged.

"Why start with that one?"

Dakota smiled.

"Why not? The question addresses the issue of whether or not there is a pattern of taking versus not taking medication. Being prescribed medication isn't enough—we need to be sure they are taking it. We can't really evaluate the effectiveness of medication in terms of curbing offending behavior if they aren't even taking the drugs."

Robin could not argue with that logic. Dakota stared at the question for a few moments, trying to consider the best possible ways to answer it. Suddenly, she snapped her fingers.

"Of course, it's so obvious. The research question is only looking at one sample, so we should use a **One-Sample Test** like the **Binomial Test**."

Robin snorted.

"Oh yea, an entire statistical procedure based on '0'and '1'; this is going to be mind-numbingly boring."

Theron shook his head and leaned towards Robin.

"No, that would be binary. She said *Binomial*."

Robin just sat quietly, staring at the board.

"It still sounds bad."

Dakota thought for a moment and tried to explain the procedure to them as best she could. Theron's face suddenly dropped as he looked over the notes he had taken.

"*One-Sample Test*? Aren't we only using one sample of sex offenders?"

Dakota shook her head.

"A *One-Sample Test* is something different. We are looking at one sample of 100 offenders and determining the *Probability* that they will take medication or not; we are not comparing the two *Probabilities*. All of the tests we have discussed have been **Two-Sample Tests**, which means we are comparing two samples, something very similar to comparing an *Experimental Group* to a *Control Group*.[1] A *One-Sample Test* is just taking your sample and comparing it to the *Mean* of the *Population*."

Michael rubbed his lower jaw, acting as if he was in pain from what Dakota just said.

"Wait a moment, how do you know what the *Population Mean* is?"

Dakota just smiled.

"The *Mean* of the *Population*, also known as μ, can be calculated from the *Population* with characteristics determined by whatever theory is specific to the hypothesis you are using."

Dakota halted for a moment, waited to see if anyone had any questions of her, and continued on with her explanation.

"Okay, there are a lot of populations that exist in **Discrete** categories. Now, *Discrete* categories are essentially constructs which have two possible outcomes. The most common example of *Discrete* categories would be 'male'/'female.'"

Michael leaned forward.

"So, all of the information we use for this test is going to essentially be *Dichotomous*? Either the person has the criteria we are looking for, or they do not."

Robin just snorted.

"Oh you are just so full of. . ."

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[1] Revisit the discussion on page 7 in Chap. 1 regarding control groups for more information.

Dakota snapped her fingers, derailing Robin's verbally abusive tirade. Once it was clear that Robin's mouth was closed for the time being, Dakota glanced over to Michael.

"You are correct; this test is designed for a *Sample* where results could take on two possible values."

Theron smiled.

"Like '1' if something does occur, and '0' if something does not."

Dakota nodded in Theron's direction, causing him to swell with pride over his own perceived mental astuteness. However, he quickly received the sharp sting of Robin smacking him in the shoulder. Theron quickly snapped away from his enraged colleague, rubbing his sore muscles as he stared at her in disbelief.

"What was that for?"

Robin just stared at him, her jaw clenched in rage.

"I just said this whole test was based on '0' and '1', and you said I was being idiotic. Now, you take credit for my idea. Prepare to feel my wrath!!!"

Michael sneered at Robin.

"Go easy on him. Most of us are not used to you being correct about anything, so I am sure that the whole experience just threw him a bit."

Needless to say, Robin's anger was no longer focused on Theron. Dakota rolled her eyes, picked up her purse, and slammed it onto the conference table. All three of her (almost) squabbling colleagues sat in stunned silence as the contents of Dakota's purse spilled across the tabletop and skittered onto the floor. Dakota just stood at the dry-erase board, smiling as she continued on with her explanation.

"Now, the *Binomial Test* is a procedure which compares the *Observed Frequencies* of two categories of a *Discrete Variable* to the *Frequencies* that would be expected with a researcher-specified *Probability* parameter under a **Binomial Distribution**. More often than not, the *Binomial Test* is based on the *Null Hypothesis* that both of the categories have an equal chance of occurring."

Michael's eyebrow arched up in a questioning way.

"*Binomial Distribution*?"

Dakota nodded.

"It's just a *Distribution* of all of the possible *Probabilities* you have for your *Discrete Variables*. Oftentimes, it's based on the number of trials you have, and what the researcher-assigned *Probability* is. More often than not, these trials are **Independent**."

Theron mouthed somewhat silently to himself.

"*Independent*?"

Robin nodded next to him.

"Yup. If one of our sex offenders is taking medication, it's not going to have an impact on whether or not another sex offender is taking medication."

The other consultants stared in silent wonder at Robin, their faces displaying the shock at what just came out of her mouth. Robin just folded her arms over her chest.

"What? I do pay attention here."

Theron (still rubbing his wounded shoulder) tried writing down some of this information. Without knowing it, he started muttering to himself.

"So, this would be like an expectant mother having a boy or a girl, or someone trying to determine whether a political campaign is a Success or a Failure."

Michael folded his hands on the conference table.

"Or, I guess in our case, whether or not one of the sex offender is taking medication or not."

Dakota went to the dry-erase board and (as she always did) cleared off a large section of it for her work:

$$p = q = \frac{1}{2}$$

"Now, the purpose of the *Binomial Test* is to determine whether or not the number of sex offenders we have falls into the categories of 'on medication' or 'not on medication' purely by chance. Now, the *Binomial Test* is an **Exact Test**; the assumption is that the derivation of the *Distribution* has been met."

Suddenly, Dakota saw a mixture of fear and horror well up in Theron's eyes; she had used way too many statistically based terms in her last sentence. She simply held up her hand in an effort to pacify him.

"All that language means is that the number of people who are *Statistically Significant* by chance will reflect your *Alpha Level*. If you are using a .05 *Alpha Level*, then you will only have 5% of your population get *Statistical Significance* purely by chance."

Michael nodded.

"Ah, I get it. Most other tests use **Approximations**, which mean that a researcher can increase the *Sample Size* in order to accommodate a 5% *Alpha Level*. This test would help you get an *Alpha* of .05 regardless of the *Sample Size*."

Robin sneered at Michael.

"That is fine and dandy that you understand this but I'm not sure I do. Are you saying that *Approximations* use the *Sample Size* to determine *Significance*?"

Michael, in an uncharacteristically helpful way, grinned at Robin.

"Yes, for most tests, the *Sample Size* is a concern because it provides some of the information for whether or not the test is *Significant*. For the *Binomial Test*, we do not have that same concern; regardless of our *Sample Size*, we can still determine *Significance* because this is an *Exact Test*."

Dakota nodded at Michael, only slightly noticing that Theron was leaning forward on his elbows.

"So, are we operating under the assumption that there is a 50/50 chance that the offenders are taking medication?"

Dakota stopped for a few moments, trying to figure out the best possible way to explain this situation.

"Honestly, the 50/50 *Probability* is a pretty common Hypothesis. We only have two groups so, by default, the *Probability* parameter for both groups is 0.5. If you don't like 0.5 or if 0.5 doesn't make theoretical sense, you can change the probabilities. For the first group, you can use whatever *Probability* you fancy or whatever you have a theoretical basis for; this is sometimes called a **Test Proportion**. After you have determined the proportion for the first group, the second group's *Probability* is easy to calculate. So, if we have reason to think that 7 out of 10 offenders will take their medication, then the *Probability*, or *Test Proportion*, of taking medication is .7, and the *Probability* of not taking medication is .3."

Dakota reached behind her and jotted some information onto the dry-erase board.

$$\mathrm{P}[X = 1] = p \text{ and } \mathrm{P}[X = 0] = 1 – p = q$$

"All right, the uppercase P tells us that we are calculating *Probability*. The $X =$ in brackets refer to whether we are calculating the *Probability* of a desired outcome or the expected outcome [X = 1] of *Probability* of an undesired outcome or the unexpected outcome [X = 0]. We only use the terms desired and undesired to differentiate the two classes; desirability does not refer to the like or dislike for something by our offenders. Since we are expecting the offenders to be taking their medication, our desired outcome is that they are taking their medication. The lowercase p is the proportion of observations that are the desired outcome and the lowercase q is the proportion of observations that are the undesired outcome."

Dakota paused just long enough to notice that there were no objections to her explanation of the equations before she continued.

"Now, if the *Sample Size* is large enough, you can use a **Continuous Probability Distribution** . . ."

Theron's hand shot into the air as a look of utter confusion crossed his face. Dakota just smiled as she stopped him cold.

". . . This is also known as a *Normal Distribution*.[2] Or, as you like to call it, a dromedary."

As quickly as his hand shot up, Theron meekly pulled it back towards the table. Dakota just continued on with her explanation.

"Now, one can also look at a **Binomial Distribution Table** to obtain the **Significance Observed** numbers based on the observations that one might have for their categories."

Theron just glared off into space; his eyes glazed over in horror at the complex stream of words which spewed out of Dakota's mouth.

"I have no idea what you just said."

Dakota just smiled reassuringly. Sometimes, that is the best possible way to go about things.

"When the sample is less than or equal to 35, instead of using an equation, you can simply look up the *Probability* in a table for the *Binomial Test*. So, if we had only 10 offenders, 7 of whom were taking their medication and 3 who were not, all we would have to do is look up the numbers to see whether the *Probability* that 7 out of 10 people do something is greater than what we would expect by chance alone. However, since we have a *Sample Size* greater than 35, the equation is what we will need to use."

---

[2] Normal distributions are considered at length beginning on page 24 in Chap. 2.

Michael cupped his face in his hands, his eyes gazing dreamily at their leader.

"What do you mean by differing *Sample Sizes*?"

Dakota glanced back to the dry-erase board, trying to think of the best way to answer his question.

"Well, it has been shown that as the *Sample Size* increases, the distribution tends to become that of a *Normal Distribution*. However, when the *Sample* is less than 35, it's often just easier to determine *Statistical Significance* based on the table value.[3] However, if you had a small sample, you would . . ."

Dakota paused for a moment, looking to see that all of the consultants were on the same page. Confident that everyone was on the same page, she continued on with her work.

"If we had something which included more *Probabilities*, like determining how a person rolls a '3' using a die, we would incorporate a 1/6 *Probability*. After all, a die has 6 sides, so there is a 1/6 chance of rolling a '3'."

Robin shoved her arm into the air, obviously perturbed by what Dakota had to say. Dakota just pointed at her colleague and hoped that she would keep her question civil.

"Whoa, you just said that the Bisexual Test would only have two possible outcomes, hence the whole 'male' and 'female' dichotomy thing. How does something with 6 sides fit into that?"

Theron shook his head.

"*Binomial Test*."

Robin nodded vigorously.

"That's what I said."

Dakota placed her hands on her hips and tossed her head back. Unfortunately, Robin could be very astute when she wanted to be. She slowly started off with her explanation.

"Well, we are actually both correct. It is possible to determine multiple *Probabilities* using something akin to the *Binomial Test*. However, this would be a **Multinomial Test**, and would be based on a **Multinomial Distribution**. That way, you would have an *Exact Test* based on whatever *Probability* you required."

Dakota pulled out her trusty marker and jotted down all of the information for the *Binomial Test*:

$$Z = \frac{(Y \pm .5) - Np}{\sqrt{Npq}}$$

N = Total number of cases.
Y = The frequency of observations for the second variable or the frequency of observations that are failures.

[3] A discussion about how to use a table (specifically the Chi-Square Table) can be found on page 85 in Chap. 4.

p = The proportion of observations expected when x = 1 or the proportion of observations that are "successes."
q = The proportion of observations expected when x = 0 or the proportion of observations that are "failures" (1 – p).
Y + .5 is used when Y < Np
Y– .5 is used when Y > Np

Dakota, in an attempt to keep her colleagues moving forward, talked as fast as she was writing on the dry-erase board.

"With the *Binomial Test*, there are two possible outcomes – a '**Success**' is a desired outcome based upon the hypothesis and a '**Failure**' is an undesired outcome based upon the *Hypothesis*. Neither outcome is really preferred over the other, it is just a way to differentiate between the two that refers back to the *Hypothesis* of the two possible outcomes, one is designated by the researcher as a '*Success*' (the desired outcome) the other as a '*Failure*' (undesired outcome).

For example, in a factory, a *Success* would be a perfectly functioning product or a defective product. So we are all talking about the same thing, let's say that a *Success* is a functioning product, and a *Failure* is a defective product. The proportion of observations that were *Successes* would be the total number of products that were perfectly functioning. The proportion of observations that are *Failures* would be the number of defective products."

Dakota looked around the room to determine the level of understanding before moving from the abstract example to the more relevant one at hand.

"So, as we look at our data, we want the sex offenders to take their medication but not just for taking their medication's sake. We want them to take their medication to hopefully stop the anti-social behaviors that lead to sexual offending. So, taking medication would be the desirable outcome—the *Success*."

Theron dutifully copied down all the information onto his legal pad, making certain that all of the components were accounted for in his work. Michael just looked slightly confused at the jumble of information which Dakota had vomited onto the dry-erase board, his mind desperate to make sense of what was written before him. Sensing his frustration, Dakota just stepped towards Michael and reassuringly patted him on the shoulder.

"I promise that this will make sense in a minute. Just give it a chance."

Michael just leaned his head forward, his eyes flashing brilliantly as he tried to wrap his head around what his lady love just told him.

"So, what does *Rejecting the Null Hypothesis* mean for the *Binomial Test*?"

Dakota turned away from the group, pointing at her drawings of the *Binomial Distribution*.

"Well, the *Null Hypothesis* is that your specified *Probability* will be the same as the *Probability* for the *Population* given the theoretical distribution you are using. By *Rejecting the Null Hypothesis*, you know that there are *Significant Differences* between your *Sample* and the *Population*."

Dakota then pointed to the data set, still housed in its cozy perch at the center of the conference table.

“Okay, our first step is to determine the **Frequencies** for those offenders taking medication, and those offenders who are not. We do not have to count the ones that are listed as ‘unknown’ because we can only assume that they fall into one of the two categories we are looking at. We are unable to definitively place them within one or the other making that data unusable for this test.”

Robin leaned forward and snatched the data set.

“So, we are counting.”

Theron pulled out his pencil.

“Looks like.”

After a few minutes of tallying numbers, Theron slid his pad over to Dakota, who dutifully placed a copy of his work on the dry-erase board.

	Number of offenders known to be currently taking medication	Number of offenders known to NOT be currently taking medication	Total
Frequency	33	54	87

Dakota then pointed to the equation on the dry-erase board.

“Not counting the ‘unknown’ offenders, we are left with 87 as our total. Now we just solve the equation. All of the components should be here, so just get to work.”

Theron looked over all of his information and compared it with what was on the dry-erase board. He sighed quietly to himself and just plugged on with his work. It only took him a few moments, and he was able to finish his work. Dakota leaned forward and was able to snatch all the information she needed to copy this work onto the dry-erase board.

$$z = \frac{(Y \pm .5) - Np}{\sqrt{Npq}} = \frac{(54 - .5) - (87)\left(\frac{1}{2}\right)}{\sqrt{(87)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}} = 2.1442$$

“Okay, given that the *Sample Size* over 35 which is too large to use the *Binomial Test* table, in order to determine *Significance*, we consult a *z-table*. When $z = 2.1442$, $a < 0.05$. Since $a < 0.05$, we *Reject the Null Hypothesis*; there is no *Difference* between the *Probability* that offenders are currently taking medication and the *Probability* that offenders are not currently taking medication. We conclude that there is a *Difference* in *Probability* between offenders who are currently taking medication and those who are not.”

Robin shook her head.

“Sorry, but this does not compute.”

Michael dropped his pen in disgust.

“Oh, big surprise there. Robin doesn’t get it. Next thing you’ll say is that the earth is round.”

Robin’s eyes narrowed at the obvious challenge by her amazingly helpful colleague. Sensing the danger at the moment, Dakota attempted to use more tact than her paramour.

“According to this result, clearly, there are more offenders not taking their medication than taking their medication, more than what you would expect by chance alone. There is a significant pattern here. Further investigation is needed as to why some offenders are more likely to *not* take their medication than to take it.”

Michael closed his eyes.

“Well, that could mean that not all of the sex offenders have a clinical diagnosis that warrants the use of medication—which in turn has implications for treatment requirements. It’s not much, but it is something which will definitely warrant further examination.”

Theron glanced over to Robin.

“Well. . . where is the sarcastic comment that always comes out of your mouth after Michael says anything which could help us do our jobs?”

Robin just shook her head.

“It’s medical in nature. Even I know when to let ‘Dr. Feelgood’ have his moment in the sun.”

Theron just smiled as he handed all of his work regarding the *Binomial Test* towards Dakota. Dakota snatched all of the paperwork from the *Binomial Test* from Theron’s outstretched hand and added it to the steadily growing pile of information which they were compiling. Even she was impressed with what they had been able to do with this data and all of the innovative research questions which they had generated. However, whether or not any of this information could be of any use to the campaign was still yet to be determined. Dakota flipped her hair out of her face and focused her attention onto the dry-erase board.

“Well, I suppose that the next logical place for us to go is to discuss whether or not there is a difference between the *Observed Frequency* for a sex offender registration category and the *Expected Frequency* of a sex offender registration category? This way, we can determine if the offenders in our dataset have similar registration requirements to those in the state.”

Robin just leaned into her hands, laying her head on the desk with a sense of exasperation.

“And why would this be the next logical place to continue?”

Dakota pointed at the question.

“Well, because we have already used a test for *One-Sample* and it’s easier to continue while it’s still fresh in our minds. In order to determine whether or not there is a difference between the *Observed Frequency* a sex offender registration category and the *Expected Frequency* a sex offender registration category, we need to use another statistical procedure which requires *One-Sample* and uses *Nominal Data*.”

Michael just smiled in her direction.

“And which test would that be?”

With a smirk on her lips, Dakota pointed to the manila folder.

“Our ‘X’ test.[4] ”

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[4] A discussion about the use of Chi-Square occurs on page 76 in Chap. 4 regarding the Cramér Coefficient.

Robin ruefully sighed and slumped down in her chair.

“Oh good, we went from talking about ‘1’ and ‘0’ to just talking about letters. Can’t wait to hear how this is going to play out.”

Dakota just shook her head.

“This one is easy. I think the best place to start is by referring to the procedure by its actual name.”

Theron suddenly crossed out all his previous work.

“Wait . . . it’s not the ‘X’ test?”

Dakota turned away from Theron and wrote something on the dry-erase board.

“Nope, the proper name of the procedure is the **Chi-Square Goodness of Fit Test**.”

Michael shrugged his shoulders.

“So, why have we been referring to it as the ‘X’ test? The name isn’t all that complicated.”

Dakota leafed through all of the *Association Test* material they have complied, holding the final stack up in the air.

“Because I was already inundating you all with new information, I didn’t want to overdo it.”

Michael’s inquisitive expression faded; it was easy to understand her logic when they were constantly learning new concepts. Dakota placed her hand on the back of her neck, her fingers pressing against the muscle tension which was quickly developing there. Instead of waiting for their questions, Dakota continued on in her explanation.

“Now, the *Chi-Square Goodness of Fit* Test is a *One-Sample* test used to determine whether or not there are *Differences* between the *Frequency* values you *Observed* versus the *Frequency* values you *Expected* to receive given whatever statistical model you wish to use. In essence, you want to see how well an *Expected* theoretical *Probability Distribution* fits with your *Observed* data.”

Theron stopped his hurried scribbling of what Dakota had said and suddenly piped up.

“Hence the phrase ‘Goodness of Fit’.”

Dakota just nodded in his general direction, allowing Theron to grab his pencil once more and continue on with his work. With Theron pacified, Dakota continued on with her explanation.

“With this test, we are trying to determine if there is a difference between the *Observed Frequency* of a registration category and *Expected Frequency* of a registration category. According to California Penal Code § 290.46, there are different categories of disclosure on the Megan’s Law Internet website, a tool for the public to search for registered sex offenders in and around the community. Registrants, depending on their offense of conviction, may be required to register under the home address category, conditional home address category, zip code category, undisclosed category, or they may apply to be part of the excluded category, which allows certain offenders to be excluded from the website if they meet certain criteria.”

Theron was quietly drinking in everything Dakota said. Still, he was left with only the obvious question.

"So why is this important?"

Dakota just silently stared at the data set splayed out in the center of the conference table.

"Because registration category is based upon severity of offense, this gives us an idea of the severity of offenses committed by the offenders in our dataset and whether or not is similar to offenders in the state."

Michael swiveled his chair in Dakota's direction, giving her his undivided attention.

"So, what do we do first?"

Dakota flashed Michael a beatific smile and resumed her now-trusty post at the dry-erase board.

"First, we need to determine what the *Expected Frequencies* will be. For the *Chi-Square Goodness of Fit* Test, the *Null Hypothesis* is that there is a high *Probability* that the *Observed Frequencies* and our *Expected Frequency* values could have been taken from the same population. For us, that means that the *Observed Frequencies* in our data set will match the *Expected Frequencies* from the California Penal Code. We need to know what is expected to be the frequency or percentage of sex offender registration requiring full home address, only requiring conditional information and ZIP code, and those that are excluded from having to register specific information. Once we have the *Expected Frequencies*, we can compare those to the data we have on our sex offenders and the *Frequencies* or percentage of sex offender registration in those same categories."

Robin bolted up in her seat, her mouth warping into a cruel smile, a smile of someone who is prepared to challenge authority and actually has thought of an intelligent way of doing so.

"So . . . how do we get *Expected Frequency* values for sex offender registration category? Is this information just going to fall out of the sky?"

Dakota was truly at a loss for words. Robin had a very good point in questioning how the group was going to come up with *Expected Frequency* values. While Dakota may have been at a loss for words, Theron was not. He pulled out his laptop, snapped open his briefcase, and pulled out his tablet.

"Yes, the information can fall out of the sky. Well, to be more specific, the information can fall out of the geniuses who designed the 'Google' search engine."

Robin narrowed her eyes, clearly miffed at the fact that Theron was about to derail her intellectual rebellion.

"Wait a minute, why do you still buy newspapers when you clearly are one of Steve Jobs' minions?"

Theron shrugged.

"Newspapers just seem classier to me. Besides, I like turning pages."

Robin just slumped back down in defeat, as Dakota craned her neck in a vain attempt to see what Theron was up to with his Internet search.

"Theron, what are you looking for?"

Theron continued sliding his fingers across the screen for the tablet.

"I saw something not too long ago that might help. Here we go, the California Department of Justice, Office of the Attorney General, provides statistics on registered sex offenders in the state. According to their February 2013 data: 41,473 offenders were required to register with their full address; 12,082 were required to register under the conditional home address and zip code categories; and 29,984 were part of the undisclosed and excluded categories.[5] Based upon this data, we know: 49.6% of offenders are required to register on the Megan's Law website under the home address category; 14.5% are required to register on the Megan's Law website under the conditional and zip code categories; and 35.9% are still registered offenders but are undisclosed or excluded from the Megan's Law website.[6] Can we use those *Expected Frequencies* with this test?"

Dakota nodded her head, as she carefully wrote down everything which Theron just told her.

"We can, that would be our theoretical basis for unequal proportions. Now, if we were hypothesizing an equal proportion of *Frequencies* in each category, we can determine *Expected Frequency* using this equation":

$$E_i = \frac{N}{k}$$

Theron quickly jotted all of the information down on his legal pad.

"Got it. Now what do we do?"

Dakota suddenly drew out a table with different segments for the *Observed Frequency* and the *Expected Frequency*.

	Registration category			
	Home address	Conditional and zip code	Undisclosed and excluded	Total
Number of observations				
Number expected				

"All right, the first thing I want us to do is to figure out the *Observed Frequency* values. Remember, we need to count out the number of offenders who fall within each of these categories."[7]

[5] California Department of Justice, Office of the Attorney General (2013a)

[6] California Department of Justice, Office of the Attorney General (2013a, 2013b); percentages calculated by dividing the number of registrants in the category by the total number of registrants in the full address, zip code/conditional, undisclosed, and excluded categories found in published statistics

[7] Descriptions of the offenses in each category can be found in California Penal Code § 290.46 subsections b (home address category), c (conditional home address category), and d (zip code category). Offenders who are convicted of offenses not specified in these subsections are still required to register with the police, but they are part of the undisclosed category and are not displayed on the Internet.

Theron reached out across the table and dragged the data set over to him. He then glanced over to Robin, his eyes pleading with her to help him with the counting aspect of this exercise. She just sighed, rolled her eyes, and started tallying up occurrences. After only a few moments, he slid his work over to Dakota, who transposed these two numbers onto the dry-erase board.

	Registration category			
	Home address	Conditional and zip code	Undisclosed and excluded	Total
Number of observations	74	25	1	100
Number expected				

Dakota then pointed to the dry-erase board.

"Now that we have the *Observed Frequencies*, we need to fill in the *Expected Frequencies*. Based on the information Theron gave us, we know that 49.6% of offenders are required to register on the website under the home address category; 14.5% under the conditional and zip code categories; and 35.9% are undisclosed or excluded from the Megan's Law website."

	Registration category			
	Home address	Conditional and zip code	Undisclosed and excluded	Total
Number of observations	74	25	1	100
Number expected	49.6	14.5	35.9	

Dakota glanced at the table and then proceeded to clear off a section of the dry-erase board to write down the equation for the *Chi-Square Goodness of Fit* Test:

$$x^2 = \sum_{i-1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

O_i = *the number of observed cases in* i^{th} *category*
E_i = *the number of expected cases in* i^{th} *category*
k = *the number of categories*

Theron glanced over all of the information which Dakota had given him. He furrowed his brow, pulled his legal pad over to him, and proceeded to work the math as best as he could. After a few moments of stillness, he slid his computations back towards Dakota. She quickly checked his work and then transposed all of this information onto the dry-erase board:

$$x^2 = \sum_{i-1}^{k} \frac{(O_i - E_i)^2}{E_i} = \frac{(74 - 49.6)^2}{49.6} + \frac{(24 - 14.5)^2}{14.5} + \frac{(1 - 35.9)^2}{35.9}$$
$$= 12 + 7.6 + 33.93 = 53.53$$

Michael glanced over all of the numbers before asking the next logical question.

"So, how do we determine *Statistical Significance*?"

Dakota bent towards the stack of papers and pulled out one particular table.

"Fortunately, there is a table which contains all of the *Critical Values* for *Chi-Square*.[8] In this instance, our *Degrees of Freedom* is simply the number of groups minus 1, or k - 1."

Dakota looked over her table and then double-checked her work against the information on the dry-erase board.

"The x^2 critical value for $df = 2$ when $a = 0.05$ is 5.99. Since $x^2 > 5.99$, we *Reject the Null Hypothesis* which states there is no difference between the *Observed* and *Expected Frequencies*. We conclude that there is a difference between the *Observed Frequency* of registration categories and the *Expected Frequency* of registration categories."

Theron's jaw dropped in horror.

"Wait a moment, if more of our offenders are required to register with a full address or zip code/conditional, doesn't that suggest a high sample of severe offenses? I mean, this is pretty unusual."

Dakota nodded somberly.

"I can see why you would say that. However, we already know that this sample is not a *Random Sample* of the population, so we know that this data is going to have some major flaws attached to it. However, we can't make speculations that this town is teaming with violent offenders. Based on our data, this is the result we have. We can't just assume something. . . "

Robin giggled.

"Well, you know what they say happens when you assume something. It makes an . . ."

Dakota sharply interrupted the obviously inappropriate tangent.

"Yes, we all know that it makes everyone involved look foolish."

Robin grinned at Dakota and held back her desire to see her comment through to the end.

"Well, that wasn't quite how I would have put it but the meaning is still there."

Robin grinned once more as Dakota pulled all of the papers together and placed them in the manila folder. Suddenly, the room jolted as a brick shattered the window pane and thundered onto the conference table. Theron and Robin dove to the carpet for cover as showers of glass shards fell about them. Michael jumped to his feet, pulled Dakota into his chest, and shielded her against the dry-erase board with his body. Dakota gazed up at Michael, who thought nothing of sacrificing his safety to protect her own. Her mouth was dry as she uttered the only words she could think to say.

"I am fine."

Instinctively, all four consultants looked at the shattered window. They were now aware of something else; they could hear chanting in the background.

[8] The Chi-Square critical value table can be found in the appendix Table L.

"We hate Nate!!! We hate Nate!!!"

Cautiously, Robin and Theron rose up from their hiding places and looked out the window. They could see a throng of angry women standing on the other side of the parking lot. Many were holding signs and banners decrying Nathanial Greenleaf. One woman even held up an effigy of the Governor. Apparently, his unintended actions in the death of Mayor Eberling's mother had come to light. Theron, who was obviously very shaken by what just happened, looked down at Robin in horror.

"This is bad."

Robin just shook her head and pointed at the effigy.

"I know. . . that doesn't even look like Greenleaf. I mean, who makes an effigy only to give him a mullet?"

Back in the far corner of the room, Michael just gazed down at Dakota.

"What are you thinking?"

Dakota said nothing, but the look on her face told him all he needed to know. Back at the window, Theron just spoke silently to himself.

"This is very bad."

Chapter Summary

- Two tests of difference are considered. Both of these tests are used when the researcher has only one sample to examine.
- The Binomial Test utilizes discrete data that are, more specifically, dichotomous.
- The Difference between one-sample and two-sample tests is discussed.
- The Differences between "Success" and "Failure" with respect to the Binomial Test are delineated. Success is the aspect of the research that is being addressed in the research question. Failure is the opposite of what is being addressed by the research question.
- The assumption of independent trials or samples is outlined.
- The Chi-Square Goodness of Fit Test also utilizes discrete data but does not have the same "two groups only" rule.

Check Your Understanding

1. How does an Exact Test differ from a test that uses approximation?
2. Describe how the Sample Size affects the Binomial Test and the Binomial Distribution.
3. The use of the terms "success" and "failure" is common when discussing Probability. Determine the "success" and "failure" in each of the following scenarios:

   a. A researcher wants to study the likelihood of recidivism for sex offenders.
   b. A consultation firm studies the affects of a treatment program for youthful offenders.

c. A researcher examines the likelihood of being selected for jury duty.
d. A group of graduate students investigates the likelihood of a defendant's case going to trial.
e. A team of political analysts assesses the likelihood of democrats being selected for political positions.

4. Which of the following should not be used to determine an Expected Frequency for research?

   a. Frequencies with a theoretical basis
   b. Results of previously conducted research on a sample from the same population
   c. Use a researcher's intuition about what the expected frequencies should be
   d. The observed frequencies for the population from which the sample comes
   e. Simply use equal proportions

5. In what ways is a Two-Sample test differentiated from a One-Sample test?

Chapter 9
Let My People Go!

Abstract This chapter introduces the reader to two tests of difference: the Permutation Test for Two Independent Samples and the Moses test for Scale Differences. Both tests require at least interval scale data and are considered amidst a journalistic firestorm. The team must work through the calculations for these two tests to make an attempt at clearing the Governor's name. Log Transformation, Dispersion Indices, and Random Number Tables are discussed in relation to the Moses test for Scale Differences. The two research questions addressed in this chapter are as follows: (1) Are two Independent samples significantly different from one another? (2) Is the Variability the same between two groups? These questions are addressed in terms of the story line by two specific research questions: (1) Are *General Aggression Scores* significantly different from one another on the basis of ethnicity/race of the offender? (2) Is the variability the same for the *General Aggression Scores* based on offense?

Robin darted into the conference room, her jacket haphazardly covering the right half of her head. Even through the heavy oak doors, she could still hear the taunting jeers of the mob which had inexplicably gathered outside. Robin tried shrugging her shoulder, attempting to shake loose the clods of mud and dirt which had been hurled at her as she made the marathon sprint from her car to the front of the building. Out past the windows, she could hear a faint tingling noise; apparently Robin's car was not going to escape the mob unscathed. Theron was plaintively seated in his usual spot at the conference table, his newspaper sprawled out before him. He slowly raised his eyes and pointed to his raincoat in the corner.

"They got me too. Fortunately, I was smart and took the bus; less risk of property damage. Besides, it would cost a small fortune to buff out the dents in a BMW."

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8_9,

Robin could see all the marks of dirt covering the back to Theron's coat, so much so that she could have sworn it was a leopard-print pattern. Exasperated by her marathon trek, Robin angrily yanked her coat off of her head.

"I feel about as popular as Salman Rushdie felt if he were doing a book tour in Iran back in the late 1980's."

Theron arched his eyebrow.

"You mean you feel like you just got knighted by Queen Elizabeth, and then you married that really attractive host from 'Top Chef'?"

Robin just let her purse and her coat drop to the floor in an echoing thud. She was in no mood to indulge his confusion.

"You know what; I am not going to explain this one to you. I am going on a limb and saying that our little welcoming committee outside possibly has something to do with the 'Midnight Rapist'."

Theron turned over his paper, flashing the banner headline to Robin. Their entire predicament was spelled out in five bold words.

"Governor Responsible for Recent Attacks."

Robin just dropped into her chair, her head buried in her hands.

"Okay, you are the politics-guru of the bunch. Just how bad is this going to get?"

Theron riffled through the paper, once again opening back up to the crossword puzzle.

"Well, do you know what the Hindenburg is?"

Robin nodded at him.

"The zeppelin disaster? What of it?"

Theron started searching his pockets for his pencil, hoping to begin his staple crossword puzzle.

"That pales in comparison to our little predicament."

Robin dropped her head onto the table as Theron methodically began filling in the little boxes of his morning routine. All she could do was turn her head and stare at Theron.

"So what does Jennifer have to say?"

Theron cackled in delight at the question.

"You mean about the press finding out that her candidate's multi-million dollar assessment screening released a sex offender, who then turned around and raped eleven women, one of whom happened to be the opposing candidate's eighty year-old mother?"

Robin just nodded at him. Theron managed to stifle his laughter and go back to his crossword puzzle.

"Well, she said that it was a brilliant campaign strategy for Mayor Eberling."

Robin felt as if someone kicked her in the ribs, until she spied another set of coats on the other side of the conference table. She bolted upright, pointing to what she knew to be Dakota and Michael's outerwear.

"When did Doctor Zhivago and Lara get here?"

Theron's eyes rose to meet Robin's.

"I thought we were feigning ignorance about their clandestine affair?"

Robin deftly shook her head.

"That was before I got pelted with dirt."

Theron lowered his eyes, returning to the safety of his morning ritual.

"No idea. Their stuff was here when I came in, but I have not seen any trace of them this morning."

Robin sat up, sliding down into her chair.

"Good. Right now, I really can't stomach any of their lovey-dovey cra. . ."

Robin was cut short by the conference doors swinging open, with Dakota and Michael marching in single file. Robin's face betrayed just how exasperated she was by the two of them, while Theron just went back to his crossword puzzle. Dakota and Michael took their seats at the conference table, both of them straining to maintain some semblance of professionalism. Dakota placed her purse under her chair and began organizing her things.

"Right. How are you two this morning?"

Robin opened her mouth to speak, when all four of them could hear the faint shattering of glass in the parking lot at the far side of the building. Robin just closed her mouth, shaking her head in disgust.

"That answer your question?"

Dakota just pulled out her dry-erase marker, smoothed out the line in her skirt, and took her place at the board.

"All right, I believe we were working on more *Tests of Difference* today, specifically those tests for *Interval Data*."

An exasperated Robin just laid her head on the conference table, as if it were a lodestone anchoring her to the bottom of some unseen ocean. Theron calmly put down his pencil, folded his hands in his lap, and leaned into his chair.

"Why?"

Dakota pointed to the notes from their session the day before.

"Well, yesterday we looked at *Tests of Difference* where only *One Sample* is involved, so today it would be prudent to examine statistics which can only be done if we have more than *One Sample*."

Theron just shook his head.

"That's not what I meant. I mean, why are we even bothering to continue on? This isn't just a run-of-the-mill political sex scandal, the Governor let out someone who brutally attacked eleven people."

Theron quietly let the gravity of the situation stew in the room for a few moments. He knew Dakota to be a brilliant woman, and he knew that she must have realized how hopeless this situation had become. Dakota let out a long sigh, as if she had been keeping her own concerns at bay for quite some time, yet with one simple observation, Theron broke down her resolve. Until that moment, not even Dakota was aware as to how much the Governor's campaign meant to her. For the first time in her life, her innate ability to understand mathematical principles was valued, and these principles could be used to actually make a difference in the world. Dakota believed in Governor Greenleaf and knew that she could do wonderful things to make their state a better place to live. She placed the cap onto the marker and looked to her colleagues across the table.

"I understand. Believe me, I understand. But all four of us were hired to do a job, and I have every intention of seeing this through."

Robin and Theron were shocked at her words. To them, it was as if she had lost all grasp of reality and was about to morph into a political version of Don Quixote. Robin sighed, her mind unable to comprehend what Dakota was trying to do. Her mouth fell open as she caught Dakota's gaze.

"Okay 'Dakota-In-The-Sky-With-Diamonds,' where is that rational woman you used to be?"

Dakota ran her hand through her hair, pushing back the errant strands which had wriggled free of their ponytail holder.

"Look, I know this situation looks bad, if not impossible. But the Governor *needs* us, now more than ever. Even though we may not like the situation, it's not as if the Governor personally released the 'Midnight Rapist'. He tried to do something to make the system better, and it backfired. We can't hold him accountable for the decisions of others, which is what that mob outside is trying to do. All we can do is look at the data we have and try to make improvements based on empirical evidence. That's all we can do, and all we should do. So who is with me?"

Dakota's voice trailed off in the room and allowed herself to emotionally falter from her well-worn pulpit by the dry-erase board. Theron's face slowly relaxed into a smile as he picked up his pencil, ready to get to work.

"So, which test shall we start with first?"

Dakota's face felt hot, as if she was flushed with a sudden fever. Instinctively, she reached her hand to her check, only to feel a bitter tear slide through her fingertips. Dakota deftly wiped her eyes and returned to the dry-erase board.

"I suppose that the most logical place to start would be the **Permutation Test for Two Independent Samples**."

Just hearing the name of this procedure caused Theron to roll his eyes and Robin to unceremoniously bang her hands on the conference table. Dakota folded her arms in front of her, patiently waiting for their temper tantrums to subside. Michael laughed with a snort at their childish behavior.

"Ah, the bright future of this campaign."

Robin pointed in Michael's direction, still refusing to lift her head up from the table.

"You stay out of this. Besides, the only reason you are in a good mood is because you actually found a woman who is willing to let you . . ."

Theron physically swung out of his chair and cupped his hand over Robin's mouth, physically stifling the last part of a sentence which would most likely infuriate Michael. Dakota remained absolutely still, finding this whole scene to be utterly ridiculous. Michael, however, turned several shades of deep crimson. Dakota quickly glanced over at him, unsure if he was enraged or just utterly embarrassed. Michael balled his hands into fists, slammed them onto the table, and rose from his seat.

"I have had it up to here with you and your condescending tone!!!"

Dakota rolled her eyes; it was rage. Robin lifted her head indifferently, glanced at the blustering medic on the other side of the room, and then lowered her head back onto the table. Dakota leaned over to the wall and began playing with the light switch, flickering the lights on and off several times in rapid succession with the palm of her hand. Even though she was surrounded by experts in their various fields, sometimes the most effective method of garnering their attention was what worked back in kindergarten. After several moments, the team members seemed calm enough to listen to her speak.

"As I was saying, the *Permutation Test for Two Independent Samples* is just a really powerful test which allows you to compare the *Mean* scores for two *Independent* sets of *Interval Data*."

Robin propped her head on her hand, trying to glean as much information from this as she could.

"And, why do we care about that?"

Dakota started writing on the dry-erase board.

"Well, the *Permutation Test for Two Independent Samples* allows you to combine your two data groups to see if both groups are from the same *Population*. For our data, the specific research question is: whether *General Aggression Scores* differ significantly from one another on the basis of Ethnicity/Race of the offender."

Robin smiled, slowly catching on to the information being presented before her.

"And this test would allow us to see if all of our sex offenders are from the exact same *Population*."

Robin rolled her eyes.

"And how does this help 'death spiral'?"

Theron craned his neck towards Robin.

"What is 'death spiral'?"

Robin just shook her head.

"It's my nickname for the campaign."

Dakota's eyes narrowed as she started to address the question.

"If we can see whether or not all of our sex offenders are from the exact same population, we can make a recommendation to Greenleaf about possibly putting money towards a diversion campaign to help address this problem to vulnerable individuals."

Robin shook her head.

"Aren't one of these diversion programs the whole reason we are in this mess?"

Theron just rolled his eyes.

"At least it gives Greenleaf something active to do to address this issue."

Dakota smiled at Theron.

"You got it."

Theron scratched his forehead, the universal sign that he was confused.

"So, it's like an **Independent t-Test**?"

Dakota nodded, mildly surprised at the question.

"Well, the *Independent t-Test* is a *Parametric Test* that examines differences among group *Means*, meaning that your data needs to fulfill all of the assumptions inherent to *Parametric Tests* in order for you to use them in your data analysis. *The Permutation Test for Two Independent Samples* does not require that your samples are *Normally Distributed*, only that they are of *Interval Scale*."

Theron leaned back in his chair, currently pacified by Dakota's simple explanation. Michael was still sulking in the corner, glowering at Robin, who was dutifully writing down everything Dakota said. Robin placed her pencil on the table, patiently waiting for Dakota to continue on with her lecture. Dakota patiently waited for a few moments to see if there were any more questions, then she continued writing her explanation on the dry-erase board.

"Okay, the first thing that you all need to know is that this test really only works with small *Sample Sizes*."

Theron furrowed his brow for a moment, submissively raising his hand.

"How small is small?"

Robin leaned in his direction, tilting her face away from Michael.

"Are you feeling inadequate?"

Theron snickered at her question.

"Well, no man can measure up to you."

Robin smiled at her playmate before returning to her dutiful position. Dakota just shook her head, pleased that at least these two could find some amusement in all the gloom. Dakota just continued writing on the dry-erase board.

"Well, you are looking at possible *Permutations*, so the larger your *Sample Size*, the more *Permutations* you will need."

Robin arched her eyebrow.

"*Permutations*? That sounds suspiciously like Calculus. Besides, didn't we already talk about these?"

Dakota pondered this question for a moment.

"Well, that's not too far off. A *Permutation* is just rearranging and listing the possible combinations of numbers for your data set."

Theron dropped his head.

"My mom used to always do that with the living room furniture. It used to drive my dad nuts, until he got her on the 'happy pills'."

Robin's mouth dropped open.

"You had an interesting childhood, didn't you?"

Theron smiled.

"It was like seeing 'Donna Reed' on speed, it was hysterical."

Dakota snapped her fingers, hoping to derail this tangent before any more damage could be done. Unfortunately, she did not have that much faith in her ability to stop these two once they got started.

"Okay you two, *Permutations* have nothing to do with interior design or mothers who are whacked out on 'happy pills'. It just concerns different ways to look at data."

Theron just flashed Dakota a "thumbs up" sign.

"Am good to go."

Michael jeered at him.

"That wasn't hard."

Dakota shot Michael an icy glare, one so threatening that it caused him to slouch back into his chair. She then continued to write on the dry-erase board.

"Let's look at the numbers 1, 2, 3, 4."

Dakota placed the numbers 1, 2, 3, 4 within brackets on the dry-erase board signifying that permutations were required.

{1, 2, 3, 4}

Dakota pointed to the four numbers on the board.

"Now, based on these four numbers, we know that there are 24 possible combinations as to how one could arrange these numbers."

Theron leaned back into his chair.

"How do you figure?"

Dakota sighed, carefully writing down all the possible *Permutations* for her example.

1, 2, 3, 4	3, 1, 2, 4	2, 1, 3, 4	4, 1, 2, 3
1, 2, 4, 3	3, 1, 4, 2	2, 1, 4, 3	4, 1, 3, 2
1, 3, 4, 2	3, 2, 1, 4	2, 3, 1, 4	4, 2, 1, 3
1, 3, 2, 4	3, 2, 4, 1	2, 3, 4, 1	4, 2, 3, 1
1, 4, 3, 2	3, 4, 1, 2	2, 4, 1, 3	4, 3, 1, 2
1, 4, 2, 3	3, 4, 2, 1	2, 4, 3, 1	4, 3, 2, 1

Once she was done, she glanced to the others, who had given up writing this long ago. Dakota placed the cap onto the dry-erase marker and gently set it upon the table.

"That's why you want small *Sample Sizes* when you do *Permutations*. The more numbers you add, the more tedious the calculations get."

Michael's chair creaked as he leaned back into it.

"So, what you are telling us is that we cannot do this test."

All three consultants looked dumbfounded by his comment. Dakota's eyes cautiously narrowed as she looked in his direction.

"Dr. O'Brien, I am not sure what you mean."

Michael took his hands and folded them behind his head, still grinning in triumph.

"Well, you said it yourself—we can only use small samples. Last time I checked, we had a *Sample Size* of 100 people. Now, I'd say that is just a tad too large for this test, don't you agree?"

Robin's jaw dropped open in shock. Michael was correct, and there was something about that fact which was very discomforting to her. Dakota just listened patiently, picking up her copy of the data set. She then slid it over to him.

"Dr. O'Brien, I assure you that we do have data that can be used in this procedure. As you can see, very few individuals in our data set are Caucasian non-Hispanic on androgens and Caucasian Hispanic on androgens, correct?"

Michael cautiously looked at the data set.

"Correct."

Dakota then leaned in close to him, pointing at the "*General Aggression Scores*" column.

"So, if we just focus on those individuals who are white and on androgens, we could examine the *General Aggression Scores* for Hispanic sex offenders and non-Hispanic sex offenders? After all, there are a large number of Hispanic individuals in California, so this information could be pertinent to the campaign."

Michael's jaw tightened.

"Yes, that is certainly something amenable to this statistical procedure."

Dakota smiled, strutting back to the dry-erase board so she could finish her explanation. She popped open the cap to the marker, looking over at Robin and Theron as she began writing.

"Okay, so what are the '*General Aggression Scores*' for those individuals who are white and those who are Hispanic?"

Robin slowly read off the corresponding numbers, as Dakota deftly wrote them onto the dry-erase board. After a few moments, their data was complete.

Participant	GAS	Ethnicity/race
2	59	White
7	54	White
12	58	Hispanic
39	50	White
53	60	White
55	76	Hispanic
56	40	Hispanic
69	51	White
70	67	Hispanic

Dakota admired the number, silently grateful that the *Permutation Test* could be conducted on this subset of the data. Underneath their smaller data set, she then wrote out the equation for the *Permutation Test for Two Independent Samples*.

"The number of *Permutations* can be calculated using this simple formula: 2^N. Once the number of *Permutations* is calculated, the rejection region is calculated by this formula: $(\alpha)(2^N)$. The *Rejection Region* tells us whether or not the two groups are from the same *Population*. If we had a data set larger than 12, we would use the **Wilcoxon Signed Ranks Test**[1,2] to avoid the cumbersome permutation calculation":

$$\frac{(m+n)!}{m!n!} = \text{all possible permutations for } m \text{ and } n \text{ observations}$$

$\alpha\left(\frac{(m+n)!}{m!n!}\right)$ = region of rejection; the number of the most extreme possible outcomes

m = number of observations in group x

n = number of observations in group y

[1] The *Wilcoxon Signed Ranks Test* is discussed in detail beginning on page 236 in Chap. 10.

[2] The Permutation Test for large samples using the *Wilcoxon Signed Ranks Test* worked example for the *General Aggression Score* data can be found in the appendix on page 380.

Group X: Caucasian/ non-Hispanic	Group Y: Caucasian/ Hispanic
59	58
54	76
50	40
60	67
51	

$m = 5$
$n = 4$

"First, the number of *Permutations* is calculated. This tells us the total number of possible combinations of the observations of x and y:

$$\frac{(m+n)!}{m!N!} = \frac{(5+4)!}{5!4!} = \frac{362880}{2880} = 126$$

Next, we calculate the region of rejection using $a = .05$; we have talked about this in other ways when we discuss our *Significance* level or the *Probability* we want in order to *Reject the Null Hypothesis*:

$$\alpha\left(\frac{(m+n)!}{m!n!}\right) = .05(126) = 6.3$$

We will specifically look at 6 particular combinations of observations in which the difference between the sum of group X, Whites, and the sum of group Y, Hispanics, is the greatest. What this means is that we are going to compare the observed differences between our two groups on *General Aggression Scores* and see what our observed difference between the two groups is."

Dakota stopped for a few moments to allow this information to be processed by her peers. Hesitantly, she opened her mouth to continue on with her explanation.

"The alternative hypothesis for the *Two-Sample Permutations* test can be directional. For example, let's say we hypothesized that Whites average General Aggression Score was larger than Hispanics average population mean. In that case, we find all 6 combinations in that one direction, where Whites are always greater than Hispanics.

In this case, we are not specifying a direction, so we will find 3 of the most extreme scores in one direction and 3 of the most extreme scores in the opposite direction (*Two-Tailed Test*). We will use the **Absolute Value** to determine the greatest difference between the scores. . ."

Robin shook her head.

"*Absolute Value*? I am hoping that this is somehow related to the vodka. I can use that."

Theron shook his head.

"You know, just because something has the word 'absolute' in front of it, it does not mean it is related to a very profitable vodka-manufacturing company."

Robin rolled her eyes.

"Spoil my fun."

Dakota just pointed to the dry-erase board.

"The *Absolute Value* just denotes how far a number is from zero if it was placed on the number line. An example would be . . ."

$|-2| = 2$

". . . as you can see, the *Absolute Value* is always going to be positive, since the distance from zero will always be the same."

Dakota quickly jotted down yet another table onto the dry-erase board.

"As I was saying, we use the *Absolute Value* to determine the greatest difference between the scores. To create the possible scores for *x* and *y*, we use the same numbers in our original subset of data":

Group X: Caucasian/ non-Hispanic	Group Y: Caucasian/ Hispanic
59	58
54	76
50	40
60	67
51	

"We then take those scores and rearrange them in the most extreme combinations. Three of these combinations have the highest scores in the *x* section and the lowest scores in *y* section. The other three combinations have the lowest scores in *x* section and the highest scores in *y* section."

	Possible scores for five *x* cases					Possible scores for four *y* cases				Σx	Σy	$\Sigma x - \Sigma y$	$\lvert \Sigma x - \Sigma y \rvert$
1	76	67	60	59	58	54	51	50	40	320	195	125	125
2	76	67	60	59	54	58	51	50	40	316	199	117	117
3	76	67	60	59	51	58	54	50	40	313	202	111	111
4	40	50	51	54	60	58	59	67	76	255	260	−5	5
5	40	50	51	54	59	58	60	67	76	254	261	−7	7
6	40	50	51	54	58	59	60	67	76	253	262	−9	9

Dakota took a step back from the dry-erase board, allowing her colleagues a chance to see what she had done.

"Within these 6 combinations, we are looking for a combination that is the same as our original sample."

Dakota drew yet another chart on the dry-erase board.

Group X: Caucasian/ non-Hispanic					Group Y: Caucasian/ Hispanic			
59	54	50	60	51	58	76	40	67

"As this combination of observed scores for Whites and Hispanics does not appear within the region of rejection, we *Fail to Reject the Null Hypothesis*, which stipulated no difference between the population means for Whites and Hispanics.

This means that the data are supporting us concluding that the samples of Whites and Hispanics were drawn from *Populations* with the same *Means*. Essentially, the scores on the General Aggression Scale are similar enough to each other to suggest that there is no effect of White versus Hispanic on general aggression. If our observed scores do appear within the region of rejection, we can calculate the p-value for a two-tailed test using the following equation where *D* is the calculated difference":

$$P_{two\ tail} = \frac{number\ of\ \left|D's\right| \geq \left|D_{obs}\right|}{\frac{(m+n)!}{m!n!}}$$

For a *One-Tailed Test*, the equation to calculate the p-value is

$$P_{uppertail} = \frac{number\ of\ D's \geq D_{obs}}{\frac{(m+n)!}{m!n!}}$$

After Dakota had finished her detailed explanation of the procedure, there wasn't a sound to be heard in the room until Theron sheepishly spoke up.

"So. . . everything makes sense but this seems like an awful lot of work for such a small *Sample Size*."

Dakota thought about his comment for a moment.

"Well, the truth of the matter is that it can be, if you have to wade through a lot of *Permutations*. However, being able to look at whether or not there are significant differences between *Population* means is certainly well worth it. This gives us some information on the *Distribution* of our data and potential differences between the groups."

Robin snickered as she continued to write down everything that was on the dry-erase board.

"Assuming that your result is *Statistically Significant*."

Dakota silently nodded in Robin's direction, but her laser-like focus was attuned to the dry-erase board and one question in particular. Dakota hurriedly wiped off the board and started down the yellow brick road of another fun statistical explanation.

"All right, the next question we should focus on is whether or not the *Variability* is the same for the *General Aggression Scores* based on offense."

Theron arched his eyebrow.

"Didn't we just do that?"

Robin shook her head.

"Nope. Which means Dakota gets to bore us senseless with a brand new statistical procedure."

Dakota smiled, happy that she found a way to contribute to their overall quality of life.

"And right now I am going to bore you with the **Moses Rank-Like Test for Scale Differences**."

Robin slid down in her chair, almost wishing that the group was going to call it a day.

"Moses, as in 'let my people go' Moses?"

Theron excitedly started jumping about in his chair.

"Please tell me that one of the requirements of this test that we do the computations on stone tablets?"

Dakota closed her eyes and shook her head, trying not to notice Theron bouncing like a small puppy waiting for a treat.

"Same spelling, not nearly as biblical. The *Moses Rank-Like Test for Scale Differences* is a terrific test which allows you to test for *Variance* differences between two groups. This allows us to see whether or not the two groups have equal *Variation*. This would give us some information on the consistency of aggression scores among the offenders, which has implications for potential treatment and supervision practices."

Michael just shook his head.

"Why don't we just look at the *Ranges* and *Standard Deviations* for each group?"

Dakota shook her head.

"Wouldn't work. Even though the numbers and the scores may be different, the *Variability* might be the same. This test allows you to examine the *Dispersion* in greater detail, as it is based off the *Medians*."

Theron's pencil slipped out of his hands. Apparently, he was on the verge of clarity, but that last sentence seemed to plunge him into another state of hopeless confusion.

"*Median*? Why not use the *Mean* for determining *Central Tendency* with *Interval Scale* data? You got the information right there."

Dakota shook her head.

"This test is really helpful because it acknowledges that you may not know what the *Means* for the data are, or you cannot make the assumption that the two different sets of data are equal."

Robin's eyes grew large.

"Wait, what do you mean 'don't know the *Means*?"

Dakota tilted her head to the ceiling, her mind buzzing with activity. Finally, an answer seemed to form within her mind.

"Okay. Let's say someone gives you a set of data which they have already **Log Transformed**. . ."

Theron arched his eyebrow.

"Sounds like a Michael Bay film."

Dakota just smiled.

"*Log Transform* just means that you take data and changing it into something else. Let's say we have a list of *Interval Data*, like ages. So, let's say our ages are Person A is 4, Person B is 7, Person C is 10, and Person D is 14. Now suppose we want to *Rank* order them, from lowest to highest. Person A would be given a *Rank* of 1, B would be given 2, C would be given 3 and D would be given 4. So, you could say that we *Log Transformed* the data."

Robin eyes brightened.

"I get it. So if you have data which has been transformed from *Interval* to *Ordinal*, and you don't have the original data points, then you could use the Bible test with the *Medians*, because you do not have the *Means*."

Dakota lifted her hand in protest.

"*Moses Rank-Like Test for Scale Differences*, not 'Bible Test'."

Robin dismissed Dakota's correction with a wave of her hand as she stretched her arms above her head, yawning at what was becoming a truly dull day at the office.

"Okay, what's the first step to the Exodus Test?"

Dakota pointed at the dry-erase board.

"*Moses Rank-Like Test for Scale Differences*."

Robin shrugged her shoulders.

"You say potato, I say 'Ten Commandments.' Just tell me: nothing is going to burst into flames, is it?"

Dakota smiled.

"Well, we do have an angry mob outside."

Theron started giggling uncontrollably as Robin cackled with laughter. Michael, however, seemed less than interested in their little interlude.

"Please Dakota, can we get on with it?"

Dakota sighed, turning her back to the group so she could write on the board.

"Okay, the first thing we need to do is divide the two groups into smaller subsets of equal size."

Theron made a move for the data set, yet froze in his tracks. He eagerly looked towards Dakota.

"Wait a moment. Is this one of those tests that will require us to use small *Samples*?"

Dakota nodded.

"Correct. The test doesn't work with larger samples."

Theron snatched the data set from the table.

"Of course it is. So, are we going to use the same data we just used; the '*General Aggression Scores*'?"

Dakota shrugged her shoulders.

"May as well, the rest of the data would be too unwieldy for this test."

Michael leaned back in his chair, his arms pensively crossed over his chest.

"And how is this information any different from the information derived from the *Permutations Test*?"

Dakota shook her head.

"The *Permutations Test for Two Independent Samples* allows you to look at the *Mean Scores* for a sample, while the *Moses Rank-Like Test for Scale Differences* allows you to look at the sample's **Dispersion**.

Robin shook her head.

"*Dispersion*? What is it with you people and the arcane multisyllabic words? Couldn't you have just said *Variance*? It means the same thing."

Theron cocked his head towards Robin, his right eyebrow arching towards the ceiling.

"Arcane?"

Robin's eyes narrowed as she met his gaze and stuck her tongue out at him. Theron erupted into a belly laugh that caught the attention of everyone in the room. Dakota methodically continued to write her equations on the dry-erase board:

$$D(X_j) = \sum_{i=1}^{k} \left(X_{ji} - \overline{X}_j\right)^2 = \text{the dispersion index for a subset of } X$$

$$D(Y_j) = \sum_{i=1}^{k} \left(Y_{ji} - \overline{Y}_j\right)^2 = \text{the dispersion index for a subset of } Y$$

j = *a particular subset*
k = *the number of observations in a subset*
m = *the number of observations in group x*
n = *the number of observations in group y*
W_x = *the sum of the x group rankings*
W_y = *the sum of the y group rankings*

Dakota dropped the dry-erase marker on the conference table and began to rifle through her bag. After finally managing to get his giggles under control, Theron looked credulously at the equation.

"So, we break this data into smaller numbers. What comes next?"

Dakota continued her quest to find her mystery item.

"Well, you need to use a **Random Numbers Table** in order to place the data into smaller groups."

Robin and Theron looked utterly confused, as Michael continued to just lean back into his chair. Robin turned to Theron, mouthing the words "*Random Numbers Table*," silently hoping he would be able to explain to her what it was. With a simple shrug of his shoulder, however, Robin quietly accepted that she would have to wait for Dakota to help clear away the confusion. Unfortunately, Dakota was still lost within the unending depths of her purse. Theron cleared his throat, speaking over the sounds of rifling papers.

"So, when do we get to know what a *Random Numbers Table is*?"

Dakota yanked out a crumpled piece of paper and began smoothing it out with her hand.

"A *Random Numbers Table* is just a table with a series of numbers on it. The objective is to get a truly *Random Sample* from your data. The hope is that by using a *Random Numbers Table*, you will be able to remove any bias that may exist in your sample."

Robin just rolled her eyes, shaking her head at her own ignorance. Dakota slid the crumpled piece of paper over to Robin and Theron.

"Okay you two, what I want you to do is take the General Aggression Score data for the two offenses sexual battery and rape by force or fear and break up this data into smaller subgroups. When we had previously examined the offenses only committed against adults, over half of the offenses were sexual battery and rape by force or fear. We have identified a *Difference* in severity between these two offenses with the *Chi-Square Goodness of Fit Test*. Now, we should look to see if there was a *Difference* in the *Variability* in terms of aggression score. Remember, each subgroup has to have at least 2 scores and the subgroups must have the same number of values."

	Group x: sexual battery (PC 243.4)	Group y: rape by force or fear (PC 261[2])
1	43	58
2	60	64
3	43	60
4	43	65
5	53	47
6	67	60
7	69	70
8	55	65

"We pulled the offense data from the large data set but we haven't yet split them into smaller groups. How do we do that again?"

Dakota nodded in an effort to encourage Robin.

"Okay, in our example, each group has 8 values. So, $m = 8$ and $n = 8$. We have to divide each of those groups into smaller subgroups, but we need to make sure each subgroup has at least 2 scores so we are able to calculate the mean. And, we need to make sure that each subgroup has the same number of scores. We can't have one subgroup with 3 scores and another with 4. They need to be equal."

Robin pointed incredulously at the dry-erase board.

"But what if we have unequal numbers in the groups?"

Dakota just grinned at her colleague, almost as if she expected this question.

"Okay, so let's say we had 9 observations in group x and 8 observations in group y. . ."

Theron's hand flew into the air, desperately hoping Dakota would notice him before this conversation left him mired down in confusion. She nodded in his direction, allowing him to speak.

"So, $m = 9$ and $n = 8$?"

Dakota flashed him one of her patented reassuring smiles.

"That's right. Now, if our $m = 9$ and $n = 8$, we could create 4 subsets of 2 for m, and 4 subsets of 2 for n."

Robin shook her head.

"Um, didn't you forget a number in m? I thought there were 9 observations, but if you break this into 4 subsets with 2 observations in it, doesn't that only leave you with 8?"

Dakota nodded.

"You got it. In our example of m, we would discard one of the observations in order to create equal subsets. It is best to use the combination that uses most of the data.

We could easily create 3 subgroups of 3 observations in the m group and use all of the data. But, if we would have created subgroups of 3 observations in a group that has 8, we can only create 2 subgroups with 2 observations left over. In that case, we would be eliminating two observations instead of just one. In the current problem, m and $n = 8$, so we will create 4 subsets of $k = 2$ for each group."

Robin appeared to have grasped the concept just as Dakota was finishing her directions for Robin and Theron.

"Now, I need you to use the *Random Numbers Table* to create those 4 subsets I mentioned."

Robin's face was a mask of confusion.

"How do we even use this thing?"

Dakota sighed as she wrote a small portion of the table on the dry-erase board.[3]

3	0	7	5	9
6	7	0	7	1
9	4	1	3	2
5	0	0	2	3
3	7	8	8	4

Theron looked incredulously at the numbers table.

"Okay, that looks mildly confusing."

Dakota pointed to the table she had sketched on the dry-erase board.

"Okay, using a Random Numbers Table is not as complicated as one might think, but there are some steps you have to keep in mind. The first thing we need to do is to determine the *Population Size* and the *Sample Size*."

Theron scratched his head.

"Okay, now what?"

Dakota closed her eyes and randomly pointed to a number on the table.

"Now you just randomly pick a point on the table to start at."

Robin shook her head.

"Well, that's certainly random."

Theron perked up.

"Hence the term '*Random Numbers Table*'."

Dakota ignored them as she continued with her explanation.

"Now we have to choose a direction. Do we want to read the table from left-to-right, right-to-left, or up-and-down?"

Robin leaned over to Theron.

"Why is she giving us options? Doesn't she know how dangerous that is?"

Michael started massaging his temples, frustration evident on his face.

"Make them shut up. Just go left-to-right."

Dakota shrugged her shoulders.

"Okay, we are going left-to-right. Now, we just have to choose a number based on the *Sample Size*."

[3] The Random Numbers Table in its entirety can be found in the appendix Table A.

Theron dropped his pencil in disgust.

"Can you vague that up a little more?"

Dakota pointed to the top of the table.

"Okay, our *Sample Size* for the *Moses Rank-Like Test for Scale Differences* included 16 individuals. Since 16 is a 2 digit number, so we are going to continue using two digit numbers to determine what is going to be included in our final *Sample Size*."

Theron pointed to the data which was being used for the data set.

"Okay, let's say we hit '0' on the *Random Numbers Table*. If we land on the '0', we would look to the right of that '0' and find a '2'. The '0', in effect, holds the tens place. So, that means we include participant number '02' in our final tally?"

Dakota smiled.

"You got it. And you just continue to arbitrarily go through the numbers table to continue gathering participants."

Robin raised her hand.

"Well, what happens if you arbitrarily choose a number that isn't in your sample? Like landing on '43', but we only have 16 people in our sample."

Dakota shrugged her shoulders.

"Well, you would just skip it and choose again. All that matters is that you do not continually start in the same place over-and-over again."

Theron and Robin both nodded in unison. Dakota just gave them a wary look.

"Got it?"

Theron and Robin both nodded hesitantly, as if they wanted to reassure themselves more than Dakota. Dakota could sense the tension in the room and flashed them the warmest smile she could muster.

"I have faith in you two."

"HA!!!!"

Michael chortled loudly at the end of the room. Dakota's skin became hot as she felt the rage boil inside of her. Theron and Robin continued to play with the *Random Numbers Table*, oblivious to the ugly scene playing out before them. Dakota leaned into her heels, forcing herself to hold her ground.

"I am sorry Dr. O'Brien, did you wish to say something?"

Michael continued to giggle, pointing to Theron and Robin.

"You really trust these two kindergarteners over here to not mess this up?"

Dakota crossed her arms over her chest, as if she were steeling herself behind some invisible coat of armor.

"I don't see you being very much help."

The smile dropped from Michael's face as he leaned forward in his chair.

"Come on, we both know that when there is a problem, I will come in and fix it."

Dakota's eyes narrowed.

"Dr. O'Brien, I believe that in order to fix something, you have to be useful at something."

Michael's smile instantly dropped. Visibly shaking with rage, the lumbering giant rose to his feet to confront the woman who called his leadership into question. Michael slowly stomped over to Dakota, his face reddened with anger. Theron and

Robin both watched as Michael inched his way towards Dakota. Theron quickly dropped his head, whispering in Robin's ear.

"Should we give them some alone time?"

Robin's eyes grew wide with shock.

"Are you kidding? This totally makes up for the mob outside trashing my car. Now be quiet so I can hear."

Both of them fell silent as Michael and Dakota stood inches apart from one another. Michael leaned his face towards Dakota, his voice erupting in a low growl.

"Maybe if you weren't such a control freak, I could help these two actually become somewhat competent in statistics."

In a flash, Dakota raised her hand and slapped him across the face; the sound of the smack echoed like thunder across the deathly silent room. Theron and Robin were stone silent, their faces pale with shock over what had just transpired. Even Michael stood before her with his face etched in confusion. Dakota then pointed to the door.

"Get out."

Dakota's voice was very low, barely a whisper. Michael turned his face to the ground, shamefully walking out of the room as if he were a dog with his tail between his legs. Dakota just stood there silently, her anger radiating off of her like heat. Theron and Robin sat quietly, stunned by what they just witnessed. Dakota pushed her hair out of her face and continued on as best she could.

"How is the *Random Numbers Table* going?"

Theron could only nod his head. Robin looked over the table and quickly filled in the last few numbers.

"How does this look? We have to select two observations for each subgroup":

	GAS	
Group *x*: sexual battery	X_{j1}	X_{j2}
Subgroup 1	43	43
Subgroup 2	53	69
Subgroup 3	43	55
Subgroup 4	60	67
Group *y*: rape by force or fear	Y_{j1}	Y_{j2}
Subgroup 1	60	65
Subgroup 2	47	58
Subgroup 3	65	64
Subgroup 4	60	70

Dakota quickly glanced over the numbers, trying to move past the rage still rolling inside her. She was actually very pleased with what they produced.

"This looks great. The next thing we do is to calculate the **Dispersion Indices**."

Theron and Robin both shifted uncomfortably in their chairs. Dakota could see the confusion etched across their faces; however, they were choosing to keep silent due to the nasty display of her personal life which just exploded before their eyes. Dakota swallowed hard, pushing away the lump in her throat, and continued to speak in the most reassuring tone she could muster.

"*Dispersion Indices* are just a way that we can assess whether or not the data we have is clustered together or spread out in a *Normal Distribution*."

Theron smiled.

"So, it looks like a bell."

Dakota felt a blush of warmth strike her cheeks, as she could imagine the rage Michael would be feeling if he were in the room. For one tiny moment, she felt a pang of guilt over how she treated Michael. The lump in her throat grew once more as she gently pulled out her marker and began to write on the dry-erase board.

"Correct. For the *Moses Test*, we determine *Dispersion Indices* by first calculating the *Mean* for each group":

$$\overline{X} = \frac{\sum_{i=1}^{k} X_{ji}}{k} = \text{the mean of a subset of } X$$

$$\overline{Y}_j = \frac{\sum_{i=1}^{k} Y_{ji}}{k} = \text{the mean of a subset of } Y$$

Robin deftly glanced over the equations and felt a question burning behind her eyes as she saw it.

"Okay, so how would that work for the data we are using?"

Dakota, without skipping a beat, quickly detailed what she was about to do.

"We calculate the mean of each subgroup of x and y":

	GAS		
Group x: sexual battery	X_{j1}	X_{j2}	$\overline{X}_j$
Subgroup 1	43	43	43
Subgroup 2	53	69	61
Subgroup 3	43	55	49
Subgroup 4	60	67	63.5
Group y: rape by force or fear	Y_{j1}	Y_{j2}	$\overline{Y}_j$
Subgroup 1	60	65	62.5
Subgroup 2	47	58	52.5
Subgroup 3	65	64	64.5
Subgroup 4	60	70	65

"So, for the first subgroup under Group x we add the first number and second number to get 86 and divide that by 2":

$$\overline{X}_1 = \frac{\sum_{i=1}^{k} X_{ji}}{k} = \frac{86}{2} = 43$$

"We continue this process for each subgroup under Group x. Then, we calculate all of the means for the subgroups under Group y":

$$\overline{Y}_1 = \frac{\sum_{i=1}^{k} Y_{ji}}{k} = \frac{125}{2} = 62.5$$

Dakota moved silently in front of the board, like a deer slipping about through the forest greenery.

"Now, we can calculate the *Dispersion Indices* for both $D(X_j)$ and $D(Y_j)$."

Dakota thought for a few moments and then started working over the numbers in her mind.

	GAS					
Group x: sexual battery	X_{j1}	X_{j2}	$\overline{X}_j$	$(X_{j1} - \overline{X}_j)^2$	$(X_{j2} - \overline{X}_j)^2$	*D* score: $D(X_j)$
Subgroup 1	43	43	43	0	0	0
Subgroup 2	53	69	61	64	64	128
Subgroup 3	43	55	49	36	36	72
Subgroup 4	60	67	63.5	12.25	12.25	24.5
Group y: rape by force or fear	Y_{j1}	Y_{j2}	$\overline{Y}_j$	$(Y_{j1} - \overline{Y}_j)^2$	$(Y_{j2} - \overline{Y}_j)^2$	*D* score: $D(Y_j)$
Subgroup 1	60	65	62.5	6.25	6.25	12.5
Subgroup 2	47	58	52.5	30.25	30.25	60.5
Subgroup 3	65	64	64.5	0.25	0.25	0.5
Subgroup 4	60	70	65	25	25	50

Theron sat in his chair, his eyes studying the chart Dakota created on the dry-erase board. Ever since Michael left the room, all of Dakota's mannerisms were much more delicate and fragile, as if she were made of glass. Still, he knew how incensed Dakota was, and chose to tread lightly.

"Okay, we have these *Dispersion Indices*. Now what?"

Dakota gestured to the numbers on the dry-erase board.

"Now we put the *Dispersion Indices* in order and assign ranks to them."

Robin just shrugged her shoulders and dutifully did as she was asked.

"Each of the *D* scores is ranked from smallest to largest":

D scores	Rank	Original group 0membership
0	1	*x*
0.5	2	*y*
12.5	3	*y*
24.5	4	*x*
50	5	*y*
60.5	6	*y*
72	7	*x*
128	8	*x*

Robin spun her legal pad around, so Dakota could examine the numbers on the page. After a few moments of careful consideration, she smiled at Robin.

"Nicely done. Now we just need to figure out the sum of the ranks, and we are almost done."

Theron suspiciously looked at Dakota out of the corner of his eye, wary that this sounded almost too easy.

"Really? That's it?"

Dakota just nodded in consent.

"That's it."

Theron and Robin slowly and methodically started adding the numbers together. After a few moments of careful calculation, they both agreed on a number.

"Using the *Rankings*, we then calculate the sum of the *Rankings*:

$$W_x = 1 + 4 + 7 + 8 = 20$$
$$W_y = 2 + 3 + 5 + 6 = 16$$

To determine significance, we must consult a *Probabilities* table for the *Wilcoxon-Mann-Whitney Rank-Sum Statistic*. Since we're using a *Two-Tailed Test*, we will double the p-value found on the table: $p = 2(0.7571) = 1.5142$."

Theron rolled his eyes.

"And that means. . ."

Dakota smiled.

"Since $p = 1.5142$ is greater than .05, we fail to *Reject the Null Hypothesis*. That means that there is no difference in *Variability* between Group x and Group y. So, we conclude that there is no difference in *Variability* of GAS between offenders charged with Sexual Battery and Rape by Force or Fear.

Dakota wrote the final number on the dry-erase board. She slowly circled the number and sat back down at the table.

"The calculations for this one were pretty straightforward because we had a smaller sample. In situations when we have larger samples, when m or $n > 10$, we can use the following equation:

$$z = \frac{W_x \pm .5 - \frac{m(N+1)}{2}}{\sqrt{\frac{mn(N+1)}{12}}}$$

"And if the rankings have ties, we use this equation":

$$z = \frac{W_x \pm .5 - \frac{m(N+1)}{2}}{\sqrt{\left[\frac{mn}{N(N-1)}\right]\left[\left(\frac{N^3-N}{12}\right) - \sum_{j=1}^{g} \frac{t_j^3 - t_j}{12}\right]}}$$

$g =$ *the number of tied groups*
$t_j =$ *number of tied ranks within the jth group*
$N = m + n$

Theron looked credulously at the equation.

"I have seen that somewhere before."

Dakota smiled at him.

"It's also used in the *Wilcoxon's T*. But we aren't there just yet.[4]"

Theron's eyes lit up like diamonds; finally something had clicked within his memory. Robin lazily swung in her chair, hoping that they could just be done with this.

"Okay, that's enough for today."

Theron and Robin both gathered their things, as Dakota stared off into space. Theron silently slid out the door, as Robin walked up to Dakota and placed a hand on her shoulder. Still lost in thought, Dakota just stared. After seeing no reaction, Robin also slid out of the conference room, leaving Dakota alone in the eerie stillness.

Chapter Summary

- Nonparametric tests that use interval scale data are introduced in this chapter.
- The two research questions raised in this chapter were:
  - For the Permutation Test for two independent samples: are two independent samples significantly different from one another?
  - For the Moses Rank-Like Test for Scale Differences: is the variability the same between two groups?
- The Permutation Test was introduced first. The Permutation Test is likened to the parametric Independent t-test. The Permutation Test does not require that the data be Normally Distributed.
- The Permutation Test involves quite extensive computations despite being fairly simple. The Permutation Test is designed and works well for small Sample Sizes.
- The Moses test was introduced second. The Moses test also utilizes interval scale data but log transforms that interval scale data into ordinal scale data.
- Dispersion indices and the random number table were mentioned regarding the Moses test.

Check Your Understanding

1. What is the main difference between the data scale used for the Permutation Test and the Moses test?
2. How do *sample sizes* effect the computation of the Permutation Test and the Moses test?

[4] *Wilcoxon's T* is discussed in detail in Chap. 10.

3. What assumptions must be met for the Permutation Test to be used and which do not need to be met?
4. Why is log transformation important for the Moses test?
5. If you were to *Reject the Null Hypothesis* in the Permutation Test and the Moses test, what is that saying about the relationship between the groups?

Chapter 10
Here's Your Sign and the Neighborhood Bowling League

Abstract The chapter opens with Dakota in an almost comatose state. The team, without Michael, must pull together to address research questions and tests of difference that deal with ranked data. Four tests are considered: Sign Test, *Wilcoxon Signed Ranks Test*, Kolmogorov-Smirnov Two-Sample Test, and the Mann-Whitney U. Each test deals with ranked data and a review of *z-scores* and normal distributions is considered. *Cumulative Relative Frequency Distributions* are addressed as they relate to the Kolmogorov-Smirnov Two-Sample Test, and a comparison is made between the nonparametric Mann-Whitney U and the parametric Independent t-test. The four research questions addressed in this chapter are as follows: (1) Is there a difference between two groups concerning two conditions? (2) When the size of the difference is important, is there a difference between a pair of data? (3) Have two independent groups been taken from the same population when the responses are cumulative? (4) Have two independent groups been taken from the same population when the responses are not cumulative? The chapter concludes with Dakota calling Michael in an effort to straighten things out between them.

There was nothing but silence.

Dakota was standing next to the dry-erase board, her body leaning into the cool smooth surface as it propped her up for support. She strained to hear anything but the chaos which had enveloped her mind. Unfortunately for her stream of consciousness, the governor was giving a speech downtown today, thus requiring the staff of the campaign office to gather around him to attend to his every want and need. It was the funny thing about politics; by choosing to be elected to a position by a vote of the democratic majority, you were given your own security detail and treated like a demigod. All the protesters who were creating a useful nuisance decided to be in attendance for the campaign stop as well; it is preferable to heckle

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someone in person over habitually pestering their underlings. The massive exodus of people resulted in a deafening silence, and it was in no danger of easing up anytime soon.

Theron and Robin both sat deathly still in their chairs, their eyes glued to the clock above Dakota's head. The hand gingerly ticked itself up to 10:30 A.M.; their meeting was to have started 90 minutes ago. Theron's hand nervously reached for the back of his neck and began kneading out the baseball-sized knot of stress what had been germinating for the past hour; his body aching to do something to break the unbearable nervous tension. Robin leaned towards him and whispered gently in his ear.

"Okay, are you getting creeped out by Miss Havisham over there?"

Theron slowly nodded his head, his eyes waiting for the comment to get some type of reaction from Dakota. After all, the Dakota he had come to admire would bristle at the thought of being compared to a lunatic spinster from a somewhat overrated Charles Dickens novel. Unfortunately, the Dakota who was standing in front of him simply remained perfectly still; oblivious to the world around her. He turned his mouth ever so slightly to respond.

"I knew she was having a rough time with this, but wow."

Robin watched in silent agony as the hands of the clock dragged itself to 10:31 A.M.

"Okay, this is getting way too 'Lifetime Movie of the Week' for me."

Robin yanked out a piece of yellow notebook paper, wadded it into a ball, and pitched it into Dakota's face. Dakota just blinked her red-rimmed eyes repeatedly, still lost in her own gloomy thoughts. Robin started waving her arms maniacally, hoping that this could somehow wrench Dakota from her stupor.

"Captain, we kind of need you back on planet earth. You know the one where our boss, the nice Governor-man, is currently being pelted by produce from angry protesters."

Theron pried his hand away from his neck and rested it on the conference table.

"Come on, you don't know that."

Robin's hand dropped into her purse and yanked out a large yellow slip of paper.

"I know they are ruining his campaign speech. Yesterday they caused $3,000 worth of damage to my car, and I am just a lackey!! What do you think they are doing to him?"

Theron looked at Robin incredulously.

"I don't believe it."

Robin handed him the paper.

"See for yourself."

Theron shook his head.

"No, I have seen what you drive around in. I honestly can't believe that your insurance company would be willing to pay this; your car can't be worth more than $250. It'd be cheaper for them to write it off as totaled."

Robin huffed as she snatched back her insurance estimate and shoved it back into her purse.

"I am so not playing with you anymore."

Theron propped his head up with his hands, continuing to stare at the clock.

"Well, I am sure Dakota would be willing to trade witty barbs with you for a little bit."

Robin pouted her lips and crossed her arms over her chest. Still, a live playmate is better than a mannequin, especially a crying mannequin at that. Realizing she had very few options left, Robin poured herself a glass of ice water, slid out of her chair, and stood next to Dakota. She put her hand on Dakota's shoulder, gently whispering in her ear.

"This is definitely not going to make you happy."

With that, Robin raised her arm over her head and dumped the icy contents over Dakota's head. Dakota yelped with shock as a few errant ice cubes made their way into her blouse. Theron jumped to his feet in shock as Robin leaned behind the dry-erase board, snatching up the roll of paper towels which was secreted away in a cupboard. Theron darted to Dakota's side and dutifully started wiping the icy contents from her forehead. His eyes were filled with a mixture of shock and horror as he chided Robin.

"What are you doing?!?! We are supposed to be doing math, not filming a music video."

Robin finished patting up the water from Dakota's blouse, only now realizing that it was white. She felt her cheeks flush with red as Theron desperately averted his gaze towards the window.

"Come on, I had to do something. On the plus side, if we were in New Orleans, Dakota would be continuously pelted with Mardi Gras beads."

With that, Dakota roared with laughter. The utter ridiculousness of the whole situation was now too humorous to be ignored. Dakota stepped away from the dry-erase board, clutching her hand to her chest in fits of laughter. Robin eased away from her hysterical colleague and slowly backed away towards Theron. Theron carefully pushed himself away from the conference table, whispering to his terrified playmate.

"I think you broke her."

Robin's eyes grew wide as she shook her head.

"Now I am scared."

Dakota flipped her hair back, shaking some stray ice cube from her hair. She reached to her purse, pulled out a dry-erase marker, and began to write on the board.

"Now, what is on the agenda for today?"

Robin just shook her head.

"This just turned into 'One flew over the cuckoo's nest'."

Dakota just gazed at the board, oblivious to Robin's comment.

"Let's see. Yesterday we discussed difference tests with only one group of data, so I believe that today we should examine statistical procedures examining the difference between two groups of data. We looked at two variables yesterday for one group where we thought one of the variables had some influence over the other variable showing a difference in one direction or another. That's great; however, it doesn't tell us whether if there is a directional difference between two groups of offenders which we need to know because comparing offenders to themselves will only give us so much information. We need to compare one group of offenders to another group of offenders to see if there is something that works for one group but is not working for the other group. Something the governor can take to his constituency."

Robin leaned forward, like a gazelle reaching out for water while a predator is watching. Her movement was painfully slow as she pulled out her chair and eased herself into it, knowing that one fast move would likely send Dakota screaming from the room. Theron watched Robin through the corner of his eye and dutifully followed suit. Dakota seemed to be back to her old self, and he was able to find some comfort in that. Dakota pulled out a manila folder from her purse and carefully laid out the data set on the conference table. After brooding over it for a few moments, she nodded to the data set and returned her attention to the dry-erase board.

"Okay, I think we should focus primarily on tests which use *Ranked Data*. We have used many tests where we rank the data but not when looking at tests that examine the difference between two groups."

Theron glanced at the data set, feeling a knot form in his stomach that matched the one in his neck.

"Not the 'Level of Meanness' data. Hasn't that gotten the Governor into enough trouble over the past few days?"

Dakota shook her head.

"Remember, you can take *Interval Data* and transform it into *Ordinal Data*. All you need to do is take your *Interval Data* and rank it."

Robin leaned back in her chair, the tension in her body melting away as the Dakota she had come to know and respect reasserted herself once more, even if she was a little wetter.

"So, where do we start?"

Dakota flipped over the dry-erase board, revealing all of the research questions which had been generated back when they started working together. Dakota's eyes narrowed as she examined each question in turn; some had been answered long ago, while others still needed to be addressed. One such question stood out, causing Dakota to circle it on the board:

In regard to sentence length, is there a difference between those charged with rape and those charged with lewd acts?

Theron dutifully jotted the question down, while Robin just stared at the question with a blank expression.

"Okay, I am going to go with 'yes'."

Theron shot her a sideways glance as he was hurriedly writing everything down.

"I think she means for us to use the data to answer the question."

Robin shook her head.

"No."

Theron dropped his pen and looked at her.

"Did you mean 'no' for us using the data to answer the question, or 'no' to whether or not there is a difference between those charged with rape and those charged with lewd acts?"

Robin opened her mouth to respond, but was cut short by Dakota snapping her fingers in an effort to derail yet another tangent. Dakota pointed to the data sheet.

"I would say that the best procedure for us to use would be the **Sign Test**."

Theron arched his left eyebrow.

"As in 'stop sign'?"

Dakota shrugged her shoulders.

"Same word, different meaning. The *Sign Test* is this interesting statistical procedure that examines the direction of the difference between two different *Continuous* sets of data."

Theron's mouth dropped open a little bit, confused as to the minor explanation he had just been given.

"So, the *Sign Test* is just going to tell you whether or not these groups are different?"

Dakota nodded.

"That's almost but not entirely correct. It is true that it will tell us if there is a difference, but it will also tell us if there is a difference in a particular direction. So, it tells us whether X >Y, not just whether X is equal to Y."

Robin shook her head.

"I call shenanigans. It sounds way too easy. What's the catch?"

Dakota just shrugged her shoulders, a smile peeking in at the corners of her mouth.

"No catch. All the *Sign Test* requires is that the *Variables* you are using be **Paired**, *Random,* and *Continuous*. Then, it will just tell you the direction of the difference, or if a difference exists at all."

Theron dropped his pen.

"Okay, so how do we do it?"

Dakota deftly jotted something onto the dry-erase board:

$$P[X_i > Y_i] = P[X_i < Y_i] = \frac{1}{2}$$

Robin looked bewildered at the equation on the board. It wasn't so much that the equation was confusing; in fact, she understood it beautifully. That fact alone was rather disheartening.

"I think I understand this. P equals the probability of one variable being greater than the other (X > Y) which we can estimate as one half. That cannot be all there is to it."

Dakota nodded.

"That's it."

Theron just shook his head.

"So, all we are doing is noting what the direction of the difference is, and counting..."

Dakota smiled.

"Well, if the first group is higher than the second group, you assign it a plus sign. If the second group is higher than the first group, you assign it a minus sign."

Theron arched his eyebrow.

"And if there is no difference?"

Robin's hand shot up in the air, anxiously trying to impress teacher by knowing the appropriate answer. Dakota silently rolled her eyes and pointed at the hyperactive young woman about to bounce out of her chair. Robin jumped to her feet.

"Let me guess, you give it a zero."

Dakota nodded in Robin's direction, and Robin triumphantly lifted her arms over her head. It was as if she had just won an Olympic gold medal. Theron gingerly wrapped his hand around her arm and guided her back into her chair.

"Thank you for reenacting 'Rocky' for me."

Robin just shot him a grin worthy of the Cheshire Cat.

"There is nothing you can say that is going to rain on this parade."

Dakota shook her head in amusement.

"And now all you do is count up the number of plus and minus signs, ignoring any ties. Then, we use this formula to compare it to a **Z-Distribution**, by turning the number into a *z-score*."

Robin's face contorted into an almost quizzical expression.

"Either this is the most depressing moment of déjà vu I have ever had, or we have talked about this before."

Dakota nodded.

"We discussed it briefly when we were discussing whether or not we had *Normal Data*."

Robin held up her hand, trying to prevent Dakota's train of thought from proceeding any further. Robin shook her head vigorously, trying to wipe her mind clean as if it were an Etch A Sketch.

"Please, I so cannot hear that conversation again."

Theron leaned forward, his curiosity clearly getting the better of him.

"Okay, so how would the *Z-Distribution* play a role here? I mean, the *Sign Test* isn't exactly looking at Normality."

Dakota's face drew itself into a rather pensive expression.

"You are correct. However, you can do much more with a *z-score* than figure out if you have a *Normal Distribution*. With the *Z-Distribution* component of the *Sign Test*, you can actually see how many *Standard Deviations* you are from the *Measures of Central Tendency*

Robin grinned.

"I get it. So, the higher the number is on the *Sign Test*, the more likely your result is to be *Statistically Significant*."

Dakota nodded.

"That's correct. If your *z-score* is higher than 1.96, you are *Statistically Significant*."

Theron arched his eyebrow.

"1.96?"

Dakota flipped to her chart of the *Normal Distribution*.

"When you go to the table which has the percentage equivalents between the *Mean* and your *z-score*, a score of 1.96 will give you 47.5%."

Robin's jaw dropped open.

"I get it. Given that 47.5% is taking into account one half of the *Bell-Shaped Curve*, we can logically extend it to the other half which means it accounts for 95% of your distribution. So anything outside of that 95% would be in the area of *Statistical Significance*."

Dakota nodded her congratulations to Robin, who was now beaming with pride. Theron just turned to another page in his notebook, patiently waiting for Dakota to continue.

"Okay, so how do we get *z-scores* with the *Sign Test*?"

Dakota quickly traced out the equation on the dry-erase board and added some explanatory notations:

$$Z = \frac{2x \pm 1 - N}{\sqrt{N}}$$

$N =$ *number of pairs that are different*
$x =$ *number of negative signs*

Robin bounced up yet again and yanked the data sheet in her direction. Theron calmly pried it from her hands so both of them could work on this together. After a few moments, they were able to generate two columns of data: one column with the sentences for those offenders charged with lewd acts (288[A] and 288[C] only) and one column for those offenders charged with other offenses, with a third column showing the direction of the difference.

"When the direction is greater than, that is, when sentences for lewd acts are greater than sentences for other offenses, we give the pair a plus (+). When the direction is less than other offenses, we give the pair a minus (−). When there is no difference or they get exactly the same sentence, we give the pair a zero (0)."

	Sentences for Lewd acts	Sentences for other offenses	Direction of difference	Sign
1	8	7	>	+
2	3	1	>	+
3	1	12	<	−
4	3	5	<	−
5	3	1	>	+
6	3	4	<	−
7	6	1	>	+
8	3	1	>	+
9	3	2	>	+
10	1	1	=	0
11	3	8	<	−
12	3	5	<	−
13	1	5	<	−
14	6	5	>	+
15	3	9	<	−
16	3	2	>	+
17	3	5	<	−

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	Sentences for Lewd acts	Sentences for other offenses	Direction of difference	Sign
18	3	5	<	−
19	8	1	>	+
20	3	1	>	+
21	3	5	<	−
22	1	2	<	−
23	3	3	=	0
24	3	1	>	+
25	6	3	>	+
26	3	6	<	−
27	3	2	>	+
28	3	3	=	0
29	3	6	<	−
30	3	1	>	+
31	3	3	=	0
32	6	1	>	+
33	1	1	=	0
34	3	1	>	+
35	1	7	<	−
36	2	3	<	−
37	3	1	>	+
38	3	3	=	0
39	3	5	<	−
40	6	2	>	+
41	3	2	>	+
42	3	16	<	−
43	6	2	>	+
44	1	1	=	0
45	3	1	>	+
46	3	3	=	0
47	1	5	<	−
48	3	6	<	−
49	3	3	=	0
50	3	1	>	+

"Now, we count the number of +'s and −'s."

Lewd acts sentence length	Number
Greater than other offenses	22
Less than other offenses	19
No difference	9

"N is the number of pairs that are different (plus or minus). In this case, $N = 41$."

Dakota glided behind Theron, subtly glancing over his shoulder to check his work. Aware of her presence, Theron leaned away from her so she could better see the equation. Dakota smiled down at the both of them.

"All right, so now we just turn this into a *z-score*."

Dakota silently ran back to the dry-erase board and began jotting down the equation.

"We use +1 when $x < \frac{N}{2}$ and -1 when $x > \frac{N}{2}$. Since $19 < 20.5$, we calculate the following equation":

$$z = \frac{2x + 1 - N}{\sqrt{N}} = \frac{2(19) + 1 - 41}{\sqrt{41}} = \frac{-2}{6.4031} = -0.3123$$

"With $z = -0.3123$, the corresponding p-value for a two-tailed test is 0.7566. Since $p = 0.7566$ is greater than $\alpha = 0.05$, we fail to *Reject the Null Hypothesis* (i.e. there is no difference in sentencing between the groups). We conclude that the sentences for offenders charged with lewd acts and other offenses are similar. This means that an offender committing a lewd sexual act against a child is not likely to get a longer sentence than an offender committing any other kind of sexual crime. Given the extensive attention to legislation designed to protect children, I think the public would be under the impression that these offenders are receiving longer sentences. However, at least for our data, this does not appear to be the case."

Dakota quickly jotted down the final answer in her notebook and then looked at her colleagues with a deftly timed sideways glance.

"Ready to move on?"

Both Theron and Robin enthusiastically nodded their heads; both of them clearly energized by their recent mastery of the *Sign Test*. Dakota looked towards their list of questions on the opposite side of the dry-erase board and moved to another one on the list.

"Okay, how about we do something a little different? I think we should answer the question 'is the size of the difference significant between Rater 1 and Rater 2 on the "Level of Meanness" scale.' What do you think? This speaks to the consistency of the instrument and how independent raters are assessing offenders in order to make treatment and supervision decisions."

With that one phrase, Dakota managed to instantaneously deflate all of their nascent enthusiasm. Theron's skin bleached to a ghostly shade of pale, while Robin's jaw almost unhinged from her mouth. Dakota watched as Robin tried to collect her thoughts.

"Are you mad? What about my car?"

Theron's pallid face suddenly bore a quizzical expression as he roused himself to turn in her direction.

"Your car has issues with the 'Level of Meanness' scales?"

Robin dramatically pointed to the window.

"They have a problem with it."

Theron just shook his head in silence. He understood that she was trying to make a point, but it probably would have been more effective had the protesters actually been there today. Dakota's affect fell flat as she called over to Robin.

"Are you through pretending to be Gloria Swanson so that we can get back to work?"

Robin tried to maintain her indignant façade, but the lines around her mouth suggested that she was actually rather proud of Dakota's pop culture reference. So, she quietly sat down and waited for Dakota to continue. Dakota spread her arms out onto the table, using them to support the weight of her body as she hung her head toward the floor.

"Look, I know that this 'Level of Meanness' scale has essentially ruined our lives. In fact, it seems like every time we mention them it is as if someone is about to crack open Pandora's Box. But, we all agreed that there may be information from these scales which we can use to do our jobs such as exactly how differently do the clinicians rate offenders on meanness. So, from here on out, I propose that we just treat this data as innocuously as possible."

Theron smirked.

"But it's so much more fun acknowledging them as radioactive."

Dakota found the energy to lift her head and shoot Theron a rather angry glare. He lifted his hands in a gesture of surrender, flashing his most charmingly boyish smile in Dakota's direction.

"Okay, you got it boss."

Dakota then turned her attention to Robin.

"What about you?"

Robin just rolled her eyes.

"Let's just get this over with. So why do you propose we examine 'rater 1' and 'rater 2'? I mean, why are we ignoring 'rater 3', or for that matter why aren't we looking at all three of them together?"

Dakota thought for a moment. Even though Robin's annoyance was palpable, she did bring up a very good point; why should they only focus on two of the three raters? Dakota placed her hands on her hips and tried to explain this as best she could.

"Hmm, that is an excellent point. Well, one of the reasons I would want to examine only 'rater 1' and 'rater 2' is to see if these two have any similarities whatsoever. That way, we can more accurately determine where the discrepancies are coming from."

Theron leaned forward, very interested in what Dakota was saying.

"I get it. If we find out that 'rater 1' and 'rater 2' are in agreement, then we will know that 'rater 3' could possibly be the odd person out. However, if 'rater 1' and 'rater 2' are really disagreeing on everything, then we would want to see whom 'rater 3' agreed with to determine which one of the 'raters' was differing from the others."

Dakota winked in his direction, acknowledging that his assertion was correct. Robin silently accepted that answer.

"Okay, so which test do you propose we use?"

Dakota just smiled at the dry-erase board.

"I say we use the *Wilcoxon Signed Ranks Test*."

Theron suddenly looked disheartened.

"I don't suppose that is anything similar to the *Wilcoxon T*?"

Dakota nodded.

"Same test, just a different name."

With that, Theron's whole body seemed to unclench.

"Oh thank heavens, a test I know about."

Robin seemed annoyed by this and just glared at him.

"Okay smart guy, so explain it to me."

Dakota just stood there, mildly interested at what type of explanation he would give. Theron nervously cleared his throat and looked down toward his paper.

"Well, I remember that *Wilcoxon T* is used to determine differences between two groups in a **Repeated-Measures Design**."

Robin quizzically looked at him.

"*Repeated-Measures*?"

Theron nodded.

"It's where you give people the same assessment over a period of time to see if your treatment is impacting their scores or you give several different assessments to the same person at the same time. You know, like a **Longitudinal Study**. It might be like having the same offender take a testosterone test and a general aggression test—the key is that the same person is taking the tests, and the results are paired for the individual not with the individual and someone else."

Robin shook her head.

"Just one problem. In our happy-fun data set, the people rating the 'Level of Meanness' scales are different people. So, how does this test apply?"

Theron nervously looked at Dakota, his eyes pleading with her to help him out. She just smiled.

"Well, you are almost right. You can also use *Wilcoxon T* to examine **Matched Pairs**."

Robin's face was still a mask of credulity.

"Like the 'Matching Game'?"

Dakota just nodded.

"Same idea. You are using two separate people, but their demographics are so similar that they can be assumed to perform the same in various situations. So, instead of spending months exposing the same person to various experimental conditions; in matched pairs you pair up your participants and expose one person to one condition, and the other person to a different condition. Since they are so similar, it is expected that these people will perform roughly the same way."

Robin nodded.

"Just like our Raters."

Dakota smiled.

"Exactly."

Robin turned her attention back to Theron.

"So, what do we do first?"

Theron thought for a moment. Then, he very hesitantly pointed to the scores for Rater 1 in the "Level of Meanness" scale.

"Well, the first thing we need to do is to get a signed difference score (d_i) between Rater 1 and Rater 2."

Robin shook her head.

"So, we need to subtract."

Theron nodded.

"Yep."

	Rater 1	Rater 2	*d*
1	4	5	−1
2	4	4	0
3	2	3	−1
4	1	1	0
5	2	2	0
6	1	1	0
7	4	3	1
8	2	2	0
9	3	3	0
10	3	3	0
11	4	4	0
12	4	4	0
13	2	2	0
14	2	2	0
15	3	3	0
16	5	5	0
17	1	1	0
18	1	2	−1
19	2	2	0
20	4	4	0
21	2	2	0
22	1	2	−1
23	5	5	0
24	1	1	0
25	5	4	1
26	4	4	0
27	2	1	1
28	1	1	0
29	5	5	0
30	3	3	0
31	4	5	−1
32	3	2	1
33	3	3	0
34	1	3	−2
35	5	5	0
36	3	3	0
37	4	5	−1
38	2	2	0
39	3	1	2
40	4	4	0
41	3	3	0
42	2	2	0
43	2	2	0
44	1	1	0
45	2	2	0
46	2	2	0
47	4	5	−1

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	Rater 1	Rater 2	*d*
48	2	2	0
49	3	2	1
50	1	1	0
51	1	1	0
52	3	3	0
53	4	4	0
54	2	2	0
55	5	5	0
56	2	2	0
57	3	3	0
58	1	2	−1
59	3	3	0
60	1	1	0
61	2	2	0
62	3	3	0
63	1	1	0
64	2	2	0
65	3	3	0
66	4	4	0
67	1	2	−1
68	1	1	0
69	1	1	0
70	4	4	0
71	5	5	0
72	3	2	1
73	3	3	0
74	4	4	0
75	3	3	0
76	3	3	0
77	2	2	0
78	4	4	0
79	4	5	−1
80	3	3	0
81	3	4	−1
82	3	3	0
83	3	3	0
84	3	3	0
85	5	5	0
86	4	3	1
87	3	3	0
88	2	3	−1
89	1	2	−1
90	1	1	0
91	3	3	0
92	5	5	0
93	5	5	0

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	Rater 1	Rater 2	d
94	2	2	0
95	3	3	0
96	1	1	0
97	3	3	0
98	2	2	0
99	3	3	0
100	2	2	0

Once all the difference scores were tallied, Robin once again turned to Theron for guidance.

"That wasn't so bad. Now what?"

Theron thought for a moment, his eyes turned up to the ceiling searching for answers. Dakota was prepared to step in and help him out, but secretly she hoped that her intervention would not be needed. Theron's eyes grew wide, and he snapped his fingers in a triumph of human memory.

"Next, we rank each of the pairs based upon d_i. Importantly, only pairs with a difference are given a rank. If $d_i = 0$, it is removed from the analysis. In our sample, $N = 22$.

Also important is that the d_i's are ranked regardless of the sign (positive or negative). A -1 is treated the same way as a +1. If there are ties, they are given the same rank.

So, if 3 people had a difference of 1, we couldn't give a ranking of first, second, and third, as they're all the same. What we would have to do is this – first put the ranks, first, second, and third, and then divide by the total number of ties. So, with our 3 tied people, we would have 1 + 2 + 3/3 = 6/3 = 2; so our 3 people would get a rank of '2'. Now, if we apply the same logic to our 20 pairs (13 negative and 7 positive) who all showed a difference of 1, we'd get a rank of:

$$= \frac{1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17+18+19+20}{20}$$

$$= 10.5$$

So for each "1" that appears, it will receive a rank of 10.5. The next possible ranking would then start with 21 (unless there were ties, as in the case with our data). Here are the pairs where there was a difference, either positive or negative."

	Rater 1	Rater 2	d	Rank of d	Rank with less frequent sign
1	4	5	−1	−10.5	
3	2	3	−1	−10.5	
7	4	3	1	10.5	10.5
18	1	2	−1	−10.5	
22	1	2	−1	−10.5	
25	5	4	1	10.5	10.5

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	Rater 1	Rater 2	d	Rank of d	Rank with less frequent sign
27	2	1	1	10.5	10.5
31	4	5	−1	−10.5	
32	3	2	1	10.5	10.5
34	1	3	−2	−21.5	
37	4	5	−1	−10.5	
39	3	1	2	21.5	21.5
47	4	5	−1	−10.5	
49	3	2	1	10.5	10.5
58	1	2	−1	−10.5	
67	1	2	−1	−10.5	
72	3	2	1	10.5	10.5
79	4	5	−1	−10.5	
81	3	4	−1	−10.5	
86	4	3	1	10.5	10.5
88	2	3	−1	−10.5	
89	1	2	−1	−10.5	

"After we determine the rankings, each ranking is given the same sign as it's the difference for the pair, namely *d*. If the pair showed a negative difference, the *d* would be negative and the ranking would get a negative sign. So, if $d = -1$, then the ranking $= -10.5$. We then calculate *T*, which is the smaller sum of like-signed ranks. In this case, as there are 14 pairs that showed a negative difference, and 8 pairs that showed a positive difference, the sum of the positive rankings is smaller than the sum of the negative rankings":

$$T = 10.5 + 10.5 + 10.5 + 10.5 + 10.5 + 10.5 + 10.5 + 21.5 = 95$$

Once that was completed, Robin dropped her pencil and folded her arms across her chest.

"Please tell me I am almost done with this."

Dakota smiled, turning toward the dry-erase board.

"Almost. We just have a few simple things to do and then all you need to do is plug the numbers into this formula":

$$z = \frac{T^{+} - \frac{N(N+1)}{4}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}}$$

N = number of matched pairs – the number of matched pairs where $d_i = 0$
T = the smaller sum of like signed ranks (can be negative or positive ranks)

Theron's eyes grew large with confusion at the sight of the foreign equation on the dry-erase board. Dakota could feel his confusion radiating from across the room, as he stammered out a question.

"W… wait a second. I don't recognize that."

Robin slouched forward, looking at her colleague as if he were a deflated balloon. Dakota quickly snapped the open dry-erase marker.

"It's all right Theron. This is the equation we use when our *Sample Size* happens to be larger than 15. I'll bet the equation you remember is the one for small samples":

$$\sum T^{+}$$

Theron's body eased as she wrote the formula on the board. It was as if he was a small child who had been given a security blanket and could now sleep through the night clutching onto his reassurance. Dakota pointed to the two different formulas.

"Essentially, both of these formulas do the exact same thing. The only difference is in how you determine the threshold for *Statistical Significance*. For small samples you look at the table specifically designed for the *Wilcoxon T*, while for large samples you look at the *Z-Table*."

Robin felt a question brewing on her lips and was hoping that Theron would be wondering the same thing. One quick glance and she knew he hadn't quite regained his composure just yet. So, she meekly stuck up her hand, signaling to Dakota that she had a question. Dakota nodded her head, allowing Robin to speak.

"Okay, but wouldn't you need to do something about tied difference scores in the large sample version of *Wilcoxon T*? I mean, since you are using the *Z-Distribution*, wouldn't any tied difference scores throw the *Variance* off?"

Robin's question hit the room like thunder. Dakota was stunned, while Theron was outright flabbergasted by what Robin just said. Robin could feel their eyes studying her and suddenly became very defensive.

"What? I can't ask a simple question."

Dakota quickly regained her composure and went back to the dry-erase board.

"No, it is an excellent question. It is true that because you are using the Z-Distribution, any ties in the difference scores will seriously impact the overall *Variance*, and that is something which needs to be accounted for in your analysis. Fortunately, there is a way to do just that."

Dakota quickly drew a third equation on the dry-erase board:

$$\sigma^2_{T^+} = \frac{N(N+1)(2N+1)}{24} - \frac{1}{2}\sum_{j=1}^{g} t_j\left(t_j - 1\right)\left(t_j + 1\right)$$

Both Robin and Theron dutifully copied down the equation, as Dakota quietly explained some of the main components of it to her colleagues.

"All that this equation does is reduce the overall spread of the distribution, or *Variance*, of your *Wilcoxon T* based upon the number of tied difference scores. Remember, this needs to be taken into account; if not, you could find *Statistical Significance* when in fact there is none."

Robin just shook her head.

"You mean you would get Type I diabetes?"

Theron shook his head.

"*Type I Error*. You would get *Type I Error*.[1]"

Dakota just shook her head and continued on with her explanation.

"As I was saying, this result occurs because the distribution has been changed in such a way that the relationship between the *Mean* and *Standard Deviation* allow an increased or decreased potential for *Significance* by chance alone. The overall increase or decrease is going to depend upon your distribution."

Robin arched her brow as she placed her pencil onto the table.

"I feel like a broken record, but is that really all there is to it?"

Dakota just pointed to the formula.

"Basically, you take that answer and compare it to 1.96. If your calculated value is higher, then your two groups are significantly different."

Robin suddenly giggled with joy.

"1.96?!?!? So, you are using the same threshold of *Significance* as the *Z-Distribution*?"

Dakota pointed her out in a quick motion of her arm, gently acknowledging that her answer was correct.

"You got it."

With a few deft motions of her wrist, Robin quickly entered the equation into her calculator. Once it was all finished, she circled her final answer and handed the paper off to Dakota.

"Now, we can calculate z:

$$z = \frac{T - \frac{N(N+1)}{4}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}} = \frac{95 - \frac{22(22+1)}{4}}{\sqrt{\frac{22(22+1)(2[22]+1)}{24}}} = \frac{-31.5}{30.8018} = -1.0227$$

With $z = -1.0227$, the corresponding p-value for a two-tailed test is 0.30773. Since $p = 0.30773$ is greater than $\alpha = 0.05$, we fail to *Reject the Null Hypothesis* (i.e. there is no difference between the raters). We conclude that the Raters 1 and 2 gave similar Level of Meanness Scores."

Dakota looked at Robin, a beam of pride erupting from her angelic face.

"See, I told you it wouldn't be so bad."

Theron leaned back into his chair, his hands cupped behind his head as a triumphant smile crossed his face. For the first time since he started working on this campaign, he actually felt like he made a generous contribution to the overall group effort. Dakota glanced to the dry-erase board, trying to see which research questions had only two groups of data which could be considered ordinal. She thought for a moment and then rose out of her chair and scanned over the data set.

"I wonder. . ."

[1] Type I and Type II Errors are discussed in detail on page 79 in Chap. 4.

Dakota's voice just trailed off into the distance, as both Theron and Robin pricked up their ears in curiosity. Dakota's body suddenly showed her pensive state as her mind played with the *Data Set*. Suddenly aware that others were watching her, Dakota lifted her head to meet their collective gaze.

"I was wondering about whether or not there is a difference in the way the Raters for the 'Level of Meanness' scale rate based on race or ethnicity? Again, this speaks to consistency among raters and whether or not they are being influenced by other factors when assessing offenders."

Theron leaned in close to the data set, hoping to see what it was which caught Dakota's eye. Unfortunately, he was not as lucky. Robin just leaned forward, resting her head on the conference table.

"I am going to go with 'yes' on that one."

Dakota just shook her head as Theron continued to be lost in a veritable sea of numbers. He finally accepted defeat and sat back into his chair, absentmindedly preparing his notebook for another guerilla session of nonparametric statistics.

"Okay, which statistical procedure are we running?"

Dakota's face was still lost in thought, her brain attempting to formulate the best test for them to use. Suddenly, a moment of insight struck her like a bolt of lightning.

"The **Kolmogorov-Smirnov Two-Sample Test**."

Robin's head shot up from the table.

"Okay, now I know she has lost her mind. We already did that test?"

Dakota smiled as she lightly waved off Robin's protestations with a flick of her wrist.

"Not quite. The test we previously ran was the *Kolmogorov-Smirnov One-Sample Test*, it's not the same thing as the *Kolmogorov-Smirnov Two-Sample Test*."

Robin tensed up.

"Kremlin-Vodka is Kremlin-Vodka. See, I even remember the idiotic nickname we gave it the last time around."

Theron reassuringly patted Robin's arm with his hand, hoping that it would calm her down. Robin met his comforting reassurance by sticking out her lower lip and pouting like a petulant child. Theron just rolled his eyes and turned his attention back to the other adult in the room.

"Okay, so what is the difference between the Kremlin-Vodka One-Shot Test and the Kremlin-Vodka Two-Shot Test?"

Dakota kept her reaction in check, trying to accept that neither one of them was ever going to remember the proper name for the procedure. Still, Dakota was pleasantly surprised that they remembered a little something about the test, even if it was just the nickname.

"The *Kolmogorov-Smirnov One-Sample Test* was looking at whether or not there was a significant difference between the Observed Frequency and the Expected Frequency of a particular response. The test is amazingly powerful for small samples, but does decrease slightly with larger samples."

Theron nodded his head slowly, trying to absorb all that Dakota was saying.

"Which is why we used it for seeing what type of distribution we had?"

Dakota nodded her head.

"Exactly. But the *Kolmogorov-Smirnov Two-Sample Test* will allow you to determine if two *Independent Groups* were taken from the same *Population*."

Theron leaned in towards Robin.

"See, they are different tests."

Robin just stuck out her tongue at him. Dakota lifted herself out of her chair, subtly rolling her eyes as she moved towards the dry-erase board. Lifting up the eraser, she cleared off the refuse from the earlier statistical endeavors. Theron looked over his notepad, a question brewing forth in his mind.

"So, what kind of *Difference* are we looking for?"

Robin's tongue instantly retracted into her mouth.

"What are you talking about? Is there such a thing as different *Difference*?"

Dakota's face contorted at Robin's painful alliteration, yet she fought her reaction to answer Theron's question.

"Oh, it is a good question. The **Two-Tailed** version of the *Kolmogorov-Smirnov Two-Sample Test* is remarkably sensitive; so much so that it can detect a larger or smaller difference between the two groups."

Theron's hand began rubbing his jaw.

"You mean, it could see if there were differences in *Measures of Central Tendency* or Measures of *Dispersion*?"

Robin rolled her eyes, muttering silently to herself.

"*Variance*. We are looking at how spread out or clustered together the data are. Why can't you just say *Variance*?"

Dakota's eyes shot Robin a disapproving look as she patiently answered Theron's question.

"That is correct. In fact, the test is so sensitive it can even look at such things as *Skew*; you know, when the distribution doesn't look like a dromedary but rather the distribution is shifted to the left or right."

Theron's eyes widened as he whistled in disbelief. Robin began scribbling out a very odd picture of what looked like a dragon with two tails. Dakota noticed this out of the corner of her eye and leaned over to her artistically inclined colleague.

"Feeling the need to color?"

Robin grinned as she shook her head.

"Nope. Just waiting for someone in the room to tell me what you all mean by *Two-Tails*."

Dakota, feeling the need to keep her colleague somewhat engaged, found herself attempting an explanation.

"Ok, you can think about two tails like the two gutters on a bowling lane. If you are fortunate enough to keep your bowling ball on the lane and avoiding the gutters, you will most likely score some points. If, however, you enjoy seeing your bowling ball follow an unwavering straight line, then you are most likely not going to score any points. In similar kind of terms, if you have avoided the tails with your **Test Statistic** then you are failing to reject your *Null Hypothesis*. On the other hand, gutter balls or a *Test Statistic* that falls within the tails means that the *Null Hypothesis* is rejected resulting in essentially accepting the alternative hypothesis."

Theron just tilted his head towards Robin, who had abandoned her dragon and was now drawing an oversized bowling pin. He let out a deep, throaty laugh.

"She gets it. However, just to continue the analogy, you want gutter balls in bowling. With our tests, we do not necessarily want to *Reject the Null Hypothesis* which is the same as getting a gutter ball. Moving out of the bowling alley and back to the conference room, the real issue is about deciding something is, or is not, different. The two tails give you boundaries for deciding about whether two things are different from each other. Things inside the tails are when two things might look different but actually aren't. Things outside the tails are when two things look different and actually are different."

Dakota just took her usual place by the dry-erase board.

"True but every analogy breaks down at some point. Anyway, the **One-Tailed** version of the *Kolmogorov-Smirnov Two-Sample Test* is used to help you determine if the randomness of the data from the population you drew your sample from is greater than what would be observed from the population."

Theron arched his eyebrow as he continued to write on his notepad.

"I don't suppose you could explain that in English."

Dakota whisked some errant strands of hair out of her face with her wrist as she thought of how to best explain this.

"Basically, you want to determine if the overall scores from one of your groups are greater than the other group. If the overall scores from one of the two groups are greater than the other group, this means they're not the same, or not from the same population. So, the comparison would fall over the boundary, in the tail region."

Theron grinned as he continued to jot down notes.

"So, all *One-Tailed* tests do is tell you only about one tail of the curve, if one group is greater than another? And, if you wanted to know if your group was less than another, you'd be looking at another *One-Tailed* test, yes?"

Dakota winked at him.

"You got it. Ready to continue?"

Theron nodded, while Robin just shrugged her shoulders. She knew that Dakota would drag her into a statistical Wonderland, regardless of whether or not she wanted to go. So, Robin just resigned herself to her fate.

"Okay, what now?"

Dakota moved away from her trusty dry-erase board and quickly glanced at the data set.

"Well, just as with the *Wilcoxon T*, there are different variations to the *Kolmogorov-Smirnov Two-Sample Test* based upon your overall *Sample Size*. However, this time your *Sample Size* must be over 25 before there is a problem; and we definitely have a group larger than 25."

Theron quietly paused while jotting things down in his notebook.

"So, what is the difference?"

Dakota gingerly snapped the cap back onto the dry-erase marker.

"Really, there is only one. The only thing which is impacted by the *Sample Size* is how you determine overall *Statistical Significance*. It determines how you assess whether the differences are *Significant* (in the tail), or not *Significant* (not in the tail)."

Robin just rolled her eyes.

"Are you kidding? There are different *Critical Values* tables for the Kremlin-Vodka Test depending upon your *Sample Size*?"

Dakota shook her head.

"There are different *Critical Values* tables and formulas for the *Kolmogorov-Smirnov Two-Sample Test* based on *Sample Size* and whether or not you are using a *One-Tailed Test* or a *Two-Tailed Test*."

Dakota took a few moments to put the equations on the dry-erase board for the team to compare side by side.

"Now, you can see that the equations are basically the same. The difference in terms of the equation is that the *Two-Tailed* test equation has absolute value symbols on the outside instead of the brackets in the *One-Tailed* test equation."

Dakota quickly jotted down some more information onto the dry-erase board.

One tailed:

$$D = max\left[Sn_1(x) - Sn_2\left(x\right)\right]$$
$$\text{Critical value} = x^2 = 4D^2\frac{n_1 n_2}{n_1 + n_2}$$

Two tailed:

$$D = max\left|Sn_1(x) - Sn_2\left(x\right)\right|$$
$$\text{Critical value}\,(\alpha = 0.05) = 1.36\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

D = the largest value (or absolute value) of Sn_1 *(x) –* Sn_2 *(x); the maximum deviation*
Sn_1 *(x) = Cumulative Relative Frequency Distribution for* n_1 *observations*
n_1 *= number of observations in the first sample*
n_2 *= number of observations in the second sample*

Robin just dropped her head in disgust.

"These people have WAY too much free time on their hands."

Theron leaned towards Robin.

"Uh, I am pretty sure the guys who invented the test are dead by now."

Robin just buried her face into her hands.

"They still have *way* too much free time."

Dakota just shook her head and popped the top off her dry-erase marker once more.

"Okay. So, we know that we are using the *Two-Tailed* version of the *Kolmogorov-Smirnov Two-Sample Test* as we want to assess for both ends of the tail, in other words, both greater and less than differences; and that our *Sample Size* is well over

25 people. Now, the next step is to arrange both groups in a ***Cumulative Relative Frequency Distribution***."

Robin's face brightened up.

"*Cumulative Relative Frequency Distribution*? That sounds boringly familiar."

Dakota nodded.

Yep, it's the same idea as with the *Kolmogorov-Smirnov One-Sample Test*, in that you still want a running total of all the *Frequencies* we are observing or expecting over a given time period. However, now we want to see the *Distribution* of the proportion of observations which are less than, or equal to, a specific value."

Robin shook her head.

"Hey, how about we try discussing this in English?"

Dakota placed her hands on her hips, mentally trying to disentangle this problem within her mind.

"All a *Cumulative Relative Frequency Distribution* does is just tell you the proportion of scores below a given level."

Dakota quickly jotted a formula on the dry-erase board:

$$Sn_1(x) = \frac{cumulative\,frequency}{sample\,size}$$

Robin and Theron both rolled their eyes, as their collective trains of thought were derailed by Dakota's exceptionally vague explanation. Sensing that they were about to go off on some non-statistic-related tangent, Dakota eased the cap on her dry-erase marker and gently sat on a corner of the conference table.

"Okay, the goal of a *Cumulative Relative Frequency Distribution* is to see if your scores fall at or below a set level. Let's say you want to study ages of sex offenders..."

Robin bolted upright in her chair listening attentively as Dakota continued.

"... So, you have two groups of offenders, all between the ages of 20 and 80; one group of offenders has a *Mean* age of 55, the other has a *Mean* age of 42. The question, then, is: are the two sets of offenders drawn from the same population, or is the second group drawn from a significantly younger population? So we divide the scores into distinct age categories: 20 – 29, 30 – 39, and so on. We then see what percentage (or frequency) of offenders fall into each age category. Importantly, all the *Frequencies* will add up to 100%—since all our participants in both groups fall within the ages of 20 – 80. That's what we mean by a *Cumulative Relative Frequency Distribution*—all *Frequencies* must accumulate to 100%. What the test is doing, though, is seeing whether the *Frequencies* or how many offenders fall into the different age categories is the same for the two groups. You got it?"

Robin and Theron both nodded in unison. Dakota then resumed her spot next to the dry-erase board and began to draw out a series of roman numerals.

"Okay, the first thing we need to do is to arrange our two sets of data into *Cumulative Relative Frequency Distributions*. In order to do this, we need to use the same classification for both distributions on these data sets and use as many classifications as possible."

Theron glanced over at the data set, mentally attempting to turn the numbers around in his mind.

"Okay. I am so lost."

Dakota stepped over to the data set, pointing at the two sets of data.

"All that means is that the more information you put into the two *Cumulative Relative Frequency Distributions*, the more information you will have to determine if your two groups are in fact different from one another. These will become points, or **Intervals**, that we will use later on."

Theron pulled out a new sheet of notebook paper and began to arrange the *Cumulative Relative Frequency Distributions* for the "Rater 1" data and the "Race / Ethnicity" data set.

Level of Meanness Score	Hispanic ($n_1 = 47$)	White ($n_2 = 22$)
1	11	5
2	11	3
3	14	7
4	7	5
5	4	2

"It is important to remember that all of our offenders fall into one of these five distinct levels or categories, just like in our age example. In this case, $n_1 = 47$ and $n_2 = 22$. For large samples ($N \geq 25$), n_1 does not have to equal n_2."

Dakota's gaze of affirmation filled the room.

"Now, we need to establish exactly what percentage, or frequency, of sex offenders fall in each of the 5 levels. We do this by taking the total number of sex offenders in each sample and dividing that by the number of offenders we observe who got each score. So, there were 11 offenders in the Hispanic group who scored 1, and there is a total of 47; that means that 23% of the offenders in the Hispanic group got a score of 1":

$$Sn_1(11) = \frac{11}{47} = .23$$

Dakota and Robin closely followed as Theron finished. Dakota patted Theron on the shoulder and returned to write out the next step on the dry-erase board.

"Wonderful. Now we need to find the difference between the two distributions."

Robin looked over Theron's notes, thoroughly confusing herself as she scanned his numbers.

"Wait, so we just subtract?"

Dakota nodded.

"That is correct. We just subtract at the different *Intervals*. That will give us difference scores."

Level of Meanness Score	Hispanic ($n_1 = 47$)	White ($n_2 = 22$)	Hispanic cumulative frequency	White cumulative frequency	$Sn_1(x)$	$Sn_2(x)$
1	11	5	11	5	0.23	0.23
2	11	3	22	8	0.47	0.36
3	14	7	36	15	0.77	0.68
4	7	5	43	20	0.91	0.91
5	4	2	47	22	1.00	1.00

Theron quickly looked over his distributions and began to subtract at the various *Interval Points*.

After he double-checked his work, Theron dropped his pencil and began to massage the cramping pain in his hands.

"Okay, this is starting to not be fun anymore. What's next?"

Dakota thought for a few moments, her eyes looking over what Theron had scribbled all over his legal pad. Dakota just pointed to a corner of the pad.

"Well, for the *Two-Tailed* test, you find the largest of these differences."

Dakota broke away from their intellectual huddle, to jot down yet another equation on the dry-erase board along with another column in the table:

$$\mathrm{D_{m,n}} = \max[\mathrm{S_m(X)} - \mathrm{S_n(X)}]$$

Level of Meanness Score	Hispanic ($n_1 = 47$)	White ($n_2 = 22$)	Hispanic Cumulative Frequency	White Cumulative Frequency	$Sn_1(x)$	$Sn_2(x)$	$\lvert Sn_1(x) - Sn_2(x) \rvert$
1	11	5	11	5	0.23	0.23	0.00
2	11	3	22	8	0.47	0.36	0.11
3	14	7	36	15	0.77	0.68	0.09
4	7	5	43	20	0.91	0.91	0.00
5	4	2	47	22	1.00	1.00	0.00

Dakota plopped the cap back onto the marker and rejoined the huddle.

"As I was saying, in the *Two-Tailed* test, the $\mathrm{D_{m,n}}$ will tell you the largest of these differences in either direction. In this case, $D = 0.11$ is the maximum difference."

Theron leaned back, muttering a question in Dakota's direction.

"And for the *One-Tailed* test, you would just get the largest difference in whatever direction you were predicting?"

Dakota smiled and patted him on the shoulder, letting him know his answer was correct.

"That's right. Then we take our calculated value and compare it to the table value. Remember, if the result is not *Statistically Significant*, that means that both of these groups could have been drawn from the same *Population*. In other words, the percentages of offenders given different meanness scores were not different between the Hispanics and the Whites; and this means that there's no evidence for any racial bias."

"Since $N \geq 25$ and this is a two-tailed test, we will use $1.36\sqrt{\frac{n_1+n_2}{n_1 n_2}}$ to determine *Significance*. If $D \geq 1.36\sqrt{\frac{n_1+n_2}{n_1 n_2}}$, we can *Reject the Null Hypothesis* at $\alpha = 0.05$:

$$1.36\sqrt{\frac{n_1 + n_2}{n_1 n_2}} = 1.36\sqrt{\frac{47 + 22}{(47)(22)}} = 0.35$$

Since $0.11 < 0.35$, we fail to *Reject the Null Hypothesis* (i.e. Hispanic and white offenders were not given different meanness scores by Rater 1). We conclude that Hispanic and white offenders were drawn from the same population based upon their Level of Meanness Scores from Rater 1."

All three of them quickly wrote out the final answers, with Theron carefully removing his sheet of legal pad and sliding it to Dakota. She checked their work over one last time and placed it into a manila folder. Once their paperwork was secure, she snatched up the eraser and began wiping away the stains of the *Kolmogorov-Smirnov Two-Sample Test*. Once the board was clean, she looked to their research questions one last time. Robin noticed what she was doing and leaned into her chair.

"Please tell me that our statistical punishment is over for the day."

Dakota smiled as she shook her head.

"Almost, just one more and we will be done."

Theron's hands were now massaging the tension which had accumulated at his temples.

"I don't know why, but I am craving vodka."

Robin started jumping about in her chair.

"I say we take a field trip to the bar."

Dakota became very still at Robin's remark. In her mind's eye, she could see Michael at the corner of the table and even hear the pompous remark he would make at Robin's expense. Suddenly, she felt very lonely and wished to reach out to him. However, she gave her word to Governor Greenleaf and to her colleagues; she would not let them down. Trying to push Michael out of her mind, she only wrote up their final research question of the afternoon.

"The last question we can look at today concerns whether or not the 'General Aggression Scores' differ between those who are in compliance of their sex offender registry and those who are in violation. If there is a link between aggression scores and violation status, those involved in supervision can use that information to perhaps design different supervision strategies for those particular offenders in order to keep them in compliance."

Theron scribbled down the question, while Robin started smacking her lips together to calm the parched sensation in her mouth. She slipped out of her chair and made her way to the sink to get a glass of water. Dakota eyed her warily as she dropped some ice in her glass and returned to her seat. Robin, aware of Dakota's gaze, just took a nonchalant sip and returned her glass to the table.

"Relax; I am not dumping this one over your head."

Theron just continued on with his writing.

"So, which statistical procedure will you be pulling out of your pocket now?"

Dakota looked over the data sheet, the research question bouncing silently upon her lips as she read.

"I am thinking that **Mann-Whitney U** would be the best approach."

Theron mouthed the test to himself, as Robin patiently waited for Dakota to begin writing on the dry-erase board. True to form, Dakota did not disappoint as she wrote out the name for the test on the dry-erase board.

"Now, the *Mann-Whitney U* test is this amazing powerful procedure which allows you to determine whether or not two sets of data are *Independent* of one another."

Robin just dropped her pencil in a defiant act of frustration.

"Wait, so how is this different from our favorite Kremlin-Vodka test?"

Dakota thought carefully for a moment.

"Well, the *Kolmogorov-Smirnov Two Sample Test* required you to use *Cumulative Relative Frequency Distributions* in order to see if the distributions were different, while the *Mann-Whitney U* doesn't require such distributional assumptions. In other words, all the different scores will add up to 100 %, like the rating scale – there's a scale from 1 to 5, and everyone has to receive a score that falls in that range. But with *Mann-Whitney U*, you're not specifying any range, particularly, so the scores don't have to add up to 100 %."

Theron just looked over his notes once more.

"So, the point of the test is just to see if two groups come from the same *Population*?"

Dakota nodded.

"Correct. You might expect that offenders who were compliant were less aggressive than offenders who were not compliant, just because aggression is a personality trait that is known to affect how prosocial you are. If an offender is more aggressive, he's likely to be more anti-social; so, he's less likely to comply."

Theron shook his head.

"Sounds like an *Independent T-Test*."

Dakota thought about this for a moment and then nodded her head.

"Well, for all intents and purposes, they do pretty much the same thing and have roughly the same assumptions."

Robin started looking back and forth at each one of them, slowly becoming more irritated as their private discussion went on. Finally, she had to interject herself into their statistical repartee.

"*Independent T-Test*? Is that like what the founding fathers did to help us gain our Independence? And how is that similar to the Mann Act?"

Theron leaned back into his chair, presenting a mixture of bemusement and confusion.

"Did you really just combine pop culture references for both the Revolutionary War and the White-Slave Traffic Act?"

Robin planted her arms on the table and cupped her chin with her hands.

"Well, we are doing statistics for a political campaign."

Once again, Dakota could see Michael's reaction in her mind's eye. It was as if his ghost was refusing to give her any sort of peace. All she could do was close her eyes, and will his memory out of the here and now. Still, it was something which would have to be dealt with.

"An *Independent T-Test* is a *Parametric Statistic* which tells you if the *Means* of two different sets of data are from the same population. So, if you find a *Statistically Significant* result, then you know that the two data sets are from disparate groups which means that two groups are different and that their difference falls in the tail region of the curve."

Robin just nodded, somewhat grasping the concepts presented to her. Dakota just continued on with her explanation.

"Now, the *Mann-Whitney U* statistic looks at two *Independent Samples* like offenders who comply and offenders who don't – they're independent because you can only be one or the other. In addition, it uses *Ordinal Data*, which in turn can severely blunt the presence of *Outliers*."

Theron just absorbed all of the information splayed out before him.

"So, I take it this is a powerful statistical procedure?"

Dakota nodded.

"Oh yes. In fact, this is one of the most powerful nonparametric procedures."

Robin just looked lazily off into the distance.

"All right, what do we do, take all the General Aggression Scores which is our measure of aggression from offenders in both groups, put them together and then rank them?"

Dakota nodded her head.

"That's correct. What we'll be doing is seeing where in the ranking offenders from the different groups are found. If there's a pattern, and one group of offenders is consistently in the bottom ranks, whilst the other group of offenders is consistently in the top ranks, we might conclude that the groups are independent. First, start with the highest number in the data set, and rank that as "1", then go to the next highest number and rank that as "2". It doesn't matter where in the columns these numbers are, just that they get ranked."

Theron methodically began to conduct the rankings, while Robin was slightly more apprehensive.

"Wait a moment. I can already see that we have multiple different versions of the same number. What do we do then?"

Dakota thought for a moment and then returned to the dry-erase board to draw out an example.

"All right, let's say your three highest numbers were all 86."

She quickly drew that onto the dry-erase board.

86
86
86

Robin nodded as Dakota kept right on giving her explanation.

"So, we know these three numbers are going to be 'rank 1', 'rank 2', and 'rank 3'. So what we do is we get the Mean for 'rank 1', 'rank 2', and 'rank 3'."

Robin quietly conducted the math in her head.

"So, all of those would receive a ranking of '2'."

Dakota wrote the ranks next to the numbers.

86	2
86	2
86	2

Dakota snapped the cap onto the dry-erase marker and continued on with her explanation.

"Now, let's say the next highest number in the sequence is '84', which would receive a ranking of '4'. Remember, whatever the number of your *Sample Size* is, that is how many rankings you will have."

Robin smiled.

"I get it. So, if you have 20 numbers in both of your columns, your rankings will be in a range from '1' to '20'."

Dakota smiled and stepped away from the dry-erase board just as Theron finished his calculations.

Dakota quickly jotted down the steps on the dry-erase board, so that they could all follow along with what was being done.

"Well, the first thing we need to do is rank both groups together. For ties, we compute the average of the assigned ranks. For example, the first 5 offenders all received a GAS of 40. The assigned rank will be 3":

$$\frac{1+2+3+4+5}{5} = 3$$

Status	GAS	Rank
Compliant	40	3
Compliant	40	3
Compliant	40	3
Compliant	40	3
In violation	40	3
Compliant	41	6.5
In violation	41	6.5
Compliant	42	10.5
Compliant	42	10.5
Compliant	42	10.5
Compliant	42	10.5
In violation	42	10.5
In violation	42	10.5
Compliant	43	17
Compliant	43	17
Compliant	43	17
In violation	43	17

(continued)

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Status	GAS	Rank
In violation	43	17
In violation	43	17
In violation	43	17
Compliant	44	23
Compliant	44	23
In violation	44	23
In violation	44	23
In violation	44	23
Compliant	45	28.5
Compliant	45	28.5
Compliant	45	28.5
Compliant	45	28.5
Compliant	45	28.5
In violation	45	28.5
In violation	46	32
Compliant	47	35.5
Compliant	47	35.5
Compliant	47	35.5
In violation	47	35.5
In violation	47	35.5
In violation	47	35.5
Compliant	48	39.5
Compliant	48	39.5
Compliant	49	41.5
In violation	49	41.5
Compliant	50	44
Compliant	50	44
In violation	50	44
Compliant	51	46.5
Compliant	51	46.5
Compliant	52	48.5
In violation	52	48.5
Compliant	53	53
Compliant	53	53
Compliant	53	53
Compliant	53	53
Compliant	53	53
In violation	53	53
In violation	53	53
Compliant	54	57.5
Compliant	54	57.5
Compliant	55	61
Compliant	55	61
In violation	55	61
In violation	55	61
In violation	55	61

(continued)

(continued)

Status	GAS	Rank
Compliant	56	66
Compliant	56	66
Compliant	56	66
Compliant	56	66
In violation	56	66
Compliant	58	70.5
Compliant	58	70.5
In violation	58	70.5
In violation	58	70.5
Compliant	59	74.5
Compliant	59	74.5
In violation	59	74.5
In violation	59	74.5
Compliant	60	78.5
Compliant	60	78.5
Compliant	60	78.5
In violation	60	78.5
Compliant	62	81
In violation	63	82
In violation	64	83
Compliant	65	85.5
In violation	65	85.5
In violation	65	85.5
In violation	65	85.5
Compliant	67	88
Compliant	68	89.5
Compliant	68	89.5
Compliant	69	91
Compliant	70	92.5
In violation	70	92.5
Compliant	73	94
Compliant	74	95.5
Compliant	74	95.5
In violation	75	97
In violation	76	98
Compliant	78	99.5
In violation	78	99.5

Theron and Robin quickly isolated the General Aggression Scores into two separate columns, one for those sex offenders who were "in compliance" and another for those sex offenders "in violation," and set those two columns next to one another. Afterwards, he added up the ranks to see if the sums of the two groups were different from each other.

Compliant Offenders

GAS	Rank
40	3
40	3
40	3
40	3
41	6.5
42	10.5
42	10.5
42	10.5
42	10.5
43	17
43	17
43	17
44	23
44	23
45	28.5
45	28.5
45	28.5
45	28.5
45	28.5
47	35.5
47	35.5
47	35.5
48	39.5
48	39.5
49	41.5
50	44
50	44
51	46.5
51	46.5
52	48.5
53	53
53	53
53	53
53	53
53	53
54	57.5
54	57.5
55	61
55	61
56	66
56	66
56	66
56	66
58	70.5
58	70.5
59	74.5
59	74.5
60	78.5
60	78.5
60	78.5
62	81
65	85.5
67	88
68	89.5
68	89.5
69	91
70	92.5
73	94
74	95.5
74	95.5
78	99.5
R_2	3049.5

In Violation Offenders

GAS	Rank
40	3
41	6.5
42	10.5
42	10.5
43	17
43	17
43	17
43	17
44	23
44	23
44	23
45	28.5
46	32
47	35.5
47	35.5
47	35.5
49	41.5
50	44
52	48.5
53	53
53	53
55	61
55	61
55	61
56	66
58	70.5
58	70.5
59	74.5
59	74.5
60	78.5
63	82
64	83
65	85.5
65	85.5
65	85.5
70	92.5
75	97
76	98
78	99.5
R_1	2000.5

Once the data was side by side, Robin scratched her head in confusion. After a quick glance to make sure Theron was correct, Dakota once again resumed her post at the dry-erase board and scribbled out an equation.

"After you have finished the rankings, you need to get the value for 'U' by using only one of these formulas, not both":

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - \Sigma R_1$$

$$U = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

n_1 = number of observations in the smaller of the two groups
n_2 = number of observations in the larger of the two groups
R_1 = sum of the assigned ranks in group n_1

Theron quickly wrote down the equation for his own benefit and then began to plug the numbers into the equation:

n_1 = number of observations in the smaller group "in violation" = 39
n_2 = number of observations in the larger group "compliant" = 61
$N = n_1 + n_2 = 100$
$R_1 = 2000.5$

"As noted above, there are two different formulas that can be used to compute U. When *Sample Sizes* are large, $n_2 > 20$, it doesn't matter which formula is used. When *Sample Sizes* are smaller, the second U is desired":

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 = (39)(61) + \frac{39(39 + 1)}{2} - 2000.5 = 1158.5$$

"Once we find U, we can then make a correction for ties. Depending on the number and length of ties, the effect of the correction may be negligible. A large number of ties can produce a more conservative test which means that it is harder to detect a difference because the two groups share so many scores with each other. In order to be sure that you really do have a *Significant* difference, then you are forced to use a more stringent, or conservative, estimate of how different the two groups need to be before you say they're from different populations, i.e., one's bigger or smaller.

In this case, we do have a large number of ties. In order to make the correction, we use the following formula:

$$T = \frac{t^3 - t}{12}$$

To do this, we must first determine the total number of observations tied for each assigned rank. In this case, we have the following ties":

5 scores of 40	6 scores of 47	7 scores of 53	4 scores of 60
2 scores of 41	2 scores of 48	2 scores of 54	4 scores of 65

(continued)

(continued)

6 scores of 42	2 scores of 49	5 scores of 55	2 scores of 68
7 scores of 43	3 scores of 50	5 scores of 56	2 scores of 70
5 scores of 44	2 scores of 51	4 scores of 58	2 scores of 74
6 scores of 45	2 scores of 52	4 scores of 59	2 scores of 78

$$\Sigma T = \frac{5^3-5}{12} + \frac{2^3-2}{12} + \frac{6^3-6}{12} + \frac{7^3-7}{12} + \frac{5^3-5}{12} + \frac{6^3-6}{12} + \frac{6^3-6}{12} + \frac{2^3-2}{12} +$$
$$\frac{2^3-2}{12} + \frac{3^3-3}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12} + \frac{7^3-7}{12} + \frac{2^3-2}{12} + \frac{5^3-5}{12} + \frac{5^3-5}{12} +$$
$$\frac{4^3-4}{12} + \frac{4^3-4}{12} + \frac{4^3-4}{12} + \frac{4^3-4}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12}\frac{2^3-2}{12} + \frac{2^3-2}{12} = 197.5$$

He spun the piece of paper around for Dakota to see and then twirled it back so he could continue working. Dakota scribbled out one final equation on the dry-erase board:

$$Zu = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 \cdot (n_1 n_2 + 1)}{12}}}$$

"Okay, the last thing we must do is that we must finish this equation. Then, we take that number and see if it is greater than 1.96."

Theron's eyes brightened.

"Wait, this is based off the *Z-Distribution* as well?"

Dakota nodded.

"In cases of large *Sample Size*, you take your calculated value and compare it to 1.96. However, if you have a smaller *Sample Size*, there is a specific table which will tell you the *Critical Values* for *Mann-Whitney U*. However, the steps for figuring out the problem are the same."

Theron just shrugged his shoulders and plugged the numbers into the final equation:

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\left(\frac{n_1 n_2}{N(N-1)}\right)\left(\frac{N^3 - N}{12} - \Sigma T\right)}} + \frac{1158.5 - \frac{(39)(61)}{2}}{\sqrt{\left(\frac{(39)(61)}{100(100-1)}\right)\left(\frac{100^3 - 100}{12} - 197.5\right)}}$$
$$= -0.2195$$

"With $z = -0.2195$, the corresponding p-value for a two-tailed test is 0.8258. Since $p = 0.8258$ is greater than $\alpha = 0.05$, we fail to *Reject the Null Hypothesis* (i.e. GAS has the same distribution for offenders who are currently 'compliant' and 'in violation'). We conclude that the distribution of General Aggression Scores is the same for offenders who are currently 'compliant' and 'in violation'. That means that offenders who are not compliant are no more, nor less aggressive than offenders who are compliant. That's interesting, as theory says that aggression affects how prosocial a person is, and complying with probation is certainly more prosocial than being in violation!"

Once all calculations were finished, Dakota once again carefully copied their final answer and set it aside for later. Robin had quickly grabbed all of her possessions and began to herd Theron out the door.

"Come on. I want to get out of here before she starts to either cry or teach us more math."

Theron looked bewildered as Robin physically began to pull on his shirt.

"What are you talking about? She seems fine."

Robin stopped for a moment, staring at Theron with bewilderment in her eyes.

"Wow, you really do not know women."

With that, she physically shoved Theron out of the conference room and into wilds of the deserted campaign office. Dakota sat in her chair, listening to their footfalls as both of them exited the campaign offices and locked the door behind them. Once again, Dakota found herself confronted by an eerie stillness. Her eyes looked towards the windows, drinking in the amber hues of what would most likely be an awe-inspiring sunset. Dakota could feel something angry welling up inside of her, angry that she was behaving like a lovelorn teenager. It was not in her nature to sit idly by and wait for things to happen. She lowered her chin to her chest and quietly muttered to herself.

"What have I got to lose?"

With that, she reached into her purse and pulled out her cell phone. After pushing a few buttons, she held the phone to her ear and heard the incessant ringing noise at the other end of the phone. An anxious knot formed in her stomach as she heard an answer from the other end of the phone.

"Dr. O'Brien here."

Dakota straightened up her posture,

"Michael, its Dakota. We need to talk."

On the other end of the phone, she heard nothing but silence for a few agonizing moments. Dakota began pulling the phone away from her ear, looking to see if he had hung up on her. Suddenly, she heard a very faint voice on the other end of the line.

"You're right. We do need to talk."

Chapter Summary

- The Sign Test is addressed through the research question: "is there a directional difference between two groups concerning two conditions?" The research question helps the researcher determine whether a difference exists between 2 groups, and if the difference does exist, in which direction that difference exists.

- Specifically, the Sign Test is used in the chapter to assess the existence and direction of difference between sentence lengths given for lewd offenses and other offenses.
- A review of *z-scores* and normal distributions is a result of considering the Sign Test and the Wilcoxon Signed Rank Test.
- The research question "when the size of the difference is important, is there a difference between a pair of data?" is addressed by using the Wilcoxon Signed Ranks Test. The Wilcoxon Signed Ranks Test assesses the size of the difference between two raters on the Level of Meanness scale.
- An analysis of the research questions "have two independent groups been taken from the same population when the responses are cumulative?" and "have two independent groups been taken from the same population when the responses are not cumulative?" leads the team to work through the Kolmogorov-Smirnov Two-Sample Test and the Mann-Whitney U Test, respectfully.
- Specifically, the Kolmogorov-Smirnov Two-Sample Test addressed cumulative frequencies of ratings for Hispanic and White offenders based upon the Level of Meanness scale. The Mann-Whitney U Test addressed the difference between offenders who were classified as compliant and in violation of parole conditions based upon their general aggression scores.
- Cumulative Relative Frequency Distributions are illuminated through the discussion of the Kolmogorov-Smirnov Two-Sample Test.

Check Your Understanding

1. What is the closest nonparametric test to the parametric Independent T Test?

   a. Sign Test
   b. Wilcoxon Signed Ranks Test
   c. Kolmogorov-Smirnov Two-Sample Test
   d. Mann-Whitney U Test

2. Can interval scale data be transformed into ordinal scale data? If so, why and how?
3. The Kolmogorov-Smirnov Two-Sample Test and the Mann-Whitney U Test are similar in their application but have one major difference. What is that difference?
4. Which level of measurement does ranked data fall into?

   a. Nominal data
   b. Ordinal data
   c. Interval data
   d. Ratio data

5. All of the following are true of the Wilcoxon Signed Ranks Test except:

   a. It is also known as the Wilcoxon T.
   b. It can be used with three or more groups.
   c. It is used in a repeated-measures design.
   d. It can be used to examine matched pairs.

Chapter 11
Geometry on Steroids

Abstract This chapter discusses two tests that use *Probability* to determine the exact likelihood of the occurrence of an event. The two research questions are as follows: (1) In terms of status, is there a difference between those who make less than $15,000 a year compared to those who make more than $15,000 a year? (2) Is the *Probability* of someone violating their parole after initially being compliant any different than someone becoming compliant after they initially violated their parole? The first research question is addressed using Fisher's Exact *Probability*, and the second research question is addressed using McNemar Change Test. The team gets an ominous phone call from the Greenleaf's right-hand woman.

All four of the consultants sat silently around the conference table, their discomfort easily apparent to all of the campaign staff mulling about the offices. While all four of them were shifting about in the awkward stillness, the heart of this uneasiness was centered on Dakota and Michael. It was not an awkwardness that was founded in antipathy towards one another, but one that was steeped in heartbreak. Theron and Robin shifted nervously towards one another, moving quickly through the awful stillness. Robin leaned her head forward and whispered at her playmate.

"So . . . this is awkward."

Theron just cracked a sly smile.

"Ain't love grand?"

Robin just shook her head.

"We should do something."

Theron shrugged his shoulders.

"Such as? How are we going to subtly alter the flow of this conversation?"

Robin closed her eyes and mentally prepared herself to say the sentence that no one ever thought she would utter. After taking a few deep breaths, her voice resonated through the room.

"How about we do some math?"

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*, DOI 10.1007/978-1-4614-9041-8_11,

Suddenly, Dakota and Michael snapped out of their awkwardness and stared incredulously at Robin. Theron just placed his elbows on the conference table and rested his head in his hands.

"Real subtle."

Robin just flashed him a look of pure exasperation.

"Well, I didn't hear you come up with anything better."

Theron just continued grinning, overtly pleased with himself and with what he orchestrated. Dakota, still thunderstruck by what Robin just said, blinked her eyes a few times and rose to take her position at the dry-erase board. She did not move with her usual dancer's grace, her body now reacting to the unease of Michael watching her every move. Dakota fought past her uneasiness and glanced over at the remaining research questions left on the dry-erase board. She quickly circled two questions in particular and returned her attention to the others in the room.

"Well, I think that we should focus on whether or not there is a difference between those participants with an annual income of less than $15,000 a year compared to those with an annual income of more than $15,000 a year and parole compliance."

Michael cleared his throat, trying to work past his uneasiness in his throat.

"So, why do we want to focus on that question?"

Dakota focused her full concentration on the dry-erase board. She was so lost in thought that she was able to force herself past the awkwardness. Dakota just leaned away from the dry-erase board and snatched up the data set, pointing to the various columns of the data set which were relevant to the research questions they were discussing.

"Well, we are still working on research questions which focus on *Differences* between two *Groups*. So, it is a logical place to continue for the moment as people who have less money are likely to resort to crime and thereby violate their parole. So, if we found a difference, we might instruct the Governor to give extra attention to those offenders making less than $15,000."

Without her knowing it, the dry-erase marker slipped out of her pocket and rolled off the conference table. Instinctively, both Michael and Dakota dropped to the floor to retrieve her fallen marker. Suddenly, both of them were on their knees in front of each other, looking into one another's eyes. All Dakota could do was pick up her pen and smile.

"Hey, are you two finished 'making up'? It's bad enough that I actually had to ask for us to do math today."

Dakota and Michael quickly rose to their feet and resumed their respective positions in the room. Dakota rose to her feet, smoothed out her skirt with her hands, and pointed at the dry-erase board.

"I am sorry about that. Now, in order for us to find whether or not there is a difference between the compliance rates of those participants with an annual income of less than $15,000 a year compared to those with an annual income of more than $15,000 a year, I propose we use the **Fisher Exact Probability** Test."

Robin turned to Theron.

"Fisher . . . like the nuts. I say we call it the peanut test."

Theron shook his head.

"I am allergic to peanuts here."

"Then this test will be no fun for you."

Dakota cleared her throat, utterly derailing the less-than-witty banter playing out before her. She snapped her fingers, instantly garnering their attention.

"Now, the *Fisher Exact Probability Test* . . ."

Robin just grinned mischievously at Dakota.

"Peanut test."

". . . Is a statistical procedure you use when you need a *Two-Sample* test and have data which are of *Nominal Scale*. The test is often only useful with *Dichotomous* data. The *Fisher Exact Probability Test* looks at an exact *Probability* of occurrence, not a *Statistical Significance*. The difference being that with *Statistical Significance* you want to see if something is fundamentally different from the population or another group, while with the *Fisher Exact Probability Test* helps you to determine the *Probability* of whether or not something will occur."

Theron pulled out his pencil, mentally preparing himself for the barrage of information about to be volleyed in his direction.

"So what do we do?"

Dakota went to the dry-erase board, jotting a table on the board.

"Well, the first thing we need to do is to create a *Contingency Table* using the 2 variables we are addressing. One variable will go in the column headings and the second will go in the row headings."

2 x 2 Contingency Table

	I	II	Totals
I	*A*	*B*	*A+B*
II	*C*	*D*	*C+D*
Totals	*A+C*	*B+D*	*N*

Theron easily drew out the *Contingency Table*, yet hesitated before he raised his head to acknowledge that he was complete with his work.

"So, I placed a 'I' and a 'II' for each of the variables but remember that each one is a different variable so the 'I' for the rows will not be the same as the 'I' for the columns. In fact in our test, one will be below $15,000 and the other will be compliant. Is *Fisher's Exact Probability* always 2x2 in nature?"

Dakota shook her head.

"No it's not only 2x2, *Fisher's Exact Probability* test is technically r X c; number of rows by number of columns. However, the most common application is a 2x2 design. The point is that the *Fisher's Exact Probability* is small; small samples and small categories. If you have anything larger, you should use a *chi-square* Statistical Procedure."

Robin instantly bristled at what Dakota had just said.

"Oh not the 'X' test again!!! I don't want to do another chai-square!!! Why do these tests seem so similar?!?"

Theron shook his head.

"*Chi-Square*."

Robin nodded at him.

"That's what I said."

Dakota folded her arms across her chest, her mind slowly formulating the best possible way to explain this to her math anxiety-ridden colleague.

"Robin, that's an excellent observation. As we are going through our explanation of *Fisher's Exact Probability*, you will see that there are several similarities between the two procedures; both tests even use *Independent* groups. However, the *Chi-Square* procedure looks for *Differences* between the *Observed Frequency* and the *Expected Frequency*, while *Fisher's Exact Probability* examines only the *Probability* of whether or not something will occur. Also, *Fisher's Exact Probability* is important for small samples, specifically to find the exact *Probability* of occurrence. *Chi-Square* may be inaccurate in this instance because *Chi-Square* is useless when *Frequencies* drop too low, such as when the expected sizes are less than 5 for any particular category. However, *Chi-Square* would certainly be appropriate when you have larger sample sizes."

Michael reached across the conference tables, grasping at the data set with his fingers. Theron leaned forward, flicking the data set towards Michael with the edge of his pencil. Michael snatched the flittering paper and dragged it over towards him. After a few moments, he grunted towards his colleagues.

"Wait a moment, we want to examine if there is a difference between the compliance rates of those sex offenders from the data with an annual income of less than $15,000 a year compared to those with an annual income of more than $15,000 a year. Is that correct?"

Robin just shook her head.

"Way to restate the obvious Dr. Moreau."

Ignoring his snarky confederate, Michael chose instead to focus on the data set.

"Well, doesn't that mean that we can only consider the participants who have a known, 90-day post-release status? After all, sex offenders often face a lot of pressure from within the community, so wouldn't many of them be moving about a lot?"

Dakota leaned over Michael's shoulder, her hand deftly caressing his shoulder as her laser-like focus aimed itself at the data set.

"You're right; many sex offenders are moving targets . . ."

Robin righted herself in her seat.

"Pun intended."

Dakota just shot Robin an angry gaze and continued on with her explanation.

". . . So, it would only make sense to focus on those sex offenders who have a known 90-day post-release status. After all, they might have to move in a few weeks because they cannot find work or are forced from their home due to social pressure from their neighbors."

Just as deftly as she touched Michael's shoulder, Dakota pinched the data set between her fingers and flung it at Theron. He haphazardly snatched the document as Dakota spoke in the background.

"Theron, how many sex offenders have a 90-day post release status?"

Theron furrowed his brow as he caught the document. Robin once again shifted uncomfortably in her seat, obviously signaling to the others in the room that she had a question. After a few moments of wriggling about in her seat, she decided it was better to just speak up.

"Why are we using a $15,000 threshold?"

Theron continued counting, oblivious to all the bouncing about next to him.

"The reason I came up with the question is because the federal threshold for the poverty level is $15,130. $15,000 was close enough[1]."

Robin quieted down, and Theron was able to finish off his tallying efforts.

"That leaves us twelve offenders overall."

Dakota marched away from her paramour and back to her post, her hands swiftly snapping of the cap of the dry-erase marker.

"All right, can you parse out those sex offenders with a 90-day post release status from the others in the data set? Make sure you get not only their incomes, but their compliance with their parole status as well."

Theron grunted his assent to her request and once again dove back into completing information.

Participant	Estimated yearly income	90-day post-release status (complying with parole conditions [0] or is in violation [1])
1	$26,104	1
12	$29,512	0
21	$29,811	0
31	$0	1
33	$0	0
34	$0	1
43	$0	1
45	$26,353	0
54	$7,612	0
62	$0	1
85	$28,575	0
94	$28,002	0

Once completed, he tore the page from his legal pad and handed it to Dakota. She mulled it over for a few moments and then placed all of the data into the blank *Contingency Table* on the dry-erase board.

"Now, compliance was scored as a '0' on the raw data sheet, while in violation was scored as a '1'; so we count the number of individuals with greater than $15,000 who received a 0 and those who received 1's and we get 5 and 1, respectively."

[1] This information is taken from the 2012 US Department of Health and Human Services Poverty Guidelines located at http://aspe.hhs.gov/poverty/12poverty.shtml

	90-day post-release status		
Estimated yearly income	In compliance	In violation	Row totals
>$15,000	5	1	6
<$15,000	2	4	6
Column totals	7	5	12

Seeing that her colleagues were simply seeing the math but not understanding the purpose, Dakota paused to shed a bit of light on the purpose in an effort to draw the team back from their day-dreaming state.

"We have a certain *Probability* that people will be either in compliance or in violation after 90 days. We want to see whether this *Probability* is different when we look at people who have different incomes, as we think that income will be likely to influence anti-social behavior. So we need to calculate the *Probability* of people being in compliance/in violation for offenders with less than $15,000; and we need to calculate the *Probability* of people being in compliance/in violation for offenders making greater than $15,000. What this test is going to do is tell us whether the exact probabilities that we have for these two populations are similar or not. If they're similar, then income does not affect parole compliance. If they're not, then income does affect parole compliance."

Dakota stepped back from the dry-erase board, quietly muttering to herself as to what the group should do next.

"So, we have a pattern that compliance is greater than in violation for greater than $15,000. From our data, we have an actual *Probability* for compliance and in violation, 5 to 1, for a total of 6 people in greater than $15,000. The only other combination of probabilities (or numbers) that our 6 people could have shown, keeping the direction of compliance and in violation, is 6 to 0. No other combination of numbers allows us to keep the total as 6, with compliance greater than in violation. We now look at less than $15,000, and we see a different pattern, compliance is less common than in violation. Here, we have an actual *Probability* from our data of 2 in compliance and 4 in violation, for a total of 6 offenders with less than $15,000. The only other combination of behavior that our 6 offenders could have shown would be 1 in compliance and 5 in violation. Now, we've got the most extreme example of how people could behave, given our total number of offenders with incomes less than or greater than $15,000, whose status is known after 90 days. From this, we are now going to see what the exact *Probability* is of getting these exact numbers. This will ultimately tell us whether the behaviors for compliance with greater than $15,000 is the same or different as the behaviors for compliance shown with less than $15,000."

Dakota paused long enough to see that the team was actually following her train of thought.

"So, we got our numbers for the most extreme example of how our offenders could behave. Now we're going to see whether, in this most extreme case, these two populations are showing the same, or different, probabilities. We're going to see, with the number we have, what the *Probability* is of these particular number occurring in this extreme pattern. This *Probability* level will tell us whether income affects compliance and violation. Now, as we look at the Contingency Table for our extreme example, we notice that '6' is our 'A', '0' is our 'B', '1' is our 'C', and '5' is our 'D'."

	90-day post-release status		
Estimated yearly income	In compliance	In violation	Row totals
>$15,000	6	0	6
<$15,000	1	5	6
Column totals	7	5	12

Dakota stepped away from the dry-erase board and pointed at both *Contingency Tables*.

"Although the *Frequencies* in the cells may change, note that the row and column totals remain the same. Since now the smallest *Frequency* is 0, we have found all possible *Contingency Tables*. If the smallest *Frequency* had been larger than 1, we would continue creating *Frequency* tables until we reached 0. Obviously, the larger the lowest number, the greater number of tables, then the greater number of calculations."

Dakota craned her neck back, looking at the faces of her peers. All three of them just sat in total stillness, with no one looking confused or flabbergasted at what she had said. Realizing that this was the best go-ahead she was going to get, Dakota returned to the dry-erase board and jotted down something else on the dry-erase board:

$$\sum p = \frac{(A+B)!(C+D)!(A+C)!(B+D)!}{N!}\left\{\sum_i \frac{1}{A_i!B_i!C_i!D_i!}\right\}$$

Dakota stepped to the side of the dry-erase board, revealing the equation to her associates. Upon seeing the equation before her, Robin started bouncing in her chair with glee.

"The equation has exclamation points!!! I love exclamation points!!! They are how I end almost all of my sentences!!! See, I am using exclamation points right now!!! It shows how excited I am to be doing math!!!"

Theron just fought to suppress his giggles, while Michael shook his head in bitter disapproval. Dakota just watched her colleague bounce around foolishly for a few moments before continuing on with her explanation.

"The exclamation points are known as **Factorials**. Factorials are the product of all positive integers, where you take the number you are using and multiply it by all the numbers less than or equal to it until you get to 1. *Factorials* are often used in many areas of mathematics, so they are pretty common. For example, in order to calculate 6!, you need to multiply 6 by 5 by 4 by 3 by 2 by 1 to get 720. You could do the same thing with larger numbers. If you want the factorial of 20, you just multiply 20 by 19 by 18 by 17 and so on to get the answer 2432902008176640000.

Obviously, the larger the initial value the greater, exponentially, the result will be. This is why it is difficult to run this test with large samples."

Robin just shook her head.

"I still like exclamation points."

Michael chimed in with a question aimed at bringing the group back on track.

"Wait a minute! I get 1!, 2!, 3! and all the rest; but, what about 0!? You can't multiple 0 by itself. What do you do in that case?"

Dakota shook her head.

"You are absolutely correct. You can't multiply 0 by itself in order to get a number that is beneficial. So, people who are far smarter than me at math have concluded that 0! is always equal to 1."

Robin turned to try and grasp this concept.

"So, we are just supposed to trust that these people know what they are doing?"

Theron attempted to assist in the explanation.

"The short answer is, yes! We are just supposed to trust these people. They spend years working through difficult math problems and proving theorems just so you can question their intellect. 0! is always 1 because it is a mathematical constant. You don't have to like it but that is the way it is."

Robin simply huffed in Theron's general direction. Dakota nodded in agreement as she continued on with her explanation.

"This equation helps us to figure out the *Probability* by using a **Hypergeometric Distribution**."

Robin just looked horrified at what Dakota just said. Theron leaned in and whispered to her in a fairly unsubdued tone.

"It's geometry on steroids."

Dakota shook her head.

"A *Hypergeometric Distribution* is just a *Discrete Probability Distribution* that is used to calculate the *Probability* of a researcher getting the observed data, as well as all data sets with even more extreme deviations, under the *Null Hypothesis* that the proportions are the same. This is important because it allows us to correct for a sampling issue that occurs when we are comparing exact probabilities. We don't really need to worry about why[2], but this is what you need to do to determine whether our data are the same when compared to the extreme example. Think of it like drawing socks out of a drawer . . ."

Robin just slid angrily in her chair.

"And now she is talking about laundry."

Dakota just continued on.

"Suppose you have a drawer full of black socks and white socks, and you are looking for white socks."

[2] The hallmark of a hypergeometric distribution is that when something is selected from a population, it is not replaced; thusly each subsequent draw is dependent and the *Probability* of success changes in each draw.

Michael leaned forward, obviously very interested as to what Dakota was saying.

"So, drawing a white sock would be a victory, while drawing a black sock would be a failure."

Dakota nodded.

"Exactly. Each time you pull out a sock, be it white or black, you alter the *Probability* that the next draw will give you a success due to the fact that you are not returning the socks to the drawer."

Theron just grinned in his chair.

"I get it. Because the *Hypergeometric Distribution* only works with *Discrete* outcomes, the *Probability* for getting one outcome is continuously going to change every time you select a participant. Or, in our case, pull out a sock."

Robin just shook her head.

"So . . . each time we pull out a sock, the *Probability* is going to alter because the overall number of socks in the drawer is continuously shrinking. I am guessing the whole point of this is that the *Probability* is continuously changing, regardless of whether or not we are dealing with socks or participants."

Theron leaned towards Robin.

"Still sounds like you should just fold laundry."

Dakota subtly rolled her eyes and moved her gaze to the person sitting next to Robin.

"Theron, do you think you can solve this?"

Theron nodded.

"Should be no problem."

He silently bent his head over his paper and started solving the math before him. Dakota went to the dry-erase board and started working through the math on her own. Both consultants finished their work at roughly the same time, and they were able to look over each other's work.

$$\sum p = \frac{(6)!(6)!(7)!(5)!}{12!}\left\{\frac{1}{5!1!2!4!}+\frac{1}{6!0!1!5!}\right\} = 0.1212$$

Michael glanced to the final calculated number.

"So, what is 0.1212?"

Dakota shared her thoughts with the rest of the team who were obviously in the same boat as Michael.

"It's the *Probability* that the particular, most extreme, combination of numbers occurs."

The team seemed pacified by Dakota's apparent understanding of the test, but Michael wasn't completely satisfied.

"So, what now?"

Dakota pointed to the final number.

"Unlike all of the other tests, there is no table with *Critical Values* for *Fisher's Exact Probability*. Instead, we need to look at the *Alpha Levels* to give us a *Critical*

Value. Since p = 0.1212 is greater than *a* = 0.05, we fail to *Reject the Null Hypothesis*; there is no difference between participants' estimated yearly income and 90-day post-release status. We conclude that for offenders who have a known 90-day post-release status, there is no difference between 90-day post-release status for those whose estimated yearly income is greater than and less than $15,000 a year. We might also say that there is no difference in the *Probability* of someone violating their parole after initially being compliant and someone becoming compliant after they initially violated their parole."

Robin just shook her head.

"Well . . . that didn't tell us anything."

Dakota smiled as she continued to gather up the paperwork from the *Fisher's Exact Probability Test*.

"On the contrary, it only leaves us with a few more research questions to address. You all ready to answer one more?"

Michael just stared at Dakota with a hint of a mischievous grin in his eyes; it was obvious that he would be willing to do whatever she asked of him. Theron dutifully readied his legal pad for all the information which was about to be bestowed upon him. Only Robin sat with utter disdain in her face; it was obvious that she was not exactly thrilled to continue on with their work. Still, after Robin noticed the eagerness on her colleagues faces, she had no choice but to halfheartedly acquiesce to the groups wishes.

"Oh let's just get this over with."

Dakota finished filing all of their paperwork and returned her attention to the dry-erase board.

"I am glad you are all so amenable to continue, and I know just the question to tackle."

Dakota situated herself at the dry-erase board and tapped one particular question with her finger.

"I think we should look at there a change in the *Probability* that offenders will be compliant or in-violation from 30 days post release to the present time."

Theron quickly jotted down the question, never letting his eyes trail away from Dakota.

"And how will we determine whether or not there is a change in the *Probability* that offenders will be compliant or in-violation from 30 days post release to the present time?"

Dakota just smiled in his direction.

"The **McNemar Change Test**."

Robin just folded her arms across her chest.

"So, we went from Fisher's nuts to fast food."

Theron shook his head.

"Seriously, what is with you and food? A nut reference and now McDonald's . . . is that really necessary? It's the *McNemar Change Test*."

Before Robin could utter a witty (or not so witty) retort to what Theron said, Michael leaned forward and tried to physically obstruct Dakota's view of the inane squabbling behind him.

"So, why is this test appropriate?"

Dakota pointed at all of the previous work they did on the *Fisher's Exact Probability Test*.

"There are a lot of similarities between *Fisher's Exact Probability Test* and the *McNemar Change Test*. Now, both tests use *Discrete* and *Dichotomous Nominal* data which is often displayed in a 2 X 2 *Contingency Table*. Fortunately for us, this test always utilizes a 2 x 2 *Contingency Table* design."

Dakota stopped for a moment, quickly checking to see if there were any questions. Once she realized there were none, she moved on.

"Now, the *Null Hypothesis* for the *McNemar Change Test* is that there is *Homogeneity*[3] between the two *Probabilities*. However, unlike *Fisher's Exact Probability Test*, the *McNemar Change Test* uses *Dependent* samples."

Robin just shook her head.

"You mean like those people who enable drug addicts to continue using?"

Theron just dropped his pencil in exasperation.

"That's co-dependent, not *Dependent*."

Dakota held up her finger, stopping the lunacy of her colleagues in their tracks.

"*Dependent* samples. Usually this means that the samples are related in some way. This is different from *Fisher's Exact Probability Test*, which uses *Independent* groups."

Robin still had a confused look across her face. Michael just shook his head and muttered under his breath.

"You usually see *Dependent* samples with *Longitudinal* studies. I see it a lot of this when I do long-term studies on drug trials. Think of it as a before-and-after test."

Robin's look of confusion faded into a look of rage. Theron shook his head as he picked up his pencil.

"You're just mad that he explained something to you, and you were able to understand it."

Dakota returned to the dry-erase board.

"I hope you are all finished so we can continue. As I was saying, with the *McNemar* test, we are interested in whether or not there is a change in *Probability* that an offender is either compliant or in-violation from 30 days post release to the present time. If it is more likely that an offender will be in violation the longer he is released, those involved in supervision may have to adapt their strategies to maintain compliance. So, we must first create a table that shows the change in compliance status for participants from 30 days post release to the present. The 2 x 2 *Contingency Table* should look like this . . ."

Dakota quickly traced a mock-up of a 2 x 2 *Contingency Table* on the dry-erase board.

[3] Page 23 in Chap. 2 contains a detailed discussion about homogeneity.

		After	
		II	I
Before	I	*A*	*B*
	II	*C*	*D*

Dakota pointed at the Contingency Table.

"Ok, so just to clarify:

A = the number of participants who changed from I to II
B = the number of participants who remained at I
C = the number of participants who remained at II
D = the number of participants who changed from II to I
A + D = the total number of participants who changed"

Dakota looked around the room with her eyebrows raised as if to inquire about the level of understanding in the room.

"Since the samples are *Dependent*, the assumption would be that the scores for all of the first group, or the pre-test, would be equal to all of the scores for the second group, or the post-test. So, the *Null Hypothesis* is that the row totals will be equivalent to the column totals."

Dakota glanced at the rest of team only to find bewilderment on their faces.

"Ok, how about this? It tells us whether people who behave differently at time A, still behave differently at time B. Essentially, it can tell us whether people change over time, and that's an important question to answer regarding compliance with parole."

Dakota stopped her explanation and just stared at Theron. Realizing that he was once again on the spot, Theron pulled out a clean piece of paper from his notepad and started tracing out a table.

"So, am I using a 2x2 *Contingency Table*?"

Dakota nodded, thus giving Theron all of the information he needed. He just shook his head, pulled over the data set, and did the work that was expected of him. Once Theron was finished with the table, he slid it over to Dakota.

	Current status[4]		
30 days post-release		In violation	Compliant
	Compliant	30	53
	In violation	9	8

After looking over the information for a few moments, Dakota then wrote out an equation underneath the *Contingency Table*:

[4] In the data set beginning on page 12 in Chap. 2, the statuses of offenders are represented as "0," compliant, and "1," in violation.

$$x^2 = \frac{\left(|A - D| - 1\right)^2}{A + D}$$

Once the equation was on the dry-erase board, Dakota just glanced over her shoulder.

"Theron, our $A = 30$ and our $D = 8$; can you do this?"

Theron just sighed and started working once again. As he was working, Dakota once again snatched up her dry-erase marker and worked on the equation in tandem with one another. After a few moments, both Theron and Dakota completed their work at the same time:

$$\begin{aligned} x^2 &= \frac{\left(|A - D| - 1\right)^2}{A + D} \\ &= \frac{\left(|30 - 8| - 1\right)^2}{30 + 8} \\ &= 11.6053 \end{aligned}$$

Robin spun in the chair.

"And these numbers mean what to us?"

Dakota placed her hands on her hips, sighing as she stared at the numbers on the board.

"Well, to determine Statistical Significance for the *McNemar test*, we consult a x^2 table with $df = 1$. . ."

Theron stuck his pencil in the air to grab Dakota's attention. She stopped her explanation and nodded in his direction.

"Do we use a $df = 1$ because there are essentially only two groups?"

Dakota nodded, easily pacifying her curious colleague. She then continued on with her explanation.

"Anyway, at $a = 0.05$, the critical value is 3.84. Since $x^2 = 11.6053$ is greater than the critical value of 3.84, we *Reject the Null Hypothesis*; the number of changes from the 30 days post release status to the current status is equally likely for compliant and in-violation. We conclude that there is a difference in *Probability* that offenders will be compliant and in-violation from the 30 days post release status to the current status. This suggests that more of the offenders who are compliant at 30 days post-release end up becoming in-violation the longer they are under supervision."

Theron smiled.

"Wow, that is actually a really interesting finding. Maybe the offenders are finding it more difficult to maintain compliance the longer they are back in the community. Or, maybe there is something about the supervision that is different as time goes by, like not as much follow-up the longer someone has been released."

Suddenly, the telephone at the end of conference table chimed to life. All four of the consultants startled at the sound of it. Robin gripped the end of the conference table, her nerves obviously shattered by the telephone experience.

"Why is it doing that?"

Theron, trying to soothe his own shock, calmed himself enough to respond to Robin's insightful comment.

"Of course it rang, it's a telephone. What did you expect it to do?"

Robin shook her head.

"I never even notice the thing. Honestly, I thought it was just for show."

Dakota shook her head and leaned toward the telephone.

"We should probably answer the phone."

Dakota scanned all of the buttons, quickly pressing the one which put the telephone on speaker. Suddenly, her mouth became dry as she realized she had no idea what she should say to the person on the other end of the phone.

"Um, can I help you?"

"Hello Dakota."

Dakota felt her stomach muscles clench when she heard the voice on the other end of the receiver. It was Jennifer Parsons, Governor Nathanial Greenleaf's amazingly intimidating Chief of Staff. For a moment, Dakota preferred the level of discomfort she felt when she had no idea who it was on the phone.

"Hello Miss Parsons, how can I help you?"

"I will meet with all of you at 6:00 tomorrow morning. Please be prompt."

Once she finished speaking, Jennifer hung up the phone. All four of the consultants sat quietly in awkward silence, listening to the dial tone beeping in the distance. Dakota lifted the receiver and replaced it in the holder, easily silencing the phone. Once the room fell silent, Robin bent over, grabbed her purse, and jumped to her feet.

"We have to be back at dawn. I am out of here."

Theron jumped to his feet, quickly grabbing all of his stuff.

"If you get to go home, I get to go home."

Both consultants hustled out of the conference room, leaving both Michael and Dakota alone in stillness. Michael bent over and snatched his briefcase. Once it was in his hand, he stopped and looked over to Dakota.

"Are we okay?"

Dakota just smiled at him.

"Yeah . . . we are okay."

Michael just smiled at her, holding out his hand in her direction. All Dakota could do was smile as she took his hand in hers.

Chapter Summary

- Two tests of difference are addressed in this chapter both of which use Probability to determine the exact likelihood of the occurrence of an event.
- The first test that was discussed was Fisher's Exact Probability used to address the research question: "In terms of status, is there a difference between those

who make less than $15,000 a year compared to those who make more than $15,000 a year?"

- The second test that was discussed was McNemar Change Test which was used to address the research question: "Is the Probability of someone violating their parole after initially being compliant any different than someone becoming compliant after they initially violated their parole?"
- The mathematical concepts of factorials and hypergeometric distributions were discussed.

Check Your Understanding

1. What are the similarities and differences between Fisher's Exact Probability and Chi-Square?
2. Find the following factorials:
   a. 10!
   b. 5!
   c. 16!
   d. 0!
   e. 21!
3. Which of the following are not characteristics of a hypergeometric distribution?
   a. It is a discrete Probability distribution.
   b. It is used to calculate the Probability of observed data compared to extreme data.
   c. It is used to determine whether proportions are the same between the data.
   d. It can be used with any kind of data.
   e. Once the data are removed from the distribution, they are not replaced.
   f. The Probability of "success" does not change with each draw.
4. What are the similarities and differences between Fisher's Exact Probability Test and McNemar Change point test?

Chapter 12
Crunch Time

Abstract In this chapter, the team is greeted early in the morning by the governor himself. He is happy with the progress thus far but is in need of a completed final report within 72 hours so that he can present his political stance to his constituency. The team scrambles to consider their final research questions which lead them through the process of the final tests of difference. The research questions considered in this chapter are as follows: (1) For those who consistently report their post-release status, is their status significantly different across all time periods (i.e., current status, 30 days post-release, and 90 days post-release)? (2) Is the distribution of race/ethnicity of the offender comparable across offense type? (3) Is there a difference between participant's income and meanness ratings? (4) Are minority ethnicities/races receiving higher General Aggression Scores? The team considers Cochran Q, Chi-Square r X k, Friedman's ANOVA, and Kruskal-Wallis ANOVA to address these questions.

"It's stuck."

Dakota once again shoved her weight against the heavy wooden door leading into the conference room, yet the massive object still would not budge. With her brow furrowed in irritation, Dakota leaned back onto her calves and thrust herself forward, easily losing her balance in her high-heeled shoes. Michael swung his arm forward, gracefully preventing her from falling into a heap on the floor. Dakota leaned into Michael, her hand gently caressing the back of his arm. But just as luck would have it, their clandestine moment was marred by the crisp rustling of a newspaper.

"Try the knob again."

The two lovers looked towards Theron, who had casually propped himself on the other side of the doorframe; his jacket draped over one arm as he strained to read through the pages of his morning ritual. Late last night, Jennifer called all four of them to request that they meet in the conference room at dawn. However, while the rest of the building was opened up for them, the doors to the conference room were not. Unfortunately, this situation resulted in Dakota and Michael attempting to open

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the door, Theron attempting to read his newspaper, and Robin balancing a tray of coffee cups in her arms. Dakota peered into the shadows next to the door knob, trying to formulate her next move. Michael nuzzled his face next to her ear.

"I'll tell you what; I'll push into the door while you twist it."

Robin rolled her eyes and slowly backed away from them.

"Ew. Just. . .ew."

Michael snarled under his breath.

"I meant the door knob."

Robin shook her head.

"Whatever helps you sleep at night."

Dakota smirked; she felt Michael propping his weight against the door. All she could do was grip the knob in the palm of her hand and chime out instructions in the clearest voice possible.

"Okay, on the count of three. One. Two. Three."

With that, Dakota wrenched the door knob as far as it would go. Just as she had hoped, the door violently swung open, allowing entrance into their sanctuary. Unfortunately, the violent force of the opening door caused Michael and Dakota to tumble onto the floor or the conference room. Dakota tried as best she could to catch herself, but gravity seemed to get the better of her.

"Good morning, I trust you are all well."

Dakota heard a strange voice coming from the opposite end of the room, yet she could not see who it was as she was splayed out on the floor. However, she did have a clear view of Robin's face as she stood in the doorway, and it was as if Robin had seen a ghost. Dakota tried to collect herself as best she could, which was aided by the use of one of the conference room chairs to help pry her off of the carpet. As she was trying to straighten herself out, she could hear Theron's halting voice in the distance.

"Gov. . .Governor Greenleaf. Why are you here?"

Dakota froze in place. It was just her luck that the she would be introduced to a major political power player by falling through a doorway with her coworker/lover almost falling on top of her. As she was trying to regain her composure, she could feel Michael standing next to her, muttering to himself.

"So much for that first impression."

Dakota swallowed past the lump in her throat, took a deep breath, and forced herself to look in Governor Greenleaf's direction. After all, there was no point being bashful; what is done is done. So, she lifted her head up and met the Governor's gaze.

"Good morning Governor Greenleaf."

Suddenly, Dakota understood why this man brought about such an astonishing reaction in everyone around him. Governor Greenleaf sat imperceptibly still in the chair at the head of the conference table, his hands folded together at impossibly perfect right angles. His eyes blazed like twin emeralds as he studied the motley crew of consultants gathering before him, while his powerfully coiffed locks of blonde hair delicately flittered in the breeze of the air conditioner. Governor Greenleaf's face was a granite mask of feigned pleasantness which was not easily discerned, save for a disarming smile that did not quite reach his eyes. Jennifer was standing behind him, dutifully guarding and consolidating all of Governor Greenleaf's activities for the day. Just for a moment, Dakota felt as if she were watching a monarch hold court over his subjects. Governor Greenleaf gracefully gestured towards the chairs around the conference room table.

"Please, sit. I won't take up much of your time."

All four consultants suddenly seemed ill at ease in this situation. For weeks they had free reign over the conference room, so much so that it almost felt like a second home for them. Now, they felt as if they needed permission just to be in his presence. Trying to move past their suspicion, the group huddled four chairs very close to each other and awkwardly sat down. Governor Greenleaf just sat perfectly still, not even twitching a muscle as he spoke.

"Thank you for agreeing to meet me at this early hour. I have heard from Jennifer that you have all been doing a wonderful job."

Not one of the consultants moved. So, Governor Greenleaf continued speaking.

"I am sure you are aware of the fact that my campaign has recently suffered some recent. . .unpleasantness."

Dakota's eyes shifted to Robin. Part of her was hoping that Robin would make some caustic remark at Greenleaf's expense. After all, only a seasoned politician would view the release of dangerous sex offenders as "unpleasant." However, Robin just sat in stony silence; apparently, there was a threshold to her caustic with that even she dared not cross. Disappointed, Dakota returned her gaze back towards the head of the table to listen to the rest of this very illuminating pep talk. Greenleaf just continued on in a slow, almost hypnotic voice.

"Given all that has transpired over the past few weeks, I feel that it is time to unveil my platform concerning how to address sex offenders in California. In two days, I have been asked to give a speech to a Victim's Services group where I will outline my position."

Dakota felt a sinking feeling in the pit of her stomach. She knew that Greenleaf's next sentence would certainly darken their meeting today.

"Because of this, Jennifer will need to see a copy of your final report tomorrow. I will come back the morning of my address to hear your findings and finalize my position."

Suddenly, Greenleaf looked at Dakota and spoke directly to her.

"I am sure you can make this happen."

With that, Governor Greenleaf gracefully rose to his feet and silently floated out the door as his lackey followed closely behind him. Just as Jennifer was about to close the door behind them, she stopped in the doorway and called out to the team.

"Remember, tomorrow."

With that, the conference room doors clicked shut and an eerie stillness enveloped the room. All four of them just sat there with looks of bewilderment plastered onto their faces; it was as if they were waiting for the other shoe to drop. The moment was interrupted by Robin, who dropped one arm to the ground, snatched her purse in her hand, snapped open the top, and unceremoniously dumped all of its contents onto the conference table. Theron just shook his head, no longer surprised at her bizarre antics.

"And you did that because..."

Robin started rifling through her belongings.

"Oh, I am just looking for my magic lamp, especially as it is the only way we are going to get all of this done by tomorrow."

Theron suppressed his giggles while Michael just shook his head. Dakota pushed herself away from the conference table and went to the dry-erase board, pulling it away from the wall so she could see the list of research questions on the back of it. After taking stock, she smirked to herself and spun the board around so the others could see.

"I don't think we need a genie. In fact, we only have four more research questions left to answer, and all of them focus on *Tests of Difference* with three or more groups. I think we can be done with all of the statistics for this *Data set* within the next few hours."

Robin's face brightened up, but Dakota could see she was still defensive. Robin just pointed at the board and then towards their fearless leader.

"Do not tease me about this, or I will cut you."

Michael just shook his head, while Theron buried his face in his hands. Dakota gripped the dry-erase board and swung it back around to the other side.

"I swear that this is not a trick."

Robin seemed placated as Dakota snapped open the dry-erase marker.

"Well, the most logical starting point for today would be to start with those individuals who consistently reported their post-release status."

Dakota leaned forward, pointing at the columns including "post-release status" with her index finger.

"Here. We should focus only on those individuals who have scores in the '30 days post release', '90 days post release', and 'current status' columns."

Robin leaned back in her chair, twirling some errant strands of hair through her fingers.

"And we care about them because..."

Dakota locked eyes with Michael, looking to him for some intellectual support. He thought quietly for a moment, speaking up only when all of the pieces of this puzzle fell into place.

"Because we might expect that time will have a factor in how people behave, and this could have serious implications for policy—which is what the Big Guy wants!"

Theron looked over the *Data Set*, counting off certain individuals with the tips of his fingers. Michael leaned towards the others, mildly curious as to what they were doing. Once Theron finished going through the list, he quickly jotted the number down on the top of a clean sheet of notebook paper.

"Okay, if we want to focus only on those individuals who have consistently reported on their status, that gives us only a dozen people."

A wide grin plastered itself across Dakota's face.

"That makes this data ideal for the **Cochran Q Test**."

Robin shook her head.

"You mean 'if the glove don't fit you must acquit'?"

Theron shook his head.

"You're thinking of the O.J. Simpson lawyer, Johnnie Cochran."

Robin nodded.

"What is your point? He was a good lawyer. If I ever killed my spouse, I'd want him to represent me."

Theron started tapping his fingers on the table.

"That is so wrong for so many reasons."

Robin leaned forward.

"How do you figure?"

Theron crossed his arms across his chest.

"Well, do I start with the fact that Johnnie Cochran is dead, or do I start with the fact that O.J. was acquitted; thus meaning there is no legal proof that he killed his wife."

Robin readied herself for a pop culture battle of wills, when Dakota's voice cascaded throughout the conference room.

"That's enough. We have work to do."

Robin and Theron turned away from one another, sheepishly sinking back into their respective corners. Michael just smiled at Dakota, impressed with how she had taken control of the situation. For her own part, Dakota was now scrawling information on the dry-erase board, hoping that she could keep the group together for one more day.

"The *Cochran Q Test* is this wonderful little test which allows you to examine whether the *Frequencies* for three or more *Related Samples* are significantly different."

Michael scratched his forehead.

"This sounds a lot like the *McNemar Change Test* we discussed not too long ago.[1] "

Dakota quietly thought about this for a moment, nodding her head as her mind analyzed what Michael had said.

"Actually, the *Cochran Q Test* could be thought of as an extension of the *McNemar Change Test*. Most people use the *McNemar Change Test* in 'pre-and-post' designs to see if a change is present with *Related Samples* of *Nominal Data* between two groups before a treatment of some kind and again after the treatment. The *Cochran Q Test* takes this one step further, by looking at *Related Samples* of *Nominal Data* for more than just two groups."

Theron leaned back in his chair, a big grin plastered across his face.

"So, this test basically works on 'yes' and 'no' questions."

Dakota thought about this for a moment.

"Well, in a sense, that is true. After all, 'yes' and 'no' are distinct categories, and the *Cochran Q* requires *Nominal Data* that is *Dichotomously Scored*."

Robin pointed to the dry-erase board.

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[1] Refer to page 272 in Chap. 11 where the McNemar Change Test was discussed in detail.

"All right, let's say I want to look at a group of people over five years to see whether or not they filed their Federal Income Tax. Could I use this Johnnie Cochran test?"

Dakota rolled her eyes.

"Yes, you can use the *Cochran Q Test* for any data of *Nominal Scale* that is *Dichotomously Scored*, provided that it concerns *Related Samples*."

Michael smirked when he could hear the emphasis in Dakota's voice, trying desperately to keep the group on task and prevent them from devolving into another pop culture detour. Dakota felt his eyes on her, and smiled as she basked in his protective gaze. Rather than look back towards him, she reached out and pointed to the data sheet.

"If you look at those twelve people who consistently reported their status post release, you can see that they either reported and were given a '1', or did not report and were given a '0'. Since these series of scores all come from the same individual, that makes it a *Related Sample*."

Theron nodded.

"So what do we do?"

Dakota looked over the data.

"Well, the first step is already done for us. Normally, you would just assign '1' and '0' to all of the data. Since the data must be *Dichotomous*, you would assign a '1' if the trait / quality you are studying is present, and a '0' if the trait / quality is absent."

Robin giggled.

"Wow, something we don't have to worry about. I like this test already!!!"

Theron shook his head as Dakota started to draw on the dry-erase board.

"Well, the next thing we need to do is put the '1's and '0's into columns and rows. Each column will be the different time periods..."

Theron's head perked up.

"Or in our case, the different post release status time periods."

Dakota nodded as she continued her explanation.

"...and each row will be the individual sex offenders. Got it?"

Robin took a deep breath and started jotting down the numbers for the offenders and their post-release status scores. After a few moments, she spun her paper around so that the group could see her work.

Offender	30 days post-release	90 days post-release	Current status
1	0	1	1
12	0	0	0
21	1	0	0
31	0	1	1
33	0	0	0
34	1	1	1
43	1	1	0
45	0	0	0
54	0	0	0

(continued)

(continued)

Offender	30 days post-release	90 days post-release	Current status
62	0	1	1
85	0	0	0
94	1	0	0

Dakota looked it over very quickly, pointing to a certain number on the table.

"Looking good so far. Now we need to determine the *Cochran Q* value."

Dakota went to the dry-erase board and wrote out an equation:

$$Q = \frac{(k-1)\left[k\sum_{j=1}^{k} G_j^2 - \left(\sum_{j=1}^{k} G_j\right)^2\right]}{k\sum_{i=1}^{N} L_i - \sum_{i=1}^{N} L_i^2}$$

N = number of cases in each of the k matched sets
k = number of related/matched samples
G_j *= total number of specified traits or qualities that are present in each column; total number of affirmative responses in each column*
L_i *= total number of specified traits or qualities that are present in each row; total number of affirmative responses in each row*

Theron quickly scribbled down everything Dakota placed on the dry-erase board, while Michael leaned back and admired her work. She was nothing if not determined. Dakota pointed to the sheet Robin created, adding another column to the end of it.

"We know that N = 12, as we have 12 offenders; and $k = 3$, since we are looking at 3 post-release times. Now, we need to sum all of the '1's and '0's together."

Offender	30 days post-release	90 days post-release	Current status	L_i
1	0	1	1	2
12	0	0	0	0
21	1	0	0	1
31	0	1	1	2
33	0	0	0	0
34	1	1	1	3
43	1	1	0	2
45	0	0	0	0
54	0	0	0	0
62	0	1	1	2
85	0	0	0	0
94	1	0	0	1
Total	$G_1 = 4$	$G_2 = 5$	$G_3 = 4$	$\sum_{i=1}^{12} L_i = 13$

Robin quickly calculated this out, smiling as she worked.

“Behold my mighty powers of addition.”

Theron giggled as Dakota looked over the numbers, silently nodding her approval. Before Robin could gloat over her mathematical victory, Dakota once again picked up her pencil and drew in one last column.

“Okay, the last thing we need to do is to take the numbers from the summation column and square them.”

Robin grunted in discontent, as Michael leaned in closer to the group.

“I hope your multiplicative powers are as good as your additive ones.”

Robin refused to even acknowledge Michael. Instead, she just put one finger to her lips and shushed him while she continued on with her work. Theron shook his head, while Dakota continued to monitor Robin’s progress. After a few moments, Robin turned the paper over for the group to see.

Offender	30 days post-release	90 days post-release	Current status	L_i	L_i^2
1	0	1	1	2	4
12	0	0	0	0	0
21	1	0	0	1	1
31	0	1	1	2	4
33	0	0	0	0	0
34	1	1	1	3	9
43	1	1	0	2	4
45	0	0	0	0	0
54	0	0	0	0	0
62	0	1	1	2	4
85	0	0	0	0	0
94	1	0	0	1	1
Total	$G_1 = 4$	$G_2 = 5$	$G_3 = 4$	$\sum_{i=1}^{12} L_i = 13$	$\sum_{i=1}^{12} L_1^2 = 27$

Dakota carefully examined the math and then continued on with her explanation.

“Now that we have all of our numbers, we just plug them into the equation”:

$$Q = \frac{(k-1)\left[k\sum_{j=1}^{k} G_j^2 - \left(\sum_{j=1}^{k} G_j\right)^2\right]}{k\sum_{i=1}^{N} L_i - \sum_{i=1}^{N} L_i^2}$$

$$= \frac{(3-1)\left[(3)\left(4^2+5^2+4^2\right) - (4+5+4)^2\right]}{(3)(13)-(27)}$$

$$= 0.3333$$

Theron quickly finished calculating the equation, showing off his final answer once he was finished. Dakota glanced over his work, her mind lost in thought as her eyes scanned over the numbers.

"Okay, for the *Cochran Q* Test, a *Statistically Significant* result means that the probability of a sex offender checking in post-release is going to vary wildly. Now, we just need to check our *Calculated Value* against the *Critical Values* for *Cochran Q*, and we are set."

Robin absentmindedly started spinning her pencil on the conference table.

"And how do we find the *Critical Value*?"

Dakota shook her head.

"*Cochran Q* uses the *Chi-Square* distribution, so we just get the corresponding *Chi-Square* value based on the number of groups we have."

Theron quickly pulled out his *Chi-Square distribution table* and handed it to Dakota. She looked it over for a moment and compared it to their answer.

"Since Q has the same approximate distribution of x^2, significance can be determined by consulting the x^2 table with $df = k - 1$. The *Critical Value* for $a = 0.05$ with $df = 2$ is 5.99. Since $Q = 0.3333 < 5.99$, we fail to *Reject the Null Hypothesis* (i.e. the probability of offenders in violation is the same across all three status times post release). We conclude that the probability of an offender being in violation 30 days post release, 90 days post release, and at his or her current status is the same; so, offenders do not seem more likely to violate as they progress away from their post-release date."

Theron jotted down their final answer, tore off the paper with their *Cochran Q* work on it, and handed it to Dakota. Dakota jotted a few notes for herself on the upper right-hand corner and slid it into her bag. Michael leaned forward towards the group, folding his hands together while he spoke.

"Okay, one down. What's next on the docket?"

Robin peered at the miniscule writing on the dry-erase board, trying to decipher what she could (which was problematic given how far back she was from the board). Dakota noticed what she was trying in vain to do and pointed to the next question on the list.

"All right, I say we address whether or not there is a difference between 'compliant' and 'in violation' status based on race and / or ethnicity?"

Michael leaned back into his chair, while Theron's eyes grew wide in shock.

"Race and / or ethnicity?!?! Isn't that a potential powder-keg?"

Dakota nodded somberly.

"That's correct."

Robin shook her head and started counting out the different offense types, while Theron continued to just stare off into the distance with a look of shock on his face.

"Um, don't we have a lot of different ethnicities?"

Robin flipped through another page on the *Data Set*.

"Well, are four different races and / or ethnicities considered to be a lot?"

Theron's jaw dropped open a little bit.

"Please tell me that there are equal numbers in all four categories?"

Robin continued to look over the *Data Set*, ignoring Theron's oncoming panic attack. She briefly lifted her eyes to the group.

"Um, no, we do not have equal numbers in all four categories. I have no idea what we are supposed to do now. But why do we keep saying 'and / or'? Since when have any of us pretended to be politically correct"

Theron wiped his hand across his face, trying in vain to force himself past the concerns he had about this potentially explosive material. In fact, he was so preoccupied with his thoughts that he completely missed Robin's verbal swipe at political correctness.

"Okay, so which statistical procedure lets us look at differences with a lot of different groups?"

Dakota lifted her eyes to the ceiling, her mind racing to see which procedure would be most appropriate. Without realizing it, she started to draw little boxes on the dry-erase board. Suddenly, the answer came to her as if it were a bolt of lightning.

"*Chi-Square*."

Theron's face took on a puzzled expression.

"Excuse me."

Dakota pointed to the little boxes on the dry-erase board.

"I think we can use the **Chi-Square r X k**."

Theron started flipping through his legal pad, while Robin closed her eyes, shook her head, and started muttering to herself.

"Oh come on, not again. Can't we 'and/or' use something else?"

Dakota smiled.

"Good memory Robin. We have discussed *Chi-Square* before, but it was in the context of the *Chi-Square Goodness of Fit Test*. The *Chi-Square r X k* test also looks at *Significant Differences* between *Independent Groups* of *Nominal Data*, but this time you can use as many groups as you need.[2] "

Michael smiled as he looked at Dakota, hoping that he could help her out in some way.

"So, this test could look at three groups against three groups, or five groups against three groups?"

Theron's ears perked up and he snapped his fingers with a blaze of enlightenment.

"Hence why it's called *Chi-Square r X k*; you can use any combination of *Nominal Variables* that you need."

Dakota nodded her assent.

"That's correct. Because this is still a *Chi-Square Test*, you are still looking for the *Frequency Observed* and comparing it to the *Frequency Expected*, it's just going to be distributed in a different way; depending upon how many variables you are looking at."

Robin leaned in over the table, looking over the numbers on the *Data Set*.

"So, this Chi-Square is going to be a 4X2?"

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[2] Refer to page 196 in Chap. 8 where the *Chi-Square Goodness of Fit Test* and related topics are discussed in detail.

Dakota thought for a moment, looking to Michael for support. Michael could see that her mind was a whirl of thought and ideas, but only he noticed the tinge of frustration in the corner of her eyes. Dakota went to the dry-erase board and started to trace out a variety of different categories on the board, still uncertain as to whether or not the 4X2 model was correct. After drawing it out a few different ways, she was appropriately pacified.

"That is correct."

Theron stuck his finger into the air.

"Okay, so we have two Nominal categories, and we know that this Chi-Square is going to be a 4X2, what do we do now?"

Dakota turned her back to the group, thinking out loud.

"Well, one of the first things we need to do is to find the *Frequency Observed* values for these Racial Categories and the 'Compliance / In Violation' Status?"

Robin grabbed her purse, readying its contents to once again be spilled in order to locate the magic lamp. Dakota shook her head at Robin.

"I don't think that will be necessary. Do we still have the original file from Jennifer?"

Theron arched his eyebrows.

"You mean the one that we got which included the entirety of our beloved *Data Set*? It's still on the back counter over there."

Dakota smoothed her hands over her skirt as she crossed the room, her gaze perpetually locked onto her intended target. As she snatched up the folder and rifled through its contents, she could feel three pairs of eyes studying the back of her head. She yanked out a particular piece of paper, tossed her hair back, and returned to her comrades in arms.

"This is where we get our *Frequency Observed* values."

The paper gently wafted in the artificial breeze of the air conditioner before sliding onto the table.

Robin rolled her eyes and shoved the paper away.

"No way. I am not playing with that *Data Set*."

Theron shook his head.

"It's just numbers. Besides, what are you going to do?"

Robin hoisted her handbag to shoulder level, deftly showing it to all of her comrades.

"Whoever shows me that *Data Set* will feel the wrath of my purse."

Theron giggled as he reached for the *Data Set*.

"Oh right, Robin and her 'purse of doom'. Come on now, it won't bite."

With disinterested looks on their faces, Dakota just stood in silence at the dry-erase board and Michael slouched down in his chair as Theron slid the *Data Set* towards Robin. Robin growled as she violently jerked her arm to the left, allowing Theron to become yet another unsuspecting victim of the "purse of doom." Theron's chair skidded back a few inches with the force of the impact, his body doubling over in pain.

"What do you have in there, bowling balls?"

A giant grin was slapped on Robin's face, making her look more like Lewis Carroll's Cheshire Cat than a political consultant.

"No, just lots and lots of make-up."

Michael sighed in the corner.

"Well, that explains why you look like a drag queen."

Robin raised the purse of doom once more, ready to once again go to war.

"Prepare to feel my wrath right in your 'Grey's Anatomy'."

Suddenly, Dakota angrily snapped her fingers in an effort to silence the utter ridiculousness splayed before her, thus halting Robin's rampage. Looking chastised, Robin slinked back to her seat and patiently waited for Dakota to resume speaking. Dakota just shook her head and returned to the dry-erase board, deftly wiping away a small corner of writing in order to clean a space for her to write. Once she had cleaned a space on the dry-erase board, she drew out three separate boxes.

"Okay, according to our *Data Set*, our *Frequency Observed* values are as follows."

	Race/ethnicity			
Current release status	Asian	Black	Hispanic	White
Compliant "0"	**6**	**14**	**29**	**12**
In violation "1"	**4**	**7**	**18**	**10**

Dakota took a step back, while Theron sat in bewilderment as he continued to look at the dry-erase board.

"How did you get these numbers from this *Data Set*?"

Dakota pointed to the *Data Set*.

"All you need to do is count the number of offenders who are compliant and in-violation in each racial/ethnic category."

Theron furrowed his brow, content to just accept this explanation at face value. Robin finally let the tension leave her body, dropping her purse to the ground with an unceremonious thud. It was much more important for her to grasp the material before her than to wage war against the egotistical medical authority seated on the opposite side of the conference table. Robin leaned forward and laid her head on the table, looking more like an exhausted child.

"Okay, we have our happy-fun *Frequency Observed* values, now what?"

Dakota was patiently standing at the dry-erase board, her mind a series of wheels and gears.

"Now we need to get the *Frequency Expected* values."

Robin shook her head.

"I feel an equation coming on."

Dakota wiped off a small portion of the board and began to write out a long string of variables.

"Don't worry, this won't hurt a bit":

$$E_{ij} = \frac{R_i C_j}{N}$$

Robin looked incredulously at the equation.

"Okay, that doesn't look too bad. This is the equation for the number of cases expected in a particular category when the *Null Hypothesis* is true. So, we just have to do this 8 times, and we can continue on?"

Dakota nodded.

"See Robin, this couldn't be simpler."

"Well, we arrange the numbers into a *Contingency Table*."

Robin's head snapped up.

"You lie. There is no way that all we have to do is just a *Contingency Table*."

Dakota shook her head.

"Nope, we just need to do your ordinary, run-of-the-mill Cross Tabulation in matrix format. With our data, it would look something like this."

	Race/ethnicity				
Current release status	Asian	Black	Hispanic	White	Total (R_i)
Compliant "0"	**6**	**14**	**29**	**12**	61
In violation "1"	**4**	**7**	**18**	**10**	39
Total (C_j)	10	21	47	22	100

Michael leaned back into his chair, his hand stroking the bottom of his chin. He thought that this gesture made him look like an erudite scholar, when in fact he looked more like the villain in a black hat that had just tied some unsuspecting young girl to the railroad tracks. He cleared his throat, readying himself to pontificate to his colleagues.

"I get it. On the *Contingency Tables*, the groups are in the columns while the rows are the different categories or *Variables*."

Dakota smiled and winked in his direction. She knew he was trying to be engaging, and he was doing this for her sake.

"True; however, we could have just as easily placed the groups in rows and the variables in columns. Next, we need to determine the expected frequency for each of the cells. For example, the expected frequency for compliant, Asian offenders can be found using the following equation where R_i is the total for the row we are looking at and C_j is the total for the column we are looking at:

$$E_{1\ 1} = \frac{R_i C_j}{N} = \frac{(61)(10)}{100} = 6.1$$

for compliant, black offenders":

$$E_{1\ 2} = \frac{R_i C_j}{N} = \frac{(61)(21)}{100} = 12.81$$

Theron shook his head, positioning himself towards Robin.

"Now I am wishing you did have a magic lamp in your purse."

Robin smiled.

“Oh come on, the worst thing that could happen is that these results could make us look just a wee bit racist. But as long as the result makes Greenleaf happy, I can live with it. Besides, we know what we are doing.”

Michael chortled with laughter, causing Robin to shake her head.

“You have no faith in us ‘regular’ people do you?”

Michael shook his head.

“I would hardly consider you ‘regular’.”

Robin was about to protest, when she felt Theron’s hand clamp over her mouth. Her eyes widened as she spun her head towards Theron, anger shooting out of her eyes. Theron’s hand held fast as he spoke to her in hushed tones.

“Why are you baiting him?”

Theron lowered his hand, allowing Robin a moment to speak. She angrily glared at Michael.

“Because I hate the fact that I agree with him. We are so not normal.”

Michael grinned from ear to ear as a look of absolute contentment crossed his face. In some ways, this was the best moment of his life, and it only came at Robin’s expense. Once he was through with his gloating, he turned his eyes towards Dakota to see what her reaction to this would be. True to form, Dakota was busily writing out one last equation and the data necessary for the equation.

	Race/ethnicity				
Current release status	Asian	Black	Hispanic	White	Total (R_i)
Compliant “0”	**6**	**14**	**29**	**12**	61
Expected	*6.1*	*12.81*	*28.67*	*13.42*	
In violation “1”	**4**	**7**	**18**	**10**	39
Expected	*3.9*	*8.19*	*18.33*	*8.58*	
Total (C_j)	10	21	47	22	100

“If you are all done bickering, we are almost done with *Chi-Square rXk*. All we need to do now is use the *Expected Frequencies* to determine the Chi-Square Value, using this equation”:

$$x^2 = \sum_{i=1}^{r} \sum_{j=1}^{k} \frac{n_{ij}^2}{E_{ij}} - N$$

n_{ij} = observed number of cases in a particular category
N = total number of independent observations

Theron smiled.

“And then we are done with the test?”

Dakota lifted her hand, holding her fingers millimeters apart.

“Not quite. But we are very, very close to being finished.”

Robin shook her head as she ripped out a piece of paper and began working on the equation:

$$\begin{aligned}x^2 &= \sum_{i=1}^{r}\sum_{j=1}^{k}\frac{n_{ij}^2}{E_{ij}} - N \\ &= \frac{6^2}{6.1}+\frac{14^2}{12.81}+\frac{29^2}{28.67}+\frac{12^2}{13.42}+\frac{7^2}{8.19}+\frac{18^2}{18.33}+\frac{10^2}{8.58} - 100 \\ &= 5.90 + 15.30 + 29.33 + 10.73 + 4.10 + 5.98 + 17.68 + 11.66 - 100 \\ &= 0.68\end{aligned}$$

Once her computation was finished, she unceremoniously shoved the piece of paper towards Dakota, who strained to grab it. After giving it a quick once over, she put the paper down and nodded towards Robin.

"Good job. Now we need to take our *Calculated Value* and compare it to the *Chi-Square Critical Value*."

Theron furrowed his brow.

"Don't we just go to the *Chi-Square Table* for that?"

Dakota nodded.

"Yes, but first we need to figure out our *Degrees of Freedom* in order to acquire the value from the table. The equation is similar to the one we used last time":

$$df = (r-1)(k-1)$$

Dakota turned to Robin, who was already punching the numbers into her calculator. Once again, she shoved a piece of notebook paper towards Dakota, who in turn snatched it up before it could slide off the table.

$$df = (r-1)(k-1) = (2-1)(4-1) = (1)(3) = 3$$

Robin, stating the obvious, spoke when no one else would.

"Now we just determine if our *Chi-Square* value is *Statistically Significant*."

Dakota smiled.

"Significance can be determined by consulting the x^2 table with $df = (r-1)(k-1)$. The *Critical Value* for $a = 0.05$ with $df = 3$ is 7.82. Since $x^2 = 0.68 < 7.82$, we fail to *Reject the Null Hypothesis* (i.e. the proportion of offenders currently in violation or compliant is the same across racial/ethnic groups). We conclude that the proportion of offenders who are currently in violation or compliant is the same between Asian, black, Hispanic, and white offenders."

Once Dakota moved away from the dry-erase board, Robin started bouncing about in her chair. The excitement was starting to become a bit too much for her.

"Only two of these stupid tests to go, and then I never have to deal with Quincy again!!!"

Theron looked towards Michael, whose face had reddened with anger to the comparison to a fictional medical examiner.

"Wow, all you did as a child was watch television didn't you?"

Robin shrugged her shoulders.

"What, it was a great show."

Dakota kept her head down, refusing to see just how angry her boyfriend had become at this pointless little tangent into classic television history. She glanced over the *Data Set*, mentally turning over the numbers in her mind. She then glanced over to the dry-erase board, staring intently at the last two research questions to remain. Silently, she muttered one of the research questions to herself.

"Is there a difference between the participant's income and their 'Level of Meanness' ratings?"

As Dakota was speaking, the room grew very silent, all of them straining to hear what was being whispered past her lips. Unfortunately, the group only heard a faint grumble from their fearless leader. Dakota's eye remained locked onto the dry-erase board, completely ignoring her surroundings as her mind attempted to wrap itself around the problem. Michael lifted himself out of his seat, deliberately moving in hopes of gaining her attention. When that failed, he waved his hands towards Dakota.

"Beg pardon beloved?"

His question was met with no response. Robin just shook her head at his vain attempt to garner Dakota's attention. She just sighed to herself as they all waited for Dakota to return to the land of the conscious.

"Come on Michael, it's obvious you broke her."

Theron instinctively dropped his head to the conference table, slamming it down in frustration. What he feared the most had come to pass; Dakota was mentally working on a problem, allowing Robin to become bored and find the best way to amuse herself. And that amusement's name was Michael. Robin leaned back in her chair, perfectly aware that Michael was once again glowering in her general direction. His voice came out in a deliberately slow manner.

"Excuse me."

Robin shook her head, pointing to Dakota as if she were an exhibit in some unseen trial.

"One clandestine rendezvous with you and a brilliant woman turns into a Stepford Wife."

Michael leapt to his feet, screaming out a string of unintelligible swears towards Robin, who just shook her head in contempt. Theron quickly took his fingers and jammed them into his ears, hoping they would be able to drown out the chaos ensuing before him. Michael lumbered towards Robin, continuing his abusive verbal tirade. Suddenly, an authoritative voice thundered across the chaos.

"**Friedman's ANOVA**."

It was Dakota, whose mind was able to find the much-needed solution after all. However, the unintended fringe benefit was that her solution was able to instantly stem the surging angry tide that was her boyfriend. Michael dutifully returned to his seat, while Robin methodically jotted down what Dakota had just said. It was as if the outburst never happened at all. Theron gingerly pried his fingers out of his ears, allowing him to hear his own voice ask the most obvious question.

"What is that?"

Dakota flashed him her beatific smile.

"*Friedman's ANOVA* is this wonderful test which allows you to look for *Significant Differences* among multiple groups of data which happen to be of a *Repeated Measures* or *Matched Pairs* Design."

Robin dropped the pencil from her hand.

"Okay, didn't we just do this like 20 minutes ago?"

Theron quickly started flipping through his notes, trying to see for himself what Dakota was talking about. Dakota gently shook her head, signaling for him to stop his quest.

"Robin, you are somewhat correct. From our earlier discussion, the *Cochran Q* test examines *Significant Differences* between *Related Samples* of *Nominal Scale*. The *Friedman's ANOVA* examines *Significant Differences* between *Related Samples* of *Ordinal Scale*. Everybody on the same page?"

Robin shrugged her shoulders and glanced at Michael.

"Hey Sigmund Fraud, were you able to follow what the nice lady had to say?"

Theron dropped his arm, staring at her.

"Okay, that reference didn't even make sense. You're pop culture references are slipping."

Robin rolled her eyes.

"I am tired, leave me alone."

Theron just grinned and looked towards Dakota.

"So, the *Null Hypothesis* for *Friedman's ANOVA* would be that all of the groups are from the same population?"

Robin's eyes grew wide.

"Excuse me?"

Theron shrugged his shoulders.

"What, I actually have been paying attention. Maybe if you spent less time making fun of the medical doctor and beating up your colleagues with a purse full of rocks..."

Robin slammed her hand on the table.

"Oh go rock yourself."

Dakota snapped her fingers again, returning their attention to her. She knew that the attention span of her colleagues was wearing dangerously thin and that she had to get as much done as possible for they devolved into utter ridiculousness.

"Good description Theron. If you find that you have *Statistical Significance* in *Friedman's ANOVA*, then your groups are different from one another. For our purposes, we want to see if there are *Significant Differences* among raters' Level of Meanness scores when offenders are grouped by income."

Michael stuck his hand in the air.

"Okay, so what do we do first?"

Dakota began hastily erasing the dry-erase board.

"Well, the first thing we need to do is create a table with the relevant data, one with the different raters in the columns and the groups listed in the rows."

Theron pulled over the *Data Set*, sketching out all of the relevant components with mercurial precision. Dakota leaned over the conference table, trying to give him enough space to finish his work. After a few minutes, Theron turned the table about and showed it to the group.

Groups (by income)	Level of Meanness		
	Rater 1	Rater 2	Rater 3
A			
B			
C			
D			

"This look to be about right?"

Dakota smiled, very pleased in what he had produced.

"That's fantastic."

Michael stuck his hand in the air. Robin stuck her finger out at him and called across the room.

"Yes pompous gas bag?"

Michael just smiled, choosing instead to keep his attention towards Dakota.

"I am assuming that we are going to run this using a *Repeated-Measures* design, instead of a *Matched-Pairs* design[3]?"

Dakota thought for a moment, giving him a subtle wink that was imperceptible to Theron or Robin.

"That's correct. Since we have the same measurement taken three separate time intervals, we're using a *Repeated-Measures* design. Theron, what do we have for the income variable?"

Theron quickly glanced at the *Data Set*.

"We have 84 different reported incomes, ranging from $0 to $34,678."

Dakota placed her hands on her hips, lancing towards the dry-erase board. Her lips moved as she silently muttered to herself as to how they can best approach this problem.

"All right. The next thing we can do is create income groups for the different offenders. I say we use income quartiles in order to create the groups."

"Okay. So now we need to create distinct categories for the income groups. Theron, what would that look like?"

Theron pulled out a fresh sheet of paper, sketching something out with a few quick flicks of his wrist.

"In this test, we are interested in whether or not there are *Significant Differences* among raters' Level of Meanness scores when offenders are grouped by income. By splitting income into quartiles, we generate four different groups. How's about this?"

[3] A more in-depth explanation of matched-pairs design can be found on page 237 in Chap. 10.

Groups (by income)
\$0–\$3,951.49
\$3,951.50–\$12,444.49
\$12,444.50–\$22,041.99
\$22,042–\$34,678

Dakota thought quietly for a moment and bean nodding her head when it was clear that this separation would work.

"That will do nicely. Now, we just write in the mean scores which coincide with the direction set forth by our lovely tables."

Robin shook her head.

"So, we just take the numbers from the data sheet and put them in the table?"

Theron shook his head.

"That's what she just said."

Dakota irritatingly shushed them, the way a librarian would, as she etched the table onto the dry-erase board. After double-checking her work, she stepped away from the board to show them all what she had done.

"Ok, so here are Means for each rater under each income group. We can see that the average level of meanness score for rater 1 in the \$0 to \$3,951.49 group is 2.8; we get that by taking all the ratings for rater 1 in the given income group and dividing that sum by the number of values we have. You can see from the data that the scores from rater 1 are 2, 2, 3, 5, 5, 5, 4, 3, 1, 4, 3, 2, 4, 3, 3, 3, 3, 1, 2, 4, 1, 1, 4, 1, 1. When we add them, we get 70. We then divide 70 by the number of values, 25, and we get 2.8. We continue the same math for the rest of the groups."

	Level of Meanness (mean scores)		
Groups (by income)	Rater 1	Rater 2	Rater 3
\$0–\$3,951.49	2.8	2.96	2.76
\$3,951.50–\$12,444.49	2.8	2.88	2.64
\$12,444.50–\$22,041.99	2.64	2.64	2.52
\$22,042–\$34,678	2.76	2.76	2.84

Robin lurched forward in her chair.

"Okay, so now that our table looks all pretty, what do we do?"

Dakota pointed to the table.

"We rank within the rows."

Robin leaned over to Theron.

"Wasn't that the big bad guy from 'Children of the Corn'?"

Theron shook his head.

"Excuse me?"

Robin just continued to stare at him.

"You know, 'Children of the Corn'. The bad movie from the 1980's where that irritating red-headed kid took Linda Hamilton hostage with a knife, and probably should have killed her in order to save us from all those terrible Lifetime movies Linda Hamilton now stars in because her career is in the toilet."

Theron dropped his head.

"Okay, two things. One, the character was called 'He who walks behind the rows'. . ."

Robin slapped him on the back.

"I knew you would understand that amazingly esoteric pop culture reference."

Theron readjusted himself and continued on.

". . .and two, I am pretty certain Dakota is going to start pelting us with dry erase markers if we don't start paying attention."

Robin instantly turned to Dakota, who was flashing an icy glare in their direction. Robin just turned forward and got very still. Once Dakota was certain that no more Stephen King references would be made, she continued on.

"Thank you. All ranking in the rows means is that we rank each number within the rows rather than the columns. For example. . ."

Dakota quickly sketched out a mock table on the board.

	Level of Meanness (mean scores)		
Groups (by income)	Rater 1	Rater 2	Rater 3
$0–$3,951.49	2.8	2.96	2.76

Dakota then drew another graph underneath the first one.

"Now, all we do when ranking within the rows is we rank from lowest-to-highest, except we do it for each group. So, our example would look something like this."

	Level of Meanness (mean scores)		
Groups (by income)	Rater 1	Rater 2	Rater 3
$0–$3,951.49	2	3	1

Dakota turned toward the group.

"Got it?"

Before she could get a hint of recognition from her comrades, Theron was already at work on the rest of the rankings for each of the rows. Within a few moments, he slipped a piece of paper to her.

"You mean, like this?"

	Level of Meanness (mean scores)		
Groups (by income)	Rater 1	Rater 2	Rater 3
$0–$3,951.49	2	3	1
$3,951.50–$12,444.49	2	3	1
$12,444.50–$22,041.99	2.5	2.5	1
$22,042–$34,678	1.5	1.5	3

Dakota just smiled in this direction.

"This is fantastic. Now we need to get the value for R_j and R_j^2."

Theron's eyes grew wide as Robin just dropped her pencil, folding her arms in disgust.

"That's not even English."

Dakota just smiled.

"In order to find R_j, you need to determine the sum of the ranks within each column. Then, you take the R_j value and square it."

Robin shrugged her shoulders.

"Oh, well why didn't you just say that to begin with?"

Michael just snickered as Theron altered the chart to include the R_j and R_j^2 values.

	Level of Meanness (mean scores)		
Groups (by income)	Rater 1	Rater 2	Rater 3
\$0–\$3,951.49	2	3	1
\$3,951.50–\$12,444.49	2	3	1
\$12,444.50–\$22,041.99	2.5	2.5	1
\$22,042–\$34,678	1.5	1.5	3
R_j	8	10	6
R_j^2	64	100	36

Robin looked over the finished chart.

"Okay, now what?"

Dakota just began writing on the board.

"Now we need to compute the F_r value."

Robin shook her head.

"You lied to me. You said this test was going to be easy, and now you are telling me that there are equations which don't even have proper names attached to them."

Theron opened his mouth, prepared to once again be Dakota's stalwart defender. However, he suddenly realized that it was going to be rather difficult to defend a fairly obtuse equation which was denoted with only two letters from the alphabet. He mournfully looked towards Dakota, who quickly sketched out the equation on the dry-erase board:

$$F_r = \left[\frac{12}{Nk(k+1)}\sum_{j=1}^{k} R_j^2\right] - 3N(k+1)$$

N = the total number of rows
k = the total number of columns
R_j = sum of the ranks in jth column

Dakota stepped away from the board, allowing the group to analyze it to the best of the ability. She lowered her head silently, letting the stress of the moment wash away from her. She decided the best way to explain this.

"Okay, even I will admit that the equation seems somewhat ridiculous. But once you get information from ranking the rows and summing the columns, you need to be able to turn that into a *Calculated Value* which you can then compare to a *Critical Value*. The equation for F_r allows us to do just that."

Michael leaned forward in his chair.

"What would you do if there were tied observations in your *Data Set*?"

Robin slammed her palms onto the arms of her chair, but the gesture was quickly stifled by Theron, who placed his forearm forcefully over hers. Apparently, the question was enough to pique his curiosity. Dakota smiled warmly towards Michael, taking it as a great compliment that he was willing to ask her a question in front of the others.

"Well, there is a separate F_r equation which takes tied observations into consideration."

After Dakota finished writing, she tilted the board to the corner of the room so that he could see it better:

$$F_r = \frac{12\sum_{j=1}^{k} R_j^2 - 3N^2k(k+1)^2}{Nk(k+1) + \frac{\left(Nk - \sum_{i=1}^{N}\sum_{j=1}^{g_i} t_{i.j}^3\right)}{(k+1)}}$$

g_i = *number of sets of tied ranks in the* i^{th} *group*
$t_{i.j}$ = *the size of the* j^{th} *set of tied ranks in the* i^{th} *group*

"Fortunately, we have tied observations in this instance so we will get to see how this equation comes out."

Theron seemed less at ease than before he had seen the first equation but was able to mask it well.

"Okay, so we take the information from the columns and just plug it into this formula, right?"

Dakota nodded.

"Just like riding a bicycle."

Robin angrily started grinding her pencil into the top of her legal pad.

"Stupid algebraic bicycle."

Theron carefully crunched the numbers together, making certain that all the components from their columns and rows found their way into the equation. After a few moments, he turned the legal pad over to Dakota.

"Since ties appear within our ranks, we must use the equation which corrects for ties. When correcting for ties, we must also take into consideration the ranks that are not tied. For example, if a group has 3 tied rankings, we would say it has a size of 3. If a group has three scores that are not tied, we would count each individual rank and say it has a size of 1 because it does not share a rank with another rater; that way the individual ranks are still accounted for in the equation. Now, in our data, we have two ties of size 2 (Ranks of 2.5 for Rater 1 and Rater 2 in the 3rd income group; Ranks of 1.5 for Rater 1 and Rater 2 in the 4th income group). They are given a

2 because there are 2 raters that are tied. We also have 8 individual ranks that are each counted as 1."

	Level of Meanness (mean scores)		
Groups (by income)	Rater 1	Rater 2	Rater 3
\$0–\$3,951.49	2	3	1
\$3,951.50–\$12,444.49	2	3	1
\$12,444.50–\$22,041.99	2.5	2.5	1
\$22,042–\$34,678	1.5	1.5	3
R_j	8	10	6
R_j^2	64	100	36

"Once we determine the total number of ties, we use the equation that corrects for ties. This is what I got":

$$\sum_{i=1}^{N}\sum_{j=1}^{g_i} t_{i,j}^3 = 1+1+1+1+1+1+1+1+8+8 = 24$$

We can now solve for F_r:

$$F_r = \frac{12\sum_{j=1}^{k} R_j^2 - 3N^2k(k+1)^2}{Nk(k+1) + \frac{\left(Nk - \sum_{i=1}^{N}\sum_{j=1}^{g_i} t_{i,j}^3\right)}{(k-1)}} = \frac{12(200) - 3(4)^2 3(3+1)^2}{(4)(3)(3+1) + \frac{[(4)(3)-24]}{(3-1)}} = 2.29$$

Dakota beatifically grinned towards Theron, long ago trusting his ability to conduct these calculations. Out of the corner of her eye, she could see Michael sitting off in the distance. He was leaning towards Theron and Robin, also interested in what they were doing. Robin folded her arms across her chest.

"And this pretty number tells us..."

Dakota just smiled.

"All we need to do is compare this number to the table which gives us the *Critical Value* for *Friedman's ANOVA*."

Dakota returned her attention to Theron and Robin, who had long ago tuned her out of their consciousness. Theron had snapped the worn manila folder towards him and was quickly rifling through all the various *Critical Values* tables which they had collected since beginning this endeavor.

"Which one is it?"

Dakota pulled two separate charts out, handing them both to her colleague.

"Well, technically there can be two. There is a chart of the *Critical Values* for *Friedman's ANOVA*, but if your sample size is too large, it won't work."

Robin started fidgeting in her seat.

"So, is this where you tell me that our sample size is too large and the test is a total wash?"

Dakota continued scanning the chart.

"Hardly. If you have a large sample size, you would look at the *Chi-Square Distribution Table*, and moderate your *Degrees of Freedom* with looking at the number of groups minus 1 ($df = \mathrm{k} - 1$).[4]"

Theron just stared at Dakota.

"So, which one do we use?"

Dakota gently placed the piece of paper onto the hard furnished wood, the lacquer from her nail polish pointing to a specific number.

"Well, we have a pretty small sample size, so we can just use the *Friedman's ANOVA* table."

Theron craned his neck, looking past the reflective surface of her nail polish.

"For $a = 0.05$, the *Critical Value* with $N = 4$ and $k = 3$ is 6.50. Since $F_r = 2.29$ is less than the *Critical Value* of 6.50, we fail to *Reject the Null Hypothesis* (i.e. all of the samples came from the same population). We conclude that raters' Level of Meanness Scores across the different income levels come from a similar population."

Robin indifferently glanced at the chart before flicking it in Michael's general direction.

"Why is it every time we solve a stats problem, I always expect it to be a fascinating triumph of the human spirit, and then it's something of a letdown?"

Michael brusquely snatched the chart, examining it for himself.

"Maybe because you have nothing else going on in your life?"

Robin glared at Michael. Dakota turned to the board, hoping that the simple elegance of the task ahead could drown out the inane foolishness of the conversation unfolding before her; at the very least, she could head it off. Dakota pointed at one small corner of the board.

"Are minority races/ethnicities receiving higher General Aggression Scores?"

Theron leaned forward in his chair, as Robin turned her penetrating glare away from Michael and towards Dakota. Robin quickly chimed in, answering Dakota's prayer of ending the ridiculous conversation.

"Was that a rhetorical question?"

Theron shook his head.

"No, I think it's one of our questions."

Dakota pulled the *Data Set* over for the group to see, pointing at the data for "race" and the data for the "General Aggression Scores."

"Okay, we need to find out whether or not these minority groups are getting higher General Aggression Scores'. . ."

Michael cleared his throat.

"Why don't we just run an *ANOVA*? It will be able to search for *Significant Differences* among multiple groups."

Dakota pensively furrowed her brow.

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[4] Chi-Square is discussed in greater detail beginning on page 196 in Chap. 8.

"Good idea, but it won't work. *ANOVA* requires at least *Interval Scale Data*. But the data from the 'General Aggression Scores' are *Ordinal*."

Michael just looked downward, actively trying to use the best of his deductive reasoning to help Dakota come up with a solution.

"Okay, so we need an ANOVA-type test that will help us to figure out whether or not *Significant Differences* exist between *Independent* groups, where the data are *Ordinal* in scale. What does that leave us with?"

Dakota's mind suddenly seized upon the solution, spilling the answer out of her lips in a hushed whisper.

"**Kruskal-Wallis ANOVA.**"

Robin recoiled in horror.

"Ew!!! That sounds like what my boyfriend gave me in college."

The room grew silent as all three consultants stared blankly at Robin. Rather than be ashamed of her minor faux pas, Robin just straightened herself up in her chair.

"What? Penicillin cleared it right up."

Theron slammed his hand onto the table, while Michael just grunted and turned his attention towards Dakota. He had to admire his beloved's ability to explain a statistical procedure after that little outburst. Dakota just wiped a portion of the dry-erase board clean, pretending to herself that the past few minutes did not take place.

"As Michael said, *Kruskal-Wallis ANOVA* is this great test which allows us to look at whether or not *Significant Differences* exist among multiple *Independent* groups."

Theron stuck his hand out, blurting out his question with lightning speed.

"Except that you use *Ordinal Scale* data?"

Dakota thought for a moment.

"Well, the test is designed for data that is *Rank-Ordered*. So, as long as your data can be numerically ranked from highest-to-lowest, this test is appropriate."

Michael leaned forward, his hands kneading the veneered wood of the conference table underneath them. If Dakota didn't know any better, she would swear that he was becoming more of a support for her than ever before.

"So, what do we do first?"

Dakota glanced over the table from the *Data Set*.

"Okay, well the first thing we are going to need to do is to divide the *Data Set* up, thus allowing us to look at how the racial groups are situated against one another. Now, I believe that we have four independent racial / minority groups: Caucasian, African-American, Hispanic, and Asian-American. Is that correct?"

Robin gave a sideways glance towards the *Data Set*.

"Yup, that is correct. . .politically correct, even."

Theron just shook his head as he pulled open a new sheet of paper from his legal pad.

"Well, can we just put each one of the racial groups into different columns and fill in the data?"

Dakota nodded.

"That would be the easiest way to go. Just make sure that you keep the proper scores in the column with the proper racial category."

Theron quickly etched out four separate columns, carefully filling in the appropriate numbers where it was needed. Once completed, he slid the legal pad towards Dakota, who carefully began to trace a copy of it on the dry-erase board.

	GAS (Scores)				
Racial/ethnic group	Asian	Black	Hispanic		White
	75	65	49	63	59
	73	53	47	76	41
	43	52	58	40	43
	65	59	45	55	54
	42	41	45	51	58
	42	60	53	46	58
	47	65	40	43	59
	50	47	60	56	64
	45	43	48	44	62
	78	42	78	42	50
		53	53	67	42
		43	50	45	60
		55	47	56	56
		59	54	68	51
		69	52	56	70
		58	74	53	47
		40	68	44	65
		74	60	56	55
		44	43	40	45
		40	44	70	45
		53	44	47	55
			53	49	55
			48	42	
			43		

Once completed, Dakota took a step back to make certain that both the information on the dry-erase board and the information on the piece of paper were synchronized. Robin just stared at Dakota, not entirely certain what to say.

"Okay, so is there a reason we made our friend with the legal pad trace out this information only to have you redo his work with 30 point font?"

Dakota smiled.

"Dividing up the data is just step one of the process. Now we need to *Rank-Order* the General Aggression Scores data from highest to lowest. We need to *Rank-Order* all 100 of our offenders together but then split them up into Race/Ethnicity after *Ranking* them."

Theron just shrugged.

"Okay, so how do we do that?"

Dakota popped open the cap to the marker.

"Well, see this score of '78'. That would be rank ordered as number 1. The next highest score would be number 2, and so on."

Theron started *rank ordering* the data, yet froze unexpectedly.

"Okay, what do we do if there are ties? We have two different scores of '78' in the *Data Set*."

Dakota just stared at the *Data Set* on the board, trying to think of the best way to explain this.

"Good eye Theron. Well, the first two rankings would be '1' and '2', correct?"

Robin nodded.

"Yes. That is known as counting."

Dakota just grinned warmly at Robin.

"Right. And we know that we have a total of 100 participants in the *Data Set*. So, the rankings are going to be from 1-100."

Robin shrugged.

"You're still not explaining ties."

Dakota pointed to the two different scores of 78 which were written on the dry-erase board.

"Okay, so we know that these two scores have to be ranked as '1' and '2'. What you do if there is a tie is you take the average of the ranking designation. So, both of these scores of '78' would be ranked as '1.5'. Then, the next highest score on the *Data Set* would be ranked as number 3."

Michael leaned back into his chair.

"So, we are basically getting the mean for the tied ranks?"

Dakota smiled.

"Couldn't be simpler."

Still sensing some confusion in the room, Dakota quickly sketched out a quick drawing of this on the dry-erase board to underscore her point.

Score	Rank
78	1.5
78	1.5
60	3
56	4
55	5
54	6

Robin looked bewildered, but Theron just nodded his head.

"I get it. So if the next two scores were also tied, you would average their rankings of '3' and '4', and the ties would be numbered as 3.5."

Dakota smiled.

"That's the idea. If there are multiple ties, you just figure out the average of the rankings for however many scores you have, and assign that number to all tied scores."

Theron let out an exasperated sigh and continued plugging away on his legal pad. Robin was now staring off into space, and Michael just leaned towards the rest of the group. After a few moments, he spun the legal pad about and slid it across the table.

	GAS (Scores)							
Racial/ethnic group	Asian	*Rank*	Black	*Rank*	Hispanic	*Rank*	White	*Rank*
	42	*11*	40	*3*	40	*3*	41	*6.5*
	42	*11*	40	*3*	40	*3*	42	*11*
	43	*17*	41	*6.5*	40	*3*	43	*17*
	45	*29*	42	*11*	42	*11*	45	*29*
	47	*36*	43	*17*	42	*11*	45	*29*
	50	*44*	43	*17*	43	*17*	47	*36*
	65	*86*	44	*23*	43	*17*	50	*44*
	73	*94*	47	*36*	43	*17*	51	*47*
	75	*97*	52	*49*	44	*23*	54	*58*
	78	*100*	53	*53*	44	*23*	55	*61*
			53	*53*	44	*23*	55	*61*
			53	*53*	44	*23*	55	*61*
			55	*61*	45	*29*	56	*66*
			58	*71*	45	*29*	58	*71*
			59	*75*	45	*29*	58	*71*
			59	*75*	46	*32*	59	*75*
			60	*79*	47	*36*	59	*75*
			65	*86*	47	*36*	60	*79*
			65	*86*	47	*36*	62	*81*
			69	*91*	48	*40*	64	*83*
			74	*96*	48	*40*	65	*86*
					49	*42*	70	*93*
					49	*42*		
					50	*44*		
					51	*47*		
					52	*49*		
					53	*53*		
					53	*53*		
					53	*53*		
					53	*53*		
					54	*58*		
					55	*61*		
					56	*66*		
					56	*66*		
					56	*66*		
					56	*66*		
					58	*71*		
					60	*79*		
					60	*79*		
					63	*82*		
					67	*88*		
					68	*90*		
					68	*90*		
					70	*93*		
					74	*96*		
					76	*98*		

(continued)

(continued)

	GAS (Scores)							
Racial/ethnic group	Asian	*Rank*	Black	*Rank*	Hispanic	*Rank*	White	*Rank*
					78	*100*		
n_j		10		21		47		22
R_j		522		1039.5		2254.5		1234
R_j		52.2		49.5		47.97		56.1

Dakota looked over the notepad.

"That looks pretty good. Now, we just need to correct it."

Robin rolled her eyes.

"Haven't we all reached the conclusion that Theron tends not to screw up on the calculation department?"

Dakota's face was a mask of confusion. Michael just swiveled in his chair, letting out a melodramatic sigh.

"She means that you have to calculate a Correction Factor to compensate for the ties."

Robin's eyes narrowed, and her jaw clenched tightly. It was one thing to have Dakota correct her mistakes, but it was another thing to have Michael do it. Through her gritted teeth, she angrily growled towards her colleague.

"The only thing you know about 'ties' is when you have to pay a wom…"

Dakota dropped the dry-erase marker down onto the conference table, instantly cutting off Robin's vituperative statement. Both Robin and Michael snapped to attention, quietly looking at Dakota. Michael's face blushed red; he was embarrassed to lose his composure in front of her. Dakota quickly etched two symbols on the legal pad, spun it around, and shoved it towards Theron.

"Okay, now we need calculate the *Sum of the Ranks* and the *Mean of the Ranks* for each of the groups. The sums are denoted by R_j, while the means are denoted by $\overline{R}_j$."

Theron just shrugged, jotted down a few numbers, and slid the pad back towards Dakota. She quickly turned and started writing a new equation on the dry-erase board. Robin slumped down in her chair.

"Now what?"

Dakota stepped out of the way, so that the group could see what she had been writing on the board.

"Now we calculate the Kruskal-Wallis ANOVA. In order to keep the equation from becoming too lengthy, we abbreviate Kruskal-Wallis with *KW*. It just makes the equation a bit cleaner":

$$KW = \frac{\left[\frac{12}{N(N+1)}\sum_{j=1}^{k} n_j\overline{R}_j^2\right] - 3(N+1)}{1 - \frac{\sum T}{N^3 - N}}$$

$N = \textit{the total number of cases}$
$n_j = \textit{the number of cases in a sample}$
$k = \textit{the number of samples}$
$R_j = \textit{sum of the ranks in } j^{th} \textit{ column}$
$R_j = \textit{average ranks of the jth column}$
$\overline{R} = \frac{(N+1)}{2} = \textit{average ranks in combined sample, the grand mean}$
$T = t^3 - t$
$t = \textit{the number of tied observations in a group of ties}$

Dakota pointed to the equation as Theron reached across the table to snatch his legal pad back.

"Should be easy enough to figure out. Since ties appear within our ranks, we again must use the equation corrected for ties. To use the corrected equation, we must first determine T for each of the 24 groupings of ties":

GAS	40	41	42	...	74	78
t	5	2	6	...	2	2
T	120	6	210	...	6	6
$\sum T$	2106					

As he was jotting things down, Michael swiveled his chair so he could face Dakota (while simultaneously blocking out Robin).

"So, there are other versions of this equation?"

Dakota smiled and began writing on the dry-erase board once more.

"Something like that. If you have no ties in the data, then you would just proceed to figure out the *sums of the rankings* and the *means of the rankings*. Whether you have a small sample size or a large sample size, you would use this equation..."

$$KW = \left[\frac{12}{N(N+1)}\sum_{j=1}^{k} n_j \overline{R}_j^2\right] - 3(N+1)$$

Theron stopped writing and once again slid the paper towards Dakota. He gingerly placed the point of his pencil against the lacquered surface of the conference table. Dakota smiled through the strands of hair which cascaded over her face.

$$KW = \frac{\left[\frac{12}{N(N+1)}\sum_{j=1}^{k} n_j \overline{R}_j^2\right] - 3(N+1)}{1 - \frac{\sum T}{N^3 - N}}$$

$$= \frac{\frac{12}{100(100+1)}\left[10(52.2)^2 + 21(49.5)^2 + 47(47.97)^2 + 22(56.1)^2\right] - 3(100+1)}{1 - \frac{2106}{1000000-100}}$$

$$= \frac{\frac{12}{10100}[27248.4 + 51455.25 + 108152.68 + 69238.62] - 303}{.9979} = 1.27$$

"Looks good. Now, if I am remembering this correctly, all we need to do is to determine *Significance*."

Michael gazed intently at Dakota.

"How do we do that?"

Dakota continued to look at Theron's calculations.

"Well, the *Degrees of Freedom* are based on the number of groups minus one or $df = k - 1$. To determine significance, we consult the x^2 table. The *Critical Value* for $a = 0.05$ with $df = 3$ is 7.82. Since $KW = 1.27 < 7.82$, we fail to *Reject the Null Hypothesis* (i.e. there is no difference in GAS based upon ethnicity/race)."

Michael leaned forward.

"So, is it *Significant*?"

Dakota, Robin and Theron simultaneously turned to look at Michael assuming that by now he would have some idea of what rejecting and failing to *Reject the Null Hypothesis* meant. Robin, with a look of defeat, addressed the question.

"No, there is no significance for this test. We conclude that GAS scores are similar across ethnic/racial groups; therefore, minority ethnic/racial groups are not receiving higher General Aggression Scores."

Dakota lowered her head, feeling a wave of relief wash over her. It was as if Governor Greenleaf's campaign was dependent upon her ability to do her job, and she was languishing in the joy of completing her task. Robin slowly started gathering her things together, stopping only for a moment to see the wave of relief in her colleague's face. She snickered, elbowing Theron in the ribcage.

"So that's what Ayn Rand meant when she said 'Atlas Shrugged'."

Theron rolled his eyes.

"Do you ever shut up?"

Robin just smiled, shaking her head like a petulant school girl. Michael silently rose from his chair and stood by Dakota, reassuring placing a hand on her shoulder. With that one gesture, she knew she had not only done her job well. Her eyes fell open for a moment, focusing on her colleagues as they were about to venture into the afternoon sunlight.

"I say we meet early to go over the report before we give it to the Governor. I will take all our work home and type it up. How about we meet at six?"

Both Theron and Robin just shrugged their shoulders and ventured out into the crowded campaign office. Although she could hear the headquarters brimming with people, Dakota just lowered her head and felt a swath of the afternoon sun flitter through the window and onto her face. She had done her best, and that felt glorious.

Chapter Summary

- In this final chapter before presenting findings to the governor, the team attempts to wrap up the few remaining research questions in order to provide the clearest picture possible for the governor.
- To address whether or not 3 or more matched samples are different from one another, the Cochran Q is explained.
- The Chi-Square r X k test is used to determine how 2 or more groups differ when the data are at least nominal.
- Friedman's ANOVA and Kruskal-Wallis ANOVA are presented to determine whether 3 or more groups come from the same population. Friedman's ANOVA addresses related samples while Kruskal-Wallis ANOVA addresses unrelated samples.
- Observed and Expected Frequencies are revisited for specific application in the Chi-Square r X k test.

Check Your Understanding

1. How does the Chi-Square r X k test differ from the previously discussed Chi-Square Goodness of Fit test?
2. You are asked to find significant differences between criminal justice, psychology, and sociology students at an Ivy League school. Are these independent or related samples? Which statistical test is most appropriate to use for this research question?
3. A pharmaceutical company asks for your help regarding a new cancer fighting drug. You gather data from 50 people over a 12-month period at various intervals to see if the drug is having any effect. If you want know if there is a significant difference between the time intervals, would you use a Friedman's or Kruskal-Wallis ANOVA? Why?
4. Which of these tests are used for related sample data?

   a. Cochran Q
   b. Chi-Square r X k
   c. Friedman's ANOVA
   d. Kruskal-Wallis ANOVA

5. Which of these tests are used with nominal or dichotomous data?

   a. Cochran Q
   b. Chi-Square r X k
   c. Friedman's ANOVA
   d. Kruskal-Wallis ANOVA

6. Which of these tests require ranked data?

   a. Cochran Q
   b. Chi-Square r X k
   c. Friedman's ANOVA
   d. Kruskal-Wallis ANOVA

Chapter 13
Presentation to the Governor

Abstract All of the team's hard work finally pays off as they present the governor with their findings. The team presents their findings to the governor in the form of a briefing report. The consultants design a platform informed by several policy goals and based upon a five-part containment approach to sex offender management which is presented in the formal briefing report. Policy recommendations are then made based upon the five-part approach. Some of the findings in the report include no difference in sentencing for those charged with an offense against a minor 15 years or younger and an offense against an adult, consistency among raters on the Level of Meanness scale, no difference in release condition compliance based upon race or upon whether or not they require medication, and an association between the General Aggression Score and length of sentence. The team is relieved to learn that Governor Greenleaf is pleased with the report and tasks Jennifer with determining some talking points for his campaign.

"It sounds like a blue jay."

Theron's neck was craned towards the window in a vain attempt to see a tiny bird through the closed slats of the vertical blinds. Robin just looked towards Theron with a look of horror on her face.

"Is a cat eating it? It sounds like the poor thing is being tormented."

Theron's lips curled in a smile as he continued to hunt for the bird.

"No, that's just a blue jay's song."

Robin just shook her head.

"And. . .are you hoping to kill it? Because I have to say, it's not making me want to own a blue jay."

Dakota smiled, glad that something was able to break the uncomfortable silence which settled itself over the room. All four of them had been seated quietly for the past hour, waiting for Nathanial Greenleaf to grace them with his presence. Dakota made sure to get there early, so that copies of their final report could be made. She felt a shiver up her spine and tilted her head to see what was behind her. It was her trusty dry-erase board, wiped clean and standing silently in the back of the

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room. For a moment, she actually felt a twinge of grief at losing what had almost become like a friend to her. However, her silent moment of goodbye towards her gleaming white companion was snatched away by the sound of the conference room doors flying open. Dakota's attention was now focused onto Nathanial Greenleaf.

"Well now, let's see what we have here."

Suddenly, Dakota's eyes snapped away from the Governor and focused onto the varnish for the conference table. For some reason, she could not bear to bring herself to look at either Nathanial Greenleaf or Jennifer, both of whom now had copies of their final report in hand. Dakota was never very good at having people read her work while she was sitting there, and today was no different. All Dakota could do was close her eyes and listen to the rustling of papers as they started to read the final result of all their hard work.

Greenleaf for U.S. Senate

PROPOSED SEX OFFENDER POLICY PLATFORM

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EXECUTIVE SUMMARY

The consultants commissioned for this report were specifically tasked with the development of a policy platform that addressed the management of sexual offenders within the State of California. The proposed platform was constructed for use by Governor Nathanial Greenleaf in his campaign for one of the State's U.S. Senate seats.

In order to develop a potential platform for Governor Greenleaf, the consultants commissioned for this report studied a group of 100 registered sex offenders within the State of California. The results of that investigation allowed the consultants to then design a platform informed by several policy goals and based upon a five-part containment approach to sex offender management: 1) victim-centered philosophy, 2) multidisciplinary collaboration, 3) containment-focused risk management, 4) informed and consistent public policies, and 5) quality control (English, 2009).

Specific policy recommendations include:

Recommendation 1:1: Comprehensive medical and mental health services for victims and their families should be included in every sex offender policy. Access to these services should be provided for little or no cost and not be limited to victims who officially reported crimes to law enforcement agencies.

Recommendation 1:2: Victim advocates need to be included as part of policy development.

Recommendation 2:1: State and parole officials should continue to collaborate with medical/clinical treatment providers to increase medication compliance and positive treatment outcomes.

Recommendation 2:2: State and parole officials should continue to collaborate with clinicians and researchers on the development of assessment materials that can be used in parole and supervision decisions.

Recommendation 3:1: All offenders' statuses should be collected at 90 days post-release and again on a uniform, consistent schedule.

Recommendation 4:1: Residency restrictions should be monitored to determine their effects (if any) on offenders becoming transient.

Recommendation 5:1: Research on the validity of the Level of Meanness and General Aggression Score (GAS) assessments needs to be conducted, along with continued evaluation of their use.

Recommendation 5:2: Measures tracking the effectiveness of State polices must be developed. The outcome of these measures should then be used to modify existing policy or to introduce new policies.

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CURRENT STUDY

Within the last several years, several high profile incidents involving registered sex offenders have brought considerable attention to the method in which the State of California manages its sex offender population. State officials involved with the management, supervision, and treatment of sex offenders began implementing new supervision and release strategies based upon outcomes of objective criminological measures (i.e. the Meanness Scale and the General Aggression Score). These measures were designed to assist professionals in determining the likelihood of re-offending and whether or not an offender was a danger to the community. Despite these new efforts, little research has been done on their efficacy.

In addition to the creation of objective release measures, greater attention has also been given to the compliance statuses of offenders. Especially given the dramatic increase in the number of transient offenders in the State, concerns have been raised by practitioners, researchers, and the public about the number of offenders who are in violation of their probation and parole conditions. Given the economic condition of the State, fewer resources are available to justice agencies in general and sex offender management officials in particular; therefore, understanding the factors that contribute to offenders violating their conditions of release is crucial.

In order to develop a potential platform for Governor Greenleaf, the consultants commissioned for this report studied a group of 100 registered sex offenders within the State of California. The results of that investigation allowed the consultants to then examine several of these aforementioned policy issues in greater detail. Based upon the results of their investigation, the consultants recommend a comprehensive policy platform that encompasses a 1) victim-centered philosophy, 2) multidisciplinary collaboration, 3) containment-focused risk management, 4) informed and consistent public policies, and 5) quality control (English, 2009).

PARTICIPANTS

Registered sex offenders who lived within the zip code and a 20-mile radius of the campaign headquarters were selected to be surveyed. One-hundred participants (N=100) were identified by the campaign staff. See Tables 1 and 2 for a summary of participants' demographic information.

INFORMATION COLLECTED

Once participants were identified by the campaign staff, information was collected about their demographic characteristics and offense history through a personal interview. Additional information was collected and/or verified by the California Department of Corrections and Rehabilitation (CDCR) Division of Adult Parole Operations (DAPO). Specific information collected included:

1) age, at the time of the interview,

2) ethnicity/race, self-identified by the participant,

3) offense that required registration as a sex offender; for offenders with multiple offenses, Count One of the listed offenses was recorded (see Table 4 for a list of offenses),

4) sentence served, in years,

5) living location, identified by the campaign staff as incorporated area, unincorporated area, or transient,

6) post-release status at 30 days, 90 days, and at the time of the interview [identified as "current status"]; post-release statuses indicated whether or not the participant was complying with or in violation of parole conditions,

7) estimated yearly income,

8) Level of Meanness Scores, provided by previous assessments from three separate raters,

9) General Aggression Score (GAS), provided by previous assessment,

10) medication status, self-identified by the participant as whether or not he/she is currently taking medication as treatment for the registered offense,

11) total testosterone level (measurements in Conventional Units),

12) type of medication (if applicable); medication types included antidepressants (SSRI), Antiandrogens, anxiolytics, and mood stabilizers; participants who were prescribed an Antiandrogen were furthered identified by the campaign staff as taking hormonal treatment,

13) and participants' testosterone level after treatment (if applicable)

RESULTS

DEMOGRAPHIC INFORMATION

Overall, the participants included in the current study are predominantly male, matching the known population of registered sex offenders. Approximately half of the participants are Caucasian/Hispanic, which is consistent with the demographics of the general population of the area; the number of Black/African American participants, however, is much greater than the community population of Black/African Americans, which is closer to 6%. The number of Asian participants is the same as the general community population, but the number of Caucasian/Non-Hispanics in the study is lower than the general community population, which is approximately 32%.

Participants range in age from 19 to 84 years, with an average age of almost 52 years. The median estimated yearly income for the participants in the current study is $12,444.50, well below that of the community's median household income which is closer to $47,000 a year. According to the 2012 Poverty Guidelines for the for the 48 Contiguous States and the District of Columbia provided by the Department of Health and Human Services, the poverty guideline for a single person household is $11,170 ("Annual Update," 2012). Assuming a single person household for the participants in the study, the average participant is just slightly above the established poverty guideline.

All of the participants included in the study are current parolees and therefore under supervision by DAPO. In 2011, the California Sex Offender Management Board (CASOMB) indicated that the number of transient parolees was close to 32%, thereby making the number of transient participants in the current study less than expected (California Sex Offender Management Board, 2011). See Tables 1 and 2 for a summary of basic participant demographic information.

Table 1
Participant Sex, Ethnicity/Race, and Living Location (N=100)

Variable	Category	Percent
Sex		
	Male	90%
	Female	10%
Ethnicity/Race		
	Asian	10%
	African American/Black	21%
	Caucasian/Hispanic	47%
	Caucasian/Non-Hispanic	22%
Living Location		
	Incorporated Area	68%
	Transient	27%
	Unincorporated Area	5%

Table 2
Participant Age and Estimated Yearly Income (N=100)

Variable	Minimum	Maximum	Mean	Median	Standard Deviation
Age (in years)	19	84	51.89	51	17.19
Estimated Yearly Income	$0	$34,678	$13,672.48	$12,444.50	$10,573.83

RACE/ETHNICITY AND LIVING LOCATION

Analysis of race/ethnicity and living location revealed no association between a participant's race/ethnicity and whether or not he/she lives in an incorporated area, unincorporated area, or is transient ($C[8, 100]=0.14$, $p>0.05$).[1]

Table 3
Race/Ethnicity by Living Location (N=100)

	Race			
Living Location	Asian	Black/African American	Caucasian/Hispanic	Caucasian/Non-Hispanic
Incorporated Area	8	15	29	16
Unincorporated Area	0	2	2	1
Transient	2	4	16	5

TYPES OF OFFENSES

[1] Chapter 4: Cramer Coefficient

Within the study sample, participants are charged with 20 different count one offenses, all of which require registration as a sex offender within the State of California. Information that appears on the Megan's Law public website, however, differs depending on the severity of the participant's offense. According to California Penal Code § 290.46, registrants may be required to register under the home address category, conditional home address category, zip code category, undisclosed category, or they may apply to be part of the excluded category, which allows certain offenders to be excluded from the website if they meet certain criteria (California Department of Justice, Office of the Attorney General, 2013b; Cal. Pen. Code § 290.46). See Table 4 for a list of participants' offenses by disclosure category.

Table 4
Offenses by Megan's Law disclosure category (N=100)

Home Address (74% of participants in the current study)		Conditional and Zip Code (25% of participants in the current study)		Undisclosed and Excluded (1% of participants in the current study)	
n	Offense	*n*	Offense	*n*	Offense
2	Continuous sexual abuse of child	1	Rape: Victim unconscious of the nature of the act	1	Indecent exposure
10	Lewd or lascivious acts with child 14 or 15 years old	5	Assault to commit rape, sodomy, or oral copulation		
40	Lewd or lascivious acts with child under 14 years	1	Oral copulation with person under 16 years		
1	Lewd or lascivious acts with child under 14 years with force	8	Sexual battery		
5	Oral copulation with person under 14 or by force	1	Entice minor female for prostitution		
2	Oral copulation with person under 18 years	1	Contacting minor with intent to commit sex offense		
8	Rape by force or fear	8	Annoy/molest child under 18 years old		
1	Rape in concert by force				
2	Rape in concert with force or violence				
1	Sexual penetration by foreign object				
1	Sexually exploit minor: depict minor in sex act				
1	Sodomy with person under 14 years or with force				

The California Department of Justice, Office of the Attorney General, provides the number of registrants per category, and according to the statistics provided in February 2013: 49.6% of offenders are required to register on the Megan's Law website under the home address category; 14.5% are required to register on the Megan's Law website under the conditional and zip code categories; and 35.9% are still registered offenders but are undisclosed or excluded from the Megan's Law website (California Department of Justice, Office of the Attorney General, 2013a). The percentage of participants in the aforementioned disclosure categories varies greatly from the percentages that would be expected, according to the data provided by the Attorney General. Analysis of the concentration of participants in each of the disclosure categories revealed statistically significant differences between the current study participants and the offenders in the State, $x^2(2, 100)=53.53$, $p<0.05$.[2]

The consultants urge extreme caution in interpreting this finding. Although it may appear that the participants' offenses in the current study are significantly more severe, it is important to remember the method in which the information was collected for the current study. The current sample is a convenience sample and had random sampling techniques been applied, the concentrations of offenses may have more closely resembled those provided by the Attorney General.

SENTENCING

Sentence length for the participants ranged from 1 to 16 years, with most offenders, $n=73$, receiving a sentence of 3 years or less. See Table 5 for a summary of sentencing information.

Table 5
Sentencing (N=100)

Variable	Minimum	Maximum	Mean	Median	Standard Deviation
Sentence (in years)	1	16	3.41	3	2.46

Sex crimes against children receive considerable attention in the media, and many of the policies currently in place at the state and national level have been named for high-profile cases involving children, i.e. Jacob Wetterling Crime Against Children and Sexually Violent Predator Program (1994), Megan's Law (1996), Jessica Lunsford Act (2005), and the Adam Walsh Child Protection and Safety Act (2006) (Terry & Ackerman, 2009). Among the many requirements of these acts are increased sentences for certain offenses involving children.

In the current study, half of the participants, $n=50$, were charged with either Lewd or Lascivious Acts with a Child under 14 years (288[A]) or Lewd or Lascivious Acts with a Child 14 or 15 years old (288[C]). To determine if there was a difference in sentencing for participants who had been charged with either of these two acts versus participants charged with other offenses, $n=50$, the consultants compared the sentence lengths of both groups. Interestingly, the consultants found no difference in sentencing between the two groups, $z(100)=-0.6247$, $p>0.05$.[3]

ASSESSMENT OF RISK AND TREATMENT OUTCOMES

Some of the more hotly debated issues within the realm of sex offender policy are the assessment of risk and treatment outcomes. Whether or not risk of re-offense can be objectively determined is greeted with much skepticism by professional clinicians and researchers, yet many state and local agencies rely heavily on assessments to make treatment and supervision decisions. Some assessments have been found to be accurate, cost effective, and easy to administer, which are all important considerations for government agencies. Two of the most prominent instruments currently in use by officials within the State of California are the Level of Meanness and General Aggression Score (GAS). Both of these instruments are used to make treatment placement and supervision decisions.

[2] Chapter 8: Chi-Square Goodness of Fit
[3] Chapter 10: Sign Test

Researchers have generally found that although treatment may positively impact recidivism, successful outcomes are greatly affected by the typology of the offender, treatment program, and the offender's social support. Pharmacological interventions are a relatively new area of treatment, but their use as part of a comprehensive treatment program has received support.

LEVEL OF MEANNESS

Each participant in the current study received a Meanness level from the Level of Meanness assessment by three separate raters. The Level of Meanness assessment was developed by clinicians in order to determine likelihood of reoffending for offenders eligible for parole. The assessment has been in use for the last three years in the State to assist parole boards in making release decisions. The cost of incorporating the instrument into the existing pre-release assessment program has been very costly. State officials believe, however, that the assessment will become more cost effective as only offenders with the lowest risk of reoffending are released into the community, thereby requiring less restrictive supervision and reducing the chances of recidivism and incarceration. Levels range from 1 to 5, with 1 indicating lower levels of meanness (higher empathy) and 5 indicating higher levels of meanness (lower empathy). A particular issue in this campaign, and resulting sex offender policy in the State, is whether or not participants' scores are consistent across different raters.

The consultants first examined the Meanness level for each participant from all three raters and determined that there was a high level of agreement among the raters, W=0.92, x^2(99,100)=273.41, p<0.05. [4] This finding was consistent when the consultants only considered the differences between Raters 1 and 2, and again in this analysis, raters were found to have given similar scores, z(100)=-1.0227, p>0.05.[5]

Prior to the current study, several clinicians who had been involved with the initial development of the Level of Meanness assessment in the State did an unofficial comparison of rankings between two clinicians with 10 of the study participants. In the trial, the clinicians were forced to choose between two of the participants, identifying which of the two was more likely to reoffend. Results of that analysis indicated that, when forced to choose between two participants at a time, there is no agreement among the raters, u=-0.1148, x^2 (59.1781, 10)=67.8848, p>0.05.[6] Although this particular test did not find agreement among the raters, the consultants urge caution in interpreting this finding. Only a small subset of the sample was included in the analysis, and further testing, as indicated in the testing among all of the raters above, did reveal consistencies among the raters.

Potential biases in ratings were also examined. Analysis revealed consistencies across Caucasian/Hispanic and Caucasian/Non-Hispanic participants, D(69)=0.11, p>0.05. [7] To examine any potential inconsistencies across income, the participants were divided into four income groups: \$0 - \$3,951.49, \$3,951.50 - \$12,444.49, \$12,444.50 - \$22,041.99, and \$22,042 - \$34,678. Again, assessed levels were found to be consistent, F_r(100)=2.29, p>0.05.[8]

[4] Chapter 7: Kendall Coefficient of Concordance
[5] Chapter 10: Wilcoxon Signed-Rank Test
[6] Kendall Coefficient of Agreement
[7] Chapter 10: Kolmogorov-Smirnov Two-Sample Test
[8] Chapter 12: Friedman's ANOVA

Overall, results provided support that the raters are generally consistent in ranking participants using the Level of Meanness assessment. Testing on the Level of Meanness assessment was limited to reliability in this particular study. As to whether or not the instrument is valid will require additional research.

GENERAL AGGRESSION SCORE

The second assessment score that was provided in the current study was the General Aggression Score (GAS). In the State of California, each convicted sex offender is evaluated and receives a GAS in order to assist officials in determining treatment program eligibility, level of supervision, and likelihood of re-offending. As with the Level of Meanness, consultants were most concerned with evaluating the overall consistency of GAS across study participants. Determining the validity of the GAS instrument will require additional research.

Scoring for this instrument can range from 20 to 80, with higher scores indicating higher levels of aggression. For a summary of participants' GAS, see Table 6.

Table 6
General Aggression Score (GAS) (N=100)

Variable	Minimum	Maximum	Mean	Median	Standard Deviation
GAS	40	78	53.34	53	10.12

GENERAL FINDINGS

One of the hypotheses considered by the consultants in the current study was the association of GAS and sentence length. Other researchers have documented relationships between aggression and violent behaviors and aggression and offending, which would therefore result in longer sentences. This hypothesis was supported by the data. The consultants did determine that there was an association between GAS and sentence length, $T(100)=0.3283$, $z=4.8397$, $p<0.05$.[9] This association was also present when the variable total testosterone was fixed for the participants, $T_{xy.z}(100)=0.3266$, $z=4.8146$, $p<0.05$.[10] A summary of participants' total testosterone levels can be found in Table 7.

Table 7
Total Testosterone Level (ng/dl) (N=100)

Variable	Minimum	Maximum	Mean	Median	Standard Deviation
Testosterone Level	31	892	545.71	572.50	205.03

The consultants also considered biases in scoring and the potential that participants may receive higher GAS due to racial/ethnic bias by the assessment administrator. According to the data, GAS is similar across racial/ethnic groups, $H(3, 100)=1.23$, $p>0.05$.[11] This finding suggests that a racial/ethnic bias does not exist.

To determine whether or not variability of GAS reflect differences in severity according to the law, GAS were examined between participants charged with Sexual Battery (PC243.4, $n=8$) and Rape by

[9] Chapter 6: Kendall Rank-Order Correlation Coefficient
[10] Chapter 6: Kendall Partial Rank-Order Correlation Coefficient
[11] Chapter 12: Kruskal-Wallis ANOVA

Force or Fear (PC261[2], *n*=8). Sexual Battery requires only a zip code disclosure on the Megan's Law website, while Rape by Force or Fear requires the disclosure of the offender's home address (see Table 4 for offenses by disclosure category). Analysis revealed no differences in variability of GAS between participants charges with Sexual Battery and Rape by Force or Fear, p=1.5142.[12]

OFFENSES AGAINST ADULTS

Throughout the last several weeks of the campaign, a serial rapist had been attacking mostly older, wealthy women, causing a great deal of public panic. The suspect, referred to as the "Midnight Rapist" by news anchors, is believed to have caused the death of 75 year-old Susan Eberling. Ms. Eberling is the mother of the opposing U.S. Senate candidate, Christina Eberling. Her status in the community, and connection to the campaign, has greatly influenced the amount of media attention given to this case, which requires special mention in the Governor's policy platform.

Although Ms. Eberling was the alleged seventh victim of the serial rapist, who allegedly attacked 11 total victims, she was the only victim to have died from her injuries. Although the suspect's arrest certainly calmed some of the public's fears, many demands for changes in sex offender management are still being made. Some of these demands stem, in part, from the fact that the suspect was previously convicted of a sex offense and was considered a low-risk release due to his score on the Level of Meanness assessment. The consultants are unable to specifically comment on the administration of the Level of Meanness assessment in this particular case, but as the previous analyses have demonstrated, there does appear to be consistency among the raters. However, further research is required to assess the instruments validity and whether or not it is a sufficient predictor of recidivism.

The consultants urge the Governor and his campaign staff to resist the tendency to support extreme policies or platforms that often follow high-profile cases. The Governor and his representatives should remember the importance of empirically-based policy and careful deliberation.

In order to address some of the concerns that have been raised during the Eberling case, the consultants specifically considered a subset of the sample in the current study – those participants who had registered offenses against adults. Within the current sample, 27 participants committed offenses against adult victims. See Tables 8 and 9 for a summary of information on these participants.

Table 8
Participants with Offenses against Adults: Sex, Ethnicity/Race, and Living Location (n=27)

Variable	Category	Percent
Sex		
	Male	96.3%
	Female	3.7%
Ethnicity/Race		
	Asian	14.8%
	African American/Black	25.9%
	Caucasian/Hispanic	33.3%
	Caucasian/Non-Hispanic	25.9%
Living Location		

[12] Chapter 9: Moses Rank-Like Test for Scale Differences

Incorporated Area	81.5%
Transient	3.7%
Unincorporated Area	14.8%

Table 9

Participants with Offenses against Adults: Age, Estimated Yearly Income, Sentencing, and GAS (n=27)

Variable	Minimum	Maximum	Mean	Median	Standard Deviation
Age (in years)	27	84	55.15	58	14.858
Estimated Yearly Income	$0	$33,426	$16,038.37	$16,871.00	$10,023.98
Sentencing	1	9	3.93	5	2.166
GAS	40	78	59.07	60	11.858
Testosterone Level (ng/dl)	67	892	608.44	607	173.42

Specifically considering the 27 participants who had offenses against adults, the consultants did not find a relationship between the participant's testosterone level and GAS, $z(27)=-0.4845$, $p>0.05$.[13] But as with the whole sample in the current study, an association was found between an offender's GAS and sentence length among offenders who were charged with crimes against adults, $t(25, 27)=3.3424$, $p<0.05$.[14]

PHARMACOLOGICAL INTERVENTIONS

In the current study, the medication statuses are known for 87 of the participants. Within those 87 participants, 33 are currently taking medication as treatment for their registered sex offense. The general types of medications taken by the participants include: antidepressants; a combination of antidepressants (SSRI) and antiandrogens (medications used to counteract male sex hormones); anxiolytics (antianxiety medication); and mood stabilizers. See Table 10 for a summary of participant medication information.

Table 10

Pharmacological Interventions among Participants (N=100)

Variable	Category	Percent
Currently Taking Medication		
	Yes	33%
	No	54%
	Unknown	13%
Type of Medication (within those currently taking medication, *n*=33)		
	Antidepressants	18.2%
	Antidepressants (SSRI) and Antiandrogens	36.4%
	Anxiolytics	12.1%
	Mood Stabilizers	33.3%
Receiving Hormonal Treatment (within those currently taking medication, *n*=33)		
	Yes	36.4%
	No	63.6%

[13] Chapter 5: Somers' Index d Statistic
[14] Chapter 5: Spearman's Rho

Of the 87 participants whose medication status is known, the consultants determined that there is a difference in probability between offenders who are currently taking medication and those who are not, $z(100) = 2.1442$, $p<0.05$.[15] The results indicate that more offenders are not taking medication than would be expected by chance; therefore, additional research is needed to assess why offenders are more likely to not be taking medication.

For the subsample on antiandrogens, GAS was compared between Caucasian/Hispanic (n=5) and Caucasian/Non-Hispanic (n=4) participants. According to the results, there is no difference in GAS based upon racial/ethnic group.[16]

COMPLIANCE STATUS

Due to the increased number of transient offenders, considerable attention has been given to the factors that influence compliance with probation and parole conditions. Since the adoption of stricter sex offender residency restrictions throughout the State, CASOMB has noted an increase in the number of transient offenders (California Sex Offender Management Board, 2011). Transient status in itself is not a violation of probation or parole conditions, but it does mean that the registered offender does not have a stable living situation, which may increase the likelihood of violating conditions and reoffending. See Table 11 for a summary of compliance information.

Table 11

30, 90, and Current Post-Release Statuses (N=100)

Variable	Category	Percent
30 Days Post Release Status		
	Complying with Conditions	83%
	In Violation of Conditions	17%
90 Days Post Release Status		
	Complying with Conditions	7%
	In Violation of Conditions	5%
	Status Unknown	88%
Current Status (at the time of study data collection)		
	Complying with Conditions	61%
	In Violation of Conditions	39%

GENERAL FINDINGS

Analyses in the current study revealed that more of the participants who are compliant at 30 days post release end up becoming in-violation the longer they are under supervision, $x^2(1, 100)=$ 11.6053, $p<0.05$.[17] However, for the offenders who have consistent reports of their post-release statuses (n=12), the probability of a participant being in-violation is the same across the different reporting times, $Q(2, 100) = 0.3333$, $p>0.05$).[18] This finding would suggest that for the participants who are consistently monitored, they are not more likely to violate the conditions of their probation or parole the longer they are released.

[15] Chapter 8: Binomial Test
[16] Chapter 9: Permutation Test for Two Independent Samples
[17] Chapter 11: McNemar Change Test
[18] Chapter 12: Cochran Q

STATUS BASED UPON PARTICIPANT VARIABLES

The consultants considered several participant variables – GAS, income, race/ethnicity, and taking medication – to determine if there was a difference in compliance based upon the second variable. The distribution of GAS is the same for offenders who are currently compliant and in-violation of their release conditions, $z(100)$=-02195, p>0.05.[19] This suggests that using the GAS to determine likelihood of compliance may not be appropriate. In addition to GAS, there is no difference in current status compliance based upon race/ethnicity ($x^2[3,100]=0.68$, p>0.05[20]); therefore, the proportion of participants who are compliant or in-violation is the same across Asian, African American/Black, Caucasian/Hispanic, and Caucasian/Non-Hispanic participants. There is also no difference in current status compliance for offenders whose medication status is known (n=87), $r_{\emptyset}$= 0.031, $x^2(1, 87)$=0.0033, p>0.05.[21]

Using the 90 day post-release statuses (n=12), the consultants considered an estimated yearly income less than $15,000 and greater than $15,000 to see if those participants with greater incomes were more likely to be in compliance. However, no difference in compliance status was found between the participants at the different income levels at 90 days post release, $p(12)$=0.1212.[22]

POLICY RECOMMENDATIONS

The consultants commissioned for this report were specifically tasked with the development of a policy platform that addressed the management of sexual offenders within the State of California. The proposed platform was constructed for use by Governor Nathanial Greenleaf in his campaign for one of the State's U.S. Senate seats.

The following platform is informed by several policy goals and based upon a five-part containment approach to sex offender management: 1) victim-centered philosophy, 2) multidisciplinary collaboration, 3) containment-focused risk management, 4) informed and consistent public policies, and 5) quality control (English, 2009). The containment approach was developed by the Colorado Department of Public Safety, Division of Criminal Justice, and focuses on protecting victims through comprehensive, individualized offender management (English, 2009). Results of the current study are incorporated as suggestions and evidence within each of these five areas.

POLICY GOALS

The purpose of any policy is to meet specified goals, and the consultants considered three policy goals when constructing the current platform: public safety, fairness, and governmental and fiscal efficiency. Existing and suggested policies can then be examined within the context of these goals. Alternatives may then be suggested for existing policies that fail to meet these goals.

GOAL 1: PUBLIC SAFETY

[19] Chapter 10: Mann-Whitney U
[20] Chapter 12: Chi-Square r x k
[21] Chapter 4: Phi Coefficient
[22] Chapter 11: Fisher Exact Probability Test

The fundamental purpose of all sex offender policy is public safety. Whether victim-specific or in general to the entire community, policies should keep known sex offenders from reoffending, and ideally, stop victimization before it occurs. The rights of victims and community members to live in a safe environment must be considered. Methods for evaluating the effectiveness of public policy in meeting this goal include examination of recidivism rates and rates of victimization provided by official arrest records and victim surveys.

GOAL 2: FAIRNESS

Public safety cannot come at the expense of the rights of the offender. An important balance needs to be achieved between the safety of victims, the safety of the general public, and the safety and rights of the offenders. Evaluating the fairness of a policy can be subjective, but legal arguments should be considered. Policies that are too stringent may be punitive and infringe upon the rights of the offenders, which in turn could result in law suits. Methods for generating fair policy may involve collaborative efforts between policy makers, victim advocates, and civil rights attorneys.

GOAL 3: GOVERNMENTAL AND FISCAL EFFICIENCY

Given the financial constraints of the State of California, policies must be constructed that are possible within budgetary limitations. Comprehensive sex offender management is expensive. The cost of assessing and housing a sexually violent predator (SVP), for example, is considerably more expensive than the cost of assessing and housing a non-SVP within the CDCR. Even though the number of SVP assessments has increased substantially since the implementation of Jessica's law, the number of offenders who are found to be SVPs has remained relatively stable (D'Orazio, Arkowitz, Adams, & Maram, 2009). Methods for evaluating the cost effectiveness of policies include comparing the cost of intensive supervision techniques for an individual offender versus system and victim costs associated with reoffending. Evaluators may also consider differences in recidivism based upon program and supervision level.

RECOMMENDED POLICY PLATFORM: *THE CONTAINMENT APPROACH*

PART 1: VICTIM-CENTERED PHILOSOPHY

Recommendation 1:1: Comprehensive medical and mental health services for victims and their families should be included in every sex offender policy. Access to these services should be provided for little or no cost and not be limited to victims who officially reported crimes to law enforcement agencies.

As demonstrated by the results in the current study, victims of sexual offenses are children and adults, and services need to be available that are specific to both populations. Researchers have consistently reported that many victims of sexual assault do not report the offense (Meloy, 2005). The reasons for failing to report are numerous and include, but are not limited to, fear, feelings of guilt, and the existing relationship between the victim and offender (i.e. offender is a parent, babysitter, etc.). Due to the grossly underreported nature of sexual offending, official arrest statistics and prosecutions, traditional measures of recidivism, may not accurately reflect true reoffending behaviors. Even if victimization is known, the road to recovery is long, painful, and expensive. Although calculating the exact costs of victimization are difficult, some studies have suggested that victims, on average, generate less income over the course of their lifetimes than non-victims (Macmillan, 2000). Insurance, if available, may only cover a portion of expenses related to the victimization.

Access to comprehensive medical and mental health services may greatly decrease future victimization and offending. Researchers have documented the repetitive, chronic nature of victimization: Victims who are victimized once have a greater chance of being victimized again (Wittebrood & Nieuwbeerta, 2000; Lauritsen & Davis Quinet, 1995). In addition, research on the history of criminal offenders has revealed that many were former victims of crime themselves. This does not suggest that all victims become later offenders, but many offenders were once victims (Spaccarelli, Coatsworth, Bowden, 1995; Ireland & Widom, 1994). Successful interventions may help decrease this relationship.

Recommendation 1:2: Victim advocates need to be included as part of policy development.

Publically posting the names, pictures, and addresses of offenders on a Megan's Law website may serve to inform the general public of the proximity of sexual offenders, but those same postings may unintentionally "out" victims to the public. This is especially true when victims are family members of the registered offender (English, 2009). With the focus on community notification and the management of the offenders, the needs and privacy of the victim may be overlooked. Advocates need to be part of policy discussions, as policies also impact the lives of the victims.

PART 2: MULTIDISCIPLINARY COLLABORATION

Recommendation 2:1: State and parole officials should continue to collaborate with medical/clinical treatment providers to increase medication compliance and positive treatment outcomes.

Participants in the current study were found to be more likely to *not* be taking medication than to be taking medication. This finding can be due to a variety of reasons, including participants not being diagnosed with a psychiatric condition related to their offending. For participants who have been diagnosed with a psychiatric condition and are on medication as part of their treatment, collaboration between supervision officials and medical treatment staff may help increase compliance with medication and treatment programs.

Recommendation 2:2: State and parole officials should continue to collaborate with clinicians and researchers on the development of assessment materials that can be used in parole and supervision decisions.

The Level of Meanness assessment was specifically developed to assist parole board officials in making release decisions. Although the assessment introduces an objective component to a highly subjective process, further research still needs to be conducted on whether or not levels are associated with future risk of reoffending. Despite these current unknowns, parole officials should be encouraged to continue working with clinicians and researchers on the development of such measures.

PART 3: CONTAINMENT-FOCUSED RISK MANAGEMENT

Recommendation 3:1: All offenders' statuses should be collected at 90 days post-release and again on a uniform, consistent schedule.

According to the data collected in the current study, the probability of a participant being in-violation is the same across the different reporting times for participants who consistently reported their 30 day, 90 day, and current post-release statuses. Analyses of the offenders who only reported their 30 day and current post-release statuses, however, revealed that more offenders were in-violation the longer they were on supervision. Decisions to require a 90 day check-in vary by each parole officer. Implementing a uniform policy that requires a 90 day check-in, thereby increasing the amount of supervision, may reduce the number of offenders who are in-violation later.

PART 4: INFORMED AND CONSISTENT PUBLIC POLICIES

Recommendation 4:1: Residency restrictions should be monitored to determine their effects (if any) on offenders becoming transient.

The number of transient offenders in the current study is lower than the state average; however, even a conservative estimate of 27% is still a considerable percentage of offenders. Numerous studies have been conducted on the impact of residency restrictions on everything from available housing and employment to offender stress, with some researchers suggesting that unstable housing situations may increase the chances of recidivism (Peckenpaugh, 2006; Walker, 2007).

A particular problem in the State of California is the variability of residency restrictions from county to county, even city to city. Such variability leads to confusion, not only for offenders, but for supervision officials and the public. Coupled with questionable empirical support for reducing recidivism, California's residency restrictions require careful examination. The proposal of a research-based, uniformly applied policy should be considered.

PART 5: QUALITY CONTROL

Recommendation 5:1: Research on the validity of the Level of Meanness and General Aggression Score (GAS) assessments needs to be conducted, along with continued evaluation of their use.

As uncovered in the current study, the Level of Meanness and GAS assessments appear to be consistently administered across several variables, including race/ethnicity and income level. While these findings are important in suggesting a uniform application of the assessments, further research needs to be done to determine if the assessments are truly measuring factors linked to recidivism. In addition to research on validity, both assessments should be continually evaluated for consistency in scoring and application.

Recommendation 5:2: Measures tracking the effectiveness of State policies must be developed. The outcome of these measures should then be used to modify existing policy or to introduce new policies.

For existing policies, such as GPS monitoring and residency restrictions, effectiveness can be examined by considering rates of victimization, recidivism rates for offenders, the number of transient offenders, and the ability of offenders to find and maintain stable housing and employment. Many of the policies introduced by the legislature and the public through the initiative process go largely unchecked. When changes are suggested, they are usually proposed after a high-profile incident. Although these types of incidents may point to glaring flaws in existing policies, they may also serve only to highlight exceptions, such as the "Midnight Rapist" case. This one case is not sufficient proof that the Level of Meanness assessment is an insufficient tool and that its use should be suspended. Further research is needed before that determination can be made. Again, the consultants urge the Governor and other policy makers to consider empirical evidence prior to making any sweeping policy decisions.

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Dakota sat very still, staring at the top of the conference room table. Ever since the Governor and Jennifer walked in 30 minutes ago and demanded to look over their report, she had been unable to look at either of them. Now, she was in the unenviable position of being in the room with someone who was reading over her work, resulting in her eyes tracing the various grains of wood on the conference table. All around her, she could feel the palpable waves of discomfort rising from her colleagues as well; each one trapped in their own private little number. Dakota remained frozen in time, wondering when this interminable ordeal would be over.

"Hmmmm."

The Governor's quizzical verbal expression almost stole the breath from her chest. Still, Dakota's hypervigilant gaze was transfixed on the conference room table; she could feel her body screaming at her to look at the Governor to try and read his facial expression. Her eyes started to rise to see the Governor's face, despite her herculean efforts to keep them in line. At once, she was met by the cold and steely gaze of a professional politician.

"This is very good work. Do you all agree with these findings?"

Suddenly, Dakota felt a surge of veracity race from her spine, filling her up with warm confidence. She knew instinctively that they had done all they could with the data they were provided, so there was nothing more than to defend their work. No longer wilting under Nathanial Greenleaf's eyes, Dakota straightened up in her seat and crossed her legs.

"Yes sir. You may of course have others review our work, but the conclusion stands."

Nathanial flipped the report closed, passing it back to his trusted right-hand woman.

"Grab a few talking points out of this. I have to go shake hands at a coffee stand."

Jennifer flipped open the report as Nathanial rose to his feet, nodded his head at Dakota, and gusted out of the conference room. Dakota remained transfixed towards the empty chair which the Governor no longer occupied; she couldn't be sure, but she had a feeling that she just made a very powerful new friend. To her right, she could hear Robin let out a great sigh.

"Oh wow that was bad. Wanna get a drink?"

Dakota tilted her head to see Theron lean in towards Robin.

"It's not even seven in the morning yet."

Robin shrugged her shoulders.

"Where are you going with this?"

Theron shook his head.

"I have to say, that is the longest I think you have gone without shooting your mouth off."

Robin shrugged.

"My paycheck hasn't cleared yet."

Bemused by her colleagues, Dakota leaned back into her chair as she felt Michael's breath touch her cheek.

"You did a good job."

Dakota just smiled. She could see her group muddling around the conference table, waiting for Dakota to walk out the door with them. Even Robin was finding it hard to say good bye.

"Dakota, may I have a word."

Dakota glanced towards Jennifer, who was motioning for her to walk over to her. Dakota gingerly placed her hand on top of Michael's.

"Be right back."

With that, Dakota gracefully pushed herself away from the table and rose to her feet. As she stood, her eyes caught a glimpse of a brightly colored turquoise box in Michael's jacket pocket. Her face took on a quizzical expression as her mind tried to figure out why he would need such a box, until the answer struck her like lightning. She stood for a moment, not quite sure how to react to such a beautiful gesture. Without even knowing it, she was tracing her fingers along the back of his neck. Michael just tilted his head towards her.

"You okay? Jennifer is waiting."

Dakota snapped from her stupor, eased herself away from Michael, and stepped towards the woman who was beelining towards her. Jennifer leaned in towards Dakota; her voice was soft and low.

"The Governor was very pleased with your work. So, I have a proposal for you."

Dakota lowered her eyes.

"Seem to be getting a lot of those today."

Jennifer looked puzzled, but continued on.

"I'd like for you to consider working with us again. Should we win the Senate race, we are going to need smart people like you."

Before she could truly think it over, Dakota was nodding her head.

"Just let me know how I can be of assistance."

Jennifer just extended her hand, and Dakota clasped it into a firm handshake. The two women stood silently for a moment, until Jennifer released her grip and followed her boss out into the campaign offices. Dakota just let her hand fall to her side, as Robin bounced next to her.

"Come on boss, let's go get hammered. Michael can buy."

Michael shook his head as Dakota followed her group out of the conference room. Once they got to the door, Robin stopped and looked back at the dry-erase board.

"I have to say, I won't miss that stupid thing."

Dakota smiled.

"So, what are you going to do now?"

Robin shrugged her shoulders.

"Tequila shots probably. What about you?"

Dakota also looked back to the dry-erase board, admiring all of the beautiful equations she so lovingly etched onto it. Suddenly, a light bulb went off in her brain.

"I think I might write a book about nonparametric statistics. You know, try to make it fun and interesting for people."

Robin shook her head.

"Impossible."

Dakota thought for a moment, letting an idea form in her brain.

"I don't know. Maybe I could explain all of the statistical procedures through a story. If the story is interesting enough, I am sure people will read it."

Robin just threw her purse over her shoulders.

"Oh come on. Who is going to read something like that?"

Appendices

Penal Code Descriptions

Offense (count one)	Offense (count one) description
220	Assault to commit rape, sodomy, or oral copulation
243.4	Sexual battery
264.1	Rape in concert with force or violence
266	Entice minor female for prostitution
288.3	Contacting minor with intent to commit sex offense
288.5	Continuous sexual abuse of child
289	Sexual penetration by foreign object
314.1	Indecent exposure
261(2)	Rape by force or fear
261(2)/264.1	Rape in concert by force
261(4)	Rape: victim unconscious of the nature of the act
286(C)	Sodomy with person under 14 years or with force
288(A)	Lewd or lascivious acts with child under 14 years
288(B)	Lewd or lascivious acts with child 14 years with force
288(C)	Lewd or lascivious acts with child under 14 or 15 years
288A(B)(1)	Oral copulation with person under 18 years
288A(B)(2)	Oral copulation with person under 16 years
288A(C)	Oral copulation with person under 14 years or by force
311.3(B)	Sexually exploit minor: depict minor in sex act
647.6(a)(1)	Annoy/molest child under 18 years old

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Tables

Table A Random numbers table

1 8 3 1 3	6 7 7 4 9	9 9 0 1 8	1 5 1 8 2	4 6 2 9 0
2 5 1 1 7	8 3 3 7 9	4 5 1 0 4	1 6 4 5 4	7 0 5 9 3
7 4 3 0 8	4 2 7 2 1	1 1 4 5 9	9 1 5 9 5	8 1 9 8 5
8 6 9 4 4	5 5 7 1 7	7 4 5 7 4	2 6 5 6 7	5 4 6 4 6
1 3 5 5 0	9 4 8 7 2	2 7 8 8 0	8 3 7 0 2	6 5 3 1 8
5 7 1 5 9	4 3 2 9 4	1 8 4 6 3	0 5 6 4 5	3 3 9 4 2
9 4 7 8 2	5 1 3 9 4	8 6 7 8 5	1 7 2 5 4	4 2 9 8 5
5 6 3 9 4	7 1 4 2 3	5 3 2 9 0	0 0 5 7 8	7 7 1 3 4
0 8 5 1 6	2 4 8 9 8	2 0 5 2 4	5 1 8 8 9	8 1 5 2 8
5 9 9 2 3	0 7 5 9 1	4 9 6 3 9	7 7 9 9 3	5 1 1 7 9
7 2 3 4 7	6 2 1 0 9	2 3 9 0 2	4 2 0 2 1	9 2 0 8 3
4 1 3 5 0	5 5 4 3 8	1 2 8 9 5	5 5 3 1 0	1 7 6 6 2
8 4 4 6 5	9 5 9 4 2	4 5 6 3 1	7 4 5 8 2	2 4 0 7 5
9 5 8 0 1	3 6 2 8 1	9 9 2 7 7	8 5 6 7 1	4 7 8 8 8
2 8 6 7 8	4 9 0 2 7	8 8 0 1 5	5 5 9 4 0	8 2 5 1 6
1 1 7 2 3	4 7 2 0 3	3 0 5 6 2	5 9 0 6 4	3 8 9 8 0
4 5 1 5 1	0 2 7 6 8	5 2 5 8 0	6 5 4 9 8	4 6 9 7 1
9 3 3 7 4	8 3 8 0 4	8 8 4 3 5	8 8 8 5 5	8 1 4 3 9
5 4 8 0 9	4 8 0 0 9	4 9 0 2 5	9 3 2 8 5	5 0 9 0 8
8 5 2 6 0	3 4 4 8 1	9 6 6 4 7	9 2 9 5 5	9 9 7 4 1
3 7 5 4 7	1 8 1 1 1	6 3 8 9 7	2 1 0 4 0	4 0 9 3 5
5 4 5 0 0	0 0 8 0 9	5 7 8 3 9	1 4 6 4 3	8 5 3 3 6
4 3 0 1 1	6 1 7 1 7	9 8 7 2 4	0 5 2 5 2	7 3 6 7 1
7 7 4 5 4	0 3 6 3 2	0 4 3 5 6	0 8 2 8 5	5 0 1 3 0
1 4 1 8 5	8 8 5 9 4	5 5 1 1 8	1 4 2 9 9	2 1 6 8 9
3 1 8 0 6	6 2 2 0 2	1 9 6 1 9	4 2 0 3 4	1 4 6 8 6
5 2 7 1 4	7 9 4 4 5	0 5 9 4 6	5 2 3 2 8	0 0 3 5 2
4 4 0 4 5	2 1 9 7 7	4 0 3 1 7	8 5 4 5 9	8 0 6 4 5
1 7 1 9 1	1 2 6 8 9	5 4 4 9 0	7 5 7 6 2	0 1 3 2 9
2 8 4 2 8	0 4 8 3 6	9 9 2 0 4	6 8 8 8 1	9 5 5 6 4

Table B Critical Values of the Spearman's rank correlation coefficient, r_s, for two-tailed and one-tailed probabilities, α(2) and α(1), respectively

α(2)	0.50	0.20	0.1	0.05	0.02	0.01	0.005	0.002	0.001
α(1)									
n	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.005
4	0.600	1.000	1.000						
5	0.500	0.800	0.900	1.000	1.000				
6	0.371	0.657	0.829	0.886	0.943	1.000	1.000		
7	0.321	0.571	0.714	0.786	0.893	0.929	0.964	1.000	1.000
8	0.310	0.524	0.643	0.738	0.833	0.881	0.905	0.952	0.976
9	0.267	0.483	0.600	0.700	0.783	0.833	0.867	0.917	0.933
10	0.248	0.455	0.564	0.648	0.745	0.794	0.830	0.879	0.903
11	0.236	0.427	0.536	0.618	0.709	0.755	0.800	0.845	0.873
12	0.224	0.406	0.503	0.587	0.671	0.727	0.776	0.825	0.860
13	0.209	0.385	0.484	0.560	0.648	0.703	0.747	0.802	0.835
14	0.200	0.367	0.464	0.538	0.622	0.675	0.723	0.776	0.811
15	0.189	0.354	0.443	0.521	0.604	0.654	0.700	0.754	0.786
16	0.182	0.341	0.429	0.503	0.582	0.635	0.679	0.732	0.765
17	0.176	0.328	0.414	0.485	0.566	0.615	0.662	0.713	0.748
18	0.170	0.317	0.401	0.472	0.550	0.600	0.643	0.695	0.728
19	0.165	0.309	0.391	0.460	0.535	0.584	0.628	0.677	0.712
20	0.161	0.299	0.380	0.447	0.520	0.570	0.612	0.662	0.696
21	0.156	0.292	0.370	0.435	0.508	0.556	0.599	0.648	0.681
22	0.152	0.284	0.361	0.425	0.496	0.544	0.586	0.634	0.667
23	0.148	0.278	0.353	0.415	0.486	0.532	0.573	0.622	0.654
24	0.144	0.271	0.344	0.406	0.476	0.521	0.562	0.610	0.642
25	0.142	0.265	0.337	0.398	0.466	0.511	0.551	0.598	0.630
26	0.138	0.259	0.331	0.390	0.457	0.501	0.541	0.587	0.619
27	0.136	0.255	0.324	0.382	0.448	0.491	0.531	0.577	0.608
28	0.133	0.250	0.317	0.375	0.440	0.483	0.522	0.567	0.598
29	0.130	0.245	0.312	0.368	0.433	0.475	0.513	0.558	0.589
30	0.128	0.240	0.306	0.362	0.425	0.467	0.504	0.549	0.580
31	0.126	0.236	0.301	0.356	0.418	0.459	0.496	0.541	0.571
32	0.124	0.232	0.296	0.350	0.412	0.452	0.489	0.533	0.563
33	0.121	0.229	0.291	0.345	0.405	0.446	0.482	0.525	0.554
34	0.120	0.225	0.287	0.340	0.399	0.439	0.475	0.517	0.547
35	0.118	0.222	0.283	0.335	0.394	0.433	0.468	0.510	0.539
36	0.116	0.219	0.279	0.330	0.388	0.427	0.462	0.504	0.533
37	0.114	0.216	0.275	0.325	0.383	0.421	0.456	0.497	0.526
38	0.113	0.212	0.271	0.321	0.378	0.415	0.450	0.491	0.519
39	0.111	0.210	0.267	0.317	0.373	0.410	0.444	0.485	0.513
40	0.110	0.207	0.264	0.313	0.368	0.405	0.439	0.479	0.507
41	0.108	0.204	0.261	0.309	0.364	0.400	0.433	0.473	0.501
42	0.107	0.202	0.257	0.305	0.359	0.395	0.428	0.468	0.495
43	0.105	0.199	0.254	0.301	0.355	0.391	0.423	0.463	0.490
44	0.104	0.197	0.251	0.298	0.351	0.386	0.419	0.458	0.484
45	0.103	0.194	0.248	0.294	0.347	0.382	0.414	0.453	0.479

(continued)

Table B (continued)

α(2)	0.50	0.20	0.1	0.05	0.02	0.01	0.005	0.002	0.001
α(1)									
n	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.005
46	0.102	0.192	0.246	0.291	0.343	0.378	0.410	0.448	0.474
47	0.101	0.190	0.243	0.288	0.340	0.374	0.405	0.443	0.469
48	0.100	0.188	0.240	0.285	0.336	0.370	0.401	0.439	0.465
49	0.098	0.186	0.238	0.282	0.333	0.366	0.397	0.434	0.460
50	0.097	0.184	0.235	0.279	0.329	0.363	0.393	0.430	0.456
52	0.095	0.180	0.231	0.274	0.323	0.356	0.386	0.422	0.447
54	0.094	0.177	0.226	0.268	0.317	0.349	0.379	0.414	0.439
56	0.092	0.174	0.222	0.264	0.311	0.343	0.372	0.407	0.432
58	0.090	0.171	0.218	0.259	0.306	0.337	0.366	0.400	0.424
60	0.089	0.168	0.214	0.255	0.300	0.331	0.360	0.394	0.418
62	0.087	0.165	0.211	0.250	0.296	0.326	0.354	0.388	0.411
64	0.086	0.162	0.207	0.246	0.291	0.321	0.348	0.382	0.405
66	0.084	0.160	0.204	0.243	0.287	0.316	0.343	0.376	0.399
68	0.083	0.157	0.201	0.239	0.282	0.311	0.338	0.370	0.393
70	0.082	0.155	0.198	0.235	0.278	0.307	0.333	0.365	0.388
72	0.081	0.153	0.195	0.232	0.274	0.303	0.329	0.360	0.382
74	0.080	0.151	0.193	0.229	0.271	0.299	0.324	0.355	0.377
76	0.078	0.149	0.190	0.226	0.267	0.295	0.320	0.351	0.372
78	0.077	0.147	0.188	0.223	0.264	0.291	0.316	0.346	0.368
80	0.076	0.145	0.185	0.220	0.260	0.287	0.312	0.342	0.363
82	0.075	0.143	0.183	0.217	0.257	0.284	0.308	0.338	0.359
84	0.074	0.141	0.181	0.215	0.254	0.280	0.305	0.334	0.355
86	0.074	0.139	0.179	0.212	0.251	0.277	0.301	0.330	0.351
88	0.073	0.138	0.176	0.210	0.248	0.274	0.298	0.327	0.347
90	0.072	0.136	0.174	0.207	0.245	0.271	0.294	0.323	0.343
92	0.071	0.135	0.173	0.205	0.243	0.268	0.291	0.319	0.339
94	0.070	0.133	0.171	0.203	0.240	0.265	0.288	0.316	0.336
96	0.070	0.132	0.169	0.201	0.238	0.262	0.285	0.313	0.332
98	0.069	0.130	0.167	0.199	0.235	0.260	0.282	0.310	0.329
100	0.068	0.129	0.165	0.197	0.233	0.257	0.279	0.307	0.326

Zar, J.H. (1972). Significance testing of the Spearman rank correlation coefficient. *Journal of the American Statistical Association, 67*, 578–580. Reprinted with permission

Table C-1 Critical Values of $D_{m,n}$ for Kolmogorov-Smirnov two-sample test (large samples, two-tailed)

Level of significance	Value of $D_{m,n}$ so large as to call for rejection of H_0 at the indicated level of significance, where $D_{m,n} = \text{maximum}\lvert S_m(X) - S_n(X)\rvert$
0.10	$1.22\sqrt{\frac{m+n}{mn}}$
0.05	$1.36\sqrt{m+nmn}$
0.025	$1.48\sqrt{\frac{m+n}{mn}}$
0.01	$1.63\sqrt{\frac{m+n}{mn}}$
0.005	$1.73\sqrt{\frac{m+n}{mn}}$
0.001	$1.95\sqrt{\frac{m+n}{mn}}$

Smirnov, N. (1948). Tables for estimating the goodness of fit of empirical distributions. *Annals of Mathematical Statistics*, 19(2), 280–28; Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table C-2 Critical Values of K_D in the Kolmogorov-Smirnov two-sample test (small samples)

	One-tailed test		Two-tailed test	
N	α=.05	α=.01	α=.05	α=.01
3	3	—	—	—
4	4	—	4	—
5	4	5	5	5
6	5	6	5	6
7	5	6	6	6
8	5	6	6	7
9	6	7	6	7
10	6	7	7	8
11	6	8	7	8
12	6	8	7	8
13	7	8	7	9
14	7	8	8	9
15	7	9	8	9
16	7	9	8	10
17	5	9	8	10
18	8	10	9	10
19	8	10	9	10
20	8	10	9	11
21	8	10	9	11
22	*9*	11	9	11
23	9	11	10	11
24	9	11	10	12
25	9	11	10	12
26	9	11	10	12
27	9	12	10	12
28	10	12	11	13
29	10	12	11	13
30	10	12	11	13
35	11	13	12	
40	11	14	13	

Goodman, L.A. (1954). Kolmogorov-Smirnov tests for psychological research. *Psychological Bulletin. 51*, 167; Massey, F.J. (1951). The distribution of the maximum deviation between two-sample cumulative step functions. *Annals of Mathematical Statistics. 22*(1), 126–127; Siegel, S. (1954). *Nonparametric Statistics for the Behavioral Sciences*. New York: McGraw Hill Book Company. Reprinted with permission

Table D Probability of obtaining a U not larger than that tabulated in comparing samples of n and m

$n = 3$

U			
0	.250	.100	.050
1	.500	.200	.100
2	.750	.400	.200
3		.600	.350
4			.500
5			.650

$n = 4$

U \ m	1	2	3	4
0	.200	.067	.028	.014
1	.400	.133	.057	.029
2	.600	.267	.114	.057
3		.400	.200	.100
4		.600	.314	.171
5			.429	.243
6			.571	.343
7				.443
8				.557

$n = 5$

U \ m	1	2	3	4	5
0	.167	.047	.018	.008	.004
1	.333	.095	.036	.016	.008
2	.500	.190	.071	.032	.016
3	.667	.286	.125	.056	.028
4		.429	.196	.095	.048
5		.571	.286	.143	.075
6			.393	.206	.111
7			.500	.278	.155
8			.607	.365	.210
9				.452	.274
10				.548	.345
11					.421
12					.500
13					.579

$n = 6$

U \ m	1	2	3	4	5	6
0	.143	.036	.012	.005	.002	.001
1	.286	.071	.024	.010	.004	.002
2	.428	.143	.048	.019	.009	.004
3	.571	.214	.083	.033	.015	.008
4		.321	.131	.057	.026	.013
5		.429	.190	.086	.041	.021
6		.571	.274	.129	.063	.032
7			.357	.176	.089	.047
8			.452	.238	.123	.066
9			.548	.305	.165	.090
10				.381	.214	.120
11				.457	.268	.155
12				.545	.331	.197
13					.396	.242
14					.465	.294
15					.535	.350
16						.409
17						.469
18						.531

(continued)

Table D (continued)

$n = 7$

U \ m	1	2	3	4	5	6	7
0	.125	.028	.008	.003	.001	.001	.000
1	.250	.056	.017	.006	.003	.001	.001
2	.375	.111	.033	.012	.005	.002	.001
3	.500	.167	.058	.021	.009	.004	.002
4	.625	.250	.092	.036	.015	.007	.003
5		.333	.133	.055	.024	.011	.006
6		.444	.192	.082	.037	.017	.009
7		.556	.258	.115	.053	.026	.013
8			.333	.158	.074	.037	.019
9			.417	.206	.101	.051	.027
10			.500	.264	.134	.069	.036
11			.583	.324	.172	.090	.049
12				.394	.216	.117	.064
13				.464	.265	.147	.082
14				.538	.319	.183	.104
15					.378	.223	.130
16					.438	.267	.159
17					.500	.314	.191
18					.562	.365	.228
19						.418	.267
20						.473	.310
21						.527	.355
22							.402
23							.451
24							.500
25							.549

(continued)

Table D (continued)

$n = 8$

U \ m	1	2	3	4	5	6	7	8	t	normal
0	.111	.022	.006	.002	.001	.000	.000	.000	3.308	.001
1	.222	.044	.012	.004	.002	.001	.000	.000	3.203	.001
2	.333	.089	.024	.008	.003	.001	.001	.000	3.098	.001
3	.444	.133	.042	.014	.005	.002	.001	.001	2.993	.001
4	.556	.200	.067	.024	.009	.004	.002	.001	2.888	.002
5		.267	.097	.036	.015	.006	.003	.001	2.783	.003
6		.356	.139	.055	.023	.010	.005	.002	2.678	.004
7		.444	.188	.077	.033	.015	.007	.003	2.573	.005
8		.556	.248	.107	.047	.021	.010	.005	2.468	.007
9			.315	.141	.064	.030	.014	.007	2.363	.009
10			.387	.184	.085	.041	.020	.010	2.258	.012
11			.461	.230	.111	.054	.027	.014	2.153	.016
12			.539	.285	.142	.071	.036	.019	2.048	.020
13				.341	.177	.091	.047	.025	1.943	.026
14				.404	.217	.114	.060	.032	1.838	.033
15				.467	.262	.141	.076	.041	1.733	.041
16				.533	.311	.172	.095	.052	1.628	.052
17					.362	.207	.116	.065	1.523	.064
18					.416	.245	.140	.080	1.418	.078
19					.472	.286	.168	.097	1.313	.094
20					.528	.331	.198	.117	1.208	.113
21						.377	.232	.139	1.102	.135
22						.426	.268	.164	.998	.159
23						.475	.306	.191	.893	.185
24						.525	.347	.221	.788	.215
25							.389	.253	.683	.247
26							.433	.287	.578	.282
27							.478	.323	.473	.318
28							.522	.360	.368	.356
29								.399	.263	.396
30								.439	.158	.437
31								.480	.052	.481
32								.520		

Mann, H.B., & Whitney, D.R. (1947). On a test of whether one of two random variables is stochastically larger than the other. *Annals of Mathematical Statistics, 18*(1), 52–54. Reprinted with permission

Table E Critical values of r in the runs test

m \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2	2	2	2	2	2	2	2	2
											–	–	–	–	–	–	–	–	–
3					2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
					–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
4				2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
				9	9	–	–	–	–	–	–	–	–	–	–	–	–	–	–
5			2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
			9	10	10	11	11	–	–	–	–	–	–	–	–	–	–	–	–
6		2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
		–	9	10	11	12	12	13	13	13	13	–	–	–	–	–	–	–	–
7		2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
		–	–	11	12	13	13	14	14	14	14	15	15	15	–	–	–	–	–
8		2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
		–	–	11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9		2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
		–	–	–	13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10		2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
		–	–	–	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11		2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
		–	–	–	13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
	–	–	–	–	13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
	–	–	–	–	–	15	16	17	18	19	19	20	20	21	21	22	22	23	23
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
	–	–	–	–	–	15	16	17	18	19	20	20	21	22	22	23	23	23	24
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
	–	–	–	–	–	15	16	18	18	19	20	21	22	22	23	23	24	24	25
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
	–	–	–	–	–	–	17	18	19	20	21	21	22	23	23	24	25	25	25
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
	–	–	–	–	–	–	17	18	19	20	21	22	23	23	24	25	25	26	26
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
	–	–	–	–	–	–	17	18	19	20	21	22	23	24	25	25	26	26	27
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
	–	–	–	–	–	–	17	18	20	21	22	23	23	24	25	26	26	27	27
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14
	–	–	–	–	–	–	17	18	20	21	22	23	24	25	25	26	27	27	28

The one-sample test is significant at $\alpha = .05$ when the observed r is less than or equal to the smaller value OR is greater than or equal to the larger value in the pair listed in the table

Swed, F.S., & Esienhart, C. (1943). Tables for testing randomness of grouping in a sequence of alternatives. *Annals of Mathematical Statistics, 14*(1), 83–86; Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table F Critical Values for the Kendall's coefficient of concordance *W*

N = 3

k	α	.05	.01
8		.376	.522
9		.333	.469
10		.300	.425
12		.250	.359
14		.214	.311
15		.200	.291
16		.187	.274
18		.166	.245
20		.150	.221

		N = 4		N = 5		N = 6		N = 7	
k	α	.05	.01	.05	.01	.05	.01	.05	.01
3		---	---	.716	.840	.660	.780	.624	.737
4		.619	.768	.552	.683	.512	.629	.484	.592
5		.501	.644	.449	.571	.417	.524	.395	.491
6		.421	.553	.378	.489	.351	.448	.333	.419
8		.318	.429	.287	.379	.267	.347	.253	.324
10		.256	.351	.231	.309	.215	.282	.204	.263
15		.171	.240	.155	.211	.145	.193	.137	.179
20		.129	.182	.117	.160	.109	.146	.103	.136

Note: For $N = 3$ and $k < 8$, no value of *W* has upper tail probability occurrence less than .05.

Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table G-1 Upper-tail probabilities for T, the Kendall's rank-order correlation coefficient ($N < 10$)

Entries are $p = P[T \geq \text{tabled value}]$

N	T	p	N	T	p	N	T	p	N	T	P
4	0.000	0.625	7	0.048	0.500	9	0.000	0.540	10	0.022	0.500
	0.333	0.375		0.143	0.386		0.056	0.460		0.067	0.431
	0.667	0.167		0.238	0.281		0.111	0.381		0.111	0.364
	1.000	0.042		0.333	0.191		0.167	0.306		0.156	0.300
				0.429	0.119		0.222	0.238		0.200	0.242
5	0.000	0.592		0.524	0.068		0.278	0.179		0.244	0.190
	0.200	0.408		0.619	0.035		0.333	0.130		0.289	0.146
	0.400	0.242		0.714	0.015		0.389	0.090		0.333	0.108
	0.600	0.117		0.810	0.005		0.444	0.060		0.378	0.078
	0.800	0.042		0.905	0.001		0.500	0.038		0.422	0.054
	1.000	0.008		1.000	0.000		0.556	0.022		0.467	0.036
							0.611	0.012		0.511	0.023
6	0.067	0.500	8	0.000	0.548		0.667	0.006		0.556	0.014
	0.200	0.360		0.071	0.452		0.722	0.003		0.600	0.008
	0.333	0.235		0.143	0.360		0.778	0.001		0.644	0.005
	0.467	0.136		0.214	0.274		0.833	0.000		0.689	0.002
	0.600	0.068		0.286	0.199		0.889	0.000		0.733	0.001
	0.733	0.028		0.357	0.138		0.944	0.000		0.778	0.000
	0.867	0.008		0.429	0.089		1.000	0.000		0.822	0.000
	1.000	0.001		0.500	0.054					0.867	0.000
				0.571	0.031					0.911	0.000
				0.643	0.016					0.956	0.000
				0.714	0.007					1.000	0.000
				0.786	0.003						
				0.857	0.001						
				0.929	0.000						
				1.000	0.000						

Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table G-2 Critical values for T, the Kendall's rank-order correlation coefficient

Entries are values of T such that $P[T \geq \text{tabled value}] \leq \alpha$

	α	0.100	0.050	0.025	0.010	0.005 (one-tailed)
N	α	0.200	0.100	0.050	0.020	0.010 (two-tailed)
11		0.345	0.418	0.491	0.564	0.600
12		0.303	0.394	0.455	0.545	0.576
13		0.308	0.359	0.436	0.513	0.564
14		0.275	0.363	0.407	0.473	0.516
15		0.276	0.333	0.390	0.467	0.505
16		0.250	0.317	0.383	0.433	0.483
17		0.250	0.309	0.368	0.426	0.471
18		0.242	0.294	0.346	0.412	0.451
19		0.228	0.287	0.333	0.392	0.439
20		0.221	0.274	0.326	0.379	0.421
21		0.210	0.267	0.314	0.371	0.410
22		0.195	0.253	0.295	0.344	0.378
23		0.202	0.257	0.296	0.352	0.391
24		0.196	0.246	0.290	0.341	0.377
25		0.193	0.240	0.287	0.333	0.367
26		0.188	0.237	0.280	0.329	0.360
27		0.179	0.231	0.271	0.322	0.356
28		0.180	0.228	0.265	0.312	0.344
29		0.172	0.222	0.261	0.310	0.340
30		0.172	0.218	0.255	0.301	0.333

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Table H Critical Values for the Friedman two-way analysis of variance by ranks statistics, F_r

k	N	$\alpha \leq .10$	$\alpha \leq .05$	$\alpha \leq .01$
3	3	6.00	6.00	–
	4	6.00	6.50	8.00
	5	5.20	6.40	8.40
	6	5.33	7.00	9.00
	7	5.43	7.14	8.86
	8	5.25	6.25	9.00
	9	5.56	6.22	8.67
	10	5.00	6.20	9.60
	11	4.91	6.54	8.91
	12	5.17	6.17	8.67
	13	4.77	6.00	9.39
	□	4.61	5.99	9.21
4	2	6.00	6.00	–
	3	6.60	7.40	8.60
	4	6.30	7.80	9.60
	5	6.36	7.80	9.96
	6	6.40	7.60	10.00
	7	6.26	7.80	10.37
	8	6.30	7.50	10.35
	□	6.25	7.82	11.34
5	3	7.47	8.53	10.13
	4	7.60	8.80	11.00
	5	7.68	8.96	11.52
	□	7.78	9.49	13.28

Hollander, M., & Wolfe, D.A. (1973). *Nonparametric statistics*. New York: J. Wiley; Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table I Critical values for $T_{xy.z}$, the Kendall's partial rank-order correlation coefficient

	α							
N	0.25	0.20	0.10	0.05	0.025	0.01	0.005	0.001
3	0.500	1.000						
4	0.447	0.500	0.707	0.707	1.000			
5	0.333	0.408	0.534	0.667	0.802	0.816	1.000	
6	0.277	0.327	.472	.600	.667	0.764	0.866	1.000
7	0.233	0.282	0.421	0.527	0.617	0.712	0.761	0.901
8	0.206	0.254	0.382	0.484	0.565	0.648	0.713	0.807
9	0.187	0.230	0.347	0.443	0.515	0.602	0.660	0.757
10	0.170	0.215	0.325	0.413	0.480	0.562	0.614	0.718
11	0.162	0.202	0.305	0.387	0.453	0.530	0.581	0.677
12	0.153	0.190	0.288	0.465	0.430	0.505	0.548	0.643
13	0.145	0.180	0.273	0.347	0.410	0.481	0.527	0.616
14	0.137	0.172	0.260	0.331	0.391	0.458	0.503	0.590
15	0.133	0.166	0.251	0.319	0.377	0.442	0.485	0.570
16	0.125	0.157	.240	.305	.361	.423	.466	.549
17	0.121	0.151	0.231	0.294	0.348	0.410	0.450	0.532
18	0.117	0.147	0.222	0.284	0.336	0.395	0.434	0.514
19	0.114	0.141	0.215	0.275	0.326	0.382	0.421	0.498
20	0.111	0.139	0.210	0.268	0.318	0.374	0.412	0.488
25	0.098	0.122	0.185	0.236	0.279	0.329	0.363	0.430
30	0.088	0.110	0.167	0.213	0.253	0.298	0.329	0.390
35	0.081	0.101	0.153	0.196	0.232	0.274	0.303	0.361
40	0.075	0.094	0.142	0.182	0.216	0.255	0.282	0.335
45	0.071	0.088	0.133	0.171	0.203	0.240	0.265	0.316
50	0.067	0.083	0.126	0.161	0.192	0.225	0.250	0.298
60	0.060	0.075	0.114	0.147	0.174	0.206	0.227	0.270
70	0.056	0.070	0.106	0.135	0.160	0.190	0.210	0.251
80	0.052	0.065	0.098	0.126	0.150	0.178	0.197	0.235
90	.049	0.061	0.092	0.119	0.141	0.167	0.185	0.221

Maghsoodloo, S. (1975). Estimated of the quartiles of Kendall's partial rank correlation coefficient. *Journal of Statistical Computing and Simulation, 4*, 155–164; Maghsoodloo, S., & Pallos, L.L. (1981). Asymptotic behavior of Kendall's partial rank correlation coefficient and additional quartile estimates. *Journal of Statistical Computing and Simulation, 13*, 41–48; Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table J Critical Values, $d_\alpha(N)$, of the maximum absolute difference between sample and population cumulative distributions [Kolmogorov-Smirnov One-Sample]

Sample size	Level of significance (α)				
(N)	0.20	0.15	0.10	0.05	0.01
1	0.900	0.925	0.950	0.975	0.995
2	0.684	0.726	0.776	0.842	0.929
3	0.565	0.597	0.642	0.708	0.828
4	0.494	0.525	0.564	0.624	0.733
5	0.446	0.474	0.510	0.565	0.669
6	0.410	0.436	0.470	0.521	0.618
7	0.381	0.405	0.438	0.486	0.577
8	0.358	0.381	0.411	0.457	0.543
9	0.339	0.360	0.388	0.432	0.514
10	0.322	0.342	0.368	0.410	0.490
11	0.307	0.326	0.352	0.391	0.468
12	0.295	0.313	0.338	0.375	0.450
13	0.284	0.302	0.325	0.361	0.433
14	0.274	0.292	0.314	0.349	0.418
15	0.266	0.283	0.304	0.338	0.404
16	0.258	0.274	0.295	0.328	0.392
17	0.250	0.266	0.286	0.318	0.381
18	0.244	0.259	0.278	0.309	0.371
19	0.237	0.252	0.272	0.301	0.363
20	0.231	0.246	0.264	0.294	0.356
25	0.21	0.22	0.24	0.27	0.32
30	0.19	0.20	0.22	0.24	0.29
35	0.18	0.19	0.21	0.23	0.27
over 35	$\frac{1.07}{\sqrt{N}}$	$\frac{1.14}{\sqrt{N}}$	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

Values of $d_\alpha(N)$ such that $\Pr[max|S_N(x) - F_0(x)| > d_\alpha(N)] = \alpha$, where $F_0(x)$ is the theoretical cumulative distribution and $S_N(x)$ is an observed cumulative distribution for a sample of N

Massey, F.J. (1951). The Kolmogorov-Smirnov test for goodness of fit. *Journal of the American Statistical Association, 46*, 70. Reprinted with permission

Table K *t*-test Critical Values

Degrees of Freedom	α	0.005 0.01	0.01 0.02	0.025 0.05	0.05 0.10	0.10 (one-tailed) 0.20 (two-tailed)
1		63.657	31.821	12.706	6.314	3.078
2		9.925	6.965	4.303	2.920	1.886
3		5.841	4.541	3.182	2.353	1.638
4		4.604	3.747	2.776	2.132	1.533
5		4.032	3.365	2.571	2.015	1.476
6		3.707	3.143	2.447	1.943	1.440
7		3.499	2.998	2.365	1.895	1.415
8		3.355	2.896	2.306	1.860	1.397
9		3.250	2.821	2.262	1.833	1.383
10		3.169	2.764	2.228	1.812	1.372
11		3.106	2.718	2.201	1.796	1.363
12		3.055	2.681	2.179	1.782	1.356
13		3.012	2.650	2.160	1.771	1.350
14		2.977	2.624	2.145	1.761	1.345
15		2.947	2.602	2.131	1.753	1.341
16		2.921	2.583	2.120	1.746	1.337
17		2.898	2.567	2.110	1.740	1.333
18		2.878	2.552	2.101	1.734	1.330
19		2.861	2.539	2.093	1.729	1.328
20		2.845	2.528	2.086	1.725	1.325
21		2.831	2.518	2.080	1.721	1.323
22		2.819	2.508	2.074	1.717	1.321
23		2.807	2.500	2.069	1.714	1.319
24		2.797	2.492	2.064	1.711	1.318
25		2.787	2.485	2.060	1.708	1.316
26		2.779	2.479	2.056	1.706	1.315
27		2.771	2.473	2.052	1.703	1.314
28		2.763	2.467	2.048	1.701	1.313
29		2.756	2.462	2.045	1.699	1.311
30		2.750	2.457	2.042	1.697	1.310
∞		2.576	2.326	1.960	1.645	1.282

Table L Critical Values for the Chi-Square distribution

	Significance level			
df	0.10	0.05	0.01	0.001
1	2.71	3.84	6.63	10.83
2	4.61	5.99	9.21	13.82
3	6.25	7.81	11.34	16.27
4	7.78	9.49	13.28	18.47
5	9.24	11.07	15.09	20.52
6	10.64	12.59	16.81	22.46
7	12.02	14.07	18.48	24.32
8	13.36	15.51	20.09	26.12
9	14.68	16.92	21.67	27.88
10	15.99	18.31	23.21	29.59
11	17.28	19.68	24.72	31.26
12	18.55	21.03	26.22	32.91
13	19.81	22.36	27.69	34.53
14	21.06	23.68	29.14	36.12
15	22.31	25.00	30.58	37.70
16	23.54	26.30	32.00	39.25
17	24.77	27.59	33.41	40.79
18	25.99	28.87	34.81	42.31
19	27.20	30.14	36.19	43.82
20	28.41	31.41	37.57	45.31
21	29.62	32.67	38.93	46.80
22	30.81	33.92	40.29	48.27
23	32.01	35.17	41.64	49.73
24	33.20	36.42	42.98	51.18
25	34.38	37.65	44.31	52.62
26	35.56	38.89	45.64	54.05
27	36.74	40.11	46.96	55.48
28	37.92	41.34	48.28	56.89
29	39.09	42.56	49.59	58.30
30	40.26	43.77	50.89	59.70
35	46.06	49.80	57.34	66.62
40	51.81	55.76	63.69	73.40
50	63.17	67.50	76.15	86.66
60	74.40	79.08	88.38	99.61
70	85.53	90.53	100.43	112.32
80	96.58	101.88	112.33	124.84
100	118.50	124.34	135.81	149.45

Rosenthal, J.A. (2012). *Statistics and data interpretation for social work*. New York: Springer

Table M Critical Values (frequencies) for the Binomial Distribution: one-tailed test, Alpha = 0.05

Sample size	Direction of alternative hypothesis	Proportion stated in null (*P*)								
		0.10	0.20	0.25	0.3333	0.5	0.6667	0.75	0.80	0.90
2	<								0	0
	>	2	2							
3	<						0	0	0	1
	>	2	3	3	3					
4	<						0	0	1	1
	>	3	3	4	4					
5	<					0	1	1	1	2
	>	3	4	4	4	5				
6	<					0	1	2	2	3
	>	3	4	4	5	6				
7	<					0	2	2	3	4
	>	3	4	5	5	7				
8	<				0	1	2	3	3	5
	>	3	5	5	6	7	8			
9	<				0	1	3	4	4	5
	>	4	5	5	6	8	9			
10	<				0	1	3	4	5	6
	>	4	5	6	7	9	10			
11	<			0	0	2	4	5	5	7
	>	4	6	6	7	9	11	11		
12	<			0	0	2	4	5	6	8
	>	4	6	7	8	10	12	12		
13	<			0	1	3	5	6	7	9
	>	4	6	7	8	10	12	13		
14	<		0	0	1	3	5	7	8	10
	>	4	6	7	9	11	13	14	14	
15	<		0	0	1	3	6	7	8	10
	>	5	7	8	9	12	14	15	15	
18	<		0	1	2	5	8	9	10	13
	>	5	8	9	10	13	16	17	18	
20	<		0	1	2	5	9	11	12	15
	>	5	8	9	11	15	18	19	20	
25	<		1	2	4	7	12	14	16	19
	>	6	9	11	13	18	21	23	24	
30	<	0	2	3	5	10	15	17	19	23
	>	7	11	13	15	20	25	27	28	30
35	<	0	2	4	6	12	18	21	23	27
	>	8	12	14	17	23	29	31	33	35
40	<	0	3	5	8	14	21	24	27	32
	>	8	13	16	19	26	32	35	37	40
50	<	1	5	7	10	18	27	31	34	40
	>	10	16	19	23	32	40	43	45	49
75	<	2	8	12	17	29	42	49	53	62

(continued)

Table M (continued)

Sample size	Direction of alternative hypothesis	Proportion stated in null (P)								
		0.10	0.20	0.25	0.3333	0.5	0.6667	0.75	0.80	0.90
	>	13	22	26	33	46	58	63	67	73
100	<	4	13	17	25	41	58	67	72	84
	>	16	28	33	42	59	75	83	87	96

Rosenthal, J.A. (2012). *Statistics and data interpretation for social work*. New York: Springer

Table N Critical values for the Kruskal-Wallis one-way analysis of variance by ranks statistic, KW

Sample sizes			α				
n_1	$_2n$	n_3	0.10	0.05	0.01	0.005	0.001
2	2	2	4.25				
3	2	1	4.29				
3	2	2	4.71	4.71			
3	3	1	4.57	5.14			
3	3	2	4.56	5.36			
3	3	3	4.62	5.60	7.20	7.20	
4	2	1	4.50				
4	2	2	4.46	5.33			
4	3	1	4.06	5.21			
4	3	2	4.51	5.44	6.44	7.00	
4	3	3	*4.71*	5.73	6.75	7.32	8.02
4	4	1	4.17	4.97	6.67		
4	4	2	4.55	5.45	7.04	7.28	
4	4	3	4.55	5.60	7.14	7.59	8.32
4	4	4	4.65	5.69	7.66	8.00	8.65
5	2	1	4.20	5.00			
5	2	2	4.36	5.16	6.53		
5	3	1	4.02	4.96			
5	3	2	4.65	5.25	6.82	7.18	
5	3	3	4.53	5.65	7.08	7.51	8.24
5	4	1	3.99	4.99	6.95	7.36	
5	4	2	4.54	5.27	7.12	7.57	8.11
5	4	3	4.55	5.63	7.44	7.91	8.50
5	4	4	4.62	5.62	7.76	8.14	9.00
5	5	1	4.11	5.13	7.31	7.75	
5	5	2	4.62	5.34	7.27	8.13	8.68
5	5	3	4.54	5.71	7.54	8.24	9.06
5	5	4	4.53	5.64	7.77	8.37	9.32
5	5	5	4.56	5.78	7.98	8.72	9.68
	Large samples		4.61	5.99	9.21	10.60	13.82

Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table O Critical Values and criteria for testing extreme values [Dixon Test]

N \ α	0.30	0.20	0.10	0.05	0.02	0.01	0.005	Criterion
3	0.684	0.781	0.886	0.941	0.976	0.988	0.994	$r_{10} = \frac{x_N - x_{N-1}}{x_N - x_1}$
4	0.471	0.560	0.679	0.765	0.846	0.889	0.926	
5	0.373	0.451	0.557	0.642	0.729	0.780	0.821	
6	0.318	0.386	0.482	0.560	0.644	0.698	0.740	
7	0.281	0.344	0.434	0.507	0.586	0.637	0.680	
8	0.318	0.385	0.479	0.554	0.631	0.683	0.725	$r_{11} = \frac{x_N - x_{N-1}}{x_N - x_2}$
9	0.288	0.352	0.441	0.512	0.587	0.635	0.677	
10	0.265	0.325	0.409	0.477	0.551	0.597	0.639	
11	0.391	0.442	0.517	0.576	0.638	0.679	0.713	$r_{21} - \frac{x_N - x_{N-2}}{x_N - x_2}$
12	0.370	0.419	0.490	0.546	0.605	0.642	0.675	
13	0.351	0.399	0.467	0.521	0.578	0.615	0.649	
14	0.370	0.421	0.492	0.546	0.602	0.641	0.674	$r_{22} = \frac{x_N - x_{N-2}}{x_N - x_3}$
15	0.353	0.402	0.472	0.525	0.579	0.616	0.647	
16	0.338	0.386	0.454	0.507	0.559	0.595	0.624	
17	0.325	0.373	0.438	0.409	0.542	0.577	0.605	
18	0.314	0.361	0.424	0.475	0.527	0.561	0.589	
19	0.304	0.350	0.412	0.462	0.514	0.547	0.575	
20	0.295	0.340	0.401	0.450	0.502	0.535	0.562	
21	0.287	0.331	0.391	0.440	0.491	0.524	0.551	
22	0.280	0.323	0.382	0.430	0.481	0.514	0.541	
23	0.274	0.316	0.374	0.421	0.472	0.505	0.532	
24	0.268	0.310	0.367	0.413	0.464	0.497	0.524	
25	0.262	0.304	0.360	0.406	0.457	0.489	0.516	

Dixon, W.J. (1953) Processing data for outliers, *Biometrics, 9*(1), p. 89. Reprinted with permission

Table P Table of probabilities associated with values as small as (or smaller than) observed values of k in the binomial test

Given in the body of the table are one-tailed probabilities under H_0 for the binomial test when $p = q = \frac{1}{2}$

Entries are $P[Y \leq k]$. Note that entries may also be read as $P[Y \geq N - k]$

	k																	
N	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
4	062	312	688	938	1.0													
5	031	188	500	812	969	1.0												
6	016	109	344	656	891	984	1.0											
7	008	062	227	500	773	938	992	1.0										
8	004	035	145	363	637	855	965	996	1.0									
9	002	020	090	254	500	746	910	980	998	1.0								
10	001	011	055	172	377	623	828	945	989	999	1.0							
11		006	033	113	274	500	726	887	967	994	999+	1.0						
12		003	019	073	194	387	613	806	927	981	997	999+	1.0					
13		002	011	046	133	291	500	709	867	954	989	998	999+	1.0				
14		001	006	029	090	212	395	605	788	910	971	994	999	999+	1.0			
15			004	018	059	151	304	500	696	849	941	982	996	999+	999+	1.0		
16			002	011	038	105	227	402	598	773	895	962	989	998	999+	999+	1.0	
17			001	006	025	072	166	315	500	685	834	928	975	994	999	999+	999+	1.0
18			001	004	015	048	119	240	407	593	760	881	952	985	996	999	999+	999+
19				002	010	032	084	180	324	500	676	820	916	968	990	998	999+	999+
20				001	006	021	058	132	252	412	588	748	868	942	979	994	999	999+
21				001	004	013	039	095	192	332	500	668	808	905	961	987	996	999
22					002	008	026	067	143	262	416	584	738	857	933	974	992	998
23					001	005	017	047	105	202	339	500	661	798	895	953	983	995
24					001	003	011	032	076	154	271	419	581	729	846	924	968	989
25						002	007	022	054	115	212	345	500	655	788	885	946	978
26						001	005	014	038	084	163	279	423	577	721	837	916	962
27						001	003	010	026	061	124	221	351	500	649	779	876	939

28	002	006	018	044	092	172	286	425	575	714	828	908
29	001	004	012	031	068	132	229	356	500	644	771	868
30	001	003	008	021	049	100	181	292	428	572	708	819
31		002	005	015	035	075	141	237	360	500	640	763
32		001	004	010	025	055	108	189	298	430	570	702
33		001	002	007	018	040	081	148	243	364	500	636
34			001	005	012	029	061	115	196	304	432	568
35			001	003	008	020	045	088	155	250	368	500

Note: Decimal points omitted, and values less than .0005 are omitted

Siegel, S. & Castellan, N.J. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Table Q-1 Positive *z*-score table

Standard normal distribution: area to the left										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50 and higher	0.9999									

Table Q-2 Negative *z*-score table

Standard normal distribution: area to the left										
Z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
−3.50 and lower	0.0001									
−3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
−3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
−3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
−3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
−3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
−2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
−2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
−2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
−2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
−2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
−2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
−2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
−2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
−2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
−2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
−1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
−1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
−1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
−1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
−1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
−1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
−1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
−1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
−0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
−0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
−0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
−0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
−0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
−0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
−0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
−0.2	0.3829	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
−0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
−0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

Table R Binomial coefficients

N	$\binom{N}{0}$	$\binom{N}{1}$	$\binom{N}{2}$	$\binom{N}{3}$	$\binom{N}{4}$	$\binom{N}{5}$	$\binom{N}{6}$	$\binom{N}{7}$	$\binom{N}{8}$	$\binom{N}{9}$	$\binom{N}{10}$
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

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Table S Critical values of T^+ for the Wilcoxon Signed Ranks Test

Table entries for a given N is $P[T^+ > c]$, the probability that T^+ is greater than or equal to the sum c.

	N						
c	3	4	5	6	7	8	9
3	.6250						
4	.3750						
5	.2500	.5625					
6	.1250	.4375					
7		.3125					
8		.1875	.5000				
9		.1250	.4063				
10		.0625	.3125				
11			.2188	.5000			
12			.1563	.4219			
13			.0938	.3438			
14			.0625	.2813	.5313		
15			.0313	.2188	.4688		
16				.1563	.4063		
17				.1094	.3438		
18				.0781	.2891	.5273	
19				.0469	.2344	.4727	
20				.0313	.1875	.4219	
21				.0156	.1484	.3711	
22					.1094	.3203	
23					.0781	.2734	.5000
24					.0547	.2305	.4551
25					.0391	.1914	.4102
26					.0234	.1563	.3672
27					.0156	.1250	.3262
28					.0078	.0977	.2852
29						.0742	.2480
30						.0547	.2129
31						.0391	.1797
32						.0273	.1504
33						.0195	.1250
34						.0117	.1016
35						.0078	.0820
36						.0039	.0645
37							.0488
38							.0371
39							.0273
40							.0195
41							.0137
42							.0098
43							.0059
44							.0039
45							.0020

(continued)

Table S (continued)

	N					
c	10	11	12	13	14	15
28	.5000					
29	.4609					
30	.4229					
31	.3848					
32	.3477					
33	.3125	.5171				
34	.2783	.4829				
35	.2461	.4492				
36	.2158	.4155				
37	.1875	.3823				
38	.1611	.3501				
39	.1377	.3188	.5151			
40	.1162	.2886	.4849			
41	.0967	.2598	.4548			
42	.0801	.2324	.4250			
43	.0654	.2065	.3955			
44	.0527	.1826	.3667			
45	.0420	.1602	.3386			
46	.0322	.1392	.3110	.5000		
47	.0244	.1201	.2847	.4730		
48	.0186	.1030	.2593	.4463		
49	.0137	.0874	.2349	.4197		
50	.0098	.0737	.2119	.3934		
51	.0068	.0615	.1902	.3677		
52	.0049	.0508	.1697	.3424		
53	.0029	.0415	.1506	.3177	.5000	
54	.0020	.0337	.1331	.2939	.4758	
55	.0010	.0269	.1167	.2709	.4516	
56		.0210	.1018	.2487	.4276	
57		.0161	.0881	.2274	.4039	
58		.0122	.0757	.2072	.3804	
59		.0093	.0647	.1879	.3574	
60		.0068	.0549	.1698	.3349	.5110
61		.0049	.0461	.1527	.3129	.4890
62		.0034	.0386	.1367	.2915	.4670
63		.0024	.0320	.1219	.2708	.4452
64		.0015	.0261	.1082	.2508	.4235
65		.0010	.0212	.0955	.2316	.4020
66		.0005	.0171	.0839	.2131	.3808
67			.0134	.0732	.1955	.3599
68			.0105	.0636	.1788	.3394
69			.0081	.0549	.1629	.3193
70			.0061	.0471	.1479	.2997
71			.0046	.0402	.1338	.2807
72			.0034	.0341	.1206	.2622
73			.0024	.0287	.1083	.2444
74			.0017	.0239	.0969	.2271
75			.0012	.0199	.0863	.2106
76			.0007	.0164	.0765	.1947
77			.0005	.0133	.0676	.1796
78			.0002	.0107	.0594	.1651

(continued)

Table S (continued)

	N		
c	13	14	15
79	.0085	.0520	.1514
80	.0067	.0453	.1384
81	.0052	.0392	.1262
82	.0040	.0338	.1147
83	.0031	.0290	.1039
84	.0023	.0247	.0938
85	.0017	.0209	.0844
86	.0012	.0176	.0757
87	.0009	.0148	.0677
88	.0006	.0123	.0603
89	.0004	.0101	.0535
90	.0002	.0083	.0473
91	.0001	.0067	.0416
92		.0054	.0365
93		.0043	.0319
94		.0034	.0277
95		.0026	.0240
96		.0020	.0206
97		.0015	.0177
98		.0012	.0151
99		.0009	.0128
100		.0006	.0108
101		.0004	.0090
102		.0003	.0075
103		.0002	.0062
104		.0010	.0051
105		.0001	.0042
106			.0034
107			.0027
108			.0021
109			.0017
110			.0013
111			.0010
112			.0008
113			.0006
114			.0004
115			.0003
116			.0002
117			.0002
118			.0001
119			.0001
120			.0000+

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Table T Lower- and upper-tail probabilities for W_x, the Wilcoxon-Mann–Whitney rank-sum statistic [Moses Rank-Like Test for scale differences]

Entries are $P[Wx < cL]$ and $P[Wx > cU]$. Wx is the rank-sum for the smaller group.

$m = 3$																				
c_L	$n = 3$	c_U	$n = 4$	c_U	$n = 5$	c_U	$n = 6$	c_U	$n = 7$	c_U	$n = 8$	c_U	$n = 9$	c_U	$n = 10$	c_U	$n = 11$	c_U	$n = 12$	c_U
6	.0500	15	.0286	18	.0179	21	.0119	24	.0083	27	.0061	30	.0045	33	.0035	36	.0027	39	.0022	42
7	.1000	14	.0571	17	.0357	20	.0238	23	.0167	26	.0121	29	.0091	32	.0070	35	.0055	38	.0044	41
8	.2000	13	.1143	16	.0714	19	.0476	22	.0333	25	.0242	28	.0182	31	.0140	34	.0110	37	.0088	40
9	.3500	12	.2000	15	.1250	18	.0833	21	.0583	24	.0424	27	.0318	30	.0245	33	.0192	36	.0154	39
10	.5000	11	.3143	14	.1964	17	.1310	20	.0917	23	.0667	26	.0500	29	.0385	32	.0302	35	.0242	38
11	.6500	10	.4286	13	.2857	16	.1905	19	.1333	22	.0970	25	.0727	28	.0559	31	.0440	34	.0352	37
12	.8000	9	.5714	12	.3929	15	.2738	18	.1917	21	.1394	24	.1045	27	.0804	30	.0632	33	.0505	36
13	.9000	8	.6857	11	.5000	14	.3571	17	.2583	20	.1879	23	.1409	26	.1084	29	.0852	32	.0681	35
14	.9500	7	.8000	10	.6071	13	.4524	16	.3333	19	.2485	22	.1864	25	.1434	28	.1126	31	.0901	34
15	1.0000	6	.8857	9	.7143	12	.5476	15	.4167	18	.3152	21	.2409	24	.1853	27	.1456	30	.1165	33
16			.9429	8	.8036	11	.6429	14	.5000	17	.3879	20	.3000	23	.2343	26	.1841	29	.1473	32
17			.9714	7	.8750	10	.7262	13	.5833	16	.4606	19	.3636	22	.2867	25	.2280	28	.1824	31
18			1.0000	6	.9286	9	.8095	12	.6667	15	.5394	18	.4318	21	.3462	24	.2775	27	.2242	30
19					.9643	8	.8690	11	.7417	14	.6121	17	.5000	20	.4056	23	.3297	26	.2681	29
20					.9821	7	.9167	10	.8083	13	.6848	16	.5682	19	.4685	22	.3846	25	.3165	28
21					1.0000	6	.9524	9	.8667	12	.7515	15	.6364	18	.5315	21	.4423	24	.3670	27
22							.9762	8	.7083	11	.8121	14	.7000	17	.5944	20	.5000	23	.4198	26
23							.9881	7	.9417	10	.8606	13	.7591	16	.6538	19	.5577	22	.4725	25
24							1.0000	6	.9667	9	.9030	12	.8136	15	.7133	18	.6154	21	.5275	24

$m = 4$																		
c_L	$n = 4$	c_U	$n = 5$	c_U	$n = 6$	c_U	$n = 7$	c_U	$n = 8$	c_U	$n = 9$	c_U	$n = 10$	c_U	$n = 11$	c_U	$n = 12$	c_U
10	.0143	26	.0079	30	.0048	34	.0030	38	.0020	42	.0014	46	.0010	50	.0007	54	.0005	58
11	.0286	25	.0159	29	.0055	33	.0061	37	.0040	41	.0028	45	.0020	49	.0015	53	.0011	57
12	.0571	24	.0317	28	.0190	32	.0121	36	.0081	40	.0056	44	.0040	48	.0029	52	.0022	56
13	.1000	23	.0556	27	.0333	31	.0212	35	.0141	39	.0098	43	.0070	47	.0051	51	.0038	55
14	.1714	22	.0952	26	.0571	30	.0364	34	.0242	38	.0168	42	.0120	46	.0088	50	.0066	54
15	.2429	21	.1429	25	.0857	29	.0545	33	.0364	37	.0252	41	.0180	45	.0132	49	.0099	53
16	.3429	20	.2063	24	.1286	28	.0818	32	.0545	36	.0378	40	.0270	44	.0198	48	.0148	52
17	.4429	19	.2778	23	.1762	27	.1152	31	.0768	35	.0531	39	.0380	43	.0278	47	.0209	51
18	.5571	18	.3651	22	.2381	26	.1576	30	.1071	34	.0741	38	.0529	42	.0388	46	.0291	50
19	.6571	17	.4524	21	.3048	25	.2061	29	.1414	33	.0993	37	.0709	41	.0520	45	.0390	49
20	.7571	16	.5476	20	.3810	24	.2636	28	.1838	32	.1301	36	.0939	40	.0689	44	.0516	48
21	.8286	15	.6349	19	.4571	23	.3242	27	.2303	31	.1650	35	.1199	39	.0886	43	.0665	47
22	.9000	14	.7222	18	.5429	22	.3939	26	.2840	30	.2070	34	.1518	38	.1128	42	.0852	46
23	.9429	13	.7937	17	.6190	21	.4636	25	.3414	29	.2517	33	.1868	37	.1399	41	.1060	45
24	.9714	12	.8571	16	.6952	20	.5364	24	.4040	28	.3021	32	.2268	36	.1714	40	.1308	44
25	.9857	11	.9048	15	.7619	19	.6061	23	.4667	27	.3552	31	.2697	35	.2059	39	.1582	43
26	1.0000	10	.9444	14	.8238	18	.6758	22	.5333	26	.4126	30	.3177	34	.2447	38	.1896	42
27			.9683	13	.8714	17	.7364	21	.5960	25	.4699	29	.3666	33	.2857	37	.2231	41
28			.9841	12	.9143	16	.7939	20	.6586	24	.5301	28	.4196	32	.3304	36	.2604	40
29			.9921	11	.9429	15	.8424	19	.7152	23	.5874	27	.4725	31	.3766	35	.2995	39
30			1.0000	10	.9967	14	.8848	18	.7697	22	.6448	26	.5275	30	.4256	34	.3418	38
31					.9810	13	.9182	17	.8162	21	.6979	25	.5804	29	.4747	33	.3852	37
32					.9905	12	.9455	16	.8586	20	.7483	24	.6334	28	.5253	32	.4308	36
33					.9952	11	.9636	15	.8929	19	.7930	23	.6823	27	.5744	31	.4764	35
34					1.0000	10	.9788	14	.9232	18	.8350	22	.7303	26	.6234	30	.5236	34

(continued)

Table T (continued)

$m = 5$												
c_L	$n = 5$	c_U	$n = 6$	c_U	$n = 7$	c_U	$n = 8$	c_U	$n = 9$	c_U	$n = 10$	c_U
15	.0040	40	.0022	45	.0013	50	.0008	55	.0005	60	.0003	65
16	.0079	39	.0043	44	.0025	49	.0016	54	.0010	59	.0070	64
17	.0159	38	.0087	43	.0051	48	.0031	53	.0020	58	.0013	63
18	.0278	37	.0152	42	.0088	47	.0054	52	.0035	57	.0023	62
19	.0476	36	.0260	41	.0152	46	.0093	51	.0060	56	.0040	61
20	.0754	35	.0411	40	.0240	45	.0148	50	.0095	55	.0063	60
21	.1111	34	.0628	39	.0366	44	.0225	49	.0145	54	.0097	59
22	.1548	33	.0887	38	.0530	43	.0326	48	.0210	53	.0140	58
23	.2103	32	.1234	37	.0745	42	.0466	47	.0300	52	.0200	57
24	.2738	31	.1645	36	.1010	41	.0637	46	.0415	51	.0276	56
25	.3452	30	.2143	35	.1338	40	.0855	45	.0559	50	.0376	55
26	.4206	29	.2684	34	.1717	39	.1111	44	.0734	49	.0496	54
27	.5000	28	.3312	33	.2159	38	.1422	43	.0949	48	.0646	53
28	.5794	27	.3961	32	.2652	37	.1772	42	.1199	47	.0823	52
29	.6548	26	.4654	31	.3194	36	.2176	41	.1489	46	.1032	51
30	.7262	25	.5346	30	.3775	35	.2618	40	.1818	45	.1272	50
31	.7897	24	.6039	29	.4381	34	.3108	39	.2188	44	.1548	49
32	.8452	23	.6688	28	.5000	33	.3621	38	.2592	43	.1855	48
33	.8889	22	.7316	27	.5619	32	.4165	37	.3032	42	.2198	47
34	.9246	21	.7857	26	.6225	31	.4716	36	.3497	41	.2567	46
35	.9524	20	.8355	25	.6806	30	.5284	35	.3986	40	.2970	45
36	.9722	19	.8766	24	.7348	29	.5835	34	.4471	39	.3393	44
37	.9841	18	.9113	23	.7841	28	.6379	33	.5000	38	.3839	43
38	.9921	17	.9372	22	.8283	27	.6892	32	.5509	37	.4296	42
39	.9960	16	.9589	21	.8662	26	.7382	31	.6014	36	.4765	41
40	1.0000	15	.9740	20	.8990	25	.7824	30	.6503	35	.5235	40

(continued)

Table T (continued)

$m = 6$										
c_L	$n = 6$	c_U	$n = 7$	c_U	$n = 8$	c_U	$n = 9$	c_U	$n = 10$	c_U
21	.0011	57	.0006	63	.0003	69	.0002	75	.0001	81
22	.0022	56	.0012	62	.0007	68	.0004	74	.0002	80
23	.0043	55	.0023	61	.0013	67	.0008	73	.0005	79
24	.0076	54	.0041	60	.0023	66	.0014	72	.0009	78
25	.0130	53	.0070	59	.0040	65	.0024	71	.0015	77
26	.0206	52	.0111	58	.0063	64	.0038	70	.0024	76
27	.0325	51	.0175	57	.0100	63	.0060	69	.0037	75
28	.0465	50	.0256	56	.0147	62	.0088	68	.0055	74
29	.0660	49	.0367	55	.0213	61	.0128	67	.0080	73
30	.0898	48	.0507	54	.0296	60	.0180	66	.0112	72
31	.1201	47	.0688	53	.0406	59	.0248	65	.0156	71
32	.1548	46	.0903	52	.0539	58	.0332	64	.0210	70
33	.1970	45	.1171	51	.0709	57	.0440	63	.0280	69
34	.2424	44	.1474	50	.0906	56	.0567	62	.0363	68
35	.2944	43	.1830	49	.1142	55	.0723	61	.0467	67
36	.3496	42	.2226	48	.1412	54	.0905	60	.0589	66
37	.4091	41	.2669	47	.1725	53	.1119	59	.0736	65
38	.4686	40	.3141	46	.2068	52	.1361	58	.0903	64
39	.5314	39	.3654	45	.2454	51	.1638	57	.1099	63
40	.5909	38	.4178	44	.2864	50	.1942	56	.1317	62
41	.6504	37	.4726	43	.3310	49	.2280	55	.1566	61
42	.7056	36	.5274	42	.3773	48	.2643	54	.1838	60
43	.7576	35	.5822	41	.4259	47	.3035	53	.2139	59
44	.8030	34	.6346	40	.4749	46	.3445	52	.2461	58
45	.8452	33	.6859	39	.5251	45	.3878	51	.2811	57
46	.8799	32	.7331	38	.5741	44	.4320	50	.3177	56
47	.9102	31	.7774	37	.6227	43	.4773	49	.3564	55
48	.9340	30	.8170	36	.6690	42	.5227	48	.3962	54
49	.9535	29	.8526	35	.7136	41	.5680	47	.4374	53
50	.9675	28	.8829	34	.7546	40	.6122	46	.4789	52
51	.9794	27	.9097	33	.7932	39	.6555	45	.5211	51

(continued)

Table T (continued)

			$m = 7$					
c_L	$n = 7$	c_U	$n = 8$	c_U	$n = 9$	c_U	$n = 10$	c_U
28	.0003	77	.0002	84	.0001	91	.0001	98
29	.0006	76	.0003	83	.0002	90	.0001	97
30	.0012	75	.0006	82	.0003	89	.0002	96
31	.0020	74	.0011	81	.0006	88	.0004	95
32	.0035	73	.0019	80	.0010	87	.0006	94
33	.0055	72	.0030	79	.0017	86	.0010	93
34	.0087	71	.0047	78	.0026	85	.0015	92
35	.0131	70	.0070	77	.0039	84	.0023	91
36	.0189	69	.0103	76	.0058	83	.0034	90
37	.0265	68	.0145	75	.0082	82	.0048	89
38	.0364	67	.0200	74	.0115	81	.0068	88
39	.0487	66	.0270	73	.0156	80	.0093	87
40	.0641	65	.0361	72	.0209	79	.0125	86
41	.0825	64	.0469	71	.0274	78	.0165	85
42	.1043	63	.0603	70	.0356	77	.0215	84
43	.1297	62	.0760	69	.0454	76	.0277	83
44	.1588	61	.0946	68	.0571	75	.0351	82
45	.1914	60	.1159	67	.0708	74	.0439	81
46	.2279	59	.1405	66	.0869	73	.0544	80
47	.2675	58	.1678	65	.1052	72	.0665	79
48	.3100	57	.1984	64	.1261	71	.0806	78
49	.3552	56	.2317	63	.1496	70	.0966	77
50	.4024	55	.2679	62	.1755	69	.1148	76
51	.4508	54	.3063	61	.2039	68	.1349	75
52	.5000	53	.3472	60	.2349	67	.1574	74
53	.5492	52	.3894	59	.2680	66	.1819	73
54	.5976	51	.4333	58	.3032	65	.2087	72
55	.6448	50	.4775	57	.3403	64	.2374	71
56	.6900	49	.5225	56	.3788	63	.2681	70
57	.7325	48	.5667	55	.4185	62	.3004	69
58	.7721	47	.6106	54	.4591	61	.3345	68
59	.8086	46	.6528	53	.5000	60	.3698	67
60	.8412	45	.6937	52	.5409	59	.4063	66
61	.8703	44	.7321	51	.5815	58	.4434	65
62	.8957	43	.7683	50	.6212	57	.4811	64
63	.9175	42	.8016	49	.6597	56	.5189	63

(continued)

Table T (continued)

			$m = 8$			
c_L	$n = 8$	c_U	$n = 9$	c_U	$n = 10$	c_U
36	.0001	100	.0000	108	.0000	116
37	.0002	99	.0001	107	.0000	115
38	.0003	98	.0002	106	.0001	114
39	.0005	97	.0003	105	.0002	113
40	.0009	96	.0005	104	.0003	112
41	.0015	95	.0008	103	.0004	111
42	.0023	94	.0012	102	.0007	110
43	.0035	93	.0019	101	.0010	109
44	.0052	92	.0028	100	.0015	108
45	.0074	91	.0039	99	.0022	107
46	.0103	90	.0056	98	.0031	106
47	.0141	89	.0076	97	.0043	105
48	.0190	88	.0103	96	.0058	104
49	.0249	87	.0137	95	.0078	103
50	.0325	86	.0180	94	.0103	102
51	.0415	85	.0232	93	.0133	101
52	.0524	84	.0296	92	.0171	100
53	.0652	83	.0372	91	.0217	99
54	.0803	82	.0464	90	.0273	98
55	.0974	81	.0570	89	.0338	97
56	.1172	80	.0694	88	.0416	96
57	.1393	79	.0836	87	.0506	95
58	.1641	78	.0998	86	.0610	94
59	.1911	77	.1179	85	.0729	93
60	.2209	76	.1383	84	.0864	92
61	.2527	75	.1606	83	.1015	91
62	.2869	74	.1852	82	.1185	90
63	.3227	73	.2117	81	.1371	89
64	.3605	72	.2404	80	.1577	88
65	.3992	71	.2707	79	.1800	87
66	.4392	70	.3029	78	.2041	86
67	.4796	69	.3365	77	.2299	85
68	.5204	68	.3715	76	.2574	84
69	.5608	67	.4074	75	.2863	83
70	.6008	66	.4442	74	.3167	82
71	.6395	65	.4813	73	.3482	81
72	.6773	64	.5187	72	.3809	80
73	.7131	63	.5558	71	.4143	79
74	.7473	62	.5926	70	.4484	78
75	.7791	61	.6285	69	.4827	77
76	.8089	60	.6635	68	.5173	76

(continued)

Table T (continued)

$m = 9$									
c_L	$n = 9$	c_U	$n = 10$	c_U	c_L	$n = 9$ (cont.)	c_U	$n = 10$ (cont.)	c_U
45	.0000	126	.0000	135	68	.0680	103	.0394	112
46	.0000	125	.0000	134	69	.0807	102	.0474	111
47	.0001	124	.0000	133	70	.0951	101	.0564	110
48	.0001	123	.0001	132	71	.1112	100	.0667	109
49	.0002	122	.0001	131	72	.1290	99	.0782	108
50	.0004	121	.0002	130	73	.1487	98	.0912	107
51	.0006	120	.0003	129	74	.1701	97	.1055	106
52	.0009	119	.0005	128	75	.1933	96	.1214	105
53	.0014	118	.0007	127	76	.2181	95	.1388	104
54	.0020	117	.0011	126	77	.2447	94	.1577	103
55	.0028	116	.0015	125	78	.2729	93	.1781	102
56	.0039	115	.0021	124	79	.3024	92	.2001	101
57	.0053	114	.0028	123	80	.3332	91	.2235	100
58	.0071	113	.0038	122	81	.3652	90	.2483	99
59	.0094	112	.0051	121	82	.3981	89	.2745	98
60	.0122	111	.0066	120	83	.4317	88	.3019	97
61	.0157	110	.0086	119	84	.4657	87	.3304	96
62	.0200	109	.0110	118	85	.5000	86	.3598	95
63	.0252	108	.0140	117	86	.5343	85	.3901	94
64	.0313	107	.0175	116	87	.5683	84	.4211	93
65	.0385	106	.0217	115	88	.6019	83	.4524	92
66	.0470	105	.0267	114	89	.6348	82	.4841	91
67	.0587	104	.0326	113	90	.6668	81	.5159	90

(continued)

Table T (continued)

$m = 10$					
c_L	$n = 10$	c_U	c_L	$n = 10$ (cont.)	c_U
55	.0000	155	81	.0376	129
56	.0000	154	82	.0446	128
57	.0000	153	83	.0526	127
58	.0000	152	84	.0615	126
59	.0001	151	85	.0716	125
60	.0001	150	86	.0827	124
61	.0002	149	87	.0952	123
62	.0002	148	88	.1088	122
63	.0004	147	89	.1237	121
64	.0005	146	90	.1399	120
65	.0008	145	91	.1575	119
66	.0010	144	92	.1763	118
67	.0014	143	93	.1965	117
68	.0019	142	94	.2179	116
69	.0026	141	95	.2406	115
70	.0034	140	96	.2644	114
71	.0045	139	97	.2894	113
72	.0057	138	98	.3153	112
73	.0073	137	99	.3421	111
74	.0093	136	100	.3697	110
75	.0116	135	101	.3980	109
76	.0144	134	102	.4267	108
77	.0177	133	103	.4559	107
78	.0216	132	104	.4853	106
79	.0262	131	105	.5147	105
80	.0315	130			

Siegel, S. & Castellan, N.J. (1988). Nonparametric Statistics for the Behavioral Sciences (2nd ed.). New York: McGraw Hill Book Company. Reprinted with permission

Additional Worked Examples

Chapter 3: Kolmogorov-Smirnov One-Sample Test (Example Using the Income Variable)

$$D = max\left|F_0(X_i) - S_N(X_i)\right|$$

D = the largest absolute value of $F_0(X_i) - S_N(X_i)$; the maximum deviation
$F_0(X_i)$ = theoretical cumulative relative frequency distribution; the theoretical distribution under H_0
$S_N(X_i)$ = observed cumulative relative frequency distribution of N observations
$i = 1, 2, \ldots N$
μ = mean
σ = standard deviation
$z = \frac{X_i - \mu}{\sigma}$ = score used to determine the theoretical relative frequency (probability)

Incomes for each of the $N = 100$ offenders:

$26,104	$17,076	$29,811	$0	$5,238	$11,232	$31,783	$10,218	$33,118	$12,419
$5,274	$29,512	$4,037	$22,078	$17,762	$0	$0	$24,866	$16,551	$17,407
$19,648	$27,949	$0	$0	$0	$12,184	$29,075	$0	$25,038	$20,504
$8,762	$0	$12,462	$0	$23,503	$7,612	$21,518	$3,380	$5,208	$28,002
$738	$0	$0	$24,013	$26,353	$24,698	$34,678	$32,705	$28,575	$12,976
$22,933	$0	$8,566	$14,703	$19,492	$20,268	$6,826	$7,746	$21,034	$144
$16,756	$3,923	$16,085	$0	$0	$5,764	$12,427	$15,577	$27,244	$11,244
$0	$20,509	$2,946	$19,936	$33,426	$2,726	$23,732	$12,495	$6,058	$5,448
$9,870	$32,706	$31,490	$0	$10,886	$0	$10,832	$1,767	$19,170	$4,471
$8,593	$16,871	$16,206	$7,823	$21,934	$3,546	$10,704	$19,077	$15,262	$23,965

$\mu = 13672.48$
$\sigma = 10573.831$

After the frequency for each income is determined, the cumulative observed frequency is calculated.

Next, the observed cumulative relative frequency is calculated by dividing the observed cumulative frequency by the sample size:

$$S_{100}(X_1) = \frac{17}{100} = .17$$

		Cumulative frequency	Cumulative relative frequency			
Income	Frequency	Observed	Observed $[S_N(X_i)]$	z-scores	Predicted $[F_0(X_i)]$	$\lvert F_0(X_i) - S_N(X_i)\rvert$
0	17	17	0.17	−1.29	0.0985	0.07
144	1	18	0.18	−1.28	0.1003	0.08
738	1	19	0.19	−1.22	0.1112	0.08
1767	1	20	0.20	−1.13	0.1292	0.07
2726	1	21	0.21	−1.04	0.1492	0.06
2946	1	22	0.22	−1.01	0.1562	0.06
3380	1	23	0.23	−0.97	0.166	0.06
3546	1	24	0.24	−0.96	0.1685	0.07
3923	1	25	0.25	−0.92	0.1788	0.07
4037	1	26	0.26	−0.91	0.1814	0.08
4471	1	27	0.27	−0.87	0.1922	0.08
5208	1	28	0.28	−0.80	0.2119	0.07
5238	1	29	0.29	−0.80	0.2119	0.08
5274	1	30	0.30	−0.79	0.2148	0.09
5448	1	31	0.31	−0.78	0.2177	0.09
5764	1	32	0.32	−0.75	0.2266	0.09
6058	1	33	0.33	−0.72	0.2358	0.09
6826	1	34	0.34	−0.65	0.2578	0.08
7612	1	35	0.35	−0.57	0.2843	0.07
7746	1	36	0.36	−0.56	0.2877	0.07
7823	1	37	0.37	−0.55	0.2912	0.08
8566	1	38	0.38	−0.48	0.3156	0.06
8593	1	39	0.39	−0.48	0.3156	0.07
8762	1	40	0.40	−0.46	0.3228	0.08
9870	1	41	0.41	−0.36	0.3594	0.05
10218	1	42	0.42	−0.33	0.3707	0.05
10704	1	43	0.43	−0.28	0.3897	0.04
10832	1	44	0.44	−0.27	0.3936	0.05
10886	1	45	0.45	−0.26	0.3974	0.05
11232	1	46	0.46	−0.23	0.409	0.05
11244	1	47	0.47	−0.23	0.409	0.06
12184	1	48	0.48	−0.14	0.4443	0.04
12419	1	49	0.49	−0.12	0.4522	0.04
12427	1	50	0.50	−0.12	0.4522	0.05
12462	1	51	0.51	−0.11	0.4562	0.05
12495	1	52	0.52	−0.11	0.4562	0.06
12976	1	53	0.53	−0.07	0.4721	0.06
14703	1	54	0.54	0.10	0.5398	0.00
15262	1	55	0.55	0.15	0.5596	0.01
15577	1	56	0.56	0.18	0.5714	0.01
16085	1	57	0.57	0.23	0.591	0.02
16206	1	58	0.58	0.24	0.5948	0.01
16551	1	59	0.59	0.27	0.6064	0.02
16756	1	60	0.60	0.29	0.6141	0.01

(continued)

(continued)

Income	Frequency	Cumulative frequency Observed	Cumulative relative frequency Observed $[S_N(X_i)]$	z-scores	Predicted $[F_0(X_i)]$	$\lvert F_0(X_i) - S_N(X_i)\rvert$
16871	1	61	0.61	0.30	0.6179	0.01
17076	1	62	0.62	0.32	0.6255	0.01
17407	1	63	0.63	0.35	0.6368	0.01
17762	1	64	0.64	0.39	0.6517	0.01
19077	1	65	0.65	0.51	0.695	0.04
19170	1	66	0.66	0.52	0.6985	0.04
19492	1	67	0.67	0.55	0.7088	0.04
19648	1	68	0.68	0.57	0.7157	0.04
19936	1	69	0.69	0.59	0.7224	0.03
20268	1	70	0.70	0.62	0.7324	0.03
20504	1	71	0.71	0.65	0.7422	0.03
20509	1	72	0.72	0.65	0.7422	0.02
21034	1	73	0.73	0.70	0.758	0.03
21518	1	74	0.74	0.74	0.7704	0.03
21934	1	75	0.75	0.78	0.7823	0.03
22078	1	76	0.76	0.79	0.7852	0.03
22933	1	77	0.77	0.88	0.8106	0.04
23503	1	78	0.78	0.93	0.8238	0.04
23732	1	79	0.79	0.95	0.8289	0.04
23965	1	80	0.80	0.97	0.834	0.03
24013	1	81	0.81	0.98	0.8365	0.03
24698	1	82	0.82	1.04	0.8508	0.03
24866	1	83	0.83	1.06	0.8554	0.03
25038	1	84	0.84	1.07	0.8577	0.02
26104	1	85	0.85	1.18	0.881	0.03
26353	1	86	0.86	1.20	0.8849	0.02
27244	1	87	0.87	1.28	0.8997	0.03
27949	1	88	0.88	1.35	0.9115	0.03
28002	1	89	0.89	1.36	0.9131	0.02
28575	1	90	0.90	1.41	0.9207	0.02
29075	1	91	0.91	1.46	0.9279	0.02
29512	1	92	0.92	1.50	0.9332	0.01
29811	1	93	0.93	1.53	0.937	0.01
31490	1	94	0.94	1.69	0.9545	0.01
31783	1	95	0.95	1.71	0.9564	0.01
32705	1	96	0.96	1.80	0.9641	0.00
32706	1	97	0.97	1.80	0.9641	0.01
33118	1	98	0.98	1.84	0.9671	0.01
33426	1	99	0.99	1.87	0.9693	0.02
34678	1	100	1.00	1.99	0.9767	0.02

Remember that the theoretical cumulative relative frequency distribution can be any distribution specified by the researcher. In this case, we want to compare the observed incomes to the normal distribution.

In order to calculate the normal cumulative distribution, we use the mean and standard deviation derived from the observed income data and calculate the z score:

$$z = \frac{X_1 - \mu}{\sigma} = \frac{1 - 13672.48}{10573.831} = -1.29$$

By consulting the standard normal distribution table, we find that the $z = -1.29$ gives us p = 0.0985. This calculation is continued for each recorded income.

The absolute value of the difference between $S_N(X)$ and $F_0(X)$ is calculated for each sentence:

$$\left|F_0(X_1) - S_N(X_1)\right| = \left|.0985 - .17\right| = 0.07$$

In this case, $D = 0.09$ is the maximum difference between the cumulative relative frequencies.

Since $N > 35$, a large sample approximation must be used to determine significance:

For $\alpha = 0.05$, $D \geq \frac{1.36}{\sqrt{N}}$ is significant:

In this case, $0.09 < \frac{1.36}{\sqrt{100}} = 0.136$

Since $D = 0.09$ is less than the critical value of 0.136, we fail to reject the null hypothesis (i.e., the observed incomes are normally distributed). We conclude that income is normally distributed.

Chapter 3: Fisher's Test for Normality of a Distribution

Skewness:

$$u_1 = \frac{K_3}{(K_2)^{3/2}} \times \left(\frac{n}{6}\right)^{1/2}$$

Kurtosis:

$$u_2 = \frac{K_4}{(K_2)^2} \times \left(\frac{n}{24}\right)^{1/2}$$

Instead of looking at skewness and kurtosis separately, you can also use a combined equation:

$$x^2 = \left[\frac{K_3}{(K_2)^{3/2}} \times \left(\frac{n}{6}\right)^{1/2}\right]^2 + \left[\frac{K_4}{(K_2)^2} \times \left(\frac{n}{24}\right)^{1/2}\right]^2$$

Fisher's *K*-statistics are used to calculate the skewness and kurtosis equations:

$$K_1 = \frac{M_1}{n}$$

$$K_2 = \frac{nM_2 - M_1^2}{n(n-1)}$$

$$K_3 = \frac{n^2M_3 - 3nM_2M_1 + 2M_1^3}{n(n-1)(n-2)}$$

$$K_4 = \frac{-6M_1^4 + 12nM_1^2M_2 - 3n(n-1)M_2^2 - 4n(n+1)M_1M_3 + n^2(n+1)M_4}{n(n-1)(n-2)(n-3)}$$

$$M_r = \sum_{i=1}^{n} x_i^r = \text{the sum of } fx \text{ to the r}^{\text{th}}\text{power}$$

Again, we'll look at the sentence data for each of the $N = 100$ offenders. First, M_1, M_2, M_3, and M_4 are calculated by finding the sum of x, x^2, x^3, and x^4:

Offender	Length of sentence (fx)	fx^2	fx^3	fx^4
1	7	49	343	2401
2	8	64	512	4096
3	3	9	27	81
4	1	1	1	1
5	1	1	1	1
6	3	9	27	81
7	12	144	1728	20736
8	3	9	27	81
9	3	9	27	81
10	6	36	216	1296
11	5	25	125	625
12	3	9	27	81
13	3	9	27	81
14	1	1	1	1
15	1	1	1	1
16	4	16	64	256
17	3	9	27	81
18	1	1	1	1
19	1	1	1	1
20	2	4	8	16

(continued)

(continued)

Offender	Length of sentence (fx)	fx^2	fx^3	fx^4
21	1	1	1	1
22	3	9	27	81
23	8	64	512	4096
24	1	1	1	1
25	5	25	125	625
26	5	25	125	625
27	6	36	216	1296
28	3	9	27	81
29	5	25	125	625
30	3	9	27	81
31	9	81	729	6561
32	2	4	8	16
33	3	9	27	81
34	3	9	27	81
35	5	25	125	625
36	5	25	125	625
37	8	64	512	4096
38	1	1	1	1
39	1	1	1	1
40	5	25	125	625
41	2	4	8	16
42	3	9	27	81
43	3	9	27	81
44	1	1	1	1
45	3	9	27	81
46	3	9	27	81
47	6	36	216	1296
48	2	4	8	16
49	3	9	27	81
50	1	1	1	1
51	3	9	27	81
52	3	9	27	81
53	6	36	216	1296
54	3	9	27	81
55	6	36	216	1296
56	3	9	27	81
57	3	9	27	81
58	3	9	27	81
59	3	9	27	81
60	3	9	27	81
61	6	36	216	1296
62	1	1	1	1
63	1	1	1	1
64	3	9	27	81
65	3	9	27	81
66	1	1	1	1

(continued)

(continued)

Offender	Length of sentence (fx)	fx^2	fx^3	fx^4
67	1	1	1	1
68	1	1	1	1
69	2	4	8	16
70	1	1	1	1
71	7	49	343	2401
72	3	9	27	81
73	1	1	1	1
74	3	9	27	81
75	3	9	27	81
76	3	9	27	81
77	3	9	27	81
78	5	25	125	625
79	6	36	216	1296
80	2	4	8	16
81	2	4	8	16
82	3	9	27	81
83	3	9	27	81
84	6	36	216	1296
85	16	256	4096	65536
86	1	1	1	1
87	3	9	27	81
88	3	9	27	81
89	2	4	8	16
90	1	1	1	1
91	1	1	1	1
92	3	9	27	81
93	5	25	125	625
94	6	36	216	1296
95	3	9	27	81
96	1	1	1	1
97	3	9	27	81
98	1	1	1	1
99	3	9	27	81
100	3	9	27	81
Σ	341	1761	13103	130941

$$M_1 = 341 M_2 = 1761 M_3 = 13103 M_4 = 130941$$

Next, each of the K-statistics is calculated:

$$K_1 = \frac{M_1}{n} = \frac{341}{100} = 3.41$$

$$K_2 = \frac{nM_2 - M_1^2}{n(n-1)} = \frac{(100)(1761) - 341^2}{100(100-1)} = \frac{176100 - 116281}{9900} = 6.0423$$

$$K_3 = \frac{n^2M_3 - 3nM_2M_1 + 2M_1^3}{n(n-1)(n-2)} = \frac{(100)^2(13103) - 3(100)(1761)(341) + 2(341^3)}{100(100-1)(100-2)}$$
$$= \frac{131030000 - 180150300 + 79303642}{970200} = 31.1104$$

$$K_4 = \frac{-6M_1^4 + 12nM_1^2M_2 - 3n(n-1)M_2^2 - 4n(n+1)M_1M_3 + n^2(n+1)M_4}{n(n-1)(n-2)(n-3)}$$
$$= \frac{\begin{array}{c}-6(341^4) + 12(100)(341^2)(1761) - 3(100)(100-1)(1761^2)\\ -4(100)(100+1)(341)(13103)100^2(100+1)(130941)\end{array}}{100(100-1)(100-2)(100-3)}$$
$$= \frac{\begin{array}{c}-81127625766 + (1200)(204770841) - (300)(307010979)\\ -(400)(451280423) + (10000)(13225041)\end{array}}{100(99)(98)(97)}$$
$$= \frac{24232330534}{94109400} = 257.4911$$

Once the *K*-statistics are calculated, use the answers to calculate skewness and kurtosis:

Skewness:

$$u_1 = \frac{K_3}{(K_2)^{3/2}} \times \left(\frac{n}{6}\right)^{1/2} = \frac{31.1104}{(6.0423)^{3/2}} \times \left(\frac{100}{6}\right)^{1/2} = \frac{31.1104}{14.8526} \times 4.0825 = 8.5512$$

The critical value at $\alpha = 0.05$ is 1.96. Since 8.5512 is greater than 1.96, we reject the null hypothesis (i.e., the sentences are normally distributed). We conclude that the sentence data are skewed; therefore, they are not normally distributed.

Kurtosis:

$$u_2 = \frac{K_4}{(K_2)^2} \times \left(\frac{n}{24}\right)^{1/2} = \frac{257.4911}{(6.0423)^2} \times \left(\frac{100}{24}\right)^{1/2} = 7.0527 \times 2.0412 = 14.396$$

Using the same critical value (1.96), we again reject the null hypothesis.

If we use the combined equation:

$$x^2 = \left[\frac{K_3}{(K_2)^{3/2}} \times \left(\frac{n}{6}\right)^{1/2}\right]^2 + \left[\frac{K_4}{(K_2)^2} \times \left(\frac{n}{24}\right)^{1/2}\right]^2 = (8.5512)^2 + (14.396)^2$$
$$= 280.3678$$

The critical value at α = 0.05 for degrees of freedom = 99 (n −1) is 123.23. Since x^2 is greater than 123.23, we reject the null hypothesis. We conclude that the data are not normally distributed.

Chapter 3: Dixon Test for Outliers (For Samples Less Than 25)

For this test, the equation used depends on the sample size:

If $3 < n \leq 7 : r = \dfrac{(x_2 - x_1)}{(x_n - x_1)}$

If $8 \leq n \leq 10 : r = \dfrac{(x_2 - x_1)}{(x_{n-1} - x_1)}$

If $11 \leq n \leq 13 : r = \dfrac{(x_3 - x_1)}{(x_{n-1} - x_1)}$

If $14 \leq n \leq 24 : r = \dfrac{(x_3 - x_1)}{(x_{n-2} - x_2)}$

n = sample size

The General Aggression Score (GAS) of the first 25 offenders was selected for analysis. Since we are most interested in whether or not the largest score is an outlier (offender 23 had a score of 78), the scores are arranged from largest to smallest. If we would have been interested in whether or not the smallest score was an outlier, we would have arranged them from smallest to largest.

x_n	Offender	General Aggression Score (GAS)
x_1	23	78
x_2	16	75
x_3	1	65
...	...	...
x_{23}	3	41
x_{24}	19	41
x_{25}	18	40

Since n = 25, we use the following equation:

$$r = \frac{(x_3 - x_1)}{(x_{n-2} - x_1)} = \frac{(65 - 78)}{(41 - 78)} = 0.3514$$

The critical value for α = 0.05 is found by consulting the Dixon table, 0.406. Since 0.406 is greater than r = 0.3514, we fail to reject the null hypothesis (i.e., the potential outlier does come from the sample). We conclude that $x_1 = 78$ is not an outlier.

Chapter 9: Large Sample Permutation Test for Two Independent Samples (Wilcoxon Signed Ranks Test)

$$z = \frac{T - \frac{N(N+1)}{4}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}}$$

$N =$ Number of matched pairs $-$ the number of matched pairs where $d_i = 0$

$T =$ the smaller sum of like $-$ signed ranks (can be negative or positive ranks)

$d_i =$ signed difference between two scores

For this test, we are looking at the General Aggression Score (GAS) in two groups matched by sex. First, we determine d_i for each pair:

	Group 1	Group 2	d_i
1	65	63	+2
2	59	60	−1
3	43	53	−10
4	53	76	−23
5	49	40	+9
6	54	55	−1
7	58	51	+7
8	47	46	+1
9	52	43	+9
10	58	43	+15
11	58	55	+3
12	45	56	−11
13	45	47	−2
14	75	59	+16
15	53	44	+9
16	40	42	−2
17	41	51	−10
18	60	67	−7
19	59	70	−11
20	78	45	+33
21	53	56	−3
22	64	69	−5
23	60	58	+2
24	50	40	+10
25	47	65	−18
26	73	68	+5
27	65	55	+10
28	43	56	−13
29	54	53	+1

(continued)

(continued)

	Group 1	Group 2	d_i
30	52	50	+2
31	47	44	+3
32	68	74	−6
33	43	56	−13
34	50	44	+6
35	60	40	+20
36	43	45	−2
37	44	45	−1
38	44	70	−26
39	42	78	−36
40	53	40	+13
41	65	47	+18
42	48	53	−5
43	43	55	−12
44	42	55	−13
45	42	42	0
46	41	42	−1
47	59	56	+3
48	48	47	+1
49	62	45	+17
50	74	49	+25

Next, we rank each of the pairs based upon d_i. Importantly, only pairs with a difference are given a rank. If $d_i = 0$, it is removed from the analysis. In our sample, $N = 49$.

Also important is that the d_i's are ranked regardless of the sign (positive or negative). A −1 is treated the same way as a +1. If there are ties, they are given the same rank.

For example, with our data there are seven 1s (4 negative and 3 positive). To determine the rank, we compute the mean rank based upon the number of rankings we need:

$$Rank = \frac{1 + 2 + 3 + 4 + 5 + 6 + 7}{7} = 4$$

So for each "1" that appears, it will receive a rank of 4. The next possible ranking would then start with 8 (unless there were ties, as in the case with our data).

	Group 1	Group 2	d_i	Rank of d_i	Rank with less frequent sign
1	65	63	+2	10.5	
2	59	60	−1	−4	4
3	43	53	−10	−29.5	29.5
4	53	76	−23	−45	45
5	49	40	+9	26	
6	54	55	−1	−4	4

(continued)

(continued)

	Group 1	Group 2	d_i	Rank of d_i	Rank with less frequent sign
7	58	51	+7	23.5	
8	47	46	+1	4	
9	52	43	+9	26	
10	58	43	+15	39	
11	58	55	+3	15.5	
12	45	56	−11	−32.5	32.5
13	45	47	−2	−10.5	10.5
14	75	59	+16	40	
15	53	44	+9	26	
16	40	42	−2	−10.5	10.5
17	41	51	−10	−29.5	29.5
18	60	67	−7	−23.5	23.5
19	59	70	−11	−32.5	32.5
20	78	45	+33	48	
21	53	56	−3	−15.5	15.5
22	64	69	−5	−19	19
23	60	58	+2	10.5	
24	50	40	+10	29.5	
25	47	65	−18	−42.5	42.5
26	73	68	+5	19	
27	65	55	+10	29.5	
28	43	56	−13	−36.5	36.5
29	54	53	+1	4	
30	52	50	+2	10.5	
31	47	44	+3	15.5	
32	68	74	−6	−21.5	21.5
33	43	56	−13	−36.5	36.5
34	50	44	+6	21.5	
35	60	40	+20	44	
36	43	45	−2	−10.5	10.5
37	44	45	−1	−4	4
38	44	70	−26	−47	47
39	42	78	−36	−49	49
40	53	40	+13	36.5	
41	65	47	+18	42.5	
42	48	53	−5	−19	19
43	43	55	−12	−34	34
44	42	55	−13	−36.5	36.5
45	42	42	0		
46	41	42	−1	−4	4
47	59	56	+3	15.5	
48	48	47	+1	4	
49	62	45	+17	41	
50	74	49	+25	46	
				T=597	

After we determine the rankings, each ranking is given the same sign as its d. So, if $d = -1$, then the ranking $= -4$.

We then calculate T, which is the smaller sum of like-signed ranks. In this case, the sum of the positive rankings is smaller than the sum of the negative rankings:

$$T = 4 + 4 + 4 + 4 + 10.5 + 10.5 + 10.5 + 15.5 + 19 + 19 + 21.5 + 23.5 + 29.5 + 29.5 + 32.5 + 32.5 + 34 + 36.5 + 36.5 + 36.5 + 42.5 + 45 + 47 + 49 = 597$$

Now, we can calculate z:

$$z = \frac{T - \frac{N(N+1)}{4}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}} = \frac{597 - \frac{49(49+1)}{4}}{\sqrt{\frac{49(49+1)(2[49]+1)}{24}}} = \frac{-15.5}{100.5298} = -0.1542$$

With $z = -0.1542$, the corresponding p-value for a two-tailed test is greater than 0.05. Since $\alpha \geq 0.05$, we fail to reject the null hypothesis (i.e., there is no difference between the groups). We conclude that the groups have similar General Aggression Scores.

Answers to "Check Your Understanding" Questions

Chapter 1

1. Association
2. Association
3. Difference
4. Examples will vary
5. Ranks from top to bottom
   a. For income: 3, 2, 4, 1, 5
   b. For age: 1, 4, 2, 5, 3
   c. For weight: 3, 5, 4, 1, 2
   d. For grade: 5, 3, 2, 1, 4
6. A covariate
7. Both deal with groups or samples in the data; related samples deal with an analysis of two responses from one person or of two responses from two people who are somehow related on some factor such as age, ethnicity, or sex; unrelated samples deal with an analysis of two completely separate groups of individuals who are not matched on any factor. Examples will vary.

Chapter 2

1. Qualitative: sex, ethnicity/race, offense, living location, 30-day post-release status, 90-day post-release status, current status, currently taking medication, type of medication, hormonal treatment. Quantitative: age, sentence, estimated yearly income, Level of Meanness, General Aggression Score, Total Testosterone Level, treatment testosterone level.

J.A. Linebach et al., *Nonparametric Statistics for Applied Research*,
DOI 10.1007/978-1-4614-9041-8,

2. Level of measurement is:

   a. Ratio
   b. Interval
   c. Nominal
   d. Ratio
   e. Ratio
   f. Nominal
   g. Ordinal

3. Measures of central tendency are:

   a. Mean, 52.6; median, 49.5; mode, 37; midrange, 51
   b. Mean, 4.7; median, 3; mode, 3; midrange, 6.5
   c. Mean, $11,867.80; median, $9,316.00; mode, no mode; midrange, $13,052.00
   d. Mean, 2.6; median, 2.5; mode, 4; midrange, 2.5
   e. Mean, 52.1; median, 52.5; mode, no mode; midrange, 53
   f. Mean, 484; median, 516; mode, no mode; midrange, 380.5

4. Measures of variability are:

   a. Variance, 540.711; range, 64; standard deviation, 23.253
   b. Variance, 12.233; range, 11; standard deviation, 3.498
   c. Variance, $82,454,543.29; range, $26,104.00; standard deviation, $9,080.45
   d. Variance, 1.378; range, 3; standard deviation, 1.174
   e. Variance, 54.989; range, 24; standard deviation, 7.415
   f. Variance, 39322; range, 659; standard deviation, 198.298

5. d. Negatively skewed

Chapter 3

1. Matching distributions:

   a. Uniform
   b. Exponential
   c. Normal
   d. Poisson

2. Randomly sampled data, Independent Sampling, At least interval data, homogeneity of variance, normally distributed data.
3. Sampling procedures are important for parameter testing because they provide a basis for determining whether parametric tests or nonparametric tests should be used. For example, if the researcher is not selecting participants by random sampling, the researcher should probably stick with nonparametric statistical tests.

4. Observed frequencies are the actual number of occurrences of a particular outcome in a contingency table usually compared to an expected frequency, and expected frequencies are the number of anticipated occurrences of a particular outcome in a contingency table.
5. Kolmogorov-Smirnov One-Sample Test: the observed frequencies are not normally distributed. One-Sample Runs Test of Randomness: the order of the data collection was not random. Test for Distributional Symmetry: the data are not from the same symmetrical distribution. Dixon Test: a particular score is an extreme value or an outlier.

Chapter 4

1. −0.26 is weak and negative, 0.98 is strong and positive, 0.58 is weak and positive, 0.11 is weak and positive, −0.87 is strong and negative, and −0.33 is weak and negative.
2. There are only 2 possible responses in the data for Phi Coefficient, while there can be more than 2 possible responses when using Cramér Coefficient.
3. False: correlation does not mean causation.
4. An observed frequency is that which can be seen, measured, or collected by the researcher. The expected frequency is based upon some theoretical perspective regarding the anticipated outcome of the data.
5. C. Chi-Square is used to determine the final result of the Cramér Coefficient.
6. D. Categorical data and contingency tables are both necessary for Cramér and Phi Coefficients. Ranked data cannot be utilized.

Chapter 5

1. C. One variable must be identified as a dependent variable.
2. An antecedent could be considered an independent variable, and a consequent could be considered a dependent variable. When a researcher is only looking at 2 variables and one is a consequence of the other, the researcher can use Somer's d to predict the consequent when the antecedent is known.
3. Concordant pairs are those that show a positive relationship while discordant pairs are those that show a negative relationship. Concordant and discordant pairs are both found by first arranging the data in a contingency table where the lowest scores are in the first row and first column and the highest scores are in the last row and last column. Concordant pairs are those that fall below and to the right of a particular cell; stated another way, concordant pairs are those that are higher scores than the particular cell being looked at. Discordant pairs are those that fall below and to the left of a particular cell; stated another way, discordant pairs are those that are lower scores based on columns and higher scores based on rows than the particular cell.

4. Ranking
5. Matching
   a. Positive correlation = +1.00
   b. Negative correlation = −1.00
   c. No relationship = 0.00
6. Rankings:

a.

Participant	Income	Rank
1	\$25,500	6
2	\$16,000	3
3	\$29,900	8
4	\$9,900	1.5
5	\$59,000	10
6	\$25,500	6
7	\$21,300	4
8	\$48,050	9
9	\$25,500	6
10	\$9,900	1.5

b.

Participant	Age	Rank
1	45	9
2	18	1
3	53	10
4	36	4
5	36	4
6	27	2
7	43	8
8	41	7
9	38	6
10	36	4

Chapter 6

1. A researcher would use Kendall's Rank-Order when he or she believed that some third variable contributed to the strength of the relationship between the first 2 variables.
2. Spearman's is more appropriate when the researcher simply wants to assess the strength of association, or correlation, between only 2 variables.
3. Income is held constant.
4. Agreements are those that show a positive relationship while disagreements are those that show a negative relationship. Agreements and disagreements are both

found by first arranging the data in a contingency table where the lowest scores are in the first row and first column and the highest scores are in the last row and last column. Agreements are those that fall below and to the right of a particular cell while disagreements are those that fall below and to the left of a particular cell. Somer's d utilizes these same concepts by different names: agreements are concordant pairs, while disagreements are discordant pairs.

5. True. When dealing with sample size for Kendall's Rank-Order, two other options exist besides the $N>30$ option. Those two options are $N \leq 10$ and $N>10$. When $N \leq 10$, the exact probability, or p-value, can be found utilizing an upper-tail probabilities for T table. When $N>10$, T can be assumed to be normally distributed a mean $= \mu T = 0$ and Variance $= \sigma T 2 = 2(2N + 5)/9N(N - 1)$. z is then found using the same formula as for $N>30$ as discussed here; however, the value for T can be found in a Critical Values table for T.
6. The difference in the two equations is found in the denominator of the equations. The equation that accounts for ties provides a correction based upon the size of the tied ranks in each set: one for the x-variable and one for the y-variable. The equation that does not account for ties only looks at the possible number of pairs based upon the overall sample size.

Chapter 7

1. Kendall's coefficient of concordance is used to analyze the association between 2 or more sets of rankings, while Kendall's coefficient of agreement is used to analyze the association between only 2 possible choices. Kendall's coefficient of agreement is a forced choice scenario, whereas Kendall's coefficient of concordance is not.
2. Slope and intercept are used to determine the line of best fit. Once the line is determined, it is going to tell you how much uncertainty there is in your line as you try to account for all of the data and random errors like outliers. We can use the line of best fit to best predict a "y" value when we know the "x" value or the "x" value when we know the "y" value.
3. Slope is what is going to tell you how steep your line is going to be when this is plotted onto a line graph. The intercept is going to tell you where the line is going to cross the Y-axis on the graph. These are then used in the regression equation to determine the line of best fit allowing a researcher to predict a likely "y" when an "x" is known. In other words, use the information from a data set to predict a specific outcome on one variable based upon the other variable.
4. A. Plane of best fit
5. The type of regression is:

   a. Nonparametric regression
   b. Nonparametric regression and parametric regression
   c. Parametric regression
   d. Parametric regression

   e. Parametric regression
   f. Parametric regression
   g. Nonparametric regression
   h. Nonparametric regression

6. C. Criterion variable

Chapter 8

1. An Exact Test is a statistical test that uses probability to determine the exact likelihood of the occurrence of an event. A test that uses approximation uses data that are incomplete or are difficult to measure exactly.
2. For most tests, the sample size is a concern because it provides some of the information for whether or not the test is significant. For the binomial test, we do not have that same concern; regardless of our sample size, we can still determine significance because this is an Exact Test.
3. Successes and Failures:

   a. Success, recidivism; failure, non-recidivism
   b. Success, treatment works; failure, treatment does not work
   c. Success, selected for jury duty; failure, not selected for jury duty
   d. Success, going to trial; failure, not going to trial
   e. Success, democrats elected; failure, democrats not elected

4. C. use a researchers intuition about what the expected frequencies should be
5. A one-sample test compares the mean of one group against the mean of the population from which that group comes, while a two-sample test compares the mean of one group against the mean of a second group.

Chapter 9

1. Both tests use interval scale data; however, the Moses test log transforms the interval scale data into ordinal scale data.
2. The Permutation Test works best with small sample sizes because large samples make the calculations cumbersome and tedious. The Moses test is less affected by sample size.
3. The assumptions that must be met include independent sampling and interval scale data. The assumptions that do not need to be met include normally distributed data, homogeneity of variance, and randomly sampled data.
4. Log transform just means that you take data and changing it into something else. Log transformation is necessary with the Moses test when the data are interval scale because the Moses test requires the data to be ordinal scale. If the data are already ordinal, there is no need for log transformation.

5. Permutation Test null hypothesis: no difference between two populations. Moses test null hypothesis: no difference between the variability between two groups or populations.

Chapter 10

1. D. Mann–Whitney U Test
2. Yes, data can be changed from interval scale to ordinal scale by ranking the data. This is known as log transformation. Interval data would be transformed when the research question and subsequent test require ordinal or ranked data instead of interval scale data.
3. The Kolmogorov-Smirnov two-sample test requires that the data be cumulative, whereas the Mann–Whitney U Test requires that the data not be cumulative.
4. B. Ordinal data
5. B. It can be used with three or more groups.

Chapter 11

1. Fisher's Exact Test is used for small samples while Chi-Square is more commonly used for larger samples. Fisher's Exact addresses an exact probability of the occurrence of an event while Chi-Square assesses the difference between observed and expected frequencies. Small frequencies of 5 or less are not an issue for Fisher's Exact like they are an issue for Chi-Square. Both tests need independent samples, nominal or ordinal data, and both utilize contingency tables.
2. Factorials are:

   a. 3,628,800
   b. 120
   c. 20,922,789,888,000
   d. 1
   e. 51,090,942,171,709,440,000

3. Two are not characteristics of hypergeometric distributions:

   a. D. It can be used with any kind of data: this is not the case because the data must fall into one of 2 discrete categories.
   b. F. The probability of "success" does not change with each draw: it does change because the "success" is not replaced after it is drawn from the sample.

4. Both tests use discrete and dichotomous nominal data which is often displayed in a 2 x 2 contingency table. The McNemar test always utilizes a 2 x 2 contingency table design. Fisher's Exact requires independent samples, whereas the McNemar test requires dependent samples.

Chapter 12

1. Both tests look at significant differences between independent groups of nominal data, but with the Chi-Square r X k, you can use as many groups as you need.
2. The three groups are independent. The best test to use would be the Chi-Square r X k test.
3. Friedman's ANOVA would be appropriate in this case because Friedman's ANOVA is meant for repeated measures or matched pairs, whereas Kruskal-Wallis ANOVA is meant for independent samples.
4. Two require related sample data:
   a. A. Cochran Q
   b. C. Friedman's ANOVA
5. Two require nominal or dichotomous data:
   a. A. Cochran Q
   b. B. Chi-Square r X k
6. Two require ranked data:
   a. C. Friedman's ANOVA
   b. D. Kruskal-Wallis ANOVA

Glossary

Absolute Value The distance from 0 on a number line; +5 is the same distance from 0 as −5

Agreement The sum of frequencies above and to the left of a given location in a contingency table when the variables are arranged in ascending magnitude

Alpha Levels The probability that the calculated value for a statistical test will fall within one of the extremities of a distribution

Antecedent A value (behavior or treatment) that comes before some resulting consequence; may also be described as an independent variable

Approximations Models used to represent data where the data are incomplete or are difficult to measure exactly

Assigned Rankings The designated rank for a particular value in a data set; see also Ranked Data

Association Analyses that are interested in the similarities between variables

Association Decision Tree A visual decision making model for nonparametric statistical tests of association

Association Tests Statistical analyses which are used to find similarities between two or more variables

Average See Mean

Asymmetry See Skew

Binomial Coefficient The algebraic expansion of the powers of a positive integer

Binomial Distribution A visual representation of the possible probabilities for discrete data

Binomial Distribution Table A table used to determine the probability of determining a certain number of successes when the sample size and probability of success is known

Binomial Test A nonparametric test of difference designed to estimate the likelihood of equal representation in two categories where the entire population falls into one of only two categories such as membership

Calculated Value A final answer after calculating a statistical equation

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Chi-Square Table A table that allows a researcher to determine the critical value for a known degrees of freedom and alpha level

Chi-Square Value The numeric result of running a Chi-Square test

Chi-Square Goodness of Fit A nonparametric test of difference designed to determine if the observed data provides a good approximation to the expected or theoretical data

Chi-Square r X k A nonparametric test of difference designed to determine if the observed data provides a good approximation to the expected or theoretical data where there are 3 or more groups for each variable

Cluster Analysis The process of placing a series of data points onto a scatter plot in such a way that groups or clusters form that are obviously different from the other groups or clusters

Cochran Q Test A nonparametric test of difference designed to determine whether 2 or more related samples of frequencies are different from one another where the data are nominal

Concordant The sum of frequencies below and to the right of a given location in a contingency table when the variables are arranged in ascending magnitude

Consequent The result or subsequent outcome following an antecedent; may also be described as a dependant variable

Contingency Table A visual representation of the frequencies observed from a qualitative study

Continuous Data Information in the form of a number that is expressed using a decimal or a fraction

Continuous Probability Distribution See Normal Distribution

Control Group A group of participants that is not given an experimental treatment and is used as a baseline against other groups which are given treatments

Convenience Sample Collecting data that is easy to obtain. Selecting participants based on accessibility rather than certain characteristics pertinent to the study

Correlation The degree to which one variable is related to another variable

Covariate A variable that is known to have an effect on the outcome of an observation but is not the effect to be tested; therefore, it is held mathematically constant

Cramér Coefficient A nonparametric test of association designed to determine the relationship between two nominal variables and not limited to only two possible responses

Criterion Variable The variable in a regression analysis that is predicted by the predictor variable; may be described as a dependant variable

Critical Value The numeric cutoff point that is used to determine significance or no significance of a test

Cross Tabulation The process used to create a table the purpose of which is to show the frequency distribution of a set of data

Cumulative Frequency An accumulation of the frequencies for a given variable over a specified time period or set of participants

Cumulative Relative Frequency An accumulation of the frequencies for a given variable over a specified time period or set of participants represented in decimal form by dividing the cumulative frequency by the sample size

Cumulative Relative Frequency Distribution A visual representation of a cumulative relative frequency that is concerned with the proportion of observations in a data set

Cumulative Response Data Frequency data that is compiled in a cumulative fashion such that the first frequency is the smallest number, and each subsequent frequency is larger than the previous

Data Points Individual pieces of data; single values or labels that are used in the compilation of a data set

Data Scale See Level of Measurement

Data Set A compilation of individual pieces of data usually compiled in the form of a table

Degrees of Freedom The value that is needed in certain types of analyses and is equivalent to $n-1$

Dependant Variable The variable observed, measured, and recorded by the researcher or experimenter

Descriptive Statistics Statistics that provide frequency data, central tendency data, and variability data

Dichotomous Data Information that is expressed in terms of 2 possible responses (i.e., "yes" and "no")

Difference Analyses that are interested in the dissimilarities between variables

Difference Decision Tree A visual decision making model for nonparametric statistical tests of difference

Difference Tests Statistical analyses which are used to find differences between two or more variables

Disagreement The sum of frequencies below and to the right of a given location in a contingency table when the variables are arranged in ascending magnitude

Discordant The sum of frequencies above and to the left of a given location in a contingency table when the variables are arranged in ascending magnitude

Discrete Data that is numeric and does not contain values to the right of a decimal point; whole numbers without the possibility of containing fractions

Dispersion The degree to which a distribution is spread out

Dispersion Indices A method of determining whether the data is clustered together or spread out

Dixon Test for Outliers A nonparametric test of violation that allows the researcher to address any values that are outside the third standard deviation of a normal distribution

Empirical Research Research based upon observations or measurements that can be verified; research that is public and replicable

Exact Test A statistical test that uses probability to determine the exact likelihood of the occurrence of an event

Expected Frequency The number of anticipated occurrences of a particular outcome in a contingency table
Experiment Research where as many variables as possible, preferably all variables, are held constant which reduces the variability in the study to the observed effect instead of other variables
Exponential Distribution A distribution that increases on a continuous basis from left to right

Factorial The multiplication of a number, n, by every positive integer less than that number but greater than 0
Failure An undesired outcome
Fisher Exact Probability A nonparametric test of difference designed to calculate an exact probability for group membership in participants based upon 2 variables when only two classifications are possible in each variable
Fisher's Test for Normality of a Distribution A nonparametric test of violation that allows the researcher to determine whether or not a sample data set is normally distributed
Fixed The state of being held constant when used to compare 2 or more other variables
Frequencies The sum of participants who match a certain criteria
Frequency Distribution The process of compiling the values of a data set into a cross tabulation table to represent the data set in terms of frequency of occurrence; the result of cross tabulation
Friedman's ANOVA A nonparametric test of difference designed to determine whether 2 related samples have been taken from the same population where the data are ordinal

Groups Categories into which participants can be separated (i.e., ethnicity, sex, or income level)

Homogeneity of Variance A parametric assumption that requires the variance of 2 groups or samples to be equivalent
Homoscedasticity See Homogeneity of Variance
Hypergeometric Distribution A discrete probability distribution that is used to calculate the probability of obtaining observed data while correcting for some sampling error and accounting for a characteristic that is selected in the population but not replaced

Independent (Groups) Groups of participants that have not been selected from the same population
Independent T-Test A parametric statistical test designed to compare the means of two independent samples
Independent Variable The variable that can be manipulated or changed by the researcher or experimenter
Intercept The location at which two lines converge on a graph
Interval Data Numeric data where the intervals between values have meaning

Intervals A grouping of data into categories which are the same size; i.e., all the groups contain 5 ages, 21–25, 26–30, etc.

Kendall's Coefficient of Agreement u A nonparametric test of association designed to force a choice of only two possible responses at one time resulting in an indication of preference for one object over another

Kendall's Coefficient of Concordance W A nonparametric test of association designed to determine the relationship between more than two sets of rankings

Kendall's Partial Rank-Order Correlation Coefficient A nonparametric test of association that is designed to identify the nature of the relationship between two ranked variables when a third variable is fixed

Kendall's Rank-Order Correlation Coefficient A nonparametric test of association that is designed to identify the nature of the relationship between two ranked variables

Kendall's tau See Kendall's Rank-Order Correlation Coefficient

Kolmogorov-Smirnov One-Sample Test A nonparametric test of difference that is designed to compare a sample data set with a theoretical distribution such as a normal distribution

Kolmogorov-Smirnov Two-Sample Test A nonparametric test of difference that is designed to determine the agreement between two independent samples analyzing whether they have both been drawn from the same population

Kruskal-Wallis ANOVA A nonparametric test of difference designed to determine whether 2 independent samples have been taken from the same population where the data are ordinal

Kurtosis A means of describing a distribution that indicates the general form of concentration around the mean

Leptokurtic A distribution where the majority of the values pile up around the mean creating a distribution that has a high peak in the middle

Level of Measurement The characteristics of the data itself such as whether it is qualitative or quantitative

Likert-Type Surveys (Likert-Type Data) Data from questions that illicit 1 of 5 possible responses to the question; true Likert data comes from questions that illicit 1 of 7 possible responses

Line of Best Fit A linear representation of the slope-intercept equation of a line for two variables; may be described as the "middle" line for a scatter plot

Linear Model Using correlation coefficients, a slope and an intercept for a data set to determine a line of best fit in order to predict an approximate outcome for a known variable value

Linear Regression A parametric test of association designed to estimate the linear relationship between two or more variables

Log Transformed A statistical method of converting data into a normally distributed data set using normal distribution theory

Longitudinal Study A research study that occurs over an extended period of time, usually years, using the same participants and measuring the same variables over the specified time period

Magnitude The degree to which something has greatness; the size of a statistical significance

Mann–Whitney U A nonparametric test of difference that is designed to compare two independent samples against one another in order to determine if both have been drawn from the same population

Matched Pairs Samples or participants that are paired with one another based on some other variable that is not part of the immediate study

McNemar Change Test A nonparametric test of difference designed to compare a treatment or time change when an individual is compared to himself or herself

Mean The arithmetic average as calculated using $\Sigma x/n$

Measures of Central Tendency Statistical analysis that addresses the center of the data. The center can be assessed by a calculated average, the middle value, or the most frequently occurring value

Measures of Variability Statistical analysis that addresses how the values compare with one another. This analysis gives an indication as to how spread out or not the data are from one another

Median The middle value when the data is ranked. In the event of an even number of pieces of data, it is the average of the 2 middle values

Mesokurtic A normal distribution; a bell-shaped, symmetrical distribution

Midrange The mathematical average of the highest and lowest scores

Mode The value (x) in a variable that occurs the most; if all values occur only once, the variable has no mode

Moses Rank-Like Test for Scale Differences A nonparametric test of difference that is designed to determine the spread or dispersion between two groups when a median is unknown or the medians cannot be assumed to be equal

Multinomial Distribution A binomial distribution that can be generalized to more than simply 2 possible outcomes

Multinomial Test A nonparametric test of difference designed to estimate the likelihood of equal representation in more than 2 categories where the entire population falls into only those categories

Multinomial The sum of two or more data points expressed algebraically using variables and constants separated by addition and subtraction signs

Multiple Regression A statistical test that uses two or more independent variables to predict one dependant variable

Negative Skew A distribution where the tail on the left is longer than the tail on the right resulting from a mean that is less than the majority of the values in the data

Nominal Data Information that is expressed in terms of categories; qualitative data

Nonparametric Procedures Statistical tests that do not require as many assumptions to be met before the test can be conducted

Nonparametric Regression A nonparametric test of association designed to estimate the relationship between two or more variables where the statistical model is determined by the sample data thus requiring a large sample size

Normal Distribution A bell-shaped, symmetrical distribution

Normal Frequency Distribution A bell-shaped, symmetrical distribution where the mean, median, and mode are equal; see also Normal Distribution

Normally Distributed See Normal Distribution

Null Hypothesis The hypothesis used in research that is used to consider the equality between two samples

Observed The actual number of occurrences of a particular outcome in a contingency table usually compared to an expected frequency

One-Sample Runs Test of Randomness A nonparametric test of violation that is designed to assess the randomness of a set of data points

One-Sample Test A comparison of the mean of one group against the mean of the population from which that group comes

One-Tailed Test A method of conducting an association or difference test that specifies a hypothesized direction in the outcome of the statistical procedure thus resulting in a rejection region on only one side of the distribution

Ordinal Data Information that is expressed in terms of categories that can be ranked

Orthogonal Data Data which is comprised of participants that are independent of one another

Orthogonality Data which is comprised of participants that are independent of one another

Outlier An extreme value that has an impact on the mean to create a positive or negative skew

Paired Variable See Matched Pairs

Pairwise The process of analyzing data one pair at a time

Parameters A term used to describe the assumptions that must be met for a researcher to utilize a parametric statistical test; in the event that the parameters are not met, the researcher should use nonparametric statistical tests

Parametric Tests Statistical tests that require that some assumptions be met before the test can be conducted such as independent sampling, random sampling, and normality.

Partial Correlation A third variable that may be causing a statistical significance between two other variables resulting in false significance

Pearson's Correlation r A parametric test of association that is designed to address the nature of the relationship between two variables with an interval level of measurement

Permutation Test for Two Independent Samples A nonparametric test of difference designed to determine significant difference between the means of two independent samples

Permutations Rearranging and listing the possible combinations of numbers for a data set

Phi Coefficient A nonparametric test of association designed to determine the relationship between two nominal variables each with only two possible responses

Plane of Best Fit A linear representation of the slope-intercept equation of a line for three variables; may be described as the "middle" plane for a scatter plot
Platykurtic A type of kurtosis in which a set of data has a wide and flattened distribution
Poisson Distribution A distribution that decreases on a continuous basis from left to right showing the increased unlikelihood of the occurrence of an event
Polynomial The sum of data points expressed algebraically using variables and constants separated by addition and subtraction signs
Population The group of people or things from which a sample is drawn
Positive Correlation The degree to which one variable is positively related to another variable; as one variable increases so does the other
Positive Skew A distribution where the tail on the right is longer than the tail on the left resulting from a mean that is greater than the majority of the values in the data
Power The degree to which a test is able to determine statistical significance
Predictor Variable The variable in a regression analysis that is being manipulated and used to predict the criterion variable; may be described as an independent variable
Probability The likelihood of an event occurring
Probability Level See Significance

Qualitative Data Analysis An analysis that usually assesses a smaller sample and is typically used to understand the reasons why a particular characteristic is employed in one's behavior
Quantitative Data Data that is comprised of numbers

r x k Contingency Table A visual representation of the frequencies observed from a qualitative study where each variable has more than two levels
Random Numbers Table A method used to select random samples
Random Sample Selecting participants in such a way that each member of the population has an equal probability of being chosen for involvement
Randomness An outcome of equal likelihood
Range A measure of variability that is calculated by finding the difference between the highest and lowest value
Ranked Data Data that is sorted from the smallest value to the largest value; the smallest values usually is signified by the rank of 1
Ratio Data Numeric data where the ratios between values have meaning
Regressions Parametric tests of association designed to estimate the relationship between two or more variables
Reject the Null Hypothesis The result of the absolute value of a calculated value being larger than the absolute value of the critical value; results in the determination of statistical significance
Related Samples See Matched Pairs
Repeated Measures Design A experimental design where the same participants are assessed two or more times on a specified set of variables

Run A series of like symbols signifying a particular attribute or response for a variable

Sample Size The number of participants in the study

Sign Test A nonparametric test of difference utilizing qualitative data and designed to determine the magnitude of difference between two variables

Significance Observed See Alpha Levels

Skew A characteristic of a distribution where one tail is longer than the other as a result of the data piling up on one side or the other of a distribution

Slope The steepness of a line in a regression analysis

Square The multiplication of a number by itself

Square Root A number whose square is the result; the square root is determined by answering the question "what number times itself equals the number below the square root symbol?"

Standard Deviation A common measure of variability that describes how spread out from the mean the data are

Significant Differences See Statistical Significance

Statistical Significance The statistical computation for an event that is statistically unlikely to occur yet occurs anyway

Success A desired outcome

Symmetrical A distribution where data points are similarly situated above and below the mean; one half of the distribution has a similar shape to the other half

t Distribution A theoretical bell-shaped, symmetrical distribution used in hypothesis testing; the shape changes based upon the degrees of freedom

t Statistic A parametric test of difference designed to compare the means of two samples; also known as a t-test

Tail The thin part of the distribution usually found on either side of the mean about three standard deviations from the mean

Test Proportion The probability that is being tested based upon some theoretical foundation in a binomial probability distribution

Test Statistic A calculated value that is used to compare against a critical value to determine statistical significance

Tests of Association See Association Tests

Tests of Distributional Symmetry Tests that are designed to determine whether or not a distribution is symmetrical around the mean or median

Tied Observations See Ties

Ties The same value is repeated two or more times within the same data set or variable

Triples Arranging data into sets of three

Two-Sample A comparison of the mean of one group against the mean of a second group

Two-Tailed Test A method of conducting an association or difference test that does not specify a hypothesized direction in the outcome of the statistical procedure thus resulting in a rejection region on both sides of the distribution

Type I Error Finding statistical significance when in fact there is none; false positive

Type II Error Not finding statistical significance when in fact there is; false negative

Uniform Distribution A distribution where there is no variation within the distribution; the distribution has the appearance of a straight line on a graph

Unrelated Samples Samples or participants that have not been paired or matched with another sample or set of participants thus resulting in independent samples

Variability The degree to which a distribution is spread out; a description of how spread out or clustered together the data are

Variable An attribute that can be observed, measured, or manipulated

Variance A measure of variability that assess how spread apart the scores are

Violation Tests Tests which are used to determine whether parametric or non-parametric tests should be used to analyze data from a study

Wilcoxon Signed Ranks Test A nonparametric test of difference designed to consider both direction of differences and the magnitude of differences

z-Distribution A distribution that is considered a normal distribution with mean $=$ 0 and standard deviation $= 1$

z-Score The number of standard deviations from the mean

z-Table A table that allows a researcher to determine the probability for a known z-score

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